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Tobias Gamp
aus Lauterbach

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Dekan:	Prof. Dr. Rainer Hüttemann
Erstreferent:	Prof. Dr. Daniel Krähmer
Zweitreferent:	Prof. Tymon Tatur Ph.D.

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Chapter 1

Search, Differentiated Products and Obfuscation

Abstract. I study a market model, where consumers may either search sequentially for suitable products or may, in contrast to the previous literature, purchase products immediately and poorly informed. In comparison to the market outcome in the absence of the option to purchase products poorly informed, market prices increase. Product differentiation is not necessarily profitable for firms anymore. Resulting concerns that firms might fail to provide the welfare optimal, rich variety of products are gratuitous if product design is endogenous. I endogenize search costs so that firms may influence the consumers' acquisition of product information through obfuscation. Although, a firm's search costs signal consumers whether its offer is good or bad, firms obfuscate product information and equilibrium search costs maximize industry profits.

1.1 Introduction

Tastes are different and consumers search for products that satisfy theirs. Searching consumers have to trade-off the gains of finding suitable products and the savings on time. Intuitively, those consumers with very high opportunity cost resolve this trade-off completely in favor of saving time, forgo the acquisition of product information and purchase products poorly informed. In this study, I examine the effect on market outcomes such as prices, product design and the information provided by firms of this richer choice of consumer strategies than the search literature has typically considered before.

This study is motivated by the observation that in many markets the acquisition of product information is voluntary and a time-consuming search is not necessary for the purchase of a product. For example, in online markets, while many consumers devote considerable amounts of time on reading product descriptions and consumer reviews, others buy goods instantly and poorly informed, with one click. Poorly informed purchases also occur outside of online markets, when consumers sign contracts without reading the fine print and purchase goods without taking neither a careful look at the product itself nor at the consumer manual. And indeed, empirical evidence supports the idea that some consumers economize on fatiguing search efforts such that they are consequently poorly informed when making their purchases.[1]

In order to examine the effect of poorly informed purchases on market outcomes, I enrich the seminal market model with consumer search by Wolinsky [1986] and Anderson and Renault [1999] (henceforth, AR) with the consumers' option to purchase products poorly informed: As in the by model by AR, rational consumers may search among the differentiated products offered for suitable ones, but, in contrast, may also purchase products immediately. Then, consumers do not acquire information about a product's characteristics and price,[2] prior to its purchase, and take the risk of a 'bad buy', as they potentially suffer from pur-

[1] For instance, Wilson and Price [2010], page 658, estimate that in the UK energy market 32 % of switching consumers lose surplus due to their choice of supplier.

[2] Clearly, the consumers' lack of information could have other causes. Firms might not disclose all relevant information or consumers might be unable to process the information available. In this study, however, I assume that it is the consumers' rational decision to remain uninformed.

chasing a product that neither fits their taste nor matches their expected price.[3] This creates a trade-off between costly inspection and the risk of a bad buy, which is the central theme of the consumers' strategic considerations. Intuitively, in particular those consumers with high opportunity costs[4] are apt to take the risk of a bad buy and exert the new option to purchase immediately.

I determine the market equilibrium if search costs and product diversity are exogenous. As my first main result, I find that in comparison to the market outcome in the absence of the option to purchase immediately, market prices increase. Intuitively, the demand from consumers who purchase products immediately is less elastic than the demand from consumers who inspect products, since immediate-purchasers only find out after their purchase whether they bought a suitable product. Therefore, the option to purchase immediately softens competition.

I derive the comparative statics results. While I replicate the common finding that higher search costs lead to higher prices, the comparative statics with respect to product diversity challenge the prevailing view that product differentiation benefits firms.[5] Instead, as my second main result, I find that market prices are u-shaped in product diversity. In other words, firms may profit if products are better substitutes for each other. The decisive ingredient to obtain decreasing prices in product diversity is the consumer's option to purchase immediately. The intuition is that if some consumers purchase products immediately, any increase in product diversity encourages some of these consumers to inspect products. This enhances competition among firms so that as a result the market price is then decreasing in product diversity. If all consumers search for suitable products and the introduced option to purchase immediately is irrelevant, the induced greater search intensity of an increase in product diversity alone cannot compensate for the increased monopoly power of firms.

[3] Prices are in this model quality adjusted prices. The consumer's inability to observe a firm's price without inspection thus captures his inability to effortlessly ascertain quality.

[4] Consumers are heterogeneous with respect to their opportunity costs of time. In addition, this study takes into account participation constraints of consumers such that the distribution of consumer types ultimately entering the market is determined endogenously.

[5] See the studies by Chamberlin [1933] on monopolistic competition, by Salop and Perloff [1985] or references in the book by Anderson et al. [1992].

I study how the information provided by firms is affected by the consumers' option to purchase immediately. I endogenize search costs so that firms may impede or simplify the acquisition of product information. As already pointed out by Ellison and Wolitzky [2012], raising search costs, through the obfuscation of product information, is collectively rational in any search model if profits are increasing in search costs. However, this does not provide a satisfying explanation for the observed obfuscation. If firms cooperate on obfuscation, it remains unclear why firms do not cooperate on prices in the first place, which is even more profitable. Hence, one is interested in whether obfuscation is individually rational, and thus, can arise in a non-cooperative market model.

In my model, firms can discourage consumers through obfuscation from inspecting their products, inducing them to purchase products immediately. This may be profitable, as the demand from immediate-purchasers is less elastic. This study however shows that there are limits to the obfuscation of product information. As firms have no commitment power to guarantee competitive prices, immediate-purchasers rely on the inspections of searching consumers to reassure that the firm offers the expected price. If a firm obfuscates too much, immediate-purchasers correctly anticipate that the firm's incentive to set low prices relaxes, and avoid purchasing the firm's product. Thus, by simplifying the consumers' information acquisition a firm encourages consumers to inspect its products and signals a low price to all other consumers.

However, despite this signaling function of search costs, I show, as my third main result, that in equilibrium firms find it individually rational to set search costs such that industry profits are maximized. Thus, although firms compete fiercely in prices, equilibrium search costs leave the impression as if firms would cooperate on impeding the information acquisition of consumers. The intuition for this result resembles the one for the well-known Diamond Paradox despite the active search of consumers. As firms have some market power in this model of monopolistic competition, they find profitable to marginally increase search costs as long as no participation constraints of consumers are binding. As a consequence, equilibrium search costs are the highest search costs such that yet all consumers participate and therefore maximize industry profits.

My last result concerns the effect of immediate purchases on product design. My previous results have shown that product differentiation does not necessarily benefit firms. This raises concerns that firms might fail to provide the rich variety of product, which is desirable from a social planner's point of view. I address this issue and endogenize the firms' choice of product design following an approach by Bar-Isaac et al. [2012] in order to examine an individual firm's incentive to offer a niche product. I find that the consumer's option to purchase immediately enhances product diversity, independent of whether search costs are exogenous or endogenous. In fact, all firms target niches such that the social planner's concerns are gratuitous.

1.1.1 Related Literature

The issue of competition with search and differentiated products has been addressed before, starting with the seminal contributions by Wolinsky [1986], Bakos [1997] and Anderson and Renault [1999].[6] I extend their model in two ways: First, consumers are heterogeneous with respect to their opportunity costs of time, which, due to participation constraints, results in the distribution of consumer types entering the market to be endogenous.[7] Second, more importantly, consumers may purchase products immediately. My study shows how this alters market outcomes. Furthermore, my richer model allows me to study the obfuscation of product information, which would otherwise not be individually rational, and provides a tractable framework to examine the firms' choice of product design.

The only study of which I am aware that also considers immediate purchases in a search

[6] Recent contributions include the ones Zhou [2011, 2014] on directed and multi-product search, Armstrong et al. [2009] on prominence and Bar-Isaac et al. [2012] on product design.

[7] Already Diamond [1971] points out that participation constraints lead to the failure of market existence if search costs are bounded away from zero. The studies by Janssen and Moraga-González [2004] and Janssen et al. [2005] are antecedents to mine in the realm of consumer search for homogeneous product who examine the effect of participation constraints on the comparative statics of market outcomes if consumers are heterogeneous with respect to their opportunity costs of time. Market existence is guaranteed in their model due the presence of shoppers. Moraga-González et al. [2014] are, along with me, the first to examine the effect of heterogeneous consumers if products are differentiated, but focus, in contrast, on the competitive effects of higher search costs.

model with differentiated products is the one by Kuksov and Villas-Boas [2010].[8] However, they abstract from prices and competition. Instead, they focus on the consumer-optimal number of products if consumers can make inferences about the locations of offered products on a Hotelling line and purchase products immediately. The theme of immediate purchases also appeared recently in the study by Wang [2014] who however studies the optimal advertising strategy of a monopolist to overcome a hold-up problem.

This study contributes to the literature on product design.[9] Most closely related is the study by Bar-Isaac et al. [2012] who examine the firm's choice of product design in a market model with consumer search. They also build on the notion of demand rotations as introduced by Johnson and Myatt [2006]. The key difference is that consumers have to inspect products prior to purchase in their model. The tenet of their analysis is that firms which have a competitive advantage try to avoid competition by producing plain vanilla products,[10] while less competitive firms target niches. This study shows that the consumer's option to purchase immediately gives firms stronger incentives to target niches, so that, in contrast to their findings, in the absence of competitive advantages all firms target niches.[11]

This paper is related to a slim literature that interprets obfuscation as raising search costs. While most studies on consumer search share the view that firms profit from collectively raising search costs, this branch of the literature puts forward arguments why it is individually rational for firms to do so. Wilson [2010] shows that in a directed search model firms split the market of heterogeneous consumers by differentiating in search costs. Ellison and Wolitzky [2012] point out that in a sequential search model firms have incentives to raise search costs in order to fatigue consumers, when search costs are convex, instead of linear. Gamp [2015b] shows how a multi-product monopolist influences the consumers' order of search by raising

[8]Bar-Isaac et al. [2010] and Armstrong and Chen [2009] study immediate purchases in markets where products are vertically differentiated.

[9]See the studies by Lewis and Sappington [1994] and Johnson and Myatt [2006] for an analysis of the choice of product design by a monopolist.

[10]Anderson and Renault [2009] find similar results with respect to the disclosure of horizontal match information.

[11]More precisely, Bar-Isaac et al. [2012] find that, even if no firm has a competitive advantage (a special case covered in their analysis), firms differentiate in product design, so that some firms target niches while others offer plain vanilla products.

search costs for less polarizing products.

More broadly, this study is therefore also related to a literature in behavioral industrial organization which explains the obfuscation of product information with the bounded rationality of consumers.[12]

This paper is organized as follows: In section 1.2, the model is introduced. In section 1.3, I examine the effect of immediate purchases on market outcomes if search costs and product diversity are exogenous. In section 1.4, I endogenize search costs in order to study the obfuscation of product information. In section 1.5, I endogenize product diversity in order to study the firm's choice of product design. Section 1.6 concludes.

1.2 A market model with consumer search

The market consists of a continuum of consumers with unit demand, and a continuum of single-product, profit-maximizing firms. Firms offer horizontally differentiated products of identical quality. Marginal costs of production are normalized to zero. If consumer i buys the product of firm k at price p_k, his quasi-linear utility absent any search costs is

$$u(\varepsilon_{ik}, p_k) = v + \mu\varepsilon_{ik} - p_k, \tag{1.1}$$

where $v > 0$ is the average valuation for the good. The parameter $\mu > 0$ is a measure for product diversity, which is yet exogenous as in AR, but will, in section 1.5, be endogenized in order to study product design following the approach by Bar-Isaac et al. [2012]. The idiosyncratic consumer-firm match-value ε_{ik} is the realization of the random variable $\hat{\varepsilon}_{ik}$ which is described by the continuously differentiable, symmetric probability density function f. The consumer-firm match-value indicates whether consumer i likes the particular variant that firm j offers. Let $\hat{\varepsilon}_{ik}$ be independent among consumers and firms. Its expectation is zero. The support of f is the interval $[\underline{\varepsilon}, \overline{\varepsilon}]$ on the real line. The cumulative density function F satisfies

[12] For example, see the studies by Gabaix and Laibson [2006], Carlin [2009], Piccione and Spiegler [2012] and Chioveanu and Zhou [2013].

the usual increasing hazard rate condition.[13]

Search protocol. While the model of utility and profits follows along the lines of the one by Wolinsky [1986] and AR, the search protocol is distinct. Most importantly, consumers may take the risk of a bad buy and purchase products immediately. Figure 1.1 illustrates the search protocol.

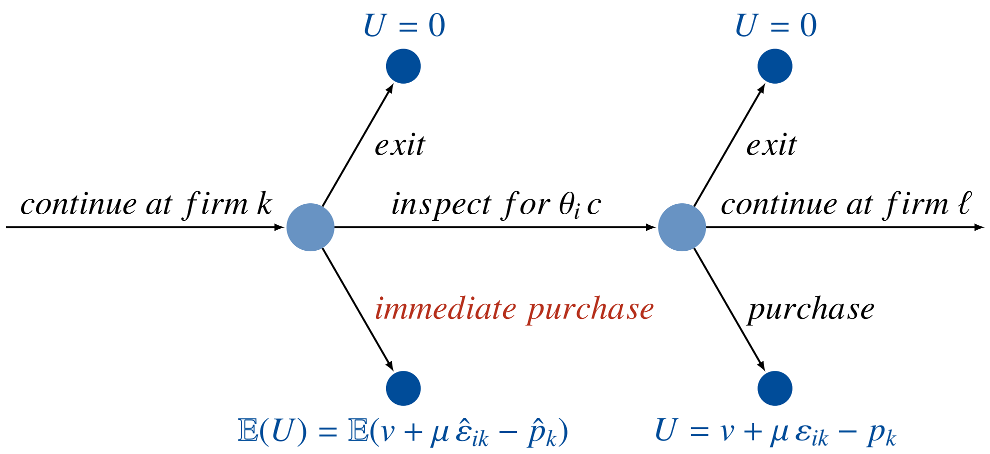

Figure 1.1: The consumer's search protocol

As in the model by AR, consumers are ex ante uninformed about the products' prices and characteristics. However, they may either inspect products privately and sequentially in order to acquire this missing information prior to purchase, or forgo the acquisition of product information and purchase products immediately. Each inspection is costly and $\theta_i c$ denote the type-dependent search costs. I interpret c as an exogenous measure for the complexity of information acquisition which affects the time necessary to inspect a product. In section 1.4, I endogenize c so that firms may create additional complexity in order to influence the time necessary to inspect their products.

The consumer's type θ_i captures the heterogeneity of consumers with respect to their opportunity costs of time. Interpreted differently, consumers with low opportunity costs are sophisticated consumers who have a greater ability to evaluate complex information. The consumer's type is an independent draw from the uniform distribution H with an interval

[13]Equivalently, the corresponding reliability function $\tilde{F} := 1 - F$ is log-concave. A sufficient condition is log-concavity of f which indeed most distributions with increasing hazard rates satisfy. For a list of log-concave p.d.f.s see the study by Bagnoli and Bergstrom [2005], p.12.

support $\Theta = [\underline{\theta}, 1]$, where $\underline{\theta} > 0$. A consumer knows his type, while firms only know the cumulative distribution of the consumers' types. Firms thus cannot target individual consumers and discriminate prices among them.

Formally, search begins at a random firm. Whenever the consumer is at some firm k and uninformed about the firm's product, he may:

i) exit: end search and obtain a utility of zero,

ii) inspect the firm's product: learn ε_k and p_k at costs $\theta_i c$,

iii) purchase immediately the firm's product: end search and obtain in expectation $\mathbb{E}(u(\hat{\varepsilon}_{ik}, \hat{p}_k))$,

where $\hat{p}_k$ denotes the consumer's belief about the firm's price. If the consumer is at firm k and informed about the firm's product, he may:

i) exit: end search and obtain a utility of zero,

ii) purchase firm k's known product: end search and obtain $u(\varepsilon_{ik}, p_k)$,

iii) continue: continue search at a random next firm l.

Search is thus without recall[14] and uniformly random so that each firm is visited next with equal probability.[15]

Detected Deviations. If an uninformed consumer forgoes a costly inspection and purchases a product immediately, firms could exploit his ignorance of prices and sell arbitrarily expensive products. Therefore, an uninformed consumer only purchases immediately if there exists an upper bound on his potential losses. To resolve this matter, I assume that an uninformed consumer realizes when an unknown product's price exceeds his expectation by more than δ. This ensures that equilibria in which consumers purchase immediately exist. Beyond that, this assumption plays no crucial role in the analysis and the obtained equilibria are independent of δ.[16]

[14]All results for exogenous search costs carry over one-to-one to a model with perfect recall; for endogenous search costs this is the case if recall is costly.

[15]The last assumption is one aspect that distinguishes this study from the ones by Armstrong et al. [2009] and Zhou [2011] who consider the other extreme case when the consumer's search is directed and the order of search depends on the consumer's rational expectations or the prominence of firms. Search models with random consumer search capture markets where consumers are ex ante uninformed firms and their offered products such that firms are ex ante identical. Then, search is per se non-directed.

[16]The analysis in the main body of this paper relies on first order conditions for undetected deviations. The interested reader is referred to appendix A4, where I examine detected and undetected deviations and rigorously prove sufficiency of first order conditions.

Timing and Equilibrium Concept. First, firms set prices simultaneously and privately. Then, each consumer searches until he exits or purchases a product. The equilibrium concept that I apply is perfect Bayesian equilibrium[17] with passive consumer beliefs: If a consumer observes a deviation by one firm, he does not change his beliefs about the behavior of others. I focus on symmetric equilibria in pure, stationary strategies.

Comment on unobserved prices. In this model, prices are ultimately markups or quality adjusted prices. The consumer's inability to observe a firm's price without inspection captures hence the consumer's inability to effortlessly ascertain quality.[18] I dispense with a model of quality choice of firms, as such a stage does not alter the analysis if firms have access to identical production technologies.

1.3 The effect of immediate purchases on market outcomes if search costs and product diversity are exogenous

1.3.1 Consumer behavior

In this subsection, I show that if consumers expect a symmetric market equilibrium their best response is monotonic in their type: Consumers with low opportunity costs invest time in costly inspections of products and search until they find satisficing products; all other consumers either take the risk of a bad buy or exit the market. If these consumers are indifferent between immediate purchase and exit, their best response is not unique and a positive measure of consumers may mix between immediate purchase and exit. Although this only occurs when the market price is equal to the average valuation for products, it will be an important case. In that case, I purify the consumers' use of mixed strategies and assume that consumers with lower opportunity costs purchase immediately, while consumers with higher opportunity costs exit.

[17] All equilibra in this paper are also sequential equilibria.

[18] This is the preferred interpretation of the author. However, unobserved prices can also be justified by the empirical findings that consumers fail to properly account for shipping costs (Della Vigna [2009] and Brown et al. [2010]) or for sales tax (Chetty et al. [2009]).

Suppose a consumer expects a symmetric market equilibrium, in which each firm's price is p^*. As the consumer's search is without recall, there are two types of decision nodes to consider: when the consumer is uninformed about the product's price and match-value, and when he is informed.

Informed consumers. An informed consumer purchases the firm's product if it supplies him with a utility which exceeds his expected utility of continued search; otherwise, w.l.o.g. he continues his search.[19] His best response is hence characterized by a reservation utility $U_{\text{res}}(\theta, p^*)$ which determines whether he purchases the product or continues search; $U_{\text{res}}(\theta, p^*)$ equals the consumer's expected utility of continued search.

Uninformed consumers. The consumer's expected utility of continued search, depends on whether he inspects a product, purchases it immediately or exits, whenever he is uninformed. First, if he purchases immediately, his expected utility is $U_{\text{I}}(p^*) = v - p^*$, as he expects each firm to charge p^* and the expectation of the match-value is zero. Second, if the consumer exits, his expected utility is obviously $U_{\text{E}} = 0$. Third, if the consumer inspects the product, his expected utility, following McCall [1970][20], is $U_{\text{S}}(\theta, p^*) = v - p^* + \mu\tilde{x}(\theta)$, where the function $\tilde{x} : \Theta \to (-\infty, \overline{\varepsilon})$ is implicitly defined by the equation

$$\mu \int_{\tilde{x}(\theta)}^{\overline{\varepsilon}} \Big(\varepsilon - \tilde{x}(\theta)\Big) f(\varepsilon) d\epsilon \overset{!}{=} c\,\theta. \tag{1.2}$$

The function $\tilde{x}$ is well-defined and decreasing in θ.[21] The interpretation for $\tilde{x}(\theta)$ is that the consumer purchases a product at p^* if the match-value exceeds the type-dependent reservation match-value $\tilde{x}(\theta)$. Therefore, equation (1.2) states the familiar result that the reservation utility equates the expected benefits of a single additional inspection (LHS) with its expected costs (RHS). As $\tilde{x}$ is decreasing in θ, consumers with lower opportunity costs have greater

[19]If the consumer prefers to exit, he can obtain the same utility if he continues and exits at the next firm. Hence, without loss of generality, I assume that the consumer does not exit if he is informed. This assumption is without loss of generality, since each firm's demand remains unaffected by how this indifference is resolved.

[20]The result is a standard result in search theory and the proof is hence omitted – i.e. see McCall [1970] on the optimality of stopping rules if the consumer expects i.i.d. product valuations.

[21]Let $g(x) := \int_x^{\overline{\varepsilon}} (\varepsilon - x) f(\varepsilon) d\epsilon$, $g : (-\infty, \overline{\varepsilon}) \to (0; \infty)$, such that $\mu g(\tilde{x}(\theta)) = c\,\theta$. The function g is strictly decreasing, differentiable, and an inverse function g^{-1} is well-defined such that $\tilde{x}$ is well-defined. Furthermore, g decreasing implies $\tilde{x}$ decreasing.

reservation match-values. A searching consumer with low opportunity costs is thus choosier and searches longer in expectation.

Among these three actions, an uninformed consumers chooses the one which maximizes his expected utility $U_{\text{res}}(\theta, p^*) = \max\{U_S(\theta, p^*), U_I(p^*), U_E\}$. Since only U_S is type-dependent and decreasing in θ, the consumer's best response is characterized by a unique cut-off type θ_S such that all consumers with lower opportunity costs inspect products. All other consumers exit if $p^* > v$, and purchase immediately if $p^* < v$. If $p^* = v$, these consumers are indifferent. Then, by assumption, those consumers with medium opportunity costs $\theta \in (\theta_S, \theta_I]$ purchase immediately and consumers with high opportunity costs $\theta \in (\theta_I, 1]$ exit. Interpreted differently, $\frac{\theta_I - \theta_S}{1-\theta_S}$ denotes the probability that an indifferent consumer takes the risk of bad buy.

Lemma 1.1. *The consumer's best response is characterized by two cut-off types θ_S and θ_I:*

i) ***Shoppers:*** *If the consumer has low opportunity costs: $\theta \leq \theta_S$, he inspects products and purchases a product if it supplies him with a utility that exceeds $U_{\text{res}}(\theta, p^*) = v - p^* + \mu\tilde{x}(\theta)$.*

ii) ***Immediate-purchasers:*** *If the consumer has medium opportunity costs: $\theta_S < \theta \leq \theta_I$, he purchases immediately and his expected utility is $U_{\text{res}}(\theta) = v - p^*$.*

iii) ***Non-participants:*** *If the consumer has high opportunity costs: $\theta_I < \theta$, he exits the market and his expected utility is zero.*

The consumers' best response is thus in line with intuition.

1.3.2 Firm behavior

Each firm faces a trade-off between the profits generated by exploiting uninformed immediate-purchasers and the gains in market share from price sensitive shoppers by offering low prices. In this subsection, I derive a candidate market price that resolves this trade-off so that no firm has an incentives to deviate given its expectations.

Suppose a firm expects a symmetric market equilibrium in which each firm's price is p^* and the consumers' behavior is characterized by $(\theta_I^*, \theta_S^*) > (\underline{\theta}, \underline{\theta})$. Assume $\theta_S^* = \min\left\{\theta_I^*, \theta_{\text{ind}}\right\}$, where $\theta_{\text{ind}} := \mu \int_0^{\bar{\varepsilon}} \varepsilon f(\varepsilon) d\epsilon / c$ denotes the consumer type that is indifferent between immediate purchase and inspect.[22] Suppose the firm changes its price marginally to p. Neither non-participants nor immediate-purchasers observe this deviation such that their demand is unaffected. In contrast, a shopper learns the firm's price and buys its product after inspection only if it supplies him with a utility that exceeds his reservation utility $v + \mu\tilde{x}(\theta) - p^*$; the probability that a shopper purchases the product after inspection is thus $\tilde{F}(\tilde{x}(\theta) + \frac{p-p^*}{\mu})$, where $\tilde{F} := 1 - F$.

In order to determine the total demand for its product, the firm has to take into account that shoppers visit more than one firm in expectation. Let $\xi(\theta)$ denote the density of consumer who visit the firm. I find:

$$\xi(\theta) = \begin{cases} \frac{h(\theta)}{\tilde{F}(\tilde{x}(\theta))}, & \text{if } \theta \leq \theta_{\text{S}}^*, \\ h(\theta), & \text{otherwise.} \end{cases} \tag{1.3}$$

The expected density of shoppers who arrive after n inspections is $F(\tilde{x}(\theta))^n h(\theta)$, where $F(\tilde{x}(\theta))^n$ is the probability that a shopper rejects n products. Thus, $\xi(\theta) = \sum_{n=0}^{\infty} F(\tilde{x}(\theta))^n h(\theta) = \frac{h(\theta)}{\tilde{F}(\tilde{x}(\theta))}$ for shoppers. All other consumers do not visit more than one firm such that $\xi(\theta) = h(\theta)$.

The firm's expected profits are therefore

$$\pi(p, p^*) = \int_{\underline{\theta}}^{\theta_{\text{I}}^*} p \left\{ 1 - \mathbb{1}_{\theta \leq \theta_{\text{S}}^*} F\left(\tilde{x}(\theta) + \frac{p - p^*}{\mu}\right) \right\} \xi(\theta) d\theta. \tag{1.4}$$

By differentiation of equation (1.4) and imposing symmetry, one obtains the unique candidate equilibrium price which satisfies the first order condition. Let $\varphi := \frac{f}{1-F}$ denote the hazard rate of F.

Lemma 1.2. *If some consumers are shoppers, the unique candidate equilibrium price is*

$$\tilde{p}(\mu, c, \theta_{\text{I}}^*) = \mu \frac{H(\theta_{\text{I}}^*)}{\int_{\underline{\theta}}^{\min\{\theta_{\text{I}}^*, \theta_{\text{ind}}\}} \varphi(\tilde{x}(\theta)) h(\theta) d\theta}. \tag{1.5}$$

[22] A derivation of θ_{ind} and $\theta_{\text{S}}^* = \min\left\{\theta_{\text{I}}^*, \theta_{\text{ind}}\right\}$ is relegated to the appendix (lemma A1.1). Note that θ_{ind} is independent of the market price, which will allow me to characterize the firm's candidate equilibrium price as a function of θ_{I}^* only.

The comparative statics of $\tilde{p}$ are as follows:

i) An increase in product diversity μ leads to

(a) a drop in $\tilde{p}$ if some consumer purchase immediately.

(b) an increase in $\tilde{p}$ if no consumer purchases immediately.

ii) An increase in search costs c leads to an increase in $\tilde{p}$.

iii) An increase in consumer participation θ_{I}^* leads to an increase in $\tilde{p}$.

