Heterogeneity in Household Portfolios and its Implication for Fiscal and Monetary Policy

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Introduction

Modern macroeconomic research originated in the 1970's in what became known as the "rational expectations revolution" (Lucas, 1972; Sargent and Wallace, 1976). The introduction of expectations that are consistent with the predictions of the model challenged the conventional debate on macroeconomic stabilization. Up until then, the government has either been considered to play an important part in stabilizing business cycles (Keynes and followers) or, opposed to this view, to amplify them (Friedman and followers). Challenging the prevailing debates, Sargent and Wallace put forward the policy ineffectiveness proposition: When economic agents do not make systematic errors in their forecasts, central banks cannot systematically surprise agents and, thus, can neither *systematically* amplify nor stabilize the business cycle. In this view, central banks should only focus on inflation because they have no systematic effect on output (Barro and Gordon, 1983). At the same time, Barro (1974) formalized the idea that economic agents internalize the government budget constraint, and government paper therefore does not constitute net worth to households. Barro's result, known as the Ricardian equivalence theorem, severely limits the role of fiscal policy in macroeconomic stabilization.

The "rational expectations revolution" also brought about a change in how the economy was modeled. In the new models aggregate relations were the result of optimal decision making by economic agents, and the economy as a whole was cast as a dynamic stochastic equilibrium. Since then, macroeconomic research has stayed faithful to this methodology but has qualified, and continues to qualify, what macroeconomic stabilization can achieve. The main thrust of this literature has been to study monetary policy under the assumption of sticky prices while leaving aside government financing decisions since they are irrelevant for the allocation under Ricardian equivalence (see, e.g., Woodford, 2001). The Ricardian equivalence theorem applies if households are linked by inter-generational altruism, taxes are lump-sum, and financial markets are perfect.

This thesis follows the literature in assuming sticky prices, but relaxes the assumption of complete financial markets in the analysis of business cycles. It builds on the so-called "incomplete markets" literature that investigates household consumption and savings behavior in the absence of full insurance. Early contributions by Bewley (1979), Huggett (1993) and Aiyagari (1994) describe the steady state properties of an economy in which households only have access to a non-contingent risk-free asset to smooth consumption in the face of idiosyncratic income risk. The major challenge in analyzing this class of models is that aggregates, including prices, depend on the entire wealth distribution. The extension by Krusell and Smith (1997, 1998) overcomes this problem by approximating the wealth distribution with a finite number of its moments and, thus, allows to compute dynamic equilibria with aggregate and idiosyncratic risk.

Equilibria in this environment are generally not Pareto optimal because real interest rates are lower than the rate of time preference, and the government may provide additional savings vehicles at low cost. Aiyagari and McGrattan (1998) quantify this trade-off in a model with capital and public debt, in which the latter has to be financed by distortionary taxation. They find a positive amount of public debt to be optimal, because, although a higher interest rate crowds out capital, it effectively loosens households' borrowing constraints and thereby enhances households' consumption-smoothing capacity. Consequently, when financial markets are incomplete, the method of government financing affects households' consumption and savings decision. The failure of Ricardian equivalence breaks with the modern tradition of business cycle analysis and puts household balance sheets at the center stage of analysis.

The economic intuition that ensues from this resembles earlier work by Tobin (1969) and Brunner and Meltzer (1972) that predates the "rational expectations revolution." Tobin thought that asset disaggregation is essential for analyzing monetary policy and financing of government deficits. In his view, government paper in particular is different from other assets because its nominal return is not determined by market forces – money has a fixed nominal return of zero and the central bank determines the nominal return on short-run treasuries. Brunner and Meltzer also understood macroeconomic imbalances through the lens of asset markets. According to this literature, equilibrium in asset markets determines the short-run response of the economy, while prices in the goods market respond with a lag. My thesis shares this view and provides a microfounded business cycle model in the rational expectations tradition that incorporates heterogeneity in household portfolios.

As such, my thesis contributes to assessing the importance of market incompleteness for business cycle dynamics and macroeconomic policy. The first chapter presents the result of joint research with Christian Bayer, Volker Tjaden, and Lien Pham-Dao, and contributes to the literature on uncertainty driven business cycles. The second chapter contributes to the literature on the transmission mechanism of monetary policy. The third chapter presents the result of joint research with Christian Bayer and Lien Pham-Dao, and contributes to the literature on fiscal policy.

CHAPTER 1. Shocks to household income are persistent and their variance changes substantially over the business cycle. Fluctuations in this risk might be an important driver of business cycles in so far as they give rise to fluctuations in precautionary savings and, thus, consumption. The joint work "*Precautionary Savings, Illiquid Assets, and* *the Aggregate Consequences of Shocks to Household Income Risk*" (Bayer et al., 2015) assesses the importance of this channel. Toward this end, we build a New Keynesian dynamic stochastic general equilibrium (DSGE) model with asset-market incompleteness, idiosyncratic income risk, and sticky prices. The novel feature of the model is to allow for portfolio choice between liquid and illiquid assets in a business-cycle framework.

In a first step, we estimate a stochastic volatility process for earnings that serves as input for the DSGE model. Households are subject to idiosyncratic productivity risk that varies according to our estimated process. We find that fluctuations in income risk generate sizable swings in precautionary savings that have significant aggregate consequences. Households respond to higher income risk by increasing their savings, but importantly do so by hording liquid paper assets. Households that participate in the capital market sell the illiquid asset because the liquid asset is superior for short-term consumption smoothing. Consequently, consumption and investment fall, which leads to sizable output losses in a demand-determined economy. The central bank can prevent this from happening by satisfying any excess demand for liquid assets.

CHAPTER 2. Monetary policy, at least in normal times, takes the form of the central bank setting the short-term interest rate and thereby affecting economic activity. The transmission mechanism relies on the effect of the real rate of interest on the optimal allocation of consumption over time. Optimal consumption plans, however, are not necessarily implementable because of borrowing constraints. These apply to a larger fraction of households than previously thought because assets differ in their degree of liquidity.

In the work "*Transmission of Monetary Policy with Heterogeneity in Household Portfolios*" (Luetticke, 2015), I assess the importance of the distribution of liquid and illiquid assets for the transmission of monetary policy in a DSGE model with sticky prices and heterogeneity in household portfolios. My main finding is that the direct effect of changes in the path of the real interest rate on consumption and savings is substantially lower than in a setup with complete markets. Monetary policy instead primarily works through indirect equilibrium effects on income.

The reversal in the importance of direct and indirect effects has important implications for the transmission of monetary policy. Control over the real interest rate is not sufficient to stabilize aggregate activity. The transmission of monetary policy to the real economy also requires a functioning labor market in this economy. Moreover, the weakening of the interest rate channel of monetary policy questions the existing results on optimal monetary policy rules.

CHAPTER 3. Discretionary transfers from the government to households with the aim of stabilizing aggregate demand have become one of the most important policy responses to recessions. These so-called "fiscal stimulus payments" have been used by the U.S. government during the last two recessions in 2001 and 2008/2009. In both periods, the sum total of all payments was about half a percentage point of GDP.

The aggregate effects of such transfers, however, are not well understood. A large empirical literature exploits random variation in the timing of payments to estimate the causal effect on household spending. This literature finds an average propensity to consume of about 1/3. We take up this evidence in "*Fiscal Stimulus Payments and Precautionary Investment*" (Bayer et al., 2016) and ask whether these empirical findings imply that "fiscal stimulus payments" are indeed expansionary?

Simulating this policy in a DSGE model with heterogeneity in household portfolios, we find that transfers do not only work through the disposable income channel but also by positively affecting household liquidity. Transfers increase individual liquidity and debt finance enhances market liquidity. This has consequences when the government retires this debt. Then households shift their savings into the physical asset to retain their enhanced consumption-smoothing capacity. This leads to a prolonged increase in capital and output. We find that this liquidity channel is stronger than wealth effects induced by government fincancing decisions and, thus, makes the aggregate effects of transfers expansionary independent of the mode of financing.

Chapter 1

Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk

Households face large income uncertainty that varies substantially over the business cycle. We examine the macroeconomic consequences of these variations in a model with incomplete markets, liquid and illiquid assets, and a nominal rigidity. Heightened uncertainty depresses aggregate demand as households respond by hoarding liquid "paper" assets for precautionary motives, thereby reducing both illiquid physical investment and consumption demand. This translates into output losses, which a central bank can prevent by providing liquidity. We show that the welfare consequences of uncertainty shocks crucially depend on a household's asset position. Households with little human capital but high illiquid wealth lose the most from an uncertainty shock and gain the most from stabilization policy.

1. Introduction

The Great Recession has brought about a reconsideration of the role of uncertainty in business cycles. Increased uncertainty has been documented and studied in various markets, but uncertainty with respect to household income stands out in its size and importance. Shocks to household income are persistent and their variance changes substantially over the business cycle. The seminal work by Storesletten et al. (2001) estimates that during an average NBER recession, income uncertainty faced by U.S. households, interpreted as income risk – i.e. the variance of persistent income shocks, is more than twice as large as in expansions.

These sizable swings in household income uncertainty lead to variations in the propensity to consume if asset markets are incomplete so that households use precautionary savings to smooth consumption. This paper quantifies the aggregate consequences of this precautionary savings channel of uncertainty shocks by means of a dynamic stochastic general equilibrium model. In this model, households have access to two types of assets to smooth consumption. They can either hold liquid money or invest in illiquid but dividend paying physical capital. This asset structure allows us to disentangle savings and physical investment and obtain aggregate demand fluctuations.[1] To obtain aggregate output effects from these fluctuations, we augment this incomplete markets framework in the tradition of Bewley (1979) by sticky prices à la Calvo (1983).

We model the illiquidity of physical capital by infrequent participation of households in the capital market, such that they can trade capital only from time to time. This can be considered as an approximation to a more complex trading friction as in Kaplan and Violante (2014), who follow the tradition of Baumol (1952) and Tobin (1956) in modeling the portfolio choice between liquid and illiquid assets.

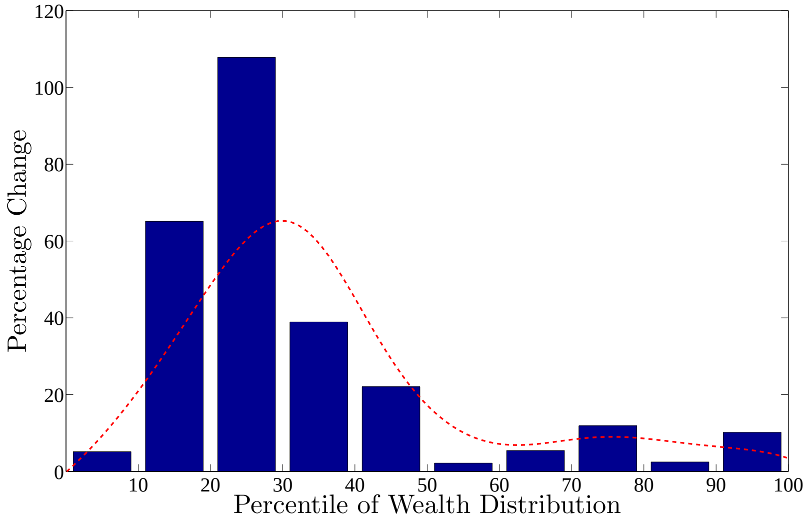
In this economy, when idiosyncratic income uncertainty increases, individually optimal asset holdings rise and consumption demand declines. Importantly, households also rebalance their portfolios toward the liquid asset because it provides better consumption smoothing. These effects are reminiscent of the observed patterns of the share of liquid assets in the portfolios of U.S. households during the Great Recession (see Figure 1.1). According to the 2010 *Survey of Consumer Finances*, the share of liquid assets in the portfolios increased relative to 2004 across all wealth percentiles, with the strongest relative increase for the lower middle-class. In our model, this portfolio rebalancing towards liquid paper reinforces, through a decline in physical investment, the decline in consumption demand caused by higher uncertainty. Consequently, aggregate demand declines even more strongly than consumption and investment and consumption co-move.

Quantitatively, we find the following: a two standard deviation increase in household income uncertainty decreases aggregate activity by roughly 0.5% on impact and 0.4% over the first year under the assumption of a monetary policy that follows a constant nominal money growth rule (Friedman's "k% rule"). This is about half the effect size that Fernández-Villaverde et al. (2015) report for a fiscal policy uncertainty at the zero lower bound. Imposing a Taylor-type rule for monetary policy as estimated in Chowdhury and Schabert (2008), we still find a 0.3% decrease in output upon the uncertainty shock. This is more than twice as large as the effect of fiscal policy uncertainty in "normal" non zero-lower-bound times reported in Fernández-Villaverde et al. (2015). Importantly, in all cases the economy recovers only sluggishly over a five-year horizon in our model.

Since the relative price of capital falls but the value of money increases upon an uncertainty shock, such a shock has not only aggregate but also rich distributional consequences. Our welfare calculations imply that households rich in physical or human capital lose the most, because factor returns fall in times of high uncertainty. In contrast, welfare losses decline in money holdings as their value appreciates. To understand the welfare consequences of systematic policy responses to uncertainty shocks, we compare a regime where monetary policy follows Friedman's k%-rule to one where monetary policy provides additional money to stabilize inflation. Since an uncertainty shock effectively works like a demand shock in our model, monetary policy is able to reduce the negative effects on output and alleviate welfare consequences. On aver-

[1] In a standard Aiyagari (1994) economy, where all savings are in physical capital, an increase in savings does not lead to a fall in total demand (investment plus consumption) because savings increase investments one-for-one.

Figure 1.1: Portfolio share of liquid assets by percentiles of wealth, 2010 vs. 2004



Notes: Portfolio share: Net liquid assets/Net total assets. Net liquid assets: cash, money market, checking, savings and call accounts, as well as government bonds and T-Bills net of credit card debt. Cash holdings are estimated by making use of the *Survey of Consumer Payment Choice* for 2008, as in Kaplan and Violante (2014). Households with negative net liquid or net illiquid wealth, as well as the top 5% by net worth, are excluded from the sample. The bar chart displays the average change in each wealth decile, and the dotted line an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15.

age, households would be willing to forgo 0.41% of their consumption over the first 20 quarters to eliminate the uncertainty shock, but this number is reduced to 0.25% with stabilization. In the latter regime, households rich in human capital pay the cost of the stabilization policy, because they save (partly in money) and thereby finance the monetary expansion. Moreover, without stabilization, these households profit from low prices of the illiquid asset in which they accumulate their long-term savings.

The remainder of the paper is organized as follows. Section 2 starts off with a review of the related literature. Section 3 develops our model, and Section 4 discusses the solution method. Section 5 introduces our estimation strategy for the income process and explains the calibration of the model. Section 6 presents the numerical results. Section 7 concludes. An Appendix follows that provides details on the properties of the value and policy functions, the numerics, the estimation of the uncertainty process from income data, and further robustness checks.

2. Related Literature

Our paper contributes to the recent literature that explores empirically and theoretically the aggregate effects of time-varying uncertainty. The seminal paper by Bloom (2009) discusses the effects of time-varying (idiosyncratic) productivity uncertainty on firms' factor demand, exploring the idea and effects of time-varying real option values of investment. This paper has triggered a stream of research that explores under which conditions such variations have aggregate effects.[2]

A more recent branch of this literature investigates the aggregate impact of uncertainty shocks beyond their transmission through investment and has also broadened the sources of uncertainty studied. The first papers in this vein highlight non-linearities in the New Keynesian model, in particular the role of precautionary price setting.[3] Fernández-Villaverde et al. (2015), for example, look at a medium-scale DSGE model à la Smets and Wouters (2007). They find that at the zero lower bound output drops by more than 1% after a two standard deviation shock to the volatility of taxes if a countervailing fiscal policy response is ruled out. Off the ZLB the drop reduces to 0.1%.[4] In a similar framework, Basu and Bundick (2012) highlight the labor market response to uncertainty about aggregate TFP and time preferences. They argue that, if uncertainty increases, the representative household will want to save more and consume less. Then, with King et al. (1988) preferences, the representative household will also supply more labor, which in a New Keynesian model depresses output through a "paradox of toil." When labor supply increases, wages and hence marginal costs for firms fall. This increases markups when prices are sticky, which finally depresses demand for consumption and investment, and a recession follows. Overall, they find similar aggregate effects as Fernández-Villaverde et al. (2015), in particular at the zero-lower bound.

While our paper also focuses on precautionary savings, it differs substantially in the transmission channel. We are agnostic about the importance of the "paradox of toil," because it crucially relies on a wealth effect in labor supply. We therefore assume Greenwood et al. (1988) preferences to eliminate any direct impact of uncertainty on labor supply to isolate the demand channel of precautionary savings instead.[5] Moreover, since we focus on *idiosyncratic* income uncertainty, we can identify the uncertainty process outside the model from the Panel Study of Income Dynamics (PSID).

This focus on idiosyncratic uncertainty and the response of precautionary savings links our paper to Ravn and Sterk (2013) and Den Haan et al. (2014). Both highlight the importance of idiosyncratic unemployment risk. In their setups, households face unemployment risk in an incomplete markets model with labor market search and nominal frictions. Both papers differ in their asset market setup and the shocks considered. Ravn and Sterk (2013) look at a setup with government bonds as a means of savings. They then study a joint shock to job separations and the share of long-term

[2]To name a few: Arellano et al. (2012), Bachmann and Bayer (2013), Christiano et al. (2010), Chugh (2012), Gilchrist et al. (2014), Narita (2011), Panousi and Papanikolaou (2012), Schaal (2012), and Vavra (2014) have studied the business cycle implications of a time-varying dispersion of firm-specific variables, often interpreted as and used to calibrate shocks to firm risk, propagated through various frictions: wait-and-see effects from capital adjustment frictions, financial frictions, search frictions in the labor market, nominal rigidities, and agency problems.

[3]With sticky prices, firms will target a higher markup the more uncertain future demand is.

[4]Born and Pfeifer (2014) report an output drop of 0.025% for a similar model and a similar policy risk shock under a slightly different calibration. Regarding TFP risk they hardly find any aggregate effect.

[5]Similarly, in a search model, higher uncertainty about match quality might translate into longer search and more endogenous separation. Thus it is not clear a priori whether labor supply would increase or decrease on impact.

unemployed. This increases income risk and hence depresses aggregate demand because of higher precautionary savings. They find that such first moment shocks to the labor market can be significantly propagated and amplified through this mechanism.

Den Haan et al. (2014) consider a model with money and equity instead, where equity is not physical capital as in our model, but is equated with vacancy-ownership. In addition, they assume wage rigidity. As in our model, poorer households, in their model the unemployed, are the marginal holders of money, the low-return asset, as they effectively discount the future more. When unemployment goes up, demand for money increases. This in turn leads to deflation, pushing up real wages because nominal wages are assumed to be sticky. This has a second-round effect on money demand. Because the labor intensity of production cannot be adjusted, higher real wages depress the equity yield on existing and newly formed vacancies, which then induces portfolio adjustments by households towards money amplifying the deflations and the related output drop.

Our transmission mechanism shares to some extent this feature, but additionally highlights the importance of liquidity. Households increase their precautionary savings in conjunction with a portfolio adjustment toward the liquid asset, because its services in consumption smoothing become more valuable to households. We find that the liquidity effect is more important than the relative return effect in our model where the labor intensity of production can be adjusted.

Finally, our work relates to Gornemann et al. (2012). We discuss the distributional consequences of uncertainty shocks and of systematic monetary policy response. We find that both differently affect households that differ in their portfolios due to differential asset price movements. This portfolio composition aspect is new in comparison to Gornemann et al., because we introduce decisions regarding nominal versus real asset holdings to the household's problem.

3. Model

We model an economy inhabited by two types of agents: (worker-)households and entrepreneurs. Households supply capital and labor and are subject to idiosyncratic shocks to their labor productivity. These shocks are persistent and have a time-varying variance. Households self-insure in a liquid nominal asset (money) and a less liquid physical asset (capital). Liquidity of money is understood in the spirit of Kaplan and Violante's (2014) model of wealthy hand-to-mouth consumers, where households hold capital, but trading capital is subject to a friction. We model this trading friction as limited participation in the asset market. Every period, a fraction of households is randomly selected to trade physical capital. All other households may only adjust their money holdings.[6] While money is subject to an inflation tax and pays no dividend, capital can be rented out to the intermediate-good-producing sector on a perfectly competitive rental market. This sector combines labor and capital services into intermediate goods and sells them to the entrepreneurs.

[6]We choose to exclude trading as a choice, and hence we use a simplified framework relative to Kaplan and Violante (2014) for numerical tractability. Random participation keeps the households' value function concave, thus making first-order conditions sufficient, and therefore allows us to use a variant of the endogenous grid method as an algorithm for our numerical calculations. See Appendix A for details.

Entrepreneurs capture all pure rents in the economy. For simplicity, we assume that entrepreneurs are risk neutral. They obtain rents from adjusting the aggregate capital stock due to convex capital adjustment costs and, more importantly, from differentiating the intermediate good. Facing monopolistic competition, they set prices above marginal costs for these differentiated goods. Price setting, however, is subject to a pricing friction à la Calvo (1983) so that entrepreneurs may only adjust their prices with some positive probability each period. The differentiated goods are finally bundled again to the composite final good used for consumption and investment.

The model is closed by a monetary authority that provides money in positive net supply and adjusts money growth according to the prescriptions of a Taylor type rule, which reacts to inflation deviations from target. All seigniorage is wasted.

3.1 Households

There is a continuum of ex-ante identical households of measure one indexed by i. Households are infinitely lived, have time-separable preferences with time-discount factor β, and derive felicity from consumption c_{it} and leisure. They obtain income from supplying labor and from renting out capital. A household's labor income $w_t h_{it} n_{it}$ is composed of the wage rate, w_t, hours worked, n_{it}, and idiosyncratic labor productivity, h_{it}, which evolves according to the following AR(1)-process:

$$\log h_{it} = \rho_h \log h_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N\left(0, \sigma_{ht}\right). \tag{1.1}$$

Households have Greenwood-Hercowitz-Huffman (GHH) preferences and maximize the discounted sum of felicity:

$$V = E_0 \max_{\{c_{it}, n_{it}\}} \sum_{t=0}^{\infty} \beta^t u\left(c_{it} - h_{it} G(n_{it})\right). \tag{1.2}$$

The felicity function takes constant relative risk aversion (CRRA) form with risk aversion ξ:

$$u(x_{it}) = \frac{1}{1-\xi} x_{it}^{1-\xi}, \quad \xi > 0,$$

where $x_{it} = c_{it} - h_{it} G(n_{it})$ is household i's composite demand for the bundled physical consumption good c_{it} and leisure. The former is obtained from bundling varieties j of differentiated consumption goods according to a Dixit-Stiglitz aggregator:

$$c_{it} = \left(\int c_{ijt}^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}}.$$

Each of these differentiated goods is offered at price p_{jt} so that the demand for each of the varieties is given by

$$c_{ijt} = \left(\frac{p_{jt}}{P_t}\right)^{-\eta} c_{it},$$

where $P_t = \left(\int p_{jt}^{1-\eta} dj\right)^{\frac{1}{1-\eta}}$ is the average price level.

The disutility of work, $h_{it} G(n_{it})$, determines a household's labor supply given the aggregate wage rate through the first-order condition:

$$h_{it}G'(n_{it}) = w_t h_{it}. \tag{1.3}$$

We weight the disutility of work by h_{it} to eliminate any Hartman-Abel effects of uncertainty on labor supply. Under the above assumption, a household's labor decision does not respond to idiosyncratic productivity h_{it}, but only to the aggregate wage w_t. Thus we can drop the household-specific index i, and set $n_{it} = N_t$. Scaling the disutilty of working by h_{it} effectively sets the micro elasticity of labor supply to zero. Therefore, it simplifies the calibration as we can calibrate the model to the income risk that households face without the need to back out the actual productivity shocks. What is more, without this assumption, higher *realized* uncertainty leads to higher productivity inequality and hence increases aggregate labor supply.[7]

We assume a constant Frisch elasticity of aggregate labor supply with γ being the inverse elasticity:

$$G(N_t) = \frac{1}{1+\gamma} N_t^{1+\gamma}, \quad \gamma > 0,$$

and use this to simplify the expression for the composite consumption good x_{it}. Exploiting the first-order condition on labor supply, the disutility of working can be expressed in terms of the wage rate:

$$h_{it}G(N_t) = h_{it}\frac{N_t^{1+\gamma}}{1+\gamma} = \frac{h_{it}G'(N_t)N_t}{1+\gamma} = \frac{w_t h_{it} N_t}{1+\gamma}.$$

In this way the demand for x_{it} can be rewritten as:

$$x_{it} = c_{it} - h_{it}G(N_t) = c_{it} - \frac{w_t h_{it} N_t}{1+\gamma}.$$

Total labor input supplied is given by:

$$\tilde{N}_t = N_t \int h_{it} di.$$

Following the literature on idiosyncratic income risk, we assume that asset markets are incomplete. Households can only trade in nominal money, $\tilde{m}_{it}$, that does not bear any interest and in capital, k_{it}, to smooth their consumption. Holdings of both assets have to be non-negative. Moreover, trading capital is subject to a friction.

This trading friction allows only a randomly selected fraction of households, ν, to participate in the asset market for capital every period. Only these households can freely rebalance their portfolios. All other households obtain dividends, but may only adjust their money holdings. For those households participating in the capital market,

[7] Without this assumption, n_{it} increases in h_{it}, and hence the aggregate effective labor supply, $\int h_{it} n_{it} di$, increases when the dispersion of h_{it} increases. While it would not change the household's problem in its asset choices and the choice of x_{it}, it would complicate aggregation.

the budget constraint reads:

$$c_{it} + m_{it+1} + q_t k_{it+1} = \frac{m_{it}}{\pi_t} + (q_t + r_t)k_{it} + w_t h_{it} N_t, \quad m_{it+1}, k_{it+1} \geq 0,$$

where m_{it} is real money holdings, k_{it} is capital holdings, q_t is the price of capital, r_t is the rental rate or "dividend," and $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate. We denote real money holdings of household i at the end of period t by $m_{it+1} := \frac{\tilde{m}_{it+1}}{P_t}$.
Substituting the expression $c_{it} = x_{it} + \frac{w_t h_{it} N_t}{1+\gamma}$ for consumption, we obtain:

$$x_{it} + m_{it+1} + q_t k_{it+1} = \frac{m_{it}}{\pi_t} + (q_t + r_t)k_{it} + \frac{\gamma}{1+\gamma} w_t h_{it} N_t, \quad m_{it+1}, k_{it+1} \geq 0. \tag{1.4}$$

For those households that cannot trade in the market for capital the budget constraint simplifies to:

$$x_{it} + m_{it+1} = \frac{m_{it}}{\pi_t} + r_t k_{it} + \frac{\gamma}{1+\gamma} w_t h_{it} N_t, \quad m_{it} \geq 0. \tag{1.5}$$

Note that we assume that depreciation of capital is replaced through maintenance such that the dividend, r_t, is the net return on capital.

Since a household's saving decision will be some non-linear function of that household's wealth and productivity, the price level, P_t, and accordingly aggregate real money, $M_{t+1} = \frac{\tilde{M}_{t+1}}{P_t}$, will be functions of the joint distribution Θ_t of (m_t, k_t, h_t). This makes Θ_t a state variable of the household's planning problem. This distribution evolves as a result of the economy's reaction to shocks to uncertainty that we model as time variations in the variance of idiosyncratic income shocks, σ^2_{ht}. This variance follows a stochastic volatility process, which allows us to separate shocks to the variance from shocks to the level of household income.

$$\sigma^2_{ht} = \bar{\sigma}^2 \exp(s_t), \quad s_t = \rho_s s_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N\left(-\frac{\sigma_s^2}{2(1-\rho_s^2)}, \sigma_s\right), \tag{1.6}$$

where $\bar{\sigma}^2$ is the steady state labor risk that households face, and s shifts this risk. Shocks ε_t to income risk are the only aggregate shocks in our model.

With this setup, the dynamic planning problem of a household is then characterized by two Bellman equations: V_a in the case where the household can adjust its capital holdings and V_n otherwise:

$$\begin{aligned} V_a(m,k,h;\Theta,s) =& max_{k',m'_a} u[x(m,m'_a,k,k',h)] \\ &+ \beta\left[\nu EV^a(m'_a,k',h',\Theta',s') + (1-\nu)EV^n(m'_a,k',h',\Theta',s')\right], \\ V_n(m,k,h;\Theta,s) =& max_{m'_n} u[x(m,m'_n,k,h)] \\ &+ \beta\left[\nu EV^a(m'_n,k,h',\Theta',s') + (1-\nu)EV^n(m'_n,k,h',\Theta',s')\right]. \end{aligned} \tag{1.7}$$

In line with this notation, we define the optimal consumption policies for the adjustment and non-adjustment cases as x^*_a and x^*_n, the money holding policies as m^*_a and m^*_n, and the capital investment policy as k^*. Details on the properties of the value functions (smooth and concave) and policy functions (differentiable and increasing in total resources), the first-order conditions, and the algorithm we employ to calculate the policy functions can be found in Appendix A.

3.2 Intermediate Goods Producers

Intermediate goods are produced with a constant returns to scale production function:

$$Y_t = \tilde{N}_t^{\alpha} K_t^{(1-\alpha)}.$$

Let MC_t be the relative price at which the intermediate good is sold to entrepreneurs. The intermediate-good producer maximizes profits,

$$MC_t Y_t = MC_t \tilde{N}_t^{\alpha} K_t^{(1-\alpha)} - w_t \tilde{N}_t - (r_t + \delta) K_t,$$

but it operates in perfectly competitive markets, such that the real wage and the user costs of capital are given by the marginal products of labor and capital:

$$w_t = \alpha MC_t \left(K_t / \tilde{N}_t \right)^{1-\alpha}, \tag{1.8}$$

$$r_t + \delta = (1-\alpha) MC_t \left(\tilde{N}_t / K_t \right)^{\alpha}. \tag{1.9}$$

3.3 Entrepreneurs

Entrepreneurs differentiate the intermediate good and set prices. They are risk neutral and have the same discount factor as households. We assume that only the central bank can issue money so that entrepreneurs participate in neither the money nor the capital market. This assumption gives us tractability in the sense that it separates the entrepreneurs' price setting problem from the households' saving problem. It enables us to determine the price setting of entrepreneurs without having to take into account households' intertemporal decision making. Under these assumptions, the consumption of entrepreneur j equals her current profits, Π_{jt}. By setting the prices of final goods, entrepreneurs maximize expected discounted future profits:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Pi_{jt}. \tag{1.10}$$

Entrepreneurs buy the intermediate good at a price equalling the nominal marginal costs, $MC_t P_t$, where MC_t is the real marginal costs at which the intermediate good is traded due to perfect competition, and then differentiate them without the need of additional input factors. The goods that entrepreneurs produce come in varieties uniformly distributed on the unit interval and each indexed by $j \in [0, 1]$. Entrepreneurs are monopolistic competitors, and hence charge a markup over their marginal costs. They are, however, subject to a Calvo (1983) price setting friction, and can only update their prices with probability θ. They maximize the expected value of future discounted profits by setting today's price, p_{jt}, taking into account the price setting friction:

$$\max_{\{p_{jt}\}} \sum_{s=0}^{\infty} (\theta\beta)^s E \Pi_{jt,t+s} = \sum_{s=0}^{\infty} (\theta\beta)^s E Y_{jt,t+s} (p_{jt} - MC_{t+s} P_{t+s}) \tag{1.11}$$

$$s.t.: Y_{jt,t+s} = \left(\frac{p_{jt}}{P_{t+s}}\right)^{-\eta} Y_{t+s},$$

where $\Pi_{jt,t+s}$ is the profits and $Y_{jt,t+s}$ is the production level in $t+s$ of a firm j that set prices in t.

We obtain the following first-order condition with respect to p_{jt}:

$$\sum_{s=0}^{\infty}(\theta\beta)^s EY_{jt,t+s}\left(\frac{p^*_{jt}}{P_{t-1}} - \underbrace{\frac{\eta}{\eta-1}}_{\mu} MC_{t+s}\frac{P_{t+s}}{P_{t-1}}\right) = 0, \tag{1.12}$$

where μ is the static optimal markup.

