Essays on Macroeconomic Consequences of Financial Frictions

Inaugural-Dissertation
zur Erlangung des Grades eines Doktors
der Wirtschafts- und Gesellschaftswissenschaften
durch die
Rechts- und Staatswissenschaftliche Fakultät
der Rheinischen Friedrich-Wilhelms-Universität
Bonn

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März 2006, Bonn

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Acknowledgements

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Chapter 1

General Introduction

Considerable empirical evidence has demonstrated strong linkages between the financial sector and the real production sector of an economy. Mishkin (1978) and Bernanke (1983) provide empirical evidence showing that financial factors (e.g., the decline in bank credit) played a more dominant role in the Great Depression than monetary factors (e.g., changes in the monetary aggregates). Using a data set for 28 manufacturing industries in over 100 countries from 1963 to 1999, Braun and Larrain (2005) show that industries with more external financing suffer more during the economic recession than those with less external financing. At the same time, industries with significant external financing are more strongly affected in recessions when they are located in countries with poorer financial contractibility, and when their assets are softer or less protective of financiers than similar industries in other countries. By summarizing the existing empirical literature on the role of financial factors in investment decisions, Hubbard (1998) finds that capital market imperfections have significant effects on firm decisions. Levine (forthcoming) surveys the relationship between financial development and economic growth.

A huge theoretical literature has emerged since 1970s for the understanding of the microeconomic origins and macroeconomic consequences of financial frictions. This dissertation presents three theoretical essays on financial frictions and macroeconomic fluctuations in a closed economy and in a small open economy. In this chapter, the relevant literature is reviewed in section 1.1. More comprehensive surveys of related issues can be found in Freixas and Rochet (1997); Reichlin (2004); Tirole (2006). The structure of this dissertation is briefly described in section 1.2.

1.1 Origins and Consequences of Financial Frictions

Today's literature derives financial constraints endogenously from optimal financial contracting and a variety of information problems in corporate finance. Entrepreneurs finance their investment using own funds and external funds. Due to endogenous financial frictions, entrepreneurs are constrained in obtaining external funds and financial constraints may vary along the business cycle. This has profound effects on macroeconomic aggregates, e.g., output, consumption, labor, etc, and their dynamics. Financial frictions can also result in endogenous business cycles in the sense that the economy fluctuates even without exogenous shocks.

1.1.1 Information Problems and Financial Contracting

Subsection 1.1.1.1 summarizes models in which the form of credit contracts is taken as given and the borrowers will repay their liabilities if they are able to do so. Subsection 1.1.1.2 summarizes models in which agents are allowed to select the best contractual arrangement and borrowers are unwilling to repay even if they can.

1.1.1.1 Information Asymmetries at the Time of Contracting

Jaffee and Russell (1976) and Stiglitz and Weiss (1981) are among the first contributions to a theory of financial frictions based on first principles. In their models, there are two types of borrowers with different probabilities of default or the projects of borrowers have different probabilities of success. Compared to outside financiers, borrowers have superior information regarding their own characteristics or their choices of projects. It could be very costly or even impossible for outside financiers to collect such information. In models with heterogenous borrowers, a rise in the interest rate may attract borrowers with high probability of default and drive away those with low probability of default; in models with heterogenous projects, a rise in the interest rate may induce borrowers to choose more risky projects. Thus, the ex post repayment might be even less than in the case of the low interest rate. As a result, the lenders have to set the interest rate at a low level and reject some identical loan applications entirely, given that the investment has a fixed size. While, as shown in Bester (1985, 1987), if lenders can actively use screening devices, e.g., collateral requirements, together with the interest rate to sort out heterogenous borrowers, credit rationing may not be a robust equilibrium.

In the model with competitive credit market, Holmstrom and Tirole (1997,

1998) consider the case in which borrowers can choose among projects with different riskiness and lenders cannot observe project choices of the borrowers. In order to induce the borrowers to choose the less risky project, the optimal financial contract must provide the borrowers with a fraction of the project outcomes as incentive. As the borrowers cannot fully pledge their project outcomes for external funds, they must put down own funds as net worth to fill in the gap between total investment and loans. In other words, loans might be proportional to entrepreneurial net worth. Thus, credit rationing in the form of denying credit to some borrowers entirely (Stiglitz and Weiss 1981) does not occur here. The size of the loan is rationed.

1.1.1.2   Information Asymmetries at the Time of Repayment

Townsend (1979) considers a two-agent model in which the borrower's project has exogenously random output and the verification of the project is costly for the lender but costless for the borrower. The borrower has the incentive to misreport the project output. The optimal contract resembles the debt contract: the borrower repays a fixed amount in the good situations and announces bankrupt in the bad situations. Gale and Hellwig (1985) analyze the impact of costly state verification in a one-period model with competitive capital market and claim that debt contracts are optimal and incentive compatible.

Hart and Moore (1994) consider the case in which the entrepreneur cannot commit not to withdraw his human capital from the project. The possibility of a default or runaway puts an upper bound on his total future indebtedness at any date. The durability and specificity of project assets serve to mitigate the entrepreneur's incentive for renegotiation during the long-term debt relationship. Lacker (2001) provides the necessary and sufficient conditions under which collateralized debt is the optimal contract: the borrower values the collateral good more than the lender does; otherwise the optimal contract does not resemble debt. Using a large sample of manufacturing firms drawn from COMPUSTAT between 1985 to 2000, Almeida and Campello (2005) show that asset tangibility increases investment-cash flow sensitivities for financially constrained firms, while no such effects are observed for unconstrained firms. This empirical result supports to some extent the role of tangible assets as collateral for mitigating enforcement problems.

1.1.2   Financial Frictions and Business Cycles

Fisher's debt-deflation theory (Fisher 1933) is among the first classic contributions showing the consequences of borrowing constraints in the Great Depression. The

idea that credit market imperfections played a key role in the transmission of cyclical shocks to the economy was an important part of the monetarist approach in the 1970s (Brunner and Meltzer 1976; Meltzer 1995). Parallel to the literature on microeconomic origins of financial frictions, a booming literature has analyzed the impact of endogenous financial frictions on business cycles since the end of 1980s.

1.1.2.1   Balance-Sheet Channel vs. Lending Channel

Bernanke and Gertler (1989) develops an overlapping generations model in which financial frictions arise from costly state verification à la Townsend (1979). They show that financial frictions can enhance the propagation of productivity shocks. More specifically, shocks that affect entrepreneurial net worth (as in a debt-deflation) can initiate fluctuations. Carlstrom and Fuerst (1997) embed the core model of Bernanke and Gertler (1989) into a computable general equilibrium framework and analyze the quantitative effects of financial frictions on business fluctuations. A principal conclusion is that their model with financial frictions replicates the empirical fact that output growth displays positive autocorrelation at short horizons (Cogley and Nason 1995), because it takes time for entrepreneurs to accumulate net worth before they can expand their production scale. Bernanke, Gertler, and Gilchrist (1999) incorporate the dynamic structure of Carlstrom and Fuerst (1997) into a new Keynesian framework with sticky prices and study the effects of monetary shocks on business cycles. They find that financial frictions help explain both the strength of the economy's response to monetary policy and the tendency for policy effects to persist even after interest rates have returned to normal, as commonly observed in the VAR analysis. However, they allow for investment delays in order to replicate a hump-shaped output response to monetary shocks. In contrast, Linnemann and Schabert (2003) show that the interactions between financial frictions and the nominal wage rigidity can help generate a smoothed hump-shaped output response to monetary shocks.

   Kiyotaki and Moore (1997) and Kiyotaki (1998) incorporate Fisher's debt deflation theory into a dynamic general equilibrium framework. Due to inalienability of human capital à la Hart and Moore (1994), borrowers cannot precommit to repay their debts and lenders are unable to appropriate the product of the borrowers' labor. Thus, physical assets are not only factors of production but also serve as collateral for loans. The dynamic interaction between asset prices and credit limits helps explain the large persistent fluctuations in output and asset prices. However, Cordoba and Ripoll (2004) use more conventional functional form (concave preference and production function) and show that collateral constraints in Kiyotaki

and Moore (1997) by themselves are not enough to account for the large output fluctuations observed in the data.

Iacoviello (2005) introduces nominal debt contracts with collateral constraints for both entrepreneurs and a subset of the households into the new-Keynesian framework (Bernanke, Gertler, and Gilchrist 1999). On the one hand, nominal debt allows the model to replicate the hump-shaped dynamics of spending to an inflation shock; on the other hand, collateral effects on entrepreneurs and some households allow matching the positive response of spending to a housing price shock. Given that Gali (2004) and Ireland (2004) have stressed the role of nontechnological and nonmonetary disturbances in understanding business fluctuations, the improvements in the model's ability to reflect short-run dynamic properties are important.

In a two-country general equilibrium model, Iacoviello and Minetti (forthcoming) focus on the difference between domestic and foreign lenders. As foreign lenders are less informed of domestic legal system and market structure than domestic lenders, their debt enforcement technologies are inferior to those of domestic lenders. Changes in the composition of foreign and domestic credit help explain one of the important empirical facts in the international business cycles: the comovement of output across countries, which standard open-economy real business cycles models (Backus, Kehoe, and Kydland 1992) fail to predict. Paasche (2001) shows in a three-country model that a crisis in a country can spread to another seemingly unrelated country via the terms-of-trade channel. A temporary negative shock to the terms of trade reduces the net worth of domestic entrepreneurs. Due to financial constraints, the foreign lending to domestic entrepreneurs declines. Then, capital flow out of the economy and the current account deteriorates. His model helps explain the adverse effect of the Asian and Russian crises on Latin America in the late 1990s, given that the direct linkages (e.g., foreign trade) between these regions are minimal.

Building on the principal-agent setting of Holmstrom and Tirole (1997), Chen (2001) analyzes the dynamic interactions among bank capital, asset prices and economic activity. In his model, entrepreneurs can choose among projects with different riskiness and finance their projects using bank loans. At the same time, banks can monitor only some of the entrepreneurs' project choices. Thus, entrepreneurial net worth is essential for loans and total investment. As the monitoring activities of banks are costly and unobservable, banks must keep a minimum amount of own capital in order to ensure their depositors that they will monitor the entrepreneurs' projects. In the event of a negative productivity shock, a fall in asset prices affects both bank capital and entrepreneurial net worth. Thus, bank loans and entrepreneurs' investment are squeezed by a higher bank capital − asset ratio for lending and a stricter collateral requirement for borrowing. The model helps

explain why banking crises often coincide with depression in the asset markets.

Kato (forthcoming) embeds the moral hazard problem à la Holmstrom and Tirole (1998) into a computable dynamic general equilibrium model. Corporate demand for liquidity from a financial intermediary, e.g., credit line, is endogenously determined and procyclical, while the degree of liquidity dependence (the ratio of liquidity demand over corporate investment) is counter-cyclical. These patterns are consistent with the empirical evidence in the "lending view" literature.

In contrast to the common wisdom that financial frictions help amplify the effects of small shocks on macroeconomic aggregates, Bacchetta and Caminal (2000) show that credit constraints might also serve to dampen the effects of shocks. Unobservable project choices result in credit constraints. Exogenous shocks can affect the allocation of funds between constrained and unconstrained firms. However, the composition effect can either amplify or dampen the shock effects, depending on the types of shocks. This helps explain the lack of systematic evidence on aggregate impacts of financial frictions.

1.1.2.2 Endogenous Credit Cycles

Models mentioned above show that financial frictions have profound effects on the dynamic responses of aggregate variables to exogenous shocks. As a complement, Suarez and Sussman (1997) present a two-period endogenous reversion mechanism in a dynamic extension of the moral hazard model of Stiglitz and Weiss (1981). Even without exogenous shocks, high quantities in boom depress prices and create the liquidity shortage that increase the propensity to default. Thus, the boom ends in a bust; vice verse.

Matsuyama (2005) analyzes how changes in the composition of the credit to heterogenous investment projects can generate endogenous credit cycles. The projects differ in productivity, in the investment requirement (the setup costs), and in the severity of agency problem. Borrowers are able to pledge only up to a fraction of their project revenues for external financing. A current movement in borrower net worth causes the composition of the credit to shift between heterogenous investment projects, which affect borrower net worth in the next period. Thus, the endogenous dynamic interaction between aggregate investment and borrower net worth can help replicate a variety of nonlinear phenomena, such as credit traps, credit collapse, leapfrogging, credit cycles, etc. Matsuyama (2004) uses the nonlinear dynamics in the aggregate investment and borrower net worth to study the causes and nature of endogenous credit cycles. The model with heterogenous investment projects can generate asymmetric fluctuations. After a long and slow recovery from a recession,

the economy enters into a rapid expansion, followed by possibly a period of high volatility. Afterwards, the economy plunges into a recession again.

1.2 Structure of the Dissertation

The main part of this dissertation consists of three chapters, each dealing with one aspect of macroeconomic consequences of financial frictions. We start in chapter two with the analysis of the dynamic interactions between financial frictions and capital reallocation in the production of intermediate goods in a closed real economy. In chapter three, we introduces financial frictions in the production of both intermediate and capital goods and show that dual financial frictions become a robust mechanism through which aggregate output responds to productivity shocks in an amplified and hump-shaped fashion. Finally, we shift our attention to a small open economy in chapter four and study how foreign borrowing can affect production efficiency and macroeconomic volatility in countries with domestic financial frictions and different degrees of foreign investor protection. We collect the main results and point out the directions for future research in chapter five.

1.2.1 Financial Frictions and Macroeconomic Fluctuations

Chapter two lays out the basic structure of the model with financial frictions which is then used in the following chapters. This chapter is motivated by the current literature on explaining empirical evidence using RBC models. Although the standard RBC models match some characteristics of the empirical data successfully, one of the well-known deficiencies of the canonical RBC model is the lack of a sufficient propagation and amplification mechanism,[1] i.e., it fails to reproduce the persistent and hump-shaped output responses to a transitory shock to total factor productivity (TFP, hereafter) commonly found in the data (Cogley and Nason 1993, 1995).

In order to enhance the internal propagation and amplification mechanism, researchers have included additional endogenous state variables by incorporating various frictions into the canonical RBC framework. However, the internal mechanism in these models is still too weak to match the empirical evidence quantitatively. In contrast, our model introduces financial frictions between agents with different production technologies and achieves a balance between propagation and amplification.

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[1] "Propagation" refers to the mechanism through which a transitory productivity shock generates positive autocorrelation in aggregate output, while "amplification" refers to the mechanism through which relatively small shocks result in large output fluctuations.

In our model, some agents are more productive than other agents and so they borrow from others. Due to moral hazard, agent with high productivity are subject to financial frictions and their net worth is essential for their borrowing capacity. In the event of a positive productivity shock, their projects become more profitable. However, it takes time for them to accumulate net worth before they can fully exploit the profit opportunity. As one kind of non-trivial real imperfections, financial frictions help explain the delayed responses of aggregate output to shocks. Meanwhile, as agents with high productivity accumulate net worth over time and borrow more to purchase capital goods, their excess demand for loans pushes up the interest rate so that agents with low productivity prefer to lend more and invest less in capital goods themselves. The additional channel of capital reallocation has profound composition effect on aggregate output and the responses of aggregate output to shocks are more amplified than in models without this additional channel. Altogether, financial frictions in our model result in a delayed and amplified output responses, which are consistent with the empirical evidence provided by Andolfatto (1996) and Cogley and Nason (1995).

The approach of quadratic capital adjustment costs is commonly adopted in the literature to capture the empirical evidence of slow adjustment of durable capital goods. However this modeling approach has an "unrealistic" feature that the price of capital goods stays persistently away from its steady state value as long as the demand for capital goods exceeds its steady state value. If we adopt this approach to model the upward-sloping capital supply curve and time-varying price of capital in our basic setting, the price of capital stays away from its steady state value for many periods in the event of productivity shocks, and the resulting capital gains speed up the capital reallocation process. Thus, output responses are more amplified but less delayed than in the case of time-constant prices of capital. More specifically, the approach of costly capital adjustment serves to exaggerate the responses of the price of capital goods. As a result, it strengthens the amplification mechanism at the expense of weakening the propagation mechanism related to financial frictions. In this sense, we should be aware of the side effect of the approach of costly capital adjustment and this approach might not be appropriate in some circumstances.

1.2.2   Dual Financial Frictions

After pointing out the side effect of the approach of costly capital adjustment, we propose financial frictions in the production of capital goods as an alternative approach to the modeling of time-varying prices of capital. Our approach helps capture the empirical feature that the supply of capital goods is quite inelastic in

the short run but becomes elastic and adapts to the demand for capital goods in the medium run. Thus, in the event of exogenous shocks, the price of capital goods stays away from its steady state value only for a few periods after the shock and converges to its steady state value faster than in the case of costly capital adjustment.

We allow financial frictions in the production of both intermediate and capital goods. In the production of intermediate goods, some agents are more productive than others. Due to moral hazard, the net worth of the more productive agents is essential for their borrowing capacity and thus their demand for capital goods. Similarly, in the production sector of capital goods, the projects of capital goods producers are profitable in the sense that their expected rate of return exceeds the interest rate. Due to moral hazard, the net worth of producers of capital goods is essential for their borrowing capacity and then the aggregate supply of capital goods. Thus, both the demand for and the supply of capital goods respond to exogenous shocks less efficiently than in the frictionless case. A rise in the price of capital goods has positive effects on the net worth of both capital goods producers and the more productive agents in the production sector of intermediate goods. In this sense, the price of capital goods responds to exogenous shocks in a non-trivial way.

In the event of a positive productivity shock, capital goods producers are subject to credit constraints and the supply of capital goods cannot meet the demand. Thus, the price of capital goods rises to clear the market. As producers of capital goods actively accumulate their net worth in order to exploit the profit opportunity, it takes only a few period that the supply of capital goods adapts to the excess demand. Therefore, the positive responses of the price of capital goods to shocks are less persistent than in the case of costly capital adjustment. Meanwhile, capital gains are small and have limited effects on the net worth of agents with high productivity in the production sector of intermediate goods and aggregate output responds to shocks in a more amplified and hump-shaped fashion. In this sense, the approach of modeling time-varying prices of capital goods via financial frictions helps reinforce the amplification mechanism without weakening the propagation mechanism.

1.2.3 Domestic and Foreign Financial Frictions

According to neoclassical models, both domestic and foreign agents can benefit from financial opening. Investors can share idiosyncratic risk globally and capital can flow to the countries with the highest productivity (Stulz 2005). However, financial opening could have uneven welfare effects on different types of domestic agents and this feature cannot be analyzed in the conventional representative-agent model.

We extend our basic model with heterogenous agents into a small-open-economy

framework. Domestic agents with high productivity borrow from domestic agents with low productivity. Meanwhile, domestic agents would like to borrow abroad, given that the foreign interest rate is smaller than the domestic interest rate. As foreign lenders are less informed of the economic activities of domestic borrowers than domestic lenders are, domestic agents have to pledge their physical assets as collateral to foreign lenders. Additionally, foreign lenders are normally less familiar with the domestic asset market than domestic lenders are. Thus, foreign lenders have to pay additional premium in liquidating the collateral assets handed over by bankrupt borrowers. Ex ante, foreign borrowing is overcollateralized in the sense that domestic borrowers pledge their physical assets to foreign investors but only get foreign funds at the amount of a fraction of the collateral value.

The degree of overcollateralization depends mainly on the efficiency of the domestic legal system and market structure. In countries with better protection of foreign investors or more efficient legal system and market structure, domestic agents can pledge their physical assets for more foreign borrowing. Cheap foreign funds facilitate the reallocation of productive assets from domestic agents with low productivity to those with high productivity. As a result, asset prices are higher, domestic production is more efficient, and aggregate output is higher in such countries. Domestic agents with high productivity benefit strictly from asset-backed foreign borrowing. While, due to substitution effect, domestic agents with low productivity, who are domestic lenders, have fewer financial assets (domestic loans) as well as fewer physical assets in the long run. In other words, their wealth is smaller in such countries and their welfare is strictly lower. In this sense, aggregate output might not be an appropriate measure for social welfare in models with heterogenous agents.

This chapter also analyzes how better protection of foreign investors can affect macroeconomic volatility via more foreign borrowing. Theory predicts that financial opening should lower consumption volatility while raise investment volatility, if most shocks are country-specific and transitory. However, the empirical literature cannot provide statistically significant evidence on the relationship between financial openness and macroeconomic volatility (Razin and Rose 1994). According to the dynamic analysis of our model, the volatilities of major macroeconomic aggregates, e.g., output, labor, consumption, and foreign trade, are non-monotonic (U-shaped) in the degree of asset-backed foreign borrowing, given exogenous shocks to productivity, the terms of trade, and the foreign interest rate. Furthermore, the volatility patterns of major macroeconomic aggregates in the degree of asset-backed foreign borrowing are more stable in the economy with domestic financial frictions than without.

Chapter 2

Financial Frictions and Macroeconomic Fluctuations

2.1 Introduction

This chapter analyzes the dynamic responses of macroeconomic aggregates to exogenous productivity shocks in a real business cycles model with financial frictions. It makes two contributions to the literature. First, the accumulation of entrepreneurial net worth and the reallocation of capital between agents with different production technologies constitute a mechanism through which aggregate output responds to exogenous productivity shocks in a hump-shaped and amplified fashion. It quantitatively replicates an important empirical fact that a standard RBC model fails to do. Second, if we model time-varying prices of capital in the approach of costly capital adjustment, the capital gains in the event of a positive productivity shock can significantly change the dynamic patterns of macroeconomic aggregates via the mechanism mentioned above. Thus, we should be aware of the side effect of this approach.

RBC models have been standing at the center of the business cycle analysis since Kydland and Prescott (1982). Although these models match some characteristics of the empirical data successfully, one of the well-known deficiencies of the canonical RBC model is the lack of a sufficient propagation and amplification mechanism, i.e., its failure to reproduce the persistent and hump-shaped output responses to a transitory TFP shock commonly found in the data (Cogley and Nason 1993, 1995). As capital is the only endogenous state variable in the standard RBC model, the dynamic structure is essentially $ARMA(1,1)$, which is responsible for this deficiency (Wen 2005). Furthermore, the standard RBC model underestimates the output

volatility around its trend in the actual U.S. economy (Andolfatto 1996).

In order to enhance the propagation and amplification mechanism, researchers have included additional endogenous state variables by introducing various frictions into the canonical RBC framework, e.g., labor adjustment costs (Cogley and Nason 1995), labor market search (Andolfatto 1996; Hashimzade and Ortigueira 2005), financial frictions (Carlstrom and Fuerst 1997, 1998; Kato forthcoming), factor hoarding (Burnside and Eichenbaum 1996), habit formation (Lettau and Uhlig 2000), and learning-by-doing (Chang, Gomes, and Schorfheide 2002). In these models, aggregate output peaks one or two periods after the shock. However, according to the empirical evidence provided by Cogley and Nason (1995), aggregate output peaks four quarters after the shock in the United States. As these models are lack of nontrivial amplification mechanism, the maximum response of aggregate output is even less than that in the frictionless RBC model. Thus, although frictions emphasized in these models account for the hump-shaped output dynamics qualitatively, more quantitative research needs to be done.

This chapter is related to the literature on the business cycle implications of financial frictions. See Bernanke, Gertler, and Gilchrist (1999) for a comprehensive survey. Compared to outside financiers, entrepreneurs have superior information regarding their own choices of projects, the effort put into these projects, or the project outcomes. As it could be very costly or even impossible for outside financiers to collect such information, they have to provide entrepreneurs with a reasonable share of the project outcomes so as to induce them to choose "good" projects, exert sufficient effort, or tell the truth. This implies that entrepreneurs can credibly pledge only part of the project outcomes for external funding. Thus, entrepreneurial net worth is required to fill in the gap between total investment and external funds. Any shock to entrepreneurial net worth affects the borrowing capacity of entrepreneurs and aggregate output along the business cycle.

Carlstrom and Fuerst (1997) and Kato (forthcoming) analyze the case of moral hazard and financial frictions in the production of capital goods. Producers of capital goods are credit-constrained and their net worth becomes another endogenous state variable in addition to the aggregate capital stock. A positive productivity shock raises the marginal product of capital goods and the boom in the aggregate demand for capital goods pushes up the price of capital. As the borrowing capacity of capital goods producers depends on their net worth, which is predetermined, the aggregate production of capital goods cannot accommodate the excess demand immediately. As it takes time for capital goods producers to accumulate net worth and expand production, aggregate investment peaks two periods after the shock. Given that capital is one of the two inputs needed for the production of final goods,

aggregate output peaks two periods after the shock. Due to the delayed and dampened responses of the aggregate capital stock, the maximum response of aggregate output is less than that in the frictionless RBC model.

We analyze instead the case of moral hazard and financial frictions in the production of intermediate goods. In contrast to other multi-sector models in the literature, e.g., Long and Plosser (1983) and Boldrin, Christiano, and Fisher (2001), we assume that households and entrepreneurs have different technologies to produce intermediate goods using capital. In the basic model, there is no friction in the production of capital goods and the price of capital is constant at unity. Intermediate goods and labor are the two factors needed for the production of final goods. Entrepreneurs are expected to be more productive than households. Due to moral hazard, entrepreneurs are subject to credit constraints. A positive TFP shock to the production of final goods raises the aggregate demand for intermediate goods. As the aggregate supply of intermediate goods depends on the investment of households and entrepreneurs made in the previous period, the price of intermediate goods rises to clear the market. Extra sales revenues improve entrepreneurial net worth so that the entrepreneurs can borrow more to expand their investments in capital goods. The rise in the entrepreneurs' demand for loans pushes up the loan rate so that the households increase their deposits and reduce their investments in capital goods. The resulting capital reallocation towards the entrepreneurs becomes an important channel through which aggregate output responds to TFP shocks more strongly than in the frictionless RBC model. However, it takes time for entrepreneurs to accumulate net worth before they can fully exploit the profit opportunity. The speed of capital reallocation determines the dynamic patterns of macroeconomic aggregates. According to our calibration, aggregate output peaks four periods after the shock and the maximum output response is larger than that in the frictionless RBC model. Thus, it reproduces the empirical evidence in the literature quantitatively.

This chapter is also related to the literature on the implications of time-varying prices of capital on macroeconomic fluctuations. The approach of costly capital adjustment is widely adopted to model time-varying prices of capital, e.g., Ireland (2003) and Linnemann and Schabert (2003). The alternative is to assume a fixed supply of durable assets, e.g., land, and analyze the amplification effects of time-varying asset prices on aggregate output (Chen 2001; Kiyotaki 1998; Kiyotaki and Moore 1997, 2005). In these models, the more productive agents borrow from the less productive agents up to endogenous credit limits. A positive productivity shock boosts the demand for productive assets and pushes up asset prices. Capital gains improve the net worth of credit-constrained agents and enables them to expand

production. As a result, output responds more strongly to the TFP shocks than in the case without credit constraints. These models emphasize the amplification and persistence mechanism but fail to generate a hump-shaped output response.

Our full model allows time-varying prices of capital in a setting with quadratic capital adjustment costs. The price of capital responds positively and persistently to a transitory TFP shock. It strengthens the amplifying effects but weakens the hump-shaped output dynamics. This also explains why models with a fixed supply of assets cannot generate hump-shaped output responses to shocks. The intuition is as follows. In the event of a positive TFP shock, extra sales revenues improve entrepreneurial net worth and entrepreneurs can borrow more to expand investment. Due to capital adjustment costs, the supply of capital can not fully accommodate the excess demand. The price of capital rises to clear the market, with two effects. First, the demand of households for capital is depressed; second, capital gains improve entrepreneurial net worth further and entrepreneurs can increase their investments more than in the case without adjustment costs. The enhanced reallocation of capital from households to entrepreneurs makes output peaks earlier than in our basic model.

The rest of this chapter is organized as follows. Section 2.2 starts with an overview of the model economy. The optimization conditions and the relevant market clearing conditions jointly describe the market equilibrium. Section 2.3 discusses the financial contracting problems in the model without financial frictions. Section 2.4 calibrates the model economy and analyzes the impulse responses of macroeconomic aggregates to productivity shocks. Section 2.5 summarizes the main findings.

2.2   The Model

2.2.1   Overview

Consider a discrete-time, closed, real economy with three goods: a capital good, an intermediate good, and a final good. The final good is chosen as the numeraire. Capital goods are durable, while intermediate goods and final goods are perishable.There are two types of agents: households and entrepreneurs. The population of each type is normalized to unity.[1] Households and entrepreneurs can invest capital goods into their respective projects at the end of each period and the projects produce intermediate goods at the beginning of the next period. Intermediate goods and labor are then employed to produce final goods contemporaneously. Final goods can be

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[1]The relative population size of agents does not matter for the results.

consumed. The aggregate capital stock depreciates at a constant rate. A continuum of competitive firms, owned by households, can transform final goods into capital goods contemporaneously. Newly-produced and existing capital goods are perfect substitutes and are traded at the same price. A continuum of competitive financial intermediaries collect deposits and provide loans.

Households are risk averse and infinitely lived. They have a safe production project for intermediate goods. They are endowed with a unit of labor each period and they supply their labor to the final goods production. At the end of the period, they invest capital goods in their projects, deposit at the financial intermediaries, and consume.

Entrepreneurs are risk neutral. As shown in subsection 2.2.3, entrepreneurs finance their project investments using own funds and loans from financial intermediaries at the end of each period, subject to credit constraints. Entrepreneurial net worth is defined as the amount of own funds they invest in their projects. Debt repayment is contingent on project outcomes. Entrepreneurs whose projects fail are released from their debt obligations and exit from the economy without consuming anything; entrepreneurs whose projects succeed repay their debts. Successful entrepreneurs have a constant probability of death.[2] In equilibrium, entrepreneurs of mass $(1 - \pi)$ exit from the economy each period and new entrepreneurs of the same mass are born with a tiny endowment,[3] e, keeping the population size of entrepreneurs constant. Our calibration guarantees that the expected rate of return on entrepreneurial net worth exceeds the cost of external funds. Thus, surviving and newly-born entrepreneurs put all own funds into their projects and borrow to the limit. Entrepreneurs with successful projects who die sell off their capital stock, consume all proceeds.

There is no moral hazard in the financial sector. Financial intermediaries can perfectly diversify their portfolios and pool the idiosyncratic project risk of the entrepreneurs.[4] According to the financial contracts specified in subsection 2.2.3,

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[2]Entrepreneurs prefer to accumulate net worth and to postpone consumption until no external funding is needed. There are two alternative ways to handle this problem in the literature. Carlstrom and Fuerst (1996) and Bernanke, Gertler, and Gilchrist (1999) assign a constant death probability to entrepreneurs, where dying means selling off the capital stock, consuming the proceeds, and exiting from the economy. Alternatively, Carlstrom and Fuerst (1997, 1998), Gomes, Yaron, and Zhang (2003) and Kato (forthcoming) assume that entrepreneurs are infinitely lived but less patient than households. In equilibrium, entrepreneurs always consume something and are never sufficiently wealthy to overcome credit constraints.

[3]Each entrepreneur must put a positive amount of own funds in the projects in order to acquire loans. Chen (2001) adopts the same approach.

[4]Chen (2001) studies the role of bank capital by making an extreme assumption that there is

they guarantee a safe rate of return on deposits in equilibrium. Due to perfect competition, the financial intermediaries break even and make no profit.
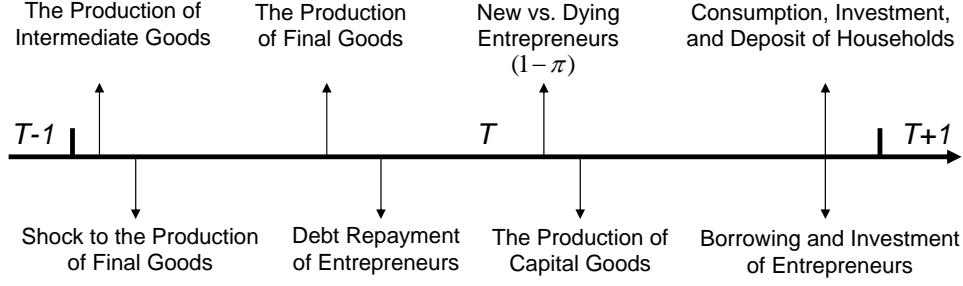
Figure 2.1: Time Sequence of Events

Figure 2.1 summarizes the time sequence of events in equilibrium. Note that an exogenous productivity shock[5] is realized at the beginning of period t. There are four endogenous factor prices: the price of capital goods, q_t, the price of intermediate goods, v_t, the wage rate, w_t, and the gross rate of return on deposits, r_t.

2.2.2   Households

Households have identical preferences over consumption and leisure. Their expected utility function takes the following form,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^h)^{1-\sigma}}{1-\sigma} + \chi \frac{(1-l_t)^{1+\psi}}{1+\psi} \right],$$

where E_t is the expectation operator based on information available in period t. $\beta \in (0,1)$ denotes the time discount factor and c_t^h and l_t denote, respectively, household consumption and hours worked, as a fraction of the total labor endowment. Given that k_{t-1}^h units of capital goods were invested in the household project at the end of period $t-1$, $G(k_{t-1}^h)$ units of intermediate goods are produced at the beginning of period t. Household sales revenues amount to $v_t G(k_{t-1}^h)$ and their wage income is $w_t l_t$. In addition, they receive $r_{t-1} d_{t-1}$ from the financial intermediaries, where d_{t-1} is the deposit made at the end of period $t-1$ and r_{t-1} is the gross rate of return on deposit. The profits of the production sector of capital goods, Π_t, are lump-sum transferred to households. At the end of period t, they invest k_t^h units of capital

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perfect correlation within the portfolio of each bank. If any one of the projects in a bank's portfolio fails, all projects financed by the bank fail together.

[5]Subsection 2.2.6 specifies the distribution of the shocks.

goods in their projects, deposit d_t units of final goods at the financial intermediaries, and consume c_t^h units of final goods. Accordingly, the flow-budget constraint is,

$$q_t[k_t^h - (1-\delta)k_{t-1}^h] + d_t + c_t^h = v_t G(k_{t-1}^h) + w_t l_t + r_{t-1} d_{t-1} + \Pi_t,$$

where $\delta \in (0,1]$ is the depreciation rate of the capital invested in the household project. The optimization over $\{c_t^h, l_t, d_t, k_t^h\}$ gives the equilibrium conditions,

$$w_t = \chi(1-l_t)^\psi (c_t^h)^\sigma, \tag{2.1}$$

$$\beta r_t = E_t \left(\frac{c_{t+1}^h}{c_t^h} \right)^\sigma, \tag{2.2}$$

$$r_t q_t = E_t \left[(1-\delta)q_{t+1} + v_{t+1} G'(k_t^h) \right]. \tag{2.3}$$

2.2.3 Entrepreneurs

Each entrepreneur can invest capital goods in one of the two projects: "Good" or "Bad", at the end of each period. At the beginning of the next period, the project generates R units of intermediate goods per unit of capital invested and the invested capital depreciates at a rate $\delta' \in (0,1]$, if the project succeeds. If the project fails, there is no output and the invested capital is fully lost. Project choices are irreversible and project outcomes are perfectly verifiable at no costs. Entrepreneurs also enjoy safe, nonpecuniary private benefits during the project process.[6] For convenience of aggregation, we assume that the private benefits are proportional to project investments in terms of the capital good. The projects differ in the probability of success and unit private benefits. See Table 2.1.

Table 2.1: Projects with Idiosyncratic Risk

Project	Good	Bad
Probability of Success	p^G	p^B
Unit Private Benefits	b^G	b^B

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[6]Our set-up resembles the principal-agent setting in Holmstrom and Tirole (1997, 1998). According to Hart (1995), private benefits may refer to any nonpecuniary benefits from running a project, e.g., large offices or luxury business cars. Private benefits are good for the project owners but may reduce the success probability of projects. The trade-off between the success probability and private benefits is a short-cut to capture divergent objectives between project owners and outside financiers.

where $0 < p^B < p^G < 1$ and $b^B > b^G > 0$ imply that project "Good" is safer than projects "Bad", but project "Bad" yields larger unit private benefits.

An entrepreneur[7] i who stays in the economy to the next period has linear preferences over consumption and private benefits. His expected utility function is,

$$E_0 \sum_{t=0}^{\tilde{T}} \beta^t \left[c_{i,t}^e + \mathcal{B} k_{i,t-1}^e \right], \qquad (2.4)$$

where $\tilde{T}$ is the stochastic time of death and $\mathcal{B} \in \{b^G, b^B\}$ denotes unit private benefits of capital invested in project "Good" or project "Bad". $c_{i,t}^e$ denotes his consumption in period t and $k_{i,t-1}^e$ denotes his project investment in terms of the capital good made at the end of period $t - 1$. Our calibration guarantees that only project "Good" has a positive expected net present value around the steady state,

$$\frac{p^G E_t[Rv_{t+1} + (1 - \delta')q_{t+1}] + b^G}{r_t} - q_t > 0 > \frac{p^B E_t[Rv_{t+1} + (1 - \delta')q_{t+1}] + b^B}{r_t} - q_t,$$

Therefore, other projects should not be financed in equilibrium. Project "Good" also has a larger expected marginal product than the household project.

At the end of period t, entrepreneur i invest $k_{i,t}^e$ units of capital goods in either project "Good" or project "Bad", using his own funds $n_{i,t}$ and inter-period loans $z_{i,t}$, i.e., $q_t k_{i,t}^e = n_{i,t} + z_{i,t}$. Thus, $n_{i,t}$ is entrepreneurial net worth in the project. The loan contract specifies a promise to repay $R_t^b k_{i,t}^e$ units of final goods in period $t + 1$ if the project succeeds. If the project fails, both parties get zero pecuniary return. There is no enforcement problem and entrepreneurs always repay their liabilities if they are able to do so. In order to motivate entrepreneur i to choose project "Good", financial intermediaries must provide him with enough incentives,

$$\{p^G E_t[Rv_{t+1} + (1 - \delta')q_{t+1} - R_t^b] + b^G\}k_{i,t}^e \geq \{p_e^B E_t[Rv_{t+1} + (1 - \delta')q_{t+1} - R_t^b] + b^B\}k_{i,t}^e.$$

The left (right) hand side denotes the expected utility of the entrepreneur if he chooses project "Good" ("Bad"). As the entrepreneur prefers to borrow to the limit, the incentive constraint is binding around the steady state and is simplified to be

$$R_t^b = E_t[Rv_{t+1} + (1 - \delta')q_{t+1}] - b, \quad \text{where} \quad b \equiv \frac{b^B - b^G}{p^G - p^B} > 0. \qquad (2.5)$$

Any promise to repay more than R_t^b is not credible, because the entrepreneur would choose project "Bad". The expected external unit value and full unit value of

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[7]Entrepreneurs differ in the end-of-period wealth and are indexed by $i \in [0, 1]$.

the capital invested in project "Good" are $p^G R_t^b$ and $p^G E_t[Rv_{t+1} + (1 - \delta')q_{t+1}]$, respectively. The difference between the two values, $p^G b$, is used to motivate the entrepreneur to choose project "Good" despite lower private benefits it promises, $b^G < b^B$.

Financial intermediaries are expected to break even in lending to the entrepreneur in period t, $r_t z_{i,t} = p^G R_t^b k_{i,t}^e$. This implies a credit constraint,

$$z_{i,t} = \Gamma_t n_{i,t}, \quad \text{where} \quad \Gamma_t \equiv \frac{p^G R_t^b}{r_t q_t - p^G R_t^b}$$

is the credit multiplier. As Γ_t is independent of $n_{i,t}$, loans are proportional to entrepreneurial net worth. Our calibration guarantees $r_t q_t > p^G R_t^b$ around the steady state and so $\Gamma_t > 0$. Note that the credit multiplier varies with q_t, r_t, $E_t q_{t+1}$, and $E_t v_{t+1}$. Ceteris paribus, a rise in the current price of capital q_t makes capital investment more expensive; similarly, a rise in the gross rate of return on deposits r_t makes external funds more expensive for entrepreneurs. In both cases, the credit multiplier falls so that less capital is allocated to entrepreneurs. Ceteris paribus, a rise in the expected prices of capital or intermediate goods in period $t+1$, $E_t q_{t+1}$ or $E_t v_{t+1}$, raises the expected external unit value of capital invested in their projects, $p^G R_t^b$. Thus, the credit multiplier is larger and entrepreneurs can expand investments.

In equilibrium, entrepreneurs of mass $(1-p^G)$ have failed projects and exit from the economy. The entrepreneurs whose projects succeed have a constant probability $\tilde{\pi}$ of surviving to the next period. In the aggregate, entrepreneurs of mass $p^G(1-\tilde{\pi})$ have successful projects and exit from the economy, and entrepreneurs of mass $p^G\tilde{\pi}$ have successful projects and live on to the next period. New entrepreneurs of mass $(1 - \pi)$ are born. We assume $\pi = p^G\tilde{\pi}$ to keep the population size of entrepreneurs constant at unity in equilibrium.

