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Introduction

The global financial crisis (2007-08) had a great influence on the banking sector and its regulation. The collapse contributed to a global recession, a credit crunch and the European sovereign-debt crisis. These effects of the crisis were accompanied by huge costs for the taxpayer. All institutions and parties with an influence on the banking sector - the banks themselves as well as regulators and researchers - have the responsibility and duty to advance and change the financial system for the better and to strive to prevent such crises in the future. To gain a thorough understanding and obtain a solid basis for decision making, a deep analysis on the financial crisis of the last years from different perspectives has to be performed.

Two of those important aspects are discussed in the following chapters: The transmission of relevant information and the continuous improvement of tools used for risk assessment and similar tasks. The existence, the transmission, and the generation of information played a crucial role in the circumstances leading to the crisis. Effective warning systems did not exist and incentives to improve the tools that banks used before the crisis, e. g. their risk models were missing. This dissertation contains an analysis of these information problems and possible solutions by pointing out effective organizational structures and regulation. Furthermore, besides improving the banking system, it is important to provide a boost to the suffering economy after the crisis. Small business growth is a key item on the agenda of many countries; however, frequently, a lack of capital available for small companies slows their development. New funding instruments can help to overcome the shortage of financial resources, e. g. partnerships with start-up accelerators or proof-of-concept centers, IP-backed financial instruments. Another new approach to obtain financing is the concept of crowdfunding. Crowdfunding has evolved within the internet community and a variety of smaller and larger projects have already been financed mostly by private investors. Again, the role of information is crucial and is analyzed in the following.

This dissertation consists of three chapters covering the development and analysis of three self-standing theoretical models. They all take an informational economics view and analyze important topics related to banking and financial markets. The first chapter discusses crowdfunding as a tool to exploit the financial wisdom of the crowd by aggregating vague and imprecise information whereas the last two chapters deal with informational problems leading to the financial crisis: the lack of early warnings and missing incentives for an improvement of risk models in banks. All models are results from joint work with Hendrik Hakenes.

Crowdfunding as a novel form of financing is in the focus of the first chapter. This funding concept has seen extensive growth over the last few years. Under crowdfunding, a firm asks for a large number of small loans from many households. However, if some predefined threshold for the aggregate loan volume is not attained, the firm cannot obtain the loans. We construct a model to determine whether this mechanism can be used to aggregate vague information. In our economy, there is a firm that needs finance for an investment project, and a number of households which are potential future consumers of the firm's product. Each household can spend an effort to produce a bit of vague information on whether they will later like the product – too vague to justify a straight loan. However, if the firm sets a high threshold, a household knows that its money will only be transferred if many other households also obtain positive information about the firm. We describe the equilibrium behavior of households and firms. In crowdfunding, from a welfare perspective, firms set both the loan rate and the threshold too low, inducing households to generate too much information. However compared with standard debt, crowdfunding enables more worthy projects to receive funding.

The second chapter constructs a theoretical model of a bank with a bank manager and lenders. It analyzes incentives to communicate warnings in case of a deteriorating situation for the loan portfolio of the bank. It is motivated by the insight that the severity and depth of the recent financial crisis hit many by surprise. Despite warning signs, the financial system seems to have been unable to aggregate existing information. As the events of fall 2008 showed, many investors were caught off guard by the large number of loans and banks collapsing worldwide. But what triggers an early warning, and what are the incentives to implement such a trigger? In our analysis, we consider a bank that is financed with debt and equity, and a bank manager monitoring the bank's loan portfolio. The manager must be incentivized to warn the bank's board before a crisis. However, we show that the board may implement a contract with insufficient incentives to communicate a warning, as refinancing conditions deteriorate when lenders notice an upcoming crisis. To obtain an equilibrium in which the bank manager monitors and discloses negative information to the board he needs to be paid a fixed wage until the disclosure date and a one-off payment at the time of the information disclosure. We discuss policies to improve information efficiency and give conditions under which regulatory measures, such as capital and liquidity regulation, increase welfare.

The third chapter examines model risk and the missing incentives to improve risk models of banks. In the recent financial crisis, risk management tools have been proven inadequate. Model risk, a key component of bank risk, has shown its negative impact. It seems that risk models did not cover the included risks comprehensively and were not kept up-to-date by banks as well as by rating agencies. Consequently, in the aftermath of the crisis banks must adjust their models to reduce model risk. But do banks have the incentive to improve their risk models? We take a close look at risk models of banks and discuss if banks generally invest enough effort to improve their risk models. The question of risk model innovation is analyzed from a managerial as well as from a welfare perspective in the context of a principal agent model - where the bank has to incentivize an agent to perform innovative improvement in the risk model technology. We show that the bank owner offers a wage that is reduced to the minimum possible amount and consists of two components: one administration fee and one bonus payment. Thus, the agent's wage depends on the quality of the applied risk model. In a next step, we show under which conditions it is more efficient to reduce model risk by costly model innovation either through an employee or through an external consultant. Hence, the third chapter provides insights for the optimal organizational structure of this improvement process of risk models in banks.

This dissertation discusses interlinked topics related to the process of communicating or aggregating information against the background of the financial system. Our results show that the relatively new tool of crowdfunding allows projects to receive large sums of money by pooling the signals of many investors or consumers. The aggregation of information brings an advantage compared to standard debt finance and enables more companies to receive funding. We also show how important it is to provide effective incentives for communicating not only positive but also negative information. As a bank might fear a negative reaction of financial markets, regulatory measures are important to induce the bank to set incentives for receiving warnings in case of deteriorating prospects. Moreover, it is analyzed how organizational structures can help to overcome informational problems related to the reduction of model risk in banks. Depending on different factors such as the market quality or effort costs for an innovation, it may be better for a bank owner to either engage an internal or an external agent to perform optimal reductions of errors in risk models.

1 Exploiting the Financial Wisdom of the Crowd — Crowdfunding as a Tool to Aggregate Vague Information

1.1 Introduction

Crowdfunding is on the rise. Consider a specific example.[1] In 2012, Michael Bohanes had an idea for a startup called *Dinnr*. Customers of the company would order cooking ingredients and recipes for meals and then cook the meals themselves. However, Michael Bohanes needed an initial investment of 60,000 GBP to start and promote his website. To raise the necessary funding, Bohanes used the crowdfunding platform *Seedrs*. He was successful: in 3 months, he obtained 100 investors who pledged an aggregate of 60,000 GBP. In exchange for their investments, the investors received shares of the newly founded company Dinnr. If the investors had pledged less than 60,000 GBP, however, the financing would have failed: Bohanes would have received no loan, and no investor would have lost money.

From a theoretical perspective, crowdfunding is not straightforward to understand. Further, from a transaction costs perspective, crowdfunding should be more expen-

[1] http://startups.co.uk/innovative-grocery-service-dinnr-raises-60000-in-seedrs-funding/

sive than, for example, bank financing because of the sheer number of contracts and relationships. Diamond (1984) argues that with costly state verification, a delegated monitor (bank) should position itself between investors and firms. Hence, what economic value does crowdfunding offer? Under what conditions can crowdfunding dominate traditional forms of financing? Moreover, is a firm's decision to use crowdfunding optimal from a welfare perspective? We develop a model to answer all of these questions.

Our model economy includes both a firm that needs financing for an investment project and a number of households. Firms can be interpreted as producers of consumption products (e. g., movies, electronic devices, computer games), and two types of firms exist: good firms and bad firms. Good firms will later have a high probability of producing a good product, creating a positive net present value (NPV) for the project, whereas bad firms have a low probability of producing a good product. Importantly, households are potential future consumers of firms' products. If they think about a product (which is costly), they can assess their later preference for it (even though it has not yet been produced). In the above example, households would consider their own preference about ordering a recipe together with its ingredients from Dinnr. Of course, this information is valuable in a firm's financing stage. If many households have a negative impression of the product, the product will likely be bad, and the firm should not receive financing.

With traditional financing, e. g., direct loan financing, households might not be willing to invest because the information that they know about a firm's prospects is too vague. Additionally, a loan officer in a bank cannot aggregate household information. As in our example of Dinnr, forecasting the success of a brand-new concept is very difficult. In such situations, crowdfunding is useful. Crowdfunding exists in various forms, which are discussed below. Let us first concentrate on lending-based crowdfunding (i. e., crowdfunding in the form of loans). Here, firms fix an interest rate and, importantly, some minimum aggregate loan volume (called the *threshold*). Households can then pledge to participate in the crowdfunding.

If enough households pledge and the threshold is reached, the pledged loans are transferred from the households to the firm. If the threshold is not reached, the funding fails, and no monetary transfers occur.

In this contractual setting, households can pledge money even if the information that they know about a firm or product is positive but very vague. They anticipate that the firm can only receive the money if enough other households also have obtained positive information about the firm; otherwise, the threshold will not be reached. However, a free-rider problem arises. A household knows that the funding will only be successful if a sufficient number of other households have obtained positive information; thus, the household may not trust the only vague signal it can receive. Anticipating this problem, the household might not gather any information at all and may thus free ride on the information gathered by others. Households' incentives to gather information, free ride, or do nothing depend on a firm's selected loan rate and its threshold.

We show that in equilibrium, firms set the contract parameters such that all households are incentivized to gather information but are motivated to participate only if that information is positive. To do so, the threshold is set sufficiently low, such that even bad firms have a significant probability of receiving financing. Thus, to avoid (or rather, mitigate) the winner's curse, a household must become informed. Accordingly, firms endogenously and completely eliminate the free-rider problem.

We then compare a firm's choice with the welfare optimum. Given that a firm uses crowdfunding, does it set the contract's parameters optimally? The answer is negative: firms set both the loan rate and the threshold too low. Accordingly, they induce *all* households to become informed, whereas the optimal number of informed households may be lower. Nevertheless, compared with standard debt financing, we find that crowdfunding enables more worthy projects to receive financing.

Examples and Institutional Background The introductory example of Dinnr is only one of many projects that have been crowdfunded. To demonstrate the broadness and importance of this financial instrument, we provide two other examples. Next, we illustrate the development of the market and the process of crowdfunding. Finally, we present a classification of the different types of crowdfunding.

In February 2014, a letter from the chairman of Roberts Space Industries announced that the company had surpassed 38 million USD in funding for its video game Star Citizen. The company was able to raise this enormous sum via an Internet campaign on its own website and via the platform Kickstarter.[2] People may receive various benefits, such as digital downloads, hard copies of books, or a CD with the game's soundtrack, in return for their funding depending on the amount they pledged.[3]

As another example, Barack Obama collected approximately 750 million USD for his 2008 presidential campaign. Most of this money was raised via the Internet from the contributions of small donors offering 200 USD or less. The crowdfunding method helped Obama surpass all of his opponents in the race for the White House.[4] In this case, funders did not receive anything in exchange for their money apart from the hope that their support would enable their favorite candidate to win the election.

In all of these examples, the projects collected a large amount of money via the Internet. This method of funding is not only useful for donations or fan support but also beneficial for companies in obtaining essential funding. For entrepreneurs, attracting outside capital remains difficult, even though many possibilities for financing exist, such as venture capital, IPOs, bootstrapping, and conventional bank financing. Before 2006, crowdfunding was mostly unknown. Initially, it was then used to support projects to make films, books, music recordings, and charitable endeavors. Today, however, many investment projects and startups are given life by crowdfunding platforms and the interest in crowdfunding is increasing and becoming broader (Cross, 2011).

[2] https://robertsspaceindustries.com/comm-link/transmission/13550-Letter-From-The-Chairman-38-Million

[3] https://www.kickstarter.com/projects/cig/star-citizen

[4] http://abcnews.go.com/Politics/Vote2008/story?id=6397572

Over the last years, the market saw a massive growth. Kickstarter, the world's largest crowdfunding platform, was founded in 2008 and has raised over 775 million USD for different projects until end of 2013. In 2010, investors pledged over 27 million USD for projects on Kickstarter. In 2011, already 99 million USD could be pledged. And from 2012 to 2013 this number further increased from 319 to 480 million USD.[5]

The literature defines crowdfunding as an "open call, essentially through the Internet, for the provision of financial resources either in form of donation or in exchange for some form of reward and/or voting rights in order to support initiatives for specific purposes" (Belleflamme, Lambert, and Schwienbacher, 2014). Another definition is provided by Cross (2011): "The term 'crowdfunding' is used to describe a form of capital raising whereby groups of people pool money, typically comprised of very small individual contributions, to support an effort by others to accomplish a specific goal."

Crowdfunding is used to attract a large group of investors. The process usually involves an online crowdfunding platform as an intermediary, such as the website *Seedrs* mentioned in our first example. The fundraiser begins with a request and provides information about the investment amount needed and what is offered in exchange (Ahlers, Cumming, Günther, and Schweizer, 2013). Potential investors are provided with detailed information about the project for which funding is required. Based on this information, interested investors decide how much they are willing to pledge. However, if a predefined minimum amount of funding cannot be reached within a certain amount of time, the funding process is deemed unsuccessful, and no funding is transferred. This feature of crowdfunding protects the individual investor. Only if enough people are convinced by the project's potential, an individual person will have the chance to make the investment. Usually, no limit is set on the amount of fundraising, but the period for fundraising is restricted. Further, the crowdfunding

[5] http://www.kickstarter.com

platform provides a technical service for exchanging information and funds and typically receives a percentage of the funding amount for providing this service.

Four different business models of crowdfunding exist: donation-based crowdfunding, reward-based crowdfunding, lending-based crowdfunding, and equity-based crowdfunding (Griffin, 2012). The determination of these four categories depends on what the investors receive in exchange for their funds (Ahlers, Cumming, Günther, and Schweizer, 2013). In donation-based crowdfunding, people are interested in supporting a special project for charitable or sponsorship purposes. They have no expectation of a monetary repayment. Investors in reward-based crowdfunding receive a product or any other non-financial benefit in exchange for their funds. For example, pre-selling a product can be designed as reward-based crowdfunding. In lending-based crowdfunding, funders receive a fixed amount as a periodic payment, as with peer-to-peer loans. Finally, equity crowdfunding involves giving small pieces of ownership to investors. For donation-based crowdfunding, a model must include non-standard preferences, which we do not discuss in this article. However, the other three types of crowdfunding can be reflected by our model.

Literature Our paper is related to three strands of literature. First, of course, a stream of both theoretical and empirical literature on crowdfunding exists. Because the phenomenon of crowdfunding is relatively new, this stream of literature is also young and not very extensive. Second, in some aspects, crowdfunding is similar to an initial public offering (IPO), which is extensively studied in the literature. Third, crowdfunding is related to crowdsourcing. In crowdfunding, financing comes from "the crowd", whereas in crowdsourcing, "the crowd" typically supplies labor.

Let us start with the *theoretical literature* focused on crowdfunding. Agrawal, Catalini, and Goldfarb (2013) and Hemer (2011) provide good overviews of the crowdfunding market. Both papers are qualitative studies that examine recent developments and the outlook for crowdfunding. Schwienbacher and Larralde (2012) also review this new method of financing and add a case study by closely examining the French startup *Media No Mad*.

A few articles analyze crowdfunding from a theoretical perspective. Belleflamme, Lambert, and Schwienbacher (2014) compare two forms of crowdfunding and their respective benefits for entrepreneurs. Depending on the initial capital requirement, an entrepreneur should decide to either engage in a form of pre-ordering or advance a fixed amount in exchange for equity. Louis (2011) studies a mechanism similar to crowdfunding. In this paper, the agents decide whether to invest, not at the same time, but in a certain order, which generates a winner's curse for those agents at the end of the order.

The crowdfunding market is also the subject of several *empirical studies*. Factors for successful projects are analyzed by Belleflamme, Lambert, and Schwienbacher (2013) and by Ahlers, Cumming, Günther, and Schweizer (2013). Further, Agrawal, Catalini, and Goldfarb (2011) closely examine the geographic distribution of investors and entrepreneurs, particularly in the music market. Using a different approach, Hildebrand, Puri, and Rocholl (2013) find evidence of adverse incentives of which the market is not aware. Specifically, certain investors' bids may be overvalued as overly positive signals.

Literature on IPOs Our paper is also related to that of van Bommel (2002) and other articles that discuss the flow of information from the market to entrepreneurs (for example Welch, 1989). Based on the famous model of Rock (1986), van Bommel (2002) provides a setting in which managers attempt to increase the information that they receive from market participants through a particular IPO price policy. Further, evidence on information production in IPOs is studied by Corwin and Schultz (2005).

Literature on Crowdsourcing A large body of literature in the computer science community has studied crowdsourcing. For a non-representative sample of studies, see Archak and Sundararajan (2009), DiPalantino and Vojnovic (2009) and Chawla, Hartline, and Sivan (2012). Our paper differs from these studies in a few ways.

First, in crowdsourcing, the cost structure may differ among different programmers (private value), whereas in our setting, the firm has the same value for all bidders (common value). Second, in crowdsourcing, one (or very few) programmers attract the work order; however, in our setting, a large number of households must participate.

The remainder of this paper is organized as follows: In Section 1.2, we introduce the model. Section 1.3 analyzes the equilibrium of crowdfunding and includes comparative statics. Section 1.4 serves as a benchmark and discusses the outcome of standard debt financing. Standard debt financing is then compared to a situation in which both forms of financing, namely, standard debt financing and crowdfunding, are possible. Section 1.5 provides a discussion of the conditions of crowdfunding with respect to welfare. Section 1.6 concludes and section 1.7 presents the appendix of the model.

1.2 The Model

In our model, a firm seeks funding to start a new project and decides between standard debt financing and crowdfunding. We discuss the perspective of investing households in the crowdfunding environment and consider conditions under debt financing. Table 1.1 summarizes the variables that are used in our model and includes the parameters of our numerical example.

Consider an economy with two types of agents: a firm and a large number N of households. Firms have a constant return to scale technology: investing I, their project yields $R\,I$ with probability q at the end of the period; otherwise (with probability $1 - q$), they receive nothing. There are two types of firms: a fraction μ consists of good firms (index G) with a success probability of q_G, and another fraction $(1 - \mu)$ consists of bad firms (index B) with a success probability of $q_B < q_G$.

Table 1.1: Index of Parameters and Variables

α	$\frac{1}{4}$	probability of obtaining a bad signal even though the firm is good
β	$\frac{1}{4}$	probability of obtaining a good signal even though the firm is bad
μ	$\frac{1}{10}$	fraction of good firms
q_G	$\frac{2}{3}$	success probability of a good firm
q_B	$\frac{1}{3}$	success probability of a bad firm
c	$\frac{1}{80}$	cost of information
R	$\frac{5}{2}$	firm's return (if successful)
N	15	number of households
r		loan rate
I		aggregate investment
ι		probability of households being ιnformed
υ		probability of households remaining υninformed yet participating

The second column shows the parametrization that we use for our numerical examples.

A number N of households exists, each owning an endowment of \$1.[6] Households have access to information technology: spending c, they obtain information about the true nature of a firm. The information, however, is noisy: with probability α, a good firm sends a bad signal; with probability β, a bad firm sends a good signal.[7] We assume that the NPV of a bad firm is negative, $q_B\, R - I < 0$.

The risk-free interest rate is normalized to zero, and all agents are risk neutral. The nature chooses the type of the firm (G or B). The *timeline* of the funding process is as follows: *at date* $t = 0$, the firm chooses the type of financing with which to seek funding. If the firm prefers standard debt financing, it sets a loan rate r. Under

[6] N is a finite number, not the measure of a continuum. If households existed on a continuum, then the law of large numbers would cancel the noise terms out of the households' information, even if only a small fraction of households became informed. Thus, there is unfortunately no alternative but to consider a finite number of households.

[7] As a possible interpretation, different firms may have products of different quality. If the quality is high, the firm is likely to be successful (q_G); if the quality is low, the firm is less likely to be successful (q_B). Households' information is then obtained because of their status as possible future consumers. They spend some time (cost c) determining whether they are likely to buy the product once it has been produced.

crowdfunding, a loan rate r and a minimum investment $I_{\min}$ are determined. If $I_{\min}$ (the threshold) can be reached, the funding will be successful, and the project can be implemented. Households choose whether to gather information about the project at private cost c. The information is independent between households and can be noisy, as defined above (with incorrect-negative rate α and incorrect-positive rate β). The households then consider whether to pledge money. Only if an aggregate amount $\geq I_{\min}$ is reached by all households, the investment will be made. In this case, the project is started. *At date* $t = 1$, the project returns $R\,I$ with probability q_G for a good firm and with probability q_B for a bad firm. Finally, the loans are repaid.

1.3 Crowdfunding

In crowdfunding, the firm defines a loan rate r and demands a minimum volume of $I_{\min}$ (called the “threshold” below). The process is stopped, and no transfers occur if the threshold $I_{\min}$ is not reached within a given amount of time. Because each household owns \$1, $I_{\min} = n_{\min}$ is also the number of households that need to participate for the financing to be successful (we will see in Section 1.3.1 that if a household participates, it invests its entire wealth).

According to auction theory, the game is a simultaneous common value, sealed bid, fixed price auction with unlimited supply. Instead of a reservation price, there is a reservation volume (the threshold $n_{\min}$). The assumption of simultaneous bids helps us to abstract from information cascades (see Bikhchandani, Hirshleifer, and Welch (1998), the book by Chamley (2004), or the recent study by Kremer, Mansour, and Perry (2013)). Even if we allowed for sequential bidding, waiting to bid until the last possible second would be rational for each household. Hence, all households use the same strategy and bid simultaneously.

We focus on parameter constellations where households acquire information, at least with positive probability (other cases will be discussed later). Households may also choose to remain uninformed. However, they may still want to make a pledge, free riding on information provided by informed households. If enough other households become informed and the threshold is sufficiently high, then the probability of a bad firm receiving a successful offer may be low. This probability cannot be zero, however: if the risk for an uninformed household to lose money on a bad firm were close to zero, then it would be a dominant strategy for *all* households to remain uninformed. Therefore, uninformed households' bids will be successful for bad firms with positive probability in equilibrium. Thus, an important property of equilibrium is that bad firms will obtain funding with a strictly positive probability.

Let ι denote the probability with which a household becomes ιnformed. Households' strategies are stochastically independent. Informed households will pledge if the information is positive; otherwise, they will not. If they pledged based on negative information, even with only a small probability, they would have to be indifferent between investing or not investing *after* receiving the information; thus, acquiring the information in the first place would not be beneficial. The same argument applies if informed investors chose not to pledge based on positive information, even with only a small probability.

The number of households N is finite. The number of *informed* households follows a binomial distribution, $n_i \sim B(N, \iota)$; thus, the probability for n_i informed households is

$$p_i(n_i) = \binom{N}{n_i} \iota^{n_i} (1 - \iota)^{N - n_i}. \tag{1.1}$$

Uninformed households may still pledge with positive probability. Let υ denote the probability that a household remains υninformed and nevertheless pledges. Thus, in equilibrium, a household chooses to become informed with probability ι, it pledges even without information with probability υ, and it remains completely inactive (i. e., it neither obtains information nor pledges money) with probability $1 - \iota - \upsilon$. A good firm receives money from informed households if those households obtained a positive signal, which occurs with probability $(1 - \alpha)$. In summary, a household pledges (with or without information) with probability $\iota\,(1-\alpha)+\upsilon$. We can now calculate the distribution of the number n_G of pledging households if the firm is *good*, which is $n_G \sim B\big(N, \iota\,(1-\alpha)+\upsilon\big)$; thus,

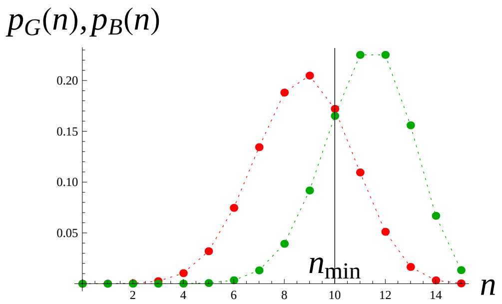
$$p_G(n_G) = \binom{N}{n_G}\big(\iota\,(1-\alpha)+\upsilon\big)^{n_G}\big(1-\iota\,(1-\alpha)-\upsilon\big)^{N-n_G}. \tag{1.2}$$

The number of non-pledging households $N-n_G$ for a good firm follows a binomial distribution with the counter-probability, $N-n_G \sim B\big(N, 1-\iota\,(1-\alpha)-\upsilon\big)$. The distribution of the number n_B of pledging households if the firm is *bad* is $n_B \sim B\big(N, \iota\,\beta+\upsilon\big)$; thus,

$$p_B(n_B) = \binom{N}{n_B}\big(\iota\,\beta+\upsilon\big)^{n_B}\big(1-\iota\,\beta-\upsilon\big)^{N-n_B}. \tag{1.3}$$

The following Figure 1.1 shows the probabilities $p_G(n)$ (green) and $p_B(n)$ (red) of a numerical simulation. Here, we have assumed a threshold of $n_{\text{min}} = 10$. Thus, whenever $n \geq n_{\text{min}}$, the funding is successful.

Figure 1.1: Probabilities of Investors n (Example)



Parameters are the same as those in Table 1.1 on page 13. Furthermore, $\upsilon = \frac{1}{2}$ and $\iota = \frac{1}{3}$ in this example. These variables will be endogenized below.

1.3.1 Households' Pledging and Information Choice

We solve the model by using backward induction. At this stage, we consider a household's information and pledging choices for a given loan rate r and threshold I_{min}. An informed household will always pledge its entire wealth of \$1 if the information it obtains is positive.[8] Without a loss of generality, we can assume that an uninformed household also pledges its entire wealth of \$1, if it pledges at all. Consequently, the aggregate investment equals the number of pledging households. The investment is canceled if this number falls below the threshold $I_{\min}$ and thus if the number of pledging households falls short of some $n_{\min}$, with $n_{\min} = I_{\min}$.

From the perspective of a single household, let P_G define the probability that a good firm has a successful funding project and P_B define the probability that a bad firm has a successful funding project (both under the condition that the household makes a pledge). P_G and P_B will depend on the firm's choices of r and $I_{\min}$ and on the ensuing households' choices of ι (the probability of becoming informed) and υ (the probability of pledging without information) in equilibrium.

We proceed as follows: First, we use the households' indifference conditions to obtain expressions for P_G and P_B as functions of r. Then, we write P_G and P_B as cumulative distribution functions, that is, as functions of ι, υ, and $n_{\min}$. Both methods must be equal, and thus, we obtain a relation among r, ι, υ, and $n_{\min}$. We then argue that $\upsilon = 1 - \iota$ and $\iota = 1$ are in equilibrium, leaving us with an implicit definition of r and $n_{\min}$ in equilibrium.

