Essays on Organization and Incentives in R&D and on Compatibility in Two-Sided Markets

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to my parents and grandma

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Contents

Preface

This dissertation covers two distinct topics. In Chapter 1 and Chapter 2 we investigate a model with a principal and two agents, where the hidden action of agents is a source of moral hazard problem. We show how competition between agents can be used to improve their incentives even if agents' production technologies are independent. In the first chapter we show that the principal is often better off financing innovation race between competing agents, rather than only one (even the most advanced) of them. In the second chapter we investigate advantages of competition as compared to team production, which is technologically more efficient. Chapter 3 deals with different topic: we study two-sided markets and develop a theory of compatibility between subsequent generations of technology.

The economic literature has since a long time realized that competition improves incentives of individuals in various settings ranging from yardstick competition (Shleifer 1985) to design of team incentives (Holmstrom 1982) to design of managerial incentives (Schmidt 1997). There are three basic channels through which the positive effect of competition is realized. First, competition on the product market induces an agent (a manager) to improve efficiency of his firm — otherwise he faces a threat of liquidation (Schmidt 1997). Second, competition provides a principal with information about agents' efforts in the situation where their production technologies are correlated, even if the effort itself is unobservable. This information can be used to design an incentive scheme that is based on relative performance of the agents. It is well known that in a static setting such relative performance evaluation dominates incentive schemes based on absolute performance (Holmstrom 1982, Lazear and Rosen 1981, Green and Stokey 1983). Third, competition has a disciplining effect on agents in the situation where only one of them can succeed (for example, in case of patent race). Even if the agents' production technologies are not correlated, competition improves incentives because each agent faces a risk that, while he shirks, his rival wins the prize. This effect of competition has not received much attention in the literature. One exception is Levitt (1995), who recognizes the value of competition even in the absence of common shock. The framework of his model, however, does not allow him to draw any concrete conclusions in this case.

The first and the second chapter of this dissertation contribute to the literature on incentives with many agents by investigating the benefits of competition in the framework where agents' production technologies are not correlated. Both chapters are motivated by an observation that it is not unusual for the principals investing in R&D (venture capital firms, grant agencies, government agencies, etc.) to finance an innovation race between competing agents (entrepreneurs). Two questions arise from this observation. First is how far the advantage of competition goes in a dynamic set-up where one of the competing agents wins a leading position, while the other remains a follower. In particular, should the principal chose the most advanced agent and abandon the financing of the follower, or can he improve his own profit by financing the innovation race? The second question is how far the advantage of competition goes when compared with team production which is technologically more efficient. Should the principal employ competing agents to perform a project or should he rather prefer a team, which is subject to synergy effects? Chapter 1 addresses the first question whereas Chapter 2 addresses the second.

In the first chapter we investigate a dynamic model of R&D where financing decisions (made by a principal or a venture capitalist) and allocation decisions (made by agents or entrepreneurs) are separated. This creates a moral hazard problem, because the agents can divert part of funds, provided by the venture capitalist, for own uses. It is shown that the venture capitalist can mitigate this moral hazard problem and hence improve his own profit by financing an innovation race between entrepreneurs.

We consider a model with two agents who work simultaneously on a project, which, if successful, generates a fixed prize. The project is developed in stages that are observable and verifiable outcomes of the R&D, such as results of test, patent, etc. The first agent who completes the second stage wins the prize. We ask, whether the venture capitalist should finance both agents or whether he should choose one of them and abandon the financing of the other. To this end we investigate two scenarios: a scenario where the agents are on the same (the last) stage of R&D and a scenario where one of them is the leader in the innovation race and the other is the follower.

We identify two effects which make the financing of competing entrepreneurs attractive to the venture capitalist. First effect is the higher probability of success (*scale effect*) and the second is the positive effect which competition has on incentives (*disciplining effect*). In order to highlight the importance of competition in the moral hazard setting, we compare it to the benchmark setting of no moral hazard. The analysis reveals, that in the scenario where both entrepreneurs are on the same stage of R&D both effects are important. Due to the scale effect financing of competing entrepreneurs is attractive in the absence of moral hazard. Under moral hazard, the disciplining effect reinforces the scale effect making the financing of competing entrepreneurs even more attractive.

However, in the scenario with a leader and a follower, the scale effect is of little importance, so that without moral hazard the follower will almost never be employed. Nevertheless, with moral hazard in place, the presence of a competitor allows to

reduce significantly the rent of the leader, which often makes competition an attractive arrangement. We also find that by improving incentives the competition allows the venture capitalist to increase the maximal research horizon during which he is willing to finance the project, making it therefore closer to the first-best (infinite) horizon.
The key finding of this chapter is that competition can be used by the venture capitalist as an effective cure against the moral hazard, in a situation where the allocation of funds by the entrepreneurs is not observable. Hence, competition serves as a "natural" mechanism that allows to improve the efficiency of research and development. The analysis of this mechanism contributes to the literature on venture capital, which up to know considered mainly contractual arrangements, based on complicated security schemes, as a mean of reducing the moral hazard problem (Sahlman 1990, Schmidt 2003). In many environments, however, the use of such securities is complicated or not possible at all. In such cases the existence of a "natural" mechanism is particulary important. Our prediction therefore is that venture capital firms, operating on developing markets (that have not yet accommodated complicated securities schemes) and grant agencies (that do not use such schemes following established tradition or due to the lack of expertise) should be inclined to finance an innovation race between the portfolio projects.
The second chapter goes a step further and asks to which extent the positive effect which competition has on incentives dominates the production gains generated by a team. Similarly to the previous chapter we investigate a model with the principal and two agents, where the principal finances an R&D project, which, if successful generates a fixed prize. The unobservable effort of agents (their investments) is a source of moral hazard in this model: the agents can divert part of funds for their own uses.
It is assumed, that the agents are symmetric and each of them is capable to perform the project himself. The principal can choose between several organizational designs. First, he can employ only one agent. Second, he can employ two agents cooperating in a team. Finally, he can employ two competing agents. As is shown in the first chapter the principal is always better off employing competing agents, rather than a single agent, if the agents are symmetric. The second chapter is focused on comparing the team production with competing agents.
As we know from the first chapter, competition has important positive effect on agents' incentives (*disciplining effect*). Team production, on the other hand, exhibits synergy effects (i.e. technological benefits resulting from specialization or from complementarity between agents' skills) and is therefore the most efficient arrangement from the technological point of view. We characterize the threshold value of synergy ef-

fects such that above this value the principal prefers to finance a team, rather than competing agents. This threshold increases with the value of the project: for more lucrative projects positive effect of competition tends to dominate productivity gains, generated by a team.

The intuition for this result stems from the fact that the principal has to balance a reward which agents receive in case of success, and the amount of investment funds allocated in the project. The larger is the reward and the smaller is the amount of investment funds in their discretion, the less tempted are the agents to consume the part of funds. However, the free-riding hazard in team weakens the incentives. The presence of synergy effect only accelerates this problem by enabling agents to achieve high success probability by investing small amounts. Hence, as the prize in stake increases, the principal is forced to limit resources allocated to the team more severely, than resources allocated to competing agents (while paying proportionally higher reward in the former case). Therefore, if the prize is sufficiently large, competing agents eventually perform better than a team.

In the second part of this chapter we show that the principal can improve incentives in the team by enforcing sequential production or (if he is unable to do it himself) by relying on the team leader to do so. There are two obstacles, however. First, it is shown in line with existing literature (Gould and Winter 2005, Ludwig 2007) that sequential production improves incentives only if the efforts of agents are strategic complements. Second, if the principal relies on the team leader to enforce sequential production, the latter is reluctant to employ a subordinate and tends to perform large part of the project himself. This leads only to the minor use of synergy effects and results in a significant loss of efficiency.

The results of this chapter lead to a number of interesting predictions about organization of R&D (or, more generally, production) process. First, the results suggest that we should observe principals switching to financing competing multiple teams, rather than a consortium of teams, as the prize in stake increases. Second, in an environment where a team is organized as a hierarchy and a team leader has difficulties verifying the effort of his subordinates, we should observe the team leader executing significantly larger effort, than his team peers.

The third paper deals with different topic. Here we develop a formal theory of compatibility choice in two-sided markets.

Rochet and Tirole (forthcoming) define two-sided markets as markets, where one or several platforms enable interaction between two distinct group of agents and the volume of transaction is affected by a price structure. Two-sided markets, in other words, are the markets which are characterized by the presence of network externalities on both sides. At such market, utility of agents on one side of the market increases with the number of agents (size of network) on the other side of the market with whom they can interact.

On a market with network externalities, the compatibility of products (technologies, platforms) affects the size of relevant network and hence the incentives of agents to buy a particular product. It is, therefore, not surprising, that any decision of a firm operating at such market crucially depends on the fact whether its product is compatible with that of a rival or with the previous generations of the same product.

The issue of compatibility has been well investigated in the literature on *simple* (i.e. there is only one group of agents) network externalities (Katz and Shapiro 1985, Katz and Shapiro 1986, Farrell and Saloner 1986, Choi 1994). This literature, however, did not pay much attention to the fact that many markets which exhibit network externalities, are two-sided markets.

The third chapter of this dissertation therefore contributes to the literature on network externalities by analyzing the compatibility choice at two-sided market. We provide a classification of the compatibility regimes which one can observe on two-sided markets and develop a theory which explains how the choice of a particular regime depends on the characteristics of the market (the size of the installed base and the market growth rate) and technological features of the new platform.

To develop a theory of compatibility choice we consider a framework with two platforms (old and new), owned by a single firm, the monopolist. The platforms enable interaction between two groups of agents, users and sellers. Some of users and sellers belong to the installed base (they are subscribed to the old platform), while there is also a number of new potential users and sellers. The monopolist earns his profit by selling the new platform to the installed base of agents and to the new agents. He also can choose a compatibility regime between the platforms and the size of per-interaction benefits, which the agents, using different platforms, derive interacting with each other.

We investigate the monopolistic market due to several reasons. First, many two-sided markets are indeed close to monopolistic (PC operating systems with Microsoft, internet auctions with eBay, etc.) Second, we want to analyze incentives to achieve compatibility rather, than those which stem from competition.

Our first important result is that the monopolist will never choose partial compatibility. He will either make technologies incompatible or will make them compatible to the extent that agents, who interact using different platforms, can enjoy maximal possible network benefits. This result allows us to concentrate our analysis on four extreme compatibility regimes: full compatibility, incompatibility and backward compatibility for each side of the market.

We show that the tradeoff which is in the heart of monopolist's decision to make technologies compatible, is the tradeoff between demand of new agents on one side of the market and demand of the old agents on the other side of the market. In particular, if the monopolist introduces backward compatibility for, say, users, he encourages new users to buy the new platform but discourages the old sellers to do so (*direct effect*). The decrease in the demand of old sellers triggers the decrease in demand of old users and of the new users (*feedback effect*). The tradeoff between these effects determines which compatibility regime will be chosen in equilibrium.

Investigating different market structures (mature market, emerging market and asymmetric market) we characterize the choice of compatibility in terms of primitives of the model. In particular, we show, that the compatibility for users will be imposed if the proportion of new users is relatively small, installed base of sellers and users is relatively small and the technological progress is moderate. Our predictions about the pattern of compatibility choice are illustrated with two examples of two-sided markets.

Chapter 1

Financing of Competing Projects with Venture Capital

1.1 Introduction

The classical approach in the literature on patent races is to model firms run by their owner and to assume away any incentive problems within these firms.[1] This approach ignores an important fact that in many cases the financing and allocation decisions in R&D process are separated. This problem arises within firms (for example, if they subcontract R&D) but it is especially important for grant agencies and venture capital funds.

Venture capital funds are usually directed to projects of uncertain quality, where neither time nor financial recourses needed for successful completion of the project are known *ex ante*. As a rule, venture capitalists are actively involved in monitoring firms in their portfolio. Nevertheless, they can rarely control perfectly whether resources are allocated efficiently, since such control requires an expertise which often only an entrepreneur himself possesses. This creates a moral hazard problem: entrepreneurs tend to misallocate the funds provided by the venture capitalist. In particular, they may divert part of funds for their own uses, or may allocate them into activities, which have high personal return but create little market value (Gompers and Lerner 2004, p. 174).

The venture capital literature has extensively discussed contractual arrangements that can be used by in order to alleviate the moral hazard problem. These are, for example, convertible securities (Sahlman 1990, Kaplan and Stromberg 2003) and monitoring

[1] Loury 1979, Lee and Wilde 1980, Reinganum 1981, Grossman and Shapiro 1987, Choi 1991, Malueg and Tsutsui 1997, etc.

mechanisms (Gompers 1995). On one hand, these mechanisms are efficient in mitigating the agency conflict. On the other hand, they are costly, complicated, and in some circumstances they are not feasible at all (most obviously, if the capital markets are not sufficiently developed to allow the use of complicated securities schemes). This creates obstacles for efficient funding of R&D.

This paper, as opposed to the existing literature on venture capital, proposes a non-contractual mechanism, namely competition between portfolio firms, that can be used to mitigate the agency conflict. The main question that we address is whether a venture capitalist can use competition between portfolio entrepreneurs to improve their incentives and thus his own profit. To answer this question we investigate a patent race in a moral hazard setting, where financing decision (made by the venture capitalist) and the allocation decisions (made by two competing agents) are separated. Comparing the patent race with a basic set up where only one agent is employed, we identify two effects of competition. Obviously, competition allows to increase the probability of success, since two agents succeed (on average) more often than one (*scale effect*). But, more importantly, the fear that the competitor wins the patent race limits the option of the agents to divert funds for own uses (*disciplining effect*). While the scale effect is important when the agents are symmetric, we show that it plays negligible role when the agents are asymmetric, so that one of them is the leader in the innovation race and the other is the follower. Nevertheless, the venture capitalist will often employ the follower together with the leader, although in terms of success probability the contribution of the former is minor. His presence, however, disciplines the leader and limits the rent which the latter can extract from the principal.

The observation that venture capital firms, grant agencies and similar institutions may find it profitable to finance an innovation race is well supported by casual empirical evidence. The venture capital division of Vulcan Inc., a corporation owned by Microsoft co-founder Paul Allen, has contracted three competing agencies for the project Halo, aimed at the development of the problem-solving software.[2] National Archives and Records Administration of the USA awarded two contracts to the competing firms to develop an Electronic Record Archives, a revolutionary system of record keeping.[3] National Institutes of Health (USA) routinely finances competing research teams working on the same problem.[4]

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[2]See `www.projecthalo.com` for details

[3]For details see `www.diglib.org/preserve/ERA2004.htm`.

[4]Recent example involves two large scale competing studies which independently revealed a gene responsible for multiple sclerosis. Both studies were supported by a grant from NIH. For details see `www.ninds.nih.gov/ news and events/press releases/`.

Despite the numerous evidence that financing of competing projects plays an important role in the venture capital and the grant funding, the literature on venture capital has up to now paid little attention to this potent mechanism of mitigating the agency problem. The only exception is Levitt (1995), who analyzes the problem of a principal employing two agents. The principal's payoff depends on the best of agents' outputs. Unlike in our model, the author investigates a static situation that does not allow to address a competition between the leader and the follower. Moreover his results rely on the fact that production technologies of the agents are interdependent, and the model is not able to deliver predictions for the case of independent technologies. We address both issues in our paper.

Analyzing the innovation race between two competing entrepreneurs we consider a research process consisting of several sequential stages that are observable and verifiable outcomes of R&D, such as a patent, results of tests, etc. Both entrepreneurs are financed by a single venture capitalist, who incurs the research costs and rewards the entrepreneurs if they succeed. He also determines the time horizon during which the project will be financed. Within this structure we investigate the effect of competition in two scenarios: when the entrepreneurs are at the same stage or at different stages of research. The simplest situation that allows to analyze the first scenario is the innovation race between two identical entrepreneurs who need to finish one stage of R&D in order to complete a project. In this case, the scale effect is important and in the moral hazard setting is re-enforced by the disciplining effect. We conclude that with identical entrepreneurs, competition is unambiguously beneficial for the venture capitalist: he will always prefers to employ two entrepreneurs rather then one.

The simplest situation which allows to analyze the second scenario is the research process consisting of two sequential stages. In particular, we assume that the venture capitalist has in his portfolio a *leader*, who has successfully completed the first stage of R&D. The venture capitalist now faces an opportunity to finance a second entrepreneur, a *follower*, who is in the initial stage of R&D. Should the venture capitalist employ both agents, or should he proceed financing the leader alone? It turns out, that the scale effect is negligible. Therefore, in the absence of the moral hazard, the venture capitalist almost always prefers to employ the most advanced agent, the leader. In the presence of the moral hazard, however, the existence of the follower is important because it relaxes the incentive compatibility constraint of the leader. Due to this disciplining effect the venture capitalist will often employ both agents.

Our modelling approach is closely related to that of Bergemann and Hege (1998, 2002, 2005). They investigate the decision of a venture capitalist who finances a single entrepreneur under uncertainty about the quality of the project and investments needed for its successful realization. Bergemann and Hege (1998) analyze a model in which the quality of a project is not known and has to be resolved through a costly experiment. Their main result is that agency costs lead to inefficiently early stopping of the project. In their paper Bergemann and Hege (2005) extend these results and analyze the difference between relationship financing and arm-length financing. Finally, in the third model Bergemann and Hege (2002) investigate the value of staged financing. The authors show that the use of financing rounds (stages) allows to increase the funding horizon and to make it closer to the socially optimal horizon. We use the framework of Bergemann and Hege to study the patent race in the moral hazard setting.

Another paper which is related to ours is Schmidt (1997). The author studies the incentives of a manager, who operates on the competitive product market. He identifies a "threat-of-liquidation" effect of competition. As the product-market competition increases, the manager is induced to spend more effort, because otherwise the profits of his firm fall below the critical value and the firm will be liquidated. This effect is somewhat similar to ours disciplining effect. In Schmidt (1997), however, competitive environment is exogenously given. We, to the contrary, assume that the venture capitalist can determine the extent of the disciplining effect by employing two agents and specifying in contracts how long each of the competitors will stay in the game.

The structure of this paper is the following. We describe the set-up of the model in Section 1.2 and derive the sequentially optimal contract in Sections 1.3 and 1.4. We introduce strategic interaction among entrepreneurs in Section 1.5 and discuss the advantage of commitment to finite horizon in Section 1.6. Section 1.7 concludes. Proofs and results of numerical simulations can be found in Appendix 1.A.

1.2 Description of the model

1.2.1 Innovation process

There are two entrepreneurs with no wealth of their own. Both have an idea (a project) how to solve a particular problem. For example, they try to find a cure against a disease. Following Bergemann and Hege (2002), we assume that the project requires each entrepreneur to complete N sequential stages. These stages are observable and verifiable outcomes, such as a patent, first version of a product, results of markets tests, etc. The stages are sequential in the sense that in order to enter the k-th stage each entrepreneur has to complete successfully previous $(k-1)$ stages. Financing of the projects is done by venture capitalist who provides necessary funds. If all stages are completed, the project generates a prize R and the prize is to be divided between the venture capitalist and the winning entrepreneur. We assume that the winner has a monopoly over the outcome of the project (by patenting the invention), hence the second entrepreneur (the looser of innovation race) does not generate any value. Entrepreneurs and the venture capitalist are risk neutral individuals with common discount rate r.

In order to successfully complete the current stage, the entrepreneur needs to allocate an amount c (provided by the venture capitalist) into the project. In that case the R&D is stochastic and we model the innovation process as a Bernoulli trial, where the stage is completed in the current period with probability p. With probability $1-p$ the entrepreneur does not succeed and needs to invest further (conditional on the fact that his rival has not yet won the race). We assume that the probability of success p is the same for both entrepreneurs. Further, following Lee and Wilde (1980) and Reinganum (1981) we assume that probability of success in each period is independent across the entrepreneurs and across time. Funds are provided by the venture capitalist, but allocation decisions are made by entrepreneurs. They can either invest funds or divert them for private uses. The venture capitalist is not able to observe the allocation decision. All he can observe is a success (completion of the current stage) or an absence of success (which can either mean that an entrepreneur has invested money but failed, or that he has diverted it).[5]

1.2.2 Moral hazard

We assume that there is a competitive market for innovative projects and a limited supply of venture capital. The venture capitalist can choose any entrepreneur from the pool of identical entrepreneurs. Therefore, the venture capitalist possesses bargaining power, which also means that after paying an entrepreneur the incentive compatible compensation, he retains the residual payoff from the project.

The allocation of funds in this model is subject to a moral hazard: In each period the entrepreneurs face a choice between allocating the funds into R&D and consuming them. The venture capitalist, however, is willing to finance R&D only if he can ensure that funds are allocated truthfully in each period of time. That is, the venture capitalist needs to suggest such reward to both entrepreneurs, so that they prefer to

[5]The innovation process in this game can be interpreted as following. Each entrepreneur owns a coin (representing a project). He tosses the coin and counts a number of "heads" (successes) and "tails" (failures). The first entrepreneur who counts N heads wins the prize R. In order to make one toss, each entrepreneur has to pay a prescribed amount of money c. The venture capitalist provides money for both entrepreneurs in exchange for a share of the prize R.

allocate the funds to R&D, rather than to divert them. Moreover, since the allocation of funds is not verifiable, the incentive scheme should reward the entrepreneurs only if a stage was successfully completed.

There are several counteracting forces that determine the size of the incentive payments. On one hand, by consuming funds the entrepreneurs receive the immediate utility c in each period. This way they also ensure themselves further financing, i.e., potential rent of c in the next period. Therefore, in each period of time the venture capitalist should promise the entrepreneurs a reward which is at least as large as the present value of all investments c which the entrepreneurs can consume. On the other hand, by consuming the funds rather than investing them, each entrepreneur faces a risk that his rival wins the prize. This lowers the expected present value of his future consumption from diverting the funds and therefore limits the option of each entrepreneur to deviate and to consume the funds. Hence, competition might make it cheaper for the venture capitalist to meet the incentive compatibility constraints of the entrepreneurs.

We analyze two scenarios: a basic scenario with entrepreneurs on the same stage of R&D, and a variation with entrepreneurs on different stages of R&D. In the scenario with entrepreneurs on the same stage of R&D, we consider the simplest case, where each entrepreneur needs to complete only one stage in order to complete the whole project. In this scenario we analyze the decision of the venture capitalist whether to employ a single agent or both agents. In the scenario with entrepreneurs on different stages of R&D, we again consider the simplest case, where in order to complete the project the first entrepreneur needs to complete one stage (the *leader*) and the second entrepreneur needs to complete two stages (the *follower*). We are, in particular, interested in the question whether the venture capitalist should in addition to more advanced leader employ the less advanced follower. Bergemann and Hege (2002) analyze a model with a single entrepreneur and N stages. However, for more than one entrepreneur the analysis of the multistage game becomes extremely complicated. In spite of this limitation, our model enables us to illustrate the importance of competition between the entrepreneurs in venture capital financing.

1.2.3 Definitions and notations

We will call a *regime* (i/j) a situation, where one entrepreneur has i successes (he has successfully completed i stages) and the other entrepreneur has j successes. Both scenarios that we analyze can be then nested within the setup with $N = 2$ (i.e., the project consist of two stages) and $i, j \in \{0, 1\}$: scenario with entrepreneurs on the

same stage of R&D corresponds to regime (1/1), and scenario with entrepreneurs different stages of R&D corresponds to regime (1/0).
We will use the following notation:

- T^{ij} denotes the financing horizon in regime (i/j). It this time elapses, the venture capitalist may abandon one or both entrepreneurs.[6]
- V_t^{ij} denotes the value of the project at time t in regime (i/j).
- E_t^L and E_t^F are the expected value of the reward of the leader and the follower respectively at time t in regime (1/0). In regime (1/1) the value function is denoted E_t^{11}.
- s_t^L and s_t^F are the rewards, which the leader, respectively the follower, earn upon successful completion of the current stage at time t in regime (1/0). In regime (1/1) the reward is denoted as s_t^{11}.

Furthermore, we will call *regime* (i) a situation, in which the venture capitalist finances only one entrepreneur, who is on i-th stage of R&D. The corresponding value of the project, value function of an entrepreneur, and his reward are denoted as V_t^i, E_t^i, and s_t^i, respectively.
Sometimes, we also denote specific contracts (candidates for the optimal contract) as $\mathcal{C}_k$, where $k = 1, 2, \ldots$. We will then use index k to denote the corresponding financing horizon, value of the project, value function of an entrepreneur, and his reward.

1.3 Innovation race between identical entrepreneurs

We start with the first scenario, where the venture capitalists faces two identical entrepreneurs, each of which is capable to perform the project. The project consist of a single stage (or equivalently, both entrepreneurs are in the last stage) and the venture capitalist has to decide whether to employ both entrepreneurs, only one of them, or none. It is assumed that the agents observe whether their rival was employed. In order to give the entrepreneurs incentives to invest in each period of time the venture capitalist has to offer them an appropriate incentive compatible contract.[7] Since the allocation of funds is not observable, the reward, which agents receive according to

[6] In some cases, we omit the superscript specifying the regime.

[7] Any contract, promising positive expected reward is assumed to satisfy the participation constraint, as the entrepreneurs' outside option is normalized to 0.

the contract, has to be conditioned on observable outcomes. Those are the event and the time of success and the identity of the winner.
In regime (i/j) a contract specifies the following terms:

1. Financing horizon T^{ij},

2. Stopping rule that is used when time T^{ij} has elapsed, but no discovery has been made,

3. Rewards for the entrepreneurs s_t^{ij}, depending on the time t when success is achieved, where $0 \leq t \leq T^{ij}$.

Following Bergemann and Hege (2002) and Neher (1999), we assume that the venture capitalist can determine the maximal financing horizon and commit to it. If this horizon have been reached but no success was achieved, then, depending on the stopping rule, either the financing of one entrepreneur or of the whole project will be irrevocably terminated. We justify this assumption in Section 1.6.
In the situation with identical entrepreneurs we limit our attention to the set of contracts, which use one of the following stopping rules:

1. Stopping rule R_1: Finance both entrepreneurs until one of them wins or until the maximal financing horizon is reached. If neither entrepreneur succeeds, abandon the financing of both.

2. Stopping rule R_2: Finance both entrepreneurs until one of them wins or until the maximal financing horizon is reached. If neither entrepreneur succeeds, abandon one entrepreneur randomly and continue financing in regime (1).

3. Stopping rule R_3: Finance a single entrepreneur until he succeeds but no longer than for T^1 periods.

These rules represent a set of deterministic stopping rules, i.e. they use the observable outcomes to decide, which entrepreneur should be financed further and which should be terminated. The set of deterministic stopping rules is not generally limited to these three stopping rules. There is an additional class of rules within this set, where the principal employs one entrepreneur in period t and the other entrepreneur in period $\tau > t$. We eliminate the latter class of stopping rules on the assumption that an entrepreneur, who is not financed for at least one period, leaves the market (he either becomes an employee or receives financing from other sources, such as bank loan, grant, friends and family, etc.)

To analyze the model we look for sequentially optimal dynamic contract which maximizes the profit of the venture capitalist in each regime of the game. In fact, such contract can be viewed as a sequence of contracts, where the new contract is signed after a regime switches. We require therefore, that in the beginning of the regime (1/1) the venture capitalist *cannot* commit to a contract which will be suboptimal in the regime (1).
In order to find the universally optimal contract we first develop an optimal contract for each of the three stopping rules and then compare the contracts across the stopping rules.

1.3.1 Value of the venture

The venture capitalist's decision whether to finance one or two entrepreneurs and the choice of the maximal horizon of R&D depends on the expected profit obtained in each case. This profit is the difference between the expected value of the project and the expected compensation of the entrepreneurs.
As our model is formulated in finite time, we can recover the value of the project recursively. Consider first the stopping rule R_3, which corresponds to a case when the principal employs a single entrepreneur - this case is extensively discussed in Bergemann and Hege (2002). In period t the expected value of the project can be written as

$$V_t^1 = Rp + \frac{(1-p)}{1+r} V_{t+1}^1 - c. \tag{1.1}$$

This value consists of three terms. The last term, c, represents funds which the venture capitalist has to distribute to the entrepreneur. With probability p the entrepreneur makes a discovery in period t. With probability $(1-p)$ the entrepreneur fails, so that the value of the project in period t is the discounted value of the project in period $t+1$, that is $\frac{1}{1+r} V_{t+1}^1$.
The sequence of values is the given by the solution of the difference equation (1.1) together with terminal condition $V_{T+1}^1 = 0$. This condition states that there is no continuation after time T and the value of the project is therefore zero. Following Bergemann and Hege (2002) we consider transition to continuous time. The innovation process becomes a Poisson process and the above difference equation equation becomes a differential equation. Solving it, we obtain the following expression for

value function in period t:

$$V_t^1 = \frac{(Rp - c)}{r + p}\left(1 - e^{-(r+p)(T-t)}\right). \tag{1.2}$$

The expression for the value function consists of two factors. The first factor represents the expected payoff from the investment, discounted with a composite discount rate which combines time discount r and the uncertain arrival of success. The second factor shows how the value of the project decreases with time of discovery.

Using the same procedure it is straightforward to derive the value function V_t^{11} for the stopping rule R_1, which corresponds to a case when the principal employs both agents (see Appendix 1.A for details). The recursively determined value of the project in period t is

$$V_t^{11} = Rp(2 - p) + \frac{(1 - p)^2}{1 + r} V_{t+1}^{11} - 2c \tag{1.3}$$

This leads to the following value function:

$$V_t^{11} = \frac{2(Rp - c)}{r + 2p}(1 - e^{-(r+2p)(T-t)}).$$

Finally, consider the stopping rule R_2. It dictates that in the case when no entrepreneur succeeds before $t = T$, one of them has to be chosen randomly and financed further for additional number of periods. In this case the value V_t^{11} satisfies equation (1.4). The expected value of the venture in the terminal period of regime (1/1) is now $V_{T+1}^{11} = V_0^1$, where V_0^1 is determined from (1.2) for $t = 0$. Solution of the corresponding differential equation leads to the following value function:

$$V_t^{11} = \left(V_0^1 - \frac{2(Rp - c)}{r + 2p}\right) \cdot e^{-(r+2p)(T-t)} + \frac{2(Rp - c)}{r + 2p}. \tag{1.4}$$

The value functions are summarized in Table 1 in Appendix 1.B.

1.3.2 Incentives of the entrepreneurs

In each period of time entrepreneurs face a choice between diverting the funds provided by the venture capitalist for private needs, and investing them into the project. In order to motivate entrepreneurs to allocate funds into research and development, the venture capitalist has to promise them a reward which is at least as large as the stream of rent that an entrepreneur can receive diverting the funds.

With our simple model of the R&D process, each entrepreneur has two available strategies: he can either "work" (that is, allocate funds into the project) or "shirk" (that is, divert all funds for private uses). For the time being, we make the assumption that the entrepreneurs do not behave strategically, i.e. each of them believes that the other entrepreneur always "works", or allocates the funds into the project in each period of time. We discuss the strategic interaction in Section 1.5 and we show that it does not change the results, obtained under the assumption of non-strategic interaction.

In each period of time, the venture capitalist has to offer each entrepreneur such reward, that he finds it incentive compatible to invest in this period, rather than consume funds. For illustration consider the stopping rule R_1. According to this rule, financing of both entrepreneurs is terminated if no success occurred before time T elapses.

The intertemporal incentive compatibility constraint for period t:

$$E_t^{11} = p(1-p)s_t^{11} + \frac{1}{2}p^2 s_t^{11} + \frac{(1-p)^2}{(1+r)}E_{t+1}^{11} \geq c + \frac{1-p}{1+r}E_{t+1}^{11}\,, \tag{1.5}$$

with terminal condition $E_{T+1}^{11} = 0$. The left-hand side of (1.5) represents the expected utility of the entrepreneur, if he allocates the funds into the project at period t. If the entrepreneur wins while his rival loses, which occurs with probability $p(1-p)$, the entrepreneur earns his share s_t^{11}. If there is a tie (i.e., both win, which occurs with probability p^2), he earns this share with probability $\frac{1}{2}$. Last, if nobody wins (with probability $(1-p)^2$), the entrepreneur will receive further financing with present value $\frac{1}{1+r}E_{t+1}^{11}$.

The right-hand side of (1.5) represents the expected payoff of the entrepreneur from diverting funds at period t. The incentive to divert funds arises from two sources. First, the entrepreneur enjoys the utility c from consuming the funds rather than investing them. Second, by consuming the funds he ensures that financing of the project will continue in the next period with probability $(1-p)$, which is the probability of the rival not making a success. Note that since $(1-p) > (1-p)^2$, by investing the entrepreneur cuts himself off the future stream of rent. If there is only one entrepreneur, as in Bergemann and Hege (2002), then by diverting funds in period t, he guarantees himself that the funding will continue in period $t+1$ with probability 1, unless it is the terminal period. In case of two entrepreneurs, however, funding of each is stochastic and depends on the fact that another entrepreneur has not yet reached success. Therefore, competition softens the incentive compatibility constraint and makes it easier for the venture capitalist to satisfy it.

The venture capitalist aims at paying each entrepreneur the minimal share which will force the latter to invest the funds rather than consume them. To determine the optimal sequence of shares in each time $t = 1, 2, \ldots, T$ the venture capitalist solves the following minimization problem:

$$\begin{aligned} E_t^{11} = \min_{\{s_t^{11}\}} \quad & p(1-p)s_t^{11} + \frac{1}{2}p^2 s_t^{11} + \frac{(1-p)^2}{(1+r)} E_{t+1}^{11} \\ \text{s.t.} \quad & p(1-p)s_t^{11} + \frac{1}{2}p^2 s_t^{11} + \frac{(1-p)^2}{(1+r)} E_{t+1}^{11} \geq c + \frac{1-p}{1+r} E_{t+1}^{11}. \end{aligned} \tag{1.6}$$

Obviously, in the optimum the incentive compatibility constraint will be binding. Considering the transition to continuous time we derive expressions for the share, which the entrepreneur receives in case of success, and the value function which describes the expected utility of the entrepreneur in each time t, given that he allocates the funds into the project(see Appendix 1.A for the derivation of a value function and of the entrepreneur's share). We obtain

$$s_t^{11} = \frac{c}{p} + E_t^{11}, \tag{1.7}$$

$$E_t^{11} = \frac{c}{r+p}\left(1 - e^{(r+p)(t-T)}\right). \tag{1.8}$$

As the entrepreneurs are ex-ante identical, in the sense that they are at the same stage of R&D and have the same probability to complete the project, the value functions (and the shares) are identical for both entrepreneurs.

The compensation scheme, described by the value function E_t^{11}, guarantees that each entrepreneur invests the funds, rather then diverting them, in each period in the regime (1/1). The above expression is very intuitive. The first factor of E_t^{11} represents the value of perpetuity which an entrepreneur would receive if he diverted the funds. The second factor represents a "punishment" for late discovery, in the sense that the share of an entrepreneur decreases over time. Analogically as in the previous case, it is easy to derive the share and the expected utility of the entrepreneurs for the stopping rule R_3:

$$\begin{aligned} s_t^1 &= \frac{c}{p} + E_t^1 \\ E_t^1 &= \frac{c}{r}(1 - e^{-r(T-t)}), \end{aligned} \tag{1.9}$$

Finally, the stopping rule R_2 differs from R_1 again only in the terminal condition. Since there is probability $\frac{1}{2}$ that the entrepreneur will be chosen to continue, we have $E_{T+1}^{11} = \frac{1}{2}E_0^1$, where E_0^1 is given by (1.9) for $t = 0$, Then it is straightforward to determine the corresponding share and the expected utility of an entrepreneur:

$$
\begin{aligned}
s_t^{11} &= \frac{c}{p} + E_t^{11}, \\
E_t^{11} &= \left(\frac{1}{2}E_0^1 - \frac{c}{r+p}\right) \cdot e^{-(r+p)(T-t)} + \frac{c}{r+p}
\end{aligned} \tag{1.10}
$$

The results are summarized in the Table 1 in Appendix 1.B.

1.3.3 Optimal stopping time

For each stopping rule the venture capitalist maximizes his profit from the project, subject to the incentive compatibility constraints. Given the stopping rule, the choice variables of the venture capitalist are the shares of entrepreneurs and the maximal time horizon. The share is the function of exogenous parameters c, r and p, time t when success is achieved, and the endogenously determined time horizon T. Hence, optimally choosing the financing horizon the principal automatically determines the sequence of shares.

Stopping rule R_1

Consider the stopping rule R_1, which requires that both entrepreneurs are financed until one of them wins or until the financing horizon elapses. The optimal time horizon is derived from the following program:

$$
\max_{T \in (0,\infty)} V_0^{11} - 2E_0^{11},
$$

where the value functions V_0^{11} and E_0^{11} are derived in the previous section and are given by (1.4) and (1.8) with $t = 0$. The first order condition yields a unique solution to the maximization problem. We will denote the *optimal* financing horizon as T_1^{11}, where

$$
T_1^{11} = -\frac{1}{p} \ln \frac{c}{Rp - c}.
$$

We denote the optimal contract, corresponding to the stopping rule R_1 as $\mathcal{C}_1$. According to this contract the venture capitalist finances both entrepreneurs until one of them succeeds but at most for T_1^{11} periods. In case of success the winner is rewarded with an appropriate share s_t^{11} as given by (1.7); see also Table 1 in Appendix 1.B for the summary of contract terms.

Stopping rule R_3

The stopping rule R_3 corresponds to the benchmark case with one entrepreneur. It is easy to show that in this case the optimal financing horizon is $T^1 = -\frac{1}{p}\ln\frac{c}{Rp-c}$ (see also Bergemann and Hege 2002). The resulting contract is denoted $\mathcal{C}_4$.[8] Since the optimal financing horizon depends on costs of R&D and on expected payoff, it is not surprising, that $T^1 = T_1^{11}$. Indeed, two entrepreneurs spend twice as much on R&D, but they also have twice as large probability of success,[9] so that the ratio of R&D costs to the expected payoff remains constant.
Note, that T^1 is positive if and only if $Rp > 2c$. The intuition behind this restriction becomes clear when we re-write inequality as $R > \frac{2c}{p}$. The R&D in our model follows a Poisson process with parameter p, so that the expected time of discovery when a single entrepreneur is employed, is $\frac{1}{p}$. Hence, the requirement $R > \frac{2c}{p}$ means that the venture capitalist will finance the project only if the value of the prize is larger than the expected cost of R&D plus the expected reward, payed to an agent. Otherwise, it is not profitable for the venture capitalist to finance the project at all. From now on we will assume, that $Rp > 2c$.

Stopping rule R_2

Let us now consider the stopping rule R_2. According to this rule both entrepreneurs will be financed until one of them wins, or until the maximal allowed time elapses. If no success was made, then one entrepreneur will be randomly chosen and financed for additional period of time. For simplicity we denote the expected profit, which the venture capitalist retains as $F(T) = V_0^{11} - 2E_0^{11}$, where the functions V_0^{11} and E_0^{11} are given by (1.4) and (1.10) with $t = 0$; see also Table 1 in Appendix 1.B. Maximizing the profit of the venture capitalist, we obtain the following first-order condition:

$$F'(T) = -(r+2p)\cdot B^{11}\cdot e^{-(r+2p)T} + (r+p)\cdot A^{11}\cdot e^{-(r+p)T} = 0,$$
$$\text{where} \qquad A^{11} = E_0^1 - \frac{2c}{r+p}, \qquad B^{11} = V_0^1 - \frac{2(Rp-c)}{r+2p}. \tag{1.11}$$

Depending on the relation of A^{11} and B^{11} the optimal time can be finite or infinite. First note that B^{11} is always negative. Indeed the inequality $B^{11} < 0$ is equivalent to

$$\frac{Rp-c}{r+p}\left(1 - e^{-T^1(r+p)}\right) < \frac{2(Rp-c)}{r+2p},$$

[8] Here, we use T^1 instead of T_4^1, since $\mathcal{C}_4$ is the only contract which arises in regime (i).

[9] Intuition for this result is explained in Section 1.3.4.

which obviously holds for all values of parameters $p, r \in (0,1)$ satisfying the feasibility condition $Rp > 2c$.
If $(r+p)A^{11} \leq (r+2p)B^{11} < 0$, then the expected profit $F(T)$ is decreasing in T and the optimal research horizon is zero, so that effectively the venture capitalist employs only one entrepreneur.[10] The resulting contract is identical to the contract $\mathcal{C}_4$.
If $(r+2p)B^{11} < (r+p)A^{11} < 0$, then the optimal research horizon is

$$T_2^{11} = -\frac{1}{p} \ln \frac{r+p}{r+2p} \frac{E_0^1 - \frac{2c}{r+p}}{V_0^1 - \frac{2(Rp-c)}{r+2p}},$$

and the corresponding contract is denoted $\mathcal{C}_2$. According to this contract, the venture capitalist commits to finance both entrepreneurs at most for T_2^{11} periods; if this time elapses without a success, then only one entrepreneur (randomly chosen) will be financed further for the maximum of T^1 periods. The terms of the contract are described in Table 1 in Appendix 1.B.
On the other hand, if $A^{11} \geq 0$, the expected profit $F(T)$ is increasing in T and the optimal research horizon is infinite i.e., the venture capitalist is willing to finance the innovation race infinitely long. The corresponding contract is denoted $\mathcal{C}_3$. This case corresponds to the favorable combinations of low costs of R&D and high probability of success. The condition $A^{11} \geq 0$ directly implies that (in expected terms) the venture capitalist would have to pay higher compensation to one entrepreneur than to two entrepreneurs, i.e., $E_0^1 > \frac{2c}{r+p}$. If this is the case, the venture capitalist always prefers a competitive arrangement to a single entrepreneur.

Remark 1.1. Note that for all contracts, the value functions and the cost functions at the optimal time are homogeneous of degree 1 in (c, R) and homogeneous of degree 0 in (c, p, r). Therefore, if we denote $W(c, p, r, R)$ the maximal value of the venture capitalist's objective function,[11] then

$$W(c,p,r,R) = R \cdot W\Big(\frac{c}{R}, p, r, 1\Big) = R \cdot W\Big(\frac{\bar{r}c}{Rr}, \frac{\bar{r}p}{r}, \bar{r}, 1\Big), \tag{1.12}$$

where $\bar{r}$ is some particular value of the discount rate. Hence any comparison of contracts for general values of parameters c, p, r, and R is equivalent to comparison for parameters c and p with an arbitrary value of r and with $R = 1$.[12] Later, without

[10]Note that $F''(T) < 0$ and $F(0) > 0$.

[11]This is, for example, $V_{0,1}^{11} - 2E_{0,1}^{11}$ at time $T = T_1^{11}$ for contract $\mathcal{C}_1$.

[12]In particular, given c, p, r, and R, we choose an arbitrary $\bar{r} > 0$ and consider new variables $\bar{c} = \frac{\bar{r}c}{Rr}$ and $\bar{p} = \frac{\bar{r}p}{r}$, which gives $W(c,p,r,R) = R \cdot W(\bar{c}, \bar{p}, \bar{r}, 1)$. We rename the variables to c and p

loss of generality, we use the value $\bar{r} = 0.05$ in numerical simulations.

1.3.4 Optimal contract

For each of the three stopping rules we can now specify a contract in terms of maximum time allowed for research and the share of the prize, which each entrepreneur receives in case of success. As we showed in the previous section for the same stopping rule the optimal contract can take several forms. In any case, the terms of the contracts depend on the probability of success and the normalized costs (that is on the ratio $\frac{c}{R}$; see Remark 1.1). For each combination of parameters, the venture capitalist will choose among three contracts, corresponding to three stopping rules. The optimal contract then is the one which maximizes the residual payoff of the venture capitalist.

Proposition 1.1. *Let $Rp > 2c$. Then, in regime (1/1) the optimal contract is to finance both identical entrepreneurs for at most $T_1^{11} = -\frac{1}{p}\ln\frac{c}{Rp-c}$ periods and abandon financing of both if no success was made (such contract is denoted $\mathcal{C}_1$).*

The proof of the proposition can be found in Appendix 1.A. The above result is based on two effects: *scale effect* and *disciplining effect*. The former means that two entrepreneurs increase the total probability of success. In particular, when the entrepreneurs' R&D processes are independent (as opposed to Levitt 1995) and are modelled as identically distributed Poisson processes, then the probability of success is exactly doubled.[13] Thus, at each moment two entrepreneurs create twice as much value as one entrepreneur. Therefore, when it is profitable to employ a single entrepreneur, then the scale effect makes competition more attractive.
At the same time, the expected reward to be paid to each of the competing entrepreneurs is less than the expected reward of a single entrepreneur:

$$E_1^{11} = \frac{c}{r+p}(1 - e^{-(r+p)(T-t)}) \quad < \quad \frac{c}{r}(1 - e^{-r(T-t)}) = E^1.$$

Hence, the competition disciplines the entrepreneurs making them working hard for smaller reward, which obviously makes competition more attractive. This effect on

by dropping the bar.

[13]In discrete time, the probability that at least one entrepreneur succeeds is in each period equal to $1-(1-p^2) = 2p-p^2$. On the other hand, in continuous time, one of the properties of Poisson process claims that the probability of two events (two successes) occurring in time interval $[t, t+\Delta]$ interval converges to zero, as $\Delta \to 0$. Therefore, after transition to continuous time, the second-order terms converge to 0 and the probability that at least one entrepreneur encounters a success in $[t, t+\Delta]$ can be approximated by $2\Delta p$. More precisely, $2\Delta p$ is the first-order approximation of the probability, which can also be written in form $2\Delta p + o(\Delta)$.

incentives we call the *disciplining effect*.

