Essays on Convergence and Synchronization

Inaugural-Dissertation
zur Erlangung des Grades eines Doktors
der Wirtschafts- und Gesellschaftswissenschaften
durch die
Rechts- und Staatswissenschaftliche Fakultät
der Rheinischen Friedrich-Wilhelms-Universität
Bonn

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Bonn 2014

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Tag der mündlichen Prüfung: 19.12.2013

Diese Dissertation ist auf dem Hochschulschriftenserver der ULB Bonn (http://hss.ulb.uni-bonn.de/diss_online) elektronisch publiziert.

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Introduction

Convergence of macroeconomic aggregates and subsequent synchronization of the cyclical features affecting these aggregates have been at the heart of economic debate for quite a while. Especially with the recent developments concerning the creation and crisis of a European common currency area, the question whether counties are similar enough to be targeted by uniform policy rules has been a crucial one. In fact, the "convergence criteria" provisions of Article 140(1) of the Treaty on the Functioning of the European Union[1], outlining desired levels of similarity and rates of adjustment, are among the most widely known and publicly discussed parts of European Union legislation. In addition to this prominent example, the policy goal of increased convergence and synchronization is the focus of various important EU institutions, such as the Stability and Growth Pact, the European Regional Development Fund, the European Social Fund, or the Cohesion Fund.

However, the issue is not only limited to a practitioner's point of view. Among the key aspects of the neoclassical growth model are the consequences regarding cross-country convergence. If economies are alike with respect to microeconomic characteristics, such as preferences and technology, then poor economies tend to grow faster than rich ones, closing the gap between them. A broad strand of literature sparked by the theoretical studies of Romer (1986) and Lucas (1988) considers the long-term behavior in the dynamic equilibrium model.

[1]Treaty on the Functioning of the European Union (Consolidated Version 2012) as given in the Official Journal of the Europen Union, CELEX number: 12012E001-12012E358

Beyond this relevance to an economist, the phenomena are also interesting from an econometric perspective. Convergence requires comovement of economic time series as the ultimate goal, but – with series displaying unequal initial levels – its analysis has to be concerned with the path leading there, too. The former aspect can be captured by well-known and widely-employed techniques such as cointegration analysis. Yet the latter part requires a more refined concept of series starting off differently but becoming more alike. In the same manner, assessing synchronization requires an advanced econometric treatment. Essentially, two dimensions of the data have to be addressed simultaneously. While the investigation whether cyclical components of output have the same length is a question alluding to the frequency domain, the localization in time is just as important.

This thesis contributes to the understanding of convergence and synchronization in various ways. The first two chapters are devoted to the task of capturing convergence more adequately than just through cointegration analysis. In the first chapter, the performance a regression test which allows for transitory divergence is assessed using artificially generated data. The second chapter empirically investigates the convergence of long-term interest rates using a methodology that allows for different initial levels of the time series. Finally, the last chapter introduces a way of reconciling time and frequency aspects for the synchronization of output time series using wavelet analysis.

A more detailed description of each of the chapters, which are self-contained, is provided in the remainder of this introduction.

Chapter 1 evaluates a novel approach to capture economic convergence as a process of transition. It considers the methodology suggested by Phillips and Sul (2007), who propose a new panel data model. Their framework explicitly allows for periods of transitional divergence by separating a common component from idiosyncratic fluctuations. In particular, they present a regression-based convergence test that relies on the loadings of a time-varying factor model. Unlike cointegration approaches, this allows for the analysis of long run convergence while still allowing for temporary heterogeneity.

The chapter investigates the performance of Phillips and Sul's test in a classical setting of time series convergence, considering both asymptotic results and Monte Carlo simulation methods. These results are put into perspective by comparing them to the cointegration-based test of convergence from Nyblom and Harvey (2000) as a benchmark.

It turns out that in a setting where the time dimension considerably exceeds the cross-sectional dimension, the performance of the regression test is inferior to that of the benchmark test, although it does have some ability to discriminate between convergent and non-convergent data. The advantage of the regression is its applicability to a wider range of applications, especially when the number of cross-sectional units goes beyond what the cointegration-based test can handle.

Chapter 2 considers a different way of incorporating transitory periods into convergence analysis. It argues that convergence is a dynamic process that can better be captured by considering changes in persistence. Namely, if two series are convergent, the gap between them should change from I(1) to I(0). Allowing for a unit root in the initial phase permits heterogeneity in the series before a long run equilibrium is reached and thus serves as a better model of the convergence implied by increasing economic integration for previously unequal economies. A further benefit of the approach is the ability to identify departures from stability by checking for persistence changes in the opposite direction. Additionally, while the traditional tests are by construction limited to pairs or very small sets of countries, the persistence change approach can easily be extended to larger groups.

Tests for changes in persistence are applied for interest rate differentials on long-term government bonds for a broad set of countries. The convergence of interest rates has been considered as a key aspect of economic integration, especially with regard to the introduction of the Euro. Overwhelming evidence for convergence in interest rates can be found when considering country pairs or groups now using the Euro. The change date towards a stable relationship is located closely before the introduction of the common currency. For many other pairs and groupings, a similar change in persistence can be dated to the early 1980s, the time popularly associated with the Great Moderation. To the contrary, very little evidence can be found for previously stable relationships beginning to diverge.

Chapter 3 puts the focus on comovement between output series. This is a key concept in macroeconomic analysis because the extent to which series are cyclically synchronized is particularly important for evaluating the feasibility of common policy measures for a group of countries. However, such an analysis is prone to many pitfalls. There are several concepts of what is actually called the cyclical component of a series, leading to different possible extraction techniques. At the same time, it is necessary to reconcile the investigation in the time and the frequency domains.

The chapter first considers various concepts based in either the time or the frequency domain to detect and describe such cyclical comovements in output data for a group of core economies. However, methods from the former category cannot account for different cycle lengths, while the statistics from the latter category fail to capture transient relationships. Therefore, the use of multivariate wavelet analysis and a modification of the cohesion statistic from Fourier analysis is suggested to simultaneously assess comovement at the frequency level and over time. The main finding is that synchronization does indeed vary with cycle length and that it has been affected by events during the time span of the sample, such as the introduction of the Euro. A further benefit of the wavelet approach turns out to be that it is hardly sensitive to the technique employed to extract the cyclical component from the output series.

Chapter 1

Analyzing the Performance of Regression-based Versus Cointegration-based Convergence Tests

1.1 Introduction

In the past years, many empirical studies have investigated the question whether poor regions of the world on average grow faster than richer regions, that is, whether they tend to catch up concerning per capita income or output levels. Barro and Sala-i-Martin (1991) point out various constellations for which this question is of particular interest, such as the South of Italy growing faster to attain the same prosperity as the North, or the East of Germany catching up to the West after its regime change and reunification. If the poorer part in these pairs is actually catching up, one should be able to witness this as a convergence process over time in the variables.

The underlying framework for studies targeted at this question has often been that developed by Barro and Sala-i-Martin (1991, 1992). These authors introduce two basic notions of convergence. Beta convergence focuses on the aspect that in the case of convergence, the initial level of income needs to be inversely related to its average growth rate and hence the coefficient β in a regression of one on the other should be negative. Sigma convergence, on the other hand, is concerned with the evolution of the cross-sectional dispersion over time, requiring it to become smaller.

Most empirical applications have considered the former definition of (beta) convergence and proceeded to test for a cointegrating relationship between the series at hand. For example, Choi (2004) conducts a study with this methodology for U.S. states, while Bernard and Durlauf (1995) do the same for OECD countries. Others have shifted the focus from output levels to other macroeconomic variables, such as the evolution of interest rates or inflation. Brüggemann and Lütkepohl (2005) test for cointegration between short- and long-term interest rates for various countries, while Kirchgässner and Wolters (1990, 1993) look at convergent inflation for major world economies. While series such as GDP are often hypothesized to involve a deterministic time trend, this is not necessarily the case for the latter time series. Thus a test striving to be applicable to a large number of situations should be able to account for both processes with a drift term and those without.

Another main issue in determining whether a process converges is the possibility of temporary divergence for some or all of the series under investigation. Essentially, it is possible that short-run dynamics are present which lead into the opposite direction of the long-run behavior of the series. Phillips and Sul (2007) suggest an alternative method to account for this aspect. Their regression test of convergence is based on a panel data model that represents the economies in transition and allows for both common and idiosyncratic factors for the individuals. The key issue of their work thus is the advanced treatment of heterogeneity in the panel data with the intuitive idea behind the test being that different idiosyncratic factor loadings should converge to a constant.

It seems worthwhile to investigate how well this concept aligns with the previous approaches based on unit root and cointegration testing and whether data which is convergent according to the traditional concepts also passes the convergence test proposed by Phillips and Sul. For that purpose, time series consisting of both a common and an idiosyncratic component are constructed and consequently subjected to the regression test. In particular, the effects of including or omitting a deterministic time trend are investigated asymptotically and using a Monte Carlo simulation for small sample properties. The results are then compared to those of a cointegration-based test introduced by Nyblom and Harvey (2000), which features identical null and alternative hypotheses.

The further setup of the chapter is as follows. In the next section, the regression test of Phillips and Sul (2007) is introduced. Section 1.3 describes the data generating process for the convergent series. Section 1.4 shows the asymptotic behavior of the regression test under the different data generating processes, while section 1.5 introduces the benchmark test of Nyblom and Harvey (2000). Empirical results based on Monte Carlo simulations for both tests are presented in section 1.6. Finally, section 1.7 concludes.

1.2 A Regression Test of Convergence

1.2.1 The Framework

Phillips and Sul (2007) propose a regression-based test for the convergence of time series in a panel of data. Their model is a time-varying factor model that includes both common and individual-specific components. Thus, idiosyncratic behavior is accounted for while at the same time a common component is maintained across the panel. To that end, they employ the widely used decomposition of panel data $X_{i,t}$ into systematic components $g_{i,t}$ and transitory components $a_{i,t}$:

$$X_{i,t} = g_{i,t} + a_{i,t}. \tag{1.1}$$

As this description permits both common and idiosyncratic components to be either in $g_{i,t}$, or $a_{i,t}$, or both, Phillips and Sul employ the transformation

$$X_{i,t} = \left(\frac{g_{i,t} + a_{i,t}}{\mu_t} \right) \mu_t = \delta_{i,t}\mu_t, \tag{1.2}$$

where $\delta_{i,t}$ is the individual distance from the common trend μ_t and hence a clear distinction between common and idiosyncratic components becomes apparent. Compared to a standard single factor model with individual-specific factor loading δ_i, the formulation in (1.2) is an extension including dynamics of factor loadings over time. Convergence then occurs if $\delta_{i,t} \longrightarrow \delta \ \forall i$ as $t \rightarrow \infty$.

This approach is claimed to be particularly useful because it requires no assumptions on the (non-)stationarity of $X_{i,t}$ or μ_t. Furthermore, a wide range of possibilities for the time path of $\delta_{i,t}$ is available. For the purpose of their test, Phillips and Sul (2007) model $\delta_{i,t}$ as

$$\delta_{i,t} = \delta_i + \sigma_i \xi_{i,t} L(t)^{-1} t^{-\alpha}. \tag{1.3}$$

$L(t)$ here represents a slowly varying function that is required to be increasing and divergent at infinity. Possible choices for $L(t)$ would hence be $\log(t+1)$ or $\log(\log(t+1))$. The purpose of including this function is to ensure that $\delta_{i,t} \longrightarrow \delta_i$ also in case $\alpha = 0$. With this setting, convergence is thus present if $\delta_i = \delta$ and $\alpha \geq 0$, while series diverge if $\delta_i \neq \delta$ or $\alpha < 0$.

As a relative measure for the evolution of $\delta_{i,t}$ Phillips and Sul (2007) furthermore introduce a transition parameter $h_{i,t}$. This parameter is directly derived from the data $X_{i,t}$ and is a functional of $\delta_{i,t}$ because the μ_t, being the common factor in equation (1.2), drops out in both numerator and denominator:

$$h_{i,t} = \frac{X_{i,t}}{\frac{1}{N}\sum_{i=1}^{N} X_{i,t}} = \frac{\delta_{i,t}}{\frac{1}{N}\sum_{i=1}^{N} \delta_{i,t}}. \tag{1.4}$$

The difference in using $h_{i,t}$ rather than $\delta_{i,t}$ is that in equation (1.4) the relative transition path for economy i is considered by comparing its transition to the panel average. Alluding to this interpretation of $h_{i,t}$, it is called the *relative transition parameter* by Phillips and Sul (2005). Since the cross-sectional average of $h_{i,t}$ is unity by construction, under convergence of factor loading coefficients $\delta_{i,t}$ to δ, it holds that $h_{i,t} \longrightarrow 1$.

1.2.2 The Testing Procedure

The test Phillips and Sul (2007) propose is a regression test of

$$\mathcal{H}_0 : \delta_i = \delta \text{ and } \alpha \geq 0 \tag{1.5}$$

as null hypothesis of convergence versus the alternative

$$\mathcal{H}_1 : \delta_i \neq \delta \text{ or } \alpha < 0. \tag{1.6}$$

The testing procedure consists of three steps.

First, the relative transition parameters $h_{i,t}$ defined in (1.4) are used to construct the ratio

$$\frac{H_1}{H_t} \text{ where } H_t = \frac{1}{N}\sum_{i=1}^{N}(h_{i,t}-1)^2.$$

Using this ratio, secondly, the following regression is run:

$$\log\left(\frac{H_1}{H_t}\right) - 2\log L(t) = \hat{a} + \hat{b}\log t + \hat{u}_t. \tag{1.7}$$

As before, $L(t)$ represents a slowly varying function. In the context of the test, it is set to $L(t) = \log(t+1)$ and serves as a penalty that helps to distinguish the null from the alternative hypothesis and to be consistent even when $\alpha = 0$. Furthermore, $\hat{b} = 2\hat{\alpha}$, with $\hat{\alpha}$ being the estimate of α in $\mathcal{H}_0$. The first 30% of observations are discarded. Phillips and Sul claim that this shifts the focus of the test towards the behavior in large samples and derive the choice of 30% from their simulations.

In the third and final step, a one sided t-test of $\hat{b} \geq 0$ against $\hat{b} < 0$ is performed using heteroscedasticity and autocorrelation consistent standard errors. With help of the standard critical values for t-tests one can thus determine whether or not to reject the null hypothesis of convergence. In particular, using a typical significance level of 5%, the convergence null has to be rejected for $t_{\hat{b}} < -1.65$.

1.3 Data Generation

The regression test proposed by Phillips and Sul (2007) and outlined in the previous section is applied to regional U.S. data, OECD data, and data from the Penn World Table in some empirical work by the same authors (Phillips and Sul, 2005). For the U.S. and OECD data sets the null of no convergence is rejected, while for the Penn World Table data set rejecting the null is not possible. Phillips and Sul (2005) attribute this to the large cross-sectional dimension for which the test is claimed to overreject. In order to further investigate the issue, this section proposes data generating processes (DGPs) to create artificial data according to a time series based definition of convergence. The data gained from these DGPs can then be used to assess the test's performance in settings involving convergence and non-convergence.

The concept of convergence introduced by Bernard and Durlauf (1995) considers series $y_{i,t}$ and $y_{j,t}$ for regions i and j. Convergence between those two series occurs if

$$\lim_{h\to\infty} \mathrm{E}(y_{i,t+h} - y_{j,t+h}|\mathfrak{F}_t) = \gamma. \tag{1.8}$$

With $\gamma = 0$, convergence thus corresponds to the equality of long-term forecasts of the series for regions i and j. This situation of equation (1.8) with the restriction to $\gamma = 0$ has been termed "strong convergence" by Bernard and Durlauf (1996), while "weak convergence" would allow γ to be a non-zero constant as well. A multivariate version of the definition is given by

$$\lim_{h\to\infty} \mathrm{E}(y_{1,t+h} - y_{n,t+h}|\mathfrak{F}_t) = 0 \qquad \forall n \neq 1. \tag{1.9}$$

The requirements for convergence are thus satisfied as soon as $y_{1,t+h} - y_{n,t+h}$ is a stationary process with mean zero. If the original series are given by unit root processes, unit root or cointegration tests can be used to test for convergence, with a cointegration relationship $y_{i,t} - y_{j,t} \sim I(0)$ and cointegrating vector $[1, -1]$ required for convergent series in the bivariate case of equation (1.8). This has been the standard practice of most empirical work on growth convergence. It should be noted that this notion of testing for convergence entirely corresponds to the beta convergence in the terminology of Barro and Sala-i-Martin (1991) since it is only concerned with the comovement of the series and not with the development of their volatility through time.

Phillips and Sul (2007) claim that their test is superior to previous approaches by explicitly allowing for transitional periods and even temporary divergence across regions through the formulation of time-varying factor loadings. Thus, the evaluation of the test's performance will consider this specific pattern by considering the following data generating processes for convergent series. Since both series with and without deterministic time drift can be relevant in practice, two distinct DGPs are used, accounting for the two options.

Convergent data $y_{i,t}^{nd}$ without a drift are generated according to

$$\begin{aligned} y_{i,t}^{nd} &= \phi r_t + u_{i,t} \\ u_{i,t} &= \rho u_{i,t-1} + \varepsilon_{i,t}. \end{aligned} \tag{1.10}$$

In the same manner, convergent data $y_{i,t}^{d}$ with a drift are generated according to

$$\begin{aligned} y_{i,t}^{d} &= \psi t + \phi r_t + u_{i,t} \\ u_{i,t} &= \rho u_{i,t-1} + \varepsilon_{i,t}. \end{aligned} \tag{1.11}$$

In both settings, r_t is a random walk with $r_0 = c < \infty$, and $\varepsilon_{i,t}$ is a white noise process with $\mathrm{E}[\varepsilon_{i,t}] = 0$ and $\mathrm{E}[\varepsilon_{i,t}]^2 = \sigma^2$. All series are thus affected by a common random walk component, but are allowed to fluctuate around it by means of the idiosyncratic AR(1) processes for u_t. In the case of DGP (1.11), an additional common component is the time trend ψt.

In a similar way, non-convergent series are generated. The structure of the DGPs remains the same, but now the random walks are individual-specific rather than common to all series. That is, the two DGPs for non-convergent data read

$$\begin{aligned} y_{i,t}^{nd*} &= \phi r_{i,t} + u_{i,t} \\ u_{i,t} &= \rho u_{i,t-1} + \varepsilon_{i,t}, \end{aligned} \tag{1.12}$$

$$\begin{aligned} y_{i,t}^{d*} &= \psi t + \phi r_{i,t} + u_{i,t} \\ u_{i,t} &= \rho u_{i,t-1} + \varepsilon_{i,t}. \end{aligned} \tag{1.13}$$

Now, $r_{i,t}$ are i different random walks, each with $r_{i,0} = c < \infty$. The conditions for $\varepsilon_{i,t}$ remain the same. Thus, the data generated according to DGP (1.12) do not contain any common component, while those following DGP (1.13) have the same time trend, but are otherwise unrelated. To reduce dependence on the starting value of the random walk, the first 30% of the generated series under all four DGPs are considered as a burn-in phase and cut off.

1.4 Asymptotic Properties

Phillips and Sul's (2007) test requires that the ratio $\log\left(\frac{H_1}{H_t}\right)$ in (1.7) diverges to infinity under the null. This occurs either as $2\log L(t) \to \infty$ when $\alpha = 0$, albeit very slowly, or as $2\alpha \log t \to \infty$ when $\alpha > 0$ in the regression equation. In contrast, under the alternative hypothesis of no convergence, this ratio is shown to converge to a positive constant as $t \to \infty$ (Phillips and Sul, 2007, Appendix B). This section investigates whether these properties actually hold for the DGPs at hand.

Recall that the relative transition parameter $h_{i,t}$ is given by

$$h_{i,t} = \frac{\delta_{i,t}}{\frac{1}{N}\sum_{i=1}^{N}\delta_{i,t}} = \frac{X_{i,t}}{\frac{1}{N}\sum_{i=1}^{N}X_{i,t}}.$$

In the case of DGP (1.10), i.e. convergent data without drift, $X_{i,t} = \phi r_t + u_{i,t}$ is the numerator, while in the denominator, ϕr_t (which does not vary over cross-sections) can be taken out of the sum: $\frac{1}{N}\sum_{i=1}^{N} X_{i,t} = \phi r_t + \frac{1}{N}\sum_{i=1}^{N} u_{i,t}$. In combination, this amounts to

$$\begin{aligned} h_{i,t} &= \frac{\phi r_t + u_{i,t}}{\phi r_t + \frac{1}{N}\sum_{i=1}^{N} u_{i,t}} \\ &= \frac{\phi r_t + u_{i,t}}{\phi r_t + \bar{u}_t}, \end{aligned}$$

where a bar above variables denotes the cross-sectional average. Consequently, the cross-sectional variance term H_t reads

$$\begin{aligned} H_t &= \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\phi r_t + u_{i,t}}{\phi r_t + \bar{u}_t} - 1\right)^2 \\ &= \frac{1}{N}\sum_{i=1}^{N}\frac{(u_{i,t} - \bar{u}_t)^2}{(\phi r_t + \bar{u}_t)^2} \\ &= \frac{\sigma_{u_t}^2}{(\phi r_t + \bar{u}_t)^2}, \end{aligned}$$

Concerning the ratio $\log\left(\frac{H_1}{H_t}\right)$ used in the test regression and considering that the variance of u_t is constant over time because the autoregressive error process is stationary, this implies

$$\log\left(\frac{H_1}{H_t}\right) = \log\left(\frac{\frac{\sigma_u^2}{(\phi r_1 + \bar{u}_1)^2}}{\frac{\sigma_u^2}{(\phi r_t + \bar{u}_t)^2}}\right) = \log\frac{(\phi r_t + \bar{u}_t)^2}{(\phi r_1 + \bar{u}_1)^2} \tag{1.14}$$

In equation (1.14), the denominator just consists of the random walk and the average error term in period $t = 1$, hence that part does not vary with t. In the numerator, however, the dominating part is r_t, which grows at rate $\sqrt{t}$ as $t \to \infty$. The entire ratio then is of order $\mathcal{O}_p(\log T)$ in the notation of Mann and Wald (1943), satisfying the requirement that it has to diverge under the null.