If the firm is good, a household's expected return is $q_G\, r - 1 > 0$. The fraction of such firms is μ, the probability that the signal is positive is $(1 - \alpha)$, and the probability that the funding succeeds is P_G. If the firm is bad, the expected return is $q_B\, r - 1 < 0$. The fraction of such firms is $1 - \mu$, the probability that the signal is positive is β, and the probability that the funding succeeds is P_B. Thus, for a household that becomes informed (and pledges if the information is positive), the aggregate expected return is

$$\Pi_i = \mu\,(1 - \alpha)\,P_G\,(q_G\, r - 1) + (1 - \mu)\,\beta\,P_B\,(q_B\, r - 1) - c. \tag{1.4}$$

[8] The NPV of a unit of investment after having obtained positive information must be positive: otherwise, obtaining the information would not be beneficial at all. Consequently, a household with positive information invests as much as possible, i. e., \$1.

A household that pledges without any information saves the cost c but pledges loans to all firms whose funding succeeds. As in the above equation, the factors P_G and P_B apply; however, the factors $(1-\alpha)$ and β are now dropped. The expected return is

$$\Pi_u = \mu \; P_G \, (q_G \, r - 1) + (1 - \mu) \, P_B \, (q_B \, r - 1). \tag{1.5}$$

The expected return of a household that does not participate in the funding at all is, of course, zero. In a mixed-strategy equilibrium, all three expected returns must be equal. We can solve for P_G and P_B,

$$P_G = \frac{c}{(1-\alpha-\beta)\,\mu\,(q_G\,r-1)} \quad \text{and} \tag{1.6}$$

$$P_B = \frac{c}{(1-\alpha-\beta)\,(1-\mu)\,(1-q_B\,r)}. \tag{1.7}$$

The two functions, P_G and P_B, are plotted in the following Figure 1.2.

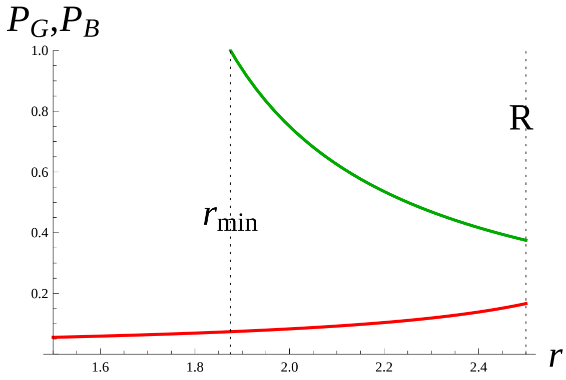
Here, the green curve represents P_G, and the red curve represents P_B. The left dotted line is at the point $r_{\min}$, where $P_G = 1$,

$$r_{\min} = \frac{1}{q_G} + \frac{c}{\mu\,(1-\alpha-\beta)\,q_G}. \tag{1.8}$$

At this point, good firms would have to succeed with their funding project with probability 1, which is technically impossible ($n_{\min}$ would have to become zero). In our numerical example, $r_{\min} = 1.875$. This number will play an important role later.

There are a number of important properties in Figure 1.2. First, the probability P_G that a good firm's funding is successful decreases with the loan rate r, and accordingly, the probability of P_B for a bad firm increases with the loan rate r. The reason for this result is that if the firm raises the loan rate r, it becomes more attractive to both households pledging after obtaining information and households pledging without information. In the mixed-strategy equilibrium, households must remain indifferent. Such indifference is only possible if both informed and uninformed participation become less attractive because the probability that a funding project is successful is lower for good firms and higher for bad firms.

Figure 1.2: Equilibrium Probabilities of Successful Funding



Parameters are as before (see Table 1.1 on page 13).

The probabilities P_G and P_B can also be calculated from the binomial distributions. P_G is an auxiliary variable. This variable measures the probability that the funding is successful (the number of participating households n_G does not fall short of the threshold $n_{\min}$) from the perspective of a single household that has already become informed and that wants to participate. P_B can be constructed analogously. Thus,

$$P_G = \sum_{n=n_{\min}-1}^{N-1} \binom{N-1}{n} \big(\iota\,(1-\alpha)+\upsilon\big)^n \big(1-\iota\,(1-\alpha)-\upsilon\big)^{N-1-n}, \tag{1.9}$$

$$P_B = \sum_{n=n_{\min}-1}^{N-1} \binom{N-1}{n} \big(\iota\,\beta+\upsilon\big)^n \big(1-\iota\,\beta-\upsilon\big)^{N-1-n}. \tag{1.10}$$

In both equations, the sums start from $n_{\min}-1$ because it is the view of one investor on decisions of the $N-1$ other households assuming that the investor itself has pledged.

We have two equations for P_G, (1.6) and (1.9), and two equations for P_B, (1.7) and (1.10). These equations give us an implicit set of equations for ι and υ. For given r and $n_{\min}$, ι and υ must be such that (1.6) = (1.9) and (1.7) = (1.10).

1.3.2 The Firm's Choice of r and $I_{\min}$

We have not yet determined whether the firm knows its own type, whether it does not, or whether something in between characterizes the firm's knowledge of its type (noisy information of a different type). The firm's choice of r and $I_{\min}$ might depend on that knowledge of its type. Let us start by discussing the first case, in which the firm has perfect information about its own type. Thus, as in many games with asymmetric information, the good type of firm will choose r and $I_{\min}$, and the bad type will be required to follow suit to avoid revealing its true type (otherwise, it would not receive any financing at all).

First, note one property. The firm will set parameters such that all households participate in some way: either they become informed (and then pledge based on that information), or they pledge without information. Formally, $\iota + \upsilon = 1$. If this condition does not hold, the good firm would leave part of the potential investment for a positive NPV project on the table. The firm can start a larger project with economies of scale in information gathering. Therefore, in equilibrium, we must have $\upsilon = 1 - \iota$. A *good* firm's expected profit is

$$\Pi_G = q_G \, (R - r) \sum_{n=n_{\min}}^{N} p_G(n) \, n. \tag{1.11}$$

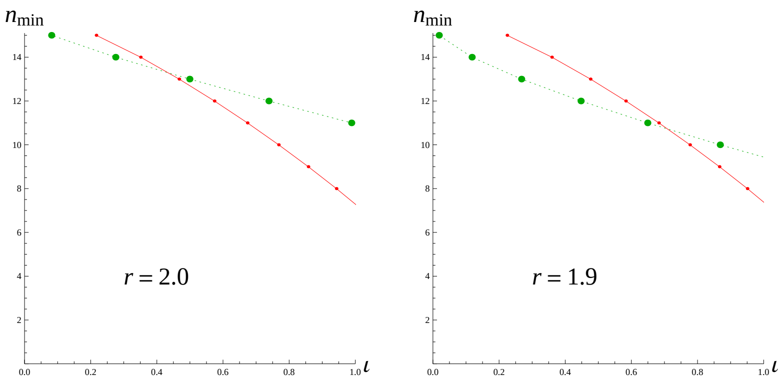
A decrease in the loan rate r will have a doubly positive effect on the good firm's profits: First, the interest margin $(R - r)$ increases. Second, we know from (1.6) that the probability P_G that the funding is successful, which equals $\sum_{n=n_{\min}}^{N} p_G(n)$, increases. Later, we will even see that an increase in r will also entail a decrease in $n_{\min}$. Therefore, the good firm sets the loan rate r as low as possible.

We can thus rewrite the firm's optimization problem:

$$\begin{aligned}
&\min_{r,n_{\min},\iota} r \\
\text{s. t.}\quad & \frac{c}{(1-\alpha-\beta)\,\mu\,(q_G\,r-1)} \\
&\quad = \sum_{n=n_{\min}-1}^{N-1} \binom{N-1}{n} \big(\iota\,(1-\alpha)+(1-\iota)\big)^n \big(1-\iota\,(1-\alpha)-(1-\iota)\big)^{N-1-n} \\
\text{and}\quad & \frac{c}{(1-\alpha-\beta)\,(1-\mu)\,(1-q_B\,r)} \\
&\quad = \sum_{n=n_{\min}-1}^{N-1} \binom{N-1}{n} \big(\iota\,\beta+(1-\iota)\big)^n \big(1-\iota\,\beta-(1-\iota)\big)^{N-1-n}. \qquad (1.12)
\end{aligned}$$

In other words, the firm minimizes r such that the two constraints still have a solution for $n_{\min}$ and ι.

Figure 1.3: Constraints on ι and $n_{\min}$ in the Optimization Problem (1.12)

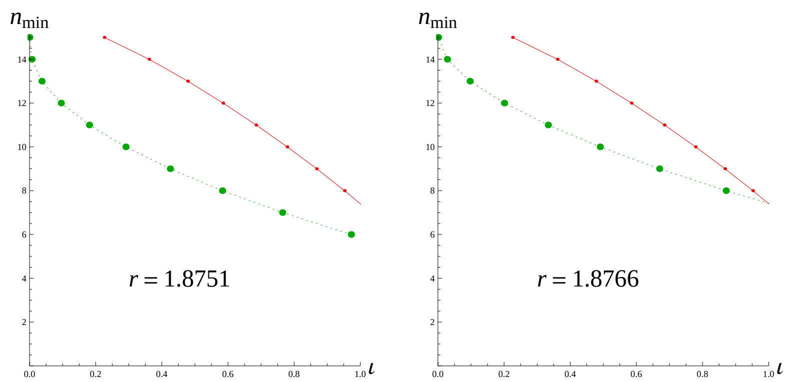


The parameters are the same as before (see Table 1.1 on page 13). In the left graph, we have set $r = 2.0$; in the right graph, $r = 1.9$. The red curve marks the combinations of $n_{\min}$ and ι where (1.7) = (1.10), the equality for the bad firm. The green curve marks combinations where (1.6) = (1.9), the equality for the good firm.

To obtain some intuition, let us plot the two constraints for our numerical example for different choices of r (Figure 1.3). First, take the left plot with $r = 2.0$. The green curve depicts all combinations of ι and $n_{\min}$ where the first constraint of (1.12) holds, and the red curve depicts those combinations where the second constraint holds. $n_{\min}$ can take only integer values, and both curves have larger dots where $n_{\min}$ takes an integer value. For clarity, we have connected these points with dotted lines. The curves intersect at $\iota = 0.4374$ and $n_{\min} = 13.27$. Abstracting from integer problems for the moment, we can interpret this result to indicate that the firm can set the loan rate to $r = 2.0$ and the threshold to $n_{\min} = 13.27$, which will then induce a probability of $\iota = 43.74\%$ for households to become informed. The other households will participate without information.

Now, we want to determine the effect of a change in r. The right plot in Figure 1.3 presents the two constraints for $r = 1.9$. A visual comparison shows that the red curve hardly changes, whereas the green curve moves downward (and becomes more curved). The new intersection point is at $\iota = 0.7108$ and $n_{\min} = 10.71$. Hence, the firm can lower the loan rate (from $r = 2.0$ to $r = 1.9$) and reduce the threshold (from $n_{\min} = 13.27$ to $n_{\min} = 10.71$). More households will thus become informed (each with probability 71.08%).

Figure 1.4: Constraints on ι and $n_{\min}$ in the Optimization Problem (1.12)



In the left graph, $r = 1.8751$. The red and green curves no longer intersect. In the right graph, $r = 1.8766$, and the curves intersect only at the extreme point of $\iota = 1$.

If the firm lowers the loan rate even further, the green curve moves downward even further. At some point, the green curve is very low, such that the red and green curves no longer intersect. This lack of intersection is evident in the left graph in Figure 1.4, for $r = 1.8751$. From this point on, there is no combination of $n_{\min}$ and ι that satisfies both (1.7) = (1.10) and (1.6) = (1.9). Economically, the firm has lowered the loan rate to such an extent that it is no longer able to attract investors.

There is equilibrium only while the curves intersect.

This property is important. As the firm tries to lower the loan rate further, the intersection point moves southeast. ι increases, and the threshold $n_{\min}$ decreases. At the point where the curves barely intersect, we have $\iota = 1$ and $0 < n_{\min} < N$. Economically, this result means that in equilibrium, the firm will set the loan rate such that it induces *all* households to become informed. Households with positive information then pledge, and those with negative information do not pledge. The threshold $n_{\min}$ is set such that good firms fail to receive funding with positive probability and bad firms successfully receive funding with positive probability. In the numerical example, the extreme case is reached for $r = 1.8766$ (right graph of Figure 1.4). The ensuing intersection point is at $\iota = 1$ and $n_{\min} = 7.4099$. From this point on, the firm cannot lower the loan rate any further.

Knowing that the firm will choose r and $n_{\min}$ such that $\iota = 1$ and $\upsilon = 0$ in equilibrium, the solution is given by (1.12), which transforms into

$$\frac{c}{(1-\alpha-\beta)\,\mu\,(q_G\, r - 1)} = \sum_{n=n_{\min}-1}^{N-1} \binom{N-1}{n} (1-\alpha)^n \, \alpha^{N-1-n} \quad \text{and} \tag{1.13}$$

$$\frac{c}{(1-\alpha-\beta)\,(1-\mu)\,(1-q_B\, r)} = \sum_{n=n_{\min}-1}^{N-1} \binom{N-1}{n} \beta^n \,(1-\beta)^{N-1-n}. \tag{1.14}$$

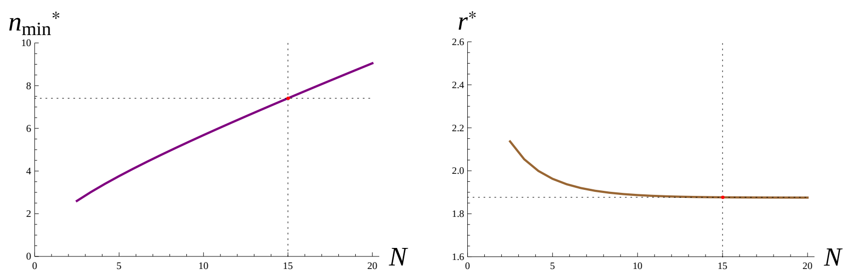
The following proposition 1.1 summarizes the main results of this analysis.

Proposition 1.1 (Equilibrium with Crowdfunding) *In equilibrium, the firm sets loan rate r and threshold $n_{\min}$ such that all households become informed and pledge money only if the information that they obtain is positive. The values for r and $n_{\min}$ are implicitly defined by* (1.13) *and* (1.14)*.*

1.3.3 Comparative Statics

In this section, to generate comparative statics, we plot a number of numerical examples and provide intuition regarding the resulting statics. Let us start by calculating the optimal $n^*_{\min}$ and r^* in the numerical example (see Table 1.1 on page 13). In this example, $n^*_{\min} = 7.409$, and $r^* = 1.876$.[9] How do these solutions react when we shift the parameters? In the following figures, we have changed one parameter at a time. The dotted lines and red dots always mark the equilibrium values for the original parameters.

Figure 1.5: Comparative Statics: The Number of Households N



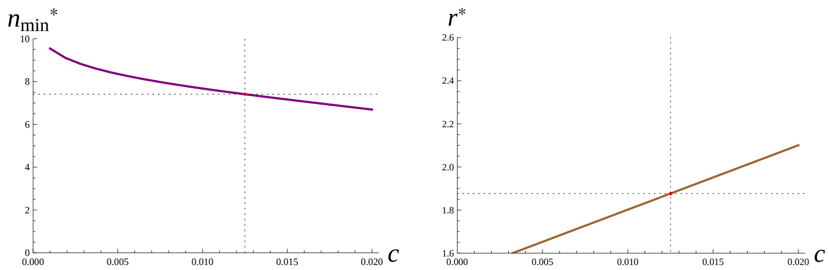
Here and in the next figures, the parameters are the same as before (see Table 1.1 on page 13).

Figure 1.5 shows the threshold ($n^*_{\min}$, left graph) and the loan rate (r^*, right graph) as functions of the number of households N. The two comparative statics are very intuitive. *First*, remember that in equilibrium, $\iota = 1$, and all households thus become informed with probability 1. Hence, the number of informed households equals the aggregate number of households N. If $n_{\min}$ were to remain constant (or even fall) for increasing N, then at some point, the probability of good firms obtaining financing P_G would converge to zero. This scenario cannot be optimal: $n^*_{\min}$ must increase in N. *Second*, because more households become informed, the probability of a bad firm obtaining financing P_B must converge to zero. Consequently, the loan rate must converge to the fair rate for a good firm, considering the information costs. We already know this number: $r_{\min} = 1.875$.

The next figure (Figure 1.6) shows the threshold ($n^*_{\min}$, left graph) and the loan rate (r^*, right graph) as functions of the fraction of the cost of information c. In

[9] A closed-form approximation for the solution of (1.13) and (1.14) for $n^*_{\min}$ and r^* is given in the appendix on page 34.

Figure 1.6: Comparative Statics: Information Costs c



our numerical example, the minimum investment n^*_{min} falls with c, but the loan rate increases with c. Again, the intuition for these comparative statics is robust. *First*, as c increases, households must be incentivized to continue to gather information and not to pledge blindly. As firms lower the threshold level n^*_{min}, the probability of pledging to a bad firm increases, and thus, the incentive to become informed increases. *Second*, the loan rate must compensate households for their information costs. Consequently, a higher information cost c entails a higher loan rate.

Figure 1.7: Comparative Statics: Fraction of Good Firms μ

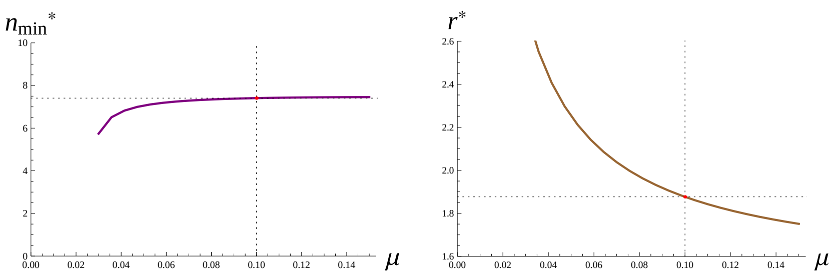
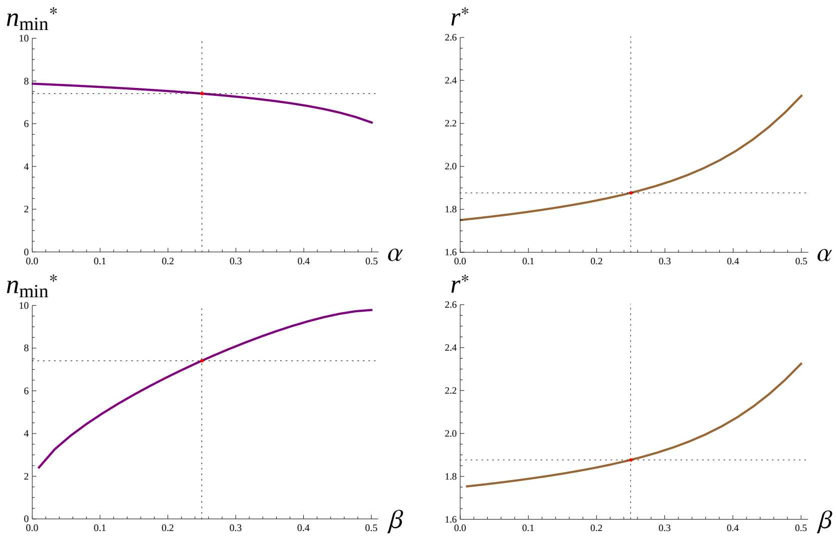


Figure 1.7 shows equilibrium values depending on the fraction of good firms, μ. *First*, the threshold n_{min} increases in μ. If there are more good firms, the households' confidence in their pledge is already relatively high. Firms do not need a high threshold n_{min} to induce further confidence. *Second*, the loan rate r falls in μ. Of course, if there are more good firms in the pool, firms need to offer a lower loan rate.

Finally, Figure 1.8 shows how equilibrium values depend on the quality of the infor-

Figure 1.8: Comparative Statics: α and β Error



mation, that is, the α-error (first line) and the β-error (second line). The reaction of the loan rate r is the same in both cases. As households make more mistakes, the aggregate financing decision (whether the issue is successful) becomes less informed. As a compensation, the firm must offer a higher loan rate. The effect on the threshold $n_{\min}$ goes in opposite directions. The threshold $n_{\min}$ decreases in α. With increasing α, households receive more bad information from good firms. Based on their information, more households do not pledge. Consequently, the threshold $n_{\min}$ must fall, otherwise at some point there would be no finance, even for good firms. Finally, the threshold $n_{\min}$ increases in β. With increasing β, households receive more good information from bad firms. Based on their information, more households pledge money, although the average quality of firms falls. To countervail this effect, the threshold $n_{\min}$ must increase.

1.4 Standard Debt Financing

This paper focuses on crowdfunding. However, we are also interested in determining the parameter constellations under which firms will benefit from crowdfunding relative to standard debt financing. Thus, this section is devoted to standard debt financing, in which a firm raises capital by borrowing money at a fixed rate of interest. In the previous section, the firm could set the loan rate r. Thus, for comparability, we assume that the firm again has market power and hence can make take-it-or-leave-it offers to households. As before, each household has an endowment of \$1.

There are three different regimes. The NPV of good and bad projects could be so low that households do not invest at all. Alternatively, good and bad projects could be so similar that households do not gather information. Let us provide a classification. (i) Households may gather information before investing by spending c. If the information is positive, they invest (otherwise, they should not have spent c at all). If the information is negative, they do not invest (owing to our assumption of a negative NPV). (ii) The average project may be so bad that households do not invest at all. (iii) The information may be so noisy or the cost of information c may be so high that households do not become informed at all but invest nonetheless. Case (ii) is the most relevant to our application. Each single piece of information is so noisy that households do not trust it enough to use it as a basis for their decision. Crowdfunding is most useful for such situations.

Case (i): Investment Only with Positive Information All firms offer the same loan rate; furthermore, all households become informed and accept offers only from the firms for which they have positive information. The expected profit of an informed household is $\mu\,(1-\alpha)\,(q_G\,r-1)+(1-\mu)\,\beta\,(q_B\,r-1)-c$, where r is the gross loan rate per unit of investment. The firm is good with probability μ, but as the signal is noisy, only with probability $(1-\alpha)$ is the firm recognized as good. $(q_G\,r-1)$ is the expected return of the household if the firm is good. With probability β, even a bad firm (fraction $1-\mu$) is declared to be good. Households must break even;

hence, the loan rate is

$$0 = \mu\,(1-\alpha)\,(q_G\,r - 1) + (1-\mu)\,\beta\,(q_B\,r - 1) - c,$$
$$r = \frac{c + \beta\,(1-\mu) + (1-\alpha)\,\mu}{\mu\,(1-\alpha)\,q_G + (1-\mu)\,\beta\,q_B}. \tag{1.15}$$

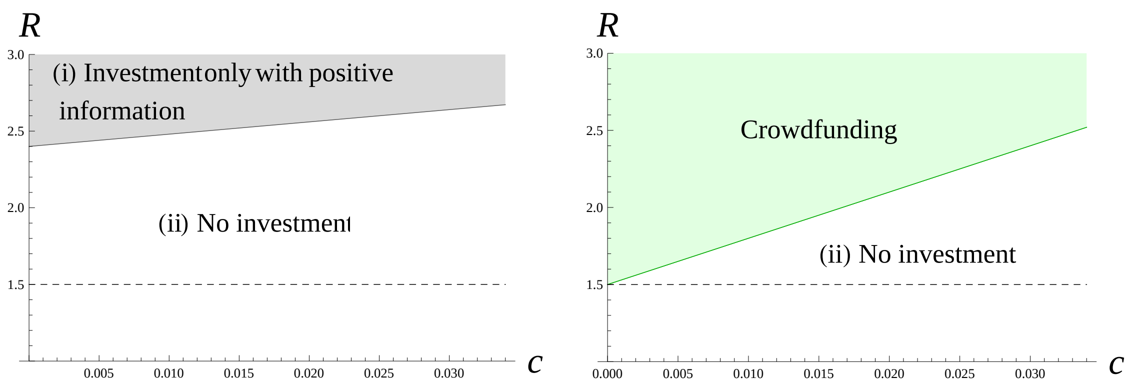
A good firm's expected financing volume is $(1-\alpha)\,(R-r)\,N$, with r as in (1.15).

Case (ii): No Investment Of course, there is an equilibrium with no lending at all and thus with zero profits for all firms.

Case (iii): Investment without Any Information Finally, there is a pooling equilibrium where all firms offer the same loan rate but where households do not become informed.

Comparison between Regimes with Standard Debt Financing Which regimes will occur under what parameter constellation? Let us return to the numerical example. The good firm always chooses the form of financing that promises the highest expected profit and sets loan rates accordingly to incentivize investors either to gather information or not. Accordingly, we can define which case of standard debt financing is preferred depending on parameters such as information costs and the firm's return.

Figure 1.9: Regimes without and with Crowdfunding



In the left graph of Figure 1.9, the white region shows the cases in which the costs are too high and the returns are too low to finance a good firm's project with standard debt financing. The dashed line shows the minimum return necessary to have a positive NPV. Above this line, there is a region where projects still are not funded. When the returns increase, projects are financed. In the gray region, inducing households to gather information before investing (case (i)) is beneficial. Case (iii) is not visible in the graph: in our numerical example, this case would require a positive NPV for bad firms.

Comparison with Crowdfunding What occurs if crowdfunding becomes available as a new form of financing? We can compare cases of debt financing with a situation in which crowdfunding is feasible. The good firm again decides on the financial instrument that generates the largest profits and sets interest rates and other parameters accordingly. Depending on project return R and information costs c, crowdfunding will be preferred over standard debt financing. In the right graph of Figure 1.9, the white region without financing (case (ii)) is considerably smaller than in the left graph. This result indicates that crowdfunding enables more projects from good firms to obtain funding. Only projects with positive NPV are financed (again, the dashed line presents the minimum return for a positive NPV). The crowdfunding possibility dominates the case of standard debt financing, where households are becoming informed as well (case (i)).

