Four Contributions to Experimental Economics: Binary Choice Experiments

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Executive Summary

This thesis is about binary decisions participants make in a laboratory environment. For this purpose, laboratory experiments are conducted to investigate effects of individual decision making in a microeconomic context. The first chapter is a short introduction and will give an overview over the next four chapters In the second chapter comparisons between theories in stationary 2x2 games and empirical data are investigated. Twelve 2x2 games have been played in the laboratory and the results have been compared with 5 stationary concepts. The third chapter reports experimental results on a simple coordination game in which two players can coordinate either on an equal distribution of payoffs or on a Pareto superior but unequal distribution of payoffs. The fourth chapter reports on simulations applied on two similar congestion games: the first is the classical minority game. The second one is an asymmetric variation of the minority game with linear payoff functions. The fifth and last chapter reports results of laboratory experiments about traffic behavior of participants with different cultural backgrounds. The minority game as an elementary traffic scenario was chosen, in which human participants of a German and Chinese subject pool had to choose over 100 periods between a road *A* and a road *B*.

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1. Introduction

The following five chapters are related to experimental economics. Every chapter consists out of an already published or submitted paper.

In the second chapter, entitled "Stationary Concepts for Experimental 2x2-Games" my coauthor Reinhard Selten and I compare experimentally five stationary concepts for completely mixed 2x2-games: Nash equilibrium, quantal response equilibrium, action-sampling equilibrium, payoff-sampling equilibrium (Osborne and Rubinstein 1998) and impulse balance equilibrium. Experiments on 12 games, 6 constant sum games and 6 non-constant sum games are run with 12 independent subject groups for each constant sum game and 6 independent subject groups for each non-constant sum game. Each independent subject group consists of four players 1 and four players 2 interacting anonymously over 200 periods with random matching. The comparison of the five theories shows that the order of performance from best to worst is as follows: impulse balance equilibrium, action-sampling equilibrium, payoff-sampling equilibrium, quantal response equilibrium, Nash equilibrium. The paper is accepted by the Journal American Economic Review and will be published in 2008. Reinhard Selten & Thorsten Chmura (2008) Stationary Concepts for Experimental 2x2 Games will appear in June 2008 *American Economic Review.* This paper was published in June 2008 in the American economic review. At the beginning of 2009 Christoph Brunner, Colin F. Camerer and Jacob K. Goeree submitted a correction of this paper "A correction and re-interpretation of 'Stationary concepts for experimental 2x2 games" to the same journal. We recalculated their corrections and included these corrections in this paper.

Chapter 3 of this thesis, entiteled „Testing (Beliefs about) Social Preferences: Evidence from an Experimental Coordination Game" was written with the following coauthors Sebastian Kube, Thomas Pitz and Clemens Puppe. This chapter reports experimental results on a simple coordination game in which two players can coordinate either on an equal distribution of payoffs or on a Pareto superior but unequal distribution of payoffs. We find that the higher the difference in individual payoffs, the less likely is a successful coordination on the Pareto superior distribution. While this is well in line with the recent models of inequity aversion, our results are best explained not by a preference for equality *per se* but rather by the *belief* that the opponent has such a preference. Thorsten Chmura, Sebastian Kube, Thomas Pitz and Clemens Puppe (2005) Testing (Beliefs about) Social Preferences : Evidence from an Experimental Coordination Game. *Economics Letters,* Vol. 88 (2), 214-220.

Chapter 4, "An Extended Reinforcement Algorithm for Estimation of Human Behavior in Experimental Congestion Games" reports simulations applied on two similar congestion games: the first is the classical minority game. The second one is an asymmetric variation of the minority game with linear payoff functions. For each game, simulation results based on an extended reinforcement algorithm are compared with real experimental statistics. It is shown that the extension of the reinforcement model is essential for fitting the experimental data and estimating the player's behavior. The paper was written with my coauthor Thomas

Pitz and published in the Journal of Artificial Societies and Social Simulations (JASSS) in 2007. Thorsten Chmura & Thomas Pitz (2007) *Journal of Artificial Societies and Social Simulation* vol. 10, no. 2.

The last chapter entitled "Are the Chinese or the Germans the better Drivers?" reports results of laboratory experiments about traffic behavior of participants with different cultural backgrounds. This paper is written together with my coauthors Thomas Pitz and Fei Fangyu. We conduct the minority game as an elementary traffic scenario in which human participants of a German and Chinese subject pool had to choose over 100 periods between a road *A* and a road *B.* In each period, the road that was chosen by the minority of players win, these participants get a payoff. The payoff in the majority group is 0. An important observation is that the number of road changes of a participant is negatively correlated with his/her cumulative payoff. In this paper, particular emphasis shall be laid on a comparison of the participants' reaction to the immediately preceding payoffs. It could be shown that Chinese participants reacted differently to the payoffs of preceding periods than the German participants. The Chinese players did not change the route after bad payoffs as often as the players of the German group. The Chinese comparison group is on average able to attain better results because "bad" payoffs are more frequent in the minority game than "good" ones. In the current draft the paper is a working paper. Thorsten Chmura, Thomas Pitz, Fanyu Fei (2008): Who are the smarter Drivers? The Germans or the Chinese? An Experimental Approach (submitted working paper).

2. Stationary Concepts for Experimental 2x2-Games

2.1. Experimental Literature and Introduction

Experimental evidence suggests that mixed Nash-equilibrium is not a very good predictor of behavior. Thus EREV AND ROTH (1998 p. 853) conclude as their first summary observation that "…in some of the games the equilibrium prediction does very badly". A normal form game is called completely mixed, if it has only one equilibrium point in which every pure strategy is used with positive probability. 2x2-games of this kind are of special interest. They are the simplest games for which mixed equilibrium is the unequivocal game theoretic prediction, if they are played as non-cooperative one-shot games.

Mixed equilibrium has several interpretations. One interpretation is that of a rational recommendation for a one-shot game. Another interpretation looks at mixed equilibrium as a result of evolutionary or learning processes in a situation of frequently repeated play with two populations of randomly matched opponents. One may speak of mixed equilibrium as a behavioral stationary concept. KEN BINMORE, JOE SWIERZBINSKI and CHRIS PROULX (Economic Journal 2001) argue in their paper that mixed Nash-equilibrium predicts reasonably well for completely mixed constant sum 2x2-games. However it is difficult to judge the goodness of fit, if there is no comparison to other stationary concepts.

This paper was published in June 2008 in the American economic review. At the beginning of 2009 Christoph Brunner, Colin F. Camerer and Jacob K. Goeree submitted a correction of this paper "A correction and re-interpretation of 'Stationary concepts for experimental 2x2 games" to the same journal. We recalculated their corrections and included these corrections in this paper.

Economic theory makes extensive use of the concept of mixed equilibrium. One of its attractions is its independence of parameters outside the structure of the game. For the purpose of analyzing theoretical models it is of great advantage to be able to rely on stationary concepts.

In this paper we will present several alternative stationary concepts for 2x2-games, which can be compared with mixed equilibrium and with each other. For this purpose we have performed experiments on 12 completely mixed 2x2-games. Six of them are constant-sum games and the other six are non-constant-sum games. Each of the constant-sum games was run with 12 independent subject groups and each of the other games with 6 independent subject groups. Each independent subject group consisted of four players 1 and four players 2 interacting in fixed roles over 200 periods with random matching.

The stationary concepts compared were:

Nash Equilibrium, Quantal Response Equilibrium, Action-sampling Equilibrium, Payoff-sampling Equilibrium and Impulse Balance Equilibrium.

Quantal response equilibrium (MCKELVEY, PALFREY 1995) assumes that players give quantal best responses to the behavior of the others (see 2.2.3). In the exponential form of quantal response equilibrium, considered here, the probabilities are proportional to an exponential with the expected payoff times a parameter in the exponent.

Payoff-sampling equilibrium (Osborne & Rubinstein 1998) envisions a stationary situation in which a player takes two samples of equal size, one for each of her pure strategies. She than compares the sum of her payoffs in the two samples and plays the strategy with the higher payoff sum. If both payoff sums are equal then both pure strategies are chosen with probability ½. Payoff-sampling equilibrium is a mixed strategy combination reflecting this picture. Here, too, the sample size is a parameter. The best fitting sample size turns out to be 6 for each of both samples. The name "payoff-sampling equilibrium" refers to the sampling of own payoffs for each pure strategy.

Action-sampling equilibrium is based on the idea that in a stationary situation a player takes a sample of 12 observations of the strategies played on the other side, and then optimizes against this sample. If a player has a unique pure best response to her sample than she plays this strategy. If both strategies are best responses then each of them is chosen with probability ½. This yields a mixed strategy depending on the probabilities of pure strategies on the other side. Action-sampling equilibrium is a mixed strategy combination consistent with this picture. The name "action-sampling equilibrium" refers to the sampling of the opponent's actions. The concept has been developed by one of the authors (R. SELTEN). As far as we know it cannot be found in the literature. However the sampling of actions of other players also appears in a paper by Osborne and Rubinstein (1993) in the context of a sampling equilibrium for a large voting game. The sample size is a parameter. Originally the sample size 7 was chosen in view of the famous paper "The Magical Number 7 Give or Take Two" by MILLER (1956). Later BRUNNER, CAMERER AND GOEREE (2009) found that 12 actually gives a better fit than other sample sizes.

Impulse balance equilibrium proposed by one of the authors (R. SELTEN) is based on learning direction theory (SELTEN, BUCHTA, 1999). This learning theory is applicable to the repeated choice of the same parameter in learning situations in which the decision maker receives feedback not only about the payoff for the choice taken, but also for the payoffs connected to alternative actions. If a higher parameter would have brought a higher payoff we speak of an upward impulse and if a lower parameter would have yielded a higher payoff we speak of a downward impulse. The decision maker is assumed to have a tendency to move in the direction of the impulse.

It is worth pointing out that impulse learning is very different from reinforcement learning. In reinforcement learning the payoff obtained for a pure strategy played in the preceding period determines the increase of the probability for this strategy. The higher this payoff is the greater is this increase. In impulse learning it is not the payoff in the preceding period which is of crucial importance. It is the difference between what could have been obtained and what has been received which moves the behavior in the direction of the higher payoff. Moreover reinforcement learning is entirely based on observed own payoffs, whereas impulse learning requires feedback on the other player's choice and the knowledge of the player's own payoff function.

In SELTEN, ABBINK and COX (2005) impulse balance theory, a semi-quantitative version of learning direction theory has been proposed. The learning process itself is not modeled, but only the stationary distribution. In the stationary distribution expected upward impulses are equal to expected downward impulses. As in prospect theory (KAHNEMANN & TVERSKY, 1979) losses are counted double in the computation of impulses (formally this involves the computation of a loss impulse).

Impulse balance equilibrium applies the idea of impulse balance theory to 2x2-games. The probability of choosing one of two pure strategies, say strategy *A,* is looked upon as the parameter to be adjusted upward or downward. It is assumed that the pure strategy maximin is the reference level determining what is perceived as profit or loss. In impulse balance equilibrium expected upward and downward impulses are equal for each of both players simultaneously.

Following a suggestion of one of the authors (R. SELTEN) impulse balance equilibrium has been successfully applied to special 2x2- and 2x2x2-games in a paper by AVRAHAMI, GÜTH and KAREEV (2005).

Remarks: Two of the stationary concepts compared in this paper, Nash equilibrium and impulse balance equilibrium, are parameter free. Action-sampling equilibrium involves one parameter, namely, the number 12. For the Payoff-sampling equilibrium this parameter is the number 6, which yields the best fit to the data. Quantal response equilibrium involves one parameter, namely, the constant multiplier of expected payoffs in the exponent. This parameter has to be adjusted to the data.

Quantal response equilibrium modifies Nash equilibrium by introducing noise into the optimization process. Thereby the best response notion is replaced by a notion of quantal response. The two sampling equilibria, action-sampling equilibrium and payoff-sampling equilibrium also involve noise produced by sampling error. However, in contrast to quantal response equilibrium this noise is endogenous and is completely determined by the sample size and the payoffs of the game.

Quantal response equilibrium is not connected to any theory which relates the noise parameter to the structure of the game. One could, of course, fit the parameter for every individual game separately. However, this does not yield a method for predicting a unique stationary mixed strategy combination for every completely mixed 2x2-game. In order to make the concept of quantal response equilibrium comparable to other theories involving at most one parameter, one has to look at the parameter of quantal response equilibrium as an unknown behavioral constant which is the same for all games. Accordingly we determine the value of the parameter which best fits all our data and base our comparison on this.

The five concepts can be thought of as stationary states of dynamic learning models. Learning models differ with respect to their requirements on prior knowledge of the game and on feedback after each period. Nash equilibrium is stationary with respect to reinforcement learning models like the ones used by Roth & Erev (1998). These models require feedback on own payoffs but not more. A player does not even have to know his or her own payoff matrix. The same knowledge and feedback requirements are sufficient for learning models with quantal response equilibrium as stationary state. The expected payoffs appearing in the formulas for quantal response equilibrium can be estimated as average past payoffs. Simple learning models yielding payoff-sampling equilibrium as stationary state immediately suggest themselves. It is clear that here, too, only feedback of a player's own period payoff is necessary.

The other two concepts seem to be more demanding with respect to learning models yielding them as stationary states. As far as we can see one needs knowledge of one's own payoff matrix as well as feedback on the other player's choice in these two cases. Clearly a player must know his or her own payoff matrix for optimizing against a sample of the other player's choices. The same kind of knowledge and feedback is necessary for perceiving impulses in learning direction theory.

The development of stationary concepts, which fit experimental data is very important for behavioral theory. With the help of such concepts theoretically interesting situations can be mathematically explored as, for example, a voting situation in a paper by Osborne and Rubinstein (2003).

Learning models could also be applied to theoretically interesting situations. However, the construction of learning models usually involves many details which may influence the outcome of computer simulations. This makes it difficult to work with learning models rather than stationary concepts. Moreover, in complex situations one may need a huge number of computer simulations in order to answer questions of comparative statics, which can be attacked mathematically on the basis of stationary concepts.

In completely mixed 2x2-games, Nash equilibrium and impulse balance equilibrium can be described by explicit formulas and therefore are easy to use in theoretical investigations. However, this is not true for quantal response equilibrium, action-sampling equilibrium and payoff-sampling equilibrium. The latter concepts can only be computed numerically with the help of a computer. Nevertheless it is maybe sometimes possible to investigate their comparative static properties by mathematical operations like implicit differentiation applied to the defining equations. A similar approach to the results of learning models seems to be almost hopeless.

In this paper all five stationary concepts will only be defined for completely mixed 2x2-games. In the literature, Nash equilibrium, quantal response equilibrium and payoff-sampling equilibrium are defined for normal form games in general. It is also clear how the concept of action-sampling equilibrium can be generalized to all normal form games. Admittedly this is less clear for impulse balance equilibrium as far as normal forms with more than 2 strategies for some players are concerned. Here different generalizations are possible. The basic principle would be that for each strategy of a player, expected incoming impulses should be equal to expected outgoing impulses unless there are no outgoing impulses as in pure Nash equilibrium. In Appendix 2.F a sketch of a generalization of impulse balance equilibrium to general n-person games in normal-form is presented.

The comparison of stationary concepts can also guide the search for adequate learning rules. In the past, many authors like Selten (1990) and Sergio Hart and MasCollel (2000) felt that a reasonable learning model should converge to Nash equilibrium or correlated equilibrium under favorable assumptions. However, if other stationary concepts better fit experimental data, one may want to look at learning processes converging to them.

As we shall see, over all 200 periods and all 108 independent subjects groups the comparison yields the following order with respect to the goodness of fit from best to worst: Impulse balance equilibrium, payoff-sampling equilibrium, action-sampling equilibrium, quantal response equilibrium, Nash equilibrium. However the difference between impulse balance equilibrium and payoff-sampling equilibrium is not statistically significant (see 2.4.8.).

In chapter 2.2 we shall present a more detailed description of the five concepts. Chapter 2.3 will explain the experimental setup and section 2.4. will describe the results. Chapter 2.5 concludes with a summary and discussion.

2.2. The Five Stationary Concepts

All the experimental 2x2-games in this paper have the structure shown by figure 2.1. The arrows around the matrix show the direction of best replies. The Parameters a_L, a_R, b_U and b_D are assumed to be non-negative. Games with negative payoffs probably would require special behavioral considerations which we want to avoid in this paper. The parameters c_L and c_R are player 1's payoff differences in favor of *U* and *D,* respectively. Similarly d_U and d_D are payoff differences of player 2 for *R* and *L,* respectively. All these payoff differences are assumed to be positive. It is clear that a game with this structure is completely mixed in the sense that it has a uniquely determined completely mixed Nash equilibrium.

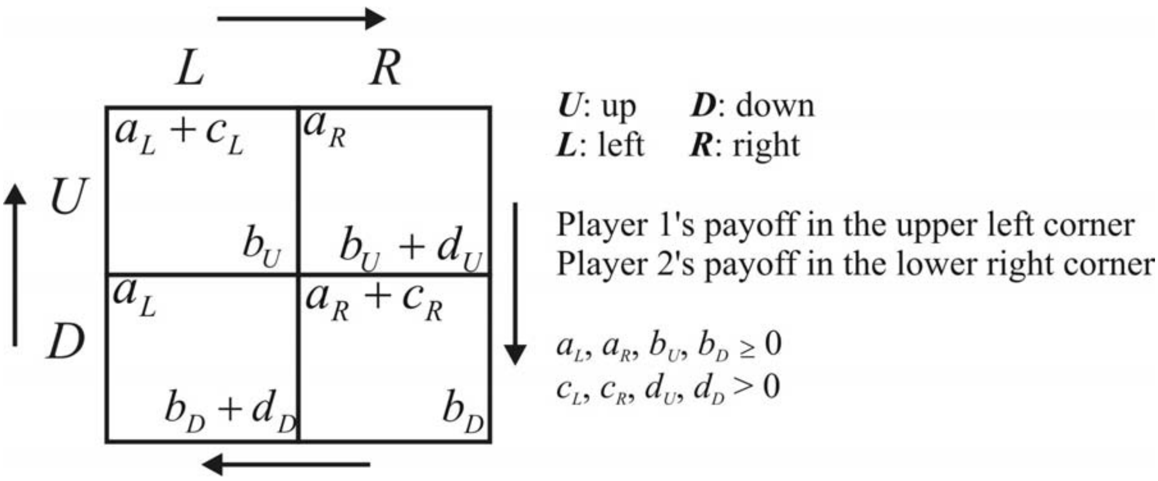

Figure 2.1: *Structure of the experimental 2x2-games.*

In a completely mixed 2x2-game the arrows may also have the opposite orientation. However, we can restrict our attention to the structure shown by figure 2.1 without any loss of generality. The case of counter-clockwise arrows can be transformed to the one shown above by an interchange of the two rows.

2.2.1. Equilibrium Conditions and Their Graphical Representation

Let $p = (p_U,p_D)$ and $q = (q_L,q_R)$ be the mixed strategies of player 1 and player 2, respectively. Here p_U and p_D are player 1's choice probabilities for *U* and *D* and q_L and q_R are player 2's choice probabilities for strategy *L* and *R*. The space of mixed strategies for a game with a structure of figure 2.1 can be described by the **(p_U,q_L)-diagram** which shows the interval $0 \leq p_U \leq 1$ horizontally and the interval $0 \leq q_L \leq 1$ vertically. Every point (p_U,q_L) in this square represents a strategy combination.

Each of the five concepts involves two equilibrium conditions. The first one describes equilibrium adjustment of player 1 for any given mixed strategy of player 2. In the same way the second condition expresses equilibrium adjustment of player 2 to any given mixed strategy of player 1. These two equilibrium conditions can be represented by curves in the (p_U,q_L)-diagram. We call the graph of the first equilibrium condition the curve for p_U and the graph for the second one the curve for q_L. The intersection of both curves is the stationary equilibrium specified by the concerning concept.

Figure 2.2 shows the curves for p_U and q_L arising in the example of our experimental game 1 (see figure 2.5 in 3.2.). With the exception of the case of Nash equilibrium, the curves for p_U are monotonically increasing and the curves for q_L are monotonically decreasing. In all five parts of figure 2.2 both curves intersect at the relevant stationary equilibrium of our experimental game 1.

We now shortly discuss the two curves in the case of the Nash equilibrium. Let p_U^N and p_L^N be the Nash equilibrium probabilities for U and L, respectively. Let us look at p_U on the curve for p_U as q_L moves from zero to 1. In the first vertical piece of the curve with $0 \le q_L \le q_L^N$ the probability p_U remains constant at $p_U = 0$. Then it moves on a horizontal piece at q_L^N from zero to one. The curve ends with a vertical piece with $q_L^N \le q_L \le 1$ at which p_U stays at $p_U = 1$. Similarly, on the curve for q_L the probability q_L stays at $q_L = 1$ in a horizontal piece with $0 \le q_U \le p_U^N$, then decreases from 1 to zero on a vertical piece with $q_L = q_L^N$, and finally comes to a horizontal piece with $q_L^N \le q_L \le 1$ and $p_U = 0$. In this sense one may say that p_U is increasing or constant along the curve for p_U and q_L is decreasing or constant along the curve for q_L.

In the case of the other four concepts the curves for p_U and q_L are continuously differentiable. For each of these concepts equations for the two curves will be given in the following sections 2.2.2, 2.2.3, 2.2.4. and 2.2.5. In these cases the value of p_U at q_L on the curve for p_U is denoted by $p_U(q_L)$. Similarly the notation $q_L(p_U)$ is used for the value of q_L at p_U on the curve for q_L.

The curves for the concepts different from Nash equilibrium reveal a considerable sensitivity with respect to the strategy of the other player. Suppose for example player 2 plays her Nash equilibrium strategy q_L^N and that player 1 chooses the strategy $p_U(q_L^N)$. The value of $p_U(q_L^N)$ for quantal response equilibrium, action-sampling equilibrium, payoff-sampling equilibrium and impulse equilibrium is .29, .52, .56, and .33, respectively, whereas p_U^N is equal to .09. It can bee seen, that in all four cases there is a considerable difference between $p_U(q_L^N)$ and p_U^N.

A look at figure 2.2 suggests a distinction of two groups of the pictures shown there. The first group consists of the two diagrams in the first row and the second group is formed by the remaining three pictures. The curves for quantal response equilibrium are near to those of Nash equilibrium. In this respect there is a close similarity within the first group. The diagrams within the second group also look very similar to each other, but there is a marked difference between the two groups.

As we shall see later, the concepts giving rise to the second group of pictures clearly outperform those connected to the first group. These three concepts yield predictions near to each other and much nearer to the observed relative frequencies.

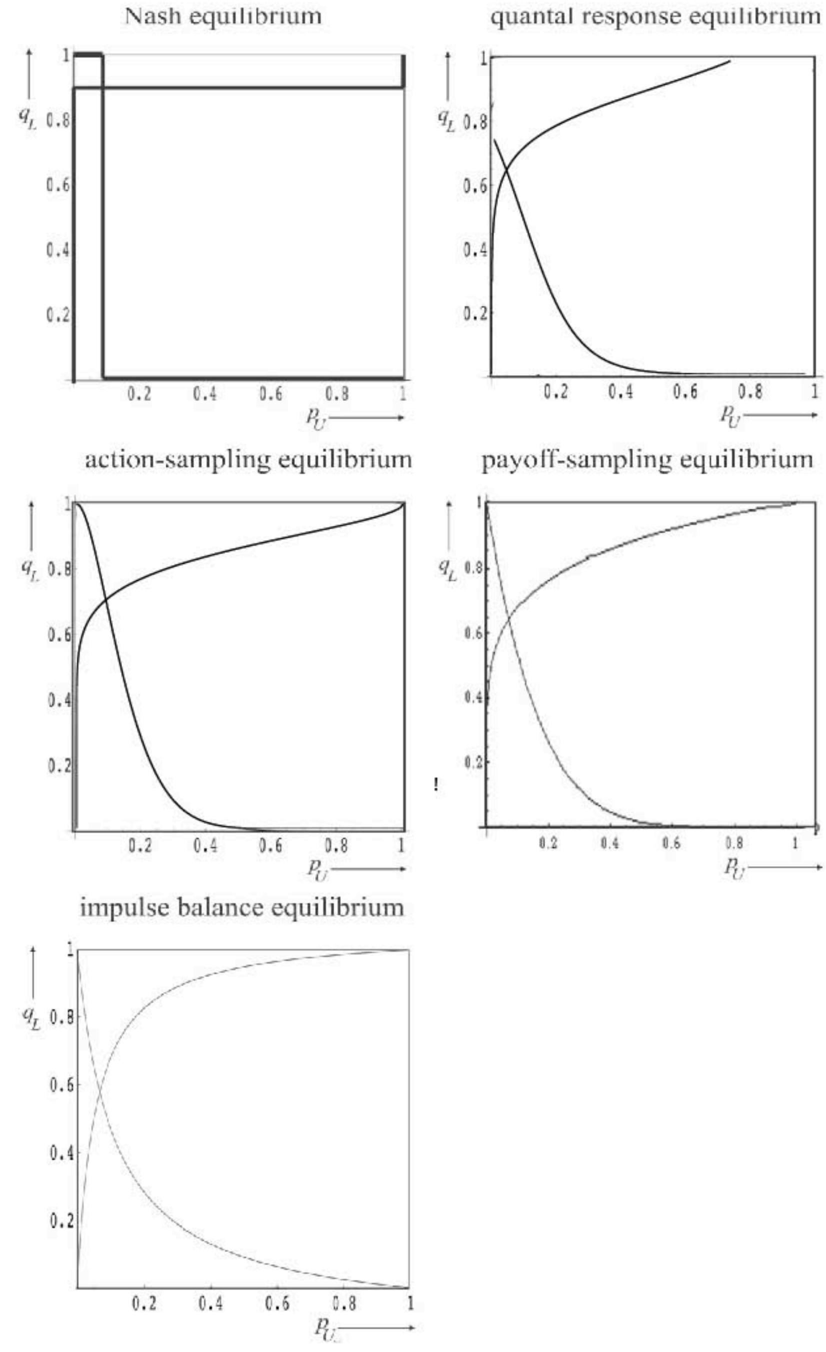

Figure 2.2: *The curves for* p_U *and* q_L *arising in the example of game 1 for each of the five concepts.*

In appendix 2.D it will be shown for each of the five stationary concepts that the curves for p_U and q_L always have a unique intersection. Therefore the stationary equilibrium exists and is uniquely determined in all five cases.

In completely mixed games the Nash equilibrium strategy of a player is independent of his own payoff. As one would intuitively expect experimental findings suggest that an increase of a players payoff in one of the four fields with all other playoffs of both players kept constant tends to increase the probability of this player's strategy used at this field. In appendix 2.E it will be shown that at equilibrium such payoff changes always increase this probability for quantal response equilibrium and for impulse balance equilibrium and, in the case of action-sampling equilibrium and payoff-sampling equilibrium, this probability is never decreased, but increased if the payoff change is big enough. The two sampling equilibria depend discontinuously on payoffs.

2.2.2. Nash Equilibrium

In the case of Nash equilibrium the curves for p_U and q_L are the graphs of the best reply correspondences for the two players (see figure 2.2).

In Nash equilibrium the choice probabilities are as follows:

(2.1) $$p_U = \frac{d_D}{d_U + d_D}, \quad p_D = \frac{d_U}{d_U + d_D}, \qquad q_L = \frac{c_R}{c_L + c_R}, \qquad q_R = \frac{c_L}{c_L + c_R}$$

The choice probabilities of a player in Nash equilibrium are independent of his own payoff. They are entirely determined by the payoff differences of the other player. This is a well known counterintuitive property of Nash equilibrium.

2.2.3. Quantal Response Equilibrium

It is assumed that players choose a "quantal best response" to the strategies of the other player. They make mistakes, taking the mistakes of the other player into account.

Let $E_U(q)$ and $E_D(q)$ be player 1's expected payoff for U and D, resp., against a strategy q of player 2. Similarly $E_L(p)$ and $E_R(p)$ are player 2's expected payoffs for L and R, resp., against a strategy p of player 1.

In quantal response equilibrium the curves for p_U and q_L are as follows.

(2.2) $$p_U = \frac{e^{\lambda E_U(q)}}{e^{\lambda E_U(q)} + e^{\lambda E_D(q)}}, \qquad q_L = \frac{e^{\lambda E_L(p)}}{e^{\lambda E_L(p)} + e^{\lambda E_R(p)}}$$

These equations yield a simultaneous equation system, which determines the choice probabilities as functions of λ. For our data λ=1.05 is the best fitting overall estimate. This value of λ minimizes the sum of mean squared distances from the actually observed relative choice frequencies for the 12 experimental games. This measure of predictive success will be explained in section 2.4.2.

The best response structure of a 2-person game is a pair of mappings (α,β). The mapping α maps the strategies q of player 2 to player 1's set $\alpha(q)$ of pure best responses to q and the mapping β maps the mixed strategies p of player 1 to the set β(p) of player 2's pure best responses to p. Nash equilibrium depends only on the best response structure of the game. However, quantal response equilibria with the same parameter λ can be different for two games with the same best response structure. If all payoffs of a 2x2-game are multiplied by the same positive factor x the best response structure remains unchanged, but quantal response equilibrium for a fixed parameter λ does change. The multiplication of all payoffs by x has the same effect as not changing payoffs and replacing λ by $\lambda' = \lambda x$.

