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Introduction

Firms often increase profits at the expense of their employees and their consumers. The firms directly exploit consumers via intransparent pricing techniques or indirectly exploit employees via shady contractual terms.

Why do firms succeed in exploiting consumers and employees? It might be the case that consumers are simply not rational. Behavioral biases can lead consumers to misunderstand the total price of a certain good which implies that they might make wrong purchasing decisions. Anticipating the bias, the firms can use partitioned pricing, that is, to split the price into a transparent headline price and an intransparent additional price to confuse consumers. Besides, consumers who are prone to addiction might not understand their future behavior which might lead them to make suboptimal choices today. Knowing this, the firms might abuse consumers' naivety about their addiction. But even if the employees are rational, they can be exploited via shady contractual terms. Non-compete clauses, which restrict an employee to work for a competitor or start their own business, decrease the worker's bargaining power benefitting firms.

The persistence of exploitation shows that the market alone seems not to be able to solve the problem. Policymakers have become aware of these problems and have implemented various policies such as banning partitioned pricing. This paternalistic approach aims at helping consumers and employees from being abused. But the question arises of what constitutes a "good" regulation and what are the adverse consequences of the current regulations that are being implemented.

This thesis consists of three independent research projects that broadly revolve around these issues. I investigate non-compete clauses and their interaction with minimum wages. Moreover, I examine partitioned pricing and its regulation when consumers underestimate the total price of a good. Finally, I study the pricing and regulation of mobile games when consumers are prone to addiction.

Chapter 1 (joint work with Thomas Kohler) investigates how firms (ab)use non-compete clauses when there are minimum wages. Many low-wage workers in the United States have signed non-compete clauses, forbidding them to work for competitors. Empirical research has found a positive correlation between the level of the minimum wage and the prevalence of non-compete clauses. We explain this link with the canonical principal-agent moral hazard model and add the possibility to costlessly reduce the agent's payoff with an NCC. If the employee is not successful in an effort task, he is laid off which activates the non-compete clause such that his employment possibilities are reduced. To avoid this case, the employee works harder by exerting more effort. By incentivizing more effort, non-compete clauses transfer utility from the agent to the principal. If the minimum wage is sufficiently high, the agent would get a rent without non-compete clauses. With a non-compete clause, the principal cannot extract the rent by lowering the wage but instead chooses a non-compete clause. To analyze welfare, we break down the total effect of non-compete clauses into an idleness effect and an incentive effect: The idleness effect measures the reduction in welfare from preventing the agent to work after a layoff. The incentive effect increases or decreases welfare by increasing the effort. If the minimum wage is low but binding, the equilibrium effort is inefficiently low. A non-compete clause brings the effort then closer to the first-best which implies that the incentive effect is positive. If the minimum wage is large, however, the agent would get a large rent. In this case, the principal uses a very severe non-compete clause to extract the agent's rent which implies that equilibrium effort will be excessive, that is, higher than the first-best. Whether non-compete clauses increase or decrease welfare depends on the magnitude of the two effects which themselves depend on the level of the minimum wage and the other parameters of the model. In any case, non-compete clauses always make the agent (at least weakly) worse off.

Chapter 2 (joint work with Simon Dato and Andreas Klümper) studies how regulating partitioned pricing affects consumer surplus and welfare. Partitioned prices consist of a (transparent) headline price and an (intransparent) additional price. Consumers are not rational because they underestimate the total price: They only consider a fraction of the additional price for their buying decision. The firms exploit this bias by setting high additional prices and low headline prices in equilibrium. We analyze two consumer protection measures that have been implemented in the past: Regulating additional prices via a price ceiling (hard intervention) and making additional prices more transparent via consumer awareness programs (soft intervention). Both measures induce a trade-off between consumer surplus and welfare. A ban of additional prices or fully transparent prices maximizes consumer surplus but renders welfare inefficiently low in imperfect competition. The reason is that imperfect competition leads to inefficiently high prices and inefficiently low demand when prices are fully transparent or partitioned pricing is prohibited. The ability to partition prices makes the consumers perceive prices to decrease which increases demand and hence brings it closer to the efficient level. We also analyze the interplay between hard and soft interventions. In markets with unrestricted headline prices, hard and soft interventions are substitutes with respect to welfare: Making prices more transparent calls for less price regulation. The reason is that more transparent prices lead to lower demand which needs to be offset by lower price regulation to keep the demand constant. But if the headline prices are restricted to be non-negative, hard and soft interventions can also be complements. Higher competition requires stricter policies to maximize welfare. The reason is that higher competition leads to lower prices and higher quantities which brings the equilibrium close to the efficient result. In this case, strict policies on price regulations or price transparency are needed to avoid inefficiently high demand.

Chapter 3 (joint work with David Zeimentz) studies in-app purchases in mobile games when consumers are prone to addiction. The firm offers a game to consumers who repeatedly choose gaming time and in-app purchases which are complements, that is, the higher the gaming time the more in-app purchases the consumers want to make. Addiction is modeled with reinforcement and tolerance. Reinforcement implies that past gaming time increases today's marginal utility of gaming and tolerance leads to past gaming time decreasing the level of today's utility. When consumers are sophisticated, i.e., they anticipate their addiction problem, the monopolist offers in-app prices that maximize welfare and uses a fixed fee to extract the complete surplus. When consumers are naive about their addiction, however, the firm makes in-app purchases inefficiently cheap in the first period and inefficiently expensive in the second period. This pricing strategy tricks the consumers into playing for many hours and making many in-app purchases firing the addiction problem to maximally exploit them afterward. Perfect competition among firms only affects the distribution of surplus but does not eliminate the inefficiency when consumers are naive. Furthermore, if there are games that differ in their degree of addictiveness, the naive consumers choose the most addictive games. Not being aware of their addiction problem, the naive consumers perceive the more addictive games to come at better contractual terms. We also analyze regulations that policymakers started to implement to fight excessive gaming time and exploitative pricing techniques. A suitably chosen playing time regulation can induce the naive consumers to play the efficient amount. This regulation, however, does not prevent the consumers from misestimating their future utility and thus from spending too much relative to the sophisticated consumers. A spending cap neither induces the efficient result nor makes the naive consumers as well off as the sophisticated ones because it cannot reduce the consumers' too high playing time. To achieve the first-best and debias the consumers, the policymaker must combine both regulations.

Chapter 1

Do Non-Compete Clauses Undermine Minimum Wages?*

Joint with Thomas Kohler

1.1 Introduction

A non-compete clause (*NCC*) is part of an employment contract that prohibits employees from working for a competitor or from starting their own business within specific geographic or temporal boundaries. A significant fraction of the US labor force is currently bound by a non-compete clause: 20% of the labor force were restricted by such a clause in 2014 and 40% had signed one in the past (Starr, Prescott, and Bishara, 2021). Moreover, many low-wage workers are bound by NCCs. 29% of the sampled workplaces that pay an average hourly salary of less than 13 dollars and 20% of the workplaces in which the typical employee has not graduated from high school have each employee sign an NCC (Colvin and Shierholz, 2019). While the public seems to accept NCCs in the contracts of CEOs, media reports about NCCs in the contracts of low-wage workers caused a public outrage.[1] As a result, there have been several attempts at restricting the use of NCCs, particularly concerning low-wage workers, in the last years.[2] Both the public and the politicians advocating

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1. The fast-food firm *Jimmy John's* made its employees sign that they were not allowed to work for "any business which derives more than ten percent (10%) of its revenue from selling submarine, hero-type, deli-style, pita and/or wrapped or rolled sandwiches and which is located with three (3) miles of either [the Jimmy John's location in question] or any such other Jimmy John's Sandwich Shop" (https://cutt.ly/tX8k15c). Jimmy John's has settled with the Attorney General in New York State and has stopped using non-compete clauses for sandwich workers in 2016 (https://cutt.ly/sX8j8i3).

2. On the federal level, President Biden issued an executive order to "curtail the unfair use of non-compete clauses and other clauses or agreements that may unfairly limit worker mobility" on July

for restrictions believe that NCCs exploit low-wage workers. The exact mechanism, however, has remained unclear.

Our main contribution is showing that effort incentives of NCCs can be such a mechanism. To show this, we use the canonical partial-market moral hazard model and add the possibility to costlessly reduce the agent's payoff with an NCC. The employer terminates the employee after a bad performance, which activates the NCC and restricts the agent's employment possibilities. To avoid this, the agent exerts more effort. Thus, both a bonus wage and an NCC provide incentives—they are substitutes in the incentive constraint. If a sufficiently large minimum wage restricts the principal's use of wages to appropriate the surplus, she resorts to an NCC. The reason for this is that a bonus wage and an NCC are opposites in the participation constraint: The bonus wage makes the participation constraint slack, as it increases the agent's payoff after a good outcome, whereas the NCC makes the participation constraint tight, as it decreases the payoff after a bad outcome. Because the minimum wage might leave the agent a rent—slacken the participation constraint—the principal might want to add an NCC to the contract. This increases the equilibrium effort at no cost to the principal; merely the agent's rent is reduced—the participation constraint gets tighter.

We compare the distribution of rents in the benchmark without NCCs and the equilibrium outcome with NCCs for different minimum wages according to Pareto dominance: When the severity of NCCs is unbounded, the profit maximizing contract makes the participation constraint bind and leaves the agent no rent, irrespective of the minimum wage's level. If the minimum wage is binding but so low that the agent gets no rent without an NCC, allowing NCCs leads to a weak Pareto improvement as the equilibrium effort increases, which increases the profit of the principal. However, because minimum wages decrease the social surplus, a policymaker who introduces minimum wages does so to grant the agent a rent and would, thus, never choose a minimum wage in this region. If the minimum wage is in the region in which the agent would get a rent without NCCs, the outcomes are not comparable by Pareto dominance: While the principal is better off, the agent is worse off.

Two possible extensions may change these results. If there is an extensive margin, NCCs reduce market exits as they increase the principal's profit. That is, minimum wages affect employment less because NCCs counteract. In Appendix 1.A, we consider bounded NCCs. We verify with an example that bounded NCCs can lead to a strict Pareto improvement over minimum wages alone. Intuitively, NCCs with a suitably chosen bound may reduce the inefficiency from minimum wages, while the bound prevents the principal from extracting the agent's rent completely.

09, 2021 (https://cutt.ly/nX8lCUw). Furthermore, the "Mobility and Opportunity for Vulnerable Employees Act", the "Workforce Mobility Act", and the "Freedom to Compete Act" have been introduced, but neither has been passed. There has also been progress on the state level: Some states now make NCCs unenforceable if the employee's salary lies below a threshold.

Further welfare results are concerned with the utilitarian welfare.[3] We decompose the total effect into an idleness effect and an incentive effect. The *idleness effect* is the direct reduction in the social surplus from reducing the agent's payoff after a failure. It always reduces the utilitarian welfare. The *incentive effect* works through the increase of the equilibrium effort, which is how NCCs transfer utility from the agent to the principal. If the minimum wage is binding but low, the incentive effect is positive as the equilibrium effort in the benchmark without NCCs is inefficiently low. The NCC brings the equilibrium effort closer to the first-best level (Proposition 1.3). If the minimum wage is large, however, the incentive effect becomes negative because the equilibrium effort with NCCs gets inefficiently large (Proposition 1.4). The principal induces an inefficiently large effort as the only way of extracting the agent's rent is through higher equilibrium efforts. The total effect of NCCs on the utilitarian welfare is positive if the minimum wage is low, because they reduce the inefficiency from minimum wages. If the minimum wage is large, NCCs decrease the utilitarian welfare as the incentive effect turns bad. Bounded NCCs put an upper limit on the equilibrium effort and can, thus, in some situations keep the total effect positive (see Appendix 1.A).

Another, related contribution explains an empirical puzzle. Hair salon owners are more likely to make their employees sign NCCs when the minimum wage increases (Johnson and Lipsitz, 2022). Johnson and Lipsitz (2022) show that this can be explained if NCCs can be used to transfer utility. We complement their study by showing that effort provision is a possible micro-foundation for the utility transfer. This micro-foundation yields additional empirically testable predictions, for example, employees with a, ceteris paribus, worse outside option should have more severe NCCs or should be more likely to have an NCC at all (for more details see Section 1.6). Furthermore, the optimal contract in our model includes an NCC only if the minimum wage is sufficiently high and from thereon the optimal NCC gets strictly more severe in the minimum wage (Proposition 1.2).

Our second main contribution is to provide a novel explanation for why rational minimum wage workers sign NCCs in the first place. Due to her market power, the principal can extract rents from the agent. Because of the minimum wage laws (or limited liability), this cannot be done via money but only through increased effort incentives. The transfer motive complements the usual four reasons for the use of NCCs in the literature; most of which are not particularly appealing for the case of low-wage workers: Firstly, employers can use NCCs to improve their bargaining power in future wage bargaining.[4] Yet, minimum wage workers rarely bargain for wage in-

3. We define the utilitarian welfare as the unweighted sum of the agent's payoff and the principal's profit.

4. The verbal argument is developed in Arnow-Richman (2006). Empirical findings from the ban of NCCs in the high-tech sector in Hawaii (Balasubramanian, Chang, Sakakibara, Sivadasan, and Starr, 2020) are consistent with the argument. Moreover, 30% of the surveyed employees that have

creases.[5] Secondly, like non-disclosure agreements and non-solicitation agreements, NCCs protect proprietary information and client lists. Yet, many low-wage workers do not possess sensitive information. Thirdly, NCCs increase the job tenure, which reduces the turnover.[6] This reduces training and hiring costs. Yet, these costs are rather low for most low-wage jobs.[7] Fourthly, NCCs mitigate the hold-up problem of investments in human capital. If workers are liquidity constrained and cannot invest in their industry-specific human capital, an NCC allows the employer to recoup her investment (Rubin and Shedd, 1981).[8] It is debatable how important human capital is in minimum wage jobs. One the one hand, it takes more than a thousand hours of training before one can become a certified hair dresser. On the other hand, many fast-food employees report that they have gotten less than three days of training. Anyway, our model does not rely on human capital or the other reasons to use NCCs. Instead, non-contractable effort is sufficient a reason to use NCCs.

This paper is organized as follows. Section 1.2 provides background information on the use of non-compete clauses and their enforcement, and discusses the related literature. Section 1.3 introduces the model. In Section 1.4, we find the profit maximizing contracts in the benchmark and with unbounded NCCs. The welfare implications of these contracts are analyzed in Section 1.5. In Section 1.6, we discuss the simplifying assumptions and summarize empirical predictions of our model. Finally, Section 1.7 concludes.

1.2 Background and Related Literature

1.2.1 Background

As the legislation on non-compete clauses is very different across the United States, we focus on the aspects that are relevant for our model. The principal uses an NCC

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been bound by an NCC have not received any advanced notice, but have been presented their NCCs on their first day at work (Starr, Prescott, and Bishara, 2021, p. 69).

5. Cahuc, Postel-Vinay, and Robin (2006) find that low-wage worker possess no significant bargaining power. Instead, the wage growth for low-wage workers often comes from changing jobs, which is also shut down by NCCs (Colvin and Shierholz, 2019).

6. A positive correlation of (the enforceability of) NCCs and the average length of job tenure has been found by Balasubramanian et al. (2020) and by Starr, Frake, and Agarwal (2019).

7. A meta-study (https://www.americanprogress.org/article/there-are-significant-business-costs-to-replacing-employees/) finds that the turnover costs average around 20% of the annual salary and are rather lower for low-skilled jobs. For the fast-food industry, reports range between \$600 and \$2000 while the turnover rate is around 150% (https://cutt.ly/9X8kQVw). Yet, many firms do not even know their turnover cost and seem to ignore them as they are not salient (https://cutt.ly/4X8dcp1)

8. Long (2005) proposes repayment agreements as a better alternative to NCCs in this case. The disadvantage of NCCs is that they usually remain in the contract even after the employer has recouped his investment, whereas repayment agreements expire.

to threaten the agent into exerting more effort. For the threat to be credible, courts have to be willing to enforce such NCCs.

There are attempts by Bishara (2011) and Garmaise (2011) to compare whether the states' courts rule in favor of rather the employees or the employers. Both use a comprehensive survey of courts' decisions (Malsberger, 2019) and questionnaires to calculate one-dimensional measures of NCCs' enforceability for all states. This allows them to order the states on a spectrum, going from states that do not enforce NCCs at all—California, North Dakota, and Oklahoma—to states in which courts are ordered to ignore hardships that NCCs cause for employees—like Florida. In many states, employers can use NCCs in the way they want to.

That NCCs might be used to provide incentives is also reflected in the enforceability questionnaire of Bishara (2011): "Question 8: If the employer terminates the employment relationship, is the covenant enforceable?" (Bishara, 2011, p. 777). The states are awarded scores on a scale from 0 to 10, where 0 means that a dismissal makes an NCC unenforceable and 10 means that a dismissal makes no difference whatsoever. Only five states score less than 6. Moreover, 15 jurisdictions score 10. That is, NCCs stay active when being dismissed for bad performance at the job in most states.

Even if the NCC became unenforceable after dismissal for bad performance, having signed an NCC might negatively affect the search for a new job. The cost of litigating an unenforceable NCC is high for low-wage workers (Colvin and Shierholz, 2019, p. 5-6), so former employees might rather adhere to an unenforceable NCC. Empirical evidence shows that unenforceable NCCs affect the employees' behavior (Starr, Prescott, and Bishara, 2020). Moreover, although California and North Dakota do not enforce NCCs, the prevalence is the same as in states that enforce NCCs (Starr, Prescott, and Bishara, 2021). Lastly, some NCCs specify that trials are not to be held by official courts but by mandatory arbitration. Since mandatory arbitrators' rulings are usually confidential, the enforceability of an NCC might differ from the expected enforceability in a given state.

Summing up, in many states, NCCs are unaffected by a dismissal due to bad performance on the job. Even if a states' law renders NCCs unenforceable after a dismissal due to bad performance, there are reasons to believe that the existence of an NCC affects the employee's job searching behavior, and, thus, also the employee's outcome.

1.2.2 Related Literature

This paper is related to multiple strands of literature. We first summarize the small literatures on the incentive effects NCCs and on utility transfers using NCCs.[9] Then,

9. We refer the reader that is interested in other theoretical and empirical articles on NCCs to the survey McAdams (2019).

we summarize two related concepts: efficiency wages and collateralized debt. Lastly, we explain our methodological contribution to the literature on moral hazard.

In Kräkel and Sliwka (2009), contrasting our model, NCCs reduce the agent's incentives. In their model, exerting more effort increases the probability of outside offers. If the agent has no NCC, an outside offer leads to a wage increase to retain the agent. If the agent has an NCC, the principal does not have to make a retention offer; reducing the payoff from exerting effort.

Cici, Hendriock, and Kempf (2021) empirically test the incentive effect of NCCs. Their identification strategy is using exogenous legislative changes in the enforceability of NCCs. The hypotheses are derived without a formal model. They find that mutual fund managers perform better when NCCs get more enforceable. This evidence suggests that the machanism in our model exists in the real world.

NCCs have been argued before to redistribute rent from the agent to the principal. Wickelgren (2018) proposes a hold-up model with investments in human capital. A minimum wage prevents the principal from extracting all rents without an NCC. By making the agent sign an NCC, the principal can prevent the agent from leaving without increasing the wage. The optimal contract does not leave a rent to the agent. In contrast to our work, this model relies on human capital investments for minimum wage workers.

Johnson and Lipsitz (2022) find in the data that higher minimum wages lead to more NCCs. They also provide a model on the use of NCCs to transfer utility if a minimum wage restricts the transfer of utility via money. If the terms of trade favor the employers, the employees have to sign NCCs to (inefficiently) transfer utility to the employers in equilibrium. When signing an NCC, employees incur an exogenous cost while employers receive an exogenous benefit. Whether NCCs are used or not is determined by the participation constraint of the least productive firm according to a "law of one price." Thus, the prediction that larger minimum wages lead to the use of NCCs. We complement their work by providing a micro-foundation for NCCs' transferring utility.

The interpretation of non-compete clauses as a means to provide incentives reminds of two similar concepts. Firstly, there are efficiency wages. In the literature started by Shapiro and Stiglitz (1984), an agent is also retained after a good outcome and dismissed after a bad outcome. The differential of the corresponding payoffs provides incentives to exert effort. The difference from our model is that efficiency wages—wages above the market-clearing level—increase the payoff in the good state. Thus, with limited liability, efficiency wages make the agent's participation constraint slack and grant him a rent. NCCs, in contrast, reduce the payoff in the bad state after a dismissal. Thus, even with limited liability, they make the agent's participation constraint slack and extract his rents.

Secondly, there is the literature on collateralized debt (e.g. Stiglitz and Weiss, 1981, Bernanke and Gertler, 1989, Chan and Thakor, 1987, Bester, 1987, Boot, Thakor, and Udell, 1991, and Tirole, 2006). An agent, who is cash constrained, might pledge an asset in order to improve his access to a credit line. After a signal for low effort (default), the asset is transferred to the bank. This both incentivizes the agent and reduces the loss of the bank. Non-compete clauses in our model are similar to collaterals in lending agreements: The agent pledges his labor. After a bad performance, the NCC is activated, and the agent is not allowed to sell his labor to someone else. One difference in these articles is the efficiency loss from using other payoff dimensions. In the one extreme, pledging a perfectly resellable asset is a perfect substitute to monetary payments because the asset is of the same value to the principal as to the agent. Thus, the friction from limited liability vanishes. In the other extreme, the principal has a negative value for the asset she has to seize (in Chwe (1990), the asset is bodily integrity, and whipping the agent also hurts the principal). Our model is in-between these extremes: An NCC costs the agent, but neither costs nor benefits the principal.

Methodologically, we contribute to the literature of agency models with moral hazard in continuous effort and with limited liability (e.g. Schmitz, 2005, Kräkel and Schöttner, 2010, Ohlendorf and Schmitz, 2012, and Englmaier, Muehlheusser, and Roider, 2014). Especially, we contribute to the agency literature with multidimensional (monetary and non-monetary) payoffs. In our model, the payoff's dimensions are present and future payoff. Minimum wages affect only present payoffs. NCCs can reduce only future payoffs via unemployment. As in the present paper, Kräkel and Schöttner (2010) show that future rents can be used to incentivize effort in the first period.

There are articles with similar models that interpret the second argument of the agent's payoffs as pain or unfriendliness. It is pain in the coerced labor settings of Chwe (1990) and Acemoglu and Wolitzky (2011). Chwe (1990) provides a model in which the principal can inflict costly pain to the agent. As in our model, inflicting pain maximizes the profit if monetary transfers are limited due to wealth constraints. Another variant of this model is used in Acemoglu and Wolitzky (2011). Besides some simplifications, the principal can pay to reduce the agent's reservation utility. Furthermore, their model is later extended from a partial market to a general equilibrium. The interpretation as unfriendliness is taken in Dur, Kvaløy, and Schöttner (2022). Under limited liability a leader might use an unfriendly leadership style to reduce the agent's payoff.

1.3 Model

We consider a moral hazard model with continuous effort, binary output, and limited liability. There is a risk-neutral principal P (*she*) who owns a project. The project can be either a success and pay off V or a failure and pay off nothing. P wants to hire a risk-neutral agent A (*he*) to work on the project for one period. The principal offers the agent a contract that consists of three items: a base wage w, which is paid unconditionally, a bonus wage b, which is paid conditionally on a success, and a non-compete clause (NCC).[10] The wages are subject to a minimum wage that introduces limited liability.

The agent's expected utility accrues in two stages: the effort provision stage and a continuation in which an NCC might come into play. For simplicity, we present a partial market model. That is, we do not micro-found the continuation payoff. Instead, we directly assume that having an NCC when losing a job reduces the expected discounted future payoff. In Section 1.6, we justify this assumption and present details about how to think about the outside option.

We now consider the effort provision stage in more detail. The agent chooses his effort $e \in [0,1]$ at a strictly convex cost of $c(e)$, where $c(0) = 0$. We assume the standard Inada conditions that $c'(0) = 0$ and $\lim_{e\to 1} c'(e) = \infty$. We also assume that $\frac{c'''(e)}{c''(e)} > \frac{1}{1-e}$ $\forall e \in [0,1)$ to get a concave objective function (see Lemma 1.3 in the Appendix).[11] Two examples are $c(e) = -\ln(1-e) - e$ and $c(e) = \frac{e^2}{1-e}$.[12] The effort level that A chooses is private information and, thus, creates a moral hazard problem. The chosen effort is the probability that the project is successful, that is, a success payoff V accrues to the principal with probability e, $\text{Prob}(\text{success}|\, e) = e$. Successes are verifiable and serve as a signal for the agent's effort. In the case of a success, the agent gets the bonus wage b.

We now consider the simplified continuation (as mentioned above, see Section 1.6 for details). After the project is completed, the agent's continuation payoff is determined. The continuation payoff can take two values. If the agent is retained, we set the continuation payoff to zero. If the agent is fired at the end of the effort provision stage, the NCC gets activated and reduces the continuation payoff. The contract's NCC directly specifies the agent's continuation payoff, $\bar{v} \leq 0$. Concerning the principal, we assume that dismissing the agent has no effect on her continuation profit. That is, hiring a replacement is costless. As we prove in Section 1.6, this implies that it is optimal for the principal to fire the agent after a failure and to retain the agent after a success.

To sum up, a contract between the principal and the agent is defined by the tuple $(w, b, \bar{v})$. These items are constrained. The minimum wage law demands that

10. Various forms of incentive pay are common in minimum wage jobs. We refer the interested reader to Section 1.6. In our model, we use explicit bonus payments as a stand-in for more complicated methods of incentive pay. The qualitative results of our model remain the same if the bonus wage is exogenously set to 0. The model is then closer to the efficiency wage literature.

11. Compared to the canonical principal agent model, the principal has an additional choice variable, the NCC. Therefore, to get a well-behaved problem, we need a stronger assumption on the cost function than the standard assumption that $c'''(e) > 0$. Chwe (1990) and Acemoglu and Wolitzky (2011) use the same assumption in their models. In the proofs in the Appendix, we will state which assumptions on the cost function we need in the respective steps. The concavity assumption is simpler and implies all of them.

12. These cost functions are only defined for $e \in [0,1)$ and $\lim_{e\to 1} c(e) = \infty$.

the agent is paid at least the minimum wage $\underline{w}$ for the effort-provision stage.[13] After a failure, the principal pays the agent $w \geq \underline{w}$, and after a success she pays him $w + b \geq \underline{w}$. The level of the minimum wage is relative to the agent's outside option that we have normalized to zero.[14] The NCC is constrained, $\bar{v} \leq 0$, because it can only reduce the agent's continuation payoff. We say that a contract does have *no non-compete clause* if $\bar{v} = 0$. The lower $\bar{v}$, the lower is the agent's continuation payoff after being dismissed. We refer to a lower $\bar{v}$ as a *more severe non-compete clause*.

If he signs the contract, the agent's expected utility is given by

$$\mathbb{E}U \ = \ w + e \cdot b + (1 - e) \cdot \bar{v} - c(e). \tag{1.1}$$

The agent takes his contract as given and chooses his effort. The base wage is paid unconditionally, the bonus wage only in the case of success, and if the agent fails, the NCC is activated.

The principal's expected profit is given by

$$\pi \ = \ -w + e \cdot (V - b). \tag{1.2}$$

The principal anticipates whether the agent will sign the offered contract and, if so, which effort the agent will exert. The principal then maximizes her expected profit by choosing the contract. We assume that the success payoff, V, is large enough such that the principal makes a profit that exceeds her outside option, and therefore ignore the extensive margin (except for a paragraph in Section 1.5).

The timing of the game is as follows. The principal offers a contract to the agent. The agent can reject or accept the offer. If he rejects, the game ends and he gets his outside option. If he accepts, the game continues. The agent then chooses his effort from the unit interval. The payoffs to the agent and the principal are determined according to the accepted contract, including that the principal dismisses the agent and activates the NCC after a failure. The solution concept is subgame perfect Nash equilibrium. We find it by backward induction.

The timeline in Figure 1.1 summarizes the game.

13. A minimum wage might not be the only friction that prompts employers to use NCCs. In general, employers want to use NCCs as soon as employee's receive some kind of rent that can be expropriated via NCCs. For example, the downward-rigidity of nominal wages might prevent the principal from reducing the wages but not from adding an NCC. Cici, Hendriock, and Kempf (2021) have shown that NCCs have incentive effects on funds managers, which suggests that funds managers receive some rent. An example for a rent that cannot be expropriated using NCCs is an information rent: Anticipating that he would be asked to sign an NCC, an employee will not reveal his private information.

14. In other models with limited liability, it is often assumed that the agents are heterogeneous in their outside options. Then, the minimum wage (or limited liability) is normalized to zero. In our model, on the contrary, we normalize the outside option to zero. This reflects our assumption that ability or human capital plays no role, thus, the agents are homogeneous in their outside options. We are interested in the effects of (an increase in) the minimum wage. Our modeling choice makes this more easily interpretable. To consider heterogeneous agents, keep in mind that a better outside option is equivalent to a lower minimum wage.

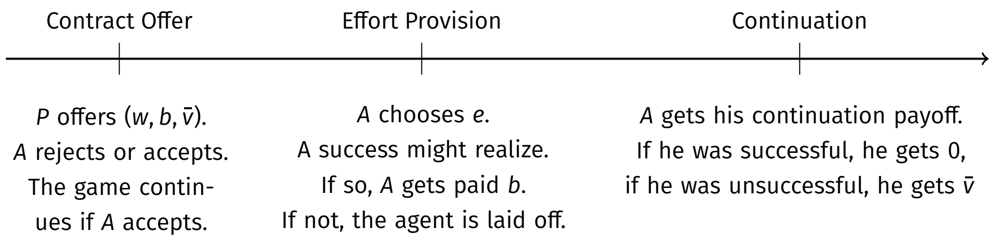

Figure 1.1. The timing of the game.

First-best welfare analysis. First, consider the benchmark without any frictions. A social planner maximizes the expected social welfare

$$W^{FB} = \max_{e\in[0,1],\ \bar{v}\leq 0} \ e \cdot V - c(e) + (1-e)\cdot \bar{v}. \tag{1.3}$$

The first-order condition shows that in the social optimum there is no NCC because due to the Inada conditions, the effort will be interior. As a result, any NCC comes into action with positive probability which forces the agent's labor to lie idle. This is inefficient. This means that $\bar{v} = 0$ maximizes the social surplus.

Given that $\bar{v} = 0$, the first-best effort equates the marginal benefit and the marginal cost, $V = c'(e^{FB})$. This is optimal due to the welfare function's concavity.

1.4 Equilibrium Contract

In this section, we characterize the profit maximizing contracts for different minimum wages. We start with analyzing how the presence of an NCC changes the incentive and the participation constraint and reformulate the participation constraint to see how an NCC affects the model. We then solve for the optimal contract in the benchmark, in which the principal is not allowed to use NCCs. Then, we allow the principal to use arbitrarily severe NCCs, and we compare the results to the benchmark. We find that the principal only wants to use NCCs if the minimum wage is sufficiently large. Moreover, there is no minimum wage for which the agent gets a rent.

To build intuition, we now look at how an NCC changes the incentive compatibility and the participation constraint. Given the contract $(w, b, \bar{v})$, the agent chooses the effort level e^* that maximizes his expected utility,

$$e^* = \arg\max_{e\in[0,1]} \ w + e\cdot b + (1-e)\cdot\bar{v} - c(e). \tag{IC}$$

This is the agent's incentive compatibility constraint. If $b - \bar{v}$ is non-negative, the agent's effort choice is characterized by the first-order condition

$$b - \bar{v} = c'(e^*). \tag{1.4}$$

The equilibrium effort is then unique because the marginal cost is strictly increasing. The first-order condition shows that the bonus wage and the NCC are perfect substitutes for giving incentives. Therefore, NCCs have an *incentive effect* as they generate higher effort incentives. P must now decide to what extent to provide incentives through an NCC and to what extent through a bonus wage.

The agent only accepts the contract if his participation constraint

$$w + e^* \cdot b + (1 - e^*) \cdot \bar{v} - c(e^*) \ \geq \ 0. \tag{PC}$$

is satisfied. The bonus wage and the severity of the NCC go into opposite directions in the participation constraint. A higher bonus wage makes the participation constraint slack.[15] A more severe NCC makes the participation constraint tight. This already hints at the use and the distributional effects of NCCs: Whenever the agent would get a rent without an NCC, the principal will add an NCC to the contract and convert the rent into more incentives. The participation constraint will always bind. In the participation constraint, the NCC enters twice. Firstly, it enters indirectly via the equilibrium effort, through the incentive effect. Secondly, the idleness effect can be seen in the participation constraint: The NCC enters directly as $(1-e^*)\bar{v} \leq 0$. That is, the NCC reduces the agent's utility (and, thus, the social surplus) because labor force has to lie idle if there is a failure.

One could also decompose the effect of an NCC in a different way. Rearranging the agent's expected utility yields $(w+\bar{v}) + e^*(b-\bar{v}) - c(e^*)$. This means that the NCC reduces the base and increases the bonus wage as perceived by the agent. In that sense, NCCs undermine minimum wage laws. The principal, however, does not profit from the reduction in the perceived base wage as the activation of the NCC, the idleness effect, burns surplus instead of transferring it. The benefit of the NCC for the principal comes from the increase in the perceived bonus wage, which increases the equilibrium effort without the principal's having to pay for it. In this reformulation, the incentive effect is hidden in the equilibrium effort and the two $\bar{v}$ in the expected utility are the idleness effect. In Section 1.5, we will take a closer look at the welfare effects of the incentive and the idleness effect.

With the possibility of imposing an NCC, the principal's problem becomes

15. That is the reason why limited liability usually causes the agent to get a rent: To provide incentives, the principal has to pay a bonus wage, slackening the participation constraint.

$$\max_{w,b,\bar{v}} \quad -w + e^* \cdot (V - b) \tag{1.5}$$

$$\text{s.t.} \quad e^* = \arg\max_{e\in[0,1]} \; w + e \cdot b + (1-e) \cdot \bar{v} - c(e), \tag{IC}$$

$$w + e^* \cdot b + (1-e^*) \cdot \bar{v} - c(e^*) \geq 0, \tag{PC}$$

$$\bar{v} \leq 0, \tag{NCC}$$

$$w \geq \underline{w}, \tag{MWC1}$$

$$w + b \geq \underline{w}. \tag{MWC2}$$

The principal maximizes her expected profit subject to the incentive-compatibility constraint, the participation constraint, the NCC feasibility constraint, and the minimum wage constraints.

The benchmark without non-compete clauses. Before we proceed and analyze the optimal contract with NCCs, we briefly consider the benchmark without NCCs. Formally, this means that $\bar{v} = 0$ is set exogenously. Hence, P can only choose the base and the bonus wage. The optimal contracts under limited liability with those two tools are well known (see for example Laffont and Martimort, 2002, and Schmitz, 2005). Proposition 1.1 derives the optimal contract that the principal offers to the agent in the benchmark.[16]

Proposition 1.1. *Consider the problem without NCCs. There exist threshold values in the minimum wage κ_1 and κ_3 such that P offers the following contract to A:*

(i) *Let $\underline{w} \leq \kappa_1$. Then P chooses the compensation scheme $(w,b) = (\kappa_1, V)$.*

(ii) *Let $\kappa_1 < \underline{w} \leq \kappa_3$. Let $e_2^{BM}(\underline{w})$ be implicitly defined by $c(e_2^{BM}) - e_2^{BM} \cdot c'(e_2^{BM}) = \underline{w}$. Then P chooses the compensation scheme $(w,b) = (\underline{w}, c'(e_2^{BM}))$.*

(iii) *Let $\kappa_3 < \underline{w}$. Let $e_3^{BM}(\underline{w})$ be implicitly defined by $c'(e_3^{BM}) + e_3^{BM} \cdot c''(e_3^{BM}) = V$. Then P chooses the compensation scheme $(w,b) = (\underline{w}, c'(e_3^{BM}))$.*

Note that the subscripts are 1 and 3. There is a specific κ_2 that lies in-between κ_1 and κ_3, but it is irrelevant in the benchmark.

The three parts of Proposition 1.1 correspond to the three cases of binding and non-binding constraints; depending on the level of the minimum wage.

Case 1. The minimum wage is lower than the wages the principal wants to sets the base wage when she ignores the minimum wage constraints. Therefore, the optimal contract is the same as with unlimited liability. The principal leaves the success payoff to the agent and uses the base wage to extract the complete surplus from the agent. Therefore, this case is commonly referred to as "selling the firm."

16. All proofs are relegated to Appendix 1.B.

Case 2. If the minimum wage is above κ_1, selling the firm violates the minimum wage condition; the principal cannot extract the full social surplus anymore. The optimal base wage is the minimum wage. For minimum wages between κ_1 and κ_3, the bonus wage that solves the principal's first-order condition of profit maximization would violate the participation constraint; it is too low. Because the profit function is concave in the bonus wage, the optimal bonus wage makes the participation constraint bind. It is below the success payoff, implying a lower than first-best effort and social surplus. Furthermore, it is decreasing in the minimum wage as a lower bonus wage is needed to keep the participation constraint satisfied when the base wage is larger. Hence, the equilibrium effort is also decreasing in the minimum wage. The binding participation constraint means that the minimum wage does not redistribute from the principal to the agent; it solely induces inefficiency.

Case 3. For minimum wages above κ_3, the bonus wage that solves the principal's first-order condition does not violate the participation constraint anymore; as the base wage is large enough. As the first-order condition does not depend on the minimum wage, neither does the optimal bonus wage; and thus nor does the equilibrium effort. The social surplus is, thus, constant. Because the participation constraint does not bind anymore, the agent gets a rent. A minimum wage now becomes a tool of perfect redistribution: An increase of the minimum wage by one unit translates into an increase of the agent's rent by one unit.

The equilibrium analysis with non-compete clauses. Proposition 1.2 summarizes the optimal contracts with NCCs.

Proposition 1.2. *Consider the problem with NCCs. There exist threshold values in the minimum wage κ_1 and κ_2. If $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2} \cdot V < 1$, there exist the threshold value in the minimum wage κ_4. P offers the following contract to A:*

(i) *Let $\underline{w} < \kappa_1$. Then P offers the contract $(w, b, \bar{v}) = (\kappa_1, V, 0)$.*

(ii) *Let $\kappa_1 \leq \underline{w} \leq \kappa_2$. Let $e_2^{BM}(\underline{w})$ be implicitly defined by $c(e_2^{BM}) - e_2^{BM} \cdot c'(e_2^{BM}) = \underline{w}$. Then P offers the contract $(w, b, \bar{v}) = (\underline{w}, c'(e_2^{BM}), 0)$.*

(iii) *Let $\kappa_2 < \underline{w} < \kappa_4$. Let $e_3^{NCC}(\underline{w})$ be implicitly defined by $c(e_3^{NCC}) + (1 - e_3^{NCC})c'(e_3^{NCC}) + e_3^{NCC}(1 - e_3^{NCC})c''(e_3^{NCC}) = V + \underline{w}$. Then P offers the contract $(w, b, \bar{v})$ $= \big(\underline{w}, (1 - e_3^{NCC})c'(e_3^{NCC}) + c(e_3^{NCC}) - \underline{w}, c(e_3^{NCC}) - \underline{w} - e_3^{NCC}c'(e_3^{NCC})\big)$.*

(iv) *Let $\kappa_4 \leq \underline{w}$. Let $e_4^{NCC}(\underline{w})$ be implicitly defined by $(1 - e_4^{NCC}) \cdot c'(e_4^{NCC}) + c(e_4^{NCC}) = \underline{w}$. Then P offers the contract $(w, b, \bar{v}) = \left(\underline{w}, 0, -\frac{\underline{w} - c(e_4^{NCC})}{1 - e_4^{NCC}}\right)$.*

The four parts of Proposition 1.2 correspond the four combinations of binding and non-binding constraints for different levels of the minimum wage. Figure 1.2 illustrates which constraints are binding in the optimum depending on the size of the minimum wage. If $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2} \cdot V \geq 1$, Combination 4 is never optimal. Importantly, the participation constraint binds in all combinations; the agent never gets a rent. If the participation constraint were slack, there would be a profitable deviation: making the NCC more severe. The equilibrium effort increases and, because the agent gets less than the success payoff, the principal profits.

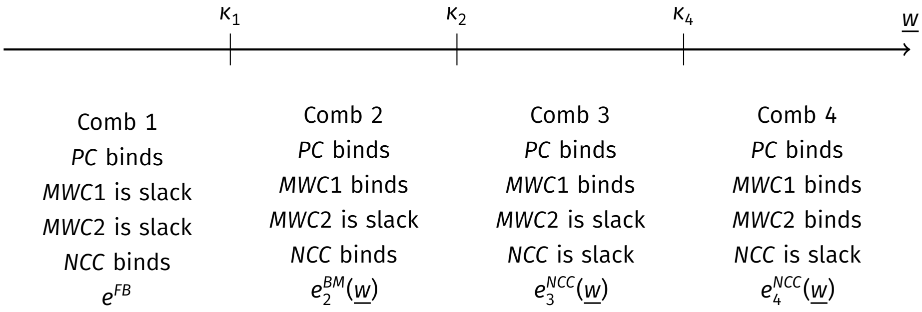

Figure 1.2. The combinations of binding and non-binding constraints that characterize the optimal contract when NCCs are allowed. The combinations are from Table 1.B.1. When there are NCCs, the cutoff κ_3 is meaningless.

Another illustration of the optimal contract for a specific cost function is given in Figure 1.3. This figures compares the optimal contract when the principal can use NCCs and when she cannot.

We will now consider each combination in more detail.

Combination 1. This combination is identical to Case 1 in the benchmark. As the principal's profit is already equal to the first-best social surplus, she cannot do any better by introducing an NCC.

Combination 2. For minimum wages between κ_1 and κ_2, it is optimal for the principal not to use NCCs. The optimal contract is the same as in the benchmark in Case 2, although it stops at a lower minimum wage, $\kappa_2 < \kappa_3$. Because the equilibrium effort is still quite high at these minimum wages and because the effort cost is convex, additional incentives increase the effort only by little. Therefore, the incentive effect of an NCC is small. On the other hand, there is the idleness effect: As the participation constraint binds, the principal would have to increase the bonus wage to compensate the agent for NCC's effect after a failure. The idleness effect is larger. Considering the small incentive effect, using an NCC is too costly.

Combination 3. When the minimum wage increases, the bonus wage and, hence, the equilibrium effort decrease. As a result, the incentive effect of NCCs gets stronger compared to the idleness effect.[17] At a minimum wage of κ_2, both effects are equally

17. Because the effort cost is convex, the first unit of NCC affects the equilibrium effort more when the effort is initially low. The negative idleness effect also gets (absolutely) larger if the equilibrium effort is initially low because it is linear in the probability of a failure. Due to the convex effort cost, the incentive effect changes more.

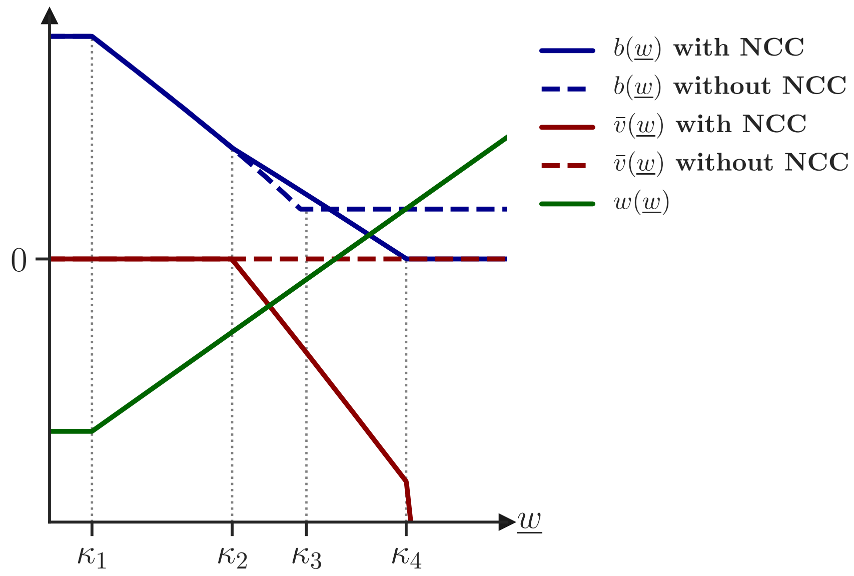

Figure 1.3. Illustration of the optimal contract for different minimum wages for $c(e) = -\ln(1-e) - e$ and $V = 11$.

strong. If the minimum wage is above κ_2, the incentive effect prevails and the principal uses an NCC. Moreover, Proposition 1.3 shows that the optimal NCC gets monotonically more severe in the minimum wage.