Entrepreneur i maximizes his expected utility (2.4), subject to his period budget constraints and credit constraints,

$$q_t k_{i,t}^e - z_{i,t} = n_{i,t}, \quad \text{where} \quad n_{i,t} \equiv \mathcal{N}_{i,t} - c_{i,t}^e, \tag{2.6}$$

$$z_{i,t} = \Gamma_t n_{i,t} \tag{2.7}$$

where $\mathcal{N}_{i,t}$ denotes his end-of-period wealth. Entrepreneurs differ in their end-of-period wealth, an issue discussed in appendix A.1. Due to the linear nature of the project technologies and the preferences of entrepreneurs, the loans and the project investment of entrepreneur i is proportional to his net worth. As a result, only the first moment of the distribution of entrepreneurial net worth matters for the

aggregate capital stock in the entrepreneurial sector. See appendix A.1. Let lower-case letters without the index i denote per capita variables of the entrepreneurial sector. Per capita consumption, per capita net worth, per capita inter-period loans, and per capita capital holding are,

$$c_t^e = (1 - \tilde{\pi})p^G[Rv_t + (1 - \delta')q_t - R_{t-1}^b]k_{t-1}^e, \tag{2.8}$$

$$n_t = \tilde{\pi}p^G[Rv_t + (1 - \delta')q_t - R_{t-1}^b]k_{t-1}^e + (1 - \pi)e, \tag{2.9}$$

$$z_t = \frac{p^G R_t^b}{r_t q_t - p^G R_t^b}n_t, \tag{2.10}$$

$$k_t^e = \frac{n_t + z_t}{q_t}. \tag{2.11}$$

For a better understanding of the model dynamics, we introduce three auxiliary variables. The first is the leverage ratio, defined as the ratio of total investment over entrepreneurial net worth, $\Omega_t \equiv \frac{q_t k_{i,t}^e}{n_{i,t}} = 1 + \Gamma_t$. The second is the entrepreneur's unit down payment of capital, defined as the amount of own funds the entrepreneur pays for each unit of capital, $u_t \equiv q_t - \frac{z_{i,t}}{k_{i,t}^e} = \frac{q_t}{\Omega_t}$. The third is the expected profitability of the entrepreneurial project, defined as the discounted expected gross rate of return on entrepreneurial net worth, $\xi_t \equiv \frac{\beta p^G E_t[Rv_{t+1} + (1-\delta')q_{t+1} - R_t^b]k_{i,t}^e}{n_{i,t}} = \frac{\beta p^G b}{u_t}$. Our calibration guarantees that the expected profitability of the entrepreneurial project exceeds the discounted cost of external funds around the steady state, $\xi_t > \beta r_t$. Thus, entrepreneurs postpone consumption and borrow to the limit in equilibrium.

For convenience of aggregation, we assume that capital depreciates faster in the household projects than in the entrepreneurial projects that turn out to be successful, $\delta = 1 - p^G + p^G\delta' > \delta'$. In equilibrium, the aggregate capital stock depreciates at the same rate in both household and entrepreneurial sectors, $1 - \delta = p^G(1 - \delta')$.

2.2.4   Financial Intermediaries

Financial intermediaries accept deposits from households and make loans to entrepreneurs in equilibrium. A deposit contract is a claim on the financial position of the intermediary. As financial intermediaries are perfectly competitive, they transfer all the loan repayments to their depositors; hence they make zero profit.

Suppose, first, that the project choice of entrepreneurs is perfectly observable so that they can pledge all the project outcomes to financial intermediaries for external funds. Due to the aggregate risk related to TFP shocks, the period-t prices of capital

and intermediate goods may differ from their expected values, i.e., $q_t \neq E_{t-1}q_t$ and $v_t \neq E_{t-1}v_t$, and so may the values of the project outcomes of entrepreneurs, $p^G[Rv_t + (1-\delta')q_t]k^e_{t-1} \neq p^G[RE_{t-1}v_t + (1-\delta')E_{t-1}q_t]k^e_{t-1}$. As a result, the rate of return on deposits is contingent on productivity shocks.

In contrast, in the case of unobservable project choices of the entrepreneurs, the loan contract described in subsection 2.2.3 implicitly provides entrepreneurs with a net unit return on capital, with a positive expected value in period $t-1$, $p^Gb > 0$. The ex post net unit return on capital to the successful entrepreneurs in period t is

$$Rv_t + (1-\delta')q_t - R^b_{t-1} = b + R(v_t - E_{t-1}v_t) + (1-\delta')(q_t - E_{t-1}q_t).$$

As long as the aggregate productivity shocks are larger than some negative threshold value, the ex post values of the project outcomes of successful entrepreneurs are larger than the promised repayments, $[Rv_t + (1-\delta')q_t]k^e_{t-1} > R^b_{t-1}k^e_{t-1}$, and the successful entrepreneurs repay their liabilities, $R^b_{t-1}k^e_{t-1}$, to the financial intermediaries. Let K^e_{t-1} and Z_{t-1} denote the aggregate capital stock held by entrepreneurs and the aggregate lending to entrepreneurs at the end of period $t-1$, respectively. The aggregate break-even condition of the financial sector is $r_{t-1}Z_{t-1} = p^GR^b_{t-1}K^e_{t-1}$. At the beginning of period t, entrepreneurs of mass p^G have successful projects and their total repayments, $p^GR^b_{t-1}K^e_{t-1}$, coincides with the expected value. Thus, the positive expected net return to entrepreneurs, $p^GbK^e_{t-1}$, absorbs the aggregate risk and the financial intermediaries pay a safe rate of return on deposits. For aggregate productivity shocks below this threshold, the prices of capital and intermediate goods are so low that the ex post values of the project outcomes of successful entrepreneurs are less than their debt obligations, $[Rv_t + (1-\delta')q_t]k^e_{t-1} < R^b_{t-1}k^e_{t-1}$. Even the successful entrepreneurs announce bankruptcy and transfer all the project outcomes to the intermediaries. Thus, given moral hazard in the entrepreneurial sector, the ex post rate of return on deposits is contingent on the productivity shock only for very large, negative shocks.

As a consequence, if productivity shocks are unbounded, the ex post rate of return on deposits as a function of productivity shocks could have a kink at the point where $Rv_t + (1-\delta')q_t = R^b_{t-1}$.

The first-order approximations used below to analyze the dynamics of our model requires that the endogenous variables are continuous and differentiable functions of the state variables. For the purpose of the approximations, therefore, we assume that TFP shocks are distributed with mean zero and a negative lower bound guaranteeing that successful entrepreneurs are always able to repay the promised amounts.

2.2.5   Costly Capital Adjustment

Let K_t and J_t denote the aggregate capital stock at the end of period t and the existing capital stock at the beginning of period t,

$$K_t \equiv k_t^h + k_t^e, \tag{2.12}$$

$$J_t \equiv (1 - \delta)k_{t-1}^h + p^G(1 - \delta')k_{t-1}^e = (1 - \delta)K_{t-1}. \tag{2.13}$$

In every period, the production sector of capital goods transforms I_t units of final goods into $\Phi(I_t; J_t)$ units of new capital goods, where $\Phi(I_t; J_t) \equiv I_t - \frac{\phi I_t^2}{2J_t}$ and $\phi \geq 0$. Appendix A.2 describes the decentralized equilibrium of the capital goods production sector. The aggregate stock of capital evolves according to

$$K_t - J_t = I_t - \frac{\phi I_t^2}{2J_t}. \tag{2.14}$$

In the basic model, we assume $\phi = 0$ so that final goods are one-to-one transformed into capital goods. The price of capital is constant at unity over time, $q_t = 1$. The capital goods production sector makes no profit, $\Pi_t \equiv q_t\Phi(I_t; J_t) - I_t = 0$.

In the full model, we assume $\phi > 0$ so that the capital goods production function includes quadratic adjustment costs. It permits a variable price of capital. The existing capital stock at the beginning of period t has positive externality on the capital goods production. In equilibrium, the price of capital is given by

$$q_t = \frac{1}{1 - \frac{\phi I_t}{J_t}}. \tag{2.15}$$

According to our calibration, $\frac{\phi I_t}{J_t} < 1$ around the steady state. The capital goods production sector transfers all profits, $\Pi_t = q_t\frac{\phi}{2}\frac{I_t^2}{J_t} \geq 0$, lump-sum to households. Appendix A.2 shows that the aggregate supply curve of capital goods has a positive slope, $\frac{d\Phi[I_t(q_t);J_t]}{dq_t} = \frac{J_t}{\phi q_t^3} > 0$. A rise in the price of capital makes the production of capital goods more profitable so that firms increase their investment expenditure and more capital goods are produced. More costly the capital adjustment is, i.e., a higher ϕ, more strongly the price of capital responds to excess demand.

2.2.6   Final Goods Production and Market Equilibrium

Final goods are produced from intermediate goods and labor,

$$Y_t = A_t M_t^\alpha L_t^{(1-\alpha)}, \tag{2.16}$$

$$A_t = \rho \log A_{t-1} + \epsilon_t, \tag{2.17}$$

where M_t and L_t denote the aggregate inputs of intermediate goods and labor.[8] Total factor productivity, A_t, is positively autocorrelated in logarithms, where $\rho \in (0, 1)$. The productivity shock has mean zero, $E_t \epsilon_{t+1} = 0$, and is distributed above a lower bound, $(-\tau, \infty)$, where $\tau > 0$ is small enough that successful entrepreneurs are always able to repay their liabilities. Final goods are produced efficiently,

$$v_t M_t = \alpha Y_t, \tag{2.18}$$

$$w_t L_t = (1 - \alpha) Y_t. \tag{2.19}$$

Markets for intermediate goods, final goods, capital, labor, and loans clear,

$$M_t = G(k_{t-1}^h) + p^G R k_{t-1}^e, \tag{2.20}$$

$$Y_t + (1 - \pi)e = c_t^h + c_t^e + I_t, \tag{2.21}$$

$$K_t = k_t^h + k_t^e, \tag{2.22}$$

$$z_t = d_t, \tag{2.23}$$

$$L_t = l_t. \tag{2.24}$$

Definition 2.1. *Market equilibrium is a set of allocations of households, $\{k_t^h, l_t, c_t^h\}$, and entrepreneurs, $\{k_t^e, n_t, c_t^e, z_t\}$, together with aggregate variables $\{M_t, Y_t, K_t, I_t, J_t\}$ given a set of prices $\{v_t, q_t, w_t, r_t, R_t^b\}$ and the exogenous process $\{A_t\}$ satisfying equations (2.1)-(2.3), (2.5), (2.8)-(2.21).*

2.3 Financial Contracting in the Frictionless Model

In this section, we assume that the project choice of entrepreneurs is perfectly observable at the date of contracting. Entrepreneurs can credibly choose project "Good" and pledge all of the project outcomes for external funding. Therefore, they do not have to put down own funds in the project, $n_t = 0$. As the expected rate of return on project "Good" is higher than that on the household project, all capital is allocated into the entrepreneurs' projects, $k_t^e = K_t$, and intermediate goods are produced only by entrepreneurs, $M_t = p^G R k_{t-1}^e$. As project "Good" has a linear technology, the capital held by an individual entrepreneur cannot be uniquely pinned down. For simplicity, we focus on a symmetric equilibrium in which all entrepreneurs invest the same amount of capital, k_t^e, in their projects. In period t, the entrepreneurs

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[8]As households and entrepreneurs are each of unit mass, aggregate variables coincide with per capita variables.

who invest in project "Good" in period $t-1$ enjoy the private benefits, $b^G k^e_{t-1}$, and transfer all the project outcomes to financial intermediaries. Newcomers consume the endowment, $c^e_t = (1-\pi)e$. The entrepreneurs who stay in the economy to period $t+1$ invest k^e_t units of capital goods using external funds, i.e., $z_t = q_t k^e_t$.

The rates of return on deposits are different in the cases with and without financial frictions. As shown in subsection 2.2.4, the entrepreneurs' expected stake in the project outcomes, $p^G b k^e_t > 0$, absorbs aggregate risk in the case of financial frictions. This enables the intermediaries to guarantee a safe rate of return on deposits. In this sense, the financial intermediaries take the form of banks. Without moral hazard, no incentive is required for entrepreneurs to engage in project "Good". The intermediaries can only diversify the idiosyncratic project risk of entrepreneurs but not the aggregate risk. Given that financial intermediaries do not accumulate reserves in our model, depositors have to bear this aggregate risk. In this sense, the financial intermediaries take the form of mutual funds and the rate of return on deposits is contingent on the productivity shock.

Households put d_{t-1} units of final goods at the intermediaries at the end of period $t-1$ for a claim on the financial position of the intermediaries in period t. The intermediaries fully finance the project investments of entrepreneurs, $z_{t-1} = q_{t-1} k^e_{t-1}$. After the project completion in period t, the intermediaries collect all the project outcomes, $p^G[Rv_t + (1 - \delta')q_t]k^e_{t-1}$, and transfer to depositors. The ex post rate of return on deposit is $\tilde{r}_t = \frac{p^G[(1-\delta')q_t + Rv_t]k^e_{t-1}}{d_{t-1}} = \frac{p^G[(1-\delta')q_t + Rv_t]}{q_{t-1}}$, differing from its expected value by the amount of $\tilde{r}_t - E_{t-1}\tilde{r}_t = \frac{p^G[(1-\delta')(q_t - E_{t-1}q_t) + R(v_t - E_{t-1}v_t)]}{q_{t-1}}$. For uniformity, we use r_t to denote the expected rate of return on deposit, $r_t = E_t\tilde{r}_{t+1}$.

Aggregate output of intermediate goods in the current period is proportional to the aggregate capital stock at the end of the previous period. Thus, it is capital accumulation that matters for aggregate output of final goods along the business cycle. In essence, the model without moral hazard is equivalent to the RBC model with a representative agent who has three production technologies: the linear technology to produce intermediate goods using capital, the Cobb-Douglas technology to produce final goods using intermediate goods and labor, and the concave technology to transform final goods into capital goods. Therefore, we call the model without moral hazard model RBC and the model with moral hazard model MH. Appendix A.3 shows the equations describing the market equilibrium of model RBC.

2.4 Dynamic Analysis

2.4.1 Calibration

For convenience of aggregation, we assume that the household project is linear,

$$G(k_t^h) = \frac{1}{2}\left(1 + \frac{k_t^e}{K_t}\right)k_t^h, \tag{2.25}$$

and the marginal product is $G'(k_t^h) = \frac{1}{2}\left(1 + \frac{k_t^e}{K_t}\right)$. This functional form implies that the entrepreneurial sector has positive production externality on the household project. The quarterly discount factor is set at $\beta = 0.99$, corresponding to an annual interest rate of 4%. By convention, the preferences of households are logarithmic in consumption ($\sigma = 1$) and in leisure ($\psi = -1$). χ is set to guarantee $l = \frac{1}{3}$, i.e., households work eight hours a day in the final goods production sector in the steady state. We set $\alpha = 0.36$ so that the household wage income accounts for 64% of aggregate output of final goods.

Following Carlstrom and Fuerst (1997, 1998), we choose $\rho = 0.95$ for the autocorrelation coefficient of TFP. Capital invested in the household project depreciates at a quarterly rate of $\delta = 2.5\%$ and capital invested in the entrepreneurs' projects that succeed depreciates at the rate of $\delta' = 1.52\%$. As in Carlstrom and Fuerst (1997), a quarterly rate of business failure at 1% implies $p^G = 0.99$. Therefore, the aggregate capital stock depreciates at the rate of $\delta = 2.5\%$ in equilibrium.

The surviving probability of the entrepreneurs with successful projects is set at $\tilde{\pi} = \frac{2}{3}$, implying that around 34% of entrepreneurs have to exit from the economy each period, $\pi = p_G\tilde{\pi} = 0.66$. We set $R = 6.04$ so that the expected marginal product of the entrepreneurial project in terms of the intermediate good always exceeds that of the household project, $p^G R > G'(0)$. It guarantees that capital is allocated to the entrepreneurial sector if their project choice is observable. Subsection 2.4.2 analyzes the impulse responses of macroeconomic aggregates under various scenarios. Together with the calibration of π and R, b is calibrated to satisfy the following conditions in the steady state: entrepreneurs hold half of the aggregate capital stock, $\frac{k^e}{K} = 0.5$; the leverage ratio, $\Omega = 2$, implies that entrepreneurs finance half of the their project investments using external funds, as in Bernanke, Gertler, and Gilchrist (1999); the entrepreneurs with successful projects keep 60% of the project outcomes for themselves, $\frac{R^b}{(1-\delta')q+vR} = 40\%$.

2.4.2   Impulse Responses to Productivity Shocks

We log-linearize the equations describing the market equilibria under various scenarios around their respective steady states and adopt the approach to the first-order approximations provided by Schmitt-Grohe and Uribe (2004). The endogenous variables are represented as the linear functions of the state variables. We analyze the impulse responses of endogenous variables with respect to a transitory TFP shock in period 0, given that relevant models are in the steady state before period 0. Subsection 2.4.2.1 compares the impulse responses of model MH and model RBC in the setting without adjustment costs. The accumulation of entrepreneurial net worth and the reallocation of capital constitute a mechanism generating the amplified and hump-shaped output dynamics. Subsection 2.4.2.2 compares the impulse responses of model MH in the settings with and without adjustment costs. The price of capital varies over time and the resulting capital gains speed up the capital reallocation process. Thus, aggregate output peaks earlier in the setting with adjustment costs than in the setting without adjustment costs. Subsection 2.4.2.3 compares the impulse responses of model MH and model RBC, in the setting with different degrees of adjustment costs. The reallocation of capital between households and entrepreneurs makes aggregate output respond more strongly in model MH than in model RBC.

2.4.2.1   The Dynamics of the Basic Model: Capital Reallocation

Figure 2.2 shows the impulse responses of model MH (solid line) and model RBC (dash-dot line) to a TFP shock in the setting without adjustment costs, $\phi = 0$. EN and HH denote entrepreneurs and households, respectively.

Consider model RBC. As capital is the only endogenous state variable, the dynamic structures is essentially $ARMA(1,1)$ and fails to generate the hump-shaped output dynamics. The supply of capital is perfectly elastic and the price of capital is constant at unity, $q_t = 1$. A 1% TFP shock raises the marginal products of labor and intermediate goods in period 0. On the one hand, the wage rate rises by 0.73%. Given the autocorrelation in TFP, as households prefer to smooth consumption over time and optimize between consumption and leisure, they increase labor supply and consumption by 0.75% and 0.40%, respectively. Given that aggregate output of intermediate goods, $M_0 = p^G RK_{-1}$, is determined by the aggregate capital stock at the end of the previous period, aggregate output of final goods rises by 1.48%. Meanwhile, the price of intermediate goods rises by 1.48% to clear the market in period 0.

Due to the autocorrelation in TFP, the marginal product of intermediate goods
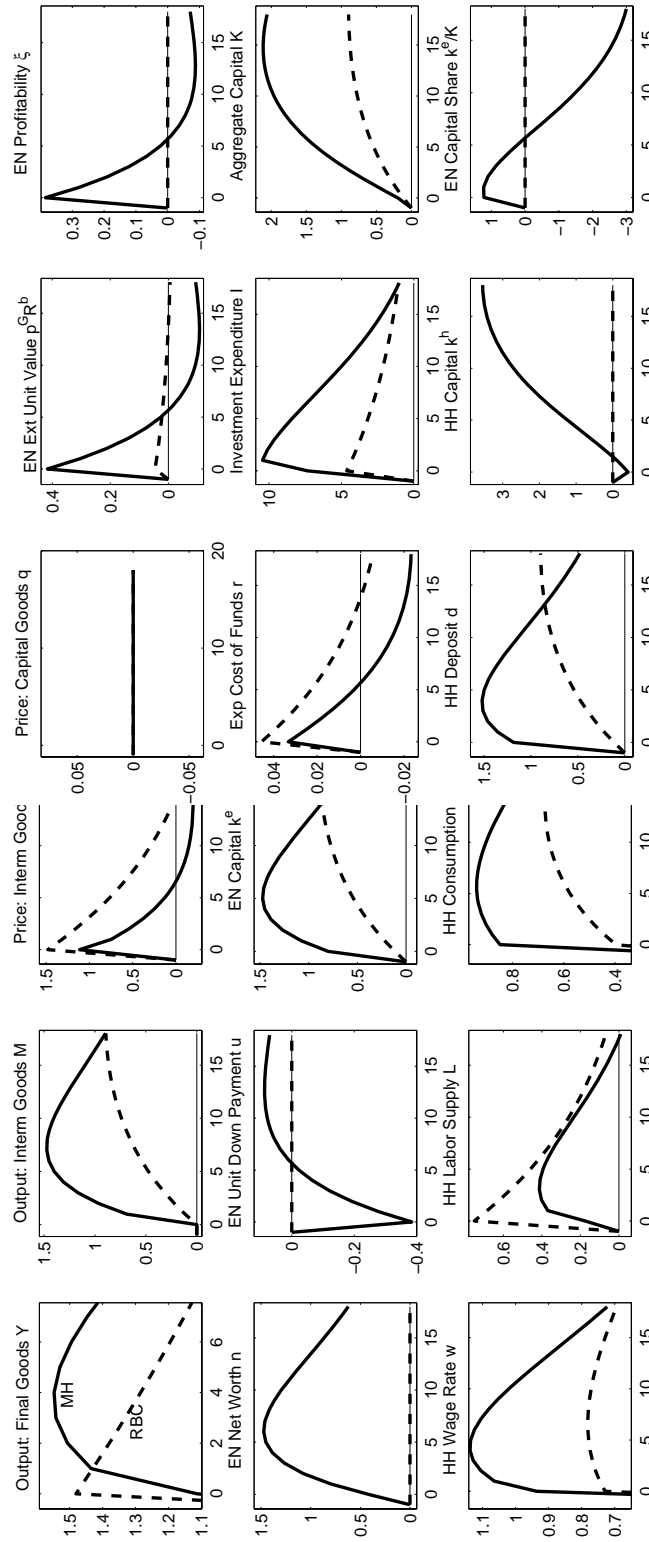
Figure 2.2: The Role of Capital Reallocation: $(\phi = 0)$

stays above its steady state value in period 1, so does the price of intermediate goods. It raises the expected unit value of capital in the entrepreneurs' projects in period 0, $p^G R_0^b = p^G[RE_0v_1 + (1 - \delta')]$, by 0.05%. Entrepreneurs can then raise more external funds and expand their project investments. On the one hand, their excess demand for external funds pushes up the expected rate of return on deposits by 0.046% and households increase their deposits by 0.12%. On the other hand, producers of capital goods increase their investment expenditure by 4.63% and fully accommodate the rise in the aggregate demand for capital by 0.16%.

From period 1 on, as the accumulation of aggregate capital stock gradually raises aggregate output of intermediate goods and TFP converges to its steady state value, the marginal product of intermediate goods falls. Meanwhile, the decline in the household labor supply and the rise in aggregate supply of intermediate goods jointly increase the marginal product of household labor, despite the convergence in TFP. As a result, the price of intermediate goods converges to its steady state value, while the wage rate has a hump-shaped pattern with the peak in period 6. In this sense, the hump-shaped dynamic pattern of household consumption results from the accumulation of aggregate capital stock, as in the standard RBC model. However, aggregate output of final goods peaks in period 0 and converges to its steady state value. Thus, model RBC cannot generate hump-shaped output responses to TFP shocks.

Consider model MH. There are three endogenous state variables $\{k_t^e, k_t^h, R_t^b\}$ and the dynamic structure can replicate the amplified and hump-shaped output behavior. The period-0 aggregate supply of intermediate goods, $M_0 = p^G R k_{-1}^e + G(k_{-1}^h)$, is determined by the investments of households and entrepreneurs made in period -1. A 1% TFP shock results in a rise in the price of intermediate goods $(v_0 > E_{-1}v_0)$. Given that the price of capital is constant at unity, $q_0 = E_{-1}q_0 = 1$, extra sales revenues improve the post-repayment wealth of entrepreneurs

$$\mathcal{N}_0 = p^G[Rv_0 + (1 - \delta') - R_{-1}^b]k_{-1}^e = p^G[b + R(v_0 - E_{-1}v_0)]k_{-1}^e > E_{-1}\mathcal{N}_0. \quad (2.26)$$

Entrepreneurial net wealth rises by 0.42%. Meanwhile, given that the price of intermediate goods is 1.11% above the the steady state value in period 1, the expected external unit value of capital invested in the projects of entrepreneurs in period 0, $p^G R_0^b = p^G E_0[(1 - \delta') + Rv_1 - b]$, rises by 0.42%, which enables entrepreneurs to acquire more loans. The entrepreneurs' excess demand for loans pushes up the gross loan rate by 0.033%. Altogether, the entrepreneur's unit down payment of capital, $u_0 = 1 - \frac{p^G R_0^b}{r_0}$, falls by 0.38%. Thus, entrepreneurs increase their capital stock, $k_0^e = \frac{n_0}{u_0}$, by 0.8%.

The rise in the deposit rate induces households to increase deposits by 1.2%, much larger than the 0.06% in model RBC. It induces households to increase their deposits to such an extent that they even reduce their capital stock by 0.42%. Extra sales revenues have the wealth effect on household consumption-leisure decision. Although the wage rate rises by 0.94% in period 0, households increase labor supply only by 0.18%, much less than the 0.75% in model RBC. Aggregate output of final goods increases only by 1.11% in period 0, much less than the 1.48% in model RBC.

The initial rise in the profitability of entrepreneurs' project by 0.44% means that entrepreneurial net worth is expected to yield higher expected return in period 1 than in the steady state. However, it takes time for entrepreneurs to accumulate net worth before they can fully exploit the profit opportunity. Given that the price of intermediate goods stays above the steady state value for six periods after the shock, the entrepreneur's unit down payment of capital, $u_t^e = 1 - \frac{p_e^G[(1-\delta')+R_e E_t v_{t+1}]}{r_t}$, stays below the steady state value for five periods after the shock, and entrepreneurs can invest more capital in their projects than in the steady state. Meanwhile, entrepreneurial net worth is positively correlated with their capital investment in the previous period, $n_t^e = \tilde{\pi} p_e^G b_e k_{t-1}^e$, where $t = 1, 2, 3, ...$ As a result, the entrepreneurs' capital investment peaks in period 5. As intermediate goods produced by the entrepreneurs' projects account for 88% of aggregate output, the dynamics of the aggregate supply of intermediate goods in the current period follow the dynamics of the entrepreneurs' capital stock in the previous period. This justifies the fact that the aggregate supply of intermediate goods peaks in period 6 and the price falls below the steady state value since period 7. Although the wage rate peaks in period 4, the household labor supply peaks in period 3. As a result, aggregate output of final goods peaks in period 4 by 1.55% above the steady state value, more than the maximum value of the output responses, 1.48%, in period 0 in model RBC.

Altogether, the accumulation of entrepreneurial net worth and the reallocation of capital among agents with different production technologies explain the amplified propagation mechanism here. The extra channel of capital reallocation is absent in the various models with financial frictions in the literature.

2.4.2.2 The Dynamics of the Full Model: Capital Gains

Before analyzing the dynamics of the full model, we first look at how costly capital adjustment affects the internal mechanism of model RBC. Figure 2.3 shows the impulse responses of model RBC to a TFP shock in the settings with moderate adjustment costs (solid line, $\phi = 3$) and without adjustment costs (dash-dot line, $\phi = 0$). The dynamics of model RBC without adjustment costs are discussed in

subsection 2.4.2.1.
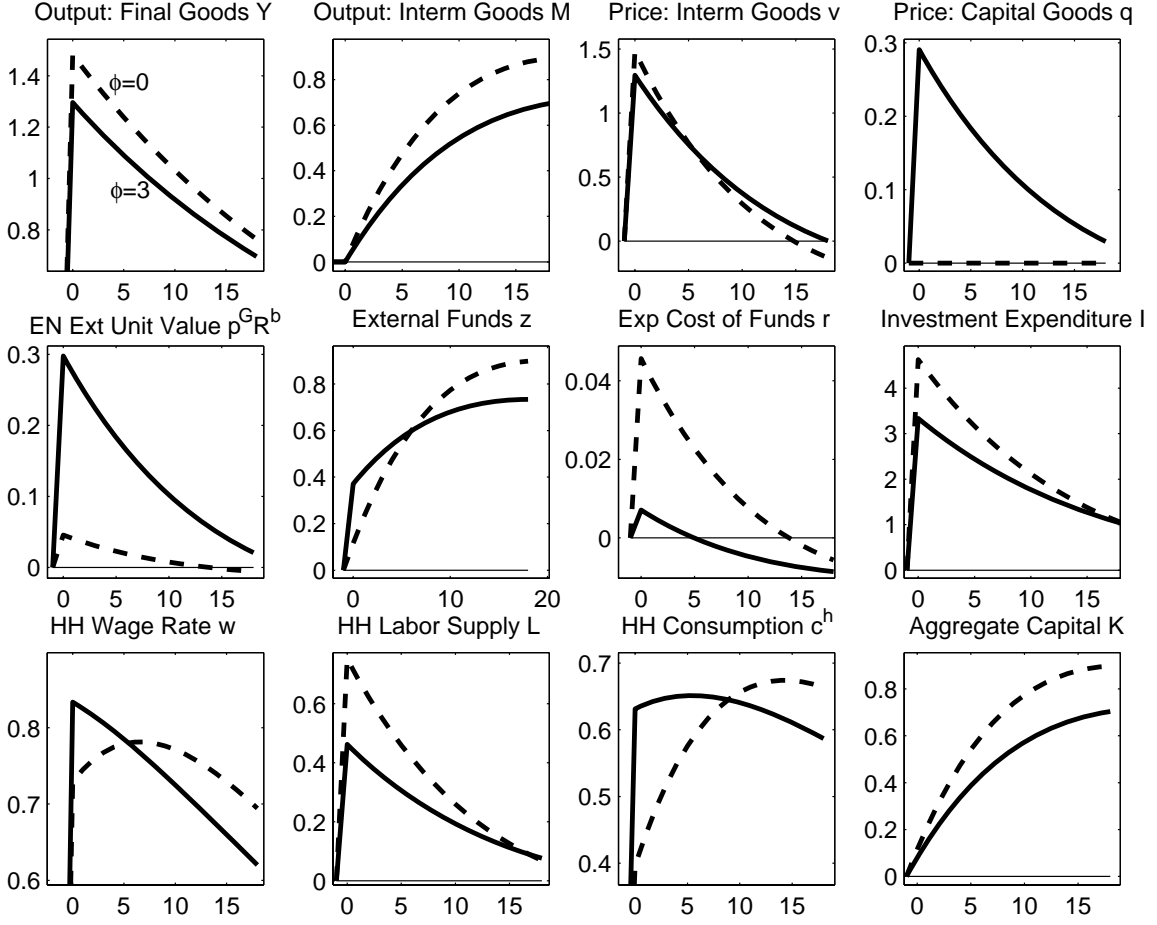
Figure 2.3: Capital Adjustment Costs in the Frictionless Models: $\phi = 0$ vs. $\phi = 3$

Consider model RBC with moderate adjustment costs. As the assumption on costly capital adjustment does not add additional endogenous state variable into the baseline model, the dynamic structures is essentially $ARMA(1,1)$, too. Similar as in the case without adjustment costs, a 1% positive TFP shock results in a rise in the price of intermediate goods in period 0. Given the price of intermediate goods stays above its steady state value in period 1, the expected unit value of capital invested in the entrepreneurs' projects rises and entrepreneurs can borrow more to expand their project investments. Due to adjustment costs, the supply of capital cannot fully accommodate the excess demand and the price of capital rises by 0.29% in period 0. Given that the financial intermediaries transfer all the project outcomes to households, extra sales revenues and capital gains, $p^G[(1 - \delta')(q_0 - E_{-1}q_0) +$

$R(v_0 - E_{-1}v_0)]K_{-1}$, have a stronger wealth effect on household consumption and deposit than in the setting without adjustment costs. Thus, households increase consumption and deposits by 0.63% and 0.37%, respectively.

Entrepreneurs finance their project investments using external funds, $K_t = \frac{z_t}{q_t}$. The rise in the price of capital partially erodes the positive effect of the rise in the external funds. As a result, entrepreneurs increase their capital stock only by 0.08% in period 0, smaller than the 0.12% in the setting without adjustment costs. In order to meet the excess demand for capital, firms producing capital goods increase their investment expenditures by 3.4%, less than the 4.63% in the setting without adjustment costs. As mentioned in section 2.3, model RBC is equivalent to a RBC model, in which households prefer to economize adjustment costs over time. So, the output dynamics are less amplified but more persistent than in the setting without adjustment costs.

Figure 2.4 compares the impulse responses of model MH to a TFP shock in the settings with moderate adjustment costs (solid lines, $\phi = 3$) and without adjustment costs (dash-dot lines, $\phi = 0$). The dynamics of model MH without adjustment costs are discussed in subsection 2.4.2.1.

Consider model MH with moderate adjustment costs. A 1% positive TFP shock leads to the rise in the prices of capital and intermediate goods, similar as in model RBC with adjustment costs. Extra sales revenues and capital gains have wealth effects on household consumption and deposit decisions. Although the wage rate rises by 0.99%, households increase their labor supply only by 0.04% in period 0, smaller than the 0.18% in the setting without adjustment costs. Given the predetermined aggregate supply of intermediate goods, aggregate output of final goods rises by 1.02%, less than the 1.11% in the setting without adjustment costs.

Meanwhile, extra sales revenues and capital gains also improve entrepreneurial net worth by the amount of $\tilde{\pi}p^G[R(v_0 - E_{-1}v_0) + (1 - \delta')(q_0 - E_{-1}q_0)]$ in period 0. Compared to the setting without adjustment costs, capital gains further enable entrepreneurs to borrow more and expand their project investments. Thus, the price of capital rises by 0.28% and entrepreneurial net worth rises by 0.74% in period 0.

Given that the prices of capital and intermediate goods stay above their steady state values in period 1, the expected external unit value of capital in the entrepreneurial sector, $p^G R_0^b = p^G E_0[(1 - \delta')q_1 + Rv_1 - b]$, rises by 1.14% in period 0, much more than the 0.30% in the setting without adjustment costs. Thus, entrepreneurs can borrow more and their excess demand for loans pushes up the gross loan rate by 0.20%, much larger than the 0.033% in the setting without adjustment costs. It induces households to increase deposits by 2.07% in period 0, much more than the 0.37% in the setting without adjustment costs. Meanwhile, the rise
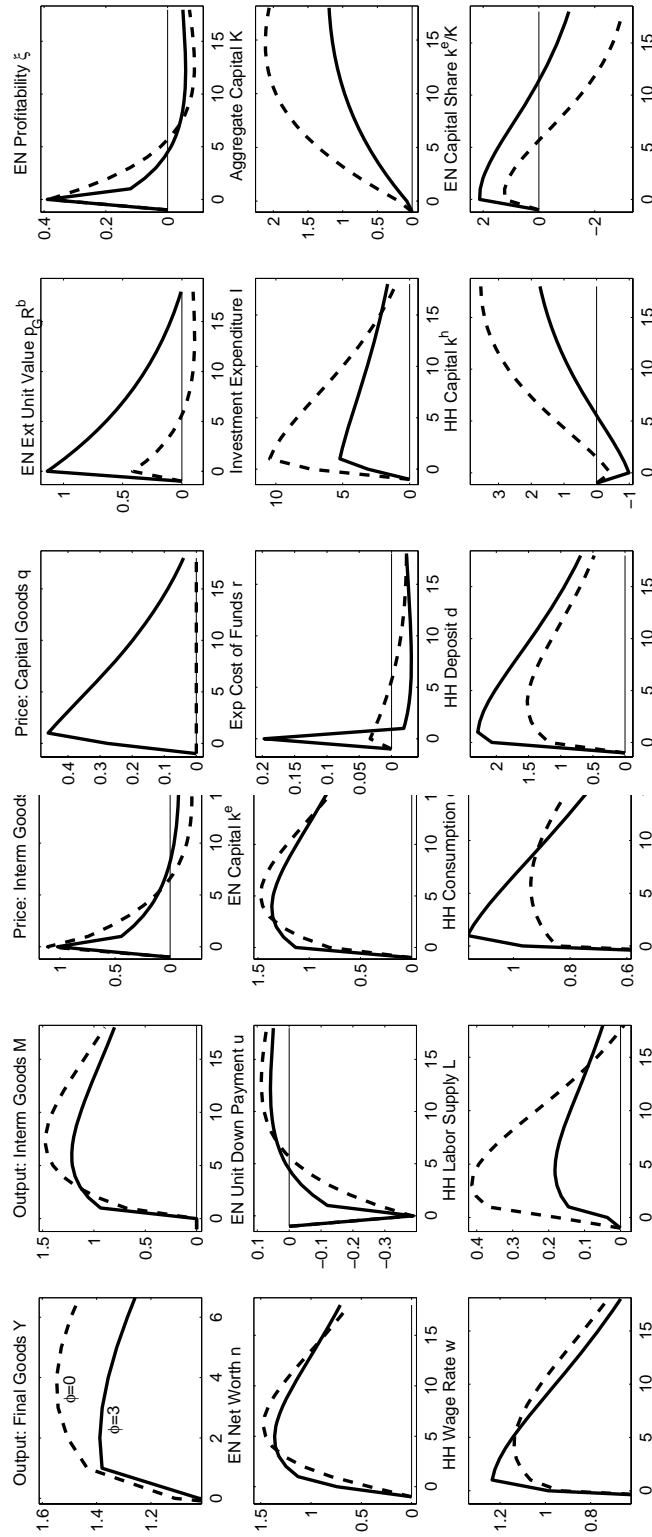
Figure 2.4: The Role of Capital Gains: $\phi = 0$ vs. $\phi = 3$

in the price of capital further reduces the capital holding of households by 0.99%, more dramatically than the 0.42% in the setting without adjustment costs. Despite the rise in the price of capital, the entrepreneur's unit down payment of capital falls by 0.39% in period 0. The rise in entrepreneurial net worth and the fall in the entrepreneur's unit down payment jointly enable entrepreneurs to increase their capital holding by 1.13%, more than the 0.80% in the setting without adjustment costs. In this sense, capital gains speed up the process of capital reallocation among heterogenous agents.

The aggregate capital stock responds to shocks in a smaller magnitude than in the setting without adjustment costs. Altogether, there are two effects at work. First, the enhanced reallocation of capital towards entrepreneurs further increases aggregate output of intermediate goods; second, the weakened response of the aggregate capital stock due to adjustment costs undermines the increase in aggregate output of intermediate goods. Altogether, aggregate output of intermediate goods rises by 0.94% in period 1, more than the 0.69% in the setting without adjustment costs.

As long as the entrepreneur's unit down payment of capital stays below its steady state value, it is cheaper for entrepreneurs to make leveraged investment than in the steady state. However, the dramatic convergence of the entrepreneur's unit down payment of capital to its steady state value in period 1 weakens their ability to expand investment in the following periods. Their capital holding peaks by 1.36% above the steady state value in period 4, earlier than in the case without adjustment costs. In this sense, time-varying prices of capital change the dynamic pattern of the entrepreneurs' capital holding. Thus, aggregate output peaks by 1.39% in period 2, earlier than in the setting without adjustment costs.

2.4.2.3 Amplification versus Propagation

Figure 2.5 compares the impulse responses of model MH (solid line) and model RBC (dash-dot line), given moderate adjustment costs, $\phi = 3$. Consider model MH. Due to capital gains, capital is reallocated faster towards entrepreneurs than in model RBC. Although the TFP in period 2 is lower than in period 0, $A_2 = 1.009 < 1.01 = A_0$, the effects of capital reallocation overcompensate the decline in TFP. As a result, aggregate output of final goods peaks by 1.39% in period 2; while, aggregate output of final goods peaks by 1.30% in period 0 in model RBC.