Suppose that the payoffs are changed by adding a constant r to all payoffs of player 1 in row R of figure 2.1 and leaving everything else unchanged. Let $E'_U(q)$ and $E'_D(q)$ be the new payoffs for U and D in the new game obtained in this way. We have

(2.3) $$E'_U(q) = E_U(q) + q_R r, \quad E'_D(q) = E_D(q) + q_R r$$

This means that the equation for p_U in the new game can be simplified by dividing numerator and denominator by the common factor $e^{q_R r}$. Therefore the equations for p_U and p_D do not really change in the transition to the new game. The same argument can be applied to the case that a constant is added to player 1's payoff in the column L or players 2's payoff in one of the two rows. We can conclude that such additive changes do not have any effect on the quantal response equilibrium, even if it does not depend on the best response structure alone.

2.2.4. Action-sampling Equilibrium

In the stationary state described by p_U, p_D, q_L, and q_R player 1 takes a sample of n choices L or R and optimizes against this sample. Player 2 behaves analogously. This concept describes a stationary state of two large populations of players 1 and 2. Every member takes a sample of n past decisions of players on the other side and optimizes against it. More precisely he chooses his best response if this is uniquely determined and plays his mixed strategy (½,½) if both pure strategies are best responses. The action-sampling equilibrium is a stationary state of this system. Here, too, p_U, p_D, p_L *and* p_R are stationary probabilities of *U, D, R* and *L*. Consider two specific players 1 and 2 in both populations. Let k be the number of L's in player 1's sample and let m be the number of D's in player 2's sample. Then player's 1 and 2 will play as follows:

player 1 plays *U, D,* (½,½) for $kc_L > (n-k)c_R$, $kc_L < (n-k)c_R$, $kc_L = (n-k)c_R$b, respectively player 2 plays *L, R,* (½,½) for $md_D > (n-m)d_U$, $md_D < (n-m)d_U$, $md_D = (n-m)d_U$, respectively

Instead of of $kc_L > (n-k)c_R$ we also can write

$$\frac{k}{n} > \frac{c_R}{c_L + c_R} \tag{2.4}$$

Let $\alpha_U(k)$ be the probability of player 1 choosing U for k. and $\alpha_L(m)$ be the probability of player 2 choosing L for m.

It can be seen immediately that we have

$$\alpha_U(k) = \begin{cases} 1 \;\; for \;\; \frac{k}{n} > \frac{c_R}{c_L + c_R} \\ \frac{1}{2} \;\; for \;\; \frac{k}{n} = \frac{c_R}{c_L + c_R}, \\ 0 \qquad\qquad else \end{cases} \qquad \alpha_L(m) = \begin{cases} 1 \;\; for \;\; \frac{m}{n} > \frac{d_U}{d_U + d_D} \\ \frac{1}{2} \;\; for \;\; \frac{m}{n} = \frac{d_U}{d_U + d_D} \\ 0 \qquad\qquad else \end{cases} \tag{2.5}$$

L is played with the probability q_L. Accordingly the number k of L's in player 1's sample is binomially distributed. An analogous statement holds for the number of D's in player 2's sample. One obtains the following equations for p_U and q_L.

(2.6) $$p_U = \sum_{k=0}^{n} \binom{n}{k} q_L^k (1-q_L)^{n-k} \alpha_U(k), \qquad q_L = \sum_{m=0}^{n} \binom{n}{m} (1-p_U)^m p_U^{\,n-m} \alpha_L(m)$$

These equations describe the curves for p_U and q_L explained in II.A.

Remarks: The functions $\alpha_U(k)$ and $\alpha_L(m)$ depend only on the payoff differences c_L, c_R, d_U and d_D. Therefore the concept of action-sampling equilibrium depends only on the best response structure.

The curves for p_U and q_L are differentiable with respect to q_L and p_U, resp., for given payoff differences c_L, c_R, d_U, and d_D however the two curves do not depend continuously on these payoff differences. If for example $c_R/(c_L+c_R)$ is equal to ½ a small change of either c_L or c_R results in a jump of $\alpha_U(k)$.

The concept of action-sampling equilibrium can easily be extended to general normal form games. In a stationary situation a player takes a sample of 7 observations of combinations of pure strategies for the other players and then optimizes against this sample. In the case of several best responses each of them is chosen with equal probability.

2.2.5. Payoff-sampling Equilibrium

The basic idea of payoff-sampling equilibrium has been explained in the introduction. Osborne and Rubinstein (1998) did not specify the probabilities of both strategies in the case that the payoff sums for the two samples are equal. In order to obtain a unique prediction we added the rule that in this case each pure strategy is chosen with probability ½.

As before p_U, p_D, q_L and q_R denote the stationary probability for the corresponding pure strategies.

Let n be the sample size and k_U and k_D be the number of L's in a player 1's sample for U and D respectively. Similarly let m_L and m_R the number of U's in a player 2's sample for L and R respectively.

Player 1's sums of payoffs H_U and H_D in the samples for U and D ,resp., are as follows

(2.7) $$H_U = k_U(a_L+c_L) + (n-k_U)a_R., \qquad H_D = k_D a_L + (n-k_D)(a_R+c_R)$$

In the same way player 2's sum of payoffs in the samples for L and R are given by

(2.8) $H_L = m_U b_U + (n\text{-}m_L)\ (b_U + d_D)$, $\qquad H_D = m_R\ (b_U + d_U) + (n\text{-}m_R)\ b_D$

Player 1's probability $\beta_U(k_U, k_D)$ of playing U if k_U and k_D are the numbers of L's in his sample as well as the probability $\gamma\ (m_L, m_R)$ of player 2 playing L if she observes the numbers m_L and m_R of U's in her samples for L and R are described below.

(2.9) $$\beta(k_U, k_D) = \begin{cases} 1 & for \quad H_U > H_D \\ \frac{1}{2} & for \quad H_U = H_D \\ 0 & else \end{cases}, \qquad \gamma(m_L, m_R) = \begin{cases} 1 & for\ H_L > H_R \\ \frac{1}{2} & for\ H_L = H_R \\ 0 & else \end{cases}$$

Since k_U and k_D as well as m_L and m_R are binominal distributed we have

(2.10) $$p_U = \sum_{k_U=0}^{n} \sum_{k_D=0}^{n} \binom{n}{k_U} \binom{n}{k_D} q_L^{k_U+k_D} (1-q_L)^{2n-k_U-k_D} \quad \beta\ (k_U, k_D)$$

(2.11) $$q_L = \sum_{m_L=0}^{n} \sum_{m_R=0}^{n} \binom{n}{m_L} \binom{n}{m_R} (1-p_U)^{m_L+m_R}\ p_U^{\ 2n-m_L-m_R} \gamma(m_L, m_R)$$

The curves for p_U and q_L.in the case of payoff-sampling equilibrium are represented by these two equations.

Remarks: The operation of adding a constant to player 1's payoffs in the column for R may change $\beta(k_U, k_D)$ and therefore the first of the two equations. Similarly adding a constant to player 2's payoffs in the row for U may change the second equation. For this reason payoff-sampling equilibrium is not invariant with respect to these operations.

As in the case of action-sampling equilibrium the curves for p_U and q_L are differentiable with respect to q_L and p_U, resp., but not continuous with respect to a small change of one payoff in the payoff-matrix for the concerning player.

2.2.6. Impulse Balance Equilibrium

As has been already explained in the introduction, impulse balance theory is not applied to the original game, but to a transformed game, in which losses with respect to a natural aspiration level get twice the weight as gains above this level.

The natural aspiration level for a player is his pure strategy maximin value, or in other words, the maximum of the lowest payoff he may obtain for using one of his pure strategies. Define:

(2.12) $$s_1 = \max\left[\min\left(a_L + c_L, a_R\right), \min\left(a_L, a_R + c_R\right)\right]$$

(2.13) $$s_2 = \max\left[\min\left(b_U, b_D + d_D\right), \min\left(b_U + d_U, b_D\right)\right]$$

From now we shall refer to s_1 and s_2 as the pure strategy maximin payoffs or shortly the security levels of players 1 and 2, respectively.

In the following it will be argued that the security level of a player is her second lowest payoff. It may happen that the lowest payoff is obtained at more than one of the four fields. In this case there is no difference between the second lowest payoff and the lowest payoff. The words "second lowest payoff" will always be understood this way.

In a completely mixed 2x2-game no pure strategy can dominate an other one (see figure 2.1). Therefore the lowest and the second lowest payoff of player 1 cannot appear in the same row. An analogous statement holds for player 2.

The second lowest payoff is always at least obtained if it is the lowest one. Otherwise the lowest payoff can be avoided by not choosing the pure strategy which may yield it. Thereby the second lowest payoff is secured. It is also clear that one cannot secure more than that by the use of a pure strategy.

The security level can be enforced, no matter what the other player does. Therefore it is natural to look at a lower payoff as a failure and its difference to the security level as a loss. It makes no sense to be satisfied with less than one could have got for sure. Loss aversion is a well known behavioural concept, used for example, in prospect theory

(Kahnemann and Tversky 1979). In the case of a payoff below the security level, there are two reasons for thinking that one should have chosen the other strategy. The first reason is that the other strategy would have yielded a higher payoff. The second reason is that the loss should be avoided. The loss counts as a part of the foregone payoff and in addition to this it counts once more by its quality of being a loss rather than merely a forgone gain.

An earlier formulation of impulse balance theory concerned an auction situation in which losses could occur only in a connection with bids appearing to be too high ex post (Selten, Abbink and Cox 2005). Therefore in the case of a loss, the decision maker experienced a downward impulse and a loss impulse. In 2x2-games losses may occur for choices of one strategy or the other depending on the structure of the game. Thus in game 1 (see figure 2.5 in 2.3.2.), player 1 at (*U*,*R*) experiences a loss of 9 and a forgone payoff of 10. Therefore a loss impulse of 9 is added to the ordinary impulse of 10 from *U* to *D* at (*U*,*R*). At (*D*,*L*) player 1 receives only an ordinary impulse of 1 from *D* to *U*.

As we shall see, the combination of ordinary impulses and loss impulses is automatically taken care of if impulses from one pure strategy to another are computed in a transformed game in which losses receive double the weight of gains. We construct this transformed game by leaving player *i*'s payoffs below and at s_i unchanged and by reducing the surplus over s_i of higher payoffs by the factor ½. Figure 2.3 shows the **impulse balance transformation** for the example of experimental game 3 (see figure 2.5 in 2.3.2.).

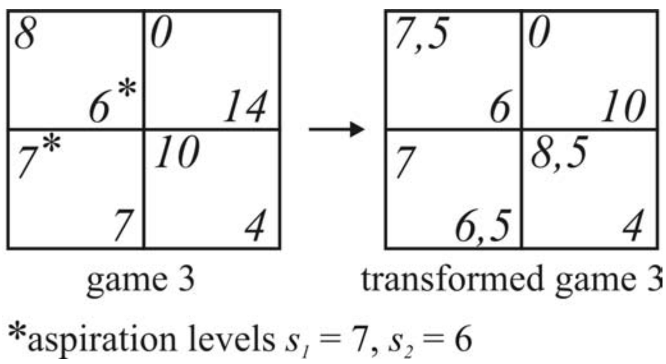

Figure 2.3: *Impulse Balance Transformation for the example of experimental game 3.*

The payoff differences in the transformed game corresponding to c_L, c_R, d_U, d_D are denoted by c_L^*, c_R^*, d_U^*, d_D^*. If after a play player *i* could have obtained a higher payoff by the choice of his other strategy, he receives an impulse in the direction of his other strategy. The size of this impulse is the forgone payoff in the transformed game. If for example player 1 chooses *U* and the other player chooses R, then player 1 receives an impulse of c_R^* = 8.5 in the direction of *D*. A player receives no impulse if the payoff for the strategy he did not choose was lower than the one he obtained. Figure 2.4 shows the impulses in the direction to the strategy not chosen, similar to a payoff table.

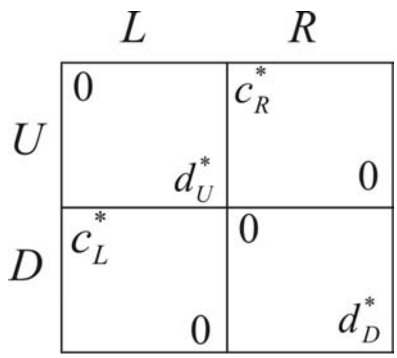

Figure 2.4: *Impulse in the direction of the strategy not chosen.*

It can now be seen without difficulty that impulses in the transformed game automatically combine ordinary impulses and loss impulses in the original game. In the case of a payoff below s_i the loss part of an impulse is fully counted and a possible forgone-gain part is reduced by the factor ½, just like an impulse in the case of a payoff above the security level. Half of the fully counted loss corresponds to the loss impulse.

Impulse balance equilibrium requires that player 1's expected impulse from *U* to *D* is equal to his expected impulse from *D* to *U*. Similarly player 2's expected impulse from L to R must be equal to her expected impulse from *R* to *L*. This yields the following two **impulse balance equations**:

(2.14) $$p_U q_R c_R^* = p_D q_L c_L^*, \qquad p_U q_L d_U^* = p_D q_R d_D^*$$

The left hand side of the first impulse balance equation is player 1's expected impulse from *U* to *D* and the right side is player 1's expected impulse from *D* to *U*. If the left hand side is greater than the right hand side then player 1 receives stronger impulses from *R* to *D* and this will decrease q_R and increase q_L. This creates a tendency in the direction of impulse balance. An analogues interpretation can be given to the second impulse balance equation. Of course this is only a heuristic argument. In this paper we do not want to explore the dynamics of impulse balance equilibrium.

The impulse balance equations yield the following equations of the curves for p_U and q_U

(2.15) $$p_U = \frac{q_L c_L^*}{q_L c_L^* + (1-q_L)c_R^*}, \qquad q_L = \frac{(1-p_U)d_D^*}{p_U d_U^* + (1-p_U)d_D^*}$$

In section 2.E4 of appendix 2.E explicit formulas will be derived for the coordinates of the intersection (p_U, q_L) of the two curves. Define $c = c_L^* / c_R^*$ and $d = d_U^* / d_D^*$: It will be shown in 2.E4 that at the intersection we have $p_U = \sqrt{c}/(\sqrt{c} + \sqrt{d})$ and $q_L = 1/(1 + \sqrt{cd})$.

In appendix 2.F a possibility of generalizing impulse balance equilibrium to *n*-person normal form games will be shortly sketched. Even if for the substance of this paper no such generalization is needed it is maybe of interest to see in which way it could be achieved.

2.3. Experimental Design

2.3.1. Procedure

The experimental data were obtained in 54 sessions with 16 subjects each and 864 altogether. The subjects were students of the University of Bonn, mainly majoring in economics or law. The experiments were run in the Bonn laboratory of experimental economics. The computer program was based on the toolbox RatImage developed by Abbink and Sadrieh (1995). Only one game was played in each session.

At the beginning of a session oral and written instructions were given to the subjects. The written instructions (in German) are shown in appendix 2.B. The subjects were informed about the game matrix including the payoffs of both players. They were told that they would interact with randomly changing opponents and always be in the same player role over 200 periods. Actually in each session there were two independent subject groups with four participants in the role of player 1 and four participants in the role of player 2. The players played against randomly chosen opponents but only within their independent group. They were not informed about the fact that there are two groups. We did not lie to them but conveyed the impression that they interact directly or indirectly with 15 other players.

After the instruction the participants were sitting in separate cubicals and made their decisions by mouse click. The decisions in a play were made without any information about the choices of the other players. After each of the 200 plays they received feedback about the other player's choice and their payoff, the period number and their cumulative payoff. No limit was imposed on the decision time. The subjects were not permitted to take notes of any kind about their playing experience. They were also not permitted to talk to each other during the experiment and they had no opportunity to see the screens of other participants. After each experiment, participants had to fill in a questionnaire. However, no use of the questionnaire data is made in this paper. Therefore the questionnaire is not shown here.

Each participants received 5 € and in addition to this a money payoff proportional to his or her game payoff accumulated over the 200 periods. The exchange rate was 1.6 €-Cent per payoff point. An experimental session took 1.5 to 2 hours and the average earning of a subject was about 24 € including the show up fee.

In some sessions a digit span test DAVIS (1931), DELLA SALA ET AL. (1999) preceded the game playing. This test is designed to measure the short time memory size. However we shall make no use of the data collected by this test in this paper. Therefore the details of the digit span test will not be explained here.

2.3.2. Experimental Games

Figure 2.5 shows the twelve games used in our Experiment. The constant sum games are shown on the left side of figure 2.5 and the non-constant sum games on the right side of figure 2.5. The non-constant sum game right next to a constant sum game in the figure 2.5 has the same best response structure. We say that the two games form a pair. The non-constant sum game in a pair is derived from the constant sum game in this pair by adding the same constant to player 1's payoff in the column for *R* and 2's payoff in the row for *U*. It is clear that this does not change the best response structure.

Nash equilibrium and action-sampling equilibrium depend only on the best response structure and therefore yield the same predictions for both games in a pair. In section 2.2.2. it has been explained that adding a constant to all payoffs of player 1 in a specific column or to all payoffs of player 2 in a row does not change the quantal response equilibrium, even if this concept does not depend only on the best response structure. Therefore quantal response equilibrium, too, yields the same prediction for the two games in a pair.

The games in a pair also have the same action-sampling equilibrium. A best response to a sample of pure strategies of the other player in one of the two games is also a best response to this sample in the other game. This is an immediate consequence of the fact that both games have the same best response structure.

In view of the remark at the end of section 2.2..4. one cannot expect that payoff-sampling equilibrium generates the same prediction for the games in a pair. In fact these predictions are different for all six pairs.

The determination of impulse balance equilibrium involves a transition from the original game to the transformed game. The pure strategy maximin payoff, which serves as a reference point for gains and losses may be different for the two games of the pair and even if this is not the case, the best response structures will usually be different. In fact, in all 6 cases the impulse balance equilibria are different for the two games in a pair.

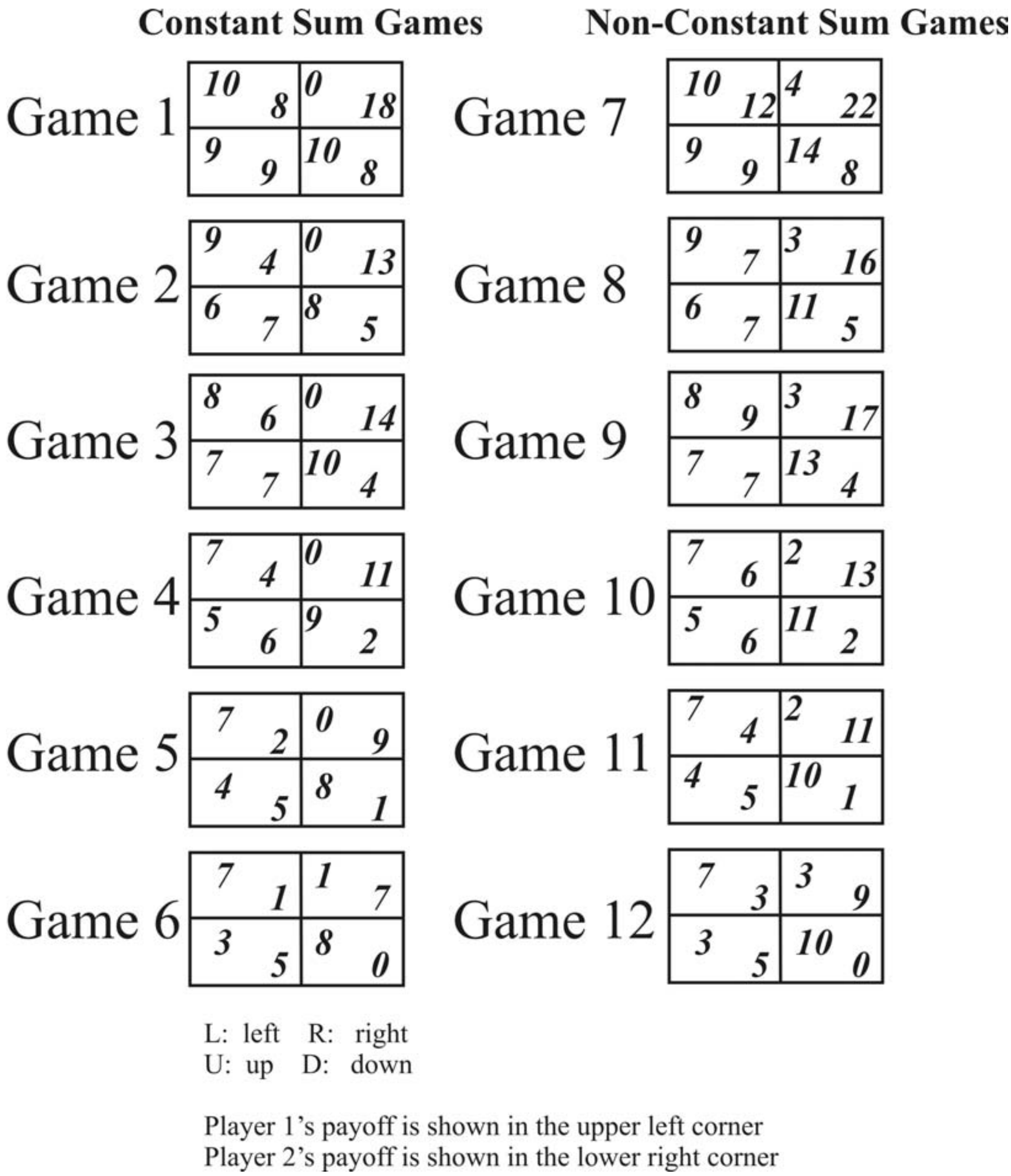

Figure 2.5: *Experimentally investigated games.*

In the selection of the experimental games we have been guided by several considerations explained in the following. Two pilot experiments were run with the games shown in figure 2.6. Game A is similar to the game played by OCHS (1995) and also by GOEREE, HOLT, CHARLES and PALFREY (2000). In the questionnaires the subjects who had played game A often reported attempts to cooperate.

Figure 2.6: *Structure of the pilot experiments.*

Even if these attempts failed they may have had an influence on the observed relative frequencies. Therefore we decided to explore constant sum games extensively. Constant sum games offer no cooperation opportunities. We wanted to contrast them with similar non-constant sum games offering some scope for cooperation.

The concepts of action-sampling equilibrium and impulse balance equilibrium have been developed on the basis of the pilot experiments with games A and B. Therefore the experimental results obtained with these games are not included in the comparison of the five theories.

The selection of the constant sum games was guided by the idea, that on the one hand a reasonably wide distribution over the parameter space should be achieved, and on the other hand the number of games should be small enough to permit a sufficiently large number of independent subject groups in every case.

The games explored here have 8 payoffs but the best response structure is characterized by two parameters. The Nash equilibrium choice probabilities p_U^N and q_L^N will serve as these two parameters in the following figure. Figure 2.7 show the six Nash equilibria for the experimental games.

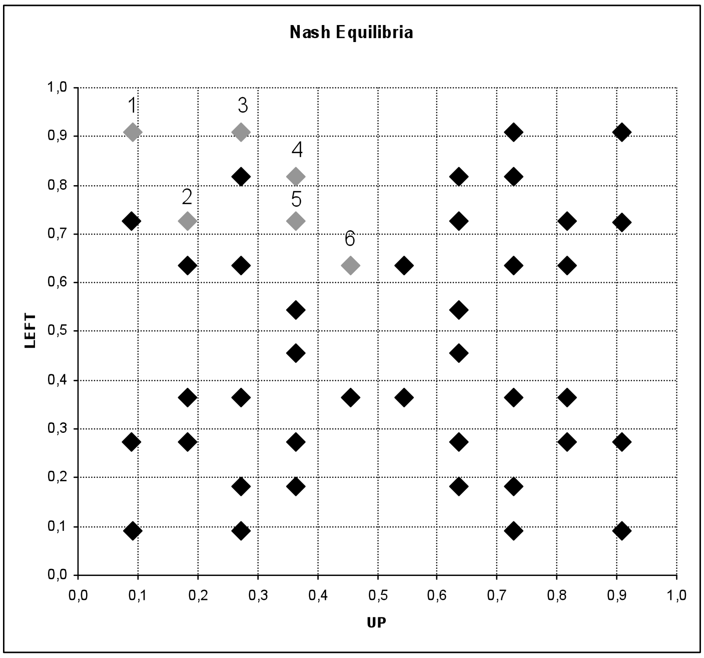

Figure 2.7: *Permutations of rows, columns, or player roles transform the 6 experimental games into 44 games with the Nash equilibria shown in the figure.*

In all six cases p_U^N is between 0 and .5 and q_L^N is between .5 and 1. Therefore only this part of the parameter space is shown in figure 7. The best reply structure remains essentially unchanged if the rows or columns or the role of both players are exchanged. Such transformations yield all the points in figure 2.7.

It can be seen, that the six games together with their automorphic transformations are widely distributed over the parameter space. However we intentionally underrepresented cases in which one of the equilibrium choice probabilities is near to .5. In our sample of 6 only game 6 has this property. In the middle of the parameter space, where both parameters are .5, every reasonable theory predicts equal probabilities for all strategies. The greater the distance from the midpoint is, the more the stationary concepts compared in this paper differ with respect to their predictions.

Since constant sum games are more basic we have run experiments with 12 independent subject groups for each of the 6 constant sum games but only 6 independent subject groups for each of the non-constant sum games.

2.4. Experimental Results

2.4.1. Predicted and Observed Relative Frequencies

We begin our descriptions of the results obtained by a number of figures showing the predictions of the five stationary concepts together with the observed overall relative frequencies for each of the experimental games. The numerical values are shown in table 2.1.

		Nash Equilibrium	Quantal response Equilibrium (λ=1.05)	Payoff-sampling Equilibrium (*n*=6)	Action-sampling Equilibrium (*n*=12)	Impulse Balance Equilibrium	Observed Average of 12 Observations
Game 1	U	0.091	0.042	0.071	0.090	0.088	0.079
	L	0.909	0.637	0.645	0.705	0.580	0.690
Game 2	U	0.182	0.154	0.184	0.193	0.172	0.217
	L	0.727	0.579	0.569	0.584	0.491	0.527
Game 3	U	0.273	0.168	0.152	0.208	0.161	0.198
	L	0.909	0.770	0.773	0.774	0.765	0.793
Game 4	U	0.364	0.275	0.285	0.302	0.259	0.286
	L	0.818	0.734	0.726	0.719	0.710	0.736
Game 5	U	0.307	0.307	0.307	0.329	0.297	0.327
	L	0.727	0.657	0.654	0.643	0.628	0.664
Game 6	U	0.455	0.417	0.427	0.426	0.400	0.445
	L	0.636	0.607	0.597	0.596	0.600	0.596
		Nash Equilibrium	Quantal response Equilibrium (λ=1.05)	Payoff-sampling Equilibrium (*n*=6)	Action-sampling Equilibrium (*n*=12)	Impulse balance Equilibrium	Observed Average of 6 Observations
Game 7	U	0.091	0.042	0.056	0.090	0.104	0.141
	L	0.909	0.637	0.691	0.705	0.634	0.564
Game 8	U	0.182	0.154	0.222	0.193	0.258	0.250
	L	0.727	0.579	0.601	0.584	0.561	0.586
Game 9	U	0.273	0.168	0.154	0.208	0.188	0.254
	L	0.909	0.770	0.767	0.774	0.764	0.827
Game 10	U	0.364	0.275	0.308	0.302	0.304	0.366
	L	0.818	0.734	0.731	0.719	0.724	0.699
Game 11	U	0.364	0.307	0.339	0.329	0.354	0.331
	L	0.727	0.657	0.651	0.643	0.646	0.652
Game 12	U	0.455	0.417	0.405	0.426	0.466	0.439
	L	0.636	0.607	0.600	0.596	0.604	0.604

Table 1.1: *Five stationary concepts together with the observed relative frequencies for each of the experimental games.*

In the first three columns of table 2.1 the theoretical values of the upper half are repeated in the lower half. This is due to the fact that Nash-equilibrium and action-sampling equilibrium depend only on the best response structure (see the remark at the end of II.D and the property of quantal response equilibrium explained at the end of 2.2.3)

In figures 2.8 and 2.14 in Appendix 2.A we show cutouts of the whole parameter space with predictions and observed averages for all 12 games. Apart from the fact that the Nash equilibrium of game 2 is nearer to (.5,.5) than that of game 3, the games 1-6 are the farther from the middle of the parameter space the lower is their order in the numbering. One can see that the discrimination between the concepts tends to be worse for games nearer to the middle of the parameter space.