1.5 Welfare

Crowdfunding clearly provides a benefit to good firms (otherwise, they would not opt to use it). However, more parties are involved in crowdfunding, i. e. bad firms and households. Thus, let us take the welfare perspective. In crowdfunding, welfare is defined as

$$W_{\text{Crowd}} = \mu\, q_G\,(R-r)\left[\sum_{n=n_{\min}}^{N} n\, p_G(n)\right] + (1-\mu)\, q_B\,(R-r)\left[\sum_{n=n_{\min}}^{N} n\, p_B(n)\right] \quad (1.16)$$

$$= \mu\,(q_G\, R-1)\left[\sum_{n=n_{\min}}^{N} n\, p_G(n)\right] + (1-\mu)\,(q_B\, R-1)\left[\sum_{n=n_{\min}}^{N} n\, p_B(n)\right] - \iota\, N\, c. \quad (1.17)$$

In (1.17), the first term consists of three factors: the fraction of good firms, μ; the positive NPV per good project, $q_G\, R - 1$; and the distribution over possible investment sizes, ranging between $n_{\min}$ and N, with the according probabilities. The second factor consists of the same factors for bad firms. The third factor equals the aggregate information costs.

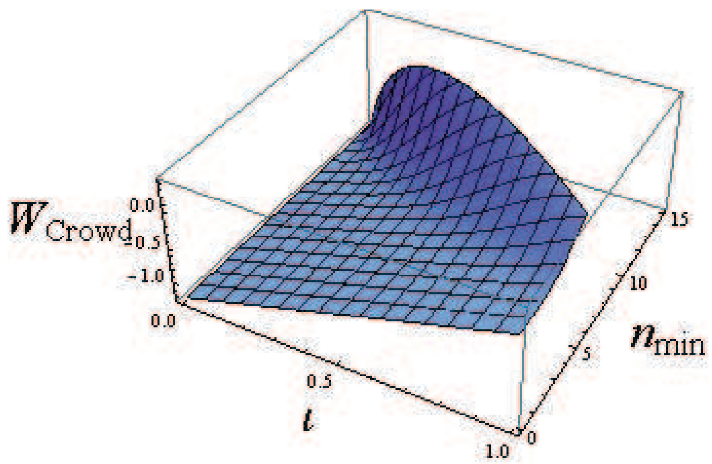
Now, we first discuss whether the introduction of crowdfunding increases welfare. The answer is ambiguous. Clearly, as Figure 1.9 shows, there are cases with no financing in the absence of crowdfunding (left picture). Here, the introduction of crowdfunding raises welfare from zero to a positive level. However, now consider a case in which the information costs are high, such that even good firms are indifferent between crowdfunding and uninformed financing (case (iii)). Households are always driven to their participation constraints, and thus, they are also indifferent. However, bad firms strictly prefer standard debt, giving them a higher chance to obtain financing. In summary, there are parameter constellations for both cases: crowdfunding may either increase or reduce welfare.

Now let us assume that a firm uses crowdfunding. In this case, does the firm set the parameters (loan rate r and threshold level $n_{\min}$) optimally? We now show that from a welfare perspective, both are set too low.

Conveniently, (1.17) does not depend on the equilibrium loan rate r^*. Thus, we can plot welfare as a function of $n_{\min}$ and the probability of informed households ι. However, there is an upper bound on $n_{\min}$. If $n_{\min}$ becomes too large, households spend information costs c, but the funding is successful with only a low probability. Households' profits become negative, and the participation constraint is violated.

To derive the upper limit for $n_{\min}$, we use (1.9) and (1.10) to calculate P_G and P_B for given ι and $n_{\min}$. We enter these values into the expected return of an informed household, given by (1.4), and solve for the minimal r that still provides informed households with a non-negative profit, $\Pi_i = 0$. We then substitute P_G, P_B, and the ensuing r into (1.5) to determine whether an uninformed household's expected profit remains non-negative. Only in that case is the combination of $n_{\min}$ and ι feasible. Figure 1.10 shows the welfare in this region.

Figure 1.10: Welfare



We find that welfare is maximized at the border, which implies that $\Pi_i = 0$ and $\Pi_u = 0$. To be precise, in the numerical example, the welfare-optimal point is at $n_{\min} = 9.2852$ and $\iota = 0.7099$. The corresponding loan rate is $r = 1.895$. However, in equilibrium, the firm makes a different choice ($n^*_{\min} = 7.4086$, $\iota = 1$, and $r^* = 1.876$). Thus, the firm sets the loan rate r and the threshold $n_{\min}$ too low, inducing too many households to become informed.

The intuition of this result is as follows: A firm chooses r and $n_{\min}$ such that at equilibrium, all households become informed ($\iota = 1$). From a welfare perspective, this situation may not be optimal (depending on the parameters): the marginal benefit of another piece of information becomes very small for large N. Thus, in the welfare optimum, we may have $\iota < 1$ such that a positive expected number of

$\upsilon = 1 - \iota$ households participates without any information. Now, a smaller fraction of informed households implies that more bad firms obtain financing. Consequently, the loan rate r must increase. Because some households make a pledge without information, the critical $n_{\min}$ must also increase (otherwise, too many bad firms will obtain financing). The following proposition summarizes these arguments.

Proposition 1.2 (Optimality of the Crowdfunding Parameters) *From a welfare perspective, firms set the interest rate r and the threshold $n_{\min}$ too low, inducing an excessive level of information.*

Crowdfunding by Uninformed Firms We have assumed that a firm knows its own type; however, in reality, a firm may not know its type. For example, a computer-game designer may not know how large the demand for his game will be in the future. Examining (1.16) reveals that owing to the binding participation constraint of households, welfare equals exactly the expected profit of a firm that does not know its own type. Numerous results immediately follow from the welfare analysis: (i) an informed firm sets a lower loan rate,(ii) sets a lower threshold level $n_{\min}$, and (iii) induces households to gather more information (higher ι) than an uninformed firm. This third point in particular may seem counterintuitive, but it is logical from a financial perspective. If the firm knows that it is good, it wants the market to know that as well, and thus, it wants the information about it in the market to be as precise as possible. Accordingly, the good firm sets a low loan rate, and very few bad firms obtain financing. If a firm does *not* know its type, it takes into account that it might be a bad firm. Further, the firm takes into account the positive expected profits that bad firms make when they obtain financing. Consequently, a lower level of information gathering ($\iota < 1$) may be optimal for an uninformed firm.

1.6 Conclusion

We have presented a microeconomic model in which crowdfunding arises endogenously. In the model, the benefit does not result from exploiting the crowd's financial resources (which could also be achieved through bank financing, in which the bank "taps" its depositors, or by market financing). The benefit of crowdfunding stems from exploiting the information obtained by the crowd. Many households have small pieces of private information. They know whether they might like a not-yet-existing product. However, this exact information is necessary for a firm's financing decision: Will consumers like the firm's future product? What will the demand look like?

The crucial feature of crowdfunding is not that the Internet or some networking platform (which merely help reduce the transaction costs) is used but that funding does not proceed if the pledged volume falls short of some threshold. Owing to this feature, households participate even if they can assess the future success of the firm's product only vaguely. Further, households participate only if many households have positive information about the firm or product. Then, this mass of vague information accumulates to relatively precise aggregate information. With this structure, crowdfunding allows more worthy projects to receive funding than standard debt financing.

Our welfare analysis has shown that firms do not set crowdsourcing parameters (the loan rate and threshold) optimally: both are set too low. There is one reason for this phenomenon: good firms make decisions without taking into account the negative externality of bad firms.

To concentrate on the basic effects, we have kept the model as stylized as possible. We will discuss a number of important extensions in a follow-up paper. *First*, we have discussed only crowdfunding in the form of equity and debt, in which participants grant a loan to a firm. In reality, two additional forms of crowdfunding are relevant: reward-based crowdfunding and donation-based crowdfunding. We want to endogenize the firm's decision particularly between the first three types of crowdfunding (donation-based crowdfunding would most likely require a model with non-standard utility functions).

Second, given that crowdfunding is primarily used by firms that produce consumption goods, particularly cultural goods, such as movies, computer games, and fashion goods, households' information may be influenced by fads. We want to analyze whether crowdfunding is more susceptible to fads than conventional financing. Specifically, a common component in households' signals may be determined by fads.

Third, in our model, the firm used a crowdfunding platform that made zero profit. In reality, many crowdfunding platforms exist, but the number is diminishing. Indeed, some consolidation is under way. As of April 2012, the top five platforms already accounted for 95% of the total funds raised in Europe and 73% of the total funds raised in North America.[10] Arguably, crowdfunding may be a natural monopoly because of network externalities, similar to online auction websites (eBay), search engines (Google), and online retailing (amazon.com). In such a case, the winner will begin to charge firms higher fees after acquiring sufficient market share. We aim to analyze how our results change under monopoly conditions.

In summary, our paper has integrated crowdfunding into the literature that views financing as a gatekeeper. Given the recent development of crowdfunding, to have a theory on crowdfunding in practice and the underlying mechanisms is important. The simplicity of the model allows for fruitful extensions to inform such theory.

1.7 Appendix

Closed-Form Approximation for (1.13) **and** (1.14) Equations (1.13) and (1.14) can be solved numerically. However, a relatively good closed-form approximation is

[10] See the article by Suw Charman Anderson from May 11, 2012, at http://www.forbes.com.

available. To see this, consider first a general property of the binomial distribution,

$$\sum_{n=n_{\min}-1}^{N-1} \binom{N-1}{n} (1-\gamma)^n \, \gamma^{N-1-n} = \sum_{n=0}^{N-n_{\min}} \binom{N-1}{n} \gamma^n \, (1-\gamma)^{N-1-n}. \qquad (1.18)$$

Here, the second term follows from the first by substituting $n \mapsto N-1-n$ and the property $\binom{N-1}{n} = \binom{N-1}{N-1-n}$. Using this, we can rewrite the system of equations (1.13) and (1.14) to

$$\frac{c}{(1-\alpha-\beta)\,\mu\,(q_G\,r-1)} = \sum_{n=0}^{N-n_{\min}} \binom{N-1}{n} \alpha^n \, (1-\alpha)^{N-1-n} \quad \text{and}$$

$$\frac{c}{(1-\alpha-\beta)\,(1-\mu)\,(1-q_B\,r)} = \sum_{n=0}^{N-n_{\min}} \binom{N-1}{n} (1-\beta)^n \, \beta^{N-1-n}. \qquad (1.19)$$

Typically, N will be large, and the information will be vague (both α and β are smaller than but close to 1/2), hence by the de Moivre-Laplace theorem, the binomials can be approximated by normal distributions. In general, we have

$$\sum_{n=0}^{n_0} \binom{N-1}{n} \gamma^n \, (1-\gamma)^{N-1-n} \approx \Phi\Big(\frac{n_0-\gamma\,(N-1)}{\sqrt{\gamma\,(1-\gamma)\,(N-1)}}\Big). \qquad (1.20)$$

In particular, this turns (1.19) into

$$\frac{c}{(1-\alpha-\beta)\,\mu\,(q_G\,r-1)} = \Phi\Big(\frac{N-n_{\min}-\alpha\,(N-1)}{\sqrt{\alpha\,(1-\alpha)\,(N-1)}}\Big) \quad \text{and} \qquad (1.21)$$

$$\frac{c}{(1-\alpha-\beta)\,(1-\mu)\,(1-q_B\,r)} = \Phi\Big(\frac{N-n_{\min}-(1-\beta)\,(N-1)}{\sqrt{\beta\,(1-\beta)\,(N-1)}}\Big). \qquad (1.22)$$

To ease the computation, we make one further observation. In equilibrium, the loan rate r^* will be close to $r_{\min}$. For example, in our numerical example, $r_{\min} = 1.875$, and $r^* \approx 1.876$. From (1.8), we see that for small c and $\mu > 0$, $q_G > 0$ and $\alpha+\beta < 1$, the equilibrium rate r^* will also be close to $1/q_G$. This implies that a change in r will have a large impact on the green curve, but not on the red curve. For practical purposes, one can approximate $r \approx r_{\min}$ in (1.22). This allows us to solve the system of equations recursively. (1.22) turns into

$$\frac{\mu}{1-\mu}\,\frac{q_G\,c}{(q_G-q_B)\,\mu\,(1-\alpha-\beta)-q_B\,c} = \Phi\Big(\frac{N-n_{\min}-(1-\beta)\,(N-1)}{\sqrt{\beta\,(1-\beta)\,(N-1)}}\Big),$$

$$\frac{\beta\,(N-1)-(n_{\min}-1)}{\sqrt{\beta\,(1-\beta)\,(N-1)}} = \Phi^{-1}\Big(\frac{\mu}{1-\mu}\,\frac{q_G\,c}{(q_G-q_B)\,\mu\,(1-\alpha-\beta)-q_B\,c}\Big),$$

$$n^*_{\min} = \beta\,N - \sqrt{\beta\,(1-\beta)\,(N-1)}\,\Phi^{-1}(\cdot), \qquad (1.23)$$

where $\Phi^{-1}(\cdot)$ is short for the longer term above. The solution n^*_{min} can be plugged into (1.21), yielding

$$\begin{aligned} \frac{c}{(1-\alpha-\beta)\,\mu\,(q_G\,r-1)} &= \Phi\Big(\frac{(N-n^*_{\text{min}}-\alpha\,(N-1)}{\sqrt{\alpha\,(1-\alpha)\,(N-1)}}\Big), \\ r^* &= \frac{1}{q_G}\Big(\frac{c}{(1-\alpha-\beta)\,\mu\,\Phi(\cdot)}+1\Big), \end{aligned} \tag{1.24}$$

where again $\Phi(\cdot)$ is short for the longer term above. Hence, we have an approximation of (1.13) and (1.14). First, use (1.24) to calculate r^*. Then, use (1.23) to calculate n^*_{min}. By using the approximation, we find for our numerical example $r^* = 1.875$ (actual value $r^* = 1.876$) and $n^*_{\text{min}} = 6.093$ (actual value $n^*_{\text{min}} = 7.409$).

2 I Spy with my Little Eye... a Banking Crisis — Early Warnings and Incentive Schemes in Banks

"Powerful few saw crash coming: I think a lot of people actually saw this train barreling down the tracks, CEOs, people in government, and they weren't telling us."

Andrew Ross Sorkin, New York Times columnist, 2009

2.1 Introduction

The financial crisis came as a surprise to financial markets and institutions. This is startling, given that warnings had been issued in a number of notes and articles (see, e. g., Shiller, 2005; Rajan, 2006).[1] As early as 2004, the chief economist of the Northern Trust Corporation, Paul Kasriel, had warned of increased risk in the housing markets and the enormous effect this could have on the banking system and the whole economy; on the organizational level, inside individual banks, many expected a credit crunch (Kasriel, 2004). However, information often was only available within the institutions, but not revealed to outsiders. When the information was publicly available, it seemed to have gone largely unnoticed. How could the

[1] For instance, Rajan (2006) pointed out: "The inter-bank market could freeze up, and one could well have a full blown financial crisis."

necessary information not have reached the relevant regulatory authorities? How did banks predominantly go into the crisis unprepared? How was it possible that a crisis of this magnitude had not been anticipated?

The propagation of information in banking plays a role in other, more specific, contexts such as banking regulation. Responding to the recent financial crisis, the U. S. Congress passed the Dodd-Frank Act, establishing the Financial Stability Oversight Council (FSOC). Its task is to identify and monitor excessive risks to the financial system. This central institution, which collects all available information regarding financial stability and provides early warnings, is meant to help prevent future crises. However, the crucial question is how to make information available to the FSOC in the first place, and how to set incentives for insiders to communicate significant information. This question is especially interesting in the current discussion regarding banking regulation and policy measures meant to avert a similar crisis in the future; such policies should emphasize the importance of making early warnings available.

We construct a theoretical model with endogenous communication of warning signals in a bank. There are three agents: the board representing the bank's owners, a bank manager, and lenders. The bank is financed with short-term debt and equity. It invests in a project, e. g., a loan portfolio. The project can be in one of three states (good, critical, and default). In the good state, it cannot fail immediately. It must first pass through the critical state before potentially defaulting. By modeling a single bank, we abstract from issues of systemic risk from individual institutional risk.

The manager is needed to monitor the project. Monitoring is costly, but reduces the probability of default and informs the manager about the state of the project. The manager can decide to use this information about the state of the project to report the transition from good to critical to the board. The board can then react to cut losses. Hence, the manager's report is needed as an early warning.

The report has endogenous negative consequences for the manager; hence, she needs to be incentivized to warn the board. But if the board reacts, this action is observed by financial markets (lenders), and the bank's refinancing conditions deteriorate. In reality, it is often the case that not only bank owners might receive available information but also financial markets. In addition, policy changes in banks are well observed by market participants and analysts. Because of this externality that interacts with financial markets, the board itself faces a reduced incentive to set up a compensation package that achieves informational efficiency. We seek to identify the conditions under which contractual arrangements will enable the propagation of critical information. In other words, we will examine which factors influence the financial market's informational efficiency.

From our analysis, we obtain a number of predictions. *First*, the functioning of the information channel depends on the bank's equity ratio. The higher the bank's leverage, the more it is affected by a deterioration of its refinancing conditions resulting from the revelation of negative information; thus, the board is less prone to implement the efficient contract. *Second*, the functioning of information transmission depends on the project's returns. If the profitability of the project is low (for example, due to high competition in the banking sector),then the board will not gain from incentivizing the manager to monitor the project. Before and during a financial crisis, both of these conditions typically arise; the bank leverage is likely to increase and bank asset value drops. These effects tend to restrict the information channel.

The model lends itself to discussing policy measures. It can be used to analyze what types of financial regulation prevents the restriction of the information channel. *First*, capital regulation has a positive effect on the propagation of critical information. The higher the capital restrictions, the more likely the manager will communicate the critical state. On the other hand, capital restrictions limit overall investment size. In our setup, both effects imply that stricter capital standards increase welfare under certain conditions; for example, welfare may increase if profitability of the project or the probability of the negative signal is not too high. Further, we argue that risk-sensitive capital regulation does not improve the system's informativeness. Under such regulation, the bank would fear an even more severe reaction to negative signals. *Second*, the presence of more liquid assets increases the bank's desire to receive early warning signals. Consequently, regulating the bank's asset's liquidity may induce the bank to implement the informative contract. *Third*, we also find that the information channel can be kept open if contingent convertible bonds are used for refinancing rather than straight bonds. These bonds are converted into shares after a drop in the bank's share price; this situation would arise after the negative information is communicated by the manager. In our setting, the conversion must come at a loss for lenders. This loss is anticipated; hence, interest rates before the conversion increase. However, the information is less detrimental to the bank; after the conversion, it no longer needs to fear an increase in refinancing rates. The negative side effect from an informative contract is reduced.

The remainder of the paper is organized as follows. After a discussion of the related literature, Section 2.2 presents the theoretical model. Section 2.3 discusses equilibria: the communication equilibrium (in 2.3.1), the no-communication equilibrium with monitoring (in 2.3.2), a mixed-strategy equilibrium (in 2.3.3) and the no-monitoring equilibrium (in 2.3.4). Subsection 2.3.5 discusses conditions required to reach the different equilibria. Section 2.4 discusses the many policy implications mentioned above, starting with a welfare analysis. Section 2.5 concludes. Proofs are given in the appendix in section 2.6.

Literature Our paper relates to many strands of the economic literature. First, the paper is connected to agency theory within corporate finance. It is related to Levitt and Snyder (1997), in which the authors analyze incentives for agents to reveal early warnings to the principal. The authors find that it is necessary for the principal to compensate the agent for being honest even if this means admitting that prospects are not good; our paper reaches a similar conclusion. Their model concentrates on incentive contracts between agents and the principal; in contrast, our model includes the financing structure and investors. Early warnings are also examined by

Povel (1999) under the consideration of bankruptcy. Eisfeldt and Rampini (2008) empirically analyze incentives for managers to reveal private information about optimal capital allocation. Aghion and Tirole (1997) discuss the benefits of delegating formal authority versus the costs of losing it in respect of the transfer or information. Benmelech, Kandel, and Veronesi (2010) find that stock-based compensation induces managers to exert costly effort and to hide bad news about future growth options. This may result in suboptimal investment. Another paper that discusses lack of information in a principal-agent setting is by Kanodia, Bushman, and Dickhaut (1989). In their model, agents do not change decisions once they are made. This arises because agents fear revealing negative information about their human capital by admitting that they did not initially choose the best strategy. Furthermore, there are a number of related empirical articles in the field of banking and compensation systems in banking (see Fahlenbrach and Stulz, 2011; Barro and Barro, 1990; John, Mehran, and Qian, 2010). The choice of compensation contracts in our model exerts a large influence on the information channel before and during a financial crisis.

The literature on financial reporting is vast (see Verrecchia, 2001, for a survey). In this literature, financial reporting is viewed as a tool to mitigate and resolve agency problems. An example is Lóránth and Morrison (2009), which is closely related to our model. The authors analyze the interdependence between internal information transmission and loan officer compensation in banks. However, for our model, the externality of information on loan rates is crucial; their focus is inside the bank. Furthermore, there is a literature on dynamic contracting under asymmetric information. For example, Quadrini (2004) and DeMarzo and Fishman (2007) attempt to find and analyze contracts that reveal full information. Our result verifies that the transfer of information is welfare-optimal if effort costs are not too high. We also discuss policy implications for reaching the equilibrium characterized by communication and full information. However, the full revelation principle does not apply in our setting because we restrict our setting to short-term contracts.

The heart of our paper is a moral hazard model in continuous time. The lively continuous-time moral hazard literature is mainly propelled by two groups of authors: Biais, Mariotti, Plantin, and Rochet (2007); Biais, Mariotti, Rochet, and Villeneuve (2010); Pagès and Possamai (2012); Pagès (2013) and DeMarzo and Sannikov (2006); Sannikov (2007, 2008). In these papers, the agent typically controls a Brownian motion. However, we keep the model as simple as possible, basing it on a three-state Markov process. This is meant to capture the deterioration of a loan's rating class, which can be influenced by the loan officer.

The communication of bad news is somewhat similar to the communication of fraud, which is the basis of the literature on whistleblowing. Dyck, Morse, and Zingales (2010) analyze data about corporate fraud that took place in the U. S. and the actors that detected the fraud. They find that employees seem to lose outright from whistleblowing. This result underlines the need for better incentives for communication. In a dynamic setting, Povel, Singh, and Winton (2007) examine the incentives of managers to commit fraud when firms seek funding from investors and investors can obtain a better signal about the true perspectives of the firm through costly monitoring. In contrast to our model, fraud is not modeled to lead to a crisis or recession; however, the incentives for fraud do change over the business cycle.

There are a number of articles that assume strategic ignorance; that is, these papers develop models in which failure to collect information may have a positive effect on an agent (Carrillo and Mariotti, 2000; Kessler, 1998). In our model, the bank owners choose to remain ignorant for a reason. The critical information is valuable to them only if they act upon it. But the action cannot be hidden, and the revelation of information creates a negative externality: financing costs increase. Therefore, bank owners may choose not to pay the manager to reveal the information; instead, they may decide to remain ignorant.

2.2 The Model

Consider a continuous-time economy with three types of agents: a bank's board of directors (the board), a bank manager (the manager), and lenders. The structure of the model is depicted in Figure 2.1. The **board** takes decisions on behalf of the owners (equity investors), who provide an endowment of E. The board and equity investors are risk-neutral and do not discount the future.

The bank can invest in a **project** (a loan portfolio) of size $I > E$ that pays a continuous return of R per unit of investment each period until it defaults. Thus, the project potentially has an infinite maturity. In reality, banks engage in maturity transformation in which they hold assets of long maturity while refinancing with short maturities. Our choices of infinite maturity and continuous time are not crucial, but they keep the model tractable. As also non-banks undertake maturity transformation, our model can also be applied in the context of other industries. However, we model the firm as a bank because the mismatch of maturities is typical for banking business and is therefore a perfect example.

The project can be in three different states: class A (good), class B (critical), and default. In state B, projects can default at any time, and the default intensity is β. In state A, projects cannot default immediately; they first deteriorate to class B and emit a negative signal only observed by the manager. The instantaneous probability of such a downgrade is $\alpha < \beta$ if the project is monitored by the manager, and $\alpha + \gamma$ if not monitored. Formally, we have a Markov chain with three states: A, B, and the default state. Transitions move only in one direction: from A to B and from B to default.[2] Default is an absorbing state. The transition probability from A to B can be controlled by the manager. The board and the lenders cannot observe whether the project is in state A or B. Of course, when the cash flow stops, they conclude

[2]One can also think of transitions in reverse directions. However, we assume here an aggregated transition rate in the direction from A over B to default, as we are interested in the events happening in the case of a deteriorating situation.

that the project is in default. They also know the parameters α, β, and γ. Initially, the project is in state A with probability $(\beta - \alpha)/\beta$, or in state B with probability α/β.[3]

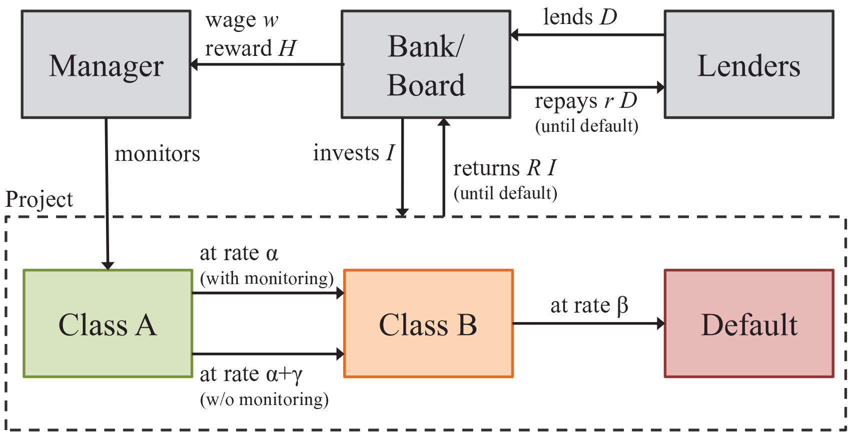
Monitoring is carried out by the **manager**; the instantaneous monitoring cost is c per unit of investment. The monitoring choice is not observable. The manager is risk-neutral and has a discount rate of ρ.[4] Discounting implies that the manager prefers her salary to be paid out early. Deferring payments comes at a cost. The manager's opportunity wage is w_0, which is normalized to $w_0 = 0$ without loss of generality.