For a better understanding of the tradeoff between amplification and propagation, we compare the impulse responses of the two models in the setting with large adjustment costs ($\phi = 10$) in figure 2.6. Consider model MH with large adjustment
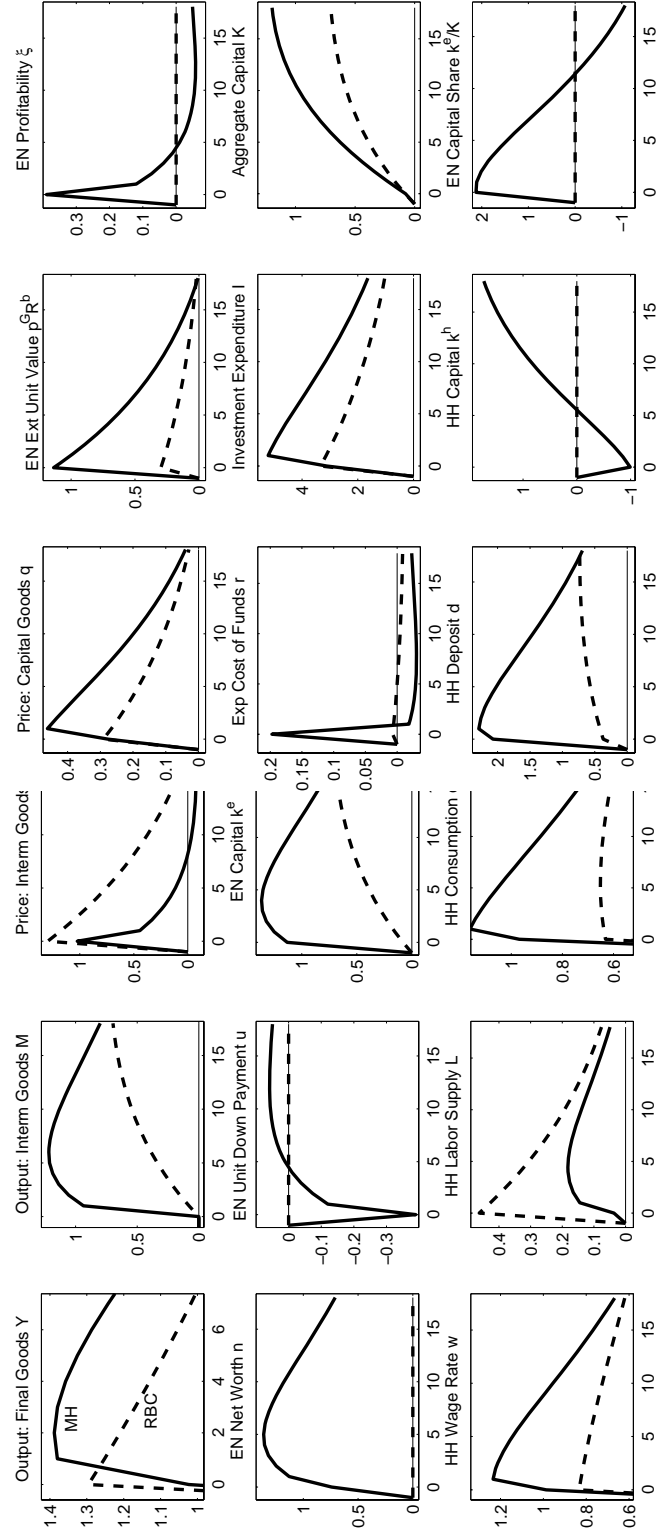
Figure 2.5: The Models with and without Moral Hazard: ($\phi = 3$)
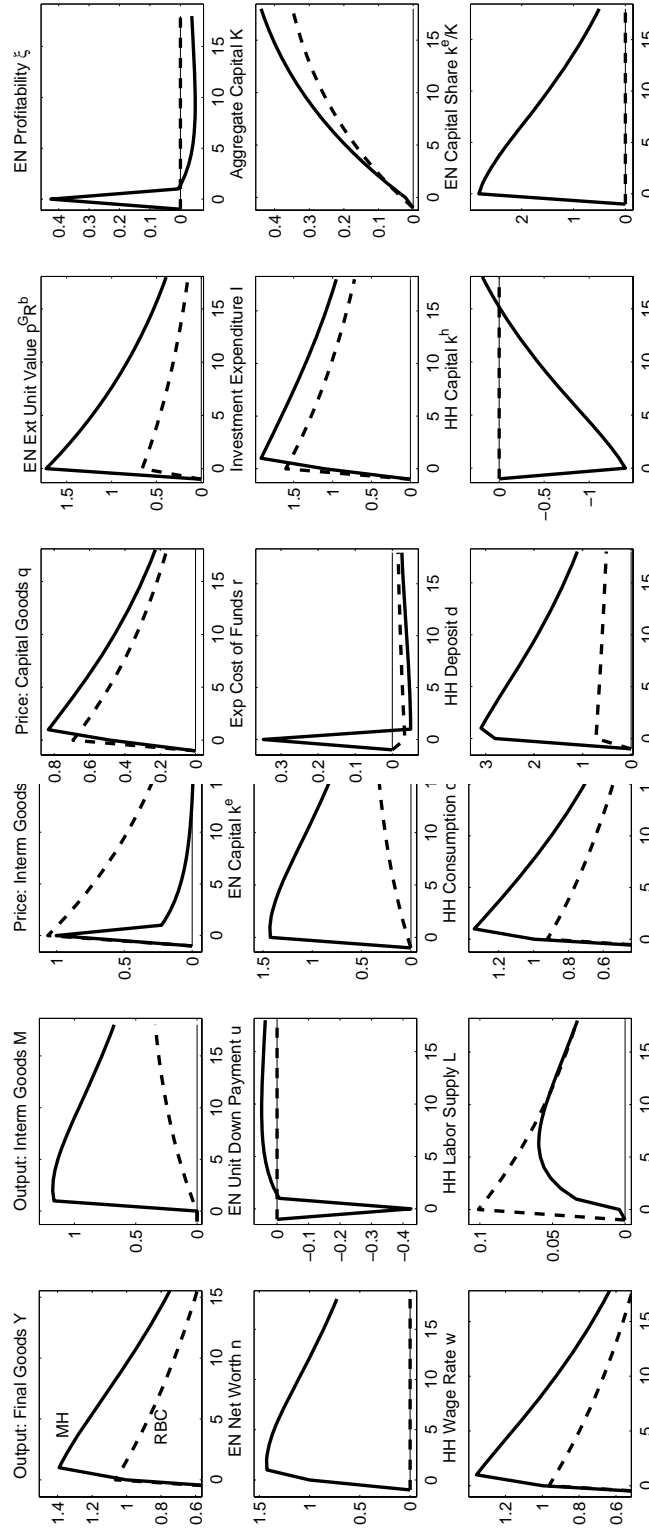
Figure 2.6: The Models with and without Moral Hazard: $(\phi = 10)$

costs. The supply of capital is even less elastic than in the setting with moderate adjustment costs ($\phi = 3$). The price of capital rises by 0.48% in period 0, larger than the 0.28% in the setting with moderate adjustment costs. Due to larger capital gains, entrepreneurial net worth improves by 1% in period 0 in comparison with 0.74% in the setting with moderate adjustment costs, $\phi = 3$. The stronger improvement in net worth enables entrepreneurs to increase capital holding by nearly 1.43%, larger than the 1.13% in the setting with moderate adjustment costs. The huge increase in the supply of intermediate goods from entrepreneurs in period $t+1$ depresses the price of the intermediate good. The entrepreneur's unit down payment of capital falls by 0.43% in period 0 but converges very closely to its steady state value in period 1. As a result, the entrepreneurs' capital holding peaks by 1.43% in period 1. Aggregate output of final goods peaks by 1.39 in period 1, larger than the 1.06% in period 0 in model RBC. In this sense, time-varying prices of capital in the current setting affect the tradeoff between amplification and propagation. This result holds under various calibrations of structural parameters.

2.5   Conclusion

This chapter analyzes how financial frictions affect macroeconomic fluctuations in a real dynamic general equilibrium model. The accumulation of entrepreneurial net worth and the reallocation of capital among agents with different production technologies explain an important empirical fact, i.e., aggregate output peaks around four quarters after the shock and in an amplified magnitude to exogenous productivity shocks.

Furthermore, we adopt the approach of quadratic capital adjustment costs to model time-varying prices of capital. For a positive productivity shock, the boom in the demand for capital pushes up the price of capital. The capital gains further improve entrepreneurial net worth and capital is reallocated to entrepreneurs faster than in the case of constant prices of capital. As entrepreneurs are more productive than other agents, aggregate output responds to productivity shocks in a more amplified but less delayed fashion than in the setting without adjustment costs. Thus, time-varying prices of capital affect the tradeoff between amplification and propagation mechanisms.

Chapter 3

Dual Financial Frictions and Macroeconomic Fluctuations

3.1 Introduction

We have shown the side effect of the approach of costly capital adjustment on the modeling of time-varying prices of capital in chapter two. Now, we propose an alternative modeling approach in which the price of capital responds to shocks in a non-trivial way. We develop a model with financial frictions on the demand for and the supply of capital goods and analyze how the price of capital, lending, investment, and output respond to a transitory TFP shock. Two contributions are made to the literature on real business cycles (RBC, hereafter). First, the dynamic interactions between the price of capital and dual financial frictions constitute a robust mechanism thought which output responds to exogenous productivity shocks in an amplified and hump-shaped fashion. Second, it addresses a methodological question: What is the proper modeling approach to the production of capital goods in a dynamic general equilibrium framework?

One of the well-known deficiencies of the canonical RBC models is the lack of a sufficient propagation and amplification mechanism, as pointed out by Cogley and Nason (1993, 1995) and Andolfatto (1996). Because the capital stock is the only endogenous state variable in these models, the dynamic structure is essentially $ARMA(1,1)$ and this fails to replicate an important empirical fact, i.e., the amplified and hump-shaped output responses to productivity shocks. Many studies in the literature introduce various frictions into the RBC framework. Additional endogenous state variables help reinforce the internal propagation mechanism. Credit market imperfections are one of these variations.

Carlstrom and Fuerst (1997, 1998) and Kato (forthcoming) analyze the case of moral hazard and financial frictions in the production of capital goods. In addition to the aggregate capital stock, the net worth of capital goods producers becomes another endogenous state variables and is essential for their borrowing capacity. A positive productivity shock raises the aggregate demand for capital, which pushes up the price of capital. Although the projects of capital goods producers become more profitable than before, they are subject to credit constraints and cannot expand their investments to fully exploit this opportunity. They accumulate net worth over time so that the supply of capital adapts to the demand in a few periods after the shock. Given that capital is one of the two inputs needed for the aggregate production of final goods, the delayed and dampened response of capital investment results in the hump-shaped and depressed output dynamics, in comparison with the frictionless RBC model. Although financial frictions in the production of capital goods in these models help generate the positive autocorrelation of aggregate output qualitatively, aggregate output peaks only two periods after the shock and in a much dampened magnitude. However, Cogley and Nason (1995) show that aggregate output peaks four quarters after the shock in the United States; Andolfatto (1996) shows that the volatility of aggregate output around its trend in the actual U.S. economy is even more than what the frictionless RBC model can predict.

In our basic model, moral hazard and financial frictions exist in the production of intermediate goods. As there is no friction in the production of capital goods, the price of capital is constant at unity. Entrepreneurs are more productive in producing intermediate goods using capital than households are. Due to moral hazard, entrepreneurs can credibly pledge only a fraction of their project outcomes for loans and they must put down own funds in their projects to fill in the gap between total investment and loans. Entrepreneurial net worth is essential for their project investments. A positive productivity shock raises aggregate demand and pushes up the price of intermediate goods. Extra sales revenues improve entrepreneurial net worth and enable them to demand for more loans and capital goods. The excess demand for loans pushes up the loan rate and induces households to lend more to the entrepreneurial sector and reduce their capital holding. As it takes time for entrepreneurs to accumulate net worth, they cannot acquire enough loans to expand their investments efficiently in the shock period. Capital reallocation from households to entrepreneurs is delayed, and the aggregate supply of intermediate goods responds to shocks in a delayed fashion, too. In contrast to Carlstrom and Fuerst (1997, 1998) and Kato (forthcoming), we show that the gradual reallocation of capital goods among agents with different production technologies amplifies the effects of shocks and aggregate output peaks four periods after the shock, in line

with the empirical evidence.

There are some alternative approaches to model the supply of capital in the literature on asset prices and business cycles. In the standard RBC model, the aggregate capital stock depreciates at a constant rate each period. Consumption goods can be transformed one-to-one to capital goods without frictions so that the price of capital is constant at unity over time. In this sense, the standard RBC models can be regarded essentially as a one-good economy. In order to analyze the role of asset prices along the business cycle, some researchers, such as Bernanke, Gertler, and Gilchrist (1999) and Aoki, Proudman, and Vlieghe (2004), derive an upward-sloping capital supply curve from convex adjustment costs. For analytical convenience, other researchers, such as Kiyotaki and Moore (1997, 2005), Kiyotaki (1998), Chen (2001), Iacoviello (2005), assume that durable assets (land) do not depreciate and have a fixed supply.

These two approaches have a common feature that an excess demand for capital leads to a large and persistent rise in the price of capital. As shown in chapter two, the persistent rise in the price of capital after the shock can exaggerate the effect of capital gains on the entrepreneurial net worth. As a result, aggregate output responds in a more amplified magnitude and peaks earlier than in our basic model.

In our full model, moral hazard and financial frictions exist in the production of both capital and intermediate goods. The assumption on credit-constrained production of capital goods captures the empirical fact that the aggregate supply of durable capital goods is rather inelastic in the short run but becomes elastic in the medium run. A positive productivity shock results in a boom in the aggregate demand for capital goods. Since producers of capital goods are subject to credit constraints, the supply of capital can not adapt to the excess demand immediately. Producers of capital goods accumulate net worth and exploit the profit opportunity over time. Thus, the supply of capital adapts to the demand in a few periods after the shock. As a consequence, the price of capital stays away from the steady state value for only a few periods after the shock and the magnitude of capital gains is rather limited. In this sense, the propagation mechanism discussed in our basic model is preserved. The dynamic interactions between the price of capital and dual financial frictions constitute a robust mechanism through which aggregate output responds to productivity shocks in an amplified and hump-shaped fashion, quantitatively consistent with the empirical evidence.

The rest of this chapter is organized as follows. Section 3.2 starts with an overview of the model economy. Section 3.3 discusses the financial contracting under other scenarios. Section 3.4 calibrates the model and analyzes the impulse responses of macroeconomic aggregates to productivity shocks. Section 3.5 summarizes.

3.2 The Model

3.2.1 Overview

Consider a discrete-time, closed, real economy with three goods: a capital good, an intermediate good, and a final good. The final good is chosen as the numeraire. Capital goods are durable, while intermediate goods and final goods are perishable. There are three types of agents: households, entrepreneurs, and producers of capital goods. The population of each type is normalized to unity. Households and entrepreneurs can invest capital goods into their respective projects at the end of each period and the projects produce intermediate goods at the beginning of the next period. Intermediate goods and labor are employed to produce final goods contemporaneously. Final goods can be consumed. The aggregate capital stock depreciates at a constant rate. Producers of capital goods can transform final goods into capital goods contemporaneously. Newly-produced and existing capital goods are perfect substitutes and are traded at the same price. Competitive financial intermediaries collect deposits and provide loans.

Households are risk averse and infinitely lived. They have a safe production project for intermediate goods. They are endowed with a unit of labor each period, which can be supplied to the final goods production. At the end of the period, they invest capital goods in their projects, make inter-period deposits, and consume.

Entrepreneurs are risk neutral. As shown in subsection 3.2.4, entrepreneurs finance their project investments using own funds and inter-period loans from financial intermediaries at the end of each period, subject to credit constraints. Entrepreneurial net worth is defined as the amount of own funds entrepreneurs invest in their projects. Debt repayment is contingent on project outcomes. Entrepreneurs whose projects fail are released from debt obligations and exit from the economy without consuming anything; entrepreneurs whose projects succeed repay their debts. Successful entrepreneurs have a constant probability of death. In equilibrium, entrepreneurs of mass $(1 - \pi)$ exit from the economy in every period and new entrepreneurs of the same mass are born with a tiny endowment, e, keeping the population size of entrepreneurs constant. Our calibration guarantees that the expected rate of return on entrepreneurial net worth exceeds the cost of external funds. Thus, the surviving and newly-born entrepreneurs put all own funds into their projects and borrow to the limit. Entrepreneurs with successful projects who die sell off their capital stock, consume the proceeds, and exit from the economy.

Producers of capital goods are risk neutral and infinitely lived. They are less patient than households and entrepreneurs. At the beginning of each period, they

supply their labor endowment to the production of final goods. As shown in subsection 3.2.5, producers of capital goods finance their projects using own funds and intra-period loans from financial intermediaries, subject to credit constraints. Their net worth is defined as the amount of own funds they invest in their projects. According to our calibration, the expected rate of return on their projects exceeds the cost of intra-period loans. As a result, they put all own funds in their projects and borrow to the limit. Debt repayment is contingent on their project outcomes. Those whose projects fail are released from debt obligations; those whose projects succeed repay the debts. At the end of the period, they make inter-period deposits at the financial intermediaries and consume the rest.

There is no moral hazard in the financial sector. Financial intermediaries have the expertise in screening loan applications, diversifying portfolios, and enforcing debt repayments, etc. In equilibrium, loans must be intermediated through the financial sector and there is no direct lending among individual agents in our model economy. By perfectly diversifying the portfolios, the financial sector pools the idiosyncratic project risk of entrepreneurs and capital goods producers. According to the financial contracts specified in subsections 3.2.4 and 3.2.5, financial intermediaries, in equilibrium, guarantee a safe rate of return on inter-period and intra-period deposits. Due to perfect competition, financial intermediaries break even and make no profit.
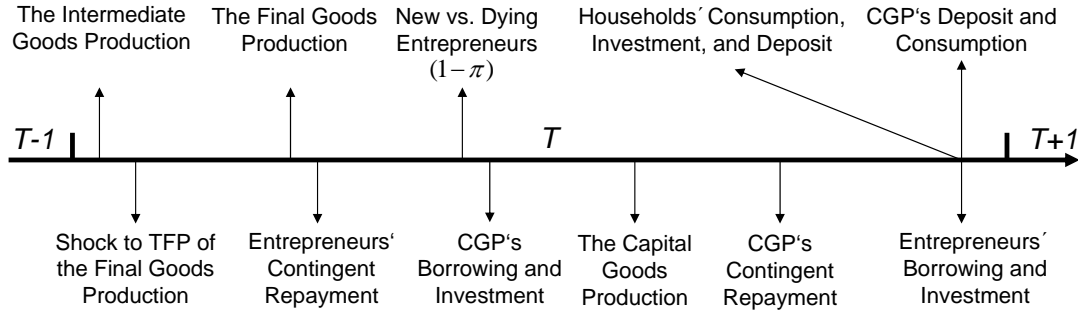
Figure 3.1: Time Sequence of Events

Figure 3.1 summarizes the time sequence of events in equilibrium, where CGP denotes capital goods producers. Note that an exogenous TFP shock[1] to the production of final goods is realized at the beginning of period t. There are five endogenous factor prices in the economy: the price of capital goods, q_t, the price of intermediate goods, v_t, the wage rate for households, w_t^h, the wage rate for capital goods produc-

---

[1]Subsection 3.2.7 specifies the distribution of the shocks.

ers, w_t^c, and the gross rate of return on inter-period deposits, r_t. As shown later, the gross rate of return on intra-period deposits is simply unity.

3.2.2   Households

Households have identical preferences over consumption and leisure. Their expected utility function takes the following form,

$$
E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^h)^{1-\sigma}}{1-\sigma} + \chi \frac{(1-l_t^h)^{1+\psi}}{1+\psi} \right],
$$

where E_t is the expectation operator based on information available in period t. $\beta \in (0,1)$ denotes the time discount factor and c_t^h and l_t^h denote, respectively, household consumption and hours worked, as a fraction of the total labor endowment. Given that k_{t-1}^h units of capital goods were invested in the household project at the end of period $t-1$, $G(k_{t-1}^h)$ units of intermediate goods are produced at the beginning of period t. The household sales revenues amount to $v_t G(k_{t-1}^h)$ and their wage income is $w_t^h l_t^h$. In addition, they receive $r_{t-1} d_{t-1}^h$ from the financial intermediaries, where d_{t-1}^h is the inter-period deposit made at the end of period $t-1$ and r_{t-1} is the gross rate of return on deposit. At the end of period t, they invest k_t^h units of capital goods in their projects, deposit d_t^h units of final goods at the financial intermediaries, and consume c_t^h units of final goods. Accordingly, the flow-budget constraint is,

$$
q_t[k_t^h - (1-\delta)k_{t-1}^h] + d_t^h + c_t^h = v_t G(k_{t-1}^h) + w_t^h l_t^h + r_{t-1} d_{t-1}^h,
$$

where $\delta \in (0,1]$ is the depreciation rate of the capital invested in the household project. The optimization over $\{c_t^h, l_t^h, k_t^h, d_t^h\}$ gives the equilibrium conditions,

$$
w_t^h = \chi(1-l_t^h)^\psi (c_t^h)^\sigma, \tag{3.1}
$$

$$
\beta r_t = E_t \left(\frac{c_{t+1}^h}{c_t^h} \right)^\sigma, \tag{3.2}
$$

$$
r_t q_t = E_t \left[(1-\delta)q_{t+1} + v_{t+1} G'(k_t^h) \right]. \tag{3.3}
$$

3.2.3   Unobservable Project Choices

Each entrepreneur can invest capital goods in one of the three projects: "Good", "Bad", or "Rotten" at the end of each period. At the beginning of the next period, the project generates R_e units of intermediate goods per unit of capital invested and the invested capital depreciates at a rate $\delta' \in (0,1]$, if the project succeeds; if the project fails, there is no output and the invested capital is fully lost.

Producers of capital goods can invest final goods in one of the three projects: "Good", "Bad", or "Rotten". The project can transform one unit of final goods into R_c units of capital goods contemporaneously, if the project succeed; if the project fails, there is no output of capital goods and the invested final goods are wasted.

Project choices are irreversible and project outcomes are perfectly verifiable at no costs. Entrepreneurs and capital goods producers also enjoy safe, nonpecuniary private benefits during the project process. For convenience of aggregation, we assume that the private benefits of the entrepreneurial projects are proportional to their project investments in terms of capital goods and the private benefits of the projects of capital goods producers are proportional to their project investments in terms of final goods. The projects differ in the probability of success and unit private benefits. See table 3.1.

Table 3.1: Projects with Idiosyncratic Risk

Project	Good	Bad	Rotten
Probability of Success	p_m^G	p_m^B	p_m^R
Unit Private Benefits	b_m^G	b_m^B	b_m^R

where $m \in \{e, c\}$ denotes the project attributes of entrepreneurs and capital goods producers, respectively. Here, $0 < p_m^R = p_m^B < p_m^G < 1$ and $b_m^R > b_m^B > b_m^G > 0$ imply that projects "Rotten" and "Bad" are riskier than project "Good" but project "Rotten" yields highest private benefits and project "Good" yields lowest private benefits to project owners. Individual agents cannot observe the project choices of entrepreneurs and capital goods producers, while the financial intermediaries have expertise in screening out project "Rotten" at no costs but cannot distinguish between project "Good" and project "Bad". The advantage of financial intermediaries over individual agents justifies the fact that loans must be intermediated through the financial sector and there is no direct lending at the credit market.

3.2.4 Entrepreneurs

An entrepreneur[2] i who stays in the economy to the next period has linear preferences over consumption and private benefits. His expected utility function is,

$$E_0 \sum_{t=0}^{\tilde{T}} \beta^s \left[c_{i,t}^e + \mathcal{B}_e k_{i,t-1}^e \right], \tag{3.4}$$

---

[2]Entrepreneurs differ in the end-of-period wealth and are indexed by $i \in [0, 1]$.

where $\tilde{T}$ is the stochastic time of death and $\mathcal{B}_e \in \{b_e^G, b_e^B\}$ denotes unit private benefits of capital invested in project "Good" or project "Bad". $c_{i,t}^e$ denotes his consumption in period t and $k_{i,t-1}^e$ denotes his project investment in terms of capital goods made at the end of period $t-1$. Our calibration guarantees that only project "Good" has a positive expected net present value around the steady state,

$$\frac{p_e^G E_t[R_e v_{t+1} + (1-\delta')q_{t+1}] + b_e^G}{r_t} - q_t > 0 > \frac{p_e^B E_t[R_e v_{t+1} + (1-\delta')q_{t+1}] + b_e^B}{r_t} - q_t,$$

Therefore, other projects should not be financed in equilibrium. Project "Good" also has a larger expected marginal product than the household project.

At the end of period t, entrepreneur i invest $k_{i,t}^e$ units of capital goods in either project "Good" or project "Bad", using his own funds $n_{i,t}^e$ and inter-period loans $z_{i,t}^e$, i.e., $q_t k_{i,t}^e = n_{i,t}^e + z_{i,t}^e$. Thus, $n_{i,t}^e$ is entrepreneurial net worth in the project. The loan contract specifies a promise to repay $R_{e,t}^b k_{i,t}^e$ units of final goods in period $t+1$ if the project succeeds. If the project fails, both parties get zero pecuniary return. In order to motivate entrepreneur i to choose project "Good", financial intermediaries must provide him with enough incentives,

$$\{p_e^G E_t[R_e v_{t+1} + (1-\delta')q_{t+1} - R_{e,t}^b] + b_e^G\}k_{i,t}^e \geq \{p_e^B E_t[R_e v_{t+1} + (1-\delta')q_{t+1} - R_{e,t}^b] + b_e^B\}k_{i,t}^e.$$

The left (right) hand side denotes the expected utility of the entrepreneur if he chooses project "Good" ("Bad"). As he prefers to borrow to the limit, the incentive constraint is binding around the steady state and is simplified to be

$$R_{e,t}^b = E_t[R_e v_{t+1} + (1-\delta')q_{t+1}] - b_e, \quad \text{where} \quad b_e \equiv \frac{b_e^B - b_e^G}{p_e^G - p_e^B} > 0. \qquad (3.5)$$

Any promise to repay more than $R_{e,t}^b$ is not credible, because he would deliberately choose project "Bad". The expected external unit value and full unit value of the capital invested in project "Good" are $p_e^G R_{e,t}^b$ and $p_e^G E_t[R_e v_{t+1} + (1-\delta')q_{t+1}]$, respectively. The difference between the two values, $p_e^G b_e$, is used to motivate him to choose project "Good" despite lower private benefits it promises, $b_e^G < b_e^B$.

Financial intermediaries are expected to break even in lending to the entrepreneur in period t, $r_t z_{i,t}^e = p_e^G R_{e,t}^b k_{i,t}^e$. This implies a credit constraint for him,

$$z_{i,t}^e = \Gamma_t^e n_{i,t}^e, \quad \text{where} \quad \Gamma_t^e \equiv \frac{p_e^G R_{e,t}^b}{r_t q_t - p_e^G R_{e,t}^b}$$

is the credit multiplier. As Γ_t^e is independent of $n_{i,t}^e$, loans are proportional to entrepreneurial net worth. Our calibration guarantees $r_t q_t > p_e^G R_{e,t}^b$ around the

steady state and so $\Gamma_t^e > 0$. Note that the credit multiplier varies with q_t, r_t, $E_t q_{t+1}$, and $E_t v_{t+1}$. Ceteris paribus, a rise in the current price of capital q_t makes capital investment more expensive; similarly, a rise in the gross rate of return on inter-period deposits r_t makes external funds more expensive for entrepreneurs. In both cases, the credit multiplier falls so that less capital is allocated to the entrepreneurial sector. Ceteris paribus, a rise in the expected prices of capital or intermediate goods in period $t + 1$, $E_t q_{t+1}$ or $E_t v_{t+1}$, improves the expected external unit value of capital invested in their projects, $p_e^G R_{e,t}^b$. As a result, the credit multiplier is larger and entrepreneurs can expand their leveraged investments.

In equilibrium, entrepreneurs of mass $(1 - p_e^G)$ have the failed projects and exit from the economy. Each of those entrepreneurs whose projects succeed have a constant probability $\tilde{\pi}$ of surviving to the next period. In the aggregate, entrepreneurs of mass $p_e^G(1 - \tilde{\pi})$ have successful projects and exit from the economy, and entrepreneurs of mass $p_e^G \tilde{\pi}$ have successful projects and live on to the next period. New entrepreneurs of mass $(1 - \pi)$ are born. We assume $\pi = p_e^G \tilde{\pi}$ to keep the population size of entrepreneurs constant at unity.

Entrepreneur i maximizes his expected utility (3.4), subject to his period budget constraints and credit constraints,

$$q_t k_{i,t}^e - z_{i,t}^e = n_{i,t}^e, \quad \text{where} \quad n_{i,t}^e = \mathcal{N}_{i,t}^e - c_{i,t}^e \tag{3.6}$$

$$z_{i,t}^e = \Gamma_t^e n_{i,t}^e. \tag{3.7}$$

where $\mathcal{N}_{i,t}^e$ denotes his end-of-period wealth. Entrepreneurs differ in end-of-period wealth, an issue discussed in appendix B.1. Due to the linear nature of the project technologies and the preferences of entrepreneurs, the loan $z_{i,t}^e$ and the project investment $k_{i,t}^e$ of entrepreneur i is proportional to his net worth $n_{i,t}^e$. As a result, only the first moment of the distribution of entrepreneurial net worth matters for the aggregate capital stock in the entrepreneurial sector. See appendix B.1. Let lowercase letters without the index i denote per capita variables of the entrepreneurial sector. Per capita consumption c_t^e, per capita net worth n_t, per capita inter-period loans z_t^e, and per capita capital holding k_t^e are,

$$c_t^e = (1 - \tilde{\pi}) p_e^G [R_e v_t + (1 - \delta') q_t - R_{e,t-1}^b] k_{t-1}^e, \tag{3.8}$$

$$n_t^e = \tilde{\pi} p_e^G [R_e v_t + (1 - \delta') q_t - R_{e,t-1}^b] k_{t-1}^e + (1 - \pi) e, \tag{3.9}$$

$$z_t^e = \frac{p_e^G R_{e,t}^b}{r_t q_t - p_e^G R_{e,t}^b} n_t^e, \tag{3.10}$$

$$k_t^e = \frac{n_t^e + z_t^e}{q_t}. \tag{3.11}$$

For a better understanding of the model dynamics, we introduce three auxiliary variables. The first is the leverage ratio, defined as the ratio of total investment over entrepreneurial net worth, $\Omega_t^e \equiv \frac{q_t k_{i,t}^e}{n_{i,t}^e} = 1 + \Gamma_t^e$. The second is the entrepreneur's unit down payment of capital, defined as the amount of own funds the entrepreneur pays for each unit of capital, $u_t^e \equiv q_t - \frac{z_{i,t}^e}{k_{i,t}^e} = \frac{q_t}{\Omega_t^e}$. The third is the expected profitability of the entrepreneurial project, defined as the discounted expected gross rate of return on entrepreneurial net worth, $\xi_t^e \equiv \frac{\beta p_e^G E_t[R_e v_{t+1} + (1-\delta')q_{t+1} - R_{e,t}^b]k_{i,t}^e}{n_{i,t}^e} = \frac{\beta p_e^G b_e}{u_t^e}$. Our calibration guarantees that the expected profitability of the entrepreneurial project exceeds the discounted cost of external funds around the steady state, $\xi_t^e > \beta r_t$. As a result, entrepreneurs prefer to postpone consumption and borrow to the limit.

For convenience of aggregation, we assume that capital depreciates faster in the household projects than in the entrepreneurial projects that turn out to be successful, $\delta = 1 - p^G + p^G \delta' > \delta'$. Thus, the aggregate capital stock depreciates at the same rate in both household and entrepreneurial sectors, $1 - \delta = p^G(1 - \delta')$.

3.2.5 Capital Goods Producers

A capital goods producer[3] j has linear preferences over consumption and and private benefits. His expected utility function is,

$$E_0 \sum_{t=0}^{\infty} (\gamma\beta)^s \left[c_{j,t}^c + \mathcal{B}_c i_{j,t}^c \right], \tag{3.12}$$

where $c_{j,t}^c$ and $i_{j,t}^c$ denote, respectively, his consumption and project investment. $\gamma \in (0,1)$ implies that capital goods producers are less patient than households and entrepreneurs. It guarantees that capital goods producers have positive consumption in equilibrium so that credit constraints are always binding around the steady state. $\mathcal{B}_c \in \{b_c^G, b_c^B\}$ denotes unit private benefits of final goods invested in project "Good" or project "Bad". As he does not care about leisure, he supplies labor endowment inelastically to the production of final goods, $l_t^c = 1$, at the wage rate, w_t^c. Our calibration guarantees that only project "Good" has a positive expected net present value around the steady state,

$$p_c^G R_c q_t + b_c^G - 1 > 0 > p_c^B R_c q_t + b_c^B - 1,$$

given that the gross rate of return on intra-period loan is unity. Therefore, only project "Good" should be financed. For simplicity, we assume $p_c^G R_c = 1$, i.e., final goods are transformed one-to-one into capital goods in the aggregate in equilibrium.

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[3]Capital goods producers differ in their end-of-period wealth and are indexed by $j \in [0,1]$.

After the final goods have been produced in period t, the total wealth of capital goods producer j consists of his wage income and the gross return on his inter-period deposit made in period $t-1$. In equilibrium, he uses own funds, $n_{j,t}^c = r_{t-1} d_{j,t-1}^c + w_t^c$ and intra-period loans $z_{j,t}^c$ to invests $i_{j,t}^c = n_{j,t}^c + z_{j,t}^c$ units of final goods in either project "Good" or project "Bad". Thus, $n_{j,t}^c$ is the net worth of the capital goods producer. The loan contract specifies a promise to repay $R_{c,t}^b i_{j,t}^c$ units of final goods if the project succeeds; if the project fails, both parties get zero return. In order to motivate the capital goods producer to choose project "Good", financial intermediaries must provide him with enough incentives,

$$p_c^G (R_c q_t - R_{c,t}^b) i_{j,t}^c + b_c^G i_{i,t}^c \geq p_c^B (R_c q_t - R_{c,t}^b) i_{j,t}^c + b_c^B i_{i,t}^c.$$

The left (right) hand side denotes the expected utility of the capital goods producer if he chooses project "Good" ("Bad"). As he prefers to borrow to the limit, the incentive constraint is binding around the steady state and is simplified to be

$$R_{c,t}^b = R_c q_t - b_c, \quad \text{where} \quad b_c \equiv \frac{b_c^B - b_c^G}{p_c^G - p_c^B} > 0. \tag{3.13}$$

Any promise to repay more than $R_{c,t}^b$ is not credible, because he would deliberately choose project "Bad". The expected external unit value and full unit value of the final goods invested in project "Good" are $p_c^G R_{c,t}^b$ and $p_c^G R_c q_t$, respectively. The difference between the two values, $p_c^G b_c$, is used to motivate him to choose project "Good" despite lower private benefits it promises, $b_c^G < b_c^B$.

Financial intermediaries are expected to break even in lending to the capital goods producer in period t, $z_{j,t}^c = p_c^G R_{c,t}^b i_{j,t}^c$. This implies a credit constraint for the capital goods producer,

$$z_{j,t}^c = \Gamma_t^c n_{j,t}^c, \quad \text{where} \quad \Gamma_t^c \equiv \frac{p_c^G R_{c,t}^b}{1 - p_c^G R_{c,t}^b}$$

is the credit multiplier. As Γ_t^c is independent of $n_{j,t}^c$, loans are proportional to the net worth of the capital goods producer. Our calibration guarantees $p_c^G R_{c,t}^b < 1$ around the steady state and so $\Gamma_t^c > 0$. Note that the credit multiplier varies with q_t. A rise in the current price of capital raises the expected external unit value of the project of the capital goods producer and he can borrow more to invest. In the aggregate, more funds flow into the production of capital goods and more capital goods are produced.

Each unit of the net worth of the capital goods producer enables him to acquire Γ_t^c units of intra-period loans and so, he invests $1 + \Gamma_t^c$ units of final goods in project

"Good". The expected gross rate of return on his net worth is

$$\xi_t^c = p_c^G(R_c q_t - R_{c,t}^b)(1 + \Gamma_t^c) = \frac{p_c^G b_c}{p_c^G b_c - (q_t - 1)}. \tag{3.14}$$

The expected one-to-one transformation of final goods into capital goods, $p_c^G R_c = 1$, implies that the price of capital must be no less than unity. Otherwise, the project would make a loss, $\xi_t^c < 1$. If the price of capital is at unity, $q_t = 1$, the project breaks even by expectation, $\xi_t^c = 1$, so that producers of capital goods do not invest own funds in the project. If the price of capital exceeds unity, $q_t > 1$, the project is profitable, $\xi_t^c > 1$, so that producers of capital goods put all own funds in the project and borrow to the limit.

Capital goods producer j maximizes his expected utility (3.12), subject to his credit constraints and period budget constraints,

$$z_{j,t}^c = \Gamma_t^c n_{j,t}^c, \quad \text{where} \quad n_{j,t}^c = r_{t-1} d_{j,t-1}^c + w_t^c \tag{3.15}$$

$$d_{j,t}^c + c_{j,t}^c = \mathcal{N}_{j,t}^c, \tag{3.16}$$

where $\mathcal{N}_{j,t}^c$ is his end-of-period wealth. Capital goods producers differ in their end-of-period wealth, an issued discussed in appendix B.2. $\mathcal{N}_{j,t}^c = (R_c q_t - R_{c,t}^b)i_{j,t}^c$, where $i_{j,t}^c = (1 + \Gamma_t^c)n_{j,t}^c$, if the project succeeds; if the project fails, $\mathcal{N}_{j,t}^c = 0$. Due to the linear nature of the project technologies and the preferences of capital goods producers, loans and the project investment is proportional to his net worth. As a result, only the first moment of the distribution of the net worth of capital goods producers matters for the aggregate capital investment. See appendix B.2. We use lower-case letters without the index j to denote per capita variables of the capital goods production sector. Period budget constraint, per capita net worth n_t^c, and per capita credit constraints are,

$$d_t^c + c_t^c = \xi_t^c n_t^c, \tag{3.17}$$

$$n_t^c = r_{t-1} d_{t-1}^c + w_t^c, \tag{3.18}$$

$$z_t^c = i_t^c - n_t^c = p_c^G R_{c,t}^b i_t^c, \tag{3.19}$$

where c_t^c and d_t^c are per capita consumption and deposit; i_t^c and z_t^c are per capita project investment and intra-period loans. Given linear preferences, their marginal utility of consuming a unit of final goods is one. If they deposit a unit of final goods at the financial intermediaries at the end of period t, they can get a safe rate of return, r_t, in period $t + 1$. They then invest the deposit return in project "Good" for the expected return of $E_t r_t \xi_{t+1}^c$. The optimization between consumption and

deposit at the end of period t gives the equilibrium condition,

$$1 = E_t \gamma \beta r_t \xi_{t+1}. \tag{3.20}$$

For a better understanding of the model dynamics, we introduce two auxiliary variables. The first is the leverage ratio, defined as the ratio of total investment over the net worth of capital goods producers, $\Omega_t^c \equiv \frac{i_{j,t}^c}{n_{j,t}^c} = 1 + \Gamma_t^c$; the second is the unit down payment of capital goods producers, defined as the amount of own funds the capital goods producer pays for each unit of final goods invested in project "Good", $u_t^c \equiv \frac{n_t^c}{i_t^c} = 1 - p_c^G R_{c,t}^b$. As capital goods are one-to-one transformed from final goods in the aggregate, $p_c^G R_c = 1$, the aggregate capital stock K_t evolves as follows,

$$K_t = (1 - \delta) K_{t-1} + i_t^c. \tag{3.21}$$

3.2.6 Financial Intermediaries

Financial intermediaries conduct intra-period and inter-period business separately in equilibrium. On the one hand, they accept intra-period deposits from households and make intra-period loans to producers of capital goods after final goods are produced. At the end of the same period, the producers of capital goods with successful projects repay their debts and households get the return on their deposits. On the other hand, the intermediaries accept inter-period deposits from households and producers of capital goods and make inter-period loans to entrepreneurs at the end of the period; at the beginning of the next period, successful entrepreneurs repay their debts and households and producers of capital goods get the return on their deposits. A deposit contract is a claim on the financial position of the intermediary.

Consider the intra-period business of the financial intermediaries. There is no aggregate uncertainty during the production of capital goods. By perfectly diversifying the portfolios of intra-period loans, the intermediaries pool the idiosyncratic project risk of capital goods producers and pay a safe rate of return on intra-period deposits at unity. Therefore, we do not have to specify the household decision on intra-period deposit explicitly. Due to perfect competition, the intermediaries transfer all of the debt repayments to depositors and make zero profit.

Consider now the inter-period business of the financial intermediaries. Suppose, first, that the project choice of entrepreneurs is perfectly observable so that they can pledge all of the project outcomes to the intermediaries for external funds. Due to the presence of the aggregate risk related to TFP shocks, the period-t prices of capital and intermediate goods may differ from their expected values, i.e., $q_t \neq E_{t-1}q_t$ and $v_t \neq E_{t-1}v_t$, and so may the values of the project outcomes of entrepreneurs,

$p_e^G[R_e v_t + (1 - \delta')q_t]k_{t-1}^e \neq p_e^G[R_e E_{t-1}v_t + (1 - \delta')E_{t-1}q_t]k_{t-1}^e$. As a result, the rate of return on inter-period deposits is contingent on productivity shocks.

In contrast, in the case of unobservable project choices of the entrepreneurs, the loan contract described in subsection 3.2.4 implicitly provides entrepreneurs with a net unit return on capital, with a positive expected value, $p_e^G b_e > 0$, in period $t - 1$. The ex post net unit return on capital to the successful entrepreneurs in period t is

$$R_e v_t + (1 - \delta')q_t - R_{e,t-1}^b = b_e + R_e(v_t - E_{t-1}v_t) + (1 - \delta')(q_t - E_{t-1}q_t).$$

As long as the aggregate productivity shocks are larger than some threshold value, the ex post values of the project outcomes of successful entrepreneurs are larger than the promised repayments, $[R_e v_t + (1 - \delta')q_t]k_{t-1}^e > R_{e,t-1}^b k_{t-1}^e$, and the successful entrepreneurs repay their liabilities, $R_{e,t-1}^b k_{t-1}^e$, to the intermediaries. Let K_{t-1}^e and Z_{t-1}^e denote the aggregate capital stock held by entrepreneurs and the aggregate lending to entrepreneurs at the end of period $t - 1$, respectively. The aggregate break-even condition of the financial sector is $r_{t-1}Z_{t-1}^e = p_e^G R_{e,t-1}^b K_{t-1}^e$. At the beginning of period t, entrepreneurs of mass p_e^G have successful projects and their total repayments, $p_e^G R_{e,t-1}^b K_{t-1}^e$, coincide with the expected value. In this sense, the positive expected net return to entrepreneurs, $p_e^G b_e K_{t-1}^e$, absorbs the aggregate risk and the financial intermediaries pay a safe rate of return on deposits in equilibrium. For aggregate productivity shocks below this threshold, the prices of capital and intermediate goods are so low that the ex post value of the project outcomes of successful entrepreneurs is less than their debt obligations, $[R_e v_t + (1 - \delta')q_t]k_{t-1}^e < R_{e,t-1}^b k_{t-1}^e$. Even the successful entrepreneurs have to announce bankruptcy and transfer all the project outcomes to the intermediaries. Thus, in the case of moral hazard in the production of intermediate goods, the ex post rate of return on inter-period deposits is contingent on the productivity shock only for very large, negative shocks.