As discussed above, the equilibrium effort is strictly decreasing in the minimum wage in Combination 2. In Combination 3, not only does the optimal NCC get more severe in the minimum wage, but also does the sum of incentives increase in the minimum wage (Proposition 1.3 (iii)). That is, the change in the NCC overcompensates the decreasing bonus wage. Thus, the equilibrium effort is strictly increasing for minimum wages above κ_2 (as we show below, the equilibrium effort remains increasing even if Combination 4 is optimal). This yields a novel result: The equilibrium effort is non-monotone in the minimum wage (Proposition 1.3). Standard models in the literature (like our benchmark) find that the equilibrium effort is decreasing and finally stays constant in the minimum wage.

Proposition 1.3 (Non-Monotonicity of Optimal Effort). *The equilibrium effort is non-monotonic in the minimum wage.*

(i) *Let $\underline{w} < \kappa_1$. Then the equilibrium effort level is constant in the minimum wage.*
(ii) *Let $\kappa_1 \leq \underline{w} \leq \kappa_2$. Then the equilibrium effort level is strictly decreasing in the minimum wage.*
(iii) *Let $\kappa_2 < \underline{w}$. Then the equilibrium effort level is strictly increasing in the minimum wage.*

The equilibrium effort is not only increasing in the minimum wage, but it also gets eventually inefficiently large, as Proposition 1.4 shows. Using more severe NCCs is the principal's only way of extracting the agent's rent. The principal accepts inducing inefficiently large efforts as some of the social surplus would otherwise end up with the agent. If the minimum wage goes to infinity, the equilibrium effort goes to one.

Proposition 1.4 (Inefficiently Large Optimal Effort). *As the minimum wage goes to infinity, the equilibrium effort goes to 1. Hence, the equilibrium effort level exceeds the first-best effort if the minimum wage is sufficiently large.*

How does Combination 3 compare to Case 2 in the benchmark? As the optimal contract in Case 2 makes the participation constraint bind already without an NCC, the bonus wage in Combination 3 has to be larger. With NCCs, the principal can provide "double incentives" by increasing the bonus wage: The higher bonus wage make the participation constraint slack. This allows for a more severe NCC, which makes the participation constraint binding again. Both the increase in the bonus wage and in the NCC's severity provide incentives. In the proof of Proposition 1.3, we also show that the optimal bonus wage nevertheless decreases in the minimum wage, although less so than in Case 2 of the benchmark. At some minimum wage above κ_3, the optimal bonus wage with NCCs falls below the optimal, constant bonus wage from Case 3. Intuitively, the minimum wage is so large that it keeps the participation constraint satisfied even with a smaller bonus wage. In Case 3, the bonus wage is so large that it makes the participation constraint slack and the agent gets a rent.

Combination 4. The effectiveness of the double incentives decides whether Combination 3 remains optimal for all minimum wages: Using a bonus wage for all minimum wages is only optimal, if the equilibrium effort reacts strongly enough to an increase in the incentives. The formal condition on the effort cost function is $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2} \cdot V \geq 1$. If this condition is not satisfied, at a minimum wage of κ_4, it becomes profitable for the principal to not pay a bonus wage anymore; Combination 4 becomes optimal. It is then better to provide incentives only through the NCC instead of using double incentives.[18] The saved cost outweighs the reduction in the equilibrium effort, as the equilibrium effort does not react much.

For the same reason, in contrast to Combination 3, the principal would even want to pay negative bonus wages—charging the agent for successes—as the equilibrium effort gets inefficiently large. This would be another way to extract the agent's rent

18. This might be one answer to the empirical question why we do not see that many explicit bonus wages in reality even in situations with verifiable performance measures: Other forms of implicit incentives or better for the principal than using bonus wages. In this case, an NCC allows the principal to increase the incentives without having to pay for it since the agent pays for the additional incentives with his rent.

than via more effort. The minimum wage condition for the case of a success, however, would be violated if the base wage is not adjusted: To use a negative bonus wage, the base wage has to be above the minimum wage. As we show in the proof of Proposition 1.2, increasing the base wage is too expensive for the principal. Therefore, the optimal bonus wage can never be negative.

1.5 Welfare Analysis

1.5.1 Utilitarian Welfare

Having characterized the profit-maximizing contracts, we can now look at the welfare effects of NCCs. The first welfare criterion that we consider is utilitarian welfare—the sum of the agent's rent and the principal's profit. From the previous section, we already know that with NCCs, the agent never gets a rent. Hence, the utilitarian welfare is equal to the principal's profit.

NCCs affect the utilitarian efficiency through two channels: the incentive and the idleness effect. The incentive effect works indirectly through the increasing equilibrium effort due to an NCC. The incentive effect formally is

$$\int_{e^{\text{No NCC}}}^{e^{NCC}} \left(V - c'(x)\right)\, dx, \tag{1.6}$$

where $e^{\text{No NCC}}$ denotes the equilibrium effort without an NCC and e^{NCC} denotes the equilibrium effort with an NCC, both of which depend on the minimum wage. The incentive effect is zero at the minimum wage κ_2, at which the principal first uses an NCC. It then increases in the minimum wage and finally decreases again: For minimum wages slightly above κ_2, without an NCC, the equilibrium effort is inefficiently low. An NCC moves the equilibrium effort closer to the first-best; the incentive effect is positive. As long as the equilibrium effort with an NCC lies below the first-best, the incentive effect is certainly positive. For large minimum wages, however, the equilibrium effort gets wastefully large as it increases above the first-best level (Proposition 1.4). At an even larger minimum wage, thus, the incentive effect starts to decrease. Finally, from some minimum wage on, the equilibrium effort is large enough to make the incentive effect negative.

The idleness effect directly affects the utilitarian inefficiency by reducing the agent's payoff. In the case of a failure, the NCC gets activated and burns $\bar{v}$ of the social surplus. Thus, this effect is unambiguously negative. It formally is

$$(1 - e^{\text{NCC}}) \cdot \bar{v}, \tag{1.7}$$

where e^{NCC} again denotes the equilibrium effort with an NCC, that depends on the minimum wage. The NCC's total effect on the utilitarian efficiency is the sum of the two effects.

We split the comparison with the benchmark into two parts: for those minimum wages below κ_3, and those above κ_3 because the agent gets no rent for minimum wages below κ_3 in the benchmark without NCCs. The principal's profit is also equal to the utilitarian welfare, which makes the comparison simple: For minimum wages between κ_2 and κ_3, the possibility to use an NCC increases the utilitarian welfare. NCCs mitigate the inefficiency that accompanies minimum wages.

For minimum wages above κ_3, the agent gets a rent in the benchmark without NCCs. Moreover, the utilitarian welfare in the benchmark is constant as the equilibrium effort is constant. The comparison of the utilitarian welfare is ambiguous. For minimum wages slightly above κ_3, an NCC improves the utilitarian welfare: It does so for the minimum wage of κ_3 and the incentive and the idleness effect are continuous in the minimum wage. If the minimum wage increases, as argued above, both the incentive and the idleness effect eventually become negative. Therefore, there is a minimum wage above which the utilitarian welfare is smaller with an NCC. The position of this minimum wage depends on the functional form of the effort cost.

1.5.2 Pareto Dominance

As the utilitarian welfare does not consider the distribution of the social surplus, it is maximized without a minimum wage. Thus, the existence of a minimum wage hints at the policymaker's putting weight on the distribution. Therefore, we also compare equilibrium outcomes using Pareto domination. This welfare criterion is relatively uncontroversial as it remains agnostic about how the policymaker aggregates profits and rents in her welfare measure. An equilibrium outcome strictly Pareto dominates another if both the agent's rent and the principal's profit are strictly larger; it weakly Pareto dominates another if either rent or profit are strictly larger and the other one is equal. An equilibrium outcome that weakly Pareto dominates another also has a strictly larger utilitarian welfare.

For minimum wages between κ_2 and κ_3, the outcome with an NCC weakly Pareto dominates the benchmark. For minimum wages above κ_3, Pareto dominance has no bite as the principal is better but the agent is worse off with an NCC.

Extensive margin. There might be a weak Pareto improvement and, thus, efficiency gain on the extensive margin. In all of the above, we have assumed that the principal wants to offer a contract to the agent irrespective of the minimum wage. That is, the success payoff is large enough such that the profit exceeds the principal's outside option for all minimum wages, with or without an NCC. For this paragraph, we drop this simplifying assumption.

Whenever the optimal contract includes an NCC, the principal's profit is strictly larger than in the benchmark. Both without and with NCCs, the principal's profit is strictly decreasing in the minimum wage. Hence, if the principal's profit at a minimum wage of κ_2 is larger than her outside option, she participates for more minimum wages when NCCs are allowed. For all minimum wages for which the principal does not participate in the benchmark but does participate with NCCs, the NCC leads to a weak Pareto improvement: The agent gets his outside utility in both cases, whereas the principal makes a profit that exceeds her outside option. Furthermore, the extensive margin corresponds to the employment effect of minimum wages: If the minimum wage drives a principal out of the game, there is one fewer job in the economy. Since the principal might participate for more minimum wages when NCCs are allowed, NCCs reduce the employment effect of NCCs (more on this empirical prediction in Section 1.6).

Bounded non-compete clauses. In the Appendix 1.A, we also consider bounded NCCs. In this case, a sufficiently large minimum wage redistributes from the principal to the agent. Therefore, Pareto dominance gets back some of its bite. Unfortunately, it is prohibitively difficult to characterize the optimal contracts analytically with bounded NCCs. Nevertheless, we provide an example in the Appendix 1.A, in which a combination of suitably chosen minimum wages and bounds on NCCs strictly Pareto dominates any outcome that can be achieved by minimum wages alone.

1.6 Discussion

In this section, we derive empirical predictions from our model that future work could take to the data. Moreover, we defend the assumptions that we made. These entail the use of incentive pay with minimum wage jobs and the partial market setting including the outside option and the continuation payoff.

Empirical predictions. In our model, the minimum wage is defined as “minimum wage minus the outside option”, because we normalized the outside option. With heterogeneous agents, thus, the same minimum wage is “low” for those with good outside options, and “high” for those with bad outside options. Therefore, our model predicts that an agent with a worse outside option, everything else equal, should be more likely to sign an NCC or have a more severe NCC. Agents might have worse outside options if they are less educated, older, less mobile, or less healthy. Surprisingly, those who would have trouble finding a new job anyway are predicted to be bound by NCCs.[19]

The same mechanism can explain why NCCs have become so frequent. The outside option of an agent might be affected by how prevalent NCCs are in the contracts

19. This effect interacts with which effect the same literal NCC has on agents with different outside options. That is, with the function that maps words into $\bar{v}$. If the same literal NCC affects the job market outcome of an agent with a worse outside option less (e.g. because the outcome is most likely unemployment anyway), this is exacerbated: To achieve the same effect as for an agent with a better outside option (the same $\bar{v}$), the principal has to offer the agent an NCC with more severe clauses.

of others. President Biden has ordered the FTC to consider banning NCCs. Being one of the authorities governing competition, the FTC might do so arguing that NCCs negatively affect parties other than the signers of the NCC, for example other employees, or on the grounds that employers collude to weaken the competition for employees. In both cases, this would mean that NCCs reduce the outside options of other employees. There is some evidence that this might be true (Starr, Frake, and Agarwal, 2019). So, if NCCs reduce the outside options of all employees in the economy, they make it more likely that the other employees will be asked to sign NCCs, too.

Concerning the wages, our model makes ambiguous predictions about the effect of NCCs. Whenever the agent gets no rent, the participation constraint means that the expected wage is equal to the effort cost plus the expected damage from the NCC (the product of the NCC's severity and the probability of a failure, $(1-E)\cdot\bar{v}$). In the benchmark, NCCs are not allowed, so this reduces to the effort cost.

Up to κ_2, the optimal contract does not entail an NCC, so the expected wage is the same whether NCCs are allowed or not. If the minimum wage is between κ_2 and κ_3, the expected wage in the benchmark is decreasing in the minimum wage because the equilibrium effort is decreasing. In contrast, if NCCs are allowed, the expected wage is increasing because the equilibrium effort is increasing. Further, the bonus component of the wage is larger and is paid more often. If the minimum wage is above κ_3, the expected wage in the benchmark increases linearly in the minimum wage as the participation constraint gets slack. The expected wage is the minimum wage plus the equilibrium effort times the bonus wage. If NCCs are allowed, the expected wage is initially still larger than in the benchmark. Above some threshold minimum wage, however, the expected wage with NCCs is lower than in the benchmark: The bonus wage goes to zero, so the expected wage goes to the minimum wage. This implies that the realized wages stop varying for high minimum wages if NCCs are allowed. Empirical research could also test whether with NCCs there is more incentive pay for low minimum wages and less incentive pay for high minimum wages.

Furthermore, our model predicts that the wages are not that informative for the well-being of employees. Although they might receive higher wages, the agent loses his rent due to an NCC because he has to exert more effort. As measured by his rent, an agent is worse off when NCCs are allowed compared to the benchmark, whenever the minimum wage lies above κ_3. Our model predicts that for such minimum wages, minimum wage workers should be happier in states in which NCCs are unenforceable compared to states in which NCCs are enforceable.[20]

20. Measures other than happiness that are interesting might be (self-reported) effort at work or stress-related health issues. In the fast food industry, work effort of minimum-wage workers could be measured as cleanliness, customer satisfaction with the service (or amount of complaints), or customer waiting time (or number of sales during peak hours).

On the macro level, the extensive margin analysis of our model predicts that the effect of minimum wages on the employment is lower when NCCs can be used. When NCCs are allowed and used (that is, if the minimum wage is above κ_2), the principal makes strictly larger profits. Therefore, when NCCs can be used, there should be fewer market exits due to the minimum wage. Johnson and Lipsitz (2022) derive the same hypothesis and test it in their Section V. They interact the enforceability measure of Bishara (2011) with the minimum wage to check whether access to NCCs moderates the employment effects of a minimum wage. They find a significant and robust effect that supports the hypothesis. This might help explain the empirical puzzle on why minimum wage increases have so little of an impact on employment.

Incentive pay. The central problem in our model is that the principal has to incentivize the agent to exert effort, which she does by using an NCC.[21] Thus, for our model to be a valid explanation for why minimum wage workers sign NCCs, it has to be the case that effort is important in minimum wage jobs. We argue that this is the case by the revealed preferences of real-world employers: There are many examples of explicit and implicit incentives in minimum wage jobs. This can only be optimal if effort is not contractible but important.

In many jobs, employees get a bonus for making a quota. Examples include salesforce agents—telemarketers often get paid the minimum wage as a base wage—shelf stackers in supermarkets, or pickers in the storehouses of e-commerce firms. Some fast food firms use explicit bonus payments.[22] A large German bakery retailer uses team bonuses (Friebel, Heinz, Krueger, and Zubanov, 2017).

Commissions are common bonuses in sales jobs (Joseph and Kalwani, 1998, p. 149). An example of minimum wage workers that receive commissions are taxi drivers. Furthermore, tips (in restaurants, at the hair dresser's, for food deliveries, and again for taxi drivers) are a kind of (stochastic) commissions.

Another kind of incentive pay are promised promotions and pay increases. Skimming job-search websites for low-wage jobs shows that many firms advertise their jobs with advancement options.[23] While promotions are often not the direct consequence of meeting a verifiable success, the literature on relational contracts shows that employers can build a reputation for rewarding high effort, which allows them to use unverifiable measures to incentivize effort. Note that pay increases do not

21. The existence of bonus wages is not crucial.

22. Chipotle Mexican Grill implemented a "bonus program that gives hourly employees the opportunity to earn up to an extra month's pay each year. To qualify for the quarterly bonus program, restaurant teams must meet certain criteria such as predetermined sales as well as cashflow and throughput goals" (https://cutt.ly/yX8fYg0).

23. "Chipotle's career trajectory begins with a path from crew member to general manager to the elite level of Restaurateur. Chipotle's focus on development shows as 80% of general managers have been promoted from within, often starting as line level crew members" (https://cutt.ly/yX8fYg0).

have to be associated with an increased productivity. Instead, the prospect of a pay increase can be a form of efficiency wages.

A last set of examples concerns non-monetary "bonuses." One example is the personal interaction between the employer and the employee: Praise can be a bonus (Dur, Kvaløy, and Schöttner, 2022). Another example are work-related perks (Marino and Zábojník, 2008). A third example are tournament incentives: Some firms let their best employees choose their favorite shifts.[24]

Firing rule. We have assumed that the principal can commit herself to a specific firing rule. We now argue that while it is important that we assume the commitment power, the firing rule we use (retain after success, fire after failure) is optimal given that the principal can commit, assuming that the principal can replace the agent at no cost. Without commitment power, there might be renegotiations.[25] An agent who has signed an NCC has a different outside option than an agent who has not signed an NCC: It is $\bar{v}$ instead of 0 because the principal can activate the NCC by firing the agent. Thus, the principal can offer another contract to the agent that includes a more severe NCC such that the agent is (almost) indifferent between the new contract and $\bar{v}$. This spiral would continue until the NCC is infinitely severe. Anticipating this, a rational agent would never sign an NCC. A principal that commits herself to a firing rule breaks the spiral as she cannot activate the NCC at will. Thus, there is no reason for the agent to accept a contract with a more severe NCC.

Reputation might be an alternative for commitment power. The principal might be infinitely-lived and embedded into a larger, infinitely repeated game with multiple short-lived agents that play one after another. If there is a small, yet strictly positive probability for the principal's being a commitment type, Proposition 2 of Fudenberg, Kreps, and Maskin (1990, p. 560) applies: If the discount factor is sufficiently large, there is a subgame perfect equilibrium in which the principal without commitment power gets almost the same payoff as the commitment type.

Given that the principal can commit herself or build a reputation for following some firing rule, it is optimal to choose to retain the agent with certainty after a success and to fire the agent with certainty after a failure, if the principal can replace the agent at no cost. Lemma 1.1 proves this by showing that the extreme firing rule maximizes the agent's expected utility for a fixed bonus wage and fixed incentives from the NCC. As the principal can extract all surplus by increasing the incentives from the NCC until the participation constraint binds, she wants to choose the firing rule that maximizes the agent's expected utility for a given equilibrium effort.

24. Anecdotal evidence suggests that in a supermarket in New Jersey in the late 1970s the best shifts were on weekend afternoons (https://cutt.ly/2X8h0es). In the fast food industry, night shifts are popular because they are usually calm.

25. We thank an anonymous referee for pointing this out.

Lemma 1.1. *Let f_f be the probability that the agent gets fired after a failure and f_s the probability that the agent gets fired after a success. In equilibrium, the principal chooses $f_s = 0$ and $f_f = 1$ if she can replace the agent at no cost.*

There are two effects at play: The more succeeding increases the probability of retention, the more incentive has the agent to exert effort. Thus, the extreme firing rule provides the most incentives. On the other hand, the extreme firing rule leads to the NCC's being activated more often which reduces the agent's expected utility. However, to still provide the same incentives, less extreme firing rules have to be paired with more severe NCCs. Lemma 1.1 shows that the negative effect from more severe NCCs outweighs the positive effect from a reduced probability of activating the NCC.

Partial market. In this subsection, we offer more detail on how to think about the agent's outside option and argue under which conditions an NCC reduces the agent's continuation payoff in general equilibrium. NCCs reduce the agent's continuation payoff if they prevent him from collecting a rent from working for another firm. In Section 1.2, we have argued that an NCC prevents employees from working for some firms. To complete the argument, we present a setting in which there is a rent associated with working for these firms.

Imagine a labor market in which firms are grouped into several sectors. A sector consists of those firms to which the NCC applies. So if an employee of a fast-food firm has an NCC that forbids him to work for any other fast-food firm, these fast-food firms are one sector. As the NCC does not rule out performing janitorial services, this is another sector. If a sector were to consist of only one firm, this solitary firm could not use an NCC to incentivize its employees as the NCC applies to no other firm on the labor market and, thus, does not change the agent's behavior.

There is friction in the labor market that causes involuntary unemployment. Being unemployed yields an exogenously fixed payoff. If an agent is unemployed, he searches for matches with any firm. The expected outcome of this search is the agent's outside option, when the agent has not signed an NCC, yet. The more firms the agent is allowed to work for, the more probable he is to find a match. If the agent has signed an NCC, he is not allowed to match with firms in the barred sector.[26]

Working for some firms in a sector yields the agent a rent. Not all minimum wage workers are asked to sign NCCs. As mentioned in the introduction, Colvin and Shierholz (2019) find that around a quarter of firms make all their low-wage workers sign an NCC. If there are no substantial differences across sectors, this means that within each sector, there are firms that use NCCs and firms that use no NCCs. Due to the minimum wage, finding a job at a firm that uses no NCCs leaves the

26. The severity of an NCC can be interpreted as the duration for which the agent is barred from matching with the firms in that sector or as how widely a sector is defined.

agent a rent.[27] Finding a job at a firm that uses NCCs makes the agent indifferent between continuing to search and taking the job, which means losing access to the firms that use no NCCs in the same sector after being fired. Thus, because not all firms use NCCs, the jobs at some firms in a sector yield a rent and an NCC reduces the probability of matching with these firms: Signing an NCC reduces the agent's continuation payoff.

Another simplification in the main part is that we set the continuation payoff of an agent without an NCC to zero, which is not a normalization as we already normalized the outside option to zero. On the one hand, this simplifies the model substantially. On the other hand, this distorts which contracts are optimal. We decided for the simplification because the qualitative results are the same, as we will now argue.

Without the simplification, in the benchmark without an NCC, the agent gets a rent in each period if he is retained. Existing work has shown that future rents can be used to provide incentives in the same fashion as NCCs are used in our model: When retention is conditioned on good performance, the agent exerts more effort Kräkel and Schöttner (2010).

If the principal can additionally use NCCs, however, she can still do better. When using future rents to incentivize the agent, the agent gets a rent in each period with a success. With NCCs, the principal can turn the rents into even more incentives by reducing the payoff after a bad performance. So, the principal can extract the future rents. Setting the continuation payoff to zero, thus, merely shifts the level of efforts.

1.7 Conclusion

We add effort incentives as a new effect of non-compete clauses to the research and the public discussion. Our simplified model shows that a single premise is sufficient to make effort incentives possible: A non-compete clause has to worsen the

27. As our simplified model implies that the principal profits from extracting the agent's rent using an NCC, we cannot answer why not all firms make their employees sign NCCs. We have, however, three suspicions. Firstly, real NCCs are bounded, so it might be impossible for some principals to extract the whole rent. Secondly, it might be that some firms have other motives than maximizing their profits: In the FTC workshop on NCCs, many comments criticized NCCs for restricting the liberty of workers, curtailing the "American dream" and being "unamerican" (see for example `Comment 15`, `Comment 96`, `Comment 196`, `Comment 271`, and `Comment 297`). Thirdly, it might be that there are losses associated with using NCCs: *Jimmy John's* experienced a public outrage after the media reported about its use of NCCs. A firm that wants to protect its image from such a disaster might prefer to leave its employees a rent.

Note that our simplified model would cause a paradox if all firms used unbounded NCCs to extract all rents: Then, no job would yield the agent a rent above the exogenous payoff from being unemployed. But then, the NCCs cannot reduce the agent's continuation payoff, the agent does not exert more effort, and his rent is not extracted. While the above three reasons solve this problem, we leave the exploration of the paradox in a general equilibrium model for future research.

employee's prospects after a dismissal. Our model shows that the effort incentives of non-compete clauses transfer utility from the agent to the principal. Without minimum wages, money does the trick and employers do not want to use non-compete clauses to provide incentives. With minimum wages—a purposefully created friction to utility transfer via money—employers use non-compete clauses to provide incentives because it is the employee's rent that pays for the incentives. Thus, non-compete clauses counteract the policymakers' attempts to transfer utility from the employer to the employee with minimum wages.

This new effect can enrich the research and public discussion. It can explain why rational employees sign non-compete clauses in the absence of other reasons, such as human capital, protection of proprietary information, bargaining, or reduction of turnovers. We argue that the other reasons are not particularly appealing for the case of minimum wage workers, among whom the prevalence of non-compete clauses is high. Effort provision can explain some observed patterns: If the minimum wage is increased, the prevalence of non-compete clauses increases (Johnson and Lipsitz, 2022). A change in the enforceability of non-compete clauses does not imply that wages change in a certain direction (as the incentive pay component changes, too). This might explain contradictory findings concerning what happens to wages when NCCs are banned: While they increased in Oregon (Lipsitz and Starr, 2022), they did not increase in Austria (Young, 2021). Furthermore, there is first evidence that our proposed channel exists: Non-compete clauses increase the effort of employees as measured by their performance (Cici, Hendriock, and Kempf (2021); albeit mutual fonds managers hardly are minimum wage workers).

Our work cannot make a straightforward policy-contribution: We have ignored all effects of non-compete clauses but one to make our argument clearer. And even within this single effect whether banning non-compete clauses for minimum wage workers is beneficial, depends on the sizes of several effects and parameters. The only thing we can say for sure is that introducing a minimum wage without taking into account the legislation on non-compete clauses is a mistake.

While our model makes empirically testable predictions, we lack suitable data to test them. Counter-intuitively, our model predicts that, ceteris paribus, those agents with the worst outside options are the most likely to being offered a non-compete clauses (because the minimum wage is effectively larger for these agents). Also, if non-compete clauses are, ceteris paribus, more enforceable in a state, this should lead to lower rents for minimum wage workers, which might be reflected in a lower job satisfaction. We leave testing these predictions for future research.

Appendix 1.A Bounded Non-Compete Clauses

As we have seen in Section 1.2, the legislation on NCC varies across the United States. No state, however, would enforce an NCC that, say, forbade the employee to ever work in the same field again: Real NCCs cannot be arbitrarily severe. In the main part, we have abstracted from that to keep the intuition simple. In this section, we assume that the severity of NCCs has an exogenous bound. Different bounds capture the differences in the legislation across states.

In the following, we will formaly define a bound on NCCs and solve for the optimal contracts with this additional constraint. We find that whenever the optimal NCC without a bound would be more severe than the bound, then the optimal NCC is equal to the bound. Moreover, there is a (large) minimum wage, for which the optimal NCC has reached the bound, and from which on the bonus wage is constant. In contrast to the case with unbounded NCCs, there is a range with constant bonus wages (i) irrespective of the cost function and (ii) the constant bonus wage might be positive. Having characterized the optimal contracts with bonus wages, we revisit the welfare analysis using Pareto dominance. The bound limits the principal's power to extract the agent's rent: From some minimum wage on, the agent is left a rent. While we cannot derive general results, we show with an example that a combination of minimum wages and bounded NCC might in some cases Pareto dominate minimum wages alone. The intuition is that redistribution with minimum wages alone causes a welfare loss due to inefficiently low effort. Redistribution with minimum wages and bounded NCCs causes a welfare loss due to the idleness effect and possibly inefficient effort (either too low or too high). Depending on the cost function and the parameters, either of those two scenarios might cause less of a loss.

1.A.1 Optimal Contracts with Bounded Non-Compete Clauses

We define $\underline{\bar{v}} < 0$ as the most severe NCC that the principal may use. The additional constraint takes the form $\bar{v} \geq \underline{\bar{v}}$.

We formalize our findings as Proposition 1.5.

Proposition 1.5 (Bounded Non-Compete Clauses). *Let $\underline{\bar{v}} < 0$ be a lower bound on the NCC.*

(i) Let, without a bound on NCCs, the optimal NCC be $\bar{v} \geq \underline{\bar{v}}$. Then, the optimal contract remains the same with a bound on NCCs.

(ii) Let, without a bound on NCCs, the optimal NCC be $\bar{v} < \underline{\bar{v}}$. Then, the optimal contract with a bound on NCCs has $\bar{v} = \underline{\bar{v}}$. If the optimal bonus wage is positive, when the bound on the NCC starts binding, the bonus wage decreases more steeply than without a bound. At some larger minimum wage, the optimal bonus wage becomes constant either at a positive level or at zero. If the optimal bonus wage is zero, when

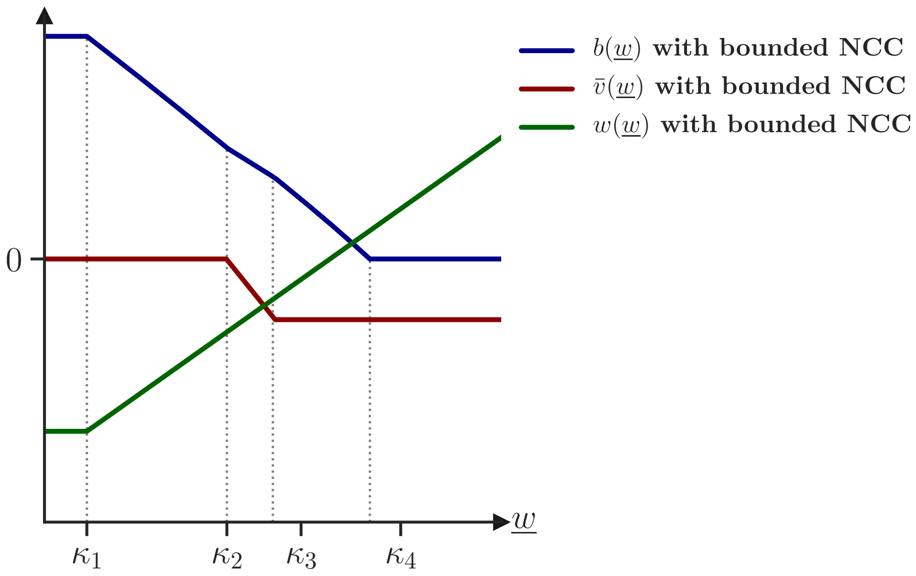

Figure 1.A.1. Illustration of the optimal contract for different minimum wages for $c(e) = -\ln(1-e) - e$, $V = 11$ and a bound on the NCC of $\underline{\bar{v}} = -3$.

the bound on the NCC starts binding, the bonus wage remains at zero for all larger minimum wages.

Some supplementary remarks: As the profit-maximizing NCC gets infinitely severe if the minimum wage goes to infinity, it will eventually reach the bound.

After the bound is reached, positive bonus wages decrease faster than without a bound because there are no more double incentives. Increasing the bonus wage means that the agent gets a rent that cannot be converted into more incentives as the NCC cannot be made more severe. This reduces the benefit of bonus wages.

As soon as the NCC has reached the bound and the bonus wage remains constant, there is redistribution as in the benchmark. The minimum wage at which the bonus wage becomes constant is larger than that in the benchmark, κ_3.

For illustration, Figure 1.A.1 shows the optimal contract with bounded NCCs for a specific effort cost function and a specific bound. In the depicted case, the optimal constant bonus wage is zero. The optimal contract is the same as without a bound up to a minimum wage slightly above κ_2. Then, the bound on the NCC starts to bind and the optimal bonus wage has a kink. Somewhere to the right of κ_3, the participation constraint gets slack and the optimal bonus wage gets constant. If the bound on the NCC were stricter, the optimal constant bonus wage might be positive.

1.A.2 Welfare Effects of Bounded Non-Compete Clauses

When NCCs are bounded, minimum wages can again redistribute from the principal to the agent. If the minimum wage increases, the profit maximizing contract eventually has a constant bonus wage and NCC that lies at the bound (Proposition 1.5). If the minimum wage increases further, the utilitarian welfare remains constant as in the benchmark for minimum wages above κ_3. In this area, a one unit increase of the minimum wage reduces the principal's profit by one unit and increases the agent's rent by one unit. Because of the NCC, this particular minimum wage is larger than in the benchmark.

It is prohibitively complicated to analytically examine the welfare effects of bounds on NCCs in general. Therefore, we use the fact that both in the benchmark and with bounded NCCs the utilitarian welfare is constant above a specific minimum wage and that minimum wages then redistribute. For an exemplary effort cost function, we show that the constant utilitarian welfare can be larger with bounded NCCs than in the benchmark, if the bound is suitably chosen. This implies that, setting the minimum wage correspondingly, bounded NCCs can lead to outcomes that strictly Pareto dominate any benchmark outcome.

We reconsider the functional form of the cost function and the parameters that we have plotted above: $c(e) = -\ln(1-e) - e$ and $V = 11$. The simplest example relies on the peculiar fact that the principal coincidentally induces first-best effort at κ_4; that is without a bonus wage, using only an NCC of $-V$. We choose this NCC as the bound, $\underline{\bar{v}} = -V$. This implies that the agent starts getting a rent at a minimum wage of κ_4 and that the equilibrium effort remains constant at the first-best level for all larger minimum wages. As the effort is at the first-best level, the incentive effect is maximized and constant in the minimum wage. The inefficiency of the minimum wage without NCCs due to reduced effort is canceled out. This leaves only the inefficiency from the idleness effect, which is also constant in the minimum wage because the equilibrium effort and the NCC are constant in the minimum wage. Thus, with the logarithmic cost function and the bound $\underline{\bar{v}} = -V$ for $\underline{w} \geq \kappa_4$, the utilitarian efficiency is $(1 - e^{FB}) \cdot V$ below the first-best utilitarian efficiency (that is achieved without a minimum wage).

Consider now the utilitarian efficiency in the benchmark for minimum wages above κ_3; minimum wages for which there is redistribution from the principal to the agent. There is one source of inefficiency: too low effort. The equilibrium effort is the same for all minimum wages above κ_3, thus, the utilitarian efficiency is also the same. It is $\int_{e_3^{BM}}^{e^{FB}} V - c'(x)\, dx$ below the first-best.

We can now compare the constant levels of utilitarian welfare with bounded NCCs and in the benchmark. With a bounded NCC, the idleness effect reduces the utilitarian welfare by $(1 - e^{FB}) \cdot V$ compared to the first-best. In the benchmark, the inefficiently low equilibrium effort reduces the utilitarian welfare by $\int_{e_3^{BM}}^{e^{FB}} V - c'(x)\, dx$ compared to the first-best. When V is large enough, the outcome with a bounded

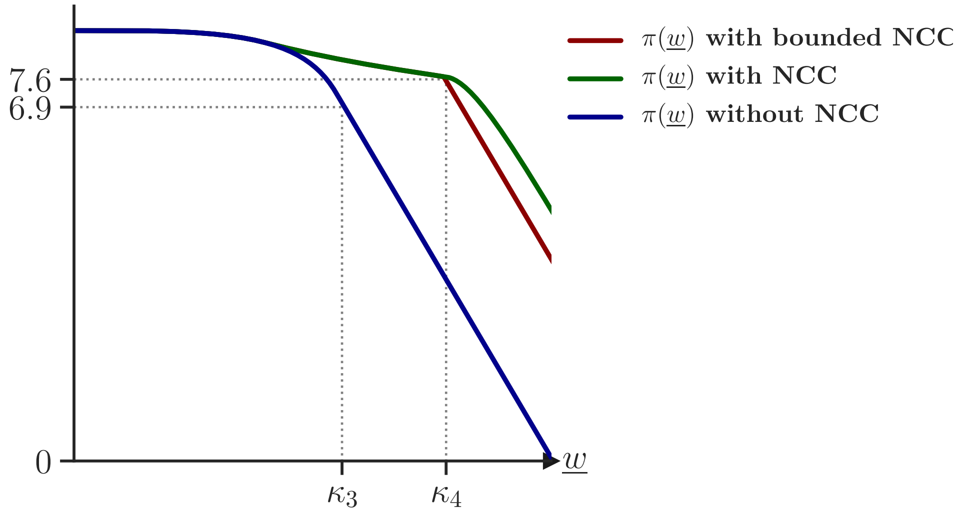

Figure 1.A.2. Bounded non-compete clauses potentially allow for strict Pareto improvements. We choose $c(e) = -\ln(1-e) - e$, $V = 11$ and $\underline{v} = -10$.

NCC is more efficient![28] This is illustrated by Figure 1.A.2. On the x-axis, the minimum wage is plotted. On the y-axis, the principal's expected profit is plotted. The utilitarian welfare is equal to the profit and decreasing up to κ_3 for the benchmark and up to κ_4 for the examplary bounded NCC. Beyond the respective thresholds, the utilitarian efficiency is constant; these levels are marked by dotted horizontal lines. The slope of the principal's profit is −1 in both cases; the agent's rent is the remainder between the utilitarian efficiency and the principal's profit. For comparison, we added the case of unbounded NCCs. In this case, the agent never gets a rent and the principal's profit always is the utilitarian efficiency. This illustrates that the utilitarian welfare decreases fast when the equilibrium effort gets close to one because of the Inada conditions.

We now argue that whenever the constant utilitarian welfare is larger with a bounded NCC than in the benchmark, as in the example, a Pareto dominating equilibrium outcome can be constructed. Intuitively, the pie that can be redistributed is the constant utilitarian welfare and the minimum wage determines the agent's

28. When $V \to 0$, both social losses go to zero. The loss with a bounded NCC is coincidentally equal to e^{FB}; it is concave in V. The loss in the benchmark is a more complicated expression, $\sqrt{1+V} - 1 - \frac{1}{2} \cdot \ln(1+V)$. It is the area between V and the marginal cost in the range from e_3^{BM} to e^{FB}; it is convex in V. When increasing V, the loss with a bounded NCC increases initially faster than the loss in the benchmark. For larger V, the loss in the benchmark increases faster. Numerically, they intersect at $V \approx 7.873$.

rent. In the benchmark, for minimum wages above κ_3, the agent's rent is $\underline{w} - \kappa_3$. With the exemplary bounded NCC, for minimum wages above κ_4, the agent's rent is $\underline{w} - \kappa_4$. Whenever the pie is larger, both principal and agent can be made strictly better off by choosing the minimum wage accordingly. To give the agent the same rent as in the benchmark, minimum wages have to be increased; in this example by $\kappa_4 - \kappa_3$. This procedure can be exported to all other effort cost functions, success payoffs, and bounds by replacing κ_4 by the minimum wage at which the utilitarian welfare becomes constant.

The welfare analysis shows that an equilibrium with bounded NCCs might strictly Pareto dominate equilibria with only minimum wages and no NCCs. It is, however, difficult to make general statements on the welfare effects of bounded NCCs. The Pareto improvement through bounded NCCs in our example depends on two aspects.

Firstly, a technological constraint: The effort cost function has to allow for Pareto improvements. Pareto improvements are only possible if there are bounds and minimum wages such that the loss in the utilitarian welfare is smaller in the redistributive range of the minimum wage. Without an NCC, a minimum wage causes a loss in the utilitarian welfare due to too little effort. With a bounded NCC, there is a welfare loss from the idleness effect. The effort, however, might be closer to the first best because of the incentive effect. In our example, this condition is satisfied. Thus, the NCC mitigates the inefficiency that is associated with redistribution.

Secondly, an informational constraint: The policymaker must have sufficient information on how to choose the bound on NCCs and the minimum wage to reap the efficiency gains. If the bound on NCCs is either too small or too large, the utilitarian welfare (the pie that can be distributed, that is) might be smaller than with minimum wages alone. The bound on NCCs has to be chosen optimally to realize the improved utilitarian welfare. If the minimum wage is too low, there is no redistribution. Importantly, the looser the bound on NCCs, the larger minimum wages have to be to redistribute at all. The policy maker has to know which minimum wage with a certain bound on NCCs redistributes a certain rent to the agent. All of this depends on the effort cost function that is unknown in the real world. Heterogeneity in agents could make it impossible to find a minimum wage that suits all. Due to all these uncertainties, an ambiguity averse (in a maxmin preferences sense) policymaker might prefer to ban NCCs.

Appendix 1.B Proofs

Proof of Proposition 1.1. First, we show that the objective function is strictly concave in the bonus wage. Let $E(b)$ be the maximizer of the agent's utility, that is, the equilibrium effort.

$$E(b) = \begin{cases} (c')^{-1}(b) & \text{if } b \geq 0, \\ 0 & \text{if } b < 0. \end{cases}$$

If the bonus wage is non-negative, the equilibrium effort is determined by the solution of the agent's first-order condition. Furthermore, $E(b)$ is strictly increasing in this range. If the bonus wage is negative, a corner solution, $E(b) = 0$, is optimal. We will use this function with a different argument again, when NCCs are allowed. The first and second derivative of $E(b)$ with respect to its positive argument are $E'(b) = \frac{1}{c''(E(b))}$ and $E''(b) = -\frac{c'''(E(b))}{(c''(E(b)))^3}$.

Remember that the expected profit is $\pi = -w + E(b) \cdot (V - b)$. The first and second derivatives with respect to the bonus wage are then given by:

$$\frac{\partial \pi}{\partial b} = E'(b) \cdot (V - b) - E(b),$$
$$\frac{\partial^2 \pi}{\partial b^2} = E''(b) \cdot (V - b) - 2E'(b).$$

Since $E''(\cdot) < 0$ and $E'(\cdot) > 0$, the second derivative is negative. This implies that P's objective function is strictly concave in the bonus wage.

Next, we look at the constraints of P's problem. We now show that $MWC2$ is always slack. Assume to the contrary that $MWC2$ binds. Rearranging $MWC2$ yields $b = \underline{w} - w$. By $MWC1$ we know that $w \geq \underline{w}$, which then implies that $b \leq 0$. A non-positive bonus wage, however, implies that the equilibrium effort is zero, which cannot be optimal.[29] Hence, $MWC2$ is always slack.

This leaves two constraints that can either bind or be slack, the PC and $MWC1$. We now show that it cannot be the case that both PC and $MWC1$ are slack. Assume to the contrary that both PC and $MWC1$ are slack. This means that there is a profitable deviation: Decreasing w by ϵ still leaves PC and $MWC1$ slack but increases P's expected profit. Therefore, we can decrease w until either PC or $MWC1$ binds.

This leaves us with the following three possible cases:
Case 1: PC binds and $MWC1$ is slack.
Case 2: PC binds and $MWC1$ binds.
Case 3: PC is slack and $MWC1$ binds.

Next, we focus on each case in more detail.

29. The maximum profit is zero for negative minimum wages and $-\underline{w}$ for positive minimum wages. As we assume that the success payoff is sufficiently large for the principal to be able to achieve a positive profit, a non-positive bonus wage cannot be optimal.

Case 1. *P*'s problem is given by:

$$\max_{w,b} \quad -w + E(b) \cdot (V - b)$$
$$\text{s.t.} \quad w + E(b) \cdot b - c(E(b)) \ = \ 0, \qquad \text{(PC)}$$
$$w \ > \ \underline{w}, \quad \text{and} \quad w + b \ > \ \underline{w}. \qquad \text{(MWC1) and (MWC2)}$$

We will ignore the slack constraints for the moment and later check for which minimum wages they are not violated. The *PC* can be rewritten such that $E(b) \cdot b = c(E(b)) - w$. We plug this into *P*'s objective function and maximize over the equilibrium effort instead of the bonus wage. The first-order condition is $V = c'(E(b)) = b$. Since the objective function is concave, we know that the first-order condition yields the global maximum. Therefore, $b = V$, $E(V) = e^{FB}$, and $w = c(e^{FB}) - e^{FB} \cdot c(e^{FB})$. Now, we check the constraints. Because $V > 0$, $MWC2$ is slack. $MWC1$ is slack if $\underline{w} < c(e^{FB}) - e^{FB} c'(e^{FB}) \equiv \kappa_1$.

Case 2. *P*'s problem is given by:

$$\max_{w,b} \quad -w + E(b) \cdot (V - b)$$
$$\text{s.t.} \quad w + E(b) \cdot b - c(E(b)) \ = \ 0, \qquad \text{(PC)}$$
$$w \ = \ \underline{w}, \quad \text{and} \quad w + b \ > \ \underline{w}. \qquad \text{(MWC1) and (MWC2)}$$

There are two unknowns and two binding constraints. Plugging $MWC1$ into *PC* implicitly characterizes $E(b)$ and the bonus wage. There are three subcases: negative minimum wages, $\underline{w} = 0$, and positive minimum wages.

For each negative $\underline{w}$, there are exactly one b and one $E(b)$ such that the participation constraint binds. The reason is the following: Rearrange the binding participation constraint to get

$$E(b) \cdot b - c(E(b)) \ = \ -\underline{w}. \tag{1.B.1}$$

The left-hand side is the part of the agent's utility that is generated by exerting effort. Graphically, it is the area above an increasing function ($c'(e)$), between 0 and $E(b)$, another increasing function. It is zero for a bonus wage of zero, and is strictly increasing in the bonus wage because $c''(e) > 0$. Therefore, there can be at most one bonus wage for each negative minimum wage such that this holds. Furthermore, for negative minimum wages, there is a bijection between b and $E(b)$. Since the right-hand side is strictly positive, so is the bonus wage, which implies $MWC2$.

