

# **Contracting in the Presence of Uncertainty**

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# Introduction

This thesis concerns the enforcement of contracts in the presence of uncertainty. Uncertainty is a basic fact of human life. The uncertainty might be exogenously given as the event of a loss in an insurance contract. In addition, there is strategic uncertainty, because the behavior of other players is unknown. Moreover, uncertainty often generates communication between the contracting parties to set up the contract and to enforce it.

This communication can influence the amount of uncertainty for the contracting parties. At the same time, it is their choice whether to communicate or not. Thus, there is an interaction between communication and uncertainty. I analyze this interaction as it changes the incentives of the contracting parties and the structure of optimal contracts. The aim is to enhance our understanding of contracting practices and to inform regulation and policy.

This thesis consists of four chapters that are linked in several ways: All chapters contribute to our understanding of contracting in the presence of uncertainty. Yet applications vary and cover topics such as insurance, competition law, industrial organization, contracting, and decision theory. From a theoretical point of view, the first two chapters form an entity as they consider costly state verification with uncertainty. Chapter 3 is concerned with communication and information transmission in order to limit the amount of uncertainty. Chapter 4 is more technical and compares different ways of modeling economic uncertainty.

Another link can be seen between Chapters 1, 2, and 4: They consider ambiguity-

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averse agents and uncertainty that cannot be specified in a statistical way. Nevertheless the overlap between the chapters is negligible; the questions scrutinized and the applications analyzed differ strongly. Hence, this thesis exemplifies the level of abstraction of economic models and theories.

Chapter 1 is based on joint work with Achim Wambach (Lang and Wambach, 2010). It shows that insurers use ambiguity about auditing strategies to fight insurance fraud. For this purpose, we study a costly state verification model with uncertainty. The insurers abstain from commitment to an auditing strategy, even if commitment is possible without incurring any costs. This contrasts with conventional wisdom, which claims that it is optimal to commit, as the credible announcement of thoroughly auditing claim reports might act as a powerful deterrent to insurance fraud. Yet, empirically it is very unusual for insurers to try to overcome the credibility issue.

We prove that it can be optimal for the insurers to maintain the ambiguity and forgo commitment. Thus, strategic ambiguity, i.e., the strategic choice to withhold information about auditing costs and strategies, is an equilibrium outcome. This finding contributes to the literature on unraveling and is also relevant for other auditing settings, like tax enforcement.

Fraudulent claims on insurance policies and fraud-detection strategies are an important issue for insurers. Previous literature suggests that there is a commitment problem. Insurers should announce the level of auditing to deter insurance fraud. Given the announced level of auditing, there are only few fraudulent claims. As auditing is costly, however, the insurer has an incentive to deviate and to audit only very few claims *ex post*. These incentives make its *ex-ante* announcement not credible. Credible commitment to a given level of auditing is a solution to this dilemma.

Yet, in reality, it is very unusual for insurers to make their level of auditing publicly available. This behavior indicates that conventional wisdom neglects some aspects of the setting. We depart from the canonical setting of insurance fraud by assuming ambiguity-averse agents and uncertainty about the insurer's auditing costs. If the costs of an audit were known, policyholders could compute auditing probabilities in equilibrium. Commitment dissolves the ambiguity about auditing as it makes the

auditing level public. Even if the market is competitive, it can be optimal for the insurers to maintain the ambiguity and abstain from commitment.

The problem of costly state verification is not limited to insurance fraud, but also appears in different settings such as tax and benefit fraud. The cause is asymmetric information between the parties of a contract. To avoid the exploitation of these asymmetries, the other side has to use costly state-verification technologies, like ticket inspections in public transport. Thus, there is a trade-off between auditing costs and losses due to the remaining information asymmetries. It is a very robust result in these models that commitment is optimal. Hence, there have been various proposals to make commitment feasible. Yet we show that it is often optimal to avoid commitment to an auditing strategy, even if this commitment were credible and comes without costs.

The second chapter considers a setting of competition law. Legal uncertainty is a major issue in competition law, as legal procedures are very complex. In addition, there has been a shift towards rules of reason in the United States and to a more economic approach in the European Union. Both approaches imply a certain degree of legal uncertainty. I prove that legal uncertainty inherent in many legal rules can be welfare-enhancing if the uncertainty is not too large.

This finding contradicts conventional wisdom that legal uncertainty necessarily decreases welfare, but is a price to pay for more selective rules. I show that legal uncertainty allows screening firms and influencing their market behavior in a beneficial way. This result holds in a model with classical expected-utility preferences. If firms are ambiguity averse, uncertainty about the enforcement of competition rules has additional deterrence effects making the enforcement of the competition authority more efficient.

It is often claimed that legal uncertainty yields disproportionate deterrence – over-detering socially beneficial actions, while under-detering socially detrimental ones. To scrutinize this claim, the second chapter formally models the legal uncertainty inherent in a legal rule.

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The analysis shows that legal uncertainty itself might advance the objectives of the policymaker and might have positive effects on social welfare if the uncertainty is not too large. Legal uncertainty allows working around the policy restrictions of the competition authority. Hence, the actual deterrence level gets closer to the optimal one that depends on aspects unobservable by the competition authority. Thus, the competition authority may use legal uncertainty as a screening device.

Consider the following two examples of legal uncertainty. Article 101 (TFEU) prohibits vertical restraints, like resale price maintenance or exclusive dealings, in the European Union. Yet, there is a block exemption so that this rule does not apply if the market shares of the involved parties are below 30%. There are guidelines available how the competition authorities calculate the relevant market shares. Nevertheless, it is extremely difficult to predict correctly the market share that the competition authorities will determine in the end. The reasons are different definitions of the relevant market, information asymmetries or imprecision in the measurement of sales, and other factors. This exemplifies legal uncertainty as scrutinized in the chapter.

The second example concerns abusive tying. In particular, Microsoft bundled its operating system with additional software, like a web browser and a media player. In both instances, the European Commission found an abuse of a dominant market position under Article 102 (TFEU). As a thought experiment, imagine a scale beginning with products where the bundling is socially beneficial, as the integration implies better performance and independent competing products are non-existent. On the other end of the scale are products where the bundling implies few efficiency gains, but competition is harmed considerably. On both ends of the scale there is legal certainty. In the middle of the scale, however, it is very difficult to exclude legal uncertainty completely.

In addition, there is uncertainty about the size of the fine that firms have to pay in case of a conviction. Suppose the uncertainty about the fine does not change the expected value of the fines and the competition authority is concerned about efficient enforcement. Then, the second chapter shows additional beneficial effects of legal uncertainty.

This chapter is also relevant for the trade-off between per-se rules and rules of reason. Per-se rules prohibit some clearly specified practices. A rule of reason, on the other hand, judges the use of a practice as illegal whenever the practice is used in an anticompetitive way. Thus, rules of reason imply a certain degree of legal uncertainty. The results in the second chapter could be understood as explaining some of the appeal of rules of reason in competition law.

The previous two chapters show that uncertainty can be beneficial. In some settings, however, uncertainty is detrimental for social welfare. Then, it is important to understand how communication allows limiting the amount of uncertainty.

The third chapter analyzes a principal-agent model, in which the performance measure of the principal is non-verifiable and unobservable by the agent. Instead, the principal has the possibility to communicate with the agent. The communication occurs at the very end of the interaction and there is no repeated interaction. Nevertheless, it is crucial for the agent's motivation that the principal gives feedback and justifies her evaluation.

Providing feedback is important, in particular, in case of bad outcomes. In addition, it is optimal to pool evaluations and to compress wages at the top. These results fit well with empirical observations, like the leniency bias and the centrality bias. Accordingly, evaluations are lenient and wage dispersion for the best evaluations is low. Chapter 3 argues that this pattern of the evaluations can be understood as a feature of the optimal contract instead of biased behavior.

Consider two parties, the principal and the agent, who may write a contract. As there is no objective standard to measure the agent's work, the agent's compensation depends on the principal's evaluation. For this purpose, the principal privately collects information about the agent's performance, like reports from colleagues, observations of the agent at work or of the agent's output. A very small part of this information is known to the agent. After the principal determines her evaluation of the agent's work, she decides about the form of communication.

She can either just tell the agent the result of her evaluation or invest some time and explain her evaluation to the agent by telling him also about the collected

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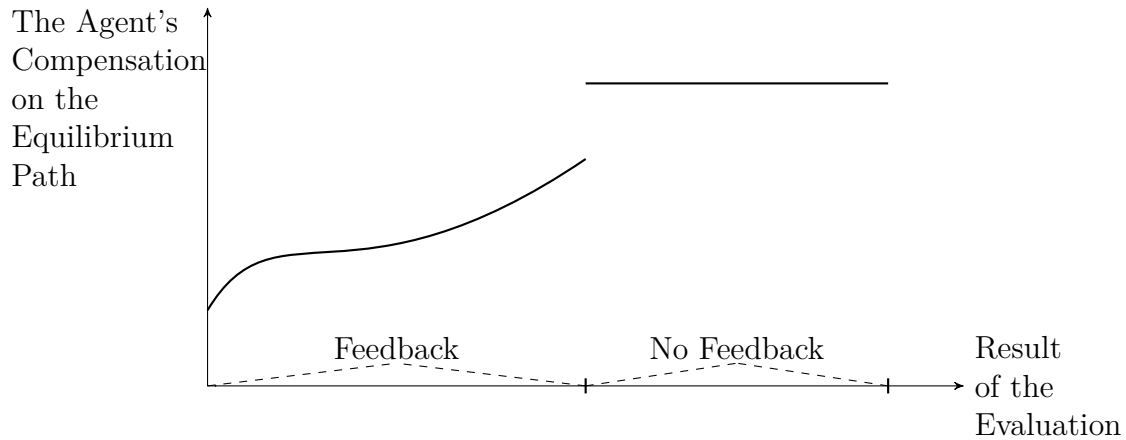


Figure 0.1: Communication Pattern

information. Messages are not necessarily truthful and providing justifications is costly. Independent of the principal's choice of communication, the agent can reply to the principal's evaluation.

Making the principal explain her evaluation requires additional incentives for the principal. This yields the following optimal contract. On the equilibrium path, the principal justifies low evaluations and adjusts the agent's compensation exactly to her evaluation of his work. For good evaluations, the principal in equilibrium just pays a high wage and abstains from providing any justifications. This yields pooling and wage compression at the top.

There is an additional, more technical contribution of this chapter. I show that it is possible to have an optimal contract that is ex-post budget-balanced. Instead of payments to third parties, stochastic contracts use differences in the risk preferences of the parties to implement the required incentives.

Finally, I turn to the way uncertainty is modeled. In contrast to risk, ambiguity denotes uncertainty that cannot be quantified exactly. There is a long literature showing that ambiguity matters for the empirical behavior of economic agents. Chapters 1 and 2 contribute to this literature. There are, however, several ways to model the ambiguity and ambiguity-sensitive preferences. Therefore, it remains to scrutinize whether results relying on ambiguity are robust to the choice of representation. The



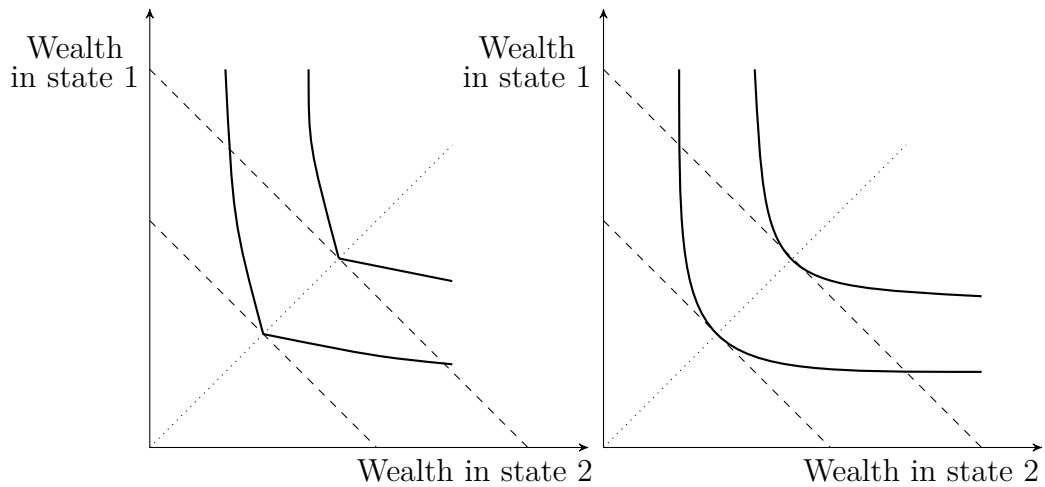


Figure 0.2: First-Order and Second-Order Ambiguity Aversion

fourth chapter studies this question.

Corresponding to the distinction between first-order and second-order *risk* aversion, I define first-order and second-order *ambiguity* aversion. Consider indifference curves in a contingent wealth space with two states of the world. The indifference curves can be kinked or smooth along the 45-degree line. I denote the former behavior first-order ambiguity aversion, and the latter second-order ambiguity aversion.

With second-order ambiguity aversion, for every ambiguity-averse agent there is an ambiguity-neutral agent so that the set of all improvement directions at an unambiguous endowment is the same for both agents. With first-order ambiguity aversion, in contrast, the set of improvement directions is a strict subset of the improvement directions of an ambiguity-neutral agent. Contrary to risk, there are tractable representations for both kinds of ambiguity aversion.

Chapter 4 provides three equivalent definitions for the distinction between first-order and second-order ambiguity aversion. For this purpose, I introduce a general ambiguity premium and a notion of reference beliefs of an ambiguity-averse agent. This distinction has direct implications for settings in finance, insurance, and contracting. In particular, I consider the validity of an adapted version of Holmström's informativeness principle under ambiguity aversion.



# The Fog of Fraud

## Mitigating Fraud by Strategic Ambiguity

### 1.1 Introduction

Fraudulent claims on insurance policies are an important issue for insurers. The extent of insurance fraud varies widely from small overstatements of claims to deliberately pretending damages that never occurred or that were intentionally arranged. Due to the nature of fraud, estimating the losses for the insurance industry is not an easy task. Nevertheless, the Insurance Information Institute, for example, estimates that in both 2004 and 2005 insurance fraud amounted to \$30 billion in the US property and casualty insurance market.<sup>1</sup> This is consistent with the estimate of \$20 billion for 1994 by the National Insurance Crime Bureau as stated in Brockett et al. (1998). According to Caron and Dionne (1997), 10% of the insurance claims in the automobile insurance are fraudulent to some extent in the Canadian province of Quebec.

Therefore the strategies of insurers to deter insurance fraud do matter. Dionne et al. (2009, p. 69), for example, estimate that in their sample, companies could save up to 41% of the costs due to fraudulent claims by implementing the optimal auditing strategy. Such a strategy has to balance auditing costs and benefits, like exposed

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<sup>1</sup>See Rees and Wambach (2008), p. 135.

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fraudulent claims. In the mass market and with small claims, it is too costly to audit each claim that is made. Consequently, claim reports are usually scanned for known patterns of fraud and only a certain fraction of these reports is verified in detail. Previous literature, like Picard (1996), who analyzes the canonical model of insurance fraud, suggests a commitment problem. Ex ante the insurers are interested in announcing a high level of auditing to deter insurance fraud. Given the announced level of auditing, the policyholders indeed report only few fraudulent claims. As auditing is costly, however, the insurer has an incentive to audit only very few claims ex post, rendering its ex-ante announcement not credible. Credible commitment to a certain level of auditing solves this dilemma. Thus, the absence of commitment implies a welfare loss. In contrast to this theoretical result, empirically it is very unusual for insurers to make their level of auditing publicly available. There are also no observable efforts to overcome the credibility issue by having an industry association scrutinize their level of auditing or using another third-party verification mechanism. Insurance firms not only announce no data on fraud detection and auditing, but even block access to it. Thus, there are very few empirical studies available.<sup>2</sup> This behavior indicates that conventional wisdom neglects some aspects of the setting.

Therefore we suggest that there is an additional issue. We depart from previous literature by assuming ambiguity-averse agents and uncertainty about the insurer's costs of an audit. We model the ambiguity on the type space, as the insured do not know which type of insurer they are facing. This leads to ambiguity about the probability of an audit. In our model, ambiguity-averse agents undertake less fraud due to this uncertainty. Yet commitment dissolves this ambiguity as it makes the level of auditing common information. We show that, even in a competitive market, it can be optimal for the insurers to maintain the ambiguity and forgo commitment.<sup>3</sup> Thus, strategic ambiguity is an equilibrium outcome.<sup>4</sup> First, we prove that holding insurers' behavior fixed, ambiguity makes fraud less appealing. Next, we endogenize the insurers' behavior. In the second step, we show that for a given contract, if the

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<sup>2</sup>A notable exception is Dionne et al. (2009). In the context of tax enforcement, the Internal Revenue Service in the U.S. defended in several court cases its right to keep auditing procedures secret.

<sup>3</sup>Notice that this result requires uncertainty about primitives of the model, here the auditing costs. Uncertainty as a purification of mixed strategies, as proposed by Harsanyi (1973), is not sufficient.

<sup>4</sup>Strategic ambiguity denotes here the strategic choice to withhold information in order to maintain the ambiguity for the other contract party, not the choice of strategic uncertainty in the sense of ambiguous strategies. The notion is discussed at the end of this section.

insurer abstains from commitment, ambiguity aversion either lowers the amount of fraud while holding the level of auditing fixed, or vice versa. Third, it will be shown that avoiding commitment is optimal if the auditing costs satisfy certain conditions discussed in the next paragraph. Finally, we also endogenize the contracts. It is shown that the utility-maximizing contracts that just break even under no commitment can be the unique equilibrium outcome.

The insurance companies have different reasons to forgo commitment. Insurance companies with high costs save on auditing costs, if they hide their type by abstaining from commitment, because the average auditing probability is higher than their own. Insurance companies with low costs also prefer the uncertainty to commitment, because a higher level of fraud due to the lower average auditing makes their auditing even more profitable. This is caused by the improved ratio between their low costs and recovered indemnities and fines imposed on the uncovered fraudsters. Risk aversion leads to different effects in the model than ambiguity aversion. If the degree of risk aversion increases, the deterrence of insurance fraud becomes easier both with and in the absence of commitment. Ambiguity aversion has only deterrence effects if there is no commitment. Therefore, only ambiguity aversion influences the balance between commitment and non-commitment. After all, it is the uncertainty that makes ambiguity-averse agents less inclined to engage in insurance fraud.<sup>5</sup>

In our model, the policyholders are ambiguity averse. Ambiguity denotes uncertainty about probabilities resulting from missing relevant information. We therefore distinguish ambiguity and risk.<sup>6</sup> In the absence of ambiguity, there is a known probability distribution, while under ambiguity the exact probabilities are unknown. Savage (1954) and Schmeidler (1989) have developed two axiomatized approaches to this problem. The Subjective Expected Utility of Savage requires the decision maker to be ambiguity neutral. This approach has been criticized for various reasons. From a normative point of view, it seems appropriate to take into account the amount

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<sup>5</sup>We were encouraged in this view when one insurance executive told us that besides being bad publicity, communicating detailed data on fighting insurance fraud, like the level of auditing, might induce more policyholders to give it a try. Moreover, according to Reinganum and Wilde (1988, p. 794), the IRS confirms that ‘one of the tools in the arsenal of the IRS which promotes voluntary compliance is the uncertainty in the minds of the taxpayers.’

<sup>6</sup>Unfortunately, the literature uses various notions. Sometimes ambiguity is called (Knightian) uncertainty or imprecision. The technical details of representations with ambiguity aversion are discussed in Appendix 1.A.

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of information on which a decision is based. This point was first made by Ellsberg (1961). In addition, there are empirical observations, like Kunreuther et al. (1995) or Cabantous (2007), which suggest that the Subjective Expected Utility approach neglects the distinction between risk and ambiguity. Insurers, which face ambiguity, usually request higher premiums and reject to offer an insurance policy in more cases than in the absence of ambiguity. The model in this chapter uses the representations of preferences with ambiguity aversion by Klibanoff et al. (2005) and Gilboa and Schmeidler (1989). In both representations, the decision maker judges situations with missing information more pessimistically than an ambiguity-neutral individual.

The problem of costly state verification considered here is not limited to insurance fraud, but also appears in different settings such as financing (Gale and Hellwig, 1985), accounting (Border and Sobel, 1987), principal-agent relationships (Strausz, 1997a) or enforcement of TV license fees (Rincke and Traxler, 2011). The main point is that there is often asymmetric information between the parties of a contract. To avoid the exploitation of these asymmetries, the other side has to use costly state verification technologies, like ticket inspections in public transport. Townsend (1979) began this analysis of the trade-off between auditing costs and losses due to the remaining information asymmetries. Commitment is optimal in these models, as discussed in, e.g., Baron and Besanko (1984). Hence, there have been various proposals to make commitment feasible and credible. Melumad and Mookherjee (1989) introduce delegation as a commitment device and Picard (1996) proposes a common agency financed by lump-sum payments to subsidize auditing costs. This lowers the variable costs of auditing claims in order to solve the credibility problem. Yet we will argue in this chapter that in some circumstances it is optimal for firms to avoid commitment to an auditing strategy, even if commitment were possible and costless.

Previous literature that combines costly state verification and uncertainty about auditing costs often uses a setting of tax evasion. Cronshaw and Alm (1995) analyze this case, but without ambiguity aversion and the possibility of commitment. Therefore, in their model, uncertainty could be counterproductive. Snow and Warren (2005), on the other hand, model ambiguity aversion by a subjective weighting of probabilities. Their paper studies the behavior of taxpayers given this ambiguity, but there is no possibility of commitment. Thus, our model is the first to consider the

strategic decision of commitment versus uncertainty.

The notion of strategic ambiguity as the strategic choice to withhold information in order to maintain the uncertainty for the other contract party has been used by Bernheim and Whinston (1998) and Baliga and Sjöström (2008) in the context of ambiguity-neutral players. In Baliga and Sjöström (2008), a country in equilibrium withholds the information about its military arsenal instead of acquiring arms with certainty and uses strategic ambiguity as a substitute for arms acquisition. In Bernheim and Whinston (1998), on the other hand, strategic ambiguity denotes the choice of an incomplete contract. Bernheim and Whinston (1998, p. 920) show “that, when some aspects of behavior are observable but not verifiable, it may be optimal to write a contract that leaves other potentially contractible aspects of the relationship unspecified.” The assumption of observable, but unverifiable aspects, while common in this literature, does not apply here. Individual fraud is either unobservable and unverifiable without an audit or becomes verifiable after an audit. Aggregate fraud levels are unobservable for the policyholders and in reality for the insurers, too. The type of an insurer is unobservable for the insured, while the occurrence of an audit is verifiable. Therefore there is no scope for negotiations that could make incomplete contracts optimal. Instead it is one party, the insurer, who decides to withhold the information about its auditing probability at a later stage after the contracting.

The optimality of incomplete contracts is confirmed by Mukerji (1998) for ambiguity-averse parties. In his paper, contractual incompleteness lessens the effects of ambiguity, because it leads to renegotiations that yield a proportional split of the surplus. This reduces the utility losses due to ambiguity, as it makes the considerations of both parties how to determine the worst distribution more similar.<sup>7</sup> In our model, avoiding

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<sup>7</sup>The reason is that the Choquet expectation is only additive for comonotonic acts. Thus, with ambiguity aversion the expected sum of the surpluses is larger than the sum of the expected surpluses, because the incentive compatibilities for the two parties require the transfers to be noncomonotonic. Therefore it is impossible to implement first-best effort. Contracts with comonotonic transfers, like incomplete contracts, cannot mitigate this, but avoid some of the ex-ante ambiguity premia and might be optimal.

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commitment enhances the effects of ambiguity.<sup>8</sup>

A second contribution of this chapter is to scrutinize a model with ambiguity aversion in a game-theoretic framework. Although many papers deal with the effects of ambiguity aversion in decision making and finance, there are few papers on games with ambiguity-averse players.<sup>9</sup> The reasons are problems with the equilibrium concepts, as addressed by Dow and Werlang (1994), Lo (1996, 1999), Eichberger and Kelsey (2000), Lo (2009), Bade (2011a), and Riedel and Sass (2011). We avoid these problems by modeling the ambiguity on the type space, i.e., the auditing costs of the insurers. This approach is also used by Lo (1998), Levin and Ozdenoren (2004), Bose et al. (2006), and Bodoh-Creed (2012) to study auctions with ambiguity-averse bidders. Bade (2011b) uses this approach, too, in order to establish the existence of equilibria in games of multidimensional political competition. It allows the use of common equilibrium concepts, like perfect Bayesian equilibria.

The third contribution is to consider whether competition makes firms provide relevant information to consumers and educate them. The argument by, e.g., Laibson and Yariv (2007) has been that competitive pressure gives consumers all the relevant information, as a competitor could always reveal the information and win market share. In our model, this is not the case. There is a market equilibrium with perfect competition where firms do not announce their information about auditing levels and ambiguity prevails that allows mitigating the effects of insurance fraud. In this respect, our results are similar to Gabaix and Laibson (2006) and Heidhues et al. (2012), where in equilibrium firms shroud the prices of some add-ons to their products.

The remainder of the chapter is organized as follows. Section 1.2 sets up a stylized model to give an intuition as to how ambiguity about the level of auditing decreases insurance fraud. In addition, it explains the decision process of the ambiguity-averse

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<sup>8</sup>Another neoclassical explanation for the withholding of the auditing information and not using commitment might be the repeated structure of the interaction. Therefore the static contracts in use by the industry might be improved by leaving room for relational contracts. Yet again this requires some observability. Either the policyholders derive the level of auditing from, e.g., income statements or the competitors observe the amount of auditing implemented. As we argued before, firms try to withhold information about auditing levels. Therefore it is difficult to get this information. Moreover it seems implausible that policyholders choose their insurer according to past auditing strategies or stochastic information about it. If competitors were to use the repeated interaction to enforce joint auditing levels, that behavior might be illegal and, in addition, their incentives are unclear. Therefore we conclude that relational contracts do not explain the observed behavior.

<sup>9</sup>See Mukerji and Tallon (2004b) and Gilboa and Marinacci (2011) for a survey of the literature.



policyholders. In Section 1.3, we take contracts as given and insurers decide on their auditing probabilities and whether or not to commit to their fraud detection strategy. We show that commitment can decrease profits and that insurers do not want to commit, even if they have the possibility to do so. In Section 1.4, insurers compete in contracts and decide on their auditing strategies. Even in this competitive market, firms in some cases want to forgo commitment. Then Section 1.5 compares the effects of ambiguity aversion with risk aversion. Finally, Section 1.6 exploits some extensions of the model and Section 1.7 contains the concluding remarks.

## 1.2 Ambiguity in Auditing

To strengthen the intuition of our results, we begin with a stylized model that shows how the ambiguity aversion of the policyholders makes them less inclined to engage in insurance fraud. The mechanism for the commitment decision of the insurers requires the full model which is set up in the next section. A risk-averse and ambiguity-averse agent takes out an insurance with a premium  $P$  and coverage  $q$  against a possible loss  $L > 0$ . Without loss of generality, we normalize the outside wealth of the agent to 0. The agent's preferences are represented by an increasing and strictly concave utility index  $u$ . A loss  $L$  occurs with probability  $\delta$  and no loss with probability  $1 - \delta$ . Given this loss distribution, a policyholder who reports a loss smaller or higher than  $L$  is immediately recognized as a fraudster. If, however, no loss occurs, the policyholder can nevertheless claim a loss of  $L$ , because the occurrence of a loss is private information of the policyholder. As it is common in the literature on costly state verification, the policyholder faces no direct costs or disutility for this behavior.

The insurer cannot observe the loss directly. It just receives the report of the policyholder. If the insurer pays out the claim, the policyholder gets  $q$  and therefore in case of fraud ends up with a final wealth of  $q - P$ . The insurer, however, has a technology to audit a fraction  $p$  of the reports for their truth. This technology is deterministic. Thus, if the insurance company audits a report, it knows for sure

whether the report is true or not.<sup>10</sup> In case the insurance company detects a fraud, it pays no indemnity and the policyholder has to pay a fine  $M$  that is determined by law. This is commonly known, but the fraction of audits  $p$  is private knowledge of the insurer. The policyholders only know that some reports will be verified. The insurer, however, may choose to disclose this fraction  $p$  to the policyholders. Without disclosure there is uncertainty about the level of auditing. We will show that the uncertainty lowers auditing costs, because it deters ambiguity-averse policyholders from fraud.

This uncertainty about probabilities due to the lack of relevant information is called ambiguity. In order to model ambiguity-averse agents, we use smooth ambiguity aversion by Klibanoff et al. (2005). A formal introduction to smooth ambiguity aversion is available in Appendix 1.A. Yet the results of this chapter do not depend on this specific representation of preferences. In Appendix 1.B, we repeat the exercise with Maxmin Expected Utility. This confirms that additional uncertainty decreases the inclination of the policyholders to engage in fraud.<sup>11</sup>

In the representation of smooth ambiguity aversion by Klibanoff et al. (2005), there is a set  $\Pi$  that contains the possible values for the first-order probability  $\hat{p}$ , here the probability of an audit. On the other hand,  $\mu(\hat{p})$  denotes the second-order probability of  $\hat{p}$  being the correct first-order probability. We assume that  $\Pi$  and  $\mu$  are such that the true value of  $p$  is contained in  $\Pi$  and equals the expected value, i.e.,  $p = \int_{\Pi} \hat{p} d\mu(\hat{p})$ . The ambiguity index  $\phi$  is continuous, strictly increasing, and concave. Thus, without a loss, the policyholder's utility is  $\phi(u(-P))$  if she makes no claim, and

$$\int_{\Pi} \phi\left((1 - \hat{p})u(-P + q) + \hat{p}u(-P - M)\right) d\mu(\hat{p})$$

for fraudulent claims. If the level of auditing is disclosed, the probabilities are known and become objective. Thus, there is no ambiguity and  $\mu$  is degenerate. Therefore

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<sup>10</sup>An alternative interpretation would be a stochastic technology in the sense that fraud is exposed only with a certain probability. Yet this does not change the analysis, because we can interpret  $p$  as the reduced probability of a claim being audited and being correctly identified if it was fraudulent.

<sup>11</sup>Gollier (2011) finds that an increase in ambiguity aversion may actually increase the demand for an ambiguous asset, in contrast to our result. The intuition for his result is similar to Rothschild and Stiglitz (1971) who show that a higher riskiness does not necessarily lower the demand of risk-averse agents for the risky asset.

the policyholder overstates the loss if the probability  $p$  of an audit is smaller than

$$p^b = \frac{u(-P + q) - u(-P)}{u(-P + q) - u(-P - M)}.$$

The following lemma compares this threshold to the case with ambiguity.

**Lemma 1.1.** *Suppose the level of auditing is fixed. If the insurer does not announce the level of auditing and the ambiguity-averse policyholders do not have all the relevant information to determine it exactly, there is less insurance fraud than in the case of available information about the auditing probability.*

The proof and all other proofs are given in Appendix 1.C. Thus, not revealing the probability of an audit decreases the level of auditing that is necessary to deter the policyholders from committing fraud. This means that withholding information about the level of auditing from the policyholders reduces their inclination to engage in insurance fraud. As the main model assumes heterogeneous policyholders with respect to the degree of ambiguity aversion, we next analyze comparative statics in the degree of ambiguity aversion. For this purpose, consider two policyholders with ambiguity index  $\phi_1$  and  $\phi_2$ . We call the second policyholder more ambiguity-averse than the first policyholder if there is an increasing and strictly concave function  $g$ , such that  $\phi_2 = g(\phi_1)$ .

**Lemma 1.2.** *Suppose the level of auditing is fixed, but ambiguous. Then the more ambiguity-averse policyholders commit less insurance fraud.*

The next section sets up the main model in the framework of Picard (1996) in order to capture the commitment decision of the insurers.

## 1.3 The Main Model

There are  $N > 3$  insurers facing a continuum of potential policyholders with mass one. The insurers make contract offers, then decide whether to commit to an auditing strategy. Finally, they choose their level of auditing. The policyholders select a contract and decide whether to make a claim.

## CHAPTER 1

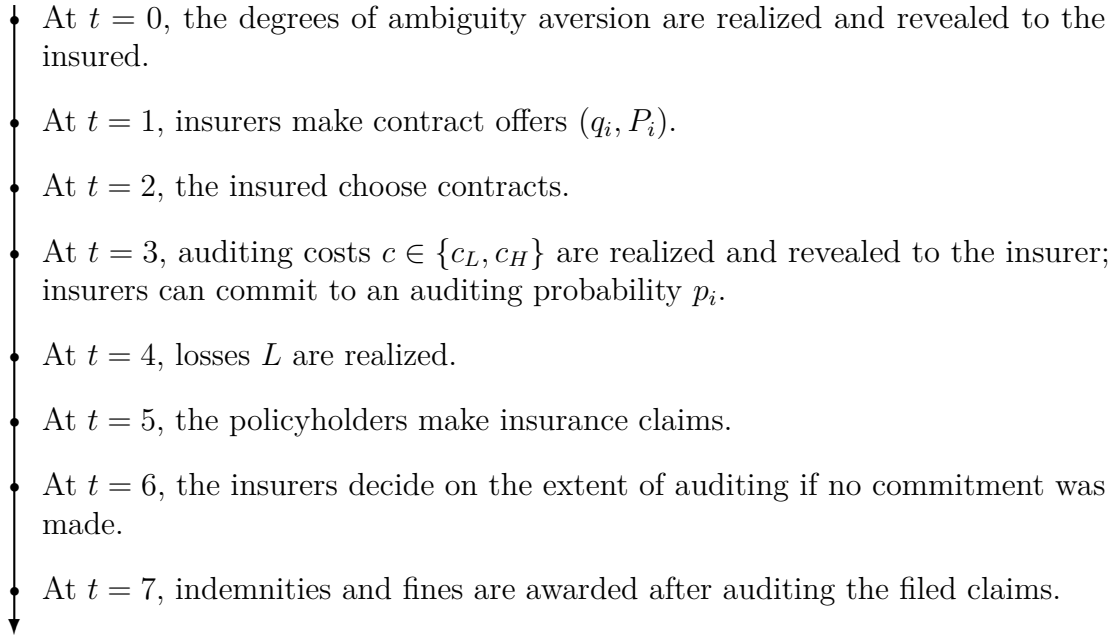


Figure 1.1: Timing of the Model

The timing is summarized in Figure 1.1. First, the degree of ambiguity aversion is assigned to the potential policyholders. Then risk-neutral and ambiguity-neutral insurers make contract offers. Each insurer  $i$  provides a quote for coverage  $q_i$  and a premium  $P_i$ , such that  $0 \leq P_i \leq q_i$ . In the next stage, the insured choose a contract from the pool of contract offers. At  $t = 3$ , nature determines the costs  $c$  of an audit for the insurer from the set  $\{c_L, c_H\}$  with  $c_H > c_L > 0$ . In the extension, we modify this timing by assuming that the insurance company knows its cost already before the contracting stage. The auditing costs are revealed only to the insurers. The policyholders only know the set of possible auditing costs, but not the distribution according to which nature is choosing. Therefore, the uncertainty is modeled, à la Harsanyi (1967), on the type space. The policyholders have no objective probabilities on the type space, but use subjective probabilities. Denote the subjective probabilities of facing a low-cost insurer by  $r$ , its non-degenerate distribution by  $\mu(r)$ , and the subjectively expected probability by  $\bar{r} = \int r d\mu(r)$ .<sup>12</sup> After observing its auditing costs, every insurance company has the possibility to commit to some auditing level. The

<sup>12</sup>This implies that auditing costs are realized independently for each insurer.

commitment could be implemented by delegation, as in Melumad and Mookherjee (1989), or by a common agency following Picard (1996). We abstract from this issue and assume that commitment is costless for the insurer to make our case as difficult as possible. If there are costs for communicating the auditing probability and making this announcement credible, it only strengthens our results. After that, at  $t = 4$ , the policyholders privately observe the occurrence of a loss  $L$  that occurs with probability  $\delta$ . Then, at  $t = 5$ , they decide whether or not to file an insurance claim. At  $t = 6$ , the insurer chooses to what extent to audit the filed claims. The auditing technology works as before. Finally, the insurer pays the indemnity  $q$  or gets a part  $m \leq M$  of the fine  $M$  a policyholder has to pay if an audited claim was fabricated. The remaining part is lost due, e.g., to litigation. As they are determined by law and legal process,  $M$  and  $m$  are exogenous in the model. This modeling choice is common in the costly state verification literature, like Picard (1996).

We restrict the analysis here to the case of smooth ambiguity aversion as proposed by Klibanoff et al. (2005).<sup>13</sup> We assume a population of agents with different degrees of ambiguity aversion. Thus, there is a family of strictly concave ambiguity functions  $\phi_{\mathcal{A}}$  indexed by  $\mathcal{A} \in [\underline{A}, \bar{A}]$ . The higher  $\mathcal{A}$ , the more ambiguity-averse the agent is, as defined in Section 1.2 above. The degree of ambiguity aversion  $\mathcal{A}$  is distributed according to a distribution function  $F$  with a density  $f > 0$ . The insurers, who know this distribution, cannot observe the degree of ambiguity aversion of a policyholder. In this section, stages 1 and 2 are taken as given. Thus, only the stages 3 to 7 of the game are considered. Section 4 solves the full model. As a first step, we determine the equilibrium of the auditing game beginning after stage 4.

### 1.3.1 Solving the Auditing Game

There are two cases to consider. First, we consider the case in which the insurer commits itself to a certain level of auditing in stage 3. We solve the model backwards. If the insurer committed to a certain level of auditing  $p$ , in stage 6 it has to stick to that decision and conduct the audits accordingly. In the next step, we analyze the decision of the insured in stage 5 whether or not to report a claim in the absence of

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<sup>13</sup>The results of this chapter are robust to other representations of preferences and, in particular, also hold with Maxmin and Choquet Expected Utility.

a loss. The level of auditing is known, so the policyholders do not care about the auditing costs of the insurer. Therefore their beliefs about the type of the insurer and the ambiguity aversion do not matter. As before, the critical value for the level of auditing is  $p^b = \frac{u(-P+q)-u(-P)}{u(-P+q)-u(-P-M)}$ . If more claims are audited, no fraud occurs. For lower levels of auditing, every policyholder makes a claim. In the third stage, the insurers choose  $p_i$ , depending on the costs of auditing  $c_i$ , to maximize their profits. The equilibrium in this game is the same as the one described in Proposition 1 of Picard (1996)<sup>14</sup> and depends on the costs of auditing  $c_i$ . If the insurer's costs are above a threshold, i.e.,  $c_i > c' = \frac{(1-\delta)q}{\delta p^b}$ , the insurer of type  $i$  does not audit any claims and all the policyholders claim a loss. If the costs of auditing are below the threshold, a fraction  $p^b$  of all claims is audited and no insurance fraud occurs.<sup>15</sup> We now turn to the case in which the insurer decides not to commit.

Solving the model backwards, the analysis begins at  $t = 6$ . As no commitment was made, the insurer will choose the level of auditing  $p$  to maximize its profits, given that a fraction  $\alpha$  of policyholders without a loss reported a false claim. A policyholder anticipates an auditing probability  $p_L$  of the low-cost insurer and  $p_H$  of the high-cost type. If the policyholder is ambiguity neutral, she expects an audit with probability  $\bar{r}p_L + (1 - \bar{r})p_H$ . Yet, the more ambiguity-averse she gets, the more averse she gets with respect to the risk of facing the low-cost insurer. Thus she reports truthfully if

$$\phi_{\mathcal{A}}(u(-P)) \geq \int \phi_{\mathcal{A}} \left( \left( r(1 - p_L) + (1 - r)(1 - p_H) \right) u(-P + q) + \left( r p_L + (1 - r) p_H \right) u(-P - M) \right) d\mu(r). \quad (1.1)$$

Therefore, the following program determines the equilibrium, in which the insurers choose the auditing probabilities  $p_L$  and  $p_H$ , after the policyholders have decided

<sup>14</sup>Picard (1996) assumes an exogenously given fraction  $\theta$  of opportunistic policyholders in an otherwise honest population. Setting  $\theta = 1$  resembles our model with credible announcement.

<sup>15</sup>To make the equilibrium unique, the insured have to abstain from fraud if the level of auditing is  $p^b$ , although they are indifferent. This seems natural, as the insurer could audit a fraction  $p^b + \epsilon$  of all insurance claims with an arbitrarily small  $\epsilon$  to make this behavior of the policyholders a unique best response. On the other hand, the insurers are indifferent for  $c = c'$ . For uniqueness, it is assumed that insurers have a preference for less fraud if profits do not change.

whether to submit fraudulent claims.

$$\begin{aligned} & \max_{p_i \in [0,1]} P - q(\delta + \alpha(1 - \delta)(1 - p_i)) + m\alpha p_i(1 - \delta) - c_i(\delta + \alpha(1 - \delta))p_i, \quad \forall i \in \{L, H\} \\ \text{subject to } & \alpha = \int_A 1 dF(\mathcal{A}) \text{ with the set} \\ & A = \left\{ \mathcal{A} \in [\underline{A}, \bar{A}] \left| \int \phi_{\mathcal{A}} \left( (r(1 - p_L^*) + (1 - r)(1 - p_H^*))u(-P + q) + \right. \right. \right. \\ & \quad \left. \left. \left. + (rp_L^* + (1 - r)p_H^*)u(-P - M) \right) d\mu(r) > \phi_{\mathcal{A}}(u(-P)) \right\} \end{aligned}$$

To calculate the optimal auditing probabilities,  $p_i^*$ , consider the reasoning of the insurer. The insurer acts after the insured reported their claims. Thus, the level of fraud  $\alpha$  is taken as given. The insurer is indifferent between auditing or not, if the costs are at the threshold  $c^*(\alpha)$ , which depends on the amount of fraud.

$$c^*(\alpha) = \frac{\alpha(1 - \delta)}{\delta + \alpha(1 - \delta)}(q + m) \quad \text{with} \quad \frac{\partial c^*(\alpha)}{\partial \alpha} > 0 \quad \forall \alpha \geq 0. \quad (1.2)$$

The fraction  $\frac{\alpha(1 - \delta)}{\delta + \alpha(1 - \delta)}$  is the insurer's belief after stage 5 about a claim to be false. Hence, at the threshold the costs of auditing equal the expected benefits of auditing, i.e., the claims  $q$  that need not to be paid and the fines  $m$  awarded to the insurer. This allows describing the unique perfect Bayesian equilibrium (modulo out-of-equilibrium beliefs and strategies) of the game after stage 4 given a contract with premium  $P$  and reimbursement  $q$ .<sup>16</sup> The following proposition distinguishes four cases, which are illustrated in Figure 1.2. If the costs of both types are very high in case (a), there will be no auditing and complete fraud. For lower costs, there are two cases, (b) and (d), in which one type will be indifferent with respect to auditing. Finally, there remains the case (c) where every type of insurer plays a pure strategy as auditing is beneficial for the low-cost type, but not for the high-cost type.

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<sup>16</sup>To have a unique equilibrium, firms have to prefer less auditing, *ceteris paribus*, in particular if it does not change the level of fraud. Moreover, while the insurer's type is unobservable, the policyholders nevertheless correctly anticipate the equilibrium strategy of each type of insurer. Hence the uncertainty only concerns the type space. Therefore the analysis does not require new equilibrium concepts, as discussed in the introduction.

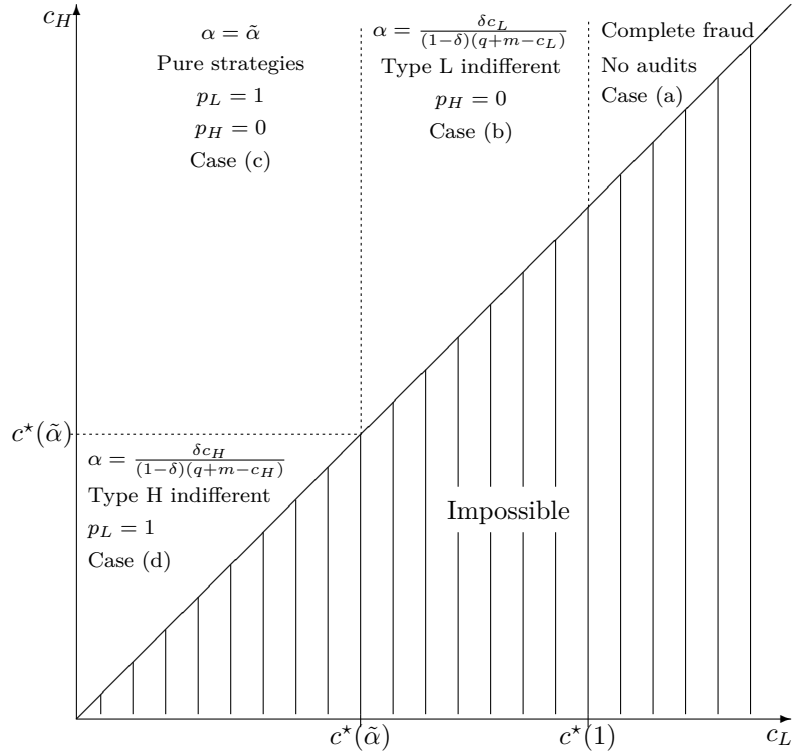


Figure 1.2: Auditing Equilibria in Proposition 1.1 for  $0 < \tilde{\alpha} < 1$

**Proposition 1.1.** *For given contracts, beliefs  $\mu(r)$  and without commitment the equilibrium has the following form:*

- (a) *If the costs of both types are above the upper threshold,  $c_L \geq c^*(1) = (1 - \delta)(q + m)$ , there is complete fraud,  $\alpha = 1$ , and no audits,  $p_H = p_L = 0$ .*
- (b) *If the costs of the low-cost type are between the two thresholds,  $c^*(\tilde{\alpha}) \leq c_L < c^*(1)$ , there is a high level of fraud  $\alpha = \frac{\delta c_L}{(1 - \delta)(q + m - c_L)} \in (\tilde{\alpha}, 1)$ , and a low level of audits  $p_H = 0$  and  $p_L = h(0, F^{-1}(\alpha))$ .*
- (c) *If the costs of both types are separated by the lower threshold  $c_L < c^*(\tilde{\alpha}) \leq c_H$ , there is some fraud  $\alpha = \tilde{\alpha}$  and partial audits of  $p_H = 0$  and  $p_L = 1$ .*
- (d) *If the costs of both types are below the lower threshold,  $c_H < c^*(\tilde{\alpha})$ , there is a low level of fraud  $\alpha = \frac{\delta c_H}{(1 - \delta)(q + m - c_H)} \in (0, \tilde{\alpha})$ , and a high level of audits  $p_H = h(1, F^{-1}(\alpha))$  and  $p_L = 1$ .*



$$\begin{aligned} & \text{with } h(x, \mathcal{A}') \text{ a solution to } \phi_{\mathcal{A}'}(u(-P)) = \\ & = \int \phi_{\mathcal{A}'} \left( u(-P+q) - \left( rx + (r + (1-2r)x)h(x, \mathcal{A}') \right) (u(-P+q) - u(-P-M)) \right) d\mu(r) \end{aligned}$$

and the level of fraud  $\tilde{\alpha} = F(\mathcal{A}^*)$  defined by case (c), such that

$$\begin{aligned} \mathcal{A}^* = \sup \left( \left\{ \mathcal{A} \in [\underline{A}, \bar{A}] \mid \int \phi_{\mathcal{A}} \left( (1-r)u(-P+q) + ru(-P-M) \right) d\mu(r) > \right. \right. \\ \left. \left. > \phi_{\mathcal{A}}(u(-P)) \right\} \cup \{\underline{A}\} \right).^{17} \end{aligned}$$

To sum up, in equilibrium smaller costs of auditing reduce the level of insurance fraud by increasing the auditing probabilities. Additionally, the level of auditing and the amount of insurance fraud depend negatively on each other. In general, insurers with high costs do not audit, except for the last case (d) of the proposition. In contrast to Picard (1996), it is possible to have auditing and the insurers employing pure strategies. Therefore the equilibrium differs if the auditing costs of both types are not too high; in particular case (c) is impossible without cost heterogeneity.

