Asymmetries in Contests and Women in Academia

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Contents

Introduction

In basic studies of economics, you start learning about models with one agent or even one representative agent. Later on, you learn what you have already known before: People are not all the same. In consequence, the agents used in models may not be symmetric or homogeneous as well. People are not homogeneous. One could say it is especially the heterogeneity in premises, characteristics or in any other way that makes a situation interesting to investigate. This fascination for heterogeneity in totally different aspects can be seen as driving force during my research and connects the three chapters of this thesis.

In the first chapter of this thesis, I investigate two players facing an asymmetric starting situation.[1] In this research we make use of aspects from behavioral economics. The findings from behavioral science translated into economic models comprise the possibility of explaining human behavior even better and in more aspects. Even findings that are puzzling at first appear to have an underlying structure and sense. Berger and Pope (2011) found that basketball teams that are slightly trailing at half time have a significantly higher probability to win the game in the end. They were able to reproduce their findings in lab experiments.

This finding appears puzzling regarding the standard tournament literature. It would be rational for the underdog to reduce his effort in order to safe energy as he has a lower chance to win. With Kahneman and Tversky (1979) introducing their prospect theory they encountered the finding that people feel losses stronger then gains. This characteristic was included the concept of reference-dependent preferences by Köszegi and Rabin (2006). These gain-loss utilities model a person's valuation in relation to a reference point. In case the person exceeds his goal, his gains will weight less compared to losses of the same amount, occurring when the target is not met.

In order to explain the puzzling results presented by Berger and Pope (2011) , I introduced the concept of reference-dependent preferences into a standard tournament

[1]This chapter is joined work with Dr. Jan Bergerhoff and has been published as Bergerhoff and Vosen (2015).

setting following Lazear and Rosen (1981). I aimed to capture the motivation effect included in the information of lying back slightly in a model. Could having a reference point really make this behavior rational? Indeed we found that giving a head start to one of the symmetric players leads to up to three different equilibria. Two of them appear to resemble exactly the pattern described by Berger and Pope (2011). In their related work, Gill and Stone (2010) were not able to find these additional equilibria due to the simplifications they made. My results reveal, that the results from the simplified model of Gill and Stone (2010) do not resemble the results from a more complex model like the model presented in this thesis or the general model used by Gill and Stone (2010) initially.

Analyzing the two new types of equilibria found, it can be noticed that they both encounter the desired characteristics but, interestingly, they appear for totally different reasons. One equilibrium (TAE2) can be seen as counterpart to the standard tournament outcome (CAE), where an increase in asymmetric makes both players reduce their efforts. In case of a TAE2 the stronger player anticipates that the underdog will choose a very high effort in order to prevent the pain of loosing and decides to resign as the effort needed to win would be to high for him; this again is anticipated by the underdog. The second equilibrium type (TAE1) is totally different. In this case, none of the players back off but both provide very high efforts. From a marketing perspective this equilibrium would be the most favorable in sports contests as they will be the most exciting ones.

The next chapter covers some marketing aspects as well. I am modeling a tournament between asymmetric firms competing for customers on the new media. As I grew up as part of the generation of digital natives I naturally entered a world with new channels of entertainment, information and advertisement. The new media such as blogs, video platforms or podcasts opened new possibilities to create content and to express your opinion on nearly everything to the public. These channels are usually hosted by one person who is in focus. He creates content on e.g. his experiences with certain products. This way of reviewing products is totally different to evaluations made by official or independent institutions who have the intention to give an objective opinion on the products being tested. In contrast, to this anonymous objective evaluation, the host is known to his followers. For this reason, subjective opinions can be used as source as well and can even capture more valuable information for consumers.

In the second chapter, I capture these components of objective and subjective criteria in a reviewing process in a contest situation starting with a standard Tullock contest. I consider only objective criteria at first combining them with the possibility of losing customers due to a bad review. Afterwards, I present a possibility to adapt the model to subjective criteria i.e. the taste of the host. Finally, I present a model where both aspects, subjective and objective, are combined. It can be seen that these contests can be an attractive opportunity to gain customers for specific firms. Furthermore, these contests may enforce high quality in products from well established companies. Especially having both criteria combined is most likely to increase the utility for the consumers. Additionally, I show that the optimization is not necessarily always fulfilled in case of using two instruments in a Tullock contest. This criterion is missing in the related work of Epstein and Hefeker (2003). Furthermore, I indicate an implication where this missing condition may change results in their work.

The essential insight, provided in this chapter, is that even these new online communities can be captured using wellknown models. Combining these with the new characteristics observed allows us to learn more about some driving mechanisms regarding the new media. As online communities will become even more important, scientific work in this field will be very important to gain insights about the modern life basing on the new media.

The third chapter can be seen as a real project of the heart as it investigates a circumstance I was confronted with repeatedly during my whole academic career. Starting my Bachelor studies in Bonn there was only one female professor who left before I got my Bachelor degree. The absence of female professors in combination with being motivated to become a professor increases once sensitivity for the fact of studying in a male dominated field. Fortunately, during my PhD studies, the number of female professors and junior professors increased. In consequence, I wanted to know what was happening in male dominated academic fields. What was changing and what could be influencing the career decisions of women on the one hand and the hiring decisions of the faculties on the other hand.

I present a dataset on PhD graduates in BA especially collected for this purpose combined with an existing dataset on BA professors in Germany. The changes over the last decades are illustrated and compared to data from the US using a variety of fields to set the collected data in context. It can be shown that the percentage of female professors exhibits a significant kink during the nineties. In this period, the majority of federal states in Germany introduced changes in laws to increase the gender equality in the labor market. For this reason, I performed an event study with the year of the change in law as period zero to even out the different years of implementation. The results suggest a connection between the changes in law and the sudden increase in female professors.

Further analysis of the data set showed a significant correlation between the gender composition of the fellow students and the decision to become a professor. The gender composition of the faculty seems to influence the decision to become a professor as well.

The presence of female professors increases the decision in favor of the professorship significantly. In contrast, a correlation between the gender composition of a faculty and the hiring decision could not be found. The gender of the supervisor does not seem to have any influence on the decision to become a professor.

Interestingly, the data reveal that the transition rate for becoming a professor has become equal for female and male PhD students during the last years. This means, the percentage of female PhDs choosing to become a professor in comparison to all female PhDs is the same as the number of male PhDs opting for this career compared to the total of male PhDs. This is a very profound finding as it contradicts the hypothesis of a glass ceiling problem in this field and it shows that the women that choose to make a PhD decide for an academic career as often as their male colleagues. In consequence, the bottleneck seems to be not between PhD and professorship but even before the stage of becoming a doctoral student. This appears to be an important insight that could be helpful for further attempts to increase the gender equality in male dominated academic fields.

Chapter 1

Can being behind get you ahead? Reference Dependence and Asymmetric Equilibria in an Unfair Tournament

Everyone remembers a plot where a disadvantaged individual facing the prospect of failure, spends more effort, turns around the game and wins unexpectedly. Most tournament theories, however, predict the opposite pattern and see the disadvantaged agent investing less effort. We show that 'turn arounds', i.e. situations where the trailing player spends more effort and becomes the likely winner of the tournament, can be the outcome of a Nash equilibrium when the initial unevenness is known and players have reference-dependent preferences. Under certain conditions, they are the only pure strategy equilibrium. If the initial unevenness is large enough the advantaged player will always invest the most effort. We also show that equilibria in which the player behind catches up without becoming the likely winner do not exist.

1. Introduction

Rank order tournaments are a common mechanism for providing social and economic order. They are somewhat special, because they tie the privilege of receiving a certain good or benefit to the effort of performing best at some productive task. Politicians need to convince their constituents to be elected and business men need to create value for their company to be considered for promotion. Especially when high stakes are involved any indication of the likely outcome of a tournament is an asset. Consider for example the large betting industry that offers bets at dynamic quotes during the progress of many publicly fought contests.

Many times tournaments are not entirely fair with one player, for example, having more information or better relations with the tournament decider. However, even more often such unevenness occurs in dynamic contests. Most real world tournaments are dynamic in the sense that they require repeated decisions by the competitors between which contestants can process new information and recalibrate their tactics and effort investment. The most general piece of new information is the intermediate score which exists in most tournament settings. Politicians obtain interims feedback through opinion polls and direct contact, students write mid-term or mock exams and sports men can usually collect a time or score signalling their relative position in the tournament. That such feedback is entirely even seems to be the exception rather than the rule.[1]

Previous research like the work on dynamic tournaments by Chan et al. (2009) or Aoyagi (2010) found that equilibria are "effort-symmetric" with respect to feedback. This means that independent of the interims feedback, both competitors invest the same level of effort and only the sum of efforts decreases the more uneven the feedback is. Without any difference in effort provisions and the corresponding changes in the relative winning probabilities these tournaments are essentially decided by the initial unevenness and chance. Providing information about the intermediate state of the game does not matter for the tournament outcome. The implicit assumption here is that the interims feedback does not affect the agent's utility directly. In such a world, a victory against all odds that follows a drastic comeback after having been far behind initially is the same as any other victory in terms of utility.

Gill and Stone (2010) were the first to account for the direct effect of feedback on utility by introducing "fairness and desert" concerns in the form of reference-dependent preferences. They investigate the influence of experiencing something as deserved on equilibrium formation. Focussing on effort-symmetric equilibria they could show that for uneven games, symmetric effort equilibria, in which the interims score is immaterial to the outcome of the tournament, do not exist. However, currently it is not clear whether asymmetric equilibria exist and if so whether they favour the victory of the player ahead or behind.[2]

Gill and Stone (2010) derive predictions for asymmetric equilibria, but only for the case where chance in the game is uniformly distributed. Uniformly distributed errors are commonly used in economic models and laboratory experiments. Indeed, without further knowledge of the situation at hand it may be as well-suited as any other distribution. However, when thinking about many examples of rank order tournaments the assumption that all random events, no matter how extreme, are equally likely and that at the same time more extreme events carry probability of zero appears odd. In many tournament applications like job or sport contests the notion that extreme events can happen, but do so with a low probability, has an intuitive appeal.[3]

[1]How humans react to feedback is not yet fully understood. One potentially related idea is the concept of cognitive dissonance Festinger (1962), which proposes that anyone who holds contradictory beliefs will try to actively reduce this dissonance. Adjusting one's reference categories could be seen as one way to overcome the dissonance between the desire for a certain prize and the naturally limited resources to obtain it.

[2]In a new article Dato et al. (2015) further explore the circumstances under which symmetric equilibria arise.

[3]Stern (1991) investigates score differences in football and cannot reject that they are normally distributed.

With uniformly distributed errors Gill and Stone (2010) find that just one class of equilibria exists, in which the player ahead always spends more effort than the trailing player. This chapter makes new and very different predictions for the same setting when uncertainty is normally distributed. We find that, depending on the strength of the reference-dependence, the tournament prize and the initial unevenness three different classes of equilibria exist. Remarkably, in two of these classes the player being behind overtakes the opponent and ends up with a higher probability of winning the tournament. In tournaments where the initial unevenness is strongly favourable for one party we find a unique equilibrium, in which the leading player extends the lead by investing more effort than the player behind. However, when the game is tight and the tournament prize is large enough to motivate the trailing player to overcome the initial disadvantage, equilibria where the player behind spends much more effort than the player ahead and obtains a higher probability of winning the tournament, always exist.

In the first class of what we call Turn Around Equilibria (TAE) the agent behind turns a marginal disadvantage ex ante, a 48 percent probability of winning, into a marginal advantage with slightly more than a 53 percent chance of winning. In the second class, the turn around can be much more pronounced. Here a trailing player starting with a winning probability of say around 30 percent may turn the game into one which yields almost certain victory with the winning probability exceeding 90 percent. We show that whenever the player behind catches up on the opponent the extra effort will be sufficient to overcome, and even exceed the entire initial disadvantage. Situations where the trailing player makes up some of the disadvantage without becoming the favourite winner do not exist in equilibrium. We show that depending on parameter values, the only possible pure strategy equilibrium is one in which the disadvantaged player turns the game. Lastly, we predict that equilibria where one agent catches up without taking the lead do not exist.

The model is set up as a version of the tournament formulated by Lazear and Rosen (1981) with the defining characteristic that the element of chance enters additively into the contest success function. Since we introduce reference-dependent preferences we use the notion of choice acclimating personal equilibrium, that was introduced by Kőszegi and Rabin (2007), in which the reference point is endogenous to maximisation process as a solution concept. This means that agents take into account that their effort choice affects their reference utility, i.e. that a high effort level makes winning more likely and, hence, increases the reference point.

We contribute to a growing literature taking an interest in dynamic and uneven tournaments. Contributions like Gill and Stone (2010) discuss that in a dynamic setting agents have time to emotionally react to events and deviate from standard rationality. How emotions within a sports game can impact the motivation and ability of players psychologically is described by Lazarus (2000). Klaassen and Magnus (2001) support this notion empirically by showing, with a large data set of tennis matches, that points in tennis are not individually and identically distributed. Gill and Prowse (2012) confirm experimentally the key economic concept of strategy functions where the effort of one agent crowds out the effort of the competitor. They introduce a dynamic frame by letting subjects choose their effort sequentially providing complete information about the choice of the first subject. Ederer (2010) studies asymmetric equilibria as a result of asymmetrically distributed ability between two agents. In his model interim feedback gives competitors the chance to update their beliefs about their opponent's ability. This leaves the agent who is ahead in the game more confident of the value of his own effort investment and results in relatively greater effort provision from the leading player.