Consider the minimum wage $\underline{w} = 0$. Since the right-hand side of Equation $1.B.1$ is zero, so is the equilibrium effort, which means that the bonus wage has to be non-positive. $MWC2$ is only slack if the bonus wage is positive. Thus, there is no bonus wage such that *PC* binds and $MWC2$ is slack.

Consider positive minimum wages. The participation constraint is always slack. That is, there are no bonus wage and no equilibrium effort that satisfy equation (1.*B*.1).

Summing up the optimal contract in Case 2: For negative minimum wages, let $e_2^{BM}(\underline{w})$ denote the effort that makes the participation constraint (1.*B*.1) binding. Then, $e_2^{BM}(\underline{w})$ is implicitly defined by $e_2^{BM}(\underline{w}) \cdot c'(e_2^{BM}(\underline{w})) - c(e_2^{BM}(\underline{w})) = -\underline{w}$. We also get that $b = c'(e_2^{BM}(\underline{w}))$ and from $MWC1$ we get $w = \underline{w}$.

Case 3. *P*'s problem is given by:

$$\max_{w,b} \quad -w + E(b) \cdot (V - b)$$

$$\text{s.t.} \quad w + E(b) \cdot b - c(E(b)) \; > \; 0, \tag{PC}$$

$$w \; = \; \underline{w}, \quad \text{and} \quad w + b \; > \; \underline{w}. \qquad \text{(MWC1) and (MWC2)}$$

We will ignore the slack constraints for the moment and later check for which minimum wages they are not violated. We plug $MWC1$ into the objective function and take the derivative. The optimal bonus wage is characterized by the marginal profit's being 0. The solution to the first-order condition implicitly defines the optimal effort in Case iii, e_3^{BM}: $c'(e_3^{BM}) + e_3^{BM} \cdot c''(e_3^{BM}) = V$. Hence, $e_3^{BM} < e^{FB}$. We also get that $w = \underline{w}$ and $b = c'(e_3^{BM})$. Next, we check the constraints. As $e_3^{BM} > 0$, $MWC2$ is slack. PC is slack if $\underline{w} > c(e_3^{BM}) - e_3^{BM} c'(e_3^{BM}) \equiv \kappa_3$.

The optimal contract. We have verified that the optimal contract from Case 1 is feasible if $\underline{w} < \kappa_1$, the optimal contract from Case 2 is feasible if $\underline{w} < 0$, and the optimal contract from Case 3 is feasible if $\underline{w} > \kappa_3$. These thresholds are $\kappa_1 = c(e^{FB}) - e^{FB} c'(e^{FB}) < 0$ and $\kappa_3 = c(e_3^{BM}) - e_3^{BM} c'(e_3^{BM}) < 0$. Because $e_3^{BM} < e^{FB}$, it follows that $\kappa_1 < \kappa_3$.

Thus, for $\underline{w} < \kappa_1$, we have two candidates: Case 1 and Case 2. The maximization problem in Case 2 has two binding constraints, while the maximization problem in Case 1 has none. As a result, the profit from the optimal contract in Case 1 is weakly larger. The concavity of the objective function and the fact that the bonus wages from Case 1 and Case 2 are different for all $\underline{w} < \kappa_1$ imply that the profit is strictly larger. For $\kappa_1 \leq \underline{w} \leq \kappa_3$, the only candidate is Case 2; thus, this contract is optimal. For $\kappa_3 < \underline{w}$, we have again two candidates: Case 2 and Case 3. Since the maximization problem in Case 3 has only one binding constraint, the profit from the optimal contract in Case 3 is weakly larger. Again, concavity and different solutions imply strictly larger profits.

□

Proof of Proposition 1.2. The proof proceeds in two main parts. The first part is about simplifying the problem. Since there are four inequality constraints, there are 16 possible combinations. First, we identify those four combinations that can be optimal. In all of those combinations, the participation constraint is binding; the agent does not get a rent. We use this fact to reduce the problem's dimensionality by using the participation constraint to express the optimal NCC in terms of the minimum wage and the bonus wage. The first combination is the same as Case 1 in the benchmark, which means that this contract is profit maximizing for $\underline{w} < \kappa_1$. For all $\underline{w} \geq \kappa_1$, the base wage has to be the minimum wage. This fact and an additional piece of notation simplify the problem further. This yields a strictly quasi-concave objective function in only the bonus wage with one inequality constraint. The optimal bonus wage and whether the inequality constraint binds shows into which combination the contract falls. In the second part, we solve this rewritten problem.

The possibly optimal combinations. The agent's first-order condition for the optimal effort with NCCs is

$$b - \bar{v} \;=\; c'(e).$$

Whenever the left-hand side is non-negative, the first-order condition yields the optimal equilibrium effort, which we, express as $E(b-\bar{v}) \equiv (c')^{-1}(b-\bar{v})$. As above, a negative left-hand side implies that the corner solution $E(b-\bar{v}) = 0$ is optimal.

The principal's problem is

$$\begin{aligned}
\max_{w,b,\bar{v}} \quad & -w + E(b-\bar{v}) \cdot (V-b) \\
\text{s.t.} \quad & w + E(b-\bar{v}) \cdot b + (1-E(b-\bar{v})) \cdot \bar{v} - c(E(b-\bar{v})) \;\geq\; 0, && \text{(PC)} \\
& \bar{v} \;\leq\; 0, && \text{(NCC)} \\
& w \;\geq\; \underline{w}, \quad \text{and} \quad w+b \;\geq\; \underline{w}. && \text{(MWC1) and (MWC2)}
\end{aligned}$$

To solve the principal's problem, one has to know which constraints bind and which are slack for different minimum wages. In total, there are 16 combinations. They are summarized in Table 1.B.1. The combinations' order in Figure 1.2 reflects their occurrence when the minimum wage increases. We will now prove that the optimal contract always falls into the Combinations 1 to 4 and never into the Combinations 5 to 16 (column six of Table 1.B.1) for three distinct reasons.

Firstly, the participation constraint has to bind. Otherwise, there is a profitable deviation: Make the NCC more severe, keeping everything else fixed. Note that the bonus wage is optimally never larger than the success. Then, the agent exerts more effort which leads to more successes and more profit.

Secondly, it cannot be that *MWC*2 and *NCC* bind simultaneously. If they did, the agent would exert no effort. Then, the principal has no revenue. This cannot be optimal by our assumption that the success payoff is sufficiently large to allow for positive profits.

Thirdly, *MWC*1 can only be slack when the NCC feasibility constraint binds. Otherwise, there is a profitable deviation. In these combinations, the principal uses an NCC and pays a larger than necessary base wage. This cannot be optimal because

Table 1.B.1. The 16 combinations of binding constraints.

No.	PC	NCC	MWC1	MWC2	Relevant?
1	binds	binds	slack	slack	$\underline{w} \leq \kappa_1$
2	binds	binds	binds	slack	$\kappa_1 < \underline{w} \leq \kappa_2$
3	binds	slack	binds	slack	$\kappa_2 < \underline{w} \leq \kappa_4$
4	binds	slack	binds	binds	$\kappa_4 < \underline{w}$
5	slack	binds	binds	binds	no, *PC*
6	slack	binds	binds	slack	no, *PC*
7	slack	binds	slack	binds	no, *PC*
8	slack	binds	slack	slack	no, *PC*
9	slack	slack	binds	binds	no, *PC*
10	slack	slack	binds	slack	no, *PC*
11	slack	slack	slack	binds	no, *PC*
12	slack	slack	slack	slack	no, *PC*
13	binds	binds	binds	binds	no, no effort
14	binds	binds	slack	binds	no, no effort
15	binds	slack	slack	binds	no, deviation
16	binds	slack	slack	slack	no, deviation

there is a profitable deviation: Decrease the base wage by one unit and increase the bonus wage and make the NCC less severe by one unit. Because bonus wage and the NCC's severity are perfect substitutes, the equilibrium effort stays the same. Furthermore, the participation constraint remains satisfied: The agent loses one unit on the base wage but gains one unit both if there is a success and if there is a failure. The principal's profit increases because he saves on the base wage one unit with certainty and loses on the bonus wage one unit with the success probability (less than one by the Inada conditions). The principal can repeat this deviation until either MWC1 or NCC binds.

When is the first combination optimal? In the benchmark, we have seen that in the first combination the optimal contract implements the first-best effort. Additionally, the principal extracts the whole surplus. Therefore, this contract is profit-maximizing whenever it is feasible.

As we have seen in the benchmark, the contract in the first combination is only feasible if $\underline{w} < \kappa_1 = c(e^{FB}) - e^{FB} c'(e^{FB}) < 0$. This implies that for all $\underline{w} \geq \kappa_1$, the optimal contract is from either the second, the third, or the fourth combination. In all of these combinations, the base wage optimally is the minimum wage; *MWC*1 binds.

From now on, $\underline{w} \geq \kappa_1$, which eliminates w from the problem. Thus, b and $\bar{v}$ remain. Furthermore, the participation constraint *PC* has to bind. This lets us express $\bar{v}$ as an implicit function of $\underline{w}$ and b:

$$\bar{v}(\underline{w}, b) \;=\; -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))}.$$

Note that $(b-\bar{v})$ is non-negative because of $MWC1$ binds, which simplifies $MWC2$ to $b \geq 0$ and NCC, $\bar{v} \leq 0$. Thus, the agent's first-order condition yields the equilibrium effort.

$\bar{v}(\underline{w}, b)$ is the most severe NCC that the agent is willing to accept given a base wage of $\underline{w}$ and a bonus wage b. Lemma 1.2 shows that the higher the minimum wage is, the more severe is this NCC for a given bonus wage. The higher the bonus wage is, the more severe is this NCC for a given minimum wage. Furthermore, due to monotonicity, the values of $\bar{v}(\underline{w}, b)$ are unique in b for a fixed $\underline{w}$ and the other way around.

Therefore, the principal's problem can also be expressed as

$$\max_{b} \quad -\underline{w} + E(b-\bar{v}(\underline{w}, b))\,(V-b)$$

$$\text{s.t.} \quad \bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b-\bar{v}(\underline{w}, b)) \cdot b - c(E(b-\bar{v}(\underline{w}, b)))}{1-E(b-\bar{v}(\underline{w}, b))}, \tag{PC'}$$

$$\bar{v} \leq 0, \tag{NCC}$$

$$b \geq 0. \tag{MWC2}$$

Whenever $\underline{w} \geq \kappa_1$, a contract is optimal if and only if it solves the simplified problem. In the second combination, $MWC2$ is slack and NCC binds. In the third combination, $MWC2$ and NCC are both slack. In the fourth combination, $MWC2$ binds and NCC is slack.

Lemma 1.2. *Consider the following:*

(i) *Fix a minimum wage. The NCC that makes the participation constraint bind* $\bar{v}(\underline{w}, b)$ *is strictly decreasing in the bonus wage:* $\frac{\partial \bar{v}(\underline{w},b)}{\partial b} < 0$.
(ii) *Fix a bonus wage. The NCC that makes the participation constraint bind* $\bar{v}(\underline{w}, b)$ *is strictly decreasing in the minimum wage:* $\frac{\partial \bar{v}(\underline{w},b)}{\partial \underline{w}} < 0$.

Proof. Rearrange the binding participation constraint to

$$Z \equiv \underline{w} + E(b-\bar{v}) \cdot (b-\bar{v}) + \bar{v} - c(E(b-\bar{v})) = 0.$$

Because this is continuously differentiable, the implicit function theorem can be used to get the derivatives of $\bar{v}$ with respect to $\underline{w}$ and b.

$$\begin{aligned}\frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} &= -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial \bar{v}}} \\ &= -\frac{1}{-E'(b-\bar{v}) \cdot (b-\bar{v}) - E(b-\bar{v}) + 1 + c'(E(b-\bar{v})) \cdot E'(b-\bar{v})} \\ &= -\frac{1}{1-E(b-\bar{v})}.\end{aligned}$$

The simplification is due to the agent's first-order constraint, $(b - \bar{v} - c'(E)) = 0$.

$$\begin{aligned}
\frac{\partial \bar{v}(\underline{w}, b)}{\partial b} &= -\frac{\frac{\partial Z}{\partial b}}{\frac{\partial Z}{\partial \bar{v}}} \\
&= -\frac{E'(b-\bar{v})\cdot(b-\bar{v}) + E(b-\bar{v}) - c'(E(b-\bar{v}))\cdot E'(b-\bar{v})}{-E'(b-\bar{v})\cdot(b-\bar{v}) + 1 - E(b-\bar{v}) + c'(E(b-\bar{v}))\cdot E'(b-\bar{v})} \\
&= -\frac{E(b-\bar{v})}{1-E(b-\bar{v})}.
\end{aligned}$$

Again, the agent's first-order constraint simplifies the expression. □

We will now define a useful term to simplify the maximization problem further. Let $b_2^{**}(\underline{w})$ denote the optimal bonus wage in Case 2 of the benchmark (binding *PC*, binding *MWC*1, slack *MWC*2). The case conditions imply a property of $b_2^{**}(\underline{w})$: It makes the participation constraint binding in the absence of an NCC.

To use this particular bonus wage to simplify the problem, we have to extend the definition of $b_2^{**}(\underline{w})$ to minimum wages above κ_3 for which it is not the optimal bonus wage. Let $b_2^{**}(\underline{w})$ denote the *minimum non-negative* bonus wage that keeps the participation constraint *satisfied* in the absence of an NCC.

$$\forall \underline{w} \geq \kappa_1 \qquad b_2^{**}(\underline{w}) \equiv \min\left\{b \in \mathbb{R}_0^+ \mid \underline{w} + E(b)\cdot b - c(E(b)) \geq 0\right\}.$$

For non-positive minimum wages, $b_2^{**}(\underline{w})$ is determined by the minimum wage that makes the participation constraint binding. For positive minimum wages the participation constraint is always slack without an NCC; there is no bonus wage that makes the participation constraint binding. Thus, if $\underline{w} \geq 0$, then $b_2^{**}(\underline{w}) = 0$. Furthermore, $b_2^{**}(\underline{w})$ has the nice property that it exists and it is strictly decreasing in the minimum wage between κ_1 and 0.

To simplify the problem, we now replace the inequality constraints using $b_2^{**}(\underline{w})$: As long as *PC*′ holds, the conditions *NCC* and *MWC*2 are equivalent to another condition, $b \geq b_2^{**}(\underline{w})$.

Consider $\underline{w} < 0$. In this case, *PC*′ and *NCC* imply *MWC*2. The bonus wage has to be at least $b_2^{**}(\underline{w})$, even without an NCC, to satisfy the participation constraint. If $\underline{w} < 0$, then $b_2^{**}(\underline{w}) > 0$, implying *MWC*2. In this case, the new constraint $b \geq b_2^{**}(\underline{w})$ is binding if and only if *NCC* is binding.

Consider $\underline{w} \geq 0$. In this case, *PC*′ and *MWC*2 imply *NCC*. If $\underline{w} \geq 0$, then $b_2^{**}(\underline{w}) = 0$; for $\underline{w} = 0$ the participation constraint is binding without an NCC, for $\underline{w} > 0$, the participation constraint is slack without an NCC. In both cases, the binding *PC* means that $\bar{v} \leq 0$, implying *NCC*. In this case, the new constraint is binding if and only if *MWC*2 is binding.

The problem is, thus, equivalent to

$$\max_b \quad -\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot (V - b) \tag{1.B.2}$$

$$\text{s.t.} \quad \bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))}, \tag{PC'}$$

$$b \geq b_2^{**}(\underline{w}). \tag{1.B.3}$$

The problem (1.B.2) is simpler because it has only one inequality constraint which is on the only argument of the objective function. Under the assumptions made in Section 1.3, moreover, the objective function is strictly concave, as Lemma 1.3 shows. We introduced this assumption because it implies all assumptions that we need in this proof. To make the proof tighter, however, we make weaker assumptions wherever possible. Thus, for determining whether the second or the third combination is optimal, we will use a weaker assumption and the notion of strict quasi-concavity that is sufficient to derive the results. In Lemma 1.4, we determine the necessary and sufficient condition that makes the objective function strictly quasi-concave in the bonus wage.

Lemma 1.3. *Equation 1.B.2 is strictly concave in b if for all bonus wages*

$$\frac{c'''(E(b, \bar{v}(\underline{w}, b)))}{c''(E(b, \bar{v}(\underline{w}, b)))} > \frac{1}{1 - E(b, \bar{v}(\underline{w}, b))}.$$

Proof. The objective function's first and second derivatives with respect to the bonus wage are

$$\frac{\partial \pi}{\partial b} = \frac{E'(b, \bar{v}(\underline{w}, b))}{1 - E(b, \bar{v}(\underline{w}, b))} \cdot (V - b) - E(b, \bar{v}(\underline{w}, b))$$

and (omitting the argument of $E(b, \bar{v}(\underline{w}, b))$ for readability)

$$\frac{\partial^2 \pi}{\partial b^2} = \left[\frac{E''}{(1-E)^2} + \frac{(E')^2}{(1-E)^3}\right] \cdot (V - b) - \frac{2E'}{1-E}.$$

Because $E'(b, \bar{v}(\underline{w}, b)) > 0$, a sufficient condition for the concavity of the objective function is that $\frac{E''}{(1-E)^2} + \frac{E'E'}{(1-E)^3} < 0$. Rearranging and simplifying shows that this is true under our assumption on the cost function.

$$E''(b, \bar{v}(\underline{w}, b)) + \frac{(E'(b, \bar{v}(\underline{w}, b)))^2}{1 - E(b, \bar{v}(\underline{w}, b))} < 0 \quad \implies \quad \frac{\partial^2 \pi}{\partial b^2} < 0.$$

Plugging in for $E'(\cdot) \equiv \frac{1}{c''(E(\cdot))}$ and $E''(\cdot) \equiv -\frac{c'''(E(\cdot))}{(c''(E(\cdot)))^3}$ yields

$$\frac{c'''(E(b, \bar{v}(\underline{w}, b)))}{c''(E(b, \bar{v}(\underline{w}, b)))} > \frac{1}{1 - E(b, \bar{v}(\underline{w}, b))}.$$

□

Lemma 1.4. *Equation 1.B.2 is strictly quasi-concave in b if for all bonus wages*

$$\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{2}{E(b-\bar{v}(\underline{w},b))}.$$

Proof. The objective function, $\pi(\underline{w},b)$, is twice continuously differentiable. It is strictly quasi-concave in b if the second derivative is negative at each critical point.

For readability, we will omit the argument of $E(b-\bar{v}(\underline{w},b))$ and its derivatives, and instead write $E(\cdot)$. The objective function's first derivative with respect to b is

$$\begin{aligned}\frac{\partial\pi(\underline{w},b)}{\partial b} &= E'(\cdot)\cdot\left(1-\frac{\partial\bar{v}(\underline{w},b)}{\partial b}\right)\cdot(V-b)-E(\cdot)\\ &= \frac{E'(\cdot)}{1-E(\cdot)}\cdot(V-b)-E(\cdot).\end{aligned} \tag{1.B.4}$$

Since $1-E(\cdot)$ is the equilibrium probability of a failure, it is positive due to the Inada conditions. Critical points are characterized by

$$V-b = \frac{E(\cdot)\cdot(1-E(\cdot))}{E'(\cdot)}. \tag{1.B.5}$$

The objective function is strictly quasiconcave in b if and only if the derivative of equation (1.B.4) is negative at every critical point. After some calculus, the sign of the derivative of equation (1.B.4) is seen equal to the sign of "expression 1":

$$E'(\cdot)\cdot(V-b)-E(\cdot)\cdot(1-E(\cdot)). \tag{Expression 1}$$

Expression 1's derivative is

$$\begin{aligned}&E''(\cdot)\cdot\left(1-\frac{\partial\bar{v}(\underline{w},b)}{\partial b}\right)\cdot(V-b)-E'(\cdot)-E'(\cdot)\cdot(1-2E(\cdot))\cdot\left(1-\frac{\partial\bar{v}(\underline{w},b)}{\partial b}\right)\\ &= \frac{E''(\cdot)}{1-E(\cdot)}\cdot(V-b)-E'(\cdot)-\frac{E'(\cdot)\cdot(1-2E(\cdot))}{1-E(\cdot)}\\ &= \frac{E''(\cdot)}{1-E(\cdot)}\cdot(V-b)-\frac{E'(\cdot)\cdot(2-3E(\cdot))}{1-E(\cdot)}.\end{aligned} \tag{1.B.6}$$

Since we only care about the sign at the critical points, we can now plug in the solution of the first-order condition (1.B.5) for $(V-b)$. This yields an expression that we would like to be negative.

$$\frac{E''(\cdot)}{1-E(\cdot)}\cdot\frac{E(\cdot)\cdot(1-E(\cdot))}{E'(\cdot)}-\frac{E'(\cdot)\cdot(2-3E(\cdot))}{1-E(\cdot)} < 0.$$

Rearranging yields

$$E''(\cdot) < \frac{(E'(\cdot))^2\cdot(2-3E(\cdot))}{E(\cdot)\cdot(1-E(\cdot))}. \tag{1.B.7}$$

Using the definition of $E(\cdot)$, this can be simplified.

$$E(\cdot) = (c')^{-1}(\cdot) \implies E'(\cdot) = \frac{1}{c''(E(\cdot))} \implies E''(\cdot) = -\frac{c'''(E(\cdot))}{(c''(E(\cdot)))^3}.$$

Therefore, Equation 1.B.7 is equivalent to our assumption

$$\frac{c'''(E(\cdot))}{c''(E(\cdot))} > \frac{1}{1-E(\cdot)} - \frac{2}{E(\cdot)}.$$

□

For equilibrium efforts below $\frac{2}{3}$, the assumption is always satisfied. For equilibrium efforts above $\frac{2}{3}$, the assumption says that the marginal cost has to be convex enough. As a result, the equilibrium effort reacts not too strongly to increased incentives and the strict quasi-concavity is preserved when introducing NCCs.

Strict quasi-concavity in the bonus wage implies that the maximum is unique if it exists. To see that the maximum exists, note that the maximum is equivalent to the maximum of the problem constraining $b_2^{**}(\underline{w}) \leq b \leq V$, since the optimal bonus wage cannot be above V. Because of the extreme value theorem we know that the latter problem has a solution ($b_2^{**}(\underline{w}) \leq b \leq V$ is a compact set and the objective function is continuous).

This last simplification concludes the first part of the proof. In the second part of the proof, we look at the three remaining combinations and determine for which minimum wages they are optimal. We first characterize the different combinations in the simplified problem. Then, we use the monotonicity of the marginal profit in the bonus wage evaluated at the bonus wage $b_2^{**}(\underline{w})$ to find the minimum wages for which the second combination is optimal. Lastly, we derive a condition under which the fourth combination is optimal for some minimum wages.

Negative minimum wages. Consider negative minimum wages first. For $\kappa_1 \leq \underline{w} < 0$, only the second or the third combination can be optimal. The sign of the derivative of the objective function with respect to the bonus wage at the lower bound $b_2^{**}(\underline{w})$ shows whether there is an inner solution or not. If the derivative is non-positive, there is a corner solution and, thus, no NCC. The second combination is optimal. If the derivative is positive, there is an inner solution and, thus, an NCC. The third combination is optimal. The monotonicity of the derivative evaluated at $b_2^{**}(\underline{w})$ in the minimum wage yields uniqueness of minimum wage at which a switch happens.

Lemma 1.5. *Assume that* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{2}{E(b-\bar{v}(\underline{w},b))}$ *for all bonus wages. There is a unique cutoff* $\kappa_2 < 0$ *in the minimum wage such that for all* $\kappa_1 \leq \underline{w} \leq \kappa_2$*, the optimal contract has* $b = b_2^{**}(\underline{w})$*, and for all* $\kappa_2 < \underline{w} < 0$*, the optimal contract has* $b > b_2^{**}(\underline{w})$.

Proof. The derivative of the profit with respect to the bonus wage evaluated at the lower bound is

$$\begin{aligned}\frac{\partial \pi(\underline{w},b)}{\partial b}\Big|_{b=b_2^{**}(\underline{w})} &= \frac{\partial E(b-\bar{v}(\underline{w},b))}{\partial b}\Big|_{b=b_2^{**}(\underline{w})} \cdot (V-b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w})) \\ &= \frac{E'(b_2^{**}(\underline{w}))}{1-E(b_2^{**}(\underline{w}))} \cdot (V-b_2^{**}(\underline{w})) - E(b_2^{**}).\end{aligned}$$

We will now look at different minimum wages and show that there is exactly one minimum wage at which the optimum switches from a corner to an inner solution. The corresponding minimum wage is the minimum wage from which on NCCs are used, κ_2. Technically, at κ_2, the objective function's first derivative evaluated at the lowest possible bonus wage $b_2^{**}(\underline{w})$ switches the sign from negative (corner solution) to positive (inner solution).

We use the same strategy as when proving quasi-concavity: We show that in all candidates for κ_2, the derivative goes from negative to positive. By continuity, there can be only one candidate.

A candidate for κ_2 is a minimum wage such that the derivative is zero:

$$\begin{aligned}\frac{\partial \pi(\underline{w},b)}{\partial b}\Big|_{b=b_2^{**}(\underline{w})} &= \frac{E'(b_2^{**}(\underline{w}))}{1-E(b_2^{**}(\underline{w}))} \cdot (V-b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w})) \overset{!}{=} 0 \\ \Leftrightarrow \quad (V-b_2^{**}(\underline{w})) &= \frac{E(b_2^{**}(\underline{w}))\cdot(1-E(b_2^{**}(\underline{w})))}{E'(b_2^{**}(\underline{w}))}. \end{aligned} \tag{1.B.8}$$

To see how the derivative of the profit with respect to the bonus wage at the lower bound changes, take the derivative with respect to the minimum wage. Note that although $\bar{v}(b,\underline{w})$ is a function of both the bonus and the minimum wage, it will not change: At $b_2^{**}(\underline{w})$, the participation constraint binds without an NCC. Thus, $\bar{v}(b_2^{**}(\underline{w}),\underline{w}) = 0$ for all negative minimum wages.

Again, we work with another expression that has the same sign as the first derivative but which is easier to work with. "Expression 2" is

$$E'(b_2^{**}(\underline{w}))\cdot(V-b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w}))\cdot(1-E(b_2^{**}(\underline{w}))). \tag{Expression 2}$$

The derivative of expression 2 with respect to the minimum wage (where we express $E(b_2^{**}(\underline{w}))$ and its derivatives as E to improve readability) is

$$\begin{aligned}\frac{\partial\left(\frac{\partial\pi}{\partial b}|_{b=b_2^{**}(\underline{w})}\right)}{\partial \underline{w}} &= E''\cdot(V-b_2^{**}(\underline{w}))\cdot\frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} - E'\cdot\frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} \\ &\quad -(1-E)\cdot E'\cdot\frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} + E'\cdot E\cdot\frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}} \\ &= \frac{\partial b_2^{**}(\underline{w})}{\partial \underline{w}}\left(E''\cdot\frac{E(1-E)}{E'} - 2E'\cdot(1-E)\right) > 0.\end{aligned}$$

The second line follows from plugging Equation 1.B.8 in. At the critical point, the derivative of the profit with respect to the bonus wage evaluated at the lower bound is increasing because $\frac{\partial b_2^{**}}{\partial \underline{w}} < 0$; the lowest bonus wage to satisfy the participation constraint is decreasing in the minimum wage because a higher minimum wage makes the participation constraint already slack. Moreover, it is globally true that $E' > 0$, and $E'' < 0$.

We have shown that any switches between corner and inner solutions have to be from corner to inner solutions. Moreover, there can be at most one switching point. That is, conditional on existence, κ_2 is unique.

To show that there is at least one critical point, we use that the derivative of the profit with respect to the bonus wage is continuous in the minimum wage. There is a minimum wage for which the derivative is negative and there is a minimum wage for which the derivative is positive. Thus, there also is a minimum wage for which the derivative is zero.

The derivative is negative for the minimum wage κ_1. The principal implements first-best effort and extracts all surplus by selling the firm. Because all of the success payoff goes to the agent, increasing the bonus wage further reduces the profit. Plugging κ_1 in, yields $b_2^{**}(\kappa_1) = V$. The derivative is

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=b_2^{**}(\kappa_1)} = -E(V) < 0.$$

The derivative is positive for the minimum wage κ_3. Following a similar argument as above, we know from the benchmark that the derivative of the profit with respect to the bonus wage without access to NCC at the minimum wage κ_3 is zero: Left of κ_3, the optimal bonus wage just satisfies the participation constraint, right of κ_3, the optimal bonus wage makes the participation constraint slack. The derivative of the profit with respect to the bonus wage without NCC is

$$\left.\frac{\partial \pi^{\text{No NCC}}}{\partial b}\right|_{b=b_2^{**}(\kappa_3),\bar{v}=0} = E'(b_2^{**}(\kappa_3)) \cdot (V - b_2^{**}(\kappa_3)) - E(b_2^{**}(\kappa_3)) = 0.$$

With NCCs, there are double incentives. Thus, the derivative with NCCs is strictly larger: The marginal benefit gets multiplied with $\frac{1}{1-E} > 1$. Therefore, the positive term is larger. The negative term is the same. Since at κ_3 the derivative without NCC is zero, the derivative with NCC is positive.

$$\left.\frac{\partial \pi(\underline{w}, b)}{\partial b}\right|_{b=b_2^{**}(\kappa_3),\bar{v}=0} = \frac{E'(b_2^{**}(\kappa_3))}{1 - E(b_2^{**}(\kappa_3))} \cdot (V - b_2^{**}(\kappa_3)) - E(b_2^{**}(\kappa_3)) > 0.$$

To sum up: The profit's first derivative evaluated at the bonus wage $b_2^{**}(\underline{w})$ is continuous and monotonically increasing. It is strictly negative at κ_1 and strictly positive at κ_3. Thus, its root, κ_2, exists and lies strictly in-between, $\kappa_1 < \kappa_2 < \kappa_3 < 0$.

□

For all minimum wages below κ_2, the optimal contract and, thus, the profit is the same as in the benchmark. For minimum wages above κ_2, an NCC is used and the principal's profits are strictly larger than in the benchmark: Strict quasi-concavity of the profit in the bonus wage means that the maximum is unique. The principal could mimic the world without NCC. He does, however, not want to. Uniqueness of the maximimum means that the optimal contract with NCC is strictly better than the optimal contract without NCC.

A minimum wage of zero. For $\underline{w} = 0$, the second combination is not feasible. The binding participation constraint with no NCC implies that the bonus wage has to be zero. In the second combination, the bonus wage has to be strictly positive. Furthermore, the fourth combination is not feasible. The binding participation constraint with no bonus wage implies that the most severe NCC is no NCC. In the fourth combination, the NCC has to be strictly negative. Thus, the optimal contract has to have both a bonus wage and an NCC.

Having established that the first, the second, and then the third combination are optimal in an increasing minimum wage, we now turn to positive minimum wages.

Positive minimum wages. For positive minimum wages, contracts from the second combination are not feasible: It is not possible to make the participation constraint binding without an NCC. In this range, only the third or the fourth combination can be optimal. We show that starting at a minimum wage of 0, the third combination is optimal. We derive one condition on the effort cost function for the existence and one condition for the uniqueness of there being a minimum wage $\kappa_4 > 0$ such that for all $\underline{w} < \kappa_4$, the third combination is optimal and for all $\underline{w} \geq \kappa_4$, the fourth combination is optimal. At κ_4, the principal stops using a bonus wage. Instead, all incentives follow from an NCC. If the condition is not met, the third combination is optimal for all positive minimum wages.

To get uniqueness of κ_4, we need an assumption on the cost function. For all bonus wages, it has to hold that $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$. While this assumption is stronger than the assumption to get strict quasi-concavity, it is also implied by our assumptions in Section 1.3 that imply strict concavity of the objective function. With this assumption, we can show that there is at most one minimum wage at which the principal switches between the third and the fourth combination. Furthermore, this assumption implies that the switch is such that for lower minimum wages there is a positive bonus wage, while for higher minimum wages, the optimal bonus wage is zero.

The strategy of the proof is to determine the sign of the marginal profit of the bonus wage, evaluated at a bonus wage of 0. If it is positive, there is an inner solution and the optimal bonus wage is positive. To make the participation constraint binding, an NCC is needed. The optimal contract is, thus, from the third combination. Using no bonus wage is optimal if it is negative. Then, the first unit of the bonus wage is not worth the marginal cost. The optimal contract is, thus, from the fourth combination. The assumption on the uniqueness implies that the every switch of the sign goes from the positive to the negative.

To prove existence, we show that the sign of the marginal profit of the bonus wage, evaluated at a bonus wage of 0, is initially positive. We assume that the condition for uniqueness is met. The marginal profit of the first unit of bonus wage is continuous in the minimum wage. Because its sign is initially positive, can switch its sign at most once, and the marginal profit's continuity, the sign in the limit is negative if and only if the switch happened for a finite minimum wage. We then derive the (necessary and sufficient) condition under which the sign is negative in the limit. This is the conition for the existence of κ_4. To determine the sign in the limit, we use L'Hôpital's rule.

Lemma 1.6. *If for all bonus wages* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$, *then there is at most one minimum wage for which* $\frac{\partial\pi}{\partial b}|_{b=0} = 0$.

Proof. Again, we will employ the same strategy of proof as above to show the uniqueness of a critical point. The critical point in the minimum wage is characterized by

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=0} = \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V - E(-\bar{v}(\underline{w},0)) \stackrel{!}{=} 0.$$

This is the marginal profit by increasing the bonus wage starting at a bonus wage of zero. That is, for which minimum wage it is optimal not to use the bonus wage $b = 0$. Since $\underline{w} > 0$, the principal will use an NCC to provide incentives. The optimal contract falls into the fourth combination.

Thus, a critical point is defined by

$$V = \frac{E(-\bar{v}(\underline{w},0)) \cdot (1-E(-\bar{v}(\underline{w},0)))}{E'(-\bar{v}(\underline{w},0))}.$$

As above, we show that this critical point is unique if it implies that the marginal profit from the first unit of bonus wage hits zero from above. Then, to the left of the critical point, it is optimal to use positive bonus wages; to the right of the critical point, it is optimal to use no bonus wages. We want to show that

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=0} \stackrel{!}{=} 0 \quad \Longrightarrow \quad \frac{\partial\left(\left.\frac{\partial \pi}{\partial b}\right|_{b=0}\right)}{\partial \underline{w}} < 0.$$

To do so, we compute this derivative (we again omit the arguments and express $E(-\bar{v}(\underline{w},0))$ as E to improve readability)

$$\frac{\partial\left(\left.\frac{\partial \pi}{\partial b}\right|_{b=0}\right)}{\partial \underline{w}} = \frac{(1-E)E'' + E' \cdot E'}{(1-E)^3} \cdot V - \frac{E'}{1-E}.$$

Plugging in the characterization of a critical point ($V = \frac{E\cdot(1-E)}{E'}$) and simplifying yields

$$\frac{c'''(E)}{c''(E)} > \frac{1}{1-E} - \frac{1}{E},$$

which holds by assumption. □

Lemma 1.7. *Assume that for all bonus wages* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$. *If*

$$\lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} \cdot V \; < \; 1, \tag{1.B.9}$$

then there is a minimum wage $\kappa_4 > 0$ such that the optimal contract uses a bonus wage for all lower minimum wages and the optimal contract uses no bonus wage for all larger minimum wages.

Proof. κ_4 exists if there is a positive minimum wage that equates the marginal benefit and the marginal cost of the first unit of bonus wage.

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=0} = \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V - E(-\bar{v}(\underline{w},0)) \; = \; 0.$$

We have shown above that there is at most one such minimum wage. Furthermore, we have shown that the intersection has to be such that the marginal benefit intersects the marginal cost from above. Now we show under which conditions there is at least one such intersection.

Initially, the marginal benefit is larger than the marginal cost. Consider the minimum wage $\underline{w} = 0$. Together with $b = 0$, this implies that $\bar{v} = 0$ to make the PC binding and that the equilibrium effort is 0. The marginal benefit is $\frac{E'(0)}{1} \cdot V$. Since $E'(\cdot) \equiv \frac{1}{c'(E(\cdot))}$, this is strictly positive for a minimum wage of 0. The marginal cost is $E(0) = 0$ at a minimum wage of 0. Hence, we showed that for $\underline{w} = 0$, the bonus wage's marginal benefit is higher than the marginal cost. By continuity, this also holds for some positive minimum wages.

Since the marginal benefit is initially larger, can intersect the marginal cost only from above, and both are continuous, it is sufficient to look at the limits of the minimum wage's going to infinity. Without a bonus wage, the non-compete clause will then become ever stronger which implies that the equilibrium effort will go to 1.

First, consider the marginal cost of increasing the bonus wage starting at $b = 0$. When the minimum wage goes to infinity, the equilibrium effort goes to 1 and the marginal cost goes to 1. Second, consider the marginal benefit of increasing the bonus wage starting at $b = 0$.. When the minimum wage goes to infinity, the equilibrium effort goes to 1 and the marginal benefit goes to $\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V$. Let us consider numerator and denominator separately. The numerator goes to zero because $\lim_{\underline{w}\to\infty} E'(-\bar{v}(\underline{w},0)) = \lim_{\underline{w}\to\infty} \frac{1}{c''(E(-\bar{v}(\underline{w},0)))}$ and $\lim_{\underline{w}\to\infty} c''(E(-\bar{v}(\underline{w},0))) = \infty$. This follows because $\underline{w} \to \infty$ implies that $E(-\bar{v}(\underline{w},0)) \to 1$ which implies that $c'(e) \to \infty$. For the same reason, the denominator also goes to zero.

Thus, we use L'Hôpital's rule to evaluate $\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V$. In order to use L'Hôpital's rule we need to check two conditions:

First, we must check that for all (positive) finite minimum wages $\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}} \neq 0$. This condition is fulfilled because $\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}} = -\frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))}$. By assumption, the numerator is positive.

Second, we must check that limit of the ratio of the derivatives exists. This condition is fulfilled because

$$\lim_{\underline{w}\to\infty} \frac{\frac{\partial E'(-\bar{v}(\underline{w},0))}{\partial \underline{w}}}{\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}}} \cdot V = \lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} \cdot V < 1.$$

by assumption and continuous on (0,1).

All in all, L'Hôpital's rule yields

$$\begin{aligned}
&\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V \\
&= \lim_{\underline{w}\to\infty} \frac{\frac{\partial E'(-\bar{v}(\underline{w},0))}{\partial \underline{w}}}{\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial \underline{w}}} \cdot V \\
&= \lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} \cdot V.
\end{aligned}$$

Therefore, there is a critical minimum wage κ_4 if and only if

$$\lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} V < 1.$$

The assumption can also be expressed in properties of the effort cost function. It is an assumption on the convergence speeds of the second and the third derivative. Note that both $c''(\cdot)$ and $c'''(\cdot)$ go to infinity when the minimum wage goes to infinity because the equilibrium effort goes to 1 and then $c'(\cdot)$ goes to infinity. Therefore, if $(c''(\cdot))^2$ goes to infinity strictly faster than $c'''(\cdot)$, the marginal benefit converges to zero. If the convergence of $(c''(\cdot))^2$ and $c'''(\cdot)$ has the same speed, the limit is some number. If this number times V is less than 1, the assumption is also satisfied. Whenever the convergence of $c'''(\cdot)$ is faster than that of $(c''(\cdot))^2$, the assumption does not hold.

□

Having characterized which constraints bind in which combination, we can now characterize the optimal contract in each combination. Note that the contract in the first (second) combination mirrors the one in Case 1 (2). The base and bonus wages are equal and the principal does not want to use a NCC. The computations of base and bonus wage is therefore identical to the computations in Case 1 and 2 in Proposition 1.1 and therefore are skipped here for clarity. We now characterize the optimal bonus wage and the optimal non-compete clause depending on the effort level that will be chosen in each combination.

Next, we consider the third combination.

Third combination. Let E be the effort level that the agent chooses given the contract. $MWC1$ binds which implies that $w = \underline{w}$. PC binds as well. We substitute IC and $MWC1$ into PC and rewrite to get

$$\bar{v} \;=\; c(E) - E \cdot c'(E) - \underline{w},$$

where we suppress the arguments of E and $\bar{v}$ for simplicity.

Combining $MWC1$, PC and IC by substituting for $\bar{v}$ gives

$$b \;=\; (1-E) \cdot c'(E) + c(E) - \underline{w}.$$

Now, we substitute for w and b in P's objective function to get.

$$\pi \;=\; E \cdot V - (1-E) \cdot \underline{w} - E \cdot (1-E) \cdot c'(E) - E \cdot c(E).$$

P maximizes over the incentive-compatible effort level and hence $E = e_3^{NCC}$ is chosen such that

$$c(e_3^{NCC}) + (1-e_3^{NCC}) \cdot c'(e_3^{NCC}) + e_3^{NCC} \cdot (1-e_3^{NCC}) \cdot c''(e_3^{NCC}) \;=\; V + \underline{w}.$$

Next, we consider the fourth combination.

Fourth combination. Let E be the effort level that the agent chooses given the contract. $MWC1$ binds which implies that $w = \underline{w}$. $MWC2$ binds which together with the binding $MWC1$ implies that $b = 0$. $\bar{v}$ is then determined by the binding participation constraint

$$\bar{v} \;=\; -\frac{\underline{w} - c(E)}{1-E}.$$

The optimal effort choice is then determined by the IC and hence $E = e_4^{NCC}$ is characterized by

$$\underline{w} + e_4^{NCC} \cdot c'(e_4^{NCC}) - c(e_4^{NCC}) \;=\; c'(e_4^{NCC}).$$

□

Proof of Proposition 1.3. We show that the equilibrium effort is constant in the minimum wage in the first combination, decreasing in the minimum wage in the second combination and increasing in the minimum wage if P uses a NCC, that is, in the third and fourth combination.

We start with the first combination. Note that we showed in Propositions 1.1 and 1.2 that P does not use an NCC and induces the first-best effort level in the first combination. First-best effort level is constant at e^{FB} and hence does not change in the minimum wage.

We proceed with the second combination. Note that we showed in Proposition 1.2 that P does not use an NCC. The equilibrium effort is hence defined by $c'(E) = b(\underline{w})$. Since the marginal cost is increasing, the equilibrium effort gets smaller if the right hand side gets smaller. Thus, we have to show that the right hand side is decreasing in the minimum wage. The binding participation constraint gives us

$$G \equiv E(b) \cdot b - c(E(b)) + \underline{w} = 0.$$

We use the implicit function theorem on the binding participation contraint. From now on, we will skip the argument of E for clarity. Since G is continuously differentiable, the implicit function theorem can be used to calculate the derivative of b with respect to $\underline{w}$.

$$\frac{\partial b}{\partial \underline{w}} = -\frac{\frac{\partial G}{\partial \underline{w}}}{\frac{\partial G}{\partial b}} = -\frac{1}{E}.$$

Hence, we get that $\frac{\partial b}{\partial \underline{w}} < 0$ which then implies that the equilibrium effort decreases in the minimum wage.

We continue with the third combination, in which the optimal contract has both a bonus wage and an NCC. We, therefore, need to evaluate their combined effect on the effort. The equilibrium effort is defined by $c'(E) = b(\underline{w}) - \bar{v}(\underline{w}, b(\underline{w}))$. Since the marginal cost is increasing, the equilibrium effort gets larger if the right hand side gets larger. Thus, we need to show that the right hand side is increasing in the minimum wage. Taking the derivative with respect to the minimum wage of the right hand side yields

$$\frac{\partial b(\underline{w})}{\partial \underline{w}} - \left(\frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial \underline{w}} + \frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial b(\underline{w})} \cdot \frac{\partial b(\underline{w})}{\partial \underline{w}} \right). \tag{1.B.10}$$

To show that this expression is positive, we look at its parts in turn. We already calculated the effect of a change in the minimum wage and in the bonus wage on the NCC that makes the participation constraint bind in Lemma 1.2. For convenience, we reproduce the result here:

$$\frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} = -\frac{1}{1 - E(b - \bar{v})}, \qquad \frac{\partial \bar{v}(\underline{w}, b)}{\partial b} = -\frac{E(b - \bar{v})}{1 - E(b - \bar{v})}.$$

It remains to characterize how the optimal bonus wage changes in the minimum wage. Again, we use the implicit function theorem on the first-order condition of the expected profit maximization problem. The FOC of *P*'s expected profit with respect to the bonus wage is

$$Z \equiv E'(b-\bar{v}) \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) \cdot (V-b) - E(b-\bar{v}) = 0.$$

We will from now on skip the argument of E for clarity. Before we apply the implicit function theorem to this equation to see how b changes in $\underline{w}$, we need two intermediary derivatives: $\frac{\partial^2 \bar{v}}{\partial b \partial \underline{w}}$ and $\frac{\partial^2 \bar{v}}{\partial b^2}$. And again, we can use Lemma 1.2, which shows that $\frac{\partial \bar{v}}{\partial b} = -\frac{E}{1-E}$.