Introducing ambiguity aversion either lowers the amount of fraud, while holding the level of auditing fixed, or vice versa. Uncertainty about the level of auditing has an additional deterrence effect. The effect discussed in Section 1.2 causes this reduction.

The level of fraud  $\tilde{\alpha}$  in case (c) is important for the structure of the proposition, because it determines the lower threshold for the costs  $c^*(\tilde{\alpha})$ . Thus, case (b) is only feasible if the low-cost type can induce some policyholders to behave honestly,  $\tilde{\alpha} < 1$ . This implies either a high expected probability  $\bar{r}$  for a low-cost insurer, a high amount of ambiguity in terms of the variance of  $\mu$  or a high degree of ambiguity aversion in the population, or else that fraud is unattractive, i.e.,  $p^b$  is low. On the other hand, if the low-cost type can induce all policyholders to behave honestly,  $\tilde{\alpha} = 0$ , the cases (c) and (d) do not arise at all.

There remain two interesting implications of Proposition 1.1. First, the insurers' profits vary continuously, as the parameters change, even if the type of equilibrium

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<sup>17</sup>As there is no continuity in  $\mathcal{A}$ , we have to consider the supremum of these values of  $\mathcal{A}$  for which fraud is optimal instead of using the indifference condition (1.1).  $\underline{A}$  captures corner solutions.

changes. Second, commitment allows eliminating insurance fraud completely, which is impossible without commitment. Nevertheless, the next section shows that ambiguity aversion in some cases allows reducing the total costs of the insurers by forgoing commitment.

### 1.3.2 Comparing Commitment with Non-Commitment

In this section conditions are derived under which non-commitment may lower the insurers' total costs. If the insurers are of the high-cost type, they need to implement less audits than under commitment and can profit from the low fraud caused by the high average auditing probability. For insurers of the low-cost type auditing is cheap. Hence they profit from the higher fraud in the population compared to a situation with commitment due to the lower average auditing if the ratio of their costs to the fines is low enough. In order to show that non-commitment can be preferred, we compare the costs due to insurance fraud,  $\alpha(1 - \delta)(1 - p_i)q$ , and auditing,  $(\delta + \alpha(1 - \delta))p_i c_i$ , minus the recovered fines,  $m\alpha p_i(1 - \delta)$ , in the absence of commitment to the costs of auditing under commitment,  $\delta p^b c_i$ . Commitment implies a loss for the insurance firms if

$$\alpha(1 - \delta)(1 - p_i)q - m\alpha p_i(1 - \delta) + (\delta + \alpha(1 - \delta))p_i c_i \leq \delta p^b c_i. \quad i \in \{L, H\} \quad (1.3)$$

The next proposition shows that this condition is feasible in the cases (a) and (c) of Proposition 1.1.

**Proposition 1.2.** *In the game beginning at stage 3, commitment has, in equilibrium, no advantage for the insurers if and only if*

- *the costs of the low-cost type are low enough, while the costs of the high-cost type are sufficiently large,  $\exists \alpha \in (0, 1]$*

$$c_L \leq \frac{m\alpha(1 - \delta)}{\delta(1 - p^b) + \alpha(1 - \delta)} \quad \text{and} \quad c_H \geq \alpha c'. \quad (1.4)$$

*If condition (1.4) holds for  $\alpha = \tilde{\alpha}$  as defined in Proposition 1.1, there is pooling with respect to the commitment decision. For other values of  $\alpha$ , there is partial pooling.*

### 1.3 THE MAIN MODEL

- the costs of auditing are high for both types,  $c_L > c' = \frac{(1-\delta)q}{\delta p^b}$ . In this case, the insurers do no auditing and therefore are indifferent with respect to the commitment decision.

Figure 1.3 summarizes the commitment decision. The most interesting case for the next section is pooling on non-commitment in the upper left corner (A). In this area, the costs of the types differ significantly and, as argued above, both types are better off by not committing. Partial pooling refers to a situation in which one type uses a mixed strategy with respect to the commitment decision. This allows adjusting the level of fraud in the absence of commitment to make non-commitment optimal for both types. For  $\alpha < \tilde{\alpha}$ , the high-cost insurer commits with some probability. For

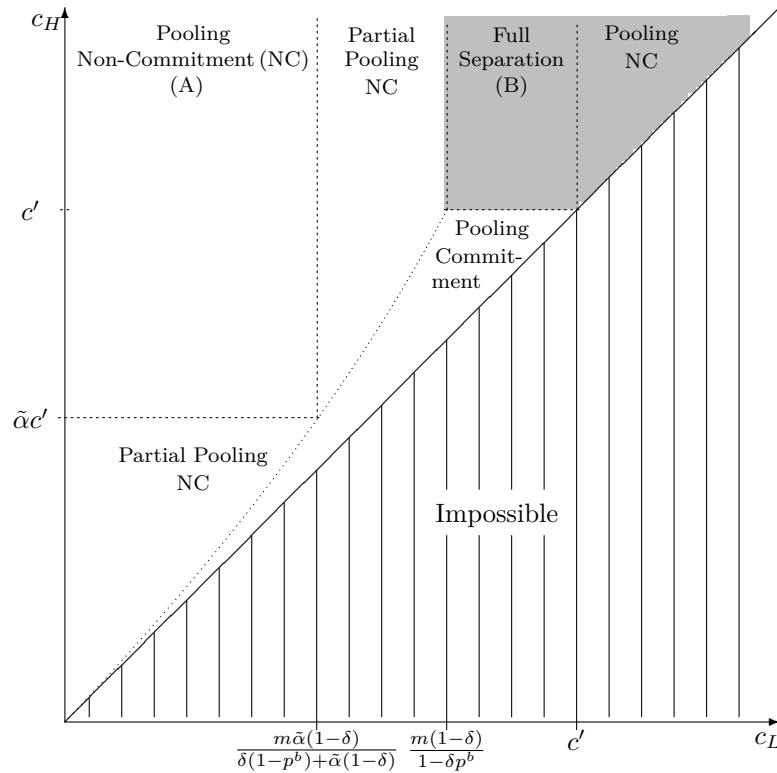


Figure 1.3: Signaling Equilibria in Proposition 1.2 for  $0 < \tilde{\alpha} < 1$

In area (A), all types avoid commitment. Adjacent to (A), there are two areas with partial pooling. Then, one type avoids commitment with a strictly positive probability, while the other type always abstains from commitment. Along the 45° degree line, there is pooling on commitment. Finally, in area (B) there is a fully separating equilibrium. Then, only the low-cost type uses commitment. Regions with multiple equilibria are shaded.

$\alpha > \tilde{\alpha}$ , the low-cost insurer commits with some probability. There is commitment to an auditing probability if the costs of an audit are close to each other for both types.<sup>18</sup> Then auditing is sufficiently cheap for the high-cost type to prefer a positive auditing probability and commitment. Alternatively, auditing costs are sufficiently high for the low-cost type to prefer commitment and the corresponding reduction in auditing levels. In area (B), it is optimal for the low-cost type to implement commitment and do some auditing. In the absence of commitment, there is a high level of fraud. Given the high costs of auditing, abstaining from commitment makes the low-cost type worse off. The high-cost type, on the other hand, implements no audits anyway and is therefore indifferent with respect to the commitment decision. Finally, if the auditing costs are sufficiently high, both types abstain from auditing independent of their commitment decision. Hence, insurers are indifferent with respect to the commitment decision. Then there are multiple equilibria. In summary, the policyholders do not know which type of insurer they face in the absence of commitment and there will be some, but not too much fraud. Both types of insurer could commit to a level of auditing  $p^b$  and completely deter the policyholders from filing fraudulent claims. In area (A) of Figure 1.3, however, they have an incentive not to do so and strictly prefer an equilibrium without commitment. We now consider the insurance market and characterize the equilibrium of the entire game starting at  $t = 0$ .

## 1.4 Market Equilibrium

So far we have analyzed the behavior of the policyholders and the insurers for given contracts. Now we endogenize these contracts according to the timing in Figure 1.1. The characterization of the equilibrium in the insurance market requires the definition of two benchmark contracts. These benchmark contracts serve the purpose to characterize the equilibrium. There are no restrictions on strategies. The first contract  $(q^{NC}, P^{NC})$  is the utility-maximizing contract that just breaks even, if the insurers avoid commitment and only the low-cost insurer audits as in case (c) of Proposition 1.1. The second contract  $(q^C, P^C)$  is defined accordingly just for the case

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<sup>18</sup>There is also an equilibrium with commitment for large costs,  $c_L \geq \frac{m(1-\delta)}{1-\delta p^b}$ , in particular if commitment does not deter fraud.

with commitment. Therefore define the contract  $(q^{NC}, P^{NC})$  as an element of the following set

$$(q^{NC}, P^{NC}) \in \arg \max_{q, P \in \mathbb{R}^+} \delta u(-L + q - P) + (1 - \delta)u(-P)$$

$$\text{with } P \geq \delta(q + \bar{r}c_L) + (1 - \delta)(\bar{r}(c_L - m) + (1 - \bar{r})q)\tilde{\alpha}[q, P] \quad (1.5)$$

and  $\tilde{\alpha}[q, P]$  as defined in Proposition 1.1.<sup>19</sup> Assume that this set is a singleton. The expected profits correspond to pooling in case (c) of Proposition 1.1. The contract  $(q^C, P^C)$  is defined analogously, but the budget constraint is this time

$$P \geq \delta(q + \bar{r}c_L p^b[q, P]) + (1 - \bar{r}) \min\{\delta c_H p^b[q, P], (1 - \delta)q\}.$$

The next proposition shows that in equilibrium firms will choose the non-commitment contract  $(q^{NC}, P^{NC})$  and there is pooling with respect to the commitment decision.

**Proposition 1.3.** *Suppose the auditing costs of both types are not excessively high,*

$$c_H < \frac{(1 - \delta)q^{NC}}{\delta p^b[q^{NC}, P^{NC}]} \quad \text{and} \quad c_L < m(1 - \delta) \quad (1.6)$$

*and condition (1.4) holds for  $\alpha = \tilde{\alpha}[q^{NC}, P^{NC}]$  and  $(q, P) \in \{(q^C, P^C), (q^{NC}, P^{NC})\}$ . In any perfect Bayesian equilibrium, firms make zero profits and avoid commitment. Furthermore,  $(q^{NC}, P^{NC})$  is the only contract accepted by policyholders in equilibrium.*

First, we show that there is a perfect Bayesian equilibrium as characterized by the proposition. By the definition of contract  $(q^{NC}, P^{NC})$  insurers make zero expected profits. Proposition 1.2 showed that insurers are worse off with commitment given condition (1.4). Conditions (1.6) ensure that there are no profitable deviations. The proof proceeds along the following lines. Fraud reduces the insurance contract to stochastic redistribution with efficiency losses. This cannot generate a positive surplus. Consequently, a deviation with a contract that only attracts policyholders anticipating fraudulent behavior is not profitable. Second, a deviation with a contract implementing commitment is unprofitable, because, given our assumptions, even the best contract with commitment,  $(q^C, P^C)$ , is less attractive than the contract

<sup>19</sup>We write  $\tilde{\alpha}[q, P]$  and  $p^b[q, P]$  to make clear that both depend on the contract  $(q, P)$ .

$(q^{NC}, P^{NC})$  for the policyholders. Third, all the policyholders anticipating honest reporting behave homogeneously and therefore receive the same expected utility in equilibrium. In case of a deviation this means that the new contract attracts either all or no honest policyholders. This leaves only the policyholders anticipating fraudulent behavior in the previous contract and yields a change in the insurers' auditing strategy, because auditing becomes more beneficial as the probability of catching a fraudulent claim increases to  $1 - \delta$ . Since the remaining policyholders anticipate this behavior by the insurers, they also move contracts and cherry-picking by the deviating insurer becomes impossible.

To show that these properties hold in any equilibrium, we prove that there is no market equilibrium in profitable contracts. Furthermore, it is impossible to offer a more attractive contract than  $(q^{NC}, P^{NC})$  and avoid losses. This concludes the analysis of the model, showing that even market pressures do not force insurers to implement commitment. They use the uncertainty created by missing commitment as a deterrence device that makes it possible to offer better contracts. The corollary summarizes this comparison.

**Corollary 1.1.** *If commitment is obligatory, insurers offer the contract  $(q^C, P^C)$  in equilibrium, which is in utility terms less attractive for the policyholders than the contract  $(q^{NC}, P^{NC})$  without commitment given the conditions of Proposition 1.3. Therefore forgoing commitment implies an ex-ante Pareto improvement.*

## 1.5 The Model in the Absence of Ambiguity Aversion

In the absence of ambiguity aversion, it does not matter whether information about aggregate behavior is available. In particular, it does not matter whether policyholders expect an average auditing level or insurers also commit to this auditing level. In equilibrium, aggregate behavior is common information. Therefore announcing this information does not change agents' behavior. This irrelevance contrasts with the case where ambiguity aversion plays a relevant part, as then the availability of auditing data matters. Ambiguity aversion has different implications from risk aversion for the

## 1.5 THE MODEL IN THE ABSENCE OF AMBIGUITY AVERSION

commitment decision. This can be seen most clearly in inequality (1.3) summarizing the firm's commitment decision. The intuition is that ambiguity aversion changes the firm's profits in the absence of commitment, i.e., the left-hand side of inequality (1.3), without touching the profits with commitment, i.e., the right-hand side. Ambiguity aversion matters only in the case of non-commitment. Risk aversion, on the other hand, affects both cases and both sides of the inequality.

Formally, in the absence of ambiguity aversion, the ambiguity index  $\phi$  is linear and can be neglected. According to Proposition 1.2, type uncertainty is a necessary condition to have insurers abstain from commitment in equilibrium. Additionally, insurers prefer to abstain from commitment only in the third case (c) of Proposition 1.1. Therefore, we focus on this case in the following. In the absence of ambiguity aversion, there is complete fraud.<sup>20</sup>

**Corollary 1.2.** *Consider given contracts, beliefs  $\mu(r)$ , a linear ambiguity index  $\phi$ , and no commitment in the game beginning in stage 4. If  $\bar{r} < p^b$  and the costs of both types are separated by the threshold  $c_L < c^*(1) \leq c_H$ , there is complete fraud,  $\alpha = 1$ , and partial audits of  $p_H = 0$  and  $p_L = 1$ .*

In contrast to Proposition 1.1, we can determine  $\tilde{\alpha}$  explicitly by comparing the probabilities  $\bar{r}$  and  $p^b$ . If  $\bar{r} \geq p^b$ , the low-cost insurer, on its own, can completely deter fraud and  $\tilde{\alpha} = 0$ . This means that case (c) is impossible. On the other hand, for  $\bar{r} < p^b$ , ambiguity aversion changes behavior, because the low-cost insurer on its own cannot deter fraud and  $\tilde{\alpha} = 1$ . Then case (c) implies complete fraud, as  $\tilde{\alpha} = 1$ . Yet, in this case, Proposition 1.2 implies that a preference for non-commitment implies complete fraud,  $\alpha = 1$ . This reduces the insurance contract to stochastic redistribution – an undesirable feature.

As we allow for heterogeneity in ambiguity aversion, the counterpart might be heterogeneity in risk aversion, which we consider next. For this purpose, assume a family of strictly concave von-Neumann-Morgenstern utility indices  $u_{\mathcal{R}}$  indexed by  $\mathcal{R} \in [\underline{R}, \bar{R}]$ . The higher  $\mathcal{R}$ , the more risk-averse the agent is. Policyholder 1 is more risk-averse than policyholder 2 if there is an increasing and strictly concave function  $g$ , such that  $u_1 = g(u_2)$ . The degree of risk aversion  $\mathcal{R}$  is distributed according to a

<sup>20</sup>In the non-generic case  $\bar{r} = p^b$ , there are multiple equilibria. Now, the low-cost insurer has to audit every claim to deter insurance fraud. Therefore the level of fraud is  $1 \geq \alpha \geq \frac{\delta c_L}{(1-\delta)(q+m-c_L)}$ .

## CHAPTER 1

distribution function  $F^\circ$  with a density  $f^\circ > 0$ . With commitment, insurer  $i \in \{L, H\}$  chooses the auditing level to maximize its profits according to

$$\begin{aligned} \sup_{R_i \in [\underline{R}, \bar{R}]} & P - q(\delta + F^\circ(R_i)(1 - \delta)(1 - p^b(R_i))) + mF^\circ(R_i)p^b(R_i)(1 - \delta) - \\ & - c_i(\delta + F^\circ(R_i)(1 - \delta))p^b(R_i) \end{aligned}$$

In general, this yields a positive level of fraud  $\alpha = F^\circ(R_i) > 0$ . In the absence of commitment, heterogeneity of the risk preferences allows case (c) with an intermediate level of fraud,  $0 < \alpha < 1$ .

**Corollary 1.3.** *Consider given contracts, beliefs  $\mu(r)$ , a linear ambiguity index  $\phi$ , no commitment, and heterogeneous risk aversion in the game beginning in stage 4. If  $\bar{r} < p^b(\underline{R})$  and the costs of both types are separated by the threshold  $c_L < c^*(\bar{\alpha}) \leq c_H$ , there is some fraud,  $\alpha = \bar{\alpha}$ , and partial audits of  $p_H = 0$  and  $p_L = 1$ . The level of fraud  $\bar{\alpha} = F^\circ(\mathcal{R}^*)$  is determined by*

$$\mathcal{R}^* = \sup \left( \left\{ \mathcal{R} \in [\underline{R}, \bar{R}] \mid (1 - \bar{r})u_{\mathcal{R}}(-P + q) + \bar{r}u_{\mathcal{R}}(-P - M) > u_{\mathcal{R}}(-P) \right\} \cup \{\underline{R}\} \right).$$

In this case, the low-cost insurer audits every claim and prefers non-commitment if its costs are low enough. Then the insurer has to pay few indemnities and earns some income from fine payments. Commitment reduces the auditing probability of the low-cost insurer. The corresponding savings on auditing costs do not compensate for the loss in indemnities and fines if the auditing costs of the low-cost type are sufficiently small. The advantage of not committing for the high-cost type is smaller than with ambiguity aversion. The insurer might still profit by reducing its auditing probability, if its costs are sufficiently high. Yet, in a market equilibrium in which insurers set contracts, the heterogeneity in the risk aversion complicates the analysis of policyholders' behavior. Even if policyholders behave honestly, they have different valuations for a given policy. These differences in valuation make it possible to screen policyholders into different contracts. Then in each contract policyholders have a similar degree of risk aversion resulting in the setting of Corollary 1.2. Thus, whether a market equilibrium similar to Proposition 1.3 exists is an open question.



## 1.6 Extensions

As already mentioned in Footnote 5, the Internal Revenue Service in the U.S. stated on several occasions that it regards uncertainty about auditing procedures as a valuable method to increase tax compliance. Furthermore, it went to great lengths to defend this approach in several court cases brought under Freedom of Information Acts. If we assume that taxpayers are mobile to some extent and counties compete for tax revenues, the model in this chapter can be modified accordingly. Instead of receiving insurance, agents have to pick one county where they pay taxes. Not declaring their income correctly would correspond to reporting a fraudulent claim. Then the mechanism in this chapter might explain why counties stick to the IRS strategy of avoiding commitment. The deviation of attracting many taxpayers with low tax rates financed by committing to an auditing regime is not profitable in the equilibrium of our model.

The next extension goes back to the initial insurance model, but shifts the realization of the cost type after the commitment decision. Therefore, firms do not know which type they are, when they have the possibility to commit to a certain level of auditing. In this case, the considerations of the firms change. If the insurance company commits and the auditing costs are high, it has to bear the high auditing costs or the costs of fraud due to the low auditing probability. This threat is weighted against the usual advantages of commitment for the insurer with low costs. The decision about commitment depends on which effect dominates in equilibrium. Another modification of the timing allows auditing costs to be realized before insurers make their contract offers. Figure 1.4 summarizes the changes. Now, insurers can signal the auditing costs by their contract offers and there is two-sided asymmetric information already at the contracting stage. Thus, at  $t = 0$  nature determines the costs of an audit for the insurer, which are the same for all firms, but uncertain.<sup>21</sup> After that, the game is the same as before. Therefore the analysis of Section 1.3 remains unchanged and there is again a perfect Bayesian equilibrium without commitment. In this equilibrium, a

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<sup>21</sup>See Jost (1996) for a model with heterogeneous costs. In the model of Jost (1996), however, the coverage  $q$  is conditional on the claim being audited, which is not a common feature of insurance contracts.

- At  $t = 0$ , auditing costs  $c$  are realized and revealed to the insurer; furthermore, the degrees of ambiguity aversion are realized and revealed to the insured
- At  $t = 1$ , insurers make contract offers  $(q_i, P_i)$
- At  $t = 2$ , the insured choose contracts
- At  $t = 3$ , insurers can commit to an auditing probability  $p_i$
- $\vdots$

Figure 1.4: Modified Timing of the Extended Model

contract  $(\tilde{q}, \tilde{P})$  is offered by both types, which is an element of the following set

$$(\tilde{q}, \tilde{P}) \in \arg \max_{q, P \in \mathbb{R}^+} \delta u(-L + q - P) + (1 - \delta)u(-P)$$

with  $P \geq \delta q + (1 - \delta)q\tilde{\alpha}[q, P]$ .

Similar to the last section, the existence of the equilibrium without commitment requires additional assumptions: we assume that condition (1.4) in Proposition 1.2 is satisfied for the contract  $(\tilde{q}, \tilde{P})$  and for the contract

$$(\hat{q}, \hat{P}) \in \arg \max \delta u(-L + q - P) + (1 - \delta)u(-P)$$

with  $P - \delta q - \delta c_L p^b[q, P] \geq \tilde{P} - \delta \tilde{q} - \delta c_L p^b[\tilde{q}, \tilde{P}]$ . Furthermore,  $\tilde{\alpha}[\tilde{q}, \tilde{P}] < 1$  and the probability of a loss should be sufficiently high (respectively low), i.e.,

$$\delta \geq (\leq) \frac{(N - 1)\tilde{P} + \tilde{\alpha}[\tilde{q}, \tilde{P}](c_L - m)}{(N - 1)\tilde{q} + \tilde{\alpha}[\tilde{q}, \tilde{P}](c_L - m) + (Np^b[\tilde{q}, \tilde{P}] - 1)c_L} \quad (1.7)$$

depending on the sign of the denominator.<sup>22</sup> This condition guarantees that the advantage of the uncertainty is sufficiently big to restrain the low-cost type from revealing itself and capturing the whole market. Intuitively, for a positive denominator there have to be enough losses to reduce the possible cases of fraud. Thus, the amount

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<sup>22</sup>It can easily be seen that the denominator is bigger than the numerator, such that the fraction is always smaller than one and the constraint set is therefore non-empty. If the denominator is positive, the fraction might be negative, and in this case, the constraint is trivially satisfied. If, on the other hand, the denominator is negative, the fraction is always positive and thus the probability of a loss can be lower than the threshold.

of claims to audit is quite high even with commitment, and commitment does not pay for the insurance company, because it loses the deterrence effect and the fine income. If, on the contrary, the denominator is negative, catching fraudulent claims is so attractive for the insurer that a low incidence of losses is necessary to stabilize the equilibrium.<sup>23</sup>

**Proposition 1.4.** *Given the discussed conditions, there is an equilibrium with every insurer offering exclusively the contract  $(\tilde{q}, \tilde{P})$  and avoiding commitment.*

The equilibrium has an interesting feature. When the insurers consider a deviation, both types want to mimic the other type. The high-cost type wants to deviate if the out-of-equilibrium beliefs are tilted towards the low-cost insurer, because there will be little fraud. If, on the other hand, the beliefs are tilted towards the high-cost type, the low-cost insurance company can by deviating increase its market share and profits due to the beliefs of the policyholders about a low auditing probability. Yet the competitors use the commitment decision to signal the type of the deviating firm. This is why the out-of-equilibrium beliefs depend on the type of the deviating firm. Hence, the deviation is no longer profitable, because once the type of the deviating firm is revealed, the insurer is worse off than before by the conditions of Proposition 1.4. This holds even though the insurer may serve the whole market after a deviation. Thus, the actions of the competitors make this equilibrium possible.

If the type is revealed before the contract stage, in the equilibrium with commitment, the insurance market can break down. This happens if the high-cost type is realized and  $c_H \geq c'$ . Then no agent has a utility higher than without an insurance. Ambiguity allows avoiding this fate by making contracts feasible that rely on the deterrence effect of the uncertainty in the absence of commitment. If there is sufficient ambiguity, the level of fraud is always smaller than 1.

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<sup>23</sup>The equilibrium is not unique. There will usually be a continuum of the equilibria, like  $(\tilde{q}, \tilde{P} + \epsilon)$ , of the type described in Proposition 1.4, depending on the parameter values. Furthermore, there is a separating equilibrium with each cost type offering the best contract that just breaks even, if the type of the insurer is known. The change in the timing yields an informed principal problem that differs fundamentally from the model considered in the previous sections.

## 1.7 Conclusion

In this chapter, we discuss a costly state verification model with ambiguity about auditing costs. For this purpose, we use an insurance fraud setting. We show that ambiguity aversion reduces the inclination to engage in insurance fraud at a given level of auditing. The insurers, on the other hand, can gain by not committing to an auditing probability and maintaining the uncertainty, even if this means abandoning the advantages of commitment. This is the main contribution of this chapter, as we prove that uncertainty can be a feasible deterrence device.

The second contribution is to study a model with ambiguity aversion in a game-theoretic framework. Although ambiguity seems even more relevant in a strategic interaction than for a single player, the literature on ambiguity aversion has so far focused on decision theory and finance with notable exceptions discussed in the introduction. We provide a game-theoretic analysis of ambiguity-averse policyholders. Modeling the ambiguity on the type space, i.e., the auditing costs of the insurers, allows the use of common equilibrium concepts.

The third contribution of this chapter is to consider whether competition forces firms to educate consumers. According to a common line of argument, competitive pressure provides consumers with all relevant information, as competitors have an incentive to reveal the information in order to increase their market shares. In our model, uncertainty prevails and on the equilibrium path no firm has an incentive to make the auditing costs public. Therefore, there is a market equilibrium with perfect competition where firms do not grant access to their information about auditing probabilities and costs and the uncertainty allows mitigating the effects of insurance fraud.

Finally, we summarize the incentives of insurers to avoid commitment. Insurers benefit from the higher perceived probability of auditing and the resulting lower level of fraud if their costs of auditing are high enough. For low costs, however, the insurers gain from non-committing, as they catch more fraudsters, thus saving indemnities and earning fines at low costs. In some cases, these effects are so strong that the costs caused by fraud and its deterrence are lower than under credible commitment to an auditing level. Consequently, the insurers will opt to implement strategic ambiguity.

## Appendix to Chapter 1

### 1.A Decision Making with Ambiguity Aversion

There are several representations of preferences that allow for ambiguity aversion, like Schmeidler (1989) or Cerreia-Vioglio et al. (2011).<sup>24</sup> Formally, ambiguity aversion is defined to be the preference of a mixture of lotteries compared to the lotteries themselves if the agent is indifferent between the lotteries.<sup>25</sup> The chapter mainly uses smooth ambiguity aversion proposed by Klibanoff et al. (2005), which goes back to Segal (1987).<sup>26</sup> The agent knows the first-order and second-order probability distributions, but does not compute the reduced lottery. The first-order probability distribution is a distribution for the states of the world, i.e., the state space. The second-order probability distribution, on the other hand, reflects the probability for a first-order distribution. In their interpretation, the first-order distribution characterizes risk and the second-order distribution ambiguity. This distinction corresponds to the assumption that the first-order and second-order probabilities are based on different information. The intuition is that the agents have some theories or models of the world, that assign probabilities to the states of the world. The trust in each model is denoted by its second-order probability. The agent's preferences are represented by

$$f \rightarrow \int_{\Pi} \phi \left( \int u \circ f dP \right) d\mu.$$

The function  $\phi$  reveals the attitude of the agent towards ambiguity. Therefore we will call it the ambiguity index. An ambiguity-neutral subject with a linear  $\phi$  simply takes the expectation and derives simple probabilities for each state of the world. With ambiguity aversion,  $\phi$  is strictly concave. The concavity of this function corresponds to the degree of ambiguity aversion. The function  $u$  is a von-Neumann-Morgenstern utility index, which determines the attitude towards risk.<sup>27</sup> In addition,  $P$  is a probability measure on the state space and  $\Pi$  is a set of first-order probability measures  $P$ .

<sup>24</sup>Gilboa and Marinacci (2011) provide an excellent survey of such representations.

<sup>25</sup>See Ghirardato and Marinacci (2002) for alternative definitions.

<sup>26</sup>Similar representations are Seo (2009), Ergin and Gul (2009), Chew and Sagi (2008) and Nau (2006).

<sup>27</sup>Ambiguity aversion is independent of the attitude towards risk. An agent may be ambiguity-averse and at the same time risk-neutral, and conversely.

$\mu$  is a probability measure that corresponds to the second-order distribution on  $\Pi$ . The preference functional may be interpreted as a double expectation. First, the expected utility for every first-order distribution  $P$  is calculated. Then the expected utility for every  $P$  is transformed by the function  $\phi$ . Finally, the mean with respect to the second-order probabilities is calculated. Yet the results do not hinge on this choice of representation.

An alternative representation is Maxmin Expected Utility of Gilboa and Schmeidler (1989), which is equivalent to Choquet Expected Utility of Schmeidler (1989) in our setting. Again there is a (finite) set  $\Pi$  of first-order probability measures, which are considered relevant. In contrast to smooth ambiguity aversion, however, agents have no second-order probabilities available. Consequently, they behave *as if* the probability distribution that yields the lowest expected utility is correct. The preferences are represented by

$$f \rightarrow \min_{P \in \Pi} \int u \circ f dP$$

The axiomatisations of both representations are based on the common decision-theoretic axioms, except that the independence axiom is restricted to specific acts. This is less restrictive than independence for all acts.

## 1.B The Model with Maxmin Expected Utility

This section shows that the results of Section 1.2 are valid also in Maxmin Expected Utility. We assume a set of relevant probability distributions  $\Pi$  such that the probability of an audit is in the interval  $[(1 - \mathcal{A})p, (1 - \mathcal{A})p + \mathcal{A}]$  with a parameter  $\mathcal{A} \in [0, 1]$ . Accordingly, the policyholders know that the auditing probability is around  $p$ , but are unaware of the exact value.<sup>28</sup> For  $\mathcal{A} = 0$ , there is no ambiguity and agents simply take the subjective probability  $p$  of Section 1.2 into consideration. On the other hand, with ambiguity,  $\mathcal{A} > 0$ , policyholders are more cautious and allow for some margin of error. Consequently, they behave *as if* the probability of getting caught were higher.

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<sup>28</sup>In another approach, Gajdos et al. (2008) propose an axiomatic foundation for such a contraction representation.

**Lemma 1.3.** *Suppose the level of auditing is fixed. If the insurer does not announce the level of auditing and the ambiguity-averse policyholders do not have all the relevant information to determine it exactly, there is less insurance fraud than with easily available information about the auditing probability.*

**Proof:** Without a loss, the Maxmin Expected Utility is

$$\left(1 - ((1 - \mathcal{A})p + \mathcal{A})\right)u(-P + q) + ((1 - \mathcal{A})p + \mathcal{A})u(-P - M)$$

for fraudulent claims and  $u(-P)$  without a claim. First, suppose the level of auditing is disclosed. Thus, there is no ambiguity and  $\mathcal{A} = 0$ . Therefore the policyholder overstates the loss if the probability  $p$  of an audit is smaller than  $p^b$ , as before.

In the second case, the insurer does not reveal the probability of auditing a claim. Then there is ambiguity. With ambiguity aversion and ambiguity,  $\mathcal{A} > 0$ , the policyholder considers the worst probability distribution in her set  $\Pi$ . So an ambiguity-averse policyholder acts as if the probability of detection were  $(1 - \mathcal{A})p + \mathcal{A}$ . Once again there is a threshold  $p^*$  for honest reporting, with

$$\begin{aligned} p^* &= \frac{(1 - \mathcal{A})u(-P + q) + \mathcal{A}u(-P - M) - u(-P)}{(1 - \mathcal{A})[u(-P + q) - u(-P - M)]} = \\ &= p^b - \frac{\mathcal{A}}{1 - \mathcal{A}} \frac{u(-P) - u(-P - M)}{u(-P + q) - u(-P - M)} < p^b. \end{aligned}$$

As the last fraction is positive, we can conclude that  $p^* < p^b$  for  $\mathcal{A} > 0$ .  $\square$

This confirms our earlier result of Lemma 1.1 and shows that it is robust to the way ambiguity aversion is modeled.

## 1.C Additional Proofs

Lemma 1.1 shows that ambiguity reduces the amount of insurance fraud, holding auditing strategies fixed.

**Proof of Lemma 1.1:** First, suppose the level of auditing is disclosed. Then there is no ambiguity. Therefore the policyholder has an incentive to engage in fraud if the

probability  $p$  of an audit is smaller than

$$p^b = \frac{u(-P + q) - u(-P)}{u(-P + q) - u(-P - M)}.$$

If  $p \geq p^b$ , the policyholder will behave honestly and report only true losses.

In the second case, the insurer does not reveal the probability of auditing a claim. Thus, the policyholder lacks relevant information. The difference to the first case depends on the ambiguity aversion and the amount of ambiguity perceived by the policyholder. An ambiguity-neutral policyholder, i.e., with a linear  $\phi$ , takes the same subjective probability into account and evaluates her possible actions as before. With ambiguity aversion  $\phi$  is strictly concave. By Jensen's inequality it holds

$$\begin{aligned} & \int_{\Pi} \phi\left((1 - \hat{p})u(-P + q) + \hat{p}u(-P - M)\right) d\mu(\hat{p}) \leq \\ & \leq \phi\left(\int_{\Pi} (1 - \hat{p})u(-P + q) + \hat{p}u(-P - M) d\mu(\hat{p})\right) = \\ & = \phi\left((1 - p)u(-P + q) + pu(-P - M)\right). \end{aligned}$$

Thus, an ambiguity-averse policyholder acts as if the probability of detection were higher. Hence if the expected probability  $p = \int_{\Pi} \hat{p} d\mu(\hat{p})$  is at least  $p^b$ , no insurance fraud occurs. If the second-order distribution is non-degenerate, this holds even for lower expected probabilities.  $\square$

**Proof of Lemma 1.2:** Suppose the second-order distribution  $\mu(\hat{p})$  is such that the first policyholder weakly prefers to abstain from fraud. Then

$$\int_{\Pi} \phi_1\left((1 - \hat{p})u(-P + q) + \hat{p}u(-P - M)\right) d\mu(\hat{p}) \leq \phi_1(u(-P)).$$

As the second policyholder is more ambiguity-averse than the first one, Jensen's inequality yields

$$\begin{aligned} & \int_{\Pi} \phi_2\left((1 - \hat{p})u(-P + q) + \hat{p}u(-P - M)\right) d\mu(\hat{p}) \leq \\ & \leq g\left(\int_{\Pi} \phi_1\left((1 - \hat{p})u(-P + q) + \hat{p}u(-P - M)\right) d\mu(\hat{p})\right) \leq \\ & \leq g(\phi_1(u(-P))) = \phi_2(u(-P)). \end{aligned}$$



If the second-order distribution is non-degenerate, the first inequality is strict. Hence, the more ambiguity-averse policyholders commit less insurance fraud.  $\square$

Proposition 1.1 characterizes the equilibrium of the game after stage 4 given a contract with premium  $P$  and reimbursement  $q$ .

**Proof of Proposition 1.1:** As the beliefs  $\mu$  about  $r$  are considered as fixed in the Proposition, it does not consider any signaling or adverse selection effects. Lemma 1.2 ensures monotonicity of the fraud decision of the policyholders in  $\mathcal{A}$ . Now define  $\tilde{\alpha}$  as the fraction of policyholders engaging in fraud if the low-cost insurer audits every claim,  $p_L = 1$ , and the high-cost insurer audits no claims,  $p_H = 0$ . Then  $\alpha = F(\mathcal{A}^*)$  and

$$\mathcal{A}^* = \sup \left( \left\{ \mathcal{A} \in [\underline{A}, \bar{A}] \mid \int \phi_{\mathcal{A}}((1-r)u(-P+q) + ru(-P-M))d\mu(r) > \right. \right. \\ \left. \left. > \phi_{\mathcal{A}}(u(-P)) \right\} \cup \{\underline{A}\} \right).^{29}$$

Solving the equilibrium backwards, we consider the insurer setting the level of auditing. As the problem for the insurer is linear, at least one type has a corner solution and audits all or none of the claims made. If for a level of fraud  $\alpha$  the costs of auditing are lower (resp. higher) than  $c^*(\alpha)$ , as defined in (1.2), all (none of the) claims are audited. Consequently, an ambiguity-neutral policyholder acts *as if* the expected probability of an audit is

$$E(p) = \begin{cases} 1 & \text{if } c_H < c^*(\alpha) \\ p \in [\bar{r}, 1] & \text{if } c_H = c^*(\alpha) \\ \bar{r} & \text{if } c_L < c^*(\alpha) < c_H \\ p \in [0, \bar{r}] & \text{if } c_L = c^*(\alpha) \\ 0 & \text{if } c_L > c^*(\alpha) \end{cases}$$

depending on the auditing costs. Thus, we distinguish the following five cases: (a) no auditing  $p = 0$ , (b) low partial auditing  $0 < p < \bar{r}$ , (c) partial auditing  $p = \bar{r}$ , (d) high partial auditing  $\bar{r} < p < 1$ , and (e) complete auditing  $p = 1$ .

(a) If policyholders expect no audits,  $p = 0$ , every policyholder will report a claim,

<sup>29</sup>See footnote 17 for an interpretation.

even if no loss occurred. Ex post it will still be optimal to abstain from auditing for the insurer if the costs of auditing  $c$  for both types of insurer are higher than the expected benefit of detecting a fraudster,  $(1 - \delta)(q + m)$ . This is the first case (a) of the proposition with  $c_L \geq c^*(1)$ . If the costs are lower, this is not an equilibrium as the insurers do some auditing.

(b) If the level of auditing is low, i.e.,  $0 < p < \bar{r}$ , the low-cost insurer is exactly indifferent between auditing claim reports or not. Therefore the high-cost insurer will abstain from auditing any claims and we can solve the equilibrium backwards by calculating

$$\alpha = \frac{\delta c_L}{(1 - \delta)(q + m - c_L)}$$

from the definition of  $c^*(\alpha)$  in equation (1.2) to make the low-cost insurer indifferent. The level of fraud determines by equation (1.1) the necessary level of auditing as a solution  $p_L^*$  to

$$\phi_{\mathcal{A}'}(u(-P)) = \int \phi_{\mathcal{A}'} \left( u(-P + q) - r p_L (u(-P + q) - u(-P - M)) \right) d\mu(r)$$

with  $\mathcal{A}' = F^{-1}(\alpha)$ . The right-hand side of this equation is decreasing in  $p_L$  and is bigger than the left-hand side for  $p_L = 0$ . Therefore,  $p_L^* > 0$ . In addition,  $p_L^* < 1$  and the low-cost insurer can on its own deter enough policyholders from filing false reports if  $\tilde{\alpha} < 1$ . This condition corresponds to a high expected probability  $\bar{r}$  for facing the low-cost insurer. Hence, the level of fraud is  $\alpha \in (\tilde{\alpha}, 1)$  depending on the auditing costs  $c_L$  of the low-cost insurer. If these costs are lower than  $c^*(\tilde{\alpha})$ , the low-cost insurer has an incentive to audit as many claims as possible. Then it is impossible to make the low-cost insurer indifferent with respect to its auditing decision. If, on the other hand, these costs are higher than  $c^*(1)$ , it would not be worthwhile to audit any claims for the insurer. Consequently, the second part (b) of the proposition requires  $c^*(\tilde{\alpha}) \leq c_L < c^*(1)$ . Notice that  $\tilde{\alpha} < 1$  if the condition is satisfied, because  $c^*(\alpha)$  is increasing in  $\alpha$ .

(c) In the next step, consider an intermediate level of auditing,  $p = \bar{r}$ . Then, the low-cost insurer audits every claim made and the high-cost insurer does not audit any claims. Therefore the costs have to be  $c_L < c^*(\tilde{\alpha}) \leq c_H$ . Otherwise one

type of insurer has an incentive to deviate. The level of fraud is  $\tilde{\alpha}$  by definition.  $0 < c_L < c^*(\tilde{\alpha})$  implies that there will be some fraud and  $\tilde{\alpha} > 0$ , as  $c^*(0) = 0$ . Part (c) of the proposition describes this equilibrium.

(d) More auditing is achieved if the low-cost insurer audits every claim and the high-cost insurer audits some claims, i.e.,  $p_L = 1$  and  $p_H > 0$ . The high-cost insurer has to be indifferent to find this level of auditing optimal. Therefore we solve equation (1.2) of the definition of the indifference costs for the corresponding level of fraud as in case (b)

$$\alpha = \frac{\delta c_H}{(1 - \delta)(q + m - c_H)}.$$

$\alpha$  is smaller than one if and only if  $c_H < (1 - \delta)(q + m) = c^*(1)$ . Equation (1.1) determines the level of auditing in equilibrium as a solution  $p_H^*$  to

$$\phi_{\mathcal{A}'}(u(-P)) = \int \phi_{\mathcal{A}'}\left(u(-P + q) - (r + (1 - r)p_H)(u(-P + q) - u(-P - M))\right) d\mu(r)$$

with  $\mathcal{A}' = F^{-1}(\alpha)$ . The right-hand side of this equation is decreasing in  $p_H$  and is smaller than the left-hand side for  $p_H = 1$ . Therefore,  $p_H^* < 1$ . In addition,  $p_H^* > 0$  if  $\tilde{\alpha} > 0$  and the low-cost insurer cannot on its own deter all policyholders from filing false reports. This condition corresponds to a low subjective probability  $\bar{r}$  for facing the low-cost insurer. Hence, the level of fraud is  $\alpha \in (0, \tilde{\alpha})$  depending on the auditing costs  $c_H$  of the high-cost insurer. If these costs are above  $c^*(\tilde{\alpha})$ , it would not be worthwhile to audit any claims for the insurer. Consequently, part (d) of the proposition requires  $c_H < c^*(\tilde{\alpha})$ . Notice that  $\tilde{\alpha} > 0$  if the condition is satisfied.

(e) Finally, if every claim is believed to be audited, only true claims are reported. Then, however, the best strategy of the insurer ex post is not to audit any reports. Therefore, in the absence of commitment, some policyholders will always report false claims in equilibrium.  $\square$

Lemma 1.4 is required for the proof of Proposition 1.2.

**Lemma 1.4.** *Assume there is no commitment and the insurer of type  $i \in \{H, L\}$  is indifferent with respect to audits by the level of fraud, as  $\alpha = \frac{\delta c_i}{(1 - \delta)(q + m - c_i)}$ . Then the insurer of type  $i$  prefers to commit to a level of auditing  $p^b$  independent of the policyholders' beliefs about its type.*

**Proof:** The costs with commitment are lower than in its absence if

$$\alpha(1 - \delta)(1 - p_i)q - m\alpha p_i(1 - \delta) + (\delta + \alpha(1 - \delta))p_i c_i \geq \delta p^b c_i.$$

Collecting the  $p_i$  terms we get

$$\alpha(1 - \delta)q - p_i [\alpha(1 - \delta)q + m\alpha(1 - \delta) - (\delta + \alpha(1 - \delta))c_i] \geq \delta p^b c_i.$$

Rearranging the terms in the square brackets gives

$$\alpha(1 - \delta)q - p_i [\alpha(1 - \delta)(q + m - c_i) - \delta c_i] \geq \delta p^b c_i.$$

As  $\alpha = \frac{\delta c_i}{(1 - \delta)(q + m - c_i)}$  the term in square brackets equals 0 and we get  $\alpha(1 - \delta)q \geq \delta p^b c_i$ . This means that the auditing costs,  $(\delta + \alpha(1 - \delta))p_i c_i$ , and cost savings due to exposed frauds, i.e., indemnities not paid out,  $q\alpha p(1 - \delta)$ , in combination with fines received by the insurer,  $m\alpha p(1 - \delta)$ , offset each other. Consequently *only* the losses due to falsely stated claims,  $\alpha(1 - \delta)q$ , matter. The indifference condition (1.2) of the insurer in equilibrium causes this effect. Inserting  $\alpha$  yields  $\frac{\delta c_i}{(1 - \delta)(q + m - c_i)}(1 - \delta)q \geq \delta p^b c_i$ .