1.3.5 The effect of competition

Now we compare our result with the first-best case. In the first-best world (without moral hazard) the principal is also always better of employing competing agents rather than a single agent. Indeed, in both cases the reward of the agents is zero. Hence, if the expected value of the project is larger than costs, i.e. $Rp > c$, the venture capitalist is willing to finance a project infinitely long. Therefore, in continuous time the value of the venture with competing agents V_{FB}^{11} and with a single agent V_{FB}^{1} is given by the following functions respectively:[14]

$$V_{FB}^{11} = \frac{2(Rp - c)}{r + 2p}, \qquad V_{FB}^{1} = \frac{Rp - c}{r + p}. \tag{1.13}$$

Obviously, $V_{FB}^{11} > V_{FB}^{1}$ for any $Rp > c$. This result is due to the scale effect: Two agents succeed two times more often than a single agent. In the situation with the moral hazard the total effect of competition is even more significant due to the effect on the incentives. Our model predicts therefore, that the venture capitalist will always choose to finance competing entrepreneurs, if they are at the same stage of innovation race. This strong conclusion is partially a result of the assumption that the entrepreneurs are considered to be identical. In the next section we show however, that the venture capitalist can often benefit from competition even if the entrepreneurs are situated on the different stages of R&D.

1.4 Innovation race between the leader and the follower

Consider now the second scenario, where in order to finish the project an entrepreneur has to complete two stages. Assume further that the venture capitalist has in his portfolio an entrepreneur (a *leader*) who has already finished one stage of R&D. Now the venture capitalist faces an opportunity to employ another entrepreneur (a *follower*) who has not yet achieved his first success. Should the venture capitalist finance the innovation race between those two agents or should he rather proceed financing the leader alone?

[14] Subscript FB stands for "first-best."

When the innovation race starts in a regime with a leader and a follower, the entrepreneurs are not identical from the venture capitalist's point of view. Indeed, the leader has a higher probability of winning a prize. Still, as we discuss in this section, competition can be beneficial, if the presence of the follower considerably limits a rent which the leader can extract from the venture capitalist. The follower has to be a credible threat in a sense that the probability that he makes a breakthrough and wins the race should be sufficiently high. On the other hand, the costs of R&D should be low, compared to the expected prize, so that the duplication of research efforts is justified.

In our notations the game with with a leader and a follower corresponds to regime (1/0). As before, to solve the game we look for sequentially optimal contract that maximizes the profit of the venture capitalist at each regime of the game. That is, we require that the venture capitalist *cannot* draft a contract in regime (1/0) that would be suboptimal in regime (1/1) and/or in regime with a single entrepreneur. Further, we will limit the set of available contracts to the contracts which use one of the following stopping rules:

1. Stopping rule R_1: Finance both entrepreneurs until one of them succeeds or until the maximal financing horizon is reached. If neither entrepreneur succeeds, abandon the financing of both.

2. Stopping rule R_2: Finance both entrepreneurs until one of them succeeds or until the maximal financing horizon is reached. If neither entrepreneur succeeds, stop financing the follower and finance the leader until he succeeds, but no longer than for T^1 periods.

3. Stopping rule R_3: Finance the leader until he succeeds, but not longer than for T^1 periods.

The set of deterministic stopping rules in the race between the leader and the follower is generally not limited to the three stopping rules described above. There is, first of all a class of rules where the venture capitalist employs one agent in period t and another agent in period $\tau > t$. Similar as in Section 1.3, we eliminate this class of rules on the assumption that an agent who is not financed at least one period leaves the market.

Further, the rules described above favor the leader in a sense that he is always financed at least as long as the follower. Potentially, the venture capitalist could use some stopping rule, which favors the follower. Intuitively, such rules are less attractive for the principal. They unambiguously decrease the probability of success (and hence

the expected value of the project) and, while, improving the incentives of the leader, they weaken the incentives of the follower. One example of such stopping rules is the counterpart of rule R_3, where the venture capitalist employs only the follower. Obviously, the expected profit of the principal in the former case is higher. The other possibility is the counterpart of a rule R_2, where the principal abandons the leader and continues financing the follower after the agents fail to deliver a success until the terminal period. It is easy to show, that the tradeoff between stopping rule R_2 and his counterpart is equivalent to the tradeoff between employing only the leader and employing only the follower (the formal argument is provided in the working paper version Goldfayn and Kováč 2005). Indeed, both stopping rules are equivalent in terms of expected value of the project and required incentives in the (1/0) regime. What matters for comparison of two stopping rules is what happens after the terminal period elapses but no success was made. In the first case (rule R_2) the venture capitalist would continue financing the leader. In the second case (counterpart of rule R_2) the venture capitalist would finance the follower, which obviously generates smaller expected profit than the former possibility. Hence, the limitation of the menu of stopping rules to those described above is well justified.
Similarly as before, in order to find the optimal contract, we develop an optimal contract for each stopping rule, and then compare the contracts across stopping rules. A contract specifies, in each regime (i/j) that can be achieved, the following terms:

1. financing horizon T^{ij},
2. stopping rule determining that is used when time T^{ij} has elapsed, but no discovery has been made,
3. rewards for the leader s_t^L, and the follower s_t^F depending on the time t when success is achieved, where $0 \leq t \leq T^{ij}$.

For each stopping rule, the value functions are derived recursively starting from the terminal period of regime (1/0) following the analogical procedure as with identical agents. Therefore, instead of providing all details about the derivation of the value functions, we will only mention the main milestones. In each period t of the game there are the following possibilities:

(a) With probability p the leader wins his second success and the game ends. Note that this also includes the case when both agents complete a current stage.

(b) With probability $p(1-p)$ the follower wins his first success and the game switches to a regime (1/1) (patent race with identical agents).

(c) With probability $(1-p)^2$ neither agent succeeds. If $t < T$ then the game continues further. If $t = T$, then according to the appropriate stopping rule either the project is terminated, or the financing of the follower is terminated and the leader is financed further for at most T^1 periods.

Further notice, that according to the stopping rule R_3 the venture capitalist finances a single agent (the leader). This situation is the same as in regime (1/1) and is analyzed in Section 1.3. Hence, we will focus on the stopping rules R_1 and R_2. For both stopping rules, the equations, that recursively determine the value of the venture, the expected reward of the leader, and the follower, satisfy:

$$V_t^{10} = pR - 2c + \frac{p(1-p)}{1+r}V_0^{11} + \frac{(1-p)^2}{1+r}V_{t+1}^{10}, \tag{1.14}$$

$$E_t^L = c + \frac{p}{1+r}E_0^{11} + \frac{1-p}{1+r}E_{t+1}^L, \tag{1.15}$$

$$E_t^F = c + \frac{1-p}{1+r}E_{t+1}^F. \tag{1.16}$$

Clearly, the terminal conditions for stopping rule R_1 are $V_{T+1}^{10} = E_{T+1}^L = E_{T+1}^F = 0$. On the other hand, the terminal conditions for stopping rule R_2 are $V_{T+1}^{10} = V_0^1$, $E_{T+1}^L = E_0^1$, and $E_{T+1}^F = 0$. Using these conditions and equations (1.14)–(1.16) it is straightforward (after transition to continuous time[15]) to derive the expected value of the venture and the expected reward of the agents. The results are summarized in Table 2 in Appendix 1.B.

An important observation is that the reward of the leader has to be higher than the reward of the follower. By diverting funds at some period of time, the leader can guarantee himself a rent $c + \frac{p}{1+r}E_0^{11}$, where $\frac{p}{1+r}E_0^{11}$ is his expected payoff in the case when the follower makes the first success.[16] Therefore, the venture capitalist has to offer the leader an incentive compatible share, i.e., such that the leader's expected reward will be at least as large as the stream of rents $c + \frac{p}{1+r}E_0^{11}$. On the other hand, if the follower consumes the funds in period t, he can only guarantee himself a rent of c in this period. Therefore, his incentive compatible share should be lower than that of the leader.

[15]Recall, that due to the property of the Poisson process the probability of two events happening at the same time period is 0. Hence, after transition to continuous time, all terms containing p^2 will become zeros.

[16]In the regime (1/1) the optimal contract is $\mathcal{C}_1$. Henceforth, for ease of notation when referring to the terms of this contract we will relax the index of a contract. That is $E_t^{11} := E_{t,1}^{11}$, $V_t^{11} := V_{t,1}^{11}$, $T^{11} := T_1^{11}$.

1.4.1 Optimal stopping time

Stopping rule R_2

Consider first the stopping rule R_2. In this case the value of the venture and expected reward of the entrepreneurs are described by the following value functions:

$$
\begin{aligned}
V_0^{10} &= \left(V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}\right)\cdot e^{-(r+2p)T} + \frac{p(R+V_0^{11})-2c}{r+2p},\\
E_0^L &= E_0^1 \cdot e^{-(r+p)T} + \frac{c+pE_0^{11}}{r+p}(1-e^{-(r+p)T}),\\
E_0^F &= \frac{c}{r+p}(1-e^{-(r+p)T}).
\end{aligned}
$$

Maximizing the expected surplus of the venture capitalist $G(T) = V_0^{10} - (E_0^L + E_0^F)$ with respect to stopping time T we obtain the first order condition:

$$G'(T) = -(r+2p)B^{10}\cdot e^{-(r+2p)T} + (r+p)A^{10}\cdot e^{-(r+p)T},$$

where

$$A^{10} = E_0^1 - \frac{pE_0^{11}+2c}{r+p}, \qquad B^{10} = V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}. \tag{1.17}$$

Depending on the relation of A^{10} and B^{10}, the optimal financing horizon can be either zero, positive finite, or infinite. The Lemma 1.1 summarizes the results; see Appendix 1.A for its proof.

Lemma 1.1. *Let $Rp > 2c$. Then in regime $(1/0)$ the following statements hold:*

(i) *If $A^{10} > 0$, then $B^{10} < 0$. In that case $G(T)$ in monotonically increasing and the optimal stoping time is infinite.*

(ii) *If $(r+2p)B^{10} < (r+p)A^{10} < 0$, then function $G(T)$ reaches maximum at time*

$$T_6^{10} = -\frac{1}{p}\ln\frac{r+p}{r+2p}\,\frac{E_0^1 - \frac{2c+pE_0^{11}}{r+p}}{V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}},$$

where $T_6^{10} > 0$.

(iii) *If $A^{10} < 0$ and $(r+p)A^{10} < (r+2p)B^{10}$, then function $G(T)$ is monotonically decreasing and the optimal stoping time is zero.*

Note that in case (iii) the venture capitalist finances a single entrepreneur, i.e., $V_0^{10} = V_0^1$ and $E_0^L + E_0^F = E_0^1$. The resulting contract is the same as in regime (1/1) and is again denoted $\mathcal{C}_4$. In case (ii) the venture capitalist finances both entrepreneurs until time T_2^{10} is reached and then abandon the follower and continue financing the leader for additional $T^1 = -\frac{1}{p} \ln \frac{c}{Rp-c}$ periods. The resulting contract is denoted $\mathcal{C}_6$. In case (i), the optimal financing horizon is infinite and the resulting contract is denoted $\mathcal{C}_7$. Detailed conditions and value function for these contracts are described in Table 2 in Appendix 1.B.

Stopping rule R_1

Consider now the stopping rule R_1. Maximizing the surplus of the venture capitalist, it is easy to establish that the optimal stopping time is finite:

$$T_5^{10} = -\frac{1}{p} \ln \frac{2c + pE_0^{11}}{p(R + V_0^{11}) - 2c}.$$

The resulting contract is denoted $\mathcal{C}_5$; see Table 2 in Appendix 1.B for details.
Note that $2c + pE_0^{11} < p(R + V_0^{11}) - 2c$ is necessary for T_5^{10} to be positive. If the reverse inequality holds, then the optimal stopping time is zero, so that the venture capitalist prefers to finance the leader alone.
Conditions, such as the one above, determine whether a particular contract is feasible. For the contracts with finite stopping time (i.e., contracts $\mathcal{C}_4$, $\mathcal{C}_5$, $\mathcal{C}_6$), these necessary conditions require, that the optimal financing horizon is positive. For contract $\mathcal{C}_7$, the necessary condition requires, that parameters are such, that the optimal financing horizon is infinite. From now on we will call these necessary conditions *feasibility conditions*. We will call a contract *feasible* in the range of parameters, where the corresponding feasibility conditions are satisfied. The range of parameters, where feasibility conditions for each contract $\mathcal{C}_k$, $k = 4, 5, 6, 7$ are satisfied, is shown in Figure 5 in Appendix 1.B.

1.4.2 Optimal contract

Given the values of parameters, we choose, out of the pool of feasible contracts, the one that maximizes the profit of venture capitalist, i.e., we look for an optimal contract with respect to stopping rules. Investigation of feasibility conditions and optimality of contracts leads to Proposition 1.2. The proof of the proposition (partly numerical) can be found in Appendix 1.A.

Proposition 1.2. *Let $Rp > 2c$. Then in regime (1/0) the following statements hold:*

(i) *If* $A^{10} > 0$*, then the feasible contracts are* $\mathcal{C}_4$*,* $\mathcal{C}_5$ *and* $\mathcal{C}_7$*. The optimal contract is* $\mathcal{C}_4$*.*

(ii) *If* $0 > A^{10}(r+p) > B^{10}(r+2p)$*, then the feasible contracts are* $\mathcal{C}_4$*,* $\mathcal{C}_5$ *and* $\mathcal{C}_6$*. The optimal contract is* $\mathcal{C}_5$*, if parameters are such that* $T_5^{10} < T_6^{10} - \frac{1}{r+p}\ln\frac{2c+pE_0^{11}-E_0^1(r+p)}{2c+pE_0^{11}}$*. Otherwise, contract* $\mathcal{C}_6$ *is optimal.*

(iii) *If* $A^{10}(r+p) < B^{10}(r+2p)$ *and* $(2c+E_0^{11}) < p(R+V_0^{11})-2c$*, then the feasible contracts are* $\mathcal{C}_5$ *and* $\mathcal{C}_4$*. The optimal contract is* $\mathcal{C}_4$*.*

(iv) *If* $A^{10}(r+p) < B^{10}(r+2p)$ *and* $(2c+E_0^{11}) > p(R+V_0^{11})-2c$*, then the only feasible (hence, the optimal) contract is* $\mathcal{C}_4$*.*

Let us denote $\mathcal{D}_i$ the domain of parameters (p,c) where contract $\mathcal{C}_i$ is optimal, where $i = 4,5,6,7$. Proposition 1.2 shows that the domain $\mathcal{D}_7$ is empty and hence the whole parameter space can be divided into three domains $\mathcal{D}_4$, $\mathcal{D}_5$, and $\mathcal{D}_6$, as shown in Figure 1.1.[17]

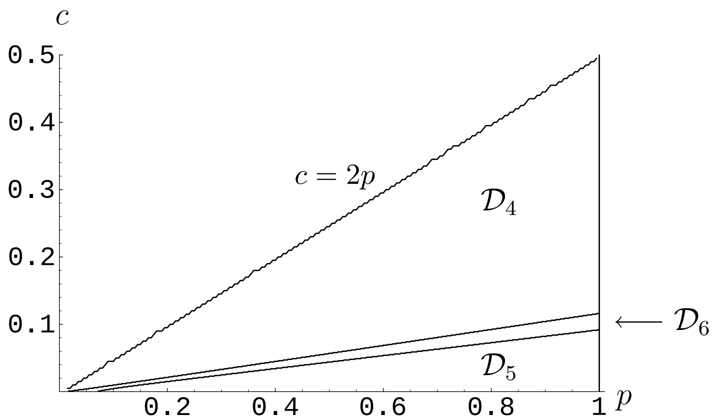


Figure 1.1: Regime (1/0): Division of the parameter space into three domains according to optimal contracts; for $r = 0.05$

The region $\mathcal{D}_5$ corresponds to the most favorable combination of costs of R&D and the probability of success. In region $\mathcal{D}_4$, on the contrary, for each success probability the costs of R&D are the highest. Finally, in region $\mathcal{D}_6$ the combination of costs and success probability is moderately favorable. It is therefore intuitive that competition

[17] To draw the domains $\mathcal{D}_4$, $\mathcal{D}_5$, and $\mathcal{D}_6$ we considered fixed values of discount rate $r = 0.05$ and prize $R = 1$ and used numerical simulations. On a grid 0.001×0.001 and for values of parameters, such that $p \in [0,1]$ and $c \in [0, p/2]$ we plotted the points where the constraints for each domain are satisfied. Due to homogeneity of the profit function the choice parameters values is without loss of generality.

is a beneficial arrangement for the venture capitalist, if values of the parameters lie in the domain $\mathcal{D}_5$. In domain $\mathcal{D}_6$ competition is beneficial if the patent race doesn't take too much time. The costs of R&D are, however, relatively high in this domain. Hence, after experimenting with patent race for some time, the venture capitalist will continue financing the leader alone, if the agents fail to deliver a success. Finally, in domain $\mathcal{D}_4$, competition is not beneficial, since the costs are too high to justify the duplication of research efforts.

1.4.3 The effect of competition

In order to investigate the effect that competition has on the decision to employ competing entrepreneurs, we compare the moral hazard setting with the benchmark case without moral hazard. In the latter case the venture capitalist can perfectly observe the allocation of funds and therefore the incentive compatible reward of both entrepreneurs is zero (this is due to the assumption that the venture capitalist has all bargaining power). Hence, the expected payoff of the venture capitalist equals the expected value of the project. Therefore, for any $Rp > 2c$ the venture capitalist is willing to finance the project infinitely long.
The value of the venture with competing agents V^{10}_{FB} and a single agent (the leader) V^{1}_{FB} are given by the following functions respectively:

$$V^{10}_{FB} = \frac{p(R + V^{11}_{FB}) - 2c}{r + 2p}, \qquad V^{1}_{FB} = \frac{Rp - c}{r + p},$$

where V^{11}_{FB} is given by (1.13). The venture capitalist will finance only the leader, if $V^{1}_{FB} \geq V^{10}_{FB}$, which is equivalent to the condition:

$$\frac{c}{Rp - c} > \frac{p \cdot r}{(r + 2p)(r + p)}. \tag{1.18}$$

Otherwise, the venture capitalist will finance both entrepreneurs.
According to condition (1.18), we divide the parameter space into two domains, as is shown in Figure 1.2. The border curve between single entrepreneur (SE) and competing entrepreneurs (CE) corresponds to the case of equality.[18] The region above the line represents combinations of costs and success probability, where (1.18) holds, i.e., where the venture capitalist finances only the leader. If the combination of costs and probability is below the line, then the venture capitalist will prefer to

[18]The areas were plotted for r is fixed at $r = 0.05$. Due to the homogeneity of the profit functions this choice of parameter is without loss of generality.

finance both entrepreneurs.

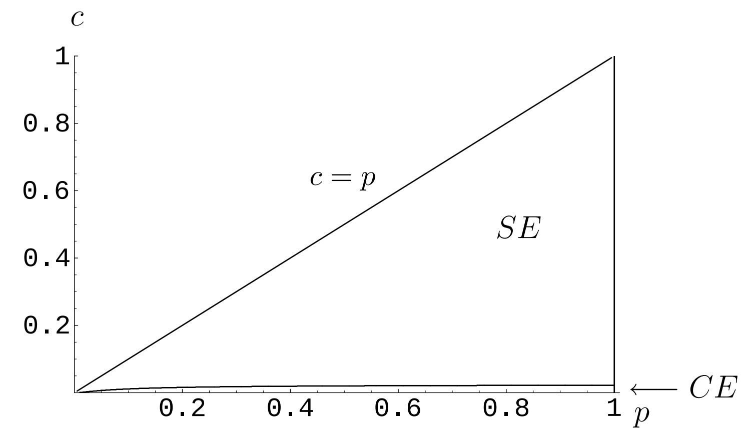


Figure 1.2: Regime (1/0), no-moral hazard case: Competing entrepreneurs (CE) vs a single entrepreneur (SE); $r = 0.05$

Recall, that in regime (1/1) the venture capitalist prefers to employ competing entrepreneurs, regardless whether the moral hazard is present or not. Without moral hazard this decision is motivated by the *scale* effect: with two entrepreneurs the probability of success is twice as large as with one entrepreneur. With moral hazard there is additional effect of competition, which we call the *disciplining* effect. This effect decreases the rent of each entrepreneur comparing to situation of no competition, so that in case of success the venture capitalist retains larger share of the prize.
The analysis of a regime with the leader and the follower allows to understand the relative importance of the scale and disciplining effects in the presence of moral hazard. A comparison of Figures 1.1 and 1.2 shows that without moral hazard the range of parameters where the competition is beneficial is significantly smaller than in a moral hazard setting (although the whole feasibility region has increased). In the absence of moral hazard the increased probability of success due to competition (scale effect) is almost always not sufficient to justify financing of both the leader and the follower. However, the disciplining effect of competition in case of moral hazard is so important, that the venture capitalist will hire both the leader and the follower, although he does not gain much in terms of success probability. The venture capitalist nevertheless gains from the reduction of rent which he has to pay to both entrepreneurs. For certain combinations of costs and probability, the decrease in compensation of the leader due to competition is large enough to justify the financing of both entrepreneurs (domains $\mathcal{D}_5$ and $\mathcal{D}_6$). Naturally, the competition can be justified only if the follower is not too expensive to finance (c should be relatively small) and the reduction in the rent of the leader due to competition is significant (p should be relatively large).

Another result of competition between entrepreneurs is the increase in the total financing horizon of the project. For the range of costs and probabilities where the venture capitalist chooses to finance both the leader and the follower (i.e., domains $\mathcal{D}_5$ and $\mathcal{D}_6$), the maximal financing horizon is longer with a competitive arrangement than with a single entrepreneur. Indeed, a single entrepreneur (the leader) would be financed for at most of T^1 periods according to contract $\mathcal{C}_4$. If both entrepreneurs are employed, then the maximum financing horizon is $T_k^{10} + T^1 > T^1$, where $k = 5, 6$. Therefore, competition helps to alleviate one of the main problems created by moral hazard — the limitation of the research horizon. We have shown that the first best solution obtained in the absence of moral hazard is to finance the project infinitely long. The same result was obtained for the case of one entrepreneur by Bergemann and Hege (2002). Since the expected value of the project increases in the research horizon, the presence of moral hazard reduces this value. Competition, however, limits the amount of rent which the entrepreneurs can extract from the venture capitalist and hence makes it profitable for the venture capitalist to set a longer financing horizon.

1.5 Strategic interaction

Up to this point we assumed that the entrepreneurs do not behave strategically, i.e., that each entrepreneur believes that his rival always invests all funds into R&D. In other words, each entrepreneur believes that by diverting the funds in each period, he faces a probability p that his rival wins the prize in the meantime. With this assumption in hand, we have shown that competition softens the incentive compatibility constraints of the entrepreneurs and makes it cheaper for the venture capitalist to provide an incentive compatible reward scheme. As we have discussed, the incentive compatible reward of each entrepreneur is lower in the case of competition, than in the case without competition.

However, if the entrepreneurs are well-trained game theorists and think strategically, they will take into account all possible strategies of the rival. Those can be either "work" (denote it w) or "shirk" (denote it s). Hence, in each period we can model the behavior of the entrepreneurs by a 2×2 game. The venture capitalist, naturally, wants to ensure the (w, w) equilibrium. Otherwise his investments are wasted. Our results already imply that under the compensation schemes considered before, it is optimal for each agent to play w, if his rival plays w (that is, w is the best response to w). In this section we will show these compensation schemes are sufficient to ensure the unique equilibrium (w, w). Note that for this it is sufficient to rule out the equilibrium (s, s).

We start with the situation where two identical entrepreneurs are involved in the innovation race (regime (1/1) in our notations). Consider the terminal period T. Let s_T be the reward of an entrepreneur if he achieves a success. In the table below we summarize payoff of one entrepreneur in four strategic situations (recall that the entrepreneurs are identical).

	w	s
w	$(p - \frac{1}{2}p^2)s_T$	ps_T
s	c	c

In order to ensure that (w, w) is a unique Nash equilibrium (in pure strategies), the reward s_T should be such that: $w \in BR(w)$ and $s \notin BR(s)$, where BR stands for best response.[19] Examining the payoffs, we receive:

$$\begin{aligned} w \in BR(w), &\iff s_T \geq \tfrac{c}{p-\frac{1}{2}p^2} =: s_T^w, \\ s \notin BR(s), &\iff s_T \geq \tfrac{c}{p} =: s_T^s. \end{aligned}$$

Since $s_T^s < s_T^w$ for all $p \in (0, 1)$, in the terminal period of the game the venture capitalist can ensure the unique equilibrium (w, w) by promising the entrepreneurs reward $s_T = s_T^w$.

Consider now some period of time $t \leq T - 1$ and assume that both entrepreneurs invest in each period $\tau = t+1, \ldots, T$. We will determine such s_t that in period t both entrepreneurs find it incentive compatible to invest funds rather than divert them. The following table shows the payoff matrix of one of the two identical entrepreneur

[19] We will assume that when the entrepreneurs are indifferent between strategies "work" and "shirk", they choose to work.

at period t.

	w	s
w	$(p - \frac{1}{2}p^2)s_t + \frac{(1-p)^2}{1+r}E^{11}_{t+1}$	$ps_t + \frac{1-p}{1+r}E^{11}_{t+1}$
s	$c + \frac{1-p}{1+r}E^{11}_{t+1}$	$c + \frac{1}{1+r}E^{11}_{t+1}$

As before, we need to determine s_t such that for each entrepreneur w is a best response to any strategy of a rival:

$$\begin{aligned} w \in BR(w), \quad &\text{iff} \quad s_t \geq \tfrac{2c}{p(2-p)} + \tfrac{2p(1-p)}{p(2-p)(1+r)}E^{11}_{t+1} =: s^w_t, \\ s \notin BR(s), \quad &\text{iff} \quad s_t \geq \tfrac{c}{p} + \tfrac{1}{1+r}E^{11}_{t+1} =: s^s_t. \end{aligned}$$

In order to ensure the unique equilibrium (w, w), the venture capitalist has to promise the entrepreneurs a share $s_t \geq \max\{s^s_t, s^w_t\}$. It can be easily shown that $s^w_t > s^s_t$, since it is equivalent to $\frac{1}{1+r}pE^{11}_{t+1} < c$, which holds since

$$pE^{11}_{t+1} = \frac{pc}{r+p}\left(1 - e^{-(r+p)\left(T^{11}-(t+1)\right)}\right) < \frac{pc}{r+p} < c.$$

Therefore, by promising the entrepreneurs a reward $s_t = s^w_t$ the venture capitalist ensures the equilibrium (w, w). Note, that in continuous time s^{11}_t converges to $\frac{c}{p} + E^{11}_t$, which is exactly the reward we have calculated before, without accounting for strategic interaction.

The result which we have established holds for any $t \leq T - 1$, therefore it holds in particular for $t = T - 1$. We have proved that in the terminal period the entrepreneurs will invest, if rewarded according to the (w, w) scheme. Therefore, they will also invest in period $(T - 1)$ if rewarded according to the (w, w) scheme. Recursively, we can prove that the result holds for any period t of regime (1/1).

It is interesting to observe that if the entrepreneurs are compensated according to the (w, w) scheme, then in regime (1/1) at each period of time the game resembles the Prisoners Dilemma game. The entrepreneurs can be better off if they divert the funds simultaneously in all periods. Indeed, in this case the expected payoff of each entrepreneur is $\frac{c}{r}(1 - e^{-rT})$, i.e., a properly discounted stream of rent c. If both entrepreneurs invest, then the expected reward of each is $\frac{c}{r+p}(1 - e^{-(r+p)T}) < \frac{c}{r}(1 - e^{-rT})$. But under the incentive scheme (w, w), "work" is always the best response to "shirk", therefore, a potentially attractive (for entrepreneurs) situation (s, s) is not a subgame perfect Nash equilibrium.

Following very similar lines of reasoning, it is easy to prove that in regime (1/0), rewarding entrepreneurs according to (w, w) scheme ensures the unique equilibrium (w, w). The following discussion applies both to contract $\mathcal{C}_5$ and contract $\mathcal{C}_6$. Consider the terminal period T. The matrix below summarizes the payoff of the follower (the leader is the row player and the follower is the column player).

	w	s
w	$p(1-p)(s_T^F + \frac{1}{1+r}E_0^{11})$	c
s	$p(s_T^F + \frac{1}{1+r}E_0^{11})$	c

Investigating the payoff of the follower, we derive the following conditions:

$$\begin{aligned} w \in BR(w), \quad &\Longleftrightarrow \quad s_T^F \geq \tfrac{c}{p(1-p)} - \tfrac{1}{1+r}E_0^{11} =: s_T^{F,w}, \\ s \notin BR(s), \quad &\Longleftrightarrow \quad s_T^F \geq \tfrac{c}{p} - \tfrac{1}{1+r}E_0^{11} =: s_T^{F,s}. \end{aligned}$$

Comparing the compensation of the follower in case when the leader works with his compensation in case when the leader shirks, we establish that $s_T^{F,w}$ is always larger than $s_T^{F,s}$. Therefore, if the venture capitalist promises the follower a reward $s_T^F = s_T^{F,w}$ he ensures that the follower will invest irrespective of a strategy of the leader. This rules out equilibrium (s, s). Hence, to enforce the (w, w) equilibrium it is enough to compensate the leader so that his best response to "work" is "work". The same logic holds for any period $t < T$. Therefore, if both entrepreneurs are compensated as if the other always invests (i.e., "work"), the unique equilibrium of the game is (w, w). This justifies our approach in Sections 1.3 and 1.4.

1.6 Finite horizon and commitment to stop

So far, we have assumed that the venture capitalist can choose the financing horizon for each regime and can commit to it. This means that if the maximum time allowed for experimentation in regime (i/j) elapses without success, then depending on terms of the contract either the project will be irrevocably abandoned, or the venture capitalist

will abandon financing of follower. In this section we provide a rationale for that assumption.
If we assume that the venture capitalist cannot commit to stop the project after the maximal allowed time has elapsed, then he will finance the entrepreneurs infinitely long. Suppose that in regime (i/j) the contract between the venture capitalist and entrepreneurs determines some (optimal) time T^{ij}. If this time elapses but no success was made by any entrepreneur, the venture capitalist is willing to start the game from the beginning, as if the world is in the first period of regime (i/j). Indeed, all costs that the venture capitalists has incurred up to time T^{ij} are sunk, and the game has not changed since the venture capitalist made his optimal decision at $t = 0$ of regime (i/j). Because of this feature of our model (sunk costs and independent probability of success in each period), the venture capitalist is willing to finance the entrepreneurs infinitely long, if he enters the game once.
If the venture capitalist cannot commit to stopping the project, he is also not able to condition further financing on successful completion of predetermined stages or benchmarks. In a world, where commitment is not credible, the venture capitalists will finance entrepreneurs until one of them wins the prize.
However, empirical literature on venture capital documents, that stage financing, which is conditional on successful completion of prescribed milestones, is one of the most important and commonly used control mechanisms in venture capital financing.[20] Therefore, the commitment assumption is not only realistic, but is essential for the ability of the venture capitalist to include the provision about the milestones into the contract.
Obviously, in our model the venture capitalist prefers to commit to finite financing horizon. Commitment to stop financing of the project is an important punishment mechanisms, that allows to decrease compensation of the entrepreneurs and therefore to increase profits of the venture capitalist, comparing to a situation with no commitment. In the model, however, there is no endogenous mechanism, which would make the ex-ante commitment credible ex-post. Hence, to justify the commitment power of the venture capitalist in our model, we make an assumption, that the venture capitalist is wealth-constrained.
This assumption is well supported by the evidence about practice of the venture-capital funds. According to Inderst and Munnich (2003), the venture capital funds are normally close-ended, which means that funds are raised once from the investors and are directed afterwards into the portfolio of projects. The partnership agree-

[20]See, for example, Kaplan and Stromberg (2003) and Sahlman (1990).

ments, which govern the venture capital funds, often contain a covenant that limits a possibility of the venture capitalist to raise further investments. Likewise, the partnership agreements restrict ability of the venture capitalist to transfer investments across projects and across different funds, run by the same partners. The wealth-constrained venture capitalist can credibly commit to limit resources directed to each of his portfolio projects and hence can commit to the finite financing horizon.

In the world described in our model, this commitment can be understood as the following. Ex ante, the venture capitalist is able to calculate the optimal period of time, during which he is willing to finance the project. He then commits a corresponding amount of money for this project and commits all other resources to his other portfolio projects. The partnership agreements restrict the ability of the venture capitalist to raise additional funds and, therefore, the commitment to stop the project is credible.[21]

1.7 Conclusion

In this paper we study innovation race in the moral hazard setting. We explore a model where two entrepreneurs simultaneously develop a project which, if successful, generates a fixed prize R. The project is developed in stages and the first entrepreneur who completes the second stage wins the prize. Research and development is financed by the venture capitalist, but the funds are allocated by the entrepreneurs. This creates a moral hazard problem: the entrepreneurs can divert the funds to their own uses. We investigate two possible scenarios: a basic scenario where both entrepreneurs are at the same (the last) stage of R&D, and its variation where one of the entrepreneurs is a leader and another is a follower.

We identify two effects which make the financing of competing entrepreneurs beneficial for the venture capitalist. First effect is the higher probability of success (scale effect) and the second is less obvious effect which competition has on incentives (disciplining effect). In order to highlight the importance of competition in the moral hazard setting, we compare it with the benchmark setting without moral hazard. The analysis reveals, that in the scenario where both entrepreneurs are on the same stage of R&D both effects are important. Due to the scale effect financing of competing entrepreneurs is attractive in the absence of moral hazard. With moral hazard,

[21] More realistic approach to model the venture capital process is to assume the venture capitalist have some prior believes about quality of the project. If the project fails to succeed, the venture capitalist becomes pessimistic and will eventually abandon the project. This mechanism ensures that the project will be stopped in finite time and is extensively discussed in Bergemann and Hege (1998, 2005).

the disciplining effect reinforces the scale effect making the financing of competing entrepreneurs even more attractive.

However, in the scenario with a leader and a follower, the scale effect is of little importance, so that without moral hazard the follower will almost never be employed. Nevertheless, with moral hazard in place, the presence of a competitor allows to reduce significantly the rent of the leader, which makes competition a beneficial arrangement for the large range of parameters. We also find that by improving incentives the competition allows the venture capitalist to increase the maximal research horizon during which he is willing to finance the project, making it therefore closer to the first-best (infinite) horizon. The prediction that the projects are financed longer in the presence of competing ventures provides a scope for the empirical test of the model since many databases on venture capital contain information about number and duration of rounds.

Our key finding is that competition can be used by the venture capitalist as an effective cure against the moral hazard, in a situation where the allocation of funds by the entrepreneurs is not observable. Hence, competition serves as a "natural" mechanism that allows to improve the efficiency of research and development. The existence of such mechanism is particulary important in those cases, where the use of complicated security schemes, developed in the venture capital literature, is difficult or not possible at all.

In terms of empirical implications this result suggests that in particular grant agencies and government agencies, as well as venture capital firms that are active on the less developed capital markets, should use competition between portfolio projects as a mechanism of mitigating the agency problems. Indeed, the former, usually do not use complicated security schemes either due to the lack of expertise or following the established practices.[22] The latter, on the other hand, might find it difficult to use such schemes on the capital market which is not sufficiently developed to accommodate them.

The idea that competition positively affects incentives of the agents (i.e., relaxes their incentive constraint) has already been applied in various settings, like yardstick competition (Tirole 1997, pp. 41–42) or design of team incentives (Holmstrom 1982). We contribute to this literature by investigating the effect of competition on incentives in the dynamic framework, where only the winner's output matters to a principal. We

[22]Consider for example the practise of NIH. The grant is usually split into several budget periods (analogy of stages in our model). Within each stage the financing is provided on a cash request basis, where the cash is transferred to the grantee's account based on his need. See `grants.nih.gov/grants/managing_awards.htm`

show, that in this framework, competition has a positive incentive effect even when the research technologies are independent (which is not the case in yardstick competition). Alternative approach to modelling a patent race in the moral hazard framework is to consider a setup where the entrepreneurs have different probability of success, in a sense that the same stage of R&D corresponds to identical probabilities of success, and a different stage of R&D corresponds to asymmetric probability of success. Then our results suggest that as asymmetry between entrepreneurs increases, the positive effect of competition becomes less pronounced. Moreover, in a setting with asymmetric entrepreneurs, the competition is beneficial if the value of the output relative to the costs is high and there is a high chance that the asymmetry will be eliminated (namely probability of success is relatively high). In our future research we aim at investigating this alternative approach in more details.

Chapter 2

Organization of R&D with Two Agents and a Principal

2.1 Introduction

On July 29, 2007 the results of two large-scale genetic studies were simultaneously published. The studies "revealed two genes that influence the risk of getting multiple sclerosis (MS) — data sought since the discovery of the only other known MS susceptibility gene decades ago. The findings could shed new light on what causes MS ... and on potential treatments for at least 350,000 Americans who have the disease".[1] As NIH reports in its press-release, the studies were conducted by two competing teams of scientists and were both financed by National Institutes of Health (NIH, the research and grant agency in the United States) and the National Multiple Sclerosis Society.

The puzzling question is why, given the complexity of the task and the importance of finding did the grant agencies that financed the research preferred to split their resources on two teams instead of using potential synergies of a research consortium? This case is by no means unique in the practice of grant agencies. A causal investigation of the NIH's web site reveals that it routinely aims at financing of competing research teams which attack the same problem.[2] Not only grant agencies, but also private venture capital firms contract competing teams to pursue a project. Vulcan Inc., which is a multi-division corporation, owned by Microsoft co-founder Paul Allen, has contracted three competing agencies for the project Halo, aimed at the development

[1] The press release of NIH `www.ninds.nih.gov/ news_and_events/press_releases/`

[2] The examples of grant projects can be found at `grants.nih.gov/grants/guide.`

of the problem-solving software.[3]

On the other hand, it is also common for financiers to grant a financial support to a consortium of co-operating independent teams, rather than to each of these team separately. NIH, for example, finances a number of consortiums, consisting of several independent research teams (most notably, Human Genome Project Consortium or Mouse Models of Human Cancer Consortium).[4] The Vulcan Inc. provides financial support to the Allen Institute for Brain Science, which is a consortium of researches working towards constructing the map of a human brain.

These casual evidence immediately suggest a question: what is the optimal organizational design of R&D activities from the principal's point of view? Several papers have addressed this issue. Levitt (1995) illustrates why it may be profitable for the principal to finance competing agents rather than a single agent. Che and Yoo (2001) analyze the attractiveness of the team production versus stand-alone production in the repeated setting. Hemmer (1995) shows, that if there are synergies from performing two tasks, then assigning a team to the subsequent tasks results in higher product quality than assigning separate agents to each of those tasks. Goldfain and Kováč (2005) compare benefits from employing competing agents, rather than a single agent in a dynamic framework with multiple stages of R&D.

This paper investigates, when (in the presence of moral hazard problem) it is in principal's interests to assign competing agents to the same task and when he prefers that agents cooperate in a team. As is well known from the literature on multi-agent incentives, in static setting competition (or more generally, relative performance evaluation, RPE) improves incentives compared to the team compensation (or joint performance evaluation, JPE). [5] The existing literature, however, is primarily interested in the effect of various compensation schemes on incentives and therefore often ignores the synergy effects of team production. The question which is not addressed is how far does the advantage of RPE goes compared with a team production which is technologically more efficient?

This paper answers this question by analyzing the tradeoff between team production and competing agents, where the team production exhibits synergy effects. I characterize a threshold value of synergy effects, such that (other parameters fixed) above this value the principal prefers to finance a team rather than competing agents. This

[3]The information about the project can be found at `www.projecthalo.com`

[4]For information about projects see `www.genome.gov` and `emice.nci.nih.gov/ mouse_models`

[5]See, for example, Lazear and Rosen (1981), Holmstrom (1982), Nalebuff and Stiglitz (1983), Mookherjee (1984)

threshold value increases with the prize in stake, so that the principal is more likely to finance competing agents if the prize is large. In other words, for larger prizes positive incentive effect due to RPE tends to dominate productivity gains generated by a team.

To analyze alternative structures of research department, I develop a framework, where research is financed by a principal, but the agents (protected by limited liability) have discretion to decide whether to allocate money into a project or to divert them for private consumption. The investment decision of the agents is not observable to the principal. Hence, to implement the desired level of investment he offers each agent an appropriate incentive compatible reward. It is assumed, that each agent is capable of performing a project himself. Hence, the principal has a choice between the following structures of research department: single agent, competing agents and team.

The project succeeds or fails with a probability that is a function of agents' investment decision and (in case of team production) synergy effects. Due to synergy effects the team is the most efficient arrangement from the technological point of view: for the fixed amount of resources devoted to the project, the team succeeds on average more often than a single agent or a pair of competing agents. The competition, on the other hand, has important positive effect on agents' incentives.

It is shown, that due to the incentive effect of the competition, the principal always prefers to employ competing agents, rather than a single agent. The paper is therefore focused on comparing team production with competing agents.

In the first part of the paper I assume that agents, cooperating in a team contribute their investments simultaneously. I show that for the fixed level of synergy effect in team, the relative advantage of competition increases with the value of prize in stake. The intuition is roughly following. To provide the agents with appropriate incentives the principal has to balance a reward they receive in case of success and the amount of investment funds allocated to the agents. The larger is the reward and the smaller is the amount of investment funds in their discretion, the less tempted are the agents to consume the part of funds. However, the free-riding hazard in team weakens the incentives. The presence of synergy effect only accelerates this problem by enabling agents to achieve high success probability by investing small amounts. Hence, as the prize in stake increases, the principal is forced to limit resources allocated to the team more severely, than resources allocated to competing agents (while paying proportionally higher reward in the former case). Therefore, if the prize is sufficiently large, competing agents eventually perform better than a team.

The existing literature on team production often suggests to use a message game to alleviate the free-riding problem in a one-shot model. The general idea behind this mechanism is that agents are required to monitor each other and to submit a report to the principal, based on their observations (Miller 1997, Ma 1988, Marx and Squintani 2002). It is well known that this mechanism requires at least some liability on agents' behalf.[6]. Moreover, the message game is difficult to implement in the environment where "spying" on the team mates is infeasible. Therefore, I suggest to utilize the ability of team peers to observe each other effort by changing the research process from simultaneous to sequential contribution of investments.

In the second part of the paper, I show that the principal can indeed improve incentives of team members by instructing them to contribute investments sequentially. However, this is only true if the investments of agents are strategic complements. In this case the sequential structure works in the favor of the principal, because a leader (the agent, who is the first to contribute) is reluctant to shirk in the fear, that so would the follower (the second mover), which will cause the failure of the project. If the investments of agents are strategic substitutes, however, the effort of one agent exerts a negative externality on his team peer: if the leader works hard, the follower is tempted to shirk. In this case the sequential team structure does not improve incentives of the agents. The comparison of competition with two alternative team structures (sequential and simultaneous) allows to characterize the optimal structure of research department for various levels of synergy effects and prize values.

In the extension of the model I discuss a possibility, which has a principal who is willing but not able to enforce sequential contribution of the effort. Through the paper I assume that the principal cannot observe the effort of the agents. Hence his ability to enforce one of them to contribute the effort before the other is likely to be limited. The principal can however organize the team as a hierarchy, with a team leader and his subordinate. In this setting the team leader has a discretion to decide whether to employ a second agent (the subordinate) and how to allocate investment funds and rewards. In addition, the team leader is assumed to be able (and is indeed willing) to enforce sequential contribution of effort.

Unlike the team without hierarchy, the hierarchical team always leads to the inefficient allocation of resources. The team leader is reluctant to involve the subordinate and prefers to perform a large part of the job himself. Hence, the hierarchical team makes only minor use of synergy effects and always leads to the loss of efficiency in terms of success probability. Still, if investments of the agents are strategic complements,

[6]Miller (1997) shows that a team can reach efficient output without punishment if one of the agents can observe the subset of other agents. This result holds, however, only if the number of partners is at least three

the hierarchical team has positive effect on incentives due to sequential contribution of efforts. If this effect is large enough to balance the loss of efficiency, the principal can use hierarchical structure to improve performance of the team.

Investigating the interaction of agents' efforts in team setting, my paper contributes to the discussion in the literature on sequential partnerships, which addresses a question whether the ability to observe co-worker's effort or output improves incentives. Nafziger (2007) shows that if the output of the co-worker is informative about agent's effort, then it might be better not to provide workers any intermediate information about each other output. Banerjee and Beggs (1989) argue that in the team the first-best solution can be achieved if the efforts are contributed sequentially, but not simultaneously. The authors however assume, that the effort of the first agent is not productive and only affects second agent's cost of effort.

My results on the tradeoff between sequential and simultaneous structure are close to Winter (2005). The author uses a framework where a group of agents works collectively on a project. The agents make a binary decision whether to contribute the effort or not. This decision may be observed by a subset of their team peers, if the effort is contributed sequentially. Winter (2005) shows that the ability to observe the effort of team peers improves incentives particulary when agents' tasks are complementary. He argues, however, that in this setting more transparency can never harm incentives, and therefore sequential production is always optimal. To the contrary, I show that with continuous choice of effort sequential production is optimal only if the efforts of agents are strategic complements.

Ludwig (2007) also investigates the tradeoff between simultaneous and sequential structures. Similar to my results, the author concludes that the sequential structure is optimal when the contributions of agents are perfect complements, while the simultaneous structure is optimal when the contributions are perfect substitutes. This result stems from the ability of the principal to partially deduce the states of the world. That enables him to save implementation costs for the follower after the leader fails to provide a high quality contribution (where "contribution" is a stochastic function of the unobservable effort). In my model, however, the driving force is the strategic effect which the effort of the leader has on the effort of the follower. This strategic effect determines the ability of the principal to save implementation costs for the leader.