The predictions of impulse balance equilibrium, payoff-sampling equilibrium, action-sampling equilibrium and quantal response equilibrium tend to be near to each other. Therefore random fluctuations make the comparisons between these four concepts difficult. The cutouts for games 7 to 12 show a similar picture. However, contrary to what happens in other games, in game 9 Nash equilibrium is slightly nearer to the observed averages than the other three concepts. As we shall see in 2.4..6 our data suggest that the results of game 9 are influenced by especially large random fluctuations.

As has been explained in 2.3.2 each of the three concepts, Nash equilibrium, action-sampling equilibrium as well as quantal response equilibrium yields the same prediction for the two games in a pair. This is not the case for payoff-sampling equilibrium and impulse balance equilibrium.

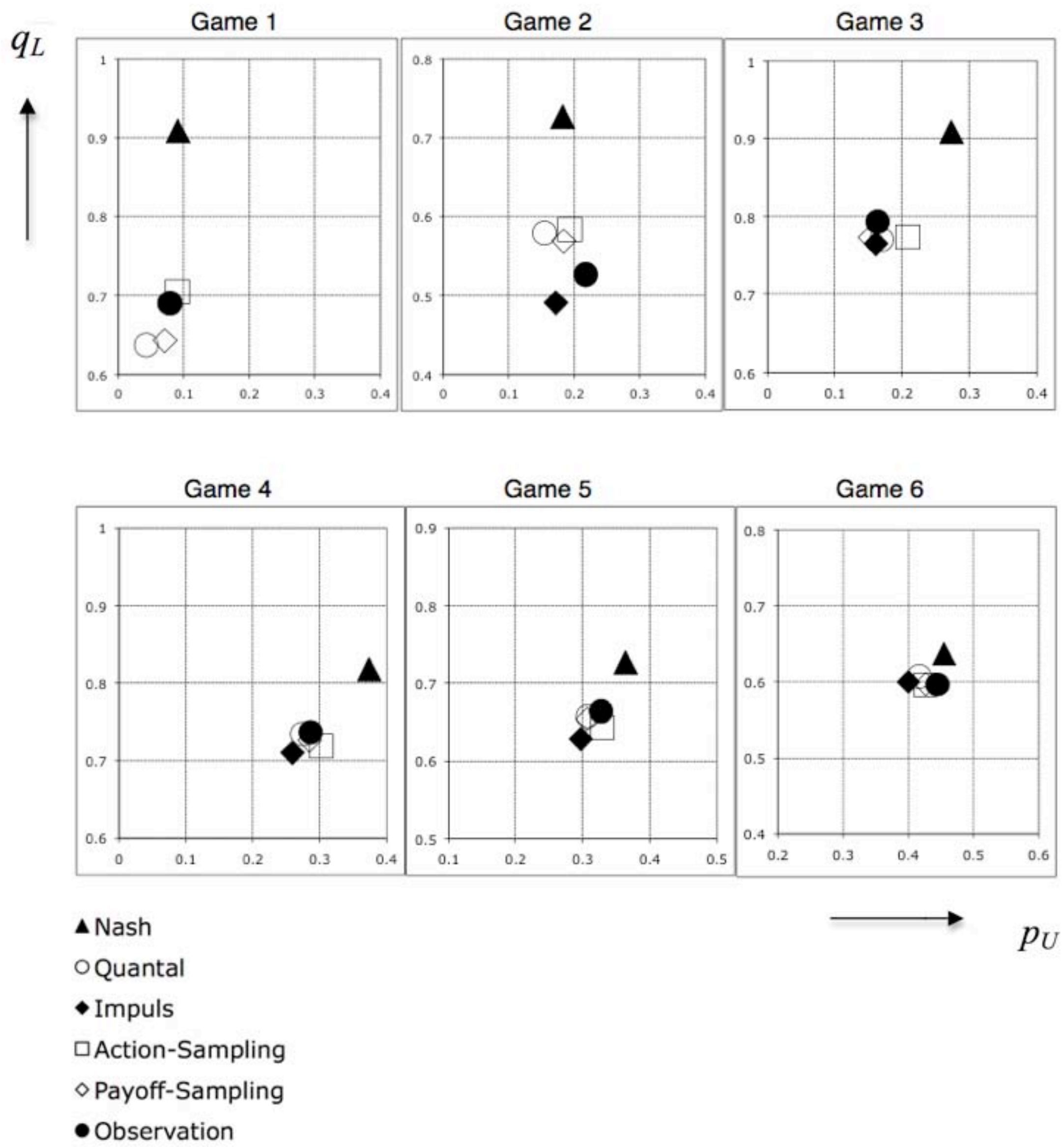

Figure 2.8: *Visualization of the theoretical equilibria and the observed average in the constant sum games.*

2.4.2. The Measure of Predictive Success

We look at the five theories compared in this paper as predictions of the relative frequencies of U and L in an independent subject group playing one of the games 1 to 12. We do not want to assert that a player uses the same mixed strategy in all 200 periods of a session and we also do not assume that all players in the same role always behave in the same way. Presumably the players are engaged in complex learning processes which differ from person to person. Nevertheless such behaviour may result in frequencies of U and L which can be predicted reasonably well by stationary concepts. It is important to know how well observed relative frequencies can be explained without going into the details of stochastic learning models.

For a theory predicting a point in an Euclidian space the squared distance of theoretical and observed values is a reasonable measure of predictive success, in the sense that the predicted success is the greater the smaller this distance is. In the following we want to explain how this measure is applied to our data. Each Game *i* with *i*=1,…,12 has been played by s_i independent subject groups with s_i=12 for *i*= 1,…,6 and s_i=6 for *i*=7,…,12

We use the index *j* with *j*=1,…,s_i for the subject groups. Let and f_{iUj} and f_{iLj} be the relative frequencies of *U* and *L* in the *j*-th independent subject group playing game *i*. Consider a prediction p_U and q_L for these relative frequencies then

(2.16) $$Q_{ij} = (f_{iUj}-p_U)^2+(f_{iLj}-q_L)^2$$

is the squared distance of the *j*-th observation for game *i* from the prediction for game *i*. The **mean squared distance** for the data of this game *i* from (p_U,p_L) is as follows

(2.17) $$Q_i = \frac{1}{s_i}\sum_{j=1}^{s} Q_{ij}$$

We shall look at the overall predicted success but also at the predicted success of the constant sum games 1 to 6 and the non-constant sum games 7 to 12 separately. Define:

(2.18) $$Q_C = \frac{1}{6}\sum_{i=1}^{6} Q_i, \qquad Q_N = \frac{1}{6}\sum_{i=7}^{12} Q_i, \qquad Q = \frac{1}{12}\sum_{i=1}^{12} Q_i$$

The indices *C* and *N* stand for constant sum and non-constant sum games. The **mean squared distances** Q_C, Q_N and *Q* will be the basis of our comparison of the five theories.

For every game *i* let f_{iU} and f_{iL} be the mean values of f_{iUj} and f_{iLj} with *j*=1,…,*s*:

(2.19) $$f_{iU} = \frac{1}{s_i}\sum_{i=1}^{s} f_{iUj} \quad \text{for } i=1,\ldots 12, \qquad f_{iL} = \frac{1}{s_i}\sum_{i=1}^{s} f_{iLj} \quad \text{for } i=1,\ldots 12$$

The expression

$$(2.20) \qquad S_i = \frac{1}{s_i}\sum_{j=1}^{n}(f_{iUj} - f_{iU})^2 + (f_{iLj} - f_{iL})^2 \qquad \text{for } i=1,\ldots 12$$

is the **sampling variance** of game *i* and

$$(2.21) \qquad T_i = (f_{iU} - p_U)^2 + (f_{iL} - q_L)^2 \qquad \text{for } i=1,\ldots 12$$

is the **theory specific component** of the mean squared distance. The mean squared distance for a game can be split into these two components:

$$(2.22) \qquad Q_i=S_i+T_i \qquad \text{for } i=1,\ldots 12$$

Define

$$(2.23) \qquad S_C = \frac{1}{6}\sum_{i=1}^{6} S_i, \qquad S_N = \frac{1}{6}\sum_{i=7}^{12} S_i, \qquad S = \frac{1}{12}\sum_{i=1}^{12} S_i$$

$$(2.24) \qquad T_C = \frac{1}{6}\sum_{i=1}^{6} T_i, \qquad T_N = \frac{1}{6}\sum_{i=7}^{12} T_i, \qquad T = \frac{1}{12}\sum_{i=1}^{12} T_i$$

The mean squared distances Q_C, Q_N and Q can also be split into two components

$$(2.25) \qquad Q_C=S_C+T_C, \qquad Q_N=S_N+T_N, \qquad Q=S+T$$

Note that each game receives equal weight in *Q*, *S*, and *T* in spite of the fact that there are twice as many observations for each constant-sum game than for each non-constant sum game. This conforms to the goal of obtaining an adequate judgment of the overall goodness of fit for the 12 games.

Since the mean sampling variances S_C, S_N and S do not depend on the theory under consideration it does not really matter whether the comparison of theories is based on Q_C, Q_N and Q or alternatively on T_C, T_N and T. However, the mean squared distances Q_C, Q_N and Q are more natural measures of predictive success. A high sampling variance limits the accuracy of prediction even if the theory specific component is very small. Therefore the mean squared distance of the individual observations from the theory is more adequate as a measure of predictive success.

For no theory the mean squared distance Q can be smaller than S. The sampling variance S is an unavoidable part of Q.

2.4.3. Comparison of Sample Sizes for Action-sampling Equilibrium

Originally action-sampling equilibrium with the sample size of 7 had been considered as a theory to be compared with the data, since this sample size finds some admittedly weak support in the psychological literature (Miller 1956). The sample size 7 seems to be connected to the average capacity of short time memory. However, it is not really clear, whether this is relevant for the behavior in our experiments. Therefore another sample size could have yielded a better fit for our data.

In order to check this we compared the predictive success for action-sampling equilibria with different sample sizes.

Figure 2.9 shows the overall mean squared distances Q for the action-sampling equilibria with the sample sizes n=1,…,15. It can be seen immediately that the average squared distance is smallest for n=12. This means that the best fit to the data is obtained with sample size 12. In our comparison of the five concepts we therefore do not have to consider other sample sizes for action-sampling equilibrium.

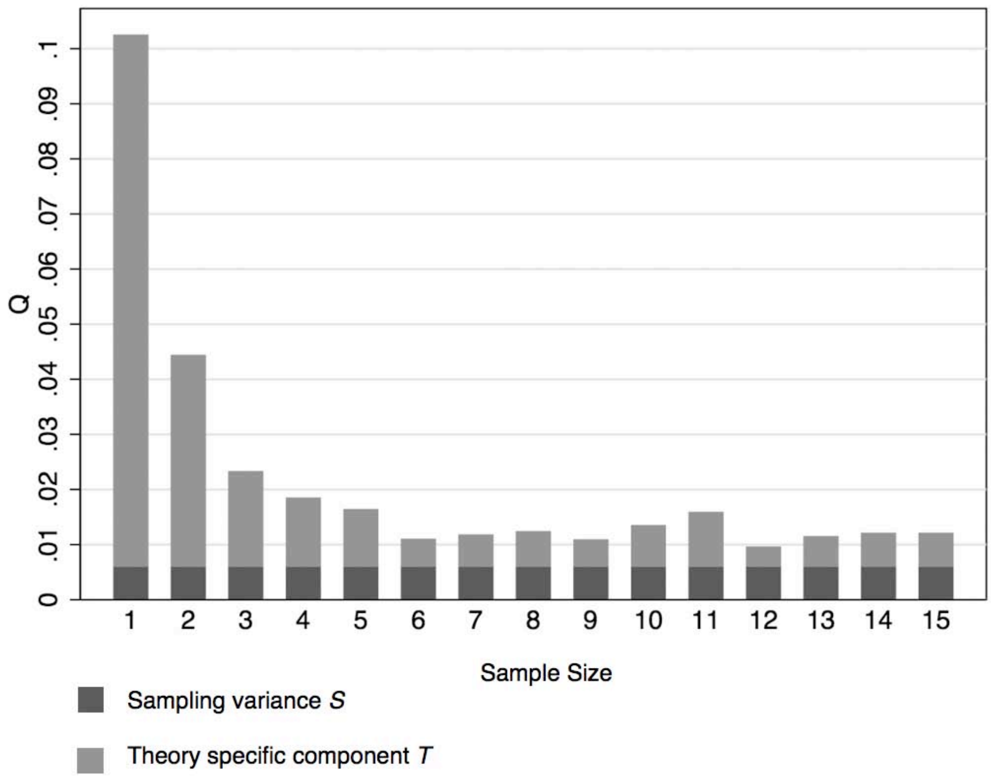

Figure2. 9: *Overall mean squared distances* Q *for the action-sampling equilibria with different sample sizes.*

The figure also shows the mean sampling variance in grey. It can be seen that for the sample size 12 the mean squared distance Q is much nearer to its unavoidable part S than for all other sample sizes.

2.4.4. Comparison of Sample Sizes for Payoff-sampling Equilibrium

Figure 2.10 shows the overall mean squared distances Q for the payoff-sampling equilibria with the sample sizes n=1,...,10. It can be seen that the sample size 6 yields the best fit to the data. Therefore our comparison of the five theories is based on the sample size 6 for payoff-sampling equilibrium.

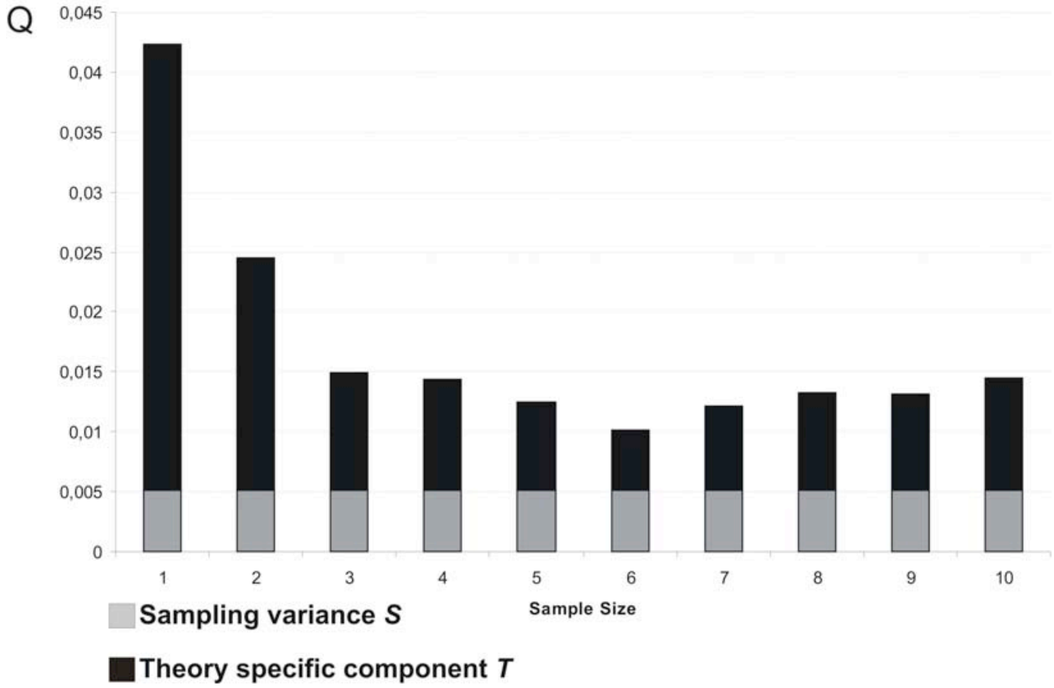

Figure 2.10: *Overall mean squared distances* Q *for the payoff-sampling equilibria with different sample sizes.*

2.4.5. Original Versus Transformed Games

The basic idea of impulse balance is applied to the transformed game rather than the original one. This idea could also be applied directly to the original game. As we shall see later, the application to the transformed game yields a better fit to the data. This was already true for the pilot study on games A and B. We therefore decided to test impulse balance theory in the form described in section 2.2.5. However, it is of interest to examine the question how the direct application compares to the concept of impulse balance equilibrium proposed here.

It could be the case that not only the predictive power of impulse balance equilibrium but also that of other concepts is increased by applying them to the transformed game rather than to the original one.

We shall examine this question for Nash equilibrium, action-sampling equilibrium and payoff-sampling equilibrium. Contrary to Nash equilibrium, quantal response equilibrium, action-sampling equilibrium and payoff-sampling fit the data quite well. It is therefore of special interest to explore whether a better fit could be obtained by applying these two concepts to the transformed game rather than the original one. If in this way one obtained a better fitting version of one of the two concepts, then this version should be compared with the other three theories.

We did not examine, what happens, if quantal response equilibrium is applied to the transformed game rather the original one.

Figure 2.11 shows the overall mean squared distances for Nash equilibrium, action-sampling equilibrium, payoff-sampling equilibrium and impulse balance equilibrium applied directly to the original game or to the transformed game. It can be seen that only impulse balance theory profits from being applied to the transformed game whereas the other three theories do not gain by being modified in this way.

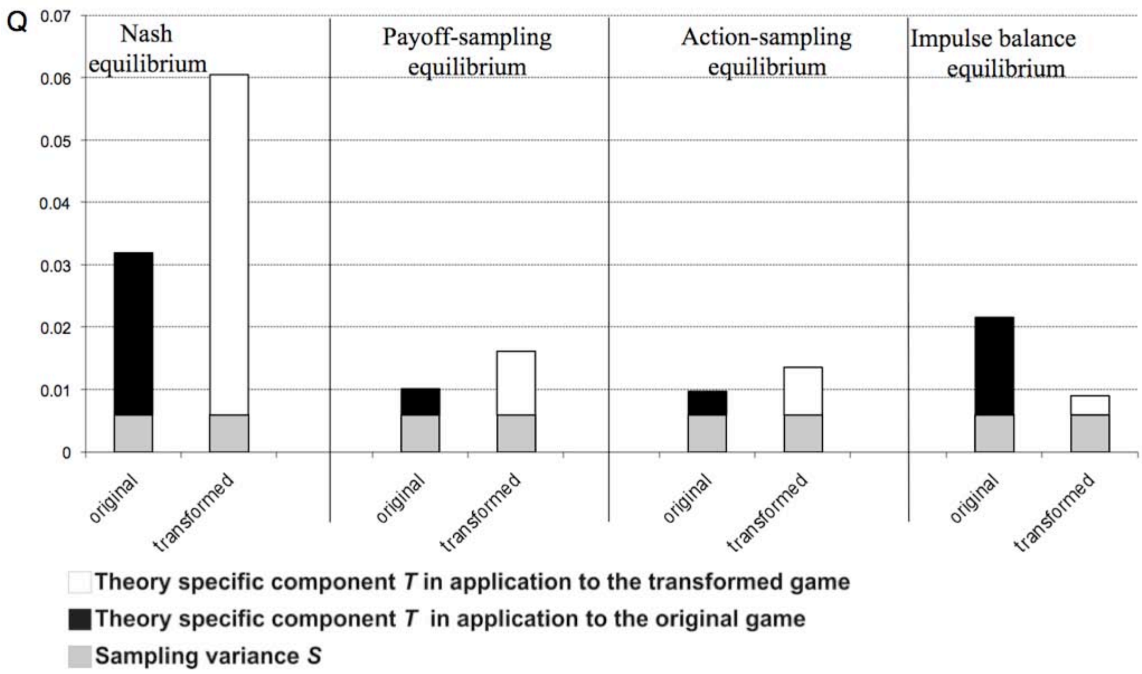

Figure 2.11: *Advantages and disadvantages of applying a concept to the transformed game rather the original one.*

The figure also shows the decomposition of the mean squared distance Q into the sampling variance S (grey) and the theory specific component T (black and white resp.). The difference between the applications to the original game and the transformed one are even more dramatic in the case of impulse balance equilibrium, if one looks at the theory specific components instead of the mean squared distance.

In view of figure 2.7 it seems to be justified not to add the modifications of Nash equilibrium, action-sampling equilibrium and payoff-sampling equilibrium to the list of the five theories which are the main focus in this paper.

As we shall see in the next section impulse balance equilibrium fits our data best. Figure 2.15 shows that this success is not mainly due to the use of the transformed game. Otherwise the predictive success of other concepts should be improved as well if they are applied to the transformed game rather than the original one. This is not the case.

2.4.6. Comparison of the Five Theories

Table 2.2 shows the mean squared distances of the five theories for the twelve games separately. It also contains the sampling variance for each game.

	Nash equilibrium	Quantal response equilibrium	Payoff-sampling equilibrium	Action-sampling equilibrium	Impulse balance equilibrium	Sampling variance
Game 1	0.0572	0.0133	0.0112	0.0103	0.0213	0.00909
Game 2	0.0483	0.0136	0.0098	0.0164	0.0102	0.00693
Game 3	0.0321	0.0058	0.0057	0.0087	0.0073	0.00523
Game 4	0.0169	0.0041	0.0041	0.0072	0.0054	0.00403
Game 5	0.0149	0.0100	0.0100	0.0115	0.0117	0.00953
Game 6	0.0042	0.0034	0.0028	0.0027	0.0045	0.00246
Game 7	0.1237	0.0171	0.0253	0.0189	0.0081	0.00178
Game 8	0.0298	0.0146	0.0063	0.0106	0.0060	0.00531
Game 9	0.0212	0.0248	0.0276	0.0332	0.0224	0.01409
Game 10	0.0208	0.0160	0.0109	0.0134	0.0111	0.00665
Game 11	0.0098	0.0037	0.0032	0.0059	0.0036	0.00307
Game 12	0.0045	0.0037	0.0047	0.0033	0.0039	0.00317

Table 2.2: *Squared distances of the five theories.*

Figure 2.12 shows the overall mean squared distances Q for the five theories compared in this paper. It can be seen that there is a order of success: Impulse balance equilibrium, action-sampling equilibrium, payoff-sampling equilibrium, quantal response equilibrium and Nash equilibrium. The figure also shows the sampling variance S in grey and the theory specific components in black.

The sampling variance for game 9 is much greater than for other games. This is probably the reason for the unusual constellation of the cutout for game 9 in figure 2.14, Appendix 2.A.

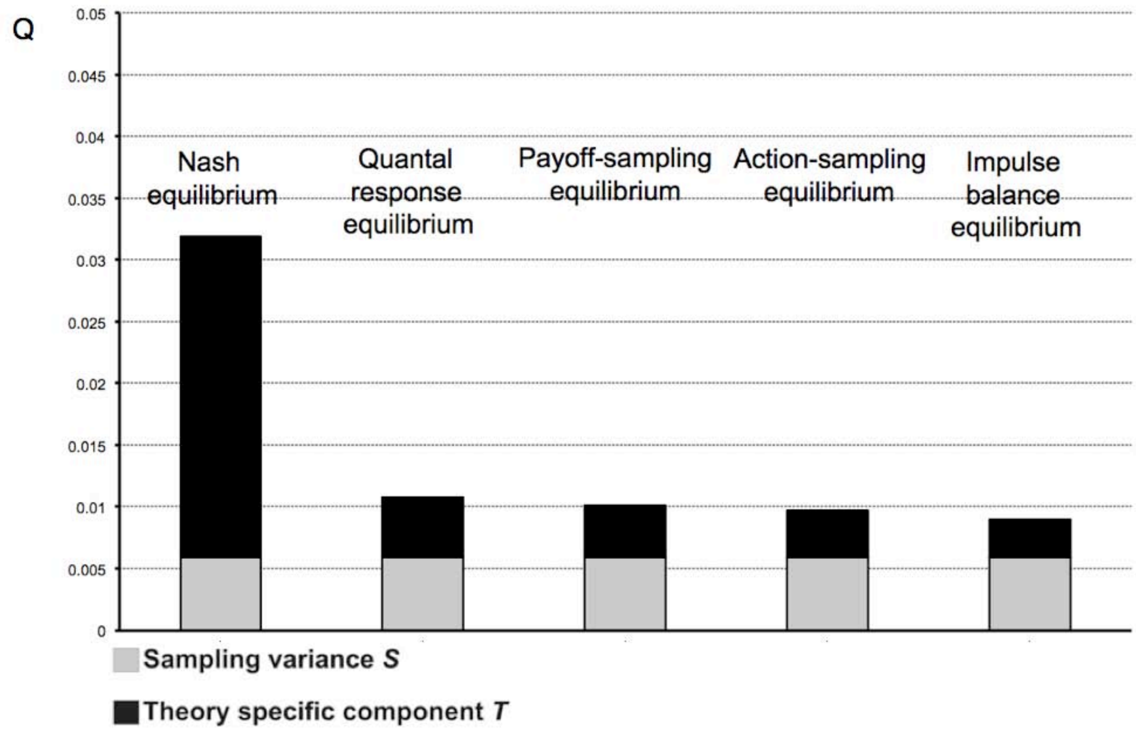

Figure 2.12: *Overall mean squared distances of the four stationary concepts compared to the observed average.*

2.4.7. Changes over Time

The question arises whether the order of predictive success of the five theories remains stable over time. Of course we can investigate this question only within the span of the 200 periods played in our experiments. For this purpose we compared the first hundred periods with the second hundred periods. Figure 2.13 shows the mean squared distances decomposed into sampling variance (grey) and the theory specific components (black and white resp.) for periods 1-100 (left) and 101-200 (right) for the five theories compared in this paper. It can be seen that in the second half of the experiments the predictive success of action-sampling equilibrium, payoff-sampling equilibrium and quantal response equilibrium is slightly greater than that of impulse balance equilibrium. The difference is not significant under the Wilcoxon signed rank test. The predictive success of impulse balance equilibrium is the same one in the first and second half. For each of the other four theories the performance is better in the second than in the first half.

The sampling variance is greater in the second half than in the first half. A two tailed matched pairs Wilcoxon signed rank test applied to the sampling variances for the first half and the second half in the twelve games shows no significant difference. Therefore we interpret the difference between the sampling variances in figure 2.17 as due to a random effect.

The improvement of predictive success in the second half of the experiment is connected to a movement of the observed relative frequencies nearer to the convex hull of the theoretical probability vectors. The relative frequencies for the first and the second half of game 4 are both inside the convex hull but for the other 11 games the relative frequencies for the first half are outside the convex hull. In the second half they are either inside (4 games) or still outside but nearer to the convex hull (7 games).

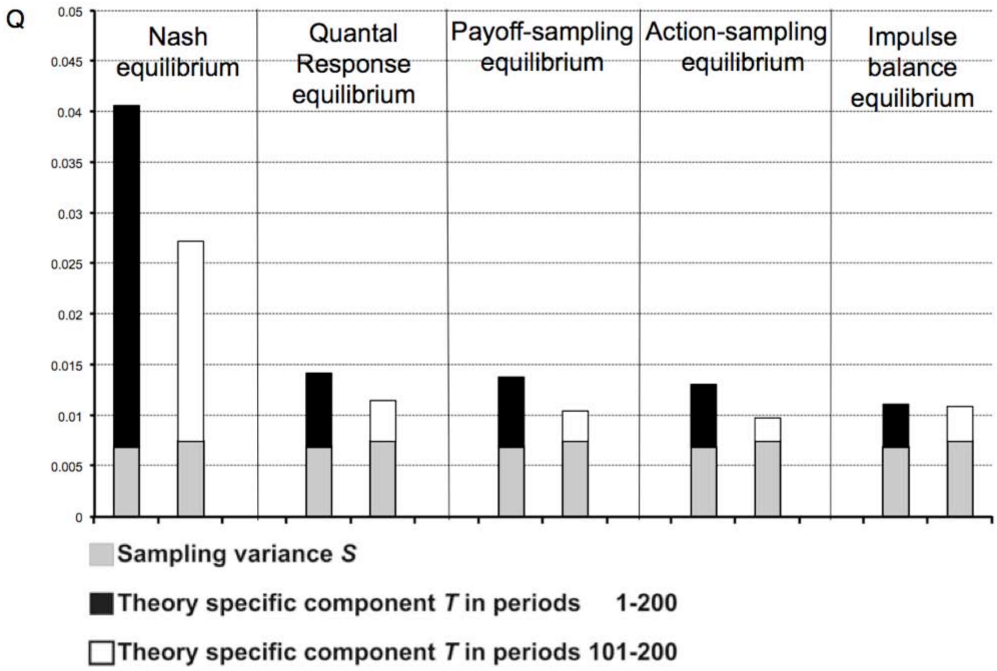

Figure 2.13: *Comparison of predictive success in the first half and second half of the experiments.*

2.4.8. Significance of the Comparisons of Predictive Success

In section 2.4.1. we have pointed out that the discrimination between the five concepts tends to be the worse the nearer the games are to the middle of the parameter space. Therefore we cannot expect significant results for the twelve or six observations for each of the games separately. It is more reasonable to apply a test to all constant sum games together and to do the same for all non-constant sum games together.

In order to compare the performance of two stationary concepts in the twelve games we apply the Wilcoxon matched pairs signed rank test to the squared distances of the theoretical values from the observed relative frequencies for the 108 independent subject groups.

In the application of this test differences of the squared distances are computed for each of the 108 observations and then ranked from 1 to 108 according to their absolute value. Smaller absolute values receive a lower rank. The test statistic is the sum of the ranks in favor of the first theory, in the sense that the squared distance for the first theory is lower than that for the second theory. This means that higher differences count more than lower ones, since they are less likely to be disturbed by random fluctuations. Therefore the fact, that games near the middle of the parameter space discriminate less among the theories, is automatically taken into account by the Wilcoxon matched pairs signed rank test.