Recall that entrepreneurs are risk neutral and that they do not interact with households in any intertemporal trades. Moreover, *aggregate* shocks to the economy are small and homoscedastic, since the only aggregate shock we consider is the shock to the variance of housdehold income shocks. Therefore, we can solve the entrepreneurs' planning problem locally by log-linearizing around the zero inflation steady state, without having to know the solution of the households' problem. This yields, after some tedious algebra (see, e.g., Galí, 2008), the New Keynesian Phillips curve:

$$\log \pi_t = \beta E_t(\log \pi_{t+1}) + \kappa(\log MC_t + \mu), \tag{1.13}$$

where

$$\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}.$$

We assume that besides differentiating goods and obtaining a rent from the markup they charge, entrepreneurs also obtain and consume rents from adjusting the aggregate capital stock. Since the dividend yield is below their time-preference rate, in equilibrium entrepreneurs never hold capital. The cost of adjusting the stock of capital is $\frac{\phi}{2}\left(\frac{\Delta K_{t+1}}{K_t}\right)^2 K_t + \Delta K_{t+1}$. Hence, entrepreneurs will adjust the stock of capital until the following first-order condition holds:[8]

$$q_t = 1 + \phi\frac{\Delta K_{t+1}}{K_t}. \tag{1.14}$$

[8]Note that we assume capital adjustment costs only on new capital (or on the active destruction of old capital) but not on the replacement of depreciation. Depreciated capital is assumed to be replaced at the cost of one-to-one in consumption goods, and replacement is forced before the capital stock is adjusted at a cost. This differential treatment of depreciation and net investment simplifies the equilibrium conditions substantially, because the user cost of capital and hence the dividend paid to households do not depend on the next period's stock of capital, and the decisions of non-adjusters are not influenced by the price of capital q_t. Quantitatively, the fluctuations in dividends that maintenance at price q_t would bring about are negligible. Upon a 2 standard deviation shock to uncertainty, q_t falls to 0.96 – hence reducing depreciation cost by 4 basis points quarterly under the alternative specification where maintenance comes at cost q_t.

3.4 Goods, Money, Capital, and Labor Market Clearing

The labor market clears at the competitive wage given in (1.8); so does the market for capital services if (1.9) holds. We assume that the money supply is given by a monetary policy rule that adjusts the growth rate of money in order to stabilize inflation:

$$\frac{M_{t+1}}{M_t} = (\theta_1/\pi_t)^{1+\theta_2}\left(\frac{M_t}{M_{t-1}}\right)^{\theta_3} \tag{1.15}$$

Here M_{t+1} is the real balances at the end of period t (with the timing aligned to our notation for the households' budget constraint). The coefficient $\theta_1 \geq 1$ determines steady-state inflation, and $\theta_2 \geq 0$ the extent to which the central bank attempts to stabilize inflation around its steady-state value: the larger θ_2 the stronger is the reaction of the central bank to deviations from the inflation target. When $\theta_2 \to \infty$ inflation is perfectly stabilized at its steady-state value. $\theta_3 \geq 0$ captures persistence in money growth. We assume that the central bank wastes any seigniorage buying final goods and choose the above functional form for its simplicity.[9]

The money market clears whenever the following equation holds:

$$\begin{aligned}(\theta_1/\pi_t)^{1+\theta_2}\left(\frac{M_t}{M_{t-1}}\right)^{\theta_3} M_t &= \int [\nu m_a^*(m,k,h;q_t,\pi_t) + (1-\nu)m_n^*(m,k,h;q_t,\pi_t)] \\ &\quad \Theta_t(m,k,h)dmdkdh, \end{aligned} \tag{1.16}$$

with the end-of-period real money holdings of the preceding period given by

$$M_t := \int m\Theta_t(m,k,h)dmdkdh.$$

Last, the market for capital has to clear:

$$\begin{aligned} q_t &= 1 + \phi\frac{K_{t+1}-K_t}{K_t} = 1 + \nu\phi\frac{K^*_{t+1}-K_t}{K_t}, \\ K^*_{t+1} &:= \int k^*(m,k,h;q_t,\pi_t)\Theta_t(m,k,h)dmdkdh, \\ K_{t+1} &= K_t + \nu(K^*_{t+1}-K_t), \end{aligned} \tag{1.17}$$

where the first equation stems from competition in the production of capital goods, the second equation defines the aggregate supply of funds from households trading capital, and the third equation defines the law of motion of aggregate capital. The goods market

[9]For the baseline calibration this is an innocuous assumption. With constant nominal money growth, the changes in seigniorage are negligible in absolute terms. Steady-state seigniorage is .64% of annual output, since money growth is 2% and the money-to-output ratio is 32%. When inflation drops, say, from 2% to 0, the real value of seigniorage increases, but only from .64% to .66% of output. As $\theta_2 \to \infty$, seigniorage occasionally turns slightly negative. It is numerically very expensive to put a constraint on M_t, and hence we abstain from doing so to keep the dynamic problem tractable. This unboundedness of seigniorage only affects the effectiveness of the stabilization policy. The central bank can commit to decrease seigniorage more in the future without the requirement of (weakly) positive seigniorage. One possible assumption to rationalize this is to assume that seigniorage is not wasted on government consumption but is used to store goods in an inefficient way.

then clears due to Walras' law, whenever both money and capital markets clear.

3.5 Recursive Equilibrium

A *recursive equilibrium* in our model is a set of policy functions $\{x_a^*, x_n^*, m_a^*, m_n^*, k^*\}$, value functions $\{V_a, V_n\}$, pricing functions $\{r, w, \pi, q\}$, aggregate capital and labor supply functions $\{N, K\}$, distributions Θ_t over individual asset holdings and productivity, and a perceived law of motion Γ, such that

1. Given $\{V_a, V_n\}$, Γ, prices, and distributions, the policy functions $\{x_a^*, x_n^*, m_a^*, m_n^*, k^*\}$ solve the households' planning problem, and given the policy functions, prices and distributions, the value functions $\{V_a, V_n\}$ are a solution to the Bellman equations (1.7).
2. The labor, the final-goods, the money, the capital, and the intermediate-good markets clear, i.e., (1.8), (1.13), (1.16), and (1.17) hold.
3. The actual law of motion and the perceived law of motion Γ coincide, i.e., $\Theta' = \Gamma(\Theta, s')$.

4. Numerical Implementation

The dynamic program (1.7) and hence the recursive equilibrium is not computable, because it involves the infinite dimensional object Θ_t.

4.1 Krusell-Smith Equilibrium

To turn this problem into a computable one, we assume that households predict future prices only on the basis of a restricted set of moments, as in Krusell and Smith (1997, 1998). Specifically, we make the assumption that households condition their expectations only on last period's aggregate real money holdings, M_t, last period's aggregate real money growth, $\Delta(\log M_t)$, the aggregate stock of capital, K_t, and the uncertainty state, s_t. The reasoning behind this choice goes as follows: (1.16) determines inflation, which in turn depends on the beginning of period money stock and last period's money growth. Once inflation is fixed, the Phillips curve (1.13) determines markups and hence wages and dividends. These will pin down asset prices by making the marginal investor indifferent between money and physical capital. If asset-demand functions, $m_{a,n}^*$ and k^*, are sufficiently close to linear in human capital, h, and in non-human wealth, m, k, at the mass of Θ_t, we can expect approximate aggregation to hold. For our exercise, the four aggregate states – s_t, M_t, $\Delta(\log M_t)$, and K_t – are sufficient to describe the evolution of the aggregate economy.[10]

While the law of motion for s_t is pinned down by (1.6), households use the following log-linear forecasting rules for current inflation and the price of capital, where the

[10] Without persistence in money growth, Equation (1.16) does not depend on $\Delta(\log M_t)$ anymore making it a redundant state. In this case, we set $\beta^4_{\pi,q} = 0$.

coefficients depend on the uncertainty state:

$$\log \pi_t = \beta_\pi^1(s_t) + \beta_\pi^2(s_t)\log M_t + \beta_\pi^3(s_t)\log K_t + \beta_\pi^4(s_t)\Delta(\log M_t), \quad (1.18)$$
$$\log q_t = \beta_q^1(s_t) + \beta_q^2(s_t)\log M_t + \beta_q^3(s_t)\log K_t + \beta_q^4(s_t)\Delta(\log M_t). \quad (1.19)$$

The law of motion for real money holdings, M_t, then follows from the monetary policy rule and is given by:

$$\log M_{t+1} = \log M_t + (1+\theta_2)(\log\theta_1 - \log\pi_t) + \theta_3\Delta(\log M_t).$$

The law of motion for K_t results from (1.17).

Fluctuations in q and π happen for two reasons: As uncertainty goes up, the self-insurance service that households receive from the illiquid capital good decreases. In addition, the rental rate of capital falls as firms' markups increase. When making their investment decisions, households need to predict the next period's capital price q' to determine the expected return on their investment. Since all other prices are known functions of the markup, only π' and q' need to be predicted.

Technically, finding the equilibrium is similar to Krusell and Smith (1997), as we need to find market clearing prices within each period. Concretely, this means the posited rules, (1.18) and (1.19), are used to solve for households' policy functions. Having solved for the policy functions conditional on the forecasting rules, we then simulate n independent sequences of economies for $t = 1, \ldots, T$ periods, keeping track of the actual distribution Θ_t. In each simulation the sequence of distributions starts from the stationary distribution implied by our model without aggregate risk. We then calculate in each period t the optimal policies for market clearing inflation rates and capital prices assuming that households resort to the policy functions derived under rule (1.18) and (1.19) from period $t+1$ onward. Having determined the market clearing prices, we obtain the next period's distribution Θ_{t+1}. In doing so, we obtain n sequences of equilibria. The first 250 observations of each simulation are discarded to minimize the impact of the initial distribution. We next re-estimate the parameters of (1.18) and (1.19) from the simulated data and update the parameters accordingly. By using $n = 20$ and $T = 750$, it is possible to make use of parallel computing resources and obtain 10.000 equilibrium observations. Subsequently, we recalculate policy functions and iterate until convergence in the forecasting rules.

The posited rules (1.18) and (1.19) approximate the aggregate behavior of the economy fairly well. The minimal within sample R^2 is above 99%. Also the out-of-sample performance (see Den Haan, 2010)) of the forecasting rules is good. See Appendix D.

4.2 Solving the Household Planning Problem

In solving for the households' policy functions we apply an endogenous gridpoint method as originally developed in Carroll (2006) and extended by Hintermaier and Koeniger (2010), iterating over the first-order conditions. We approximate the idiosyncratic productivity process by a discrete Markov chain with 17 states and time-varying transition probabilities, using the method proposed by Tauchen (1986). The stochastic volatility process is approximated in the same vein using 7 states.[11] Details on the algorithm can be found in Appendix A.4.

5. Calibration

We calibrate the model to the U.S. economy. The behavior of the model in steady state without fluctuations in uncertainty does not correspond to the time-averages of the simulated variables in the model with uncertainty shocks. Hence we cannot use the steady state to calibrate the model, but instead iterate over the full model to match the calibration targets.[12] The aggregate data used for calibration spans 1980 to 2012. One period in the model refers to a quarter of a year. The choice of parameters as summarized in Tables 1 and 2 is explained next. We present the parameters as if they were individually changed in order to match a specific data moment, but all calibrated parameters are determined jointly of course.

5.1 Income Process

We estimate the income process and hence uncertainty faced by households from income data in the Cross-National Equivalent File (CNEF) of the Panel Study of Income Dynamics (PSID), excluding the low-income sample. We construct household income as pre-tax labor income plus private and public transfers minus all taxes, and control for observable household characteristics in a first stage regression. We use the residual income to estimate the parameters governing the idiosyncratic income process $\rho_s, \rho_h, \bar{\sigma}$, and σ_s.

In a first stage regression for log-income, we control nonparametrically for the effects of age, household size, and educational attainment and parametrically with up to squared-order terms in age for the age-education interaction. We then generate variances and first and second order auto-covariances of residual income by age groups for the years 1970-2009. Based on these age-year variances and covariances, the parameters of interests are estimated by generalized method of moments (GMM). We find that the implied quarterly autocorrelation of the persistent component of income, ρ_h, is 0.976 and the average standard deviation of quarterly persistent income shocks is $\bar{\sigma} = 0.078$. The implied quarterly persistence of income risk, ρ_s, is 0.903 and thus in line with business cycle frequencies. The annual coefficient of variation for income risk, $\frac{\sigma_s}{\bar{\sigma}}$, is 0.62, which is consistent with the estimates in Storesletten et al. (2004).[13] Table 1.1 summarizes the parameter estimates, where the values are adapted to the quarterly frequency of our model. Details on data selection and the estimation procedure can be found in Appendix B.

[11] We solve the household policies for 30 points on the grid for money and 50 points on the grid for capital using equi-distant grids on log scale plus outliers. For aggregate money and capital holdings we use a relatively coarse grid of 3 points each. We experimented with changing the number of gridpoints without a noticeable impact on results. See Appendix D.

[12] As this is very expensive computational-wise, we match the target-ratios within +/- 1%.

[13] Storesletten et al. estimate the variance of persistent shocks to annual income to be 126% higher in times of below average GDP growth than in times of above average GDP growth. This implies that the unconditional *annual* coefficient of variation of s is roughly 0.5.

Table 1.1: Estimated parameters of the income process

Parameter	Value	Description
ρ_h	0.976	Persistence of income
$\bar{\sigma}$	0.078	Average STD of innovations to income
ρ_s	0.903	Persistence of the income-innovation variance, σ_h^2
σ_s	0.277	Conditional STD (log scale) of σ_h^2

Notes: All values are adapted to the quarterly frequency of the model. For details on the estimation see Appendix B.

5.2 Preferences and Technology

While we can estimate the income process directly from the data, all other parameters are calibrated within the model. Table 1.2 summarizes our calibration. In detail, we choose the parameter values as follows.

Households

For the felicity function, $u = \frac{1}{1-\xi}x^{1-\xi}$, we set the coefficient of relative risk aversion $\xi = 4$, as in Kaplan and Violante (2014). The time-discount factor, β, and the asset market participation frequency, ν, are jointly calibrated to match the ratios of liquid and illiquid assets to output. We equate illiquid assets to all capital goods at current replacement values. This implies for the total value of illiquid assets relative to nominal GDP a capital-to-output ratio of 286%. In our baseline calibration, this implies an annual real return for illiquid assets of 3.2%. We equate liquid assets to claims of the private sector against the government and not to inside money, because the net value of inside claims does not change with inflation. Specifically, we look at average U.S. federal debt for the years 1980 to 2012 held by domestic private agents plus the monetary base. This yields an annual money-to-output ratio of 32%. For details on the steady-state asset distribution, see Appendix C. The calibrated participation frequency $\nu = 4.25\%$ is close to Kaplan and Violante's estimate for working households in their state-dependent participation framework. We take a conservative value for the inverse Frisch elasticity of labor supply, $\gamma = 2$, corresponding to the estimates by microeconometric studies. We provide a robustness check with an estimate of the inverse Frisch elasticity of labor supply, $\gamma = 1$, which follows the New Keynesian literature (Chetty et al. (2011)).

Table 1.2: Calibrated parameters

Parameter	Value	Description	Target
Households			
β	0.987	Discount factor	$K/Y = 286\%$ (annual)
ν	4.25%	Participation frequency	$M/Y = 32\%$ (annual)
ξ	4	Coefficient of rel. risk av.	Kaplan and Violante (2014)
γ	2	Inverse of Frisch elasticity	Standard value
Intermediate Goods			
α	0.73	Share of labor	Income share of labor of 2/3
δ	1.35%	Depreciation rate	NIPA: Fixed assets
Final Goods			
κ	0.09	Price stickiness	Mean price duration of 4 quarters
μ	0.10	Markup	10% markup (standard value)
Capital Goods			
ϕ	220	Capital adjustment costs	Relative investment volatility of 3
Monetary Policy (Friedman's k% rule)			
θ_1	1.005	Money growth	2% p.a.
θ_2	0	Reaction to inflation deviations	
θ_3	0	Persistence in money growth	

Intermediate, Final, and Capital Goods Producers

We parameterize the production function of the intermediate good producer according to the U.S. National Income and Product Accounts (NIPA). In the U.S. economy the income share of labor is about 2/3. Accounting for profits we hence set $\alpha = 0.73$.

To calibrate the parameters of the entrepreneurs' problem, we use standard values for markup and price stickiness that are widely employed in the New Keynesian literature. The Phillips curve parameter κ implies an average price duration of 4 quarters, assuming flexible capital at the firm level. The steady-state marginal costs, $exp(-\mu) = 0.91$, imply a markup of 10%. The entrepreneurs' and households' discount factor are equal.

We calibrate the adjustment cost of capital, $\phi = 220$, to match an investment to output volatility of 3.

Table 1.3: Alternative monetary policy rules

Parameter	Value	Description	Target
Inflation Stabilization			
θ_1	1.005	Money growth	2% p.a.
θ_2	1000	Reaction to inflation deviations	No deviations from target
θ_3	0	Persistence in money growth	
Fed Policy Rule (Post-1980)			
θ_1	1.005	Money growth	2% p.a.
θ_2	0.35	Reaction to inflation deviations	Chowdhury and Schabert (2008)
θ_3	0.9	Persistence in money growth	Chowdhury and Schabert (2008)

Notes: For the Fed policy rule as well as all robustness checks, we recalibrate the discount factor and the participation frequency of households to match the targeted capital and money to output ratios and the capital adjustment costs to match a relative investment volatility of 3.

Central Bank

We set the average growth rate of money, θ_1, such that our model produces an average annual inflation rate of 2%, in line with the usual inflation targets of central banks and roughly equal to average inflation in the U.S. between 1980 and 2012. To simplify the dynamics of the model and for expositional purpose, we assume in our baseline setup that the central bank follows Friedman's $k\%$ rule and hence set θ_2 and θ_3 to 0. Alternatively, we consider two additional policy rules, see Table 1.3. First, we set $\theta_2 = 1000$ and $\theta_3 = 0$, to examine uncertainty shocks without movements in the price level. Second, we calibrate towards the post-1980s money supply rule of the Federal Reserve as estimated in Chowdhury and Schabert (2008) to quantify the contribution of uncertainty shocks to the U.S. business cycle over this period. This implies $\theta_2 = 0.35$ and $\theta_3 = 0.9$.[14]

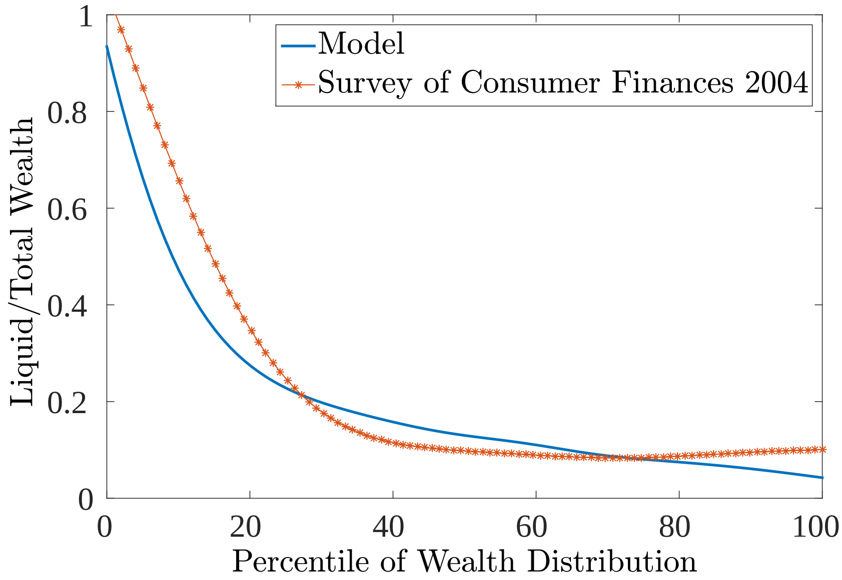
6. Quantitative Results

6.1 Household Portfolios and the Individual Response to Uncertainty

In our model, households hold money because it provides better short-term consumption smoothing than capital, as the latter can only be traded infrequently. This value of

[14] Originally, Chowdhury and Schabert report Taylor rules for money including a reaction to the output gap. We obtain $\theta_2 = 0.35$ by using the Phillips curve from our model to eliminate the output gap.

Figure 1.2: Share of liquid assets in total net worth against percentiles of wealth

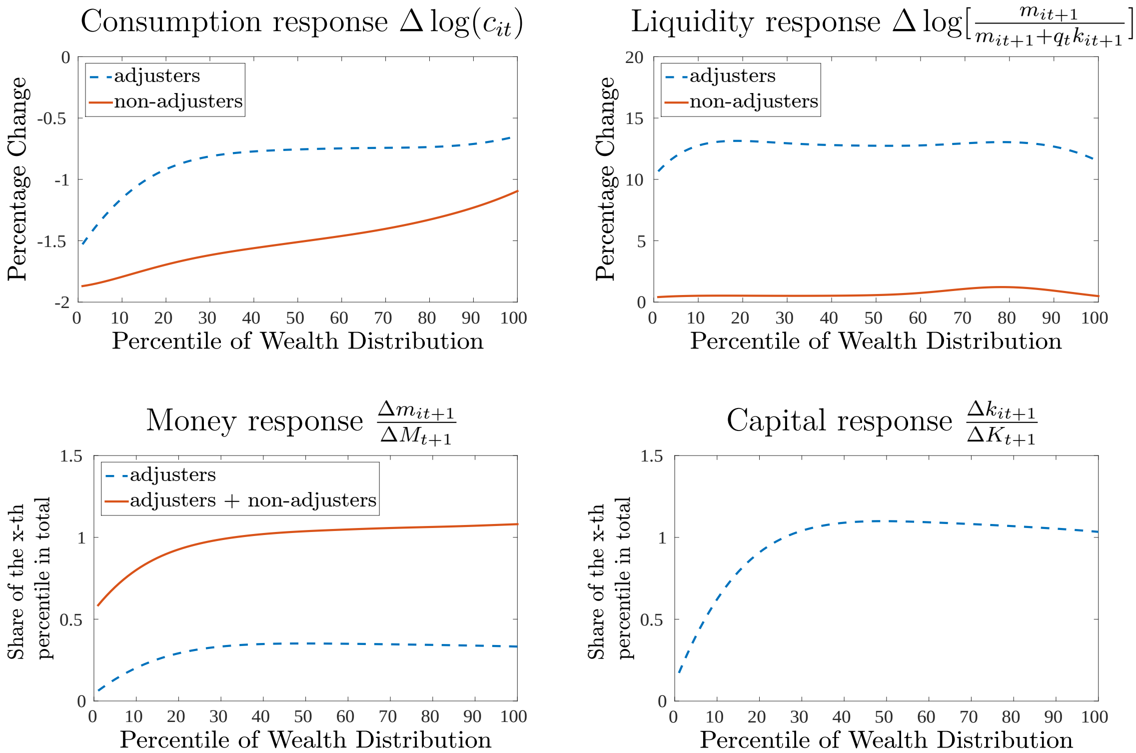


Notes: For graphical illustration we make use of an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15. For the definition of net liquid assets see Figure 1.1.

liquidity decreases in the amount of money a household holds, because a household rich in liquid assets will likely be able to tap into its illiquid wealth before running down all liquid wealth. For this reason, richer households, who typically hold both more money and more capital, hold less liquid portfolios. The poorest households, on the contrary, hold almost all their wealth in the liquid asset. This holds true in the actual data as well as in our model. While our model matches relatively well the shape of the actual liquidity share of household portfolios at all wealth percentiles, it slightly underestimates the share of liquid assets for the lowest deciles; see Figure 1.2, which compares our model to the *Survey of Consumer Finances* 2004.

So what happens to total savings and its composition when uncertainty increases? In response to the increase in income uncertainty, households aim for higher precautionary savings to be in a better position to smooth their consumption. Since the liquid asset is better suited to this purpose, households first increase their demand for this asset – in fact, they even reduce holdings of the illiquid asset to increase the liquidity of their portfolio. Figure 1.3 shows how households' portfolio composition and consumption policy react to an increase in uncertainty without imposing any market clearing. The top panels displays the relative change in the consumption and portfolio liquidity compared to the average uncertainty state. For this exercise, we evaluate households' consumption policies and the portfolio choice of adjusters and non-adjusters after a 2 standard deviation shock to uncertainty, increasing the variance of idiosyncratic income shocks by 55%. We here perform a partial equilibrium analysis and compute the policies under the expectation that all prices are at their steady-state values isolating hence the

Figure 1.3: Partial equilibrium response – Change in individual policy upon an uncertainty shock keeping prices and expectations constant at steady-state values

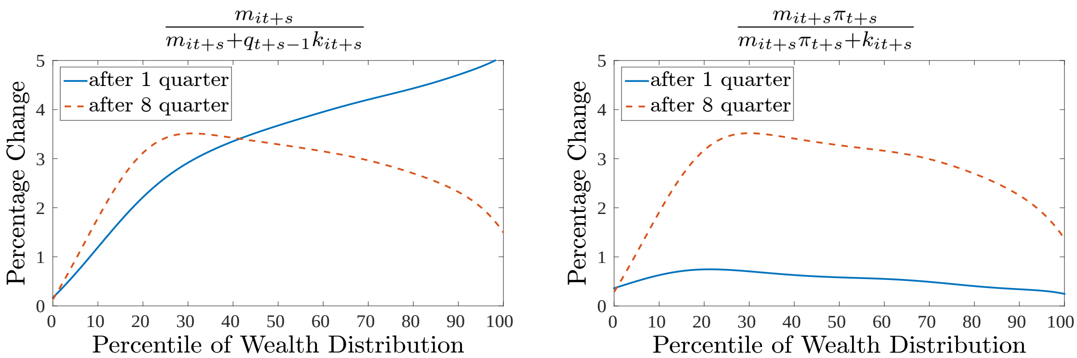


Notes: Top Panels: Reaction of individual consumption demand and portfolio liquidity of adjusters and non-adjusters at constant prices and price expectations relative to the respective counterpart at average uncertainty. The policies are averaged using frequency weights from the steady-state wealth distribution and reported conditional on a household falling into the x-th wealth percentile. High uncertainty corresponds to a two standard deviation shock, which is equal to a 55% increase in uncertainty.
Bottom Panels: Fraction of total demand change for money and capital accounted for by all households in a given percentile of the wealth dristribution.
As with the data, we use an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15.

direct effect of income uncertainty. Across all wealth levels, households wish to increase their savings (i.e., decrease their consumption) as well as the liquidity of their portfolios when uncertainty goes up. Adjusters can do so by tipping into their capital account and thus their consumption falls less. This flight to liquidity leads to falling demand for capital even though total savings increase.

The bottom panels of Figure 1.3 display the contribution of each wealth percentile to the total change in demand for money and capital. Values above (below) one imply that a certain percentile of the wealth distribution is contributing more (less) than proportionally. We find that almost all wealth groups are equally important for the change in total asset demand. In other words, poorer households, while making up a smaller fraction of total asset demand, observe larger changes in their asset positions

Figure 1.4: General equilibrium response – Change in the liquidity of household portfolios



Notes: Change in the distribution of liquidity at all percentiles of the wealth distribution at equilibrium prices and price expectations for $s = \{1, 8\}$ quarters after a two standard deviation shock to income uncertainty. The liquidity of the portfolios is averaged using frequency weights from the steady-state wealth distribution and reported conditional on a household falling into the x-th wealth percentile. The left-hand panel shows the change including changes in prices; the right-hand panel shows the pure quantity responses. As with the data, we use an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15.

and hence are as important as richer households for the aggregate demand changes.

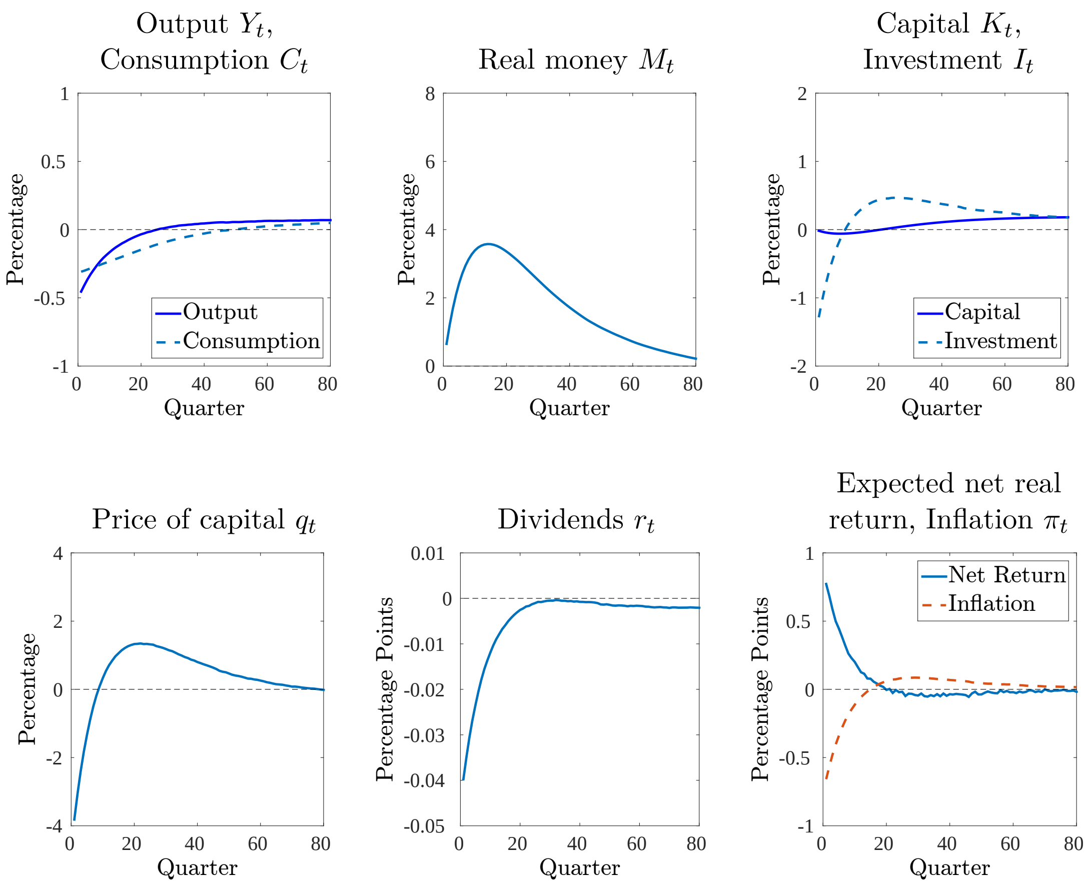
The change in the liquidity of household portfolios in general equilibrium is displayed in Figure 1.4; the left-hand panel shows the change in value terms; the right-hand panel shows the change in quantities, i.e., at constant prices. Portfolio liquidity initially increases at all wealth levels – in particular in value terms because the price of illiquid assets drops sharply as we will see in the next section. The increase in the share of liquid assets is least pronounced for the poorest, because of the negative income effect. After two years, the increase in liquidity is concentrated at households somewhat below median wealth. By then, rich households have partially reversed their portfolio shares as they also increase their savings in physical capital, exploiting lower capital prices. Interestingly, this picture is exactly what we found in Figure 1.1, where the increase in the liquidity of the portfolios is strongest for the lower middle class. Only the magnitude of changes in the liquidity of household portfolios during the Great Recession is much more dramatic.

6.2 Aggregate Consequences of Uncertainty Shocks

Main Findings

This simultaneous decrease in the demand for consumption and capital upon an increase in uncertainty leads to a decline in output. Figure 1.5 displays the impulse responses of output and its components, real balances and the capital stock as well as asset prices and returns for our baseline calibration. The assumed monetary policy follows a strict money growth rule, i.e., it is not responsive to inflation. After a two standard deviation

Figure 1.5: Uncertainty shock under constant money growth



Notes: Expected net real return: $\frac{Eq_{t+1}+r_t}{q_t}$.
Impulse responses to a 2 standard deviation increase in the variance of idiosyncratic productivity. We generate these impulses by averaging over 100.000 independent simulations of the law of motions, Equations 1.18 and 1.19, that simultaneously receive the shock in $T = 500$. All rates (inflation, dividends, etc.) are *not* annualized.

increase in the variance of idiosyncratic productivity shocks, output drops on impact by 0.5% and only returns to the normal growth path after roughly 20 quarters. Over the first year the output drop is 0.37% on average.

The output drop in our model results from households increasing their precautionary savings in conjunction with a portfolio adjustment toward the liquid asset. In times of high uncertainty, households dislike illiquid assets because of their limited use for short-run consumption smoothing. Conversely, the price of capital decreases on impact by 4%. Since the demand for the liquid asset is a demand for paper and not for (investment) goods, demand for both consumption and investment goods falls.

This decrease in demand puts pressure on prices. Inflation falls by 65 basis points on impact, increasing the average markup in the economy. Thus, the marginal return on capital, r_t, and consequently investment demand decline, while the return on money goes up. Thereby, the flight to liquidity increases the relative return of money, which further amplifies the portfolio adjustment. In line with the excess stock volatility puzzle, uncertainty shocks move capital prices and expected returns much more (and in the opposite direction) than they move dividends (65 vs. -4 basis points, quarterly).