The structure of this paper is the following. The basic framework of the model is described in Section 2.2. The setup with a single agent and competing agents are discussed and compared in Sections 2.4 and 2.5. In Sections 2.6 and 2.7 I discuss team production and characterize the optimal structure of the research department. The extension to the hierarchical team is discussed in Section 2.8. Section 2.9 concludes.

Proofs and figures can be found in Appendix 2.A and Appendix 2.B.

2.2 Basic framework of the model

There are two identical risk-neutral agents (entrepreneurs), each of which could be employed to perform a research project. If a project succeeds, it yields a prize of size R. For example, if the project is to find a cure against a disease, then R may represent a discounted stream of all future payoff, generated by sales of this cure.
It is assumed that the agents have no wealth. The necessary funds for research and development are provided by a principal (venture capitalist, grant agency or a firm which has subcontracted research and development to the agents) who owns the project and rewards agents for their effort. Although finances are provided by the principal, allocation decisions are made by agents. They can either invest funds or divert them for private uses. The principal is not able to observe the allocation decision. All he can observe is a success or a failure of the project. Following Holmstrom (1982), in this situation the incentive compatible reward must be a function of a single observable and verifiable outcome in this model, namely a success of the project. I will also assume that limited liability prevents the principal from imposing a monetary punishment on agents in case of failure. Therefore, I limit my attention to the investigation of share contracts, where principals rewards a success by transferring agents a share of the prize. Note, that such limitation is without loss of generality because in this framework the share contract is equivalent to a wage contract, where agents receive positive wage in case of success and receive nothing in case of failure.
The principal is risk-neutral and maximizes his expected payoff from the project. I assume that the principal has all bargaining power, which means that after paying the agents their contractual payoffs, he retains all the residual surplus. The principal offers agents a contract, which specifies the share of each agent in case of success and the size of investment. Moreover, the principal decides whether the agents should to compete for the patent or should form a research team and join their research efforts. In the former case the principal shares the prize with a winning agent. In the latter case, the prize is shared between the team and the principal.
The project succeeds with a probability, which depends on efforts of both agents and (if agents form a team) on synergy effects. The efforts of agents in this model are equivalent to their monetary investments into R&D - therefore I will use the terms "investment" and "effort" interchangeably.[7]

[7]Notice, that I make a normalization assuming that x monetary units invested in the project

Consider the case with one agent. Let the principal transfer amount c to this agent. The agent allocates amount $x \leq c$ to the research and consumes $c - x$. I assume, that the probability of success is given by:

$$p^A(x) = 1 - e^{-x}. \tag{2.1}$$

A possible justification for this functional form is that a research horizon T is limited exogenously, so that $T = 1$. The project succeeds only if it is finished before this time elapses.[8] Hence, if the time of discovery is an exponentially distributed variable with mean $\frac{1}{x}$, then the probability of success is given by $p^A(x)$.
In line with the literature on patent races (Loury 1979, Lee and Wilde 1980, Reinganum 1982) I assume that success probabilities of two competing agents are independent. Therefore, given the above specification, a probability that at least one of two competing agents succeeds can easily be calculated:

$$p^C(x, y) = 1 - e^{-(x+y)}, \tag{2.2}$$

where x and y are investments, allocated by first and second agents respectively.
Finally, I assume that compared to other organizational structures a team may have technological benefits. These technological benefits (further referred to as *synergy effect*) allow the team to generate higher success probability for fixed amount of investments, than each agent in stand-alone situation can achieve. I model the team production by assuming that joint investments influence the probability of success in the following way:

$$p^T(x, y) = 1 - e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}. \tag{2.3}$$

The parameter $\alpha \in [0, 1)$, assumed to be a common knowledge, characterizes a degree of complementarity between skills of team members, which is the source of synergies in this model. If $\alpha = 0$, then there are no technological benefits from employing a team. It succeeds with probability $p^T(x, y) = 1 - e^{-(x+y)}$, which is the same as if a single agent invests $(x+y)$. Note, that this probability also equals the probability that

result in the probability $1 - e^{-x}$. I could alternatively assume that x euros lead to a a probability of $1 - e^{-\theta x}$, where θ is the productivity of 1 euro. However, normalizing $\theta = 1$ does not change any results.

[8] The assumption of limited financing horizon is quite realistic, since it is common for the venture capital firms or grant agencies to set time limits within which the research must be completed. For theoretical justification see Goldfain and Kováč (2005).

at least one of two competing agents succeeds. If $\alpha > 0$, i.e. there are synergies in team production, then for the same amount of investment x and y the team succeeds with higher probability, than a single agent, who invests the same amount (or, for that matter, than at least one of two competing agents).

Remark 2.1. It is possible to show that the choice of the function $(x^{1-\alpha} + y^{1-\alpha})^{\frac{1}{1-\alpha}}$ for modelling of synergy effects is justified in the framework where the probability of success depends on two skills or production factors, and each of two agents is "talented" in different skill. The formal model is provided in the earlier version of this paper (see Goldfain 2005).

The research and development is modelled as a one-shot game. After the contract is signed the agents make one-time decision on how much to invest and the probability of success is a function of this investment.

2.3 Optimal structure in the absence of moral hazard

In this section I establish the results of a benchmark model where actions of agents are observable and verifiable. In the absence of moral hazard the principal can write a contract specifying the level of investments, which agents should allocate into the project (let me denote these investments as c and d for first and second agent respectively). The agents' reward is zero and the whole payoff from the projects is retained by the principal. Therefore, the profit of the principal in the set-up with single agent, competing agents and team is given by Π_P^A, Π_P^C and Π_P^T respectively:

$$\begin{aligned}
\Pi_P^A &= R(1 - e^{-c}) - c, \\
\Pi_P^C &= R(1 - e^{-(c+d)}) - (c + d), \\
\Pi_P^T &= R(1 - e^{-(c^{1-\alpha}+d^{1-\alpha})^{\frac{1}{1-\alpha}}}) - (c + d).
\end{aligned}$$

Note, that for any $\alpha > 0$ and given the amount of invested resources $(c + d)$ the probability of success in team is maximized if $c = d$. Therefore, whenever speaking about team production, I will call the allocation of resources *"efficient"* if the team members allocate equal amounts into the project.
Straightforward maximization of the principal profit in different regimes allows to establish the equilibrium level of investments.

Proposition 2.1. *The equilibrium level of investments in the respective organizational structures is given by:*

(i) *single agent:* $c = 0$ *if* $R \leq 1$ *and* $c = \ln R$ *if* $R > 1$

(ii) *competing agents:* $c + d = 0$ *if* $R \leq 1$ *and* $c + d = \ln R$ *if* $R > 1$

(iii) *team:* $c = d = 0$ *if* $R \leq 2^{\frac{\alpha}{\alpha-1}}$ *and* $c = d = 2^{\frac{1}{\alpha-1}} \ln \left(2^{\frac{\alpha}{1-\alpha}} R\right)$ *if* $R > 2^{\frac{\alpha}{\alpha-1}}$

Further, investigation of the principal's profit under alternative structures leads to several observations. The principal is indifferent between employing competing agents, single agent or team of agents if $\alpha = 0$. For any $\alpha > 0$ the principal is strictly better off employing the team, since it generates larger probability of success.

Corollary 2.1. *For* $\alpha = 0$ *and* $R > 1$ *the principal is indifferent between alternative organizational structures. He is strictly better off employing a team if* $\alpha > 0$ *and* $R > 2^{\frac{\alpha}{\alpha-1}}$.

2.4 Single agent

Let us start the investigation of the principal's problem in the presence of moral hazard with the simplest case, where a single agent is employed. The game has two stages: In the first stage the principal offers a contract, where he determines amount of investment funds c and the agent's share β^A. In the second stage the agent allocates $x \leq c$ into R&D and consumes $c - x$. In case of success the agent receives a reward $R\beta^A$. I will denote the profit of the agent Π^A, where A stays for "single agent". The game is solved backwards, starting from the agent's problem:

$$\max_{x \in [0,c]} \Pi^A = R\beta^A(1 - e^{-x}) + c - x. \tag{2.4}$$

Agent's profit consists of two parts. First, he enjoys a reward $R\beta^A$ in case of success (which happens with probability $1 - e^{-x}$). Second, he consumes part of funds at his discretion, so that $c - x \geq 0$. The agent's problem has the following solution:

1. $x = 0$, if $R\beta^A \leq 1$,

2. $x \in (0, c)$: is such that $R\beta^A = e^x$, if $1 \leq R\beta^A \leq e^c$,

3. $x = c$, if $R\beta^A \geq e^c$.

The principal chooses the terms of the contract, namely the amount of investment c and the reward of the agent $R\beta^A$, taking into account the solution of Problem 2.4. If the principal chooses $R\beta^A \leq 1$, the agent will consume all funds, which leaves a principal with a negative profit $\Pi_P^A = -c$. He can do better by not investing in the project at all. Therefore, if the principal decides to invest in the project, he will never choose $R\beta^A \leq 1$. Hence, we can limit our attention to the investigation of the strategies, which dominate $R\beta^A \leq 1$:

$$\begin{aligned} \max_{c,\beta^A} \quad & \Pi_P^A = R(1-\beta^A)(1-e^{-x}) - c \\ s.t. \quad & (IC_A)\ R\beta^A \geq e^x, \\ & (RC_A)\ x \leq c, \\ & (CS_A)\ (R\beta^A - e^x)(x-c) = 0. \end{aligned} \tag{2.5}$$

The incentive compatibility constraint (IC_A) ensures, that the agent invests in R&D. According to resource constraint (RC_A) he can only invest as much as c. Finally, according to complimentary slackness condition (CS_A) at least one of the two other constrains should be binding, as follows from the equilibrium conditions above. If the incentive constraint does not bind, the agent invests all available funds, so that (RC_A) binds. If the resource constraint does not bind, then the incentive constraint will necessarily be binding.

Both the incentive compatibility constraint and the recourse constraint will bind in the optimum. To prove this important result that I will repeatedly use, assume that (RC_A) does not bind, so that $x < c$, where x is the equilibrium choice of the agent. If the principal marginally decreases c, the agent's investment does not change (the probability of success stays unaltered), but the investment expenditures decline, so that the profit of the principal increases. Hence, in optimal solution the principal always chooses c so, that (RC_A) binds. The same intuition justifies why the incentive compatibility constraint should be binding. Indeed, assume that the constraint does not bind, so that $R\beta^A > e^x$. Then the principal can decrease a share of the agent (hence, increase his own share) without altering the probability of success. So, in optimum the principal will choose such β^A, that (IC_A) constraint binds.

With binding constrains the solution to the principal's problem is immediate. It is formalized in the proposition below.

Proposition 2.2. *Assume that the principal employs one agent. Then in SPNE the following statements hold:*

(i) *the amount of investment resources is given by* $c = \ln \frac{1}{2}(-1 + \sqrt{1+4R})$, *if* $R > 2$

and $c = 0$*, if* $R \leq 2$*.*

(ii) *the agent allocates all funds into the project:* $x = c$*.*

(iii) *the reward of the agent is* $R\beta^A = e^c$*.*

The proof of the fist part is given above. The second and the third part follow directly from the solution of (2.5).
Both the equilibrium amount of investment c and the reward of the agent $R\beta^A$ increase in the value of R. This is the essence of the tradeoff which the principal faces. He is willing to increase his investment, if the project promises a lucrative payoff. However, in order to ensure that the agent does not divert funds to private consumption, the principal has to balance the incentive constraint of the latter by promising him a larger share of the prize.
Note further, that the $c = 0$ as $R = 2$. Since the equilibrium investment expenditures of the principal increase in R, for any $R \leq 2$ the principal will not employ a stand-alone agent in equilibrium. Naturally, the equilibrium level of c is smaller in the presence of the moral hazard, than in the benchmark model, while the value of R which makes financing of a single agent feasible is larger in the former case.

2.5 Competing agents

In a setting with competing agents, the prize is shared between the winning agent and the principal. After the terms of a contract (i.e., agents' reward in case of success and the amount of investments) are announced, the agents simultaneously decide which part of funds they allocate to R&D and which part they consume.
Let the principal transfer amount c to the first agent and amount d to the second agent. Let $x \leq c$ be the funds which the first agent allocates to the project and $c - x \geq 0$ be the funds that he diverts to the private consumption. Likewise, I define y and $d - y$. The second agent wins the prize, if he successfully completes the project at time t_y, such that $t_y \leq t_x$ and $t_y \leq 1$, where t_x is a time, when the first agent completes his project. Hence, the probability that second agent succeeds is:

$$P(t_y \leq t_x \;\wedge\; t_y \leq 1) = P(t_y \leq 1 \leq t_x) + P(t_y \leq t_x < 1) =$$
$$= e^{-x}(1 - e^{-y}) + \int_0^1 \int_0^t xe^{-xt} ye^{-yu} du dt = \left(1 - \frac{x}{x+y}\right)\left(1 - e^{-(x+y)}\right).$$

Then, the expected payoff of the second agent is Π_2^C, where C stands for "competition" is :

$$\Pi_2^C(x,y) = R\beta_2^C \frac{y}{x+y}\left(1-e^{-(x+y)}\right) + d - y,$$

where $R\beta_2^C$ is a reward which the second agent receives according to a contract. Analogically, the expected payoff of the first agent is

$$\Pi_1^C(x,y) = R\beta_1^C \frac{x}{x+y}\left(1-e^{-(x+y)}\right) + c - x,$$

where $R\beta_1^C$ is a reward which the first agent receives according to a contract. Note, that conditional on the fact, that at least one agent succeeds, the probability that first agent succeeds is $\frac{x}{x+y}$ and the probability that the second agent succeeds is $\frac{y}{x+y}$. This result is typical for the literature on contests and patent races (Tullock 1980, Dixit 1987, Loury 1979).

In equilibrium, each agent plays his best response to the rival's strategy by choosing amount of investment x (respectively, y) and taking $R\beta_1^C$, $R\beta_2^C$, c and d as given. Let us consider the best response correspondence for the first agent. The derivative of Π_1^C is given by the following function:

$$\frac{\partial \Pi_1^C}{\partial x} = R\beta_1^C\left(\frac{1-e^{-(x+y)}}{(x+y)^2}y + \frac{x}{x+y}e^{-(x+y)}\right) - 1.$$

Denoting $k_1(x,y) := \dfrac{e^{x+y}(x+y)^2}{x(x+y)+y(e^{x+y}-1)}$ I can write the best response correspondence for the first agent:

1. $x = 0$, if $R\beta_1^C \le \dfrac{y}{1-e^{-y}}$,
2. $x \in (0,c)$ such that $R\beta_1^C = k_1(x,y)$, if $\dfrac{y}{1-e^{-y}} \le R\beta_1^C \le k_1(c,y)$
3. $x = c$, if $R\beta_1^C \ge k_1(c,y)$.

The best response of the second agent can be derived similarly. Depending on parameters, there are following equilibrium candidates in the last stage of the game: $(0,0)$, $(x^*,0)$, $(0,y^*)$, (x^*,y^*), (c,y^*), (x^*,d), (c,d). Here $x^* \in [0,c]$ and $y^* \in [0,c]$ denote the equilibrium level of effort, given by respective incentive compatibility constraint. Incentive compatibility constraints for each equilibrium candidate, are summarized in Table 3 in Appendix (there (IC_c^1) and (IC_c^2) denote the incentive compatibility constraint for first and second agent respectively).

The problem of the principal is to choose the terms of the contract so that the residual expected payoff (gross payoff net of agents compensation) is maximized. I first derive

the optimal contract for each equilibrium candidate and then choose the one, which delivers the principal the highest profit.
If in equilibrium the agents choose $x \in (0,c]$, $y = 0$ or $x = 0$, $y \in (0,d]$, then the outcome of the game is equivalent to the game with a single agent. Solution of the problem in this case is described in previous section. If, given the terms of the contract, the agents invest $(x,y) = (0,0)$ then the principal is better off not financing a project at all, in which case he earns zero profit. For $R > 2$ the principal can do better than that by employing a single agent.
Finally, if $x > 0$ and $y > 0$ in equilibrium, then the problem of principal is to maximize his profit subject to the appropriate incentive compatibility constraints (see Table 3). The principal receives his share of the prize if at least one of the agents wins, which happens with probability $(1 - e^{-(x+y)})$. Since the agents are identical, in equilibrium the principal is going to treat them symmetrically, so that $\beta_1^C = \beta_2^C$ and $c = d$ (this result is formally proved in Proposition 2.3). Further, the optimal contract will be such, that the agents find it just incentive compatible to allocate all recourses which they receive to R&D. In other words, they will receive exactly a share which makes them to invest $x = y = c$ into the project. The intuition behind this result was already discussed in the previous section.
In the reduced form[9], i.e. with binding constrains and symmetric agents the problem of the principal is as follows:

$$\begin{aligned} \max_{\beta^C,\ c} \quad & \Pi_P^C = R(1-\beta^C)(1-e^{-2x}) - 2c \\ s.t. \quad & R\beta^C := R\beta_1^C = R\beta_2^C = \frac{e^{2x}4x}{2x - 1 + e^{2x}}, \\ & x = c. \end{aligned}$$

The solution to this problem leads to the optimal contract and is formalized in Proposition 2.3.

Proposition 2.3. *Let competing agents be employed. Then in equilibrium the following statements hold:*

(i) *Equilibrium level of investment c increases in R and is given by*

$$R = \frac{e^{2c}\left[4c(e^{2c}-1) + 3(e^{2c}-1)^2 + c^2(4+8e^{2c})\right]}{(e^{2c}+2c-1)^2} \qquad \textit{if} \quad R > 2,$$

$$c = 0 \qquad \textit{if} \quad R \leq 2.$$

[9] For the general form of the principal's problem see Proof of Proposition 2.3 in Appendix

(ii) *The agents allocate all funds in R&D:* $x = c$, $y = d$.

(iii) *The agents are treated symmetrically:* $\beta_1^C = \beta_2^C$, $c = d$.

(iv) *The reward of each agent is given by* $R\beta^C = \dfrac{4ce^{2c}}{e^{2c} - 1 + 2c}$.

Remark 2.2. The optimal contract for the competing agents is developed under the assumption, that only the winner of the patent race receives a reward. It is easy to see that the equilibrium outcome of any contract, where the follower also receives some reward, will be strictly worse from the principal's point of view. Indeed, for each amount of investment funds the principal allocates to the project, such contract increases the reward of the agents without altering the probability of success. Hence, in a more general contract, where an agent earns a reward $R\beta_L$ if he wins the patent race, and a reward $R\beta_F$, if he looses this race ("L" and "F" stay for the "Leader" and the "Follower" respectively), the principal will optimally choose $R\beta_F = 0$.

Finally, it remains to verify whether the principal indeed prefers to employ competing agents instead of a single agent. The competition has a twofold effect in this model. Due to the independent success probabilities (let us call this *scale* effect) two competing agents investing x and y succeed with the same probability as a single agent, who invests $(x+y)$. In addition, the competition improves incentives of agents making the diversion of funds for the own uses less attractive (the *disciplining effect*). Indeed, by diverting funds in the competing setting an agent suffers twice: The diversion of funds decreases a probability that the agent wins and increases a conditional probability that his rival wins. Hence the presence of a rival disciplines the agents and makes it cheaper for the principal to provide them with required incentives.

Corollary 2.2. *Let $R > 2$. Then the principal is always better off employing competing agents, rather than a single agent.*

This result conforms to the intuition in the first chapter of this thesis, where it is shown that competition is always beneficial for the principal, when the agents are identical and there are no fixed costs of employing an agent.

2.6 Team production: simultaneous choice of effort

An alternative organization of R&D, which the principal might use, is a research team. In the team the agents join their efforts in order to complete the project. If the project is successful, the prize is divided according to the contract between the principal and the team. I assume, that the principal observes neither individual nor the joint contribution of agents to the project. Hence, each agent's reward must be conditional upon the success of the project. The problem inherited in the team production is a free-riding. If the team wins the prize, each agent receives his share, no matter how much he has invested in the research. Hence, each agent faces a tradeoff between increasing a joint probability of success by investing and increasing his own payoff by consuming the funds.

2.6.1 Optimal contract

The principal signs two separate contracts with both agents, in which he determines investment funds (denoted c and d) allocated to each agent and a reward which an agent receives in case of success (denoted $R\beta_1^T$ and $R\beta_2^T$ for the first and second agent respectively). As before, the game is solved backwards starting from the last stage, where the agents choose x and y, given the terms of contract.
In this section I assume that agents choose their efforts simultaneously. For this reason I will often refer to this setting as the *simultaneous team*. Each member of the team maximizes his own profit (labelled "T" for team), by playing the best response to the investment decision of his teammate, taking the terms of contract as given. The decision problem of the first and second agent respectively are:

$$\max_{x\in[0,c]} \quad \Pi_1^T = R\beta_1^T(1 - e^{-(x^{1-\alpha}+y^{1-a})^{\frac{1}{1-a}}}) + c - x, \tag{2.6}$$

$$\max_{y\in[0,d]} \quad \Pi_2^T = R\beta_2^T(1 - e^{-(x^{1-\alpha}+y^{1-a})^{\frac{1}{1-a}}}) + d - y. \tag{2.7}$$

Consider the problem of the first agent. The first derivative of the payoff function is the following:

$$\frac{\partial \Pi_1^T}{\partial x} = R\beta_1^T(x^{1-\alpha} + y^{1-\alpha})^{\frac{\alpha}{1-\alpha}} x^{-\alpha} e^{-(x^{1-\alpha}+y^{1-a})^{\frac{1}{1-a}}} - 1. \tag{2.8}$$

Let me define $k_2(x,y) := \dfrac{x^{\alpha} e^{(x^{1-\alpha}+y^{1-a})^{\frac{1}{1-\alpha}}}}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}}$. The best response of the first agent to any choice y of the second agent is:

1. $x = 0$, if $R\beta_1^T \leq 1$ and $y = 0$,

2. $x \in (0, c)$ such that $R\beta_1^T = k_2(x, y)$, if $1 \leq R\beta_1^T \leq k_2(c, y)$

3. $x = c$ if $R\beta_1^T \geq k_2(c, y)$.

The best response for the second agent is defined analogically. Depending on parameters of the contract there are following equilibrium candidates in the last stage of the game: $(0,0)$, (x^*, y^*), (c, y^*), (x^*, d), (c, d), where x^* and y^* denote a level of effort corresponding to the appropriate incentive compatibility constraint. The incentive compatibility constraints are summarized in Table 4 in Appendix (there ICT_1 and ICT_2 are the incentive compatibility constraints for first and second agent respectively).

Solving for SPNE, there are several considerations to be taken into account. First, in equilibrium the principal will give the agents exactly the amount of money, which they are willing to invest. Hence, in equilibrium $x = c$ and $y = d$. Further, the agents' rewards must be just sufficient to ensure that in equilibrium they allocate all resources into the project. Hence, in equilibrium the incentive compatibility constrains of both agents are going to be binding. The intuition behind this result was discussed in Section 2.4. Finally, as I have already mentioned in Section 2.3, the probability of success in team is maximized if agents allocate equal amounts to R&D. Hence, the principal will offer symmetric contracts to both agents, so that $x = y = c = d$ (this result is proved in the Proposition 2.4). The problem of the principal can be written in the following reduced form:[10]

$$\max_{\beta^T, c} \quad \Pi_P^T = R(1 - 2\beta^T)(1 - e^{-2^{\frac{1}{1-\alpha}} x}) - 2c \tag{2.9}$$

$$s.t. \quad R\beta^T := R\beta_1^T = R\beta_2^T = \frac{e^{2^{\frac{1}{1-\alpha}} x}}{2^{\frac{\alpha}{1-\alpha}}},$$

$$x = c.$$

Proposition 2.4. *Assume that agents in a team contribute their investments simultaneously. Than, in SPNE the following holds:*

(i) *if* $R > 1 + 2^{\frac{1}{1-\alpha}}$*, then there is a unique equilibrium (full-investment equilibrium). In this equilibrium agents allocate all funds into R&D.*

(ii) *if* $3 \cdot 2^{\frac{\alpha}{\alpha-1}} < R \leq 1 + 2^{\frac{1}{1-\alpha}}$ *then there are two SPNEs: full-investment equilibrium with* $(x, y) = (c, c)$ *and no-investment equilibrium with* $(x, y) = (0, 0)$.

(iii) *if* $R \leq 3 \cdot 2^{\frac{\alpha}{\alpha-1}}$ *the project is not financed.*

[10] The general form is given in the proof to Proposition 2.4

In the full investment equilibrium the optimal contract has the following features:

(i) *the contracts are symmetric:* $\beta_1^T = \beta_2^T$, $c = d$.

(ii) *each agent's reward is* $R\beta^T = \frac{e^{2^{\frac{1}{1-\alpha}}c}}{2^{\frac{\alpha}{1-\alpha}}}$.

(iii) *amount of investment funds is* $c = 2^{\frac{1}{\alpha-1}} \ln \frac{1}{4}\left(-1+\sqrt{1+2^{\frac{1}{1-\alpha}}4R}\right)$.

The most surprising result of the Proposition 2.4 is that teams with very high synergy effects (α close to 1) might end up investing nothing to the research and development. From the proposition it follows, that the higher is the synergy effect, the higher should be the price in stake to ensure full-investment equilibrium. Actually, for α close to 1, the prize should be infinitely large to ensure unique (c, c) equilibrium. For smaller prizes there is a second equilibrium $(0, 0)$. The intuition is that for high synergy effects it is sufficient to invest a small amount in order to have a success with high probability. Hence, the principal will allocate relatively small c to the project and consequently will promise low reward to the agents. If one of the team peers decides to divert funds, the other has low probability of winning the prize alone, and the share is not large enough to justify the effort.

However, the equilibrium $(0, 0)$ is Pareto-dominated by equilibrium (c, c). Indeed using the Proposition 2.4, the expected profit of each agent in the (c, c) equilibrium is $\Pi_i^T(c, c) = 2^{\frac{\alpha}{\alpha-1}}(e^{2^{\frac{1}{1-\alpha}}c} - 1)$, where $i = \{1, 2\}$. In equilibrium $(0, 0)$ each agent earns $\Pi_i^T(0, 0) = c$. It is then straightforward, that $\Pi_i^T(c, c) > \Pi_i^T(0, 0)$ for any $\alpha \in [0, 1)$ and $c > 0$. In line with the theoretical literature (see for example Harsanyi and Selten 1992) I will consider the Pareto-dominant equilibrium a natural focal point and will therefore assume that the agents are able to coordinate on the full - investment equilibrium.[11]

Note, that this equilibrium selection is also in line with the experimental literature. On one hand this literature shows, that in the games with Pareto-ranked equilibria (called coordination games), the Pareto-dominant Nash equilibrium is not always the unique outcome. However, according to Cooper, DeJong, Forsythe and Ross (1990) the coordination failure is likely to happen when decisions of the agents are influenced by the presence of a cooperative dominated strategy, which gives the agents larger payoff, than the Pareto-dominant equilibrium. Their results suggest, that otherwise the agents are likely to choose the Pareto-dominant equilibrium.

[11] According to Harsanyi and Selten (1992) p.221-223, among two equilibria U and V, equilibrium U dominates equilibrium V if it results in strictly higher payoff for both players. The authors take a point of view, that "there is no risk involved in a situation, where expectations can be coordinated by common payoff interests of the relevant players".

It is easy to show in my model that, given the optimal contract, the joint profit of a team is maximized if agents invest $(x, y) = (c, c)$. Therefore, there is no cooperative strategy which would give the agents a higher outcome than the equilibrium (c, c). Hence also the argument of Cooper et al. (1990) is in favor of selecting the equilibrium (c, c) over $(0, 0)$.

2.6.2 Team versus competition

The compensation in a setting with competing agents is a special case of relative performance evaluation scheme (RPE), which is used to penalize a member of a team, who performs worse than his pears (see Holmstrom 1982, Mookherjee 1984, Che and Yoo 2001). This feature is also present in my model: if the agents compete, each of them is rewarded only if he has better result than his rival (which also means that he wins the prize). On the other hand, the compensation scheme in a team rewards each entrepreneur if the whole team performs well; this is so-called joint performance evaluation (JPE). The insights from the optimal contract literature suggest that in a one-shot game the optimal payment scheme for teams is RPE. Intuitively, this conclusion should also hold if we compare the competing agents and team without synergy effects. However, in the presence of synergies the team could potentially become an attractive arrangement, if the increase in success probability due to synergies is sufficiently high.
I show in Proposition 2.1 that in the absence of moral hazard the principal is better off employing a team for any $\alpha > 0$. Moral hazard, however, changes this result significantly. According to Propositions 2.3 and 2.4 the reward and amount of investment funds which agents receive in equilibrium are such that both agents find it incentive compatible to invest entire funds in the project. Hence, the principal has two mechanisms how to induce the most efficient investment decision: the size of funds and the size of reward. For given prize R and synergy effect α, the larger are the funds which agents receive, the larger is the amount which they can potentially divert from investing. On the other hand, the larger is their reward, the more prone are the agents to invest in the project. Therefore, the principal always faces a tradeoff between increasing his investment (hence increasing the probability of success) and increasing the reward of the agents in order to balance their incentive constraint. Intuitively, because of the inherited free-riding problem it is more difficult to provide required incentives for team, than for competing agents. Hence, one should expect that the team will never be a preferred arrangement if the technological benefits of team production are moderate. This intuition is confirmed in Proposition 2.5.

Proposition 2.5. *Let $R \geq 2$, so that $\Pi_P^C \geq 0$. Then the following statements hold.*

1. *The principal prefers to employ competing agents, rather than a team for any $\alpha < \alpha_1$, where $\alpha_1 = \frac{\log 3 - \log 2}{\log 3} \approx 0.37$.*

2. *For any $R \geq 2$, let $\hat{\alpha}(R)$ denote a solution of $\Pi_P^T(\alpha, R) - \Pi_P^C(R) = 0$. Then, $\hat{\alpha}(R)$ is an increasing function and converges to 1 as $R \to \infty$.*

On Figure 2.1 the line, labelled $\hat{\alpha}(R)$, shows combinations of α and R, such that the principal is indifferent between team and competition. According to the corollary above, $\hat{\alpha}(R)$ converges to 1 as R becomes infinite.

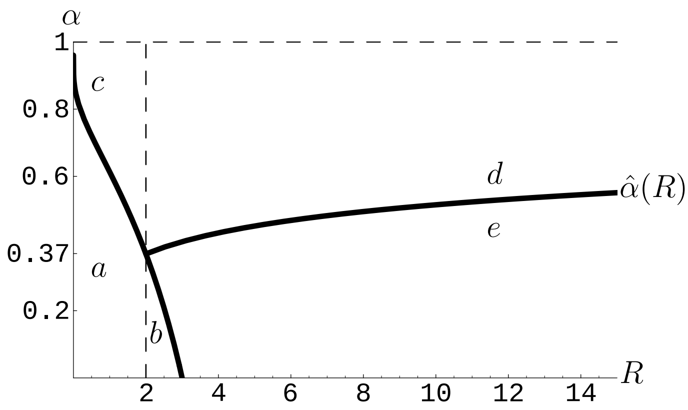


Figure 2.1: Competing agents versus team.

Competing agents will never be employed, if $R < 2$ (on the Figure 2.1 the corresponding regions are labelled a and c). On the other hand, the team will never be employed, if $R \leq 3 \cdot 2^{\frac{\alpha}{\alpha-1}}$ (the region corresponds to the union of a and b on the picture). If $\alpha > \alpha_1$, then $3 \cdot 2^{\frac{\alpha}{\alpha-1}} < 2$ and for all $3 \cdot 2^{\frac{\alpha}{\alpha-1}} < R < 2$ the profit of the principal is positive if he employs team, but is negative if competing agents are employed (region c). The principal will not finance a project, if $3 \cdot 2^{\frac{\alpha}{\alpha-1}} > R$ (region a). For all values of R, such that $2 < R < 3 \cdot 2^{\frac{\alpha}{\alpha-1}}$ the profit of the principal is positive, if he employs competing agents, but is negative, if he employs a team (the corresponding region is b). When the parameters R and α are such that both the team and competing agents generate a positive profit (regions d and e), the principal chooses an arrangement, which maximizes his surplus from the project: team in region d and competition in region e.

Let $R \geq 2$, so that financing of competing agents is feasible. According to the Corollary 2.5, the team is never a preferred arrangement, if the synergy effects are moderate ($\alpha \leq \alpha_1$). If $\alpha \to 1$, than team is always a preferred arrangement, because it generates success with almost certainty. However, for $\alpha_1 < \alpha < 1$ the competition becomes more beneficial from the principal's point of view, as R increases. This result is due to the strict incentive compatibility constraint which the principal faces financing a team. In particular, if the synergy effects are high in team, the agents can generate high probability of success by investing small amounts. Hence, it is very tempting for them to divert fraction of investment resources. To balance the incentive compatibility constraint the principal is forced to limit his investments in team. On the other hand the disciplining effect of competition enables the principal to increase invested resources significantly, while increasing the incentive compatible reward only marginally. Therefore, for sufficiently large R, competition becomes more attractive arrangement for the principal, even when team production is beneficial from the technological point of view.

2.7 Team production: Sequential choice of effort

The literature has extensively discussed mechanisms, which can reduce a moral hazard within the team and hence increase the surplus of the principal. In one way or another many of these mechanisms utilize an ability of team members to observe their peers' effort.

Che and Yoo (2001) show that repeated interaction reduces the moral hazard in a team if after each period each agent can observe whether his team peer has shirked. In the present model, however, I concentrate on one-time interaction between the principal and the agents. Namely, the interaction will be terminated, as soon as the project succeeds or as soon as the maximal financing horizon elapses. In such static setting the ability of the team members to monitor each other can be useful, if it creates a potential for peer pressure, such as mental or physical harassment (Barron and Gjerde 1997, Kandel and Lazear 1992). Alternatively, the ability of agents to observe each other effort may be utilized by a principal through a mechanism which requires agents to report the effort executed by their team peers (Marx and Squintani 2002).

I will assume that a contract between the principal and an agent, which requires the latter to monitor his team peers and to report on them is infeasible.[12] I will also disregard the effect of psychological peer pressure. Abstracting from these two mechanism allows me to investigate the strategic effects which arise from the ability of agents to observe the effort of their peers in the environment where the agents are instructed to contribute their efforts sequentially.

[12] This may be the case if agents face moral costs "spying" on their team peers.

Consider a game where agents, cooperating in the team, execute their effort sequentially (the team is henceforth referred to as *sequential team*). The game has three stages. In the first stage the principal offers a contract $(R\beta_1, c)$ to the first agent (the leader) and a contract $(R\beta_2, d)$ to the second agent (the follower). In the second stage, the leader executes an effort $x \leq c$; the follower observes an effort of the leader (but not the outcome of this effort) and executes an effort $y \leq d$. In the third stage payoffs are realized. As before, I assume that the principal observes neither individual nor the joint contribution of agents to the project. For the time being I assume that the principal nevertheless is able to ensure that the efforts are contributed sequentially.[13]
The decision problem of the follower is given by (2.7). In equilibrium the principal always chooses contracts, where first-order conditions of the leader's and the follower's problem are satisfied with equality and the rewards, promised to both agents are just sufficient to induce an effort $x = c$ and $y = d$. The intuition behind this observation was already discussed in Section 2.4.
Taken this consideration into account, one can obtain the incentive compatibility constraint (IC) of the follower's problem:

$$R\beta_2 = \frac{y^\alpha e^{(x^{1-\alpha}+y^{1-a})^{\frac{1}{1-\alpha}}}}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}} = \frac{y^\alpha e^t}{t^\alpha}, \tag{2.10}$$

where $t = (x^{1-\alpha} + y^{1-\alpha})^{\frac{1}{1-\alpha}}$ is the team contribution to the project.
Using Equation (2.10), it is possible to express x and y as follows:

$$y = t\left(\frac{R\beta_2}{e^t}\right)^{\frac{1}{\alpha}}, \quad x = t\left(1 - \left(\frac{R\beta_2}{e^t}\right)^{\frac{1-\alpha}{\alpha}}\right)^{\frac{1}{1-\alpha}}. \tag{2.11}$$

Making his investment decision, the leader takes into account the reaction function of the follower, which is implicitly given by equation (2.10)

[13] The assumption that the follower can observe the effort of the leader, but not the outcome of his task is in the spirit of Winter (2005) and Strausz (1998).

$$\max_{x\in[0,c]} \quad \Pi_1^S = R\beta_1(1-e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}) + c - x \tag{2.12}$$
$$s.t. \quad R\beta_2 = \frac{y^\alpha e^t}{t^\alpha},$$
$$x = t\left(1-\left(\frac{R\beta_2}{e^t}\right)^{\frac{1-\alpha}{\alpha}}\right)^{\frac{1}{1-\alpha}},$$
$$x\in[0,t], t\in[0,(c^{1-\alpha}+d^{1-\alpha})^{\frac{1}{1-\alpha}}].$$

Note, that $x \in [0, t]$ implies $0 \le (\frac{R\beta_2}{e^t})^{\frac{1-\alpha}{\alpha}} \le 1$.
As discussed above, in equilibrium the first order condition of the leader's maximization problem is satisfied in the interior of the feasible set. Hence, the first order condition can be written as follows:

$$R\beta_1 = e^t(1-B)^{\frac{\alpha}{1-\alpha}}\left(1+B\frac{t-\alpha}{\alpha}\right), \text{ where } B := \left(\frac{R\beta_2}{e^t}\right)^{\frac{1-\alpha}{\alpha}}, B\in[0,1] \tag{2.13}$$

The IC constraint resulting from the follower's problem can also be re-written as a function of B and t:

$$R\beta_2 = B^{\frac{\alpha}{1-\alpha}}e^t. \tag{2.14}$$

Finally, the principal's decision is described by the following maximization problem:

$$\max_{c,d,R\beta_1,R\beta_2} \quad \Pi_P^S = (R-R\beta_1-R\beta_2)(1-e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}) - (c+d) \tag{2.15}$$
$$\text{s.t.} \quad x=c, y=d,$$
$$x+y = t(B^{\frac{1}{1-\alpha}} + (1-B)^{\frac{1}{1-\alpha}}),$$
$$(2.13), (2.14).$$

The problem of the principal can be expressed in terms of B and t. Note, that if the principal chooses $B = 0$ or $B = 1$, he effectively employs only one agent.
The first order conditions to this problem are complicated and generally can not be solved analytically.[14] Nevertheless, it is possible to show that for some parameters the principal can improve performance of the team if he instructs agents to contribute

[14]In order to characterize the optimal structure of research department on Figure 2.2 I solve the problem numerically using the first order condition for variable t and the grid of $0.01 \times 0.01 \times 0.01$ for parameters α and R and variable B.

effort sequentially, rather than simultaneously.

Proposition 2.6. *There exist an open set of parameters, where the principal is better off if agents contribute their investments sequentially, rather than simultaneously.*

On the Figure 2.2 I characterize the optimal organization of the research department depending on parameters (α, R). The solid lines divide regions, corresponding to the different structures of research department. The dashed lines are used to provide comparison to Figure 2.1. In the shaded region (labelled "S") the principal employs the team and instructs the agents to contribute effort sequentially. In the region "T" the principal employs a team and instructs agents to contribute their effort simultaneously. In the region"C" the principal employs competing agents. Finally, in the region "N" the project is not financed.

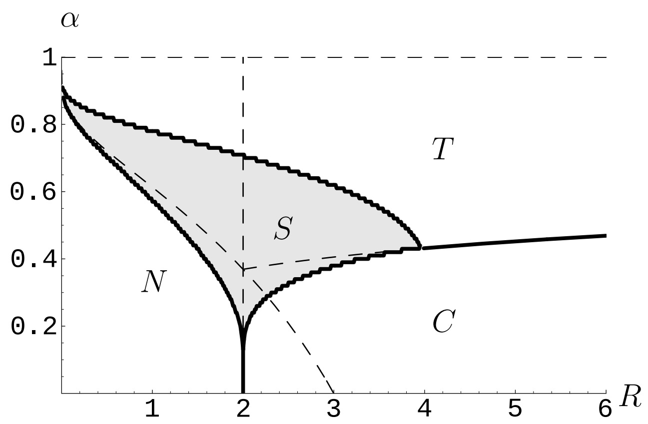


Figure 2.2: Optimal structure of research department.

Let us discuss the intuition behind the decision of the principal to use sequential team for some range of parameters. Applying the implicit function theorem to Equation (2.10), I obtain the derivative of the follower's reaction function (let me call it $R_y(x)$):

$$\frac{dR_y(x)}{dx} = \frac{y^{1+\alpha}(\alpha - (x^{1-\alpha} + y^{1-\alpha})^{\frac{1}{1-\alpha}})}{\alpha x y^{\alpha} + x^{\alpha} y (x^{1-\alpha} + y^{1-\alpha})^{\frac{1}{1-\alpha}})} = \frac{y^{1+\alpha}(\alpha - t)}{\alpha x y^{\alpha} + x^{\alpha} y t}.$$

The reaction function of the follower increases in the effort of the leader if $\alpha > t$. To illustrate the intuition let $\bar{x}$ and $\bar{y} = y(\bar{x})$ be such that $t(\bar{x}, \bar{y}) = \alpha$ and $t(x, y(x)) > \alpha$ for any $x > \bar{x}$. If x is small ($x < \bar{x}$) the follower is willing to increase his investments in response to an increase in x. Indeed, for small x he can considerably improve the probability of success by allocating funds into the project. Therefore, his marginal gain from additional monetary unit invested in the project is larger, than from consuming it. The incentives of the follower are exactly opposite if x is large. If the leader invests significant amount of resources, it is sufficient to generate a high probability of success. The follower now can afford to free-ride on the effort of the leader and is prompted to reduce his investments in response to an increase in x.

The sequential structure does not affect the incentives of the follower but alters significantly the incentives of the leader. In the sequential setting increasing function $R_y(x)$ makes the leader reluctant to shirk in the fear that so would do the follower, which would cause the failure of the project. Hence, when the parameters (α, R) are such that $\alpha > t$ in equilibrium (the efforts of agents are *strategic complements*), the sequential structure works in the favor of the principal making it easier to provide incentives for the leader. In particular, as I demonstrate in the proof of Proposition 2.6, the leader may even be ready to contribute the same effort as the follower for smaller reward.

On the contrary, if optimal contract results in $\alpha < t$ (the efforts are *strategic substitutes*), the reaction function of the follower decreases in the effort of the leader. In this case the sequential structure works against the principal by creating what Gould and Winter (2005) call a "negative peer effect": a high effort of one agent decreases the effort of another agent.

Note, that $\alpha > t$ is a necessary but not a sufficient condition for the principal to prefer the sequential setting. Although the sequential setting improves incentives on behalf of the leader, it may also result in asymmetric allocation of funds. Since efficient allocation of resources requires $c = d$, the principal may still prefer the simultaneous setting for $\alpha > t$. I use the numerical calculations to illustrate this fact on the Figure 9 (see Appendix). The set of parameters with $\Pi_P^S > \Pi_P^T$ and $\alpha > t$ are almost identical (up to the region "B", where $\Pi_P^S < \Pi_P^T$, but $\alpha > t$).[15]

The result that sequential structure improves incentives of agents only if their efforts are complementary is in line with evidence provided by Gould and Winter (2005). Using data on professional baseball they show that performance of the agents with complementary tasks increases with the performance of their fellows. On the contrary, performance of agents with substitutable tasks decreases in the performance of their colleagues.

Another paper, which proves a similar result, is Ludwig (2007). The author shows that the principal is better off under sequential structure if the contributions of agents are perfect complements. The author, however, provides different intuition for this result.

[15]Note, that region "S" on the Figure 9 is a subset of the region "A" on the Figure 2.2. The former characterize a range of parameters, where $\Pi_P^S > \Pi_P^T$ and $\Pi_P^S > \Pi_P^C$, while the latter characterizes a larger set, where $\Pi_P^S > \Pi_P^T$.

In the model the principal gains from the sequential structure because it makes easier for him to provide incentives for the follower. In my model, however, the principal gains because strategic complementarity makes it easier for him to provide incentives for the leader.

2.8 Hierarchical team

In the previous section I showed that the principal can benefit from the sequential team production because it improves incentives of the team leader if efforts of the agents are strategic complements. The sequential team however has an important drawback. The principal can not observe efforts of the agents, therefore his ability to coordinate who is the first to exert effort and who is the second may be limited. Since the contracts in sequential team are generally not symmetric, the agents themselves will fail to coordinate the sequence of moves. For example, if the contract is such, that the follower benefits more than the leader, both agents will not agree on who is going to be the leader.[16] In this situation to enforce the sequential contribution of effort the principal may appoint a team leader who has a discretion to decide on allocation of funds and tasks between himself and his subordinate and can enforce the sequential contribution of effort. In this section I analyze how such hierarchical structure alters the incentives of agents and the production efficiency.

It is clear that the profit of the principal is at least as large in the sequential team setting as in the setting with hierarchical team. Indeed, if the principal is able to enforce a sequential production, then (using the appropriate incentives) he can replicate the equilibrium outcome of the hierarchical team. Nevertheless, teams where one of the members has a higher level of hierarchy are common. Therefore, it is useful to understand the limits of such set up. As I show below, hierarchical team is inefficient. This inefficiency stems from the reluctance of the team leader to employ a subordinate. He prefers to complete a large part of the project himself, making therefore only minor use of synergy effect.