The candidate equilibrium price is equal to the inverse of the elasticity of demand. Equation (1.5) captures that only shoppers create competition among firms. Among shoppers, those with low opportunity costs create more competition, since they have greater reservation match-values and since hazard rates are increasing by assumption. Intuitively, these consumers are choosier and search longer in expectation. The comparative statics of $\tilde{p}$ are repeatedly used in the following. A discussion is postponed to the discussion of the comparative statics results of equilibrium market outcomes.

1.3.3 Market equilibrium

A market equilibrium is fully characterized by a triple $(\theta_{\mathrm{I}}^*, \theta_{\mathrm{S}}^*, p^*)$. I focus on market equilibria in which trade occurs. Other equilibria, trivial equilibria, exist for all parameter values.[23] Assume $v > \frac{\mu}{\varphi(0)}$, which guarantees that the market price drops below v if only shoppers enter the market. Otherwise, the introduced option to purchase immediately is irrelevant. I identify four equilibrium regimes, which are illustrated in the left panel of figure 1.2. In order to simplify the notation fix c and v.[24]

Proposition 1.1. *There exists a unique market equilibrium in which trade occurs. There are four regimes which are separated by three cut-off values in product diversity $\mu_{\mathrm{A}}, \mu_{\mathrm{B}}, \mu_{\mathrm{C}}$:*

i) ***Market Failure:*** *For very low product diversity: $\mu \leq \mu_{\mathrm{A}}$, all consumers exit the market and no trade occurs. The market price exceeds the average valuation v.*

[23] In trivial equilibria, all consumers exit the market in expectation of excessively high prices such that firms are indifferent between all pricing strategies such that excessively high prices are also optimal.

[24] For each c and v four equilibrium regimes with corresponding cut-off values exist.

ii) ***Partial Participation Regime:*** *For low product diversity: $\mu_A < \mu \leq \mu_B$, consumers with low opportunity costs inspect products; the participation of all other consumers adjusts such that the market price is equal to v.*

iii) ***Full Participation Regime:*** *For medium product diversity: $\mu_B < \mu < \mu_C$, all consumers purchase products, but only consumers with low opportunity costs inspect products. The market price is $p^* = \tilde{p}(\mu, c, 1)$.*

iv) ***Search Regime:*** *For high product diversity: $\mu_C \leq \mu$, all consumers inspect products, and the market price is $p^* = \tilde{p}(\mu, c, 1)$.*

In comparison to the market outcome in the absence of the option to purchase immediately, market prices increase.

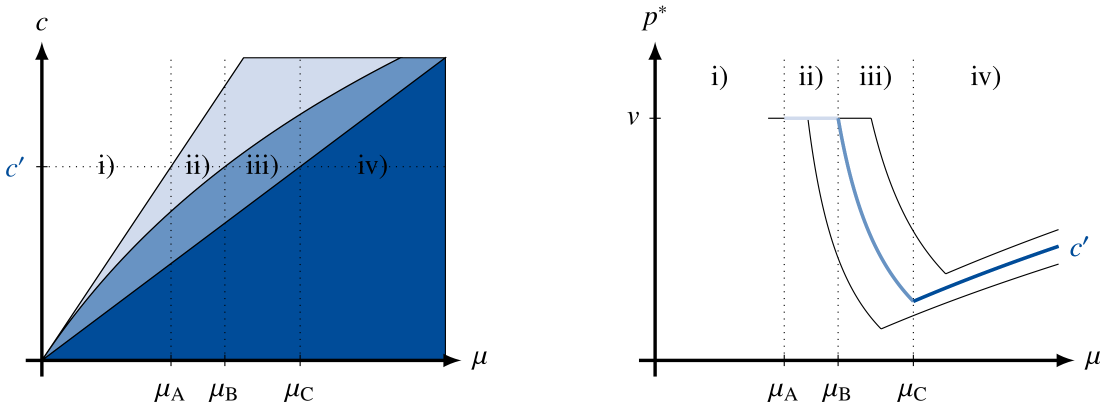

Figure 1.2: A market with uniformly distributed match-values.
(left): The four equilibrium regimes: *i)* market failure, *ii)* partial participation, *iii)* full participation, *iv)* search. **(right)**: The market price as a u-shaped function of product diversity for distinct search costs. Curves to the upper right correspond to greater search costs.

As in the model by AR, for very low product diversity a market fails to exist. The reason for this no trade result is that otherwise, if trade occurred, a firm's demand would be perfectly inelastic, inducing it to raise its price without bound. The demand would perfectly inelastic, since, for very low product diversity, all consumers who participated in trade would prefer immediate purchase to inspect. It is noteworthy that for some values of product diversity below μ_A, trade would occur if consumers could not purchase products immediately.[25] This

[25]More precisely, the resulting lower bound in product diversity for market existence is greater than the one

means that the consumers' option to purchase products immediately makes market existence less likely.

For low product diversity trade occurs, but not the whole market is covered. The reason for the consumers' partial participation is a commitment problem of firms that is due to the consumer's inability to observe prices without inspection. As firms cannot commit to low prices, immediate-purchasers correctly anticipate that firms would raise prices in the absence of competition. Therefore, immediate-purchasers rely on the competitive pressure that is generated by shoppers to reassure that firms offer low prices. In other words, the active search of shoppers exerts a positive externality on all other consumers, on which immediate-purchasers free-ride. In the partial participation regime there are not enough shoppers present to generate sufficient competition for the whole market to be covered. Consequently, consumer participation adjusts such that firms still can credibly commit to prices weakly below the average valuation. Only in the full participation regime and the search regime, the market is sufficiently competitive to ensure prices below v even if all consumers participate.

1.3.4 Comparative statics results

Proposition 1.2. *The comparative statics results of market outcomes are:*

Regime:	Partial Participation			Full Participation		Search	
	p^*	θ_{I}^*	π^*	p^*	π^*	p^*	π^*
Product diversity μ	-	↑	↑	↓	↓	↑	↑
Search costs c	-	↓	↓	↑	↑	↑	↑
Average valuation v	↑	↑	↑	-	-	-	-

Table 1.1: Comparative statics results

An upward (downward) arrow tells whether the up-sided variable is strictly increasing (decreasing) in the left-sided variable. '-' means that there is no effect.

Product diversity. The main result in this subsection is that the market price is a u-shaped

that is identified by AR, although in both models market existence depends on whether demand is elastic or inelastic. However, in the absence of the option to purchase immediately, demand is elastic only if shoppers refrain to buy misfits and continue search. This occurs if product diversity is sufficiently high so that those shoppers with the lowest opportunity costs continue search if they find their most disliked variants. That is if $\tilde{x}(\underline{\theta}) > \underline{\varepsilon}$. In contrast, in my model, elasticity is determined by whether these consumers prefer inspect to immediate purchase, and hence, by $\tilde{x}(\underline{\theta}) = 0$. Consequently, I find that the lower bound in product diversity for market existence is greater.

function of product diversity, as is illustrated in the right panel of figure 1.2. This result challenges the common wisdom that firms benefit from greater product differentiation. The comparative statics results are the outcome of the interplay of four distinct effects of which the first two are known from AR: a *niche-product-effect*, a *search-intensity-effect*, an *information-acquisition-effect*, and a *participation-effect*.

Let us briefly revisit the first two effects which pin down the effect of an increase in product diversity in the search regime. In the search regime consumers neither exert the option to purchase immediately nor are participation constraints binding so that the analysis coincides with AR, leaving aside the consumer's heterogeneity. If products are strongly differentiated, buying a suitable product is more valuable to consumers such that products are worse substitutes for each other. This softens competition among firms, and allows firms to extract some of the additional surplus that is due to the provision of niche products. Therefore, the *niche-product-effect* raises prices and captures the common argument why firms profit from product differentiation. The *search-intensity-effect* is negative and is generated by greater search intensity of shoppers. An increase in product diversity increases reservation match-values. Then, consumers are choosier and compare more products, which enhances competition among firms due to the assumption of increasing hazard rates.

The novel effects are the *information-acquisition-effect* and the *participation-effect*. The *information-acquisition-effect* is negative and captures the effect of an increase in the number of shoppers – thus, an increase of θ_S^*. If product diversity increases, more consumers find it worthwhile to search for suitable products. Consequently, the demand becomes more elastic and enhances competition among firms. These first three effects determine the comparative statics in the full participation regime. In this regime, prices are unambiguously decreasing in product diversity. As a consequence, firms do not necessarily from product differentiation in this regime.

The *participation-effect* is positive and is due to an increase in consumer participation – an increase of θ_I^* – that results in an increase of inelastic demand from immediate-purchasers. In the partial participation regime, the *participation-effect* prohibits that the market price is decreasing in product diversity. Not before all consumers participate, market prices begin to fall after an increase in product diversity.

Search costs. I replicate the standard result within the literature on consumer search that higher search costs lead to higher prices, since a corresponding *search-intensity-effect* and a *information-acquisition-effect* after an increase in search costs both point to greater prices. Only if consumers participate partially, these two effects are compensated by a *participation-effect*.

Welfare. Total welfare equals the participating consumers' expected utility net of prices: $W = \int_{\underline{\theta}}^{\theta_{\mathrm{I}}^*} [v + \mathbb{1}_{\theta \leq \theta_{\mathrm{S}}^*} \mu \tilde{x}(\theta)] d\theta$. Welfare is increasing in product diversity, since an increase in product diversity has a positive effect on consumer participation, θ_{I}^*, on the number of shoppers, θ_{S}^*, and on each shopper's utility net of prices: $\mu\tilde{x}(\theta)$ is increasing in μ, since $\tilde{x}(\theta)$ is increasing in μ.[26] From a social planner's point of view product differentiation is therefore desirable. This raises concerns that the market might fail to provide the desired, rich variety of products, as my previous result have shown that firms do not necessarily benefit from product differentiation. In section 1.5, I attend this issue and discuss a single firm's incentive to offer a niche product.

1.4 Do firms obfuscate product information?

1.4.1 A search cost model of obfuscation

In this section, I endogenize search costs with the purpose to study the firms' incentives to obfuscate product information. Each firm chooses an information strategy $s_j \in [\underline{s}, \overline{s}]$ which affects the consumer's search costs $\theta(c + s_j)$ that he incurs if he inspects its product. Each firm may thus either simplify the acquisition of product information, a transparency policy, or impede the inspection of its product through obfuscation. Let $c + \underline{s} > 0$ such that $\theta(c + s_j) > 0$ for any $s_j \in [\underline{s}, \overline{s}]$.

[26] With respect to an increase in search costs, exactly the opposite is the case so that welfare is decreasing in search costs as usually.

Timing. Each firm chooses its information strategy privately in a first stage. Then, each firm chooses its price. A firm's pricing strategy thus specifies a price for each possible information strategy.[27] Hence, let $p, p^* : [\underline{s}, \overline{s}] \to \mathbb{R}$ throughout this section. Afterwards, time elapses as before, with one slight difference – namely, a consumer observes s_j upon arrival at firm j. This assumption is crucial, since otherwise, a firm cannot affect the consumer's decision whether or not to inspect its product.[28] It captures the idea that, for example, a consumer observes the length or complexity of the contract terms and hence observes whether the contract offered is more or less difficult to evaluate, without however observing what is written in the contract.

Solution approach. In order to derive equilibrium market outcomes, it suffices to determine the equilibrium information strategy s^*. Then, market outcomes follow directly from my previous analysis, as the continuation game coincides with the one analyzed in section 1.3. However, in order to determine s^* the behavior of firms and consumers off the equilibrium path, after a deviation $s \neq s^*$, has to be determined.

1.4.2 Consumer behavior

I find, analogously, that the consumer's best response is monotonic in his type. While the intuition remains the same, the analysis is slightly more evolved. Sill, a consumer's strategy consists of two parts: First, a reservation utility that specifies for an informed consumer whether to purchases the firm's product or to continue search. Second, a plan that specifies for an uninformed consumer whether, upon observing some s, to exit, to inspect or to purchase the firm's product. In the following, I determine the consumer's best response if he expects a symmetric market equilibrium (s^*, p^*).

[27]The timing is key, as it pins down the consumer's belief about the firm's price after observing s_j. The idea beyond this choice is that the consumer should not belief that the firm unintentionally chose to make it difficult to inspect its product. To the contrary, I want the consumer to infer that the firm possibly has something to hide. Note that otherwise it would be clearly profitable for firms to prevent consumers from inspecting their products.

[28]If s_j was unobservable to consumers, then consumer behavior would remain unchanged, and only sunk search costs would increase or decrease. This could affect the consumer's future behavior if search costs are non-linear. Indeed, this is the main tenet of Ellison and Wolitzky [2012] who show that if search costs are convex, firms have an incentive to obfuscate information in order to fatigue consumers.

Informed consumers. Analogously, the consumer's reservation utility is his expected utility of continued search. As the consumer's beliefs are passive, he expects, also after observing a deviation, all following firms to choose (s^*, p^*). Thus, the consumer's expected utility of continued search coincides with his expected utility of continued search in the game with exogenous search costs $c + s^*$ and expected market price $p^*(s^*)$, which was examined in section 1.3. Therefore, his reservation utility follows directly from my previous analysis: $U_{\text{res}}(\theta, s^*, p^*) = \max\left\{0, v - p^*(s^*), v - p^*(s^*) + \mu\tilde{x}(\theta, s^*)\right\}$, where $\tilde{x}(\theta, s^*)$ denotes the consumer's reservation match-value if search costs are exogenous and equal to $c + s^*$.[29]

Uninformed consumers. Recall that upon arrival an uninformed consumer observes a firm's information strategy. If he then exits, his utility is $U_{\text{E}} = 0$. If he purchases the firm's product immediately, his expected utility is $U_{\text{I}}(s, p^*) = v - p^*(s)$, where his expectation about the firm's price $p^*(s)$ depends on the observed information strategy s and on the firm's expected pricing strategy p^*. If he inspects the firm's products, his expected utility is

$$U_{\text{S}}(\theta, s, s^*, p^*) = \int_{\left\{\varepsilon\in[\underline{\varepsilon},\overline{\varepsilon}]:\, u(\varepsilon,p^*(s))-U_{\text{res}}(\theta,s^*,p^*)\geq 0\right\}} \left\{u(\varepsilon, p^*(s)) - U_{\text{res}}(\theta, s^*, p^*(s^*))\right\} f(\varepsilon)d\epsilon$$
$$- \theta\,(c + s) + U_{\text{res}}(\theta, s^*, p^*).$$

The first term is the expected gain which the consumers obtains if finds a product which supplies him with a utility greater than his reservation utility; the second term are the costs of inspecting the firm's product; the third term is his expected utility if he continues search after inspection – his reservation utility.

Among these three actions, the consumer chooses the action which maximizes his expected utility. Analogously, the consumer's best response is monotonic in his type such that it can be described by two cut-off types $\theta_S(s), \theta_I(s)$, where $\theta_S, \theta_I : [\underline{s}, \overline{s}] \to \mathbb{R}$ throughout this section. The reason is that only U_{S} is type-dependent and strictly decreasing in θ due to two effects: a direct effect, as consumers with lower opportunity costs incur lower search costs $\theta\,(c + s)$; an indirect effect, as U_{S} is strictly increasing in the consumer's expected utility U_{res},

[29] Thus, $\tilde{x}(\theta, s^*)$ is implicitly defined by $\mu \int_{\tilde{x}(\theta,s^*)}^{\overline{\varepsilon}} (\varepsilon - \tilde{x}(\theta, s^*)) f(\varepsilon) d\epsilon \overset{!}{=} (c + s^*)\,\theta$.

which in turn is decreasing in his type θ.

Lemma 1.3. *An uninformed consumer's best response, upon observing the firm's information strategy s, is characterized by two cut-off types $\theta_S(s)$ and $\theta_I(s)$:*

i) If the consumer has low opportunity costs: $\theta \leq \theta_S(s)$, he inspects the product.

ii) If the consumer has medium opportunity costs: $\theta_S(s) < \theta \leq \theta_I(s)$, he purchases the firm's product immediately.

iii) If the consumer has high opportunity costs: $\theta_I(s) < \theta$, he exits.

An informed consumer's best response is to purchase the firm's product if it supplies him with a utility greater than his reservation utility.

i) If the consumer has low opportunity costs: $\theta \leq \theta_S(s^*)$, his reservation utility is $U_{\text{res}} = v - p^*(s^*) + \mu\tilde{x}(\theta, s^*)$.

ii) If the consumer has medium opportunity costs: $\theta_S(s^*) < \theta \leq \theta_I(s^*)$, his reservation utility is $U_{\text{res}} = v - p^*(s^*)$.

iii) If the consumer has high opportunity costs: $\theta_I(s^*) < \theta$, his reservation utility is zero.

As before the consumer's behavior is in line with intuition: Consumers with low opportunity costs inspect the firm's product, while all other consumers purchase immediately or exit.

1.4.3 Firm behavior

A firm's strategy consist of an information strategy s, which influences whether a consumer inspects its product, purchases it immediately or exits, and a pricing strategy p, which specifies a price for every s and which trades off the profits generated by exploiting immediate-purchasers and the gains in market share form consumers who inspect the firm's product.

In the following, I derive the firm's profits after a deviation from a symmetric market equilibrium in which trade occurs. Suppose the firm expects each other firm's strategy to be (s^*, p^*) and each consumer's strategy to be $(\theta_S^*, \theta_I^*) > (\underline{\theta}, \underline{\theta})$. Let $\xi(\theta, s^*)$ denote the density of consumer types that arrive at the firm. As before, $\xi(\theta, s^*) = \sum_{n=0}^{\infty} F(\tilde{x}(\theta, s^*))^n h(\theta) = \frac{h(\theta)}{\bar{F}(\tilde{x}(\theta, s^*))}$ for $\theta \leq \theta_S(s^*)$, and $\xi(\theta, s^*) = h(\theta)$ otherwise. Equivalent to equation (1.4), the firm's strategy

(s, p) therefore generates the profits

$$\pi(s,p) = p(s)\int_{\underline{\theta}}^{\theta_{\mathrm{I}}^*(s)} \Big\{1 - \mathbb{1}_{\theta\leq\theta_{\mathrm{S}}^*(s)} F\Big(\max\{\tilde{x}(\theta,s^*),0\} + \frac{p(s)-p^*(s^*)}{\mu}\Big)\Big\}\xi(\theta,s^*)d\theta, \tag{1.6}$$

where the term in curly brackets denotes the probability that a consumer purchases the firm's product conditionally on inspecting it.[30] Notice that π is increasing in $\theta_{\mathrm{I}}^*(s)$ and decreasing in $\theta_{\mathrm{S}}^*(s)$, since an increase in $\theta_{\mathrm{I}}^*(s)$ and a decrease in $\theta_{\mathrm{S}}^*(s)$ cause an upward shift of the demand curve. This is in line with intuition: A firm profits if more consumer purchase its product immediately either instead of exiting the market or instead of inspecting it.

1.4.4 Market equilibrium

Multiplicity of equilibria. There exists for two reasons a multiplicity of perfect Bayesian equilibria in which trade occurs.[31] First, if product diversity is very high, all consumers inspect products independent of the information strategies of firms. Then, firms cannot affect the search behavior of consumers by increasing or lowering search costs, so that they are in effect indifferent among all information strategies. The second reason resembles the reason why there exist trivial equilibria in this game. If all consumers exit after observing a deviation of one firm in its information strategy, as they expect the firm to charge an excessively high price, it is indeed also optimal for the firm to charge such a price. This discourages firms from deviations and creates a multiplicity of equilibria and equilibrium market outcomes.

Refinement. I resolve this multiplicity by focusing on those equilibria which generate the highest industry profits. As the resulting equilibrium market outcome is unique, this also allows me to do comparative statics exercises. This choice seems to be appropriate, as it is plausible that firms coordinate on their preferred equilibrium.[32]

[30] The reservation utility of any consumer is $v - p^*(s^*) + \mu\ \max\{\tilde{x}(\theta, s^*), 0\}$ in any equilibrium in which trade occurs. A product supplies the consumer thus with a utility greater than his reservation utility if $\mu\,\varepsilon - p(s) - \mu\ \max\{\tilde{x}(\theta, s^*), 0\}) + p^*(s^*)$ exceeds zero.

[31] As before, there always exist trivial equilibria.

[32] A reasonable alternative is a refinement that captures the idea of forward induction. Then, consumers anticipate that a firm only deviates if it expects the deviation to be profitable, and consequently, off the equilibrium path those consistent beliefs are chosen that render deviation profitable to firms and encourage deviations. The proof of proposition 1.3 is a constructive one. However, whenever there exist consistent beliefs for which trade

Proposition 1.3. *The firm-optimal market equilibrium has five regimes and is characterized by four cut-off values of product diversity.*

i) ***Market Failure:*** *For very low product diversity: $\mu \leq \mu^S_A$, the firm's information strategy is arbitrary. All consumers exit and no trade occurs; the market price exceeds the average valuation v.*

ii) ***Transparency Regime:*** *For low product diversity: $\mu^S_A < \mu \leq \mu^S_B$, firms choose a transparency policy and simplify the acquisition of product information: $s^* = \underline{s}$. Consumers with low opportunity costs inspect products; while the participation of all other consumers adjusts such that the market price is equal to v.*

iii) ***Intermediate Regime:*** *For intermediate product diversity: $\mu^S_B < \mu < \mu^S_C$, all consumers purchase products. The firms' information strategy adjusts such that the market price is equal to v.*

iv) ***Obfuscation Regime:*** *For medium product diversity: $\mu^S_C \leq \mu < \mu^S_D$, firms obfuscate product information: $s^* = \overline{s}$. All consumers purchase products, but only consumers with low opportunity costs inspect products. The market price is $\tilde{p}(\mu, c + \overline{s}, 1)$.*

v) ***Search Regime:*** *For high product diversity: $\mu^S_D \leq \mu$, firms obfuscate product information: $s^* = \overline{s}$. All consumers inspect products and the market price is $\tilde{p}(\mu, c + \overline{s}, 1)$.*

Equilibrium search costs $c + s^*$ maximize industry profits.

The main task in the constructive proof which establishes the existence of the described market equilibrium is to construct the pricing behavior of firms off the equilibrium path in order to verify that there exists no profitable deviation for firms. Recall that, given s^*, market outcomes follow immediately from the analysis in section 1.3.

Discussion. Beginning from low product diversity, I replicate the previously found no trade result with a slightly modified lower bound in product diversity. For $\mu \leq \mu^S_A$, no consumer inspects products for any $s^* \in S$. Hence, there only exist equilibria in which no trade occurs.

occurs off the equilibrium path, then these beliefs are in fact unique and coincide with the ones constructed in the proof. Therefore, the firm-optimal market equilibrium is stable with respect to a notion of forward induction. However, a refinement that captures the idea of forward induction does not help to overcome the first reason for the multiplicity of equilibria.

In the transparency regime, firms choose a transparency policy and simplify the acquisition of product information as much as possible. Still, consumers only participate partially. If a firm deviates to higher search costs, less consumers inspect the firm's product, which seems to be desirable at first sight. However, consumers that consider to purchase the firm's product immediately anticipate that the firm's incentive to set low prices relaxes, and refrain from purchasing the firm's product immediately. I find that the loss in demand from immediate-purchasers outweighs the gains of having less consumers inspect the firm's product. In other words, by encouraging consumers to inspect its product, a firm can signal to consumers who consider to purchase its product immediately that they can rely on the competitive pressure that is induced by informed consumers. This renders a transparency policy to be profitable, as it generates additional demand from consumers who purchase the firm's product immediately and who would otherwise exit.

This effect dominates until all consumers participate. For greater product diversity, in the intermediate regime, firms gradually begin to obfuscate product information such that still all consumers participate. The intuition is that if lowering search costs does not generate additional demand from otherwise exiting consumers, it is profitable for firms to raise the costs of inspecting its product, since then less consumers become informed and potentially continue search if they find the firm's product not to be suitable.

In the obfuscation regime, all consumers participate although firms impede the inspection of products as much as possible. Lowering search costs is not profitable, as it does not generate any additional demand. For even greater product diversity, in the search regime, all consumers inspect products. In this regime, each firms is indifferent between all information strategies, as it cannot prevent that the consumer inspects its product. Nevertheless, firms profit if they collectively obfuscate product information, since this increases each firm's monopoly power, as a consumer's option to continue search becomes less attractive. As a result, in the firm-optimal market equilibrium all firms obfuscate information.

1.4.5 Comparative statics results for endogenous search costs

Only in the intermediate regime the information strategy of firms varies with product diversity μ, search costs c and the average valuation v. Therefore, in all other regimes the comparative statics effects on market outcomes follow those for exogenous search costs. Hence, I only discuss the intermediate regime.

Comparative statics results in the intermediate regime

Proposition 1.4. *In the intermediate regime, the information strategy s^* is strictly increasing in product diversity μ and average valuation v, and strictly decreasing in search costs c.*

Thus, the more variety there exist among products, the more firms obfuscate product information in order to discourage the acquisition of product information. Furthermore, any decrease in exogenous search costs c is obviously offset by an increase in obfuscation. The intuition for the first result is that if the firms' information strategies remained unchanged the equilibrium price would decrease as a result of an increase in product diversity. Then, the participation constraint of no consumer would be binding anymore. Hence, a firm could marginally increase its search costs, which would render demand slightly less elastic and increase its profits. However, if it increases its search costs just slightly, it could still credibly commit to a price below the average valuation, so that it does not face any loss in demand from immediate-purchasers.

Welfare. Since any increase in product diversity comes along with further obfuscation of product information, consumers might not benefit from product differentiation. To address this issue, I first show that the reservation match-value $\tilde{x}(\theta, s^*)$ is increasing in product diversity μ.

Lemma 1.4. *In the intermediate regime, the reservation match-value $\tilde{x}(\theta, s^*)$ is strictly increasing in product diversity.*

Therefore, despite more obfuscation, shoppers are choosier and search on average longer after an increase in product diversity. As the equilibrium prices is constant at v and a shopper's utility is $v - p^* + \mu\tilde{x}(\theta)$, each shopper's surplus is strictly increasing in product diversity, as reservation match-values increase. Furthermore, before the increase in product diversity, all non-participants and immediate-purchasers obtained a utility of zero. Their expected utility therefore unambiguously weakly increases, as it may not drop below zero.

Corollary 1.1. *In the intermediate regime, each consumer's welfare is weakly increasing in product diversity, and total welfare is strictly increasing in product diversity.*

Therefore, in the intermediate regime an increase in product diversity is, despite further obfuscation, not only desirable form the social planner's point of view, but as well benefits each firm and each consumer. Obfuscation only allows firms to maintain their profits.

I conclude with a short remark on the effect of obfuscation on welfare. As welfare is decreasing in search costs by proposition 1.2, obfuscation of product information clearly reduces welfare. Hence, the welfare analysis provides a rationale for policy intervention that aim to reduce the obfuscation of product information.