Stabilization Policy

How much of this is driven by the increased value of liquidity, and how much by the differential impact of disinflation on the return of money and on dividends? We can isolate the flight to liquidity from the effect of the change in relative returns by looking at a monetary policy that is stabilizing the economy – setting $\theta_2 = 1000, \theta_3 = 0$. Under this policy, inflation is fixed and output barely moves. Also dividends are virtually constant. Thus, the relative-return effect vanishes in the case of strict inflation targeting. The corresponding impulse responses are displayed in Figure 1.6. As a consequence of the stabilization, the price of capital falls less, but it still falls by more than 2%. The expected return on capital increases by about 50 basis points. The total income of households almost stays constant in the first 5 years and hence money demand peaks at an even higher level than without stabilization.

In other words, the portfolio adjustment is to a large extent driven by a flight to liquidity. After roughly 2.5 years, real balances have increased to a point where households are well insured and want to increase their holdings of the illiquid asset again.

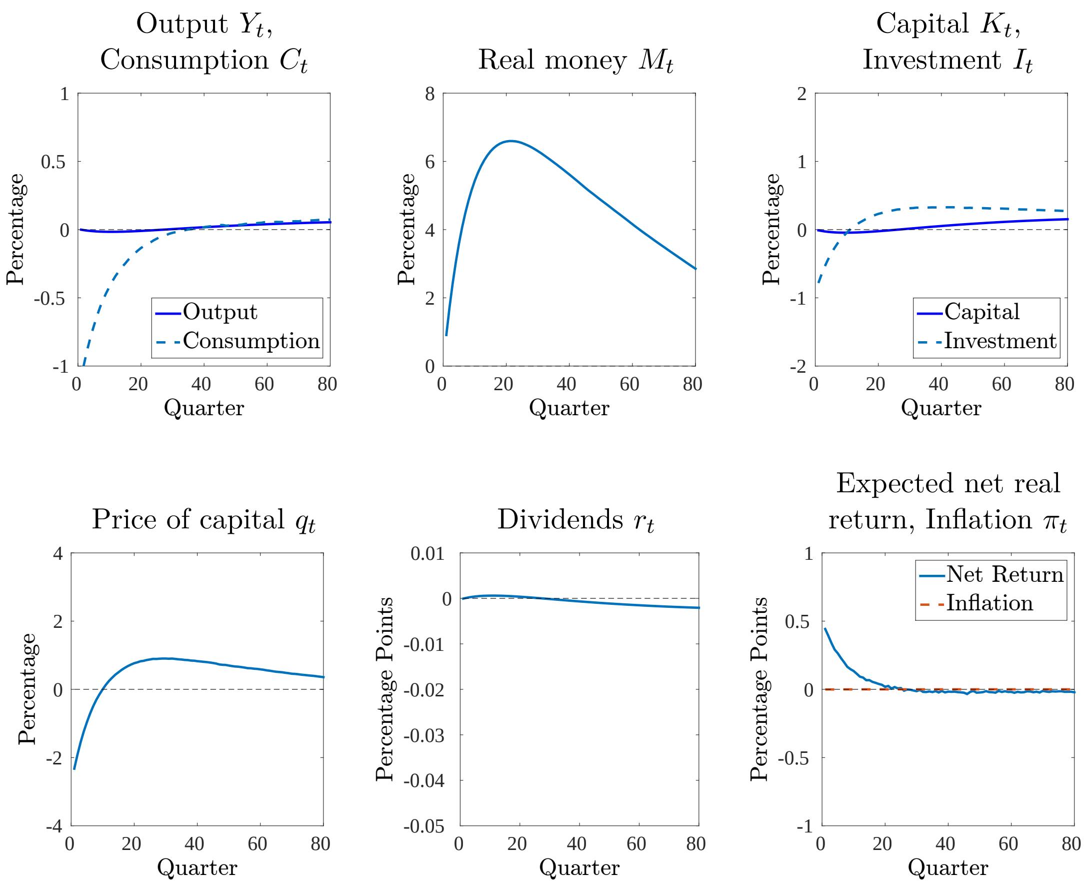
Quantitative Importance: Fed Policy Rule

Figure 1.7 displays the aggregate consequences of shocks to household income risk using the Fed's post-1980's money supply reaction function as estimated by Chowdhury and Schabert (2008). The results are roughly half way between perfect stabilization and constant money growth.

How Important Is the (Il)liquidity of Capital?

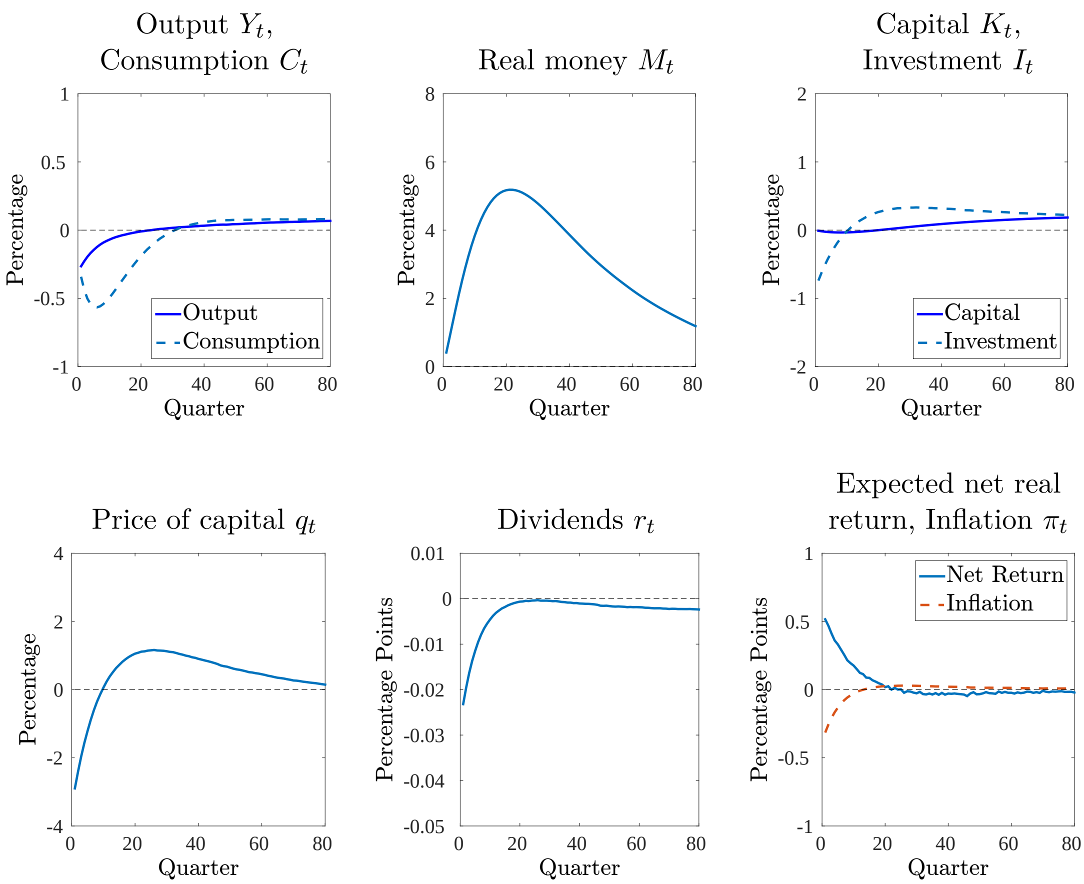
Our calibration suggests that households can adjust their capital holdings on average every 23.5 quarters. This restricted access to savings in capital limits its use for short-run consumption smoothing considerably. If capital were easier to access, it would

Figure 1.6: Uncertainty shock under inflation stabilization



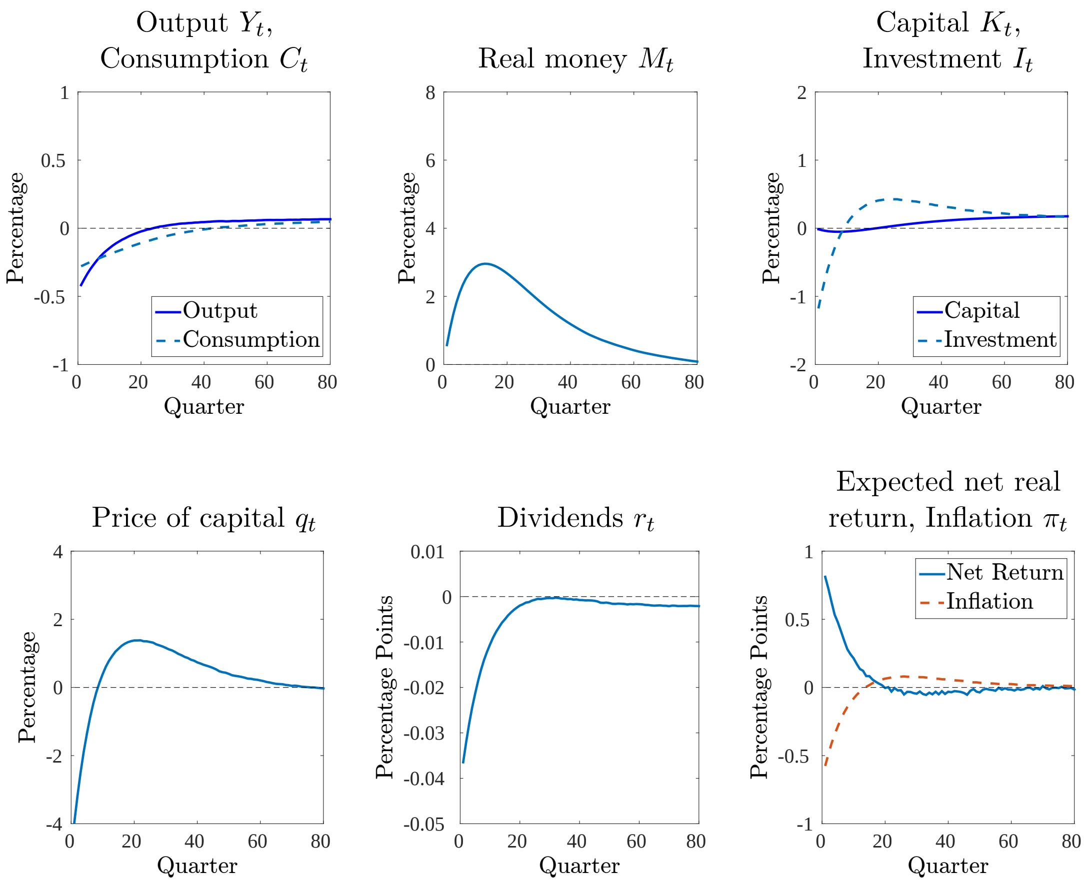
Notes: Expected net real return: $\frac{Eq_{t+1}+r_t}{q_t}$.
Impulse responses to a 2 standard deviation increase in the variance of idiosyncratic productivity. We generate these impulses by averaging over 100.000 independent simulations of the law of motions, Equations 1.18 and 1.19, that simultaneously receive the shock in $T = 500$. All rates (inflation, dividends, etc.) are *not* annualized.

Figure 1.7: Uncertainty shock under Fed's post-80's reaction function as estimated in Chowdhury and Schabert (2008)



Notes: Expected net real return: $\frac{Eq_{t+1}+r_t}{q_t}$.
Impulse responses to a 2 standard deviation increase in the variance of idiosyncratic productivity. We generate these impulses by averaging over 100.000 independent simulations of the law of motions, Equations 1.18 and 1.19, that simultaneously receive the shock in $T = 500$. All rates (inflation, dividends, etc.) are *not* annualized.

Figure 1.8: Uncertainty shock with liquid capital ($\nu = 35\%$)



Notes: Expected net real return: $\frac{Eq_{t+1}+r_t}{q_t}$.
Impulse responses to a 2 standard deviation increase in the variance of idiosyncratic productivity. We generate these impulses by averaging over 100.000 independent simulations of the law of motions, Equations 1.18 and 1.19, that simultaneously receive the shock in $T = 500$. All rates (inflation, dividends, etc.) are *not* annualized.

become more and more of a substitute for money in terms of its use for consumption smoothing. Hence, aggregate money holdings decline as ν increases. Figure 1.8 plots the impulse responses for an average adjustment frequency of less than a year ($\nu = 35\%$). In this case money holdings are only 8.5% of annual output on average, which corresponds to the U.S. monetary base.[15]

Figure 1.8 shows that the output drop is very similar with a higher portfolio adjustment frequency, although the share of money in the economy is significantly smaller and capital is very liquid in comparison to the baseline calibration. Money demand reacts more elastically to uncertainty as more households are able to adjust their portfolio. Consequently, the flight to liquidity is stronger and happens faster than with more illiquid capital – in the build-up and in the reverse.

[15] We use the St. Louis Fed adjusted annual monetary base from 1980 to 2012.

In summary, the macroeconomic effects of uncertainty shocks are robust to changes in ν. While in the limit with perfectly liquid capital money is driven out of the economy, the economy seems to not converge toward the "Aiyagari" economy without money and perfectly liquid capital. In the "Aiyagari" case, investment replaces consumption demand one-for-one when uncertainty hits. As long as households hold even tiny amounts of money for liquidity-consumption smoothing reasons, the value of money increases with income uncertainty and money demand is higher in uncertain times, which creates deflationary pressures.

In other words, and more generally speaking, uncertainty shocks will affect aggregate demand negatively only if they trigger precautionary savings in paper and not in real assets. In our model, it is the increased value of liquidity that is responsible for the portfolio adjustment toward money.

6.3 Redistributive and Welfare Effects

So far we have described the aggregate dimension of an uncertainty shock and its repercussions. Since such shocks affect the price level, asset prices, dividends, and wages differently, our model predicts that not all agents (equally) lose from the decline in consumption upon an uncertainty shock. For example, if capital prices fall, those agents that are rich in human capital but hold little physical capital could actually gain from the uncertainty shock. These agents are net savers. They increase their holdings of physical capital and can do so now more cheaply.

To quantify and understand the relative welfare consequences of the uncertainty shock and of systematic policy response, one would normally just look at the change in a household's value function. However, since solving directly for the value function is prohibitively time consuming in our model, we instead simulate and compare two sets of economies: one where the uncertainty state simply evolves according to its Markov chain properties and another set where, at time T, we exogenously increase income uncertainty, σ_{ht}^2, by setting the shock to uncertainty to $\epsilon_T = 2\sigma_s$, a 2 standard deviation increase. We then let the economies evolve stochastically. We trace agents over the next S periods for both sets of economies, and track their period-felicity u_{iT+t} to calculate for each agent with individual state (h, m, k) in period T the discounted expected felicity stream over the next S periods as:

$$v_S(h, m, k) = E\left[\sum_{t=0}^{S} \beta^t u_{T+t} \middle| (h_T, m_T, k_T) = (h, m, k)\right],$$

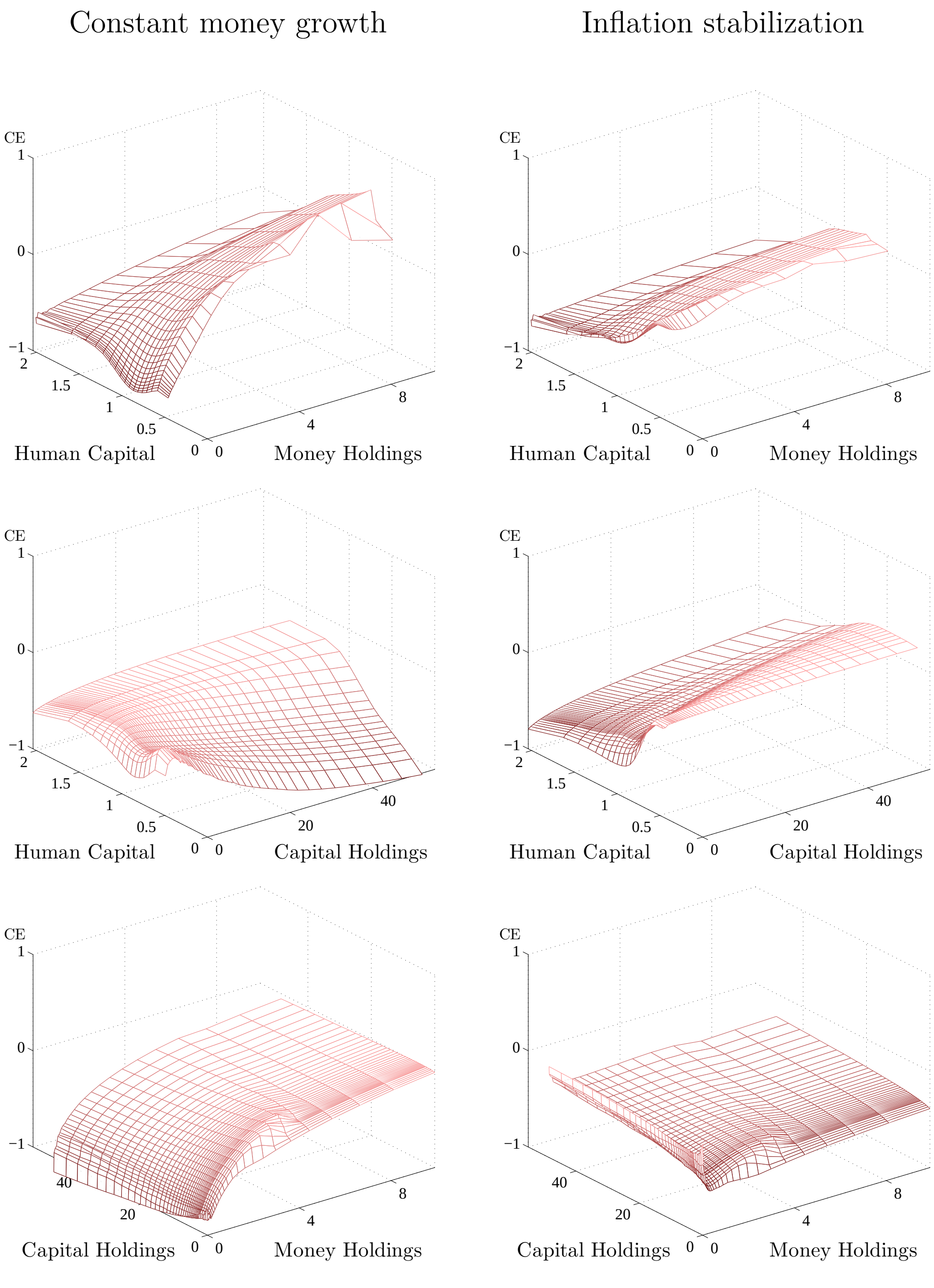
where u_{T+t} is the felicity stream in period $T + t$ under the household's optimal saving policy. For large S, v_S approximates the actual household's value function.

We then determine an equivalent consumption tax that households would be willing to face over the next S quarters in order to eliminate the uncertainty shock at time T as:

$$\mathtt{CE} = \left(\frac{v_S^{\mathtt{shock}}}{v_S^{\mathtt{no\ shock}}}\right)^{1/(1-\xi)} - 1. \qquad (1.20)$$

Figure 1.9 displays the relative differences in v_S for $S = 20$ quarters in terms of consumption equivalents, CE, between the two sets of simulations of the economy. This

Figure 1.9: Welfare after 5 years



Notes: Welfare costs in terms of consumption equivalents (CE) as defined in (1.20). The graphs refer to the conditional expectations of CE with respect to the two displayed dimensions, respectively. The missing dimension has been integrated out. Capital and money are reported in terms of quarterly income.

Table 1.4: Welfare after 5 years

Policy regime: Constant money growth										
	Quintiles of money holdings					Quintiles of capital holdings				
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
Conditional	-0.92	-0.73	-0.55	-0.38	-0.18	-0.53	-0.47	-0.51	-0.55	-0.61
Median	-0.96	-0.78	-0.57	-0.35	-0.02	-0.63	-0.59	-0.61	-0.64	-0.69
	Quintiles of Human Capital									
Conditional	-0.62	-0.62	-0.58	-0.45	-0.43					
Median	-0.59	-0.61	-0.60	-0.52	-0.58					

Policy regime: Inflation stabilization										
	Quintiles of money holdings					Quintiles of capital holdings				
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
Conditional	-0.21	-0.39	-0.39	-0.35	-0.31	-0.43	-0.36	-0.32	-0.29	-0.25
Median	-0.41	-0.42	-0.37	-0.29	-0.14	-0.57	-0.45	-0.37	-0.31	-0.20
	Quintiles of human capital									
Conditional	-0.08	-0.22	-0.35	-0.40	-0.53					
Median	-0.09	-0.22	-0.39	-0.45	-0.62					

Policy regime: Fed reaction function										
	Quintiles of money holdings					Quintiles of capital holdings				
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
Conditional	-0.56	-0.55	-0.47	-0.36	-0.26	-0.48	-0.42	-0.41	-0.42	-0.43
Median	-0.71	-0.61	-0.47	-0.32	-0.09	-0.60	-0.52	-0.50	-0.47	-0.45
	Quintiles of human capital									
Conditional	-0.34	-0.42	-0.46	-0.42	-0.48					
Median	-0.32	-0.42	-0.49	-0.48	-0.60					

Notes: Welfare costs in terms of consumption equivalents (CE) as defined in (1.20). Conditional refers to integrating out the missing dimensions, whereas Median refers to median asset holdings of the respective other assets. We track households over 20 quarters and average over 100 independent model simulations.

time horizon captures the welfare consequences of the recession following the uncertainty shock. See Appendix E for an assessment of welfare after more than 75 years, when the initial position, (h_T, m_T, k_T), has washed out in the sense that the conditional and the unconditional distributions are almost identical. Of course, in the long run there are no differences between the two sets of economies.

On average, households would be willing to forgo roughly 0.41% of their consumption over 5 years to eliminate the uncertainty shock. This average loss masks heterogeneous effects across households with different asset positions and human capital. While monetary policy can reduce the cost to roughly 0.25% on average, it also shifts the burden of the shock between households. Figure 1.9 displays the expected welfare costs of households conditioning on two of the three dimensions of the (h, m, k)-space – integrating out the missing dimension.

Without stabilization, money rich and physical asset poor households lose the least. These are households that typically acquire physical capital in exchange for their money holdings, and they can do so at favorable capital prices after the uncertainty shock. For a similar reason, the steepness of the gradient in human capital is relatively modest. After the shock, human capital rich households suffer from lower wages, but as savers they are partly compensated, because they can acquire physical capital at lower prices. Table 1.4 summarizes the figures numerically. In this table, we condition on just one dimension of the households' portfolio, and display the average relative welfare gains. We do so in two ways: First, we calculate welfare conditional on one asset taking the conditional distribution of the other two assets into account. Second, we also report welfare effects at median asset holdings of the respective other assets. The latter isolates the direct effect in the dimension of interest.

Table 1.4 and Figure 1.9 shows that the intervention of the central bank helps households with high amounts of physical assets. In particular wealthy agents with low human capital profit the most from stabilization. Conversely, the capital poor but human-capital rich households profit the least from stabilization, because it is them who finance the increased money supply and they comparatively suffer from stable prices for the physical asset.

7. Conclusion

This paper examines how variations in uncertainty about household income affect the macroeconomy through precautionary savings. For this purpose we develop a novel and tractable framework that combines nominal rigidities and incomplete markets in which households choose portfolios of liquid paper and illiquid physical assets – merging incomplete markets with wealthy hand-to-mouth consumers and New Keynesian modeling. In this model, higher uncertainty about income triggers a flight to liquidity because it is superior for short-run consumption smoothing. This reduces not only consumption but also investment and hence depresses economic activity.

Calibrating the model to match the evolution of uncertainty about household income in the U.S., we find that a spike in income uncertainty can lead to substantive output, consumption, and investment losses. This may help us to understand the slow recovery of the U.S. economy during the Great Recession, for which we document a shift toward liquid assets across all percentiles of the U.S. wealth distribution. We find that a two standard deviation increase in household income uncertainty generates output losses that are sizable.

The welfare effects of such uncertainty shocks crucially depend on a household's asset position and the stance of monetary policy. Monetary policy that drastically increases the money supply in times of increased uncertainty limits the negative welfare effects of uncertainty shocks but redistributes from the asset poor to the asset rich.

Appendices

A. Dynamic Planning Problem with Two Assets

The dynamic planning problem of a household in the model is characterized by two Bellman equations, V_a in the case where the household can adjust its capital holdings and V_n otherwise

$$\begin{aligned} V_a(m,k,h;\Theta,s) =& max_{k',m'_a \in \Gamma_a} u[x(m,m'_a,k,k',h)] \\ &+ \beta\left[\nu EV^a(m'_a,k',h',\Theta',s') + (1-\nu)EV^n(m'_a,k',h',\Theta',s')\right] \\ V_n(m,k,h;\Theta,s) =& max_{m'_n \in \Gamma_n} u[x(m,m'_n,k,k,h)] \\ &+ \beta\left[\nu EV^a(m'_n,k,h',\Theta',s') + (1-\nu)EV^n(m'_n,k,h',\Theta',s')\right] \end{aligned} \tag{1.21}$$

where the budget sets are given by

$$\Gamma_a(m,k,h;\Theta,s) = \{m',k' \geq 0 | q(\Theta,s)(k'-k) + m' \leq \frac{\gamma}{1+\gamma} w(\Theta,s)hN + r(\Theta,s)k + \frac{m}{\pi(\Theta,s)}\} \tag{1.22}$$

$$\Gamma_n(m,k,h;\Theta,s) = \{m' \geq 0 | m' \leq \frac{\gamma}{1+\gamma} w(\Theta,s)hN + r(\Theta,s)k + \frac{m}{\pi(\Theta,s)}\} \tag{1.23}$$

$$x(m,m',k,k',h) = \frac{\gamma}{1+\gamma} w(\Theta,s)hN + r(\Theta,s)k + \frac{m}{\pi(\Theta,s)} - q(\Theta,s)(k'-k) - m' \tag{1.24}$$

To save on notation, let Ω be the set of possible idiosyncratic state variables controlled by the household, let Z be the set of potential aggregate states, let $\Gamma_i : \Omega \to \Omega$ be the correspondence describing the feasibility constraints, and let $A_i(z) = \{(\omega, y) \in \Omega \times \Omega : y \in \Gamma_i(\omega, z)\}$ be the graph of Γ_i. Hence the states and controls of the household problem can be defined as

$$\Omega = \{\omega = (m,k) \in R^2_+ : m,k \leq \infty\} \tag{1.25}$$

$$z = \{h, \Theta, s\} \tag{1.26}$$

and the return function $F : A \to R$ reads:

$$F(\Gamma_i(\omega,z),\omega;z) = \frac{x_i^{1-\gamma}}{1-\gamma} \tag{1.27}$$

Define the value before the adjustment/non-adjustment shock realizes as

$$v(\omega,z) := \nu V_a(\omega,z) + (1-\nu)V_n(\omega,z).$$

Now we can rewrite the optimization problem of the household in terms of the definitions above in a compact form:

$$V_a(\omega, z) = \max_{y \in \Gamma_a(\omega,z)} [F(\omega, y; z) + \beta_w Ev(y, z')] \tag{1.28}$$

$$V_n(\omega, z) = \max_{y \in \Gamma_n(\omega,z)} [F(\omega, y; z) + \beta_w Ev(y, z')]. \tag{1.29}$$

Finally we define the mapping $T : C(\Omega) \rightarrow C(\Omega)$, where $C(\Omega)$ is the space of bounded, continuous and weakly concave functions.

$$\begin{aligned} (Tv)(\omega, z) &= \nu V_a(\omega, z) + (1 - \nu)V_n(\omega, z) \qquad (1.30) \\ V_a(\omega, z) &= \max_{y \in \Gamma_a(\omega,z)} [F(\omega, y; z) + \beta_w Ev(y, z')] \\ V_n(\omega, z) &= \max_{y \in \Gamma_n(\omega,z)} [F(\omega, y; z) + \beta_w Ev(y, z')]. \end{aligned}$$

A.1 Properties of Primitives

The following properties of the primitives of the problem obviously hold:

P 1. *Properties of sets* $\Omega, \Gamma_a(\omega, z), \Gamma_n(\omega, z)$

1. Ω *is a convex subset of* R^3.

2. $\Gamma_i(\cdot, z) : \Omega \rightarrow \Omega$ *is non-empty, compact-valued, continuous, monotone and convex for all* z.

P 2. *Properties of return function* F

F is bounded, continuous, strongly concave, C^2 differentiable on the interior of A, and strictly increasing in each of its first two arguments.

A.2 Properties of the Value and Policy Functions

Lemma 1. *The mapping T defined by the Bellman equation for v fulfills Blackwell's sufficient conditions for a contraction on the set of bounded, continuous and weakly concave functions $C(\Omega)$.*

a) It satisfies discounting.

b) It is monotonic.

c) It preserves boundedness (assuming an arbitrary maximum consumption level).

d) It preserves strict concavity.

Hence, the solution to the Bellman equation is strictly concave. The policy is a single-valued function in (m, k), and so is optimal consumption.

Proof. The proof proceeds item by item and closely follows Nancy L. Stokey (1989) taking into account that the household problem in the extended model consists of two Bellman equations.

a) Discounting
Let $a \in R_+$ and the rest be defined as above. Then it holds that:

$$\begin{aligned}(T(v+a))(\omega,z) =&\nu \max_{y\in\Gamma_a(\omega,z)}[F(\omega,y,z)+\beta_w Ev(y,z')+a]\\ &+(1-\nu)\max_{y\in\Gamma_n(\omega,z)}[F(\omega,y,z)+\beta_w Ev(y,z')+a]\\ =&(Tv)(\omega,z)+\beta_w a\end{aligned}$$

Accordingly, T fulfills discounting.

b) Monotonicity
Let $g:\Omega\times Z\to R^2$, $f:\Omega\times Z\to R^2$ and $g(\omega,z)\geq f(\omega,z)\ \forall\omega,z\in\Omega\times Z$, then it follows that:

$$\begin{aligned}(Tg)(\omega,z) =&\nu \max_{y\in\Gamma_a(\omega,z)}[F(\omega,y,z)+\beta_w Eg(y,z')]\\ &+(1-\nu)\max_{y\in\Gamma_n(\omega,z)}[F(\omega,y,z)+\beta_w Eg(y,z')]\\ \geq&\nu \max_{y\in\Gamma_a(\omega,z)}[F(\omega,y,z)+\beta_w Ef(y,z')]\\ &+(1-\nu)\max_{y\in\Gamma_n(\omega,z)}[F(\omega,y,z)+\beta_w Ef(y,z')]\\ =&Tf(\omega,z)\end{aligned}$$

The objective function for which Tg is the maximized value is uniformly higher than the function for which Tf is the maximized value. Therefore, T preserves monotonicity.

c) Boundedness
From properties **P1** it follows that the mapping T defines a maximization problem over the continuous and bounded function $[F(\omega,y)+\beta_w Ev(y,z'))]$ over the compact sets $\Gamma_i(\omega,z)$ for $i=\{a,n\}$. Hence the maximum is attained. Since F and v are bounded, Tv is also bounded.

d) Strict Concavity
Let $f\in C''(\Omega)$, where $C^{''}$ is the set of bounded, continuous, strictly concave functions on Ω. Since the convex combination of two strictly concave functions is strictly concave, it is sufficient to show that $T_i[C''(\Omega)]\subseteq C''(\Omega)$, where T_i is defined by

$$T_i v = \max_{y\in\Gamma_i(\omega,z)}[F(\omega,y,z)+\beta_w Ev(y,z')], i\in\{a,n\}$$

Let $\omega_0\neq\omega_1, \theta\in(0,1), \omega_\theta=\theta\omega_0+(1-\theta)\omega_1$.
Let $y_j\in\Gamma_i(\omega_j,z)$ be the maximizer of $(T_if)(\omega_j)$ for $j=\{0,1\}$ and $i=\{a,n\}$,

$y_\theta = \theta y_0 + (1-\theta)y_1$.

$$\begin{aligned}(T_i f)(\omega_\theta, z) \geq & [F(\omega_\theta, y_\theta, z) + \beta_w Ef(y_\theta, z')] \\ > & \theta[F(\omega_0, y_0, z) + \beta_w Ef(y_0, z'))] + (1-\theta)[F(\omega_1, y_1, z) + \beta_w Ef(y_1, z')] \\ = & \theta(Tf)(\omega_0, z) + (1-\theta)(Tf)(\omega_1, z)\end{aligned}$$

The first inequality follows from y_θ being feasible because of convex budget sets. The second inequality follows from the strict concavity of f. Since ω_0 and ω_1 are arbitrary, it follows that $T_i f$ is strictly concave, and since f is arbitrary that $T[C''(\Omega)] \subseteq C''(\Omega)$.