I will assume, that in the case when a team is structured as a hierarchy the principal signs a contract only with the team leader. The contract determines the total amount of investment expenditures c, distributed to the team, and the total reward $R\beta_1 \leq R$ which the team receives in case of success. The team leader decides whether to employ a second agent (a subordinate) and offers him a contract. Since the leader

[16]This result is in the spirit of Dowrick (1986), who shows that in the Stackelberg duopoly model the agents may disagree over the choice of roles of the leader and the follower.

has no wealth of his own, he can only reward the subordinate by distributing him a share $R\beta_1\beta_2 \leq R\beta_1$ of the total reward and allocating him the part of total investment funds $d \leq c$.
I will assume that the agents can observe but not verify the effort of each other.[17] Further, it is natural to assume that the team leader can (unlike the principal) induce the subordinate to perform investment before, after or simultaneously with himself. The team leader, obviously, is at least as well off in a sequential setting, where he has the role of a Stackelberg leader, as in a simultaneous setting. Hence, he is indeed willing to induce the sequential contribution of effort.
In the following I will concentrate on the case where the team leader has a position of a Stackelberg leader at the investment stage of the game (he exerts effort before his subordinate). I will then briefly discuss the alternative situation where the team leader has the position of a Stackelberg follower.
The timing of the game is as follows:

1. The principal signs a contract $(R\beta_1, c)$ with the team leader.
2. The team leader decides whether to employ a subordinate. If the subordinate is employed, then he and the leader sign a contract $(R\beta_1\beta_2, d)$, where $d \leq c$.
3. The team leader allocates $x \leq c - d$ into the project.
4. The subordinate observes x and chooses the level of investment $y \leq d$.
5. The outcome is realized and the payoffs are distributed

The game is solved by backward induction starting from the problem of the subordinate. He chooses the optimal level of investment y, such that $y \leq d$:

$$\max_{y\in[0,d]} \Pi_2^H = R\beta_1\beta_2(1 - e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}) + d - y.$$

The derivative of the profit function is

$$\frac{\partial \Pi_2^H}{\partial y} = R\beta_1\beta_2 y^{-\alpha}(x^{1-\alpha} + y^{1-\alpha})^{\frac{\alpha}{1-\alpha}} e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} - 1.$$

I have already discussed, why the principal always chooses such a contract, that the agents find it just incentive compatible to invest all funds which they receive into

[17]If the team leader is able to observe and verify the effort of his subordinate, the moral hazard on behalf of the latter is eliminated. This, obviously, improves the performance of the team.

R&D. The same argument applies in the case when the first agent acts as a principal to the second agent. The team leader will choose such combination $(R\beta_1\beta_2, d)$, that for any given x the second agent finds it just incentive compatible to allocate all funds into R&D. Hence, the first order condition to the subordinate's problem will be satisfied at the boundary $y = d$ of the set $[0, d]$. Defining $t := (x^{1-\alpha} + y^{1-\alpha})^{\frac{1}{1-\alpha}}$, I can re-write the first order condition of the subordinate's problem in the following form:

$$R\beta_1\beta_2 = \frac{y^\alpha}{t^\alpha}e^t. \tag{2.16}$$

Note that in equilibrium the team leader always invests at least as much as his subordinate, so that $x \geq y$. Indeed, the probability of success $p(x, y) = 1 - e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}$ is symmetric in x and y. On the other hand, a share which has to be paid to the subordinate increases in y. Hence, if the team leader wants to implement the total amount of investment $x + y$, he is better off choosing such contract and level of investment, that $x \geq y$.

The presence of synergies creates a complicated tradeoff for the team leader. He can employ the second agent and allow him to allocate part of investment funds into the project, which will increase the probability of success due to synergy effect. However, he also has to promise this agent a share of the prize, large enough to deter the latter from consuming the investment funds. Hence, by employing the subordinate the team leader suffers twice: he has to give away part of the funds, which he otherwise could consume himself, and he has to give up part of the reward in case of success. It is therefore intuitive, that the team leader should be reluctant to employ the subordinate and if he does employ him, he allocates to the latter only a small part of total investment funds.

Taking into account the reaction function of the subordinate, the team leader solves the following problem:

$$\begin{aligned} \max_{\{d,\beta_2,x\}} \quad & \Pi_1^H = R\beta_1(1-\beta_2)(1-e^{-t}) + c - (d+x) \\ s.t. \quad & t = (x^{1-\alpha} + y^{1-\alpha})^{\frac{1}{1-\alpha}}, y = d, \\ & 0 \leq d \leq t,\ x \geq 0, \\ & d + x \leq c, \\ & R\beta_1\beta_2 = \frac{y^\alpha}{t^\alpha}e^t. \end{aligned} \tag{2.17}$$

In the solution to the team leader's maximization problem (2.17), the derivatives with respect to d and t become:

$$\begin{aligned}\frac{\partial \Pi_1^H}{\partial d} &= -1-\alpha d^{\alpha-1}(e^t-1)t^{-\alpha}+d^{-\alpha}(t^{1-\alpha}-d^{1-\alpha})^{\frac{\alpha}{1-\alpha}},\\ \frac{\partial \Pi_1^H}{\partial t} &= e^{-t}t^{-(1+\alpha)}\left(\alpha d^{\alpha}e^t(e^t-1)-t\left(-R\beta_1 t^{\alpha}+e^t(d^{\alpha}e^t+(t^{1-\alpha}-d^{1-\alpha})^{\frac{\alpha}{1-\alpha}})\right)\right).\end{aligned}$$

Investigation of the first-order conditions allows to make a number of propositions about the investment decisions of the agents.

Proposition 2.7. *Let the team leader have a position of the Stackelberg leader at the investment stage of the game. Then the following statements hold.*

(i) *The team leader will not employ agent the second agent, whenever $\alpha \leq \alpha_2$, where $\alpha_2 \approx 0.43$ solves the equation $1+\alpha=(1-\alpha)^{\frac{1-\alpha}{2\alpha-1}}(1-2\alpha)$.*

(ii) *If the team leader employs the second agent, then the allocation of investment resources between the agents is suboptimal, i.e., $d < x$.*

It follows from Proposition 2.7 that the hierarchical team is an inefficient arrangement. Due to the suboptimal allocation of resources between the agents, it generates the smaller probability of success for given c, then the simultaneous team. On Figure 8 in Appendix 2.B I illustrate the allocation of funds in case of hierarchical team for $\alpha = \frac{2}{3}$. This example shows, that the team leader is willing to transfer the second agent only a small fraction of the investment recourses, which he receives from the principal. In other words, the team leader makes only a minor use of the synergy effects, compared with efficient allocation $x = d$. Therefore, if the principal opts for a hierarchical team, then in order to achieve the same probability of success as in simultaneous team, he has to invest more in the former case.
According to the Proposition 2.7, for any $\alpha < \alpha_2$ the hierarchical structure is infeasible for the principal in a sense that in equilibrium the team leader will not employ the subordinate. Consider therefore such (α, R), that the hierarchical team is feasible. Following the intuition of Section 2.7, the sequential nature of the research process will have a positive effect on the leader's incentives, if the efforts of agents are complementary. Since in the hierarchical team the resources are allocated inefficiently, the principal can replicate any probability of success in the simultaneous setting incurring smaller costs. He, however, might still benefit from the hierarchical team, if it allows sufficient reduction in the reward distributed to the agents.

Corollary 2.3. *Let the team leader have the position of the Stackelberg leader at the investment stage of the game. Then there exists an open set of parameters (α, R),* *such that the hierarchical team is feasible for the principal and he prefers it to a simultaneous team and competing agents.*

The natural question which arises in the framework of hierarchical team is whether the team leader prefers a position of a Stackelberg leader or Stackelberg follower in the investment stage of the game. The analysis of the latter situation is untractable in the framework of this model. It is possible to show, however, that the inefficiency resulting from the suboptimal allocation of resources between the leader and the follower is present also in this case.

2.9 Conclusion

In this paper I investigate the alternative organizational designs of a stochastic production process (i.e. R&D) in a framework where financing decisions (made by a principal) and allocation decisions (made by agents) are separated. The allocation decisions are not observable to the principal, which creates a moral hazard problem. The common implication for the contracts between the principal and the agents is that the principal has to increase a reward which the agents obtain in the case of success, when he decides to increase the amount of financial resources invested in the project. Otherwise, the agents will consume part of funds, instead of contributing them into R&D.

Different structures have, however, different effect on the incentives, which leads to decrease or increase in the rent, allocated to the agents. Comparison of alternative structures leads to several conclusions about the optimal organisation of the production process.

First conclusion deals with the tradeoff between team production and production by competing agents. While the team is technologically more efficient, competition has advantages from the incentives' point of view. The team production is shown to be optimal when the technological gains (synergy effect) are significant and the prize in stake is not too large. For any level of synergy effects, however, as the prize in stake increases, the competition becomes more attractive arrangement from the principal's point of view.

The second conclusion concerns the tradeoff between the sequential and the simultaneous team production. The sequential team production allows agents to observe the effort of their team peers, which may have positive effect on the incentives. Yet, the sequential structure improves the incentives only if the efforts of agents are strategic complements. If the efforts are strategic substitutes, then the high level of effort,

contributed by one agent makes the other agent more prone to shirk. This result is in line with empirical evidence by Gould and Winter (2005). Investigating the performance of professional baseball players the authors conclude, that the performance of players with substitutable tasks is negatively correlated and the performance players with complementary tasks is positively correlated.
The last conclusion is related to the tradeoff between hierarchical team and team without hierarchy. The hierarchical structure allows the principal to induce the sequential contribution of effort in the environment where he is unable to do it himself. The hierarchical team, however, always leads to the suboptimal allocation of resources, being therefore less efficient, than the simultaneous team. Still, for the range of parameters where the agents' investments are strategic complements, the sequential nature of production process has a positive effect on the incentives. If this positive effect counterbalances the loss of efficiency, the principal can use hierarchical structure to improve performance of the team.
The paper has a number of interesting implications for the organization of production process. It suggests that we should observe principals switching to financing competing teams, rather than a consortium of teams, as the prize in stake increases. The casual examples, such as the one cited in introduction, indicate that this indeed might be the case. Sometimes the necessary degree of competition is provided by the market, as was the case with Human Genome Project Consortium, the success of which was fuelled by the competition with the private firm Celera.[18] But it is easy to see that (at least in the framework of this model) a profit - maximizing principal would prefer to keep competing teams "in house", rather than rely on the competition with the external team (in which case he is left empty-handed whenever the other team wins).
The paper also implies, that in an environment where a team is organized as a hierarchy and a team leader has difficulties verifying the effort of his subordinates, we should observe the team leader executing significantly larger effort, than his team peers. This result is similar to Hermalin (1998) who concludes that a team leader should work harder than his subordinates. His result relies, however, on the fact that the leader has an information which is not available to the other team members and signals this information by "leading by example". I provide yet another mechanism which explains why team leaders tend to work harder than their subordinates. This mechanism is important if a team leader can decide on the allocation of reward be-

[18] In 2003 HGP Consortium and Celera simultaneously and independently published the sequence of human genome, see `news.bbc.co.uk/1/hi/sci/tech/2940601.stm`

tween himself and his subordinate (which is often the case in R&D). Notice, that in this case unlike in Hermalin (1998), the tendency of the team leader to exert higher effort than his subordinate reduces the efficiency of production.

The obvious question that arises in connection to the results presented in the paper, is what should be the optimal structure of a production process with more than two agents? There is a rich set of possibilities from N competing agents to the multiple levels of hierarchy. The investigation of this interesting problem is a direction for a future research.

Chapter 3

On Compatibility in Two-Sided Markets

3.1 Introduction

The problem of compatibility choice in the framework of markets with network externalities has received much attention in the literature. This is not surprising, since the compatibility of products in this environment affects the size of relevant network and hence the incentives of agents to buy a particular product. Any decision of a firm operating in such market, from R&D to the introduction of upgrades, crucially depends on the fact whether its product is compatible with those of a rival or/and with the previous generations of the same product. It is surprising, however, that investigating compatibility choice, the literature did not pay much attention to the fact that many of the markets exhibiting network externalities are two-sided markets.[1]

Indeed, examples of two-sided markets are numerous. First of all, they include many industries of the classical economy: newspapers and TV-channels, commercial fairs, dating agencies and night clubs, shopping malls, etc. The most prominent examples, however, are related to the New Economy in general and to software platforms in particular. Operating systems, video-game consoles, payment cards, smart phones and PDA's all share features of two-sided (or, more generally, multi-sided) markets. In a recent book, Evans, Hagiu and Schmalensee (2006) describe multi-sided software platforms as *invisible engines* that "are in the process of transforming industries ranging from automobiles to home entertainment" (p. vii).

[1]Rochet and Tirole (forthcoming) define two-sided markets as markets, where one or several platforms enable interaction between two distinct group of agents and the volume of transaction is affected by a price structure.

In this paper we investigate the choice of compatibility between two generations of platforms (old and new) in the framework of two-sided markets. We provide classification of the compatibility regimes that one can observe on two-sided markets and develop a theory that explains how the choice of a particular regime depends on the characteristics of the market (the size of the installed base and the market growth rate) and technological features of the new platform. We show that the driving force which determines the choice of a compatibility regime is the tradeoff between incentives of the new agents on one side of the market and the incentives of the installed base on the other side of the market.

This paper is motivated by two observations. First, compatibility of technologies on two-sided markets has several regimes. Obviously, platforms may be incompatible with each other. GameCube, a video game console of Nintendo, is incompatible with its predecessor, N 64. Further, platforms may be backward compatible for one side of the market. Sony PlayStation 3, for example, is backward compatible with Sony PlayStation 2, its predecessor: a user of the former can play any games designed for the latter. Finally, platforms may be fully compatible with each other, as is the case with Palm OS. Not only a user of Palm OS can run on it any program designed for the older version of this operation system, but also any program designed for the new version of operation system can be run on the older version.

The second observation is that the choice of compatibility not only differs across industries (as illustrated by examples above) but (for the same firm) across time periods. As an example, consider Nintendo, which, after producing generations of incompatible game consoles, made its new game console, Wii, backward compatible with its predecessor, GameCube.

To explain these observations and to provide a theory of compatibility choice in two-sided markets we consider a framework with two platforms owned and operated by a single firm (referred to as the *monopolist*). The platforms enable interaction between two groups of agents, labelled as *users* and *sellers*. One of the platforms represents an old generation of technology and the other platform represents a new generation of technology. The new platform is superior to the old one in the extent of network benefits (which we also call *per-interaction benefits*) that it confers to users and sellers. In addition it has some intrinsic benefits, that are independent on the size of network, and reflect fashion or alternative uses of the platform. The size of network benefits and stand-alone benefits determine the extent of technological progress.

The old platform has an installed base: some users and sellers are already subscribed to this platform and can use it to interact with each other. In addition, there is a number of new users and sellers entering the market (their measure represents market growth rate). These new agents cannot subscribe to the old platform. To interact with agents on the other side of the market they have to, therefore, subscribe to the new platform. This indeed reflects the situation on many markets of interest, where the old generation of the platform (for example, an outdated operation system or an old generation of a game console) is no longer available (unless in a secondary market). The users and sellers are assumed to be heterogeneous with respect to net costs which they incur when adopting a new platform.

The price-discriminating monopolist earns profit by selling the new platform to the installed base and to the new agents and charging them a subscription fee. In addition the monopolist is free to choose among four compatibility regimes: making the new platform incompatible with the old one, fully compatible, or only backward compatible for agents on one side of the market. In the absence of any form of compatibility, only agents subscribed to the same generation of platform can interact. By imposing compatibility, the monopolist enables an interaction between users and sellers subscribed to different generations of platform.

Finally, deciding on compatibility, the monopolist can also determine the quality of interaction between agents, subscribed to the new platform and the agents on the other side of the market, subscribed to the old platform. The minimal quality which the monopolist can choose is zero, which corresponds to the situation where the new platform and the old platform are incompatible. It is assumed that the quality of interaction between agents subscribed to different platforms can never exceed the quality of interaction between agents subscribed to the old platform. In other words, the people who play a game designed for PlayStation 2 on PlayStation 3 can only enjoy the graphic and sound to the extent they would enjoy it using PlayStation 2. Any quality of interaction between zero and the maximal value corresponds to *partial compatibility*, because it only confers a part of maximal network benefits to agents on both sides od the market.

Our first crucial result is that the monopolist will never choose partial compatibility. He either will make technologies incompatible for one side on the market or will make them compatible to the extent that agents can enjoy the maximal network benefits. This result is new to the literature on network externalities, which up to now *assumed* that the compatibility is a yes/no decision (Katz and Shapiro 1985, Katz and Shapiro 1986, Farrell and Saloner 1986, Katz and Shapiro 1992, Doganoglu and Wright 2006),[2] although there always was an unease about this assumption (see, for example, Choi

[2]One exception from this rule is Farrell and Saloner (1992), who assume that compatibility is provided through the use of converter, which can be imperfect. However, the quality of converter in their model is exogenously determined and is not chosen by firms, who provide converter.

1994). Our result provides a justification for this assumption and allows to concentrate our analysis on four extreme compatibility regimes (incompatible platforms, fully compatible platforms and two types of backward compatibility).

Analyzing the choice of the compatibility regime, we showed that at the heart of monopolist's decision to make technologies compatible is the tradeoff between demand of new agents on one side of the market and demand of the old agents on the other side of the market. In particular, if the monopolist introduces backward compatibility for, say, users, he encourages new users to buy the new platform but discourages the old sellers to do so (*direct effect*). For illustration consider a case, where monopolist makes platforms backward compatible for users. This improves incentives of new users to buy the new platform. Indeed, now, using this platform, they can access the installed base of sellers. On the other hand, the sellers, who belong to the installed base have less incentives to buy the new platform. Indeed, now, using their old platform they can interact with all users, subscribed to the new platform.

The decrease in the demand of old sellers triggers the decrease in demand of old users and of the new users (*feedback effect*). The negative feedback effect becomes more important if the technological progress is revolutionary, while the direct positive effect less so. Indeed, if the new platform is very advanced, then the new users have large incentives to buy it even if it does not allow them to access the installed base of sellers. The compatibility therefore will bring only moderate improvement in their demand.

The tradeoff between *direct* and *feedback* effects determines which type of compatibility will be chosen on the market. To provide trackable analysis of compatibility choice, in the second part of the paper we concentrate on several market structures which are characterized by extreme values of one or several parameters and are observed in reality. These are mature market (the market growth rate is small), emerging market (the installed base is small) and the asymmetric market (the installed base exists only on one side of the market).

We characterize the optimal choice of compatibility for these chosen market structures. As follows from our analysis, the monopolist is more likely to make platforms compatible if the technological progress is moderate. Further, the compatibility for, say, users is likely to be imposed if the installed base of sellers is relatively small, the installed base of users is relatively small and the growth rate of their installed base is moderate.

Although our model is static we are able to provide some intuition about dynamics of compatibility choice as the market develops from emerging to mature or as the monopolist, who treated his market as one-sided business, embraces a two-sided model. We illustrate our predictions with examples from video game console market and market

for personal digital assistants.
The set-up of our model shares common features with the literature on two-sided markets (Rochet and Tirole 2002, Caillaud and Jullien 2003, Armstrong forthcoming, Armstrong and Wright 2004, Rochet and Tirole forthcoming). Our results, however, are novel to this literature, which up to now typically ignored compatibility. The exception is Doganoglu and Wright (2006), who investigate the incentives of competing firms to make their platforms compatible given that consumers of their products may (or may not) multihome, i.e. subscribe to both platforms. The authors mainly investigate markets with simple network externalities (i.e. there is only one group of agents). They, however, also discuss implications of their model for two-sided markets. The focus of this model is very different from ours. First, the incentives to make platforms compatible stem from competition. Second, the ability of consumers to multihome in their model is the driving force of the result, while in our model this is the tradeoff between incentives of old and new agents. Finally, the authors do not distinguish between different compatibility regimes and view compatibility as a yes/no decision (full compatibility/incompatibility).
The literature on compatibility in the presence of simple network externalities may be divided into two groups. The first group of papers investigate compatibility of technologies on perfectly competitive or oligopolistic market. The incentives of firms to make technologies compatible stem mostly from competition. Katz and Shapiro (1986) show that in a dynamic framework the competing firms have incentives to achieve compatibility of the products in order to soften the price competition on the early stage of the industry development. Kristiansen (1998) shows that compatibility may also be used to reduce the R&D competition at the stage of product introduction. Katz and Shapiro (1992) study a dynamic model, where consumers entering at each date choose between buying a incumbent technology or waiting until the entrant introduces a more advanced technology. The authors show that, depending on the size of the installed base, market growth rate, and consumers' beliefs, either entrant or incumbent (but seldom both of them) would prefer to make both technologies compatible.
Unlike this strand of literature, we study the situation where both old and new platform (technology) are owned by a monopolist. We do this for two reasons. First, the structure on many industries involving multi-sided markets indeed is monopolistic (or close to monopolistic), for instance, PC operating systems with Microsoft, internet auctions with eBay, etc. Second, we want to analyze the incentives for achieving compatibility other than those which are related to competition. We show in the paper that incentives of the monopolist to make platforms compatible are determined by

the extent to which he looses the demand on the behalf of the existing agents from one side of the market, which free-ride on the compatibility of platforms for agents on the other side of the market.

The second group of papers in the literature on network externalities is a literature on planned obsolescence. The paper which shares a number of similarities with our model in this literature is Choi (1994). This paper considers a decision of the monopolist in a two-period model. The monopolist sells a technology in the first period, forming an installed base, and a new generation of this technology in the second period. He has a choice between making the technologies compatible or incompatible with each other. Choi (1994) shows that the decision to introduce an incompatible technology crucially depends on the fact whether the monopolist intends to sell this technology to both installed base and new agents or only to new agents. In the former case the monopolist will make technologies incompatible, while in the latter case he will make them compatible. The first strategy (incompatible technology, sell to both groups of agents) is shown to be optimal if the new technology has sufficiently high stand-alone benefits and the first group of agents (installed base) is sufficiently large, compared to the number of new agents.

The intuition, underlying these results, that the tradeoff between the demand of old and new agents, is determinant for the compatibility decision, is similar to ours. Important difference, however, between this paper and Choi (1994) is that in our framework it is demand of the new agents on one side of the market and the demand of the old agents on the other side of the market which matters for compatibility choice. Further, in the framework of two-sided markets we are able to characterize the richer set of compatibility regimes than Choi (1994). Finally, we also investigate how the choice of compatibility regime depends on the extent of network benefits that the new technology confers to the agents on both sides of the market. Turns out that higher network benefits intensify the negative feedback effect while making the positive direct effect less important. This analysis allows us to predict how the choice of compatibility changes with the technological progress.

The remainder of the paper is organized as follows. In Section 3.2 we describe the setup of the model and provide a classification of compatibility regimes. Section 3.3 analyzes compatibility of platforms under a general demand specification. In Section 3.4 we introduce assumption of linear demand function and investigate three market structures: mature market, emerging market and asymmetric market. In Section 3.5 we illustrate our predictions about compatibility choice with two examples. Section 3.6 concludes. Appendix 3.A contains proofs of all lemmas and propositions. Figures and tables are given in Appendix 3.B.

3.2 Description of the model

There are two types of agents on the market: agents of type x and agents of type y. For simplicity we will often refer to the x-agents as *users* and to the y-agents as *sellers*. In line with the literature on two-sided markets, we assume that agents of each type derive utility from interacting with the agents of the other type, but not from interacting with the agents of their own type. The utility of each agent increases with the number of agents he can interact with.

In order to *interact with* (*connect to*) an agent of type j, an agent of type i (where $i, j \in \{x, y\}$, $i \neq j$) needs to be subscribed to a *platform*. There are two different platforms available: Platform 0 and Platform 1. Platform 0 represents the old (default) technology and Platform 1 represents the new technology.[3] Both platforms are operated by a single monopolistic firm, which also retains all profits generated by the platforms. The extent to which these platforms differ will become clear later.

We present our model in a general setting that allows to derive general results for two-sided markets. In the Introduction we have described numerous examples of such two-sided markets. Although, each specific example may involve some properties not captured by the model, we believe that the mechanisms described in this paper is robust in most cases. As a typical example to illustrate the assumptions on the technology, we will use the market for video-game consoles. Agents of type x (users) are represented by the players of video-games and agents of type y (sellers) are represented by software developers. A platform in such a market is a video-game console, which enables users to play games, developed by software developers. Old technology then corresponds to an old generation of the console (e.g., Sony PlayStation 2) and the new technology corresponds to a new generation of the console (e.g., Sony PlayStation 3).

We assume that there are non-negative measures b^x and b^y of users and sellers respectively, who are already subscribed to the old platform (Platform 0). We will refer to these agents as *existing members*, *old agents*, or *installed base*. In addition, there are measures c^x and c^y of *new agents* (of types x and type y respectively) who are not subscribed to any platform. We assume that these agents cannot subscribe to the old technology. Their only way to connect to the agents of the other type is to subscribe to the new technology. This assumption reflects the situation where the old technology is discontinued (no longer available) and has been replaced by the new technology. For example, in case of video-game consoles, after introducing a new console, the old one cannot be purchased (unless in a secondary market).

[3] We will often use the words *platform* and *technology* interchangeably.

The existing members, already subscribed to Platform 0, may also subscribe in addition to Platform 1. In this case, the agents retain also the old technology[4] and may use either the new or the old technology to interact with the agents of the other type. This assumption is justified in the situation where the parallel use of two platforms generates no or only negligible additional costs. Note that if there is no possibility of resale and the use of the old platform does not involve any additional costs, agents who subscribed to the new platform have no incentives to stop using the old one.
Subscription to the new technology may be beneficial due to two reasons. First, it may involve some technological advantages (better graphics, sound, etc.) or alternative uses (as a DVD player), that increase the utility from interaction. Second, it may enable interaction with agents, not subscribed to the default technology.
If two agents interact using the old technology, their benefit from this interaction is normalized to 1. This implies that users and sellers that are subscribed to the old technology, are guaranteed to receive the benefit of b^y (respectively b^x) by interacting with the agents from the opposite group, who are also subscribed to this technology. We will further assume that when interaction is realized through the new platform (Platform 1), the benefit from this interaction is scaled up by a constant factor $s \geq 1$. Hence, new technology is beneficial for both users and sellers, because it allows to extract a higher utility from the same number of interactions. Finally, if the two technologies are compatible, also agents using different technologies can interact. We assume that in this case the benefits of interaction are determined by the lowest technology which enables the interaction. For example, when old games can be played in the new console, there is usually no additional benefit compared to the old console.[5]
The profit-maximizing monopolist, who owns and operates both platforms, makes profit by charging per-subscription prices for Platform 1. It is intuitive to think about this situation as a three-stage game: in the first stage the monopolist chooses the compatibility regime, in the second stage he chooses the prices, and in the third stage the agents simultaneously decide whether to subscribe to Platform 1. As a solution concept we use subgame-perfect Nash equilibrium.[6] We denote A_0^i the price

[4]This is often called *multihoming*.

[5]If we define something like quality of the platform, then this amounts to assuming that benefit from the interaction is determined by the minimal quality of platforms used. This is an assumption on the technology of interactions. Alternatively, one could consider the benefits from interactions to be determined by the maximum of qualities, some convex combination of qualities, or by own type's quality.

[6]Further, we will call it only *equilibrium.* We will also refer to the monopolist's choice in the second stage given the compatibility regime (i.e., equilibrium in the second stage) as *optimum* or *maximum.*

charged to installed base, A_1^i the price charged to new agents ($i \in \{x, y\}$). Observe that we allow for price discrimination, i.e., the monopolist can charge different prices to old and new agents of the same type. This can be achieved by selling the new platform in form of an *update* to the old platform with a different price than the stand-alone platform. The assumption of price discrimination is in line with existing literature on network externalities (Ellison and Fudenberg 2000, Choi 1994). The most prominent example are the rebates for the users of operating systems for updates.

For simplicity, we will assume that there is no cost of operating a platform. The monopolist cannot charge a price for Platform 0 (it is not any more available for sale) and cannot make any profit on those agents which use the old technology. Let $m^i \in [0,1]$ denote the share of the existing members that subscribe to Platform 1 and let $n^i \in [0,1]$ denote the share of the new agents that subscribe to Platform 1. Then $b^i m^i$ and $c^i n^i$ are their demands for Platform 1. Since the monopolist has no costs and charges per subscription prices, his profit is

$$\Pi = A_0^x b^x m^x + A_0^y b^y m^y + A_1^x c^x n^x + A_1^y c^y n^y. \tag{3.1}$$

The model we presented so far is a modification of the traditional model of two-sided markets (Rochet and Tirole 2004). Our main contribution is the analysis of monopolist's decision about compatibility between the old and the new technology. There are four possible *compatibility regimes*. The new technology may be *incompatible* (NC) with the old technology. It may be *backward compatible* with the old technology for x-agents (BCx); it may be backward compatible for y-agents (BCy) or it may be *fully compatible* with the old technology (FC).

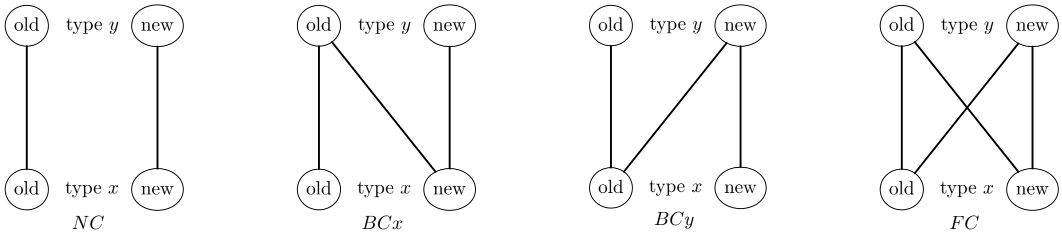


Figure 3.1: Interactions in various compatibility regimes

Under incompatibility, the new and old technologies cannot be interconnected. Backward compatibility for i-agents means that an i-agent, who is subscribed to the new technology may use it to interact with an j-agent subscribed to the old technology (see Figure 3.1). In the example of game consoles, backward compatibility for users means that games produced for the old console (PlayStation 2) can be played on the new console (PlayStation 3). In technical language, this form of compatibility is simply called "backward compatibility." A related notion of "forward compatibility" means that games written for the new console can be played using the old console.[7] In our setting of two-sided markets, forward compatibility is equivalent to backward compatibility for sellers — it simply means that sellers subscribed to the new technology can interact with users subscribed to the old technology. Finally, if the new technology is backward compatible for both sides of the market, we say that the technology is fully compatible. Example of full compatibility is the USB standard: USB 2.0 is fully compatible with USB 1.1.

All these compatibility regimes can be easily nested within one general framework. Towards this end let us assume that the benefit from interaction of a new x-agent with an old y-agent is γ^x and the benefit from interaction of a new y-agent with an old x-agent is γ^y.[8] Thus, γ^x and γ^y can interpreted as *degrees of backward compatibility* for x-agents and y-agents. The value $\gamma^x = 0$ means that the benefit from interaction (between a new x-agent and an old y-agent) is 0, i.e., the new platform is not backward compatible for x-agents. On the other hand, the value $\gamma^x = 1$ means this benefit is 1, i.e., the new platform is backward compatible for x-agents. The regime NC then corresponds to the case $\gamma^x = \gamma^y = 0$, regime BCx to $\gamma^x = 1$ and $\gamma^y = 0$, regime BCy to $\gamma^x = 0$ and $\gamma^y = 1$, and regime FC to $\gamma^x = \gamma^y = 1$. We will refer to the case when γ^x or γ^y belong to $(0, 1)$ as *partial compatibility.* As will be shown below (Proposition 3.1), partial compatibility is never chosen by the monopolist, even if he is free to choose any $\gamma^x, \gamma^y \in [0, 1]$. This provides justification for analyzing only the polar cases NC, BCx, BCy, and FC.

If an agent of type $i \in \{x, y\}$ does not subscribe to the new platform, his utility is simply equal to his benefit from interactions. We denote U_0^i the utility of the old agent not subscribed to the new platform; the utility of the new agents not subscribed to any platform is normalized to zero. If the agent subscribes to the new platform, his utility depends positively on the per-interaction benefits, negatively on the subscription price A_k^i (where $k \in \{0, 1\}$) and on intrinsic benefits or costs of acquiring the new platform.[9] We will summarize those by θ^i, $i \in \{x, y\}$ that represents the *net costs* of acquiring

[7]For more details see `en.wikipedia.org/wiki/Backward_compatibility` and `en.wikipedia.org/wiki/Forward_compatibility`.

[8]Alternatively this can mean that a new x-agent is able to connect only to a share of γ^x of the old y-agents and *vice versa.*

[9]The intrinsic benefits may reflect alternative uses of the platform (Sony PlayStation can be used as a DVD player) or fashion.The cost may represent switching costs.

the new platform. The utility function is assumed to be additive and, thus, equal to

$$\text{(benefit from interactions)} - \theta^i - A^i_k.$$

Call U^i_1 and V^i_1 the utility of old and new agents respectively who are subscribed to the new platform.

Benefits which an agent derives from interactions depend on the degree of compatibility. For illustration, consider an old agent of type x. If he joins Platform 1, he can interact with $b^y m^y + c^y n^y$ agents using Platform 1 (with per-interaction benefit s) and with the remaining $b^y(1 - m^y)$ agents using Platform 0 (with per-interaction benefit 1). Thus, his benefit from interactions is $s(b^y m^y + c^y n^y) + b^y(1 - m^y)$. Here, degree of compatibility plays no role. On the other hand, if he does not join Platform 1, he can interact with b^y old agents using Platform 0 (with per-interaction benefit 1) and also with $c^y n^y$ new y-agents (with per-interaction benefit γ^y). In that case, his benefit from interactions is $b^y + \gamma^y c^y n^y$. Formally, the agent's utilities are

$$U^x_1 = s(b^y m^y + c^y n^y) + b^y(1 - m^y) - \theta^x - A^x_0, \qquad U^x_0 = b^y + \gamma^y c^y n^y.$$

A new agent of type x can stay out of the market in which case he has no access to the agents of type y and receives zero benefits from interactions. Alternatively, he can subscribe to the new platform. Platform 1 enables him to interact with $c^y n^y$ new agents (with per-interaction benefit s) and with additional $b^y m^y$ old agents (with per-interaction benefit γ^x). Formally,

$$V^x_1 = \gamma^x b^y + (s - \gamma^x) b^y m^y + s c^y n^y - \theta^x - A^x_1.$$

The demand for Platform 1 is given by the number (measure) of existing members for which $U^i_1 > U^i_0$ and the number (measure) of new agents for which $V^i_1 > 0$. In particular,

$$U^x_1 > U^x_0 \iff (s-1) b^y m^y + (s - \gamma^y) c^y n^y - A^x_0 > \theta^x, \tag{3.2}$$

$$V^x_1 > 0 \iff \gamma^x b^y + (s - \gamma^x) b^y m^y + s c^y n^y - A^x_1 > \theta^x. \tag{3.3}$$

All existing x-agents with θ^x satisfying the former inequality and all new x-agents with θ^x satisfying the latter inequality will subscribe to Platform 1.

Comparison across different compatibility regimes reveals the twofold effect which compatibility has on the incentives of agents. BCx as compared to NC (or in general increase in γ^x), for example, increases incentives of new users to subscribe to the

Platform 1 by enabling them to access the larger population of agents on the other side of the market. On the other hand, BCx regime (or increase in γ^x) discourages existing sellers to buy the new technology. Indeed, in this regime they can interact with users using their old platform. This tradeoff between incentives of the new agents on one side of the market and old agents on the other side of the market will be determinant for the choice of the compatibility regime.
The agents are assumed to be heterogenous with respect to the net costs θ^i; let $F(\theta^i)$ be its cumulative distribution function. We assume that the distribution of agents' net costs has a finite support $[\underline{\theta}, \bar{\theta}]$. In addition, function F is increasing and twice continuously differentiable on $[\underline{\theta}, \bar{\theta}]$, and the following assumption hold.

Assumption 3.1. $F'(\bar{\theta}) = 0$ and $\lim\limits_{\theta \to \underline{\theta}^+} \dfrac{F(\theta)}{F'(\theta)} < -\underline{\theta}$.

Note that under the introduced specification, the net costs of the old and new agents have the same distribution, reflecting the fact that the new agents are a "copy" of the old agents. This setup allows to analyze monopolist's decision depending on size of b^i and c^i. It is straightforward to modify the model in order to allow for different distributions for old and new agents (assuming still that Assumption 3.1 holds). All results remain valid, however, at the expense of simplicity of some conditions.[10]
As will be shown later, Assumption 3.1 guarantees existence of interior solution to the monopolist's maximization problem.[11] The first inequality implies that there is no kink at point $\bar{\theta}$ and hence we may use first-order conditions to find the maximal profit.[12] The second inequality requires $\underline{\theta} < 0$, which means that there is some group of agents who derive (positive) net benefits from the new technology. These agents then ensure that all demands are positive in equilibrium. Note that the second inequality holds whenever $\underline{\theta} < 0$ and $\lim_{\theta \to \underline{\theta}^+} F'(\theta) > 0$.
It follows from (3.2) and (3.3) that the demands of old and new x-agents and by a

[10]To analyze the model with different distributions for each type of agents, fix the markets sizes and analyze the decision with respect to demand elasticities.

[11]This holds for all values of other parameters. In Section 3.4 we consider F linear on $[\underline{\theta}, \bar{\theta}]$ that satisfies the second inequality, but violates the first inequality. Thus, some corner solutions arise.

[12]Note that this assumption is by no means restrictive. Indeed, any function can be "smoothed" in a small neighborhood of $\bar{\theta}$ so that the first and second derivatives become continuous.

symmetric argument also of old and new y-agents are given by

$$m^x = F\big((s-1)b^y m^y + (s-\gamma^y)c^y n^y - A_0^x\big), \tag{3.4}$$

$$m^y = F\big((s-1)b^x m^x + (s-\gamma^x)c^x n^x - A_0^y\big), \tag{3.5}$$

$$n^x = F\big(\gamma^x b^y + (s-\gamma^x)b^y m^y + sc^y n^y - A_1^x\big). \tag{3.6}$$

$$n^y = F\big(\gamma^y b^x + (s-\gamma^y)b^x m^x + sc^x n^x - A_1^y\big). \tag{3.7}$$

Note, that if there is no entry to the market (i.e., $c^x = c^y = 0$), then all compatibility regimes result in the same demand for Platform 1. Therefore, in the absence of new agents, the monopolist is indifferent between four compatibility regimes.[13]

3.3 General demand function

In this section we analyze the general model introduced above. Denote the inverse function of F as $G : [0,1] \to [\underline{\theta}, \bar{\theta}]$ the inverse function to F. By assumptions, G is increasing and twice continuously differentiable. Our assumption on the distribution of θ^i implies that $G(m^i)$ and $G(n^i)$ represent the characteristic θ^i of the indifferent old agent and new agent respectively. Note that $G(0) = \underline{\theta}$ and $G(1) = \bar{\theta}$. In order to simplify the notation, it will be convenient to use function $H : [0,1] \to \mathbb{R}$ such that

$$H(z) = \tfrac{1}{2} z G(z) \qquad \text{for all } z \in [0,1]. \tag{3.8}$$

With this notation, we obtain inverse demands:

$$\begin{aligned}
A_0^x &= -G(m^x) + (s-1)b^y m^y + (s-\gamma^y)c^y n^y, \\
A_0^y &= -G(m^y) + (s-1)b^x m^x + (s-\gamma^x)c^x n^x, \\
A_1^x &= -G(n^x) + \gamma^x b^y + (s-\gamma^x)b^y m^y + sc^y n^y, \\
A_1^y &= -G(n^y) + \gamma^y b^x + (s-\gamma^y)b^x m^x + sc^x n^x.
\end{aligned}$$

and the monopolist's profit becomes

$$\begin{aligned}
\Pi = -[b^x m^x G(m^x) + b^y m^y G(m^y) + c^x n^x G(c^x) + c^y n^y G(c^y)] + \\
+2(s-1)b^x b^y m^x m^y + 2(s-\gamma^x)b^y c^x m^y n^x + 2(s-\gamma^y)b^x c^y m^x n^y + 2sc^x c^y n^x n^y + \\
+\gamma^x b^y c^x n^x + \gamma^y b^x c^y n^y.
\end{aligned} \tag{3.9}$$

[13]If imposing compatibility involves some small fixed costs, then in the absence of entry, the monopolist has no incentives to make platforms compatible.

This is to be maximized with respect to $m^x, m^y, n^x, n^y \in [0,1]$. We may immediately observe that the "coefficient" at γ^x is $b^y c^x n^x(1-2m^y)$. As will become clear below, the comparison of m^y to $\frac{1}{2}$ will be important for monopolist's decision whether to impose backward compatibility for x-agents.
The following lemmas provide sufficient conditions for existence and uniqueness of the maximum.

Assumption 3.2. Function $H''(z)$ is bounded from below on $[0,1]$ and its minimum $\Delta = \min_{z\in[0,1]} H''(z)$ satisfies the following conditions:

(i) $\Delta > 0$;

(ii) $\Delta^2 > (s-1)^2 b^x b^y + s^2 b^y c^x$; and

(iii) $\Delta^4 + s^2 b^x b^y c^x c^y > [(s-1)^2 b^x b^y + s^2 b^y c^x + s^2 b^x c^y + s^2 c^x c^y]\Delta^2$.

Lemma 3.1. *If Assumption 3.2 is satisfied, then monopolist's profit* (3.9) *is concave in m^x, m^y, n^x, n^y for all $\gamma^x, \gamma^y \in [0,1]$.*

Lemma 3.2. *If Assumptions 3.1 and 3.2 are satisfied, then for all $\gamma^x, \gamma^y \in [0,1]$, the monopolist's profit* (3.9) *has a unique maximum. This maximum is achieved for m^x, m^y, n^x, and n^y from $(0,1)$ that satisfy the first-order conditions:*

$$\begin{aligned}
H'(m^x) &= (s-1)b^y m^y + (s-\gamma^y)c^y n^y,\\
H'(m^y) &= (s-1)b^x m^x + (s-\gamma^x)c^x n^x,\\
H'(n^x) &= \tfrac{1}{2}\gamma^x b^y + (s-\gamma^x)b^y m^y + s c^y n^y,\\
H'(n^y) &= \tfrac{1}{2}\gamma^y b^x + (s-\gamma^x)b^x m^x + s c^x n^x.
\end{aligned}$$

Throughout this section we assume that both Assumptions 3.1 and 3.2 are satisfied. We will focus on the analysis of the optimum. A natural question arises, what would be the optimal choice of γ^x and γ^y if the monopolist could choose any values from $[0,1]$. The answer is surprisingly that the monopolist would never choose $\gamma^x \in (0,1)$ and is formulated in Proposition 3.1. Furthermore, Proposition 3.2 provides sufficient conditions for comparisons of compatibility regimes. We will use the relation "$\prec$" to denote the comparison of compatibility regimes.

Proposition 3.1. *Partial compatibility is never optimal.*

Proposition 3.2. *The following statements hold for comparison of optimums in compatibility regimes:*

(i) *If* $m^x \leq \frac{1}{2}$ *in* NC *(resp.* BCx*) regime, then* $NC \prec BCy$ *(resp.* $BCx \prec FC$*).*

(ii) *If* $m^x \geq \frac{1}{2}$ *in* BCy *(resp.* FC*) regime, then* $NC \succ BCy$ *(resp.* $BCx \succ FC$*).*

(iii) *If* $m^y \leq \frac{1}{2}$ *in* NC *(resp.* BCy*) regime, then* $NC \prec BCx$ *(resp.* $BCy \prec FC$*).*

(iv) *If* $m^y \geq \frac{1}{2}$ *in* BCx *(resp.* FC*) regime, then* $NC \succ BCx$ *(resp.* $BCy \succ FC$*).*

Proposition 3.1 provides a justification for analyzing only the polar cases: NC, BCx, BCy, and FC. The intuition behind Proposition 3.1 is based on a stronger statement. Namely, the monopolist has incentives to reduce the degree of backward compatibility for x-agents, if sufficiently many old y-agents subscribe to the new platform. Indeed, in this case the backward compatibility is less important for new agents of type x, while they can interact with many old agents of type y, using the new platform.

Sufficiently many here means that the median old y-agent, i.e., agent with costs θ that satisfy $F(\theta) = \frac{1}{2}$, will in optimum subscribe to the new platform. This reminds on the *Median Voter Theorem* in the sense that the median agent is determinant for the compatibility choice. On the other hand, if the median old y-agent does not subscribe to the new platform, the monopolist has incentives to increase the degree of compatibility for x-agents. Thus, the only candidate for optimum that remains is when the median old y-agent is exactly indifferent between subscribing and not subscribing to the new platform. However, in that case the reduction in γ^x has a positive effect on old y-agents' incentives to subscribe to the new platform and the monopolist is again willing to reduce the degree of compatibility. Therefore, the monopolist never chooses a partial degree of compatibility for x-agents.

The remaining sufficient conditions in Proposition 3.2 follow the same intuition. Note that the sufficient conditions are formulated in terms of the median old y-agent. This is intuitive, since the measure of old y-agents who join the new platform determines the incentives of new x-agents. Analogous statements can be also made for backward compatibility for y-agents.