The manager's contract can depend only on observable parameters. There can be a wage level w before the manager communicates the downgrade and a one-time payment H when the manager communicates the downgrade. Because of the structure of the model, w is constant over time. Hence, the contract consists of two variables: a wage w and a reward H. The setting contains two problems of asymmetric information: hidden action (the manager is able to shirk instead of monitoring the project) and hidden information (the manager may conceal the negative signal). The incentive wage w can be conceptualized as a bonus payment, whereas H can be paid as part of an employee suggestion system or on the basis of target agreements. If the project is in class B, the bank can save wages by terminating incentive wages. For a class B project, monitoring has no positive effect. Therefore, the information about a class transition is valuable because it enables the bank manager to concentrate on other projects and allows the bank to save wage costs.

[3]This assumption is not critical, but simplifies the analysis. With this initial setting, the expected project quality is unchanged as long as the manager monitors; the fraction of projects moving from A to B is as large as the fraction moving from B to default. Hence, the proportion of A-projects to B-projects is constant, and so is the probability of default. Mathematically, we start on an eigenvalue of the dynamic system.

[4]The assumption that managers discount more heavily is standard in the corporate finance literature, see for example Tirole (2006), based on Aghion, Bolton, and Tirole (2004). This assumption is used to endogenize short-term compensation.

Figure 2.1: Structure of the Model



Lenders are risk-neutral and do not discount. They have an endowment $\geq I - E$; thus, they can provide financing for the project. There is debt finance, where $D = I - E$ is the amount of debt. The lenders lend at a competitive rate, hence the risk-free rate is zero. For the actual interest rate r, lenders take into account the current default risk. The lenders observe when the manager is assigned new tasks or at least is removed from her initial duties. They do not observe the actual wage contract for the manager. All contracts (debt and labor contracts) are short-term; there is no long-term commitment. Consequently, the contracting space is incomplete. Short-term contracting is consistent with the maturity transformation that banks carry out given a long-term project.

One can interpret this model as follows. The manager is a bank employee who is incentivized to monitor a loan portfolio. She can observe the quality of the loan (A or B). Loans in class B can default at any time, so the manager can exert monitoring effort to reach the lower transition rate α instead of $\alpha + \gamma$ and keep the loans in class A as long as possible. Because the effort choice is not observable, the manager collects an additional rent as incentive, which she does not want to lose. The board would like to know the class of the loans. It wants to act upon that information; for example, the board might restructure a loan that deteriorates, or it might increase reserves. In our model, the advantage of early information is endogenous. The manager discounts at a higher rate, so she wants her salary as early as possible. As long as the manager needs to be incentivized, wages must be paid continuously so that monitoring does not break down. When the project is in class B, the manager's effort is no longer needed. Thus, the board benefits from paying only the opportunity costs of the manager, $w_0 = 0$, after the transition date. There is no need to promise future incentive wages.

In sum, a compensation package is offered to incentivize the manager. However, when negative information arrives, this package is modified, to the detriment of the manager. Therefore, the manager reports the loan's deterioration only reluctantly. She needs to be compensated to do so; for example, she may be offered a one-off reward H for communicating.

The revelation of private information by the manager is desirable for the bank because the bank is able to save money by saving wage costs. In recent years, many banks reduced bonuses and salaries in response to the financial crisis.[5] In reality, many other benefits from the uncovering of a deteriorated situation exist. One of those additional benefits could be a reduction of exposure of the project that is discussed in section 2.4.3 about liquidity regulation. Further reactions are conceivable. From a welfare perspective, the revelation of signals is also preferable in our model. Managers are compensated with a communication reward for the reduction in wage payments. In total, the economy may be able to save money by an early reaction to negative signals.

The model thus contains the following features. An insider in a leveraged bank has preferential access to information about the bank's assets. She needs to be incentivized to communicate this information. She may see the crisis coming, but not let others know. Giving her the right incentives is costly. The costs are born

[5] See The Huffington Post, 26.01.2012: "BofA, Credit Suisse the Latest Banks to Reduce Banker Bonuses."

entirely by the bank, but the bank's lenders also benefit because they can raise interest rates. As a consequence, the board might have insufficient incentive to implement a contract that informs markets. Using this model, we can analyze what factors drive information propagation. It is possible that the manager foresees an upcoming crisis, but decision makers and markets are left ignorant.

2.3 Equilibrium

There are three potential types of equilibrium, each characterized by different wage contracts. *First*, the board can pay the manager zero incentive wages. As a consequence, the manager will not monitor the project, the decay rate of class A projects is $\alpha + \gamma$ rather than just α. This equilibrium is indexed with '0', denoting no monitoring and no communication. *Second*, the board can pay the manager an incentive wage until the project defaults, $w > 0$, but no reward H for communicating the downturn, $H = 0$. In this case, the manager has no incentive to inform the board about the true class of the project, because the board would stop paying incentive wages as soon as it discovered that the project had downgraded to class B. Therefore, the wage cannot depend on the class of the project. In this equilibrium, the wage is just high enough to induce the manager to exert effort (efficiency wage). This equilibrium is indexed with 'NC' for no communication. *Third*, the board can promise the manager $w > 0$ and a one-off reward $H > 0$ for revealing when the project moves to class B. Once this information is revealed, the board reacts by reducing the competence and wage of the manager. In this equilibrium, information flows to the board and, as a consequence, to the capital market. Lenders take the board's reaction as negative information and increase interest rates. The advantage of such a reward H is to get the negative information as soon as possible and to be able to react at that time. Otherwise, the incentive payments to the manager would only add to the wage bill. This equilibrium is indexed with a 'C' for communication. In an additional equilibrium, the board mixes between 'C' and 'NC', and there is a parameter range with no investment at all (and thus no bank). We start by discussing the most interesting case: equilibrium C.

2.3.1 The Communication Equilibrium (C)

In this equilibrium, the board pays a positive efficiency wage $w > 0$ to the manager until the date when the manager admits that the project class has deteriorated. At that date (called t_A), the manager is promised a one-off reward $H > 0$ as a compensation. We now calculate the equilibrium values for w and H and expected profits in equilibrium.

The Lenders In this equilibrium, the lenders are always informed about the project class. As long as there is no news about the project, it is in class A. When the manager receives the reward and is assigned new tasks, this can only have happened because the class has switched to B. Interest rates r_A and r_B are set accordingly.

As long as the project is in class A, the lenders know that the instantaneous default rate is zero. Because the opportunity cost of lending is zero they demand a zero interest rate, $r_A = 0$. After a negative signal, the instantaneous probability of default is β, so the lenders must be compensated with an interest rate of $r_B > 0$. The repayment after dt periods is then $e^{r_B \, \mathrm{d}t} D$, leading to an expected repayment of $e^{r_B \, \mathrm{d}t} e^{-\beta \, \mathrm{d}t} D = e^{(r_B - \beta) \, \mathrm{d}t} D$. The participation constraint is binding for lenders. The interest rate r_B is just sufficient to compensate the lenders for their zero opportunity cost. Hence, $r_B = \beta$.

The Manager In the communication equilibrium C, the manager works on the project until she receives and transmits information about a deterioration of the loan's class. At that point, she receives a reward H, but her future wage is reduced. Wage w and reward H must be chosen such that the manager behaves as required. Assume for a moment that the manager is making a decision at date t_A, which is the moment just after the transition from class A to B has occurred. The manager now decides whether to communicate the bad news. If she does, she gets a one-off reward H. If she does not, she gets the incentive wage w until the project defaults. Her expected utility is then

$$\begin{aligned} U_B &= \int_0^\infty \Big(\int_0^{t_B} w\, e^{-\rho\, t} \mathrm{d}t \Big) \beta\, e^{-\beta\, t_B}\, dt_B \\ &= \frac{w}{\beta + \rho}. \end{aligned} \tag{2.1}$$

This utility consists of several parts. The project is already in class B, hence the date t_B at which the loan finally defaults is distributed with density $f(t_B) = \beta\, e^{-\beta\, t_B}$. Until this date, the manager collects her wage w, discounted by the factor $e^{-\rho\, t}$. The reward H must be at least as large as this term. In equilibrium, the inequality is binding, so $H = w/(\beta + \rho)$. Note that the optimal reward H is proportional to the wage w. The reward compensates the manager for forgone wages. Hence, the higher the wages, the higher H must be in equilibrium.

The wage w must be high enough to motivate the manager to monitor the project. If she does monitor a loan in class A, the manager's expected utility is

$$\begin{aligned} U_A &= \int_0^\infty \Big(e^{-\rho\, t_A}\, H + \int_0^{t_A} (w - c\, I)\, e^{-\rho\, t}\, \mathrm{d}t \Big)\, \alpha\, e^{-\alpha\, t_A}\, \mathrm{d}t_A \\ &= \frac{\alpha\, H + w - c\, I}{\alpha + \rho}. \end{aligned} \tag{2.2}$$

Again, there are several parts to this expression. If the project is monitored, the stochastic time of transition to class B is distributed with density $f(t_A) = \alpha\, e^{-\alpha\, t_A}$. At date t_A, the manager collects the reward H, discounted by the factor $e^{-\rho\, t_A}$. Until that date (from now to t_A), she receives the wage w, but she exerts effort at cost $c\, I$; both of these quantities are discounted by the factor $e^{-\rho\, t}$.

Now assume the manager decides whether to monitor in the next period of duration $\mathrm{d}t$. If she does monitor, her expected utility is

$$\alpha\, \mathrm{d}t\, H\, e^{-\rho\, \mathrm{d}t} + (1 - \alpha\, \mathrm{d}t)\, U_A\, e^{-\rho\, \mathrm{d}t} + (w\, e^{-\rho\, \mathrm{d}t} - c\, I)\, \mathrm{d}t. \tag{2.3}$$

With probability $\alpha\,\mathrm{d}t$, the class switches from A to B, and the manager collects her reward H, which is discounted by $e^{-\rho\,\mathrm{d}t}$. With probability $(1-\alpha\,\mathrm{d}t)$, the project remains in class A, and the aggregate future utility is given by U_A as expressed in (2.2). Over the period $\mathrm{d}t$, the wage w is collected and effort costs $c\,I$ are paid. However, if the manager chooses not to monitor, her expected utility becomes

$$(\alpha+\gamma)\,\mathrm{d}t\,H\,e^{-\rho\,\mathrm{d}t} + (1-(\alpha+\gamma)\,\mathrm{d}t)\,U_A\,e^{-\rho\,\mathrm{d}t} + w\,\mathrm{d}t, \tag{2.4}$$

with U_A defined in (2.2). The transition probability increases from $\alpha\,\mathrm{d}t$ to $(\alpha+\gamma)\,\mathrm{d}t$, but the manager avoids incurring the effort cost $c\,I$. In equilibrium, the board sets the wage w just high enough to induce effort, such that (2.3) = (2.4). Solving for w^* and H^* and taking the limit $\mathrm{d}t \to 0$, we get the following lemma.

Lemma 2.1 *The optimal wage and reward in the communication equilibrium are*

$$w^* = \frac{(\beta+\rho)\,(\alpha+\gamma+\rho)}{\gamma\,\beta}\,c\,I \quad \textit{and} \tag{2.5}$$

$$H^* = \frac{\alpha+\gamma+\rho}{\gamma\,\beta}\,c\,I. \tag{2.6}$$

The Board Finally, we calculate the bank's expected profit in this equilibrium. The board implements a contract that pays a wage $w>0$ to the manager. It also pays the manager a reward $H>0$ when a downgrade in the project class is reported. However, the project continues to pay off after the report. The default rate is β, and while the project does not default, it pays a continuous $R\,I$. The expected aggregate payoff to the bank, net of interest payments, is

$$\begin{aligned}
\Pi_{\mathrm{C}} = \frac{\beta-\alpha}{\beta}\int_0^\infty \Bigg[\int_0^{t_A} (R\,I - r_A\,D - w^*)\,\mathrm{d}t - H \\
+ \int_{t_A}^\infty \Big(\int_{t_A}^{t_B} (R\,I - r_B\,D)\,\mathrm{d}t\Big)\,\beta\,e^{-\beta\,(t_B-t_A)} dt_B\Bigg]\,\alpha\,e^{-\alpha\,t_A}\,\mathrm{d}t_A \\
+ \frac{\alpha}{\beta}\Bigg[-H + \int_0^\infty \Big(\int_0^{t_B} (R\,I - r_B\,D)\,\mathrm{d}t\Big)\,\beta\,e^{-\beta\,t_B}\,dt_B\Bigg] \\
= \frac{R\,I}{\alpha} - H^* - \frac{\alpha\,r_B\,D + (\beta-\alpha)\,(w^* + r_A\,D)}{\alpha\,\beta} \\
= \frac{R\,I}{\alpha} - D - \frac{(\alpha+\gamma+\rho)\,(\beta^2+\rho\,(\beta-\alpha))}{\gamma\,\alpha\,\beta^2}\,c\,I.
\end{aligned} \tag{2.7}$$

There are several parts. The board does not know whether the project is in class A or B at the start. With probability $(\beta - \alpha)/\beta$, it starts in class A. The date of transition to class B, t_A, is stochastic, with density $f(t_A) = \alpha\, e^{-\alpha\, t_A}$. Until t_A, the bank receives $R\, I$ from the project, but pays $r_A\, D$ to lenders and the wage w to the manager. At date t_A, the manager reports the transition and the board pays the reward H. The information becomes public, raising the refinancing rate from $r_A = 0$ to $r_B = \beta$. The project may now default at any time; at date t_A, the default date t_B is distributed with density $f(t_B) = \beta\, e^{-\beta\,(t_B - t_A)}$. With probability α/β, the project starts in class B. The board immediately pays the reward, recalls part of the loan, and collects $R\, I$ and pays $r_B\, D$ for refinancing until the project defaults. Substituting w^*, H^*, $r_A = 0$ and $r_B = \beta$, we get equation (2.7).

However, it may not be optimal for the board to implement a contract that induces the manager to monitor and communicate information. Out of equilibrium, the board may profit from low refinancing conditions, but save the reward H or even the efficiency wage w. However, lenders anticipate this behavior and equilibrium C breaks down. For example, if the monitoring cost c is high in comparison to γ, then it may be optimal to pay the manager lower incentive wages w (specifically, zero wages). The optimal reward H would then also be zero. If the difference between R and the default intensity in class B (β) is very small, there is little incentive to reduce the bank's outstanding debt. In addition, the board might want to induce the manager to monitor the project, but to not communicate the deterioration of project quality. Examining the board's incentives out of equilibrium, we derive the following conditions (with proof in the appendix). The parameter range is plotted in Figure 2.2 below on page 59.

Proposition 2.1 *The equilibrium 'C' with efficiency wage* $w > 0$ *and reward* $H > 0$ *exists if and only if*

$$D \leq \frac{\rho\,(\alpha + \gamma + \rho)}{\gamma\,\beta^2}\, c\, I, \tag{2.8}$$

otherwise the board deviates and leaves the reward out of the contract ($H = 0$), and

$$D \leq \frac{\gamma\,(\beta-\alpha)}{\alpha\,(\alpha+\gamma)\,\beta}\,R\,I - \frac{(\alpha+\gamma+\rho)\,\big(\beta^2+\rho\,(\beta-\alpha)\big)}{\gamma\,\alpha\,\beta^2}\,c\,I, \tag{2.9}$$

otherwise the board deviates and sets the wage to zero ($w = 0$).

2.3.2 The No-Communication Equilibrium (NC)

We now discuss equilibrium NC, in which the board induces the manager to monitor the loan, but does not pay a reward when informed about a rating transition. Without the reward, the manager is not incentivized to communicate. We follow the same structure as above in analyzing this equilibrium.

The Lenders The lenders anticipate remaining uninformed about the project's current class. They must therefore set interest rates according to their beliefs. Initially, the project is in class A with probability $(\beta-\alpha)/\beta$ and class B with probability α/β. In the first period of duration $\mathrm{d}t$, the expected return is thus

$$D\,(1+r\,\mathrm{d}t)\,e^{-(\frac{\beta-\alpha}{\beta}\cdot 0+\frac{\alpha}{\beta}\cdot\beta)\,\mathrm{d}t} = D\,(1+r\,\mathrm{d}t)\,(1-\alpha\,\mathrm{d}t) = D + D\,(r-\alpha)\,\mathrm{d}t. \tag{2.10}$$

The participation constraint is binding if $r = \alpha < \beta$ initially. As time elapses, the beliefs about the class of the project might change. We argue that these beliefs are constant in our setting. Let us call $p_A(t)$ the probability that the project is in class A at date t, $p_B(t)$ the probability that it is in class B, and $p_D(t)$ the probability that it has already defaulted. Then if the loan is monitored, the following differential equations describe the evolution of probabilities.

$$\dot{p}_A(t) = -\alpha\,p_A(t), \quad \dot{p}_B(t) = \alpha\,p_A(t) - \beta\,p_B(t), \tag{2.11}$$

with $p_D(t) = 1 - p_A(t) - p_B(t)$, and $p_A(0) = (\beta-\alpha)/\beta$, $p_B(0) = \alpha/\beta$, $p_D(0) = 0$. This linear ordinary differential equation has the following solutions:

$$p_A(t) = \frac{\beta-\alpha}{\beta}\,e^{-\alpha\,t}, \quad \text{and} \quad p_B(t) = \frac{\alpha}{\beta}\,e^{-\alpha\,t}. \tag{2.12}$$

Both probabilities decrease at the same rate α. The probability of a class A project decreases at rate α anyway, and class B projects diminish at rate β, but new class B projects arrive from class A all the time, so the aggregate growth rate is also $-\alpha$. The reason this arises is that we have chosen an eigenvector of the dynamic system as initial condition, $p_A(0) = (\beta - \alpha)/\beta$ and $p_B(0) = \alpha/\beta$. This makes the evolution of probabilities especially simple. The probability of being in class A, conditional on not being in default, is constant at $(\beta - \alpha)/\alpha$.

The Manager If the board does not want to induce the manager to communicate, it sets $H = 0$. Consequently, we need to calculate the manager's behavior as a function of the wage w only. Assume that the project is currently in class A. The manager's discounted expected utility is then

$$\begin{aligned} U_A &= \int_0^\infty \left[\int_{t_A}^\infty \Big(\int_0^{t_A} (w - c\,I)\, e^{-\rho t}\, \mathrm{d}t + \int_{t_A}^{t_B} w\, e^{-\rho t}\, \mathrm{d}t \Big) \beta\, e^{-\beta\,(t_B - t_A)}\, dt_B \right] \alpha\, e^{-\alpha\, t_A}\, dt_A \\ &= \frac{\alpha + \beta + \rho}{(\alpha + \rho)\,(\beta + \rho)}\, w - \frac{c\,I}{\beta + \rho}. \end{aligned} \tag{2.13}$$

Let us give some intuition. The date t_A of transition from A to B is distributed with density $f(t_A) = \alpha\, e^{-\alpha\, t_A}$. For a given t_A, the final default date t_B is distributed with density $\beta\, e^{-\beta\,(t_B - t_A)}$. Between date 0 and t_A, the managers receives the wage w net of $c\,I$, discounted by $e^{-\rho t}$. Between t_A and t_B, she receives the wage but no longer exerts effort. From a project in class B, the expected utility would only be

$$\begin{aligned} U_B &= \int_0^\infty \Big\{ \int_0^{t_B} w\, e^{-\rho t}\, \mathrm{d}t \Big\} \beta\, e^{-\beta\, t_B}\, dt_B \\ &= \frac{w}{\beta + \rho}. \end{aligned} \tag{2.14}$$

Now assume the manager consider deviating from the equilibrium behavior by not monitoring for a short period $\mathrm{d}t$. If she monitors, the expected utility is

$$\alpha\, \mathrm{d}t\, U_B\, e^{-\rho\, \mathrm{d}t} + (1 - \alpha\, \mathrm{d}t)\, U_A\, e^{-\rho\, \mathrm{d}t} + (w\, e^{-\rho\, \mathrm{d}t} - c\,I)\, \mathrm{d}t. \tag{2.15}$$

If she shirks, the expected utility becomes

$$(\alpha + \gamma)\, \mathrm{d}t\, U_B\, e^{-\rho\, \mathrm{d}t} + (1 - (\alpha + \gamma)\, \mathrm{d}t)\, U_A\, e^{-\rho\, \mathrm{d}t} + w\, e^{-\rho\, \mathrm{d}t}\, \mathrm{d}t. \tag{2.16}$$

In equilibrium, the incentive condition is binding. Setting (2.15) = (2.16), taking the limit $\mathrm{d}t \to 0$ and solving for w yields

$$w^* = \frac{(\alpha+\gamma+\rho)\,(\beta+\rho)}{\gamma\,\beta}\,c\,I. \tag{2.17}$$

The wage is exactly the same as it was in equilibrium C. This is not surprising, given that the manager was just indifferent between taking the reward or not.

The Board In equilibrium, the bank's expected profit is

$$\begin{aligned}\Pi_{\mathrm{NC}} = \frac{\beta-\alpha}{\beta}\int_0^\infty \Bigg[\int_0^{t_A}&(R\,I - r\,D - w^*)\,\mathrm{d}t \\ &+ \int_{t_A}^\infty \left(\int_{t_A}^{t_B}(R\,I - r\,D - w^*)\,\mathrm{d}t\right)\beta e^{-\beta\,(t_B-t_A)}dt_B\Bigg]\,\alpha e^{-\alpha\,t_A}dt_A \\ + \frac{\alpha}{\beta}\int_0^\infty &\left(\int_0^{t_B}(R\,I - r\,D - w^*)\,\mathrm{d}t\right)\beta e^{-\beta\,t_B}dt_B \\ = \frac{R\,I - r\,D - w^*}{\alpha} &= \frac{R\,I}{\alpha} - D - \frac{(\gamma+\alpha+\rho)\,(\beta+\rho)}{\gamma\,\alpha\,\beta}\,c\,I.\end{aligned} \tag{2.18}$$

The earnings $R\,I$ and the payments $r\,D$ and w always remain the same as there is no new information revealed until default; however, the probabilities of default differ. With probability $(\beta-\alpha)/\beta$, the project is initially in class A; with probability α/β the project is in class B. Date t_A denotes the date at which the transition from class A to class B occurs and is characterized by its density, $f(t_A) = \alpha\,e^{-\alpha\,t_A}$. Date t_B denotes the time of default, and is characterized by its density, $f(t_B) = \beta\,e^{-\beta\,(t_B-t_A)}$. Inserting w^* and $r = \alpha$ yields (2.18). The proof of the following proposition is in the appendix in section 2.6.

Proposition 2.2 *The equilibrium 'NC' with efficiency wage as in (2.17) but no reward exists if and only if*

$$D \le \frac{R\,I}{\alpha} - \frac{(\alpha+\gamma)\,(\beta+\rho)\,(\gamma+\alpha+\rho)}{\gamma^2\,\alpha\,(\beta-\alpha)}\,c\,I, \tag{2.19}$$

otherwise the board deviates and sets the wage to zero ($w = 0$), and

$$D \ge \frac{\rho\,(\gamma+\alpha+\rho)}{\gamma\,\beta\,(\beta-\alpha)}\,c\,I, \tag{2.20}$$

otherwise the board prefers to have a reward for communication ($H > 0$).

If the monitoring costs c are very high, it may be too expensive for the board to pay the incentive wage w. However, if the difference between refinancing costs β of class B and the returns of the project is large while the difference between the two transition rates β and α is small, the board prefers to receive the information so it may reduce the costly outstanding debt.

2.3.3 The Mixed-Strategy Equilibrium

So far, we have defined two types of equilibrium. However, a mixture of these equilibria can be another type. The boundaries of the two equilibria C and NC are not identical; there is an equilibrium in which both strategies are chosen from randomly that exists in the space between the previous two equilibria.

The higher the debt level and the greater the increase in refinancing costs caused by the warning, the more profitable it is for the board not to pay a reward. However, the increase in refinancing costs caused by the warning is reduced if the board does not pay the reward. Thus, there exists a mixed-strategy equilibrium in which the board randomizes between contracts with and without a reward. In the mixed-strategy equilibrium, we assume the board chooses strategy C with probability p_{C} and strategy NC with probability $p_{\mathrm{NC}} = 1 - p_{\mathrm{C}}$. The lenders anticipate the mixed strategy and require the interest rate r_{Mix} if they have no information about a transition of the project from class A to B, where

$$\begin{aligned} r_{\mathrm{Mix}} &= \frac{0\, p_{\mathrm{C}}\, \frac{\beta-\alpha}{\beta} + 0\,(1-p_{\mathrm{C}})\, \frac{\beta-\alpha}{\beta} + \beta\,(1-p_{\mathrm{C}})\frac{\alpha}{\beta}}{p_{\mathrm{C}}\, \frac{\beta-\alpha}{\beta} + (1-p_{\mathrm{C}})\, \frac{\beta-\alpha}{\beta} + (1-p_{\mathrm{C}})\, \frac{\alpha}{\beta}} \\ &= \frac{(1-p_{\mathrm{C}})\,\alpha\,\beta}{\beta - \alpha\, p_{\mathrm{C}}}. \end{aligned} \tag{2.21}$$

The interest rate takes the expected probabilities of default into account. With probability $\frac{\beta-\alpha}{\beta}$, the project is initially in class A, and with probability $\frac{\alpha}{\beta}$ it is in class B. The lenders and the board have exact information about the probability of default only if strategy C is chosen (with probability p_{C}) and the project is in class B; in this case, the probability of default is β. The other three cases have to be summed and weighted by the sum of their probabilities to calculate r. As the respective probabilities of being in class A or B decay at the same rate α, the proportions of probabilities remain constant. Therefore, the interest rate r remains constant during this time as well.

The mixed-strategy equilibrium exists if the board is indifferent between playing strategy C and playing strategy NC. Therefore, we can calculate the probability p_{C} by setting $\Pi_{\mathrm{NC}} = \Pi_{\mathrm{C}}$ and substituting w^*, H^*, $r = r_{\mathrm{Mix}}$ and $r_B = \beta$,

$$\frac{R\,I - r\,D - w}{\alpha} = \frac{R\,I}{\alpha} - H - \frac{\alpha\,r_B\,D + (\beta - \alpha)\,(w + r\,D)}{\alpha\,\beta},$$

$$p_{\mathrm{C}} = \frac{\beta}{\alpha}\left(1 - \frac{\beta - \alpha}{\rho\,\frac{\alpha+\gamma+\rho}{\gamma\,\beta}\,c}\,\frac{D}{I}\right). \tag{2.22}$$

In this mixed-strategy equilibrium, the board plays strategy C with probability p_{C} and strategy NC with probability $(1 - p_{\mathrm{C}})$. Substituting the expressions for r and p_{C} given by (2.21) and (2.22), the bank's expected return is

$$\begin{aligned}\Pi_{\mathrm{Mix}} &= p_{\mathrm{C}}\,\Pi_{\mathrm{C}} + (1 - p_{\mathrm{C}})\Pi_{\mathrm{NC}} = \Pi_{\mathrm{C}} \\ &= \frac{1}{\alpha}\left(R\,I - \beta\,D - \frac{\alpha + \gamma + \rho}{\gamma}\,c\,I\right).\end{aligned} \tag{2.23}$$

Proposition 2.3 *Outside the parameter ranges for equilibrium C and NC, if*

$$D \leq \frac{R\,I}{\beta} - \left(\alpha + \gamma + \frac{\alpha\,(\beta + \gamma)\,\rho}{\beta^2}\right)\frac{\alpha + \gamma + \rho}{(\beta - \alpha)\,\gamma^2}\,c\,I, \tag{2.24}$$

then there is a mixed-strategy equilibrium that is a mixture of strategies C and NC. Strategy C is played with probability p_C given by (2.22) and strategy NC is played with probability $(1 - p_C)$.