The same calculation with DGP (1.11) and hence a drift term included in $X_{i,t}$ yields

$$\begin{aligned} h_{i,t} &= \frac{\psi t + \phi r_t + u_{i,t}}{\psi t + \phi r_t + \bar{u}_t}, \\ H_t &= \frac{\sigma^2_{u_t}}{(\psi t + \phi r_t + \bar{u}_t)^2}, \quad \text{and} \\ \log\left(\frac{H_1}{H_t}\right) &= \log \frac{(\psi t + \phi r_t + \bar{u}_t)^2}{(\psi + \phi r_1 + \bar{u}_1)^2} \end{aligned} \tag{1.15}$$

Here, the dominating term in the numerator is the time trend, which grows at rate t rather than $\sqrt{t}$ in the situation without drift. Combined with the denominator, which is the realization at $t = 1$ again, the entire ratio is of order $\mathcal{O}_p(\log T^2)$. Since $\log T^2 = 2\log T$, however, this differs from $\log T$ only by a constant factor, which is disregarded for the limiting behavior. Hence, divergence occurs in the same manner and at the same speed as for the case in equation (1.14). The inclusion of a drift in the DGP should thus not affect the test's performance under the null.

In the same manner, the properties of the ratio $\log\left(\frac{H_1}{H_t}\right)$ can be analyzed when non-convergent data without a drift, that is, data generated according to DGP (1.12) are used. The relative transition parameter then reads

$$h_{i,t} = \frac{\phi r_{i,t} + u_{i,t}}{\frac{1}{N}\sum_{i=1}^{N}(\phi r_{i,t} + u_{i,t})} = \frac{\phi r_{i,t} + u_{i,t}}{\phi \bar{r}_t + \bar{u}_t}$$

since both $r_{i,t}$ and $u_{i,t}$ in the denominator can be averaged over the cross-section. Consequently,

$$\begin{aligned} H_t &= \frac{1}{N}\sum_{i=1}^{N} \frac{(\phi r_{i,t} + u_{i,t} - \phi \bar{r}_t - \bar{u}_t)^2}{(\phi \bar{r}_t + \bar{u}_t)^2} \\ &= \frac{\phi^2 \sigma^2_{r_t} + 2\sigma_{r_t u_t} + \sigma^2_{u_t}}{(\phi \bar{r}_t + \bar{u}_t)^2}. \end{aligned}$$

Unlike for the stationary u_t process, the variance of the random walk does increase over time, so while $\sigma^2_{u_1} = \sigma^2_{u_t} = \sigma^2_u$, the same does not hold for $\sigma^2_{r_t}$. For the ratio used in the test regression, this implies

$$\log\left(\frac{H_1}{H_t}\right) = \log\frac{\phi^2\sigma^2_{r_1} + 2\sigma_{r_1 u} + \sigma^2_u}{(\phi\bar{r_1} + \bar{u_1})^2} - \log\frac{\phi^2\sigma^2_{r_t} + 2\sigma_{r_t u} + \sigma^2_u}{(\phi\bar{r_t} + \bar{u_t})^2}. \tag{1.16}$$

The expression representing H_1 in equation (1.16) still tends to a constant as $t \to \infty$. Yet now also the part constituting H_t does, since both $\sigma^2_{r_t}$ and $\bar{r}^2_t$ grow at rate t. Thus, in total, $\log\left(\frac{H_1}{H_t}\right) \to c < \infty$ as $t \to \infty$ as required by Phillips and Sul under the alternative.

For the final case, non-convergent data with a drift following DGP (1.13), the quantities relevant to Phillips and Sul's test are

$$\begin{aligned}
h_{i,t} &= \frac{\psi t + \phi r_{i,t} + u_{i,t}}{\psi t + \phi\bar{r}_t + \bar{u}_t}, \\
H_t &= \frac{\phi^2\sigma^2_r + 2\sigma_{ru} + \sigma^2_u}{(\psi t + \phi\bar{r}_t + \bar{u}_t)^2}, \quad \text{and} \\
\log\left(\frac{H_1}{H_t}\right) &= \log\frac{\phi^2\sigma^2_{r_1} + 2\sigma_{r_1 u} + \sigma^2_u}{(\psi + \phi\bar{r_1} + \bar{u_1})^2} - \log\frac{\phi^2\sigma^2_{r_t} + 2\sigma_{r_t u} + \sigma^2_u}{(\psi t + \phi\bar{r}_t + \bar{u}_t)^2}.
\end{aligned} \tag{1.17}$$

The inclusion of a drift changes the convergence properties of the denominator of H_t which is now $\mathcal{O}_p(T^2)$, while the numerator of H_t remains $\mathcal{O}_p(T)$ as in equation (1.16). When applying the logarithm, however, the discrepancy again reduces to a constant factor, so both numerator and denominator grow at the same rate and thus also in the case with a drift $\log\left(\frac{H_1}{H_t}\right) \to c < \infty$ as $t \to \infty$ holds.

1.5 Benchmark Tests

To put the performance of the test by Phillips and Sul into perspective, this section considers a benchmark test that both the convergent and the non-convergent data will be subjected to in order to compare size and power properties. Various variants of unit root and cointegration tests are available for this purpose. The standard cointegration rank test of Johansen (1991), which is widely used, does not consider the null of a specific cointegrating rank but rather tests the null of up to a specified number of cointegrating relationships against the alternative that all series converge. Hence, it is not well-suited to the question at hand. The concept of testing a null hypothesis of nonstationarity against the alternative of stationarity has to be turned around in order to achieve comparability to Phillips and Sul's test. In the context of unit root testing, this is achieved by Kwiatkowski et al.'s (1992) test of stationarity. Examples of such tests with a reverse set of hypotheses in a cointegration setting are the tests of Nyblom and Harvey (2000) and Breitung and Trenkler (2002). Because they consider convergence of all series as null hypothesis versus the alternative of at least one non-convergent series, they have the same null and alternative hypothesis as the regression test.

Both Nyblom and Harvey's and Breitung and Trenkler's test are based on eigenvalue problems and follow the rationale that if the actual rank is higher than the hypothesized one, large eigenvalues will enter the statistic and lead to rejection of the null. It should be noted that these tests are more specialized for a time series setting than the regression test, since they require to T to be much larger than N and can only deal with small cross-sectional dimensions of N altogether. With regard to possible applications, they are thus less versatile than the test of Phillips and Sul (2007), which puts little restrictions on the choice of T and N. However, they still provide a valuable comparison for settings where both can be used. Because Breitung and Trenkler (2002) suggest that Nyblom and Harvey's test may have the most favorable size and power properties, the latter is used as a benchmark. It will be outlined in the remainder of this section.

1.5.1 Nyblom and Harvey

Nyblom and Harvey (2000) suggest a test for common stochastic trends that checks the validity of a specific rank k for the covariance matrix of the disturbances in a multivariate random walk or, equivalently, tests the validity of the hypothesis that there are k common trends in the series, against the alternative that there are more. This is the same as testing the hypothesis of $N-k$ cointegrating vectors against the alternative of a cointegrating rank smaller than $N-k$. Hence, setting $k=1$, the benchmark test should not be able to reject the null for the convergent data, while it should for the non-convergent data.

The test statistic for this test is constructed from the ordered eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N$ arising from the eigenvalue problem

$$|C - \lambda_j S(m)| \qquad j = 1, \ldots, N, \tag{1.18}$$

where

$$C = T^{-2} \sum_{i=1}^{T} \left[\sum_{t=1}^{i} (y_t - \bar{y}) \right] \left[\sum_{t=1}^{i} (y_t - \bar{y}) \right]' \tag{1.19}$$

and $S(m)$ is an estimate of the long-run covariance matrix obtained as

$$S(m) = \sum_{\tau=-m}^{m} w_{\tau m} \hat{\Gamma}(\tau) \text{ with } \hat{\Gamma}(\tau) = T^{-1} \sum_{\tau+1}^{T} (y_t - \bar{y})(y_t - \bar{y})'. \tag{1.20}$$

$w_{\tau m}$ is weighting function based on a lag window m satisfying $m \to \infty$ as $T \to \infty$ and $m = \mathcal{O}(T^{\delta})$ for $0 < \delta < 0.5$. Here, the Bartlett kernel with $w_{\tau m} = 1 - \frac{\tau}{m+1}$ will be used. The exact choice of m is somewhat difficult, as automatic bandwidth estimation is only useful for stationary variables. Breitung and Trenkler (2002) note that the test is rather sensitive regarding the choice of m, making it difficult to balance size and power properties. For the simulation part in this chapter, a value of $m = 2$ will be used.

The test of the hypothesis that there are k common trends is then based on the $N-k$ smallest eigenvalues, resulting in the test statistic

$$\zeta_{k,N} = \lambda_{k+1} + \lambda_{k+2} + \ldots + \lambda_N \qquad k = 1, \ldots, N-1. \tag{1.21}$$

If a linear time trend is included in the data, the test statistic in (1.21) is computed using ordinary least squares residuals from regressing the original series on a constant and a time trend. Nyblom and Harvey (2000) provide critical values for this statistic, which depend on both the hypothesized number of common trends k and the number of series N.

1.6 Monte Carlo Results

$\phi = 0.5$			$\phi = 1.0$			$\phi = 1.5$		
ρ	size	power	ρ	size	power	ρ	size	power
.1	0.233	0.367	.1	0.237	0.374	.1	0.247	0.376
.2	0.223	0.368	.2	0.242	0.378	.2	0.238	0.383
.3	0.224	0.370	.3	0.233	0.379	.3	0.240	0.374
.4	0.229	0.364	.4	0.233	0.383	.4	0.242	0.383
.5	0.226	0.362	.5	0.226	0.380	.5	0.234	0.386
.6	0.215	0.364	.6	0.232	0.373	.6	0.232	0.379
.7	0.219	0.355	.7	0.225	0.375	.7	0.227	0.373
.8	0.219	0.359	.8	0.224	0.382	.8	0.227	0.385
.9	0.236	0.363	.9	0.237	0.376	.9	0.243	0.381
.95	0.268	0.373	.95	0.261	0.374	.95	0.260	0.381
.99	0.345	0.372	.99	0.341	0.378	.99	0.343	0.374

Table 1.1: Size and power of the regression test for a panel with dimensions $T = 50$ and $N = 4$ using the DGP without drift term

This section presents findings on the small sample properties of the regression and benchmark tests based on Monte Carlo simulations. They are based on the four DGPs with $T = 50$ and $N = 4$ as well as $T = 100$ and $N = 4$. The choice of N is determined by the maximum cross-sectional dimension which the benchmark test can handle. Additionally, various combinations of the

$\phi = 0.5$			$\phi = 1.0$			$\phi = 1.5$		
ρ	size	power	ρ	size	power	ρ	size	power
.1	0.008	0.480	.1	0.011	0.780	.1	0.012	0.842
.2	0.012	0.483	.2	0.013	0.774	.2	0.015	0.842
.3	0.019	0.475	.3	0.023	0.777	.3	0.025	0.826
.4	0.036	0.476	.4	0.039	0.784	.4	0.045	0.838
.5	0.067	0.479	.5	0.070	0.773	.5	0.077	0.842
.6	0.131	0.473	.6	0.151	0.773	.6	0.156	0.845
.7	0.274	0.478	.7	0.285	0.777	.7	0.289	0.840
.8	0.506	0.483	.8	0.521	0.770	.8	0.532	0.842
.9	0.756	0.468	.9	0.763	0.777	.9	0.773	0.841
.95	0.845	0.484	.95	0.846	0.777	.95	0.846	0.843
.99	0.888	0.478	.99	0.889	0.782	.99	0.889	0.837

Table 1.2: Size and power of Nyblom and Harvey's test for a panel with dimensions $T = 50$ and $N = 4$ using the DGP without drift term

parameters ϕ and ρ are considered. The magnitude by which the common random walk in case of convergent data or the individual random walk in case of non-convergent data enter the series is adjusted by choosing $\phi \in \{0.5, 1, 1.5\}$. For each of these choices, the autoregressive parameter in the error term is considered from $\rho = 0.1$ to $\rho = 0.9$ in increments of 0.1. Furthermore, the near unit root cases of $\rho = 0.95$ and $\rho = 0.99$ are investigated. For each setting, $10,000$ replications are conducted.

In the following tables, the proportion of false rejections of the null hypothesis of convergence is reported under the "size" heading, while the other column reports the power of the test, that is, the proportion of rightful rejections when the DGP is of the type in (1.12) or (1.13). The slowly varying function $L(t)$ has been set to $\log(t + 1)$ for the results presented below; yet findings are robust to the exact choice of $L(t)$, since changing it to $\log(\log(t))$ or other alternatives suggested by Phillips and Sul leads to no notable differences.

$\phi = 0.5$			$\phi = 1.0$			$\phi = 1.5$		
ρ	size	power	ρ	size	power	ρ	size	power
.1	0.118	0.248	.1	0.111	0.268	.1	0.117	0.277
.2	0.116	0.242	.2	0.115	0.266	.2	0.116	0.281
.3	0.113	0.246	.3	0.117	0.266	.3	0.114	0.289
.4	0.110	0.243	.4	0.108	0.274	.4	0.117	0.278
.5	0.108	0.241	.5	0.107	0.277	.5	0.114	0.283
.6	0.108	0.253	.6	0.108	0.278	.6	0.107	0.274
.7	0.116	0.251	.7	0.111	0.282	.7	0.115	0.273
.8	0.121	0.246	.8	0.115	0.272	.8	0.124	0.274
.9	0.153	0.244	.9	0.150	0.272	.9	0.144	0.275
.95	0.176	0.251	.95	0.183	0.279	.95	0.168	0.276
.99	0.286	0.252	.99	0.260	0.280	.99	0.265	0.269

Table 1.3: Size and power of the regression test for a panel with dimensions $T = 50$ and $N = 4$ using the DGP including a drift term

Table 1.1 presents these results for a panel with dimensions $T = 50$ and $N = 4$. Clearly, the size of the test is nowhere near the nominal size of 5% for any of the combinations. The highest size values are observed for values of ρ that are close to unity, namely $\rho = 0.95$ and $\rho = 0.99$. This comes as no surprise since in the case of an idiosyncratic component that is stationary but comes close to having a unit root, the danger of false positives is intuitively considerably larger than for individual-specific components that are clearly I(0). Surprisingly, the lowest size values are not found for the smallest instances of the autoregressive parameter, but rather for $\rho \in [.6, .8]$. Still, all size values for $\rho \leq 0.9$ range between 21% and 25%, far above the nominal level as well. The choice of ϕ appears to have little influence on the size of the test. Likewise, the power of the test is hardly affected by either ϕ or ρ; it always lies between 0.35 and 0.39, which is quite far from adequate levels. Nevertheless, the test

$\phi = 0.5$			$\phi = 1.0$			$\phi = 1.5$		
ρ	size	power	ρ	size	power	ρ	size	power
.1	0.019	0.504	.1	0.024	0.827	.1	0.026	0.895
.2	0.040	0.497	.2	0.045	0.831	.2	0.048	0.893
.3	0.066	0.498	.3	0.085	0.832	.3	0.088	0.894
.4	0.133	0.497	.4	0.149	0.833	.4	0.143	0.902
.5	0.228	0.492	.5	0.261	0.830	.5	0.265	0.895
.6	0.390	0.499	.6	0.409	0.832	.6	0.418	0.901
.7	0.574	0.487	.7	0.608	0.827	.7	0.622	0.896
.8	0.779	0.499	.8	0.787	0.827	.8	0.798	0.900
.9	0.905	0.496	.9	0.908	0.828	.9	0.908	0.900
.95	0.934	0.494	.95	0.938	0.829	.95	0.938	0.898
.99	0.946	0.502	.99	0.948	0.825	.99	0.942	0.891

Table 1.4: Size and power of Nyblom and Harvey's test for a panel with dimensions $T = 50$ and $N = 4$ using the DGP including a drift term

– though not explicitly designed for this situation – does have some ability to distinguish between convergent and non-convergent data.

Table 1.2 reports the results of the benchmark test in the same setting for comparison. There are several striking differences. The size depends much more on the autoregressive parameter ρ. It attains values that are better than for the regression test as long as $\rho \leq 0.6$, while for larger values of ρ it deteriorates fast. Being taken from a time-series setting, the benchmark test is thus much more susceptible to near unit root processes for the error term. The choice of ϕ, to the contrary, does not appear to have much impact on the size, while the power improves with higher ϕ and is generally substantially higher than for the regression test.

The effects obtained from including a drift term, and thus using DGP (1.11) rather than (1.10), on the regression test are displayed in Table 1.3. The general pattern, with size being higher for values of ρ that come close to a unit root,

$\phi = 0.5$			$\phi = 1.0$			$\phi = 1.5$		
ρ	size	power	ρ	size	power	ρ	size	power
.1	0.254	0.399	.1	0.261	0.381	.1	0.248	0.367
.2	0.249	0.390	.2	0.250	0.370	.2	0.261	0.368
.3	0.247	0.390	.3	0.250	0.379	.3	0.248	0.367
.4	0.253	0.394	.4	0.245	0.378	.4	0.255	0.366
.5	0.239	0.391	.5	0.246	0.379	.5	0.243	0.366
.6	0.236	0.390	.6	0.235	0.388	.6	0.239	0.363
.7	0.228	0.399	.7	0.230	0.387	.7	0.229	0.363
.8	0.220	0.398	.8	0.215	0.387	.8	0.230	0.372
.9	0.212	0.397	.9	0.213	0.372	.9	0.217	0.369
.95	0.230	0.392	.95	0.228	0.373	.95	0.225	0.358
.99	0.291	0.393	.99	0.298	0.380	.99	0.289	0.361

Table 1.5: Size and power of the regression test for a panel with dimensions $T = 100$ and $N = 4$ using the DGP without drift term

remains the same as in the case without drift, as does the finding that the value of ϕ has little effect on size and power. However, across all constellations, size and power are substantially smaller than in Table 1.1. Thus, with a trend included, the regression test is more successful in detecting convergent data, but at the cost of classifying more non-convergent data as convergent, too.

For the benchmark test, including a drift term also changes results as can be seen in Table 1.4. Interestingly, the effect is rather contrary to that of the regression test. For Nyblom and Harvey's test, both size and power are generally higher than in the case without drift. The test thus fails to detect convergence more often but the fraction of false negatives becomes smaller. Additionally, the influence of ρ on the size is even more pronounced than in the case without drift. Values of ρ exceeding 0.4 lead to values which are no longer adequate and even approach unity as the error term comes closer to being a unit root process.

$\phi = 0.5$			$\phi = 1.0$			$\phi = 1.5$		
ρ	size	power	ρ	size	power	ρ	size	power
.1	0.029	0.972	.1	0.048	0.993	.1	0.038	0.997
.2	0.050	0.981	.2	0.053	0.995	.2	0.051	0.997
.3	0.076	0.974	.3	0.075	0.992	.3	0.068	0.999
.4	0.115	0.983	.4	0.127	0.998	.4	0.137	0.993
.5	0.193	0.971	.5	0.194	0.995	.5	0.206	0.999
.6	0.328	0.975	.6	0.323	0.996	.6	0.331	0.999
.7	0.531	0.975	.7	0.568	0.992	.7	0.570	0.998
.8	0.803	0.973	.8	0.826	0.997	.8	0.823	0.997
.9	0.972	0.973	.9	0.982	0.998	.9	0.968	0.999
.95	0.994	0.978	.95	0.997	0.996	.95	0.995	0.996
.99	0.998	0.983	.99	0.998	0.997	.99	0.999	0.997

Table 1.6: Size and power of Nyblom and Harvey's test for a panel with dimensions $T = 100$ and $N = 4$ using the DGP without drift term

A further relevant question is how strongly the length of the time dimension affects the performance of the tests. The results of the simulations with $T = 100$ are presented in Tables 1.5 and 1.7 for the regression test without and with drift as well as Tables 1.6 and 1.8 for the corresponding benchmark tests. As Table 1.5 indicates, there is little effect on the results of the regression test for data without a drift. Size and power are very similar for $T = 50$ and $T = 100$; also the finding that size is not best for very small or very large values of ρ, but rather for intermediate values around 0.8 is obtained again. However, the benchmark test – which is time series based – shows notably improved power, which still increases with ϕ and attains values close to unity when T is larger, while the size only increases to a lesser degree. For small values of the autoregressive parameter, the size is close to its nominal level. This test thus clearly benefits from the extended time span.

