Three essays in International Macroeconomics

Developments in the euro area crisis

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Chapter 0

Introduction

With the onset of the Great Recession in 2007, the euro area has plunged into its deepest crisis since its inception in 1999. Ten years on, the effects are still visible: euro-area GDP in 2017 is below its 2007 level, the European Central Bank is conducting large-scale Quantitative Easing to fight the spectre of deflation, and there is a continued debate whether the euro area will persist in its current form into the future.

While the dimensions of the crisis are manifold, three aspects appear particularly noteworthy and are therefore investigated more rigorously in this thesis: the inherent role of debt and deleveraging, both public and private, the effects of expectations about euro area break-up, and the role of competitiveness gaps among euro 'core' (e.g. Germany and the Netherlands) and euro 'periphery' (e.g. Greece and Spain).

The first chapter of the thesis entitled 'Downward wage rigidity and wage restraint' formalizes the intuition that wages ought to rise slowly in environments where they are restricted in their ability to fall. The theory developed helps interpret the joint observation of large wage increases in euro periphery countries pre 2008 and the particular deep recession in those countries post 2008: the former may have fuelled the latter because outright wage deflation in crisis times is hard to achieve. Indeed, the calibrated model suggests that lacking wage restraint in Greece has contributed significantly to its crisis, implying an unemployment rate post 2008 that was too high by about 15 percentage points, creating welfare losses of about 0.4 percent of permanent consumption.

The second chapter of the thesis entitled 'Exit expectations and debt crises in currency unions', which is joint work with Alexander Kriwoluzky and Gernot Müller, zooms into the Greek sovereign debt crisis and the so called 'Grexit' debate—the idea that Greece may exit the euro area. In the paper we rely on a Markov-switching rational expectations framework to develop a model of a small open economy which (still) operates within a currency union and which experiences a sovereign debt crisis. We analyze how expectations of an exit from the currency union impact macroeconomic outcomes. We find that exit expectations destabilize the economy i) by reinforcing the sovereign debt crisis and ii) by harming macroeconomic stability in general. We estimate an extended version of the model on Greek time series and find that exit expectations did indeed contribute significantly to the adverse dynamics in Greece during the period 2009—2012.

The third and last chapter of the thesis entitled 'Deleveraging, deflation and depreciation in the euro area', which is joint work with Dmitry Kuvshinov and Gernot Müller, starts from the observation that the euro periphery has experienced a significant amount of private sector debt deleveraging since 2008. Our contribution is to show that within a two-country currency union model of deleveraging, this observation can be used to explain, both qualitatively and quantitatively, the fact that the real exchange rate among euro core and periphery has remained flat even though the recession has been disproportionately deep in the euro periphery. Intuitively, deleveraging in one group of countries generates deflationary spillovers which cannot be contained by monetary policy as it becomes constrained by the zero lower bound. As a result, the real exchange rate response becomes muted, and the output collapse—concentrated in the deleveraging economies.

Chapter 1

Downward wage rigidity and wage restraint

1.0 Abstract

This paper formalizes the intuition that wages ought to rise slowly in environments where they are restricted in their ability to fall. I show that such 'wage restraint' is the socially efficient outcome in neoclassical labour markets where wages are downward rigid, and that the laissez-faire equilibrium generally creates socially inefficient wage restraint due to monopolistic wage mark-ups charged by labour unions and due to a wage setting externality. The open economy version of the model calibrated to Greece suggests that the rise in wages and the real exchange rate experienced by Greece 2001-2008 were inefficiently high—exacerbating its 2009-2016 recession.

1.1 Introduction

Many observers see slowly falling wages as contributing to the deep recession in some countries of the euro zone in the aftermath of the Great Recession (see e.g. the papers in Baldwin and Giavazzi, 2015). In turn, the fact that there is a strong need for wages to fall to begin with is taken to reflect the high wage inflation that has characterized those countries in the *run-up* to the Great Recession. As shown by Figure 1.1, this narrative accords well with the empirical evidence. The figure shows the evolution of real GDP and unit labour costs in the euro area as a whole and in a group of countries commonly known as the 'euro periphery'. Clearly, the euro periphery experienced an above-average rise in unit labour costs pre 2008, and slowly falling labour costs post 2008 despite a particularly deep recession.

This paper formalizes the intuition that wages ought to rise slowly in environments where they are restricted in their ability to fall. I show that such 'wage restraint' is the socially efficient outcome in neoclassical labour markets where nominal wages are downward rigid. I also show that the laissez-faire equilibrium generally creates socially inefficient wage restraint due to monopolistic wage mark-ups charged by labour unions and due to a wage setting externality. In the open economy version of the model I inspect the case of Greece. In line with the above narrative, the model suggests that the rise in wages experienced by Greece pre 2008 was inefficiently large: It caused an unemployment rate post 2008 that was too high by 15 percentage points, associated with large welfare losses.

I start the analysis by discussing a stylized model of wage restraint. An economy is populated by households which derive utility from consumption out of current income and disutility from labour supplied. Wages are implied by the allocation via the marginal product of labour, yet it is assumed they are also downward rigid. To derive an efficiency condition, I consider a social planner with restricted planning abilities (e.g. Bianchi, 2016; Lorenzoni, 2008). Specifically, the planner chooses the allocation subject to technological constraints, yet in such a way that the downward wage rigidity is respected.

I show that as an efficiency condition, in periods where the rigidity is slack, real wages must be set at a mark-down below the households' marginal rate of substitution between consumption and leisure. The mark-down is endogenous and time-varying, and, inter alia, it is larger i) the lower the expected price inflation and ii) the higher the underlying volatility in the economy. I show that the mark-down smooths the amount of working hours over the cycle, with the purpose to i) smooth costly variation in the level of production and thereby consumption, ii) smooth costly variation in the disutility of labour supplied. I interpret the mark-down as 'wage restraint', and, as in the literature on time-varying risk, as an insurance against nominal rigidities (e.g. Fernández-Villaverde et al., 2015).

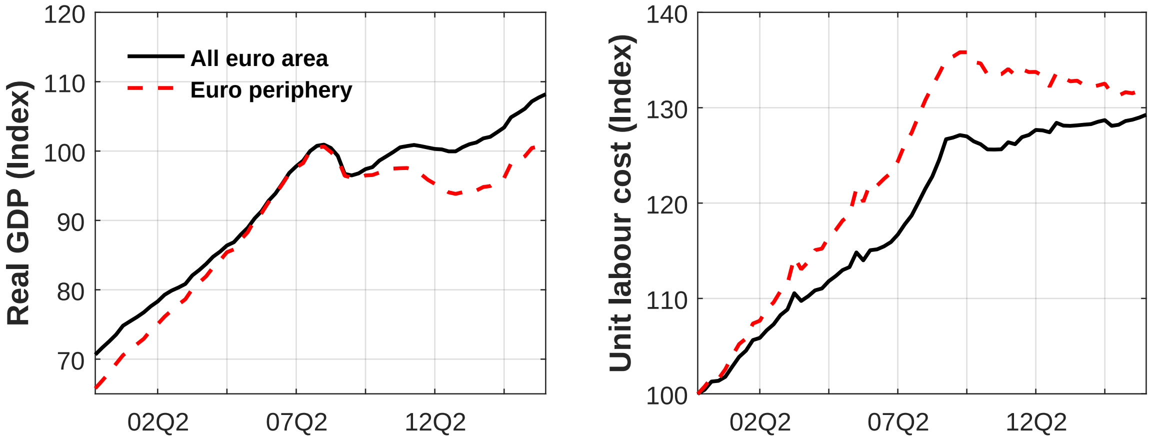

Figure 1.1: Averages using 2007-GDP-weights. Euro periphery: GRC, IRL, ITA, POR, ESP. All data used in the paper are taken from the OECD and from Eurostat.

I next turn to a model of an open economy where wages are downward rigid and where the nominal exchange rate is fixed. Importantly, the open economy dimension allows me to speak to developments in the euro zone. As discussed above, the wage mark-down required for wage restraint is downward-sloping in expected price inflation. As a result, to the extent that the fixed nominal exchange rate rules out (permanently) high inflation rates, wage restraint is particularly important in this environment. Aside a large sector for nontradeable goods, the open economy is characterised by a sector which is integrated in the world market, where households can trade goods and assets, and by a labour supply side in which households organize into labour unions (as in Alesina and Perotti, 1997).

I contrast the allocation implied by the markets (the 'laissez-faire equilibrium') to the allocation implied by a constrained social planner (see above), in order to uncover the inefficiencies that generate insufficient wage restraint in this economy. I document two such inefficiencies. First, the labour unions charge monopolistic wage mark-ups which are 'doubly inefficient' in this environment, because, as explained earlier on, efficiency requires wages to be characterized by mark-*downs*. Second, the strategic wage setting interaction between the unions suffers from an externality. Specifically, each union ignores that by reducing the wages of its members thereby engaging in wage restraint individually, it creates an incentive for the other unions to also engage in wage restraint as their members have now become relatively more expensive. As a result, while the unions insure against downward wage rigidity in principle, they do so to a socially less-than-efficient extent.

In an open economy, the wages and the real exchange rate are intimately linked, such that lacking wage restraint feeds through to inefficient real exchange rate dynamics. As explained in Schmitt-Grohé and Uribe (2016), the combination of downward wage rigidity and the fixed nominal exchange rate then leads to unemployment in the nontradeable sector, creating large welfare losses. To assess the magnitude of these effects, I study wage restraint in the laissez-faire equilibrium calibrated to Greece. I find that the average wage mark-down required for wage restraint is 14 percent, whereas Greece generates an average wage mark-*up* of about 12 percent. This lacking wage restraint implies welfare losses of about 0.4 percent of permanent consumption on average, reflected in an average unemployment rate in the nontradeable sector that is too high by more than 10 percentage points.

I also consider a counter factual: How much of the rise in wages and the real exchange rate in Greece pre 2008 were inefficient, and to what extent did this lacking wage restraint contribute to the subsequent recession? The model suggests that the Greek real appreciation was inefficient in its entirety. It also suggests that this is driven by the fact that wages had been too high already in 2001—when Greece incepted the euro in the first place. Regarding the subsequent recession, the lacking wage restraint triggered the unemployment rate post 2008 to be too high by about 15 percentage points on average—e.g. 28 percent at its peak in 2013 versus 13 percent under the constrained efficient allocation.

My results have also implications for labour market institutions. Namely, echoing the centralized bargaining literature (e.g. Blanchard and Wolfers, 2000; Calmfors and Driffill, 1988; Ortigueira, 2013), I show that larger-sized unions may make the allocation more efficient, as larger unions may internalize strictly better the externality characterizing their wage setting interaction described above. The force going against this is that larger unions typically charge larger monopolistic mark-ups. Hence larger unions may raise welfare in economies in which ex-ante insurance against downward wage rigidity is especially relevant. This holds for example once shocks are highly volatile: As argued above, the required insurance against downward wage rigidity increases as volatility increases. Because larger unions may provide strictly more of this insurance—while volatility does not affect the size of the monopolistic mark-up—larger unions may raise welfare in more volatile economies. Yet quantitatively, in my calibration these forces are dominated by the distortion due to monopolistic mark-ups, such that welfare rises as union size is reduced—reflecting that mark-ups are 'doubly inefficient' when wages are downward rigid.

The remainder of the paper is structured as follows. The remainder of the introduction discusses related literature. Section 1.2 formalizes the notion of wage restraint in a stylized model economy. Section 1.3 introduces the open economy model, and Section 1.4 discusses its quantitative implications. Section 1.5 discusses implications for centralized wage bargaining. Finally, Section 1.6 concludes.

Related literature. The paper relates to the literature studying the implications of wage rigidities for business cycle dynamics. A classic contribution on downward nominal wage rigidity is Bewley (1999), who argues that reducing real wages through nominal wage deflation is more difficult to achieve than through price inflation. This 'greasing the wheels' is part of optimal monetary policy (e.g. Abo-Zaid, 2013; Fahr and Smets, 2010), yet impossible to achieve in open economies where the nominal exchange rate is fixed because in this case the long-run price level is pinned down by purchasing power parity (Corsetti et al., 2013b; Farhi and Werning, 2012).[1]

As a result, un-internalized wage increases are particularly damaging in this environment—which I also highlight in this paper. Schmitt-Grohé and Uribe (2016) consider a similar model environment, however focus on inefficient current account dynamics, i.e. an 'overborrowing externality', while labour supply in their analysis is exogenous. In contrast, inefficient labour supply decisions are key in my analysis, while I abstract from an endogenous current account. Elsby (2009) studies a model where workers are resistant to wage cuts, and—similar as under wage restraint in this paper—finds the repercussions of this rigidity to be small to the extent that firms refrain from pay rises in the first place.

While I assume wage rigidities to be hard-wired in this analysis, other papers have studied ways to ameliorate such rigidities directly. For instance, the 'Fiscal Devaluations' literature considers internal devaluation via taxation (Farhi et al., 2016). However, as argued in Schmitt-Grohé and Uribe (2012), those taxes would in general have to vary at business cycle frequency, which may render them impractical to actually implement. Capital controls are an alternative (Schmitt-Grohé and Uribe, 2016), yet possibly infeasible to implement in advanced economies which are members of a monetary union (such as the euro zone).[2]

One strand of literature that is conceptually related is the literature on time-varying risk (Born and Pfeifer, 2014, 2016; Fernández-Villaverde et al., 2015). In this literature it is emphasized how forward-looking firms cope with uncertainty by charging endogenous price and wage mark-ups (over and above possible monopolistic mark-ups). In my analysis, I show that endogenous wage mark-downs are required for efficiency in an environment where uncertainty about downward wage rigidity is pervasive.

[1] An alternative is temporary inflation in the nominal anchor country: Schmitt-Grohé and Uribe (2013) and Fahr and Smets (2010) make this case for the euro zone, arguing that price inflation in the euro core may help overcome downward wage rigidity in the euro periphery.

[2] Proposals to introduce permanent capital controls in the euro zone have been consistently met with the deepest scepticism, on the grounds that restricting the mobility of international capital would undermine the single monetary system and therefore the unity of the euro zone. Even the case of temporary capital controls has been fiercely debated, and is not clear to be compatible with EU law (see for example the debate on capital controls in Cyprus, Markets Insight, 2013).

The paper also relates to work which stresses intra-euro-area 'misalignment'. Gilchrist et al. (2015) point to financial frictions as explaining the differential development in cost competitiveness among euro core and periphery. Kuvshinov et al. (2016) explore how wage rigidity and the zero lower bound may have paralysed real exchange rate adjustment following a period of deleveraging in the euro periphery. Alternative views are provided in Berka et al. (2014), who argue that the euro periphery wage inflation may represent productivity growth, and in Blanchard and Giavazzi (2002), who argue that intra-euro-area capital flows may reflect convergence inside a monetary union.

A last strand of related literature links wage bargaining centralization and macroeconomic efficiency. One part of the Calmfors and Driffill (1988) hypothesis is that larger labour unions internalize the feedback of their action into the economy which may be welfare improving. Ortigueira (2013) makes this case in a search and matching environment, linking the welfare improvement to an externality which may be present once the labour market is decentralized. Evidence for a positive link between union-size and employment is contained in Blanchard and Wolfers (2000) and Daveri and Tabellini (2000). Alesina and Perotti (1997) stress a similar channel as Calmfors and Driffill, yet in the context of the welfare state and external competitiveness, while Cukierman and Lippi (1999) show that the benefits of centralized wage bargaining depend on the conduct of monetary policy.

1.2 Wage restraint—Intuition in a stylized model

As a starting point for the analysis of wage restraint, consider the following stylized economy. An economy is populated by a continuum of identical households who consume and work. There is a neoclassical labour market in which real wages equate the marginal product of labour. Furthermore, nominal wages are assumed to be rigid downwards. There is no savings technology, output is produced using only labour, utility is linear in consumption, and monetary policy has direct control over the price level.

1.2.1 Defining wage restraint

Households derive utility from consumption, and disutility from supplying labour. Their lifetime utility at time 0 is

$$\mathcal{U}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{c_t - V(h_t)\}, \tag{1.2.1}$$

where E_0 is the mathematical expectation with respect to information at time 0, where c_t and h_t are consumption and hours worked at time t, where $V(h) = (h^{1+\varphi})/(1+\varphi)$ with inverse Frisch elasticity $\varphi \geq 0$, and where $\beta < 1$ is the subjective discount factor.

I denote the production technology $F(h) = h^{\alpha}$, where $\alpha \in (0, 1)$ is the labour share parameter. Output is produced via

$$y_t = a_t F(h_t) \tag{1.2.2}$$

where a_t is a productivity shock following an exogenous process specified below.
I assume that the real wages in this economy correspond to the marginal product of labour. Denoting w_t the nominal wages and p_t the price level, this is

$$w_t/p_t = a_t F'(h_t). \tag{1.2.3}$$

The intuition is that, in a labour market where wage rigidity operates purely downwards, an excess demand for labour can always be corrected by a wage increase. As a result, firms must always be on their labour demand schedule (compare Section 1.3 below where firms are explicitly modelled).
Specifically, I assume that nominal wages are rigid downwards of degree $\gamma \leq 1$. Wages can therefore fall by at most $(1 - \gamma) * 100$ percent per period

$$w_t \geq \gamma w_{t-1}. \tag{1.2.4}$$

This way of modelling downward wage rigidity borrows from e.g. Benigno and Ricci (2011) and Schmitt-Grohé and Uribe (2016).
The model is closed by $c_t = y_t$ as I assume that there is no savings technology. I may therefore insert equation (1.2.2) into equation (1.2.1) to isolate h_t as the single endogenous variable. In particular, recall that $\{p_t\}$ is exogenous, as the price level is under the direct control of the central bank.

Definition 1. [EQUILIBRIUM STYLIZED MODEL] *An equilibrium is a path for hours worked $\{h_t\}$ such that, for given processes for $\{a_t, p_t\}$, the implied nominal wages $\{w_t\}$ from equation* (1.2.3) *satisfy downward wage rigidity from equation* (1.2.4).

The key question is: How should the path for $\{h_t, w_t\}$ be chosen, such that the utility of the household is maximized? This is a constrained planning problem in a dynamic context, as in Lorenzoni (2008) and Bianchi (2011, 2016).

Definition 2. [CONSTRAINED EFFICIENCY STYLIZED MODEL] *The constrained efficient equilibrium is the solution to the following Bellman equation*

$$\mathcal{V}(w_{t-1}) = \max \{a_t F(h_t) - V(h_t) + \beta E_t \mathcal{V}(w_t)\}$$

s.t.

$$i) \qquad w_t/p_t = a_t F'(h_t) \quad \textit{(labour demand)}$$

$$ii) \qquad w_t \geq \gamma w_{t-1} \quad \textit{(downward wage rigidity)}$$

for given exogenous $\{a_t, p_t\}$.

I solve this problem in Appendix A. The solution method consists of attaching a Lagrange multiplier $\psi_t \geq 0$ to inequality constraint (1.2.4). Intuitively, this multiplier measures the utility loss suffered by the household implied by the fact that wages cannot fall frictionlessly.[3] The relevant first order condition is[4]

Proposition 1. [WAGE RESTRAINT STYLIZED MODEL] *Assume that the downward wage rigidity is not binding in the current period ($\psi_t = 0$). The constrained efficient equilibrium is then characterized by the following first order condition*

$$w_t/p_t + \gamma E_t \Lambda_{t,t+1} \psi_{t+1} = V'(h_t), \tag{1.2.5}$$

where $\Lambda_{t,t+1} \equiv \beta(p_t/p_{t+1})(\Omega_{t+1}/\Omega_t) > 0$ is a (nominal) stochastic discount factor, with $\Omega_t \equiv -(\partial h_t/\partial(w_t/p_t)) = 1/(1-\alpha)\ h_t/(w_t/p_t) > 0$ the slope of labour demand at time t.

Equation (1.2.5) is the key equation in the paper. It *defines* wage restraint in that it formalizes the intuition that, in a rigid labour market where wages cannot fall, wages ought to be kept low 'ex-ante'—in periods where the rigidity is slack—in order to alleviate the dislocations produced by the rigidity 'ex-post'—in periods where the rigidity is binding.

To see this, first note that $\gamma E_t \Lambda_{t,t+1} \psi_{t+1} \geq 0$ because of $\psi_{t+1} \geq 0$ and because the stochastic discount factor is strictly positive. As a result, $w_t/p_t \leq V'(h_t)$, and the inequality is strict when there is a strictly positive probability that the wage rigidity binds in the next period. Optimality thus requires that, in periods where the rigidity is slack, real wages be kept at a mark-down below the households' marginal rate of substitution between consumption and leisure.[5] Following the literature on time-varying risk, I interpret this mark-down as an insurance premium (e.g. Fernández-Villaverde et al., 2015). Note that the mark-down is endogenous, and therefore in general time-varying.

What determines its size? First note that the mark-down is zero if the labour market is not rigid ($\gamma = 0$). Here we recover the conventional efficiency condition whereby the real wage equates the marginal rate of substitution in the neoclassical labour market. More generally, the mark-down is increasing in the degree of rigidity in the labour market (as γ increases). Second, the mark-down is larger to the extent that the utility loss associated with dislocations in hours worked, as measured by ψ_{t+1}, is larger.

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[3] To be precise, it measures the utility loss due to working hours departing from their frictionless level—'due to dislocations in hours worked'—that result from the wage rigidity. See Appendix A for details.

[4] The proofs of all propositions are in the Appendix A.

[5] The marginal disutility of labour supply $V'(h_t)$ corresponds to the households' marginal rate of substitution between consumption and leisure in this model, because utility is linear in consumption. Otherwise, the corresponding condition would be $w_t/p_t \leq V'(h_t)/U'(c_t)$, where $U'(c_t)$ is the marginal utility of consumption (see the open economy model in Section 1.3 below, which has a curved consumption utility). I abstract from a curved utility for consumption in the stylized model in order to demonstrate that wage restraint is independent of the traditional motive for consumption smoothing.

Parameter		*Value assigned*
β	Time discount factor	.99
p_t/p_{t-1}	Gross price inflation	1
γ	Downward nominal wage rigidity	.99
α	Labour share	.66
ρ	Autocorrelation technology shock	.95
φ	Inverse Frisch elasticity of labour supply	$\{2,\ 3.5,\ 5\}$
σ	Volatility technology shock	$\{.015,\ .02,\ .025\}$

Table 1.1: Parameters used for numerical illustration.

As for the stochastic discount factor $\Lambda_{t,t+1}$, note that it is decreasing in the expected price inflation in the next period. This is classical 'greasing the wheels': Price inflation erodes the real wages, such that the effects of a given constraint on the nominal wages are effectively dampened (Tobin, 1972). This also implies that wage restraint is particularly important in environments where (permanent) price inflation is impossible, such as when the central bank pursues price level targeting or when in an open economy, the central bank pursues a fixed exchange rate target (see Section 1.3 below). Second, the stochastic discount factor is increasing in the slope of labour demand in the next period, but decreasing in the slope of labour demand in the current period.[6] This is intuitive: To the extent that the labour demand curve in the next period is steep (h_{t+1} moves strongly with changes in w_{t+1}/p_{t+1}), the dislocation in hours worked produced by wages being 'stuck' above their frictionless level is large. On the other hand, if the labour demand curve is steep in the *current* period, the mark-down *itself* produces a large dislocation in hours worked, in that by keeping wages below their frictionless level today, hours worked today rise far above their frictionless level. These two opposing forces must be weighed against each other, which is what the stochastic discount factor achieves.

1.2.2 Numerical illustration

Here I illustrate the properties of the mark-down numerically. I establish that its size depends on model parameters, most notably the Frisch elasticity of labour supply and the degree of volatility in the economy, as well as on the economy's current state.

Given its simplicity, the stylized model can be solved easily numerically: The problem in Definition 2 can be solved via standard value function iteration. I base my analysis on the

[6] 'Labour demand' is the function $h(w_t/p_t)$ implictly defined in equation (1.2.3): $w_t/p_t = a_t F'(h(w_t/p_t))$. Given the assumptions imposed on F, labour demand is positive, but strictly falling in w_t/p_t—i.e. the slope of labour demand is $-1/(1-\alpha)\ h_t/(w_t/p_t)$, see Proposition 1.

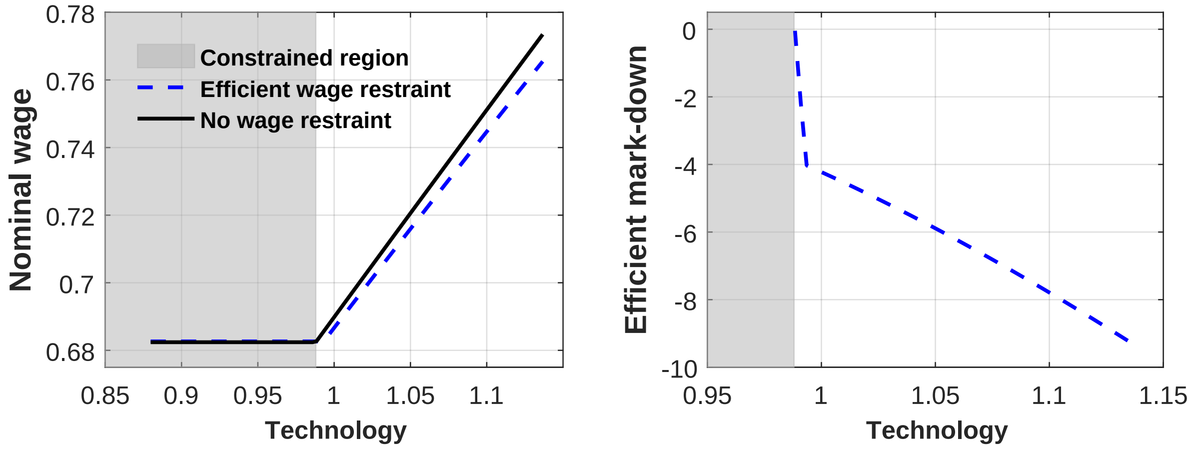

Figure 1.2: Policy function for nominal wages w_t and the mark-down $\mathcal{M}_t$ in technology a_t. I only plot $\mathcal{M}_t$ in the region where it is negative, i.e. in the 'slack' region. The other state variable w_{t-1} is kept at its non-stochastic steady state. Parameters used: $\varphi = 3.5, \sigma = .02$.

parameters that are summarized in Table 1.1: I consider a time discount factor of $\beta = .99$ and zero net inflation: $p_t/p_{t-1} = 1$. The latter is without loss of generality, given that in the model a higher inflation target is isomorphic to a lower wage rigidity. Specifically, I use $\gamma = .99$ but consider alternate values for this parameter in Appendix B. The labour share α is set to the conventional value of two thirds, and variation in this parameter is also discussed in Appendix B.

Below, I vary the inverse of the Frisch elasticity of labour supply φ within the set $\{2,\ 3.5,\ 5\}$, and I use

$$\log(a_t) = 0.95 \times \log(a_{t-1}) + \sigma \times \epsilon_t$$

as a shock process for technology shocks, varying the volatility σ within the set $\{.015,\ .02,\ .025\}$ but keeping the autocorrelation ρ fixed at .95.

The left panel in Figure 1.2 shows the policy for nominal wages as a function in technology shocks. It contrasts the equilibrium under constrained efficiency to an equilibrium where wage restraint is entirely absent: The equilibrium where $w_t/p_t = V'(h_t)$ in all periods.[7] The difference between the two policy functions therefore measures wage restraint. This is further depicted as 'efficient mark-down' $\mathcal{M}_t$ in the right panel of Figure 1.2, which I define as $w_t = (1 + \mathcal{M}_t/100)V'(h_t)$ and which I therefore plot in percent.[8,9]

[7] As discussed above, this would be the condition for efficiency in the absence of the rigidity ($\gamma = 0$). At the same time this would be the condition for labour supply obtained in decentralised markets, that is, under perfect labour market competition where households are wage takers (e.g. Galí, 2008).

[8] Throughout the paper, I call $\mathcal{M}_t$ a 'mark-down' whenever it is negative, a 'mark-up' whenever it is positive.

[9] The kink in the mark-down policy is the point where the planner hits downward wage rigidity. As wages

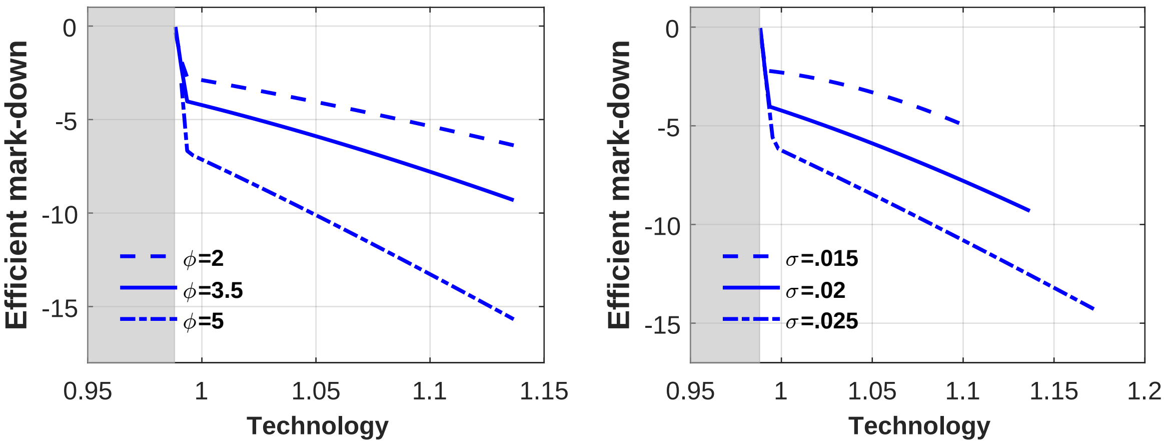

Figure 1.3: Policy function for the mark-down $\mathcal{M}_t$ in technology a_t for different parameters φ and σ. The other state variable w_{t-1} is kept at its non-stochastic steady state. The parameter that is not varied is kept at $\varphi = 3.5, \sigma = .02$.

The first observation is that wages increase in high technology shocks (when the economy is in a 'boom'), yet that they fall by only so much in low technology shocks due to the downward wage rigidity (when the economy is in a 'bust'). In turn, the mark-down increases in the size of the technology shocks. Intuitively, larger technology shocks are associated with stronger wage-inflationary pressure such that, in the next period, there is a higher probability that a negative shock will make the wage rigidity bind. As a result, the ex-ante insurance through wage mark-downs must be larger.

Figure 1.3 provides insights into the sources of the endogenous mark-down in the first place. The left panel shows that the efficient mark-down is larger when the Frisch elasticity is lower. To understand this result, note that period felicity $a_tF(h_t) - V(h_t)$ in Definition 2 is curved (hump-shaped) in working hours h_t, because both the production technology F and the disutility of labour supply V are curved in h_t. As a result, variation in h_t is costly: It implies both losses in average consumption and in the average utility of enjoying leisure—where the size of the latter is determined by the Frisch elasticity. Hence the source of wage restraint in the first place: The efficient mark-down smooths the amount of hours worked, with the two goals i) to provide a high level of average production thus consumption and ii) to avoid costly variation in the disutility of labour supply.[10]

cannot be lowered frictionlessly to the left of this point, the mark-down quickly shrinks to zero. The 'constrained region' is the region where also the equilibrium under $w_t/p_t = V'(h_t)$ hits downward wage rigidity (which occurs at slightly lower technology shocks than under the planner allcation, see Figure 1.2 left panel). In this region, $\mathcal{M}_t$ is positive due to nominal wages being 'stuck' above their frictionless level—it is therefore a mark-*up*. For clarity, I only plot $\mathcal{M}_t$ in the slack region where it is indeed negative.

[10] For this reason, a set-up where unemployment bears an additional utility loss ($V'(h_t) < w_t/p_t$ is not

The right panel of Figure 1.3 displays how the mark-down depends on the parameter underlying variation in h_t: The volatility of the technology shocks. As expected, a higher volatility for technology shocks necessitates a larger mark-down—that is, more insurance. This reflects the inherent asymmetry of the wage rigidity operating purely downwards: While volatility generates both wage-inflationary and wage-deflationary pressure symmetrically, the fact that wage deflation cannot occur frictionlessly necessitates a larger mark-down in wage-inflationary periods. Formally, this follows from $\gamma E_t \Lambda_{t,t+1} \psi_{t+1}$ (from Proposition 1) being an expectation over a convex function (ψ_{t+1} is kinked and non-negative). Indeed, in the absence of uncertainty ($\sigma = 0$), the mark-down is identically zero.

1.3 Excessive wage inflation—An open economy model

I turn to a business cycle model of a small open economy that is characterized by a fixed nominal exchange rate, by labour unions and by downward wage rigidity. The goal is to assess the performance of this economy vis-à-vis an efficient benchmark, where the benchmark is provided by a constrained social planner (recall Section 1.2). Intuitively, the planner pins down the inefficiencies that generate insufficient wage restraint in this economy. The open economy dimension makes the model quantitatively applicable to euro area countries—a natural application given that wage restraint is particularly important under fixed exchange rates (recall Section 1.2), and given the high wage inflation that has been observed in some euro area countries in the run-up to the Great Recession.

In this section I introduce the model details: I lay out the interaction on markets between the three central agents households, firms, and labour unions (the 'laissez-faire equilibrium'). I also define this economy's efficient benchmark in the spirit of Definition 2, and I discuss the model's key theoretical insights.

1.3.1 Laissez-faire equilibrium

Households:

The economy is populated by a continuum of households that trade goods on international markets. There is also a nontradeable sector such that households derive utility from both tradeable and nontradeable consumption. Households trade a complete set of assets with the rest of the world. Households are organized into $J \geq 1$ labour unions that take the labour supply decision on their behalf. Finally, the business cycle is driven by demand and cost-push shocks, which constitute the stochastic structure of the model.

perceived as 'additional leisure') would also be associated with a larger ex-ante endogenous mark-down.

A household of labour type / organized in union $j \in \{1, ..., J\}$ maximizes

$$\mathcal{U}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{U(A(c_t^T(j), c_t^N(j))) - V(h_t(j))\}$$

subject to the period budget constraint

$$p_t^T c_t^T(j) + p_t^N c_t^N(j) + E_t s_{t,t+1} d_{t+1}(j) = p_t^T y^T + w_t(j) h_t(j) + \Phi_t + d_t(j),$$

and subject to a no-Ponzi constraint.

Here $U = (c^{1-\kappa})/(1-\kappa)$ is consumption utility with risk aversion $\kappa > 1$, where consumption is an aggregate $c = A(c^T, c^N) = [\omega (c^T)^{1-1/\zeta} + (1-\omega)(c^N)^{1-1/\zeta}]^{1/(1-1/\zeta)}$ with $\omega \in (0,1)$ a share parameter and with elasticity parameter $\zeta > 0$. Superscripts 'T' and 'N' distinguish tradeable and nontradeable consumption and price indexes, respectively, and d_t are (cross-border) state contingent claims with associated discount factor $s_{t,t+1}$. Households receive an endowment of y^T tradeable goods each period—which, without loss of generality, I assume to be constant—as well as wage income $w_t(j)h_t(j)$ and firm profits Φ_t.[11]

I assume that the law of one price holds for tradeable goods, such that $p_t^T = \bar{e} p_t^{T,*}$, where $\bar{e}$ is the fixed nominal exchange rate and $p_t^{T,*}$ is the price of tradeable goods in the rest of the world. For ease of exposition, I normalize the level of the fixed exchange rate to unity, and equivalently I assume that the price of tradeables in the rest of the world is constant and equal to unity. As a result, also $p_t^T = 1$.

First order conditions with respect to state contingent claims link the marginal utility of tradeable consumption to its counterpart in the rest of the world: $U_{c_t^T} = U^*_{c_t^{T,*}}$—the 'Backus Smith condition' (Backus and Smith, 1993).[12] Note that, as in Galí and Monacelli (2016), I assume exogenous variation in the marginal utility of tradeable consumption abroad—$U^*_{c_t^{T,*}}$ is exogenous and stochastic—with the interpretation of a demand shock.[13] I also make the following parametric assumption

Assumption 1. *The intertemporal and intratemporal consumption elasticity relate to each other via* $1/\kappa = \zeta$.

The combination of complete international asset markets and Assumption 1 are key ingredients in my analysis. The combination of both guarantees that tradeable consumption and thus the current account become effectively exogenous, allowing me to abstract from 'over-

[11] As I show below, households' consumption of tradeables is independent of y^T via the insurance received in complete international asset markets, making possible variations in y^T irrelevant for the allocation (variation in y^T would only be reflected in the current account). Thus for simplicity, I assume y^T to be constant.

[12] In the general case, also the (tradeable-goods) real exchange rate would appear in this equation. However, because of my normalizations above, $p_t^{T,*}\bar{e}/p_t^T = 1$.

[13] This shock generates exogenous variation in tradeable consumption and therefore captures changes in external borrowing conditions or the terms of trade (as in Schmitt-Grohé and Uribe 2016).

borrowing externalities' that are the key mechanism in related work and that are therefore well understood. Rather, my analysis will be focussed purely on inefficient *wage* dynamics for a given size of the current account.[14]

Technically, Assumption 1 guarantees that substitution and income effects between tradeable and nontradeable consumption exactly balance, such that the Backus Smith condition is

$$\omega(c_t^T)^{-\kappa} = U^*_{c_t^{T,*}}. \tag{1.3.1}$$

Tradeable consumption thus becomes effectively exogenous.[15] Note that in equation (1.3.1), I have already omitted household index j by anticipating that, in equilibrium, all households will behave identically.

First order conditions with respect to tradeable and nontradeable consumption link the respective marginal utilities to their relative price: $U_{c_t^N}/U_{c_t^T} = p_t^N$, where I have used that $p_t^T = 1$. This can be written as

$$(1-\omega)(c_t^N)^{-\kappa}/U^*_{c_t^{T,*}} = p_t^N, \tag{1.3.2}$$

where I have used Assumption 1 and the Backus Smith condition (1.3.1) above. Equation (1.3.2) constitutes a demand function for nontradeable consumption.

Finally, the labour supply decision is discussed below, given that households are organized into labour union which take the labour supply decision on their behalf.

Firms:

A continuum of identical firms hire labour, maximize profits, and sell the goods produced on the domestic market. That is, I assume that firms operate in the nontradeable sector only.[16] While operating under perfect competition in the goods market, firms face monopolistic unions in the labour market. I assume that firms require workers from all the unions for their operation, and that the bundling technology is of the CES type with elasticity of substitution $\theta_t > 1$. I assume that this elasticity is stochastic, with the interpretation of a cost-push shock (see below). Aside the demand shock above, this cost push shock is the second (and last) source of uncertainty in the model.

[14] Indeed, without Assumption 1 or with incomplete international asset markets, my model would feature the overborrowing externality documented in Schmitt-Grohé and Uribe (2016). As a result, the constrained social planner in Definition 4 would fix both this externality as well as the inefficient wage dynamics which are the focus of my analysis, making the results much less transparant. Another open economy business cycle model that relies on inefficient current account dynamis is Bianchi (2011).

[15] I derive equation (1.3.1) in Appendix A. Assumption 1 is also known as 'Cole-Obstfeld condition' (Cole and Obstfeld, 1991). In general, U_{c^T} is a function in both c^T and h, however with Assumption 1 the dependence on h disappears. In my calibration to Greece, $\zeta = .44$ such that the implied risk aversion is $\kappa = 2.27$—well within the range of plausible values for this parameter.

[16] This simplifying assumption reflects that in most (developed) economies, nontradeables are the largest share of domestic output. For example in my calibration to Greece below, the share of nontradeables is 74 percent.

The bundling technology is $h_t = (\sum 1/J\ h_t(j)^{(\theta_t-1)/\theta_t})^{\theta_t/(\theta_t-1)}$ such that union-type labour demand curves are $h_t(j) = (w_t(j)/w_t)^{-\theta_t} h_t$ with the corresponding cost-minimizing wage index $w_t = (\sum 1/J\ w_t(j)^{1-\theta_t})^{1/(1-\theta_t)}$.[17] Firms also maximize profits $\Phi_t = \max\ \{p_t^N F(h_t) - w_t h_t\}$, with first order condition

$$p_t^N F'(h_t) = w_t. \tag{1.3.3}$$

This first order condition must always hold with equality, given that firms demand the marginal hour as long as $p_t^N F'(h_t) > w_t$—which is always true for small enough h_t if wages are free in their ability to rise. As a functional form for the production technology, I use $F(h) = h^\alpha$ as in the stylized model above.

Labour unions:

The labour supply side is characterised by a total number of $J \geq 1$ equal-sized labour unions. As a result, $1/J$ denotes the share of households organized into union $j \in \{1, ..., J\}$. In terms of modelling assumptions, here I follow closely the literature on centralized wage bargaining (see for example Alesina and Perotti 1997 or Guzzo and Velasco 1999). In addition, I impose the following key friction: I assume that nominal wages are downward rigid at the union level. This assumption, in combination with the fixed nominal exchange rate that pins down the price of tradeable consumption (see above), generates the (real) rigidities in the labour market that give relevance to wage restraint.[18]

The problem of individual union $j \in \{1, ..., J\}$ is summarized in Definition 3. To understand this problem, it is key to recall that unions have market power over the wage of their members if the labour types are imperfectly substitutable ($\theta < \infty$). As a result, the unions' problem is dynamic, subject to downward wage rigidity (condition i), and subject to union-type labour demand curves (condition ii). It is also key that large unions ($J < \infty$) internalize the effect of their individual on the aggregate wage (condition iii). The aggregate wage w_t matters for the unions, because an increase in w_t reduces the demand for aggregate labour h_t via conditions iv)-v) and thereby the demand for union-type labour $h_t(j)$ via h_t appearing in condition ii). Specifically, as the aggregate wage w_t rises, the firms marginal product of labour $F'(h_t)$ must rise, such that h_t must fall (condition iv).

[17] See the Appendix A for a derivation.

[18] Frictions in the tradeable goods sector such as consumption home-bias would allow for short-run departures of the price of tradeable consumption from unity (i.e. changes in the terms of trade). However this would not affect that, under a fixed nominal exchange rate, tradeable goods prices in the long run are pinned down by the nominal anchor country (e.g. Corsetti et al., 2013b). Wage restraint would therefore be relevant even in this extended environment.

Definition 3. [INDIVIDUAL UNION PROBLEM] *The problem of individual union $j \in \{1, ..., J\}$ is to solve the following Bellman equation*

$$\mathcal{W}(w_{t-1}(j)) = \max \{U_{c_t^T} w_t(j) h_t(j) - V(h_t(j)) + \beta E_t \mathcal{W}(w_t(j))\}$$

s.t.

i) $\quad w_t(j) \geq \gamma w_{t-1}(j) \quad$ (downward wage rigidity)

ii) $\quad h_t(j) = (w_t(j)/w_t)^{-\theta_t} h_t \quad$ (union-type labour demand)

iii) $\quad w_t = (\sum 1/J \; w_t(i)^{1-\theta_t})^{1/(1-\theta_t)} \quad$ (the wage index)

iv) $\quad p_t^N F'(h_t) = w_t \quad$ ((aggregate) labour demand I)

v) $\quad p_t^N = (1-\omega)(F(h_t))^{-\kappa} / U^*_{c_t^{T,*}} \quad$ ((aggregate) labour demand II)

for given $\{\{w_t(-j)\}\}, \{\{h_t(-j)\}\}$ and for given exogenous $\{\theta_t, U^*_{c_t^{T,*}}\}$ and thereby $\{U_{c_t^T}\}$ from Backus Smith condition (1.3.1) *above.*[19]

However, there is an opposing effect. The fall in h_t implies less production in the nontradebale sector (where firms operate), such that the relative price of nontradeable goods p_t^N rises due to their increased scarcity (condition v).[20] This, in turn, reduces the drop in employment in the nontradeable sector because firms are able to sell their products expensive (condition iv). Lastly, note that I am also assuming that the individual union takes the action of the other unions as given. That is, to keep the analysis tractable, I abstract from dynamic strategic interaction (e.g. Maskin and Tirole, 2001).
I solve this problem in Appendix A. The solution method consists of attaching a Lagrange multiplier $\psi_t(j) \geq 0$ to downward wage rigidity at the union level (condition i). In equilibrium, all unions behave identically such that index j disappears and only aggregates matter.

Proposition 2. [LABOUR UNIONS] *Assume that the downward wage rigidity is not binding in the current period ($\psi_t = 0$). The labour supply side in the laissez-faire equilibrium is then characterized by the following first order condition*

$$(\tilde{\theta}_t - 1)/\tilde{\theta}_t \; U_{c_t^T} w_t + \gamma E_t \tilde{\Lambda}_{t,t+1} \psi_{t+1} = V'(h_t), \tag{1.3.4}$$

where $\tilde{\Lambda}_{t,t+1} \equiv \beta(\tilde{\Omega}_{t+1}/\tilde{\Omega}_t) > 0$ is a stochastic discount factor, and where $\tilde{\theta}_t$ is a time-varying elasticity. The expressions for $\tilde{\Omega}_t$ and $\tilde{\theta}_t$ are given in the main text below.

Along with equations (1.3.1)-(1.3.3), the definition for $\tilde{\Omega}_t$ and $\tilde{\theta}_t$ given below, market clearing $F(h_t) = c_t^N$ and inequality constraints and slackness conditions as implied by the downward wage rigidity, equation (1.3.4) fully determines the equilibrium of the model. The full set of equilibrium conditions is summarized in Appendix A.

[19] Here I summarize $\{w_t(-j)\} = \{w_t(1), ..., w_t(j-1), w_t(j+1), ..., w_t(J)\}$ and similarly for $\{h_t(-j)\}$.
[20] Here it is used that $c_t^N = F(h_t)$ in equilibrium, that is, all nontradeable output must be consumed within the same period. See below the summary of equilibrium conditions.

1.3.2 Defining the efficient benchmark

Define now the benchmark for efficient wage restraint in this economy, which turns out to be close-to-identical to the constrained efficient allocation obtained in Section 1.2 above. The social planner first allocates the same amount of union-type labour $h_t(j) = h_t \forall j$ to all the unions, as the unions are symmetric and the bundling technology is of the CES type. This also implies that $w_t(j) = w_t \forall j$. Thereafter, the planner chooses aggregate working hours $\{h_t\}$ and aggregate wages $\{w_t\}$ according to

Definition 4. [CONSTRAINED EFFICIENCY] *The constrained efficient equilibrium is the solution to the following Bellman equation*

$$\mathcal{V}(w_{t-1}) = \max \{U(A(c_t^T, F(h_t))) - V(h_t) + \beta E_t \mathcal{V}(w_t)\}$$

s.t.

$$i) \quad p_t^N F'(h_t) = w_t \quad \textit{((aggregate) labour demand I)}$$

$$ii) \quad p_t^N = (1-\omega)(F(h_t))^{-\kappa}/U^*_{c_t^{T,*}} \quad \textit{((aggregate) labour demand II)}$$

$$iii) \quad w_t \geq \gamma w_{t-1} \quad \textit{(downward wage rigidity)}$$

for given exogenous $\{U^*_{c_t^{T,*}}\}$ *and thereby* $\{c_t^T\}$ *from Backus Smith condition* (1.3.1) *above.*

I solve this problem in Appendix A. Again, the solution method consists of attaching a Lagrange multiplier $\psi_t \geq 0$ to the inequality constraint on wages (condition iii). The relevant first order condition is

Proposition 3. [WAGE RESTRAINT] *Assume that the downward wage rigidity is not binding in the current period ($\psi_t = 0$). The constrained efficient equilibrium is then characterized by the following first order condition*

$$U_{c_t^T} w_t + \gamma E_t \Lambda_{t,t+1} \psi_{t+1} = V'(h_t), \tag{1.3.5}$$

where $\Lambda_{t,t+1} \equiv \beta(\Omega_{t+1}/\Omega_t) > 0$ *is a stochastic discount factor, with* $\Omega_t \equiv -(\partial h_t / \partial w_t) = 1/(1-\alpha+\alpha/\zeta)\ h_t/w_t > 0$ *the slope of (aggregate) labour demand at time* t.

Compare equations (1.3.4) and (1.3.5) to see that labour supply decisions are not efficient in the laissez-faire equilibrium. This equilibrium, therefore, generates an inefficient amount of wage restraint. Or equivalently, it generates an inefficient amount of wage inflation when the economy is in an expansion, as studied next.

1.3.3 What undermines wage restraint?

Inspecting equation (1.3.4) reveals that the laissez-faire equilibrium generates wage restraint, yet to a socially inefficient extent. It generates wage restraint, because an insurance term against downward wage rigidity appears in the first order condition for labour supply. Yet, the extent of wage restraint is not optimal for two reasons.

The first is a standard monopolistic distortion—yet with non-standard consequences. Specifically, the term $(\tilde{\theta}_t - 1)/\tilde{\theta}_t$ distorts the equilibrium in that, ceteris paribus, it induces a mark-up of the real wage above the marginal rate of substitution. As argued above, when wages are downward rigid it is required that real wages be set at a mark-*down* below the marginal rate of substitution. Monopolistic mark-ups therefore push wages in the 'exact wrong direction', and are in this sense 'doubly inefficient' when wages are downward rigid.

The monopolistic mark-up that is charged by the unions is governed by elasticity $\tilde{\theta}_t$, which is given by the following expression

$$\tilde{\theta}_t \equiv \frac{J-1}{J}\theta_t + \frac{1}{J}(1/(1-\alpha+\alpha/\zeta)), \tag{1.3.6}$$

where $1/(1-\alpha+\alpha/\zeta) = -(\partial h_t/\partial w_t)(w_t/h_t) \equiv \epsilon^{h,w}$ is the elasticity of aggregate labour demand with respect to the aggregate wage. This is intuitive: When unions are small, $\tilde{\theta}_t = \theta_t$, reflecting monopolistic competition in the labour market. Instead in the case of one single union, $\tilde{\theta}_t = \epsilon^{h,w}$, reflecting a monopolist on the labour supply side who fully exploits the aggregate labour demand curve.

In terms of size of the mark-up, it is instructive to observe that for values of the intratemporal consumption elasticity $\zeta < 1$—which I consider in my calibration below—the elasticity of aggregate labour demand is strictly below unity: $\epsilon^{h,w} < 1$.[21] Because $\theta_t > 1$ is required to be above unity, and because the effective elasticity $\tilde{\theta}_t$ that determines the monopolistic mark-up is a weighted average between θ_t and $\epsilon^{h,w}$, it follows that larger unions charge strictly larger monopolistic mark-ups[22]

Remark 1. *Assume that $\zeta < 1$. Then in the laissez-faire equilibrium, the larger the labour unions (the larger $1/J$), the larger the monopolistic wage mark-ups $\tilde{\theta}_t/(\tilde{\theta}_t - 1) \geq 1$.*

Turn now to the second source of inefficiency. The second source of inefficiency is more

[21] Intuitively, as mentioned above, an increase in w_t has an effect on h_t through two channels. First, it increases the marginal product of labour, lowering the amount of hours worked. This has an elasticity of $1/(1-\alpha) > 1$. However, there is a partial offsetting effect: As hours drop, the price of nontradeables rises due to less production in the nontradeable sector. This allows firms to sell their products expensive, keeping hours high. If ζ is low, the price movement is strong and the offsetting effect is large. Therefore, in this case the combined elasticity $1/(1-\alpha+\alpha/\zeta) < 1$.