The same test has also been applied to the 72 observations on constant-sum games and the 36 observations on non-constant sum games separately.

Table 2.3 shows the two tailed significances in favor of the row concept.

	Action sampling equilibrium	**Payoff sampling equilibrium**	**Impulse balance equilibrium**	**Quantal response equilibrium**	**Nash equilibrium**
Action sampling equilibrium		n.s. n.s. n.s.	n.s. 5 percent n.s.	2 percent 10 percent 10 percent	.1 percent .1 percent .1 percent
Payoff sampling equilibrium			n.s. .1 percent reversed	5 percent .1 percent n.s.	.1 percent .1 percent .1 percent
Impulse balance equilibrium		5 percent		n.s. reversed 5 percent	.1 percent .1 percent .1 percent
Quantal response equilibrium			.1 percent		.1 percent .1 percent 1 percent
Nash equilibrium					

Above: all games. **Middle:** games 1-6. **Below:** games 7-12
“ **n.s.** “ means “ not significant “
“ **reversed“** indicates that the pairwise comparison yields an entry below the main diagonal

Table 2.3: Significances in favor of row concepts, two tailed matched pairs Wilcoxon signed rank test, rounded to the next higher level among .1 percent, .2 percent, .5 percent, 1 percent, 2 percent, 5 percent, and 10 percent.

In order to judge how well the five concepts do in the revised table 3 we form and “significant entry index” for every concept. This index is the number of significance entries in the row of a concept minus those in the column of the same concept. (Entries “n.s.” and “reversed” do not count). Significances in the row of a concept indicate a comparison in favor of this concept and those in the column of the same concept stand for comparisons in favor of another concept. The significance entry index of the five concepts is as follows:

Action-sampling equilibrium:	+7
Payoff-sampling equilibrium:	+5
Impulse balance equilibrium:	+2
Quantal response equilibrium:	-2
Nash equilibrium:	-12

20 of the 22 significance levels shown in table 3 are in favor of the concept with the higher significance entry index. The two entries below the main diagonal concern comparisons between adjacent concepts with respect to the order given by the significance entry index and therefore represent relatively mild violations of this order.

It is also of interest that the index is positive for action sampling equilibrium, payoff sampling equilibrium and impulse balance equilibrium, but negative for quantal response equilibrium and Nash equilibrium. Of course the index is not more than a descriptive device for conveying a condensed impression of the information contained in table 3.

Nash equilibrium and impulse balance equilibrium are parameter free whereas the other three concepts involve one parameter estimated from the data. The possibility of adjusting a parameter to the observations adds's a degree of freedom not available to parameter free theory. A fair comparison between different concepts should take this advantage into account and level the playing field. However, it is not clear how this should be done. Therefore we did not try to remove this advantage of the one-parameter concepts.

Moreover, when a comparison is significant it is not always consistent across data sets (for example, impulse balance equilibrium does significantly better than quantal response equilibrium for non-constant sum games but quantal response equilibrium does significantly better than impulse balance equilibrium for constant sum games). According to the Wilcoxon signed rank test all other concepts beat Nash equilibrium.

2.5. Summary and Discussion

Five stationary concepts for completely mixed 2x2-games have been compared in this paper. For this purpose experiments on 12 games have been run, 6 constant sum games with 12 independent subject groups each and 6 non-constant sum games with 6 independent subject groups each.

The games were selected in such a way, that the constant sum games were reasonably well distributed over the parameter space. Each non-constant sum game had the same best reply structure as an associated constant sum game.

Each subject group consisted of 8 participants, four playing on one side and four on the other. Each subject group played only one game over 200 periods with random matching.

The literature reports about similar experiments with 2x2-games (MCKELVEY, PALFREY, WEBER (2000), GOEREE, HOLT, and PALFREY (2000); BINMORE, SWIERZBINSKI, AND PROULX. (2001); GOEREE, HOLT and PALFREY (2000); OCHS (1995)). Usually the number of periods played is much lower and more than one game has been played by the same subjects in one session. Thus in the Experiments by GOEREE, HOLT, and PALFREY (2000) the number of periods was 40. We wanted a greater number of periods because it is doubtful whether a stationary state can be reached within only relatively few periods. Play must be long enough to wash out initial effects.

An exception with respect to the number of plays is the paper by BINMORE, SWIERZBINSKI, AND PROULX (2001). They report experiments about several games played 150 times. However there was only one completely mixed 2x2-game (game 1) among them. Each subject played 7 games (including two practice games). If several games are played one after the other by the same subjects transfer of experience may occur from earlier to later games. Moreover data from different games played by the same subject are not statistically independent from each other. In our experiment each subject participated in only one independent subject group and played only one game. This is necessary for an appropriate application of statistical tests.

In the literature usually only two of the stationary concepts are confronted with experimental data, Nash-equilibrium and quantal response equilibrium. An exception is the paper by Avrahami, Kareev and Güth (2005). They successfully compared impulse balance equilibrium with their data, following the suggestion of one of the authors (R. Selten). The new concept of action-sampling equilibrium was never examined before. The same is true for payoff-sampling equilibrium.

Our measure of predictive success forms mean square deviations of observed relative frequencies from predicted probabilities for every game separately and then takes the average over the 12 games. The comparison of the five theories over the entire time span of 200 periods yields an order of predictive success from best to worst:

Impulse balance equilibrium
Action-sampling equilibrium
Payoff-sampling equilibrium
Quantal response
Nash equilbrium.

A remarkable result can be seen in the fact that the newer concepts of impulse balance equilibrium, payoff-sampling equilibrium and action-sampling equilibrium and quantal response equilibrium clearly outperform the established concept of Nash equilibrium. It can bee seen in figure 12 that impulse balance equilibrium, action-sampling equilibrium, payoff-sampling equilibrium and quantal response equilibrium are near to each other with respect to their predictive success. Moreover the predictive success of these three newer and the one more established theories is strikingly better than Nash equilibrium (see figure 12).

It is of great importance that even for completely mixed constant sum 2x2-games Nash equilibrium equilibrium fails in comparison to other concepts.

In this paper we concentrated on games played repeatedly with random matching by two populations. The literature reports also experiments on 2x2-games played repeatedly by the same two opponents. Behavior in such games may very well be different from that in games played by populations. If two subjects play the same 2-person zero-sum game hundred times against each other, they will be concerned about not being predictable. This may drive them nearer to maximin strategies. The experimental investigation by O'NEILL (1987) and an empirical paper by WALKER & WOODERS (2005) on "Minimax Play at Wimbledon" suggests that this may be the case.

It would be desirable to complement quantal response equilibrium by a theory which permits the computation of the noise parameter λ as a function of the payoffs of the game. Extended in this way quantal response equilibrium could become a much more powerful stationary concept. Since at the moment no theory of λ is available we have applied quantal response theory with λ interpreted as a natural constant which is the same one for all games.

In the same way as Nash equilibrium and quantal response equilibrium, action-sampling equilibrium is still a concept based on best replies, even if these are not best replies to the equilibrium strategies of the others, but to a random sample of strategies on the other side. Payoff-sampling equilibrium is not based on best replies but rather on the comparison of samples of payoffs obtained for own choices.

Impulse balance equilibrium is very different from the four other concepts since it is neither based on best responses nor on payoffs obtained for own choices. Unlike the other four concepts it cannot be considered to be a modification of Nash equilibrium. Impulse balance is different from optimization even in one-person decision problems (SELTEN, ABBINK AND COX (2001), OCKENFELS AND SELTEN (2005)). Moreover impulse balance equilibrium is applied to a transformed game. The transformation is based on the idea that losses relative to a natural reference point (the pure strategy maximin payoff) count double.

Impulse balance theory could also be applied to the original game but the application to the transformed game improves its performance. If Nash equilibrium, action-sampling equilibrium or payoff-sampling equilibrium is applied to the transformed game rather the original one, the performance of these concepts becomes worse. The transformation is an important part of impulse balance theory but it is not the only reason for its success.

It is not easy to understand why the predictions of the four newer concepts are not very far apart, in spite of the fact, that they are based on very different principles. This is maybe peculiar to our sample. It would be desirable to devise experiments, which permit a better discrimination between the four concepts.

In this paper we look at stationary concepts without any discussion of learning processes. The comparison of our data with learning processes will be the subject matter of a later paper. As far as movement over time is concerned we looked only at differences between periods 1-100 and 101-200. We have seen that the order of predictive success of impulse balance theory and payoff-sampling theory reverses from the first half to the second half of the experiments. The reversal is not statistical significant. No other changes of the order of predictive success from the first half to the second half are observed. In the second half the sampling variance is slightly increased. The predictive success of impulse balance equilibrium is the same in the second half and in the first half but the other four concepts perform much better in the second half than in the first half. The mean frequencies of individual observations seem to move nearer to the convex hull of the theoretical predictions, even if within a game the variance of the relative frequencies in independent subject groups does not change significantly. One cannot know whether the stationary distribution is reached within the 200 periods but the evidence conveys the impression that one comes near to it.

Stationary concepts are of great importance especially if they do not depend on parameters, which have to be adjusted to the data. Impulse balance theory does not involve any such parameters and can be used in theoretical investigations just like Nash equilibrium. It is possible to generalize impulse balance theory to general games in normal form (see Appendix F). It would certainly be desirable to gain experiences with games with more than two strategies or more than two players.

3. Testing (Beliefs about) Social Preferences: Evidence from an Experimental Coordination Game

3.1. Introduction

By now there seems to be broad agreement that in many contexts the traditional model of narrowly self-interested individuals is not the most useful description of economic agents. There is overwhelming experimental evidence that even in simple situations individual behavior involves more than just the maximization of one's own material payoff. In response to this evidence, a focus of recent research in behavioral economics has been the question of how to model the "social preferences" of agents, i.e. the preferences over *distributions* of payoffs. Two influential approaches are the "inequity aversion models" of BOLTON AND OCKENFELS (2000) and FEHR AND SCHMIDT (1999) on the one hand, and the "quasi-maximin" model of CHARNESS AND RABIN (2002) on the other hand. The inequity aversion models presume that *ceteris paribus* agents prefer more equal distributions of payoffs, while the quasi-maximin model emphasizes both the role of the worst-off individual and of the aggregate payoff for the group. There are several studies that test the two approaches against each other. ENGELMANN AND STROBEL (2004), for instance, find that in their simple "dictator" experiments the influence of both efficiency concerns and maximin preferences is stronger than that of inequ(al)ity aversion; similar evidence is reported in KRITIKOS AND BOLLE (2001). By contrast, GÜTH, KLIEMT AND OCKENFELS (2003) and FEHR, NAEF AND SCHMIDT (2004) find that fairness concerns dominate efficiency concerns. HERREINER AND PUPPE (2004) study the relevance of efficiency versus equity considerations in a free-form bargaining context.

The purpose of the present study is to shed further light on the relative importance of fairness concerns versus efficiency concerns. Specifically, in our experimental design (to be described in the next section) "fairness concerns" are represented by an aversion to payoff differences between two players, while "efficiency concerns" correspond to a preference for (possibly unequal) distributions with higher total payoff; in most cases considered here, higher total payoff in fact means Pareto improvement, i.e. *both* players' payoff increases. Two main conclusions can be drawn from our results. First, the coordination on Pareto superior allocations is the more difficult the greater the asymmetry between the two players, i.e. the more unequal the resulting payoff distributions. In light of the evidence reported in CHARNESS AND RABIN (2002) and ENGELMANN AND STROBEL (2004), this re-establishes and confirms the importance of inequity aversion as modeled by FEHR AND SCHMIDT (1999) and BOLTON AND OCKENFELS (2000). Secondly, and perhaps even more importantly, our results suggest that it is not so much inequity aversion *per se* but rather the belief that others are driven by fairness concerns that best explains our observed behavior.

3.2. Experimental Setting and Design

The game underlying our experiments is the following two-person normal form game. Both players simultaneously choose either strategy *E* or *F*. The resulting payoffs are common knowledge.

		Player 2	
		E	F
Player 1	E	x_2 x_1	0 0
	F	0 0	225 225

Figure 2.14: *Payoff matrix.*

We conducted two experiments. The first consisted of seven treatments ([T1] – [T7]) with different values for the payoff vector (x_1,x_2) resulting from the choice of *(E,E)*. Specifically, the treatments involved the following values: (x_1,x_2) = (375,200) [T1], (375,225) [T2], (250,250) [T3], (325,250) [T4], (375,250) [T5] (400,250) [T6] and (475,250) [T7]. The second experiment featured the same game, but there we held x_2 fixed at 250 and let x_1 steadily increase from 175 to 475 in steps of 5 units, resulting in 61 different distributions.

If both players are purely selfish (and if this is common knowledge), the game represents a simple coordination game with two pure strategy Nash equilibria, namely (*F,F*) (both choosing the "fair" outcome, i.e. the equal distribution (225,225)) and (*E,E*) (both choosing the "efficient" outcome, i.e. the distribution (x_1,x_2) which in almost all cases[1] maximizes the sum of the payoffs). Note that in [T2] – [T7] the (*E,E*) equilibrium is in fact Pareto superior to the (*F,F*) equilibrium; the same holds in the second experiment for all $x_1 > 225$.

Our experiments were computerized[2] and were conducted at the University of Bonn. Participants were recruited from the campus mensa and had no previous training in economics or game theory. Each participant played only one game and had to make exactly one decision. In total, 402 persons participated. In the first experiment, each treatment consisted of 20 games with 2 players, thus a total number of 7x20x2=280 subjects participated in the first experiment. In the second experiment, each of the games corresponding to the different payoff distributions was played only once, hence 61x2=122 subjects participated in the second experiment. Each participant was informed about the game and his/her role as *player 1* or *player 2*, but not about who the other player was. The game was given in a matrix form, strategies were

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[1] This is not true in the second experiment for distributions with $x_1 < 200$.

[2] The program was written in PASCAL using RATImage by Abbink and Sadrieh (1995).

labeled *A* and *B*, and the instructions[3] were given in a neutral language to avoid framing effects.

After making their decision, subjects had also to fill out a questionnaire. Each subject received a lump sum payment of 1 € plus the individual payoff converted at a rate of 1 point = 0.01 €.[4] If people failed to coordinate, they nevertheless got 1 €. The average earning in the first experiment was 2,49 €, with its minimum in [T3] at 1,88 € and its maximum in [T6] at 2,80 €. The average earning in the second experiment was 2,94 €. There were statistically significant differences in payoffs between the treatment pairs [T1][T5], [T1][T6], [T2][T5], [T2][T6], [T3][T4], [T3][T5], [T3][T6] and [T3][T7].

3.3. Experimental results

The experimental results from our first experiment are summarized in Tables 3.4-3.6. In the following, special attention will be given to treatments [T3]–[T7] because in each of these the distribution resulting from (*E,E*) ("efficient" outcome) is a strict Pareto improvement relative to the "fair" outcome (225,225). *Player 2* always gets a payoff of 250. *Player 1's* payoff increases from 250 in [T3] to 475 in [T7], so we have an increasing inequality in payoffs between *players 1* and *2*.

Under inequity aversion this has the following behavioral implications. Specifically, assume as in Fehr and Schmidt (1999) (henceforth: [F/S]), that *player i*'s utility function u_i is given by $u_i(x_i, x_j) = x_i - \alpha_i \max(x_j - x_i, 0) - \beta_i \max(x_i - x_j, 0)$, where x_i and x_j are the payoffs of player *i* resp. *j*, and α_i and β_i are parameters that measure *i*'s degree of aversion against disadvantageous resp. advantageous inequality. As in [F/S], we assume that $\alpha_i \geq \beta_i$ and $0 \leq \beta_i \leq 1$. Simple calculations show that in [T3]–[T7] *player 1*'s best response is *E* to *E* and *F* to *F*, and that he/she always prefers the efficient distribution to the fair distribution. By contrast, whether *player 2* prefers the efficient distribution depends on her/his individual α and the specific payoff difference in *(E,E)*. The larger the difference, the less likely it is that a *player 2* will prefer *(E,E)* over *(F,F)* in [T4]–[T7]. If $\alpha > 250/(x_1 - 250)$, strategy *E* is strictly dominated by *F* for *player 2*, leaving the game with only one pure strategy Nash equilibrium, namely *(F,F)*.[5] Similar considerations apply to [T1] and [T2].

Tables 3.4 and 3.5 show the relative frequency of the observed strategy choices.

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[3] Instructions and screenshots are available from the authors upon request.

[4] Thus, for instance, a payoff of 275 points corresponds to 2,75 + 1,00 = 3,75 €.

[5] Thanks to an anonymous referee for pointing this out. Note, however, that for a situation with just one equilibrium to arise, one needs a large α. In [T4]–[T7], the required α value would imply a rejection of a $250/(x_1+250)$ share in an ultimatum bargaining game when the outside option equals 0. For example, the α needed in [T7] would lead to a rejection of an offer of ≤ 34,46 % in the ultimatum game, the α needed in [T4] to a rejection of an offer of ≤ 43,47 %. But rejections of such offers are rarely observed in experiments on ultimatum bargaining (see, e.g., **[F/S]**).

Player 1	[T1]	[T2]	[T3]	[T4]	[T5]	[T6]	[T7]
X_1	*375*	*375*	*250*	*325*	*375*	*400*	*475*
X_2	*200*	*225*	*250*	*250*	*250*	*250*	*250*
E	40%	55%	70%	80%	50%	60%	65%
F	60%	45%	30%	20%	50%	40%	35%

Table 3.4: *Relative frequency of decisions (player 1).*

In [T3], 70% of *player 1* choose *E*, in [T4] it is even 80%. But then it drops significantly[6] down to 50% in [T5], and although it rises again, it does not get beyond 65% in [T7]. On average, 65% of *player 1* choose strategy *E* in [T3]–[T7]. In light of the [F/S]-predictions, this is a relatively small percentage.

Player 2	[T1]	[T2]	[T3]	[T4]	[T5]	[T6]	[T7]
X_1	*375*	*375*	*250*	*325*	*375*	*400*	*475*
X_2	*200*	*225*	*250*	*250*	*250*	*250*	*250*
E	40%	70%	65%	65%	70%	70%	75%
F	60%	30%	35%	35%	30%	30%	25%

Table 3.5: *Relative frequency of decisions (player 2).*

Looking at the behavior of *player 2*, we see a slightly different trend. In [T3] it starts with only 65% of subjects playing *E*, but then it rises steadily to 75% in [T7]. On average, 69% of *player 2* choose strategy *E* in [T3]–[T7]. Even if there are no significant differences between these treatments[7], the number of *player 2* choosing *E* nonetheless rises with increasing inequity in *(E,E)*. This is surprising in view of the [F/S] model, which would predict a declining number of choices of *E* by *player 2*. Note that even in the presence of inequity aversion, *(E,E)* can still be a Nash equilibrium of our simple coordination game, as long as player 2's α does not get too large. Thus, the [F/S] model certainly does not always rule out choices of strategy *E* by either player. However, inequality aversion implies that the probability of the choice of *E* by *player 2* should decrease with increasing payoff difference, leaving the game with just one pure strategy Nash equilibrium *(F,F)* in the extreme case.

Table 3.6 lists the observed distributions in the first experiment. The first row shows the number of games resulting in the efficient distribution, the second row the number of games resulting in the fair distribution. The third and fourth rows give the

[6] Significant at 5% level using a Fisher-Test. Other significant differences in behavior of *player 1* are between [T1][T3] (5% level), [T1][T4] (1% level), [T1][T7] (10% level) and [T2][T4] (10% level).
[7] Significance (Fisher) of 5% in [T1][T2], [T1][T5], [T1][T6], [T1][T7], and of 10% in [T1][T3], [T1][T4].

number of coordination failures. Obviously, more games result in the efficient distribution than in the fair distribution. This holds for all treatments except for [T1]. In [T3] – [T7], we observe 43 efficient endings and only 9 fair ones. This might give the impression that people are driven mainly by efficiency concerns, but it is in fact not evident. If we look at the number of games in which one player chooses *E* while the other chooses *F*, we see that nearly half of all games fall into this category (67 out of 140). Thus, efficiency concerns can at least not be common knowledge.

	[T1]	[T2]	[T3]	[T4]	[T5]	[T6]	[T7]	[T3-7]	[T1-7]
(E,E)	*4*	*7*	*7*	*10*	*8*	*9*	*9*	*43*	*54*
(F,F)	*8*	*2*	*0*	*1*	*4*	*3*	*1*	*9*	*19*
(E,F)	*4*	*4*	*7*	*6*	*2*	*3*	*4*	*22*	*30*
(F,E)	*4*	*7*	*6*	*3*	*6*	*5*	*6*	*26*	*37*

Table 3.6: *Resulting distributions.*

Taking a closer look, one can distinguish two different cases of coordination failure. Either *player 1* or *player 2* can be made "responsible" for not reaching the efficient distribution by choosing strategy *F*. Remarkably, the pattern of coordination failures changes from [T3] to [T7]. In [T3] we observe 6 instances of (*F,E*) versus 7 instances of (*E,F*), and in [T4] 3 instances of (*F,E*) versus 6 instances of (*E,F*), thus in these two treatments the failure to reach the efficient outcome is more often due to *player 2's* choice. By contrast, as is evident from Table 3.6, in [T5] - [T7] it is more often *player 1* who is responsible for not reaching the efficient outcome. Note that in the [F/S] model one would expect the opposite pattern of behavior.

Inequity aversion does seem to influence players' choices, but in a complex way. A clue can be found by analyzing the questionnaires. When asked for the reason of their decision, many subjects wrote that they tried to anticipate the other player's choices and determined their own strategy based on that belief. In our simple coordination game, we may thus take the actual strategy choice as an estimator of a player's belief. In view of this, our results suggest that it is not so much inequity aversion *per se* that matters but rather the belief that the other player is inequity averse.[8] Not surprisingly, the assessment of the situation seems to be sensitive to the magnitude of the payoff difference in the efficient distribution. In [T3] and [T4], when the difference is small, many *player 1* think that *player 2* will choose *E* because they both can earn more by doing so, so we see a high percentage of *player 1* choosing *E*. Around [T5], there seems to be a turning point. *Player 1* now seems to think that the other player regards the efficient distribution as unfair, so we see many *player 1*

[8] Unfortunately, the players are often wrong in their estimation of the other player's behavior, so they frequently fail to coordinate.

choosing *F*. In the extreme, if *player 1* believes that *player 2*'s inequity aversion is large enough to prefer (0,0) over (x_1,x_2), then in equilibrium *player 1* must choose *F* because for *player 2 F* is a dominant strategy now. By contrast, *player 2* seems to think that *player 1* will choose *E* because his/her payoff increases significantly, so in order to coordinate *player 2* chooses *E*. Thus, people appear to think too "badly" about the other player's attitude, and the strength of this effect seems to be influenced by the size of the payoff difference resulting from (*E,E*).

To further examine this, we conducted the second experiment already described above. The results are summarized by Figures 2 and 3.

Figure 3.15 shows the choices of *player 1* for each x_1 ($175 \leq x_1 \leq 475$), Figure 3.16 shows the corresponding choices of *player 2*. A dot at the top marks a choice of strategy *E*, a dot at the bottom a choice of *F*.[9]

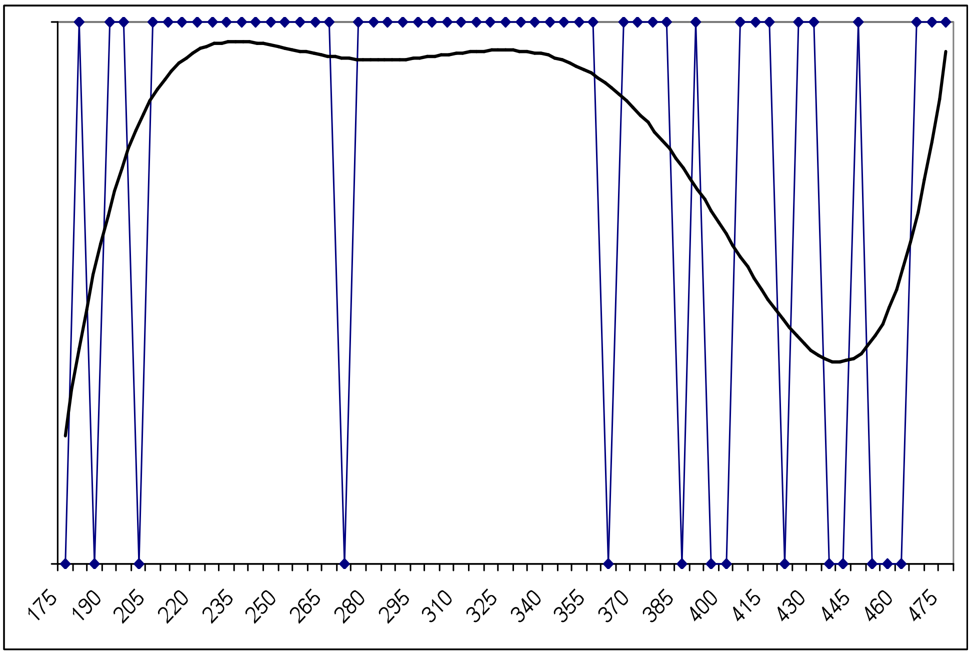

Figure 3.15: *Choice player 1.*

[9] The curves represent a polynomial trend (of fifth degree); they only serve for visualization of the results.

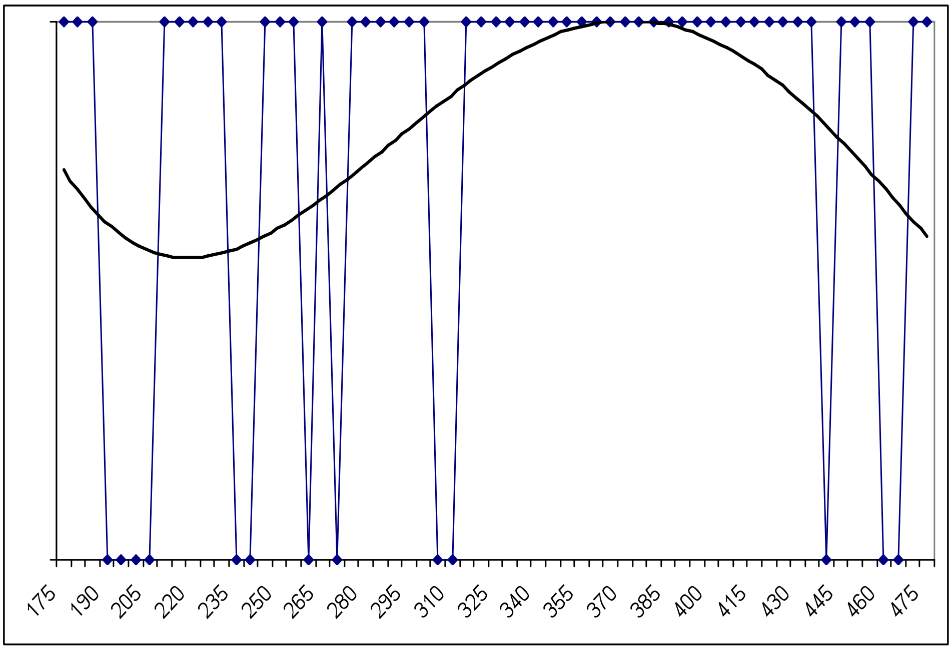

Figure 3.16: *Choice player 2.*

Qualitatively, the trend for *player 1* decisions is similar to the behavior observed in the first experiment. For low x_1 values they mostly choose *E*. Then, around $x_1 = 360$, they seem to start to think that the other player may choose *F* and play *F* more often.[10] For high x_1 values, fairness concerns are no longer dominant and the choice of *E* is observed more often. The behavior of *player 2* corresponds with the results from our first experiment as well. After a variable beginning there is a remarkably long period (from $x_1 = 325$ to $x_1 = 440$) with the constant choice of *E.* At the end the inequality seems to get too large and *F* is again chosen sometimes. For both players, some choices of *F* are observed at high x_1 values after a period of constant choice of *E*. Remarkably, this period ends much later for *player 2* than for *player 1,* which again confirms the conjecture that the qualitative nature of our observations is due to *player 1's* beliefs rather than *player 2's* actual social preferences.

Combining the choices, we see that in 62% of the cases the games result in the efficient distribution. Only in 7% of the cases the fair outcome results, and in 31% of all cases the players fail to coordinate and earn zero payoff. We thus have a much lower number of coordination failures than in our first experiment, and a much higher number of efficient endings.[11]

Summarizing, we find that efficiency concerns are important. But if inequality gets significant, difference aversion hampers the coordination on an efficient and even Pareto superior outcome. Thus, the disregard of equality in favor for unanimous improvement is at least not common knowledge.

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[10] One could think that they care for fairness, but in the questionnaires many individuals explicitly wrote that they chose *F* because they thought that the other player may do so.

[11] However, each x_1 value was played only once in our second experiment, so the data basis is much weaker than in the first experiment.