$\phi = 0.5$			$\phi = 1.0$			$\phi = 1.5$		
ρ	size	power	ρ	size	power	ρ	size	power
.1	0.063	0.260	.1	0.061	0.265	.1	0.063	0.257
.2	0.060	0.254	.2	0.058	0.253	.2	0.058	0.255
.3	0.065	0.259	.3	0.059	0.246	.3	0.063	0.253
.4	0.056	0.260	.4	0.065	0.256	.4	0.059	0.257
.5	0.065	0.258	.5	0.061	0.252	.5	0.059	0.251
.6	0.061	0.256	.6	0.059	0.246	.6	0.055	0.242
.7	0.056	0.255	.7	0.055	0.248	.7	0.058	0.248
.8	0.060	0.261	.8	0.059	0.252	.8	0.060	0.254
.9	0.075	0.260	.9	0.074	0.255	.9	0.068	0.251
.95	0.099	0.254	.95	0.104	0.247	.95	0.102	0.261
.99	0.199	0.253	.99	0.185	0.255	.99	0.180	0.252

Table 1.7: Size and power of the regression test for a panel with dimensions $T = 100$ and $N = 4$ using the DGP including a drift term

When a drift is included, the regression test once again features both a lower size and power. However, with the increased T, the size of the test is better than for $T = 50$, while the power is about the same. For the benchmark test, the inclusion of a drift term means that the power is very close to unity for any parameter constellation. The size is worse than for the case without drift and rather similar to that of the situation with $T = 50$ and a drift term in the data. Overall, for small choices of ρ, the benchmark test delivers very convincing results as the time dimension increases, while the regression test performs considerably more poorly.

While for small cross-section dimensions as considered so far, the regression test for convergence faces competition from the time series based alternatives, as pointed out before, the latter cannot handle settings where N exceeds four. In the following, the performance of Phillips and Sul's test will be investigated for larger N and for settings where the time and cross-section dimensions are

$\phi = 0.5$			$\phi = 1.0$			$\phi = 1.5$		
ρ	size	power	ρ	size	power	ρ	size	power
.1	0.055	0.982	.1	0.048	0.099	.1	0.059	1.000
.2	0.083	0.982	.2	0.096	0.999	.2	0.107	0.998
.3	0.166	0.979	.3	0.162	0.999	.3	0.182	0.999
.4	0.275	0.986	.4	0.278	0.998	.4	0.281	1.000
.5	0.453	0.992	.5	0.421	0.998	.5	0.431	1.000
.6	0.658	0.983	.6	0.649	0.999	.6	0.667	1.000
.7	0.863	0.982	.7	0.859	0.998	.7	0.845	1.000
.8	0.962	0.990	.8	0.974	0.997	.8	0.981	1.000
.9	0.998	0.979	.9	0.999	0.997	.9	0.997	1.000
.95	1.000	0.982	.95	0.999	0.998	.95	1.000	1.000
.99	1.000	0.983	.99	1.000	0.999	.99	0.999	1.000

Table 1.8: Size and power of Nyblom and Harvey's test for a panel with dimensions $T = 100$ and $N = 4$ using the DGP including a drift term

of comparable range, even though these results cannot be confronted with corresponding benchmark tests.

Tables with simulated size and power for these constellations can be found in the appendix. Tables 1.9 and 1.10 summarize results for a panel with $T = 50$ and $N = 25$, that is, a time dimension as in the original setting, but with a much bigger cross section. Compared to the situation with $N = 4$, both size and power increase only very slightly for the DGP without a drift term. For the DGP with a drift term, the difference is more noticeable. While the size increases a bit compared to the result using a cross-sectional dimension of four, the power is considerably lower and barely exceeds the size in most instances. Notably, unlike for the previous settings, the power also appears to depend on the choice of ϕ. In any case, both size and power are considerably lower than for the DGP without drift as was the case with smaller cross-sectional dimensions.

For a panel with $T = N = 25$, results are shown in Tables 1.11 and 1.12. Compared to the situation with $T = 50$, the size is slightly higher for the DGP without drift and more notably so for the DGP including one. The power in the situation without a drift remains roughly the same for $T = 50$ and $T = 25$, though the dependence of ϕ is greater for the shorter time period. In the case including a drift, the power is higher for $T = 25$ than for $T = 50$, yet it still depends on the choice of ϕ and is clearly below satisfactory levels.

In summary, the results indicate that the regression test introduced by Phillips and Sul (2007) is very robust to changes in T and especially in N. For analyzing convergence in settings with many countries and short time spans, it could thus potentially serve as an alternative to time series based tests such as Nyblom and Harvey (2000) or Breitung and Trenkler (2002). Furthermore, the results indicate that Phillips and Sul's test may have some advantage if there is a strong autoregressive component in the error term. With respect to power, the performance of the regression test is poor in most cases and for a limited cross-section dimension over a reasonably long time span it is clearly inferior to the alternative test specialized in this situation.

1.7 Conclusion

This chapter has investigated the performance of the regression test for convergence suggested by Phillips and Sul (2007) in a setting of artificially created convergent series consisting of a common trend and an idiosyncratic component. While the asymptotic properties of Phillips and Sul's statistics under these conditions are as required by the authors for the test to function, it turns out that the test does not perform convincingly in a small sample study with the time dimension considerably larger than the cross-section dimension.

Furthermore, the Monte Carlo simulations reveal that the test has severe size distortions when applied to data generated from a DGP without drift term while the size properties are somewhat better if a drift term is included. Although still not too convincing, the power in the setting without a time trend is higher than in the situation including one.

A comparison to the cointegration-based test of Nyblom and Harvey (2000) indicates that the latter performs best for many settings. Especially when the autoregressive order of the error term is small and as the time dimension increases, it seriously outperforms the regression test. Bearing in mind the deficiencies when a time trend is present, Phillips and Sul's test however does have some merit when cointegration tests cannot be applied. That is, it can provide some guidance concerning convergence of series in a panel of similar time and cross-section dimensions.

Appendix to Chapter 1

$\phi = 0.5$			$\phi = 1.0$			$\phi = 1.5$		
ρ	size	power	ρ	size	power	ρ	size	power
.1	0.269	0.385	.1	0.265	0.409	.1	0.273	0.400
.2	0.267	0.377	.2	0.274	0.404	.2	0.272	0.406
.3	0.264	0.382	.3	0.272	0.396	.3	0.267	0.406
.4	0.272	0.385	.4	0.263	0.393	.4	0.269	0.403
.5	0.257	0.372	.5	0.258	0.391	.5	0.266	0.403
.6	0.262	0.383	.6	0.259	0.409	.6	0.265	0.396
.7	0.259	0.375	.7	0.278	0.403	.7	0.264	0.394
.8	0.265	0.373	.8	0.259	0.401	.8	0.271	0.396
.9	0.276	0.374	.9	0.272	0.403	.9	0.266	0.406
.95	0.280	0.381	.95	0.277	0.401	.95	0.295	0.410
.99	0.345	0.385	.99	0.359	0.407	.99	0.357	0.400

Table 1.9: Size and power of the regression test for a panel with dimensions $T = 50$ and $N = 25$ using the DGP without drift term

$\phi = 0.5$			$\phi = 1.0$			$\phi = 1.5$		
ρ	size	power	ρ	size	power	ρ	size	power
.1	0.141	0.158	.1	0.146	0.213	.1	0.149	0.225
.2	0.145	0.165	.2	0.141	0.216	.2	0.147	0.229
.3	0.150	0.155	.3	0.153	0.224	.3	0.147	0.231
.4	0.149	0.166	.4	0.144	0.217	.4	0.143	0.217
.5	0.137	0.158	.5	0.131	0.220	.5	0.145	0.217
.6	0.142	0.155	.6	0.142	0.215	.6	0.149	0.229
.7	0.136	0.160	.7	0.148	0.212	.7	0.143	0.219
.8	0.139	0.158	.8	0.141	0.202	.8	0.152	0.227
.9	0.142	0.156	.9	0.157	0.216	.9	0.157	0.223
.95	0.155	0.166	.95	0.161	0.209	.95	0.167	0.219
.99	0.266	0.160	.99	0.259	0.210	.99	0.264	0.215

Table 1.10: Size and power of the regression test for a panel with dimensions $T = 50$ and $N = 25$ using the DGP including a drift term

$\phi = 0.5$			$\phi = 1.0$			$\phi = 1.5$		
ρ	size	power	ρ	size	power	ρ	size	power
.1	0.278	0.351	.1	0.273	0.401	.1	0.290	0.431
.2	0.276	0.351	.2	0.294	0.407	.2	0.283	0.417
.3	0.268	0.337	.3	0.278	0.402	.3	0.289	0.407
.4	0.270	0.341	.4	0.271	0.405	.4	0.279	0.422
.5	0.261	0.347	.5	0.274	0.400	.5	0.272	0.417
.6	0.272	0.330	.6	0.280	0.406	.6	0.281	0.423
.7	0.275	0.341	.7	0.269	0.399	.7	0.278	0.397
.8	0.281	0.355	.8	0.279	0.406	.8	0.278	0.419
.9	0.281	0.344	.9	0.299	0.402	.9	0.293	0.412
.95	0.324	0.332	.95	0.343	0.404	.95	0.328	0.417
.99	0.401	0.346	.99	0.404	0.407	.99	0.395	0.419

Table 1.11: Size and power of the regression test for a panel with dimensions $T = 25$ and $N = 25$ using the DGP without drift term

$\phi = 0.5$			$\phi = 1.0$			$\phi = 1.5$		
ρ	size	power	ρ	size	power	ρ	size	power
.1	0.210	0.230	.1	0.206	0.333	.1	0.212	0.355
.2	0.201	0.220	.2	0.217	0.334	.2	0.209	0.368
.3	0.208	0.219	.3	0.201	0.321	.3	0.211	0.354
.4	0.209	0.227	.4	0.213	0.335	.4	0.204	0.355
.5	0.201	0.224	.5	0.210	0.347	.5	0.211	0.350
.6	0.203	0.239	.6	0.218	0.338	.6	0.216	0.358
.7	0.208	0.227	.7	0.210	0.334	.7	0.212	0.363
.8	0.222	0.234	.8	0.218	0.335	.8	0.214	0.358
.9	0.238	0.231	.9	0.226	0.329	.9	0.236	0.358
.95	0.271	0.223	.95	0.269	0.327	.95	0.260	0.353
.99	0.357	0.230	.99	0.360	0.328	.99	0.372	0.359

Table 1.12: Size and power of the regression test for a panel with dimensions $T = 25$ and $N = 25$ using the DGP including a drift term

Chapter 2

Convergence and Orders of Integration for Interest Rates

2.1 Introduction

The relation between interest rates of different countries is a key element of many international economic models. Ties between interest rates are an indicator of monetary integration and as institutions around the world have implemented measures to stabilize monetary policy, the question arises whether there is a tendency towards some global level of the long-term interest rate. The idea of a "world interest rate" has repeatedly caught the attention of researchers and has been discussed extensively (e.g. Blanchard et al., 1984; Mishkin, 1984; Barro and Sala-i-Martin, 1990). The main finding has been an increasing dominance of global factors for the determination of interest rates. The theory behind this effect is that increasing deregulation has led to higher international capital mobility with respect to both the speed and volume of movements.

In a flexible exchange rate setting under the assumption of perfect substitutability of bonds independent of the currency they are denominated in, the no-arbitrage condition in international financial markets implies that interest rate differentials between domestic and foreign assets with the same characteristics and maturity m (years) must be reflected in corresponding differences between spot and forward foreign exchange rates. Assuming risk-averse behavior in the foreign exchange market implies that the forward rate is the expected future spot rate $\mathrm{E}_t[s_{t+m}]$ plus a country-specific and time-varying risk premium ρ_t. Furthermore, the expected spot rate differs from the actual rate by a forecast error ε_t. This can be summarized as the uncovered interest rate parity (UIP) relationship

$$r_t - r_t^* = \frac{1}{m}(s_{t+m} - s_t - \varepsilon_{t+m} + \rho_t). \tag{2.1}$$

where r_t denotes the domestic and r_t^* the foreign interest rate.

Since the order of integration of the interest rate differential $r_t - r_t^*$ has to be the same as that for the right-hand side of equation (2.1), the stochastic properties of the following three components are relevant: the change in the exchange rate, the rational expectations error ε_{t+m}, and the risk premium ρ_t. Various empirical studies find the exchange rate to be a random walk (Meese and Singleton, 1983; Meese and Rogoff, 1983), which implies stationarity of its change. The rational expectations error is by definition stationary and independent of information available at time t; in fact, assuming that expectations are on average correct and hence $\varepsilon_{t+m} = 0$ is not very restrictive. Finally, the risk premium remains the only part whose properties cannot be anticipated. It might hence either be stationary, which is the standard assumption in most models of UIP; however, it could also be integrated of order one. The latter is argued by Evans and Lewis (1994), who argue that nonstationary risk premia are required to explain excess bond returns and Crowder (1994), who also demonstrates possible non-stationarity of the forward risk premium.

Caporale et al. (1996) and Harvey and Carvalho (2002) have pointed out that convergence toward a stable interest rate differential requires a transition phase, during which time series approach each other, and a stability phase, representing a situation in which convergence has been achieved. However, most tests currently used for interest rate convergence are based on a cointegrating relationship between interest rates and hence only focus on the latter phase. In the context of the UIP framework, this would require a risk premium that is stationary throughout the sample. However, the formation of a “world interest rate” at some point during the sample for countries previously in the process of transition or, at least, a tendency towards a common movement of interest rates within the European Monetary Union (EMU) would be reflected in a change of the order of integration of the forward risk premium and hence the interest rate differential from $I(1)$ to $I(0)$.

The contribution of this chapter to the existing literature is threefold. First, it allows for the joint treatment of the transition and the stability phase of convergence using a test for a change in persistence recently proposed and shown to outperform other tests (Harvey et al., 2006). Second, it remedies the problem of exogenously specifying a starting date for convergence, since the breakpoints can be endogenously determined with appropriate estimators. Third, it uses data over a larger set of countries and a sufficiently long time span to investigate patterns both within and outside the EMU on a pair-wise basis as well as for entire groups of countries seen as one entity.

The chapter proceeds as follows. The next section discusses policy measures targeted at higher monetary integration and reviews previous attempts to model interest rate convergence. Section 2.3 provides a definition of convergence as a change in persistence, while section 2.4 describes the methodology used to detect such changes. Section 2.5 presents and discusses the results. Finally, section 2.6 concludes.

2.2 Indicators of Convergence

2.2.1 Policy Measures and Monetary Integration

Investigating the pattern of interest rates evolving over time has been of particular interest for countries within the European Union and under the impact of its coordinated monetary policy. A number of politically motivated steps toward economic convergence have been taken since the collapse of the Bretton Woods agreement in 1973 and the ensuing possibility for exchange rates to fluctuate freely. Right from the start, a number of central banks in Europe adopted a “snake in the tunnel” approach, limiting exchange rate fluctuations to a small band, although initially these bands underwent frequent readjustments.

The process was formalized in 1979 through the formation of the European Monetary System (EMS) by Belgium, Denmark, France, Germany, Ireland, Italy, Luxemburg, and the Netherlands with an Exchange Rate Mechanism (ERM) that prescribed currency bands as a key element. The system was later joined by Spain and Great Britain and remained in place until 1999, with a notable interruption in 1992, when Great Britain and Italy were forced to leave the ERM temporarily because their central banks were unable to maintain the fixed band.

In 1999, the European Monetary Union (EMU) replaced the previous system and fixed exchange rates invariably for those countries adopting the euro as a common currency. Among the four criteria required for joining this group, one explicitly targeted interest rate convergence, mandating that the interest rate on long-term government bonds be at most two percentage points above the average of the three countries with the highest price stability. Other criteria focused on price stability, the amount and growth of public debt, and a two-year participation in ERM II, which was introduced as a successor to the original ERM for those countries not yet matching all criteria for the euro.

However, evidence of convergence in interest rates goes beyond the European Union. Since the first half of the 1980s, economists have noted a marked decline in the volatility of aggregate economic activity for most industrialized countries. Due to its comprehensiveness and impact, Stock and Watson (2003) coined the term “Great Moderation”, by which this phenomenon has since become known. Research on this development has stressed its international dimension (Blanchard and Simon, 2001) and has also sought for possible explanations. Clarida et al. (2000) find evidence of a significant change in U.S. monetary policy, namely that it began to respond to deviations from desired GDP and inflation growth more pronouncedly after 1979. They argue that this shift, initiated by the installation of Paul Volcker as chairman of the Federal Reserve, has had a stabilizing impact. This is supported by Fase and Vlaar (1998) who note driving forces toward higher integration such as the gradual abolishment of capital controls and the ensuing higher international diversification of portfolios.

Regarding interest rates, this should imply convergence since inflation became more predictable and country-specific risk became less relevant.

From an economic perspective, there is thus reason to expect convergence in interest rates both within EMU and worldwide. A common feature of both is the gradual movement toward the current situation. While there are certain key dates associated with the development, convergence does not appear all of a sudden but rather constitutes a sustained process toward the goal of higher financial integration.

2.2.2 Previous Approaches to Assess Convergence

Due to the topic's relevance, in particular with the advancement of a monetary union in Europe, the degree of financial integration and the movement of interest rates toward a common level have been the focal point of many previous studies which strive to provide a framework to assess convergence.

The vast majority of the literature on financial integration considers the comovement of interest rates as an indicator of convergence. This is usually done by means of cointegration tests for interest rate series or, equivalently, stationarity tests on their spreads. As some of the first authors to make use of this approach, Katsimbris and Miller (1993) find little evidence of convergence between German interest rates and those of other EMS countries. Likewise, Throop (1994) finds no cointegration between government bond yields for the United States, Canada, Germany, Japan, and the United Kingdom. Similar investigations with variations in frequency of the data, the selection of countries, or the time period considered (e.g. Poghosyan, 2009, who also provides an overview of the literature using this method) generally come to the same conclusion.

Several papers extend this framework by not only considering the relationship between long-term interest rates of two countries but also the connection between long- and short-term rates within each country. The theoretical foundation for the comovement of the latter is provided by the expectations hypothesis of the term structure and cointegration tests are then carried out in a four-dimensional system including both long- and short-term rates for each country. Consequently, one should expect to find three linearly independent cointegration relationships. With this setting, however, Kremer (1999) finds no evidence for cointegration between interest rates for Germany and the United States. Brüggemann and Lütkepohl (2005) compare the United States and EMU, using an artificially constructed rate consisting of the German rate prior to 1999 and a weighted average of the EMU rates afterwards for the latter. Confining themselves to a particular sample period starting in the middle of the 1980s, they indeed find the three theoretically expected relationships; yet, for different choices of the starting point, their results cannot be confirmed.

Other authors have noted that the development of financial and economic cooperation has caused shifts which imply structural breaks in the cointegration relationship. Zhou (2003) exogenously establishes three sub-periods of his entire sample from 1979–1998 and finds evidence for convergence of interest rates of countries within the European Monetary System. Arghyrou et al. (2009) endogenously determine two breaks and test for stationarity of interest rate differentials against the EMU average but find only limited evidence for convergence of European countries' interest rates. The reason could be that endogenous break tests as well as the test for cointegration breakdown introduced by Andrews and Kim (2006) may fail to detect breaks appropriately due to the gradual nature of the adjustment (Brada et al., 2005).

Because of this deficiency, a rolling cointegration approach has been used to capture the development. Poghoysan and de Haan (2007) perform rolling threshold cointegration analysis for interest rates from various financial market segments in European countries. Besides deposits, loans, and mortgages, they also investigate ten-year government bonds. Despite the dynamic nature of their test, they only find evidence for financial integration in a few instances; none of them involving the latter segment. A common problem of all these cointegration-based tests is that their results strongly hinge on the starting point of the sample.

A new approach is considered by Frömmel and Kruse (2009), who use a persistence change test to assess convergence. Restricting themselves to the short-term interest rate differentials of four key EMU economies versus Germany and a 1983–2007 sample, they find evidence of convergence for three of them using a test proposed by Leybourne et al. (2007). The convergence dates vary for the different countries, but generally fall into the years before the introduction of the Euro.

2.3 Convergence in Interest Rates

2.3.1 Definition of Convergence

As seen in the previous section, most studies consider convergence to be present if a cointegrating relationship exists between the two interest rates, which obviously implies a stationary risk premium in the UIP relationship from equation (2.1). The motivation for this definition of convergence is often linked to the work of Bernard and Durlauf (1995, 1996), who provide two definitions of convergence.

When adapted to the situation here, the first one views convergence as catching up: the rates r_t and r_t^* of two countries with $r_{t_1} > r_{t_1}^*$ converge between dates t_1 and $t_2 > t_1$ if

$$\mathrm{E}(r_{t_2} - r_{t_2}^* | \mathfrak{F}_{t_1}) < r_{t_1} - r_{t_1}^*, \tag{2.2}$$

where $\mathrm{E}_t[.]$ is the mathematical expectations operator and $\mathfrak{F}_{t_1}$ denotes all information available up to time t_1. The second definition refers to convergence as the equality of long-term forecasts for the interest rates at a fixed time:

$$\lim_{k\to\infty} \mathrm{E}(r_{t+k} - r_{t+k}^* | \mathfrak{F}_t) = \gamma. \tag{2.3}$$

A further distinction is made between strong convergence, which requires long-term forecasts of the series to be identical, so that $\gamma = 0$ in equation (2.3), and weak convergence, allowing for a non-zero but constant γ.