□

Lemma 2. *The value function is C^2 and the policy function C^1 differentiable.*

Proof. The properties of the choice set **P1**, of the return function **P2**, and the properties of the value function proven in (1) fulfill the assumptions of Santos's (1991) theorem on the differentiability of the policy function. According to the theorem, the value function is C^2 and the policy function C^1 differentiable.
Note that strong concavity of the return function holds for CRRA utility, because of the arbitrary maximum we set for consumption. □

Lemma 3. *The total savings $S_i^* := m_i^*(\omega, z) + q(z)k_i^*(\omega, z)$ and consumption $c_i^*, i \in \{a, n\}$ are increasing in ω if $r(z)$ is positive. In the adjustment case total savings and consumption are increasing in total resources $R_a = [q(z)+r(z)]k+m/\pi(z)$ for any r(z).*

Proof. Define $\tilde{v}(S, z) := \max_{\{m,k|m+q(z)k\leq S\}} Ev(m, k; z')$ and resources in the case of no adjustment $R_n = r(z)k + m/\pi(z)$. Since v is strictly concave and increasing, so is $\tilde{v}$ by the line of the proof of Lemma 1.d). Now we can (re)write the planning problem as

$$V_a(m, k; z) = \max_{S\leq\frac{\gamma}{1+\gamma}w(z)hN+R_a}[u(\frac{\gamma}{1+\gamma}w(z)hN + [q(z) + r(z)]k + m/\pi(z) - S) + \beta_W\tilde{v}(S, z)]$$
$$V_n(m, k; z) = \max_{m'\leq\frac{\gamma}{1+\gamma}w(z)hN+R_n}[u(\frac{\gamma}{1+\gamma}w(z)hN + r(z)k + m/\pi(z) - m') + \beta_W Ev(m', k; z')].$$

Due to differentiability we obtain the following (sufficient) first-order conditions:

$$\frac{\partial u\left(\frac{\gamma}{1+\gamma}w(z)hN + [q(z) + r(z)]k + m/\pi(z) - S\right)}{\partial c} = \beta_W\frac{\partial\tilde{v}(S, z)}{\partial S}$$
$$\frac{\partial u\left(\frac{\gamma}{1+\gamma}w(z)hN + r(z)k + m/\pi(z) - m'\right)}{\partial c} = \beta_W\frac{\partial v(m', k; z)}{\partial m'}. \tag{1.31}$$

Since the left-hand sides are decreasing in $\omega = (m, k)$, and increasing in S (respectively m'), and the right-hand side is decreasing in S (respectively m'), $S_i^* = \begin{cases} qk' + m' \text{ if } i = a \\ qk + m' \text{ if } i = n \end{cases}$ must be increasing in ω.
Since the right-hand side of (1.31) is hence decreasing in ω, so must be the left-hand

side of (1.31). Hence consumption must be increasing in ω.
The last statement follows directly from the same proof. □

A.3 Euler Equations

Denote the optimal policies for consumption, for money holdings and capital as $x_i^*, m_i^*, k^*, i \in \{a, n\}$ respectively. The first-order conditions for an inner solution in the (non-)adjustment case read:

$$k^* : \frac{\partial u(x_a^*)}{\partial x} q = \beta E \left[\nu \frac{\partial V_a(m_a^*, k^*; z')}{\partial k} + (1-\nu) \frac{\partial V_n(m_a', k'; z')}{\partial k} \right] \quad (1.32)$$

$$m_a^* : \frac{\partial u(x_a^*)}{\partial x} = \beta E \left[\nu \frac{\partial V_a(m_a^*, k^*; z')}{\partial m} + (1-\nu) \frac{\partial V_n(m_a^*, k^*; z')}{\partial m} \right] \quad (1.33)$$

$$m_n^* : \frac{\partial u(x_n^*)}{\partial x} = \beta E \left[\nu \frac{\partial V_a(m_n^*, k; z')}{\partial m} + (1-\nu) \frac{\partial V_n(m_n^*, k; z')}{\partial m} \right] \quad (1.34)$$

Note the subtle difference between (1.33) and (1.34), which lies in the different capital stocks k' vs. k in the right-hand side expressions.

Differentiating the value functions with respect to k and m, we obtain:

$$\frac{\partial V_a(m,k;z)}{\partial k} = \frac{\partial u[x_a^*(m,k;z)]}{\partial x}(q(z) + r(z)) \quad (1.35)$$

$$\frac{\partial V_a(m,k;z)}{\partial m} = \frac{\partial u[x_a^*(m,k;z)]}{\partial x}\pi(z)^{-1} \quad (1.36)$$

$$\frac{\partial V_n(m,k;z)}{\partial m} = \frac{\partial u[x_n^*(m,k;z)]}{\partial x}\pi(z)^{-1} \quad (1.37)$$

$$\begin{aligned}\frac{\partial V_n(m,k;z)}{\partial k} &= r(z)\frac{\partial u[x_n^*(m,k;z)]}{\partial x} \\ &+ \beta E\left[\nu \frac{\partial V_a[m_n^*(m,k;z), k; z']}{\partial k} + (1-\nu)\frac{\partial V^n[m_n^*(m,k;z), k; z']}{\partial k}\right] \\ &= r(z)\frac{\partial u[x_n^*(m,k;z)]}{\partial x} + \beta\nu E \frac{\partial u\{x_a^*[m_n^*(m,k;z),k;z],k;z'\}}{\partial x}(q(z') + r(z')) \\ &+ \beta(1-\nu)E\frac{\partial V_n\{[m_n^*(m,k;z),k;z],k;z'\}}{\partial k}\end{aligned} \quad (1.38)$$

such that the marginal value of capital in non-adjustment is defined recursively.

Now we can plug the second set of equations into the first set of equations and obtain the following Euler equations (in slightly shortened notation):

$$\frac{\partial u[x_a^*(m,k;z)]}{\partial x}q(z) = \beta E\left[\nu\frac{\partial u[x_a^*(m_a^*,k^*;z')]}{\partial x}[q(z')+r(z')] + (1-\nu)\frac{\partial V^n(m_a^*,k';z')}{\partial k'}\right] \tag{1.39}$$

$$\frac{\partial u[x_a^*(m,k;z)]}{\partial x} = \beta E\pi'(z')^{-1}\left[\nu\frac{\partial u[x_a^*(m_a^*,k^*;z')]}{\partial x} + (1-\nu)\frac{\partial u[x_n^*(m_a^*,k';z')]}{\partial x}\right] \tag{1.40}$$

$$\frac{\partial u[x_n^*(m,k,;z)]}{\partial x} = \beta E\pi'(z')^{-1}\left[\nu\frac{\partial u[x_a^*(m_n',k;z')]}{\partial x} + (1-\nu)\frac{\partial u[x_n^*(m_n^*,k;z')]}{\partial x}\right] \tag{1.41}$$

A.4 Algorithm

The algorithm we use to solve for optimal policies given the Krusell-Smith forecasting rules is a version of Hintermaier and Koeniger's (2010) extension of the endogenous grid method, originally developed by Carroll (2006).

It works iteratively until convergence of policies as follows: Start with some guess for the policy functions x_a^* and x_n^* on a given grid $(m,k) \in M \times K$. Define the shadow value of capital

$$\begin{aligned}\beta^{-1}\psi(m,k;z) :=& \nu E\left\{\frac{\partial u\{x_a^*[m_n^*(m,k,z),k;z']\}}{\partial x}[q(z')+r(z')]\right\} \\ &+ (1-\nu)E\frac{\partial V_n[m_n^*(m,k,z),k;z']}{\partial k} \\ &= \nu E\left\{\frac{\partial u\{x_a^*[m_n^*(m,k,z),k;z']\}}{\partial x}[q(z')+r(z')]\right\} \\ &+ (1-\nu)E\left\{\frac{\partial u\{x_n^*(m_n^*(m,k,z),k;z')]}{\partial x}r(z')\right\} \\ &+ (1-\nu)E\left\{\psi[m_n^*(m,k,z),k;z']\right\}.\end{aligned} \tag{1.42}$$

Guess initially $\psi = 0$. Then

1. Solve for an update of x_n^* by standard endogenous grid methods using equation (1.41), and denote $m_n^*(m,k;z)$ as the optimal money holdings without capital adjustment.

2. Find for every k' on-grid some (off-grid) value of $\tilde{m}_a^*(k';z)$ such that combining (1.40) and (1.39) yields:

$$\begin{aligned}0 =& \nu E\left\{\frac{\partial u[x_a^*(\tilde{m}_a^*(k',z),k';z')]}{\partial x}\left[\frac{q(z')+r(z')}{q(z)} - \pi(z')^{-1}\right]\right\} \\ &+ (1-\nu)E\left\{\frac{\partial u[x_n^*(\tilde{m}_a^*(k',z),k';z')]}{\partial x}\left[\frac{r(z')}{q(z)} - \pi(z')^{-1}\right]\right\} + (1-\nu)E\left[\frac{\psi(\tilde{m}_a^*(k',z),k';z')}{q(z)}\right]\end{aligned} \tag{1.43}$$

N.B. that $E\psi$ takes the stochastic transitions in h' into account and does not replace the expectations operator in the definition of ψ. If no solution exists, set $\tilde{m}_a^* = 0$. Uniqueness (conditional on existence) of $\tilde{m}_a^*$ follows from the strict concavity of v.

3. Solve for total initial resources, by solving the Euler equation (1.40) for $\tilde{x}^*(k', z)$, such that:

$$\tilde{x}^*(k', z) \\ = \frac{\partial u}{\partial x}^{-1} \left\{ \beta E \pi(z')^{-1} \left[\nu \frac{\partial u\{x_a^*[m_a^*(k', z), k'; z']\}}{\partial x} + (1-\nu) \frac{\partial u\{x_n^*[m_a^*(k', z), k'; z']}{\partial x} \right] \right\} \tag{1.44}$$

where the right-hand side expressions are obtained by interpolating $x_a^*(m_a^*(k', z), k', z')$ from the on-grid guesses $x_a^*(m, k; z)$ and taking expected values with respect to z'.

This way we obtain total non-human resources $\tilde{R}_a(k', z)$ that are compatible with plans $(m^*(k'), k')$ and a consumption policy $\tilde{\tilde{x}}_a^*(\tilde{R}_a(k', z), z)$ in total resources.

4. Since (consumption) policies are increasing in resources, we can obtain consumption policy updates as follows: Calculate total resources for each (m, k) pair $R_a(m, k) = (q+r)k + m/\pi$ and use the consumption policy obtained before to update $x_a^*(m, k, z)$ by interpolating at $R_a(m, k)$ from the set $\left\{ (\tilde{\tilde{x}}_a^*(\tilde{R}_a(k', z), z), R_a(k', z)) \middle| k' \in K \right\}$.[16]

5. Update ψ: Calculate a new value of ψ using (1.38), such that:

$$\begin{aligned} \psi^{new}(m, k, z) = & \beta \nu E \left\{ \frac{\partial u\{x_a^*[m_n^*(m, k, z), k; z']\}}{\partial x} [q(z') + r(z')] \right\} \\ & + \beta(1-\nu) E \left\{ \frac{\partial u\{x_n^*(m_n^*(m, k, z), k; z')]}{\partial x} r(z') \right\} \\ & + \beta(1-\nu) E \left\{ \psi^{old}[m_n^*(m, k, z), k; z'] \right\}. \end{aligned} \tag{1.45}$$

making use of the **updated** consumption policies.

B. Estimation of the Stochastic Volatility Process for Household Income

B.1 Income Process

We assume that the observed log-income of a household, $y_{i,a,t}$, is composed of four components: a deterministic part $f(o_{i,a,t})$, a transitory part $\tau_{i,a,t}$, a persistent part

[16]If a boundary solution $\tilde{m}^*(0) > 0$ is found, we use the "n" problem to obtain consumption policies for resources below $\tilde{m}^*(0)$.

$h_{i,a,t}$, and a permanent part μ_i such that:

$$\begin{aligned} y_{i,a,t} &= f(o_{i,a,t}) + y^*_{i,a,t}, && (1.46)\\ y^*_{i,a,t} &= \tau_{i,a,t} + h_{i,a,t} + \mu_i, && (1.47)\\ h_{i,a,t} &= \rho_h h_{i,a-1,t-1} + \epsilon_{i,a,t}, && (1.48)\end{aligned}$$

where $o_{i,a,t}$ is observable characteristics of the household's head, $y^*_{i,a,t}$ is the stochastic component of a household's income ("residual income"), t is calendar time, and a is the household's years of labor market experience. We assume that all households start with $h_{i,0,t} = 0$ when they enter the labor market.

For the shocks $\epsilon_{i,a,t}$ to the persistent part h we assume them to be Gaussian, $\epsilon_{i,a,t} \sim N(0, \sigma_t^{\epsilon,2})$, with a time-varying variance that follows an AR(1) process (in logs) plus quadratic trend.

$$\begin{aligned} \log \sigma_t^{\epsilon,2} &= (1-\rho_s)\mu_s + \xi_1 t + \xi_2 t^2 + \rho_s \log \sigma_{t-1}^{\epsilon,2} + \varepsilon_t, && (1.49)\\ \varepsilon_t &\sim N(0, \sigma_s^2). && (1.50)\end{aligned}$$

For the variances of the fixed effect μ_i we assume them to be cohort specific, such that $\mu_i \sim N(0, \sigma_{t-a}^{\mu,2})$, where $t-a$ denotes the birth cohort. We assume the transitory component, $\tau_{i,a,t} \sim N(0, \sigma_\tau^2)$, to have a constant variance.

Income Variances

Under the above assumptions, the variance of residual income, $y^*_{i,a,t}$, is given by

$$\begin{aligned} \sigma_{a,t}^{y,2} &= \sigma_\tau^2 + \sigma_{\mu,t-a}^2 + \sigma_{a,t}^{h,2}, && (1.51)\\ \sigma_{a,t}^{h,2} &= \rho_h^2 \sigma_{a-1,t-1}^{h,2} + \sigma_t^{\epsilon,2}; \quad \sigma_{0,t}^{h,2} = 0, && (1.52)\\ \log \sigma_t^{\epsilon,2} &= (1-\rho_s)\mu_s + \xi_1 t + \xi_2 t^2 + \rho_s \log \sigma_{t-1}^{\epsilon,2} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_s^2). && (1.53)\end{aligned}$$

We use the above equations to identify the parameters governing the stochastic volatility process, Equation (1.6), $\{\rho_h, \rho_s, \bar{\sigma}, \sigma_s^2\}$ from the data.[17]

B.2 Data

We take the 1970-2009 Cross-National Equivalent File (CNEF) of the Panel Study of Income Dynamics (PSID) and drop the low income sample. We keep all households in the sample that have at least two but no more than 10 household members who work combined at least 1040 hours per year and a male household head no younger than 25 and not older than 55. We focus on the age of 25 to 55 to abstract from the effects of household formation and retirement. We construct household income as pre-tax labor income plus private and public transfers minus all taxes. These selection criteria yield a sample that has on average about 1815 observations for each year of the survey.

[17]Where $\bar{\sigma} = sqrt(exp(\mu_s + \frac{\sigma_s^2}{(1-\rho_s^2)})$ corresponds to the level-mean of Equation (1.53).

B.3 Estimation

Our estimation procedure proceeds in two steps. First we estimate the deterministic component, $f(o_{i,a,t})$ (Eq. (1.54)), by running an OLS regression of log household income on time dummies, age dummies, schooling dummies interacted with up to a quadratic age trend, and household size dummies.

$$f(o_{i,a,t}) = \theta_0 + \theta_1^T \mathbf{D}_t + \theta_2^T \mathbf{x}_{i,a,t}, \tag{1.54}$$

where $\mathbf{D}_t$ is a vector of year dummy variables, $t = \{1970, ..., 1997, 1999, 2001, 2003, 2005, 2007, 2009\}$, and $\mathbf{x}_{i,a,t}$ is a vector containing all remaining regressors for household i with a years of labor market experience at date t. We eliminate any observation where the residual of this regression, $y^*_{i,a,t}$, belongs to the bottom or top per percent of all residuals for an age group.

From the residuals of this regression, we then calculate the sample variance within an age-year cell, $s^2_{a,t}$, across ages, $a = \{1, \ldots, 31\}$, and times t as well as covariances $c^1_{a,t} = cov(y^*_{i,a,t}, y^*_{i,a-1,t-1})$ and $c^2_{a,t} = cov(y^*_{i,a,t}, y^*_{i,a-2,t-2})$. This yields 992 sample-variance and 1798 sample-covariance estimates, where each estimate is constructed from on average 55 observations on the log-income residual.

Given the income process as laid out above, we can derive the moment conditions corresponding to the estimates for empirical variance s^2 and first and second order auto covariances in residual household income c^1, c^2 for each age-year combination.

$$s^2_{a,t} = \sigma^{\mu,2}_{t-a} + \sigma^2_\tau + \sum_{j=1}^{a-1} \rho_h^{2j} \sigma^{h,2}_{a-j,t-j} + \psi^s_{a,t} \tag{1.55}$$

$$c^1_{a,t} = \sigma^{\mu,2}_{t-a} + \rho_h \sigma^{h,2}_{a-1,t-1} + \psi^{c1}_{a,t} \tag{1.56}$$

$$c^2_{a,t} = \sigma^{\mu,2}_{t-a} + \rho_h^2 \sigma^{h,2}_{a-2,t-2} + \psi^{c2}_{a,t} \tag{1.57}$$

where $\sigma^{h,2}_{a,t}$ obeys Equations (1.52) and (1.53) and ψ are the residuals.

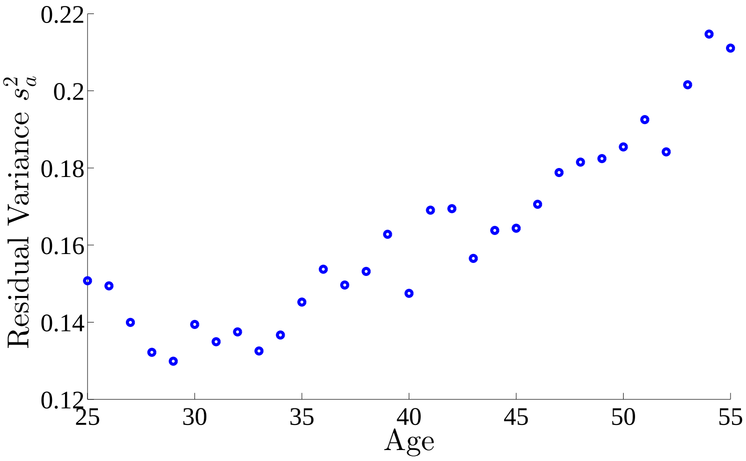
B.4 Results

First-Stage Regression

The first-stage regression, Equation (1.54), controls for observable household characteristics and hence filters out the deterministic cross-sectional variation in household income. The results are comparable to existing studies, implying a concave earnings function in age and education. The inclusion of age-education interactions as well as controlling for age, education, and household size nonparametrically considerably raises the $R^2 = 0.6$.

The residuals of this regression yield the idiosyncratic component of income, $y^*_{i,a,t}$, from which we obtain the idiosyncratic cross-sectional variation in household income. Figure 1.10 depicts the variance of idiosyncratic income by age averaged across 1970-2009. The variance at labor market entry is already substantial and it increases by about 50% after 30 years of labor market participation. The initial dispersion helps to identify $\sigma^2_\tau + \sigma^{\mu,2}_{t-a}$, whereas the rate of increase contains information on $\sigma^{h,2}_{a,t}$.

Figure 1.10: Idiosyncratic cross-sectional variance by age



Notes: Cross-sectional variance averaged across time calculated from the residuals of the first-stage regression.

Parameter Estimates

We estimate the following parameters by the generalized method of moments (GMM) minimizing the sum of squared residuals ψ^2 given the moment conditions in Equations (1.55), (1.56), and (1.57):

$$\begin{pmatrix} \rho_h & \rho_s & \mu_s & \xi_1 & \xi_2 & \sigma_t^{\mu,2} & \sigma_\tau^2 & \sigma_t^{\epsilon,2} \end{pmatrix}, \tag{1.58}$$

for $t = \{1939, ..., 1969, \ldots, 2009\}$.

We can track the history of the variance of persistent income shocks, $\sigma_t^{\epsilon,2}$, back to the year when the oldest cohorts at the start of the survey in 1970 entered the labor market. This way we obtain a time series for income uncertainty going back to 1939, see Figure 1.11.

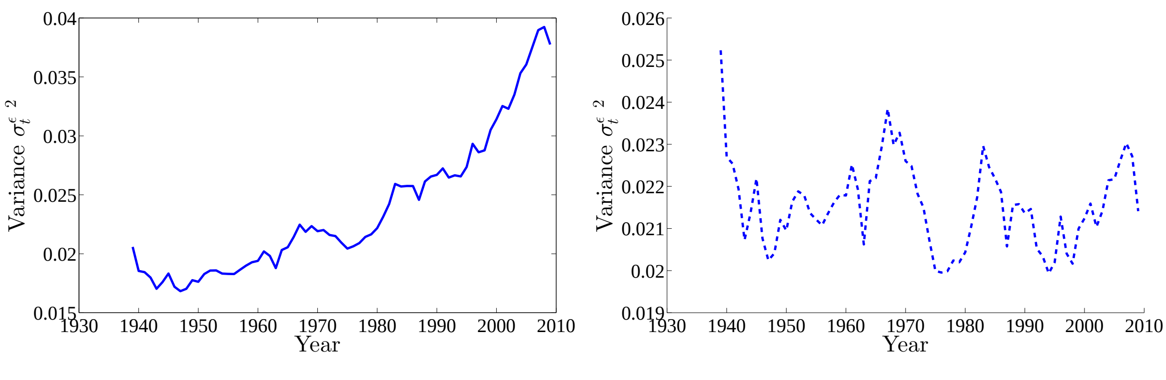
Table 1.5 summarizes the parameters values of interest. Persistent shocks to idiosyncratic income have an annual autocorrelation of $\rho_h = 0.9069$ and an average standard deviation of $\bar{\sigma} = sqrt(exp(\mu_s + \frac{var(\sigma_t^{\epsilon,2})}{2(1-\rho_s^2)}) = 0.1483$ – similar to the estimates by Storesletten et al. (2004). For shocks to the variance of persistent income shocks, we estimate an annual autocorrelation of $\rho_s = 0.6651$ and a coefficient of variation of $\frac{\sigma_s}{\sqrt{1-\rho_s^2}} = sqrt(exp(\frac{var(\sigma_t^{\epsilon,2})}{(1-\rho_s^2)}) - 1) = 0.607$.

Table 1.5: Parameter estimates

ρ_h	ρ_s	$\bar{\sigma}$	$\frac{\sigma_s}{\sqrt{1-\rho_s^2}}$
0.9069	0.6651	0.1483	0.6070

Notes: Where $\bar{\sigma}$ in Equation 1.6 corresponds to the (level-)mean of the persistent component, $sqrt(exp(\mu_s + \frac{var(\sigma_t^{\epsilon,2})}{(1-\rho_s^2)})$, and the risk-shifting parameter s follows from the coefficient of variation implied by the variation in the persistent component, $sqrt(exp(\frac{var(\sigma_t^{\epsilon,2})}{(1-\rho_s^2)}) - 1)$. These annual estimates are then converted to quarterly frequency.

Figure 1.11: Idiosyncratic income uncertainty 1939-2009



Notes: Constructed time series for the variance of persistent idiosyncratic income shocks based on PSID data. The second panel is without the linear-quadratic trend.

C. Asset Distribution

Table 1.6 summarizes the wealth distribution implied by our model (i.e., for the baseline calibration without fluctuations in uncertainty). As with any incomplete markets model that does not resort to heterogeneity in preferences or extremely skewed processes for idiosyncratic productivity, we fail to match the skewness in wealth documented for the U.S. Whereas the fraction of wealth held by the richest quintile is about 80% in the U.S., the top quintile in our model holds only 41% of total wealth. The same discrepancy holds for the Gini coefficient, where our model falls short as well – 0.38 versus 0.8 in the data.

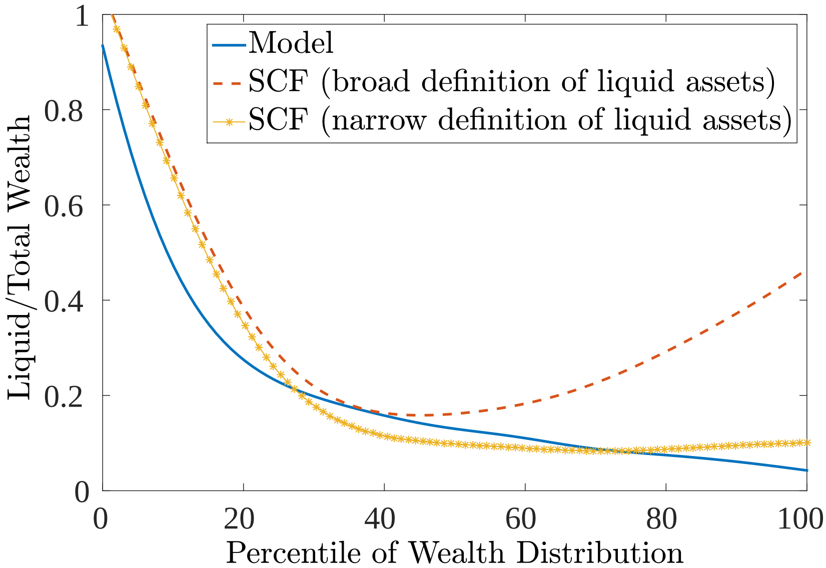
Table 1.6: Asset distribution

	Quintiles					Gini-Coeff.
	1st	2nd	3rd	4th	5th	
Fraction of total wealth	3.62	10.93	17.98	26.16	41.31	0.38
...share held in money	31.99	18.54	12.56	8.32	5.54	
...share held in capital	68.01	81.46	87.44	91.68	94.46	
Fraction without money	0.87	1.00	1.61	2.42	2.65	
Fraction without capital	4.30	0.12	0.02	0.00	0.00	

These shortcomings are, however, not of great importance for our transmission mechanism. The top quintile is well insured, because they hold a sizable amount of liquid assets. Hence, they are least affected by ups and downs in uncertainty. The lower quintiles are the ones building up precautionary savings and thus the ones that react strongest to changes in uncertainty. In this dimension our model replicates the data fairly well. The poorest quintile in the U.S. has about zero wealth on average – including indebted households. The poorest households in our model hold only few assets – 3.6% of total wealth.

Our model also has implications for the ratio of liquid to illiquid assets conditional on how rich households are in total. Households save in money because it provides better short-term consumption smoothing than capital. This value of liquidity decreases in the amount of money a household holds. Hence, our model implies that the share of liquid assets in the portfolio declines in total wealth. Figure 1.12 plots the prediction of our model and the data equivalent taken from the Survey of Consumer Finances 2004 (SCF) according to the definitions by Kaplan and Violante (2014). The poorest households in the U.S. and in our model predominantly hold liquid assets. The share of liquid assets then rapidly falls below 20% in both graphs, but rises again in the SCF for the richest households. This is because stocks, mutual funds, and non-governmental bond holdings are concentrated at the top quintiles as can be seen by comparing the broad liquidity measure, which includes all of those, to the narrow definition. If we also exclude those assets that usually induce some transaction cost (e.g., a commission)

Figure 1.12: Share of liquid assets in total net worth against percentiles of total wealth in 2004



Notes: We compare our measure of liquid net worth (see Figure 1.1) to a broader definition of liquid assets that includes mutual funds, stocks, and non-governmental bonds as in Kaplan and Violante (2014). For graphical illustration we make use of an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15.

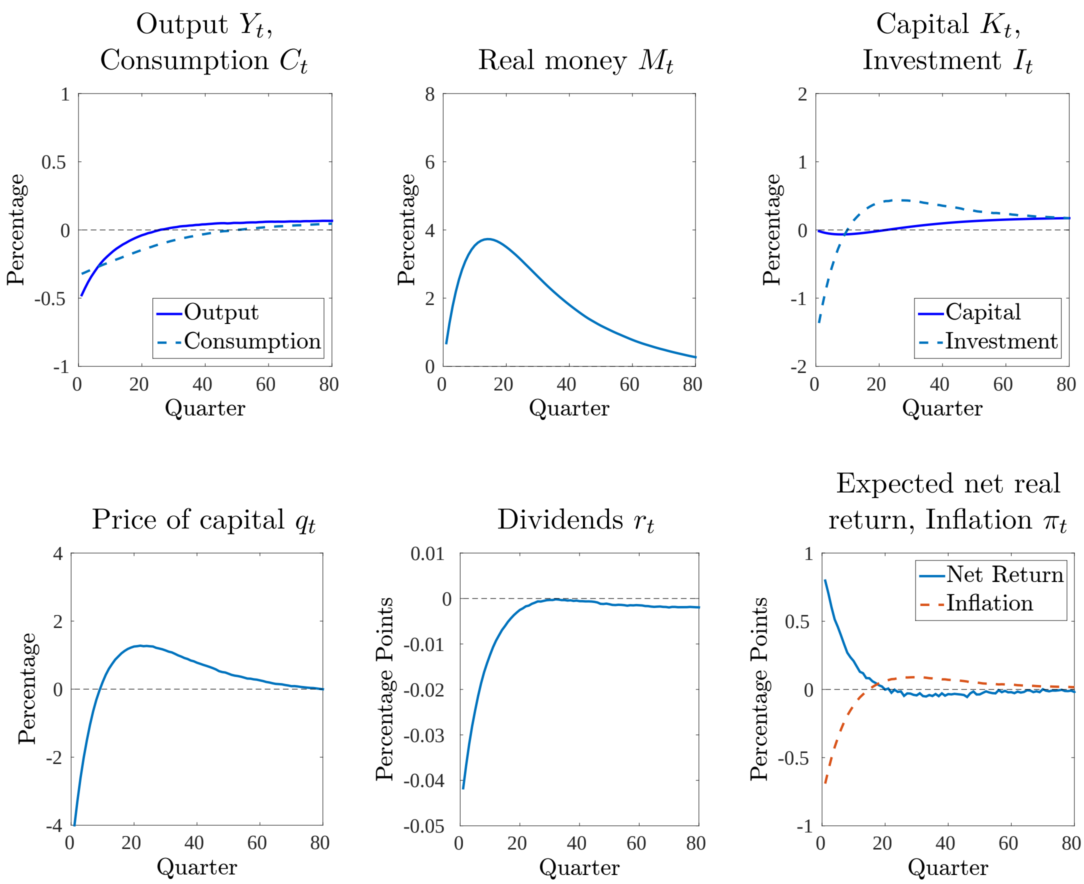
when acquiring them from a bank or broker, the share of liquid assets is substantially reduced for the asset rich.

D. Quality of the Numerical Solution

The equilibrium forecasting rules are obtained by regressing them in each iteration of the algorithm on 10.000 observations. We generate the observations by simulating the model in parallel on 20 machines, letting each economy run for 750 periods and discarding the first 250 periods. The R^2 is generally above 99% for all calibrations; see Tables 1.9 and 1.10. In the case of perfect stabilization, π_t is virtually constant, such that the R^2 of the π-forecasting is a nonsensical statistic. Figure 1.13 shows that our results are robust to increasing the resolution for the aggregate state variables.

Following Den Haan (2010), we also test the out-of-sample performance of the forecasting rules. For this we initialize the model and the forecasting rules at steady state values, feed in the same shock sequence, but otherwise let them run independently. Figure 1.14 plots time series of the prices q and π as well as the states K and M taken from the simulation of the model and the forecasting rules. The equilibrium forecasting rules track the evolution of the underlying model without any tendency of divergence. Table 1.7 summarizes the mean and maximum difference between the series generated

Figure 1.13: Uncertainty shock with higher resolution in K and M



Notes: See figure 1.5. We increase the number of gridpoints to 5 for capital and 7 for money.