[22] Indeed there is a critical value for $1/J$ above which the monopolistic mark-up becomes infinity. This is when $\tilde{\theta}_t = 1$ or $1/J = (1-\theta_t)/(\epsilon^{h,w} - \theta_t)$—or, in my calibration below, for values above $1/J \approx .88$.

nuanced than the first. Technically, it arises from the fact that the market uses a different stochastic discount factor than the constrained social planner to insure against downward wage rigidity: $\tilde{\Lambda}_{t,t+1} \neq \Lambda_{t,t+1}$ in general (compare equations (1.3.4) and (1.3.5)) Plainly, the market therefore assesses the ex-ante risk that is associated with the rigidity differently than the constrained social planner.

Furthermore, this difference can be traced to $\tilde{\Omega}_t \neq \Omega_t$ in general. Recalling from Proposition 3 that Ω_t measures the slope of aggregate labour demand, the expression for $\tilde{\Omega}_t$ is

$$\tilde{\Omega}_t \equiv \frac{J-1}{J}\left(-\partial h_t(j)/\partial w_t(j)\right) + \frac{1}{J}\Omega_t, \tag{1.3.7}$$

a weighted average between the slope of *individual* and aggregate labour demand.[23] Hence, the differential discount factor results from small unions facing a different labour demand curve than the whole economy.

This is an externality, with the following intuition. By engaging in wage restraint individually, a small union may have little impact on the aggregate loss in hours worked if the rigidity binds in the next period. The union in this case faces a 'flat' (individual) expected labour demand curve. What the union ignores, however, is that by engaging in wage restraint individually thereby lowering the wage index, it creates an incentive for the other unions to *in turn* engage in wage restraint as their workers have now become relative more expensive. Moreover, as the unions *collectively* engage in wage restraint, the impact on the aggregate loss in hours worked if the rigidity binds in the next period may be very large. The whole economy in this case faces a 'steep' (aggregate) expected labour demand curve.

In a nutshell, the laissez-faire equilibrium therefore underinsures against downward wage rigidity to the extent that the individual union perceives the aggregate consequences of its keeping wages low individually to be small. This also implies that larger-sized unions insure strictly better against downward wage rigidity, as they face strictly less of this coordination problem. Indeed, equation (1.3.7) shows that the laissez-faire equilibrium insures efficiently in the case of one single union in the labour market ($\tilde{\Omega}_t = \Omega_t$ as $J = 1$).

Remark 2. *In the laissez-faire equilibrium, the larger the labour unions (the larger $1/J$), the better the insurance $\gamma E_t \tilde{\Lambda}_{t,t+1}\psi_{t+1}$ against downward wage rigidity.*

Figure 1.4 visualizes these insights in an overview diagram. Under flexible wages it is optimal that the wage equates the households' marginal rate of substitution between consumption and leisure. Yet this is inefficient when wages are downward rigid. Instead in this case, in

[23] Expression $-\partial h_t(j)/\partial w_t(j)$ denotes the derivative of individual labour demand, $h_t(j) = (w_t(j)/w_t)^{-\theta_t} h_t$, with respect to the individual wage, $w_t(j)$, for given (constant) w_t and h_t, i.e. the slope of labour demand that is faced by an infinitisimal union. This derivative can be computed explicitly: I obtain $\theta_t h_t(j)/w_t(j)$, or, by ommitting index j, $\theta_t h_t/w_t$.

periods when the wage rigidity is slack, it is optimal that wages be set below the marginal rate of substitution. I call this the wage restraint channel. Monopolistic wage mark-ups due to market power in the labour market are 'doubly inefficient', as they push the wages in the exact wrong direction—namely, above the marginal rate of substitution. Finally, while (forward-looking) wage setters may provide insurance against downward wage rigidity in principle, they may do so to an inefficient extent because of an externality. I have also shown that the relative importance of the two inefficiencies depends on the size of the unions, with the effects on wage restraint of unions growing larger-sized going in opposite directions (Remarks 1-2). Similar results have been emphasized by the literature on centralized wage bargaining (where the original contribution is Calmfors and Driffill, 1988), and I provide some further discussion of this in Section 1.5.

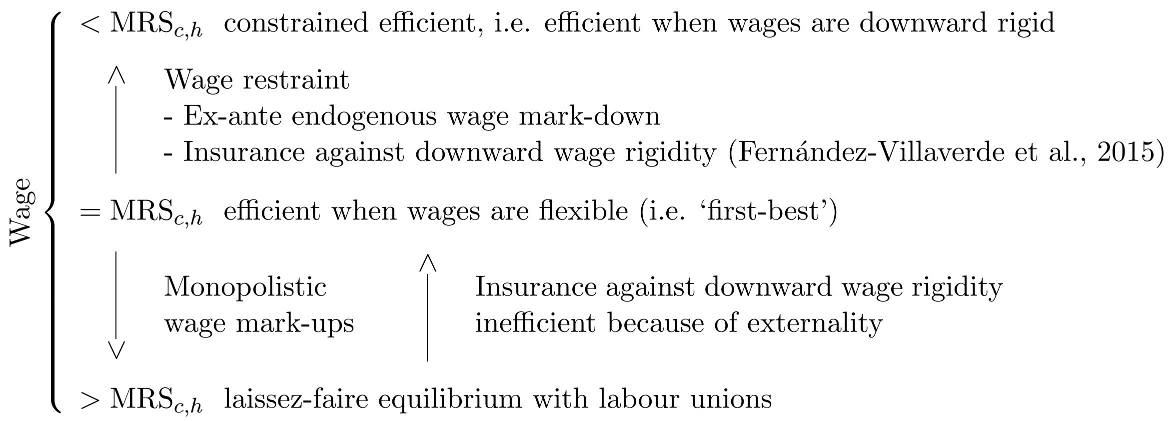

Figure 1.4: Wage restraint: An overview diagram.

1.4 Quantitative analysis

The countries in the euro periphery have recovered particularly slowly following the Great Recession (the 'post-crisis slump', Kollmann et al., 2016). One popular narrative for the causes of this poor performance is accumulated imbalances in the *run-up* to the Great Recession—in the boom years of the euro periphery since the early 2000s. In particular, it is argued that pre-2008 wage inflation has made the euro periphery overly expensive thereby has brought it into a vulnerable position before the crisis struck (e.g., Baldwin and Giavazzi, 2015; Chen et al., 2013; Eichengreen, 2010; Krugman, 2012).

In this section I turn to quantitative analysis of the open economy model. First, while in the numerical illustration of the stylized model in Section 1.2 I have explored wage restraint, here I explore the inefficiencies that create *insufficient* wage restraint.

Second, as a result, I quantitatively assess the above narrative of the euro periphery crisis. To do so I inspect the case of Greece. This reflects that Greece was amongst the euro periphery countries that experienced the largest amount of wage inflation before the crisis and subsequently also the deepest recession (see below). My main question is the following: To what extent has lacking wage restraint during 2001-2008 contributed to the Greek downturn 2009-2016? I find that lacking wage restraint had a sizeable impact: The Greek real appreciation pre 2008 was inefficient in its entirety, pushing the Greek unemployment rate post 2008 15 percentage points above its (constrained) efficient level, creating a welfare loss of 0.4 percent of permanent consumption.

1.4.1 Solution method

The model features an occasionally binding constraint, making numerical analysis challenging. Moreover, unlike in most models of occasionally binding constraints, the multiplier associated with the constraint appears explicitly in a conditional expectation—see equation (1.3.4). As a result, conventional methods of dealing with occasionally binding constraints, such as Guerrieri and Iacoviello (2015), fail in this environment. Instead I am using a version of the parameterized expectations algorithm (PEA), developed by den Haan and Marcet (1990) and applied by Christiano and Fisher (2000) in an environment of occasionally binding constraints, in order to approximate well the conditional expectation. Instead, the constrained planning problem is solved using value function iteration (as in Section 1.2). The Appendix C contains further details on the numerical implementation.

1.4.2 Model calibration

I set $\beta = .99$ because the model is quarterly frequency. I set the steady-state marginal utility of tradeable consumption $U^*_{c^T,*} = .3756$ such that, in steady state, $c^T = c^N = h^\alpha$. This implies that ω measures the share of tradeable in total consumption in steady state. I therefore set $\omega = .26$ as well as the intratemporal consumption elasticity $\zeta = .44$ according to an estimate for Greece from Schmitt-Grohé and Uribe (2016). Note that $\zeta = .44$ implies a risk aversion parameter $\kappa = 2.27$ from Assumption 1—a number in line with conventional values for risk aversion used in open economy business cycle models (e.g. Bianchi (2011) uses a value of $\kappa = 2$). I also set $y^T = c^T$ such that the current account to GDP ratio $1 - c^T/y^T$, which I am studying further below, is zero in steady state.

Concerning the labour market I am using an inverse Frisch elasticity of $\varphi = 2$, following the calibration to the euro periphery in Eggertsson et al. (2014). Recall from Section 1.2 that this is a conservative value for φ, in that it implies a rather small role for wage restraint. Again

Parameter		*Value*	*Target/Source*
κ	Risk aversion	2.27	Assumption 1 with $\zeta = 0.44$
ω	Share tradeable consumption	.26	Schmitt-Grohé and Uribe (2016)
ζ	Intratemp consumption elast	.44	Schmitt-Grohé and Uribe (2016)
β	Time discount factor	.99	Quarterly model
y^T	Tradeable endowment	.851	Zero current account in steady state
α	Labour share	.75	Schmitt-Grohé and Uribe (2016)
φ	Inverse Frisch elasticity	2	Eggertsson et al. (2014)
γ	Downward wage rigidity	.99	Schmitt-Grohé and Uribe (2016)
$1/J$	Share households per union	.249	OECD Trade union density

Table 1.2: Calibration table. Upper part: households and firms. Lower part: labour market. The stochastic structure of the model is described in the main text.

in line with Schmitt-Grohé and Uribe (2016), I am using a rigidity parameter $\gamma = .99$ (such that wages can fall by at most 4 percent per year) and a labour share $\alpha = .75$. This implies an aggregate labour demand elasticity of $1/(1-\alpha+\alpha/\zeta) = .51$, a value in line with available empirical estimates (Lichter et al., 2015).

A key parameter is the number of labour unions J. As a baseline value, I am using $1/J = .249$ in line with the OECD Economic Outlook's Trade Union Density for Greece in 2001. Trade Union Density measures the ratio of wage and salary earners that are trade union members, divided by the total number of wage and salary earners. This measure does therefore not exactly correspond to $1/J$ as defined in the paper. However, it gives a sense of the degree of Greek unionisation and therefore serves as a useful baseline. I discuss the implications of different values for $1/J$ in Section 1.5. Regarding the elasticity between labour types, I follow the calibration in Galí and Monacelli (2016) for the euro periphery and set a steady state value for the monopolistic wage mark-up of 30 percent (this is also in line with the estimate for the euro zone from Smets and Wouters (2003), who obtain a mark-up of 28.9 percent). From equation (1.3.6) for $\tilde{\theta}_t$, this yields $\theta = 5.67$ in steady state.

Recall that the stochastic structure of the model consists of demand and cost-push shocks, reflecting my interest in wage dynamics at business cycle frequency.[24] Demand shocks arise from fluctuations in the marginal utility of tradeable consumption abroad $U^*_{c_t^{T,*}}$, cost push shocks arise from fluctuations in the elasticity of substitution between labour types θ_t. As for the cost-push shocks, Smets and Wouters (2003) estimate an iid process for the log monopolistic wage mark-up for the euro area, and obtain a volatility of .026. I therefore simulate a

[24] I thus abstract from other factors which drive wage and real exchange rate dynamics in open economies, such as productivity trends in the traded goods sector giving rise to the 'Balassa Samuelson' effect (see Berka et al., 2014, for a recent study).

series $-\log[(\tilde{\theta}_t - 1)/\tilde{\theta}_t]$ iid normal with volatility .026, back out the resulting series for $\log(\theta_t)$ and compute the volatility of the resulting series. This yields

$$\log(\theta_t) = \log(5.67) + 0.084 \times \epsilon_t.$$

As for the demand shocks, I assume an AR(1)-process for (the log of) $U^*_{c_t^{T,*}}$ with autocorrelation .95 and calibrate the associated volatility such that the mean unemployment rate in the laissez-faire equilibrium is 16.8 percent, where the unemployment rate u_t is defined in $U_{c_t^T} w_t = V'(h_t(1 + u_t/100))$ (see below). This mean unemployment rate corresponds to the mean unemployment rate in Greece from 2001Q1 (when Greece joined the euro) until 2016Q3. I obtain

$$\log(U^*_{c_t^{T,*}}) = (1 - 0.95) \times \log(0.3756) + 0.95 \times \log(U^*_{c_{t-1}^{T,*}}) + 0.052 \times \epsilon_t.$$

The parameters underlying this calibration are given in Table 1.2 above.

1.4.3 The Greek boom and bust 2001-2016

During its economic expansion 2001-2008, the Greek economy had witnessed a 10-15 percentage points higher increase in nominal labour costs than the rest of the euro area during the same period.[25] This has been accompanied by an appreciation of the Greek real exchange rate vis-à-vis all euro area by about 10 percent, by large current account deficits, and by a decline in the Greek unemployment rate. In turn, starting with the crisis in 2008, nominal labour costs in Greece have hardly declined and there has hardly been any depreciation in the Greek real exchange rate. Yet current account deficits have plummeted and the unemployment rate has quickly exceeded 20 percent.

To show that the model can account for these dynamics, and once this is established, to assess to what extent lacking wage restraint during the boom has contributed to the crisis, I now conduct the following experiment.

Simulating the shock processes described above, I simulate the calibrated economy over a long horizon. I isolate the periods where over a time span of 64 quarters (16 years), two criteria are satisfied. First, after 32 quarters (8 years) wages must have increased by 10-15 percent. Second, in quarter 52 (end of year 13) the unemployment rate must exceed 20 percent. This procedure therefore isolates model-implied boom-bust cycles as the one in Greece 2001-2016 described above. Next, I save the state variables at the onset of each of the cycles as well as the shocks that drive the entire cycle, and apply policy functions from the constrained social

[25] As in Figure 1.1, here and in the following I am focussing on nominal unit labour costs, defined as $w_t/(y_t/h_t)$ (the nominal wage divided by output per worker), as a measure of an economy's nominal wage bill. Note that if the labour share is large, $w_t/(y_t/h_t) \approx w_t$. However, a similar pricture obtains for other measures of nominal labour cost, like for example Eurostat's nominal labour cost index.

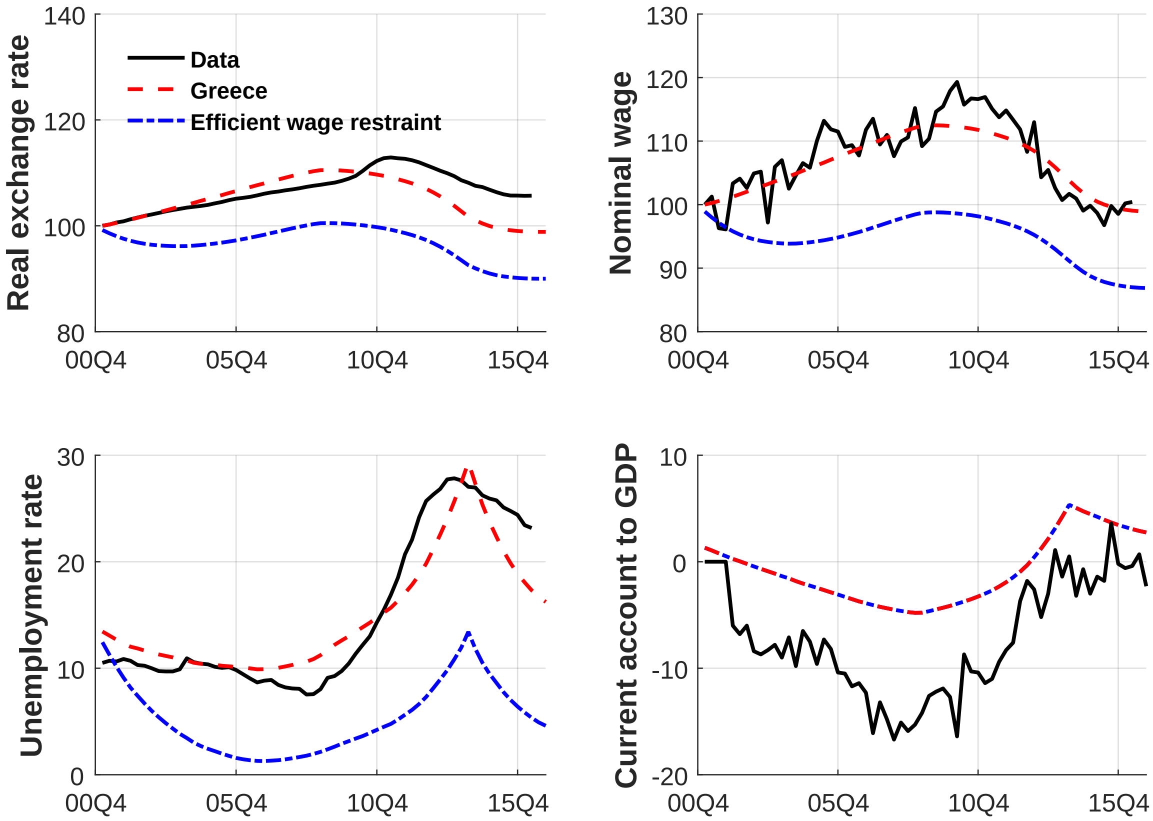

Figure 1.5: Real exchange rate: Greek over all-euro-area HICP. Nominal wage: Greek over all-euro-area nominal unit labour cost. Both are normalized to 100 in 2001Q1. Unemployment rate in percent. Current account to GDP in percent. Quarters on x-axis: 2001Q1-2016Q4. In the model, the real exchange rate is $(\omega + (1-\omega)(p_t^N)^{1-\zeta})^{1/(1-\zeta)}$, the unemployment rate is $U_{c_t^T} w_t = V'(h_t(1+u_t/100))$, and the current account to GDP ratio is $(1 - c_t^T/y^T) \times 100$.

planner—yielding a counter factual. Finally, I average over the isolated periods in order to eliminate initial conditions, i.e. get an 'average' cycle.

Figure 1.5 shows the result.[26] The black solid line is Greek data, the red dashed line is the calibrated laissez-faire equilibrium, the blue dashed-dotted line is the counter factual. First focus on the black solid and the red dashed lines to contrast data and model predictions. By construction, the model captures the rise in wages until 2008 well, and also the peak unemployment rate in 2013. Yet the model performs well also along non-targeted margins. The dynamics of the drop in wages following 2008 are captured well, as are the unemployment

[26] In the simulation, cycles of this type occur about 1.8 percent of the time, or, given quarterly calibration, about once every 15 years.

dynamics before and after 2013. Note that the combination of high unemployment and slowly declining labour costs after 2008 is per se puzzling, motivating the choice of downward wage rigidity as the key model friction in the first place.

The model-implied real exchange rate also follows its empirical counterpart, while the model underpredicts the amplitude of the current account dynamics—more on this below. In the Appendix B I show the shock series underlying the cycle. It turns out that the cycle is driven by an increase then reversal in tradeable absorption, mirroring the development of the current account in Figure 1.5, while cost-push shocks play a subordinate role. This is consistent with studies which have found current account dynamics ('capital inflows') to have contributed to the euro area crisis (Martin and Philippon, 2016).

Turning to the counter factual, note that there are dramatic differences in terms of model dynamics. Under efficient wage restraint, the rise in wages (in excess of the euro average) and the real appreciation are cut in their entirety. Moreover, both variables display non-monotonic dynamics: They first fall to a strictly lower level before they start to increase throughout the remainder of the boom—however not by enough to make up for the shortfall accumulated in the first periods, such that overall the wages and the real exchange rate reach lower levels in 2008 than in 2001. Moreover, the unemployment rate during the boom declines quickly in the counter factual, falling to about 2-3 percent.

During the bust 2009-2016, the unemployment rate rises to 13 percent and therefore spikes at about 15 percentage points less than in actuality. This is the consequence of wage restraint during the boom: The bust following 2008 turns out much milder. In turn, the dynamics of the drop in wages and the real exchange rate during the bust in the counter factual are similar as in actuality, yet both at an overall lower level.

Lastly, note that throughout the entire cycle, the planner leaves the dynamics of the current account unaffected. As discussed in Section 1.3, this reflects a deliberate modelling choice: I abstract from inefficient current account dynamics to make stark the implications of lacking wage restraint for macroeconomic outcomes. The price to pay is that, as a result, the model underpredicts the amplitude of the current account dynamics. That is, the model does not capture amplification via endogenous current account contraction (a 'sudden stop', see Bianchi 2011 for an example of this mechanism).

1.4.4 Inspecting the mechanism: Inefficient wage dynamics

To understand the dynamics in Figure 1.5, consider Figure 1.6 which shows the policy functions in tradeable absorption c_t^T for nominal wages and the endogenous mark-down (or 'mark-

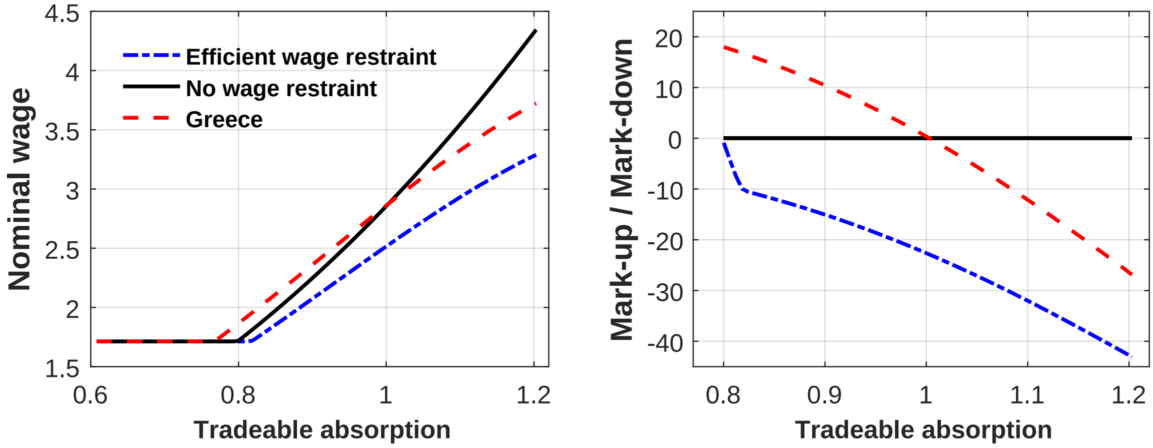

Figure 1.6: Policy functions for nominal wages w_t and the mark-up / mark-down $\mathcal{M}_t$ in tradeable absorption c_t^T. The other two state variables w_{t-1} and θ_t are kept at their non-stochastic steady states.

up' when positive) $\mathcal{M}_t$.[27,28] Figure 1.6 thus is the equivalent of Figure 1.2 from Section 1.2. Intuitively, exogenous variation in tradeable absorption in the open economy ('demand shock') plays a similar role as the technology shock in the stylized economy: Positive shocks induce (wage-)inflationary pressure and an increase in hours worked in the nontradeable sector, such that the economy enters a 'boom' associated with a strengthening in the real exchange rate (compare Schmitt-Grohé and Uribe, 2016).[29]

In the figure I distinguish the constrained efficient outcome (dashed-dotted in blue), the laissez-faire equilibrium (dashed in red) and, as an informative benchmark, the 'no wage restraint' scenario defined by $U_{c_t^T} w_t = V'(h_t)$ (solid in black). Intuitively, the latter captures the (first-best) allocation if wage rigidities were absent (recall Section 1.2 and Figure 1.4 above). Comparing the benchmark black solid line and the social planner, the first observation is that the amount of wage restraint required in this economy is quantitatively remarkable: For large demand shocks the efficient mark-down exceeds 40 percent (the average is 14 percent—see Section 1.4.5 which presents summary statistics).

Now consider the red dashed line which represents the calibration to Greece. Here we see that, for intermediate-sized demand shocks, the level of wages is even higher than under

[27] Recall the one-to-one mapping between the marginal utility of tradeable consumption abroad and tradeable consumption, equation (1.3.1). For ease of interpretation, I therefore plot the policies against c_t^T.

[28] As in Section 1.2, the mark-up / mark-down $\mathcal{M}_t$ is given in $U_{c_t^T} w_t = (1 + \mathcal{M}_t/100)V'(h_t)$ and therefore defined in percent.

[29] The policies for hours worked and the real exchange rate are shown in the Appendix B. The policies for all four variables in cost-push shocks are also shown in the Appendix B. In contrast to demand shocks, cost-push shocks are stagflationary—inducing wage inflation and a strengthening in the real exchange rate, however associated with less employment in the nontradeable sector.

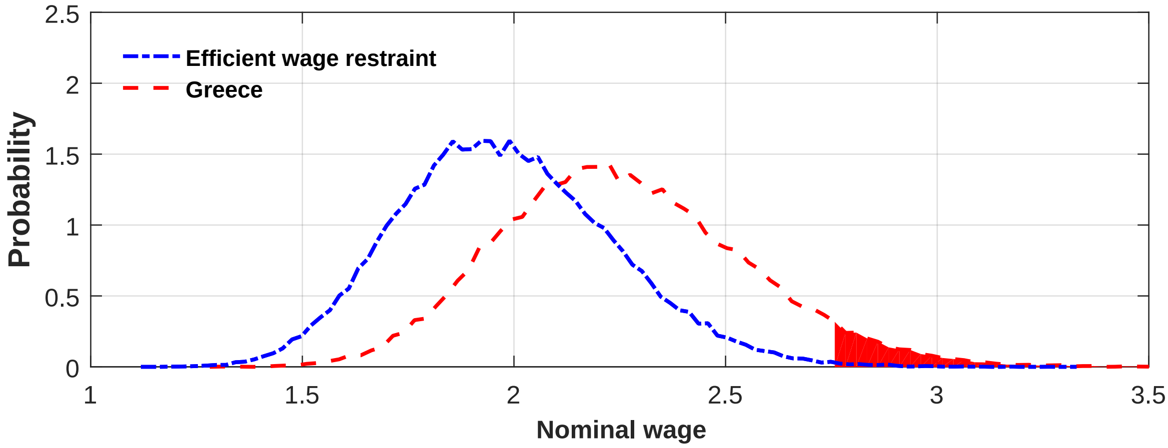

Figure 1.7: Density estimate of the ergodic nominal wage distribution in the laissez-faire and the constrained efficient equilibrium. The red filled area has a probability mass of 6.5 percent.

the benchmark where wage restraint is absent. As discussed in Section 1.3, this reflects the monopolistic wage mark-up which pushes wages in the 'exact wrong direction'. This becomes especially clear in the right panel, which shows that the implied mark-down is in fact positive in this region (i.e. a 'mark-up')—exceeding the efficient mark-down by between 20 and 30 percentage points. Yet, forward-looking unions insure strongly against downward wage rigidity as the demand shocks grow larger. While characterized by an externality (recall Section 1.4), this insurance becomes still large enough for the wage mark-up to at some point grow *negative*—as under the constrained efficient allocation.

The way in which the differential policy functions translate into different wage dynamics is shown in Figure 1.7, giving a density estimate for the ergodic wage distribution. Remarkable here is that the difference among the calibrated laissez-faire and the constrained efficient equilibrium arises primarily in different *means* for the distributions. The constrained efficient equilibrium features a lower level for wages on average, however only marginally so a tighter distribution. Wage restraint therefore operates primarily through a 'mean' effect, and less so through a 'volatility' effect. As a result, the laissez-faire equilibrium has a 6.5 percent chance of reaching wage levels that have zero probability in the constrained efficient allocation (the red filled area in Figure 1.7). Hence intuitively, what is crucial is that the social planner reduces the exposure to wage *levels* that make the economy vulnerable to a severe unemployment crisis when the economy is hit by an adverse shock.

The implications for the counter factual in Figure 1.5 are the following. Recall first that in the counter factual, the planner starts the cycle from the initial state inherited from the laissez-faire equilibrium, which includes in particular the level for inherited wages. If those wages are *too high*—because Greece fails to generate the efficient wage level *on average* (Figure 1.7)—the planner first seeks to bring wages to a *lower level* before allowing for wage inflation given the positive macroeconomic conditions. This explains the U-shaped dynamics for wages and the real exchange rate during 2001-2008 in the counter factual.

To confirm this, in the Appendix B I provide a second counter factual in which at the start of the cycle, the planner starts from her own average for inherited wages, thereby controlling for the 'mean' effect. I find that in this case, during the boom period, the efficient wage inflation and the wage inflation in the laissez-faire equilibrium are literally identical. Confirming the previous insight, wage restraint therefore does not operate via a reduction *per se* of the amount of wage inflation when the economy is in an expansion.

In sum, the finding that—under efficient wage restraint—wages should have declined during the Greek boom rather than increased (recall Figure 1.5) reflects that wages had been 'too high' already when Greece joined the euro in 2001: With the exchange rate fixed, wages should be at a mark-down below the households' marginal rate of substitution. Second, remarkably, despite the fact that the planner first needs to undo the 'initial level effect' in the counter factual in Figure 1.5, the recession already turns out substantially milder: As wages and the real exchange rate are at lower levels once deflationary pressure arises post 2008, the hike in the unemployment rate turns out significantly smaller.

1.4.5 Welfare effects and summary statistics

Lacking wage restraint entails large welfare losses. As a starting point for the discussion, consider Figure 1.8 below, which plots period felicity (the consumption value minus the disutility of work, $U(c_t) - V(h_t)$) as well as the value function as a welfare measure, against the amount of tradeable absorption.[30] Here I distinguish the calibrated laissez-faire equilibrium and the constrained efficient equilibrium as before, however additionally the 'first-best' allocation which is the allocation by the constrained social planner under $\gamma = 0$—that is, in case the planner is in fact unconstrained.[31]

As must be the case, the first-best allocation performs best along both margins. In contrast, it is instructive to observe that the laissez-faire equilibrium performs *better* than the constrained efficient equilibrium in terms of felicity when tradeable absorption is large (when the economy is in a 'boom'). Yet, welfare is still higher in the constrained efficient equilibrium independently of the state of the economy.

[30] The value function for the laissez-faire equilibrium is defined in Appendix A. The value function for the constrained social planner is given in equation (1.3.5).

[31] Here I consider the first-best allocation where wage rigidities are truly absent. In contrast in the 'no wage restraint' cases in Figures 1.2 and 1.6, I plotted policies that arise under first-best in the unconstrained region, yet still imposed downward wage rigidity when the implied wages would have violated $w_t \geq \gamma w_{t-1}$.

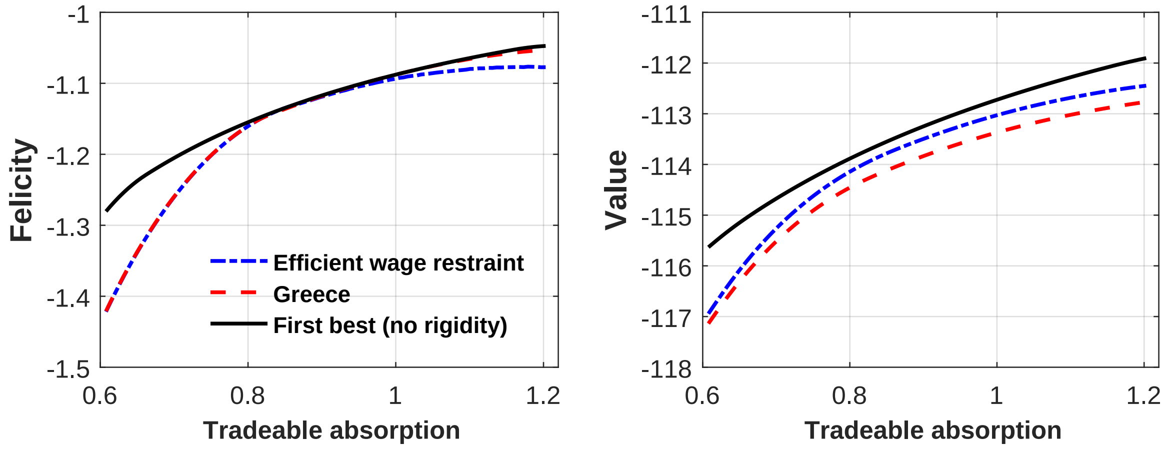

Figure 1.8: Felicity and value in the laissez-faire equilibrium, the constrained efficient equilibrium, and the constrained efficient equilibrium under $\gamma = 0$. The policies are plotted against the amount of tradeable absorption c_t^T. The other two state variables w_{t-1} and θ_t are kept at their non-stochastic steady states.

This again reflects that the inefficiencies discussed are *dynamic*: The planner endures 'short-run costs' by demanding higher wage restraint in booms than the laissez-faire equilibrium—which explains the lower felicity in this region. In return, the planner reaps 'long-run gains' by facing less deep recessions. Equivalently, the planner visits *more often* the high-felicity regions of the state space, and outright avoids the regions where felicity drops are substantial (Figure 1.7). On average, the gains strictly dominate the costs (by definition of constrained efficiency), such that the value function is strictly higher.

The laissez-faire equilibrium produces a loss in terms of permanent consumption, which I define as the (negative of) the percentage increase in aggregate consumption in each period that is necessary for the households to be indifferent between staying in this equilibrium and moving to the constrained efficient equilibrium. Formally, by denoting $\mathbf{s}_t \equiv (w_{t-1}, U^*_{c_t^T,*}, \theta_t)$ the state of the economy, the consumption loss in percent $\lambda^{\%}(\mathbf{s}_t)$ is defined by

$$E_t \sum_{j=0}^{\infty} \beta^j \{U(c_{t+j} \times (1 + \lambda^{\%}(\mathbf{s}_t)/100)) - V(h_{t+j})\} \equiv \mathcal{V}(\mathbf{s}_t),$$

where the value function $\mathcal{V}$ is welfare in the constrained efficient equilibrium (Definition 4), and where c_{t+j} and h_{t+j} are (policy functions for) consumption and hours worked in the laissez-faire equilibrium at time $t + j$. In the Appendix A, I derive a formula to compute $\lambda^{\%}(\mathbf{s}_t)$ conveniently.

As does the value function, $\lambda^{\%}(\mathbf{s}_t)$ depends on the state of the economy. In order to obtain an average loss, I weight $\lambda^{\%}(\mathbf{s}_t)$ with the ergodic distribution of the state variables. This yields

	$\lambda_t^{\%}$	$\mathcal{M}_t$	$\mathcal{M}_t \vert \psi_t = 0$	$u_t \vert \psi_t > 0$	$\Delta c_t^N \vert \psi_t > 0$
Greece	-	30.30	11.76	16.82	-4.89
Efficient wage restraint	.39	-.15	-14.19	2.37	-
First best	.55	0	-	-	-

Table 1.3: Summary statistics. Column 1: average permanent consumption loss. 2: average mark-up/mark-down. 3: average mark-up/mark-down conditional on the wage rigidity being slack. 4: average unemployment conditional on the wage rigidity being binding. 5: average drop in nontradeable consumption once the rigidity becomes binding.

a value of .39 percent—a large number compared to traditional estimates of costs of business cycle fluctuations (compare the discussion in Schmitt-Grohé and Uribe 2016). Interestingly, I also obtain that the loss relative to first-best is around .55 percent of permanent consumption, thus only slightly higher than the loss relative to 'second-best'. Echoing Elsby (2009), this reflects that the wage rigidity *per se* is not overly detrimental in terms of welfare. It is only the 'markets' that deal inefficiently with the rigidity, such that the welfare loss in the laissez-faire equilibrium is correspondingly higher.

The consumption loss is summarized in Table 1.3 which also contains summary statistics. Inspect first the mark-up / mark-down $\mathcal{M}_t$ defined earlier above: It is on average 30.30 percent in the laissez-faire equilibrium, but -.15 percent in the constrained efficient equilbrium. Hence the laissez-faire equilibrium generates insufficient wage restraint on average. Because this average contains periods where the wage rigidity is binding, and thus where wages are 'stuck' above their market clearing levels which biases up $\mathcal{M}_t$, the table also contains the average $\mathcal{M}_t$ conditioned on $\psi_t = 0$. This yields an average efficient mark-down of 14.19 percent, versus an average mark-*up* of 11.76 percent. Hence also in this statistic, the lack in wage restraint is quantitatively sizeable: The actual mark-up exceeds the efficient mark-down by about 25 percentage points on average.

Finally, the table also contains the source of the welfare loss in the first place: The large unemployment that frequently arises in the nontradeable sector and the associated low amounts of nontradeable consumption driven by the low levels of production in the nontradeable sector during these periods (recall the discussion in Section 1.2).

As in Figure 1.5, I define unemployment u_t in $U_{c_t^T} w_t = V'(h_t(1 + u_t/100))$, and in Table 1.3 I report the mean for u_t conditional on the wage rigidity being binding ($\psi_t > 0$). Without the conditioning, periods where the mark-down is negative would be counted as 'overemployment' ($u_t < 0$), and thereby bias downwards the average unemployment rate. The result is stark: The constrained social planner reduces the average unemployment rate from 16.82 percent (recall that an unemployment rate of 16.8 percent was a calibration target) to a mere 2.37 percent. In turn, the level of nontradeable consumption is lower by about 4.89 percent in the average recession in the laissez-faire compared to the constrained efficient equilibrium, as indicated by $\Delta c_t^N|\psi_t > 0 = -4.89$ in Table 1.3 above.

1.5 Implications for wage bargaining centralization

Before concluding the paper, I briefly discuss the implications of union size $1/J$ for wage restraint. The literature on centralized wage bargaining has emphasized that countries may benefit from having 'larger-sized' unions (where size is taken as a proxy for more centralization of wage bargaining)—despite larger unions possibly charging larger monopolistic wage mark-ups—because larger unions may better internalize the effects of their actions and hence moderate their wage claims in the economy.[32]

For example, Cukierman and Lippi (1999) and Guzzo and Velasco (1999) consider a set-up in which labour unions interact strategically with the central bank. The unions charge higher wage mark-ups (attempt to increase the real wage of their members) whenever the inflationary feedback from the central bank is expected to be small. The central bank is inflation averse, yet also cares about economic activity and therefore uses ex-post inflation in order to erode the too-high real wages resulting from the mark-ups. In this set-up, an equilibrium with small unions has excessive amounts of ex-post inflation because small unions charge excessively high mark-ups ex-ante, given that their *perceived* individual impact on the aggregate real wage and thereby on the amount of ex-post inflation, is small.

In the present paper, larger unions may be welfare improving for a similar reason. As summarized in Remark 2, larger unions may provide better insurance against downward wage rigidity because they may better internalize that higher wage inflation in the current period raises the probability of a deeper recession in the following period if the wage rigidity becomes binding. In this sense, I add to this literature by having a model with implications for wage *dynamics* resulting from differential degrees of wage centralization.

In my model, larger-sized unions improve welfare whenever the effects of better ex-ante insurance against downward wage rigidity (Remark 2) dominate the effects of larger monopolistic mark-ups (Remark 1). To gain further insights into the insurance term, note that the slope of labour demand faced by the unions, $\tilde{\Omega}_t$ from equation (1.3.7), is just the product of the *elasticity* of labour demand faced by the unions, $\tilde{\theta}_t$ from equation (1.3.6), times the amount of hours worked h_t over the wage w_t: $\tilde{\Omega}_t = \tilde{\theta}_t \times h_t/w_t$ (this holds by definition). As a result,

[32] The original contribution is Calmfors and Driffill (1988), later contributions and empirical evidence are discussed in the related literature section.

the stochastic discount factor in equation (1.3.4) can be written as

$$\tilde{\Lambda}_{t,t+1} = \beta(\tilde{\Omega}_{t+1}/\tilde{\Omega}_t) = \beta(\tilde{\theta}_{t+1}/\tilde{\theta}_t)(h_{t+1}/h_t)(w_t/w_{t+1}), \tag{1.5.1}$$

whereas the corresponding expression in the constrained efficient equilibrium is $\Lambda_{t,t+1} = \beta(\Omega_{t+1}/\Omega_t) = \beta(\epsilon_{t+1}^{h,w}/\epsilon_t^{h,w})(h_{t+1}/h_t)(w_t/w_{t+1})$—recall equation (1.3.5)—where $\epsilon_t^{h,w}$ again denotes the elasticity of aggregate labour demand.

Therefore the laissez-faire equilibrium provides efficient insurance against downward wage rigidity ($\tilde{\Lambda}_{t,t+1} = \Lambda_{t,t+1}$) whenever the elasticity of labour demand faced by the unions and the elasticity of aggregate labour demand, coincide: $\epsilon_t^{h,w} = \tilde{\theta}_t$ at all times. Now recall that above I had argued that the elasticity of aggregate labour demand is in fact constant in this economy, $\epsilon^{h,w} = 1/(1-\alpha+\alpha/\zeta)$. Efficient insurance therefore requires that also $\tilde{\theta}_t$ is constant. From equation (1.3.6) we know that $\tilde{\theta}_t = (J-1)/J\ \theta_t + 1/J\ \epsilon^{h,w}$, such that efficient insurance is obtained once cost-push shock are absent ($\theta_t = \theta$ at all times).

Remark 3. *In the absence of cost-push shocks ($\theta_t = \theta$), the laissez-faire equilibrium insures efficiently against downward wage rigidity ($\tilde{\Lambda}_{t,t+1} = \Lambda_{t,t+1}$) independently of the degree of wage bargaining centralization $1/J$. Hence in this case, larger-sized unions are strictly welfare deteriorating (as they still charge larger monopolistic mark-ups, Remark 1).*

This is a useful benchmark as it defines a case of 'divine coincidence': The laissez-faire equilibrium yields the efficient outcome in terms of ex-ante insurance against downward wage rigidity even if wage setting is highly decentralized.[33,34]

What breaks the 'divine coincidence', and hence makes it possible for larger-sized unions to be strictly welfare improving? At a general level, it is necessary that the two elasticities $\epsilon_t^{h,w}$ and $\tilde{\theta}_t$ fall apart in a time-varying way. I achieve this by allowing for cost-push fluctuations in $\tilde{\theta}_t$. However, even in this case, the ratio $\tilde{\theta}_{t+1}/\tilde{\theta}_t$ which matters for the discount factor in equation (1.5.1) is often close to one, such that the equilibrium is still 'close to' divine coincidence. The case for larger-sized unions therefore remains limited, unless the volatility underlying the fluctuations in $\tilde{\theta}_t$ becomes very large.[35] As a result, in the calibrated model, welfare is

[33] However, of course the equilibrium is still inefficient because of the monopolistic mark-up.

[34] To understand this result intuitively, recall the discussion in Section 1.2 on how the slope of labour demand in the current versus the following period shapes the stochastic discount factor and thereby wage restraint. Assume for instance that $\tilde{\theta} > \epsilon^{h,w}$, i.e. the unions face a 'steeper' labour demand curve than the planner ('steep' meaning that h moves strongly with changes in w). This leads to two opposing effects. First, the union would insure strictly *more* against downward wage rigidity than the planner, because it anticipates larger dislocations in working hours if the rigidity binds in the following period. At the same time, it insures strictly *less* than the planner, because it faces larger dislocations in working hours *in the current period* resulting from the endogenous mark-down. If both elasiticities are constant, these two forces exactly balance, such that on net the unions insure in the exact same way as the planner.

[35] And even in this case, cost-push shocks would merely add noise to the insurance by the unions, not lead to a biased insurance term on average.

strictly *deteriorating* in union size, with the best outcomes obtained under decentralized wage bargaining where monopolistic mark-ups are small.[36]

Hence the most promising way of overturning the divine coincidence appears to be frameworks where fluctuations in the elasticity of aggregate labour demand, $\epsilon_t^{h,w}$, are possible. For instance, if the labour share is cyclical this would also give rise to cyclicality in the elasticity. Ríos-Rull and Santaeulàlia-Llopis (2010) and Mangin and Sedlacek (2016) document empirically that the labour share is counter-cyclical, which here would imply elasticity $\epsilon_t^{h,w}$ to be counter-cyclical as well (recall that $\epsilon_t^{h,w} = 1/(1 - \alpha + \alpha/\zeta)$ where $\zeta < 1$ and where the labour share is α.) This would imply that Ω_{t+1}/Ω_t is pro-cyclical, i.e. the required mark-down for efficient wage restraint would be larger in larger-sized booms. Since small unions would fail to deliver this increased need for insurance in booms, this would break the divine coincidence. While this provides a promising additional channel for larger unions to provide better wage restraint (and thereby to possibly improve welfare), since the present model cannot deliver this channel, I leave this for future research.

1.6 Conclusion

This paper formalizes the intuition that wages ought to rise slowly in environments where they are restricted in their ability to fall. The empirical motivation is to assess the rise in wages and the real exchange rate in the euro periphery in the early 2000s, which many perceive as having been 'too high' and thus as having paved the way for the deep crisis in these countries in the aftermath of the Great Recession.

To this end, the paper first formalizes the notion of 'wage restraint' as an ex-ante endogenous wage mark-down in a standard neoclassical labour market where nominal wages are downward rigid. I show that the size of the mark-down depends on specifics of the economy as well as on the state of the cycle. Formally, the wage mark-down is the result of a constrained planning problem as in Lorenzoni (2008) and Bianchi (2011).

Further, I develop an open economy model with labour unions which may speak quantitatively to developments in the euro zone. Within the open economy, I study the inefficiencies which undermine wage restraint, uncovering two such inefficiencies. First, monopolistic wage mark-ups charged by the unions are 'doubly inefficient', because, as argued before, efficiency requires that wages be characterized by mark-*downs*. Second, the wage setting interaction between the unions suffers from an externality.

[36] However, note that even as $1/J \to 0$, monopolistic mark-ups would remain (unless also $\theta = \infty$), such that the equilibrium would remain inefficient. This would be a set-up of households which are heterogenous with respect to their labour type, and which interact in monopolistic competition in the labour market subject to downward wage rigidity, as in Benigno and Ricci (2011).

Quantitatively, the model suggests that welfare costs from lacking wage restraint are large: in the order of 0.4 percent of permanent consumption in my calibration to Greece, reflected in unemployment rates that are on average too high by more than 10 percentage points. Further, the required efficient wage mark-down for Greece is 14 percent, whereas Greece produces an average wage mark-*up* of 12 percent. Lastly, in a counter factual, the model suggests that the increase in wages and the real exchange rate in Greece 2001-2008 were inefficiently high, that this is driven by the fact that both had been too high already in 2001, and that this substantially deepened the 2009-2016 Greek recession.

Appendices

A Appendix: Analytical derivations

The Appendix A contains analytical derivations, as well as the proofs of all propositions.

A.1 Proof of Proposition 1

Here give the proof of Proposition 1 from Section 1.2. Recall the maximization problem

$$
\begin{aligned}
&\mathcal{V}(w_{t-1}) = \max \left\{a_t F(h_t) - V(h_t) + \beta E_t \mathcal{V}(w_t)\right\} \\
s.t.\quad & i) \qquad w_t/p_t = a_t F'(h_t) \\
& ii) \qquad w_t \geq \gamma w_{t-1}
\end{aligned}
$$

for given exogenous $\{a_t, p_t\}$.

To solve this problem, attach multiplier λ_t to constraint i), and multiplier $\tilde{\psi}_t \geq 0$ to constraint ii). Set up the Lagrangian

$$\mathcal{L} = a_t F(h_t) - V(h_t) + \beta E_t \mathcal{V}(w_t) + \lambda_t (w_t/p_t - a_t F'(h_t)) + \tilde{\psi}_t (w_t - \gamma w_{t-1}).$$

The first order conditions are

$$
\begin{aligned}
\partial h_t \qquad & a_t F'(h_t) - V'(h_t) - \lambda_t \partial(w_t/p_t)/\partial(h_t) = 0 \\
\partial w_t/p_t \qquad & \beta E_t \partial/\partial(w_t/p_t)\mathcal{V}(w_t) + \lambda_t + \tilde{\psi}_t p_t = 0
\end{aligned}
$$

and the Envelope condition is

$$\partial/\partial(w_{t-1}/p_{t-1})\mathcal{V}(w_{t-1}) = -\gamma \tilde{\psi}_t p_{t-1}.$$

Combine these equations and use condition i) to obtain

$$w_t/p_t + \partial(w_t/p_t)/\partial(h_t)\ (\tilde{\psi}_t - \beta\gamma E_t \tilde{\psi}_{t+1})\ p_t = V'(h_t).$$

Now define $\psi_t \equiv -\partial(w_t/p_t)/\partial(h_t)\ \tilde{\psi}_t\ p_t \geq 0$ to arrive at

$$w_t/p_t + \gamma E_t \beta(\Omega_{t+1}/\Omega_t)(p_t/p_{t+1})\psi_{t+1} - \psi_t = V'(h_t)$$

with $\Omega_t \equiv -1/(\partial(w_t/p_t)/\partial(h_t)) = -\partial(h_t)/\partial(w_t/p_t) > 0$. From $w_t/p_t = a_t \alpha h_t^{\alpha-1}$ (labour demand using $F(h) = h^\alpha$), we further obtain $\Omega_t = 1/(1-\alpha)\ h_t/(w_t/p_t)$. This is the expression in the main text, where in the main text I have stated this condition under $\psi_t = 0$ (the rigidity is slack in period t).

A.2 The consequences of Assumption 1

Here I show that once Assumption 1 is satisfied, that is $1/\kappa = \zeta$, the marginal utility of tradeable consumption is $U_{c_t^T} = \omega(c_t^T)^{-\kappa}$, such that the Backus Smith condition is equation (1.3.1). Note that the same reasoning also implies that $U_{c_t^N}$ in equation (1.3.2) is given by $U_{c_t^N} = (1-\omega)(c_t^N)^{-\kappa}$.

From the utility function $U(A(c^T, c^N))$ we obtain the following partial derivative

$$U_{c_t^T} = U'((A(c_t^T, c_t^N)))A_{c^T}(c_t^T, c_t^N)$$

or with the functional forms imposed

$$U_{c_t^T} = (A(c_t^T, c_t^N))^{-\kappa}[\omega(c_t^T)^{1-1/\zeta} + (1-\omega)(c_t^N)^{1-1/\zeta}]^{1/(1-1/\zeta)-1}\omega(c_t^T)^{-1/\zeta}$$

By recognizing that $A(c^T, c^N) = [\omega(c^T)^{1-1/\zeta} + (1-\omega)(c^N)^{1-1/\zeta}]^{1/(1-1/\zeta)}$ the term $[\cdot]^{1/(1-1/\zeta)-1}$ can be written as $(A(c_t^T, c_t^N))^{1/\zeta}$. Therefore the whole expression becomes

$$U_{c_t^T} = (A(c_t^T, c_t^N))^{1/\zeta-\kappa}\omega(c_t^T)^{-1/\zeta}.$$

We see that under $1/\kappa = \zeta$ the first term drops out such that we obtain

$$U_{c_t^T} = \omega(c_t^T)^{-\kappa}$$

as claimed.

A.3 Derivation of the union-type labour demand curves

Here I derive the union-type labour demand curves $h_t(j) = (w_t(j)/w_t)^{-\theta_t} h_t$ and the cost-minimizing wage index $w_t = (\sum 1/J\ w_t(j)^{1-\theta_t})^{1/(1-\theta_t)}$ that appear in Definition 3 in the main text.