Unfortunately, given the general form of distribution of agents' costs, it is not possible to provide a complete characterization of the monopolist's compatibility choice in terms of the primitives of the model (i.e., market sizes b^x, b^y, c^x, c^y, quality of connection s and distribution of agents' costs represented by function F). We are able, however, to derive some comparative statics results. In particular, we are interested in comparative statics with respect to γ^x (with symmetric results for γ^y). It provides an important intuition on the effects that drive a choice of compatibility regimes. We can easily evaluate that all derivatives $\mathrm{d}m^x/\mathrm{d}\gamma^x$, $\mathrm{d}m^y/\mathrm{d}\gamma^x$, $\mathrm{d}n^x/\mathrm{d}\gamma^x$, $\mathrm{d}n^y/\mathrm{d}\gamma^x$ can be written in the form

$$b^y(1-2m^y)\beta_1 + 2c^x n^x \beta_2, \qquad \text{where } \beta_1 > 0 \text{ and } \beta_2 < 0$$

(coefficients β_1 and β_2 differ across variables, but their signs are always as indicated; see the proof of Proposition 3.3 for technical details). Thus, the total effect of an increase in γ^x can be decomposed in two effects. The first effect, captured by the term $2c^x n^x \beta_2$, stems from the change in incentives of old y-agents (*direct effect* of compatibility) and is always negative. The second effect, captured by $b^y(1-2m^y)\beta_1$, is ambiguous. It stems from the change in incentives of new x-agents and is negative or positive depending on whether the median old y-agent subscribes to the new platform or not.

This ambiguity is due to interaction of two effects. The *direct effect* of increase in γ^x is positive for new x-agents and is negative for old y-agents. The reduction in demand of old y-agents due to direct effect triggers the reduction in demand of new x-agents (*negative feedback effect*).[14] Intuitively, if m^y is large the negative effect should overweight the positive effect. Indeed, in this case sufficiently many old y-agents purchase the new platform to make it attractive for the new x-agents even in the absence of compatibility. Hence, as compatibility is improved, the direct positive effect is negligible and is dominated by negative effect. Since the direct effect of compatibility on the incentives of the old y-agents is always negative, the total effect will be negative as well. This intuition is formalized in the following proposition.

Proposition 3.3. *For any $\gamma^y \in [0,1]$, all demands (m^x, m^y, n^x, and n^y) and also the monopolist's profit are decreasing in γ^x whenever $m^y \geq \frac{1}{2}$ in optimum.*

In a similar way we may derive comparative statics result with respect to γ^y.

3.4 Compatibility choice with linear demand functions

To investigate the choice of compatibility in more details we will make here an additional assumption that θ^i (where $i \in \{x, y\}$) is uniformly distributed on the interval $[-B, 1-B]$, where $0 < B < 1$. This means, that there are some agents who derive net benefit and some agents who derive net costs from the new platform. The value of B then corresponds to the maximal net benefit (derived by agents with $\theta = -B$). In

[14] It becomes clear later that demand of the old x-agents is also subject to negative feedback effect.

the notation of Section 3.2 the corresponding distribution function is $F(\theta) = \theta + B$ for $\theta \in [-B, 1-B]$. The demands are then linear in prices and are given by (3.4)–(3.7). To ensure that the demand functions are decreasing in prices, we will impose two conditions on parameters.

Assumption 3.3.

(i) $b^x \leq 1, b^y \leq 1, c^x \leq 1, c^y \leq 1$,

(ii) $1 - b^x b^y (s-1)^2 - s^2(b^y c^x + c^x c^y + b^x c^y - b^x b^y c^x c^y) > 0$.

These conditions are sufficient conditions on parameters and (for linear demand function) imply conditions in Lemma 3.1. Indeed, for linear demand function $\Delta = 1$. Hence, condition (iii) in Lemma 3.1 becomes identical to condition (i) of Assumption 3.3. Further, for linear demand function Assumptions 3.3 implies condition (ii) of Lemma 3.1.

Since in equilibrium each demand can be either interior or corner, we have multiple candidates for equilibrium allocation. However, it is possible to show, that the monopolist never chooses prices, such that $m^i = 0$ or $n^i = 0$ or both. Indeed, if the prices are such that, for example, $m^i = 0$, the monopolist can decrease the price A_0^i marginally to gain a positive demand on behalf of old agents of i-type. Since the prices for other groups of agents remain unchanged, the profit of the monopolist will be strictly larger. Moreover, the interior solution maximizes the principal's profit, whenever feasible.

Lemma 3.3. *Consider the maximization of monopolist's profit with respect to m^x, when keeping $m^y, n^x, n^y \in [0,1]$ fixed. Let $m^{x*} \in \mathbb{R}$ solve the first-order condition $\partial \Pi^r / \partial m^x = 0$, where $r \in \{NC, BCx, BCy, FC\}$. Then, the following statements hold:*

(i) $m^{x*} > 0$.

(ii) *If $m^{x*} \in [0,1]$, then $m^x = m^{x*}$ maximizes monopolist's profit when keeping m^y, n^x, n^y fixed.*

(iii) *If $m^{x*} > 1$, then $m^x = 1$ maximizes monopolist's profit when keeping m^y, n^x, n^y fixed.*

Analogous statements holds for (partial) maximization with respect to any of the variables m^y, n^x, n^y.

Based on the Lemma 3.3, we can eliminate allocations where either $m^i = 0$ or $n^i = 0$ for any $i \in \{x, y\}$ from the set of equilibrium candidates. This leaves us sixteen allocations, which are candidates for equilibrium. These allocations are summarized in Table 9 in Appendix 3.B. Notations are as follows: $E_{i_1 i_2 i_3 i_4}$ denotes a particular type of allocation. Indexes $i_1 \in \{0, 1\}$, $i_2 \in \{0, 1\}$, $i_3 \in \{0, 1\}$ and $i_4 \in \{0, 1\}$ indicate, whether in this allocation the demands m^x, m^y, n^x and n^y respectively are interior ($i_k = 0$) or corner ($i_k = 1$). For example, the allocation where $m^x \in (0, 1)$, $m^y \in (0, 1)$, $n^x = 1$, and $n^y \in (0, 1)$ is denoted as E_{0010}.

The use of linear demand functions allows us to derive equilibrium prices, demands and the monopolist's profit in closed form. However, with six parameters the presentation of results in general case (that is $b^i > 0$, $c^i > 0$, $s > 1$, and $0 < B < 1$) does not reveal the underlying intuition. Therefore, in what follows we discuss several specific market structures that one can observe in reality, and that are characterized by extreme values of one or several parameters.

3.4.1 Mature market

We define a mature market (or satiated market) as a market with low growth rate, i.e. we assume that c^x and c^y are close to zero. An example of such market is the market for Microsoft Windows. The operation system is installed on more than 90% of all computers, hence there are very few users who do not belong to the installed base of Microsoft and very few software developers who do not adapt their applications for Windows.[15]

Recall, that in our model at the market where $c^x = c^y = 0$ all compatibility regimes are equivalent in terms of monopolist's profit, agents' demands and prices for the new technology. Using this result we can compare different compatibility regimes at the market where the number of new agents is small, investigating the monopolist's profit in the neighborhood of $(c^x, c^y) = (0, 0)$. The comparison is summarized in the following proposition. Let us first define:

$$g_1(z) = \frac{1 - B[1 + z(s-1)]}{z(s-1)^2}.$$

Proposition 3.4. *Assume that c^x, c^y are sufficiently small (close to zero). Then the*

[15] See `en.wikipedia.org/wiki/Microsoft_Windows`.

following implications hold:

$$\begin{aligned}
b^x < \frac{1-B}{2(s-1)} \quad &\textit{or} \quad b^y < g_1(b^x) &\Longrightarrow\quad & NC \prec BCx \quad \textit{and} \quad BCy \prec FC,\\
b^x > \frac{1-B}{2(s-1)} \quad &\textit{and}\ b^y > g_1(b^x) &\Longrightarrow\quad & NC \succ BCx \quad \textit{and} \quad BCy \succ FC,\\
b^y < \frac{1-B}{2(s-1)} \quad &\textit{or} \quad b^x < g_1(b^y) &\Longrightarrow\quad & NC \prec BCy \quad \textit{and} \quad BCx \prec FC,\\
b^y > \frac{1-B}{2(s-1)} \quad &\textit{and}\ b^x > g_1(b^y) &\Longrightarrow\quad & NC \succ BCy \quad \textit{and} \quad BCx \succ FC.
\end{aligned}$$

Using the above proposition we can define curves[16] $\mathcal{I}^x$ and $\mathcal{I}^y$ which represent the monopolist's indifference between providing and not providing backward compatibility for agents of x-type and y-type respectively:

$$\mathcal{I}^y = \begin{cases} \frac{1-B}{2(s-1)}, & \text{if } b^y \leq \frac{1-B}{2(s-1)},\\ g_1(b^y), & \text{if } b^y > \frac{1-B}{2(s-1)}. \end{cases} \tag{3.10}$$

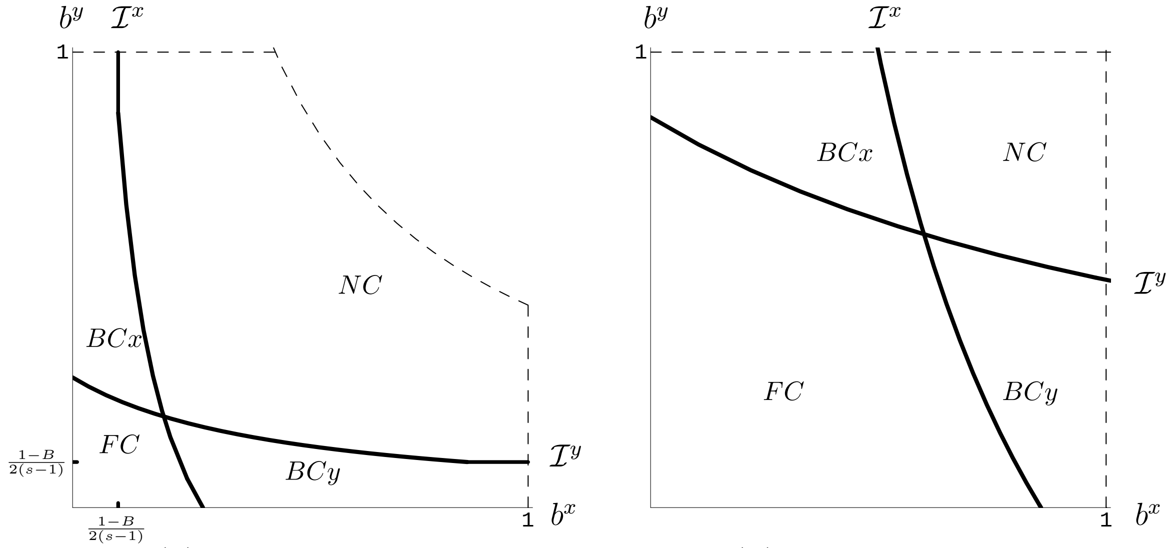
$\mathcal{I}^x$ is analogically defined.[17] Notice, that $\mathcal{I}^y$ decreases in B, s and is non-increasing in b^y (analogical result holds for $\mathcal{I}^x$). The curves $\mathcal{I}^x$ and $\mathcal{I}^y$ and optimal choice of compatibility regime is illustrated on the Figure 3.2 (the dashed line shows the area where Assumption 3.3 is satisfied).

There are several observations to be made about the choice of compatibility regime. As we have already mentioned, backward compatibility for, say, y-agents improves incentives of new agents of this type to buy the new technology, while it discourages the old x-agents to buy the new technology (*direct effect*). The decrease in demand on behalf of old x-agents triggers the decrease in demand of new y-agents and old y-agents (*negative feedback effect*).

Therefore, whether the monopolist is willing to make technologies compatible for y-agents depends on whether the positive effect on behalf of new y-agents overweighs the negative effects. This tradeoff explains the fact that for each b^x there exist a cutoff value of b^y (defined by $\mathcal{I}^y$), such that for all b^y above this value the monopolist will make technologies not compatible for y-agents. Moreover, this cutoff value is non-increasing in b^x, since the larger is b^x the more important is the negative effect

[16]With some abuse of notation we use the same notation for the curve and for the function describing it.

[17]These curves need to be interpreted properly. Consider, for example, curve $\mathcal{I}^y$. If (b^x, b^y) lies above (resp. below) the curve $\mathcal{I}^y$, then there exists $\delta > 0$ such that the monopolist prefers NC to BCy (resp. prefers BCy to NC) for all $c^x, c^y \in (0, \delta)$.



(a) $s = 2.5$, $B = 0.7$

(b) $s = 1.5$, $B = 0.7$

Figure 3.2: Optimal choice of compatibility regimes at the mature market

of backward compatibility for the monopolist's profit. On the other hand, if b^y is sufficiently small (when $b^y < \frac{1-B}{2(s-1)}$), the feedback effect on old y-agents becomes negligible and the monopolist is willing to make technologies backward compatible for any value of b^y. In particular, for $s = 1$ the negative feedback effect vanishes. Indeed, if $s = 1$ the demand of old y-agents does not depend on demand of old x-agents (one can readily see this from the definition of demand functions). In this case FC regime will be always chosen in equilibrium.

Another observation is that the larger is the technological progress, the less willing is the monopolist to make technologies compatible. This follows directly from the fact, that $\mathcal{I}^x$ and $\mathcal{I}^y$ decrease in B (value of stand-alone benefits) and s (per-interaction benefits). The underlying intuition is that the better is the new technology, the more incentives have the new agents to purchase it even in the absence of compatibility. On the other hand, the reduction in demand on behalf of old agents becomes more important for the principal's profit as s or/and B increase. This observation is formalized in the following corollary.

Corollary 3.1. *Consider some $b^x, b^y > 0$. Let $\bar{s} > 1$ and $\bar{B} \in (0, 1)$ satisfy Assumption 3.3 (where $c^x \to 0$ and $c^y \to 0$). If the monopolist makes technologies compatible for $s = \bar{s}$ and $B = \bar{B}$, then he will make technologies compatible for all $s \leq \bar{s}$ and $B \leq \bar{B}$.*

Finally, as follows from the Proposition 3.4, the decision of the monopolist whether to make technologies compatible for agents of type x does not depend on the fact, whether they are already compatible for the agents of type y. In other words, if the monopolist decides to switch from NC regime to BCx regime, he would also switch from BCy regime to FC regime (the symmetric argument holds the other side of the market). This result amounts to saying that the change in demand of new agents of type y is negligible and does not play a role for the decision of the monopolist to introduce BCx regime. Indeed, for $c^y \to 0$ this effect is insignificant and is dominated by the direct effect of BCx regime (negative effect on the incentives of old agents of type y and positive effect on the incentives of new agents of type x) and indirect feedback effect.

3.4.2 Emerging market

We define an emerging market as the market with very small installed base ($b^x \to 0$, $b^y \to 0$) and potentially high growth rate ($c^x > 0, c^y > 0$). The examples of emerging markets are numerous: Video game industry in early 80's, PDA's in early 90's, currently smart phones.

Clearly, if $b^x = b^y = 0$ then in our model all compatibility regimes are equivalent in terms of monopolist's profit, equilibrium demands and prices. We can use this result to analyze the optimal compatibility choice in a situation where b^x and b^y are small, by investigating profits of the monopolist in the neighborhood of $(b^x, b^y) = (0, 0)$. The comparison is summarized in the following proposition. Let us first define:

$$g_2(z) = \frac{2(1-B) - Bz(2s-1)}{zs(2s-B)}.$$

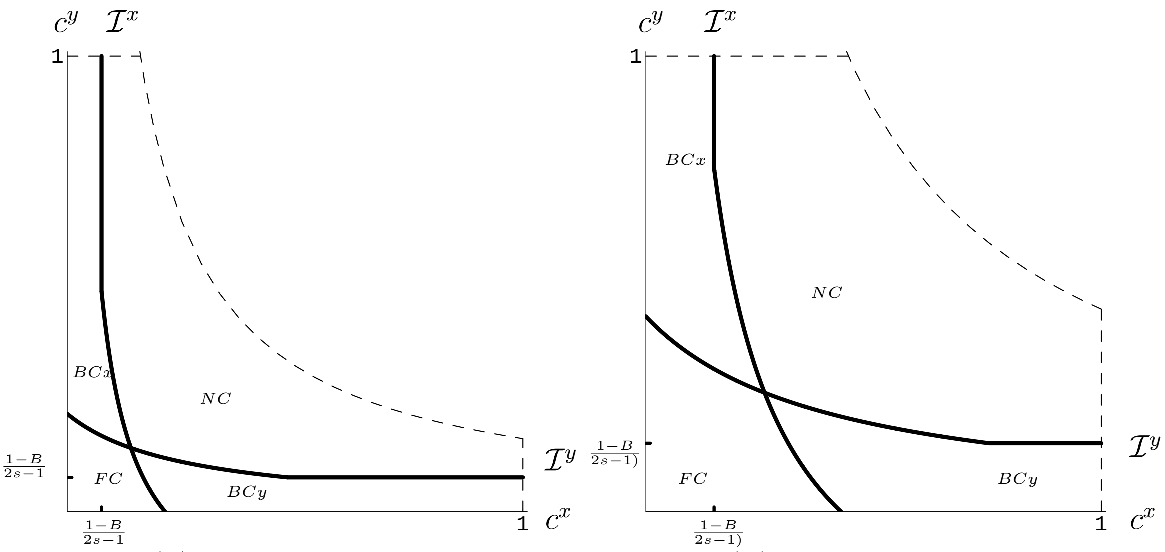
Proposition 3.5. *Assume that b^x, b^y are sufficiently small (close to zero). Then the following implications hold:*

$$\begin{array}{llcll}
c^x < \frac{1-B}{2s-1} & \text{or} \quad c^y < g_2(c^x) & \Longrightarrow & NC \prec BCx \quad \text{and} & BCy \prec FC, \\
c^x > \frac{1-B}{2s-1} & \text{and } c^y > g_2(c^x) & \Longrightarrow & NC \succ BCx \quad \text{and} & BCy \succ FC, \\
c^y < \frac{1-B}{2s-1} & \text{or} \quad c^x < g_2(c^y) & \Longrightarrow & NC \prec BCy \quad \text{and} & BCx \prec FC, \\
c^y > \frac{1-B}{2s-1} & \text{and } c^x > g_2(c^y) & \Longrightarrow & NC \succ BCy \quad \text{and} & BCx \succ FC.
\end{array}$$

Similarly as before we can define curves $\mathcal{I}^x$ and $\mathcal{I}^y$ representing the indifference of the monopolist between making technologies compatible for agents of the respective type and making them incompatible.

$$\mathcal{I}^y = \begin{cases} \frac{1-B}{2s-1}, & \text{if } c^y \le \frac{1-B}{2s-1}, \\ g_2(c^y), & \text{if } c^y > \frac{1-B}{2s-1}. \end{cases} \tag{3.11}$$

$\mathcal{I}^x$ is analogically defined. It is clear from the definition of the $\mathcal{I}^y$ and $g_2(z)$, that $\mathcal{I}^y$ is decreasing in B, s and is non-increasing in c^y (similar result holds for $\mathcal{I}^x$). The curves and the optimal choice of compatibility regime are illustrated on the Figure 3.3 (the dashed line shows the area where Assumption 3.3 is satisfied).



(a) $s = 2.5$, $B = 0.7$ **(b)** $s = 1.5$, $B = 0.7$

Figure 3.3: Optimal choice of compatibility regimes at the emerging market

As shown on the figure above, the backward compatibility for agents of type $i \in \{x, y\}$ is chosen if there are few new agents of this type. In particular, it is always optimal to make technologies compatible for i-agents if $c^i < \frac{1-B}{2s-1}$. If $c^i > \frac{1-B}{2s-1}$, then according to Proposition 3.5 there exist a cutoff value of c^i (which is the decreasing function of c^j), such that technologies will be incompatible for i-agents for all c^i above this value. This result is due to the tradeoff between incentives of new i-agents and old j-agents. Consider for example BCy regime and assume that parameters other than c^y are fixed. If c^y is small then introduction of BCy regime has no significant effect on the incentives of old x-agents to buy the new technology (the direct negative effect is small). Indeed, it provides the agents who belong to the installed base with few additional connections. Their decision to purchase the new technology depends therefore on its characteristics and the demand of old y-agents, rather than on the number of new agents subscribed to it. As c^y increases, however, the access to the new agents starts to play more important role for the decision of the installed base to buy the new technology. In

this case introduction of BCy leads to the significant reduction of the demand on behalf of old x-agents and the resulting feedback effect becomes also pronounced. In this case the monopolist is better off making technologies incompatible.
The threshold value of c^y is non-increasing in c^x. Indeed, if c^x is large the new y-agents would buy the technology even if it does not provide them an access to the installed base of x-agents. Hence introduction of compatibility will only moderately increase the demand on their behalf, while still discouraging the old agents from buying the new technology.
As follows from the Proposition 3.5, the decision of the principal to introduce backward compatibility for agents of type y does not depend on the compatibility of platforms for agents of type x. The intuition is similar to the case of mature markets. The monopolist ignores any implications which the change in demand of new x agents due to BCy has on the demand of old y agents. Indeed, this indirect effect is insignificant, because the installed base of y-agents is small.
Finally, we should observe that similarly to the case of mature market, the increase in the stand-alone benefits (B) or in the per-interaction benefits (s) shifts the curve $\mathcal{I}^y$ downwards and $\mathcal{I}^y$ to the left. In other words the monopolist is less likely to make technologies compatible if the technological progress is revolutionary. This observation is formalized in the following corollary.

Corollary 3.2. *Consider some $c^x, c^y > 0$. Let $\bar{s} > 1$ and $\bar{B} \in (0, 1)$ satisfy Assumption 3.3 (where $b^x \to 0$ and $b^y \to 0$). If the monopolist makes technologies compatible for $s = \bar{s}$ and $B = \bar{B}$, then he will make technologies compatible for all $s \leq \bar{s}$ and $B \leq \bar{B}$.*

3.4.3 Asymmetric market

We define asymmetric market as a market where there is an installed base only on one side, that is, for example, a market where $b^x > 0$ and $b^y = 0$. Such asymmetry can exist because the market in question, although having characteristics of two-sided market, was treated by the monopolist as a "single-sided" business. One example is iPod/iTunes music platform. The (potential) two sides of the market in this case are users who download the music and publishers who provide it. However, as is documented in Evans et al. (2006, Ch. 6, pp. 213–244 and pp. 257), the two sides of the market in the case of iPod/iTunes platform do not interact with each other; in fact the publishers have no access to platform at all. Instead, Apple follows a vertically integrated strategy buying the music by paying publishers royalties and distributes this music to customers who want it. Also PDA's and smart phones

evolved from a product which provided an integrated solution (hardware, operation system and applications) to the two-sided platforms, where applications and hardware are provided by the third-party developers (Evans et al. 2006, Ch. 6 and Ch. 9).
In this section we investigate which compatibility regime is going to be chosen at the market, where the monopolist who treated a platform as a single-sided business switches to the two-sided model. We assume therefore, that $b^x > 0$, $b^y = 0$, $c^x > 0$, $c^y > 0$. Notice, $b^y = 0$ implies, that in terms of monopolist's profit, prices and demands, NC regime is equivalent to BCx regime and FC regime is equivalent to BCy regime. We will consider therefore only the choice between NC and BCy regimes. The logic, applied in previous cases holds also here. If we fix parameters s, B and b^x, then we can define a curve (let us denote it $\mathcal{I}^y$), such that for the combination of (c^x, c^y) below this curve the monopolist would make technologies compatible and he would prefer them to be incompatible for the parameter range above this curve.

Proposition 3.6. *Let $b^y = 0$ and $b^x > 0$. Then for any c^x there exists a unique $\mathcal{I}^y = g_3(c^x)$ such that $NC \prec BCy$ if and only if $c^y < g_3(c^x)$.*

The curve $\mathcal{I}^y$ and the optimal choice of compatibility regimes is illustrated on Figure 3.4.

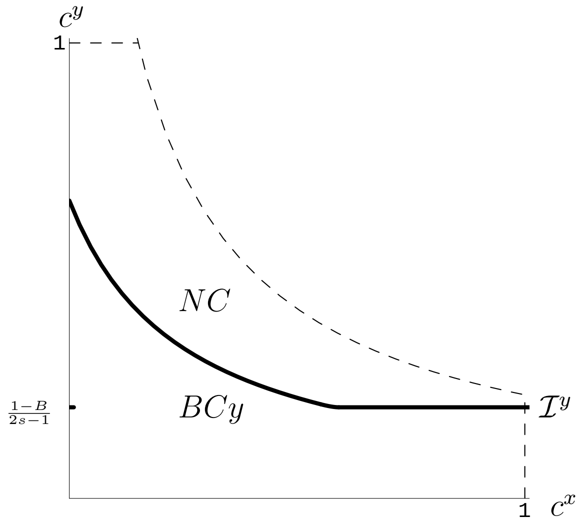


Figure 3.4: Optimal choice of compatibility regimes at the asymmetric market($s = 2$, $B = 0.4$, $b^x = 0.1$)

As the intuition discussed for the previous market structures suggests, the curve $\mathcal{I}^y$ must be non-increasing in c^x. The tradeoff between NC and BCy regimes is driven by direct positive effect on the incentives of new agents of type y and the direct negative effect on the incentives of old agents of type x. If c^x is large, than the new agents of type y find the platform attractive even if it is not compatible with the old platform. In this case the positive direct effect of BCy is relatively unimportant, and hence NC will be chosen. If c^x is small, however, then introduction of BCy regime significantly improves the demand on behalf of new agents of type y — and this effect offsets the negative effect on demand of old agents of type x.

Improvement in technological characteristics (increase of s or B) of the new platform has similar effect on incentives of new y-agents as increase in c^x. On the other hand, any loss of demand on behalf of old agents of type x is more important for the monopolist's if s or B is large. Introduction of BCy regime not only discourages some agents from installed base to buy the new platform, but also the reduction in their demand triggers the reduction in the demand of new agents of type y. These two effects become more important as s or B increases. Hence, for larger s or B the monopolist is less willing to make technologies compatible.

Finally, if the size of installed base is large then the monopolist is less willing to introduce BCy regime. Indeed, if b^x is large compared to c^y, then the monopolist should be more concerned with the reduction in demand of the installed base, than with the increase in demand of new agents.

Although the intuition for comparative statics above does not depend on particular values of parameters, it is not possible to provide the analytical comparative statics for a general case. Therefore, let us assume, that the parameters of the model are such, that the interior solution is feasible.

Corollary 3.3. *Let $c^x, c^y > 0$, $\bar{b}^x > 0$, $\bar{s} > 1$, and $\bar{B} \in (0,1)$ be such that the interior solution is feasible. Then the curve $\mathcal{I}^y = g_3(c^x)$ is downward sloping. Moreover, if the monopolist makes technologies compatible for some $s = \bar{s}$, $B = \bar{B}$, and $b^x = \bar{b}^x$, then (other things equal) he will make them compatible for all $s \leq \bar{s}$, $B \leq \bar{B}$, and $b^x \leq \bar{b}^x$.*

3.5 Discussion

We have identified two effects that backward compatibility for, say, agents of y type has on two-sided market (the argument is naturally the same for the backward compatibility for agents of x type). First effect is a *direct effect* that is positive for new y-agents (backward compatibility improves their incentives) and is negative for old x-agents (backward compatibility discourages them from buying the new technology). Second effect is a negative *feedback effect.* Namely, decrease in the demand of old agents of type x leads to the decrease in the demand of old agents of type y and to the decrease in the demand of new agents of type y. The negative feedback effects become more important if the technological progress is revolutionary (s and B are large), while the direct positive effect less so. Indeed, if the new platform is very advanced, then the new agents have large incentives to buy it even if it does not allow them to access the installed base of agents on the other side of the market. The compatibility therefore will bring only moderate improvement in their demand.

The tradeoff between direct effect and feedback effects determines the optimal compatibility choice. In particular, as follows from our analysis, the backward compatibility is more likely to be imposed on the market where the technological progress is moderate.[18] Further (other things equal), the compatibility for agents of type y is more likely to be imposed if their installed base is relatively small, the growth rate of the installed base is moderate and the installed base of x-agents is small.

Our model provides several predictions about patterns of compatibility choice. On the emerging market, where technological progress is rapid and the entry of agents on both sides of the market is significant, we should often observe the subsequent generations of technologies being not compatible with each other. As, however, the pace of technological improvement slows down and the growth of installed base decelerates (the market becomes mature), we should expect some degree of compatibility between subsequent generations of technology. In particular, technologies are likely to be backward compatible for some side of the market if the installed base on this side of the market is relatively small. Technologies are likely to be fully compatible if the both sides of the market are symmetrically represented. Only if the technological progress is significant and the installed base on both sides of the market are very large should the technologies remain incompatible.

The predicted pattern of the compatibility regimes as a market develops from emerging to mature is nicely illustrated by the experience of video game console industry. The following discussion is adopted mainly from the Evans et al. (2006). The start of the video game console industry dates back to the earlier 70's. However, the industry was emerging at the slow pace. The leader of the industry, Atari, at the pick of it success sold only around 5 mln units of video game consoles. Moreover, the game industry crashed in 1983 due to the overproduction of poor quality games. The credit for the revival of the industry goes to Nintendo. Around 1983 Nintendo introduced its first console (Nintendo Entertainment System, NES) which has revolutionized the way how the video console business was done. Nintendo actively pursued a two-sided

[18] Note, that this argument does not rely on the costs of achieving compatibility, which are naturally higher if the platforms belong to the very distant generations of technology. Higher costs provide another reasons why the platforms should be incompatible if the technological progress is revolutionary.

market strategy. It drafted licensing agreements with third party providers to ensure the quality of the games and the critical mass of the games for the new system. The sales of the NES and related games skyrocketed. It sold around 60 mln consoles world wide.

Ninetendo operated at a clearly emerging market, where the pace of technological growth was rapid and the installed base of users of consoles and game developers was relatively small. The future generations of Nintendo video game consoles were incompatible with the previous version.[19] Super NES (introduced in 1990) was incompatible with its predecessor NES; Nintendo 64 (introduced in 1996) was incompatible with Super NES and Game Cube (introduces in 2001) was incompatible with Nintendo 64. Presently, in Japan, USA and Europe, the video game console market has reached its mature state. According to estimation of Nielsen, a market research company, 41.1% of US households own a game console and the rate of console penetration has slowed down[20]. In line with our predictions Nintendo made its new console, Wii (introduced in 2006) backward compatible with its predeccessor, Game Cube.

Our analysis also indicates how the compatibility of platforms should evolve on the asymmetric market where a monopolist, who previously treated his market as a one-sided business decides to disintegrate and to embrace a two-sided model. Following Section 3.4.3 let us assume that there is an installed base only on the x side of the market and the growth rate of the market are c^x and c^y. Then we would expect subsequent generations of platforms be compatible for agents of type y, if c^y is small and/or if the pace of technological progress is moderate.

To illustrate this prediction, consider the case of Palm company.[21] Palm started as a software company but soon integrated in a hardware. Although it did not produced the hardware itself, it controlled all stages of the process and treated the involved firms as subcontractors. PalmPilot, produced in 1996, was a hardware with integrated operation system. However, in late 1997 Palm switched to a two-sided model. It has concentrated on the development of Palm OS operation system, which it was licensing to the hardware makers, such as Sony, Kyocera, Nokia, Handspring etc. (y agents, in the terminology of our model). On the other hand, to ensure the popularity

[19]Interesting enough, there are add-ons, unlicensed by Ninetendo, which allow to make subsequent generations of Nintendo consoles compatible. It indicates that Nintendo decided to make platforms incompatible not because it was technically impossible, but because it was more profitable strategy. The information about backward compatibility is taken from `en.wikipedia.org/wiki/Backward_compatibility`.

[20]See report of The Nielsen Company (2006)

[21]The example is taken from Evans et al. (2006, Ch. 6 and Ch. 9). The data about compatibility of Palm OS is available at `www.access-company.com/developers/documents/docs/palmos`.

of Palm OS, Palm has intensively courted the developers of applications (the x side of the market) from the time of introduction of Palm Pilot. It already had significant installed base of third party developers when it decided to switch to the two-sided model. The efficient courting strategy ensured that this base was growing fast. However, due to some management failures and the market trends, Palm had less success in ensuring the cooperation of third party providers of hardware. Sony, for example, has stopped selling PDA's which run Palm OS. Handspring was purchased by PalmOne (a hardware company, independent from PalmSource, provider of Palm OS). In line with our prediction, Palm, eager to improve attractiveness of its operation system for the hardware developers, made it backward compatible. Any version of Palm OS, installed on hardware device, is not only able to run the applications, written for this version but also applications written for the older versions of the operation system.

3.6 Conclusion

In this paper we developed a theory of compatibility choice at two-sided market. This theory contributes to the literature on two-sided markets which up to now did not devoted much attention to the issues of compatibility.

Our first important result is that the monopolist will never choose partial compatibility. He will either make technologies incompatible or will make them compatible to the extent that agents, who interact using different platforms, can enjoy maximal possible network benefits. This result allows us to concentrate our analysis on four extreme compatibility regimes: full compatibility, incompatibility and backward compatibility for each side of the market.

We showed that the tradeoff, which is at the heart of monopolist's decision to make technologies compatible, is the tradeoff between demand of new agents on one side of the market and demand of the old agents on the other side of the market. In particular, if the monopolist introduces backward compatibility for, say, users, he encourages new users to buy the new platform but discourages the old sellers to do so (*direct effect*). The decrease in the demand of old sellers triggers the decrease in demand of old users and of the new users (*feedback effect*). The tradeoff between these effects determines which compatibility regime will be chosen in equilibrium.

Investigating different market structures (mature market, emerging market and asymmetric market) we characterized the choice of compatibility in terms of primitives of the model. In particular, we showed, that the compatibility for users will be imposed if the proportion of new users is relatively small, installed base of sellers and users is relatively small and the technological progress is moderate. We illustrate our predictions about the pattern of compatibility choice with two examples.

Our model can be modified in several ways. First, we assume that both sides of the market are symmetric in terms of per-interaction benefits. This is not necessarily the case on two-sided markets. We could modify the model by introducing some asymmetry between agents. This modification, however, would not change the underlying intuition and therefore the basic results.

Throughout the paper we assume that the quality of interaction between users and sellers is fixed and depends on the lowest technology that enables this interaction. For some markets other technological assumptions can be more realistic; for example, the quality of interaction may be determined by the best of two technologies. It would be useful to see how the choice of compatibility regime depends on technological assumptions.

Appendices

1.A Appendix: Proofs

Derivation of the value functions in Section 1.3.1. For illustration consider regime (1/1) and stopping rule R_1. When Δ is the length of the time period, Equation 1.3 can be rewritten as:

$$V_t^{11} = 2(R\Delta p - \Delta c) - R(\Delta p)^2 + \frac{1 - 2\Delta p + (\Delta p)^2}{1 + \Delta r} V_{t+\Delta}^{11},$$

Dividing the expression by Δ and taking $\Delta \to 0$ we receive the following differential equation:

$$V_t^{11}(r + 2p) = 2(Rp - c) + \dot{V}_t^{11}.$$

Solving the differential equation with terminal condition, which translates to $V_T^{11} = 0$, we receive the expression for V_t^{11}:

$$V_t^{11} = \frac{2(Rp - c)}{r + 2p}(1 - e^{-(r+2p)(T-t)}).$$

The derivation of the value function for the case, where stopping rule R_2 applies is identical except for the boundary condition, which now translates to $V_T^{11} = V_0^1$ in continuous time. Recall that V_0^1 is the value of the project in regime (1), i.e., when only a single entrepreneur is employed.

To derive value functions of entrepreneurs E_t^{11} and their incentive compatible shares s_t^{11} we use the same approach. Consider again regime (1/1), stopping rule R_1. The minimization program, which allows us to determine the optimal share s_t and expected reward E_t^{11} of the entrepreneur is given in Section 1.3 by problem (1.6). With incentive compatibility constraint being binding this problem results in the following expression for a share s_t^{11}:

$$ps_t - \frac{1}{2}p^2 s_t = c + \frac{p(1-p)}{1+r} E_{t+1}^{11}. \tag{12}$$

Considering transition of equality (12) to continuous time, we receive:

$$s_t = \frac{c}{p} + E_t^{11}$$

Since incentive compatibility constraint is binding in equilibrium, we can derive solution to the minimization problem (1.6) from the following equality:

$$E_t^{11} = c + \frac{1-p}{1+r} E_{t+1}^{11}. \tag{13}$$

Considering transition of the equation (13) to continuous time we obtain again a differential equation. After solving the differential equation, with the terminal condition which translates to $E_T^{11} = 0$, we receive the expression for the value function of an entrepreneur:

$$E_t^{11} = \frac{c}{r+p} \left(1 - e^{(r+p)(t-T)}\right).$$

□

Proof of Proposition 1.1. The proof is divided into two parts depending on the sign of A^{11}. If the parameters are such that $A^{11} > 0$, then the feasible contracts are $\mathcal{C}_1$, $\mathcal{C}_3$ and $\mathcal{C}_4$. On the other hand, if $A_{11} \leq 0$, then the available contracts are $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_4$. We will show that in both cases contract $\mathcal{C}_1$ is optimal.
First we show that contract $\mathcal{C}_1$ is always (regardless of A^{11}) preferred to contract $\mathcal{C}_4$. Translated into profits, this is equivalent to the inequality $V_{0,1}^{11} - 2E_{0,1}^{11} > V_0^1 - E_0^1$, with $V_{0,1}^{11}$, $E_{0,1}^{11}$, V_0^1, and E_0^1 given in Table 1. After substitution, this can be rewritten as

$$\frac{2(Rp-c)}{r+2p} - \frac{Rp+c}{r+p} + \frac{c}{r} - \left(\frac{2(Rp-c)}{r+2p}e^{-2pT} - \frac{Rp+c}{r+p}e^{-pT} + \frac{c}{r}\right) e^{-rT} > 0.$$

Note that the optimal stopping time T is the same for both contracts is $T^1 = -\frac{1}{p} \ln \frac{c}{Rp-c}$. Therefore, $e^{-pT} = \frac{c}{Rp-c}$. Using a substitution

$$x = \frac{c}{Rp-c}, \tag{14}$$

or equivalently $c = Rp\frac{x}{1+x}$, we the rewrite the above inequality as

$$\frac{Rp}{1+x}\left[\frac{2}{r+2p} - \frac{1+2x}{r+p} + \frac{x}{r} - \left(\frac{2x^2}{r+2p} - \frac{x(1+2x)}{r+p} + \frac{x}{r}\right) x^{r/p}\right] > 0.$$

Note that $e^{-pT} = x$ and the assumption $Rp > 2c > 0$ implies that $x \in (0,1)$.

Multiplying the last inequality by $(r+2p)(r+p)r(1+x)/(Rp)$ yields

$$r^2 + (p-r)(2p+r)x + p(2rx - 2p - r)x^{1+r/p} > 0.$$

Denote the left-hand side of this inequality as $f(x)$.[22] Then

$$\begin{aligned} f'(x) &= (r+2p)[2rx^{1+r/p} - (p+r)x^{r/p} + (p-r)], \\ f''(x) &= (r+2p)\left[2r\left(1+\frac{r}{p}\right)x^{r/p} - (p+r)\frac{r}{p}x^{-1+r/p}\right]. \end{aligned}$$

First observe that $f(0) = r^2 > 0$, $f(1) = 0$, $f'(1) = 0$, $f''(1) = r(r+p)(r+2p)/p > 0$. Moreover, for $p \leq r$, the function f is decreasing on interval $(0,1)$, since $f'(x) < (r+2p)[2rx^{r/p} - (p+r)x^{r/p} + (p-r)] = (r+2p)(p-r)(1-x^{r/p}) < 0$. Hence, $f(x) > f(1) = 0$, for $p \leq r$.

On the other hand, for $p > r$ we have $f'(0) = (r+2p)(p-r) > 0$. Therefore, $f(x) > f(0)$ in some neighborhood of 0. Now, assume by contradiction that $f(x_0) = 0$ for some $x_0 \in (0,1)$. Then by continuity there exists some $x_1 \in (0, x_0)$ such that $f(x_1) = f(0)$, which (according to *Rolle's theorem*) implies that there exist some $x_2 \in (0, x_1)$ and $x_3 \in (x_0, 1)$ such that $f'(x_2) = f'(x_3) = 0 = f'(1)$. Therefore, the equation $f''(x) = 0$ has at least two solutions in interval $(0,1)$, which is a contradiction, since $f''(x) = 0$ only if $x = \frac{1}{2}$. This proves that contract $\mathcal{C}_1$ is preferred to contract $\mathcal{C}_4$.

Now, we will show that for $A^{11} > 0$, contract $\mathcal{C}_1$ is preferred to $\mathcal{C}_3$. Obviously the latter contract is a limiting case of the former, when the research horizon is infinity. However, for contract $\mathcal{C}_1$ the optimal time $T_1^{11} = -\frac{1}{p}\ln\frac{c}{Rp-c}$ is finite. Hence, contract $\mathcal{C}_1$ with research horizon T_1^{11} is more profitable for the venture capitalist than contract $\mathcal{C}_1$ with any other research horizon, including infinite research horizon.[23] Therefore, contract $\mathcal{C}_1$ is better than contract $\mathcal{C}_3$.

It remains to prove that contract $\mathcal{C}_1$ is preferred to contract $\mathcal{C}_2$, i.e., that $V_{0,1}^{11} - 2E_{0,1}^{11} > V_{0,2}^{11} - 2E_{0,2}^{11} > 0$, with $V_{0,1}^{11}$, $E_{0,1}^{11}$, $V_{0,2}^{11}$, and $E_{0,2}^{11}$ given in Table 1. This can be rewritten as follows:

$$\begin{aligned} &-\frac{2(Rp-c)}{r+2p}e^{-(r+2p)T_1^{11}} + \frac{2c}{r+p}e^{-(r+p)T_1^{11}} - \\ &-\left(V_0^1 - \frac{2(Rp-c)}{r+2p}\right)e^{-(r+2p)T_1^{11}} + \left(E_0^1 - \frac{2c}{r+p}\right)e^{-(r+p)T_1^{11}} > 0. \end{aligned}$$

[22]Note that f is $\mathcal{C}^2$ on $(0,1]$.

[23]One can easily see that for the stopping rule R_1: $\frac{\mathrm{d}}{\mathrm{d}T}(V_0^{11} - 2E_0^{11}) < 0$, when $T > T_1^{11}$.

Using again the substitution (14), we obtain

$$e^{-pT_1^{11}} = x, \qquad \text{and} \qquad e^{-pT_2^{11}} = \frac{x(p+r)}{r} \cdot \frac{r-p+(r+p)x^{r/p}}{r+(r+2p)x^{1+r/p}}.$$

Then, the above inequality can be, after multiplying by $r(p+r)(2p+r)(1+x)/(Rpx^{2+r/p})$, rewritten as follows:[24]

$$2r - [r-p+(r+p)x^{r/p}]\left[\frac{p+r}{r} \cdot \frac{r-p+(r+p)x^{r/p}}{r+(r+2p)x^{1+r/p}}\right]^{1+r/p} > 0.$$

Similarly as in the first part of this proof, denote the left-hand side of this inequality as $g(x)$. Observe that $g(1) = 0$ and that

$$g'(x) = \frac{(r+p)(r+2p)}{p^2}\left[\frac{p+r}{r} \cdot \frac{r-p+(r+p)x^{r/p}}{r+(r+2p)x^{1+r/p}}\right]^{1+r/p} x^{-1+r/p} \times \\ \times \frac{r^2(x-1)+p^2x(x^{r/p}-1)}{r+(r+2p)x^{1+r/p}},$$

which is negative, since $0 < x < 1$. Therefore, $g(x) > g(1) = 0$ for all $x \in [0,1)$, which completes the proof. $\square$

Proof of Lemma 1.1. 1. Using the expressions for V_0^{11}, E_0^{11}, V_0^1, and E_0^1 from Table 1 in Appendix 1.B, we obtain

$$\begin{aligned} A^{10} &= \frac{c}{r(p+r)^2}\big(p^2 - pr - r^2 - (r+p)^2 e^{-rT} + pre^{-(r+p)T}\big), \\ B^{10} &= \frac{1}{(p+r)(r+2p)^2}\big[c(r+p)(r+2p) - pr(Rp-c) + \\ &\quad +(Rp-c)\big(-(r+2p)^2 e^{-(r+p)T} + 2p(r+p)e^{-(r+2p)T}\big)\big], \end{aligned}$$

with T being the optimal stopping time for contracts $\mathcal{C}_1$ and $\mathcal{C}_4$ from regime (1/1), which is the same, i.e., $T = T^1 = T_1^{11} = -\frac{1}{p}\log\frac{c}{Rp-c}$.

Similarly as in the Proof of Proposition 1.1 we use the substitution (14), or $c = Rp\frac{x}{1+x}$. In addition, to simplify the expressions, we use another substitution

$$z = \frac{r}{p},$$

or $r = zp$. Given the conditions on parameters, we have $x \in (0,1)$ and $z > 0$. With this substitution, e^{-rT} simplifies to a nice form x^z and the above

[24] Note that $r-p+(r+p)x^{r/p} > 0$, since $A^{11} = -Rpx/[r(r+p)(1+x)] \cdot [r-p+(r+p)x^{r/p}]$.

expressions can be rewritten as follows:

$$\begin{aligned} A^{10} &= \frac{x[1-z-z^2-(1+z)^2x^z+zx^{1+z}]}{(1+x)z(1+z)^2}, \\ B^{10} &= \frac{-z+(1+z)(2+z)x-(2+z)^2x^{1+z}+2(1+z)x^{2+z}}{(1+x)(1+z)(2+z)^2}. \end{aligned}$$

For simplicity denote $a(x)$ and $b(x)$ the numerators of A^{10} and B^{10}, respectively. Note that since their denominators are positive, the signs of A^{10} and B^{10} are the same as the signs of $a(x)$ and $b(x)$, respectively.