Our model provides a theoretical explanation for the existence of turn arounds. Our results can explain the puzzling empirical evidence presented by Berger and Pope (2011), who investigate data from $18,060$ American basketball games and find that teams which are slightly behind at half time have a significantly higher probability to win the game. As basketball is a complex sport it could be argued, for example, that their results are not directly linked to effort investment. However, they consolidate their finding by running an experiment in a controlled laboratory environment where participants had to compete in a real effort task that involved fast clicking and were told an intermediate score at half time. Those who were slightly behind at half time showed a marked increase in clicks in the second half compared to those who were ahead or to the no feedback control group. Previous literature was not able to explain this pattern.[4]

2. The Model

The model studies a contest with two players $j \in \{A, B\}$ who exert effort e^j. The initial unevenness is given by δ_1 which represents an advantage for Player A when positive and vice versa. The parameter δ_1 is exogenous and observable. The unobservable random noise parameter ϵ is not affected by effort and follows a normal distribution with mean 0 and variance σ^2.[5] The initial unevenness δ_1, the shock term ϵ and the two choice variables e^A and e^B constitute the final outcome which is determined by $\delta_2 = \delta_1 + e^A - e^B + \epsilon$.

The prize received by Player j is given by z_j. If the player wins the tournament the player receives the winner prize w, while the loser prize is normalised to zero. Therefore: Player A wins if $\delta_2 > 0$ and receives $z_A = w$, while Player B obtains $z_B = 0$ and vice versa. In this setting, the probability that Player A wins the contest equals $Prob(\delta_2 > 0)$ which implies $Prob(\epsilon > -\delta_1 + e^B - e^A)$. Using the fact that ϵ is normally distributed we can rewrite this as $1 - F(-\delta_1 + e^B - e^A)$ where $F(\cdot)$ is the cumulative distribution function of the normal distribution. From the symmetry of the normal

[4] In Ederer (2010) and Gill and Stone (2010), for example, the only type of asymmetric equilibrium is one, where the leading player exerts more effort than the disadvantaged opponent.

[5] To ensure pure-strategy equilibria the variance has to be sufficiently large as described in Lazear and Rosen (1981).

distribution it follows that:

$$Prob_{\{\text{A wins}\}} = Prob(\delta_2 > 0) = 1 - F(-\delta_1 + e^B - e^A) = F(\delta_1 + \Delta e)$$
where $\Delta e = e^A - e^B$

2.1 Utility with reference-dependent preferences

In the first part of our analysis we make no assumptions about how the reference points $\{r^A, r^B\}$ are formed. Instead, we study the additional incentives reference-dependence imposes on the players. Afterwards, we investigate how a reference point contributes to determine the tournament winner assuming that it is formed endogenously as described by Köszegi and Rabin (2006).

A player's utility under a reference point r^j is given by:

$$U^j = v(z_j) + m(z_j|r^j) - c(e^j) \text{ where } m(z_j|r^j) = \begin{cases} \eta(w - r^j) & \text{if j wins} \\ \eta(1+\theta)(0 - r^j) & \text{if j loses} \end{cases}$$

and $v(z_j) = z_j$, $c(e^j) = \frac{1}{2}(e^j)^2$, $\eta \geq 0$, $\theta \geq 0$

We assume $r^j \in [0, w]$ as the reference point for the tournament prize should give us a value between the lowest possible outcome and the highest possible outcome of the tournament. The utility is composed of a convenient consumption part v, for which a linear specification is used, the cost of effort provision $c(e^j)$ and a reference-dependent term. The weight of the reference utility is calibrated by η, such that setting $\eta = 0$ returns the model without reference-dependence. The parameter θ introduces loss aversion. It represents the difference between the disutility of falling short of the reference point and the utility of exceeding it by one unit. We assume that losses loom larger than gains and consequently take θ to be positive. We use quadratic costs of effort for simplicity.

Both players choose an effort level to maximise their expected utility given the unevenness δ_1. Player A maximises expected utility with respect to e^A. Consequently, the optimisation problem for Player A can be written as:

$$\max_{e^A} F(\delta_1 + \Delta e)(w + \eta(w - r^A)) + (1 - F(\delta_1 + \Delta e))(-\eta(1+\theta)r^A) - c(e^A).$$

The first term $F(\delta_1 + \Delta e)(w + \eta(w - r^A))$ represents the utility in case the agent wins the tournament. It is added to the utility of losing $(1 - F(\delta_1 + \Delta e))(-\eta(1+\theta)r^A)$ and the costs of effort which have to be paid independent of the outcome. We define P^j as winning-probability of player j, i.e. $P^A = F(\delta_1 + \Delta e)$ and $P^B = 1 - F(\delta_1 + \Delta e)$. The contribution of reference-dependent utility lies in adding the term below to the standard objective function $wF(\delta_1 + \Delta e) - c(e^A)$:

$$R^j := \eta\Big(P^j w - r^j\big[1 + \theta\big(1 - P^j\big)\big]\Big)$$

Except for the potentially different reference points and the individual winning probability the term R^j is the same for both players. While the sign of R^j depends on the actual parameter values, it becomes apparent that a greater reference point reduces the agents' utility. This is intuitive as a higher reference point renders a victory less sweat, but a defeat all the more bitter. Moreover, reference-dependence contributes an incentive effect which is given by

$$\frac{\partial R^A}{\partial e^A} = \eta\Big(f(\delta_1 + \Delta e)(w + r^A\theta) - \frac{\partial r^A}{\partial e^A}\big[1 + \theta(1 - P^A)\big]\Big).$$

The expression reveals the delicate nature of the effect which may take different sizes locally over the decision space. The first term $\eta f(\delta_1 + \Delta e)(w + r^A\theta)$ adds a positive incentive, that is caused by an increase of the effective prize spread. Since Lazear and Rosen (1981) it has been known that when there are no participation constraints an agent's effort decision is not affected by the absolute level of prizes, but by the spread between the winner and loser prize. reference-dependence increases the effective prize spread, making the valuation of both tournament outcomes more extreme. The strength of its impact, however, depends on the reference point r^A which may take different values for different $\{e^A, e^B, \delta_1\}$. The second term reduces to 0 in case of an exogenous reference point as the derivative $\frac{\partial r^A}{\partial e^A}$ remains 0.

2.2 Endogenous Reference Points

In the following we endogenise the reference points and employ the choice-acclimating personal equilibrium concept of Köszegi and Rabin (2006) to derive the player's first and second order conditions. After establishing a necessary and sufficient condition for the interiority of all solutions in Lemma 1, we show in Lemma 2 and Lemma 3 that both first and second order conditions can be reduced to one equivalent expression. We proceed to define the three classes of equilibria and derive conditions for their existence in Proposition 1 to Proposition 3. In Proposition 4 we give conditions for the uniqueness of a Turn Around Equilibrium. Finally, in Proposition 5 we discuss a fourth class of equilibria and interpret our results.

Modelling the reference point formation explicitly makes the precise effect of reference-dependence tractable. We assume expectation based reference points, but remain agnostic about whether expectations are formed as in the reference-dependence theory of Köszegi and Rabin (2006) or as in disappointment aversion theory developed by Bell (1985) and Loomes and Sugden (1986). Additionally, we will allow the reference point to adjust in the process. As solution concept we use choice-acclimating personal equilibria (CPE) that are defined "as a decision that maximises expected utility given that it determines both the reference lottery and the outcome lottery" (p.1049, Kőszegi and Rabin (2007)). In consequence, the reference points are taken to be the endogenous winning probability of each player multiplied by the winner prize, which constitutes the expected gain of each player. Explicitly, the reference points are modelled as $r^A = F(\Delta e + \delta_1)w$ for Player A and $r^B = (1 - F(\Delta e + \delta_1))w$ for Player B.[6] The explicit reference point enables us to rewrite the contribution term R^A for both players to $-w\eta\theta F(\Delta e + \delta_1)(1 - F(\Delta e + \delta_1))$. The negative sign shows that each player has an incentive to minimise this term. For player A this results in the following incentive effect[7]:

$$\frac{\partial R^A}{\partial e^A} = w\eta\theta f(\delta_1 + \Delta e)(2F(\delta_1 + \Delta e) - 1)$$

The derivative above is positive if $\delta_1 + \Delta e > 0$ and negative if $\delta_1 + \Delta e < 0$. The absolute value of R^A is highest for close games when $\delta_1 + \Delta e$ is zero and falls steadily when the game gets less tight. In other words, players have an incentive to flee the middle and avoid the uncertainty associated with close games, which has also been described by Gill and Stone (2010). Note that the incentive does not point the player into a particular direction. Whether the player "gets ahead" or "falls behind" is not important. Evenness at the end of the period is unattractive for agents with reference points since it jointly maximises the size of the disutility from falling short of the reference point weighted by the probability of its occurrence. With normally distributed chance in the game, this opens up the possibility for multiple equilibria.

To understand this last point better consider Figure 1.1 which sketches both player's marginal costs and benefits.[8] In the upper graph, which depicts the standard Lazear and Rosen (1981) tournament without reference-dependence, both player's marginal benefit curves coincide with the equilibrium being reached at their peak. The effect of reference-dependence in the lower graph of Figure 1.1 is to steepen and drive apart both player's marginal benefits. The peak of the marginal benefits of both players is now located in the area where they themselves are more likely to win. Intuitively, both players benefit most from their effort when they can use it not only to increase their own winning probability, but also to decrease the uncertainty of the game. Here, without further asymmetries (i.e. $\delta_1 \neq 0$) the same symmetric equilibrium continues to exist.

This can also be seen in two top panels of Figure 1.2 which plots both players' best response functions along with a 45 degree line for the given parameter values $\eta = 1, \theta = 1, \delta_1 = 0.2, \sigma = 2$ and $w = 3\pi$. Moving from the top panel to the middle one with reference-dependence the symmetric equilibrium is preserved. However, we can now see that also two other potential asymmetric equilibria on either side do exist. Again both players peak best response effort lies on the side of the 45 degree line where they are more likely to win. When we introduce asymmetry in favour of Player A (i.e. $\delta_1 > 0$) we can see that A's peak best response effort moves towards the 45 degree line while Player B's moves away from it. From the intersections of the two functions we can thus identify three potential equilibrium candidates, two of whom lie above the 45

[6] Like Gill and Stone (2010) we do not model a reference point in the effort domain. We believe that further conceptual work on what a reference point in the effort domain could be is interesting and could yield a valuable addition to this and other models. Yet with all its psychological and technical implications it exceeds the scope of this chapter.

[7] The corresponding term for Player B is $\frac{\partial R}{\partial e^B} = w\eta\theta f(\delta_1 + \Delta e)(1 - 2F(\delta_1 + \Delta e))$.

[8] The marginal benefits are given by $MB^A = MB^B = w * f(\delta_1 + \Delta e)$.

degree line which implies that the disadvantaged player behind spends more effort than the advantaged player.The best response functions have the simple structure:

$$e^A = wf(\delta_1 + \Delta e)\Big[1 + \eta\theta\Big(2F(\delta_1 + \Delta e) - 1\Big] = wf(x)\Big[1 + \gamma G(x)\Big]$$
$$e^B = wf(\delta_1 + \Delta e)\Big[1 - \eta\theta\Big(2F(\delta_1 + \Delta e) - 1)\Big] = wf(x)\Big[1 - \gamma G(x)\Big]$$

We define $x = \delta_1 + \Delta e$, $\gamma = \eta\theta$ and $G(x) = 2F(x) - 1$. The variable x, thus, represents the state of the game just before the random shock ϵ is realised. Since the two conditions for e^A and e^B must be fulfilled in equilibrium they provide information about when equilibria are interior, i.e. when both agents provide strictly positive effort. From $wf(x) > 0$ we know that there is an interior solution whenever $(1 + \gamma G(x))$ and $(1 - \gamma G(x))$ are both strictly greater than zero. Small rearrangement implies that both conditions are fulfilled whenever $\gamma < |\frac{1}{G(x)}|$. Since the set of possible values of $|G(x)|$ which is bounded above by one,[9] a simple corollary is that for $\gamma \leq 1$ the condition is fulfilled and the corresponding equilibrium must be interior. This leads to the first lemma.

[9] $|G(x)| = |2F(x) - 1|$ converges to 1, since the cdf of the normal distribution converges to 0 for $x \to -\infty$ and to 1 for $x \to \infty$.