To obtain the demand curves for union-type labour, consider the cost minimization problem of the firm (minimize costs subject to employing an aggregate of h_t hours worked)

$$\mathcal{L} = \sum_{j=1}^{J} w_t(j)h_t(j) + \vartheta_t \left(h_t - \left(\sum_{j=1}^{J} \frac{1}{J} h_t(j)^{\frac{\theta_t - 1}{\theta_t}} \right)^{\frac{\theta_t}{\theta_t - 1}} \right),$$

where ϑ_t is a Lagrange multiplier. Take the derivative with respect to $h_t(j)$

$$w_t(j) = \vartheta_t \frac{1}{J} \left(\sum_{j=1}^{J} \frac{1}{J} h_t(j)^{\frac{\theta_t - 1}{\theta_t}} \right)^{\frac{\theta_t}{\theta_t - 1} - 1} h_t(j)^{\frac{\theta_t - 1}{\theta_t} - 1}$$

which by using the definition of h_t can be written as

$$w_t(j) = \vartheta_t \frac{1}{J} \left(\frac{h_t(j)}{h_t} \right)^{-\frac{1}{\theta_t}}.$$

To eliminate the multiplier ϑ_t, raise both sides of this equation to the power of $1 - \theta_t$, then multiply through with $1/J$

$$\frac{1}{J} w_t(j)^{1-\theta_t} = (\vartheta_t \frac{1}{J})^{1-\theta_t} h_t^{\frac{1-\theta_t}{\theta_t}} \frac{1}{J} h_t(j)^{\frac{\theta_t - 1}{\theta_t}}.$$

Now take the sum on both sides, and raise left and right hand side to the power of $1/(1 - \theta_t)$

$$w_t \equiv \left(\sum_{j=1}^{J} \frac{1}{J} w_t(j)^{1-\theta_t} \right)^{\frac{1}{1-\theta_t}} = \vartheta_t \frac{1}{J}.$$

This defines the wage index w_t. Replace the multiplier above with the wage index and rearrange to obtain the labour demand curves

$$h_t(j) = \left(\frac{w_t(j)}{w_t} \right)^{-\theta_t} h_t.$$

A.4 Proof of Proposition 2

To prove Proposition 2, it will be convenient to first rewrite the union-type labour demand curves above such that $w_t(j)$ only appears once on the right hand side, not also indirectly through aggregator w_t. I do this in the following section. Thereafter, I use the rearranged union type labour demand curves to solve the union problem in Definition 3.

Rewrite the labour demand curves

Rearrange the union-type labour demand curve as

$$\sum_{j=1}^{J} \frac{1}{J} w_t(j)^{1-\theta_t} = w_t(j)^{1-\theta_t} \left(\frac{h_t(j)}{h_t}\right)^{\frac{1-\theta_t}{\theta_t}}$$

$$\iff w_t(j)^{1-\theta_t} \left(\frac{1}{J} - \left(\frac{h_t(j)}{h_t}\right)^{\frac{1-\theta_t}{\theta_t}}\right) = -\frac{J-1}{J} \sum_{\neq j} \frac{1}{J-1} w_t(i)^{1-\theta_t}$$

$$\iff w_t(j) = \left(\frac{J-1}{J}\left(\left(\frac{h_t(j)}{h_t}\right)^{\frac{1-\theta_t}{\theta_t}} - \frac{1}{J}\right)^{-1}\right)^{\frac{1}{1-\theta_t}} w_t(-j)$$

such that

$$w_t(j) = \left(\frac{J}{J-1}\left(\frac{h_t(j)}{h_t}\right)^{\frac{1-\theta_t}{\theta_t}} - \frac{1}{J-1}\right)^{-\frac{1}{1-\theta_t}} w_t(-j)$$

where I define the wage index that includes all individual wages except for $w_t(j)$ as

$$w_t(-j) \equiv \left(\frac{1}{J-1}\sum_{\neq j} w_t(i)^{1-\theta_t}\right)^{1/(1-\theta_t)}.$$

Here $\sum_{\neq j}$ indicates summation over all indices $i \in \mathcal{J}$ except for index j.

Solving for $h_t(j)$ yields the final expression

$$h_t(j) = \left(\frac{J-1}{J}\left(\frac{1}{J-1} + \left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta_t)}\right)\right)^{\frac{\theta_t}{1-\theta_t}} h_t.$$

Solving the problem of the unions

With this rewritten labour demand curve, the problem in Definition 3 can be reformulated as follows

$$\mathcal{W}(w_{t-1}(j)) = \max \{U_{c_t^T} w_t(j) h_t(j) - V(h_t(j)) + \beta E_t \mathcal{W}(w_t(j))\}$$

s.t.

i) $$h_t(j) = \left(\frac{J-1}{J}\left(\frac{1}{J-1} + \left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta_t)}\right)\right)^{\frac{\theta_t}{1-\theta_t}} h_t$$

ii) $$w_t = \left(\sum_{j=1}^{J} \frac{1}{J} w_t(j)^{1-\theta_t}\right)^{\frac{1}{1-\theta_t}}$$

iii) $$(1-\omega)(F(h_t))^{-\kappa} F'(h_t) = U_{c_t^T} w_t$$

iv) $$w_t(j) \geq \gamma w_{t-1}(j).$$

for given exogenous $U_{c_t^T}$.

The Lagrangian for this problem is

$$\begin{aligned}\mathcal{L} = U_{c_t^T} w_t(j)\left(\frac{J-1}{J}\left(\frac{1}{J-1}+\left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta_t)}\right)\right)^{\frac{\theta_t}{1-\theta_t}} h_t(w_t(j))\\ - V\left(\left(\frac{J-1}{J}\left(\frac{1}{J-1}+\left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta_t)}\right)\right)^{\frac{\theta_t}{1-\theta_t}} h_t(w_t(j))\right)\\ + \beta E_t \mathcal{W}(w_t(j)) + \tilde{\psi}_t(j)(w_t(j)-\gamma w_{t-1}(j)).\end{aligned}$$

where I have made the dependence of h_t on $w_t(j)$ explicit by writing $h_t(w_t(j))$. This dependence arises from h_t depending on w_t through aggregate labour demand (condition iii above), which in turn depends on $w_t(j)$ through the wage index (condition ii above). Intuitively, a higher union-type wage also raises the aggregate wage which feeds back adversely into aggregate labour demand, as explained in the main text. Therefore, a more precise writing would be $h_t(w_t(w_t(j)))$, with corresponding partial derivative

$$\begin{aligned}\frac{\partial}{\partial w_t(j)} h_t(w_t(w_t(j))) &= \frac{\partial}{\partial w_t(j)} w_t \times \frac{\partial}{\partial w_t} h_t(w_t)\\ &= -\frac{1}{J}\left(\frac{w_t(j)}{w_t}\right)^{-\theta_t} \times \Omega_t\end{aligned}$$

where I have denoted Ω_t the (negative of the) derivative of aggregate labour demand h_t with respect to the aggregate wage w_t as in the main text (recall Proposition 1).
As another intermediate step, consider the derivative of $w_t(j)h_t(j)$ with respect to $w_t(j)$ by using condition i above but by keeping aggregate hours h_t fixed

$$\begin{aligned}&\left(\frac{\partial}{\partial w_t(j)} w_t(j)\left(\frac{J-1}{J}\left(\frac{1}{J-1}+\left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta_t)}\right)\right)^{\frac{\theta_t}{1-\theta_t}}\right) h_t\\ &= \left((\cdot)^{\frac{\theta_t}{1-\theta_t}} - \theta w_t(j)\,(\cdot)^{\frac{\theta_t}{1-\theta_t}-1}\frac{J-1}{J}\left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta_t)-1}\frac{1}{w_t(-j)}\right) h_t\\ &= h_t(j) - \theta_t \frac{J-1}{J}\left(\frac{h_t(j)}{h_t}\right)^{\frac{2\theta_t-1}{\theta_t}} h_t\left(\frac{J}{J-1}\left(\frac{h_t(j)}{h_t}\right)^{\frac{1-\theta_t}{\theta_t}} - \frac{1}{J-1}\right)\\ &= h_t(j) - \theta_t\left(h_t(j) - \frac{1}{J}\left(\frac{h_t(j)}{h_t}\right)^{\frac{2\theta_t-1}{\theta_t}} h_t\right)\\ &= h_t(j)\left(1-\theta_t\left(1-\frac{1}{J}\left(\frac{h_t(j)}{h_t}\right)^{\frac{\theta_t-1}{\theta_t}}\right)\right).\end{aligned}$$

Now the derivative of $w_t(j)h_t(j)$ with respect to $w_t(j)$ by only considering the variation in h_t

$$w_t(j)\left(\frac{J-1}{J}\left(\frac{1}{J-1}+\left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta_t)}\right)\right)^{\frac{\theta_t}{1-\theta_t}}\frac{\partial}{\partial w_t(j)}h_t(w_t(j))$$
$$=-w_t(j)\left(\frac{J-1}{J}\left(\frac{1}{J-1}+\left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta_t)}\right)\right)^{\frac{\theta_t}{1-\theta_t}}\frac{1}{J}\left(\frac{w_t(j)}{w_t}\right)^{-\theta_t}\Omega_t$$
$$=-\frac{1}{J}w_t(j)\frac{h_t(j)}{h_t}\left(\frac{w_t(j)}{w_t}\right)^{-\theta_t}\Omega_t,$$

where I have used the results on the impact of $w_t(j)$ on h_t from above.

Using similar steps, we obtain the derivative of $V(h_t(j))$ with respect to $w_t(j)$

$$\frac{\partial}{\partial w_t(j)}V\left(\left(\frac{J-1}{J}\left(\frac{1}{J-1}+\left(\frac{w_t(j)}{w_t(-j)}\right)^{-(1-\theta_t)}\right)\right)^{\frac{\theta_t}{1-\theta_t}}h_t(w_t(j))\right)$$
$$=V'(h_t(j))\left(-\theta_t\frac{h_t(j)}{w_t(j)}+\theta_t\frac{1}{J}\left(\frac{h_t(j)}{h_t}\right)^{\frac{2\theta_t-1}{\theta_t}}\frac{h_t}{w_t(j)}-\frac{1}{J}\frac{h_t(j)}{h_t}\left(\frac{w_t(j)}{w_t}\right)^{-\theta_t}\Omega_t\right)$$
$$=V'(h_t(j))\left(-\theta_t\frac{h_t(j)}{w_t(j)}\left(1-\frac{1}{J}\left(\frac{h_t(j)}{h_t}\right)^{\frac{\theta_t-1}{\theta_t}}\right)-\frac{1}{J}\frac{h_t(j)}{h_t}\left(\frac{w_t(j)}{w_t}\right)^{-\theta_t}\Omega_t\right).$$

Finally, consider the partial derivative

$$\frac{\partial}{\partial w_t(j)}\ \beta E_t\mathcal{W}(w_t(j))+\tilde{\psi}_t(j)(w_t(j)-\gamma w_{t-1}(j))$$
$$=\beta E_t\frac{\partial}{\partial w_t(j)}\mathcal{W}(w_t(j))+\tilde{\psi}_t(j),$$

which, once combined with the Envelope condition

$$\frac{\partial}{\partial w_{t-1}(j)}\mathcal{W}(w_{t-1}(j))=-\gamma\tilde{\psi}_t(j)$$

becomes

$$=\tilde{\psi}_t(j)-\beta\gamma E_t\tilde{\psi}_{t+1}(j).$$

We have computed all partial derivatives which are necessary to take first order conditions of the above Lagrangian with respect to $w_t(j)$.

Doing so and putting the pieces together, and using that, in the symmetric equilibrium, $w_t(j)=w_t$, $h_t(j)=h_t$, and $\tilde{\psi}_t(j)=\tilde{\psi}_t$, I obtain

$$V'(h_t)\left(\frac{J-1}{J}\theta_t+\frac{1}{J}\Omega_t\frac{w_t}{h_t}\right)=U_{c_t^T}w_t\left(\frac{J-1}{J}\theta_t+\frac{1}{J}\Omega_t\frac{w_t}{h_t}-1\right)-\frac{w_t}{h_t}(\tilde{\psi}_t-\beta\gamma E_t\tilde{\psi}_{t+1})$$

Define now $\tilde{\theta}_t\equiv(J-1)/J\ \theta_t+1/J\epsilon_t^{h,w}$ with $\epsilon_t^{h,w}\equiv\Omega_t w_t/h_t$ as well as $\tilde{\Omega}_t\equiv(J-1)/J\ \theta_t\ h_t/w_t+$

$1/J\ \Omega_t$ to rewrite this as

$$V'(h_t) = U_{c_t^T} w_t \frac{\tilde{\theta}_t - 1}{\tilde{\theta}_t} - \frac{1}{\tilde{\Omega}_t}(\tilde{\psi}_t - \beta\gamma E_t \tilde{\psi}_{t+1}).$$

Lastly, define $\psi_t \equiv \tilde{\psi}_t/\tilde{\Omega}_t$ to obtain the final expression

$$\frac{\tilde{\theta}_t - 1}{\tilde{\theta}_t}\ U_{c_t^T} w_t + \gamma E_t \beta(\tilde{\Omega}_{t+1}/\tilde{\Omega}_t)\psi_{t+1} - \psi_t = V'(h_t).$$

The slope and elasticity of labour demand

Note that in the main text, I have defined $\tilde{\Omega}_t \equiv (J-1)/J\ (-\partial h_t(j)/\partial w_t(j)) + 1/J\ \Omega_t$. To see that this is equivalent to the $\tilde{\Omega}_t$ defined here, consider that

$$-\partial h_t(j)/\partial w_t(j) = \theta_t h_t(j)/w_t(j)$$

in the case of $J = \infty$, which is $\theta_t\ h_t/w_t$ as claimed.

Finally, note that in the main text I have claimed that $\epsilon_t^{h,w} = 1/(1-\alpha+\alpha/\zeta)$ and thus that the elasticity of aggregate labour demand $\epsilon^{h,w}$ is in fact constant.

To see this, recall that aggregate labour demand is given by

$$\begin{aligned} w_t &= (1-\omega)(h_t^{\alpha})^{-\kappa}\alpha h_t^{\alpha-1}/U_{c_t^T} \\ &= (1-\omega)\alpha/U_{c_t^T} h_t^{-\alpha\kappa+\alpha-1}. \end{aligned}$$

It is easy to see that $w_t(h_t)$ has elasticity $-\alpha\kappa+\alpha-1$ such that the inverse function $h_t(w_t)$ has elasticity $1/(1-\alpha+\alpha\kappa)$ (in absolute value). But then from Assumption 1, this corresponds to $1/(1-\alpha+\alpha/\zeta)$ as claimed.

A.5 Equilibrium conditions laissez-faire equilibrium

The laissez-faire equilibrium in the extended model is completely described by the following set of equations

$$\begin{aligned} &i) && p_t^N \alpha h_t^{\alpha-1} = w_t \\ &ii) && p_t^N = (1-\omega)(h_t^{\alpha})^{-\kappa}/U_{c_t^T} \\ &iii) && U_{c_t^T} = \omega(c_t^T)^{-\kappa} = U^*_{c_t^{T,*}} \\ &iv) && (\tilde{\theta}_t - 1)/\tilde{\theta}_t\ U_{c_t^T} w_t + \gamma E_t\beta(\tilde{\Omega}_{t+1}/\tilde{\Omega}_t)\psi_{t+1} - \psi_t = h_t^{\varphi}, \end{aligned}$$

along with the inequality constraints

$$v)\ w_t \geq \gamma w_{t-1}$$
$$vi)\ \psi_t \geq 0$$

and the slackness condition

$$vii)\ \psi_t(w_t - \gamma w_{t-1}) = 0,$$

and where $\tilde{\theta}_t$ and $\tilde{\Omega}_t$ are given by

$$viii)\ \tilde{\theta}_t \equiv (J-1)/J\ \theta_t + 1/J\ 1/(1-\alpha+\alpha/\zeta)$$
$$ix)\ \tilde{\Omega}_t = \tilde{\theta}_t\ h_t/w_t.$$

Recall that the two exogenous variables are $\{U^*_{c_t^{T,*}}\}$ and $\{\theta_t\}$. This defines a process for endogenous variables $\{p_t^N, h_t, w_t, U_{c_t^T}, \tilde{\theta}_t, \tilde{\Omega}_t, \psi_t\}$. In the non-stochastic steady state, the wage rigidity is not binding such that $\psi = 0$. As a result, the steady state is characterized by

$$h_{ss} = [(1-\omega)\alpha(\tilde{\theta}-1)/\tilde{\theta}]^{1/(1+\varphi-\alpha+\kappa\alpha)}$$

where I have combined conditions i), ii) and iv) from the previous section. The implied wage is given by

$$w_{ss} = h_{ss}^{\varphi}\tilde{\theta}/((\tilde{\theta}-1)U_{c^T})$$

from condition iv). As explained in the main text, I calibrate U_{c^T} such that $c_{ss}^T = h_{ss}^{\alpha}$. This yields, from condition iii), a value for U_{c^T} of

$$U_{c^T} = \omega[(1-\omega)\alpha(\tilde{\theta}-1)/\tilde{\theta}]^{-\alpha\kappa/(1+\varphi-\alpha+\kappa\alpha)}.$$

This completes the description of the steady state of the (laissez-faire equilibrium of the) model.

A.6 Proof of Proposition 3

The problem of the constrained planner can be written as Lagrangian

$$\mathcal{L} = U(A(c_t^T, F(h_t)))) - V(h_t) + \beta E_t \mathcal{V}(w_t) + \lambda_t((1-\omega)(F(h_t))^{-\kappa}/U^*_{c_t^{T,*}} F'(h_t) - w_t) + \tilde{\psi}_t(w_t - \gamma w_{t-1})$$

where I have combined $p_t^N F'(h_t) = w_t$ and $p_t^N = (1-\omega)(F(h_t))^{-\kappa}/U_{c_t^T}$, and for given exogenous c_t and $U_{c_t^T}$ as both are implied by Backus Smith equation $U_{c_t^T} = \omega(c_t^T)^{-\kappa} = U^*_{c_t^{T,*}}$, $U^*_{c_t^{T,*}}$ exogenous and stochastic.

The first order conditions are

$$\partial h_t \qquad\qquad U_{c_t^N} F'(h_t) - V'(h_t) - \lambda_t \partial w_t/\partial h_t = 0$$

$$\partial w_t \qquad\qquad \beta E_t \partial/\partial w_t \mathcal{V}(w_t) + \lambda_t + \tilde{\psi}_t = 0$$

and the Envelope condition is

$$\partial/\partial(w_{t-1})\mathcal{V}(w_{t-1}) = -\gamma\tilde{\psi}_t.$$

Now use that $p_t^N = (1-\omega)(F(h_t))^{-\kappa}/U_{c_t^T}$ which is just $p_t^N = U_{c_t^N}/U_{c_t^T}$ and combine the previous equations to obtain

$$U_{c_t^T} p_t^N F'(h_t) - V'(h_t) + \partial w_t/\partial h_t(\tilde{\psi}_t - \gamma\beta E_t\tilde{\psi}_{t+1})$$

Recognizing that $p_t^N F'(h_t) = w_t$, defining $\Omega_t \equiv -\partial h_t/\partial w_t$ and defining $\psi_t \equiv \tilde{\psi}_t/\Omega_t$ yields the final expression

$$U_{c_t^T} w_t + \gamma E_t \beta(\Omega_{t+1}/\Omega_t)\psi_{t+1} - \psi_t = V'(h_t).$$

Note that above in Appendix Section A.4 I had shown that the elasticity of aggregate labour demand is $1/(1-\alpha+\alpha/\zeta)$. Because Ω_t is the slope of aggregate labour demand, it can be computed from the elasticity via $\Omega_t = 1/(1-\alpha+\alpha/\zeta)\ h_t/w_t$. This completes the proof of Proposition 3.

A.7 Permanent consumption equivalent

Denote $c^{lf}(\mathbf{s}_t)$ the consumption policy function in the laissez-faire equilibrium in state $\mathbf{s}_t \equiv (w_{t-1}, U^*_{c_t^{T,*}}, \theta_t)$, and equivalently for $\mathbf{s}^{lf}_{t+1}(\mathbf{s}_t) = (w_t^{lf}(\mathbf{s}_t), U_{c_{t+1}^{T,*}}, \theta_{t+1})$ and $h^{lf}(\mathbf{s}_t)$. Define welfare implied by this allocation via

$$\mathcal{V}^{lf}(\mathbf{s}_t) = U(c^{lf}(\mathbf{s}_t)) - V(h^{lf}(\mathbf{s}_t)) + \beta E_t \mathcal{V}^{lf}(\mathbf{s}^{lf}_{t+1}(\mathbf{s}_t)).$$

In turn, welfare under the constrained efficient allocation is defined in

$$\mathcal{V}^{ce}(\mathbf{s}_t) = U(c^{ce}(\mathbf{s}_t)) - V(h^{ce}(\mathbf{s}_t)) + \beta E_t \mathcal{V}^{ce}(\mathbf{s}^{ce}_{t+1}(\mathbf{s}_t))$$

with the corresponding policies having superscript *ce*.[37] Note that by definition of constrained efficiency it must hold that $\mathcal{V}^{lf}(\mathbf{s}_t) \leq \mathcal{V}^{ce}(\mathbf{s}_t)$ in all states $\mathbf{s}_t$.

The permanent consumption equivalent is a function $\lambda(\mathbf{s}_t)$ multiplying consumption policy $c^{lf}(\mathbf{s}_t)$ such that $\mathcal{V}^{lf}(\mathbf{s}_t) = \mathcal{V}^{ce}(\mathbf{s}_t)$ in all states $\mathbf{s}_t$. Formally, we may therefore write

$$\mathcal{V}^{ce}(\mathbf{s}_t) = U(\lambda(\mathbf{s}_t)c^{lf}(\mathbf{s}_t)) - V(h^{lf}(\mathbf{s}_t)) + \beta E_t \mathcal{V}^{ce}(\mathbf{s}^{lf}_{t+1}(\mathbf{s}_t)),$$

[37] In the main text I had denoted value function $\mathcal{V}^{ce}$ as $\mathcal{V}$, see Definition 4. However, for clarity I am now making the superscript specific.

where I have inserted function $\lambda(\mathbf{s}_t)$ and replaced value function $\mathcal{V}^{lf}$ by $\mathcal{V}^{ce}$. Importantly, note that the policy $\mathbf{s}^{lf}_{t+1}(\mathbf{s}_t)$ from the laissez-faire equilibrium still appears in the argument of the expectation of the value function on the right hand side. This equation can be further rewritten as

$$\mathcal{V}^{ce}(\mathbf{s}_t) = \lambda(\mathbf{s}_t)^{1-\kappa}U(c^{lf}(\mathbf{s}_t)) - V(h^{lf}(\mathbf{s}_t)) + \beta E_t \mathcal{V}^{ce}(\mathbf{s}^{lf}_{t+1}(\mathbf{s}_t)),$$

where I have used that function U is of the CRRA type (and thus homothetic). Rearranging and solving for $\lambda(\mathbf{s}_t)$ I obtain

$$\lambda(\mathbf{s}_t) = \{[\mathcal{V}^{ce}(\mathbf{s}_t) - \beta E_t \mathcal{V}^{ce}(\mathbf{s}^{lf}_{t+1}(\mathbf{s}_t)) + V(h^{lf}(\mathbf{s}_t))]/U(c^{lf}(\mathbf{s}_t))\}^{1/(1-\kappa)},$$

equation which I solve numerically. Note that the permanent consumption gain in percent follows from $\lambda(\mathbf{s}_t)$ via

$$\lambda^{\%}(\mathbf{s}_t) = (\lambda(\mathbf{s}_t) - 1) * 100\%,$$

which is what I use in the main text. The consumption equivalent relative to the first best allocation is computed in the same way, by using the constrained efficient equilibrium where I additionally impose $\gamma = 0$.

B Appendix: Additional Figures

The Appendix B contains additional figures.

B.1 Stylized model: Sensitivity of the mark-down

Here I complement the insights in Section 1.2 by showing how the ex-ante endogenous wage mark-down depends on parameters α and γ. Recall that α is the labour share and therefore the curvature of the production function, whereas γ is the wage rigidity coefficient. In turn, a lower value for γ is isomorphic to a higher inflation target p_t/p_{t-1}.
The result is shown in Figure 1.9. Not surprisingly, a higher rigidity coefficient requires a larger endogenous mark-down (left panel). Indeed if wages can fall only 2 percent per year ($\gamma = .995$), the endogenous mark-down reaches 20 percent for two standard deviation positive technology shocks. As for the labour share coefficient α, a higher α is associated with larger mark-downs. This may be surprising, because a larger α is associated with less curvature in period felicity $a_tF(h_t) - V(h_t)$ (recall the discussion in Section 1.2). However there is a second effect: a higher α makes labour demand defined in $a_tF'(h_t) = w_t/p_t$ 'steeper', i.e. h_t fluctuates more strongly with movements in w_t/p_t. As a result, the dislocations in working hours produced by a given constraint on w_t are larger with a larger α, such that the required ex-ante insurance against such dislocations must be larger.

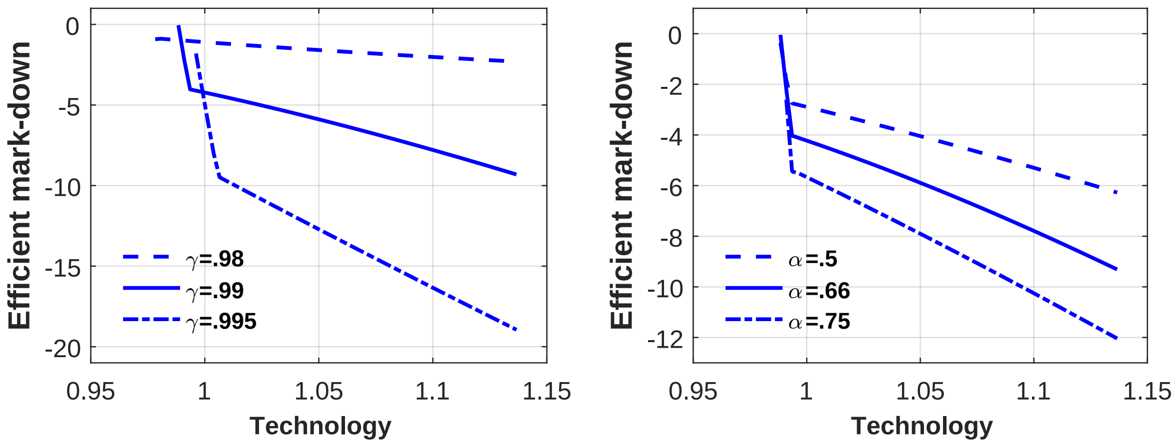

Figure 1.9: Policy function for the mark-down $\mathcal{M}_t$ in technology a_t for different parameters γ and α. The other state variable w_{t-1} is kept at its non-stochastic steady state. The parameter that is not varied is kept at $\varphi = 3.5, \sigma = .02, \alpha = .66, \gamma = .99$.

B.2 Open economy model: Policy functions for demand shocks

Figure 1.10 shows, in addition to the policies for w_t and $\mathcal{M}_t$ shown in Figure 1.6, the policies for the real exchange rate $(\omega + (1-\omega)(p_t^N)^{1-\zeta})^{1/(1-\zeta)}$ and for working hours h_t. The result is as expected: the real exchange rate appreciates with the rising wages, however falls by only so much with wage-deflationary pressure. In turn, in intermediate regions of tradeable absorption, the real exchange rate exceeds the benchmark scenario of no-wage-restraint, reflecting the monopolistic mark-up which pushes wages in the 'exact wrong direction'.

In contrast, hours worked are higher in the constrained efficient allocation than in the laissez-faire equilibrium. This reflects the inverse relationship between wages and working hours, or equivalently, that the social planner demands wage restraint by pushing hours in the nontradeable sector.

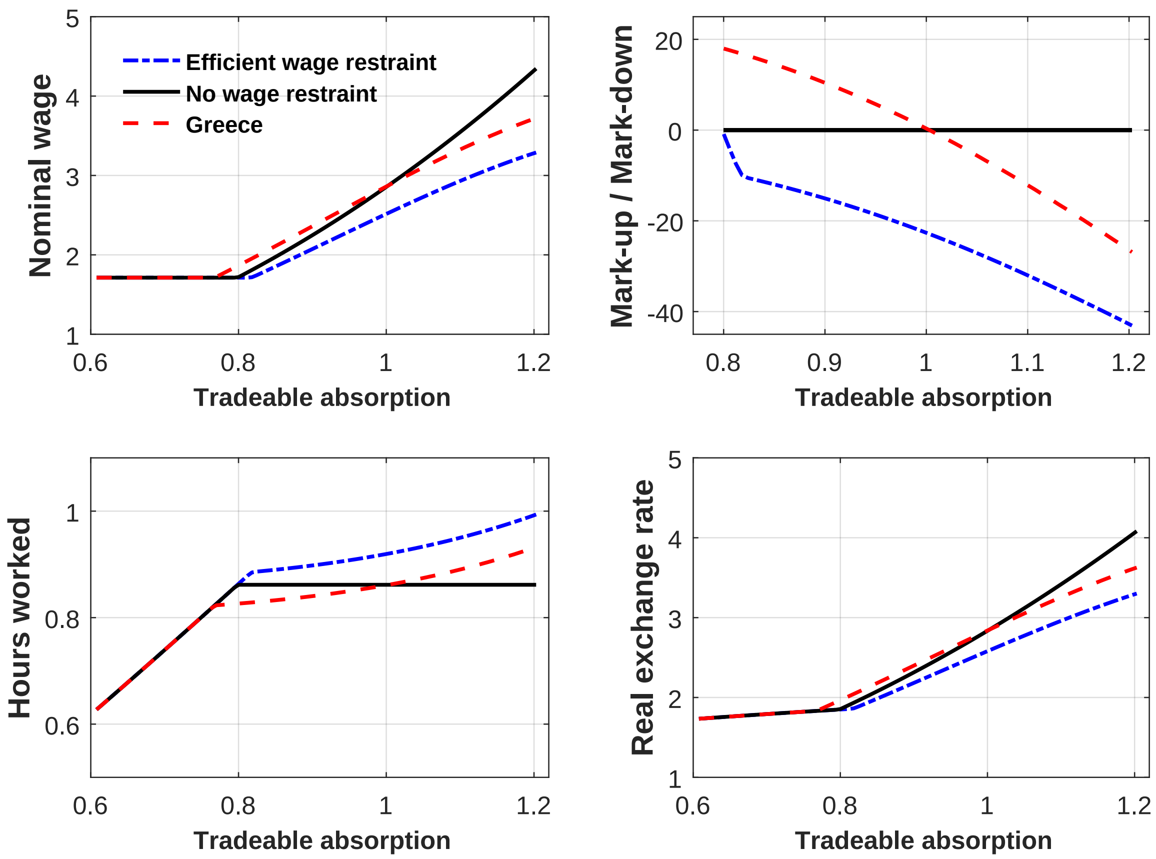

Figure 1.10: Policy functions for endogenous variables in tradeable absorption c_t^T. The other two state variables w_{t-1} and c_t^T are kept at their non-stochastic steady states. The mark-up / mark-down and the real exchange rate are defined in the main text.

B.3 Open economy model: Policy functions for cost-push shocks

Mirroring Figure 1.10, Figure 1.11 show policies in the elasticity between labour types θ_t—the 'cost-push' shock. Yet, this shock matters only in the laissez-faire equilibrium, while it plays no role in the constrained efficient equilibrium. As Definition 4 makes clear, this is because the planner is indifferent between the degree of substitutability between the labour types θ_t, charging no monopolistic mark-up. Recognizing that cost-push pressure arises as θ_t is low (the monopolistic mark-up $\tilde{\theta}_t/(\tilde{\theta}_t - 1)$ falls in θ_t), the result is as expected: cost push pressure raises wages and the real exchange rate, making the endogenous wage mark-up more positive. At the same time, the cost-push shock is contractionary: as θ_t becomes smaller, hours worked in the nontradeable sector fall. Yet, the quantitative impact of the cost-push shocks is small relative to the impact of the demand shocks, see above Figure 1.10.

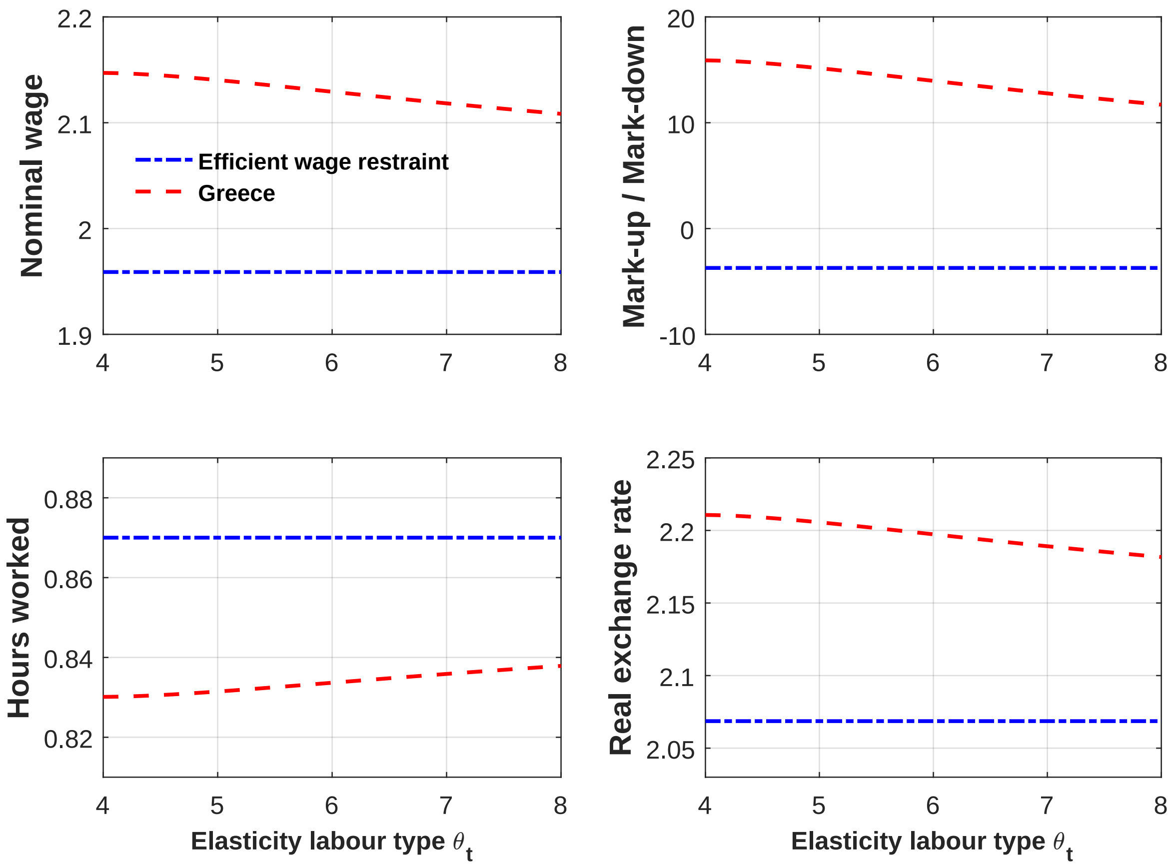

Figure 1.11: Policy functions for endogenous variables in the elasticity of labour types θ_t. The other two state variables w_{t-1} and c_t^T are kept at their non-stochastic steady states. The mark-up / mark-down and the real exchange rate are defined in the main text.

B.4 Application to Greece 2001-2016: Second counter factual

Following up on Section 1.4.3, in the second counter factual I control for the fact that the constrained efficient allocation generates a different average level for wages as a result of the efficient mark-down. Specifically, I simulate both the laissez-faire equilibrium and the constrained efficient equilibrium over a long horizon with the same sequence of shocks. Next, I isolate and average the same periods in the laissez-faire equilibrium as in the first counter factual. However, instead of computing the response of the variables under constrained efficiency anew by using the initial states of the cycle, here I merely plot the evolution of the variables in the constrained efficient allocation during the isolated periods.

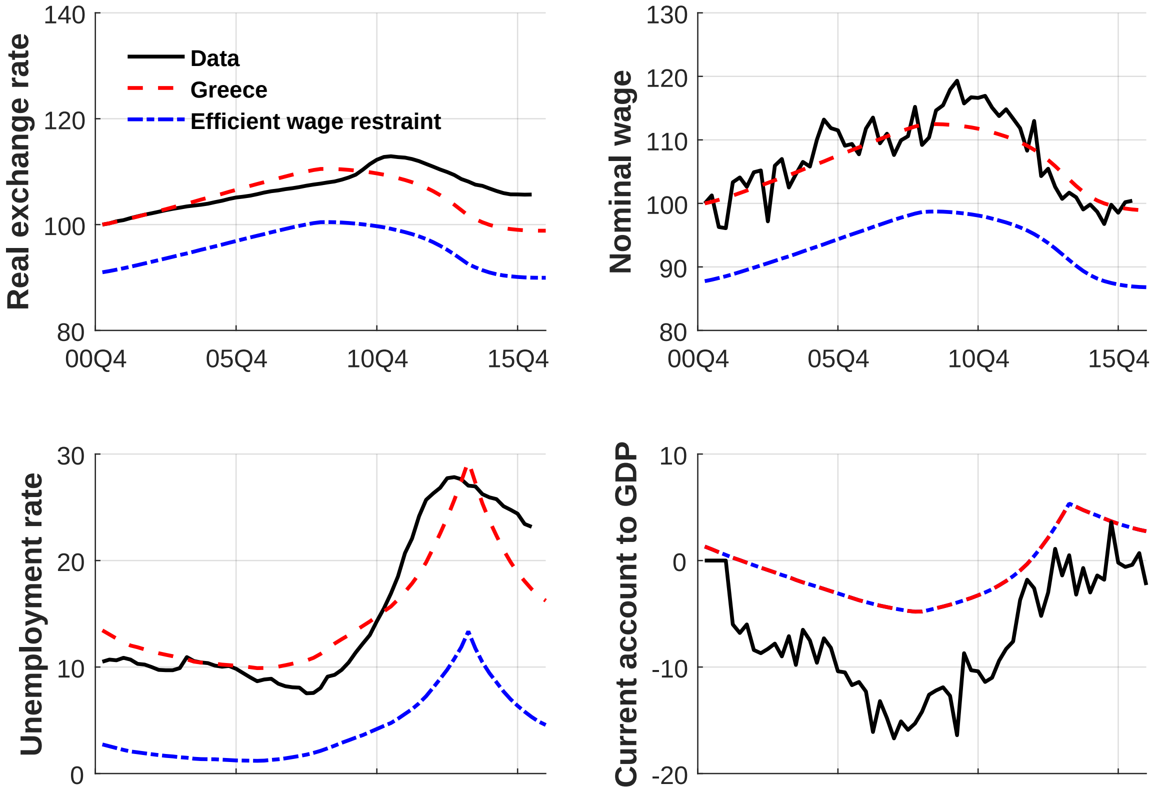

Figure 1.12: Real exchange rate, HICP-based, vis-à-vis all euro area. Nominal unit labour cost index vis-à-vis all euro area. Both are normalized to 100 in 2001Q1. Unemployment rate in percent. Current account to GDP ratio in percent. Quarters on x-axis: 2001Q1-2016Q4. In the model, the real exchange rate is $(\omega + (1-\omega)(p_t^N)^{1-\zeta})^{1/(1-\zeta)}$, the unemployment rate is $U_{c_t^T} w_t = V'(h_t(1+u_t/100))$, and the current account to GDP ratio is $(1 - c_t^T/y^T) \times 100$.

Figure 1.12 shows the result. As it turns out, controlling for the 'mean' effect induced by the efficient mark-down, the dynamics among the constrained efficient and the laissez-faire equilibrium during the boom period are close to identical. That is, also in the constrained efficient equilibrium do wages rise by 10-15 percent during the boom, such that strong wage inflation *per se* does not indicate any inefficiency. What is crucial is that the wage inflation occurs at an overall lower level, such that the unemployment rate in the subsequent recession is still much lower than in actuality.

B.5 Application to Greece 2001-2016: Implied shock series

Here I show the implied (average) shock series that generates the cycles in the first and second counter factual. As it turns out, the cycles are generated by a series of positive then negative demand shocks (a gradual rise then decline in tradeable absorption), while cost-push shocks play a subordinate role. See Figure 1.13 for the result.

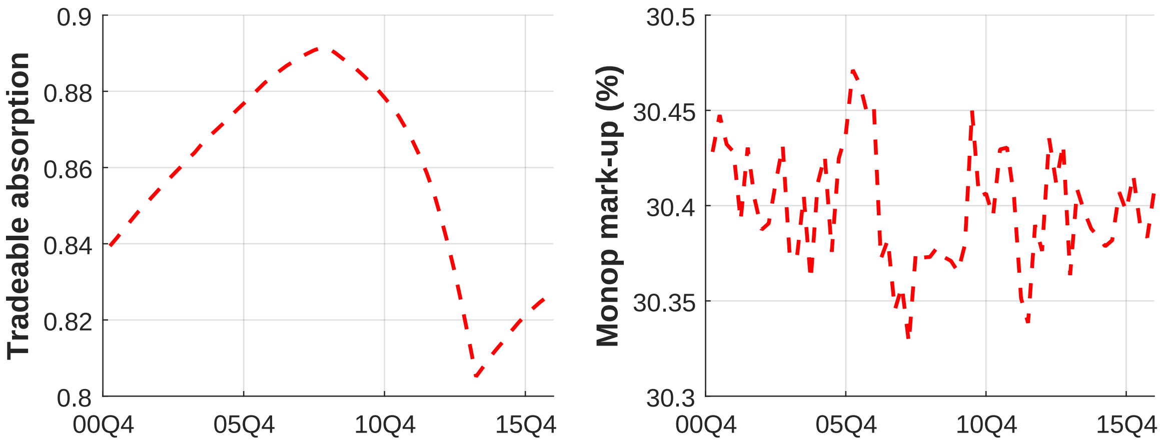

Figure 1.13: Implied shocks for the two counter factuals. Quarters on the x-axis. Tradeable absorption c_t^T in levels, the monopolistic mark-up $(\tilde{\theta}_t/(\tilde{\theta}_t - 1) - 1) \times 100$ in percent.

C Appendix: Numerical implementation

The Appendix C contains details on the numerical implementation of the laissez-faire equilibrium introduced in Section 1.3.

The relevant system of equations is given in Appendix Section A.5. To solve this system of equations, I divide the state space into two regions, one where the wage rigidity is binding ($\psi_t > 0$) and one where the rigidity is slack ($\psi_t = 0$). I define a polynomial for the conditional expectation in equation (1.3.4) for each of these two regions

$$i) \qquad \gamma E_t \tilde{\Lambda}_{t,t+1}\psi_{t+1} = P_n^{\psi=0}(w_{t-1}, \mathbf{s}_t; \eta_n^{\psi=0}) \quad \text{(slack region)} \tag{C.1}$$

$$ii) \qquad \gamma E_t \tilde{\Lambda}_{t,t+1}\psi_{t+1} = P_n^{\psi>0}(w_{t-1}, \mathbf{s}_t; \eta_n^{\psi>0}) \quad \text{(constrained region)} \tag{C.2}$$

where $\mathbf{s}_t \equiv (U^*_{c_t^{T,*}}, \theta_t)$ summarizes the shock space and where $P_n(w_{t-1}, \mathbf{s}_t; \eta_n)$ are polynomials in the variables w_{t-1} and $\mathbf{s}_t$ of degree n with coefficients η_n.[38] I am using two polynomials to approximate the conditional expectation, because the latter has a kink at the point where the wage rigidity is starting to bind. As a result, the two parameter vectors $\eta_n^{\psi=0} \neq \eta_n^{\psi>0}$ in general (e.g. Christiano and Fisher, 2000).

The algorithm proceeds as follows. Combine the equations in Appendix Section A.5 to

$$(\tilde{\theta}_t - 1)/\tilde{\theta}_t \; U_{c_t^T} w_t + P_n(w_{t-1}, \mathbf{s}_t; \eta_n) - \psi_t = \left[w_t U_{c_t^T} / ((1-\omega)\alpha)\right]^{\varphi/(\alpha-1-\alpha\kappa)}, \tag{C.3}$$

which, for the given polynomial $P_n(w_{t-1}, \mathbf{s}_t; \eta_n)$, for given ψ_t, and for given (exogenous) $\tilde{\theta}_t$ and $U_{c_t^T}$, defines implicitly a value for w_t. Simulate a series of shocks $\{U_{c_t^T}, \theta_t\}$.

Guess initial values for $\eta_n^{\psi=0}$ and $\eta_n^{\psi>0}$ (I am using a vector of ones). Start with a value for w_0 (I am using the non-stochastic steady state). Guess that in the first period, the wage rigidity is not binding. Therefore $P_n(w_0, \mathbf{s}_1; \eta_n) = P_n^{\psi=0}(w_0, \mathbf{s}_1; \eta_n^{\psi=0})$ in equation (C.3) as well as $\psi_1 = 0$, such that one can solve for the implied value of w_1 (this has to be done numerically). Check whether indeed, the wage rigidity is not binding in the first period: that is, check if $w_1 \geq \gamma w_0$. If yes, proceed to period 2. If not, set $w_1 = \gamma w_0$ and use equation (C.3) with $P_n(w_0, \mathbf{s}_1; \eta_n) = P_n^{\psi>0}(w_0, \mathbf{s}_1; \eta_n^{\psi>0})$ to back out the implied $\psi_1 > 0$. Once this is done, proceed to period 2.[39]

[38] I am using $P_n(w_{t-1}, \mathbf{s}_t; \eta_n) = \exp(p_n(\log(w_{t-1}), \log(\mathbf{s}_t); \eta_n))$, that is, the exponential of a tensor polynomial p_n in logs. I am using $n = 1$, which turns out to be good enough for a decent approximation (to check this, I am comparing the solution of the constrained efficient allocation under parameterized expectations and under value function iteration).

[39] Note that with this procedure one complication arises: it could be that the implied ψ_1 from the last step—in case the rigidity turned out to be violated—is negative. For this problem to arise it would be required that both $P_n^{\psi>0}(w_0, \mathbf{s}_1; \eta_n^{\psi>0}) < P_n^{\psi=0}(w_0, \mathbf{s}_1; \eta_n^{\psi=0})$, that is, the 'constrained polynomial' must be smaller than the 'slack polynomial', and that at the same time $w_1 < \gamma w_0$. If this problem arises, I am using the implied

Using the preceding method repeatedly results in a sequence $\{w_t\}$ and $\{\psi_t\}$ for the given sequence of exogenous shocks. This brings us to the updating step of the coefficients $\eta_n^{\psi=0}$ and $\eta_n^{\psi>0}$ for the polynomial. To do the updating, define a sequence of $\{y_{t+1}\}$ according to $y_{t+1} = \gamma\beta\tilde{\Lambda}_{t,t+1}\psi_{t+1}$, and split this sequence into two subsequences. The first subsequence defines the 'slack region' where $\psi_t = 0$, that is $\{y_{t+1}^{\psi=0}\} \equiv \{y_{t+1}\}_{\psi_t=0}$. Similarly, define a subsequence for the constrained region $\{y_{t+1}^{\psi>0}\}$. Now use that

$$i) \qquad y_{t+1}^{\psi=0} = P_n^{\psi=0}(w_{t-1}, \mathbf{s}_t; \eta_n^{\psi=0}) + u_{t+1} \quad \text{(slack region)}$$

$$ii) \qquad y_{t+1}^{\psi>0} = P_n^{\psi>0}(w_{t-1}, \mathbf{s}_t; \eta_n^{\psi=0}) + u_{t+1} \quad \text{(constrained region)}$$

where u_{t+1} should have (approximately) the properties of a prediction error such that a regression of the previous equations is feasible. That is, back out the $\eta_n^{\psi=0}$ and $\eta_n^{\psi>0}$ that give the best fit according to

$$i) \qquad \eta_n^{\psi=0} = \text{argmin}_{\eta_n^{\psi=0}} \sum_{t=\underline{T}}^{T} \left(y_{t+1}^{\psi=0} - P_n^{\psi=0}(w_{t-1}, \mathbf{s}_t; \eta_n^{\psi=0}) \right)^2 \quad \text{(slack region)}$$

$$ii) \qquad \eta_n^{\psi>0} = \text{argmin}_{\eta_n^{\psi>0}} \sum_{t=\underline{T}}^{T} \left(y_{t+1}^{\psi>0} - P_n^{\psi>0}(w_{t-1}, \mathbf{s}_t; \eta_n^{\psi>0}) \right)^2 \quad \text{(constrained region),}$$

where $\underline{T} >> 1$ to allow for a burn-in period of the simulation.[40] Once the update for $\eta_n^{\psi=0}$ and $\eta_n^{\psi>0}$ is complete, proceed back to the simulation step for $\{w_t\}$ and $\{\psi_t\}$. Iterate until convergence of $\eta_n^{\psi=0}$ and $\eta_n^{\psi>0}$.

w_1 from equation (C.3) with $P_n^{\psi=0}(w_0, \mathbf{s}_1; \eta_n^{\psi=0})$ and set $\psi_1 = 0$, that is, I am accepting that for this period the rigidity is violated $w_1 < \gamma w_0$. This is because, *in equilibrium*, with the correct $\eta_n^{\psi=0}$ and $\eta_n^{\psi>0}$ obtained, $P_n^{\psi>0}(w_0, \mathbf{s}_1; \eta_n^{\psi>0}) < P_n^{\psi=0}(w_0, \mathbf{s}_1; \eta_n^{\psi=0})$ will only materialize in the slack region where $w_1 > \gamma w_0$, such that *in equilibrium*, this problem does not arise. Therefore, as the algorithm converges to the correct $\eta_n^{\psi=0}$ and $\eta_n^{\psi>0}$, the problem gradually disappears (which can easily be checked ex-post).

[40] I am using $T = 500.000$ and a burn-in period of $\underline{T} = 500$.

Chapter 2

Exit expectations and debt crises in currency unions

2.1 Abstract

Membership in a currency union is not irreversible. Exit expectations may emerge during sovereign debt crises because exit allows countries to reduce their liabilities through a currency redenomination. As market participants anticipate this possibility, sovereign debt crises intensify. We establish this formally within a small open economy model of changing policy regimes. The model permits explosive dynamics of debt and sovereign yields inside currency unions and allows us to distinguish between exit expectations and those of an outright default. By estimating the model on Greek data, we quantify the contribution of exit expectations to the crisis dynamics during 2009–2012.

2.2 Introduction

Countries may join, but also exit currency unions, and market developments foreshadow such events. The euro area is a case in point. Figure 2.1 displays monthly yields on public debt in Italy, Spain, Ireland, and Greece relative to Germany. They fell strongly in the run up to the creation of the euro in 1999—in sync with expected inflation and depreciation—and stayed close to zero for about a decade. This episode illustrates not only that currency unions provide a nominal anchor to inflation-prone countries (Alesina and Barro, 2002); it also shows that credibility gains materialize prior to the adoption of the common currency. Yet the reverse holds as well: the mere expectation of an exit from a currency union may push up yields, as securities are expected to be redenominated into a new, weaker currency. In fact, "fears of a reversibility of the euro" are arguably an important driver of rising yield spreads after 2009 (ECB, 2013).