As a consequence, if productivity shocks are unbounded, the ex post rate of return on inter-period deposits as a function of productivity shocks could have a kink at the point where $R_e v_t + (1 - \delta')q_t = R_{e,t-1}^b$. The first-order approximations used below to analyze the dynamics of our model require that the endogenous variables should be continuous and differentiable functions of the state variables. For the purpose of the the approximations, we assume that TFP shocks are distributed with mean zero and a negative lower bound guaranteeing that successful entrepreneurs are always able to repay the promised amount. See subsection 3.2.7 for details.

3.2.7 Final Goods Production and Market Equilibrium

Final goods are produced from intermediate goods and labor,

$$Y_t = A_t M_t^\alpha (L_t^h)^{(1-\alpha-\alpha')} (L_t^c)^{\alpha'}, \tag{3.22}$$

$$\log A_t = \rho \log A_{t-1} + \epsilon_t, \tag{3.23}$$

where M_t, L_t^h, and L_t^c denote the aggregate inputs of intermediate goods, the labor of households, and the labor of capital goods producers[4]. Total factor productivity, A_t, is positively autocorrelated in logarithms, where $\rho \in (0, 1)$. The productivity shock has mean zero, $E_t \epsilon_{t+1} = 0$, and is distributed above a lower bound, $(-\tau, \infty)$, where $\tau > 0$ is small enough that the successful entrepreneurs are always able to repay their liabilities. The production of final goods takes place at the efficient level,

$$v_t M_t = \alpha Y_t, \tag{3.24}$$

$$w_t^h L_t^h = (1 - \alpha - \alpha') Y_t, \tag{3.25}$$

$$w_t^c L_t^c = \alpha' Y_t. \tag{3.26}$$

Markets for intermediate goods, final goods, capital, labor, and loans clear,

$$M_t = G(k_{t-1}^h) + p_e^G R_e k_{t-1}^e, \tag{3.27}$$

$$Y_t + (1 - \pi)e = c_t^h + c_t^c + c_t^e + i_t^c, \tag{3.28}$$

$$K_t = k_t^h + k_t^e, \tag{3.29}$$

$$z_t^e = d_t^c + d_t^h, \tag{3.30}$$

$$L_t^h = l_t^h, \tag{3.31}$$

$$L_t^c = l_t^c = 1. \tag{3.32}$$

Definition 3.1. *Market equilibrium is a set of allocations of households, $\{k_t^h, l_t^h, c_t^h, d_t^h\}$, entrepreneurs, $\{k_t^e, n_t^e, c_t^e, z_t^e\}$, and capital goods producers, $\{n_t^c, i_t^c, c_t^c, d_t^c\}$, together with aggregate variables $\{M_t, Y_t, K_t\}$ given a set of prices $\{v_t, q_t, w_t^h, w_t^c, r_t, \xi_t^c, R_{e,t}^b, R_{c,t}^b\}$ and the exogenous process $\{A_t\}$ satisfying equations (3.1)-(3.3), (3.5), (3.8)-(3.11), (3.13)-(3.14), (3.17)-(3.30).*

3.3 Financial Contracting in Other Scenarios

As shown in subsection 3.2.6, the financial intermediaries accept inter-period and intra-period deposits and make inter-period and intra-period loans in the model with

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[4]As each type of agents is of unit mass, aggregate variables coincide with per capita variables.

dual financial frictions (Model DF). In order to understand macroeconomic fluctuations in the setting with dual financial frictions, we briefly discuss three alternative scenarios for the dynamic analysis in section 3.4.

3.3.1 Model SFE

In this subsection, we assume that the project choice of capital goods producers is observable but that of entrepreneurs is not. Thus, capital goods producers can credibly choose project "Good" and financial frictions exist only in the entrepreneurial sector. This scenario captures the basic amplification and propagation mechanism and we call it model SFE.

Since producers of capital goods can pledge all the expected project outcomes for external funding, $z_t^c = q_t p_c^G R_c i_t^c$, they do not have to put down own funds in their projects, $n_t^c = 0$. Capital goods are priced at $q_t = 1$. Because the deposit rate is less than their time preference rate around the steady state, $r_t < \frac{1}{\gamma \beta}$, they do not deposit, $d_t^c = 0$. They consume their wage income each period, $c_t^c = w_t^c$. For simplicity, we focus on a symmetric equilibrium in which all producers of capital goods invest the same amount of final goods i_t^c in project "Good" and enjoy private benefits, $b_c^G i_t^c$. Other sectors are same as in the setting with dual financial frictions.

3.3.2 Model SFC

In this subsection, we assume that the project choice of entrepreneurs is observable but that of capital goods producers is not. Thus, entrepreneurs can credibly choose project "Good" and financial frictions exist only in the capital goods production sector. We call it model SFC.

Since entrepreneurs can pledge all of the expected project outcomes for external funding, $z_{i,t}^e = \frac{p_e^G [R_e v_{t+1} + (1-\delta')q_{t+1}] k_{i,t}^e}{r_t}$, they do not have to put down own funds in their projects, $n_{i,t}^e = 0$. As the expected rate of return on project "Good" is higher than that on the household project, all capital is allocated to the entrepreneurial projects and intermediate goods are produced by entrepreneurs only. We focus on a symmetric equilibrium in which all entrepreneurs use external funds to invest the same amount of capital goods k_t^e in their projects and enjoy private benefits, $b_e^G k_t^e$ in period $t+1$. Newcomers consume their endowment each period, $c_t^e = (1-\pi)e$.

The rates of return on inter-period deposits are different in the settings with and without the unobservable project choice of entrepreneurs. As shown in subsection 3.2.6, the positive expected stake of entrepreneurs in the project outcomes, $p_e^G b_e k_t^e > 0$, absorbs the aggregate risk due to TFP shocks in the setting with financial frictions

in the entrepreneur sector. This enables the financial sector to guarantee a safe rate of return on inter-period deposits, r_t. In this sense, the intermediaries take the form of banks. Without moral hazard, no incentive is required for entrepreneurs to engage in project "Good". The intermediaries can only diversify the idiosyncratic project risk of entrepreneurs but not the aggregate risk. Given that the intermediaries do not accumulate reserves in our model, inter-period depositors have to bear this aggregate risk. In this sense, the intermediaries take the form of mutual funds and the rate of return on deposit is contingent on the productivity shock.

Households and capital goods producers, respectively, put d_{t-1}^h and d_{t-1}^c units of final goods at the intermediaries at the end of period $t-1$ for the claim of the financial position of the intermediaries in period t. The intermediaries fully finance the project investments of entrepreneurs, $q_{t-1}k_{t-1}^e = z_{t-1}^e = d_{t-1}^h + d_{t-1}^c$. After the project completion in period t, the intermediaries collect all the project outcomes, $p_e^G[R_e v_t + (1-\delta')q_t]k_{t-1}^e$, and transfer to depositors pro rata. The ex post rate of return on deposits,

$$\tilde{r}_t = \frac{p_e^G[R_e v_t + (1-\delta')q_t]k_{t-1}^e}{z_{t-1}^e} = \frac{p_e^G[R_e v_t + (1-\delta')q_t]}{q_{t-1}},$$

differs from its expected value by $\tilde{r}_t - E_{t-1}\tilde{r}_t = \frac{p_e^G[R_e(v_t - E_{t-1}v_t) + (1-\delta')(q_t - E_{t-1}q_t)]}{q_{t-1}}$ due to the unexpected changes in the prices of capital and intermediate goods. For uniformity, we use r_t to denote the expected rate of return on deposits, $r_t \equiv E_t\tilde{r}_{t+1}$. Other sectors remain the same as in the setting with dual financial frictions.

3.3.3 Model *RBC*

In this subsection, we assume that the project choices of entrepreneurs and capital goods producers are observable. Thus, both entrepreneurs and capital goods producers can credibly choose project "Good" and pledge all the expected outcomes of their projects for external funding. They can use external funds to fully financed their project investments, $q_t k_t^e = z_t^e$ and $i_t^c = z_t^c$ and do not have to put down own funds in their projects, $n_t^e = n_t^c = 0$. The price of capital is constant at unity, $q_t = 1$ and the projects of capital goods producers earn zero profits, $\xi_t^c = 1$. Capital is all allocated to entrepreneurs and only entrepreneurs produce intermediate goods.

The model economy is equivalent to a RBC model with a representative agent who has three production technologies: a linear technology to produce intermediate goods using capital, a Cobb-Douglas technology to produce final goods using intermediate goods and labor, and a linear technology to transform final goods into capital goods. So, we call it model *RBC*. The market equilibrium can

be defined as the set of two state variables $\{k_t^e, A_t\}$ and nine control variables $\{c_t^h, l_t^h, w_t^h, c_t^c, i_t^c, w_t^c, v_t, M_t, Y_t\}$ satisfying equations from (3.33) to (3.43),

$$1 = \beta E_t \left(\frac{c_{t+1}^h}{c_t^h} \right)^{-\sigma} p_e^G [(1-\delta') + R_e v_{t+1}], \tag{3.33}$$

$$w_t^h = \chi(1 - l_t^h)^\psi (c_t^h)^\sigma, \tag{3.34}$$

$$M_t = p_e^G R_e k_{t-1}^e, \tag{3.35}$$

$$Y_t = A_t M_t^\alpha (l_t^h)^{1-\alpha-\alpha'}, \tag{3.36}$$

$$v_t M_t = \alpha Y_t, \tag{3.37}$$

$$w_t^h l_t^h = (1 - \alpha - \alpha')Y_t, \tag{3.38}$$

$$w_t^c = \alpha' Y_t, \tag{3.39}$$

$$\log A_t = \rho \log A_{t-1} + \epsilon_t, \tag{3.40}$$

$$c_t^h + c_t^c + i_t^c = Y_t, \tag{3.41}$$

$$c_t^c = w_t^c, \tag{3.42}$$

$$k_t^e = p_e^G (1 - \delta')k_{t-1}^e + i_t^c. \tag{3.43}$$

3.4 Dynamic Analysis

3.4.1 Calibration

For convenience of aggregation, we assume that the household project is linear,

$$G(k_t^h) = \frac{1}{2} \left(1 + \frac{k_t^e}{K_t} \right) k_t^h, \tag{3.44}$$

and the marginal product is $G'(k_t^h) = \frac{1}{2} \left(1 + \frac{k_t^e}{K_t} \right)$. This functional form implies that the entrepreneurial sector has positive production externality on the household project. The quarterly discount factor is set at $\beta = 0.99$, corresponding to an annual interest rate of 4%, while the relative impatience of capital goods producers versus households is set at $\gamma = 0.95$, as in Carlstrom and Fuerst (1997, 1998) and Kato (forthcoming). By convention, households have logarithmic preferences in consumption ($\sigma = 1$) and in leisure ($\psi = -1$). We set $\chi = 1.95$ so that households work eight hours a day in the final goods production sector in the steady state, $l^h = \frac{1}{3}$. We set $\alpha' = 0.001$ and $\alpha = 0.36$ so that capital goods producers always have positive wealth to start the projects and the household wage income accounts for 63.9% of aggregate output of final goods.

Following Carlstrom and Fuerst (1997, 1998), we choose $\rho = 0.95$ for the auto-correlation coefficient of TFP. Capital invested in the household project depreciates at a quarterly rate of $\delta = 2.5\%$ and capital invested in the entrepreneurs' projects that become successful depreciates at the rate of $\delta' = 1.52\%$. As in Carlstrom and Fuerst (1997), a quarterly rate of business failure at 1% implies $p_e^G = p_c^G = 0.99$. Thus, the aggregate capital stock depreciates at the rate of $\delta = 2.5\%$ in equilibrium. By assumption, $R_c = \frac{1}{p_c^G} = 1.01$.

The expected profitability of the projects of capital goods producers is $\xi = \frac{1}{\gamma} > 1$ in the steady state so that they invest all own funds in their projects and borrow to the limit. We set $b_c = 0.55$ so that the leverage ratio is $\Omega^c = 2$, implying that capital goods producers finance half of their project investments using intra-period loans, as in Bernanke, Gertler, and Gilchrist (1999).

The surviving probability of the entrepreneurs with successful projects is set at $\tilde{\pi} = \frac{2}{3}$, implying that around 34% of entrepreneurs have to exit from the economy each period, $\pi = p_e^G \tilde{\pi} = 0.66$. We set $R_e = 6.04$ so that the expected marginal product of the entrepreneurial project in terms of intermediate goods always exceeds that of the household project, $p_e^G R_e > G'(0)$. It guarantees that capital is allocated to the entrepreneurial sector if their project choice is observable. Together with the calibration of π and R_e, we set $b_e = 0.78$ to satisfy the following conditions in the steady state: entrepreneurs hold half of the aggregate capital stock, $\frac{k^e}{K} = 0.5$; the leverage ratio, $\Omega^e = 2$, implies that entrepreneurs finance half of the their project investments using external funds; the entrepreneurs with successful projects can keep 60% of the project outcomes for themselves, $\frac{R_e^b}{(1-\delta')q + vR_e} = 40\%$.

3.4.2 Impulse Responses to Productivity Shocks

We log-linearize the equations describing the market equilibria of the four models (DF, SFE, SFC, and RBC) around their respective steady states and adopt the approach to the first-order approximations provided by Schmitt-Grohe and Uribe (2004). The endogenous variables are represented as the linear functions of the state variables. We analyze the impulse responses of endogenous variables with respect to a transitory TFP shock in period 0, given that relevant models are in their steady states before period 0. Subsections 3.4.2.1 and 3.4.2.2 discuss, respectively, how financial frictions in the entrepreneurial sector and in the capital goods production sector can change the dynamic responses of macroeconomic aggregates to TFP shocks. Then, subsections 3.4.2.3 and 3.4.2.4 investigate the interactions between time-varying prices of capital and dual financial frictions.

3.4.2.1   Financial Frictions in the Capital Goods Production

Figure 3.2 shows the impulse responses of model RBC (dash line) and model SFC (solid line) to a TFP shock, where EN, HH, and CGP denote households, entrepreneurs, and capital goods producers, respectively.

Consider model RBC. As capital is the only endogenous state variable in model RBC, its dynamic structure is essentially $ARMA(1,1)$ and fails to generate the hump-shaped output dynamics. A 1% TFP shock raises the aggregate demand for labor and intermediate goods in period 0. On the one hand, the rise in the marginal product of labor pushes up the household wage rate by 0.73% contemporaneously. Given the autocorrelation in TFP, as households prefer to smooth consumption over time and optimize between consumption and labor, they increase labor supply by 0.75%. Given that the aggregate supply of intermediate goods is determined by the project investments of entrepreneurs made in period -1, aggregate output of final goods rises by 1.48% in period 0. Meanwhile, the price of intermediate goods jumps by 1.48% to clear the market.

Due to the autocorrelation in TFP, the marginal product of intermediate goods stays above its steady state value in period 1, so does the price of intermediate goods. It improves the expected external unit value of capital invested in the entrepreneurial projects in period 0, $p_e^G R_{e,0}^b = p_e^G E_0[R_e v_1 + (1 - \delta')]$, by 0.05%. Entrepreneurs can then raise more external funds and expand their project investments. On the one hand, their excess demand for external funds pushes up the expected rate of return on deposits by 0.046% contemporaneously and induces households to raise their inter-period deposits by 0.12%; on the other hand, producers of capital goods increase their investment expenditure by 4.62% to fully accommodate the entrepreneurs' extra demand for capital goods. In equilibrium, household consumption rises by 0.4% in period 0 and producers of capital goods simply consume their wage income. Essentially, the model dynamics are driven by the fact that households smooth consumption over time by saving in the form of capital goods, as in the standard RBC model.

Consider model SFC. There are three endogenous state variables, $\{k_t^e, d_t^c, z_t^e\}$ and the dynamic interactions between the price of capital and financial frictions in the production of capital goods help generate the hump-shaped output responses to productivity shocks. Similar as in model RBC, a 1% TFP shock leads to the entrepreneurs' excess demand for capital. As the production of capital goods is constrained by the aggregate net worth of capital goods producers, the entrepreneurs' excess demand for capital cannot be fully accommodated and the price of capital goods rises by 0.63% in equilibrium. On the one hand, the project of capital
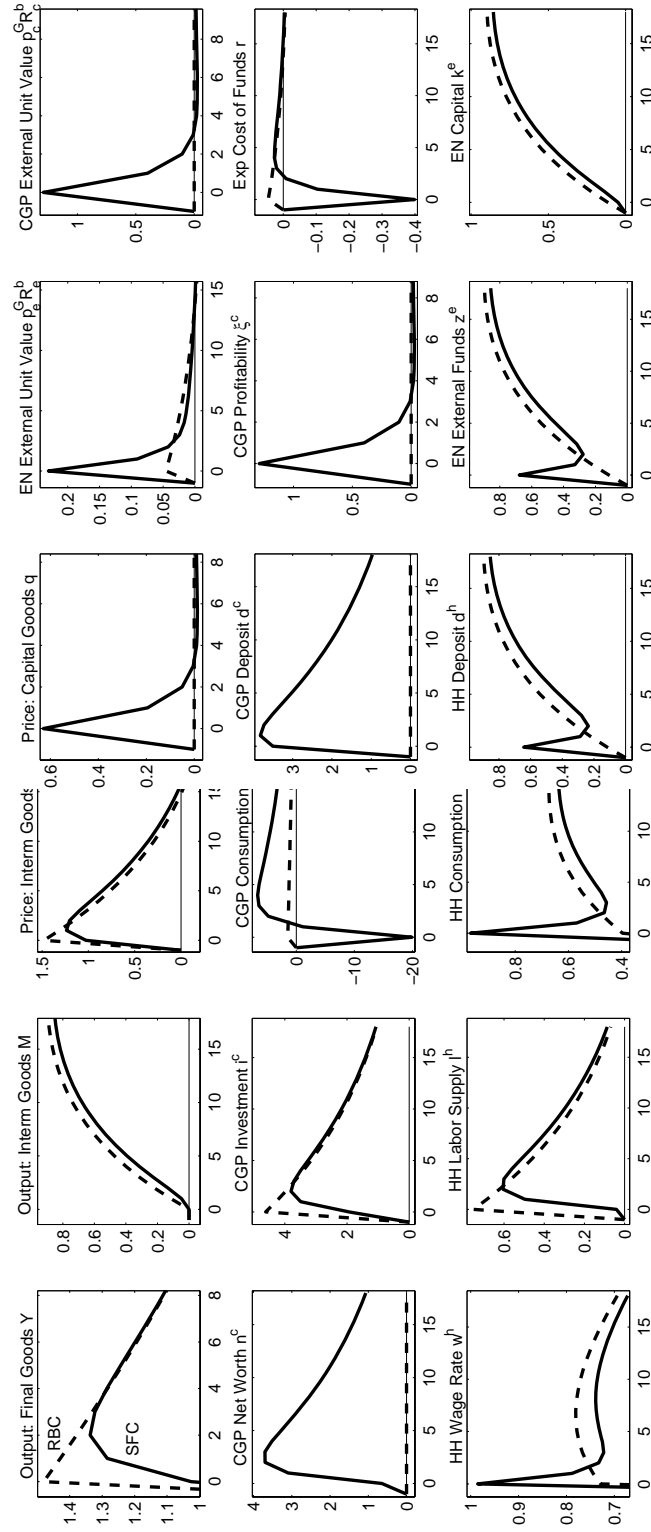
Figure 3.2: Model *SFC* vs. Model *RBC*

goods producers becomes more profitable so that the credit multiplier rises; on the other hand, the realized rate of return on deposits exceeds its expected value by the amount of $\tilde{r}_0 - E_{-1}\tilde{r}_0 = \frac{p_e^G[(1-\delta')(q_0 - E_{-1}q_0) + R_c(v_0 - E_{-1}v_0)]}{q_{-1}} > 0$ so that the net worth of capital goods producers[5], $n_0^c = \tilde{r}_0 d_{-1}^c + w_0^c$, rises by 0.65%. As a result, capital goods producers can expand their project investments, $i_0^c = (1 + \Gamma_0^c)n_0^c$, by 1.94%. Compared with the rise in aggregate output of capital goods by 4.6% in model RBC, the output of capital goods is much less price-elastic and more depressed due to credit constraints. It then justifies the rise in the price of capital goods.

The rise in the ex post rate of return on deposits increases the return on household deposits $\tilde{r}_0 d_{-1}^h$ by 0.65%. The wealth effect induces households to increase inter-period deposits and consumption by 0.64% and 0.96%, more than the 0.12% and the 0.4% in model RBC, respectively. Although the wage rate rises by 0.99%, households increase their labor supply only by 0.04%, because of the intratemporal substitution between labor and consumption. Thus, aggregate output of final goods rises by 1.03% in period 0, much less than the 1.48% in model RBC.

Given that the price of capital stays above its steady state value in period 1, the projects are still more profitable for capital goods producers than in the steady state. In order to expand their production in period 1, they reduce consumption by nearly 20% and increase inter-period deposits by 3.5% in period 0. Excess deposits of households and capital goods producers reduce the expected rate of return on deposits by 0.4%. The net worth of capital goods producers increases by 3.1% in period 1. As the price of capital is still above the steady state value by 0.19% in period 1, the project profitability of capital goods producers is 0.4% above the steady state value. They expand their investments by 3.49% and partially mitigates the entrepreneurs' excess demand for capital. As their aggregate net worth is still insufficient in period 1, the constrained capital goods production justifies the fact that the price of capital is around 0.19% above the steady state level in period 1.

Due to the financially-constrained production of capital goods, the aggregate capital stock rises only by 0.05% in period 0, less than the 0.12% in model RBC; so is aggregate output of intermediate goods in period 1. Meanwhile, the household wage rate is 0.79% above the steady state value in period 1. As the deposit return improves household wealth in period 1, households raise their consumption and labor supply by 0.57% and 0.5%, respectively. As intermediate goods and labor are the two inputs needed for the final goods production, aggregate output is around 1.28% above the steady state value, still lower than the 1.43% in model RBC.

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[5]As the wage income accounts for only 0.8% of the net worth of capital goods producers, the change in their wage income has a negligible effect on their net worth.

It takes two periods before capital goods producers can accumulate sufficient net worth and accommodate the excess demand for capital. The price of capital converges very close to the steady state value from period 2 on. The interaction between the price of capital and the accumulation of the net worth of capital goods producers in model SFC constitutes a dampened propagation mechanism through which output peaks by 1.34% in period 2, later and smaller than in model RBC.

3.4.2.2 Financial Frictions in the Entrepreneurial Sector

Figure 3.3 shows the impulse responses of model SFE (solid line) and model RBC (dash line). There are three endogenous state variables, $\{k_t^e, k_t^h, R_{e,t}^b\}$ in model SFE. Different from the dampened propagation mechanism in model SFC, it is now the reallocation of capital between entrepreneurs and households that generates the amplified and hump-shaped output responses to TFP shocks. Given that capital goods are one-to-one transformed from final goods in the aggregate, the price of capital is constant at unity, $q_t = 1$.

Consider model SFE. A 1% TFP shock in period 0 raises the aggregate demand for intermediate goods and the price rises to clear the market. Extra sales revenues improve per capita post-repayment wealth of entrepreneurs,

$$\mathcal{N}_0^e = p_e^G \left[b_e + R_e(v_t - E_{-1}v_0)\right] k_{-1}^e > p_e^G b_e k_{-1}^e = E_{-1}\mathcal{N}_0^e.$$

Entrepreneurial net worth rises by 0.42%, as $n_0^e - E_{-1}n_0^e = \tilde{\pi}(\mathcal{N}_0^e - E_{-1}\mathcal{N}_0^e)$. Meanwhile, given that the price of intermediate goods is 0.77% above the steady state value in period 1, the expected external unit value of capital invested in the projects of entrepreneurs in period 0, $p_e^G R_{e,0}^b = p_e^G E_0[(1 - \delta') + R_e v_1 - b_e]$, is around 0.44% above the steady state value. Entrepreneurs then demand more loans, which pushes up the loan rate by 0.033%. Thus, the entrepreneur's unit down payment, $u_0^e = 1 - \frac{p_e^G R_{e,0}^b}{r_0}$, falls by 0.39% below the steady state value in period 0. Altogether, entrepreneurs increase their project investments, $k_0^e = \frac{n_0^e}{u_0^e}$, by 0.81%.

The rise in the loan rate induces households to increase inter-period deposits by 1.22% and to reduce their project investments by 0.33% in period 0. Extra sales revenues of intermediate goods have the wealth effect on the household consumption and leisure decision. Although the household wage rate rises by 0.92% in period 0, they increase their labor supply only by 0.21%, much less than the 0.75% in model RBC. As the aggregate supply of intermediate goods is predetermined, output of final goods increases only by 1.13% in period 0, less than the 1.48% in model RBC.

Given that the price of intermediate goods stays above the steady state value for six periods after the shock, the entrepreneur's unit down payment of capital,
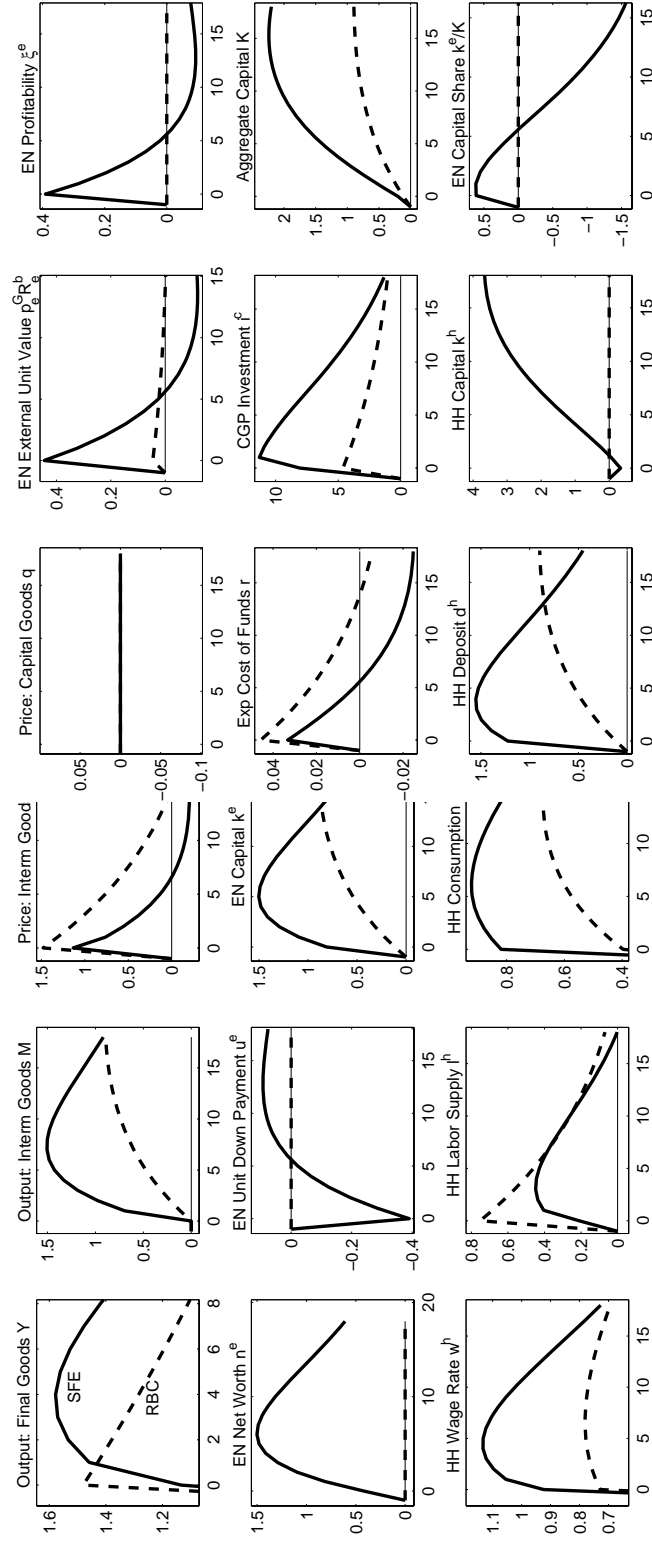
Figure 3.3: Model *SFE* vs. Model *RBC*

$u_t^e = 1 - \frac{p_e^G[(1-\delta')+R_e E_t v_{t+1}]}{r_t}$, stays below the steady state value for five periods after the shock, and entrepreneurs can invest more capital in their projects than in the steady state. Meanwhile, entrepreneurial net worth is positively correlated with their capital investment in the previous period, $n_t^e = \tilde{\pi} p_e^G b_e k_{t-1}^e$, where $t = 1, 2, 3, ...$. Thus, their capital investments peak in period 5, $\frac{k_t^e}{k_{t-1}^e} = \frac{\tilde{\pi} p_e^G b_e}{u_t^e}$. As intermediate goods produced by the projects of entrepreneurs account for 87% of the aggregate output, the dynamics of the aggregate supply of intermediate goods in the current period follow the dynamics of the entrepreneurs' capital investment in the previous period. This justifies the fact that the aggregate supply of intermediate goods peaks in period 6 and the price falls below the steady state value since period 7. Although the household wage rate peaks in period 4, the household labor supply peaks in period 3. As a result, aggregate output of final goods peaks in period 4 by 1.58% above the steady state value, more than the maximum value of the output responses, 1.48%, in period 0 in model RBC.

Altogether, the accumulation of entrepreneurial net worth and the reallocation of capital between agents with different production technologies constitute the amplified propagation mechanism. Aggregate output of final goods peaks in period 4 by 1.58% above the steady state value in model SFE, while aggregate output of final goods peaks in period 2 by only 1.34% above the steady state value in the model SFC. In this sense, model SFE dominates model SFC in generating more amplified and delayed responses of aggregate output to TFP shocks.

3.4.2.3 Dual Financial Frictions: Model DF vs. Model SFE

Figure 3.4 shows the impulse responses of model DF (solid line) and model SFE (dash-dot line). Both the demand and the supply of capital goods are subject to financial constraints and there are six endogenous state variables, $\{k_t^h, k_t^e, z_t^e, R_{e,t}^b, r_t, d_t^c\}$ in model DF. The dynamic interactions between time-varying prices of capital and dual financial frictions in model DF reinforce the amplification mechanism and preserve the propagation mechanism of model SFE shown in subsection 3.4.2.2.

Consider model DF. A 1% TFP shock in period 0 pushes up the aggregate demand for intermediate goods and the price rises to clear the market. Extra sales revenues improve entrepreneurial net worth and they can make more leveraged investment. Due to the constrained production of capital goods, the excess demand of entrepreneurs for capital pushes up the price of capital, $q_0 > E_{-1} q_0$. Capital gains,
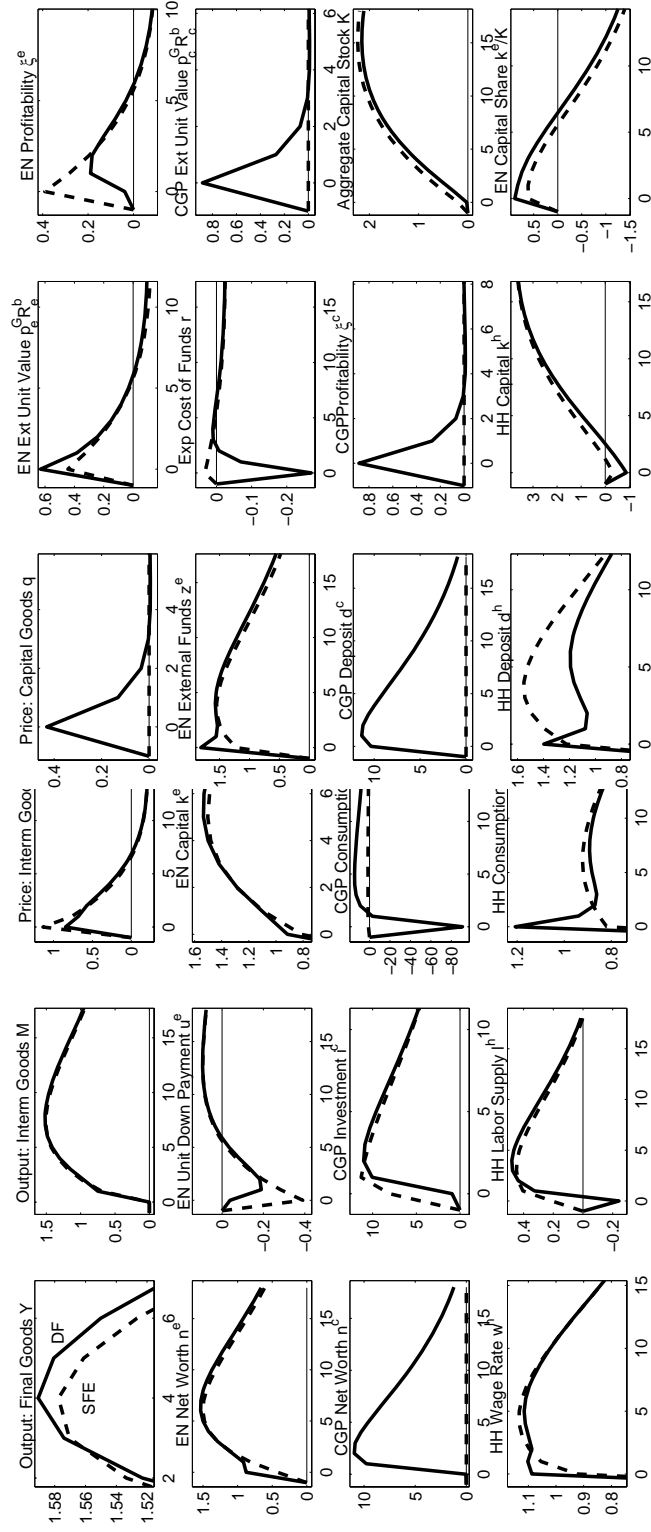
Figure 3.4: Model *DF* vs. Model *SFE*

absent in model SFE, further improves entrepreneurial net worth by,

$$n_0^e - E_{-1}n_0^e = \tilde{\pi}p_e^G\left[(1-\delta')(q_0 - E_{-1}q_0) + R_e(v_0 - E_{-1}v_0)\right]k_{-1}^e,$$

which enables them to expand their leveraged investments to a larger extent. In equilibrium, the price of capital rises by 0.43%. Entrepreneurial net worth rises by 0.88% in period 0, much higher than the 0.42% in model SFE. Given that the prices of capital and intermediate goods are above their steady state values in period 1, the expected external unit value of capital in the entrepreneurial projects, $p_G^e R_{e,0}^b = p_G^e E_0[(1-\delta')q_1 + R_e v_1 - b_e]$, rises to a large magnitude than in model SFE.

Extra sales revenues and capital gains have the wealth effect on the optimization decisions of households in period 0. They increase consumption and deposits by 1.21% and 1.4%, larger than the 0.82% and the 1.22% in model SFE, respectively. Meanwhile, the rise in the price of capital depresses the households' capital investment by 0.87%, more dramatically than the 0.33% in model SFE. Although the household wage rate rises by 1.09% in period 0, due to the consumption-leisure substitution, they reduce labor supply by 0.24%. Thus, aggregate output of final goods rises by 0.85%, even less than the magnitude of the TFP shock.

As the rise in the price of capital improves the expected external unit value of the project of capital goods producers, they can borrow more and expand their investments in period 0. Given that the price of capital is 0.13% above the steady state value in period 1, the project of capital goods producers is still more profitable than in the steady state. As a result, capital goods producers reduce consumption by 91% and raise their deposits by 10.4% in period 0 in order to have more net worth and expand the project investments in period 1. The rise in inter-period deposits of households and capital goods producers reduces the gross deposit rate by 0.27%.

Due to the rise in the price of capital and the expected external unit value of capital and the fall in the deposit rate, the entrepreneur's unit down payment of capital, $u_0^e = q_0 - \frac{p_G^e R_{e,0}^b}{r_0^d}$, falls only by 0.04%, smaller than the 0.39% in model SFE. Thus, they increase their capital investment by 0.91%, slightly more than the 0.81% in model SFE. Altogether, due to the constrained capital goods production, the excess demand for capital pushes up the price of capital and the resulting capital gains speed up the reallocation of capital towards entrepreneurs in period 0.

Capital goods producers increase their inter-period deposits in period 0 so that their net worth rises by 9.8% in period 1 and they can expand their project investments by 10%. It partially accommodates the entrepreneurs' excess demand for capital and the price of capital is only 0.13% above the steady state value. As the price of capital is still 0.03% above the steady state value in period 2, the project of

capital goods producers is still more profitable than in the steady state. Therefore, they increase deposits by 11.4% and reduce consumption by 2.8% in period 1.

The deposit return improves household wealth in period 1. On the one hand, household consumption is above the steady state value by 0.94%, more than the 0.85% in model SFE; on the other hand, the gross rate of return on inter-period loans stays below the steady state value by 0.07% in period 1 and household deposits stay above the steady state value by 1.08%, less than the 1.4% in model SFE. The household wage rate is above the steady state value by 1.1% in period 1. The consumption-leisure substitution induces households to increase labor supply by 0.33%, still lower than the 0.4% in model SFE.

In contrast to the full model of chapter 2,the price of capital converges toward the steady state value much fast in model DF. From period 3 on, the price of capital deviates from its steady state value by at most 0.01%. In this sense, the period-0 price of capital is not much affected by future prices of capital and the initial capital gains in period 0 have only limited effects on entrepreneurial net worth. As a result, the pattern and magnitude of the capital stock held by entrepreneurs do not differ much from those in model SFE from period 3 on. Aggregate output of final goods peaks by 1.591% in period 4, slightly higher than the 1.578% in model SFE. Here, we allow time-varying prices of capital in the case of the financially-constrained capital goods production and we find a balance between amplification and propagation.

3.4.2.4   Dual Financial Frictions: Model DF vs. Model SFC

This subsection shows that financial frictions in the entrepreneurial sector limit their demand for capital so that the price of capital responds to shocks in a dampened magnitude in model DF in comparison with model SFC. Figure 3.5 shows the impulse responses of model DF (solid line) and model SFC (dash-dot line).

Consider model DF. A 1% TFP shock in period 0 raises the sales revenues of entrepreneurs and improves their net worth. The excess demand of entrepreneurs for capital goods pushes up the price of capital. The resulting capital gains further improves entrepreneurial net worth. Thus, the price of capital rises by 0.43% in period 0. However, as entrepreneurs are subject to credit constraints and cannot raise sufficient external funds to fully exploit the project profitability, their demand for capital is constrained in period 0, too. As a result, the price of capital responds to shocks less than in model SFC. The smaller increase in the price of capital only raises the expected external unit value of the investment of capital goods producers, $p_c^G R_{c,0}^b = p_c^G(R_c q_0 - b_c)$, by 0.88%, less than the 1.29% in model SFC. Given the

Figure 3.5: Model *DF* vs. Model *SFC*
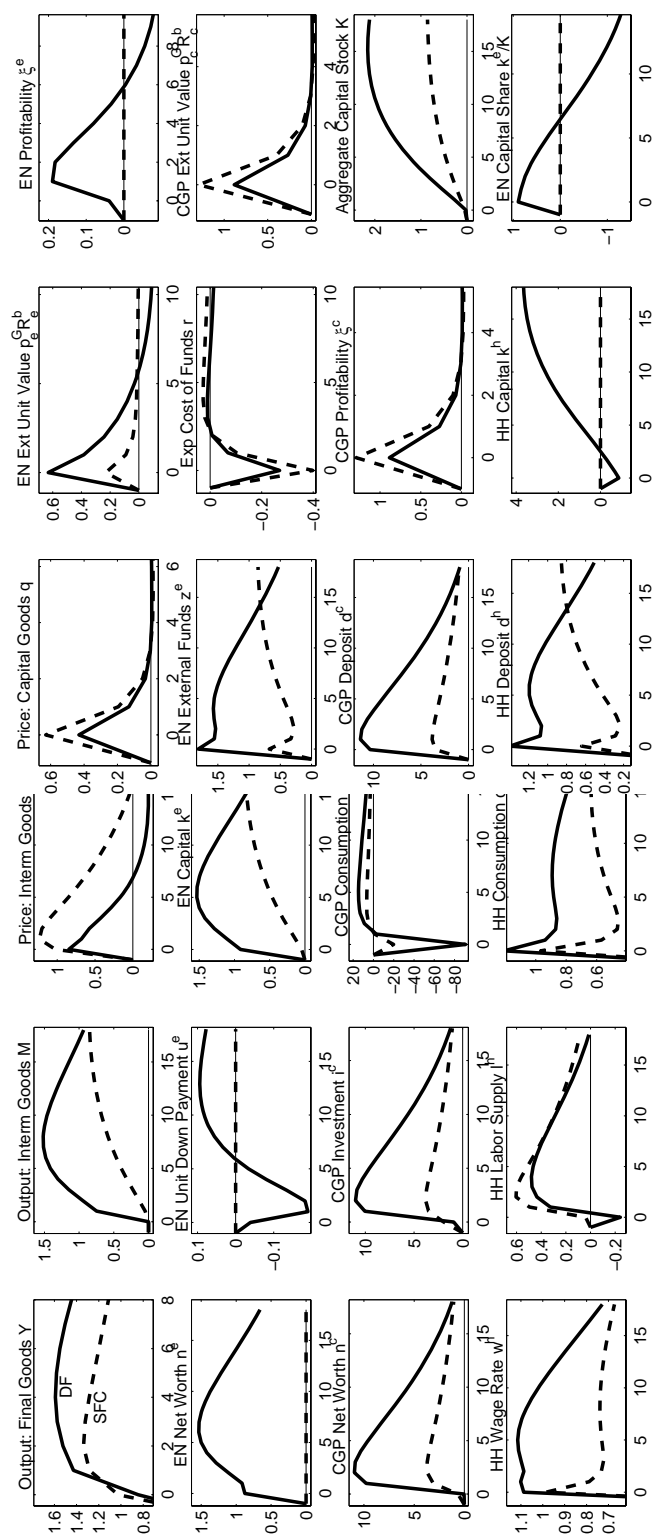
predetermined deposit return and net worth in period 0, capital goods producers increase their project investments by 0.9%, less than the 1.94% in model SFC.