1.5 Do firms offer niche or plain vanilla products?

In this section, I examine product design, by endogenizing product diversity, such that, figuratively speaking, each firm chooses whether to offer a plain vanilla product or a niche product.[33] The notion of product design builds on the notion of demand rotation, as introduced by Johnson and Myatt [2006] and is also used by Bar-Isaac et al. [2012]. Formally, let $\mu_j \in [\underline{\mu}, \overline{\mu}]$ denote the product design choice of firm j. Thus, with its choice of μ_j the firm affects the variance of the valuations of consumers for its product. Intuitively, a low μ_j represents a plain vanilla product; a high μ_j represents a niche product. I include product design choice in the two previously presented models by adding a first stage to the game, in which all firms simultaneously and privately choose their product design. Then, time elapses

[33] Intuitively, it is irrelevant whether product differentiation is real or spurious. More broadly, the chosen approach also relates to other models of information disclosure if one considers undisclosed information about match-values to be equivalent to a plain vanilla design. See also the work by Johnson and Myatt [2006] on the equivalence of information disclosure and product design.

as before. I assume that the product design choice is unobservable to consumers, but that consumers learn μ_j if they inspect the product.

Proposition 1.5. *If product design is endogenous, independent of whether search costs are exogenous or endogenous, then in any market equilibrium, in which trade occurs, each firm offers a niche product. That is* $\mu^* = \overline{\mu}$.

I find thus a strong result in favor of maximal product differentiation on the individual firm level. While this result is in line with the prevailing tenet in industrial organization that firms should seek to differentiate, recent studies suggest that individual firms might profit from offering plain vanilla products if they have a competitive advantage.[34] The intuition for why I obtain a different result is simple and also prevails in models of vertically differentiated products. The key insight is that the firm's choice of product design only affects the demand of shoppers who inspect its product and learn about the firm's choice of product design. As shoppers never purchase a misfit at p^*, a product with a negative match-value, it is irrelevant for firms whether shoppers just slightly dislike their products or hate them, since, if the product is misfit, they have no intention to purchase the product in either case. The underlying reason why shoppers do not purchase misfits at p^* is the consumer's option to purchase immediately. If consumers may purchase immediately, then a shopper's reservation match-value must exceed zero, $\tilde{x}(\theta) \geq 0$, as otherwise, he would prefer to purchase immediately. To continue, it is, on the other hand, decisive whether a shopper just slightly likes a product or loves it. Consequently, each firm offers a niche product in order to increase the variance in the consumer's valuation for its product and by no means can commit to offering a plain vanilla product in order to discourage the information acquisition of consumers. The general quintessence is hence that the option to purchase products immediately enhances product diversity.

[34] In example, Bar-Isaac et al. [2012] demonstrate that in a consumer search model, if firms are vertically differentiated, those firms that have a competitive advantage choose plain vanilla designs, while the remaining firms focus on targeting niches. A similar argument is made by Anderson and Renault [2009].

1.6 Conclusion

This study examines poorly informed purchases in a market where consumers search for suitable products. Intuitively, poorly informed purchases raise market prices. Moreover, firms do not necessarily benefit from product differentiation anymore, as it induces more consumers to inspect products. If firms however may choose their product designs, each firm targets a niche. This study also illustrates that the consumer's option to purchase poorly informed provides firms with an incentive to obfuscate product information, but points out that there are limits to the obfuscation of product information. If firms obfuscate too much, consumers correctly anticipate that the firm's incentive to offer a good deal relaxes and they consequently avoid purchasing the firm's product. Nevertheless, in equilibrium firms find it individually rational to obfuscate product information such that industry are profits maximized. My analysis of this model of monopolistic competition thus suggests that informational frictions emerge endogenously even if firms only have little market power.

Chapter 2

Guided Search

Abstract. Is it profitable for a multi-product monopolist to support her consumers in finding their preferred products? I study a monopolist who may influence the information acquisition of a consumer who inspects her products by raising wasteful search costs, which I interpret as the obfuscation of product information. I show that obfuscation is a profitable sales technique, as it allows the monopolist to influence the consumer's order of search, so that, at an optimum, the consumer purchases the most expensive product which supplies him with positive utility. In equilibrium, polarizing products are sold at the highest prices and the monopolist obtains the second-best profits. The consumer's equilibrium utility, on the other hand, does not necessarily exceed zero. Nevertheless, obfuscation may lead to welfare improvements. My results suggest that informational frictions emerge endogenously if firms have market power.

2.1 Introduction

Finding a suitable product can be tedious and time-consuming. How difficult it is to acquire information about products also depends on how this information is presented to customers. In this paper, I address the question whether it is a profitable sales technique for firms to support their customers in finding suitable products by offering easy access to product information, or whether it is more profitable to impede the consumers' information acquisition by creating artificial and wasteful search frictions.

This question is motivated by the presence of obfuscated products in markets – products which are unnecessarily difficult to evaluate such that it appears as if they are intentionally designed this way. I contribute to explaining their existence by showing that it is profitable for firms to strategically raise the time necessary to evaluate their products. A case in point are complex contracts offered such as insurance policies and mobile phone contracts, where firms impede the consumer's evaluation of contracts by using complex payment schedules. In these case, firms apparently prefer not to support their customers in finding suitable contracts through the use of particular simple payment schedules which would help consumers to evaluate and compare the contracts offered. For similar motives one might suspect that firms write unnecessary and excessively long contract terms and hide important information in the fine print, both increasing the time necessary to evaluate contracts. Comparable strategic situations emerge outside of contracting settings if retailers can influence how difficult it is to judge whether a product is suitable and thus also can influence the time necessary to evaluate products. For example, retailers of consumer electronics decide whether or not to provide understandable and comprehensive product descriptions, whether or not to provide the opportunity to test the product at the store, and whether and where to locate helpful staff within the store.[1]

In order to examine a firm's incentive to influence the consumer's information acquisi-

[1] That some products are unnecessarily difficult to evaluate has also come to the attention of policy makers. For example, the directive 2014/92/EU of the European Parliament aims to improve the comparability of fees related to payment accounts, and finds that "Consumers would benefit most from information that is concise, standardized and easy to compare". My welfare analysis illustrates that such policy intervention can backfire and actually be harmful if equilibrium effects are not taken into account.

tion, I study a monopolist who offers several products. A consumer searches sequentially, product by product, and upon inspection of a product privately learns whether it is a suitable one. Crucially, search costs, which the consumer incurs for each product inspection, are product-specific and a choice variable of the monopolist. The monopolist may thus impede the consumer's information acquisition by strategically raising search costs which captures that firms may raise the time necessary to evaluate their products.[2] While the consumer is, prior to his search, uninformed about the utility which each product supplies, he observes the monopolist's choices of prices and search costs. For example, without observing what is written in the contract offered, the consumer can observe the length or complexity of the contract terms and hence observe which contracts are more or less difficult to evaluate. In effect, by choosing prices and search costs, the monopolist can strategically influence the order in which the consumer searches for a suitable product. This is the strategic consideration at the heart of this paper.

As my first main result, I show that, for an arbitrary, exogenous profile of product prices, the monopolist can set search costs so that the consumer is induced to purchase the most expensive product[3] which supplies him with positive utility. Since the consumer's valuation upon inspection is his private information, this is clearly an upper bound on the profits that the monopolist can obtain for a given price profile. I show that the monopolist achieves this upper bound by setting for the majority of products search costs at a level so that the consumer is just indifferent between inspecting any of these products and terminating her search.[4] In this sense, the monopolist guides the consumer exclusively through her choice of search costs, not through her choice of prices, and the monopolist's problem can be separated into two methodologically distinct steps: choosing optimal search costs for given prices and choosing prices which generate the highest profits if the consumer purchases the most expensive product which supplies him with positive utility.

[2]To the contrary, the monopolist may also support the consumer in finding a suitable product by setting search costs equal to zero such that the consumer can inspect all products at no search costs.

[3]As marginal costs are normalized to zero, more expensive products are more profitable ones.

[4]The monopolist's motives to obfuscate are therefore independent of her power to set prices. In example, an estate agent has consequently incentives to influence the consumer's search for a suitable flat by only providing near-term appointments for more profitable flats if she obtains a share of the predetermined rent.

To show this result, I exploit a well-known search theoretic insight that goes back to Weitzman which says that, when faced with a given profile of prices and search costs, the consumer's optimal search strategy is characterized by a product's reservation utility which depends only on this product's price, search costs, and valuation distribution (and not on the characteristics of other products). The consumer then inspects products in descending order of reservation values. By adjusting search costs, product by product, so as to equalize all reservation values, the monopolist makes it optimal for the consumer to inspect products in a descending order of prices. Intuitively, obfuscation plays the dual role of inducing the consumer to inspect expensive products first but also to reduce the benefits of continued search, inducing the consumer to terminate search sooner in expectation and to purchase one of the expensive items.

My second main result is that obfuscation is not necessarily harmful but may lead to welfare improvements. Intuitively, for the consumer the equilibrium outcome is bad news. In fact, there even exists an equilibrium in which the consumer's expected utility is zero. In such an equilibrium the consumer does not derive any utility in excess of the utility which his outside option supplies and his expected information rent only compensates for the search costs that he incurs in expectation. For the monopolist the equilibrium outcome is, on the other hand, good news as he obtains the second-best profits. Overall obfuscation may increase welfare in comparison to the welfare generated in the absence of search frictions. The underlying reason is that obfuscation allows the monopolist to discriminate between those consumers who have found an expensive products to be suitable and those who have not. This enables her to give the latter consumers a discount on the remaining products which improves market coverage. The associated gains in welfare may outweigh the welfare losses due to search frictions. In terms of policy implications, my analysis thus does not provide a rationale for policy interventions which aim to reduce obfuscation.

My third contribution is the detailed characterization of the profit-maximizing profile of product prices. As the monopolist's problem can be separated into two methodologically distinct steps, this characterization boils down to determining which prices generate the highest profits if the consumer purchases the most expensive product which supplies him with positive utility.[5] Most interesting is the ordering of the optimal prices. Not only as I provide a recursive formula to determine prices if the ordering is known, but more importantly, as the ordering of prices determines to which products the monopolist guides the consumer first. One might suspect that the quality[6] of products pins down the ordering of prices and hence the consumer's search order. I find, however, that this is not the case. The property which instead shapes prices, and thus the consumer's search order, is how *polarizing*[7] a product is. Intuitively, a product is polarizing if extreme valuations, very high and very low ones, are likely to occur. I provide conditions under which the consumer is guided to more polarizing products first. For instance, these conditions are met if the monopolist offers an arbitrary number of products with uniformly distributed valuations. Then, those products with the higher variance in the consumer's valuation are more polarizing and are inspected first in any equilibrium.

The intuition for why the monopolist guides the consumer to polarizing products first is that the monopolist attempts, to first sell products at high prices as part of a long-shot strategy. In order to increase the likelihood that the consumer finds a product to be suitable despite its high price, the monopolist guides the consumer to polarizing products first. In contrast, the consumer's valuations for less polarizing products are less likely to be extreme. This guarantees the monopolist that she can sell less polarizing products at moderate prices with high probability. The monopolist guides the consumer hence to less polarizing product last as these products are particularly suited to be used as part of a last resort safety strategy, which ensures that the consumer purchases some product before exiting.

2.1.1 Related Literature

Ellison and Ellison [2009] provide empirical evidence of obfuscation which shows that on-

[5]From a theoretical point of view, this problem and its solution is of interest in itself.

[6]A product is *better* than another one if the corresponding distribution function of valuations first order stochastic dominates its counterpart.

[7]The notion of a polarizing product builds on a single-crossing property of a family of distribution functions. This property has been used before in Courty and Li [2000] and is a special case of a demand rotation (Johnson and Myatt [2006]).

line retailer intentionally present product information in complex and misleading ways. Recent approaches explain the obfuscation of product information with the bounded rationality of consumers: Through obfuscation consumers do not notice shrouded add-on costs (Gabaix and Laibson [2006], Heidhues et al. [2012]), misconceive the utility a product supplies (Salop and Perloff [1985], Spiegler [2006], Salant and Siegel [2015]) and fail to compare products (Carlin [2009], Piccione and Spiegler [2012], Chioveanu and Zhou [2013]).[8] I explain obfuscation in the presence of rational consumers as a sales technique which aims to increase the costs of information acquisition in order to influence the consumers' search among her products.[9]

The paper is not the only one to interpret obfuscation as raising search costs in an environment of costly information acquisition. This branch of the literature seeks to put forward arguments why it is individually rational for firms to raise search costs.[10] Most closely related is an alternative approach to a multi-product monopolist who sets prices and search costs by Petrikaite [2015]. In contrast to my paper, in her model the consumer can neither observe prices nor search costs. Therefore, the consumer's search order depends only on his expectations and cannot be influenced by the monopolist, which is the key point of my paper.[11] Wilson [2010] shows that in a directed search model firms split the market of heterogeneous consumers by differentiating in search costs. Ellison and Wolitzky [2012] point out that in a sequential search model firms have incentives to raise search costs in order to fatigue consumers, when search costs are convex, instead of linear.[12] Gamp [2015a] illustrates that firms

[8]Also, firms obfuscate product information in order to discriminate among naive and sophisticated consumers (Rubinstein [1993], Eliaz and Spiegler [2006], Carlin and Manso [2011]).

[9]That the profitability of a firm crucially depends on its ability to 'manipulate' the consumer's search order has been acknowledged before within the literature on advertising. For instance, Chen and He [2011] and Athey and Ellison [2011] study position auctions, where firms bid for sponsored links at the top of search engine results.

[10]In classic oligopoly search models as studied by Diamond [1971], Burdett and Judd [1983], Wolinsky [1986] and Stahl [1989] it is not individually rational for firms to raise search costs. However, obfuscation is collectively rational since profits are, typically, increasing in search costs. Consequently, a monopolistic search engine has therefore an incentive to encourage low-relevance advertisers to enter its search pool in order to raise the consumers' expected costs to find a relevant advertiser (Eliaz and Spiegler [2011b]).

[11]The equilibrium profile of my model which is generically unique up to the search costs of the most expensive product, is one of many equilibria in her model.

[12]Related, Armstrong and Zhou [2016] show that it is profitable sales technique to deter the consumer from continuing search by making it harder or more costly to return.

also impede the information acquisition of consumers in a competitive environment, when consumers search sequentially but may also purchase products without prior information acquisition.

This paper also contributes to the classic literature on consumer search. It builds on the seminal analysis of Gittins and Jones [1974] and Weitzman [1979] who determine a consumer's optimal search rule if the consumer may inspect products in his preferred order. This paper provides the corresponding equilibrium analysis if prices and search costs are endogenous. Very few other studies have examined pricing in directed search models. Noteworthy exceptions are the studies by Armstrong et al. [2009] and Zhou [2011],[13] who examine oligopolies, in which firms are visited in an exogenously given order. The tenet of these papers is that, in contrast to my analysis, later inspected products are more expensive, as firms correctly anticipate that the consumer's valuations for previously inspected products are low, conditionally on being visited.

More broadly, the paper is related to the literature on information provision and advertising. The main difference is that the monopolist offers several products and that the consumer's information acquisition is sequential and costly.

The paper is organized as follows: In section 2.2, the model is introduced. In section 2.3, the consumer's optimal search rule is determined. The main results are presented in section 2.4. Section 2.5 concludes. All proofs are relegated to the appendix.

2.2 Model

The market consists of a searching consumer (he) with unit demand and a profit-maximizing monopolist (she). The monopolist offers several heterogeneous products. If the consumer buys product $k \in \{1, \ldots, K\}$ at price p_k, his quasi-linear utility absent any search costs is

$$u_k = \theta_k - p_k, \tag{2.1}$$

[13] The homogeneous product case is studied by Arbatskaya [2007].

where the match-value θ_k is the consumer's valuation for the k-th product. Each θ_k is the realization of an independent random variable $\hat{\theta}_k$ which is described by a valuation distribution F_k. The interval support of F_k is $\text{Supp}(F_k) = [\underline{\theta}_k, \overline{\theta}_k]$. In order to simplify the exposition, assume that any product, absent prices, may be preferred to any other such that the supports of any two valuation distributions overlap. Furthermore, any F_k has no mass points and is twice continuously differentiable on the open interval $(\underline{\theta}_k, \overline{\theta}_k)$. Denote the corresponding reliability functions with $\tilde{F}_k := 1 - F_k$.

Search protocol. Ex ante, the consumer only knows the products' prices and search costs, but does not know realized match-values. He may, however, inspect products privately and sequentially in order to learn these. Each inspection is costly and $s_k \geq 0$ denotes the product-specific search costs. The consumer's search ends, once he purchases a product or exits. Formally, the consumer can at any time:

i) inspect one product of his choice: learn θ_k at costs s_k,

ii) purchase any previously inspected product k: end search and obtain $u_k = \theta_k - p_k$,

iii) exit: end search and obtain a utility of zero.

Monopolist. The monopolist's marginal costs of production are normalized to zero for each product. She chooses publicly a price profile and a search cost profile: $(\vec{p}, \vec{s}) \in \mathbb{R}_+^K \times \mathbb{R}_+^K$ with the aim to maximize profits. Crucially, the product-specific search costs are thus a choice variable of her.

Timing of events and solution concept. First, the monopolist chooses publicly a price profile and a search cost profile. Second, the consumer searches until he exits or purchases a product. The equilibrium concept is Perfect Bayesian Equilibrium.

2.3 The consumer's search rule

The consumer faces a so called Pandora's problem (Weitzman [1979]). He has to decide in which order to inspect products, possibly dependent on learned match-values, and when to end search in order to either purchase a product or to exit. As match-values are the realizations of independent random variables, the consumer's optimal search rule satisfies Pandora's rule (Weitzman [1979]).[14] To begin with, I introduce the Weitzman reservation utility.

Definition 2.1 (Weitzman reservation utility)**.** *For every (p_k, s_k), the Weitzman reservation utility $U_k^{\text{res}}(p_k, s_k)$ is implicitly defined by:*

$$s_k \stackrel{!}{=} \mathbb{E}\left[\max\left\{(\hat{\theta}_k - p_k) - U_k^{\text{res}}(p_k, s_k), 0\right\}\right] \tag{2.2}$$

The Weitzman reservation utility $U_k^{\text{res}}(p_k, s_k)$ equates the benefits of inspecting product k with its costs. It is thus equal to the highest utility which a hypothetical outside option supplies such that the consumer yet prefers to inspect the considered product.

Pandora's rule states that the consumer inspects products with higher reservation utilities first, and ends his search once the utility which his so far most preferred product supplies exceeds the highest reservation utility of all remaining products. In the following, treat the consumer's option to exit as an inspected product that supplies zero utility.

Definition 2.2 (Pandora's rule)**.** *A search rule satisfies Pandora's rule if:*

i) Selection rule: If the consumer inspects a product, he inspects a product with the highest reservation utility among those which have not been inspected before.

ii) Stopping rule: If the consumer terminates search, he purchases a product with the highest utility among all inspected products, and this utility exceeds the reservation utility of all products which have not been inspected before.

Lemma 2.1 (Weitzman [1979])**.** *A search rule is optimal if and only if it satisfies Pandora's rule.*

The consumer's search rule is unique up to how indifferences are resolved. In appendix A4, I show that in any equilibrium all payoff-relevant indifferences, those that have strictly positive probability mass, are resolved in favor of the monopolist. In particular, the consumer inspects

[14]As has been acknowledged before, the optimality of a search rule that satisfies Pandora's rule also follows from the Gittins index theorem (Gittins and Jones [1974]), where the Weitzman reservation utilities represent the Gittins indices of a corresponding bandit problem.

more expensive products first if several products have equal reservation utilities. Without loss of generality, in the following all indifferences are resolved in favor of the monopolist.

The consumer's search order. For each profile $(\vec{p}, \vec{s})$, a search rule induces a *search order* that determines in which order the consumer inspects products. In order to satisfy Pandora's rule, products are ordered by their reservation utilities.

Definition 2.3 (Search order)**.** *A search order is a bijection $\phi : K \to K$, where $\phi(k)$ denotes the k-th product which the consumer inspects if he does not end his search earlier. A profile $(\vec{p}, \vec{s})$ induces the search order ϕ if $\{U^{\text{res}}_{\phi(k)}(p_{\phi(k)}, s_{\phi(k)})\}_{k\in K}$ is decreasing.*

Vice versa, product k is thus the $\phi^{-1}(k)$-th product to be inspected.

2.4 The monopolist's behavior

2.4.1 The monopolist's search costs strategy

The consumer never purchases a product which supplies him with strictly less utility than his outside option supplies. For a price profile $\vec{p}$ the best which the monopolist can hence achieve is that the consumer purchases her most expensive product which supplies him with positive utility given the realized match-values. The corresponding expected profits $\overline{\pi}(\vec{p})$ are therefore an upper bound on her profits generated by the price profile $\vec{p}$.

Definition 2.4 (Upper-bound profits)**.** *For a price profile $\vec{p}$, the upper-bound profits $\overline{\pi}(\vec{p})$ are defined as the expected profits that the monopolist obtains if the consumer purchases a most expensive product which supplies him with positive utility:*

$$\overline{\pi}(\vec{p}) := \mathbb{E}\left[\max_{k\in K}\left\{p_k | p_k \leq \hat{\theta}_k\right\}\right]$$

My first result is that the monopolist can influence the consumer's search and reduce the benefits of continued search in a manner so that for any price profile she achieves the corresponding upper-bound profits $\overline{\pi}(\vec{p})$. Beforehand, I introduce the obfuscation strategy of the monopolist.

Definition 2.5 (Obfuscation). *Product k is obfuscated if its search costs are $s_k^0(p_k)$ which is defined as:*

$$s_k^0(p_k) := \mathbb{E}\left[\max\left\{(\hat{\theta}_k - p_k), 0\right\}\right] \tag{2.3}$$

Equivalently, $s_k^0(p_k)$ solves $U_k^{\text{res}}\big(p_k, s_k^0(p_k)\big) = 0$. A product is thus referred to as obfuscated in the following if its reservation utility is zero. Recall that the consumer never inspects a product whose reservation utility is strictly below zero, the utility which his outside option supplies. A product is hence obfuscated if the monopolist chooses, conditional on the product's price, the highest search costs such that the consumer yet prefers inspecting the product to exiting.

Theorem 2.1. *For any price profile $\vec{p}$, the monopolist obtains the upper-bound profits $\overline{\pi}(\vec{p})$ if all products are obfuscated.*

There are two reasons why obfuscation is profitable for the monopolist and enables her to achieve the upper-bound profits. The first reason is that by obfuscating products she influences the consumer's search in her favor: If products are obfuscated, their reservation utilities are equal. The consumer is therefore indifferent about the order of search, so that it is also optimal for him to inspect more expensive products first.

The second reason is that it is profitable to reduce the consumer's benefits of continued search. Intuitively, if the consumer inspects more expensive products first, the monopolist wants to discourage intense consumer search. By obfuscating products, she makes search as costly as possible under the constraint that the consumer is yet willed to inspect products. Formally, if products are obfuscated, their reservation utilities are equal to zero. As a consequence, as soon as the consumer finds a product for which his valuation is at least equal to its price, he purchases the product, since the utility it supplies exceeds the reservation utility of all remaining products. He thus purchases the first product that he finds, the most expensive one, which supplies positive utility. As a result, the monopolist obtains the upper-bound profits.

Another interpretation is that by raising search costs the monopolist discriminates between those consumers who have found early inspected, expensive products to be suitable and those who have not. Due to obfuscation, consumers who are willed to purchase these expensive products after inspection reveal this information to the monopolist such that only all other consumers continue search and purchase one of the remaining cheaper products.

With theorem 2.1 in mind, the approach taken here to find the monopolist's equilibrium strategy profile becomes evident. I identified an upper bound on the monopolist's expected profits for each price profile and established that there exists a search cost strategy that enables the monopolist to achieve these upper-bound profits. It follows that in any equilibrium the monopolist must obtain the highest upper-bound profits among all price profiles. This observation allows me to characterize the monopolist's strategy further.

2.4.2 Equilibrium characterization

First-best. Consider an informed monopolist and consumer who both know the consumer's valuations for the monopolist's products. Intuitively, in any equilibrium of this game without search the consumer purchases his, absent prices, preferred product at a price equal to his valuation for the product. The equilibrium outcome is thus a first-best allocation of products and the monopolist extracts the whole surplus generated from trade.

Second-best. In my model, the monopolist is initially uninformed and may only post prices.[15] As learned match-values are the consumer's private information, the highest, expected profits attainable for the monopolist, denoted as second-best profits, are the highest upper-bound profits which are generated by some price profile.

[15]Robustness. Suppose that the consumer and the monopolist could sign a binding contract prior to the consumer's search such that a mechanism design approach is applicable. The monopolist could then offer a contract which charges the expected value of the highest match-value among all products as a lump-sum upfront fee, such that the consumer, upon payment, learns all match-values for free (search costs equal to zero) and may pick his preferred product. With such a contract the monopolist could extract the whole expected surplus due to trade and obtain the first-best profits. The underlying reason is that the consumer is initially uninformed, so that the consumer consequently obtains no information rent.

Definition 2.6 (Second-best profits)**.** *The second-best profits π^{2nd} are defined as:*

$$\pi^{2nd} := \max_{\vec{p}\in\mathbb{R}_+^K} \bar{\pi}(\vec{p}).$$

The first-best profits exceed the second-best profits. As will be discussed later on in greater detail, one reason is that the consumer obtains an information rent, since learned match-values are his private information, so that the monopolist cannot extract the full surplus due to trade.[16]

As indicated before, a corollary to theorem 2.1 is that a monopolist's strategy is an equilibrium strategy if and only if it yields the second-best profits.

Corollary 2.1. *There exists an equilibrium. A monopolist's strategy profile is an equilibrium strategy profile if and only if it generates the second-best profits π^{2nd}.*

One equilibrium strategy of the monopolist is to choose a price profile that maximizes $\bar{\pi}(\vec{p})$ and to obfuscate each product. This equilibrium has further noteworthy properties. Not only does the consumer purchase a most expensive product which supplies positive utility, but he also inspects more expensive products first, never returns to a previously inspected product, and obtains an expected utility of zero. The latter holds, since the Weitzman reservation utility of all obfuscated product is equal to zero so that the consumer is ex ante indifferent about whether to exit or not. The next proposition clarifies to what extent these properties hold in general.

Theorem 2.2. *The following properties hold in any equilibrium:*

i) The consumer buys a most expensive product which supplies positive utility.

ii) The consumer buys each product with strictly positive probability.

iii) The consumer inspects more expensive products first.

iv) The monopolist obfuscates each product, except a most expensive one.

v) The consumer never returns in order to purchase a strictly more expensive product which he inspected before.

[16]The other reason is that the equilibrium outcome is not a first-best allocation of products.

Furthermore, there exists an equilibrium in which the consumer's expected utility is zero.

Clearly, the consumer purchases in any equilibrium a most expensive product which supplies positive utility, since otherwise the monopolist would not obtain the upper-bound profits which would in turn contradict that the monopolist obtains the second-best profits. Jointly with the assumption that each product, absent prices, may be preferred to any other, it follows furthermore that each product is bought in equilibrium, since each product may be the most expensive product which supplies the consumer with positive utility. The technical, underlying reason is that the price of each product, but the cheapest one, is necessarily an interior solution such that each product supplies strictly negative utility with strictly positive probability.

The consumer does not return in equilibrium, since almost all products are obfuscated. If each product is obfuscated, then the consumer purchases a product after inspection if it supplies positive utility. Since the consumer never purchases a product which supplies strictly negative utility, the consumer thus never returns. This result correctly suggests that all results would not change if the consumer had imperfect recall.[17]

Finally, let me emphasize that there exists an equilibrium in which the consumer's expected utility is zero – equal to the utility that his outside option supplies.[18][19] In my model this cannot be interpreted as usually as that the monopolist extracts the whole surplus from

[17]Robustness. If the consumer had imperfect recall, so that he could not return to any previously inspected product, all results remain valid. This is the case, since first the consumer evidently has no incentive to return in equilibrium, and second, by the definition of the second-best profits, there cannot exists a profitable deviation for the monopolist.