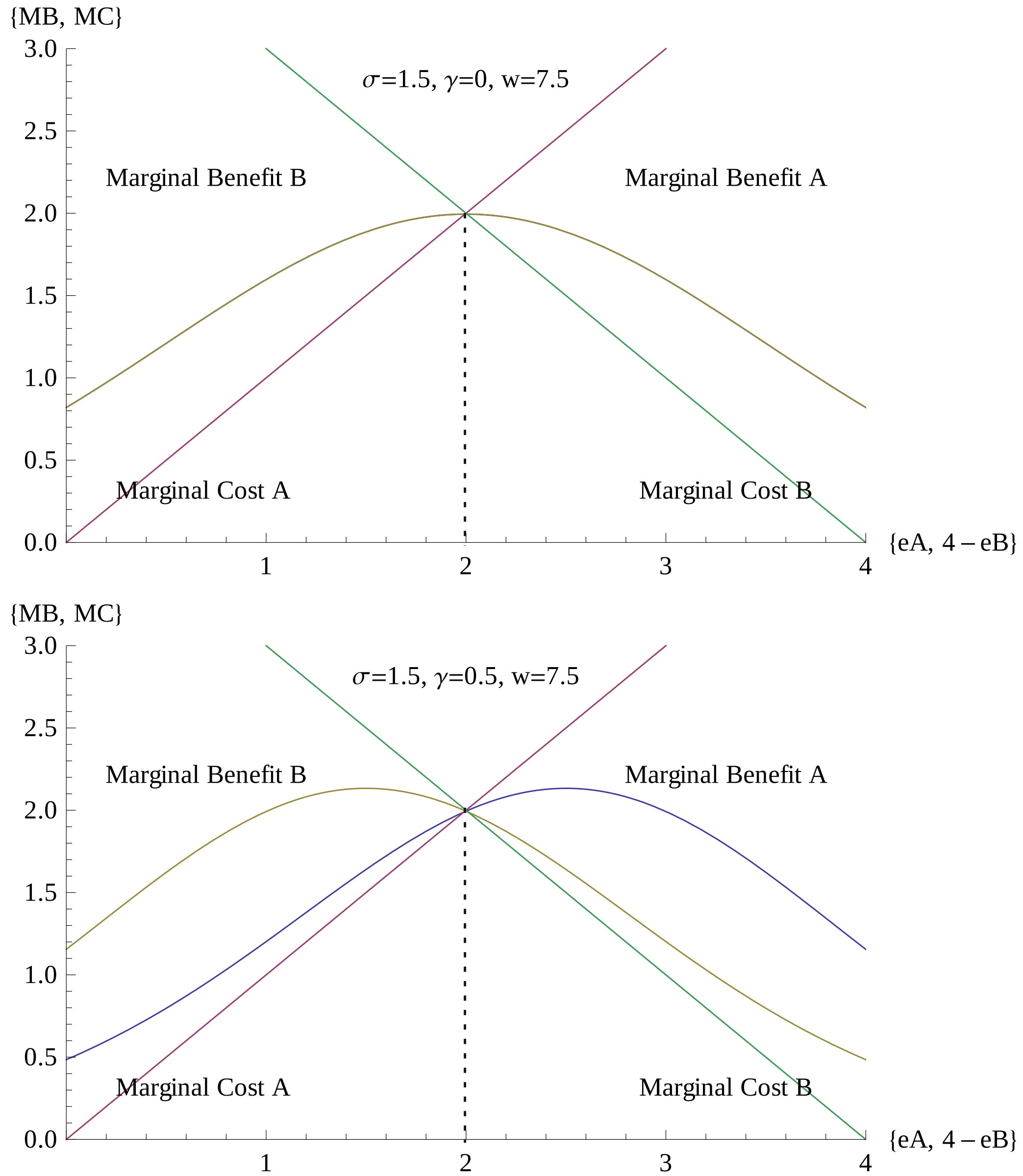


Figure 1.1: The Figure shows the **Marginal Costs and Benefits** of both players. Player B's effort increases from left to right. In the top panel without reference-dependence ($\gamma = 0$) marginal benefits of both players are identical. Introducing reference-dependence changes that. The Marginal Benefit curves are now only equal at the equilibrium effort levels, which without initial asymmetry are still symmetic (they are equal to two in this case).

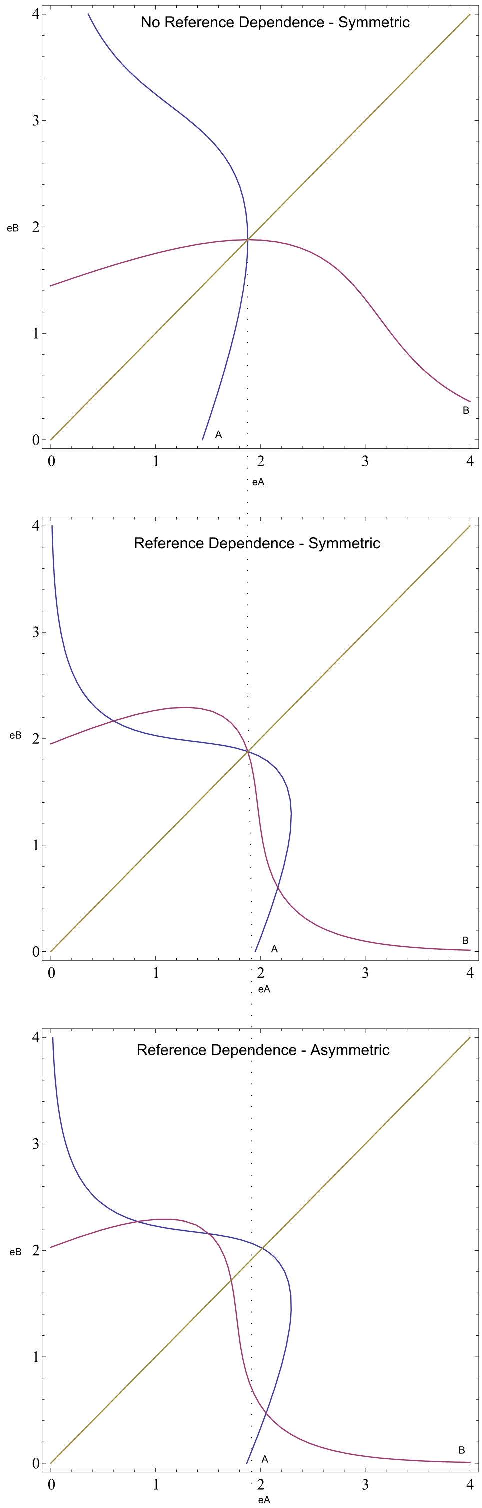


Figure 1.2: Effort combinations which fulfil each players First Order Condition. All intersections are potential equilibrium candidates. Below the 45 line Player A exerts greater effort, above it Player B exerts more. The top panel assumes $w = 3\pi$ and $\sigma = 2$. In the middle panel the reference-dependence parameter $\gamma = 1$. In the lower panel Player A is additionally given an advantage of $\delta_1 = 0.2$.

Lemma 1. *An equilibrium is interior if* $\gamma < |\frac{1}{G(x)}|$. *Therefore all equilibria are interior whenever* $\gamma \leq 1$.

All lemmas and propositions are proven formally in Appendix 1. The term $\frac{1}{G(x)}$ is always defined as $G(x) \neq 0$ for all x that describe equilibria. To ensure that all equilibria are interior, we will assume $\gamma < |\frac{1}{G(x)}|$. This is not a restrictive assumption as for any x, $|G(x)|$ is always between zero at the origin and one as x becomes arbitrarily small or large. Hence, all moderate forms of loss aversion where $\gamma \leq 1$ are covered as well as many stronger versions depending on the degree of the state of the game x.

For simplification we proceed by combining both first order conditions as well as both second order conditions to obtain two new functions we term candidate and maximum condition function.

Lemma 2. *The system of first order conditions can be expressed as the candidate function* $\delta_1 = x - 2w\gamma f(x)G(x)$. *All combinations of* $\{e^A, e^B, \delta_1\}$ *which fulfil this equation are referred to as candidate points.*

Lemma 3. *If* x *fulfils the maximum condition* $0 < \frac{1}{wf(x)} - \gamma(2f(x) - \frac{x}{\sigma^2}G(x)) - \frac{|x|}{\sigma^2}$ *then at the corresponding vector* $\{e^A, e^B, \delta_1\}$ *both second order conditions are fulfilled.*

We call the function that describes the border of the inequality given in Lemma 3, $\frac{|x|}{\sigma^2} = \frac{1}{wf(x)} - \gamma(2f(x) - \frac{x}{\sigma^2}G(x))$, the maximum condition function. We can plot the candidate function and the maximum condition functions into one system as illustrated in Figure 1.3. Both graphs depend on x which is given on the horizontal axis. The candidate function is depicted by the blue curve and every point on it represents an equilibrium in case the second order conditions are fulfilled for the same x-value. The second order conditions are jointly represented by the maximum condition function in red. In case this function has a positive value for a certain x both second order conditions are fulfilled. Remember that x was initially defined as $e^A - e^B + \delta_1$. For this reason we know that Player A has a higher winning probability for positive and Player B for negative x, but we also know that Player B must have chosen a significantly higher effort than A in case of a negative x-value and $\delta_1 > 0$. We can now read Figure 1.3 in a convenient way. The vertical axis is also a scale for δ_1; hence we can choose a particular initial unevenness δ_1, take the corresponding x-value from the candidate function and evaluate it using the maximum condition function. When it is positive at that point, the combination of x, δ_1 must be an equilibrium. Lemma 2 shows that with the help of the first order conditions the unique pair of $\{e^A, e^B\}$ can be retrieved.

3. Multiple Equilibria

In the following, we leave most technical details to the appendix but provide some intuition verbally and graphically for why asymmetric equilibria exist. We will assume throughout that $\delta_1 > 0$, i.e. Player A is ahead and benefits from the initial unevenness.[10]

[10]Due to the symmetry of the problem all results also apply in case Player B is ahead.

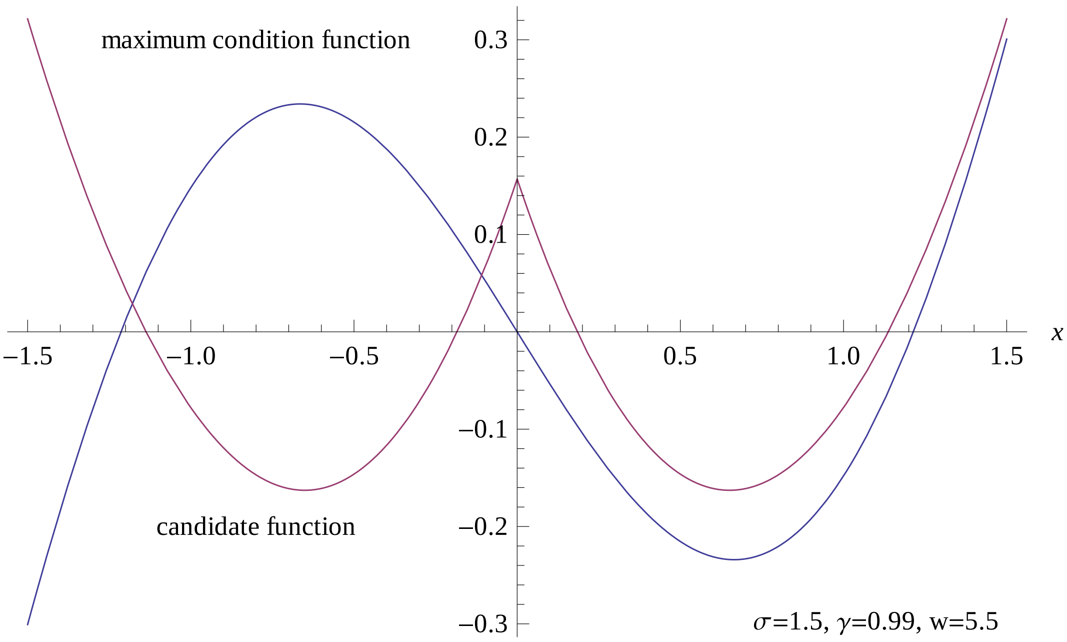


Figure 1.3: maximum condition and candidate function with positive intersections

3.1 Confirming Asymmetric Equilibria

As explained earlier, introducing reference-dependence renders the middle ground, i.e. when $\delta_1+\Delta e$ is close to zero, unattractive to both players. Without reference-dependence Player B and Player A would always choose the same level of effort since both players have the same marginal costs and benefits are also the same due to the symmetry of the normal distribution's density. Therefore, the player ahead always maintains the same advantage in the relative winning probability. An extra incentive rewarding more unequal winning probabilities like reference-dependent preferences, in this setting, would just widen the already existing probability spread. To achieve this the player ahead needs to put in relatively more effort than the player behind. Thus, when reference-dependence increases the effective prize spread, both players will invest more effort, but the player ahead claims a larger share of the extra contribution. In the following this is referred to as Confirming Asymmetric Equilibrium.

Definition 1. *Confirming Asymmetric Equilibrium (CAE)*
A Confirming Asymmetric Equilibrium is an equilibrium where the advantaged player spends more effort than the other player.