4. An Extended Reinforcement Algorithm for Estimation of Human Behaviour in Experimental Congestion Games

4.1. The Investigated Games

4.1.1. Congestion Game I (CI) – The Minority Game

The first discussed congestion game (CI) is a well known minority game CHALLET (1997 and 1998). The minority game is an important example of a Congestion Game. The game can be applied on different situations with social and economic contexts. One can analyse the minority game exemplarily as an elementary traffic scenario in which human participants had to choose several times between a road *A* and a road *B.* In each period, the road which was chosen by the minority of players won. This paper reports about the results of laboratory experiments of minority games and a learning algorithm witch simulates the observed human behaviour in these games.

The minority game is the most important example for a classic non-zero-sum-game and can be applied on different situations with social and economic contests.

Imagine two big and famous gold fields in South Africa, near Cape Town and Johannesburg. The diggers heard that a big gold-nugget was found in Johannesburg. From now on every digger went to Johannesburg to dig gold, the city got overcrowded and there was not enough space for all of them, so the profit was very small. The diggers who stayed in Cape Town on the other hand had enough space for their claims. The profit in Cape Town was very high for everybody. This is an example of the minority game, the people who choose the majority got no payoffs, but the people on the minority in Cape Town found enough gold for all of them, so everybody got a payoff.

The minority game which is also called the El Farol Bar Problem (EFPB) was introduced by Arthur 1981. The setup of the minority game is the following: a number of agents *n* have to choose in several periods whether to go in room *A* or *B*. Those agents who have chosen the less crowded room win, the others lose.

Later on, the EFBP was put in a mathematical framework by Challet and Zhang, the so-called Minority Game (MG). An odd number *n* of players has to choose between two alternatives (e.g., yes or no, *A* or *B*, or simply 0 or 1). In the Literature are many examples, where the MG is discussed [CHALLET 1997, 1998, JOHNSON ET AL. 1998].

In this paper we transferred the minority problem into a route choice context. We did minority game experiments at the Laboratory of Experimental Economics (University of Bonn). In these Experiments subjects are told that in each of 100 periods they have to make a choice between a road *A* and road *B* for traveling from X to Y.

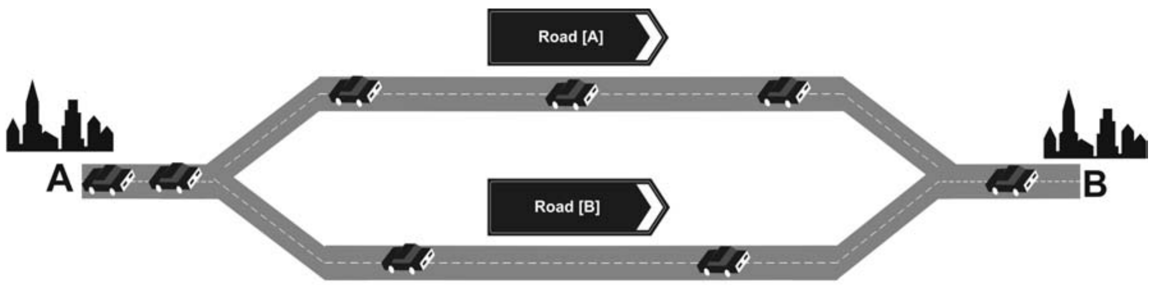

Figure 4.17: *Participants had to choose between a road [A] and a road [B].*

The set-up of the minority game was introduced by BRIAN ARTHUR (1991). Newer approaches were done by CHALLET (1997 and 1998). The experimental setup is the following: a number of players n have to choose in several periods whether to go to a place A or B. Those players who have chosen the less crowded place win, the others lose. The number of players in each Simulation was 9, the number of periods was 100. The players get a payoff t_A or t_B depending on the numbers n_A and n_B of participants choosing *A* and *B,* respectively:

(4.1) $$t_A = 1, t_B = 0 \Leftrightarrow n_A < n_B$$

(4.2) $$t_B = 1, t_A = 0 \Leftrightarrow n_A > n_B.$$

The period payoff was t_A if *A* was chosen and t_B if *B* was chosen. There are no pure equilibria in this game. The pareto-optimum can be reached by 4 players on one and 5 players at the other place.

4.1.2. Asymmetric Congestion Games (CII)

The second congestion game (CII) is a variation of the minority game: the number of agents in this game was 18, 36, 54, 72 and 90. The number of played periods was in each game 100.

The period payoff for the 18 player setting was 40 – *t* with $t = t_A$ if *A* was chosen and $t = t_B$ if *B* was chosen, where t_A and t_B depend on the numbers n_A and n_B of participants choosing *A* and *B,* respectively:

(4.3) $$t_A = 6 + 2n_A \quad \text{and} \quad t_B = 12 + 3n_B .$$

In the route choice scenario *A* represents a main road and *B* a side road. *A* is faster if *A* and *B* are chosen by the same number of people SCHRECKENBERG, SELTEN, PITZ, CHMURA (2003).

All pure equilibria of the game are characterized by $n_A = 12$ *and* $n_B = 6$*. The equilibrium payoff is 10 units per player and period. The pareto-optimum can be reached by*

(4.4) $$n_A = 11 \text{ and } n_B = 7.$$

The modified payoff functions for the experiments with 36, 54, 72 and 90 agents are

(4.5) $$18\lambda,\ \lambda = 2,\ldots,5,$$

where

(4.6) $$p_A = 40\lambda - [6\lambda + 2n_A]$$

(4.7) $$p_B = 40\lambda - [12\lambda + 3n_B]$$

Table 4.7. shows all pure equilibria in the CII depending on the number of players.

Number of Players	**Equilibrium**	
	A	**B**
18	12	6
36	24	12
54	36	18
72	48	24
90	60	30

Table 4.7: *Pure equilibria in CII. The equilibria depend on the number of participating agents.*

In the case of CII, place *A* and place *B* are understand as a road with high capacity (main road) and a road with low capacity (side road) and t_A and t_B as travel times.

4.1.3. Experimental Set-up of CI and CII

Each of the games CI and CII with 9 and 18 persons were played 6 times with students at the Laboratory of Experimental Economics in Bonn. Additionally CII was played 1 time with 36, 54, 72 and 90 students. Subjects are told that in each period they have to make a choice between A and B. The subjects of the CII set-up did not know the payoff function. They were told that if *A* and *B* are chosen by the same number of people, subjects who had chosen *A* get a better payoff than subjects who had chosen *B*. At the end of an experiment, each participant was paid an amount in Euro proportional to his cumulated payoff sum he had reached over the 100 periods.

The experimental data statistics are listed and compared with simulation results in chapter 4.4.

4.2. Reinforcement Learning

4.2.1. Reinforcement Algorithm with Pure Strategies

The reinforcement algorithm with pure strategies already described by HARLEY (1981) has been used extensively by EREV and ROTH (1995) in the experimental economics literature. The convergence in games with pure strategies was analyzed by LASLIER and WALLISER (2005). Figure 4.18 explains the original reinforcement algorithm.

We are looking at player *i* who has to choose among *n* pure strategies 1,…,*n* over a number of periods *t, t*=1..*T*. The probability that "strategy *x* is chosen by player *i*" is proportional to its "propensity" $q_{i,x}^{t}$. In period 1 these propensities are exogenously determined parameters. Whenever the strategy *x* is used in period *t*, the resulting payoff a_x^t is added to the propensity if this payoff is positive. If all payoffs are positive, then the propensity is the sum of all previous payoffs for this strategy plus its initial propensity. Therefore one can think of a propensity as a payoff sum.

Initialisation: For each player *i* let $[q_{i,1}^1,...,q_{i,n}^1]$ be the initial propensity, where *n* is the number of strategies which are used in the simulations.

1. period: Each player *i* chooses strategy *x* with probability $\frac{q_{i,x}^1}{\sum_{y=1}^{n} q_{i,y}^1}$.

t+1. period: For each player *i*, let a_i^t be the payoff of player *i* in period *t*, *x* the number of the chosen strategy in period *t*.

CASE I:: $a_i^t \geq 0$:

$$q_{i,y}^{t+1} := \begin{cases} q_{i,y}^t + a_i^t & for \ \ y = x \\ q_{i,y}^t & for \ \ y \neq x \end{cases}$$

CASE II $a_i^t < 0$:

$$q_{i,y}^{t+1} := \begin{cases} q_{i,y}^t & for \ \ y = x \\ q_{i,y}^t + \left| a_i^t \right| & for \ \ y \neq x \end{cases}$$

Each player *i* chooses strategy *x* with probability $\frac{q_{i,x}^{t+1}}{\sum_{y=1}^{n} q_{i,y}^{t+1}}$.

Figure 4.18: *Reinforcement algorithm.*

4.2.2. The Empirical Foundation for an Extended Reinforcement Model

The only pure strategies in CI and CII are "place A" and "place B". These strategies do not represent a player's belief about the other participant's behaviour. In our extended model we add two further strategies which include the consideration of players about the others based on the last period's payoff.

Direct: A participant who had a good (bad) payoff may stay on the last period's place (change his last choice). We call this *direct response mode*. A change is more probable the worse the payoff was. The *direct* response mode is the prevailing one but there is also a *contrarian* response mode.

Contrarian: Under the *contrarian response mode* a change of the last choice is more likely the better the payoff was. The contrarian participant expects that a high payoff will attract many others in the next period.

In CI a "*bad*" payoff could obviously be defined by 0 and a "*good*" payoff by 1. In CII with 18λ, λ=1,…,5 players, the pure equilibrium payoff is $\varepsilon = 10\lambda$. Payoffs perceived as "*bad*" tend to be below ε and payoffs perceived as "*good*" tend to be above ε. Accordingly we classified the strategy of a subject as *direct* if there is a change (stay) after a payoff smaller (greater) than 10λ. The opposite strategy is classified as *contrarian*.

4.3.2. Measuring Direct and Contrarian Strategies

For each subject let c_- (c_+) be the number of times in which a subject changes from *A* to *B* or from *B* to *A* when there was a bad (good) payoff in the period before. And for each subject let s_- (s_+) be the number of times in which a subject stays on the same place when there was a bad (good) payoff in the period before.

For each subject in the experiments CI and CII, a Yule coefficient *Q* has been computed as follows:

(4.8) $$Q = \frac{c_- \cdot s_+ - c_+ \cdot s_-}{c_- \cdot s_+ + c_+ \cdot s_-}\ ,\quad c_- \cdot s_+ + c_+ \cdot s_- \neq 0\,.$$

The Yule coefficient has a range from –1 to +1. In the rare cases that a subject never (in each period) changes his last choice, we defined $Q = 0$ because the decision of such a subject does not depend on the last period payoff. A subject with Yule coefficients below –.5 could be understood to be classified as direct and subjects above +.5 as contrarian.

4.2.4. Extended Reinforcement Learning

In our simulations of CI 9 agents, respectively of CII 18, 46, 54, 72, 90, agents interact for 100 periods just like in our experiments described in section 3. In CI and CII each player has two pure strategies:

Place A: This strategy consists in taking A.

Place B: This strategy consists in taking B.
After the first period in each of the two games (CI) and (CII) the two extended strategies direct and contrarian are available:

(CI) *direct*: If the payoff of a player is 1, then the player stays on the same place last chosen. If his payoff is 0, the players changes (from *A* to *B* or from *B* to *A*).

(CI) *contrarian*: If the payoff of a player is 1, then the player changes (from *A* to *B* or from *B* to *A*). If his payoff is 0, the players will stay on the same place.

(CII) *direct*: This strategy corresponds to the direct response mode. The payoff of a player is compared to his median payoff among his payoffs for all periods up to now. If the present payoff is lower then this median payoff, then the place is changed. If the payoff is greater than this median payoff, the player stays on the same place as before. It may also happen that the current payoff is equal to the median payoff. In this case, the place is changed if the number of previous payoffs above the median is greater than the number of previous payoffs below the median. In the opposite case, the place is not changed. In the rare cases where both numbers are equal, the place is changed with probability ½.

(CII) ***contrarian*****:** A player who takes this strategy stays on the last chosen place if his current payoff is smaller then the median payoff among this payoffs for all previous periods, and he changes the place in the opposite case. If the current payoff is equal to this median payoff, then he changes the place if the number of previous payoffs below the median payoff is greater then the number above the median payoff. If the numbers of previous payoff below and above the median payoff are equal, the place is changed with probability ½.

The strategies *direct* and *contrarian* are necessary to be represented in the simulations for fitting the experimental data. They appear in the simulations as the result of an endogenous learning behaviour by which initially homogeneous subjects become differentiated over time.

4.2.5. Initial Propensity

The difficulty arises that the initial propensities must be estimated from the empirical data. For each game CI and CII we did this by varying the initial propensities for the strategies *place A* and *place B* over all integer values from 1 to 120 and the initial propensities for the strategies *direct* and *contrarian* over all integer values from 0 to 120.

For each initial propensity we tested 1000 simulations. To show the general behaviour of the simulations, Figures 4.20-4.23 show several selected statistical parameters depending on the initial propensities listed in figure 4.19. The numbers refer to the strategies *place A*, *place B*, *direct* and *contrarian* in this order.

$$I_0 := \{q,q,0,0] : 1 \le q \le 120\},\ I_1 := \{q,q,q,0] : 1 \le q \le 120\}, I_2 = \{q,q,q,q] : 1 \le q \le 120\}$$

$$I_3 := \{q,q,0,q] : 1 \le q \le 120\},\ I_4 := \{0,0,q,q] : 1 \le q \le 120\}$$

Figure 4.19*: Initial Propensities.*

One could see in figure 4.4 that, for each simulation run and each initial propensity the mean number of agents on place A is close to 4.5. The convergence to the theoretical mixed equilibrium was already observed in the simulation data of Roth & Erev (1995).

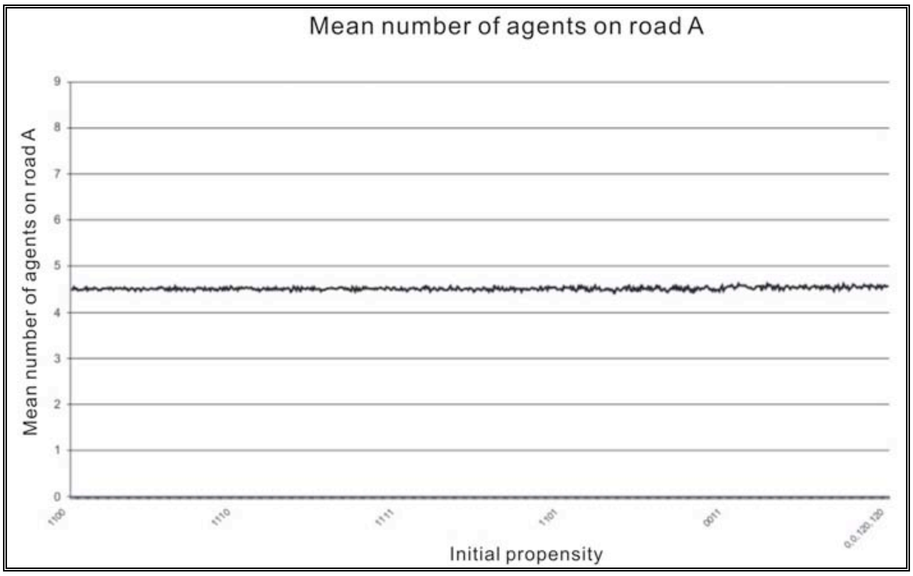

Figure 4.20: *Number of players on A.*

The standard deviation of the number of players on place A per period (figure 4.5) is correlated to the number of changes (figure 4.6) per periods. It got the highest values with propensities from the set I_1. In this cases the strategy direct is present and contrarian is absent. The strategy directly forces changes after a “bad” payoff 0, which is the most frequent in the majority game.

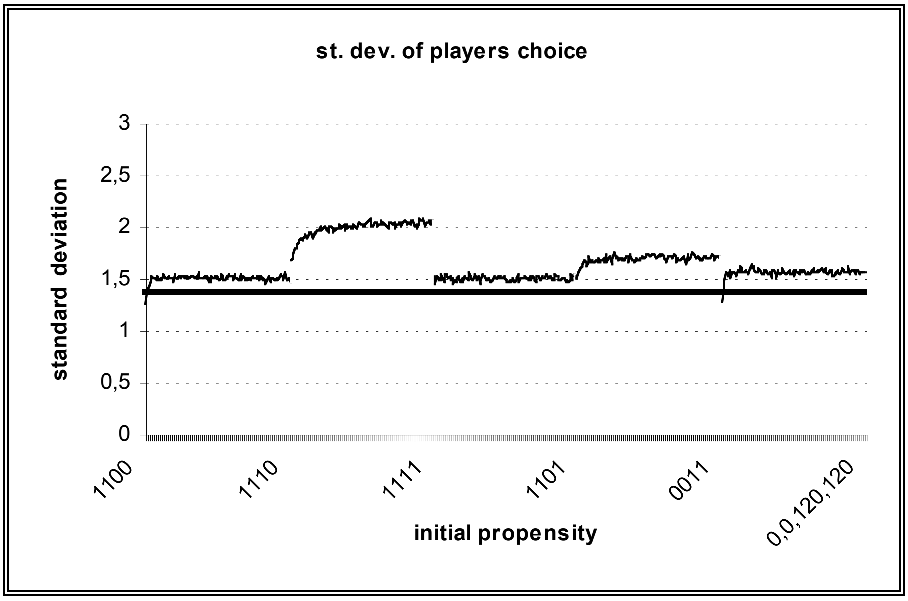

Figure 4.21: *Standard deviation of number of players on A.*

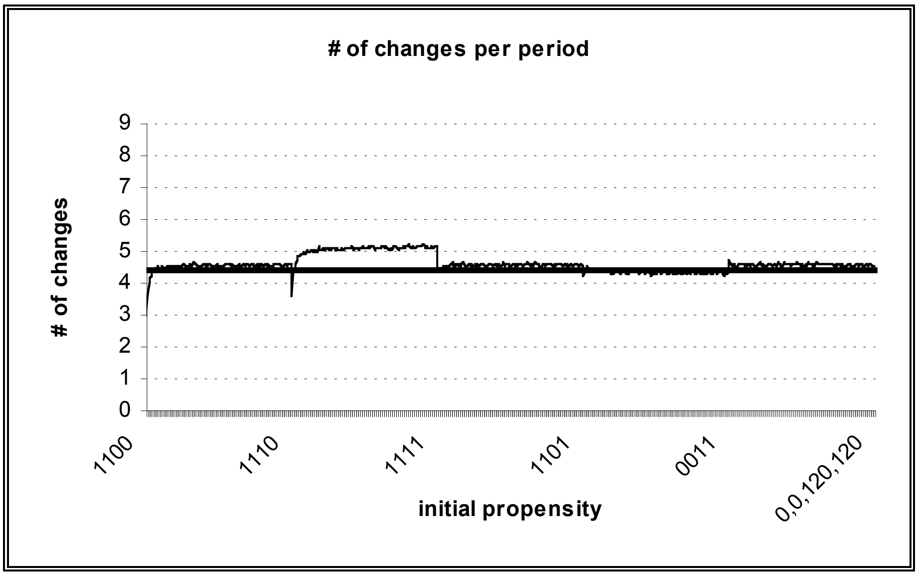

Figure 4.22: *Number of changes per period.*

Players with high Yule-coefficients in the experiments are assigned to the direct type; this appears also in the figure 4.7. For the initial propensities, in which no contrarian change behaviour is implemented, for example (1110), step high Yule-coefficients up. For the initial propensities, in which the contrarian behaviour is favoured, for example (1101), all values of the Yule-coefficients are negative.

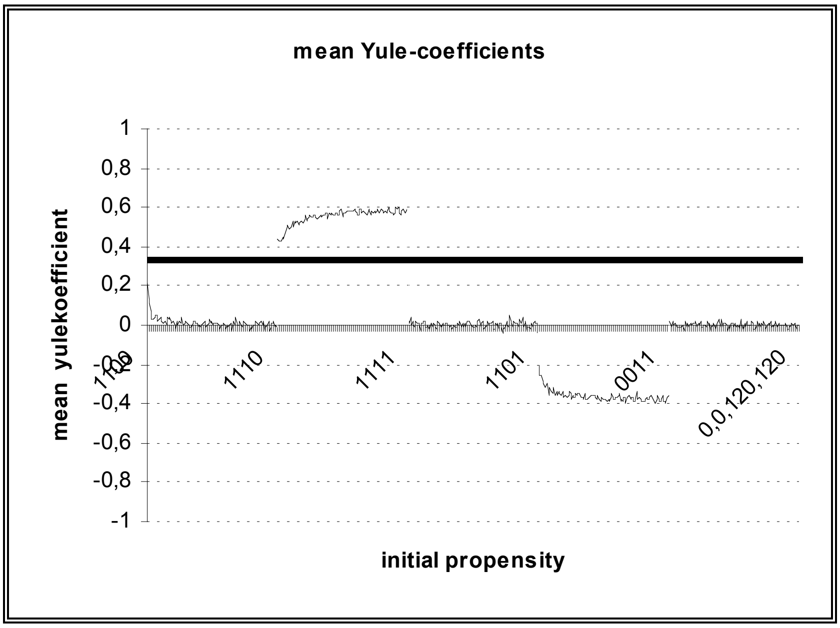

Figure 4.23: *Mean Yule-coefficients.*

Similar results could be obtained by investigations of the initial propensities for simulations of CII.

4.3. Experimental Statistics and Simulation Results

4.3.1. CI with 9 Players

For each propensity vector $[q_1,...,q_4] \in \{1,...,120\}^2 \times \{0,...,120\}^2$ we ran 1000 simulations according to the experiments with 100 periods. The numbers of the propensity vector refer to the strategies *place A*, *place B*, *direct* and *contrarian* in this order. We compared the mean values of each of the 1000 simulations of 6 statistical variables which are listed in table 4.8 with minimum and maximum values of the experimental data.

There were three parameter combinations which satisfied the requirement of yielding means for the six variables between the minimal and maximal experimentally observed values. These were the parameter combinations *(1,1,2,1)* and *(2,2,1,1)* and *(3,3,4,2)*.

CI	Experiment Minimum	Simulations {1,1,2,1}	Simulations {2,2,1,1}	Simulations {3,3,4,2}	Experiment Maximum
Player on A [mean]	4,19	4.48	4.50	4.54	4.74
Player on A [standard deviation]	0.67	1.45	1.48	1.50	1.50
Changes [mean]	0.59	4.32	4.18	4.51	5.17
Period of last Change	54.44	96.11	97.67	97.44	98.11
Yule Q [mean]	-0.01	0.10	0.04	0.14	0.87
Yule Q [standard deviation]	0.33	0.50	0.40	0.35	0.76

Table 4.8: *CI – 9 Players - Experimental minima & maxima vs. simulation means.*

Additionally we could show that the vector (1,1,2,1) minimizes the sum of normalized quadratic deviations of experimental data and simulation results of the six variables. The quadratic deviations where normalized by division by the standard deviations of the experimental results over the treatments. Figure 4.24 shows the quadratic deviations of the best initial vectors from the average experimental data.

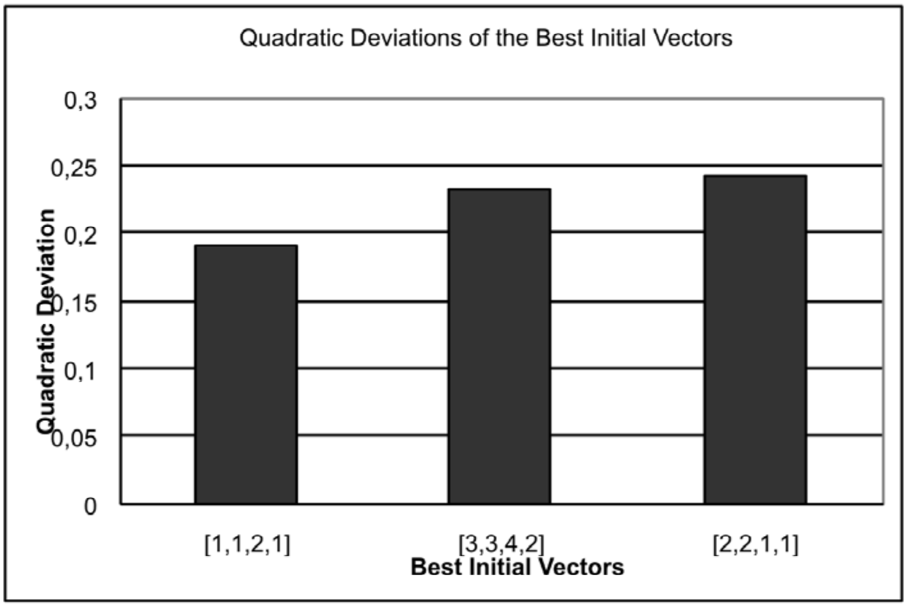

Figure 4.24: *Quadratic deviations of the best initial vectors from the average experimental data.*

Remark: The parameter combinations seem to be reasonable vectors of initial propensities. There is no difference between place *A* and place B. It is clear to see that the vectors have the same propensities for both places. In two of the three vectors the propensity of the direct mode is greater than the value of the other propensities. The higher initial value for the direct strategy and the smaller value of the contrarian strategy represent the ratio of the experimental data referring to the player types (Chmura & Pitz 2006).

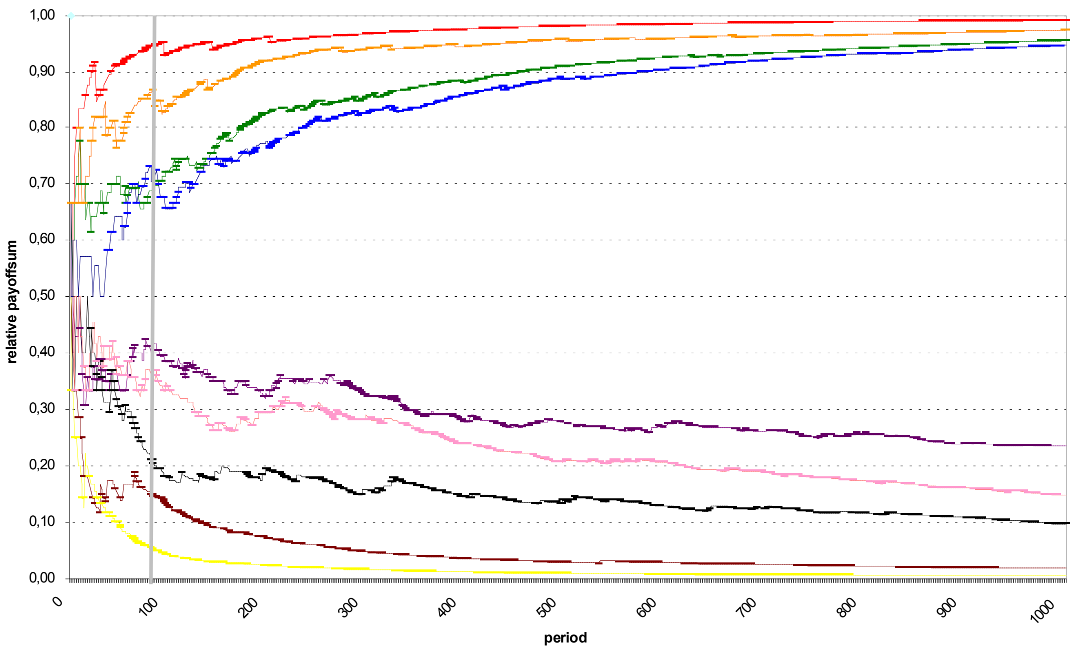

Figure 4.25: *Example simulation shows the relative payoff-sum for each of the 9 players over 1000 periods.*

It is remarkable that no initial propensities which contain only pure strategies fit the experimental data. We want the simulation model as easy as possible, therefore all experimental players start with the same propensity vector combination in one simulation. As in the experiments the agents become differentiated over time (see figure 4.25).

4.3.2. CII with 18 Players

In set-up CII with 18 players, we got only one parameter combination from the set $\{0,...,120\}^2 \times \{0,...,120\}^2$ which satisfied the requirement of yielding means for the six variables between the minimal and maximal experimentally observed values. This was the parameter combination *(4,3,3,2)*. In table 4.9, we compared the mean values of each of the 1000 simulations of 6 statistical variables which are listed in table 4.9 with minimum and maximum values of the experimental data.

Additionally we could show that the vector (4,3,3,2) minimizes the sum of normalized quadratic deviations of experimental data and simulation results of the six variables. The quadratic deviations where normalized by division by the standard deviations of the experimental results over the treatments.

Figure 4.26 shows the distribution of the mean player on *B* for the simulated vector (4,3,3,2) in 1000 simulations.