Thus,

$$\frac{\partial^2 \bar{v}}{\partial b \partial \underline{w}} = -\frac{E' \cdot (1-E) \cdot \frac{\partial \bar{v}}{\partial \underline{w}} + E' \cdot E \cdot \frac{\partial \bar{v}}{\partial \underline{w}}}{(1-E)^2} = -\frac{E'}{(1-E)^3},$$

and

$$\frac{\partial^2 \bar{v}}{\partial b^2} = \frac{-E' \cdot (1-E) \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) - E' \cdot E \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right)}{(1-E)^2} = -\frac{E'}{(1-E)^3}.$$

Since Z is continuously differentiable, the implicit function theorem can be used to get the derivative of b with respect to $\underline{w}$.

$$\begin{aligned}\frac{\partial b}{\partial \underline{w}} &= -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial b}} \\ &= -\frac{-E'' \cdot \frac{\partial \bar{v}}{\partial \underline{w}} \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) \cdot (V-b) - E' \cdot \frac{\partial^2 \bar{v}}{\partial b \partial \underline{w}} \cdot (V-b) + E' \cdot \frac{\partial \bar{v}}{\partial \underline{w}}}{E'' \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right)^2 \cdot (V-b) - E' \cdot \frac{\partial^2 \bar{v}}{\partial b^2} \cdot (V-b) - 2E' \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right)} \\ &= -\frac{\left(\frac{1}{1-E} - \frac{c'''(E)}{c''(E)}\right) \cdot \frac{V-b}{(1-E)^2 \cdot (c''(E))^2} - \frac{1}{(1-E) \cdot c''(E)}}{\left(\frac{1}{1-E} - \frac{c'''(E)}{c''(E)}\right) \cdot \frac{V-b}{(1-E)^2 \cdot (c''(E))^2} - \frac{2}{(1-E) \cdot c''(E)}}. \end{aligned} \tag{1.B.11}$$

Since $E(\cdot) < 1$, $c''(\cdot) > 0$, $c'''(\cdot) > 0$, $b \leq V$ and concavity $\left(\frac{c'''(E)}{c''(E)} > \frac{1}{1-E}\right)$, we get that $\frac{\partial b}{\partial \underline{w}} < 0$. Hence, a higher minimum wage implies a lower bonus wage.

Let us recap what we have shown so far. On the one hand, we found that a higher minimum wage leads to a lower bonus wage which provides less incentives. On the other hand, we found that a higher minimum wage implies a more severe NCC which provides more incentives. It remains to show that the effect on the NCC is stronger than on the bonus wage. Rearranging the marginal change of the incentives in the minimum wage (Equation 1.B.10) and plugging in yields

$$
\begin{aligned}
&-\frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial \underline{w}} + \frac{\partial b(\underline{w})}{\partial \underline{w}} \cdot \left(1 - \frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial b}\right) \\
&= \frac{1}{1-E} + \frac{\partial b(\underline{w})}{\partial \underline{w}} \cdot \left(1 + \frac{E}{1-E}\right) \\
&= \frac{1}{1-E} \cdot \left(1 + \frac{\partial b(\underline{w})}{\partial \underline{w}}\right).
\end{aligned}
$$

To show that this is positive, it now suffices to show that the bracket is positive. That is, $\frac{\partial b(\underline{w})}{\partial \underline{w}} > -1$.

Consider $-\frac{\partial b}{\partial \underline{w}}$ as it is characterized in Equation 1.B.11. For simplicity, let

$$
x \equiv \left(\frac{1}{1-E} - \frac{c'''(E)}{c''(E)}\right) \frac{V-b}{(1-E)^2 (c''(E))^2} \quad \text{and} \quad y \equiv \frac{1}{(1-E)c''(E)}.
$$

We have that $x < 0$ and $y > 0$. It is then easy to check that $-\frac{\partial b}{\partial \underline{w}} = \frac{x-y}{x-2y} < 1$. Which was to be shown. Therefore, the equilibrium effort is increasing in the minimum wage in the third combination.

We now show that in the fourth combination the equilibrium effort is also increasing in the minimum wage. The principal does not use a bonus wage anymore. Lemma 1.2 shows that $\frac{\partial \bar{v}}{\partial \underline{w}} = -\frac{1}{1-E}$ where $E(-\bar{v}(\underline{w}))$ is the solution to the agent's incentive problem. This shows that higher minimum wages lead to more severe NCCs which then leads to higher effort through the incentive constraint.

To sum up, if $\underline{w} > \kappa_2$, then higher minimum wages lead to more effort incentives, and, thus a non-monotonicity of the equilibrium effort.

□

Proof of Proposition 1.4. We show that the principal induces a higher effort level than first-best effort if the minimum wage is sufficiently large. Due to the Inada conditions, the first-best effort level will be strictly smaller than 1. We show that the equilibrium effort in the third (in case the fourth combination does not exist) and in the fourth combination must go to 1. This directly implies that the equilibrium effort level will be higher than the first-best effort level if the minimum wage is large enough. We start with the third combination. Formally, we want to show that

$$
\lim_{\underline{w} \to \infty} E(b(\underline{w}) - \bar{v}\big((\underline{w}), b(\underline{w})\big) \;=\; 1,
$$

where E is continuous and monotonically increasing in the bonus wage, in the severity of the NCC, and in the minimum wage (Proposition 1.3). Therefore, we can rewrite the limit such that

$$\begin{aligned}
&\lim_{\underline{w}\to\infty} E(b(\underline{w}) - \bar{v}((\underline{w}), b(\underline{w}))) \\
=&\lim_{\underline{w}\to\infty} (c')^{-1}(b(\underline{w}) - \bar{v}((\underline{w}), b(\underline{w}))) \\
=&(c')^{-1}\left(\lim_{\underline{w}\to\infty} b(\underline{w}) - \lim_{\underline{w}\to\infty} \bar{v}((\underline{w}), b(\underline{w}))\right) \\
=&(c')^{-1}(\infty) \\
=&1,
\end{aligned}$$

due to the Inada conditions.

We proceed with the fourth combination. Formally, we want to show that

$$\lim_{\underline{w}\to\infty} E(-\bar{v}(\underline{w})) = 1,$$

where E is continuous and monotonically increasing in the severity of the NCC, and in the minimum wage (Proposition 1.3). Therefore, we can rewrite the limit such that

$$\begin{aligned}
&\lim_{\underline{w}\to\infty} E(-\bar{v}(\underline{w})) \\
=&\lim_{\underline{w}\to\infty} (c')^{-1}(-\bar{v}(\underline{w})) \\
=&(c')^{-1}\left(-\lim_{\underline{w}\to\infty} \bar{v}(\underline{w})\right) \\
=&(c')^{-1}(\infty) \\
=&1,
\end{aligned}$$

due to the Inada conditions.

□

Proof of Lemma 1.1. Let f_f be the probability that the agent gets fired after a failure and f_s the probability that the agent gets fired after a success. Plugging this general firing rule into the agent's problem changes his incentive constraint:

$$e^* = \arg\max_{e\in[0,1]} \; w + e\cdot b + e\cdot f_s\cdot\bar{v} + (1-e)\cdot f_f\cdot\bar{v} - c(e). \qquad \text{(IC')}$$

The agent's first-order condition (the Inada conditions ensure an interior solution) is then

$$b - (f_f - f_s)\bar{v} - c'(e) = 0.$$

The agent's incentives from the NCC are now given by $-(f_f - f_s)\cdot\bar{v}$ instead of $-\bar{v}$. Therefore, all combinations of b and of f_f, f_s, and $\bar{v}$ that have the same product induce the same effort. Since the principal has to make the agent willing to participate, she chooses the firing rule and NCC that reduces the agent's expected utility as little as possible. To implement a fixed effort, given a fixed base and bonus wage, the principal, thus, chooses the firing rule that solves

$$\begin{aligned} \max_{f_f, f_s, \bar{v}} \quad & w + e \cdot b + e \cdot f_s \cdot \bar{v} + (1-e) \cdot f_f \cdot \bar{v} - c(e) \\ \text{s.t.} \quad & 0 \leq f_f \leq 1, \\ & 0 \leq f_s \leq 1, \\ & \bar{v} \leq 0, \\ & -(f_f - f_s)\bar{v} = K \geq 0. \end{aligned}$$

K is the "amount of incentives" from the NCC. The principal only wants to use NCCs at all, if she wants to provide more incentives than with the bonus wage alone. That is why K is positive. Ignoring constants and rearranging the constraints, this problem simplifies to $\bar{v} = -\frac{K}{f_f - f_s}$ and

$$\begin{aligned} \max_{f_f, f_s} \quad & -\frac{f_f}{f_f - f_s} \\ \text{s.t.} \quad & 0 \leq f_s < f_f \leq 1. \end{aligned}$$

The derivative with respect to f_f is globally positive, given the constraint. Therefore, it is optimal to set $f_f = 1$. The derivative with respect to f_s is globally negative, given the constraint. Therefore, it is optimal to set $f_s = 0$. The optimal $\bar{v}$ is then given by $-K$. The firing rule that makes the agent's participation constraint as slack as possible is to fire him if and only if he fails. □

Proof of Proposition 1.5. Let the principal's expected profit be strictly quasi-concave in the bonus wage, that is,

$$\frac{c'''(E(b - \bar{v}(\underline{w}, b)))}{c''(E(b - \bar{v}(\underline{w}, b)))} > \frac{1}{1 - E(b - \bar{v}(\underline{w}, b))} - \frac{2}{E(b - \bar{v}(\underline{w}, b))},$$

holds for all minimum wages.

With a bounded NCC we have the additional constraint that $\bar{v} \geq \underline{v}$. This changes P's maximization problem to

$$\begin{aligned} \max_{b} \quad & -\underline{w} + E(b - \bar{v}) \cdot (V - b) \\ \text{s.t.} \quad & \bar{v} = \max\left\{\bar{v}(\underline{w}, b), \underline{v}\right\}, \qquad \text{(NCC)} \\ & b \geq b_2^{**}(\underline{w}), \end{aligned}$$

where again $\bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))}$. The NCC condition already uses that the profit is increasing in more severe non-compete clauses (because the optimal bonus wage is smaller than the success payoff). Thus, the principal would never use a NCC that is less severe than the one NCC that makes the PC binding ($\bar{v}(\underline{w}, b)$), except this would violate the bound on NCCs ($\underline{\bar{v}}$). As a result, the optimal NCC is determined by which constraint binds first: the participation constraint ($\bar{v}(\underline{w}, b)$) or the bound on NCCs ($\underline{\bar{v}}$).

We now split the minimum wages into two ranges. One for which the bound on NCCs is insubstantial and one for which the bound on NCCs makes the formerly optimal contracts infeasible. This is possible because the optimal $\bar{v}$ without a bound decreases continuously and strictly monotonically in the minimum wage above κ_2. Moreover, $\bar{v}$ lies between zero and minus infinity such that any bound binds for some minimum wages. We define $\underline{w}_{bound}$ as the minimum wage for which the optimal contract without a bound on NCCs uses an NCC that is exactly the bound. That is, the optimal contract is $(\underline{w}_{bound}, b(\underline{w}_{bound}), \underline{\bar{v}})$. As argued above, $\underline{w}_{bound}$ exists and is unique for each bound $\underline{\bar{v}}$.

Case i) $\underline{w} < \underline{w}_{bound}$. For these minimum wages, the optimal contract without a bound on the NCCs does not violate the bound on the NCCs. Since the bound only introduces another constraint, these contracts remain optimal. The bound on NCCs can be ignored.

Case ii) $\underline{w} \geq \underline{w}_{bound}$. For all minimum wages above $\underline{w}_{bound}$, the optimal contracts without a bound on the NCCs are not feasible anymore: they violate the bound on NCCs. In the simplified problem, the only choice variable of the principal is the bonus wage. Thus, the optimal NCC is implicitly defined by the optimal bonus wage.

For $\underline{w} \geq \underline{w}_{bound}$, the constraint $b \geq b_2^{**}(\underline{w})$ can be ignored. The constraint said that, firstly, the participation constraint must not be violated if $\bar{v} = 0$, and, secondly, that the bonus wage must be non-negative. Since the optimal NCC at $\underline{w}_{bound}$ is strictly negative (and because of its comparative statics), we know that the participation constraint without an NCC would be satisfied. Furthermore, the optimal bonus wage can never be negative because there is a profitable deviation, as argued in the proof of Proposition 1.2; this deviation exists independently of a bound on the NCC.

For minimum wages $\underline{w} \geq \underline{w}_{bound}$, the optimal contract without a bound is either the one from the third combination or the one from the fourth combination. We can distinguish these as different cases. For each case, we show that once the binding NCC is optimal, it will remain optimal for all larger minimum wages, and we characterize the optimal bonus wage.

a) The optimal contract for the minimum wage $\underline{w}_{bound}$ is from the third combination. That is, the optimal bonus wage without a bound is strictly positive. Thus, the optimal bonus wage is determined by the first-order condition; the bonus wage for which the marginal profit gets zero. It is unique because the objective function is quasi-concave by assumption. For $\underline{w} = \underline{w}_{bound}$, the optimal contract remains optimal and just makes the bound on the NCC binding. Thus, the marginal profit at the

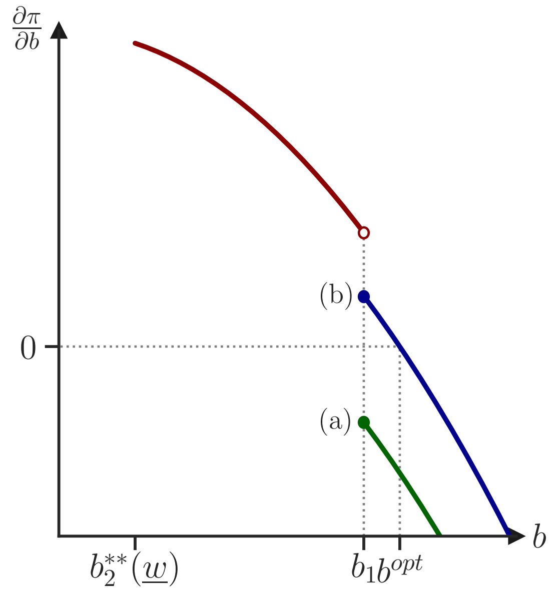

Figure 1.B.1. The derivative of the profit with respect to the bonus wage jumps down as soon as the bound on the NCC binds. If (a) the jump ends below zero, the agent gets no rent. If (b) the jump ends above zero, the agent gets a rent and the optimal bonus wage is the same for higher minimum wages. Drawn for a concave objective function.

bonus wage $b(\underline{w}_{bound})$ is 0. We will reconsider this particular minimum wage after describing the marginal profit in the bonus wage in general.

How does the marginal profit with respect to the bonus wage behave for a fixed minimum wage $\underline{w} > \underline{w}_{bound}$? For a sketch of the marginal profit, see Figure 1.B.1.

As mentioned above, starting at $b_2^{**}(\underline{w})$, the marginal profit is positive. When increasing the bonus wage, it keeps being positive. Moreover, it has the same values as in the problem without a bound. Then, the bonus wage, $b_{bound}(\underline{w})$, is reached that allows the principal to reach the bound $\bar{v}(\underline{w}, b_{bound}(\underline{w})) \overset{!}{=} \underline{\bar{v}}$. Importantly, at this minimum wage, the derivative is still positive: The optimal bonus wage is $b(\underline{w})$ and by the case assumption it is true that $\bar{v}(\underline{w}, b(\underline{w})) < \underline{\bar{v}}$. Because $\bar{v}(\underline{w}, b)$ is decreasing in the bonus wage, and because the root of the first-order condition is unique, we know that $b_{bound}(\underline{w}) < b(\underline{w})$. From $b_{bound}(\underline{w})$ on, the principal cannot make the NCC more severe when increasing the bonus wage. Therefore, there are no double incentives anymore. The marginal profit, thus, jumps downwards to its level in the benchmark; formally

$$\lim_{b' \to b_{bound}(\underline{w})^-} \frac{\partial \pi}{\partial b}(\underline{w}, b')$$
$$= \frac{E'(b_{bound}(\underline{w}) - \underline{\bar{v}})}{1 - E(b_{bound}(\underline{w}) - \underline{\bar{v}})} \cdot (V - b_{bound}(\underline{w})) - E(b_{bound}(\underline{w}) - \underline{\bar{v}}) \; > \; 0,$$

$$\lim_{b' \to b_{bound}(\underline{w})^+} \frac{\partial \pi}{\partial b}(\underline{w}, b')$$
$$= E'(b_{bound}(\underline{w}) - \underline{\bar{v}}) \cdot (V - b_{bound}(\underline{w})) - E(b_{bound}(\underline{w}) - \underline{\bar{v}}).$$

For bonus wages after the jump, the profit function is strictly concave in the bonus wage, as in the benchmark.[30] The marginal profit is, thus, strictly decreasing.

The optimal bonus wage is now either $b_{bound}(\underline{w})$, if the marginal profit jumps (weakly) below zero, or a higher bonus wage if the marginal profit remains positive after the jump. In any case, this implies that $\bar{v}(\underline{w}, b) \leq \underline{\bar{v}}$ in the optimum. Therefore, the bound on the NCC is the binding constraint; thus $\underline{\bar{v}}$ is the optimal NCC.

To find the optimal bonus wage, we have to find out which constraints will bind. This depends on whether the optimal bonus wage is at the jump point or not. If it is at the jump point, the participation constraint binds ($\bar{v}(\underline{w}, b_{bound}(\underline{w})) = \underline{\bar{v}}$); which implies that the agent gets no rent. If it is to the right of the jump point, the participation constraint is slack because the NCC that would make the participation constraint binding lies outside the bound. Therefore, it is slack; which implies that the agent gets a rent.

For the other constraints (MWC_1, MWC_2, NCC), the same reasoning as above, in the proof of Proposition 1.2, applies. The minimum wage condition on the base wage binds. Otherwise, there is a profitable deviation. Due to the case assumption, the optimal bonus wage without a bound is positive, thus MWC_2 is slack. With a bound, it might also be that MWC_2 binds if ignoring the constraint leads to a violation. Due to the case assumption, an NCC is used, which means that the NCC feasibility constraint is slack. As a result, the optimal base wage always is the minimum wage and, as shown above, the optimal NCC is the binding NCC.

Firstly, we now determine the optimal bonus wage depending on where the jump ends and then, secondly, we show that there always is a range of minimum wages for which the jump ends in the negative.

We start with the case in which the marginal profit's jump ends in the non-positive. In this case, the optimal bonus wage is at the jump point and makes the participation constraint binding. Thus, the participation constraint pins down the optimal bonus wage. How does the optimal bonus wage change in the minimum wage? We use the implicit function theorem to show that the bonus wage that makes the

30. The second derivative is $E''(\cdot) \cdot (V - b) - 2E'(\cdot) < 0$. $E''(\cdot)$ is globally negative and $E'(\cdot)$ is globally positive.

participation constraint binding is strictly decreasing in the minimum wage. Rearrange the binding participation constraint to

$$Z \equiv \underline{w} + E(b_{bound} - \underline{v}) \cdot (b_{bound} - \underline{v}) + \underline{v} - c(E(b_{bound} - \underline{v})) = 0.$$

Because this is continuously differentiable, the implicit function theorem can be used to get the derivatives of b_{bound} with respect to $\underline{w}$.

$$\begin{aligned}\frac{\partial b_{bound}(\underline{w})}{\partial \underline{w}} &= -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial b_{bound}}} \\ &= -\frac{1}{E'(\cdot) \cdot (b_{bound} - \underline{v}) + E(\cdot) - c'(E(\cdot)) \cdot E'(\cdot)} \\ &= -\frac{1}{E(\cdot)},\end{aligned}$$

where we suppress the argument of E for clarity. The simplification is due to the agent's first-order constraint, $b_{bound} - \underline{v} - c'(E) = 0$. Since $E(b_{bound} - \underline{v}) > 0$, the bonus wage that makes the participation constraint binding is strictly decreasing in the minimum wage.

Further, we can say that the optimal bonus wage with a bound lies below the optimal bonus wage without a bound on the NCC. In both cases, the participation constraint is binding and the bonus wage is positive (due to the case assumption). Without a bound on the NCC, the optimal NCC is weakly more severe than the bound because $\underline{w} \geq \underline{w}_{bound}$; strictly more severe if $\underline{w} > \underline{w}_{bound}$. For a fixed minimum wage, a strictly more severe NCC needs a strictly larger bonus wage to keep the participation constraint satisfied. Thus, with a bound on the NCC, the optimal bonus wage is smaller.

When the optimal bonus wage hits zero, it stays at zero for all larger minimum wages. It can never become negative because of the profitable deviation. Note that when the bonus wage hits zero, for all larger minimum wages the participation constraint is slack and the agent gets a rent.

We now look at the optimal bonus wage if the jump in the marginal benefit ends in the positive and the participation constraint can be ignored. The optimal bonus wage is constant because the minimum wage does not enter the problem anymore. The optimal bonus wage is determined by the marginal profit's being zero or the minimum wage condition on the bonus wage. We define b_3 as the root.

$$\frac{\partial \pi}{\partial b} \overset{!}{=} 0 \quad \Leftrightarrow \quad b_3 : \quad E'(b_3 - \underline{v}) \cdot (V - b_3) - E(b_3 - \underline{v}) = 0.$$

Note that $E'(\cdot)$ is decreasing in its arguments because $E''(\cdot) < 0$. Furthermore, $E(\cdot)$ is increasing in its arguments. Therefore, compared to the third case in the benchmark, the marginal benefit of the bonus wage is smaller and the marginal cost is larger for all bonus wages. We shift $E'(\cdot)$ to the left and $E(\cdot)$ to the right. Thus, $b_3 < b^{***}$. If the marginal profit is zero for a negative bonus wage, the optimal bonus wage is zero because of the minimum wage condition. Thus, the optimal bonus wage is $b_3^+ \equiv \max\{0, b_3\}$.

What is the relation between the solution when the jump ends in the negative and when it ends in the positive? The maximization problem when ignoring the participation constraint yields a weakly larger maximum than taking into account the participation constraint. Therefore, the profit with b_3^+ is weakly larger than the profit with $b_{bound}(\underline{w})$. b_3^+ is optimal whenever it does not violate the participation constraint.

We now show that there are some minimum wages for which b_3^+ does violate the participation constraint, such that $b_{bound}(\underline{w})$ is the optimal solution. Reconsider the minimum wage $\underline{w}_{bound}$. The optimal contract is $(\underline{w}_{bound}, b(\underline{w}_{bound}), \underline{\bar{v}})$. By the case assumption, $b(\underline{w}_{bound}) > 0$. Thus, without a bound on NCCs, the marginal profit of an additional unit of bonus wage is 0 at $b(\underline{w}_{bound})$. With a bound on NCCs, this is the bonus wage at which the jump from double incentives to incentives (only through bonus wage) happens. The jump, thus, has to end in the negative. Thus, this is one minimum wage for which the participation constraint would be violated for b_3^+. Furthermore, the point at which the jump ends, moves continuously in the minimum wage: The marginal profit is a continuous function in the bonus wage and the bonus wage at which the jump happens is a continuous function of the minimum wage. Thus, the jump also ends in the negative for some larger minimum wages.

b) The optimal contract for the minimum wage $\underline{w}_{bound}$ is from the fourth combination, that is, the optimal bonus wage is 0. With a bound on the NCC, the optimal contract now is $(\underline{w}, 0, \underline{\bar{v}})$. A positive bonus wage cannot increase the profits. The optimal contract only falls into the fourth combination if the marginal profit from the first unit of bonus wage is negative. Because the binding NCC does not violate the participation constraint even without a bonus wage, there never are double incentives. Thus, the marginal profit is smaller than without a bound on the NCC (intuitively, the jump happened for a negative bonus wage). Since the marginal profit was negative with double incentives, the marginal profit is still negative. It is optimal not to use a positive bonus wage. A negative bonus wage cannot increase the profits because this means increasing the base wage above the minimum wage (otherwise the minimum wage constraint on the bonus would be violated). Then, there is a profitable deviation (making the NCC less severe, the bonus wage larger and the base wage lower by one marginal unit).

□

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Chapter 2

Consumer Protection or Efficiency? The Case of Partitioned Pricing*

Joint with Simon Dato and Andreas Klümper

2.1 Introduction

Partitioned pricing refers to a firm's practice to split the price of a good or service into two or more components (Morwitz, Greenleaf, and Johnson, 1998).[1] Amplified by the increased prevalence of e-commerce, it seems to be the norm nowadays that consumers face substantial additional charges for shipping, handling, or payment methods (Mohammed, 2019). Empirical evidence documents that consumers underestimate the total price when being confronted with multiple prices (Greenleaf, Johnson, Morwitz, and Shalev, 2016; Voester, Ivens, and Leischnig, 2017), such that partitioned pricing is likely to deceive consumers into overbuying. Consequently, it has come under increased scrutiny: evaluating several possible price frames, the UK Office of Fair Trading concluded that partitioned pricing has the greatest potential to cause harm for consumers (Office of Fair Trading, 2010). Furthermore, competition authorities have penalized firms from several industries for having engaged in partitioned or drip pricing.[2]

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1. Partitioned pricing initially referred to a practice in which prices were presented simultaneously. Following Friedman (2020), a broader definition of partitioned pricing also contains pricing strategies in which consumers observe prices with a delay such as drip pricing and pricing of unavoidable add-ons as sub-cases.

2. The Australian Competition and Consumer Commission imposed sanctions on the two largest airlines (https://cutt.ly/shLcMbb). Likewise, the Canadian Competition Bureau fined rental car agencies for charging hidden fees (https://cutt.ly/ohLvtl1).

For partitioned pricing to exploit consumers by affecting their purchasing decisions, it needs to hold that (i) firms are allowed to partition prices and (ii) consumers mistakenly undervalue the total price. Accordingly, authorities can and actually do engage in two types of policies that address each requirement to protect consumers. Regarding the first type, they have invoked policies that regulate extra fees or additional prices, even up to a ban of partitioned pricing. In recent legislation, consumer protection agencies have either banned (UK, USA[3], EU[4]) or limited (Australia[5]) payment surcharges for the use of credit or debit cards. In Australia, agencies have ruled that surcharges must not be excessive, i.e., above the transaction's actual costs to the merchant. Similarly, while many countries have banned fuel-surcharges in the airline industry, Japan still allows airlines to add a fuel-surcharge to their price. However, the surcharge is tied to the actual costs of fuel two months before the flight.[6] In a similar vein, several lawsuits dealing with excessively high interest rates for car loans bought at a car dealer combined with a car were settled by capping the mark-up a dealer can add to the actual interest rate (Cohen, 2012). As this first type of policy limits the firms' choice set, i.e., the set of prices to choose from, it can be labeled as a hard intervention (Heidhues and Kőszegi, 2018).

The second type of policy does not regulate or ban partitioned pricing. Instead, it aims to lower the degree to which consumers underestimate additional prices by increasing either (i) consumers' awareness or (ii) the degree of the additional prices' transparency. One possibility to increase price transparency is to restrict firms' ability to hide additional prices. The Australian Competition and Consumer Commission (ACCC), for instance, requires that whenever firms advertise prices, the total price must be at least as visible as prices that do not include additional fees and charges.[7] However, the second type of policy does not necessarily intervene on the firms' side but may also educate consumers to reduce the impact of deceptive pricing strategies. For example, in 2003, the US Congress passed the *Fair and Accurate Credit Transactions Act* (FACTA), which provided better consumer financial literacy. It brought forth the *Financial Literacy and Education Commission* (FLEC), which is concerned with setting a national strategy to increase consumer awareness of credit scores and their impact on their financial decisions (Cohen, 2012). Contrary to the first type of policy, the second type of policy does not affect the set of feasible prices firms may offer. In line with the terminology used by Heidhues and Kőszegi (2018), it can be labeled as a soft intervention.

This paper analyzes the consequences of hard and soft interventions that aim at limiting the impact of partitioned pricing on consumer surplus and welfare. Since

3. For the UK and the USA, see https://cutt.ly/jhLilo7.

4. See, the Directive (EU) 2015/2366 of the European Parliament and of the Council https://cutt.ly/ehLiQby.

5. See, https://cutt.ly/xhLiYxm.

6. See, https://cutt.ly/AxNexGQ.

7. See, https://cutt.ly/rhLiVdp.

the first is arguably the measure that is mostly applied by competition authorities when designing policies, with Canada as a notable exception (Heyer, 2006), one might argue that our results on the first measure might be most relevant for practitioners. However, analyzing the effects of policy interventions on both measures is particularly relevant in light of a longstanding debate among antitrust scholars whether consumer surplus or welfare should be considered by regulators when intervening in markets. It goes back at least to Bork (1978) and was recently addressed in Wilson (2019). In particular, critics argue that using consumer surplus as a measure to evaluate policy interventions "must therefore be counted as 'distributive' to the extent that it produces outcomes that shift wealth or resources in favor of consumers even though an alternative outcome would produce greater total wealth." (Hovenkamp, 2012, p. 2472). To account for these arguments, we study the effects of policies on both measures and identify circumstances under which they lead to identical or very different policy implications.

For this purpose, we incorporate partitioned pricing and consumer naivety into the framework of Singh and Vives (1984) with differentiated products. Consequently, we allow firms to partition their prices into a headline price and an additional price. Moreover, to account for the mounting evidence that consumers underestimate partitioned prices, we assume that they are naive and underestimate the additional prices, as in, e.g., Gabaix and Laibson (2006), Chetty (2009) or Heidhues, Kőszegi, and Murooka (2016b). We model a hard intervention as imposing an upper bound on the additional price and a soft intervention as an increase in price transparency, i.e., a decrease in the degree to which consumers underestimate a given additional price. Comparative static in these parameters allows us to evaluate each policy's consequences on consumer surplus and welfare.

Our analysis reveals that firms have an incentive to charge high additional and low headline prices in markets with naive consumers and the possibility to partition prices. The reason is that naive consumers react less sensitive to additional prices than to headline prices. If the upper bound on the additional price is very high, equilibrium headline prices tend to become overly low. While very low or even negative headline prices might be feasible in some markets, they might not work well in others. For example, consumers might become suspicious and abstain from buying at all if deals look too good to be true. Likewise, in markets in which firms charge the additional price for an unavoidable add-on, arbitrage traders might force firms to charge positive prices (Heidhues, Kőszegi, and Murooka, 2016a,b). To account for both situations, we analyze two different scenarios. First, we analyze markets in which firms can freely choose their headline prices. Second, we analyze markets in which firms are restricted to charge non-negative headline prices.

Our first main result demonstrates that there exists a fundamental trade-off between consumer protection and efficiency: Independent of whether the headline price is restricted or not, the strongest possible consumer protection policy maximizes consumer surplus but is never welfare-optimal. Consumer protection is maximized with the strongest possible hard intervention, i.e., a ban of partitioned pricing, or the strongest possible soft intervention, i.e., making additional prices fully transparent. In both cases, firms' pricing strategies cannot be deceptive, and consumers can make fully informed choices. Due to the imperfect competition between firms arising from imperfect substitutability of products, it directly follows that equilibrium demand is inefficiently low.

In contrast, allowing firms to engage in partitioned pricing and leaving consumers with some positive degree of naivety renders consumer protection imperfect. In equilibrium, consumers perceive total prices to be lower than they actually are and, therefore, demand higher quantities. Via this channel, imperfect consumer protection boosts demand and brings equilibrium quantities closer to efficiency. Accordingly, maximizing welfare calls for imperfect consumer protection.

Although the trade-off between consumer protection and efficiency arises with hard and soft interventions and the presence and the absence of a price floor on the headline price, the strength of the welfare-maximizing intervention and the resulting welfare level strongly depend on the scenario and the type of intervention considered. First, regarding soft interventions, it holds that the welfare-maximizing degree of price transparency induces efficient demand and, hence, achieves first-best welfare only if the regulation of the additional price is sufficiently weak. However, under a rather strict additional price regulation, firms need to set comparably high headline prices to attain sufficient revenues per unit sold. Accordingly, even if prices are fully intransparent and firms can offer negative headline prices, firms do not find it optimal to offer such low prices. Hence, irrespective of whether the headline price is restricted, it is impossible to reach efficient demand if the additional price regulation is strict.

Whether it is possible to induce efficient demand with a hard intervention, i.e., a regulation of additional prices, may crucially depend on the scenario analyzed. If headline prices are unrestricted, the welfare-maximizing upper bound on the additional price induces first-best. Intuitively, the less regulated the additional price is, the stronger firms can trick consumers into overbuying by increasing the additional price and, at the same time, decreasing the headline price. It is then always possible to relax the regulation of additional prices just as much as necessary to increase demand up to efficiency. Furthermore, our results reveal that not to regulate partitioned pricing is the worst possible option: every bound on the additional price leads to an improvement in both welfare and consumer surplus.

If the headline price is restricted to be non-negative, it is not always possible to counteract an increase in the additional price with a decrease in the headline price as the price floor starts to be binding. If consumers are sufficiently sophisticated and, hence, per se do not strongly fall victim to overbuying, the binding price floor prevents equilibrium demand from strictly increasing with the upper bound on the additional price. Accordingly, even with unregulated partitioned pricing, equilibrium demand is inefficiently low. We also show that compared to an unregulated additional price, capping the additional price at the firms' marginal costs is welfare neutral but leads to a higher consumer surplus. This result is robust to all parameter constellations of the model. In the case of doubt, this policy is a safe course of action.

Moreover, our results allow us to shed light on the interplay between hard and soft interventions. It turns out that the strategic dependence of the two types of interventions is strongly affected by market characteristics, i.e., whether a floor for the headline price exists or not. With unrestricted headline prices, consumer protection via (i) increased price transparency and (ii) stronger price regulations are strategic substitutes with respect to welfare: an increase in the degree of price transparency calls for weaker price regulations. The negative effect of an increase in price transparency on demand has to be offset by a less strict regulation of additional prices, allowing firms to maintain high demand. However, in markets with restricted headline prices, the two types of interventions may also constitute strategic complements. If the price floor for the headline price is binding, equilibrium demand decreases in the upper bound on the additional price: contrary to the case of a non-binding price floor, consumers correctly assess that total prices are increasing in the upper bound. Accordingly, the demand-decreasing effect of an increase in price transparency needs to be offset by stronger regulations of partitioned pricing.

We contribute to a strand of the literature that discusses the effects of consumers' misperceptions of prices or products on market outcomes (for instance, Gabaix and Laibson, 2006; Chetty, 2009; Heidhues, Kőszegi, and Murooka, 2016b). In particular, this paper relates to those studies that analyze the effects of hard interventions (e.g., Heidhues, Kőszegi, and Murooka, 2016a; Heidhues, Johnen, and Kőszegi, 2020), soft interventions (e.g., Glaeser and Ujhelyi, 2010; de Meza and Reyniers, 2012; Kosfeld and Schüwer, 2016), or both (Armstrong and Vickers, 2012) on consumer surplus or welfare.

Heidhues, Johnen, and Kőszegi (2020) compare the effects of two hard interventions, regulating total prices and regulating additional prices in which consumers can choose between browsing and studying. In this model, consumers have limited attention such that they can only examine a few products in detail, including their additional prices, and have to browse other products superficially. If additional prices are constrained, consumers can engage in more browsing and, hence, compare more options. However, when total prices are regulated, the consumer is tempted to search less, which might leave him worse off. In our model, we adopt the notion that consumers are naive, as in Heidhues, Kőszegi, and Murooka (2016b). More precisely, even if they observe all prices, they still underestimate the total price as long as prices are partitioned.

In models of additional prices and hidden fees, more similar to our approach, Armstrong and Vickers (2012) and Kosfeld and Schüwer (2016) find that transparency-enhancing innovations in the form of consumer education might decrease welfare. These studies differentiate between sophisticated and naive consumers and endow sophisticates with the possibility to avoid a product's additional price at some cost. As (i) this cost is assumed to be socially wasteful, and (ii) sophisticated consumers make use of this option in equilibrium, educating naifs to sophisticates might be welfare-decreasing. Moreover, Armstrong and Vickers (2012) show that in contrast to a transparency-enhancing intervention, regulation of additional prices is at least weakly welfare increasing. Because sophisticated consumers only exert socially wasteful effort if additional prices are sufficiently high, tight regulation of additional prices avoids socially wasteful effort and welfare increases. However, Heidhues, Kőszegi, and Murooka (2016a) show that this might lead to higher innovation incentives on new types of hidden charges, which then lowers the policy's welfare effect. While these studies focus on inefficiencies that arise due to costs that sophisticated consumers bear to avoid additional charges, welfare effects, in our model, are solely driven by inefficiently low or high demand.

Glaeser and Ujhelyi (2010) analyze a model of Cournot competition with positive profits and inefficiently low demand in equilibrium. They show that consumers' underestimation of future health costs caused by consumption is detrimental to consumer surplus but leads to a more efficient demand. Accordingly, the mechanism at work is similar to the channel through which the negative effect of price transparency on welfare emerges in our paper. However, in their model, consumers perceive prices correctly and only underestimate the cost they have to bear. Thus, they remain silent about potential hard interventions.

Most closely related to our analysis is de Meza and Reyniers (2012). They show that when consumers underestimate additional prices in a Cournot model with constant elasticity of demand, consumer surplus and, with it, welfare might decrease in transparency. Full transparency leads to an increased and, hence, more inefficient total price. However, while full transparency also leads to an inefficiently high total price in our model, their result crucially hinges on the assumption of isoelastic demand and competition in quantities. We complement their study by discussing partitioned pricing and consumer naivety in the classical framework by Singh and Vives (1984). Moreover, we discuss the effects of hard and soft interventions, their interplay, and identify a trade-off between consumer surplus and efficiency.

The paper is organized as follows. Section 2.2 introduces the model. In Section 2.3, we solve for the equilibrium and do the comparative statics analysis for the case of unrestricted headline prices (2.3.1) and when non-negative headline prices are not feasible (2.3.2). Finally, Section 2.4 concludes.

2.2 Model

We adopt the differentiated duopoly framework by Singh and Vives (1984), in which each of the two firms $i \in \{1, 2\}$ produce quantity q_i of good i at constant marginal cost c. There is a continuum of consumers with mass one who derive utility from consuming quantities $q = (q_1, q_2)$ via

$$U(q) = \omega \sum_{i=1}^{2} q_i - 0.5 \left(\sum_{i=1}^{2} q_i^2 + 2\gamma q_i q_j \right) \text{ for } i \in \{1,2\} \text{ and } j \neq i,$$

where q_i denotes the quantity purchased at firm i, $\omega > 0$ is a measure of product quality, and $\gamma \in (-1,1)$ measures the degree of substitutability between the two products. The higher the value of γ, the more alike the products are.

Firms compete via prices to attract consumers. We extend the original framework by incorporating partitioned pricing and consumer naivety. First, we allow firms to partition prices, i.e., we assume that the total price of good i is given by the sum of a headline price p_i and an additional price $\hat{p}_i$. Among others, we follow Gabaix and Laibson (2006) in assuming bounded additional prices, i.e. $\hat{p}_i \leq \bar{p}$. Essentially, the assumption limits firms' ability to exploit naive consumers as they cannot be tricked into buying an infinite amount of a good. In our model, we treat $\bar{p}$ as a policy measure that can be influenced by policymakers.[8] We label the two extreme cases $\bar{p} = 0$ and $\bar{p} \to \infty$ as a ban of partitioned pricing and unregulated additional prices, respectively.

Second, consumer base their buying decision on a perceived price $p_i + \beta \cdot \hat{p}_i$ with $\beta \in [0,1]$ instead of the actual total price $p_i + \hat{p}_i$. Note that β measures the degree of sophistication of the consumers in our model, i.e., the higher β, the closer is the consumers' perceived price to the actual price $p_i + \hat{p}_i$. Therefore, we can interpret β as a policy parameter that influences firms' ability to charge intransparent additional prices, e.g., a minimum font size of an additional mandatory fee on a price comparison website. We refer to the extreme cases $\beta = 1$ and $\beta = 0$ as fully transparent and fully intransparent prices, respectively.

Since consumers underestimate the total price if firms engage in partitioned pricing they maximize their *perceived* net-utility function

$$\tilde{V}(q,p,\hat{p}) = U(q) - \sum_{i=1}^{2} q_i \left(p_i + \beta \hat{p}_i\right).$$

with $p = (p_i, p_j)$, and $\hat{p} = (\hat{p}_i, \hat{p}_j)$, when making their buying decisions. Consumer behavior is defined by the first-order conditions[9] given by

$$\frac{\partial \tilde{V}(q,p,\hat{p})}{\partial q_i} = \omega - q_i - \gamma q_j - p_i - \beta \hat{p}_i \overset{!}{=} 0 \qquad \forall\, i \in \{1,2\}.$$

8. Although we treat $\bar{p}$ as a policy variable, such an upper bound could also be imposed by other players in the market. For example, car loan companies allow car dealers to mark-up loans up to some percentage points but not above (Grunewald, Lanning, Low, and Salz, 2020).

9. Since $\frac{\partial^2 \tilde{V}(q,p,\hat{p})}{\partial q_i^2} = \frac{\partial^2 \tilde{V}(q,p,\hat{p})}{\partial q_j^2} = -1$, $\frac{\partial^2 \tilde{V}(q,p,\hat{p})}{\partial q_j \partial q_i} = -\gamma$, and $det(H) = 1 - \gamma^2 > 0$, where $det(H)$ denotes the Hessian matrix, the consumers' perceived utility is a strictly concave function.

The resulting symmetric demand functions are given by

$$q_i\left(p,\hat{p}\right) = \frac{1}{1-\gamma^2}\left[(1-\gamma)\omega - (p_i + \beta\hat{p}_i) + \gamma(p_j + \beta\hat{p}_j)\right] \quad i \neq j, i,j \in \{1,2\}$$

Firm profits are, therefore, given by

$$\pi_i(p,\hat{p}) = q_i\left(p,\hat{p}\right)\cdot\left(p_i + \hat{p}_i - c\right) \qquad \forall\, i \in \{1,2\}. \tag{2.1}$$

We follow the usual definition of producer surplus as the sum of profits, i.e.

$$\mathscr{PS} = \sum_{i=1}^{2}\pi_i(p,\hat{p}). \tag{2.2}$$

While the definition of producer surplus is standard in our model, we need to take a stance on how consumer surplus is measured because they are naive. Consumers base their buying decision on the perceived total price. As they end up, however, paying the actual total price, we follow Glaeser and Ujhelyi (2010) and measure consumer surplus as the experienced net-utility, i.e.,

$$\mathscr{CS} = U(q) - \sum_{i=1}^{2} q_i\left(p_i + \hat{p}_i\right).$$

Welfare is defined as the sum of producer and consumer surplus such that

$$\mathscr{W} = \omega\sum_{i=1}^{2} q_i - 0.5\left(\sum_{i=1}^{2}(q_i)^2 + 2\gamma q_i q_j\right) - \sum_{i=1}^{2} q_i\cdot c.$$

Maximizing welfare[10] over quantities yields the first-best quantities, given by $q_i^{FB} = \frac{\omega - c}{1+\gamma}$, $\forall i \in \{1,2\}$, which will serve as a benchmark in the following analysis.

As argued in the introduction, authorities have mainly engaged in two policy measures to protect consumers from being deceived: increasing price transparency (increasing β) and decreasing additional prices (decreasing $\bar{p}$). These policies aim at reducing the wedge between the perceived and the actual prices and, thereby, allow consumers to make more informed choices. In line with the rationale behind these policies, we think of consumer protection as a measure inversely related to the differences between the perceived and the actual prices, $(1-\beta)\hat{p}_i$. [11] Intuitively, the higher the level of consumer protection, the closer are the perceived prices to the actual prices. Full consumer protection refers to the case when the perceived prices equal the actual prices, which occurs under full price transparency ($\beta = 1$) or a ban of partitioned pricing ($\bar{p} = 0$).

10. Note that welfare is a strictly concave function since $\frac{\partial^2\mathscr{W}}{\partial q_i^2} = \frac{\partial^2\mathscr{W}}{\partial q_j^2} = -1$, $\frac{\partial^2\mathscr{W}}{\partial q_j \partial q_i} = -\gamma$, and $det\,(H) = 1-\gamma^2 > 0$, where $det\,(H)$ denotes the determinant of the Hessian matrix.