Anticipating the accumulation of entrepreneurial net worth and the resulting dramatic increase in the demand of entrepreneurs for capital goods in period 1, producers of capital goods increase their inter-period deposits in period 0 in order to accumulate more net worth and expand their project investments in period 1. The joint effects of the demand for and the supply of capital goods make the price of capital stay above its steady state value by 0.13% in period 1, less than the 0.19% in model SFC. Altogether, due to the constrained demand of entrepreneurs for capital goods, the price of capital responds to shocks in a dampened fashion.

3.5   Final Remarks

This chapter introduces financial frictions on the demand and the supply of capital goods in a dynamic general equilibrium model. The dynamic interactions between dual financial frictions and time-varying prices of capital constitute a robust mechanism through which aggregate output responds to productivity shocks in an amplified and hump-shaped fashion, in line with the empirical evidence in the literature.

We also address a methodological question: Is the widely adopted costly capital adjustment a proper modeling approach to an upward-sloping capital supply curve? As an inherent feature of this approach, the price of capital converges rather slowly towards the steady state value after exogenous shocks. In the full model of chapter 2,the resulting capital gains reinforce capital reallocation among agents with different production technologies. It enhances the amplification mechanism but weakens the propagation mechanism discussed in our basic model (model SFE).

The production of capital goods is subject to financial frictions in the full model. It captures the empirical feature that the supply of capital goods is relatively inelastic in the short run but is elastic in the medium run. The price of capital stays away from the steady state for only a few periods after the shock. Capital gains are rather limited so that we achieve a balance between amplification and propagation.

In this sense, we argue that the effects of capital gains can be exaggerated if one adopts the approach of costly capital adjustment. Similar arguments also apply to the assumption that durable assets, e.g., land, have a fixed stock and do not depreciate.

Chapter 4

Domestic and Foreign Borrowing in a Small Open Economy

4.1 Introduction

This chapter analyzes how better protection of foreign investors can affect production efficiency, social welfare, and macroeconomic fluctuations in a small, open, real economy. More specifically, it addresses two questions: Who benefits from better protection of foreign investor in the long run? Are macroeconomic aggregates less volatile in countries with better protection of foreign investors?

According to neoclassical models, the economic benefits of financial opening are significant. Investors are able to share risk globally and capital can flow to the countries with the highest productivity (Stulz 2005). In the past two decades, many countries have deregulated financial markets and reduced explicit barriers to foreign investors. In addition to financial regulations, the differences in the legal system and market efficiency may affect the ex post repayment to foreign investors and thus their ex ante lending behaviors. In countries with better protection of foreign investors, domestic agents are able to borrow ex ante more abroad. In this sense, institutional differences in the protections of foreign investors can affect the actual financial openness. However, the increase in foreign borrowing might have uneven welfare implications for domestic agents with different production technologies. This issue cannot be addressed in the conventional representative agent models.

Theory predicts that financial opening should lower consumption volatility while raising investment volatility, if most shocks are country-specific and transitory. However, the empirical literature cannot provide statistically significant evidence on the relationship between financial openness and macroeconomic volatility (Razin

and Rose 1994). Using a panel dataset for OECD countries for the past 40 years, Buch, Doepke, and Pierdzioch (2005) find that the implications of financial openness for business cycle volatility depend on the nature of the shocks and the link between macroeconomic policy, financial openness, and business cycle volatility actually changes over time.

There is a huge literature concerning foreign borrowing and its macroeconomic implications. Caballero and Krishnamurthy (2001, 2003) investigate the dynamic interactions between domestic and international collateral constraints and show that limited financial development reduces the incentives for foreign lenders to enter emerging markets. Aoki, Benigno, and Kiyotaki (2005) analyze the medium-run adjustment process after capital account liberalization and show that the allocation of domestic assets and production efficiency depends on the degree of capital account liberalization. Alessandria and Qian (2005) examine the impact of foreign borrowing on both welfare and the structure of lending contracts. The entry of foreign investors to the domestic financial market may improve or worsen the efficiency of financial intermediaries, leading to an improvement or worsening of the aggregate composition of investment projects.

We address the two questions mentioned above in a real dynamic general equilibrium model of a small open economy. In our basic setting, two types of domestic agents, entrepreneurs and households, use durable assets, e.g., land, to produce domestic goods. Entrepreneurs are more productive than households and households lend to entrepreneurs via mutual funds. Due to the unobservable project choices, the entrepreneurs are subject to credit constraints. Thus, some of the durable productive assets are allocated to the less productive households. The degree of moral hazard determines the severity of the credit constraints and the efficiency of domestic production. Entrepreneurial net worth helps mitigate moral hazard and changes in entrepreneurial net worth can amplify macroeconomic fluctuations. Foreign trade is perfectly liberalized and the terms of trade are exogenously determined. Due to financial regulation or very bad protection of foreign investors, domestic agents cannot borrow abroad, although the foreign interest rate is lower than the domestic interest rate. Therefore, foreign trade must balance each period.

Consider a positive transitory shock to total factor productivity or the terms of trade. Extra sales revenues improve entrepreneurial net worth and the rise in the entrepreneurs' demand for assets pushes up asset prices. Capital gains improve their net worth further and the spiral process continues. As a result, output and asset prices respond to shock more strongly than in the setting without moral hazard.

In the full model, we introduce foreign investors. We assume that domestic mutual funds can perfectly verify the output of domestic borrowers, while foreign

investors cannot. Thus, domestic productive assets, e.g., land, must be pledged to the foreign investors as collateral to mitigate the debt enforcement problem. In comparison to domestic agents, the foreign investors are normally less familiar with the domestic asset market, or they have the inferior liquidation technology, or the domestic legal system is biased against them. Thus, foreign borrowing has to be overcollateralized in the sense that domestic agents can pledge only a fraction of the value of their assets to the foreign investors. We call this fraction the degree of collateralization. Domestic financial regulations can also affect this degree to some extent. The difference between the market value and the pledgeable value of assets can be regarded as the premium the foreign investors have to pay when they liquidate the assets handed over by the bankrupted borrowers.

In countries with better protection of foreign investors, domestic agents can pledge their assets for more foreign funds. The demand for land is higher and so is the land price in these countries. Households can pledge their assets only to the foreign investors and deposit at the mutual funds, while entrepreneurs can pledge some of their project revenues to the mutual funds as well as their assets to the foreign investors. Thus, higher asset prices enable entrepreneurs to invest more assets into their projects and thus domestic production is more efficient in countries with better protection of foreign investors. However, this has uneven welfare implications for domestic agents in the long run. Entrepreneurs own a larger share of the aggregate asset stock and are wealthier in these countries. Suppose that the consumption of domestic agents is proportional to their wealth. Thus, entrepreneurs have more consumption in these countries and their welfare is higher. In contrast, households own fewer productive assets and their deposits are lower due to the substitution effect. Thus, households are less wealthy in these countries; their consumption and welfare are lower. Whether the long-run social welfare is higher in these countries depends on the weights the social planner assigns to households and entrepreneurs. The fact that households lose and entrepreneurs benefit in the long run in countries with better protection of foreign investors results mainly from the substitution effect, given that the foreign interest rate is smaller than the domestic interest rate. This assumption is justified in many small developing economies.

Better protection of foreign investors also has ambiguous implications for macroeconomic volatility. We investigate the dynamics of the model with and without domestic financial frictions to shocks to total factor productivity, to the terms of trade, and to the foreign interest rate. The volatility of major macroeconomic aggregates are non-monotonic (U-shaped) in the degree of collateralization for each type of shocks. Thus, if we pool the empirical data of countries with different degrees of openness, we might not find a clear relationship between financial openness

and macroeconomic volatility in a simple OLS regression, because the "underlying" relationship is highly nonlinear. Furthermore, this U-shaped volatility patterns of macroeconomic aggregates, e.g., output, consumption, labor, foreign trade, are flatter in the model with domestic financial frictions than without. In other words, for countries with domestic financial frictions, it is even more difficult to find a clear relationship between financial openness and macroeconomic volatility from the empirical data than for countries without domestic financial frictions. In this sense, our model helps explain the empirical evidence provided by Buch, Doepke, and Pierdzioch (2005) and domestic financial frictions may reinforce our arguments.

The rest of this chapter is organized as follows. Section 4.2 lays out the basic model and domestic financial frictions arise from unobservable project choices of the entrepreneurs. Asset reallocation between households and entrepreneurs explains the amplified responses of macroeconomic aggregates to exogenous shocks. Section 4.3 introduces foreign investors and specifies the financial contracts between foreign investors and domestic agents. We analyze the implications of changes in the degree of collateralization for the long-run welfare of domestic agents and to macroeconomic volatility. Section 4.4 summarizes the main findings. Appendix collects some derivations.

4.2 The Basic Model

4.2.1 Overview

Consider a discreet-time, small, open, real economy. There is a domestic durable asset (land) with a fixed supply, K. There are three perishable goods: a domestic intermediate good, a domestic final good, and a foreign final good. There are two types of domestic agents, households and entrepreneurs. The population of each type is normalized to unity. Households and entrepreneurs have projects for the production of domestic intermediate goods using land and it takes one period for them to complete their projects. Domestic intermediate goods and labor are then employed to produce domestic final goods contemporaneously. Domestic final goods can be consumed, invested, or exported. Foreign trade is perfectly liberalized.

Domestic and foreign final goods are imperfect substitutes for the consumption of domestic agents. We choose the consumption composite of domestic agents as the numeraire. See subsection 4.2.2.1 for the definition of consumption composite. Let v_t and p_t denote the prices of domestic intermediate and final goods, respectively. For simplicity of notation, let s_t denote the inverse of the terms of trade, i.e., the relative price of foreign final goods with respect to domestic final goods. The domestic

economy is small enough that the terms of trade are exogenously determined abroad. Thus, the domestic price of foreign final goods is $p_t s_t$. Let q_t and w_t denote the land price and the wage rate, respectively. A continuum of competitive domestic mutual funds accept deposits and provide loans. A deposit contract is a claim on the financial position of the mutual funds. Let the domestic interest rate, r_t, denote the expected rate of return on mutual funds. Foreign borrowing and lending are not allowed in our basic model. Thus, foreign trade must balance each period.

Households are risk averse and infinitely lived. They have a safe production project for intermediate goods. They have labor endowments each period and work for the production of domestic final goods. At the end of the period, they invest land in their projects, make deposits at the mutual funds, and consume the rest.

Entrepreneurs are risk neutral. As shown in subsection 4.2.2.2, they finance their projects using own funds and loans from the mutual funds at the end of each period, subject to credit constraints. Entrepreneurial net worth is defined as the amount of own funds they invest in their projects. Debt repayment is contingent on project outcomes. Entrepreneurs whose projects fail hand over their land stock to the mutual funds and exit from the economy without consuming anything; entrepreneurs whose projects succeed repay their liabilities to the mutual funds. Successful entrepreneurs have a constant probability of death. 5In equilibrium, entrepreneurs of mass $(1 - \pi)$ exit from the economy each period and new entrepreneurs of the same mass are born with a tiny endowment, e, in terms of domestic final goods, keeping the population size of entrepreneurs constant. Our calibration guarantees that the expected rate of return on entrepreneurial net worth exceeds the domestic interest rate around the steady state. Surviving and newly-born entrepreneurs put all own funds into their projects and borrow to the limit. Entrepreneurs with successful projects who die sell off their land stock, consume all proceeds.

There is no moral hazard in the financial sector. The mutual funds can perfectly pool the idiosyncratic project risk of the entrepreneurs.Due to perfect competition, the mutual funds transfer all proceeds to their depositors and make no profit.

Figure 4.1 summarizes the time sequence of events in equilibrium, where DIG, DFG, and FFG denote domestic intermediate goods, domestic final goods, and foreign final goods, respectively. We focus on exogenous shocks to the production of domestic final goods (TFP shocks) and to the terms of trade (ToT shocks). Note that all exogenous shocks are realized at the beginning of every period.
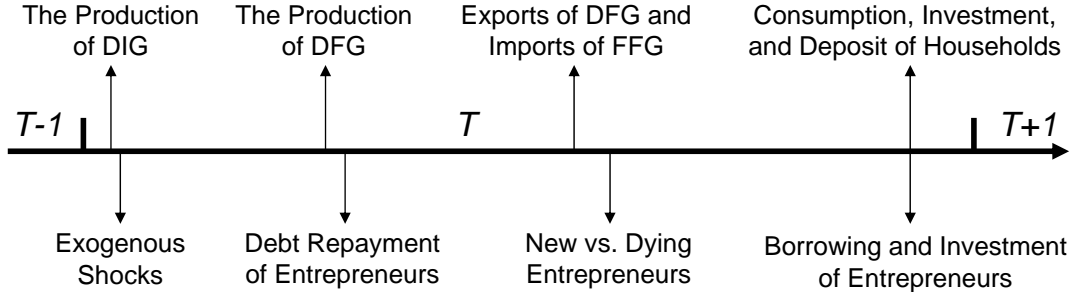
Figure 4.1: Time Sequence of Events

4.2.2   Efficiency Conditions

4.2.2.1   Households

Households have identical preferences over consumption and leisure. Their expected utility function takes the following form,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^h)^{1-\sigma}}{1-\sigma} + \chi \frac{(1-l_t)^{1+\psi}}{1+\psi} \right], \tag{4.1}$$

where E_t is the expectation operator based on information available in period t. $\beta \in (0,1)$ denotes the time discount factor and l_t denotes the household labor supply, as a fraction of their total time endowment. Households have composite consumption,

$$c_t^h \equiv (c_{D,t}^h)^\gamma (c_{F,t}^h)^{1-\gamma} \tag{4.2}$$

where $c_{D,t}^h$ and $c_{F,t}^h$ denote their consumption of domestic and foreign final goods, respectively. See Clarida, Gali, and Gertler (2002). Given that k_{t-1}^h units of land were invested in the household project at the end of period $t-1$, $G(k_{t-1}^h)$ units of domestic intermediate goods are produced at the beginning of period t. Household sales revenues amount to $v_t G(k_{t-1}^h)$ and their wage income is $w_t l_t$. In addition, they receive $\tilde{r}_t d_{t-1}$ from the mutual funds, where d_{t-1} is the household deposit made at the end of period $t-1$ and $\tilde{r}_t$ is the ex post rate of return on mutual funds in period t. Due to the aggregate risk related to TFP shocks and ToT shocks, $\tilde{r}_t$ could differ from its expected value, an issue discussed in subsection 4.2.2.3. By definition, $r_t = E_t \tilde{r}_{t+1}$. At the end of period t, households invest k_t^h units of land in their projects, deposit d_t at the mutual funds, and consume c_t^h. Accordingly, the flow-budget constraint is,

$$q_t(k_t^h - k_{t-1}^h) + d_t + c_t^h = v_t G(k_{t-1}^h) + w_t l_t + \tilde{r}_t d_{t-1},$$

The optimization over $\{c_t^h, c_{D,t}^h, c_{F,t}^h, l_t, d_t, k_t^h\}$ gives the equilibrium conditions,

$$p_t c_{D,t}^h = \gamma c_t^h, \tag{4.3}$$

$$p_t s_t c_{F,t}^h = (1 - \gamma) c_t^h, \tag{4.4}$$

$$w_t = \chi (1 - l_t)^\psi (c_t^h)^\sigma, \tag{4.5}$$

$$1 = \beta r_t \left(\frac{E_t c_{t+1}^h}{c_t^h} \right)^{-\sigma}, \tag{4.6}$$

$$q_t = \beta E_t \left(\frac{c_{t+1}^h}{c_t^h} \right)^{-\sigma} [q_{t+1} + v_{t+1} G'(k_t^h)]. \tag{4.7}$$

Equations (4.2), (4.3), and (4.4) imply that the price of domestic final goods (foreign final goods) is positively (negatively) related to the terms of trade. See appendix C.1 for details. Recall that s_t denotes the inverse of the terms of trade.

$$p_t = \gamma^\gamma \left(\frac{1 - \gamma}{s_t} \right)^{1-\gamma}, \tag{4.8}$$

$$p_t s_t = (\gamma s_t)^\gamma (1 - \gamma)^{1-\gamma}. \tag{4.9}$$

4.2.2.2 Entrepreneurs

Each entrepreneur can choose one of two projects: "Good" or "Bad". The projects have a Leontief technology, i.e., a units of domestic final goods are required for each unit of land investment at the end of the period. At the beginning of the next period, the project generates R units of domestic intermediate goods per unit of land invested, if the project succeeds. Otherwise, there is no output. Land does not depreciate, while the invested domestic final goods fully depreciate. Project choices are irreversible and the mutual funds can perfectly verify project outcomes at no costs. Entrepreneurs also enjoy safe, nonpecuniary private benefits during the project process. For convenience of aggregation, we assume that private benefits are proportional to land investment. The projects differ in the probability of success and unit private benefits. See Table 4.1.

Table 4.1: Projects with Idiosyncratic Risk

Project	Good	Bad
Probability of Success	p^G	p^B
Unit Private Benefits	b^G	b^B

where $0 < p^B < p^G < 1$ and $b^B > b^G > 0$ imply that project "Good" is safer than projects "Bad", but project "Bad" yields larger unit private benefits.

An entrepreneur[1] i who stays in the economy to the next period has linear preferences over consumption and private benefits. His expected utility function is,

$$E_0 \sum_{t=0}^{\tilde{T}} \beta^t \left[c_{i,t}^e + \mathcal{B} k_{i,t-1}^e \right],   (4.10)$$

where $\tilde{T}$ is the stochastic time of death and $\mathcal{B} \in \{b^G, b^B\}$ denotes private benefits per unit of the land invested in project "Good" or project "Bad". $c_{i,t}^e$ denotes his composite consumption in period t and $k_{i,t-1}^e$ denotes his land investment at the end of period $t - 1$. Our calibration guarantees that only project "Good" has a positive expected net present value around the steady state,

$$\frac{E_t(p^G R v_{t+1} + q_{t+1}) + b^G}{r_t} - (q_t + ap_t) > 0 > \frac{E_t(p^B R v_{t+1} + q_{t+1}) + b^B}{r_t} - (q_t + ap_t).$$

Therefore, project "Bad" should not be financed in equilibrium. Project "Good" also has a larger expected marginal return than the household project.

At the end of period t, entrepreneur i invest $k_{i,t}^e$ units of land and $ak_{i,t}^e$ units of domestic final goods into either project "Good" or project "Bad", using his own funds, $n_{i,t}$, and loans from the mutual funds, $z_{i,t}^{e,m}$, i.e., $(q_t + ap_t)k_{i,t}^e = n_{i,t} + z_{i,t}^{e,m}$. Thus, $n_{i,t}$ is the entrepreneur's net worth in the project. The loan contract specifies a promise to repay $R_t^m k_{i,t}^e$ units of the consumption composite in period $t + 1$ if the project succeeds. If the project fails, the entrepreneur hands over his land stock to the mutual funds. There is no debt enforcement problem between entrepreneurs and mutual funds, i.e., entrepreneurs always repay their liabilities if they are able to do so. In order to motivate entrepreneur i to choose project "Good", the mutual funds must provide him with enough incentives,

$$\left[p^G E_t(R v_{t+1} + q_{t+1} - R_t^m) + b^G \right] k_{i,t}^e \geq \left[p^B E_t(R v_{t+1} + q_{t+1} - R_t^m) + b^B \right] k_{i,t}^e.$$

The left (right) hand side denotes the expected utility of the entrepreneur if he chooses project "Good" ("Bad"). As he prefers to borrow to the limit, the incentive constraint is binding around the steady state and is simplified to,

$$R_t^m = E_t(R v_{t+1} + q_{t+1}) - \tilde{b}, \quad \text{where} \quad \tilde{b} \equiv \frac{b^B - b^G}{p^G - p^B} > 0.   (4.11)$$

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[1]Entrepreneurs differ in the end-of-period wealth and are indexed by $i \in [0, 1]$.

Each unit of the land invested in project "Good" has an expected value of $E_t(p^G R v_{t+1} + q_{t+1})$ in period t. Any promise to repay more than $R_t^m k_{i,t}^e$ in the case of success is not credible, because the entrepreneur would choose project "Bad" in the first place. Thus, the entrepreneur can only pledge $p^G R_t^m + (1 - p^G) E_t q_{t+1}$ per unit of land invested to the mutual funds in period t. We define $E_t(p^G R v_{t+1} + q_{t+1})$ and $p^G R_t^m + (1 - p^G) E_t q_{t+1}$ as the expected full unit value and the expected external unit value of the land invested in project "Good", respectively. The difference between the two values, $p^G \tilde{b}$, is used to motivate the entrepreneur to choose project "Good" despite the lower private benefits it promises, $b^G < b^B$.

The mutual funds are expected to break even in their lending to the entrepreneur in period t, $r_t z_{i,t}^{e,m} = [p^G R_t^m + (1 - p^G) E_t q_{t+1}] k_{i,t}^e$. This implies a credit constraint,

$$z_{i,t}^{e,m} = \Gamma_t n_{i,t}, \quad \text{where} \quad \Gamma_t \equiv \frac{p^G(RE_t v_{t+1} - \tilde{b}) + E_t q_{t+1}}{(q_t + ap_t)r_t - [p^G(RE_t v_{t+1} - \tilde{b}) + E_t q_{t+1}]}. \quad (4.12)$$

Γ_t is the credit multiplier. Our calibration guarantees $(q_t + ap_t)r_t > p^G(RE_t v_{t+1} - \tilde{b}) + E_t q_{t+1}$ around the steady state and so $\Gamma_t > 0$. As Γ_t is independent of $n_{i,t}$, loans are proportional to the entrepreneur's net worth. Note that the credit multiplier varies with q_t, p_t, r_t, $E_t q_{t+1}$, and $E_t v_{t+1}$. Ceteris paribus, a rise in the current prices of land or domestic final goods, q_t or p_t, makes the project investment more expensive; similarly, a rise in the domestic interest rate, r_t, makes loans more expensive for entrepreneurs. In both cases, the credit multiplier Γ_t falls so that the entrepreneur can get fewer loans and less land is allocated to the entrepreneurial sector. Ceteris paribus, a rise in the expected prices of land or domestic intermediate goods in period $t + 1$, $E_t q_{t+1}$ or $E_t v_{t+1}$, improves the expected external unit value of the land invested in their projects, $p^G R_t^m$. Thus, the credit multiplier is larger and entrepreneurs can expand their land investment.

In equilibrium, entrepreneurs of mass $(1 - p^G)$ have failed projects and exit from the economy. Entrepreneurs whose projects succeed have a constant probability $\tilde{\pi}$ of surviving. In the aggregate, entrepreneurs of mass $p^G(1 - \tilde{\pi})$ have successful projects and exit from the economy, and entrepreneurs of mass $p^G \tilde{\pi}$ have successful projects and live on to the next period. New entrepreneurs of mass $(1 - \pi)$ are born. We assume $\pi = p^G \tilde{\pi}$ to keep the population size of entrepreneurs constant.

At the end of period t, entrepreneur i maximizes his expected utility (4.10), subject to his period budget constraints and credit constraints,

$$(q_t + ap_t)k_{i,t}^e - z_{i,t}^{e,m} = n_{i,t}, \quad \text{where} \quad n_{i,t} \equiv \mathcal{N}_{i,t} - c_{i,t}^e, \quad (4.13)$$

$$z_{i,t}^{e,m} = \Gamma_t n_{i,t} \quad (4.14)$$

where $\mathcal{N}_{i,t}$ denotes his end-of-period wealth. Entrepreneurs differ in their end-of-period wealth, an issue discussed in appendix C.2. Due to the linear nature of the project technologies and the preferences of entrepreneurs, the loan and the project investment of entrepreneur i are proportional to his net worth. In this sense, only the first moment of the distribution of entrepreneurial net worth matters for the aggregate land stock in the entrepreneurial sector. See appendix C.2. Let lower-case letters without the index i denote per capita variables of the entrepreneurial sector. Per capita consumption c_t^e, net worth n_t, loans $z_t^{e,m}$, and land holding k_t^e are,

$$c_t^e = (1 - \tilde{\pi})p^G[Rv_t + q_t - R_{t-1}^m]k_{t-1}^e, \tag{4.15}$$

$$n_t = \tilde{\pi}p^G[Rv_t + q_t - R_{t-1}^m]k_{t-1}^e + (1 - \pi)p_t e, \tag{4.16}$$

$$z_t^{e,m} = \Gamma_t n_t = \frac{p^G(RE_t v_{t+1} - \tilde{b}) + E_t q_{t+1}}{(q_t + ap_t)r_t - [p^G(RE_t v_{t+1} - \tilde{b}) + E_t q_{t+1}]}n_t, \tag{4.17}$$

$$k_t^e = \frac{n_t + z_t^{e,m}}{q_t + ap_t}. \tag{4.18}$$

Per capita consumption of domestic and foreign final goods of entrepreneurs are

$$c_{D,t}^e = \frac{\gamma c_t^e}{p_t}, \tag{4.19}$$

$$c_{F,t}^e = \frac{(1 - \gamma)c_t^e}{p_t s_t}. \tag{4.20}$$

We introduce three auxiliary variables. The first is the leverage ratio, defined as the ratio of total investment over the entrepreneur's net worth, $\Omega_t \equiv \frac{(q_t + ap_t)k_{i,t}^e}{n_{i,t}} = 1 + \Gamma_t$. The second is the entrepreneur's unit down payment, defined as the amount of own funds the entrepreneur pays for each unit of land and the required investment of domestic final goods, $u_t^e \equiv (q_t + ap_t) - \frac{z_{i,t}^{e,m}}{k_{i,t}^e} = \frac{q_t + ap_t}{\Omega_t}$. The third is the expected profitability of the entrepreneurs' project, defined as the expected gross rate of return on the entrepreneur's net worth, $\xi_t \equiv \frac{p^G E_t[Rv_{t+1} + q_{t+1} - R_t^m]k_{i,t}^e}{n_{i,t}} = \frac{p^G \tilde{b}}{u_t^e}$. Our calibration guarantees that the expected profitability of entrepreneurs' project exceeds the domestic interest rate around the steady state, $\xi_t > r_t$. Thus, entrepreneurs postpone consumption and borrow to the limit.

4.2.2.3   Mutual Funds

Mutual funds accept deposits from households and make loans to entrepreneurs in equilibrium. The loan contract described in subsection 4.2.2.2 implicitly provides entrepreneurs with a net unit return, with a positive expected value, $p^G \tilde{b} > 0$, in period $t - 1$. The net unit return to the successful entrepreneurs in period t is

$$Rv_t + q_t - R^m_{t-1} = \tilde{b} + R(v_t - E_{t-1}v_t) + (q_t - E_{t-1}q_t).$$

In equilibrium, the prices of land and domestic intermediate goods are positively correlated to exogenous TFP or ToT shocks, as shown in subsection 4.2.5. As long as exogenous shocks are larger than some negative threshold values, the ex post value of the project outcomes of successful entrepreneurs is larger than the promised repayment, $(Rv_t+q_t)k^e_{t-1} > R^m_{t-1}k^e_{t-1}$. Then, the successful entrepreneurs repay their liabilities, $R^m_{t-1}k^e_{t-1}$, to the mutual funds. Let K^e_{t-1} and Z_{t-1} denote the aggregate land stock held by entrepreneurs and the aggregate lending to entrepreneurs at the end of period $t-1$, respectively. The aggregate expected break-even condition of the financial sector is $r_{t-1}Z_{t-1} = [p^G R^m_{t-1} + (1-p^G)E_{t-1}q_t]K^e_{t-1}$. At the beginning of period t, entrepreneurs of mass p^G have successful projects and their total repayment is $p^G R^m_{t-1}K^e_{t-1}$; entrepreneurs of mass $(1-p^G)$ have failed projects and hand over their land stock with the market value of $(1-p^G)q_t K^e_{t-1}$ to the mutual funds. In this sense, the positive expected net return to entrepreneurs, $p^G \tilde{b}K^e_{t-1}$, helps absorb aggregate risk partially. The ex post rate of return on mutual funds is

$$\tilde{r}_t = \frac{[p^G R^m_{t-1} + (1-p^G)q_t]K^e_{t-1}}{Z_{t-1}} = r_{t-1}\left[1 + \frac{(1-p^G)(q_t - E_{t-1}q_t)}{p^G(RE_{t-1}v_t - \tilde{b}) + E_{t-1}q_t}\right]. \quad (4.21)$$

which differs from its expected value due to capital gains or capital losses on the land stock of failed entrepreneurs. For the TFP or ToT shocks below these thresholds, the prices of land and intermediate goods are so low that the ex post values of the project outcomes of successful entrepreneurs are less than their liabilities, $(Rv_t + q_t)k^e_{t-1} < R^m_{t-1}k^e_{t-1}$. Thus, even the successful entrepreneurs have to announce bankruptcy and transfer all the project outcomes to the mutual funds. As a consequence, if aggregate shocks are unbounded, the ex post rate of return on mutual funds as a function of aggregate shocks could have a kink at the point where $Rv_t + q_t = R^m_{t-1}$.

The first-order approximations used below to analyze the model dynamics requires that the endogenous variables should be continuous and differentiable functions of the state variables. For the purpose of the approximations, we assume that the aggregate shocks are distributed with mean zero and negative lower bounds guaranteeing that the successful entrepreneurs are always able to repay their debts.

4.2.2.4   Domestic Final Goods Production and Foreign Trade

Domestic final goods are produced from domestic intermediate goods and labor,

$$Y_t = A_t M_t^\alpha L_t^{(1-\alpha)}, \tag{4.22}$$

$$\log A_t = \rho^a \log A_{t-1} + \epsilon_t^a, \tag{4.23}$$

where M_t and L_t denote aggregate inputs of domestic intermediate goods and labor; total factor productivity, A_t, is an $AR(1)$ in logarithms with the autocorrelation coefficient $\rho^a \in (0,1)$. The TFP shock, ϵ_t^a, has mean zero, $E_t \epsilon_{t+1}^a = 0$, and is distributed above a lower bound, $(-\tau^a, \infty)$, where $\tau^a > 0$ is small enough that successful entrepreneurs are always able to repay their liabilities. Domestic final goods are produced efficiently and the inputs are priced by their marginal products,

$$v_t M_t = \alpha p_t Y_t, \tag{4.24}$$

$$w_t L_t = (1 - \alpha) p_t Y_t. \tag{4.25}$$

Let X_t and I_t denote the exports in terms of domestic final goods and the imports in terms of foreign final goods in period t, respectively. As foreign borrowing or lending is not allowed, foreign trade must balance each period,

$$s_t I_t = X_t, \tag{4.26}$$

$$\log \frac{1}{s_t} = \rho^s \log \frac{1}{s_{t-1}} + \epsilon_t^s, \tag{4.27}$$

where the terms of trade, $\frac{1}{s_t}$, is an $AR(1)$ in logarithms with the autocorrelation coefficient $\rho^s \in (0,1)$. The ToT shock, ϵ_t^s, has mean zero, $E_t \epsilon_{t+1}^s = 0$, and is distributed above a lower bound, $(-\tau^s, \infty)$, where $\tau^s > 0$ is small enough that successful entrepreneurs are always able to repay. ToT shocks can be interpreted as changes in the foreign demand for domestic final goods, i.e., preference shocks.

Assumption 4.1. $\lim_{s \to \infty} E_t(r_{t+s}^{-s} q_{t+s}) = 0$.

Assumption 4.1 helps rule out exploding bubbles and the economy converges to its steady state along a locally unique equilibrium path after it is hit by small shocks.

4.2.2.5 Market Equilibrium

The markets of domestic intermediate goods, domestic final goods, foreign final goods, land, labor, and domestic lending clear each period,

$$M_t = G(k_{t-1}^h) + p^G R k_{t-1}^e, \tag{4.28}$$

$$Y_t + (1 - \pi)e = c_{D,t}^h + c_{D,t}^e + a k_t^e + X_t, \tag{4.29}$$

$$I_t = c_{F,t}^h + c_{F,t}^e, \tag{4.30}$$

$$K = k_t^h + k_t^e, \tag{4.31}$$

$$L_t = l_t, \tag{4.32}$$

$$z_t^{e,m} = d_t. \tag{4.33}$$

Definition 4.1. *Market equilibrium is a set of allocations of households, $\{k_t^h, l_t, c_t^h, c_{D,t}^h, c_{F,t}^h\}$, and entrepreneurs, $\{k_t^e, n_t, z_t^{e,m}, c_t^e, c_{D,t}^e, c_{F,t}^e\}$, together with the aggregate variables $\{M_t, Y_t, I_t, X_t\}$ given a set of prices $\{v_t, p_t, q_t, w_t, r_t, R_t^m\}$ and the exogenous process $\{A_t, s_t\}$ satisfying equations (4.2)- (4.7), (4.11), (4.15)-(4.20), (4.22)-(4.31).*

4.2.3 Calibration

The household project takes the following form,

$$G(k_t^h) = \frac{\epsilon K}{1 + \lambda} \left[1 - \left(1 - \frac{k_t^h}{K} \right)^{1+\lambda} \right], \tag{4.34}$$

and the marginal product, $G'(k_t^h) = \epsilon K \left(1 - \frac{k_t^h}{K} \right)^{\lambda}$, is decreasing in the household land holding, where $\lambda = 8$. The quarterly discount factor is set at $\beta = 0.98$, corresponding to an annual interest rate of 8%. By convention, we set $\sigma = 2$ and $\psi = -5$. We set $\chi = 0.15$ so as to keep $l = \frac{1}{3}$ in the steady state, i.e., households work eight hours a day in the production of domestic final goods. We set $\alpha = 0.36$ so that the household wage income accounts for 64% of aggregate output of domestic final goods. By convention, we set the autocorrelation coefficient of total factor productivity at $\rho^a = 0.9$. For simplicity, we set $\gamma = 0.5$ and $s = 1$ in the steady state so that the price of domestic final goods is $p = 0.5$ and domestic agents consume the equal amounts of domestic and foreign final goods. Following Devereux, Lane, and Xu (forthcoming), we set the autocorrelation coefficient of the terms of trade at $\rho^s = 0.77$.

The surviving probability of the entrepreneurs with successful projects is set at $\tilde{\pi} = \frac{2}{3}$, implying that around 34% of entrepreneurs have to exit from the economy

each period, $\pi = p_G \tilde{\pi} = 0.66$. $\{R = 3084, \tilde{b} = 1.75, \epsilon = 326, a = 2.61\}$ are calibrated jointly to satisfy the following conditions in the steady state: the aggregate land stock is unity, $K = 1$; the land price is $q = 1$; entrepreneurs hold half of the aggregate land stock, $\frac{k^e}{K} = 0.5$; the leverage ratio, $\Omega = 2$, implies that entrepreneurs finance half of the their project investments using loans, as in Bernanke, Gertler, and Gilchrist (1999). In the steady state, the successful entrepreneurs keep 60% of the project outcomes for themselves, $\frac{R^m}{Rv+q} = 40\%$. We normalize $e = 0$.

4.2.4 The Degree of Moral Hazard and the Frictionless Model

Let model MH denote the model with moral hazard. The degree of moral hazard can be measured by the entrepreneurs' expected unit return on land, $p^G \tilde{b}$. In the first half of this subsection, we assume away aggregate risk and study how the degree of moral hazard can affect the steady state values of macroeconomic aggregates. Let $\Upsilon = \frac{p^G \tilde{b}}{p^G Rv + q}$ denote the entrepreneurs' expected share of the project outcomes in the steady state. Let $\Psi^h \equiv \frac{q + vG'(k^h)}{q}$ and $\Psi^e \equiv \frac{q + p^G Rv}{q + ap}$ denote, respectively, the expected marginal returns on the projects of households and entrepreneurs in the steady state. Take our calibration of $\tilde{b} = 1.75$ as the baseline value. Ceteris paribus, figure 4.2 shows the steady state values of relevant variables with respect to various degrees of moral hazard, $\Delta p^G \tilde{b}$. The horizontal axis denotes $\Delta \in [0.01, 1.15]$. EN and HH denote entrepreneur and household, respectively.

The moral hazard problem becomes less severe, as Δ decreases from unity to zero. The pledgeable value per unit of the land invested in the projects that become successful, R^m, rises. Entrepreneurs can ex ante acquire more loans from the mutual funds and expand their project investments. Given the fixed aggregate land stock, the rise in the entrepreneurs' demand for land pushes up the land price, which further raises R^m. As the land investment becomes more expensive for households, they reduce their project investment and increase deposits at the mutual funds. In contrast, it is the unit down payment, u^e, that matters for the project investment of entrepreneurs. The fall in the degree of moral hazard increases the leverage ratio and reduces the unit down payment of entrepreneurs. As a result, entrepreneurs expand their land investment. As project "Good" of entrepreneurs has a larger expected marginal rate of return than the household project, $\Psi^e > \Psi^h$, more domestic intermediate goods are produced. Given that domestic intermediate goods and labor are two imperfect substitutes for the production of domestic final goods, the price of domestic intermediate goods falls and the wage rate rises. Aggregate output
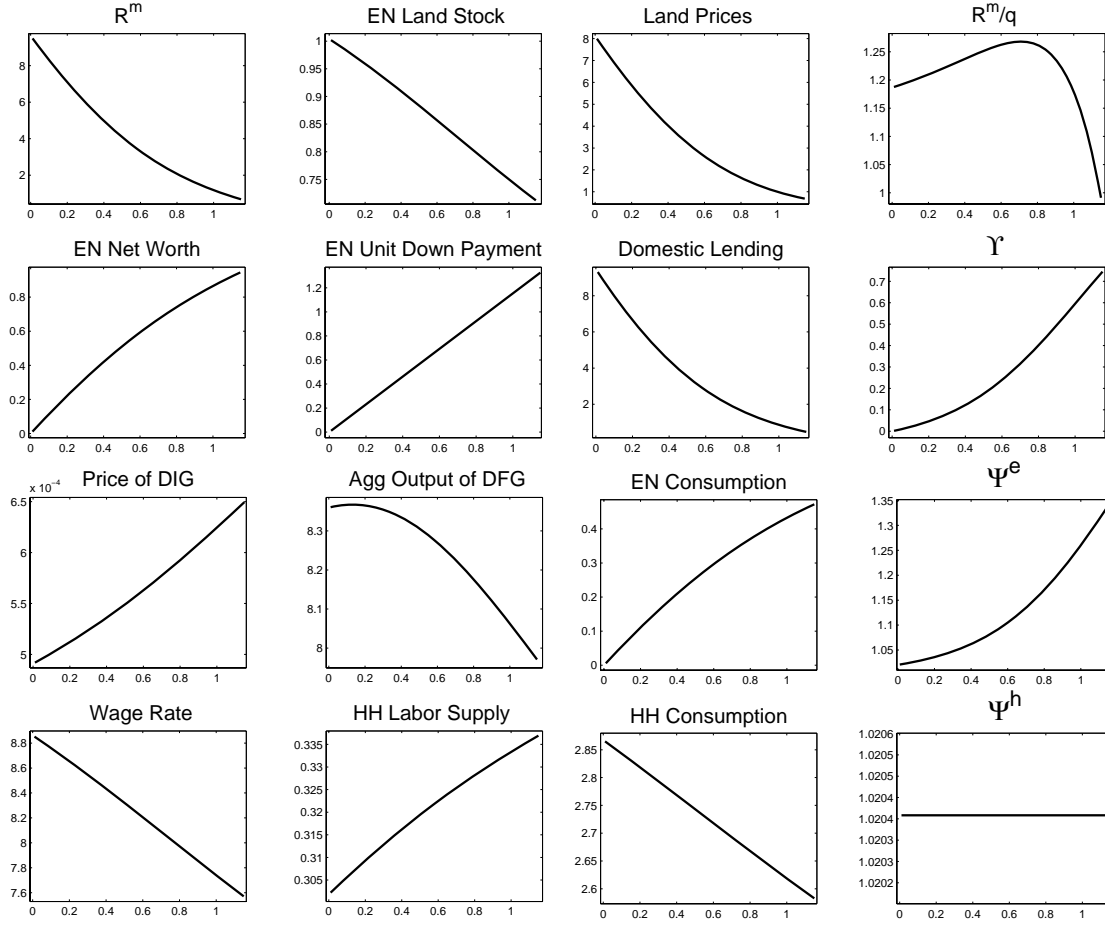
Figure 4.2: The Steady State Effect of Moral Hazard

of domestic final goods rises. As the moral hazard problem becomes less severe, a smaller share of the project outcomes is required to motivate entrepreneurs to choose project "Good" and the successful entrepreneurs have less post-repayment wealth. The consumption and net worth of entrepreneurs are proportional to their post repayment wealth. The investment in terms of domestic final goods ak^e required in project "Good" of entrepreneurs rises proportionally to the land investment of entrepreneurs. Altogether, household consumption $c^h = p(Y - ak^e) - c^e$ rises, given the constant price of domestic final goods. Due to the consumption-leisure substitution, households reduce their labor supply.