Yet these spreads, observed during a sovereign debt crisis, are understood to also provide compensation for the possibility of outright sovereign default (e.g., Lane, 2012). It is perhaps no coincidence that default premia and redenomination premia emerge jointly. After all, public debt, even if issued in nominal terms, is effectively real for an individual member country of a currency union as it lacks control of inflation. Without support from the union, a member state will have to repudiate its debt if it runs out of funds or faces a rollover crisis (Aguiar et al., 2013, 2015). By exiting the currency union and introducing a new currency, on the other hand, a country regains control of inflation: debt becomes nominal—provided it is issued under domestic law and can be redenominated by fiat. The real value of debt may then be reduced through inflation and depreciation.

How does the possibility of exit and currency redenomination impact the dynamics of sovereign debt crises? To address this question, we develop a model of a small open economy which is (initially) operating within a currency union. We assume that the government cannot commit to repaying its debt obligations in all states of the world. As sovereign default looms, the country experiences a sovereign debt crisis: a vicious circle of ever rising debt levels and sovereign yields. Moreover, the country may exit the currency union at any time, and in the process convert exiting liabilities at par value into a new currency. Market participants are fully aware of this possibility and ask for a redenomination premium because they expect the new currency to depreciate. Expected depreciation after exit is larger, the more severe the sovereign debt crisis, because it is through redenomination and depreciation that the health of public finances can be restored.

Formally, we specify policies through simple feedback rules which we permit to change over time in a way consistent with agents' expectations. Transitions are governed by a Markov

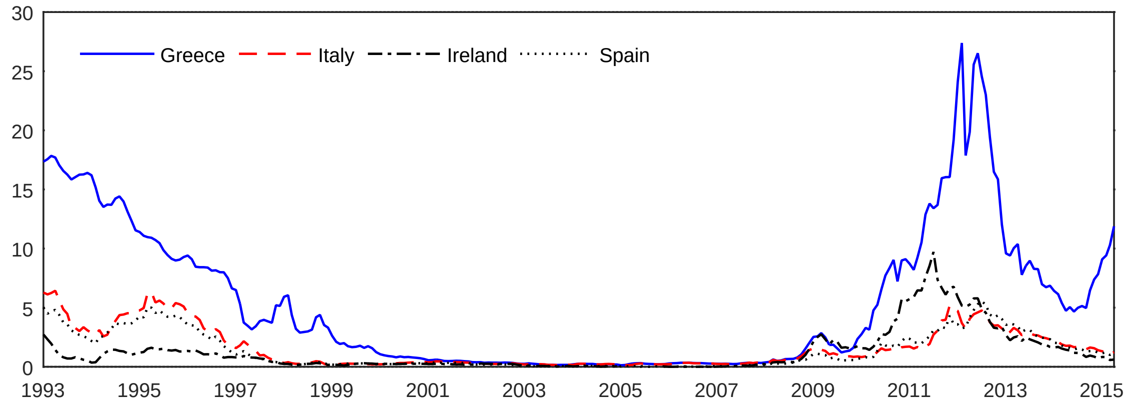

Figure 2.1: Interest rate spread vis-à-vis Germany for selected euro-area countries (percentage points). Data: monthly observations (1993M1–2015M4) for long-term interest rates for convergence purposes; source: ECB.

chain, such that exit and default are determined according to exogenous probabilities. Upon exit, monetary policy is expected to be "passive", thereby accommodating an "active" fiscal policy (Leeper, 1991). As a result, the initial price level and, importantly, the value of the new currency are expected to be determined upon exit by the need to align the real value of outstanding liabilities and future primary surpluses—an instance of the fiscal theory of the price level.[1] Hence, investors suffer losses upon exit which are proportional to outstanding government debt, such that redenomination premia prior to exit fluctuate endogenously over time.

In terms of methodological contribution, we introduce changing exchange-rate regimes in an otherwise standard New Keynesian small-open economy framework (Galí and Monacelli, 2005). While New Keynesian models are frequently used to study the properties of alternative exchange-rate regimes, the possibility of regime change as part of the equilibrium process and the expectation thereof are commonly ignored, even though policy regime changes have been analyzed in other contexts (Bianchi, 2013; Davig and Leeper, 2007a).[2] Because regime change is exogenous in our Markov-switching model, we maintain a high degree of tractability. In fact, we obtain closed-form expressions which allow us to inspect the mechanism by which

[1] For the fiscal theory in an open-economy context, see Woodford (1996), Sims (1997), Bergin (2000), Dupor (2000) and Daniel (2001). Our regime switching model builds on these contributions to the extent that the fiscal theory becomes operative after exit. Yet our focus is on the spillovers of the exit regime on the equilibrium outcome while the economy still operates in the currency union. Bianchi and Melosi (2014) rely on a similar mechanism, yet in a closed-economy context, as they explain the lack of disinflation in ths US during the Great Recession. Uribe (2006) studies sovereign default in a fiscal-theory environment.

[2] See also Bianchi and Ilut (2014); Davig and Leeper (2007b, 2011). These authors put forward models where monetary and fiscal policy rules change over time. Andolfatto and Gomme (2003) consider changes in money-growth rules under imperfect information. All these studies analyze closed-economy models.

exit expectations impact equilibrium outcomes in some detail. Moreover, exogenous regime change ensures that estimation of the model remains feasible.

We establish two results on how expectations of exit and default impact equilibrium dynamics. First, exit expectations reinforce the adverse dynamics of a sovereign debt crisis while the country still operates within the currency union. Such a crisis arises if public debt is high and fiscal policy fails to generate sufficiently high budget surpluses, for given beliefs of a regime change. Beliefs about regime change matter, because they determine—for given levels of debt—the size of redenomination (and default) premia which, in turn, impact public finances adversely. As a result, a sovereign debt crisis is reinforced—or may even be caused—by an adverse shift in beliefs about exit. The effect of such a shift is stronger, the more monetary policy is expected to tolerate inflation after exit in order to revalue the debt stock.[3]

Second, exit expectations which emerge during a sovereign debt crisis harm macroeconomic stability more generally. If public debt is high, expectations about exit drive up interest rates, not only for the sovereign, but also for private borrowers. This, in turn, has adverse effects on economic activity if nominal rigidities persist beyond exit. Moreover, in this case, inflation takes off already before the actual exit takes place due to forward-looking price-setting decisions. As a result, competitiveness deteriorates leading to a further drop in economic activity. Importantly, in order to establish these results, we permit the frequency of price adjustment to change with exit. This gives rise to a generalized Phillips curve. Here we find that, unless inflation moves one-for-one with the nominal exchange rate upon exit, exit expectations induce public debt and deficits to have stagflationary effects on the economy.

By way of contrast, if exit is ruled out, public debt and deficits are neutral in the baseline version of the model. Importantly, this is true even if there are expectations about outright default. Thus, expectations about exit and outright default affect the economy very differently in our analysis. Debt and deficits become recessionary in the presence of expectations about outright default, once we assume that sovereign default premia spill over into the private sector and impact borrowing conditions adversely (Bocola, 2015; Corsetti et al., 2013a), or once we assume taxes to be distortionary. Yet, even in this case, default and exit expectations generally impact macroeconomic dynamics differently. This allows us to identify exit and

[3] Note that our model does not feature full-fledged self-fulfilling crises, because we assume that probabilities of regime change are exogenous. Drazen and Masson (1994) consider a stylized two-period model in which exchange rate regimes may change depending on both the credibility of policy makers and the state of the economy. Calvo (1988) and Cole and Kehoe (2000) are classic references for self-fulfilling debt crises in the context of outright default. Aguiar et al. (2013, 2015) model self-fulfilling debt crises while highlighting the role of the monetary/exchange rate regime. Obstfeld (1996) analyzes self-fulfilling currency crises. Yet our analysis reiterates a theme which features prominently in classic studies of the stability of exchange rate regimes, namely that expectations about regime change destabilize an existing regime (Flood and Garber, 1984; Krugman, 1979).

default premia in actual time-series data.

We do so in the second part of our analysis, as we estimate the model on Greek data. Our sample starts in 2009, because the sizeable upward revision of the 2009-fiscal deficit in October 2009 arguably triggered the Greek debt crisis. In due course, with rising bond yields and a spiralling public debt-to-GDP ratio, the macroeconomic outlook deteriorated further, and discussions of Greece exiting the Euro area started to look serious.[4] Eventually, debt was restructured and converted in English-law bonds in early 2012. Our sample ends at this point, because once under English law, exit and depreciation would raise rather than lower the real value of debt. When we interpret the time series through the lens of the model, we find that redenomination premia account for a significant fraction of sovereign yields and for the bulk of the rise in yields in the private sector. Moreover, exit expectations account for about some ten percent of the output collapse as well as for a sizeable part of the (lack of) real exchange rate adjustment during our sample period. Overall, we thus find that the Greek crisis intensified considerably because of exit expectations.

In our analysis, we consider outright default and exit as alternative outcomes of a sovereign debt crisis in a currency union. Of course, debt repudiation and devaluation often occur jointly (Reinhart, 2002). Na et al. (2014) rationalize this observation in a model where default and exchange rates are determined optimally. Central to their analysis is the assumption that governments are indebted in foreign currency, the "original sin" of many emerging market economies. As a result, inflation and devaluation are ineffective in reducing the real value of debt. In our analysis, instead, we assume that public debt is governed by domestic law, in line with actual practice in the euro area (Chamon et al., 2015).[5] We also note that our analysis reestablishes—within the New Keyensian framework—two results of earlier work on currency crises, namely that expected devaluation may raise the refinancing cost of governments (Obstfeld, 1994) and induce a loss in competitiveness due to forward-looking price setting behaviour (Obstfeld, 1997).

Moreover, our paper relates to work which accounts for important aspects of the recent euro-area crisis, but with a focus on outright sovereign default (Bi, 2012; Daniel and Shiamptanis, 2012; Lorenzoni and Werning, 2013). Related empirical studies, instead, also focus on exit and redenomination premia. De Santis (2015) seeks to identify redenomination risk on the basis of

[4] The term "Grexit" has been widely used only since February 2012 (Buiter and Rahbari, 2012); around that time the German Ifo-think tank prepared a report on "Greece's exit from European Monetary Union" (Born et al., 2012). Still, the possibility of a Greek exit from the euro area has been discussed earlier (see, for instance, Feldstein, 2010). Shambaugh (2012) reports evidence on exit expectations from online betting markets.

[5] During the period 2003–2014 most (many) European countries issued more than 60-70 (90) percent of its debt under domestic law. Exceptions are the Baltic countries and Cyprus as well as Greece after the restructuring of its debt in 2012.

CDS spreads, thereby de facto conditioning his findings on default taking place simultaneously with exit. Krishnamurthy et al. (2014), in turn, decompose yield spreads into redenomination and default premia while accounting for market segmentation as well. According to their measure redenomination premia account for a very small fraction of yield spreads in those countries where sufficient data are available, namely, Italy, Spain, and Portugal.

Finally, we stress that there are a number of influential studies on how government debt can be or has been reduced through inflation. On the theory side, we mostly borrow from the fiscal theory of the price level (Cochrane, 2001; Sims, 2013; Woodford, 1995). On the empirical side, Reinhart and Sbrancia (2011) provide an account of how advanced economies have actually relied on inflation to rebase outstanding nominal liabilities. Hilscher et al. (2014) study this possibility for the US as of today, and conclude that it is unlikely that future inflation will contribute significantly to a reduction in the US debt-to-GDP ratio.

The remainder of the paper is organized as follows. Section 2 presents the baseline model. Section 3 develops our results regarding the destabilizing effect of exit expectations. We discuss several model extensions as well as details and results of the estimation of the model in Section 4. Section 5 concludes.

2.3 The model

We consider an open economy which is sufficiently small so as to have a negligible impact on the rest of the world. There are a representative household and monopolistically competitive firms, possibly restricted in their ability to adjust prices.[6] Households supply labor to firms, purchase goods produced domestically and in the rest of the world, and trade assets with the rest of the world. The government sells nominal debt and levies taxes on domestic households and firms. Government debt carries a default premium as the government reneges on its debt obligations in some states of the world. The economy either forms a currency union with the rest of the world or operates an independent monetary policy.

We capture the behavior of monetary and fiscal policy through simple feedback rules. Importantly, in order to analyze the effect of exit expectations, we permit policy rules to change over time in a way consistent with agents' expectations. Formally, we draw on recent contributions which analyze discrete changes in structural parameters as well as in the conduct of policy within Markov-switching linear rational expectations models (Bianchi, 2013; Davig and Leeper, 2007a). This framework is well suited to analyze the extent to which market beliefs regarding regime change impact equilibrium outcomes before actual regime change

[6] We consider a New Keynesian environment which has been studied extensively in a small-open economy context, for instance, in Kollmann (2001), Galí and Monacelli (2005) or Corsetti et al. (2013c).

takes place.

In what follows, we outline the structure of the baseline model which features complete international financial markets, lump-sum taxation, one-period government debt and abstracts from spillovers of sovereign risk into the private sector. We use the baseline model to illustrate how exit expectations impact macroeconomic dynamics and contrast their effects to those of default expectations. The mechanisms which we identify remain operative, once we relax several simplifying assumptions in our empirical analysis below.

2.3.1 Setup

The household problem is standard and not directly affected by the possibility of regime change. The representative household has preferences over consumption, C_t and aggregate hours worked, H_t:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{H_t^{1+\varphi}}{1+\varphi} \right),$$

where φ^{-1} parameterizes the Frisch elasticity of labor supply. $E_0(\cdot)$ is the expectation operator which accounts for uncertainty due to fundamental shocks as well as possible changes of the policy regime.

Consumption is a composite of goods produced at home, $C_{H,t}$, and imports, $C_{F,t}$, defined as follows

$$C_t = \left[(1-\omega)^{\frac{1}{\sigma}} C_{H,t}^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}} C_{F,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where σ denotes the elasticity of intratemporal substitution; $1-\omega$ measures the degree of home bias in consumption. Domestically produced goods and imports are both CES aggregates defined over different varieties, each produced by a firm $j \in [0,1]$:

$$C_{H,t} = \left[\int_0^1 C_{H,t}(j)^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{\gamma}{\gamma-1}}, \qquad C_{F,t} = \left[\int_0^1 C_{F,t}(j)^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{\gamma}{\gamma-1}},$$

where $\gamma > 1$ measures the elasticity of substitution. $P_{H,t} = \left[\int_0^1 P_{H,t}(j)^{1-\gamma} dj \right]^{\frac{1}{1-\gamma}}$ and $P_{F,t} = \left[\int_0^1 P_{F,t}(j)^{1-\gamma} dj \right]^{\frac{1}{1-\gamma}}$ correspond to the price indices for goods produced at home and imported from abroad, respectively. $P_t = \left[(1-\omega) P_{H,t}^{1-\sigma} + \omega P_{F,t}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$ denotes the consumer price index.

Household maximization is subject to a sequence of budget constraints of the type

$$E_t \left\{ \rho_{t,t+1} \Xi_{t+1} \right\} + P_t C_t = W_t H_t + \mathcal{Y}_t - T_t + \Xi_t,$$

where W_t denotes the nominal wage rate and $\mathcal{Y}_t = \int_0^1 \mathcal{Y}_t(j) dj$ are aggregate firm profits. T_t are taxes collected by the government in a lump-sum manner and Ξ_{t+1} is a portfolio of state-

contingent assets traded on international financial markets. Finally, $\rho_{t,t+1}$ is the one-period nominal stochastic discount factor. For future reference we define the nominally risk-free interest rate as the yield on a bond which pays one unit of *domestic currency* in all states of the world: $R_t \equiv 1/\{E_t\rho_{t,t+1}\}$, and say this bond is issued under *domestic law* (see below).
Households in the rest of the world face a symmetric problem such that in equilibrium, complete risk sharing ties relative consumption to the real exchange rate (see, for instance, Chari et al., 2002). Formally, using $\mathcal{E}_t$ to denote the nominal exchange rate—the price of one unit of foreign currency in terms of domestic currency—and an asterisk to denote variables in the rest of the world, we define the real exchange rate as the price of foreign consumption in terms of domestic consumption: $Q_t = \mathcal{E}_t P^*/P_t$. Assuming symmetric initial conditions across the two regions implies

$$Q_t = \frac{C_t}{C^*}.$$

Firms operate in a monopolistically competitive environment and rely on a linear production technology: $Y_t(j) = H_t(j)$. Moreover, while we assume that firms face price adjustment frictions à la Calvo, we permit the frequency of price adjustment to change with the monetary-policy/exchange-rate regime. This accommodates concerns that the Calvo parameter is not invariant vis-à-vis such fundamental policy changes and, hence, that there might be a structural break in the Phillips curve. In our rational expectations model, firms are fully aware of these complications.
Formally, at any time t, a resetting firm j chooses price $P_{H,t}(j)$ to satisfy the following objective

$$\max E_t \sum_{k=0}^{\infty} \left(\prod_{i=0}^{k} \xi_{\varsigma_{t+i}} \right) \rho_{t,t+k} \mathcal{Y}_{t+k}(j).$$

In this expression, $\xi_{\varsigma_{t+i}}$ is the per-period probability of not being able to reset a posted price. It is indexed to the policy regime in place at time $t+i$ by variable ς_{t+i}, the evolution of which is specified below. Prices are sticky in producer currency and the law of one price holds at the level of varieties. $\mathcal{Y}_{t+k}(j) = Y_{t,t+k}(j)(P_{H,t}(j) - W_{t+k})$ denotes profits in period $t+k$. Here $Y_{t,t+k}(j)$ is domestic and import demand for variety j at time $t+k$, given by[7]

$$Y_{t,t+k}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t+k}} \right)^{-\gamma} \left(\frac{P_{H,t+k}}{P_{t+k}} \right)^{-\sigma} \left[(1-\omega)C_{t+k} + \omega Q_{t+k}^{\sigma} C^* \right].$$

We define aggregate working hours as $H_t = \int_0^1 H_t(j)dj$ which—up to a factor capturing price dispersion—are linear in aggregate domestic output $Y_t = \int_0^1 Y_t(j)P_{H,t}(j)/P_{H,t}dj$.

[7] Here we use that the domestic country is small, which implies that $P_F^* = P^*$, that is, the consumption basket in the rest of the world is made up entirely of foreign-produced goods (see, e.g., De Paoli, 2009a).

The fiscal authority sells nominal debt to international investors. Its flow budget constraint is given by

$$(I_t)^{-1}D_t = D_{t-1}(1-\theta_t) - T_t.$$

Here, I_t denotes the gross yield of nominal government debt and D_{t-1} is debt which comes due in period t. The government reneges on its debt obligations in some states of the world. In the event, it applies a haircut to its outstanding liabilities of size $\theta_t \in [0, 1]$, which depends on the policy regime currently in place (see below).

International investors are risk neutral such that the absence of arbitrage possibilities requires the following condition for gross yields of government debt to be satisfied

$$(I_t)^{-1} = E_t\left((1-\theta_{t+1})\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right)(R^*)^{-1}.$$

Here, R^* is the opportunity cost of funds for international investors, namely the nominal yield on a bond which pays one unit of *foreign currency* in all states of the world—we say it is issued under *foreign law*. When the domestic economy is part of a currency union, the currency denomination of foreign and domestic law securities coincides: both are issued in the common currency. By contrast, whenever the domestic economy is operating an independent monetary policy, assets issued under domestic (foreign) law are denominated in domestic (foreign) currency. By the same token, if exit from a currency union is possible, the law under which assets are issued cannot be ignored, as the currency in which they pay off is contingent on whether the economy remains part of the currency union in the future.[8]

The model is closed by regime-dependent rules for monetary and fiscal policy which, given the other variables, pin down R_t, $\mathcal{E}_t$, θ_t and T_t as specified below.

2.3.2 Equilibrium with changing policy regimes

We conduct our analysis within a Markov-switching linear rational expectations (MS-LRE) model. For this purpose, we first specify the Markov chain which determines the evolution of policy regimes over time. In a second step, we characterize the policy regime in terms of linear policy rules, and present the linearized equilibrium conditions which describe the behaviour of the private sector. In a last step, we define the equilibrium. Throughout we refer to variables in terms of (log-)deviations from steady state using lower-case letters. A lower-case letter with a hat indicates deviations from steady state measured in percentage points of

[8] In the recent euro area crisis, market participants expected securities issued under Greek law to be converted into new currency upon exit (see, for example, Buiter and Rahbari 2012). As for Greek government debt, we note that more than 90% of Greek debt were issued under Greek law prior to the restructuring in 2012 (see, e.g., Buchheit et al. 2013; Chamon et al. 2015). Similarly, historical examples of "forcible conversions" of debt issued in foreign currency, but under home law highlights the role of jurisdiction for currency conversions (Reinhart and Rogoff 2011).

steady-state output. The steady state is assumed to be independent of policy regimes. There is no outright default and zero inflation in steady state and purchasing power parity holds.

Policy regimes are governed by the Markov chain $\{\varsigma_t\}$ which consists of the four states:

$$\varsigma_t \in \{Union,\ Union\ Default,\ Union\ Permanent,\ Exit\}.$$

Regimes differ in terms of parameters, as well as in terms of their (expected) duration. Formally, we define the transition matrix $\mathcal{P} = [\mathcal{P}_{lm}] = [Prob(\varsigma_t = m; \varsigma_{t-1} = l)]$ with

$$\mathcal{P} = \begin{pmatrix} \lambda & \delta & 0 & \epsilon \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where $\epsilon \in [0, 1]$, $\delta \in [0, 1]$ and $\lambda := 1 - \epsilon - \delta$ denote the transition probabilities between policy regimes. Assuming $\varsigma_0 = Union$, we represent the sequence of regime transitions as follows

$$\begin{array}{lll} & \nearrow_{\delta} & Union\ Default\ \rightarrow_1\ Union\ Permanent_{\circlearrowleft 1} \\ Union_{\circlearrowleft \lambda} & & \\ & \searrow_{\epsilon} & Exit_{\circlearrowleft 1}. \end{array}$$

Hence, initially the economy is part of a currency union. *Union* persists with probability $\lambda = 1 - \epsilon - \delta$, where δ denotes the probability of moving to *Union Default* in the next period. As specified further below, a haircut on government debt is applied in this regime. Immediately thereafter the economy moves to *Union Permanent* for good: further regime change is ruled out. By contrast, ϵ denotes the probability of moving to *Exit*, that is, of leaving the currency union and subsequently operating an independent monetary policy. We assume that, just like *Union Permanent*, *Exit* is an absorbing state of the Markov chain.[9]

We specify policies in terms of simple rules and parameterize how they change across policy regimes. The government raises lump-sum taxes in order to service debt as follows

$$\hat{t}_t = \psi_{\varsigma_t} \hat{d}_{t-1} - \mu_t. \tag{2.1}$$

Here, μ_t denotes a "deficit shock", a one-time transfer of resources from the government to the representative household. Rules of this type have been popularized by Leeper (1991) and are also recently used to characterize fiscal policy in the context of a sovereign debt crisis (e.g., Lorenzoni and Werning, 2013). The parameter ψ_{ς_t} captures the fiscal capacity of the

[9] Assuming absorbing states allows us to keep the analysis tractable. At the same time we acknowledge that reentering a monetary union or another haircut in the future cannot be ruled out in practice. Yet we abstract from these possibilities as their effect on the equilibrium outcome in the initial regime is bound to be small.

country and/or its willingness to raise taxes in response to a build up of public debt. It varies across policy regimes. We do not restrict this parameter in regimes *Union* and *Exit* but require $\psi_{\text{Union Default}} > 1-\beta$ as well as $\psi_{\text{Union Permanent}} > 1-\beta$. In the terminology of Leeper (1991), fiscal policy is "passive" in these regimes, as taxes are sufficiently responsive to debt.[10] Similarly, we posit a simple feedback rule for outright default

$$\theta_t = \zeta^{-1}\theta_{\varsigma_t}\hat{d}_{t-1}, \tag{2.2}$$

such that default only takes place in regimes where $\theta_{\varsigma_t} > 0$. Here parameter θ_{ς_t} captures the haircut applied to government debt in excess of its steady-state level. We allow for default in *Union Default*, such that $\theta_{\text{Union Default}} \geq 0$, and rule out default in all other regimes. Turning to monetary policy, we specify the following rule

$$\mathbb{1}_{\varsigma_t}e_t + (\mathbb{1}_{\varsigma_t} - 1)(r_t - \phi_\pi\pi_{H,t}) = 0. \tag{2.3}$$

Here, $\mathbb{1}_{\varsigma_t}$ is an indicator function which takes on the value of one in regimes where the country is part of a currency union, and of zero if monetary policy is independent. In the first case, there is no independent monetary policy, and the exchange rate is fixed exogenously at its steady-state value. In the second case, the central bank follows a Taylor-type rule which targets producer price inflation with a feedback coefficient $\phi_\pi \geq 0$. Note that our assumptions regarding *Exit* imply that, in the period of exit, domestic prices (as well as domestic-law securities) are converted at par into new currency. At the same time, the nominal exchange rate adjusts to clear the foreign exchange market upon exit.

We close the model by describing linearized equilibrium conditions which determine the behaviour of the private sector (see Section 2.3.1). Appendices A and B provide details on the derivation. Using $\varpi := 1 + \omega(2-\omega)(\sigma-1)$, we obtain a dynamic IS relation:

$$y_t = E_t y_{t+1} - \varpi(r_t - E_t\pi_{H,t+1}). \tag{2.4}$$

Under complete international financial markets output is tied to the real exchange rate q_t, the price of foreign consumption in terms of domestic consumption

$$(1-\omega)y_t = \varpi q_t, \tag{2.5}$$

$$q_t = (1-\omega)(e_t - p_{H,t}). \tag{2.6}$$

[10] Intuitively, given that *Union Permanent* is an absorbing state of the Markov chain, for equilibrium to exist it is required that fiscal policy ensures intertemporal solvency in this regime (see below the definition of stability). Hence, strictly speaking it is only required that $\psi_{\text{Union Permanent}} > 1-\beta$ for equilibrium to exist given that *Union Permanent* is an absorbing state of the Markov chain. By contrast, *Union Default* is purely transitory such that the size of $\psi_{\text{Union Default}}$ does not impact equilibrium dynamics and stability. For simplicity, then, we restrict it to be the same as in *Union Permanent*.

In introducing the firms' problem above, we explicitly considered the possibility that the frequency of price setting changes with a change in the monetary-policy/exchange-rate regime. Given our assumptions regarding regime transitions, this implies that parameter ξ_{Union} may differ from ξ_{Exit}.[11] We thus obtain a generalized New Keynesian Phillips curve for *Union*:

$$\begin{aligned}\pi_{H,t} &= \beta\left[\lambda E_t(\pi_{H,t+1}|\text{Union}) + \delta\Omega_1 E_t(\pi_{H,t+1}|\text{U Def}) + \epsilon\Omega_2 E_t(\pi_{H,t+1}|\text{Exit})\right] \\ &\quad + \kappa\left(\varphi + \varpi^{-1}\right)\Omega_1 y_t,\end{aligned} \tag{2.7}$$

where $\kappa := (1-\beta\xi)(1-\xi)/\xi$. The two factors Ω_1 and Ω_2 are given by

$$\Omega_1 = \frac{(1-\beta\lambda\xi)(1-\beta\xi_{\text{Exit}})}{(1-\beta\xi)(1-\beta\xi_{\text{Exit}}) + (1-\beta\xi)\beta\epsilon\xi_{\text{Exit}} + (1-\beta\xi_{\text{Exit}})\beta\delta\xi}$$

$$\Omega_2 = \frac{\xi_{\text{Exit}}}{\xi}\frac{1-\xi}{1-\xi_{\text{Exit}}}\frac{1-\beta\xi}{1-\beta\xi_{\text{Exit}}}\Omega_1.$$

In expression (2.7), operator $E_t(\cdot|\cdot)$ denotes the expectation conditional on a particular regime being in place in the next period. In all other regimes, the Phillips curve is standard:

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_{\varsigma_t}\left(\varphi + \varpi^{-1}\right) y_t, \tag{2.7'}$$

where $\kappa_{\varsigma_t} = \kappa$ in *Union Default* and *Union Permanent*, and where $\kappa_{Exit} = (1-\beta\xi_{Exit})(1-\xi_{Exit})/\xi_{Exit}$.

The ratio of public debt to GDP evolves as

$$\beta\hat{d}_t = \hat{d}_{t-1} + \zeta(\beta i_t - \pi_{H,t} - \Delta y_t - \theta_t) - \hat{t}_t, \tag{2.8}$$

where i_t denotes the sovereign bond yield, $\hat{t}_t$ denote taxes in units of GDP and ζ parameterizes the public debt-to-GDP ratio in steady state. Lastly, the yield is related to the nominal interest rate and expected default as follows

$$i_t = r_t + E_t\theta_{t+1}. \tag{2.9}$$

We are now in the position to define an equilibrium. First, we restate equations (2.1) - (2.9) more compactly as follows

$$\mathit{\Gamma}_{\varsigma_t} x_t = \mathit{\Phi}_{\varsigma_t} E_t(x_{t+1}|\varsigma_{t+1}) + \Lambda_{\varsigma_t}\mu_t, \tag{2.10}$$

where $x_t = (y_t, r_t, i_t, \theta_t, \pi_{H,t}, p_{H,t}, e_t, q_t, \hat{d}_t, \hat{t}_t)'$ and $\pi_{H,t} = p_{H,t} - p_{H,t-1}$. The matrices $\mathit{\Gamma}_{\varsigma_t}$, $\mathit{\Phi}_{\varsigma_t}$ and Λ_{ς_t} contain the parameters of the model and ς_t indicates that they are regime dependent. Our equilibrium definition follows Farmer et al. (2011).

Definition 5. A rational expectations equilibrium is a mean square stable (MSS) stochastic process that, given the Markov chain $\{\varsigma_t\}$, satisfies (2.10).

[11] To simplify the exposition, in the following we omit subscripts *Union*, *Union Default* and *Union Permanent* for ξ, with the understanding that $\xi := \xi_{\text{Union}} = \xi_{\text{Union Default}} = \xi_{\text{Union Permanent}}$.

Definition 6. An $n-$dimensional process $\{x_t\}$ is MSS if there exists an $n-$vector x_∞ and an $n \times n$ matrix Σ_∞ such that in all regimes

- $\lim_{n \to \infty} E_t[x_{t+n}] = x_\infty$
- $\lim_{n \to \infty} E_t[x_{t+n}\ x_{t+n}'] = \Sigma_\infty$.

Note that the concept of stability as defined above differs from stability as it is commonly applied in fixed-regime models. Intuitively, explosive trajectories in some regimes are not an issue if the economy does not stay in these regimes for too long. What matters is that trajectories are not globally explosive which is ruled out by MSS. The expected duration of regimes is thus key for stability.[12]

2.4 Inspecting the mechanism

We now inspect the mechanism by which exit expectations destabilize the economy while the country still operates in the initial regime. In this regime, the country is part of a currency union but membership is imperfectly credible. First, we show that exit expectations may reinforce or even induce a sovereign debt crisis—a vicious circle of ever growing debt and yield spreads. In particular, we establish conditions under which the model exhibits locally explosive dynamics. Second, we illustrate that exit expectations harm macroeconomic stability more generally. Throughout, we contrast the effects of exit expectations and default expectations.

2.4.1 How exit expectations reinforce sovereign debt crises

In this subsection, we assume prices to be perfectly flexible, both before and after exit ($\xi = \xi_{Exit} = 0$). This assumption, combined with our assumptions on the transition probabilities, permits us to solve the model in closed form. Specifically, because the two target regimes are absorbing, we solve the model backwards using the method of undetermined coefficients.[13]

[12] In general, a minimum state variable solution is mean square stable whenever the eigenvalues of $(\mathcal{P}' \otimes I_{n^2})\text{diag}(F_{\varsigma_1} \otimes F_{\varsigma_1}, ..., F_{\varsigma_h} \otimes F_{\varsigma_h})$ are all inside the unit circle, where h denotes the number of regimes, $\otimes$ is the Kronecker product and the F are solution matrices in the respective regimes, i.e. $x_t = F_{\varsigma_h} x_{t-1} + G_{\varsigma_h} \mu_t$ (Farmer et al. 2009). Note that MSS collapses to the conventional criterion of stability applied in fixed-regime models (see, for instance, Blanchard and Kahn, 1980) in absorbing states of the Markov chain. Thus, we require bounded dynamics in *Union Permanent* and *Exit*, while locally explosive dynamics are (in principle) possible in all other regimes.

[13] All derivations can be found in Appendix C. The analytical solution presented in Section 2.4.1 is the unique mean square stable minimum state variable solution of the model, provided $\lambda\left((1-\psi_{\text{Union}})\Theta^d\right)^2 < 1$, where Θ^d is defined below. If the latter condition is violated, no solution exists. The condition holds unless either ϵ or $\delta\theta$ are close to unity, and unless δ and ϵ are both close to zero while $\psi_{\text{Union}} < 1 - \beta$.

In all regimes, flexible prices imply constant output $y_t = 0$ by equations (2.7) and (2.7'). Given equation (2.4), this implies a constant real interest rate, $r_t - E_t\pi_{H,t+1} = 0$, and a constant real exchange rate, $q_t = 0$ (see equation (2.5)). The latter, in turn, requires $p_{H,t} = e_t$ by (2.6), such that prices move one-for-one with the nominal exchange rate after exit. Prior to exit, yields carry a redenomination premium which, in turn, affects public finances adversely. To see this, start from the observation that interest rates reflect expectations of future policies via a version of the uncovered interest parity (UIP) condition. Combine equations (2.4), (2.5) and (2.6) to obtain

$$r_t = E_t\Delta e_{t+1}. \tag{2.11}$$

In the initial regime, $e_t = 0$, while $e_{t+1} \neq 0$ is possible only if the country exits the currency union. Condition (2.11) holds in equilibrium and reflects the absence of arbitrage possibilities, as market participants are able to trade securities issued under both domestic and under foreign law. Imagine that exit from the currency union cannot be ruled out and that, upon exit, the newly created domestic currency is expected to depreciate ($E_t\Delta e_{t+1} > 0$). In this case, a domestic-law bond must offer a higher interest rate, because a foreign-law bond pays off strictly better (in terms of new domestic currency) in those states of the world where exit and depreciation occur. Given that r_t corresponds to the yield of a one-period bond issued under domestic jurisdiction (expressed in terms of deviation from steady state), it represents the "redenomination premium", that is, the spread of the yield of a domestic-law bond relative to that of a bond issued under foreign jurisdiction.[14]

To determine the redenomination premium, we solve for the change of the exchange rate in *Exit*. As it turns out, the degree of nominal depreciation upon exit depends primarily on how strongly the newly independent monetary policy raises nominal interest rates in response to inflation, as captured by parameter ϕ_π. Recall that, as we assume conversion at par, inflation *in the period of exit* is given by the initial price level in new currency, determined in general equilibrium, minus the price level which prevailed in terms of old currency, the period before exit. We obtain the following solution for nominal depreciation in *Exit*

$$\Delta e_t = \Theta^e \left[(1 - \psi_{\text{Exit}})\hat{d}_{t-1} + \mu_t\right], \tag{2.12}$$

$$\text{where} \quad \Theta^e = \begin{cases} 0 & \text{if } \phi_\pi > 1 \\ \dfrac{1 - \psi_{\text{Exit}} - \beta\phi_\pi}{\zeta(1-\beta\phi_\pi)(1-\psi_{\text{Exit}})} > 0 & \text{if } 0 \leq \phi_\pi \leq 1. \end{cases}$$

Hence, in case monetary policy satisfies the Taylor principle ($\phi_\pi > 1$), the exchange rate

[14] Recall that the latter pays one unit of common currency in all states of the world. It represents the spread, because variables are expressed in terms of deviation from steady state and we only consider shocks originating in the domestic economy, such that yields on foreign securities are constant.

will remain unchanged upon exit. By contrast, if monetary policy adjusts nominal rates only weakly in response to inflation ($0 \leq \phi_\pi \leq 1$), the exchange rate depreciates, and more so the lower the central bank's feedback coefficient (note that Θ^e attains a maximum at $\phi_\pi = 0$).[15] Intuitively, as $\phi_\pi \leq 1$ monetary policy permits inflation to adjust in order to stabilize public debt in real terms, such that nominal depreciation is larger, the larger the amount of outstanding debt and the larger the current budget deficit. The fiscal theory of the price level applies, such that the initial price level as well as the exchange rate adjust after exit in order to align the real value of debt with the expected sequence of real primary surpluses. Note that $\phi_\pi \leq 1$ is required for equilibrium to exist if ψ_{Exit} is sufficiently small; and that, conversely, $\phi_\pi > 1$ is possible only if the fiscal authority adjusts taxes sufficiently strongly after exit.[16]

Combining (2.11) and (3.4.1) determines the redenomination premium in *Union* which, in turn, impacts public finances adversely through sovereign yields. Sovereign yields carry a redonomination premium, because government debt is issued under domestic law, see equation (2.9). Higher debt service, all else equal, contributes to rising debt levels, see equation (2.8). As a result, there may be a vicious circle: rising debt levels raise expectations of a depreciation upon exit and vice versa.

To see this formally, we state the solution for public debt in *Union*

$$\hat{d}_t = \Theta^d \left[(1 - \psi_{\text{Union}}) \hat{d}_{t-1} + \mu_t \right], \tag{2.13}$$

$$\text{where } \Theta^d = \frac{1}{\beta} \left(1 - \epsilon \left(\frac{1 - \psi_{\text{Exit}} - \beta \phi_\pi}{1 - \beta \phi_\pi} \right) - \delta\theta \right)^{-1} \geq \frac{1}{\beta}.$$

We note that $\Theta^d(1 - \psi_{\text{Union}})$, the autoregressive root on debt in equation (2.13), may be either above or below unity. Moreover, Θ^d increases in ϵ. Hence, if—for a given fiscal policy parameter ψ_{Union}—public debt is on explosive trajectory, the rate at which debt accumulates increases further as $\epsilon > 0$.[17] Moreover, debt may be on a stable trajectory in the absence of exit expectations, but become explosive as ϵ is raised sufficiently.[18] In this regard, exit

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[15] Furthermore, one can show that the solution for public debt in *Exit* is given by the following expression

$$\hat{d}_t = \frac{\phi_\pi}{1 - \psi_{\text{Exit}}} \left[(1 - \psi_{\text{Exit}}) \hat{d}_{t-1} + \mu_t \right] \qquad \text{if } \; 0 \leq \phi_\pi \leq 1.$$

Thus, (the real value of) public debt is wiped out completely within one period after exit if monetary policy does not respond to the resulting inflation and nominal depreciation at all ($\phi_\pi = 0$).

[16] More formally, for uniqueness and stability of equilibrium, it is required that fiscal policy ensures intertemporal solvency ($\psi_{\text{Exit}} > 1 - \beta$) in case $\phi_\pi > 1$, an instance of "active monetary, passive fiscal" policy. Instead it is required that $\psi_{\text{Exit}} < 1 - \beta$ in case of $\phi_\pi \leq 1$, an instance of "active fiscal, passive monetary" policy (Leeper 1991). As we vary ϕ_π, we assume ψ_{Exit} satisfies these assumptions throughout.

[17] In case regime change is ruled out ($\epsilon = \delta = 0$), or if exit is ruled out and no haircut is expected ($\epsilon = \theta = 0$), we have $\Theta^d = \beta^{-1}$, that is, debt is mean reverting provided $\psi_{\text{Union}} > 1 - \beta$. In the reverse case of $\psi_{\text{Union}} < 1 - \beta$, debt is on an explosive trajectory even in the absence of expectations about regime change.

[18] Note also that for any given probability of exit, debt becomes more explosive as monetary and fiscal policy

expectations impact public finances in a way which is comparable to expectations about outright default: as with ϵ, Θ^d increases in $\delta\theta$, that is, as the expected losses due to a haircut become larger. Finally note that, for given expectations about exit or default, a sufficiently aggressive fiscal stance in *Union* may shield the economy from explosive dynamics.[19]
As public debt settles on a (locally) explosive path in *Union*, its price collapses and yields take off

$$i_t = \left(\Theta^\theta + \Theta^r\right) \left[(1 - \psi_{\text{Union}})\hat{d}_{t-1} + \mu_t\right], \tag{2.14}$$

$$\text{where } \Theta^\theta = \delta\theta\frac{\Theta^d}{\zeta} > 0 \quad \text{and} \quad \Theta^r = \epsilon\left(\frac{1 - \psi_{\text{Exit}} - \beta\phi_\pi}{1 - \beta\phi_\pi}\right)\frac{\Theta^d}{\zeta} > 0.$$

This closes the vicious circle described above: as debt builds up, expected losses to be realized in some states of the world increase. Investors are compensated by lower bond prices, but this raises debt levels further. Both exit and default expectations may drive such a vicious circle, as can be observed from equation (2.14): it decomposes sovereign yields into a redenomination premium and a default premium, as captured by the two parameters Θ^r and Θ^θ.[20]

2.4.2 How exit expectations harm macroeconomic stability

With sticky prices ($\xi > 0$ and $\xi_{\text{Exit}} > 0$), exit expectations matter for how debt dynamics feed back into the economy. To illustrate this and to contrast the effect of exit expectations to that of default expectations, we rely on model simulations using a version of the algorithm developed in Farmer et al. (2011). We use the same parameter values as in our application of the model to Greece. Section 2.5 provides details. An exception are the parameters ϵ, δ, θ and ξ_{Exit} which we vary in what follows. Figure 2.2 displays impulse responses of selected variables to a purely transitory deficit shock. We show results for the two polar cases: a scenario with exit expectations but without outright default ($\epsilon = 0.1, \delta = 0.1, \theta = 0$), represented by solid lines, and a scenario with default expectations but without exit ($\epsilon = 0, \delta = 0.2, \theta = 0.5$), represented by dashed lines. In both cases, we assume price stickiness is not expected to change with an exit from the currency union ($\xi_{\text{Exit}} = \xi$).
The upper left panel displays the deficit shock. The shock is assumed to be purely transitory and equal to one percent of annual steady-state output. In response to the shock, public debt and sovereign yield spreads rise steadily, irrespectively of whether there are only exit

are expected to be more accommodative upon exit (as ϕ_π or ψ_{Exit} decline).

[19] In related work, Lorenzoni and Werning (2013) consider default and slow moving debt crises and find that sufficiently responsive fiscal policy may shield the economy from explosive dynamics. Our results show that this insight carries over to the case of exit expectations.

[20] One can further show that, in the initial regime, $r_t = \Theta^r\left[(1 - \psi_{\text{Union}})\hat{d}_{t-1} + \mu_t\right]$ and $E_t\theta_{t+1} = \Theta^\theta\left[(1 - \psi_{\text{Union}})\hat{d}_{t-1} + \mu_t\right]$, thus the superscripts 'r' and 'θ'.

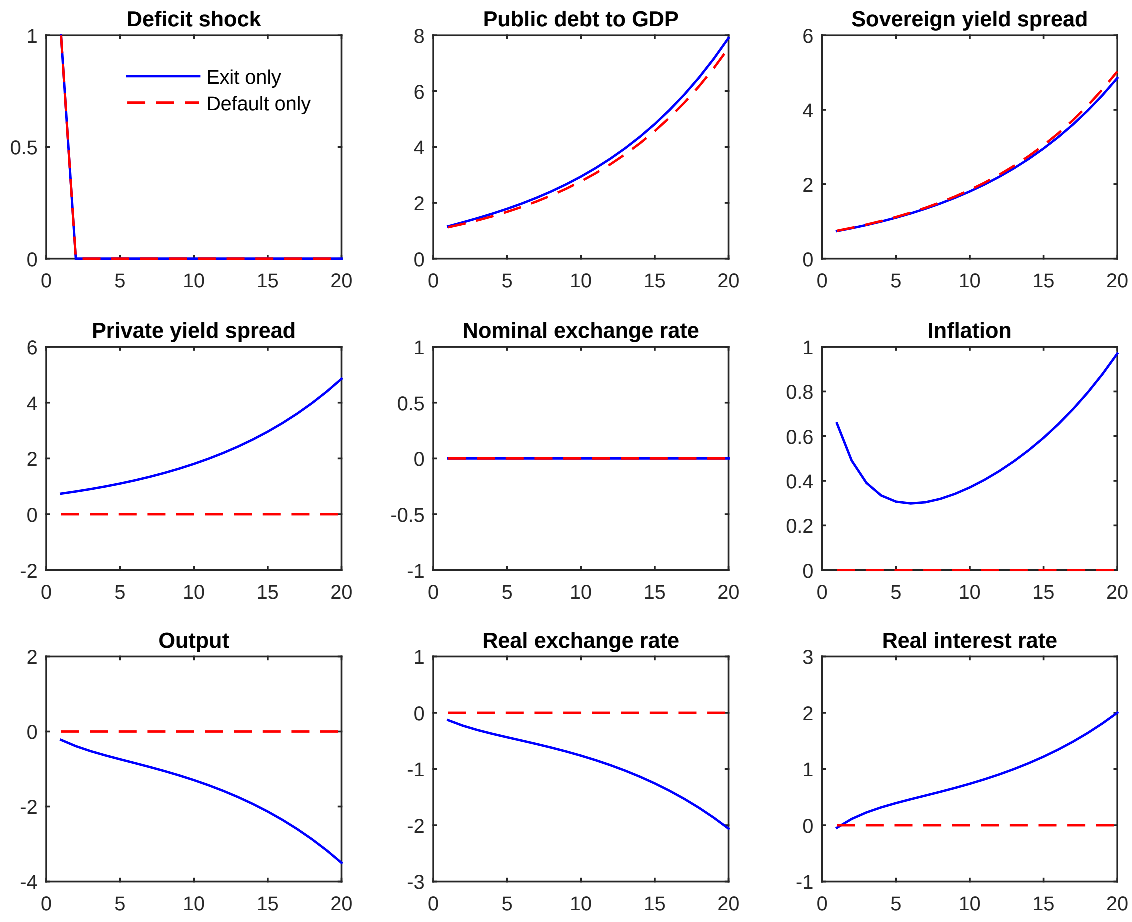

Figure 2.2: Impulse responses to a deficit shock in *Union*, conditional on staying in *Union*. Notes: Solid (dashed) lines represent exit-only (default-only) scenario; horizontal axes measure time in quarters; vertical axes measure deviations from steady state in percent, and percentage points in case of debt to GDP and the deficit shock (annual steady-state GDP in all cases); (producer-price) inflation and interest rates are annualized.

expectations or expectations about default. Thus, exit and default premia induce explosive dynamics in this example. This is because—in the initial regime—neither taxes nor the price level adjust (sufficiently) to stabilize the real value of public debt. As such, we note that a transitory deficit shock induces long-lasting effects—the model generates substantial internal propagation.

The dynamic adjustment of the economy differs fundamentally, however, depending on whether there are exit expectations or default expectations. In the presence of exit expectations (solid lines), deficits harm macroeconomic stability. Private yield spreads rise along with sovereign yield spreads. As the ex ante real interest rate rises, output collapses with domestic demand.

At the same time, (producer-price) inflation rises, while the nominal exchange rate remains flat. This appreciates the real exchange rate thereby making the domestic economy less competitive, which contributes to a further drop in domestic output.[21] Overall, exit expectations destabilize the economy by making debt and deficits stagflationary.

Instead, in the presence of default expectations (dashed lines) the deficit shock has no bearing on the economy other than on public finances. In particular, in the absence of exit expectations, the private yield spread r_t is zero. Thus, while the government's refinancing costs rise with expected default, private-sector interest rates remain unaffected. Intuitively, while yields on government debt increase notionally in expected losses due to a sovereign default, the *effective* ex ante interest rate remains unchanged. This holds irrespectively of whether government debt is held domestically or by international investors. In fact, Ricardian equivalence obtains either way.[22]

To illustrate the mechanism by which exit expectations harm macroeconomic stability in the initial regime, we conduct an additional experiment where exit actually realizes in period 10 after the deficit shock. To simplify the discussion, we rule out outright default for this experiment ($\epsilon = 0.1, \delta = 0.1, \theta = 0$). By contrast, we now allow for the possibility that price stickiness changes with an exit from the currency union. In particular, we contrast the case of unchanged rigidity ($\xi_{\text{Exit}} = \xi$) to a scenario of flexible prices after exit ($\xi_{\text{Exit}} = 0$).[23] Formally, note that in case of $\xi_{\text{Exit}} = 0$, $\Omega_1 = \frac{1-\beta\lambda\xi}{1-\beta(\lambda+\epsilon)\xi}$ and $\Omega_2 = 0$ such that (2.7) collapses to

$$\pi_{H,t} = \beta \left[\lambda E_t(\pi_{H,t+1}|\text{Union}) + \delta \frac{1-\beta\lambda\xi}{1-\beta(\lambda+\epsilon)\xi} E_t(\pi_{H,t+1}|\text{U Def}) \right] + \kappa \left(\varphi + \varpi^{-1}\right) \frac{1-\beta\lambda\xi}{1-\beta(\lambda+\epsilon)\xi} y_t. \quad (2.15)$$

That is, the Phillips curve in *Union* becomes steeper, the larger the probability of an exit (as ϵ increases)—as this effectively reduces price stickiness. At the same time, firms' pricing decisions in *Union* are unaffected by developments after exit, as firms anticipate that once the exit occurs, they will be able to optimally re-adjust their prices. Figure 2.3 shows results of this additional experiment, namely the response of selected variables to the deficit shock as considered before. In the figure, solid lines correspond to the case of unchanged rigidity, and dashed lines correspond to the case of flexible prices after exit.

The upper-left panel shows the response of the nominal exchange rate. In case of unchanged

[21] The nominal exchange rate (relative to steady state) is zero in the initial regime, $e_t = 0$, such that the real exchange rate appreciates one for one with a rise in producer prices, $q_t = -(1-\omega)p_{H,t}$, from equation (2.6).

[22] Intuitively, if bonds are priced actuarially fair, the possibility of sovereign default does not alter the present value of expected future taxation, see, for instance, Uribe (2006).

[23] As far as the dynamics in *Union* are concerned, this is equivalent to a scenario of a one-time reset of prices upon exit and renewed stickiness thereafter.