Present studies thus focus on the second definition, presented in equation (2.3). As Bernard and Durlauf (1996) note in their Proposition 5, this definition is violated if the difference of the series under consideration contains a unit root. They hence conclude that "time series tests may have poor power properties when applied to data from economies in transitions" (Bernard and Durlauf, 1996, p. 171); a view that is shared by Caporale et al. (1996), who take issue with the fact that such tests only address convergence as a state but do not consider the process leading there.

To illustrate this point, consider a simple example with I(1) variables x_t and y_t. If their difference $x_t - y_t$ is I(1) as well, which has been taken as rejection of the convergence hypothesis by cointegration-based tests, there are actually two possible explanations. Either the variables are indeed completely unrelated or they are gradually moving toward one another, implying a monotonically decreasing difference series. This latter case, however, corresponds to Bernard and Durlauf's first definition of convergence from equation (2.2).

In the same way, an I(0) difference $x_t - y_t$ need not imply convergence. If the starting values of the two series are far apart, a stationary difference merely means that the variables persist at these unequal levels. Harvey and Carvalho (2002) thus propose to call the tests based on cointegration *stability* tests rather than convergence tests. While they may be a powerful tool once transition toward a common level has been achieved, they fail to capture the preceding phase during which transition is actually taking place.

2.3.2 A Persistence Change Test for Convergence

An actual test of convergence should thus allow for both a *transition* and a *stability* phase. The concept of series first approaching and later on comoving can be represented as the switch from an I(1) to an I(0) process in the gap between them. Requiring a stationary gap for converged series picks up the notion of the cointegration-based tests for the stability phase.

For the case of interest rates, a test of changes in persistence, i.e. changes in the order of integration, applied to the interest rate differentials can thus serve as a test of convergence. Under the alternative, series change from a transition phase characterized by a unit root in the risk premium – and hence the differential – to a stability phase characterized by stationarity. The null, on the other hand comprises two distinct cases of non-convergence. Either the differential has a unit root throughout, implying that a process of transition is still ongoing or there is no relation at all; or the differential is stationary during the entire sample, which means there is no room for a transition period and series remain as far apart as they have been initially.

Hence, a test proposed by Harvey et al. (2006) is applied to check for changes in persistence. While various tests of changing persistence have been proposed in the literature, e.g. by Kim (2000) or Busetti and Taylor (2004), the test by Harvey et al. performs superiorly. It allows for a null of constant integration of unspecified degree, while the other tests require a specific null of constant I(0) processes and may spuriously overreject if the series is actually a constant I(1) process. The exact implementation is outlined in the following section.

2.4 Detecting Persistence Changes

2.4.1 Methodology

To assess changes in persistence, an autoregressive integrated moving average ($ARIMA$) process given by

$$\alpha(L)\Delta^d x_t = \mu + \theta(L)\varepsilon_t \tag{2.4}$$

is considered, where L is the lag and $\Delta \equiv 1 - L$ the difference operator. ε_t is assumed to be white noise, while $\alpha(L)$ and $\theta(L)$ are the p-th order autoregressive and q-th order moving-average polynomials, respectively, with the stability condition that all roots lie outside the unit circle. The value of d represents the number of times that the process in equation (2.4) must be differenced to obtain a stationary, invertible $ARMA(p, q)$ process. For the interest rate series at hand, the only economically justifiable values for d are zero and one, so the investigation is restricted to these two possibilities.[2] A change in persistence hence means a change in the value of d at some point of the sample.

Similar to Harvey et al. (2006), the following hypotheses will be considered to test for such change in persistence. The first constitutes the null hypothesis of no change, that is, a degree of integration that remains unity throughout the sample.

$$\mathcal{H}_{0a} : d = 1 \qquad \text{for } t = 1, \ldots, T.$$

On the other hand, the alternative hypothesis of a change in persistence implying convergence of interest rates is given by

$$\mathcal{H}_{10} : d = \begin{cases} 1 & \text{for } t = 1, \ldots, \lfloor \tau T \rfloor \\ 0 & \text{for } t = \lfloor \tau T \rfloor + 1, \ldots, T, \end{cases}$$

where $\tau \in (0, 1)$ and $\lfloor x \rfloor$ denotes the largest integer not greater than x.

Independently of each other, Kim (2000) and Busetti and Taylor (2004) develop various statistics for testing $\mathcal{H}_{0a}$ against the alternative that the process changes from $I(1)$ to $I(0)$ at some unknown breakpoint in the sample. All of them are based on the ratio statistic

$$K_{\lfloor \tau T \rfloor} = \frac{(\lfloor \tau T \rfloor)^{-2} \sum_{t=1}^{\lfloor \tau T \rfloor} \left(\sum_{s=1}^{t} \hat{\varepsilon}_{0,s} \right)^2}{(T - \lfloor \tau T \rfloor)^{-2} \sum_{t=\lfloor \tau T \rfloor + 1}^{T} \left(\sum_{s=\lfloor \tau T \rfloor + 1}^{t} \hat{\varepsilon}_{1,s} \right)^2}, \tag{2.5}$$

where $\hat{\varepsilon}_{0,t}$ are the residuals from an OLS regression of y_t on a constant over $t = 1, \ldots, \lfloor \tau T \rfloor$ and, similarly, $\hat{\varepsilon}_{1,t}$ are the residuals from this regression over $t = \lfloor \tau T \rfloor + 1, \ldots, T$. Since the point τ^* at which a persistence change occurs is a priori unknown, the entire sequence of statistics $\{K_{\lfloor \tau T \rfloor}, \tau \in \Lambda\}$ is examined. Following Harvey et al. (2006), tests are conducted over the central 60% of the sample, so $\Lambda = [\tau_l, \tau_u] = [0.2, 0.8]$. The three statistics proposed by Kim

[2] When assuming integer orders of integration, $I(2)$ and higher are not considered plausible in empirical analyses. Further options for the value of d would arise when considering fractional integration, which is investigated by Frömmel and Kruse (2009).

(2000) and Busetti and Taylor (2004) are based on the mean score statistic from Hansen (1991), the mean-exponential statistic from Andrews and Ploberger (1994), and the maximum over the sequence of statistics from Andrews (1993), respectively. They are

$$\begin{aligned}
MS &= (1 + \lfloor \tau_u T \rfloor - \lfloor \tau_l T \rfloor)^{-1} \sum_{t=\lfloor \tau_l T \rfloor}^{\lfloor \tau_u T \rfloor} K_t, \\
ME &= \ln \left\{ (1 + \lfloor \tau_u T \rfloor - \lfloor \tau_l T \rfloor)^{-1} \sum_{t=\lfloor \tau_l T \rfloor}^{\lfloor \tau_u T \rfloor} \exp\left(\frac{1}{2} K_t\right) \right\}, \\
MX &= \max_{t \in \{\lfloor \tau_l T \rfloor, \ldots, \lfloor \tau_u T \rfloor\}} K_t.
\end{aligned}$$

Harvey et al. (2006) perform Monte Carlo simulations to show that the aforementioned statistics cannot be used to distinguish between a change in persistence and a constant I(0) process as opposed to the constant $I(1)$ process assumed under $\mathcal{H}_{0a}$. They propose various modifications based on the approach of Vogelsang (1998) that, for some given significance level, yield statistics with the same critical value as for the unmodified test under $\mathcal{H}_{0a}$, but where this critical value is also appropriate under stationarity throughout the sample, denoted by $\mathcal{H}_{0b}$. The modified statistic that performs best as a replacement for MS follows as

$$MS_m = \exp(-bJ_{min})MS, \tag{2.6}$$

where b is a finite constant chosen such that the critical values of the statistic under $\mathcal{H}_{0a}$ and $\mathcal{H}_{0b}$ coincide and $J_{min} = \min_{\tau \in \Lambda} J_{\lfloor \tau T \rfloor + 1, T}$. $J_{\lfloor \tau T \rfloor + 1, T}$ represents T^{-1} times the Wald statistic for testing the hypothesis $\gamma_1 = \ldots = \gamma_9 = 0$ in the regression $y_t = \beta + \sum_{i=1}^{9} \gamma_i t^i + u_t, \quad t = \lfloor \tau T \rfloor + 1, \ldots, T$. The modified statistics for ME and MX follow from entirely similar steps and are subsequently denoted as ME_m and MX_m. Relevant values of b for each of these statistics are tabulated in Harvey et al. (2006).

For those countries outside the European Monetary Union and thus not immediately affected by political measures to foster convergence, an opposite development to that described before is also possible, that is, series starting to diverge at some point again. This change from $I(0)$ to $I(1)$ can be captured by the reverse of hypothesis $\mathcal{H}_{10}$:

$$\mathcal{H}_{01} : d = \begin{cases} 0 & \text{for } t = 1, \ldots, \lfloor \tau T \rfloor \\ 1 & \text{for } t = \lfloor \tau T \rfloor + 1, \ldots, T, \end{cases}$$

with the corresponding null hypothesis

$$\mathcal{H}_{0b} : d = 0 \qquad \text{for } t = 1, \ldots, T.$$

This set of hypotheses can essentially be tested using the same framework as above. However, as Busetti and Taylor (2004) demonstrate, the statistics MS, MX, and ME are inconsistent for testing whether a change of persistence in the opposite direction occurs. They introduce statistics MS^R, MX^R, and ME^R which differ from the previous ones only in that the nominator and the denominator of $K_{\lfloor \tau T \rfloor}$ in equation (2.5) are switched. Busetti and Taylor show these reverse statistics to be consistent against a change from I(1) to I(0), but not against one in the opposite direction. The corresponding modified reverse statistic for MS is obtained as $MS_m^R = \exp(-bJ_{min}^R)MS^R$ where b is chosen in the same way as before and $J_{min}^R = \min_{\tau \in \Lambda} J_{1,\lfloor \tau T \rfloor}$. Again, the modified versions ME_m^R and MX_m^R are constructed in the same way.

An advantage of the persistence change approach over other approaches is that the break date does not have to be provided exogenously but can instead be inferred from a statistic. For a change from I(1) to I(0), both Kim (2000) and Busetti and Taylor (2004) propose

$$\hat{\tau^*} = \arg \min_{\tau \in [\tau_l, \tau_u]} \frac{(\lfloor \tau T \rfloor)^2 \sum_{t=\lfloor \tau T \rfloor + 1}^{T} \hat{\varepsilon}_{1,s}^2}{(T - \lfloor \tau T \rfloor)^2 \sum_{t=1}^{\lfloor \tau T \rfloor} \hat{\varepsilon}_{0,s}^2}, \tag{2.7}$$

with $\hat{\varepsilon}_{0,t}$ and $\hat{\varepsilon}_{1,t}$ defined as before. As with the K statistic from equation (2.5), a change from I(0) to I(1) can be dated by switching nominator and denominator.

Following an argument by Halunga et al. (2009), breaks in persistence are only considered when they occur strictly inside the search interval Λ.

2.4.2 Pairwise and Average Measures

In their seminal work on convergence, Bernard and Durlauf (1995) suggest two primary approaches that can be used to investigate convergence across groups of N countries. The first one is to compute results for the $N-1$ gaps with respect to a benchmark country and thus immediately arises as an extension of the pairwise tests. One issue with this approach is its dependence on a benchmark that is selected arbitrarily. Thus with, say, the United States as a benchmark, it may fail to capture convergence between Germany and France, because only the relationship of each country towards the benchmark is considered. A meaningful analysis would thus extend Bernard and Durlauf's suggestion to all possible combinations of countries.

Let $\mathbb{1}(\cdot)$ denote the indicator function and consider $Z_{ij} = \mathbb{1}(MS_{ij} > \lambda_{MS,\alpha})$ where MS_{ij} is the test in equation (2.6) conducted for $y_t = y_{it} - y_{jt}$ and $\lambda_{MS,\alpha}$ is the critical value corresponding to this test, that is, $\lim_{T\to\infty} \Pr(MS_m > \lambda_{MS,\alpha}|\mathcal{H}_0) = \alpha$. Throughout, the relevant critical value is set to $\alpha = 5\%$. Then the number of pairs out of a set of N countries for which the null of constant integration is rejected and thus convergence takes place is given by

$$Z_{MS} = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} Z_{ij}, \tag{2.8}$$

or, equivalently, the fraction of rejections by $\frac{2}{N(N-1)} Z_{MS}$. In the same way, the number of rejections obtained using ME_m or MX_m can be summarized as Z_{ME} and Z_{MX}.

While this approach is simple to implement, it should be noted that it can only be used to test whether any convergence has taken place during the period under consideration. For different pairs, the actual change from I(1) to I(0) may have occurred at very different points in time. With this method, it is thus impossible to pinpoint a date at which the system of countries has converged.

Since the latter is a major limitation to pairwise approach, a second suggestion made in Bernard and Durlauf (1995) seems more appealing. It directly uses the entire system of N time series and performs a test with the null hypothesis of up to $N-1$ cointegrating vectors between them. Two drawbacks of this approach are the limitation to relatively few countries and that it cannot easily be adapted to the persistence test methodology used in this chapter.

To nevertheless be able to address a more comprehensive set of countries, Pesaran (2007) suggests a multivariate approach that concentrates the data from various time series into an average and then tests for convergence in this combined series. Two measures are introduced for this purpose; one comparing each series against the overall average and one consisting of the average of the absolute gaps. The former, D_t^2, is constructed as

$$\begin{aligned} D_t^2 &= \frac{1}{N(N-1)} \sum_{i+1}^{N} \sum_{j+1}^{N} [y_{it} - y_{jt}]^2 \\ &= \frac{1}{N(N-1)} \sum_{i+1}^{N} \sum_{j+1}^{N} [(y_{it} - \bar{y_t}) - (y_{jt} - \bar{y_t})]^2 \\ &= 2 \left(\frac{\sum_{i+1}^{N} (y_{it} - \bar{y_t})^2}{N-1} \right) = 2s_t^2, \end{aligned} \tag{2.9}$$

with $\bar{y_t}$ denoting the cross-section average of all series under consideration. That is, D_t^2 represents twice the square of the cross-sectional standard deviation of y_{it}, s_t. This measure is exactly what Barro and Sala-i-Martin (1992) introduce as σ-convergence in their seminal analysis of output series.

The second measure used by Pesaran (2007) also roots in the literature on output dispersion and is known as the absolute mean difference coefficient

$$\Delta_t = \frac{1}{N(N-1)} \sum_{i+1}^{N} \sum_{j+1}^{N} |y_{it} - y_{jt}|. \tag{2.10}$$

The appeal of this statistic is that it directly relates to the gaps between the individual series as in the pairwise analysis. It is also related to the Gini coefficient as a well-known measure of statistical dispersion given by $G_t = \Delta_T / \bar{y_t}$.

However, for the purpose of assessing convergence, the scaling by $\bar{y}_t$ yields no advantage (Pesaran, 2007), so Δ_t is used.

2.4.3 Data

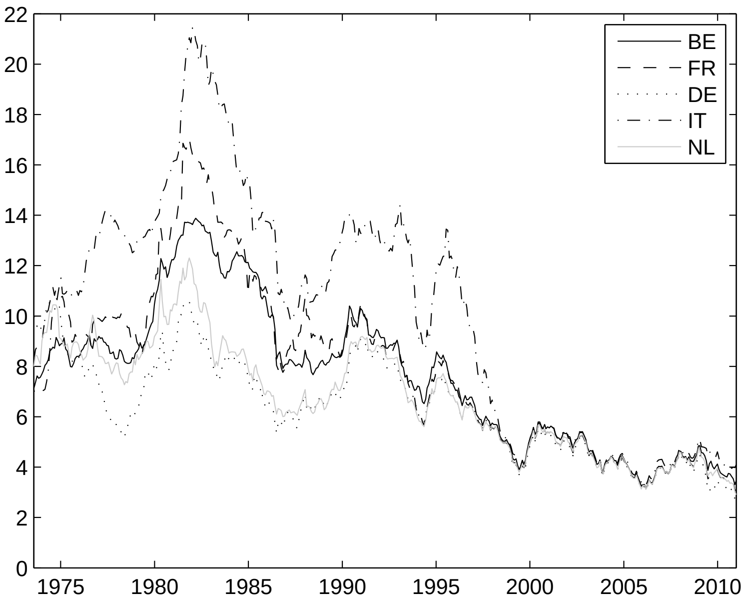

Figure 2.1: Long-term interest rates for EMU countries 1973–2010

The empirical investigation considers nominal interest rates on long-term government bonds for the period from July 1973 to June 2010. All data were retrieved form the International Monetary Fund's International Financial Statistics (IFS) database and are at a monthly frequency. Thus, each interest rate series has $T = 444$ observations, which corresponds to 37 years. The choice of start date follows from the collapse of the Bretton-Woods system of fixed exchange rates in the first half of 1973 and the ensuing possibility for exchange rates to fluctuate freely. In choosing nominal interest rates, this chapter follows previous literature (e.g. Fase and Vlaar, 1998; Brüggemann and Lütkepohl, 2005; Poghoysan and de Haan, 2007).

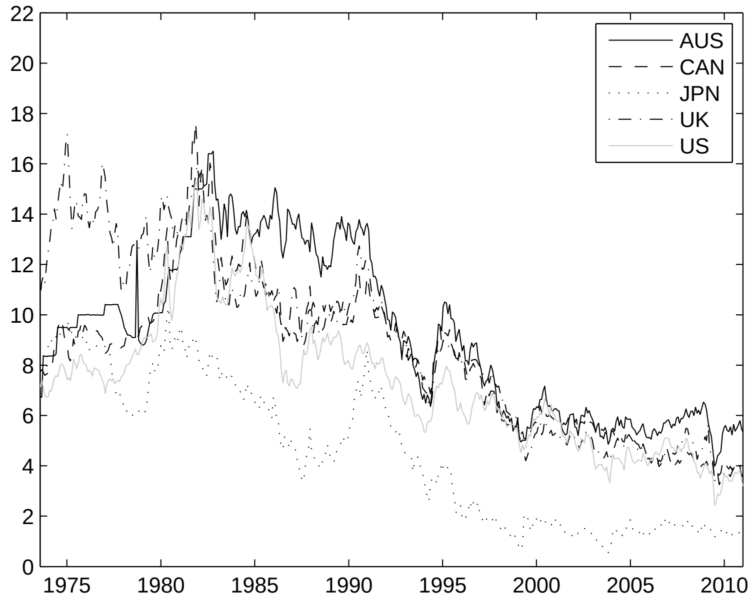

Figure 2.2: Long-term interest rates for non-EMU countries 1973–2010

The above tests are carried out for all possible bivariate pairings of Germany, France, Italy, Belgium, and the Netherlands to assess convergence of interest rates within the European Monetary System. All of them are major economic powers of the Euro area while Belgium is also host to the union's key political institutions. The development of interest rates for this group is depicted in Figure 2.1. The same analysis can be performed for the U.S., the U.K., Canada, Australia, and Japan, whose interest rates over time can be seen in Figure 2.2. This yields another ten bivariate combinations, that, together with the 25 pairs consisting of one EMU and one non-EMU member each, shed light on the issue whether convergence has also occurred for long-term interest rates of key economies worldwide.

Beyond the pairwise tests, to investigate whether there has been an effect of the Maastricht convergence criteria, or the actual adoption of the Euro, three different sets of countries will be considered for possible group convergence. Set *all* denotes the aggregate of all countries and can hence be used to check for overall convergence. Splitting this group into sets *EMU* and *non-EMU* allows to check for different patterns between members and non-members of the currency union.

2.5 Results

2.5.1 Preliminary Analysis

Visual inspection of the interest rate graphs in Figures 2.1 and 2.2 already provides some hints regarding convergence. In the EMU, rates were generally very high during the early 1980s with no clear pattern of comovement. Rates were slightly lower in the late 1980s and early 1990s, but still ranged from around 6% to almost 15%. On the contrary, they have been virtually identical and at a much lower level since the late 1990s.

For the set of non-EMU countries in Figure 2.1, Japan has always had the lowest interest rates except for the very first years of the sample. Generally, there is also a movement toward lower rates during the 1990s, however, series appear to comove already well before the end of that decade, especially those for Canada, the U.K., and the U.S.