2.3.4 The No-Monitoring Equilibrium

In the no-monitoring equilibrium (indexed with '0'), the board offers low wages and no reward. Consequently, the manager shirks, the loan is not monitored, and neither the board nor the lenders know whether the project is in class A or B.

The Lenders Consider equilibrium 0, in which the board pays neither the reward nor an efficiency wage. Accordingly, the manager does not monitor the project. Starting from a project that is in class A with probability $(\beta-\alpha)/\beta$ and in B with probability α/β, the average quality deteriorates continuously over time. Given that the project is not monitored, the transition from class A to B happens relatively quickly compared to a monitored project. This quick transition is anticipated by the lenders. Formally, the decay rate in class A increases to $\alpha+\gamma$. The probabilities $p_A(t)$ and $p_B(t)$ are no longer an eigenvector of the dynamic system, so the evolution is

$$
\begin{aligned}
p_A(t) &= \frac{\beta-\alpha}{\beta}\, e^{-(\alpha+\gamma)\,t}, \quad \text{and} \\
p_B(t) &= \frac{\beta-\alpha}{\beta}\,\frac{\alpha+\gamma}{\beta-\alpha-\gamma}\, e^{-(\alpha+\gamma)\,t} - \frac{\gamma}{\beta-\alpha-\gamma}\, e^{-\beta\,t}.
\end{aligned}
\tag{2.25}
$$

The instantaneous probability of default is given by

$$
\frac{0\cdot p_A(t)+\beta\cdot p_B(t)}{p_A(t)+p_B(t)} = \frac{(\alpha+\gamma)\,(\beta-\alpha)-\beta\,\gamma\, e^{-(\beta-\alpha-\gamma)\,t}}{\beta-\alpha-\gamma\, e^{-(\beta-\alpha-\gamma)\,t}}. \tag{2.26}
$$

In order to break even at each point in time, the interest rate $r(t)$ must be equal to this rate. Finally, the probability that the project has defaulted at date t_B is

$$
F(t_B) = 1-\big(p_A(t)+p_B(t)\big) = 1-\frac{(\beta-\alpha)\, e^{-(\alpha+\gamma)\,t_B}-\gamma\, e^{-\beta\,t_B}}{\beta-(\alpha+\gamma)}. \tag{2.27}
$$

This is the probability distribution function of the default date t_B. Thus, the density function of the default date t_B is

$$
f(t_B) = \frac{(\gamma+\alpha)\,(\beta-\alpha)\, e^{-(\alpha+\gamma)\,t_B}-\gamma\,\beta\, e^{-\beta\,t_B}}{\beta-(\alpha+\gamma)}. \tag{2.28}
$$

The Board The bank's expected profit in equilibrium 0 is

$$
\begin{aligned}
\Pi_0 &= \int_0^\infty \int_0^{t_B} \big(R\,I-r(t)\,D\big)\mathrm{d}t\, f(t_B)\, dt_B \\
&= \int_0^\infty \int_0^{t_B} (R\,I)\ \mathrm{d}t\, f(t_B)\, dt_B - D \\
&= \int_0^\infty (R\,I)\ t_B\, f(t_B)\, dt_B - D = \frac{\gamma+\beta}{(\gamma+\alpha)\,\beta}\, R\,I - D.
\end{aligned}
\tag{2.29}
$$

Until default at date t_B, the bank earns returns $R\,I$ and pays interest on its debt D; the interest rate is increasing and is given by $r(t) =$ (2.26). The second equality holds because lenders anticipate the correct default rate; hence, in aggregate, they must be repaid exactly D.

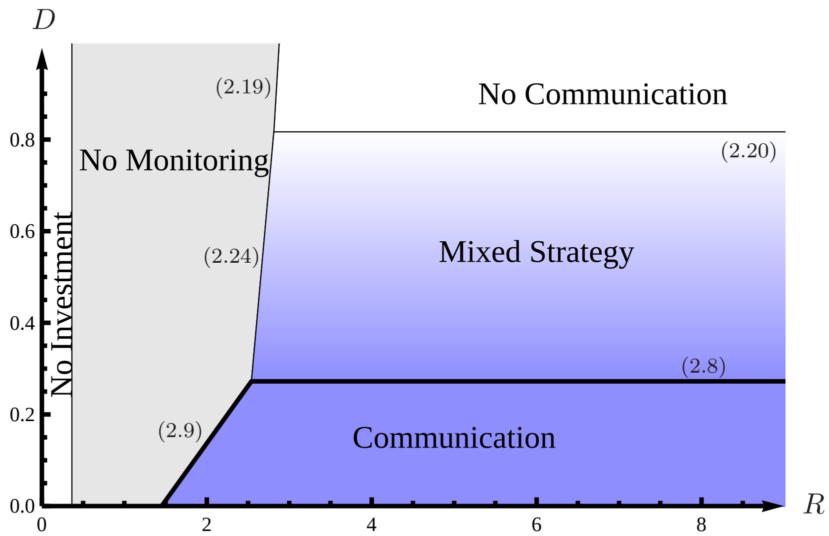
Considering out-of-equilibrium behavior by the board, we can now derive conditions under which equilibrium 0 exists. The exact condition and the proof are given in the appendix in section 2.6. The condition differs from those in Propositions 2.2 and 2.4, and there can be multiple equilibria. In this case, we concentrate on the Pareto-dominant equilibrium, which is the communication equilibrium C.

Proposition 2.4 *Whenever neither C nor NC nor a mixture between them exists, then either equilibrium 0 (with neither incentive wage nor reward) exists or the bank does not invest at all.*

2.3.5 Discussion of Factors Influencing Communication

We have identified three different types of pure-strategy equilibria and one mixed-strategy equilibrium. In the communication equilibrium (C), contracts consist of a reward $H > 0$ and an efficiency wage $w > 0$. In the no-communication equilibrium (NC), there is no reward, and the information about the deterioration does not become public. In the mixed-strategy equilibrium (Mix), both contracts C and NC are implemented with positive probability; therefore, the transmission of information is uncertain. In the no-monitoring equilibrium (0), the manager does not play an active role. The deterioration of project quality progresses quickly. Proposition 2.4 shows that the parameter space is completely covered with equilibria. Figure 2.2 depicts the equilibria given parameters $\alpha = 1/3$, $\beta = 1/2$, $\gamma = 1/9$, $\rho = 9/10$, $c = 1/160$ and $I = 1$ as the debt D and the project return R vary. For very low R, there is no investment in the project. For low R, there is investment in the project, but the project is not monitored (gray, equilibrium 0). In the remaining

Figure 2.2: Equilibria for Different Parameter Constellations



Numbers in brackets correspond to the relevant inequalities.

parameter space, either equilibrium C, Mix or NC is played. The shade of the color in the figure indicates the probability of information transmission. Blue means full communication (C), white means no communication (NC), and light blue means some communication (Mix). The intensity of the blue stands for the probability of a communicative contract.

The conditions in the above propositions show that the general structure of this figure is independent of parameter choices. The equilibrium with communication exists if D is sufficiently low. If D is high, the board fears the information will become public and that financing costs will jump up, which is more costly if D is high. Hence, for high D, the board does not write a reward into the contract. The same intuition applies for the mixed-strategy equilibrium. As D gets higher, the equilibrium probability that a warning is communicated gets lower. Because $D + E = I$, D also measures the leverage of the bank; hence, it is negatively related to the equity ratio. We discuss the implications of capital requirements in Section 2.4.2.

The influence of the project return R is also intuitive. For extremely low R, the project is not undertaken in the first place. For slightly higher values of R, the project is carried out, but it is too expensive for the bank to pay the manager an efficiency wage. R measures the income from the project, hence it can be influenced by different factors. For example, R could be higher during economic upswings or if competition between banks (not explicitly modeled here) is low. In a banking sector with high competition, returns would be lower, which would lead to less informational efficiency. As already mentioned, in difficult times, returns R drop even more; this decrease increases the probability of no communication and exacerbates the problem.

Let us discuss some additional comparative statics that are not immediately visible in the figure. The effect of monitoring costs c on the communication probability is positive: as the manager's monitoring cost c increases, the rent that she collects also increases. Because of the manager's discount rate ρ, the bank can economize by offering the manager a one-time payment when the monitoring effort is less important. For that reason, c and ρ enter the expression only as a product. The effect of increasing γ is unambiguously negative. Again, this effect arises because of the relationship with the manager's rent. For small γ, it is difficult to incentivize the manager, so her rent is extremely small. Therefore, paying her off early is a profitable decision for the bank. Finally, both α and β influence the communication probability p_C through many channels; hence, it is more difficult to derive an unambiguous intuition for these comparative statics.

2.4 Policy Implications

We can now use the model to discuss policy implications. We concentrate on three of many possible applications: capital regulation as the fundamental form of banking regulation, liquidity regulation, and convertible bonds as one new regulatory approach implemented in Basel III. We start with a small welfare analysis, showing that information-efficient contracting enhances welfare.

2.4.1 Welfare Analysis

Our analysis has no yet determined whether the communication equilibrium is actually the preferred equilibrium for the economy. We define social welfare as the sum of the lenders' profit, the bank's profit Π and the manager's utility U. Because we assume perfect competition in the market, the lenders make zero profits in any equilibrium. Thus, welfare is only composed of bank's profit and manager's utility because they may receive positive profits and wages. Welfare in the communication equilibrium is

$$\begin{aligned} W_{\mathrm{C}} &= \Pi_{\mathrm{C}} + U_{\mathrm{C}} \\ &= \Pi_{\mathrm{C}} + \frac{\beta-\alpha}{\beta} \int_0^{\infty} \left[\int_0^{t_A} (w - c\,I)\, e^{-\rho\,t}\,\mathrm{d}t + H\, e^{-\rho\,t_A} \right] \alpha\, e^{-\alpha\,t_A}\,\mathrm{d}t_A + \frac{\alpha}{\beta}\,H \\ &= \frac{R\,I}{\alpha} - D - \frac{(\beta-\alpha)\,(\gamma\,(\beta+\rho) + \rho\,(\alpha+\beta+\rho))}{\alpha\,\gamma\,\beta^2}\,c\,I. \end{aligned} \tag{2.30}$$

The manager receives her utility by earning w and incurring cost $c\,I$ as long as the loan is in class A. After communicating the negative signal, she receives reward H. This utility together with the bank's profit results in (2.30).

Similarly, we calculate welfare in the no-communication equilibrium as

$$\begin{aligned} W_{\mathrm{NC}} &= \Pi_{\mathrm{NC}} + U_{\mathrm{NC}} \\ &= \Pi_{\mathrm{NC}} + \frac{\beta-\alpha}{\beta} \int_0^{\infty} \Bigg[\int_0^{t_A} (w - c\,I)\, e^{-\rho\,t}\,\mathrm{d}t \\ &\qquad + \int_{t_A}^{\infty} \left(\int_{t_A}^{t_B} w\, e^{-\rho\,t}\,\mathrm{d}t \right) \beta\, e^{-\beta\,(t_B - t_A)}\mathrm{d}t_B \Bigg] \alpha\, e^{-\alpha\,t_A}\,\mathrm{d}t_A \\ &\qquad + \frac{\alpha}{\beta} \int_0^{\infty} \left(\int_0^{t_B} w\, e^{-\rho\,t}\,\mathrm{d}t \right) \beta\, e^{-\beta\,t_B}\,\mathrm{d}t_B \\ &= \frac{R\,I}{\alpha} - D - \frac{\rho\,(\beta+\rho) + \gamma\,(\beta-\alpha+\rho)}{\alpha\,\gamma\,\beta}\,c\,I. \end{aligned} \tag{2.31}$$

Accordingly, welfare in the mixed-strategy equilibrium is

$$\begin{aligned} W_{\mathrm{Mix}} &= p_{\mathrm{C}}\,(\Pi_{\mathrm{C}} + U_{\mathrm{C}}) + (1 - p_{\mathrm{C}})\,(\Pi_{\mathrm{NC}} + \Pi_{\mathrm{NC}}) \\ &= \frac{R\,I}{\alpha} - \frac{\beta}{\alpha}\,D - \frac{(\beta-\alpha)\,(\gamma+\rho)}{\alpha\,\gamma\,\beta}\,c\,I. \end{aligned} \tag{2.32}$$

By calculating the difference between (2.30) and (2.31), one can show that W_{C} is greater than W_{NC} for all parameters,

$$W_{\mathrm{C}} - W_{\mathrm{NC}} = \frac{\rho\,(\alpha+\gamma+\rho)}{\gamma\,\beta^2}\,c\,I. \tag{2.33}$$

We find that communication is always preferred from a welfare perspective if there are positive wages w that set monitoring incentives for the manager. This is intuitive: the lenders make no profits and the manager is indifferent between the two equilibria because the incentive compatible wage w^* is chosen by setting $U_{\mathrm{C}} = U_{\mathrm{NC}}$. Therefore, the difference between W_{C} and W_{NC} is simply the difference between the bank's profits in the two equilibria. The bank owners receive a higher profit in the communication equilibrium because they are able to reduce the costly debt. In additional, they pay the smaller one-time payment H^* in equilibrium C instead of the larger sum of wages w^* from date t_A to t_B in equilibrium NC; this difference arises because of the manager's discount rate. Thus, W_{C} is always higher than W_{NC}.

Welfare in equilibrium 0, the no-monitoring equilibrium, is

$$W_0 = \Pi_0 + U_0 = \frac{\gamma+\beta}{(\alpha+\gamma)\,\beta}R\,I - D. \tag{2.34}$$

The manager receives neither incentive wage nor reward. Therefore, her utility equals 0 and the total welfare is simply the bank's profit. Whether the welfare in the communication equilibrium is greater than W_0 depends on the exogenous parameters, as we can see by calculating the difference between (2.30) and (2.34),

$$W_{\mathrm{C}} - W_0 = \frac{\gamma\,(\beta-\alpha)}{\alpha\,\beta\,(\alpha+\gamma)}\,R\,I - \frac{(\beta-\alpha)\big(\gamma\,(\beta+\rho)+\rho\,(\alpha+\beta+\rho)\big)}{\alpha\,\gamma\,\beta^2}\,c\,I. \tag{2.35}$$

As the cost of monitoring loans become very high, they reach a level at which it is too expensive to set incentives for monitoring. With costs

$$c \le \frac{\gamma^2\,\beta}{(\alpha+\gamma)\,(\gamma(\beta+\rho)+\rho\,(\alpha+\beta+\rho))}\,R, \tag{2.36}$$

the equilibrium with communication yields higher welfare than the no-monitoring equilibrium. In summary, it may be possible that no monitoring is better than communication from a welfare perspective depending on the parameters; for example, the ordering varies with the costs c. However, with monitoring and low costs c, incentives for providing information are always welfare-optimal.

2.4.2 Capital Regulation

When discussing potential policy measures in the banking sector, capital adequacy standards are a good point of departure. We need to slightly reinterpret the original model to discuss this class of policies. Originally, the initial investment into the project was I, of which the debt D was provided by lenders and the remaining $E = I - D$ was the equity stake. Let us now assume that E is fixed and that the maximum investment is given by $E = \kappa\, I$. Thus, $I = \frac{1}{\kappa}\, E$. The parameter κ identifies the (required) equity ratio. We assume that bank owners cannot reinvest profits; therefore, equity does not increase over time. The profits of the bank are distributed to the equity owners; however, equity invested in the current project of the bank remains fixed. We can now discuss the role of an increase in κ. The question of interest is whether communication can be induced by introducing capital regulation. In addition, we are interested in whether welfare can be enhanced by stricter capital requirements. Therefore, we start our analysis in the mixed strategy region of the parameter space to examine whether the probability of communication increases.

We concentrate on the mixed-strategy equilibrium because a marginal increase in capital regulation can only reduce welfare in any of the other equilibria. This reduction occurs because project size is reduced, but the probability of communication is unchanged. In the mixed-strategy equilibrium, the probability of choosing contract C is changes from equation (2.22) to

$$p_{\mathrm{C}} = \frac{\beta}{\alpha}\Big(1 - \frac{(\beta - \alpha)\,(1 - \kappa)}{\rho\,\frac{\alpha+\gamma+\rho}{\gamma\,\beta}\,c}\Big). \tag{2.37}$$

It can be seen that an increase in κ leads to a higher value of p_{C}. Hence, higher capital regulation leads to a higher probability of communication. Welfare in the mixed-strategy equilibrium under capital regulation is

$$W_{\mathrm{Mix}} = \Big(\frac{R}{\alpha\,\kappa} - \frac{\beta\,(\frac{1}{\kappa} - 1)}{\alpha} - \frac{(\beta - \alpha)\,(\gamma + \rho)}{\alpha\,\gamma\,\beta\,\kappa}\,c\Big)\,E. \tag{2.38}$$

The various terms in this expression are readily interpretable. The first term gives the income from the project as long as it is in class A, which depends negatively on κ. The second term represents refinancing costs. The third term is the cost of incentivizing the manager, which includes the savings from paying her off early. Taking the derivative,

$$\frac{\partial W_{\text{Mix}}}{\partial \kappa} = \Big(\frac{\beta - R}{\alpha\,\kappa^2} + \frac{(\beta-\alpha)\,(\gamma+\rho)}{\alpha\,\gamma\,\beta\,\kappa^2}\,c\Big)\,E. \tag{2.39}$$

Depending on the relative sizes of α, β and R, this expression could be positive or negative. However, we have assumed that $\beta > \alpha$ to ensure that the transition probability from B to default is greater than the transition probability from A to B. Consequently, the derivative is positive if $\beta > R$; that is, the welfare effect of capital requirements is positive despite the fact that aggregate investment is reduced. The benefit of increased transparency dominates the cost of reduced investment. The preceding analysis establishes that a marginal increase in capital requirements is welfare-positive in the mixed-strategy equilibrium. It is welfare-negative in all other equilibria because it reduces investment volume.

In this setting, it would also be possible to discuss *risk-sensitive* capital requirements, as introduced in Basel II and retained in Basel III. The asset portfolio of the bank would have to be rated, but this rating could depend only on the available information. Hence, in equilibrium C, the rating would be high (A) before the transition, and lower (B) afterward. In other equilibria, the rating would be somewhere in between because of the lack of available information. Let us assume that the better the rating, the lower the κ. This would imply that part of the project would have to be sold after a deterioration (deleveraging), possibly at a large discount.[6] The implications for a bank's incentive to implement an informative contract (C) are largely negative. In reaction to a warning from a manager, markets will realize and react resulting in increased refinancing costs. However, rating agencies will also react by downgrading the bank, which forces it to deleverage at unfavorable prices. In sum, capital regulation is good for the system's informativeness, but requirements

[6] These prices are difficult to endogenize in our setting without further assumptions. Thus, we have left this discussion informal.

must be risk-insensitive.

2.4.3 Liquidity Regulation

In the model, the bank can react to the negative information, but only in one way: it can restructure the manager's wage contract. It could not sell part of its assets, because we had assumed perfectly illiquid assets. This assumption keeps the model simple, but it is not necessarily realistic. In reality, the bank will want to restructure (or sell) its asset portfolio after the negative information is revealed. As a consequence, the bank has an additional incentive to implement the communication contract. As the information channel is likely unobstructed, the financial market becomes more efficient. As a consequence, there is scope for regulation: if the bank does not choose the optimal degree of liquidity itself, the regulator may force it to do so. Of course, the initial intention and most important feature of liquidity regulation is the protection of banks against liquidity shocks. However, as can be seen in our model, liquidity regulation also influences capital and communication incentives.

First, we analyze the effect of asset liquidity on information efficiency. To analyze this relationship, we expand our model slightly. Assume that a fraction λ of assets can be liquidated at any time at zero cost. The remaining fraction $1-\lambda$ cannot be liquidated. Furthermore, assume that after the project is downgraded, it is optimal to liquidate; that is, assume that $\beta > R$. We discuss conditions under which the communication equilibrium is implemented. The discussion of the other equilibria proceeds along the same lines.

The incentive conditions for the manager are unchanged: H^* and w^* are still given by (2.5) and (2.6), respectively. The expected profits of the bank change because part of the portfolio can be liquidated after the negative information is revealed. For example, (2.7) becomes

$$
\begin{aligned}
\Pi_{\mathrm{C}} = \frac{\beta-\alpha}{\beta} \int_0^\infty \Bigg[\int_0^{t_A} (R\,I - r_A\,D - w^*)\,\mathrm{d}t - H^* \\
+ \int_{t_A}^\infty \Big(\int_{t_A}^{t_B} (R\,I\,(1-\lambda) - r_B\,(D-\lambda\,I))\,\mathrm{d}t \Big)\,\beta\,e^{-\beta\,(t_B-t_A)} dt_B \Bigg]\,\alpha\,e^{-\alpha\,t_A}\,\mathrm{d}t_A \\
+ \frac{\alpha}{\beta}\Bigg[-H^* + \int_0^\infty \Big(\int_0^{t_B} (R\,I\,(1-\lambda) - r_B\,(D-\lambda\,I))\,\mathrm{d}t \Big)\,\beta\,e^{-\beta\,t_B}\,dt_B \Bigg] \\
= \frac{R\,I}{\alpha} + \Big(1-\frac{R}{\beta}\Big)\,\lambda\,I - D - \frac{(\alpha+\gamma+\rho)\,\big(\beta^2+\rho\,(\beta-\alpha)\big)}{\gamma\,\alpha\,\beta^2}\,c\,I.
\end{aligned}
\tag{2.40}
$$

As for Proposition 2.1, we now compare these expected profits with the corresponding profits out of equilibrium. We derive conditions for the existence of the communication equilibrium that are very similar to those given by Proposition 2.1, (2.8) and (2.9). The bank will prefer the contract with communication to that without communication only if

$$
D \leq \frac{\rho\,(\alpha+\gamma+\rho)}{\gamma\,\beta^2}\,c\,I + \Big(1-\frac{R}{\beta}\Big)\,\lambda\,I. \tag{2.41}
$$

It will prefer the contract with communication to that without monitoring only if

$$
D \leq \frac{\gamma\,(\beta-\alpha)}{\alpha\,(\alpha+\gamma)\,\beta} - \frac{(\alpha+\gamma+\rho)\,\big(\beta^2+\rho\,(\beta-\alpha)\big)}{\gamma\,\alpha\,\beta^2}\,c\,I + \Big(1-\frac{R}{\beta}\Big)\,\lambda\,I. \tag{2.42}
$$

Both inequalities are weaker than (2.8) and (2.9). In each case, the limiting D is larger by $\big(1-\frac{R}{\beta}\big)\,\lambda\,I$. As the degree of liquidity λ of the assets increases, the communicative contract is more likely to be implemented and the financial market is more information-efficient.

We want to make three more points informally. *First*, if the bank could choose the degree of liquidity for its assets, what would it choose? Clearly, it would choose $\lambda = 1$ if there were no downside to doing so. Assume that the expected return of an asset depends negatively on its liquidity, $R = R(\lambda)$ with $R'(\lambda) < 0$, such that the problem has an interior solution. Then the bank would choose some degree of liquidity λ^*. If lenders could not observe the asset's liquidity, the bank would choose a λ that is too low. In this case, there would be scope for liquidity regulation. *Second*, in (2.41) and (2.42), one can see that as assets become more liquid, the bank can take on more leverage. Therefore, in our context, capital regulation and liquidity regulation are substitutes. *Third*, we have assumed that a fraction λ of assets can be liquidated at no cost. In reality, the saleability of assets will depend on whether the source of distress is idiosyncratic or systemic. In a systemic crisis, many banks will want to sell their assets, and the price will be low. Consequently, liquidity regulation is less beneficial with respect to the communication of crisis warnings.

2.4.4 Convertible Bonds

We have assumed that the bank can finance its loan portfolio solely with debt and inside equity. If we allow for more general financial tools, the equilibria might look different. For example, let us discuss the role of contingent convertible bonds (coco bonds) as an innovative source of finance; this source is considered in Basel III. We show that equilibrium C can be reached when it is welfare-optimal for the following reason: the coco debt is converted into shares at a predefined conversion rate after the value of equity has dropped below some threshold. The original reason for why the board may not want to implement a contract inducing communication is the negative market reaction after a bad signal. In other words, there is a positive externality on investors that is not internalized by the bank. However, by adjusting the conversion rate of the cocos to the right level, this externality can be taken into account.

Without loss of generality, assume there is no straight debt; there are only coco bonds. The volume that needs to be financed by lenders is $D = I - E$ as before. We show that the face value can be different and that debt may have to be issued below par. We denote $\bar{D}$ the face value and r as the short-term interest rate.