	Experiment	Simulations	Experiment
CII	**Minimum**	**{4,3,3,2}**	**Minimum**
Player on B [mean]	5.85	5.95	6.17
Player on B [standard deviation]	1.59	1.65	1.99
Changes [mean]	4.62	5.17	5.38
Period of last Change	64.78	83.73	90.39
Yule Q [mean]	0.11	0.14	0.39
Yule Q [standard deviation]	0.53	0.61	0.75

Table 4.9: *CII – 18 Players - experimental minima & maxima vs. simulation means.*

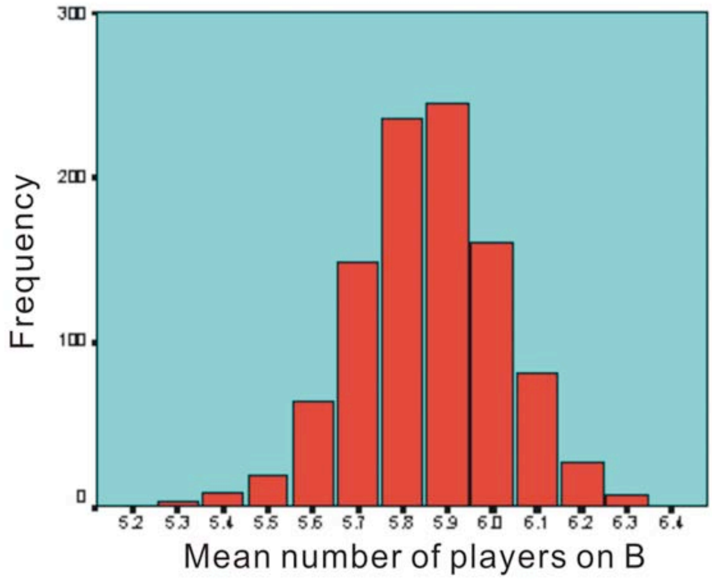

Figure 4.26: *Distribution of the mean number of players on* B *for the simulated vector (4,3,3,2) in 1000 simulations.*

Remark: At the beginning of the game the players know that the capacity of A is greater than the capacity of *B*. It seems to be reasonable to suppose that at least in the beginning the pure strategies *A* and *B* have a greater propensity sum than *direct* and *contrarian*. Like in CI, no initial propensity which contains only pure strategies fits the experimental data. Further on it seems reasonable that the initial value for *A* is greater than the initial value for *B*. The reason for this is the game theoretical equilibrium in the experiments with human players. The equilibrium shows a higher value for *A* (equilibrium in the experiments with 18 players was *A*:12 *B*:6). In the experiments occur more direct player types 40% and less contrarian player types 20% (SELTEN ET AL. (2004)). This ratio could be found in the simulations, where the 3 represents the initial value for the direct strategy and 2 represents the initial value for the contrarian strategy.

4.3.3. Simulations of CII with 18, 36, 54, 72, and 90 Players

Finally we compared the mean of six statistical variables of 1000 simulations with 18, 36, 54, 72, and 90 players with experiments of the same number of players. For simulations with 18λ players we used the initial propensities $\lambda \cdot (4,3,3,2)$, $\lambda = 2,\ldots,5$. The vector *(4,3,3,2)* has been determined in section 4.3.2.

In the transition from 18 to 18λ = 36, 54, 72, 90 players the road capacity is also increased by λ:

$$t_A = 6 + 2\frac{n_A}{\lambda} \qquad t_A = 12 + \frac{3n_B}{\lambda}$$

Moreover the payoffs in points have also be miltiplied by λ in order to obtain integer numbers of pointsfor all pairs (n_A, n_B)

$$p_A = \lambda\ (40 - t_A) \qquad p_B = \lambda\ (40 - t_B)$$

.
The initial propensities may be thought of as „prior sums“ and should therefore also be multilplied by λ in the same way as the payoff.

Additionally we could show that the vector λ (4,3,3,2) minimizes the sum of normalized quadratic deviations of experimental data and simulation results of the six variables. The quadratic deviations where normalized by division by the standard deviations of the experimental results over the treatments.

Statistical Data CII	Data Source	Number of Players				
		18	36	54	72	90
Mean (# players on B)	E	5.98	12.21	17.98	24.2	30.02
	S	5.95	11.91	17.9	23.83	29.02
st. Dev. (# players on B)	E	1.78	2.64	3.24	4.54	5.02
	S	1.65	2.39	3.04	3.78	4.58
Mean (# of place changes)	E	4.82	11.35	15.57	22.76	26.02
	S	5.17	10.07	15.98	21.32	23.04
Mean (last place change	E	81	82	86	89	88
	S	84	89	84	88	90
Mean (Yule-coefficient)	E	0.28	0.14	0.22	0.2	0.24
	S	0.14	0.16	0.15	0.15	0.16
st. Dev. (Yule-coefficient)	E	0.58	0.58	0.58	0.57	0.6
	S	0.61	0.54	0.54	0.52	0.56

Table 4.10: *CII –Experimental means (E) vs. Simulation means (S).*

4.4. Conclusion

We have run simulations based on a payoff sum reinforcement model. We applied this model on two similar experimental set ups CI and CII. Simulated mean values of six variables have been compared with the experimentally observed minimal and maximal of these variables. The simulated means were always in this range. Only four parameters of the simulation model, the initial propensities, were estimated from the data. In view of the simplicity of the model, it is surprising that one obtains a quite close fit to the experimental data. With the appropriate linear transformation of the initial propensity, the simulations fit experimental results with a higher number of players.

Two response modes can be found in the experimental data, a *direct* one in which changes follow bad payoffs and a *contrarian* one in which changes follow good payoffs. One can understand these response modes as due to different views of the causal structure of the situation. If one expects that *A* is crowded in period t, and *A* is likely to be crowded in period t+1 one will be in the direct response mode. But if one thinks that many people will change in the next period because it was crowded today, one has reason to be in the contrarian response mode.

The strategies *direct* and *contrarian* are necessary to be represented in the simulations for fitting the experimental data. They appear in the simulations as the result of an endogenous learning behaviour by which initially homogeneous subjects become differentiated over time. A sample simulation for 9 players over 1000 periods is shown in figure 4.25. Each player has a specific colour. The grey line indicates the separation of the player's payoff sums at period 100.

It is surprising that a very straightforward reinforcement model reproduces the experimental data as well as shown by table 4.4. Even the mean Yule coefficient is in the experimentally observed range in spite of the fact that at the beginning of the simulation the behaviour of all simulated players is exactly the same. It is not assumed that there are different types of players.

5. Who are the smarter drivers? The Chinese or the Germans? An Experimental Approach

5.1. Introduction

This paper reports about laboratory experiments concerning traffic behaviour of participants with different cultural backgrounds. We used a classfication system for behavioural types, which was introduced by (SELTEN ET AL. 2007). It can be shown that different cultural backgrounds may have an influence on the cognitive decision process in binary choice situations; we used a route choice scenario. Two subject pools with 54 participants each were analysed:

1. German students at the University of Bonn (Germany).
2. Chinese students at the Shanghai Jiao Tong University and Nankai University (China).

Obviously the traffic situation in Chinas densely populated cities differs from the German and most Central European areas. The traffic in China's cities is much more heterogeneous. Especially in Shanghai are more bikes, motorcycles, pedestrians, cars and busses on the road at the same time. In Germany as in most other countries of the European Union there are often extra lanes for busses, taxies and bikes. Our approach is not comparing the traffic situations inherently, but it could give a better understanding whether traffic participants in China act more anticipatory in view of the more complex situation on the roads, than the German traffic participants. It seems necessary to react in a different way in China.

Cross Cultural Studies have become an important field in experimental economics. The most common experimental setups deal with various specifications of the ultimatum game, the trust game, the dictator game as well as public good games was extensively discussed for example in BUCHAN (1997), BOTELHO (2000), 2004), BURNS (2004), CARPENTER (2004), CHUAH (2005), HENRICH (2000, 2001, 2004, 2005), OOSTERBEEK (2004) and ROTH (1991).

Characteristic for a traffic situation worldwide is that many subjects have to interact without a negotiation procedure. Since there is an inherent lack of communication, optimal coordination is rather unlikely. The only way to increase individual benefit, what means to decrease individual travel time, is to adapt individual decisions to the behavior of the other participants; which could be observed in the past. To model such a situation we used a simple Minority Game.

The Minority Game is an example of a *n*-person game with no strict pure equilibria and can be applied on different situations including their specific social and economic contexts. The Minority Game, which is also called the El Farol Bar Problem and was introduced by (ARTHUR 1994) and theoretically analysed in detail by (CHALLET, ZHANG, 1997, 1998). There is already some literature about experimental studies of the game. HELBING ET AL. (2005), RENAULT ET AL. (2005), CHMURA & PITZ (2006), BOTTAZZI & DEVETAG (2007) and KETS & VOORNFELD (2007).

The rules of the Minority Game can be described in a short way: a number of agents *n* have to choose during several periods whether to enter a given room A or a room B. Those agents who choose the less crowded room win whereas the others lose. Our aim is to present Minority Game experiments with a large number of periods and with sufficiently many independent observations for meaningful applications of non-parametric significance tests.

Market entry games (RAPOPORT ET AL 2002, EREV AND RAPOPORT 1998) are another kind of games found in experimental literature, which can be compared in some aspects with the Minority Game. In these types of games players usually have the choice either to enter a market or to stay out of it. The payoff for entering the market is a decreasing function of the number of entrants. The payoff for staying out is a constant opportunity cost. One may say that the route choice game is similar to a market entry game with two markets instead of one. However, the players do not have the choice to stay out of both markets.

5.2. *Experimental setup*

The experiments were conducted during September and November 2006. The German sessions were run at the BonnEconLab at the University of Bonn, Germany. The first three Chinese sessions are located at the Reinhard Selten Lab at Nankai University of Tianjin, China and the Chinese sessions 4, 5, 6 are located at the Vernon Smith Experimental Lab of the Shanghai Jiao Tong University. At the 3 universities students from several departments participated.

Experiments were run by local helpers comprehensively instructed and supported by the authors, who stayed in the background. We are aware that this might result in an experimenter effect. We decided to choose this procedure to avoid self-presentation and face-saving effects (BOND & HWANG, 1986) of inexperienced subjects resulting from the presence of people from foreign countries. Since we are interested in the pure presentation effect this procedure seems to be justified. Instructions were written in neutral language.

To avoid translation errors regarding the task and the cadence instructions were translated by natural speakers from German into the corresponding language and afterwards translated back into German applying the back-translation method (BRISLIN, 1970).

For this survey, the Minority Game was transferred to a route choice context (CHMURA & PITZ 2006). In these experiments, subjects were told that in each of the periods 0 to 100, they had to make a choice between a road *A* and a road *B* for travelling from X to Y. Six sessions were run with German and six sessions with Chinese participants. The number of subjects in each session was 9. They were told that the travel times t_A and t_B on road *A* and *B* depended on the numbers n_A and n_B of participants choosing *A* and *B* respectively:

$$(5.1) \qquad t_A = 1, t_B = 0 \;\Leftrightarrow\; n_A < n_B \text{ and } t_B = 1, t_A = 0 \;\Leftrightarrow\; n_A > n_B .$$

The period payoff was t_A if *A* was chosen and t_B if *B* was chosen. The total payoff of a subject was the sum of all period payoffs (Taler) converted proportionally to money payoffs in Euro respectively RMB. No further information was given to the subjects. The conversion rate was 1 Taler = 0.4 € in Germany and 1 Taler = 2 RMB in China. The difference of the conversion rate can be explained by the Laboratory standard payoff in each country. The experimental data were obtained in 12 sessions with 9 subjects each and 108 altogether. The computer program was based on the toolbox RatImage developed by ABBINK & SADRIEH (1995).

A number of experiments on route choice behaviour could be found in the literature (e.g. BONSALL 1992, MAHMASSANI & LIU 1999, SELTEN ET AL. 2007, CHMURA & PITZ 2006). Here, we focus on the route choice behaviour in a generic two route scenario, which has already been investigated in the scientific literature (e.g. IIDA ET AL. 1992). In HELBING ET AL (2002) volatile dynamics of decisions independent of an optimal payoff distribution were observed in route choice experiments. It could be shown that specific guidance strategies are able to increase the performance of all users by reducing overreaction and stabilizing the decision dynamics. In DE MARTINO (2004) a model for analysing the emergent collective behaviour of drivers in a city was discussed. The results proved that in absence of information noise, inductive drivers turn out to behave in a more effective way than random drivers during periods of low car density, while high car densities produce the opposite effect.

In this paper, special emphasis shall be laid on a comparison of the participants' reactions to the immediately preceding payoffs. The results showed that differences in behaviour are observed between the culturally divergent groups.

5.3. Experimental results

In this section, we explain the main statistical findings while later in the subchapters we will explain the results in view of the response modes and the cumulative payoff.

5.3.1. Descriptive statistics for the Chinese and the German treatment

The basic statistical findings are shown in table 5.11. Figure 5.27 shows the number of participants on road A as a function of time for a typical observation of the Chinese participants and the German group. The mean number of players on road A is 4.5 in the Chinese group and 4.49 in German group. That the mean is so close to the mixed equilibrium was the expected outcome since the experimental setup does not suggest a preference for one road. The Minority Game with 9 players has $2\binom{9}{4} = 252$ (non strict) Nash equilibria in pure strategies.

The lack of strict pure strategy equilibria poses a coordination problem that may be one of the reasons for non-convergence and the persistence of fluctuations in both treatments. The mean number of players for the Chinese and the German observations are shown in figure 5.28. There is no significant difference between the German and the Chinese treatment for the mean numbers of players choosing the road A.

		cumulative payoff (mean)	number of players on A (mean)	number of road changes (mean)	Yule (mean)	Spearman rank correlation road changes vs. cumulative payoff
German treatment	sess. I 01	37	4.33	5.08	.1369	-.48
	sess. I 02	36	4.74	3.87	.1468	.34
	sess. I 03	36	4.41	5.16	.2694	-.44
	sess. I 04	38	4.4	5.19	.0122	-.7
	sess. I 05	37	4.65	5.28	.1128	-.18
	sess. I 06	38	4.44	4.35	-.0083	-.18
	treat. I	**37**	**4.50**	**4.82**	**.1116**	**-.27**
Chinese treatment	sess. II 01	38	4.23	3.99	-.1295	-.49
	sess. II 02	38	4.46	3.68	.1281	-.35
	sess. II 03	37	4.49	4.97	-.0245	-.42
	sess. II 04	37	4.57	5.57	.0916	-.63
	sess. II 05	38	4.59	3.39	-.0029	-.35
	sess. II 06	39	4.59	3.36	-.0747	-.52
	treat. II	**37.83**	**4.49**	**4.16**	**-.0020**	**-.46**

Table 5.11: *Statistical data of the experiments.*

It seems that there is no outstandingly advisable strategy for the participants to enhance their payoffs because due to the symmetry of the game, each road has the same properties. However, one can see in the next section that in some cases, certain types of reactions to former payoffs are more successful than others.

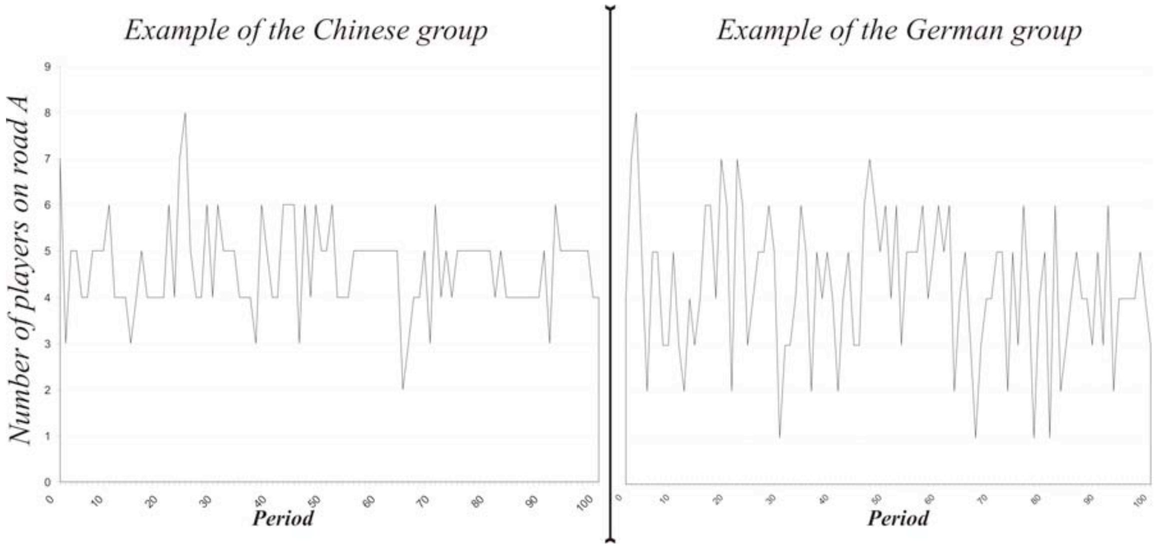

Figure 5.27: *Number of participants on* A: *a typical session of the German and the Chinese group*

Table 5.1 also shows the mean number of road changes, the mean Yule-coefficient and cumulative payoff as well as the spearmen rank correlation coefficient for the number of road changes versus the cumulative payoff. All these values for the German treatment are significantly different from the Chinese treatment. We will try to explain this in section 5.3.2 and 5.3.3

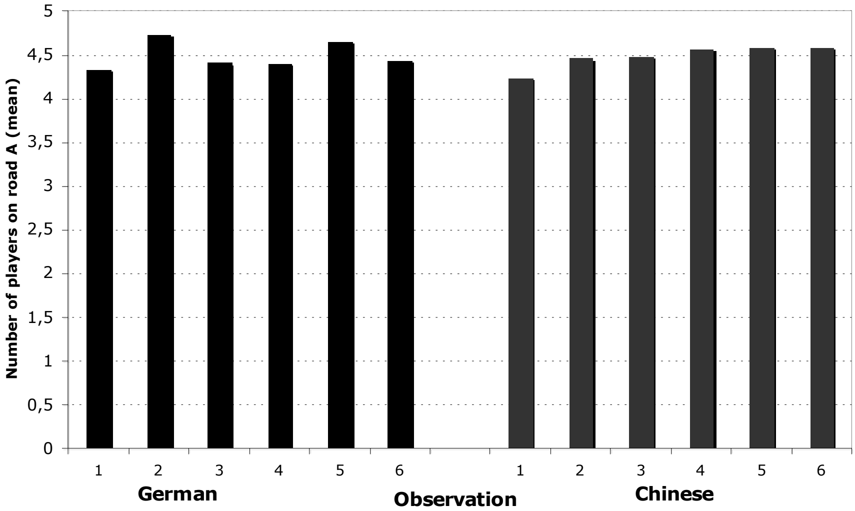

Figure 5.28: *Number of participants on road A for the German and the Chinese treatment.*

5.3.2. A classifier system of response modes

We used a classifier system for behavioural types introduced by (SELTEN ET AL. 2007) to describe reactions of former payoffs. The classifier system can be described as follows: A participant who had a payoff 0 (1) on the road chosen may change the road (stay on the same road) in the next period in order to travel on a less crowded route. We call this the *direct* response mode. The *direct* response mode is the prevailing one but there is also a *contrarian* response mode. The contrarian participant expects that a payoff 1 will attract (deter) many others and that therefore the road chosen will be crowded (free) in the next period.

For each subject, let c_- (c_+) be the number of times in which a subject changes the roads when the payoff in the period before was p=0 (p=1). And for each subject let s_- (s_+) be the number of times in which a subject stays on the road when there was a payoff s=0 (s=1) in the period before.

	change	stay
p=0	c_-	s_-
p=1	c_+	s_+

Table 5.12*: 2x2 table for the computation of Yule-coefficients.*

For each subject such a 2x2 table has been determined and a Yule-coefficient Q has been computed as follows.

(5.2) $$Q = \frac{c_- \cdot s_+ - c_+ \cdot s_-}{c_- \cdot s_+ + c_+ \cdot s_-}$$

The Yule coefficient has a range from −1 to +1. Participants with a "high" Yule-coefficient near to 1 (-1) tend to be direct (contrarian).

5.3.3. Observed Response mode

To classify behavioural types we used the Yule-coefficient we described this already in section 5.3.1. The mean Yule-coefficients are significantly higher in the German treatment (see figure 5.30). The null-hypothesis for both treatments is rejected by a Wilcoxon-Mann-Whitney-Test on the significance level of 5% (one-sided). That means that there are less contrarian response modes in the German treatment.

The reason for the smaller Yule-coefficients in the Chinese treatment lies in the fact that contrarian reactions to former payoffs occur more frequently in this group. One can see in table 5.11. that the number of road changes per round in the German treatment is significantly higher than in the Chinese treatment. The null-hypothesis for both treatments is rejected by a Wilcoxon-Mann-Whitney-Test on the significance level of 1% (one-sided). Since the players' mean payoff (for all the experiments) is 37.41 and since therefore a player receives more „bad" than „good" payoffs on average, the decline in road changes in the treatment of Chinese participants is another indicator for an increase of contrarian behavioural types. The number or road changes for both treatments is graphically shown in figure 5.29.

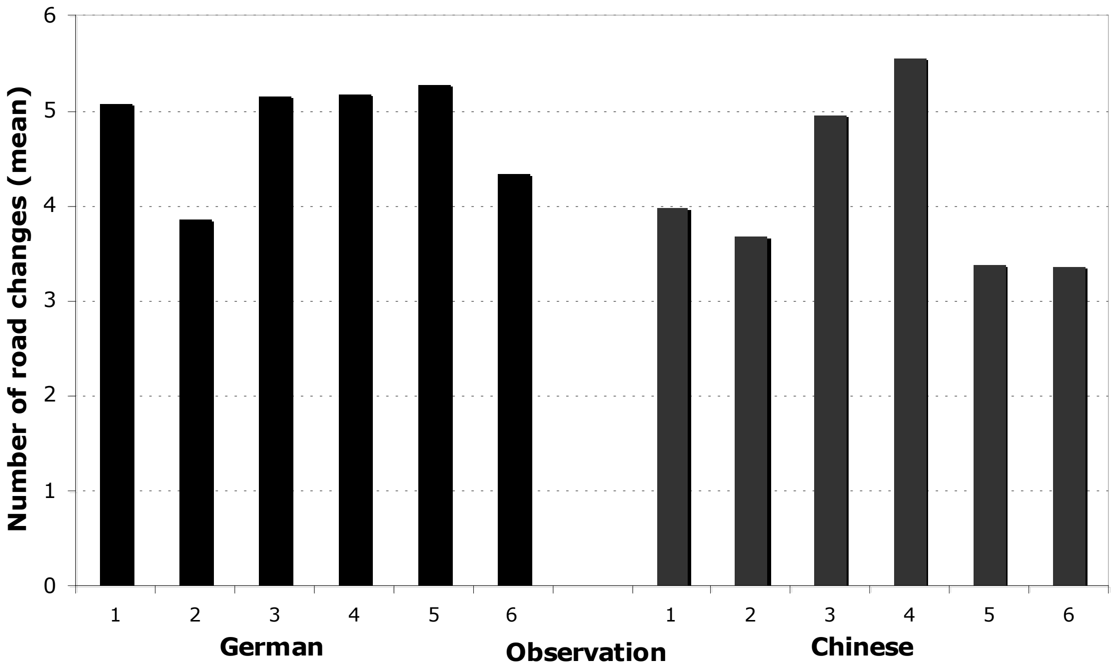

Figure 5.29: *Mean number of road changes for the German and the Chinese treatment.*

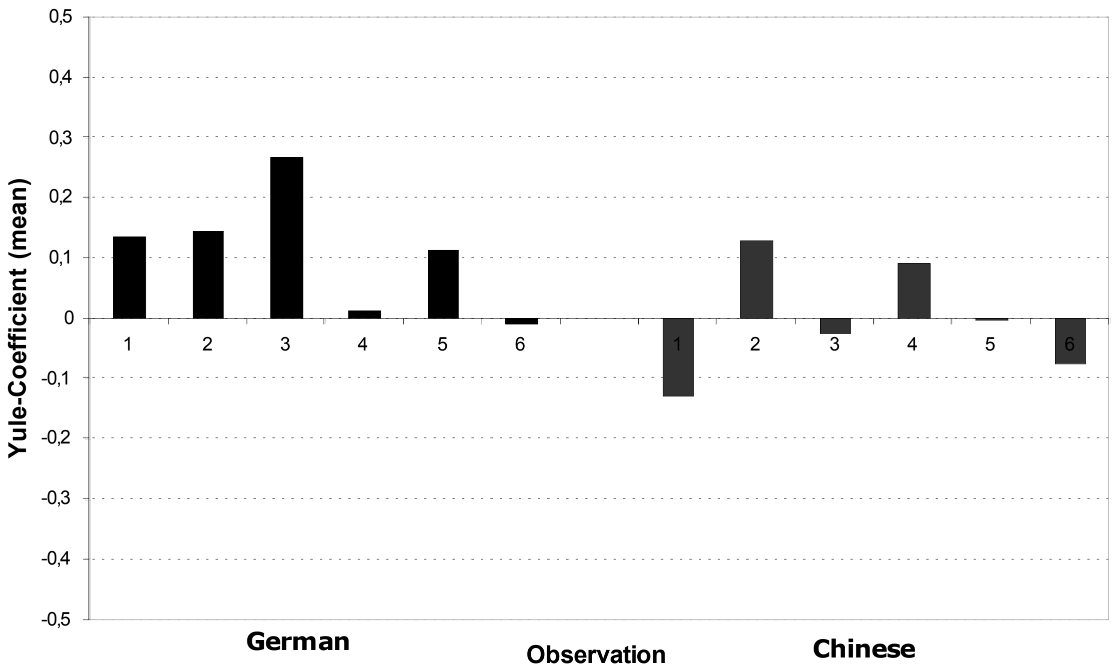

Figure 5.30: *Yule-Coefficient for the German and the Chinese treatment.*

5.3.4. Cumulative Payoff

In (CHMURA & PITZ 2006), it was already pointed out that a negative correlation exists between the cumulative payoff and the frequency of road changes of a player. Figure 5.31. shows the mean cumulative payoff for the German and the Chinese treatment. As shown in table 5.11., the Spearman rank correlation coefficient is negative for all Chinese sessions and 5 German sessions. This also is shown in figure 5.32. Since the contrarian response mode could be observed more frequent in the Chinese treatment and thus, the number of "good" payoffs was on average higher than of the "bad" payoffs, it could be expected that the Chinese players would on average receive better results than the group with German participants. Indeed, table 5.11 shows that the mean payoff per session is higher in Chinese observations than in the German observations. The related null-hypothesis was rejected by a Wilcoxon-Mann-Withney-test on the significance level of 5% (one-sided). In the case of the Minority Game, the contrarian response mode of the Chinese participants is the more promising strategy.