2.5.2 Persistence Test Results

Table 2.1 shows the results for the MS_m, ME_m, and MX_m tests in the sample of EMU countries. Using the asymptotic critical values of Harvey et al. (2006), the null of a constant order of integration can be rejected in favor of a change from I(1) to I(0) for all of the bivariate combinations at a significance level of 5%, regardless of the choice of statistic. The startlingly high values of the statistics for the pair France–Netherlands result from the fact that their rates are virtually identical during the stability phase, and hence the differential is very close to zero. The break dates estimated for the changes in persistence fall into a narrow range of years immediately preceding the introduction of the euro. They range from mid-1995 for Germany and Belgium to early 1998 for Italy versus Belgium, the Netherlands, and France, respectively. The only exception is the pair Germany–Netherlands, for which the persistence change occurs in October 1983 already. From this time on, the close comovement of the interest rates asserted by the I(0) differential can also be evidenced by looking at the dotted and the gray line in Figure 2.1 that almost coincide. Another observation concerning the timing of the persistence change is that for pairs involving Italy, it is generally slightly later than for the others.

country pair	MS_m	ME_m	MX_m	date
BE–FR	245.31	389.02	791.07	February 1996
BE–DE	54.39	75.40	161.68	July 1995
BE–IT	33.28	166.44	344.71	January 1998
BE–NL	186.10	401.10	814.66	November 1996
DE–FR	219.44	300.56	615.30	April 1996
DE–IT	29.48	59.20	129.63	October 1997
DE–NL	40.09	37.58	82.44	October 1983
FR–IT	13.14	42.59	96.39	January 1998
FR–NL	7046.73	6553.50	6993.11	January 1997
IT–NL	44.94	117.63	249.38	January 1998
5% crit. val.	4.60	5.11	17.85	

Table 2.1: Convergence test results for EMU country pairs

These results correspond very well to those of Frömmel and Kruse (2009), who only investigate pairs involving Germany using different variations of the methodology in Leybourne et al. (2007) and a 3-month interest rate. They find the Belgium–Germany pair to achieve stability in May 1995, just two months earlier than the date estimated here; and also support the finding that the persistence change occurs later when Italy is involved. Only for the pair Germany–Netherlands, they find a constant I(0) process rather than evidence for a change in persistence. However, this is perfectly in line with my results, since the breakpoint found lies before the start of their sample in 1987.

Economic reasoning strongly supports the findings in Table 2.1. All EMS countries underwent considerable effort to meet the convergence criteria set forth in the Maastricht treaty. Yet von Hagen et al. (2001) note that, of the countries examined here, Italy struggled most and was only able to meet the criteria through measures such as an explicit "Euro tax". Until shortly before the actual introduction of the common currency, it was debated whether Italy would be able to join. This is reflected by the comparatively late convergence date found in the statistical analysis. Belgium, France, and Germany did not face similar problems to the same extent, so stability was achieved up to 2.5 years earlier in bivariate combinations of those three countries. Finally, the early breakpoint for the Germany–Netherlands pair can also be related to economic history, since in 1983 the last realignment of the Dutch guilder against the German mark occurred. From that point on, a close peg was maintained by the Dutch central bank, even throughout the 1992 EMS crisis. In a sense, a situation coming close to a monetary union had already be achieved by unilaterally mimicking key monetary policy decisions made by the German central bank.

Convergence results for the second group of countries – major economies not member of EMU – are provided in Table 2.2. Those countries did not see coordinated activity toward convergence; hence both convergence and divergence may occur.[3] Results of the latter are reported in Table 2.3. For a majority of pairs, a persistence change from $I(1)$ to $I(0)$ cannot be rejected at the 5% level. Exceptions are the pairs Australia–Canada, Canada–United States, and Japan–United States. On the other hand, the opposite change from $I(0)$ to $I(1)$ is rejected for all pairs besides Australia–United States, Canada–United States, and Japan–United Kingdom.

Turning first to the break dates of those cases where exactly one persistence change is found in either direction, it should be noted that the dates for convergence are spread over a greater time span than those for the EMU

[3] For completeness, the test for a change from $I(0)$ to $I(1)$ has also been applied to the EMU pairs; since no such change was found, results remain unreported here.

country pair	MS_m	ME_m	MX_m	date
AUS– CAN	2.83	2.76	10.87	—
AUS– JAP	98.96	411.35	834.39	August 1997
AUS– UK	77.92	107.30	224.73	April 1990
AUS– US	6.13	7.20	19.84	May 1991
CAN– JAP	5.49	4.32	17.53	September 1982
CAN– UK	102.88	103.60	217.61	November 1984
CAN– US	0.98	1.12	6.15	—
JAP– UK	0.81	1.21	8.21	—
JAP– US	16.00	19.79	47.37	September 1982
UK – US	82.92	269.24	553.52	January 1985
5% crit. val.	4.60	5.11	17.85	

Table 2.2: Convergence test results for non-EMU country pairs

countries. Except for the pairs including Australia, they fall into the first half of the 1980s. This is exactly the time pinpointed to be the start of the "Great Moderation" by Stock and Watson (2003). The case of Australia may be different for two reasons. Firstly, as its economy is very dependent on highly volatile exports of natural resources, the country may be facing a different business cycle. Secondly, the Reserve Bank of Australia has been found to implement measures toward greater transparency later than other major central banks (Eijffinger and Geraats, 2006), which may have had detrimental effects on interest rate convergence (Eijffinger et al., 2006).

Two pairs only exhibit a change from stationarity to a unit root process, that is, they departed from the state of having a stable relationship at some point. For the pair Canada–United States the $I(0)$ phase up to October 2002 may be a reflection of long-lasting economic ties between the two countries. Furthermore, as Ceglowski (1998) notes, each is the largest trading partner of the other with restrictions and barriers on free economic interaction lifted well

country pair	MS_m^R	ME_m^R	MX_m^R	date
AUS– CAN	1.79	2.50	10.28	—
AUS– JAP	0.43	0.77	4.88	—
AUS– UK	0.13	0.51	1.55	—
AUS– US	4.10	10.39	27.84	November 1983
CAN– JAP	0.23	0.54	1.34	—
CAN– UK	0.01	0.43	0.09	—
CAN– US	19.42	42.09	94.16	October 2002
JAP– UK	6.19	9.30	24.90	April 1995
JAP– US	0.10	0.51	0.42	—
UK – US	0.06	0.40	0.27	—
5% crit. val.	4.60	5.12	17.80	

Table 2.3: Divergence test results for non-EMU country pairs

before the CUSFTA and NAFTA free trade agreements. Nevertheless, 2002 is pinpointed by various studies (Mair, 2005; Beine et al., 2012) to be the start of a strong appreciation of the Canadian dollar against its U.S. counterpart, reflected here in the departure from a stable interest rate relationship in this year. For the Japan–United Kingdom pair, which changed from $I(0)$ to $I(1)$ in April 1995, there appears no such compelling reason.

country pair	lags	*DF-GLS*	5% c.v.	conclusion
AUS– CAN	1	-1.60	-1.94	I(1)

Table 2.4: Order of integration for the country pair without persistence change during the sample

Two cases require special attention. The Australia–Canada pair was found to have no change in persistence at all. Because the null hypothesis just implies a constant degree of integration, it may either still be in the transition phase or those countries had achieved stability at the beginning of the sample already. In the latter case, the gap should be I(0) throughout, while the former case implies a unit root. This can be investigated through a standard unit root test. Table 2.4 reports the outcomes of GLS-detrended augmented Dickey-Fuller unit root tests (Elliott et al., 1996) for this interest rate differential.[4] A constant is included and the optimal lag length is determined by the Schwarz info criterion. Since the null of a unit root cannot be rejected over the entire sample, this leads to the conclusion that either no convergence is prevalent or the countries are still in the transition phase.

country pair	MS_m^R	ME_m^R	MX_m^R	date
AUS– US	8.55	14.94	37.45	July 1978
5% crit. val.	4.58	5.06	17.18	
country pair	MS_m	ME_m	MX_m	date
AUS– US	14.46	23.04	53.35	April 1991
5% crit. val.	4.63	5.17	17.73	

Table 2.5: Persistence test results after resampling

According to two of the three test statistics, the Australia–United States pair has a change in persistence in both directions; indicating divergence in 1983 followed by convergence in 1991. A potential problem with this result is that the sample used in testing for an I(0) to I(1) change also contains the I(1) to I(0) changes and vice versa. A possible remedy is the sample repartitioning procedure suggested by Bai (1997) to estimate multiple structural breaks. The idea underlying the procedure is to conduct the tests on the entire sample as done in Tables 2.2 and 2.3 and then split the sample at each of the estimated

[4]Using a standard Dickey-Fuller test instead of the GLS-detrended version qualitatively yields the same result.

dates of the persistence change. That is, MS^R, ME^R and MX^R tests for divergence are conducted on a subsample ranging from the beginning until May 1991, while MS, ME and MX tests for convergence use data from the November 1983 to the end of the whole sample. Table 2.5 reports test results using this sample repartitioning technique. Note that due to the shortened sample, critical values change slightly.

country group		MS	ME	MX	date
all	D_t^2	12.82	225.45	462.09	March 2003†
	Δ_t	43.24	449.11	910.23	March 2003†
Euro	D_t^2	1009.76	12719.24	3884.56	August 1997
	Δ_t	35.53	58.10	124.84	November 1997
non-Euro	D_t^2	2.22	26.55	64.27	March 2003†
	Δ_t	9.70	16.48	40.74	March 2003†
5% crit. val.		4.60	5.11	17.85	

† denotes an estimated break date at the upper extreme of the trimmed sample

Table 2.6: Group convergence test results

The results for the individual pairs of countries are supported by those for the country groups. When aggregating the pairwise results in the form of the Z statistic, all out of the ten EMU pairs appear convergent, regardless whether the MS_m, ME_m or MX_m tests are used. Out of the ten non-EMU pairs, the values are $Z_{MS} = 7$, $Z_{MS} = 6$, and $Z_{MS} = 6$, respectively. The group persistence change statistics based on averaging are presented in Table 2.6. It shows results of tests for interest rates using both the D_t^2 and the Δ_t methods of averaging and the standard trimming value of $\tau = .2$. While the test statistics are significant at the 5% level for all instances except the MS for the D_t^2 average of the non-Euro group, the break is found to be at the upper border of the search interval – that is, at point $\lfloor \tau_u T \rfloor$ – for this group as well as that consisting of all countries. Since this means very different implied samples before and

country pair	MS_m	ME_m	MX_m	date
BE – AUS	14.67	31.62	74.07	December 1990
BE – CAN	0.94	0.90	3.56	—
BE – JAP	13.27	79.43	170.05	July 1985
BE – UK	72.10	95.87	201.36	February 1985
BE – US	1.02	0.92	2.84	—
DE – AUS	32.47	70.44	150.43	May 1991
DE – CAN	13.40	12.45	30.31	December 1990
DE – JAP	9.58	65.13	141.48	May 1982
DE – UK	—	—	—	March 2003†
DE – US	39.94	45.16	98.20	July 1989
FR – AUS	35.41	102.47	216.59	December 1990
FR – CAN	7.05	5.76	15.85	April 1984
FR – JAP	6.55	20.79	52.71	September 1984
FR – UK	52.81	65.65	140.85	November 1984
FR – US	10.20	20.16	47.32	November 1984
IT – AUS	16.32	68.54	145.94	January 1998
IT – CAN	2.31	3.60	14.20	August 1997
IT – JAP	—	—	—	March 2003†
IT – UK	46.97	144.80	300.29	November 1996
IT – US	5.35	13.55	32.61	July 2000
NL – AUS	49.22	138.39	289.04	February 1991
NL – CAN	7.36	5.27	14.28	November 1989
NL – JAP	8.20	82.94	177.12	May 1982
NL – UK	53.68	146.96	303.88	October 1998
NL – US	23.54	32.33	73.59	June 1989
5% crit. val.	4.60	5.11	17.85	

† denotes an estimated break date at the upper extreme of the trimmed sample

Table 2.7: Convergence test results for mixed country pairs

after the break, following Harvey et al. (2006) and Halunga et al. (2009), they cannot actually be considered as changes in persistence. Hence, when looking at groups of countries, convergence can be confirmed in the second half of 1997 for the European Monetary Union, but not elsewhere. The test has also been performed to check for a change in the opposite direction but found no evidence of divergence for any of the groups.

country pair	lags	*DF-GLS*	5% c.v.	conclusion
BE– CAN	0	-5.56	-1.94	I(0)
BE– US	2	-3.71	-1.94	I(0)
DE– UK	3	-1.51	-1.94	I(1)
JAP– US	1	-2.79	-1.94	I(0)

Table 2.8: Order of integration for mixed country pairs without persistence change during the sample

Finally, combining the data for EMU and non-EMU economies, pairs with one member each from the former and the latter group are considered to shed light on the issue of convergence across the borders of the single-currency area. Overall, the results presented in Table 2.7 are similar to those for the purely non-EMU pairs. In two cases, the statistics cannot reject the null of a changing degree of integration, while in two further cases a break is found at the upper extreme of the search interval and are hence disregarded. For those four pairs without a persistence change strictly inside the search interval, Table 2.8 provides Dickey-Fuller[5] tests results to differentiate between the purely I(1) and purely I(0) cases. Except for the differential between the United Kingdom and Germany, which contains a unit root, all pairs are found to be stationary.

The fact that both the combinations of Belgium with the U.S. and Belgium with Canada are stationary again stresses the long-lasting close relationship between the two North American countries mentioned earlier, yet there is no

[5]Again, the reported values are results of the GLS-detrended version of the augmented Dickey-Fuller test, while the standard test yields qualitatively similar results.

apparent economic interpretation for the inclusion of Belgium in this group. The close U.S.–Canada ties are also witnessed through break dates not far apart for the combination of any country with the United States, and the same country with Canada. The remaining pairs support the notion of the "Great Moderation", with many of the break dates falling into the first half of the 1980s. Again, the combinations involving Australia constitute an exception, as none of those persistence changes occur before 1990. Another group for which the transition phase was still ongoing in the early 1990s consists of Italy, the Netherlands, and Germany, each paired with the United Kingdom. For those combinations, the persistence change from I(1) to I(0) occurred in 1996, 1998, and not at all within the sample, respectively. This may be a consequence of the EMS crisis in 1993 and the ensuing struggle that caused British interest rates to leave the desired range.

Tests for possible divergence have also been carried out for the mixed country pairs. The only combinations for which the null of a constant order of integration is rejected in favor of a switch from $I(0)$ to $I(1)$ are Australia–Belgium with a change date in May 1985 and Australia–Netherlands with a change date in December 1981. However, when the sample repartitioning technique of Bai (1997) is applied as described before, both of these findings cannot be upheld. Hence, there is no evidence of departure from convergence here.

2.5.3 Robustness

Aside from disregarding the transition phase, a common drawback of cointegration-based convergence tests has been the high sensitivity against the exact beginning of the sample period. To check whether this also applies for the persistence test, the analysis has also been performed for samples starting in January 1968, the earliest point from which on data is available for all countries considered, as well as January 1980, because the Great Moderation has widely been reported as taking its beginnings in the early 1980s. The results for these exercises are available upon request. Overall, they turn out very similar to those for the sample starting at the collapse of Bretton-Woods.

The same holds for the exact choice of countries. For the sake of clarity, the presentation has focused on just ten countries, namely the most relevant ones in terms of the size of their economy. However, the results are qualitatively the same when different countries are included in the EMU and non-EMU groups. For most bivariate pairs, a persistence change takes place; for pairs of EMU members, the breakpoint date is typically shortly before the introduction of the common currency, while for many of the remaining pairs, the date falls into the period of the Great Moderation.

2.6 Conclusion

This chapter presented persistence change tests for convergence of long-term interest rates understood as a transition toward a common level with an ensuing stability phase. Both for countries that participate in EMU and for other major economies, there is ample evidence of an interest rate differential that is integrated of order one initially and changes to stationarity during the sample.

For country pairs within EMU, this change can be dated to years immediately preceding the introduction of the Euro. It hence confirms compliance with the criteria of the Maastricht Treaty which explicitly include interest rate convergence. For country pairs involving non-EMU countries, there is also clear indication for changes in persistence, yet it is not as profound as for the EMU country pairs. When changes in persistence take place, they occur earlier, providing support for the notion of a "Great Moderation" since the early 1980s. Furthermore, the breakpoints for the converging non-EMU pairs are spread over a longer period and not clustered around a single date. For this reason, convergence cannot be confirmed if all countries are considered as a whole; while for average measures combining just EMU countries, a switch from transition to stability takes place in late 1997.

On the other hand, in neither set there is any indication that interest rates may begin to diverge again after having reached a stable relationship. This result is merely found for a single one of the non-EMU pairs.

Exceptions to these general results can be linked to specific economic circumstances for the respective country pairs. Overall, the persistence change tests for convergence, unlike earlier cointegration-based tests, are thus able to adequately capture the behavior of interest rates over the last decades.

A possible extension to the present work is the inclusion of countries that adopted the Euro later than 1999 or that are currently considered as candidates. Using the same methodology, insights could be gained whether their interest rates have sufficiently converged toward those of current EMU countries and whether the break date coincides with the introduction of the common currency.

Furthermore, allowing for near-integration besides purely I(0) and I(1) processes might clarify whether a "Great Moderation" effect in the early 1980s has also been present for EMU countries and is only hidden by the much stronger effect stemming from the introduction of the Euro.

Chapter 3

Synchronization of Output Cycles

3.1 Introduction

With the creation and ongoing extension of the European Monetary Union, a lot of research has focused on the long-run convergence of member countries to common levels for important macroeconomic aggregates. The Maastricht convergence criteria as a prerequisite for admission to the group of countries using the Euro reflect several of these aspects, such as price stability, similar long-term interest rates, and exchange rates limited to a narrow band. These criteria have – among others – also been put forward as requirements for an optimum currency area (OCA) by Mundell (1961).

However, suitability for a monetary union does not only require long-term convergence between countries but also common characteristics in the cyclical components of their economies. That is, a sufficient degree of business cycle comovement is required to conduct a common monetary policy effectively. If asymmetric shocks were affecting member countries of a currency union and as a consequence business cycles were not synchronized, a common policy measure could not yield favorable outcomes in all member countries; with those adversely affected being bereaved of their own tools to achieve stability for their particular situation. To evaluate whether the benefits of a monetary union – namely lower transaction costs and possibly more transparent pricing – outweigh these costs of giving up the option of individual policy intervention, the linkage of business cycles thus turns out to be an important issue in addition to looking at the long-term convergence goals.

The definition of a business cycle dates back to Burns and Mitchell (1946), who describe a cycle as a recurrent sequence of expansions and contractions in aggregate economic activities which is not periodic like a seasonal pattern and which cannot be divided into shorter cycles with similar amplitude and characteristics. When using Gross Domestic Product data as a proxy for economic activity, the business cycle thus captures those components of output with higher frequencies than long-term growth components, but lower ones than short-term noise. It is generally agreed that business cycles typically range in length from approximately two to eight years.

Various methods have been introduced to measure business cycles. Approaches in the literature involve categorizing output series into periods of sustained growth and decline as advocated by Bry and Boschan (1971) or Harding and Pagan (2006), considering growth rates over a specific interval, or applying various filtering techniques that are able to extract cycles of a specified length. Among these, the Hodrick-Prescott filter (Hodrick and Prescott, 1997) is most widely known and used in economic applications, yet other methods are able to cut off frequencies corresponding to business cycles more precisely, such as the bandpass filter due to Christiano and Fitzgerald (2003). Results from different synchronization statistics will be compared to assess in which cases the selection of filtering procedure is crucial to the findings.

The chapter will provide an overview of methods to determine business cycle synchronization based in the time domain as well as in the frequency domain. They have been traditionally used for this purpose, yet with rather vague results, making it worthwhile to consider alternative approaches to evaluate comovement. The first alternative has been suggested by Stoffer et al. (1993) outside of an economic context. It is also based in the frequency domain and investigates cyclical components of categorical time series by computing the so-called spectral envelope. The concept is extended to real-valued time series by McDougall et al. (1997). This approach may prove useful for the analysis of output cycles, because it provides a means to analyze whether combinations of series with close economic ties have higher spectral power at business cycle frequencies. Second, another concept widely used in a broad variety of scientific applications, but not very often in economics, wavelet analysis, and in particular the continuous wavelet transform, is introduced because it allows to consider both aspects pertaining to the time domain and to the frequency domain simultaneously. As an addition to the existing literature, a new measure for synchronization of groups of countries which is localized in both time and scale is presented. The different measures for synchronization of output data are applied to output series from various countries representing members of the European common currency zone as well as major economies from outside that group.

The further setup is as follows. The next section provides a brief overview of previous literature on business cycle synchronization. Afterwards, the characteristics of the output data used for the empirical investigation are presented and the filtering methods to extract the cycle are discussed. Section 3.4 introduces and applies several methods to capture synchronization in the time domain, while section 3.5 does the same for approaches in the frequency domain. Section 3.6 presents the concept of the spectral envelope as a novel way of looking at comovement of economic time series. Synchronization across the time and the frequency domain using wavelet analysis is investigated in section 3.7. Finally, section 3.8 concludes.

3.2 Literature Review

Many previous studies have suggested a variety of different measures to capture business cycle synchronization for pairs or groups of countries. Just using regular contemporaneous correlation coefficients between growth cycles of EU-12 members, Fatas (1997) finds higher values in the sub-sample after the creation of the European Monetary System (EMS) than in the pre-EMS period. Döpke (1998) also considers contemporaneous correlation for the cyclical component obtained from applying the Hodrick-Prescott filter to output data of five core Euro countries using a rolling window approach. He observes increases in correlation for most countries, yet with several exceptions pointing in the other direction. Looking at pairwise correlations and the band-pass filter introduced by Baxter and King (1999), Wynne and Koo (2000) find that there is some synchronization between founding members of the European Union, while evidence is weaker for newer member states.