Multiplying the inequality by  $q + m - c_i$  leads to  $q \geq p^b(q + m - c_i)$ . Finally, arranging the terms for  $c_i$  and dividing by  $p^b$  gives us

$$-\frac{q}{p^b} + q + m \leq c_i. \tag{1.8}$$

$m \leq M$  and  $\epsilon = [u(-P) - u(-P - M)]q - [u(-P + q) - u(-P)]M > 0$ , because the utility index  $u$  is strictly concave. Therefore,

$$q - p^b(q + m) = (1 - p^b)q - p^b m \geq (1 - p^b)q - p^b M = \epsilon [u(-P + q) - u(-P - M)]^{-1} > 0.$$

This ensures that the left-hand side of inequality (1.8) is negative. Then, the inequalities are satisfied and the respective insurer can make itself better off by committing to an auditing level  $p^b$ .  $\square$

**Proof of Proposition 1.2:** We begin by considering pooling equilibria with both types avoiding commitment. Then the policyholders' beliefs remain unchanged at  $\mu$  if no commitment is observed. In the case of commitment, the beliefs are irrelevant. In the following, we consider different cost ranges. For high costs of auditing, the

insurers abstain from auditing and this is common knowledge. Consequently, they are indifferent on the commitment issue and the beliefs do not matter. Due to Proposition 1.1, Lemma 1.4 and the fact that  $c' > c^*(1)$ , this is the case for  $c_L > c'$ . Below  $c'$ , under commitment, audits become worthwhile and there is no insurance fraud. Without commitment, auditing is still too expensive. Therefore, commitment is necessary to avoid complete fraud and at least one type has an incentive to commit.<sup>30</sup> Yet, once the auditing costs of the low-cost type drop below  $c^*(1)$ , there is auditing even without commitment. We now distinguish the following cases according to Proposition 1.1. In case (b) or (d), commitment is always preferable to no commitment, because the insurer which does partial auditing has an incentive to commit itself. The reason is the same as in Picard (1996). As the indifference of the insurer determines the level of fraud, the insurer's costs are independent of its level of auditing. Therefore replicating the auditing level  $p^b$  of the commitment case does not change profits. Without commitment, the insurer still faces fraud causing additional costs for indemnities and audits that are not balanced by income from fines. The details can be found in Lemma 1.4. Consequently, there is an incentive to commit to an auditing level in these cases and no pooling equilibrium exists with both types avoiding commitment. Now suppose case (c) of Proposition 1.1 with audits of  $p_H = 0$  and  $p_L = 1$ . Then the low-cost type prefers not to commit if and only if equation (1.3) is valid for  $p_L = 1$  or  $-m\alpha(1 - \delta) + [\delta + \alpha(1 - \delta)]c_L \leq \delta p^b c_L$ . Rearranging the terms yields

$$c_L \leq \frac{m\alpha(1 - \delta)}{\delta(1 - p^b) + \alpha(1 - \delta)}.$$

The fraction on the right-hand side is positive and does not depend on  $c_L$ . Furthermore the threshold is smaller than  $c^*(\tilde{\alpha})$ , because by Lemma 1.4

$$\begin{aligned} & q - p^b(q + m) > 0 \\ \Leftrightarrow & \alpha(1 - \delta)q + \delta[q - p^b(q + m)] > 0 \\ \Leftrightarrow & \alpha(1 - \delta)q[\delta(1 - p^b) + \alpha(1 - \delta)] - mp^b\delta\alpha(1 - \delta) > 0 \\ \Leftrightarrow & \alpha(1 - \delta)(q + m)[\delta(1 - p^b) + \alpha(1 - \delta)] > m(\delta + \alpha(1 - \delta))\alpha(1 - \delta) \end{aligned}$$

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<sup>30</sup>Due to the definition of  $p^b$  and the concavity of  $u$  it holds  $c' > c^*(1)$ . Formally, this is equivalent to  $q/(\delta p^b) > q + m$ . Consequently, it is enough to show that  $q - p^b(q + m) > 0$ . This is done in Lemma 1.4.

$$\Leftrightarrow \frac{m\alpha(1-\delta)}{\delta(1-p^b) + \alpha(1-\delta)} < \frac{\alpha(1-\delta)(q+m)}{\delta + \alpha(1-\delta)} = c^*(\alpha).$$

Therefore condition (1.4) on  $c_L$  guarantees case (c). Consequently, for  $c_L$  small enough the low-cost type of insurer forgoes commitment. By equation (1.3), the high-cost type, on the other hand, avoids to commit if  $\alpha(1-\delta)q \leq \delta p^b c_H$ , as  $p_H = 0$ . This leads to

$$c_H \geq \frac{\alpha(1-\delta)q}{\delta p^b} = \alpha c'.$$

Moreover, this threshold is higher than the threshold for  $c_H$  in case (c) as seen by Lemma 1.4 and

$$\begin{aligned} & q - p^b(q+m) > 0 \\ \Leftrightarrow & \frac{q}{\delta p^b} > \frac{q+m}{\delta} > \frac{q+m}{\delta + \alpha(1-\delta)} \\ \Leftrightarrow & \alpha c' = \alpha \frac{(1-\delta)q}{\delta p^b} > \frac{\alpha(1-\delta)(q+m)}{\delta + \alpha(1-\delta)} = c^*(\alpha). \end{aligned}$$

Thus, the high-cost insurer has no incentive to commit if its costs are high enough. In summary, we have found a range of parameters such that, in equilibrium, the insurers choose not to commit to an auditing level, even if they have the possibility to do so credibly and free of charge. Area (A) in Figure 1.3 on page 25 illustrates this range of parameters. So far, we have considered only complete pooling with respect to the commitment decision. Yet by including partial pooling, it is possible to increase the parameter range for  $c_L$  and  $c_H$ , because the line of argument does not depend on the specific level of fraud  $\alpha$ . If condition (1.4) holds only for an  $\alpha < \tilde{\alpha}$  with  $\tilde{\alpha}$  as defined in Proposition 1.1, it is possible to choose  $\alpha$ , such that the high-cost type is indifferent with respect to commitment. Thus, it plays a mixed strategy and commits to an auditing level with some probability  $\sigma_H$ . This changes equilibrium beliefs if no commitment was observed. Then, the probability of a low-cost insurer is  $r + (1-r)(1-\sigma_H)$  with subjective probability  $\mu(r)$ . Hence, the equilibrium level of fraud decreases. As shown before, this behavior is sequentially rational. If, on the other hand, condition (1.4) holds only for  $\alpha > \tilde{\alpha}$ , choose  $\alpha$ , such that the low-cost type plays a mixed strategy with respect to the commitment decision. This decreases equilibrium beliefs if no commitment was observed. Hence, the equilibrium level

of fraud increases. Figure 1.3 depicts the bounds on the costs of the two types. Now consider pooling equilibria with both types committing. For  $c_L \geq c^*(1)$  this equilibrium exists independently of the out-of-equilibrium beliefs. For costs below this threshold, we distinguish three cases corresponding to cases (b), (c), and (d) in Proposition 1.1, depending on the out-of-equilibrium beliefs. Suppose the beliefs given that no commitment was observed are such that the insurer of type  $L$  is indifferent with respect to audits as in case (b). Then the high-cost insurer abstains from auditing,  $p_H = 0$ , if it does not commit to an auditing level. By the same reasoning as before, commitment is only optimal in this case if  $c_H \leq \frac{\alpha(1-\delta)q}{\delta p^b} = \frac{qc_L}{p^b(q+m-c_L)}$  and  $c_L < c^*(1)$ . If, on the other hand, the beliefs are such that the insurer of type  $H$  is indifferent as in case (d), the low-cost insurer will do complete auditing,  $p_L = 1$ , in the absence of commitment. Then commitment is only optimal for both types if  $c_H \leq c^*(1)$  and  $c_L \geq \frac{m\alpha(1-\delta)}{\delta(1-p^b)+\alpha(1-\delta)}$  with  $\alpha = \frac{\delta c_H}{(1-\delta)(q+m-c_H)}$ . Finally, the beliefs could be such that both insurers have a corner solution as in case (c) of Proposition 1.1. Analogously, this is sequentially optimal only if there exists an  $\alpha \in (0, 1]$  such that

$$c^*(\alpha) < c_H \leq \frac{\alpha(1-\delta)q}{\delta p^b} \quad \text{and} \quad c^*(\alpha) > c_L \geq \frac{m\alpha(1-\delta)}{\delta(1-p^b)+\alpha(1-\delta)}.$$

In addition, there is a corner solution for  $\alpha = 1$  with  $c_H > c^*(1)$  and  $c^*(1) > c_L \geq \frac{m(1-\delta)}{1-\delta p^b}$ . Finally, consider separating equilibria with type  $i$  committing and the other type  $j$  avoiding commitment. Then there is no ambiguity. Without ambiguity, however, commitment is at least weakly optimal. Policyholders' beliefs in the absence of commitment are degenerate at  $r = 1$  or  $0$ , respectively. According to Proposition 1.1 with  $\tilde{\alpha} = 0$  or  $1$ , respectively, this yields two cases for the non-committing insurer. First, there might be some fraud,  $\alpha = \frac{\delta c_j}{(1-\delta)(q+m-c_j)}$  for low costs,  $c_j < c^*(1)$ . In this case and in all other cases where the indifference of the insurer determines the level of fraud, Lemma 1.4 shows that commitment is preferable to no commitment, as it decreases the insurer's costs. Second, there is complete fraud,  $\alpha = 1$ , for high costs,  $c_j \geq c^*(1)$ . To ensure non-commitment is optimal for type  $j$ , it has to hold  $c_j \geq c'$ . Above this threshold, the non-committing insurer  $j$  is indifferent with respect to the commitment decision and avoiding commitment is sequentially optimal. Yet the committing insurer  $i$  might profit from a deviation to avoid commitment.

This deviation is only profitable if the committing insurer  $i$  is the low-cost type.  $-m(1 - \delta) + c_L \geq \delta p^b c_L$  makes this deviation unprofitable, as the low-cost type will audit every claim. Rearranging the terms gives  $c_L \geq \frac{m(1-\delta)}{1-p^b\delta}$ , which is smaller than  $c'$ , as  $\frac{q}{\delta p^b} > q + m$  by Lemma 1.4. Together with  $c_H \geq c'$ , this allows for a fully separating equilibrium in area (B) of Figure 1.3. Moreover, we have shown that in every fully separating equilibrium at least one type is indifferent with respect to the commitment decision.  $\square$

Proposition 1.3 considers the market equilibrium.

**Proof of Proposition 1.3:** First, we show that the strategy profile in the proposition is an equilibrium of the game. Given that the other insurers offer the contract  $(q^{NC}, P^{NC})$ , each insurer makes zero expected profits in equilibrium, because by Proposition 1.1 and 1.2 in combination with condition (1.4) it is optimal to avoid commitment and have the low-cost type doing the auditing, i.e.,  $p_L = 1$  and  $p_H = 0$ . Therefore there is pooling with respect to the commitment decision. At the time of contracting, auditing costs have not been realized yet. Thus, signaling is impossible. Yet the commitment decision allows for signaling. The equilibrium beliefs are  $\mu$ , as no commitment is observed. Off the equilibrium path, beliefs about types are given by the behavior characterized in Proposition 1.2 if they are on the equilibrium path of the continuation games beginning with the realization of insurer's types. Otherwise set them to  $\mu$ .

Consider insurer  $j$  deviating by offering a less appealing contract, denoted by  $(\hat{q}, \hat{P})$ , where the appeal or the attractiveness of a contract is given by

$$\delta u(-L + q - P) + (1 - \delta)u(-P).$$

By definition no policyholder who behaves honestly with probability one will accept  $(\hat{q}, \hat{P})$  independent of her beliefs. Thus, only policyholders who behave fraudulently might opt for the contract  $(\hat{q}, \hat{P})$ . Yet, compared to the equilibrium contract, fraud implies stochastic redistribution financed by the policyholders themselves with efficiency losses due to the auditing costs and the difference  $M - m$  in the fine payments. As agents are risk averse and ambiguity averse, any profitable contract



with this property offers less utility than the contract  $(q^{NC}, P^{NC})$ .<sup>31</sup> Consequently, the contract  $(\hat{q}, \hat{P})$  will either make a loss or attract no demand at all. Hence, this deviation is not profitable.

Now consider a deviation with a (weakly) more attractive contract  $(\bar{q}, \bar{P})$ . In this case all consumers who are made weakly better off switch contracts. Then the insurer makes a loss with every policyholder unless the insurer succeeds in lowering its costs due to auditing and fraudulent claims by changing the level of fraud in this contract. The next two paragraphs show that it is impossible to do so.

First, assume that the new contract  $(\bar{q}, \bar{P})$  implements commitment for some types of the insurer to reduce the costs related to fraudulent behavior. Given condition (1.4), however, commitment makes contracts more expensive for the insurer according to Proposition 1.2. Therefore even the best available contract  $(q^C, P^C)$  with commitment is less attractive than  $(q^{NC}, P^{NC})$ . If the deviating insurer anticipates to use commitment independent of its type, the condition on  $c_L$  in (1.4) and  $(1 - \delta)\tilde{\alpha}q < \min\{\delta c_H p^b[q^C, P^C], (1 - \delta)q\}$  by the condition on  $c_H$  result in

$$\begin{aligned}
 c_L &< \frac{m\tilde{\alpha}(1 - \delta) + \frac{1-\bar{r}}{\bar{r}}(\min\{\delta c_H p^b[q^C, P^C], (1 - \delta)q\} - (1 - \delta)\tilde{\alpha}q)}{\delta(1 - p^b[q^C, P^C]) + \tilde{\alpha}(1 - \delta)} \\
 \bar{r}\delta c_L + (1 - \delta)(\bar{r}(c_L - m) + (1 - \bar{r})q)\tilde{\alpha} &< \\
 &< \bar{r}\delta c_L p^b[q^C, P^C] + (1 - \bar{r})\min\{\delta c_H p^b[q^C, P^C], (1 - \delta)q\}. \quad (1.9)
 \end{aligned}$$

The right-hand side of the inequality calculates the costs of fighting fraud with commitment. It is higher than the costs in the absence of commitment. If, on the other hand, the deviating insurer makes the commitment decision dependent on its type, the following cases are feasible by Proposition 1.2. The fully separating equilibrium for  $(\bar{q}, \bar{P})$  is never profitable, because it implies complete fraud for the high-cost type and condition (1.9) ensures that the insurer is worse off. Moreover, it is impossible due to  $c_L < m(1 - \delta)$ . Now consider contracts that result in partial pooling, i.e., one type of insurer plays a mixed strategy with respect to the commitment decision. As the mixing type of insurer is indifferent between commitment and non-

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<sup>31</sup>The condition  $c_L < m(1 - \delta)$  ensures that a fully separating equilibrium is impossible, as  $m(1 - \delta) < \frac{m(1 - \delta)}{1 - \delta p^b}$  for all  $p^b > 0$ . If  $\tilde{\alpha}[q^{NC}, P^{NC}] + p^b[q^{NC}, P^{NC}] < 1$ , condition (1.4) already implies  $c_L < m(1 - \delta)$ .

commitment, its profits are the same in both cases and the budget constraint (1.5) is still binding.<sup>32</sup> By the definition of contract  $(q^{NC}, P^{NC})$  the contract  $(\bar{q}, \bar{P})$  cannot be profitable. Consequently, the contract  $(\bar{q}, \bar{P})$  makes losses with commitment and the deviating insurer will not implement commitment.

Second, the deviating insurer engages in cherry-picking and the policyholders with a low degree of ambiguity aversion are attracted to the contract  $(q^{NC}, P^{NC})$  offered by the remaining insurers. Thus, fraud will be low in contract  $(\bar{q}, \bar{P})$ . This yields a change in the auditing regime in the contracts  $(q^{NC}, P^{NC})$ . Due to the assumptions on  $c_L$  and  $c_H$ , complete fraud is never optimal in contract  $(q^{NC}, P^{NC})$ , as the insurers adapt their auditing strategies accordingly. Thus, some policyholders will report honestly, although they have chosen the contract  $(q^{NC}, P^{NC})$ , which is a contradiction. Hence, this deviation is not profitable. Together with Propositions 1.1 and 1.2, this completes the first part of the proof and shows that offering the contract  $(q^{NC}, P^{NC})$  without commitment and the low-cost type doing the auditing, i.e.,  $p_L = 1$  and  $p_H = 0$ , is a perfect Bayesian equilibrium of the game.

In the second part of the proof, we show that any equilibrium satisfies the properties stated in the proposition. For this purpose, assume to the contrary that there is an equilibrium with different contracts accepted by the policyholders. If in expectation insurers make profits on their contracts in this alternative equilibrium, we show a contradiction in the next three steps. First, assume to the contrary that there are at least two profitable contracts with complete fraud. If  $c_L \geq c'$  in the corresponding contract and the contract is profitable, no one will accept the contract. Therefore the only remaining case is pooling on non-commitment with  $\tilde{\alpha} = 1$ . Then it is a profitable deviation to propose a contract that does not attract any honest policyholders, but is preferred by the fraudsters from the first two contracts. This is always feasible and decreases profits per policyholder, but increases total profits due to the gain in market share. Therefore there is at most one contract with complete fraud.

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<sup>32</sup>The change in the level of fraud caused by the partial pooling makes it more difficult to satisfy the budget constraint (1.5). If the low-cost insurer is using partial commitment, the level of fraud increases in the case without commitment compared to both types not committing. Yet partial pooling is only implemented if in the contract  $(\bar{q}, \bar{P})$  the level of fraud with pooling is lower than in a corresponding contract where complete pooling is optimal. Therefore fraud is still lower than in the pooling contract. If such a contract is more profitable than  $(q^{NC}, P^{NC})$ , this contradicts the definition of  $(q^{NC}, P^{NC})$ . The argumentation is analogous if the high-cost insurer uses partial commitment.

Second, assume to the contrary that in equilibrium there are at least two profitable contracts with commitment and some policyholders who always behave honestly. This implies partial pooling or complete pooling on commitment. Now reduce the premium  $P$  by a small  $\epsilon > 0$  and implement commitment as in the initial contract. The honest policyholders now choose the new contract and increase the market share of the insurer, making this deviation profitable. Therefore there is at most one profitable contract with commitment and honest policyholders in equilibrium.<sup>33</sup>

Third, take one of the profitable contracts with honest policyholders and no commitment,  $(q^1, P^1)$ . By the previous steps, there exists at least two of them, as full separation is impossible by  $c_L < m(1 - \delta)$ . Moreover, by Proposition 1.2, the auditing regime corresponds to pooling on case (c) of Proposition 1.1. In these contracts, beliefs about types and auditing probabilities for each type of insurer are identical at the contracting stage. This allows for a profitable deviation by offering a contract that is slightly more attractive than  $(q^1, P^1)$  instead of  $(q^1, P^1)$ . The modified contract attracts all the policyholders from the contracts in this class.

Therefore, in equilibrium, some insurers make zero profits on their contracts. If these contracts are less attractive than the contract  $(q^{NC}, P^{NC})$ , an insurer may deviate by offering the contract  $(q^{NC}, P^{NC} + \epsilon)$  with  $\epsilon > 0$ , such that the contract is still more attractive than the equilibrium contracts. Then all policyholders, who before anticipated behaving honestly independently of the commitment decision or were in a contract with complete fraud, opt for the new contract, because it increases their utility. Other policyholders follow suit, as they anticipate that auditing regimes are changing due to the different distribution of ambiguity aversion in the previous contracts. This guarantees positive profits for the deviating insurer.

Assume to the contrary that the equilibrium contracts are (weakly) more attractive than contract  $(q^{NC}, P^{NC})$ . As shown in the first part of the proof, the contracts make losses if they use commitment or if there is complete fraud. Therefore, by Proposi-

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<sup>33</sup>In equilibrium it is infeasible to have partial pooling with respect to commitment and every policyholder filing a claim in the absence of commitment. The reason is that policyholders get a higher utility the higher the probability is of facing a high-cost type in the absence of commitment. This can be achieved by changing the probability of commitment of the types. The level of fraud remains unchanged, as  $\alpha = 1$ . Furthermore, profits remain unchanged due to the indifference condition of the mixing type of insurer. Consequently, the policyholders would be willing to enter a more profitable contract. This is a profitable deviation and shows why such an auditing regime is impossible.

tion 1.2, the only remaining auditing regime is pooling in case (c) of Proposition 1.1 and all equilibrium contracts offer the same expected utility for an honest policyholder. By the definition of contract  $(q^{NC}, P^{NC})$ , insurers make a loss if the policyholders are split up equally between insurers. Now assume the distribution of policyholders into contracts is heterogeneous, so that the amount of fraud differs between contracts. In some contracts, it is above  $\tilde{\alpha}[q, P]$ , while in others it is below  $\tilde{\alpha}[q, P]$ . Yet it is impossible to reduce the costs due to auditing and fraudulent claims and screen the policyholders according to their ambiguity aversion. The reason is the following. If  $\bar{r}/(1 - \bar{r}) = q/(m - c_L)$ , profits do not change in the amount of fraud and any contract except  $(q^{NC}, P^{NC})$  that is (weakly) more attractive than  $(q^{NC}, P^{NC})$  makes a loss. If  $\bar{r}/(1 - \bar{r}) < q/(m - c_L)$ , profits are decreasing in the level of fraud and contracts with fraud above  $\tilde{\alpha}[q, P]$  make losses. Given the high indemnity, however, these contracts attract the policyholders anticipating fraudulent behavior, as the auditing regime remains unchanged. If, on the other hand,  $\bar{r}/(1 - \bar{r}) > q/(m - c_L)$ , the contracts with fraud below  $\tilde{\alpha}[q, P]$  make losses. Again, the low indemnity deters the fraudsters from those contracts generating a loss for the insurer. Yet, in equilibrium there are no insurers with loss-making contracts. Consequently,  $(q^{NC}, P^{NC})$  is the only accepted contract in any equilibrium of the game.  $\square$

Proposition 1.4 considers the market equilibrium in the extension of the game.

**Proof of Proposition 1.4:** The beliefs of the insured about the type of insurer are  $\mu$  if they observe the contract  $(\tilde{q}, \tilde{P})$ . If, on the other hand, they observe a different contract and at least  $N - 2$  of the insurers commit, they update their beliefs to  $\mu(1) = 1$ . Otherwise beliefs remain at  $\mu$ . If a deviation at the contracting stage occurs, firms with low costs  $c_L$  commit at  $t = 3$ . The beliefs of the insurer about the ambiguity aversion of its policyholders are according to the distribution  $F$ .

The low-cost type makes positive profits with the contract  $(\tilde{q}, \tilde{P})$ , because according to Proposition 1.2 in combination with condition (1.4) auditing is profitable and the premium is set, such that no auditing gives zero profits and auditing is profitable for the low-cost type. If a firm  $j$  of the low-cost type tries to capture the whole market by offering a more attractive contract  $(\hat{q}, \hat{P})$ , due to the out-of-equilibrium beliefs agents know its type, since the behavior of the competitors reveals it. Consequently, insurer  $j$  always wants to commit to an auditing level in its contract  $(\hat{q}, \hat{P})$ . No matter

whether the insured go to the deviating insurer or stay with the equilibrium contract, we show that the deviation is not profitable.<sup>34</sup> The profits with the new contract  $(\hat{q}, \hat{P})$  are lower, because by assumption  $\hat{P} - \delta\hat{q} - \delta p^b[\hat{q}, \hat{P}]_{c_L} \leq \tilde{P} - \delta\tilde{q} - \delta p^b[\tilde{q}, \tilde{P}]_{c_L}$  and condition (1.7) yields

$$\begin{aligned}
 & \tilde{P} - \delta\tilde{q} - \delta p^b c_L \leq \frac{1}{N} \left[ \tilde{P} - \delta\tilde{q} + m\alpha(1 - \delta) - (\delta + \alpha(1 - \delta))c_L \right] \\
 \Leftrightarrow & (\tilde{P} - \delta\tilde{q})(N - 1) - N\delta p^b c_L \leq m\alpha(1 - \delta) - (\delta + \alpha(1 - \delta))c_L \\
 \Leftrightarrow & (N - 1)\tilde{P} + \alpha(c_L - m) \leq \delta \left[ \tilde{q}(N - 1) + Np^b c_L - m\alpha - (1 - \alpha)c_L \right] \\
 \Leftrightarrow & \delta \geq (\leq) \frac{(N - 1)\tilde{P} + \alpha(c_L - m)}{(N - 1)\tilde{q} + \alpha(c_L - m) + (Np^b - 1)c_L}.^{35}
 \end{aligned}$$

The direction of the inequality in the last line depends on the sign of the denominator, as discussed before. The strategy of the other insurers is sequentially optimal, as commitment is optimal for an insurer offering contract  $(\tilde{q}, \tilde{P})$  given the beliefs  $\Pr(c_L) = 1$ . Therefore it is a best response for the low-cost insurer to offer  $(\tilde{q}, \tilde{P})$  in this equilibrium.

The high-cost type, on the other hand, has no incentive to deviate either, because by offering the contract  $(\tilde{q}, \tilde{P})$  with commitment, the insurer would make a loss according to Proposition 1.2. Similarly, the insurer would incur a loss if it offered a more attractive contract by the definition of contract  $(\tilde{q}, \tilde{P})$ . Given the beliefs  $\mu$ , the other insurers have no incentive to commit. Therefore no profitable deviation is possible. In equilibrium, both types of insurers decide to avoid commitment and every firm offers the contract  $(\tilde{q}, \tilde{P})$ .  $\square$

<sup>34</sup>Indeed, for  $c_H \leq \frac{(1-\delta)\tilde{q}}{\delta p^b[\tilde{q}, \tilde{P}]}$ , all insured opt for the new contract  $(\hat{q}, \hat{P})$ , because the new contract is more attractive and the insured behave honestly. If  $c_H$  is higher, some policyholders may stay with the old contract.

<sup>35</sup>Here we suppress the dependency of  $\alpha$  and  $p^b$  on  $[\tilde{q}, \tilde{P}]$  for notational convenience.



# Chapter 2

## Legal Uncertainty

### An Effective Deterrent in Competition Law?

#### 2.1 Introduction

Given the complexity of legal procedures in competition law, legal uncertainty is a major issue.<sup>1</sup> Previous literature has shown that legal uncertainty yields disproportionate deterrence – over-deterring socially beneficial actions, while under-deterring socially detrimental ones.<sup>2</sup> This chapter sets up a formal model to study whether the legal uncertainty inherent in a legal rule can advance the objectives of the policymaker. The analysis shows that legal uncertainty itself might have positive effects if it is not too large. The reason is that it allows mitigating the policy restrictions of the competition authority. Legal uncertainty allows getting closer to the optimal deterrence level that is contingent on aspects unobservable by the competition authority. Thus, the competition authority may use legal uncertainty as a screening device. A certain business practice can be pro- or anticompetitive depending on the circumstances and the competition authority cannot perfectly distinguish between them. In this case, uncertainty about the threshold of legality deters firms with few gains from the

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<sup>1</sup>With legal uncertainty, I refer here to circumstances where it is not clear whether a certain business practice is legal. This is similar to the notion of D’Amato (1983).

<sup>2</sup>See for instance Calfee and Craswell (1984); Craswell and Calfee (1986), Schinkel and Tuinstra (2006) or Katsoulacos and Ulph (2012).

## CHAPTER 2

business practices that are close to the threshold. Yet, firms with large gains still pursue the new business practices, even if they are near the legal threshold. This allows screening firms according to some unobservable characteristics and increases social welfare, as the inherent legal uncertainty makes the rules more selective. If a consumer welfare standard is pursued, the result still holds, as legal uncertainty increases the probability of a conviction and thus reduces the enforcement costs of the competition authority.

There are different reasons for this kind of legal uncertainty. According to Calfee and Craswell (1984, p. 968) ‘it is difficult to predict . . . how an antitrust court will distinguish between ‘predatory’ and ‘competitive’ price cuts.’ Alternative reasons are the existence of different procedures, measurement errors by the competition authority, different assessments of efficiency defenses or uncertainty about what kind of evidence will be allowed. Consider two examples. First, in the European Union vertical restraints, like resale price maintenance or exclusive dealings, are prohibited under Article 101 (TFEU), formerly Article 81 (EC).<sup>3</sup> There is a Block Exemption, however, so that this rule does not apply if the market shares of the involved parties are below 30%. Although the European Commission gives guidelines how the relevant market shares are to be determined, it is extremely difficult to predict correctly the market share determined by the competition authorities. The causes are discrepancies in the definition of the relevant market, information asymmetries or imprecision in the measurement of sales, and other factors. This creates the kind of uncertainty analyzed in the model.

The second example is the case of Microsoft tying its operating system with additional software, in particular, a web browser and a media player.<sup>4</sup> In both instances the European Commission found an abuse of a dominant market position under Article 102 (TFEU), formerly Article 82 (EC). Think of a scale beginning with products where the bundling with the operating system is socially beneficial, as the integration allows for new features or higher performance and independent competing products are non-existent. On the other end of the scale are products where the bundling yields few or no efficiency gains, but competition is harmed considerably.

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<sup>3</sup>See European Commission (2010a) for details.

<sup>4</sup>These are the cases COMP/39.530 and T-201/04 Microsoft vs. Commission. The commission summarizes its findings in the former case in European Commission (2010b).



While there is legal certainty on both ends of the scale, in the middle there is a region where it is very difficult to exclude legal uncertainty completely. According to the model in this chapter this legal uncertainty might actually be socially beneficial.<sup>5</sup>

In an extension of the model I analyze uncertainty about the size of the fine that is imposed on firms in case of a conviction. After a conviction by the competition authority, firms might turn to the courts to change the fine. If the competition authority is more concerned about enforcement costs than its income from fines, this additional uncertainty is beneficial, as long as there is no change in the expected value of the fines. Legal uncertainty about the fine makes enforcement easier for the competition authority.

A caveat applies here. Although this model points out positive effects of legal uncertainty, too much legal uncertainty actually decreases social welfare. Furthermore, there may be negative effects of legal uncertainty that are not captured in our analysis.<sup>6</sup> Consequently, legal uncertainty is no panacea. The policymakers, however, might positively influence the effects of legal uncertainty and the direction of the deterrence effects towards anticompetitive behavior by complementing a rule of reason with per-se exceptions, like safe harbors, or detailed information with respect to some aspects of the procedure.<sup>7</sup>

The effects of legal uncertainty discussed in this chapter might also influence the trade-off between per-se rules and rules of reason. With per-se rules, some clearly specified practices, like, e.g., certain rebates or resale price maintenance, are prohibited. A rule of reason, on the other hand, judges the use of a practice as illegal whenever the practice is used in an anticompetitive way. Thus, the test of legality is whether competition was promoted or hindered.<sup>8</sup> Therefore a business practice may be legal in some cases, but not in others, depending on its consequences. Hence, rules of reason typically imply a certain degree of legal uncertainty. Recently, there

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<sup>5</sup>Although both examples are from Europe, the results of this chapter are also valid for U.S. competition law. Yet in the United States courts have frequently interpreted legal uncertainty in favor of the investigated party, thereby reducing overall deterrence.

<sup>6</sup>For instance, legal uncertainty may reduce the possibility to control the competition authority, as it becomes more difficult to detect incompetent or corrupt behavior. In addition, legal uncertainty might result in socially wasteful expenses in safeguards and evidence production.

<sup>7</sup>Ahlborn et al. (2004) and Christiansen and Kerber (2006) propose such modified or structured rules of reason.

<sup>8</sup>Kaplow and Shapiro (2007, p. 54ff) provide a good discussion of rules of reason in antitrust.

has been a major shift away from per-se rules — exemplified by the case *Leegin vs. PSKS*, as the court’s decision allowed resale price maintenance if it does not impede competition.<sup>9</sup> Also the competition authorities in the European Union aim to pursue a more economic approach. This approach focuses more on the market effects of the business practice under consideration. An example is the discussion of the European Commission about the enforcement of Article 102 (TFEU), formerly Article 82 (EC).<sup>10</sup> Previous literature has argued that rules of reason allow differentiating the competition law in a more selective way at the price of some inherent legal uncertainty, because firms sometimes do not know whether their conduct is legal. Katsoulacos and Ulph (2012, p. 3) summarize this issue as follows: ‘legal uncertainty induced by effects-based procedures [i.e., rules of reason] should lead the CA [competition authority] to favor per-se procedures.’ This chapter shows that the conclusion depends very much on the amount of legal uncertainty. If it is sufficiently small, it could even improve the balance in favor of rules of reason.

The rest of the chapter is organized as follows. Section 2.2 discusses the relevant literature. Section 2.3 sets up the model with the competition authority choosing a policy that determines which actions will be punished if the competition authority detects a firm implementing them. The competition authority can observe only one of two dimensions of the action, and thus cannot distinguish pro- and anticompetitive actions perfectly. Section 2.4 provides the analysis why legal uncertainty may be beneficial. The reason is a screening effect, as legal uncertainty allows influencing firms’ behavior depending on unobservable aspects of their type. Section 2.5 discusses an extension of the model, allowing firms to turn to the courts to repeal the decision of the competition authority. Finally, Section 2.6 contains the concluding remarks.

## 2.2 Related Literature

Legal uncertainty inherent in legal rules is closely related to the trade-off between rules of reason and per-se rules that has been studied before. Ehrlich and Posner

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<sup>9</sup>Supreme Court of U.S. ‘*Leegin Creative Leather Products, Inc. vs. PSKS, Inc.*’ Decision No. 06-480, June 28th, 2007.

<sup>10</sup>Cf. European Commission (2009), Kroes (2006), European Commission (2005) or Gual et al. (2005).

## 2.2 RELATED LITERATURE

(1974) discuss the advantages and disadvantages of having per-se rules replaced by rules of reason, as these could better distinguish beneficial from harmful actions, but provide less guidance for the concerned parties. Yet, they do not analyze the overall effects on welfare. Kaplow (1995) assumes mutual ignorance about the nature of the considered action, because firms do not know the exact rules and the competition authority does not know the specific circumstances of the firm. Therefore both parties have to invest if they want to get the information. Thus, Kaplow (1995) models the trade-off between compliance costs and selectivity of rules. He shows that compliance costs are often low, even for quite complex rules. There is no legal uncertainty, however, if a firm decides to invest in order to learn the rules.

Katsoulacos and Ulph (2009) model the trade-off between different procedures in competition law. They characterize the conditions, when the distinction of pro- and anticompetitive procedures by rules of reason is welfare-enhancing compared to per-se rules. In many circumstances rules of reason are welfare superior. The extension of Katsoulacos and Ulph (2012) scrutinizes, in particular, the arising legal uncertainty by introducing a second dimension of uncertainty about the nature of the considered business practice similarly to my model. They find that the selectivity of a rule of reason often outweighs the losses due to the arising legal uncertainty. I concentrate on legal uncertainty and do not consider the comparison between per-se rules and rules of reason. My model allows varying continuously the legal uncertainty inherent in legal rules. I show that the uncertainty itself might increase social welfare if the amount of uncertainty is sufficiently small.

Calfee and Craswell (1984) discuss the kind of legal uncertainty I consider here and Craswell and Calfee (1986) formalize it. In their model, however, there is no information asymmetry about the nature of the considered action. Therefore, the legal uncertainty only hinders the implementation of the optimal legal threshold and either causes too much or too little deterrence. I show that legal uncertainty is beneficial and has positive effects on welfare.

The beneficial effects of legal uncertainty have appeared in different contexts. Strausz (2011) points out that regulatory risk might be advantageous and studies the necessary market structures. Choné and Linnemer (2008) study the effect of uncertain efficiency gains on merger control. They characterize the market structure

and demand elasticities that make such uncertainty beneficial. Furthermore, the deterrence effect of uncertainty is already used in tax enforcement. According to Reinganum and Wilde (1988, p. 794), the Internal Revenue Service (IRS) in the U.S. confirms that ‘one of the tools in the arsenal of the IRS which promotes voluntary compliance is the uncertainty in the minds of the taxpayers.’ There are additional aspects to consider, however, as individuals as well as firms are affected.<sup>11</sup>

Finally, there is a literature on costly state verification. Besanko and Spulber (1989) use such a model to analyze optimal enforcement of antitrust laws, but do not touch on the issue of legal uncertainty. The problem of costly state verification (Townsend, 1979) considered here is not limited to competition law, but also appears in different settings, like regulation (Baron and Besanko, 1984), financing (Gale and Hellwig, 1985), or accounting (Border and Sobel, 1987).

## 2.3 The Firms Facing the Competition Authority

In the model there is a competition authority facing a continuum of firms with mass one.<sup>12</sup> Its objective is to maximize total welfare, i.e., the sum of consumer surplus and firms’ profits, weighting the firms’ profits by  $\alpha$  with  $0 \leq \alpha < 1$ .<sup>13</sup> The competition authority sets competition policy by choosing two enforcement parameters  $\hat{x}$  and  $p$ .  $\hat{x} \in [0, 1]$  captures the threshold of legality. In the first example of vertical restraints this is a specific market structure, e.g. a market share of 30%.  $p$  is the fraction of firms audited. Every firm has the binary choice whether to implement a specific, new business practice, like, e.g., bundling, retail price maintenance or rebates, or to abstain from it. Depending on its choice, I refer to a firm as active or deterred.

The nature of the available action depends on the firm’s type  $(x, b)$  that is two-dimensional as in Katsoulacos and Ulph (2012). The firm knows its type  $(x, b)$  in both dimensions. The first dimension  $x$  captures the aspects that the competition authority can observe with its auditing technology at a cost  $c$  per audit. Returning to

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<sup>11</sup>For a discussion, see Cronshaw and Alm (1995), Snow and Warren (2005), Osofsky (2011), and Gergen (2011). In a different realm, Teitelbaum (2007) discusses the effects of ambiguity on individuals in tort law, while Harel and Segal (1999) consider criminal law.

<sup>12</sup>This is equivalent to a single firm. Both interpretations are valid.

<sup>13</sup>Besanko and Spulber (1993) and Neven and Roller (2005) study the relative merits of different welfare standards.

## 2.3 THE FIRMS FACING THE COMPETITION AUTHORITY

my examples from the introduction, this refers to the market structure, for example, market shares in the case of vertical restraints. In the case of Microsoft  $x$  denotes the kind of software added to the operation system and whether the integration is socially beneficial or harmful. The value  $x$  of the firm's type is drawn from a uniform distribution  $G$  on the interval  $[0, 1]$ .<sup>14</sup> The value  $b$  is unobservable by the competition authority. As  $b$  influences the decision of the firm, I will call  $b$  the private benefits of the company, which are independently and uniformly distributed on  $[0, \bar{b}]$  with  $\bar{b} > 0$ .<sup>15</sup>

Social welfare remains unchanged if the firm continues its previous market behavior. If a firm of type  $(x, b)$  adopts the new business practice, it generates negative externalities of  $h(x)$  given by the function  $h(\cdot)$  and private benefits of  $b$ .<sup>16</sup> Thus, weighted welfare changes by  $\alpha b - h(x)$ . The first dimension of the firm's type  $x$  is ordered in such a way that a higher  $x$  signifies higher social harm, i.e.,  $h'(x) > 0$ . Yet, for some types implementing the business practice is socially beneficial and for some it is socially harmful, such that  $h(0) < 0$  and  $h(1) > 0$  to make the problem interesting. Thus, the practice can be pro- or anticompetitive depending on the firm's type. There are many examples for business practices that can have pro- as well as anticompetitive effects. Price reductions, e.g., might reflect lower costs or an attempt at predatory pricing. The same holds for bidding patterns in procurement contests or standardization efforts, which might have beneficial effects or be part of some collusive agreement in order to harm other market participants. In the case of vertical restraints, a simplification would be to consider only the market shares. If these are very low, the restraints do not harm other market participants,  $h(0) < 0$ . The vertical restraints, however, could be socially very harmful,  $h(1) > 0$ , if the firms involved dominate their respective markets. In the case of Microsoft,  $h(1) > 0$  corresponds to implementing a web browser in order to acquire a dominant position in the browser market by abusing its dominance in the market for operating systems.  $h(0) < 0$ , on

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<sup>14</sup>This assumption is only a small restriction, as I could redefine the units of  $x$  to match any distribution that admits a density and has connected support.

<sup>15</sup>The model is robust to the introduction of some correlation between  $b$  and  $x$ . As long as the correlation with  $x$  is not too strong, the mechanism in this model works. With perfect correlation the competition authority could infer the value of  $b$  from the  $x$  value and therefore does not need legal uncertainty as a screening device.

<sup>16</sup>The externality function  $h(x)$  should be differentiable on the whole domain, i.e.,  $|h'(x)| < \infty$ .

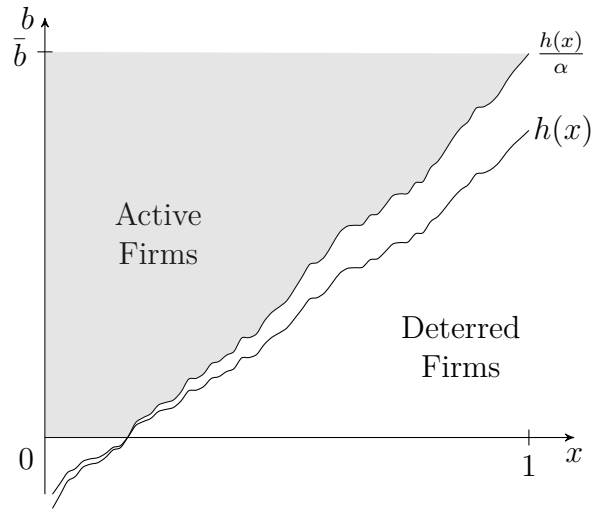


Figure 2.1: Active Firms in the First-best Policy

the other hand, corresponds to integrating new and socially beneficial features, like a basic firewall, touchscreen support or improved USB drivers.

As a benchmark consider the first-best policy, where the firm's type is observable and verifiable. In this case a firm of type  $(x, b)$  should be active, whenever  $\alpha b - h(x) \geq 0$ . Then only the firms depicted in Figure 2.1 are active and implement the new business practice. In the model the competition authority cannot perfectly observe and verify the firms' type. In particular, the competition authority might prohibit the business practice for some firms depending on the observable aspects, i.e., the first dimension, of their type. Due to the monotonicity of  $h(x)$ , it is optimal for the competition authority to forbid all actions above a threshold, i.e.  $x \geq \hat{x}$ . Therefore restricting competition policy to setting a threshold of legality  $\hat{x}$  is without loss of generality. If the competition authority finds the firm to have  $x$  below  $\hat{x}$ , its actions may well be socially efficient and therefore the competition authority allows the firm to continue. Above  $\hat{x}$ , however, actions are judged as anticompetitive and are prohibited. If the competition authority detects a firm violating this policy, it can make the firm pay a fine  $f > \bar{b}$ . Yet for this purpose, the competition authority has to invest resources to produce evidence. The competition authority can make the observed  $x$  verifiable at

## 2.3 THE FIRMS FACING THE COMPETITION AUTHORITY

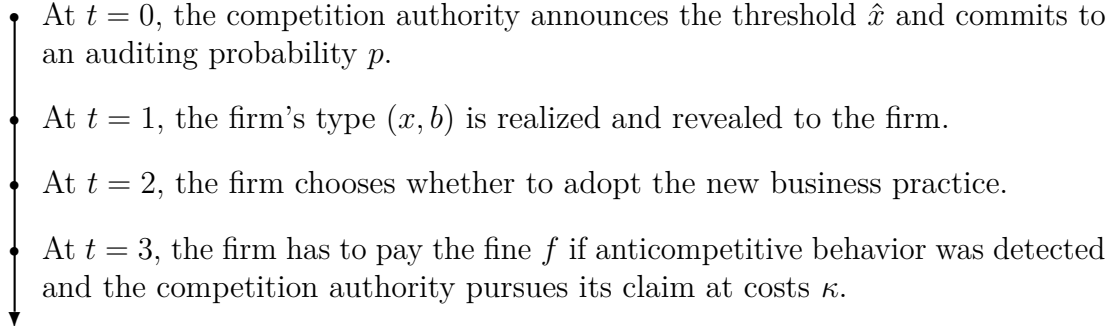


Figure 2.2: Timing of the Model

costs  $\kappa$  with  $\kappa < (1 - \alpha)f$ .<sup>17</sup> The costs  $\kappa$  capture experts' testimonies, reports and other expenses to prove the competition authority's case. Figure 2.2 summarizes the timing of the model.

Legal uncertainty makes the auditing technology of the competition authority imperfect. By an audit, the competition authority does not learn the first dimension  $x$  of the type exactly, but receives only a noisy signal  $x^M = x + \delta$  with  $\delta$  uniformly distributed on  $[-\Delta, \Delta]$  with a small  $\Delta$ . In the case of vertical restraints this captures the difficulty in determining, whether the market share is 29.8% or 30.1%. With Microsoft the uncertainty might arise for products, like anti-virus software, where tying might offer great benefits, but also has the potential to harm other market participants considerably. This uncertainty about the legal threshold or this measurement error is implied by the structure of the legal rules and is exogenous to the competition authority.<sup>18</sup> Therefore the case without legal uncertainty, i.e.,  $\Delta = 0$ , serves only as a benchmark. Proposition 2.1 will show that legal uncertainty increases welfare compared to a rule of reason without legal uncertainty.

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<sup>17</sup>The fine  $f$  is exogenous. Yet, making the fine endogenous does not change the model. The results in this chapter just require a jump in the fine at  $\hat{x}$  which is optimal, even if the fine is completely endogenous. Appendix 2.A discusses this case. For a discussion of the setting of fines in Europe see European Commission (2006) and Wils (2007).