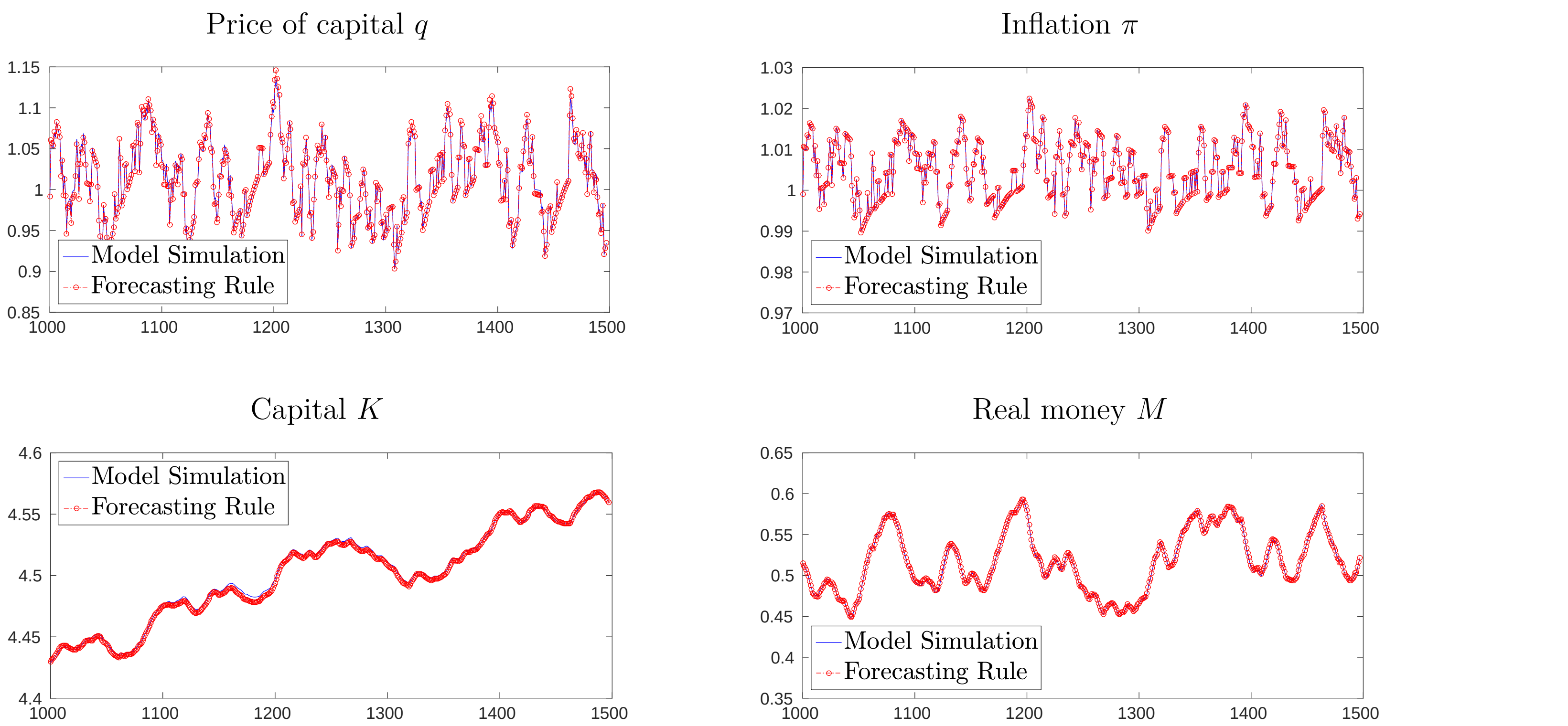
by the model and the forecasting rules. The mean error for all four time series is less than 0.3%. The maximum errors are small, too.

Table 1.7: Forecasting errors

	Price of capital q_t	Capital K_t	Inflation π_t	Real money M_t
Mean Error	0.28	0.07	0.01	0.22
Max Error	1.20	0.20	0.07	0.69

Notes: Percentage differences in out-of-sample forecasts between forecasting rules and model for $t = \{1, ..., 1.500\}$; see Den Haan (2010).

Figure 1.14: Out-of-sample performance of the forecasting rules



Notes: Out-of-sample comparison between law of motions and model zoomed in at $t = \{1000, ..., 1.500\}$ for visibility; see Den Haan (2010).

E. Welfare

Table 1.8 provides the long run welfare effects with and without stabilization after 75 years when the economy is back at its steady state.

Table 1.8: Welfare after 75 years

	Policy regime: Constant money growth									
	Quintiles of money holdings					Quintiles of capital holdings				
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
Conditional	-0.20	-0.18	-0.15	-0.11	-0.05	-0.11	-0.10	-0.12	-0.14	-0.18
Median	-0.23	-0.18	-0.13	-0.08	0.02	-0.14	-0.13	-0.14	-0.16	-0.18
	Quintiles of Human Capital									
Conditional	-0.09	-0.13	-0.14	-0.14	-0.16					
Median	-0.06	-0.11	-0.15	-0.16	-0.23					
	Policy regime: Inflation stabilization									
	Quintiles of money holdings					Quintiles of capital holdings				
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
Conditional	-0.03	-0.06	-0.06	-0.06	-0.07	-0.06	-0.05	-0.06	-0.06	-0.07
Median	-0.06	-0.05	-0.04	-0.02	-0.00	-0.07	-0.05	-0.04	-0.04	-0.03
	Quintiles of human capital									
Conditional	0.01	-0.01	-0.05	-0.08	-0.14					
Median	0.02	-0.01	-0.05	-0.08	-0.17					
	Policy regime: Fed reaction function									
	Quintiles of money holdings					Quintiles of capital holdings				
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
Conditional	-0.11	-0.11	-0.10	-0.08	-0.07	-0.08	-0.08	-0.09	-0.10	-0.11
Median	-0.14	-0.11	-0.08	-0.05	0.00	-0.10	-0.09	-0.09	-0.09	-0.10
	Quintiles of human capital									
Conditional	-0.04	-0.07	-0.09	-0.10	-0.15					
Median	-0.01	-0.06	-0.09	-0.12	-0.20					

Notes: Welfare costs in terms of consumption equivalents (`CE`) as defined in (1.20). Conditional refers to integrating out the missing dimensions, whereas Median refers to median asset holdings of the respective other assets. We track households over 300 quarters and average over 100 independent model simulations.

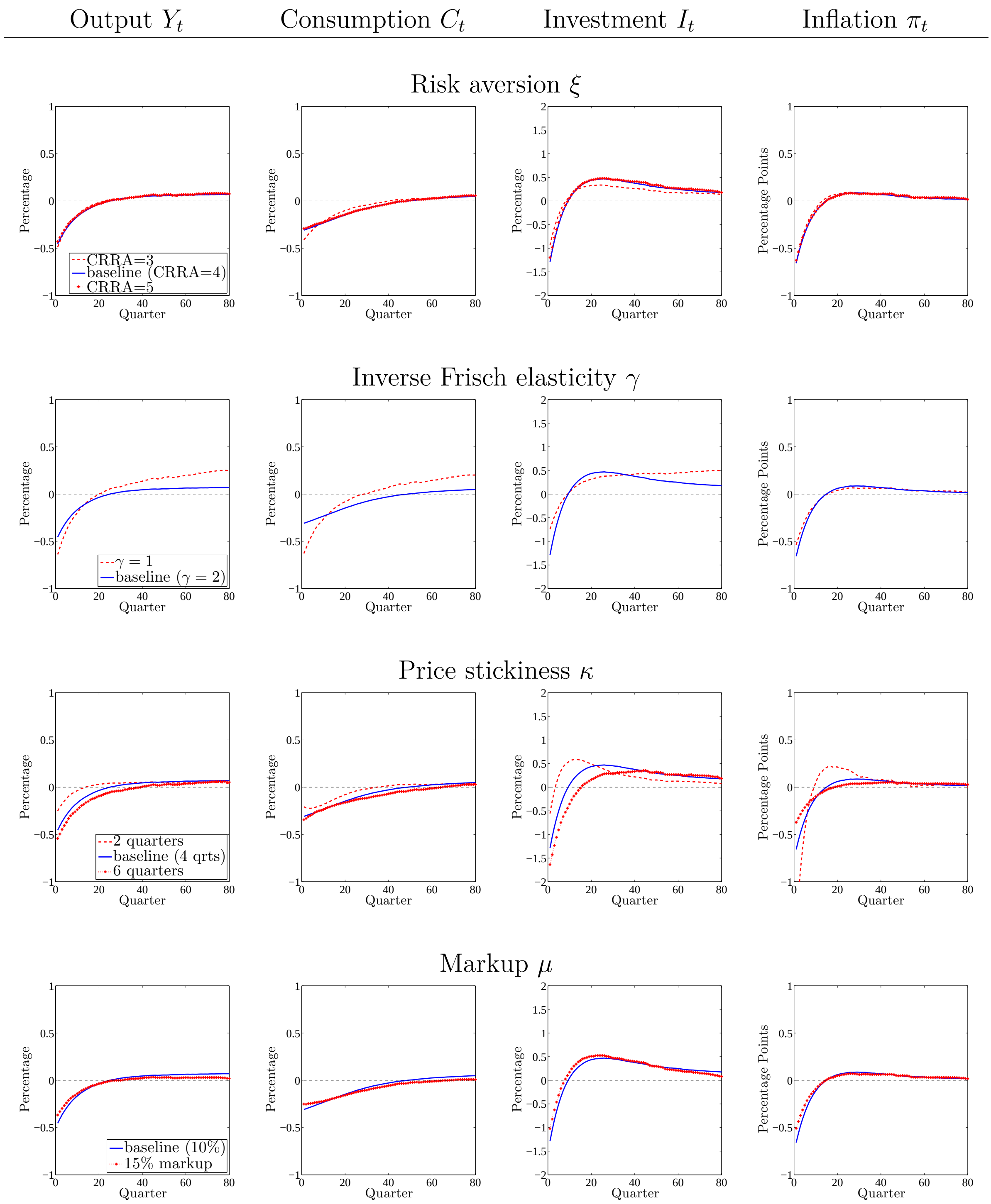
F. Robustness Checks

For the risk aversion parameter, the Frisch elasticity of labor supply, the average markup, and the frequency of price adjustment, we take standard values from the literature as there is no direct counterpart in the data. To account for this loose calibration strategy, we check the robustness of our findings with respect to the assumed parameter values. We do so by varying one of the parameters at a time while recalibrating the discount factor and the participation frequency of households to match the targeted capital and money to output ratios and the capital adjustment costs to match a relative investment volatility of 3.[18]

We find our results to be robust to all the considered parameter variations. The impulse response functions for output, consumption, investment, and inflation are displayed in Figure 1.15. The output drop on impact always remains around 0.5% – the result of our baseline calibration. Key for this robustness is the recalibration of other parameters. For example, if households are assumed to be less risk averse, then capital must be less liquid to match the observed holdings of liquid assets. Therefore, while a lower risk aversion makes the increase in precautionary savings less pronounced, the inferred lower liquidity of capital intensifies the liquidity effect. This leaves the output effect almost unchanged. In other words, the stability of our results stems from the model inherent trade-offs.

[18]For the robustness check where we set the inverse Frisch elasticity to 1, physical capital becomes so attractive to households, that the calibration forces capital to become very illiquid in order to match the observed money to output ratio. Then, however, investment moves so little (as close to nobody can trade) that there is no positive adjustment cost such that the relative investment volatility is 3. In these cases we assume no capital adjustment costs.

Figure 1.15: Uncertainty shock – Robustness



Notes: See Figure 1.5 or 1.6.

G. Equilibrium Forecasting Rules

Tables 1.9 to 1.11 display the equilibrium laws of motion for the Krusell-Smith equilibrium.

Table 1.9: Laws of motion for the price of capital

	Baseline			ξ		κ		μ		γ
	$\theta_2 = 0$ $\theta_3 = 0$	$\theta_2 = 10^3$ $\theta_3 = 0$	$\theta_2 = .35$ $\theta_3 = .9$	3^*	5	0.04	0.5	5^*	15	1^*
β_q^1										
s_1	1.71	1.39	1.75	0.79	2.03	3.46	-0.44	0.15	2.36	5.18
s_2	1.71	1.41	1.75	0.87	2.01	3.30	-0.36	0.67	2.34	5.70
s_3	1.70	1.50	1.77	0.89	1.99	3.19	-0.39	0.53	2.33	5.85
s_4	1.69	1.55	1.78	0.90	2.00	3.10	-0.37	0.35	2.33	5.96
s_5	1.70	1.62	1.79	0.91	2.03	3.09	-0.35	0.22	2.34	6.04
s_6	1.72	1.66	1.80	0.93	2.05	3.03	-0.39	0.33	2.35	6.29
s_7	1.82	1.75	1.82	1.00	2.14	3.06	-0.25	0.51	2.40	6.58
β_q^2										
s_1	0.50	0.12	0.22	0.14	0.63	0.65	0.46	0.08	0.74	1.02
s_2	0.52	0.13	0.24	0.16	0.65	0.68	0.47	0.10	0.75	1.17
s_3	0.53	0.14	0.26	0.17	0.66	0.69	0.48	0.12	0.77	1.30
s_4	0.54	0.15	0.27	0.19	0.68	0.69	0.50	0.13	0.78	1.45
s_5	0.55	0.16	0.29	0.22	0.69	0.69	0.51	0.15	0.78	1.60
s_6	0.58	0.18	0.31	0.24	0.71	0.69	0.54	0.17	0.78	1.80
s_7	0.61	0.20	0.34	0.29	0.74	0.73	0.57	0.20	0.79	2.04
β_q^3										
s_1	-0.86	-0.83	-1.03	-0.45	-1.00	-1.97	0.55	-0.04	-1.22	-2.87
s_2	-0.86	-0.85	-1.03	-0.49	-0.99	-1.86	0.49	-0.36	-1.21	-3.18
s_3	-0.87	-0.90	-1.04	-0.51	-0.99	-1.79	0.49	-0.26	-1.21	-3.25
s_4	-0.87	-0.94	-1.05	-0.51	-1.00	-1.74	0.48	-0.15	-1.22	-3.29
s_5	-0.88	-0.99	-1.06	-0.50	-1.03	-1.74	0.46	-0.07	-1.23	-3.30
s_6	-0.90	-1.02	-1.07	-0.51	-1.05	-1.72	0.48	-0.13	-1.25	-3.42
s_7	-0.98	-1.08	-1.10	-0.55	-1.13	-1.74	0.38	-0.23	-1.31	-3.56
R^2										
	99.55	98.54	99.70	98.52	99.74	99.58	99.40	98.77	99.57	99.00

* For readability the coefficients of the law of motion are mutiplied by 10.000.

Table 1.10: Laws of motion for inflation

	Baseline			ξ		κ		μ		γ
	$\theta_2 = 0$ $\theta_3 = 0$	$\theta_2 = 10^3$ $\theta_3 = 0$	$\theta_2 = .35$ $\theta_3 = .9$	3	5	0.04	0.5	5	15	1
β_π^1										
s_1	2.80	0.48	0.08	12.46	2.65	44.20	-19.49	40.74	3.76	6.07
s_2	2.37	0.49	0.50	13.19	2.40	43.59	-19.30	33.41	3.54	6.34
s_3	2.16	0.49	1.20	14.36	2.44	44.14	-19.31	27.85	3.56	6.57
s_4	1.67	0.49	1.49	14.33	2.17	41.17	-19.21	21.63	3.26	6.55
s_5	1.19	0.49	1.82	14.08	1.86	37.87	-19.55	18.86	2.87	6.63
s_6	1.00	0.49	2.20	13.99	1.41	34.76	-19.92	2.06	2.54	6.56
s_7	0.59	0.49	2.18	13.54	0.77	28.95	-20.62	-39.60	2.29	6.28
β_π^2										
s_1	4.19	-0.00	0.53	4.18	3.87	10.06	2.83	3.36	4.03	2.79
s_2	4.31	0.00	0.71	4.64	3.97	11.44	2.85	3.87	4.12	3.09
s_3	4.39	0.00	0.87	5.04	4.06	11.97	2.89	4.30	4.23	3.37
s_4	4.47	0.00	1.02	5.42	4.14	12.07	2.96	4.69	4.28	3.65
s_5	4.57	0.00	1.19	5.85	4.26	12.19	3.06	5.13	4.36	3.98
s_6	4.76	0.00	1.38	6.39	4.42	12.34	3.21	5.64	4.49	4.39
s_7	5.17	0.00	1.67	7.27	4.80	13.19	3.49	6.44	4.88	5.03
β_π^3										
s_1	1.17	0.01	0.78	-5.25	1.14	-22.92	14.99	-22.73	0.71	-1.78
s_2	1.27	0.01	0.49	-5.76	1.11	-22.25	14.73	-18.23	0.74	-2.01
s_3	1.22	0.01	0.00	-6.58	0.92	-22.81	14.62	-14.83	0.61	-2.22
s_4	1.36	0.01	-0.22	-6.59	0.92	-21.24	14.46	-11.04	0.68	-2.26
s_5	1.48	0.01	-0.47	-6.46	0.95	-19.50	14.60	-9.37	0.81	-2.34
s_6	1.40	0.01	-0.78	-6.43	1.05	-18.03	14.75	0.91	0.88	-2.34
s_7	1.48	0.01	-0.83	-6.08	1.29	-14.71	15.13	26.52	0.97	-2.13
R^2										
	99.87	88.77	99.66	99.89	99.84	99.08	99.81	99.89	99.73	99.87

Notes: For readability all values are mutiplied by 100.

Table 1.11: Laws of motion for Fed policy rule continued

	Baseline	
	$\theta_2 = .35$ $\theta_3 = .9$	$\theta_2 = .35$ $\theta_3 = .9$
	β_q^4	β_π^4
s_1	5.65	19.43
s_2	25.18	20.84
s_3	33.51	20.71
s_4	36.83	20.32
s_5	47.80	19.39
s_6	70.02	18.78
s_7	116.69	19.15

Notes: For readability all values are mutiplied by 100.

Chapter 2

Transmission of Monetary Policy with Heterogeneity in Household Portfolios

This paper assesses the importance of heterogeneity in household portfolios for the transmission of monetary policy in a New Keynesian business cycle model with incomplete markets and portfolio choice under liquidity constraints. In this model, the consumption response to changes in interest rates depends on the joint distribution of labor income, liquid and illiquid assets. The presence of liquidity-constrained households weakens the direct effect of changes in the real interest rate on consumption, but at the same time makes consumption more responsive to equilibrium changes in labor income. The redistributive consequences, including debt deflation, amplify the consumption response, whereas they dampen the investment response. Market incompleteness has important implications for the conduct of monetary policy as it relies to a larger extent on indirect equilibrium effects in comparison to economies with a representative household.

1. Introduction

A household's portfolio generally consists of non-tradable and tradable assets. The most important non-tradable asset is human capital. It is the primary source of income for most households and at the same time subject to substantial idiosyncratic shocks. The presence of such shocks gives rise to both precautionary savings and cross-sectional differences in holdings of tradable assets when markets are incomplete. Importantly, tradable assets vary in their degree of liquidity. In fact, a large fraction of households in the United States holds low levels of liquid assets relative to their income, although most households exhibit considerable positive net worth.[1] This has important implications for the transmission of monetary policy, because consumption of low-liquidity households is less sensitive to interest rates but responds more strongly to current income. While the monetary authority has *direct* control over the interest rate, changes in income only arise *indirectly* out of second-round general equilibrium effects.

This paper assesses quantitatively the implications of heterogeneity in household

[1] See Kaplan et al. (2014) for a documentation of this fact for the U.S. and other industrialized countries.

portfolios for the transmission of monetary policy and the relative importance of the direct and indirect transmission channels. Toward this end, I build a New Keynesian dynamic stochastic general equilibrium (DSGE) model with asset-market incompleteness, idiosyncratic income risk, and sticky prices. The novel feature of the model is to allow for portfolio choice between liquid and illiquid assets in a business-cycle framework. The illiquid asset is real capital. It can only be traded with a certain probability each period but pays a higher return than the liquid asset, which comprises nominal government and household debt and can be traded without frictions. These characteristics make sure that the model endogenously generates the distribution of portfolio shares and marginal propensities to consume across households as documented for the United States.[2] In particular, illiquidity of the real asset leads to "wealthy hand-to-mouth" households that explain the high aggregate propensity to consume out of current income (cf. Kaplan and Violante, 2014).

My main finding is that, when markets are incomplete, the direct response to changes in the interest rate makes up only 35% of the consumption response to monetary shocks, while indirect effects account for the remaining 65%. Indirect effects mainly work through equilibrium changes in labor income (about 85%), which represents the most important income source for the majority of households. The revaluation of nominal claims, including debt deflation, adds another indirect channel of monetary policy, which roughly accounts for 15% of the total response.

The importance of indirect effects contrasts sharply with the standard New Keynesian model that builds on a representative household. In the latter, the direct effect explains close to all of the consumption response. The indirect effect is quantitatively unimportant, because it works exclusively through changes in life-time income that monetary shocks hardly affect. With complete markets, savings adjust to undo any temporary mismatch between income and consumption. In an economy with incomplete markets, by contrast, borrowing constraints and precautionary motives are important. They make savings and, thus, consumption less sensitive to interest rate changes. This renders the direct effect of monetary policy less potent. At the same time, indirect effects become stronger because current income is a binding constraint for households at or close to the borrowing constraint. What is more, revaluation of nominal assets impacts on the tightness of borrowing constraints as inflation changes the real value of debt. This amplifies the effect of borrowing constraints through Fisher (1933) debt deflation.

The indirect effect through changes in income is, therefore, the key determinant of the consumption response to monetary shocks in an economy with incomplete markets. This reversal of the importance of direct and indirect effects explains how monetary policy may have sizable effects on aggregate consumption while interest rate elasticities at the household level are low.[3]

While the consumption response to monetary shocks works through different channels and is also stronger in total, a monetary shock moves output to a similar extent

[2] See the empirical literature on the consumption response to transfers; e.g. Johnson et al. (2006), Parker et al. (2013), or Misra and Surico (2014).

[3] See for example the handbook chapter on monetary policy by Christiano et al. (1999) for evidence on the aggregate consequences of monetary policy shocks.

in the representative and heterogeneous agent version of the model. Investment falls by 25% less in the incomplete-markets setting and, thus, cancels out the stronger consumption response. The reason for this smaller reaction of investment relates to the fact that monetary policy has non-trivial redistributive consequences that interact with heterogeneity in household portfolios. In line with the empirical findings by Coibion et al. (2012), a monetary tightening increases inequality and makes households at the top of the wealth distribution, who primarily hold real assets, richer. Thereby they stabilize investment demand after a contractionary monetary policy shock.

With these results, my paper contributes to the recently evolving literature that incorporates market incompleteness and idiosyncratic uncertainty into New Keynesian models.[4] As such it builds on the New Keynesian literature with its focus on nominal rigidities. This literature has proven successful in replicating the impulse responses to monetary policy shocks as identified from time-series data (cf. Christiano et al., 2005). What my paper and other recent contributions add to this literature is the attempt to endogenize heterogeneity in wealth.[5] In this class of models, the response of consumption and savings depends on the distribution of wealth, which evolves in response to aggregate shocks.

Relative to this literature, my paper is the first to analyze monetary policy in a business cycle framework with portfolio choice. My work is most closely related to Kaplan et al. (2015), which originated in parallel. They also decompose the effects of monetary policy into direct and indirect effects but focus on the consumption response to a one-time unexpected monetary shock. My model, in contrast, is calibrated to match business cycle statistics and, thus, adds to their analysis by considering the response of consumption, investment, and output in unison.

This paper also contributes to the assessment of debt deflation as transmission mechanism of monetary policy. My analysis shows that redistribution through inflation is of secondary importance relative to the effect of monetary policy on aggregate income in models with sticky prices.[6]

The remainder of the paper is organized as follows. Section 2 presents the model, and Section 3 discusses the solution method. Section 4 explains the calibration of the model. Section 5 presents the quantitative results. Section 6 concludes.

2. Model

The model economy consists of households, firms, and a government/monetary authority. Households consume, supply labor, obtain profit income, accumulate physical capital, and trade in the bonds market. Firms combine capital and labor services to produce goods. The government issues bonds, raises taxes, and purchases goods, while the monetary authority sets the nominal interest rate. Let me describe each agent in turn.

[4] See Guerrieri and Lorenzoni (2011), Oh and Reis (2012), Gornemann et al. (2012), McKay and Reis (2016), Ravn and Sterk (2013), Den Haan et al. (2014), Auclert (2014), Bayer et al. (2015), McKay et al. (2015), Werning (2015) and Kaplan et al. (2015).

[5] Exogenous heterogeneity is well-established in New Keynesian models. See for example Iacoviello (2005) and Galí et al. (2007).

[6] See Doepke et al. (2015) for an analysis of this channel in a flexible-price model.

Households face idiosyncratic income risk, but insurance markets are incomplete. They self-insure by trading nominal bonds and illiquid physical capital. I model this illiquidity as infrequent participation in the capital market. Every period a fraction of households is randomly selected to trade physical capital.[7] Households are either workers or entrepreneurs with a certain probability. Worker-households supply labor on a perfectly competitive market and are subject to idiosyncratic shocks to their labor productivity. Entrepreneur-households do not supply labor, but instead receive an equal share of economy-wide profits.[8,9,10]

There are three types of firms. Perfectly competitive intermediate-goods producers hire capital and labor from households and sell the homogeneous intermediate good at marginal costs. Monopolistically competitive resellers then differentiate the intermediate good and set prices above marginal costs. They may, however, only adjust their prices with some positive probability each period as in Calvo (1983). As a result, demand determines output in the short-run, because a fraction of firms has to satisfy demand at given prices. The differentiated goods are finally bundled again by perfectly competitive final-good producers to final goods used for consumption and investment.

Monetary policy follows a Taylor (1993)-type rule that determines the nominal interest rate at which the government and households may borrow and lend to each other. It thereby affects the real rate of interest because of sticky prices. The balance sheet of the central bank is not modeled explicitly. The government collects proportional income taxes to finance its interest expenses and government purchases. The latter follow a simple rule to stabilize debt.

The model economy is subject to aggregate shocks as in Krusell and Smith (1998). The shocks affect total factor productivity of intermediate-goods production and the Taylor-rule. I next describe the model in more detail.

2.1 Households

There is a continuum of ex-ante identical households of measure one indexed by $i \in [0, 1]$. Households are infinitely lived, have time-separable preferences with time-discount factor β, and derive felicity from consumption c_{it} and leisure. Households can be entrepreneurs ($s_{it} = 0$) or workers ($s_{it} = 1$). Transition between both types is exogenous and stochastic, but the fraction of households that are entrepreneurs at any given time $t = 0, 1, 2, ...$ is constant.

Workers supply labor. Their labor income $w_t h_{it} n_{it}$ is composed of the wage rate, w_t,

[7]This setup builds on Chapter 1. Where there is overlay the exposition closely follows Chapter 1. We choose to exclude trading as a choice and hence use a simplified framework relative to Kaplan et al. (2015) for numerical tractability. Random participation keeps the households' value function concave, thus makes first-order conditions sufficient, and therefore allows us to use a variant of the endogenous grid method as algorithm for our numerical calculations. See Chapter 1 for proofs.

[8]According to the Congressional Budget Office, the top 1% of the income distribution receives about 30% of their income from financial income, a much larger share than any other segment of the population.

[9]For reasons of tractability, I abstract from tradable shares in monopoly profits and instead introduce an exogenous employment state that receives all profits.

[10]Fixed types of workers and entrepreneurs (or capitalists) without stochastic transitions can be found in Walsh (2014) or Broer et al. (2015), while Romei (2014) uses stochastic transitions.

hours worked, n_{it}, and idiosyncratic labor productivity, h_{it}, which evolves according to the following first-order autoregressive process:

$$\log h_{it} = \rho_h \log h_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_h). \tag{2.1}$$

Entrepreneurs have zero productivity on the labor market, but instead receive an equal share of the economy's total profits Π_t. They pay the same tax rate as workers, $1 - \tau$.

Households have GHH preferences (cf. Greenwood et al., 1988) and maximize the discounted sum of felicity:

$$V = E_0 \max_{\{c_{it}, n_{it}\}} \sum_{t=0}^{\infty} \beta^t u(x_{it}), \tag{2.2}$$

where $x_{it} = c_{it} - h_{it}G(n_{it})$ is household i's composite demand for the physical consumption good c_{it} and leisure.

The disutility of work, $h_{it}G(n_{it})$, determines a workers' labor supply given the aggregate wage rate through the first-order condition:

$$h_{it}G'(n_{it}) = \tau w_t h_{it}. \tag{2.3}$$

Under the above assumption, a workers' labor decision does not respond to idiosyncratic productivity h_{it}, but only to the net aggregate wage τw_t. Thus I can drop the household-specific index i, and set $n_{it} = N_t$.

The Frisch elasticity of aggregate labor supply is constant with γ being the inverse elasticity:

$$G(N_t) = \frac{1}{1+\gamma} N_t^{1+\gamma}, \quad \gamma > 0.$$

Exploiting the first-order condition on labor supply, the disutility of working can be expressed in terms of the net wage rate:

$$h_{it}G(N_t) = h_{it}\frac{N_t^{1+\gamma}}{1+\gamma} = \frac{h_{it}G'(N_t)N_t}{1+\gamma} = \frac{\tau w_t h_{it} N_t}{1+\gamma}.$$

In this way the demand for x_{it} can be rewritten as:

$$x_{it} = c_{it} - h_{it}G(N_t) = c_{it} - \frac{\tau w_t h_{it} N_t}{1+\gamma}.$$

Total labor input supplied is given by:

$$\tilde{N}_t = N_t \int s_{it} h_{it} di.$$

Asset markets are incomplete. Households may only self-insure in nominal bonds, $\tilde{b}_{it}$, and in capital, k_{it}. Holdings of capital have to be non-negative, but households may issue nominal bonds up to an exogeneously specified limit $-\underline{b} \in (-\infty, 0]$. Moreover, trading capital is subject to a friction.

This trading friction only allows a randomly selected fraction of households, ν, to participate in the market for capital each period. All other households obtain dividends, but may only adjust their holdings of nominal bonds. For those households participating in the capital market, the budget constraint reads:[11]

$$
\begin{aligned}
c_{it} + b_{it+1} + q_t k_{it+1} =& \frac{R^B_{t-1}}{\pi_t} b_{it} + (q_t + r_t) k_{it} + \tau \left[s_{it} w_t h_{it} N_t + (1 - s_{it}) \Pi_t \right], \\
& k_{it+1} \geq 0, b_{it+1} \geq -\underline{b} \qquad (2.4)
\end{aligned}
$$

where b_{it} is the real value of nominal bond holdings, k_{it} are capital holdings, q_t is the price of capital, r_t is the rental rate or "dividend", R^B_{t-1} is the gross nominal return on bonds, and $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate. I denote real bond holdings of household i at the end of period t by $b_{it+1} := \frac{\tilde{b}_{it+1}}{P_t}$.

For those households that cannot trade in the market for capital the budget constraint simplifies to:

$$
\begin{aligned}
c_{it} + b_{it+1} =& \frac{R^B_{t-1}}{\pi_t} b_{it} + r_t k_{it} + \tau \left[s_{it} w_t h_{it} N_t + (1 - s_{it}) \Pi_t \right], \\
& b_{it+1} \geq -\underline{b}. \qquad (2.5)
\end{aligned}
$$

Note that I assume that the depreciation of capital is replaced through maintenance such that the dividend, r_t, is the net return on capital.

A household's optimal consumption-savings decision is a non-linear function of that household's asset portfolio $\{b_{it}, k_{it}\}$ and employment type $\{s_{it}, h_{it}\}$. Accordingly, the price level P_t and aggregate real bonds $B_{t+1} = \frac{\tilde{B}_{t+1}}{P_t}$ are functions of the joint distribution Θ_t over idiosyncratic states (b_t, k_t, h_t, s_t). This makes the distribution Θ_t a state variable of the households' planning problem. The distribution Θ_t fluctuates in response to aggregate monetary and total factor productivity shocks. Let Ω stand in for aggregate shocks.

With this setup, two Bellman equations characterize the dynamic planning problem of a household; V_a in case the household can adjust its capital holdings and V_n otherwise:

$$
\begin{aligned}
V_a(b, k, h, s; \Theta, \Omega) =& \max_{k', b'_a} u[c(b, b'_a, k, k', h, s)] + \beta[\nu E V^a(b'_a, k', h', s', \Theta', \Omega') \\
& + (1 - \nu) E V^n(b'_a, k', h', s', \Theta', \Omega')], \\
V_n(b, k, h, s; \Theta, \Omega) =& \max_{b'_n} u[c(b, b'_n, k, h, s)] + \beta[\nu E V^a(b'_n, k, h', s', \Theta', \Omega') \\
& + (1 - \nu) E V^n(b'_n, k, h', s', \Theta', \Omega')]. \qquad (2.6)
\end{aligned}
$$

In line with this notation, I define the optimal consumption policies for the adjustment and non-adjustment cases as c^*_a and c^*_n, the nominal bond holding policies as b^*_a and b^*_n, and the capital investment policy as k^*. See Appendix A for the first order conditions.