Instead of using output data, which is typically only available at a quarterly frequency, several authors also resort to using industrial production data, which is provided as a monthly series. Among these, Artis and Zhang (1997, 1999) find that synchronization has increased for country pairs within the EMS, while it has diverged for others. Their finding, however, cannot be replicated by Inklaar and de Haan (2001) with the same data but a slightly longer sample. Massmann and Mitchell (2004), using rolling windows rather than just a pre-EMS and a post-EMS subsample on the same data, finally conclude that the Euro area has switched between periods of close comovement and divergence throughout the sample. For recent years, they record evidence of increasing synchronization.

Harding and Pagan (2002, 2006) propose a way of modelling the synchronization of cycles that is quite different from the correlation measure used in previous studies. They suggest analyzing a constructed binary variable, which is set equal to unity in case of an upward movement in a series and to zero when the direction is downward. This allows them to capture expansions and contractions with much more emphasis on the turning points of the series and hence the classical view of the business cycle by Burns and Mitchell (1946). Correlations are then computed between these binary series, leading to only low values for individual Euro member countries against the group average.

Another main branch of the literature is based on frequency domain analysis. Several studies have adopted this tool to study relationships at the frequency level (e.g. A'Hearn and Woitek, 2001; Breitung and Candelon, 2006). Measures to quantify a comovement relationship between variables at the frequency level have been suggested by Croux et al. (2001) who introduce the dynamic correlation coefficient and a multidimensional counterpart termed cohesion. Using these statistics, Croux et al. find that cycles of U.S. states are more similar than those of European countries, while Valle e Azevedo (2002) also finds high dynamic correlations between European countries and the Eurozone average. Allowing for time-varying coherence, Hughes Hallett and Richter (2006), however, conclude that the coherence between the United Kingdom and the Eurozone is unstable at best, while it is even decreasing for Germany and the Eurozone.

In recent years, some studies have appreciated the advantages wavelet analysis offers for the study of business cycles. Among these, Jagrič and Ovin (2004) compare different wavelet types to measure synchronization of industrial production for Slovenia and Germany and find evidence of increasing synchronization over time. On the other hand, Crowley and Mayes (2008) use wavelet analysis for quarter-on-quarter growth rates of France, Germany, and Italy with the result that cycles continue to differ for each of the pairs. Rua (2010) introduces a refined version of the cross-wavelet spectrum to find that the amount of comovement depends on the frequency and changes over time. Finally, Aguiar-Conraria and Soares (2011) compare industrial production data for each EU-15 country against the weighted EU average and find the highest degree of synchronization for France and Germany, with more peripheral countries being more detached. All wavelet approaches present in the literature so far only allow to check pairs of series for comovement, not larger groups.

3.3 Data and Cycle Extraction

The data set used in this chapter comprises output data for a number of key European economies, namely France, Germany, Italy, the Netherlands, Spain, Switzerland, and the United Kingdom. For the following analyses, besides this group of countries, two further groups are considered for comparison purposes. One leaves out the latter two countries of the above list, hence focusing only on countries which share the Euro as a common currency, while the other one adds Japan and the United States representing important economies outside Europe. In summary, the country groups thus are

- *Euro:* FR, DE, IT, NL, ES;
- *European:* FR, DE, IT, NL, ES, CH, UK;
- *all:* FR, DE, IT, NL, ES, CH, UK, US, JPN.

All data are obtained from the OECD and represent annual levels of seasonally adjusted output data measured at current prices. Data is available for each quarter from the beginning of 1961 to the end of 2010, covering a total of 50 years.

To investigate the cyclical properties of the data, a variety of measures are computed. These include growth rates as well as the cyclical components obtained from different filtering techniques. A problem of using growth rates is that they severely amplify high-frequency components and consequently attenuate lower frequencies (Baxter and King, 1999). This increases the noise in the extracted cycle and thus limits the usefulness of the approach. Nevertheless, they are widely used as a proxy for business cycles, so that both quarter-on-quarter and year-on-year growth rates will be considered.

In addition to them, two further cycle extraction methods will be used. The Hodrick-Prescott filter (Hodrick and Prescott, 1997) which separates a series into a cyclical and a trend component and allows different degrees of smoothing by adjusting a penalty parameter for deviations from the trend. Here, this parameter will be set to 1600, the standard for quarterly data in the literature. This filter is probably the most widely used in economics, yet King and Rebelo (1993) stress that it may seriously alter measures of persistence, variability, and comovement.

Lastly, Christiano and Fitzgerald (2003) propose a band-pass filter using an asymmetric weighting scheme that avoids having to cut off values at the beginning and end of the sample, which would otherwise be the case for a symmetric version. It is devised as a combination of a low-pass and a high-pass filter and designed to pass through cycles with a length between 8 and 32 quarters without modification, but eliminate movements that have a different frequency.

It is clear that the choice of cycle extraction technique affects any result on cycle synchronization, because the approaches are dissimilar in nature. Yet as pointed out in section 3.2, they are often used interchangeably with the same purpose in mind. Also, there is disagreement on how strongly the choice actually influences results. Canova (1998) points out that alternative filters extract different types of information from the original series and asserts that the idea of having just one method corresponding to the exact definition of a business cycle is misleading. As a consequence, he explicitly suggest to subject data to various filtering methods. Burnside (1998) does not consider this to be a problem, yet acknowledges that different filtering techniques may provide different insights concerning the cycle. Massmann and Mitchell (2004) raise the point that the choice of filtering technique can affect the exact shape of what is then termed the "cyclical component", but may at the same time have no impact regarding convergence or synchronization. To account for these observations, it is worthwhile to compare the different filtering methods when checking for possible synchronization.

3.4 Synchronization Measures in the Time Domain

Most early studies on cyclical comovement have considered the standard contemporaneous correlation coefficient as a measure of alignment. That is,

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \tag{3.1}$$

has been computed for two series x_t and y_t, with σ_{xy} denoting the covariance between x and y. Exemplarily, this exercise is considered for the cyclical component of the HP filter here. Table 3.1 reports the resulting correlations for each combination of countries.

It is apparent that the correlation is stronger between countries of the Eurozone, as shown in the upper left corner of Table 3.1 up to the dashed lines. For that group, the average correlation is 0.624, with some subgroups such as France–Italy–Spain showing even higher figures. While across the European country group, delimited by the next set of dashed lines, the average drops to 0.562, this stems notably from the poor alignment of the United Kingdom's cycle with that of the Euro countries, while Switzerland – being entirely surrounded

	FR	DE	IT	NL	ES	CH	UK	US	JPN
FR	1.000	0.549	0.710	0.699	0.705	0.641	0.472	0.328	0.410
DE		1.000	0.468	0.654	0.523	0.643	0.385	0.393	0.629
IT			1.000	0.619	0.706	0.627	0.362	0.342	0.438
NL				1.000	0.610	0.641	0.439	0.392	0.437
ES					1.000	0.641	0.412	0.299	0.439
CH						1.000	0.295	0.290	0.497
UK							1.000	0.546	0.443
US								1.000	0.352
JPN									1.000

Table 3.1: Static correlations for the cyclical component of the HP filter at business cycle frequencies

by Euro members – has much higher correlations with each of them. Finally, looking at the whole set of countries, the data indicate that the United States' cycle is only correlated weakly with those of all other countries except the U.K. For the combinations involving Japan, correlations are also below 0.5 with the notable exception of the Japan–Germany pair with $\rho = 0.629$. The average of correlations for the entire set of countries amounts to 0.501 and is thus lower than for both of the smaller groups.

A rolling window approach can be used to investigate changes that have occurred in this quantity. Instead of considering the entire time series, the correlation coefficient is computed for ten-year windows moving across the entire sample from 1960 to 2010. Figure 3.1 illustrates this development over time for the example of the correlation between the German and the French cycle and compares results for the different methods of cycle extraction. The upper left panel is based on the quarter-on-quarter growth rates and the upper right panel considers the yearly growth data. In the lower half of the figure, the left panel shows the results using the cyclical component of the HP filter, the right panel those based on the CF filter. The years on the x-axis denote the centers of the respective ten-year intervals for which the correlation measure is computed.

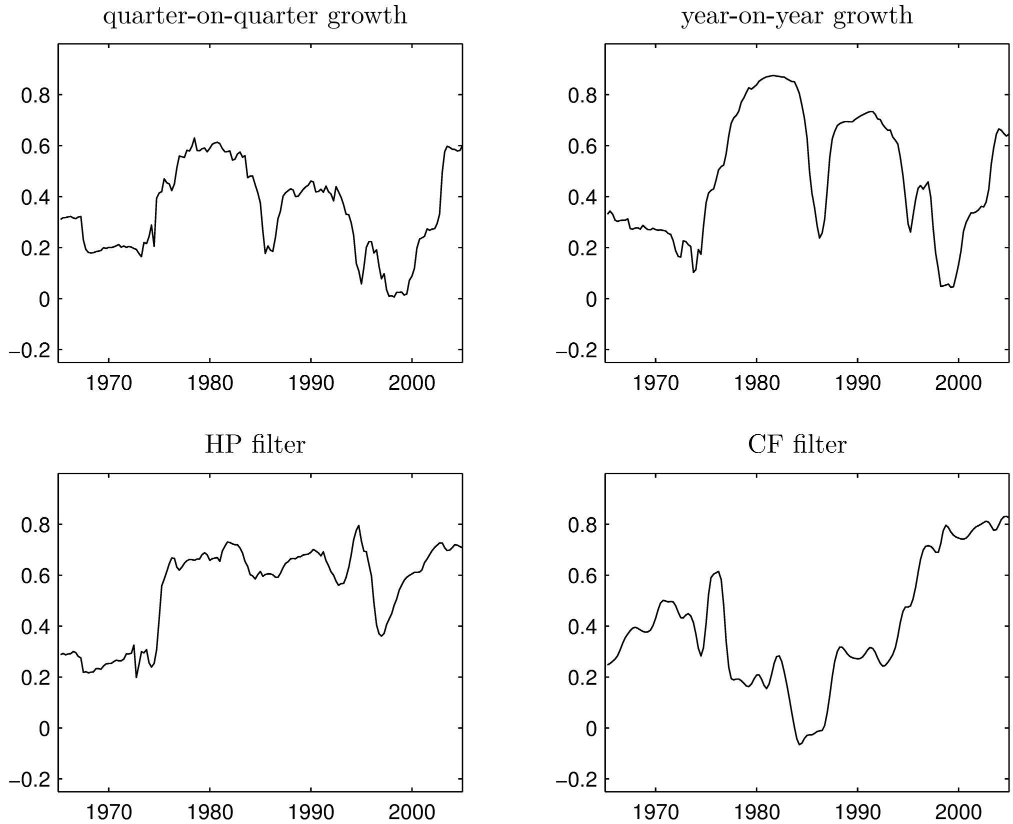

Figure 3.1: Development of the correlation between the cyclical components of output for France and Germany over time. The years on the x-axis of each panel denote the centers of rolling ten-year intervals.

An immediate observation from Figure 3.1 is that the results depend strongly on the method that is used to extract the cycle, which is an immediate consequence of the extraction methods being conceptually different. Furthermore, the correlation substantially varies over time, following no apparent pattern except for some increase towards the very end of the sample in all four cases. In any case, from these statistics no inference can be drawn concerning the length of common cycles leading to correlation, so the following section will venture into the frequency domain in order to better capture periodicities of a particular frequency.

3.5 Synchronization Measures in the Frequency Domain

In order to obtain appropriate comovement indices or measures for common cyclical features, this sections shifts the focus to frequency-domain approaches. The goal is to extract short-run and long-run properties of the relationship between series by considering cyclical components of a particular frequency.

Let the spectral density functions of two time series x and y be given by $S_x(\omega)$ and $S_y(\omega)$, respectively, while the cross spectrum for the two is denoted by $S_{xy}(\omega)$. As a widely-used concept in the literature, coherency between x and y is defined as

$$h_{xy}(\omega) = \frac{S_{xy}(\omega)}{\sqrt{S_x(\omega)S_y(\omega)}}. \tag{3.2}$$

Croux et al. (2001) argue that a slightly different quantity is a better choice for the analysis of comovements. They consider just the real part of coherency, which they name the dynamic correlation between x and y at frequency ω. Dynamic correlation can also be specified as

$$\rho_{xy}(\omega) = \frac{C_{xy}(\omega)}{\sqrt{S_x(\omega)S_y(\omega)}}, \tag{3.3}$$

where $C_{xy}(\omega)$ denotes the cospectrum. A useful feature of dynamic correlation is that it can not only be computed for a specific choice of frequency ω, but also be defined for an entire frequency band $\Lambda = [\omega_1, \omega_2]$ by integrating the spectra over the individual frequencies; that is,

$$\rho_{xy}(\Lambda) = \frac{\int_\Lambda \rho_{xy}(\omega)\sqrt{S_x(\omega)S_y(\omega)}d\omega}{\sqrt{\int_\Lambda S_x(\omega)d\omega \int_\Lambda S_y(\omega)d\omega}}. \tag{3.4}$$

If the entire range of frequencies is covered, the dynamic correlation coefficient is the same as the static correlation coefficient from the previous section. By choosing Λ to represent only a sub-interval of the entire range of frequencies, however, a special focus can be put on correlations pertaining to cycles of a particular length. Because interest predominantly lies in the synchronization of series at business cycle frequencies, the frequency band to be considered is chosen to represent cycles with a period between 2 and 8 years. This approximately means $\Lambda_{bc} = [.20, .79]$. Results for this choice of Λ using the cyclical component of the HP filter as data input are summarized in Table 3.2.

	FR	DE	IT	NL	ES	CH	UK	US	JPN
FR	1.000	0.561	0.749	0.762	0.799	0.663	0.518	0.364	0.402
DE		1.000	0.497	0.695	0.586	0.660	0.440	0.462	0.667
IT			1.000	0.704	0.735	0.666	0.398	0.365	0.389
NL				1.000	0.699	0.729	0.462	0.398	0.482
ES					1.000	0.681	0.430	0.316	0.430
CH						1.000	0.328	0.332	0.467
UK							1.000	0.571	0.534
US								1.000	0.431
JPN									1.000

Table 3.2: Dynamic correlations for the cyclical component of the HP filter at business cycle frequencies.

The results resemble those of the contemporaneous correlation coefficients from the previous section. Those connections between countries that appeared especially weak or strong in Table 3.1 also do in Table 3.2. On average, the dynamic correlations at business cycle frequencies are slightly higher than the correlations considering all frequencies. The averages for $\rho(\Lambda)$ are 0.679 for the Euro group, 0.608 for the European group, and 0.538 when looking at all combinations of the nine countries. This gives some indication that cyclical comovement may be higher at business cycle frequencies than elsewhere.

While dynamic correlation can only capture a cyclical relationship between two variables, it is possible to construct weighted averages of dynamic correlations for all possible combinations of two countries from a larger set $y_t = (y_{1t} \dots y_{nt})'$. The corresponding quantity introduced by Croux et al. is termed cohesion and computed as

$$\mathrm{coh}_y(\omega) = \frac{\sum_{i \neq j} w_i w_j \rho_{y_i y_j}(\omega)}{\sum_{i \neq j} w_i w_j}, \tag{3.5}$$

where w_i and w_j are the weights assigned to variables y_i and y_j, respectively. While choosing $w_i = 1$ for all i is possible, the countries within the dataset are very dissimilar in terms of inhabitants and economic power, so weighting by population or GDP may be more appropriate. For the empirical results in this section, the w_i represent the population in millions of the countries under consideration.

Again, it is possible to construct the cohesion measure for a frequency band Λ, yielding

$$\mathrm{coh}_y(\Lambda) = \frac{\sum_{i \neq j} w_i w_j \rho_{y_i y_j}(\Lambda)}{\sum_{i \neq j} w_i w_j}, \tag{3.6}$$

Table 3.3 reports cohesion at business cycle frequencies for the three subgroups and all types of cycle extraction methods.

The column corresponding to the cyclical component of the HP filter confirms the result from the pairwise analysis using dynamic correlations. The general pattern, namely cohesion being highest for the Euro countries, followed by the European group and lastly the whole set of countries, remains the same regardless of the filtering technique considered. However, the actual values for

subgroup	qtr. growth	yr. growth	HP filter	CF filter
all	.516	.555	.467	.484
European	.552	.607	.574	.592
Euro	.602	.664	.646	.648

Table 3.3: Cohesion at business cycle frequencies.

cohesion are somewhat different depending on the detrending method. For the HP and CF filters, they are spread across a bigger interval, with the value for the full group being smaller than those obtained using growth rates, but vice versa for the Euro group.

While the cohesion measure allows to look at the frequencies of interest more closely, it does not provide information concerning the development of comovement patterns over time. To this end, a rolling window approach is used again. The cohesion measure from equation (3.6) is computed for ten-year windows moving across the entire sample from 1960 to 2010. Figure 3.2 shows the development of the measure over time. The four filtering techniques are arranged in the same way as before. In each panel, the solid line corresponds to the Euro group, the dotted one represents the European group, and the dashed one shows results for all countries considered together.

According to all filtering methods, there is little evidence of increasing business cycle cohesion for the entire group of countries. Only for the HP-filtered data, a small rise over the years is observable; for the growth rate data, cohesion even falls considerably below its value during the initial window (1965–1975) for some time. For the other two groups, and particularly for the adopters of the Euro, cohesion increases towards the end of the sample. Except for the very early part of the quarterly growth series, cohesion for the Euro countries always exceeds that of the European group, which is in line with the results for the full sample. However, the patterns produced by the different filtering techniques are rather distinct. When looking at either of the growth rate panels, cohesion

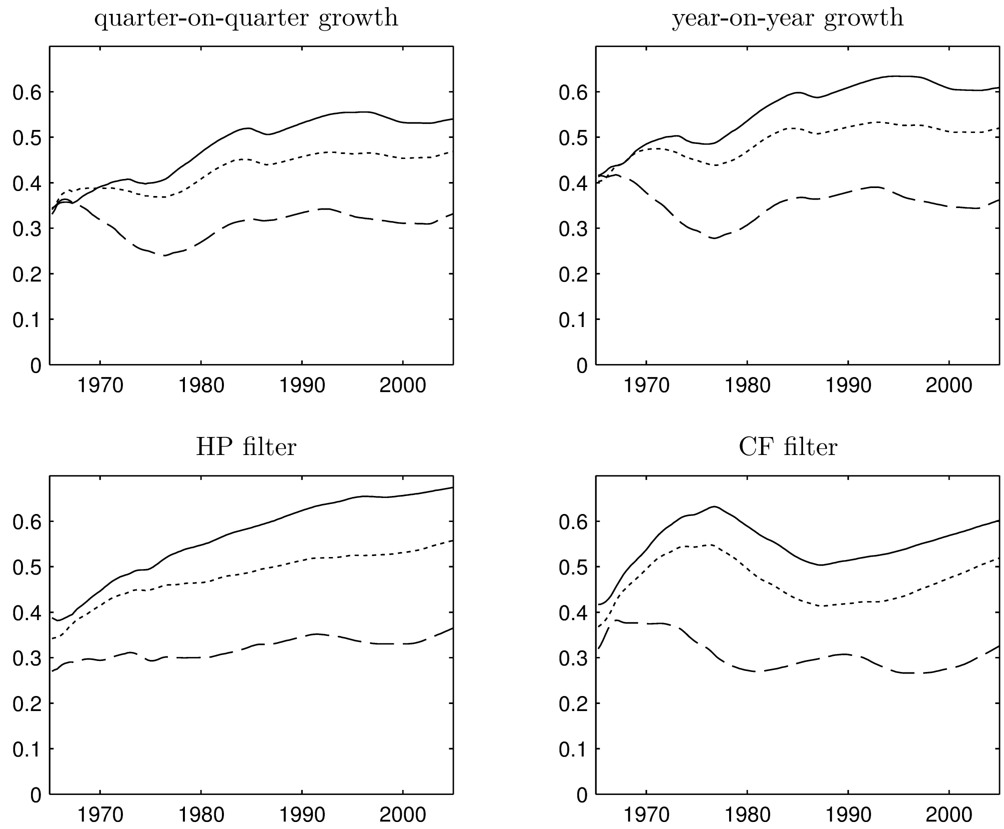

Figure 3.2: Development of cohesion at business cycle frequencies over time. The solid lines represent data from the Euro countries, the dotted lines correspond to the European group of countries, while the dashed line is based on data for all countries. The years on the x-axis of each panel denote the centers of rolling ten-year intervals.

only rises during the first half of the sample and remains at about the same level afterwards. Using the cyclical component of the HP filter to the contrary yields cohesion values that increase throughout the sample. The result using the CF filter is especially surprising as it indicates a steep rise in cohesion for the first 15 years, followed by a decline during a period of approximately equal length and finally increasing values for the last 20 years again.