After the negative signal, the project is in class B, hence the aggregate value is

$$\int_0^\infty R\, I\, t_B\, \beta\, e^{-\beta\, t_B}\, \mathrm{d}t_B = \frac{R\, I}{\beta}. \tag{2.43}$$

A fraction $1/(1+\eta)$ goes to the bank, and the remaining fraction $\eta/(1+\eta)$ goes to lenders. Now remember that the project may be in class B right away, with probability α/β. The lender then loses part of his investment immediately, and he wants to be compensated for that fact. Therefore,

$$\begin{aligned} D &= \frac{\beta-\alpha}{\beta}\,\bar{D} + \frac{\alpha}{\beta}\cdot\frac{R\,I}{\beta}, \\ \bar{D} &= \frac{\beta}{\beta-\alpha}\,D - \frac{\eta}{1+\eta}\,\frac{\alpha}{\beta-\alpha}\,\frac{R\,I}{\beta}. \end{aligned} \tag{2.44}$$

After this initial period, the interest rate r adjusts such that lenders break even:

$$\begin{aligned} \bar{D} &= \alpha\,\mathrm{d}t\,\frac{\eta}{1+\eta}\cdot\frac{R\,I}{\beta} + (1-\alpha\,\mathrm{d}t)\,(1+r\,\mathrm{d}t)\,\bar{D}, \\ r &= \alpha\Big(1 - \frac{\eta}{1+\eta}\,\frac{R\,I}{\beta\,n}\Big). \end{aligned} \tag{2.45}$$

The second line obtains by solving for r and taking the limit $\mathrm{d}t \to 0$. We can now calculate the bank's expected profit within equilibrium C, i. e., the equilibrium with a reward and the efficiency wage. This quantity consists of two parts: the expected profits before the negative signal and the profits after the signal.

$$\begin{aligned} \Pi_{\mathrm{C}} &= \frac{\beta-\alpha}{\beta}\int_0^\infty (R\,I - w - r\,\bar{D})\,t_A\,\alpha\,e^{-\alpha\,t_A}\,\mathrm{d}t_A - H + \frac{1}{1+\eta}\,\frac{R\,I}{\beta} \\ &= \frac{R\,I}{\alpha} - D - \frac{(\alpha+\gamma+\rho)\,\big(\beta^2 + \rho\,(\beta-\alpha)\big)}{\gamma\,\alpha\,\beta^2}\,c. \end{aligned} \tag{2.46}$$

The second line obtains by inserting the optimal w^* and H^* from (2.5) and (2.6), respectively, and the equilibrium $\bar{D}$ from (2.44). Note that this equilibrium profit is identical to that in the informative equilibrium with debt financing only, as given by (2.7). This is not surprising because the expected profits of lenders are always zero and those of the manager are unchanged. In addition, η drops out of the equation. A higher η is exactly compensated by lower interest rates r. We now need to check the conditions under which the board wants to drop the reward. Out of equilibrium, the board's expected profit is

$$\begin{aligned} \Pi'_{\mathrm{C}} &= \frac{\beta-\alpha}{\beta}\int_0^\infty (R\,I - w - r\,\bar{D})\,t_A\,\alpha\,e^{-\alpha\,t_A}\,\mathrm{d}t_A + \int_0^\infty (R\,I - w - r\,\bar{D})\,t_B\,\alpha\,e^{-\alpha\,t_B}\,\mathrm{d}t_B \\ &= \frac{1}{\beta-\alpha}\Big(\big(\frac{\beta}{\alpha} - \frac{1}{1+\eta}\big)\,R\,I - \beta\,D\Big) - \frac{(\beta+\rho)(\alpha+\gamma+\rho)}{\gamma\,\beta}\,c. \end{aligned} \tag{2.47}$$

For a given η, the board chooses to implement the reward if $\Pi_{\mathrm{C}} \geq \Pi'_{\mathrm{C}}$; that is, the reward is implemented if

$$D \leq \frac{\eta}{1+\eta}\,\frac{R\,I}{\alpha} - \frac{\beta-\alpha}{\beta}\,\frac{\rho\,(\alpha+\gamma+\rho)}{\alpha\,\beta\,\gamma}\,c. \tag{2.48}$$

In Section 2.4.1, we showed that the equilibrium with a reward always dominates the equilibrium with monitoring only. Hence, in order to achieve the communication equilibrium C, one needs to set η high enough such that (2.48) is binding. Taking the limit as $\eta \to \infty$, the reward is implemented if $D \leq R/\alpha$. But R/α is the expected return from the project; a project with $D > R/\alpha$ would not be financed in the first place. As a result, the communication equilibrium can always be obtained. The following proposition sums up these arguments.

Proposition 2.5 *Starting from the no-communication equilibrium (NC), financing with appropriate contingent convertible bonds induces the board to implement contracts with a reward such that the communication equilibrium obtains.*

Also in the no-monitoring equilibrium, one can show that finance through coco bonds can lead to the communication equilibrium. For a given η, the profit without efficiency wage is

$$\Pi''_{\mathrm{C}} = \frac{\beta+\gamma}{\beta-\alpha}\left(\frac{\beta\,(1+\eta)-\alpha}{\beta\,(1+\eta)\,(\alpha+\gamma)}\,R - \frac{\alpha}{\alpha+\gamma}\,D\right). \tag{2.49}$$

The board chooses to implement efficiency wages and reward if $\Pi_{\mathrm{C}} > \Pi''_{\mathrm{C}}$:

$$\begin{aligned} D \leq \frac{(\beta-\alpha)\,(\alpha+\gamma)}{\alpha^2+\gamma\,(2\,\alpha-\beta)} &\left(\left(\frac{(\beta+\gamma)\,(\beta\,(1+\eta)-\alpha}{(\beta-\alpha)\,\beta\,(\alpha+\gamma)\,(1+\eta)} - \frac{1}{\alpha}\right) R\right. \\ &+ \left.\frac{(\alpha+\gamma+\rho)\,(\beta^2+(\beta-\alpha)\,\rho)}{\alpha\,\beta^2\,\gamma}\,c\right). \end{aligned} \tag{2.50}$$

The derivative of this term with respect to η is positive. This means that as η increases, the bank can take on more debt without destroying the communication channel.

2.5 Conclusion

We have constructed a microeconomic model of a bank in which communication of negative information plays a crucial role. The board would like to react to bad news by eliminating the manager's monitoring duties. However, the board needs to persuade the manager to report the news in the first place. The refinancing markets take notice of the board's reaction to the bad news; hence, the news is incorporated into market prices. This means higher refinancing costs for the bank. From a welfare perspective, the board has insufficient incentives to implement an informative contract; that is, financial markets are always semi-strong form efficient, and they become strong-form efficient when the informative contract is chosen. The degree of information efficiency is endogenous.

The model matches a number of stylized facts from the recent financial crisis. First and foremost, it explains how it is possible that crucial information could remain hidden for such a long time. Because many financial institutions were highly leveraged, the effect on refinancing costs would have been disastrous. So even if individuals within financial institutions could have foreseen the crisis, the institutions would not have wanted to incentivize them to report this information. Referring back to Figure 2.2, this situation is specific to highly leveraged institutions. Furthermore, many assets of financial institutions (e. g., mortgage loans) seemed to be highly liquid before the crisis, but proved to be illiquid during the crisis. It was impossible to cancel or reverse housing loans, because borrowers would have simply defaulted. In our model, a low level of asset liquidity results in a low level of informativeness.

We have modeled the bank as a single institution, and so there are no systemic effects. Modeling a banking system would have many consequences that would push outcomes in different directions. For example, the information about the deterioration of the loan portfolio may trigger further allocative decisions. In the communication equilibrium, the capital market is strongly information-efficient. In the other equilibria, it is not. This implies that financial markets serve their informational function less effectively in the uninformative equilibria.

As another example, if the loan portfolios of several banks are stochastically dependent, a manager's contract will contain information about other banks. If one bank gets into trouble, the probability that another bank's project deteriorates increases. Consequently, the optimal reward decreases. This might induce some form of competition between managers to be the first to report the deterioration.

Even abstracting from systemic effects, we can discuss a number of important implications. A higher equity ratio means that the bank fears the deterioration of credit conditions less; hence, it is incentivized to implement a communicative contract that results in early warnings to the markets. Capital requirements force banks to deleverage and thus reduce aggregate investment. But the benefits of increased transparency, including the responses taken by the banks themselves, dominate the costs of reduced investment. In other words, it is better to invest less if this enables banks to optimally react to negative news that would otherwise have been suppressed. Introducing a risk-sensitive capital requirement is detrimental. A possible liquidation of assets has a positive effect on the probability of implementation of the communicative contract; hence, liquidity regulation can be beneficial to the economy. Contingent convertible bonds can also increase welfare if the conversion rate is fixed low enough such that it reduces pressure from refinancing markets. In all applications, we stress that policies should be designed such that the communication channel is not obstructed.

2.6 Appendix

Proof of Lemma 2.1 To calculate the optimal wage w_A* and the optimal reward H^*, we set the manager's expected utility when monitoring equal to her expected utility when she is not monitoring, that is (2.3) = (2.4). In the next step, we use our result that the reward H must equal the expected utility U_B as in (2.1).

The condition is binding because the board will pay only the minimum amount required to incentivize the manager to communicate. In this case, the manager is just indifferent between telling bad news to the board and not communicating. By solving these conditions for w_A^* and H^* and taking the limit $\mathrm{d}t \to 0$, we receive (2.5) and (2.6).

Proof of Proposition 2.1 When does the board prefer to (out of equilibrium) implement contracts with lower wages or a lower reward? We calculate the board's expected profits for these strategies. If, out of equilibrium, the board would write the manager a contract without reward, then the board would have to pay the manager until the project defaults. However, refinancing costs would never adjust from r_A to r_B. Consequently, the aggregate payoff to the bank would be

$$\begin{aligned}
\Pi'_{\mathrm{C}} = \frac{\beta-\alpha}{\beta} \int_0^\infty \Big[\int_0^{t_A} (R\,I - r_A\,D - w)\,\mathrm{d}t \\
+ \int_{t_A}^\infty \Big(\int_{t_A}^{t_B} (R\,I - r_A\,D - w)\,\mathrm{d}t \Big)\,\beta\,e^{-\beta\,(t_B - t_A)} dt_B \Big]\,\alpha\,e^{-\alpha\,t_A}\,dt_A \\
+ \frac{\alpha}{\beta} \Big[\int_0^\infty \Big(\int_0^{t_B} (R\,I - r_A\,D - w)\,\mathrm{d}t \Big)\,\beta\,e^{-\beta\,t_B}\,dt_B \Big] \\
= \frac{R\,I - w^* - r_A\,D}{\alpha} \\
= \frac{R\,I}{\alpha} - \frac{(\beta+\rho)\,(\gamma+\alpha+\rho)}{\gamma\,\beta\,\alpha}\,c\,I.
\end{aligned} \tag{2.51}$$

The last line is obtained by substituting w^* and setting $r_A = 0$.

The board chooses a reward and a positive wage only if $\Pi'_{\mathrm{C}} \leq \Pi_{\mathrm{C}}$; that is, if

$$\begin{aligned}
\frac{R\,I}{\alpha} - \frac{(\beta+\rho)\,(\gamma+\alpha+\rho)}{\gamma\,\beta\,\alpha}\,c\,I \leq \frac{R\,I}{\alpha} - D - \frac{(\alpha+\gamma+\rho)\,\big(\beta^2 + \rho\,(\beta-\alpha)\big)}{\gamma\,\alpha\,\beta^2}\,c\,I \\
D \leq \frac{\rho\,(\alpha+\gamma+\rho)}{\gamma\,\beta^2}\,c\,I.
\end{aligned} \tag{2.52}$$

If, again out of equilibrium, the board would set up a contract with neither reward nor efficiency wage, the aggregate payoff to the board would be

$$\begin{aligned}
\Pi''_{\mathrm{C}} = \frac{\beta-\alpha}{\beta} \int_0^\infty \Big[\int_0^{t_A} (R\,I - r_A\,D)\,\mathrm{d}t \\
+ \int_{t_A}^\infty \Big(\int_{t_A}^{t_B} (R\,I - r_A\,D)\,\mathrm{d}t \Big)\, \beta\, e^{-\beta\,(t_B - t_A)} dt_B \Big]\, (\alpha+\gamma)\, e^{-(\alpha+\gamma)\,t_A}\, dt_A \\
+ \frac{\alpha}{\beta} \Big[\int_0^\infty \Big(\int_0^{t_B} (R\,I - r_A\,D)\,\mathrm{d}t \Big)\, \beta\, e^{-\beta\,t_B}\, dt_B \Big] \\
= \frac{(R\,I - r_A\,D)\,(\gamma+\beta)}{\beta\,(\alpha+\gamma)} = \frac{(\gamma+\beta)}{\beta\,(\alpha+\gamma)}\, R\,I. \qquad (2.53)
\end{aligned}$$

The manager chooses a reward plus a positive wage only if $\Pi''_{\mathrm{C}} \leq \Pi_{\mathrm{C}}$:

$$\begin{aligned}
\frac{(\gamma+\beta)}{\beta\,(\alpha+\gamma)}\, R\,I \leq \frac{R\,I}{\alpha} - D - \frac{(\alpha+\gamma+\rho)\,\big(\beta^2 + \rho\,\big(\beta-\alpha\big)\big)}{\gamma\,\alpha\,\beta^2}\, c\,I \\
D \leq \frac{\gamma\,(\beta-\alpha)}{\alpha\,(\alpha+\gamma)\,\beta}\, R\,I - \frac{(\alpha+\gamma+\rho)\,\big(\beta^2 + \rho\,(\beta-\alpha)\big)}{\gamma\,\alpha\,\beta^2}\, c\,I. \qquad (2.54)
\end{aligned}$$

We have to prove one more proposition. Up to now, we have implicitly assumed that the board implements the contract once and for all. However, given that all contracts are only short-term, the board may change the contract at any time. For example, it may start with a low wage, but increase the wage after some time. However, one can show that the reward does not depend on the probability with which the board expects the project to be in class A or class B because both the reward and reduced wage costs apply only under the condition that a transition occurs; hence, the probability cancels out. Furthermore, because the board is always informed about the project's class, the efficiency wage does not change over time. Therefore, when (2.52) and (2.54) hold, the board writes a contract with an efficiency wage and a positive reward.

Proof of Proposition 2.2 The board can deviate from the equilibrium in two ways. First, it can pay zero wages and the manager shirks. The expected return to the board is then

$$
\begin{aligned}
\Pi'_{\mathrm{NC}} = \frac{\beta-\alpha}{\beta} \int_0^\infty \Big[\int_0^{t_A} (RI - rD)\,\mathrm{d}t \\
+ \int_{t_A}^\infty \Big(\int_{t_A}^{t_B} (RI - rD)\mathrm{d}t\Big) \beta e^{-\beta(t_B - t_A)} dt_B \Big] (\alpha+\gamma) e^{-(\alpha+\gamma)t_A} dt_A \\
+ \frac{\alpha}{\beta} \int_0^\infty \Big(\int_0^{t_B} (RI - rD)\mathrm{d}t \Big) \beta e^{-\beta t_B} dt_B \\
= \frac{(RI - \alpha\, D)(\gamma+\beta)}{\beta\,(\gamma+\alpha)}. \qquad (2.55)
\end{aligned}
$$

Comparing this expression with (2.18), the board implements the efficiency wage if $\Pi_{\mathrm{NC}} > \Pi'_{\mathrm{NC}}$:

$$
D \leq \frac{R\,I}{\alpha} - \frac{(\alpha+\gamma)\,(\beta+\rho)\,(\gamma+\alpha+\rho)}{\gamma^2\,\alpha\,(\beta-\alpha)}\,c\,I. \qquad (2.56)
$$

Second, the board can deviate by paying the wage and the reward. The manager then monitors and communicates all news. In this case, the bank's expected profit is

$$
\Pi''_{\mathrm{NC}} = \frac{RI}{\alpha} - \frac{2\beta-\alpha}{\beta}\,D - \frac{(\gamma+\alpha+\rho)(\beta^2+\rho\,(\beta-\alpha))}{\gamma\alpha\beta^2}\,c\,I. \qquad (2.57)
$$

The board pays efficiency wages but no reward only if $\Pi_{\mathrm{NC}} > \Pi''_{\mathrm{NC}}$:

$$
D \geq \frac{\rho\,(\gamma+\alpha+\rho)}{\gamma\,\beta\,(\beta-\alpha)}\,c\,I. \qquad (2.58)
$$

Hence, when (2.56) and (2.58) hold, there is an equilibrium with a positive wage, $w > 0$, and a zero reward, $H = 0$.

Proof of Proposition 2.3 The boundaries of the mixed-strategy equilibrium are defined by the conditions under which the board would deviate from it. Instead of playing a mixed strategy, the board could decide to implement a contract with incentive wage and reward for certain. The expected return to the board is then

$$
\begin{aligned}
\Pi'_{\mathrm{Mix}} &= \frac{R\,I}{\alpha} - H^* - \frac{\alpha\,r_B\,D + (\beta-\alpha)\,(w^* + r\,D)}{\alpha\,\beta} \\
&= \frac{R\,I}{\alpha} - D - \frac{(\alpha+\gamma+\rho)\,(\beta^2 + \rho\,\big(\beta-\alpha)\big)}{\gamma\,\alpha\,\beta^2}\,c\,I. \qquad (2.59)
\end{aligned}
$$

The second line is obtained by inserting w^*, H^* and setting $r = r_{\text{Mix}}$, $r_B = \beta$. Because strategy C is definitely chosen, $p_{\text{C}} = 1$. The board plays the mixed strategy only if $\Pi_{\text{Mix}} > \Pi'_{\text{Mix}}$:

$$D \geq \frac{\rho\,(\alpha+\gamma+\rho)}{\gamma\beta^2}\,c\,I. \tag{2.60}$$

The board could instead decide to play strategy NC for certain and pay only an incentive wage but no reward to the manager. In that case, its profit would be

$$\begin{aligned}\Pi''_{\text{Mix}} &= \frac{R\,I - r\,D - w}{\alpha}\\ &= \frac{R\,I}{\alpha} - D - \frac{(\gamma+\alpha+\rho)\,(\beta+\rho)}{\gamma\,\alpha\,\beta}\,c\,I.\end{aligned} \tag{2.61}$$

Again, the second line is obtained by inserting w^* and setting $r = r_{\text{Mix}}$. In this case, strategy NC is definitely chosen so that $p_{\text{C}} = 0$. The board plays the mixed strategy only if $\Pi_{\text{Mix}} > \Pi''_{\text{Mix}}$:

$$D \leq \frac{\rho\,(\alpha+\gamma+\rho)}{\gamma\,\beta\,(\beta-\alpha)}\,c\,I. \tag{2.62}$$

Conditions (2.60) and (2.62) are identical to equations (2.8) and (2.20) from Propositions 2.1 and 2.2, respectively. Thus, the mixed-strategy equilibrium is located exactly between equilibrium C and equilibrium NC.

There is one more possible deviation from the mixed-strategy equilibrium. If the board decides to pay neither an incentive wage nor a reward, then its profit would be

$$\begin{aligned}\Pi'''_{\text{Mix}} &= \int_0^\infty \int_0^{t_B} \big(R\,I - r\,D\big)\mathrm{d}t\, f(t_B)\,dt_B\\ &= \frac{\beta+\gamma}{\alpha+\gamma}\left(\frac{R\,I}{\beta} - D + \frac{\rho\,(\alpha+\gamma+\rho)}{\beta^2\,\gamma}\,c\,I\right).\end{aligned} \tag{2.63}$$

Inserting p_{C} and setting $r = r_{\text{Mix}}$ leads to the second line. The board prefers the mixed strategy to a contract without incentive wage and reward if $\Pi_{\text{Mix}} > \Pi'''_{\text{Mix}}$:

$$D \leq \frac{R\,I}{\beta} - \left(\alpha+\gamma+\frac{\alpha\,(\beta+\gamma)\,\rho}{\beta^2}\right)\frac{\alpha+\gamma+\rho}{(\beta-\alpha)\,\gamma^2}\,c\,I. \tag{2.64}$$

If equations (2.60), (2.62) and (2.64) hold, an mixed-strategy equilibrium with positive probabilities on strategies C and NC exists.

Proof of Proposition 2.4 We need to establish for which parameters out of equilibrium behavior may be optimal for the board. For example, the board may want to pay the manager an efficiency wage, in which case the deterioration of the project is not as fast. We must calculate the expected profits in this case. The density function of the default date t_B is simply

$$\hat{f}(t_B) = \alpha\, e^{-\alpha\, t_B}. \tag{2.65}$$

The bank's expected profit consists of three parts: the expected returns from the project, the expected wages, and the expected refinancing costs. The first two parts are simply

$$\int_0^\infty \big(R\,I - w\big)\, t_B\, \hat{f}(t_B)\, dt_B = \frac{R\,I - w}{\alpha}. \tag{2.66}$$

The third part is

$$\begin{aligned}
&\int_0^\infty \Big[\int_0^{t_B} r(t)\, D\, \mathrm{d}t\Big]\, \hat{f}(t_B)\, dt_B \\
&= D \cdot \int_0^\infty \Big[\int_0^{t_B} \frac{(\alpha+\gamma)\,(\beta-\alpha) - \beta\,\gamma\, e^{-(\beta-\alpha-\gamma)\, t}}{(\beta-\alpha) - \gamma\, e^{-(\beta-\alpha-\gamma)\, t}}\, \mathrm{d}t\Big]\, \alpha\, e^{-\alpha\, t_B}\, dt_B \\
&= D \cdot \int_0^\infty \Big[(\alpha+\gamma)\, t_B - \log\Big(\frac{\beta-\alpha-\gamma\, e^{-(\beta-\alpha-\gamma)\, t_B}}{\beta-\alpha-\gamma}\Big)\Big]\, \alpha\, e^{-\alpha\, t_B}\, dt_B \\
&= D \cdot \int_0^\infty \Big[(\alpha+\gamma)\, t_B + \log(\beta-\alpha-\gamma) - \log\big(\beta-\alpha-\gamma\, e^{-(\beta-\alpha-\gamma)\, t_B}\big)\Big]\, \alpha\, e^{-\alpha\, t_B}\, dt_B \\
&= D \cdot \Big[\frac{\alpha+\gamma}{\alpha} + \log(\beta-\alpha-\gamma) - \int_0^\infty \log\big(\beta-\alpha-\gamma\, e^{-(\beta-\alpha-\gamma)\, t_B}\big)\, \alpha\, e^{-\alpha\, t_B}\, dt_B\Big] \\
&= D \cdot \Big[\frac{\alpha+\gamma}{\alpha} - \frac{\gamma}{\beta-\alpha} \cdot \Phi\Big(\frac{\gamma}{\beta-\alpha}, 1, \frac{\beta-\gamma}{\beta-\alpha-\gamma}\Big)\Big],
\end{aligned} \tag{2.67}$$

where the Lerch transcendent Φ is defined by $\Phi(z, 1, a) = \sum_{n=0}^{\infty} z^n/(a+n)$. The aggregate expected profit consists of (2.66) net of (2.67):

$$\Pi_0' = \frac{R\,I - w^*}{\alpha} - D \cdot \Big[\frac{\alpha+\gamma}{\alpha} - \frac{\gamma}{\beta-\alpha} \cdot \Phi\Big(\frac{\gamma}{\beta-\alpha}, 1, \frac{\beta-\gamma}{\beta-\alpha-\gamma}\Big)\Big] \tag{2.68}$$

with w^* as defined in (2.5). The board implements the naked contract (without efficiency wage or reward) only if $\Pi_0 \geq \Pi_0'$:

$$D \geq \frac{\beta-\alpha}{\gamma^2\,(\gamma+\alpha)\,\beta} \cdot \frac{\gamma^2\,(\beta-\alpha)\,R\,I - (\gamma+\alpha)(\beta+\rho)(\gamma+\alpha+\rho)\,c\,I}{\beta-\alpha\,\big[1+\Phi(\frac{\gamma}{\beta-\alpha}, 1, \frac{\beta-\gamma}{\beta-\alpha-\gamma})\big]}. \tag{2.69}$$

Finally, we must calculate potential out of equilibrium profits if the board wants to implement a contract with both an efficiency wage and a reward. As long as the manager does not reveal the information, lenders believe they are financing a project of mixed quality; hence, they demand the loan rate $r(t)$ as defined in (2.26). Once the negative signal is communicated, lenders learn they have had wrong beliefs; they then charge the rate $r = \beta$, which is consistent with the correct instantaneous probability of default. The profit function can be expressed as several parts. First, with probability $(\beta - \alpha)/\beta$, the project starts in class A and the interest rate is $r(t)$. The date of the transition to class B is exponentially distributed with parameter α. The profit is

$$\begin{aligned}&\int_0^\infty \Big[\int_0^{t_A} (R\,I - w - r(t)\,D)\,\mathrm{d}t\Big]\alpha\, e^{-\alpha\, t_A}\, dt_A \\ &= \frac{R\,I - w^*}{\alpha} - D \cdot \Big[\frac{\alpha+\gamma}{\alpha} - \frac{\gamma}{\beta-\alpha}\cdot \Phi\Big(\frac{\gamma}{\beta-\alpha}, 1, \frac{\beta-\alpha}{\beta-\alpha-\gamma}\Big)\Big], \end{aligned} \tag{2.70}$$

which is analogous to (2.67). Then at date t_A, the reward H is paid, and the project continues with reduced investment size until it defaults completely. This happens with instantaneous probability β; hence, the interest rate is also β.

$$\begin{aligned}&\int_0^\infty \Big[\int_0^{t_B} (R\,I\, - r_B\,D)\,\mathrm{d}t\Big]\alpha\, e^{-\alpha\, t_B}\, dt_B \\ &= \frac{R\,I}{\beta} - D. \end{aligned} \tag{2.71}$$

With probability α/β, the loan starts in class B right away, and the profit is as given by (2.71). The aggregate expected profit is then $1 \cdot (2.71) + (\beta-\alpha)/\beta \cdot (2.70)$, which is

$$\begin{aligned}\Pi_0'' &= \frac{R\,I}{\beta} - D \\ &\quad + \frac{\beta-\alpha}{\beta}\cdot\Big[\frac{R\,I - w^*}{\alpha} - D\cdot\Big[\frac{\alpha+\gamma}{\alpha} - \frac{\gamma}{\beta-\alpha}\cdot\Phi\Big(\frac{\gamma}{\beta-\alpha}, 1, \frac{\beta-\alpha}{\beta-\alpha-\gamma}\Big)\Big]\Big] \\ &= \frac{\beta}{\alpha\,\beta}\,R\,I - \Big(1 + \frac{(\beta-\alpha)\,(\alpha+\gamma)}{\alpha\,\beta}\Big)\,D \\ &\quad + \frac{(\alpha\,\beta - (\beta-\alpha)\,(\beta+\rho))\,\,(\alpha+\gamma+\rho)}{\gamma\,\alpha\,\beta^2}\,c\,I + \frac{\gamma}{\beta}\,D\,\Phi\Big(\frac{\gamma}{\beta-\alpha}, 1, \frac{\beta-\gamma}{\beta-\alpha-\gamma}\Big). \end{aligned} \tag{2.72}$$

Hence, the board implements the naked contract only if $\Pi_0 \geq \Pi_0''$:

$$\frac{D}{I} \geq \frac{(\beta-\alpha)\gamma^2\beta R - (\alpha+\gamma)\big((\beta-\alpha)(\beta+\rho) - \alpha\beta\big)(\alpha+\gamma+\rho)c}{\gamma(\alpha+\gamma)\beta\Big[(\beta-\alpha)\,(\alpha+\gamma) - \alpha\,\gamma\,\Phi\Big(\frac{\gamma}{\beta-\alpha}, 1, \frac{\beta-\alpha}{\beta-\alpha-\gamma}\Big)\Big]}. \tag{2.73}$$

The board only conducts the project if the bank's profit suffices to pay back at least the invested equity. Therefore, the returns of the project must be high enough:

$$\Pi_0 \geq I - D \qquad \Longrightarrow \qquad R \geq \frac{\beta\,(\alpha+\gamma)}{\beta+\gamma}. \tag{2.74}$$

3 Model Risk — an Agency Theoretic Approach

3.1 Introduction

The recent financial crisis has raised a host of questions concerning the quantitative models used by financial market participants: Why did rating systems and banks' risk models not warn against failing projects, especially in the real estate market? Have these systems not been good enough? Which role did organizational structures play? It seems that the risk models did not cover all risks and were not kept up-to-date by banks.