[11]The household problem can be expressed in terms of composite good x_{it} by making use of $c_{it} = x_{it} + \frac{\tau w_t h_{it} N_t}{1+\gamma}$.

2.2 Intermediate Good Producer

Intermediate goods are produced with a constant returns to scale production function:

$$Y_t = Z_t \tilde{N}_t^{\alpha} K_t^{(1-\alpha)},$$

where Z_t is total factor productivity (TFP). It follows a first-order autoregressive process:

$$\log Z_t = \rho_Z \log Z_{t-1} + \epsilon_t^Z, \quad \epsilon_t^Z \sim N\left(0, \sigma_Z\right). \tag{2.7}$$

Let MC_t be the relative price at which the intermediate good is sold to resellers. The intermediate-good producer maximizes profits,

$$MC_t Y_t = MC_t Z_t \tilde{N}_t^{\alpha} K_t^{(1-\alpha)} - w_t \tilde{N}_t - (r_t + \delta) K_t,$$

but it operates in perfectly competitive markets, such that the real wage and the user costs of capital are given by the marginal products of labor and capital:

$$w_t = \alpha MC_t Z_t \left(K_t / \tilde{N}_t\right)^{1-\alpha}, \tag{2.8}$$

$$r_t + \delta = (1-\alpha) MC_t Z_t \left(\tilde{N}_t / K_t\right)^{\alpha}. \tag{2.9}$$

2.3 Resellers

Resellers differentiate the intermediate good and set prices. They are risk neutral and have the same discount factor as households. For tractability reasons, I assume that resellers obtain an arbitrarily small share of profits and do neither participate in the bond nor capital market. This assumption separates the resellers' price setting problem from the households' saving problem.

By setting prices of final goods, resellers maximize expected discounted future profits:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Pi_{jt}. \tag{2.10}$$

Resellers buy the intermediate good at a price equaling the nominal marginal costs, $MC_t P_t$, where MC_t are the real marginal costs at which the intermediate good is traded due to perfect competition, and then differentiate them without the need of additional input factors. The goods that resellers produce come in varieties uniformly distributed on the unit interval and each indexed by $j \in [0, 1]$. Resellers are monopolistic competitors, and hence charge a markup over their marginal costs. They are, however, subject to a Calvo (1983) price setting friction, and can only update their prices with probability θ. They maximize the expected value of future discounted profits by setting today's price, p_{jt}, taking into account the price setting friction:

$$\max_{\{p_{jt}\}} \sum_{s=0}^{\infty} (\theta\beta)^s E \Pi_{jt,t+s} = \sum_{s=0}^{\infty} (\theta\beta)^s E y_{jt,t+s} (p_{jt} - MC_{t+s} P_{t+s}) \tag{2.11}$$

$$s.t.: y_{jt,t+s} = \left(\frac{p_{jt}}{P_{t+s}}\right)^{-\eta} y_{t+s},$$

where $\Pi_{jt,t+s}$ are profits and $y_{jt,t+s}$ is the production level in $t+s$ of a firm j that set prices in t.

I obtain the following first-order condition with respect to p_{jt}:

$$\sum_{s=0}^{\infty}(\theta\beta)^s E y_{jt,t+s}\left(\frac{p_{jt}^*}{P_{t-1}} - \underbrace{\frac{\eta}{\eta-1}}_{\mu} MC_{t+s}\frac{P_{t+s}}{P_{t-1}}\right) = 0, \tag{2.12}$$

where μ is the static optimal markup.

Recall that resellers are risk neutral, and that they do not interact with households in any intertemporal trades. Therefore, I can solve the resellers' planning problem locally by log-linearizing around the zero-inflation steady state, without having to know the solution of the households' problem. This yields the New Keynesian Phillips curve, see e.g. Galí (2008):

$$\log \pi_t = \beta E_t(\log \pi_{t+1}) + \kappa(\log MC_t + \mu), \tag{2.13}$$

where

$$\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}.$$

Besides differentiating intermediate goods, I assume that resellers also obtain rents from adjusting the aggregate capital stock. The cost of adjusting the stock of capital is $\frac{\phi}{2}\left(\frac{\Delta K_{t+1}}{K_t}\right)^2 K_t$. Hence, resellers will adjust the stock of capital until the following first-order condition holds:

$$q_t = 1 + \phi\frac{\Delta K_{t+1}}{K_t}. \tag{2.14}$$

2.4 Final Good Producer

Perfectly competitive final good producers use differentiated goods as input taking input and sell price as given. Final goods are used for consumption and investment. The problem of the representative final good producer is as follows:

$$\max_{Y_t, y_{jt}\in[0,1]} P_t Y_t - \int_0^1 p_{jt} y_{jt} dj \tag{2.15}$$

$$s.t.: Y_t = \left(\int_0^1 y_{jt}^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}},$$

where y_{jt} is the demanded quantity of differentiated good j as input. From the zero-profit condition, the price of the final good is given by $P_t = \left(\int_0^1 p_{jt}^{1-\eta} dj\right)^{\frac{1}{1-\eta}}$.

2.5 Central Bank and Government

Monetary policy sets the gross nominal interest rate, R_t^B, according to a Taylor (1993)-type rule that reacts to inflation deviations from target and exhibits interest rate smoothing:

$$\frac{R_t^B}{R^B} = \left(\frac{R_{t-1}^B}{R^B}\right)^{\rho_{RB}} \left(\frac{1+\pi_t}{1+\pi}\right)^{\theta_\pi} \epsilon_t^D, \tag{2.16}$$

where $\log \epsilon_t^D \sim N(0, \sigma_D)$ are monetary policy shocks. All else equal, the central bank raises the nominal rate above its steady-state value R^B whenever inflation exceeds its target value. It does so by more than one-to-one to guarantee a non-explosive price path ($\theta_\pi > 1$). The parameter ρ_{R^B} captures "intrinsic policy inertia", a feature supported by empirical evidence, see Nakamura and Steinsson (2013).

The fiscal authority decides on government purchases, G_t, raises tax revenues, T_t, and issues nominal bonds. Let B_{t+1} denote their time t real value. The government budget constraint reads:

$$B_{t+1} = \frac{R_{t-1}^B}{1+\pi_t} B_t + G_t - T_t, \tag{2.17}$$

where real tax revenues are given by:

$$T_t = (1-\tau)\left[(N_t W_t \int s_i h_i \Theta_t(b,k,h,s)) + \Pi_t\right]. \tag{2.18}$$

I assume that government purchases stabilize the debt level,

$$G_t = \gamma_1 - \gamma_2 (B_t - B), \tag{2.19}$$

with B equal to steady state debt, while the tax parameter τ remains constant. Adjustment via government purchases is the baseline formulation because changing taxes would directly redistribute across households. This also applies to lump-sum taxes in this environment. Government purchases, in contrast, do not have any direct distributional consequences.

2.6 Bonds, Capital, Goods, and Labor Market Clearing

The labor market clears at the competitive wage given in (2.8); so does the market for capital services if (2.9) holds. The nominal bonds market clears, whenever the following equation holds:

$$B_{t+1} = \int \left[\nu b_a^*(b,k,h,s;q,\pi) + (1-\nu) b_n^*(b,k,h,s;q,\pi)\right] \Theta_t(b,k,h,s) dbdkdhds. \tag{2.20}$$

Last, the market for capital has to clear:

$$\begin{aligned} q_t &= 1 + \phi \frac{K_{t+1} - K_t}{K_t} = 1 + \nu\phi \frac{K_{t+1}^* - K_t}{K_t}, \\ K_{t+1}^* &:= \int k^*(b,k,h,s;q_t,\pi_t)\Theta_t(b,k,h,s) dbdkdhds, \\ K_{t+1} &= K_t + \nu(K_{t+1}^* - K_t), \end{aligned} \tag{2.21}$$

where the first equation stems from competition in the production of capital goods, the second equation defines the aggregate supply of funds from households trading capital, and the third equation defines the law of motion of aggregate capital. The goods market then clears due to Walras' law, whenever both, bonds and capital markets, clear.

2.7 Recursive Equilibrium

A *recursive equilibrium* in this model is a set of policy functions $\{c_a^*, c_n^*, b_a^*, b_n^*, k^*\}$, value functions $\{V_a, V_n\}$, pricing functions $\{r, R^B, w, \pi, q\}$, aggregate bonds, capital, and labor supply functions $\{B, K, N\}$, distributions Θ_t over individual asset holdings, types, and productivity, and a perceived law of motion Γ, such that

1. Given V_a, V_n, Γ, prices, and distributions, the policy functions $\{c_a^*, c_n^*, b_a^*, b_n^*, k^*\}$ solve the households' planning problem, and given prices, distributions, and the policy functions $\{c_a^*, c_n^*, b_a^*, b_n^*, k^*\}$, the value functions $\{V_a, V_n\}$ are a solution to the Bellman equations (2.6).

2. The labor, the final-goods, the bonds, the capital, and the intermediate-good markets clear, i.e. (2.8), (2.13), (2.20), and (2.21) hold.

3. The actual law of motion and the perceived law of motion Γ coincide, i.e. $\Theta' = \Gamma(\Theta, \Omega')$.

3. Numerical Implementation

The dynamic program (2.6) and hence the recursive equilibrium is not computable, because it involves the infinite dimensional object Θ_t.

3.1 Krusell-Smith Equilibrium

To turn this problem into a computable one, I assume that households predict future prices only on the basis of a restricted set of moments, as in Krusell and Smith (1997, 1998). Specifically, I make the assumption that households condition their expectations on last period's aggregate real bond holdings, B_t, last period's nominal interest rate, R_{t-1}^B, and the aggregate stock of capital, K_t. If asset-demand functions, $b_{a,n}^*$ and k^*, are sufficiently close to linear in human capital, h, types, s, and in non-human wealth, b and k, at the mass of Θ_t, I can expect approximate aggregation to hold. For my exercise, the three aggregate states – B_t, R_{t-1}^B, K_t – are sufficient to describe the evolution of the aggregate economy conditional on the aggregate shocks Ω.

While the law of motion for Z_t is pinned down by (2.7), households use the following log-linear forecasting rules for current inflation and the price of capital:

$$\log \pi_t = \beta_\pi^1(\Omega) + \beta_\pi^2(\Omega)\hat{B}_t + \beta_\pi^3(\Omega)\hat{K}_t + \beta_\pi^4(\Omega)\hat{R}_{t-1}^B, \tag{2.22}$$

$$\log q_t = \beta_q^1(\Omega) + \beta_q^2(\Omega)\hat{B}_t + \beta_q^3(\Omega)\hat{K}_t + \beta_q^4(\Omega)\hat{R}_{t-1}^B, \tag{2.23}$$

where $\hat{(.)}$ refers to log-differences from the steady state value of each variable and Ω indicates the dependence on aggregate shocks. The law of motion for aggregate real bonds, B_t, then follows from the government budget constraint (2.17). The Taylor-rule (2.16) determines the motion of the nominal interest rate, R_t^B. The law of motion for K_t results from (2.21).

Technically, finding the equilibrium is similar to Krusell and Smith (1997), as I need to find market clearing prices within each period. Concretely, this means the posited rules, (2.22) and (2.23), are used to solve for households' policy functions. Having solved for the policy functions conditional on the forecasting rules, I then simulate n independent sequences of economies for $t = \{1, \ldots, T\}$ periods, keeping track of the actual distribution Θ_t. In each simulation the sequence of distributions starts from the stationary distribution implied by the model without TFP and monetary policy shocks. I then calculate in each period t the optimal policies for market clearing inflation rates and capital prices assuming that households resort to the policy functions derived under rule (2.22) and (2.23) from period $t+1$ onward. Having determined the market clearing prices, I obtain the next period's distribution Θ_{t+1}. In doing so, I obtain n sequences of equilibria. The first 250 observations of each simulation are discarded to minimize the impact of the initial distribution. I next re-estimate the parameters of (2.22) and (2.23) from the simulated data and update the parameters accordingly. By using $n = 20$ and $T = 1250$, it is possible to make use of parallel computing resources and obtain 20.000 equilibrium observations. Subsequently, I recalculate policy functions and iterate until convergence in the forecasting rules.

The posited rules (2.22) and (2.23) approximate the aggregate behavior of the economy well. The minimal within sample R^2 is above 99.9%. Out-of-sample performance as defined by Den Haan (2010) is also good. See Appendix B.

3.2 Solving the household planning problem

In solving for the households' policy functions I apply an endogenous grid point method as originally developed in Carroll (2006) and extended by Hintermaier and Koeniger (2010), iterating over the first-order conditions. I approximate the idiosyncratic productivity/employment state process by a discrete Markov chain with 4 states, using the method proposed by Tauchen (1986).[12,13]

4. Calibration

I calibrate the model to the U.S. economy over the time period 1984Q1 to 2008Q3 as my focus lies on conventional monetary policy. One period in the model is a quarter. Table 2.1 summarizes the calibration. In detail, I choose the parameter values as follows.

4.1 Households

I assume that the felicity function is of constant-relative-risk-aversion form: $u(x) = \frac{1}{1-\xi}x^{1-\xi}$, where $\xi = 2$, a standard value. The time-discount factor, $\beta = 0.985$, and the capital market participation frequency, $\nu = 0.075$, are jointly calibrated to match the ratio of capital and government bonds to output.[14] I equate capital to all capital goods

[12] I solve the household policies for 40 points on the grid for bonds and 40 points on the grid for capital. For aggregate bonds, aggregate capital, and last period's nominal interest rate I use a grid of 3 points each, while for TFP I use 7 points and 7 for the *iid* monetary shock.

[13] Details on the algorithm can be found in Chapter 1.

[14] The participation frequency of 7.5% is higher than in the optimal participation framework of Kaplan and Violante (2014). They find a participation frequency of 4.5% for working households given a fixed-adjustment cost of $500.

relative to nominal GDP. The annual capital-to-output ratio is therefore 290%. This implies an annual real return on capital of about 4%. I equate government bonds to the outstanding government debt held by private domestic agents, which implies an annual bonds-to-output ratio of 31%.

I set the borrowing limit in bonds, $\underline{b}$, such that 20% of households have negative net worth as in the *Survey of Consumer Finances* (2007). This implies a relatively tight borrowing limit that equals the average quarterly income.

I calibrate the stochastic process for the employment state to capture the distribution of wealth in the U.S. economy. In particular, I determine the share of entrepreneur-households to match a Gini coefficient of 0.82. For simplicity, I assume that the probability of becoming an entrepreneur is the same for workers independent of their labor productivity and that, once they become a worker again, they draw their labor productivity from a uniform distribution. I set the quarterly standard deviation of persistent shocks to idiosyncratic labor productivity to 0.08. The quarterly autocorrelation is 0.987 – a standard value in the literature. This implies for the baseline calibration that on average 1% of households are entrepreneurs.

4.2 Intermediate, Final, and Capital Goods Producers

The labor and capital share including profits (2/3 and 1/3) align with long-run U.S. averages. The persistence of the TFP shock is set to $\rho_Z = 0.9$. The standard deviation of the TFP shock, $\sigma_Z = 0.005$, is calibrated to make the model match the standard deviation of H-P-filtered U.S. output.

To calibrate the parameters of the resellers' problem, I use standard values for markup and price stickiness that are widely employed in the New Keynesian literature (c.f. Christiano et al., 1999). The Phillips curve parameter κ implies an average price duration of 4 quarters, assuming flexible capital at the firm level. The steady state marginal costs, $exp(-\mu) = 0.95$, imply a markup of 6%. I calibrate the adjustment cost of capital, $\phi = 10$, to match the relative investment volatility in the United States.

4.3 Central Bank and Government

I target an average annual inflation rate of 2% according to the Federal Reserve System's inflation objective. I set the real return on bonds to 3.5% in line with the average federal funds rate in the U.S in real terms from 1984 to 2008.[15] Hence, the nominal return is $R^B = 1.0136$ quarterly. Nakamura and Steinsson (2013) provide an estimate for the parameter governing interest rate smoothing, $\rho_{R^B} = 0.95$, while the central bank's reaction to inflation deviations from target is standard, $\theta_\pi = 1.5$. The standard deviation of the monetary policy shock, σ_D, is 71 basis points annually (c.f. Christiano et al., 1999).

The government levies a proportional tax on labor income and profits to finance government purchases and interest expense on debt. I adjust $1 - \tau = 0.3$ to close the budget constraint given the interest expense and a government-spending-to-GDP ratio of 20% in steady state. Government purchases, in turn, react to debt deviations from

[15] I obtain real returns by subtracting the GDP deflator from the Effective Federal Funds Rate. Both time series are retrieved from the FRED database, Federal Reserve Bank of St. Louis.

Table 2.1: Calibrated parameters

Parameter	Value	Description	Target
Households			
β	0.985	Discount factor	$K/Y = 290\%$ (annual)
ν	7.5%	Participation frequency	$B/Y = 23\%$ (annual)
ξ	2	Coefficient of rel. risk av.	Standard value
γ	0.5	Inv. Frisch elasticity	Standard value
Intermediate Goods			
α	72%	Share of labor	Income share of labor of 66%
δ	1.35%	Depreciation rate	NIPA: Fixed assets & durables
ρ_Z	0.9	Persistence of TFP shock	Standard value
σ_Z	0.03	STD of TFP shock	STD(Y)=1
Final Goods			
κ	0.08	Price stickiness	Avg. price duration of 4 quarters
μ	0.06	Markup	6% markup (standard value)
Capital Goods			
ϕ	10	Capital adjustment costs	STD(I)/STD(Y)=3.5
Fiscal Policy			
$1-\tau$	0.3	Tax rate	Budget balance
γ_1	0.05	G in steady state	$G/Y = 20\%$
γ_2	0.1	G reaction function	Small value
Monetary Policy			
Π	1.005	Inflation	2% p.a.
R^B	1.0136	Nominal interest rate	5.5% p.a.
θ_π	1.5	Reaction to inflation	Standard value
ρ_{R^B}	0.95	Interest rate smoothing	Nakamura and Steinsson (2013)
σ_D	18e-3	STD of monetary shock	Christiano et al. (1999)
Income Process			
ρ_h	0.987	Persistence of productivity	Standard value
$\bar{\sigma}$	0.08	STD of innovations	Standard value

steady state such that the debt level remains bounded. Specifically, I choose $\gamma_2 = 0.1$, which ensures that the reaction of government spending builds up very slowly and, thus, interference with the aggregate consequences of monetary shocks is minimized.

4.4 Model Fit

Table 2.2 reports the business cycle statistics implied by the model. The volatility of output and investment are calibrated to U.S. data, while the remaining statistics and variables are not targeted. The most striking fact is the low volatility of government spending. This is the result of the passive nature of government spending in the model as it only moderately reacts to stabilize debt and does not feature any shocks. The volatility of government spending is deliberately low to keep interference with monetary shocks minimal.

Table 2.2: Business cycle statistics

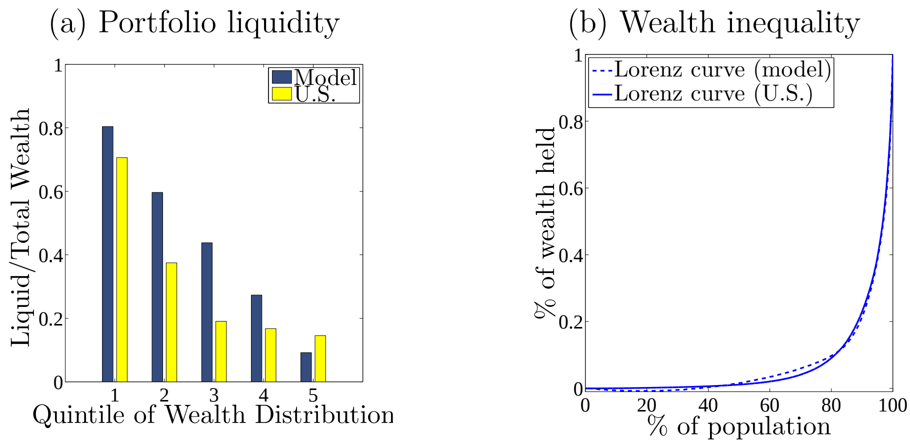
		Model			Data	
	STD	CORR	AC(1)	STD	CORR	AC(1)
GDP	1.02	1.00	0.74	0.97	1.00	0.72
Consumption	0.77	0.99	0.74	0.85	0.88	0.68
Investment	3.50	0.94	0.71	4.42	0.87	0.79
Gov. spending	0.43	0.15	0.96	1.22	-0.08	0.49

Notes: Standard deviation, correlation with GDP, and autocorrelation after log-HP(1600)-filtering. Standard devation is multiplied by 100.

Figure 2.1 (a) shows the liquidity of portfolios across the wealth distribution. I measure this as the total amount of liquid assets a household might withdraw (up to the borrowing constraint) relative to total wealth. The poorest households hold almost all their wealth in liquid assets. As households become richer the share of liquid assets in their portfolios falls. As the wealth-to-income ratio increases, the optimal share of liquid assets falls because the marginal value of liquidity declines for given income and households rather invest in the high-return illiquid asset. The declining share of liquid assets in household portfolios generated by the model approximately matches U.S. data from the *Survey of Consumer Finances* (2007).

The model also performs well in matching U.S. wealth inequality. Figure 2.1 (b) compares the Lorenz curve of wealth implied by the model to U.S. data. The U.S. Gini coefficient of 0.82 is matched by construction, but the model also generates realistic shares in total wealth across all percentiles of the wealth distribution.

Figure 2.1: Household portfolios



Notes: U.S. data corresponds to the *Survey of Consumer Finances* (2007).
(a): Maximum withdrawal of liquid assets relative to total wealth. U.S.: Liquid assets include all financial assets except for stocks (incl. mutual funds that primarily hold stocks) minus unsecured credit. Illiquid assets include all non-financial assets plus stocks minus secured credit. I assume the same borrowing limit as in the model ($20.000) and exclude all households with more unsecured credit.
(b): Wealth Lorenz curve in the model (dashed line) against Lorenz curve of wealth defined as financial plus nonfinancial assets minus debt for the U.S. (solid line).

5. Results

This section discusses the transmission of monetary policy shocks to the aggregate economy and, in particular, the transmission channels. I first discuss the theoretical channels through which monetary policy affects aggregates in this model and then compare the aggregate effects in the economy with heterogeneity in household portfolios to the same economy with a representative household.[16] I elaborate on heterogeneity in the savings response across the wealth distribution to highlight the importance of heterogeneity in household portfolios for aggregate outcomes. The section concludes with an assessment of redistribution through inflation as a potential transmission channel of monetary policy.

5.1 Transmission Channels of Monetary Policy

Key for understanding the transmission of monetary policy in any DSGE model is the household consumption-savings decision. The decision problem of households in an incomplete-markets setting differs from that of a representative household in that

[16]The representative household version of the model does not feature limited participation in the capital market because households are perfectly insured through state-contingent claims. I keep the parameters of the model unchanged to isolate the effect of heterogeneity in household portfolios on the transmission of monetary policy.

borrowing constraints apply. This gives rise to differences in optimal decisions as households take the existence of borrowing constraints into account or might actually be at the constraint. The response of consumption and savings to monetary shocks hence differs between an economy with and without complete markets. The effect of monetary policy on household decisions, in turn, can be split into direct and indirect effects along the lines of Table 2.3.

Table 2.3: Monetary policy transmission mechanism in the model

Decision	Variable	Determined by	Relevant prices	Effect is
intertemporal consumption -savings	$\{X_t\}_{t=0\ldots\infty}$	sequence of Euler equations	$\{R^B_{t-1}/\pi_t\}$	direct
		life-time budget	$\{w_t, r_t, \pi_t\}$	indirect
		borrowing constraints	$\{w_t, r_t, \pi_t, q_t\}$	
intratemporal labor-leisure	$\{N_t\}\{C_t = X_t + G(N_t)\}$	marginal disutility of work	$\{w_t\}$	

Notes: The table breaks the household problem down into inter- and intratemporal decisions. The gray shaded block represents the effects of monetary policy through general equlibrium changes in prices, i.e. the indirect effects. **Borrowing constraints** (in bold) **only** bind **in** the **incomplete markets** version of the model.

Consider a contractionary monetary policy shock. All else equal, an increase in the nominal interest rate increases the real return on nominal assets and, thus, the intertemporal relative price of composite consumption of leisure and goods, X_t, today vs. tomorrow.[17] I refer to the interest rate channel as the direct effect of monetary policy. Since prices are sticky, the decrease in consumption is not completely offset by lower prices, and output falls. Lower output, in turn, decreases income and consumption, which again reduces income and so forth. I refer to the equilibrium changes in income and prices as the indirect effects of monetary policy.

In the complete markets economy, these indirect effects matter for composite consumption only in so far as they change life-time income, because the consumption path is determined by a sequence of Euler equations and a single life-time budget constraint. The consumption of final goods, C_t, and labor supply, N_t, follows then through the intratemporal consumption-leisure trade-off that solely depends on the wage rate. With

[17]Recall that the household problem can be expressed in terms of composite consumption X_t with GHH preferences: $x_{it} = c_{it} - \frac{\tau_1 w_t h_{it} N_t}{1+\gamma}$. It is therefore the intertemporal allocation of composite consumption that matters for the household in this model.

incomplete markets, however, current income becomes an important determinant of composite consumption and, thus, consumption of final goods because of borrowing constraints.

The indirect effects of monetary policy, therefore, work through the (life-time) budget constraint and the complementarity of consumption and hours worked inherent in GHH preferences in this model. This paper is about the effect of borrowing constraints on household decisions through the budget constraint channel. For this purpose, GHH preferences and the specific form of the disutility of labor adopted are helpful. They rule out wealth effects on labor supply and more generally make labor supply independent of all idiosyncratic states. As a result, labor supply only depends on the aggregate wage rate in both versions of the model and, thus, does not contribute to differences in the response to monetary shocks.

The complementarity of leisure and hours worked does matter for the total response, of course, as this is an important factor in the determination of consumption of final goods.[18] It is hence the difference in the response of composite consumption between both economies that identifies the effect of borrowing constraints. The quantitative assessment of the differences between both economies comes next.

5.2 Aggregate Effects of a Monetary Policy Shock

In the following, I consider the effect of a monetary surprise that, all else equal, would increase the nominal interest rate by one-standard deviation, i.e. 18 basis points (quarterly), in period 0. Figure 2.2 compares the responses of the economy with and without heterogeneity in household portfolios.

What stands out immediately is that the aggregate responses of both economies are very similar. In particular the responses of employment and wages are nearly identical due to the specification of the preferences. The initial drop in output is 0.54 percent in the full model and 0.48 percent in the representative household version of the model. The composition of the output drop, however, is quite different. The fall in consumption is steeper and more persistent in the economy with heterogeneous households, while the reverse is true for investment.[19] Consumption falls by 0.13 percentage points more and the total consumption loss over 4 years is 0.31 percentage points higher with incomplete markets. Looking at composite consumption X_t, which leaves out the effect of GHH preferences, makes this more evident.

To assess the importance of the direct effect on consumption in both economies, I compare the response of composite consumption to interest rate changes keeping all other prices at steady state values. With complete markets, changing the path of the real interest rate to the path in Figure 2.2 lowers composite consumption by 0.065 percent.[20] This number reduces to 0.056 percent with incomplete markets.[21]

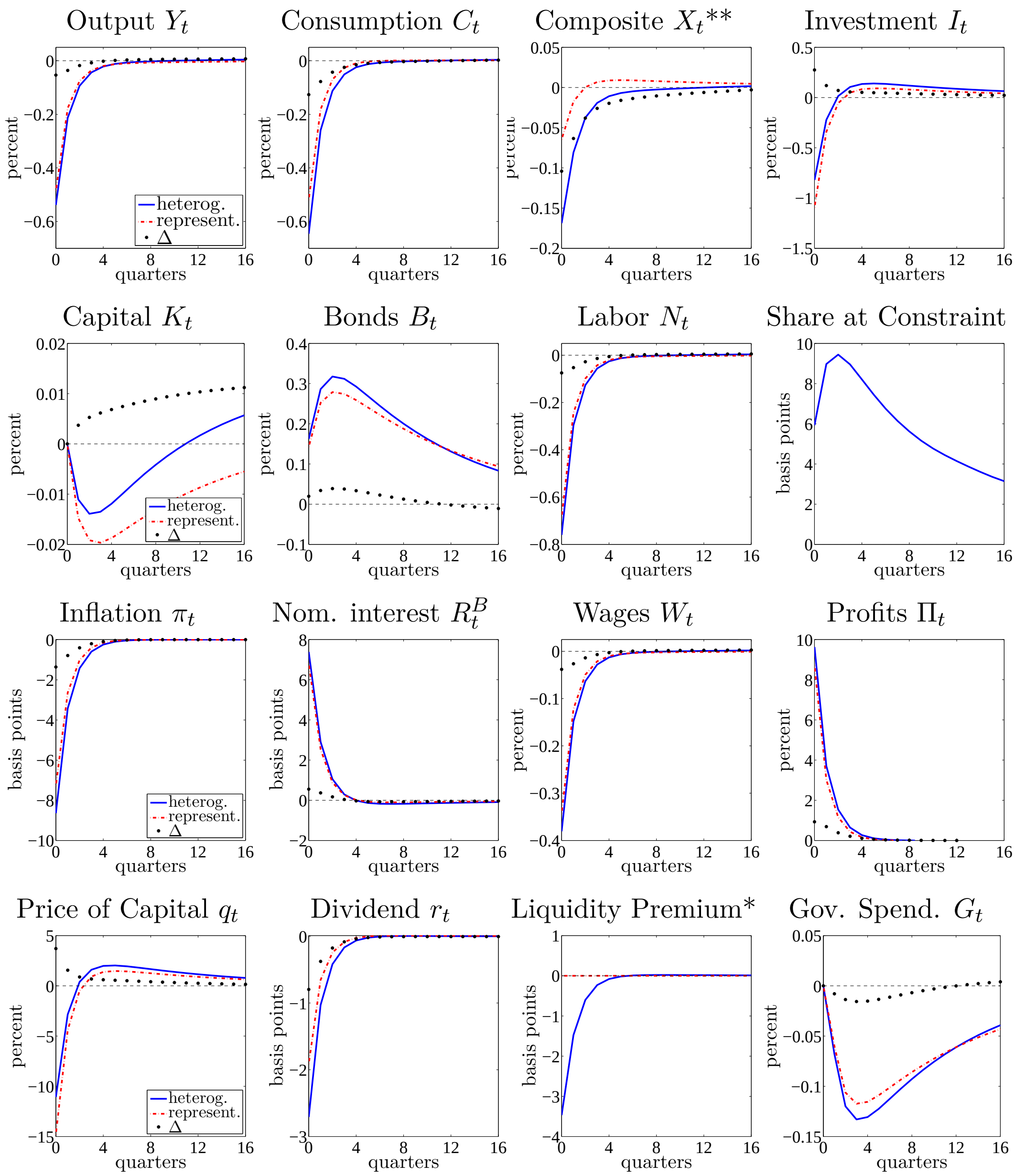
[18]More leisure time decreases the marginal utility of consumption with GHH preferences such that, all else equal, consumption falls with labor supply N_t.

[19]Government spending does not respond in period 0. The responses differ at the maximum by 0.02 percentage points and, thus, are not of importance for differences in the output responses.

[20]I feed the path of the real interest rate into the Euler equation and budget constraint of the representative household without changing any other prices to determine the partial consumption response.

[21]The path of the real interest rate differs between both economies. Assuming the same path as in the

Figure 2.2: Response to a one-standard deviation monetary shock



Notes: Impulse responses to a one-standard deviation monetary policy shock, $\epsilon^D =$ 18 basis points. Solid line: the model with heterogeneous households. Dashed line: same calibration with a representative household. Dots: solid-dashed. The y-axis shows either percent or basis points deviations from the no-shock path. The x-axis shows time since the shock in quarters. $*LP = (q_{t+1} + r_{t+1})/q_t - R_t^B/\pi_{t+1}$ $**X_t = \int (c_{it} - h_{it}\frac{n_{it}^{1+\gamma}}{1+\gamma})di$

Comparing these numbers to the equilibrium response of composite consumption in Figure 2.2 identifies the indirect effect through income. Composite consumption falls almost three times more in the economy with incomplete markets relative to the direct response, whereas it barely changes with complete markets. What matters for composite consumption of the representative household is life-time income as savings adjust to undo any effect of temporary income losses on consumption. Therefore, the indirect effect through the life-time budget constraint is of minor importance. Current income, however, responds strongly and so does composite consumption with market-incompleteness because of borrowing constraints. The indirect effect through tighter borrowing constraints, therefore, more than outweighs the muted direct effect of interest rate changes on consumption in the full model.