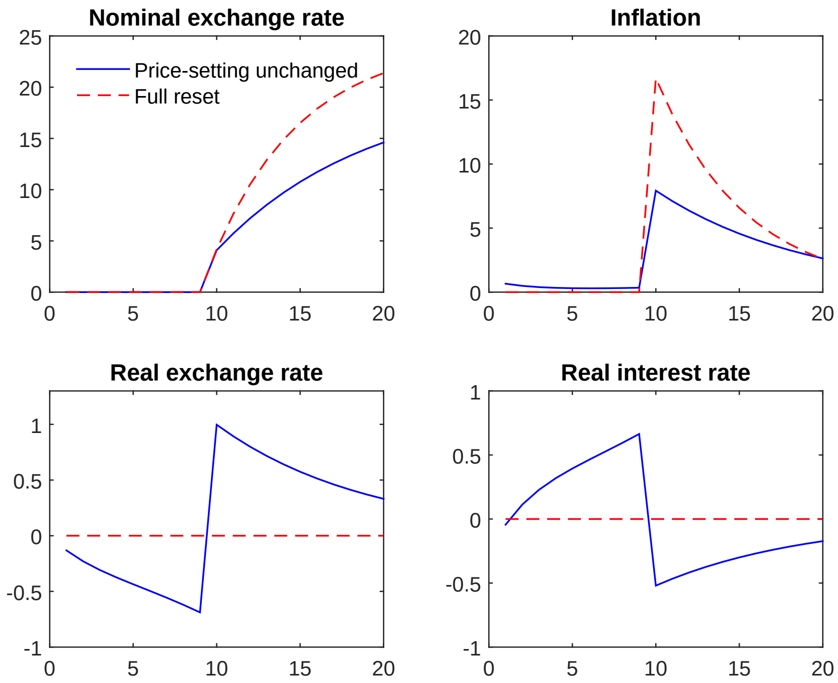

Figure 2.3: Impulse responses to a deficit shock in *Union* and actual exit in period 10 for different levels of rigidity in *Exit*. Solid line corresponds to unchanged rigidity ($\xi = \xi_{\text{Exit}} = 0.85$), dashed line assumes flexible prices after exit; horizontal axes measure time in quarters; vertical axes measure deviations from steady state in percent, (producer-price) inflation and interest rates are annualized.

rigidity, there is a discrete upward shift upon exit and more gradual depreciation thereafter. Overall, the response of the exchange rate is quite similar under flexible prices after exit, yet in the long run it depreciates by more in this case.[24] The response of inflation (upper-right panel) is highly dependent on the degree of rigidity: it increases sharply in case prices are flexible after exit. While inflation also takes up in the case of unchanged rigidity, its response is muted relative to the scenario of flexible prices. Moreover, if prices are flexible after exit, the real exchange rate does not adjust after exit (bottom-left panel). Instead, in case of unchanged rigidity, the sluggish response of inflation after exit induces the real exchange rate to depreciate upon exit along with the nominal exchange rate.

The lower-right panel shows the ex ante real interest rate which governs the intertemporal allocation of private domestic expenditure and hence, the recessionary impact of the deficit shock in the presence of exit expectations illustrated in Figure 2.2 above. The ex ante real

[24] Under $\phi_\pi = 0$, the exchange rate jumps to its long-run value straight away upon exit. By contrast, as $\phi_\pi > 0$ in our simulations, nominal interest rates increase with inflation in *Exit* such that as a result, the nominal exchange rate adjusts gradually—in line with UIP condition (2.11).

rate relates to the real exchange rate as follows

$$r_t - E_t\pi_{H,t+1} = E_t(\Delta e_{t+1} - \pi_{H,t+1}) = (1-\omega)^{-1}E_t\Delta q_{t+1}, \tag{2.16}$$

where the above relation follows from combining the UIP condition (2.11) and the definition of the real exchange rate (2.6). Thus, equilibrium requires that an expected real depreciation is met by increased real interest rates. If prices are flexible throughout (see Section 2.4.1), the above expression is zero because—upon exit—inflation is expected to adjust one-for-one with the depreciation of the nominal exchange rate. In other words, while market participants expect *nominal* depreciation upon exit, which raises *nominal* interest rates in the initial regime, they do not expect *real* depreciation such that *real* interest rates in the initial regime are unchanged. As Figure 2.3 shows, it is enough for price rigidity to disappear *upon exit* for the same result to obtain in the sticky-price model.

We conclude that exit expectations harm macroeconomic stability to the extent that (some) price stickiness is expected to persist beyond exit.[25] Under the same condition, inflation rises (somewhat) already prior to exit, implying an appreciation of the real exchange rate in the initial regime (see Figure 2.3). This is because forward looking firms tend to raise prices if they expect real depreciation upon exit which, in turn, will raise marginal costs. We note that empirically, large devaluations tend to be associated with sizeable real depreciations (Burstein et al., 2005). To allow for this possibility, but also for the possibility of a structural break in the Phillips curve upon exit, we let the parameter ξ_{Exit} be determined in the estimation in our empirical analysis below.

2.5 Greece 2009–2012

In this section we quantify the contribution of exit expectations to the actual crisis dynamics in Greece. For this purpose, we estimate a variant of the model on time-series data for the period 2009Q3–2012Q1. The sovereign debt crisis in Greece started in earnest in 2009Q4, shortly after the newly elected Papandreou government announced a substantial overshooting of the previous government's projection for the 2009-budget deficit, from 6 to 12.7 percent of GDP (Gibson et al. 2012). We limit our analysis to the period prior to the restructuring of Greek public debt in March/April 2012, because we are interested in the repercussions of expectations of exit and default, rather than of the event itself. Recall that before the restructuring Greek public debt—in line with our modelling assumption—was issued almost exclusively under Greek jurisdiction (Buchheit et al., 2013; Chamon et al., 2015).

Two properties of the model are essential for the estimation. First, the model allows us

[25] For $0 < \xi_{\text{Exit}} < \xi$, the responses of all variables fall in between the cases displayed in Figure 2.3.

to tell redenomination and default premia apart, because they impact the transmission of shocks in distinct ways. Second, our Markov-switching linear rational expectation model permits equilibria which feature (locally) explosive dynamics. This is important, because Greek time series for debt and yields appear to follow explosive trajectories in our sample period. However, our baseline model abstracts from a number of complications which appear essential for a serious quantitative assessment of the macroeconomic developments in Greece. Hence, we introduce a number of model extensions before turning to the data.

2.5.1 Extended model

First, note that in the baseline model public debt is non-neutral in the presence of expectations of an exit, but neutral in the presence of expectations of an outright default. The latter property may seem inadequate to the extent that output growth tends to be reduced if default looms (Yeyati and Panizza, 2011). We therefore allow for the possibility that sovereign default premia spill over to the private sector via a "sovereign risk channel" (Bocola, 2015; Corsetti et al., 2014). In order to do so, we relax the assumption that international financial markets are complete. In the extended model, the household budget constraint is given by

$$\Psi_{B,t} B_t + \Psi_{B^*,t} B_t^* \mathcal{E}_t + P_t C_t = W_t H_t + \mathcal{Y}_t + B_{t-1} + B_{t-1}^* \mathcal{E}_t + \mu_t,$$

where B_t and B_t^* are nominally non-contingent bonds issued under domestic and foreign law, respectively, both of which are traded with the rest of the world.[26] We also allow for taxes to be distortionary, namely proportional to the output of firms.[27]

In order to allow for the possibility that sovereign default risk spills over to bond prices in the private sector we postulate the following relationships

$$\Psi_{B,t} = E_t(1 - \chi\theta_{t+1})(R_t)^{-1}, \quad \Psi_{B^*,t} = E_t(1 - \chi\theta_{t+1})(R^*)^{-1}.$$

Here R_t (R^*), as before, denotes the nominally risk-free interest rate on a bond issued under domestic (foreign) law, that is, on a bond that pays one unit of domestic (foreign) currency in all states of the world.[28] Following Corsetti et al. (2013a) we rationalize a value of χ larger than zero by the observation that private-sector contracts may not be fully enforced in the event of a sovereign default. Importantly, however, we assume that even though lenders may

[26] In the absence of complete international financial markets small open economy models generally feature non-stationary dynamics. To avoid this property, we assume an endogenous discount factor (Schmitt-Grohe and Uribe, 2003). Also, we assume that B_t is in zero net supply, that is, all (cross-border) private saving is under foreign law. This roughly corresponds to actual practice in Greece during 2009–2012 (Buiter and Rahbari, 2012).

[27] Hence the term T_t does not appear in the household budget constraint any longer. The deficit shock, μ_t, however, continues to appear as it represents a lump-sum transfer to the household.

[28] If the sovereign risk channel is operative, R_t really is a "shadow" interest rate, as securities are not actually traded at this interest rate.

not be fully serviced in the event of sovereign default, borrowers do not retain resources in due course.[29]

The dynamics of sovereign debt are a key feature of our analysis. Therefore, in the extended model, we account for the fact that public debt is long-term following Woodford (2001). The government's flow budget constraint changes to

$$\Psi_t D_t + \tau_t P_{H,t} Y_t + Z_t = (1 + \iota \Psi_t) D_{t-1}(1 - \theta_t) + \mu_t,$$

where $\iota \geq 0$ parameterizes the maturity of debt. Ψ_t denotes the price of government debt, which solves

$$\Psi_t = E_t \left((1 + \iota \Psi_{t+1})(1 - \theta_{t+1}) \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right) (R^*)^{-1},$$

and which relates to the (gross) sovereign bond yield via

$$I_t = \frac{1 + \iota \Psi_t}{\Psi_t}.$$

In the government's budget, τ_t is the tax rate proportional to output, which, as before, may depend on the size of public debt through the feedback parameter ψ_{ς_t}. Furthermore, we allow the rest of the world to subsidize the domestic government through a transfer payment Z_t, which we model as an exogenous process. Such a subsidy may result from favorable borrowing conditions granted to Greece by official lenders such as the IMF or the EFSM. We measure it as the difference between interest rate payments on sovereign debt implied by market rates and actual interest payments.

Finally, we introduce four additional shocks. We introduce a world-demand shock, because world demand falls rather dramatically in the wake of the global financial crisis, presumably contributing to the recession in Greece during our sample period. We capture frictions in the labour market by driving a wedge between the household's marginal rate of substitution between consumption and leisure and the economy's marginal rate of transformation. This shock re-appears in the Phillips curve, thus equivalently has the interpretation of a "cost-push shock". We thereby also account for the possibility that Greece loses competitiveness vis-à-vis its euro area partners during our sample period (see, e.g., Born et al. 2012).[30] Moreover, we account for financial frictions affecting the consumption-savings choice at the household level, by introducing a "spread shock" in the consumption Euler equation. Lastly, we allow for sovereign yields to fluctuate independently of default and redenomination premia and other

[29] Hence, an actual default has no direct bearing on the household's budget constraint. Otherwise, borrowers' interest rate would rise with sovereign risk only *notionally*, not affecting behaviour up to first order, as explained in Curdia and Woodford (2010). Bocola (2015) models the pass-through of sovereign risk while explicitly accounting for financial intermediation.

[30] In principle, a series of positive cost-push shocks could generate an "overvalued" real exchange rate, thereby contributing to the recession. Cost-push shocks are also an important factor when it comes to accounting for inflation dynamics (Smets and Wouters, 2007).

fundamentals due to "liquidity shocks". This addresses concerns that "market segmentation" is an important factor driving yield spreads during the recent euro area crisis (Krishnamurthy et al., 2014). All shocks in the model are assumed to be i.i.d. We provide more details on the extended model in Appendix A.

2.5.2 Data and estimation

We estimate the model using a Bayesian approach (see, e.g., Fernández-Villaverde et al., 2016). For this purpose we rely on quarterly observations for six variables: output, CPI inflation, sovereign and private-sector yield spreads, the primary budget surplus as well as transfers from abroad. The data is obtained from ECB and Eurostat and described in more detail in Appendix D. Given the short sample period under consideration, we thus rely on 60 observations to estimate the model. For this reason, we keep a number of parameters fixed and estimate only four parameters and the six standard deviations of the shocks. We verify that all parameters are identified locally, using the method by Iskrev (2010). We also find that the data contains sufficient information for the posterior distributions of the estimated parameters to tighten considerably relative to the prior distributions (see below).
Figure 2.9 displays the data. Both sovereign and private yield spreads are measured relative to their German counterparts. Private-sector yield spreads are measured using interest rates earned on short-term deposits of non-financial institutions and households with domestic banks; results based on loan rates are very similar. Sovereign yield spreads are measured using yields on ten-year government debt, because the average maturity of public debt during the sample period is quite high (see below). Both, private and public spreads follow apparently explosive trajectories. Our measures for CPI inflation and output growth are also computed in terms of differences relative to their German counterpart. While output growth is persistently negative throughout, inflation is particularly high during the first half of the sample. The primary budget surplus is persistently negative throughout the sample period. Finally, transfers are measured in percent of output, using secondary-market interest rates and actual interest payments on public debt. They start to rise sharply from 2011 onwards, but are negative during the first half of the sample. This reflects high actual financing costs relative to secondary-market rates during the early stage of the crisis, because substantial amounts of short term debt had to be refinanced.
As discussed above, we only estimate a subset of model parameters. Specifically, we estimate the probability of exit and default, ϵ and δ, as well as parameter χ which captures the strength of the sovereign risk channel. Moreover, we estimate the degree of price rigidity after exit, captured by ξ_{Exit}. As discussed above, this parameter also determines how strongly exit

	Prior distribution			Posterior distribution			
	Distribution	Mean	Std	Mean	Std	10 %	90 %
ϵ	Gamma	0.055	0.05	0.034	0.011	0.022	0.048
δ	Gamma	0.055	0.05	0.058	0.051	0.008	0.126
χ	Beta	0.2	0.1	0.123	0.067	0.048	0.213
ξ_{Exit}	Beta	0.66	0.1	0.836	0.095	0.692	0.925
σ_{deficit}	Inverse-G.	0.01	∞	0.097	0.022	0.073	0.125
$\sigma_{\text{cost-push}}$	Inverse-G.	0.01	∞	0.008	0.003	0.006	0.013
$\sigma_{\text{world-demand}}$	Inverse-G.	0.01	∞	0.073	0.019	0.052	0.097
$\sigma_{\text{sov-liqu}}$	Inverse-G.	0.01	∞	0.013	0.003	0.009	0.017
σ_{spread}	Inverse-G.	0.01	∞	0.003	0.001	0.002	0.004
$\sigma_{\text{transfers}}$	Inverse-G.	0.01	∞	0.032	0.007	0.025	0.041

Notes: exit probability measured by ϵ, probability of outright default measured by δ, χ parameterizes pass-through of sovereign risk into private yields, ξ_{Exit} captures price rigidities after exit. The remaining six parameters measure the standard deviations of the shocks. The posterior distributions are computed on the basis of the Metropolis-Hastings algorithm. Other parameters are held fixed in the estimation, see main text for details.

Table 2.1: Prior and posterior distribution of estimated model parameters

expectations impact the allocation in the initial regime. Lastly, we estimate the standard deviation of all six disturbances.

As prior distributions we choose a Gamma distribution with mean 0.055 and standard deviation 0.05 for both ϵ and δ. The mean implies a probability of either exit or default of 25 percent within the next 18 months, in line with views maintained by market participants during our sample period (e.g., Buiter and Rahbari, 2012, UBS, 2010 and Shambaugh, 2012).[31] Regarding χ, we choose a Beta distribution with mean 0.2 and standard deviation 0.1. We thereby try to account for results from a variety of empirical studies. While Neri (2013) finds that the pass-through of sovereign risk into bank lending rates is quite low in Greece (about 0.07), other studies find that the adverse effect of sovereign risk on borrowing conditions is quite a bit stronger (Harjes, 2011; Zoli, 2013). Regarding ξ_{Exit} we maintain as prior a Beta distribution centered around 0.66 with standard deviation 0.1. Given that we assume $\xi = 0.85$ (see below), this accommodates the notion that prices should become more flexible upon and after exit. However, under our prior they are unlikely to become fully flexible, given that large devaluations are typically associated with strong movements in real exchange rates (Burstein et al. 2005). Finally, we employ an Inverted-Gamma distribution with mean 0.01 and an

[31] Shambaugh (2012) reports share prices on online-betting platforms regarding an exit of at least one member state from the euro zone until December 2012. His sample starts in September 2010, when the share price stood at 40%. This roughly corresponds to a 5% per-quarter probability of an exit of a member state from the union, in line with our prior mean for parameter ϵ.

infinite variance for the standard deviations of all shocks. Table 2.1 summarizes our priors in the left panel.

The remaining parameters are kept fixed in the estimation procedure. The discount factor β is set to 0.99. We set $\varphi = 4$, implying a moderate Frisch elasticity of labor supply (Chetty et al., 2011). The trade-price elasticity σ is set to 2, in line with estimates for Southern European countries reported by European Commission (2014). For ω we assume a value of 0.2, corresponding to the 2009 export-to-GDP ratio in Greece. We set $\gamma = 11$ such that the steady-state mark up is equal to 10 percent. Moreover, we assume $\iota = 0.9648$ which implies an average maturity of debt of 7.1 years (Krishnamurthy et al., 2014). To account for a relatively flat Phillips curve during the recent crisis period, we set $\xi = 0.85$ (see, e.g., IMF 2013). Furthermore, we set $\zeta = 2.4$, such that public debt in steady state amounts to 60% of annual GDP. Recall that steady-state debt is not subject to the haircut if default takes place. At the time of restructuring, Greek debt held by official institutions (EFSF, ECB/NCB and IMF) amounted to about 60 Percent of GDP and was indeed exempted from the restructuring. Private investors, instead, accepted a haircut of approximately 64 percent (Zettelmeyer et al., 2013). We thus set $\theta = 0.64$.

Regarding fiscal policy, we assume $\psi_{\text{Union}} = \psi_{\text{U Per}} = 0.015$. Given $\beta = 0.99$, this value ensures that explosive dynamics in the initial regime are driven by exit and default expectations. Moreover, we assume an inflationary monetary-fiscal mix after exit by setting $\phi_\pi = 0.83$ and $\psi_{\text{Exit}} = 0$. This choice is guided by estimates for the pre-Volker period in the U.S. The seminal study of Clarida et al. (2000), for instance, reports a value of $\phi_\pi = 0.83$. Similarly, Traum and Yang (2011) estimate of a full-fledged business cycle model and report a value of $\phi_\pi = 0.84$. They also report values for the debt-feedback of taxes and public expenditures very close to zero. Finally, given ζ, we set $B^*\mathcal{E}/PY = -1.056$ in order to match the Greek net foreign asset to GDP position in 2009 equal to -86.4% according to estimates by ECB (2013).

We approximate the posterior distribution of the estimated parameters using a standard Metropolis-Hastings algorithm. We run two chains with 1,000,000 draws each. In order to assess convergence of the chains, we compute several measures following Brooks and Gelman (1998), see Appendix F for details. We find that the interval of the posterior distribution which is covered by the chains as well as the second moment of the posterior distribution are stable after approximately 700,000 draws. We report results based on every second draw of the last 100,000 draws of each chain.

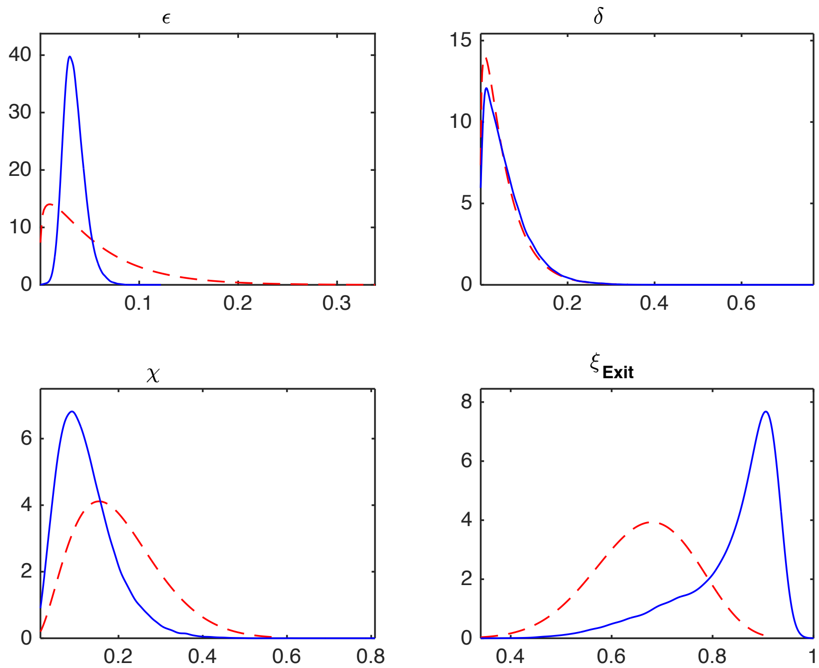

Figure 2.4: Prior (dashed) and posterior (solid) distribution of model parameters. Notes: exit probability measured by ϵ, probability of outright default measured by δ, χ parameterizes pass-through of sovereign risk into private yields, ξ_{Exit} captures price rigidities after exit.

2.5.3 Estimation results

Turning to the estimation results, we report key statistics in the right panel of Table 2.1. We note that the posterior mean has shifted somewhat above the prior mean in the case of δ and below the prior mean in the case of ϵ. At the same time, probability bands are quite large for δ. The posterior mean for χ implies that only 12% of sovereign risk spills over into the private sector. This finding, in line with Neri (2013), suggests that the role of the sovereign risk channel is limited in the Greek debt crisis. Lastly, the estimate of ξ_{Exit} suggests that nominal rigidities are expected to decline only moderately upon exit. Figure 2.4 displays prior and posterior distributions for these parameters illustrating the extent of identification (see Figure 2.10 in Appendix E for the distributions of the standard errors of the shocks). We find that for three of the four parameters the posterior distribution tightens considerably. We apply a Kalman smoother to reconstruct the sequences of unobserved shocks at the posterior mean and show the results in Figure 2.11 of Appendix E.

We now turn to the central issue, namely the quantitative contribution of exit (and default) expectations to the crisis dynamics in Greece. For this purpose, we simulate the model using the estimated shock sequences and the posterior mean of the estimated parameters and

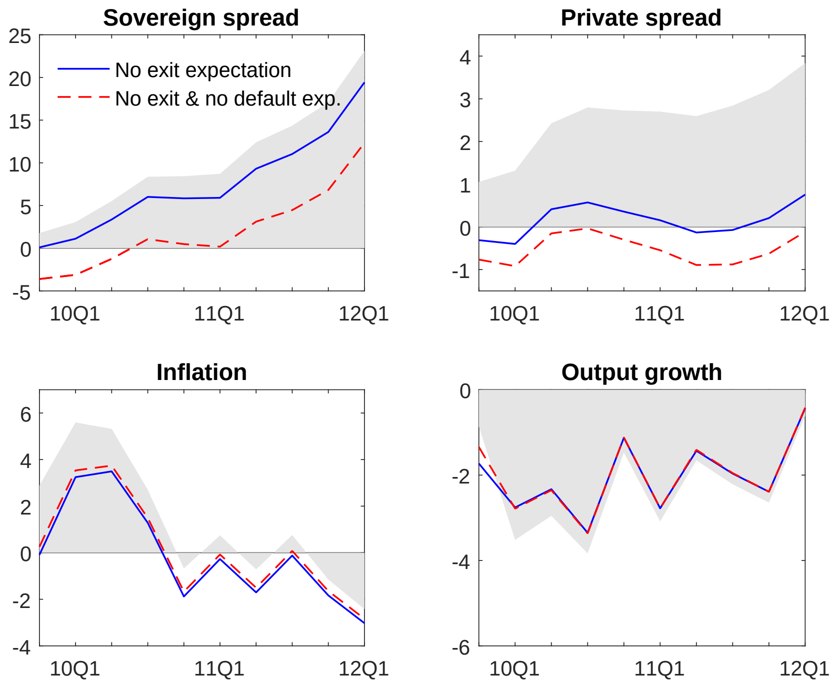

Figure 2.5: Counterfactual (vs actual) outcome of observed variables. Notes: shaded area indicates actual time series/prediction of estimated model; counterfactuals: solid line corresponds to scenario where exit is ruled out ($\epsilon = 0$), dashed line represents dynamics in the absence of exit and default expectations ($\epsilon = \theta = 0$). Interest rates are annualized and in percentage points. Inflation is annualized and measured in percent, quarterly output growth is measured in percent.

contrast the outcome to two counterfactuals. First, we isolate the effect of exit expectations by running a simulation as in the estimated model, except that expectations of exit are ruled out ($\epsilon = 0$). Second, we also rule out outright default by setting the haircut parameter to zero ($\theta = 0$). Figure 2.5 shows the result. In the figure, the grey area corresponds to the actual outcome, predicted by the estimated model. The solid blue and dashed red line, in turn, correspond to the counterfactual scenario where either exit or both exit and default expectations are absent. Given initial conditions in 2009Q3, we compute the counterfactual outcome for the period 2009Q4–2012Q1.

We find that exit expectations substantially impact the crisis dynamics during this period. Consider, first, the sovereign yield spread (upper left panel). Absent exit expectations, spreads would have been lower by some 1.5 to 3.5 percentage points at the beginning and the end of the sample, respectively. At the height of the crisis, the redenomination premium thus accounts for more than 15 percent of the yield spread. Our result that exit expectations reinforce sovereign debt crises is thus quantitatively relevant for Greece. At the same time, sovereign yields carry a substantial premium which compensates for the possibility of an outright default. Without expectations of exit *and* default yields would have been lower by some 5 to 10 percentage points. The remainder, that is, roughly one-half of the spread is explained by liquidity shocks. This finding is in line with Krishnamurthy et al. (2014), who find “market segmentation” is important when accounting for sovereign yield spreads in several crisis countries (although Greece is not included in their sample).

Our finding of a significant redenomination premium lends support to the view expressed by ECB president Mario Draghi in his “Whatever-it-takes”-speech on July 26, 2012. Regarding sovereign yield spreads, he remarks: “These premia have to do, as I said, with default, with liquidity, but they also have to do more and more with convertibility, with the risk of convertibility.” In fact, this consideration provides the rationale for what later becomes known as the “Outright Monetary Transactions” Program of the ECB. In this regard, it is crucial that these premia also show up in private-sector yields. Draghi emphasizes: “To the extent that the size of these sovereign premia hampers the functioning of the monetary policy transmission channel, they come within our mandate” (ECB, 2012).

The upper right panel of Figure 2.5 shows the decomposition of private-sector yields according to our counterfactuals. Results are clear cut: redenomination premia basically account for almost all of the private-sector spread observed during our sample period. If, in addition to exit, default is ruled out as well, there is a further reduction in private yield spreads, but the effect is small. This reflects the low estimate of the sovereign risk channel (parameter χ). Note that spreads may be negative because of our inclusion of financial friction “spread” shocks.

Exit expectations harm macroeconomic stability more generally. We contrast actual and counterfactual outcomes of CPI inflation and output growth in the bottom panels of Figure 2.5. We find that in the absence of exit expectations inflation is strongly reduced and particularly so in the early stage of the crisis period, that is, exit expectations are inflationary—in line with the discussion above. The effect of exit expectations on output turns out to be sizeable as well: the cumulative effect on output growth amounts to about 2.5 percentage points during our sample period and hence for some 12 percent of the total output loss.[32]

[32] During the first quarter of our sample, output growth would have been lower in the absence of exit expectations. This is surprising in light of our discussion from Section 2.4.2, where we found exit expectations to be unambiguously contractionary. However, as we assume international financial markets to be incomplete in the estimated model, this is not necessarily the case. Exit, in this case, reduces the real value of public debt in the hands of international investors and entails a wealth transfer to domestic tax payers. This implication of an exit, all else equal, stimulates domestic demand prior to exit and may (partly) offset the adverse effect of exit expectations on private demand via increased yields. The first effect dominates in the first quarter. Starting in 2010Q1, however, the latter effect dominates: growth would have been higher in the absence of exit expectations.

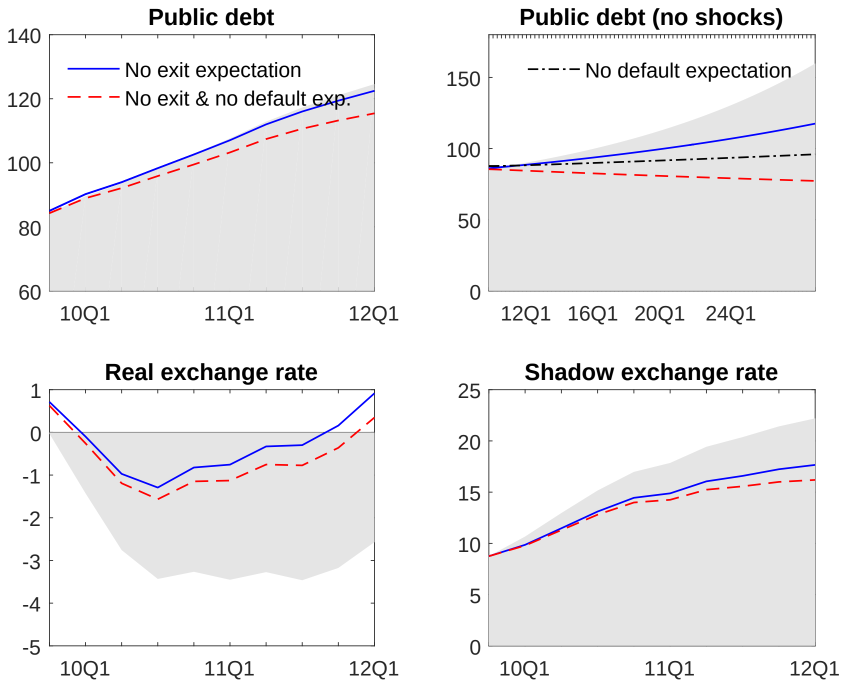

Figure 2.6: Counterfactual (vs actual) outcomes of additional variables. Notes: shaded areas indicate time series predicted by estimated model; counterfactuals: solid (dashed, dashed-dotted) line captures scenario w/o exit (w/o exit and w/o default, w/o default). Upper row shows public debt (percentage points of GDP), w/ and w/o shocks. Lower row shows shadow exchange rate—nominal exchange rate if exit were to take place—and real exchange rate, both in percent.

In order to assess the contribution of exit (and default) expectations to the sovereign debt crisis in Greece, we compute the model prediction of the debt-to-GDP ratio for the estimated model as well as for the two counterfactual scenarios. The upper-left panel of Figure 2.6 shows the results. The shaded area corresponds to the prediction of the estimated model, which captures the actual increase by some 40 percentage points during the sample period very well.[33] At the same time, our counterfactual simulations show that expectations of exit and default contributed only moderately to the build-up of debt during 2009–2012 (solid and dashed line). In fact, during our sample period the bulk of the debt increase is due to persistently negative primary surpluses and the strong drop in output at the time.

The destabilizing role of exit and default expectations becomes clear, once we abstract from shocks, and let the simulation run over a somewhat longer horizon, namely over 15 years. This is illustrated in the upper-right panel of Figure 2.6. It turns out that—given conditions

[33] Public debt in Greece amounted to some 130 percent of GDP in 2009Q4 and to 170 Percent in 2012Q1. Hence the model underestimates the level of debt at the beginning of the sample period.

in 2009Q3 for the estimated model (shaded area)—public debt is on an explosive trajectory. Exit expectations are to a large extent responsible, as our counterfactual simulation (solid line) illustrates. More than that, exit expectations alone suffice to generate an explosive trajectory in debt-to-GDP (dashed-dotted line). Only if both exit and default expectations are ruled out (dashed line), we observe that debt converges back to its steady state level in the long run.

Finally, we also report model predictions for the exchange rate. The lower-left panel of Figure 2.6 shows the real exchange rate, predicted to appreciate in the early stage of the crisis (as before, the shaded area corresponds to the prediction of the estimated model). We find that competitiveness does not start to improve before 2011 in line with actual developments. To a large extent this is due to exit expectations. In the counterfactual simulation without exit expectations (solid line), the real exchange rate hardly moves. The effect of default expectations on the exchange rate is small and of the opposite sign.

The lower right panel of Figure 2.6 shows the shadow exchange rate: the nominal exchange rate which would clear the foreign exchange market, were the country to exit the union in the respective period (see also Flood and Garber, 1984). According to our estimates, the nominal exchange rate would have depreciated by more than 20 percent had exit taken place at the end of our sample period. Absent exit or default expectations, the shadow exchange rate is lower than in the estimated model. Note that in principle, depreciation in the event of exit can be due to an appreciated real exchange rate or due to an accommodating monetary-fiscal policy mix after exit whenever public debt is high. Our analysis has highlighted the mechanisms which underlie the second channel and, indeed, according to our estimates, debt-induced depreciation accounts for the bulk of the depreciation of the shadow exchange rate—because according to our estimates, the appreciation of the real exchange rate is fairly moderate.

2.6 Conclusion

Countries may join, as well as exit currency unions. Expectations of an exit, in particular, may arise in the context of a sovereign debt crisis because by exiting, countries can redenominate their liabilities. The real value of debt will then decline with the value of the new currency. Against this background, we ask how exit expectations impact the dynamics of a sovereign debt crisis within a currency union. We put forward a small open economy model with changing policy regimes. In particular, we focus on a country which operates inside a currency union, but which may exit or, alternatively, apply a haircut to its outstanding liabilities while remaining part of the union.

Market participants are aware of these possibilities and expectations of exit and default matter for the equilibrium outcome. In particular, exit expectations drive up yields of securities issued under domestic law, both public and private, provided that the new currency is expected to depreciate upon exit. As a result, the sovereign debt crisis intensifies in the presence of exit expectations along two dimensions. First, exit expectations reinforce the adverse debt dynamics through their impact on yields and public finances. Second, exit expectations make public debt and deficits stagflationary.

In order to assess the quantitative importance of exit expectations, we estimate an extended version of the model on Greek times series for the period 2009–2012. We find that the estimated model performs rather well: we obtain plausible estimates for exit and default probabilities as well as for the dynamics of public debt and the real exchange rate, both not included in the vector of observables. Importantly, we find that exit expectations have an adverse and sizeable impact on economic outcomes in Greece during our sample period. Redenomination premia account for a significant fraction of sovereign yield spreads, and for almost all the spread observed in private-sector yields. Exit expectations also account for a large fraction of the loss in competitiveness and for more than 10 percent of the output loss during our sample period.

While our analysis is silent on the benefits and costs of an actual exit, it makes transparent how the adverse dynamics of a sovereign debt crisis within a currency union may intensify in the presence of exit expectations. Our findings are thus in line with a more general insight: policy frameworks which lack credibility tend to generate inferior outcomes.

Appendices

A Model appendix

A.1 Baseline model

Here we present details on the baseline model outlined in Section 2.3. In the following, lowercase letters denote the percentage deviation of a variable from its steady-state value, "hats" denote (percentage point) deviations from steady state scaled by nominal output. Variables in the rest of the world are assumed to be constant. The steady state is the same across regimes and characterized by zero net inflation, purchasing power parity, and zero default. We allow for non-zero public debt to GDP in steady state.

Households' first order conditions are given by an Euler equation

$$(C_t)^{-1} = \beta R_t E_t (C_{t+1})^{-1} \frac{P_t}{P_{t+1}}$$

and by a consumption-leisure condition

$$\frac{W_t}{P_t} = C_t H_t^{\varphi}.$$

Log-linearization of these two conditions, as well as of the risk-sharing condition stated in the main text, yields

$$c_t = E_t c_{t+1} - (r_t - E_t \pi_{t+1}) \tag{A.1}$$

$$w_t^r := w_t - p_t = c_t + \varphi h_t, \tag{A.2}$$

$$c_t = q_t, \tag{A.3}$$

where $\pi_t = p_t - p_{t-1}$ is CPI inflation.

Intermediate good firms face the demand function

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\gamma} Y_t,$$

so that

$$\int_0^1 Y_t(j)dj = \Delta_t Y_t,$$

where $\Delta_t = \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\gamma} dj$ measures price dispersion. Aggregation gives

$$\Delta_t Y_t = \int_0^1 H_t(j)dj = H_t.$$

A first order approximation is given by $y_t = h_t$. The derivation of the New Keynesian Phillips curve in *Union* is delegated to Appendix B. In all other regimes, the first order condition of the price setting problem is given by

$$E_t \sum_{k=0}^{\infty} \xi_{\varsigma_t}^k \rho_{t,t+k} Y_{t,t+k}(j) \left[P_{H,t}(j) - \frac{\gamma}{\gamma-1} W_{t+k}\right] = 0.$$

By linearizing this expression and using the definition of price indices, one obtains a variant of the New Keynesian Phillips curve (see, e.g., Galí and Monacelli, 2005):

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_{\varsigma_t} mc_t^r, \tag{A.4}$$

where $\kappa_{\varsigma_t} := (1-\xi_{\varsigma_t})(1-\beta\xi_{\varsigma_t})/\xi_{\varsigma_t}$, for $\xi_{\varsigma_t} = \xi_{\text{Exit}}$ in regime *Exit*, and $\xi_{\varsigma_t} = \xi$ in regimes *Union Default* and *Union Permanent*. Marginal costs defined in real terms, deflated with the domestic price index, are given by

$$mc_t^r = w_t - p_{H,t} = w_t^r - (p_{H,t} - p_t). \tag{A.5}$$

The real exchange rate and the relation between the producer and consumer price indices can be written as

$$q_t = e_t - p_t \tag{A.6}$$

$$p_t = (1-\omega)p_{H,t} + \omega p_{F,t} = (1-\omega)p_{H,t} + \omega e_t, \tag{A.7}$$

where in the last line we have used the law of one price, that is, $P_{F,t} = \mathcal{E}_t P_F^*$ such that $p_{F,t} = e_t$.

Goods market clearing in linear terms can be written as

$$y_t = -\sigma(p_{H,t} - p_t) + (1-\omega)c_t + \omega\sigma q_t,$$

which, combined with (A.6) and (A.7), can be written as

$$y_t = (1-\omega)c_t + \omega\sigma(2-\omega)/(1-\omega)q_t. \tag{A.8}$$

The key equations in the main text are obtained as follows. Combining equations (A.6) and (A.7) yields equation (2.6). Insert risk sharing (A.3) into goods market clearing (A.8) to obtain equation (2.5) in the main text. Rewrite the Euler equation (A.1)

$$\begin{aligned} c_t &= E_t c_{t+1} - (r_t - E_t[(1-\omega)\pi_{H.t+1} + \omega\Delta e_{t+1})] \\ &= E_t c_{t+1} - (r_t - E_t\pi_{H,t+1} - \frac{\omega}{\varpi}E_t\Delta y_{t+1}), \end{aligned}$$

where we use (A.7) in the first line and (2.5) and (2.6) from the main text in the second line. Combine (A.3) and (2.5) from the main text to obtain

$$c_t = \frac{1-\omega}{\varpi}y_t.$$

Use this expression to substitute for consumption in the Euler equation above to obtain

$$y_t = E_t y_{t+1} - \varpi(r_t - E_t\pi_{H,t+1}),$$

which is (2.4) in the main text. Use (A.2), (A.3), (A.6), (A.7) and production technology $y_t = h_t$ to rewrite marginal cost

$$mc_t^r = w_t^r - (p_{H,t} - p_t) = c_t + \varphi h_t - (p_{H,t} - p_t) = (\varpi^{-1} + \varphi)y_t.$$

Insert this into the Phillips curve to obtain (2.7)-(2.7') in the main text.

Sovereign yields and debt. The government's flow budget constraint can be written as

$$\beta\frac{(I_t)^{-1}}{\beta}\frac{D_t}{P_{H,t}Y_t}\frac{D_{t-1}}{P_{H,t-1}Y_{t-1}}\frac{Y_{t-1}}{Y_t}\frac{P_{H,t-1}}{P_{H,t}}(1-\theta_t) - \frac{T_t}{P_{H,t}Y_t}.$$

We linearize the flow constraint and denote $\hat{d}_t$ the deviation of debt to GDP from steady state, $\hat{t}_t$ the deviation of taxes to GDP from steady state, and ζ the level of debt to GDP in steady state. Furthermore, we denote i_t the log-deviation of the gross yield I_t from steady state (which is $1/\beta$).

Linearize the bond price schedule from the main text to obtain

$$i_t = E_t \Delta e_{t+1} + E_t \theta_{t+1},$$

where we have used that $R^* = 1/\beta$ and that $\theta = 0$ in steady state. Insert (2.11) from the main text to obtain (2.9) from the main text.

A.2 Extended model

Here we present details on the extended model which we estimate in Section 2.5. We provide the non-linear model equations, along with first order conditions, and details on the linearization. The steady state is the same across regimes and characterized by zero net inflation, purchasing power parity and zero default. However, we allow for non-zero public debt to GDP, as well as for non-zero net foreign assets to GDP in steady state.

Household preferences are now given by

$$E_0 \sum_{t=0}^{\infty} \beta_t \left(\log C_t - \eta_t \frac{H_t^{1+\varphi}}{1+\varphi} \right).$$

where the discount factor is endogenous and assumed to depend on the country's (aggregate) net foreign asset position, scaled by nominal output, in deviation from steady state, ζ_{B^*}:

$$\beta_{t+1} = \beta \left(1 + \alpha \left[\frac{\mathcal{E}_t \tilde{B}_t^*}{P_{H,t} Y_t} - \zeta_{B^*} \right] \right)^{-1} \beta_t, \quad \beta_0 = 1.$$

Households maximize utility subject to the budget constraint stated in Section 2.5.1. We note that in equilibrium, $B_t^* = \tilde{B}_t^*$, and that $B_t = 0$ is in zero net supply. η_t is a shock affecting the household's disutility of labour, which drives a wedge between the household's marginal rate of substitution between consumption and leisure and the economy's marginal rate of transformation. It thereby captures frictions in the labour market, while at the same time, acting as a "cost-push shock" to firms. α is a (small) positive constant, which induces stationarity to the model.

First order conditions are given by

$$\Psi_{B,t} = \frac{\beta_{t+1}}{\beta_t} E_t \frac{(C_{t+1})^{-1}}{(C_t)^{-1}} \frac{P_t}{P_{t+1}}, \qquad \Psi_{B^*,t} = \frac{\beta_{t+1}}{\beta_t} E_t \frac{(C_{t+1})^{-1}}{(C_t)^{-1}} \frac{P_t}{P_{t+1}} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$$

as well as the consumption leisure condition

$$\frac{W_t}{P_t} = \eta_t C_t H_t^{\varphi}.$$

As stated in the main text, we postulate the bond-prices to be affected by a "sovereign risk channel" as follows

$$\Psi_{B,t} = \nu_t E_t(1 - \chi\theta_{t+1})(R_t)^{-1}, \qquad \Psi_{B^*,t} = \nu_t E_t(1 - \chi\theta_{t+1})(R^*)^{-1}.$$

In these expressions, ν_t is a shock affecting directly the household's intertemporal consumption-savings choice. We shall label it "spread shock", as it drives a spread between the risk free rate of interest and the rate relevant for household decision making.

Linearizing and combining the previous equations yields an Euler equation and an uncovered interest parity (UIP) condition[34]

$$c_t = E_t c_{t+1} - (r_t - E_t \pi_{t+1} + \chi E_t \theta_{t+1} + \nu_t - \alpha \hat{b}_t^*) \tag{A.9}$$

$$r_t = E_t e_{t+1} - e_t. \tag{A.10}$$

Note that the effective ex ante real interest rate depends on sovereign risk if $\chi > 0$, by the spread shock ν_t, and by the stock of net foreign assets—a positive stock of net foreign assets reduces the ex ante real interest rate, making the household more impatient. Moreover, the leisure-consumption trade-off becomes

$$w_t - p_t = c_t + \varphi h_t + \eta_t. \tag{A.11}$$

We rewrite the household budget constraint as

$$\beta \frac{\Psi_{B^*,t}}{\beta} \frac{B_t^* \mathcal{E}_t}{P_{H,t} Y_t} + \frac{P_t C_t}{P_{H,t} Y_t} = (1 - \tau_t) + \frac{B_{t-1}^* \mathcal{E}_{t-1}}{P_{H,t-1} Y_{t-1}} \frac{P_{H,t-1}}{P_{H,t}} \frac{Y_{t-1}}{Y_t} \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} + \mu_t,$$

where we use that $W_t H_t + \mathcal{Y}_t = (1 - \tau_t) P_{H,t} Y_t$ (see the firm's problem below), and that $B_t = 0$ in equilibrium. Here, τ_t denotes the sales-tax rate at time t applied to firms. As mentioned above, we allow for non-zero net foreign assets in steady state. At the same time, we still assume purchasing power parity in steady state. This implies $P = P_H$ and thus (from the previous constraint) generally requires that $C \neq Y$. In the following, let $\zeta_c := C/Y$.[35] Linearization gives

$$\beta \hat{b}_t^* + \zeta_c (c_t - y_t + (p_t - p_{H,t})) = -\tilde{\tau}_t + \hat{b}_{t-1}^* + \zeta_{B^*} (\beta(\chi E_t \theta_{t+1} + \nu_{B,t}) + \Delta e_t - \pi_{H,t} - \Delta y_t) + \mu_t, \tag{A.12}$$

where $\tilde{\tau}_t$ denotes the deviation of the sales tax rate τ_t from steady state, and where we have

[34] Here and below, we slightly abuse notation by giving the shock in the non-linear model the same name as the relative deviation of the shock from steady state.

[35] From the budget constraint, we see that $\zeta_c = 1 - \tau + (1 - \beta)\zeta_{B^*}$, where τ is made explicit in the government's problem below (it is given by $\tau = (1 - \beta)\zeta$, where ζ is public debt to GDP in steady state).

linearized the bond price schedule for $\Psi_{B^*,t}$ above to replace

$$-\log\left(\frac{\Psi_{B^*,t}}{\Psi_{B^*}}\right) = \chi E_t\theta_{t+1} + \nu_{B,t}.$$

Intermediate good firms face the same problem as in the baseline model, with the exception that profits now comprise sales taxes τ_t as follows

$$\mathcal{Y}_t(j) = Y_t(j)(1-\tau_t)(P_{H,t}(j) - W_t).$$

As a result, the derivation of the Phillips curves is unchanged from before, but marginal costs now read

$$mc_t^r = w_t^r - (p_{H,t} - p_t) - \tilde{\tau}_t/(1-\tau). \tag{A.13}$$

Similarly, technology is the same as in the baseline model above, such that up to first order, output corresponds to working hours of the households

$$y_t = h_t. \tag{A.14}$$

In turn, aggregate period profits are given by

$$\begin{aligned}
\mathcal{Y}_t &= \int_0^1 ((1-\tau_t)P_{H,t}(j)Y_t(j) - W_tH_t(j))dj \\
&= (1-\tau_t)\int_0^1 P_{H,t}(j)Y_t(j)dj - W_tH_t \\
&= \frac{Y_t}{P_{H,t}^{-\gamma}}(1-\tau_t)\int_0^1 P_{H,t}(j)^{1-\gamma}dj - W_tH_t \\
&= (1-\tau_t)P_{H,t}Y_t - W_tH_t,
\end{aligned}$$

where we have used $\int_0^1 H_t(j)dj = H_t$ in the second equality, demand function $Y_t(j) = (P_{H,t}(j)/P_{H,t})^{-\gamma}\, Y_t$ in the third equality, and the definition of price index $P_{H,t}^{1-\gamma} = \int_0^1 P_{H,t}(j)^{1-\gamma}$ in the last equality.

Market clearing requires the same condition to be satisfied as in the baseline model, except that i) C_t^* is allowed to be time varying and stochastic (the "foreign demand shock") and ii) the steady state level for $C^* \neq C$. Rather, we have $\zeta_{c^*} := C^*/Y = (1-(1-\omega)\zeta_c)/\omega$.[36] Linearization gives

$$y_t = -\sigma(p_{H,t} - p_t) + (1-\omega)\zeta_c c_t + \omega\zeta_{c^*}(\sigma q_t + c_t^*). \tag{A.15}$$

The price of government debt is given by

$$\Psi_t = \varkappa_t E_t\left((1+\iota\Psi_{t+1})(1-\theta_{t+1})\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right)(R^*)^{-1},$$

where $\varkappa_t$ is a shock affecting the price of government debt directly, which we call a "sovereign

[36] This follows from $Y = (1-\omega)C + \omega C^*$ in steady state, thus $1 = (1-\omega)\zeta_c + \omega\zeta_{c^*}$.

liquidity shock". Define the gross yield of government debt as

$$I_t = \frac{1 + \iota \Psi_t}{\Psi_t}$$

and linearize to obtain

$$i_t = -(1 - \iota\beta) \log(\Psi_t/\Psi), \tag{A.16}$$

where $\Psi = \frac{\beta}{1-\iota\beta}$ is the price of debt in steady state (this follows from the bond price schedule above, using that $R^* = 1/\beta$ and that $\varkappa_t = 1$ in steady state). Linearize the bond price schedule, and combine with (A.16) to obtain

$$i_t = (1 - \iota\beta)(r_t + E_t\theta_{t+1} + \varkappa_t) + \iota\beta E_t i_{t+1}, \tag{A.17}$$

where we have used (A.10) to replace $E_t \Delta e_{t+1}$.

The budget constraint, stated in Section 2.5.1, can be written as

$$\beta \left(\frac{\Psi_t}{\Psi}\right) \left(\frac{\Psi}{\beta} \frac{D_t}{P_{H,t}Y_t}\right) + \tau_t = \beta \left(\frac{1}{\Psi} + \iota \frac{\Psi_t}{\Psi}\right) \left(\frac{\Psi}{\beta} \frac{D_{t-1}}{P_{H,t-1}Y_{t-1}}\right) \frac{Y_{t-1}}{Y_t} \frac{P_{H,t-1}}{P_{H,t}} (1 - \theta_t) + Z_t - \mu_t,$$

where Z_t is an exogenous (stochastic) transfer from abroad to the domestic government, called a "transfer shock", and where μ_t is the deficit shock as before (a lump-sum transfer to the domestic household). Note that in steady state, $\tau = (1 - \beta)\zeta$. Linearization gives

$$\beta \hat{d}_t = \hat{d}_{t-1} + \zeta(\beta \frac{1 - \iota}{1 - \iota\beta} i_t - \pi_{H,t} - \Delta y_t - \delta_t) - \tilde{\tau}_t - Z_t + \mu_t, \tag{A.18}$$

where we have used (A.16) to replace the price of government debt by the sovereign yield i_t. Lastly, we posit a policy rule for the tax rate equivalent to the one in our baseline model, such that

$$\tilde{\tau}_t = \psi_{\varsigma_t} \hat{d}_{t-1}, \tag{A.19}$$

where the feedback parameter ψ_{ς_t} may vary with the policy regime. Similarly, the policy for default is the same as before

$$\theta_t = \zeta^{-1} \theta_{\varsigma_t} \hat{d}_{t-1}, \tag{A.20}$$

where θ_{ς_t} may vary with the policy regime.

Equilibrium conditions include rules for monetary policy ($r_t = \phi_\pi \pi_{H,t}$ or $e_t = 0$). The extended model can be summarized by equations (A.9)-(A.20), along with (A.6) and (A.7). This gives a system of 14 equations in the 14 unknowns $\{c_t, y_t, h_t, w_t, p_t, p_{H,t}, e_t, q_t, i_t, r_t, \hat{d}_t, \tilde{\tau}_t, \theta_t, \hat{b}^*_t\}$. There are exogenous processes for $\{\mu_t, Z_t, \varkappa_t, \nu_t, \eta_t, c^*_t\}$.

B Generalized Phillips curve

Here we provide details on the derivation of the generalized Phillips curve, which we refer to in the main text. We consider a Calvo setup and denote with ξ_{Exit} the probability that a firm may not adjust its price in *Exit*, while ξ denotes this probability in all other regimes.