<sup>18</sup>Even in very formalistic approaches such uncertainty can arise. An example is the case of Michelin II. The European Commission fined Michelin for using pure quantity rebates, although most scholars at that time believed these to be legal. The details can be found in European Commission 'Manufacture Française des pneumatiques Michelin vs. Commission' Decision 2002/405, June 20th, 2001. See Motta (2006) for a discussion.

CHAPTER 2

To sum up, the expected change in weighted welfare is

$$w(x, b) = \begin{cases} -cp & \text{if the firm } (x, b) \text{ is deterred} \\ \alpha b - h(x) - cp & \text{if the active firm } (x, b) \text{ is not fined} \\ \alpha b - h(x) + (1 - \alpha)pf - (c + \kappa)p & \text{if the active firm } (x, b) \text{ is fined.} \end{cases}$$

On the other hand, the expected profits of the firm are

$$\pi(x, b) = \begin{cases} 0 & \text{if the firm } (x, b) \text{ is deterred} \\ b & \text{if the firm } (x, b) \text{ is active and not fined} \\ b - pf & \text{if the firm } (x, b) \text{ is active and fined.} \end{cases}$$

Thus, the firm's type is two-dimensional. Yet the competition authority can only observe a costly and imprecise signal about one dimension of the firms' types. Given its policy constraints, it cannot enforce the first-best policy. To make the problem interesting, I assume that on average the new behavior is harmful

$$\alpha \frac{\bar{b}}{2} < \int_0^1 h(x) dG(x). \quad (2.1)$$

Therefore a social planner restricted to a binary rule, i.e.,  $\hat{x} = 0$  or  $1$ , would prohibit all the business practices in the class considered here. This assumption is not crucial for the results, but allows avoiding a corner solution with the competition authority approving all actions. Furthermore, I assume that the costs of auditing are not excessively high, so that auditing is worthwhile. That is

$$c < \frac{f}{\bar{b}} \max_{\hat{x}} \int_0^{\hat{x}} \alpha \frac{\bar{b}}{2} - h(x) dG(x). \quad (2.2)$$

To guarantee an interior solution of the competition authority's optimization, another, more technical assumption is required

$$\int_{x'-\Delta}^{x'+\Delta} 2f^2(1 - x' - \frac{1}{3}\Delta)h'(x) - fh(x) - \bar{b}((1 - \alpha)f - \kappa)dx > 0, \quad (2.3)$$

for all  $x' \in [\Delta, 1 - \Delta]$ , which is equivalent to the slope of the externality function



being sufficiently high, such that the policy of the competition authority matters. The next section shows that legal uncertainty allows mitigating the limitations of policy enforcement.

## 2.4 The Effects of Legal Uncertainty

First, we consider the decision process of the firm. The firm faces the fine  $f$  if it is caught by the competition authority implementing the controversial business practice and  $x^M \geq \hat{x}$ . Therefore the firm will only take action if its private benefits  $b$  are high enough. Thus, there is a cut-off level  $\hat{b}(x)$ , such that only firms above  $\hat{b}(x)$  will become active. The cut-off for the private benefit  $\hat{b}(x)$  will vary with  $x$ . If the firm's type is low,  $x < \hat{x} - \Delta$ , the firm will always implement the new business practice, as long as the private benefits are positive. Beginning at  $x = \hat{x} - \Delta$ ,  $\hat{b}(x)$  will be increasing in  $x$ . Consequently, the cut-off level will be

$$\hat{b}(x) = \begin{cases} pf & \text{if } x > \hat{x} + \Delta \\ \frac{x - \hat{x} + \Delta}{2\Delta} pf & \text{if } \hat{x} - \Delta \leq x \leq \hat{x} + \Delta \\ 0 & \text{if } x < \hat{x} - \Delta. \end{cases} \quad (2.4)$$

Therefore legal uncertainty created by the imprecise measurement allows some screening of firms. If they are close to the policy threshold  $\hat{x}$ , firms with low private benefits will abstain from taking action for lower values of  $x$  than firms with a high value of private benefits. If the rule of reason would provide complete legal certainty, the measure of the competition authority would be perfect, i.e.,  $\Delta = 0$ , and the policy threshold would be sharp. Then below  $\hat{x}$ , all firms will take action. Above  $\hat{x}$ , only those with a private value above the cut-off  $\hat{b}(x) = pf$  will implement the new business practice. Figure 2.3 depicts this situation. It is easy to recognize that the right-hand side is closer to the optimal schedule of Figure 2.1 than the left-hand side, as there is no discontinuous jump at the threshold  $\hat{x}$ . Consequently, a rule of reason with some inherent legal uncertainty about the policy threshold is beneficial, as this makes the behavior of the firm more gradual around the threshold. Before the next proposition formalizes this argument, we turn to the competition authority. The competition

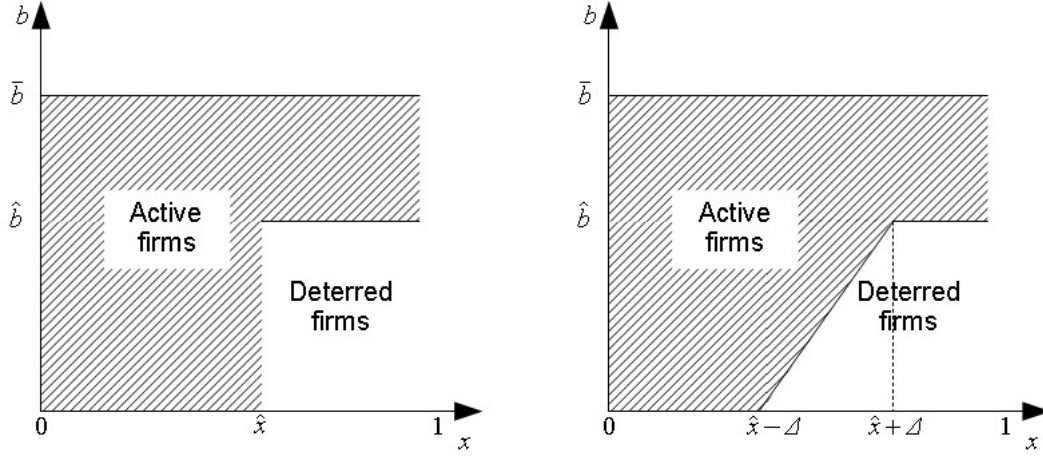


Figure 2.3: Changes in Active Firms due to Legal Uncertainty

authority chooses the threshold  $\hat{x}$  and the auditing probability  $p$  to maximize total welfare  $W(\hat{x}, p)$  which is given by

$$\begin{aligned}
 & \int_0^{\hat{x}-\Delta} \alpha \frac{\bar{b}}{2} - h(x) dG(x) + \\
 & + \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} \frac{\bar{b} - \hat{b}(x)}{\bar{b}} \left( \alpha \frac{\bar{b} + \hat{b}(x)}{2} - h(x) + \frac{x - \hat{x} + \Delta}{2\Delta} (f(1 - \alpha) - \kappa)p \right) dG(x) + \\
 & + \int_{\hat{x}+\Delta}^1 \frac{\bar{b} - pf}{\bar{b}} \left( \alpha \frac{\bar{b} + pf}{2} - h(x) + (f(1 - \alpha) - \kappa)p \right) dG(x) - cp. \quad (2.5)
 \end{aligned}$$

The first term in equation (2.5) captures the region where the competition authority judges all business practices as procompetitive. In the intermediate region, there is legal uncertainty which decreases the probability of a firm implementing the new business practices while increasing its expected benefits. The second integral represents this behavior. Finally, in the anticompetitive region, activity is limited to the firms with the highest benefits. The intuition is that the active firms, under the policy implemented by the competition authority, match more closely the active firms in the first-best policy, because firms' behavior allows some inference about the second dimension. Payments of the fine enter the considerations of the competition authority if the weight of firms' profit  $\alpha$  in the objective function of the competition authority is smaller than one. As legal uncertainty increases the probability of a

## 2.4 THE EFFECTS OF LEGAL UNCERTAINTY

conviction, this effect can make legal uncertainty attractive if the weight  $\alpha$  is very low.<sup>19</sup> Proposition 2.1 summarizes these arguments.

**Proposition 2.1.** *Legal uncertainty with  $\Delta > 0$ , i.e., uncertainty about the legal threshold à la Craswell and Calfee (1986), increases social welfare compared to a regime with  $\Delta = 0$  and legal certainty if the uncertainty is not too large.*

**Proof:** Taking the derivative with respect to  $\Delta$  results by the envelop theorem in

$$\begin{aligned} \frac{\partial W(\hat{x}, p)}{\partial \Delta} &= -\alpha \frac{\bar{b}}{2} + h(\hat{x} - \Delta) + \alpha \bar{b} - \alpha \frac{(pf)^2}{3\bar{b}} - h(\hat{x} + \Delta) \frac{\bar{b} - pf}{\bar{b}} + \\ &+ \frac{\bar{b} - pf}{\bar{b}} h(\hat{x} + \Delta) - h(\hat{x} - \Delta) - \int_{-\Delta}^{\Delta} h(\hat{x} + x) \frac{x}{2\Delta^2} \frac{pf}{\bar{b}} dx - \alpha \frac{\bar{b}^2 - (pf)^2}{2\bar{b}} + \\ &+ (f(1 - \alpha) - \kappa) \frac{p^2 f}{3\bar{b}}. \end{aligned}$$

Rearranging gives

$$\begin{aligned} \alpha \frac{\bar{b}}{2} + (f(1 - \alpha) - \kappa) p \frac{pf}{3\bar{b}} - \alpha \frac{(pf)^2}{3\bar{b}} - \int_{-\Delta}^{\Delta} h(\hat{x} + x) \frac{x}{2\Delta^2} \frac{pf}{\bar{b}} dx - \alpha \frac{\bar{b}^2 - (pf)^2}{2\bar{b}} &= \\ = \alpha \frac{(pf)^2}{6\bar{b}} + ((1 - \alpha)f - \kappa) \frac{fp^2}{3\bar{b}} - \frac{pf}{2\Delta^2 \bar{b}} \int_{-\Delta}^{\Delta} h(\hat{x} + x) x dx. \end{aligned}$$

In order to approximate the integral, a second-order Taylor extension around  $x = 0$  is used for  $h(\hat{x} + x)x$ . Accordingly, the term  $h(\hat{x} + x)x$  equals

$$h(\hat{x} + x)x = h(\hat{x})0 + x(h'(\hat{x})0 + h(\hat{x})) + x^2 h'(\hat{x} + \epsilon) = xh(\hat{x}) + x^2 h'(\hat{x} + \epsilon) \quad (2.6)$$

with  $\epsilon \in (0, x)$ . We define  $h'_{\max} = \max_{\epsilon \in [-\Delta, \Delta]} h'(\hat{x} + \epsilon) > 0$  to derive an upper bound in the next step. Consequently, the integral is bounded by

$$\int_{-\Delta}^{\Delta} h(\hat{x} + x)x dx \leq \int_{-\Delta}^{\Delta} xh(\hat{x}) + x^2 h'_{\max} dx = \frac{2}{3} \Delta^3 h'_{\max}. \quad (2.7)$$

Therefore we get a lower bound for the derivative.

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<sup>19</sup>A lump-sum payment of all active or all firms, in general, cannot replicate this effect. Revenues might be equivalent but ex-ante incentives for firms to implement the business practice change.

$$\frac{\partial W(\hat{x}, p)}{\partial \Delta} \geq \frac{pf}{3\bar{b}} \left( p\left(1 - \frac{\alpha}{2}\right)f - \kappa \right) - \Delta h'_{\max} \quad (2.8)$$

Under a per-se rule or with a perfect auditing technology, there would be no measurement error and  $\Delta = 0$ . Yet having some legal uncertainty and some imprecision in the measurement increases social welfare as the derivative is positive for small  $\Delta$ .

$$\Delta < \frac{(2 - \alpha)f - 2\kappa}{2h'_{\max}} p \Rightarrow \frac{\partial W(\hat{x}, p)}{\partial \Delta} > 0. \quad (2.9)$$

Notice that the assumption  $\kappa < (1 - \alpha)f$  ensures that  $(2 - \alpha)f - 2\kappa$  is positive and the condition is feasible.<sup>20</sup> This concludes the proof and shows how legal uncertainty might increase social welfare.  $\square$

The next step considers the effects of legal uncertainty on the policy threshold  $\hat{x}$ . Suppose there is some legal uncertainty  $\Delta$  about the threshold of legality inherent in the legal rules. This allows a lower policy threshold without hurting too many procompetitive business practices if the externality function is convex.

**Proposition 2.2.** *If the externality function,  $h(x)$ , is convex, legal uncertainty decreases the policy threshold  $\hat{x}$ . With a concave externality function legal uncertainty raises the policy threshold.*

**Proof:** For social welfare as stated in (2.5), the first derivative with respect to the policy threshold  $\hat{x}$  equals

$$\begin{aligned} \frac{\partial W(\hat{x}, p)}{\partial \hat{x}} &= \alpha \frac{\bar{b}}{2} - h(\hat{x} - \Delta) - h(\hat{x} + \Delta) \frac{\bar{b} - pf}{\bar{b}} + h(\hat{x} - \Delta) - \int_{\hat{x} - \Delta}^{\hat{x} + \Delta} h(x) \frac{pf}{2\Delta\bar{b}} dx - \\ &- \alpha \frac{\bar{b}^2 - (pf)^2}{2\bar{b}} - ((1 - \alpha)f - \kappa) p \frac{\bar{b} - pf}{\bar{b}} + \frac{\bar{b} - pf}{\bar{b}} h(\hat{x} + \Delta). \end{aligned} \quad (2.11)$$

Rearranging the terms delivers

$$\frac{\partial W(\hat{x}, p)}{\partial \hat{x}} = \alpha \frac{(pf)^2}{2\bar{b}} - p((1 - \alpha)f - \kappa) \frac{\bar{b} - pf}{\bar{b}} - \frac{pf}{2\Delta\bar{b}} \int_{\hat{x} - \Delta}^{\hat{x} + \Delta} h(x) dx. \quad (2.12)$$

<sup>20</sup>Finally, consider the second derivative  $\frac{\partial^2 W(\hat{x}, p)}{\partial \Delta^2} =$

$$\frac{pf}{\Delta^3\bar{b}} \int_{-\Delta}^{\Delta} h(\hat{x} + x) x dx - \frac{pf}{2\Delta\bar{b}} (h(\hat{x} + \Delta) - h(\hat{x} - \Delta)) \leq \frac{pf}{\Delta\bar{b}} \int_{-\Delta}^{\Delta} \frac{1}{3} h'_{\max} - \frac{1}{2} h'(\hat{x} + x) dx. \quad (2.10)$$

It is negative if, e.g.,  $h(x)$  is linear. Then the benefits of legal uncertainty decrease in its size.

Then the first-order condition corresponds to

$$\alpha p f \Delta - 2 \frac{(1 - \alpha) f - \kappa}{f} \Delta (\bar{b} - p f) = \int_{\hat{x} - \Delta}^{\hat{x} + \Delta} h(x) dx, \quad (2.13)$$

if an interior solution exists.<sup>21</sup> As, the externality function  $h$  is increasing, there is a unique solution to (2.13). Due to assumption (2.1) and the increasing externality function, equation (2.13) shows that there is consequently never a corner solution at the right end and  $\hat{x} < 1 - \Delta$ . Yet there might be a corner solution at the left end with  $\hat{x} = 0$  if the business practice under consideration is very harmful. In this case the competition authority uses a per-se prohibition. Using Taylor's theorem there is a function  $\epsilon: [-\Delta, \Delta] \mapsto (\hat{x} - \Delta, \hat{x} + \Delta)$ , such that the integral equals

$$\int_{\hat{x} - \Delta}^{\hat{x} + \Delta} h(x) dx = \int_{-\Delta}^{\Delta} h(\hat{x}) + h'(\hat{x})x + \frac{1}{2}h''(\epsilon(x))x^2 dx = 2\Delta h(\hat{x}) + \frac{1}{2} \int_{-\Delta}^{\Delta} h''(\epsilon(x))x^2 dx.$$

The sign of the last term corresponds to the sign of the second derivative of the externality function. Consequently, the curvature of the externality function determines the effect of legal uncertainty and for the policy threshold  $\hat{x}$  it holds that

$$h(\hat{x}) = \frac{1}{2} \alpha p f - \frac{(1 - \alpha) f - \kappa}{f} (\bar{b} - p f) - \frac{1}{4\Delta} \int_{-\Delta}^{\Delta} h''(\epsilon(x))x^2 dx. \quad (2.14)$$

On the other hand, the condition for the policy threshold is

$$h(\hat{x}) = \frac{1}{2} \alpha p f - \frac{(1 - \alpha) f - \kappa}{f} (\bar{b} - p f), \quad (2.15)$$

if there is no legal uncertainty,  $\Delta = 0$ , and the measurement is sharp. The same policy threshold results if the externality function  $h$  is linear. Making the externality function concave increases the policy threshold  $\hat{x}$  above (2.15). On the contrary, a convex externality function results in a lower threshold.  $\square$

Depending on the curvature of the externality function, the policy threshold  $\hat{x}$  is changed by legal uncertainty in the legal rule. If the externality function is convex, there is a low threshold of legality prohibiting more actions than in the case with an exact measurement and legal certainty about the policy threshold. Thus, the

<sup>21</sup>Appendix 2.D shows that the conditions of a maximum are satisfied.

competition authority adapts its policy to the uncertainty. Finally, I scrutinize the optimal auditing probability and show that there will always be some auditing. In contrast to the common costly state verification models, even deterministic auditing is optimal if auditing costs are small. Lemma 2.1 in Appendix 2.C determines the optimal auditing level for interior solutions.

**Proposition 2.3.** *Auditing occurs with positive probability. Moreover, complete auditing is optimal if the auditing costs are sufficiently low,  $c < \frac{1}{3}\Delta((1 - 5\alpha)f - \kappa)$ .*

**Proof:** To show that it is never optimal to have no auditing, I compare social welfare with complete deterrence to welfare without auditing. Without any auditing, social welfare is independent of the policy threshold  $\hat{x}$

$$\alpha \frac{\bar{b}}{2} + \int_0^1 h(x)dx < 0, \quad (2.16)$$

which is negative by assumption (2.1). With complete deterrence,  $p \geq \frac{\bar{b}}{f}$ , the policy threshold  $\hat{x}$  matters and social welfare  $W(\hat{x}, p)$  equals

$$\int_0^{\hat{x}-\Delta} \frac{\alpha \bar{b}}{2} - h(x)dx + \Delta \frac{2\alpha \bar{b}^2}{3pf} - \int_{-\Delta}^{\Delta(2\frac{\bar{b}}{pf}-1)} \frac{\Delta - x}{2\Delta} h(\hat{x} + x)dx + ((1 - \alpha)f - \kappa) \frac{\Delta p}{3} - cp.$$

By assumption (2.2) the last expression is positive for the optimal  $\hat{x}$  and  $\Delta = 0$ . Therefore social welfare is lower without auditing,  $W(\hat{x}, 0) < W(\hat{x}, \frac{\bar{b}}{f})|_{\Delta=0}$ . Increasing  $\Delta$  raises social welfare by Proposition 2.1. Consequently, having a small  $\Delta > 0$  increases the objective function of the competition authority further. Therefore social welfare is higher with auditing,  $W(\hat{x}, 0) < W(\hat{x}, \frac{\bar{b}}{f})$ , and it is never optimal to abstain from auditing,  $p = 0$ .

On the other hand, it is sometimes optimal to implement complete auditing,  $p = 1$ . Increasing the value of the externality function in the interval  $[1 - \Delta, 1]$  increases the optimal  $p$  according to Lemma 2.1. Therefore using complete deterrence,  $p \geq \frac{\bar{b}}{f}$ , is optimal if some actions with positive mass are very harmful. This does not imply complete auditing. Yet if, in addition, the auditing costs are small,  $c < \frac{1}{3}\Delta((1 - 5\alpha)f - \kappa)$ , the competition authority prefers to audit every firm.<sup>22</sup> The

<sup>22</sup>This result is derived in Lemma 2.2 in Appendix 2.C.

## 2.5 EXTENSIONS - AMBIGUITY AND THE COURTS

reason is that complete deterrence just holds for firms in the illegal region,  $x > \hat{x} + \Delta$ , in contrast to firms close to the threshold of legality,  $\hat{x} - \Delta \leq x \leq \hat{x} + \Delta(2\frac{\bar{b}}{pf} - 1)$ , that are still active, although they might face a fine. This extreme case, however, is impossible if legal uncertainty is sufficiently small. In that case decreasing  $p$  from  $p = 1$  slightly allows the competition authority to save on auditing costs, while at the same time the deterrence effect does not change for  $x \notin (\hat{x} - \Delta, \hat{x} + \Delta)$ , because  $f > \bar{b}$ . □

This completes the analysis of the model.

## 2.5 Extensions - Ambiguity and the Courts

So far I abstracted from the interaction between the judicial system and the competition authority. In the following I want to model it more explicitly. Thus, a convicted firm might turn to the courts. If the courts decide in favor of the firm and repeal the assessment of the competition authority, the fine is reassessed.<sup>23</sup> Therefore there is an additional stage,  $t = 5$ , at which the courts can overturn the decision of the competition authority and change the fine  $f$  to  $f_L$  or  $f_H$  with  $f_L < f \leq f_H$ . It seems plausible that there are no objective probabilities available of the courts setting the fine because of few precedents and inconsistent decisions. This creates ambiguity, as it is unclear how severely the courts assess the infringement of the firm.<sup>24</sup> The firm exhibits ambiguity aversion à la Gilboa and Schmeidler (1989). Therefore it has a set of priors  $\Pi$  and maximizes its expected utility under the worst possible distribution. The preferences of the firm are represented by

$$g \mapsto \min_{\pi \in \Pi} \int u \circ g d\pi. \tag{2.17}$$

Due to the risk neutrality of the firm, the von-Neumann Morgenstern utility index  $u$  is linear. I assume that the subjective probability of a high fine  $f_H$  can take one of the following values  $\{q_1, \dots, q_n\}$ . An ambiguity-neutral agent is not concerned

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<sup>23</sup>In Lagerlöf and Heidhues (2005), firms can produce efficiency defenses to influence the decision of the competition authority.

<sup>24</sup>The notion of ambiguity is introduced in Appendix 2.B which also discusses the assumption of ambiguity-averse firms.

by this ambiguity and behaves as if she anticipates the high fine with a subjective probability  $q_N$ , such that

$$\min_{1 \leq i \leq n} q_i < q_N < \max_{1 \leq i \leq n} q_i.$$

I write  $E(f) = q_N f_H + (1 - q_N) f_L$  for the fine expected by an ambiguity-neutral agent. This completes the model set-up. Now turn to the analysis. Ambiguity-averse firms will turn to the courts whenever  $\max_i (q_i f_H + (1 - q_i) f_L) = f^A < f$ . Assume this is the case. Then the cut-off level for firms to be active is

$$\hat{b}(x) = \begin{cases} \hat{p} f^A & \text{if } x > \hat{x} + \Delta \\ \frac{x - \hat{x} + \Delta}{2\Delta} p f^A & \text{if } \hat{x} - \Delta \leq x \leq \hat{x} + \Delta \\ 0 & \text{if } x < \hat{x} - \Delta. \end{cases} \quad (2.18)$$

The competition authority takes this modified cut-off level into account when setting its policy. The next proposition shows that the additional uncertainty surrounding the judicial decision has positive welfare effects, because the ambiguity about the fine makes the firm implement less anticompetitive business practices than in the absence of ambiguity.

**Proposition 2.4.** *Having ambiguous procedures increases social welfare compared to a regime without ambiguity, if auditing costs are higher than the expected income due to fines collected by the competition authority*

$$c > ((1 - \alpha)E(f) - \kappa) \frac{1}{\bar{b}} \left( \frac{1}{3} p f^A \Delta + (1 - \hat{x})(\bar{b} - p f^A) \right).$$

**Proof:** The competition authority can implement any policy  $\hat{x}, p$  with a strictly lower probability of auditing than in the absence of ambiguity. As it is easier to enforce compliance, the competition authority can save on auditing costs to the extent of  $cp(1 - E(f)/f^A)$ . The reason is that, at a given probability of auditing, an ambiguity-averse firm will have a higher cut-off value  $\hat{b}(x)$  than an ambiguity-neutral firm in the region with intervention of the competition authority  $x > \hat{x} - \Delta$ . This is caused by overvaluing the fine. Notice that the ambiguity aversion does not change the beliefs, but the firms only act *as if* the probability for a high fine  $f_H$  were higher.



Yet at the same time the fine income of the competition authority decreases by

$$\frac{(1 - \alpha)\mathbb{E}(f) - \kappa}{\bar{b}} \left( \frac{\Delta}{3} p f^A + (1 - \hat{x})(\bar{b} - p f^A) \right) p \left( 1 - \frac{\mathbb{E}(f)}{f^A} \right) \quad (2.19)$$

In total, the effect is positive if the change in income of fines imposed by the competition authority is smaller than the marginal auditing costs

$$\frac{(1 - \alpha)\mathbb{E}(f) - \kappa}{\bar{b}} \left( \frac{\Delta}{3} p f^A + (1 - \hat{x})(\bar{b} - p f^A) \right) < c.$$

Social welfare is not affected by ambiguity premia. The reason is the following. The fraction of active firms and their utilities remains unchanged by the ambiguity if the competition authority reduces its auditing correspondingly. In addition, ambiguity does not affect the utility of deterred firms. This gives a lower bound for the welfare gains due to ambiguity, because the competition authority can reoptimize its policies. Therefore ambiguous procedures increase social welfare compared to a regime without ambiguity.  $\square$

To understand this result, notice that the ambiguity has no negative effects on firms' behavior, because the competition authority can always balance the ambiguity by reducing the auditing probability and the firms can avoid the ambiguity.<sup>25</sup> Moreover the ambiguity only affects firms violating the rules of competition law. If the firm is in the procompetitive region,  $x \leq \hat{x} - \Delta$ , it does not care about the ambiguity in the fine. If it is in the prohibited region, it has always the possibility to abstain from the controversial business practice and avoid the ambiguity.

## 2.6 Conclusion

This chapter sets up a costly state verification model to scrutinize the effects of legal uncertainty in competition law. A rule of reason approach is often more selective than a per-se rule and is hence welfare-enhancing. Yet the inherent legal uncertainty of the rule of reason might be a drawback. This chapter points out why legal uncertainty itself might further raise social welfare.

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<sup>25</sup>The first part of the argument requires homogeneous attitudes towards ambiguity of the firms.

## CHAPTER 2

The uncertainty about the threshold of legality might be due to imprecision in the measurement of the competition authority, missing precedents or unclear rules. The model considered here shows that this uncertainty may be welfare-enhancing as the deterrence becomes more selective without having more selective rules. Firms self-select according to the nature of the business practice under consideration. This is beneficial, as firms have better knowledge and information to assess the nature of the business practice than the competition authority.

Yet there are limitations to the benefits of legal uncertainty. If there is too much legal uncertainty, this will decrease social welfare, because it will deter procompetitive business practices and encourage anticompetitive ones. In a dynamic framework additional effects may appear. Legal uncertainty might give firms incentives to experiment and therefore implement more anticompetitive business practices than under legal certainty. On the other hand, the costs of such behavior, e.g., possible fines, are incurred by a single firm, while the benefits spill over to all market participants, as they learn the court decision reducing legal uncertainty. Therefore competition might decrease the experimentation.

## Appendix to Chapter 2

### 2.A Endogenous Fines

Assume the fine  $f$  is endogenous and a function of the observed dimension of the type  $x$ . Then the competition policy consists of the auditing probability and the fine, as the threshold of legality is implicitly defined by the schedule of the fine. To guarantee existence of a solution, there has to be an upper bound  $\bar{f}$  for the fines.<sup>26</sup> Otherwise fines would be raised to infinity to save on auditing costs. For a discussion of this effect, see, e.g., Kaplow and Shavell (1994, p. 586) and Polinsky and Shavell (1979). The upper bound corresponds to wealth constraints of the firm or legal considerations. If  $\bar{f}$  is sufficiently high, pointwise optimization yields an optimal fine of

$$f(x) = \begin{cases} 0 & \text{for } x < \hat{x} \\ \frac{\kappa}{(1-\alpha)} & \text{for } x \in [\hat{x}, x_1] \\ \frac{(1-\alpha)\bar{b}+h(x)+p\kappa}{(2-\alpha)p} & \text{for } x \in (x_1, x_2) \\ \bar{f} & \text{for } x \geq x_2 \end{cases}$$

with  $\hat{x} \leq x_1 \leq x_2 < 1$ .<sup>27</sup>  $x_1$  is determined by  $h(x_1) = \frac{p\kappa}{(1-\alpha)} - (1-\alpha)\bar{b}$  and  $x_2$  by  $h(x_2) = (2-\alpha)p\bar{f} - (1-\alpha)\bar{b} - p\kappa$ . Thus, for socially beneficial types, i.e., low  $x$ , the competition authority tolerates active firms by setting the fine to zero. As the threshold of legality  $\hat{x}$  is passed, a fine of  $\kappa/(1-\alpha)$  is imposed, because lower fines do not justify spending the costs  $\kappa$  to make  $x$  verifiable. Then there is a region  $x \in (x_1, x_2)$  where  $f$  is strictly increasing, until it reaches  $\bar{f}$  at  $x_2$ . Finally, the fine equals the upper bound,  $f(x) = \bar{f}$  for all  $x \geq x_2$ .

### 2.B Ambiguity and Why It Matters

The ambiguity in Section 2.5 captures the vagueness of procedures in legal rules. In general, ambiguity denotes uncertainty about probabilities resulting from missing

<sup>26</sup>The upper bar could correspond to a principle of proportionality. Accordingly, “the severity of sanctions cannot simply be raised to excessive levels.” (Frese, 2011, p. 426)

<sup>27</sup>This result holds for the case of a rule of reason without legal uncertainty.

## CHAPTER 2

relevant information. Thus, ambiguity aversion describes a preference for lotteries where the firm has more confidence in its probability assessment. A classical example is due to Ellsberg (1961) and considers two urns. In both urns there are red and black balls. Yet the distribution of the balls is only known in the first urn. The ratio of red to black balls in the second urn is unknown. Individuals can place bets on either a red ball or a black ball drawn from one of the urns. Most subjects prefer to take the bet on balls drawn from the first urn, the familiar one, no matter what color was specified. Thus, under ambiguity, the exact probabilities are unknown. Missing precedents might imply this lack of reliable probabilities for a conviction. Savage (1954) and Schmeidler (1989) have developed two axiomatized approaches to this problem. The Subjective Expected Utility of Savage requires agents to be ambiguity-neutral. The representation of Schmeidler (1989) allows agents to have preferences with respect to ambiguity.<sup>28</sup>

In order to guarantee that ambiguity is relevant in competition law, firms have to be ambiguity-averse. Yet in contrast to risk aversion, ambiguity-averse firms seem more plausible. First, there are empirical observations, like Kunreuther et al. (1995) or Cabantous (2007), that even professionals in firms behave in an ambiguity-averse way. Ambiguity makes insurers usually more restrictive, i.e., they request higher premiums and reject to offer an insurance policy in more cases than in the absence of ambiguity. Second, even from a theoretical point of view firms may be ambiguity-averse. Marinacci (1999) shows that ambiguity changes the law of large numbers and the ambiguity does not vanish when different ambiguous random variables are combined. Therefore even perfect diversification does not protect an investor from ambiguity in the underlying assets. Third, joint decision-making on boards or on committees does not mitigate the effects of ambiguity aversion. On the contrary, Keller et al. (2007) show that collaboration even amplifies ambiguity aversion. Thus, joint decision making exhibits a higher degree of ambiguity aversion than the average member. Consequently, the assumption of ambiguity-averse firms seems plausible.<sup>29</sup>

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<sup>28</sup>Following Schmeidler (1989) different representations with ambiguity preferences have been proposed by, e.g. Gilboa and Schmeidler (1989) or Klibanoff et al. (2005).

<sup>29</sup>Previous examples of the assumption include, e.g., Mukerji (1998) for incomplete contracting and Tor and Rinner (2011) for retail price maintenance.

## 2.C Additional Proofs

Lemma 2.1 determines the optimal auditing level for interior solutions.

**Lemma 2.1.** *The optimal auditing probability is determined by (2.21).*

**Proof:** For social welfare stated in (2.5) the first derivative with respect to the auditing probability  $p$  equals

$$\begin{aligned} \frac{\partial W(\hat{x}, p)}{\partial p} = & -\alpha\Delta\frac{2pf^2}{3\bar{b}} + f\int_{-\Delta}^{\Delta}\frac{x+\Delta}{2\Delta\bar{b}}h(\hat{x}+x)dx - \alpha(1-\hat{x}-\Delta)\frac{pf^2}{\bar{b}} + \\ & + \frac{f}{\bar{b}}\int_{\hat{x}+\Delta}^1h(x)dx + ((1-\alpha)f - \kappa)\left(\frac{2pf}{3\bar{b}}\Delta + (1-\hat{x})\frac{\bar{b}-2pf}{\bar{b}}\right) - c. \end{aligned} \quad (2.20)$$

Consequently, the first-order condition for an interior solution equals

$$p = \frac{((1-\alpha)f - \kappa)(1-\hat{x})\bar{b} + f\int_{-\Delta}^{\Delta}\frac{x+\Delta}{2\Delta}h(\hat{x}+x)dx + f\int_{\hat{x}+\Delta}^1h(x)dx - c\bar{b}}{f((2-\alpha)f - 2\kappa)(1-\hat{x} - \frac{1}{3}\Delta)}. \quad (2.21)$$

By Proposition 2.3 the optimal  $p$  is always positive. Therefore only the corner solution  $p = 1$  exists for values of equation (2.21) bigger than 1.  $\square$

The next lemma shows the optimality of complete auditing for small costs.

**Lemma 2.2.** *If the costs of auditing are sufficiently small,  $c < \frac{1}{3}\Delta((1-5\alpha)f - \kappa)$ , the competition authority implements complete auditing,  $p = 1$ .*

**Proof:** Proposition 2.3 shows that with complete deterrence,  $p \geq \frac{\bar{b}}{f}$ , social welfare  $W(\hat{x}, p)$  equals

$$\int_0^{\hat{x}-\Delta}\frac{\alpha\bar{b}}{2} - h(x)dx + \Delta\frac{2\alpha\bar{b}^2}{3pf} - \int_{-\Delta}^{\Delta(2\frac{\bar{b}}{pf}-1)}\frac{\Delta-x}{2\Delta}h(\hat{x}+x)dx + ((1-\alpha)f - \kappa)\frac{\Delta p}{3} - cp.$$

The derivative of the social welfare  $W(\hat{x}, p)$  with respect to  $p$  is

$$-\Delta\frac{4\alpha\bar{b}^2}{3p^2f} + \left(1 - \frac{\bar{b}}{pf}\right)h\left(\hat{x} + \Delta\left(2\frac{\bar{b}}{pf} - 1\right)\right)4\Delta\frac{\bar{b}}{p^2f} + ((1-\alpha)f - \kappa)\frac{\Delta}{3} - c \quad (2.22)$$

for  $p \geq \frac{\bar{b}}{f}$ . Rearranging yields

$$\frac{1}{3}\Delta \left( (1-\alpha)f - \kappa - \frac{4\alpha\bar{b}^2}{p^2f} \right) + \left( 1 - \frac{\bar{b}}{pf} \right) h \left( \hat{x} + \Delta \left( 2\frac{\bar{b}}{pf} - 1 \right) \right) 4\Delta \frac{\bar{b}}{p^2f} - c. \quad (2.23)$$

$\frac{4\alpha\bar{b}^2}{p^2f} \leq 4\alpha f$  for  $p \geq \frac{\bar{b}}{f}$ . Therefore the first term is bigger than  $\frac{1}{3}\Delta((1-5\alpha)f - \kappa)$ , which is positive for small welfare weights  $\alpha$  of firms' profits. Moreover, the second term in the derivative is positive. This shows that (2.23) and the derivative are positive if  $c < \frac{1}{3}\Delta((1-5\alpha)f - \kappa)$ .  $\square$

## 2.D Second-Order Conditions

Given the two-dimensional optimization, there are three conditions in order to assure that the chapter characterizes the optimum. First, I compute the second derivatives. From (2.12) it follows

$$\frac{\partial^2 W(\hat{x}, p)}{\partial \hat{x}^2} = -\frac{pf}{2\Delta\bar{b}}(h(\hat{x} + \Delta) - h(\hat{x} - \Delta)) < 0, \quad (2.24)$$

and differentiating (2.20) with respect to  $p$  yields

$$\frac{\partial^2 W(\hat{x}, p)}{\partial p^2} = -\frac{1}{b} \left( 1 - \hat{x} - \frac{1}{3}\Delta \right) f((2-\alpha)f - 2\kappa) < 0 \quad (2.25)$$

for small  $\Delta$ . Finally, the following steps prove that the discriminant is positive in the relevant range. Differentiating (2.12) with respect to  $p$  results in the cross-derivative

$$\begin{aligned} \frac{\partial^2 W(\hat{x}, p)}{\partial \hat{x} \partial p} &= \alpha \frac{pf^2}{b} - ((1-\alpha)f - \kappa) \frac{\bar{b} - 2pf}{b} - \frac{f}{2\Delta\bar{b}} \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} h(x) dx = \\ &= \frac{pf^2}{2b} \alpha + \frac{pf}{b} ((1-\alpha)f - \kappa) = \frac{pf}{2b} ((2-\alpha)f - 2\kappa). \end{aligned} \quad (2.26)$$

The second equality here follows from the first-order condition (2.13). The determinant of the Hessian equals

$$\begin{aligned}
 & \frac{\partial^2 W(\hat{x}, \hat{b})}{\partial \hat{x}^2} \frac{\partial^2 W(\hat{x}, \hat{b})}{\partial \hat{b}^2} - \left( \frac{\partial^2 W(\hat{x}, \hat{b})}{\partial \hat{b} \partial \hat{x}} \right)^2 = \\
 & = \frac{pf}{2\Delta \bar{b}^2} (h(\hat{x} + \Delta) - h(\hat{x} - \Delta)) \left(1 - \hat{x} - \frac{1}{3}\Delta\right) f((2 - \alpha)f - 2\kappa) - \\
 & \quad - \left( \frac{pf}{2\bar{b}} ((2 - \alpha)f - 2\kappa) \right)^2 = \\
 & = \frac{pf^2}{2\bar{b}^2} ((2 - \alpha)f - 2\kappa) \left( \frac{1}{\Delta} (h(\hat{x} + \Delta) - h(\hat{x} - \Delta)) \left(1 - \hat{x} - \frac{1}{3}\Delta\right) - \right. \\
 & \quad \left. - \frac{p}{2} ((2 - \alpha)f - 2\kappa) \right) = \\
 & = \frac{p}{4\Delta \bar{b}^2} ((2 - \alpha)f - 2\kappa) \int_{\hat{x} - \Delta}^{\hat{x} + \Delta} 2f^2 \left(1 - \hat{x} - \frac{1}{3}\Delta\right) h'(x) - fh(x) - \bar{b}((1 - \alpha)f - \kappa) dx.
 \end{aligned}$$

For the simplification in the third line again the first-order condition (2.13) is used. The assumption  $\kappa < (1 - \alpha)f$  ensures that  $(2 - \alpha)f - 2\kappa$  is positive. Finally, the determinant is positive if the slope of the externality function  $h$  is sufficiently high, as this lowers  $\hat{x}$  and increases  $h'$ . By assumption (2.3) this is satisfied. The assumption holds, for example, if the harm function  $h(\cdot)$  is linear and the slope is sufficiently high. The reason is that  $\hat{x}$  is decreasing in the slope, while  $h'(\cdot)$  is increasing. Therefore  $(h(\hat{x} + \Delta) - h(\hat{x} - \Delta))(1 - \hat{x} - \frac{1}{3}\Delta)$  in the second line increases, while  $\frac{p}{2}((2 - \alpha)f - 2\kappa)$  is bounded from above by  $\frac{1}{2}((2 - \alpha)f - 2\kappa)$ .





# Chapter 3

## Contracting with Subjective Evaluations and Communication

### 3.1 Introduction

This chapter analyzes a principal-agent model in which the performance measure of the principal is nonverifiable and unobservable by the agent, but the principal has the possibility to communicate with the agent. Such subjective or nonverifiable measures of performance are widely used, as verifiable, i.e., objective, performance measures are often unavailable.<sup>1</sup> Examples for subjective measures of performance are the evaluations by supervisors, co-workers, and others. Their subjectivity, however, makes it the principal's choice whether to disclose and justify her evaluation of the agent's work.

In the model, the agent works for the principal who privately receives information about the agent's performance, like reports from colleagues, observations of the agent at work or of the agent's output. In addition, the principal and the agent receive some common noise resulting from random encounters or joint observations. These shared signals, however, are uninformative about the agent's effort or performance.

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<sup>1</sup>The extensive use of subjective performance measures is confirmed by Dessler (2008, p. 339), Porter et al. (2008, p. 148), MacLeod and Parent (1999), and Murphy (1993). The reason is that agents can manipulate objective performance measures or multitask problems. Consequently, Gibbons (1998, p. 120) concludes that "objective performance measures typically cannot be used to create ideal incentives."

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After the principal determines her evaluation of the agent's work, she has two options. Either she reports only the result of her evaluation or she justifies her evaluation by telling the agent also about the information she collected.<sup>2</sup> Her message is not necessarily truthful, and providing a justification is costly. The agent replies with a cheap-talk message about the shared signals. As the messages are the only third-party enforceable information, the contract just depends on these messages. The chapter studies the resulting communication pattern and characterizes the optimal contract: on the equilibrium path the principal justifies bad evaluations and pays a fully contingent wage that is increasing in the evaluation. For good evaluations, the principal in equilibrium saves the hassle of explaining them and simply pays a high wage. This yields pooling and wage compression at the top.

These results fit well with empirical observations that evaluations are lenient and wage dispersion for the best evaluations is low.<sup>3</sup> Those observations are typically referred to as leniency bias and centrality bias. This chapter argues that this pattern can be understood as a feature of the optimal contract instead of biased behavior. In addition, many studies show that principals evaluating for developmental or feedback purposes are more likely to differentiate among subordinates than they are when the evaluation is used for administrative purposes, like merit increases or promotions.<sup>4</sup> In the latter case, evaluations are more compressed and show less variation between employees. The finding goes back to Taylor and Wherry (1951, p. 39) who compare ratings for different purposes. They find more lenient evaluations for administrative purposes “with considerably poorer discrimination at the top.” This observation is in line with the predictions of this chapter. The principal must be given explicit incentives to report her evaluation truthfully. These incentives cause pooling of the

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<sup>2</sup>Justifications of subjective evaluations are a common HR practice: “92% require a review and feedback session as part of the appraisal process.” (Dessler, 2008, p. 366)

<sup>3</sup>According to Bretz et al. (1992), usually 60–70% of all employees get an evaluation from the best or second-best category. Moreover, “Medoff and Abraham (1980) found in two companies that, among the 99% of employees in the same position who received the top three performance ratings, the difference in salary between the highest and lowest rated employees was about 5%.” (Gibbs, 1991, pp. 4-5) Similarly, Murphy (1993, p. 56) reports that the top 1% of employees at the pharmaceutical company Merck receive a pay raise just 3% higher than the median employee in 1985.

<sup>4</sup>This effect is found in Dessler (2008, p. 356), Milkovich et al. (2008, p. 351), a meta-study by Jawahar and Williams (1997), Jawahar and Stone (1997); Harris et al. (1995); McDaniel et al. (1994), Milkovich and Wigdor (1991, pp. 3, 72), and Landy and Farr (1980).

best evaluations. If the evaluation is for developmental or feedback purposes, these incentives are unnecessary, as the preferences of the principal and the agent are likely to be better aligned. Managers at Merck, for example, experienced that “the salary link made discussions on performance improvement difficult.” (Murphy, 1993, p. 58) Psychological costs of supervisors to give bad evaluations to their subordinates yield no straightforward explanation of this pattern, since those costs apply to evaluations for all purposes similarly.

The intuition for the compression at the top is the following. First, it is never optimal to justify all evaluations, because communication is costly. Second, if the principal provides no feedback, the agent cannot verify the evaluation. Then the principal has an incentive to choose the evaluation yielding the lowest wage payment. Hence, no wage dispersion is feasible. Additionally, abstaining from feedback should not allow the principal to save on wage and communication costs. Thus, the principal’s payments have to be higher than in the case following a justification. Together with the first step, this yields pooling of the highest wages. Finally, the monotone likelihood ratio property of the subjective performance measure ensures that, with regard to communication, a threshold strategy is optimal. Such a strategy is the most efficient way to give the agent incentives to implement a certain level of effort. For bad performance, the principal has to bear the communication costs, but pays a lower wage. For good performance, on the other hand, she pays a higher wage instead of giving feedback.<sup>5</sup>

The chapter explores the joint implication of subjectivity and communication costs. It provides a framework to discuss a range of personnel policies, in particular, the value of feedback and communication. This value is well known if communication gives the agent instructions or helps him in a learning process. If communication reveals additional information about the agent’s effort, Holmström (1979) shows that it is beneficial to make the contract contingent on the additional information. Here, the feedback is uninformative in the sense of Holmström (1979). Yet, the agent learns how the principal has derived her evaluation, allowing him to verify the evaluation. Hence, the optimal contract makes the agent’s remuneration contingent on the content

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<sup>5</sup>Murphy (1993, p. 49) summarizes the reasoning as follows: Principals have “nonpecuniary costs [here, communication costs] associated with performance appraisal, which leads them to prefer to assign uniform ratings rather than to carefully distinguish employees by their performance.”

of the feedback. Accordingly, ex-ante the principal wants to explain her evaluation to the agent ex-post. Nevertheless, ex-post she might withhold this information to save on wage and communication costs. Institutional details, like multi-source feedback, might be used as commitment devices in addition to the mechanism proposed here to solve this problem.