Kiyotaki and Moore (1997) assume that the human capital of entrepreneurs is essential for the project outcomes and is inalienable. Thus, entrepreneurs can always renegotiate their liabilities ex post to the value of physical assets. As a

result, the total liabilities of entrepreneurs are then limited by the expected value of their physical assets in the due period. In contrast, it is unobservable project choice of the entrepreneurs that gives rise to financial frictions in our model and entrepreneurs always repay if they are able to do so. Thus, entrepreneurs repay more in the case of project success than in the case of project failure, $R^m > q$. In this sense, the moral hazard problem in our model is less severe than in models with collateral constraints. We will revisit this issue in subsection 4.3.1.

Note that if $\Delta \to 0$ or the project choice of entrepreneurs is perfectly observable at the time of contracting, entrepreneurs can credibly choose project "Good" and pledge all of the project outcomes for loans. Therefore, they do not have to put down own funds in the project, $n_t = 0$. As the expected marginal return on project "Good" is higher than that on the household project, all land is allocated into the entrepreneurs' projects, $k^e = K$, and domestic intermediate goods are produced only by entrepreneurs, $M = p^G R K$. In period t, the entrepreneurs who invest in project "Good" in period $t-1$ enjoy the private benefits, $b^G k^e$, and transfer all the project outcomes to the mutual funds. Newcomers consume the endowment, $c_t^e = (1-\pi)p_t e$. The entrepreneurs who stay in the economy to period $t+1$ invest k^e units of land using loans from the mutual funds, i.e., $z_t^{e,m} = (q_t + a p_t)k^e$.

In the case of aggregate risk related to TFP or ToT shocks, the ex post rate of return on mutual funds is different in the models with and without financial frictions. As shown in subsection 4.2.2.3, the entrepreneurs' expected stake in the project outcomes, $p^G \tilde{b} k_t^e > 0$, absorbs part of aggregate risk in the model with financial frictions. Without moral hazard, no incentive is required for entrepreneurs to engage in project "Good". The mutual funds can only diversify the idiosyncratic project risk of entrepreneurs but not aggregate risk. Given that the mutual funds do not accumulate reserves, depositors have to bear all aggregate risk. In both cases, the ex post rate of return on mutual funds is contingent on the aggregate shocks.

Consider the model without moral hazard. Households put d_{t-1} units of consumption composites at the mutual funds at the end of period $t-1$ for the claim on the financial position of the mutual funds in period t. The mutual funds use deposits to finance the project investment of entrepreneurs, $z_{t-1}^{e,m} = (q_{t-1} + a p_{t-1})k^e$. After the project completion in period t, the mutual funds collect all the project outcomes, $[p^G R v_t + q_t]k^e$, and transfer them to depositors. The ex post rate of return on mutual funds is

$$\tilde{r}_t = \frac{(p^G R v_t + q_t)k^e}{d_{t-1}} = r_{t-1}\left[1 + \frac{p^G R(v_t - E_{t-1}v_t) + (q_t - E_{t-1}q_t)}{p^G R E_{t-1}v_t + E_{t-1}q_t}\right]. \qquad (4.35)$$

Thus, depositors have to bear the risk of unexpected changes in the prices of domestic

intermediate goods and land.

Aggregate input for and output of the production of domestic intermediate goods are proportional to the aggregate land stock, aK and $M = p^G RK$. In essence, the model without moral hazard is equivalent to a standard RBC model with a representative agent who has two production technologies: the linear technology to produce intermediate goods employing land K and domestic final goods aK, and the Cobb-Douglas technology to produce domestic final goods using domestic intermediate goods M and labor L_t. In this sense, aggregate output of domestic final goods, $Y_t = A_t M^\alpha L_t^{(1-\alpha)}$, depends on the aggregate labor supply and total factor productivity. Let model RBC denote the model without moral hazard. Appendix C.3 shows the equations describing the market equilibrium of model RBC.

4.2.5 Dynamic Analysis

We log-linearize the equations describing the market equilibria of model MH and model RBC around their respective steady states. The endogenous variables are approximated to the first order as the linear functions of the state variables, which we solve using the MATLAB codes provided by Schmitt-Grohe and Uribe (2004). We analyze the impulse responses of endogenous variables with respect to a transitory TFP shock and a transitory ToT shock in period 0, respectively, given that models are in the steady state before period 0.

4.2.5.1 Impulse Responses to Transitory TFP Shocks

Figure 4.3 shows the impulse responses of model MH (solid line) and model RBC (dashed line) to a transitory TFP shock in period 0. For simplicity, we set the inverse of the terms of trade constant at $s_t = 1$. Thus, the prices of domestic and foreign final goods are constant at $p_t = p_t s_t = 0.5$.

Consider model RBC first. As there is no endogenous state variable in model RBC, the dynamic structure is essentially $AR(1)$. The distinction between households and entrepreneurs does not matter substantially for economic allocation. A 1% positive TFP shock raises the marginal products of domestic intermediate goods and labor in period 0. The price of domestic intermediate goods rises by 0.73% to clear the market, given that aggregate output of domestic intermediate goods is fixed at $M = p^G RK$. Meanwhile, the wage rate rises by 1.15%. In addition, given the autocorrelation in TFP, the marginal product of domestic intermediate goods stays above its steady state value in period 1 and so does the price of domestic intermediate goods. It improves the expected unit value of the land invested
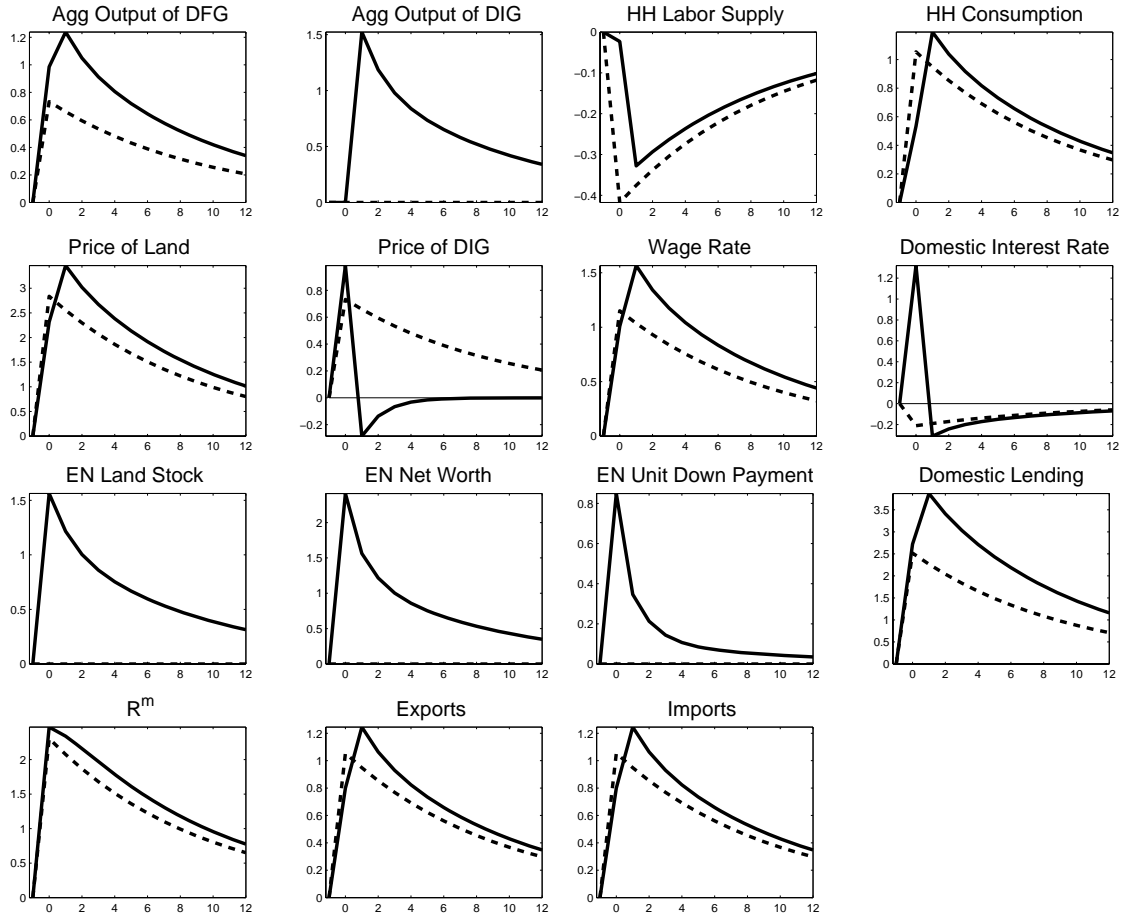
Figure 4.3: Impulse Responses to a TFP Shock: Model MH vs Model RBC

in the entrepreneurs' projects in period 0, $E_0(p^G R v_1 + q_1)$, and entrepreneurs are able to demand more loans and expand their project investment. Given the fixed aggregate land stock, the price of land rises by 2.84% to clear the market. Thus, the positive responses of the prices of land and domestic intermediate goods to the TFP shock in period 0 improves the ex post rate of return on mutual funds. See equation (4.35). Thus, the positive TFP shock improves household wealth in period 0. As households prefer to smooth consumption over time and optimize between consumption and labor, they reduce labor supply by 0.42% in period 0 and increase their deposits at the mutual funds by 2.52%. The decline in household labor supply partially offset the rise in TFP and thus aggregate output of domestic final goods rises only by 0.73%. The rise in the deposits reduces the expected rate of return on mutual funds by 0.21%.

As the amount of domestic final goods invested in the projects of entrepreneurs

is fixe at aK, the domestic final goods used for exports and consumption amount to $Y_t - aK$. Given $\gamma = 0.5$ and $p_t = p_t s_t = 0.5$, households consume equal amounts of domestic and foreign final goods, $c_{F,t}^h = I_t = X_t = 0.5(Y_t - ak) = c_{D,t}^h$. Thus, household consumption rises by 1.05% in period 0 and so do imports and exports.

Consider now model MH. There are two endogenous state variables, $\{k_t^e, R_t^m\}$, in model MH. A 1% positive TFP shock raises the marginal products of domestic intermediate goods and labor in period 0. Given that the aggregate supply of domestic intermediate goods has been predetermined by the project investments of households and entrepreneurs at the end of period -1, $M_0 = p^G R k_{-1} + G(k_{-1}^h)$, the price of domestic intermediate goods rises by 0.99% in equilibrium to clear the market. Extra sales revenues improve the post-repayment wealth of entrepreneurs,

$$\mathcal{N}_0 - E_{-1}\mathcal{N}_0 = p^G[R(v_0 - E_{-1}v_0) + (q_0 - E_{-1}q_0)]k_{-1}^e. \tag{4.36}$$

The rise in entrepreneurial net worth enables entrepreneurs to borrow more from the mutual funds and expand their project investment. Given the fixed aggregate land stock, the rise in their demand for land pushes up the land price in period 0. The capital gains further improves their net worth, as shown in equation (4.36). Altogether, the land price rises by 2.32%. Benefiting from the positive responses of the prices of both domestic intermediate goods and land, entrepreneurial net worth rises by 2.41%. Meanwhile, given that the period-1 land price is above the steady state value by 3.46%, R_0^m rises by 2.47%. Thus, the entrepreneurs' unit down payment rises only by 0.85%, less than the rise in the period-0 land price. Altogether, the land holding of entrepreneurs, $k_0^e = \frac{n_0}{u_0^e}$, rises by 1.56%.

As entrepreneurs bear most of the aggregate risk related to TFP shocks in model MH, the ex post return on mutual funds exceeds its expected value by a smaller amount than in model RBC; compare equations (4.21) and (4.35). In addition to extra ex post return on deposits, the capital gains and extra sales revenues improve household wealth by the amount of $(q_0 - E_{-1}q_0)k_{-1}^h + (v_0 - E_{-1}v_0)G(k_{-1}^h) > 0$ in period 0. Due to the rise in the entrepreneurs' demand for loans, the domestic interest rate rises by 1.32% and it induces households to increase their deposits by 2.72% and reduce their land holding in period 0. Due to the consumption-leisure substitution, households raise their consumption by 0.53% and reduce labor supply by 0.02%, less dramatically than in model RBC. Thus, aggregate output of domestic final goods rises by 0.99%, more than the 0.73% in model RBC. Note that the distinction between entrepreneurs and households matters for aggregate output in model MH. The capital gains on the entrepreneurs' land stock which are transferred to households in model RBC are now enjoyed by entrepreneurs. Thus, household

wealth increases in a smaller magnitude in model MH than in model RBC. The wealth effect explains the dynamics of household labor supply and aggregate output.

Given that entrepreneurs produce more than 98% of domestic intermediate goods in the steady state, the dynamics of aggregate output of domestic intermediate goods approximately follow the dynamics of the land holding of entrepreneurs with a one-period lag. Thus, aggregate output of domestic intermediate goods rises by 1.52% in period 1. So, the price of domestic intermediate goods falls below the steady state value by 0.28%, and meanwhile, the wage rate rises by 1.57%. As the deposits made in period 0 improve household wealth significantly in period 1, the wealth effects induce households to increase their period-1 consumption by 1.19%, larger than in period 0. They also increase period-1 deposits by 3.87% for the consumption smoothing. The rise in the supply of deposits reduces the domestic interest rate by 0.31%. In the meantime, households reduce labor supply by 0.33% due to the consumption-leisure substitution. Altogether, aggregate output of domestic final goods rises by 1.24% above its steady state value in period 1, much more than the 0.73% in period 0 in model RBC. The hump-shaped patterns of consumption, labor, and aggregate output are common in models with financial frictions.

Given that the prices of land and domestic intermediate goods are 3% above and 0.14% below their respective steady state values in period 2, R_1^m is 2.34% above the steady state value. As entrepreneurial net worth is proportional to their land holding in the previous period from period 1 on, $n_t = \tilde{\pi} p^G \tilde{b} k_{t-1}^e$, where $t = 1, 2, 3, ...$, their period-1 net worth is 1.56% above the steady state value. Since the wealth of both households and entrepreneurs gets improved in period 1, the rise in their demand for land further pushes up the land price by 3.46% in period 1. Altogether, the entrepreneurs' unit down payment rises by 0.35%. Thus, their period-1 land holding is 1.22% above the steady state value, less than the 1.56% in period 0.

As loan contracts specify a non-contingent liabilities for successful entrepreneurs, entrepreneurs bear unexpected price changes. Thus, the reallocation of land between households and entrepreneurs is further enhanced. It constitutes a mechanism through which the effects of a transitory TFP shock are amplified.

4.2.5.2   Impulse Responses to Transitory ToT Shocks

Figure 4.4 shows the impulse responses of model MH (solid line) and model RBC (dashed line) to a transitory ToT shock in period 0. For simplicity, we set total factor productivity constant at $A_t = 1$.

Consider model RBC. A 1% negative ToT shock raises the relative price of foreign final goods with respect to domestic final goods in period 0. According to
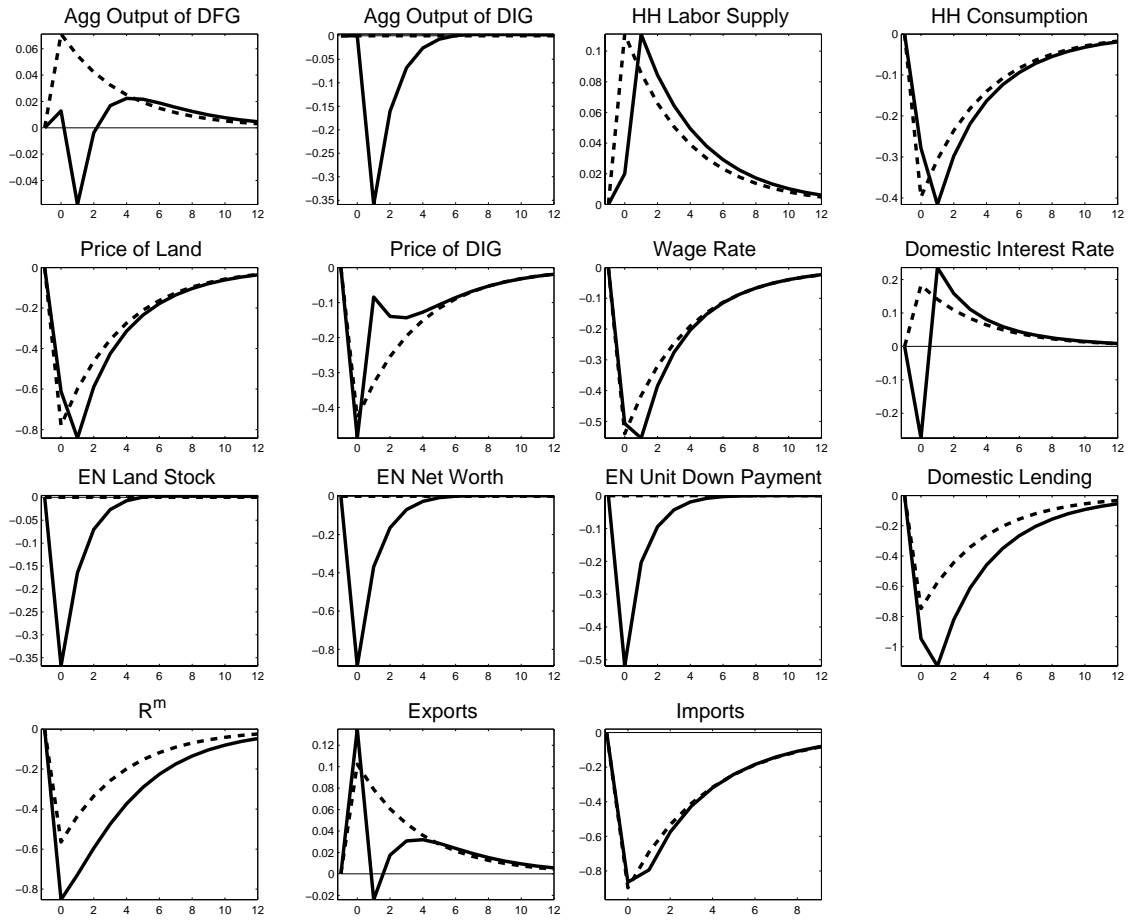
Figure 4.4: Impulse Responses to a ToT Shock: Model MH vs Model RBC

equations (4.8) and (4.9), the price of domestic final goods falls by 0.5% and the price of foreign final goods rises by 0.5% in period 0. The fall in the price of domestic final goods reduces the marginal products of labor and domestic intermediate goods. Therefore, the wage rate and the price of domestic intermediate goods fall by 0.54% and 0.43%, respectively. In this sense, a negative ToT shock has similar effect as a negative TFP shock. Due to the decline in the value of all the project outcomes, the ex post rate of return on mutual funds falls below its expected value and so does the household wealth. As a result, households increase their labor supply by 0.11% and reduce their composite consumption and deposits by 0.4% and 0.75% in period 0. The increase in household labor supply pushes up aggregate output of domestic final goods by 0.07%. The fall in household deposits raises the domestic interest rate by 0.18%. With less loans available for the project investment, entrepreneurs reduce their demand for land. In equilibrium, the land price falls by 0.78%.

The fall in the price of domestic final goods induces households to raise their consumption of domestic final goods by 0.1% and the rise in the price of foreign final goods induces households to reduce their consumption of foreign final goods by 0.9%. Thus, imports fall by 0.9%. As foreign trade must balance, $X_0 = s_0 I_0$, exports rises by 0.1%.

Consider model MH. A 1% negative ToT shock reduces the price of domestic final goods and raises the price of foreign final goods. As entrepreneurs bear most of the aggregate uncertainty using their net worth, the ex post rate of return on mutual funds does not fall as much as in model RBC; compare equations (4.21) and (4.35). Thus, despite the fall in the wage rate by 0.51% in period 0, households increase their labor supply only by 0.02%, much less than the 0.11% in model RBC. Thus, aggregate output of domestic final goods rises only by 0.013%.

The fall in the price of domestic intermediate goods reduces the sales revenues of entrepreneurs. Due to the decline in entrepreneurial net worth, entrepreneurs cannot borrow as much as before and have to reduce their land holding. The fall in the demand for land leads to the fall in the land price. This capital loss further reduces entrepreneurial net worth. In equilibrium, the net worth and the land holding of entrepreneurs fall by 0.89% and 0.37%, respectively. Due to the fall in the demand of entrepreneurs for loans, the domestic interest rate falls by 0.28%.

The fall in household wealth forces households to reduce their consumption by 0.28%. Entrepreneur consumption and entrepreneurial net worth are both proportional to their post-repayment wealth. Given that household consumption is around 6 times as much as entrepreneur consumption in the steady state, imports, $I_0 = \frac{(1-\gamma)(c_0^h + c_0^e)}{p_0 s_0}$, fall by 0.86%. As foreign trade must balance each period, $s_0 I_0 = X_0$, exports rise by 0.14%.

Due to the fall in the entrepreneurs' period-0 land stock, aggregate output of domestic intermediate goods falls by 0.36% in period 1. As the land price is still below the steady state value in period 1, household wealth is below its steady state value. Thus, households reduce their consumption and deposits by 0.42% and 1.13%, respectively. The fall in the supply of deposits pushes up the domestic interest rate by 0.24%. Due to the consumption-leisure substitution, households raise their labor supply by 0.11%. Altogether, aggregate output of domestic final goods falls by 0.058% in period 1, in contrast to the rise by 0.055% in model RBC.

From period 1 on, the consumption and the net worth of entrepreneurs are proportional to their land holding in the previous period, $c_t^e = (1 - \tilde{\pi}) p^G \tilde{b} k_{t-1}^e$ and $n_t = \tilde{\pi} p^G \tilde{b} k_{t-1}^e$, where $t = 1, 2, 3, \ldots$. Thus, the consumption and net worth of entrepreneurs are both below their respective steady state values by 0.37% in period 1. Aggregate demand for foreign final goods falls by 0.79% and so do imports. Due

to autocorrelation in the terms of trade, the inverse of the period-1 terms of trade, s_1, is above its steady state value by 0.77%. Thus, the balanced foreign trade requires that exports, $X_1 = s_1 I_1$, be below the steady state value by 0.02%.

The distinction between households and entrepreneurs matters for business volatility. A deterioration in the terms of trade (a rise in s_t) reduces the price of domestic final goods. As domestic loans are written in terms of the domestic consumption composite, the fall in the price of domestic final goods essentially raises the liabilities of entrepreneurs to the mutual funds. Thus, entrepreneurs with successful projects have to pay even more in terms of domestic intermediate goods. Therefore, entrepreneurs bear the aggregate risk related to ToT shocks via domestic loan contracts. In comparison with the case of a transitory TFP shock, the asset reallocation in the case of a transitory ToT shock actually results from debt deflation.

4.3 The Full Model

The economy described by the basic model in section 4.2 is actually under international financial autarky. This section considers the case where the economy opens up to foreign investors.

4.3.1 Financial Frictions on Foreign Borrowing

A continuum of risk-neutral foreign investors supply funds in terms of foreign final goods inelastically at the expected rate of return, r_t^*. Let the foreign interest rate denote the expected rate of return on foreign funds. The domestic economy is small enough that the foreign interest rate is exogenously determined abroad. The foreign interest rate is an $AR(1)$ in logarithms,

$$\log r_t^* = (1 - \rho^*) \log \bar{r}^* + \rho^* \log r_{t-1}^* + \epsilon_t^*, \tag{4.37}$$

where ρ^* denotes the autocorrelation coefficient of the foreign interest rate and the $\bar{r}^*$ denotes the non-stochastic steady state value of the foreign interest rate. Following Devereux, Lane, and Xu (forthcoming), we set $\rho^* = 0.46$. The shock to the foreign interest rate (FIR shock), ϵ_t^*, has mean zero, $E_t \epsilon_{t+1}^* = 0$, and is distributed within an interval, $(-\tau^*, \tau^*)$, where $\tau^* > 0$ is small enough that successful entrepreneurs are always able to repay their liabilities and r_t^* always exceeds unity. In the meantime, the foreign interest rate is always smaller than the domestic interest rate around the steady state, $r_t^* < r_t$.

A unit of foreign final goods borrowed abroad has the domestic value of $p_t s_t$ and its required repayment is expected to be $r_t^* E_t p_{t+1} s_{t+1}$ in terms of the domestic

consumption composite. For convenience of notation, let

$$r_t^f = \frac{r_t^* E_t(p_{t+1} s_{t+1})}{p_t s_t}, \tag{4.38}$$

denote the effective foreign interest rate in terms of the domestic consumption composite, which, according to our calibration, is smaller than the domestic interest rate around the steady state, $r_t^f < \frac{1}{\beta}$.

The mutual funds have the exclusive technology to perfectly verify the ex post project outcomes of entrepreneurs and they can liquidate at no discount the land handed over by the entrepreneurs whose projects failed. The foreign investors do not have the required verification technology. Therefore, entrepreneurs cannot credibly commit any project output against a loan from foreign investors. However, they can use part of their land stock as collateral for foreign borrowing. We assume that the foreign investors are less familiar with the domestic land market than domestic mutual funds, or have the inferior liquidation technology for collateral handed over by the entrepreneurs with failed projects, or the domestic legal system is biased against the foreign investors. Either way, foreign borrowing is overcollateralized in the sense that each unit of land has an expected domestic value of $E_t q_{t+1}$ in period t and entrepreneurs can only pledge $\theta E_t q_{t+1}$ to the foreign investors for $\frac{E_t \theta q_{t+1}}{r_t^* E_t p_{t+1} s_{t+1}}$ units of foreign final goods, where $\theta \in (0, 1]$ denotes the degree of collateralization and $r_t^* E_t p_{t+1} s_{t+1}$ denotes the foreign interest rate adjusted by the expected price of foreign final goods at the time of repayment, i.e., period $t+1$. $(1-\theta)$ can be regarded as a premium the foreign investors have to pay to the domestic land buyers when they liquidate the land handed over by failed entrepreneurs ex post.[2] For simplicity, we assume that θ is constant. Our basic model in section 4.2 can be regarded as a special case of the full model with $\theta = 0$.

The mutual funds do not have any physical assets pledgable to foreign investors as collateral. Thus, the foreign investors do not make deposits directly at the mutual funds. By the same logic, the households cannot use their deposit certificates as collateral for foreign funds.

4.3.1.1   Foreign and Domestic Borrowing of Entrepreneurs

Entrepreneurs can pledge their land stock to the foreign investors and their collateral constraints are

$$r_t^* z_t^{e,*} E_t(p_{t+1} s_{t+1}) \leq \theta E_t q_{t+1} k_t^e, \tag{4.39}$$

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[2]This premium may change along the business cycle and so does θ. See Iacoviello and Minetti (forthcoming) for a detailed discussion.

where $z_t^{e,*}$ denotes the land-backed foreign borrowing of entrepreneurs. As the effective foreign interest rate specified in equation (4.38) is smaller than the domestic interest rate around the steady state, entrepreneurs borrow first from the foreign investors to the limit and then borrow from the mutual funds. Thus, the collateral constraints are binding in equilibrium. As entrepreneurs and the foreign investors are both risk neutral, the financial contract is a contract sharing aggregate risk between them. The ex post repayment to foreign investors in period $t+1$ is $\theta q_{t+1} k_t^e$ and thus, the ex post rate of return to foreign investors is

$$r_{t+1}^{e,*} = r_t^* \left[\frac{q_{t+1} E_t(p_{t+1} s_{t+1})}{p_{t+1} s_{t+1} E_t q_{t+1}} \right], \tag{4.40}$$

which differs from its expected value r_t^* due to unexpected changes in the prices of land and foreign final goods. For the project investment of each unit of land and a units of domestic final goods, entrepreneurs can borrow $\frac{z_t^{e,m}}{k_t^e}$ units of domestic funds in terms of the domestic consumption composite and $\frac{z_t^{e,*}}{k_t^e}$ units of foreign funds in terms of foreign final goods. Thus, entrepreneurs have to use own funds to fill in the gap between total investment and external funds. Their unit down payment is

$$u_t^e = (q_t + ap_t) - \frac{z_t^{e,m}}{k_t^e} - \frac{p_t s_t z_t^{e,*}}{k_t^e}. \tag{4.41}$$

Given that the entrepreneurs have pledged the fraction θ of their land stock to the foreign investors, the financial contract between the entrepreneurs and the mutual funds in period t specifies a fixed repayment of

$$R_t^m = E_t[Rv_{t+1} + (1-\theta)q_{t+1}] - \tilde{b}, \tag{4.42}$$

if the projects succeed in period $t+1$; if the projects fail, the entrepreneurs first hand over θk_t^e units of land to foreign investors and transfer the remaining $(1-\theta)k_t^e$ to the mutual funds. The expected break-even condition of the mutual funds in period t is

$$r_t z_t^{e,m} = [p^G R_t^m + (1 - p^G)(1-\theta)E_t q_{t+1}]k_t^e. \tag{4.43}$$

The ex post rate of return on mutual funds in period $t+1$ is

$$\tilde{r}_{t+1} = r_t \left\{ 1 + \frac{(1-p^G)(1-\theta)(q_{t+1} - E_t q_{t+1})}{E_t[p^G(Rv_{t+1} - \tilde{b}) + (1-\theta)q_{t+1}]} \right\}, \tag{4.44}$$

which differs from its expected value r_t due to unexpected changes in the land price. In other words, the mutual funds have to bear capital gains or losses of the

land handed over by the failed entrepreneurs ex post. According to our calibration, $1 - p^G = 0.01$ and thus, the ex post return on mutual funds and household deposits does not differ much from its expected value. Furthermore, as the foreign investors also bear a fraction of capital gains or losses, the difference between the ex post rate of return on deposits and its expected value is decreasing in the degree of collateralization.

Per capita consumption c_t^e, net worth n_t, and the land holding k_t^e of the entrepreneurs are as follows,

$$c_t^e = (1 - \tilde{\pi})p^G[Rv_t + (1 - \theta)q_t - R_{t-1}^m]k_{t-1}^e, \qquad (4.45)$$

$$n_t = \tilde{\pi}p^G[Rv_t + (1 - \theta)q_t - R_{t-1}^m]k_{t-1}^e + (1 - \pi)p_t e, \qquad (4.46)$$

$$u_t^e k_t^e = n_t, \qquad (4.47)$$

4.3.1.2   Foreign Borrowing of Households

As the effective foreign interest rate is smaller than the domestic interest rate around the steady state, households borrow abroad and deposit at the mutual funds to take advantage of the interest rate differentials. Thus, households pledge their land stock to foreign investors and their collateral constraints are

$$r_t^* z_t^{h,*} E_t p_{t+1} s_{t+1} \le \theta E_t q_{t+1} k_t^h, \qquad (4.48)$$

where $z_t^{h,*}$ denotes the land-backed foreign borrowing of households. For each unit of land invested in their projects, households can acquire $\frac{\theta E_t q_{t+1}}{r_t^* E_t(p_{t+1}s_{t+1})}$ units of foreign funds in terms of foreign final goods in period t. We define the household unit down payment as the amount of own funds they pay for each unit of land,

$$u_t^h = q_t - \frac{\theta E_t q_{t+1}}{r_t^f}. \qquad (4.49)$$

The household expected marginal rate of return on land is $E_t[(1-\theta)q_{t+1}+v_{t+1}G'(k_t^h)]$. Households optimize between their project investment and deposits,

$$u_t^h = \frac{E_t[(1 - \theta)q_{t+1} + v_{t+1}G'(k_t^h)]}{r_t}. \qquad (4.50)$$

As households are risk averse and foreign investors are risk neutral, the financial contract between them provides households with perfect insurance against unexpected changes in the land price. Suppose that households invest k_t^h units of land in their project and pledge the fraction θ of the land stock to the foreign investors in period t. The ex post returns to households and the foreign investors are $(1 - \theta)E_t q_{t+1}k_t^h$

and $q_{t+1}k_t^h - (1-\theta)E_t q_{t+1} k_t^h$ in period $t+1$. Thus, the ex post rate of return to the foreign investors in period $t+1$ is

$$r_{t+1}^{h,*} = r_t^* \left[\frac{E_t(p_{t+1}s_{t+1})}{p_{t+1}s_{t+1}} \right] \left[1 + \frac{q_{t+1} - E_t q_{t+1}}{\theta E_t q_{t+1}} \right], \tag{4.51}$$

which differs from its expected value r_t^* due to the unexpected changes in the prices of land and foreign final goods. Household period-by-period budget constraints are,

$$u_t^h k_t^h + c_t^h + d_t = (1-\theta)E_{t-1}q_t k_{t-1}^h + v_t G'(k_{t-1}^h) + \tilde{r}_t d_{t-1} + w_t l_t. \tag{4.52}$$

4.3.2 Balance of Payment

The aggregate collateral constraints of the domestic economy are

$$r_t^* Z_t^* E_t(p_{t+1}s_{t+1}) = \theta E_t q_{t+1} K, \tag{4.53}$$

where $Z_t^* = z_t^{h,*} + z_t^{e,*}$ denote the aggregate foreign borrowing in terms of foreign final goods. Foreign funds are overcollateralized by the aggregate land stock. Thus, the aggregate foreign borrowing in the current period depends on the current foreign interest rate and the expected prices of land and foreign final goods in the next period. The interest payment of foreign borrowing is covered by the trade surplus,

$$NX_t + Z_t^* = r_t^{h,*} z_{t-1}^{h,*} + r_t^{e,*} z_{t-1}^{e,*}, \tag{4.54}$$

$$NX_t = \frac{X_t}{s_t} - I_t, \tag{4.55}$$

where NX_t denotes net exports in terms of foreign final goods. As we rule out exploding bubbles in the land price, the foreign borrowing backed by the domestic land is sustainable. In this sense, the domestic economy, as a whole, is solvent and does not run into the problem of Ponzi games.

4.3.3 Market Equilibrium

Definition 4.2. *Market equilibrium in the model with domestic and foreign financial frictions is a set of allocations of households, $\{k_t^h, l_t, z_t^{h,*}, c_t^h, c_{D,t}^h, c_{F,t}^h\}$, and entrepreneurs, $\{k_t^e, n_t, z_t^{e,m}, z_t^{e,*}, c_t^e, c_{D,t}^e, c_{F,t}^e\}$, together with aggregate variables $\{M_t, Y_t, I_t, X_t, NX_t, Z_t^*\}$ given a set of prices $\{v_t, p_t, q_t, w_t, r_t, \tilde{r}_t, r_t^{h,*}, r_t^{e,*}, u_t^h, u_t^e, R_t^m\}$ and the exogenous processes $\{A_t, s_t, r_t^*\}$ satisfying equations (4.2)- (4.6), (4.19)-(4.20), (4.22)-(4.25), (4.27)-(4.31), (4.37), (4.39)-(4.51), (4.53)-(4.55),*

Model MH in section 4.2 is a special case of $\theta = 0$ here. For consistency, we still call our full model model MH.

### 4.3.4 The Benchmark: Foreign Financial Frictions Only

This chapter focuses on the macroeconomic implications of domestic financial frictions. Thus, the benchmark model is defined as the model with foreign financial frictions only: project choices of the entrepreneurs are perfectly observable to the mutual funds but foreign borrowing must be backed by land. As model RBC discussed in subsection 4.2.4, all land is allocated to entrepreneurs, $k_t^e = K$; domestic final goods are produced by entrepreneurs only, $M_t = p^G R K$. In fact, model RBC in subsection 4.2.4 is a special case ($\theta = 0$) of the benchmark model here. For consistency, we still call the benchmark model model RBC. As the foreign interest rate is smaller than the domestic interest rate, entrepreneurs first borrow abroad to the limit and then pledge the rest of their project outcomes to the mutual funds. Entrepreneurs use domestic and foreign borrowing to finance all of their project investment; they do not have to provide own funds. Appendix C.3 shows the equations describing the market equilibrium in the benchmark model.

4.3.5 Long-Run Effects of Foreign Financial Frictions

This subsection analyzes how the degree of collateralization can affect macroeconomic aggregates, production efficiency, and social welfare in the long run. To this end, we assume away aggregate uncertainty. Figure 4.5 shows the steady state values of endogenous variables of model MH with respect to θ, in the cases of $r^* = 1.01$ (solid line) and $r^* = 1.0025$ (dashed line), corresponding to the annual interest rates of 4% and 1%, respectively. The horizontal axis denotes $\theta \in [0, 1]$.

Consider first the case of $r^* = 1.01$. The domestic interest rate, $r = \frac{1}{\beta}$, is unaffected by θ. In comparison with the case of $\theta = 0$, households and entrepreneurs now can borrow cheap foreign funds and expand their project investment. The rise in their demand for land pushes up the land price. As show in subsection 4.2.2.2, households can only pledge a fraction of the value of their land stock for foreign funds, while entrepreneurs can pledge not only their land stock to foreign investors but also some revenues of their projects to the mutual funds. Thus, the land holding of entrepreneurs rises in θ and so does their consumption, $c^e = (1 - \tilde{\pi})\mathcal{N} = (1 - \tilde{\pi})p^G \tilde{b} k^e$. The welfare of entrepreneurs, defined as the discounted sum of their consumption and private benefits, is linear in their land stock. Thus, entrepreneurs benefit strictly from borrowing abroad. The rise in the land stock of entrepreneurs increases aggregate output of domestic intermediate goods.

As foreign funds are cheaper than domestic loans, entrepreneurs borrow first from foreign investors to the limit. Due to the substitution effect, the domestic
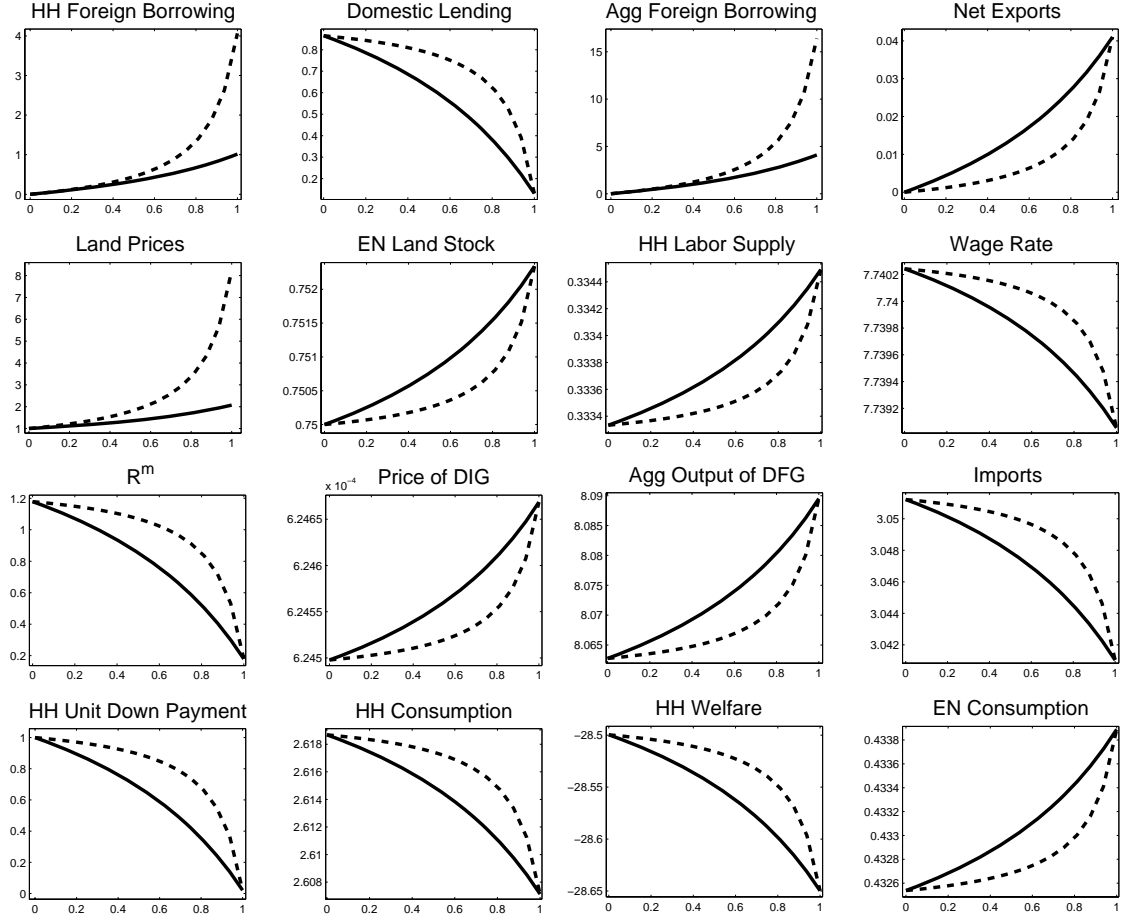
Figure 4.5: Long-Run Effects of the Degree of Collateralization to Model MH

lending to entrepreneurs falls in θ and so does the deposit return to households. Household wealth consists of their net land holding $(1 - \theta)qk^h$, deposit return, rd, sales revenues, $vG(k^h)$, and wage income, wl, as shown in equation (4.52). The first three components fall in θ. As a result, households increase their labor supply to partially offset the fall in their wealth. The rise in the supply of household labor reduces the wage rate. Altogether, household wealth and consumption fall in θ; so does household welfare defined as the discounted sum of their period utility from consumption and leisure.

Thus, entrepreneurs benefit strictly and households lose strictly from land-backed foreign borrowing. Whether or not cheap foreign funds improve the long-run social welfare depends on the relative weights the social planner puts on households and entrepreneurs. Due to the rise in domestic intermediate goods and labor, aggregate output of domestic final goods rises in the degree of collateralization and

so does the efficiency of domestic production. However, aggregate output is not a good indicator for social welfare in our model with heterogenous agents. At a first glance, household consumption falls by 0.004 while entrepreneurs' consumption rises by 0.0005 as θ rises from 0 to 0.5. It seems that cheap foreign funds reduce social welfare in the long run. Our preliminary investigation shows that, due to the wealth effect, households benefit strictly during the transition from international financial autarky to financial opening. Thus, cheap foreign funds may improve social welfare in the short run.