[18]Robustness. This results remains valid if the consumer's outside option supplies a non-zero utility. In that case, the monopolist chooses search costs such the reservation utility of all products is equal to the utility that the consumer's outside option supplies. Thus, more generally, there always exists an equilibrium in which the consumer does not obtain any utility in excess of the utility which his outside option supplies. This also illustrates that similar results are obtained if one considers monopolistic competition which results in a non-zero outside option of consumers.

[19]Robustness. I assume that the consumer must inspect a product prior to its purchase. This assumption can be relaxed if one only considers identically distributed match-values. Then, the last product to be inspected and to be potentially purchased without prior inspection imposes a lower bound on the reservation utility of all other products similarly to an outside option which supplies a non-zero utility (footnote 18). Analogously, the monopolist would then choose search costs such that the reservation utility of all other, more expensive products is equal to the expected utility which the last product in the consumer's search order supplies if the consumer purchases it immediately without prior inspection. Qualitatively all results would hence carry over. For non-identical distributed valuations the optimal search rule if consumers may purchase immediately is the subject of on-going research (Doval [2014]).

trade, which is here equal to the match-value of the product that the consumer purchases. As the consumer purchases a product whose match-value strictly exceeds its price with probability one, the consumer obtains some part of the surplus. As this depends on that learned match-values are the consumer's private information, this part of the surplus can be interpreted as the consumer's information rent. Consequently, an equilibrium in which the consumer expected utility is zero is an equilibrium in which the consumer's information rent is equal to his expected search costs.

Multiplicity of equilibria. The equilibrium is not unique. Theorem 2.1, however, implies that the equilibrium is generically unique up to the search costs of the most expensive product.[20] Each product, but the most expensive one, is obfuscated; its search costs are thus uniquely determined. To the contrary, the search costs of the most expensive product are arbitrary to the extent that they only satisfy that the corresponding reservation utility is positive. This suffices to ensure that it is inspected first. Note, however, that these search costs pin down the consumer's expected utility. Only if the monopolist obfuscates all products, then the consumer's expected utility is zero.

2.4.3 Welfare

So far, I have shown that obfuscation allows the monopolist to obtain the second-best profits while the consumer's expected surplus does not necessarily exceed zero. In line with intuition, the second observation suggests that the creation of wasteful search frictions reduces overall welfare. This would provide a rationale for policy interventions which aim to reduce obfuscation. In order to examine the question whether obfuscation is harmful from a social planner's point of view, I compare the market outcome when search costs are endogenous to the market outcome in the absence of search frictions: search costs are exogenous and zero. The following theorem shows that obfuscation is not necessarily harmful but might be desirable from a social planner's point of view.

[20] The reason is that generically $\arg\max_{\vec{p}\in\mathbb{R}_+^K} \overline{\pi}(\vec{p})$ is a singleton and the maximizing price profile satisfies that no two products have identical prices.

Theorem 2.3. *Obfuscation may increase welfare.*

The reason why obfuscation may improve welfare is that obfuscation allows the monopolist to discriminate between those consumers who have found expensive products to be suitable and those who have not. In the absence of search costs, the monopolist is forced to charge high prices for all other products in order to ensure that some consumers purchase her most expensive products. If search costs are endogenous, the monopolist can ensure through obfuscation that only those consumers who have found expensive products to be unsuitable prefer to continue search. As a consequence, the monopolist can offer the remaining products at a discount, without sabotaging the demand for expensive products, which results in greater market coverage. In the proof of the theorem, it is shown via an example that the associated gains in welfare may outweigh the losses in welfare due to search costs, so that ultimately obfuscation may enhance welfare.

2.4.4 The monopolist's pricing strategy

In equilibrium, there is a close link between the monopolist's price profile and the consumer's search order. Since the consumer inspects more expensive products first, his search order follows immediately from the price profile. Vice versa, the search order is a partial characterization of the price profile, as it pins down the order of prices. In this subsection, I show that equilibrium prices can be readily determined given the equilibrium search order.

To begin with, I introduce some further notation. Define the monopoly profits $\pi^{\mathrm{M}}(p, F, \pi_{\mathrm{cont}})$ as the profits which a monopolist obtains if she sells a product at p, whenever the product supplies positive utility, and otherwise obtains the continuation profits π_{cont}:

$$\pi^{\mathrm{M}}(p, F, \pi_{\mathrm{cont}}) := p\tilde{F}(p) + F(p)\,\pi_{\mathrm{cont}}.$$

Proposition 2.1. *Let ϕ^* be the consumer's equilibrium search order. Then, the equilibrium price profile $\vec{p}^*$ is a solution to the recursive formula which is defined by the equations (2.4),* *(2.5) and (2.6):*

$$\pi^*_{K+1} := 0, \tag{2.4}$$

$$\pi^*_k := \pi^{\mathrm{M}}(p^*_{\phi(k)}, F_{\phi(k)}, \pi^*_{k+1}), \tag{2.5}$$

$$p^*_{\phi(k)} \in \arg\max_{p \in \mathbb{R}_+} \pi^{\mathrm{M}}(p, F_{\phi(k)}, \pi^*_{k+1}). \tag{2.6}$$

The monopolist's profits are $\pi^* = \pi^*_1$.

The profits π^*_k denote the monopolist's expected equilibrium continuation profits if the consumer does not purchase any of the first $k-1$ inspected products. If the consumer purchases no product at all, then the monopolist's profits are zero: $\pi^*_{K+1} = 0$, as stated in equation (2.4). This is the initial condition of the recursive equation (2.5), which determines the continuation profits. Its meaning becomes clear if one assumes that all products are obfuscated. Then, whenever the consumer does not purchase any of the first $k-1$ inspected products, the monopolist either sells the k-th inspected product at $p^*_{\phi(k)}$, whenever it supplies positive utility, and otherwise obtains the continuation profits π^*_{k+1}. Thus, $\pi^*_k := \pi^{\mathrm{M}}(p^*_{\phi(k)}, F_{\phi(k)}, \pi^*_{k+1})$. Finally, if all products are obfuscated, the price of the k-th inspected product does not affect the consumer's demand for the first $k-1$ inspected products. The monopolist chooses hence that price for the k-th inspected product which maximizes the continuation profits π^*_k, as stated in equation (2.6).

Proposition 2.1 not only provides a recursive formula to find the equilibrium price profile given the equilibrium search order, but also suggests an algorithm to determine the equilibrium search order in the first place. For any arbitrary search order ϕ there exists a well-defined solution to the corresponding recursive formula, where ϕ replaces ϕ^* in the equations (2.4), (2.5) and (2.6). The equilibrium search order can then be identified as the search order which yields the highest profits, as the following two-product example illustrates.[21]

Example: Two products with uniformly distributed match-values

[21] A proof of this claim is in the appendix A4. More general, this procedure is not analytically tractable, since the number of candidate search orders grows at factorial speed.

Consider a monopolist who offers two products. Suppose that the consumer's valuation for each product is distributed uniformly: $F_k(\theta_k) = \frac{1}{2} + a_k\theta_k$*. Suppose* $a_1 < a_2 < 8a_1$[22]*; product 1 is then inspected first in equilibrium. Solving the recursive formula of proposition 2.1 yields the equilibrium prices and profits:*

$$p_2^* = \frac{1}{4a_2} \quad (2.7) \qquad \pi_2^* = \frac{1}{16a_2} \quad (2.9)$$

$$p_1^* = \frac{1}{32a_2} + \frac{1}{4a_1} \quad (2.8) \qquad \pi_1^* = \frac{a_1^2 + 32a_1a_2 + 192a_2^2}{1024a_1a_2^2} \quad (2.10)$$

where $\pi^* = \pi_1^*$ *are the equilibrium profits of the monopolist.*

In this simple two-product example, product 1 *is inspected first, since the equilibrium profits exceeds the profits obtained if product* 2 *is inspected first. These latter profits are readily obtained by solving for the corresponding, recursive formula of proposition 2.1.*[23]

In the considered two product example with uniformly distributed match-values the product with the higher variance in the consumer's valuation is inspected first. This is not a coincidence, as will be shown in the following subsection. The notion of a more polarizing product will be introduced and conditions are provided which ensure that more polarizing products are inspected first. For instance, for an arbitrary number of products with uniformly distributed match-values, those products with a higher variance in the consumer's valuation are more polarizing and are always inspected first.

2.4.5 To which products guides the monopolist the consumer first?

So far no restrictive assumptions with regard to the distribution of match-values have been made. In this subsection, I impose some assumptions which allow me to characterize the equilibrium search order.

[22]A technical remark: The assumption is a necessary and sufficient assumption which ensures that the maximization problem of the monopolist has an interior solution.

[23]The profits if product 1 is inspected first are given by equation (2.10). Relabeling of a_1 and a_2 in equation (2.10) yields the profits if product 2 is inspected first. Some algebra then yields the desired inequality.

Two products

I begin with an analysis of a monopolist who offers two products of which one product is ex ante *better* than the other one in the sense that its valuation distribution first order stochastic dominates its counterpart. Later on, I examine more general cases and consider also an arbitrary number of products. These later results build on the main ideas presented in the following.

Lemma 2.2. *Suppose the monopolist offers two products of which product* 1 *is ex ante better than product* 2. *Let* $(\vec{p}^*, \vec{s}^*)$ *be the monopolist's equilibrium strategy profile. Suppose* $p_1^*, p_2^* \in \mathrm{Supp}(F_1) \cap \mathrm{Supp}(F_2)$ *with* $p_1^* \neq p_2^*$.

i) The consumer inspects the better product first if the reversed hazard rate $f_k(\theta)/F_k(\theta)$ *is strictly increasing in k.*

ii) The consumer inspects the better product second if the hazard rate $f_k(\theta)/\bar{F}_k(\theta)$ *is strictly decreasing in k and increasing in* θ. [24]

Let me explain lemma 2.2 on the basis of two distinct two-product examples which are illustrated in figure 2.1. The example on the LHS satisfies the conditions of *i*) in lemma 2.2, so that the better product is inspected first. The example on the RHS satisfies the conditions of *ii*) in lemma 2.2, so that the better product is inspected second.

[24] There exists an elegant interpretation of the latter condition. Suppose that the monopolist's demand for the two products is independent and described by the demand functions F_i. Suppose furthermore that hazard rates are increasing and that the monopolist faces equal marginal costs for both products. Then, the ordered hazard rate condition is a necessary and sufficient condition to guarantee that the monopolist sells the better product at a lower price under the assumption of an interior solution. Recall that the underlying reason is that hazard rates determine markups in a standard monopoly setup. Hence, the conditions which ensure that the better product is inspected second and sold at a lower price, are as well sufficient and necessary conditions which guarantee that the monopolist sells the better product at a lower price if the demand for the two products was independent.

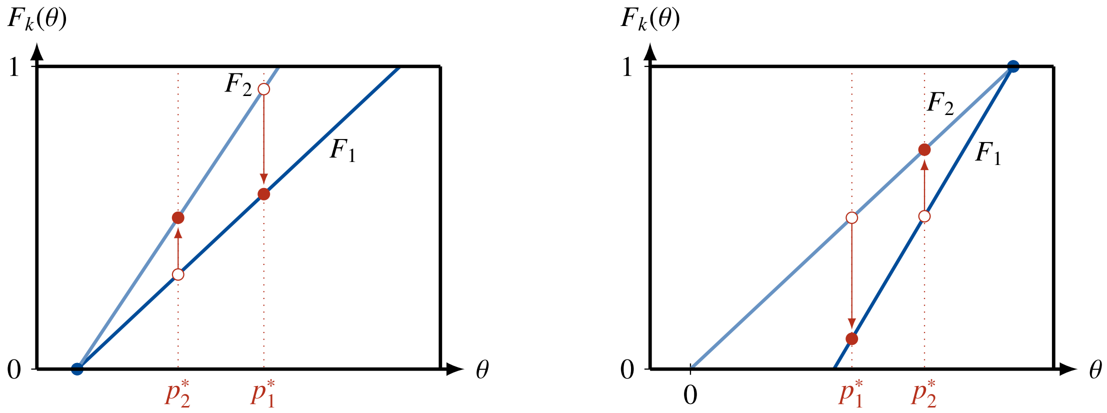

Figure 2.1: Uniform distributions & first order stochastic dominance

Both plots show the distribution function of two products, for which the consumer's valuations are uniformly distributed. In either example, product 1 is ex ante better than product 2. Equilibrium prices are shown on the horizontal axis and are indicated by full dots. The example shown on the LHS satisfies the conditions of *i*) in lemma 2.2, so that the better product is inspected first. The example shown on the RHS satisfies the conditions of *ii*) in lemma 2.2, so that the better product is inspected second. In either example, empty circles illustrate a particular non-equilibrium price profile for which arrows illustrate a profitable deviation of the monopolist, a switch of prices, which is at the heart of the proof of lemma 2.2. Note that in either example the product whose distribution function is flatter is inspected first.

Consider the example on the LHS of figure 2.1. Suppose that the better product was inspected second and hence sold at a lower price. For instance, as is indicated by empty circles, the true equilibrium prices were interchanged. Such a price profile would be part of a safety strategy which ensures that the consumer purchases the product which he inspects last with a high conditional probability. In contrast, after a switch of prices, indicated by arrows, the better product is sold at a higher price as part of a of long-shot strategy which increases the chances of selling the first inspected product at a high price. Whether this switch of prices – a switch from a safety strategy to a long-shot strategy – is profitable, intuitively, depends on whether $F_2(\theta) - F_1(\theta)$ is increasing or decreasing in θ.

The example on the LHS of figure 2.1 illustrates the case in which $F_2(\theta) - F_1(\theta)$ is sufficiently increasing such that a switch from a safety strategy to a long-shot strategy is profitable. The monopolist guides the consumer then to the better product first. More precisely, I find that $F_2(\theta) - F_1(\theta)$ is sufficiently increasing if F_2 is relatively more increasing than F_1: $F_2(\theta)/F_1(\theta)$ strictly increasing in θ.

The example on the RHS of figure 2.1 illustrates the opposite case. A switch from a long-shot strategy to a safety strategy is profitable if $F_2(\theta) - F_1(\theta)$ is sufficiently decreasing. This is the case if the reliability function $\tilde{F}_2$ is relatively more decreasing than its counterpart $\tilde{F}_1$: $\tilde{F}_2(\theta)/\tilde{F}_1(\theta)$ strictly increasing in θ. The monopolist guides the consumer then to the better product second.

The provided conditions imply, loosely speaking, that the distribution function of one product is relatively steeper. For this product, a decrease in its price results thus in a higher relative increase in probability that the consumer's valuations exceeds its price. As a consequence, it is more profitable to lower the price of the product whose distribution function is steeper such that this product is sold in equilibrium at a lower price.

Lemma 2.2 already points at results which are yet to come. Note that a product whose valuation distribution is flatter is a more polarizing product. An alternative interpretation of lemma 2.2 is thus that monopolist guides the consumer to the more polarizing product first.

Multiple products

In this subsection, I generalize the idea that those products whose valuation distributions are flatter are inspected first. First, I introduce the notion of a polarizing product which builds on a single-crossing property of several distribution functions. This single-crossing property has already appeared before in Courty and Li [2000] and is closely related to the concept of a (not necessarily mean-preserving) spread as proposed by Johnson and Myatt [2006] to describe demand rotations.[25]

Definition 2.7 (Single-Crossing Property). *A family of distribution functions $\{F_k\}_{k\in K}$ satisfies the single-crossing property if there exists a single-crossing $(\theta_{\text{cross}}, F_{\text{cross}})$ such that for $k < l$:*

$$\theta \gtreqless \theta_{\text{cross}} \Leftrightarrow F_k(\theta) \lesseqgtr F_l(\theta)$$

This means that for each $k < l$ the distribution F_l can be generated by a counter clockwise rotation of F_k, which is illustrated in figure 2.2.

[25]For a thorough discussion of the property and its relation to the single-crossing property by Diamond and Stiglitz [1974], I refer to the discussion in Johnson and Myatt [2006].

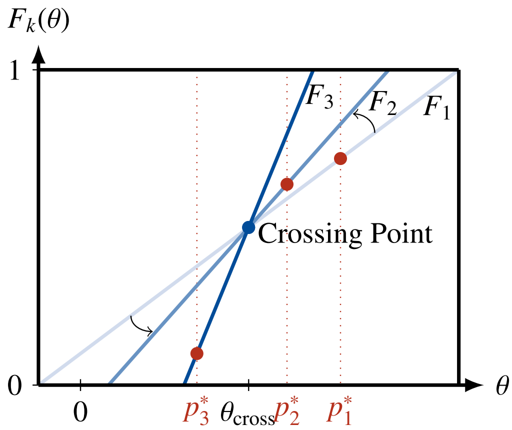

Figure 2.2: Uniform distributions

The figure shows three uniform distribution functions: $F_k(\theta_k) = F_{\text{cross}} + a_k(\theta_k - \theta_{\text{cross}})$, from which the consumer's valuations for three heterogeneous products are drawn. a_k is increasing in k, so that $\{F_k\}_{k\in K}$ satisfies the single-crossing property, which is indicated by arrows. This example satisfies the conditions of theorem 2.4 such that more polarizing products are inspected first and are thus more expensive.

Polarizing product. I refer to a product whose valuation distribution is flatter at the point of crossing as more polarizing.

Definition 2.8 (Polarizing product)**.** *Suppose $\{F_k\}_{k\in K}$ satisfies the single-crossing property. Then, product k is more polarizing than product l if $k < l$.*

This definition captures that for a more polarizing product very high and very low match-values are more likely to occur.

In the following theorem, I provide conditions which guarantee that polarizing products are inspecteded first.

Theorem 2.4. *Let $\{F_k\}_{k\in K}$ be a generic family of distribution functions which satisfies the single-crossing property. If the following two conditions hold:*

i) The reversed hazard rate $f_k(\theta)/F_k(\theta)$ is strictly increasing in k for $\theta \geq \theta_{\text{cross}}$,

ii) The hazard rate $f_k(\theta)/\tilde{F}_k(\theta)$ is strictly increasing in k and in θ for $\theta \leq \theta_{\text{cross}}$,

the monopolist guides the consumer to more polarizing products first.

The proof of theorem 2.4 consists of two steps, which I will outline in the following and which will provide intuition for the theorem. A discussion follows at the end of this subsection after an example.

The first step of the proof is to show that each product whose price exceeds θ_{cross} is more polarizing than each other product whose price is below θ_{cross}. The intuition is simple. Among two products the less polarizing product is better for prices below θ_{cross}, while the more polarizing one is better for prices above θ_{cross}. As a consequence, if the price of the less polarizing product was above θ_{cross} and the price of the more polarizing one below θ_{cross}, it would be profitable to switch the prices of the two products, and thus switch their positions in the consumer's search order.

The second step builds on the main ideas used to prove lemma 2.2. Consider only those products whose prices exceed θ_{cross}. Note that for prices above θ_{cross}, more polarizing products are better. If more polarizing products were not inspected first, then there would exist two products which have 'neighboring' positions in the consumer's search order and which satisfy that the less polarizing product is inspected first. Then, a more general version of lemma 2.2 is applicable, which considers pairwise switches of prices and positions in the consumer's search order for an arbitrary number of products.[26] Therefore, among those products whose prices exceed θ_{cross}, better ones, more polarizing ones, are inspected first if $f_k(\theta)/F_k(\theta)$ is strictly increasing in k for $\theta \geq \theta_{\text{cross}}$. Similarily, among those products whose prices are below θ_{cross}, more polarizing ones are inspected first. The reason is that for prices below θ_{cross}, less polarizing products are better and inspected last if $f_k(\theta)/\tilde{F}_k(\theta)$ is strictly increasing in k.

The following example describes a class of families of distribution functions which satisfy the conditions of theorem 2.4, so that the more polarizing products are inspected first.

Example cont': An arbitrary number of heterogeneous products with uniformly distributed match-values.

Consider a monopolist that sells an arbitrary, finite number of products, for which the consumer's valuations are uniformly distributed and satisfy the single-crossing property. Formally, let the family of valuation distributions $\{F_k\}_{k \in K}$ *be defined by* $F_k(\theta_k) = F_{\text{cross}} + a_k(\theta_k - \theta_{\text{cross}})$*, where* a_k *is strictly increasing in k. Special cases are illustrated in figure 2.2 and in figure 2.1. The latter example illustrates that the family encompasses distribution functions which are ordered by first order stochastic dominance. The family* $\{F_k\}_{k \in K}$ *satisfies the condi-*

[26] In fact, lemma 2.2 is proven as a corollary to this auxiliary lemma, which is relegated to the appendix.

tions of theorem 2.4: First, the reversed hazard rate is strictly increasing in k, since $F_k(\theta_k)$ *is strictly log-supermodular:* $\frac{d^2}{da_k\, d\theta_k} \log F_k(\theta_k) = \frac{F_{\text{cross}}}{[F_k(\theta_k)]^2} > 0$. *Second, the hazard rate is increasing in k, since* $\tilde{F}_k(\theta_k)$ *is strictly log-submodular:* $\frac{d^2}{da_k\, d\theta_k} \log \tilde{F}_k(\theta_k) = \frac{-(1-F_{\text{cross}})}{[\tilde{F}_k(\theta_k)]^2} < 0$. *Finally, the uniform distribution has increasing hazard rates in* θ*, since its probability density function is log-concave.*[27] *Consequently, theorem 2.4 applies and more polarizing products are thus unambiguously inspected first.*

The intuition for theorem 2.4 is that the monopolist attempts to first sell products at high prices as part of a long-shot strategy. In order to increase the likelihood that the consumer finds a product to be suitable despite its high price, the monopolist guides the consumer to polarizing products first. In contrast, the consumer's valuations for less polarizing products is less likely to be extreme. This guarantees the monopolist that she can sell less polarizing products at moderate prices with high probability. The monopolist guides the consumer hence to less polarizing product last as these products are particularly suited to be used as part of a last resort safety strategy, which ensures that the consumer purchases some product before leaving the store. The takeaway from this last subsection is that, maybe in contrast to a first guess, the consumer should be neither guided to the best products nor the worst products first, but to those products which are polarizing and which have a high variance in the consumer's valuation.

2.5 Conclusion

This paper has shown that a multi-product monopolist prefers not to support its customers in finding their preferred products. Instead, it is a profitable sales technique for the monopolist to impede the consumer's information acquisition through obfuscation in order to influence the consumer's search. The results hence suggest that informational frictions emerge endogenously if firms have market power. Although the associated search frictions create welfare losses, the overall effect of obfuscation on welfare may be positive due to an increase in mar-

[27] For instance, see Bagnoli and Bergstrom [2005].

ket coverage. In terms of policy implications, my analysis therefore questions the existence of a clear-cut rationale for policy interventions which aim to reduce obfuscation.

Chapter 3

Consideration Sets and Competitive Marketing: Corrigendum

3.1 Introduction

Eliaz and Spiegler [2011a] (henceforth, ES) proposed a model of competitive marketing when consumers have limited propensity to consider all feasible market alternatives. A key result in the paper (Proposition 2) stated that there always exists a symmetric equilibrium in which firms earn the max-min profit. This statement turns out to be incorrect, and in this corrigendum we provide a necessary condition on the "consideration function" for the existence of an equilibrium with max-min payoffs for any "admissible" cost structure. Interestingly, this condition is based on the well-known mathematical concept of a "Helly family". We illustrate that the necessary condition is not sufficient, and also that the sufficient condition provided in Proposition 4 of ES is not necessary.

In this corrigendum we also address another, more minor mistake in ES. When costs are sufficiently small, firms earn max-min payoffs in any symmetric equilibrium, for essentially any consideration function. Proposition 6 in ES provided a bound on costs, below which firms earn max-min payoffs. The proof that appeared in ES contained a mistake, and here we restate the result with a slightly modified bound.

We begin by recalling the model of ES, using slightly different notation that will be useful for our current objective. Let X be a finite set of products, and let D be a finite set of "marketing devices". Two firms facing a single consumer play the following simultaneous-move game with complete information. A pure strategy for a firm is a pair (x, M), where $x \in X$ and $M \subseteq D$. Let $c_x > 0$ and $c_m > 0$ denote the fixed costs of x and $m \in D$, respectively, and let $c(x, M) \equiv c_x + \sum_{m \in M} c_m$ be the fixed cost of (x, M). Faced with the strategy profile $(x^i, M^i)_{i=1,2}$, the consumer chooses according to a procedure based on two primitives: a strict preference relation $\succ$ on X and a *consideration function* $f : D \to 2^{X \setminus \{x^*\}}$, where x^* is the $\succ$-maximal element in X, and $f(m)$ is interpreted as the set of products from which m attracts attention. The consumer initially draws a firm i at random. He switches to firm j (and subsequently consumes x^j) if and only if (x^j, M^j) "beats" (x^i, M^i), which occurs whenever $x^i \in \cup_{m \in M^j} f(m)$ and $x^j \succ x^i$. Each firm tries to maximize its probability of being chosen minus the fixed cost of its strategy. We focus on symmetric Nash equilibria. Given an equilibrium σ let $Supp(\sigma)$ denote its support. Let $\beta_\sigma(x)$ denote the probability that x is played in σ, namely $\beta_\sigma(x) = \sum_M \sigma(x, M)$.

The following conditions are imposed on the primitives. First, $c_x \geq c_y$ whenever $x \succ y$, with a strict inequality when $x = x^*$. Second, $c(x^*, D) < \frac{1}{2}$. Third, $\cup_{m \in D} f(m) = X \setminus \{x^*\}$. These conditions imply that the max-min payoff in the game is $\frac{1}{2} - c_{x^*}$. This naturally raises the question of whether firms are able to earn payoffs above this level in symmetric Nash equilibrium. This is an important question, for two reasons. First, the max-min payoff is also the equilibrium payoff that firms earn if consumers are fully rational in the sense that they always consider the entire feasible set of market alternatives, independently of the firms' marketing strategies. Second, max-min equilibrium payoffs imply an interesting corollary regarding consumers' conversion rates on the equilibrium path, a property referred to as the Effective Marketing Property.

Proposition 2 in ES stated that for a tuple $(c, f, \succ)$ with the above properties, there exists a symmetric mixed-strategy Nash equilibrium in which firms earn max-min payoffs. However, the constructive proof of this claim failed to take into account certain deviations to pure strategies outside the support of the putative equilibrium strategy.

3.2 Necessary condition

For every non-empty $Y \subseteq X$, denote $f_Y(m) = f(m) \cap Y$. A collection $\{X^k\}_{k=1,...,K}$ of subsets of X is a Helly family, if whenever $X^k \cap X^{k'} \neq \emptyset$ holds for any $k, k' \in \{1, ..., K\}$, then $\cap_{k=1,...,K} X^k \neq \emptyset$.[1] We say that a consideration function f satisfies the **Helly** property if for every non-empty subset of marketing devices $\{m^1, ..., m^K\} \subseteq D$ and every non-empty subset of products $Y \subseteq X \backslash \{x^*\}$, the collection of subsets $\{f_Y(m^k)\}_{k=1,...,K}$ is a Helly family.