The rest of the chapter is organized as follows. Section 3.2 discusses the related literature. Section 3.3 sets up the model and characterizes the optimal contract. Section 3.4 implements stochastic contracts and makes the optimal contract ex-post budget-balanced. Therefore the contract requires no payments to third parties in contrast to previous models. Instead, stochastic contracts use differences in the risk preferences of the parties to implement the required incentives. Then Section 3.5 points out a more familiar implementation of the optimal contract by an indirect mechanism. Section 3.6 contains the concluding remarks. All proofs are relegated to the appendix.

## 3.2 Related Literature

There is a long literature on subjective performance measures. Usually, it is assumed that evaluations are observable and relationships are long-term. This yields implicit contracts, like for example in Compte (1998), Kandori and Matsushima (1998), Baker et al. (1994), MacLeod and Malcomson (1989), and Bull (1987). Then reputation effects created by the continuation value for both parties allow subjective performance measures to gain credibility and to be used as the basis for the agent's incentives. Levin (2003) drops the assumption that the subjective performance measure is perfectly observable by both contracting parties. Then optimal contracts often have a termination form, i.e., the contract ends after observing a bad performance. In contrast to these repeated interactions, subjective evaluations are also used in static settings. MacLeod (2003) was the first to implement subjective performance measures in a static setting. He assumes that the agent has a signal that is correlated with the principal's evaluation and introduces a message game. Each party reports their information by sending a public message. This enables the parties to condition their contract on these messages, which essentially solves the credibility problem. As the

## 3.2 RELATED LITERATURE

information structure is exogenously given, the principal cannot decide, depending on the performance measure, whether to reveal her evaluation. Thus, the results correspond to two special cases of my model. If the agent's and the principal's signal are correlated, MacLeod (2003) achieves the common second-best solution. This corresponds to obligatory or costless communication in my model. If the signals are uncorrelated, the optimal contract in MacLeod (2003) resembles the case of prohibitively expensive communication in my model. The case of imperfect correlation in combination with a binding upper limit on wage payments shares some features with the optimal contract here, but the reasoning and the proofs are different. First, I do not assume an upper limit on payments. Second, the agent receives no private signals telling him that he received no information. Instead, it is the principal's incentive – resulting from the contract and the communication costs – to withhold and distort her evaluation that yields the compression at the top result. Economically, the main difference between this chapter and MacLeod (2003) is that I consider the principal's decision whether to justify her evaluation. Moreover, a justification is meaningful, as the agent realizes any distortions, although he learns new information by the justification.

In the current chapter, I follow a static approach. Some justification can be found in Fuchs (2007) who considers a finitely repeated principal-agent model. He shows that it is optimal for the principal to announce her subjective evaluation only once at the end of the interaction. In this case, the agent does not learn whether a good performance has already occurred. Hence, it is sufficient to penalize only the worst outcome, while paying a constant wage following all other terminal histories. Brown and Heywood (2005) and Addison and Belfield (2008) provide additional justification for a static approach. They show empirically that performance evaluations are more likely to be used for employees with shorter expected tenure.

This chapter also relates to the literature on endogenous contracts, like Kvaløy and Olsen (2009). Yet, I do not assume any cost for writing specific contractual arrangements. The contract can be any functions of the messages, but there are costs for communicating. The evaluation is free of charge in contrast to Rahman (2011). As the communication allows verifying the performance measure, there is a parallel to the literature on costly state verification, like Hart and Moore (1998),

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Gale and Hellwig (1985), and Townsend (1979). These models allow the investor to verify the firm's performance by a costly audit. They show the optimality of debt contracts, which are similar to the optimal contract in this chapter, as there are no audits for high payments. In this literature, however, the firm learns its performance, while the investor chooses whether to perform an audit. I assume the better-informed party decides on the information exchange. In addition, the communication need not be truthful and cannot be verified directly by one of the parties, while the result of an audit is truthful and verifiable. Strausz (2006) analyses the incentives for the principal to communicate with the agent in an adverse selection setting. The agent is privately informed about her effort costs and effort is observable. There is, however, always communication and the principal truthfully reports her signal about the agent's type. My model studies when it is optimal to communicate depending on the principal's signal about the agent's effort. Ex ante, the agent has no private information. Strausz (1997b) studies a moral hazard setting. The principal can delegate the costly monitoring of the agent's effort to a supervisor. Strausz (1997b) shows that delegation is optimal. Delegation allows the principal to fine-tune the monitoring incentives and to commit to revealing the result of the monitoring. In my model, the principal's evaluation is subjective and nonverifiable. Therefore making the communication decision verifiable or delegating it to a third party has no advantages. Moreover, a subjective evaluation is by definition unobservable by any third party.

Following truthful communication, the performance measure becomes observable, but unverifiable – similarly to a hold-up setting. Aghion et al. (2012), Hart and Moore (1988), and Grossman and Hart (1986) discuss solutions to this problem. In my model, preferences are independent of the evaluation, while in the hold-up setting the preferences depend on the types or the effort of the parties. Therefore I cannot replicate the solutions of these models. In contrast to the literature on informed principals, the principal's information arises during the principal-agent relationship and is unavailable at the contracting stage.

Furthermore, the credibility of the promised incentives is sometimes discussed under the notion of fairness and trust. According to Bernardin and Orban (1990, p. 197) the “trust in appraisal accounted for a significant proportion of variance in performance ratings.” In my model, this trust is established by communication. In

Fehr et al. (2007), Aghion and Tirole (1997), and Rotemberg and Saloner (1993), for example, this trust is created by the extent to which the principal's preferences incorporate the agent's well-being and does not depend on communication. In my model, the principal and the agent have opposing preferences.

Finally, the present chapter concerns stochastic contracts and ex-post budget balance. Previous literature, like MacLeod (2003) or Fuchs (2007), requires payments to third parties. This allows the contracting parties to renegotiate in order to avoid paying money to an outsider – as already discussed by Hart and Moore (1988). If stochastic contracts are possible, I show how to establish ex-post budget balance. An example of a stochastic contract is a mediation process with an uncertain outcome or wages in shares or options, whose valuation is influenced by external random forces. Maskin and Tirole (1999) use a similar mechanism to implement incomplete contracts in an investment setting. Rasmusen (1987) shows that stochastic wage payments ensure ex-post budget balance in a team-production setting. He does not consider differences in risk aversion between the principal and the agent, as the principal's payment is complete deterministic, only the sharing between the agents is stochastic. In addition, joint effort is perfectly observable in his model. In my model, the principal's payment has to be stochastic to guarantee budget balance. Moreover, the principal receives only a noisy signal about the agent's effort.

## 3.3 Justify Bad Evaluations

### 3.3.1 Actions

Consider a risk-averse agent working for a risk-neutral principal. The principal proposes a contract that specifies the agent's wage  $W$  depending on any information that is available at the time of the wage payments and enforceable by a third party. After signing such a contract, the agent chooses his work effort  $e \in [0, 1)$ , which is unobservable by any other person. Then, the principal collects subjective information  $I(t) \in \{0, 1\}$  about the agent's work from different sources  $t \in T = [0, 1]$ , like direct and indirect observations of the output or of the agent at work. The information of each source indicates either a success, 1, or a failure, 0, and depends stochastically on the agent's work effort  $e$  as will be specified in Section 3.3.3. This closely captures

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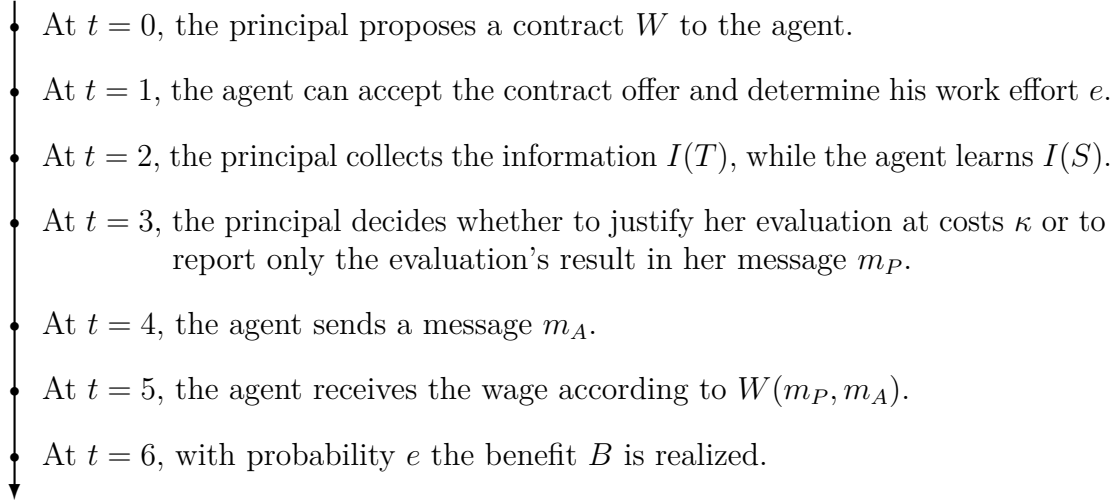


Figure 3.1: Timing of the Model

a practical evaluation process, as “an appraiser would use evidence from direct observation of the employee, or by reports from others, to make judgment about the appraisee’s performance.” (Porter et al., 2008, p. 149) An alternative interpretation is that the agent produces a number of widgets  $T$  whose quality is either good or bad. The principal then inspects the widgets’ quality. Define the result of the subjective evaluation by the principal as the average  $\mu = \int I(t)dt$ .

The agent does not learn the principal’s information  $I(\cdot)$ . She just observes  $I(\cdot)$  on a finite subset  $S \subset T$  with  $|S| = n$  sources. Nature randomly chooses the subset  $S$ , as Section 3.3.3 will explain below.  $n$  is common knowledge, while the set  $S$  is private information of the agent. This assumption ensures that the principal does not know which information is observed by both parties. The specification of the distributions also guarantees that the agent does not learn anything about the result  $\mu$  of the principal’s evaluation.  $S$  includes only sources that report pure noise. Nevertheless, the agent notices with strictly positive probability any distortion in the result of the evaluation if the principal explains and justifies her evaluation of the agent’s work. For this purpose, the principal can expend communication effort  $\kappa$  to tell the agent all information  $I(\cdot)$  upon which her evaluation is based. She might lie and send any message  $m_P \in \mathcal{I} = \{0, 1\}^T$ .<sup>6</sup> The costs of communication are positive,  $\kappa > 0$ .

<sup>6</sup> $\{0, 1\}^T$  denotes the set of functions  $T \rightarrow \{0, 1\}$  and  $I(S)$  denotes the function  $t \mapsto I(t)$  for  $t \in S$  and 0 otherwise.



They capture the opportunity costs of the principal having to justify her evaluation and to spend time writing a report or talking instead of doing other tasks. Let  $\beta \in \{0, 1\}$  denote the principal's communication decision. For  $\beta(I) = 0$  she tells the agent only the result of the evaluation,  $\int I(t)dt$ . This message is cheap talk and from the restricted message set  $R = [0, 1]$ .<sup>7</sup> The agent replies with a cheap-talk message  $m_A = (m_A^1, m_A^2) \in T^n \times \mathcal{I}$  about his sources  $S$  and their reports  $I(S)$ .

### 3.3.2 Payoffs

Then, the contract  $W$  is performed according to the available enforceable information, i.e., the messages  $m_P$  and  $m_A$ . Thus, the contract is formally a function  $W: \mathcal{I} \times T^n \times \mathcal{I} \rightarrow \mathbb{R}_+$ .<sup>8</sup> Finally, with probability equal to the agent's effort  $e$ , a benefit  $B$  is realized for the principal. This delayed realization of  $B$  corresponds to a benefit that the principal cannot observe earlier. This is a natural assumption in many investment settings. Figure 3.1 summarizes the timing.

The agent is represented by a utility function  $U(W, e) = u(W) - d(e)$  if he chooses effort  $e$ . The function  $u(\cdot)$  is increasing and strictly concave with the limit  $\lim_{w \rightarrow 0} u(w) = -\infty$  and the derivative  $u'(\cdot) > \epsilon > 0$ . On the other hand, the function  $d(\cdot)$ , the disutility of performing effort, is increasing and strictly convex with the limit  $\lim_{e \rightarrow 1} d(e) = \infty$ . Both functions are twice differentiable. The agent receives a reservation utility  $\bar{u}$  if he rejects the principal's offer. The expected benefit of the principal is  $eB - \mathbb{E}[W + \kappa\beta]$  given work effort  $e$  of the agent.

### 3.3.3 Distribution of Information

The information is distributed as follows. The principal's information  $I(t)$  is derived from informative and uninformative sources  $t \in T$ . With probability  $1 > q > 0$  a source  $t$  is informative, but being informative is unobservable. Uninformative sources

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<sup>7</sup>In an alternative specification that yields the same optimal contract, the principal only learns the average of the information  $I$ . Then she decides whether to spend  $\kappa$  to acquire the entire information  $I$ . After that, the principal can send a cheap-talk message in  $\mathcal{I}$  independent of her choice  $\beta$ ; see the discussion in the concluding section.

<sup>8</sup>The optimal contract remains unchanged if the wage also depends on the choice of communication  $\beta$ . The reason is that  $\beta$  is observable by both parties and the message spaces are sufficiently rich to use a shoot-the-liar mechanism. Therefore I neglect an explicit dependency on  $\beta$ .

report success or failure with probability  $1/2$  each. Examples are results corrupted by a computer crash. An informative source declares success with probability  $p$  and failure with probability  $1 - p$ . The probability  $p$  is drawn from the distribution  $F(p|e) = eF^H(p) + (1 - e)F^L(p)$  and therefore depends on the agent's effort  $e$ . Thus, the average  $\mu$  of the principal's information is a sufficient statistics for the agent's effort, as  $\mu = \int I(t)dt = qp + \frac{1-q}{2}$ .<sup>9</sup> The cumulative distribution functions  $F^H(p)$  and  $F^L(p)$  shall admit continuous densities  $f^H(p), f^L(p) > \epsilon > 0$ . In addition, the probability  $p$  satisfies the monotone likelihood ratio property, i.e.,

$$f^H(p)/f^L(p) \text{ is strictly increasing in } p. \quad (\text{MLRP})$$

The monotone likelihood ratio property ensures that a higher average  $\mu$  indicates higher work effort. Therefore a higher (lower, resp.) wage for good (bad) evaluations eases the agent's incentive compatibility. Assume that the agent's sample  $S$  is large and consists of a random draw from the uninformative sources with full support and no atoms.<sup>10</sup> This ensures that the agent's information is uninformative and pure noise. Hence, the principal cannot use the agent's signal to save on communication costs. Denote by  $P(S|I)$  the conditional distribution and by  $P(I, S)$  the joint distribution of the principal's information  $I$  and the agent's sample  $S$ .

### 3.3.4 Analysis

For  $\beta(I) = 0$  the principal's message space is restricted to  $R$ .<sup>11</sup> It is crucial here that the principal has to pay the communication costs  $\kappa$  to transmit the information  $I$ . Technically, the model ensures this by making it impossible to encode the information  $I$  in a message from the restricted message set  $R$ .<sup>12</sup> Thus, a truthful message by the

<sup>9</sup>I assume a law of large number here. Judd (1985) constructs a probability measure that allows avoiding measurability problems in formulating a law of large numbers for a continuum of random variables. Sun (2006) proves such a law of large numbers assuming essential pairwise independence.

<sup>10</sup>The model does not change if the agent also learns from a small number of informative sources. All results are such that  $\exists \bar{N} \in \mathbb{N}$  and the result is valid for all  $n > \bar{N}$ .

<sup>11</sup>I identify a message  $m_P \in R$  with the step function:  $T \rightarrow \{0, 1\}$  with  $1$  for  $t \leq m_p$  and  $0$  otherwise.

<sup>12</sup>According to Cantor's theorem, the set of all subsets of a set  $A$  has a strictly greater cardinality than the set  $A$ . Here, the cardinality of  $\{0, 1\}^T$  equals the cardinality of the power set of  $T$  and is bigger than the cardinality of  $T$  and the one of  $R$ . Hence, it is impossible to encode the information  $I$  into a message in  $R$ .

### 3.3 JUSTIFY BAD EVALUATIONS

principal corresponds to

$$\theta(I, \beta) = \begin{cases} I & \text{if } \beta = 1 \\ \theta'(I) & \text{if } \beta = 0 \end{cases} \quad \text{with a function } \theta' : \mathcal{I} \rightarrow R.$$

Due to the restriction of the message space for  $\beta = 0$ , the classical revelation principle does not apply here. See Green and Laffont (1986) for an example. Lemma 3.2 in the appendix proves that nevertheless truthful revelation is optimal. Thus, in the optimal contract both parties send a truthful message that reveals their private information, the principal's information  $I$  or its average  $\mu$  and the agent's sample  $(S, I(S))$  respectively.<sup>13</sup> As Proposition 2 in MacLeod (2003) and Proposition 1 in Fuchs (2007) demonstrate, some surplus has to be destroyed in this kind of model to implement positive effort of the agent. To account for this, I denote by  $W(m_P, m_A)$  the wage paid by the principal after the parties sent messages  $m_P$  and  $m_A$ . On the other hand,  $c(m_P, m_A)$  is the wage received by the agent. Proposition 3.5 shows how stochastic payments make the optimal contract ex-post budget-balanced. Grossman and Hart (1983) prove that the model can be solved in two steps. First, for every level of effort  $e$ , the optimal wage schedule  $W$  and its expected costs  $C(e)$  for the principal are computed. The second step determines the optimal effort level  $e$  by

$$\max_{e \in [0,1]} eB - C(e).$$

Now, returning to the first step, Program A below determines the optimal contract that implements effort  $e$  by choosing wages payments  $W(m_P, m_A)$ ,  $c(m_P, m_A) \geq 0$  and when to communicate,  $\beta(I)$ . To simplify the exposition, define the equilibrium payments  $W_{eq}(I, S) = W(\theta(I, \beta(I)), S, I(S))$  and  $c_{eq}(I, S) = c(\theta(I, \beta(I)), S, I(S))$ .

$$C(e) = \inf \int W_{eq}(I, S) + \kappa\beta(I) dP(I, S), \quad (\text{A})$$

$$\text{subject to } \int u(c_{eq}(I, S)) dP(I, S) - d(e) \geq \bar{u}, \quad (\text{PC})$$

$$e \in \arg \max \int u(c_{eq}(I, S)) dP(I, S) - d(e), \quad (\text{ICA})$$

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<sup>13</sup>Yet it is impossible to have the agent reveal his work effort  $e$  truthfully and make the wage dependent on his message about the effort in order to implement  $e > 0$ . Therefore it is without loss of generality for the messages to contain only the information the parties collected at  $t = 2$ .

$$\begin{aligned}
\kappa\beta(I) + \int W_{eq}(I, S)dP(S|I) &\leq \tilde{\kappa} + \int W(\hat{I}, S, I(S))dP(S|I) \\
&\forall I \in \mathcal{I}, \forall (\tilde{\kappa}, \hat{I}) \in \{\{0\} \times R, \{\kappa\} \times \mathcal{I}\}, \quad (\text{IC}_P) \\
u(c(I, S, I(S))) - u(c(I, \hat{S}, \hat{r})) &\geq 0 \quad \forall I, \hat{r} \in \mathcal{I}, \forall S, \hat{S} \in T^n, \quad (\text{IC}_{m_A}) \\
W(I, S, r) &\geq c(I, S, r) \quad \forall I, r \in \mathcal{I}, \forall S \in T^n. \quad (3.1)
\end{aligned}$$

The objective is to minimize the expected wage payment subject to several conditions. The participation constraint (PC) makes the agent willing to accept the proposed contract. The agent's incentive compatibility ( $\text{IC}_A$ ) guarantees that the agent chooses the desired level of effort. The principal's incentive compatibility ( $\text{IC}_P$ ) gives her an incentive to justify her evaluation if communication is desired. In addition, sending a true message has to be incentive compatible for the principal ( $\text{IC}_P$ ) and the agent ( $\text{IC}_{m_A}$ ). Finally, the principal's payment has to be higher than the wage received by the agent.

If the principal's information  $I$  were observable and contractible, as in common principal-agent models, the principal's and the agent's incentive for sending truthful messages ( $\text{IC}_P$ ) and ( $\text{IC}_{m_A}$ ) can be neglected. Denote this problem by  $A^*$ , the solution, the optimal complete wage, by  $w^*(\mu, e)$ , and the expected costs by  $C^*(e)$ .

**Lemma 3.1.** *If the principal's information  $I$  is contractible, the optimal contract offers a wage  $w^*(\mu, e)$ , that only conditions on the average  $\mu$  of the principal's information. Furthermore, the wage scheme  $w^*(\mu, e)$  is almost surely continuous and increasing in  $\mu$  for positive effort,  $e > 0$ .*

Thus, the results of Holmström (1979) are valid here. The principal conditions the contract only on the sufficient statistics  $\mu$  instead of the entire information  $I$  and a better performance measure implies a higher wage. If the principal's information is subjective and communication is a choice variable of the principal, the additional incentive constraints for the messages do matter. First, it is not optimal to communicate always. With full communication, the optimal contract implements wage payments  $w^*(\mu, e)$  on the equilibrium path. Problem  $A^*$  determines the wage-costs minimizing way to implement effort  $e$ . Yet, I can modify this wage scheme leaving the incentives in place and save on communication costs, because the agent is happy to accept a high wage and does not demand an explanation. Therefore it is suboptimal to

### 3.3 JUSTIFY BAD EVALUATIONS

implement the optimal complete wage payments  $w^*(\mu, e)$ . Denote the communication set by  $\mathcal{I}_C = \{I \in \mathcal{I} | \beta(I) = 1\}$ .

**Proposition 3.1.** *Complete feedback is never optimal, i.e., in the optimum  $\Pr(\mathcal{I}_C) < 1$ . The expected costs of the optimal contract are below  $C^*(e) + \kappa$ .*

The proof in the appendix shows that the principal's costs decrease if the principal refrains from communicating the highest wages. To further determine the communication set, it is necessary to know more about the structure of the payments in the optimal contract. The next proposition provides a solution to Program A and characterizes the optimal contract.

**Proposition 3.2.** *In the optimal contract the wage is constant if no justification takes place. Otherwise, the wage depends on the principal's evaluation and the principal is punished for disagreements in the messages,*

$$c^{**}(m_P, m_A) = \begin{cases} w^{**} (\int m_P(t) dt) & \text{if } m_P \in \mathcal{I}_C \\ w^{**} & \text{if } m_P \notin \mathcal{I}_C \end{cases}$$

and

$$W^{**}(m_P, m_A) = \begin{cases} w^{**} (\int m_P(t) dt) & \text{if } m_P \in \mathcal{I}_C \text{ and } m_P(m_A^1) = m_A^2(m_A^1) \\ w^{**} & \text{if } m_P \notin \mathcal{I}_C \\ w^{**} + \kappa & \text{otherwise.} \end{cases}$$

*The principal communicates to justify low wage payments, as  $w^{**} (\int m_P(t) dt) < w^{**}$  for all  $m_P \in \mathcal{I}_C$ . The values of  $w^{**}$  and  $w^{**}(\mu)$  are determined in the proof.*

Making the agent's wage contingent on his message imposes additional risks on him without easing the incentive constraint for sending a truthful message. Therefore in the optimal contract the agent's wage does not depend on his message  $m_A$ , but only on the average of the principal's one  $m_P$ . Additionally, the principal's payments have to be high in the absence of communication. Otherwise the principal would deviate and abstain from communication, because the agent cannot verify such a deviation. Furthermore, the agent's wage equals the principal's payments in the absence of communication. This results from the interaction of the participation constraint with the agent's incentive compatibility.

If the principal provides a justification, the agent can detect deviations by the principal, as a mismatch of the messages occurs. Therefore payments can vary in the principal's message  $m_P$  and a disagreement in the messages is punished by making the principal pay the highest wage. If the messages agree, the principal's payment equals the agent's wage. This structure of the wage payments allows characterizing the communication set, that takes an interval form. There will only be communication for low values of the performance measure in the optimal contract. In these cases, the agent suspects a distortion by the principal and insists on a justification for the low wages.

**Proposition 3.3.** *In the optimal contract there is a threshold  $\delta$ , such that the principal communicates for subjective evaluations  $\mu$  below  $\delta$ , while she abstains from justifying subjective evaluations  $\mu$  above  $\delta$ . Moreover,  $\frac{1-q}{2} \leq \delta < \frac{1+q}{2}$  and, on the equilibrium path, the wage is increasing in the agent's performance for evaluations below  $\delta$ .*

The proposition exhibits the communication pattern described in the introduction and summarized by Figure 3.2. There will be partial communication. The principal will use communication only as a justification of bad evaluations and low wages, while she remains silent on good performances. This confirms empirical observations, like the leniency bias and the centrality bias that there is less distinction in subjective evaluations than in the underlying performance measure, in particular at the top. Yet this behavior is not the result of a bias, but part of the optimal contract,

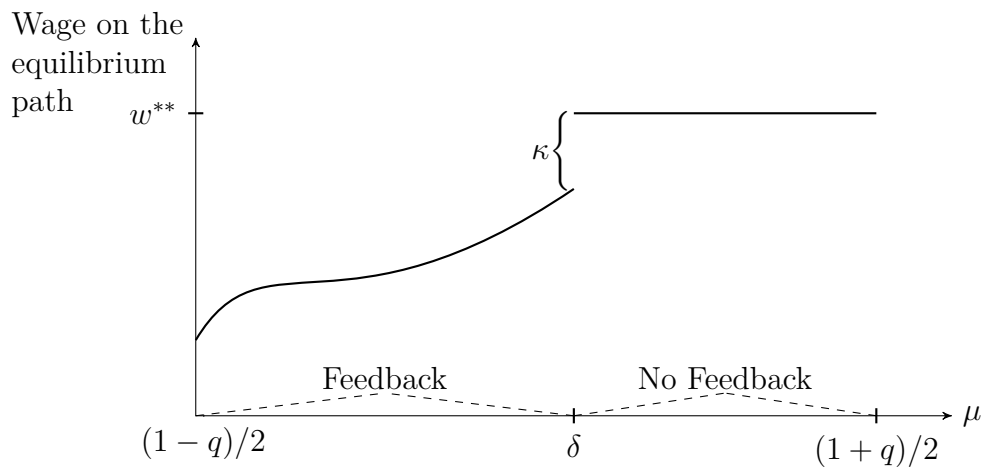


Figure 3.2: Proposition 3.3 and the Equilibrium Wage

which pools several evaluations and rewards them with the same wage. Thus, the contract eliminates wage differences that the principal would have to justify. If the evaluation is for developmental or feedback purposes only, the principal's and the agent's preferences are not completely opposed, but better aligned. Therefore the agent has less reason to assume a deviation and the principal can distinguish more finely the agent's performance.

The last proposition shows that communication is optimal if communication costs are not prohibitively high. Consequently, the optimal contract makes the principal invest some effort into communication to explain her evaluation of the agent's work. The communication is beneficial, although it is costly and conveys no additional information about the agent's effort. Yet, communication allows the principal to ensure the agent that her evaluation is not distorted.

**Proposition 3.4.** *In the optimal contract there is communication with positive probability,  $\Pr(\mathcal{I}_C) > 0$ , if the principal wants to implement positive effort  $e > 0$  of the agent and the communication costs are not too high, i.e.,*

$$\kappa \leq u^{-1} \left( \bar{u} + d(e) + \frac{f(0|e)}{f^L(0) - f^H(0)} d'(e) \right) - \int w^* \left( qp + \frac{1-q}{2}, e \right) dF(p|e). \quad (3.2)$$

Proposition 3.4 proves that I have identified an additional reason, why communication is valuable. Here, the communication is not about the principal collecting information for her decision making or giving the agent instructions in the sense of learning or which tasks to perform. Instead, communication serves the purpose to make the principal's promise of incentives to the agent credible. Thus, it is in the principal's interest to be open about her evaluations, even if communication is costly and takes place after the agent's effort choice. Finally, consider two extensions to simplify the contract and make it ex-post budget balanced.

### 3.4 Stochastic Contracts

Ex-post budget balance requires stochastic contracts. For stochastic payments, the expected value for the principal is higher than the agent's certainty equivalent. Therefore it is possible to replace payments to a third party by stochastic payments.

This does not require a risk-neutral principal. As long as there is a difference in the degree of risk aversion between the principal and the agent, the optimal contract can achieve ex-post budget balance.

**Proposition 3.5.** *It is possible to make the optimal contract ex-post budget-balanced if stochastic contracts are feasible. The optimal contract is*

$$\bar{W}^{**}(m_P, m_A) = \begin{cases} w^{**}(\int m_P(t)dt) & \text{if } m_P \in \mathcal{I}_C \text{ and } m_P(m_A^1) = m_A^2(m_A^1) \\ w^{**} & \text{if } m_P \notin \mathcal{I}_C \\ w^{**} + \Lambda(z(\int m_P(t)dt)) & \text{otherwise.} \end{cases}$$

The lotteries  $\Lambda$  have a mean of  $\mathbb{E}(\Lambda(z)) = \kappa$  and a certainty equivalent for the agent of  $u^{-1}(\mathbb{E}[u(w^{**} + \Lambda(z(\mu)))]) = w^{**}(\mu)$ . The values of  $w^{**}$  and  $w^{**}(\mu)$  are determined in Proposition 3.2.

Stochastic contracts ensure ex-post budget balance. Examples are stock options whose valuation is influenced by external shocks to the financial sector or uncertain arbitration procedures. The contracting parties might be uncertain how a disagreement is interpreted and which wage payment is appropriate.

### 3.5 Indirect Mechanism

The mechanism described in Proposition 3.2 can be simplified by changing the message spaces. The principal proposes a wage  $w'$  from the set  $\{w^{**}(\mu) | \mu \leq \delta\} \cup \{w^{**}\}$ . The agent can either accept the proposed wage  $w'$  or reject it. If he accepts  $w'$ , the principal pays him the wage  $w'$ . If he rejects, the principal has to pay  $w^{**} + \Lambda(z(\mu))$ . The principal values this payment at  $\mathbb{E}(w^{**} + \Lambda(z(\mu))) = w^{**} + \kappa$ , while the agent's certainty equivalent is  $w'$ .

Formally, the contract is now a function  $W: [\frac{1-q}{2}, \frac{1+q}{2}] \times \{Y, N\} \rightarrow \mathbb{R}_+$  and depends on the principal's proposal,  $m_P \in [\frac{1-q}{2}, \frac{1+q}{2}]$ , and the agent's decision,  $m_A \in \{Y, N\}$ . Thus, the agent has the possibility to object to the principal's evaluation. This conflict resolution might be quite realistic, as Bretz et al. (1992, p.332) state that "most organizations report having an informal dispute resolution system (e.g., open door



policies) that employees may use to contest the appraisal outcome. About one-quarter report having formalized processes available for this purpose (e.g., binding decisions made by a third party).”

The indirect mechanism leaves the wage and the incentives of the parties unchanged. The reason is the following. If the principal receives information in the communication set, any deviation, e.g., proposing a different wage and/or abstaining from communication, makes her worse off, as the deviation increases her payments to at least  $w^{**} \geq \kappa + \sup_{I \in \mathcal{I}_C} w^{**}(\int I(t)dt)$ . For evaluations outside the communication set, it is also not profitable to deviate, as any lower wage proposal will be rejected. For the agent, on the other hand, the following strategy is a best reply: accept a proposed wage if and only if the principal proposed  $w^{**}$  or she justified her evaluation and the justification matches the agent’s information. Consequently, the modified setting also implements effort  $e$  of the agent at optimal costs. Hence, the relevant part of the model is the principal’s decision to justify her evaluation to the agent at  $t = 3$ .

### 3.6 Conclusion

This chapter discusses a principal-agent model with a private and subjective performance measure allowing for communication. The principal can explain her evaluation of agent’s work to the agent. Giving justifications is costly, does not convey additional information about the agent’s effort, and does not serve a learning or instructing purpose. Nevertheless, in the optimal mechanism the principal provides justifications. This allows the agent to detect inappropriate or distorted evaluations by the principal if her explanations are truthful. Therefore providing a justification makes the incentives for the agent to expend effort credible. In the optimal contract, the principal justifies only bad evaluations. This communication pattern results in pooling and wage compression at the top, as illustrated in Figure 3.2 on page 92. These results fit well with empirical observations, often referred to as leniency bias and centrality bias.<sup>14</sup> The chapter argues that this pattern of evaluations is a feature of the optimal contract with unbiased agents and no proof of biased behavior per se.

The principal’s justifications convince the agent that the principal evaluates her

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<sup>14</sup>“The distribution of ratings is typically both concentrated and biased.” (Gibbs, 1991, p. 5)

## CHAPTER 3

appropriately ex-post. In addition, they motivate him ex-ante to implement the specified work effort. Compare this to a naive contract that does not give the principal an incentive to justify her evaluation. In this naive contract, the principal abstains from justifications and always reports the evaluation associated with the lowest wage. Anticipating this behavior the agent is unmotivated to implement any positive work effort. This partially explains the concern of the management literature to ensure credible feedback provision. In addition, the problem of credible evaluations provides a partial answer to Fuchs (2007, p. 1446), who emphasizes the importance of exploring “possible reasons for the existence of communication” between agents and principals. Communication at the interim stages might be explained by training and instruction reasons, but credibility problems are responsible for the communication in the final stage of the principal-agent relation according to my model.

The results of this chapter are important for the design of incentives systems. First, the systems have to ensure the credible provision of appropriate feedback by institutionalizing the feedback process or using multi-source feedback. Second, the pooling at the top could cause the costs of an incentives scheme to be substantial if there is a bonus attached to receiving a positive evaluation and many employees receive a positive evaluation due to the compression at the top.<sup>15</sup> Third, my result provides a rationale, why forced distribution systems, requiring supervisors to match a given distribution with their evaluations, are an uncommon response to lenient evaluations.<sup>16</sup> These systems ensure dispersion in the results of the evaluation, but are suboptimal, as they require too much communication.

This chapter assumes that the principal incurs costs for communicating with the agent. I would get similar results if I instead assumed that the principal’s costs concerned the acquisition of information. In this case, the principal only learns the result of the evaluation, i.e., the average of the information  $I$ . Then she decides whether to spend  $\kappa$  to acquire the entire information  $I$ . Independently of her choice of

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<sup>15</sup>Bernardin and Orban (1990, p. 199) provide the example of the Small Business Administration and NASA introducing a bonus scheme based on subjective evaluations. After more than 50% of eligible employees should receive a bonus, Congress responded with the requirement that no more than 25% of employees shall receive a bonus.

<sup>16</sup>Bretz et al. (1992) and Gibbs (1991) show that the use of forced distributions is very limited. According to Murphy (1993, p. 47), forced distribution systems “mitigate managerial tendencies to assign uniform ratings but may generate important counterproductive side effects.”

## 3.6 CONCLUSION

information acquisition, the principal can send a cheap-talk message in the unrestricted message set  $\mathcal{I}$ . Both settings have some merits; in reality, there could be a mixture of these two polar cases.

An avenue for future research is the role of communication for rewarding good performance. Many people regard communication as an appreciation of their work and are genuinely happy about positive feedback. Therefore communication, such as praise or commendation, might enter the agent's utility function. Then there is a trade-off between communicating good outcomes as a reward and the motive for communication discussed in this chapter.

## Appendix to Chapter 3

### 3.A The Optimal Complete Contract

Lemma 3.1 characterizes the optimal complete contract and states the solution to Program A\* if the principal's information  $I$  is public and verifiable. This yields a benchmark solution  $w^*(\mu, e)$ , the optimal complete wage. Additionally, the lemma shows that every effort  $e \in [0, 1)$  is implementable at finite costs  $C^*(e)$ .

**Proof of Lemma 3.1:** Holmström (1979) shows that the optimal wage only conditions on  $\mu = \int I(t)dt$ , because the average of the principal's information  $I$  is a sufficient statistics for the agent's effort,  $\Pr(I, S|e, \int I(t)dt) = \Pr(I, S|\int I(t)dt)$ . In order to implement no effort,  $e = 0$ , the optimal contract sets  $w^*(\mu, 0) = u^{-1}(\bar{u} + d(0))$  for all  $\mu$ . If, on the other hand, the desired effort is positive,  $e > 0$ , the agent's incentive compatibility matters. The first-order approach is valid here, because  $F(p|e)$  is a linear combination of distribution functions. This implies that the convex distribution function condition is satisfied. According to Grossman and Hart (1983) and Rogerson (1985), the convex distribution function condition in combination with the convexity of  $d(\cdot)$  and the monotone likelihood ratio property guarantees the validity of the first-order approach. Thus, the agent's incentive compatibility reads

$$\int u(w(\mu)) \frac{f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu))}{q} d\mu = d'(e) \quad (\text{IC}_A)$$

with the normalization  $\hat{\mu}(\mu) = \frac{\mu - (1-q)/2}{q}$ . For positive effort, the constraint set is also nonempty. Take for example any  $\bar{w} > 0$  and the contract

$$w(\mu) = \begin{cases} \bar{w} & \text{if } f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu)) \geq 0 \\ h(\bar{w}) & \text{otherwise} \end{cases}$$

with  $h(\bar{w})$  positive, but small enough, such that the incentive compatibility ( $\text{IC}_A$ ) is satisfied. This implicitly defines an increasing function  $h(\cdot)$ , as  $\partial h(\bar{w})/\partial \bar{w} > 0$ . Consequently, there is a  $\bar{w}$  fulfilling the participation constraint (PC) with equality. Therefore the constraint set of Program A\* is nonempty. Moreover, the costs of the contract given by  $\bar{w}$  are  $h(\bar{w})F(\zeta|e) + \bar{w}(1 - F(\zeta|e)) < \bar{w} < \infty$  with  $\zeta = \inf\{p \in [0, 1] | f^H(p) - f^L(p) \geq 0\}$ .

Holmström (1979) proves that the Lagrange multipliers of the participation constraint  $\lambda_1$  and of the incentive compatibility  $\lambda_2$  are positive. Pointwise optimization<sup>17</sup> determines the optimal contract as

$$f(\hat{\mu}(\mu)|e) - \lambda_1 u'(w(\mu))f(\hat{\mu}(\mu)|e) - \lambda_2 u'(w(\mu))(f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu))) = 0 \quad \text{a.s.},$$

$$\frac{1}{u'(w(\mu))} = \lambda_1 + \lambda_2 \frac{f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu))}{f(\hat{\mu}(\mu)|e)} = \lambda_1 + \lambda_2 \frac{\frac{f^H(\hat{\mu}(\mu))}{f^L(\hat{\mu}(\mu))} - 1}{e^{\frac{f^H(\hat{\mu}(\mu))}{f^L(\hat{\mu}(\mu))}} + 1 - e} \quad \text{a.s.}$$

Since the fraction  $\frac{l-1}{el+1-e}$  is increasing in  $l$ , the right-hand side of above equation is increasing in  $\mu$  due to the monotone likelihood ratio property. Therefore the concavity of  $u(\cdot)$  implies that  $w^*(\mu, e)$  is increasing in  $\mu \in [\frac{1-q}{2}, \frac{1+q}{2}]$  almost surely. Moreover,  $w^*(\mu, e)$  is continuous almost surely and any discontinuity is removable, because the densities  $f^H$  and  $f^L$  are continuous.  $\square$

### 3.B The Optimal Contract

According to Proposition 3.1, it is suboptimal to provide justifications almost surely.

**Proof of Proposition 3.1:** Suppose the principal communicates almost surely, i.e.,  $\Pr(\mathcal{I}_C) = 1$ . Then the expected communication costs are  $\kappa \mathbb{E}(\beta(I)) = \kappa$  and it just remains to minimize the wage costs. Yet it is possible to implement payments  $w^*(\mu, e)$  defined in Program A\* by the following contract

$$\bar{W}(m_P, m_A) = \begin{cases} w^*(\mu, e) & \text{if } m_P(m_A^1) = m_A^2(m_A^1) \\ w^*(1, e) + 2\kappa & \text{if } m_P(m_A^1) \neq m_A^2(m_A^1) \end{cases}$$

and  $\bar{c}(m_P, m_A) = w^*(\mu, e)$  with  $\mu = \int m_P(t) dt$ . There is a  $\bar{N} \in \mathbb{N}$ , such that a deviation is unprofitable for the principal for all  $n > \bar{N}$ , because the probability of a mismatch in the messages is sufficiently close to 1. Hence, the additional incentive constraints  $(IC_P)$  and  $(IC_{m_A})$  are satisfied. In addition, the realized wage payments

<sup>17</sup>This technique allows for piecewise continuous functions, as Kamien and Schwartz (1991) show in Part II, Section 12. Therefore bonus wages are possible and there is no restriction to continuous wage schemes.

### CHAPTER 3

remain unchanged, as the parties' messages agree in equilibrium. It is possible, however, to implement a certain work effort  $e$  of the agent even cheaper by partial communication. For this purpose, I modify the contract  $\bar{W}$  to

$$\bar{W}(m_P, m_A) = \begin{cases} w^*(\mu, e) & \text{if } m_P(m_A^1) = m_A^2(m_A^1) \\ w^*(1, e) + 2\kappa & \text{if } \mu < \delta \text{ and } m_P(m_A^1) \neq m_A^2(m_A^1) \\ w^*(\delta, e) + \kappa & \text{if } \mu \geq \delta \text{ and } m_P(m_A^1) \neq m_A^2(m_A^1) \end{cases}$$

with a  $\delta < 1$ , such that

$$f^H\left(\frac{\delta - \frac{1-q}{2}}{q}\right) - f^L\left(\frac{\delta - \frac{1-q}{2}}{q}\right) \geq 0 \quad \text{and} \quad w^*(\delta, e) + \kappa \geq w^*(1, e). \quad (3.3)$$

Lemma 3.1 proves that  $w^*(\mu, e)$  is almost surely continuous and any discontinuity is removable. Consequently, there exists a continuous function that almost surely equals  $w^*(\mu, e)$ . Replacing  $w^*(\mu, e)$  by that function in the definition of  $\bar{W}'$  also yields a solution to Program A. This procedure guarantees that the conditions (3.3) on  $\delta$  are feasible.

In the contract  $\bar{W}'$ , for  $n > \bar{N}$ , the principal will reveal all evaluations except the highest ones and the communication set is

$$\mathcal{I}_C = \left\{ I \in \mathcal{I} \mid \frac{\int I(t)dt - \frac{1-q}{2}}{q} \in [0, \delta] \right\}.$$

If the principal's information indicates a very good performance,  $\mu > \delta$ , Lemma 3.1 has shown that  $w^*(\delta, e) + \kappa < w^*(\mu, e) + \kappa$ . Thus, in these cases communication would increase her total costs consisting of wage and communication costs. In addition, the conditions in (3.3) guarantee that constraints (PC) and (IC<sub>A</sub>) are still satisfied by choosing the agent's wage appropriately. Therefore the contract  $\bar{W}'$  implements effort  $e$  of the agent and is cheaper than the contract  $\bar{W}$ . This shows that the principal will not explain her evaluation to the agent with probability 1.  $\square$

Lemma 3.2 shows that I can concentrate on truthful messages without loss of generality.

### 3.B THE OPTIMAL CONTRACT

**Lemma 3.2.** *For every contract  $W$  there is a contract  $W'$ , such that  $W'$  implements the same effort  $e$  at (weakly) lower costs than  $W$  and gives the agent and the principal an incentive to send truthful messages. In addition, contract  $W'$  has the following structure*

$$c'(m_P, m_A) = \begin{cases} c(m_P) & \text{if } m_P \in \mathcal{I}'_C \\ \bar{w} & \text{if } m_P \notin \mathcal{I}'_C \end{cases} \quad (3.4)$$

$$W'(m_P, m_A) = \begin{cases} w(m_P) & \text{if } m_P \in \mathcal{I}'_C \text{ and } m_P(m_A^1) = m_A^2(m_A^1) \\ \bar{w} + \kappa & \text{if } m_P \in \mathcal{I}'_C \text{ and } m_P(m_A^1) \neq m_A^2(m_A^1) \\ \bar{w} & \text{if } m_P \notin \mathcal{I}'_C \end{cases} \quad (3.5)$$

with the communication set  $\mathcal{I}'_C = \{I \in \mathcal{I} | \beta'(I) = 1\}$ .

**Proof:** The proof consists of four parts. The first part characterizes the equilibrium utilities in the contract  $W$ . In the second part, the contract  $W'$  is determined in such a way that the parties have an incentive to send truthful messages. The third part analyzes the agent's incentive compatibility for his work effort. The fourth part ensures that the new contract  $W'$  satisfies the agent's incentive compatibility and participation constraint.

**Step 1** Denote the expected utilities given equilibrium strategies in contract  $W$  by  $w(I)$  for the principal and by  $u(c(I))$  for the agent. I consider certainty equivalents with respect to the agent's sample  $S$  given equilibrium strategies. If contract  $W$  specifies no communication for  $I \notin \mathcal{I}_C$ , then in equilibrium the principal provides no justifications and the agent cannot verify the evaluation. Therefore the principal's payments have to be constant or  $w(I) = w(I')$  for all  $I, I' \notin \mathcal{I}_C$ . Moreover, they have to be higher than the principal's payments in the communication set including communication costs.

$$w(I) \geq \kappa + \sup_{I' \in \mathcal{I}_C} w(I') \quad \forall I \notin \mathcal{I}_C$$

Otherwise, the principal would abstain from communication and act as if  $I \notin \mathcal{I}_C$ , because the agent could not observe this deviation. Finally, notice that the agent detects a deviation in the principal's message with probability 0 if the principal

deviates only in the reports of sources with mass 0. Therefore  $w(I) = w(\hat{I})$  for all  $\hat{I}(t), I(t) \in \mathcal{I}$  with  $\hat{I}(t) = I(t)$  almost surely.