Consider now the case of $r^* = 1.0025$. As foreign funds are cheaper than in the case of $r^* = 1.01$, the demand of domestic agents for land is more enhanced given the same degree of land-backed foreign borrowing. As a result, the land price rises more dramatically than in the case of $r^* = 1.01$.

Figure 4.6 shows the steady state values of endogenous variables of model RBC with respect to the degree of collateralization, given $r^* = 1.01$ (solid line) and $r^* = 1.0025$ (dashed line). The horizontal axis denotes $\theta \in [0, 1]$.

Consider first the case of $r^* = 1.01$. As θ rises from 0 to 1, entrepreneurs can borrow more abroad and the substitution effect reduces the deposit returns to households. As discussed above in model MH, households increase labor supply in order to partially offset the fall in their wealth. Household consumption falls in the long run and so does their welfare. Meanwhile, entrepreneurs only consume their tiny endowment and their welfare depends mainly on private benefits from running project "Good", $b^G K$, which is independent of the degree of collateralization. Altogether, social welfare falls strictly in θ in the long run. Thus, in the case of a positive interest rate differential between domestic and foreign funds, foreign borrowing reduces the long-run social welfare of the domestic economy strictly in model RBC.

In the case of $r^* = 1.0025$, changes in θ result in the similar patterns of macroeconomic aggregates in model RBC as in model MH. We exclude explosive bubble in the land price in subsection 4.2.2.4 and foreign borrowing is backed by the aggregate land stock. Thus, although the value of aggregate foreign borrowing may exceed aggregate output of domestic final goods in the domestic economy, e.g., in the case of $\theta = 1$, the domestic economy is still solvent and use the trade surplus to pay the interest on foreign borrowing.

Note that changes in the degree of collateralization have rather tiny effects on macroeconomic aggregates, compared to changes in the degree of moral hazard in subsection 4.2.4. It results from the fact that changes in the degree of collateralization (θ) affect only the average cost of external funds for entrepreneurs but not the moral hazard problem between mutual funds and entrepreneurs in the domestic economy. The incentive for entrepreneurs to invest a unit land in project "Good" is
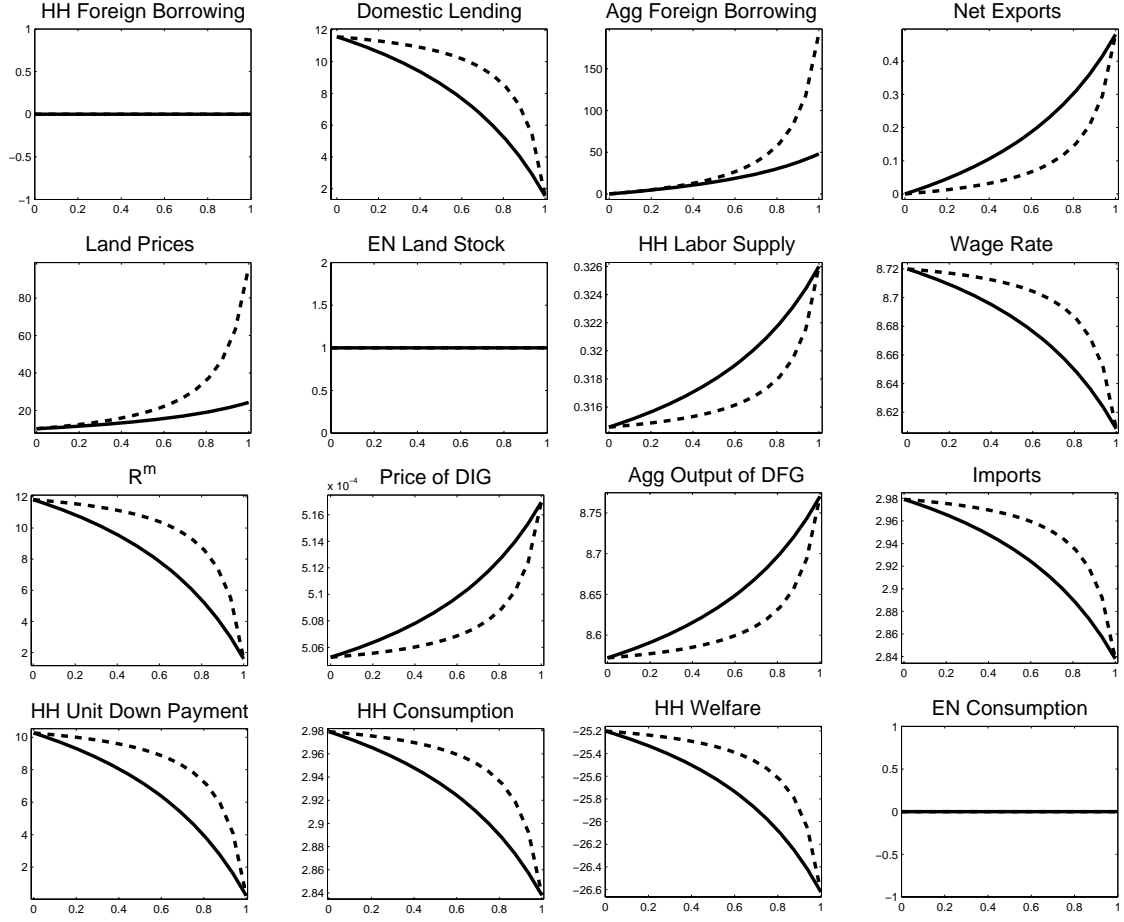
Figure 4.6: Long-Run Effects of the Degree of Collateralization to Model RBC

independent of the degree of collateralization and constant at $p^G\tilde{b}$ by expectation. As a result, the entrepreneurs' total capacity of external financing does not change much in θ and so is their land holding. In other words, changes in θ only result in substitution between domestic and foreign lending to entrepreneurs.

Iacoviello and Minetti (forthcoming) explain the comovement of output across countries in a model with domestic and foreign borrowing similar as in our model. In their model, only entrepreneurs borrow from abroad in equilibrium, while households do not. We can also exclude households from borrowing abroad by assuming other information frictions. If so, the household unit down payment of land is simply q_t instead of $u_t^h = q_t - \frac{\theta E_t q_{t+1}}{r_t^f}$ in the case of the positive foreign borrowing of households. As a result, a rise in θ makes the overall cost of external funds cheaper for entrepreneurs only and their excess demand for land pushes up the land price.

As land is more expensive for households, the household land holding declines more strongly than in the case of the positive foreign borrowing of households. While, the qualitative results in this subsection do not change. As we prefer to keep the model simple and do not introduce more restrictions on households, we allow households to borrow abroad for analytical completeness.

4.3.6 Dynamic Analysis

This subsection analyzes how the degree of collateralization can affect macroeconomic volatility via foreign borrowing in the domestic economy. As in subsection 4.2.5, we log-linearize the equations describing the market equilibria of relevant models around their respective steady states and use the first-order approximations. In comparison with subsection 4.2.5, we analyze the model dynamics with respect to transitory TFP and ToT shocks in the case of $\theta \in (0, 1]$ in subsections 4.3.6.1 and 4.3.6.2, respectively. As foreign borrowing is not allowed in the economy described in section 4.2, changes in the foreign interest rate do not affect the domestic economy. In the case of $\theta \in (0, 1]$, changes in the foreign interest rate can affect the domestic economy via foreign borrowing. We analyze the model dynamics with respect to transitory FIR shocks in the case of $\theta \in (0, 1]$ in subsection 4.3.6.3.

4.3.6.1 Impulse Responses to Transitory TFP Shocks

As specified in equation (4.23), total factor productivity, A_t, is an AR(1) in logarithms. Figure 4.7 shows the impulse responses of model RBC in the case of $\theta = 0.5$ (solid line) and in the case of $\theta = 0$ (dashed line) to a transitory TFP shock. For simplicity, we set the foreign interest rate and the inverse of the terms of trade constant at $r_t^* = 1.01$ and $s_t = 1$. Thus, the prices of domestic and foreign final goods are constant at $p_t = p_t s_t = 0.5$. The impulse responses of model RBC in the case of $\theta = 0$ have been discussed in subsection 4.2.5.1.

Consider model RBC in the case of $\theta = 0.5$. As there is no endogenous state variable in model RBC for $\theta \in [0, 1]$, the dynamic structure is essentially $AR(1, 1)$. A 1% positive TFP shock pushes up the prices of domestic intermediate goods and land in period 0. Meanwhile, the wage rate rises by 1.09%. Due to autocorrelation in TFP, the marginal product of domestic intermediate goods is above its steady state value in period 1 and so does the price of domestic intermediate goods. The expected unit value of entrepreneurs' projects, $E_0(p^G R v_1 + q_1)$, rises and entrepreneurs can demand more external funds from domestic and foreign lenders. As the aggregate land stock is fixed, the price of land rises by 1.57% to clear the market in period
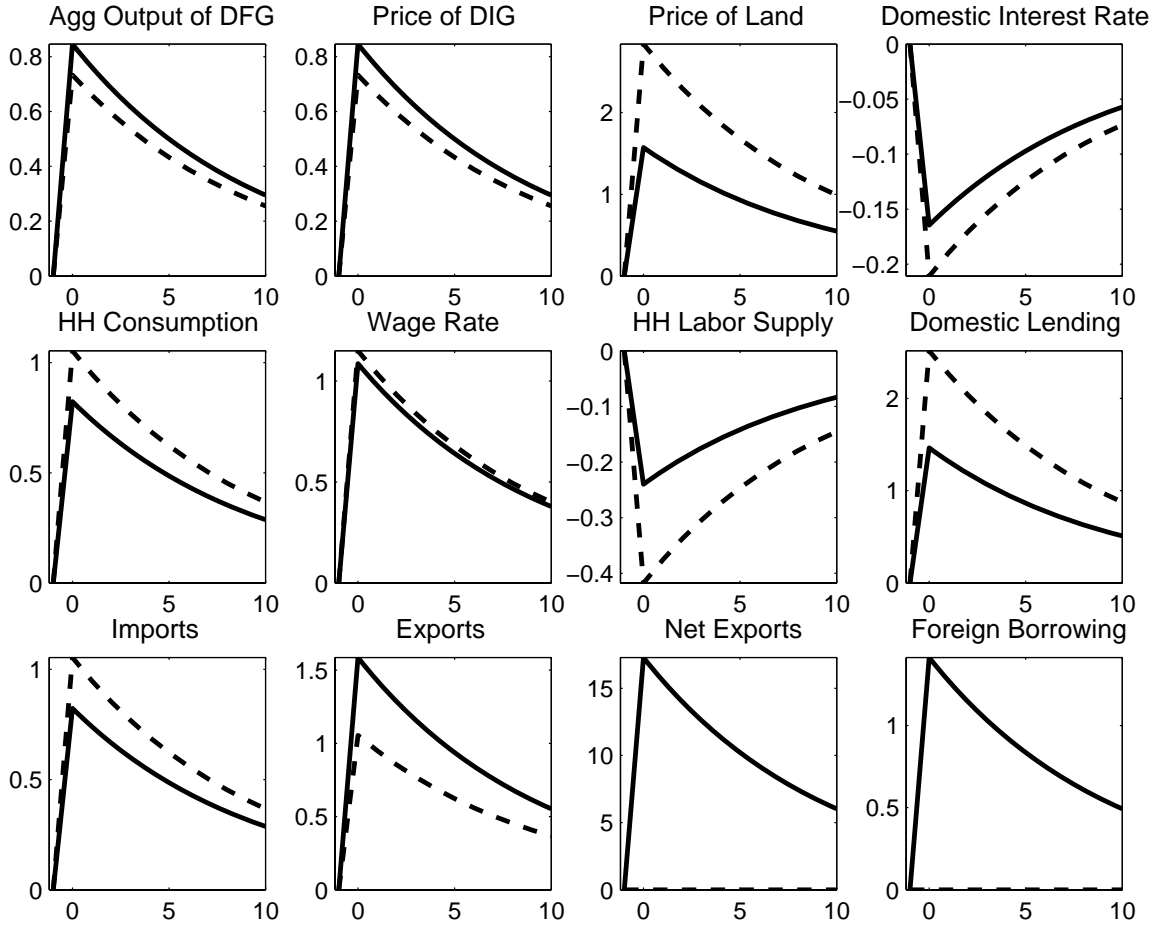
Figure 4.7: Impulse Responses to a TFP shock: Model RBC

0, much less than the 2.84% in the case of $\theta = 0$. This can be understood as follows. Household wealth consists of their deposit return and wage income. In the case of $\theta = 0.5$, entrepreneurs and foreign investors jointly share capital gains or capital losses related to aggregate risk, according to the financial contract specified in subsection 4.3.1.1. Due to the leakage of capital gains to the foreign investors, the entrepreneurs' ex post repayment to the mutual funds, $p^G R v_0 + (1-\theta)q_0$, exceed the expected value by a smaller amount in the case of $\theta = 0.5$ than in the case of $\theta = 0$, so does the ex post rate of return on household deposits. See equation (4.44). Due to the wealth effect, households raise their deposits and consumption only by 1.46% and 0.82% in the case of $\theta = 0.5$, less than the 2.52% and the 1.05% in the case of $\theta = 0$. The weaker rise in the supply of deposits reduces the domestic interest rate by 0.16% in period 0 and the land price rises in a smaller magnitude than in

the case of $\theta = 0$. The land price in period 1 is expected to be above its steady state value by 1.42% and so is the aggregate foreign borrowing, $Z_0^* = \frac{\theta E_0 q_1 K}{r^*}$, in period 0. As foreign investors benefit from capital gains, net exports rise by 17.3% in period 0. See equation (C.19). Given that imports follow the pattern of household consumption, exports rise by 1.59%. See equation (C.17).

Due to the wealth effect, households reduce their labor supply only by 0.24%, less than the 0.42% in the case of $\theta = 0$. Given fixed aggregate supply of domestic intermediate goods, aggregate output of domestic final goods rises by 0.85%, larger than the 0.73% in the case of $\theta = 0.5$.

Schmitt-Grohe (2005) use first-order approximations and show that the unconditional standard deviations of endogenous variables are proportional to the standard deviations of exogenous shocks. Figure 4.8 shows the unconditional standard deviations of some endogenous variables in model MH (solid line) and in model RBC (dashed line) normalized by the standard deviation of TFP shocks. The horizontal axis denotes $\theta \in [0, 1]$.

As the degree of collateralization rises from 0 to 0.8, the foreign investors bear more and more capital gains (losses) in the case of positive (negative) TFP shocks. Thus, the difference between the ex post repayment of entrepreneurs to the mutual funds and its expected value becomes smaller. Due to the wealth effect, household consumption and labor supply respond to TFP shocks in a smaller magnitude. As a result, imports respond less while aggregate output of domestic final goods responds more to TFP shocks. Meanwhile, the weaker responses of household deposits to TFP shocks result in the weaker responses of the domestic interest rate and the land price, and so does the foreign borrowing. Aggregate output of domestic final goods are either consumed by households, or invested in the projects of entrepreneurs, or exported. Given that investment of domestic final goods in the projects of entrepreneurs is constant, aK, the rise in the volatility of aggregate output of domestic final goods and the fall in the volatility of household consumption jointly imply that exports respond more to TFP shocks. As shown in figure 4.6, the rise in θ leads to the decline in the domestic lending due to the substitution of foreign borrowing. Thus, the share of household deposits in household wealth falls in θ, too. As long as θ is below 0.8, the wealth effects still dominate. However, if θ exceeds 0.8, household deposits account for a less significant share of household wealth. Thus, the wealth effects related to deposit returns have less impacts on household consumption and labor decision. As a result, the rise in the degree of collateralization can have opposite effects on some macroeconomic aggregate.

Consider model MH now. Figure 4.9 shows the impulse responses of model MH in the case of $\theta = 0.5$ (solid line) and in the case of $\theta = 0$ (dashed line) to
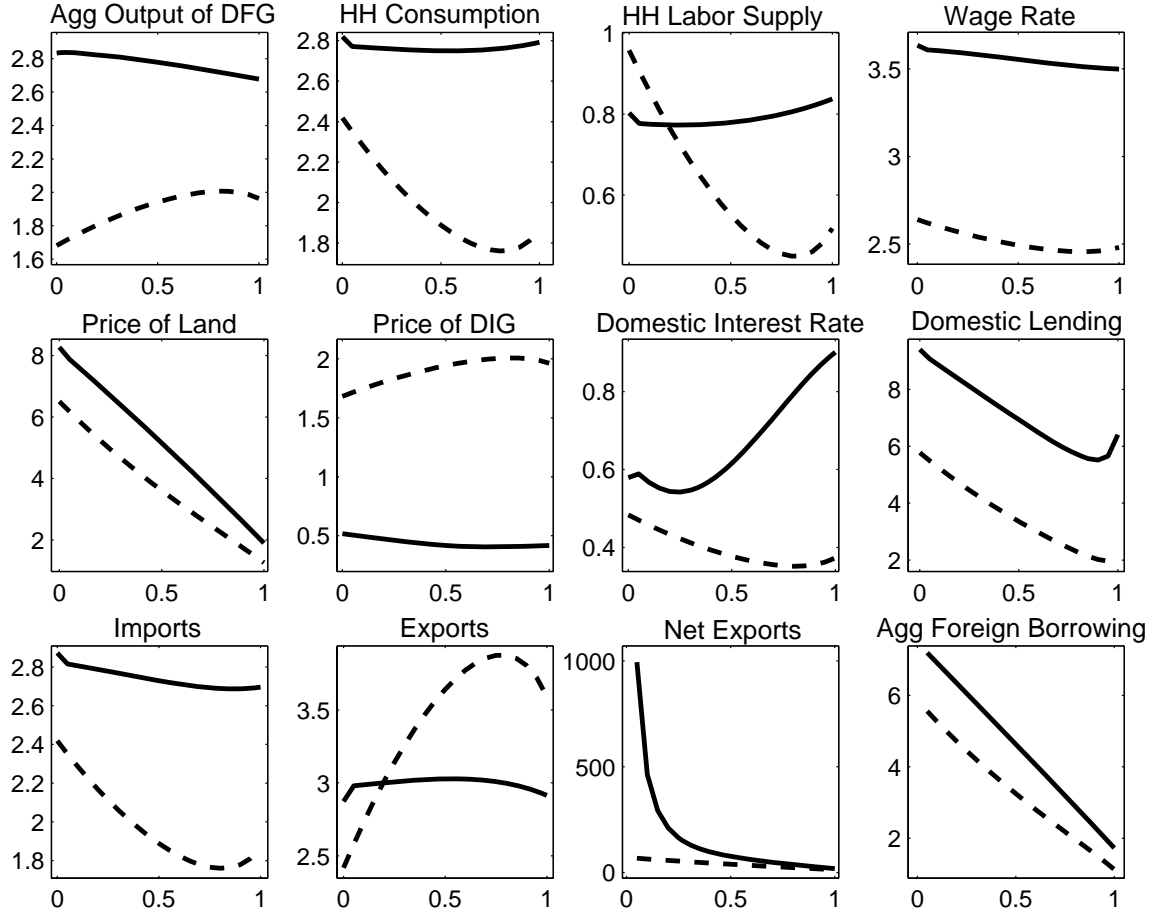
Figure 4.8: Foreign Borrowing and Macroeconomic Volatility: TFP shocks

a transitory TFP shock, given $\theta = 0.5$. For simplicity, we set the foreign interest rate and the inverse of the terms of trade constant at $r_t^* = 1.01$ and $s_t = 1$. Thus, the prices of domestic and foreign final goods are constant at $p_t = p_t s_t = 0.5$. The impulse responses of model MH in the case of $\theta = 0$ have been discussed in subsection 4.2.5.1.

Consider model MH in the case of $\theta = 0.5$. Domestic agents pledge half of the expected value of their land stock to the foreign investors. Additionally, entrepreneurs can pledge part of the expected value of their output for domestic loans from the mutual funds. A 1% positive TFP shock leads to the rise in the wage rate and the price of domestic intermediate goods in period 0. Extra sales revenues improve entrepreneurial net worth and entrepreneurs are able to demand more external funds and land. The entrepreneurs' excess demand for land pushes up the land
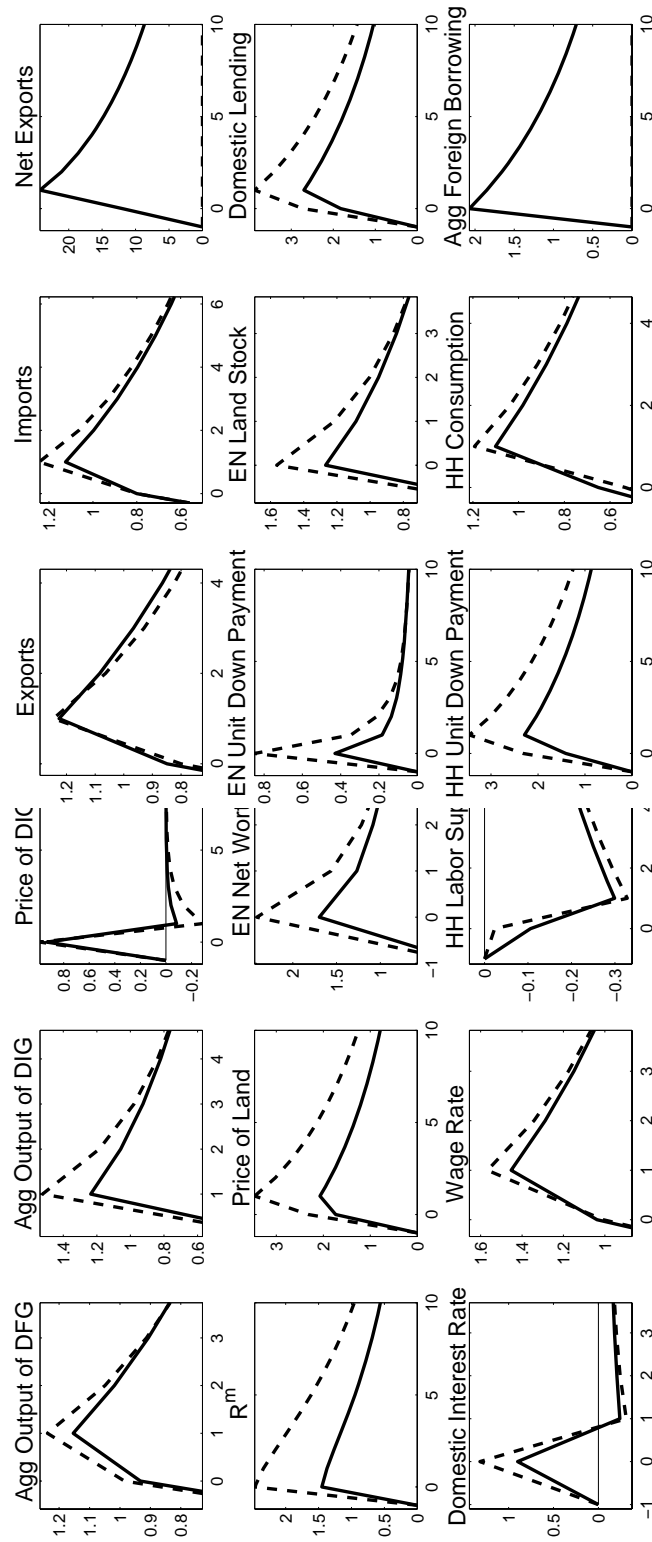
Figure 4.9: Impulse Responses to a TFP shock: Model MH

price. According to the financial contracts specified in subsection 4.3.1.1, foreign investors and entrepreneurs share the capital gains on the entrepreneurs' land stock on a pro rata basis. Thus, capital gains improve entrepreneurial net worth further and enhance the entrepreneurs' demand for land and loans. In all, the land price rises by 1.74%, less than the 2.32% in the case of $\theta = 0$, due to the leakage of the capital gains to the foreign investors. Meanwhile, the entrepreneurs' excess demand for the domestic loans from the mutual funds pushes up the domestic interest rate by 0.9%, less than the 1.32% in the case of $\theta = 0$.

According to the financial contracts between households and foreign investors specified in subsection 4.3.1.2, the foreign investors take all of the capital gains on the land stock of households. Household unit down payment for land rises by 1.41%, larger than the rise in the entrepreneurs' unit down payment by 0.43%. Thus, the entrepreneurs' land stock rises by 1.27%.

According to equation (4.52), household wealth consists of the value of their net land stock, sales revenues, deposit returns, and wage income. The first component is unaffected by TFP shocks. The second and the third components exceed their expected values in period 0, due to the ex post rise in the price of domestic intermediate goods and the rate of return on deposits. The rise in the domestic interest rate induces households to increase their deposits by 1.82%. However, as the rise in the domestic interest rate is smaller than in the case of $\theta = 0$, the rise in household deposits is also smaller. The wealth effects and the weaker rise in the domestic interest rate induce households to increase consumption by 0.65%, larger than the 0.53% in the case of $\theta = 0$. Due to the consumption-leisure substitution, households reduce labor supply by 0.11%, larger than the 0.023% in the case of $\theta = 0$. Thus, aggregate output of domestic final goods rises by 0.93% in period 0, less than the 0.99% in the case of $\theta = 0$. According to equation (4.53), the responses of the aggregate foreign borrowing has a one-period lead to those of the land price.

As in the case of $\theta = 0$, the responses of entrepreneurial net worth has a one-period lag to those of the land stock of entrepreneurs from period 1 on. Thus, the period-1 entrepreneurial net worth is above its steady state value by 1.27%. The return on household deposits improve household wealth in period 1. As both households and entrepreneurs increase their demand for land, the land price is above its steady state value by 2.07%. Given that the entrepreneurs' unit down payment is still above its steady state value by 0.18%, the entrepreneurs' land stock is above its steady state value by 1.08%. Meanwhile, households increase their period-1 consumption by 1.1%, less than the 1.19% in the case of $\theta = 0$. Note that the rise in the degree of collateralization enhances the responses of household consumption and labor in the shock period but weakens their responses in the following periods.

Let us look at how the degree of collateralization can affect macroeconomic volatility to TFP shocks via foreign borrowing in model MH. See figure 4.8. Due to the leakage of capital gains (losses) to the foreign investors, the entrepreneurs' demand for land and loans responds less to TFP shocks as θ rises from 0 to 1. It has three effects. First, the land price becomes less volatile and so does the aggregate foreign borrowing; second, aggregate output of domestic intermediate goods responds less to TFP shocks; third, the domestic interest rate also responds less to TFP shocks in the shock period. Thus, household consumption responds more in the shock period but less in the following periods. The overall effects of θ on the volatility of household consumption can be non-monotonic. Similarly, due to the consumption-leisure substitution, household labor supply responds more strongly to TFP shocks in the shock period and less in the following periods. The volatility of household labor supply is also non-monotonic in the degree of collateralization. As a result, aggregate output of domestic final goods becomes less volatile and so does the wage rate.

4.3.6.2   Impulse Responses to Transitory ToT Shocks

As specified in equation (4.27), the terms of trade, $\frac{1}{s_t}$, is an AR(1) in logarithms. Figure 4.10 shows the impulse responses of model RBC in the case of $\theta = 0.5$ (solid line) and in the case of $\theta = 0$ (dashed line) to a transitory ToT shock. For simplicity, we set the foreign interest rate and total factor productivity constant at $r_t^* = 1.01$ and $A_t = 1$. The impulse responses of model RBC in the case of $\theta = 0$ have been discussed in subsection 4.2.5.2.

Consider model RBC in the case of $\theta = 0.5$. As in the case of $\theta = 0$, a 1% negative ToT shock leads to the fall in the price of domestic final goods by 0.5% and the rise in the price of foreign final goods by 0.5%. On the one hand, the wage rate and the price of domestic intermediate goods fall by 0.50% and 0.51%, respectively; on the other hand, the effective foreign interest rate falls by 0.12%, according to equation (4.38), which is absent in the case of $\theta = 0$. Thus, entrepreneurs can get cheaper foreign funds. Despite of the fall in the entrepreneurs' land-backed foreign borrowing by 0.4%, the effective foreign borrowing in terms of the domestic consumption composite, $p_0 s_0 Z_0^*$, actually rises by 0.1%. Meanwhile, the domestic lending falls by 0.19% and the required investment of domestic final goods in the entrepreneurs' project, $ap_t K$, falls by 0.5%. In all, the land price, $q_t = \frac{z_0^{e,m} + p_0 s_0 Z_0^*}{K} - ap_0$, falls only by 0.01%, much less than the 0.78% in the case of $\theta = 0$. Intuitively, in the case of a negative ToT shock, the foreign investors not only share the capital losses but also provide cheaper funds in terms of the domestic consumption composite. The
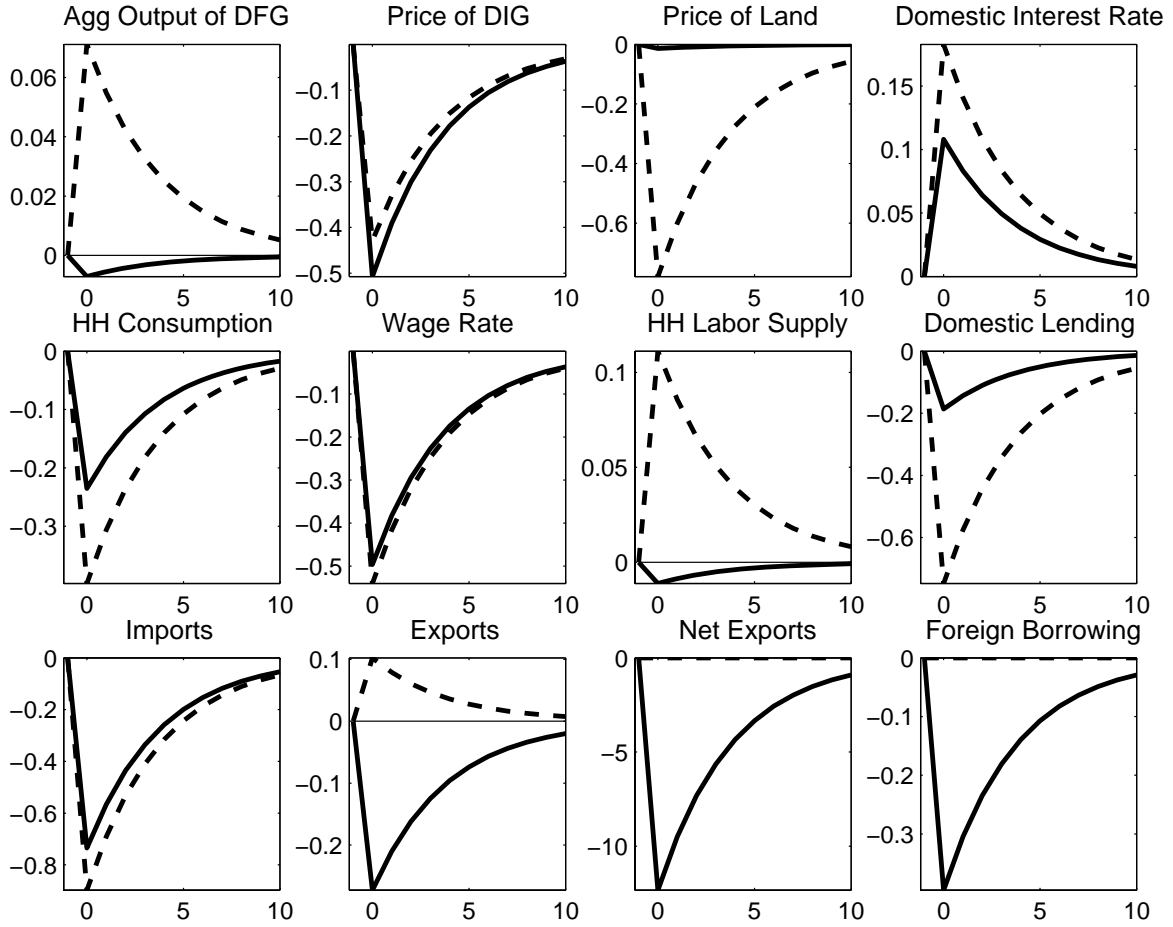
Figure 4.10: Impulse Responses to a ToT shock: Model *RBC*

two factors weaken the fall in the land price.

Household wealth consists of their deposit returns and the wage income. Due to the fall in the prices of land and domestic intermediate goods, the period-0 return on household deposits is below its expected value. Thus, households reduce their consumption by 0.24% and thus, the period-0 imports, $I_0 = \frac{(1-\gamma)c_0^h}{p_0 s_0}$, falls by 0.74%. Due to the consumption-leisure substitution, household labor supply falls by 0.01% and thus, aggregate output of domestic final goods falls by 0.007%. Although the domestic value of the ex post repayment of foreign liabilities, $\theta q_0 K$, falls only by 0.01%, its value in terms of foreign final goods falls by 0.51%. Thus, according to the balance of payment specified in equations (C.19) and (C.17), trade surplus falls by 12.3% and exports falls by 0.27%. In all, changes in the effective foreign interest rate and the leakage of the capital losses to the foreign investors partially offset the

effects of negative ToT shocks.

Figure 4.11 shows the unconditional standard deviations of some endogenous variables in model MH (solid line) and in model RBC (dashed line) normalized by the standard deviation of ToT shocks. The horizontal axis denotes $\theta \in [0, 1]$.
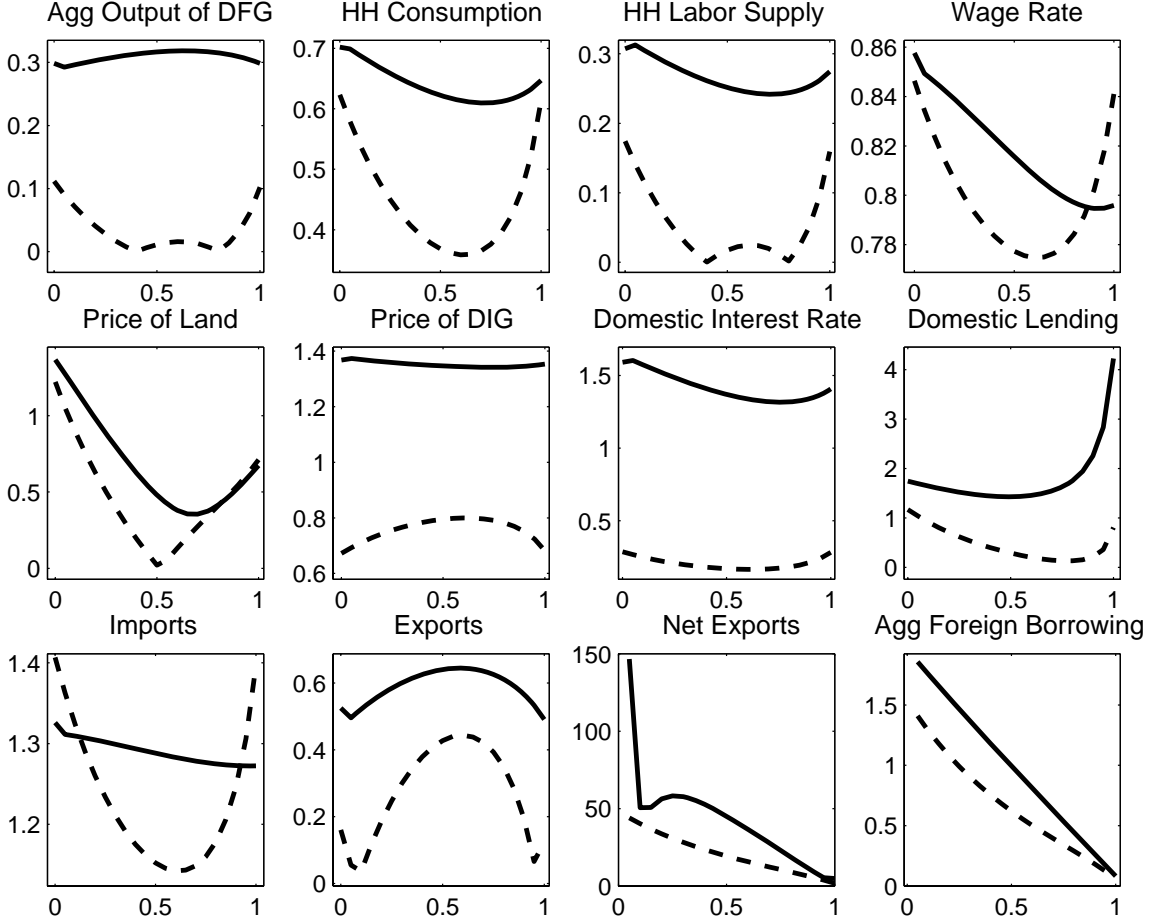
Figure 4.11: Foreign Borrowing and Macroeconomic Volatility: ToT shocks

Let us look at how the degree of collateralization can affect macroeconomic volatility to ToT shocks via foreign borrowing in model RBC. As the degree of collateralization rises from 0 to 1, entrepreneurs finance their project investment using more and more cheap foreign funds. Meanwhile, foreign investors bear a larger faction of capital gains or losses. As the effects of ToT shocks are partially offset by changes in the effective foreign interest rate, household consumption, labor, aggregate output, and the land price become less volatile as θ rises from 0 to 0.4. As θ rises from 0.4 to 0.6, the volatility of household consumption becomes further

smaller. Due to the substitution between consumption and leisure, household labor supply responds in the same direction as ToT shocks and it becomes more volatile.

Note that as θ exceeds 0.55, entrepreneurs have more foreign borrowing than domestic borrowing, $p_t s_t Z_t^* > z_t^{e,m}$. Thus, the overall cost of external funds become smaller in the case of negative ToT shocks. Thus, entrepreneurs increase their project investment and the land price responds in the opposite direction to that in the case of $\theta \in [0, 0.55)$. Therefore, the land price becomes more volatile as θ rises from 0.55 to 1.

As θ rises from 0.6 to 1, more land is pledged to foreign investors. The mutual funds benefit less from capital gains in the case of negative ToT shocks. Thus, household wealth falls more strongly in the case of negative ToT shocks. As a result, households reduce consumption in the shock period more strongly. As long as $\theta \in (0.6, 0.8)$, the consumption-leisure substitution effect dominates and households still reduce their labor supply in the shock period in the case of negative ToT shocks. As θ rises from 0.8 to 1, the wealth effect dominates and households increase their labor supply more and more so as to partially offset the fall in their wealth. Thus, household labor supply becomes more volatile in θ.

Figure 4.12 shows the impulse responses of model MH in the case of $\theta = 0.5$ (solid line) and in the case of $\theta = 0$ (dashed line) to a transitory ToT shock, given $\theta = 0.5$. For simplicity, we set the foreign interest rate and total factor productivity constant at $r_t^* = 1.01$ and $A_t = 1$. The impulse responses of model MH in the case of $\theta = 0$ have been discussed in subsection 4.2.5.2.

Consider model MH in the case of $\theta = 0.5$. A 1% negative ToT shock reduces the price of domestic final goods and raises the price of foreign final goods by 0.5% in period 0. Thus, the wage rate and the price of domestic intermediate goods fall by 0.52% and 0.47%, respectively. The fall in the sales revenues reduces entrepreneurial net worth. Due to debt deflation mentioned in subsection 4.2.5.2, entrepreneurs have to reduce their project investment. The decline in their land demand results in the fall in the land price. As foreign investors share the capital losses with entrepreneurs, entrepreneurial net worth falls in a smaller magnitude than in the case of $\theta = 0$ and so does the land price. In all, entrepreneurial net worth falls only by 0.57%. Due to the decline in the effective foreign interest rate, entrepreneurs can get cheaper foreign funds than in the steady state. Given that the entrepreneurs' unit down payment falls by 0.33%, the land stock of entrepreneurs falls only by 0.24% in period 0, less than the 0.37% in the case of $\theta = 0$. Accordingly, their demand for domestic lending falls only by 0.72% and then the domestic interest rate falls by 0.08%, both are smaller than in the case of $\theta = 0$.