Depending on the sign of $1-z-z^2$, we discuss two cases. First, when $1-z-z^2 \leq 0$, then $a(x) < 0$, since $-(1+z)^2x^z + zx^{1+z} = [-(1+z+z^2) - z(1-x)]x^z < 0$. Second, when the inequality $1-z-z^2 > 0$ holds, we will prove a stronger statement that this inequality already implies $b(x) < 0$, regardless of the sign of $a(x)$. Note that for $z > 0$, the condition $1-z-z^2 > 0$ is equivalent to $0 < z < \frac{1}{2}(\sqrt{5}-1) \approx 0.6180$. Obviously $b(0) = -z$ and $b(1) = 0$. Taking the derivatives of $b(x)$ we obtain

$$\begin{aligned} b'(x) &= (1+z)(2+z)\big[1-\big(z+2(1-x)\big)x^z\big], \\ b''(x) &= (1+z)(2+z)x^{-1+z}[2(1+z)x - z(2+z)]. \end{aligned}$$

Then $b'(0) = (1+z)(2+z) > 0$ and $b'(1) = (1+z)(2+z)(1-z) > 0$. The second derivative implies that b is concave in the interval $(0, x_1)$ and convex on $(x_1, 1)$, where $x_1 = \frac{z(2+z)}{2(1+z)} < \frac{1}{2}$, due to assumption $1-z-z^2 > 0$. Therefore, b has a local maximum (denote it x_2) on interval $(0, x_1)$ and a local minimum on $(x_1, 1)$. Its possible shape is illustrated on Figure 6 in Appendix 1.B. Hence, in order to prove that $b(x) < 0$ on $(0, 1)$ it remains to show that $b(x_2) < 0$. Although it is not possible to find a closed formula for x_2, we know that

$$x_2^z = \frac{1}{z+2(1-x_2)}.$$

Using this, we obtain

$$\begin{aligned}
&2[z+2(1-x_2)]b(x_2) = \\
&\quad = \; 2[-z+x_2(1+z)(2+z)]\,[z+2(1-x_2)] - \\
&\qquad -2(2+z)^2x_2+4(1+z)x_2^2 = \\
&\quad = \; -4(1+z)^2x_2^2+z(z^2+4z+6)x_2-z(2+z) = \\
&\quad = \; -[2(1+z)x_2-z(2+z)]^2+z[z+2(1-x_2)](z^2+2z-2) < \\
&\quad < \; 2z[z+2(1-x_2)](z^2+z-1) < 0.
\end{aligned}$$

As a consequence, $A^{10} > 0$ implies that $G'(T) > 0$ for all $T \geq 0$. Hence the optimal stoping time is infinite.

2. The optimality condition $G(T) = 0$ can be rewritten as $e^{-pT} = \frac{(r+p)A^{10}}{(r+2p)B^{10}}$. The condition $(r+2p)B^{10} < (r+p)A^{10} < 0$ implies that $e^{-pT_2^{10}} \in (0,1)$, i.e., T_2^{10} is positive and finite. Moreover, we have $G''(T) = (r+2p)^2B^{10}e^{-(r+2p)T} - (r+p)^2A^{10}e^{-(r+p)T}$, which yields $G''(T_2^{10}) = (r+p)pA^{10}e^{-(r+p)T} < 0$.

3. We consider two cases. If $B^{10} \geq 0$, then obviously $G'(T) < 0$. If $B^{10} < 0$, then $G'(T) < [-(r+2p)B^{10} + (r+p)A^{10}]e^{-(r+p)T} < 0$ for all $T \geq 0$. Hence, $G(T)$ is monotonically decreasing and the optimal stoping time is zero.

□

Proof and numerical simulations for Proposition 1.2.

1. Contract $\mathcal{C}_4$ is feasible whenever $Rp > 2c$. If the second stopping rule is applied, the optimal stopping time is infinity (see discussion in Section 1.4) and contract $\mathcal{C}_7$ is feasible.

   The conditions $A^{10} > 0$ and $B^{10} < 0$ imply that $pE_0^{11} + 2c < (r+p)E_0^1$ and $(r+2p)V_0^1 < p(R+V_0^{11}) - 2c$ respectively. Moreover, from $V_0^1 > E_0^1 > 0$ we get $(r+p)E_0^1 < (r+2p)V_0^1$. Combining the inequalities, we obtain that

$$\frac{pE_0^{11}+2c}{r+p} < \frac{p(R+V_0^{11})-2c}{r+2p}, \quad \text{and hence} \quad T_1^{10} = -\frac{1}{p}\ln\frac{2c+pE_0^{11}}{p(R+V_0^{11})-2c} > 0,$$

   which means that contract $\mathcal{C}_5$ is feasible. We have proved that if the feasibility condition $A^{10} > 0$ is satisfied, then the pool of available contracts is $\mathcal{C}_4$, $\mathcal{C}_5$, $\mathcal{C}_7$. Further we will compare the surplus which the venture capitalist retains with each contract, in order to choose the optimal one.

Consider contracts $\mathcal{C}_4$ and $\mathcal{C}_7$. From the Proof of Lemma 1.1 we know that contract for $A^{10} > 0$, the contract $\mathcal{C}_7$ is optimal among all contracts with stoping rule R_2. As contract $\mathcal{C}_4$ is a degenerate case of this stoping rule (when the research horizon is zero), condition $A^{10} > 0$ then implies that $\mathcal{C}_7 \succ C_4$.[25]

Further, let us compare contract $\mathcal{C}_5$ and contract $\mathcal{C}_7$. In case of contract $\mathcal{C}_5$ the surplus of the venture capitalist is maximized at finite stopping time, $T_5^{10} = -\frac{1}{p}\ln\frac{2c+pE_0^{11}}{p(R+V_0^{11})-2c}$. However, if the financing horizon is infinite, then $\mathcal{C}_5$ is identical to contract $\mathcal{C}_7$. Hence, the former contract is always preferred to the latter.

In summary we get $\mathcal{C}_5 \succ C_7 \succ C_4$. Hence, the optimal contract is $\mathcal{C}_5$. Note that condition $A^{10} > 0$ implies that $E_0^1 > E_{0,5}^F + E_{0,5}^L$. In other words, competing entrepreneurs together require less compensation, than would a single entrepreneur.

2. Assume that $0 > A^{10}(r+p) > B^{10}(r+2p)$. According to Lemma 1.1, contract $\mathcal{C}_6$ is feasible. Recall, that

$$A^{10} = E_0^1 - \frac{pE_0^{11} + 2c}{r+p}, \qquad B^{10} = V_0^1 - \frac{p(R+V^{11}) - 2c}{r+2p}.$$

Hence, the inequality $A^{10}(r+p) > B^{10}(r+2p)$ implies that

$$0 < (pE_0^{11} + 2c) - E_0^1(r+p) < [p(R+V_0^{11}) - 2c] - V_0^1(r+2p).$$

Since $E_0^1(r+p) < V_0^1(r+2p)$, it necessarily must be that $pE_0^{11} + 2c < p(R + V_0^{11}) - 2c$. Hence, $T_1^{10} > 0$ and contract $\mathcal{C}_5$ is feasible as well. Therefore, the pool of contracts consists of $\mathcal{C}_5$, $\mathcal{C}_6$ and $\mathcal{C}_4$.

Let us first compare contracts $\mathcal{C}_5$ and $\mathcal{C}_6$. The former contract is preferred to the latter, if and only if

$$V_{0,5}^{10} - V_{0,6}^{10} > \left(E_{0,5}^{(10),L} + E_{0,5}^{(10),F}\right) - \left(E_{0,6}^{(10),L} + E_{0,6}^{(10),F}\right), \tag{15}$$

where all value functions are given in Table 2 in Appendix 1.B. After straightforward but tedious calculations we conclude that inequality (15) is equivalent to

$$T_5^{10} < T_6^{10} - \frac{1}{r+p}\ln\frac{2c + pE_0^{11} - E_0^1(r+p)}{2c + pE_0^{11}}.$$

[25] The relation "$\succ$" is used to denote preferences between contracts from the viewpoint of the venture capitalists, i.e., that one contract generates a larger profit for the venture capitalist than another one.

In that case, contract $\mathcal{C}_5$ is optimal. Otherwise, the optimal contract is $\mathcal{C}_6$.

Note, that now it is sufficient to prove, that $\mathcal{C}_6$ is preferred to contract $\mathcal{C}_4$, always when the feasibility condition $0 > A^{10}(r+p) > B^{10}(r+2p)$ holds. If this is the case, then $\mathcal{C}_5$ will be optimal, when $\mathcal{C}_5 \succ C_6 \succ C_4$ and $\mathcal{C}_6$ will be optimal, when $\mathcal{C}_6 \succ C_5$ and $\mathcal{C}_6 \succ C_4$.

Contract $\mathcal{C}_6$ is better, than contract $\mathcal{C}_4$, if and only if the following inequality holds:

$$V_0^1 e^{-(r+2p)T} + \frac{p(1+V_0^{11}) - 2c}{r+2p}\left(1 - e^{-(r+2p)T}\right) -$$
$$-E_0^1 e^{-(r+p)T} - \frac{2c + pE_0^{11}}{r+p}\left(1 - e^{-(r+p)T}\right) > V_0^1 - E_0^1,$$

This can be re-written in the form

$$A^{10}\left(1 - e^{-(r+p)T}\right) > B^{10}\left(1 - e^{-(r+2p)T}\right), \tag{16}$$

where A^{10} and B^{10} are defined above. Consider now two cases:

(a) If $0 > A^{10} > B^{10}$, then inequality (16) obviously holds, since

$$0 < \left(1 - e^{-(r+p)T}\right) < \left(1 - e^{-(r+2p)T}\right).$$

(b) If $A^{10} \leq B^{10} < 0$ we show numerically that (16) holds. In the numerical simulations we considered without loss of generality (see Remark 1.1) values $r = 0.05$ and $R = 1$. Using a grid 0.001×0.001 on the set of all positive (p, c), such that $p > 2c$ and $\frac{r+2p}{r+p}B^{10} < A^{10} \leq B^{10} < 0$, we plotted points where profit of the venture capitalist under contract $\mathcal{C}_6$ exceeds his profit under contract $\mathcal{C}_4$. The simulations show that this is the case everywhere in the defined domain. Figure 7 illustrates the case for $r = 0.05, p = 0.5$, where $\Delta := A^{10}\left(1 - e^{-(r+p)T}\right) - B^{10}\left(1 - e^{-(r+2p)T}\right)$.

3. According to Lemma 1.1, condition $A^{10}(r+p) < B^{10}(r+2p)$ implies that contract $\mathcal{C}_6$ is not feasible. Moreover, condition $(2c + E_0^{11}) < p(R + V_0^{11}) - 2c$ implies that $\mathcal{C}_5$ is feasible. Therefore, we choose the optimal contract between $\mathcal{C}_5$ and $\mathcal{C}_4$. Using numerical simulations, we have verified that in domain R_4, given that the feasibility conditions are satisfied for contract $\mathcal{C}_5$, the venture capitalist prefers to finance the leader alone (contract $\mathcal{C}_4$ is better than contract $\mathcal{C}_5$). Again, the

numerical simulations were performed for $r = 0.05$ and $R = 1$, using a grid of 0.001×0.001 for parameters (p, c).

4. If $(2c + E_0^{11}) > p(R + V_0^{11}) - 2c$, the only feasible (hence, optimal) contract is $\mathcal{C}_4$.

$\square$

1.B Appendix: Tables and figures

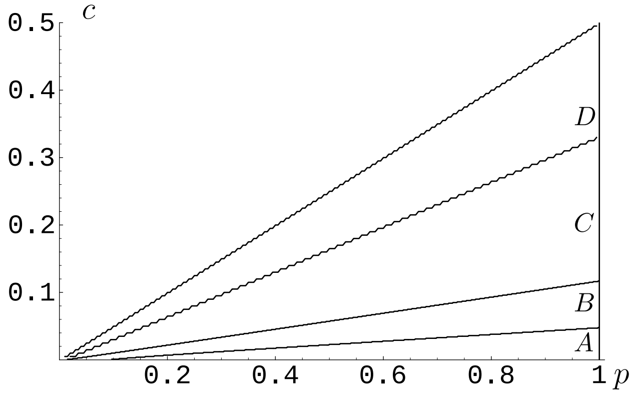


Figure 5: Feasibility of contracts in regime (1/0)

Notes to Figure 5:

1. Contract $\mathcal{C}_4$ is feasible in domains A,B,C and D;
2. Contract $\mathcal{C}_5$ is feasible in domains A, B and C;
3. Contract $\mathcal{C}_6$ is feasible in domain B;
4. Contract $\mathcal{C}_7$ is feasible in domain A.

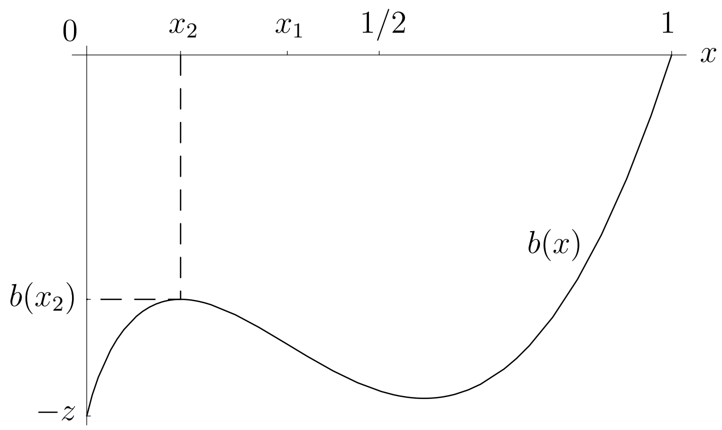


Figure 6: Shape of function $b(x)$

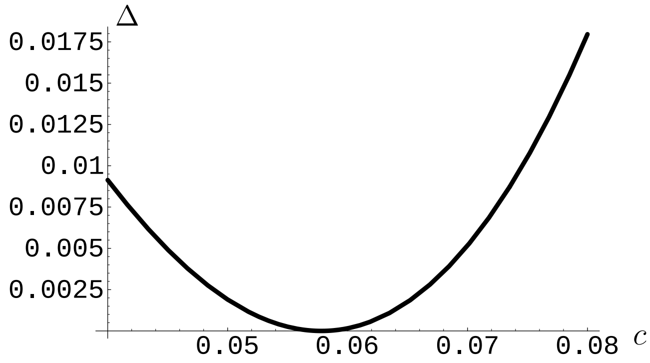


Figure 7: Regime (1/0): Illustration for Case 2; $p = 0.5$, $r = 0.05$.

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
Stoping rule	R_1	R_2	R_2	R_3
Share of entrep.	$s_{t,1}^{11} = \frac{c}{p} + E_t^{11}$	$s_{t,2}^{11} = \frac{c}{p} + E_t^{11}$	$s_{t,3}^{11} = \frac{c}{p} + E_t^{11}$	$s_t^1 = \frac{c}{p} + E_t^1$
Value fnct. of entrep.	$E_{t,1}^{11} = \frac{c}{r+p} \cdot \left(1 - e^{-(r+p)(T-t)}\right)$	$E_{t,2}^{11} = \left(\frac{1}{2}E_0^1 - \frac{c}{r+p}\right) \cdot$ $\cdot e^{-(r+p)(T-t)} + \frac{c}{r+p}$	$E_{t,3}^{11} = \frac{c}{r+p}$	$E_t^1 = \frac{c}{r}\left(1 - e^{-r(T-t)}\right)$
Value of the venture	$V_{t,1}^{11} = \frac{2(Rp-c)}{r+2p} \cdot \left(1 - e^{-(r+2p)(T-t)}\right)$	$V_{t,2}^{11} = \left(V_0^1 - \frac{2(Rp-c)}{r+2p}\right) \cdot$ $\cdot e^{-(r+2p)(T-t)} + \frac{2(Rp-c)}{r+2p}$	$V_{t,3}^{11} = \frac{2(Rp-c)}{r+2p}$	$V_t^1 = \frac{Rp-c}{r+p} \cdot$ $\cdot \left(1 - e^{-(r+p)(T-t)}\right)$
Optimal time	$T_1^{11} = -\frac{1}{p} \ln \frac{c}{Rp-c}$	$T_2^{11} = -\frac{1}{p} \ln \frac{r+p}{r+2p} \frac{E_0^1 - \frac{2c}{r+p}}{V_0^1 - \frac{2(Rp-c)}{r+2p}}$	$T_3^{11} \to \infty$	$T^1 = -\frac{1}{p} \ln \frac{c}{Rp-c}$
Feasibility condit.		$0 > A^{11}(r+p) > B^{11}(r+2p)$	$A^{11} > 0$	

Table 1: Optimal contracts and corresponding expected values in regime (1/1)

	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$
Stoping rule	R_3	R_1	R_2	R_2
Share of the L.	$s_t^L = s_t^1 = \frac{c}{p} + E_t^1$	$s^L_{t,5} = \frac{c}{p} + E^F_{t,5} - E^{11}_0$	$s^L_{t,6} = \frac{c}{p} + E^L_{t,6}$	$s^L_{t,7} = \frac{c}{p} + E^L_{0,7}$
Value fnct. of the L.	$E_t^1 = \frac{c}{r}\left(1 - e^{-r(T-t)}\right)$	$E^L_{t,5} = \frac{c+pE_0^{11}}{r+p} \cdot$ $\cdot \left(1 - e^{-(r+p)(T-t)}\right)$	$E^L_{t,6} = \left(E_0^1 - \frac{c+pE_0^{11}}{r+p}\right) \cdot$ $\cdot e^{-(r+p)(T-t)} + \frac{c+pE_0^{11}}{r+p}$	$E^L_{t,7} = \frac{c+pE_0^{11}}{r+p}$
Share of the F.		$s^F_{t,5} = \frac{c}{p} + E^F_{t,5} - E^{11}_0$	$s^F_{t,6} = \frac{c}{p} + E^F_{t,6} - E^{11}_0$	$s^F_{t,7} = \frac{c}{p} + E^F_{t,7} - E^{11}_0$
Value fnct. of the F.		$E^F_{t,5} = \frac{c}{r+p} \cdot$ $\cdot \left(1 - e^{-(r+p)(T-t)}\right)$	$E^F_{t,6} = \frac{c}{r+p}\left(1 - e^{-(r+p)(T-t)}\right)$	$E^F_{t,7} = \frac{c}{r+p}$
Value of the venture	$V_t^1 = \frac{Rp-c}{r+p} \cdot$ $\cdot \left(1 - e^{-(r+p)(T-t)}\right)$	$V^{10}_{t,5} = \frac{p(R+V_0^{11})-2c}{r+2p} \cdot$ $\cdot \left(1 - e^{-(r+2p)(T-t)}\right)$	$V^{10}_{t,6} = \left(V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}\right) \cdot$ $\cdot e^{-(r+2p)(T-t)} + \frac{p(R+V_0^{11})-2c}{r+2p}$	$V^{10}_{t,7} = \frac{p(R+V_0^{11})-2c}{r+2p}$
Optimal time	$T^1 = -\frac{1}{p} \ln \frac{c}{Rp-c}$	$T^{10}_5 = -\frac{1}{p} \ln \frac{2c+pE_0^{11}}{p(R+V_0^{11})-2c}$	$T^{10}_6 = -\frac{1}{p} \ln \frac{r+p}{r+2p} \frac{E_0^1 - \frac{2c+pE_0^{11}}{r+p}}{V_0^1 - \frac{p(R+V_0^{11})-2c}{r+2p}}$	$T^{10}_7 \to \infty$
Feasibility cond.		$2c + E_0^{11} < p(R + V_0^{11}) - 2c$	$0 > A^{10}(r+p) > B^{10}(r+2p)$	$A^{10} > 0$

Table 2: Optimal contracts and corresponding expected values in regime (1/0)

2.A Appendix: Proofs

Proof of Proposition 2.3. The problem of the principal is as follows

$$
\begin{aligned}
\max_{\beta_1,\ \beta_2,\ c,\ d} \quad & \Pi_P^C = R(1 - \tfrac{x}{x+y}\beta_1 - \tfrac{y}{x+y}\beta_2)(1 - e^{-(x+y)}) - (c+d) \\
s.t. \quad & (IC_c^1)\ R\beta_1 \geq \frac{e^{x+y}(x+y)^2}{x^2 - y + e^{x+y}y + xy}\ , \\
& (IC_c^2)\ R\beta_2 \geq \frac{e^{x+y}(x+y)^2}{y^2 - x + e^{x+y}x + xy}\ , \\
& (RC_c^1)\ x \leq c, \\
& (RC_c^2)\ y \leq d, \\
& (CS_c^1) \left(R\beta_1 - \frac{e^{x+y}(x+y)^2}{x^2 - y + e^{x+y}y + xy} \right)(c - x) = 0, \\
& (CS_c^2) \left(R\beta_2 - \frac{e^{x+y}(x+y)^2}{y^2 - x + e^{x+y}x + xy} \right)(d - y) = 0.
\end{aligned}
$$

I will develop the proof in several steps.

Step 1. All constrains are binding. The argument is identical to the one provided in Section 2.4.

Step 2. With all constrains being binding, we can re-write the principal problem in the following form:

$$
\max_{x,y} \left(R - \frac{e^{x+y}(x+y)^2}{x^2 - y + e^{x+y}y + xy} - \frac{e^{x+y}(x+y)^2}{y^2 - x + e^{x+y}x + xy} \right)(1 - e^{-(x+y)}) - (x+y).
$$

Taking the first-order condition with respect to x and y and subtracting resulting equations from each other, I receive the following expression:

$$
\frac{e^{x+y}(e^{x+y} - 1)^2(x-y)(x+y)^3(-1 + e^{x+y} - (x+y))(e^{x+y} + x + y - 1)}{(y^2 + x(y + e^{x+y} - 1))^2((-1 + e^{x+y})y + x(x+y))^2} = 0.
$$

The above equality holds, if each of the following conditions are satisfied:

1. $[y^2 + x(y + e^{x+y} - 1)]^2[(-1 + e^{x+y})y + x(x+y)]^2 \neq 0,$
2. $e^{x+y}(e^{x+y} - 1)(x-y)(x+y)^3[-1 + e^{x+y} - (x+y)]^2(e^{x+y} + x + y - 1) = 0$

From the first condition it follows, that $x \neq -y$. The second condition holds if at least one of the following conditions is satisfied:

1. $x = y$,

2. $x = -y$

3. $-1 + e^{x+y} + x + y = 0$,

4. $-1 + e^{x+y} - (x + y) = 0$.

Condition $2, 3, 4$ are ruled out based on the result that $x \neq -y$. Hence, Condition 1 necessarily holds, i.e. $x = y$. This automatically implies, that $R\beta_1 = R\beta_2$ and $c = d = x = y$. Hence the agents are offered a symmetric contract.

Step 3. With binding constrains and symmetric contracts the problem of the principal can be written in the following form:

$$\max_{\beta^C,\ c} \quad \Pi_P^C = R(1-\beta^C)(1-e^{-2x}) - 2c$$
$$s.t \quad R\beta^C = \frac{e^{2x}4x^2}{2x^2 - x + e^{2x}x},$$
$$x = c.$$

The solution to the problem is given by

$$R = \frac{e^{2c}\left(4c(e^{2c}-1) + 3(e^{2c}-1)^2 + c^2(4+8e^{2c})\right)}{(e^{2c}+2c-1)^2}. \tag{17}$$

Taking the derivative of the right-hand side of the Equation (17), one can see that R is increasing in c if

$$-7 + 3e^{3t} + 13e^{2t}(t-1) + t[5 + (t-3)t] + e^t[17 + t(2+t)(4t-9)] > 0,$$

where $t = 2c$. If $t = 0$, then the left-hand side of this inequality is 0. Since the left-hand side obviously increases in t, the inequality is strictly satisfied for $t > 0$. Hence, R increases in c.

Applying the L'Hospital rule to the (17) it is easy to establish, that $R \to 2$ as $c \to 0$. Since R increases in c the project will not be financed for $R < 2$.

Step 4. Assume $R > 2$. Let us verify, that the optimal contract, where

$$c = d, \qquad R\beta_1 = R\beta_2 := R\beta^C = \frac{4ce^{2c}}{e^{2c} - 1 + 2c},$$

and c is implicitly given by equation (17) leads to unique SPNE. For this, it is sufficient to show that the reaction functions of the agents cross only once.

Let us denote the reaction function of the first and second agents as $R_x(y)$ and $R_y(x)$ respectively. The functions $R_x(y)$ and $R_y(x)$ are symmetric and are implicitly given by the Equations (18) and (19) respectively:

$$R\beta_1 = \frac{e^{x+y}(x+y)^2}{x^2 - y + e^{x+y}y + xy}, \tag{18}$$

$$R\beta_2 = \frac{e^{x+y}(x+y)^2}{y^2 - x + e^{x+y}x + xy}. \tag{19}$$

Notice, that $R_x(0) = R_y(0) = \log R\beta_1$. Hence, to prove that reaction functions cross only once it is sufficient to prove that $\frac{dR_x(y)}{dy} \neq -1$ and $\frac{dR_y(x)}{dx} \neq -1$. Consider $R_x(y)$. Applying implicit function theorem to (18) we obtain $\frac{dR_x(y)}{dy} = -\frac{A(x,y)}{B(x,y)}$, where

$$A(x,y) = x^3 - y(1 - e^{x+y} + y) + x^2(1+2y) + x(1 - e^{x+y} + y^2)$$
$$B(x,y) = x^3 + 2x^2y - xy(2-y) - 2y(1 - e^{x+y} + y)$$

It is easy to show, that $A(x,y) < B(x,y)$ is equivalent to $x + y + 1 - e^{x+y} < 0$, which is always true for $x > 0, y > 0$. Hence, $\frac{dR_x(y)}{dy} \neq -1$. The argument for $R_y(x)$ is symmetric.

□

Proof of Corollary 2.2. Recall the incentive compatibility constraints in a setting with a single agent and competing agents:

$$R\beta^A = e^x, \qquad R\beta_1^C = \frac{e^{x+y}(x+y)^2}{x^2 - y + e^{x+y}y + xy}, \qquad R\beta_2^C = \frac{e^{x+y}(x+y)^2}{y^2 - x + e^{x+y}x + xy}.$$

Assume that the principal wants to implement the following probability of success: $p(t) = 1 - e^{-t}$. This probability is achieved in the setting with a single agent if the latter allocates $x = t$ into the project. In the setting with competing agents this probability is achieved if both agents allocate amount $x + y = t$ into the project. The incentive compatibility constraints then can be written as follows:

$$e^{-t} = \frac{1}{R\beta^A}, \tag{20}$$

$$e^{-t} + \frac{e^{-t}y(e^t - 1 - t)}{t^2} = \frac{1}{R\beta_1^C}. \tag{21}$$

Obviously, for any positive t and y the left-hand side of the equality (21) is larger than the left-hand side of the equality (20). Hence the required incentive compatible share $R\beta_1^C$ is smaller than $R\beta^A$ (the same argument holds for the second competing agent). Recall, that if competing agents are employed only the winner receives a reward. Hence, to implement the same probability of success, the principal has to pay smaller reward if he employs competing agents than if he employs a single agent (while investing the same amount t in both cases). It is clear therefore, that in SPNE the principal is better off employing competing agents.

□

Proof of Proposition 2.4. The problem of the principal in general form is:

$$\max_{\beta_1,\ \beta_2,\ c,\ d} \quad \Pi_P^T = R(1-\beta_1-\beta_2)(1-e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}}) - (c+d)$$

$$s.t. \quad (IC_t^1)\ R\beta_1 \geq \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} x^\alpha}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}},$$

$$(ICT_2)\ R\beta_2 \geq \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} y^\alpha}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}},$$

$$(RCT_1)\ x \leq c,$$

$$(RCT_2)\ y \leq d,$$

$$(CST_1)\ \left(R\beta_1 - \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} x^\alpha}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}}\right)(x-c) = 0,$$

$$(CST_2)\ \left(R\beta_2 - \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} y^\alpha}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}}\right)(y-d) = 0.$$

The proof is developed in several steps.

Step 1. All constrains for the principal's problem are binding. See the argument in Section 2.4.

Step 2. With all constrains being binding, we can re-write the problem of the principal in the following form:

$$\max_{x,y} \Pi_P^T = R\left(1 - \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} x^\alpha}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}} - \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} y^\alpha}{(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}}}\right)(1-e^{-(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}})-(x+y)$$

Taking the first-order condition with respect to x and y and subtracting the second one from the first one I receive the following equality:

$$xy(x^\alpha - y^\alpha) = \alpha\left(e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} - 1\right)(x^{1-\alpha}+y^{1-\alpha})^{\frac{\alpha}{1-\alpha}} xy(y^{2\alpha-1} - x^{2\alpha-1}).$$

The above equation obviously holds if and only if $x = y$. This implies, that $c = d$ and $\beta_1 = \beta_2$.

Step 3. Given the results of two previous steps, I can re-write the problem of the principal in the following form:

$$\max_{\beta^T,c,x} \quad \Pi_P^T = R(1-2\beta^T)(1-e^{-2^{\frac{1}{1-\alpha}}x}) - 2c \tag{22}$$
$$s.t. \quad R\beta^T = \frac{e^{2^{\frac{1}{1-\alpha}}x}}{2^{\frac{\alpha}{1-\alpha}}},$$
$$x = c.$$

Solving this problem, I receive the optimal amount of investments c:

$$R = 2^{\frac{\alpha}{\alpha-1}} e^{2^{\frac{1}{1-\alpha}}c}(1+2e^{2^{\frac{1}{1-\alpha}}c}) \Leftrightarrow c = 2^{\frac{1}{\alpha-1}} \ln\frac{1}{4}\left(-1+2^{\frac{1}{1-\alpha}}\sqrt{4^{\frac{1}{\alpha-1}}+2^{2+\frac{1}{\alpha-1}}R}\right). \tag{23}$$

Step 4. According to (23), c is increasing in R. Let us determine the threshold $\hat{R}$, such that $c > 0$ if $R > \hat{R}$. Solving (23) for $c = 0$, I obtain $\hat{R} = 3 \cdot 2^{\frac{\alpha}{\alpha-1}}$. So, for $R \leq 3 \cdot 2^{\frac{\alpha}{1-\alpha}}$ the team generates negative profit for the principal and will never be employed. For $R > 3 \cdot 2^{\frac{\alpha}{1-\alpha}}$ the team generates positive profit. To see this, notice that $\Pi_P^T(\hat{R}) = 0$ and Π_P^T is increasing in R, if $R > \hat{R}$. Indeed, substituting the expression for optimal c into the Π_P^T, I receive:

$$\Pi_P^T = \frac{1}{4}\left(4R - 8\sqrt{4^{\frac{1}{\alpha-1}}+2^{2+\frac{1}{\alpha-1}}R} + 2^{3+\frac{1}{\alpha-1}}(2+\ln 4) - \right.$$
$$\left. -2^{3+\frac{1}{\alpha-1}} \ln\left(-1+2^{\frac{1}{\alpha-1}}\sqrt{4^{\frac{1}{\alpha-1}}+2^{2+\frac{1}{\alpha-1}}R}\right)\right).$$

Taking the derivative of the profit I establish the following equivalence:

$$\frac{d\Pi_P^T}{dR} = \frac{-5\cdot 2^{\frac{1}{\alpha-1}} + \sqrt{4^{\frac{1}{\alpha-1}}+2^{2+\frac{1}{\alpha-1}}R}}{-2^{\frac{1}{\alpha-1}} + \sqrt{4^{\frac{1}{\alpha-1}}+2^{2+\frac{1}{\alpha-1}}R}} > 0 \Longleftrightarrow R > \hat{R}$$

In addition, $\Pi_P^T(\hat{R}) = 0$. Together with the equivalence above it implies that for any $R > \hat{R}$, the profit of the principal is positive.

Step 5. Since agents are offered symmetric contracts, we can concentrate our attention on symmetric equilibria. This leaves us with two equilibrium candidates: $(0, 0)$ and (c, c), where c was derived on Step 3.

According to the incentive compatibility constraints (see Table 4), an equilibrium

$(0,0)$ emerges if $R\beta_1 = R\beta_2 \leq 1$. From 22, the optimal contract results in $R\beta_1 = R\beta_2 = \frac{e^{2^{\frac{1}{1-\alpha}}c}}{2^{\frac{\alpha}{1-\alpha}}}$. Therefore, $(0,0)$ is not an equilibrium, if

$$\frac{e^{2^{\frac{1}{1-\alpha}}c}}{2^{\frac{\alpha}{1-\alpha}}} > 1 \Longleftrightarrow c > 2^{\frac{1}{\alpha-1}} \ln 2^{\frac{\alpha}{1-\alpha}} \Longleftrightarrow R > 1 + 2^{\frac{1}{1-\alpha}}.$$

Hence, if $3 \cdot 2^{\frac{\alpha}{1-\alpha}} < R \leq 1 + 2^{\frac{1}{1-\alpha}}$, then there are two equilibria (c,c) and $(0,0)$. Otherwise, there is a unique equilibrium (c,c).

□

The following lemma will be useful for proof of Proposition 2.5.

Lemma 2.2. *Consider the difference $F(\alpha, R) = \Pi_P^T(\alpha, R) - \Pi_P^C(R)$. The following statements hold:*

1. *For fixed R, as $\alpha \to 1$ the $F(\alpha, R)$ converges to a positive constant.*
2. *For any fixed R, function $F(\alpha, R)$ increases in α.*
3. *For fixed $\alpha < 1$, as $R \to \infty$ the function $F(\alpha, R)$ converges to $-\infty$.*

Proof of Lemma 2.2.

1. As $\alpha \to \infty$ the team succeeds with certainty for arbitrary small investment. Hence, $\Pi^{sim} \to R$. On the other hand for any R, $\Pi_P^C < R$. Therefore, $\Pi_P^T(\alpha, R) > \Pi_P^C(R)$ for $\alpha \to 1$.

2. $F(\alpha, R)$ increases in α if $\Pi_P^T(\alpha, R)$ increases in α. Using Proposition 2.4 I obtain the following expression for the profit function:

$$\Pi_P^T(\alpha, R) = \frac{1}{4}(4R - 8A + 2^{3+\frac{1}{\alpha-1}}(2 + \log 4) - 2^{3+\frac{1}{\alpha-1}} \log(-1 + 2^{\frac{1}{1-\alpha}} A)),$$

where $A = \sqrt{4^{\frac{1}{\alpha-1}} + 2^{2+\frac{1}{\alpha-1}} R}$. From Proposition 2.4 follows, that for all (α, R) where $c > 0$ holds $2^{\frac{1}{1-\alpha}} A - 1 > 4$.

The derivative of $\Pi_P^T(\alpha, R)$ is positive if the following inequality is satisfied:

$$2^{\frac{1}{\alpha-1}} + 2^{\frac{1}{1-\alpha}}(4^{\frac{\alpha}{\alpha-1}} + 2^{2+\frac{1}{\alpha-1}} R) + (A - 2^{\frac{1}{\alpha-1}})(\log(2^{\frac{1}{1-\alpha}} A - 1) - \log 4) > 3A \quad (24)$$

Both sides of the above inequality are positive. Moreover, for any $\alpha \in [0, 1)$:

$$2^{\frac{1}{1-\alpha}}(4^{\frac{\alpha}{\alpha-1}} + 2^{2+\frac{1}{\alpha-1}} R) > 2^{\frac{1}{1-\alpha}}(4^{\frac{1}{\alpha-1}} + 2^{2+\frac{1}{\alpha-1}} R) = 2^{\frac{1}{1-\alpha}} A^2.$$

Hence, to prove that inequality (24) holds it is sufficient to show, that $2^{\frac{1}{1-\alpha}}A^2 > 3A$. The latter inequality is satisfied due to the result $2^{\frac{1}{1-\alpha}}A - 1 > 4$, which is equivalent to $A > 3 \cdot 2^{\frac{1}{\alpha-1}}$.

3. $\Pi_P^T(\alpha, R)$ is explicitly defined in terms of exogenous parameters (α, R). On the contrary, $\Pi_P^C(R)$ can not be explicitly expressed in terms of R. However, from Proposition 2.3 follows, that there is a a functional relationship between R and c:

$$R = \frac{e^{2c}\left(4c(e^{2c}-1) + 3(e^{2c}-1)^2 + c^2(4+8e^{2c})\right)}{(e^{2c}+2c-1)^2} \tag{25}$$

From (25) follows that $R \to \infty$ as $c \to \infty$. Using this expression in the place of R, I receive the following result:

$$\Pi_P^T(\alpha, R) - \Pi_P^C(R) = k_1 A(c) - k_2\sqrt{B(c)} - k_3 \log(-1 + k_4\sqrt{B(c)}) + k_5,$$

where $A(c)$ and $B(c)$ are functions of c, while $k_1 - k_5$ are constants, independent of c.

$$A(c) = \frac{4(8c^3 + 3(e^{2c}-1)^2 + 12c^2(2e^{2c}-1) + c(2 - 8e^{2c} + 6e^{4c}))}{(e^{2c}+2c-1)^2}, \tag{26}$$

$$B(c) = 4^{\frac{1}{\alpha-1}} + \frac{2^{2+\frac{1}{\alpha-1}} e^{2c}\left(4c(e^{2c}-1) + 3(e^{2c}-1)^2 + c^2(4+8e^{2c})\right)}{(e^{2c}+2c-1)^2}. \tag{27}$$

Notice, $B(c) \to \infty$ as $c \to \infty$. Using the fact that $log(x-1) < x$ for any $x > 2$ we can estabish the following inequality:

$$k_1 A(c) - k_2\sqrt{B(c)} - k_3 \log(-1 + k_4\sqrt{B(c)}) + k_5 > k_1 A(c) - k_6\sqrt{B(c)} + k_5. \tag{28}$$

It easy to show, that $\frac{A(c)}{c} \to const$ and $\frac{B(c)}{e^{2c}} \to const$ as $c \to \infty$. Finally, let me re-write the right-hand side of Inequality (28):

$$e^c\left(\frac{c}{e^c}k_1\frac{A(c)}{c} - k_6\sqrt{\frac{B(c)}{e^{2c}}} + \frac{k_5}{e^c}\right).$$

To complete the proof, notice, that as $c \to \infty$ the expression in brackets converges to a negative constant, so that the whole expression converges to $-\infty$.

Proof of Proposition 2.5. To proof the first statement, notice that $\alpha_1 = \frac{\log 3 - \log 2}{\log 3}$ implies $3 \cdot 2^{\frac{\alpha}{\alpha-1}} = 2$. From Propositions 2.2, 2.3 and 2.4 follows, that for any $R > 2$ and

α such that $3 \cdot 2^{\frac{\alpha}{\alpha-1}} \leq 2$, $\Pi_P^A(R) > 0$, $\Pi_P^C(R) > 0$ and $\Pi_P^T(\alpha, R) > 0$.
It is immediate, that $\Pi_P^A(R) \geq \Pi_P^T(R)$ for any R if $\alpha = 0$. Indeed, in the absence of synergy effects the principal is at least as well off employing a single agent, as employing a team. However, by Corollary 2.2, $\Pi_P^C(R) > \Pi_P^A(R)$ for any R. Hence, for $\alpha = 0$ holds $\Pi_P^C(R) > \Pi_P^T(\alpha, R)$.
Further, $\Pi_P^C(R) > \Pi_P^A(R) > \Pi_P^T(\alpha_1, R)$. To see this, consider profit functions $\Pi_P^A(R)$ and $\Pi_P^T(\alpha, R)$, which can be defined from Proposition 2.2 and Proposition 2.4 respectively.
Evaluating $\Pi_P^T(\alpha, R)$ at α_1 and subtracting the resulting expression from $\Pi_P^A(R)$, I receive the following function:

$$\begin{aligned} f(R) &= \Pi_P^A(R) - \Pi_P^T(\alpha_1, R) = \frac{1}{3}\Big(-1 - 3\sqrt{1+4R} + \\ &+ 2\sqrt{1+12R} - \log 2 - 3\log(-1+\sqrt{1+4R}) + 2\log(-1+\sqrt{1+12R})\Big). \end{aligned}$$

The function $f(R)$ increases in R and $f(2) = 0$. Hence, $\Pi_P^A(R) > \Pi_P^T(\alpha, R)$ for any $R > 2$, which implies that $\Pi_P^C(R) > \Pi_P^T(\alpha, R)$.
Finally, by Lemma 2.2 the difference $\Pi_P^T(R) - \Pi_P^C(\alpha, R)$ increases in α for fixed R. Hence, for all $\alpha \in [0, \alpha_1)$ this difference is negative, so that $\Pi_P^T(R) < \Pi_P^C(\alpha, R)$.
Consider now $\alpha \geq \alpha_1$ and let me prove, that $\hat{\alpha}(R)$ increases in R. Taking into account the functional relation between R and c, given by (25), $\hat{\alpha}(R)$ can be written as $\hat{\alpha}(R(c))$. Therefore,

$$\frac{d\hat{\alpha}}{dc} = \frac{d\hat{\alpha}}{dR} \cdot \frac{dR}{dc}. \tag{29}$$

From Proposition 2.3 it is known, that $R(c)$, given by (25) is an increasing function. Hence, to prove that $d\hat{\alpha}/dR > 0$ it is sufficient to show, that $\mathrm{d}\hat{\alpha}/dc > 0$.
It is possible to calculate the derivative $\mathrm{d}\hat{\alpha}/dc$ by applying the implicit function theorem to $F(\alpha, R)$, where $R(c)$ is given in (25) and $F(\alpha, R)$ is defined in Lemma 2.2:

$$\frac{d\hat{\alpha}}{dc} = -\frac{\partial F/\partial c}{\partial F/\partial \alpha}. \tag{30}$$

From Lemma 2.2, $\partial F/\partial \alpha > 0$. Hence, to show that $d\hat{\alpha}/dc > 0$ it is sufficient to proof that $\partial F/\partial c < 0$. Substituting $R(c)$ into $F(\alpha, R)$ and taking the derivative of this function with respect to c, one can derive the following equivalence:

$$\frac{\partial F}{\partial c} < 0 \Longleftrightarrow B(c) < (2^{\frac{1}{\alpha-1}} + 2^{2+\frac{1}{1-\alpha}} e^{2c})^2, \tag{31}$$

where $B(c)$ is given in (27).
The inequality above can be re-written as

$$2^{\frac{\alpha}{\alpha-1}} > \frac{4c(e^{2c}-1)+3(e^{2c}-1)+c^2(4+8e^{2c})}{(2c+e^{2c}-1)^2(1+2e^{2c})}. \tag{32}$$

Let me denote the right-hand side of this inequality $K(c)$. Taking the derivative $K'(c)$, it is easy to show that this derivative is negative if

$$4-(7-26c+4c^2+8c^3)e^{2c}+5(1-2c)^2e^{4c}+(16c^2-5-6c)e^{6c}+3e^{8c}>0.$$

For simplicity, let me denote the left-hand side of this inequality as $k(c)$. Note, that $k(c)=0$ if $c=0$. Hence, for any $c>0$, $k'(c)>0$ implies $k(c)>0$. Note, that $k'(0)=k''(0)=0$. Further, $k'''(c)>0$ for any $c>0$:

$$-4c^3+4c^2(-5+20e^{2c}+54e^{4c})+c(40e^{2c}-11+135e^{4c})+ \\ +2(5-5e^{2c}-36e^{4c}+48e^{6c})>0.$$

Hence, $k''(c)>0$ for any $c>0$, which implies that $k'(c)>0$ for any $c>0$.
Since $K(c)$ is a decreasing function, for any α there exist $\hat{c}$, such that $2^{\frac{\alpha}{\alpha-1}}>K(c)$ if $c<\hat{c}$ and $2^{\frac{\alpha}{\alpha-1}}<K(c)$ if $c>\hat{c}$. Therefore, function $F(R(c),\alpha)$ reaches its maximum in $c=\hat{c}$.
From Proposition 2.3 it follows that $R(c)\to 2$ as $c\to 0$. For any $R\leq 2$ competing agents are not employed. On the other hand, from Proposition 2.4, team is employed (and generates positive profit) whenever $R>3\cdot 2^{\frac{\alpha}{\alpha-1}}$. Since $2\geq R>3\cdot 2^{\frac{\alpha}{\alpha-1}}$ implies $\alpha\geq\alpha_1$, the team generates positive profit for any $R\leq 2$ and $\alpha\geq\alpha_1$. Hence, $F(\alpha,R(\alpha))>0$ as $c\to 0$.
Finally, from Lemma 2.2, it follows that $F(\alpha,R)\to-\infty$ as $R\to\infty$. Since $R(c)$ given by (25) is an increasing function, which converges to infinity as $c\to\infty$, this also implies $F(\alpha,R(c))\to-\infty$ as $c\to\infty$.
Combining all results, I conclude, that $F(\alpha,R(c))$ reaches its maximum in $\hat{c}$; for any $c\in[o,\hat{c}]$, $F(\alpha,R(c))>0$ and $F(\alpha,R(c))\to-\infty$ as $c\to\infty$. Therefore, there exist $c^*>\hat{c}$, such that $F(\alpha,R(c^*))=0$. Further $\partial F/\partial c(c=c^*)<0$, which implies $\mathrm{d}\alpha/dc>0$.
To proof the result that $\hat{\alpha}(R)\to 1$ as $R\to\infty$, notice that the first statement of this Corollary implies, that $F(0,R)<0$ for any $R>2$. This, together with continuity of $F(\alpha,R)$ and statements (1) and (4) of the Lemma 2.2 implies the existence of $\hat{\alpha}(R)$. As was shown above, $\hat{\alpha}(R)$ is an increasing function.