Proposition 3.1. *If f violates the Helly property, then the rational-consumer payoff $\frac{1}{2} - c_{x^*}$ is unsustainable in symmetric Nash equilibrium for generic permissible cost structures.*

***Proof*:** We construct a proof by contradiction that proceeds in three steps. Let σ be a symmetric Nash equilibrium.

***Step 1*:** If f violates the Helly property, then there exists a set of three marketing devices $M^3 = \{m^1, m^2, m^3\} \subseteq D$ and a set of three inferior products $X^3 = \{x^1, x^2, x^3\} \subseteq X \backslash \{x^*\}$ such that $\{f_{X^3}(m^k)\}_{k=1,2,3}$ is *not* a Helly family.

***Proof*:** If f violates the Helly property, then there is a set of marketing devices $M' \subseteq D$ and a set of products $Y' \subseteq X$ such that $f_{Y'}(m) \cap f_{Y'}(m') \neq \emptyset$ holds for every $m, m' \in M'$, but $\cap_{m \in M'} f_{Y'}(m) = \emptyset$. Among these pairs (M', Y'), select a pair (M, Y) with a minimal M — that is, there exists no (M', Y') as defined above such that $M' \subset M$. Therefore, $\cap_{m' \in M \backslash \{m\}} f_Y(m) \neq \emptyset$ for every $m \in M$. Clearly, $|M|, |Y| \geq 3$. Impose an arbitrary enumeration on M, such that $M = \{m^1, ..., m^K\}$, $K \geq 3$. By the minimality of M, for every $m^k \in M$ there is $x^k \in Y$ such that $x^k \notin f_Y(m^k)$ and $x^k \in \cap_{m \in M \backslash \{m^k\}} f_Y(m)$. Define $M^3 = \{m^1, m^2, m^3\} \subseteq M$ and $X^3 = \{x^1, x^2, x^3\}$. By definition, $f_{X^3}(m^1) = \{x^2, x^3\}$, $f_{X^3}(m^2) = \{x^1, x^3\}$ and $f_{X^3}(m^3) = \{x^1, x^2\}$, hence the collection $\{f_{X^3}(m^k)\}_{k=1,2,3}$ is not a Helly family. □

[1] See http://en.wikipedia.org/wiki/Helly_family.

Let $\alpha_\sigma(m)$ denote the probability that a marketing device m is played in σ, i.e.,

$$\alpha_\sigma(m) \equiv \sum_{(x,M)\in Supp(\sigma)|\, m\in M} \sigma(x, M).$$

***Step 2*:** For any $\epsilon > 0$, there exists a generic permissible cost structure such that for any symmetric Nash equilibrium strategy σ that induces max-min payoffs, $\alpha_\sigma(m) \leq \epsilon$ for every $m \notin M^3$.

***Proof*:** Denote $X_3^{\complement} = X\backslash(X^3 \cup \{x^*\})$, and denote $M_3^{\complement} = D\backslash M^3$. Let $c_{x^k} = \bar{c} + \epsilon_k$ for every $x^k \in X^3$, where $\epsilon_1 + \epsilon_2 + \epsilon_3 = 0$, $c_{x^*} > \bar{c} + \frac{1}{3} + \epsilon|D|$ and let $c_x \geq c_{x^*} - \epsilon/2$ for every $x \in X_3^{\complement}$, where $\epsilon > 0$. Set $c_{m^k} = \tilde{c} + \epsilon_k$ for every $m^k \in M^3$, and let $c_m > 3\tilde{c}$ for every $m \in M_3^{\complement}$. Clearly, $\epsilon, \epsilon_1, \epsilon_2, \epsilon_3$ must all be sufficiently small in order to ensure the cost structure is permissible, namely that $c(x^*, D) < \frac{1}{2}$ and that $c_{x^*} > c_x$ for all $x \neq x^*$.

Assume there exists a symmetric Nash equilibrium σ in which firms earn max-min payoffs. Assume that $\alpha_\sigma(m) > \epsilon$ for some $m \in D$. Suppose there exists $(x', M') \in Supp(\sigma)$ such that $x' \in f_{X_3^{\complement}}(m)$. By the Effective Marketing Property (Proposition 5 in ES), (x', M') is beaten by any $(x'', M'') \in Supp(\sigma)$ with $m \in M''$. Hence, by playing (x^*, M') instead of (x, M') a player would increase his market share by at least $\frac{1}{2}\alpha_\sigma(m)$, while increasing his cost by less than $\frac{1}{2}\epsilon < \alpha_\sigma(m)$. It follows that for any marketing device m with $\alpha_\sigma(m) > \epsilon$, the only products in $f(m)$ that are played with positive probability in σ are those in X^3. Suppose $\alpha_\sigma(m) > \epsilon$ for some $m \in M_3^{\complement}$ and consider some $(\hat{x}, \hat{M}) \in Supp(\sigma)$ for which $m \in \hat{M}$. Then, by switching from $(\hat{x}, \hat{M})$ to $(\hat{x}, (\hat{M}\backslash\{m\}) \cup M^3)$, a firm reduces its cost without lowering its market share. □

***Step 3*:** Firms earn more than $\frac{1}{2} - c_{x^*}$ in any symmetric Nash equilibrium for a generic permissible cost structure.

***Proof*:** Consider the cost structure assumed at the beginning of the proof of Step 2, and assume that there exists a symmetric Nash equilibrium strategy σ that induces max-min payoffs. Then, $(x^*, \emptyset)$ is a best response to σ. Thus, for any $x^k \in X^3$, the strategy $(x^k, \emptyset)$ cannot achieve a higher payoff against σ than $(x^*, \emptyset)$. This means that if a player switched from playing

$(x^*, \emptyset)$ against σ to playing $(x^k, \emptyset)$, the expected loss in market share would be weakly greater than the savings in costs. Therefore,

$$\frac{1}{2} \sum_{m \in M^3 \setminus \{m^k\}} \alpha_\sigma(m) + \frac{1}{2} \sum_{m \in M_3^\complement} \epsilon \geq c_{x^*} - \bar{c} - \epsilon_k$$

To see why this inequality holds, recall that by Step 2, only marketing devices in M^3 are chosen with a probability strictly greater than ϵ in any equilibrium with max-min payoffs. Therefore, the left hand side of the above inequality is an upper bound for the probability that a consumer's attention is attracted by the opponent's marketing strategy, and hence, the left side is an upper bound for the loss in market share. Summing up these inequalities over all $x^k \in X^3$ yields

$$\sum_{m^k \in M^3} \alpha_\sigma(m^k) + \frac{3}{2} \sum_{m \in M_3^\complement} \epsilon \geq 3(c_{x^*} - \bar{c})$$

Since $c_{x^*} - \bar{c} > \frac{1}{3} + \epsilon |D|$, it follows that $\sum_{m^k \in M^3} \alpha_\sigma(m^k) > 1 + \frac{3}{2}\epsilon |D|$. This means that there exists some M with $\sum_{x \in X | M \subseteq M'} \sigma(x, M') > \epsilon$ and $\left| M \cap M^3 \right| \geq 2$ - otherwise, the sum of all elements in $Supp(\sigma)$ would be strictly greater than one, a contradiction. Assume w.l.o.g. that $\{m^1, m^2\} \subseteq M$. Recall that in the proof of Step 2, we showed that for any marketing device m with $\alpha_\sigma(m) > \epsilon$, the only products in $f(m)$ that are played with positive probability in σ are those in X^3. Since $f_{X^3}(m^1) \cup f_{X^3}(m^2) = X^3$, it follows that $M = \{m^1, m^2\}$ - otherwise, a firm could deviate from (x, M) to $(x, \{m^1, m^2\})$ and lower its cost without lowering its market share.

As firms earn by assumption max-min payoffs, and $x^3 \in f(m^1)$ and $x^3 \in f(m^2)$ holds, Proposition 3 in ES implies that x^3 is not played at all in σ. But this means that a firm could deviate from $(x, \{m^1, m^2\})$ to $(x, \{m^3\})$ and lower its cost without lowering its market share. Hence $(x, \{m^1, m^2\})$ is not a best-reply to σ, a contradiction.■

Let us now illustrate that the necessary condition for max-min equilibrium payoffs is not sufficient, and that the sufficient condition provided by Proposition 4 in ES, namely that the consideration function is partitional, is not necessary.

Necessary condition is not sufficient

Let $X = \{x^1, \ldots, x^K\} \cup \{x^*\}$ and $D = \{m^1, \ldots, m^K\}$. Define the consideration function f_K as follows:

$$f_K(m^k) = \{x^{k \bmod K}, x^{(k+1) \bmod K}\} \tag{3.1}$$

Note that f_3 is an example of a consideration function that violates the Helly property. Hence, symmetric equilibrium profits exceed the max-min for generic permissible cost structures.

This example also illustrates the non-monotonicity of equilibrium profits with respect to consumer attention. Fix $X = \{x^1, x^2, x^3\} \cup \{x^*\}$ and $D = \{m^1, m^2, m^3\}$. As pointed out in ES, one could imagine a scale that measures consumers' resistance to considering new alternatives. At one end of the scale is the rational consideration function $f_R(m) = \{x^1, x^2, x^3\}$ for all $m \in D$, and at the other end of the scale there is the partitional consideration function f_P defined by $f_P(m^k) = \{x^k\}$. In both cases, symmetric equilibrium profits are equal to the max-min. The function f_3 is "in between" these two extremes (in terms of the consumer's propensity to consider new alternatives), and yet it induces equilibrium payoffs above the max-min for generic permissible cost structures.

Now consider f_5. This consideration function vacuously satisfies the Helly property, and yet it can be shown that symmetric equilibrium payoffs must exceed the max-min for generic permissible cost structures, using a construction similar to that in the proof of Proposition 3.1.

Sufficient condition is not necessary

Proposition 4 in ES shows that if $\{f(m)\}_{m \in M}$ is a partition of $X \backslash \{x^*\}$, then firms earn max-min payoffs in every symmetric Nash equilibrium. Consider the following specification of the model. Let $X \backslash \{x^*\} = \{1, 2, 3, 4\}$, $D = \{m_1, m_2, m_3\}$, $f(m_k) = \{k, 4\}$ for every $k = 1, 2, 3$. Assume $x \succ 4$ for every $x \neq 4$. The consideration function is clearly non-partitional, but it satisfies the Helly property. Let us now show that firms earn the max-min in every symmetric Nash equilibrium, for any permissible cost structure.

Fix a symmetric equilibrium strategy σ. By Lemma 1 in ES, $\beta_\sigma(x^*) > 0$. If $(x^*, \varnothing) \in$

$Supp(\sigma)$, we are done. Suppose $(x^*, \emptyset) \notin Supp(\sigma)$. If $\beta_\sigma(4) > 0$, then 4 is beaten by every $(x^*, M) \in Supp(\sigma)$. Since 4 is the $\succ$-minimal product, $(4, \emptyset) \in Supp(\sigma)$ and this alternative does not beat any element in $Supp(\sigma)$. It follows that a firm deviates from $(4,\emptyset)$ to (x^*, D), it increases its market share by at least $\frac{1}{2}\beta_\sigma(x^*) + \frac{1}{2}(1 - \beta_\sigma(x^*)) = \frac{1}{2} > c(x^*, D) - c(4, \emptyset)$, hence the deviation is profitable. Therefore, $\beta_\sigma(4) = 0$. But this means that the equilibrium must be the same as if 4 were eliminated from X, in which case f would be partitional. By Proposition 4 in ES, symmetric equilibrium payoffs in this case are equal to the max-min.

3.3 Max-min payoffs for sufficiently small costs

Proposition 6 of ES stated that if $c(x^*, D) < 1/(2^{|D|} + 2)$, then firms would earn the max-min payoffs in any symmetric equilibrium. The proof pointed out that if firms earn above the max-min payoff at some equilibrium σ, then any $(x^*, M) \in Supp(\sigma)$ must beat some $(x', M') \in Supp(\sigma)$. Since (x', M') is a best-reply to σ, it cannot be profitable to deviate from (x', M') to (x^*, M). In ES, we translated this observation to the following inequality:

$$\frac{1}{2}\sigma(x^*, M) + \frac{1}{2}\sum_{x \prec x^*}\beta_\sigma(x) \leq c(x^*, M) - c(x', M')$$

This inequality, however, is incorrect because it ignores the possibility that some strategies are beaten by *both* (x^*, M) *and* (x', M').

The following result is a restatement of Proposition 6 in ES, with a slightly lower upper bound on costs. When costs are below this bound, firms earn the max-min payoff in any symmetric equilibrium.

Proposition 3.2. *Let m^* be the most costly marketing device. If*

$$(2^{|D|} - 1) \cdot c_{x^*} + (|X| - 1) \cdot c_{m^*} < \frac{1}{2} \tag{3.2}$$

then firms earn the rational-consumer payoff in any symmetric Nash equilibrium.

***Proof*:** Assume (3.2) holds. Let σ be a symmetric Nash equilibrium in which firms earn above the max-min payoff. By Step 1 of Proposition 3.1, for every $(x, M) \in Supp(\sigma)$ with $x \neq x^*$, there exists $(x^*, M') \in Supp(\sigma)$ such that (x^*, M') does not beat (x, M). By assumption (P2) in ES, there exists some $m(x) \in D$ such that $x \in f(m)$ and hence, $(x^*, M' \cup \{m(x)\})$ would beat (x, M). Since $(x^*, M') \in Supp(\sigma)$, it follows that

$$\frac{1}{2}\beta_\sigma(x) \leq c_{m(x)}$$

since otherwise, it would be strictly profitable to deviate from (x^*, M') to $(x^*, M' \cup \{m(x)\})$. Summing these inequalities over all $x \prec x^*$ yields:

$$\frac{1}{2}\sum_{x \prec x^*}\beta_\sigma(x) \leq \sum_{x \prec x^*} c_{m(x)} \leq (|X| - 1)c_{m^*} \tag{3.3}$$

Let $A(x^*, M)$ denote the set of strategies $(x', M') \in Supp(\sigma)$ that are beaten by (x^*, M). Let $a(x', M') \in A(x^*, M)$. Because firms earn above the max-min payoffs, $A(x^*, M) \neq \emptyset$ for *all* $(x^*, M) \in Supp(\sigma)$. In addition, for each $(x^*, M) \in Supp(\sigma)$, it is not profitable to deviate from any $a(x^*, M)$ to (x^*, M), hence

$$\frac{1}{2}\sigma(x^*, M) \leq c_{x^*} - c_{a(x^*,M)}$$

Since by assumption, $(x^*, \emptyset) \notin Supp(\sigma)$ (firms earn above max-min payoffs), summing over all strategies $(x^*, M) \in Supp(\sigma)$ we obtain

$$\frac{1}{2}\beta_\sigma(x^*) < (2^{|D|} - 1) \cdot c_{x^*} \tag{3.4}$$

Summing (3.3) and (3.4) yields:

$$\frac{1}{2} < (2^{|D|} - 1) \cdot c_{x^*} + (|X| - 1) \cdot c_{m^*}$$

a contradiction. ■

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Appendices

A1 Chapter 1: Formal results

***Proof of lemma 1.1*:** All arguments are given in the text.

Lemma A1.1 ($\boldsymbol{\theta_{\text{ind}}}$). *Let $\theta_{\text{ind}} := \mu \int_0^{\bar{\varepsilon}} \varepsilon f(\varepsilon) d\epsilon / c$. Then, $\theta_{\text{S}} = \min\left\{\theta_{\text{I}}, \theta_{\text{ind}}\right\}$ if $\theta_{\text{S}} \geq \underline{\theta}$. Furthermore, θ_{ind} is strictly increasing in μ and strictly decreasing in c.*

***Proof of lemma A1.1*:** Let me show first that $\theta_{\text{ind}} \in \mathbb{R}_+$ is the unique, hypothetical consumer type that is indifferent between immediate purchase and inspect. Thus, θ_{ind} is implicitly defined by $U_{\text{S}}(\theta_{\text{ind}}, p^*) \overset{!}{=} U_{\text{I}}(p^*)$, and therefore, by $\tilde{x}(\theta_{\text{ind}}) \overset{!}{=} 0$. I find $\theta_{\text{ind}} = \mu g(0)/c$, which implies $\theta_{\text{ind}} = \mu \int_0^{\bar{\varepsilon}} \varepsilon f(\varepsilon) d\epsilon / c$, as $g(x) = \int_x^{\bar{\varepsilon}} (\varepsilon - x) f(\varepsilon) d\epsilon$. Obviously, θ_{ind} is strictly increasing in μ and strictly decreasing in c, which is used repeatedly in the following. $\boldsymbol{\theta_{\text{S}} = \min\left\{\theta_{\text{I}}, \theta_{\text{ind}}\right\}}$**:** Assume that some consumers inspect products such that $\theta_{\text{S}} \geq \underline{\theta}$. Then, either $\theta_{\text{S}} = \theta_{\text{I}}$ or $\theta_{\text{S}} < \theta_{\text{I}}$, as $\theta_{\text{S}} \leq \theta_{\text{I}}$. If $\theta_{\text{S}} < \theta_{\text{I}}$, the consumer type θ_{S} must be indifferent between immediate purchase and inspect. Thus, $\theta_{\text{S}} = \theta_{\text{ind}}$ if some consumer purchase immediately such that $\theta_{\text{S}} < \theta_{\text{I}}$. ■

***Proof of lemma 1.2*:** Let me first provide the missing step in the derivation of the candidate equilibrium price. Differentiation of equation 1.4 with respect to p yields

$$\frac{d}{dp}\pi(p, p^*) = H(\theta_{\text{I}}^*) - H(\theta_{\text{S}}^*) + \int_{\underline{\theta}}^{\theta_{\text{S}}^*} \frac{h(\theta)}{\tilde{F}(\tilde{x}(\theta))} \left\{ \tilde{F}\Big(\tilde{x}(\theta) + \frac{p - p^*}{\mu}\Big) - \frac{p}{\mu} f\Big(\tilde{x}(\theta) + \frac{p - p^*}{\mu}\Big) \right\} d\theta.$$

Imposing symmetry, one obtains the following first order condition:

$$0 \overset{!}{=} H(\theta_I^*) - \int_{\underline{\theta}}^{\theta_S^*} \frac{h(\theta)}{\tilde{F}(\tilde{x}(\theta))} \frac{p^*}{\mu} f\left(\tilde{x}(\theta)\right) d\theta,$$

which yields the unique candidate equilibrium price after substitution of $\theta_S^* = \min\left\{\theta_I^*, \theta_{\text{ind}}\right\}$, which holds by lemma A1.1. □

***The comparative statics of $\tilde{p}$*:** Define $\tilde{\varphi}(\theta) := \varphi(\tilde{x}(\theta))$.

***i) The effect on an increase in product diversity μ*:**

$\theta_S^* < \theta_I^*$**:** If some consumer purchase immediately: $\theta_S^* < \theta_I^*$, $\theta_S^* = \theta_{\text{ind}} < \theta_I^*$ follows by lemma A1.1. I show that then $\frac{d}{d\mu}\frac{1}{\tilde{p}(\mu,c,\theta_I^*)} > 0$.

$$\frac{d}{d\mu}\frac{1}{\tilde{p}(\mu, c, \theta_I^*)} = \frac{1}{\mu H(\theta_I^*)}\left\{\frac{-1}{\mu}\int_{\underline{\theta}}^{\theta_{\text{ind}}} h(\theta)\,\tilde{\varphi}(\theta)\,d\theta + \frac{d\theta_{\text{ind}}}{d\mu} h(\theta_{\text{ind}})\,\tilde{\varphi}(\theta_{\text{ind}}) + \int_{\underline{\theta}}^{\theta_{\text{ind}}} h(\theta)\,\frac{d}{d\mu}\varphi(\tilde{x}(\theta))\,d\theta\right\}.$$

Rewriting the derivative of the hazard rate with respect to product diversity yields[2]

$$\frac{d}{d\mu}\frac{1}{\tilde{p}(\mu, c, \theta_I^*)} = \frac{1}{\mu^2 H(\theta_I^*)}\left\{-\int_{\underline{\theta}}^{\theta_{\text{ind}}} h(\theta)\,\tilde{\varphi}(\theta)\,d\theta + \theta_{\text{ind}}\,h(\theta_{\text{ind}})\,\tilde{\varphi}(\theta_{\text{ind}}) - \int_{\underline{\theta}}^{\theta_{\text{ind}}} \theta\,h(\theta)\frac{d}{d\theta}\tilde{\varphi}(\theta)\,d\theta\right\}.$$

A partial integration of the last summand yields

$$\frac{d}{d\mu}\frac{1}{\tilde{p}(\mu, c, \theta_I^*)} = \frac{1}{\mu^2 H(\theta_I^*)}\left\{\underline{\theta}\,h(\underline{\theta})\,\tilde{\varphi}(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_{\text{ind}}} \theta\,h'(\theta)\,\tilde{\varphi}(\theta)\,d\theta\right\}.$$

The first summand is strictly positive. The second is zero for the uniform distribution. □

$\theta_S^* = \theta_I^*$**:** If no consumer purchases immediately: $\theta_S^* = \theta_I^*$, $\theta_{\text{ind}} \geq \theta_I^*$ follows by lemma A1.1. I show that then $\frac{d}{d\mu}\frac{1}{\tilde{p}(\mu,c,\theta_I^*)} < 0$.

$$\frac{d}{d\mu}\frac{1}{\tilde{p}(\mu, c, \theta_I^*)} = \frac{1}{\mu H(\theta_I^*)}\left\{\frac{-1}{\mu}\int_{\underline{\theta}}^{\theta_I^*} h(\theta)\,\tilde{\varphi}(\theta)\,d\theta + \int_{\underline{\theta}}^{\theta_I^*} h(\theta)\,\frac{d}{d\mu}\varphi(\tilde{x}(\theta))\,d\theta\right\}.$$

[2] $\frac{d}{d\mu}\varphi(\tilde{x}(\theta)) = -\frac{\theta}{\mu}\frac{d}{d\theta}\tilde{\varphi}(\theta)$

A partial integration of the last summand, after a substitution of $\frac{d}{d\mu}\varphi(\tilde{x}(\theta)) = -\frac{\theta}{\mu}\frac{d}{d\theta}\tilde{\varphi}(\theta)$, yields

$$\frac{d}{d\mu}\frac{1}{\tilde{p}(\mu,c,\theta_{\mathrm{I}}^{*})} = -\frac{1}{\mu^{2}H(\theta_{\mathrm{I}}^{*})}\,\theta\, h(\theta)\,\tilde{\varphi}(\theta)\Big|_{\underline{\theta}}^{\theta_{\mathrm{I}}^{*}},$$

since $h'(\theta) = 0$. Then, a sufficient condition for $\frac{d}{d\mu}\frac{1}{\tilde{p}(\mu,c,\theta_{\mathrm{I}}^{*})} < 0$ is $\frac{d}{d\theta}\,\theta\,\tilde{\varphi}(\theta) \geq 0$ for every $\underline{\theta} < \theta \leq \theta_{\mathrm{I}}^{*}$. A substitution of $\theta = \frac{\mu}{c}\,(\tilde{x}(\theta))$ yields

$$\frac{d}{d\theta}\theta\tilde{\varphi}(\theta) = \frac{\partial\tilde{x}(\theta)}{\partial\theta}\frac{\partial}{\partial\tilde{x}(\theta)}\left\{\frac{\mu}{c}\, g(\tilde{x}(\theta))\,\varphi(\tilde{x}(\theta))\right\}.$$

Simple analysis shows that $\tilde{F} = -g'$. This implies

$$\frac{d}{d\theta}\theta\tilde{\varphi}(\theta) = \frac{\mu}{c}\frac{\partial\tilde{x}(\theta)}{\partial\theta}\frac{\partial}{\partial\tilde{x}(\theta)}\left\{\frac{-g(\tilde{x}(\theta))}{g'(\tilde{x}(\theta))}\, f(\tilde{x}(\theta))\right\}.$$

Recall that consumers with higher opportunity costs have lower reservation values: $\partial\tilde{x}(\theta)/\partial\theta < 0$. Thus, $\frac{\partial}{\partial\tilde{x}(\theta)}\left\{\frac{-g(\tilde{x}(\theta))}{g'(\tilde{x}(\theta))} f(\tilde{x}(\theta))\right\} \leq 0$ completes the proof. The "first" term of the derivative is negative, since $f(\tilde{x}(\theta))$ is strictly positive, and the function g is log-concave, which follows from simple analysis. The "second" term of the derivate is negative if $f'(\tilde{x}(\theta)) \leq 0$, since $-g(\tilde{x}(\theta))/g'(\tilde{x}(\theta)) > 0$ is positive. As $\theta \leq \theta_{\mathrm{I}}^{*}$ and $\theta_{\mathrm{I}}^{*} \leq \theta_{\mathrm{ind}}$, $\theta \leq \theta_{\mathrm{ind}}$. By the definition of θ_{ind}, this implies $\tilde{x}(\theta) \geq 0$. Since f is symmetric and log-concave, $f'(x) \leq 0$ for $x \geq 0$. Thus, $f'(\tilde{x}(\theta)) \leq 0$ for $\theta \leq \theta_{\mathrm{I}}^{*}$. □

***ii) The effect on an increase in search costs c*:** Suppose $\theta_{\mathrm{ind}} < \theta_{\mathrm{I}}^{*}$. If $\theta_{\mathrm{ind}} \geq \theta_{\mathrm{I}}^{*}$, then the first term in the derivative vanishes and the comparative statics remain unaffected.

$$\begin{aligned}\frac{d}{dc}\frac{1}{\tilde{p}(\mu,c,\theta_{\mathrm{I}}^{*})} &= \frac{1}{\mu\, H(\theta_{\mathrm{I}}^{*})}\left\{\frac{d\theta_{\mathrm{ind}}}{dc}\, h(\theta_{\mathrm{ind}})\,\tilde{\varphi}(\theta_{\mathrm{ind}}) + \int_{\underline{\theta}}^{\theta_{\mathrm{ind}}}\frac{d}{dc}\,\tilde{\varphi}(\theta)\, h(\theta)\, d\theta\right\}\\ &= \frac{1}{\mu\, H(\theta_{\mathrm{I}}^{*})}\left\{-\frac{\mu}{c^{2}} g(0)\, h(\theta_{\mathrm{ind}})\,\tilde{\varphi}(\theta_{\mathrm{ind}}) + \int_{\underline{\theta}}^{\theta_{\mathrm{ind}}}\frac{d\varphi(\tilde{x}(\theta))}{d\tilde{x}(\theta)}\frac{d\tilde{x}(\theta)}{dc}\, h(\theta)\, d\theta\right\}\end{aligned}$$

Obviously, the first summand is strictly negative. The second summand is strictly negative, since the hazard rate φ is strictly increasing by assumption and since the reservation value

$\tilde{x}(\theta)$ is strictly decreasing in search costs by equation (1.2). □

iii) The effect on an increase in θ_I^*: If $\theta_{\text{ind}} \leq \theta_{\text{I}}^*$, then $\theta_{\text{S}}^* = \theta_{\text{I}}^*$. Then, the price is obviously increasing in θ_{I}^*, as more consumers purchase immediately. If $\theta_{\text{ind}} > \theta_{\text{I}}^*$, then $\theta_{\text{S}}^* = \theta_{\text{ind}}$. Then, $\tilde{p}$ is the inverse of the weighted average of hazard rates evaluated at the consumer's reservation match-values $\tilde{x}(\theta)$. As consumers with higher opportunity costs have lower reservation values, $\tilde{p}$ is strictly increasing in θ_{I}^*, due to the assumption of increasing hazard rates. ■

***Proof of proposition 1.1*:** Obviously, market prices increase in comparison to the market outcome, when consumers may not purchase products poorly informed, as immediate-pruchasers would otherwise either exit or inspect products, which both results in demand to become more elastic.