11. More formally, we define consumer protection as a differentiable function $\mathscr{P} : [0,\bar{p}]^2 \to \mathbb{R}$, with strictly negative partial derivatives and the arguments being the difference between the actual and perceived total prices, $(1-\beta)\hat{p}_i\ \forall\ i$.

2.3 Policy Analysis

To analyze the welfare consequences of changing the regulatory requirements for additional prices and price transparency, we first analyze markets in which firms can partition prices and face no restriction on headline prices. We derive optimal policies for consumers, producers, and welfare in these types of markets. Afterward, we consider markets where headline prices are restricted to be non-negative and perform the same analysis. Our results hinge on crucial similarities and differences between consumer optimal and welfare optimal policies in both types of markets.

2.3.1 Unrestricted Headline Prices

In markets in which firms can freely choose their headline prices, firms only take the restriction on the additional price into account when choosing their prices. Therefore, anticipating the demand of its consumers, firm i chooses its prices to solve

$$\max_{p_i,\hat{p}_i} \pi_i(p,\hat{p}) \;=\; q_i\big(p,\hat{p}\big)\cdot\big(p_i+\hat{p}_i-c\big) \;\; s.t. \;\; \hat{p}_i \;\leq\; \bar{p}.$$

Since consumers are less sensitive to an increase in the additional price than to a corresponding decrease in the headline price, firms will always choose the highest possible additional price, i.e., in any equilibrium $\hat{p}_i^* = \bar{p}$. To see this, note that for any combination of prices $(p_i, \hat{p}_i)$ of firm i with $\hat{p}_i < \bar{p}$, there exists a feasible combination of prices $(p_i^{'}, \hat{p}_i^{'})$ with $\bar{p} \geq \hat{p}_i^{'} > \hat{p}_i$ and $p_i^{'} = p_i + \hat{p}_i - \hat{p}_i^{'}$ that leads to a strictly higher profit: revenue per unit sold remains constant but demand strictly increases. Consequently, firm i's maximization problem reduces to

$$\max_{p_i} q_i\big(p,\hat{p}\big)\cdot\big(p_i+\bar{p}-c\big).$$

The best-response function of firm $i \in \{1,2\}$ is then given by

$$p_i \;=\; \frac{\big(1-\gamma\big)\,\omega}{2} + \frac{\gamma}{2}\cdot p_j + \frac{c-\big[1+\big(1-\gamma\big)\beta\big]\bar{p}}{2}$$

for $i \in \{1,2\}$ and $j \neq i$. Calculating the equilibrium prices leads to

$$p_i^* \;=\; p_j^* \;=\; \frac{\big(1-\gamma\big)\,\omega + c - \big[1+\big(1-\gamma\big)\beta\big]\bar{p}}{2-\gamma}, \tag{2.3}$$

which yields the following proposition:

Proposition 2.1. *The unique equilibrium is symmetric with* $\hat{p}^* = \bar{p}$,

$$p^* \;=\; \frac{\big(1-\gamma\big)\big(\omega-\beta\bar{p}\big)+c-\bar{p}}{2-\gamma}, \; \textit{and} \; q^* \;=\; \frac{\omega-c+\big(1-\beta\big)\bar{p}}{\big(1+\gamma\big)\big(2-\gamma\big)}. \tag{2.4}$$

Proposition 2.1 implies that an increase in the upper bound on the additional price $\bar{p}$ leads to lower headline prices p^*, higher additional prices $\hat{p}^*$, and a higher quantity q^*. As we have already shown, firms always charge the highest possible additional price. Accordingly, relaxing the upper bound, $\bar{p}$, leads to higher additional prices, which puts higher competitive pressure on the headline prices. Intuitively, the higher the additional price, the more profit can be extracted from naive consumers. Consequently, attracting consumers with low headline prices becomes more valuable, and headline prices decrease. However, as we consider differentiated products, the competitive pressure is not strong enough to fully compete away these additional profits. Accordingly, the decrease in the headline prices is smaller than the increase in the additional prices such that an increase in $\bar{p}$ leads to higher total prices $p^* + \hat{p}^*$. Consumers correctly take into account the decrease in the headline price but undervalue the increase in the additional price. It turns out that the decrease in the headline price outweighs the consumers' perceived increase in the additional price such that the perceived total price $p^* + \beta\hat{p}^*$ decreases. Hence, consumers demand higher quantities, which explains the counter-intuitive result that equilibrium quantities and total prices increase as a response to an increase in $\bar{p}$.

A higher degree of transparency, i.e., an increase in β, does not affect the additional price but leads to lower headline prices and a lower quantity. Clearly, the additional price remains constant as it depends only on the upper bound. Higher transparency makes consumers more sensitive to a change in the additional prices. Hence, competitive forces become fiercer and lead to lower headline prices. As an increase in β only leads to a lower headline price, the total price decreases such that one would expect demand to increase. Due to higher transparency of prices, however, consumers take the additional price more strongly into account and, via this channel, perceive the good to become more expensive. This demand-decreasing effect dominates the demand-increasing effect of a lower headline price such that the perceived total price increases. Although an increase in transparency leads to lower actual prices, equilibrium quantities decrease as well.

We will continue by analyzing the effects of (i) a hard intervention via tighter regulation of additional prices (i.e., a decrease in $\bar{p}$) and (ii) a transparency-enhancing soft intervention (i.e., an increase in β). The effects of an increase in the degree of transparency given by β and of a decrease in the upper bound, $\bar{p}$ can be summarized as an increase in consumer protection: although the total price decreases, consumers perceive the good to be more expensive such that they demand lower quantities in equilibrium. Thus, both effects lead to a decrease in the wedge between actual and perceived prices, $(1-\beta)\bar{p}$. The following proposition summarizes its effects on consumer surplus, producer surplus, and welfare. We denote the welfare-maximizing degree of price transparency by $\beta^{SB}\left(\bar{p}\right)$ and, correspondingly, the welfare-maximizing upper bound on the additional price by $\bar{p}^{SB}\left(\beta\right)$.[12]

Proposition 2.2. *Full consumer protection maximizes consumer surplus, while no consumer protection maximizes producer surplus. Full consumer protection is not welfare-maximizing. For every $\beta < 1$, $\bar{p}^{SB}\left(\beta\right)$ induces first-best welfare. First-best can be achieved via $\beta^{SB}\left(\bar{p}\right) \in (0,1)$ if and only if $\bar{p} > \left(1-\gamma\right)(\omega - c)$. Otherwise, $\beta^{SB}\left(\bar{p}\right) = 0$.*

Confirming intuition, consumer surplus is globally increasing in the degree of consumer protection. Decreasing the wedge between actual and perceived additional prices by either decreasing $\bar{p}$ or increasing β allows consumers to make more informed choices. Therefore, they benefit from stronger consumer protection. Likewise, producer surplus is globally decreasing in the degree of consumer protection. Consumer protection mitigates the extent to which consumers underestimate the additional price, limiting the firms' possibility to exploit consumers profitably, and, hence, decreases their profits.

Importantly, Proposition 2.2 demonstrates a fundamental trade-off between consumer protection and efficiency. Protecting consumers completely from being exploited – either by banning additional prices or by eliminating price intransparency – maximizes consumer surplus. However, at the same time, such a policy renders welfare inefficiently low. In the absence of possible consumer exploitation, imperfect substitutability between the firms' products implies imperfect competition, which leads to inefficiently high equilibrium prices and eventually to inefficiently low demand. A marginal increase in $\bar{p}$ or a marginal decrease in β renders consumer protection imperfect and allows firms to take advantage of consumer naivety. Although the actual total price increases, consumers mistakenly perceive the product to become less expensive and demand higher quantities. Via this channel, imperfect consumer protection boosts demand and is strictly welfare-increasing as it mitigates the inefficiency arising from imperfect competition.

Although the effects of changes in the upper bound on the additional price and changes in the degree of price transparency are similar, an important distinction between the two policies has to be made: Whereas for every β the upper bound $\bar{p}$ can be adjusted to induce first-best welfare, the reverse is not true. For any degree of price transparency, $\beta \in [0,1)$, equilibrium demand is increasing in $\bar{p}$. Firms react to a decrease in consumer protection by adapting equilibrium prices such that consumers mistakenly perceive the total price to decrease and demand higher quantities. Hence, relaxing the upper bound on the additional price leads to an increase in equilibrium quantities. In particular, it is always possible to select $\bar{p}$ large enough to induce exactly the demand that leads to first-best welfare.

However, the same logic does not apply regarding the impact of a change in price transparency for a given upper bound on the additional price. Even if con-

12. We relegate the proofs of all following propositions and lemmas to the appendix.

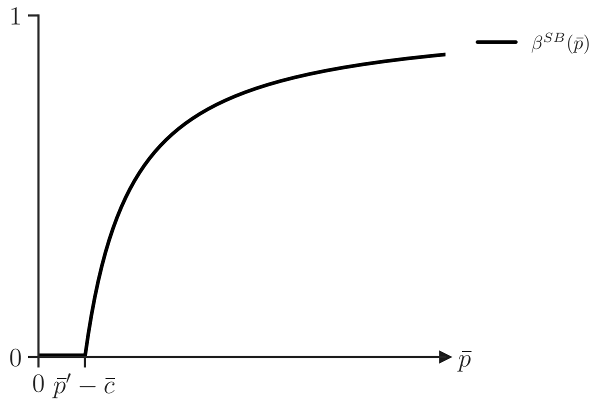


Figure 2.1. The equilibrium welfare-maximizing degree of price transparency (β^{SB}) as a function of the upper bound on the additional price ($\bar{p}$) in markets with unrestricted headline prices.

sumers completely neglect the additional price, i.e., $\beta = 0$, extractable profits per unit via the additional price are bounded by $\bar{p}$. Accordingly, in equilibrium, firms are not willing to offer arbitrarily low headline prices. The perceived total price is bounded from below, and the consumers' demand is, therefore, bounded from above. If $\bar{p}$ is sufficiently large, the upper bound on demand exceeds efficient demand, and the welfare-maximizing degree of price transparency $\beta^{SB}\left(\bar{p}\right) \in (0,1)$ induces first-best. However, with a strongly regulated additional price it might not be possible to adjust the degree of price transparency to achieve the first-best welfare. In equilibrium, firms can charge only low additional prices such that they need to charge comparably high headline prices. Accordingly, even with minimal price transparency, the perceived total price is so high that equilibrium demand is inefficiently low. The trade-off between consumer surplus and efficiency then globally holds as every policy designed to increase price transparency leads to an increase in consumer surplus but is detrimental to welfare. Complementing Proposition 2.2, the following lemma characterizes the welfare-maximizing regulations.

Lemma 2.1. *If $\beta < 1$, then the equilibrium welfare-maximizing upper bound on the additional price is given by $\bar{p}^{SB}\left(\beta\right) = \frac{(1-\gamma)(\omega-c)}{1-\beta}$. Similarly, if $\bar{p} > 0$, then $\beta^{SB}\left(\bar{p}\right) = \max\left\{0, 1 - \frac{(1-\gamma)(\omega-c)}{\bar{p}}\right\}$ denotes the welfare-maximizing degree of price transparency.*

The equation for $\bar{p}^{SB}\left(\beta\right)$ reveals that if prices are rather intransparent, i.e., β is low, only moderate additional prices should be allowed to induce first-best: consumers perceive total prices to be low and demand relatively high quantities per se such that a high upper bound on the additional price would result in excessive demand. Ceteris paribus, the more transparent prices are, the lower the quantities consumers demand. Accordingly, the more transparent prices are, the less regulated additional prices should be because an increase in $\bar{p}$ leads to lower perceived total prices, which offsets the decrease in demand caused by higher price transparency. Therefore, $\bar{p}^{SB}(\beta)$ is increasing in the degree of price transparency. In analogy, the welfare-maximizing degree of price transparency $\beta^{SB}(\bar{p})$ is increasing in $\bar{p}$. The relationship is strict for $\bar{p}$ being sufficiently large. However, if the upper bound on the additional price is so low that first-best welfare is infeasible, the welfare-maximizing degree of transparency is zero and, hence, independent of $\bar{p}$.

Interestingly, our model predicts that in markets with fiercer competition via a higher degree of substitutability between the products (higher γ), ensuring welfare-maximizing demand requires stricter regulations regarding the additional price or price transparency. Fiercer competition leads to lower prices, higher quantities, and, thus, higher welfare even in the absence of strict regulations. Consequently, equilibrium quantities are close to being efficient, and strong consumer protection policies are needed to prevent excessive demand. When the degree of substitutability between the products and the degree of competition is low, ensuring efficient demand requires some leeway for firms regarding deceptive pricing practices and calls for a weaker consumer protection policy to ensure efficiency. An immediate implication of these observations is that the trade-off between consumer surplus and welfare is less severe in more competitive markets. In these markets, agencies should always regulate additional prices strongly independent of which standard they apply.

2.3.2 Non-Negative Headline Prices

A crucial feature of the equilibrium described in Proposition 2.1 is that a higher degree of price transparency or a weaker upper bound on the additional price leads to a lower headline price. Accordingly, with very weak or non-existing regulations of the additional prices, the equilibrium headline price can be arbitrarily low. While this might be possible in some markets, in others, negative headline prices might prove infeasible. For example, in the case in which firms' pricing strategy involves a base product and an unavoidable add-on product, Heidhues, Kőszegi, and Murooka (2016a,b) argue that arbitrage traders, i.e., consumers that only buy the base good, can prevent firms from charging negative headline prices. They also argue that customers might become suspicious and abstain from buying a product when confronted with overly low headline prices. Moreover, an effective price floor may also arise as a consequence of legal restrictions, e.g., a ban of below-cost pricing, anti-dumping duties, or anti-dilution clauses for mutual funds.[13]

To account for these restrictions, we will extend the model analyzed in section 2.3.1 and follow Armstrong and Vickers (2012), Grubb (2014), and Heidhues, Kőszegi, and Murooka (2016a,b) by assuming that headline prices have to be non-negative, i.e., $p_i \geq 0$ for all i. As the firms' ability to sell higher quantities with higher

13. See, for instance, https://cutt.ly/ChLg1Pi on a ban of below-cost pricing in the United States.

total prices crucially depends on the possibility of decreasing headline prices while increasing additional prices, one might be tempted to conclude that imposing a floor on the headline price changes our results from the previous sections. While the results in the following section will prove the robustness of our main insights, as the fundamental trade-off between consumer surplus and welfare prevails, a price floor has important implications for welfare-optimal policies. For example, the price floor's existence sometimes renders the implementation of first-best welfare with a suitable upper bound on the additional price infeasible and may drastically affect the design of a regulatory intervention that maximizes equilibrium welfare. Therefore, it proves necessary for regulators to distinguish between markets in which firms face a lower price bound for headline prices and markets in which headline prices are essentially unrestricted when designing welfare-optimal policies.

Deriving equilibrium prices is more subtle in this variant of the model because it is not immediately clear that firms choose to price at the same bounds of the prices. However, as in the previous section, firms never find it optimal to set two interior prices. Hence, the best-response function of firm i either involves the highest possible additional price, the lowest possible headline price, or both. The following proposition characterizes the equilibrium with a lower bound for the headline price.

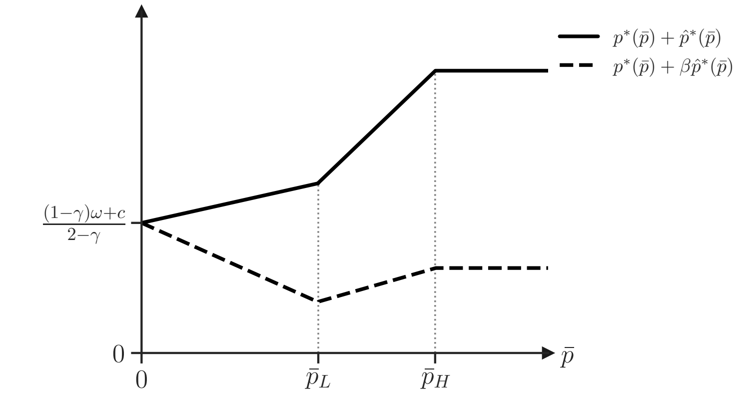
Proposition 2.3. *Let* $\bar{p}_L = \frac{(1-\gamma)\omega+c}{1+(1-\gamma)\beta}$ *and* $\bar{p}_H = \frac{(1-\gamma)\omega+\beta c}{\beta(2-\gamma)}$ *for* $\beta > 0$. *Then the unique equilibrium is symmetric and given by*

(i) $p^* = \frac{(1-\gamma)\omega+c-(1+(1-\gamma)\beta)\bar{p}}{2-\gamma}$, $\hat{p}^* = \bar{p}$, *and* $q^* = \frac{\omega-c+(1-\beta)\bar{p}}{(1+\gamma)(2-\gamma)}$ *if* $\bar{p} < \bar{p}_L$,

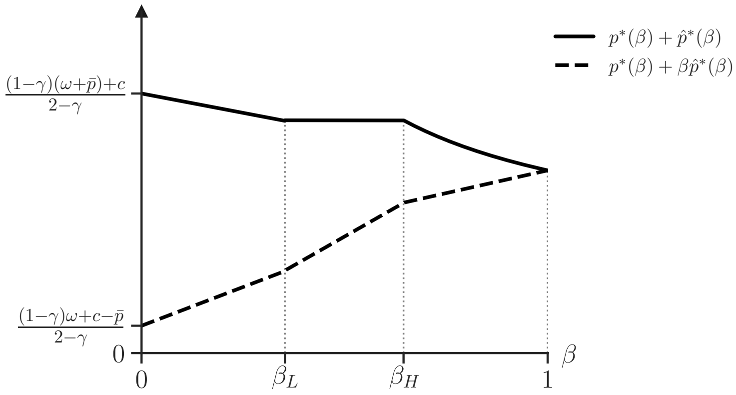
(ii) $p^* = 0$, $\hat{p}^* = \bar{p}$, *and* $q^* = \frac{\omega-\beta\bar{p}}{1+\gamma}$ *if* $\bar{p} \in [\bar{p}_L, \bar{p}_H]$,

(iii) and $p^* = 0$, $\hat{p}^* = \frac{(1-\gamma)\omega+\beta c}{(2-\gamma)\beta}$, *and* $q^* = \frac{\omega-\beta c}{(1+\gamma)(2-\gamma)}$ *if* $\beta > 0$ *and* $\bar{p}_H < \bar{p}$.

The proposition reveals that we have to distinguish between three different equilibrium outcomes depending on the upper price bound. If the upper bound on the additional price is sufficiently low, the equilibrium is identical to the case with unrestricted headline prices: the additional price is at its upper bound, and the headline price is positive and decreasing in $\bar{p}$. However, if the upper bound exceeds $\bar{p}_L$, the optimal unrestricted headline price is negative such that the lower bound on the headline price starts to be binding. It is then optimal to charge both prices at their respective bounds. In analogy to the previous analysis, further relaxing the upper bound on the additional price leads to higher additional prices in equilibrium. In this case, contrary to the equilibrium described in Proposition 2.2, an increase in the additional price is no longer accompanied by a decrease in the headline price. Consequently, the actual total prices and the perceived total prices increase, which implies that equilibrium quantities decrease. The less-regulated the additional prices are, the lower quantities consumers then demand. If the additional price regulation is sufficiently weak, i.e., $\bar{p} > \bar{p}_H$, the additional revenues per unit sold associated with a higher additional price can no longer compensate for the negative effect on



(a) $\omega = 2$, $\bar{c} = 1$, $\gamma = 0.5$, and $\beta = 0.3$



(b) $\omega = 2$, $\bar{c} = 1$, $\gamma = 0.5$, and $\bar{p} = 1.7$

Figure 2.2. The actual total equilibrium prices ($p^* + \hat{p}^*$) and the perceived total equilibrium prices ($p^* + \beta\hat{p}^*$) as functions of the upper bound on the additional price (2.2a) and price transparency (2.2b) in markets with restricted headline prices.

profits via lower demand. Firms prefer not to charge the maximum additional price anymore. The equilibrium is then not affected by further relaxing the upper bound on the additional price. The following proposition summarizes the implications for the effects of consumer protection on consumer surplus, producer surplus, and welfare.

Proposition 2.4. *Full consumer protection maximizes consumer surplus, while no consumer protection maximizes producer surplus. Full consumer protection is never welfare-maximizing.* $\bar{p}^{SB}(\beta) > 0$ *induces first-best if and only if* $\beta \leq \frac{c}{(1-\gamma)\omega+\gamma c}$. $\beta^{SB}(\bar{p}) \in (0,1)$ *induces first-best if and only if* $\bar{p} > (1-\gamma)(\omega - c)$.

Imposing a lower bound on the headline price does not change our results from the previous section regarding consumer protection's impact on consumer and producer surplus. Protecting consumers via increased price transparency or stronger regulation of additional prices reduces the extent to which they misperceive prices and allows them to demand quantities closer to actual utility-maximizing demand. Clearly, firms suffer from consumer protection as it narrows their ability to trick consumers into overbuying profitably.

Even with a regulated headline price, it holds that full consumer protection is never welfare-maximizing. In contrast to the set-up with unrestricted headline prices, however, there does not always exist an upper bound on the additional price that leads to first-best welfare. The crucial effect of a lower bound on the headline price is that it prevents equilibrium demand from increasing monotonically with $\bar{p}$. If $\bar{p} \geq \bar{p}_L$, equilibrium demand is (weakly) decreasing in $\bar{p}$ as the headline price is at its lower bound. Accordingly, equilibrium demand is maximized with $\bar{p} = \bar{p}_L$, the lowest upper bound on the additional price that leads to a zero headline price. If imposing this upper bound leads to inefficiently low demand, achieving first-best welfare is not possible. Perversely, this is true if and only if prices are sufficiently transparent and consumers' demand is determined by rather sophisticated decision-making. This result demonstrates an essential interaction between the two policies analyzed: the higher the degree of price transparency, the less likely it is that additional prices can be regulated in a way that induces first-best welfare.

A closer look at the welfare-optimal policies facilitates our understanding of the interaction between the two policy variables. It turns out that whether a suitably-chosen degree of price transparency can achieve first-best welfare is not affected by a lower bound on the headline price. It is still valid that first-best welfare can only be achieved if the upper bound on the additional price is sufficiently large. Otherwise, demand is inefficiently low even with fully intransparent prices. In that case, the already described global trade-off between consumer surplus and welfare arises. Although the restriction on the headline price does not affect *if* it is possible to achieve first-best, it strongly affects *how* it is achieved as the critical value of β that induces first-best, changes and differently depends on the upper bound on the additional price. The next lemma summarizes these insights.

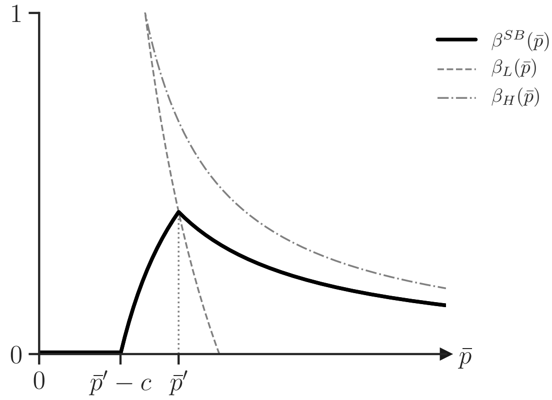
Lemma 2.2. *Let* $\bar{p}' \equiv \left(1-\gamma\right)\omega + \gamma c$.

(i) $\beta^{SB}\left(\bar{p}\right)$ *is unique for all* $\bar{p} > 0$.

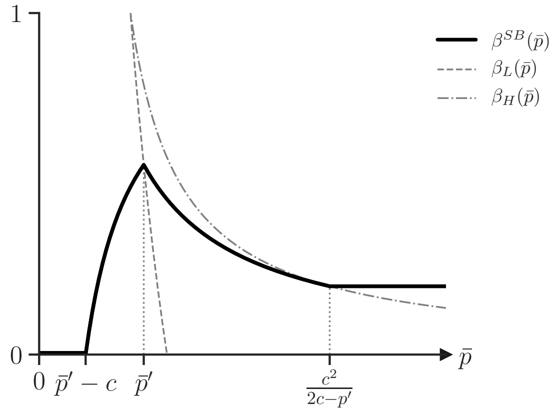
(ii) *If* $0 < \bar{p} \leq \bar{p}'$ *then* $\beta^{SB}\left(\bar{p}\right)$, *increases in* $\bar{p}$. *The relation is strict if* $\bar{p}' - c < \bar{p}$.

(iii) *If* $\bar{p} > \bar{p}'$, $\beta^{SB}\left(\bar{p}\right)$ *decreases in* $\bar{p}$. *The relation is strict unless* $\bar{p}' < 2c$ *and* $\bar{p} > \frac{c^2}{2c - \bar{p}'}$.

As for any $\bar{p} > 0$ the equilibrium quantities are monotonically decreasing in β and does not affect the first-best quantity the equilibrium welfare-maximizing degree of price transparency is unique. The two panels of Figure 2.3 illustrate how $\beta^{SB}\left(\bar{p}\right)$ depends on $\bar{p}$. The left panel depicts $\beta^{SB}(\bar{p})$ for $\bar{p}' > 2c$, whereas the right panel depicts it for the opposite case. The dotted lines $\beta_L(\bar{p})$ and $\beta_H(\bar{p})$ delineate



(a) $\bar{p}' > 2c$ with $\omega = 3$, $\bar{c} = 1$, and $\gamma = 0.3$



(b) $\bar{p}' < 2c$ with $\omega = 3$, $\bar{c} = 1$, and $\gamma = 0.6$

Figure 2.3. The equilibrium welfare-maximizing degree of price transparency (β^{SB}) as a function of the upper bound on the additional price ($\bar{p}$) for $\bar{p}' > 2c$ (2.3a) and $\bar{p}' < 2c$ (2.3b) in markets with restricted headline prices.

the different cases described in Proposition 2.3. Firms play the equilibrium (i) described in Proposition 2.3 if $\beta < \beta_L(\bar{p})$, equilibrium (ii) if $\beta_L(\bar{p}) \leq \beta \leq \beta_H(\bar{p})$, and equilibrium (iii) if $\beta > \beta_H(\bar{p})$.

First, if regulation of the additional price is sufficiently strong, i.e., $\bar{p} \leq \bar{p}'$, the results are identical to the case with unrestricted headline price. As $\beta^{SB}(\bar{p})$ is below $\beta_L(\bar{p})$, equilibrium prices are identical to the case with unrestricted headline prices. With a very low bound on the additional price, $\bar{p} \leq \bar{p}' - c$, equilibrium demand is inefficiently low irrespective of the exact degree of price transparency. Accordingly, welfare is maximized with fully intransparent prices, i.e., $\beta^{SB}\left(\bar{p}\right) = 0$. With a moderate bound on the additional price, $\bar{p}' - c < \bar{p} \leq \bar{p}'$, it is possible to induce first-best welfare with $\beta^{SB} \in (0, 1)$. If the degree of transparency is set so as to maximize welfare, the perceived total equilibrium price is decreasing in $\bar{p}$. To offset increase in demand associated with an increase in $\bar{p}$, transparency needs to be increased to maintain first-best welfare. Accordingly, $\beta^{SB}\left(\bar{p}\right)$ is strictly increasing in $\bar{p}$.

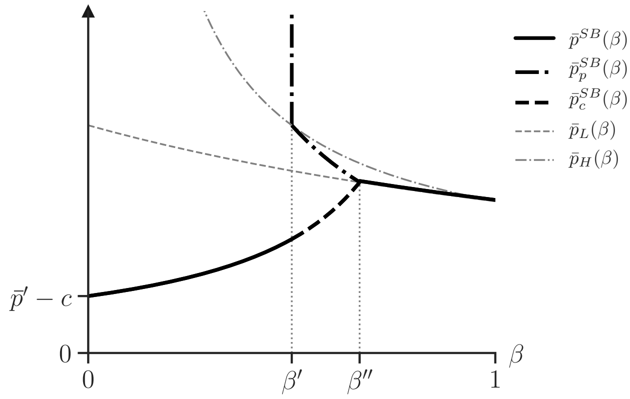
However, contrary to the case with unrestricted headline prices the welfare-maximizing degree of transparency is not monotonically increasing in $\bar{p}$. If $\bar{p} > \bar{p}'$, then $\beta^{SB}(\bar{p}) > \beta_L(\bar{p})$. Under the welfare-maximizing degree of price transparency, firms then choose a headline price at the lower bound in equilibrium. Consumers correctly anticipate that the total equilibrium price increases with $\bar{p}$. Then, equilibrium demand is decreasing in the upper bound, and price transparency needs to be reduced to maintain the demand that induces first-best welfare. The welfare-maximizing degree of price transparency, β^{SB}, is then decreasing in $\bar{p}$.

It remains to analyze the limit case, i.e., how β^{SB} evolves if $\bar{p} \to \infty$. One might be tempted to conclude that there needs to exist a critical value of $\bar{p}$ from which on β^{SB} is constant and does not change with a further relaxation of the additional prices' regulation because the third case of Proposition 2.3 is reached. This scenario is depicted in the right panel of Figure 2.3: for $\bar{p}' \leq 2c$, $\beta^{SB}(\bar{p}) > \beta_H(\bar{p})$ if $\bar{p} > \frac{c^2}{2c-\bar{p}'}$. The intuition is that with a restricted headline price, firms are not willing to offer arbitrarily high additional prices. If $\bar{p}$ is sufficiently large, firms play equilibrium (iii) described in Proposition 2.3. Accordingly, they prefer to offer an interior additional price such that the regulation of additional prices is ineffective. A further increase then does not change equilibrium prices and, hence, also the welfare-maximizing degree of price transparency, β^{SB}, does not depend on $\bar{p}$.

For $\bar{p}' > 2c$, however, β^{SB} is strictly decreasing in $\bar{p}$ even for very high values of $\bar{p}$. Contrary to the previously described case, $\beta^{SB}(\bar{p}) < \beta_H(\bar{p})$ for all values of $\bar{p}$. Even with a very large bound on the additional price, the degree of price transparency that induces first best welfare is so low that firms charge maximum additional prices in this equilibrium. It then clearly holds that β^{SB} is strictly decreasing in $\bar{p}$ and approaches zero as $\bar{p} \to \infty$.

The crucial difference is that only if $\bar{p}' < 2c$, the first-best quantity is low enough such that it can be reached in equilibrium in case (iii) of Proposition 2.3. Note that for any given $\bar{p}$ the equilibrium quantity is strictly decreasing in β, because higher price transparency makes consumers more aware of additional prices and, hence, decreases their demand. As case (iii) of Proposition 2.3 is only reached if $\beta > \beta_H\left(\bar{p}\right)$, the equilibrium quantity is rather low in this equilibrium. Consequently, first-best welfare can only be achieved in this case if the first-best quantity is sufficiently low. As $\bar{p}' < 2c$ is more likely to be fulfilled if the gains from trade, $\omega - c$, are low, and the degree of substitutability, γ, is high, the inequality ensures that the first-best quantities are low enough such that they are reached in case (iii) of Proposition 2.3.

Overall, it becomes evident that the welfare-maximizing degree of price transparency strongly depends on whether headline prices are restricted or not. With unrestricted headline prices and a large bound on the additional price, a regulator needs to implement (almost) fully transparent prices to induce efficient demand. This is not the case if headline prices are restricted. Instead, quite the opposite might



(a) $\omega = 2$, $\bar{c} = 1$, and $\gamma = 0.5$

Figure 2.4. The equilibrium welfare-maximizing upper bounds on the additional price ($\bar{p}^{SB}$, $\bar{p}_c^{SB}$, and $\bar{p}_p^{SB}$) as functions of price transparency (β) in markets with restricted headline prices.

be true as we have shown that inducing efficient demand calls for (nearly) fully intransparent additional prices if $\bar{p}' > 2c$.

We will now analyze how the welfare-optimal upper bound on the additional price, $\bar{p}^{SB}(\beta)$, depends on the degree of price transparency. Note that $\bar{p}^{SB}(\beta)$ is not necessarily unique. Whenever the welfare maximizing upper bound is not unique, we will, in a slight abuse of notation, refer to the set of welfare-maximizing upper bounds as $\bar{p}^{SB}(\beta)$ as well. To see that there might exist several welfare maximizing upper bounds recall that equilibrium demand is increasing in $\bar{p}$ only if $\bar{p} \leq \bar{p}_L$ and decreasing otherwise. This implies that if some $\bar{p}$ below $\bar{p}_L$ maximizes welfare, there might exist at least one additional upper bound above this threshold that leads to the same demand and, accordingly, maximizes welfare as well. The following lemma characterizes the instances under which the welfare-optimal upper bound on the additional price is unique or not as well as the effects of a change in the degree of price transparency on the welfare-optimal upper bound.

Lemma 2.3. *Let* $\beta' \equiv (2-\gamma) - (1-\gamma)\frac{\omega}{c}$ *and* $\beta'' \equiv \frac{c}{(1-\gamma)\omega + \gamma c}$.

(i) *If* $\beta \geq \beta''$ *then* $\bar{p}^{SB}(\beta)$ *is unique and decreasing in* β.

(ii) *If* $\beta \in (\beta', \beta'')$ *then* $\bar{p}^{SB}(\beta) = \{\bar{p}_c^{SB}(\beta), \bar{p}_p^{SB}(\beta)\}$ *and* $\bar{p}_c^{SB}(\beta) < \bar{p}_p^{SB}(\beta)$*, also* $\bar{p}_c^{SB}(\beta)$ *increases and* $\bar{p}_p^{SB}(\beta)$ *decreases in* β.

(iii) *If* $\beta = \beta'$ *then* $\bar{p}^{SB}(\beta) = \{\bar{p} \mid \bar{p} = \bar{p}_c^{SB}(\beta') \vee \bar{p} \geq \bar{p}_H(\beta')\}$.

(iv) *If* $\beta < \beta'$ *then* $\bar{p}^{SB}(\beta)$ *is unique and increasing in* β.

Figure 2.4 and Lemma 2.3 show that the optimal upper price bound is nonmonotonic in β. To understand the intuition behind this result, note that by the discussion after Proposition 2.3 the welfare-optimal upper bound on the additional price either achieves or falls short of the first-best quantity. If price transparency increases, the equilibrium quantities always decrease because consumers become more aware of the additional prices. Consequently, the welfare-optimal upper price bound has to adjust such that it counteracts the decrease in the equilibrium quantity due to an increase in price transparency. Whether an increase or a decrease of the upper bound leads to an increase in the equilibrium quantities depends on the question which of the three cases of Proposition 2.3 is at play.

Suppose first that the first-best quantities are not achievable in any of the three cases in Proposition 2.3, which is the case if $\beta > \beta''$. Intuitively, price transparency and first-best quantities are so high under these circumstances that firms cannot fool consumers into buying the first-best quantities because this would require negative headline prices in equilibrium. Therefore, the welfare optimal upper price bound is unique and given by $\bar{p}_L$ because the equilibrium quantities increase for $\bar{p} < \bar{p}_L$ and (weakly) decreases otherwise. Moreover, $\bar{p}_L$ decreases in β such that the welfare optimal upper bound decreases for $\beta > \beta''$. The reason why $\bar{p}_L$ decreases in β is that with higher price transparency, firms have to compensate consumers for higher additional prices via a stronger decrease in the headline prices such that the lower bound on the headline price is reached for lower values of $\bar{p}$.

Suppose that the first-best quantities are achievable in one of the three cases in Proposition 2.3 now, which is the case if $\beta \leq \beta''$. As argued above, the welfare-optimal upper bound is then not necessarily unique because the equilibrium quantity is a non-monotonic function of the upper bound on the additional price. When price transparency is in an intermediate range, i.e., $\beta \in \left(\beta', \beta''\right)$, then first-best quantities can be either achieved by a restrictive upper bound on the additional price, $\bar{p}_c^{SB}\left(\beta\right)$, or a less-restrictive upper bound on the additional price, $\bar{p}_p^{SB}\left(\beta\right)$. While firms charge an additional price at the respective upper bound in each of the two cases, firms charge positive headline prices in the first case and headline prices at their lower bound in the second case. In the first case, firms can still counteract an increase in the additional price by decreasing the headline price, which leads consumers to increase their demand. However, this is not feasible in the second case, where firms already charge headline prices at their lower bound such that demand decreases in the upper bound on the additional price. Consequently, the restrictive upper price bound that achieves first-best welfare increases with an increase in price transparency, while the less-restrictive upper price bound that achieves first-best welfare decreases with an increase in price transparency for intermediate degrees of price transparency.

However, if $\beta < \beta'$, i.e. the price transparency is relatively low, then the optimal upper price bound is again unique. The reason is that price transparency is so low in this case that whenever firms would charge headline prices at their lower bound, equilibrium demand would exceed the first-best quantities. Therefore, only upper-bounds that lead to positive headline prices may induce first-best welfare. Again, the logic for the restrictive upper-bound on the additional price described in the previous paragraph applies, and the welfare-optimal bound on the additional price increases in price transparency in this case.

In the knife-edge case, when $\beta = \beta'$ Lemma 2.3 and Figure 2.4 show that there exist infinitely many upper price bounds that maximize equilibrium welfare. The reason for this result is that the equilibrium quantity is a constant function of the upper price bound on the additional price for $\bar{p} > \bar{p}_H(\beta')$. When $\beta = \beta'$ the equilibrium quantity in case (iii) of Proposition 2.3 is equal to the first-best quantity and, therefore, achieves first-best welfare. However, because firms do not find it optimal to set an additional price at the upper price bound in this case, equilibrium quantities do not depend on the upper bound on the additional price. Hence, any upper-bound on the additional price that induces case (iii) of Proposition 2.3 maximizes welfare in this case. Moreover, as the equilibrium quantity is non-monotonic in $\bar{p}$, the same logic as in the case of two welfare-maximizing upper bounds applies. Hence, there also exist a restrictive upper bound, $\bar{p}_c^{SB}(\beta')$, that maximizes equilibrium welfare.

Note that whenever there exist two equilibrium-welfare maximizing upper price bounds on the additional price the different bounds lead to the same level but to a different distribution of welfare. While $\bar{p}_c^{SB}(\beta)$ is the consumer-optimal choice among the two, $\bar{p}_p^{SB}(\beta) > \bar{p}_c^{SB}(\beta)$ is producer-optimal. Intuitively, because demand is equal in both scenarios, perceived prices must be equal as well. However, actual total prices differ in the two cases. As actual total prices only redistribute welfare from consumers to firms, firms profit from higher actual total prices, while consumers profit from lower actual total prices. From the discussion of Proposition 2.3 we now that actual total prices are strictly increasing in the upper bound on the additional prices such that firms get a larger share of the same surplus under $\bar{p}_p^{SB}(\beta)$ compared to $\bar{p}_c^{SB}(\beta)$. This observation is important for regulators who want to promote consumer surplus but do not want to sacrifice welfare. In the case of intermediate ranges of price transparency, they should aim at regulating firms more strictly by introducing the tighter upper bound $\bar{p}_c^{SB}(\beta)$ instead of $\bar{p}_p^{SB}(\beta)$.

We complete the analysis with another observation of strong practical relevance for consumer protection and competition authorities.

Proposition 2.5. *If $\beta < 1$, compared to effectively unregulated partitioned pricing ($\bar{p} \geq \bar{p}_H$), an upper bound on the additional prices that equals the firms' marginal costs, i.e., $\bar{p} = c$, leaves welfare unaffected but yields a strictly higher consumer surplus.*

The perceived prices, which determine the consumers' purchasing decision, are decreasing in $\bar{p}$ up to $\bar{p} = \bar{p}_L$, then increase with $\bar{p}$ and converge to $\frac{(1-\gamma)\omega+\beta c}{2-\gamma}$ as the upper bound on the additional prices becomes arbitrarily large, which is identical to the perceived price in the equilibrium with $\bar{p} = c$. Hence, consumers demand exactly the same quantity if $\bar{p} = c$ or $\bar{p} \geq \bar{p}_H$. Importantly, this result is independent of the degree of substitutability between products, the gains from trade, and the degree of price transparency. Note that the actual total prices are globally increasing in $\bar{p}$. Therefore, consumers strictly benefit from a regulation on the additional prices tied to the firm's actual cost. As the effect of such a regulation is independent of the exact parameter constellations and the regulation itself is easy to formulate and implement, we consider it to be a highly relevant regulatory intervention designed to protect consumers and not to harm welfare.

2.4 Concluding Remarks

We study the impact of consumer naivety and partitioned pricing in the differentiated duopoly framework of Singh and Vives (1984) and examine the interplay between hard and soft interventions and how they affect consumer surplus and welfare. Full consumer protection maximizes consumer surplus but is never welfare-maximizing. This demonstrates a trade-off between consumer protection and efficiency, which is essential for evaluating regulatory interventions to increase transparency or regulate pricing strategies. We show that this trade-off's strength depends on the gains from trade and the degree of substitutability between the two goods.

However, our results show that any regulation on the additional price increases welfare and consumer surplus, compared to no regulation in the case of unrestricted headline prices. If headline prices are restricted to be non-negative, capping additional prices at the firms' marginal costs renders welfare unaffected but makes consumers better off. Moreover, we elucidate the interplay between the welfare-optimal degree of price transparency and additional price regulation. For unregulated headline prices, these policies are substitutes. If prices are relatively transparent, additional price regulation should be sufficiently loose to reach the first-best welfare level. On the contrary, when headline prices cannot be negative, whether these policies are substitutes or complements depends on whether the bound on the headline price binds in equilibrium or not.

Given our results, a natural question for policymakers arises: How should the trade-off between consumer surplus and welfare be resolved? The majority of antitrust agencies, with Canada as a notable exception (Heyer, 2006), have primarily focused on consumer surplus when evaluating potential interventions. However, this also implies that agencies only intervene when consumers' interests are at stake and do not intervene to increase efficiency. Scholars have argued that efficiency should play a more important role when agencies decide on suitable policy measures in a recent and ongoing debate (Wilson, 2019). We do not take a stance in this important discussion, but we think that our analysis is informative for both sides, as it identifies optimal policies from a consumer and from a welfare perspective. Moreover, whenever an agency decides to give weight to both principles, our results on policies that increase either both or increase consumer surplus and do not sacrifice welfare might be especially informative.

Incorporating partitioning of prices into the influential framework of Singh and Vives (1984) paves the way for several promising avenues of future research. For example, how deceptive pricing strategies influence collusive behavior is an open question. A repeated version of our model could be used to analyze the stability of tacit collusion. Furthermore, as partitioned prices affect demand, it certainly influences firms' incentives to invest in innovation. This could be investigated by adding a preceding stage where firms may invest in, e.g., cost-reducing R&D. We leave these important and interesting questions for future research.

Appendix 2.A Proofs

Proof of Proposition 2.2. In equilibrium, using the first-order conditions of the firms which yield $(1-\gamma^2)q^* = \bar{p} + p^* - c$, producer surplus is given by $\mathscr{PS}^* = 2(1-\gamma^2)q^{*2}$. Since $\frac{\partial q^*}{\partial \bar{p}} > 0$ and $\frac{\partial q^*}{\partial \beta} < 0$, we get that

$$\frac{\partial \mathscr{PS}^*}{\partial \bar{p}} = 4(1-\gamma^2)q^* \frac{\partial q^*}{\partial \bar{p}} > 0 \quad \text{and} \quad \frac{\partial \mathscr{PS}^*}{\partial \beta} = 4(1-\gamma^2)q^* \frac{\partial q^*}{\partial \beta} < 0.$$

Accordingly, no consumer protection via (i) $\beta = 0$ or (ii) $\bar{p} \to \infty$ maximizes producer surplus.

In equilibrium, again using the first-order conditions of the firms and plugging in equilibrium quantities, consumer surplus is given by $\mathscr{CS}^* = 2\omega q^* - \big(3-2\gamma\big)\big(1+\gamma\big)q^{*2} - 2q^*c$. Taking the derivatives with respect to $\bar{p}$ and β yields

$$\begin{aligned}\frac{\partial \mathscr{CS}^*}{\partial \bar{p}} &= 2\frac{\partial q^*}{\partial \bar{p}}\big(\omega - \big(3-2\gamma\big)\big(1+\gamma\big)q^* - c\big) \text{ and}\\ \frac{\partial \mathscr{CS}^*}{\partial \beta} &= 2\frac{\partial q^*}{\partial \beta}\big(\omega - \big(3-2\gamma\big)\big(1+\gamma\big)q^* - c\big).\end{aligned}$$

Hence, $\frac{\partial \mathscr{CS}^*}{\partial \bar{p}} < 0$ and $\frac{\partial \mathscr{CS}^*}{\partial \beta} > 0$ if and only if $q^* > \frac{\omega - c}{(3-2\gamma)(1+\gamma)}$, which always holds. Thus, full consumer protection via (i) $\beta = 1$ or (ii) $\bar{p} = 0$ maximizes consumer surplus.