All banks are usually confronted with several types of risk: market risk, credit risk, operational risk et cetera. Model risk — which is in our focus here — can be classified as part of operational risk. Typically, model risk is analyzed from an empirical or statistical perspective. However, to improve a risk model — and simultaneously reduce model risk —, two components are necessary: the institutional component and the technical component; the latter implies to have technical prerequisites, e. g. statistics or IT. However, even with perfect technical components it is crucial to choose the right incentives to use these models correctly, and hence, reduce model risk. The present paper focuses on the institutional component represented by the organizational and financial structures of firms that influence the incentives for innovation. Offering right incentives may enable an improvement of risk model accuracy.

Our paper contributes a further aspect to the understanding why existing risk models could not prevent the financial crisis. In the context of a principal agent model, a bank has to incentivize an agent to perform innovations in the risk model technology. The paper deals with the optimal organizational structure of the bank: under which conditions is it preferable to engage an external agent for the task of risk model innovation compared to the situation where an internal employee works on the improvement? Finally, the question of risk model improvement is analyzed from a managerial as well as from a welfare perspective. We discuss if banks undertake enough effort to improve their risk models.

Risk management strategies are based on the assumption of a certain risk model. To determine the future ability of a firm to repay credit, the bank's credit department will use a risk model which incorporates any relevant information it has available to assign the applicant to a certain risk class. However, the risk models are rarely reliable in representing the reality. It is possible that on the basis of a risk model credit-worthy firms are rejected and loan seeking firms that are not worth a credit are accepted. Given this model imperfection, the bank strives for the best achievable risk model.

In an article by Rebonato (2002), model risk is defined as the risk "of occurrence of a significant difference between the mark-to-model value of a complex and/or illiquid instrument, and the price at which the same instrument is revealed to have traded in the market". Sibbertsen, Stahl, and Luedtke (2008) define model risk as every risk induced by application, choice, specification, and estimation of a statistical model. Thus, in a broader sense, model risk can be seen as the risk of losing money because of a failing model.

We apply a principal agent setting, where a bank owner has to incentivize its employee, i. e. , the loan officer, to decide about credit approval of loan seeking projects in the bank owner's interest. We show that the bank owner offers a wage that is reduced to the minimum possible amount and consists of two components: one administration fee and one bonus payment depending on the success of the projects.

As a result, the loan officer's wage depends not only on the quality of the market and the costs for a credit check but also on the quality of the applied risk model.

In a next step, we add to the analysis a further agency problem resulting from the effort undertaken to make improvements of the risk model. This effort is private information of the agent as the bank owner may notice a modification of the risk model but is not able to recognize if this modification is a change for the better.

We regard both types of possible judgment errors of risk models: rejection of credit-worthy firms, which we call type-I-error or alpha-error, and the acceptation of firms unworthy of credits, called type-II-error or beta-error. As it is most important for banks to reduce their default rates – which are resulting from the beta-error, we predominantly concentrate on this failure. However, implications and interpretation of the analysis can be seen as general results.

By undertaking innovations, the bank is then able to reduce alpha- or beta-error. In this setting, the bank owner may outsource the work on the innovation process to an external agent, e. g. a risk manager or consultant. An external agent can be imagined as any person that is outsider to the credit check process. This could be a risk manager, a consultant or also an employee of the risk department of the bank. In the following, we will call this external person a consultant. We examine the three different possibilities of the bank: (i) to abandon a possible improvement or process, (ii) to incentivize the internal employee, e. g. the loan officer, to do an innovation or (iii) to engage the external consultant in order to improve the bank's internal risk model.

We show that the decision about internal or external contracting for the innovation is depending on several factors, as market quality, effort costs and extent of the innovation. With lower market quality, e. g. a low share of good projects, the probability for an internal agent to perform the innovation is rising. With high market quality, the bank owner decides to delegate the innovation process to an external agent. If the innovation planned by the bank's owner is of a smaller extent, he prefers to engage the external consultant. The decision in favor of the external consultant will also be taken in the case of high effort costs required for the innovation.

These results are intuitive if we imagine situations in which it is more difficult to implement an innovation. This is the case with subtleties as a very high market quality or a very small extent of innovation. High effort costs also make the improvement of a risk model more difficult. Due to the different incentives for credit check and innovation, in these cases, the principal benefits from the possibility of diversification and engages one agent for each task. This is a realistic result as, in reality, we also observe that consultants are typically retained for specialized and difficult tasks.

In some cases, for example when market quality or effort costs are very high, it may even be optimal for the bank owner to have no improvement process at all. Incentives, and therefore wages for internal or external agent, would be too costly for the bank. In our welfare analysis, we show conditions under which an innovation would not be profitable. However, in a comparison of the welfare optimal situation and the bank decision, it can be seen that innovation happens too rarely. Externalities keep banks from investing in innovation at certain levels of market and risk model quality. We therefore can conclude that regulatory standards for risk models and for the improvement of risk models are necessary in the economy in order to reduce model risk whenever a reduction is welfare optimal.

The remainder of the paper is organized as follows. After a short presentation of the institutional background and a discussion of the related literature, Section 3.2 introduces our theoretical model. The next sections deal with the extension of the basic model. Section 3.3 discusses the reduction of the beta-error of the risk model, starting with the structure of the innovation (in 3.3.1), followed by a discussion about conditions for the different regimes (in 3.3.2), and a welfare analysis with specific examples for the decision of the bank (in 3.3.3). Section 3.4 concludes. Proofs are presented in the appendix.

Institutional Background Responding to the crisis and the ongoing discussions about risk management, the use of models in banks was extended more and more and the complexity of these models increased. Consulting firms (such as Oliver Wyman or Navigant) are specialized in offering individual solutions in an advisory capacity to financial institutions. Customized models are developed and can be implemented, tested, and improved by those companies. If banks do not want to hire consultancy firms, they also have the possibility to buy software packages (e. g. SAS products as SAS Risk Management for Banking). Those packages offer models, analyses, and reports that can be adjusted to the needs of the respective user. In some banks, however, risk management is part of the own activities and employees model particular systems to deal with risk management. In our article, we are interested in the reasons and the benefits of the decision for one of the two approaches: engaging external experts or developing own solutions.

The Board of Governors of the Federal Reserve System together with the Office of the Comptroller of the Currency recently underlined the importance of model risk and published a document that provides practical guidance on effective model risk management (OCC, 2011). This guide highlights the critical role of model risk for banks and the importance to validate risk models on a periodic basis. By giving useful hints to establish a well working model risk management, the document emphasizes the benefit of independence between model development and usage. At the same time, it also discusses the challenges of the use of vendor and other third-party products regarding data, parameter values and complete models. In our paper, we consider challenges and benefits from an incentive perspective.

Literature Our paper relates to many strands of the economic literature. Most articles about model risk use a statistical analysis (for example Rebonato, 2002; Derman, 1996; Alexander, 2005; Sibbertsen, Stahl, and Luedtke, 2008). In addition to this empirical literature, theoretical literature exists as well. The articles that are most closely related to our model are dealing with type-I- or type-II-error.

In his famous article, Broecker (1990) discusses imperfect credit check tests and relates this to the competition between banks. A similar subject is analyzed in the paper of Hauswald and Marquez (2003). The authors show how better information in credit screening decreases interest rates and the returns from screening. In a further article, Hauswald and Marquez (2006) are interested again in the interaction of information acquisition and banking competition. They find that investment in information acquisition is falling as competition increases.

In addition, there is a theoretical literature on innovation. Hellmann and Thiele (2011) discuss innovation as an unplanned activity whereas in our model it is intended. In the context of different industries (no banking), interactions between planned and unplanned activities are considered and the conditions are discussed under which agents decide to pursue innovative and unplanned work. The authors show that it helps to reduce incentives for planned activities if desired innovations are very firm specific. A further article in the literature about innovation is from Manso (2011) who also looks at incentive problems for managers to innovate in a long-term structure. Aghion and Tirole (1994) discuss organizational questions regarding innovations made by R&D departments.

Crouhy, Galai, and Mark (2000) compare different approaches of credit risk models. They analyze models that use distinct measures to decide the credit worthiness of a firm or a project. In another context, Figlewski (1998) considers model risk: empirical evidence shows the impact of this risk type in the valuation of derivatives as theoretical models are also used in derivatives trading.

In contrast to our model, several papers exist that discuss risk models that are intentionally specified in a wrong way to communicate an overoptimistic view about the bank (see e. g. Colliard, 2012; Danelsson, 2008; Mariathasan and Merrouche, 2014). We assume here that the initial alpha- and beta-error are not strategically chosen by the bank.

3.2 The Basic Model

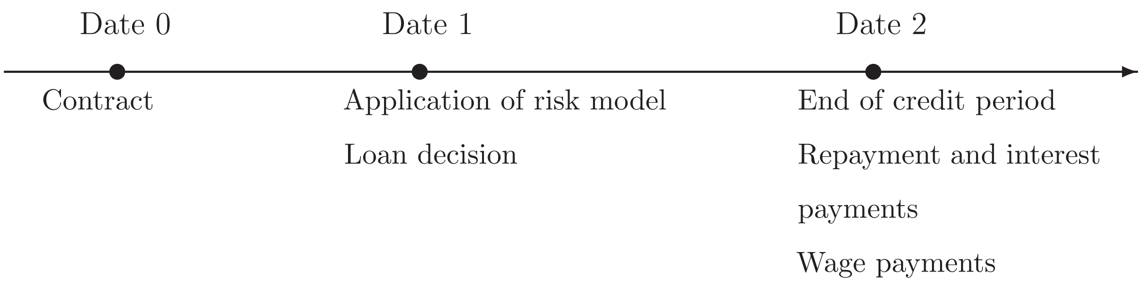
Consider a principal agent problem where a loan officer (agent) decides about credit approval of loans by means of a risk model in the interest of a bank owner (principal). In our model, there is one representative firm seeking a loan. If the firm receives the loan, its project may either succeed or fail. For simplicity, we suppose that this happens with probability 1. Accordingly, there are two types of firms: a firm may either be good with success probability $p_{success} = 1$ or bad with success probability $p_{success} = 0$. All agents are risk neutral and maximize their profits.

The decision process of the bank is conducted by the loan officer. He uses the risk model as basis for his decision about approval or rejection by running a credit check on the loan seeking firm. This task causes effort costs c per credit check for the loan officer. As the bank owner is not able to recognize directly if the loan officer is working on the credit decision as contracted, he needs to incentivize his employee by a certain wage structure.

The time structure of the model can be seen in Figure 3.1: at date 0, bank owner and loan officer conclude a contract. At date 1, the loan officer decides about lending based on the prediction of the risk model. At date 2, the credit period is over. The bank owner receives repayment of the credit and interest payments, the loan officer obtains his wage.

With probability $\gamma \in (0, 1)$, a loan seeking firm is a good firm and creditworthy. With probability $1 - \gamma$, a firm will not be able to repay the loan amount. Thus, γ represents the quality of the market. The risk model is now used as an instrument to decide between firms that are worthy of credit or not. However, no such risk model is able to distinguish perfectly between the two classes of firms. It may recommend to reject a good firm (alpha- or type-I-error) or to accept a bad firm (beta- or type-II-error). In general, $\alpha \in (0, 1)$ is the risk of an incorrect rejection and $\beta \in (0, 1)$ the risk of an incorrect acceptation. We assume $\alpha + \beta < 1$. The structure of this risk model is illustrated in Figure 3.2. The white region shows the projects of a good

Figure 3.1: Time Structure

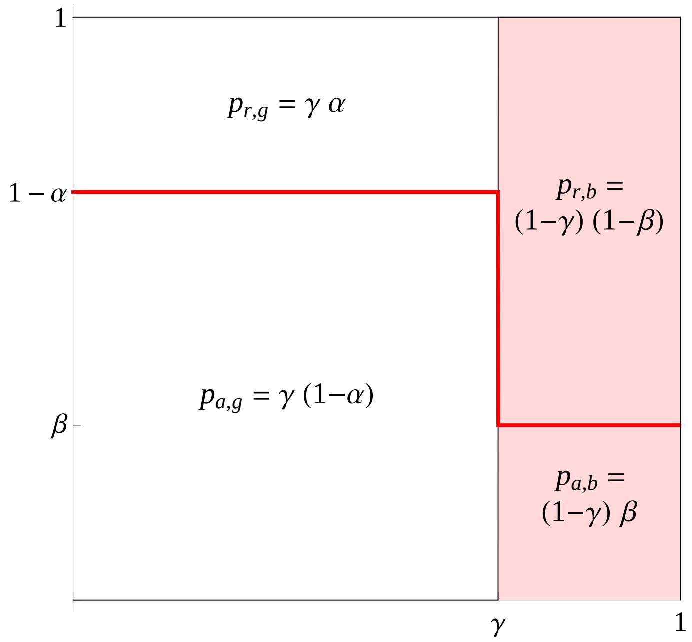


firm, the light red region consists of projects of a bad firm. Loans are accepted with probability $p_a = \gamma\,(1-\alpha) + (1-\gamma)\,\beta$ (area below the red line in the graph) and rejected with probability $p_r = \gamma\,\alpha + (1-\gamma)\,(1-\beta)$ (area above the red line) by the risk model or rather the loan officer who works with it.

If the firm, i. e. its project, is accepted by the bank, the bank has to pay the loan amount I to the firm at date 1. At the end of the credit period, the bank receives repayment of I and interest rate r from the firm. Probabilities $p_{r,g}$ (rejected, but good firm) and $p_{a,b}$ (accepted, but bad firm) mark alpha- and beta-error in the model.

To incentivize the loan officer to work and to tell the truth about the signal he receives from the risk model, the bank owner compensates him by paying a wage. The loan officer has the possibilities to work properly and to use the risk model or simply to shirk by rejecting or accepting all credit requests. In order to avoid the two shirking possibilities, the bank has to pay a wage that consists of two components. In our model, the loan officer receives an administration fee $a \geq 0$ for every rejected project (i. e. with proability $p_r = \gamma\,\alpha + (1-\gamma)\,(1-\beta)$) and a bonus fee $b \geq 0$ for creditworthy and accepted loans (i. e. with probability $p_{a,g} = \gamma\,(1-\alpha)$). Thus, part of the loan is conditional on the realization at date 2. Such a payment structure is used for example in banks where a bonus is paid at the end of the year depending of the outcome of projects. If less accepted loans default the bonus for the loan officer

Figure 3.2: Structure of the Model



will be higher. Structures like this often are determined in a target agreement. The wage scheme results in a payment for all loan assignments worked on besides those who fail at the end.

The expected profit of the bank owner consists of the repayment and interest rate of loans he receives and is reduced by the loan amounts granted to firms and by the wage he has to pay to the loan officer. This is

$$E\Pi B = p_{a,g}\,(1+r)\,I - p_a\,I - (p_r\,a + p_{a,g}\,b). \tag{3.1}$$

The loan officer receives his wage and has credit check costs, thus his expected profit is

$$E\Pi L = p_r\,a + p_{a,g}\,b - c. \tag{3.2}$$

The bank owner now calculates the optimal values for the wage parameters a and b in order to incentivize the loan officer to work. The loan officer is not shirking if his utility equates at least to the amount he can get if he just accepts every project or no project at all. Incentive constraints are then:

$$E\Pi L \geq a \tag{3.3}$$

$$E\Pi L \geq \gamma\, b \tag{3.4}$$

The bank owner is willing to pay only the minimum necessary to fulfill incentive constraints. Therefore, both constraints are binding which leads to the following Lemma:

Lemma 3.1 *Optimal wage for the loan officer is determined by the parameters*

$$a = \frac{c}{(1-\alpha-\beta)\,(1-\gamma)} \quad \textit{and} \tag{3.5}$$

$$b = \frac{c}{(1-\alpha-\beta)\,\gamma\,(1-\gamma)}. \tag{3.6}$$

The proof of Lemma 3.1 is presented in the appendix.

By inserting wage parameters (3.5) and (3.6) in profit functions (3.1) and (3.2), equilibrium profits can be calculated. Expected profits of the bank owner and the loan officer are

$$E\Pi B^{BasicModel} = [r\,(1-\alpha)\,\gamma + (1-\gamma)\,\beta]\, I - c - \frac{c}{(1-\gamma)\,(1-\alpha-\beta)} \quad \text{and}$$

$$E\Pi L^{BasicModel} = \frac{c}{(1-\alpha-\beta)\,(1-\gamma)}.$$

Evidently, the wage of the loan officer must depend on costs c, as the bank owner has to compensate his employee for his effort, respectively costs. The higher the costs, the higher is the wage. The quality of the risk model as well as the quality of the market also have a positive influence on the loan officer's wage.

3.3 Innovations of Risk Model: The Reduction of the Beta-Error

3.3.1 Structure of the Innovation

In order to improve the risk model, the bank owner may want to install an innovation process to improve alpha- and beta-error. In this section, we concentrate on the latter which is more relevant in recessions. The beta-error is responsible for losses in the bank whereas a reduction of the alpha-error allows raising profits. If there are high default rates in the bank, it is absolute priority to reduce them. This can be done by lowering beta. In our analysis of the beta-error-reduction, we assume for simplicity that $\alpha = 0$.

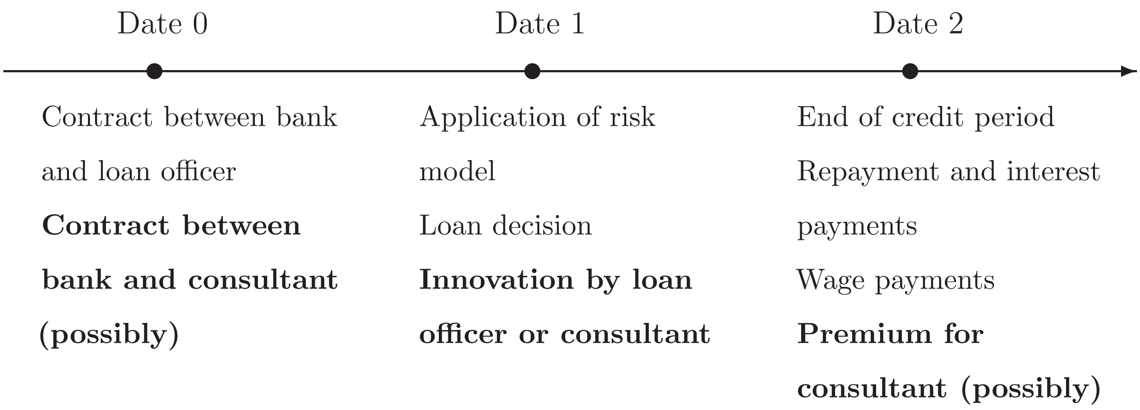
The risk model improvement is a further agency problem, as the bank owner cannot directly observe the innovation process. In order to get the costly innovation done, he has to incentivize the loan officer. The bank owner may observe a modification of the risk model. However, as he is not working on a daily basis with the risk model, he cannot notice if the modification is indeed a change for the better.

We assume now that the bank owner has two possibilities to get the innovation done: he can engage either an internal employee, i. e. the loan officer, or an external innovator, i. e. a risk manager or consultant. In a managerial examination, we analyze whether the bank will innovate with the loan officer, with an external consultant or not at all.

As can be seen in Figure 3.3, the time structure remains similar and is extended by a possible mandate for an external consultant.

The improvement of the beta-error reduces β by the multiplier ψ, with $0 < \psi < 1$. Therefore, a high ψ stands for a small extent of innovation and vice versa. Each innovation or improvement process costs some one-time effort costs e independent of the person who innovates.

Figure 3.3: Time Structure with Innovation (Beta-Error)



After the innovation is done and with the assumption $\alpha = 0$, probabilities in the risk model (labeled as p^{β}) are changed to

$$\begin{aligned} p^{\beta}_{r,b} &= (1-\gamma)\,(1-\psi\,\beta) \\ p^{\beta}_{a,b} &= (1-\gamma)\,\psi\,\beta \\ p^{\beta}_{r} &= p_{r,g} + p^{\beta}_{r,b} = 0 + (1-\gamma)\,(1-\psi\,\beta) \\ p^{\beta}_{a} &= p_{a,g} + p^{\beta}_{a,b} = \gamma + (1-\gamma)\,\psi\,\beta. \end{aligned}$$

Innovation by the loan officer. If the loan officer is responsible for the implementation of the innovation, the structures of the bank owner's profit and the loan officer's wage are similar as before. The bank owner has to anticipate the possible improvement of the beta-error and the effort costs of the loan officer when incentivizing the agents.

The changes result in amended profits for the bank and the loan officer. Probabilities are adjusted to the new models and the loan officer has to take additional effort costs e for the innovation into account.

$$E\Pi L^{\beta} = p^{\beta}_{r}\,a + p_{a,g}\,b - c - e \tag{3.7}$$

$$E\Pi B^{\beta} = p_{a,g}\,(1+r)\,I - p_a\,I - (p^{\beta}_{r}\,a + p_{a,g}\,b). \tag{3.8}$$

In order to put new incentive constraints, all possible actions of the loan officer have to be considered: the loan officer can (a) exert effort for credit check and innovation, (b) exert effort for credit check but not for innovation, (c) do the innovation but shirk regarding the credit check or (d) omit the credit check as well as the innovation. In omitting the credit check, the loan officer can either simply accept all loan requests or reject all credit applications.

It can easily be seen that (c) never is optimal for the loan officer as (c) is dominated by (d). It makes no sense for the loan officer to costly improve the risk model if he will not apply it and shirk in the next step. The incentive constraints need to ensure that (a) is preferred over (b) and (d). As (d) can be split in (d1) (acceptance of all loans) and in (d2) (rejection of all loans), there are three constraints:

$$E\Pi L^{\beta} \geq E\Pi L \tag{3.9}$$

$$E\Pi L^{\beta} \geq a \tag{3.10}$$

$$E\Pi L^{\beta} \geq \gamma\, b \tag{3.11}$$

With two wage variables (a and b), only two of the three constraints simultaneously bind. Therefore, two possible contracts between loan officer and bank emerge, contract 1 and contract 2 (see appendix). The proof of the following Lemma is in the appendix in section 3.5.

Lemma 3.2 *If* $e \leq \frac{\beta\,(1-\psi)\,c}{1-\beta}$, *contract 1 is used and wage parameters are*

$$a = \frac{c+e}{(1-\psi\,\beta)\,(1-\gamma)} \quad and \tag{3.12}$$

$$b = \frac{c+e}{(1-\psi\,\beta)\,\gamma\,(1-\gamma)}. \tag{3.13}$$

If $e \geq \frac{\beta\,(1-\psi)\,c}{1-\beta}$, *contract 2 is used and wage parameters are*

$$a = \frac{e}{(1-\psi)\,\beta\,(1-\gamma)} \quad and \tag{3.14}$$

$$b = \frac{c\,(\beta\,(1-\psi)\,(1-\gamma) + e\,(\gamma + \beta\,(1-\gamma))}{(1-\psi)\,\beta\,\gamma\,(1-\gamma)}. \tag{3.15}$$

As in Section (3.2), we are able to calculate profit functions by using the results for the wage parameters in equilibrium. The profits are labeled with $i1$ respectively $i2$ as the beta-error is performed by the $i$nternal agent under contract 1 or 2.

If $e \leq \frac{\beta\,(1-\psi)\,c}{1-\beta}$ (contract 1), profits of the bank owner and the loan officer are

$$E\Pi B^{\beta,i1} = (r\,\gamma - (1-\gamma)\,\psi\,\beta)\,I - e - c - \frac{c+e}{(1-\gamma)\,(1-\psi\,\beta)}$$
$$E\Pi L^{\beta,i1} = \frac{c+e}{(1-\gamma)(1-\psi\,\beta)}.$$

If $e \geq \frac{\beta\,(1-\psi)\,c}{1-\beta}$ (contract 2), profits of the bank owner and the loan officer are

$$E\Pi B^{\beta,i2} = (r\,\gamma - (1-\gamma)\,\psi\,\beta)\,I - e - c - \frac{e}{(1-\gamma)\,(1-\psi)\,\beta}$$
$$E\Pi L^{\beta,i2} = \frac{e}{(1-\gamma)(1-\psi)\,\beta}.$$

It can be seen that the profit of the bank with both contracts differ only in the last term which is equivalent to the respective profit of the loan officer. What the bank owner earns from disbursed loans remains the same with both contracts. However, the wage he has to pay is different because of different incentives he has to account for. In both cases, the loan officer's wage is depending on effort costs, the extent of the innovation, and the quality of the market and the risk model. Also in both cases, the loan officer invests credit check costs. However, these costs do not appear directly in the wage function of contract 2, as innovation costs e are above a certain level.

Innovation by the external consultant. The bank owner may want to engage an external consultant to reduce the beta-error. If this is the case and the innovation is done by the consultant, the structures of the bank owner's profit and the loan officer's wage are similar as before. The loan officer is incentivized to using the improved risk model instead of shirking and the bank owner now has to pay a premium to the consultant.

After the bank owner engaged the consultant, he will adjust the wage of the loan officer to the new situation of the improved risk model. If the wage would not be adjusted, the bank owner would waive some additional profit, he otherwise could get for himself. The loan officer must be incentivized only to do the credit check, not the innovation. The wage therefore is similar to the one of the basic model as in Lemma 3.1, adjusted to the new situation of an improved risk model. Loan parameters from (3.5) and (3.6) are changed to

$$a = \frac{c}{(1-\psi\,\beta)\,(1-\gamma)} \quad \text{and} \tag{3.16}$$

$$b = \frac{c}{(1-\psi\,\beta)\,\gamma\,(1-\gamma)}. \tag{3.17}$$

by inserting $\alpha = 0$ and using the new risk model.