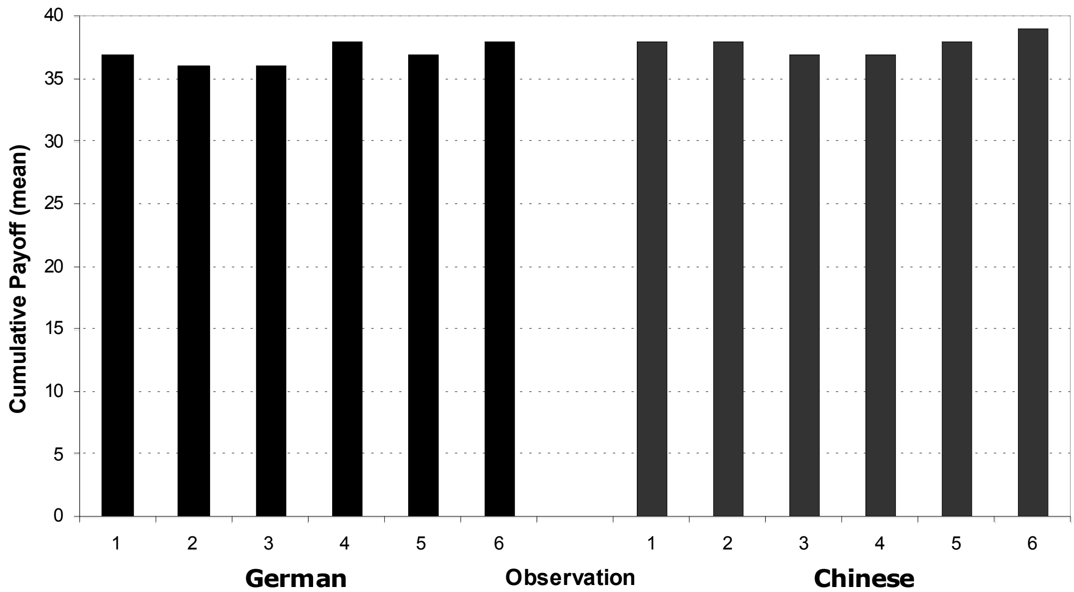

Figure 5.31: *Mean cumulative payoff for the German and the Chinese treatment.*

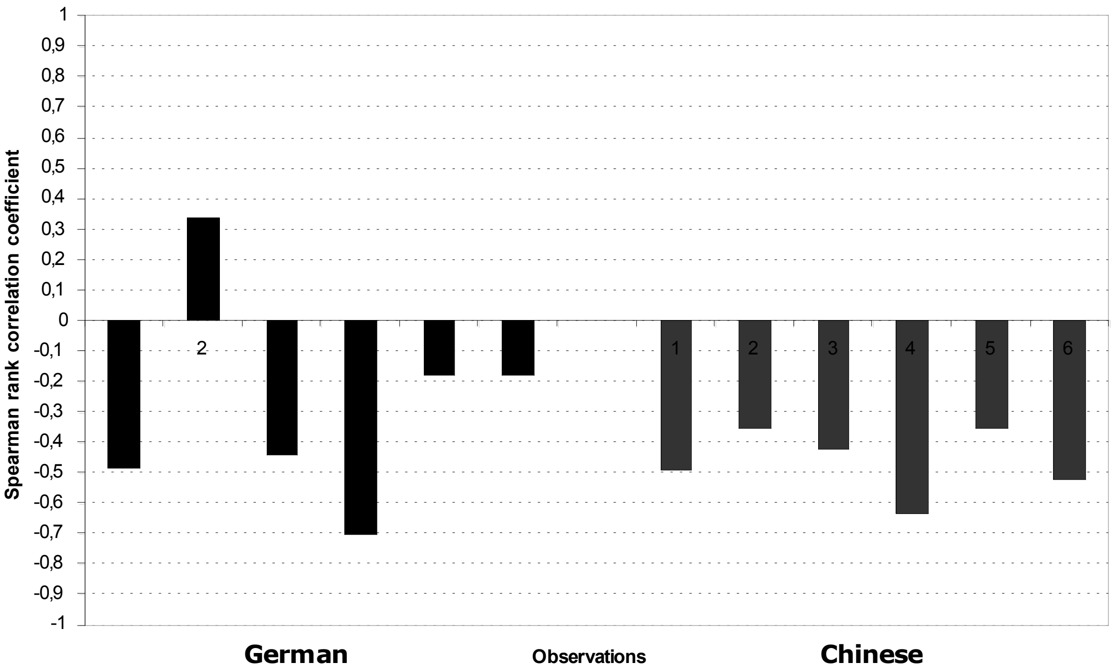

Figure 5.32: *Spearmen rank correlation coefficient for the cumulative payoff vs. the number of road changes.*

5.4. Conclusion

In this paper we discussed an elementary traffic scenario, modelled as a minority game with subjects of different cultural backgrounds. We found two response modes using the Yule-coefficient. The first response mode is a direct response and the second a contrarian response to the received payoff in the last period. The reactions of participants of the two investigated groups were significantly different. The German subjects reacted in a more direct way than the Chinese, i. e. by the above definition of direct, that they chose the same road after good payoffs and changed after bad payoffs. Due to the different behaviour and the structure of the minority game the average payoff of the German subjects in this game was lower than the average payoff of the Chinese. The less direct reactions of the Chinese participants may be caused by their different experience in their daily traffic situation. In a crowded inhomogeneous traffic situation a contrarian reaction, which anticipates, the possible reactions of the other participants more severely than the direct response mode, seems to be reasonable.

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Appendix

Appendix 2.A: Table of Realative Frequencies

Observation	Game 1		Game 2		Game 3		Game 4		Game 5		Game 6	
	U	*L*	*U*	*L*	*U*	*L*	*U*	*L*	*U*	*L*	*U*	*L*
1	0.104	0.716	0.255	0.583	0.218	0.836	0.291	0.748	0.154	0.873	0.453	0.604
2	0.079	0.640	0.175	0.510	0.154	0.716	0.230	0.818	0.378	0.690	0.439	0.621
3	0.091	0.794	0.156	0.431	0.210	0.778	0.320	0.714	0.358	0.676	0.430	0.591
4	0.109	0.688	0.210	0.616	0.217	0.844	0.245	0.748	0.276	0.648	0.398	0.604
5	0.085	0.571	0.240	0.409	0.154	0.700	0.318	0.684	0.341	0.635	0.444	0.619
6	0.059	0.730	0.151	0.601	0.232	0.785	0.360	0.718	0.320	0.659	0.389	0.654
7	0.184	0.575	0.286	0.591	0.081	0.856	0.283	0.723	0.295	0.689	0.463	0.574
8	0.044	0.770	0.195	0.580	0.170	0.795	0.284	0.661	0.329	0.659	0.421	0.544
9	0.048	0.750	0.225	0.563	0.093	0.723	0.371	0.750	0.353	0.561	0.438	0.626
10	0.056	0.755	0.229	0.448	0.133	0.873	0.249	0.805	0.328	0.651	0.535	0.594
11	0.034	0.524	0.206	0.551	0.164	0.829	0.266	0.741	0.366	0.583	0.428	0.560
12	0.056	0.768	0.275	0.441	0.130	0.778	0.213	0.720	0.431	0.640	0.505	0.566
Mean of 12	**0.079**	**0.690**	**0.217**	**0.527**	**0.163**	**0.793**	**0.286**	**0.736**	**0.327**	**0.664**	**0.445**	**0.596**

Observation	Game 7		Game 8		Game 9		Game 10		Game 11		Game 12	
	U	*L*	*U*	*L*	*U*	*L*	*U*	*L*	*U*	*L*	*U*	*L*
1	0.151	0.531	0.199	0.571	0.164	0.744	0.451	0.745	0.274	0.645	0.441	0.653
2	0.103	0.563	0.180	0.665	0.105	0.793	0.416	0.711	0.289	0.659	0.414	0.653
3	0.176	0.596	0.246	0.529	0.188	0.839	0.299	0.634	0.336	0.688	0.431	0.559
4	0.178	0.575	0.341	0.610	0.299	0.869	0.365	0.729	0.410	0.631	0.463	0.568
5	0.090	0.530	0.314	0.585	0.355	0.844	0.416	0.713	0.378	0.678	0.458	0.664
6	0.149	0.586	0.220	0.559	0.413	0.874	0.246	0.665	0.301	0.611	0.428	0.529
Mean of 6	**0.141**	**0.564**	**0.250**	**0.586**	**0.254**	**0.827**	**0.366**	**0.699**	**0.331**	**0.652**	**0.439**	**0.604**

Table A1.13: *Relative frequencies of* U *and* L *in the 108 independent subject groups for games 1-12.*

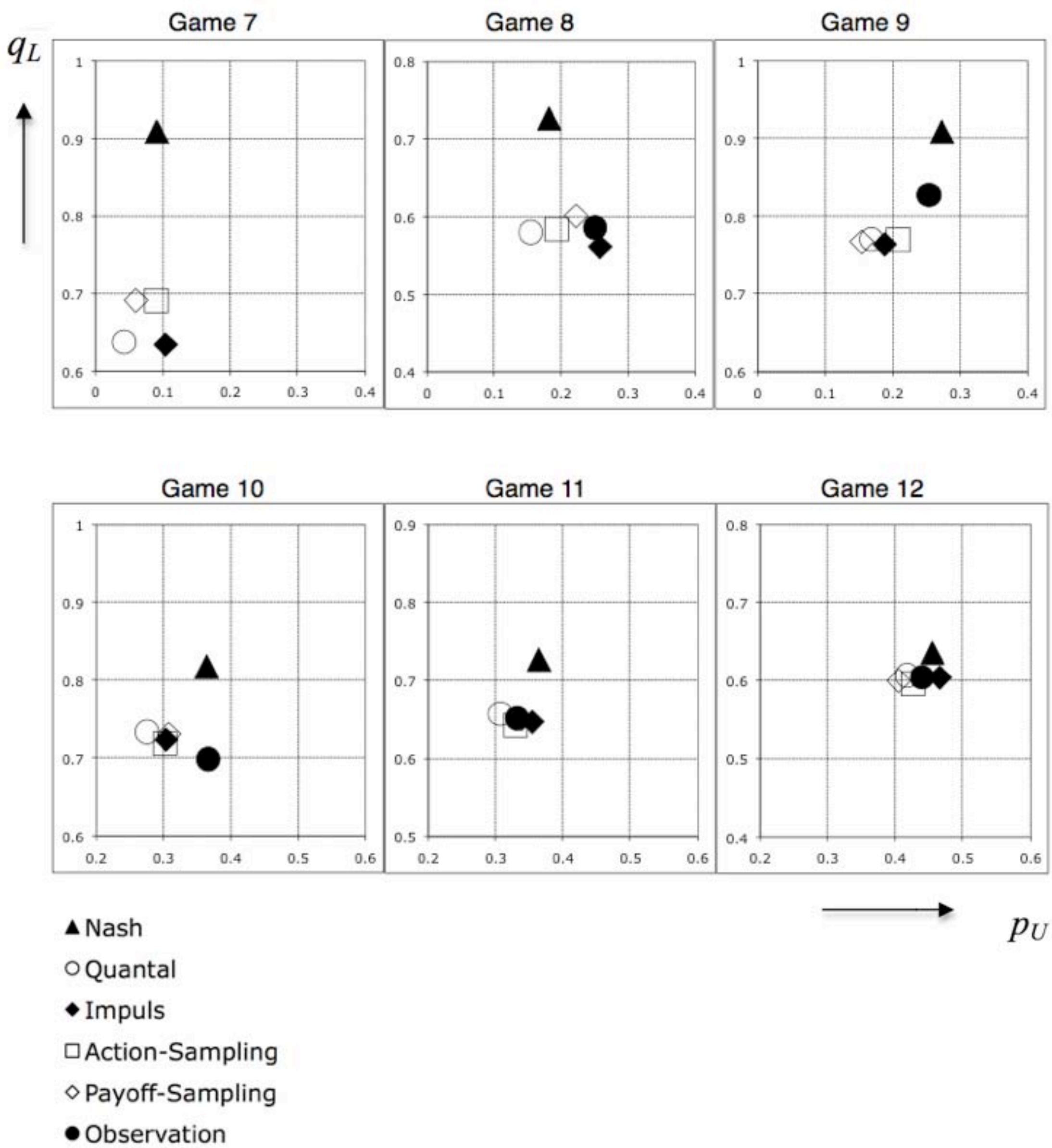


***Figure 2.A1.33:** Visualization of the theoretical equilibria and the observed average in the non-constant sum games.*

* In the cutout for game 11 the symbol for payoff sampling equilibrium is covered by the symbols for observation and Nash equilibrium

Appendix 2.B: Written instructions

Merkblatt zum Matrixexperiment

An diesem Experiment nehmen 16 Personen teil. Jeder Teilnehmer ist entweder ein Spieler 1 oder ein Spieler 2. Diese Rolle behalten Sie über die ganze Dauer des Experimentes bei.

Das Spiel erstreckt sich über 200 Runden.

In jeder Runde spielt jeder Spieler 1 mit einem Spieler 2. Die 8 Spielerpaare werden in jeder Runde neu zufällig zusammengestellt.

Auf dem Bildschirm sehen sie eine Matrix mit vier Feldern.

In jeder Runde haben sie die Möglichkeit zwischen Zeile A oder Zeile B zu wählen.

Ihre eigene Auszahlung ist auf dem Bildschirm umrandet dargestellt. Ihre Auszahlung hängt von ihrer eigenen Wahl und der Wahl des anderen Spielers ab. Nachdem Sie diese Wahl getroffen haben, färbt sich Ihre gewählte Zeile rot. Nach der Wahl des anderen färbt sich das Feld gelb, in dem der Betrag steht, der ihnen ausgezahlt wird.

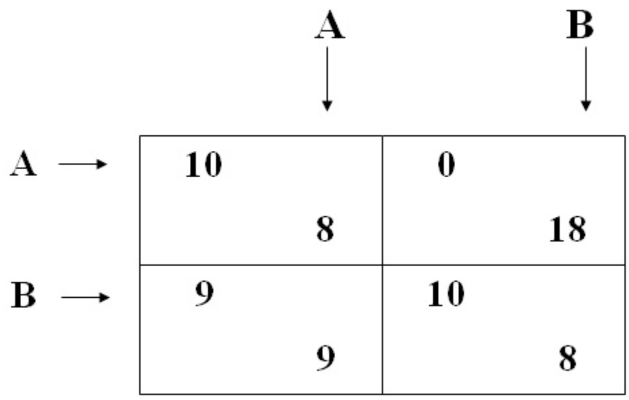


Figure 2.B1.34: *Schematics of game-matrix.*

Es gibt zwei Gruppen von Spielern. In jeder Gruppe hat jeder Spieler dieselbe Matrix, aber die Matrizen sind für beide Gruppen verschieden. Sie spielen immer mit einem Spieler aus der anderen Gruppe.
In jeder Runde werden 8 Spielerpaare zufällig zusammengestellt. Ihnen wird also in jeder Runde ein neuer Mitspieler zugelost. Ihre Mitspieler haben immer dieselbe Matrix.

Nach jeder Runde wird Ihnen mitgeteilt welche Auszahlung sie in der letzten Runde erhielten. Der Umrechnungskurs für ihre Auszahlung wird Ihnen auf dem Bildschirm

bekannt gegeben.

Appendix 2.C: Screenshot of Game

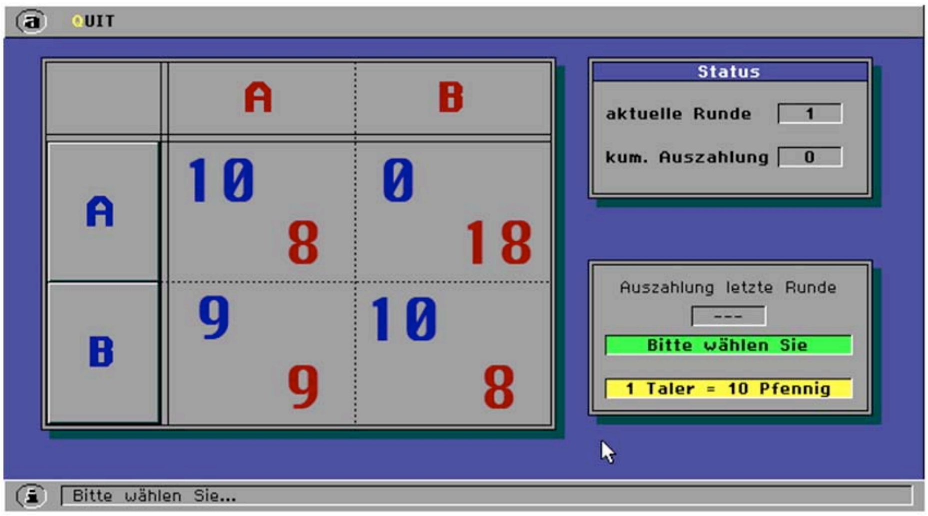


Figure 2.C1.35: *Screenshot of the RatImage Program.*

Appendix 2.D: Monotonicity, Existence and Uniqueness

Nash equilibrium is uniquely determined in completely mixed 2x2-games. This is clear from what has been said in II.A. However, it is not obvious that each of the four other concepts determines a unique stationary equilibrium for every completely mixed 2x2 game. In the following it will be shown that this is the case.

As we shall see for each of the five concepts with the exception of Nash equilibrium the curve for p_U is monotonically increasing and the curve for q_L is monotonically decreasing and both curves have a unique intersection. In the following this will be discussed for every concept separately.

2.D1 Quantal Response Equilibrium

In the following we shall drop the arguments q and p of $E_U(q)$, $E_L(p)$ and $E_D(p)$ $E_R(p)$. This can be done without any danger of confusion. The curves for p_U and q_L can the be written as follows

(2.D1) $$p_U = \frac{e^{\lambda E_U}}{e^{\lambda E_U} + e^{\lambda E_D}}$$

(2.D2) $$q_L = \frac{e^{\lambda E_L}}{e^{\lambda E_L} + e^{\lambda E_R}}$$

As we have seen in II.B. the constants a_L, a_R, b_U and b_D have no influence on the right hand sides of the two equations. Therefore we can assume that all these four constants are zero. This leads to the following formulas for the expected payoffs E_U, E_D, E_L and E_R.

(2.D3) $E_U=q_Lc_L$ $\quad$ $E_D=(1-q_L)c_R$ $\quad$ $E_L=(1-p_U)d_D$ $\quad$ $E_R=p_Ud_U$

Define

(2.D4) $X = e^{\lambda(E_D-E_U)} = e^{\lambda[c_R-(c_L+c_R)q_L]}$, $\quad$ $Y = e^{\lambda(E_R-E_L)} = e^{\lambda[(d_U+d_D)p_U-d_U]}$

With the help of the auxiliary variables X and Y the two equations for p_U and q_L can be rewritten as follows

(2.D5) $$p_U = \frac{1}{1+X} \qquad q_L = \frac{1}{1+Y}$$

We have

(2.D6) $$\frac{\partial X}{\partial q_L} = -\lambda(c_L + c_R)X, \qquad \frac{\partial Y}{\partial q_L} = \lambda(d_U + d_D)Y$$

This yields

(2.D7) $$\frac{\partial p_U}{\partial q_L} = \lambda(c_L + c_R)\frac{X}{(1+X)^2}, \qquad \frac{\partial q_L}{\partial p_L} = -\lambda(d_U + d_D)\frac{Y}{(1+Y)^2}$$

Since $\lambda, c_L, c_R, d_U, d_D$ as well as X and Y are positive it follows that p_U is an increasing function of q_L and q_L is a decreasing function of p_U. We have

(2.D8) $$p_U(0) = \frac{1}{1+e^{\lambda c_R}}, \qquad p_U(1) = \frac{1}{1+e^{-\lambda c_L}}$$

The formulas for $p_U(0)$ and $p_U(1)$ permit the conclusion that

(2.D9) $$0 < p_U(0) < \frac{1}{2} < p_U(1) < 1$$

holds. This means that the curve for p_U goes from the left border of the (p_U,q_L)-diagram to the right one. Similarly it can be seen that

(2.D10) $$1 > q_L(0) > ½ > q_L(1) > 0.$$

holds. Therefore the curve for q_L goes from the upper border of the (p_U,q_L)-diagram to the lower one. In view of the monotonicity properties of the curves it is clear that they have a unique intersection.

2.D2 A property of the binomial distribution

Consider a binomial distribution. Let q be the probability of a success in one trial. We use the notation $B(k,n,q)$ for the probability of at least k successes in n trials. This probability is as follows

(2.D11) $$B(k,n,q) = \sum_{j=k}^{n} \binom{n}{j} q^j (1-q)^{n-j}$$

For k=0,…,n and $0<q<1$. For the interpretation of the right hand side of this equation we adopt the convention

(2.D12) $$0^0=1$$

With this convention the formula also holds for q=0 and q=1.

We now show that for n=1,2,… and k=1,2,…,n the probability $B(k,n,q)$ is a monotonically increasing function of q in the interval $0\le q\le 1$.

In order to do this we look at n continuous random variables $R_1,\ldots,R_n$ independently and uniformly distributed over the interval [0,1]. The probability that at least k of the realizations $r_1,\ldots,r_n$ of $R_1,\ldots,R_n$, resp., satisfy $0\le r_i\le q$ with $q\le 1$ is $B(k,n,q)$.

Consider two numbers q and q' with $0\le q<q'\le 1$. A realization vector $(r_1,\ldots,r_n)$ with at least k components satisfying $0\le r_i\le q$ also satisfies $0\le r_i\le q'$ for these components. Moreover there is a positive probability for realization vectors $(r_1,\ldots,r_n)$ with at most k-*1* components satisfying $0\le r_i\le q$ but at least k components with $0\le r_i\le q$. This shows that $B(k,n,q)$ is monotonically increasing in q for k=1,…,n.

Of course, the case k=0 is different. The probability of at least zero successes is always 1. We have $B(0,n,q)=1$ regardless of the value of q.

It can be seen immediately that we have

(2.D13) $B(k,n,0)=0$ and $B(k,n,1)=1$, for $k=1,\ldots,n$ and $n=1,2,\ldots$

2.D3 Action-sampling Equilibrium

The equations of the curves for p_U and q_L in the case of action-sampling equilibrium and the functions α_U and α_L have been described in II.D.. Let k^* be the smallest integer k with $\alpha_U(k)\geq 0$. Similarly let m^* be the smallest integer with $\alpha_L(m^*)\geq 0$. We cannot have $k^*=0$ since this would imply $0\geq nc_R$ contrary to $c_R>0$. A similar argument excludes $m^*=0$. We have

(2.D14) $k^* \geq 1$ and $m^* \geq 1$

The two equations of the curves for p_U and q_L can be written as follows:

(2.D15)
$$p_U = \begin{cases} B(k^*,n,q_L) & for\ \alpha_U(k^*)>0 \\ \frac{1}{2}B(k^*,n,q_L)+\frac{1}{2}B(k^*+1,n,q_L) & for\ \alpha_U(k^*)=0 \end{cases}$$

(2.D16)
$$q_L = \begin{cases} B(m^*,n,1-p_U) & for\ \alpha_L(m^*)>0 \\ \frac{1}{2}B(m^*,n,1-p_U)+\frac{1}{2}B(m^*+1,n,1-p_U) & for\ \alpha_L(m^*)=0 \end{cases}$$

Since $B(k,n,0)$ is increasing in q for $k>1$ we can conclude that the curve for p_U is monotonically increasing in q_L. Similarly the curve for q_L is monotonically decreasing.

In view of $B(k,n,0)=0$ and $B(k,n,1)=1$ for $k>0$ it is clear that the curve for p_U begins at $(p_U,q_L)=(0,0)$ and ends with $(p_U,q_L)=(1,1)$. Similarly the curve for q_L begins at $(p_U,q_L)=(0,1)$ and ends at $(p_U,q_L)=(1,0)$. Obviously the two curves have exactly one intersection. Consequently the action-sampling equilibrium for sample size n is uniquely determined for completely mixed 2x2-games.

2.D4 Payoff-sampling Equilibrium

The equations of the curves for p_U and q_L in the case of payoff-sampling equilibrium and the functions β and γ appearing there have been described in II.E. Let k_U and k_D be the numbers of L's in player 1's sample for U and D, respectively. Similarly m_L and m_U are the numbers of D's in player 2's sample for L and R, respectively.

Let H_U and H_D be player 1's payoff sums for his samples for U and D, respectively. Similarly let K_L and K_R be players 2's payoff sums for her samples for L and R, respectively. We have

(2.D17) $$H_U = k_U(a_L+c_L) + (n-k_U)a_R, \qquad H_D = k_D a_L + (n-k_D)(a_R+c_R)$$

(2.D18) $$K_L = m_L(b_D+ d_D) + (n-m_L)b_U, \qquad K_R = m_R b_D + (n-m_R)(b_U+d_U)$$

As in the case of the action-sampling equilibrium the right hand sides of the two equations can be rewritten as a linear combination of binomial probabilities of the form $B(k,n,q)$ with positive coefficients. This has to be shown. Afterwards the monotonicity of the right hand side with respect to q will be a simple consequence of our result in D2.

We shall first look at the curve for p_U. The function β can be described by a (k_U,k_D)-diagram which shows the interval $0 \le k_U \le n$ horizontally and $0 \le k_D \le n$ vertically. We have

(2.D19) H_U-$H_D = -na_R$ for $k_U = k_D = 0$ and H_U-$H_D = na_L$ for $k_U = k_D = n$

Therefore regardless of the payoff parameters the function β has the following properties:

(2.D20) $$\beta\,(0,0) = 0, \qquad \beta\,(n,n) = 1$$

The equation $H_U = H_D$ determines a line in the (k_U,k_D)-diagram. In view of the equations for $\beta\,(0,0)$ and $\beta\,(n,n)$ it is clear that we have H_U-H_D>0 above this line and H_U-H_D<0 below it. Therefore we obtain $\beta(k_U,k_D) = 1$ for pairs (k_U,k_D) above the line and $\beta(k_U,k_D)$ = ½ for such points on the line. Below the line $\beta(k_U,k_D) = 0$ holds.

Define:

(2.D21) $$V(k_D,q_L) = \sum_{k_U=0}^{n} \binom{n}{k_U}\binom{n}{k_D} q_L^{\,k_U+k_D}(1-q_L)^{2n-k_U-k_D}\beta(k_U,k_D)$$

For every k_D let $h(k_D)$ be the smallest k_U with $\beta(k_U,k_D)>0$. It can be seen without difficulty that for every $k_D=0,\ldots,n$ we either have

(2.D22) $$V(k_D,q_L) = B(h(k_D),n,q_L) \qquad \text{or}$$

(2.D23) $$V(k_D,q_L) = \frac{1}{2}B(h(k_D),n,q_L) + \frac{1}{2}B(h(k_D)+1,n,q_L) \qquad \text{or}$$

(2.D24) $$V(k_D,q_L) = \frac{1}{2}B(h(k_D),n,q_L)$$

The first case arises if $(h(k_D),k_D)$ is above the line $H_U=H_D$. The second equation holds, if $(h(k_D),k_D)$ is on this line and $h(k_D)<n$ holds. The third form of $V(k_D,q)$ is valid for $h(k_D)=n$ if $(h(k_D),k_D)$ is on the line $H_U=H_D$.

It follows by the result of D2 that in all three cases $V(k_D,q)$ is monotonically increasing in q_L. In view of

(2.D25) $$p_U = \sum_{k_D=0}^{n} V(k_D,q_L)$$

It is clear that we have $\frac{\partial p_U}{\partial q_L} > 0$

Analogous arguments have to be used for proving that the curve for q_L is monotonically decreasing in p_U. In this proof one has to make use of the fact that the probability of at most k-1 successes in n trials is

(2.D26) $$1-B(k,n,q)$$

If q is the success probability for a single trial. It follows by the results of D2 that for k=1,…,n and 0≤q≤1 this probability is decreasing in q. Apart from this difference the proof is analogues to the one showing that the curve for p_U is monotonically increasing. It is not necessary to work out the details.

In view of $\beta(0,0) = 0$ and $\beta(1,1) = 1$ it is clear that the curve for p_U begins at $(p_U,q_L)=(0,0)$ and ends at $(p_U,q_L)=(1,1)$. Similarly the curve for q_L begins at $(p_U,q_L)=(0,1)$ and ends at $(p_U,q_L)=(1,0)$. It is clear that the two curves have a unique intersection and that therefore the payoff-sampling equilibrium of Osborne and Rubinstein (with the slight modification introduced here) is uniquely determined for completely mixed 2x2-games.

2.D5 Impulse Balance Equilibrium

In II.F the curves for p_U and q_L in the case of impulse balance equilibrium have been described. It can be seen immediately that the curve for p_U begins at $(p_U,q_L)=(0,0)$ and then increases until it ends at $(p_U,q_L)=(1,1)$. Similaryly the curve for q_L begins at $(p_U,q_L)=(0,1)$ and then decreases until it ends at $(p_U,q_L)=(1,0)$. It follows that both intersect in exactly one point.

Appendix 2.E Responsiveness to Own Payoff Parameters

It is the purpose of this appendix to examine how changes of a player's payoff parameter for one of his strategies influence the equilibrium probability of this strategy under the five stationary concepts examined here. It will always be assumed that the change of a payoff parameter is sufficiently small to make sure that the new game resulting by the change is still completely mixed. If a change is too big, it may result in a new game which has a pure equilibrium. Without loss of generality we can restrict our attention to changes of player 1's payoffs for (U,L) and (U,R).

In the case of Nash equilibrium the equilibrium probability p_U^N is not influenced by such changes, since it only depends on payoffs of the other player. It will be shown that for each of the four other stationary concepts the equilibrium probability for p_U is increased or at least not decreased by such a change.

2.E1 Quantal Response Equilibrium

Consider a sufficiently small change of player 1's payoff for (U,L) or (U,R). In both cases $E_U(q)$ will be increased for all q whereas $E_D(q)$ remains unchanged. This results in an upward shift of the curve for p_U. The curve for q_L remains unchanged; therefore the equilibrium probability for U is increased.

2.E2 Action-sampling Equilibrium

We first consider the case of a small increase of player 1's payoff for (U,L). Since a_L, the payoff for (D,L), remains unchanged such a change results in a decrease of $c_R/(c_L+c_R)$. In view of the formula for $\alpha_U(k)$ in II.D the quantity $\alpha_U(k)$ cannot be decreased by the change but it may increase or stay constant. This results in an upward shift of the curve for p_U.

The quantity $\alpha_U(k)$ depends discontinuously on c_L and c_R. It may happen for very small changes that the equilibrium is not affected but if it is affected the curve for p_U is shifted upwards and the equilibrium probability for U is increased.

Now consider a small change of player 1's payoff a_R for (U,R). Such an change will decrease c_R and therefore also $c_R/(c_L+c_R)$. The change works in the same direction as a small increase of c_L. Here, too, the equilibrium probability for U may not be affected but if it is affected it is increased.

2.E3 Payoff-sampling Equilibrium

Consider a small increase of player 1's payoff for (U,L) or (U,R). It can be seen immediately that such a payoff change increases H_U-H_D. Therefore $\beta(k_U,k_D)$ either remains unaffected or is increased. We can conclude that for every fixed q_L the associated probability p_U on the curve for p_U either is unaffected or increased. An increase results in an upward shift of the curve for p_U. As in the case of action-sampling equilibrium a small increase of a payoff for U either increases the equilibrium probability for U, or leaves it unaffected.