Plugging in the equilibrium quantities into the welfare function yields equilibrium welfare. Note that equilibrium welfare is a strictly concave function in equilibrium quantities, which is maximized whenever $q^* = \frac{\omega-c}{1+\gamma} = q^{FB}$. Comparing first-best and equilibrium quantities reveals that

$$q^* = q^{FB} \iff \bar{p} = \frac{(1-\gamma)(\omega-c)}{1-\beta} \iff \beta = 1 - \frac{(1-\gamma)(\omega-c)}{\bar{p}}.$$

The welfare-maximizing upper bound $\bar{p}^{SB} = \frac{(1-\gamma)(\omega-c)}{1-\beta}$ induces first-best and is finite for every $\beta < 1$. Regarding the welfare-maximizing degree of price transparency, β^{SB}, it holds that $1 - \frac{(1-\gamma)(\omega-c)}{\bar{p}} \geq 0 \iff \bar{p} \geq (1-\gamma)(\omega-c)$. If, however, $\bar{p} < (1-\gamma)(\omega-c)$, first-best is not feasible. It then holds that $q^* < q^{FB} \forall \beta \in [0,1]$. As welfare is concave in q^* and $\frac{\partial q^*}{\partial \beta} < 0$, welfare is maximized with $\beta = 0$. It directly follows that $\beta^{SB} = \max\left\{0, 1 - \frac{(1-\gamma)(\omega-c)}{\bar{p}}\right\}$. □

Proof of Proposition 2.3. The proof of the proposition evolves in four steps. First, we argue that in any equilibrium each firm charges at least one price at its bound. Second, we show that in any equilibrium firms will choose to charge at least one price at the same bound. Third, we derive the unique equilibrium candidates for any value of the upper bound $\bar{p}$. Lastly, we prove that these candidates indeed constitute an equilibrium.

Step 1: First, note that for every firm at least one price bound is binding in equilibrium. Suppose, to the contrary, that both constraints are slack, i.e., $p_i > 0$ and $\hat{p}_i < \bar{p}$. It is possible to decrease p_i by ϵ and increase $\hat{p}_i$ by ϵ so that both constraints remain slack. This leads to a strictly higher profit: revenue per unit sold remains constant, but demand increases strictly as consumers are less sensitive to an increase in the additional price than a corresponding decrease in the headline price. Accordingly, whenever both constraints are slack, there exists a profitable deviation. It follows that the best-response function of firm i either involves the highest possible additional price, the lowest possible headline price, or both.

Step 2: (i) Suppose firm i chooses the lowest headline price, i.e. $p_i = 0$. The corresponding optimal additional price $\hat{p}_i^*(p_j, \hat{p}_j)$ solves the maximization problem

$$\max_{\hat{p}_i} \pi_i(p, \hat{p}) \;=\; q_i\left(p, \hat{p}\right) \cdot \left(\hat{p}_i - c\right) \;\; s.t.\; \hat{p}_i \;\leq\; \bar{p}. \tag{2.A.1}$$

The first-order condition is given by

$$\frac{-\beta}{1-\gamma^2}\left(\hat{p}_i - c\right) + \frac{1}{1-\gamma^2}\left[(1-\gamma)\omega - (p_i + \beta\hat{p}_i) + \gamma(p_j + \beta\hat{p}_j)\right] \;=\; 0.$$

As π_i is strictly concave in $\hat{p}_i$, it follows that

$$\hat{p}_i^*(p_j, \hat{p}_j) \;=\; \min\left\{\frac{\left(1-\gamma\right)\omega}{2\beta} + \frac{\gamma}{2}\hat{p}_j + \frac{\gamma}{2\beta}p_j + \frac{c}{2}, \bar{p}\right\}. \tag{2.A.2}$$

(ii) Suppose firm i chooses the highest possible additional price, i.e. $\hat{p}_i = \bar{p}$. The corresponding optimal headline price $p_i^*(p_j, \hat{p}_j)$ solves the maximization problem

$$\max_{p_i} \pi_i(p, \hat{p}) \;=\; q_i\left(p, \hat{p}\right) \cdot \left(p_i + \bar{p} - c\right) \;\; s.t.\; p_i \;\geq\; 0. \tag{2.A.3}$$

The first-order condition is given by

$$-\frac{1}{1-\gamma^2}\left(p_i + \bar{p} - c\right) + \frac{1}{1-\gamma^2}\left[(1-\gamma)\omega - (p_i + \beta\hat{p}_i) + \gamma(p_j + \beta\hat{p}_j)\right] \;=\; 0.$$

As π_i is strictly concave in p_i, it follows that

$$p_i^*(p_j, \hat{p}_j) \;=\; \max\left\{0, \frac{\left(1-\gamma\right)\omega}{2} - \frac{1+\beta}{2}\bar{p} + \frac{\gamma}{2}\left(p_j + \beta\hat{p}_j\right) + \frac{c}{2}\right\}. \tag{2.A.4}$$

Lemma 2.4. *If $\bar{p} > \bar{p}_L$ then in any equilibrium either $p_i^* = p_j^* = 0$ or $p_i^*; p_j^* > 0$.*

Proof. Suppose to the contrary that $\bar{p} > \bar{p}_L$ and there exists an equilibrium with $p_i^* > 0$ and $p_j^* = 0$. This implies

$$p_i^* \;=\; \frac{\left(1-\gamma\right)\omega}{2} - \frac{1+\beta}{2}\bar{p} + \frac{\gamma}{2}\beta\hat{p}_j^* + \frac{c}{2}.$$

As $\hat{p}_j^* \leq \bar{p}$, $p_i^* \leq \frac{(1-\gamma)\omega}{2} - \frac{1+\beta}{2}\bar{p} + \frac{\gamma}{2}\beta\bar{p} + \frac{c}{2}$. Accordingly, $p_i^* > 0$ requires $\frac{(1-\gamma)\omega}{2} - \frac{1+\beta}{2}\bar{p} + \frac{\gamma}{2}\beta\bar{p} + \frac{c}{2} > 0 \;\Rightarrow\; \bar{p} < \bar{p}_L$, a contradiction. □

Lemma 2.5. *If $\bar{p} \leq \bar{p}_L$ then in any equilibrium either $\hat{p}_i^* = \hat{p}_j^* = \bar{p}$ or $\hat{p}_i^*, \hat{p}_j^* < \bar{p}$.*

Proof. Suppose to the contrary that $\bar{p} \leq \bar{p}_L$ and there exists an equilibrium with $\hat{p}_j^* = \bar{p}$ and $\hat{p}_i^* < \bar{p}$. It then needs to hold that

$$\hat{p}_i^* = \frac{(1-\gamma)\omega}{2\beta} + \frac{\gamma}{2}\bar{p} + \frac{\gamma}{2\beta}p_j^* + \frac{c}{2} < \bar{p}.$$

Accordingly, $\hat{p}_i^* < \bar{p}$ requires $\frac{(1-\gamma)\omega}{2\beta} + \frac{\gamma}{2}\bar{p} + \frac{c}{2} < \bar{p} \iff \bar{p} > \frac{(1-\gamma)\omega+\beta c}{\beta(2-\gamma)}$. This, however, contradicts the assumption $\bar{p} \leq \bar{p}_L$, as $\frac{(1-\gamma)\omega+\beta c}{\beta(2-\gamma)} > \frac{(1-\gamma)\omega+c}{1+\beta(1-\gamma)} \iff \omega > \beta c$, which always holds. □

Step 3: From the two lemmas and the observation that at least one bound is binding it directly follows that (i) $p_i^* = p_j^* = 0$, (ii) $\hat{p}_i^* = \hat{p}_j^* = \bar{p}$, or (iii) both.

(i) Suppose $p_i^* = p_j^* = 0$. Firm $i \in \{1,2\}$ solves the maximization problem in (2.A.1) given $p_j = 0$, such that the best-response function of firm i is given by

$$\hat{p}_i^*(\hat{p}_j) = \min\left\{\frac{(1-\gamma)\omega}{2\beta} + \frac{\gamma}{2}\hat{p}_j + \frac{c}{2}, \bar{p}\right\}.$$

Note that $\hat{p}_i^* = \bar{p}$ if and only if $\frac{(1-\gamma)\omega}{2\beta} + \frac{\gamma}{2}\hat{p}_j^* + \frac{\gamma}{2\beta}p_j^* + \frac{c}{2} \geq \bar{p}$. Now suppose $\hat{p}_j^* = \bar{p}$ and $\hat{p}_i^* < \bar{p}$. This implies that $\hat{p}_i^* = \frac{(1-\gamma)\omega}{2\beta} + \frac{\gamma}{2}\bar{p} + \frac{c}{2} < \bar{p}$ and $\hat{p}_j^* = \frac{(1-\gamma)\omega}{2\beta} + \frac{\gamma}{2}\hat{p}_i^* + \frac{c}{2} > \bar{p}$ such that $\frac{(1-\gamma)\omega}{2\beta} + \frac{\gamma}{2}\hat{p}_i^* + \frac{c}{2} > \frac{(1-\gamma)\omega}{2\beta} + \frac{\gamma}{2}\bar{p} + \frac{c}{2}$, which yields a contradiction. Hence, only symmetric combinations $\hat{p}_1^* = \hat{p}_2^* = \hat{p}^*$ can constitute mutual best responses. There exists exactly one such combination with $p^* = \bar{p}$ if and only if $\hat{p}_i^*(\bar{p}) = \bar{p}$, i.e., $\frac{(1-\gamma)\omega}{2\beta} + \frac{\gamma}{2}\bar{p} + \frac{c}{2} \geq \bar{p} \iff \bar{p} \leq \frac{(1-\gamma)\omega+\beta c}{(2-\gamma)\beta} = \bar{p}_H$. Otherwise, the intersection of the best-response functions is determined by $\hat{p} = \frac{(1-\gamma)\omega}{2\beta} + \frac{\gamma}{2}\hat{p} + \frac{c}{2} \iff \hat{p} = \frac{(1-\gamma)\omega+\beta c}{(2-\gamma)\beta}$.

(ii) Now suppose $\hat{p}_i^* = \hat{p}_j^* = \bar{p}$. Firm $i \in \{1,2\}$ solves the maximization problem in (2.A.3) given $\hat{p}_j = \bar{p}$, such that the best-response function of firm i is given by

$$p_i^*(p_j) = \max\left\{0, \frac{(1-\gamma)\omega}{2} - \frac{1+\beta}{2}\bar{p} + \frac{\gamma}{2}\left(p_j + \beta\bar{p}\right) + \frac{c}{2}\right\}.$$

Note that $p_i^* = 0$ if and only if $\frac{(1-\gamma)\omega}{2} - \frac{(1+\beta)}{2}\bar{p} + \frac{\gamma}{2}\left(p_j^* + \beta\hat{p}_j^*\right) + \frac{c}{2} \leq 0$. Now suppoes $p_j^* = 0$ and $p_i^* > 0$. This implies that $p_i^* = \frac{(1-\gamma)\omega}{2} - \frac{(1+\beta)}{2}\bar{p} + \frac{\gamma}{2}\beta\bar{p} + \frac{c}{2} > 0$ and $p_j^* = \frac{(1-\gamma)\omega}{2} - \frac{(1+\beta)}{2}\bar{p} + \frac{\gamma}{2}\left(p_i^* + \beta\bar{p}\right) + \frac{c}{2} < 0$, such that $\frac{(1-\gamma)\omega}{2} - \frac{(1+\beta)}{2}\bar{p} + \frac{\gamma}{2}\left(p_i^* + \beta\bar{p}\right) + \frac{c}{2} < \frac{(1-\gamma)\omega}{2} - \frac{(1+\beta)}{2}\bar{p} + \frac{\gamma}{2}\beta\bar{p} + \frac{c}{2}$, which yields a contradiction. Therefore, again, only symmetric combinations $p_i^* = p_j^* = p^*$ can constitute mutual best responses and there exists exactly one such combination with $p^* = 0$ if and only if $p_i^*(0) = 0$, i.e., $\frac{(1-\gamma)\omega}{2} - \frac{1+\beta}{2}\bar{p} + \frac{\gamma}{2}\beta\bar{p} + \frac{c}{2} \leq 0 \iff \bar{p} \geq \frac{(1-\gamma)\omega+c}{1+(1-\gamma)\beta} = \bar{p}_L$. Otherwise, the intersection of the best-response functions is determined by $p = \frac{(1-\gamma)\omega}{2} - \frac{1+\beta}{2}\bar{p} + \frac{\gamma}{2}\left(p + \beta\bar{p}\right) + \frac{c}{2} \Leftrightarrow p = \frac{(1-\gamma)\omega + c - \left[1+(1-\gamma)\beta\right]\bar{p}}{(2-\gamma)}$.

For $\bar{p}_L \leq \bar{p} \leq \bar{p}_H$, the equilibrium candidate is $(p,\hat{p}) = (0,\bar{p})$ in case (i) as well as in case (ii) and, hence, unique. If $\bar{p} < \bar{p}_L$, the candidate $(p,\hat{p}) = (0,\bar{p})$ in case (i) was feasible also in case (ii) but not selected. Accordingly, the candidate from case (ii), $(p,\hat{p}) = \left(\frac{(1-\gamma)\omega + c - \left[1+(1-\gamma)\beta\right]\bar{p}}{(2-\gamma)}, \bar{p}\right)$, is the unique equilibrium candidate. Likewise, for $\bar{p} > \bar{p}_H$, the candidate $(p,\hat{p}) = (0,\bar{p})$ in case (ii) was feasible also in case (i) but not selected. It follows that the candidate from case (i), $(p,\hat{p}) = \left(0, \frac{(1-\gamma)\omega + \beta c}{(2-\gamma)\beta}\right)$, is the unique equilibrium candidate. Hence, for any $\bar{p}$ and each of the two cases (i) $p^* = 0$ and (ii) $\hat{p}^* = \bar{p}$, we have derived the unique equilibrium candidate.

Step 4: Finally, we need to check that no profitable deviation from the equilibrium candidates exists. From Step 1, we know that the most profitable deviation $(p_i^d, \hat{p}_i^d)$ entails either $p_i^d = 0$ or $\hat{p}_i^d = \bar{p}$.

If $\bar{p} < \bar{p}_L$, the equilibrium candidate $(p,\hat{p}) = \left(\frac{(1-\gamma)\omega + c - \left[1+(1-\gamma)\beta\right]\bar{p}}{(2-\gamma)}, \bar{p}\right)$ was derived by maximizing π_i over p_i given that $\hat{p}_i = \bar{p}$. Accordingly, no profitable deviation with $\hat{p}_i^d = \bar{p}$ can exist. Therefore, a profitable deviation needs to entail $p_i^d = 0$ with additional price being given by (2.A.2). As the equilibrium candidate entails $p_j > 0$ and $\hat{p}_j = \bar{p}$, it holds that $\frac{(1-\gamma)\omega}{2\beta} + \frac{\gamma}{2}\bar{p} + \frac{\gamma}{2\beta}p_j + \frac{c}{2} > \frac{(1-\gamma)\omega}{2\beta} + \frac{\gamma}{2}\bar{p} + \frac{c}{2} > \bar{p}$ if $\bar{p} < \bar{p}_L$, which implies $\hat{p}_i^d = \bar{p}$. As we have shown that no profitable deviation with $\hat{p}_i^d = \bar{p}$ exists, we can conclude that no profitable deviation exists.

If $\bar{p}_L \leq \bar{p} \leq \bar{p}_H$, the unique equilibrium candidate is given by $(p,\hat{p}) = (0,\bar{p})$. The derivation has shown that no firm has an incentive to deviate (i) to an interior headline price given $\hat{p} = \bar{p}$ and (ii) to an interior additional price given $p = 0$. Accordingly, no profitable deviation exists.

If $\bar{p} > \bar{p}_H$, the equilibrium candidate $(p,\hat{p}) = \left(0, \frac{(1-\gamma)\omega + \beta c}{(2-\gamma)\beta}\right)$ was derived by maximizing π_i over $\hat{p}_i$ given $p_i = 0$. Accordingly, no profitable deviation with $p_i = 0$ exists. Therefore, a profitable deviation needs to entail $\hat{p}_i^d = \bar{p}$ with the headline price being given by (2.A.4). As the equilibrium candidate entails $p_j = 0$ and $\hat{p}_j < \bar{p}$, it holds that $\frac{(1-\gamma)\omega}{2} - \frac{1+\beta}{2}\bar{p} + \frac{\gamma}{2}\beta\hat{p}_j + \frac{c}{2} < \frac{(1-\gamma)\omega}{2} - \frac{1+\beta}{2}\bar{p} + \frac{\gamma}{2}\beta\bar{p} + \frac{c}{2} < 0$ if $\bar{p} > \bar{p}_H$, which implies $p_i^d = 0$. As we have shown that no profitable deviation with $p_i^d = 0$ exists, we can conclude that no profitable deviation exists. □

Proof of Proposition 2.4: First, we will show that full consumer protection maximizes consumer surplus. First, note that case (i), i.e. $\bar{p} < \bar{p}_L$ in Proposition 2.3 yields the same prices and quantities as Proposition 2.1. Therefore, the comparative statics remain unchanged such that $\frac{\partial \mathscr{CS}^*}{\partial \beta} > 0$ for $\bar{p} < \bar{p}_L$.

Second, consider case (ii), i.e. $\bar{p} \in [\bar{p}_L, \bar{p}_H]$, now. Plugging the equilibrium prices and quantities into the consumer surplus results in

$$\mathscr{CS}^* = 2\omega q^* - \left(1+\gamma\right)q^{*2} - 2q^*\bar{p} = \frac{\omega^2 - 2\omega\bar{p} + \beta\left(2-\beta\right)\bar{p}^2}{1+\gamma}.$$

Taking the derivative with respect to β yields $\frac{\mathscr{CS}^*}{\partial\beta} = \frac{2(1-\beta)\bar{p}^2}{1+\gamma} > 0$ such that $\frac{\mathscr{CS}^*}{\partial\beta} > 0$ for $\bar{p} \in [\bar{p}_L, \bar{p}_H]$.

Consider case (iii), i.e. $\bar{p} > \bar{p}_H$, now. Plugging equilibrium quantities into consumer surplus results in

$$\mathscr{CS}^* = 2\omega q^* - \left(1+\gamma\right)q^{*2} - 2q^*\hat{p}^* = \frac{\omega^2 - 2\omega\hat{p}^* + \beta\left(2-\beta\right)\hat{p}^{*2}}{1+\gamma}.$$

Taking the derivative with respect to β yields

$$\frac{\partial\mathscr{CS}^*}{\partial\beta} = \frac{-2\omega\frac{\partial\hat{p}^*}{\partial\beta} + 2\beta\left(2-\beta\right)\hat{p}^*\frac{\partial\hat{p}^*}{\partial\beta} + 2\left(1-\beta\right)\hat{p}^{*2}}{1+\gamma}.$$

Inserting $\hat{p}^*$ and $\frac{\partial\hat{p}^*}{\partial\beta}$ into the derivative and rearranging terms yields $\frac{\partial\mathscr{CS}^*}{\partial\beta} > 0 \iff (1-\gamma)\left(\omega^2 - \beta^2\omega c\right) + \left(1-\beta\right)\beta^2c^2 > 0$, which is always the case since $c < \omega$. Therefore, we can conclude that $\frac{\partial\mathscr{CS}^*}{\partial\beta} > 0$ for $\bar{p} > \bar{p}_H$. We have shown that $\frac{\partial\mathscr{CS}^*}{\partial\beta} > 0$ in all three cases. As consumer surplus is a continuous function of β, we can conclude that it is monotonically increasing in β. It follows that for all values of $\bar{p}$, consumer surplus is maximized at $\beta = 1$.

To show that consumer surplus is maximized with $\bar{p} = 0$, we will derive the upper bound that maximizes consumer surplus in each of the cases in Proposition 2.3, and then compare these candidates against each other.

Note that for $\bar{p} < \bar{p}_L$, i.e., case (i) of Proposition 2.3, equilibrium prices and quantities are equal to equilibrium prices and quantities in Proposition 2.1. Hence, it remains to hold that $\frac{\partial\mathscr{CS}^*}{\partial\bar{p}} < 0$ for $\bar{p} < \bar{p}_L$. Therefore, the only candidate for the consumer optimal upper bound on the interval $[0, \bar{p}_L]$ is given by $\bar{p} = 0$.

Consider case (ii) now. In equilibrium consumer surplus for $\bar{p} \in [\bar{p}_L, \bar{p}_H]$ equals

$$\mathscr{CS}^* = 2\omega q^* - \left(1+\gamma\right)q^{*2} - 2q^*\bar{p} = \frac{\omega^2 - 2\omega\bar{p} + \beta\left(2-\beta\right)\bar{p}^2}{1+\gamma}.$$

Taking the derivative with respect to $\bar{p}$ yields $\frac{\mathscr{CS}^*}{\partial\bar{p}} = \frac{-2\omega + 2\beta\left(2-\beta\right)\bar{p}}{1+\gamma}$ such that $\frac{\mathscr{CS}^*}{\partial\bar{p}} < 0$ if and only if $\bar{p} < \frac{\omega}{\beta\left(2-\beta\right)}$. Depending on parameter constellations, it may hold that $\bar{p}_L < \frac{\omega}{\beta\left(2-\beta\right)} < \bar{p}_H$. Consequently, $\bar{p}_L$ and $\bar{p}_H$ are the two potential candidates for the consumer surplus optimal upper bound in the interval $[\bar{p}_L, \bar{p}_H]$.

Lastly, consider case (iii). As neither equilibrium quantities nor prices are affected by a change in $\bar{p}$ in this case, equilibrium consumer surplus is not affected by a change in $\bar{p}$ either. Hence, the optimal upper bound from a consumer surplus perspective is given by $\bar{p}_H$.

As we have identified a unique candidate for each of the three cases, we may now compare the resulting consumer surplus $\mathscr{CS}^*\left(\bar{p}\right)$. As consumer surplus is a continuous function of $\bar{p}$ and strictly decreasing on the interval $\left[0, \bar{p}_L\right)$, $\mathscr{CS}^*(0) > \mathscr{CS}^*\left(\bar{p}_L\right)$ follows. To prove the claim of the proposition, it suffices to show $\mathscr{CS}^*(0) > \mathscr{CS}^*(\bar{p}_H)$. Note that $c < \bar{p}_L$ and $q^*(\bar{p} = c) = q^*(\bar{p} = \bar{p}_H)$. The perceived total prices are identical for these two upper bounds such that consumers demand identical quantities. From the facts that (i) perceived total prices $p^* + \beta\hat{p}^*$ are identical and (ii) the equilibrium additional price is strictly larger with $\bar{p} = \bar{p}_H$ than with $\bar{p} = c$, it follows that actual total prices $p^* + \hat{p}^*$ are higher with $\bar{p} = \bar{p}_H$ than with $\bar{p} = c$. To see this, consider two tuple of prices $(p', \hat{p}')$ and $(p'', \hat{p}'')$ with $p' + \beta\hat{p}' = p'' + \beta\hat{p}''$ and $\hat{p}' < \hat{p}''$. Then $p' + \hat{p}' = p'' + \beta\hat{p}'' + (1-\beta)\hat{p}' < p'' + \hat{p}''$. As consumers end up paying less for the same quantities with $\bar{p} = c$ than with $\bar{p} = \bar{p}_H$, it follows that $\mathscr{CS}^*(0) > \mathscr{CS}^*(c) > \mathscr{CS}^*(\bar{p}_H)$. As we have shown that consumer surplus is maximized with $\beta = 1$ as well as with $\bar{p} = 0$, we can conclude that full consumer protection maximizes consumer surplus.

Next, we will show that no consumer protection maximizes producer surplus. First, by the same argument as before, the comparative statics result from Proposition 2.2 carry over for $\bar{p} < \bar{p}_L$ such that $\frac{\partial \mathscr{PS}^*}{\partial \beta} < 0$.

Plugging equilibrium prices and quantities of case (ii) into the equation for producer surplus results in

$$\mathscr{PS}^* = 2\frac{\omega - \beta\bar{p}}{1+\gamma} \cdot \left(\bar{p} - c\right).$$

Taking the derivative with respect to β yields $\frac{\partial \mathscr{PS}^*}{\partial \beta} = \frac{-2\bar{p}(\bar{p}-c)}{1+\gamma} < 0$ such that $\frac{\mathscr{PS}^*}{\partial \beta} < 0$ for $\bar{p} \in [\bar{p}_L, \bar{p}_H]$.

Using the firms' first-order conditions, producer surplus in case (iii) is given by

$$\mathscr{PS}^* = \sum_{i=1}^{2} \pi_i(p^*, \hat{p}^*) = \frac{2(1-\gamma^2)}{\beta} q^{*2},$$

From $\frac{\partial q^*}{\partial \beta} < 0$ it follows that $\frac{\partial \mathscr{PS}^*}{\partial \beta} = -\frac{2(1-\gamma^2)}{\beta^2} q^{*2} + 4q^* \frac{\partial q^*}{\partial \beta} \frac{1-\gamma^2}{\beta} < 0$. Therefore, we can conclude that $\frac{\partial \mathscr{PS}^*}{\partial \beta} < 0$ for $\bar{p} > \bar{p}_H$. As $\mathscr{PS}^*$ is a continuous function of β, it follows that $\mathscr{PS}^*$ is monotonically decreasing in β and maximized with $\beta = 0$.

Again, the comparative statics result from Proposition 2.2 carry over for $\bar{p} < \bar{p}_L$ such that $\frac{\partial \mathscr{PS}^*}{\partial \bar{p}} > 0$.

Plugging equilibrium prices and quantities of case (ii) into the equation for producer surplus results in

$$\mathscr{PS}^* = 2\frac{\omega - \beta\bar{p}}{1+\gamma} \cdot \left(\bar{p} - c\right).$$

Taking the derivative with respect to $\bar{p}$ yields $\frac{\partial \mathscr{PS}^*}{\partial \bar{p}} = 2\left(\frac{\omega - 2\beta\bar{p} + \beta c}{1+\gamma}\right) > 0 \iff \bar{p} < \frac{\omega+\beta c}{2\beta}$. Note that $\frac{\omega+\beta c}{2\beta} > \bar{p}_H$, such that $\frac{\partial \mathscr{PS}^*}{\partial \bar{p}} > 0$ for all $\bar{p} \in [\bar{p}_L, \bar{p}_H]$.

Consider the third case now. Since prices and quantities are not affected by a change in the upper bound of additional prices, producer surplus is unaffected by an increase of $\bar{p}$ whenever $\bar{p} > \bar{p}_H$. As $\mathscr{PS}^*$ is a continuous function of $\bar{p}$, it follows that $\mathscr{P}\mathscr{S}^*$ is monotonically increasing in $\bar{p}$ and maximized with $\bar{p} \to \infty$. As we have shown that producer surplus is maximized with $\beta = 0$ as well as with $\bar{p} \to \infty$, we can conclude that no consumer protection maximizes producer surplus.

Finally, we will derive the welfare results. Equilibrium welfare $\mathscr{W}^*$ is continuous in β as it is continuous in equilibrium quantities, which in turn are continuous in β. Regarding equilibrium quantities $q^*(\beta)$, it holds that $q^*(1) = \frac{\omega-c}{(1+\gamma)(2-\gamma)} < q^{FB}$. Note that $\frac{\partial q^*}{\partial \beta} < 0$ for all $\bar{p}$ at which q^* is differentiable. Hence, q^* is a monotonically decreasing function in β for $\beta \in [0,1]$. Accordingly, there exists a $\beta^{SB} \in (0,1)$ such that $q^*\left(\beta^{SB}\right) = q^{FB}$ if and only if $q^*(0) > q^{FB}$. It holds that $q^*(0) = \min\left\{\frac{\omega-c+\bar{p}}{(1+\gamma)(2-\gamma)}, \frac{\omega}{1+\gamma}\right\} > q^{FB} \iff \bar{p} > (1-\gamma)(\omega-c)$.

Equilibrium welfare $\mathscr{W}^*$ is continuous in $\bar{p}$ as it is continuous in equilibrium quantities, which in turn are continuous in $\bar{p}$. Regarding equilibrium quantities $q^*(\bar{p})$, it holds that $q^*(0) = \frac{\omega-c}{(1+\gamma)(2-\gamma)} < q^{FB}$. Furthermore $\frac{\partial q^*}{\partial \bar{p}} > 0 \iff \bar{p} < \bar{p}_L$. Hence, there exists at least one $\bar{p}^{SB} > 0$ with $q^*\left(\bar{p}^{SB}\right) = q^{FB}$ if and only if $q^*(\bar{p}_L) \geq q^{FB} \iff \beta \leq \frac{c}{(1-\gamma)\omega+\gamma c}$. □

Proof of Lemma 2.2. First note that by Proposition 2.4, regarding the equilibrium quantities $q^*(\beta)$ it holds that $q^*(0) < q^{FB} \iff \bar{p} \leq (1-\gamma)(\omega-c)$. As q^* is monotonically decreasing in β and equilibrium welfare is concave in quantities, it follows that $\beta^{SB}(\bar{p}) = 0$ if $\bar{p} \leq (1-\gamma)(\omega-c)$. Hence, $\frac{\partial \beta^{SB}(\bar{p})}{\partial \bar{p}} = 0$ for $\bar{p} \leq (1-\gamma)(\omega-c)$.

Now, suppose $\bar{p} > (1-\gamma)(\omega-c)$. In this case, Proposition 2.4 reveals that there exists a unique welfare-maximizing degree of price transparency $\beta^{SB}(\bar{p})$ that induces first-best.

If $\bar{p} < \bar{p}_L$, or equivalently, $\beta < \frac{(1-\gamma)\omega+c-\bar{p}}{(1-\gamma)\bar{p}} \equiv \beta_L$, $\beta^{SB}(\bar{p})$ is implicitly defined by

$$\frac{\omega-c}{1+\gamma} = \frac{\omega-c+\left(1-\beta_I^{SB}(\bar{p})\right)\bar{p}}{(1+\gamma)(2-\gamma)} \iff \beta_I^{SB}(\bar{p}) = 1-\frac{(1-\gamma)(\omega-c)}{\bar{p}},$$

which is feasible if and only if $\beta_I^{SB} < \beta_L \iff \bar{p} < (1-\gamma)\omega+\gamma c$.

If $\bar{p}_L \leq \bar{p} \leq \bar{p}_H$, or equivalently, $\beta_L \leq \beta \leq \frac{(1-\gamma)\omega}{(2-\gamma)\bar{p}-c} \equiv \beta_H$, $\beta^{SB}(\bar{p})$ is implicitly defined by

$$\frac{\omega-c}{1+\gamma} = \frac{\omega-\beta_{II}^{SB}(\bar{p})\bar{p}}{1+\gamma} \iff \beta_{II}^{SB}(\bar{p}) = \frac{c}{\bar{p}}.$$

It needs to hold that $\beta_L \leq \beta_{II}^{SB}(\bar{p}) \leq \beta_H$, which is true if and only if $\bar{p} \geq (1-\gamma)\omega+\gamma c$ and $\bar{p}\left[(2-\gamma)c-(1-\gamma)\omega\right] \leq c^2$. Note that the last inequality holds if either $\frac{\omega}{c} \geq \frac{2-\gamma}{1-\gamma}$ or $\frac{\omega}{c} < \frac{2-\gamma}{1-\gamma}$ and $\bar{p} \leq \frac{c^2}{(2-\gamma)c-(1-\gamma)\omega}$.

If $\bar{p} > \bar{p}_H$, or equivalently $\beta > \beta_H$, $\beta^{SB}(\bar{p})$ is implicitly defined by

$$\frac{\omega-c}{1+\gamma} = \frac{\omega-\beta_{III}^{SB}(\bar{p})c}{(1+\gamma)(2-\gamma)} \iff \beta_{III}^{SB}(\bar{p}) = (2-\gamma)-(1-\gamma)\frac{\omega}{c}.$$

It needs to hold that $\beta_{III}^{SB}\left(\bar{p}\right) > \beta_H$, which is true if and only if $\frac{\omega}{c} < \frac{2-\gamma}{1-\gamma}$ and $\bar{p} > \frac{c^2}{(2-\gamma)c-(1-\gamma)\omega}$.

We have shown that $\beta^{SB}(\bar{p}) = \beta_I^{SB}(\bar{p})$ such that $\frac{\partial \beta^{SB}(\bar{p})}{\partial \bar{p}} > 0$ if $\bar{p} < \left(1-\gamma\right)\omega + \gamma c$. Furthermore, $\beta^{SB}(\bar{p}) = \beta_{II}^{SB}(\bar{p})$ such that $\frac{\partial \beta^{SB}(\bar{p})}{\partial \bar{p}} < 0$ if $\bar{p} \geq \left(1-\gamma\right)\omega + \gamma c$ and $\bar{p}\left[\left(2-\gamma\right)c - \left(1-\gamma\right)\omega\right] \leq c^2$. Finally, $\beta^{SB}(\bar{p}) = \beta_{III}^{SB}(\bar{p})$ such that $\frac{\partial \beta^{SB}(\bar{p})}{\partial \bar{p}} = 0$ if $\frac{\omega}{c} < \frac{2-\gamma}{1-\gamma}$ and $\bar{p} > \frac{c^2}{(2-\gamma)c-(1-\gamma)\omega}$. □

Proof of Lemma 2.3. The proof of Proposition 2.4 has revealed that $q^*(\bar{p}) < q^{FB}$ for all values of $\bar{p}$ if $\beta > \frac{c}{(1-\gamma)\omega+\gamma c} \equiv \beta''$ and that equilibrium quantities are maximized if $\bar{p} = \bar{p}_L$. As equilibrium welfare is concave in $q^*\left(\bar{p}\right)$, $\bar{p}^{SB}\left(\beta\right) = \bar{p}_L$. Hence, $\frac{\partial \bar{p}^{SB}(\beta)}{\partial \beta} = \frac{\partial \bar{p}_L}{\partial \beta} < 0$ if $\beta \geq \beta''$.

If $\beta < \beta''$, it follows from the proof of Proposition 2.4 that there exists at least one $\bar{p}$ that induces first-best welfare.

The following four observations will help to prove the remaining claims of the lemma:

(i) $\frac{\partial q^*}{\partial \bar{p}} > 0 \Leftrightarrow \bar{p} < \bar{p}_L$, $\frac{\partial q^*}{\partial \bar{p}} < 0 \Leftrightarrow \bar{p}_L < \bar{p} < \bar{p}_H$, and $\frac{\partial q^*}{\partial \bar{p}} = 0 \Leftrightarrow \bar{p} > \bar{p}_H$.

(ii) $q^*(\bar{p} = 0) = \frac{\omega-c}{(1+\gamma)(2-\gamma)} < q^{FB}$.

(iii) q^* strictly decreases in β for $\bar{p} > 0$.

(iv) $q^*(c) = q^*(\bar{p}_H)\ \forall \beta$.

(v) $q^*(c) = q^{FB} \Leftrightarrow \beta = (2-\gamma) - (1-\gamma)\frac{\omega}{c} \equiv \beta' < \beta''$.

First, by observation (i) there exist either one, two, or infinitely many $\bar{p}^{SB}(\beta)$ that induce first-best welfare if $\beta < \beta''$, as equilibrium quantities have to be equal to the first-best quantities to induce first-best welfare.

Second, let $\beta \in [0, \beta')$. Due to observations (ii) and (v) we know that $q^*(0) < q^{FB} < q^*(c)$ in this case. By observation (i) and the fact that $c < \bar{p}_L$ this implies that there can only be one intersection between the first-best quantities and the equilibrium quantities. The resulting unique welfare-maximizing upper bound is implicitly defined by

$$\frac{\omega - c + \left(1-\beta\right)\bar{p}^{SB}\left(\beta\right)}{\left(1+\gamma\right)\left(2-\gamma\right)} = \frac{\omega - c}{1+\gamma} \quad \Leftrightarrow \quad \bar{p}^{SB}\left(\beta\right) = \frac{\left(1-\gamma\right)(\omega - c)}{\left(1-\beta\right)}.$$

Its derivative is given by

$$\frac{\partial \bar{p}^{SB}}{\partial \beta} = \frac{\left(1-\gamma\right)(\omega - c)}{\left(1-\beta\right)^2} > 0.$$

Third, let $\beta \in (\beta', \beta'')$. Due to observations (iii), (iv), and (v) it holds that $q^*(c) < q^{FB} < q^*(\bar{p}_L)$ and, equivalently, $q^*(\bar{p}_H) < q^{FB} < q^*(\bar{p}_L)$. Then, observation (i) implies that there are exactly two welfare-maximizing upper bounds, one in case (i) and one in case (ii) of Proposition 2.3. Denote the welfare-maximizing upper bound with the lower (higher) additional price as $\bar{p}_c^{SB}(\beta)$ ($\bar{p}_p^{SB}(\beta)$). Hence, $\bar{p}_c^{SB}(\beta)$ is implicitly defined by

$$\frac{\omega - c + (1-\beta)\bar{p}_c^{SB}(\beta)}{(1+\gamma)(2-\gamma)} = \frac{\omega - c}{1+\gamma} \quad \Leftrightarrow \quad \bar{p}_c^{SB}(\beta) = \frac{(1-\gamma)(\omega - c)}{(1-\beta)}$$

such that

$$\frac{\partial \bar{p}_c^{SB}}{\partial \beta} = \frac{(1-\gamma)(\omega - c)}{(1-\beta)^2} > 0.$$

Similarly, $\bar{p}_p^{SB}(\beta)$ is implicitly defined by

$$\frac{\omega - \beta \bar{p}_p^{SB}(\beta)}{(1+\gamma)} = \frac{\omega - c}{1+\gamma} \qquad \Leftrightarrow \qquad \bar{p}_p^{SB}(\beta) = \frac{c}{\beta}.$$

Its derivative is given by

$$\frac{\partial \bar{p}^{SB}}{\partial \beta} = -\frac{c}{\beta^2} < 0.$$

Fourth, consider the knife-edge case $\beta = \beta'$ now. Due to observation (v), it holds that $q^{FB} = q^*(c)$. We also have that $q^*(c) = q^*(\bar{p})\ \forall \bar{p} \in [\bar{p}_H, \infty)$ due to observation (i) and (iv). This means that the welfare-optimal upper bounds are elements of the set $\{\bar{p} \mid \bar{p} = \bar{p}_c^{SB}(\beta') \vee \bar{p} \geq \bar{p}_H(\beta')\}$.

□

Proof of Proposition 2.5. It holds that $q^*(c) = \frac{\omega - \beta c}{(1+\gamma)(2-\gamma)} = q^*(\bar{p})\ \forall \bar{p} \geq \bar{p}_H$. Accordingly, $\bar{p} = c$ leads to the same level of welfare as all upper bounds $\bar{p} \geq \bar{p}_H$. As perceived total prices are identical but $\bar{p} = c$ leads to an equilibrium with strictly lower additional prices than any $\bar{p} \geq \bar{p}_H$, the resulting total equilibrium prices are strictly lower and, hence, consumer surplus is strictly higher with $\bar{p} = c$ than with $\bar{p} \geq \bar{p}_H$. □

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Chapter 3

Can Regulation Ameliorate Mobile Gaming Addiction?*

Joint with David Zeimentz

3.1 Introduction

Mobile gaming addiction has become a problem for society. On the one hand, medical practitioners are concerned about excessive gaming. The American Psychiatric Association (2013) lists "internet gaming disorder" as a condition worth further study and the World Health Organization (2019) incorporated "[digital] gaming disorder" in their most recent classification of diseases (ICD-13). On the other hand, consumer advocates and the media are concerned about excessive spending on mobile games.[1] This suspicion is supported by the fact that the majority of the profit in the entire gaming market is generated by in-app purchases (IAP) of free-to-play mobile games.[2] Some gamers manage to spend hundreds or even thousands of dollars and regret their choices afterward, although single transactions are typically inexpensive (with a common price of 0.99 USD).[3]

Policymakers implemented various regulations to fight excessive gaming time and exploitative pricing. The Chinese government, for instance, announced a re-

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1. See, https://cutt.ly/OC0z1hj
2. The gaming market accounted for approximately 126.6 billion USD in 2020 (SuperData, 2020). The majority of the revenues are generated by in-app purchases of mobile games (73.8 billion USD or 58.3%).
3. A survey by Finance (2020) finds that 30% of male young adults in Japan regret having made in-app purchases. Further, there is a plethora of consumer reports on regret and overspending such as https://cutt.ly/NC0xqAX or https://cutt.ly/NC0xugt.

striction of gaming time for young adults to three hours per week in August 2021.[4] Thailand[5] and South Korea[6] passed bills to prohibit young adults from playing during the night. To limit exploitative pricing, researchers recommend a spending cap of 100 USD.[7] The classical argument of economists is that competition alleviates the exploitative tendencies in the market, or as the former Chairman of the Federal Trade Commission, Timonthy Muris, put it: "robust competition is the best single means for protecting consumer interests" (Muris, 2002, p. 7). But does this remedy work in the mobile gaming market? For an effective regulation, policymakers require answers to the following questions. How do firms exploit consumers with in-app purchases? Do consumers benefit from more competition, a playing-time regulation, or a spending cap?

In this paper, we investigate how firms use in-app purchases to exploit consumers and if a more competitive environment, a playing-time regulation, or a spending cap can eliminate the exploitative practices.

We propose a three-period model of optimal contract design. First, the firm offers a contract that consists of a fixed fee and prices for in-app purchases in the two subsequent periods. After accepting the contract, the consumers choose a gaming time and if they want to make an IAP in each of the two subsequent periods. Gaming time and IAP are complements, that is, the higher the gaming time, the more likely are the consumers to make an IAP and vice versa. We model addiction by incorporating the *reinforcement effect* and the *tolerance effect* into the consumers' utility proposed by Becker and Murphy (1988). Tolerance means that past gaming time decreases the level of today's utility, whereas reinforcement means that past gaming time increases today's marginal utility of gaming. We consider sophisticated consumers, who anticipate their addiction when accepting the contract, and naive consumers who do not anticipate their addiction when accepting the contract.

We derive the profit-maximizing contracts in a monopoly. When the consumers are sophisticated, the monopolist offers in-app prices that maximize welfare and uses the fixed fee to completely extract the agents' utility. Naive consumers make two mistakes. (1) They choose an inefficiently high effort in the first period which aggravates addiction in the second period and (2) they spend too much on the game relative to the sophisticated consumers. The firm sets prices so that the naive consumers do not anticipate to make an IAP in the second period. The firm makes IAP in the first period cheap to get the consumers hooked. Then, in the second period, the firm exploits the increased willingness to pay by setting higher prices. This is particularly harmful to the consumers, as the purchase is not anticipated.

4. See, https://cutt.ly/FC0xPNa
5. See, https://cutt.ly/4C0x54U
6. See, https://cutt.ly/EC0cic6
7. See, https://cutt.ly/jC0czFh

Perfect competition among firms only affects the distribution of surplus but does not eliminate the inefficiency, that is, inefficiently high gaming time. Furthermore, if there are games with different degrees of addictivess, e.g. a game with a low addictiveness and a game with a high addictiveness, then the naive consumers choose the game with the high addictiveness. The reason is that the naive consumers do not anticipate the addiction and thus perceive the more addictive game to come at better contractual terms. Therefore, perfect competition does not eliminate the inefficiency but might even exacerbate it.

A playing-time regulation can achieve the first-best result. This regulation, however, does not prevent the naive consumers from misanticipating their future utility and consequently from accepting an exploitative contract. Hence, the regulation cannot make naive consumers as well off as the sophisticated ones. A spending cap neither induces the first-best result in equilibrium nor makes the naive consumers as well off as the sophisticated ones. The spending cap does not reduce the consumers' inefficiently high playing time and therefore cannot mitigate the addiction problem. To achieve the first-best and debias the consumers, the policymaker must combine both policies. Only both policies at the same time can eliminate both mistakes of the naive consumers.