Further, assume by contradiction, that $\hat{\alpha}(R)$ does not converge to 1 as $R \to \infty$. Then, there exists $\varepsilon > 0$, such that for all $M > 0$ there exists $R > M$ such that $\hat{\alpha}(R) < 1 - \varepsilon$. Since $F(\alpha, R)$ is increasing in α, then $\hat{\alpha}(R) < 1 - \varepsilon$ is equivalent to $F(1 - \varepsilon, R) > F(\hat{\alpha}(R), R) = 0$. Using mathematical induction I will construct a sequence $R_1, R_2, \ldots,$ converging to $+\infty$, such that $F(1 - \varepsilon, R_i) > 0$ for all $i = 1, 2, \ldots$ This would contradict statement (3) of the Lemma 2.2, according to which $F(1 - \varepsilon, R_i) \to -\infty$ as $i \to \infty$.

It remains to construct such sequence. Let $M_1 = 2$. Then there exists $R_1 > M_1$, such that $\hat{\alpha}(R_1) < 1 - \varepsilon$ or equivalently $F(1 - \varepsilon, R_1) > 0$. For a sequence $R_1, R_2, \ldots, R_i$ let $M_{i+1} = R_i + 1$. Then, there exists $R_{i+1} > M_{i+1}$, such that $F(1 - \varepsilon, R_{i+1}) > 0$. This way it is possible to construct a sequence $R_1, R_2, \ldots$. This sequence is increasing, since $R_{i+1} > M_{i+1} = R_i + 1 > R_i > i$ and converges to infinity as $i \to \infty$. This completes the proof.

□

Proof of Proposition 2.6. The proof of is done by the mean of example. Let $\alpha = 0.5$. Then the first order conditions of the principal's problem are:

$$\begin{aligned} \frac{d\Pi_P^S}{dB} &= (2B - 1)(1 - 4t + e^t(2t - 1)) = 0, \\ \frac{d\Pi_P^S}{dt} &= e^{-t}(-(1 - 2B)^2 e^t + R + e^{2t}(B^2(1 + 2t) - B(1 + 2t) - 1)) = 0. \end{aligned}$$

The first of these conditions is satisfied, if $B = 0.5$ or if $t = \hat{t}$, where where $\hat{t} \approx 1,06$ is a solution of equation $1 - 4t + e^t(2t - 1) = 0$. Assume, $B \neq 0.5$. The sign of Hessian matrix depends on the following derivatives:

$$\begin{aligned} \frac{\partial^2 \Pi_P^S}{\partial t^2} &= e^{-t}(e^{2t}((3 + 2t)(B^2 - B) - 1) - R), \\ \frac{\partial^2 \Pi_P^S}{\partial B^2} &= 2(1 - 4t + e^t(2t - 1)), \\ \frac{\partial \Pi_P^S}{\partial B \partial t} &= (2B - 1)(-4 + 2e^t + e^t(2t - 1)). \end{aligned}$$

$\frac{\partial^2 \Pi_P^S}{\partial B^2} = 0$ if $1 - 4t + e^t(2t - 1) = 0$. Hence, if $t = \hat{t}$, the determinant of Hessian matrix is non-positive since $\left(\frac{\partial \Pi_P^S}{\partial B \partial t}\right)^2 \geq 0$. For any $B \neq 0.5$ the determinant of Hessian matrix is negative and therefore function $\Pi_P^S(B, t)$ does not reach its maximum at $t = \hat{t}$.

Hence, the maximum is determined by the following conditions:

$$B = 0.5, \quad R = \frac{e^{2t}}{4}(5 + 2t). \tag{33}$$

$B = 0.5$ implies $x = y = \frac{t}{4}$, $R\beta_1 = \frac{e^t}{2}$ and $R\beta_2 = \frac{e^t}{2}(\frac{1}{2}+t)$. Note, that although agents contribute the same effort, the leader receives smaller reward ($R\beta_1 < R\beta_2$).
Optimal value of t is implicitly given by (33). Using Equations (2.11),(2.13) and (2.14) the profit of the principal in the sequential team setting can be written as

$$\Pi_P^S = \left(R - \frac{e^t}{4}(2t-1)\right)(1 - e^{-t}) - \frac{t}{2}. \tag{34}$$

From (33), $R \to 1.25$ as $t \to 0$. Since R increases in t, $\Pi_P^S > 0$ for any $R > 1.25$. At the same time, Proposition 2.4 implies, that given $\alpha = 0.5$, $\Pi_P^T < 0$ if $R \leq 3 \cdot 2^{\frac{\alpha}{\alpha-1}} = 1.50$. Hence, $\Pi_P^S > \Pi_P^T$ for any $R \in (1.25, 1.50]$. Moreover, since profits are increasing in R, there exist a set of $R > 1.50$, where both teams generate positive profit, but the principal is better off under sequential arrangement.

□

The following lemma will be useful for the proof of Proposition 2.7.

Lemma 2.3. *If the team leader employs the second agent in equilibrium, the optimal solution to the maximization problem (2.17) is reached in the interior of the feasible set, so that* $\frac{\partial \Pi_1^H}{\partial d} = 0$ *and* $\frac{\partial \Pi_1^H}{\partial t} = 0$.

Proof. Recall, that the domain of d and t is such, that $d \in [0, t]$ and $t \in [d, \left((c - d)^{1-\alpha} + d^{1-\alpha}\right)^{\frac{1}{1-\alpha}}]$. Let me first consider the derivative $\frac{\partial \Pi_1^H}{\partial d} = -1 - \alpha d^{\alpha-1}(e^t - 1)t^{-\alpha} + d^{-\alpha}(t^{1-\alpha} - d^{1-\alpha})^{\frac{\alpha}{1-\alpha}}$. If in the SPNE the team leader employs a subordinate this derivative must be increasing in $d = 0$. Further, for $d = t$ the derivative $\frac{\partial \Pi_1^H}{\partial d}$ takes the value $\frac{\partial \Pi_1^H}{\partial d} = -1 - \frac{\alpha}{t}(e^t - 1) < 0$. Hence, $d = t$ cannot be the optimal solution. Therefore, the optimum must be reached in the interior of the interval $[0, t]$, so that $\frac{\partial \Pi_1^H}{\partial d} = 0$.
Consider now the derivative $\frac{\partial \Pi_1^H}{\partial t}$. I have already discussed, that it is optimal for the principal to provide such incentives, that agents invest all funds in their discretion into the R&D. Hence, the principal will choose such c, that the optimal choice of x and d satisfies $c = d + x = d + (t^{1-\alpha} - d^{1-\alpha})^{\frac{1}{1-\alpha}}$, which is equivalent to $t = ((c - d)^{1-\alpha} + d^{1-\alpha})^{\frac{1}{1-\alpha}}$. But, for this t to be an equilibrium choice of the first agent, it must be, that $\frac{\partial \Pi_1^H}{\partial t} \geq 0$, or, equivalently

$$R\beta_1 \geq e^t t^{-(1+\alpha)} \left(-\alpha d^\alpha (e^t - 1) + t(d^\alpha e^t + (t^{1-\alpha} - d^{1-\alpha})^{\frac{\alpha}{1-\alpha}})\right).$$

Since $t = ((c-d)^{1-\alpha} + d^{1-\alpha})^{\frac{1}{1-\alpha}}$ for any β_1, which satisfies the inequality above, the principal will choose β_1, such that the inequality above is just satisfied, so that $\frac{\partial \Pi_1^H}{\partial t} = 0$.

□

Proof of Proposition 2.7. Consider the first statement of the proposition. The team leader will not employ a subordinate, if derivative $\frac{\partial \Pi_1^H}{\partial d} < 0$ for any d. This condition is satisfied, if

$$-1 - \alpha d^{\alpha-1}(e^t - 1)t^{-\alpha} + \left(\left(\frac{t}{d}\right)^{1-\alpha} - 1\right)^{\frac{\alpha}{1-\alpha}} < 0.$$

Using the fact, that $e^t - 1 > t$, the following inequality holds:

$$-1 - \alpha d^{\alpha-1}(e^t - 1)t^{-\alpha} + \left(\left(\frac{t}{d}\right)^{1-\alpha} - 1\right)^{\frac{\alpha}{1-\alpha}} < \quad (35)$$

$$< -1 - \alpha d^{-1+\alpha}t^{1-\alpha} + \left(\left(\frac{t}{d}\right)^{1-\alpha} - 1\right)^{\frac{\alpha}{1-\alpha}}$$

Let me substitute $z = \left(\frac{t}{d}\right)^{1-\alpha}$. Note, that $z > 1$ since $d < t$. Then, I can re-write the right-hand side of the inequality (35), as:

$$-1 - \alpha d^{-1+\alpha}t^{1-\alpha} + \left(\left(\frac{t}{d}\right)^{1-\alpha} - 1\right)^{\frac{\alpha}{1-\alpha}} = -1 - \alpha z + (z-1)^{\frac{\alpha}{\alpha-1}}.$$

If $f(z) = 1 + \alpha z - (z-1)^{\frac{\alpha}{\alpha-1}}$ is positive for some α, then the right-hand side of inequality (35) is negative, which implies $\frac{\partial \Pi_1^H}{\partial d} < 0$. The function $f(z)$ reaches its minimum in $z^* = 1 + (1-\alpha)^{\frac{1-\alpha}{2\alpha-1}}$. The value of function in $z = z^*$ is $f(z^*) = 1 - (1-\alpha)^{\frac{\alpha}{2\alpha-1}} + (1 + (1-\alpha))^{\frac{1-\alpha}{2\alpha-1}}$. This value is positive, if $\alpha < \bar{\alpha}$, where $\bar{\alpha} \approx 0.43$. Hence, for all $\alpha \leq \bar{\alpha}$, $f(z) \geq 0$. This implies that the derivative $\frac{\partial \Pi_1^H}{\partial d}$ is strictly negative, so that the team leader will never employ the subordinate.

The second statement of the proposition follows directly from the Lemma 2.3. If the team leader employs the subordinate, then the first order condition with respect to d is $\frac{\partial \Pi_1^H}{\partial d} = 0$. Substituting $x = (t^{1-\alpha} - d^{1-\alpha})^{\frac{\alpha}{1-\alpha}}$ I receive the equivalent condition:

$$x^\alpha = d^\alpha + \alpha d^{2\alpha-1}(e^t - 1)t^{-\alpha}.$$

Given $t > 0$, this condition directly implies $x > d$.

□

Proof of Corollary 2.3. The proof is done by a mean of example. Let $\alpha = \frac{2}{3}$. Using Lemma 2.3, the first order conditions of the team leader's problem are:

$$d = \frac{27t^4}{8(3t + e^t - 1)^3}, \tag{36}$$

$$R\beta_1 = \frac{e^t}{3t^{\frac{5}{3}}} \left(-6d^{\frac{1}{3}} t^{\frac{4}{3}} + 3t^{\frac{5}{3}} + d^{\frac{2}{3}} (2 + 3t + e^t(3t - 2)) \right). \tag{37}$$

The problem of the principal is

$$\max_{\{c,\beta_1\}} \quad \Pi_P^H = R(1 - \beta_1)(1 - e^{-t}) - c$$

$$s.t. \quad c = x + d = (t^{1-\alpha} - d^{1-\alpha})^{\frac{1}{1-\alpha}} + d$$

$$(36), (37).$$

The first order condition results in the following equation:

$$R = \frac{1}{4(3t + e^t - 1)^3} e^t \Big(14 + 4e^t - 27t^2 + 27t^3 + e^{3t}(54t - 26) + $$
$$+ e^t(-46 + 59t - 9t^2) + 18e^{2t}(3 - 6t + 2t^2 + 3t^3) \Big). \tag{38}$$

It is tedious, but relatively straightforward to verify, that R increases in t.
The fact, that $t = ((c - d)^{1-\alpha} + d^{1-\alpha})^{\frac{1}{1-\alpha}}$ and $d < c$ implies, that $t = 0$ iff $c = 0$. Hence, we can use Equation (38) to find R, such that $c(R) = 0$. Applying L'Hospital rule to the Equation (38), one can establish, that $R \to \frac{47}{64} \approx 0.73$ as $t \to 0$. Since R is increasing in t, the hierarchical team generates positive profit, if $R > \frac{47}{64} \approx 0.73$. For $R \leq 0.75$, the simultaneous team will not be financed (this follows from the Proposition 2.4). The profit functions Π_P^H and Π_P^T are increasing in R, which implies, that $\Pi_P^H > \Pi_P^T$ if $R \in (0.73, 0.75]$. Moreover, due to continuity that there exists a range of $R > 0.75$, where both teams generate a positive profit, but the principal is better off under hierarchical arrangement. Finally, for any $R \in [0.73, 2]$ competing agents generate negative profit (and are therefore never financed). Hence, for this range of parameters hierarchical team also performs better than competing agents.

□

2.B Appendix: Tables and figures

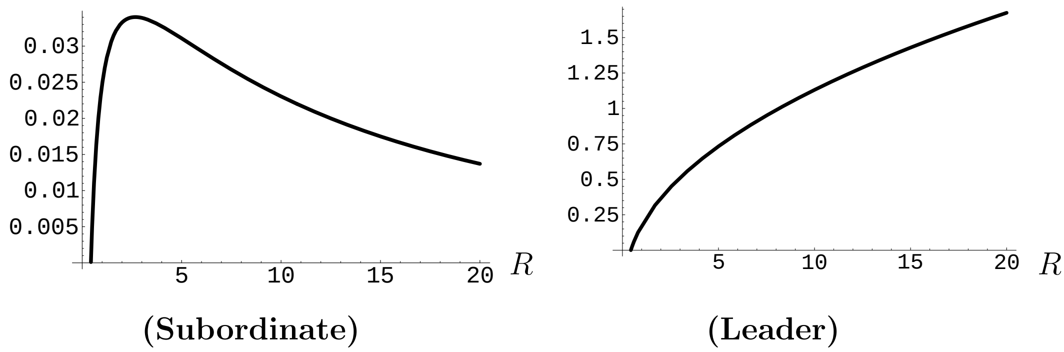


Figure 8: Equilibrium investments of the subordinate and the team leader in the hierarchical team: $\alpha = \frac{2}{3}$

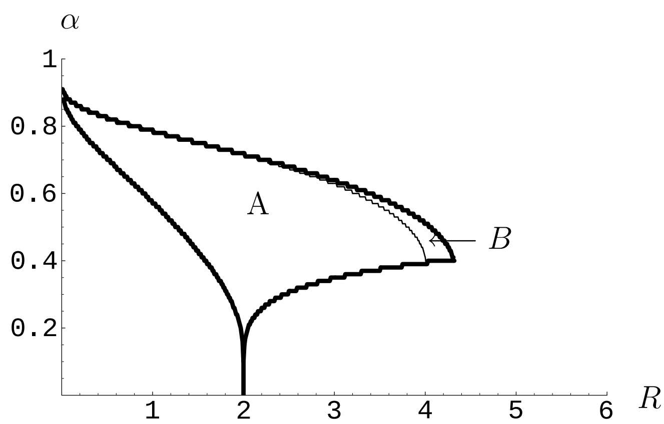


Figure 9: The set of parameters where $\Pi_P^S > \Pi_P^T$ (region A) and $\alpha > t$ (region A $\cup$ B)

	(IC_C^1)	(IC_C^2)
$(0,0)$	$R\beta_1 \leq 1$	$R\beta_2 \leq 1$
$(x^*,0)$	$R\beta_1 = e^x$	$R\beta_2 \frac{1-e^{-x}}{x} \leq 1$
$(0,y^*)$	$R\beta_1 \frac{1-e^{-y}}{y} \leq 1$	$R\beta_2 = e^y$
(x^*,y^*)	$R\beta_1 = \frac{e^{x+y}(x+y)^2}{x(x+y)+y(e^{x+y}-1)}$	$R\beta_2 = \frac{e^{x+y}(x+y)^2}{y(x+y)+x(e^{x+y}-1)}$
(c,y^*)	$R\beta_1 \geq \frac{e^{c+y}(c+y)^2}{c(c+y)+y(e^{c+y}-1)}$	$R\beta_2 = \frac{e^{c+y}(c+y)^2}{y(c+y)+c(e^{c+y}-1)}$
(x^*,d)	$R\beta_1 = \frac{e^{x+d}(x+d)^2}{x(x+d)+d(e^{x+d}-1)}$	$R\beta_2 \geq \frac{e^{x+d}(x+d)^2}{d(x+d)+x(e^{x+d}-1)}$
(c,d)	$R\beta_1 \geq \frac{e^{c+d}(c+d)^2}{c(c+d)+d(e^{c+d}-1)}$	$R\beta_2 \geq \frac{e^{c+d}(c+d)^2}{d(c+d)+c(e^{c+d}-1)}$

Table 3: Competition: Equilibrium candidates and corresponding incentive compatibility constraints

	(IC_T^1)	(IC_T^2)
$(0,0)$	$R\beta_1 \leq 1$	$R\beta_2 \leq 1$
(x^*,y^*)	$R\beta_1 = \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot x^\alpha}{(x^{1-\alpha}+y^{1-a})^{\frac{\alpha}{1-\alpha}}}$	$R\beta_2 = \frac{e^{(x^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot y^\alpha}{(x^{1-\alpha}+y^{1-a})^{\frac{\alpha}{1-\alpha}}}$
(c,y^*)	$R\beta_1 \geq \frac{e^{(c^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot c^\alpha}{(c^{1-\alpha}+y^{1-a})^{\frac{\alpha}{1-\alpha}}}$	$R\beta_2 = \frac{e^{(c^{1-\alpha}+y^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot y^\alpha}{(c^{1-\alpha}+y^{1-a})^{\frac{\alpha}{1-\alpha}}}$
(x^*,d)	$R\beta_1 = \frac{e^{(x^{1-\alpha}+d^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot x^\alpha}{(x^{1-\alpha}+d^{1-a})^{\frac{\alpha}{1-\alpha}}}$	$R\beta_2 \geq \frac{e^{(x^{1-\alpha}+d^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot d^\alpha}{(x^{1-\alpha}+d^{1-a})^{\frac{\alpha}{1-\alpha}}}$
(c,d)	$R\beta_1 \geq \frac{e^{(c^{1-\alpha}+d^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot c^\alpha}{(c^{1-\alpha}+d^{1-a})^{\frac{\alpha}{1-\alpha}}}$	$R\beta_2 \geq \frac{e^{(c^{1-\alpha}+d^{1-\alpha})^{\frac{1}{1-\alpha}}} \cdot d^\alpha}{(c^{1-\alpha}+d^{1-a})^{\frac{\alpha}{1-\alpha}}}$

Table 4: Simultaneous team: Equilibrium candidates and corresponding incentive compatibility constraints

3.A Appendix: Proofs

Proof of Lemma 3.1. Taking the second derivatives, we obtain the *Hessian matrix*:

$$H = \begin{bmatrix} -b^x H''(m^x) & (s-1)b^x b^y & 0 & (s-\gamma^y)b^x c^y \\ (s-1)b^x b^y & -b^y H''(m^y) & (s-\gamma^x)b^y c^x & 0 \\ 0 & (s-\gamma^x)b^y c^x & -c^x H''(n^x) & sc^x c^y \\ (s-\gamma^y)b^x c^y & 0 & sc^x c^y & -c^y H''(n^y) \end{bmatrix} \tag{39}$$

In order to obtain concavity, we need to show that $(-1)^j D_j > 0$, where D_j is the leading principal minor of order $j = 1, 2, 3, 4$.
Let us now fix some $\gamma^x, \gamma^y \in [0, 1]$. We will show that the following three conditions are sufficient for concavity:

(i') $\Delta > 0$;

(ii') $\Delta^2 > (s-1)^2 b^x b^y + (s-\gamma^x)^2 b^y c^x$; and

(iii') $\Delta^4 + \Gamma^2 b^x b^y c^x c^y > [(s-1)^2 b^x b^y + (s-\gamma^x)^2 b^y c^x + (s-\gamma^y)^2 b^x c^y + s^2 c^x c^y]\Delta^2$,

where $\Gamma = s(s-1) - (s-\gamma^x)(s-\gamma^y)$. Note that these conditions reduce to (i)–(iii) when $\gamma^x = \gamma^y = 0$.
Because $\Delta > 0$, then $H''(z) > 0$ for all $z \in [0, 1]$. Consequently, $D_1 < 0$. Condition (ii') implies that $\Delta^2 > (s-1)^2 b^x b^y$ and, thus, $D_2 > 0$. Moreover, it follows from condition (ii') that $\Delta^2 > (s-\gamma^x)^2 b^y c^x$ and

$$\begin{aligned} &\Delta^2[\Delta^2 - (s-1)^2 b^x b^y - (s-\gamma^x)^2 b^y c^x] = \\ &= [\Delta^2 - (s-1)^2 b^x b^y]\,[\Delta^2 - (s-\gamma^x)^2 b^y c^x] - (s-1)^2 b^x b^y (s-\gamma^x)^2 b^y c^x \leq \\ &\leq [\Delta H''(m^x) - (s-1)^2 b^x b^y]\,[\Delta H''(n^x) - (s-\gamma^x)^2 b^y c^x] - (s-1)^2 b^x b^y (s-\gamma^x)^2 b^y c^x = \\ &= \Delta[\Delta H''(m^x) H''(n^x) - (s-1)^2 b^x b^y H''(n^x) - (s-\gamma^x)^2 b^y c^x H''(m^x)] \end{aligned}$$

Now,

$$-\frac{D_3}{b^x b^y c^x} = H''(m^x)H''(m^y)H''(n^x) - (s-1)^2 b^x b^y H''(n^x) - (s-\gamma^x)^2 b^y c^x H''(m^x)$$

and the inequality $D_3 < 0$ follows from the fact that $H''(m^y) \geq \Delta$.
In order to prove that $D_4 > 0$, consider first the matrix H when $H''(m^x) = H''(m^y) = H''(n^x) = H''(n^y) = \Delta$. It follows from conditions (i')–(iii') that this matrix is negative definite. Thus, we obtain similar conditions as (ii') also for other principal

minors of order 3:

$$\Delta^2 \geq (s-1)^2 b^x b^y + (s-\gamma^y)^2 b^x c^y, \qquad \Delta^2 \geq (s-\gamma^x)^2 b^y c^x + s^2 c^x c^y,$$
$$\Delta^2 \geq (s-\gamma^y)^2 b^y c^x + s^2 c^x c^y.$$

By the same procedure as above, we may show that all principal minors of order 3 of matrix H are non-negative. Direct computation reveals that

$$\begin{aligned}\frac{D_4}{b^x b^y c^x c^y} &= H''(m^x)H''(m^y)H''(n^x)H''(n^y) + \Gamma^2 b^x b^y c^x c^y - \\ &- [(s-1)^2 b^x b^y H''(n^x)H''(n^y) + (s-\gamma^x)^2 b^y c^x H''(m^x)H''(n^y) + \\ &+ (s-\gamma^y)^2 b^x c^y H''(m^y)H''(n^x) + s^2 c^x c^y H''(m^x)H''(m^y)].\end{aligned}$$

Now, it can be easily shown that the value of D_4 does not increase when subsequently substitute $H''(n^y) = \Delta$, $H''(n^x) = \Delta$, $H''(m^y) = \Delta$, and $H''(m^x) = \Delta$. At the end we obtain a positive expression due to (iii').

Now it remains to show that conditions (i)–(iii) imply conditions (i')–(iii'). Condition (i') is identical to (i) and clearly, the condition (ii') follows from (ii), since its right-hand side is decreasing in γ^x. Now, let is rewrite the condition (iii') as $\Delta^4 + \Gamma^2 b^x b^y c^x c^y - [(s-1)^2 b^x b^y + (s-\gamma^x)^2 b^y c^x + (s-\gamma^y)^2 b^x c^y + s^2 c^x c^y]\Delta^2 > 0$. It's derivative with respect to γ^x is $2b^y c^x \big[b^x c^y s(s-1)(s-\gamma^y) + (s-\gamma^x)\big(\Delta^2 - (s-\gamma^y)^2 b^x c^y\big)\big]$, which is non-negative. Similarly, we may show that the derivative with respect to γ^y is non-negative. □

Proof of Lemma 3.2. The uniqueness of the maximizer follows from concavity. Taking the first derivative of the profit with respect to m^x we obtain

$$\frac{\partial \Pi}{\partial m^x} = b^x[-H'(m^x) + (s-1)b^y m^y + (s-\gamma^y)c^y n^y].$$

If follows from the definition of H that $H'(z) = \frac{1}{2}G(z) + \frac{1}{2}zG'(z)$. By Assumption 3.1 we have $\lim_{z\to 1^-} G'(z) = +\infty$. Thus, $\lim_{z\to 1^-} H'(z) = +\infty$ and $\partial\Pi/\partial m^x|_{m^x=1} < 0$. Moreover, when $G(z) = \theta$, then $2H'(z) = \theta + F(\theta)/F'(\theta)$ and $2H'(0) = \underline{\theta} + \lim_{\theta\to\underline{\theta}^+} F(\theta)/F'(\theta)$, which is negative by Assumption 3.1. Therefore, $\partial\Pi/\partial m^x|_{m^x=0} > 0$. This implies, that $0 < m^x < 1$ in maximum. Such m^x then satisfies the first-order condition $\partial\Pi/\partial m^x = 0$. The proofs for m^y, n^x, and n^y are analogous. □

Proof of Proposition 3.1. The proposition follows from part (i) of Lemma 3.4. □

Proof of Proposition 3.2. The proposition follows immediately from Lemma 3.4, parts (ii) and (iii). □

Lemma 3.4. *For any $\gamma^y \in [0,1]$ the following statements hold:*

(i) *If the monopolist is free to choose any $\gamma^x \in [0,1]$, he would choose either $\gamma^x = 0$ or $\gamma^x = 1$.*

(ii) *If $m^y \le \frac{1}{2}$ in optimum for $\gamma^x = 0$, then the monopolist would choose $\gamma^x = 1$.*

(iii) *If $m^y \ge \frac{1}{2}$ in optimum for $\gamma^x = 1$, then the monopolist would choose $\gamma^x = 0$.*

Proof of Lemma 3.4. Taking the partial derivative of monopolist's profit (3.9), we obtain

$$\frac{\partial \Pi}{\partial \gamma^x} = b^y c^x n^x (1 - 2m^y). \tag{40}$$

For $\gamma^x \in [0,1]$, let $\tilde{m}^x(\gamma^x)$, $\tilde{m}^y(\gamma^x)$, $\tilde{n}^x(\gamma^x)$, and $\tilde{n}^y(\gamma^x)$ be the solution of the first-order conditions from Lemma 3.2 and let $\tilde{\Pi}(\gamma^x) = \Pi\big(\tilde{m}^x(\gamma^x), \tilde{m}^y(\gamma^x), \tilde{n}^x(\gamma^x), \tilde{n}^y(\gamma^x)\big)$. Using the *Envelope Theorem* we obtain that $\mathrm{d}\tilde{\Pi}(\gamma^x)/\mathrm{d}\gamma^x = b^y c^x \tilde{n}^x(\gamma^x)\big(1 - 2\tilde{m}^y(\gamma^x)\big)$. Now we will show that if $\tilde{m}^y(\bar{\gamma}^x) \ge \frac{1}{2}$ for some $\bar{\gamma}^x > 0$, then $\tilde{m}^y(\gamma^x) > \frac{1}{2}$ for all $\gamma^x \in [0, \bar{\gamma}^x)$. Using the *Implicit Function Theorem* for the first-order conditions, we obtain that $\tilde{m}^y(\gamma^x)$ is continuous and differentiable with derivative

$$\begin{aligned}\frac{\mathrm{d}\tilde{m}^y}{\mathrm{d}\gamma^x}(\gamma^x) = {} & \frac{b^x b^y c^x c^y}{2D_4} c^x \big[b^y (1 - 2m^y)\big((s-\gamma^x)H''(m^x)H''(n^y) + (s-\gamma^y)\Gamma b^x c^y\big) + \\ & + 2n^x\big(- H''(m^x)H''(n^x)H''(n^y) + s^2 c^x c^y H''(m^x) + (s-\gamma^y)^2 b^x c^y H''(n^y)\big)\big],\end{aligned} \tag{41}$$

where D_4 is the determinant of the *Hessian matrix* H defined in (39) and $\Gamma = s(s-1) - (s-\gamma^x)(s-\gamma^y)$. Observe that the coefficient at n^x is actually a third minor of matrix H multiplied by a positive factor and is, thus, negative. In addition, as $(s-\gamma^x)(s-\gamma^y)^2 \ge -(s-\gamma^y)\Gamma$ and $H''(m^x)H''(n^y) > (s-\gamma^y)^2 b^x c^y$, then the coefficient at $(1-2m^y)$ is positive. Therefore, if $\tilde{m}^y(\gamma^x) \ge \frac{1}{2}$, then $\mathrm{d}\tilde{m}^y(\gamma^x)/\mathrm{d}\gamma^x < 0$, which yields the desired statement.

Consequently, if $\tilde{m}^y(1) \ge \frac{1}{2}$, then $\tilde{m}^y(\gamma^x) > \frac{1}{2}$ for all $\gamma^x \in [0,1)$. Thus, $\tilde{\Pi}(\gamma^x)$ is decreasing on $[0,1]$ and the monopolist would choose $\gamma^x = 0$. This proves (iii). On the other hand, if $\tilde{m}^y(0) \le \frac{1}{2}$, then it is not possible that $\tilde{m}^y(\gamma^x) \ge \frac{1}{2}$ for some $\gamma^x \in (0,1]$. Thus, $\tilde{m}^y(\gamma^x) < \frac{1}{2}$ for all $\gamma^x \in (0,1]$ and $\tilde{\Pi}(\gamma^x)$ is decreasing on $[0,1]$. Therefore, the monopolist would choose $\gamma^x = 1$, which proves (ii). Finally, if $\tilde{m}^y(0) > \frac{1}{2} > \tilde{m}^y(1)$, then there exists unique $\tilde{\gamma}^x \in (0,1)$ such that $\tilde{m}^y(\tilde{\gamma}^x) = \frac{1}{2}$. Then also $\tilde{m}^y(\gamma^x) > \frac{1}{2}$ when $\gamma^x \in [0, \tilde{\gamma}^x)$ and $\tilde{m}^y(\gamma^x) < \frac{1}{2}$ when $\gamma^x \in (\tilde{\gamma}^x, 1]$. Thus, $\tilde{\Pi}(\gamma^x)$ is decreasing on $[0, \tilde{\gamma}^x)$ and increasing on $(\tilde{\gamma}^x, 1]$. This means, that the monopolist's profit can achieve its maximum only when $\gamma^x = 0$ or $\gamma^x = 1$, which together with the previous statements proves (i). □

Proof of Proposition 3.3. The derivatives can be obtained from the first-order conditions in Lemma 3.2 using the *Implicit Function Theorem*. The derivative $\mathrm{d}m^y/\mathrm{d}\gamma^x$ is computed in the proof of Lemma 3.4 and is given by (41). The coefficients β_1 and β_2 can be written as $\beta_1 = \tilde{\beta}_1 b^x b^y c^x c^y/(2D_4)$, and $\beta_2 = \tilde{\beta}_2 b^x b^y c^x c^y/(2D_4)$, where the expressions for $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are summarized in the Table 8 in Appendix 3.B. Using analogous arguments as in the proof of Lemma 3.4, we can verify that $\beta_1 > 0$ and $\beta_2 < 0$. The effect on profit also follows from that proof. □

Proof of Lemma 3.3. To prove the first statement, consider a case where monopolist sets prices so that the demands are given by $m^x = 0, m^y \in [0,1], n^y \in [0,1]$ and $n^x \in [0,1]$. This allocation is never an equilibrium. Indeed, the monopolist can marginally decrease price A_0^x to gain a positive demand on behalf of old x-agents. Since the prices for other groups of agents remain unchanged, the profit of the monopolist will be strictly larger.
The proof of the second and the third statement results from the fact that, given fixed m^y, n^x and n^y, the profit of the monopolist is quadratic, concave function of A_0^x. Hence, if the solution of the first-order condition with respect to A_0^x leads to $m^{x*} \in [0,1]$, then $m^x = m^{x*}$ maximizes monopolist's profit. If $m^{x*} > 1$, then the profit of the monopolist increases in $m^x = 1$ and therefore on the interval $m^x \in [0,1]$ the profit is maximized if $m^x = 1$. □

Corollary 3.4. *Assume that $i_k, j_k \in \{0,1\}$ are such that $i_k \leq j_k$ for $k = 1,2,3,4$ and at least one of these inequalities is strict. If both equilibria $E_{i_1 i_2 i_3 i_4}$ and $E_{j_1 j_2 j_3 j_4}$ feasible, then monopolist's profit in equilibrium $E_{i_1 i_2 i_3 i_4}$ is higher.*

Proof of Corollary 3.4. The proof follows directly from Lemma 3.3. □

Proof of Proposition 3.4. We provide comparison between NC and BCx regimes. The comparison of other regimes is done by analogously.
Before proceeding with comparison of NC and BCx regimes we need to describe equilibrium allocations for each regime. Here we will derive the equilibrium allocations for NC regime. The equilibrium allocations for BCx regimes are derived by analogy. Let us denote

$$h_1 = \frac{2-B}{Bs + b^x(s-1)(2s-2+B)}, \qquad h_2 = \frac{2-B-Bb^x s}{b^x(s-1)(2s-2+B)},$$
$$h_3 = \frac{2-B}{[B+2b^x(s-1)](s-1)}, \qquad h_4 = \frac{2-B(1-b^x+b^x s)}{2b^x(s-1)^2}.$$

Further, let $b^y = h_5(b^x)$ be implicitly given by the equation $b^x = \frac{2-b^y-B[1+b^y(s-1)]}{(2-b^y)b^y(s-1)^2}$. Obviously, functions h_i, $i = 1,\ldots,5$ are decreasing in b^x.

The feasibility conditions for each equilibrium candidates are summarized in Table 10 in Appendix 3.B in the first and third column for NC and BCx regimes respectively. Notice, that each allocation $E_{i_1i_2i_3i_4}$ is feasible only for some range of parameters. The relevant feasibility conditions result from the requirement that in equilibrium demands must be such, that $m_i \in [0,1]$ and $n_i \in [0,1]$ for any $i \in \{x,y\}$. With some abuse of notation and terminology, we will say that allocation $E_{i_1i_2i_3i_4}$ *dominates* allocation $E_{j_1j_2j_3j_4}$ ($E_{j_1j_2j_3j_4} \prec E_{i_1i_2i_3i_4}$) if the profit of the monopolist is larger in the former case, than in the later.

We call conditions which ensure that a particular allocation is an equilibrium *optimality conditions*. The optimality conditions for NC and BCx regimes are summarized in the second and the fourth column of Table 10 respectively.

Consider NC regime, allocation E_{1000}:

$$m^x = 1, \qquad n^x = \frac{-2b^x b^y(s-1)s - 2b^x c^y s^2 - B(1+b^y s + c^y s)}{2(-1+b^y c^x s^2 + c^x c^y s^2)},$$
$$m^y = \frac{-B(1+c^x s) + b^x(2-2s-2c^x c^y s^2)}{2(-1+b^y c^x s^2 + c^x c^y s^2)}, \; n^y = \frac{B(-1-c^x s) - 2b^x s(1-b^y c^x s)}{2(-1+b^y c^x s^2 + c^x c^y s^2)}.$$

It is easy to verify that given $c^x \to 0$, $c^y \to 0$ this allocation is feasible if

$$b^x < \frac{2-B}{2s} \quad \text{and} \quad b^y < \frac{2-B}{s[B+2b^x(s-1)]}.$$

From Table 10, allocation E_{0000} is feasible if $b^y < \min[h_1, h_2]$. It is clear, that $h_1 > \frac{2-B}{s(B-2b^x+2b^x s)}$. Further, h_1 and h_2 are decreasing in b^x and $h_1 = h_2$ if $b^x = \frac{2-B}{2s}$. Hence, whenever E_{1000} is feasible, E_{0000} is also feasible. By Corollary 3.4, $E_{1000} \prec E_{0000}$. Similar argument allows to establish, that allocations E_{1001}, E_{0100}, E_{0110}, E_{1100}, E_{1110}, and E_{1101} never occur in equilibrium.

Therefore, the set of equilibrium candidates is limited to nine allocations E_{0000}, E_{0010}, E_{0011}, E_{0001}, E_{1010}, E_{1011}, E_{0111}, E_{0101}, and E_{1111}. Using Lemma 3.3 we can immediately see, that whenever E_{0000} is feasible, it is optimal. By Corollary 3.4, $E_{0011} \prec E_{0010}$ and $E_{0011} \prec E_{0001}$ whenever the allocations are simultaneously feasible. Further, $E_{1010} \prec E_{0010}$, $E_{0101} \prec E_{0001}$, $E_{1011} \prec E_{1010}$, $E_{0111} \prec E_{0101}$, $E_{0111} \prec E_{0011}$, and $E_{1011} \prec E_{0011}$, whenever allocations are simultaneously feasible. Combining these results, we establish that the feasibility conditions divide the whole range of parameters into nine domains as characterized in the Table 10.

Having established the characterization of equilibrium allocation, we now proceed with comparison of profits in BCx and NC regimes. As follows from Table 10, if we fix B and s then the whole range of parameters $b^x \in [0,1]$, $b^y \in [0,1]$ can be divided into several domains where for each compatibility regime a particular allocation is optimal. These domains are illustrated on Figure 10 in Appendix 3.B. Note, that the dashed line shows the range of parameters where Assumption 3.3 is satisfied. In the proof of the proposition we will assume that these conditions are satisfied for all domains. This assumption is without loss of generality, because if the conditions are not satisfied for some domain, then this domain is not feasible.

To prove the proposition we need to compare profits of the principal in NC and BCx regimes in each of those domains. The domains and corresponding equilibrium allocations are summarized in Table 5. We analyze each domain separately.

domain	feasibility condition	type of allocation NC	BCx
a	$b^y < \min[h_1, h_2, h_5]$	E_{0000}	E_{0000}
b	$h_5 < b^y < h_1$	E_{0000}	E_{0010}
c	$h_1 < b^y < h_5$	E_{0010}	E_{0000}
d	$\max[h_1, h_5] < b^y < \min[h_2, h_3]$	E_{0010}	E_{0010}
e	$b^y > h_3, b^x < \frac{2-B}{2s}$	E_{1010}	E_{1010}
f	$b^y > h_3, \frac{2-B}{2s} < b^x < \frac{2-B}{2(s-1)}$	E_{1011}	E_{1011}
g	$\max[h_2, h_5] < b^y < \min[h_3, h_4]$	E_{0011}	E_{0011}
h	$\max[h_1, h_2] < b^y < \min[h_4, h_5]$	E_{0011}	E_{0001}
i	$h_2 < b^y < \min[h_1, h_4]$	E_{0001}	E_{0001}
j	$h_4 < b^y < \frac{2-B}{2s}$	E_{0101}	E_{0101}
k	$\max[\frac{2-B}{2s}, h_4] < b^y < \frac{2-B}{2s-1}$	E_{0111}	E_{0101}
l	$\max[\frac{2-B}{2s-1}, h_4] < b^y < \frac{2-B}{2(s-1)}$	E_{0111}	E_{0111}
m	$\min[b^y, b^x] > \frac{2-B}{2(s-1)}$	E_{1111}	E_{1111}

Table 5: Equilibrium allocations on the mature market

▷ *Domains* **a**, **d**, **g**, **i**

Consider first domain **a**. In this domain the interior solution (allocation of the type E_{0000}) is an equilibrium both in NC and BCx regimes. Given $c^x \to 0$, $c^y \to 0$, the equilibrium prices and allocations in NC regime are as follows:

$$m^x = \frac{B}{2} \cdot \frac{1 + b^y(s-1)}{1 - b^x b^y (s-1)^2}, \qquad n^x = \frac{B}{2} \cdot \frac{1 + b^x b^y(s-1) + b^y s}{1 - b^x b^y (s-1)^2},$$
$$m^y = \frac{B}{2} \cdot \frac{1 + b^x(s-1)}{1 - b^x b^y (s-1)^2}, \qquad n^y = \frac{B}{2} \cdot \frac{1 + b^x b^y(s-1) + b^x s}{1 - b^x b^y (s-1)^2},$$
$$A_0^x = A_0^y = A_1^x = A_1^y = \frac{B}{2}.$$

In BCx regime the equilibrium allocation and prices are

$$m^x = \frac{B}{2} \cdot \frac{1 + b^y(s-1)}{1 - b^x b^y (s-1)^2}, \qquad n^x = \frac{B}{2} \cdot \frac{1 + b^y(s-1)}{1 - b^x b^y (s-1)^2} + \frac{b^y}{2},$$
$$m^y = \frac{B}{2} \cdot \frac{1 + b^x(s-1)}{1 - b^x b^y (s-1)^2}, \qquad n^y = \frac{B}{2} \cdot \frac{1 + b^x b^y(s-1) + b^x s}{1 - b^x b^y (s-1)^2},$$
$$A_0^x = A_0^y = A_1^y = \frac{B}{2}, \; A_1^x = \frac{B + b^y}{2}.$$

Calculating the respective profits of the monopolist Π^{NC} and Π^{BCx}, it is straightforward to establish, that $\Pi^{NC} < \Pi^{BCx}$ if and only if $B < \frac{1-b^x b^y (s-1)^2}{1+b^x(s-1)}$ or equivalently $b^y < g_1(b^x)$. The same result holds for domains **d**, **g** and **i**.

▷ *Domains* **e** *and* **f**

Consider now domain **e**. Here the equilibrium allocation is of the type E_{1011} in NC and BCx regimes. Comparing the respective profits, we establish, that

$$\Pi^{NC} < \Pi^{BCx} \iff -1 + B - c^x + 2b^x(s-1) + 2c^x s < 0.$$

Given c^x and c^y close to zero the above inequality implies that $NC \prec BCx$ if and only if $b^x < \frac{1-B}{2(s-1)}$. The same result holds in domain **f**.

▷ *Domains* **j**, **l** *and* **m**

In domains **j**, **l** and **m**, NC regime is clearly preferred to BCx regime. Here in both compatibility regimes $m^y = 1$ in equilibrium. Therefore, whether technologies compatible or not, new x-agents can interact with old y-agents since all of them are subscribed to the new technology. BCx regime therefore has no positive effect on the incentives of new x-agents.

▷ *Domain* **k**

In this domain the equilibrium allocation in BCx and NC regimes are of the type E_{0101} and E_{0111} respectively. Comparing the profits of the monopolist we receive the following equivalence:

$$\Pi^{NC} > \Pi^{BCx} \iff 4 + B^2 + (b^y)^2(2s-1)^2 - 8b^y s + B[-4 + b^y(4s-2)] < 0.$$

The inequality above is satisfied if $b_1 < b^y < b_2$, where

$$b_{1,2} = \frac{B + 4s - 2Bs \mp 2\sqrt{4s - 1 + B - 2Bs}}{1 - 4s + 4s^2}.$$

Notice, that $4s - 1 + B - 2Bs > 0$ for any $s > 1$, $B \in (0,1)$. In addition, domain **k** is feasible if $\frac{2-B}{2s} < b^y < \frac{2-B}{2s-1}$. It is easy to show that $b_1 < \frac{2-B}{2s}$ and $b_2 > \frac{2-B}{2(s-1)}$. Hence, for any $\frac{2-B}{2s} < b^y < \frac{2-B}{2s-1}$, NC regime is preferred to the BCx regime.

$\triangleright$ *Domains* **c**, **b** *and* **h**

Consider now domain **c**. Here the optimal allocation in NC regime is of the type E_{0010} and in BCx regime is of the type E_{0000}. Comparing the respective profits, we receive, that $\Pi^{BCx} \prec \Pi^{NC}$ if $\varphi(b^x)/\psi(b^x) > 0$, where

$$\psi(b^x) = -1 + b^x b^y (s-1)^2 + b^y c^x (s-1)^2 + c^x c^y s^2 + b^x c^y s^2.$$

Due to Assumption 3.3, $\psi(b^x) > 0$. We do not list the expression for $\varphi(b^x)$ here because this is a rather complicated. Function $\varphi(b^x)$ is quadratic in b^x and the coefficient at $(b^x)^2$ is $-4B(b^y)^2(s-1)^3 - (b^y)^2[4 + (b^y)^2](s-1)^4 < 0$. Hence, $\varphi(b^x) > 0$ if and only if $b_1 < b^x < b_2$, where $b_{1,2}$ are roots of the quadratic equation $\varphi(b^x) = 0$.

$$b_{1,2} = \frac{[4 + (b^y)^2](s-1) + B[(b^y)^2(s-1)^2 + 4 - 2s - b^y(s-1)(2s-1) \mp 2k(1 + b^y(s-1))]}{b^y[4B + (4 + (b^y)^2)(s-1)](s-1)^2}$$

where $k = \sqrt{-1 + B - Bs + (2 + b^y)s - b^y s^2}$. In addition, domain **c** is feasible if $h_1 < b^y < h_5$, which is equivalent to

$$\frac{-2 + b^y + B[1 + b^y(s-1)]}{(-2 + b^y)b^y(s-1)^2} < b^x < \frac{2 - B - Bb^y s}{b^y(s-1)(2s - 2 + B)}. \tag{42}$$

Given $b^x \geq 0$, the inequalities above are satisfied if and only if $b^y < \frac{2-B}{s}$. It is easy to show, that $b_1 < \frac{-2+b^y+B[1+b^y(s-1)]}{(-2+b^y)b^y(s-1)^2}$ and $b_2 > \frac{2-B-Bb^y s}{b^y(s-1)(2s-2+B)}$ for any $b^y < \frac{2-B}{s}$. Hence, $\varphi(b^x) > 0$ for any $h_1 < b^y < h_5$, which implies that $BCx \prec NC$.

By analogy we can show, that $BCx \prec NC$ in domain **h** and $NC \prec BCx$ in domain **b**. Summarizing the results, we receive that $BCx \prec NC$ if $b^x < \frac{1-B}{2(s-1)}$ or $b^y < g_1(b^x)$. $\square$

Proof of Proposition 3.5. Similarly as in the proof of Proposition 3.4, we provide comparison between NC and BCx regimes. The comparison of other regimes is done by analogy.