Figure 1.4 shows CAEs explicitly when Player A being initially advantaged. When δ_1 is positive, Player A ends up with a higher winning probability and the CAEs are located in the upper right quadrant of the coordinate system. We see that each $\{x, \delta_1\}$ combination, for which the candidate function lies in this quadrant, is a CAE in case the maximum condition function is positive for this x-value as well. In the depicted case there exists a CAE for all values of δ_1. However, this does not need to be the case.[11] While there will always be candidate CAEs for all values of δ_1, the maximum condition is not necessarily fulfilled. We prove the following proposition:

Proposition 1. *For δ_1 large enough there always exists one Confirming Asymmetric Equilibrium (CAE) that is a unique equilibrium.*

For tournaments without reference-dependence, Lazear and Rosen (1981) show that symmetric equilibria do not necessarily exist and depend on the wage-schedule as well as the degree of uncertainty inherent to the tournament.[12] Proposition 1 shows that strong unevenness at the start of the tournament curbs the first point. For sufficient uncertainty, it eventually guarantees the existence of a pure strategy equilibrium. While equilibria in which a leading player extends the lead are not uncommon in the literature, we now introduce two further types of equilibria.

3.2 Type One Turn Around Equilibria

Reference-dependence as described above introduces an incentive to "flee the middle", but this can be done in yet another way. As an alternative to the CAE the player behind may decide to outspend the leading player. Such an equilibrium is called Turn Around Equilibrium.

Definition 2. *Turn Around Equilibrium (TAE)*
A Turn Around Equilibrium is an equilibrium where the initially disadvantaged player spends so much more effort that this player has higher probability to win the game than the opposing player.

Definition 3. *Type one Turn Around Equilibria (TAE1)*
Type one Turn Around Equilibria (TAE1) are TAEs that exist over an interval for $\{e^A - e^B + \delta_1\}$ that is open and bounded above by 0.

Suppose that Player B is initially disadvantaged and considers investing more effort than Player A. For Player A this could be an equilibrium since the player is willing to settle at a point where the marginal benefits together with the marginal reduction of the reference-dependence cost meets the marginal costs. The key to understanding this intuition is to see that the incentive effect of reference-dependence changes sign at $x = \delta_1 + \Delta e = 0$. When Player A backs off, the incentive effect $\frac{\partial R^A}{\partial x} = w\gamma f(x)(2F(x)-1)$ flips and the player will accept an equilibrium where the lower marginal benefit minus the reference cost of increasing effort equals the marginal cost. This intuition is intact as long as the unevenness is rather small and the wage level is high enough to motivate Player B to overcome the initial disadvantage, but not so high as to make it intolerable for Player A to back off.

[11] It can happen, that the candidate function produces combinations of x and δ_1 at which the maximum condition function is still negative. In consequence CAEs are guaranteed for great x and δ_1, but given parameter values they may not exist for the whole range of δ_1.

[12] Imagine there was no uncertainty in the tournament. Then, each player would try to marginally overbid the opponent and no equilibrium in pure strategies would exist. Besides there would of course exist a symmetric mixed strategy equilibrium.

This leads to the following proposition:

Proposition 2.

i) If $w > \frac{1}{4\gamma f(0)^2}$ *and* $w < \frac{1}{2\gamma f(0)^2}$*, a type one Turn Around Equilibrium (TAE1) always exists.*

ii) TAE1s are always interior.

The condition provided formulates a parameter range for the exogenous tournament prize w and the reference-dependence variables $\gamma = \eta\theta$. Under the conditions of Proposition 2 no symmetric equilibria exist.[13]

In case of an initial disadvantage for Player B, TAEs are defined as equilibrium points where Player B spends sufficiently more effort than Player A to become the favourite for winning the tournament. In consequence, TAEs for positive δ_1 can be found in the upper left quadrant of Figure 1.4. When $\delta_1 > 0$, as we assume throughout without loss of generality, TAE1s are equilibria located in the negative x-domain bordering zero. Depending on the parameter values of w and γ these equilibria exist since the curvature of the candidate function is strong enough to reach into the positive range of δ_1 while the maximum condition function is still fulfilled for those x-values as can be seen in Figure 1.4.[14]

The TAE1s that follow from Proposition 2 occur only for tight games and the magnitude of the turn around is generally small. For illustration consider the example where the tournament prize $w = 5.5$, chance has standard deviation of $\sigma = 1.5$, the experience of losses is twice as strong as that of gains ($\theta = 1$), and reference utility is weighted equally strongly as consumption utility $\eta = 1$ such that $\gamma = 1$. Then, in a game where Player A is ahead by 0.033 standard deviations initially, Player B can overtake in equilibrium turning around a disadvantage of 0.06 standard deviations into a lead of roughly 0.06 standard deviations. In terms of winning probabilities Player B starts the tournament with a chance of winning of about 48.6 percent and ends it with about 52.4 percent. So the leading player has a 3.8 percentage points lower probability to win the game in the end. This is similar to the empirical evidence of Berger and Pope (2011) who conduct an experiment where participants compete against each other over two periods in a real effort task. They find that their subjects inserted most effort in the second period when being told that they were slightly behind their opponent and were more likely to win as a result. Berger and Pope (2011) also find a significant increase of winning probability for basketball teams that are slightly behind before the break compared to the leading team. Instead of having a lower probability to win, the

[13] This is also shown in Gill and Stone (2010).

[14] To verify that TAE1s are not only pathological cases, but appear over a range of x, we estimate an interval of x values over wich TAE1s exist. For this we use a linear approximation of the maximum condition function. Because of the convexity of the maximum condition function we can evaluate a conservative estimation guarantees us TAE1 for $x \in \left[\frac{(w\sigma\gamma - 2\sigma^3\pi)}{\sqrt{2\pi}w}, 0\right)$. The maximum condition function is convex for the whole range of w used in this proposition. The proof is given in Lemma 5. The boundaries for the set are derived in the proof of Proposition 2.

team being behind by one point is more likely to win the game. In case of the NBA data the trailing team has 1.1 percentage points higher probability to win the game than the leading team. For the NCAA the result is even stronger: 5.6 percentage points. The difference in winning probability at the breakpoint is significant. Naturally, this field data result can have various explanations, one of which would be to describe it as a TAE1 under the premises of this model.

3.3 Type Two Turn Around Equilibria

While the TAE1s described above are tight in the sense that the initially disadvantaged player increases his winning probability only marginally above fifty percent, there can also be TAEs where the lagging player outspends the opponent sufficiently to increase the winning probability to much more than fifty percent.

Definition 4. *Type two Turn Around Equilibria (TAE2)*
Type two Turn Around Equilibria (TAE2) are TAEs that exist over intervals for $\{e^A - e^B + \delta_1\}$ *that are bounded above by some* $x_\delta \leq 0$.

In this second class of TAEs the leading player backs off much to benefit from the following reference point reduction. This equilibrium may also exist for greater values of w, which becomes apparent once we remember that the weight of the reference-dependence effect, $\frac{\partial R^A}{\partial e^A} = \eta f(\delta_1 + \Delta e) w(\theta(2F(\delta_1 + \Delta e) - 1))$, increases in w. The stronger impact of reference-dependence makes it more important in the turn around case for the leading player to reduce the effort and flee the middle. As a result, even for high w, TAE2s exist.

To construct the formal criterion we will use the point where the candidate function and the maximum condition function intersect. This point is given by $x^s = \frac{(2f(x^s)^2 w\gamma - 1 - 2f(x^s)^2 G(x^s) w^2 \gamma)\sigma^2}{f(x^s) w(1 + G(x^s)\gamma - \sigma^2)}$. As x^s is exogenously determined by the parameters of the model the conditions for w and γ provided in the proposition are exogenous as well.

Proposition 3.

i) When $w \in \left(\frac{1}{4\gamma f(0)^2}, \frac{1}{2\gamma f(x^s)^2 + B}\right)$ *where* $\gamma \in \left[0.54, -\frac{1}{G(x^s)}\right)$, σ *sufficiently large and* $B = \sqrt{-\gamma f(x^s)^2 \frac{2G(x^s)(1+\gamma G(x^s))}{\sigma^2}} \geq 0$ *, a type two Turn Around Equilibrium (TAE2) in which the agent behind spends much more effort than the agent ahead exists. The parameter* x^s *determines the intersection between candidate function and maximum condition function exogenously.*

ii) If there exist TAE2s there also exist Confirming Asymmetric Equilibria (CAEs) for small δ_1.

iii) If the maximum condition function and the candidate function intersect but there are no TAE2s also CAEs for small δ_1 *do not exist.*

The conditions in Proposition 3 appear more complex than they are. Unlike Proposition 2, Proposition 3 requires a minimum strength of reference-dependence γ. If this condition is not met, it is never optimal for the leading player to back off as much as required in the TAE2. To illustrate this consider the following example: Suppose the tournament prize is $w = 10$, chance again enters with a standard deviation of $\sigma = 1.5$, experience of losses is twice as strong as that of gains ($\theta = 1$) and reference utility enters fully with $\eta = 1$ such that $\gamma = 1$. Then, TAE2 exist for any unevenness that is smaller or equal to 0.07 standard deviations. From an initial probability of winning of around 47.3 percent the lagging player in this equilibrium improves his chances to 87.8 percent. This will only be optimal for the player ahead if it is possible to benefit sufficiently from lowering the reference point and hence γ must exceed a certain value. The new condition for w has a similar spirit. While the lower bound coincides with the one in Proposition 2, the upper bound is tightened by $B \geq 0$ which added to the denominator. Again, the reason is that for large tournament prizes it is never optimal for the leading player to allow the other player to overtake. Imagine for example a student who is competing with a class mate over relative grades in a course that is not too important to both. After beating his mate in the midterms that student could still decide not to prepare much for the final exam. He knows that he will probably not come in first. Yet, that would not be too bad, because he also knows that it happened because he was not really trying and could not expect to do any better given his effort.

The definition of TAE2s includes all TAE1s, but TAE2s are potentially located further away from zero than TAE1s as illustrated in Figure 1.4. Due to the symmetry properties of the candidate and maximum condition function they can be seen as mirror images of certain CAEs. The maximum condition function is axis-symmetric whereas the candidate function is point-symmetric. Consequently, any intersection of the candidate function with the x-axes on the negative domain also exists in the positive domain and vice versa. For x-values larger than the positive root of the candidate function there are CAEs while for x-values above the negative root there exist TAE2s given that the maximum condition function to be positive. Due to axis-symmetry of the maximum condition function it returns the same value at both outer roots of the candidate function. Therefore, if the one equilibrium exist for small δ_1 the other does as well.[15]

3.4 Unique Turn Around Equilibria

The equilibria described spark questions about why the leading player may allow the other player to overtake. One conceivable explanation would be that Turn Arounds are somewhat “lazy equilibria” where the agent ahead has discovered that he greatly benefits from lowering its reference point. However, such an intuition does not truly capture the dynamics of the model. When there are three equilibria, TAE1s are the equilibria with the highest total effort investment. Only for the CAE and TAE2s large asymmetries are possible because one player benefits from lowering his reference point. Moreover, we show that for certain parameter values where the CAEs do not exist a TAE1 is the unique equilibrium.

[15] Because of continuity this is at least the case for an ϵ-ball around the root of the candidate function.

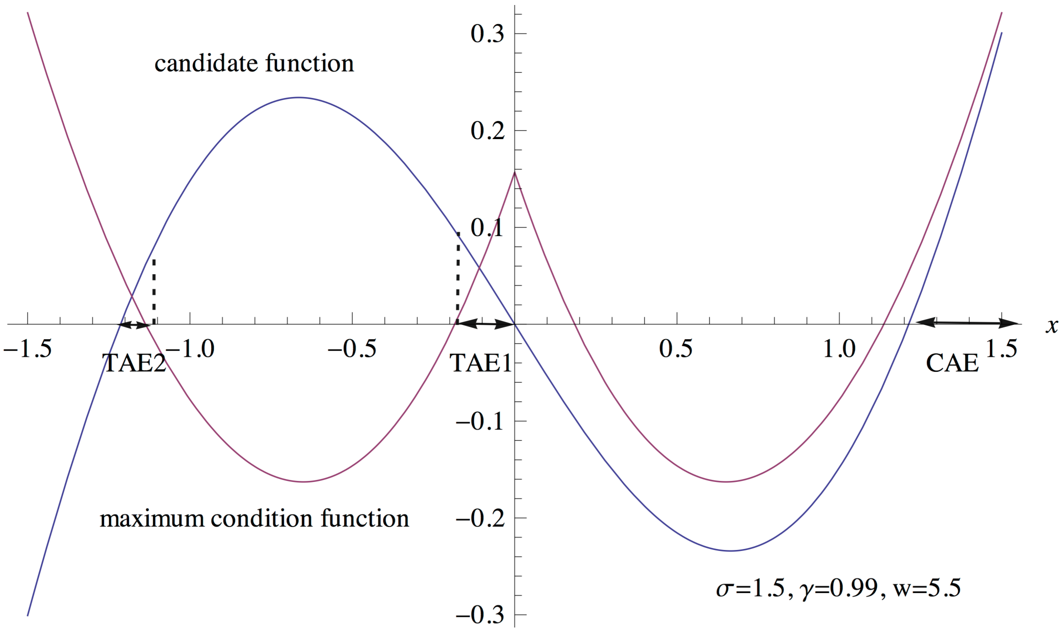


Figure 1.4: maximum condition and candidate function with all equilibria

Proposition 4. *When* $\frac{1}{4f(0)^2\gamma} < w < \frac{1}{2f(0)^2\gamma}$ *and* $\gamma \leq -\frac{f(x^s)^2 G(x^s)\pi^2}{\frac{2}{\sigma^2}+f(x^s)^2\pi(-2+G(x^s)^2\pi)}$ *then for small unevenness the unique equilibrium in pure strategies is a type one Turn Around Equilibrium (TAE1), where* x^s *exogenously determines the intersection between candidate function and maximum condition function.*

The condition for w ensures that TAE1s exist, while the condition for γ rules out the existence of TAE2s and even of CAEs for the particular interval of unevenness over which TAE1s exist. This surprising result is made possible by the missing guarantee for the existence of equilibria in Lazear and Rosen (1981) type tournaments. In a region where the second order conditions do not allow CAEs to exist, the TAE1 candidate point close to, but smaller than zero, satisfies them as illustrated in Figure 1.5.