We contribute to different strands of the literature. First, we relate to the literature on digital addiction. Ichihashi and Kim (2022) consider addictive platforms that can choose the addictiveness of their services and rational consumers who can choose more than one platform to join. Unlike them, we do not consider how addictive the firm wants to make its product. Rather, we focus on how firm chooses its pricing strategy for a given addictive product when consumers are naive. Furthermore, since we focus on pricing decisions we can also consider how different regulations affect the pricing, the consumer surplus, and the overall welfare. Allcott, Gentzkow, and Song (2022) develop a model of social media addiction and estimate their model with data from a field experiment. Similarly to Ichihashi and Kim (2022), they model how consumers get addicted but do not focus on monetization with prices. We complement their analyses because we consider contract design and regulation.

Most closely related to our analysis is Triviza (2020) who considers optimal contract design with habit-forming consumers. She only models spending but does not consider the gaming time. Furthermore, she does not consider regulation. We go beyond her analysis because we can also consider how spending interacts with gaming time, which are both crucial features in the gaming market, and unlike her we consider the effects of regulation.

Second, we contribute to the literature on exploitative contracting when firms use contracts to exploit consumers who mispredict their behavior towards a product[8] (DellaVigna and Malmendier, 2004; Eliaz and Spiegler, 2006; Heidhues and Kőszegi, 2010). Closest to our paper is Heidhues and Kőszegi (2010) who consider loan-repayment behavior and welfare in a competitive credit market model when consumers are present-biased. They find that naive consumers overborrow, pay penalties for deferring amounts of payments, and suffer welfare losses due to a difference between perceived and experienced utility. Unlike them, we consider misprediction of addiction in mobile games and show that this induces naive consumers to make suboptimal consumption choices also because perceived utility differs from the experienced one.

The paper is structured as follows. Section 3.2 introduces the model. Section 3.3 analyzes the monopoly equilibrium and considers the welfare analysis. Section 3.4 considers perfect competition and Section 3.5 examines an effort regulation and a spending cap. Finally, section 3.6 concludes.

3.2 Model

Timing. We employ a model with three periods, $t = 0, 1, 2$. In period $t = 0$ a risk-neutral firm provides a mobile game (e.g. Candy Crush Saga) that might contain in-app purchases (IAP). The consumers receive some benefit from the game and the perks obtained through the IAP. The firm sets the up-front fee of the game and the pricing scheme of the IAP.[9]

Firm. The firm offers the consumers a contract at $t = 0$ that consists of the following elements: A lump-sum payment L that must be paid in $t = 0$ and prices for IAP p_t that must be paid in stage $t \in \{1, 2\}$ if the consumers make an IAP. The firm's cost of providing the game and the cost of the IAP are normalized to zero.[10] The firm designs the contract to maximize expected profits, i.e.

$$\pi = \max_{L, p_1, p_2} \quad L + p_1 a_1 + p_2 a_2.$$

where a_t denotes the consumers' decision to make IAP.

8. Heidhues and Kőszegi (2018) provide a review on behavioral industrial organization and exploitative contracting.

9. Instead of committing to a pricing scheme in $t = 0$, the firm could choose the prices in each period (short-run contract). In this case, the consumers form beliefs about future prices. Although this may be an interesting extension, we restrict our analysis, for simplicity, to long-run contracts (similar to DellaVigna and Malmendier, 2004).

10. We neglect fixed costs because the developing costs of games such as Candy Crush are considerably lower than the costs of blockbuster games such as Call of Duty. Furthermore, the marginal costs of IAP are negligible in practice.

Consumers. In $t = 0$, the consumers are offered a contract and decide whether they wish to accept it. In $t = 1$ and $t = 2$, the consumers choose effort (gaming time) $e_t \in [0, 1]$ and decide if they want to make an IAP $a_t \in \{0, 1\}$.

The consumers' instantaneous utility at time t is denoted as u_t and given by

$$u_t(e_t, a_t) = e_t(xa_t + v) - c(e_t) - \theta e_{t-1}(1 - e_t) - p_t a_t.$$

If the consumers choose effort e_t, they get a value of $e_t v$ if they do not make the IAP and $e_t(v + x)$ if they make the IAP. We assume that $v > x$, that is, the value of the game is higher than that of the in-app features. The effort costs are denoted by $c(e_t) = \frac{c}{2} e_t^2$.

The parameter $\theta \in [0, 1]$ measures addiction that the gamer may be prone to. We assume that $\theta < x$ because we want to focus on the case that in-app purchases are efficient. We formalize addiction as proposed by Becker and Murphy (1988). The addicted consumers' utility function is affected by past behavior (gaming time) in two ways. First, the consumers' utility in the current period is adversely affected by past gaming time (tolerance). Second, the consumers' incentive to play is increasing in past gaming time (reinforcement). Formally, we define tolerance and reinforcement as

$$\frac{\partial u_t(\cdot)}{\partial e_{t-1}} = -\theta \left(1 - e_t - e_{t-1} \frac{\partial e_t}{\partial e_{t-1}}\right) < 0 \qquad \text{(Tolerance)}$$

$$\frac{\partial^2 u_t(\cdot)}{\partial e_{t-1} \partial e_t} = \theta > 0. \qquad \text{(Reinforcement)}$$

Formally, we must assume that the effort cost is sufficiently high such that both tolerance and reinforcement hold in equilibrium. We assume that[11]

$$c^2 > (c + 2\theta)(x + v).$$

The addiction term of the utility function can be motivated in the following way: The consumers are endowed with 1 unit of time that they must allocate between time spent playing e_t and time spent not playing the game (e.g. time spent with family and friends) $(1 - e_t)$. The time that they do not play the game gives them an outside utility value of $\bar{u}$ that is normalized to 0. The value of the outside option deteriorates with past consumption for addicted consumers. Hence, the value of the outside option is reduced by each hour spent gaming in the previous period.

We will consider different consumer types that are denoted by superscript $i \in \{s, n\}$: Sophisticated consumers (s) who anticipate addiction, and naive consumers n who do not anticipate addiction. We consider markets either with sophisticated or naive consumers only.

11. This assumption can be rewritten such that $c > \frac{x+v+\sqrt{x+v}\sqrt{x+v+8\theta}}{2}$. This implies that $c > x + v$. The assumption also ensures that equilibrium effort will be interior.

Therefore, we must distinguish *experienced* utility which the consumers experience and *perceived* utility (beliefs are denoted by ˆ variables) which the consumers believe to experience.

The *experienced* utility viewed from stage 0 is given by

$$\begin{aligned} u_0^i(L^i, p_1^i, p_2^i) &= \sum_{t=1}^{2} u_t^i(e_t^i, a_t^i; e_{t-1}^i) - L^i \\ &= \sum_{t=1}^{2} \left(e_t^i(xa_t^i + v) - c(e_t^i) - \theta e_{t-1}^i(1 - e_t^i) - p_t^i a_t^i \right) - L^i. \end{aligned}$$

The *perceived* utility viewed from stage 0 is given by

$$\begin{aligned} \hat{u}_0^i(L^i, p_1^i, p_2^i) &= \sum_{t=1}^{2} \hat{u}_t^i(\hat{e}_t^i, \hat{a}_t^i; \hat{e}_{t-1}^i) - L^i \\ &= \sum_{t=1}^{2} \left(\hat{e}_t^i(x\hat{a}_t^i + v) - c(\hat{e}_t^i) - \hat{\theta} \hat{e}_{t-1}^i(1 - \hat{e}_t^i) - p_t^i \hat{a}_t^i \right) - L^i. \end{aligned}$$

The sophisticated consumers anticipate the addiction: $\theta = \hat{\theta}^s > 0$. The naive consumers do not anticipate the addiction: $\theta > \hat{\theta}^n = 0$. This implies that the sophisticates correctly anticipate their utility, that is, experienced and perceived utility is equal.

Equilibrium. We solve for *perception-perfect strategies* (see O'Donoghue and Rabin, 1999, 2001). Borrowing from Herweg and Müller (2011) we define perception-perfect strategies as follows.

Definition 3.1 (Perception-perfect strategies)**.** A perception-perfect strategy for an agent type i is given by $(e_1^{i\,*}, a_1^{i\,*}, e_2^{i\,*}, a_2^{i\,*}, \hat{e}_2^i, \hat{a}_2^i)$ such that

(i) $(\hat{e}_2^{i*}, \hat{a}_2^{i*}) \in \arg\max_{\hat{e}_2^i, \hat{a}_2^i} \ \hat{u}_2^i(\hat{e}_2^i, \hat{a}_2^i, e_1^i)$

(ii) $(e_2^{i\,*}, a_2^{i\,*}) \in \arg\max_{e_2^i, a_2^i} \ u_2^i(e_2^i, a_2^i, e_1^i)$

(iii) $(e_1^{i\,*}, a_1^{i\,*}) \in \arg\max_{e_1^i, a_1^i} \ u_1^i(e_1^i, a_1^i) + \hat{u}_2^i(\hat{e}_2^i, \hat{a}_2^i, e_1^i)$

where * indicates equilibrium behavior and ˆ denotes beliefs.

In a perception-perfect equilibrium, each agent plays a perception-perfect strategy and the firm maximizes profits.

Welfare. We follow O'Donoghue and Rabin (1999, 2001) and define consumer surplus as the experienced utility viewed from stage 0.

$$CS^i = u_0^i(L^i, p_1^i, p_2^i) = \sum_{t=1}^{2} u_t^i(e_t^i, a_t^i; e_{t-1}^i) - L^i.$$

We define producer surplus as profits.

$$PS = \pi = \sum_{t=1}^{2} p_t^i a_t^i + L^i.$$

Welfare is the sum of consumer surplus and producer surplus.

$$\begin{aligned} W &= \sum_{t=1}^{2} u_t^i(e_t^i, a_t^i; e_{t-1}^i) + \sum_{t=1}^{2} p_t^i a_t^i \\ &= \sum_{t=1}^{2} \left(e_t^i(xa_t^i + v) - c(e_t^i) - \theta^i e_{t-1}^i (1 - e_t^i) \right). \end{aligned}$$

3.3 Monopoly

We analyze the simple model to understand why a firm develops a game with in-app purchases We solve the model by backward induction. First, we consider a market with sophisticated consumers. Second, we consider a market with naive consumers.

3.3.1 Sophisticated Consumers

This section considers a market with sophisticated consumers only who are aware of their addiction. In $t = 2$, the consumers choose effort and decide whether to make an IAP. The incentive compatibility constraint is given by

$$(e_2^{s*}, a_2^{s*}) = \operatorname*{arg\,max}_{e_2^s, a_2^s} \; u_2^s(e_2^s, a_2^s, e_1^s)$$

The equilibrium effort is characterized by the first-order condition

$$e_2^{s*} = \frac{a_2^s x + v + \theta e_1^s}{c}.$$

The equilibrium effort is unique because the marginal cost is strictly increasing.

The first-order condition shows that effort and IAP are complements. If the agent makes an IAP, he chooses a higher effort. Intuitively, if the agent buys a fancy costume in the game, he enjoys playing the game more and hence plays a longer time. Similarly, the incentive to make an IAP increases, when the consumers play more.

The first-order condition also shows reinforcement: A higher effort in stage 1 increases effort in stage 2. Intuitively, if the consumers play the game for a long time in stage 1, they also want to play it for a long time in stage 2 because they become addicted. Furthermore, addicted consumers are more likely to make in-app purchases.

In $t = 1$, the consumers choose effort and decide whether to make in-app purchases. The incentive compatibility constraint is given by

$$(e_1^{s*}, a_1^{s*}) = \underset{e_1^s, a_1^s}{\arg\max} \; u_1^s(e_1^s, a_1^s, e_1^s) + \hat{u}_2^s(\hat{e}_2^{s*}(e_1^s), \hat{a}_2^{s*}(a_1^s), e_1^s).$$

By definition, the sophisticated consumers anticipate the addiction: The perceived utility equals the experienced utility. Still, the two stages are not independent of each other because the choices in stage 1 affect the ones in stage 2 via the addiction.

In stage 0, the consumers accept the contract if the participation constraint is satisfied

$$u_1^{s*}(e_1^{s*}, a_1^{s*}) + u_2^{s*}(e_2^{s*}(e_1^{s*}), a_2^{s*}(a_1^{s*}), e_1^{s*}) - L^s \geq 0.$$

The firm maximizes profits subject to the incentive compatibility constraints and the participation constraint.

$$\begin{aligned}
\max_{L^s, p_1^s, p_2^s} \; \pi^s &= L^s + p_1^s a_1^{s*} + p_2^s a_2^{s*} \\
\text{s.t. } (e_1^{s*}, a_1^{s*}) &= \underset{e_1^s, a_1^s}{\arg\max} \; u_1^s(e_1^s, a_1^s) + u_2^s(e_2^{s*}(e_1^s), a_2^{s*}(a_1^s), e_1^s) \\
(e_2^{s*}, a_2^{s*}) &= \underset{e_2^s, a_2^s}{\arg\max} \; u_2^s(e_2^s(e_1^s), a_2^s(a_1^s), e_1^s) \\
0 &\leq u_1^{s*}(e_1^{s*}, a_1^{s*}) + u_2^{s*}(e_2^{s*}(e_1^{s*}), a_2^{s*}(a_1^{s*}), e_1^{s*}) - L^s
\end{aligned}$$

Proposition 3.1 derives the profit-maximizing contract that the firm offers to the sophisticated consumers.[12]

Proposition 3.1. *In equilibrium, the firm chooses* $L^{s*} = u_1^{s*}(\cdot) + u_2^{s*}(\cdot)$, $p_1^{s*} \leq \frac{x[c(2v+x)+2\theta(v+x-c)]}{2(c^2-\theta^2)}$, *and* $p_2^{s*} \leq \frac{x[c(2v+x)+2\theta(v+x-\theta)]}{2(c^2-\theta^2)}$. *The consumers accept the contract and choose* $a_1^{s*} = a_2^{s*} = 1$, $e_1^{s*} = \frac{c(x+v)+\theta(x+v-c)}{c^2-\theta^2}$, *and* $e_2^{s*} = \frac{c(x+v)+\theta(x+v-\theta)}{c^2-\theta^2}$.

The firm is indifferent between infinitely many contracts and chooses the fixed fee such that the participation constraint binds. The firm completely extracts the consumers' utility: The consumers do not get a rent.

Since the firm can extract the complete utility, the firm wants to maximize the consumers' utility. Making an in-app purchase has two consequences. On the one hand, it increases utility because it provides a value of x to the consumers. On the other hand, it leads to higher effort and therefore increases the addiction problem which lowers utility. The first effect outweighs the second one and the firm, therefore, wants to induce in-app purchases in both stages.

The firm is indifferent between infinitely many contracts as long as the fixed fee is chosen to make the participation constraint bind and the in-app prices are

12. All proofs are relegated to the appendix.

low enough such that the agent makes an IAP in every stage. Lower in-app prices would then translate into a higher fixed fee. If the prices were too high, the consumers would not make an IAP anymore and the principal would get lower profits. Furthermore, the upper bounds on the IAP prices differ because the consumers will be addicted in the second stage. Non-addicted consumers would face equal in-app prices in both stages. Addicted consumers, however, choose a higher effort and are more likely to make an IAP. Therefore, the highest price under which the consumers make an IAP is higher in the second stage than in the first one when the consumers are not yet addicted.

Note that the consumers choose a different effort in each stage because of the addiction. Non-addicted consumers would choose the same playing time in each stage. First-stage effort decreases in the addiction parameter θ whereas second-stage effort increases in the addiction parameter. The reason is that the consumers anticipate their addiction. If the addiction parameter increased, the consumers would choose a lower first-stage effort because they want to prevent themselves from getting too addicted. Intuitively, if the game is very addictive, sophisticated consumers might only play for a limited amount of time to counteract the addiction.

But even though they reduce their first-stage effort, addiction leads to a higher second-stage effort. This means that the reduction in the first-stage effort is not enough to completely offset the addiction effect on the second-stage effort.

3.3.2 Naive Consumers

This section considers a market with naive consumers only who do not anticipate their addiction. In $t = 2$, the consumers choose effort and decide whether to make an IAP to maximize utility given the price. The utility maximization problem is the same as the one of the sophisticated consumers. The important difference is, however, that the naive consumers do not anticipate the addiction. They mistakenly think at $t = 1$ that they will not be addicted in $t = 2$. Therefore, they believe to choose effort and IAP such that the *perceived* utility is maximized. The perceived incentive constraint is given by

$$(\hat{e}_2^{n*}, \hat{a}_2^{n*}) = \arg\max_{\hat{e}_2^n, \hat{a}_2^n} \hat{u}_2^n(\hat{e}_2^n, \hat{a}_2^n)$$

The equilibrium effort is characterized by the first-order condition

$$\hat{e}_2^{n*} = \frac{x\hat{a}_2^n + v}{c}.$$

Comparing the perceived effort with the actual effort shows that the consumers think that they will choose a lower effort than they actually will. Intuitively, the naive consumers ends up playing the game for more hours than they originally thought. Due to the wrong future beliefs they choose first-stage effort and IAP to maximize

$$(e_1^{n*}, a_1^{n*}) = \underset{e_1^n, a_1^n}{\arg\max} \ u_1^n(e_1^n, a_1^n, e_1^n) + \hat{u}_2^n(\hat{e}_2^{n*}, \hat{a}_2^{n*}).$$

In stage 0, the consumers accept the contract if the perceived participation constraint is satisfied

$$u_1^{n*}(e_1^{n*}, a_1^{n*}) + \hat{u}_2^{n*}(\hat{e}_2^{n*}, \hat{a}_2^{n*}) - L^n \geq 0.$$

This shows that the naive consumers make two mistakes. First, on the intensive margin, they choose a higher effort than originally anticipated. Second, on the extensive margin, they choose to participate although they might better not participate because the experienced utility is lower than the perceived utility.

The firm anticipates the consumers' mistakes and takes them into account when maximizing profits.

$$\begin{aligned}
\max_{L^n, p_1^n, p_2^n} \quad \pi^n &= L^n + p_1^n a_1^{n*} + p_2^n a_2^{n*} \\
\text{s.t. } (e_1^{n*}, a_1^{n*}) &= \underset{e_1^n, a_1^n}{\arg\max} \ u_1^n(e_1^n, a_1^n) \\
(e_2^{n*}, a_2^{n*}) &= \underset{e_2^n, a_2^n}{\arg\max} \ u_2^n(e_2^n, a_2^n) \\
(\hat{e}_2^{n*}, \hat{a}_2^{n*}) &= \underset{\hat{e}_2^n, \hat{a}_2^n}{\arg\max} \ \hat{u}_2^n(\hat{e}_2^n, \hat{a}_2^n) \\
0 &\leq u_1^{n*}(e_1^{n*}, a_1^{n*}) + \hat{u}_2^{n*}(\hat{e}_2^{n*}, \hat{a}_2^{n*}) - L^n
\end{aligned}$$

Proposition 3.2 derives the profit-maximizing contract that the firm offers to the naive consumers.

Proposition 3.2. *In equilibrium, the firm chooses* $L^{n*} = u_1^{n*}(\cdot) + \hat{u}_2^{n*}(\cdot)$, $p_1^{n*} \leq \frac{x(x+2v)}{2c}$, *and* $p_2^{n*} = \frac{x[c(x+2v)+2v\theta]}{2c^2}$. *The consumers accept the contract and choose* $a_1^{n*} = 1$, $\hat{a}_2^{n*} = 0$, $a_2^{n*} = 1$, $e_1^{n*} = \frac{x+v}{c}$, *and* $e_2^{n*} = \frac{(c+\theta)(x+v)}{c^2}$.

The naive consumers do not anticipate their addiction and therefore make two mistakes. First, the consumers accept exploitative contracts and end up paying "too" much because they perceive their future utility as higher than it really is. Second, the consumers play too much time in the first stage which aggravates the addiction problem in the future. We now consider each of these mistakes in detail.

The firm chooses the fixed fee to make the perceived participation constraint binding. This allows the firm to extract the perceived utility and the consumers believe that they would not get a rent. Apart from extracting the perceived utility, the firm can increase profits by inducing unplanned in-app purchases in the second stage. Given a fixed fee, a low in-app price in the second stage means that the naive consumers make an IAP but that they also anticipate it. In this case, the firm would be indifferent between using an in-app price or the fixed fee because the decision is anticipated. If the in-app price, however, is higher the consumers make an IAP but do not anticipate that beforehand. In this case, the firm would not be able to extract this unanticipated surplus with a fixed fee and thus must use in-app prices. The highest price such that the consumers make an IAP in the second stage is $p_2^{n*} = \frac{x[c(x+2v)+2v\theta]}{2c^2}$ which is higher than the price that the sophisticated consumers are charged. Taken the prices together, the consumer ends up paying “too” much relative to sophisticated consumers

Furthermore, the naive consumers choose a “too” high effort in the first stage which exacerbates the addiction problem in the future. In particular, the naive consumers choose a higher first-stage effort than the sophisticated consumers who reduce effort in the first stage to counteract the addiction. As a result, the second-stage effort of the naive consumers will also be higher. Higher effort leads to being more likely to make an IAP because of the complementarity between gaming time and IAP. This means that the naive consumers are more likely to make an IAP, or in other words, are willing to pay a higher price for an IAP.

The firm recognizes the consumers’ mistakes and offers a contract with high in-app prices that maximally exploit the consumers. The consumers regret their choices ex-post. If they had known their future behavior at the beginning, they would not have accepted the contract in the first place.

The welfare implications of the sophisticated and naive consumers’ choices are of particular interest. It is not surprising that naive consumers are worse off than the sophisticated ones. The next section considers a welfare analysis by deriving the welfare-optimal choices and comparing these to the equilibrium results.

3.3.3 Welfare

This section considers the welfare-maximizing outcome (first-best) and compares it to the profit-maximizing contracts that the firm chooses in equilibrium. Furthermore, we compare the consumer surplus of the naive consumers to the consumer surplus of the sophisticated ones.

The first-best outcome follows from the welfare maximization problem

$$\max_{e_1,e_2,a_1,a_2} \; W = \sum_{t=1}^{2} u_t^i(e_t^i, a_t^i; e_{t-1}^i) + \sum_{t=1}^{2} p_t^i a_t^i.$$

Proposition 3.3 characterizes the first-best outcome and compares welfare and consumer surplus in a market with sophisticated consumers to a market with naive ones.

Proposition 3.3. *The following holds:*

(i) *The profit-maximizing contract for sophisticates induces the first-best welfare outcome whereas the profit-maximizing contract for naifs does not induce the first-best* *welfare outcome for $\theta > 0$. The naive consumers choose inefficiently high effort for $\theta > 0$.*

(ii) *Furthermore, the consumer surplus of the naive consumers is lower than that of the sophisticated ones, i.e. $CS^{n*} < 0 = CS^{s*}$ for $\theta > 0$.*

The Proposition shows that the equilibrium contract with sophisticated consumers achieves the first-best outcome. The sophisticated consumers perfectly anticipates their behavior. The firm, therefore, cannot increase profits by an exploitative contract, as the consumers would anticipate this. Because the firm can extract the entire consumer surplus, her optimal behavior maximizes the sum of consumer and producer surplus, i.e. the welfare. This means that the firm chooses the IAP prices to set the first-best incentives. Then, it sets the fixed fee to completely extract the consumers' utility. The consumers are indifferent between accepting and not accepting and the consumer surplus is zero.

The profit-maximizing contract for naive consumers, on the other hand, does not achieve the first-best result because the naive consumers exert too much effort relative to the first-best. The reason is the misanticipation of addiction. The firm cannot use the fixed fee to extract the experienced surplus because of the consumers' misanticipation. Instead, the firm can do better and extract a (larger) perceived surplus and use high in-app prices to induce unplanned IAP.

The firm's exploitative strategy increases profits by lowering the total surplus at the expense of the naive consumers who would later regret their choices. Due to the misanticipation, they consider a contract with a "too" high fixed fee and play "too" much time as compared to the sophisticated consumers. The naive consumers' bad choices lead then to a negative consumer surplus which implies that the consumers would be better off if they had not accepted the contract.

The next section investigates whether more competition can solve the inefficiency with naive consumers.

3.4 Competition

The previous sections shows that the firm wants to rip naive consumers off through exploitative contracts. An argument often put forward is that competition helps to protect consumers. The former Chairman of the Federal Trade Commission, Timothy Muris, said "robust competition is the best single means for protecting consumer interests" (Muris, 2002, p. 7). The claim reflects the idea that a competitive market supplies more efficient products than less competitive markets. This is broadly correct with fully-rational consumers. If consumers misperceive certain elements, however, competition might not solve the inefficiency (Heidhues and Kőszegi, 2010). We contribute to this literature and show that addiction and its misanticipation yields similar results. In particular, we show that competition can even aggravate the problem if naive consumers choose more addictive games.

We consider the same framework as above but with perfect competition (1) when there is only one type of firm and (2) when there are two types of firms. The firm type in this case refers to how addictive the game is, that is, we consider firms with different values for θ. Intuitively, some firms offer a game that is not addictive at all $\theta_1 = 0$ whereas other firms offer a game that is addictive ($\theta_2 > 0$). The idea is that we want to examine which kinds of firms survive under (perfect) competition.

3.4.1 Perfect Competition with One Firm Type

Let us assume that there are no entry costs and there is only one firm type. Then, there are enough firms in the market such that no firm has any market power. Since we consider perfect competition in $t = 0$, the firm maximization problem boils down to maximizing the perceived consumer surplus subject to a zero-profit constraint. The reason is that there is Bertrand competition. The firms compete by offering better contracts to the consumers until their profits go to zero.

For sophisticated consumers, this means

$$\begin{aligned}
\max_{L^s,p_1^s,p_2^s} \quad & u_1^{s*}(e_1^{s*},a_1^{s*}) + u_2^{s*}(e_2^{s*}(e_1^{s*}),a_2^{s*}(a_1^{s*}),e_1^{s*}) - L^s \\
\text{s.t. } (e_1^{s*},a_1^{s*}) &= \arg\max_{e_1^s,a_1^s} \; u_1^s(e_1^s,a_1^s) + u_2^s(e_2^{s*}(e_1^s),a_2^{s*}(a_1^s),e_1^s) \\
(e_2^{s*},a_2^{s*}) &= \arg\max_{e_2^s,a_2^s} \; u_2^s(e_2^s(e_1^s),a_2^s(a_1^s),e_1^s) \\
0 &= L^s + (p_1^s - \kappa)a_1^{s*} + (p_2^s - \kappa)a_2^{s*}
\end{aligned}$$

For naive consumers, the maximization problem changes to

$$\begin{aligned}
\max_{L^n,p_1^n,p_2^n} \quad & u_1^{n*}(e_1^{n*},a_1^{n*}) + \hat{u}_2^{n*}(\hat{e}_2^{n*},\hat{a}_2^{n*}) - L^n \\
\text{s.t. } (e_1^{n*},a_1^{n*}) &= \arg\max_{e_1^n,a_1^n} \; u_1^n(e_1^n,a_1^n) \\
(e_2^{n*},a_2^{n*}) &= \arg\max_{e_2^n,a_2^n} \; u_2^n(e_2^n,a_2^n) \\
(\hat{e}_2^{n*},\hat{a}_2^{n*}) &= \arg\max_{\hat{e}_2^n,\hat{a}_2^n} \; \hat{u}_2^n(\hat{e}_2^n,\hat{a}_2^n) \\
0 &= L^n + (p_1^n - \kappa)a_1^{n*} + (p_2^n - \kappa)a_2^{n*}
\end{aligned}$$

The maximization problems show that perfect competition leads to the same equilibrium effort and equilibrium IAP choices in perfect competition as in the monopoly. The only difference is the distribution of the surplus. Under perfect competition, the consumers receive the entire surplus because the firms make zero profits.[13] As in the monopoly case, the naive consumers make inefficient choices under

13. Theoretically, the fixed fee might be negative. It is not uncommon that consumers receive a payoff for playing the game. Mobile games frequently give free items to players on special occasions. In

perfect competition. Hence, the inefficiency, which is due to the misperception of future utility, persists under perfect competition. The reason is that perfect competition occurs at $t = 0$ which is before naive consumers become aware of their bias.

Proposition 3.4 derives the equilibrium contract under perfect competition.

Proposition 3.4. *Perfect competition only affects the distribution of surplus but not the amount of surplus being generated. The equilibrium effort and in-app purchases are the same as in a monopoly; the fixed fee, however, is different because firms make zero profits.*

The reason that the inefficiency survives under perfect competition is the disparity between the perceived and the experienced utility. The naive consumers maximize their perceived utility and therefore their preferred contract does not maximize their experienced utility.

Similar results have been found in the literature. Triviza (2020) shows a similar result when consumers make a binary purchase decision. Heidhues and Kőszegi (2010) consider a competitive credit market when borrowers have a taste for immediate gratification. We contribute to this literature because we show that this result is robust as it also occurs in a market for mobile games. The question remains as to what consumers would do if they had to choose between different games some of which would be more addictive than others.

3.4.2 Perfect Competition with Two Firm Types

This section considers perfect competition with two firm types and hence goes beyond the related literature that only focuses on perfect competition with identical types. For simplicity, assume that firm type 1 offers a game that is not addictive at all $\theta_1 = 0$ whereas firm type 2 offers an addictive game $\theta_2 > 0$.[14] There are no entry costs which implies that there are many firms of each type in the market such that no firm has any market power. This implies again that firms make zero profits and each firm type maximizes perceived consumer surplus. The consumers compare the contracts of both firm types and choose the desired contract. Basically, we take the equilibrium perceived consumer surplus that we derived above and maximize it again over the addiction parameter θ.

Proposition 3.5 derives the equilibrium under perfect competition with two firm types.

Pokemon GO, each player gets one "raid pass" for free every day to participate in raid battles which are events in which the players fight against a powerful Pokemon. Otherwise, the player must purchase a raid pass.

14. The same analysis would also hold if we consider a more general setting with different values of θ as long as $\theta_1 < \theta_2$.

Proposition 3.5. *Consider perfect competition and two firm types with* $0 = \theta_1 < \theta_2$. *In equilibrium, the sophisticated consumers choose the non-addictive game whereas the naive ones choose the addictive game.*

The sophisticated consumers choose the non-addictive game because they are aware of their bias. They anticipate that the addictive product makes them addicted which will ultimately harm them. Hence, they are able to accurately compare the offers of the two firms and makes informed choices that they will not regret later. In general, for any two levels of addictiveness, the sophisticated consumers will always choose the less addictive product because this generates a higher surplus.

The naive consumers, on the contrary, choose the addictive game because they are not aware of their bias. They mistakenly think that both firm types offer an identical game but one firm type at better contractual terms, i.e. cheaper anticipated total payments. However, they do not understand that the game with the better terms appears to be better exactly because it makes the consumers addicted. The more addictive game yields a higher profit by distorting prices such that the game is perceived as being cheaper as future IAP are not anticipated. The adverse effects of addiction outweigh the better price such that the consumers end up with a lower consumer surplus than the sophisticated ones.

This reason for this result is the completely reversed bargaining power and the utility misperception. In the monopoly, the firm makes a take-it-or-leave-offer that the consumers accept. Under perfect competition, the consumers have all the bargaining power and the firms are forced to maximize the consumers' perceived surplus. Since the perceived surplus differs from the experienced one when consumers are naive, the naive consumers make suboptimal choices. Therefore, competition does not help because the naive consumers are not aware of their bias.

The Proposition also shows that, contrary to Muris' quote, perfect competition does not always eliminate the market failure. Of course, the naive consumers are better off than under a monopoly because they receive the surplus but the addiction still prevails or might even be worse. From an ex-post perspective, the naive consumers regret that they did not choose the other less-addictive game. The question again arises if there are other simple interventions in this market that can help consumers. In this manner, we consider regulation henceforth.

3.5 Regulation

The previous analyses show that naive consumers make suboptimal decisions and therefore get exploited. Contrary to conventional wisdom, perfect competition cannot solve the problem. Another instrument to improve the well-being of consumers is regulation. In particular, we consider two simple interventions that each address one of the naive consumers' mistakes. First, we consider a regulation of the time spent playing to ensure that consumers do not play an inefficiently high level in the first period. Second, we consider a spending cap to prevent the naive consumers from spending too much via fraudulent in-app purchases compared to the sophisticated ones.

3.5.1 Effort Regulation

Many countries have recently become aware of (mobile) gaming addiction and want to tackle this problem. One option that some countries are experimenting with is the regulation of playing time. In 2003, Thailand enacted a law that prevented adolescents to play online games between 10:00 pm and 6:00 am.[15] In 2011, South Korea implemented a similar law which was abolished again in 2021 due to enforcement problems among other reasons.[16] The most prominent example is probably China which announced in the summer of 2021 that children can only play video games for at most 3 hours a week.[17] These laws are meant to protect naive consumers from getting addicted. But do consumers really profit from such regulation and if yes to what extent?

To answer these questions, we consider an extension to our model in which a policymaker implements an upper bound on effort, that is, the consumers might not be able to play as much time as they would like to. We denote the effort bound as $\bar{e}(a_1^n)$ which means that the bound is not fixed but depends on the IAP decision. The bound corresponds to the equilibrium effort of the sophisticated consumers respectively with or without in-app purchases. We use this effort bound and not a fixed one because of two reasons. First, it makes sense that the consumers can play more time if they invest more resources into the game. Second, this makes the computations easier because otherwise, we must additionally distinguish whether the naive consumers' effort without in-app purchases exceeds the bound. Proposition 3.6 shows the results with an upper bound.

Proposition 3.6. *There exists an upper bound on effort, $\bar{e}(a_1^n)$, which is equal to the equilibrium effort level of the sophisticated consumers, such that the first-best result is achieved in equilibrium with naive consumers. There is no upper bound on effort, however, such that the naive consumers are as well off as the sophisticated ones for $\theta > 0$.*

Proposition 3.6 shows that there exists a bound that implements the first-best result in equilibrium. To achieve that, the policymaker must use the effort of the sophisticated consumers as an upper bound. Since the naive consumers would ideally like to choose a higher effort than the sophisticated ones and the utility function is concave, the naive consumers end up choosing the effort of the sophisticated consumer. The idea is to force the naive consumers to act like sophisticated ones. This

15. See, https://cutt.ly/4C0x54U
16. See, https://cutt.ly/EC0cic6
17. See, https://cutt.ly/FC0xPNa

bound is insofar successful as it produces the first-best outcome: the naive consumers choose the first-best effort level.

Even though the first-best result can be implemented, there is no upper bound that makes the naive consumers as well off as the sophisticated ones. The reason is that effort regulation only corrects one of the consumers' mistakes. Via an upper bound on effort, the policymaker can directly control reinforcement and tolerance because their severity depends on the time spent playing. The policy, however, does not prevent the consumer from misanticipating the addiction which implies that the consumers accept a contract with a "too" high fixed fee.

Furthermore, to achieve the first-best result, it is sufficient to only dictate an effort bound in the first stage. The reason is that the consumers' high addiction stems from a "too" high choice of first-stage effort. If the naive consumers are forced to choose the same effort as the sophisticated ones in the first stage, they become as addicted as the sophisticated ones and would therefore like to choose the same effort level in the second stage.

An additional bound on the second-stage effort,[18] however, would not increase consumer surplus. The reasons is that such a bound does not have an effect. The actual effort choice in the second stage is equal to the sophisticated consumers' effort independent if there is the bound on the second stage effort or not. Furthermore, the bound does not affect the perceived effort because the naive consumers think that they are non-addicted and therefore believe to choose a lower effort than the sophisticated consumers.

The results imply that a policymaker can achieve the first-best allocation and can make the naive consumers better off with effort regulation. But it is not possible to make the naive consumers as well off as the sophisticated ones. The reason is again that the naifs will not learn about their addiction via this regulation and therefore accept a contracts with too high prices.

Even if the effort regulation is perfect, it is impossible to achieve first-best and debias consumers at the same time. In practice, there are most certainly three frictions when it comes to regulation. First, the policymaker might not know the first-best and therefore the right threshold for effort. Second, the threshold to achieve first-best might also differ substantially across individuals because they have different preference parameters. Third, even if the policymaker knew everything, it might be difficult to enforce and monitor the regulation. If the maximal playing time is reached for a certain game, the consumers might just switch to another game and play again until the playing time is reached. Therefore, this strategy could even backfire if the consumers start to play more games and subsequently might become addicted to more games than without the bound.

18. Since the sophisticates choose a higher effort level in the second than in the first stage, this bound would also have to be higher than the other bound to make the naive consumers choose the effort of the sophisticated ones.

3.5.2 Spending Cap

This section considers a further simple regulation that could address the second mistake of the consumers, that is, that they spend too much money relative to the sophisticated ones because of the unanticipated addiction. A spending cap is an upper bound on the total amount of money that the consumers spend on the game. The idea is exactly to prevent that some consumers spend huge amounts of money because they become addicted. For online gambling, some consumer advocates recommend a spending cap of £100 per month because it "is more than what the majority of gamblers spend, while also being a threshold that ensures (according to our analysis of income and living standards) that gambling activities do not amount to serious financial harm."[19] Although there have not yet been similar policy proposals for mobile games, we analyze this type of regulation such that policymakers can make informed decisions.

To analyze this type of regulation, we consider a spending cap κ such that the consumers' total spending is bounded by the cap, i.e. $a_1^n p_1^i + a_2^n p_2^i + L^i \leq \kappa$. Proposition 3.7 shows the welfare and distributional consequences of the cap.

Proposition 3.7. *There is no spending cap, κ such that the first-best result can be achieved for the naive consumers in equilibrium or such that the naive consumers are as well off as the sophisticated ones for $\theta > 0$.*

The first part of the Proposition shows that first-best cannot be achieved in equilibrium in a market with naive consumers. The spending cap solely restricts how much money the consumers can spend on the game but not how much time they play it. The naive consumers still play "too" much from an efficiency perspective. The spending cap therefore cannot influence reinforcement or tolerance.

Furthermore, a spending cap cannot make the consumers as well off as the sophisticates which would have been one of the aims of such a policy. The cap definitely makes the naive consumers better off because they must spend less which leads to less exploitative contracts. However, the spending cap does not prevent the consumers from choosing too high effort in period 1 which then translates into a higher addiction in the future. This means that the spending cap cannot influence reinforcement or tolerance. As a result, the cap cannot completely eliminate the difference between perceived and experienced utility. The regulation fails to induce the naive consumers to make optimal choices and therefore they are worse off than the sophisticated ones.

Similar to effort regulation, even if the policymakers have all relevant information for the spending cap and can perfectly enforce it, it is impossible to achieve the first-best and debias the naive consumers. Enforcement remains a problem here.

19. See, https://cutt.ly/VC0vMt0

Across games, it might be difficult to enforce the cap because consumers might easily create a new account for a new game and might then become addicted to more games than without the cap.

3.5.3 Combined Regulation

Each regulation that we consider above addresses one of the consumers' mistakes. Considered separately, they cannot achieve the first-best result and debias the naive consumers at the same time. Effort regulation ensures that the naive consumers choose the first-best effort. It fails to achieve that the consumers only spend a moderate amount of money. The spending cap ensures that the naive consumers do not spend more than the sophisticated ones but cannot achieve the first-best effort. The question arises if both regulations considered together do the trick and make naive consumers as well off as sophisticated ones while achieving first-best. Proposition 3.8 shows that the answer to this question is yes.

Proposition 3.8. *If the policymaker can combine an upper bound on effort and a spending cap, then the equilibrium in a market with naive consumers achieves first-best and naive consumers are as well off as sophisticated ones.*

The upper bound on effort ensures allocative efficiency. The consumers do not play the game for a longer time than the sophisticated ones although they would like to do that initially. Given the allocative efficiency, the spending cap ensures a consumer-friendly distribution. The consumers receive the same utility as the sophisticated ones. Since they do not anticipate their addiction, the naive consumers first believe that the spending cap does not help them. In the second period, however, they realize that the spending cap helps them not to spend too much money because they have become addicted.

The proposition shows that only a combination of suitable regulations on playing time and a spending cap achieves first-best and makes the naive consumers as well off as the sophisticated ones. In other words, there is no optimal regulation that only restricts effort or spending. This result is in line with the Tinbergen rule (Tinbergen, 1952) which states that to achieve m independent policy targets, one must use at least use m independent instruments.

It is debatable how practical this result is for policymakers who might already find it difficult to implement either of those regulations. They must have detailed information on both playing time and spending to compute the optimal thresholds. At the same time, it remains difficult to monitor and enforce the policies because consumers might start to play more games with new accounts which might exacerbate the addiction problem again.

3.6 Conclusion

This paper studies how a firm uses in-app purchases to exploit consumers and if gaming time regulation or a spending cap can eliminate the problem. We propose a contracting model of a firm that offers a mobile game to consumers. The consumers might become addicted and might not anticipate that. We find that the firm offers an exploitative contract to naive consumers who do not anticipate their addiction. The naive consumers play the game inefficiently much and spend too much money relative to sophisticated ones.

Perfect competition does not solve this inefficiency because naive consumers when offered several games always choose the most addictive one because they mistakenly think that this game comes at the best price. Regulation on the gaming time induces the first-best outcome but does not make the naive consumers as well off as the sophisticated ones because they do not anticipate that they becomes addicted. A spending cap neither induces first-best nor makes the consumers as well off as the sophisticated consumers. To achieve the first-best and debias consumers, the policymaker must combine both regulations.

This paper is a first step to guiding policymakers on how to handle games with in-app purchases. A next step might be to investigate a case in which the firm cannot price discriminate between different kinds of consumers. A certain policy might then be beneficial for some consumers but harmful to others. Furthermore, future research can identify more interventions that policymakers could use to debias naive consumers. One potential policy could be consumer education programs to make consumers aware of their addictive behavior. We leave these and further questions for future research.

Appendix 3.A Proofs

Proof of Proposition 3.1. We solve the game by backward induction. The consumers choose a_t^s and e_t^s given the price p_t^s in each period $t \in \{1, 2\}$. We rewrite the problem such that the consumers choose only e_t^s. The firm can effectively choose a_t^s via the price. Hence, we consider that the firm chooses a_t^s and later show that this choice is incentive compatible via the price.

In $t = 2$, the sophisticated cosumers choose effort to maximize their utility.

$$\max_{e_2^s \in [0,1]} \; e_2^s(a_2^s x + v) - c(e_2^s) - \theta e_1^s(1 - e_2^s) - p_2^s a_2^s.$$

The first-order condition with respect to e_2^s is given by

$$\begin{aligned} a_2^s x + v + \theta e_1^s - c'(e_2^s) &\overset{!}{=} 0 \\ \frac{a_2^s x + v + \theta e_1^s}{c} &= e_2^{s*}(e_1^s). \end{aligned}$$

The second-order condition is satisfied because

$$-c''(e_2^s) < 0.$$

In $t = 1$, the sophisticated consumers choose effort to maximize their utility.

$$\begin{aligned} \max_{e_1^s \in [0,1]} \; & e_1^s(a_1^s x + v) - c(e_1^s) - p_1^s a_1^s + e_2^{s*}(e_1^s)(a_2^{s*} x + v) - c(e_2^{s*}(e_1^s)) \\ & - \theta e_1^s(1 - e_2^{s*}(e_1^s)) - p_2^s a_2^{s*}. \end{aligned}$$

The first-order condition with respect to e_1^s is given by

$$\begin{aligned} a_1^n x + v - c'(e_1^s) + \frac{\partial e_2^{s*}}{\partial e_1^s}[a_2^s x + v + \theta e_1^s - c'(e_2^{s*})] - \theta(1 - e_2^{s*}) &\overset{!}{=} 0 \\ \frac{x a_1^s + v - \theta(1 - e_2^{s*})}{c} &= e_1^{s*}. \end{aligned}$$

The term in the square bracket is the FOC from $t = 2$ and therefore cancels.

The second-order condition is fulfilled if

$$\begin{aligned} & -c + \frac{\partial^2 e_2^{s*}}{\partial e_1^{s*2}}\left[x a_2^{s*} + v + \theta e_1^s - c'(e_2^{s*})\right] + \frac{\partial e_2^{s*}}{\partial e_1^{s*}}\left[-c\frac{\partial e_2^{s*}}{\partial e_1^{s*}} + \theta\right] + \theta\frac{\partial e_2^{s*}}{\partial e_1^{s*}} \\ =& \frac{-c^2 + \theta^2}{c} < 0. \end{aligned}$$

Since $c > \theta$, this is fulfilled and the utility is concave in e_1^s.