Before comparing NC and BCx regimes we need to describe equilibrium allocations for each regime. Here we will derive the equilibrium allocations for NC regime. The equilibrium allocations for BCx regimes are derived by analogy. For that, let us denote

$$h_1 = \frac{2-B}{Bs + 2c^x s^2}, \qquad h_2 = \frac{2-B-Bc^x s}{2c^x s^2}, \qquad h_3 = \frac{2-B-Bc^x(s-1)}{2c^x s(s-1)}.$$

The feasibility conditions for each equilibrium candidates are summarized in Table 11 (in Appendix 3.B) in the first and third column for NC and BCx regimes respectively. The optimality conditions for NC and BCx regimes are summarized in the second and the fourth column of Table 11 respectively.

We can directly eliminate the following allocations from the set of equilibrium candidates: E_{1000},E_{0100}, E_{1100}, E_{0010} and E_{0001}. Indeed, as demonstrated in Table 11, all those allocations are feasible if and only if allocation E_{0000} is feasible, which (according to Lemma 3.3) always generates larger profit. The same argument readily allows to establish that $E_{1011} \prec E_{1010}$, $E_{0111} \prec E_{0101}$, $E_{1110} \prec E_{1010}$, and $E_{1101} \prec E_{0101}$. Further notice, that $h_1 = h_2 = \frac{2-B}{2s}$ if $c^x = \frac{2-B}{2s}$. Hence, allocation E_{0000} is feasible whenever $\frac{2-B}{2s} > \max[c^x, c^y]$. Since feasibility conditions for E_{0011} require $\frac{2-B}{2s} > \max[c^x, c^y]$, we conclude that $E_{0011} \prec E_{0000}$. Finally, allocation E_{1001} is feasible if $c^y < \min[h_1, \frac{2-B}{2s}]$. Since $\min[h_1, \frac{2-B}{2s}] < \min[h_1, h_2]$, allocation E_{0000} is feasible whenever E_{1001} is feasible and $E_{1001} \prec E_{0000}$ by Lemma 3.3.

This leaves us with four candidates for equilibrium: E_{0000}, E_{1010}, E_{0101} and E_{1111}. Whenever allocation E_{0000} is feasible, it is optimal. Moreover, by Lemma 3.3, whenever E_{1010} and E_{1111} are simultaneously feasible, $E_{1111} \prec E_{1010}$ (the symmetric argument holds for allocation E_{0101}). Therefore, the whole range of feasible parameters c^x and c^y is divided into four domains, as summarized in Table 11.

Having the characterization of equilibrium allocation, we now proceed with comparison of profits in BCx and NC regimes. As follows from Table 11, if we fix B and s then the whole range of parameters $b^x \in [0, 1]$, $b^y \in [0, 1]$ can be divided into several domains where for each compatibility regime a particular allocation is optimal. These domains are illustrated on Figure 11 in Appendix 3.B. Note, that the dashed line shows the range of parameters where Assumption 3.3 is satisfied. In the proof of the proposition we will assume that these conditions are satisfied for all domains. This assumption is without loss of generality, because if the conditions are not satisfied for some domain, then this domain is not feasible.

To prove the proposition we need to compare profits of the principal in NC and BCx regime in each of the domains. The domains and corresponding equilibrium allocations for NC and BCx regimes are summarized in Table 6. We analyze each domain separately.

domain	feasibility condition	type of allocation NC	BCx
a	$c^y < \min[h_1, h_2]$	E_{0000}	E_{0000}
b	$h_1 < c^y, c^x < \frac{2-B}{2s}$	E_{1010}	E_{1010}
c	$c^y > \frac{2-B}{2s}, \frac{2-B}{2s} < c^x < \frac{2-B}{2(s-1)}$	E_{1111}	E_{1011}
d	$c^y > \frac{2-B}{2s}, c^x > \frac{2-B}{2(s-1)}$	E_{1111}	E_{1111}
e	$h_3 < c^y < \frac{2-B}{2s}$	E_{0101}	E_{0101}
f	$h_2 < c^y < \min[h_3, \frac{2-B}{2s}]$	E_{0101}	E_{0001}

Table 6: Equilibrium allocations on the emerging market

▷ *Domains* **d** *and* **e**

We can immediately establish, that $BCx \prec NC$ in domains **d** and **e** . Indeed, for both compatibility regimes the allocation in these domains is such, that $m^y = 1$. In this case BCx regime has, clearly, no positive effect on the incentives of new x-agents, and hence NC regime is optimal.

▷ *Domain* **a**

Here the equilibrium is of the type E_{0000} in both NC and BCx regimes. Let us denote

$$K_1 = 1 - b^x b^y (s-1)^2 - s^2(b^y c^x + c^x c^y + b^x c^y - b^x b^y c^x c^y),$$
$$K_2 = 1 - b^x b^y (s-1)^2 - b^y c^x (s-1)^2 - s^2(c^x c^y + b^x c^y).$$

Notice, that due to Assumption 3.3, $K_1 > 0, K_2 > 0$. The equilibrium prices and demands in NC regime are as follows:

$$m^x = \frac{B}{2} \cdot \frac{1 + c^y s + b^y(s - 1 - c^x s)}{K_1}, \qquad m^y = \frac{B}{2} \cdot \frac{1 + c^x s + b^x(s - 1 - c^y s)}{K_1},$$
$$n^x = \frac{B}{2} \cdot \frac{1 + b^y s + c^y s + b^x b^y(s - 1 - c^y s)}{K_1}, \quad n^y = \frac{B}{2} \cdot \frac{1 + b^x s + c^x s + b^x b^y(s - 1 - c^x s)}{K_1}$$
$$A_0^x = A_0^y = A_1^x = A_1^y = \frac{B}{2}.$$

In BCx regime the prices and allocations are

$$\begin{aligned}
m^x &= \frac{B[1+b^y(s-1)+c^y s]+b^y c^x[b^y(s-1)^2+c^y s^2]}{2K_2},\\
m^y &= \frac{B[1+b^x(s-1)+c^x(s-1)-c^y s(b^x+c^x)]+b^y c^x(s-1)}{2K_2},\\
n^x &= \frac{B[1+b^y(s-1)+c^y s]+b^y[1-b^x b^y(s-1)^2-b^x c^y s^2]}{2K_2},\\
n^y &= \frac{B[1+b^y c^x(s-1)+c^x s+b^x b^y(s-1)+b^x s]+b^y c^x s}{2K_2},\\
A_0^x &= A_0^y = A_1^y = \frac{B}{2}, A_1^x = \frac{B+b^y}{2}.
\end{aligned}$$

Calculating profits Π^{NC} and Π^{BCx} and comparing them for $b^x \to 0, b^y \to 0$, it is easy to establish, that $NC \prec BCx$ if

$$c^y < \frac{1-B[1+c^x(s-1)]}{c^x(s-1)^2} \equiv g_2(c^x).$$

Note, that $g_2(c^x) < h_1$ if $c^x > (1-B)/(2s-1)$

▷ *Domain* **b**

Comparison of equilibrium profits in domain **b** leads to the conclusion, that $\Pi^{NC} < \Pi^{BCx}$ if $B - 1 + 2b^x(s-1) + 2c^x s < 0$. Given $b^x \to 0$ the inequality is satisfied if $c^x < \frac{1-B}{2s-1}$.

▷ *Domain* **c**

In domain **c**, $\Pi^{NC} > \Pi^{BCx}$ is equivalent to

$$4(c^x)^2(s-1)^2 + c^x[4+4B(s-1)-8s] + 4 - 4B + B^2 < 0.$$

Let us denote the left-hand side of the inequality as $\varphi(c^x)$. Since $\varphi(c^x)$ is a quadratic, convex function, $\varphi(c^x) < 0$ if $c_1 < c^x < c_2$, where

$$c_{1,2} = \frac{-1-B(s-1)+2s \mp \sqrt{4s-3+2B-2Bs}}{2(s-1)^2}$$

In addition, domain **c** is feasible if $\frac{2-B}{2s} < c^x < \frac{2-B}{2(s-1)}$. It is easy to see, that $c_2 > \frac{2-B}{2(s-1)}$ and $c_1 < \frac{2-B}{2s}$. Hence, $BCx \prec NC$ for any $\frac{2-B}{2s} < c^x < \frac{2-B}{2(s-1)}$.

▷ *Domain* **f**

This domain is feasible, if $h_2 < c^y < h_3$. Let us denote $\psi(b^y) = \Pi^{NC} - \Pi^{BCx}$. From the specification of demand functions (see Section 3.4), it is clear, that if $b^y = 0$, then

BCx regime is equivalent to NC regime in terms of equilibrium demands, prices and monopolist's profits. Therefore, in order to establish whether $\Pi^{NC} > \Pi^{BCx}$ for b^y and b^x in the neighborhood of zero, we will investigate the derivative of $\psi(b^y)$. In particular,

$$\frac{\partial \psi}{\partial b^y}|_{c^y=h_2} = \frac{(2-B)(2s-B)}{4s^2} > 0, \qquad \frac{\partial \psi}{\partial b^y}|_{c^y=h_3} = \frac{(2-B)}{2s-1} > 0.$$

Therefore, $BCx \prec NC$ for any $c^y = h_2$ and $c^y = h_3$. It follows that $BCx \prec NC$ also for $h_2 < c^y < h_3$.
Summarizing the results, we conclude, that $NC \prec BCy$ for any $c^x < \frac{1-B}{2s-1}$ or $c^y < g_2(c^x)$. □

Proof of Proposition 3.6. Since $b^y = 0$, we will omit the second subscript in the notations for equilibria we have used so far. The candidates for equilibrium allocations, feasibility and optimality conditions are summarized in Table 12 in Appendix 3.B. The derivation of equilibrium allocations is analogical to the case of mature and emerging market and is therefore omitted. Again, let us start with some notation:

$$h_1 = \frac{2-B}{s[B+2s(b^x+c^x)]}, \qquad h_2 = \frac{2-B-Bb^xs-Bc^xs}{2s^2(b^x+c^x)},$$
$$h_3 = \frac{2-B}{s(B+2c^xs)+b^x[2+B(s-1)-3s-2s^2]},$$
$$h_4 = \frac{2-b^x-B[1+b^x(s-1)+c^xs]}{2[b^x(s-1)^2+c^xs^2]}, \qquad h_5 = \frac{2-B}{(s-1)[B+2c^xs+b^x(2s-1)]},$$
$$h_6 = \frac{2-b^x-B[1+b^x(s-1)]-2c^xs}{2b^x(s-1)^2}.$$

In addition, let $k_1 = \frac{2-B-2b^xs}{2s}$ and $k_2 = \frac{2-B-2b^xs+b^x}{2s}$. If we fix B, s and b^x then, using optimality conditions in Table 12, the whole range of parameters (c^x, c^y) can be divided into several domains. These domains are summarized in the Table 7 and are illustrated on Figure 12 in Appendix 3.B. We split the proof of the proposition in a number of cases.

▷ *Domains* **i** *and* **k**
From the table we can readily see that in domains **i** and **k**, $BCy \prec NC$. Indeed, in both domains equilibrium allocation is such, that $m^x = 1$. In this case BCy does not improve incentives of new agents of type y and hence has no advantages compared with NC regime.

▷ *Domain* **l**

domain	feasibility condition	type of allocation NC	BCy
a	$c^y < \min[h_1, h_2, h_4]$	E_{000}	E_{000}
b	$h_2 < c^y < h_4$	E_{000}	E_{001}
c	$\max[h_2, h_4] < c^y < \frac{2-B}{2s}$	E_{001}	E_{001}
d	$\max[\frac{2-B}{2s}, h_4] < c^y < h_2$	E_{001}	E_{000}
e	$\frac{2-B}{2s} < c^y < h_4, c^x > k_1$	E_{111}	E_{000}
f	$h_1 < c^y < h_3, c^x < k_1$	E_{110}	E_{000}
g	$h_3 < c^y < h_5, c^x < k_1$	E_{110}	E_{010}
h	$h_3 < c^y < [h_5, h_6], c^x > k_1$	E_{111}	E_{010}
i	$h_5 < c^y, c^x < k_1$	E_{110}	E_{110}
j	$h_5 < c^y, k_1 < c^x < k_2$	E_{111}	E_{000}
k	$cy > \frac{2-B}{2s}, c^x > k_2$	E_{111}	E_{111}
l	$\max[\frac{2-B}{2s}, h_6] < c^y < \frac{2-B}{2(s-1)}$	E_{111}	E_{011}

Table 7: Equilibrium allocations on the asymmetric market

Calculating the respective profits and comparing them we receive, that

$$\Pi^{NC} > \Pi^{BCy} \iff B^2 + 4B[-1 + c^y(s-1)] + 4[1 + c^y + (c^y)^2(s-1)^2 - 2c^y s] < 0.$$

Let us denote the expression on the left-hand side of the inequality as $\varphi_1(c^y)$, which is a quadratic, convex function of c^y. Hence, $\varphi_1(c^y) < 0$ if $c_1 < c^y < c_2$, where

$$c_{1,2} = \frac{-1 + B + 2s - Bs \mp \sqrt{-3 + 2B + 4s - 2Bs}}{2(s-1)^2}.$$

In addition, feasibility constrains for domain **l** require $\frac{2-B}{2s} < c^y < \frac{2-B}{2(s-1)}$. Since $c_1 < \frac{2-B}{2s}$ and $c_2 > \frac{2-B}{2(s-1)}$, for all range of parameters in domain **l**, $\varphi_1(c^y) < 0$, which implies $BCy \prec NC$.

▷ *Domains* **j**, **g** *and* **h**
The inequality $\Pi^{NC} > \Pi^{BCy}$ is equivalent to

$$4 - 4B + B^2 + (b^x)^2(1-2s)^2 - 8b^x s +$$
$$+4(c^x)^2 s^2 + Bb^x(4s-2) + c^x[4s(B-2) + 4b^x s(2s-1)] < 0.$$

Let us denote an expression on the left-hand side of the inequality as $\varphi_2(c^x)$, which is a quadratic, convex function of c^x. Hence, $\varphi_2(c^x) < 0$ if $c_1 < c^x < c_2$, where

$$c_{1,2} = \frac{2 - B \mp 2\sqrt{b^x} + b^x - 2b^x s}{2s}.$$

In addition, feasibility constrains requires that $k_1 < c^x < k_2$. It is easy to check, that $c_1 < k_1$ and $c_2 > k_2$. Hence, $\varphi_2(c^x) < 0$ in domain **j**, which implies $BCy \prec NC$. Using very similar line of argument it is easy to show, that $BCy \prec NC$ also in domains **g** and **h**.

▷ *Domain* **b**

Consider now domain **b**, which is feasible if $h_2 < c^y < h_4$. Observe, that $h_2 < h_4$ if $c^x > \frac{4s-2-b^x s^2+B[1-(2-b^x)s-b^x s^2]}{s[B(s-1)+s]} \equiv k_3$. Notice further, that h_2 and h_4 are decreasing functions, and the value of these functions at $c^x = k_3$ is $(1-B)/[2(2s-1)]$. Hence, everywhere in domain **b**, $c^y < (1-B)/[2(2s-1)]$. Finally, $h_2 < c^y < h_4$ is equivalent to

$$\frac{2 - b^x(1 + 2c^y(s-1)^2) + B(-1 + b^x + b^x s)}{s(B + 2c^y s)} < c^x < \frac{2 - B - Bb^x s - 2b^x c^y s^2}{s(B + 2c^y s)}.$$

Calculating the profits and comparing them, we receive, that

$$\Pi^{NC} < \Pi^{BCy} \iff \frac{\varphi_3(c^x)}{4[-1 + b^x(c^y)^2 + c^x c^y s^2]} < 0$$

(the expression for function φ_3 is not listed here as it is rather complicated). Due to Assumption 3.3, we have $b^x(c^y)^2 + c^x c^y s^2 < 1$. Hence, $\Pi^{NC} < \Pi^{BCy}$ if and only if $\varphi_3(c^x) > 0$. Function $\varphi_3(c^x)$ is quadratic in c^x with coefficient on $(c^x)^2$ being $-B^2 c^y s^2 - 4B(c^y)^2 s^3 - 4(c^y)^3 s^4 < 0$. Hence, $\varphi_3(c^x) > 0$ if and only if $c_1 < c^x < c_2$:

$$c_{1,2} = -\frac{1}{s(B + 2c^y s)^2}\big[B^2(1 + b^x s) + 2B\big(-1 + c^y s(1 - b^x + 2b^x s)\big) + \\ + 2c^y s\big(-2 + b^x(1 + c^y - 2c^y s + 2c^y s^2)\big) \pm \sqrt{K_1}\big].$$

where $K_1 = -b^x[-1+B+c^y(2s-1)]\,[b^x(c^y)^2 s^2(1+c^y-2c^y s)+B^2(1+c^y s)+Bc^y s(2+(2-b^x)c^y s)]$.

It is left to verify that $c_1 < \frac{2-b^x(1+2c^y(s-1)^2)+B(-1+b^x+b^x s)}{s(B+2c^y s)}$ and $c_2 > \frac{2-B-Bb^x s-2b^x c^y s^2}{s(B+2c^y s)}$. Both this inequalities hold if $c^y < \frac{1-B}{2s-1}$, which is true for any $c^y < \frac{1-B}{2(2s-1)}$. Hence, in domain **b**, $NC \prec BCy$.

▷ *Domain* **a**

In domain **a** the profits in NC and BCx regime are as follows:

$$\Pi^{NC} = \frac{B^2\big[c^x + c^y + 2c^x c^y s + b^x(1+2c^y s)\big]}{4(1-b^x c^y s^2 - c^x c^y s^2)},$$

$$\Pi^{BCy} = \frac{1}{4(1-b^x c^y s^2 - c^x c^y s^2)}\big[(b^x)^2 c^y + 2Bb^x c^y\big(1+b^x(s-1)+c^x s\big)+ \\ + B^2\big(c^x + c^y + 2c^x c^y s + b^x(1-2c^y - c^x c^y + 2c^y s)\big)\big].$$

Comparing profits we receive, that $NC \prec BCy$ if

$$c^y < \frac{(B-1)[t - c^x + B(2+c^x - t + 2st)]}{s^2(c^x - t)t - 2Bs^2 t[1+c^x+t(s-1)] + B^2[1+2s(t-1)-(2-c^x)s^2 t]},$$

where $t \equiv b^x + c^x$. Let us denote the right-hand side of this inequality as $\mathcal{I}_1$.

▷ *Domain* **d**

In domain **d** feasibility constrains require $h_2 < c^x < h_4$, which is equivalent to

$$\frac{2-B-Bb^x s - 2b^x c^y s^2}{s(B+2c^y s)} < c^x < \frac{2-b^x[1+2c^y(s-1)^2]+B(-1+b^x+b^x s)}{s(B+2c^y s)}.$$

Comparing profits of the monopolist, we establish that $\Pi^{NC} < \Pi^{BCy}$ if

$$\frac{\varphi_4(c^x)}{-1+b^x c^y(s-1)^2 + c^x c^y s^2} < 0.$$

The denominator in the above expression in negative by Assumption 3.3. The nominator $\varphi_4(c^x)$ is a quadratic function where the coefficient on $(c^x)^2$ equals $s^2(B+2c^y s)^2$. Hence, $\varphi_4(c^x) > 0$ if $c^x < c_1$ or $c^x > c_2$:

$$c_{1,2} = -\big[B^2\big(1+b^x(s-1)\big) + 2c^y s\big(-2+b^x c^y(1-2s+2s^2)\big)+ \\ B\big(-2+2c^y s + b^x(1+c^y(2-4s+4s^2))\big) \mp 2\sqrt{K_2}\big]/[s(B+2c^y s)^2].$$

where

$$K_2 = b^x[-1+B+c^y(2s-1)]\big[B^2\big(1+b^x c^y(s-1)+c^y s\big) \\ + Bc^y s\big(2+b^x+2c^y s - b^x c^y(3s-2)\big) + b^x(c^y)^2 s^2\big(1+c^y(2s-1)\big)\big].$$

$K_2 > 0$ if $c^y > \frac{1-B}{2s-1}$. Further, $c_2 > \frac{2-b^x(1+2c^y(s-1)^2)+B(-1+b^x+b^x s)}{s(B+2c^y s)}$ for $c^y > \frac{1-B}{2s-1}$. Hence, $\varphi_4(c^y) > 0$ if $c^x < c_1$. Let us denote $\varphi_4^{-1}(c^y)$ as $\mathcal{I}_2$.

▷ *Domains* **e** *and* **f**

Using the same argument, we can show that $NC \prec BCy$ if $c^y < \mathcal{I}_4$ in domain **e** and $NC \prec BCy$ if $c^y < \mathcal{I}_5$ in domain **f**. Expressions for $\mathcal{I}_4$ and $\mathcal{I}_5$ can be easily derived as the roots of the quadratic equation (similar as in previous case). The expressions, however, are rather complicated and therefore are not presented here.

$\triangleright$ *Domain* **c**

Finally, consider domain **c**. Calculating the respective profits and comparing them we receive, that $\Pi^{NC} < \Pi^{BCy}$ if $B - 1 + c^y(2s-1) < 0$, which is equivalent to $c^y < \frac{1-B}{2s-1}$. Let us denote $\mathcal{I}_3 \equiv \frac{1-B}{2s-1}$.

Combining the results we can describe a curve $\mathcal{I}^y$ as a function of c^x which shows the indifference of the monopolist between making technologies compatible versus not compatible with each other.

$$\mathcal{I}^y = \begin{cases} \mathcal{I}_1, & \text{if } \mathcal{I}_1 < \min[h_1, h_2, h_4], \\ \mathcal{I}_2, & \text{if } h_2 < \mathcal{I}_2 < \min[\frac{2-B}{2s}, h_4], \\ \mathcal{I}_3, & \text{if } \frac{1-B}{2s-1} > h_4, \\ \mathcal{I}_4, & \text{if } \frac{2-B}{2s} < \mathcal{I}_4 < h_3 \text{ and } k_1 < c^x < k_2, \\ \mathcal{I}_5, & \text{if } h_3 < \mathcal{I}_5 < h_1 \text{ and } c^x < k_1. \end{cases}$$

We have shown, that $BCy \prec NC$ in domains **g**, **h**, **i**, **j**, **k**, **l** and $NC \prec BCy$ in domain b. Further, for each domain **a**, **c**, **d**, **e**, **f** there is a single curve, such that $NC \prec BCy$ below this curve and the inequality is reversed otherwise. It remains to show, that for each c^x there is a unique cutoff value of c^y, such that below this value $NC \prec BCy$ and above this value the inequality is reversed.

Assume, to the contrary, that for some $c^x = \hat{c}$ there are two distinct values of c^y, $c^y = c_1$ and $c^y = c_2$, such that $NC \prec BCy$ for all $c^y < c_1$ and $c^y < c_2$ and $BCy \prec NC$ for all $c^y > c_1$ and $c^y > c_2$. Note that since for each domain there exist a unique curve, points $(\hat{c}, c_1)$ and $(\hat{c}, c_2)$ must belong to the different domains. Assume first, that these domains have a common border (which is the case for domains **a** and **f**, **a** and **d**, **d** and **e**). Observe, that assumption $c_1 \neq c_2$ implies, that the curve, describing the indifference of the monopolist between NC and BCy, cannot be identical with the border between the domains. For clarity of notations, consider for example domains **a** and **f** (the argument is exactly the same for other pairs of domains).

The border between two domains gives the values of (c^x, c^y), such that the feasibility constrains for both domains are satisfied in the limit cases. Hence, if we denote the difference of profits in domain **a** as $\psi^a \equiv \Pi^{NC} - \Pi^{BCx}$ and in domain **f** as $\psi^f \equiv \Pi^{NC} - \Pi^{BCx}$, then on the border between two domains must hold $\psi^a \equiv \psi^f$. Then the existence of distinct c_1 and c_2 is possible only if $\psi^a = \psi^f = 0$. But this contradicts the assumption, that there is a unique curve, describing indifference of the monopolist between two compatibility regimes, for each domain.

Consider now the case, where c_1 and c_2 belong to the two domains which do not have a common border (domain **a** and **e**). Then the function $\psi \equiv \Pi^{NC} - \Pi^{BCx}$ must change sign on the border of the domain **d**. Since domains **e** and **d** are neighbor domains, see discussion above. □

Proof of Corollary 3.3. If the parameters of the model are such, that interior solution is feasible, then curve, describing the indifference of the monopolist between two compatibility regimes, is given by $\mathcal{I}^y = \mathcal{I}_1$, where $\mathcal{I}_1$ is given in the proof of Proposition 3.6.

First we prove that this curve is downward sloping. Consider the derivative $\partial \mathcal{I}_1/\partial c^x = \varphi_1(B)/[\psi_1(B)]^2$, where

$$\psi_1(B) = b^x s^2 t + 2Bs^2 t(1 - b^x + st) + B^2[-1 - 2s(t-1) + s^2(2 + b^x - t)t].$$

Hence, $\partial \mathcal{I}_1/\partial c^x < 0$ is equivalent to $\varphi_1(B) < 0$. Function $\varphi_1(B)$ is polynomial of the form $\varphi_1(B) = \alpha_0 B^4 + \alpha_1 B^3 + \alpha_2 B^2 + \alpha_3 B + \alpha_4$, where $\alpha_i, i \in \{1,2,3,4\}$ are coefficients that depend on b^x, c^x, and s. The function $\varphi_1(B)$ has the following properties:

$$\varphi_1(0) = -(b^x s) < 0, \qquad \varphi_1'(0) = -4b^x s^2[1 + b^x(s-1) + c^x s] < 0,$$
$$\varphi_1(1) = 0, \qquad \varphi_1'(1) = 2s(2s-1)(1 + b^x s + c^x s)^2 > 0.$$

Hence, to prove that $\varphi_1(B) < 0$ for any $B \in (0,1)$ it is sufficient to show, that $\varphi_1''(B)$ changes sign at most once on the interval $[0,1]$. This is indeed the case, since $\varphi_1''(B)$ is a quadratic function which has the following properties:

$$\varphi_1''(0) = 4s[-2(b^x)^2(s-1)^2 s - 2s(1 + c^x s)^2 - b^x(1 + c^x s)(4s^2 - 4s - 1)] < 0,$$
$$\varphi_1''(1) = 4s[(4s-3)(1 + c^x s)^2 + (b^x)^2 s(1 - 5s + 4s^2) + b^x(1 + c^x s)(1 - 8s + 8s^2)] > 0.$$

Thus, $\varphi_1(B) < 0$ on the interval $B \in (0,1)$, which implies that $\partial \mathcal{I}_1/\partial c^x < 0$.

Second, we prove the statement for $\bar{s}$. Towards this end consider the derivative $\partial \mathcal{I}_1/\partial s = \varphi_2(B)/[\psi_1(B)]^2$. By the same argument as above, $\partial \mathcal{I}_1/\partial s < 0$ is equivalent to $\varphi_2(B) < 0$. Function $\varphi_2(B)$ is polynomial of the form $\varphi_2(B) = \beta_0 B^4 + \beta_1 B^3 + \beta_2 B^2 + \beta_3 B + \beta_4$, where $\beta_i, i \in \{1,2,3,4\}$ are coefficients which depend on

b^x, c^x and s. The function $\varphi_2(B)$ has the following properties:

$$\varphi_2(0) = -2(b^x)^2 st < 0, \qquad \varphi_2'(0) = -8b^x st(1 - b^x + st) < 0,$$
$$\varphi_2(1) = 0, \qquad \varphi_2'(1) = 2(1 + st)^2[2 + (4s - 1)t] > 0.$$

Given this properties, to prove that $\varphi_2(B) < 0$ for any $B \in (0, 1)$ it is sufficient to show, that $\varphi_2''(B)$ changes sign at most once on the interval $[0, 1]$. This is indeed the case, since $\varphi_2''(B)$ is a quadratic function which has the following properties. The function is concave, since the coefficient at B^2 is

$$b^x st[b^x - t(1 + s)] - b^x t - (2 - t)(1 + st)^2 < 0,$$

which is negative for any $t = b^x + c^x < 2$. Further $\varphi_2''(1) < 0$ for any $b^x < 1$:

$$\varphi_2''(1) = -4(1 + st)\big[- 6 + (3 - 14s)t + (3 - 8s)st^2 + b^x\big(1 + (3s - 1)t\big)\big] < 0.$$

Hence, $\varphi_2(B) < 0$ on the interval $B \in (0, 1)$, which implies that $\partial \mathcal{I}_1/\partial s < 0$.
Third, we prove the statement for $\bar{B}$. For any $s > 1$ and $B \in (0, 1)$:

$$\frac{\partial \mathcal{I}^y}{\partial B} = -\frac{2B(2s - 1)(2 + st)^2[B + b^x(1 - B) + Bst]}{[\psi_1(B)]^2} < 0.$$

Fourth, the proof of the statement for $\bar{b}^x$ follows the same logic as the proof of the first and the second statement and is therefore abandoned. □

3.B Appendix: Tables and figures

$\tilde{\beta}_1$	$\mathrm{d}m^x/d\gamma^x$	$c^x[(s-1)(s-\gamma^x)b^y H''(n^y) + s(s-\gamma^y)c^y H''(m^y)]$
	$\mathrm{d}m^y/\mathrm{d}\gamma^x$	$c^x[(s-\gamma^x)H''(m^x)H''(n^y) + (s-\gamma^y)\Gamma b^x c^y]$
	$\mathrm{d}n^x/\mathrm{d}\gamma^x$	$H''(m^x)H''(m^y)H''(n^y) - (s-1)^2 b^x b^y H''(n^y) - (s-\gamma^y)^2 b^x c^y H''(m^y)$
	$\mathrm{d}n^y/\mathrm{d}\gamma^x$	$c^x[sH''(m^x)H''(m^y) - (s-1)\Gamma b^x b^y]$
$\tilde{\beta}_2$	$\mathrm{d}m^x/d\gamma^x$	$b^y[-(s-1)H''(n^x)H''(n^y) + s\Gamma c^y c^y]$
	$\mathrm{d}m^y/\mathrm{d}\gamma^x$	$-H''(m^x)H''(n^x)H''(n^y) + s^2 c^x c^y H''(m^x) + (s-\gamma^y)^2 b^x c^y H''(n^x)$
	$\mathrm{d}n^x/\mathrm{d}\gamma^x$	$b^y[-(s-\gamma^y)^2 H''(m^x)H''(n^y) - (s-\gamma^y)\Gamma b^x c^y]$
	$\mathrm{d}n^y/\mathrm{d}\gamma^x$	$b^y[-s(s-\gamma^x)c^x H''(m^x) - (s-1)(s-\gamma^y)b^x H''(n^x)]$

Table 8: Coefficients $\tilde{\beta}_1$ and $\tilde{\beta}_2$

E_{0000}	$m^x \in (0,1)$	$m^y \in (0,1)$	$n^x \in (0,1)$	$n^y \in (0,1)$
E_{0010}	$m^x \in (0,1)$	$m^y \in (0,1)$	$n^x = 1$	$n^y \in (0,1)$
E_{0011}	$m^x \in (0,1)$	$m^y \in (0,1)$	$n^x = 1$	$n^y = 1$
E_{0001}	$m^x \in (0,1)$	$m^y \in (0,1)$	$n^x \in (0,1)$	$n^y = 1$
E_{1000}	$m^x = 1$	$m^y \in (0,1)$	$n^x \in (0,1)$	$n^y \in (0,1)$
E_{1010}	$m^x = 1$	$m^y \in (0,1)$	$n^x = 1$	$n^y \in (0,1)$
E_{1011}	$m^x = 1$	$m^y \in (0,1)$	$n^x = 1$	$n^y = 1$
E_{1001}	$m^x = 1$	$m^y \in (0,1)$	$n^x \in (0,1)$	$n^y = 1$
E_{0100}	$m^x \in (0,1)$	$m^y = 1$	$n^x \in (0,1)$	$n^y \in (0,1)$
E_{0110}	$m^x \in (0,1)$	$m^y = 1$	$n^x = 1$	$n^y \in (0,1)$
E_{0111}	$m^x \in (0,1)$	$m^y = 1$	$n^x = 1$	$n^y = 1$
E_{0101}	$m^x \in (0,1)$	$m^y = 1$	$n^x \in (0,1)$	$n^y = 1$
E_{1100}	$m^x = 1$	$m^y = 1$	$n^x \in (0,1)$	$n^y \in (0,1)$
E_{1110}	$m^x = 1$	$m^y = 1$	$n^x = 1$	$n^y \in (0,1)$
E_{1111}	$m^x = 1$	$m^y = 1$	$n^x = 1$	$n^y = 1$
E_{1101}	$m^x = 1$	$m^y = 1$	$n^x \in (0,1)$	$n^y = 1$

Table 9: Candidates for equilibrium allocations with linear demand function

	NC		*BCx*	
	feasibility conditions	optimality conditions	feasibility conditions	optimality conditions
E_{0000}	$b^y < \min[h_1, h_2]$	$b^y < \min[h_1, h_2]$	$b^y < \min[h_2, h_5]$	$b^y < \min[h_2, h_5]$
E_{0010}	$b^y < \min[h_2, h_3]$	$h_1 < b^y < \min[h_2, h_3]$	$b^y < \min[h_2, h_3]$	$h_5 < b^y < \min[h_2, h_3]$
E_{0011}	$b^y < \min[h_3, h_4]$	$\max[h_1, h_2] < b^y < \min[h_3, h_4]$	$b^y < \min[h_3, h_4]$	$\max[h_2, h_5] < b^y < \min[h_3, h_4]$
E_{0001}	$b^y < \min[h_1, h_4]$	$h_2 < b^y < \min[h_1, h_4]$	$b^y < \min[h_4, h_5]$	$h_2 < b^y < \min[h_4, h_5]$
E_{1000}	$b^y < \frac{2-B}{s[B+2b^x(s-1)]}, b^x < \frac{2-B}{2s}$	$\times$	$b^y < \frac{2-B}{1+B(s-1)+2b^x(s-1)^2}, b^x < \frac{2-B}{2s}$	$\times$
E_{1010}	$b^x < \frac{2-B}{2s}$	$b^y > h_3, b^x < \frac{2-B}{2s}$	$b^x < \frac{2-B}{2s}$	$b^y > h_3, b^x < \frac{2-B}{2s}$
E_{1011}	$b^x < \frac{2-B}{2(s-1)}$	$b^y > h_3, \frac{2-B}{2s} < b^x < \frac{2-B}{2(s-1)}$	$b^x < \frac{2-B}{2(s-1)}$	$b^y > h_3, \frac{2-B}{2s} < b^x < \frac{2-B}{2(s-1)}$
E_{1001}	$b^y < \frac{2-B}{s[B+2b^x(s-1)]}, b^x < \frac{2-B}{2(s-1)}$	$\times$	$b^x < \min[\frac{2-B}{2(s-1)}, \frac{2-B}{1+B(s-1)+2b^x(s-1)^2}]$	$\times$
E_{0100}	$b^y < \min[\frac{2-B}{2s}, \frac{2-B-Bb^x s}{2b^x s(s-1)}]$	$\times$	$b^y < \min[\frac{2-B}{2s-1}, \frac{2-B-Bb^x s}{2b^x s(s-1)}]$	$\times$
E_{0110}	$b^y < \min[\frac{2-B}{2(s-1)}, \frac{2-B-Bb^x s}{2b^x s(s-1)}]$	$\times$	$b^y < \min[\frac{2-B}{2(s-1)}, \frac{2-B-Bb^x s}{2b^x s(s-1)}]$	$\times$
E_{0111}	$b^y < \frac{2-B}{2(s-1)}$	$\max[\frac{2-B}{2s}, h_4] < b^y < \frac{2-B}{2(s-1)}$	$b^y < \frac{2-B}{2(s-1)}$	$\max[\frac{2-B}{2s-1}, h_4] < b^y < \frac{2-B}{2(s-1)}$
E_{0101}	$b^y < \frac{2-B}{2s}$	$h_4 < b^y < \frac{2-B}{2s}$	$b^y < \frac{2-B}{2s-1}$	$h_4 < b^y < \frac{2-B}{2s-1}$
E_{1100}	$\frac{2-B}{2s} < \min[b^x, b^y]$	$\times$	$b^y < \frac{2-B}{2s-1}, b^x < \frac{2-B}{2s}$	$\times$
E_{1110}	$b^x < \frac{2-B}{2s}$	$\times$	$b^x < \frac{2-B}{2s}$	$\times$
E_{1111}	$b^x \in [0, \infty), b^y \in [0, \infty)$	$\frac{2-B}{2(s-1)} < \min[b^x, b^y]$	$b^x \in [0, \infty), b^y \in [0, \infty)$	$\frac{2-B}{2(s-1)} < \min[b^x, b^y]$
E_{1101}	$b^y < \frac{2-B}{2s}$	$\times$	$b^y < \frac{2-B}{2s-1}$	$\times$

Table 10: Mature market: feasibility and optimality conditions for equilibrium candidates in *NC* and *BCx* regimes

	NC		BCx	
	feasibility conditions	optimality conditions	feasibility conditions	optimality conditions
E_{0000}	$c^y < \min[h_1, h_2]$	$c^y < \min[h_1, h_2]$	$c^y < \min[h_1, h_2]$	$c^y < \min[h_1, h_2]$
E_{0010}	$c^y < h_1, c^x < \frac{2-B}{2s}$	$\times$	$c^y < h_1, c^x < \frac{2-B}{2s}$	$\times$
E_{0011}	$\frac{2-B}{2s} > \max[c^x, c^y]$	$\times$	$c^y < \frac{2-B}{2s}$	$\times$
E_{0001}	$c^y < \min[h_2, \frac{2-B}{2s}]$	$\times$	$c^y < \min[\frac{2-B}{2s}, h_3]$	$h_2 < c^y < \min[\frac{2-B}{2s}, h_3]$
E_{1000}	$c^y < \min[h_1, h_2]$	$\times$	$c^y < \min[h_1, h_2]$	$\times$
E_{1010}	$c^x < \frac{2-B}{2s}$	$c^y > h_1, c^x < \frac{2-B}{2s}$	$c^x < \frac{2-B}{2s}$	$c^x < \frac{2-B}{2s}, c^y > h_1$
E_{1011}	$c^x < \frac{2-B}{2s}$	$\times$	$c^x < \frac{2-B}{2(s-1)}$	$c^y > \frac{2-B}{2s}, \frac{2-B}{2s} < c^x < \frac{2-B}{2(s-1)}$
E_{1001}	$c^y < \min[h_2, \frac{2-B}{2s}]$	$\times$	$c^y < \min[\frac{2-B}{2s}, h_3]$	$\times$
E_{0100}	$c^y < \min[h_1, h_2]$	$\times$	$c^y < [h_1, h_2]$	$\times$
E_{0110}	$c^y < h_1, c^x < \frac{2-B}{2s}$	$\times$	$c^y < h_1, c^x < \frac{2-B}{2s}$	$\times$
E_{0111}	$c^y < \frac{2-B}{2s}$	$\times$	$c^y < \frac{2-B}{2s}$	$\times$
E_{0101}	$c^y < \frac{2-B}{2s}$	$h_2 < c^y < \frac{2-B}{2s}$	$c^y < \frac{2-B}{2s}$	$h_3 < c^y < \frac{2-B}{2s}$
E_{1100}	$c^y < \min[h_1, h_2]$	$\times$	$c^y < \min[h_1, h_2]$	$\times$
E_{1110}	$c^x < \frac{2-B}{2s}$	$\times$	$c^x < \frac{2-B}{2s}$	$\times$
E_{1111}	$b^x \in [0, \infty), b^y \in [0, \infty)$	$\frac{2-B}{2s} < \min[c^x, c^y]$	$b^x \in [0, \infty), b^y \in [0, \infty)$	$c^y > \frac{2-B}{2s}, c^x > \frac{2-B}{2(s-1)}$
E_{1101}	$c^y < \frac{2-B}{2s}$	$\times$	$c^y < \frac{2-B}{2s}$	$\times$

Table 11: New market: feasibility and optimality conditions for equilibrium candidates in NC and BCx regimes

	NC		BCx	
	feasibility conditions	optimality conditions	feasibility conditions	optimality conditions
E_{000}	$c^y < \min[h_1, h_2]$	$c^y < \min[h_1, h_2]$	$c^y < \min[h_3, h_4]$	$c^y < \min[h_3, h_4]$
E_{010}	$c^y < \min[h_1, \frac{2-B-Bb^x s-2c^x s}{2b^x s^2}]$	$\times$	$c^y < \min[h_5, h_6]$	$h_3 < c^y < \min[h_5, h_6]$
E_{011}	$c^y < \frac{2-B}{2s}$	$\times$	$c^y < \frac{2-B}{2(s-1)}$	$\max[\frac{2-B}{2s}, h_5] < c^y < \frac{2-B}{2(s-1)}$
E_{001}	$c^y < \frac{2-B}{2s}$	$h_2 < c^y < \frac{2-B}{2s}$	$c^y < \frac{2-B}{2s}$	$h_4 < c^y < \frac{2-B}{2s}$
E_{100}	$c^y < \min[h_1, \frac{2-B(1+c^x s)-2b^x s}{2c^x s^2}]$	$\times$	$c^y < \min[\frac{2-B}{s(B+2c^x s+b^x(2s-1))}, \frac{2-B(1+c^x s)-b^x(2s-1)}{2c^x s^2}]$	$\times$
E_{110}	$c^x < \frac{2-B-2b^x s}{2s}$	$c^y > h_1, c^x < \frac{2-B-2b^x s}{2s}$	$c^x < \frac{2-B-b^x(2s-1)}{2s}$	$c^y > h_6, c^x < \frac{2-B-b^x(2s-1)}{2s}$
E_{111}	$c^x \in [0, \infty), c^y \in [0, \infty)$	$c^y > \frac{2-B}{2s}, c^x > \frac{2-B-2b^x s}{2s}$	$c^x \in [0, \infty), c^y \in [0, \infty)$	$c^y > \frac{2-B}{2(s-1)}, c^x > \frac{2-B-b^x(2s-1)}{2s}$
E_{101}	$c^y < \frac{2-B}{2s}$	$\times$	$c^y < \frac{2-B}{2s}$	$\times$

Table 12: Assymmetric market: feasibility conditions for equilibrium candidates

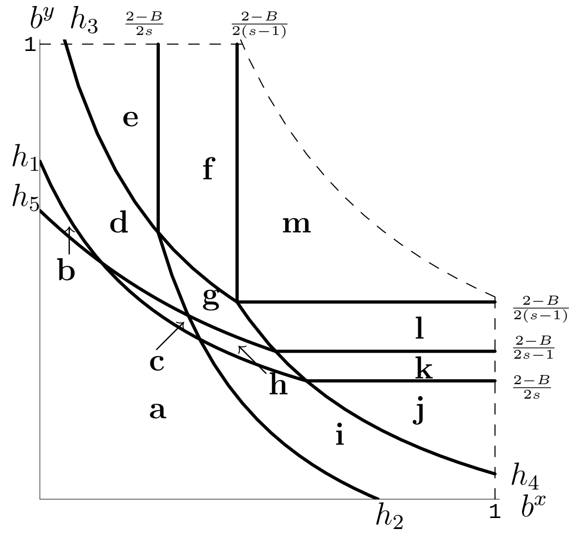


Figure 10: Equilibrium allocations for NC and BCx regimes at the mature market ($s = 2.5,\ B = 0.7$)

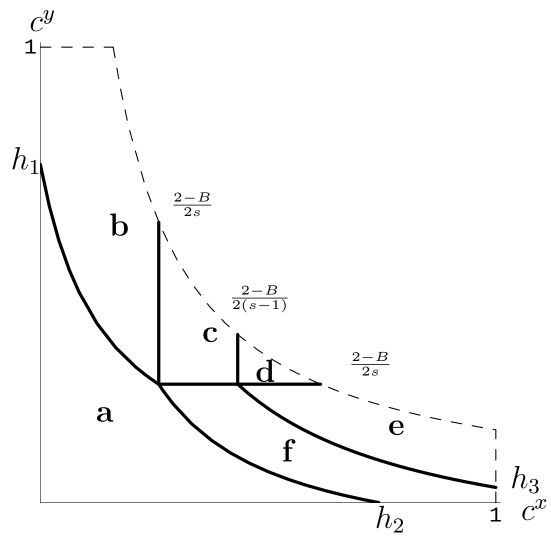


Figure 11: Equilibrium allocations for NC and BCx regimes at the emerging market ($s = 2.5,\ B = 0.7$)

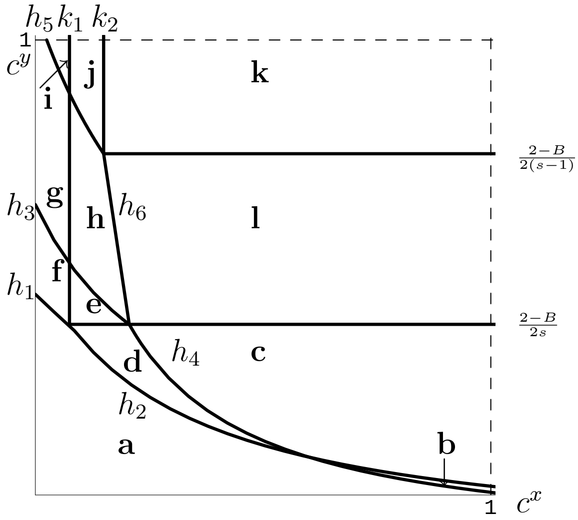


Figure 12: Equilibrium allocations for NC and BCy regimes at the asymmetric market ($s = 2$, $B = 0.5$, $b^x = .3$)

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