Proposition 4 demonstrates that a TAE1 can be the only equilibrium in pure strategies. While we do not engage in equilibrium selection, this shows that at least among pure strategies there are situations where TAE1s must be played, as no other equilibria exist. Thus, we show that it does not need differences in ability or imperfect information to obtain the unambiguous prediction that a trailing player wins a tournament. Having expectation based reference-dependent preferences can be sufficient for given parameter constellations.

3.5 Catching Up Equilibria

At first glance the notion of Turn Around Equilibria maybe appears (too) strict. It would have been possible to define TAEs as all asymmetric equilibria in which the initially disadvantaged player spends more effort than the advantaged player irrespective of whether the difference is significant enough to turn the game. We call this broader

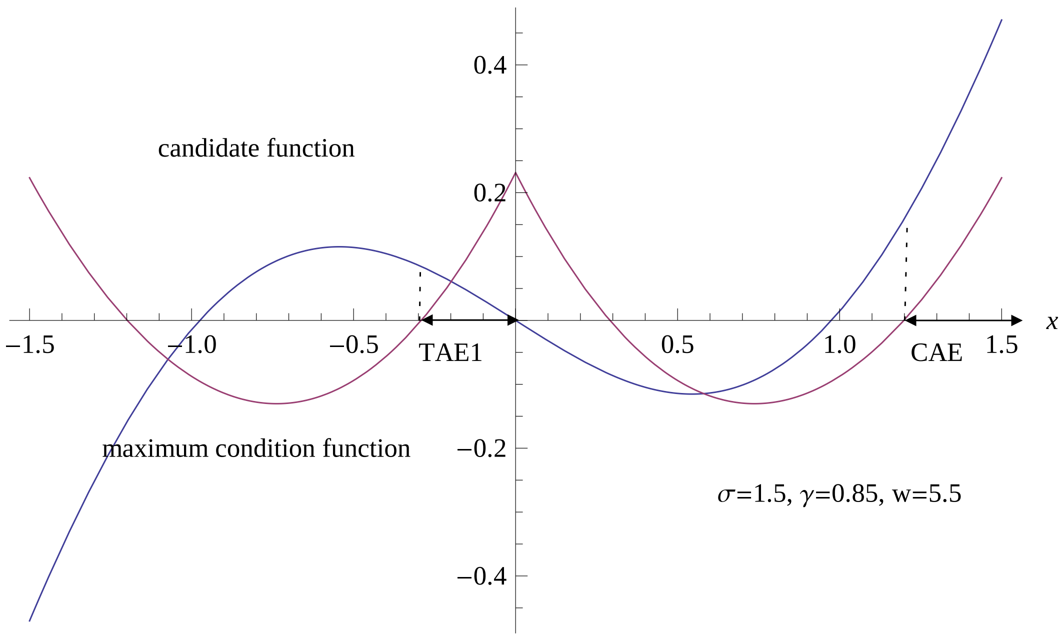


Figure 1.5: maximum condition and candidate function with only TAE1

class of equilibria Catching Up Equilibria (CUE).

Definition 5. *Catching Up Equilibrium (CUE)*
A Catching Up Equilibrium is an equilibrium where the initially disadvantaged player spends more effort than the opposing player.

From the definition it is apparent that every TAE must also be a CUE. However, we show that the converse holds as well. Every equilibrium in which the player being behind invests more effort than the opponent is also a TAE. In other words, situations where trailing player catches up a little without turning the game do not exist.

Proposition 5. *Every Catching Up Equilibrium is also a Turn Around Equilibrium.*

For an intuition consider again the equilibrium in the model without reference-dependence. Although one player is advantaged at the start of the tournament both players pick the same effort. Compared to the trailing player in this set-up, a player who tries to catch up, but not overtake, in the model with reference-dependent preferences faces greater marginal effort costs, larger marginal benefits[16] and a more negative marginal utility from reference comparison as the game becomes more even. If the agent had favoured the greater marginal benefits over the marginal effort cost, it would have chosen to insert more effort ex ante. Introducing an additional marginal cost in the form of reference-dependence cannot motivate the agent to try catching up. Only

[16] Since the probability density function of the normal distribution is single peaked at $x = 0$.

when the sign of the marginal effect of reference-dependence changes, as it is the case once one agent overtakes the other, this can be an equilibrium.

4. Conclusion

Which factors motivate players to invest in contest success is still a vibrant topic of debate. While classical tournament theory as introduced by Lazear and Rosen (1981) focuses on the higher probability of winning as benefit and the unpleasantness of effort as a cost, a large recent literature indicates that players evaluate outcomes also along certain reference points. Such reference-dependent preferences are an economically powerful concept, as they can imply that an otherwise positive event causes negative utility if the reference category was even more positive and vice versa. As a result, theoretical predictions can change drastically once a model is augmented by reference-dependence. In the context of tournaments, predicting a different winner could be considered such a change.

We add to the work of Gill and Stone (2010), who focus on symmetric equilibria when the half time score is even. For the large class of non even scores Gill and Stone (2010) show that symmetric equilibria do not exist. We find that depending on the strength of the reference-dependence, the tournament prize and the initial unevenness three different classes of equilibria exist. In games where the initial unevenness is strongly favourable for one party we find a unique equilibrium, in which the leading player invests more effort than the player behind. However, when the game is tight and the tournament prize is large enough to motivate the lagging player to overcome the initial disadvantage, Turn Around Equilibria, where the player initially behind spends much more effort than the player ahead and has a higher probability of winning the tournament, always exist.

Our results can help to explain tournament outcomes that so far have been economically puzzling as presented by Berger and Pope (2011). Our results generate further testable predictions. We find that for all equilibria where the player behind spends more effort than the opponent, this player also has a greater chance of winning the tournament. Thus, we show that equilibria, in which the player behind merely catches up with the leading player do not exist. Furthermore, we can show that under certain conditions the TAE is the unique pure strategy equilibrium. While dynamic implications of this framework were only touched upon, future research adding a further optimisation period may provide interesting insights into how the anticipation of possible TAEs affects agents' incentives in an initial period.

5. Appendix 1

In this appendix we prove all propositions and the lemmas stated in the main section. The proofs will make use of additional lemmas which are proven within the proposition where they are used first. Throughout we will assume without loss of generality that $\delta_1 > 0$, which implies that Player A is at an advantage. However, due to the symmetry of both players all results are also valid when $\delta_1 < 0$. All equilibria described assume that a solution to the tournament exists. As described by Lazear and Rosen (1981) this is always the case when precision of the random term given by $\frac{1}{\sigma}$ is sufficiently small.[17] The following proofs hold for $\sigma^2 \geq 1$.

Lemma 1. *An equilibrium is interior if* $\gamma < |\frac{1}{G(x)}|$. *Therefore all equilibria are interior whenever* $\gamma \leq 1$.

Proof. This follows directly from the first order conditions. Using $x = \Delta e + \delta$ and $\gamma = \eta\theta$ the first order conditions yield:

$$e^A = wf(x)(1 + \gamma G(x))$$
$$e^B = wf(x)(1 - \gamma G(x))$$

Since $wf(x)$ must be positive we will obtain interior solutions whenever $(1+\gamma G(x))$ and $(1 - \gamma G(x))$ are also greater than zero. This implies that both conditions are fulfilled whenever $\gamma < |\frac{1}{G(x)}|$.

The term $G(x)$ will never be 0 for any equilibrium with $\delta_1 > 0$:
Suppose: $G(x) = 0 \Rightarrow 2F(x) - 1 = 0 \Leftrightarrow F(x) = \frac{1}{2} \Leftrightarrow 0 = x = \delta_1 + \Delta e$. From the first order conditions we know that in case of $x = 0$ $e^A = e^B = wf(0) \Rightarrow \Delta e = 0$. This leads to a contradiction with $\delta_1 > 0$.[18]

Since the function $|G(x)|$ is bounded above by one and open there, a simple corollary is that for $\gamma \leq 1$ the condition is fulfilled and the corresponding equilibrium must be interior. □

Lemma 2. *The system of first order conditions can be expressed as the candidate function* $\delta_1 = x - 2w\gamma f(x)G(x)$. *All combinations of* $\{e^A, e^B, \delta_1\}$ *which fulfil this equation are referred to as candidate points.*

Proof. Using $x = \Delta e + \delta$ and $\gamma = \eta\theta$ the first order conditions yield:

$$e^A = wf(x)(1 + \gamma(2F(x) - 1))$$

$$e^B = wf(x)(1 - \gamma(2F(x) - 1))$$

Subtracting both equations and substituting $G(x) = 2F(x) - 1$ leads to:

$$e^A - e^B = 2wf(x)\gamma G(x) \tag{1.1}$$

[17] See Lazear and Rosen (1981) p.845 for more information.

[18] This also reveals that there cannot exist symmetric equilibria with initial unevenness.

Since $x - \delta_1 = e^A - e^B$ we can reformulate the above expression as:

$$\delta_1 = x - 2wf(x)\gamma G(x)$$

The variable x describes an equilibrium uniquely whereas the exact corresponding effort combination can be revealed by inserting x back into the first order conditions.

□

Lemma 3. *If x fulfils the maximum condition $0 < \frac{1}{wf(x)} - \gamma(2f(x) - \frac{x}{\sigma^2}G(x)) - \frac{|x|}{\sigma^2}$ then at the corresponding vector $\{e^A, e^B, \delta_1\}$ both second order conditions are fulfilled.*

Proof. The second order conditions for a local maximum are given by:

$$wf'(x) - wf'(x)\gamma + 2\gamma w\left[f'(x)F(x) + f(x)^2\right] - 1 < 0 \tag{1.2}$$

$$wf'(x)(-1) - wf'(x)\gamma + 2\gamma w\left[f'(x)F(x) + f(x)^2\right] - 1 < 0 \tag{1.3}$$

We use the following property of the normal distribution:

$$f'(x) = \frac{-x}{\sigma^2}f(x) \tag{1.4}$$

By substituting (1.4) into (1.2) and (1.3) we can derive new inequalities which include only the density and the distribution function of the normal distribution. Using that $G(x) = 2F(x) - 1$ we can solve for:

$$wf(x)\left\{2\gamma f(x) - \frac{x}{\sigma^2}\left[1 + \gamma G(x)\right]\right\} - 1 < 0$$

$$wf(x)\left\{2\gamma f(x) + \frac{x}{\sigma^2}\left[1 - \gamma G(x)\right]\right\} - 1 < 0$$

We will use the symmetry of the above two statements to condense their informational content into a single condition. Using $a = 2\gamma wf(x)^2$, $b = w\frac{x}{\sigma^2}f(x)$ and $c = \gamma G(x)$ we can reformulate the statements to:

$$a - b(1 + c) - 1 < 0$$

$$a + b(1 - c) - 1 < 0$$

which can be rewritten as:

$$-b < 1 - a + bc$$

$$b < 1 - a + bc$$

It is now clear that both conditions must be fulfilled whenever $|b| < 1 - a + bc$ holds. Substituting back we obtain $0 < \frac{1}{wf(x)} - \gamma(2f(x) - \frac{x}{\sigma^2}G(x)) - \frac{|x|}{\sigma^2}$.

□

5.1 Proof of Proposition 1:

Proposition 1.

For δ_1 large enough there always exists one Confirming Asymmetric Equilibrium (CAE) that is a unique equilibrium.

Proof. We first showed in Lemma 2 that we can rewrite the system of first order conditions to a simpler, but equivalent representation. Afterwards, using symmetry we derived a single bound from the second order conditions which will be necessary and sufficient to identify equilibria in Lemma 3.

We make use of the candidate function from Lemma 2 and the maximum condition derived in Lemma 3.

$$0 < \frac{1}{wf(x)} - \gamma(2f(x) - \frac{x}{\sigma^2}G(x)) - \frac{|x|}{\sigma^2}$$

$$\delta_1 = x - 2w\gamma f(x)G(x)$$

We know that $f(x)G(x) \to 0$ for $x \to \infty$ since $f(x) \to 0$ and $G(x) \to 1$. For this reason letting δ_1 go towards ∞ implies that $x \to \infty$.