Quantitatively, the indirect effect explains 65% of the drop in composite consumption with market-incompleteness, while the direct effect through interest rate changes accounts for only 35%. This difference becomes substantially higher when the indirect effect through GHH preferences is included. Looking at consumption of final goods, indirect effects make up for more than 90% of the total response. The GHH effect, however, is also present in the complete-markets setting. It also accounts for 87.5% of the response in consumption of final goods there. This is driven by the adopted preference specification, of course, and vanishes with additively separable preferences in consumption and leisure. With such preferences, the response of composite consumption applies, which is completely determined by the direct effect with complete markets.

The stronger reaction of consumption to monetary shocks is not reflected in output because investment falls by 25% less with incomplete versus complete markets. The smaller reaction of investment is a consequence of the redistributive effects of monetary policy in the full model. A tightening of monetary conditions increases inequality because it redistributes from borrowers to lenders and from households that earn wage-income to those that earn profit-income. Both channels transfer from the bottom to the top of the wealth distribution and hence increase inequality.[22] Wealthy households hold relatively more high-return real assets in their portfolios, recall Figure 2.1, and, thus, stabilize investment demand as they get richer through redistribution.

This points to the importance of heterogeneity in portfolios for aggregate outcomes. The redistributive consequences of monetary policy are discussed next.

5.3 Importance of Heterogeneity for the Transmission

With incomplete markets, the transmission of monetary policy also works through redistributive effects. Household portfolios in the model differ in net nominal positions, real asset holdings, and human capital. This section quantifies the relative importance of gains and losses on these three dimensions for the transmission. Let me discuss the channels in turn. A higher real rate of interest benefits bondholders at the expense of debtors. Both lenders and borrowers, however, lose on their real asset holdings as asset prices and dividends fall. Labor income declines as well, while income from profit increases.

complete markets benchmark shows that consumption falls by 25% less with market-incompleteness.

[22]These findings mirror recent empirical evidence by Coibion et al. (2012).

Table 2.4: Exposure to monetary shocks by wealth holdings

	Income gains/losses			Capital gains/losses
By wealth percentiles	Interest $\Delta(R^B_{t-1}/\pi_t)$	Dividends Δr_t	Labor/Profit $\Delta(W_t N_t + \Pi_t)$	on real assets Δq_t
0-10	-0.23	-0.00	-1.62	-0.00
10-20	-0.10	-0.01	-1.57	-0.04
20-30	-0.03	-0.03	-1.53	-0.13
30-40	0.02	-0.05	-1.51	-0.21
40-50	0.04	-0.08	-1.49	-0.31
50-60	0.06	-0.11	-1.45	-0.43
60-70	0.08	-0.14	-1.40	-0.56
70-80	0.10	-0.20	-1.28	-0.80
80-90	0.15	-0.52	0.01	-2.11
90-100	0.29	-1.27	1.39	-5.18

Notes: Gains and losses in percent of within group consumption in period 0 to a one-standard deviation monetary policy shock, $\epsilon^D = 18$ basis points. Results are expressed in terms of steady-state consumption of each decile and averaged by using frequency weights from the steady-state wealth distribution.

Table 2.4 summarizes the gains and losses on each of the three portfolio dimensions across the wealth distribution relative to average consumption of each wealth bracket. The sizable fall in labor income represents the single largest loss for the bottom 80% of the wealth distribution. Households in the top quintile of the wealth distribution, in contrast, enjoy higher returns on their human capital on average, because an over-proportionate share are entrepreneurs. As such, they receive profit income, which increases. The top quintile incurs the highest losses on the real asset position. However, most of it is due to lower asset prices that are not completely realized. In addition, those households are also the largest bondholders and, thus, gain on that account from higher real returns. All in all, the top 10% of households in terms of net worth stands to gain from a monetary tightening as long as they do not realize more than 8% of their capital losses. This is clearly the case as Figure 2.3 shows.

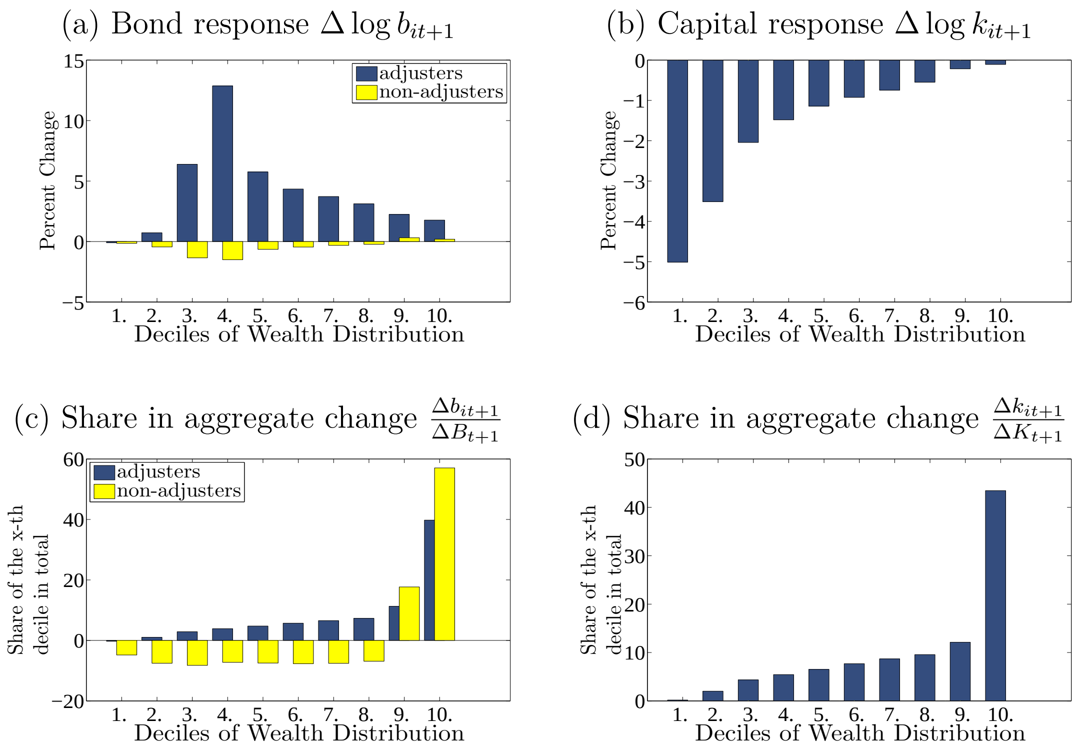
Investment Response

Figure 2.3 provides an overview of the portfolio adjustments to the monetary shock across the wealth distribution. The charts in the first row show the change in bond and capital holdings, whereas the second row depicts the contribution of each decile of the wealth distribution to the total change.

Two results stand out. First, the response of the top 20% in terms of net worth explains more than 50% of the change in aggregate savings in bonds and capital. Second, the response of capital declines in wealth holdings. The wealthiest households liquidate only a very small fraction of their capital holdings, while capital holdings by the poorest decile falls 5 times more than aggregate capital. Wealthy households do not only adjust their capital holdings little, but they also account for the majority of the aggregate capital response. As a consequence, higher inequality stabilizes investment as it broadens the difference in the capital response between the top and the bottom. It is therefore the redistributive consequences of monetary policy that explain the significantly weaker response of investment in the incomplete markets economy relative to the complete markets setting, in which no redistribution occurs by definition.

In fact, the investment response would be even more muted if the liquidity premium, i.e. the return on capital relative to the return on bonds, remained at its steady-state value. The relative return on bonds, however, increases to absorb the additional supply of government bonds created by the shortfall in tax revenues. In equilibrium, the liquidity premium falls 3 by basis points (cf. Figure 2.2). This makes wealthy household invest more in bonds as they would otherwise do.

Figure 2.3: Portfolio adjustments to a monetary shock

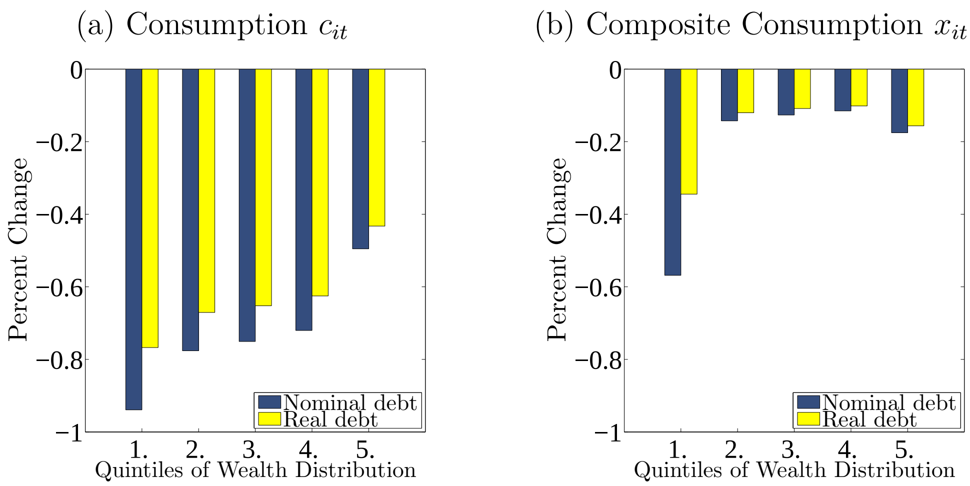


Notes: Change in savings in bonds and capital to a one-standard deviation monetary policy shock, $\epsilon^D = 18$ basis points in period 0. The first row shows the average savings response of all households in a given decile of the wealth distribution. The second row shows how much each decile contributes to the aggregate change in bond and capital holdings. The policies are averaged using frequency weights from the steady-state wealth distribution.

Consumption Response

The redistributive consequences of monetary policy do not only matter for the investment response, but also for the consumption response. Debt deflation is a prominent channel, going back at least to Fisher (1933), that has potentially strong effects on consumption as indebted households are closer to the borrowing constraint. In the following, I assess the importance of this channel for consumption across households and aggregate outcomes.

Figure 2.4: Consumption response to a monetary shock with nominal vs. real debt



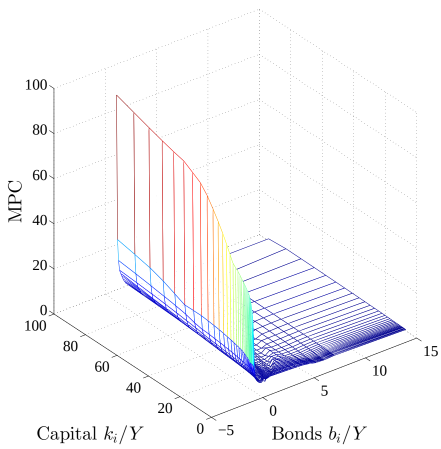
Notes: Average consumption response of all households in a given quintile of the wealth distribution to a one-standard deviation monetary policy shock, $\epsilon^D = 18$ basis points, in period 0. The left columns correspond to the full model with nominal debt and right ones to the same model with real debt. The policies are averaged using frequency weights from the steady-state wealth distribution.

Heterogeneity in portfolios and, thus, in the exposure to monetary shocks generates sizable heterogeneity in household consumption. The left bars in Figure 2.4 plot the change in consumption by wealth holdings in the baseline economy with nominal debt. The effect on consumption of final goods includes both the effect of lower income and less hours worked through the complementarity of consumption and leisure. Consumption by households in the first quintile declines by 2 times more than consumption by the top quintile in terms of net worth. This difference becomes more pronounced by considering consumption of the composite good X_t as it leaves out the GHH effect that applies to all households. The ratio of consumption between top and bottom quintile of the wealth distribution increases from 2 to 3 in this case.

Households in the first quintile of the wealth distribution suffer not only from substantially lower earnings, but also from a higher real rate on nominal debt (c.f. Table 2.4). The effect of the latter goes only through surprise inflation in period 0. By assumption, the monetary shock affects the nominal interest rate tomorrow but not today.

This timing assumption allows to shut down the initial redistribution through differences in net nominal positions by considering an economy with real debt (inflation-indexed bonds). The right bars in Figure 2.4 show the response of consumption in the same economy but with real debt. Indebted households gain the most, but also lenders gain because the indirect effects of monetary policy become weaker. Considering consumption of composite goods identifies the importance of indirect effects through the budget constraint. Clearly, this effect is the dominant one for the first quintile of the wealth distribution, while richer households gain little as they are further away from the borrowing constraint.

Figure 2.5: Marginal propensity to consume



Notes: $\Delta c_{it}/\Delta Y_t$ with $\Delta Y_t = 0.01Y^{SS}$ across capital and bond holdings expressed relative to quarterly income. Human capital is integrated out using the steady-state joint-distribution.

All in all, redistribution through inflation explains about 15% of the total output loss in Figure 2.2. Redistribution through inflation amplifies the indirect effects of monetary policy because it is strongly correlated with marginal propensities to consume (MPC). Figure 2.5 shows the MPCs across capital and bond holdings. Clearly, consumption becomes more sensitive to current income the less liquid bonds a household holds. Lower than expected inflation therefore redistributes from households with high MPCs to households with low MPCs. This depresses aggregate consumption and, thus, output.

6. Conclusion

Heterogeneity in household portfolios has important implications for the transmission of monetary policy. This paper quantifies the consumption and savings response to monetary shocks in a New Keynesian business cycle model with incomplete markets and assets with different degrees of liquidity. When markets are incomplete, the direct effect of changes in interest rates explains less than half of the consumption response to monetary policy shocks. The response of consumption is primarily driven by indirect equilibrium changes in income that strongly affect consumption by liquidity-constrained households. The equilibrium effects, in turn, mainly work through changes in labor income. Redistribution through the revaluation of nominal claims, including Fisher (1933) debt deflation, reinforces the effect on consumption. At the same time, the redistributive consequences of monetary policy imply a muted investment response. The share of real assets in household portfolios increases in household wealth such that second-round changes in inequality affect the investment response.

This is in stark contrast to the transmission mechanism in standard New Keynesian models that build on a representative household. When borrowing constraints do not apply, temporary changes in income are not of importance and monetary shocks do hardly affect life-time income. Consequently, consumption responds solely to the changes in interest rates. Savings, in contrast, react strongly and undo any temporary mismatch between income and consumption.

The reversal of the importance of direct and indirect effects in the transmission mechanism has important implications for the conduct of monetary policy. When markets are incomplete, the power of interest rates to affect aggregate economic activity relies to a large extent on equilibrium effects on labor income. The response of labor income, in turn, depends on a functioning labor market. In particular, how much does demand for labor respond to changes in aggregate demand? Labor market frictions or financial frictions on the side of firms might, therefore, impede the transmission of monetary policy. Provided the transmission works, mistakes in the setting of the interest rate still imply larger consumption volatility. Therefore, welfare costs of monetary policy shocks might be substantially higher than previously thought. Moreover, the weakening of the interest rate channel questions the existing results on optimal monetary policy rules. It is thus important to reassess the optimality of the properties of the Taylor-rule in a New Keynesian model with incomplete markets.

Appendices

A. First Order Conditions

Denote the optimal policies for consumption, bond holdings, and capital holdings as $x_i^*, b_i^*, k^*, i \in \{a, n\}$ respectively. Let $\mathbf{z}$ be a vector of potential aggregate states. The first-order conditions for an inner solution in the (no-)adjustment case read:

$$k^* : \frac{\partial u(x_a^*)}{\partial x} q = \beta E\left[\nu \frac{\partial V_a(b_a^*, k^*; \mathbf{z}')}{\partial k} + (1-\nu)\frac{\partial V_n(b_a', k'; \mathbf{z}')}{\partial k}\right] \tag{2.24}$$

$$b_a^* : \frac{\partial u(x_a^*)}{\partial x} = \beta E\left[\nu \frac{\partial V_a(b_a^*, k^*; \mathbf{z}')}{\partial b} + (1-\nu)\frac{\partial V_n(b_a^*, k^*; \mathbf{z}')}{\partial b}\right] \tag{2.25}$$

$$b_n^* : \frac{\partial u(x_n^*)}{\partial x} = \beta E\left[\nu \frac{\partial V_a(b_n^*, k; \mathbf{z}')}{\partial b} + (1-\nu)\frac{\partial V_n(b_n^*, k; \mathbf{z}')}{\partial b}\right] \tag{2.26}$$

Note the subtle difference between (2.25) and (2.26), which lies in the different capital stocks k' vs. k in the right-hand side expressions.

Differentiating the value functions with respect to k and b, I obtain the following:

$$\frac{\partial V_a(b, k; \mathbf{z})}{\partial k} = \frac{\partial u[x_a^*(b, k; \mathbf{z})]}{\partial x}(q(\mathbf{z}) + r(\mathbf{z})) \tag{2.27}$$

$$\frac{\partial V_a(b, k; \mathbf{z})}{\partial b} = \frac{\partial u[x_a^*(b, k; \mathbf{z})]}{\partial x}\frac{R^B(\mathbf{z})}{\pi(\mathbf{z})} \tag{2.28}$$

$$\frac{\partial V_n(b, k; \mathbf{z})}{\partial b} = \frac{\partial u[x_n^*(b, k; \mathbf{z})]}{\partial x}\frac{R^B(\mathbf{z})}{\pi(\mathbf{z})} \tag{2.29}$$

$$\begin{aligned}\frac{\partial V_n(b, k; \mathbf{z})}{\partial k} &= r(\mathbf{z})\frac{\partial u[x_n^*(b, k; \mathbf{z})]}{\partial x} \\ &+ \beta E\left[\nu \frac{\partial V_a[b_n^*(b, k; \mathbf{z}), k; \mathbf{z}']}{\partial k} + (1-\nu)\frac{\partial V^n[b_n^*(b, k; \mathbf{z}), k; \mathbf{z}']}{\partial k}\right] \\ &= r(\mathbf{z})\frac{\partial u[x_n^*(b, k; \mathbf{z})]}{\partial x} + \beta\nu E\frac{\partial u\{x_a^*[b_n^*(b, k; \mathbf{z}), k; \mathbf{z}], k; \mathbf{z}'\}}{\partial x}(q(\mathbf{z}') + r(\mathbf{z}')) \\ &+ \beta(1-\nu)E\frac{\partial V_n\{[b_n^*(b, k; \mathbf{z}), k; \mathbf{z}], k; \mathbf{z}'\}}{\partial k}\end{aligned} \tag{2.30}$$

The marginal value of capital in the case of non-adjustment is defined recursively.

Substituting the second set of equations into the first set of equations I obtain the following Euler equations (in slightly shortened notation):

$$\frac{\partial u[x_a^*(b,k;\mathbf{z})]}{\partial x}q(\mathbf{z}) = \beta E\left[\nu\frac{\partial u[x_a^*(b_a^*,k^*;\mathbf{z}')]}{\partial x}[q(\mathbf{z}')+r(\mathbf{z}')]+(1-\nu)\frac{\partial V^n(b_a^*,k';\mathbf{z}')}{\partial k'}\right] \tag{2.31}$$

$$\frac{\partial u[x_a^*(b,k;\mathbf{z})]}{\partial x} = \beta E\frac{R^B(\mathbf{z}')}{\pi(\mathbf{z}')}\left[\nu\frac{\partial u[x_a^*(b_a^*,k^*;\mathbf{z}')]}{\partial x}+(1-\nu)\frac{\partial u[x_n^*(b_a^*,k';\mathbf{z}')]}{\partial x}\right] \tag{2.32}$$

$$\frac{\partial u[x_n^*(b,k,;\mathbf{z})]}{\partial x} = \beta E\frac{R^B(\mathbf{z}')}{\pi(\mathbf{z}')}\left[\nu\frac{\partial u[x_a^*(b_n',k;\mathbf{z}')]}{\partial x}+(1-\nu)\frac{\partial u[x_n^*(b_n^*,k;\mathbf{z}')]}{\partial x}\right] \tag{2.33}$$

B. Equilibrium Forecasting Rules

The equilibrium forecasting rules are obtained by regressing them in each iteration of the algorithm on 20.000 observations. I generate the observations by simulating the model in parallel on 20 machines, letting each economy run for 1250 periods and discarding the first 250 periods. The R^2 is generally above 99.99%. Table 2.5 shows the equilibrium forecasting rules for monetary policy shocks at steady state TFP.

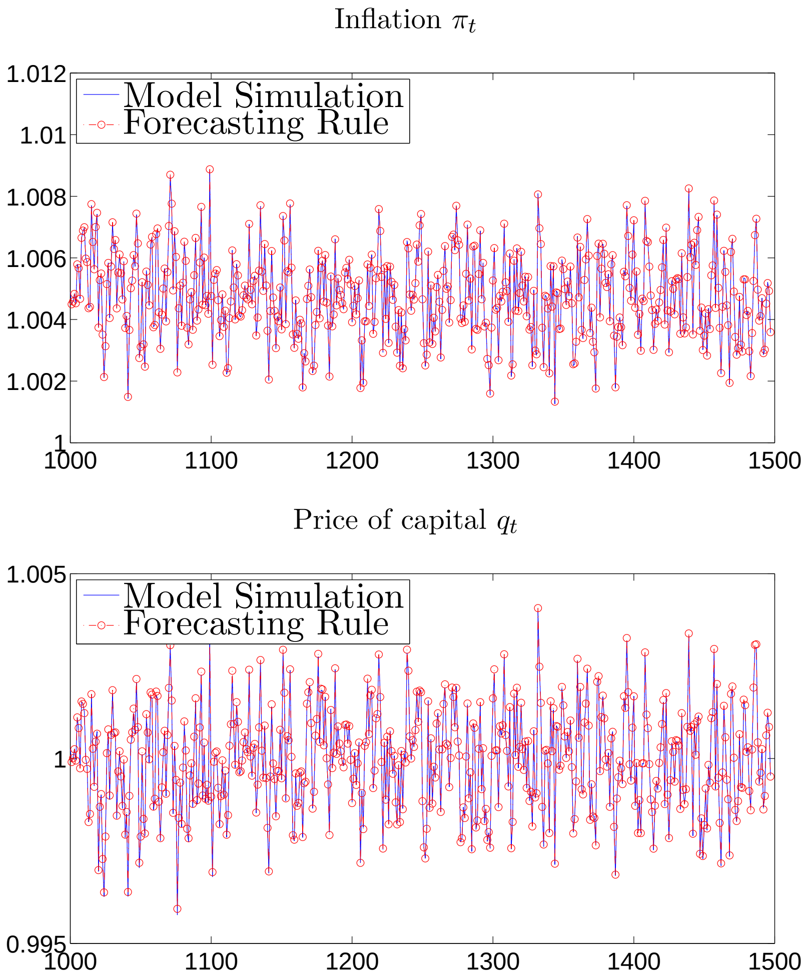
Table 2.5: Laws of motion

	Law of motion for π ($R^2 = 99.99$)						
	s^1	s^2	s^3	s^4	s^5	s^6	s^7
β_π^1	0.66	0.60	0.54	0.49	0.43	0.37	0.32
β_π^2	-0.33	-0.33	-0.34	-0.36	-0.38	-0.40	-0.43
β_π^3	-1.58	-1.65	-1.63	-1.63	-1.67	-1.65	-1.77
β_π^4	-34.82	-35.06	-35.31	-35.54	-35.80	-35.97	-36.03
	Law of motion for q ($R^2 = 99.99$)						
	s^1	s^2	s^3	s^4	s^5	s^6	s^7
β_q^1	0.24	0.16	0.09	0.01	-0.07	-0.15	-0.23
β_q^2	7.09	7.15	7.18	7.24	7.30	7.34	7.39
β_q^3	8.34	8.55	8.90	9.19	9.46	9.54	9.82
β_q^4	-43.22	-43.00	-42.75	-42.68	-42.93	-43.47	-43.85

Notes: All values are multiplied by 100 for readability.

Following Den Haan (2010), I also test the out-of-sample performance of the forecasting rules. For this I initialize the model and the forecasting rules at steady state values, feed in the same shock sequence, but otherwise let them run independently. Figure 2.6 plots time series of the prices q and π taken from the simulation of the model and the forecasting rules. The equilibrium forecasting rules track the evolution of the underlying model without any tendency of divergence. Table 2.6 summarizes the mean

Figure 2.6: Out-of-sample forecast performance of forecasting rules



Notes: Out-of-sample comparison between forecasting rules and model zoomed in at $t = \{1000, ..., 1.500\}$ for visibility; see Den Haan (2010).

and maximum difference between the series generated by the model and the forecasting rules. The mean error for all four time series is less than 0.005%. The maximum errors are small, too.

Table 2.6: Forecasting errors

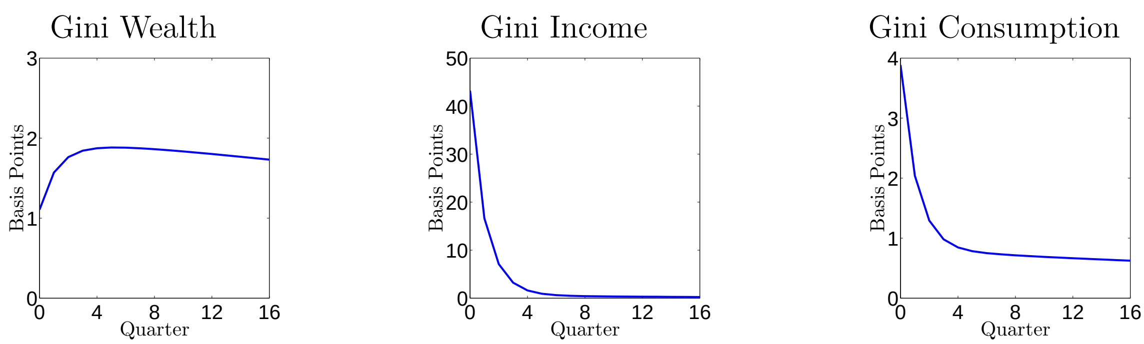
	Price of Capital q_t	Inflation π_t
Mean Error	0.004%	0.002%
Max Error	0.016%	0.007%

Notes: Percentage differences in out-of-sample forecasts between forecasting rules and model; see Den Haan (2010).

C. Distributional Consequences: Gini Indexes

Figure 2.7 displays the Gini indexes for total wealth, income, and consumption. Inequality in income and consumption instantaneously react to the expansionary monetary policy shock, whereas wealth inequality slowly falls. The initial decrease in the Gini index for income is about 5 times larger than the decrease in the Gini index for consumption. This points to substantial consumption smoothing. The dynamics of income inequality follow the response of inflation, which quickly returns to its steady state value and with it profits as well. The decline in consumption inequality, by contrast, is more persistent because of a prolonged time of lower wealth inequality.

Figure 2.7: Monetary policy shock: Gini indexes

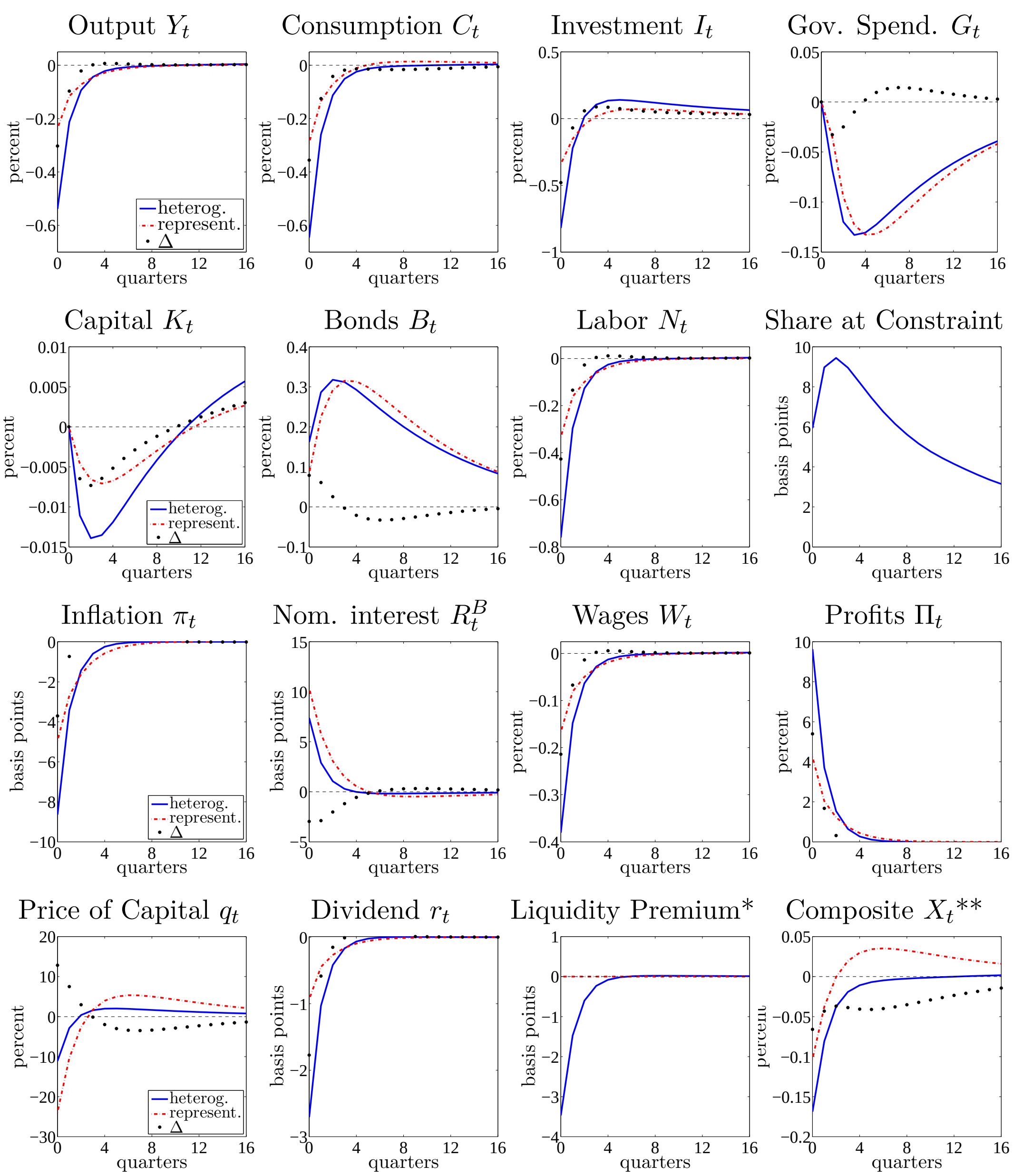


Notes: Impulse responses of Gini indexes of wealth, income, and consumption to an 18 basis points monetary policy shock, ϵ^D. The y-axis shows basis point changes (an increase of "100" implies an increase in the Gini index from, say, 0.81 to 0.82).

D. Recalibration of Investment Volatility

Figure 2.8 shows the impulse responses to a one-standard deviation monetary tightening for the economies with incomplete and complete markets. In this section, I have recalibrated the capital adjustment costs in the latter economy to make both versions match the same business cycle statistics.

Figure 2.8: Response with recalibrated business cycle statistics



Notes: Impulse responses to a one-standard deviation monetary policy shock, $\epsilon^D =$ 18 basis points. Solid line: the model with heterogeneous households. Dashed line: same economy with a representative household and recalibrated capital adjustment costs. The y-axis shows either percent or basis points deviations from the no-shock path. The x-axis shows time since the shock in quarters. $^*LP = (q_{t+1} + r_{t+1})/q_t - R_t^B/\pi_{t+1}$ $^{**}X_t = \int (c_{it} - h_{it}\frac{n_{it}^{1+\gamma}}{1+\gamma})di$

Chapter 3

Fiscal Stimulus Payments and Precautionary Investment

Deficit-financed government transfers to households have been an important part of the fiscal response to the last two recessions in the United States. This paper assesses the aggregate effects of this type of fiscal intervention in a New Keynesian business cycle model with incomplete markets and portfolio choice between liquid public debt and illiquid physical assets. In this environment, transfers do not only work through the disposable income channel but also by affecting household liquidity. Transfers increase individual liquidity and debt finance enhances market liquidity. This has consequences when the government retires this debt. Then households shift their savings into the physical asset to smooth consumption. This leads to a prolonged increase in capital and output. This precautionary investment channel dominates negative wealth effects of distortionary taxes making aggregate effects expansionary independent of the mode of financing.