**Step 2** Consider the contract  $W'$  given by equations (3.4) and (3.5) with  $\mathcal{I}'_C = \mathcal{I}_C \setminus R$  and  $\bar{w} = w(I)$  for a  $I \notin \mathcal{I}_C$ .<sup>18</sup> The agent's wage does not depend on his message. Therefore he is indifferent between sending any message and the incentive compatibility for his message is satisfied. If the principal should communicate,  $I \in \mathcal{I}_C$ , any disagreement in the messages shows a deviation by the principal and the payment  $W'(I, S, r)$  with  $r(S) \neq I(S)$  and  $I \in \mathcal{I}_C$  matters only for the right-hand side of the principal's incentive compatibility ( $IC_P$ ). Therefore I can increase this payment to satisfy ( $IC_P$ ) without affecting any other constraint or the objective function. Accordingly, there will be a penalty for  $I \in \mathcal{I}_C$  and  $I(S) \neq r(S)$ . By setting the principal's payment in this case to

$$W(I, S, r) = \bar{w} + \kappa$$

there is a  $\bar{N} \in \mathbb{N}$  such that for  $n > \bar{N}$  the principal will never deviate to another message in the communication set  $\mathcal{I}_C$  independent of her communication choice  $\beta(I)$ . The reason is that the deviation either does not influence payments or the probability of a mismatch in the messages is sufficiently close to 1 and

$$\bar{w} + \kappa > \int W'(\hat{I}, S, I(S))dP(S|I) \geq \bar{w} > w(I) = \int W'(I, S, I(S))dP(S|I)$$

for all  $I, \hat{I} \in \mathcal{I}_C$  and  $\hat{I}(t) \neq I(t)$  almost surely.

**Step 3** For every  $\mu$  denote the set of all information with the average  $\mu$  by  $M(\mu) = \{I \in \mathcal{I} \mid \int I(t)dt = \mu\}$ . For the agent's incentives only the expected wage in  $M(\mu)$  matters, because the agent's information,  $S$  and  $I(S)$ , does not depend on her effort choice and the average of the principal's information is a sufficient statistics for the agent's effort,  $\Pr(I, S|e, \int I(t)dt) = \Pr(I, S \mid \int I(t)dt)$ . In addition, Lemma 3.1 shows that the first-order approach is valid here. Therefore, the agent's incentive compatibility ( $IC_A$ ) equals

$$\int_{\frac{1-q}{2}}^{\frac{1+q}{2}} \left( \int u(c(I))dP(I, S \mid I \in M(\mu)) \right) \frac{f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu))}{q} d\mu = d'(e) \quad (3.6)$$

with the normalization  $\hat{\mu}(\mu) = \frac{\mu - (1-q)/2}{q}$ . Consequently, the agent's incentives remain

<sup>18</sup>If  $\mathcal{I}_C = \mathcal{I}$ , set  $\bar{w} = \kappa + \sup_{\hat{I} \in \mathcal{I}_C} w(\hat{I})$ .



unchanged if

$$\int u(c(I))dP(I, S|I \in M(\mu)) = \int u(c'(\theta(I, \beta(I)), S, I(S)))dP(I, S|I \in M(\mu)) \quad (3.7)$$

for all  $\mu \in [\frac{1-q}{2}, \frac{1+q}{2}]$ . By the definition of  $c'$  in (3.4), the right-hand side equals

$$\int \beta'(I)u(c(I))dP(I, S|I \in M(\mu)) + u(\bar{w}) \int 1 - \beta'(I)dP(I, S|I \in M(\mu))$$

Hence, the equality in (3.7) is not guaranteed for all  $\mu$ , as  $c(I) \leq c'(I, m_A) = \bar{w}$  for  $I \notin \mathcal{I}'_C$ .

**Step 4** If (3.7) is not satisfied, it is necessary to reduce the expected wage of the agent. For this purpose, let the principal communicate every information  $I \notin \mathcal{I}_C \cup R$  with  $c(I) \leq \bar{w} - \kappa$ . Set  $\mathcal{I}'_C = \mathcal{I}_C \cup \{I\}$ ,  $c'(I, m_A) = c(I)$  and

$$W'(I, m_A) = \begin{cases} c(I) & \text{if } I(m_A^1) = m_A^2(m_A^1) \\ \bar{w} + \kappa & \text{if } I(m_A^1) \neq m_A^2(m_A^1) \end{cases} \quad \forall m_A.$$

Thus, for any remaining information that is not communicated,  $I \notin \mathcal{I}'_C \cup R$ , the agent's wage is  $\bar{w} - \kappa < c(I) \leq \bar{w}$  in contract  $W$ . Finally, increase the communication set and make the principal communicate a fraction  $\alpha$  of the information  $M(\mu) \setminus \mathcal{I}'_C$ , such that

$$\begin{aligned} \int u(c(I))dP(I, S|I \in M(\mu)) &= \int \beta'(I)u(c(I))dP(I, S|I \in M(\mu)) + \\ &+ (\alpha u(\bar{w} - \kappa) + (1 - \alpha)u(\bar{w})) \int 1 - \beta'(I)dP(I, S|I \in M(\mu)) \end{aligned}$$

As the agent's wage is  $\bar{w} - \kappa < c(I) \leq \bar{w}$  for all  $I \notin \mathcal{I}'_C \cup R$ , it is possible to find such an  $\alpha \in [0, 1]$ . Set  $\alpha = 1$  if  $\alpha$  is not uniquely determined, as  $M(\mu) \setminus \mathcal{I}'_C$  has mass 0. Denote the addition information that is communicate by  $\mathcal{I}''$ . To make communication optimal, adjust contract  $W'$  by  $\mathcal{I}'_C = \mathcal{I}'_C \cup \mathcal{I}'''$ ,  $\mathcal{I}''' = \mathcal{I}'' \setminus R$ ,  $c'(I, m_A) = \bar{w} - \kappa$  and

$$W'(I, m_A) = \begin{cases} \bar{w} - \kappa & \text{if } I(m_A^1) = m_A^2(m_A^1) \\ \bar{w} + \kappa & \text{if } I(m_A^1) \neq m_A^2(m_A^1) \end{cases} \quad \forall I \in \mathcal{I}''', \forall m_A.$$

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Repeat these steps for every  $\mu$ . Then the agent's incentives in the new contract  $W'$  are the same as in contract  $W$ . In addition, the agent's participation constraint is also satisfied in contract  $W'$ , so that contract  $W'$  implements effort  $e$  at (weakly) lower costs than contract  $W$ .  $\square$

Lemma 3.3 proves that contracts in which the agent's wage payment depends only on the average of the principal's information are no loss of generality.

**Lemma 3.3.** *For every contract  $W$  there is a contract  $W'$  implementing the same effort  $e$  at the same costs as  $W$ . Moreover, in contract  $W'$  the communication choice  $\beta$  and the agent's wage just depends on the average of the principal's information, i.e.,  $c'(m_P, m_A) = c'(m'_P, m_A)$  for all  $m_A, m_P, m'_P$  with  $\int m_P(t)dt = \int m'_P(t)dt$ .*

**Proof:** Lemma 3.2 shows that the agent's wage does not depend on his message  $m_A$ . In addition, according to equation (3.6), the agent's incentive compatibility ( $IC_A$ ) depends only on the expected utility of the agent given the average of the principal's information. The same is valid for the agent's participation constraint in Program A. Therefore it is possible without violating these constraints to set  $c'(m_P, m_A) = \tilde{c}(\mu)$  for all  $m_P \in \mathcal{I}_C$  and  $m_A$  with

$$\mu = \int m_P(t)dt, \quad u(\tilde{c}(\mu)) = \int u(c(\hat{I}, S, \hat{I}(S)))dP(\hat{I}, S | \hat{I} \in M(\mu) \cap \mathcal{I}_C)$$

and  $M(\mu) = \{I \in \mathcal{I} | \int I(t)dt = \mu\}$ . This reduces at least weakly the expected wage, because the agent is risk-averse and

$$\tilde{c}(\mu) \leq \int c(\hat{I}, S, \hat{I}(S))dP(\hat{I}, S | \hat{I} \in M(\mu) \cap \mathcal{I}_C). \quad (3.8)$$

Yet, the agent's new wage  $c'(m_P, m_A) = \tilde{c}(\mu)$  might be higher than the principal's payment  $W(m_P, m_A)$  for some  $m_A$  and  $m_P \in \mathcal{I}_C$ . To make the contract feasible and satisfy constraint (3.1) in Program A, set

$$W'(m_P, m_A) = \int W(\hat{I}, S, \hat{I}(S))dP(\hat{I}, S | \hat{I} \in M(\mu) \cap \mathcal{I}_C) \quad \forall m_P \in \mathcal{I}_C, \forall m_A$$

with  $\mu = \int m_P(t)dt$ . This ensures that  $W'(m_P, m_A) \geq \tilde{c}(\mu)$  for all  $m_P, m_A$ , because  $W(m_P, m_A) \geq c(m_P, m_A)$  in contract  $W$  and the new wage,  $\tilde{c}(\mu)$ , is lower than the previous expected wage, as shown in equation (3.8). In addition, the expected

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payments of the principal in the new contract  $W'$  are the same as in contract  $W$ , as

$$\int W'_{eq}(I, S)dP(I, S|I \in M(\mu)) = \int W_{eq}(I, S)dP(I, S|I \in M(\mu)) \quad \forall \mu.$$

Consequently, there exists a contract that gives the principal and the agent the same utility as the old contract  $W$ , but makes the agent's utility depend only on the average of the principal's information, i.e.,  $c(m_P, m_A) = \tilde{c}(\mu)$  for all  $m_A$  and all  $m_P \in \mathcal{I}_C$ . It remains to ensure constraint  $(IC_P)$  in contract  $W'$ . For  $I \in \mathcal{I}_C$  in the communication set,  $W'(I, S, r)$  with  $I(S) \neq r(S)$  matters only on the right-hand side of constraint  $(IC_P)$ . Therefore increasing  $W'(I, S, r)$  to  $\kappa + \sup_{\bar{I}, \bar{S}} W(\bar{I}, \bar{S}, \bar{I}(\bar{S}))$  does not affect the objective function or the other constraints, but gives the principal incentives to communicate truthfully. Additionally, the wage in the communication set is lower than outside this set including the communication costs,  $W'(I, S, I(S)) + \kappa < W'_{eq}(\hat{I}, \hat{S})$  for all  $I \in \mathcal{I}_C, \hat{I} \notin \mathcal{I}_C$  and all  $S, \hat{S}$ , because contract  $W$  meets this condition according to Lemma 3.2. This guarantees that there is a  $\bar{N} \in \mathbb{N}$  such that for all  $n > \bar{N}$  any deviation in the communication choice,  $\beta(I)$ , and/or the message  $m_P$  makes the principal worse off. Therefore also the new contract  $W'$  satisfies the principal's incentive compatibility.

It remains to prove that the communication choice does not change within the set  $M(\mu)$  for any  $\mu$ . Suppose there is a  $\mu$  such that  $\hat{c}(\mu) \leq \bar{w} - \kappa$  with  $\bar{w}$  the wage outside the communication set according to Lemma 3.2 and  $\hat{c}$  defined by

$$u(\hat{c}(\mu)) = \int u(c(\hat{I}, S, \hat{I}(S)))dP(\hat{I}, S | \hat{I} \in M(\mu)).$$

Then it is possible to communicate all  $I \in M(\mu)$  by setting  $\mathcal{I}'_C = \mathcal{I}_C \cup M(\mu)$  and by adapting the wage as in the first part of the proof. If, on the other hand,  $\hat{c}(\mu) = \bar{w}$ , the principal can abstain from communication for all  $I \in M(\mu)$  by setting  $\mathcal{I}'_C = \mathcal{I}_C \setminus M(\mu)$  and  $W(I, m_A) = c(I, m_A) = \bar{w}$  for all  $m_A$ .

Finally, in the last case  $\bar{w} - \kappa < \hat{c}(\mu) < \bar{w}$ . Denote by  $A$  the set of all  $\mu$  with this property,

$$A = \left\{ \mu \in \left[ \frac{1-q}{2}, \frac{1+q}{2} \right] \mid \bar{w} - \kappa < \hat{c}(\mu) < \bar{w} \right\}.$$

If the set  $A$  has no mass,  $\int_A f(\hat{\mu}(\mu)|e)d\mu = 0$ , set  $\mathcal{I}'_C = \mathcal{I}_C \cup M(\mu)$  and  $\hat{c}(\mu) = \bar{w} - \kappa$  for

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all  $\mu \in A$  and adapt the wage as in the first part of the proof. If  $\int_A f(\hat{\mu}(\mu)|e)d\mu > 0$ , there exists a unique  $\delta$ , such that communicating only information with an average below  $\delta$  does not change the agent's expected utility, i.e.,

$$\int_A u(\hat{c}(\mu))dF(\hat{\mu}(\mu)|e) = u(\bar{w} - \kappa) \int_{A^1} dF(\hat{\mu}(\mu)|e) + u(\bar{w}) \int_{A^2} dF(\hat{\mu}(\mu)|e)$$

with the sets  $A^1 = \{\mu \in A | \mu \leq \delta\}$  and  $A^2 = \{\mu \in A | \mu > \delta\}$ . For this purpose, modify the contract to

$$\beta'(m_P) = \begin{cases} 1 & \text{if } \mu \in A^1 \\ 0 & \text{if } \mu \in A^2 \end{cases} \quad c'(m_P, m_A) = \begin{cases} \bar{w} - \kappa & \text{if } \mu \in A^1 \\ \bar{w} & \text{if } \mu \in A^2 \end{cases}$$

$$W'(m_P, m_A) = \begin{cases} \bar{w} & \text{if } \mu \in A^2 \\ \bar{w} - \kappa & \text{if } \mu \in A^1 \text{ and } m_P(m_A^1) = m_A^2(m_A^1) \\ \bar{w} + \kappa & \text{if } \mu \in A^1 \text{ and } m_P(m_A^1) \neq m_A^2(m_A^1) \end{cases}$$

for all  $m_P \in M(\mu)$ , all  $m_A$  and all  $\mu \in A$ . As the agent is risk-averse and  $W(m_P, m_A) \geq c(m_P, m_A) \in \{\bar{w}\} \cup (0, \bar{w} - \kappa]$ , this reduces the expected wage.

$$\int_A \left( \int W(\hat{I}, S, \hat{I}(S))dP(\hat{I}, S | \hat{I} \in M(\mu)) \right) dF(\hat{\mu}(\mu)|e) \geq$$

$$\geq (\bar{w} - \kappa) \int_{A^1} dF(\hat{\mu}(\mu)|e) + \bar{w} \int_{A^2} dF(\hat{\mu}(\mu)|e)$$

In addition, the difference in the agent's incentive compatibility is positive.

$$\int_{A^1} (u(\bar{w} - \kappa) - u(\hat{c}(\mu))) \Delta^f(\mu) d\mu + \int_{A^2} (u(\bar{w}) - u(\hat{c}(\mu))) \Delta^f(\mu) d\mu =$$

$$= \int_{A^1} \underbrace{(u(\bar{w} - \kappa) - u(\hat{c}(\mu)))}_{<0} \frac{\Delta^f(\mu) f(\hat{\mu}(\mu)|e)}{f(\hat{\mu}(\mu)|e)} d\mu +$$

$$+ \int_{A^2} \underbrace{(u(\bar{w}) - u(\hat{c}(\mu)))}_{>0} \frac{\Delta^f(\mu) f(\hat{\mu}(\mu)|e)}{f(\hat{\mu}(\mu)|e)} d\mu >$$

$$> \frac{\Delta^f(\delta) q}{f(\hat{\mu}(\delta)|e)} \left( \int_{A^1} u(\bar{w} - \kappa) dF(\hat{\mu}(\mu)|e) + \int_{A^2} u(\bar{w}) dF(\hat{\mu}(\mu)|e) - \int_A u(\hat{c}(\mu)) dF(\hat{\mu}(\mu)|e) \right) = 0$$

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with  $\Delta^f(\mu) = \frac{1}{q}(f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu)))$ . The monotone likelihood ratio property ensures the strict inequality and the constant value of the participation constraint yields the final equality. This shows that it is possible to satisfy the agent's incentive compatibility ( $\text{IC}_A$ ).

Consequently, contract  $W'$  satisfies all the constraints of Program A and the communication decision just depends on the average of the principal's information, except for the null set  $R$ , as  $\mathcal{I}'_C = \mathcal{I}'_C \setminus R$  and  $W'(m_P, m_A) = c(m_P, m_A) = \bar{w}$  for all  $m_P \in R$  and all  $m_A$ .  $\square$

Proposition 3.2 characterizes the optimal contract.

**Proof of Proposition 3.2:** Lemma 3.2 proves the basic structure of the optimal contract and this proof uses the notation introduced there. Lemma 3.3 shows that the communication decision and the agent's wage just depend on the average of the principal's message. It remains to calculate the agent's wage. First, I prove that there are no payments to third parties on the equilibrium path. For this purpose, simplify Program A according to Lemma 3.2 and 3.3 to

$$\begin{aligned} & \inf \int w(I) + \kappa\beta(I)dP(I, S), & (\text{B}) \\ \text{subject to } & \int u(c(\mu))dF(\hat{\mu}(\mu)|e) - d(e) \geq \bar{u}, & (\text{PC}) \\ & \int u(c(\mu))\frac{f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu))}{q}d\mu = d'(e), & (\text{IC}_A) \\ & \bar{w} \geq w(I) + \beta(I)\kappa \geq (1 - \beta(I))\bar{w} \quad \forall I \in \mathcal{I}, & (\text{IC}_P) \\ & w(I) \geq c\left(\int I(t)dt\right) \quad \forall I \in \mathcal{I} & (3.9) \end{aligned}$$

with the normalization  $\hat{\mu}(\mu) = \frac{\mu - (1-q)/2}{q}$ . ( $\text{IC}_P$ ) ensures that the wage is constant outside the communication set. Furthermore, the condition requires that the principal's costs in the communication set, accounting for the communication costs, is lower than outside this set. According to Lemma 3.2 this is equivalent to condition ( $\text{IC}_P$ ) in Program A. Condition (3.9) is the equivalent to (3.1) in the initial program and guarantees that payments to third parties are nonnegative. Assume to the contrary that in the optimal contract there is a  $I \in \mathcal{I} \setminus R$ , such that  $w(I) > c(\int I(t)dt)$ . Then  $\beta(I) = 1$ , because Lemma 3.2 and 3.3 show that  $c(\int I(t)dt) = \bar{w} = w(I)$  for  $I \notin \mathcal{I}'_C$ .

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In addition, it is possible to (weakly) decrease the objective function without violating any constraint by setting  $w(I) = c(\int I(t)dt)$ . Therefore the principal's consensus payment just depends on the average of her message and  $w(I) = c(\int I(t)dt)$  for all  $I \in \mathcal{I} \setminus R$ . This results in payments to a third party of

$$W(m_P, m_A) - c\left(\int m_P(t)dt\right) = \begin{cases} \bar{w} - w(m_P) + \kappa & \text{for } m_P(m_A^1) \neq m_A^2(m_A^1) \text{ and } m_P \in \mathcal{I}_C \\ 0 & \text{otherwise.} \end{cases}$$

As the principal's consensus payments and her communication decision just depend on the average of the principal's information, I write them as functions of the average  $\mu$  instead of the information  $I$  in the following.

Finally, it remains to determine the values of  $w^{**}(\mu)$  and  $w^{**}$ , which solve the following program

$$C^c(e) = \inf \int w(\mu) + \kappa\beta(\mu)dF(\hat{\mu}(\mu)|e), \quad (\text{C})$$

$$\text{subject to } \int u(w(\mu))dF(\hat{\mu}(\mu)|e) - d(e) \geq \bar{u}, \quad (\text{PC})$$

$$\int u(w(\mu)) \frac{f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu))}{q} d\mu = d'(e), \quad (\text{IC}_A)$$

$$(1 - \beta(\mu))(w(\mu) - w) = 0 \quad \forall \mu, \quad (3.10)$$

$$w(\mu) + \kappa\beta(\mu) \leq w \quad \forall \mu. \quad (3.11)$$

As before, the objective is to minimize the expected costs, here communication costs and the principal's payments. The participation constraint (PC) and incentive compatibility (IC<sub>A</sub>) of the agent remain unchanged. Constraints (3.10) and (3.11) replace condition (IC<sub>P</sub>). Program C completes the proof and determines the optimal contract.  $\square$

### 3.C The Optimal Communication Pattern

Proposition 3.3 verifies that a threshold strategy is optimal, with justifications below a threshold  $\delta$  and pooling above  $\delta$ . Its proof follows the intuition given in the introduction.

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**Proof of Proposition 3.3:** Proposition 3.1 proves that complete feedback is suboptimal and  $\Pr(\mathcal{I}_C) < 1$ . In order to show that there will only be communication of bad evaluations, assume to the contrary that in the optimal contract there is a  $\mu^*$  with the following properties. With positive probability communication takes place for  $\mu \geq \mu^*$  and with positive probability the principal does not reveal her performance measure for  $\mu \leq \mu^*$ . Denote the corresponding sets by  $A^K = \{\mu \in [\frac{1-q}{2}, \frac{1+q}{2}] | \mu \geq \mu^* \text{ and } \beta(\mu) = 1\}$  for communicated evaluations above  $\mu^*$  and by  $A^N = \{\mu \in [\frac{1-q}{2}, \frac{1+q}{2}] | \mu \leq \mu^* \text{ and } \beta(\mu) = 0\}$  the set of evaluations without communication below  $\mu^*$ . By assumption,  $\Pr(A^K), \Pr(A^N) > 0$ . Then rewrite program C in the following way. Change constraint (3.10) to

$$\frac{1}{2q} f(\hat{\mu}(\mu)|e) (1 - \beta(\mu)) (w(\mu) - w)^2 \leq 0 \quad (3.12)$$

for all  $\mu$ . (3.12) is equivalent to (3.10), but simplifies the next steps of the proof. This condition guarantees that the wage is constant outside the communication set, in particular, in  $A^N$ . In the communication set and thus in the subset  $A^K$ , condition (3.12) is trivially fulfilled, as  $1 - \beta(\mu) = 0$ . Furthermore, multiply constraint (3.11) by  $f(\hat{\mu}(\mu)|e)/q$  for all  $\mu$  to get

$$\frac{1}{q} f(\hat{\mu}(\mu)|e) (w(\mu) + \kappa\beta(\mu) - w) \leq 0. \quad (3.13)$$

(3.13) is equivalent to (3.11) and guarantees that it is optimal to communicate if communication is required by the contract, i.e.,  $\beta(\mu) = 1$ . Together, both constraints make communication optimal, whenever  $\beta(\mu) = 1$  and vice versa. In addition, they ensure that the wage has to be lower in  $A^K$  than in  $A^N$ , as

$$w(\mu) < w = w(\mu') \quad \forall \mu \in A^K, \mu' \in A^N. \quad (3.14)$$

The inequality follows from condition (3.13), while the equality is given by condition (3.12). Define  $\lambda_1, \lambda_2, \nu_1(\mu)$  and  $\nu_2(\mu)$  to be the Lagrange multipliers of the constraints (PC), (IC<sub>A</sub>), (3.12) and (3.13) respectively. Pointwise optimization<sup>19</sup> with respect to  $w(\mu)$  yields

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<sup>19</sup>Cf. footnote 17 for the generality of this method.

$$\begin{aligned}
 1 - u'(w(\mu)) \left( \lambda_1 + \lambda_2 \frac{f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu))}{f(\hat{\mu}(\mu)|e)} \right) + \nu_1(\mu)(1 - \beta(\mu))(w(\mu) - w) + \nu_2(\mu) &= 0 \\
 \Leftrightarrow 1 + \nu_2(\mu) = u'(w(\mu)) \left( \lambda_1 + \lambda_2 \frac{f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu))}{f(\hat{\mu}(\mu)|e)} \right), & \quad \text{a.s.} \tag{3.15}
 \end{aligned}$$

because constraint (3.12) guarantees  $(1 - \beta(\mu))(w(\mu) - w) = 0$  for all  $\mu$ .  $\hat{\mu}(\mu)$  is increasing in  $\mu$  and the monotone likelihood ratio property ensures that

$$\frac{f^H(\mu) - f^L(\mu)}{f(\mu|e)} = \frac{x(\mu) - 1}{ex(\mu) + 1 - e} \quad \text{with } x(\mu) = \frac{f^H(\mu)}{f^L(\mu)}$$

is also increasing in  $\mu$ . Together with (3.14) this proves that the right-hand side of equation (3.15) is higher for  $\mu \in A^K$  than for  $\mu' \in A^N$ , because  $\mu' < \mu$ . To match this increase,  $\nu_2(\mu)$  almost surely has to be positive in  $A^K$ . This results in  $w(\mu) = w - \kappa$  almost surely for  $\mu \in A^K$ .

The next step modifies the wage contract to implement effort  $e$  cheaper than before. For this purpose, determine the median  $\alpha$  of  $A^K$ , such that

$$\int_{A_1^K} f(\hat{\mu}(\mu)|e) d\mu = \int_{A_2^K} f(\hat{\mu}(\mu)|e) d\mu \tag{3.16}$$

for  $A_1^K = A^K \cap [\mu^*, \alpha)$  and  $A_2^K = A^K \cap [\alpha, 1]$ . For the elements of  $A^K$  above  $\alpha$ , the principal abstains from communication in the new contract,  $\beta(\mu) = 0$ , while below  $\alpha$ , there is communication as before. Then, I add (resp. subtract) the monetary equivalent of  $\hat{u} = u(w) - u(w - \kappa)$  to the wage, so that

$$w'(\mu) = \begin{cases} u^{-1}(u(w - \kappa) + \hat{u}) = w & \text{for } \mu \in A_2^K \\ u^{-1}(u(w - \kappa) - \hat{u}) = u^{-1}(2u(w - \kappa) - u(w)) & \text{for } \mu \in A_1^K. \end{cases} \tag{3.17}$$

This means that the wage increases by  $\kappa$  for values in  $A^K$  above  $\alpha$  and decreases by an amount adjusted for the changes in marginal utility below  $\alpha$ . The modification of the wage still satisfies the condition (PC). Additionally, the left-hand side of condition (IC<sub>A</sub>) is now strictly bigger than the marginal cost of effort,  $d'(e)$ , because the difference in the condition (IC<sub>A</sub>) equals



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$$\begin{aligned}
& \hat{u} \left( - \int_{A_1^K} \frac{f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu))}{q} d\mu + \int_{A_2^K} \frac{f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu))}{q} d\mu \right) = \\
& = \hat{u} \left( - \int_{A_1^K} \frac{f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu))}{qf(\hat{\mu}(\mu)|e)} f(\hat{\mu}(\mu)|e) d\mu + \right. \\
& \quad \left. + \int_{A_2^K} \frac{f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu))}{qf(\hat{\mu}(\mu)|e)} f(\hat{\mu}(\mu)|e) d\mu \right) > \\
& > \hat{u} \frac{f^H(\alpha) - f^L(\alpha)}{qf(\alpha|e)} \left( - \int_{A_1^K} f(\hat{\mu}(\mu)|e) d\mu + \int_{A_2^K} f(\hat{\mu}(\mu)|e) d\mu \right) = 0.
\end{aligned}$$

The last equality follows from the definition of  $\alpha$  in (3.16). The main inequality follows from the increasing likelihood ratio. By reducing the variance of the wage payments until condition (IC<sub>A</sub>) holds with equality, condition (PC) becomes slack. This allows decreasing the wage and shows a contradiction to the existence of  $\mu^*$ . Therefore the optimal contract does not require communication of good evaluations, as the agent is pleased to accept these evaluation results. On the other hand, the principal has to justify bad evaluations. The set  $A^K$  can be replaced by any set with communication and a wage of  $w - \kappa$ . Thus, in the optimal contract any set with these properties has no mass. These sets have no influence on the optimal contract. Hence, the wage and communication pattern can be adjusted according to the proposition. Combining these results with Program C, allows me to prove analogously to Lemma 3.1 that  $w(\mu)$  is strictly increasing in  $\mu \in [\frac{1-q}{2}, \delta]$ .  $\square$

**Proof of Proposition 3.4:** The proof proceeds along the following lines. If  $e > 0$  and there is almost surely no communication, e.g., due to high communication costs  $\kappa$ , the optimal contract does not exist.<sup>20</sup> Yet, for every  $\epsilon$  there is a contract whose costs are at most  $\epsilon$  higher than the infimum costs to implement effort  $e > 0$  by the agent. Finally, the proof will show that condition (3.2) ensures that in the optimal contract there is communication with positive probability.

According to Lemma 3.2, the principal's payments have to be constant, as it is impossible to verify her message  $m_P$ . Denote her payments by  $\bar{w}$ . The next steps calculate the infimum costs to implement positive effort  $e > 0$  of the agent. For this purpose, the set of feasible contracts is reduced step by step, as I show that contracts

<sup>20</sup>There is an optimal contract if the distributions  $F^H$  and  $F^L$  have atoms at 0.

with specific characteristics are suboptimal. First consider a contract with  $c(\mu) < \bar{w}$  almost surely. In this case, it is possible to reduce the principal's payments  $\bar{w}$  without violating any constraint. Therefore I only have to take contracts into account with a positive probability for  $\{\mu \in [\frac{1-q}{2}, \frac{1+q}{2}] | c(\mu) = \bar{w}\}$ .

Second, consider an optimal contract  $W$  with a  $\mu^*$ , such that there are payments to a third party for  $\mu > \mu^*$  with positive probability, but with positive probability there are no such payments for  $\mu \leq \mu^*$ . Denote the corresponding sets by  $A^S = \{\mu \in [\frac{1-q}{2}, \frac{1+q}{2}] | \mu > \mu^* \text{ and } c(\mu) < \bar{w}\}$  with third-party payments above  $\mu^*$  and by  $A^D = \{\mu \in [\frac{1-q}{2}, \frac{1+q}{2}] | \mu \leq \mu^* \text{ and } c(\mu) = \bar{w}\}$  the set of evaluations without such payments below  $\mu^*$ . By assumption  $\int_{A^S} f(\hat{\mu}(\mu)|e)d\mu, \int_{A^D} f(\hat{\mu}(\mu)|e)d\mu > 0$ . Now reduce the bigger set, until both sets have the same mass,  $\int_{A^S} f(\hat{\mu}(\mu)|e)d\mu = \int_{A^D} f(\hat{\mu}(\mu)|e)d\mu$ . In the next step, I modify the wage scheme to

$$c'(\mu) = \begin{cases} u^{-1} \left( \frac{1}{\int_{A^D} f(\hat{\mu}(\mu)|e)d\mu} \int_{A^S} u(c(\mu)) f(\hat{\mu}(\mu)|e)d\mu \right) & \text{for } \mu \in A^D \\ \bar{w} & \text{for } \mu \in A^S \\ c(\mu) & \text{otherwise.} \end{cases} \quad (3.18)$$

On  $A^D$  the agent's wage is reduced to the average wage on  $A^S$  in contract  $W$ , while on  $A^D$  the wage increases to  $\bar{w}$ . Otherwise the wage scheme remains unchanged. In order to check whether this contract is feasible, I analyze the remaining constraints (PC) and  $(IC_A)$ . By the definition of the wage modification  $W'$  in (3.18), the agent's participation constraint (PC) still holds. On the other hand, the difference in the agent's incentive compatibility  $(IC_A)$  is positive, as

$$\begin{aligned} & \int_{A^D} \underbrace{(u(c'(\mu)) - u(\bar{w}))}_{<0} \Delta^f(\mu) d\mu + \int_{A^S} \underbrace{(u(\bar{w}) - u(c(\mu)))}_{>0} \Delta^f(\mu) d\mu > \\ & > \frac{\Delta^f(\mu^*)}{f(\hat{\mu}(\mu^*)|e)} \left( \int_{A^D} u(c'(\mu)) f(\hat{\mu}(\mu)|e) d\mu - \int_{A^S} u(c(\mu)) f(\hat{\mu}(\mu)|e) d\mu \right) = 0. \end{aligned}$$

with  $\Delta^f(\mu) = (f^H(\hat{\mu}(\mu)) - f^L(\hat{\mu}(\mu)))/q$ . The monotone likelihood ratio property ensures the strict inequality and the constant value of the participation constraint yields the equality. Therefore the modified wage  $W'$  in (3.18) makes the left-hand side of condition  $(IC_A)$  strictly bigger than the marginal cost of effort,  $d'(e)$ . In addition,

### 3.C THE OPTIMAL COMMUNICATION PATTERN

the contract satisfies condition (PC). By reducing the variance of the wage payments, until condition (IC<sub>A</sub>) holds with equality, condition (PC) becomes slack. This allows decreasing the wage. Consequently, restrict attention to contracts with a  $\delta < \frac{1+q}{2}$ , such that  $c(\mu) = \bar{w}$  for all  $\mu > \delta$ .

Third, consider such a contract  $W$  with  $\hat{\delta}$ . To satisfy the agent's incentive compatibility, there has to be a payment to a third party, i.e.,  $c(\mu) < \bar{w}$ , with positive probability. Denote the corresponding set by  $A^S = \{\mu \in [\frac{1-q}{2}, \frac{1+q}{2}] | c(\mu) < \bar{w}\}$  and its median by  $\alpha$ . By the previous remarks, these values are below  $\hat{\delta}$ , i.e.,  $\mu \leq \hat{\delta}$  for all  $\mu \in A^S$ . Now, modify the wage scheme to

$$c''(\mu) = \begin{cases} u^{-1}(\hat{u}(\mu)) & \text{for } \mu \leq \alpha \text{ and } \mu \in A^S \\ \bar{w} & \text{otherwise} \end{cases}$$

$$\hat{u}(\mu) = u(c(\mu)) - u(\bar{w}) + \frac{2}{\int_{A^S} f(\hat{\mu}(\bar{\mu})|e)d\bar{\mu}} \int_{\bar{\mu} \in A^S \text{ and } \bar{\mu} > \alpha} u(c(\bar{\mu}))f(\hat{\mu}(\bar{\mu})|e)d\bar{\mu}.$$

For evaluations above the median  $\alpha$ , the agent's wage increases to  $\bar{w}$ , while below the median the wage is reduced to balance the utility gain above the median. Otherwise, the wage scheme remains unchanged. The proof is now analogous to the last case. Therefore it is possible to improve the contract, as long as  $\delta > 0$ . Yet the contract with a constant wage for the agent and  $\delta = 0$  does not satisfy the agent's incentive compatibility. Therefore an optimal contract does not exist and I have to consider a sequence of contracts.

For this purpose, construct a sequence of feasible contracts that satisfy the properties derived in this proof. For those contracts, I derive upper and lower bounds for their costs. Consider the following contracts for a small  $\delta > 0$ :  $W_\delta(m_P, m_A) = \bar{w}(\delta)$  and

$$c_\delta(m_P, m_A) = \begin{cases} \bar{w}(\delta) & \text{if } \int m_P(t)dt > \delta \\ \bar{w}(\delta) - \Lambda_\delta(\int m_P(t)dt) & \text{if } \int m_P(t)dt \leq \delta, \end{cases}$$

with  $\bar{w}(\delta)$  and third-party payments  $\Lambda_\delta(\mu)$  such that the agent's incentive compatibility and his participation constraint are satisfied. This requires

$$\int_{(1-q)/2}^{\delta} u(c_{\delta}(\mu)) \frac{1}{q} f(\hat{\mu}(\mu)|e) d\mu + (1 - F(\hat{\mu}(\delta)|e)) u(\bar{w}(\delta)) = \bar{u} + d(e) \quad (\text{PC})$$

$$\int_{(1-q)/2}^{\delta} u(c_{\delta}(\mu)) \Delta^f(\mu) d\mu - u(\bar{w}(\delta)) \int_{(1-q)/2}^{\delta} \Delta^f(\mu) d\mu = d'(e) \quad (\text{IC}_A)$$

as  $\int_{\delta}^{(1+q)/2} \Delta^f(\mu) d\mu = -\int_{(1-q)/2}^{\delta} \Delta^f(\mu) d\mu$ . The constraint (IC<sub>A</sub>) implies  $u(c_{\delta}(\mu)) < 0$  for all  $\mu \leq \delta$  and  $\delta$  sufficiently small, because  $\int_{(1-q)/2}^{\delta} \Delta^f(\mu) d\mu \rightarrow 0$  for  $\delta \rightarrow (1-q)/2$  and  $\Delta^f(\mu) < 0$  for all  $\mu \leq \delta$ . Rearranging the agent's incentive compatibility and approximating it from above results in

$$d'(e) \leq -u(\bar{w}(\delta)) \int_{(1-q)/2}^{\delta} \Delta^f(\mu) d\mu + \frac{f^H(0) - f^L(0)}{qf(0|e)} \int_{(1-q)/2}^{\delta} u(c_{\delta}(\mu)) f(\hat{\mu}(\mu)|e) d\mu.$$

In the next step, insert the participation constraint for  $\int_{(1-q)/2}^{\delta} u(c_{\delta}(\mu)) f(\hat{\mu}(\mu)|e) d\mu$  to get

$$u(\bar{w}(\delta)) \geq \frac{d'(e) + \frac{f^L(0) - f^H(0)}{f(0|e)} (\bar{u} + d(e))}{-\int_{(1-q)/2}^{\delta} \Delta^f(\mu) d\mu + \frac{f^L(0) - f^H(0)}{f(0|e)} (1 - F(\hat{\delta}|e))}$$

as a lower bound or

$$u(\bar{w}(\delta)) \leq \frac{d'(e) + \frac{f^L(\hat{\delta}) - f^H(\hat{\delta})}{f(\hat{\delta}|e)} (\bar{u} + d(e))}{-\int_{(1-q)/2}^{\delta} \Delta^f(\mu) d\mu + \frac{f^L(\hat{\delta}) - f^H(\hat{\delta})}{f(\hat{\delta}|e)} (1 - F(\hat{\delta}|e))}$$

as an upper bound with  $\hat{\delta} = \hat{\mu}(\delta)$ . For  $\delta \rightarrow (1-q)/2$ , both bounds converge to

$$\bar{u} + d(e) + \frac{f(0|e)}{f^L(0) - f^H(0)} d'(e).$$

This also coincides with the optimal contract for the case of atoms at 0, which guarantee existence of an optimal contract. To ensure that the set of these contracts  $W_{\delta}$  is nonempty, consider the following contract:

$$\bar{w}(\delta) = u^{-1} \left( \bar{u} + d(e) + \frac{F(\hat{\delta}|e)}{-\int_{(1-q)/2}^{\delta} \Delta^f(\mu) d\mu} d'(e) \right) \quad \text{and}$$

$$u(\bar{w}(\delta) + \Lambda_{\delta}(\bar{\mu})) = \bar{u} + d(e) - \frac{1 - F(\hat{\delta}|e)}{-\int_{(1-q)/2}^{\delta} \Delta^f(\mu) d\mu} d'(e), \quad \forall \bar{\mu} \leq \delta.$$

### 3.D BUDGET-BALANCED CONTRACTS

This contract satisfies all the properties derived in this proof as well as constraints (PC) and (IC<sub>A</sub>) with equality. The costs of this contract are  $\bar{w}(\delta)$  which by l'Hôpital's rule converges to  $C^0(e) = u^{-1}\left(\bar{u} + d(e) + \frac{f'(0|e)}{f^L(0) - f^H(0)}d'(e)\right)$  for  $\delta \rightarrow (1 - q)/2$ . The convergence is also monotone in the costs of the contract, because  $\bar{w}(\delta)$  is strictly increasing in  $\delta$ . As the densities are continuous, for every  $\epsilon > 0$  there exists a  $\delta' > 0$ , such that the costs of the contract  $W_\delta$  for all  $(1 - q)/2 < \delta \leq \delta'$  are lower than  $C^0(e) + \epsilon$ . Yet it is impossible to approximate first-best, because  $C^0(e) > u^{-1}(\bar{u} + d(e))$ .

For the final step of the proof, compare the principal's costs with and without communication. If communication happens with probability 0 and  $\Pr(\mathcal{I}_C) = 0$ , the principal's costs are  $C^0(e)$ . On the other hand, the expected wage costs are

$$C^*(e) = \int w^*(\mu, e) dF(\hat{\mu}(\mu)|e)$$

with communication almost surely,  $\Pr(\mathcal{I}_C) = 1$ , as shown in Proposition 3.1 in combination with Proposition 3.2. Then the optimal contract implements the second-best benchmark wage defined by Program A\*. It is now possible to show that assumption (3.2) is feasible, which is equivalent to the difference between  $C^0(e)$  and  $C^*(e)$  being positive. Neglecting the communication costs, the costs for implementing the communication contract  $\bar{W}$  are lower than for any contract without communication. The reason is the informativeness principle of Holmström (1979). This proves that condition (3.2) is indeed feasible and provides a lower bound for the savings, i.e.,  $C^c(e) - C^0(e)$ , which are possible with communication. Hence, as long as the communication costs are lower than this bound, there will be communication with positive probability.  $\square$

## 3.D Budget-Balanced Contracts

It is possible to make the optimal contract ex-post budget-balanced.

**Proof of Proposition 3.5:** In order to capture stochastic payments, change the interpretation of the notation. Now  $w(m_P, m_A)$  denotes the expected wage after the parties sent messages  $m_P$  and  $m_A$ . On the other hand,  $c(m_P, m_A)$  is the agent's certainty equivalent of the wage payment. Finally, lotteries with the corresponding mean and certainty equivalent will be specified. This formalization captures any

## CHAPTER 3

stochastic payment without loss of generality. The principal could pay the agent a lottery or could discard certain messages with some probability by ‘turning a blind eye’.<sup>21</sup>

Program A still describes the problem. Yet constraint (3.1) now captures the agent’s risk aversion. Proposition 3.2 states the solution to Program A. The solution  $W^{**}$  gives the principal and the agent the same utilities as the contract  $\bar{W}^{**}$  in Proposition 3.5 if the lotteries are chosen accordingly. For the principal this is obvious, as the lotteries have mean  $\kappa$ . The agent’s expected utility also remains unchanged, because the certainty equivalent equals his former wage.

Finally, specify a lottery with the desired properties. Let  $\Lambda(z)$  denote a lottery that pays  $\kappa + z$  and  $\kappa - z$  with probability  $1/2$ , respectively.<sup>22</sup>  $z(\mu)$  is determined, such that

$$\mathbb{E}u(w^{**} + \Lambda(z(\mu))) = \frac{1}{2}u(w^{**} + \kappa + z(\mu)) + \frac{1}{2}u(w^{**} + \kappa - z(\mu)) = u(w^{**}(\mu)).$$

It is possible to find such a  $z(\mu)$  and  $z(\mu)$  is unique for every  $\mu$  due to the strict concavity of  $u$ . Adding the lottery does not change the principal’s expected payments, but reduces the agent’s certainty equivalent. The mean preserving spread introduced by the lottery is the reason for this loss of utility. Once the lottery is realized, the party who gains in the lottery has an incentive to avoid renegotiations. Therefore the lottery ought to be realized as soon as the messages are available in order to make the contract renegotiation-proof.<sup>23</sup>  $\square$

<sup>21</sup>This is optimal for example in Herweg et al. (2010).

<sup>22</sup>If the agent has limited liability, the lottery  $\Lambda(z)$  can be redefined to limit the negative realization, but to assign it a higher probability.

<sup>23</sup>Rasmusen (1987) and Maskin (2002) provide a good discussion of this problem.

# Distinguishing First-Order and Second-Order Ambiguity Aversion

## 4.1 Introduction

Following the seminal contribution by Schmeidler (1989), there have been many representations of ambiguity-averse preferences. Recently, Cerreia-Vioglio et al. (2011) have axiomatized a general representation encompassing a wide set of convex preferences, in particular, Maxmin-Expected Utility (MEU), Choquet-Expected Utility (CEU), Smooth Ambiguity Aversion (KMM), Variational and Multiplier Preferences. Although all these models can now be traced to the same axioms, there is an important distinction when it comes to economic applications. Consider a contingent wealth space with two states of the world and the indifference curves depicted in Figure 4.1. The indifference curves can be kinked or smooth along the 45-degree line. I denote the former behavior first-order ambiguity aversion, and the latter second-order ambiguity aversion. This chapter clarifies the distinction and shows that it is relevant for behavior, like the distinction between first-order and second-order risk aversion by Segal and Spivak (1990). Second-order risk aversion implies approximately risk-neutral behavior, when small risks are concerned.<sup>1</sup> First-order risk

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<sup>1</sup>The Arrow-Pratt risk premium is approximately zero if the stakes are small. This risk premium is the transfer that makes the agent indifferent between a risky gamble with zero mean and a sure zero payoff.

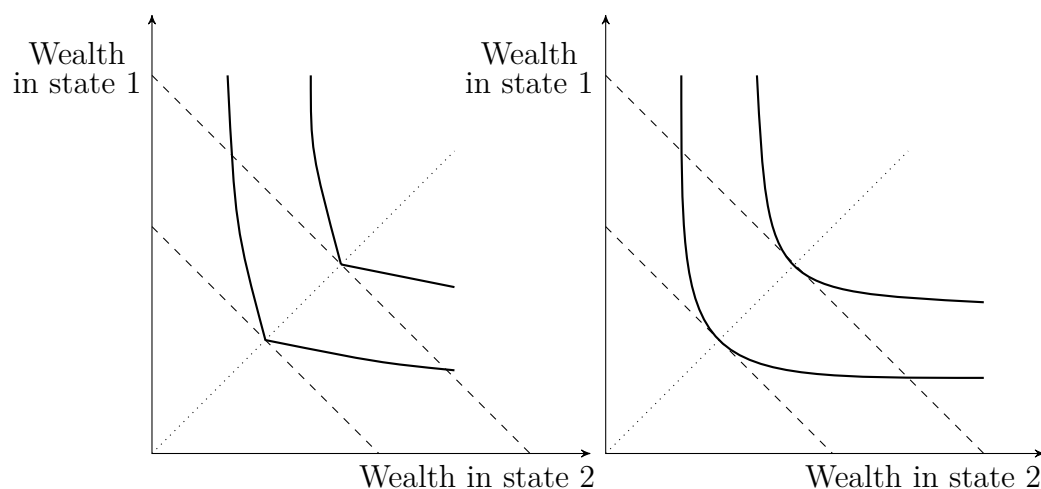


Figure 4.1: The Distinction between First-Order and Second-Order Ambiguity Aversion

aversion allows for risk-averse behavior, even if the stakes are small. Correspondingly, I define first-order and second-order ambiguity aversion. Some representations with ambiguity aversion implement first-order ambiguity aversion, such as MEU, CEU, or Multiplier Preferences, while others feature second-order ambiguity aversion, like KMM.