As $x > 0$ we can simplify the maximum condition to:

$$1 < \frac{\sigma^2}{f(x)wx} - \frac{\sigma^2\gamma(2f(x) - \frac{x}{\sigma^2}G(x))}{x}$$

The second term on the RHS will converge to the constant γ as $x \to \infty$. The first term can be reformulated as

$$\frac{\sigma^2}{f(x)wx} = \frac{\sigma^3\sqrt{2\pi}e^{\frac{x^2}{2\sigma^2}}}{wx}$$

Following L'Hôpital's rule

$$\frac{\sigma\sqrt{2\pi}xe^{\frac{x^2}{2\sigma^2}}}{w * 1} \to \infty \qquad \Rightarrow \qquad \frac{\sigma^2}{f(x)wx} \to \infty$$

So the maximum condition will be fulfilled for sufficiently large δ_1. It is not only unique in the class of asymmetric equilibria but for all equilibria as symmetric equilibria cannot exist for $\delta_1 \neq 0$ (see Proposition 4 in Gill and Stone (2010)).

□

5.2 Proof of Proposition 2:

To prove Proposition 2 we first show that under certain conditions candidate points in the sense of Lemma 2 exist that are potentially type one Turn Around Equilibria. We proceed by showing that the maximum condition function introduced in Lemma 3 is strictly convex over some interval.

Lemma 4. *For* $w > \frac{1}{4f(0)^2\gamma}$*, there always exist a positive* δ_1 *such that its corresponding extreme points include candidate Turn Around Equilibria (i.e.* $x < 0$*).*

Proof. We show that under the condition TAE candidates (i.e. points where both player's First Order Conditions are fulfilled s.t. $x < 0$) exist for small positive values of δ_1. The inverse of the candidate function Lemma 2 would yield the equilibrium candidates for each value of δ_1. Since it is not possible to express the inverse explicitly we show that the function produces a small positive $\delta_1(x)$ when given a small negative value for x as an argument. Note that for $x < 0$ the function $G(x) < 0$ as well.

$$\delta_1(x) = x - 2w\gamma f(x)G(x) \tag{1.5}$$

The derivative of this function with respect to x yields:

$$\frac{\partial \delta_1(x)}{\partial x} = 1 - 4w\gamma f(x)^2 + \frac{x}{\sigma^2} 2w\gamma f(x)G(x)$$

When $x = 0$ and $w = \frac{1}{4f(0)^2\gamma}$ the above expression equals zero and is negative for any w larger than $\frac{1}{4f(0)^2\gamma}$. Given this negative slope at $x = 0$ the function must be positive for some small negative x.

□

Lemma 5. *The maximum condition function* $\frac{1}{wf(x)} - \gamma(2f(x) - xG(x)) - \frac{|x|}{\sigma^2}$ *is strictly convex for all* $w \in \left[\frac{1}{4f(0)^2\gamma}, \frac{1}{f(0)^2\gamma}\right]$.

Proof. The maximum condition $0 < \frac{1}{wf(x)} - \gamma(2f(x) - \frac{x}{\sigma^2}G(x)) + \frac{x}{\sigma^2}$ for $x < 0$ is convex if the second derivative is positive:

$$\frac{\sigma^2 + x^2}{2f(x)^2 w\sigma^2} + \left(3 - \frac{2x^2}{\sigma^2}\right)\gamma > 0 \tag{1.6}$$

To find the prize w for which this condition is always fulfilled we substitute $w = \frac{1}{f(0)^2\gamma * a}$ and obtain $\frac{af(0)^2}{2f(x)^2\sigma^2}(\sigma^2 + x^2) + (3 - 2\frac{x^2}{\sigma^2}) > 0$. Solving as an equality for a yields

$$a = \frac{4x^2 - 6\sigma^2}{\frac{f(0)^2}{f(x)^2}(\sigma^2 + x^2)} \tag{1.7}$$

We then find the maximum value for 1.7 using the following first order condition,

$$8\sigma^2 + x^2 - 2\frac{x^4}{\sigma^2} = 0$$

which is fulfilled whenever $x_{max} = -\frac{1}{2}\sigma\sqrt{1 + \sqrt{65}}$.[19] Then, at the maximum σ drops out and we obtain $a(x_{max}) = (9 - \sqrt{65})e^{-0.25(1+\sqrt{65})} \approx 0.97$. Consequently the second order condition must be fulfilled when $w < \frac{1}{f(0)^2\gamma}$.

[19]The Second Order Condition at x_{max} is negative and yields $\frac{(7\sqrt{65}-65)8e^{-0.25(1+\sqrt{65})}}{5\sigma^2} \approx \frac{-1.42}{\sigma^2}$.

□

Proposition 2.

i) If $w > \frac{1}{4\gamma f(0)^2}$ *and* $w < \frac{1}{2\gamma f(0)^2}$*, a type one Turn Around Equilibrium (TAE1) always exists.*

ii) TAE1s are always interior.

Proof. We showed in Lemma 4 that for certain values of w extreme point couples (for values of $\{e^A, e^B, \delta_1\}$) exist that could be type one Turn Around Equilibria (TAE1). Lemma 5 gives us the convexity of the maximum condition for certain values of w.

We will execute the proof of part $i)$ by using Lemma 4 and by showing that given $w < \frac{1}{2\gamma f(0)^2}$ the maximum condition derived in Lemma 3 is fulfilled. From Lemma 3 we know that both second order conditions will be fulfilled whenever

$$0 < \frac{1}{wf(x)} - \gamma(2f(x) - \frac{x}{\sigma^2}G(x)) - \frac{|x|}{\sigma^2} \tag{1.8}$$

Since $w < \frac{1}{2\gamma f(0)^2}$ we know that

$$\frac{2\gamma f(0)^2}{f(x)} - \gamma(2f(x) - \frac{x}{\sigma^2}G(x)) - \frac{|x|}{\sigma^2} < \frac{1}{wf(x)} - \gamma(2f(x) - \frac{x}{\sigma^2}G(x)) - \frac{|x|}{\sigma^2}$$

Now suppose $x = 0$. We obtain:

$$2\gamma f(0) - 2\gamma f(0) + 0 - 0 = 0 < \frac{1}{wf(0)} - 2\gamma f(0)$$

Therefore we know that for all $w < \frac{1}{2\gamma f(0)^2}$ the maximum condition function $\frac{1}{wf(x)} - \gamma(2f(x) - \frac{x}{\sigma^2}G(x)) - \frac{|x|}{\sigma^2}$ will take up a value greater than zero when $x = 0$. Then, it follows by the continuity of the maximum condition function that for any such w there exist some ϵ close to zero such that $0 < \frac{1}{wf(\epsilon)} - \gamma(2f(\epsilon) - \frac{\epsilon}{\sigma^2}G(\epsilon)) - \frac{|\epsilon|}{\sigma^2}$.

To obtain a conservative estimate of an interval in which the TAE1s lie, we use the strict convexity of the maximum condition function shown in Lemma 5. Now we can derive the first order Taylor approximation around $x = 0$ for $x \leq 0$ which yields:

$$T_1(0) = \left(\frac{\sigma\sqrt{2\pi}}{w} - \frac{1}{\sigma\sqrt{2\pi}}\gamma\right) + \frac{x}{\sigma^2}$$

Given the positive slope and the convexity of the maximum condition function we know, that the intersection of the approximation with the abscissa will provide a conservative lower bound for the interval. The resulting interval of x-values in which TAE1s exist can be expressed as:

$$\left[\frac{(w\sigma\gamma - 2\sigma^3\pi)}{\sqrt{2\pi}w}, 0\right)$$

As $G(x) \to 0$ for $x \to 0$ all TAE1s close to zero are interior as stated in part *ii*). □

5.3 Proof of Proposition 3:

To prove Proposition 3 we first show in Lemma 6 that there is only one convex interval for x over which candidate TAEs exist. We continue by showing in Lemma 7 that the candidate function and the maximum condition function have an intersection where $\delta_1 > 0$ or the maximum condition is always fulfilled. Then, we show in Lemma 8 that the maximum condition function cannot intersect the horizontal axis more than twice. Lastly, we establish in Lemma 9 that the maximum condition function may not have these two roots over an interval over which it is strictly greater than the candidate function.

Definition 6. *Intersection in positive/negative range*
We say that two function intersect in positive/negative range, when they return a positive/negative value at that intersection.

Lemma 6. *For $w > \frac{1}{4f(0)^2\gamma}$, the candidate function $\delta_1(x) = x - 2w\gamma f(x)G(x)$ has exactly one maximum on the domain $x \in (-\infty, 0)$. At this maximum the candidate function is positive. There exists some $x^* < 0$ such that $\delta_1(x^*) = 0$.*

Proof. We know from Lemma 4 that when $w > \frac{1}{4f(0)^2\gamma}$ Turn Around candidates with $x < 0$ and $\delta_1(x) > 0$ exist for some x close to zero. Moreover, it is easy to see that $\delta_1(x) \to -\infty$ when $x \to -\infty$ and that $\delta_1(0) = 0$. Since the candidate function is continuous there must be at least one maximum point for negative x. In the following we will show that there is only one. Consider the first and second derivative of the candidate function:

$$\frac{\partial \delta_1(x)}{\partial x} = 1 + \frac{2w\gamma x f(x)G(x)}{\sigma^2} - 4w\gamma f(x)^2 \tag{1.9}$$

$$\frac{\partial \delta_1(x)^2}{\partial^2 x} = \frac{8x\gamma w f(x)^2}{\sigma^2} + \frac{2w\gamma f(x)}{\sigma^2}(G(x) + x(2f(x) - \frac{xG(x)}{\sigma^2})) \tag{1.10}$$

Note that $|G(x)| < 0$ for $x < 0$ so that (1.10) is strictly negative and hence the first derivative is monotonously decreasing as long as $\frac{G(x)x}{\sigma^2} \leq 2f(x)$. This is fulfilled as long as $\frac{2f(x)\sigma^2}{G(x)} \leq x$ and $x < 0$. Inserting the boundary case $x = \frac{2f(x)\sigma^2}{G(x)}$ in (1.9) simplifies it to:

$$1 - 2w\gamma(-2f(x)^2 + 2f(x)^2) = 1 > 0$$

However when $x = 0$ equation (1.9) is smaller than zero if $w > \frac{1}{4f(0)^2\gamma}$. Thus the first derivative of the candidate function is below zero for $x = 0$ and greater than zero when $x = \frac{2f(x)\sigma^2}{G(x)}$ and it is monotonously decreasing over the interval $[\frac{2f(x)\sigma^2}{G(x)}, 0)$. Thus, the first derivative intersects the abscissa exactly once over that interval. Furthermore, when $x < \frac{2f(x)\sigma^2}{G(x)}$ condition (1.9) is always positive and therefore does not have another root for negative x. □

Lemma 7. *When* $w \in \left(\frac{1}{4\gamma f(0)^2}, \frac{1}{\gamma f(x^s)^2+\sqrt{\gamma f(x^s)^2(\gamma f(x^s)^2-\frac{2G(x^s)(1+\gamma G(x^s))}{\sigma^2})}}\right)$,
$\gamma \in \left[0.54, -\frac{1}{G(x^s)}\right)$ *and* σ *large enough*

- *either the maximum condition function and the candidate function intersect and do so for* $x < 0$ *when* $\delta_1 > 0$ *only*
- *or in case of no intersection the maximum condition is fulfilled for all* x *where the candidate function has positive values.*

Proof. To derive the conditions for when the intersection is within positive range (as illustrated in Figure 1.3) we begin by setting both functions equal. The intersection point is endogenously described by a value for x that is $x^s = \frac{(2f(x^s)^2w\gamma-1-2f(x^s)^2G(x^s)w^2\gamma)\sigma^2}{f(x^s)w(1+G(x^s)\gamma-\sigma^2)}$ and is used as an argument for the maximum condition which, then, yields $0 < -\frac{2f(x^s)^2w\gamma(G(x^s)w+G(x^s)^2w\gamma-\sigma^2)+\sigma^2}{f(x^s)w(1+G(x^s)\gamma-\sigma^2)}$. Using $\sigma^2 \geq 1$ we can derive the following condition. The latter expression is larger than zero whenever either of the following hold:

$$w < \frac{1}{\gamma f(x^s)^2 - \sqrt{\gamma f(x^s)^2(\gamma f(x^s)^2 - \frac{2G(x^s)(1+\gamma G(x^s))}{\sigma^2})}} \tag{1.11}$$

$$w < \frac{1}{\gamma f(x^s)^2 + \sqrt{\gamma f(x^s)^2(\gamma f(x^s)^2 - \frac{2G(x^s)(1+\gamma G(x^s))}{\sigma^2})}} \tag{1.12}$$

To ensure that the equilibrium is interior we assume $\gamma < -\frac{1}{G(x)}$. When $\gamma < -\frac{1}{G(x)}$, (1.11) is always negative and is therefore neglected. Instead we employ (1.12) as an upper bound. To ensure that the lower bound $w > \frac{1}{4\gamma f(0)^2}$ is below (1.12) another restriction for γ is required which is obtained by solving the following for γ:

$$\frac{1}{4\gamma f(0)^2} < \frac{1}{\gamma f(x^s)^2 + \sqrt{\gamma f(x^s)^2(\gamma f(x^s)^2 - \frac{2G(x^s)(1+\gamma G(x^s))}{\sigma^2}}}$$