Let $\mu_{\text{A}} := \underline{\theta} c/g(0)$ such that $\theta_{\text{ind}}(\mu_{\text{A}}, c) = \underline{\theta}$; let $\mu_{\text{C}} := c/g(0)$ such that $\theta_{\text{ind}}(\mu_{\text{C}}, c) = 1$; define μ_{B} as the unique solution to $\tilde{p}(\mu_{\text{B}}, c, 1) \overset{!}{=} v$ that satisfies $\mu_{\text{B}} < \mu_{\text{C}}$. Uniqueness and existence is shown in part *ii*) of the proof.

In order to show that the triple θ_{S}^*, θ_{I}^*, p^* is a market equilibrium in which trade occurs, it suffices to show first, that $\theta_{\text{S}}^* = \min\left\{\theta_{\text{I}}^*, \theta_{\text{ind}}\right\}$ by lemma A1.1, and second, that $p^* = \tilde{p}(\mu, c, \theta_{\text{I}}^*)$ by lemma 1.2, and third, that θ_{I}^* is optimal given p^*.

***i) Market Failure*:** If $\mu < \mu_{\text{A}}$, then $\theta_{\text{ind}} < \underline{\theta}$, and no consumer prefers inspect to immediate purchase. Then, there does not exist a non-trivial equilibrium, as otherwise, if trade occurred, the firms' demand is perfectly inelastic.[3] Thus, only trivial equilibria exist, in which all consumers exit, and $p^* \geq v$ holds, so that no consumers strictly prefers to purchase a product immediately. □

***ii) Partial Participation Regime*:** Define $q(\mu, c, \theta) := \mu/\tilde{\varphi}(\theta)$. This means that $q(\mu, c, \theta)$ is the symmetric equilibrium price if only consumers of type θ participated and inspected products.

***Existence*:** The proof proceeds in four steps.

***Step 1*:** If $\mu \in (\mu_{\text{A}}, \mu_{\text{C}}]$, then $\tilde{p}(\mu, c, \theta_{\text{ind}}) < v$ holds.

[3]For $\mu = \mu_{\text{A}}$ there does not exist a non-trivial symmetric equilibrium in pure strategies, but in mixed strategies, in which only consumer $\underline{\theta}$ participates and randomizes between search and immediate purchase.

***Proof*:** If $\mu \in (\mu_{\mathrm{A}}, \mu_{\mathrm{C}}]$, then the indifferent consumer satisfies $\theta_{\mathrm{ind}} \in (\underline{\theta}, 1]$ by definition. Furthermore, $\tilde{p}(\mu, c, \theta_{\mathrm{ind}}) < q(\mu, c, \theta_{\mathrm{ind}})$ holds for any μ, since $\tilde{x}$ is decreasing in θ and hazard rates are increasing. Moreover, $q(\mu, c, \theta_{\mathrm{ind}}) = \mu/\varphi(0) < v$, since the reservation match-value of the indifferent consumer is zero and $\mu/\varphi(0) < v$ by assumption. □

***Step 2*:** There exists an unique $\mu_{\mathrm{B}} \in (\mu_{\mathrm{A}}, \mu_{\mathrm{C}})$ such that $v = \tilde{p}(\mu_{\mathrm{B}}, c, 1)$. Furthermore, if $\mu \in (\mu_{\mathrm{A}}, \mu_{\mathrm{B}}]$, then $\tilde{p}(\mu, c, 1) \geq v$.

***Proof*:** Proof by the intermediate value theorem. First, consider $\tilde{p}(\mu_{\mathrm{C}}, c, 1)$, the candidate equilibrium price if all consumers participate. By definition, $\theta_{\mathrm{ind}}(\mu_{\mathrm{C}}, c) = 1$ holds. Thus, $\tilde{p}(\mu_{\mathrm{C}}, c, 1) = \tilde{p}(\mu_{\mathrm{C}}, c, \theta_{\mathrm{ind}}) < v$ by step 1. Second, $\lim_{\mu \to \mu_{\mathrm{A}}^{+}} \tilde{p}(\mu, c, 1) = \infty$, since then the mass of shoppers vanishes and $\varphi(0)$ is bounded. Since, $\tilde{p}$ is continuous, the intermediate value theorem applies, and the existence of some $\mu_{\mathrm{B}} \in (\mu_{\mathrm{A}}, \mu_{\mathrm{C}})$ that satisfies $v = \tilde{p}(\mu_{\mathrm{B}}, c, 1)$ follows. Furthermore, as $\tilde{p}(\mu, c, 1)$ is strictly decreasing in μ for $\mu < \mu_{\mathrm{C}}$ by lemma 1.2, μ_{B} is unique and $\tilde{p}(\mu, c, 1) \geq v$ holds if $\mu \in (\mu_{\mathrm{A}}, \mu_{\mathrm{B}}]$. □

***Step 3*:** For $\mu \in (\mu_{\mathrm{A}}, \mu_{\mathrm{B}}]$ there exists $\theta_{\mathrm{I}}^{*} \in (\theta_{\mathrm{ind}}, 1]$ that solves $\tilde{p}(\mu, c, \theta_{\mathrm{I}}^{*}) = v$.

***Proof*:** Proof by the intermediate value theorem. If $\mu \in (\mu_{\mathrm{A}}, \mu_{\mathrm{B}}]$, then $\tilde{p}(\mu, c, \theta_{\mathrm{ind}}) < v$ by step 1 and $\tilde{p}(\mu, c, 1) \geq v$ by step 2. By continuity and monotonicity of $\tilde{p}$ in θ_{I}, there exists an unique θ_{I}^{*} that solves $v = \tilde{p}(\mu, c, \theta_{\mathrm{I}}^{*})$. □

***Step 4*:** For $\mu \in (\mu_{\mathrm{A}}, \mu_{\mathrm{B}}]$, θ_{I}^{*}, as defined in step 3, $\theta_{\mathrm{S}}^{*} = \theta_{\mathrm{ind}}$ and $p^{*} = v$ describe a market equilibrium.

***Proof*:** Any $\theta_{\mathrm{I}}^{*} \in [\theta_{\mathrm{ind}}, 1]$ is optimal if $p^{*} = v$; $\theta_{\mathrm{S}}^{*} = \theta_{\mathrm{ind}} < \theta_{\mathrm{I}}^{*}$ by step 3, such that $\theta_{\mathrm{S}}^{*} = \min\left\{\theta_{\mathrm{I}}^{*}, \theta_{\mathrm{ind}}\right\}$; $p^{*} = \tilde{p}(\mu, c, \theta_{\mathrm{I}}^{*})$ by step 3. □

***Uniqueness*:** Proof by contradiction. First, assume that there exists an equilibrium with $p^{*} < v$. Then, all consumers strictly prefer to participate, so that $\theta_{\mathrm{I}}^{*} = 1$. Since $p^{*} = \tilde{p}(\mu, c, 1)$ must hold, it follows, by step 2, that $p^{*} \geq v$ - a contradiction. Second, assume that there exists an equilibrium with $p^{*} > v$. Then, no consumer buys immediately. Hence, $\theta_{\mathrm{I}}^{*} \leq \theta_{\mathrm{ind}}$ holds, which implies $p^{*} = \tilde{p}(\mu, c, \theta_{\mathrm{I}}^{*}) \leq \tilde{p}(\mu, c, \theta_{\mathrm{ind}}) < v$ by lemma 1.2 and step 1. A contradiction to $p^{*} > v$. □

***iii) Full Participation Regime*:** By definition of μ_B, $\tilde{p}(\mu_B, c, 1) = v$. Furthermore, by definition of μ_C, $\theta_{ind} < 1$ for $\mu < \mu_C$. Then, by lemma 1.2, $\tilde{p}(\mu, c, 1)$ is decreasing in μ on (μ_B, μ_C). Jointly with $\tilde{p}$ increasing in θ_I, this implies $\tilde{p}(\mu, c, \theta_I) < v$ for any $\theta_I \leq 1$ and $\mu \in (\mu_B, \mu_C)$. Consequently, in any non-trivial equilibrium $\theta_I^* = 1$. Thus, $\theta_I^* = 1$, $p^* = \tilde{p}(\mu, c, 1)$ and $\theta_S^* = \theta_{ind}$ characterize the unique market equilibrium, as $\theta_{ind} < \theta_I^*$ such that $\theta_S^* = \min\left\{\theta_I^*, \theta_{ind}\right\}$ for $\mu \in (\mu_B, \mu_C)$. □

***iv) Search Regime*:** If $\mu \geq \mu_C$, then $\theta_{ind} \geq 1$ by the definition of μ_C. This means that all consumers who purchase a product prefer inspect to immediate purchase. Then, all consumers must participate in equilibrium, since for any $\theta_I \leq 1$ and any $\mu > \mu_C$, $\tilde{p}(\mu, c, \theta_I) \leq \tilde{p}(\mu, c, 1) \leq q(\mu, c, 1) \leq q(\mu, c, \theta_{ind}) = \frac{\mu}{\varphi(0)} < v$, where the last inequality holds by assumption and previous inequalities by lemma 1.2. Thus, $\theta_I^* = 1$, $p^* = \tilde{p}(\mu, c, 1)$ and $\theta_S^* = \theta_{ind}$ characterize the unique market equilibrium. ■

***Proof of proposition 1.2*:** In the partial participation regime the market price is v by proposition 1.1, and consumer participation adjusts such that $v = \tilde{p}(\mu, c, \theta_I^*)$ holds. If θ_I^* was constant, then the market price would be decreasing in product diversity by lemma 1.2, as some consumers purchase immediately. Thus, in order for p^* to remain constant, θ_I^* has to be increasing in product diversity, as $\tilde{p}$ is increasing in θ_I. It follows that $\pi^*(\mu, c, v) = H(\theta_I^*)\, p^*$ is increasing in product diversity. Analogously, one finds that θ^* is decreasing in c and increasing v.

In the full participation and search regime, $\theta_I^* = 1$ holds. However, only in the full participation regime some consumer purchase immediately. Hence, in the full participation regime p^* is unambiguously decreasing in μ and increasing in c by lemma 1.2. In the search regime, all consumers inspect products. Then, p^* is increasing in c and increasing in μ by lemma 1.2. ■

***Proof of lemma 1.3*:** All arguments are given in the text. ■

Lemma A1.2 ($\hat{\theta}_{ind}$). *Define $\hat{\theta}_{ind}(s, p^*(s), s^*, p^*(s^*))$ by $U_S(\hat{\theta}_{ind}, s, s^*, p^*) \stackrel{!}{=} U_I(s, p^*)$ such that it denotes the indifferent consumer type that is indifferent between immediate purchase and* *inspect upon observing s.*[4] *Then,* $\theta_{\mathrm{S}}(s) = \min\{\hat{\theta}_{\mathrm{ind}}(s, p^*(s), s^*, p^*(s^*)), \theta_{\mathrm{I}}(s)\}$ *if* $\theta_{\mathrm{S}}(s) \geq \underline{\theta}$*. Furthermore,* $\hat{\theta}_{\mathrm{ind}}(s, p^*(s), s^*, p^*(s^*))$ *is strictly increasing in* $p^*(s)$ *and strictly decreasing in* s*, holding* $p^*(s)$ *fixed.*

***Proof of lemma A1.2*:** The proof of $\theta_S(s) = \min\{\hat{\theta}_{ind}(s, p^*(s), s^*, p^*(s^*)), \theta_I(s)\}$ if $\theta_S(s) \geq \underline{\theta}$ follows along the same lines as the corresponding result in section 1.3 and is hence omitted.

$\hat{\theta}_{\mathrm{ind}}$ ***strictly increasing in*** $p^*(s)$**:** Suppose $s \neq s^*$ and $\hat{\theta}_{\mathrm{ind}} > 0$.[5] Consider an increase in the expected price $p^*(s)$. Then, the utility of immediate purchase U_{I} decreases more than the expected utility of inspecting the product U_{S}. This is the case, as a consumer that purchases the firm's product immediately pays the higher price with probability one, while, to the contrary, a consumer that inspects the firm's product only pays the higher price if he chooses to purchase the firm's product after inspection, which the consumer expects to occur with a probability strictly lower than one. This means that inspecting becomes in comparison to immediate purchase more attractive such that more consumers prefer inspect to immediate purchase. Hence, $\hat{\theta}_{\mathrm{ind}}$ is strictly increasing in $p^*(s)$. In other words, an increase in the consumer's expectation of the firm's price encourages inspections.

$\hat{\theta}_{\mathrm{ind}}$ ***strictly decreasing in*** s**:** Consider an increase in s, holding $p^*(s)$ fixed. Since only a consumer who inspects the firm's product incurs the higher costs, only U_{S} is affected and decreases strictly. Thus, $\hat{\theta}_{\mathrm{ind}}$ is strictly decreasing in s. Intuitively, firms can impede the information acquisition of consumers by obfuscation. ■

***Proof of proposition 1.3*:** Recall that, given s^*, the behavior of firms and consumers on the equilibrium path, and hence the described market outcomes, follow immediately from proposition 1.1. In a nutshell, the proposition thus states that the firm-optimal information strategy is:

---

[4] A technical remark. In order for $\hat{\theta}_{\mathrm{ind}}$ to exist, let $\hat{\theta}_{\mathrm{ind}} \in \mathbb{R}_0^+$. Set $U_{\mathrm{res}}(\theta, s^*, p^*(s^*)) = v - p^*(s^*) + \mu\bar{\varepsilon}$ for $\theta = 0$, so that U_{res} and $\hat{\theta}_{\mathrm{ind}}$ are well-defined.

[5] A technical remark. The following results are valid if $\hat{\theta}_{\mathrm{ind}}(s, p^*(s), s^*, p^*(s^*)) \neq 0$. The excluded case occurs, when $p^*(s)$ is so low such that the consumer expects to purchase the product after inspection for sure. Then, there is no expected gain of evaluating the product, and only the hypothetical consumer that satisfies $\theta = 0$, that can inspect products for free, is indifferent between inspect and immediate purchase.

$$s^* \quad = \quad \begin{cases} \text{arbitrary}, & \text{if } \mu \leq \mu^{\text{S}}_{\text{A}}, \\ \underline{s}, & \text{if } \mu^{\text{S}}_{\text{A}} < \mu \leq \mu^{\text{S}}_{\text{B}}, \\ \text{such that } \tilde{p}(\mu, (c+s^*), 1) = v, & \text{if } \mu^{\text{S}}_{\text{B}} < \mu \leq \mu^{\text{S}}_{\text{C}}, \\ \overline{s}, & \text{if } \mu^{\text{S}}_{\text{C}} < \mu, \end{cases} \tag{A1.1}$$

where $\mu^{\text{S}}_{\text{A}} := \underline{\theta}(c+\underline{s}/g(0)$ such that $\theta_{\text{ind}}(\mu^{S}_{\text{A}}, c) = \underline{\theta}$; $\mu^{\text{S}}_{\text{D}} := (c+\overline{s})/g(0)$ such that $\theta_{\text{ind}}(\mu^{S}_{\text{D}}, c+\overline{s}) = 1$; $\mu^{\text{S}}_{\text{B}}$ such that $\tilde{p}(\mu^{\text{S}}_{\text{B}}, (c+\underline{s}), 1) \overset{!}{=} v$; $\mu^{\text{S}}_{\text{C}}$ sucht $\tilde{p}(\mu^{\text{S}}_{\text{C}}, (c+\overline{s}), 1) \overset{!}{=} v$. Existence and uniqueness of the described cutoff-values in product diversity, and the existence and uniqueness of s^* that solves $p^*(\mu, c+s^*, 1) = v$ for $\mu \in (\mu^{\text{S}}_{\text{B}}, \mu^{\text{S}}_{\text{C}}]$, can be shown by application of the intermediate value theorem, analogously to the proof of proposition 1.1, and is hence omitted.

Thus, what remains to be shown is that s^* is indeed the information strategy of firms in the firm-optimal perfect Bayesian equilibrium and that s^* maximizes industry profits. In the following, I construct a quadruple $(s^*, p^*, \theta^*_{\text{S}}, \theta^*_{\text{I}})$, where s^* is as described above, and each firm's and consumer's strategy is optimal if beliefs are consistent. The proof proceeds in 6 steps. Step 1 and 2 are auxiliary results. The decisive steps of the proof are the steps 3 and 4. The main result, which is established in step 5, follows directly from step 3 and 4. Firm-optimality and industry profit maximization are shown in step 6.

Before, let me introduce some further notation. Let $\hat{\pi}\Big(p(s), \theta^*_{\text{S}}(s), \theta^*_{\text{I}}(s), s^*, p^*(s^*)\Big)$ also denote the firm's profits obtained after a deviation (s, p), as given in equation 1.6, however, this time expressed as function of distinct arguments. Henceforth, let subindizes denote partial derivatives.

Step 1**:** For $\underline{\theta} < \theta^*_{\text{S}}(s) < \theta^*_{\text{I}}(s) < 1$, $\hat{\pi}$ is strictly submodular with respect to $p(s)$ and $\theta^*_{\text{S}}(s)$, and strictly supermodular with respect to $p(s)$ and $\theta^*_{\text{I}}(s)$.

Proof**:** The first derivative of profits with respect to prices is

$$
\begin{aligned}
&\hat{\pi}_1(p(s), \theta^*_S(s), \theta^*_I(s), s^*, p^*(s^*)) \\
&= \int_{\underline{\theta}}^{\theta^*_S(s)} \tilde{F}\Big(\max\{\tilde{x}(\theta, s^*), 0\} + \frac{p(s) - p^*(s^*)}{\mu} \Big) \\
&* \left\{ 1 - \frac{p(s)}{\mu} \varphi\Big(\max\{\tilde{x}(\theta, s^*), 0\} + \frac{p(s) - p^*(s^*)}{\mu} \Big) \right\} \xi(\theta, s^*) d\theta \\
&+ \int_{\theta^*_S(s)}^{\theta^*_I(s)} \xi(\theta, s^*) d\theta.
\end{aligned}
$$

First, $\hat{\pi}_2$ is obviously strictly increasing in $\theta^*_I(s)$. Second, $\hat{\pi}_2$ is strictly decreasing in $\theta^*_S(s)$, since $\tilde{F}(x) \leq 1$ for every x and $\varphi(x) > 0$ for every x. □

In other words, the more consumers purchase immediately, the more profitable is an increase in prices; the less consumers inspect products, the more profitable is an increase in prices.

Step 2**:** If $\underline{\theta} < \theta^*_S(s) = \theta^*_I(s)$, then $\hat{\pi}_1(v, \theta^*_S(s), \theta^*_I(s), s^*, v) < 0$.

Proof**:** Substitutes $\theta^*_S(s) = \theta^*_I(s)$, $p^*(s^*) = v$ and $p(s) = v$ in $\hat{\pi}_1$, as given in step 1. Then

$$
\pi_1(v, \theta^*_I(s), \theta^*_I(s), s^*, v) = \int_{\underline{\theta}}^{\theta^*_I(s)} \tilde{F}(\tilde{x}(\theta, s^*)) \left\{ 1 - \frac{v}{\mu} \varphi(\tilde{x}(\theta, s^*)) \right\} \xi(\theta, s^*) d\theta < 0,
$$

where the last inequality holds due to the assumption of increasing hazard rates, $\tilde{x}(\theta, s^*) \geq 0$ and $v > \frac{\mu}{\varphi(0)}$. □

In other words, if the market price is v, then $p^*(s) = v$ is not a fixpoint off or on the equilibrium path if no consumer purchases immediately.

Step 3**:** Suppose there exist an equilibrium path $(s^*, p^*(s^*))$ and $(\theta^*_S(s^*), \theta^*_I(s^*)) > (\underline{\theta}, \underline{\theta})$, where $p^*(s^*) = v$. Then, for any $s > s^*$ there exist $(\theta^*_S(s), \theta^*_I(s))$ such that $p^*(s) = v$ and $(\theta^*_S(s), \theta^*_I(s))$ are optimal after s. Furthermore, the firm's profits are lower after s than after s^*.

Proof**:** Set $p^*(s) = v$, set consider $\hat{\theta}_{ind}(s, v, s^*, v)$. There are two cases to consider.

Case 1**:** Suppose $\hat{\theta}_{ind}(s, v, s^*, v) \leq \underline{\theta}$, such that no consumer strictly prefers to inspect the firm's product. Then, $\theta^*_S(s) = \theta^*_I(s) = 0$ (all consumers exit) and $p^*(s) = v$ are optimal after s. Furthermore, the firms profits are zero.

Case 2**:** Suppose $\hat{\theta}_{ind}(s, v, s^*, v) > \underline{\theta}$, such that some consumers strictly prefer to inspect the firm's product. Set $\theta^*_S(s) = \hat{\theta}_{ind}(s, v, s^*, v)$. Let's first verify that $\theta^*_S(s)$ does not exceed 1.

This is the case, because $\theta_S^*(s) = \hat{\theta}_{\text{ind}}(s, v, s^*, v) < \hat{\theta}_{\text{ind}}(s^*, v, s^*, v) = \theta_S^*(s^*) < 1$. The second inequality holds, as $\hat{\theta}_{\text{ind}}$ is decreasing in search costs. The third and fourth relation follow from $\theta_S^*(s^*) < \theta_I^*(s^*)$, which holds, since otherwise, if $\theta_S^*(s^*) = \theta_I^*(s^*)$, $p^*(s^*) = v$ is not optimal after s^* by step 2. Notice that moreover $\theta_S^*(s) < \theta_S^*(s^*)$ has been established.

To continue, $\hat{\pi}_1(v, \theta_S^*(s), \theta_S^*(s), s^*, v) < 0$ holds by step 2. On the other hand, I find $0 < \hat{\pi}_1(v, \theta_S^*(s), \theta_I^*(s^*), s^*, v)$. This holds, as $\hat{\pi}_1(v, \theta_S^*(s^*), \theta_I^*(s^*), s^*, v) = 0$, by the optimality of $p^*(s^*) = v$, and due to the supermodularity of $\hat{\pi}$, established in step 1, in combination with $\theta_S^*(s) < \theta_S^*(s^*)$. Notice that $\theta_S^*(s) < \theta_I^*(s^*)$, since $\theta_S^*(s) < \theta_S^*(s) \leq \theta_I^*(s^*)$. Thus, the intermediate value theorem applies and establishes the existence of $\theta_I^*(s) \in (\theta_S^*(s), \theta_I^*(s^*))$ such that $\hat{\pi}_1(v, \theta_S^*(s), \theta_I^*(s), s^*, v) = 0$. Then, $p^*(s) = v$ and $(\theta_S^*(s), \theta_I^*(s))$ are optimal after s. □

***Profits*:** Less consumer inspect the firm's product upon observing s, as $\theta_S^*(s^*) > \theta_S^*(s)$. Thus, the firm's demand from shoppers is strictly lower after s. Furthermore, the firm generates lower profits from consumers that purchase the firm's product immediately if

$$\int_{\theta_S^*(s)}^{\theta_I^*(s)} \xi(\theta, s^*)d\theta \leq \int_{\theta_S^*(s^*)}^{\theta_I^*(s^*)} \xi(\theta, s^*)d\theta. \tag{A1.2}$$

In order to show that inequality (A1.2) holds, note that the first order conditions with respect to the firm's profits after s: $\hat{\pi}_1(v, \theta_S^*(s), \theta_I^*(s), s^*, v) = 0$ can be rewritten as

$$\int_{\theta_S^*(s)}^{\theta_I^*(s)} \xi(\theta, s^*)d\theta = -\int_{\underline{\theta}}^{\theta_S^*(s)} \tilde{F}\Big(\tilde{x}(\theta, s^*)\Big)\left\{1 - \frac{v}{\mu}\varphi\Big(\tilde{x}(\theta, s^*)\Big)\right\}\xi(\theta, s^*)d\theta.$$

Analogously, the corresponding first order condition after s^* is

$$\int_{\theta_S^*(s^*)}^{\theta_I^*(s^*)} \xi(\theta, s^*)d\theta = -\int_{\underline{\theta}}^{\theta_S^*(s^*)} \tilde{F}\Big(\tilde{x}(\theta, s^*)\Big)\left\{1 - \frac{v}{\mu}\varphi\Big(\tilde{x}(\theta, s^*)\Big)\right\}\xi(\theta, s^*)d\theta,$$

Inequality (A1.2) follows then from $\theta_S^*(s^*) > \theta_S^*(s)$ and $\tilde{F}(\tilde{x}(\theta, s^*))\left\{1 - \frac{v}{\mu}\varphi(\tilde{x}(\theta, s^*))\right\}\xi(\theta, s^*) < 0$ for every θ, which holds, since $1 - \frac{v}{\mu}\varphi(\tilde{x}(\theta, s^*)) \leq 1 - \frac{v}{\mu}\varphi(0) < 0$. □

***Step 4*:** Suppose there exists an equilibrium path $(s^*, p^*(s^*))$ and $(\theta_S^*(s^*), \theta_I^*(s^*)) > (\underline{\theta}, \underline{\theta})$, where all consumers participate $\theta_I^*(s^*) = 1$. Then, for any $s < s^*$ there exist $p^*(s) < p^*(s^*)$

and $(\theta_S^*(s), \theta_I^*(s))$ such that $p^*(s)$ and $(\theta_S^*(s), \theta_I^*(s))$ are optimal after s. Furthermore, the firm's profits are lower after s than after s^*.

***Proof*:** Consider some arbitrary $p(s)$ and $s < s^*$. Set $\theta_S^*(s) = \hat{\theta}_{\text{ind}}(s, p(s), s^*, p^*(s^*))$ and $\theta_I^*(s) = 1$. Again, I want to apply the intermediate value theorem. Suppose first that the firm sets off the equilibrium path the same price as on the equilibrium price such that $p(s) = p^*(s^*)$. Then, $\hat{\pi}_1(p^*(s^*), \hat{\theta}_{\text{ind}}(s, p^*(s^*), s^*, p^*(s^*)), 1, s^*, p^*(s^*)) < 0$, by the submodularity of profits with respect to $p(s)$ and $\theta_S^*(s)$ in combination with $\hat{\theta}_{\text{ind}}(s, p^*(s^*), s^*, p^*(s^*)) > \hat{\theta}_{\text{ind}}(s^*, p^*(s^*), s^*, p^*(s^*)) \geq \theta_S^*(s^*)$ and the first order condition $\hat{\pi}_1(p^*(s^*), \theta_S^*(s^*), 1, s^*, p^*(s^*)) = 0$. Recall that $\hat{\theta}_{\text{ind}}(s, p(s), s^*, p^*(s^*))$ is strictly increasing in $p(s)$ such that there exists $p'(s)$ that satisfies $\hat{\theta}_{\text{ind}}(s, p'(s), s^*, p^*(s^*)) = \underline{\theta}$. Then, $\hat{\pi}_1(p'(s), s^*, p^*(s^*), \hat{\theta}_{\text{ind}}(s, p'(s), s^*, p^*(s^*)), 1) > 0$. The intermediate applies value theorem applies, so that there exists $p^*(s) \in (p'(s), p^*(s^*))$ that solves first order condition of the monopolist $\hat{\pi}_1(p^*(s), \hat{\theta}_{\text{ind}}(s, p^*(s), s^*, p^*(s^*)), 1, s^*, p^*(s^*)) = 0$. Furthermore, if $\theta_I^*(s^*) = 1$, then $s < s^*$ and $p^*(s) < p^*(s^*)$ imply that $\theta_I^*(s) = 1$ is optimal for the consumer after s. □

***Profits*:** Due to the weak concavity of $\hat{\pi}$ with respect to $p(s)$, which is formally proven in the appendix B, and due to the submodularity of $\hat{\pi}$ with respect to $p(s)$ and $\theta_S^*(s)$, which is proven in step 1, $p^*(s) < p^*(s^*)$ and $\theta_I^*(s^*) = \theta_I^*(s)$ imply $\theta_S^*(s^*) < \theta_S^*(s)$. Then, the firm's profits are lower after s than after s^*, since $\theta_I^*(s^*) = \theta_I^*(s)$ and $\theta_S^*(s^*) < \theta_S^*(s)$. □

***Step 5*:** There exists a perfect Bayesian equilibrium such that s^*, as described in equation A1.1, is the firm's optimal information strategy.