For simplicity, we only present the case without distortionary taxation (Section 2.3). The firm maximization problem in *Union* can be written as

$$\max_{P_{H,t}(j)} \sum_{k=0}^{\infty} (\lambda\xi)^k E_t \left(\rho_{t,t+k} Y_{t,t+k}(j) \left[P_{H,t}(j) - W_{t+k}\right] \,|\text{Union}\right)$$
$$+ \sum_{i=1}^{\infty}\sum_{k=i}^{\infty} (\lambda\xi)^{i-1}\delta\xi^{k-i+1} E_t \left(\rho_{t,t+k} Y_{t,t+k}(j) \left[P_{H,t}(j) - W_{t+k}\right] \,|\text{U Def in } t+i\right)$$
$$+ \sum_{i=1}^{\infty}\sum_{k=i}^{\infty} (\lambda\xi)^{i-1}\varepsilon\xi_{\text{Exit}}^{k-i+1} E_t \left(\rho_{t,t+k} Y_{t,t+k}(j) \left[P_{H,t}(j) - W_{t+k}\right] \,|\text{Exit in } t+i\right),$$

where we have split the expectation operator into expectations conditional on realizations of the Markov chain. More precisely, expectations are conditional on still being in the first regime, or on having switched regimes at time $t+i$. The maximization is subject to the conventional demand constraints given in the main text. Keeping track of the time of the switch is important, since it determines when the shift in rigidity occurs.
The first order condition can be written as

$$0 = \sum_{k=0}^{\infty} (\lambda\xi)^k E_t \left(\beta^k C_{t+k}^{-1} \frac{P_{H,t-1}}{P_{t+k}} Y_{t,t+k}(j) \left[\frac{P_{H,t}(j)}{P_{H,t-1}} - \frac{\gamma}{\gamma-1}\frac{P_{H,t+k}}{P_{H,t-1}}\frac{W_{t+k}}{P_{H,t+k}}\right] \,|\text{Union}\right)$$
$$+ \sum_{i=1}^{\infty} (\lambda\xi)^{i-1}\delta\xi^{1-i} \sum_{k=i}^{\infty} \xi^k E_t \left(\beta^k C_{t+k}^{-1} \frac{P_{H,t-1}}{P_{t+k}} Y_{t,t+k}(j) \left[\frac{P_{H,t}(j)}{P_{H,t-1}} - \frac{\gamma}{\gamma-1}\frac{P_{H,t+k}}{P_{H,t-1}}\frac{W_{t+k}}{P_{H,t+k}}\right] \,|\text{U Def in } t+i\right)$$
$$+ \sum_{i=1}^{\infty} (\lambda\xi)^{i-1}\varepsilon\xi_{\text{Exit}}^{1-i} \sum_{k=i}^{\infty} \xi_{\text{Exit}}^k E_t \left(\beta^k C_{t+k}^{-1} \frac{P_{H,t-1}}{P_{t+k}} Y_{t,t+k}(j) \left[\frac{P_{H,t}(j)}{P_{H,t-1}} - \frac{\gamma}{\gamma-1}\frac{P_{H,t+k}}{P_{H,t-1}}\frac{W_{t+k}}{P_{H,t+k}}\right] \,|\text{Exit in } t+i\right).$$

We linearize the expressions inside the three sums running over k to obtain

$$0 = \frac{p_{H,t}^* - p_{H,t-1}}{1-\beta\lambda\xi} - \sum_{k=0}^{\infty} (\beta\lambda\xi)^k E_t (mc_{t+k}^r + p_{H,t+k} - p_{H,t-1} \,|\text{Union})$$
$$+ \sum_{i=1}^{\infty} (\lambda\xi)^{i-1}\delta\xi^{1-i} \left(\frac{(\beta\xi)^i (p_{H,t}^* - p_{H,t-1})}{1-\beta\xi} - \sum_{k=i}^{\infty} (\beta\xi)^k E_t (mc_{t+k}^r + p_{H,t+k} - p_{H,t-1} \,|\text{U Def in } t+i)\right)$$
$$+ \sum_{i=1}^{\infty} (\lambda\xi)^{i-1}\varepsilon\xi_{\text{Exit}}^{1-i} \left(\frac{(\beta\xi_{\text{Exit}})^i (p_{H,t}^* - p_{H,t-1})}{1-\beta\xi_{\text{Exit}}} - \sum_{k=i}^{\infty} (\beta\xi_{\text{Exit}})^k E_t (mc_{t+k}^r + p_{H,t+k} - p_{H,t-1} \,|\text{Exit in } t+i)\right),$$

where we write $mc_t^r := w_t - p_{H,t}$ for brevity and denote $P_{H,t}^* = P_{H,t}(j)$, the latter using the fact that all resetting firms will choose the same reset price.

We note that

$$\frac{1}{1-\beta\lambda\xi}+\frac{1}{1-\beta\xi}\sum_{i=1}^{\infty}(\lambda\xi)^{i-1}\delta\xi^{1-i}(\beta\xi)^{i}+\frac{1}{1-\beta\xi_{\text{Exit}}}\sum_{i=1}^{\infty}(\lambda\xi)^{i-1}\varepsilon\xi_{\text{Exit}}^{1-i}(\beta\xi_{\text{Exit}})^{i}$$
$$=\frac{(1-\beta\xi)(1-\beta\xi_{\text{Exit}})+(1-\beta\xi)\beta\varepsilon\xi_{\text{Exit}}+(1-\beta\xi_{\text{Exit}})\beta\delta\xi}{(1-\beta\lambda\xi)(1-\beta\xi)(1-\beta\xi_{\text{Exit}})}=\frac{1}{(1-\beta\xi)\Omega_1},$$

where Ω_1 is defined as in the main text. This allows us to factor out $p^*_{H,t}-p_{H,t-1}$ from the linearized first order condition above, leading to

$$\begin{aligned}p^*_{H,t}-p_{H,t-1}=(1-\beta\xi)\Omega_1\ \{&\\ &\sum_{k=0}^{\infty}(\beta\lambda\xi)^{k}E_t(mc^r_{t+k}+p_{H,t+k}-p_{H,t-1}\ |\text{Union})\\ &+\sum_{i=1}^{\infty}(\lambda\xi)^{i-1}\delta\xi^{1-i}\sum_{k=i}^{\infty}(\beta\xi)^{k}E_t(mc^r_{t+k}+p_{H,t+k}-p_{H,t-1}\ |\text{U Def in } t+i)\\ &+\sum_{i=1}^{\infty}(\lambda\xi)^{i-1}\varepsilon\xi_{\text{Exit}}^{1-i}\sum_{k=i}^{\infty}(\beta\xi_{\text{Exit}})^{k}E_t(mc^r_{t+k}+p_{H,t+k}-p_{H,t-1}\ |\text{Exit in } t+i)\ \}.\end{aligned}\qquad (\text{B.1})$$

We now write (B.1) recursively. In order to see how this works, assume that regime change occurs at time $t+1$. Consider the example of shifting to *Exit*. In this case, conditional on the regime having changed, we obtain at $t+1$

$$p^*_{H,t+1}-p_{H,t}=(1-\beta\xi_{\text{Exit}})\sum_{k=0}^{\infty}(\beta\xi_{\text{Exit}})^{k}E_{t+1}(mc^r_{t+1+k}+p_{H,t+1+k}-p_{H,t}\ |\text{Exit in } t+1)$$

and therefore, using the law of iterated expectations at time t,

$$\begin{aligned}&E_t(p^*_{H,t+1}-p_{H,t}\ |\text{Exit in } t+1)\\ &\qquad=(1-\beta\xi_{\text{Exit}})\sum_{k=0}^{\infty}(\beta\xi_{\text{Exit}})^{k}E_t(mc^r_{t+1+k}+p_{H,t+1+k}-p_{H,t}\ |\text{Exit in } t+1).\end{aligned}$$

A similar equation holds for a shift to *Union Default*. Use this to rewrite (B.1) as

$$
\begin{aligned}
p^*_{H,t} - p_{H,t-1} = \pi_{H,t} + (1-\beta\xi)\Omega_1 \; \{ & mc^r_t \\
+ \frac{\delta\beta\xi}{1-\beta\xi}[E_t(p^*_{H,t+1} - p_{H,t} \; | \text{U Def in } t+1)] + & \frac{\varepsilon\beta\xi_{\text{Exit}}}{1-\beta\xi_{\text{Exit}}}[E_t(p^*_{H,t+1} - p_{H,t} \; | \text{Exit in } t+1)] \\
+ \beta\lambda\xi \; \{ & \sum_{k=0}^{\infty}(\beta\lambda\xi)^k E_t(mc^r_{t+1+k} + p_{H,t+1+k} - p_{H,t} \; | \text{Union}) \\
+ \sum_{i=1}^{\infty}(\lambda\xi)^{i-1}\delta\xi^{1-i} & \sum_{k=i}^{\infty}(\beta\xi)^k E_t(mc^r_{t+1+k} + p_{H,t+1+k} - p_{H,t} \; | \text{U Def in } t+1+i) \\
+ \sum_{i=1}^{\infty}(\lambda\xi)^{i-1}\varepsilon\xi_{\text{Exit}}^{1-i} & \sum_{k=i}^{\infty}(\beta\xi_{\text{Exit}})^k E_t(mc^r_{t+1+k} + p_{H,t+1+k} - p_{H,t} \; | \text{Exit in } t+1+i) \; \} \; \}.
\end{aligned}
$$

Focus on the last three lines of this expression, more precisely on the sums multiplying $\beta\lambda\xi$. One can see that these sums correspond to the ones in (B.1), only at time $t+1$ and with a conditional time-t expectations operator in front. Because (B.1) is conditional on being in *Union* at time t, we can write

$$
\begin{aligned}
p^*_{H,t} - p_{H,t-1} = \pi_{H,t} + \beta\lambda\xi E_t(p^*_{H,t+1} - p_{H,t} \; | \text{Union}) + (1-\beta\xi)\Omega_1 \; \{ mc^r_t \\
+ \frac{\delta\beta\xi}{1-\beta\xi}[E_t(p^*_{H,t+1} - p_{H,t} \; | \text{U Def})] \\
+ \frac{\varepsilon\beta\xi_{\text{Exit}}}{1-\beta\xi_{\text{Exit}}}[E_t(p^*_{H,t+1} - p_{H,t} \; | \text{Exit})] \; \}, \quad \text{(B.2)}
\end{aligned}
$$

where we have omitted the "in $t+i$" because all expectations are now conditional on the shift ocurring (or not ocurring) at time $t+1$. In a last step, we use a standard property of Calvo pricing, which is that

$$
\pi_{H,t} = (1-\xi_{\text{Exit}})(p^*_{H,t} - p_{H,t-1}), \quad \pi_{H,t} = (1-\xi)(p^*_{H,t} - p_{H,t-1}),
$$

the first equation in *Exit*, the second in all other regimes. Insert this into (B.2) and rearrange to obtain the final expression

$$
\begin{aligned}
\pi_{H,t} = \beta\,[\lambda E_t(\pi_{H,t+1}|\text{Union}) + \delta\Omega_1 E_t(\pi_{H,t+1}|\text{U Def}) + \epsilon\Omega_2 E_t(\pi_{H,t+1}|\text{Exit})] \\
+ \frac{(1-\beta\xi)(1-\xi)}{\xi}\Omega_1 mc^r_t, \quad \text{(B.3)}
\end{aligned}
$$

where we define

$$
\Omega_2 = \frac{\xi_{\text{Exit}}}{\xi}\frac{1-\xi}{1-\xi_{\text{Exit}}}\frac{1-\beta\xi}{1-\beta\xi_{\text{Exit}}}\Omega_1
$$

as in the main text.

C Closed-form solution of special case (Section 2.4.1)

Here we provide details on the closed-form solution of the special case which we study in detail in Section 2.4.1. In this case, we consider the baseline model, but let $\xi = \xi_{\text{Exit}} = 0$. To solve the model we exploit the property that *Exit* and *Union Permanent* are absorbing states of the Markov chain. This allows us to solve the model backwards using the method of undetermined coefficients.

If $\xi = \xi_{\text{Exit}} = 0$, the model collapses to

$$r_t = E_t \pi_{H,t+1} \tag{C.1}$$

$$e_t = p_{H,t} \tag{C.2}$$

$$\beta \hat{d}_t = (1 - \psi_{\varsigma_t})\hat{d}_{t-1} + \zeta(\beta i_t - \pi_{H,t} - \theta_t) + \mu_t \tag{C.3}$$

$$i_t = r_t + E_t \theta_{t+1} \tag{C.4}$$

$$\theta_t = \zeta^{-1} \theta_{\varsigma_t} \hat{d}_{t-1} \tag{C.5}$$

as well as $y_t = q_t = 0$ and policy $r_t = \phi_\pi \pi_{H,t}$ or $e_t = 0$.

Target regimes. In *Union Permanent*, $e_t = 0$, such that from (C.2) $p_{H,t} = 0$ and therefore $\pi_{H,t} = 0$. Since default is not possible in this regime, and further regime change is ruled out, $r_t = i_t = 0$ from (C.1) and (C.4). Debt to GDP evolves according to

$$\beta \hat{d}_t = (1 - \psi_{\text{UPer}})\hat{d}_{t-1} + \mu_t,$$

and is mean-reverting provided $\psi_{\text{UPer}} > 1 - \beta$ (which holds by assumption). Dynamics are identical in *Union Default*, except for the fact that

$$\begin{aligned}\beta \hat{d}_t &= (1 - \psi_{\text{UDef}})\hat{d}_{t-1} - \zeta\theta_t + \mu_t \\ &= (1 - \psi_{\text{UDef}} - \theta)\hat{d}_{t-1} + \mu_t,\end{aligned}$$

where we have used (C.5). This is true because we assume *Union Default* to be purely transitory, such that expected default is equal to zero, thus $i_t = 0$ also in this regime.

In *Exit*, both default and expected default are equal to zero, thus $i_t = r_t$ from equation (C.4). By contrast, generally $e_t = p_{H,t} \neq 0$ in this regime. The system (C.1)-(C.5) collapses to

$$\begin{aligned}\phi_\pi \pi_{H,t} &= E_t \pi_{H,t+1} \\ \beta \hat{d}_t &= (1 - \psi_{\text{Exit}})\hat{d}_{t-1} + \zeta(\beta\phi_\pi - 1)\pi_{H,t} + \mu_t.\end{aligned}$$

It features one forward looking ($\pi_{H,t}$), one backward looking variable ($\hat{d}_t$). As can be easily checked, the system exhibits bounded (and determinate) dynamics to the extent that either

i) $\psi_{\text{Exit}} > 1-\beta$ along with $\phi_\pi > 1$ or ii) $\psi_{\text{Exit}} < 1-\beta$ along with $\phi_\pi < 1$, as in Leeper (1991).

A guess and verify approach yields for case i)

$$\begin{aligned}\pi_{H,t} &= 0\\ \beta\hat{d}_t &= (1-\psi_{\text{Exit}})\hat{d}_{t-1} + \mu_t\end{aligned}$$

and for case ii)

$$\begin{aligned}\pi_{H,t} &= \frac{1-\psi_{\text{Exit}}-\beta\phi_\pi}{\zeta(1-\beta\phi_\pi)(1-\psi_{\text{Exit}})}[(1-\psi_{\text{Exit}})\hat{d}_{t-1}+\mu_t]\\ \hat{d}_t &= \frac{\phi_\pi}{1-\psi_{\text{Exit}}}[(1-\psi_{\text{Exit}})\hat{d}_{t-1}+\mu_t].\end{aligned}$$

Initial regime. In *Union*, which is the initial regime of the Markov chain, $p_{H,t} = 0$ and thus $\pi_{H,t} = 0$ from equation (C.2). However, generally $r_t \neq 0$ because of expected changes in inflation and nominal depreciation (equations (C.1) and (C.2)), and $i_t \neq 0$ because of (in addition to the variation in r_t) expected outright default (equation (C.4)). Moreover, movements in i_t feed back into $\hat{d}_t$ through equation (C.3).

We assume that $\psi_{\text{Exit}} < 1-\beta$ along with $\phi_\pi < 1$, such that inflation moves with the level of debt in *Exit*. By applying the law of iterated expectations we can then write equation (C.3) as

$$i_t = \left[\varepsilon\frac{1-\psi_{\text{Exit}}-\beta\phi_\pi}{\zeta(1-\beta\phi_\pi)} + \delta\zeta^{-1}\theta\right]\hat{d}_t, \tag{C.6}$$

where ε denotes the probability of moving to *Exit*, and δ denotes the probability of moving to *Union Default*, see sequence (2.3.2) in the main. Insert this into (C.3)

$$\beta\hat{d}_t = (1-\psi_{\text{Union}})\hat{d}_{t-1} + \zeta\beta i_t + \mu_t$$

and rearrange for $\hat{d}_t$ to obtain (2.13) from the main text. Substitute back the result for $\hat{d}_t$ into (C.6) to obtain (2.14) from the main text.

D Data Appendix

The frequency of all data used is quarterly. The data has been obtained in August 2015.

Sovereign bond yields Long-term interest rates for convergence purposes. Reference area: Greece, Italy, Ireland, Spain and Germany. Spreads are computed as differences in yields (all vis-à-vis Germany). Quarterly data are obtained by taking averages of monthly

data. Source: ECB Statistical Data Warehouse.
http://sdw.ecb.europa.eu.

Private sector yields MFI interest rate statistics. Reference area: Greece and Germany. Credit and other institutions (MFI except MMFs and central banks); Balance sheet item: Deposits with agreed maturity; Original maturity: Up to 1 year; Amount category: Total ; BS counterpart sector: Non-Financial corporations and Households; IR business coverage: New business. Spreads are computed as differences in yields. Source: ECB Statistical Data Warehouse.
http://sdw.ecb.europa.eu

Real GDP Growth Real GDP growth rates are computed as the difference between GDP growth rates in Greece and Germany. For both countries we obtain GDP at market prices, chain-linked volumes, reference year 2005. Source: ECB's Statistical pocket book, Section 11.3.
http://sdw.ecb.europa.eu.

Consumer Price Inflation Harmonized index of consumer prices. CPI inflation is computed as the difference between CPI growth rates in Greece and Germany. For both countries we obtain HICP data as 'prc hicp midx96' from Eurostat. We adjust the data for seasonal effects before computing growth rates. Source: Eurostat.
http://epp.eurostat.ec.europa.eu.

Primary surplus This series is computed as the sum of Net Lending in units of GDP and Interest, payable, in units of GDP. Both are taken from the Quarterly non-financial accounts for general government [gov_q_ggnfa]. Source: Eurostat.
http://epp.eurostat.ec.europa.eu.

Transfers We compute transfers in units of GDP from the two time series Interest, payable, in units of GDP (see item Primary surplus above) and the sovereign yield spread (see item Sovereign bond yields above). The (model-implied) quarterly interest payment to GDP (which is the interest payment that would be implied by market interest rate) is given by $\zeta *$sovereign yield spread/4, where ζ measures the debt-to-quarterly-GDP ratio (see main text). Transfers are thus given by the difference between actual interest payments to GDP and market-rate implied interest payments to GDP: 'Transfer = $\zeta *$sovereign yield spread/4 - Interest, payable, in units of GDP'.

Further adjustments A few further adjustments are required in order to make the data model consistent. Annualized data are divided by four to obtain quarterly data (Sovereign bond yields and Private sector yields). We correct the primary surplus to GDP ratio for its model-implied value in steady state, given by $(1-\beta)\zeta$.

E Additional Figures

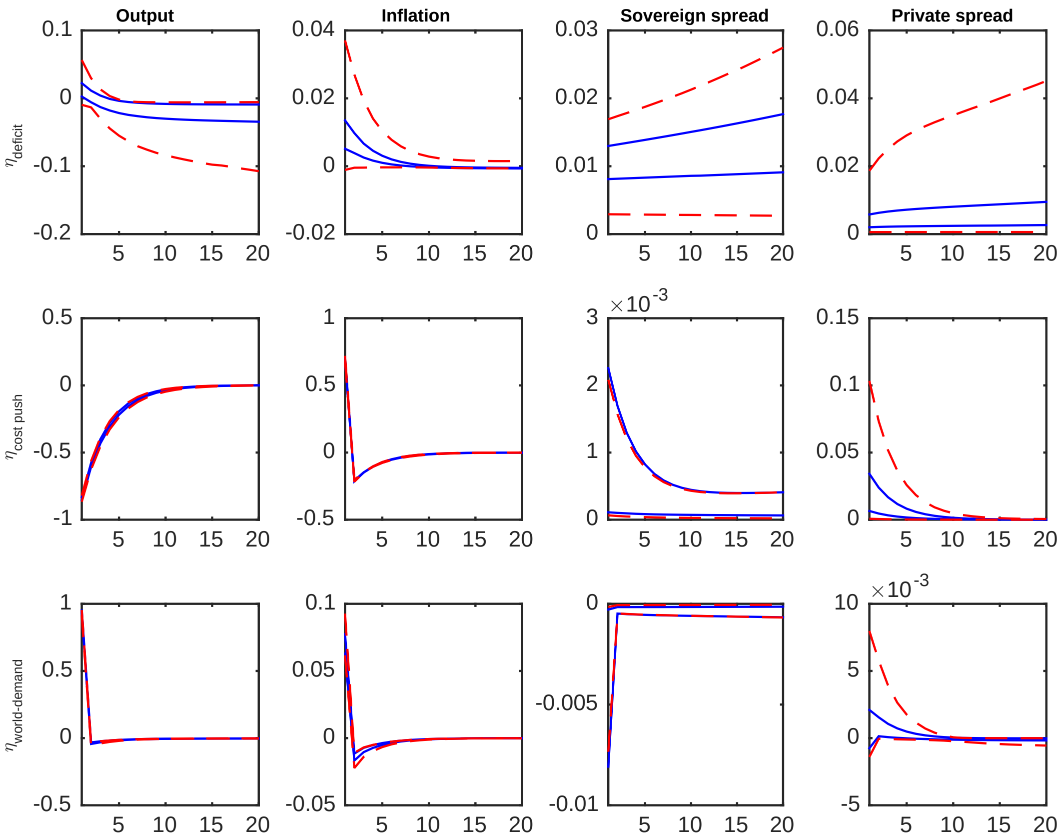

Figure 2.7: Impulse responses of selected variables, given prior (dashed) and posterior (solid) distributions of model parameters. Notes: lines indicate maximum and minimum in each period. We consider 50.000 draws from the distributions of ϵ, δ, χ and ξ_{Exit}; shocks are normalized to one percent.

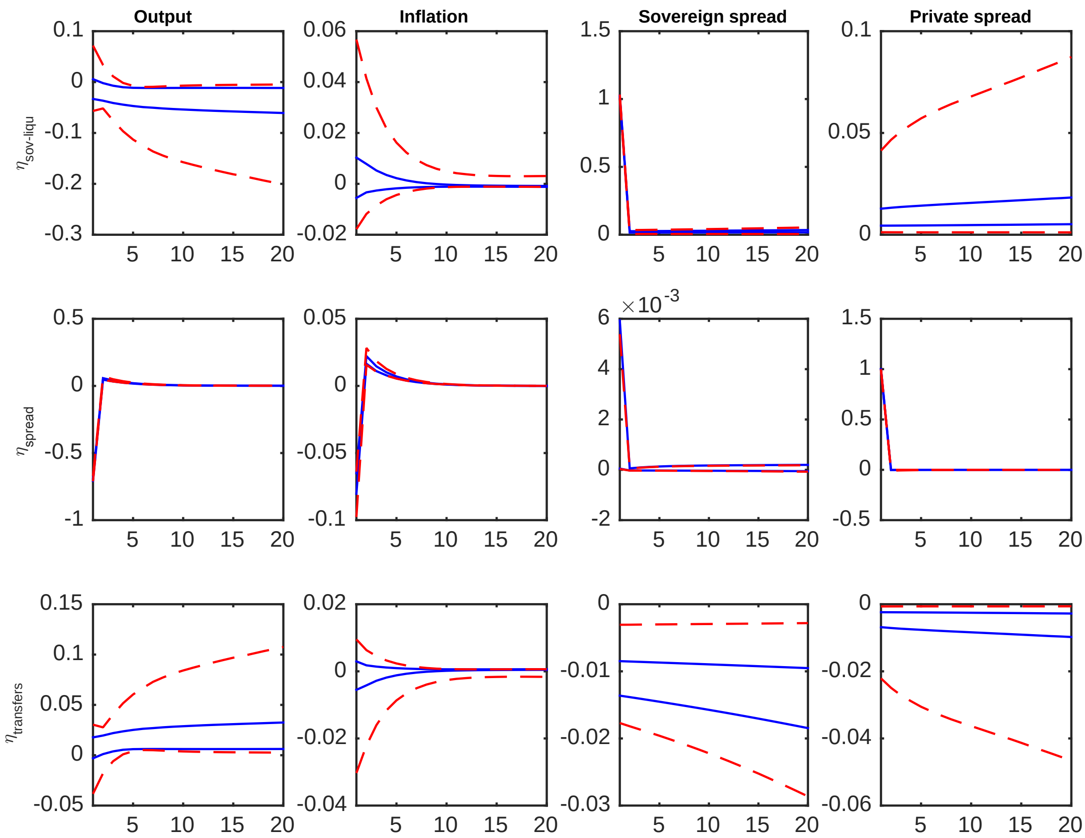

Figure 2.8: Impulse responses of selected variables, given prior (dashed) and posterior (solid) distributions of model parameters. Notes: lines indicate maximum and minimum in each period. We consider 50.000 draws from the distributions of ϵ, δ, χ and ξ_{Exit}; shocks are normalized to one percent.

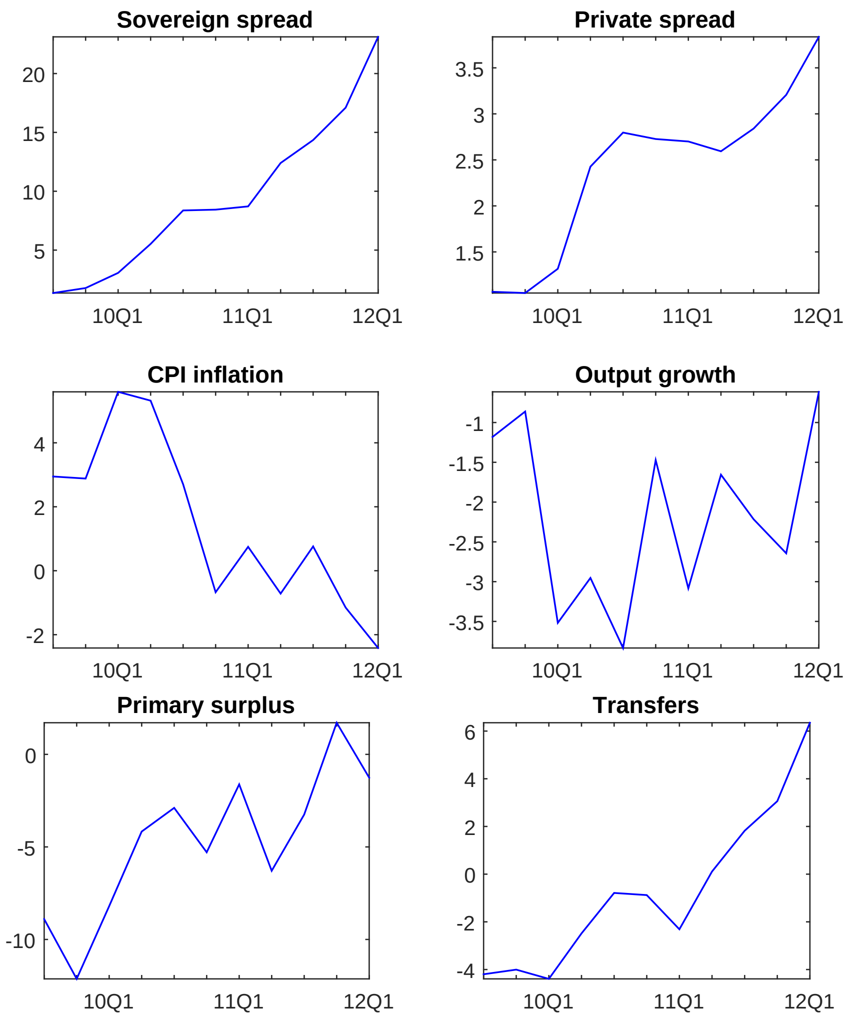

Figure 2.9: Greek time-series data 2009Q3–2012Q1. Notes: vertical axes measure percent/percentage points; spreads (annualized), inflation (CPI-based, annualized), and output growth all measured relative to Germany; primary surplus is measured relative to GDP; transfers are computed as difference between market interest rates and actual interest payments.

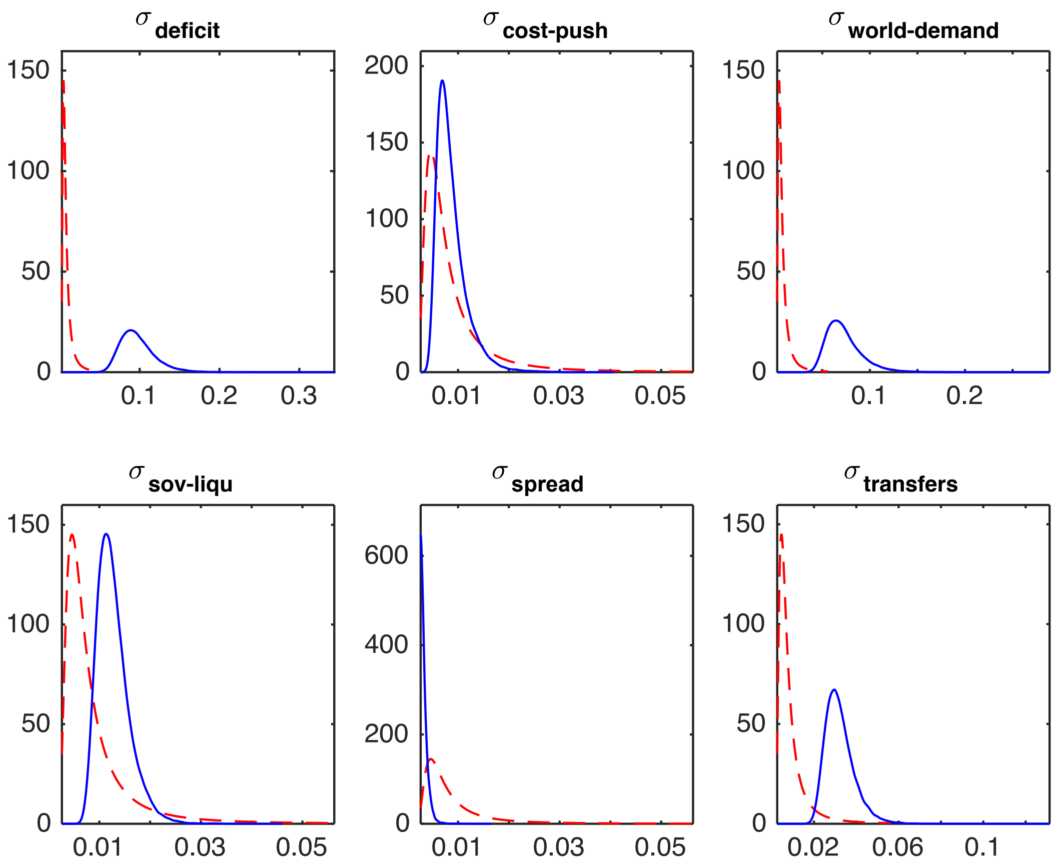

Figure 2.10: Prior (dashed) vs. posterior (solid) distribution of standard errors of shocks.

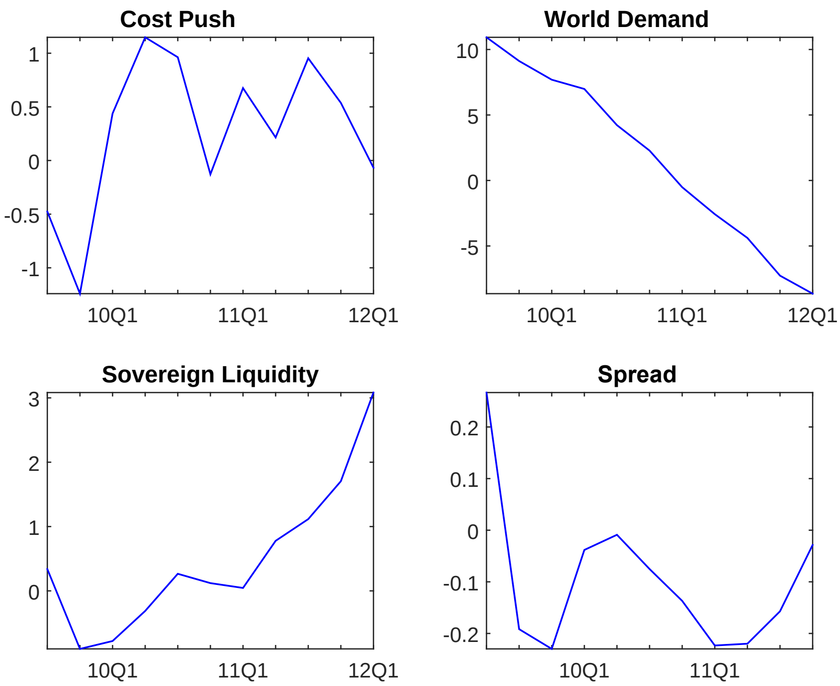

Figure 2.11: Estimated sequence of unobserved shocks 2009Q3–2012Q1. Note: shock sequences are obtained by applying Kalman smoother at the posterior mean; vertical axes measure percent.

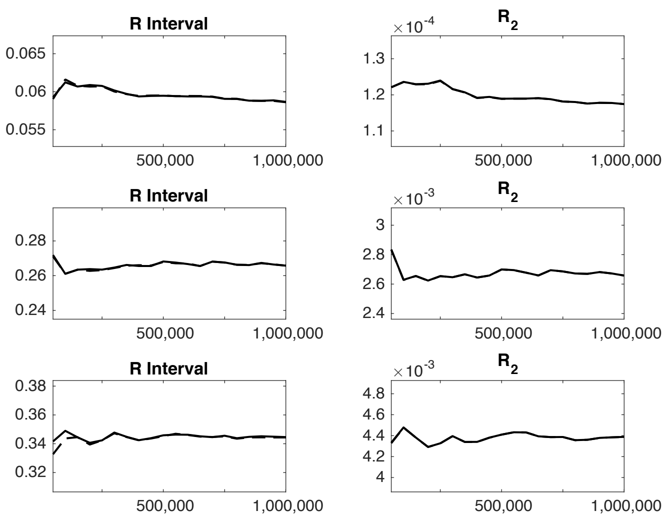

Figure 2.12: Convergence statistics for ϵ (first row), δ (second row), and χ (third row). The lines indicate the evolution of numerator (solid line) and the denominator (dashed line) of the statistics (F.7) and (F.8) along the number of draws.

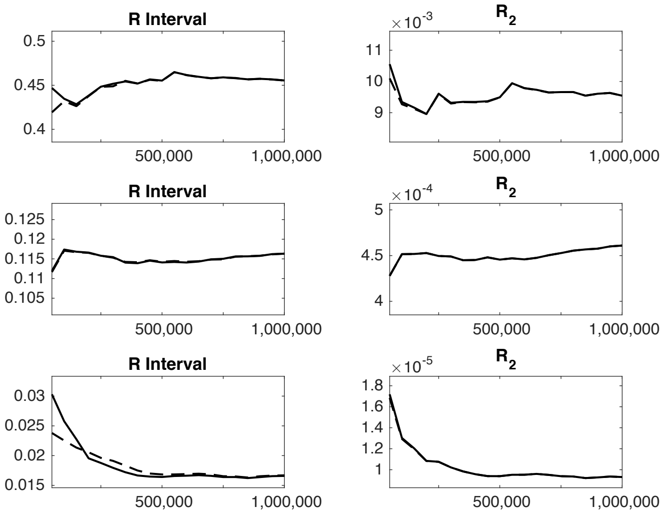

Figure 2.13: Convergence statistics for ξ_{Exit} (first row), $\sigma_{deficit}$ (second row), and $\sigma_{cost-push}$ (third row). The lines indicate the evolution of numerator (solid line) and the denominator (dashed line) of the statistics (F.7) and (F.8) along the number of draws.

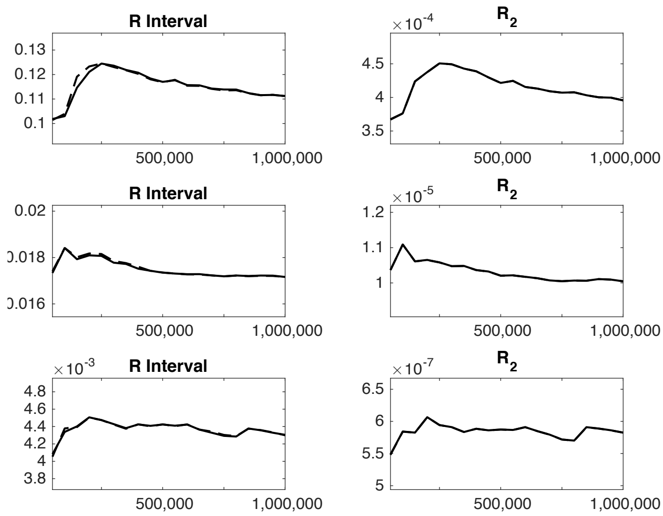

Figure 2.14: Convergence statistics for $\sigma_{world-demand}$ (first row), $\sigma_{sov-liqu}$ (second row), and σ_{spread} (third row). The lines indicate the evolution of numerator (solid line) and the denominator (dashed line) of the statistics (F.7) and (F.8) along the number of draws.

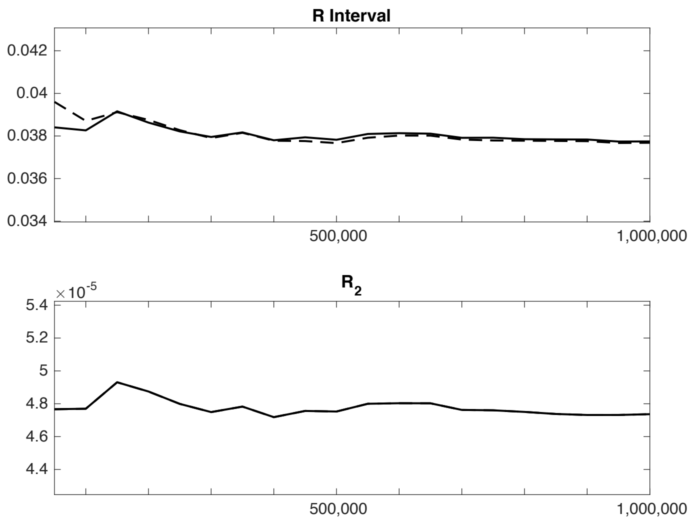

Figure 2.15: Convergence statistics for $\sigma_{transfers}$. The lines indicate the evolution of numerator (solid line) and the denominator (dashed line) of the statistics (F.7) and (F.8) along the number of draws.

F Convergence statistics

We report convergence statistics in Figures 2.12–2.15. Following Brooks and Gelman (1998) we first compute a measure which assesses the interval covered the by chains:

$$R_{Interval} = \frac{length \quad of \quad total \quad sequence \quad interval}{mean \quad length \quad of \quad the \quad within \quad sequence \quad intervals}. \tag{F.7}$$

A second measure monitors the evolution of the second moment of the posterior distribution:

$$R_2 = \frac{\frac{1}{mn-1}\sum_{j=1}^{m}\sum_{t=1}^{n}|\psi_{jt} - \bar{\psi}_{..}|^2}{\frac{1}{m(n-1)}\sum_{j=1}^{m}\sum_{t=1}^{n}|\psi_{jt} - \bar{\psi}_{j.}|^2}. \tag{F.8}$$

In this context, m denotes the number of chains (2) and n the number draws per chain. ψ_{jt} denotes the t-th realization from chain j, $\bar{\psi}_{..}$ the mean across chains, and $\bar{\psi}_{j.}$ the mean of chain j.

Chapter 3

Deleveraging, deflation and depreciation in the euro area

3.0 Abstract

During the post-crisis period, economic performance has been highly heterogenous across the euro area. While some economies rebounded quickly after the 2009 output collapse, others are undergoing a protracted further decline as part of an extensive deleveraging process. At the same time, inflation has been subdued throughout the whole of the euro area and intra-euro-area exchange rates have hardly moved. We interpret these facts through the lens of a two-country model of a currency union. We find that deleveraging in one country generates deflationary spillovers which cannot be contained by monetary policy, as it becomes constrained by the zero lower bound. As a result, the real exchange rate response becomes muted, and the output collapse—concentrated in the deleveraging economies.

3.1 Introduction

Following the onset of the global financial crisis, the euro area has experienced a protracted economic slump, with aggregate output in 2015 still below the 2008 level. The dynamics of the slump among individual countries, however, have been highly heterogenous. Whilst all countries underwent a deep recession in 2009, some rebounded quickly whereas others have experienced a persistent and protracted further decline. These most adversely affected economies are also undergoing a severe deleveraging process (Martin and Philippon, 2014; Reinhart and Rogoff, 2014).[1] Meanwhile, inflation in the euro area has been subdued, both during the initial downturn and the subsequent years. Moreover this subdued inflation has taken hold throughout the union, both in countries experiencing a deep slump and those doing relatively well. As a consequence, intra-euro-area real exchange rates have hardly moved during the post-crisis period.

In this paper we ask why—despite the heterogenous deleveraging and output performance across the euro area—there has been no significant adjustment of intra-euro-area real exchange rates. To provide an answer, we put forward a stylised two-country model of a currency union which accounts for the key features of the crisis in the euro area. The two countries represent the two heterogenous groups in the euro area, namely the "stressed" economies undergoing a severe slump and the "non-stressed" economies which have performed relatively well. Each country specialises in production of differentiated goods. The real exchange rate fluctuates with the relative price of these goods because goods markets are imperfectly integrated. Whilst goods prices are flexible in both economies, wages are downwardly rigid as in Schmitt-Grohé and Uribe (2016). Monetary policy aims to stabilise union-wide inflation, but may be constrained by the zero lower bound on nominal interest rates.

Drawing on Eggertsson and Krugman (2012), we assume that a group of households in the stressed economy are forced to reduce their debt because their borrowing limit tightens exogenously. We study how the repercussions of this deleveraging shock transmit through the entire currency union. The model is able to account for key features of the data. First, the output performance is heterogenous. Output collapses in the stressed economy, but is hardly affected in the rest of the union. Second, there is—at the same time—union-wide deflationary pressure such that, third, the real exchange rate hardly moves. Our analysis reveals that size matters for these results: both the size of the country which is subject to the deleveraging shock and the size of the shock itself.

[1] For an early account of how deleveraging in the global banking sector helped the financial crisis in the United States to morph into a global crisis, see Kollmann et al. (2011).

To establish this analytically, we first consider the case when the stressed economy is generically small. In this case, deleveraging does not impact the rest of the union at all and depreciates the stressed economy's real exchange rate. In line with conventional wisdom, we find that the extent of depreciation is limited by the extent of downward wage rigidity in the stressed economy (Friedman, 1953).[2] By the same token, the recession turns out to be less severe, the more flexible wages are.

Once we turn to the other polar case and assume that the stressed economy is large, the effects of the shock turn out to be fundamentally different. This holds in particular if the shock, in addition to the economy, is large as well. In this case, monetary policy finds itself pushed to the zero lower bound, and hence unable to contain the deflationary effects of the shock—not only in the stressed economy, but in the entire union. Whilst relative wage rigidities in the two countries still play a role in the adjustment process, deflationary spillovers from the stressed to the non-stressed economy will generally dampen the extent of real depreciation. Under specific conditions the real exchange rate may even appreciate, echoing earlier findings of a "perverse" response of real exchange rates at the zero lower bound when exchange rates are flexible (Cook and Devereux, 2013).

Furthermore in this scenario—somewhat paradoxically—the real exchange rate may depreciate less, the more flexible wages in the stressed economy are. The reason for this can be traced back to the root cause of the crisis itself—debt, or more precisely debt deflation à la Fisher (1933). As prices in the domestic economy decline, the real value of debt increases, thwarting the initial efforts to reduce the debt. Increased wage flexibility gives rise to more debt deflation, amplifying the recession and dampening the real exchange rate response further—an instance of the "paradox of flexibility", as established by Eggertsson and Krugman (2012) for the closed economy. We offer an important qualification to this paradox in the currency union setting, however: it only applies if both the shock and the size of the domestic economy are large. In particular, if the domestic economy is small, the drop in domestic prices depreciates the real exchange rate, stabilising the economy. Moreover in this case, long-run purchasing power parity implies that the decline in domestic prices is met by future inflation, which is also stabilising (Corsetti et al., 2013b).

What remains to be determined is whether the size of stressed economies in the euro area, and the magnitude of deleveraging are sufficiently large to push the union to the zero lower bound and generate enough deflationary spillovers for the real exchange rate response to be muted. In our quantitative analysis, we find this to be the case for plausible parametrisations

[2] Kollmann (2001) and Monacelli (2004) perform analyses of how nominal rigidities impact real exchange rate volatility under flexible and fixed exchange rates in small open economies. Broda (2004) provides evidence for developing countries in support of the received wisdom.

of the model.[3] As monetary policy becomes constrained by the zero lower bound, domestic output and prices decline, as do prices in the entire monetary union. Importantly, the decline in prices in the foreign economy implies that foreign output remains close to full employment. The model thus generates a heterogenous output response across the two countries. At the same time, deflationary spillovers ensure that the real exchange rate remains basically flat. Counterfactual simulations suggest that greater wage flexibility in the stressed economies is unlikely to be stabilising, and may instead deepen the recession.

The effects of debt deleveraging in an open economy context have been analysed in a number of other studies. Benigno and Romei (2014) examine the implications of deleveraging by one country within the world economy and study how monetary policy should be optimally set at a global level. However they do not consider the case of a monetary union. Fornaro (2015) studies the implications of deleveraging within a part of a monetary union under the zero lower bound constraint, as in our paper. But he abstracts from internal debt and debt deflation within countries, and does not focus his attention on relative price movements across stressed and non-stressed economies, which is the main focus of our paper. Gilchrist et al. (2015) also study the lack of real exchange rate adjustment across the euro area, but focus on financial constraints of firms rather than households. In their account, adverse financing conditions induce firms in stressed economies to keep prices high relative to what would be optimal under benign conditions. Firms in non-stressed economies, in turn, find it optimal to reduce prices in order to cannibalise the market share of stressed firms.

The remainder of the paper is organised as follows. The next section provides a number of basic facts regarding the post-2008 dynamics in the euro area. Section 3.3 introduces the model. We discuss analytical results for the limiting cases in Section 3.4. Section 5 presents results obtained from model simulations. A final section concludes.

3.2 Some facts

In this section we present time series evidence that highlights important aspects of the post-crisis slump in the euro area and provides some background for our model-based analysis. Our focus is on two regions, each consisting of a group of countries. The "stressed" region comprises countries where the crisis was particularly severe: Greece, Italy, Ireland, Portugal and Spain. The "non-stressed" region consists of Austria, Belgium, Finland, France, Germany and the Netherlands. For each group we aggregate time series using 2007 GDP weights. The stressed economies make up roughly 37% of GDP of all the countries in our sample.[4] Our

[3] Stressed economies make up 37% of the union and deleveraging is 34% of annual GDP per borrower, based on eurozone data.

[4] The 37% figure corresponds to pre-crisis levels. During the crisis, the share falls to around 34%.

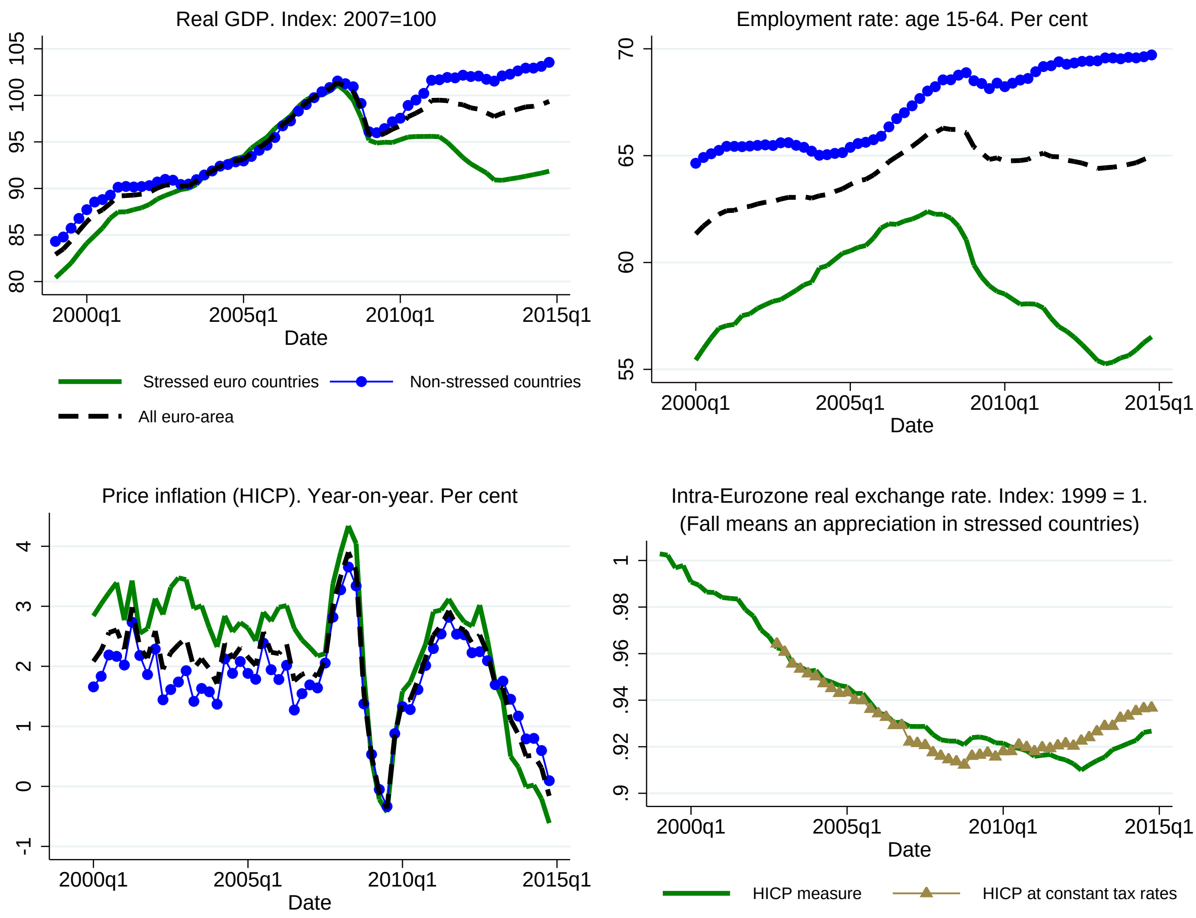

Figure 3.1: Development of macroeconomic aggregates. Source: Eurostat. Averages are weighted by 2007 nominal GDP. Irish and Finnish tax-adjusted exchange rates series start in 2013 and 2005 respectively; unadjusted data used beforehand.

sample, in turn, covers about 97% of euro-area GDP.