1. Introduction

Deficit-financed government transfers to households, so called fiscal stimulus payments, have become part and parcel of the fiscal response to recessions. In the last two recession of 2001 and 2007-2009, U.S. households received one-off payments between \$500 to \$1000 amounting to fiscal outlays of $0.4 - 0.7\%$ of annual GDP. Household-level data from both episodes reveal that households spent on average around 25% of those payments on consumption.[1] The average size and the distribution of the consumption response can be rationalized through liquidity constraints (see Kaplan and Violante, 2014). Whether this type of fiscal intervention is successful in stabilizing output, however, depends on the joint-response of consumption and investment. A key argument against government transfers is that government deficits may crowd out private invest-

[1] See e.g. Johnson et al. (2006) and Parker et al. (2013). These studies have exploited the fact that the timing of receipt of payment was based on the last two digits of individual Social Security Numbers and, thus, effectively random. This randomization allows to estimate the causal effect of receipt of payment on consumption relative to the control group of households that received the payment in a different quarter.

ment potentially offsetting any increase in consumption.

This paper accounts for both household consumption and portfolio decisions in assessing the aggregate effects of fiscal stimulus payments. Toward this end, we build a New Keynesian business cycle model with incomplete markets and portfolio choice between liquid public debt and illiquid physical assets. Public debt can be traded without frictions, whereas physical capital can only be adjusted with a fixed probability each period. As in Aiyagari and McGrattan (1998), households value public debt as an additional means of smoothing consumption.

Despite of public debt and capital being substitutes, we find that deficit-financed government transfers may actually lead to a crowding-in of investment. Key for this result is the transient nature of the increase in public debt. On impact, when fiscal transfers and the increase in public debt take place, public debt only partly replaces capital while total savings increase. When - foreseeable - the government starts to retire this debt, households would like to hang on to their improved consumption-smoothing capacity. This crowds in private investment. We find that a faster reversal in public debt leads to a stronger boom in investment.

What is more, this precautionary investment channel is little affected by how public debt is financed. This is in stark contrast to an economy without the liquidity effect of public debt, in which financing by tax hikes or spending cuts lead to opposite investment responses. When transfers are financed by distortionary taxes, the net wealth effect of transfers is negative and, thus, savings fall in an economy with complete markets. When financed by government spending cuts, transfers imply an increase in wealth and, thus, a positive investment response. When markets are incomplete, however, the liquidity effect dominates the wealth effect such that investment always increases independent of the financing.

With these results, this paper contributes to the literature on the aggregate effects of fiscal stimulus payments by highlighting the liquidity channel of public debt. Oh and Reis (2012) build a model with incomplete markets and sticky prices, but abstract from public debt and instead look into the effects of redistribution across households within a period. Other studies feature Ricardian households to simplify the role of public debt. Giambattista and Pennings (2013) compare the multiplier of government purchases and transfer in a model with Ricardian and rule-of-thumb households. In a similar vein, Mehrotra (2014) compares both multipliers in a borrower-lender economy. McKay and Reis (2016) assess automatic fiscal stabilizers in a model in which Ricardian households own the capital stock.

We also contribute to the literature that discusses the effects of public debt by raising the supply of assets. We share with Aiyagari and McGrattan (1998) the focus on precautionary motives, but go beyond their steady state analysis. We show that transitory increases in public debt do not crowd-out capital because of an investment boom during the transition back to steady state. In a model without precautionary motives, Woodford (1990) shows that higher public debt may crowd-in investment through loosening liquidity constraints. The spirit of the analysis is closest to Challe and Ragot (2011). They investigate a similar liquidity channel in the case of deficit-financed government spending shocks with a focus on the consumption response. They find as well that liquidity effects dominate wealth effects leading to a crowding-in of consumption.

The remainder of the paper is organized as follows. Section 2, 3, and 4 explain changes made to the model, solution method, and calibration relative to Chapter 2. Section 5 presents the quantitative results, and Section 6 concludes.

2. Model

The model economy builds on Chapter 2 and, thus, we only describe the changes made to the model presented in Chapter 2 in the following. The economic environment differs as we consider different shocks and modify the fiscal rules. In particular, the economy is only subject to deficit-financed transfer shocks. We next describe the modified household problem and government in more detail. The problem of firms is unchanged.

2.1 Households

The household problem is modified in two ways: First, lump-sum transfers from the government are included in the budget constraint and, second, the tax rate might change over time depending on the fiscal rule in place.

For those households participating in the capital market, the modified budget constraint reads:[2]

$$\begin{aligned} c_{it} + b_{it+1} + q_t k_{it+1} =& \frac{R^B_{t-1}}{\pi_t} b_{it} + (q_t + r_t) k_{it} + \tau_t \left[s_{it} w_t h_{it} N_t + (1 - s_{it}) \Pi_t \right] + \tau_0, \\ & k_{it+1} \geq 0, b_{it+1} \geq 0, \end{aligned} \tag{3.1}$$

where b_{it} is the real value of nominal bond holdings, k_{it} are capital holdings, q_t is the price of capital, r_t is the rental rate or "dividend", R^B_{t-1} is the gross nominal return on bonds, and $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate. τ_0 is a lump-sum transfer paid out in period $t = 0$ and τ_t is a proportional tax rate on labor income $w_t h_{it} N_t$ and profit income Π_t. Households stochastically transit between receiving labor and profit income according to idiosyncratic shock s_{it}. We denote real bond holdings of household i at the end of period t by $b_{it+1} := \frac{\tilde{b}_{it+1}}{P_t}$.

For those households that cannot trade in the market for capital the modified budget constraint simplifies to:

$$\begin{aligned} c_{it} + b_{it+1} = & \frac{R^B_{t-1}}{\pi_t} b_{it} + r_t k_{it} + \tau_t \left[s_{it} w_t h_{it} N_t + (1 - s_{it}) \Pi_t \right] + \tau_0, \\ & b_{it+1} \geq 0. \end{aligned} \tag{3.2}$$

With this setup, two Bellman equations characterize the dynamic planning problem of a household; V_a in case the household can adjust its capital holdings and V_n

[2]The household problem can be expressed in terms of composite good x_{it} by making use of $c_{it} = x_{it} + \frac{\tau_t w_t h_{it} N_t}{1+\gamma}$.

otherwise:[3]

$$
\begin{aligned}
V_a(b,k,h,s;\Theta) = & \max_{k',b'_a} u[c(b,b'_a,k,k',h,s)] + \beta[\nu EV^a(b'_a,k',h',s',\Theta') \\
& + (1-\nu)EV^n(b'_a,k',h',s',\Theta')], \\
V_n(b,k,h,s;\Theta) = & \max_{b'_n} u[c(b,b'_n,k,h,s)] + \beta[\nu EV^a(b'_n,k,h',s',\Theta') \\
& + (1-\nu)EV^n(b'_n,k,h',s',\Theta')]. \qquad (3.3)
\end{aligned}
$$

2.2 Central Bank and Government

Monetary policy follows the same Taylor (1993)-type rule as in Chapter 2 but is not subject to shocks:

$$\frac{R_t^B}{R^B} = \left(\frac{R_{t-1}^B}{R^B}\right)^{\rho_{R^B}} \left(\frac{1+\pi_t}{1+\pi}\right)^{\theta_\pi}. \qquad (3.4)$$

All else equal, the central bank raises the nominal rate above its steady-state value R^B whenever inflation exceeds its target value. It does so by more than one-to-one to guarantee a non-explosive price path ($\theta_\pi > 1$). The parameter ρ_{R^B} captures "intrinsic policy inertia".

The fiscal authority pays lump-sum transfers $\tau_0 > 0$ to households in period $t = 0$ that are financed by debt issuance. Let B_{t+1} denote time t real value of public debt. The government budget constraint reads:

$$B_{t+1} = \frac{R_{t-1}^B}{1+\pi_t} B_t + G_t + \tau_0 - T_t, \qquad (3.5)$$

where real tax revenues are given by:

$$T_t = (1-\tau_t)\left[(N_t W_t \int s_i h_i \Theta_t(b,k,h,s)) + \Pi_t\right]. \qquad (3.6)$$

The government either adjusts taxes τ_t or government spending G_t to bring debt back to its steady state value from $t = 1$ onwards. We assume simple linear rules similar to the ones estimated by Leeper et al. (2010):

$$G_t = \gamma_1 - \gamma_2(B_t - B), \qquad (3.7)$$

$$\tau_t = \gamma_3 + \gamma_4 \log(B_t/B), \qquad (3.8)$$

with B equal to the steady state debt level. The parameters $\gamma_2, \gamma_4 > 0$ measure the speed at which public debt returns to its steady state value.

2.3 Recursive Equilibrium

The *recursive equilibrium* and market clearing conditions are unchanged.

[3] No conditioning on aggregate shocks is required. The transfer shock only occurs at $t = 0$ and then the economy deterministically reverts back to steady state.

3. Numerical Implementation

We compute the transitional dynamics after an unexpected one-off fiscal stimulus shock with the help of Krusell and Smith (1998)-rules. We consider an economy that is in steady state before period $t = 0$. In $t = 0$, all households receive an unexpected fiscal transfer τ_0. There are no more shocks from $t = 1$ onwards. From then on, households anticipate how prices evolve on the path back to the long-run equilibrium of the economy. These prices are, of course, a function of all states including the joint distribution $\Theta_t(b, k, h)$. Hence, we assume that households predict future prices on the basis of a restricted set of moments as in Krusell and Smith (1997, 1998).

Specifically, we make the assumption that households condition their expectations on last period's aggregate real bond holdings, B_t, last period's nominal interest rate, R^B_{t-1}, and the aggregate stock of capital, K_t. If asset-demand functions, $b^*_{a,n}$ and k^*, are sufficiently close to linear in human capital, h, and in non-human wealth, b and k, at the mass of Θ_t, we can expect approximate aggregation to hold. For this exercise, the three aggregate states – B_t, R^B_{t-1}, K_t – are sufficient to describe the evolution of the aggregate economy.

Households use the following log-linear forecasting rules for current inflation and the price of capital:

$$\log \pi_t = \beta^1_\pi + \beta^2_\pi \hat{B}_t + \beta^3_\pi \hat{K}_t + \beta^4_\pi \hat{R}^B_{t-1}, \tag{3.9}$$

$$\log q_t = \beta^1_q + \beta^2_q \hat{B}_t + \beta^3_q \hat{K}_t + \beta^4_q \hat{R}^B_{t-1}, \tag{3.10}$$

where $\hat{(.)}$ refers to log-differences from the steady state value of each variable. The law of motion for aggregate real bonds, B_t, then follows from the government budget constraint (3.5). The Taylor-rule (3.4) determines the motion of the nominal interest rate, R^B_t. The law of motion for K_t is the same as in Chapter 2.

To find the deterministic law of motion in response to a zero-probability fiscal stimulus payment shock, we need to solve for the market clearing prices each period. Concretely, this means the posited rules, (3.9) and (3.10), are used to solve for the households' policy functions. Having solved for the policy functions conditional on the forecasting rules, we then simulate the model for $t = 0, \ldots, T$ periods, keeping track of the actual distribution Θ_t. The simulation starts in steady state and the transfer shock hits in $t = 0$. We then calculate in each period t the optimal policies for market clearing inflation rates and asset prices assuming that households resort to the policy functions derived under rule (3.9) and (3.10) from period $t + 1$ onwards. Having determined the market clearing prices, we obtain next period's distribution Θ_{t+1}. We next re-estimate the parameters of (3.9) and (3.10) from the simulated data and update the parameters accordingly. Subsequently, we recalculate policy functions and iterate until convergence in the forecasting rules.

The posited rules (3.9) and (3.10) approximate the aggregate behavior of the economy well. The within sample R^2 is well above 99%. See Appendix A.

4. Calibration

One period in the model is a quarter. We adopt the calibration of Chapter 2 for the household and firm side. The model of Chapter 2 is calibrated to U.S. business cycle statistics and asset holdings by U.S. household. Fiscal policy is described either by a tax or spending rule reacting to public debt deviations from steady state. We choose the reaction parameters such that the path of debt is the same under both rules. Table 3.1 summarizes the calibration.

5. Results

In the following, we consider a policy experiment that consists of an unexpected, one-off payment of about \$500 (0.5% of annual output) to each household in the economy paid out in $t = 0$. We assume that the policy is deficit financed in $t = 0$ and that from $t = 1$ onwards either labor taxes or government spending react to debt deviations from steady state according to a fiscal rule that brings debt back to its steady state value.

We first show that the model replicates the distribution of marginal propensities to consume across households as documented by the empirical literature. We then assess the aggregate effects of this type of fiscal intervention under both financing schemes while keeping the path of debt constant. We do so in the full model in which public debt affects household liquidity and in a representative agent version of the model to highlight the importance of liquidity effects for aggregate outcomes.

5.1 Individual Consumption Response

Figure 3.1 plots the marginal propensity to consume (MPC) as a function of a household's asset position. Throughout most of the asset-space households consume very little out of the \$500 payment by the government. This includes the diagonal along which all households would be clustered if we were to consider net worth only. With two assets, however, a large fraction of wealthy households prefers to hold high-return illiquid over liquid assets (capital k_i over bonds b_i). This makes them potentially constrained in their consumption. In addition, these "wealthy hand-to-mouth" households have a higher MPC than their poor counterparts without any assets. The MPC increases in capital because richer households have a higher target for their consumption path. For households very rich in capital but without any liquid assets it actually reaches 90%, whereas households with no assets at all consume around 25% out of extra cash.

These patterns are reminiscent of recent empirical findings on household consumption behavior. Misra and Surico (2014), who use quantile regressions to identify heterogeneity in consumption responses, document that the households with the highest MPCs hold little liquid assets. The model is able to replicate this finding because of two features: First, markets are incomplete and households face idiosyncratic income risk making them ex-post heterogeneous. Second, portfolio choice between liquid and illiquid assets renders a large fraction of households constrained in their consumption each period. Short-run fluctuations in marginal utility are less costly then foregoing

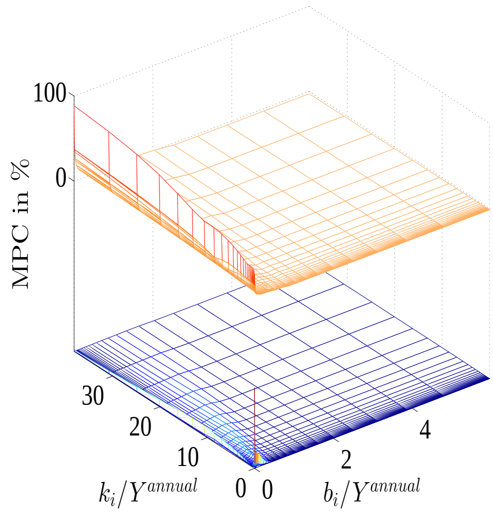
Table 3.1: Calibrated parameters

Parameter	Value	Description	Target
Households			
β	0.985	Discount factor	$K/Y = 290\%$ (annual)
ν	4.5%	Participation frequency	$B/Y = 23\%$ (annual)
ξ	1.5	Coefficient of rel. risk av.	Standard value
γ	0.5	Inv. Frisch elasticity	Standard value
Intermediate Goods			
α	72%	Share of labor	Income share of labor of 66%
δ	1.35%	Depreciation rate	NIPA: Fixed assets & durables
Final Goods			
κ	0.08	Price stickiness	Avg. price duration of 4 quarters
μ	0.06	Markup	6% markup (standard value)
Capital Goods			
ϕ	10	Capital adjustment costs	STD(I)/STD(Y)=3.5
Fiscal Policy			
γ_1	0.05	G in steady state	$G/Y = 20\%$
γ_2	0.2	G reaction function	Path of debt
γ_3	0.3	Tax rate in steady state	Budget balance
γ_4	0.2	τ reaction function	Path of debt
Monetary Policy			
Π	1.005	Inflation	2% p.a.
R^B	1.01	Nominal interest rate	4% p.a.
θ_π	1.5	Reaction to inflation	Standard value
ρ_{R^B}	0.95	Interest rate smoothing	Nakamura and Steinsson (2013)
Income Process			
ρ_h	0.987	Persistence of productivity	Standard value
$\bar{\sigma}$	0.08	STD of innovations	Standard value

the higher return on the illiquid asset.[4]

When it comes to the total effect of transfers, incomplete markets imply that it is important to take into account the path of public debt. An increase in public debt enhances household liquidity by effectively loosening borrowing constraints (see Aiyagari and McGrattan, 1998). When markets are incomplete, this has first order effects on consumption and savings. In the following section, we discuss the role of public debt for the aggregate effects of transfers.

Figure 3.1: Individual consumption response to a transfer shock



Notes: Marginal propensities to consume in partial equilibrium with fixed prices. The bottom of the graph depicts the distribution of households over capital and bond holdings relative to annual income in steady state.

5.2 Aggregate Effects of Transfers Payments

This section assesses the aggregate effects of deficit-financed transfer payments in a model in which public debt affects household liquidity. We compare two different financing scenarios with opposing wealth effects while keeping the path of public debt the same. We first discuss the effects of transfer payments with government spending adjusting from $t = 1$ onwards. Under this scenario, the present-value wealth effect of the fiscal intervention is positive.

Figure 3.2 shows the response of aggregate prices and quantities to the transfer in the case of lower government spending in the future. The first row of Figure 3.2 depicts output and its components (consumption, investment, government spending) as a percent of transfer. The solid line corresponds to the economy with endogenous heterogeneity, whereas the dashed line shows the response of an economy with two types

[4]The idea that small deviations from optimal consumption imply negligible utility costs goes at least back to Cochrane et al. (1989).

of households: Ricardian households and hand-to-mouth households without access to financial markets. We determine the share of hand-to-mouth households (about 30%) by matching the consumption response of the full model in the first period.

Accordingly, consumption increases by 50% of the transfer in both economies. This is about twice as much as the partial equilibrium response of consumption discussed in the previous section. With sticky prices the initial increase in consumption is met by higher production and, thus, higher income amplifying the direct effect of the transfer on consumption. This disposable income channel, which relies on the presence of households with high marginal propensities to consume, drives the output response in the first period.

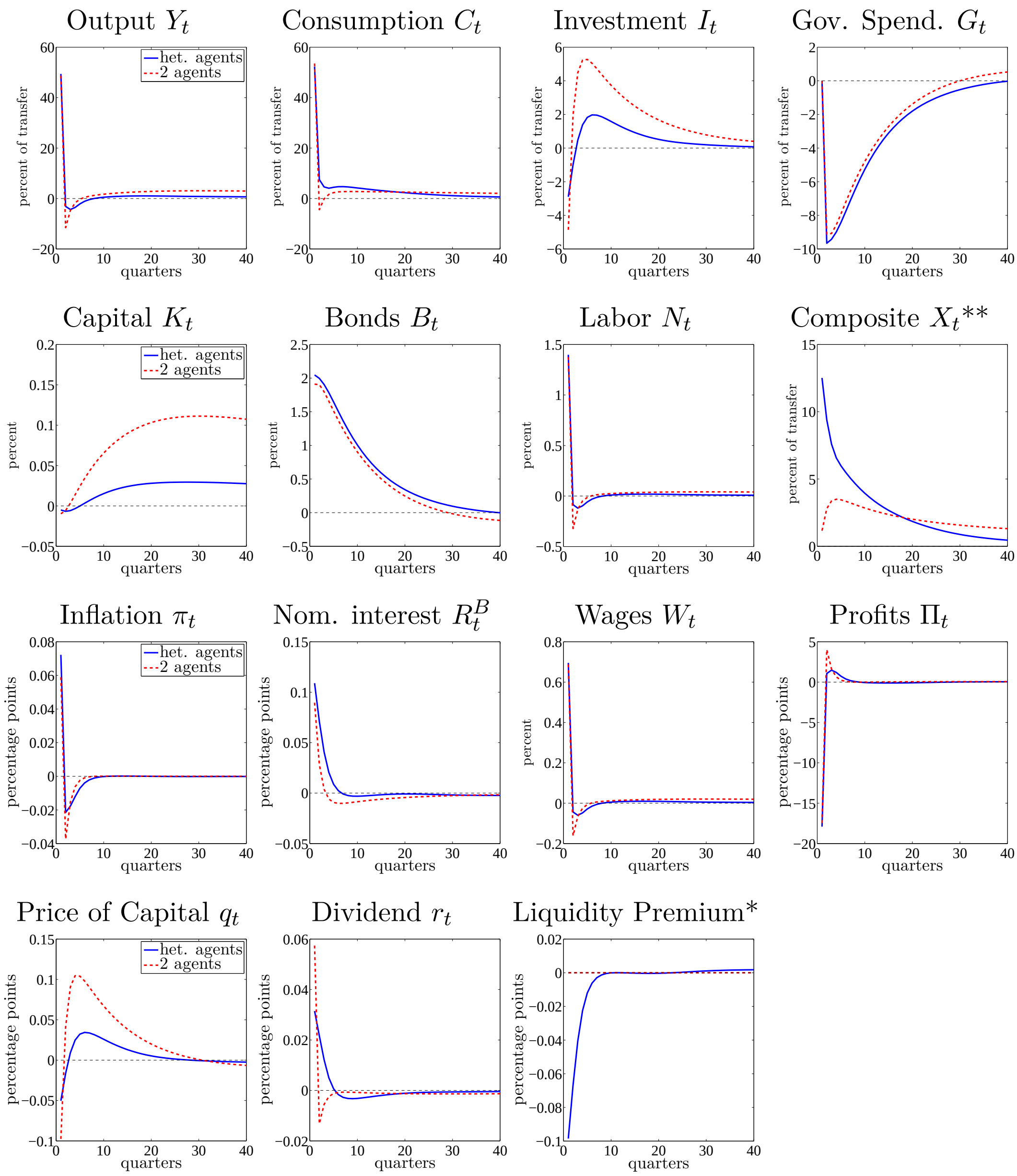
After the first period, the response of unconstrained households becomes central for the path of output. Unconstrained households respond to the positive wealth effect by increasing consumption permanently. To do so, households move wealth into the future. In particular, they increase their holdings of physical assets as the government reduces the amount of outstanding debt at the same time. This investment boom is stronger with complete markets. Under incomplete markets, household portfolio choices are influenced by precautionary motives, and investment therefore responds less strongly because the portfolio composition matters for consumption smoothing (see also Chapter 2 for a similar argument in the case of monetary policy shocks).

The role of precautionary motives in shaping the investment response becomes more evident in the case of higher future taxes (see Figure 3.3). Under this scenario, the present-value wealth effect of the fiscal intervention is negative because of higher distortionary taxes. In the economy with limited heterogeneity, Ricardian households react to the negative wealth effect by lowering their consumption and savings. This leads to a crowding out of investment given the expansion of public debt. In the economy with endogenous heterogeneity, however, households still save in physical capital because of precautionary motives. Households would like to hang on to their improved consumption-smoothing capacity that the government brought about by increasing the aggregate supply of savings devices. As the government retires its debt, this crowds in private investment. The precautionary motives are also reflected in a lower liquidity premium.

This precautionary investment channel breaks the downward spiral of lower capital and lower labor supply, which occurs under complete markets. As a result, the output response is positive despite the negative wealth effect. Without precautionary savings, by contrast, increasing labor taxes to finance transfers leads to a long lasting recession by crowding out capital.

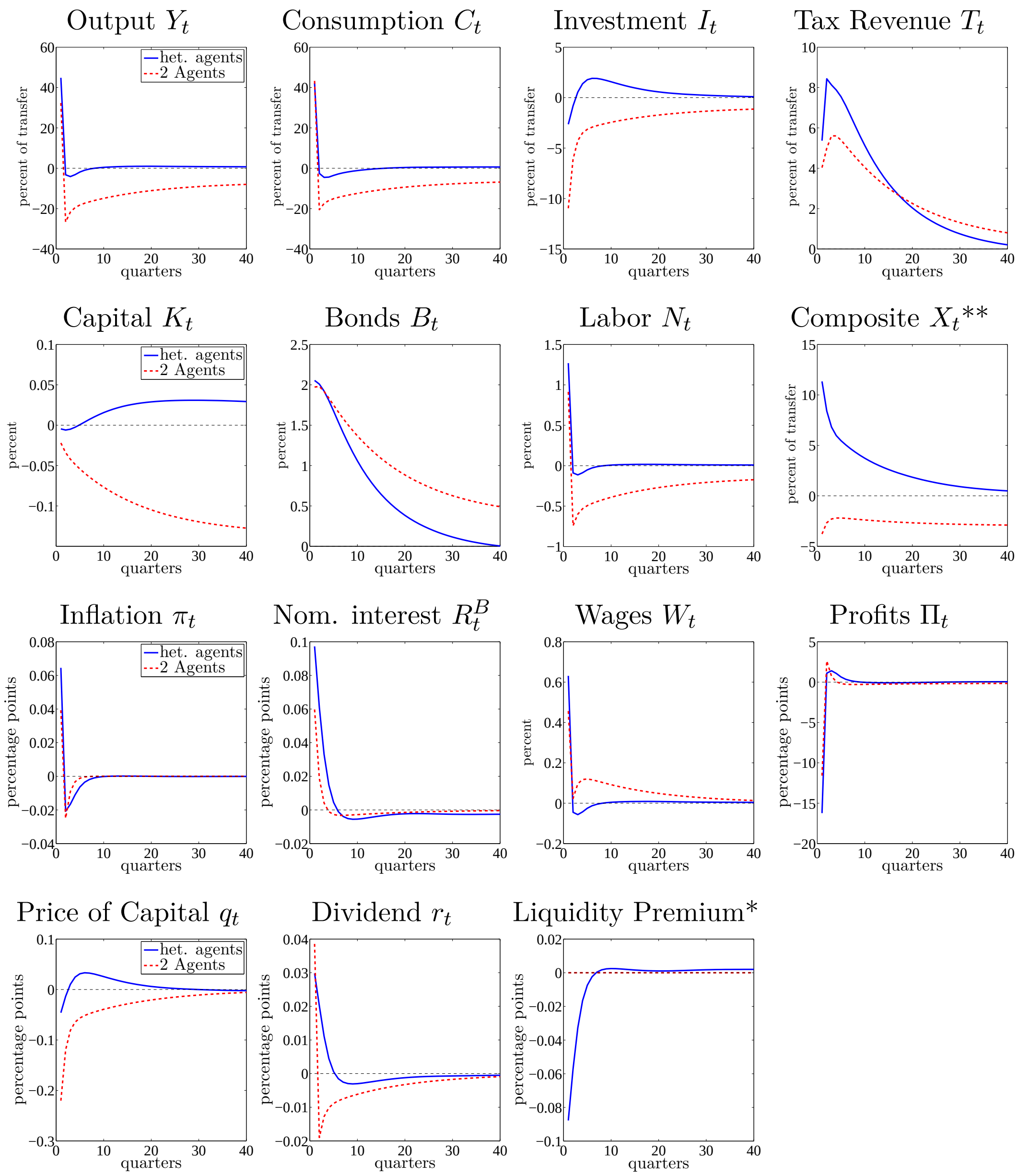
Key for the crowding in of capital is the transient nature of the increase in public debt. As in Aiyagari and McGrattan (1998), a permanent increase in public debt would displace capital while total savings increase. Figure 3.2 and 3.3 show that a similar logic applies to deficit-financed transfers. Investment initially falls but by substantially less than the increase in bond holdings so that total savings increase significantly. In contrast to a permanent increase in public debt, however, investment immediately recovers as households respond to the reversal in public debt by shifting savings from bonds to capital. After 40 quarters public debt is back at its steady state value and the boom in investment comes to an end.

Figure 3.2: Response to transfer shock (0.5% of annual output) under spending rule



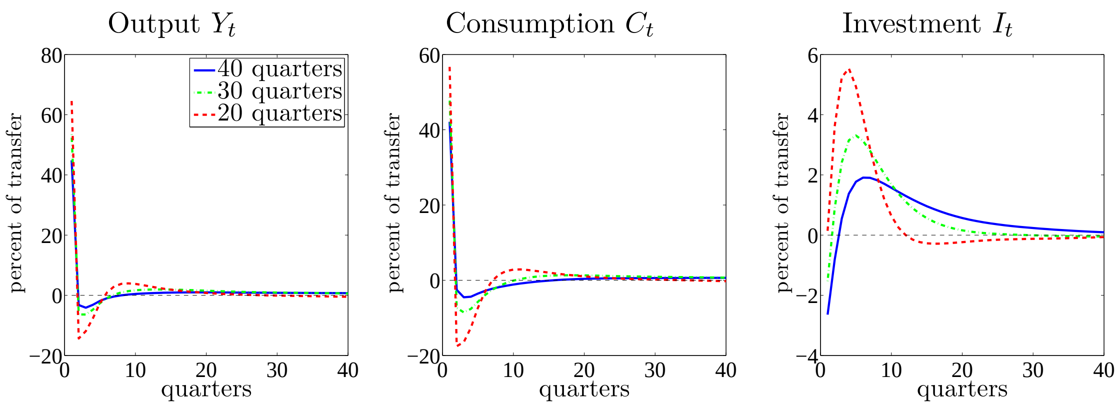
Notes: Impulse responses to a deficit-financed one-off transfer of 0.5% of annual output with tax rule stabilizing debt. Solid line: the model with heterogeneous households. Dashed line: same calibration with a representative household and rule-of-thumb households. $*LP = (q_{t+1} + r_{t+1})/q_t - R_t^B/\pi_{t+1}$ $**X_t = \int (c_{it} - h_{it} \frac{n_{it}^{1+\gamma}}{1+\gamma}) di$

Figure 3.3: Response to transfer shock (0.5% of annual output) under tax rule



Notes: Impulse responses to a deficit-financed one-off transfer of 0.5% of annual output with tax rule stabilizing debt. Solid line: the model with heterogeneous households. Dashed line: same calibration with a representative household and rule-of-thumb households. $*LP = (q_{t+1} + r_{t+1})/q_t - R_t^B/\pi_{t+1}$ $**X_t = \int (c_{it} - h_{it} \frac{n_{it}^{1+\gamma}}{1+\gamma}) di$

Figure 3.4: Response to transfer shock (0.5% of annual output) for different paths of public debt



Notes: Impulse responses to a deficit-financed one-off transfer of 0.5% of annual output for different parameterizations of the tax rule stabilizing debt. Solid line: Debt back at steady state in 40 quarters. Uneven-dashed line: Debt back at steady state in 30 quarters. Even-dashed line: Debt back at steady state in 20 quarters.

Figure 3.4 compares the output, consumption, and investment response for different paths of public debt. We find that the investment boom is more pronounced for a faster reversal in debt as implied by the precautionary investment motive. Households rely on their savings to smooth consumption in the presence of idiosyncratic income shocks. Hence, households react to a faster reversal in public debt by faster accumulating physical assets. In the case of a return to the steady state debt level in 20 quarters, investment does not fall in the first period and the output response is 40% larger than in the baseline.

6. Conclusion

Deficit-financed fiscal stimulus payments have become an important policy measure to counteract recessions. In this paper, we ask whether the empirical evidence on a sizable consumption response to such transfers at the household level implies that this type of fiscal intervention is indeed expansionary? We do so by building a New Keynesian business cycle model with heterogeneous households that takes into account the financing of transfers and matches the empirical evidence on the individual consumption response. Importantly, in this environment, transfers not only affect the aggregate economy through the disposable income channel but also by enhancing household and market liquidity because of debt finance.

To highlight the importance of this liquidity channel for the aggregate economy, we contrast our model results to a two-agent model with Ricardian and hand-to-mouth households, which replicates the consumption response to transfers but lacks the liquidity effect of public debt. In the two-agent model, Ricardian households would like to reduce their consumption and savings in response to the negative wealth effect of higher future distortionary taxes, inducing a persistent decline in investment and a prolonged recession. In contrast, in the presence of potentially binding borrowing constraints, a precautionary investment motive overturns this result as households would like to hang on to their improved consumption smoothing capacity and, thus, shift their savings into the physical asset when the government starts to retire its debt.

We find that this liquidity channel is stronger than wealth effects induced by government fincancing decisions and, thus, makes the aggregate effects of transfers expansionary independent of the mode of financing.

Appendices

A. Equilibrium Forecasting Rules

Tables 3.2 displays the equilibrium laws of motion for the Krusell-Smith equilibrium.

Table 3.2: Laws of motion

	β^1	β^2	β^3	β^4	R^2
Spending Rule					
β_π	0.50	-1.23	2.10	-23.69	99.73
β_q	0.01	5.05	-43.21	-38.59	99.95
Tax Rule					
β_π	0.50	-0.70	0.27	-24.09	99.76
β_q	0.00	3.77	-28.61	-41.56	99.95

All values are multiplied by 100 for readability.

The equilibrium forecasting rules are obtained by regressing them in each iteration of the algorithm on the response of the economy to the transfer shock. The R^2 is above 99%.

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