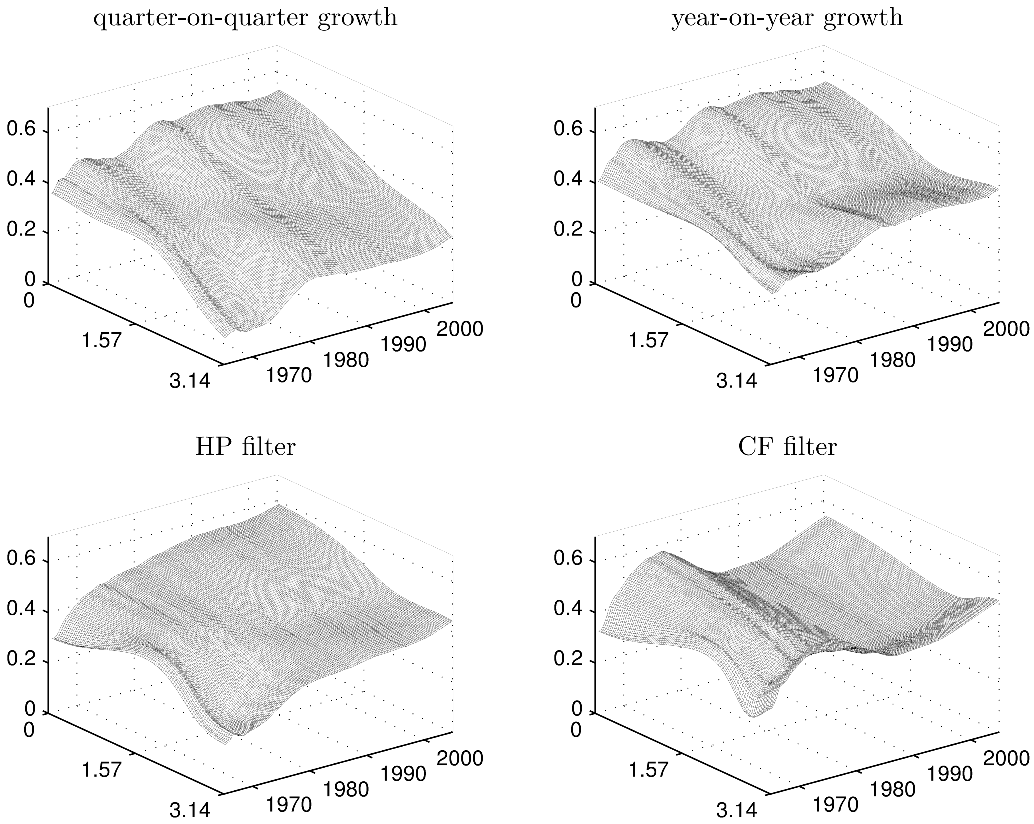

Figure 3.3: Comparison of cohesion results for the European country group. Cohesion is shown for all possible time-frequency combinations, with the frequencies on the x-axis and the years denoting the centers of rolling ten-year intervals on the y-axis.

To consider the question whether cohesion is the same for cycles of different length, the previous summary statistic is disentangled. Rather than considering the range of business cycle frequencies as a whole, Figure 3.3 displays the pattern of cohesion across both time and frequencies in a three-dimensional plot. Again, ten-year rolling windows are used. For the sake of clarity, only the European country group is considered in this figure, however, the respective graphs for the whole set of countries and for the Euro group are similar. In the three-dimensional plot of Figure 3.3, the x-axis denotes the frequencies, while the y-axis depicts the years representing the centers of the ten-year intervals.

When looking at the changes over time, it is notable that the measure increases during the first 20 years of the sample for most frequencies, but then levels off. It can be noted that cohesion is generally highest for the low frequencies, representing long-term growth effects. Also within the range of business cycle frequencies, that is for ω approximately between 0.20 and 0.79, the longer cycles display higher cohesion than the shorter ones. The decline in cohesion during the 1980s when looking at the CF-filtered data is present at all frequencies.

As a whole, the results presented in this section give some indication that comovement between output cycles for key Euro countries is more pronounced compared to other major economies. Furthermore, especially for the Euro countries a tendency towards increasingly strong common cycles is apparent over time.

3.6 The Spectral Envelope

Another method for evaluating common movements that is based in the frequency domain has been suggested by Stoffer et al. (1993) and McDougall et al. (1997). The original approach tackles a problem from molecular biology – namely periodicities in DNA sequences – and outlines a way to conduct harmonic analysis for these data. Since the DNA sequence data is a categorical time series, this requires assigning numerical values to each of the categories present, yielding a time series which can then be investigated using spectral analysis. Stoffer et al.'s proposal for choosing the numerical values associated with the categories is to select numbers such that any periodic elements possibly present in the categorical process are highlighted. For a variety of such scalings, the spectral density can be considered, with the so-called spectral envelope being the maximum standardized spectral density attainable across all possible scalings. It thus provides a way to identify scalings that emphasize relevant periodic features.

While the approach for categorical data is suitable to explain the concept behind the spectral envelope, the GDP data used here is real-valued. An extension of the methodology to such time series is introduced by McDougall et al. (1997). Instead of just looking at the original time series, it is also possible to consider transformations thereof, the spectra of which will vary according to the particular transformation. Similar to the situation for categorical data, the spectral envelope encompasses the spectral densities for all possible transformations, with the most relevant periodic feature revealed through the transformation which yields the highest standardized spectral density. In case of a multivariate time series, a natural choice for the class of transformations to consider are the linear combinations of the elements.

Formally, the spectral envelope is obtained as follows: Let $y_t = (y_{1t} \dots y_{nt})'$ be a vector of n time series with spectral densities $f_y(\omega)$. This process can be scaled by any $n \times 1$ vector of real or complex constants to yield $x_{t,\beta} = \boldsymbol{\beta}^* y_t$, where * denotes the transpose and conjugate. For each of these scaled processes, the standardized spectral density is given by $f_x(\omega, \boldsymbol{\beta})/\sigma^2_{\boldsymbol{\beta}}$, where $\sigma^2_{\boldsymbol{\beta}} = \text{var}[x_{t,\beta}]$. The optimal scaling at frequency ω is then obtained as

$$\lambda(\omega) = \sup_{\beta \neq 0} \left\{ \frac{f_x(\omega, \boldsymbol{\beta})}{\sigma^2_{\boldsymbol{\beta}}} \right\}. \tag{3.7}$$

$\lambda(\omega)$ is thus the largest proportion of the spectral power that can be obtained at frequency ω for any scaling of the time series in y_t. It is called the spectral envelope because it provides an upper bound for all standardized spectra at frequency ω, with the spectral density equaling $\lambda(\omega)$ only if $\boldsymbol{\beta}$ is proportional to the optimal scaling.

With respect to the original, unscaled time series y_t, the spectral envelope is given by

$$\lambda(\omega) = \sup_{\beta \neq 0} \left\{ \frac{\boldsymbol{\beta}^* f_y(\omega) \boldsymbol{\beta}}{\boldsymbol{\beta}^* \mathbf{V}_y \boldsymbol{\beta}} \right\}. \tag{3.8}$$

across the entire range of frequencies $-\pi < \omega \leq \pi$. In this expression, $\mathbf{V}_y = \text{var}(y_t)$, i.e. the variance-covariance matrix of y_t, for which positive definiteness is assumed. By considering only real-valued scalings and denoting the real part of $f_y(\omega)$ by $f_y^{\text{re}}(\omega)$, it can be noted that $\boldsymbol{\beta}^* f_y(\omega)\boldsymbol{\beta} = \boldsymbol{\beta}^* f_y^{\text{re}}(\omega)\boldsymbol{\beta}$ for all $\boldsymbol{\beta} \in \mathbb{R}^n$ and hence $f_y^{\text{re}}(\omega)$ may replace $f_y(\omega)$ in equation (3.8). Furthermore, symmetry of $\lambda(\omega)$ is asserted from $f_y^{\text{re}}(\omega) = f_y^{\text{re}}(-\omega)$, so it is sufficient to consider the spectral envelope just for $0 < \omega \leq \pi$. The $\lambda(\omega)$ that maximizes the problem in equation (3.8) can then be obtained by solving

$$f_y^{\text{re}}(\omega)\boldsymbol{\beta}(\omega) = \lambda(\omega)\mathbf{V}_y\boldsymbol{\beta}(\omega) \tag{3.9}$$

for $\boldsymbol{\beta} \neq 0$. Because $\mathbf{V}_y$ has full rank, this means $\lambda(\omega)$ is the largest eigenvalue of

$$\left| f_y^{\text{re}}(\omega) - \lambda\mathbf{V}_y \right| = 0. \tag{3.10}$$

For the analysis of output cycles, the spectral envelope may prove a useful tool because of its ability to highlight common cyclical components at a particular frequency. If there are frequencies for which a certain linear combination of GDP cycles stand out, the spectral envelope at these frequencies should be higher than elsewhere. By considering the specific linear combination that led to the spectral envelope, inference can be drawn concerning the contribution of the individual countries' cycles to a common one. To quantify the notion of standing out from the surroundings, Stoffer (1999) presents a significance threshold level which the spectral envelope should exceed to be meaningful.

The graph in Figure 3.4 depicts the spectral envelopes for the cyclical component of output according to the four filtering techniques. Interestingly, there is no indication of any significant maxima for the spectral envelope when considering quarterly growth rates as shown in the upper left panel. No combination of cycles from different countries thus exhibits particular spectral power, neither at business cycle frequencies nor anywhere else. If yearly growth rates are used instead, as shown in the upper right panel there is clear indication of some sort of long-range dependence as the spectral envelope attains very high values for the low frequencies corresponding to cycles with length well above one decade.

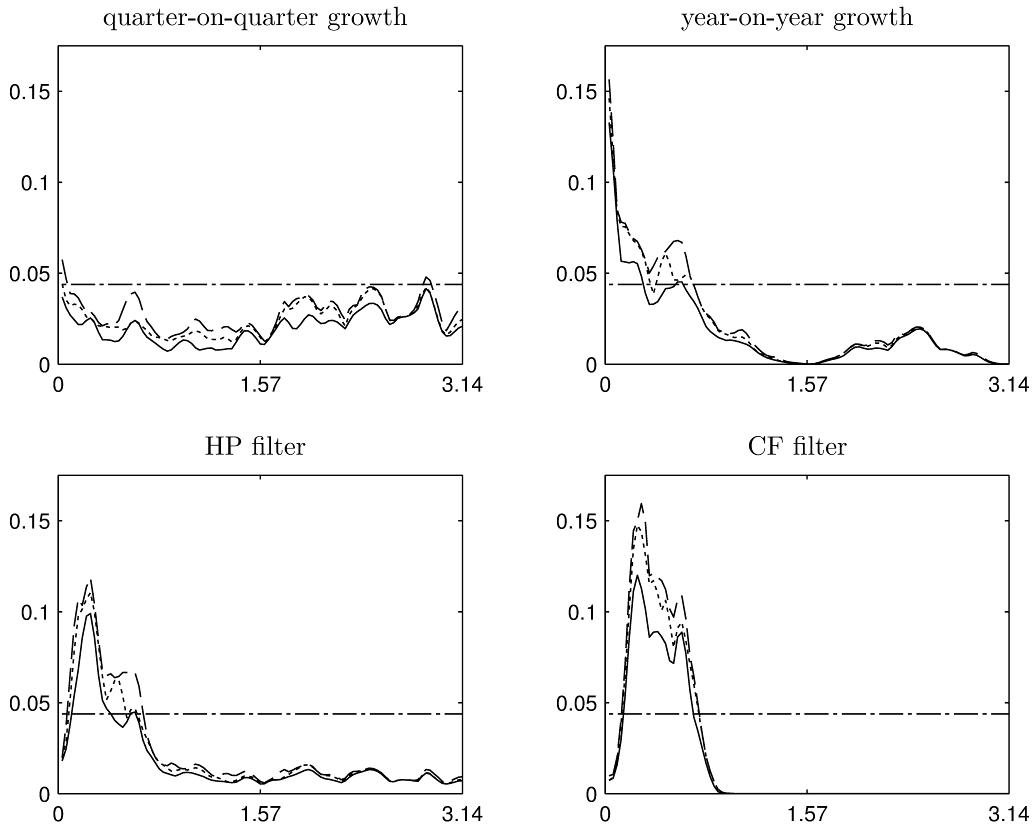

Figure 3.4: Results for the spectral envelope at different frequencies (denoted on the x-axis). The solid lines represent data from the Euro countries, the dotted lines correspond to the European group of countries, while the dashed line is based on data for all countries. The horizontal line represents the significance threshold.

Evidence of such behavior even in detrended economic time series has been mentioned by several studies before, such as Ding et al. (1993). At the same time, there is some very limited indication of spectral power around frequency 0.6, that is, for cycles with length 10 quarters. However, the peak in the spectral envelope is barely significant and tiny compared to the values at the lowest frequencies.

The situation is entirely different for the series subjected to a filtering procedure. Results are shown in the bottom two panels, with the HP filtered data on the left and the CF filtered data on the right. Rather obviously, there is little to no evidence of spectral power outside the passband of the filter, because these areas are cut off as far as possible. However, the comparison of the spectral envelope for frequencies above or below those corresponding to business cycles gives a nice illustration of the superior performance of the CF filter in removing these components. While the spectral envelope is far from significant, but nevertheless noticeable for the HP-filtered data, it is completely non-existent when the CF filter is used. Yet the main result for both cases is the strong indication of common spectral power at business cycle frequencies, with the maximum value of the spectral envelope corresponding to cycles of length approximately 7 years. The finding furthermore supports the notion that the Euro countries are the main driver behind this common periodicity, because the spectral envelope increases only slightly when the two further European countries are added, and the additional gain is close to zero for the two non-European ones.

For the data in the lower two panels that exhibit a clear peak for the spectral envelope at a frequency around 0.22, the corresponding scaling at that frequency for the Euro country group is $\boldsymbol{\beta}(\omega) = (0.68, -0.87, 1.00, 0.22, 0.23)'$ in case of the cyclical component from the HP filter and $\boldsymbol{\beta}(\omega) = (0.98, -0.97, 1.00, -0.08, -0.03)'$ when using the CF filter. For the latter one, this can be approximated as $\boldsymbol{\beta}(\omega) = (1, -1, 1, 0, 0)'$, indicating that the last two countries (the Netherlands and Spain) contribute little to the common periodicity, while it is mainly driven by the cycles of France, Germany, and Italy. The results obtained with the HP filter are not as clear-cut, but point into the same direction.

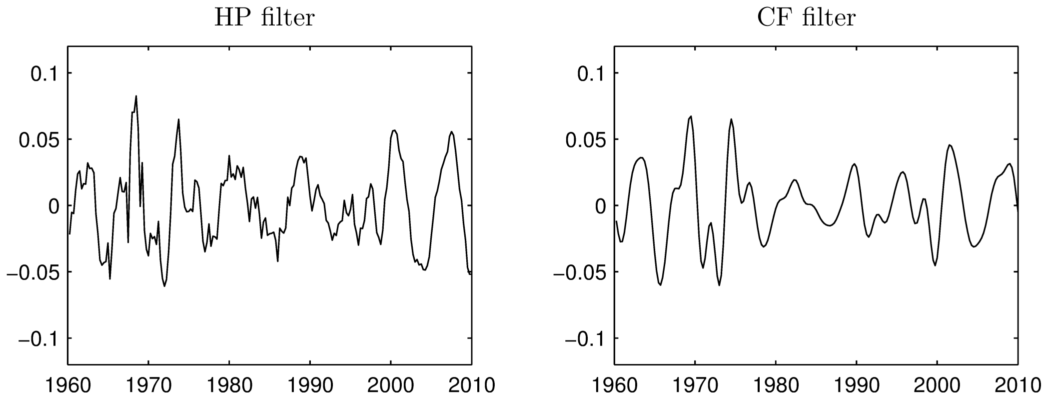

Figure 3.5: Common cycle obtained from the linear combination corresponding to the spectral envelope.

It appears counterintuitive that the elements of the $\boldsymbol{\beta}$ vectors do not all have the same sign. However, as Figure 3.5 shows, the common cycle that follows from multiplying the scaling vectors with the corresponding filtered series leads to a rather reasonable representation.

Interestingly, the results relating to the spectral envelope do not change much when only a sub-sample of the data is considered. A rolling window approach similar to that in Section 3.5 indicates little development over time.

3.7 Wavelets

A common feature of all approaches discussed so far is their exclusive focus on either the time or the frequency domain. While regular correlation coefficients can capture relationships over time, no indication regarding the length of cycles is given. Similarly, considering dynamic correlation, cohesion, or the spectral envelope, the time information included in the series is lost. Rolling window approaches as presented in the previous sections do provide some insights as to the development of these measures over time, yet it is desirable to have a combined measure involving both the time and the frequency domain. One way to reconcile these two is by the use of wavelet analysis.

3.7.1 The Wavelet Transform

The basic notion of a wavelet transform, as outlined by Daubechies (1988, 2006), is to consider a mapping from a given time series into a function of time and frequency. Two main types of wavelet transforms can be distinguished. The Discrete Wavelet Transform (DWT) limits itself to select discrete parameter values to consider along the time and frequency dimensions. Because the result of the transformation from one to two dimensions is extremely redundant, this still allows to recover the original series from the DWT. The characteristic of just looking at specific values makes the DWT widely used in applications such as image processing, where compression of data is an essential objective. DWTs have also been considered in economics, e.g. by Ramsey and Lampart (1998) to investigate relationships between money supply and nominal income, or Shik Lee (2004) to analyze price and volatility spillovers in stock markets.

However, for the task of detecting business cycle synchronization, the second type of wavelet transforms – the Continuous Wavelet Transform (CWT) – is more appropriate. It is computationally more demanding, but provides the full redundant outcome of the transformation from a single time series into a set of time and frequency values. Therefore, the interpretation of results is made much easier.

Wavelet analysis addresses the issue that in Fourier analysis the time information included in a series is no longer available after transformation. It is not possible to pinpoint when an event took place or whether any of the cyclical components changed over time. While this is not much of a problem if series exhibit similar properties throughout the sample, analysis becomes problematic as soon as different regimes and events begin and end within the time span under investigation. For the question at hand, the creation and enhancement of EMU ties constitute numerous such changes during the sample. All these transient dynamics are lost when relying on pure frequency analysis.

The key idea behind using wavelets thus is to look at the spectrum as a function of time and hence capture temporary developments as well. This is achieved by not considering waves of infinite duration, such as the sine or cosine in Fourier analysis, but rather “little waves” (or “ondelettes” in the original French literature). They consist of just a brief oscillation whose amplitude goes towards zero very fast as the function approaches $\pm\infty$. Intuitively, a wavelet can be compared to the recording of an earthquake by a seismograph.

To cover the entire real line despite the decay property required for wavelet functions, sets or families of wavelets are considered. They are derived from a mother wavelet ψ by scaling and shifting. A family $\psi_{\tau,s}$ of daughter wavelets would then read

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right). \tag{3.11}$$

The scaling parameter s influences the width of the wavelet through stretching ($s > 1$) or compressing ($s < 1$) and the translation parameter τ controls the location of the wavelet by shifting its position in time. A wavelet created with a specific frequency and duration will then resonate if the signal embedded in the data contains components of this particular frequency.

The mother wavelet has to fulfill several technical conditions as pointed out by Daubechies et al. (1992). Its mean, $\int_{-\infty}^{\infty}\psi(t)dt$, must equal zero, while the integral of its square, $\int_{-\infty}^{\infty}(\psi(t))^2dt$, has to be one. The latter requirement yields the limitation of the wavelet to a certain time interval. Additionally, the admissibility condition $0 < C_\psi = \int_0^\infty \frac{|\hat{\psi}(\omega)|}{\omega}d\omega < \infty$, where $\hat{\psi}(\omega) = \int_{-\infty}^{\infty}\psi(t)e^{-i\omega\tau}dt$ denotes the Fourier transform of ψ, has to be met. The simplest choice of wavelet is the Haar wavelet (Haar, 1910) defined by

$$\psi_H(t) = \begin{cases} 1 & \text{for } 0 \leq t < 0.5, \\ -1 & \text{for } 0.5 \leq t < 1, \\ 0 & \text{otherwise.} \end{cases} \tag{3.12}$$

which can be obtained by combining two rectangular functions. Its disadvantage is the obvious non-continuity of the function.

The vast majority of recent studies using CWTs instead focus on one group of continuous mother wavelets known as Morlet wavelets (Goupillaud et al., 1984) and given by

$$\psi_{\omega_0}(t) = \pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2}. \tag{3.13}$$

The Morlet wavelet depends on one parameter, ω_0. It is usually chosen to be $\omega_0 = 6$, which is also the value that will be used throughout the analysis here.

The popularity of the Morlet wavelets arises from a number of favorable properties. When considering the desired localization in both time and frequency, the Heisenberg uncertainty principle asserting that both cannot be determined to arbitrary precision simultaneously has to be considered. The Morlet wavelet minimizes the size of this Heisenberg window of uncertainty around a point, thereby reaching the lower bound for the inevitable uncertainty. Furthermore, the concentration of ψ in time is the same as in frequency, providing the best balance with respect to the two dimensions. Finally, as Lilly and Olhede (2009) outline, there are several ways to relate the scale parameter s, responsible for stretching the wavelet and thus corresponding to the space between oscillations, to Fourier wavelengths. For the Morlet wavelet with $\omega_0 = 6$, all these associated frequencies coincide, so the scale parameter can be treated like the frequency in Fourier analysis.