Consider an ambiguity-averse agent Anna choosing between an ambiguous act and an unambiguous endowment. In the case of second-order ambiguity aversion, denote her by  $Anna_2$ , while  $Anna_1$  exhibits first-order ambiguity aversion. Suppose the stakes of the ambiguous act are sufficiently small. Then there exists an ambiguity-neutral agent Nathan, so that  $Anna_2$  chooses the ambiguous act if and only if Nathan strictly prefers the ambiguous act to the endowment. Conversely, for every ambiguity-neutral Nathan there are some acts that  $Anna_1$  rejects, but Nathan strictly prefers, no matter how small the stakes are. This difference is relevant in many applications. First, in an investment setting  $Anna_2$  invests (at least a small amount) in an asset if and only if Nathan invests in the asset. Moreover, if every asset has the same price and short selling is possible,  $Anna_2$  generically buys or sells some amount of every asset. Yet, there are assets that  $Anna_1$  does not want to buy or sell. Second, consider an agent that can buy any amount of insurance coverage at a constant premium per unit of indemnity.  $Anna_2$  demands full insurance coverage if and only if Nathan



does so. Nathan's behavior, meanwhile, is characterized by Mossin's (1968) Theorem.  $\text{Anna}_1$ , however, may demand full insurance coverage, even if Nathan demands a lower coverage. Third, in contracting, first-order ambiguity aversion could make the contracting parties abstain from conditioning the contract on informative signals. Sometimes even constant payments without incentives are optimal. This is impossible with second-order ambiguity aversion, because an adjusted version of Holmström's (1979) informativeness principle is valid in this case.

The rest of the chapter is organized as follows. Section 4.2 defines the distinction between second-order and first-order ambiguity aversion. In addition, it introduces a general ambiguity premium and the reference beliefs of an ambiguity-averse agent. Then, Section 4.3 explores the consequences of this distinction for the willingness to participate in ambiguous acts under ambiguity aversion and discusses some applications of this result. After that, Section 4.4 gives some extensions of the chapter. Section 4.5 discusses the related literature. Finally, Section 4.6 contains the concluding remarks. Appendix 4.A provides closed-form solutions of a principal-agent model in the KMM representation. Technical details regarding decision-making under ambiguity and the distinction between first-order and second-order risk aversion are found in the appendix to this chapter.

## 4.2 Defining the Distinction

This section provides three equivalent characterizations for the distinction between first-order and second-order ambiguity aversion. These definitions are independent of any specific representation.

### 4.2.1 Decision-Theoretic Framework

Consider an agent with ambiguity-averse preferences. I operate on an Anscombe and Aumann (1963) domain with a finite state space  $\Omega$ , an algebra  $\Sigma$  containing subsets of  $\Omega$ , a set of consequences  $X = \mathbb{R}$  and the set of simple lotteries over consequences  $\Delta X$ . An act in this framework is a measurable mapping from the state

space into the simple lotteries,  $\Omega \rightarrow \Delta X$ .<sup>2</sup> Denote by  $\mathcal{F}$  the set of all acts. Notice that any constant act is unambiguous in the sense that the act yields the same simple lottery  $l \in \Delta X$  in every state of the world,  $\omega \mapsto l$  for all  $\omega \in \Omega$ .

As in Cerreia-Vioglio et al. (2011), assume that the agent's preferences satisfy the axioms of weak order, monotonicity, convexity, risk independence, and continuity. The convexity axiom ensures that the preferences are ambiguity averse. Accordingly, the agent (weakly) prefers any mixture of acts to the acts themselves if she is indifferent between the acts. The reason is that the mixture allows the agent to hedge some of the ambiguity about the state of the world. Additionally, these axioms ensure the existence of a certainty equivalent for any simple lottery. See Maccheroni et al. (2006, Lemma 28) for a proof. Thus, it is possible to replace the simple lotteries by their certainty equivalents and interpret an act as a mapping  $\Omega \rightarrow X$ . Consequently, there is a representing utility function  $U: X^\Omega \rightarrow \mathbb{R}$  that is continuous and quasi-concave.  $X^\Omega$  denotes the set of all functions  $\Omega \rightarrow X$  and  $X^\Omega$  is homeomorphic to  $\mathbb{R}^{|\Omega|}$ . The upper contour set of  $U$  at  $f_0$  is denoted  $B(f_0) = \{f \in X^\Omega | U(f) \geq U(f_0)\}$ . The set  $B(f_0)$  is nonempty, compact and convex for all  $f_0$ . Finally, the axioms guarantee that the agent's preferences over constant acts can be represented by a von-Neumann-Morgenstern utility index  $u: X \rightarrow \mathbb{R}$ . I assume the utility index to be differentiable.

$\mathbf{1}$  denotes the vector  $(1, 1, \dots, 1)$  and  $\|x\|$  denotes the Euclidean norm of a vector  $x$ . The notation  $\epsilon f$  means that the payoffs of an act  $f$  are rescaled by a factor  $\epsilon \in \mathbb{R}$ .

### 4.2.2 Definitions

The first definition is based on the differentiability of the indifference curves in the contingent wealth space in which simple lotteries are represented by their certainty equivalents. This definition follows the distinction in Figure 4.1 in the introduction. For this purpose, define a notion of differentiability of an indifference surface. An indifference surface  $I$  is *differentiable* at an act  $f$  if there is a function  $\eta: \mathbb{R}^{|\Omega|} \rightarrow \mathbb{R}$ , so that  $I$  is the level set of  $\eta$  at the value  $\eta(f)$  and  $\eta$  is differentiable at  $f$  or if there

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<sup>2</sup>Simple lotteries with a finite number of outcomes are written as vectors of the form  $(x_1, p_1; \dots; x_n, p_n)$ . In a common abuse of notation, I denote degenerate lotteries by their outcome  $x$  instead of  $(x, 1)$ .

is an  $\epsilon > 0$  with  $f \sim f - \epsilon \mathbf{1}$ . If an indifference surface is not differentiable at  $f$ , I call it kinked at  $f$ .

**Definition 4.1.** Ambiguity aversion is *second order*<sub>(1)</sub> at a wealth level if the indifference surface is differentiable at the constant act with this wealth level. *First-order*<sub>(1)</sub> ambiguity aversion features a kinked indifference surface for the constant act at this wealth level.

The second definition is based on the agent's reference beliefs that will be defined below. As a motivation consider the following comparative statics with respect to ambiguity aversion. The aim is to compare the behavior of an ambiguity-averse agent with that of an ambiguity-neutral agent. For this purpose, the preferences of the ambiguity-neutral agent should be similar to the preferences of the ambiguity-averse agent. I define similarity of preferences in terms of improvement directions. For any act  $f \in \mathcal{F}$ , an *improvement direction*  $d$  at  $f$  is an act  $d \in \mathcal{F}$  such that there is an  $\epsilon > 0$  with  $f + \epsilon d \succ f$ .<sup>3</sup> Denote the set of all improvement directions at  $f$  by  $D(f)$ . Thus, I match the ambiguity-averse preferences  $\preceq$  with ambiguity-neutral preferences  $\preceq_N$  approximating the initial preferences in a neighborhood of the constant act  $f$  in the sense that  $D_{\preceq}(f) \subseteq D_{\preceq_N}(f)$ . I construct these ambiguity-neutral preferences in two steps. First, the certainty independence axiom ensures that the agent's preferences restricted to the constant acts can be represented by a von-Neumann-Morgenstern utility index. Use this utility index for the corresponding ambiguity-neutral preferences. Second, I define an object to be interpreted as a set of beliefs of the ambiguity-averse agent and show that it shares many properties with similar notions in the literature, like the subjective beliefs by Rigotti et al. (2008) or the plausible priors by Siniscalchi (2006). For this purpose, consider an act  $f_0$  and the function  $W_{f_0}: X^{|\Omega|} \rightarrow \mathbb{R}$  with

$$W_{f_0}(f) = U(f_0) - \inf \left\{ \|f - g\| \mid g \in B(f_0) \right\}.$$

Define the *reference beliefs* for a wealth level  $w_0$  as the projection to the  $|\Omega|$ -dimensional

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<sup>3</sup>Rubinstein (2006, p. 60) introduces a concept of differentiability of consumer preferences in terms of improvement directions.

simplex of the subdifferential of  $W_{w_0\mathbf{1}}$  at the constant act  $w_0\mathbf{1}$ .<sup>4</sup> Lemma 4.1 ensures the existence of these reference beliefs, as the function  $W_{f_0}$  is well-behaved and preserves the upper contour set of  $U$  at  $f_0$ .

**Lemma 4.1.** *For all  $w_0 \in \mathbb{R}$ , the reference beliefs at  $w_0$  are a nonempty set.*

**Proof:** The set  $B(f_0)$  is convex, because it is the upper contour set of the quasi-concave function  $U$ . The distance function of a convex set is convex. Therefore  $W_{w_0\mathbf{1}}$  is concave. In addition,  $W_{w_0\mathbf{1}}(f)$  is finite for all  $f$ . Then Rockafellar (1970, Theorem 23.4) ensures that the subdifferential is a nonempty set.  $\square$

Intuitively, the reference beliefs are elicited from the marginal rate of substitution between states evaluated at the constant acts. This follows the approach of de Finetti (1937), Ramsey (1931), and Savage (1954), identifying the decision maker's beliefs as the odds at which she is willing to make small bets.<sup>5</sup> Accordingly, the reference beliefs of an subjective expected-utility maximizer are her prior beliefs. As a motivating example, consider the KMM representation.<sup>6</sup> There is a set  $\Pi$  that contains the possible first-order probability measures  $P$  on the state space  $\Omega$ . On the other hand,  $\mu$  denotes a second-order probability measure defined on the set of priors  $\Pi$ . In addition, there is an ambiguity index  $\phi$  and a von-Neumann-Morgenstern utility index  $u$ . See Appendix 4.C for the details of this representation. The representation has the following form

$$\int_{\Pi} \phi \left( \int_{\Omega} u \circ f dP \right) d\mu.$$

In the KMM representation, the uniquely determined reference belief equals the expected probabilities of the states of the world, i.e.,  $\mathbb{E}_{\mu}(P)$ , the expected value of the first-order probability measure  $P$  under the second-order measure  $\mu$ . A proof follows

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<sup>4</sup>The subdifferential at point  $f_0$  is the convex set of all subgradients  $\partial_i$  at  $f_0$ :  $W_{f_0}(f) \leq W_{f_0}(f_0) + \langle \partial_i, f - f_0 \rangle$  for all  $f$ . (Rockafellar, 1970, p. 308) The projection to the simplex normalizes each subgradient, a vector in  $\mathbb{R}^{|\Omega|}$ , to a length of 1 according to the  $L_1$  norm.  $\mathbf{0}$  is projected to another normalized subgradient  $\partial_i$  if there is a  $\partial_i \neq \mathbf{0}$ , and otherwise to  $\frac{1}{|\Omega|}\mathbf{1}$ . It is impossible to consider the subdifferential of  $U$ , because  $U$  is only quasi-concave. Even using a generalized subdifferential, like Aussel et al. (1994), Clarke (1975), Crouzeix (1981), or Kruger (2003), does not yield the desired results.

<sup>5</sup>In the absence of ambiguity, Yaari (1969) formalizes these considerations.

<sup>6</sup>Similar representations are Seo (2009), Ergin and Gul (2009), Chew and Sagi (2008), and Nau (2006).

after Proposition 4.2. As a final example, consider MEU preferences. Then the set of reference beliefs equals the convex hull of the set of priors in the representation.

Related notions in the literature are the relevant beliefs by Klibanoff et al. (2011), the subjective beliefs by Rigotti et al. (2008), the plausible priors by Siniscalchi (2006), and the uncertainty-free beliefs by Epstein (1999).<sup>7</sup> These notions compare in the following way. If uncertainty-free beliefs exist, the set of reference beliefs equals the set of *uncertainty-free beliefs* for MEU, CEU and KMM preferences. In addition, the set of reference beliefs is the convex hull of the *plausible priors* for MEU and CEU preferences. The set of reference beliefs coincides with the set of *subjective beliefs* at the constant acts. Finally, the set of reference beliefs equals the set of *relevant beliefs* for MEU and CEU preferences. For KMM preferences, however, the uniquely determined reference belief is the average of the *relevant beliefs* weighted by the second-order measure  $\mu$ . The reference beliefs defined above serve a reference purpose, as they allow calculating a subjective expectation. Yet, here I am more interested in their uniqueness.

**Definition 4.2.** Ambiguity aversion is *second order*<sub>(II)</sub> at a wealth level if there is a unique reference belief for this wealth level. If there are several reference beliefs for the wealth level, ambiguity aversion is *first order*<sub>(II)</sub> there.

The third definition relies on the ambiguity premium of an ambiguous act and makes the distinction depending on its limit for small payoffs. The challenge lies in disentangling the risk premium and the ambiguity premium. Therefore it is impossible to follow the approach by Pratt (1964) directly. The idea is to use a benchmark lottery that captures the risk and to define the ambiguity premium as the transfer that makes an ambiguity-averse person indifferent between this benchmark lottery and the ambiguous act in combination with the transfer.

For this purpose, consider a wealth level  $w_0$ , a reference belief  $\nu$  at  $w_0$  and an ambiguous act  $f_A \in \mathcal{F}$ . Now construct the constant act  $f_U: \Omega \mapsto l_U$  with the lottery  $l_U \in \Delta X$ , such that the reference belief  $\nu$  assigns any prize  $x \in X$  the same probabilities under  $f_A$  and  $l_U$ , i.e.,  $l_U(x) = \int_{\Omega} f_A(\omega|x) d\nu(\omega)$  for all  $x \in X$ . Thus, any ambiguity-neutral agent with the reference belief  $\nu$  is indifferent between both acts.

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<sup>7</sup>See Section 4.5 for details and a discussion.

Define the ambiguity premium as the transfer  $\pi_A^\nu: \Omega \mapsto \bar{\pi}_A^\nu$  with  $\bar{\pi}_A^\nu \in X$ , such that at the wealth level  $w_0$  it makes the ambiguity-averse agent indifferent between her endowment and the act  $f_A - f_U$  in combination with the transfer  $\pi_A^\nu$ .<sup>8</sup>

$$w_0 \mathbf{1} \sim w_0 \mathbf{1} + \pi_A^\nu + f_A - f_U$$

The benchmark lottery  $f_U$  ensures that the ambiguity premium does not cover the risk, but only captures the ambiguity. Yet  $f_U$  depends on the chosen reference belief  $\nu$ . Therefore the ambiguity premium  $\pi_A^\nu$  also depends on  $\nu$ . To vary the amount of ambiguity, define the ambiguity premium  $\pi_A^\nu(t)$  as a function of a parameter  $t \in [0, 1]$ , so that

$$w_0 \mathbf{1} \sim w_0 \mathbf{1} + \pi_A^\nu(t) + t(f_A - f_U). \quad (4.1)$$

The ambiguity premium vanishes for  $t \rightarrow 0$ , as the ambiguity disappears. In order to distinguish between first-order and second-order ambiguity aversion, consider the following limit

$$L = \lim_{t \rightarrow 0} \frac{\pi_A^\nu(t)}{t}.^9$$

**Definition 4.3.** If  $L = 0$  for a reference belief  $\nu$  at a wealth level  $w_0$  and all acts  $f_A$ , the ambiguity aversion is *second order*<sub>(III)</sub> at  $w_0$ . If, on the other hand,  $L > 0$  for a reference belief  $\nu$  at  $w_0$  and some acts  $f_A$ , the ambiguity aversion is *first order*<sub>(III)</sub> there.

The proof for the equivalence shows that the orders of ambiguity aversion according to the last definition are mutually exclusive. Therefore it is sufficient to require  $L = 0$  for a reference belief  $\nu$  at the wealth level instead of all reference beliefs for second-order<sub>(III)</sub> ambiguity aversion.

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<sup>8</sup>See Jewitt and Mukerji (2011) for an alternative definition that differs with respect to the reference belief, but yields the same limit results. Other available definitions are only defined for a given representation.

<sup>9</sup>The proof of Theorem 4.1 ensures existence of this limit. If the ambiguity premium is differentiable, it is possible to use the first derivative of the premium  $(\pi_A^\nu)'(t)$  in the limit  $L$  instead of  $\pi_A^\nu(t)/t$ .

### 4.2.3 Equivalence of the Definitions

It remains to show the equivalence of the different definitions of the distinction between first-order and second-order ambiguity aversion presented in this section.

**Theorem 4.1.** *Consider a wealth level  $w_0$ . The following statements are equivalent:*

- *Ambiguity aversion is second order<sub>(I)</sub> at  $w_0$ .*
- *Ambiguity aversion is second order<sub>(II)</sub> at  $w_0$ .*
- *Ambiguity aversion is second order<sub>(III)</sub> at  $w_0$ .*

*The same equivalence holds for first-order<sub>(.)</sub> ambiguity aversion.*

**Proof:** First, consider Definition 1 and 2. Assume ambiguity aversion is second order<sub>(II)</sub> at a given wealth level  $w_0$ . Denote the constant act for the wealth level  $w_0$  by  $f_0 = w_0 \mathbf{1}$ . Without loss of generality suppose  $0 \notin B(f_0)$ . If the subdifferential of  $W_{f_0}$  at  $f_0$  is  $\{\mathbf{0}\}$ , the agent is indifferent between  $f_0$  and  $(w_0 - \epsilon) \mathbf{1}$  for some  $\epsilon > 0$ . Then the indifference surface is differentiable at  $f_0$ . Otherwise, define the function  $\bar{W}_{f_0}: X^{|\Omega|} \rightarrow \bar{\mathbb{R}}$  with

$$\bar{W}_{f_0}(f) = \begin{cases} 1 - \inf\{\alpha \in \mathbb{R}_+ | \alpha f \in B(f_0)\} & \text{if } \exists \alpha \in \mathbb{R}_+ : \alpha f \in B(f_0) \\ -\infty & \text{otherwise.} \end{cases}$$

To verify that  $\bar{W}_{f_0}$  is concave, consider  $f_1, f_2 \in X^{|\Omega|}$  and assume  $\bar{W}_{f_0}(f_1), \bar{W}_{f_0}(f_2) > -\infty$ . Define  $\alpha_i = 1 - \bar{W}_{f_0}(f_i) = \inf\{\alpha \in \mathbb{R}_+ | \alpha f_i \in B(f_0)\}$  for  $i = 1, 2$ . Since  $B(f_0)$  is a closed and convex set,  $\alpha_i f_i \in B(f_0)$  for  $i = 1, 2$  and

$$(\lambda \alpha_1 + (1 - \lambda) \alpha_2) (\lambda' f_1 + (1 - \lambda') f_2) = \lambda \alpha_1 f_1 + (1 - \lambda) \alpha_2 f_2 \in B(f_0)$$

with  $\lambda' = \frac{\lambda \alpha_1}{\lambda \alpha_1 + (1 - \lambda) \alpha_2} \in [0, 1]$  for all  $\lambda \in [0, 1]$ . Notice that  $\lambda \geq \lambda'$  if and only if  $\alpha_2 \geq \alpha_1$ .

$$\lambda' \bar{W}_{f_0}(f_1) + (1 - \lambda') \bar{W}_{f_0}(f_2) = 1 - \lambda' \alpha_1 - (1 - \lambda') \alpha_2 \leq 1 - \lambda \alpha_1 - (1 - \lambda) \alpha_2 \leq$$

$$\leq 1 - \inf\{\alpha \in \mathbb{R} | \alpha(\lambda'f_1 + (1 - \lambda')f_2) \in B(f_0)\} = \bar{W}_{f_0}(\lambda'f_1 + (1 - \lambda')f_2)$$

The inequality,  $\lambda'\bar{W}_{f_0}(f_1) + (1 - \lambda')\bar{W}_{f_0}(f_2) \leq \bar{W}_{f_0}(\lambda'f_1 + (1 - \lambda')f_2)$ , obviously holds if  $\bar{W}_{f_0}(f_i) = -\infty$  for  $i = 1$  or  $2$ . Consequently,  $\bar{W}_{f_0}$  is a proper concave function.

This ensures that the subdifferential of  $\bar{W}_{f_0}$  at  $f_0$  exists. In addition, the upper contour set of  $\bar{W}_{f_0}$  at  $f_0$  equals  $B(f_0)$ . According to Rockafellar (1970, Theorem 23.7), the closure of the convex cone generated by the subdifferential at  $f_0$  is the normal cone to the upper contour set of a concave function at  $f_0$ . Therefore the closure of the convex cone generated by the subdifferential of  $\bar{W}_{f_0}$  and the one of  $W_{f_0}$  at  $f_0$  are the same. By second-order<sub>(II)</sub> ambiguity aversion, the subgradients of  $W_{f_0}$  at  $f_0$  are parallel. Additionally, the two-sided directional derivative of  $\bar{W}_{f_0}$  at  $f_0$  with respect to  $\mathbf{1}$  exists and equals  $w_0^{-1}$ . Therefore any parallel subgradient has the same length and  $\bar{W}_{f_0}$  has a unique subgradient at  $f_0$ . Rockafellar (1970, Theorem 25.1) shows that a concave function has a unique subgradient at  $f_0$  if and only if the function is differentiable at  $f_0$ . Therefore the function  $\bar{W}_{f_0}$  is differentiable at  $f_0$  and continuous in a neighborhood of  $f_0$ .<sup>10</sup> In addition, the indifference surface is the level set of  $\bar{W}_{f_0}$  at the value  $0 = \bar{W}_{f_0}(f_0)$ . Thus, the indifference surface is differentiable at  $f_0$  and ambiguity aversion is second order<sub>(I)</sub>.

Now assume ambiguity aversion is first order<sub>(II)</sub>. Then there are several reference beliefs at  $w_0$ . Therefore the subdifferential of  $W_{f_0}$  at  $f_0$  has several elements that are not parallel. Assume to the contrary that there is a function  $\eta$ , so that the indifference surface is the level set of  $\eta$  at the value  $\eta(f_0)$  and  $\eta$  is differentiable at  $f_0$ . Then the gradient  $\nabla$  of  $\eta$  at  $f_0$  is orthogonal to the indifference surface. On the other hand, the non-parallel subgradients of  $W_{f_0}$  define supporting hyperplanes at  $f_0$  and corresponding half-spaces. The indifference surface is contained in the union of these half-spaces. Yet this yields a contradiction to the definition of the gradient  $\nabla$ . Therefore there is a kink in the indifference surface at  $f_0$ . Accordingly, ambiguity aversion is first order<sub>(I)</sub>.

The second step concerns Definition 2 and 3. Consider first second-order<sub>(II)</sub> ambiguity aversion at  $w_0$  and a uniquely determined reference belief  $\nu$ . If the subdifferential

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<sup>10</sup>By an implicit function theorem, like Kumagai (1980), in combination with the convexity and monotonicity of preferences, this allows constructing a topological manifold that covers the indifference surface in a neighborhood of  $f_0$ .



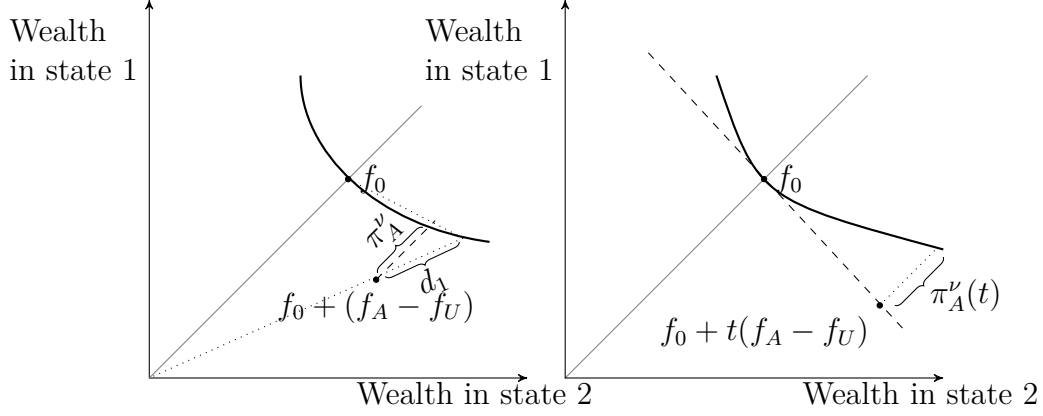


Figure 4.2: The Ambiguity Premium

of  $W_{f_0}$  at  $f_0$  is  $\{\mathbf{0}\}$ , the agent is indifferent between  $f_0$  and  $(w_0 - \epsilon)\mathbf{1}$  for some  $\epsilon > 0$ . Then  $L = 0$ . Otherwise, the ambiguity premium equals the distance between  $f_0 + t(f_A - f_U)$  and the upper contour set  $B(f_0)$  in the direction  $\mathbf{1}$  and  $\pi_A^\nu(t) = \inf\{\beta \in \mathbb{R}_+ | \beta\mathbf{1} + f_0 + t(f_A - f_U) \in B(f_0)\}$ . Figure 4.2 depicts this distance. Compare this to the distance  $d_1$  between  $f_0 + t(f_A - f_U)$  and the upper contour set  $B(f_0)$  in the direction  $f_0 + t(f_A - f_U)$ , i.e., for  $t$  sufficiently small

$$\begin{aligned} d_1 &= \inf \left\{ \beta \in \mathbb{R}_+ \left| \beta \frac{f_0 + t(f_A - f_U)}{\|f_0 + t(f_A - f_U)\|} + f_0 + t(f_A - f_U) \in B(f_0) \right. \right\} = \\ &= \inf \left\{ \beta \in \mathbb{R}_+ \left| \left( \frac{\beta}{\|f_0 + t(f_A - f_U)\|} + 1 \right) (f_0 + t(f_A - f_U)) \in B(f_0) \right. \right\} = \\ &= \left( \inf \left\{ \alpha \in \mathbb{R}_+ | \alpha(f_0 + t(f_A - f_U)) \in B(f_0) \right\} - 1 \right) \|f_0 + t(f_A - f_U)\| = \\ &= -\bar{W}_{f_0}(f_0 + t(f_A - f_U)) \|f_0 + t(f_A - f_U)\| \end{aligned}$$

By the intersection theorem

$$\pi_A^\nu(t) \|\mathbf{1}\| \leq \frac{d_1 \|f_0\|}{d_1 + \|f_0 + t(f_A - f_U)\|} < 2d_1$$

for  $t$  sufficiently small. The first part of the proof showed that  $\bar{W}_{f_0}$  is differentiable at  $f_0$  with a gradient  $\nabla$ . Now use the gradient  $\nabla$  to construct a hyperplane  $I_0$  that contains  $f_0$ . This hyperplane  $I_0$  is the indifference surface of a risk-neutral,

subjective expected-utility maximizer with the reference belief  $\nu$ . The subjective expected-utility maximizer is, in particular, indifferent between the ambiguous act  $tf_A$  and the constant act  $tf_U$  by construction of the act  $f_U$ . Now consider a simple lottery  $l \in \Delta X$  with a scaling parameter  $t$ . According to Nielsen (1999, Theorem 1), the differentiability of the von-Neumann-Morgenstern utility index  $u$  guarantees that the difference between the certainty equivalent and the expected value of the lottery  $tl$  divided by  $t$  goes to 0 for  $t \rightarrow 0$ . Consequently,  $d_2/t \rightarrow 0$  for  $t \rightarrow 0$  with the distance  $d_2$  between  $f_0 + t(f_A - f_U)$  and the hyperplane, i.e.,

$$d_2 = \inf \left\{ \|f_0 + t(f_A - f_U) - g\| \mid g \in I_0 \right\} = \inf \left\{ \beta \in \mathbb{R}^+ \mid f_0 + t(f_A - f_U) + \beta \frac{\nabla}{\|\nabla\|} \in I_0 \right\}.$$

Thus,

$$\lim_{t \rightarrow 0} \frac{\bar{W}_{f_0}(f_0 + t(f_A - f_U))}{t} = \lim_{t \rightarrow 0} \frac{\langle \nabla, f_0 + t(f_A - f_U) - f_0 \rangle}{t} = \lim_{t \rightarrow 0} \frac{-\|\nabla\|d_2}{t} = 0$$

because  $\bar{W}_{f_0}$  is differentiable at  $f_0$  and  $\bar{W}_{f_0}(f_0) = 0$ .<sup>11</sup> This ensures that  $L = 0$  for the reference belief  $\nu$  and all acts  $f_A$ . Therefore ambiguity is second order<sub>(III)</sub>. Moreover,  $L = 0$  for all reference beliefs and all acts  $f_A$ , because the reference belief is unique.

Finally, consider the case of first-order<sub>(II)</sub> ambiguity aversion at  $w_0$ . Then there are several reference beliefs and some subgradients of  $W_{f_0}$  at  $f_0$  are not parallel. Pick a reference belief  $\nu$ , a corresponding subgradient  $x_1^*$  and an act  $f_A \in \mathcal{F}$ . Denote another subgradient of  $W_{f_0}$  at  $f_0$  that is not parallel to  $x_1^*$  by  $x_2^*$ . Now use these subgradients to construct two hyperplanes  $I_1$  and  $I_2$  that contain  $f_0$ . The ambiguity premium is bigger than the distance between the act  $f_0 + t(f_A - f_U)$  and the upper contour set  $B(f_0)$ ,

$$\pi_A^\nu(t)\|\mathbf{1}\| \geq \inf \{ \|f_0 + t(f_A - f_U) - g\| \mid g \in B(f_0) \} = U(f_0) - W_{f_0}(f_0 + t(f_A - f_U))$$

for  $t$  sufficiently small. Moreover,

$$\begin{aligned} U(f_0) - W_{f_0}(f_0 + t(f_A - f_U)) &= W_{f_0}(f_0) - W_{f_0}(f_0 + t(f_A - f_U)) \geq \\ &\geq \langle -x_i^*, f_0 + t(f_A - f_U) - f_0 \rangle \end{aligned}$$

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<sup>11</sup> $\langle x, y \rangle = \sum_i x_i y_i$  denotes the inner product of the two vectors  $x$  and  $y$ .

by the definition of the subgradients (Rockafellar, 1970, p. 308). Thus,

$$\lim_{t \rightarrow 0} \frac{W_{f_0}(f_0) - W_{f_0}(f_0 + t(f_A - f_U))}{t} \geq \lim_{t \rightarrow 0} \frac{\langle -x_i^*, f_0 + t(f_A - f_U) - f_0 \rangle}{t}$$

for  $i = 1, 2$ . The first limit exists, because the one-sided directional derivative of the concave function  $W_{f_0}$  exists everywhere. (Rockafellar, 1970, Theorem 23.1) As before, by the construction of  $f_U$  the distance  $d_2^1$  between  $f_0 + t(f_A - f_U)$  and the first hyperplane  $I_1$  converges to 0 faster than  $t$ ,  $d_2^1/t \rightarrow 0$  for  $t \rightarrow 0$ . This is valid for any act  $f_A \in \mathcal{F}$  adjusting  $f_U$  accordingly. Denote the distance between  $f_0 + t(f_A - f_U)$  and the second hyperplane  $I_2$  by  $d_2^2$ . By the triangle inequality,  $d_2^2$  is bigger than the distance between the orthogonal projections of  $f_0 + t(f_A - f_U)$  onto the two hyperplanes minus  $d_2^1$ . Moreover, the subgradients  $x_1^*$  and  $x_2^*$  are not parallel. Therefore, it is possible to find an act  $f_A \in \mathcal{F}$ , for which the distance  $d_2^2$  between  $f_0 + t(f_A - f_U)$  and the second hyperplane  $I_2$  decreases slowly, i.e.,  $d_2^2/t \rightarrow \epsilon > 0$  for  $t \rightarrow 0$ . Additionally, either  $\langle -x_i^*, f_0 + t(f_A - f_U) - f_0 \rangle$  or  $\langle -x_i^*, f_0 - t(f_A - f_U) - f_0 \rangle$  is positive for  $t$  sufficiently small. Consequently,  $L > 0$  for either  $f_A$  or  $-f_A$  and ambiguity aversion is first order<sub>(III)</sub> at  $w_0$ .  $\square$

Since the definitions are equivalent, I skip the subscripts in the following. Before applying the distinction, consider a couple of specific representations.

#### 4.2.4 Examples

The order of ambiguity aversion is defined locally at a given wealth level, but most representations exhibit either first-order ambiguity aversion or second-order ambiguity aversion. I begin with the representation of  $\alpha$ -Maxmin-Expected Utility by Ghirardato et al. (2004).  $\Pi$  is a closed set of first-order probability measures  $P$  on the state space  $X$ . In addition, there is an increasing von-Neumann-Morgenstern utility index  $u$  and a parameter of ambiguity attitude  $\alpha \in [0, 1]$ . The representation has the following form

$$\alpha \min_{P \in \Pi} \int_X u \circ f dP + (1 - \alpha) \max_{P \in \Pi} \int_X u \circ f dP.$$

Special cases of this representation are Maxmin-Expected Utility for  $\alpha = 1$  by Gilboa and Schmeidler (1989) and Choquet-Expected Utility with ambiguity aversion by Schmeidler (1989). Choquet-Expected Utility exhibits ambiguity aversion if the capacity is convex. Yet, if the capacity is convex, CEU preferences can be represented by a MEU representation with the set of priors given by the core of the capacity. See Appendix 4.D for the details of this representation.

**Proposition 4.1.** *Suppose the set of priors has at least two elements. Then  $\alpha$ -MEU preferences exhibit first-order ambiguity aversion.*

**Proof:** Consider the closure  $\bar{H}$  of the convex hull of the set of priors  $\Pi$ . A vertex of  $\bar{H}$  is a prior  $P \in \Pi$  such that  $P$  is not contained in the convex hull of  $\Pi \setminus \{P\}$ . Define the set  $H$  as the convex hull of  $\{\alpha P + (1 - \alpha)Q \mid P \neq Q \text{ and } P, Q \text{ are vertices of } \bar{H}\}$ . For the constant act  $f_0$  at a wealth level  $w_0$ , the one-sided directional derivative of  $W_{f_0}$  at  $f_0$  with respect to  $-h/\|h\|$  is  $-1$  for any  $h \in H$ . Then the subdifferential of  $W_{f_0}$  at  $f_0$  is  $\{\gamma u'(w_0)h \mid \gamma \in [0, \frac{1}{u'(w_0)\|h\|}], h \in H\}$ . The projection into the simplex yields the reference beliefs  $H$ . Consequently, the  $\alpha$ -MEU representation implements first-order ambiguity aversion by Definition 2.  $\square$

The intuition is the following. Whenever the ordering of the payoffs changes, this might yield different priors minimizing (maximizing, resp.) expected utility. Hence, the indifference curves are kinked and there are several reference beliefs. The next step turns to smooth ambiguity aversion by Klibanoff et al. (2005) with a utility index  $u$  and a concave ambiguity index  $\phi$ . See the beginning of this section for a brief introduction and Appendix 4.C for details.

**Proposition 4.2.** *Suppose  $u$  and  $\phi$  are differentiable. Then KMM preferences exhibit second-order ambiguity aversion.*

**Proof:** The representing utility function

$$U(f) = \int_{\Pi} \phi \left( \int_X u \circ f dP \right) d\mu$$

is differentiable. Hence, there is a unique reference belief for any wealth level  $w_0$ , because all subgradients of  $W_{w_0 \mathbf{1}}$  at  $w_0 \mathbf{1}$  are parallel. Therefore the preferences exhibit second-order ambiguity aversion according to Definition 2.  $\square$

Maccheroni et al. (2011) study the certainty equivalent of lotteries for smooth ambiguity aversion. They show that smooth ambiguity aversion has only a second-order effect confirming my results here.

Next, I determine the reference beliefs explicitly. The gradient of  $U$  at the constant act  $f_0$  with a wealth level  $w_0$  is  $\nabla = \phi'(u(w_0))u'(w_0)\mathbb{E}_\mu(P)$ . The function  $W_{f_0}$  equals  $W_{f_0}(f) = \phi(u(w_0)) - \inf\{\|f - g\| \mid g \in B(f_0)\}$ . In addition, the one-sided directional derivative of  $W_{f_0}$  at  $f_0$  with respect to  $-\nabla/\|\nabla\|$  is  $-1$ . Then the subdifferential of  $W_{f_0}$  at  $f_0$  is  $\{\alpha\nabla \mid \alpha \in [0, 1/\|\nabla\|]\}$ . The projection into the simplex yields the reference beliefs  $\mathbb{E}_\mu(P)$ .

Multiplier Preferences by Hansen and Sargent (2001) are based on a probability measure  $Q$  on the state space  $\Omega$ .<sup>12</sup> In addition, there is a von-Neumann-Morgenstern utility index  $u$  and a parameter of ambiguity attitude  $\alpha \in \mathbb{R}_+$ .  $R(P||Q)$  is the relative entropy, also known as Kullback-Leibler divergence, of  $P$  with respect to  $Q$  measuring a ‘distance’ between the probability measures. Then the representation is

$$\min_{P \in \Delta\Omega} \left( \int_X u \circ f dP + \alpha R(P||Q) \right).$$

These preferences feature second-order ambiguity aversion, because  $Q$  is the unique reference belief for any wealth level. Finally, Variational Preferences by Maccheroni et al. (2006) exhibit first-order or second-order ambiguity aversion depending on the cost function  $c: \Delta\Omega \rightarrow \mathbb{R}$  for choosing a prior. These preferences are represented by

$$\min_{P \in \Delta\Omega} \left( \int_X u \circ f dP + c(P) \right).$$

Variational Preferences feature second-order ambiguity aversion if and only if the equation  $c(P') = 0$  has a unique solution  $P'$ . In addition, Maccheroni et al. (2006, Theorem 18) show that the preference functional is everywhere differentiable if and only if the cost function  $c$  is essentially strictly convex. MEU and Multiplier Preferences are special cases of Variational Preferences.

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<sup>12</sup>See Strzalecki (2011) for an axiomatization.

### 4.3 Application of the Distinction

Consider an act  $f$  and a scaling parameter  $t \in [0, 1]$  to model a perfectly divisible act. An ambiguity-averse agent with an endowment of  $w_0 \in \mathbb{R}$  can choose her optimal  $t$ . To avoid less interesting cases of indifference, assume that there is no  $\bar{t} \in (0, 1]$  such that

$$w_0 \mathbf{1} + tf \sim w_0 \mathbf{1} \quad \forall t \in [0, \bar{t}]. \quad (4.2)$$

If condition (4.2) is violated, the agent is indifferent between all  $t \in [0, \bar{t}]$ . Then the following theorem is valid if preferences are locally non-satiated and a choice of  $t^* > 0$  is replaced by a strict preference for  $t^* > 0$ . Condition (4.2) is violated, e.g., for ambiguity-neutral and risk-neutral preferences, risk-neutral MEU preferences, or if the indifference surfaces are thick.

**Theorem 4.2.** *Assume (4.2). With second-order ambiguity aversion at  $w_0 \mathbf{1}$ , the agent chooses  $t^* > 0$  if and only if there is a reference belief  $\nu$  at  $w_0$  that yields a positive expectation of the act,  $\mathbb{E}_\nu(f) > 0$ . This condition is necessary, but not sufficient for first-order ambiguity aversion.*

**Proof:** Consider an ambiguity-averse agent with second-order ambiguity aversion at the constant act  $f_0$  with the wealth level  $w_0$ . Then there is a uniquely determined reference belief  $\nu$  at  $w_0$ . Assume the agent prefers  $f_0 + t^*f$  to  $f_0$  with  $t^* > 0$ . Then  $f_0 + tf \in B(f_0)$  and  $\bar{W}_{f_0}(f_0 + tf) > 0$  for all  $t \in [0, t^*)$ , because the preferences are convex and monotone in addition to (4.2). The proof of Theorem 4.1 shows that the concave function  $\bar{W}_{f_0}$  is differentiable at  $f_0$  with a gradient  $\nabla$ . Therefore

$$\bar{W}_{f_0}(f_0 + tf) = \bar{W}_{f_0}(f_0 + tf) - \bar{W}_{f_0}(f_0) \leq \langle \nabla, tf \rangle \quad (4.3)$$

and  $\langle \nabla, f \rangle > 0$ .  $\nabla$  and  $\nu$  are parallel and  $\langle \nu, \nabla \rangle = \|\nabla\| \|\nu\|$ . Hence  $\langle \nu, f \rangle > 0$ . Yet  $\langle \nu, f \rangle > 0$  if and only if  $\mathbb{E}_\nu(f) > 0$ . This proves that the condition is necessary.

To prove the other direction, assume  $\mathbb{E}_\nu(f) > 0$ . Then  $\langle \nu, f \rangle > 0$  and  $\langle \nabla, f \rangle > 0$ . As

$$\lim_{t \rightarrow 0} \frac{\bar{W}_{f_0}(f_0 + tf)}{t} = \lim_{t \rightarrow 0} \frac{\bar{W}_{f_0}(f_0 + tf) - \bar{W}_{f_0}(f_0)}{t} = \lim_{t \rightarrow 0} \frac{\langle \nabla, tf \rangle}{t} = \langle \nabla, f \rangle,$$

there is a  $t^* > 0$ , such that  $\bar{W}_{f_0}(f_0 + tf) > 0$  and  $f_0 + tf \in B(f_0)$  for all  $t \in [0, t^*]$ . Consequently, the agent prefers  $f_0 + t^*f$  to  $f_0$ . This concludes the sufficiency part.

Now consider an ambiguity-averse agent with first-order ambiguity aversion at wealth  $w_0$ . Assume the agent prefers  $f_0 + t^*f$  to  $f_0$  with  $t^* > 0$ . Then  $f_0 + tf \in B(f_0)$  and  $\bar{W}_{f_0}(f_0 + tf) > 0$  for all  $t \in [0, t^*]$ , as before. The proof of Theorem 4.1 shows that the concave function  $\bar{W}_{f_0}$  has a subgradient  $x^*$  at  $f_0$ . Therefore equation (4.3) is still valid after replacing the gradient  $\nabla$  with the subgradient  $x^*$ . This yields  $\langle x^*, f \rangle > 0$ . In addition, there is a reference belief  $\nu$  at  $w_0$  that is parallel to  $x^*$ . Hence  $\langle \nu, f \rangle > 0$  and  $\mathbb{E}_\nu(f) > 0$ . Thus, the condition is necessary.

Finally, according to the proof of Theorem 4.1 there is a subgradient  $\bar{x}^*$  of  $\bar{W}_{f_0}$  at  $f_0$  that is not parallel to  $x^*$ . Consequently, there is an act  $f$  with  $\langle x^*, f \rangle > 0$  and  $\langle \bar{x}^*, f \rangle < 0$ . The necessity part shows that the agent prefers  $f_0$  to  $f_0 + tf$  for all  $t \in (0, 1]$ , although  $\mathbb{E}_\nu(f) > 0$ .<sup>13</sup>  $\square$

Consider an ambiguity-neutral agent Nathan whose subjective beliefs are given by a reference belief of the ambiguity-averse agent Anna at  $w_0$ . By convexity, Anna's set of improvement directions at  $w_0$  is a subset of Nathan's set of improvement directions. In the case of second-order ambiguity aversion, the sets are equal. In the case of first-order ambiguity aversion, Anna is willing to participate in the ambiguous act if the act has a positive expectation for all reference beliefs at  $w_0$ . Hence, Anna's set of improvement directions at  $w_0$  is the intersection of Nathans' sets of improvement directions where there is a Nathan for every reference belief at  $w_0$ . Notice that ambiguity aversion changes the agent's optimal exposure  $t$  to ambiguity, independently of the order of ambiguity aversion. I now turn to some applications of the distinction.

### 4.3.1 Investment

Assume the agent's preferences are locally non-satiated. The agent with an endowment of  $w_0 \in \mathbb{R}$  can buy or sell  $t \in \mathbb{R}$  units of an asset  $f \in X^\Omega$  at a exogenously given price per unit,  $P$ . Suppose that the remaining part of the agent's wealth is invested in a risk-free and unambiguous asset. Now hold the price  $P$  and the agent's endowment fixed and vary the asset  $f$ .

<sup>13</sup>For a specific counterexample to the equivalence, see the end of Appendix 4.D.

**Lemma 4.2.** *With second-order ambiguity aversion, the agent generically buys or sells a strictly positive amount of the asset. First-order ambiguity aversion makes the agent avoid any investment for assets with a strictly positive mass under the Lebesgue measure.*

The proof of the lemma is based on the following corollary of Theorem 4.2.<sup>14</sup>

**Corollary 4.1.** *With second-order ambiguity aversion, the agent invests some fraction of her wealth in an ambiguous asset  $f$  if and only if the expected return under a reference belief of the ambiguous asset is higher than the return of the risk-free asset. First-order ambiguity aversion can make the agent avoid any investment in the ambiguous asset, even if the condition is satisfied.*

**Proof of Lemma 4.2:** Suppose  $\nu$  is a reference belief of the agent at her endowment. The expected return under the reference belief  $\nu$  of the asset  $f$  at price  $P$  is either higher, equal or lower than the the return of the risk-free asset. According to the corollary, the agent either buys or sells a fraction of the asset  $f$  at price  $P$  with second-order ambiguity aversion whenever the expected returns are unequal. The set of assets whose expected return equals the return of the risk-free asset is a null set in the asset space  $X^\Omega$  with the Lebesgue measure. Hence, the agent generically buys or sells a strictly positive amount of each asset. With first-order ambiguity aversion, there is a set of assets with strictly positive mass under the Lebesgue measure so that the agent avoids any investment in these assets. In addition, the agent strictly prefers not to invest in these assets.  $\square$  With second-order ambiguity aversion, the set of assets that the agent does not buy coincides with the agent's indifference surface at her endowment. This set has no mass in the asset space. Hence, ambiguity aversion in general does not yield no-trade results.<sup>15</sup> There are natural implications for asset prices.