This can be rearranged as condition for γ:

$$\gamma > -\frac{f(x^s)^2G(x^s)\pi^2}{\frac{2}{\sigma^2} + f(x^s)^2\pi(-2+G(x^s)^2\pi)} \tag{1.13}$$

This expression appears to be complicated and restrictive. However, it can be simplified at little cost in terms of accuracy. Using that $G(x) < 0$ for negative x and that $f(x)^2 < \frac{1}{2\pi\sigma^2}$, one can quickly see that the denominator will always be larger than one. The numerator contains $G(x)$ which equals $2F(x) - 1 = Erf(\frac{x}{\sqrt{2}\sigma}) = \frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{\sqrt{2}\sigma}} e^{-t^2}\mathrm{d}t$. It must hold that the actual area underneath the integrated function is smaller than the area of the rectangle formed by the global maximum of the function over the x-interval. The largest value e^{-t^2} may assume is one. Thus, it holds for negative x that $-G(x) = -Erf(\frac{x}{\sqrt{2}\sigma}) \leq -\frac{2}{\sqrt{2\pi}\sigma}x * 1$. For the entire numerator this implies that $-f(x)^2G(x)\pi^2 \leq -\frac{\sqrt{\pi}}{\sqrt{2}\sigma}xe^{-\frac{x^2}{\sigma^2}}$, the maximum of which is at $x = -\frac{\sigma}{\sqrt{2}}$. Hence, the numerator will not exceed $\frac{\sqrt{\pi}}{\sqrt{4}}e^{-\frac{1}{2}} \approx 0.53$ and whenever $\gamma \geq 0.54$, condition (1.13) will also

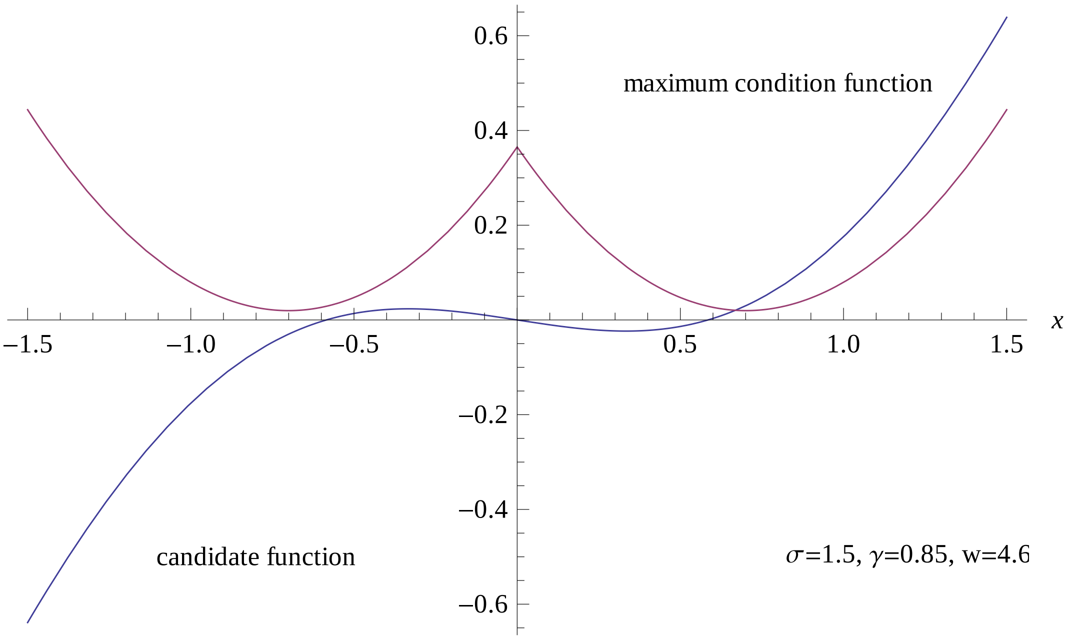


Figure 1.6: maximum condition and candidate function having no intersection points in the negative domain

be satisfied. When both conditions are fulfilled any intersection between the maximum condition and the candidate function occurs in positive range.

Suppose no intersection between the candidate function and the maximum condition function and hence no x^s exists (as illustrated in Figure 1.6). For sufficiently small x we know that the maximum condition function is always positive while the candidate function is strictly negative. Without an intersection the continuity of both functions implies that the maximum condition function lies above the candidate function for all $x < 0$. However, when $w > \frac{1}{4f(0)^2\gamma}$ it is known from Lemma 4 that there are always values for $x < 0$ where the candidate function is positive. Since the maximum condition function must return greater values than the candidate function it also must be positive. □

Lemma 8. *The maximum condition function has no more than two roots when* $x < 0$ *and* $w < \frac{1}{2f(x)^2\gamma}$.

Proof. Setting the maximum condition function equal to zero and solving for x yields $x^R = \frac{\sigma^2(2w\gamma f(x)^2-1)}{wf(x)(1+\gamma G(x))} = x^R(x)$. This equation must be fullfilled for every root of the maximum condition function. We show that the maximum condition function has at most two roots by showing that this equation has at most two solutions for $x < 0$. For this we demonstrate in the remainder of the proof that the function $x^R(x) = \frac{\sigma^2(2w\gamma f(x)^2-1)}{wf(x)(1+\gamma G(x))}$ is strictly concave and can, thus, have at most one maximum. To understand why this implies the statement in the lemma, consider the following: We want to know for how many x the equation $x^R = x^R(x)$ can be fulfilled. We also know that x^R (the left-hand-side of the equation) is a straight line with slope one. If we now knew that $x^R(x)$ was strictly concave, we would know that it cannot intersect the straight line x^R more than twice (and hence that the maximum condition function may not have more than two roots). Thus, in the remainder of the proof we show that the second derivative of $x^R(x)$ ist strictly smaller than zero for $x < 0$. The second derivative of $x^R(x)$ is given by:[20]

$$\begin{aligned}\frac{\partial x^R(x)^2}{\partial^2 x} =& \frac{8\gamma^2\sigma^2 f(x)(2w\gamma f(x)^2 - 1) + 2x\gamma(1+\gamma G(x))(6w\gamma f(x)^2 + 1)}{w(1+\gamma G(x))^3} \\ & - \frac{(\frac{1}{f(x)}(1 + 2w\gamma f(x)^2) + \frac{x^2}{\sigma^2 f(x)}(1 - 2w\gamma f(x)^2))}{w(1+\gamma G(x))} < 0\end{aligned}$$

We now show that the term above is strictly negative. For this it suffices to look at the numerator of both fractions as the denominators are strictly positive under the assumption of Lemma 1 that $\gamma < |\frac{1}{G(x)}|$. The numerator of the first fraction is a sum of two elements. The first element must be negative, since $(2w\gamma f(x)^2 - 1) < 0$ when $w < \frac{1}{2f(x)^2\gamma}$. The second element is all positive except for the x which is taken to be smaller than zero. Thus, we know that the first fraction is negative. The second fraction, which gets substracted, is positive. It is also composed of two elements, the first of which is unambiguously positive while the second is positive as long as $w < \frac{1}{2f(x)^2\gamma}$. In consequence, the second derivative of $x^R(x)$ is strictly smaller than zero.

Therefore, the equation $x^R(x) = \frac{\sigma^2(2w\gamma f(x^R)^2-1)}{wf(x^R)(1+\gamma G(x^R))}$ has at most two solutions and the maximum condition function has at most two roots. □

Lemma 9. *The maximum condition function cannot have two roots within an interval over which it is strictly larger than the candidate function for* $w < \frac{1}{2f(x)^2\gamma}$.

Proof. Consider again the root of the maximum condition function as given by $x^R(x) = \frac{\sigma^2(2w\gamma f(x^R)^2-1)}{wf(x^R)(1+\gamma G(x^R))}$. We will show that its first derivative is strictly positive if the maximum condition function lies above the candidate function. The latter is true whenever:

$$\frac{1}{wf(x)} - \gamma\left(2f(x) - \frac{x}{\sigma^2}G(x)\right) + \frac{x}{\sigma^2} > x - 2w\gamma f(x)G(x)$$

which can be rewritten as an upper bound for w:

$$w < \frac{\sigma^2}{f(x)^2\gamma\sigma^2 - \frac{f(x)x((1+G(x)\gamma)-\sigma^2)}{2} + \frac{\sqrt{f(x)^2(-8G(x)\gamma\sigma^4 + (x+G(x)x\gamma - (x+2f(x)\gamma)\sigma^2)^2)}}{2}} = \tilde{w} \tag{1.14}$$

Now consider the first derivative of the root function $x^R(x)$:

$$\frac{\partial x^R(x)}{\partial x} = \frac{-x(1+\gamma G(x))(2wf(x)\gamma + \frac{1}{f(x)}) - 4w\sigma^2\gamma^2 f(x)^2 + 2\gamma\sigma^2}{w(1+\gamma G(x))^2} \tag{1.15}$$

[20]The first derivative is given by $\frac{\partial x^R(x)}{\partial x} = \frac{-x(1+\gamma G(x))(2wf(x)\gamma+\frac{1}{f(x)})-4w\sigma^2\gamma^2 f(x)^2+2\gamma\sigma^2}{w(1+\gamma G(x))^2}$

As the denominator is positive it remains to show that the numerator is strictly positive. We start by rewriting the term to the following inequality:

$$\sigma^2(1-2w\gamma f(x)^2) - \frac{x}{2f(x)\gamma}(1+\gamma G(x)) - wxf(x)(1+\gamma G(x)) > 0 \tag{1.16}$$

The last two subtrahends of the numerator are negative for all $x < 0$ whereas the first summand is positive in case $w < \frac{1}{2\gamma f(x)^2}$. Thus, if condition (1.14) implies $w < \frac{1}{2\gamma f(x)^2}$, the lemma must be true. Consequently, we verify in the following that $w < \frac{1}{2\gamma f(x)^2}$ holds if the maximum condition function is bigger than the candidate function.

We begin by considering the large term under the root in the denominator of $\tilde{w}$ in condition (1.14):

$$\sqrt{f(x)^2(-8G(x)\gamma\sigma^4 + (x + G(x)x\gamma - (x+2f(x)\gamma)\sigma^2)^2)} =$$

$$\sqrt{f(x)^2(4f(x)^2\gamma^2\sigma^4 + x^2((1+\gamma G(x)) - \sigma^2)^2 + C)}$$

Firstly, we show that the term $C = -8G(x)\gamma\sigma^4 - 4f(x)\gamma\sigma^2 x((1+G(x)\gamma) - \sigma^2)$ is positive.

$$0 < -4(2G(x)\gamma\sigma^4 + f(x)\gamma\sigma^2 x((1+G(x)\gamma) - \sigma^2))$$

$$\Leftrightarrow \qquad 0 < -4\gamma\sigma^2(\sigma^2(2G(x) - f(x)x) + f(x)x(1+\gamma G(x)))$$

$$\Leftrightarrow \qquad 0 > \sigma^2(2G(x) - f(x)x) + f(x)x(1+\gamma G(x))$$

It is easy to verify that $(2G(x) - f(x)x)$ is strictly negative[21] for all $x < 0$. Since the term $(1+\gamma G(x))$ is positive by the assumptions on γ, the statement above must be true and C is indeed positive. Having established that C is positive we can now overestimate $\tilde{w}$ by dropping C. Thus,

$$\tilde{w} < \frac{1}{f(x)^2\gamma\sigma^2 - \frac{f(x)x((1+G(x)\gamma)-\sigma^2)}{2} + \frac{\sqrt{f(x)^2[x((1+G(x)\gamma)-\sigma^2)+2f(x)\gamma\sigma^2]^2}}{2}}$$

which can be simplified to:

$$\tilde{w} < \frac{1}{f(x)^2\gamma\sigma^2 - \frac{f(x)x((1+G(x)\gamma)-\sigma^2)}{2} + \frac{2f(x)^2\gamma\sigma^2 + f(x)x((1+G(x)\gamma)-\sigma^2)}{2}} = \frac{1}{2f(x)^2\gamma\sigma^2}$$

Hence, $\tilde{w} < \frac{1}{2\gamma f(x)^2}$ is true as well. Consequently, (1.16) holds for all $x < 0$ where the maximum condition function lies above the candidate function. □

Proposition 3.

i) When $w \in \left(\frac{1}{4\gamma f(0)^2}, \frac{1}{2\gamma f(x^s)^2 + B}\right)$ *where* $\gamma \in \left[0.54, -\frac{1}{G(x^s)}\right)$, σ *sufficiently large and* $B = \sqrt{-\gamma f(x^s)^2 \frac{2G(x^s)(1+\gamma G(x^s))}{\sigma^2}} \geq 0$ *, a type two Turn Around Equilibrium (TAE2)*

[21] Its derivative $f(x)(3 + \frac{x^2}{\sigma^2})$ is strictly positive. Moreover it is zero when $x = 0$ and approaches -2 when $x \to -\infty$.

in which the agent behind spends much more effort than the agent ahead exists. The parameter x^s determines the intersection between candidate function and maximum condition function exogenously.

ii) If there exist TAE2s there also exist Confirming Asymmetric Equilibria (CAEs) for small δ_1.

iii) If the maximum condition function and the candidate function intersect, but there are no TAE2s, also no CAEs for small δ_1 exist.