Thus focusing on this particular choice of wavelet, the CWT for some time series $x(t)$ with respect to the wavelet ψ is given by

$$W_{\psi,x}(\tau, s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-\tau}{s} \right) dt, \tag{3.14}$$

where the asterisk denotes complex conjugation. The CWT maps the original one-variable function into a function of both τ and s, making it possible to conduct inference on both time and frequency simultaneously. The main difference of the CWT compared to a Fourier transform is the use of the wavelet instead of sine and cosine functions and the appearance of τ as a localization parameter in the time domain. The wavelet power spectrum corresponding to the CWT is denoted as

$$WPS_{\psi,x}(\tau, s) = |W_{\psi,x}(\tau, s)|^2 . \tag{3.15}$$

3.7.2 Cross-Wavelet Analysis

While the measures described so far aim at detecting time-frequency patterns in a single time series, the question of whether business cycles are aligned requires checking for common patterns in both. Measures for this purpose can be derived from their counterparts in pure frequency analysis.

For two time series x and y, the cross-wavelet transform (XWT) has been introduced by Hudgins et al. (1993) as

$$W_{xy}(\tau, s) = W_x(\tau, s)W_y(\tau, s)^*, \tag{3.16}$$

suppressing the ψ in the index from now on, since the investigation is only concerned with the Morlet wavelet. It represents the covariance between two series at each possible combination of time and frequency and can hence serve as an indication of how similarities are distributed in this space.

This is visualized in Figure 3.6 for the pairs that can be constructed from the HP-filtered cyclical components of the members in the Euro country group. Dark areas depict time-frequency combinations spotting a high cross wavelet transform, while lower values are presented in lighter shades.

For convenience, the y-axis denotes the length of the cycles in years. While the results differ for the individual pairs, there is a general tendency that higher cross-wavelet transforms can be observed for cycles with a period of at least four years. Concerning the development over time, cross-wavelet transforms are generally lower in the middle of the sample than in the initial or final years. The highest values can be observed for the last decade, after the introduction of the Euro. Only for a few country pairs, such as Germany–Italy or Germany–Netherlands, the indication of comovement for cycle lengths around five years

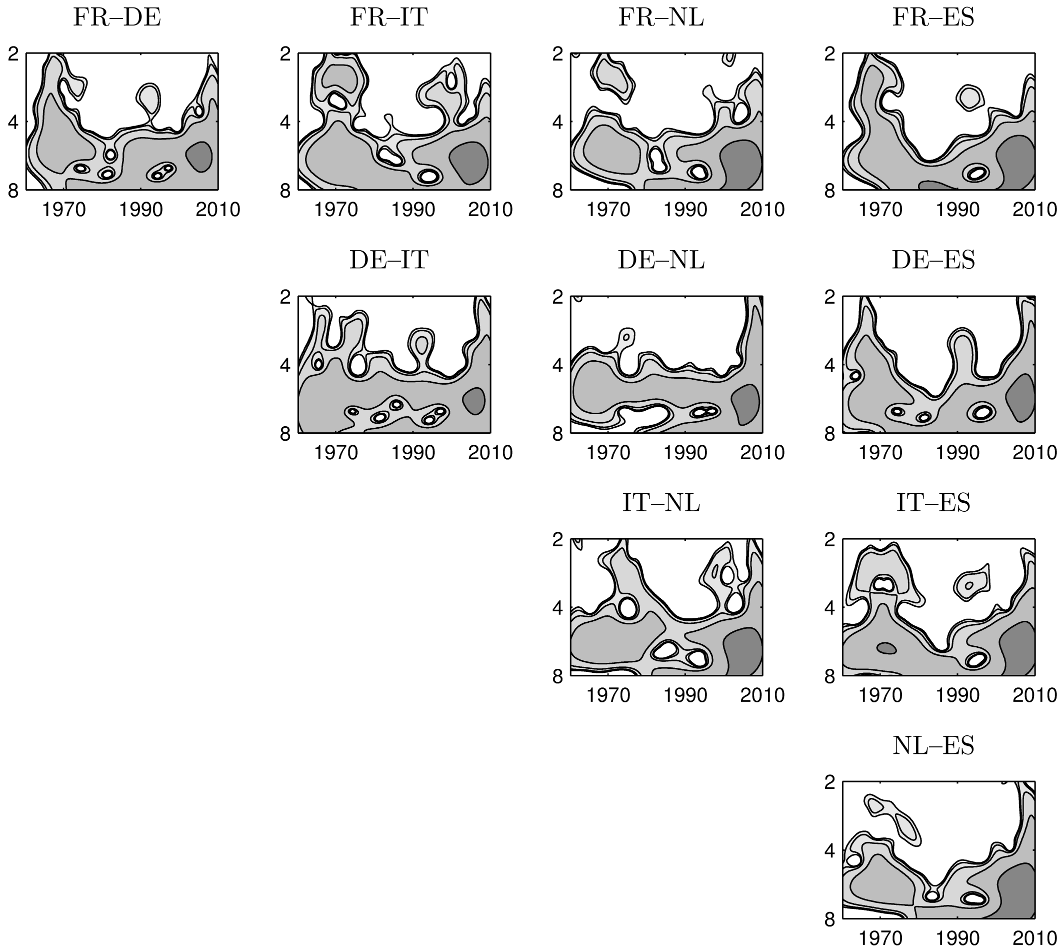

Figure 3.6: Cross Wavelet Transforms (XWT) at business cycle lengths for pairs of Euro countries. For each panel, the y-axis shows the length of the cycle in years and the x-axis shows the localization in time. Time-scale combinations with a higher XWT are shaded darker.

is uninterrupted throughout the sample. Furthermore, there is some evidence of synchronization at higher frequencies, that is, for shorter cycles, during the 1970s and towards the end of the sample for most country pairs.

A serious drawback of the cross-wavelet transform as introduced in equation (3.16) is the lack of normalization. The wavelet coherency measure, due to Torrence and Webster (1999), addresses this concern through normalizing

$W_{xy}(\tau, s)$ by the spectrum of each series. The resulting statistic is very similar to the concept of coherency in pure frequency analysis and is defined as

$$R_{xy}(\tau, s) = \frac{|S(W_{xy}(\tau, s))|}{\sqrt{S(|W_x(\tau, s)|^2) S(|W_y(\tau, s)|^2)}}, \tag{3.17}$$

where $S(\cdot)$ denotes a smoothing operator. The smoothing operator is applied with regard to both frequencies and time. Suitable choices for the operator are discussed in Torrence and Webster (1999) and Torrence and Compo (1998). Basically, smoothing can be obtained by a convolution with a window function along both the time and scale dimensions:

$$S(W(\tau, s)) = \int_{\tau-\Delta_1/2}^{\tau+\Delta_1/2} \int_{s-\Delta_2/2}^{s+\Delta_2/2} W(t, \varsigma) f_{\Delta_1,\Delta_2}(t, \varsigma) dt d\varsigma,$$

where f_{Δ_1,Δ_2} satisfies $\int\int f_{\Delta_1,\Delta_2}(t, \varsigma) dt d\varsigma = 1$. Wavelet coherency thus is unity if at a particular time and scale, a perfect linear relation exists between the two time series; while at the other extreme, it is zero if the series are independent. Although the exact choice of the smoothing function f is somewhat arbitrary, this is not different to the situation in Fourier analysis, where coherency is based on the smoothed periodogram.[6]

Just as for the coherency measure in pure frequency analysis, the appeal of the statistic in (3.17) is its similarity to the standard correlation coefficient, so that it can be considered as a correlation coefficient localized in time-frequency space (Grinsted et al., 2004).

Because of the similarity to its Fourier counterpart, the approach can easily be extended to provide insight for entire groups of countries, allowing to compare the group of Euro countries with broader sets. Consider a vector $y_t = (y_{1t} \dots y_{nt})'$ with $n \geq 2$ and positive weights $w = (w_1 \dots w_n)'$ attached to each element of y_t. The proposed measure is motivated in the same manner as Croux et al.'s

[6] Both in the Fourier and in the wavelet case, the coherency measure would be unity everywhere without smoothing.

(2001) measure of cohesion in the frequency domain and will hence be referred to as wavelet cohesion. It equals

$$\text{wavecoh}_y(\tau, s) = \frac{\sum_{i \neq j} w_i w_j R_{y_i y_j}(\tau, s)}{\sum_{i \neq j} w_i w_j}. \tag{3.18}$$

To simplify the measure, weights can be chosen as $w_i = 1$ for all i. While this may be appropriate if each element in y_t were of the same importance, with countries in the dataset greatly varying in size, it is more reasonable to take account of this fact by a suitable weighting scheme. Hence, like previously for the cohesion measure in pure frequency analysis, the w_i are chosen to be the population of the countries in 2010, i.e. at the end of the sample.

With this new wavelet cohesion measure, it is possible to capture synchronization of business cycles for groups of countries. Figures 3.7 to 3.9 show graphical representations of the $\text{wavecoh}_y(\tau, s)$ statistic using the county groups consisting of all, the European, and the Euro member states, respectively. The contour plots indicate wavelet cohesion for the entire time span of the sample and for cycle lengths between two and eight years. Areas colored in the darkest shade of gray indicate a cohesion statistic between 0.8 and 1.0 and thus the highest possible degree of synchronization. Each contour line and switch to a lighter shade then represents a decrease in cohesion by a 0.2 increment, leaving areas with little cohesion – $\text{wavecoh}_y(\tau, s)$ between 0 and 0.2 – entirely white.

Figure 3.7 shows cohesion for all countries together. It indicates that except for the very beginning of the sample, synchronization is present for cycles with a length of approximately five years, as cohesion at this scale exceeds surrounding higher and lower frequencies. During the late 1980s and early 1990s, the link is least substantial. Also within that time span, cohesion is particularly low for shorter cycles between two and four years. More generally, cohesion is low for cycles shorter than four years everywhere except at the very end of the sample, where all frequencies show a clear increase in cohesion. Another notable aspect is that, unlike for the purely time- and frequency-based measures of the previous sections, there is only very little discrepancy between the results for

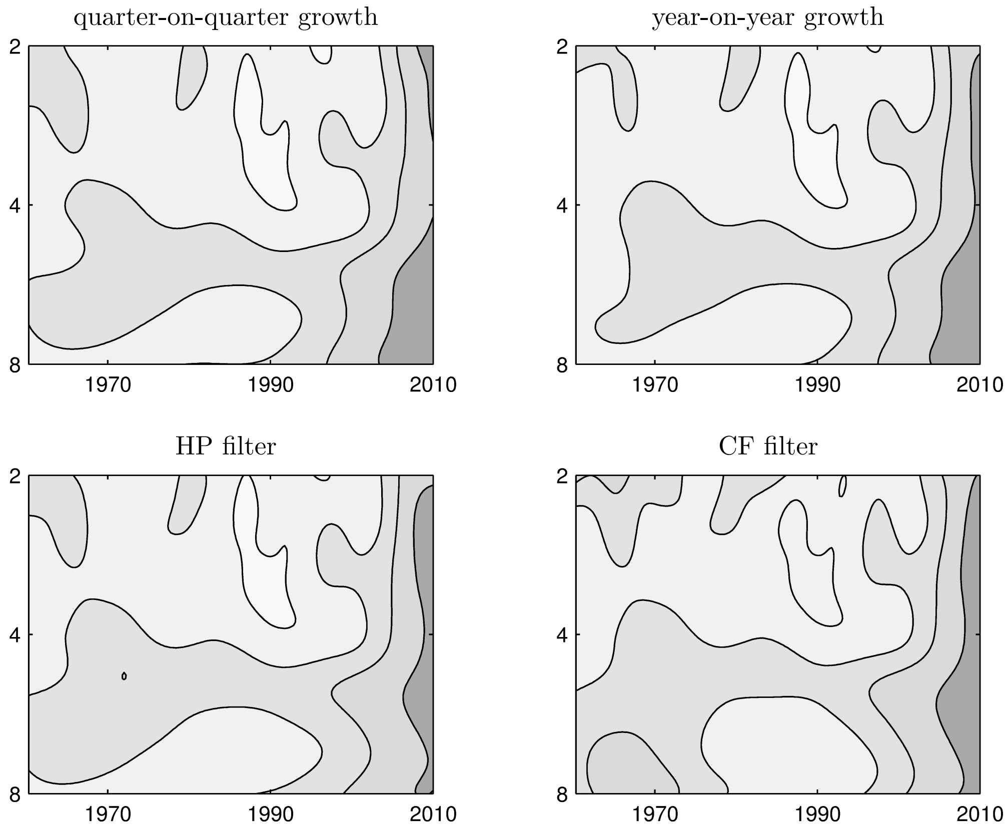

Figure 3.7: Wavelet cohesion for country group *all* at business cycle lengths. The y-axis shows the length of the cycle in years and the x-axis shows the localization in time. Time-scale combinations with higher wavelet cohesion are shaded darker.

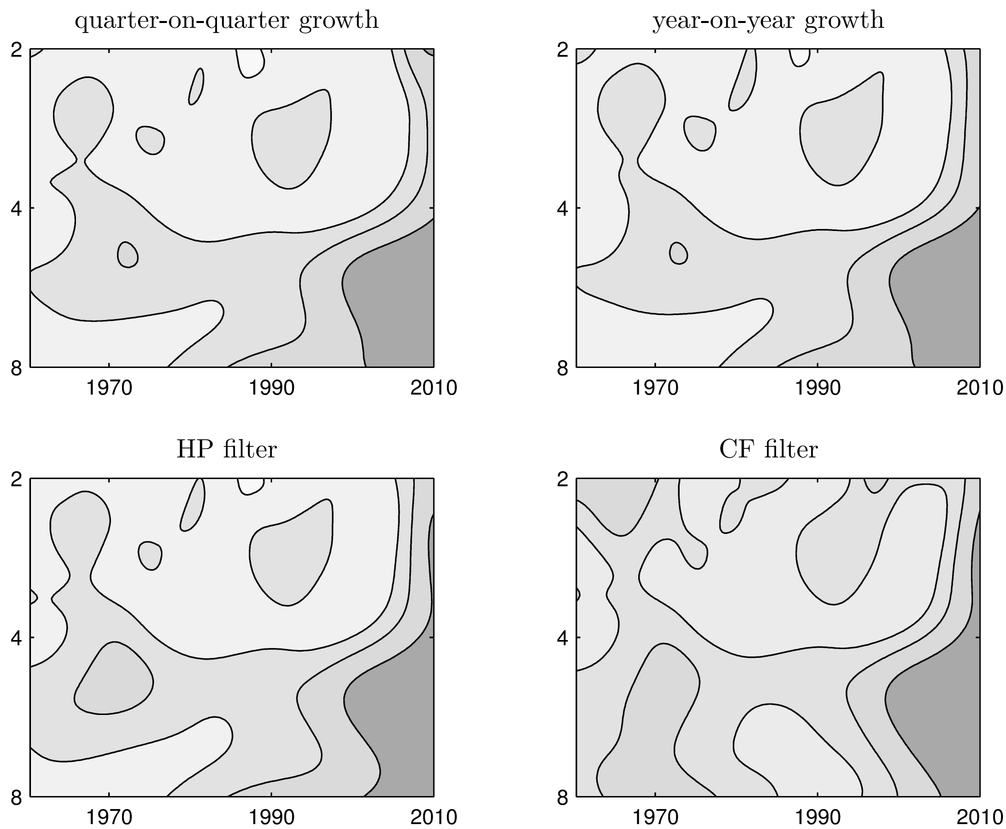

Figure 3.8: Wavelet cohesion for country group *Europe* at business cycle lengths. The y-axis shows the length of the cycle in years and the x-axis shows the localization in time. Time-scale combinations with higher wavelet cohesion are shaded darker.

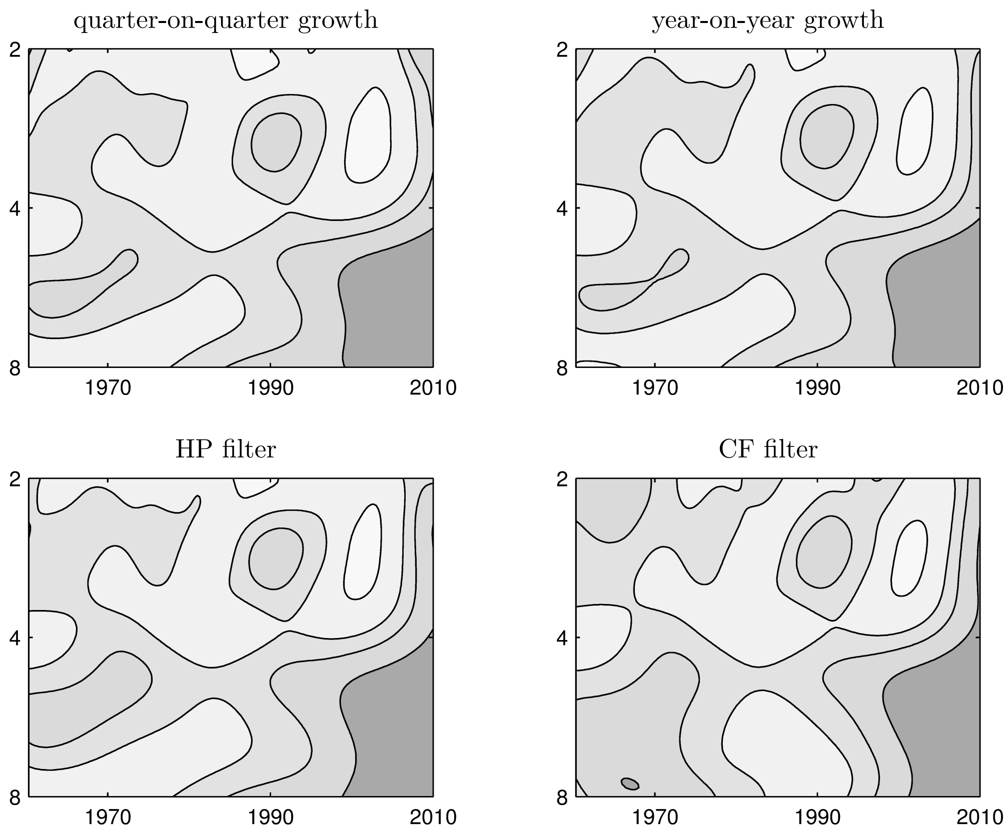

Figure 3.9: Wavelet cohesion for country group *Euro* at business cycle lengths. The y-axis shows the length of the cycle in years and the x-axis shows the localization in time. Time-scale combinations with higher wavelet cohesion are shaded darker.

the different cycle extraction methods and the conclusions concerning cyclical comovement behavior hold regardless of the filtering method.

For the European country group, whose wavelet cohesion results are depicted in Figure 3.8, the overall picture is rather similar to that for the complete set of countries. Evidence of cohesion is a bit stronger than before and again it is most pronounced for cycles of around five years length. Since around 2000, those five-year cycles have very high cohesion values of above 0.8. Interestingly, the time-scale combination of two- to three-year cycles around 1990, which spotted the smallest cohesion in Figure 3.7 now exhibits a somewhat stronger synchronization link than its surroundings.

Finally, Figure 3.9, considering only the Euro member countries, confirms the presence of synchronization for five-year cycles throughout the sample. Unlike for the other groups, however, there is considerable cohesion already during the early years. As in the previous groups, cohesion is generally somewhat lower in the 1980s than before and after. The "island" of high cohesion for shorter cycles around 1990 is again present and more substantial than for the European group. For the years subsequent to the introduction of the Euro in 1999 and for cycles longer than five years, there is very extensive cohesion throughout. The results of the different filtering approaches are not as similar as for the group consisting of all countries, yet unlike for the purely time- or frequency-based approaches, all important features are qualitatively the same across methods.

The use of wavelets thus does not only introduce the advantage of combining the analysis of time and frequency dimensions, but it also turns out that the statistics are much more robust to the choice of filtering technique. While both the contemporaneous correlation coefficients and the different approaches in the frequency domain yield results that vary depending on which measure is used to extract the cycle, this is not the case for wavelet analysis.

3.8 Conclusion

This chapter has compared a variety of methods targeted at measuring business cycle synchronization. It has shown that it is insufficient to consider the time domain and the frequency domain separately and established the sensitivity of statistics to the choice of cycle extraction technique. This holds in particular for the spectral envelope. As a more refined approach, the chapter has pointed out that wavelets are a valuable tool for the analysis of business cycles, because they allow to consider a localization of common periodicities in scale and time. In particular, a new measure of comovement between several – rather than just a pair of – series has been introduced for a wavelet setting.

The wavelet cohesion statistic asserts that cyclical components in output with a length of approximately five years are synchronized to a certain degree for various sets of countries. This lies well within the range typically considered as business cycles and corresponds exactly to the finding of Artis et al. (1997) who pinpoint the typical business cycle length to be between five and six years. Cohesion is stronger for countries sharing the Euro, in particular since the actual introduction of the currency, but also in earlier years already. These findings align with those of previous studies such as Rua (2010), who considers fewer countries and just bivariate relationships over a shorter sample. Furthermore, using the wavelet approach, Canova's (1998) criticism of arbitrariness in the choice of cycle extraction technique is remedied, because the deliberate choice of filter does not affect the result eventually obtained for synchronization.

The wavelet-based measures turn out to be very useful because results show comovements both hinge on the frequency of cycles and develop over time. These two dimensions are easily incorporated in the wavelet cohesion statistic, while other approaches have to resort to auxiliary tools to provide insight beyond a single one of the dimensions.

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