**Corollary 4.2.** *With second-order ambiguity aversion, asset prices are uniquely determined. First-order ambiguity aversion can yield indeterminacies in equilibrium.*

<sup>14</sup>The result of the corollary for the special case of KMM corresponds to Lemma 1 in Gollier (2011).

<sup>15</sup>See also Epstein and Schneider (2010, pp. 316, 337) who discuss the robustness of no-trade results depending on the representation.



This result implies that the incompleteness of financial markets in Mukerji and Tallon (2001) requires first-order ambiguity aversion. Rigotti and Shannon (2012), however, show in a general equilibrium model that equilibrium prices will be generically determined, even in a representation with first-order ambiguity aversion. The reason is that CEU or MEU preferences feature only kinks at non-generic endowments. Thus, for generic endowments indifference curves are differentiable in a neighborhood of the endowment. Mandler (2011) endogenizes the endowment values by adding an ex-ante investment stage. In his model, asset prices are generically indeterminate with MEU preferences. Intuitively, this result probably extends to any preferences with first-order ambiguity aversion. The next section turns to an insurance setting.

### 4.3.2 Insurance

Consider an agent who faces a potential loss  $\tilde{L} \in \mathcal{F}$  that may be ambiguous. Denote the expected loss under a reference belief  $\nu$  by  $L_\nu = \mathbb{E}_\nu(\tilde{L})$  and the agent's endowment by  $w_0 \in \mathbb{R}$ . She can buy an insurance contract covering an fraction  $\alpha \in \mathbb{R}_+^0$  of the loss to protect her from the loss. The insurer requests a constant premium  $P$  per unit of coverage  $\alpha$ . Therefore buying an insurance contract with a coverage  $\alpha$  corresponds to the act  $w_0\mathbf{1} + (1 - \alpha)\tilde{L} - \alpha P\mathbf{1}$ . Assume that the agent's preferences are locally non-satiated.

**Lemma 4.3.** *With second-order ambiguity aversion, full insurance coverage is demanded at a unique premium. With first-order ambiguity aversion, demand for full insurance coverage is consistent with an interval of premium levels.*

Again the proof uses a corollary of Theorem 4.2.

**Corollary 4.3.** *With second-order ambiguity aversion, (at least) full insurance coverage is demanded if and only if the premium  $P$  is lower than  $L_\nu$ . Exactly full insurance coverage is demanded at the premium  $P = L_\nu$ . With first-order ambiguity aversion, demand for full insurance coverage is consistent with premium levels  $P$  above  $L_\nu$ .*

The proof of Lemma 4.3 is analogous to the one of Lemma 4.2 and therefore I omit the proof here. Notice that according to the corollary Mossin's (1968) Theorem on

the optimality of full coverage versus partial coverage still holds with second-order ambiguity aversion. Similar results are valid if the agent can self-insure or invest into prevention effort. I assume that prevention effort is costly with strictly increasing and convex costs.

**Corollary 4.4.** *With second-order ambiguity aversion, the risk of a potential loss will be completely eliminated if and only if the marginal costs of the last prevention effort are lower than  $L_\nu$ . With first-order ambiguity aversion, the agent might be willing to avoid the loss completely, even if the marginal costs are higher than  $L_\nu$ .*

Finally, consider the effects of the distinction on contracting.

### 4.3.3 Contracting

Ghirardato (1994) was the first to point out that more informative signals might hurt the principal if the agent is ambiguity-averse in a CEU model. This result violates the informativeness principle of Holmström (1979). I will show that the violation of the informativeness principle hinges on first-order ambiguity aversion. The chapter proceeds in two steps. First, it will establish that constant wages can only be optimal under first-order ambiguity aversion. In the second step, an adapted version of the informativeness principle is proven for the representation of KMM. Coming back to the first step, consider a risk-averse and ambiguity-averse agent working for a risk-neutral and ambiguity-neutral principal. They can enter a contract that specifies the agent's wage  $W(y)$  depending on his performance  $y \in Y \subset [0, 1]$  with  $Y$  finite. Then the principal's objective function is  $\mathbb{E}_\mu(y - W(y))$  under the beliefs  $\mu$  of the ambiguity-neutral principal. After signing such a contract, the agent chooses effort  $e \in \mathbb{R}_+^0$ , which is unobservable by the principal. The agent's preferences satisfy the conditions of subsection 4.2.1 and are locally non-satiated. His payoff is given by  $W(y) - c(e)$  if he chooses effort  $e$ . The function  $c(\cdot)$ , the disutility of performing effort, is increasing and strictly convex with the limits  $\lim_{e \rightarrow 0} c'(e) = 0$ ,  $\lim_{e \rightarrow \infty} c'(e) = \infty$  and  $c(0) = 0$ .  $c$  is twice differentiable. The agent receives a reservation utility of  $\bar{u}$  if he rejects the principal's offer. Define the reservation wage  $\bar{w} = u^{-1}(\bar{u})$ .

There is a stochastic and ambiguous relationship between the agent's effort  $e$  and the performance  $y$ . Effort positively influences performance in the sense of

### 4.3 APPLICATION OF THE DISTINCTION

$\frac{\partial \mathbb{E}_\mu(y)}{\partial e}(e) > \bar{\epsilon} > 0$  for all  $e \in [0, \bar{e})$  with a  $\bar{e} > 0$  under the principal's prior  $\mu$ . To avoid issues of shifting supports, assume that the principal's prior  $\mu$  has full support on  $Y$  for all  $e \in \mathbb{R}_+^0$ . For consistency, I assume that the principal's prior  $\mu$  is contained in the agent's reference beliefs for all wealth levels  $w \in [\bar{w} - \hat{\epsilon}, \bar{w} + \hat{\epsilon}]$  with  $\hat{\epsilon} > 0$ . Moreover, the ambiguity-averse agent shares the principal's view, i.e.,  $\exists \delta \in Y: \forall w \in \mathbb{R}, \forall b, e \in \mathbb{R}_+, \forall \epsilon \in (0, e]$

$$f(e) \succ f(e - \epsilon) \text{ with the act } f(e) = \begin{cases} w & \text{if } y(e) \leq \delta \\ w + b & \text{if } y(e) > \delta. \end{cases} \quad (4.4)$$

Hence, the agent expects effort to influence performance positively. Neglecting the disutility of effort, this effect increases his valuation of pay-offs conditional on good performances.

**Proposition 4.3.** *The principal pays a performance-related wage if the ambiguity aversion is second order. With first-order ambiguity aversion a constant wage can be optimal.*

**Proof:** A constant wage of  $\bar{w}$  implements zero effort,  $e = 0$ . In order to give the agent incentives to implement a positive effort  $e$ , let the agent participate in the performance. Consider, for example, the following wage

$$W(y) = \begin{cases} \tilde{w} & \text{if } y(e) \leq \delta \\ \tilde{w} + \epsilon & \text{if } y(e) > \delta \end{cases}$$

with  $\tilde{w} = \bar{w}$ ,  $\delta$  given by (4.4) and  $\epsilon > 0$ . By the definition of  $\bar{w}$ , the agent prefers to accept the contract  $W(y)$ . Now decrease  $\tilde{w}$  until the agent is indifferent between accepting the contract  $W(y)$  and rejecting it. Ignoring the effort costs for the moment, the agent prefers more effort to less effort under this wage schedule by the choice of  $\delta$ . In addition, there is a reference belief such that choosing a positive effort level in expectation increases income even including effort costs, as  $\frac{\partial \mathbb{E}_\mu(W(y))}{\partial e}(e) - c'(e) > 0$  for  $e$  sufficiently small. Then, according to Theorem 4.2, second-order ambiguity aversion ensures that the wage schedule  $W(y)$  implements positive effort. This defines an implicit function  $e(\epsilon)$ .<sup>16</sup> Now consider the limit of the change in the principal's

<sup>16</sup>In cases of indifference, pick a value of  $e$  suitably.

expected profits

$$\lim_{\epsilon \rightarrow 0} \frac{\mathbb{E}_\mu(y(e(\epsilon))) - c(e(\epsilon)) - \pi(\epsilon) - \pi_A(\epsilon) - \mathbb{E}_\mu(y(0))}{\epsilon}. \quad (4.5)$$

According to Nielsen (1999, Theorem 1),  $\pi(\epsilon)/\epsilon \rightarrow 0$  for  $\epsilon \rightarrow 0$ , as the utility index is differentiable. In addition,

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \frac{\mathbb{E}_\mu(y(e(\epsilon))) - c(e(\epsilon)) - \mathbb{E}_\mu(y(0)) - c(0)}{\epsilon} &= \\ &= \lim_{\epsilon \rightarrow 0} \left( \frac{\partial \mathbb{E}_\mu(y(e))}{\partial e}(e(\epsilon)) - c'(e(\epsilon)) \right) \frac{e(\epsilon)}{\epsilon} \geq 0 \end{aligned}$$

because  $\frac{\partial \mathbb{E}_\mu(y)}{\partial e} > \bar{\epsilon} > 0$  and  $\lim_{e \rightarrow 0} c'(e) = 0$ . By Definition 3, for  $\epsilon \rightarrow 0$  second-order ambiguity aversion yields  $\pi_A(\epsilon)/\epsilon \rightarrow 0$ . Consequently, the change in the principal's expected profits (4.5) is positive for  $\epsilon$  sufficiently small if the agent exhibits second-order ambiguity aversion. In this case the wage is performance-related and the optimal effort is positive.

With first-order ambiguity aversion, however,  $\pi_A(\epsilon)/\epsilon \rightarrow b > 0$ . Then the change in the principal's expected profits (4.5) is negative if the agent's performance is sufficiently ambiguous. Consider the following example with MEU preferences. Let  $Y = \{0, 1\}$  and  $c(e) = e^2/2$ . Assume that the set of the agent's subjective probabilities for  $y = 1$  are  $\Pi(e) = [\frac{1}{2}\mu(e), \frac{1}{2}\mu(e) + \frac{1}{2}]$  with the principal's prior  $\mu(e) = 1 - \tilde{p} \exp(-e)$  and  $\tilde{p} \in (0, 1)$ . The optimal wage scheme is  $w_L = c(e) + \bar{w} - \mu(e)\frac{e}{\mu'(e)}$  for low performances,  $y = 0$ , and  $w_H = w_L + \frac{2e}{\mu'(e)}$  for high performances,  $y = 1$ . The principal's expected profits are  $\mu(e)(1 - \frac{e}{\mu'(e)}) - c(e) - \bar{w}$ . Hence, profits are decreasing in  $e$  if  $\tilde{p} < \frac{\sqrt{5}-1}{2}$ . Then a constant wage is optimal and the principal does not give the agent any incentives. This optimality of a constant wage requires first-order ambiguity aversion.<sup>17</sup>  $\square$

Mukerji and Tallon (2004a) analyze a similar cause for incomplete contracts in the presence of ambiguity aversion. They consider wage indexation with aggregate

<sup>17</sup>Mukerji (2003) shows that with MEU incomplete contracts can be optimal and informative signals are neglected in the optimal contract. He proves that MEU can make it optimal to pay a constant wage independent of the performance. Weinschenk (2010) confirms this finding for the widely-used LEN setting under MEU. Lopomo et al. (2011) show the optimality of a bonus wage. Yet, they assume incomplete preferences. These preference, however, feature a kinked indifference surface, as well. This allows a comparison to first-order ambiguity aversion.

and idiosyncratic price shocks. Aggregate price shocks affect the prices of all goods in the economy, while idiosyncratic price shocks just affect a good not consumed or produced by the firm and the agents. Agents have CEU preferences. If the idiosyncratic price shock is sufficiently ambiguous and the aggregate price shock is sufficiently small, wages are contracted in absolute terms and no wage indexation takes places. This result requires a representation with kinks like MEU or CEU according to the previous considerations. In KMM, it is always optimal to have some wage indexation. The reason is that wage indexation allows hedging some ambiguity about aggregate price shocks and the ambiguity premium for doing so is negligible. Second-order ambiguity aversion as defined here is not sufficient for wage indexation, because the results require differentiability of the indifference surfaces at all acts  $f$  where the agent is indifferent between the outcome of the act in at least two different states, i.e.,  $\{f \in \mathcal{F} | f(\omega_1) \sim f(\omega_2) \text{ for } \omega_1 \neq \omega_2 \in \Omega\}$ .

There is another line of reasoning in the literature why ambiguity aversion makes incomplete contracts optimal. Mukerji (1998) considers a bilateral hold-up setting with CEU preferences. Although first-best is implementable in a complete contract for risk-neutral agents, it is unattainable for ambiguity-averse agents. This result is robust with respect to the representation of ambiguity-averse preferences. A formal proof with second-order ambiguity aversion would proceed along the lines of Section 5 in Williams and Radner (1988), who show that first-best is unattainable with second-order risk aversion in the absence of ambiguity aversion.

Appendix 4.A contains an closed-form solution to a principal-agent problem in a KMM representation. More generally, an adapted version of Holmström's (1979) informativeness principle still holds for KMM, such that all relevant information is used in any contract.

#### 4.3.4 Informativeness

For this purpose, adjust the assumptions of the previous section. The principal can contract on the output  $y \in Y \subseteq \mathbb{R}$  and on a signal  $s \in \bar{S} \subseteq \mathbb{R}^n$  about the agent's effort  $e \in \bar{E} \subseteq \mathbb{R}$ . With first-order ambiguity aversion, in particular for MEU or CEU preferences, it is possible to construct examples where it is optimal to condition the wage only on the output  $y$  and to neglect any signal  $s$  if the signal is sufficiently

ambiguous and the output is not too ambiguous. This is impossible with KMM preferences. To make the contrast as stark as possible, assume that the output  $y$  is unambiguously distributed and only the signal  $s$  is ambiguous. Then there is a joint distribution  $F(y, s; e, \delta)$  of the output  $y$  and the signal  $s$  given effort  $e$ . The distribution depends on a parameter  $\delta \in D$ . This parameter  $\delta$  is some aspect of the world that is ambiguous. The agent and the principal have a subjective second-order distribution  $G$  about  $\delta$ . Both distributions have full support and admit densities  $f$  and  $g$ . In addition,  $\partial f(y, s; e, \delta)/\partial e$  should be well-defined for all  $y, s, e$ , and  $\delta$ . The output  $y$  is unambiguously distributed. Therefore the marginal distribution  $F_Y(y; e)$  is constant in  $\delta$ . The agent's effort  $e$  positively influences output with  $\partial F_Y(y; e)/\partial e \leq 0$  for all  $e \in \bar{E}$  and  $y \in Y$ . The inequality shall be strict for some  $y$  with positive mass for every  $e$ . The principal maximizes the expected output minus the wage payments,  $\mathbb{E}_G \mathbb{E}_F(y - W(y, s))$ . On the other hand, the agent's preferences are separable in effort and represented by

$$U_A(W(y, s), e) = \int_D \phi \left( \int_{Y \times \bar{S}} u(W(y, s)) dF(y, s; e, \delta) \right) - c(e) dG(\delta).$$

The assumptions on the function  $c$  remain unchanged.<sup>18</sup> The risk index  $u$  and the ambiguity index  $\phi$  are increasing, concave and twice differentiable. It remains to define *informativeness*. A signal  $s$  is uninformative if  $y$  is a sufficient statistic for  $(y, s)$  with respect to  $e$  under the agent's reference belief. Then it is possible to find functions  $i$  and  $h$ , such that for all  $e \in \bar{E}$

$$\int_D f(y, s; e, \delta) dG(\delta) = i(y, s) h(y, e) \quad \forall y \in Y, s \in \bar{S}. \quad (4.6)$$

This means that the signal  $s$  provides no (additional) information about the agent's effort  $e$ . The next proposition shows that it is optimal to condition the contract on  $s$  – no matter how ambiguous the signal  $s$  is – if  $s$  is informative.

**Proposition 4.4.** *Consider the optimal contract  $W(y)$  with a unique and interior best response  $\hat{e}$  of the agent if the signal  $s$  is unavailable. Suppose signal  $s$  is informative. Then there is a contract  $W(y, s)$  that makes both parties strictly better off.*

<sup>18</sup>The proposition also holds if the preferences are not separable in effort. The assumption is made for ease of exposition.

### 4.3 APPLICATION OF THE DISTINCTION

**Proof:** Define  $f_e(y, s; \hat{e}, \delta) = \frac{\partial f(y, s; e, \delta)}{\partial e}(\hat{e})$ . Now consider a fixed  $y$  and a subset  $S \subset \bar{S}$  with the following properties.  $\int_D \int_S f(y, s; \hat{e}, \delta) ds dG(\delta) = f(y, S, \hat{e}) > 0$  and  $f(y, S^c, \hat{e}) > 0$  for the complementary set  $S^c = \bar{S} \setminus S$ . In addition,  $S$  shall satisfy

$$\frac{f_e(y, S, \hat{e})}{f(y, S, \hat{e})} > \frac{f_e(y, S^c, \hat{e})}{f(y, S^c, \hat{e})}. \quad (4.7)$$

Hence a signal from the set  $S$  indicates a better outcome than a signal from  $S^c$ . It is possible to find such an  $S$ , because the signal  $s$  is informative about the agent's effort  $e$ . Then (4.6) is false and  $\int_D f_e(y, s; e, \delta) dG(\delta) / \int_D f(y, s; e, \delta) dG(\delta)$  is not constant, but varies with the signal  $s$ . For any  $\epsilon > 0$ , I define

$$\epsilon^c = \frac{\epsilon f(y, S, \hat{e})}{f(y, S^c, \hat{e})} > 0.$$

Now change the wage  $W(y)$  by  $\Delta^\epsilon W(y, s)$ . This adds  $\epsilon$  for all  $s \in S$  and subtracts  $\epsilon^c$  for all  $s \in S^c$ .

$$\Delta^\epsilon W(y, s) = \begin{cases} \epsilon & \text{if } s \in S \\ -\epsilon^c & \text{if } s \in S^c \end{cases}$$

The steps so far can be repeated for a set  $\bar{Y}$  of  $y$  with positive mass under  $F_Y(y; \hat{e})$ , because the signal  $s$  is informative. Denote the aggregate change in the wage also by  $\Delta^\epsilon W(y, s)$ . The definition of each  $\epsilon^c$  ensures that the expected wage does not change and

$$\int_D \int_{\bar{S}} \Delta^\epsilon W(y, s) f(y, s; \hat{e}, \delta) ds dG(\delta) = 0 \quad \forall y \in Y.$$

In addition, the expectation of the derivative of  $\Delta^\epsilon W(y, s)$  with respect to  $\epsilon$  is 0. Since  $\hat{e}$  is a unique and interior best response of the agent, the introduction of  $\Delta^\epsilon W(y, s)$  does not change the agent's effort choice on the margin. Thus, the introduction of  $\Delta^\epsilon W(y, s)$  yields a marginal change in the agent's utility from the contract equal to

$$\begin{aligned} & \frac{dU_A(W(y) + \Delta^\epsilon W(y, s), \hat{e})}{d\epsilon}(0) = \\ & = \int_D \phi' \left( \int_{Y \times \bar{S}} u(W(y)) dF(y, s; \hat{e}, \delta) \right) \int_{Y \times \bar{S}} u'(W(y)) \frac{\partial \Delta^\epsilon W(y, s)}{\partial \epsilon} dF(y, s; \hat{e}, \delta) dG(\delta) = \end{aligned}$$

$$\begin{aligned}
&= \phi' \left( \int_Y u(W(y)) dF_Y(y; \hat{e}) \right) \int_D \int_{Y \times \bar{S}} u'(W(y)) \frac{\partial \Delta^\epsilon W(y, s)}{\partial \epsilon} dF(y, s; \hat{e}, \delta) dG(\delta) = \\
&= \phi' \left( \int_Y u(W(y)) dF_Y(y; \hat{e}) \right) \int_Y u'(W(y)) \int_D \int_{\bar{S}} \frac{\partial \Delta^\epsilon W(y, s)}{\partial \epsilon} f(y, s; \hat{e}, \delta) ds dG(\delta) dy = \\
&= \phi' \left( \int_Y u(W(y)) dF_Y(y; \hat{e}) \right) \int_Y u'(W(y)) 0 dy = 0. \tag{4.8}
\end{aligned}$$

On the other hand, the marginal change in the principal's utility from the contract is the change in the expected wage plus additional terms from the constraints. As the agent's utility does not change according to (4.8), there is no effect on the participation constraint.<sup>19</sup> Yet there is an effect on the incentive compatibility. This delivers

$$\begin{aligned}
&- \mathbb{E}_G \mathbb{E}_F \left( \frac{\Delta^\epsilon W(y, s)}{\epsilon} \right) + \\
&+ \mu \int_D \phi' \left( \int_Y u(W(y)) \frac{\partial f_Y(y; \hat{e})}{\partial e} dy \right) \int_{Y \times \bar{S}} u'(W(y)) \frac{\partial \Delta^\epsilon W(y, s)}{\partial \epsilon} f_e(y, s; \hat{e}, \delta) d(y, s) dG(\delta) = \\
&= \mu \phi' \left( \int_Y u(W(y)) \frac{\partial f_Y(y; \hat{e})}{\partial e} dy \right) \int_Y u'(W(y)) \int_{D \times \bar{S}} \frac{\partial \Delta^\epsilon W(y, s)}{\partial \epsilon} f_e(y, s; \hat{e}, \delta) ds dG(\delta) dy
\end{aligned}$$

where  $\mu$  is the Lagrange multiplier of the incentive compatibility. The multiplier is strictly positive,  $\mu > 0$ , because a constant wage is the cheapest way to satisfy the agent's participation constraint. In addition,  $\phi'(\cdot) > 0$  and  $u'(\cdot) > 0$ , as the agent's ambiguity and risk index are increasing. Rewrite the last term with the help of the definition of  $\epsilon^c$

$$\begin{aligned}
&\int_{D \times \bar{S}} \frac{\partial \Delta^\epsilon W(y, s)}{\partial \epsilon} f_e(y, s; \hat{e}, \delta) ds dG(\delta) = \tag{4.9} \\
&= \frac{1}{\epsilon} (\epsilon f_e(y, S, e) - \epsilon^c f_e(y, S^c, e)) = f(y, S, e) \left( \frac{f_e(y, S, e)}{f(y, S, e)} - \frac{f_e(y, S^c, e)}{f(y, S^c, e)} \right) > 0
\end{aligned}$$

for all  $y \in \bar{Y}$ . Here (4.7) yields the inequality. The term (4.9) equals 0 for  $y \in Y \setminus \bar{Y}$ . This shows that the principal can be made strictly better off. Redistributing some of the principal's gains to the agent ensures that both parties are strictly better off for a sufficiently small  $\epsilon$ .  $\square$

<sup>19</sup>Kellner (2010) shows that in KMM the participation constraint need not be binding in the optimal contract.



As there is second-order ambiguity aversion, giving the agent some small additional incentives makes both parties better off without increasing the ambiguity premium or the risk premium necessary to compensate the agent for the risk and ambiguity that she has to bear. With first-order ambiguity aversion, equation (4.8) is not valid, because the agent requires a positive ambiguity premium. In this case, giving the agent the appropriate incentives might be too expensive for the principal. Thus the principal might find it optimal to neglect some informative signals.

## 4.4 Extension

First-order aversion to uncertainty is not restricted to ambiguity aversion, but there are several other non-expected utility representations that incorporate behavioral insights and feature a first-order aversion to uncertainty. For example, Quiggin (1982) proposes rank-dependent preferences. This approach uses probability weights instead of probabilities to account for behavior associated with the distortion of subjective probabilities. Thus, individuals seem to overweight small probabilities and underweight high-probability events. Yet this kind of preferences can also imply first-order risk aversion. Chew et al. (1987) explore the implication of rank-dependent preferences on the order of risk aversion. Epstein and Zin (1990) use this approach, in particular the dual theory of Yaari (1987), in order to implement first-order risk aversion into their model.

Another approach is to use reference-dependent preferences, like Kőszegi and Rabin (2006). These preferences allow for loss-averse behavior and an endogenously determined reference point. In their model, agents consider not only absolute wealth or consumption, but also differences to anticipated outcomes. If the difference is negative, they feel a loss and consider this loss to be more important than a gain of equal size. Kőszegi and Rabin (2007) show that these preferences feature first-order risk aversion. The optimal contract with loss-averse agents often takes the form of a bonus contract according to Herweg et al. (2010). Thus, much performance-related information is neglected when determining the wage.

## 4.5 Related Literature

There is some literature discussing the existence of beliefs of ambiguity-averse agents and defining such beliefs. Klibanoff et al. (2011) define relevant beliefs as all the beliefs about the state of the world that the agent deems relevant. The relevant beliefs are, e.g., the set of priors in MEU and the support of the second-order distribution in KMM. Rigotti et al. (2008), on the other hand, define subjective beliefs for an act as the set of (normalized) supporting hyperplanes of the upper contour set of the act.<sup>20</sup> This requires a more restrictive monotonicity axiom than the one in Cerreia-Vioglio et al. (2011), not allowing for null events. The approach of using the (normalized) supporting hyperplanes goes back to Debreu (1972). The plausible priors of Siniscalchi (2006) are defined as the beliefs of a subjective expected-utility maximizer whose preferences coincide with the preferences of the ambiguity-averse agent on a suitable defined subset of the action space. Plausible priors are not defined in the KMM representation. Epstein (1999) also derives uncertainty-free beliefs from approximating ambiguity-neutral preferences. He works on a Savage domain and only implicitly derives these beliefs. Existence of the uncertainty-free beliefs requires additional assumptions.

The chapter also relates to the discussion initiated by Rabin (2000). As classical expected utility implies second-order risk aversion, he points out that classical expected utility seriously limits the amount of risk aversion for small stakes. Most risks we face are small compared to lifetime earnings and total wealth. Nevertheless, empirically uncertainty-averse behavior is very common, even for small stakes. Calibrating a model with second-order risk aversion with this data implies that very favorable gambles ought to be rejected if the stakes are sufficiently high. Yet, this is inconsistent with observed behavior.<sup>21</sup> First-order ambiguity aversion may be one way to describe this behavior consistently if the objects of choice are ambiguous.

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<sup>20</sup>Alternatively, Chambers and Quiggin (2007) use the subdifferential of the benefit (translation, resp.) function to define beliefs in the case of generalized risk preferences. The definitions are equivalent.

<sup>21</sup>Safra and Segal (2008) show that the argument is also valid for many non-expected utility theories. Loomes and Segal (1994), e.g., experimentally confirm this tension between actual behavior and the predictions of classical expected utility.

## 4.6 Conclusion

This chapter applies the distinction between first-order and second-order aversion by Segal and Spivak (1990) to ambiguity aversion, as Klibanoff et al. (2005, p. 1873) suggest. I provide three equivalent definitions for the distinction between first-order and second-order ambiguity aversion. For this purpose, I introduce the notion of reference beliefs of an ambiguity-averse agent. In addition, I define an ambiguity premium of an ambiguous act as the monetary equivalent of the agent's ambiguity aversion towards the act.

Then I show that second-order ambiguity aversion does not change the decision whether to accept some exposure to ambiguity compared to an ambiguity-neutral agent. This general result is applied to an investment and an insurance setting. Moreover, I study the implication of this result for contracting problems. The optimality of a constant wage and a violation of the Informativeness Principle requires first-order ambiguity aversion. In the case of second-order ambiguity aversion, some incentives will always be given. Furthermore, an adapted version of the Informativeness Principle is still valid for KMM preferences.

## Appendix to Chapter 4

### 4.A Moral Hazard in KMM

This section provides an explicit solution for a principal-agent model in KMM. Consider the setting of Section 4.3.3. The agent's output  $y$  is determined by her effort  $e \in \mathbb{R}_0^+$  and a noise term  $s \in \mathbb{R}$  with  $y = e + s$ . The random variable  $s$  is distributed according to a normal distribution with mean  $\mu$  and variance  $\sigma_1^2$ ,  $s \sim N(\mu, \sigma_1^2)$ . The expected value  $\mu$  of  $s$  is unknown, but the agent subjectively expects it to be normally distributed,  $\mu \sim N(0, \sigma_2^2)$ . The random variable might represent cyclical noise or random elements in the business cycle that have an influence on the agent's performance, but cannot be verified and the agent is not able to affect them. Effort is unverifiable and hence not contractible in contrast to output. In a general setting with KMM preferences, Kellner (2010) shows that the contract need not be monotone. In order to make closed-form solutions possible, restrict possible contracts to the class of linear wage payments,  $W(y) = \delta + \gamma y$ .<sup>22</sup> Given work effort  $e$  of the agent, the expected benefit of the principal is

$$(1 - \gamma)(e + \mathbb{E}[s]) - \delta.$$

The agent's preferences are represented by the KMM model. See Section 4.C for further details. I assume constant absolute risk aversion (CARA),  $u(x) = -\frac{1}{A} \exp(-Ax)$  with  $A > 0$ , and constant relative ambiguity aversion (CRAA),  $\phi(u) = -\frac{1}{1+B}(-u)^{1+B}$  with  $B > 0$ . This setting with CARA risk preferences and CRAA ambiguity preferences was introduced by Gollier (2011) and makes closed-form solutions possible.<sup>23</sup> On the other hand, the function  $c(\cdot)$ , the disutility of performing effort is increasing, convex and thrice differentiable with  $c'''(\cdot) \geq 0$  and the limits  $\lim_{e \rightarrow 0} c'(e) = 0$  and  $\lim_{e \rightarrow \infty} c'(e) = \infty$ , as before. The agent receives a reservation utility of  $\bar{u}$  if he rejects the principal's offer. Denote  $u^{-1}(\bar{u})$  by  $\bar{w}$ .

<sup>22</sup>Linear wages are used in many contracts, as their simplicity makes them appealing. Yet the optimal contract in this framework is not linear. Holmström and Milgrom (1987), however, show that linear contracts are optimal in a repeated setting.

<sup>23</sup>Gollier (2011) studies the comparative statics of an ambiguity-averse investor and shows that in some circumstances an increase in ambiguity aversion raises the demand for the ambiguous asset. It is an interesting question whether such a result is also possible in a principal-agent framework.

Grossman and Hart (1983) show that the model can be solved in two steps. First, for every level of effort  $e$ , the optimal wage schedule  $W^e(y)$  and its costs  $C^*(e)$  are computed. This is done in the following program

$$C^*(e) = \min_{\delta, \gamma \in \mathbb{R}} \delta + \gamma(e + \mathbb{E}[s]) \quad (4.10)$$

$$\text{subject to } \mathbb{E}\phi(\mathbb{E}u[\delta + \gamma(e + s) - c(e)]) \geq \phi(\bar{u}) \quad (\text{PC})$$

$$e \in \arg \max_{e' \in \mathbb{R}_+} \mathbb{E}\phi(\mathbb{E}u[\delta + \gamma(e' + s) - c(e')]). \quad (\text{IC})$$

Due to the participation constraint (PC), the agent is willing to accept the contract. The incentive compatibility (IC) guarantees that the agent chooses the desired level of effort. Finally, the program

$$\max_{e \in \mathbb{R}_+} e - C^*(e) \quad (4.11)$$

determines the optimal effort. Solving Program (4.11), shows that it is never optimal to pay a constant wage.

**Proposition 4.5.** *The principal pays a performance-related wage with*

$$\gamma^* = c'(e^*) > 0 \quad \text{and} \quad \delta^* = \bar{w} - c'(e^*)e^* + c(e^*) + \frac{1}{2}Ac'(e^*)^2(\sigma_1^2 + (1+B)\sigma_2^2).$$

*The optimal effort level  $e^*$  is determined by equation (4.14). Furthermore, an increase in the ambiguity or in the agent's ambiguity aversion reduces the level of incentives.*

**Proof:** First, determine the agent's utility

$$\mathbb{E}\phi(\mathbb{E}u[\delta + \gamma(e + s) - c(e)]) = \mathbb{E}\phi\left(-\frac{1}{A}\mathbb{E}\exp(-A[\delta + \gamma(e + s) - c(e)])\right).$$

Due to the properties of the exponential function, her utility equals

$$\mathbb{E}\phi\left(-\frac{1}{A}\exp(-A[\delta + \gamma e - c(e)])\mathbb{E}\exp(-A\gamma s)\right).$$

Additionally, for the normally distributed random variable  $s$ ,

$$\mathbb{E}\exp(a + bs) = \exp\left(a + b\mu + \frac{1}{2}b^2\sigma_1^2\right), \quad (4.12)$$

because the Arrow-Pratt approximation is exact for the combination of constant absolute risk aversion and normally distributed random variables. Then the agent's utility is

$$\mathbb{E}\phi\left(-\frac{1}{A}\exp(-A[\delta + \gamma e - c(e)])\exp(-A\gamma\mu + \frac{1}{2}A^2\gamma^2\sigma_1^2)\right).$$

Rearranging and using the functional form of  $\phi$  leads to

$$\mathbb{E}\left(-\frac{1}{1+B}\frac{1}{A}^{1+B}\exp[-A(1+B)(\delta + \gamma e - c(e) - \frac{1}{2}A\gamma^2\sigma_1^2)]\exp(-A(1+B)\gamma\mu)\right).$$

To compute the expectation, use equation (4.12) again. This results in

$$-\frac{1}{1+B}\frac{1}{A}^{1+B}\exp\left(-A(1+B)(\delta + \gamma e - c(e) - \frac{1}{2}A\gamma^2\sigma_1^2 - \frac{1}{2}A(1+B)\gamma^2\sigma_2^2)\right). \quad (4.13)$$

The agent maximizes her utility with respect to  $e$ . Therefore the agent chooses his effort  $e$ , such that  $c'(e) = \gamma^*$ . Consequently, every  $e$  can be implemented by setting  $\gamma^* = c'(e)$ . The principal determines  $\delta^*$ , so that the agent is indifferent between accepting and rejecting the contract. Therefore she chooses

$$\delta^* = \bar{w} - c'(e)e + c(e) + \frac{1}{2}Ac'(e)^2(\sigma_1^2 + (1+B)\sigma_2^2)$$

This completes the first step, because the proof has now determined the cost-minimizing  $\gamma^*$  and  $\delta^*$  to implement any  $e$ . In summary, the costs for the principal of implementing effort  $e$  are

$$C^*(e) = \delta^* + \gamma^*e = \bar{w} + c(e) + \frac{1}{2}Ac'(e)^2(\sigma_1^2 + \sigma_2^2 + B\sigma_2^2).$$

The second step calculates the optimal  $e^*$  by solving Program (4.11). This results in

$$1 = c'(e^*)(1 + c''(e^*)A(\sigma_1^2 + \sigma_2^2 + B\sigma_2^2)). \quad (4.14)$$

The assumptions on  $c(\cdot)$  and  $A(\sigma_1^2 + \sigma_2^2 + B\sigma_2^2) > 0$  guarantee that there is an interior solution for  $e^*$ . Moreover, the second-order condition is satisfied, as  $-c''(e^*) - (c''^2(e^*) + c'(e^*)c'''(e^*))A(\sigma_1^2 + (1+B)\sigma_2^2) < 0$ . Consequently,  $e^* > 0$  and this results

in a performance-related wage with  $\gamma^* = c'(e) > 0$ , which is the first part of the proposition.

The more ambiguity there is, the higher is  $\sigma_2^2$  and correspondingly the lower is  $e^*$ , because it becomes more expensive to compensate the agent. To show this, notice that  $\text{sgn}(\partial\gamma^*/\partial\sigma_2^2) = \text{sgn}(\partial e^*/\partial\sigma_2^2)$  and by the implicit function theorem

$$\partial e^*/\partial\sigma_2^2 = -\frac{c'(e^*)c''(e^*)A(1+B)}{c''(e^*) + (c''^2(e^*) + c'(e^*)c'''(e^*))A(\sigma_1^2 + \sigma_2^2 + B\sigma_2^2)} < 0.$$

This proves the second part of the proposition. The same comparative statics hold with respect to the degree of ambiguity aversion  $B$ .  $\square$

It is interesting to scrutinize the agent's certainty equivalent revealed by equation (4.13). The first part  $\delta + \gamma e - c(e)$  is the expected wage minus the effort costs. Yet this is reduced by two terms. The first is the risk premium  $\pi = \frac{1}{2}A\gamma^2(\sigma_1^2 + \sigma_2^2)$ , as the wage is risky. The second term corresponds to  $\pi_A = \frac{1}{2}AB\gamma^2\sigma_2^2$ , the ambiguity premium. The ambiguity premium is proportional to  $\sigma_2^2$ . This confirms the second-order ambiguity aversion. If the ambiguity gets very small and  $\sigma_2 \rightarrow 0$ , the derivative of the ambiguity premium with respect to  $\sigma_2$  goes to 0. This is different to a model with first-order ambiguity aversion, like MEU preferences. There the limit of the derivative of the ambiguity premium is positive.

For better illustration, assume MEU preferences,  $\sigma_2^2 = 0$  and the mean of the random variable  $s$  from the interval  $[\underline{\mu}, \bar{\mu}]$  with  $\underline{\mu} < 0 < \bar{\mu}$ .<sup>24</sup> Then with a linear wage the certainty equivalent of the wage payment to the agent is

$$\delta + \gamma e - c(e) - \frac{1}{2}A\gamma^2\sigma_1^2 + \gamma\underline{\mu}.$$

Thus, the ambiguity premium is  $\pi_A = -\gamma(\underline{\mu} - \mu^N)$  with  $\mu^N$  the expectation of the noise term under a reference belief. This premium is of first order and sometimes a constant wage is optimal.

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<sup>24</sup>See Weinschenk (2010), Proposition 1, for a nice exposition of such a model.

## 4.B Orders of Risk Aversion

Consider a classical expected utility representation with a twice differentiable von-Neumann-Morgenstern utility index and a lottery  $l$ . Then the risk premium is proportional to the variance for small risks, as already recognized by Pratt (1964, p. 126).<sup>25</sup> Consequently, the risk premium only contains terms of second order. This yields risk-neutral behavior for small stakes and is called second-order risk aversion. Segal and Spivak (1990) introduce the notion of first-order risk aversion if the risk premium is proportional to the standard deviation.

Their definition works as follows. The lottery  $l$  is distributed according to  $F$  and  $\mathbb{E}_F(l) = 0$ . Assume a risk-averse individual with the choice between a zero payoff and the lottery  $l$ . Given her risk aversion, the agent prefers the certain payoff to the lottery,  $0 \succ l$ . If preferences are continuous in wealth, there exists a payment  $\pi$ , such that the individual is indifferent between  $-\pi$  and the lottery  $l$ .

$$0 - \pi \sim l$$

$\pi$  is the risk premium of the lottery  $l$ .<sup>26</sup> To capture small risks, consider  $t \geq 0$  and the function  $\pi(t): \mathbb{R}_+^0 \mapsto \mathbb{R}_+^0$  implicitly defined by  $0 - \pi(t) \sim tl$ . Thus, the payment  $\pi(t)$  makes the individual indifferent between the lottery  $tl$  and the zero payoff. If the risk disappears, the risk premium vanishes and  $\pi(t) \rightarrow 0$  for  $t \rightarrow 0$ . According to Segal and Spivak (1990), the agent exhibits first-order risk aversion if

$$\lim_{t \rightarrow 0} \pi(t)/t > 0.$$

On the other hand, she is second-order risk-averse if the risk premium is proportional to the variance and declines quadratically. Then

$$\lim_{t \rightarrow 0} \pi(t)/t = 0 \quad \text{and} \quad \lim_{t \rightarrow 0} \pi(t)/t^2 > 0.$$

It is common to subsume all higher risk orders under second-order risk aversion and

<sup>25</sup>See Hellwig (2004) for the multidimensional case.

<sup>26</sup>There are different definition of the risk premium. Sometimes it is the transfer that is added to the lottery in order to make the agent indifferent to the zero payoff. See also Schlesinger's (1997) footnote 2 for a discussion of the different definitions.



neglect the second condition. Consequently, an individual is second-order risk-averse if and only if she behaves in a risk-neutral way with regard to infinitesimal risks. If the risk premium is differentiable for  $t \in (0, a)$  with  $a > 0$ , the definition can be stated equivalently in terms of derivatives.

$$\lim_{t \rightarrow 0} \pi'(t) \begin{cases} > 0 & \text{for first-order risk aversion} \\ = 0 & \text{for second-order risk aversion} \end{cases}$$

Segal and Spivak (1990) prove that expected utility theory with a differentiable utility index implies second-order risk aversion. If the utility index is nondifferentiable at a wealth  $w$ , expected utility theory features first-order risk aversion. Risk aversion implies a concave utility index  $u$ . Hence, there exist the right and the left derivative of  $u$  at  $w$  and

$$\pi'(t)|_{t=0^+} = \left(1 - \frac{u'_+(w)}{u'_-(w)}\right) \int_{\{x \in X | l(x) > 0\}} l(x) dF(x).$$

The same holds in the setting of Machina (1982), where local utility functions allow approximating non-standard behavior in an expected-utility framework. Segal and Spivak (1997) show that also in this case, non-differentiability of the local utility functions is equivalent to first-order risk aversion.

## 4.C Smooth Ambiguity Aversion

Klibanoff et al. (2005) propose a model to represent preferences with second-order ambiguity aversion. Their approach goes back to Segal (1987). The agent knows the first-order and second-order probability measures, but does not compute the reduced lottery. The first-order probability measure is a measure for the states of the world, i.e., the state space. The second-order probability measure, on the other hand, reflects the probabilities for a first-order measure. In their interpretation, the first-order measure characterizes risk, and the second-order measure ambiguity. This distinction corresponds to the assumption that the first-order and second-order probabilities are

based on different information. The preferences  $\succeq$  of the agent are represented by

$$U(f) = \int_{\Pi} \phi \left( \int_X (u \circ f) dP \right) d\mu.$$

The function  $\phi$  reveals the attitude of the agent towards ambiguity. An ambiguity-neutral individual with a linear function  $\phi$  calculates the expectation and derives simple probabilities for each state of the world. With ambiguity aversion, the function  $\phi$  is concave. The function  $u$  is a von-Neumann-Morgenstern utility index and determines the attitude towards risk. In addition,  $P$  is a probability measure on the state space and  $\Pi$  is a set of first-order probability measures.  $\mu$  is a probability measure that represents the second-order measure. The preference functional may be interpreted as a double expectation. First, the expected utility for every first-order measure  $P$  is calculated. Then, the expected utility for every  $P$  is transformed by the function  $\phi$ . Finally, the expectation regarding the second-order probabilities  $\mu$  is calculated.

## 4.D Choquet Expected Utility

CEU of Schmeidler (1989) exhibits first-order ambiguity aversion. This representation is based on capacities. Capacities mathematically model probability intervals and are therefore non-additive.<sup>27</sup> Let  $\Omega$  be a non-empty set and  $\mathcal{A}$  a  $\sigma$ -algebra of subsets of  $\Omega$ . A capacity  $v$  is a function  $\mathcal{A} \rightarrow \mathbb{R}$  which maps sets into the real numbers with the following properties:

- $v(\emptyset) = 0$  and  $v(\Omega) = 1$
- $A \subset B \Rightarrow v(A) \leq v(B) \quad \forall A, B \in \mathcal{A}$

Consequently, capacities have values between 0 and 1. Ambiguity aversion corresponds to the convexity of a capacity. A capacity  $v$  is convex if

$$v(A \cup B) \geq v(A) + v(B) - v(A \cap B) \text{ for all } A, B \in \mathcal{A}.$$

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<sup>27</sup>Sometimes capacities are also called non-additive probability measures or Choquet measures.

Calculating expectations regarding a capacity requires integrals regarding a capacity. Choquet (1953/4) introduced these integrals. Let  $f$  be a  $\mathcal{A}$ -measurable and bounded function  $\Omega \rightarrow \mathbb{R}$ . Then the Choquet integral of  $f$  regarding  $(\Omega, \mathcal{A}, v)$  is defined as

$$\int_{\Omega} f dv = \int_0^{\infty} v(f \geq t) dt + \int_{-\infty}^0 v(f \geq t) - 1 dt.$$

$v(f \geq t)$  stands for  $v(\{\omega \in \Omega | f(\omega) \geq t\})$  and is a function  $\mathbb{R} \rightarrow [0, 1]$  of  $t$ . Accordingly, the Lebesgue integrals on the right-hand side are well defined. If  $v$  is a probability measure, the Choquet integral equals the Lebesgue integral regarding the measure  $v$ .

Schmeidler (1989) was the first to propose an axiomatization of the CEU representation. He uses the common decision-theoretic axioms, except that the independence axiom has to hold only for comonotonic functions. This is less restrictive than independence for all acts. So, according to the representation, there exists a utility function  $u$  and a capacity  $v$ , such that for two acts  $f$  and  $g$

$$f \succeq g \iff \int (u \circ f) dv \geq \int (u \circ g) dv.$$

To see the first-order ambiguity aversion in this representation, consider the following example. Assume an ambiguity-averse, but risk-neutral individual. She can bet on a coin flip of an unknown coin. With heads up, she wins  $t$  consumption goods. For tails she loses  $t$  goods. As the individual has no information regarding the coin, her capacity assigns each side the same value, i.e.,  $v(\text{heads}) = v(\text{tails}) = (1 - \alpha)\frac{1}{2}$ .  $\alpha \in [0, 1]$  captures the suspicion about the fairness of the coin, which corresponds to the amount of ambiguity and the degree of ambiguity aversion at the same time. Then the Choquet Expected Utility of the act is

$$t(1 - \alpha)\frac{1}{2} + ((1 - \alpha)\frac{1}{2} + \alpha)(-t) = -\alpha t$$

Consequently, the ambiguity premium  $\pi_A(t)$  is  $(\alpha - 2p + 1)t$  with a reference belief  $p \in [(1 - \alpha)\frac{1}{2}, (1 + \alpha)\frac{1}{2}]$  according to Definition (4.1). Suppose the agent is ambiguity averse and the coin is ambiguous, i.e.,  $\alpha > 0$ . Then, the derivative of the ambiguity premium is positive  $\pi'_A(t) = \alpha - 2p + 1 > 0$ , except for the reference belief  $p = (1 - \alpha)\frac{1}{2}$ . In this case, the opposite act yields a positive derivative.



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