***Proof*:** The consumers' and firms' behavior on the equilibrium path and its optimality follow from proposition 1.1. Thus, what remains to be done is to construct the behavior off the equilibrium path and to verify its optimality.

Market failure regime: For $\mu < \mu_A^S$, even if $s^* = \underline{s}$, no consumer inspects products on the equilibrium path. Clearly, then also no trade occurs off the equilibrium path and $p(s) \geq v$ and $\theta_S^*(s) = \theta_I^*(s) = 0$ (all consumers exit) for all $s \in S$ is optimal.□

All other regimes: Note that either $p^*(s^*) = v$ or $\theta_I^*(s^*) = 1$ or both hold and trade occurs on the equilibrium path such that either step 3 or step 4 or both are applicable. Then, the behavior of consumers and firms off the equilibrium path and its optimality follows from step 3 and 4. □

Step 6: The information strategy s^*, as described in equation A1.1, is firm-optimal and maximizes industry profits.

Proof: Obviously, if s^* maximizes industry profits, then s^* is firm-optimal. Thus, it suffices to show that $s^* \in \arg\max_{s \in S} \pi^*(\mu, c+s, v)$, where π^* denote the firm's equilibrium profits for exogenous search costs $c + s$. Recall from proposition 1.2 that each firm's profits are increasing in search costs if all consumers enter the market and purchase some product, and that profits are decreasing in search costs if consumers participate only partially. Therefore, $s^* \in \arg\max_{s \in S} \pi^*(\mu, c+s, v)$ holds, as s^* is, whenever full consumer participation is feasible, the greatest value in S such that all consumers participate, and otherwise, whenever full consumer participation is not feasible, the lowest value in S: $s^* = \underline{s}$. ■

Proof of proposition 1.4: In the intermediate regime, $p^* = v$ and s^* solves $\tilde{p}(\mu, c+s^*, 1) = v$. Recall that $\tilde{p}$ is strictly increasing in c and strictly decreasing in μ by lemma 1.2, where the latter holds as some consumers purchase immediately in the intermediate regime. Consequently, for $\tilde{p}(\mu, c+s^*, 1) = v$ to hold, s^* must be strictly increasing in product diversity μ and average valuation v, and strictly decreasing in search costs c. ■

Proof of lemma 1.4: The reservation match-value $\tilde{x}(\theta, s^*)$ solves $\mu\, g(\tilde{x}(\theta, s^*)) = \theta(c+s^*)$ by equation (1.2). As g is strictly decreasing, it suffices to show that $\frac{\mu}{c+s^*}$ is strictly increasing in μ.

Suppose upon an increase in μ, s^* would adjust such that $\frac{\mu}{c+s^*}$ would remain constant. Then, $\tilde{x}(\theta, s^*)$ and, consequently, $\tilde{\theta}_{\text{ind}}(\mu, c+s^*)$ also would remain constant, so that $p^* = \tilde{p}(\mu, c+s^*, 1)$ would be linear in μ, since the nominator of $\tilde{p}(\mu, c+s^*, 1)$ in equation (1.5) would remain constant. Thus, the equilibrium price would be strictly increasing in μ. However, in the intermediate regime the equilibrium price is constant. As $\tilde{p}$ is increasing in $c+s^*$, this implies that $\frac{\mu}{c+s^*}$ must be increasing in μ. ■

Proof of proposition 1.5: Without loss of generality, assume that search costs are exogenous.

Assume the contrary, namely that there exists a market equilibrium p^*, μ^* that satisfies $\mu^* < \overline{\mu}$. Consider a deviation of a firm to (p^*, μ). Then first, the demand from immediately purchasing consumers remains unaffected, as they do not observe the deviation. Second, the demand from exiting consumers remains unaffected, as they do not observe the deviation and still exit. Third, the demand of shoppers is strictly increasing in μ. Consider a consumer that prefers to inspect products. Then, his reservation utility is $U^*(\theta) = v - p^* + \mu^* \tilde{x}(\theta)$. Hence, the consumer purchases the firm's product if and only if the match-value ε satisfies $\mu\varepsilon \geq \mu^* \tilde{x}(\theta)$. Therefore, $\tilde{F}(\frac{\mu^*}{\mu}\tilde{x}(\theta))$ is the conditional probability that the consumer purchases the firm's product after inspection. Note that the first derivate of the conditional probability is strictly positive, since $\tilde{x}(\theta)$ is weakly positive and strictly positive for some consumers if there exists a positive measure of consumers that inspect products. Hence, a deviation to $\mu = \overline{\mu}$ is always profitable if $\mu^* < \overline{\mu}$ and trade occurs in equilibrium. ■

A2 Chapter 1: Sufficiency of first order conditions

In this this section, I show that the firm's profits are maximized, whenever the first order condition holds. In order to derive the desired result, I show first that among all undetected deviations the first order condition is sufficient, and second, that a firm's deviations which are detected by consumers who purchase immediately are never profitable. In the following, I revisit the firm's profits for exogenous search costs, so that the notation follows the one from section 1.3. With respect to endogenous search costs, the following proof implies the sufficiency of first order conditions on the equilibrium path. A proof of sufficiency of first order conditions off the equilibrium path, that is after a deviation $s \neq s^*$, follows along the same lines, and is hence omitted.

Before, let me briefly point out why sufficiency is an issue here beyond the necessary distinction between detected and undetected deviations. The natural approach, that is adopted by Anderson and Renault [1999], is to show quasi-concavity of the firm's profit function. The problem is that, due to the consumers' heterogeneity with respect to search costs, it does not suffice to show quasi-concavity of the profits from a particular consumer θ, since the sum of quasi-concave functions is per se not quasi-concave.

Upper bound on profits for detected deviations. Suppose a firm chooses a price that exceeds the consumer's belief p^* by more than δ. Then, each consumer notices that p exceeds $p^* + \delta$, even if he does not inspect the firm's product. As a result, no consumer purchases the firm's product immediately, as otherwise, firms would raise prices without bounds. Therefore, the firm's best scenario is that those consumers that intended to purchase the firm's product immediately and those consumers that intended to inspect the firm's product, inspect the firm's product. This is the case, since a consumer that intended to exit, strictly prefers to exit upon detecting a deviation, given the higher price. Hence, the firm's profits for undetected deviations are bounded from above by

$$\bar{\pi}^{\mathrm{D}}(p) = \int_{\underline{\theta}}^{\theta_{\mathrm{S}}^*} p\tilde{F}\Big(\tilde{x}(\theta) + \frac{p - p^*}{\mu}\Big)\xi(\theta)\, d\theta + \int_{\theta_{\mathrm{S}}^*}^{\theta_{\mathrm{I}}^*} p\tilde{F}\Big(\frac{p - p^*}{\mu}\Big)\xi(\theta)\, d\theta. \tag{A2.1}$$

The second term are the profits generated by the demand from those consumers that intended to purchase the firm's product immediately, but inspect the firm's product upon detecting the firm's deviation. Note that their reservation utility is $v - p^*$, and thus, such a consumer purchases the firms product if the match-value exceeds $\frac{p-p^*}{\mu}$.

In the following, I show that first order conditions are sufficient if consumers are sufficiently cautious and the search cost heterogeneity among consumers is not too great. The main idea, as outlined in step 4, is to show weak concavity of the firm's profits for undetected deviations, and to derive an upper bound on profits for detected deviations. To avoid any confusion, let π^{U} denote the firm's profit for undetected deviations.

Lemma A2.3. *There exists $\delta, \underline{\theta} < 1$ such that, if $\frac{f_1}{f}$ is bounded from below by $\frac{f_1}{f} \geq -2\frac{\mu}{v+\mu\bar{\varepsilon}}$ and f is log-concave, then $\pi_1^{\mathrm{U}}(p^*) = 0$ implies that p^* is a global maximum.*

***Proof*:** The proof proceeds in four steps.

***Step 1*:** For any δ there exists $\underline{\theta} < 1$ such $\tilde{x}(\underline{\theta}) - \tilde{x}(\theta) < \frac{\delta}{\mu}$ for every $\theta \in [\underline{\theta}, 1]$.

***Proof*:** The claim is equivalent to left-sided continuity of g^{-1} at $\frac{c}{\mu}$, and thus follows from continuity of g^{-1}. □

Set $\delta = \frac{\bar{\varepsilon}-\tilde{x}(\theta)}{2}$, and set $\underline{\theta} < 1$ such that $\tilde{x}(\underline{\theta}) - \tilde{x}(\theta) < \frac{\delta}{\mu}$ for every $\theta \in [\underline{\theta}, 1]$.

***Step 2*:** The profit function is weakly concave for $p \leq p^* + \delta$, and strictly concave at $p = p^*$.

***Proof*:** A deviation $p \leq p^* + \delta$ is only detected by consumers who inspect the firm's product. The firm's profits for undetected deviations are given by equation (1.4)

$$\pi^{\mathrm{U}}(p) = \int_{\underline{\theta}}^{\theta_{\mathrm{I}}^*} \underbrace{p\left\{1 - \mathbb{1}_{\theta \leq \theta_{\mathrm{S}}^*} F\left(\tilde{x}(\theta) + \frac{p - p^*}{\mu}\right)\right\}}_{\pi^{U,\theta}(p)} \xi(\theta) d\theta. \tag{A2.2}$$

Then, the profit function is weakly concave if $\pi^{U,\theta}$ is weakly concave for every $\theta \leq \theta_{\mathrm{I}}^*$, which I show in the following.

First, for $\theta > \theta_{\mathrm{S}}^*$, for those agents that purchase the firm's product immediately, $\pi^{U,\theta}$ is linear, and thus weakly concave.

Second, consider $\theta \leq \theta^*_S$. Then, $\pi^{U,\theta}$ is linear for $p \leq \mu\big(\underline{\varepsilon} - \tilde{x}(\theta)\big) + p^*$, since then $F\left(\tilde{x}(\theta) + \frac{p-p^*}{\mu}\right) = 0$ and the consumer purchases the product after inspection. At $p = \mu\big(\underline{\varepsilon} - \tilde{x}(\theta)\big) + p^*$ it holds that $\pi^{U,\theta}$ is continuous, but has a kink. However, $\pi^{U,\theta}$ is steeper on the left side of the kink,

$$1 = \lim_{p \to \{\mu(\underline{\varepsilon} - \tilde{x}(\theta)) + p^*\}^-} \pi_1^{U,\theta}(p) \geq \lim_{p \to \{\mu(\underline{\varepsilon} - \tilde{x}(\theta)) + p^*\}^+} \pi_1^{U,\theta}(p) = 1 - \frac{p}{\mu} f(\underline{\varepsilon}),$$

such that weak concavity is not violated. Finally, what remains to be shown is that $\pi^{U,\theta}(p)$ is weakly concave on $\big(\mu(\underline{\varepsilon} - \tilde{x}(\theta)\big) + p^*, p^* + \delta)$. In this regime $\pi^{\mathrm{U}}(p, \theta)$ is twice continuously differentiable as $\tilde{x}(\theta) + \frac{p-p^*}{\mu} \in (\underline{\varepsilon}, \overline{\varepsilon})$. The second derivate of $\pi^{U,\theta}$ with respect to prices is

$$\pi_{11}^{U,\theta}(p) = \frac{f\left(\tilde{x}(\theta) + \frac{p-p^*}{\mu}\right)}{\mu} \left\{ -2 - \frac{p}{\mu} \frac{f_1\left(\tilde{x}(\theta) + \frac{p-p^*}{\mu}\right)}{f\left(\tilde{x}(\theta) + \frac{p-p^*}{\mu}\right)} \right\}.$$

If f is weakly increasing, then the second derive of $\pi^{U,\theta}$ is strictly negative which completes the proof. Thus, suppose that f is not weakly increasing in some regime such that for some p' it holds that $f_1\left(\tilde{x}(\theta) + \frac{p-p^*}{\mu}\right) < 0$. Consider an arbitrary p'. Then,

$$\pi_{11}^{U,\theta}(p') \leq \frac{f\left(\tilde{x}(\theta) + \frac{p'-p^*}{\mu}\right)}{\mu} \left\{ -2 - \frac{v + \mu\overline{\varepsilon}}{\mu} \lim_{\varepsilon \to \overline{\varepsilon}} \frac{f_1(\varepsilon)}{f(\varepsilon)} \right\} < 0.$$

The first inequality holds, since $p' < v + \mu\overline{\varepsilon}$, and since f_1/f is weakly decreasing, which holds as f is log-concave. The last inequality holds by assumption.

Step 3: For $p > p^* + \delta$ the firm's profits are bounded from above by $\pi^{\mathrm{U}}(p^* + \delta)$.

Proof: The firm's profits for a detected deviations $p > p^* + \delta$ are bounded from above by $\bar{\pi}^D(p)$ as defined in equation (A2.2). Then, first, $\bar{\pi}^D(p^* + \delta) \leq \pi^{\mathrm{U}}(p^* + \delta)$, and second, $\bar{\pi}^D(p)$ weakly decreasing for $p \geq p^* + \delta$ complete the proof.

The first claim follows immediately from the definition of π^{U} in equation (A2.2) and $\bar{\pi}^D$ in equation (A2.1).

For the second claim it suffices to show that $\bar{\pi}^{D,\theta}$ is weakly decreasing for every θ on

$p \geq p^* + \delta$. First, note that $\bar{\pi}^{D,\theta}$ is continuous and bounded from below by zero. Furthermore, $\bar{\pi}^{D,\theta}$ is differentiable whenever $\bar{\pi}^{D,\theta}(p) \neq 0$. Thus, it suffices to show that $\bar{\pi}_1^{D,\theta}(p) \leq 0$ whenever $\bar{\pi}^{D,\theta}$ is differentiable at p. I find

$$\frac{\bar{\pi}_1^{D,\theta}(p)}{\tilde{F}\left(\tilde{x}(\theta) + \frac{p-p^*}{\mu}\right)} = 1 - \frac{p}{\mu}\varphi\left(\tilde{x}(\theta) + \frac{p-p^*}{\mu}\right) < 1 - \frac{p^*}{\mu}\varphi\left(\tilde{x}(\underline{\theta}) + \frac{p-p^*}{\mu} + \left[\tilde{x}(\theta) - \tilde{x}(\underline{\theta}) + \frac{p-p^*}{\mu}\right]\right),$$

where the inequality follows from $p > p^*$. Furthermore, since φ is weakly increasing, and $\left[\tilde{x}(\theta) - \tilde{x}(\underline{\theta}) + \frac{p-p^*}{\mu}\right] \geq \left[\tilde{x}(\theta) - \tilde{x}(\underline{\theta}) + \frac{\delta}{\mu}\right] > 0$ by step 1. Hence,

$$\frac{\bar{\pi}_1^{D,\theta}(p)}{\tilde{F}\left(\tilde{x}(\theta) + \frac{p-p^*}{\mu}\right)} < 1 - \frac{p^*}{\mu}\varphi\left(\tilde{x}(\underline{\theta}) + \frac{p^*-p^*}{\mu}\right) \leq 0.$$

The last inequality holds as $1 - \frac{p^*}{\mu}\varphi\left(\tilde{x}(\underline{\theta}) + \frac{p^*-p^*}{\mu}\right) = \pi_1^{U,\underline{\theta}}(p^*)$. Then, $\pi_1^{U,\theta}(p^*) \leq 0$ holds, since $\pi_1^{\mathrm{U}}(p^*) = 0$ implies $\pi_1^{U,\theta}(p^*) \leq 0$ for some θ, which implies $\pi_1^{U,\underline{\theta}}(p^*) \leq 0$, since the demand generated by shoppers is more elastic by assumption of increasing hazard rates. □

***Step 4*: $\pi_1^{\mathrm{U}}(p^*) = 0$** implies that p^* is a global maximum

***Proof*:** By step 2, the profit function is weakly concave for $p \leq p^* + \delta$, and strictly concave at p^*. By step 3, the firm's profits for $p > p^* + \delta$ are bounded from above by $\pi^{\mathrm{U}}(p^* + \delta)$, and thus strictly lower than $\pi^{\mathrm{U}}(p^*)$, by step 2. ■

A3 Chapter 2: Formal results

***Proof of lemma 2.1*:** A proof can be found in Weitzman [1979]. ■

***Proof of theorem 2.1*:** Consider an arbitrary price profile $\vec{p}$, and suppose that the monopolist obfuscates each product: $\vec{s} = \vec{s}^0(\vec{p})$. Thus, the reservation utility of each product is zero: $\vec{U}^{\text{res}}(\vec{p}, \vec{s}) = \vec{0}$, and the consumer is indifferent about in which order to inspect products or whether to exit. Since indifferences are resolved in favor of the monopolist, the consumer inspects more expensive products first and the resulting price sequence, induced by the consumer's search order, is decreasing. If the consumer finds then a product that supplies positive utility, he ends his search and purchases the product. This holds, as the product supplies a utility that exceeds the highest reservation utility of all products that have not been inspected, which is equal to zero. Therefore, the consumer purchases the most expensive product that supplies positive utility, so that the monopolist's expected profits are $\overline{\pi}(\vec{p})$. ■

***Proof of corollary 2.1*:** The claim is obvious, except that it remains to be shown that π^{2nd} is well-defined, so that $\arg\max_{\vec{p}\in\mathbb{R}^K_+} \overline{\pi}(\vec{p})$ exists. Then, for any $\vec{p} \in \arg\max_{\vec{p}\in\mathbb{R}^K_+} \overline{\pi}(\vec{p})$, $(\vec{p}, \vec{s}^0(\vec{p}))$ constitutes an equilibrium strategy profile. Let $\overline{\theta} := \max_{k\in K}\{\overline{\theta}_k\}$. Define $\mathcal{P}^{2\overline{\theta}} := \{\vec{p} \in \mathbb{R}^K_+ | p_k \leq 2\overline{\theta}$ for every $k\}$ and note that it is a compact set. Then, since $\overline{\pi}$ is continuous, a maximum of $\overline{\pi}$ on $\mathcal{P}^{2\overline{\theta}}$ exists by the Bolzano-Weierstrass theorem. Moreover, the maximum is a global maximum, since $\overline{\pi}(\vec{p}) = 0$ for every $\vec{p} \in \mathbb{R}^K_+ \backslash \mathcal{P}^{2\overline{\theta}}$. ■

***Proof of theorem 2.2*:** Let $(\vec{p}^*, \vec{s}^*)$ be a monopolist's equilibrium strategy profile.

***i)*:** By corollary 2.1, the monopolist obtains the second-best profits in equilibrium. The monopolist thus obtains the upper-bound profits $\overline{\pi}(\vec{p}^*)$ and therefore the consumer purchases one of the most expensive products which supply positive utility. □

***ii)*:** Assume the contrary – i.e. that for some $(\vec{p}^*, \vec{s}^*)$ product l is not bought in equilibrium. Since under $\vec{p}^*$ product l is not bought in equilibrium, it is never the only most expensive product that supplies positive utility by *i*) in theorem 2.2. If there exists a price profile $\vec{p}$ that only alters the price of product l and under $\vec{p}$ product l is the only most expensive product that supplies positive utility with strictly positive probability, then this price profile generates higher upper-bound profits $\bar{\pi}(\vec{p}) > \bar{\pi}(\vec{p}^*)$ – a contradiction to $\bar{\pi}(\vec{p}^*) = \pi^{2nd}$.

Consider the following price profile $\vec{p}$ which only alters the price of product l: set $p_k = p_k^*$ for all $k \neq l$ and $p_l = \left(\min_{\{k\in K\}}(\overline{\theta}_k) + \max_{\{k\in K\}}(\underline{\theta}_k)\right)/2$. Recall that by assumption $\text{Supp}(F_m) \cap \text{Supp}(F_n)$ has positive measure for any m, n, which implies $\min_{\{k\in K\}} \overline{\theta}_k < \max_{\{k\in K\}} \underline{\theta}_k$. First, $\tilde{F}_l(p_l) > 0$, so that product l supplies positive utility with strictly positive probability. Second, if $p_k \geq p_l$, then $\tilde{F}_k(p_k) < 1$ by the definition of p_l. Thus, all more expensive products yield strictly negative utility with strictly positive probability. As a consequence, product l is with strictly positive probability the most expensive product which supplies positive utility. □

***iii)*:** Assume the contrary – i.e. that there exist product k and l such that product k is more expensive and product l is inspected first: $p_k^* > p_l^*$ and $U_k^{\text{res}}(p_k^*, s_k^*) < U_l^{\text{res}}(p_l^*, s_l^*)$. I want to show that product is not always purchased, whenever it is the most expensive product that supplies positive utility, in contradiction to *i*) in theorem 2.2. Suppose that product k is the most expensive product that supplies positive utility. By *ii*) in theorem 2.2 this occurs with strictly positive probability. By the definition of the reservation utility in equation (2.2), $\overline{\theta}_l - p_l^* \geq U_l^{\text{res}}(p_l^*, s_l^*)$. Consequently, the utility that product l supplies exceeds the reservation utility of product k with strictly probability: $\theta_l - p_l^* > U_k^{\text{res}}(p_k^*, s_k^*)$ with strictly positive probability. Then, the consumer ends his search and purchases product l if he searches product l – a contradiction to product k being purchased. □

***iv)*:** Assume the contrary – i.e. that there exists a product k and a product l such that product k is strictly cheaper, but product k is not obfuscated: $p_k^* < p_l^*$ and $s_k^* \neq s_k^0(p_k)$. Equivalently, $U_k^{\text{res}}(p_k^*, s_k^*) \neq 0$. Since product k is bought with strictly positive utility by *ii*) in theorem 2.2, $U_k^{\text{res}}(p_k^*, s_k^*) > 0$ must hold, as otherwise the consumer would never inspect the product. The strictness of the inequality follows from $s_k^* \neq s_k^0(p_k)$. Now, with strictly positive probability the following three events occur jointly: product l is the most expensive product that supplies positive utility, product k is the second most expensive product that supplies positive utility, and product k supplies a greater utility to the consumer than product l. There are two cases to consider. First, if product k is inspected before l, then the consumer purchases product k –

a contradiction to *i*) in theorem 2.2. Second, product l is inspected first. Then, with strictly positive probability the utility that l supplies is strictly lower than $U_k^{\text{res}}(p_k^*, s_k^*)$, in which case the consumer inspects k and purchases k – a contradiction to *i*) in theorem 2.2. □

***v*):** Suppose $p_l^* < p_k^*$. By *iii*) in theorem 2.2, product k is inspected first. By *iv*) in theorem 2.2, product l is obfuscated: $U_{\phi^*(l)}^{\text{res}} = 0$. Consequently, the consumer would never inspect product l, whenever product k supplies positive utility. Thus, if product l is inspected, product k supplies strictly negative utility and the consumer therefore never returns in order to purchase product k. □

The existence of an equilibrium in which the consumer's expected utility is zero is equivalent to the existence of an equilibrium in which each product is obfuscated. By corollary 2.1, there exists an equilibrium strategy profile $(\vec{p}^*, \vec{s}^*)$. By theorem 2.1, $\vec{p}^*$ generates the upper-bound profits if all products are obfuscated. Since $\bar{\pi}(\vec{p}^*) = \pi^{2nd}$, this constitutes an equilibrium by corollary 2.1. ■

***Proof of theorem 2.3*:** I proof the claim via an example. Suppose the monopolists offers two product for which the consumer's valuations are independently and identically distributed: the consumer's valuation is v with probability $(1 - \alpha)$ and $\frac{v}{2}$ otherwise. In the following, I show that if $\frac{1}{2} < \alpha \leq \frac{2}{3}$, then welfare increases if search search costs are endogeneous and not necessarily zero.

***Absent search costs*:** Simple algebra shows that an optimal price profile is $\vec{p}^{*,NS} = (v, v)$ if $\alpha \leq \frac{2}{3}$. Equilibrium welfare equals then the monopolist's profits, as the consumer's surplus is zero: $W^{*,NS} = \pi^{*,NS} = (1 - \alpha^2)v$.

***Endogeneous search costs*:** Simple algebra shows that an optimal price profile is $\vec{p} = (v, \frac{v}{2})$ and an optimal search costs profile is $\vec{s}^* = (0, (1 - \alpha)\frac{v}{2})$ if $\alpha \geq \frac{1}{2}$. The reservation utility of both products is then zero. Again, equilibrium welfare equals the monopolist's profits, as the consumer's surplus is zero: $W^* = \pi^* = (2 - \alpha)v$.

***Comparison*:** Simple algebra yields $W^* > W^{*,NS}$ if $\alpha > \frac{1}{2}$. ■

Proof of proposition 2.1: The proof consists of two steps.

***Step 1*:** Suppose the consumer inspects products in the equilibrium search order and purchases the first product which he encounters that supplies positive utility. Then, each price profile which generates the highest profits must solve the recursive formula defined by the equations (2.4), (2.5) and (2.6).

***Proof*:** The argument argument is given in the text. □

***Step 2*:** Any equilibrium price profile $\vec{p}^*$ generates the highest profits if the consumer inspects products in the equilibrium search order and purchases the first product which he encounters that supplies positive utility.

***Proof*:** Suppose the consumer inspects products in the equilibrium search order and purchases the first product which he encounters that supplies positive utility. Recall that the equilibrium price profile $\{p^*_{\phi^*(k)}\}_{k\in K}$ is decreasing by theorem 2.2. Therefore, the consumer purchases the most expensive product that supplies positive utility and $\vec{p}^*$ generates the upper-bound profits $\bar{\pi}(\vec{p}^*)$. Since $\vec{p}^*$ is an equilibrium price profile, $\bar{\pi}(\vec{p}^*) = \pi^{2nd}$. This implies that $\vec{p}^*$ generates the highest profits, since the profits for any price profile are bounded from above by π^{2nd}, as the consumer's behavior implies that the consumer never purchases a product which supplies strictly negative utility. ■

***Proof of lemma 2.2*:** The result is obtained as a corollary to lemma A3.4. ■

***Comment on lemma A3.4*:** The auxiliary lemma A3.4 is a generalization of lemma 2.2. For reasons of exposition, lemma 2.2 only considers the two product case. In contrast, lemma A3.4 considers an arbitrary number of products, and provides conditions that determine which of two products, that have neighboring positions in the consumer's search order, is inspected first in equilibrium. This particular exposition is chosen such that lemma A3.4 can be used in the proof of theorem 2.4.