2.E4 Impulse Balance Equilibrium

In the following the formulas mentioned at the end of II.F. for the impulse balance equilibrium values will be derived. The point of departure are the impulse balance equations: With the help of easy algebraic transformations the two impulse balance equations can be solved for p_U and q_L, respectively. One obtains

(2.E1) $$p_U q_R c^*_R = p_D q_L c^*_L, \qquad p_U q_L d^*_U = p_D q_R d^*_D$$

In order to solve the impulse balance equation system we introduce the following definitions:

(2.E2) $$u = \frac{p_U}{p_D} \qquad v = \frac{q_L}{q_R} \qquad c = \frac{c^*_L}{c^*_R} \qquad d = \frac{d^*_U}{d^*_D}$$

We divide the first impulse balance equation by p_D, q_L and c^*_R and the second impulse balance equation by p_D, q_L *and* d^*_D with the help of the definitions of *u*, *v*, *c* and *d* the impulse balance equations can be rewritten as follows:

(2.E3) $$u = cv, \qquad duv = 1$$

Replacing *u* by *cv* in the second equation yields

(2.E4) $$v = \frac{1}{\sqrt{cd}}$$

With the help of the first equation we obtain

(2.E5) $$u = \sqrt{\frac{c}{d}}$$

The definition of u together with $p_D = 1 - p_U$ yields

(2.E6) $$p_U = \frac{u}{1+u}$$

In the same way we can conclude that

(2.E7) $$q_L = \frac{v}{1+v}$$

holds.

Together with the formulas for u and v this leads to the following result:

(2.E8) $$p_U = \frac{\sqrt{c}}{\sqrt{c}+\sqrt{d}},\ p_D = \frac{\sqrt{d}}{\sqrt{c}+\sqrt{d}},\ q_L = \frac{1}{1+\sqrt{cd}}, \qquad q_R = \frac{\sqrt{cd}}{1+\sqrt{cd}}.$$

Explicit formula in terms of $c = c_L^* / c_R^*$ and $d = d_U^* / d_D^*$ have been derived for the impulse balance equilibrium probabilities in II.F. Before the effects of an increase of player 1's payoff at (U,L) or (U,R) can be determined it is necessary to examine how the transformed payoff differences c_L^* and c_R^* depend c_L, c_R, a_L and a_R.

Player 1's security level s_1 is his second lowest payoff. This payoff can be obtained at each of the four fields of the bimatrix. We have:

(2.E9) $$s_1 = \begin{cases} a_R + c_R & \textit{for} \quad a_R + c_R \qquad a_L \\ a_L & \textit{for} \quad a_R \qquad a_L \leq a_R + c_R \\ a_R & \textit{for} \quad a_L \qquad a_R \leq a_L + c_L \\ a_L + c_L & \textit{for} \quad a_L + c_L \qquad a_R \end{cases}$$

The conditions on c_L, c_R, a_L and a_R in this formula for s_1 can be expressed as intervals for $a_R - a_L$. We obtain

(2.E10)
$$s_1 = \begin{cases} a_R + c_R & for \qquad\qquad a_R - a_L \qquad -c_R \\ a_L & for \qquad\qquad a_R - a_L \qquad 0 \\ a_R & for \qquad\qquad a_R - a_L \qquad c_L \\ a_L + c_L & for \qquad\qquad a_R - a_L \end{cases}$$

Obviously we have if s_1 is at most a_L. This is the case for $s_1=a_R+c_R$ in view of $a_R+c_R \leq a_L$ and for $s_1=a_L$. Therefore we have

(2.E11)
$$c_L^* = \frac{1}{2}c_L \qquad \text{for} \qquad a_R - a_L \leq 0$$

in the interval

(2.E12)
$$0 \leq a_R - a_L \leq c_L$$

we have $s_1=a_R$. The payoff difference c_L can be split into two parts, one below a_R, and the other above a_R.

(2.E13)
$$c_L = a_R - a_L + (a_L + c_L - a_R)$$

In the transition to the transformed game the first part remains unchanged and the second one is multiplied by ½. This leads to

(2.E14)
$$c_L^* = \frac{c_L}{2} + \frac{1}{2}(a_R - a_L) \qquad for \qquad 0 \leq a_R - a_L \leq c_L$$

For $c_L \leq a_R - a_L$ we have $s_1=a_L+c_L$. Consequently the impulse c_L is fully counted in the transformed game. Therefore we have

(2.E15)
$$c_L^* = c_L \qquad for \qquad = c_L \leq a_R - a_L$$

With the help of the notation

(2.E16) $$|x|_+ = \max\,[o,x]$$

our results about c_L^* can be expressed by the first of the following four equations. The equations for c_R^*, d_U^* and d_D^* can be derived analogously.

(2.E17) $$c_L^* = \min\left[\frac{1}{2}c_L + \frac{1}{2}|a_R - a_L|_+, c_L\right]$$

(2.E18) $$c_R^* = \min\left[\frac{1}{2}c_R + \frac{1}{2}|a_L - a_R|_+, c_R\right]$$

(2.E19) $$d_U^* = \min\left[\frac{1}{2}d_U + \frac{1}{2}|b_D - b_U|_+, d_U\right]$$

(2.E20) $$d_D^* = \min\left[\frac{1}{2}d_D + \frac{1}{2}|b_U - b_D|_+, d_D\right]$$

With the help of these formulas we now discuss the influence of a sufficiently small increase of player 1's payoff at (*U*,*L*) or (*U*,*R*) on the impulse balance equilibrium probability for *U*.

Suppose that player 1's payoff at (*U*,*L*) is increased. This results in an increase of c_L. The constants a_L and a_R as well as c_R remain unchanged. This means that c_L^* is increased and c_R^* is not changed. Consequently in this case $c = c_L^* / c_R^*$ is increased and $d = d_U^* / d_D^*$ remains unaffected. It immediately follows by the formula for p_U in (60) that p_U is increased.

Now assume that player 1's payoff for (*U*,*R*) is increased. Thereby a_R is increased but not a_R+c_R, player 1's payoff at (*D*,*R*). Consequently c_R is decreased. Moreover $|a_L - a_R|_+$ is not increased. It follows that c_R^* becomes smaller. In addition to this c_L^* may or may not increase but it cannot decrease. It follows that $c = c_L^* / c_R^*$ must increase. Since $d = d_U^* / d_D^*$ remains unaffected the formula for p_U in (60) leads to the conclusion that this probability is increased.

It is now clear that a ceteris paribus increase of a player's payoff at one of the four fields of the bimatrix leads to an increase of the impulse balance equilibrium probability of the strategy used by this player at this field.

Appendix 2.F: A Possibility of Generalizing Impulse Balance Equilibrium

In this paper we only look at impulse balance equilibrium in completely mixed 2x2-games. The concept can be extended to general normal form games. However, this can be done in different ways. In the following we shall sketch one of the possibilities.

The transition to the transformed game proceeds in the same way as in the 2x2-case. The pure strategy maximin s_i is the reference level of player *i*. Gains above s_i are counted half. In the following all our explanations refer to the transformed game.

Suppose that player *i* has used π_i in the preceding period and another pure strategy ρ_i would have yielded a higher payoff against the pure strategies played by the other players in this period. Then the surplus of the payoff for ρ_i which would have been receivable against these strategies over the payoff actually received for π_i is an **impulse from π_i to ρ_i**. Impulses from π_i to other pure strategies of player *i* are called **outgoing** and those from other pure strategies of player *i* to π_i are **incoming** for π_i.

The basic principle of **impulse balance** requires that for every pure strategy used with positive probability in impulse balance equilibrium the expected sum of outgoing impulses is either zero or equal to the expected sum of incoming impulses. Moreover every pure strategy with a positive expected sum of incoming impulses must be used with positive probability in impulse balance equilibrium.

According to this definition pure strategy equilibria are special impulse balance equilibria. In pure strategy equilibrium there are no outgoing impulses for equilibrium strategies and no incoming impulses for other pure strategies.

Appendix 3. Leaflet to Matrix Experiment

Merkblatt

An diesem Experiment nehmen mehrere Personen teil. Jeder Teilnehmer ist entweder ein Spieler 1 oder ein Spieler 2.

Sie sind Spieler 1.

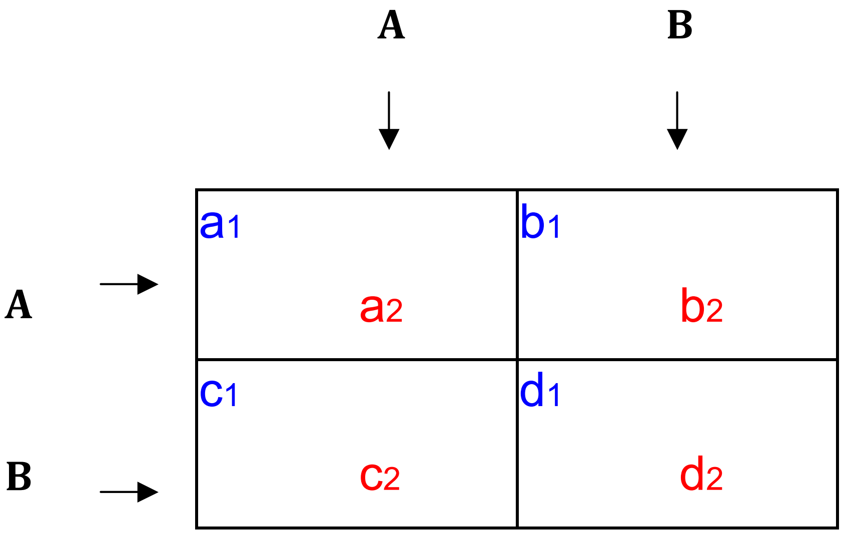
Auf dem Bildschirm sehen sie eine Matrix mit vier Feldern. In jedem Ergebnisfeld steht ihre eigene Auszahlung in blau (a1 b1 c1 oder d1) und die des Spielers 2 in rot (a2 b2 c2 oder d2).

Ihre Auszahlung hängt von ihrer eigenen Wahl und der Wahl des anderen Spielers ab.

Sie haben die Möglichkeit zwischen Zeile A oder Zeile B zu wählen. Spieler 2 wählt zwischen Spalte A oder Spalte B.

Die Zahlen in den Feldern entsprechen Cent-Beträgen.

Beispiel : Sie wählen A. Spieler 2 wählt B. In diesem Fall erhalten Sie eine Auszahlung von b1 Cent und Spieler 2 erhält b2 Cent.



Spielerpaare werden zufällig zusammengestellt.

Nachdem genügend viele Entscheidungen gesammelt wurden, werden diese zufällig einander zugeordnet. Deswegen erhalten Sie ihr Auszahlung nicht sofort.

Sie erhalten zu Beginn des Experiments eine Teilnahmepauschale von 1€.

Viel Erfolg!

Appendix 4. Graphical Presentation of the Statistical Results

Figures A4.36-A4.41 illustrate the experimental means in comparison to the simulated means of table 4.10. Black boxes represent the simulated values and white boxes, the empirical data.

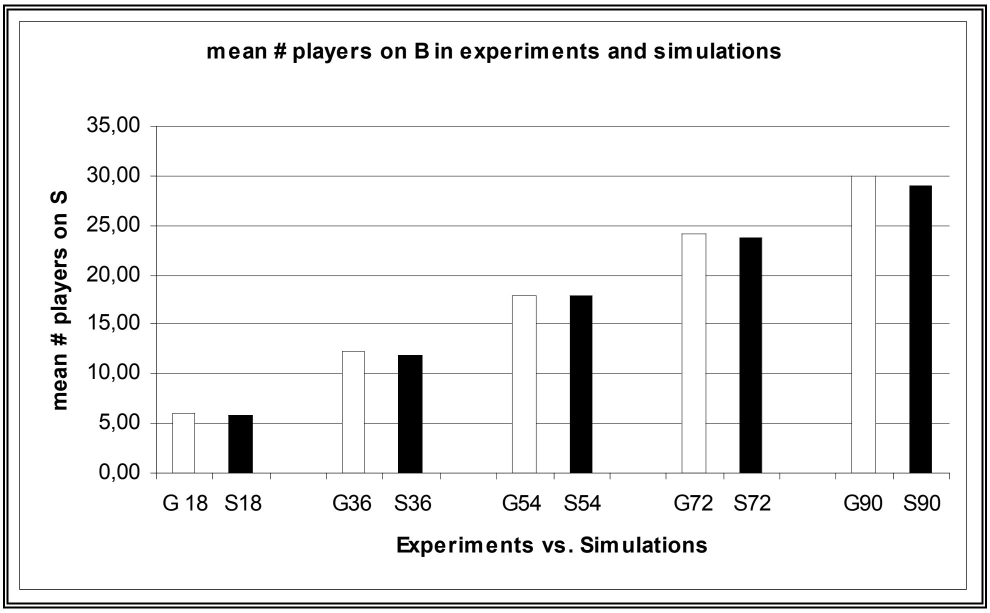


Figure A4.36: *Mean Number of Players on B in Experiments and Simulations.*

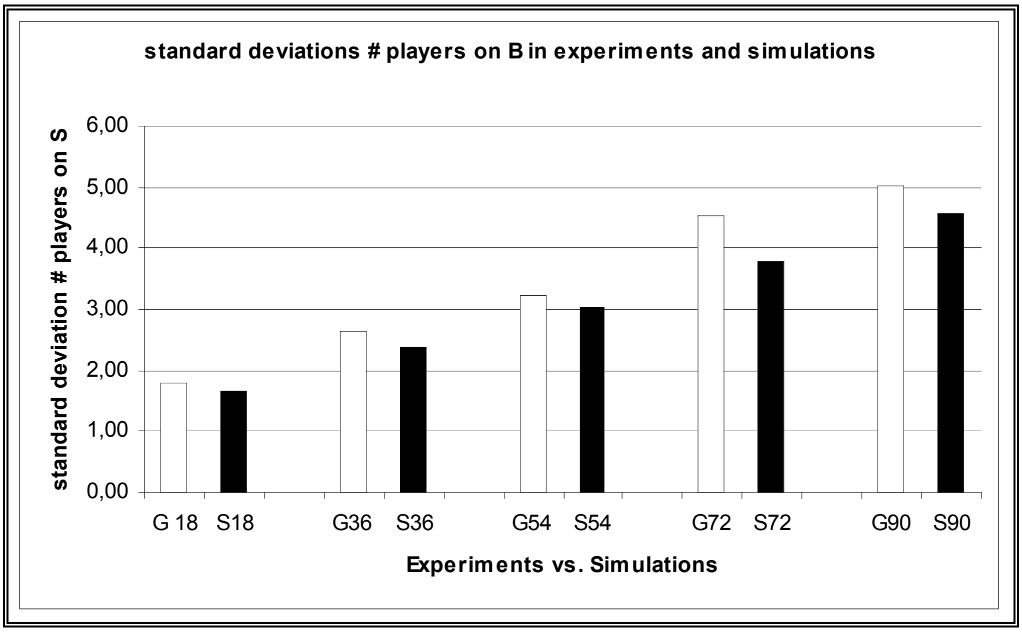


Figure A4.37: *Standard Deviation number of Players on B in Experiments and Simulations.*

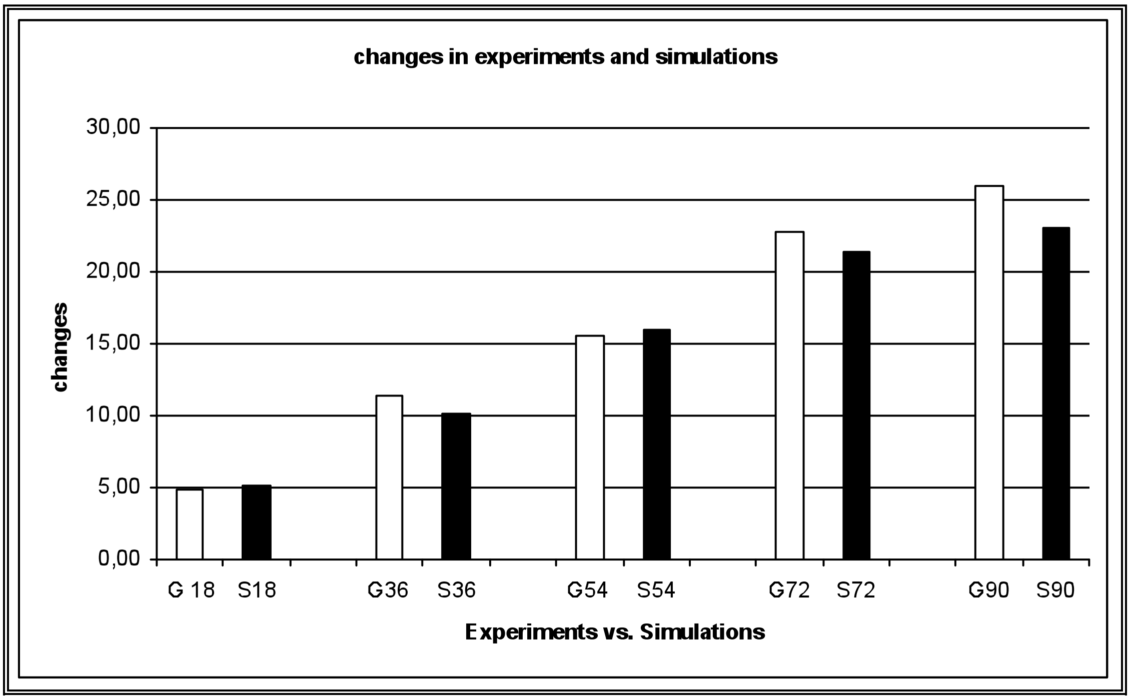


Figure A4.38: *Number of Changes in Experiments and Simulations.*

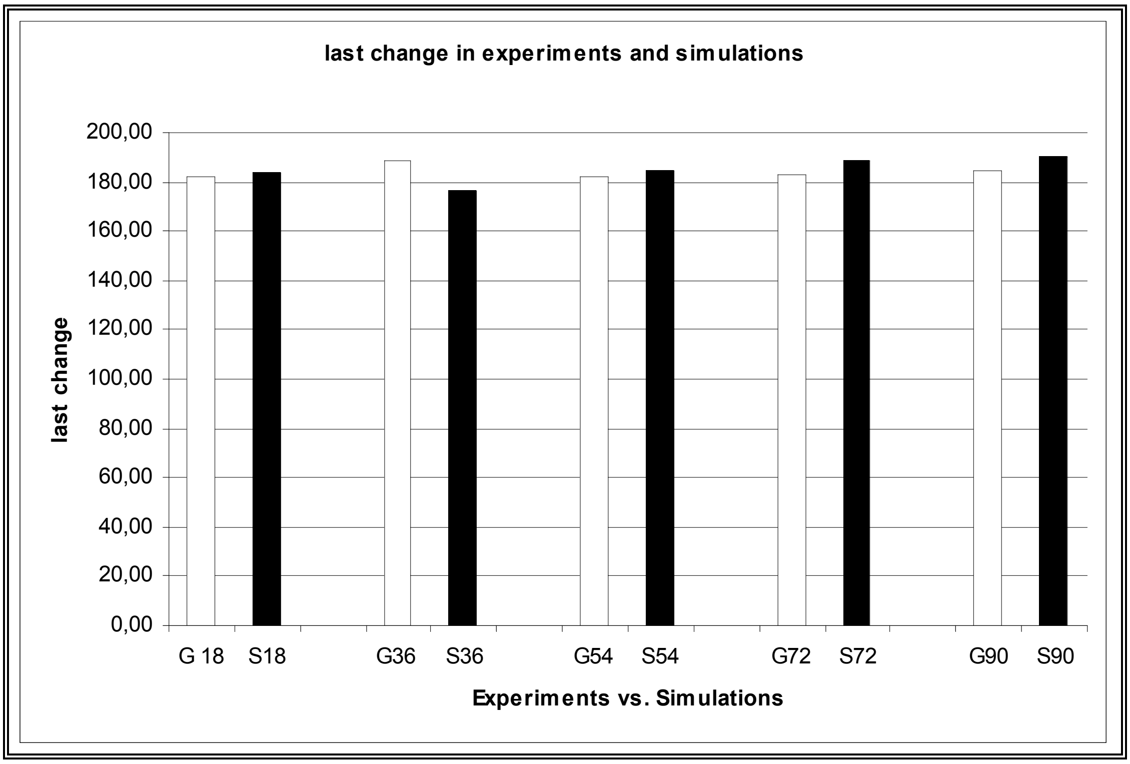


Figure A4.39: *Last change in experiments and simulations.*

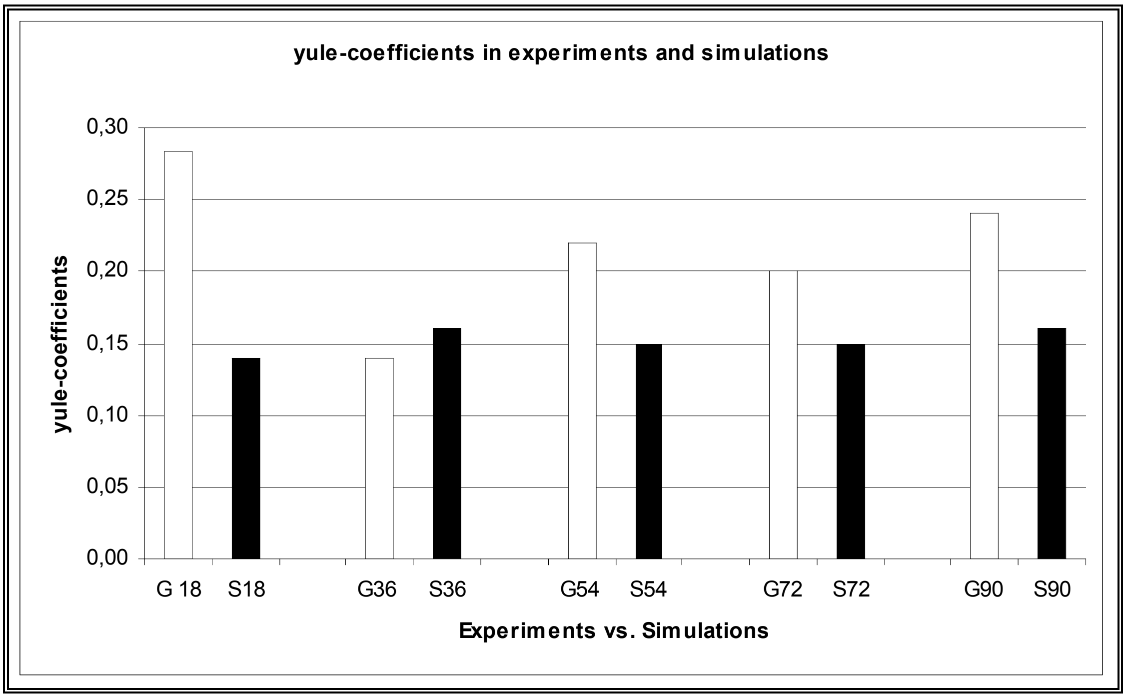


Figure A4.40: *Mean Yule-coefficients in experiments and simulations.*

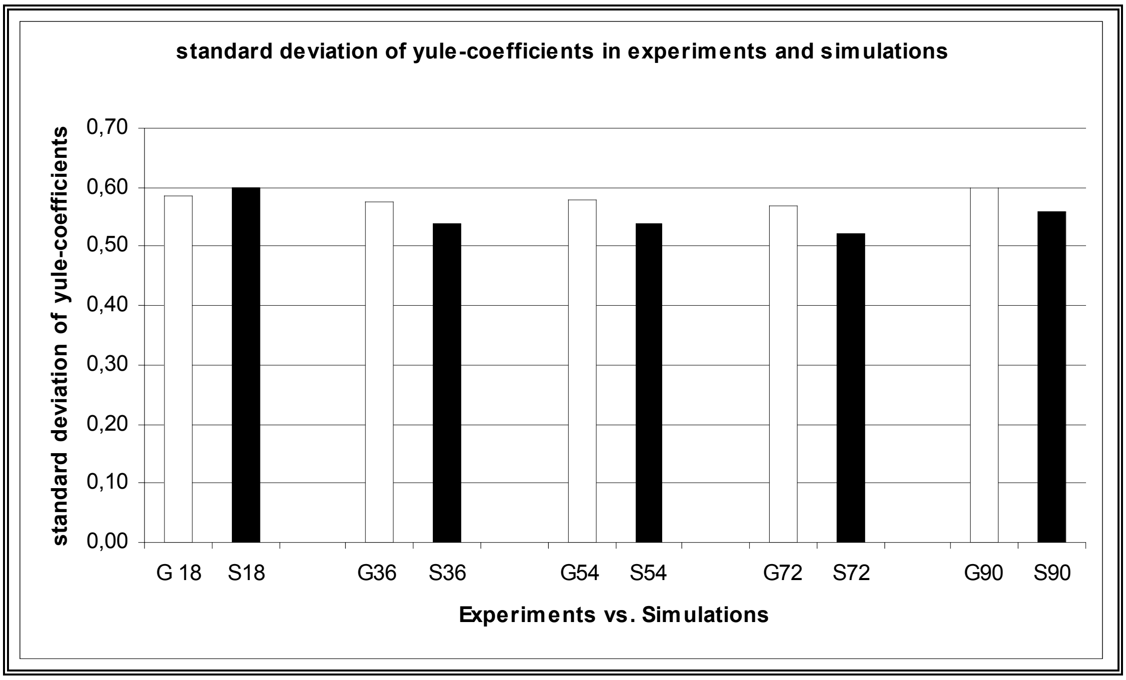


Figure A4.41: *Standard Deviation of Yule-coefficients in Experiments and Simulations.*

Appendix 5.: Who are the Smarter Drivers Leaflet and Screenshot

Appendix 5.A: Leaflet to minority experiment

Welcome to the experiment

Procedure:

- Altogether 9 persons are participating in this experiment. The game situation is the same for every participant.
- The experiment consists of **100 periods**.
- In each period you are travelling from a **starting point X** to an **arrival point Y**. You can either choose **road A** or **road B** to get from **X** to **Y** (see drawing).

Road A

X  Y

Road B

- After your decision which road you choose, you will get a payoff if you are on the road, which the minority has chosen. In this game 9 players interact with each other. An example would be:

- 3 participants choose road A and 6 participants choose road B, then each of the 3 participants on road A get the payoff of 1 Taler and the 6 participants on road B get the payoff 0 Taler.

  5 participants choose road A and 4 participants choose road B, then each of the 4 participants on road B get the payoff 1 Taler and the 5 participants on road A get the payoff 0 Taler.

- You can make a new route choice in every period.

- **The following information you will get after each period:**
  - Your route chosen in the preceding period.
  - Your period payoffs in the preceding period in **Talers**.
  - Your cumulated payoffs before the route choice in **Talers**.
  - Number of the current period.

- The exchange rate is **0,40 € (2 RMB in the Chinese treatment)** per Taler.

Thank you for participating!

Appendix 5.B: Screenshot Of The Program

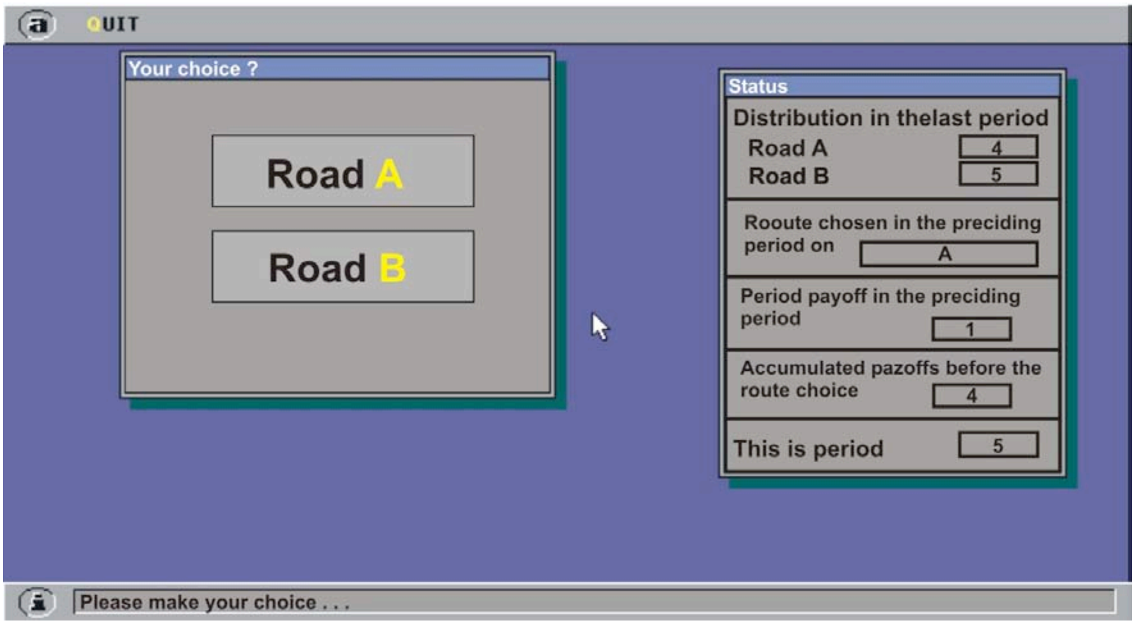


Figure A5.42: *Screenshot of the Program.*