Figure 3.1 displays quarterly time series data for macroeconomic aggregates, covering the period 1999Q1–2014Q4. Dashed lines correspond to data for the euro area as a whole, solid lines correspond to the stressed economies and dotted lines—to the non-stressed economies. The upper-left panel shows real GDP normalised to 100 in the pre-crisis year 2007. In the run-up to the crisis output growth was quite synchronised across the two regions. Even during the early stages of the crisis, the GDP collapse in stressed and non-stressed countries was roughly the same. But since 2010 the growth performance has been quite distinct. The decline in GDP in the stressed economies seemed to bottom out in 2011–2012, only to decline further afterwards. While there is a small recovery at the end of our sample, stressed-economy GDP is still almost 10 percent below its pre-crisis level. In contrast, GDP recovered relatively quickly in the non-stressed countries, surpassing the pre-crisis level in early 2011. A similar picture emerges for employment data (top-right panel).

The intra-euro-area real exchange rate is displayed in the bottom-right panel of Figure 3.1. The solid line indicates the exchange rate measure based on the harmonised index of consumer prices. The line with markers (triangles) corresponds to a series which controls for tax changes. In both instances, we normalise the exchange rate to unity in 1999 and define it such that a decline corresponds to an appreciation for the stressed economies. In the years prior to the crisis the real exchange rate appreciated by about 8 percent, but has moved little after 2008. The HICP-based measure indicates that there was a further, if very mild, appreciation in the early stage of the crisis and an equally mild depreciation after 2013. The relative price adjustment is somewhat more pronounced but still muted for the series which is purged of the effect of tax changes. Within the euro area, changes in the real exchange rate are the result of differential inflation developments across the two regions, which are shown in the lower left panel. Prior to the crisis inflation in the stressed economies always exceeded inflation in the non-stressed region. Since the start of the crisis, however, inflation dynamics across the two regions have been markedly similar: inflation in both regions briefly turned negative in 2009, recovered afterwards but declined again after 2012.

In the model-based analysis that follows we explore the adjustment of the real exchange rate to a deleveraging process which takes place in one region of the currency union. That such a process contributed to the post-crisis slump has been suggested by many observers (see, for instance, Martin and Philippon, 2014; Reinhart and Rogoff, 2014) and is hardly controversial in light of the facts. The upper panels of Figure 3.2 display real credit growth (top row) and credit volumes relative to GDP (middle row) in the euro area, again distinguishing between area-wide developments and those in the stressed and non-stressed economies. In the left and right panels we show private and household debt respectively.

We observe that prior to the crisis the stressed economies experienced a particularly rapid expansion of credit. In fact their real credit growth averaged close to 13% per year, roughly three times the corresponding rate in the non-stressed economies. The advent of the crisis in 2008–09 coincided with a collapse in lending growth. Credit growth in both regions was close to zero in 2009 and then turned negative in the stressed economies whilst recovering somewhat in the non-stressed economies. In the former, it is still negative at the end of our sample. Overall, credit volumes thus declined considerably during the crisis—as far as the stressed economies are concerned. Expressed relative to GDP, overall private credit (household debt) peaked at some 170 (65) percent of GDP in the stressed economies in 2009. Since then the decline in credit relative to GDP has been muted by the sizeable decline in GDP.

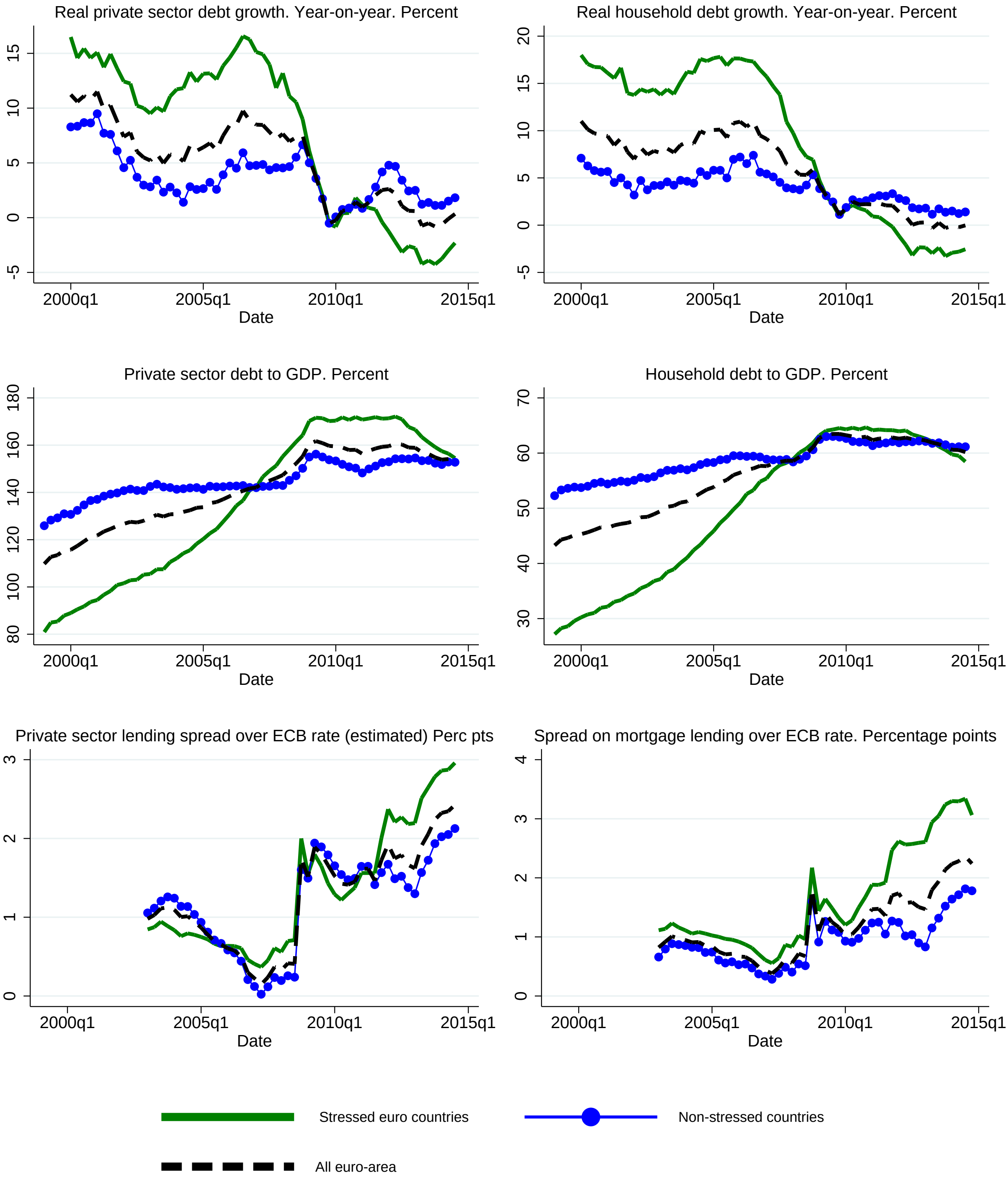

Figure 3.2: Development of debt and spreads. Sources: Eurostat, ECB and BIS. Averages are weighted by 2007 nominal GDP. Irish household debt data estimated before 2001Q4.

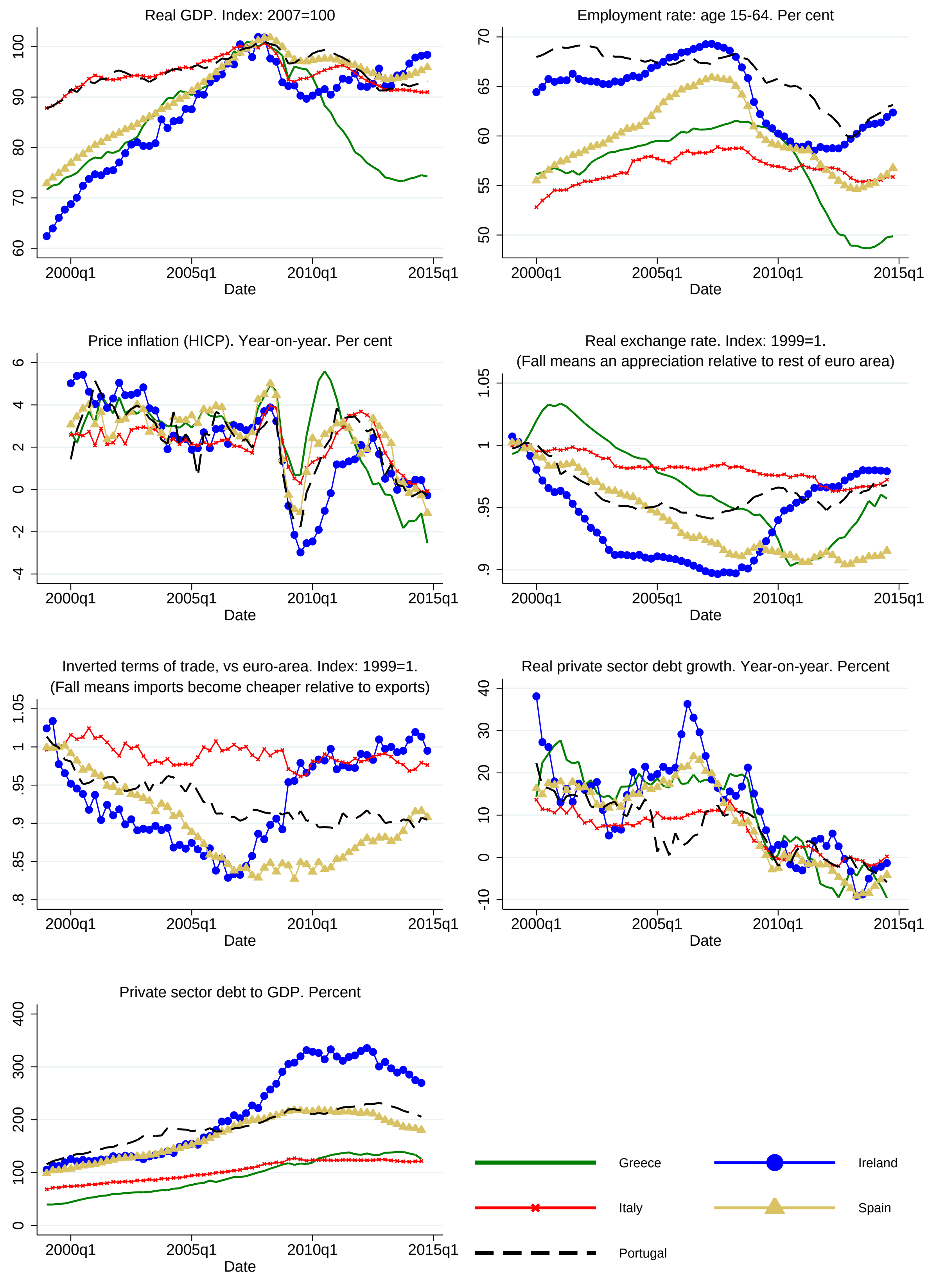

Figure 3.3: Developments in individual countries of the stressed region. Sources: Eurostat, BIS and OECD Economic Outlook.

In the bottom panels of Figure 3.2 we show the dynamics of interest rates. Specifically, we compute the difference of private-sector lending (left panel) and mortgage (right panel) rates relative to the ECB main refinancing rate. The spread on mortgages is higher in the stressed economies throughout the sample period. There is, however, a marked widening in the gap between stressed and non-stressed economies after 2010, reaching some 2 percentage points. The picture is less clear-cut for the spread on private sector lending rates. However, private sector spreads also tightened by considerably more in the stressed economies after 2009.

Figure 3.3 zooms in on the developments of individual countries of the stressed-economies aggregate. As we might expect, the disaggregated picture is more nuanced, with somewhat different dynamics from country to country. Ireland stands out with the largest lending boom, and also the most significant real exchange rate adjustment. Italy, on the contrary, did not experience much of a boom in lending or a significant real exchange rate appreciation before the crisis. Greece, in turn, shows the most dismal performance in terms of post-crisis output and employment.

Despite these differences, we find sufficiently strong similarities across the stressed euro area countries for our stressed/non-stressed country classification to be justified. First, the top panels of Figure 3.3 (real GDP and employment) show that all countries are experiencing a prolonged slump and are yet to fully recover. Second, inflation dynamics are quite synchronised (second row, left panel). Third, in all countries real exchange rates are still appreciated relative to the beginning of the sample (second row, right panel). For our sample, we also observe that the movements in real exchange rates reflect to a large extent movements of the terms of trade (displayed in the left panel of the third row).[5] This observation squares well with formal results of earlier studies which decompose real exchange rate movements (e.g., Engel, 1999).[6] Fourth and finally, note that all countries experienced a lending surge before the crisis and a lending slowdown after 2009, albeit to different degrees.

In sum, a simple inspection of the facts supports the view that the stressed economies of the euro area are experiencing a fully-fledged balance sheet recession. A sizeable build-up of debt was followed by a lengthy period of deleveraging, with very adverse consequences for economic activity. On the contrary, there is a recovery in the non-stressed region. Against this background it may be surprising that inflation is subdued not only in the stressed economies, but in the non-stressed region, too. Equivalently, the lack of real deprecation in the stressed

[5] For ease of comparison, the graph depicts "inverted" terms of trade: the price of imports relative to exports. That way it is directly comparable to our real exchange rate series. No observations are available for Greece.

[6] However, in a recent study Berka et al. (2015) find that for a pre-crisis sample of euro area countries, cross-country productivity differentials in the traded and non-traded good sector matter a great deal for real exchange rate movements, even in the short run. According to the authors this result is likely due to the fact that the countries in their sample maintain a common currency.

economies relative to the non-stressed economies may appear puzzling. We investigate this issue further by means of a model-based analysis.

3.3 The model

Our analysis is based on a simple two-country model of a currency union. Countries specialise in the production of specific goods which are traded across countries. Good market integration is incomplete, however, as countries' consumption is biased towards domestically produced goods. The real exchange rate may therefore deviate from unity. Given that the real exchange rate and the terms of trade co-move strongly in our sample (see Section 3.2), we abstract from the production of non-traded goods. Countries may differ in size and we assume "Home" makes up a mass $[0, n)$ of the total union population, where $n \in [0, 1]$. Each country is populated by a unit mass of agents, who supply labour inelastically to domestic firms. Within Home we distinguish between households with high and low discount factors as in Eggertsson and Krugman (2012). We refer to these households as "savers" and "borrowers" respectively. The rest of the union ("Foreign") is populated by savers only. Savers in one country can trade a nominally non-contingent bond with the savers in the other country. Savers in Home can, in addition, lend funds to domestic borrowers. Prices are fully flexible, but downward adjustment of nominal wages is restricted as in Schmitt-Grohé and Uribe (2016).

3.3.1 Households

In Home, borrowers account for a fraction $\chi \in (0, 1)$ of the population. They are less patient than savers, which account for the rest. A typical saver in Home maximises

$$\max_{\{C_t^s\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta^s)^t \ln(C_t^s)$$

subject to

$$P_t C_t^s + R_t^{-1} B_t^s + R_t^{*-1} D_t = W_t L_t + B_{t-1}^s + D_{t-1} \tag{3.3.1}$$

and a constraint which rules out Ponzi games. Here $\beta^s < 1$ is the discount factor which exceeds the discount factor of the borrower: $\beta^s > \beta^b$. C_t^s denotes savers' per capita consumption and P_t is the consumer price level in Home. Savers earn rate R_t by lending to borrowers at home (B_t^s), and rate R_t^* by saving abroad (D_t). All debt and interest rates are denominated in nominal terms. W_t is the nominal wage rate in Home, and L_t are hours worked. Optimality requires the following Euler equation to hold

$$(C_t^s)^{-1} = \beta^s R_t \left(C_{t+1}^s\right)^{-1} \frac{P_t}{P_{t+1}}, \tag{3.3.2}$$

as well as a transversality condition. The absence of arbitrage possibilities between domestic and foreign assets requires that

$$R_t = R_t^*. \tag{3.3.3}$$

In turn, a typical borrower maximises

$$\max_{\{C_t^b\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta^b)^t \ln(C_t^b)$$

subject to

$$P_t C_t^b + B_{t-1}^b = R_t^{-1} B_t^b + W_t L_t \tag{3.3.4}$$

$$B_t^b \leq \bar{B}_t. \tag{3.3.5}$$

Here, B_t^b denotes nominal debt vis-à-vis the savers which may not exceed an exogenous, potentially time-varying, debt limit $\bar{B}_t$. First order conditions imply that (3.3.5) holds with equality at all times.[7] Aggregate consumption in Home is given by

$$C_t = (1-\chi)C_t^s + \chi C_t^b. \tag{3.3.6}$$

In the rest of the union, all households are savers and we suppress the superscript s for simplicity. The objective is

$$\max_{\{C_t^*\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta^s)^t \ln(C_t^*)$$

subject to

$$P_t^* C_t^* + R_t^{*-1} D_t^* = W_t^* L_t^* + D_{t-1}^*, \tag{3.3.7}$$

where stars denote variables in the rest of the union. First order conditions imply

$$(C_t^*)^{-1} = \beta^s R_t^* \left(C_{t+1}^*\right)^{-1} \frac{P_t^*}{P_{t+1}^*}. \tag{3.3.8}$$

We allow for home bias in consumption in both countries, and the elasticity of substitution between domestic and imported goods is unity. Specifically, aggregate consumption is a composite of two goods in both countries:

$$C_t = \frac{C_{H,t}^{\lambda} C_{F,t}^{1-\lambda}}{\lambda^{\lambda}(1-\lambda)^{1-\lambda}}, \qquad C_t^* = \frac{C_{H,t}^{\lambda^*} C_{F,t}^{1-\lambda^*}}{\lambda^{*\lambda^*}(1-\lambda^*)^{1-\lambda^*}},$$

where $\lambda = 1-(1-n)\omega$ and $\lambda^* = n\omega$. Here, $C_{H,t}$ is the locally produced good, $C_{F,t}$ is the good produced in the rest of the union, and $\omega \in [0,1]$ determines the degree of home bias in consumption, which we assume is symmetric across countries. If $\omega = 1$ there is no home bias.

[7] More precisely, optimality requires that $\left(C_t^b\right)^{-1} \geq \beta^b R_t \left(C_{t+1}^b\right)^{-1} \frac{P_t}{P_{t+1}}$, holding with equality whenever $B_t^b < \bar{B}_t$, and requiring $B_t^b = \bar{B}_t$ whenever holding with strict inequality. In the steady state of the model as well as during the deleveraging phase, the latter case always obtains, such that we omit this case distinction from the main text.

This formulation of relative consumption weights follows Sutherland (2005) and De Paoli (2009b). Note that both country size n and home bias $1-\omega$ affect the consumption shares of Home- and Foreign-produced goods; however they do so in different ways. As the size of the domestic country grows (as n increases), both domestic and foreign households consume a bigger share of the good produced in Home. By contrast, an increase in $1-\omega$ implies that both domestic and foreign households consume a bigger share of goods produced in their own country (Home and Foreign respectively)—the classic notion of "home bias".[8]

The local good sells at price $P_{H,t}$, while the foreign good sells at price $P_{F,t}$. Expenditure minimisation implies

$$P_t = P_{H,t}^{\lambda} P_{F,t}^{1-\lambda}, \qquad P_t^* = P_{H,t}^{\lambda^*} P_{F,t}^{1-\lambda^*}, \tag{3.3.9}$$

that is, the consumer price indices domestically and abroad are a weighted average of the producer prices of the two goods.

Furthermore, we define the real exchange rate Q_t as the price of foreign consumption in terms of domestic consumption,

$$Q_t = \frac{P_t^*}{P_t}, \tag{3.3.10}$$

such that an increase in Q_t indicates a depreciation of Home's real exchange rate.

3.3.2 Firms

Firms operate in competitive goods and labour markets. They maximise profits $P_{H,t}Y_t - W_t L_t$ in Home, $P_{F,t}Y_t^* - W_t^* L_t^*$ in Foreign, subject to

$$Y_t = L_t, \qquad Y_t^* = L_t^* \tag{3.3.11}$$

respectively, and their first order conditions imply

$$P_{H,t} = W_t, \qquad P_{F,t} = W_t^*. \tag{3.3.12}$$

As in Schmitt-Grohé and Uribe (2016), the labour market is characterised by downward nominal wage rigidity. In each period, a maximum of $\bar{L}$ hours can be sold to firms

$$L_t \leq \bar{L}, \qquad L_t^* \leq \bar{L} \tag{3.3.13}$$

while wages may fall by at most $(1-\gamma)$ in Home, $(1-\gamma^*)$ in Foreign, in proportion to their previous level

$$W_t \geq \gamma W_{t-1}, \qquad W_t^* \geq \gamma^* W_{t-1}^*. \tag{3.3.14}$$

[8] This in turn implies that the real exchange rate is independent of the distribution of wealth *only if* home bias is zero ($\omega = 1$), regardless of the value of n. A formal analysis of this issue is available on request. Note that furthermore, the size of the economy n impinges on the supply side of the model, as more goods are being produced in a country that is larger (formally, this is implied from equation (3.3.21)).

We require that $1 \geq \gamma > 0$ and $1 \geq \gamma^* > 0$, where $\gamma, \gamma^* \to 0$ characterises flexible wages, and $\gamma, \gamma^* = 1$—full rigidity. The labour markets are closed by complementary slackness conditions of the form

$$(L_t - \bar{L})(W_t - \gamma W_{t-1}) = 0, \qquad (L_t^* - \bar{L})(W_t^* - \gamma^* W_{t-1}^*) = 0, \tag{3.3.15}$$

which imply that, as long as wages are free to adjust, the economy must operate at potential. Conversely, involuntary unemployment is possible as (3.3.14) becomes a binding constraint.

3.3.3 Monetary policy

We assume that monetary policy is characterised by a strict inflation targeting rule, adjusting the nominal interest rate such that area-wide inflation is zero, subject to a zero lower bound constraint. It targets

$$\Pi_t^u = 1 \text{ subject to } R_t \geq 1, \tag{3.3.16}$$

where $\Pi_t^u = (P_t)^n (P_t^*)^{1-n} / (P_{t-1})^n (P_{t-1}^*)^{1-n}$ is area-wide inflation, and sets

$$R_t = 1 \tag{3.3.17}$$

if due to deflationary pressure, the inflation target cannot be reached.

3.3.4 Market clearing

Goods market clearing requires that the supply of domestically produced goods equals domestic as well as export demand

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-1} \left(\lambda C_t + \frac{\lambda^*(1-n)}{n} Q_t C_t^*\right). \tag{3.3.18}$$

Equivalently, we require for the Foreign-produced good[9]

$$Y_t^* = \left(\frac{P_{F,t}}{P_t^*}\right)^{-1} \left(\frac{(1-\lambda)n}{1-n} Q_t^{-1} C_t + (1-\lambda^*) C_t^*\right). \tag{3.3.19}$$

Moreover, asset market clearing requires

$$(1-\chi) B_t^s = \chi B_t^b \tag{3.3.20}$$

within Home and

$$(1-\chi) n D_t + (1-n) D_t^* = 0 \tag{3.3.21}$$

across the two countries.

An equilibrium is a sequence of endogenous variables $\{Y_t, Y_t^*, L_t, L_t^*, C_t, C_t^*, C_t^s, C_t^b, B_t^s, B_t^b, \ldots$ $D_t, D_t^*, R_t, R_t^*, P_t, P_t^*, P_{H,t}, P_{F,t}, W_t, W_t^*, Q_t, \Pi_t^u\}$ solving equations (3.3.1)—(3.3.21), for given

[9] Note that for the limiting cases we discuss in Section 3.4, $\frac{\lambda^*(1-n)}{n} \to \omega$ as $n \to 0$ and $\frac{(1-\lambda)n}{1-n} \to \omega$ as $n \to 1$.

parameters and initial conditions, and exogenous $\{\bar{B}_t\}$.

3.3.5 Steady state

We assume that initially the economy is in a symmetric steady state: the real exchange rate, consumer and producer price indices are equal to unity, $P_H = P = P^* = P_F = 1$, which from (3.3.12) implies that $W = W^* = 1$. Moreover, we let $Y = Y^* = \bar{L}$ and $C^* = Y^*$. This implies $C = Y$ from equations (3.3.18) and (3.3.19). We obtain $R = R^* = 1/\beta^s$ from equations (3.3.2) and (3.3.8). Borrowers are up against the borrowing constraint, hence $C^b = Y - (1 - \beta^s)\bar{B}$. Savers in Home consume $C^s = Y + (1 - \beta^s)(B^s + D)$, and savers in Foreign $C^* = Y^* + (1-\beta^s)D^*$. This combined with the fact that $C^* = Y^*$ yields $D^* = 0$, and from (3.3.21) $D = 0$. That is, net foreign assets must equal zero in the initial steady state. Note that the economy is characterised by non-stationary dynamics, that is, it will generally not revert back to its initial steady state, once it departs from it.[10]

3.4 Relative prices in a crisis

We now investigate how the economy adjusts to a deleveraging shock. More specifically, we consider a one-off tightening of the debt limit in Home from $\bar{B}_t = \bar{B}^H$ to a permanently lower level $\bar{B}^L$ in time period t. Our setup mimics Eggertsson and Krugman (2012), except that we consider a two-country model and restrict the tightening of the debt limit to take place in one country only. The adjustment will then depend on the size of this country. We illustrate this by first focusing on two limiting cases: $n \to 0$ and $n \to 1$. For these cases we obtain closed-form results, and develop an intuition of the underlying mechanisms. We discuss numerical results for intermediate n in Section 3.5. Throughout, our main interest is how the real exchange rate in period t responds to the shock.

3.4.1 Deleveraging in a small union member

If $n \to 0$, Home is effectively a small open economy (see De Paoli, 2009b; Galí and Monacelli, 2005). The Home-good consumption weights are $\lambda \to 1-\omega$ and $\lambda^* \to 0$ in Home and Foreign, respectively. In this case, $P_t^* = P_{F,t}$ from equation (3.3.9), $Y_t^* = C_t^*$ from (3.3.19) and $D_t^* = 0$ from (3.3.21). In other words, the rest of the union resembles a closed economy and is not affected by the deleveraging shock in Home. It therefore remains in the initial steady state during the whole deleveraging process. This has important implications for monetary policy:

[10] The economy is non-stationary for two reasons. First, international financial markets are incomplete, and second, households are heterogenous. The distribution of wealth, both across agents and across countries is a state of the economy which induces unit-root behaviour in some variables.

from (3.3.16) we obtain $\Pi_t^u = P_t^*/P_{t-1}^* = 1$, such that average inflation in the union is zero, and as a result the nominal interest rate remains unchanged as well: $R_t^{-1} = \beta^s$. We summarise our main result in what follows.

Proposition 4. *Consider the economy defined in Section 3.3 and let $n \to 0$. Suppose that in period t, the debt limit in Home is unexpectedly and permanently reduced from $\bar{B}^H$ to $\bar{B}^L$. The real exchange rate at time t is then given by*

$$Q_t = P_t^*/P_t = \min\left(\gamma^{\omega-1}, [1 - \eta(\bar{B}^H - \bar{B}^L)/Y]^{\omega-1}\right) \geq 1, \tag{3.4.1}$$

where $\eta = (\beta^s(1-\omega)\chi)/(1-(1-\omega)\chi) > 0$. Therefore, there is no depreciation ($Q_t = 1$) if wages are completely rigid ($\gamma = 1$), and a greater depreciation, the more flexible wages are, where the upper threshold $[1 - \eta(\bar{B}^H - \bar{B}^L)/Y]^{\omega-1} > 1$ is reached once (3.3.14) *ceases to bind.*

Proof. See Appendix. □

Intuitively, the deleveraging shock forces borrowers to cut consumption in order to repay their debts. This reduces aggregate demand and puts downward pressure on prices, resulting in a real exchange rate depreciation, as long as wages are allowed to adjust sufficiently. We now establish a second result.

Proposition 5. *Consider again the economy defined in Section 3.3 with $n \to 0$. In the period of deleveraging, output, saver consumption, borrower consumption and real wage income all decline strictly less if wages are more flexible (that is, if γ is reduced) up until* (3.3.14) *ceases to bind, point beyond which they do not vary further. The recession is deepest if wages are completely downwardly rigid ($\gamma = 1$).*

Proof. See Appendix. □

Propositions 4 and 5 establish that wage flexibility and the associated movements in the real exchange rate dampen the response to country-specific shocks, as the received wisdom—going back to at least Friedman (1953)—suggests. Still, it is interesting to analyse this case for the following reasons. First, it will serve as a useful benchmark once we consider $n > 0$. Second, the fact that a real depreciation plays a stabilising role is actually not obvious during a deleveraging recession. The fall in Home prices required to bring about the depreciation increases the real value of debt, giving rise to debt deflation à la Fisher (1933).

To see this, consider how the borrowers respond to the deleveraging shock. When the shock hits, nominal debt of $\bar{B}^H - \beta^s \bar{B}^L$ has to be repaid to satisfy the new, lower, borrowing limit. By rearranging the budget constraint (3.3.4) as follows

$$C_t^b = -\frac{\bar{B}^H - \beta^s \bar{B}^L}{P_t} + \frac{W_t L_t}{P_t}, \tag{3.4.2}$$

we see that borrowers' consumption depends on debt repayment as well as on real wage incomes $W_t L_t / P_t$. For a given real income, a lower price level increases real debt and reduces consumption. Still, recall that borrower consumption declines less in general equilibrium with greater wage flexibility and real depreciation (Proposition 5). The reason is that in this case, overall economic activity is higher, which helps sustain the real wage income of the borrowers. First, a weaker real exchange rate crowds in foreign demand for the domestic good. Second, since long-run prices are pinned down by purchasing power parity, a temporary drop in the price level generates expected inflation, which reduces the real interest rate and increases spending by savers.[11] It turns out that these two mitigating factors necessarily outweigh the adverse impact of debt deflation. In other words, while rigidity in wages may rule out debt deflation altogether, the resulting drop in real wage income (which operates via a drop in working hours L_t) depresses borrower consumption all the same—in fact, depresses it by more, the more rigid the wages.

In sum, in the case of a small open economy, a lack of relative price adjustment reflects the presence of nominal rigidities, in our case downwardly sticky wages. We show the adjustment dynamics in Figure 3.4, which contrasts results for the cases of fully rigid ($\gamma = 1$, solid lines) and flexible ($\gamma = 0.75$, dashed lines) wages. We discuss the parameter choices which underlie the model simulations in Section 3.5 below. Importantly, we assume that a deleveraging of 17% GDP is undertaken over one year, in period 5.[12] Because the share of borrowers is 0.5, this amounts to a deleveraging of 34% GDP per borrower. Figure 3.4 echoes our discussion above: if wages are sticky, the exchange rate response is flat, while output collapses. If wages are fully flexible, the exchange rate depreciates strongly, while output remains constant.[13]

The response of consumption also differs across the two scenarios: both saver and borrower consumption are higher under flexible wages, relative to the rigid-wage scenario. This implies, in the context of our model, that welfare is higher under more flexible wages. Galí and Monacelli (2013), in contrast, find that higher wage flexibility may reduce welfare whenever monetary policy seeks to stabilise the exchange rate. This is because Galí and Monacelli (2013) assume a monopolistically competitive labour market and staggered wage setting. As a result, higher wage inflation induces wage dispersion which is detrimental to welfare. We do not consider this possibility. Moreover, recall that in our set-up labour effort has no direct bearing on household utility.

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[11] Thus, implicit in fixed exchange rate regimes is an element of price level targeting, which has been emphasised in previous work (Corsetti et al., 2013b). A more detailed discussion is provided in Section 3.4.2.

[12] Here and in the rest of the paper, we refer to deleveraging in terms of nominal GDP before the crisis, which equals 1 in our parametrisation.

[13] In our calibration, $\gamma = 0.75$ provides sufficient flexibility for (3.3.14) not to bind in the period of deleveraging. A further increase in wage flexibility would then leave results unaltered.

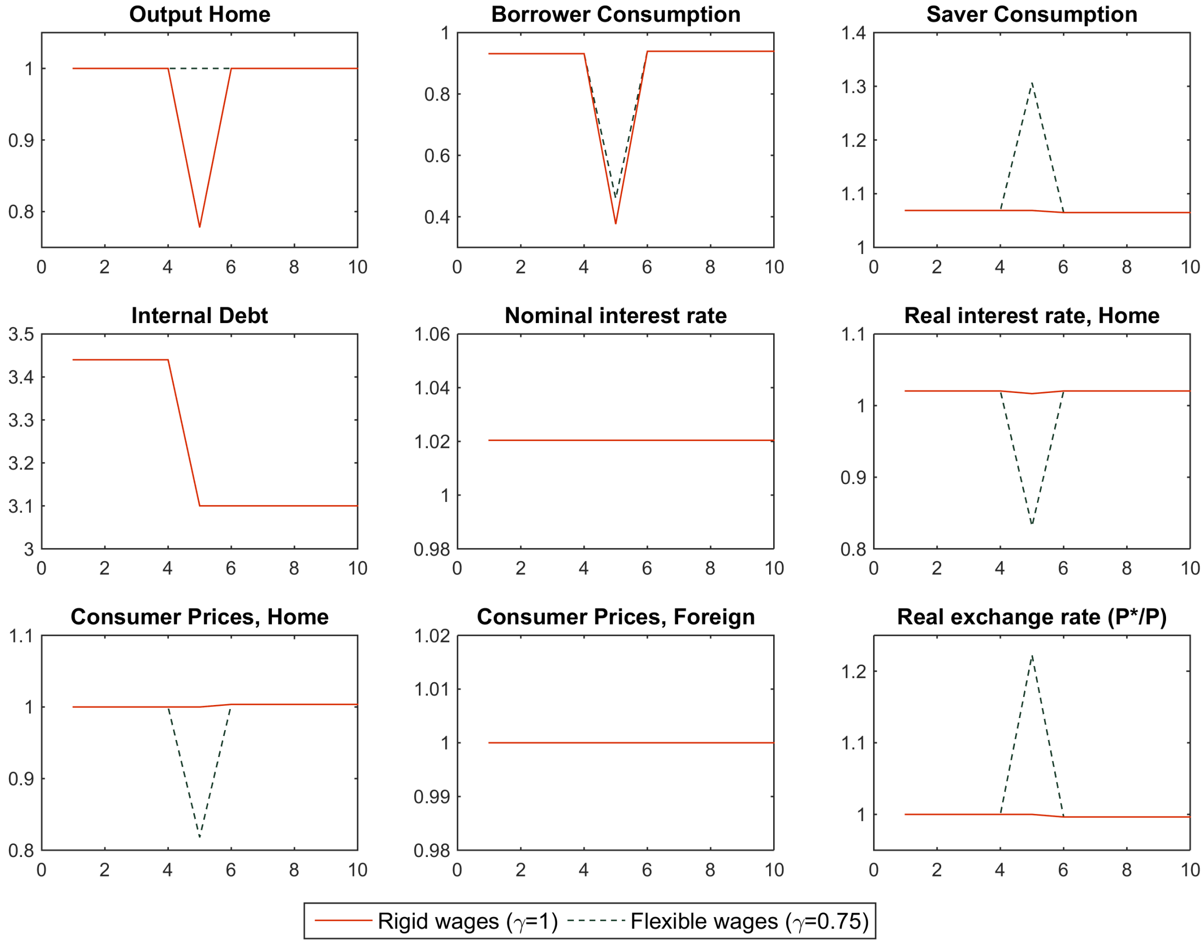

Figure 3.4: Deleveraging in a small member of a currency union (n → 0). Parameter values: $\bar{L} = 1, \bar{B}^H = 3.44, \bar{B}^L = 3.1, \chi = 0.5, \omega = 0.2, \beta^s = 0.98, \beta^b = 0.97$.

Martin and Philippon (2014) also study a deleveraging shock within a small member country of a currency union. They find their model to perform well in accounting for the dynamics of nominal GDP, employment and net exports in a number of euro area countries during the period 2000–2012. Through counterfactual simulations they explore the role of macro-prudential and fiscal policy in influencing macroeconomic outcomes. Their analysis does not consider movements in relative prices, and in particular real exchange rates, which is the main focus of our paper. Furthermore, focussing on shocks to a small union member—as we do in this section—implies that a deleveraging shock does not generate spillovers on the rest of the union. As we show next, allowing for such spillovers has important implications for the area-wide response to the shock and, hence, for the adjustment of the real exchange rate.

3.4.2 Deleveraging in a (very) large union member

If $n \to 1$, the Home-good consumption weights are $\lambda \to 1$ and $\lambda^* \to \omega$ in Home and Foreign respectively. We obtain $Y_t = C_t$ from equation (3.3.18), $P_t = P_{H,t}$ from (3.3.9) and $D_t = 0$ from (3.3.21). Thus, Home accounts for almost the entire currency union and behaves like a closed economy. Foreign, in turn, effectively becomes a small open economy. Again, this has important implications for monetary policy: it is entirely geared towards developments in Home, as Foreign has a negligible effect on average union-wide inflation: $\Pi_t^u = P_t/P_{t-1}$ from (3.3.16).

As we now show, in this case the real exchange rate response hinges critically on the size of the shock, and the related monetary policy response. We establish that for a large shock, monetary policy becomes constrained by the zero lower bound (3.3.17) and, as a result, the real exchange rate response becomes muted, provided wages in Home are not much more flexible than in Foreign. Moreover, under certain conditions the real exchange rate may in fact appreciate rather than depreciate, reversing the usual dynamics.

Proposition 6. *Consider the economy defined in Section 3.3 and let $n \to 1$. Suppose that in period t, the debt limit in Home is unexpectedly and permanently reduced from $\bar{B}^H$ to $\bar{B}^L$. The real exchange rate response at time t depends on whether this shock is large enough to push the union to the zero lower bound.*

(a) If the deleveraging shock is small, $\beta^s \bar{B}^H - \bar{B}^L < \underline{\zeta}$, monetary policy is unconstrained by the zero lower bound and the real exchange rate depreciates. Formally, we have

$$Q_t \ (:= Q_t^{NoZLB}) = \left[(1-\omega)\left(1-\beta^s\left(1-\frac{(1-\chi)Y+\chi\bar{B}^H}{(1-\chi)Y+\chi\bar{B}^L}\right)\right)+\omega\right]^{1-\omega} > 1.$$

(b) If the deleveraging shock is large, $\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$, the zero lower bound binds in the period of deleveraging. If wages in Home are not much more flexible than in Foreign, $\gamma^*/\gamma < 1 + \kappa$, where $\kappa > 0$, then the following inequality holds

$$Q_t \ (:= Q_t^{ZLB}) = \max\left(\left[\frac{\gamma^*}{\gamma}\right]^{1-\omega}, \left[1-\omega+(1-(1-\omega)\beta^s)\frac{Y_t}{Y}\right]^{1-\omega}\right) < Q_t^{NoZLB}.$$

That is, the zero lower bound generally dampens the real depreciation. Moreover, for a sufficiently large shock, $\beta^s \bar{B}^H - \bar{B}^L > \bar{\zeta} \geq \underline{\zeta}$, the second part in $\max(\cdot,\cdot)$ above is below one such that the real exchange appreciates ($Q_t^{ZLB} < 1$), provided wages in Home are less flexible than in Foreign, $\gamma^*/\gamma < 1$.

Proof. See Appendix. □

The critical values $\underline{\zeta}$, $\bar{\zeta}$ and κ are provided in the appendix along with the proof of Proposition

6. The solution for Y_t is given in equation (3.4.4) below; note that $Y_t < Y$, the full-employment output level. The first part of the proposition establishes that country size *per se* does not alter the sign of the real exchange rate response. Intuitively, if monetary policy is not constrained in pursuing the inflation target, it reduces the nominal interest rate sufficiently in response to the deleveraging shock. Average inflation in the currency union remains at zero, because Home inflation is zero ($n \to 1$). Lower interest rates, in turn, raise consumption in Foreign. In the presence of home bias, this pushes up the price level in Foreign. Thus the Home real exchange rate depreciates.

The second part of the proposition shows that if Home is large, the real exchange rate response is generally hampered whenever monetary policy becomes constrained by the zero lower bound. In this case, monetary policy is unable to stabilise Home prices, as this would require pushing nominal interest rates into negative territory. As Home demand collapses, nominal wages (and as a result: prices from (3.3.12)) decline up to the floor set by downward rigidity, parametrised by γ. This has implications for Foreign, too. As before, Foreign consumption will tend to increase to the extent that monetary policy reduces interest rates. However, there is now a second effect: the demand for Foreign-produced goods falls with Home consumption. This exerts downward pressure on the Foreign price level: there are deflationary spillovers which dampen the real exchange rate depreciation. In fact, if the shock is large enough ($\beta^s \bar{B}^H - \bar{B}^L > \bar{\zeta}$) and if Foreign wages are more flexible than Home wages, the Foreign price level may decline more strongly than the Home price level: the Home real exchange rate appreciates.[14]

This finding qualifies a result obtained by Cook and Devereux (2014). They use a two-country model to contrast the effect of a negative demand shock under flexible exchange rates with that under a common currency. In case the zero lower bound binds, they find the Home real exchange rate to appreciate, but only if the nominal exchange rate is flexible. Under a common currency, instead, there is no such "perverse adjustment" of the real exchange rate. As we allow for differential degrees of stickiness in the two countries, we find that, for a large enough shock, the real exchange rate response may in fact also reverse its usual pattern under a common currency.

Having established that real exchange rate movements are dampened at the zero lower bound, we now turn to the role of wage rigidity in the adjustment process. It turns out that at the zero lower bound, increasing wage flexibility is actually destabilising, in line with the findings of Eggertsson and Krugman (2012) for the closed economy. We summarise this result in the

[14] Thus a sufficient condition for the real exchange rate depreciation to be dampened for an intermediate-sized shock, and even to be reversed for a larger shock, is given by $\gamma^* < \gamma$—that is, Foreign wages are more flexible than Home wages.

following Proposition.

Proposition 7. *Paradox of flexibility. Consider the economy defined in Section 3.3 and let $n \to 1$. Assume the zero lower bound is binding in the period of deleveraging ($\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$). In this case, if wages become more flexible domestically (that is, as γ is reduced)*

(a) Output, borrower consumption and real wage income all decline strictly more.

(b) The real exchange rate depreciates strictly less (or, if $\beta^s \bar{B}^H - \bar{B}^L > \bar{\zeta} \geq \underline{\zeta}$, appreciates strictly more), provided the wage rigidity condition (3.3.14) *is not binding in Foreign.*

Proof. See Appendix. □

The reason why more flexibility is harmful is simple: debt deflation. To see this, we again rearrange the borrower budget constraint (3.3.4) to obtain

$$C_t^b = -\frac{\bar{B}^H - \bar{B}^L}{P_t} + \frac{W_t L_t}{P_t}, \tag{3.4.3}$$

where we have used that $R_t = 1$ in the period of deleveraging. As before, the shock exerts downward pressure on domestic prices, making real debts harder to repay. And again, the general equilibrium response of real wage income is crucial for how the borrower responds to the shock.

As $n \to 1$, real wage income corresponds to domestic output, $W_t L_t / P_t = Y_t$, which follows from combining (3.3.11) with (3.3.12), and using that $P_t = P_{H,t}$ (see above). In turn, domestic output in the period of deleveraging solves[15]

$$Y_t = (\beta^s)^{-1} \left[Y - \frac{\chi}{1-\chi} \left(\frac{\beta^s \bar{B}^H - \bar{B}^L}{\gamma} \right) \right] < Y. \tag{3.4.4}$$

The second part of this expression is negative, and more so, the more flexible the domestic wages (the lower the γ). That is, in contrast to the case of $n \to 0$ analysed earlier, economic activity (and therefore real wage incomes) now decline with more flexible wages.

The intuition here is that while lower prices still increase the real value of debts, they no longer stimulate economic activity by crowding in foreign demand or by lowering the real interest rate. For foreign demand, at the zero lower bound deflationary spillovers imply that the real exchange rate response is dampened (Proposition 6).[16] Real interest rates could fall because of either a cut in nominal rates, or expected inflation. But the zero lower bound implies that nominal interest rates are stuck at zero, and under inflation targeting a drop in prices does not generate expected inflation.

[15] See the appendix, the proof of Proposition 7.

[16] Incidentally, in the case of $n = 1$, foreign demand would not be crowded in even if the real exchange rate depreciated substantially, given Foreign's negligible size. However, as we turn to $n < 1$ next, we believe the deflationary spillovers are the more relevant intuition.

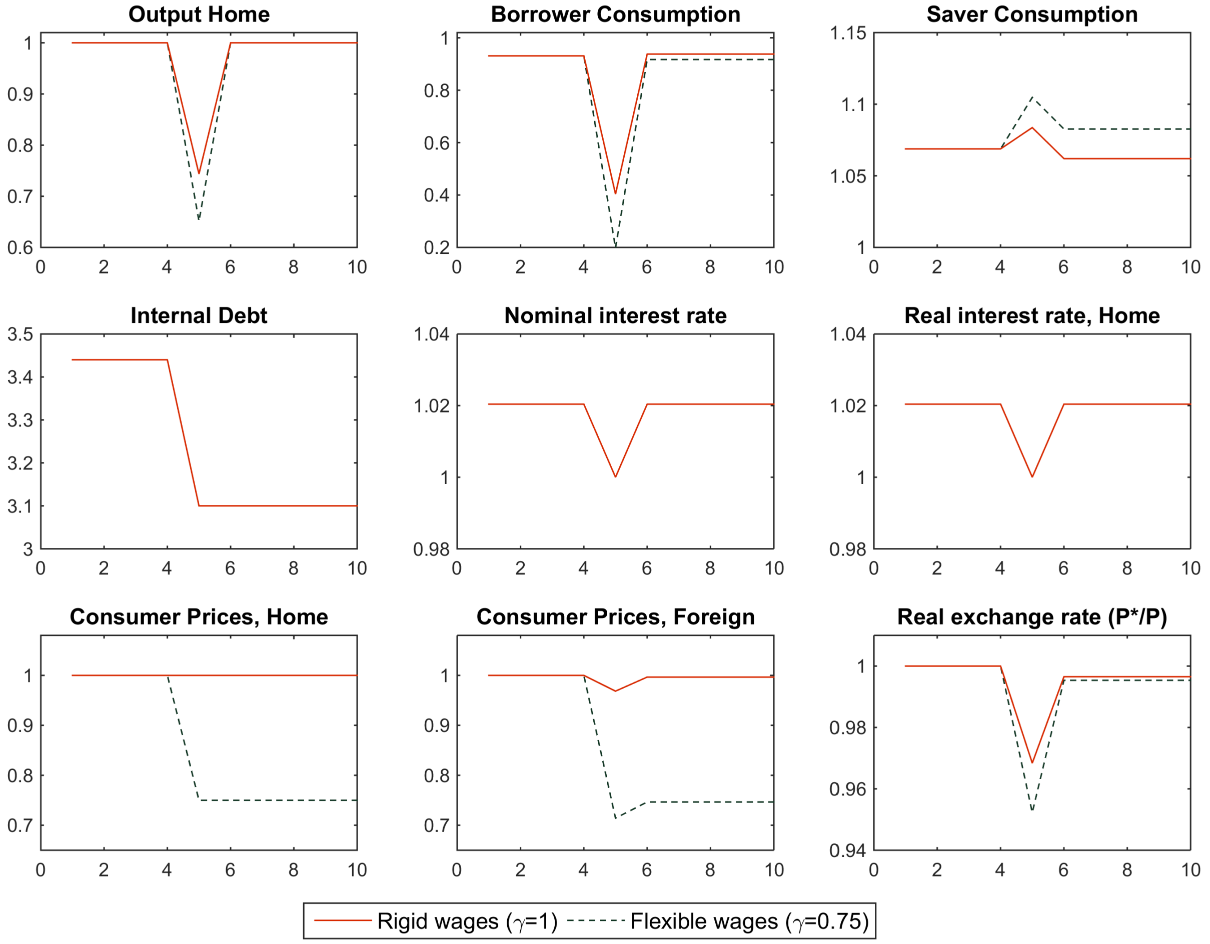

Figure 3.5: Deleveraging in a large union member (n $\to$ 1). Parameter values: $\bar{L} = 1, \bar{B}^H = 3.44, \bar{B}^L = 3.1, \chi = 0.5, \omega = 0.2, \beta^s = 0.98, \beta^b = 0.97,\ \gamma^* = 0.7$.

As pointed out in Proposition 6, for a large enough shock the fall in Home consumption creates large enough spillovers such that Foreign prices fall by more than Home prices, to the extent that the former are more flexible than the latter. Part a) of Proposition 7, in turn, states that these spillovers can only *increase* as domestic rigidities are reduced. Thus, somewhat paradoxically, more flexible wages in Home may lead to an even larger appreciation of its real exchange rate (part b) of Proposition 7).[17] Figure 3.5 shows the full adjustment dynamics for precisely such a case, by comparing fully rigid ($\gamma = 1$, solid lines) and more flexible wages ($\gamma = 0.75$, dashed lines).

From a conceptual point of view, we also note that there is a critical level of wage flexibility

[17] Note that $\gamma^* < \gamma$—the condition for the real exchange to appreciate if the shock is large (part b) of Proposition 6)—is implied by the wage rigidity condition (3.3.14) not binding in Foreign, as assumed in part b) of Proposition 7. See the appendix, the proofs of the two propositions for further details.

beyond which the model has no solution—or equilibrium—because the real value of debt repayments exceeds the level of output under full employment, which would imply $Y_t < 0$ in equation (3.4.4). Moreover, there is no solution even as wages become fully flexible. Intuitively, for a large deleveraging shock, the flexible-wage equilibrium requires a real interest rate below unity to induce savers to consume all of the repayment made by the borrowers. At the zero lower bound, this requires expected inflation—which is inconsistent with the central bank's inflation targeting mandate.[18] A flexible-wage equilibrium would exist, however, under an alternative monetary policy rule which generates the required expected inflation, such as a price level targeting rule. Moreover, such an alternative monetary policy rule could maintain full employment even under partially rigid wages, avoiding both the paradox of flexibility and the deleveraging recession itself.

The above insight relates to a point made by Cochrane (2015), who maintains that in New Keynesian models equilibria involving deep recessions and paradoxes of flexibility at the ZLB are the result of monetary policy rules containing an element of equilibrium selection. In our model, we account for alternative monetary policy rules as we vary the country size n. When $n \to 0$, monetary policy in Home corresponds to a simple exchange rate peg. As discussed in Corsetti et al. (2013b), such a peg provides an implicit price level commitment—in our case, to prices in Foreign—which is stabilising because it generates expected inflation. As n increases, a Foreign country with more rigid wages can still provide such a commitment mechanism, albeit to a lesser extent. Instead, for $n \to 1$ the price level commitment disappears and we are left with a simple inflation targeting rule.

In the next section we turn to analyse the numerical solution of our model for an empirically relevant, intermediate value of n. Generally speaking though, the insights gained from looking at the limiting cases $n \to 0$ and $n \to 1$ carry over to our quantitative analysis. Most importantly, as long as the deleveraging shock is large enough to push the union to the zero lower bound, deflationary spillovers will generally mute the adjustment of relative prices, and may sometimes even reverse them. This lack of relative price adjustment, and the lack of movement in the real interest rate, imply that Home could enter a deep recession even if wages are quite flexible.