Lemma A3.4. *Suppose the monopolist offers a finite number of products. Let* $(\vec{p}^*, \vec{s}^*)$ *be the monopolist's equilibrium strategy profile and* ϕ^* *the consumer's equilibrium search order. Suppose there exists two products,* $m < n$*, that have neighboring positions in the consumer's* *search order.*[6] *Suppose* $\vec{p}_m^* \neq \vec{p}_n^*$ *and let* $\mathcal{P} \subset Supp(F_m) \cap Supp(F_n)$ *be a price interval such that* $\vec{p}_m^*, \vec{p}_n^* \in \mathcal{P}$. *Suppose that product m is better than product n in the sense that* $F_m(p) \leq F_n(p)$ *for any* $p \in \mathcal{P}$.

i) Product m is inspected first if the reversed hazard rate $f_k(\theta)/F_k(\theta)$ *is strictly increasing in k.*

ii) Product n is inspected first if the hazard rate $f_k(\theta)/\tilde{F}_k(\theta)$ *is strictly decreasing in k and increasing in* θ.

***Proof*:** The proof proceeds in three steps.

***Step 1*:** Let $\vec{p}^*$ be an equilibrium price profile. Let $l < K - 1$. There exists a search cost profile such that a switch of prices of the products $\phi^*(l)$ and $\phi^*(l+1)$ is profitable if:

$$\begin{aligned}\pi_l^* - \pi_l = &\underbrace{\Big(F_{\phi^*(l+1)}(p^*_{\phi^*(l)}) - F_{\phi^*(l)}(p^*_{\phi^*(l)})\Big)}_{\Xi_1}\underbrace{\Big[p^*_{\phi^*(l)} - p^*_{\phi^*(l+1)}\Big]}_{\Xi_2} \\ &+ \underbrace{\Big(F_{\phi^*(l+1)}(p^*_{\phi^*(l)})F_{\phi^*(l)}(p^*_{\phi^*(l+1)}) - F_{\phi^*(l)}(p^*_{\phi^*(l)})F_{\phi^*(l+1)}(p^*_{\phi^*(l+1)})\Big)}_{\Xi_3}\underbrace{\Big[p^*_{\phi^*(l+1)} - \pi^*_{l+2}\Big]}_{\Xi_4}.\end{aligned} \tag{A3.1}$$

is strictly negative.

***Proof*:** Let $\vec{p}$ be the price profile that is generated by a switch of prices of the products $\phi^*(l)$ and $\phi^*(l+1)$, which by definition have neighboring positions in the consumer's search order. Thus, set $p_{\phi^*(l)} = p^*_{\phi^*(l+1)}$, set $p_{\phi^*(l+1)} = p^*_{\phi^*(l)}$ and $p_k = p_k^*$ for each $k \notin \{\phi^*(l), \phi^*(l+1)\}$. Let ϕ denote the search order that is induced by $\vec{p}$ if each product is obfuscated. Since $\{p_{\phi(k)}\}_{k \in K}$ and $\{p^*_{\phi(k)}\}_{k \in K}$ are both decreasing, they must coincide: $p_{\phi(k)} = p^*_{\phi^*(k)}$. Furthermore, $\phi^*(k) = \phi(k)$ for each $k \notin \{l, l+1\}$, $\phi^*(l) = \phi(l+1)$ and $\phi^*(l+1) = \phi(l)$.[7]

Since, $\phi^*(k) = \phi(k)$ for each $k \notin \{l, l+1\}$, a deviation is profitable if the expected equilibrium continuation profits which the monopolist obtains if the consumer has not purchased

[6] Formally, there exists some l such that either $\phi^*(l) = m$ and $\phi^*(l+1) = n$ or $\phi^*(l) = n$ and $\phi^*(l+1) = m$.

[7] Note that it is payoff-irrelevant which product is inspected first if two products have equal prices. Hence, it is without loss of generality to consider the particular search order

any of the first $l-1$ inspected products are lower than their counter part. Thus, if

$$\pi_l^* = \tilde{F}_{\phi^*(l)}(p^*_{\phi^*(l)})p^*_{\phi^*(l)} + F_{\phi^*(l)}(p^*_{\phi^*(l)})\Big[\tilde{F}_{\phi^*(l+1)}(p^*_{\phi^*(l+1)})p^*_{\phi^*(l+1)} + F_{\phi^*(l+1)}(p^*_{\phi^*(l+1)})\pi^*_{l+2}\Big]$$

is lower than the continuation profits that are obtained by choosing $\vec{p}$ and obfuscating each product:

$$\pi_l = \tilde{F}_{\phi(l)}(p_{\phi(l)})p_{\phi(l)} + F_{\phi(l)}(p_{\phi(l)})\Big[\tilde{F}_{\phi(l+1)}(p_{\phi(l+1)})p_{\phi(l+1)} + F_{\phi(l+1)}(p_{\phi(l+1)})\pi_{l+2}\Big]$$

Recall that $p_{\phi(k)} = p^*_{\phi^*(k)}$ for each k, $\phi^*(l) = \phi(l+1)$ and $\phi^*(l+1) = \phi(l)$, and $\phi^*(k) = \phi(k)$ for each $k \notin \{l, l+1\}$. The latter implies that the continuation profits coincide if none of the first $l+1$ inspected products is purchased: $\pi_{l+2} = \pi^*_{l+2}$. Substitution and simple algebra yields:

$$\begin{aligned}\pi_l^* - \pi_l = & \underbrace{\Big(F_{\phi^*(l+1)}(p^*_{\phi^*(l)}) - F_{\phi^*(l)}(p^*_{\phi^*(l)})\Big)}_{\Xi_1}\underbrace{\Big[p^*_{\phi^*(l)} - p^*_{\phi^*(l+1)}\Big]}_{\Xi_2} \\ & + \underbrace{\Big(F_{\phi^*(l+1)}(p^*_{\phi^*(l)})F_{\phi^*(l)}(p^*_{\phi^*(l+1)}) - F_{\phi^*(l)}(p^*_{\phi^*(l)})F_{\phi^*(l+1)}(p^*_{\phi^*(l+1)})\Big)}_{\Xi_3}\underbrace{\Big[p^*_{\phi^*(l+1)} - \pi^*_{l+2}\Big]}_{\Xi_4}.\end{aligned}$$

Thus, the considered deviation is profitable if equation (A3.1) is strictly negative. □

***Step 2*:** Given the assumption of lemma A3.4, product m is inspected first if the reversed hazard rate $f_k(\theta)/F_k(\theta)$ is strictly increasing in k.

***Proof*:** Proof by contradiction. Assume that there exist two neighboring products, $m < n$, where product m is better than product n. Furthermore, assume $f_k(\theta)/F_k(\theta)$ increasing in k, however, in contradiction to lemma A3.4, assume that product n is inspected first.

Consider a switch of prices of the products m and n. Recall that product n is inspected first by assumption. By step 1, there exists a search cost profile such that this switch is profitable if equation (A3.1) is negative after substitution of $\phi^*(l) = n$ and $\phi^*(l+1) = m$.

First, Ξ_1 is negative. Substitution yields $\Xi_1 = F_m(p^*_{\phi^*(l)}) - F_n(p^*_{\phi^*(l)})$, which is negative, since by assumption product m is better than product n. Second, Ξ_2 is positive, since $\{p^*_{\phi^*(k)}\}_{k\in K}$ is decreasing. Third, Ξ_3 is strictly negative. This holds, since a sufficient and necessary condition for $F_m(p^*_{\phi^*(l)})F_n(p^*_{\phi^*(l+1)}) - F_n(p^*_{\phi^*(l)})F_m(p^*_{\phi^*(l+1)})$ to be strictly negative for any arbitrary strictly decreasing price sequence is that $F_k(\theta)$ is strictly log-supermodular with respect to k and θ, which is equivalent to the imposed assumption $f_k(\theta)/F_k(\theta)$ strictly increasing in k. Fourth, Ξ_4 is strictly positive, since the continuation profits π^*_{l+2} cannot exceed $p^*_{\phi^*(l+1)}$ if $\{p^*_{\phi^*(k)}\}_{k\in K}$ is decreasing. Thus, the difference is strictly negative such a switch of prices is a profitable deviation – a contradiction. □

***Step 3*:** Given the assumption of lemma A3.4, product n is inspected first if $f_k(\theta)/\tilde{F}_k(\theta)$ is strictly decreasing in k and increasing in θ.

***Proof*:** Proof by contradiction. Assume that there exist two neighboring products, $m < n$, where product m is better than product n. Furthermore, assume $f_k(\theta)/\tilde{F}_k(\theta)$ is strictly decreasing in k, however, in contradiction to lemma A3.4, assume that product m is inspected first.

Again, consider a switch of prices of the products m and n. Recall that this time product m is inspected first by assumption. By step 1, there exists a search cost profile such that this switch is profitable if equation (A3.1) is negative after substitution of $\phi^*(l) = m$ and $\phi^*(l+1) = n$. This time, however, it is necessary to take a slight detour in order to show that equation (A3.1) is indeed negative.

Consider the auxiliary function Δ that is defined as:

$$\begin{aligned}\Delta(p_L) = &\Big(F_{\phi^*(l+1)}(p^*_{\phi^*(l)}) - F_{\phi^*(l)}(p^*_{\phi^*(l)})\Big)\Big[p^*_{\phi^*(l)} - p_L\Big] \\ &+ \Big(F_{\phi^*(l+1)}(p^*_{\phi^*(l)})F_{\phi^*(l)}(p_L) - F_{\phi^*(l)}(p^*_{\phi^*(l)})F_{\phi^*(l+1)}(p_L)\Big)\Big[p_L - \pi^*_{l+2}\Big],\end{aligned} \tag{A3.2}$$

Note that for $p^*_{\phi^*(l+1)}$, Δ takes the value of equation (A3.1): $\Delta(p^*_{\phi^*(l+1)}) = \pi^*_l - \pi_l$. Furthermore, $\Delta(p^*_{\phi^*(l)}) = 0$. Hence, a sufficient condition for equation (A3.1) to be negative is that the derivate of Δ is positive: $\Delta'(p_L) > 0$ for $p_L \in (p^*_{\phi^*(l+1)}, p^*_{\phi^*(l)})$, since $p^*_{\phi^*(l+1)} < p^*_{\phi^*(l)}$. Simple algebra yields

$$\Delta'(p_L) = F_{\phi^*(l)}(p^*_{\phi^*(l)})\tilde{F}_{\phi^*(l+1)}(p_L) - F_{\phi^*(l+1)}(p^*_{\phi^*(l)})\tilde{F}_{\phi^*(l)}(p_L)$$
$$+ \underbrace{\Big(F_{\phi^*(l+1)}(p^*_{\phi^*(l)})f_{\phi^*(l)}(p_L) - F_{\phi^*(l)}(p^*_{\phi^*(l)})f_{\phi^*(l+1)}(p_L)\Big)}_{\chi}\Big[p_L - \pi^*_{l+2}\Big]. \tag{A3.3}$$

The remainder proceeds in two steps. First, I show that χ is strictly positive. Then, I derive the desired result: $\Delta'(p_L) > 0$.

$\chi > 0$**:** Recall that by assumption $\phi^*(l) = m$ and $\phi^*(l+1) = n$ such that $\chi = F_n(p^*_{\phi^*(l)})f_m(p_L) - F_m(p^*_{\phi^*(l)})f_n(p_L)$. First, $F_n(p^*_{\phi^*(l)}) \geq F_m(p^*_{\phi^*(l)})$, since product m is better by assumption. Second, by assumption, $f_k(\theta)/\tilde{F}_k(\theta)$ is strictly decreasing in k, such that in particular it holds that $f_m(p_L)/\tilde{F}_m(p_L) > f_n(p_L)/\tilde{F}_n(p_L)$. Since $F_n(p_L) \geq F_m(p_L)$ implies $\tilde{F}_n(p_L) \leq \tilde{F}_m(p_L)$, it must hold that $f_m(p_L) > f_n(p_L)$. Jointly, this implies $\chi > 0$. □

$\Delta'(p_L) > 0$**:** Consider again equation (A3.3). For $p_L \in (p^*_{\phi^*(l+1)}, p^*_{\phi^*(l)})$, it holds that $p_L - \pi^*_{l+2} > p^*_{\phi^*(l+1)} - \pi^*_{l+2} = 1/\varphi_{\phi^*(l+1)}(p^*_{\phi^*(l+1)})$, where $\varphi_k = f_k/\tilde{F}_k$ is the usual hazard rate. The second equality is the usual first order condition that is implied by an interior solution of the maximization problem from proposition 2.1.[8]

Due to the assumption of increasing hazard rates: $1/\varphi_{\phi^*(l+1)}(p^*_{\phi^*(l+1)}) \geq 1/\varphi_{\phi^*(l+1)}(p_L)$, so that $1/\varphi_{\phi^*(l+1)}(p_L)$ is a lower bound for the last term in the second line of equation (A3.3): $p_L - \pi^*_{l+2} > 1/\varphi_{\phi^*(l+1)}(p_L)$. Since $\chi > 0$, this yields the following lower bound for $\Delta'(p_L)$:

$$\Delta'(p_L) > F_{\phi^*(l)}(p^*_{\phi^*(l)})\tilde{F}_{\phi^*(l+1)}(p_L) - F_{\phi^*(l+1)}(p^*_{\phi^*(l)})\tilde{F}_{\phi^*(l)}(p_L)$$
$$+ \Big(F_{\phi^*(l+1)}(p^*_{\phi^*(l)})f_{\phi^*(l)}(p_L) - F_{\phi^*(l)}(p^*_{\phi^*(l)})f_{\phi^*(l+1)}(p_L)\Big)\frac{1}{\varphi_{\phi^*(l+1)}(p_L)}.$$

Above inequality can be rewritten as:

$$\Delta'(p_L) > \frac{F_{\phi^*(l+1)}(p^*_{\phi^*(l)})\tilde{F}_{\phi^*(l)}(p_L)\tilde{F}_{\phi^*(l+1)}(p_L)}{f_1(p_L)}\Big(\varphi_{\phi^*(l)}(p_L) - \varphi_{\phi^*(l+1)}(p_L)\Big) > 0,$$

[8] A technical remark: Note that if the solution to the maximization problem is not an interior one and $p^*_{\phi^*(l+1)} = \underline{\theta}_{\phi^*(l+1)}$, then $p^*_{\phi^*(l+1)} - \pi^*_{l+2} \geq \lim_{p \searrow p^*_{\phi^*(l+1)}} 1/\varphi_{\phi^*(l+1)}(p)$ must hold, which is sufficient for the purpose of the proof.

The last inequality holds, since by assumption the hazard rate is strictly decreasing with respect to k and $\phi^*(l) = m < n = \phi^*(l+1)$.■

Proof of theorem 2.4: The proof proceeds in three steps. Define $k^\dagger$ such that the product $\phi^*(k^\dagger)$ is the last product in the consumer's search order whose price exceeds θ_{cross}. Formally, let $k^\dagger = \max\left\{0, k \in K | p^*_{\phi^*(k)} \geq \theta_{\text{cross}}\right\}$.

Step 1: $\phi^*(k) \leq k^\dagger$ for $k \leq k^\dagger$.

Proof: Proof by contradiction. Assume the contrary, namely, that there exists $n' \leq k^\dagger$ such that $\phi^*(n') = n > k^\dagger$. Then, there must exist as well $m' > k^\dagger$ that satisfies $\phi^*(m') = m \leq k^\dagger$. Consider a switch of the prices of the two products m, n, where $m < n$, which implies a switch of positions in the consumer's search order. Let $\vec{p}^{\text{dev}}$ denote the resulting price profile and ϕ^{dev} the consumer's search order. Recall that a switch is profitable if $\bar{\pi}(\vec{p}^*) < \bar{\pi}(\vec{p}^{\text{dev}})$.

Before, I introduce a more tractable expression for the upper-bound profits. Let $\vec{p}$ be an arbitrary price profile and ϕ be a search order such that $\{p_{\phi(k)}\}_{k \in K}$ is decreasing. The upper-bound profits are then given by:

$$\begin{aligned}\overline{\pi}(\vec{p}) &= \mathbb{E}\left(\max_{k \in K}\left\{p_k | p_k \leq \theta_k\right\}\right) \\ &= \sum_{l \in K}\left\{\prod_{k<l} F_{\phi(k)}\big(p_{\phi(k)}\big)\right\}\tilde{F}_{\phi(l)}\big(p_{\phi(l)}\big)p_{\phi(l)}.\end{aligned} \tag{A3.4}$$

The equation states that the upper-bound profits $\overline{\pi}(\vec{p})$ are the sum of the profits obtained if the l-th inspected product is purchased, where the l-th inspected product is purchased if all more expensive products $k < l$, which are inspected earlier, supply strictly negative utility and product l supplies positive utility.

First, note that $\{p^*_{\phi^*(k)}\}_{k \in K}$ and $\{p^{\text{dev}}_{\phi^{\text{dev}}(k)}\}_{k \in K}$ are both decreasing and coincide: $p^*_{\phi^*(k)} = p^{\text{dev}}_{\phi^{\text{dev}}(k)}$. Second, as the two search order coincide up to a switch of positions of two product, the respective expression for the upper-bound profits coincide except that $F_{\phi^*(m')} = F_{\phi^{\text{dev}}(n')}$ and $F_{\phi^*(n')} = F_{\phi^{\text{dev}}(m')}$. Note that the upper-bound profits, as given in equation (A3.4), are increasing in $F_{\phi(k)}(p_{\phi(k)})$, the probability that the consumer's valuation for a product exceeds the demanded price. This holds, as this raises the demand for the respective product and only steals demand from products which are less expensive. Hence, a switch is profitable if $\tilde{F}_m(p_n^*) \geq \tilde{F}_n(p_n^*)$ and $\tilde{F}_n(p_m^*) \geq \tilde{F}_m(p_m^*)$, where at least one inequality must be strict.

By the definition of $k^\dagger$, $p_n^* \geq \theta_{\text{cross}}$ and $p_m^* < \theta_{\text{cross}}$. Furthermore, by assumption product m is more polarizing than product n such that $F_m(p) \lesseqgtr F_n(p) \Leftrightarrow p \gtreqless \theta_{\text{cross}}$. This implies $\tilde{F}_m(p_n^*) \geq \tilde{F}_n(p_n^*)$ and $\tilde{F}_n(p_m^*) > \tilde{F}_m(p_m^*)$. □

***Step 2*:** $\phi^*(k) = k$ for $k \leq k^\dagger$.

***Proof*:** Proof by contradiction. Assume the contrary – i.e. that there exist $l' \leq k^\dagger$ such that $\phi^*(l') \neq l'$. Then, step 1 implies that there exist $l < k^\dagger$ such that $\phi^*(l) > \phi^*(l+1)$. Let $\phi^*(l) = n$ and $\phi^*(l+1) = m$, such that $m < n$.

What remains to be shown is that we can apply lemma A3.4. First, product m and n are neighbors in the consumer's search order. Second, product m is better than product n for all prices in $[p_m^*, p_n^*]$. This holds, since $m < n$ implies that product m is more polarizing such that for prices that exceed θ_{cross} product m is better. The prices of the two product exceed θ_{cross}, since by $\phi^*(l) = n$ and $\phi^*(l+1) = m$ and $l < k^\dagger$. Finally, by assumption $f_k(\theta)/F_k(\theta)$ is strictly increasing in k for $\theta \geq \theta_{cross}$ such that $f_k(\theta)/F_k(\theta)$ is increasing on $[p_m^*, p_n^*]$. Hence, *i*) in lemma A3.4 applies and the better product m must be inspected first – a contradiction. □

***Step 3*:** $\phi^*(k) = k$ for $k \leq k^\dagger$.

***Proof*:** Proof by contradiction. Assume the contrary – i.e. that there exist $l' > k^\dagger$ such that $\phi^*(l') \neq l'$. Then, step 1 implies that there exist $l > k^\dagger$ such that $\phi^*(l) > \phi^*(l+1)$. Let $\phi^*(l) = n$ and $\phi^*(l+1) = m$, such that $m < n$.

What remains to be shown is that we can apply lemma A3.4. First, product m and n are neighbors in the consumer's search order. Second, product n is better than product m for all prices in $[p_m^*, p_n^*]$. This holds, since $m < n$ implies that product m is more polarizing such that for prices below θ_{cross} product n is better. The prices of the two product are below θ_{cross}, since by $\phi^*(l) = n$ and $\phi^*(l+1) = m$ and $l > k^\dagger$. Finally, by assumption $f_k(\theta)/\tilde{F}_k(\theta)$ is strictly increasing in k for $\theta \leq \theta_{cross}$ such that $f_k(\theta)/F_k(\theta)$ is increasing on $[p_m^*, p_n^*]$. Note that here the better product has the higher indice. Hence, *ii*) in lemma A3.4 applies and the better product n must be inspected second – a contradiction. □

Step 4**:** The consumer is guided to more polarizing products first.

Proof**:** By step 2 and 3, $\phi^*(k) = k$ for all $k \in K$. Furthermore, since $\{F_k\}_{k\in K}$ satisfies the single-crossing property, product k is more polarizing than l if $k < l$. Thus, more polarizing products are inspected first by the consumer. ■

A4 Chapter 2: Equilibrium search rule

Claim A4.1. *In any equilibrium, at any decision node which is reached with strictly positive probability, the consumer chooses the monopolist's preferred action in case of indifference.*

***Proof*:** First, recall that no distribution function has any mass point. Consequently, the only decision nodes which are reached with strictly positive probability, where the consumer is indifferent among several actions, are those, where the reservation utilities of several products are equal, and those where the reservation utility of one, or several products, is equal to the utility that the consumer's outside option supplies. Clearly, the best action for the monopolist at any of these decision nodes is that the consumer inspects among these products a most expensive one, which I want to show in the following. The proof consists of three steps.

***Step 1*:** If there exists a decision node which is reached with strictly positive probability, if the consumer's search rule satisfies Pandora's rule, and the consumer does not choose the monopolist's preferred action in case of indifference, then the monopolist's expected profits are strictly below π^{2nd}.

***Proof*:** Let $(\vec{p}, \vec{s})$ be a strategy profile such that there exists a decision node which is reached with strictly positive probability, if the consumer's search rule satisfies Pandora's rule, and the consumer does not choose the monopolist's preferred action in case of indifference. Thus, either the consumer exits in case of indifference or does not inspect a most expensive product among several products, which have identical reservation utilities.

First, the profile $(\vec{p}, \vec{s})$ cannot generate profits that strictly exceed π^{2nd} by the definition of the second-best profits. Second, in order to generate the second-best profits it must hold that with probability one a most expensive product which supplies positive utility is bought. Carefully note that the proof of *ii*) in theorem 2.2 remains valid, which shows that in order to achieve π^{2nd}, each product must be bought with strictly positive probability. Consequently, each product is the unique most expensive product with strictly positive probability. Third, by the definition of the Weitzman reservation utility each product generates a utility that strictly exceeds its reservation utility with strictly positive utility. Hence, each product is purchased immediately after it is inspected with strictly positive probability.

Now, if a decision node is reached at which it is a consumer's optimal action to inspect a product, then all previously inspected products must supply a utility that is lower than the reservation utility of this product. If the consumer does not inspect a most expensive product with this particular reservation utility, then the consumer either exits or ends his search after the inspection of a strictly cheaper product with strictly positive probability utility. As in either case, the most expensive product, among the considered ones, can be the most expensive product that supplies positive utility with strictly positive probability, the monopolist's profits must be strictly below π^{2nd}. □

***Step 2*:** For any price profile $\vec{p}$ and any $\delta > 0$, there exists a search cost profile $\vec{s}$ such that, if the consumer's search rule satisfies Pandora's rule, the monopolist obtains profits that exceed $\bar{\pi}(\vec{p}) - \delta$.

***Proof*:** Without loss of generality, suppose that $\{p_k\}_{k\in K}$ is decreasing and satisfies $p_k < \bar{\theta}_k$ for all k. Let $s_k^\varepsilon(p_k)$ solve $U_k^{\text{res}}\big(p_k, s^\varepsilon(p_k)\big) \overset{!}{=} \frac{\varepsilon}{k}$. A solution exists for ε sufficiently small by the definition of the Weitzman reservation utility and $p_k < \bar{\theta}_k$ for all k. Now, if the consumer's search rule satisfies Pandora's rule, he inspects more expensive first such that as ε approaches zero he purchases the most expensive product that supplies positive utility. Thus, $\lim_{\varepsilon\to 0^+} \pi(\vec{p}, \vec{s}^\varepsilon(\vec{p})) = \pi^{2nd}$. □

***Step 3*:** In any equilibrium, at any decision node which is reached with strictly positive probability, the consumer chooses the monopolist's preferred action in case of indifference.

***Proof*:** Suppose the contrary. By lemma 2.1, the consumer's search rule satisfies Pandora's rule. By step 1, the monopolist's expected profits π^* are thus strictly below π^{2nd}. As there exists a price profile that satisfies $\bar{\pi}(\vec{p}) = \pi^{2nd}$, there exists, by step 2, a strategy profile for the monopolist which generates profits that strictly exceed π^* – a contradiction. ■

Claim A4.2. *For any ϕ, let $\vec{p}^\phi$ denote the solution to the corresponding recursive formula, where ϕ^* is replaced by ϕ in the equations (2.4), (2.5) and (2.6). Let π^ϕ denote the corresponding profits. Then, $\phi' \in \arg\max_{\phi\in\Phi} \pi^\phi$ implies that ϕ' is the equilibrium search order of some equilibrium.*

***Proof*:** Suppose $\phi' \in \arg\max_{\phi \in \Phi} \pi^{\phi}$. By step 1 in the proof of proposition 2.1, $\vec{p}^{\phi}$ is a price profile that generates the highest profits if the consumer inspect products in the search order ϕ' and purchases the first product that he encounters, which supplies positive utility.

Furthermore, $\pi^{\phi'} \leq \pi^{2nd}$, since the consumer only purchases products that supply positive utility, so that the profits are bounded from above by second-best profits. And $\pi^{\phi'} \geq \pi^{2nd}$, since $\pi^{\phi'} \geq \pi^{\phi^*} = \pi^{2nd}$. The latter holds, since there must exists some equilibrium with equilibrium search ϕ^* by corollary 2.1, for which $\pi^{\phi^*} = \pi^{2nd}$ by an argument along the lines of step 2 in the proof of proposition 2.1. Thus, $\pi^{\phi'} = \pi^{2nd}$.

If $\pi^{\phi'} = \pi^{2nd}$, then $\pi^{\phi'} = \bar{\pi}(\vec{p}^{\phi'})$. Thus, if the consumer inspects products in the search oder ϕ' and purchases the first product which supplies positive utility, he purchases the most expensive product which supplies positive utility. Therefore, $\{p^{\phi'}_{\phi'(k)}\}_{k \in K}$ must be decreasing. Now, $\pi^{\phi'} = \pi^{2nd}$ implies that $\vec{p}^{\phi'}$ is an equilibrium price profile if all products are obfuscated by theorem 2.1 and corollary 2.1. The consumer is indifferent about which products to inspect first, however, in equilibrium he must inspect more expensive products first. Thus, ϕ' is a corresponding equilibrium search order, since $\{p^{\phi'}_{\phi'(k)}\}_{k \in K}$ is decreasing. ■