Next, we compute the optimal effort levels. Substituting the FOC from $t = 1$ into the FOC from $t = 2$ gives

$$\begin{aligned}
e_2^{s*} &= \frac{xa_2^s + v + \theta e_1^{s*}}{c} \\
\Leftrightarrow e_2^{s*} &= \frac{xa_2^s + v + \theta \frac{xa_1^s + v - \theta(1-e_2^{s*})}{c}}{c} \\
\Leftrightarrow e_2^{s*}(a_1^s, a_2^s) &= \frac{c(xa_2^s + v) + \theta(xa_1^s + v - \theta)}{c^2 - \theta^2} \\
e_1^{s*}(a_1^s, a_2^s) &= \frac{c(xa_1^s + v) + \theta(xa_2^s + v - c)}{c^2 - \theta^2}.
\end{aligned}$$

The firm maximizes its profits given the incentive constraints and the participation constraint. The profit maximization problem is

$$\begin{aligned}
\max_{L^s, p_1^s, p_2^s} \; & L^s + p_1^s a_1^s + p_2^s a_2^s \\
\text{s.t. } (e_1^{s*}, a_1^{s*}) &= \arg\max_{e_1^s, a_1^s} u_1^s(e_1^s, a_1^s) + u_2^s(e_2^s, a_2^s), \\
(e_2^{s*}, a_2^{s*}) &= \arg\max_{e_2^s, a_2^s} u_2^s(e_2^s, a_2^s), \\
0 &\leq u_1^{s*}(e_1^s, a_1^s) + u_2^{s*}(e_2^s, a_2^s) - L^s.
\end{aligned}$$

We consider a rewritten version in which the firm chooses a_t^s and then chooses p_t^s to make the choices incentive compatible.

$$\begin{aligned}
\max_{L^s, a_1^s, a_2^s} \; & L^s + p_1^s a_1^{s*} + p_2^s a_2^{s*} \\
\text{s.t. } e_1^{s*} &= \arg\max_{e_1^s} u_1^s(e_1^s, a_1^s) + u_2^s(e_2^s, a_2^s), \\
e_2^{s*} &= \arg\max_{e_2^s} u_2^s(e_2^s, a_2^s), \\
0 &\leq u_1^{s*}(e_1^s, a_1^s) + u_2^{s*}(e_2^s, a_2^s) - L^s.
\end{aligned}$$

The participation constraint must be binding. If it were not binding, the firm could increase L^s to increase profits. Hence, we get that

$$\begin{aligned}
L^{s*} &= e_1^{s*}(a_1^s x + v) - c(e_1^{s*}) + e_2^{s*}(a_2^s x + v) - c(e_2^{s*}) \\
&\quad - \theta e_1^{s*}(1 - e_2^{s*}) - a_1^s p_1^s - a_2^s p_2^s.
\end{aligned}$$

Substituting the binding participation constraint into the objective function gives

$$\begin{aligned}
\max_{p_1^s, p_2^s} \; & e_1^{s*}(a_1^s x + v) - c(e_1^{s*}) + e_2^{s*}(a_2^s x + v) - c(e_2^{s*}) \\
& - \theta e_1^{s*}(1 - e_2^{s*}).
\end{aligned}$$

Depending on how the firm sets a_1^s and a_2^s, the firm can basically induce the profit-maximizing choice of the IAP. There are 4 cases that could be profit maximizing: (i) $a_1^{s*} = 0$ and $a_2^{s*} = 0$; (ii) $a_1^{s*} = 0$ and $a_2^{s*} = 1$; (iii) $a_1^{s*} = 1$ and $a_2^{s*} = 0$; and (iv) $a_1^{s*} = 1$ and $a_2^{s*} = 1$. We now show that Case (iv) maximizes profits. To do that we consider as if the firm could directly choose the IAP. Later, we verify that there exist prices to induce the IAP.

Since there are no costs of IAP, it is profit-maximizing to choose $a_1^s = a_2^s = 1$ given the same effort decisions.

$$\begin{aligned}
&\pi(a_1^s = 1, a_2^s = 1, e_1^{s\prime}, e_2^{s\prime}) \\
> &\max\{\pi(a_1^s = 1, a_2^s = 0, e_1^{s\prime}, e_2^{s\prime}), \pi(a_1^s = 0, a_2^s = 1, e_1^{s\prime}, e_2^{s\prime}), \\
&\pi(a_1^s = 0, a_2^s = 0, e_1^{s\prime}, e_2^{s\prime})\} \quad \forall\ e_1^{s\prime}, e_2^{s\prime}.
\end{aligned}$$

The consumers' effort choices also maximize the profit given that the firm can choose the fixed fee to extract the utility.

$$\max_{e_1^s, e_2^s} \pi(a_1^s = 1, a_2^s = 1, e_1^s, e_2^s) \geq \pi(a_1^s = 1, a_2^s = 1, e_1^{s\prime}, e_2^{s\prime}) \quad \forall\ e_1^{s\prime}, e_2^{s\prime}.$$

This shows that Case (iv) maximizes profits.

Given $a_1^{s*} = 1$, $a_2^{s*} = 1$ is incentive compatible iff

$$\begin{aligned}
&e_1^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)(x+v) - c(e_1^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)) \\
&+ e_2^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)(x+v) - c(e_2^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)) \\
&- \theta e_1^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)(1 - e_2^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)) - p_1^s - p_2^s \\
&\geq e_1^{s*}(a_1^{s*} = 1, a_2^{s*} = 0)(x+v) - c(e_1^{s*}(a_1^{s*} = 1, a_2^{s*} = 0)) \\
&+ e_2^{s*}(a_1^{s*} = 1, a_2^{s*} = 0)v - c(e_2^{s*}(a_1^{s*} = 1, a_2^{s*} = 0)) \\
&- \theta e_1^{s*}(a_1^{s*} = 1, a_2^{s*} = 0)(1 - e_2^{s*}(a_1^{s*} = 0, a_2^{s*} = 0)) - p_1^s \\
&\Leftrightarrow p_2^{s*} \leq \frac{x[c(2v+x) + 2\theta(v+x-\theta)]}{2(c^2-\theta^2)}.
\end{aligned}$$

Given $a_2^{s*} = 1$, $a_1^{s*} = 1$ is incentive compatible iff

$$\begin{aligned}
&e_1^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)(x+v) - c(e_1^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)) \\
&+ e_2^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)(x+v) - c(e_2^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)) \\
&- \theta e_1^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)(1 - e_2^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)) - p_1^s - p_2^s \\
&\geq e_1^{s*}(a_1^{s*} = 0, a_2^{s*} = 1)v - c(e_1^{s*}(a_1^{s*} = 0, a_2^{s*} = 1)) \\
&+ e_2^{s*}(a_1^{s*} = 0, a_2^{s*} = 1)(x+v) - c(e_2^{s*}(a_1^{s*} = 0, a_2^{s*} = 1)) \\
&- \theta e_1^{s*}(a_1^{s*} = 0, a_2^{s*} = 1)(1 - e_2^{s*}(a_1^{s*} = 0, a_2^{s*} = 1)) - p_2^s \\
&\Leftrightarrow p_1^{s*} \leq \frac{x[c(2v+x) + 2\theta(v+x-c)]}{2(c^2-\theta^2)}.
\end{aligned}$$

The firm's profit is then given by

$$\begin{aligned}\pi &= e_1^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)(x+v) - c(e_1^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)) \\ &+ e_2^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)(x+v) - c(e_2^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)) \\ &- \theta e_1^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)(1 - e_2^{s*}(a_1^{s*} = 1, a_2^{s*} = 1)) \\ &= \frac{c\theta^2 - 2c\theta v - 2c\theta x + 2cv^2 + 4cvx + 2cx^2 - 2\theta^2 v - 2\theta^2 x}{2(c^2 - \theta^2)} \\ &+ \frac{2\theta v^2 + 4\theta vx + 2\theta x^2}{2(c^2 - \theta^2)}.\end{aligned}$$

□

Proof of Proposition 3.2. We solve the game by backward induction. The consumers choose a_t^n, e_t^n, $\hat{a}_t^n$, $\hat{e}_t^n$ given the price p_t^n in each period $t \in \{1,2\}$. We rewrite the problem such that the consumers choose only e_t^n and $\hat{e}_t^n$. The firm can effectively choose a_t^n and $\hat{a}_t^n$ via the price, that is, that these choices are incentive compatible.

In $t = 2$, the naive consumers choose effort to maximize their utility.

$$\max_{e_2^n \in [0,1]} \; e_2^n(a_2^n x + v) - c(e_2^n) - \theta e_1^n(1 - e_2^n) - p_2^n a_2^n.$$

The first-order condition with respect to e_2^n is given by

$$\begin{aligned} a_2^n x + v + \theta e_1^n - c'(e_2^n) &\overset{!}{=} 0 \\ \frac{a_2^n x + v + \theta e_1^n}{c} &= e_2^{n*}.\end{aligned}$$

The second-order condition is satisfied because

$$-c''(e_2^n) < 0.$$

In $t = 1$, the naive consumers choose e_1^n to maximize their utility.

$$\max_{e_1^n \in [0,1]} \; e_1^n(a_1^n x + v) - c(e_1^n) - p_1^n a_1^n.$$

The first-order condition with respect to e_1^n is given by

$$\begin{aligned} a_1^n x + v - c'(e_1^n) &\overset{!}{=} 0 \\ \frac{a_1^n x + v}{c} &= e_1^{n*}.\end{aligned}$$

The second-order condition is satisfied because

$$-c''(e_1^n) < 0.$$

The consumers have wrong beliefs about the future because they do not anticipate the addiction. Before $t = 2$, they *believe* to choose effort in $t = 2$ to maximize

$$\max_{\hat{e}_2^n \in [0,1]} \hat{e}_2^n(\hat{a}_2^n x + v) - c(\hat{e}_2^n) - p_2^n \hat{a}_2^n.$$

The first-order condition with respect to $\hat{e}_2^n$ is given by

$$\begin{aligned} x\hat{a}_2^n + v - c'(\hat{e}_2^n) &\overset{!}{=} 0 \\ \frac{x\hat{a}_2^n + v}{c} &= \hat{e}_2^{n*}. \end{aligned}$$

The second-order condition is satisfied because

$$-c''(\hat{e}_2^n) < 0.$$

Firm Choice. The firm maximizes profits given the incentive constraints and the participation constraint. The profit maximization problem is

$$\begin{aligned} &\max_{L^n, p_1^n, p_2^n} L^n + p_1^n a_1^{n*} + p_2^n a_2^{n*} \\ \text{s.t. } (e_1^{n*}, a_1^{n*}) &= \arg\max_{e_1^n, a_1^n} u_1^n(e_1^n, a_1^n), \\ (e_2^{n*}, a_2^{n*}) &= \arg\max_{e_2^n, a_2^n} u_2^n(e_2^n, a_2^n), \\ (\hat{e}_2^{n*}, \hat{a}_2^{n*}) &= \arg\max_{e_2^n, a_2^n} \hat{u}_2^n(\hat{e}_2^n, \hat{a}_2^n), \\ 0 &\leq u_1^n(e_1^{n*}, a_1^{n*}) + \hat{u}_2^n(\hat{e}_2^{n*}, \hat{a}_2^{n*}) - L^n. \end{aligned}$$

We solve the rewritten version in which the firm chooses a_t^n and $\hat{a}_t^n$ and then chooses p_t^n to make the choices incentive compatible.

$$\begin{aligned} &\max_{L^n, a_1^n, a_2^n, \hat{a}_2^n} L^n + p_1^n a_1^n + p_2^n a_2^n \\ \text{s.t. } e_1^{n*} &= \arg\max_{e_1^n} u_1^n(e_1^n, a_1^n), \\ e_2^{n*} &= \arg\max_{e_2^n} u_2^n(e_2^n, a_2^n), \\ \hat{e}_2^{n*} &= \arg\max_{\hat{e}_2^n} \hat{u}_2^n(\hat{e}_2^n, \hat{a}_2^n), \\ 0 &\leq u_1^n(e_1^{n*}, a_1^n) + \hat{u}_2^n(\hat{e}_2^{n*}, \hat{a}_2^n) - L^n. \end{aligned}$$

The participation constraint must be binding. If it were not binding, the firm could increase L^n to increase profits. Hence, we get that

$$L^{n*} = e_1^{n*}(a_1^n x + v) - c(e_1^{n*}) + \hat{e}_2^{n*}(\hat{a}_2^n x + v) - c(\hat{e}_2^{n*}) - a_1^n p_1^n - \hat{a}_2^n p_2^n.$$

Substituting the binding participation constraint into the objective function gives

$$\max_{a_1^n, a_2^n, \hat{a}_2^n} \; e_1^{n*}(a_1^n x + v) - c(e_1^{n*}) + \hat{e}_2^{n*}(\hat{a}_2^n x + v) - c(\hat{e}_2^{n*}) + (a_2^n - \hat{a}_2^n)p_2^n.$$

Depending on how the firm sets p_1^n and p_2^n, there are 8 cases that could be profit maximizing as shown in Table 3.A.1:

Table 3.A.1. The 8 cases that are candidates for profit maximization

Case No.	a_1^{n*}	a_2^{n*}	$\hat{a}_2^{n*}$
i	0	0	0
ii	0	0	1
iii	0	1	0
iv	0	1	1
v	1	0	0
vi	1	0	1
vii	1	1	0
viii	1	1	1

We now show that either Case (vii) or Case (viii) maximizes profits, that is, the consumer makes IAP both in both periods. Since there are no costs of IAP, it is profit-maximizing to choose $a_1^n = a_2^n = 1$ given the same effort decisions.

$$\begin{aligned} \pi(a_1^n = 1, a_2^n = 1, \hat{a}_2^{n\prime}, \hat{e}_2^{s\prime}, e_1^{s\prime}) > \max\; \{&\pi(a_1^n = 1, a_2^n = 0, \hat{a}_2^{n\prime}, \hat{e}_2^{s\prime}, e_1^{s\prime}) \\ &\pi(a_1^n = 0, a_2^n = 1, \hat{a}_2^{n\prime}, \hat{e}_2^{s\prime}, e_1^{s\prime}), \\ &\pi(a_1^n = 0, a_2^n = 0, \hat{a}_2^{n\prime}, \hat{e}_2^{s\prime}, e_1^{s\prime})\} \quad \forall\; \hat{a}_2^{n\prime}, \hat{e}_2^{s\prime}, e_1^{s\prime}. \end{aligned}$$

The consumers' perceived effort choices also maximize the profit given that the firm can choose the fixed fee to extract the utility.

$$\begin{aligned} &\max_{e_1^n, \hat{e}_2^n} \; \pi(a_1^n = 1, a_2^n = 1, \hat{a}_2^{n\prime}, \hat{e}_2^n, e_1^n) \\ &\geq \pi(a_1^n = 1, a_2^n = 1, \hat{a}_2^{n\prime}, \hat{e}_2^{s\prime}, e_1^{s\prime}) \quad \forall\; \hat{a}_2^{n\prime}, \hat{e}_2^{s\prime}, e_1^{s\prime}. \end{aligned}$$

This shows that either Case (vii) or Case (viii) maximize profits.

Case 7: $a_1^{n*} = 1$, $a_2^{n*} = 1$, $\hat{a}_2^{n*} = 0$:. Given $a_1^{n*} = 1$, $a_2^{n*} = 1$ is incentive compatible iff

$$\begin{aligned} &e_2^{n*}(a_2^{n*} = 1)(x + v) - c(e_2^{n*}(a_2^{n*} = 1)) - \theta e_1^{n*}(1 - e_2^{n*}(a_2^{n*} = 1)) - p_2^n \\ &\geq e_2^{n*}(a_2^{n*} = 0)v - c(e_2^{n*}(a_2^{n*} = 0)) - \theta e_1^{n*}(1 - e_2^{n*}(a_2^{n*} = 0)) \\ &\Leftrightarrow p_2^n \leq \frac{x[xc + 2cv + 2\theta v + 2\theta x]}{2c^2}. \end{aligned}$$

$\hat{a}_2^{n*} = 0$ is incentive compatible iff

$$\begin{aligned}
&\hat{e}_2^{n*}(\hat{a}_2^n = 1)(x+v) - c(\hat{e}_2^{n*}(\hat{a}_2^n = 1)) - p_2^n \\
&< \hat{e}_2^{n*}(\hat{a}_2^n = 0)v - c(\hat{e}_2^{n*}(\hat{a}_2^n = 0)) \\
&\Leftrightarrow p_2^n > \frac{x(x+2v)}{2c}.
\end{aligned}$$

There exist incentive-compatible prices in the second stage because

$$\begin{aligned}
&\frac{x(x+2v)}{2c} \leq \frac{x[xc + 2cv + 2\theta v + 2\theta x]}{2c^2} \\
&\Leftrightarrow 0 \leq \frac{\theta x(v+x)}{c^2}.
\end{aligned}$$

$a_1^{n*} = 1$ is incentive compatible iff

$$\begin{aligned}
&e_1^{n*}(a_1^n = 1)(x+v) - c(e_1^{n*}(a_1^n = 1)) - p_1^n \\
&\geq e_1^{n*}(a_1^n = 0)v - c(e_1^{n*}(a_1^n = 0)) \\
&\Leftrightarrow p_1^n \leq \frac{x(x+2v)}{2c}.
\end{aligned}$$

The firm's profit is then given by

$$\begin{aligned}
\pi_7^n &= e_1^{n*}(x+v) - c(e_1^{n*}) + \hat{e}_2^{n*}v - c(\hat{e}_2^{n*}) + p_2^{n*} \\
&= \frac{v^2 + (x+v)^2}{2c} + \frac{x[xc + 2cv + 2\theta(x+v)]}{2c^2}.
\end{aligned}$$

Note that as long as the consumers make IAP in the second period, the profit is strictly increasing in the price. Therefore, the firm sets the highest possible price that makes the consumers indifferent between making IAP and not in the second period. This means that $p_2^{n*} = \frac{x(xc+2cv+2\theta(x+v))}{2c^2}$ maximizes the profit.

Case 8: $a_1^{n*} = 1$, $a_2^{n*} = 1$, $\hat{a}_2^{n*} = 1$:. Given $a_1^{n*} = 1$, $a_2^{n*} = 1$ is incentive compatible iff

$$p_2^n \leq \frac{x[xc + 2cv + 2\theta v + 2\theta x]}{2c^2}.$$

$\hat{a}_2^{n*} = 1$ is incentive compatible iff

$$p_2^n \leq \frac{x(x+2v)}{2c}.$$

$a_1^{n*} = 1$ is incentive compatible iff

$$p_1^n \leq \frac{x(x+2v)}{2c}.$$

The firm's profit is then given by

$$\begin{aligned}\pi_8^n &= e_1^{n*}(x+v) - c(e_1^{n*}) + \hat{e}_2^{n*}(x+v) - c(\hat{e}_2^{n*})\\ &= \frac{(x+v)^2}{c}.\end{aligned}$$

Case 7 (weakly) dominates Case 8 because

$$\begin{aligned}\frac{v^2+(x+v)^2}{2c} + \frac{x[xc+2cv+2\theta(x+v)]}{2c^2} &> \frac{(x+v)^2}{c}\\ \Leftrightarrow 2\theta x(x+v) &\geq 0.\end{aligned}$$

If $\theta > 0$ and $x > 0$, the inequality is strict. □

Proof of Proposition 3.3. The proof proceeds in three steps. First, we compute the first-best welfare outcome. Second, we compare the equilibrium outcome to the first-best outcome. Third, we compare the consumer surplus of the naive consumers to the one of the sophisticated consumers. First, consider the first-best welfare result. The first-best welfare result follows from maximizing welfare.

$$\max_{e_1,e_2,a_1,a_2} W = u_1(\cdot) + u_2(\cdot) + p_1a_1 + p_2a_2.$$

The first-order conditions are given by

$$\begin{aligned}\frac{\partial W}{\partial e_1} &= xa_1 + v - ce_1 - \theta(1-e_2) \stackrel{!}{=} 0,\\ \frac{\partial W}{\partial e_2} &= xa_2 + v - ce_2 + \theta e_1 \stackrel{!}{=} 0.\end{aligned}$$

The second order conditions are given by

$$\begin{aligned}\frac{\partial^2 W}{\partial e_1^2} &= -c < 0,\\ \frac{\partial^2 W}{\partial e_2^2} &= -c < 0,\\ \frac{\partial^2 W}{\partial e_1 \partial e_2} &= \theta.\end{aligned}$$

The determinant of the Hesse matrix is given by $c^2 - \theta^2 > 0$. It follows that the welfare function is a strictly concave function. This implies that the first-best effort choices are unique.

The FOCs are identical to the ones in the profit-maximization problem for sophisticates. Solving for $e_1^{fb}(a_1,a_2)$ and $e_2^{fb}(a_1,a_2)$ and substituting these into welfare gives

$$\max_{a_1,a_2} W = u_1(e_1^{fb}(a_1,a_2), e_2^{fb}(a_1,a_2), a_1) + u_2(e_1^{fb}(a_1,a_2), e_2^{fb}(a_1,a_2), a_2).$$

This shows that the welfare-maximization problem is identical to the profit-maximization problem with sophisticated consumers. Hence, the profit-maximization problem for sophisticates gives the first-best welfare results.

Second, we compare the first-best with the equilibrium. We showed that the equilibrium for sophisticates achieves the first-best welfare. To compare the profit-maximizing contract for naifs with the first-best result, we compare the effort choices and the IAP decision. The naive consumers make IAP in both periods, as do the sophisticated consumers. We show that the naive consumers choose higher efforts in both periods than the sophisticated consumers.

The naive consumer choose higher effort in $t = 1$ for $\theta > 0$ because

$$\begin{aligned} & e_1^{s*} \leq e_1^{n*} \\ \Leftrightarrow & \frac{c(x+v) + \theta(x+v-c)}{c^2 - \theta^2} \leq \frac{v+x}{c} \\ \Leftrightarrow & \theta(\theta + c)(x+v) \leq \theta c^2. \end{aligned}$$

Since $c^2 > (c + 2\theta)(x + v)$ by assumption, the above inequality holds true. The inequality is strict for $\theta > 0$.

The naive consumers choose higher effort in $t = 2$ for $\theta > 0$ because

$$\begin{aligned} & e_2^{s*} \leq e_2^{n*} \\ \Leftrightarrow & \frac{c(v+x) + \theta(x+v-\theta)}{c^2 - \theta^2} \leq \frac{(v+x)(c+\theta)}{c^2} \\ \Leftrightarrow & \theta^2(c+\theta)(x+v) \leq \theta^2 c^2. \end{aligned}$$

which is true since $c^2 > (c + 2\theta)(x + v)$ by assumption. The inequality is strict for $\theta > 0$.

Third, we compare the consumer surplus of naive and sophisticated consumers. It remains to show that the naive consumers are worse off than the sophisticated ones. Note that the consumer surplus of the sophisticated consumers is equal to the outside option, that is, 0. Therefore, it remains to show that the consumer surplus of the naive consumers is negative.

$$\begin{aligned} CS^{n*} &= u_1^n(e_1^{n*}, a_1^{n*}) + u_2^n(e_2^{n*}, a_2^{n*}) - L^{n*} \\ &= u_2^n(e_1^{n*}, a_1^{n*}) - \hat{u}_2^n(\hat{e}_2^{n*}, \hat{a}_2^{n*}) \\ &= \frac{\theta\left[-2c^2v - 2c^2x + 2cv^2 + 2cvx + 2cx^2 + \theta v^2 + 2\theta vx + \theta x^2\right]}{2c^3} \\ &< \frac{\theta\left[-2c^2v - 2c^2x + 2cv^2 + 2cvx + 2cx^2 + \theta v^2 + 2\theta vx + \theta x^2 + 2cvx\right]}{2c^3} \\ &= \frac{\theta(x+v)\left[-2c^2 + (2c+\theta)(x+v)\right]}{2c^3} < 0. \end{aligned}$$

which holds because $c^2 > (2c + \theta)(x + v)$ by assumption. □

Proof of Proposition 3.4. The proof proceeds in two steps. First, we characterize the equilibrium for sophisticated consumers and second we characterize the equilibrium for naive consumers.

Consider the sophisticated consumers. The rewritten firms' maximization problem is then given by

$$\begin{aligned}\max_{L^s,a_1^s,a_2^s} \quad & u_1^s(e_1^{s*},a_1^s) + u_2^s(e_2^{s*},a_2^s) - L^s \\ \text{s.t. } e_1^{s*} &= \arg\max_{e_1^s} \; u_1^s(e_1^s,a_1^s) + u_2^s(e_2^s,a_2^s), \\ e_2^{s*} &= \arg\max_{e_2^s} \; u_2^s(e_2^s,a_2^s), \\ 0 &\le L^s + p_1^s a_1^s + p_2^s a_2^s.\end{aligned}$$

As there is perfect competition, the constraint is binding and firms make zero profits. Substituting the binding constraint into the objective function gives

$$\max_{a_1^s,a_2^s} e_1^{s*}(a_1^s x + v) - c(e_1^{s*}) + e_2^{s*}(a_2^s x + v) - c(e_2^{s*}) - \theta e_1^{s*}(1 - e_2^{s*}).$$

Note that this is the same optimization problem than in the monopoly case with different fixed fee, L^{s*}. Hence, the equilibrium effort and IAP decision are the same as in the monopoly case.

Next, consider the naive consumers. The rewritten firms' maximization problem is given by

$$\begin{aligned}\max_{L^n,a_1^n,a_2^n} \quad & u_1^n(e_1^{n*},a_1^n) + \hat{u}_2^n(\hat{e}_2^{n*},\hat{a}_2^n) - L^n \\ \text{s.t. } e_1^{n*} &= \arg\max_{e_1^n} \; u_1^n(e_1^n,a_1^n) + \hat{u}_2^n(\hat{e}_2^n,\hat{a}_2^n), \\ e_2^{n*} &= \arg\max_{e_2^n} \; u_2^n(e_2^n,a_2^n), \\ \hat{e}_2^{n*} &= \arg\max_{\hat{e}_2^n} \; u_1^n(e_1^n,a_1^n) + \hat{u}_2^n(\hat{e}_2^n,\hat{a}_2^n), \\ 0 &\le L^n + p_1^n a_1^n + p_2^n a_2^n.\end{aligned}$$

As there is perfect competition, the constraint is binding and firms make zero profits. Substituting the binding constraint into the objective function gives

$$\max_{a_1^n,a_2^n,\hat{a}_2^n} e_1^{n*}(a_1^n x + v) - c(e_1^{n*}) + \hat{e}_2^{n*}(\hat{a}_2^n x + v) - c(\hat{e}_2^{n*}) + (a_2^n - \hat{a}_2^n)p_2^n.$$

Note that this is the same optimization problem than in the monopoly case with different fixed fee, L^{n*}. Hence, the equilibrium effort and IAP decision are the same as in the monopoly case. □

Proof of Proposition 3.5. The proof proceeds in two steps. First, we characterize the equilibrium for sophisticated consumers and second, for naive consumers.

First, consider sophisticated consumers. Under perfect competition, we showed above that we get the same allocation as in the monopoly but a different fixed fee. The firm maximizes the social surplus which then completely goes to the consumers. We now show which level of θ maximizes social surplus, that is, which firm type the consumers would ideally choose.

$$\max_{\theta}\ e_1^{s*}(x+v) - c(e_1^{s*}) + e_2^{s*}(x+v) - c(e_2^{s*}) - \theta e_1^{s*}(1-e_2^{s*}).$$

The first-order condition is given by

$$\frac{\partial e_1^{s*}}{\partial \theta}\left[x+v-c'(e_1^{s*}) - \theta(1-e_2^{s*})\right] + \frac{\partial e_2^{s*}}{\partial \theta}\left[x+v-c'(e_2^{s*}) + \theta e_1^{s*}\right] - e_1^{s*}(1-e_2^{s*})$$
$$= -e_1^{s*}(1-e_2^{s*}) \ \leq\ 0.$$

The inequality is strict if $e_1^{s*} > 0$. This means that for $e_1^{s*} > 0$ the sophisticated consumer would choose the firm type with the lowest θ and ideally $\theta = 0$.

Similar to the case of sophisticated consumers, we now show which level of θ maximizes the perceived consumer surplus when consumers are naive, that is, which firm type the naive consumers would ideally choose. Note that the objective function under perfect competition is the same than in a monopoly and the equilibrium allocations are also the same. Maximizing the objective function wrt to θ gives.

$$\max_{\theta}\ e_1^{n*}(x+v) - c(e_1^{n*}) + \hat{e}_2^{n*}v - c(\hat{e}_2^{n*}) + p_2^{n*}.$$

The first-order condition is given by

$$\frac{\partial e_1^{n*}}{\partial \theta}\left[x+v-c'(e_1^{n*})\right] + \frac{\partial \hat{e}_2^{n*}}{\partial \theta}\left[v-c'(\hat{e}_2^{n*})\right] + \frac{\partial p_2^{n*}}{\partial \theta}$$
$$= \frac{\partial p_2^{n*}}{\partial \theta}$$
$$= \frac{x(x+v)}{c^2} \ >\ 0.$$

Hence, the perceived utility is strictly increasing in θ. The consumer would like to choose the firm type with the highest θ and ideally $\theta = 1$. □

Proof of Proposition 3.6. The proof proceeds in two steps. First, we show that for any positive effort bound, the consumer surplus of the naive consumers is lower than the one of the sophisticated consumers. Second, we show that the effort bound that is equal to the equilibrium effort level of the sophisticated consumers induces the first-best result in equilibrium.

Note that the consumer surplus of the sophisticated consumer is equal to the outside option, that is, 0. It remains to show that the consumer surplus of the naive consumer is negative. The firm chooses the contract such that the perceived participation constraint binds which means that the perceived utility is 0. Hence, it is sufficient to show that the experienced utility, i.e. the consumer suprlus, is lower than the perceived utility.

Consider any positive upper bound $\bar{e}$. For any e_1^n and any a_1^n, the naive consumers believe to choose $\hat{e}_2^n$ and $\hat{a}_2^n$ to maximize utility.

This maximum always exists but might be equal to the bound. By the definition of a maximum the following holds.

$$\max_{\hat{e}_2^n,\hat{a}_2^n} \hat{u}_2^n(\hat{e}_2^n,\hat{a}_2^n) \geq \hat{u}_2^n(\hat{e}_2^n,\hat{a}_2^n) \ \forall \ \hat{e}_2^n,\hat{a}_2^n.$$

Furthermore, for the same IAP and effort choice, the perceived utility is higher than the experienced utility because of the additional addiction term in the experienced utility as long as $\theta > 0$ and $e_1^n > 0$.

$$\hat{u}_2^n(\hat{e}_2^n,\hat{a}_2^n) > u_2^n(\hat{e}_2^n,\hat{a}_2^n) \ \forall \ \hat{e}_2^n,\hat{a}_2^n.$$

The naive consumers chooses e_2^n and a_2^n to maximize experienced utility. Let e_2^{n*} and a_2^{n*} be values that maximize the experienced utility. We get that

$$0 = \max_{\hat{e}_2^n,\hat{a}_2^n} \hat{u}_2^n(\hat{e}_2^n,\hat{a}_2^n) \geq \hat{u}_2^n(e_2^{n*},a_2^{n*}) > u_2^n(e_2^{n*},a_2^{n*}) = \max_{e_2^n,a_2^n} u_2^n(e_2^n,a_2^n).$$

However, note that if $e_1^n = 0$ or if $\theta = 0$, then the perceived and the experienced utility collapse because the addiction term disappears. In this case, the naive consumers are as well off as the sophisticated ones.

Next, consider the effort bound that is equal to the equilibrium effort level of the sophisticated consumers. Note that the utility maximization problem in the second stage is equal to the one without the effort bound because the effort bound only affects first-stage effort. Note that we consider a bound that is contingent on the IAP decision. If the naive consumer makes an IAP, we consider the bound being equal to the sophisticated effort choice making the same IAP decision. This means that the effort bound differs depending on if the naif makes an IAP or not. In particular, the bound is defined as

$$\bar{e}(a_1^n = 0) = \frac{vc + (x+v)\theta - \theta c}{c^2 - \theta^2},$$
$$\bar{e}(a_1^n = 1) = \frac{(x+v)(c+\theta) - \theta c}{c^2 - \theta^2}.$$

In $t = 1$, the naive consumers would like to choose e_1^n by maximizing utility such that

$$\frac{xa_1^n + v}{c} = e_1^{n*}.$$

We have that $\bar{e}(a_1^{n*}) < e_1^{n*}$ irrespective of the IAP decision. Since the naifs' utility function is concave (as has been show in the Proof of Proposition 3.2), we then get that $e_1^{n*} = \bar{e}$.

Again we rewrite the maximzation problem such that the firm chooses a_t and $\hat{a}_t$ and we show that there are prices such that these choices are incentive compatible. The rewritten profit maximization problem is

$$\begin{aligned}
\max_{L^n, a_1^n, a_2^n} \quad & L^n + p_1^n a_1^n + p_2^n a_2^n \\
\text{s.t. } e_1^{n*} &= \bar{e}(a_1^n), \\
e_2^{n*} &= \arg\max_{e_2^n} \; u_2^n(e_2^n, a_2^n), \\
\hat{e}_2^{n*} &= \arg\max_{e_2^n} \; \hat{u}_2^n(\hat{e}_2^n, \hat{a}_2^n), \\
0 &\leq u_1^n(e_1^{n*}, a_1^n) + \hat{u}_2^n(\hat{e}_2^{n*}, \hat{a}_2^n) - L^n.
\end{aligned}$$

Note that the participation constraint must be binding. If it were not binding, the firm could increase L^n to increase profits. Substituting the binding participation constraint into the objective function gives

$$\max_{a_1^n, a_2^n} e_1^{n*}(a_1^n x + v) - c(e_1^{n*}) + \hat{e}_2^{n*}(\hat{a}_2^n x + v) - c(\hat{e}_2^{n*}) + (a_2^n - \hat{a}_2^n)p_2^n.$$

Depending on how the firm sets a_1^n and a_2^n, Table 3.A.1 shows the 8 cases that could be profit-maximizing. Using the same argument as in Proposition 3.2, Case (vii) or Case (viii) are profit-maximizing.

Case 7: $a_1^{n*} = 1$, $a_2^{n*} = 1$, $\hat{a}_2^{n*} = 0$:. Given $a_1^{n*} = 1$, $a_2^{n*} = 1$ is incentive compatible iff

$$\begin{aligned}
& e_2^{n*}(a_2^{n*} = 1)(x+v) - c(e_2^{n*}(a_2^{n*} = 1)) - \theta e_1^{n*}(1 - e_2^{n*}(a_2^{n*} = 1)) - p_2^n \\
\geq\ & e_2^{n*}(a_2^{n*} = 0)v - c(e_2^{n*}(a_2^{n*} = 0)) - \theta e_1^{n*}(1 - e_2^{n*}(a_2^{n*} = 0)) \\
\Leftrightarrow\ & p_2^n \leq \frac{x(2c^2 v + c^2 x - 2c\theta^2 + 2c\theta v + 2c\theta x + \theta^2 x)}{2c(c^2 - \theta^2)}.
\end{aligned}$$

$\hat{a}_2^{n*} = 0$ is incentive compatible iff

$$\begin{aligned}
& \hat{e}_2^{n*}(\hat{a}_2^n = 1)(x+v) - c(\hat{e}_2^{n*}(\hat{a}_2^n = 1)) - p_2^n \\
<\ & \hat{e}_2^{n*}(\hat{a}_2^n = 0)v - c(\hat{e}_2^{n*}(\hat{a}_2^n = 0)) \\
\Leftrightarrow\ & p_2^n > \frac{x(x+2v)}{2c}.
\end{aligned}$$

There exist incentive-compatible prices in the second stage because

$$\begin{aligned}
\frac{x(x+2v)}{2c} &< \frac{x[xc + 2cv + 2\theta v + 2\theta x]}{2c^2} \\
\Leftrightarrow 0 &< \frac{\theta x(c(v + x - \theta) + \theta(v + x))}{2c(c^2 - \theta^2)}.
\end{aligned}$$

which holds true when $x > 0$ and $\theta > 0$.

$a_1^{n*} = 1$ is incentive compatible iff

$$\begin{aligned} & e_1^{n*}(a_1^n = 1)(x+v) - c(e_1^{n*}(a_1^n = 1)) - p_1^n \\ & \geq e_1^{n*}(a_1^n = 0)v - c(e_1^{n*}(a_1^n = 0)) \\ & \Leftrightarrow p_1^n \leq \frac{c(2v+x)(c^2-2\theta^2)+2\theta^3(c-v-x)}{2(c-\theta)^2(c+\theta)^2}. \end{aligned}$$

The firm's profit is then given by

$$\begin{aligned} \pi_7^n &= e_1^{n*}(x+v) - c(e_1^{n*}) + \hat{e}_2^{n*}v - c(\hat{e}_2^{n*}) + p_2^{n*} \\ &= \frac{-c^2(c\theta-(c+\theta)(x+v))^2 - 2c(c^2-\theta^2)(x+v)(c\theta-(c+\theta)(x+v))}{2c(c^2-\theta^2)^2} \\ &+ \frac{v^2(c^2-\theta^2)^2 + x(c^2-\theta^2)(2c^2v+c^2x-2c\theta^2+2c\theta v+2c\theta x+\theta^2 x)}{2c(c^2-\theta^2)^2}, \end{aligned}$$

where the firm sets the highest possible price: $p_2^{n*} = \frac{x(2c^2v+c^2x-2c\theta^2+2c\theta v+2c\theta x+\theta^2 x)}{2c(c^2-\theta^2)}$ to maximize the profits.

Case 8: $a_1^{n*} = 1$, $a_2^{n*} = 1$, $\hat{a}_2^{n*} = 1$:. Given $a_1^{n*} = 1$, $a_2^{n*} = 1$ is incentive compatible iff

$$p_2^n \leq \frac{x(2c^2v+c^2x-2c\theta^2+2c\theta v+2c\theta x+\theta^2 x)}{2c(c^2-\theta^2)}.$$

$\hat{a}_2^{n*} = 1$ is incentive compatible iff

$$p_2^n \leq \frac{x(x+2v)}{2c}.$$

$a_1^{n*} = 1$ is incentive compatible iff

$$p_1^n \leq \frac{c(2v+x)(c^2-2\theta^2)+2\theta^3(c-v-x)}{2(c-\theta)^2(c+\theta)^2}.$$

The firm's profit is then given by

$$\begin{aligned} \pi_8^n &= e_1^{n*}(x+v) - c(e_1^{n*}) + \hat{e}_2^{n*}(x+v) - c(\hat{e}_2^{n*}) \\ &= \frac{c^2(-c^2+\theta^2)(c\theta-(c+\theta)(x+v))^2(c^2-\theta^2)^3(v+x)^2}{2x(c^2-\theta^2)^3} \\ &- \frac{2c(c^2-\theta^2)^2(x+v)(c\theta-(c+\theta)(v+x))}{2x(c^2-\theta^2)^3}. \end{aligned}$$

Case 7 (weakly) dominates Case 8 because

$$\begin{aligned} & \pi_7^n - \pi_8^n \\ =& \frac{\theta x(-c\theta+cv+cx+\theta v+\theta x)}{c(c^2-\theta^2)} \geq 0. \end{aligned}$$

If $x > 0$ and $\theta > 0$, the inequality is strict. The consumers then choose the efficient efforts in both periods

$$\begin{aligned} e_1^{n*} &= \frac{c(x+v) + \theta(x+v-c)}{c^2 - \theta^2} = e_1^{fb}, \\ e_2^{n*} &= \frac{c(x+v) + \theta(x+v-\theta)}{c^2 - \theta^2} = e_2^{fb}. \end{aligned}$$

□

Proof of Proposition 3.7. This proof proceeds in two steps. First, we show that a spending cap cannot induce the first-best in equilibrium when consumers are naive. Second, we show that a spending cap cannot make naive consumers equally well off as the sophisticated ones. First, let $\kappa \geq p_1^{n*} + p_2^{n*} + L^{n*}$ which is the equilibrium profit when consumers are naive. Then, the spending cap does not have any effect and the first-best cannot be achieved. Second, let $\kappa < p_1^{n*} + p_2^{n*} + L^{n*}$. The firm wants to maximize its profit and therefore the spending cap becomes binding. As long as the spending cap is binding, the firm is indifferent between the contracts it offers to the consumers. Hence, we consider the contract that maximizes the experienced utility because this maximizes the consumer surplus. The rewritten maximization problem then becomes

$$\begin{aligned} \max_{L^n, a_1^n, a_2^n, \hat{a}_2^n} \quad & u_1^n(e_1^n, a_1^n) + u_2^n(e_2^n, a_2^n) - L^n \\ \text{s.t. } e_1^{n*} &= \arg\max_{e_1^n} \; u_1^n(e_1^n, a_1^n), \\ e_2^{n*} &= \arg\max_{e_2^n} \; u_2^n(e_2^n, a_2^n), \\ \hat{e}_2^{n*} &= \arg\max_{\hat{e}_2^n} \; \hat{u}_2^n(\hat{e}_2^n, \hat{a}_2^n), \\ 0 &\leq u_1^n(e_1^{n*}, a_1^n) + \hat{u}_2^n(\hat{e}_2^{n*}, \hat{a}_2^n) - L^n, \\ \kappa &= L^n + p_1^n a_1^n + p_2^n a_2^n. \end{aligned}$$

Substituting the binding profit constraint into the objective function then gives

$$\max_{a_1^n, a_2^n, \hat{a}_2^n} \quad e_1^{n*}(a_1^n x + v) - c(e_1^n) + \hat{e}_2^n(\hat{a}_2^n x + v) - c(\hat{e}_2^n) + (a_2^n - \hat{a}_2^n)p_2^n - \kappa.$$

This is the same objective function than in the Proof of Proposition 3.2 shifted down by κ. therefore, the equilibrium efforts are the same than in the monopoly without the spending cap which implies that the spencing cap does not induce first-best effort for $\theta > 0$.

Second, we show that a spending cap cannot make naive consumers equally well off than sophisticated ones. Note that the firm makes weakly higher profits with a spending cap with naive consumers than with sophisticated ones. We showed that the sophisticated consumers choose first-best effort such that the first-best welfare is achieved whereas first-best is not achieved for naive consumers. Since welfare equals producer surplus (profits) plus consumer surplus, this implies that the naive consumers must be worse off than the sophisticated ones. □

Proof of Proposition 3.8. The proof proceeds in two steps. First, we show that naive consumers choose the effort of the sophisticated one. Second, we show that the profit maximization problem is identical to the one of the sophisticated consumer.

Let the effort bound be equal to the equilibrium effort of the sophisticated consumer.

$$\begin{aligned}\bar{e}(a_1^n = 0) &= \frac{vc + (x+v)\theta - \theta c}{c^2 - \theta^2},\\ \bar{e}(a_1^n = 1) &= \frac{(x+v)(c+\theta) - \theta c}{c^2 - \theta^2}.\end{aligned}$$

As in Proposition 3.6, this implies that the naive consumers choose the first-best effort in both periods. The profit maximization problem then becomes.

$$\begin{aligned}\max_{L^n, a_1^n, a_2^n, \hat{a}_2^n} \quad & u_1^n(e_1^n, a_1^n) + u_2^n(e_2^n, a_2^n) - L^n\\ \text{s.t.} \quad e_1^{n*} &= \bar{e}(a_1^{n*}),\\ e_2^{n*} &= \arg\max_{e_2^n} \; u_2^n(e_2^n, a_2^n),\\ \hat{e}_2^{n*} &= \arg\max_{\hat{e}_2^n} \; \hat{u}_2^n(\hat{e}_2^n, \hat{a}_2^n),\\ 0 &\leq u_1^n(e_1^{n*}, a_1^n) + \hat{u}_2^n(\hat{e}_2^{n*}, \hat{a}_2^n) - L^n,\\ \kappa &\geq L^n + p_1^n a_1^n + p_2^n a_2^n,\end{aligned}$$

where $\kappa = L^{s*} + p_1^{s*} + p_2^{s*}$ is the equilibrium spending of the sophisticated consumers.

Proposition 3.6 shows that this effort regulation leads to the first-best but cannot make naive consumers as well off as the sophisticated ones. As a consequence, firms make higher profits when consumers are naive than when they are sophisticated. Since the firm can make higher profits in a market with naive consumers than in a market with sophisticated ones, the spending cap becomes binding. Substituting the binding spending cap into the objective function gives

$$\max_{a_1^n, a_2^n, \hat{a}_2^n} \bar{e}(a_1^n x + v) - c(\bar{e}) + e_2^{n*}(a_2^n x + v) - c(e_2^{n*}) - \theta \bar{e}(1 - e_2^{n*}) - \kappa.$$

Since the effort choices are the same than for sophisticated consumers, the maximization problem is the same than the one with sophisticated consumers. Hence, the equilibrium is first best. Furthermore, since the consumers choose the same allocations and the same spending as the sophisticated consumers, the naive consumers are as well off as the sophisticated ones. □

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