3.5 Quantitative analysis

The analytical results established in the previous section show that country size matters for the adjustment dynamics to a region-specific deleveraging process. In 2009, credit growth stalled in most countries of the euro area. However only the stressed economies of the euro

[18] We show this formally in note which is available on request.

area experienced a full-fledged deleveraging process in the years thereafter (see Section 3.2). In the following we perform a quantitative assessment of our model and explore to what extent it can account for key features of the post-crisis slump in the euro area. Precisely, we examine the non-linear impulse response to a deleveraging shock in two settings. First, we consider the baseline model introduced in Section 3.3, where deleveraging takes place over one period. Second, we use a modified version of the model, where borrowers are allowed to deleverage gradually and choose the optimal path of deleveraging over a number of periods.

3.5.1 Baseline model

We assign parameter values in order to solve the model numerically. The specific values are summarised in Table 3.1. The parameters which govern the size of the stressed economy and the size of the shock are determined by the following observations. First, we set $n = 0.37$ to match the share of stressed countries in euro-area GDP on the eve of the crisis. Second, we observe that the level of total private sector debt in the stressed economy declines from 172% of GDP to 155% of GDP, that is, by 17 percentage points of GDP. The debt limit in our model, $\bar{B}_t$, is debt *per borrower*, which means that we have to scale up the economy-wide debt-to-GDP values by a factor $1/\chi$, in our case 2. This gives us deleveraging from 344% GDP per borrower ($\bar{B}^H = 3.44$) to 310% GDP per borrower ($\bar{B}^L = 3.1$), a total of 34% GDP.[19] We use the borrower share parameter χ from Martin and Philippon (2014), which captures the share of liquidity-constrained households in the data from Eurosystem Household Finance and Consumption Survey (HFCS).

The other parameters are standard. Assuming a discount factor β^s of 0.98 for the patient households implies an annual real interest rate of 2% in steady state. We set the home-bias parameter $\omega = 0.2$. Given $n = 0.37$ this implies an import share of 0.12, the average GDP weight of imports from the rest of the euro area in the stressed economies.[20] Finally, we vary the parameter γ which captures downward wage rigidities. Specifically, we consider a range of values between 0.93 and 0.99, which is equivalent to maximum wage deflation of between 7% and 1% per year respectively. The high downward wage rigidity implied by $\gamma = 0.99$ is broadly in line with estimates for the euro area (Schmitt-Grohé and Uribe, 2016).

Figure 3.6 shows the response of this economy to the deleveraging shock, contrasting three scenarios. Solid lines represent our baseline, characterised by high wage rigidities ($\gamma = 0.99$) in Home. In addition we consider two counterfactuals. The lines with crosses correspond to a

[19] The upper limit is the peak value, observed in 2012Q2, whilst the trough value is the latest observation in our sample, 2014Q3. As before, in our model nominal GDP equals 1 in the pre-crisis steady state, and a deleveraging shock of 34% is equivalent to 34% of pre-crisis nominal GDP.

[20] Source: OECD, Monthly Foreign Trade Statistics, period: 1999–2006.

Parameter:	n	$\bar{B}^H$	$\bar{B}^L$	β^s	ω	χ	γ
Value:	0.37	3.44	3.1	0.98	0.2	0.5	0.93–0.99

Table 3.1: Parameters for quantitative analysis: baseline model

scenario where Home rigidities are reduced, but remain above Foreign ($\gamma = 0.96$ vs $\gamma^* = 0.95$). The dashed lines correspond to a case where rigidity in Home is significantly lower, to the extent that Home becomes more flexible than Foreign ($\gamma = 0.93$ vs $\gamma^* = 0.95$).

The deleveraging shock is displayed in the bottom-left panel. Under our baseline scenario, it pushes the economy into a deep but asymmetric recession: while Home output collapses, Foreign output remains unaffected, even though monetary policy becomes constrained by the zero lower bound (first row of Figure 3.6). The dynamics of consumption also differ. While borrowers reduce consumption to repay their debts, Home savers' and Foreign households' consumption increases (second row of Figure 3.6), reflecting a reduced real interest rate (bottom-right panel for the case of Home).

The third row shows the responses of variables of particular interest. For the baseline case we find a mild decline of the price level in Home, reflecting the presence of downward wage rigidities. At the same time, Foreign prices decline—if only by little—due to deflationary spillovers. As a result, the real exchange rate remains broadly unchanged in the baseline scenario.

Turning to the counterfactuals, consider first the case of somewhat increased wage flexibility ($\gamma = 0.95$, crossed lines) in Home. Despite the sharper drop in prices in Home, the real exchange rate movement is virtually identical to that under the baseline scenario of higher wage rigidity (third row). This is because more flexible wages increase debt deflation and deepen the recession in Home—an instance of the paradox of flexibility (see Proposition 7 for the $n \to 1$ case). Deflationary spillovers increase, such that prices in Foreign decline by more to stabilise output in the face of an export collapse.

Under our second counterfactual, wage rigidities in Home are relaxed yet further, to the point where wages are more downwardly flexible than in the rest of the union ($\gamma = 0.93$, $\gamma^* = 0.95$, dashed lines). Following the shock, both countries find themselves against the wage rigidity constraint in (3.3.15), but because Home wages are more flexible, prices in Home fall by more (third row). The real exchange rate depreciates, more so than in the baseline scenario. Turning to output (top row), Foreign now enters a mild recession because its prices cannot fall by enough to fully offset the negative demand spillover from Home. Even though aggregate

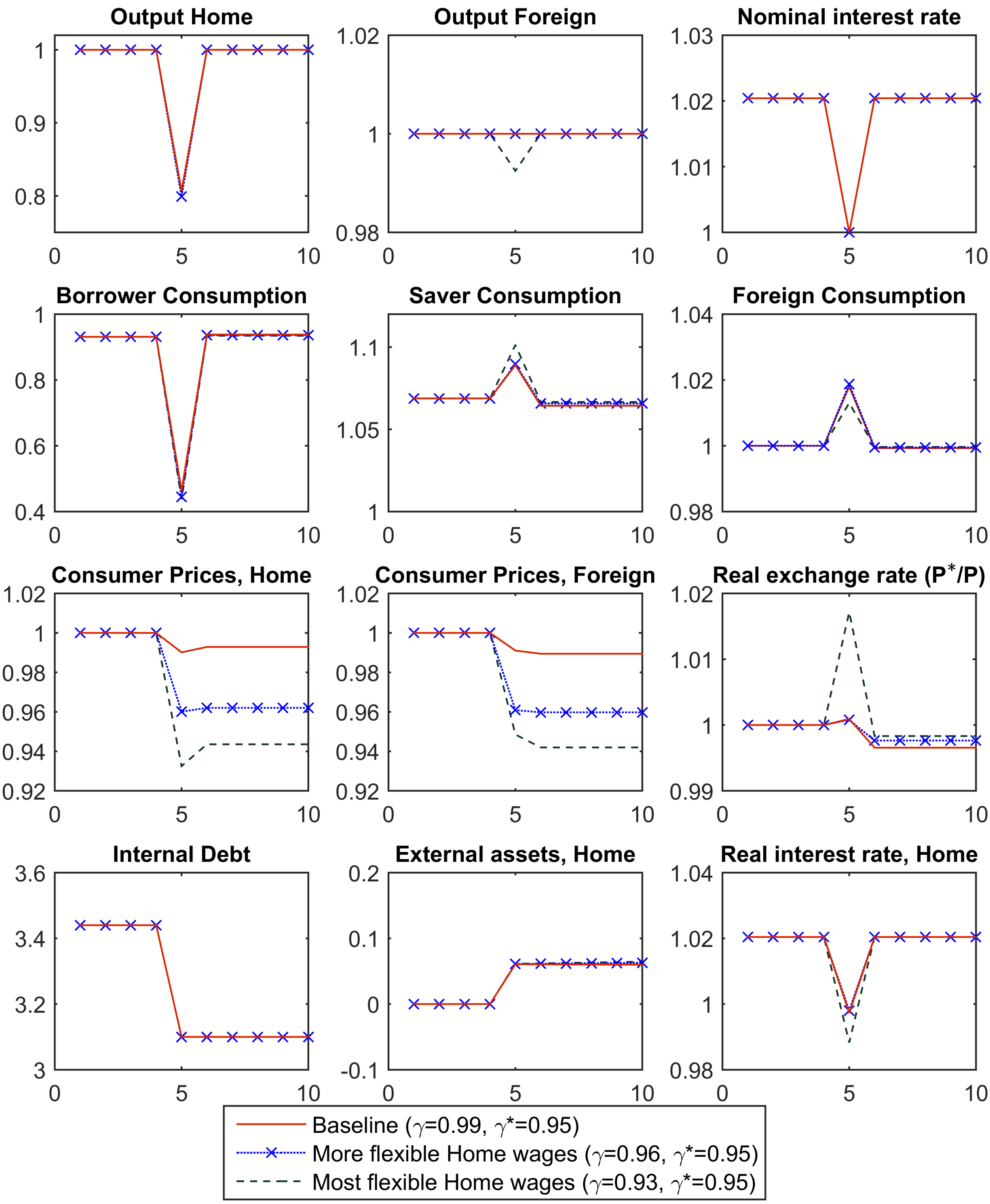

Figure 3.6: Deleveraging in a medium-sized union member, $n = 0.37$. Baseline model. Parameter values in Table 3.1. One period is a year.

demand and Home output are boosted by the real depreciation and a lower real interest rate, the benefits of these are roughly offset by higher debt deflation, and Home output falls by roughly the same amount as in the baseline scenario.

On reflection, the results in this section resemble the $n \to 1$ case of Section 3.4.2 more than they do the $n \to 0$ case of Section 3.4.1. The deleveraging is associated with a large output drop in Home and deflationary spillovers to Foreign, which increase in size with higher wage flexibility. Unlike Home output and prices, the real exchange rate and Foreign output change little in all three of our simulated scenarios.

In the next section, we examine a fuller model which allows us to relate our quantitative results to the dynamics of the post-crisis slump in the euro area. To do this, we allow the deleveraging to take place gradually, and over a longer time period.

3.5.2 Dynamic Deleveraging

Euro-area deleveraging has been taking place for the best part of six years, and the pace of the deleveraging has been quite gradual. To account for this, we modify our model in one key way. Borrowers no longer face a fixed borrowing limit $\bar{B}_t$. Instead, when debts in the economy exceed some level perceived as "safe"—denoted $\bar{B}_{S,t}$—the savers (through financial intermediaries) start charging higher interest rates on borrowing. A deleveraging shock is then a reduction in this safe level of debt, which increases interest spreads and incentivises borrowers to reduce their indebtedness. In this regard we mimic the set-up of Benigno et al. (2014) who consider a closed economy.[21]

Formally, borrowers now maximise

$$\max_{\{C_t^b\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta^b)^t \ln(C_t^b)$$

subject to

$$P_t C_t^b + B_{t-1}^b = (R_t^b)^{-1} B_t + W_t L_t, \tag{3.5.1}$$

where R_t^b is the gross borrowing rate determined according to

$$R_t^b = R_t \left(B_t^{\text{Av}} / \bar{B}_{S,t}\right)^{\phi} \tag{3.5.2}$$

$$R_t^b \geq R_t. \tag{3.5.3}$$

Note that the spread is determined by the average debt per borrower throughout the economy, rather than individual debt, even though in equilibrium $B_t^{\text{Av}} = B_t^b$. Here, ϕ is the elasticity of the gross borrowing rate with respect to excessive debt. As argued in Benigno et al. (2014), one

[21] Our set-up differs only slightly from Benigno et al. (2014), in the following ways. First, we examine two open economies. Second, to improve tractability, the spread is a function of nominal, not real, debt. Last, we provide a full non-linear solution of the model under perfect foresight.

Parameter:	n	$\bar{B}^H$	$\bar{B}^L$	β	ω	χ	γ	ϕ	φ^π
Value:	0.37	13.76	12.4	0.995	0.2	0.5	0.98–0.99	0.38	1.5

Table 3.2: Parameters for quantitative analysis: dynamic deleveraging model

can interpret this set-up as capturing financial intermediation in a very stylised way: banks lend to borrowers at rate R_t^b and pay the savers R_t. The premium for excessive borrowing can be interpreted as a charge for default risk in the presence of asymmetric information, or as compensating for fraud. And the profits of these transactions are distributed to savers, who own the banks.[22]

Optimality requires that borrowers satisfy an Euler equation

$$\left(C_t^b\right)^{-1} = \beta^b R_t^b \left(C_{t+1}^b\right)^{-1} \frac{P_t}{P_{t+1}}. \tag{3.5.4}$$

Furthermore, as in Benigno et al. (2014), we let $\beta^b \to \beta^s = \beta$, such that in steady state $B^b = \bar{B}_S$ from combining (3.5.2) and (3.5.4), and so banks make zero profits.

Lastly, for our numerical analysis of the gradual deleveraging model, we assume that the central bank implements its inflation target via a Taylor-type rule of the form

$$R_t = \left(\beta^s\right)^{-1} \left(\Pi_t^u\right)^{\varphi^\pi} \text{ subject to } R_t \geq 1, \tag{3.5.5}$$

where we assume $\varphi^\pi > 1$.

We measure periods in quarters, and adjust the model parameters accordingly. All parameter values are listed in Table 3.2. Values for n, ω and χ are unchanged from before. The quarterly discount factor $\beta = 0.995$ is equivalent to a 2% annualised real interest rate in steady state. Debt limits of 344% and 310% annual GDP per borrower translate to 1376% and 1240% of quarterly GDP respectively. We set the Taylor rule parameter φ^π to the conventional value of 1.5. Wage rigidity occupies a range of values, allowing for maximum wage falls of between 4% and 8% per year.[23] Finally, we set the spread elasticity to 0.38 in order to target a zero-lower-bound episode which lasts 6 quarters.

We now turn to our quantitative analysis. We assume that at time t, the safe debt limit unexpectedly and permanently tightens from $\bar{B}^H$ to $\bar{B}^L$. From (3.5.2), this opens up a spread between borrowing and lending rates, since the level of debt becomes judged as "excessive":

[22] The savers' budget constraint is now given by $P_t C_t^s + R_t^{-1} B_t^s + R_t^{*-1} D_t = W_t L_t + B_{t-1}^s + D_{t-1} + \frac{\chi}{1-\chi}\left(R_t^b - R_t\right) B_t^b$, where the last term are banking profits distributed to savers in a lump-sum manner.

[23] Note that we examine a slightly narrower range than in Section 3.5.1, to preserve stability of the numerical solution. Because of dynamic feedback effects, large changes in wage flexibility can have dramatic effects on model outcomes: they trigger a deflationary spiral, such that a stable model solution does not exist.

$\bar{B}^H > \bar{B}^L = \bar{B}_{S,t}$, triggering a deleveraging from equation (3.5.4), until the new safe debt level $\bar{B}^L$ is reached in the long run. The dynamic response to the shock is depicted in Figure 3.7. We compare the response in a baseline scenario of rigid wages in stressed countries (red solid lines) to a counterfactual scenario of more flexible wages (black dashed lines). Throughout this experiment, we assume that wages in Foreign are flexible enough to support full employment. From the top row, we can see that the deleveraging shock generates a recession and pushes the union to the zero lower bound. Looking at the mechanism in more detail, the second row shows the associated increase in spreads which triggers a slow and gradual debt reduction towards the new, lower, safe level. The debt repayment acts as a long-lasting drag on borrower consumption (third row, left), which lowers aggregate demand and generates deflationary pressures. In the early stages of the adjustment, the central bank is unable to cut interest rates sufficiently in order to offset these pressures, and the economy enters a recession.

Turning to the focus of our experiment, the bottom row depicts the movements in relative prices. In every period of the recession, Home wages fall by the maximum amount permitted by the rigidity constraint, which leads to a fall in Home prices. Under the baseline scenario, Foreign prices fall slightly due to the deflationary spillovers, dampening the movement in the real exchange rate. Prices in both countries fall by around 1% in the first quarter such that there is no real depreciation. Thereafter prices in Home fall by around 5.5% and in Foreign—by 3%, so just over half of the price fall in Home is dampened by the deflationary spillover to Foreign, and the resulting real depreciation is rather modest in size.

The absolute price movements under the counterfactual scenario of more flexible wages (bottom row, dashed lines) are much more pronounced. However this does not mean that *relative* prices adjust by more—in fact, they adjust by less and even move in the "wrong" direction. This is because the sharp fall in Home prices triggers a destabilising deflationary spiral and increases the negative demand spillover to Foreign, and with it the pressure on Foreign prices to drop *relative* to Home prices. Debt deflation in Home forces borrowers to reduce consumption further (third row, left). At the same time, expected deflation raises real interest rates (second row, right), such that even savers cut back on consumption (third row, middle). This in turn reduces real incomes throughout the Home economy and deepens the recession. The resulting fall in demand for Foreign exports pushes down Foreign prices to the extent that the real exchange rate appreciates sharply at first, further depressing real incomes and exacerbating the deleveraging in Home. The paradox of flexibility is in full force, and the recession is deeper than under the baseline scenario of more rigid wages.

The discussion above supports the conclusions of our analytical model in Section 3.4 and the one-period deleveraging experiment in Section 3.5. Further, the impact of higher wage

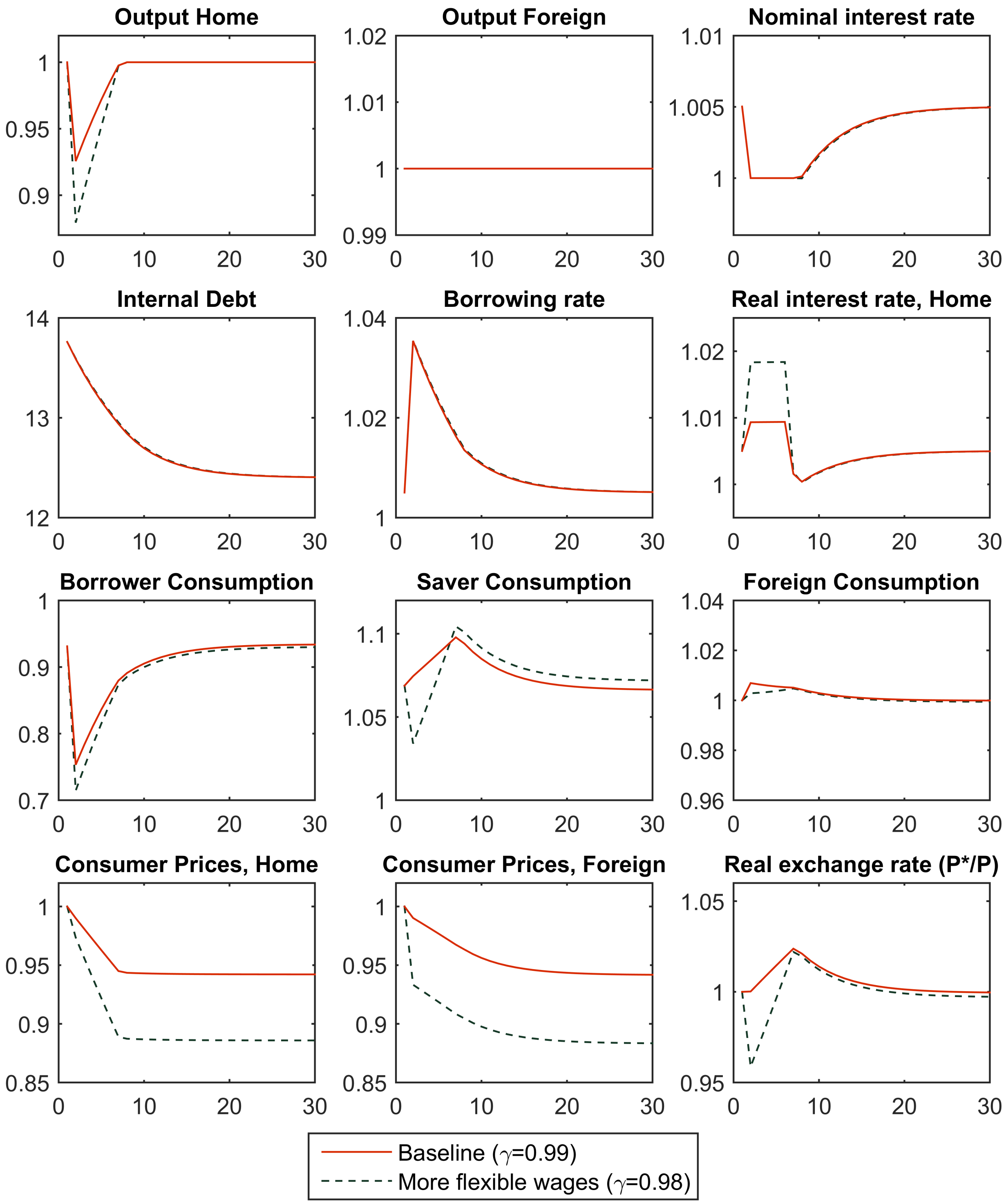

Figure 3.7: Dynamic deleveraging in a medium-sized union member, $n = 0.37$. Parameter values in Table 3.2. One period is a quarter. Interest rates shown on a quarterly basis.

flexibility—and the associated paradoxes—seem to be much more pertinent in the dynamic setting. More precisely, the differences in output and real exchange rate movements between the baseline and flexible-wage counterfactuals are much larger. Whilst under one-period deleveraging higher wage flexibility was not helpful but largely harmless, under dynamic deleveraging it can be highly destabilising. The reason for this is twofold. First, because deleveraging is more gradual, the adverse effects of debt deflation can persist for longer. Second, because some of the deflation is anticipated, it raises real interest rates and reduces consumption and demand further.

Even though the model in this section remains fairly stylised, the impulse responses for our baseline scenario match a number of facts which characterise the post-crisis slump in the euro area (Section 3.2). Just like the stressed euro-area economies, there is a deep recession in the Home country, triggered by a long and gradual deleveraging process. The recession persists for several periods and pushes the union to the zero lower bound. Prices in Home and Foreign move in a synchronised manner, which means that the real depreciation is small, around 2.4% at its peak. This is still somewhat larger than the "raw" 0.6% real depreciation we observe for the stressed euro countries in the data, but broadly in line with the 2% depreciation calculated using tax-adjusted price data.[24] The fall in the Foreign price level helps the rest of the union remain at full employment, so whilst prices move together, the output performance of the two country groups diverges. Our counterfactual simulation suggests that had wages in stressed economies been more flexible, we would have still seen little relative price adjustment within the euro area. Indeed, the real exchange rate may have even appreciated, deepening the recession.

3.6 Conclusion

Why—given the heterogenous economic performance across the euro area after 2009—has there been no significant adjustment of intra-euro-area real exchange rates? In this paper, we address the question by setting up a model of a currency union in which one of the two member countries experiences a large deleveraging shock. A key feature of the model is that the size of countries may differ. This allows us to study polar cases for which we are able to solve the model in closed form. In doing so we clarify aspects of the transmission mechanism, which also turn out to be relevant in the quantitative assessment of the model.

In case the domestic economy is small, there are no spillovers from the deleveraging shock into the rest of the union. Also, monetary policy remains unchanged. In this case, in line with much of the received wisdom, the extent of real deprecation is inversely related to the degree

[24] Figures quoted are for 2008Q4—2014Q4.

of wage rigidities. In the context of our model, this is simply the result of prices fluctuating one-for-one with wages. More flexibility in wages not only makes the real exchange rate more flexible, it also makes the economy more resilient in the face of adverse country-specific shocks. This result is notable because more wage flexibility also leads to more debt deflation in response to the deleveraging shock. Still, this effect is not strong enough to offset the benefits from increased flexibility for a small member country of a currency union.

A different picture emerges once we consider a large currency union member. Monetary policy lowers interest rates in response to the shock, and reaches the zero lower bound if the shock is large enough. The output response in the domestic economy and in the rest of the union will generally be different, in particular if there is sufficient wage flexibility in the rest of the union: while domestic output collapses, it may stay at its initial level in the other countries. At the same time there is deflationary pressure across the entire union, and the response of the real exchange rate to the deleveraging shock is dampened if the zero lower bound binds. In fact, the real exchange rate may even appreciate if the shock is large and/or wages are relatively flexible in the rest of the union. More wage flexibility in the domestic economy can also be destabilising and induce an even stronger real appreciation—an instance of the paradox of flexibility, established by Eggertsson and Krugman (2012) in a closed-economy context.

In our quantitative assessment we assume that the domestic economy accounts for 37 percent of the currency union, corresponding to the GDP weight of the stressed economies prior to the crisis. In response to a deleveraging shock of 34 percent GDP, we find the model able to account for some of the basic facts of the post-crisis slump in the euro area: there is large divergence in terms of output performance, there is deflationary pressure in the entire union, and the real exchange rate response is muted.

We also find that the paradox of flexibility reigns in our calibrated model. In a counterfactual scenario where wages are more flexible, the real exchange rate appreciates and the output loss is even larger than in the baseline case. Hence, we think that our analysis has some relevance for the ongoing policy debate in the euro area. Namely, similarly to Eggertsson et al. (2014), we find that measures geared towards raising the flexibility of the economies hit hardest by the crisis are likely to be ineffective—unless they are accompanied by expansionary monetary policy measures which may make up for the zero-lower-bound constraint on interest rates.

Appendices

This Appendix presents proofs of Propositions 1—4.

A Proof of Proposition 1

For $n \to 0$, we can reduce the system of equations (3.3.1)—(3.3.21) to the following 7 equations:

$$P_t C_t^s = P_{t+1} C_{t+1}^s \tag{A.1}$$

$$P_t C_t^b = \beta^s \bar{B}_t - \bar{B}_{t-1} + P_{H,t} Y_t \tag{A.2}$$

$$C_t = (1-\chi) C_t^s + \chi C_t^b \tag{A.3}$$

$$(1-\chi)(\beta^s D_t - D_{t-1}) = P_{H,t} Y_t - P_t C_t \tag{A.4}$$

$$P_t = P_{H,t}^{1-\omega} \tag{A.5}$$

$$P_{H,t} Y_t = (1-\omega) P_t C_t + \omega Y \tag{A.6}$$

$$0 = (Y_t - Y)(P_{H,t} - \gamma P_{H,t-1}), \tag{A.7}$$

as well as two inequalities:

$$Y_t \leq Y \tag{A.8}$$

$$P_{H,t} \geq \gamma P_{H,t-1} \tag{A.9}$$

The first equation is derived from the savers' Euler equation (3.3.2), the second—from the borrowers' budget constraint (3.3.4) (combined with (3.3.11), (3.3.12)), and the third—from aggregate consumption (3.3.6). Equation (A.4) is the country's budget constraint, obtained by combining the saver's and borrower's budget constraints (3.3.1) and (3.3.4), and substituting for nominal incomes from (3.3.11) and (3.3.12). The remaining equations are the price index (from 3.3.9), Home market clearing (from 3.3.18), and the complementary slackness condition (from 3.3.15). The inequality in (A.9) is obtained by combining (3.3.14) and (3.3.12). Additionally, we have used that $R_t = (\beta^s)^{-1}$, $P_t^* = P_{F,t} = 1$, and $C_t^* = Y_t^* = Y$.
The perfect foresight solution is a sequence of endogenous variables $\{C_t^s, C_t^b, C_t, Y_t, P_t, P_{H,t}, D_t\}$ that solves equations (A.1)—(A.9), given the initial conditions (stated in Section 3.3.5), and an exogenous path for $\{\bar{B}_t\}$, known in the initial period.
We can solve for real and nominal variables separately. The system in nominal variables can be reduced further to three equations

$$P_t C_t^s = P_{t+1} C_{t+1}^s \tag{A.10}$$

$$(1-\chi)(\beta^s D_t - D_{t-1}) = (1-\chi) P_{H,t} Y_t - (1-\chi) P_t C_t^s - \chi(\beta^s \bar{B}_t - \bar{B}_{t-1}) \tag{A.11}$$

$$(1-(1-\omega)\chi) P_{H,t} Y_t = (1-\omega)(1-\chi) P_t C_t^s + (1-\omega)\chi(\beta^s \bar{B}_t - \bar{B}_{t-1}) + \omega Y \tag{A.12}$$

in three unknowns $(P_t C_t^s), (P_{H,t} Y_t)$ and D_t.
We solve this system forward analytically. The full solution is presented in the proof of

Proposition 5. Here, we only present the solution for Home nominal income ($P_{H,t}Y_t$):

$$P_{H,t}Y_t = Y + (1-\beta^s)\frac{(1-\chi)(1-\omega)}{\omega}D_{t-1} - \eta(\bar{B}_{t-1} - \bar{B}_t),$$

where $\eta = (\beta^s(1-\omega)\chi)/(1-(1-\omega)\chi) > 0$.

In the period of deleveraging, $D_{t-1} = 0$, $\bar{B}_{t-1} = \bar{B}^H$ and $\bar{B}_t = \bar{B}^L$, which yields

$$P_{H,t}Y_t = Y - \eta(\bar{B}^H - \bar{B}^L), \tag{A.13}$$

Under full employment, $Y_t = Y$, and hence

$$P_{H,t} = 1 - \eta(\bar{B}^H - \bar{B}^L)/Y. \tag{A.14}$$

If there is unemployment, $Y_t < Y$ and from (A.7)

$$P_{H,t} = \gamma P_{H,t-1} = \gamma$$

From (A.9),

$$P_{H,t} = \max(\gamma, 1 - \eta(\bar{B}^H - \bar{B}^L)/Y \leq 1. \tag{A.15}$$

The inequality holds because $\gamma \leq 1$.

From (3.3.10) and (A.5), $Q_t = P_{H,t}^{\omega-1}$, where $\omega - 1 < 0$. Combining this with (A.15) gives us the real exchange rate formula in Propostion 4:

$$Q_t = P_t^*/P_t = \min\left(\gamma^{\omega-1}, [1 - \eta(\bar{B}^H - \bar{B}^L)/Y]^{\omega-1}\right) \geq 1.$$

B Proof of Proposition 2

We first present the full solution to the system of equations (A.10)—(A.12):

$$\begin{aligned}
P_tC_t^s &= Y + (1-\beta^s)\left(\frac{\chi}{1-\chi}\bar{B}_{t-1} + \frac{1-(1-\omega)\chi}{\omega}D_{t-1}\right)\\
P_tC_t^b &= Y - (1-\beta^s)\left(\bar{B}_{t-1} - \frac{(1-\chi)(1-\omega)}{\omega}D_{t-1}\right) - \frac{\beta^s}{1-(1-\omega)\chi}(\bar{B}_{t-1} - \bar{B}_t)\\
P_{H,t}Y_t &= Y + (1-\beta^s)\frac{(1-\chi)(1-\omega)}{\omega}D_{t-1} - \frac{\beta^s(1-\omega)\chi}{1-(1-\omega)\chi}(\bar{B}_{t-1} - \bar{B}_t)\\
D_t &= D_{t-1} + \frac{\omega\chi}{(1-(1-\omega)\chi)(1-\chi)}(\bar{B}_{t-1} - \bar{B}_t).
\end{aligned}$$

In the period of deleveraging, $D_{t-1} = 0$, $\bar{B}_{t-1} = \bar{B}^H$ and $\bar{B}_t = \bar{B}^L$. Substituting for this gives the following expressions for nominal spending and incomes:

$$\begin{aligned} P_t C_t^s &= Y + (1-\beta^s)\frac{\chi}{1-\chi}\bar{B}^H \\ P_t C_t^b &= Y - (1-\beta^s)\bar{B}^H - \frac{\eta}{(1-\omega)\chi}(\bar{B}^H - \bar{B}^L) \\ P_{H,t} Y_t &= Y - \eta(\bar{B}^H - \bar{B}^L), \end{aligned}$$

where $\eta = (\beta^s(1-\omega)\chi)/(1-(1-\omega)\chi) > 0$, as above.

We can immediately see that nominal spending and incomes are independent of wage flexibility γ. Savers' nominal spending is independent of the deleveraging shock, whilst borrowers' nominal spending and incomes fall proportionately with the deleveraging shock $\bar{B}^H - \bar{B}^L$. Because of this, the smaller the nominal adjustment in prices, the larger the adjustment of real spending and incomes will be, which gives us some intuition for the benefits of higher wage flexibility. We establish this more formally below.

Suppose first that the wage rigidity condition (A.9) is binding. Then $P_{H,t} = \gamma$ and $P_t = \gamma^{1-\omega}$ from (A.5). Real spending and output, as well as real incomes $W_t L_t / P_t$, are given below

$$C_t^s = \gamma^{\omega-1}\left[Y + (1-\beta^s)\frac{\chi}{1-\chi}\bar{B}^H\right] \tag{B.1}$$

$$C_t^b = \gamma^{\omega-1}\left[Y - (1-\beta^s)\bar{B}^H - \frac{\eta}{(1-\omega)\chi}(\bar{B}^H - \bar{B}^L)\right] \tag{B.2}$$

$$Y_t = \gamma^{-1}\left[Y - \eta(\bar{B}^H - \bar{B}^L)\right] \tag{B.3}$$

$$W_t L_t/P_t = P_{H,t}Y_t/P_t = \gamma^{\omega}Y_t = \gamma^{\omega-1}\left[Y - \eta(\bar{B}^H - \bar{B}^L)\right], \tag{B.4}$$

where we have used (3.3.11) and (3.3.12) in the last equation. Note that all increase with lower γ, or higher wage flexibility (which follows from $\omega - 1 < 0$).

When γ is low enough such that the wage rigidity condition in (A.9) is not binding, from (A.15), prices are at their lowest level given by (A.14), output is at full employment level, $Y_t = Y$, and consumption and real incomes are at their maximum level.[25] If γ is lowered further beyond this point, the real variables no longer change. This concludes the proof of Proposition 5.

[25] That is, the maximum when varying wage flexibility and taking all other parameters as given.

C Proof of Proposition 3

For $n \to 1$, we can reduce the system of equations (3.3.1)—(3.3.21) to the following 11 equations:

$$\beta^s R_t P_t C_t^s = P_{t+1} C_{t+1}^s \tag{C.1}$$

$$P_t C_t^b = R_t^{-1} \bar{B}_t - \bar{B}_{t-1} + P_t Y_t \tag{C.2}$$

$$C_t = Y_t \tag{C.3}$$

$$C_t = (1-\chi) C_t^s + \chi C_t^b \tag{C.4}$$

$$\beta^s R_t P_t^* C_t^* = P_{t+1}^* C_{t+1}^* \tag{C.5}$$

$$R_t^{-1} D_t^* - D_{t-1}^* = \omega (P_t C_t - P_t^* C_t^*) \tag{C.6}$$

$$P_{F,t} Y_t^* = (1-\omega) P_t^* C_t^* + \omega P_t C_t \tag{C.7}$$

$$P_t^* = P_{F,t}^{1-\omega} P_t^{\omega} \tag{C.8}$$

$$0 = (R_t - 1)(P_t - P_{t-1}) \tag{C.9}$$

$$0 = (Y_t - Y)(P_t - \gamma P_{t-1}), \tag{C.10}$$

$$0 = (Y_t^* - Y)(P_{F,t} - \gamma^* P_{F,t-1}), \tag{C.11}$$

as well as these inequalities:

$$Y_t \leq Y, \qquad Y_t^* \leq Y \tag{C.12}$$

$$P_t \geq \gamma P_{t-1}, \qquad P_{F,t} \geq \gamma^* P_{F,t-1} \tag{C.13}$$

$$R_t \geq 1 \tag{C.14}$$

The first five equations are the Home savers' Euler equation (3.3.2), the borrowers' budget constraint (3.3.4), Home goods market clearing (3.3.18), Home aggregate consumption (3.3.6), and Foreign saver's Euler equation (3.3.8) respectively. Equation (C.6) is the Foreign country's budget constraint, which we obtain by combining the Foreign saver's budget constraint (3.3.7) with the Foreign goods market clearing condition (3.3.19), and using the expressions in (3.3.10), (3.3.11) and (3.3.12). The remaining equations are, in order, the Foreign goods market clearing (3.3.19), Foreign price index (3.3.9), monetary policy (equations 3.3.16 and 3.3.17), and the complementary slackness conditions in Home and Foreign, in (3.3.15). Inequalities refer to the full employment constraint on output, downward wage rigidity and the zero lower bound. Throughout, we have used $P_t^u = P_t = P_{H,t} = W_t$, and $D_t = 0$ due to the large size of Home.[26]

[26] $P_t^u = (P_t)^n (P_t^*)^{1-n}$ is the union-wide price level.

Additionally, we write down a simplified expression for the savers' budget constraint in Home, which is not necessary to compute the solution, but provides a useful stepping stone in parts of the proof:

$$P_t C_t^s = \frac{\chi}{1-\chi}(\bar{B}_{t-1} - R_t^{-1}\bar{B}_t) + P_t Y_t. \tag{C.15}$$

The perfect foresight solution is a sequence of endogenous variables $\{C_t^s, C_t^b, C_t, Y_t, P_t, R_t, P_t^*, C_t^*, D_t^*, Y_t^*, P_{F,t}\}$ that solves equations (C.1)—(C.14), given the initial conditions (stated in Section 3.3.5), and an exogenous path for $\{\bar{B}_t\}$, known in the initial period. The solution will depend on whether the union is at the zero lower bound or not. We consider each of these two cases in turn.

(a) Outside of the zero lower bound

For the remainder of the proof, we adopt the same time notation as in Proposition 3: rather than using the general time subscript t, we denote t as the period of deleveraging, and $t-1$ as the initial steady state. Equation (C.9) then yields $P_t = P_{t-1} = 1$, and hence $Y_t = Y$ from equation (C.10): we have zero inflation and full employment in Home during the deleveraging period. At this point, it is helpful to write down the savers' budget constraint (C.15) at t and $t+1$:

$$C_t^s = Y + \frac{\chi}{1-\chi}\left(\bar{B}^H - R_t^{-1}\bar{B}^L\right)$$
$$C_{t+1}^s = Y + \frac{\chi}{1-\chi}(1-\beta^s)\bar{B}^L$$

where we have used that $Y_t = Y_{t+1} = Y$, $\bar{B}_{t-1} = \bar{B}^H$ and $\bar{B}_t = \bar{B}^L$.

Plugging the above expressions into the savers' Euler equation (C.1), we obtain an expression for the nominal interest rate

$$R_t = (\beta^s)^{-1}\frac{(1-\chi)Y + \chi\bar{B}^L}{(1-\chi)Y + \chi\bar{B}^H} \tag{C.16}$$

Intuitively, the central bank cuts the nominal interest rate sufficiently such that the savers consume all of the debt repayments they receive from the borrowers to maintain full employment.

We now turn to the developments in Foreign. In the period before deleveraging $D_{t-1}^* = 0$, and thereafter $D_{t+1}^* = D_t^*$ and $R_{t+1} = (\beta^s)^{-1}$. Also, there is no downward pressure on Foreign prices, and it remains at full employment, $Y^* = Y$.[27] We can then write down a 4x4 equation

[27] One can verify this ex post by checking the real exchange rate formula in (C.16). Since $Q_t > 1$, $P_{F,t} > 1$ and thus $P_{F,t} > \gamma^* P_{F,t-1}$ since $\gamma^* \leq 1$ and $P_{F,t-1} = 1$. From (C.11), this implies $Y_t^* = Y$.

system which allows us to solve for the Foreign price level, and hence, the real exchange rate.

$$R_t P_t^* C_t^* = (\beta^s)^{-1} P_{t+1}^* C_{t+1}^* \tag{C.17}$$

$$R_t^{-1} D_t^* = \omega(P_t C_t - P_t^* C_t^*) \tag{C.18}$$

$$(1-\beta^s) D_t^* = \omega(P_{t+1}^* C_{t+1}^* - P_{t+1} C_{t+1}) \tag{C.19}$$

$$P_{F,t} Y = (1-\omega) P_t^* C_t^* + \omega P_t C_t \tag{C.20}$$

The first equation is Foreign saver's Euler in (C.5), second and third—the Foreign country budget constraint (C.6) at t and $t+1$ respectively, and the last equation is the Foreign goods market clearing in (C.7). R_t is exogenous to Foreign and given by (C.16). Using that $P_t C_t = Y$, we first combine (C.17)—(C.19) to solve for $P_t^* C_t^*$

$$P_t^* C_t^* = Y(1 - \beta^s + R_t^{-1}), \tag{C.21}$$

and then substitute this expression into (C.20) to yield

$$Q_t = P_t^*/P_t = P_{F,t}^{1-\omega} = \left[(1-\omega)(1-\beta^s + R_t^{-1}) + \omega\right]^{1-\omega}, \tag{C.22}$$

which, combined with the interest rate expression in (C.16), gives us the real exchange rate formula in Proposition 3(a):

$$Q_t \;(:= Q_t^{\text{NoZLB}}) = \left[(1-\omega)\left(1 - \beta^s\left(1 - \frac{(1-\chi)Y + \chi \bar{B}^H}{(1-\chi)Y + \chi \bar{B}^L}\right)\right) + \omega\right]^{1-\omega}. \tag{C.23}$$

Because $\bar{B}^H > \bar{B}^L$, $Q_t^{\text{NoZLB}} > 1$, hence the real exchange rate depreciates.

(b) At the zero lower bound

Suppose that stabilising union-wide inflation (and therefore economic activity from (C.10)) would require $R_t < 1$. From equation (C.16), this requires a shock large enough, such that

$$\beta^s \bar{B}^H - \bar{B}^L > \frac{(1-\chi)}{\chi}(1-\beta)Y =: \underline{\zeta}.$$

In this case, we know that union-wide inflation cannot be stabilised (for if it were, output would be at potential as argued above, and thus the implied R_t would be negative). We shall demonstrate in Section D that in equilibrium, the price level will fall to its lower bound provided by downward wage rigidity, from (C.10), $P_t = \gamma P_{t-1} = \gamma$, and that $Y_t < Y$ so that output strictly drops below potential. Further note that in period $t+1$, the economy is in steady state, hence $R_{t+1} = (\beta^s)^{-1}$, $P_{t+1} = P_t = \gamma$, and $Y_{t+1} = Y$.

Before doing so, however, we establish the response of the real exchange rate by taking for granted the equilibrium at the zero lower bound described above, and to be established in the following section. To do so, first focus on the developments in Foreign. Suppose first that Foreign wages are fully flexible. This means Foreign remains at full employment from (C.11), and $Y_t^* = Y$. Furthermore, we have $R_t = 1$ from (C.9), $P_t C_t = \gamma Y_t$ and $P_{t+1} C_{t+1} = \gamma Y$ (where we use $P_t = P_{t+1} = \gamma$ in (C.3), and that $Y_{t+1} = Y$). Plugging these values into the 4x4 system of equations in (C.17)—(C.20) yields an expression for Foreign nominal consumption

$$P_t^* C_t^* = \gamma Y (1 + (1 - \beta^s) \frac{Y_t}{Y}).$$

Combining this with the market clearing condition in (C.7) gives us an expression for the real exchange rate under flexible Foreign prices:

$$Q_t = \left[\frac{P_{F,t}}{P_t} \right]^{1-\omega} = \left[1 - \omega + (1 - (1 - \omega)\beta^s) \frac{Y_t}{Y} \right]^{1-\omega} \tag{C.24}$$

We refer the reader to equation (3.4.4) and the proof of Proposition 7 for the precise expression for Y_t.

Suppose now that Foreign wages are downwardly rigid, and furthermore, the rigidity is sufficiently high such that (C.11) binds and hence $P_{F,t} = \gamma^*$. Then, trivially,

$$Q_t = \left[\frac{\gamma^*}{\gamma} \right]^{1-\omega}.$$

We now combine the cases of rigid and flexible wages in Foreign. From (C.13), we know that Foreign price level falls to either the flex-price level, or the maximum amount permitted by downward wage rigidity:

$$P_{F,t} = \max \left(\gamma^*, 1 - \omega + (1 - (1 - \omega)\beta^s) \frac{Y_t}{Y} \right),$$

Putting this expression into the real exchange rate formula in (3.3.10) yields

$$Q_t \ (:= Q_t^{\text{ZLB}}) = \max \left(\left[\frac{\gamma^*}{\gamma} \right]^{1-\omega}, \left[1 - \omega + (1 - (1 - \omega)\beta^s) \frac{Y_t}{Y} \right]^{1-\omega} \right), \tag{C.25}$$

which is the formula in Proposition 3(b).

To establish the inequality $Q_t^{\text{ZLB}} < Q_t^{\text{NoZLB}}$, we require that both parts of the max() operator above are smaller in magnitude than Q_t^{NoZLB}. This first of all requires that Foreign wages are not excessively rigid relative to Home:

$$\left[\frac{\gamma^*}{\gamma} \right]^{1-\omega} < Q_t^{\text{NoZLB}},$$

which from (C.23) requires

$$\gamma^*/\gamma < 1 + \kappa, \text{ where}$$
$$\kappa = (1 - \omega)\beta^s \left(\frac{(1 - \chi)Y + \chi \bar{B}^H}{(1 - \chi)Y + \chi \bar{B}^L} - 1 \right) > 0$$

Turning to the second part of the max() operator, since $Y_t < Y$ and $1 - (1 - \omega)\beta^s > 0$, this is bounded above by

$$\left[1 - \omega + (1 - (1 - \omega)\beta^s)\,\frac{Y_t}{Y}\right]^{1-\omega} < [2 - \omega - (1 - \omega)\beta^s]^{1-\omega}\,.$$

Because $\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$, we know that the counterfactual $R_t < 1$ in (C.16), and hence $R_t^{-1} > 1$. Applying this inequality to equation (C.22), we get

$$Q_t^{\text{NoZLB}} > [2 - \omega - (1 - \omega)\beta^s]^{1-\omega}\,,$$

Therefore, the inequality is satisfied for both parts of the max() operator, and $Q_t^{\text{ZLB}} < Q_t^{\text{NoZLB}}$.

To get a real appreciation, we need both parts of the max() operator to be less than 1. For the first part, we simply require $\gamma^*/\gamma < 1$. For the second part, we require

$$\beta^s \bar{B}^H - \bar{B}^L > \frac{(1-\chi)}{\chi}\left(\frac{1-\beta}{1-(1-\omega)\beta}\right)\gamma Y =: \widetilde{\zeta}$$

which is obtained by substituting for Y_t in (C.24) (using the formula in (3.4.4), derived in the Proposition 7 proof), and setting the resulting expression to be less than 1. Finally, a real appreciation requires that the deleveraging shock is both large enough to push the union to the zero lower bound ($\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$), and large enough to trigger a real appreciation once at the zero lower bound ($\beta^s \bar{B}^H - \bar{B}^L > \widetilde{\zeta}$). Thus it is required that

$$\beta^s \bar{B}^H - \bar{B}^L > \max(\widetilde{\zeta},\, \underline{\zeta}) =: \bar{\zeta}.$$

D Proof of Proposition 4

Recall that from our definition of monetary policy, Section 3.3.3, monetary policy cuts the nominal interest rate all the way to $R_t = 1$ if due to deflationary pressure, the inflation target $\Pi_t^u = 1$ cannot be reached. Here we show in a first step that for a large enough shock ($\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$), $P_t = P_{t+1} = \gamma$ along with $Y_t < Y$ is such an equilibrium, and in a second step, that it is the only one possible at the ZLB. Finally, we provide a proof of Proposition 4.

First note that as $n = 1$, real wage income and economic activity are directly related (as shown in the main text)

$$\frac{W_t L_t}{P_t} = Y_t.$$

Substituting this and the fact that $P_t = P_{t+1} = \gamma$ into the borrowers' budget constraint in

(C.2), and the savers' Euler equation in (C.1) gives

$$C_t^b = -\frac{\bar{B}^H - \bar{B}^L}{\gamma} + Y_t \tag{D.1}$$

$$C_t^s = (\beta^s)^{-1} C_{t+1}^s \tag{D.2}$$

Turning to savers' budget constraint (equation C.15) at $t+1$, and knowing that $\bar{B}_t = \bar{B}^L$ in the new steady state, we get

$$C_{t+1}^s = Y + \frac{\chi}{1-\chi}\left(\frac{(1-\beta^s)\bar{B}^L}{\gamma}\right) \tag{D.3}$$

Combining (C.3) in (C.4) yields

$$Y_t = \chi C_t^s + (1-\chi) C_t^b,$$

and substituting for saver and borrower consumption using equations (D.1)—(D.3) gives us the expression for output in (3.4.4):

$$Y_t = (\beta^s)^{-1}\left[Y - \frac{\chi}{1-\chi}\left(\frac{\beta\bar{B}^H - \bar{B}^L}{\gamma}\right)\right]. \tag{D.4}$$

From this equation, it follows that $Y_t < Y$ whenever

$$\beta^s \bar{B}^H - \bar{B}^L > \gamma\frac{(1-\chi)}{\chi}(1-\beta)Y = \gamma\underline{\zeta},$$

which must always hold, because $\gamma \leq 1$ and $\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$ at the ZLB. Thus we have established that $P_t = P_{t+1} = \gamma$ along with $Y_t < Y$ is an equilibrium.

To see that no other deflationary equilibrium at the ZLB exists, assume that prices fall to some level $1 = P_{t-1} > P_t = P_{t+1} (=: \tilde{P}) > \gamma$. In this case, from slackness condition (C.10), output must be at potential in the period of deleveraging, $Y_t = Y$. However as prices fall to $\tilde{P}$ equation (3.4.4), by setting $Y_t = Y$, can be written as

$$\beta^s \bar{B}^H - \bar{B}^L = \tilde{P}\underline{\zeta}.$$

The fact that at the ZLB, $\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$, then leads to a contradiction because under deflationary pressure, $\tilde{P} < 1$ as mentioned before. Thus, there can be no other equilibrium at the ZLB where due to deflationary pressure, the central bank cuts its interest rate to $R_t = 1$.[28]

We can see that from equation (3.4.4), output Y_t falls with lower γ. This also means that real incomes fall, and from (D.1), that borrower consumption falls—since real incomes Y_t are lower and real debt repayments $(\bar{B}^H - \bar{B}^L)/\gamma$, which enter negatively, are higher. This completes

[28] Note that an inflationary equilibrium at the ZLB exists, $\tilde{P} > 1$, such that $Y_t = Y$ and $R_t = 1$ in the period of deleveraging. Thus we rule out this equilibrium by maintaining that the central bank would only cut its interest rate to $R_t = 1$ in the case of deflationary pressure in the period of deleveraging. This would be strictly implied if—as we do in our numerical implementation of the model—the central bank *implemented* its strict inflation target via a Taylor-type feedback rule.

the proof of Proposition 4(a). As a side note, from (D.2) and (D.3) we can see that saver consumption increases slightly in the period of deleveraging with more flexibility (and thus lower prices), because saver consumption becomes higher in the new steady state. This is because the value of saver assets (equal to borrower debt) rises in real terms as prices decline by more. However, this increase in saver consumption comes at the expense of the borrowers (both during the deleveraging period and in the new steady state), and is not enough to offset the fall in borrower consumption during the deleveraging period (because output falls, from equation 3.4.4).

We now turn to part (b) of Proposition 4. If the Foreign wage rigidity constraint is not binding, the real exchange rate is given by expression (C.24). Then γ only enters this expression via Y_t. We can see that higher wage flexibility lowers output Y_t, and, since $1-(1-\omega)\beta^s > 0$, lowers the real exchange rate Q_t. If $\bar{\zeta} > \beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$, $Q_t > 1$ and the real exchange rate depreciates by less as γ declines. If $\beta^s \bar{B}^H - \bar{B}^L > \bar{\zeta}$, $Q_t < 1$ and the real exchange rate appreciates by more as γ declines. This completes the proof of the Proposition.

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