The external consultant is compensated by a premium that increases with the reduction of the beta-error. Hence, he receives a premium A for all rejected projects. With this compensation, the external consultant will be incentivized to exert effort for the innovation. His incentive constraint is the following:

$$p_r^\beta\, A - e \geq p_r\, A. \tag{3.18}$$

As the bank owner is only willing to pay the lowest possible amount to the external consultant, the above condition (3.18) is binding, which results in the premium

$$A = \frac{e}{\beta\,(1-\psi)\,(1-\gamma)}. \tag{3.19}$$

With these wage components, expected profits (indexed with e as the innovation is done by the $e$xternal consultant) of bank owner, loan officer and external consultant are

$$E\Pi B^{\beta,e} = [r\,\gamma - (1-\gamma)\,\psi\,\beta]I - e - c - \frac{1-\beta}{(1-\psi)\,\beta}\,e - \frac{c}{(1-\gamma)\,(1-\psi\,\beta)}$$

$$E\Pi L^{\beta,e} = \frac{c}{(1-\gamma)\,(1-\psi\,\beta)}$$

$$E\Pi C^{\beta,e} = \frac{1-\beta}{(1-\psi)\,\beta}\,e.$$

As in $E\Pi B^{\beta,i1}$ and $E\Pi B^{\beta,i2}$, the bank owner again earns the same amount from allowed credits. This amount is reduced by the last four terms as he has to compensate loan officer and external consultant for credit check costs c and for the effort e of the innovation.

3.3.2 Discussion of Factors Influencing the Innovation Decision

So far, two different possibilities to perform the innovation are defined. The bank owner may assign the improvement process either internally to the loan officer or externally to a consultant. The bank owner maximizes his income, and therefore chooses the option that leads to the highest profit.

Hence, the innovation decision of the bank is depending on different factors of the risk model and the market.

Proposition 3.1 *If γ is above a certain level, the bank owner engages the external consultant to perform the innovation. Otherwise, the loan officer will do the improvement of the beta-error.*

With contract 1 ($e \leq \frac{\beta\,(1-\psi)\,c}{1-\beta}$), $E\Pi B^{\beta,e} > E\Pi B^{\beta,i1}$, if and only if

$$\gamma > \frac{1 - 2\,\beta + \beta^2\,\psi}{(1-\psi\,\beta)\,(1-\beta)}. \tag{3.20}$$

With contract 2 ($e \geq \frac{\beta\,(1-\psi)\,c}{1-\beta}$), $E\Pi B^{\beta,e} > E\Pi B^{\beta,i2}$, if and only if

$$\gamma > \frac{\beta\,[(1-\psi)\,c - (1-\psi\,\beta)\,e]}{(1-\psi\,\beta)\,(1-\beta)\,e}. \tag{3.21}$$

A high γ means a high market quality. Many good projects are available which can be taken as a sign for a boom situation in the economy. In this case, the bank owner earns higher profits, if he does not need to incentivize the loan officer for both – credit check and innovation –, but is able to differentiate between the two tasks. Obviously, the bank owner will decide for the alternative that promises higher profits.

There is a robust intuition for this result. The wage of the loan officer is composed of the bonus b and the administration fee a. In case of a high γ, the bonus accounts for a large share of the total wage, and it is more difficult to incentivize the loan officer to do an additional effort for the innovation as the incentive for the improvement of the beta-error will be achieved mainly by the administration fee.

It can also be seen that the wage of the loan officer (in all cases, i. e. $E\Pi L^{\beta,i1}$, $E\Pi L^{\beta,i2}$ and $E\Pi L^{\beta,e}$) is increasing with γ whereas the consultant's profit ($E\Pi C^{\beta,e}$) is independent of γ. The marginal increase of the loan officer's wage is higher in the case of the innovation by the loan officer than in the case of the innovation by the consultant. Thus, at a certain level, it is more favorable to engage the external consultant as it becomes more expensive to incentivize the loan officer.

Proposition 3.2 *If ψ is below a certain level, the bank owner engages the external consultant to perform the innovation. Otherwise, the loan officer will do the improvement of the beta-error.*

With contract 1 ($e \leq \frac{\beta\,(1-\psi)\,c}{1-\beta}$), $E\Pi B^{\beta,e} > E\Pi B^{\beta,i1}$, if and only if

$$\psi < \frac{\beta - (1-\beta)\,(1-\gamma)}{\beta\,[1-(1-\beta)\,(1-\gamma)]}. \tag{3.22}$$

If the bank owner wants to have a small effect of the innovation – that means he orders a low improvement of the risk model (high level of ψ), he prefers to engage the loan officer with contract 1. With a small reduction of the beta-error, it is easier to incentivize the loan officer as the loan officer benefits from a better risk model himself in reaching better results of the also enhanced credit check. Only for a high extent of the innovation, it is better for the bank owner to differentiate between the two tasks of innovation and credit check. With contract 2, the decision dependent on ψ is not unambiguous as there is a second effect of higher wages for the loan officer with an improved risk model. For a major innovation, there is no general result which effect outweighs the other.

3.3.3 Decision of the Bank and Welfare Analysis

The bank owner not only compares the two different possibilities for innovation but also takes into account to have no improvement of the risk model. Depending on the different parameters responsible for market quality, risk model quality, credit check costs and effort, he may decide to not innovate at all. We are able to find examples of parameters for each possible decision of the bank owner: no improvement (as in the *b*asic *m*odel, labeled as BM), innovation by internal loan officer and innovation by the external consultant. After a general welfare analysis, this section shows the concrete decision of the bank and the respective welfare optimal decision for specific parameter constellations of the risk model.

In a welfare analysis, we now answer the question if the decision of the bank is pareto optimal for the economy. To analyze the welfare implications, we define welfare as the sum of the rents of the bank owner, loan officer, external consultant and in addition the rents of all projects that are accepted for loans. Let us assume that good projects have a return of Y and consequently, that the rent of borrowers consists of $Y - (1 + r)\, I$. Only borrowers who are successful in their projects (i. e. borrowers with probability $p_{a,g}$) receive the return and are able to pay interest rates. Other borrowers earn 0 and their loans default or they did not receive any loan. Thus, welfare is

$$W = E\Pi B + E\Pi L + E\Pi C + p_{a,g}\,[Y - (1 + r)\, I]. \tag{3.23}$$

By inserting the profit equations shown above, we find two different results for welfare: one of the basic model without an innovation and one of the model with improvement of the beta-error. Obviously, there is no difference in the welfare whether loan officer or external consultant are contracted to do the innovation. This leads only to different allocations of rents; the sum of rents remains the same though. Thus, we focus on these two cases: without innovation (W^{BM}) and with innovation

(W^{β}). Results for welfare functions are

$$W^{BM} = \gamma\,(Y - I) - \beta\,(1-\gamma)\,I - c \text{ and} \tag{3.24}$$

$$W^{\beta} = \gamma\,(Y - I) - \psi\,\beta\,(1-\gamma)\,I - c - e. \tag{3.25}$$

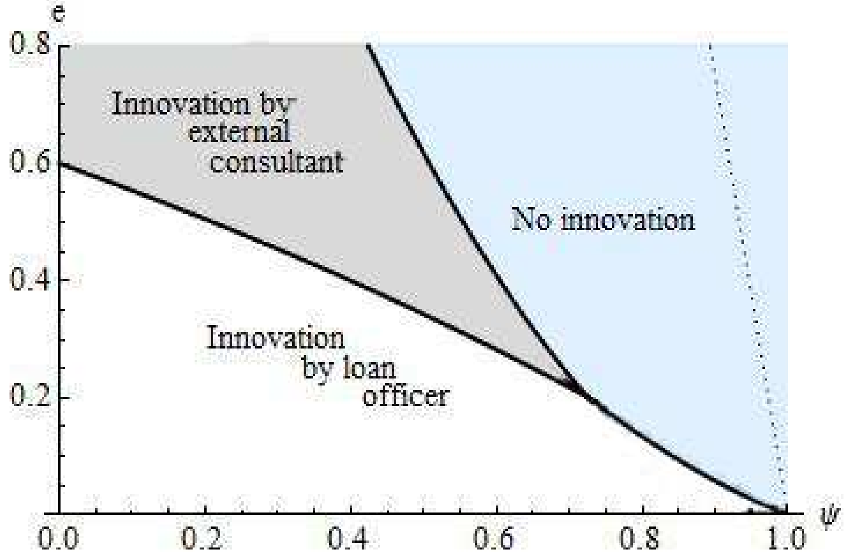
It can be seen that it is better from a welfare point of view not to exert effort for the innovation if $e > (1-\psi)\,\beta\,(1-\gamma)\,I$. Intuitively, the bank should refrain from improving the risk model if effort costs for this task are very high. The same decision should be taken if the market quality γ is very high, as incentivizing loan officer or external consultant will become more expensive. If the risk model has already a high quality, i. e. β is low, the costs for a further improvement of the risk model will be expensive and the probability for a welfare optimal reduction of the beta-error is decreasing.

Example Will the bank always incentivize agents for an improvement of the risk model if it is welfare optimal? And will the bank omit the innovation whenever $W^{BM} > W^{\beta}$? To answer these questions, the profits of the bank under the different possibilities have to be compared.

In Figure 3.4, the decisions of the bank under parameters $\gamma = 1/2$, $\beta = 1/4$, $c = 3/2$, $r = 7/8$, and $I = 60$ can be seen.

If effort costs are not too high, the bank owner will assign the task of innovation to the internal loan officer (white region). With increasing effort costs it is better for the bank owner to entrust the external consultant with the improvement of the risk model (gray region). In addition, we can see that the higher the effort costs (and the higher ψ) the higher is also the probability for the bank owner to decide for no innovation at all (blue region). This is very intuitive as high effort costs make it expensive for the bank owner to remunerate an agent for the innovation task. The dotted line marks the welfare result. To the left of the dotted line, an innovation is welfare optimal; to the right of it, no innovation should be done.

Figure 3.4: Decision of the Bank



Considering the x-Axis, it can be seen that, with a lower level of ψ, the external consultant will perform the innovation. In contrast, with a higher level of ψ, the risk model remains in its original condition. Therefore, a planned innovation will only be realized, if it is of a larger extent.

Intuitively, we can see that easy bits are done within the bank whereas the last bits are done by an external agent. In the second case, the bank benefits from the possibility of diversification between the two tasks of credit check and innovation process.

On the upper right side, we see a large area in which the bank decides for no innovation. However, only in a small part of it – the part on the right of the dotted line, it is also welfare optimal to do so. Because of externalities, the bank bears the costs of asymmetric information and chooses a policy that is not optimal for the economy. Hence, we find that under certain conditions, the bank does not put enough effort in the improvement of risk models. A possible measure to solve this problem would be regulation concerning risk models. The regulatory authority could set standards that must be kept by banks. Another idea is to conduct periodic checks or to call for regular reports about measures taken in order to improve risk models.

As another interesting fact we find that it is beneficial for the economy to have the possibility to engage external consultants. If the innovation could be done only by the loan officer, the 'no innovation' regime would be even larger under conditions where it would be welfare optimal to have the improvement process for the risk model. Therefore, it is optimal to let the bank choose about who should be engaged for the tasks.

3.4 Conclusion

This paper proposes a framework to study the relation between organizational structures and model risk and the incentives for an improvement of a risk model. We have constructed a microeconomic model of a bank in which incentives for innovation play a crucial role. The bank uses a risk model as credit test that is assumed to be imperfect. As no accidental innovation exists, the bank owner has to incentivize an agent to improve the inaccurate risk model. If it is too difficult or expensive to incentivize an internal agent, the bank owner also has the possibility to engage an external consultant for the innovation. He will make use of this possibility if there are many good projects available, effort costs for the innovation are high or if the planned innovation is of a smaller extent. In short, the external agent is preferred from the bank owner in extreme cases.

The paper also provides a reason for ongoing discussions about consultancy and its high costs. It arguments that outsourcing tasks may be reasonable. In some cases that are discussed in this article, the bank owner benefits from the chance to differentiate between incentives for diverse tasks. Also from a welfare perspective, we find that under certain conditions external agents should be preferred over an internal solution as otherwise too less incentives for innovation may exist.

In special cases, we find that banks undertake too less effort to incentivize both, internal or external service workers for an improvement of the bank's risk model. This is a clear argument for regulation. Regulatory standards for risk models could drive banks to innovate even under conditions that would not lead banks to decide for innovations on their own.

The Federal Reserve System and the Office of the Comptroller of the Currency have already shown their concern on this topic (OCC, 2011). They have given practical advisory to deal with model risk and to balance internal and external impact on the development and improvement of risk models used in banks. Another discussion on valuation requirements is driven by the European Banking Authority also requesting banks to consider model risk by making additional valuation adjustments (EBA, 2012).

3.5 Appendix

Proof of Lemma 3.1 Incentive constraints (3.3) and (3.4) are binding and can be rewritten as

$$(1-\beta)\,a - c + (1-\alpha)\,\gamma\,b - (1-\alpha-\beta)\gamma\,a - c = a \;\text{ and}$$
$$(1-\beta)\,a - c + (1-\alpha)\,\gamma\,b - (1-\alpha-\beta)\gamma\,a - c = \gamma\,b.$$

Solving these two equations for a and b leads to the result of Lemma 3.1, i. e. the wage parameters of (3.5) and (3.6).

Proof of Lemma 3.2 *For contract 1*: If incentive constraints (3.10) and (3.11) are binding, the constraint (3.9) is fulfilled if and only if $e \leq \frac{\beta\,(1-\psi)\,c}{1-\beta}$. Parameters a and b can be calculated by solving (3.10) and (3.11) for a and b.

Incentive constraints (3.10) and (3.11) can be rewritten as:

$$(1-\gamma)\,(1-\psi\,\beta)\,a+\gamma\,b-c-e\geq a \text{ and}$$
$$(1-\gamma)\,(1-\psi\,\beta)\,a+\gamma\,b-c-e\geq\gamma\,b.$$

If those constraints are binding, they can be solved for

$$a=\frac{c+e}{(1-\psi\,\beta)\,(1-\gamma)} \text{ and}$$
$$b=\frac{c+e}{(1-\psi\,\beta)\,\gamma\,(1-\gamma)}$$

which are equations (3.12) and (3.13) from Lemma 3.2. If we now insert (3.12) and (3.13) in the incentive condition (3.9), this condition can be simplified to

$$e\leq\frac{\beta\,(1-\psi)\,c}{1-\beta}.$$

For contract 2: If incentive constraints (3.9) and (3.10) are binding, the constraint (3.11) is fulfilled if and only if $e\geq\frac{\beta\,(1-\psi)\,c}{1-\beta}$. Parameters a and b can be calculated by solving (3.9) and (3.10) for a and b.

Incentive constraints (3.9) and (3.10) can be rewritten as:

$$(1-\gamma)\,(1-\psi\,\beta)\,a+\gamma\,b-c-e\geq(1-\gamma)\,(1-\beta)\,a+\gamma\,b-c \text{ and}$$
$$(1-\gamma)\,(1-\psi\,\beta)\,a+\gamma\,b-c-e\geq a.$$

If those constraints are binding, they can be solved for

$$a=\frac{e}{(1-\psi)\,\beta\,(1-\gamma)} \text{ and}$$
$$b=\frac{c\,(\beta\,(1-\psi)\,(1-\gamma)+e\,(\gamma+\beta\,(1-\gamma))}{(1-\psi)\,\beta\,\gamma\,(1-\gamma)}.$$

which are equations (3.14) and (3.15) from Lemma 3.2. If we now insert (3.14) and (3.15) in the incentive condition (3.11), this condition can be simplified to

$$e\geq\frac{\beta\,(1-\psi)\,c}{1-\beta}.$$

A third possible contract does not exist as binding incentive constraints (3.9) and (3.11) do not have a solution for a and b.

Bibliography

Aghion, P., P. Bolton, and J. Tirole (2004): "Exit Options in Corporate Finance: Liquidity vs. Incentives," *Review of Finance*, 8(3), 327–353.

Aghion, P., and J. Tirole (1994): "The Management of Innovation," *The Quarterly Journal of Economics*, 109(4), 1185–1209.

——— (1997): "Formal and Real Authority in Organizations," *Journal of Political Economy*, 105(1), 1–29.

Agrawal, A., C. Catalini, and A. Goldfarb (2011): "The Geography of Crowdfunding," Working Paper, NBER 16820.

——— (2013): "Some Simple Economics of Crowdfunding," Working Paper, NBER 19133.

Ahlers, G. K., D. Cumming, C. Günther, and D. Schweizer (2013): "Signaling in Equity Crowdfunding," Working Paper.

Alexander, C. (2005): "The Present and Future of Financial Risk Management," *Journal of Financial Econometrics*, 3(1), 3–25.

Archak, N., and A. Sundararajan (2009): "Optimal Design of Crowdsourcing Contests," *ICIS Proceedings*.

Barro, J. R., and R. J. Barro (1990): "Pay, Performance, and Turnover of Bank CEOs," *Journal of Labor Economics*, 8(4), 448–481.

Belleflamme, P., T. Lambert, and A. Schwienbacher (2013): "Individual Crowdfunding Practices," *Venture Capital: An International Journal of Entrepreneurial Finance*, 15(4), 313–333.

——— (2014): "Crowdfunding: Tapping the Right Crowd," *Journal of Business Venturing*, 29(5), 585–609.

Benmelech, E., E. Kandel, and P. Veronesi (2010): "Stock-Based Compensation and CEO (Dis)Incentives," *Quarterly Journal of Economics*, 125(4), 1769–1820.

Biais, B., T. Mariotti, G. Plantin, and J.-C. Rochet (2007): "Dynamic Security Design: Convergence to Continuous Time and Asset Pricing Implications," *Review of Economic Studies*, 47(4), 345–390.

Biais, B., T. Mariotti, J.-C. Rochet, and S. Villeneuve (2010): "Large Risks, Limited Liability, and Dynamic Moral Hazard," *Econometrica*, 78(1), 73–118.

Bikhchandani, S., D. Hirshleifer, and I. Welch (1998): "Learning from the Behavior of Others: Conformity, Fads, and Informational Cascades," *Journal of Economic Perspectives*, 12(3), 151–170.

Broecker, T. (1990): "Credit-Worthiness Tests and Interbank Competition," *Econometrica*, 58(2), 429–452.

Carrillo, J. D., and T. Mariotti (2000): "Strategic Ignorance as a Self-Disciplining Device," *The Review of Economic Studies*, 67(3), 529–544.

Chamley, C. P. (2004): *Rational Herds*. Cambridge University Press.

Chawla, S., J. D. Hartline, and B. Sivan (2012): "Optimal Crowdsourcing Contests," *Proceedings of the twenty-third annual ACM-SIAM symposium on Discrete Algorithms*, pp. 856–868.

Colliard, J.-E. (2012): "Rational Blinders: Is it Possible to Regulate Banks Using Their Internal Risk Models?," Working Paper, SSRN 2170459.

Corwin, S. A., and P. Schultz (2005): "The Role of IPO Underwriting Syndicates: Pricing, Information Production, and Underwriter Competition," *Journal of Finance*, 60(1), 443–486.

Cross, M. B. (2011): "Testimony on Crowdfunding and Capital Formation," U.S. Securities and Exchange Commission, available online at https://www.sec.gov/news/testimony/2011/ts091511mbc.htm.

Crouhy, M., D. Galai, and R. Mark (2000): "A Comparative Analysis of Current Credit Risk Models," *Journal of Banking and Finance*, 24(1-2), 59–117.

Danelsson, J. (2008): "Blame the Models," *Journal of Financial Stability*, 4(4), 321–328.

DeMarzo, P., and Y. Sannikov (2006): "Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model," *Journal of Finance*, 61(6), 2681–2724.

DeMarzo, P. M., and M. J. Fishman (2007): "Agency and Optimal Investment Dynamics," *The Review of Financial Studies*, 20(1), 151–188.

Derman, E. (1996): "Model Risk," *Quantitative Strategies Research Notes*, Goldman Sachs.

Diamond, D. W. (1984): "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies*, 51(3), 393–414.

DiPalantino, D., and M. Vojnovic (2009): "Crowdsourcing and All-pay Auctions," *Proceedings of the 10th ACM conference on Electronic Commerce*, pp. 119–128.

Dyck, A., A. Morse, and L. Zingales (2010): "Who Blows the Whistle on Corporate Fraud?," *The Journal of Finance*, 65(6), 2213–2253.

EBA (2012): "Discussion Paper on Draft Regulatory Technical Standards on Prudent Valuation, under Article 100 of the Draft Capital Requirements Regulation (CRR)," Discussion Paper, European Banking Authority.

Eisfeldt, A. L., and A. A. Rampini (2008): "Managerial Incentives, Capital Reallocation, and the Business Cycle," *Journal of Financial Economics*, 87, 177–199.

Fahlenbrach, R., and R. M. Stulz (2011): "Bank CEO Incentives and the Credit Crisis," *Journal of Financial Economics*, 99(1), 11–26.

Figlewski, S. (1998): "Derivatives Risks, Old and New," NYU Working Paper FIN-98-033, New York University.

Griffin, Z. (2012): "Crowdfunding: Fleecing the American Masses," *Case Western Reserve Journal of Law, Technology & the Internet*, forthcoming.

Hauswald, R., and R. Marquez (2003): "Information Technology and Financial Services Competition," *The Review of Financial Studies*, 16(3), 921–948.

——— (2006): "Competition and Strategic Information Acquisition in Credit Markets," *The Review of Financial Studies*, 19(3), 967–1000.

Hellmann, T., and V. Thiele (2011): "Incentives and Innovation: A Multitasking Approach," *American Economic Journal: Microeconomics*, 3(1), 78–128.

Hemer, J. (2011): "A Snapshot on Crowdfunding," Fraunhofer ISI, Working Paper No. R2/2011.

Hildebrand, T., M. Puri, and J. Rocholl (2013): "Adverse Incentives in Crowdfunding," Working Paper.

John, K., H. Mehran, and Y. Qian (2010): "Outside Monitoring and CEO Compensation in the Banking Industry," *Journal of Corporate Finance*, 16(4), 483–399.

Kanodia, C., R. Bushman, and J. Dickhaut (1989): "Escalation Errors and the Sunk Cost Effect: An Explanation Based on Reputation and Information Asymmetries," *Journal of Accounting Research*, 27(1), 59–77.

Kasriel, P. L. (2004): "Collateral Damage from a U.S. Housing Bust," *Positive Economic Commentary, The Northern Trust Company*, July, 30.

Kessler, A. S. (1998): "The Value of Ignorance," *The Rand Journal of Economics*, 29(2), 339–354.

Kremer, I., Y. Mansour, and M. Perry (2013): "Implementing the 'Wisdom of the Crowd'," The Warwick Economics Research Paper Series (TWERPS).

Levitt, S. D., and C. M. Snyder (1997): "Is no News Bad News? Information Transmission and the Role of "Early Warning" in the Principal-Agent Model," *The RAND Journal of Economics*, 28(4), 641–661.

Lóránth, G., and A. Morrison (2009): "Internal Reporting Systems, Compensation Contracts, and Bank Regulation," EFA Bergen Meetings Paper.

Louis, P. (2011): "Standing in Line: Demand for Investment Opportunities with Exogenous Priorities," Working Paper.

Manso, G. (2011): "Motivating Innovation," *The Journal of Finance*, 66(5), 1326–1860.

Mariathasan, M., and O. Merrouche (2014): "The Manipulation of Basel Risk-Weights," *Journal of Financial Intermediation*, 23(3), 300–321.

OCC (2011): "OCC Bulletin 2011-12: Supervisory Guidance on Model Risk Management," Discussion Paper, Office of the Comptroller of the Currency and Board of Governors of the Federal Reserve System.

Pagès, H. (2013): "Bank Monitoring Incentives and Optimal ABS," *Journal of Financial Intermediation*, 22(1), 30–54.

Pagès, H., and D. Possamai (2012): "A Mathematical Treatment of Bank Monitoring Incentive," *Banque de France Document de Travail No. 378*.

Povel, P. (1999): "Optimal 'Soft' or 'Tough' Bankcruptcy Procedures," *Journal of Law, Economics, and Organization*, 15(3), 659–684.

Povel, P., R. Singh, and A. Winton (2007): "Booms, Busts, and Fraud," *Review of Financial Studies*, 20(4), 1219–1254.

Quadrini, V. (2004): "Investment and Liquidation in Renegotiation-proof Contracts with Moral Hazard," *Journal of Monetary Economics*, 51(4), 713–751.

Rajan, R. G. (2006): "Has Finance Made the World Riskier?," *European Financial Management*, 12(4), 499–533.

Rebonato, R. (2002): "Theory and Practice of Model Risk Management," Working Paper, QuaRC (Quantitative Research Center), available at www.quarchome.org.

Rock, K. (1986): "Why New Issues are Underpriced," *Journal of Financial Economics*, 15(1-2), 187–212.

Sannikov, Y. (2007): "Games with Imperfectly Observable Actions in Continuous Time," *Econometrica*, 75(5), 1285–1329.

——— (2008): "A Continuous-Time Version of the Principal-Agent Problem," *Review of Economic Studies*, 75(3), 957–984.

Schwienbacher, A., and B. Larralde (2012): "Crowdfunding of Small Entrepreneurial Ventures," in *Handbook of Entrepreneurial Finance*, ed. by D. Cumming. Oxford University Press.

Shiller, R. J. (2005): *Irrational Exuberance*. University Presses of Ca, 2 edn.

Sibbertsen, P., G. Stahl, and C. Luedtke (2008): "Measuring Model Risk," *Journal of Risk Model Validation*, 2(4), 65–81.

TIROLE, J. (2006): *The Theory of Corporate Finance*. Princeton University Press.

VAN BOMMEL, J. (2002): "Messages from Market to Management: The Case of IPOs," *Journal of Corporate Finance*, 8, 123–138.

VERRECCHIA, R. E. (2001): "Essays on Disclosure," *Journal of Accounting and Economics*, 32(1-3), 97–180.

WELCH, I. (1989): "Seasoned Offerings, Imitation Costs, and the Underpricing of Initial Public Offerings," *The Journal of Finance*, 44(2), 421–449.