As foreign investors bear all capital losses in the land stock of households, house-
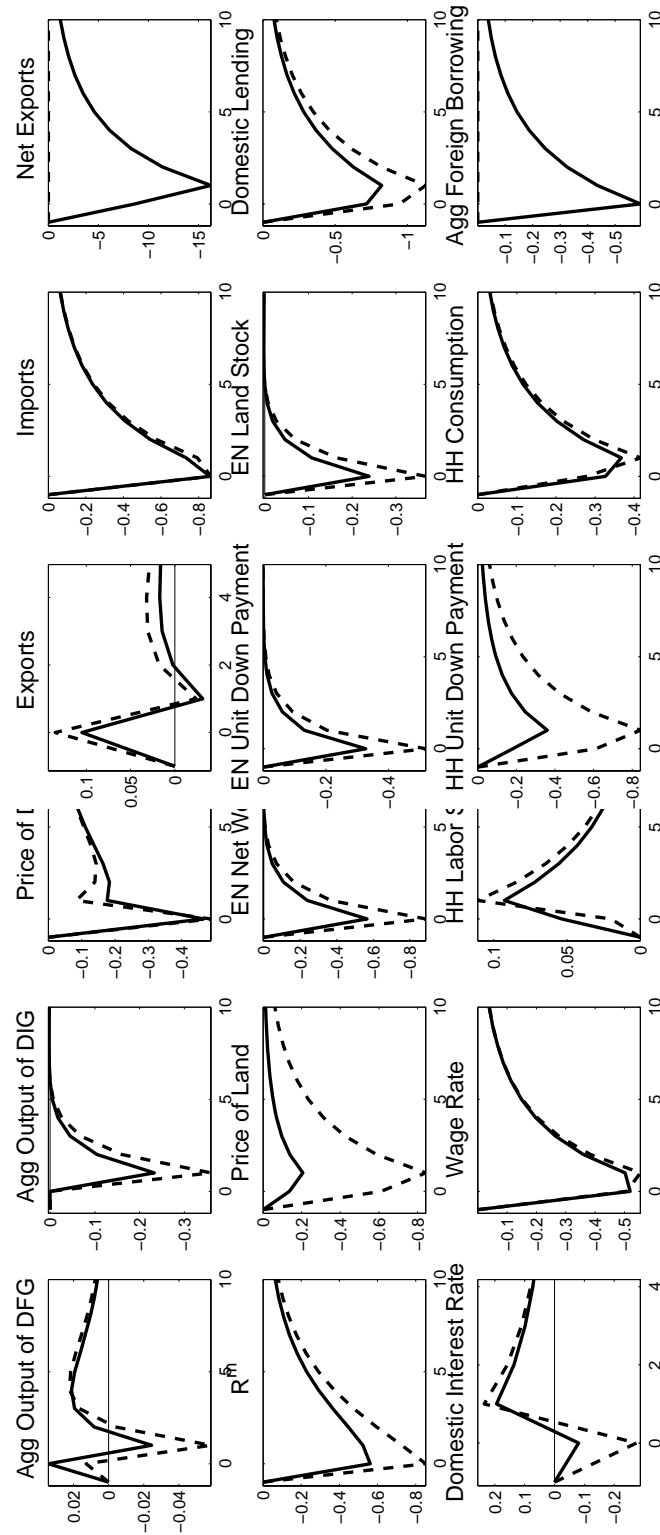
Figure 4.12: Impulse Responses to a ToT shock: Model MH

hold wealth does not fall as much as in the case of $\theta = 0$. Meanwhile, households also benefit from the fall in the effective foreign interest rate in period 0 and thus they increase their land investment. As the domestic interest rate falls less than in the case of $\theta = 0$, households reduce their deposits also in a smaller magnitude. Meanwhile, they increase their labor supply and reduce consumption by 0.05% and 0.33%, larger than the 0.02% and 0.28% in the case of $\theta = 0$. As a result, aggregate output of domestic final goods rises by 0.034% in period 0.

As aggregate output of domestic intermediate goods is mainly determined by the project investment of entrepreneurs in the previous period, aggregate output of domestic intermediate goods falls by 0.23% in period 1, less than the 0.36% in the case of $\theta = 0$. As a result, aggregate output of domestic final goods is below its steady state value by 0.024% in period 1, less than the 0.058% in the case of $\theta = 0$.

Let us look at how the degree of collateralization can affect macroeconomic volatility to ToT shocks via foreign borrowing in model MH. See figure 4.11. The sharing of capital gains or losses by foreign investors and the effective foreign interest rate partially offset the effects of ToT shocks. As θ rises from 0 to 0.7, household labor supply responds to ToT shocks more strongly in the shock period but less strongly in the following periods. The overall volatility of household labor supply falls in θ and so does the volatility of household consumption.

As the rise in θ enables both households and entrepreneurs to borrow more abroad. On the one hand, the steady state value of the household net land holding, $(1 - \theta)E_{t-1}q_t k^h_{t-1}$, falls in θ; on the other hand, entrepreneurs borrow less from the mutual funds and thus household deposits fall in θ. Household net land holding and household deposits are mainly unaffected by ToT shocks. As θ exceeds 0.7, the weights of these two components in household wealth are so low that households increase their labor supply both in and after the shock periods to a larger magnitude in the case of negative ToT shocks in order to partially offset the fall in their wealth. As a result, household labor supply becomes more volatile.

As θ rises from 0 to 1, foreign investors share more capital gains or losses with entrepreneurs, the land stock of entrepreneurs becomes less volatile monotonically in θ and so does aggregate output of domestic intermediate goods. Thus, as θ rises from 0 to 0.65, the effect of households labor supply dominates so that aggregate output becomes more volatile in θ; as θ rises from 0.65 to 1, the effect of the entrepreneurs' land stock dominates so that aggregate output becomes less volatile in θ.

4.3.6.3   Impulse Responses to Transitory FIR Shocks

Figure 4.13 shows the impulse responses of model MH (solid line) and model RBC (dashed line) to a transitory FIR shock in the case of $\theta = 0.5$. For simplicity, we set the inverse of the terms of trade and total factor productivity constant at $s_t = 1$ and $A_t = 1$. The prices of domestic and foreign final goods are constant at $p_t = p_t s_t = 0.5$.

Consider first model RBC in the case of $\theta = 0.5$. A 1% positive FIR shock increases the foreign interest rate by 1% and foreign borrowing is more expensive than in the steady state. The actual foreign borrowing of entrepreneurs in terms of the domestic consumption composite, $p_0 s_0 Z_0^* = \frac{\theta E_0 q_1 K}{r_0^*}$, is less than its steady state value in period 0 and so is the entrepreneurs' demand for land. Thus, the land price falls by 1.34%. Given that the period-1 land price is below the steady state value by 0.61%, the entrepreneurs' foreign borrowing falls 1.61%. Although the capital losses are shared by entrepreneurs and foreign investors, the period-0 return on household deposits is less than its expected value. Households increase their labor supply by 0.28% to partially offset the decline in their wealth. Thus, aggregate output of domestic final goods rises by 0.18% and so does the price of domestic intermediate goods. The wage rate falls by 0.1% in period 0. Note that extra sales revenues of entrepreneurs partially offset the fall in the ex post return on household deposits. Meanwhile, due to the wealth effect, households reduce their consumption and deposits by 0.38% and 0.9%, respectively. The fall in the supply of household deposits pushes up the domestic interest rate by 0.41%.

Figure 4.14 shows the unconditional standard deviations of some endogenous variables in model MH (solid line) and in model RBC (dashed line) normalized by the standard deviation of FIR shocks. The horizontal axis denotes $\theta \in [0, 1]$.

Let us look at how the degree of collateralization can affect macroeconomic volatility to FIR shocks via foreign borrowing in model RBC. As θ rises from 0 to 1, entrepreneurs finance their project investment more by foreign funds and changes in the foreign interest rate have larger effects on the entrepreneurs' demand for land. Thus, the land price responds more strongly to FIR shocks and so does the land-backed foreign borrowing. As long as θ is below 0.55, the deposit return accounts for a significant share of household wealth. The resulting capital gains or losses then have larger effect on household deposits and households adjust their labor supply to an larger extent to partially offset changes. When θ is above 0.55, the deposits account for only a smaller fraction of household wealth. Therefore, the household labor supply responds to FIR shocks to an smaller extent. The volatilities of aggregate output, imports, exports, wage, and consumption have the
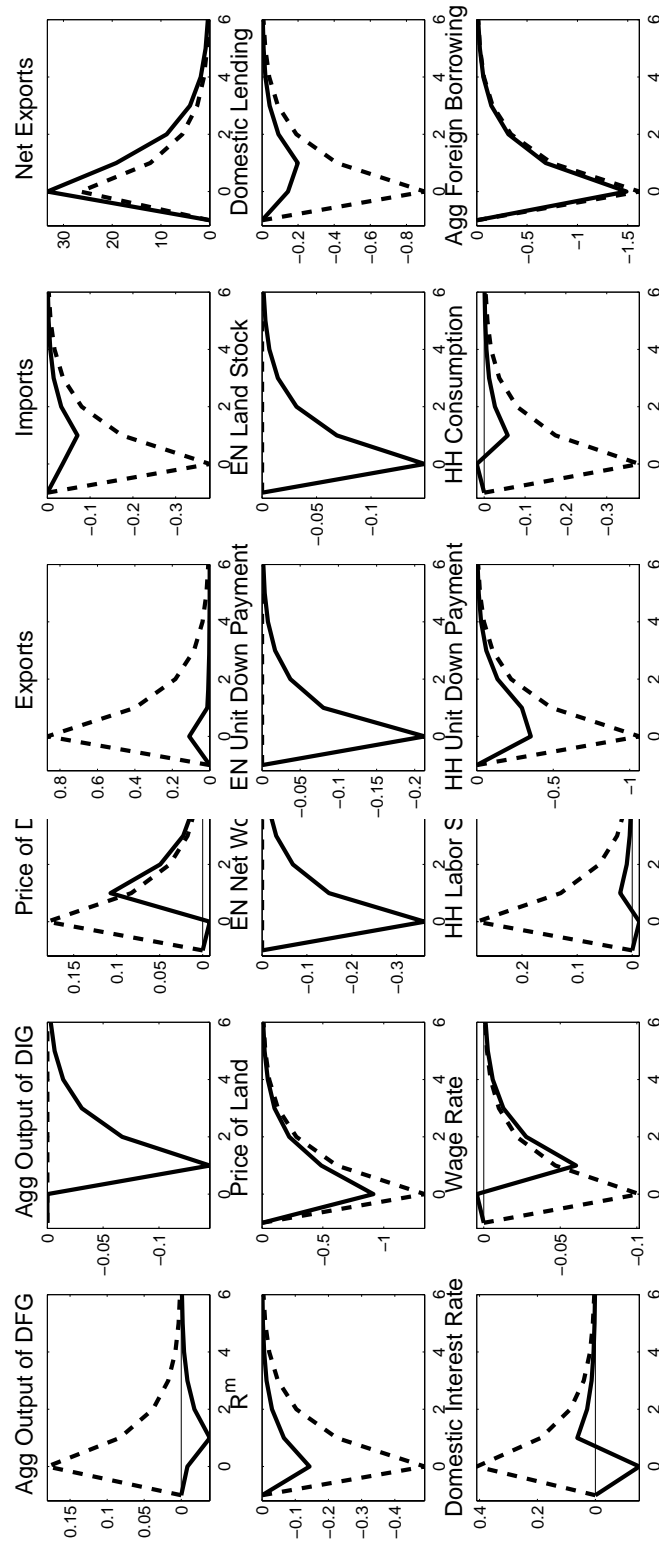
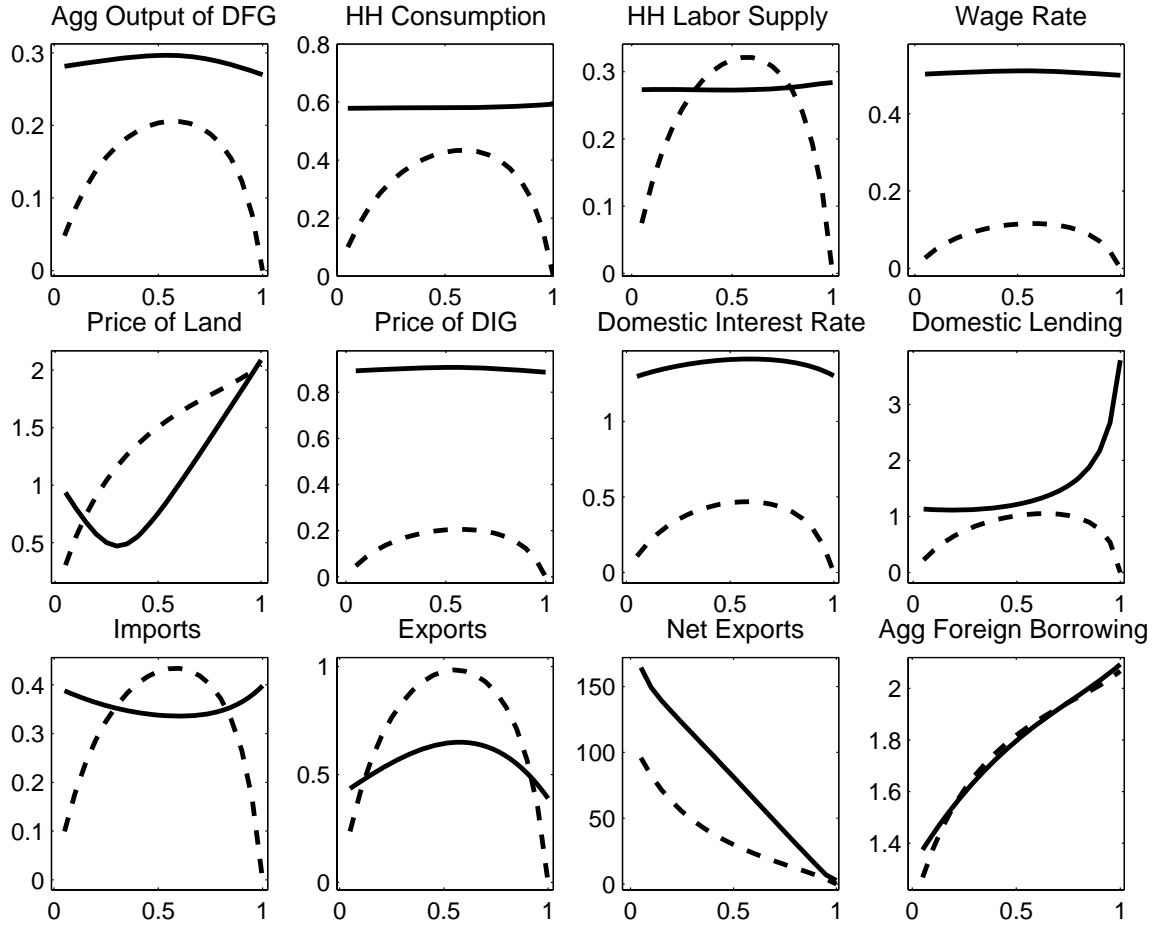Figure 4.13: Impulse Responses to a FIR Shock: Model *MH* vs Model *RBC*

Figure 4.14: Foreign Borrowing and Macroeconomic Volatility: FIR shocks

similar hump-shaped patterns with respect to the degree of collateralization.

Consider the impulse responses of model MH in the case of $\theta = 0.5$. See figure 4.13. A 1% positive FIR shock makes foreign funds more expensive for households and entrepreneurs. The decline in the land demand reduces the land price. Although foreign investors share the capital losses with entrepreneurs, entrepreneurial net worth still falls by 0.36%. Given that entrepreneurs' unit down payment falls by 0.21%, the entrepreneurs' land stock falls by 0.15%. The fall in the entrepreneurs' demand for domestic loans reduces the domestic interest rate by 0.15%, in contrast to the rise in the domestic interest rate by 0.41% in model RBC.

According to equation (4.52), household wealth has four components, the value of their net land stock, their sales revenues, their deposit returns, and their wage income. The first three components are mainly unaffected by the FIR shock. Given

that the household unit down payment falls by 0.35%, households increase their land stock. Due to the fall in the domestic interest rate, households reduce their deposits by 0.14% in period 0. Altogether, households increase their consumption by 0.018% and reduce their labor supply by 0.013%. Thus, aggregate output of domestic final goods falls by 0.008% and so does the price of domestic intermediate goods, given the predetermined aggregate output of domestic intermediate goods. As the consumption of households is five times more than that of entrepreneurs, the increase in household consumption by 0.018% and the fall in entrepreneur consumption by 0.36% jointly result in the fall in imports by 0.036%.

Due to the fall in the entrepreneurs' land stock in period 0, the period-1 aggregate output of domestic intermediate goods falls by 0.15%. As the domestic interest rate is above its steady state value by 0.062% in period 1, households reduce their consumption by 0.058% and increase their labor supply by 0.022% so as to take advantage of it. As a result, aggregate output of domestic final goods is below its steady state value by 0.038% in period 1 and the price of domestic intermediate good is above its steady state value by 0.11%.

Let us look at how the degree of collateralization can affect macroeconomic volatility to FIR shocks via foreign borrowing in model MH. See figure 4.14. As θ rises from 0 to 1, entrepreneurs and households finance their project investment using more foreign funds. However, the value of the net land stock of entrepreneurs, $p^G(1-\theta)q_t k_{t-1}^e$ is affected by FIR shocks in a non-monotonic way as θ rises from 0 to 1 and so is entrepreneurial net worth, $p^G[Rv_t + (1-\theta)q_t - R_{t-1}^m]k_{t-1}^e$. As long as θ is below 0.55, changes in FIR have more and more effects on the project investment of entrepreneurs in the sense that the land stock of entrepreneurs falls more strongly to a rise in the foreign interest rate as θ rises. However, if θ exceeds 0.55, capital gains or losses are born more by foreign investors and thus changes in the land price related to FIR shocks have smaller and smaller effects on entrepreneurial net worth and their land stock. Aggregate output of domestic intermediate and final goods has a similar volatility pattern with respect to FIR shocks.

4.4 Final Remarks

This chapter analyzes the macroeconomic implications of foreign borrowing in a small, open, real economy. Due to debt enforcement problem, foreign borrowing must be collateralized by domestic assets and the degree of collateralization may differ in countries with different legal system or liquidation costs. Foreign investors are better protected in countries with efficient legal systems and market structures.

Thus, a larger fraction of asset value can be pledged for foreign funds in these countries. In addition, domestic agents with high productivity can pledge some of their project outcomes for domestic borrowing from other domestic agents. In this sense, the moral hazard problem at the root of domestic financial constraints is less severe than that at the root of foreign financial constraints.

Better protection of foreign investors may have uneven welfare implications for domestic agents with different production technologies. Given that the foreign interest rate is below the domestic interest rate, domestic agents can borrow more cheap foreign funds in countries with better protection of foreign investors. As domestic agents with high productivity can borrow additional funds from other domestic agents, they invest more productive assets into their projects and thus, domestic production becomes more efficient. Domestic agents with high productivity benefit strictly, while, due to the substitution of foreign lending for domestic lending, domestic agents with low productivity lose in the long run from better protection of foreign investors. Thus, aggregate output might not be a good indicator for social welfare in models with heterogenous agents.

Better protection of foreign investors may have ambiguous implications for macroeconomic volatility. We look at three types of exogenous shocks. The standard deviations of major macroeconomic aggregates with respect to each type of shocks are non-monotonic in the degree of foreign investor protection, which are consistent with the empirical evidence.

Some of our assumptions deserve further attention. The fact that better protection of foreign investors have uneven long-run welfare implications for domestic households and entrepreneurs actually results from the leakage of the interest payment to foreign investors. In addition to our analysis in a small-open-economy framework, we may conduct similar analysis in a closed-economy model. The welfare implications of better protection of domestic investors might be different from our conclusion here.

For simplicity, we assume in this chapter that the economy is small enough so that the terms of trade and the foreign interest rate are exogenously determined abroad. Meanwhile, the foreign interest rate is assumed to be lower than the domestic interest rate such that domestic agents prefer to borrow abroad. In addition, foreign investors are assumed to be risk neutral and they share capital gains or losses with domestic agents. We can endogenize the terms of trade and the foreign interest rate in a two-country general equilibrium framework in which the foreign lenders could be risk averse. Whether our results still hold remains the subject of future research.

Chapter 5

Concluding Remarks

5.1   Main Results

This dissertation contains a theoretical study on financial frictions and macroeconomic fluctuations in a closed economy and in a small open economy. Our main results can be summarized as follows.

First, we analyze how financial frictions and time-varying prices of capital can serve to amplify and propagate the shock effects on macroeconomic aggregates. Due to moral hazard in the production of intermediate goods, entrepreneurs are subject to credit constraints. The accumulation of entrepreneurial net worth and the reallocation of capital goods between agents with different production technologies explain the amplified and hump-shaped output responses to productivity shocks. We adopt the conventional approach of costly capital adjustment to model time-varying prices of capital. In the event of a positive productivity shock, due to adjustment costs, the supply of capital goods cannot adapt to the boom in the demand for capital goods and the price of capital goods rises to clear the market. In addition to extra revenues, the capital gains improve entrepreneurial net worth further and capital goods are reallocated from agents with low productivity to agents with high productivity more quickly. As a result, the output responses are more amplified but less delayed, in comparison with the case of costless capital adjustment. In this sense, the tradeoff between amplification and propagation in our model can be significantly affected if we model time-varying prices of capital in the approach of costly capital adjustment.

Second, we propose an alternative approach to the modeling of time-varying prices of capital, in contrast to the approach of costly capital adjustment commonly used in the literature. Besides financial frictions in the production of intermediate

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goods, we assume that financial frictions also exist in the production of capital goods. This assumption helps capture the empirical fact that the supply of durable capital goods is relatively price-inelastic in the short run but accommodates the boom in the demand for capital in the medium run. Thus, in a model with dual financial frictions, this assumption helps balance the tradeoff between amplification and propagation. The dynamic interactions between the price of capital and dual financial frictions constitute a robust mechanism through which aggregate output responds to productivity shocks in an amplified and hump-shaped fashion, in line with the empirical evidence.

Third, we addresses two questions concerning foreign borrowing in a small open economy: Who benefits from better protection of foreign investors in the long run? How can better protection of foreign investors change macroeconomic volatility? Private foreign borrowing depends on the efficiency of the domestic legal system, market structure, financial regulations, etc. Ex post better protection of foreign investors raises the ex ante willingness of foreign investors to lend to domestic agents. Given that the foreign interest rate is smaller than the domestic interest rate, domestic agents with high productivity borrow more from foreign investors and less from domestic lenders in countries with better protection of foreign investors. Domestic production is more efficient and the welfare of domestic borrowers is also higher. However, the welfare of domestic lenders is lower due to the substitution of foreign lending for domestic lending and the reallocation of productive assets from domestic lenders to domestic borrowers. In this sense, better protection of foreign investors may have uneven welfare implications for different agents. Meanwhile, better protection of foreign investors may have ambiguous implications for macroeconomic volatility. More specifically, the volatilities of major macroeconomic aggregates are non-monotonic in the degree of foreign investor protection. It helps explain why the empirical literature cannot find a significant relationship between financial openness and macroeconomic volatility.

5.2   Directions for Further Research

We can extend our future research into the following directions.

First, we may analyze how the interactions among different aspects of financial liberalization can affect production efficiency in the long run. Private foreign borrowing depends on the domestic financial regulation as well as the efficiency of domestic legal system and market structure. In many developing economies, public financial regulators, in consideration of financial security and stability, normally set

upper limits for the fraction of domestic physical assets (e.g., land) or financial assets (e.g., deposits) that is allowed to be pledged to foreign investors as collateral. Thus, financial liberalization can be modeled as the increase in these upper limits. Our preliminary investigation shows that productive assets are allocated to domestic agents with high productivity and thus domestic production becomes strictly more efficient if only land-backed foreign borrowing or only deposit-backed foreign borrowing is deregulated. However, if the deposit-backed foreign borrowing is already deregulated to a high degree, an increase in the degree of land-backed foreign borrowing may reduce production efficiency in the long run. While, if the land-backed foreign borrowing is already deregulated to a high degree, increasing the degree of deposit-backed foreign borrowing strictly improve production efficiency in the long run. In this sense, different aspects of financial liberalization may have countervailing effects. Thus, policy coordination and proper sequencing have profound implications for production efficiency.

Second, we may analyze how financial liberalization should be implemented in a small open economy. News on future productivity is immediately embedded in the current asset prices. If there are domestic financial frictions, changes in asset prices can lead to asset reallocation among heterogenous agents and the current aggregate production is affected, even if current technologies are actually unchanged. Thus, asset prices are more volatile in the economies with financial frictions than in the economies without financial frictions, due to the inherent two-way interactions between prices and quantities. Furthermore, asset prices can overshoot in the short run, similar as in Dornbusch (1976). Note that the internal mechanism has its root in financial frictions instead of price rigidity. Similarly, in the economy with domestic financial frictions, financial liberalization, e.g., the increase in the degree of asset-backed foreign borrowing, should be implemented gradually in order to avoid huge macroeconomic fluctuations and welfare loss. Furthermore, to let everyone know the whole path of future liberalization policy helps achieve a smooth transition.

Third, we may analyze how financial liberalization and asset prices can affect macroeconomic fluctuations in a two-country general equilibrium model. As the foreign interest rate and the terms of trade are endogenously determined in the two-country model, exogenous shocks to domestic and foreign total factor productivity may result in different model dynamics through the channels of the terms of trade and the foreign interest rate which are absent in the model of small open economy. Whether the results of chapter four still hold in the two-country framework deserves further research.

Fourth, we may analyze optimal monetary policy in the two-country general equilibrium model with financial frictions. As shown in Blanchard and Gali (2005),

the standard new Keynesian framework implies no trade-off between stabilizing inflation and stabilizing the welfare-relevant output gap. This feature results from the an inherent property of new Keynesian framework: the absence of nontrivial real imperfections. Blanchard and Gali (2005) introduce real wage rigidities and show that central banks indeed face a trade-off between stabilizing inflation and stabilizing output gap. In contrast, we analyze whether domestic and foreign financial frictions can help justify the active management of central banks over aggregate demand via counter-cyclical monetary policy. Meanwhile, the policy coordination between domestic and foreign central banks is an important issue to be analyzed in such a framework.

Appendix A

Financial Frictions and Macroeconomic Fluctuations

A.1 Heterogenous Entrepreneurs

Entrepreneurs are heterogenous in equilibrium and can be categorized into four groups: group F of mass $(1 - p^G)$ includes those with failed projects; group X of mass $p^G(1 - \tilde{\pi})$ includes those with successful projects but exiting from the economy; group N of mass $(1 - \pi)$ includes the newcomers; group V of mass $p^G\tilde{\pi}$ include those who have successful projects and can survive to the next period.[1] We analyze the economic behaviors of the entrepreneurs in each group.

The entrepreneurs in group F get zero pecuniary return from the failed projects and simply exit from the economy without consumption, $c_{f,t}^e = 0$.

After repaying the debts, the entrepreneurs in group X liquidate their capital stock and consume all their wealth, $c_{x,t}^e = [Rv_t + (1 - \delta')q_t - R_{t-1}^b]k_{x,t}^e$.

The entrepreneurs in group N is born with a tiny endowment e. They maximizes their expected utility (2.4), subject to their period budget constraints and credit constraints (equations 2.6 and 2.7). They invest their endowment in project "Good" $n_{n,t}^e = e$ and borrows to the limit. They do not consume, $c_{n,t}^e = 0$.

After repaying the debts, the entrepreneurs in group V maximizes their expected utility (2.4), subject to their period budget constraints and credit constraints (equations 2.6 and 2.7). They invest all own funds in the project, $n_{v,t}^e = [Rv_t + (1 - \delta')q_t - R_{t-1}^b]k_{v,t-1}^e$, and borrows to the limit. They do not consume, $c_{v,t}^e = 0$.

Let K_{t-1}^e denote the aggregate capital stock in the entrepreneurial sector in

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[1]The entrepreneurs in group V are heterogenous in net worth, so are those in group X.

period $t-1$. In the aggregate, the consumption, the net worth, the loans, and the capital holding of the entrepreneurial sector in period t are

$$
\begin{aligned}
C_t^e &= (1-\tilde{\pi})p^G[Rv_t + (1-\delta')q_t - R_{t-1}^b]K_{t-1}^e, \\
N_t &= \tilde{\pi}p^G[Rv_t + (1-\delta')q_t - R_{t-1}^b]K_{t-1}^e + (1-\pi)e, \\
Z_t &= \Gamma_t N_t, \\
K_t^e &= \frac{N_t + Z_t}{q_t} = \frac{1+\Gamma_t}{q_t}N_t.
\end{aligned}
$$

A.2 Decentralizing the Capital Goods Production

There are m homogeneous firms which produce capital goods and are owned by households. They behave identically in a symmetric equilibrium. They invest i_t units of final goods to produce y_t^k units of capital goods and the production function is, $y_t^k = i_t - m\frac{\phi}{2}\frac{i_t^2}{J_t}$, where $\phi \geq 0$ denotes the degree of costly adjustment. As $m \to \infty$, there is perfect competition in the capital goods production sector and each firm takes the price of capital as given.

If $\phi = 0$, final goods are one-to-one transformed into capital goods. The price of capital is constant at unity, $q_t = 1$. Firms earn zero profits, $\pi_t \equiv q_t y_t^k - i_t = 0$.

If $\phi > 0$, the capital goods production function includes the term of quadratic adjustment costs. Let $I_t \equiv mi_t$ and $Y_t^k \equiv my_t^k$ denote the aggregate investment expenditure and the aggregate newly-produced capital goods. The aggregate capital stock evolves as follows

$$
K_t - J_t = Y_t^k = I_t - \frac{\phi}{2}\frac{I_t^2}{J_t}.
$$

Taking the price of capital as given, each firm maximizes the profit with respect to its investment expenditure,

$$
\max_{\{i_t\}} \ \pi_t \equiv q_t y_t^k - i_t = q_t\left(i_t - m\frac{\phi}{2}\frac{i_t^2}{J_t}\right) - i_t.
$$

The optimization condition gives $\frac{1}{q_t} = 1 - m\phi\frac{i_t}{J_t}$, which justifies the equilibrium price of capital, $q_t = \frac{1}{1-\phi\frac{I_t}{J_t}}$.

The profit of each firm is, $\pi_t = q_t m\frac{\phi}{2}\frac{i_t^2}{J_t} \geq 0$. The aggregate profit of the capital goods production sector, $\Pi_t = m\pi_t = q_t\frac{\phi}{2}\frac{I_t^2}{J_t}$, is lump-sum transferred to households.

Given $\phi > 0$, a rise in the price of capital makes the production of capital goods more profitable so that firms increase their investment expenditure and more capital goods are produced. The aggregate supply curve of capital goods is

$$Y_t^k = \frac{J_t}{2\phi}\left(1 - \frac{1}{q_t^2}\right),$$

which has a positive slope, $\frac{dY_t^k}{dq_t} = \frac{J_t}{\phi q_t^3} > 0$. More costly the capital adjustment is, i.e., a larger ϕ, more strongly the price of capital responds to excess demand.

A.3 Market Equilibrium in the Frictionless Model

Given that the project choice of entrepreneurs are perfectly observable, the market equilibrium is the set of two state variables $\{K_t, A_t\}$ and nine control variables $\{c_t^h, l_t, M_t, Y_t, I_t, J_t, q_t, w_t, v_t\}$ satisfying equations (2.1), (2.14)-(2.19), (A.1)-(A.4),

$$q_t = \beta E_t \left(\frac{c_{t+1}^h}{c_t^h}\right)^{-\sigma} p^G[(1-\delta')q_{t+1} + Rv_{t+1}], \tag{A.1}$$

$$M_t = p^G R K_{t-1}, \tag{A.2}$$

$$c_t^h + I_t = Y_t, \tag{A.3}$$

$$J_t = p^G(1-\delta')K_{t-1}, \tag{A.4}$$

$$z_t = q_t k_t^e, \tag{A.5}$$

$$r_t = \frac{1}{\beta} E_t \left(\frac{c_{t+1}^h}{c_t^h}\right)^{\sigma}, \tag{A.6}$$

$$R_t^b = E_t[(1-\delta')q_{t+1} + Rv_{t+1}]. \tag{A.7}$$

The variables $\{z_t, r_t, R_t^b\}$ are inessential to the market equilibrium and can be determined separately by equations (A.5)-(A.7).

Appendix B

Dual Financial Frictions and Macroeconomic Fluctuations

B.1 Heterogenous Entrepreneurs

Entrepreneurs are heterogenous in equilibrium and can be categorized into four groups: group F of mass $(1 - p_e^G)$ includes those with failed projects; group X of mass $p_e^G(1 - \tilde{\pi})$ includes those with successful projects but exiting from the economy; group N of mass $(1 - \pi)$ includes the newcomers; group V of mass $p_e^G \tilde{\pi}$ include those who have successful projects and can survive to the next period.[1] We analyze the economic behaviors of the entrepreneurs in each group.

The entrepreneurs in group F get no pecuniary return from the failed projects and simply exit from the economy without consumption, $c_{f,t}^e = 0$.

After repaying the debts, the entrepreneurs in group X liquidate their capital stock and consume all their wealth, $c_{x,t}^e = [R_e v_t + (1 - \delta')q_t - R_{e,t-1}^b]k_{x,t}^e$.

The entrepreneurs in group N is born with a tiny endowment e. They maximizes their expected utility (3.4), subject to their period budget constraints and credit constraints (equations 3.6 and 3.7). They invest their endowment in project "Good" $n_{n,t}^e = e$ and borrows to the limit. They do not consume, $c_{n,t}^e = 0$.

After repaying the debts, the entrepreneurs in group V maximizes their expected utility (3.4), subject to their period budget constraints and credit constraints (equations 3.6 and 3.7). They invest all own funds in the project, $n_{v,t}^e = [R_e v_t + (1 - \delta')q_t - R_{e,t-1}^b]k_{v,t-1}^e$, and borrows to the limit. They do not consume, $c_{v,t}^e = 0$.

Let K_{t-1}^e denote the aggregate capital stock in the entrepreneurial sector in

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[1]The entrepreneurs in group V are heterogenous in net worth, so are those in group X.

period $t - 1$. In the aggregate, the consumption, the net worth, the loans, and the capital holding of the entrepreneurial sector in period t are

$$C_t^e = (1 - \tilde{\pi})p_e^G[R_e v_t + (1 - \delta')q_t - R_{e,t-1}^b]K_{t-1}^e,$$
$$N_t^e = \tilde{\pi}p_e^G[R_e v_t + (1 - \delta')q_t - R_{e,t-1}^b]K_{t-1}^e + (1 - \pi)e,$$
$$Z_t^e = \Gamma_t^e N_t^e,$$
$$K_t^e = \frac{N_t^e + Z_t^e}{q_t} = \frac{1 + \Gamma_t^e}{q_t}N_t^e.$$

B.2   Heterogenous Capital Good Producers

In equilibrium, producers of capital goods are heterogenous in their end-of-period wealth and can be categorized into two groups: group F of mass $(1 - p_c^G)$ include those with failed projects; group V of mass p_c^G include those with successful projects[2]. We analyze the economic behaviors of capital goods producers in each group.

Capital goods producers in group F get zero pecuniary return and have no consumption, $c_{f,t}^c = 0$. They have to wait until period $t + 1$ and supply labor to the production of final goods. They then invest their wage income in project "Good".

After repaying their liabilities, the capital goods producers in group V have net return $\mathcal{N}_{v,t}^c = (R_c q_t - R_{c,t}^b)i_{v,t}^c$. At the end of period t, they allocate their wealth between consumption and inter-period deposit, $c_{v,t}^c + d_{v,t}^c = \mathcal{N}_{v,t}^c$.

The aggregate net worth, N_t^c, deposit D_t^c, investment I_t^c, and consumption C_t^c of the capital goods production sector in period t are determined as follows,

$$N_t^c = r_{t-1}^d D_{t-1}^c + w_t^c,$$
$$D_t^c = Z_t^e - D_t^h,$$
$$I_t^c = (1 + \Gamma_t^c)(r_{t-1}^d D_{t-1}^c + w_t^c),$$
$$C_t^c = \xi_t^c N_t^c - D_t^c,$$

where Z_t^e and D_t^h denote the aggregate inter-period lending to the entrepreneurial sector and the aggregate inter-period deposits of the household sector.

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[2]The capital goods producers in group V are heterogenous among themselves.

Appendix C

Domestic and Foreign Borrowing in a Small Open Economy

C.1 Price Index of Composite Consumption

We now choose domestic final goods as the numeraire. The price of foreign final goods is s_t. Suppose, households have W_t units of domestic final goods and they consume $c_{D,t}^h$ and $c_{F,t}^h$ units of domestic and final consumption goods, respectively. Given the definition of household consumption (4.2), households maximize their composite consumption with respect to the budget constraints,

$$\max_{\{c_{D,t}^h, c_{F,t}^h\}} \quad (c_{D,t}^h)^\gamma (c_{F,t}^h)^{1-\gamma}, \quad \text{s.t.} \quad c_{D,t}^h + s_t c_{F,t}^h = W_t. \tag{C.1}$$

The solution is

$$c_{D,t}^h = \gamma W_t, \tag{C.2}$$

$$c_{F,t}^h = \frac{(1-\gamma)W_t}{s_t}, \tag{C.3}$$

$$c_t^h = (\gamma W_t)^\gamma \left[\frac{(1-\gamma)W_t}{s_t} \right]^{1-\gamma} = (\gamma)^\gamma \left(\frac{1-\gamma}{s_t} \right)^{1-\gamma} W_t. \tag{C.4}$$

Let P_t denote the price index of composite consumption in terms of domestic final goods, i.e., $P_t c_t^h = W_t$. It is positively related to the inverse of the terms of trade,

$$P_t = \left(\frac{1}{\gamma} \right)^\gamma \left(\frac{s_t}{1-\gamma} \right)^{1-\gamma}. \tag{C.5}$$

A rise in the terms of trade (a fall in s_t) means that foreign final goods become cheaper than before. Given that household wealth W_t is unchanged, households can

consume more foreign final goods and the same amount of domestic final goods. Thus, their composite consumption rises and the price index falls accordingly.

Domestic agents eventually care about their composite consumption. It is reasonable to choose the consumption composite as the numeraire. If so, the price of domestic final goods is simply the inverse of P_t, i.e., $p_t = \frac{1}{P_t}$. The choice of the numeraire does not affect agents' intratemporal decisions but can have significant effects on agents' intertemporal decisions, e.g., inter-period borrowing and lending (Zhang 2003).

C.2 Heterogenous Entrepreneurs

Entrepreneurs are heterogenous in equilibrium and can be categorized into four groups: group F of mass $(1 - p^G)$ includes those with failed projects; group X of mass $p^G(1 - \tilde{\pi})$ includes those with successful projects but exiting from the economy; group N of mass $(1 - \pi)$ includes the newcomers; group V of mass $p^G\tilde{\pi}$ include those who have successful projects and can survive to the next period.[1] We analyze the economic behaviors of the entrepreneurs in each group.

The entrepreneurs in group F get no pecuniary return from the failed projects and simply exit from the economy without consumption, $c_{f,t}^e = 0$.

After repaying the debts, the entrepreneurs in group X liquidate their land stock and consume all their wealth, $c_{x,t}^e = [Rv_t + (1 - \delta')q_t - R_{t-1}^b]k_{x,t}^e$.

The entrepreneurs in group N is born with a tiny endowment e. They maximizes their expected utility (4.10), subject to their period budget constraints and credit constraints (equations 4.13 and 4.14). They invest their endowment in project "Good" $n_{n,t}^e = e$ and borrows to the limit. They do not consume, $c_{n,t}^e = 0$.

After repaying the debts, the entrepreneurs in group V maximizes their expected utility (4.10), subject to their period budget constraints and credit constraints (equations 4.13 and 4.14). They invest all own funds in the project, $n_{v,t}^e = [Rv_t + (1 - \delta')q_t - R_{t-1}^b]k_{v,t-1}^e$, and borrows to the limit. They do not consume, $c_{v,t}^e = 0$.

Let K_{t-1}^e denote the aggregate land stock in the entrepreneurial sector in period $t - 1$. In the aggregate, the consumption, the net worth, the loans, and the land holding of the entrepreneurial sector in period t are

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[1]The entrepreneurs in group V are heterogenous in net worth, so are those in group X.

$$C_t^e = (1 - \tilde{\pi})p^G[Rv_t + (1 - \delta')q_t - R_{t-1}^b]K_{t-1}^e, \tag{C.6}$$

$$N_t = \tilde{\pi}p^G[Rv_t + (1 - \delta')q_t - R_{t-1}^b]K_{t-1}^e + (1 - \pi)e, \tag{C.7}$$

$$Z_t = \Gamma_t N_t, \tag{C.8}$$

$$K_t^e = \frac{N_t + Z_t}{q_t} = \frac{1 + \Gamma_t}{q_t}N_t. \tag{C.9}$$

C.3 Market Equilibrium in Model *RBC*

Consider the case in which the mutual funds can observe project choices of entrepreneurs and domestic agents can borrow foreign funds against their land stock. Given that the foreign interest rate is smaller than the domestic interest rate, entrepreneurs prefer to borrow abroad to the limit and pledge the rest of their project outcomes to the mutual funds to finance their project investment. Our calibration guarantees that the expected marginal rate of return on project "Good" of entrepreneurs exceeds that on the household project. Thus, all land is allocated to entrepreneurs and domestic final goods are produced by entrepreneurs only. Model *RBC* in section 4.2 is a special case of $\theta = 0$.

The market equilibrium is defined as the set of three exogenous state variables $\{A_t, s_t, r_t^*\}$ and thirteen control variables $\{r_t, c_t^h, z_t^{e,m}, l_t, w_t, Z_t^*, v_t, p_t, q_t, Y_t, I_t, X_t, NX_t\}$ satisfying equations (4.5)-(4.6), (4.8), (4.23), (4.27), (4.37), and (C.10)-(C.19),

$$r_t z_t^{e,m} = E_t[p^G Rv_{t+1} + (1 - \theta)q_{t+1}]K, \tag{C.10}$$

$$(q_t + ap_t)K = z_t^{e,m} + p_t s_t Z_t^*, \tag{C.11}$$

$$Y_t = A_t(p^G RK)^\alpha l_t^{1-\alpha}, \tag{C.12}$$

$$p^G RKv_t = \alpha p_t Y_t, \tag{C.13}$$

$$l_t w_t = (1 - \alpha)p_t Y_t, \tag{C.14}$$

$$p_t X_t = p_t(Y_t - aK) - \gamma c_t^h, \tag{C.15}$$

$$p_t s_t I_t = (1 - \gamma)c_t^h, \tag{C.16}$$

$$X_t = s_t(NX_t + I_t), \tag{C.17}$$

$$r_t^* Z_t^* E_t(p_{t+1}s_{t+1}) = \theta E_t q_{t+1}K, \tag{C.18}$$

$$p_t s_t(NX_t + Z_t^*) = \theta q_t K. \tag{C.19}$$

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