Proof. *i)* To establish the existence of TAE2s, i.e. TAEs over an x-interval which is not necessarily adjacent to zero, one needs to show that over such an interval and under some conditions both the candidate function and maximum condition function return positive values. In Lemma 6 it was established that the candidate function has exactly one maximum and no other extreme points over the domain of strictly negative x. We also know from Lemma 4 that when $w > \frac{1}{4\gamma f(0)^2}$ the candidate function always returns positive values over the interval given by the roots of candidate function $(x^*, 0)$ where $x^* = 2w\gamma f(x^*)G(x^*)$. Lemma 7 implies that when candidate and maximum condition function do not intersect for $x < 0$ the maximum condition derived in Lemma 3 is fulfilled for all x where the candidate function is positive. Especially at the left root of the candidate function this leads to TAE2s that are rather 'far away' from $x = 0$. Additionally, given its conditions Lemma 7 implies that if intersections between the candidate and the maximum condition function exist for some $x < 0$, then both the maximum condition and the candidate function are positive at the intersection as illustrated in figure 1.4. It follows that around this intersection TAE2s exist.

Before we continue with part *ii)* we show that Lemma 8 and Lemma 9 also hold for the upper bound of w given in Proposition 3. We will proceed with the proof for the general case using x so it will also hold for a specific x^s. We want to show that $\frac{1}{2f(x)^2\gamma} > \frac{1}{2f(x)^2\gamma+B}$ which holds if B is strictly positive. If this holds the upper bound used in Proposition 3 is always smaller than the one used in Lemma 8 and Lemma 9 meaning they also hold for this proposition. First we show that the upper bound for w given in Proposition 3 is tighter than the upper bound of Lemma 7. The latter's denominator can be reformulated as follows:

$$\gamma f(x)^2 + \sqrt{\gamma f(x)^2(\gamma f(x)^2 - \frac{2G(x)(1+\gamma G(x))}{\sigma^2})}$$

$$= \gamma f(x)^2 + \sqrt{\gamma^2 f(x)^4 - \frac{2\gamma f(x)^2 G(x)(1+G(x)\gamma)}{\sigma^2}} = \gamma f(x)^2 + \sqrt{\gamma^2 f(x)^4 - A}$$

$$\leq 2\gamma f(x)^2 + \sqrt{-A} = 2\gamma f(x)^2 + B$$

This last step was established by using Jensen's inequality. Since the numerators of both upper bounds are equal this step shows that the upper bound given in Proposition 3 is tighter than that of Lemma 7. Solving for B leads to:

$$B \geq -\gamma f(x)^2 + \sqrt{\gamma^2 f(x)^4 - A} > 0$$

This holds as A is negative for $\gamma \leq |\frac{1}{G(x)}|$.

Since $B > 0$ it must be that $\frac{1}{2f(x)^2\gamma} > \frac{1}{2f(x)^2\gamma+B}$. This also holds for the case $x = x^s$.

ii) To study the relationship between strong TAEs and CAEs for small δ_1 we make use of the symmetry property of the candidate function $\delta_1(x)$ as well as the maximum condition function $maxcond(x)$:

$$\delta_1(x) = -\delta_1(-x)$$

$$maxcond(x) = maxcond(-x)$$

Having CAEs means that $\delta_1(x) > 0$ for $x > 0$ and the $maxcond(x) > 0$. Using the symmetry this is equivalent to $\delta_1(-x) < 0$ while $maxcond(-x) < 0$.

From Lemma 6 we know that the candidate function has only one maximum on the negative domain and in Proposition 1 we derived that the candidate function approaches infinity if $x \to \infty$. By symmetry this implies that the candidate function goes towards minus infinity if $x \to -\infty$. Since $\delta_1(0) = 0$ it follows from continuity that candidate CAEs (not necessarily CAEs) exist for all values of δ_1.

To find CAEs we have to insure that the maximum condition is fulfilled. We use the previously derived lemmas to make a statement about the maximum condition for all $x < x_\delta$, where x_δ is the negative root of the candidate function, and, then, use the symmetry properties from above to apply it to the candidate CAEs. We know that the maximum condition as derived in Lemma 3 goes to infinity when $x \to -\infty$ and since we have shown the existence of TAE2s in part *i*) there also exist some $x < 0$ where maximum condition and candidate function are both positive.

In consequence for the maximum condition to become negative over $x < 0$ it has to have at least two roots on the same domain. Moreover, we know from Lemma 8 that the maximum condition function cannot not have more than 2 roots for $x < 0$.

One possibility would be that the maximum condition function could have one root below x_δ and one above. In this case there would be a negative intersection of the maximum condition function with the candidate function as the candidate function must be negative for $x < 2w\gamma f(x)G(x)$ by Lemma 4. Since our conditions for TAE2s ensure that all intersection points are positive for $x < 0$ this case can be excluded. Secondly, the roots of the maximum condition function could both be below x_δ. This, however, directly contradicts Lemma 9 as the maximum condition has to be bigger than the candidate function at x_δ. Otherwise this would be equivalent to the previous example. Using symmetry this implies the existence of CAEs for small δ_1.

Lastly we address part *iii*). Following the same argument we know that in cases where no TAE2s exist, but the candidate and maximum condition functions still intersect, the intersection point must lie in negative range. Since the candidate function is negative for sufficiently small x, we know from Lemma 6 that the candidate function will not have an intersection with the abscissa for $x < 0$. This implies, that, because of the symmetry property of the candidate function, CAEs for small δ_1 do also not exist.

□

5.4 Proof of Proposition 4:

Proposition 4. *When* $\frac{1}{4f(0)^2\gamma} < w < \frac{1}{2f(0)^2\gamma}$ *and* $\gamma \leq -\frac{f(x^s)^2G(x^s)\pi^2}{\frac{2}{\sigma^2}+f(x^s)^2\pi(-2+G(x^s)^2\pi)}$ *then for small unevenness the unique equilibrium in pure strategies is a type one Turn Around Equilibrium (TAE1), where* x^s *exogenously determines the intersection between candidate function and maximum condition function.*

Proof. We know from Proposition 3 and Lemma 7 that TAE2s only exist when:

$$\gamma > -\frac{f(x^s)^2G(x^s)\pi^2}{\frac{2}{\sigma^2}+f(x^s)^2\pi(-2+G(x^s)^2\pi)} \tag{1.17}$$

Moreover, we know from Proposition 3 that for δ_1 small enough CAEs only exist if TAE2s exist as well. TAE1s as described in Proposition 2 on the other hand, always exist when

$$\frac{1}{4f(0)^2\gamma} < w < \frac{1}{2f(0)^2\gamma}$$

Since the lower bound for γ (1.17) is strictly positive and lower and upper bound for w cannot intersect, we know that when the condition for γ is not satisfied there is yet a prize level w for which a TAE1 exists and is the only equilibrium for small enough δ_1.

□

5.5 Proof of Proposition 5:

Proposition 5. *Every Catching Up Equilibrium (CUE) is also a Turn Around Equilibrium (TAE).*

Proof. Remember that x was defined as $x = \Delta e + \delta_1$. Suppose again without loss of generality that player 1 is initially ahead, i.e. $\delta_1 > 0$, and that at the CUE player 2 spends more effort than player 1 with $\Delta e < 0$, but not enough to turn the game, i.e. $\Delta e + \delta_1 > 0$. From Lemma 2 we know that the candidate function provides all possible equilibrium candidate points:

$$\delta_1(x) = x - 2f(x)w\gamma G(x)$$

To find a CUE that is no TAE we need to show that there exist candidate points where for $x > 0$ and $\Delta e = x - \delta_1 < 0$. We show that this can never be the case:

$$x - \delta_1 = 2f(x)w\gamma G(x)$$

For $x > 0$ the RHS cannot be negative since $G(x)$ is positive for all $x > 0$ and the other terms are always positive. So $x - \delta_1$ will be positive for all $x > 0$.

□

6. Appendix 2 – Technicalities

We provide these pages as an additional aid for the verification of some expressions.

6.1 Derivation of (1.11) and (1.12) in Lemma 7

We want to find a condition for w that ensures that the intesection between the candidate and the maximum condition function occurs in positive range. We show in Lemma 7 that this must be the case when:

$$0 < \frac{-2f(x^s)^2 w\gamma(G(x^s)w + G(x^s)^2 w\gamma w - \sigma^2) + \sigma^2}{f(x^s)w(1 + G(x^s)\gamma - \sigma^2)}$$

Note that as $G(x^s)\gamma < 0$ and $\sigma^2 \geq 1$ the denominator is smaller zero. Collecting the w terms and multiplying with the negative denominator yields:

$$0 > w^2(-2f(x^s)^2\gamma G(x^s)(1 + G(x^s)\gamma)) + 2f(x^s)^2 w\gamma\sigma^2 - \sigma^2$$

Next, we solve the above inequality as a quadratic equation for w. This gives:

$$w = \frac{\sigma^2(2\gamma f(x^s)^2 \pm \sqrt{f(x^s)^2\gamma(f(x^s)^2\gamma - \frac{2G(x^s)(1+\gamma G(x^s))}{\sigma^2}})}{2f(x^s)^2\gamma G(x^s)(1 + \gamma G(x^s))} = \frac{\sigma^2(A \pm \sqrt{B})}{C}$$

We now get to (1.11) and (1.12) by recognising that $C = \sigma^2(A + \sqrt{B})(A - \sqrt{B})$. Thus

$$w = \frac{\sigma^2(A \pm \sqrt{B})}{\sigma^2(A + \sqrt{B})(A - \sqrt{B})} = \frac{1}{(A - \sqrt{B})} \text{ or } \frac{1}{(A + \sqrt{B})}$$

The first possible solution is equivalent to (1.11), the second to (1.12) in Lemma 7.

6.2 Derivation of (1.13) in Lemma 7

We want to derive the lower bound for γ given in (1.13). Starting with the inequality

$$\frac{1}{4\gamma f(0)^2} = \frac{\pi\sigma^2}{2\gamma} < \frac{1}{\gamma f(x^s)^2 + \sqrt{\gamma f(x^s)^2(\gamma f(x^s)^2 - \frac{2G(x^s)(1+\gamma G(x^s))}{\sigma^2}})}$$

first rearranging leads to

$$\gamma f(x^s)^2 + \sqrt{\gamma f(x^s)^2(\gamma f(x^s)^2 - \frac{2G(x^s)(1 + \gamma G(x^s))}{\sigma^2}}) < \frac{2\gamma}{\pi\sigma^2}$$

Suqaring the next rearrangement

$$\sqrt{\gamma f(x^s)^2(\gamma f(x^s)^2 - \frac{2G(x^s)(1 + \gamma G(x^s))}{\sigma^2}}) < \frac{2\gamma}{\pi\sigma^2} - \gamma f(x^s)^2$$

gives us

$$
\begin{aligned}
& \gamma f(x^s)^2(\gamma f(x^s)^2 - \frac{2G(x^s)(1+\gamma G(x^s))}{\sigma^2}) < \gamma^2(\frac{2}{\pi\sigma^2} - f(x^s)^2)^2 \\
\Leftrightarrow \quad & f(x^s)^4 - \frac{2G(x^s)f(x^s)^2}{\sigma^2\gamma} - \frac{2f(x^s)^2G(x^s)^2}{\sigma^2} < (\frac{2}{\pi\sigma^2} - f(x^s)^2)^2 \\
\Leftrightarrow \quad & -\frac{2G(x^s)f(x^s)^2}{\sigma^2\gamma} < (\frac{2}{\pi\sigma^2} - f(x^s)^2)^2 - f(x^s)^4 + \frac{2G(x^s)f(x^s)^2}{\sigma^2\gamma}
\end{aligned}
$$

In the next step, we need to solve for γ.

$$
\frac{2G(x^s)f(x^s)^2}{\sigma^2((\frac{2}{\pi\sigma^2} - f(x^s)^2)^2 - f(x^s)^4 + \frac{2f(x^s)^2G(x^s)^2}{\sigma^2})} < \gamma
$$

Simplifying the the LHS reveals (1.13).

$$
\begin{aligned}
& -\frac{2G(x^s)f(x^s)^2}{\sigma^2(\frac{4}{\pi^2\sigma^4} - \frac{4f(x^s)^4}{\pi\sigma^2} + f(x^s)^4 - f(x^s)^4 + \frac{2f(x^s)^2G(x^s)^2}{\sigma^2})} = \\
& \qquad -\frac{f(x^s)^2G(x^s)\pi^2}{\frac{2}{\sigma^2} + f(x^s)^2\pi(-2+G(x^s)^2\pi)} < \gamma
\end{aligned}
$$

6.3 Derivation of (1.14) in Lemma 9

$$
\frac{1}{wf(x)} - \gamma\left(2f(x) - \frac{x}{\sigma^2}G(x)\right) + \frac{x}{\sigma^2} > x - 2w\gamma f(x)G(x)
$$

can be rewritten as:

$$
w < \frac{\sigma^2}{f(x)^2\gamma\sigma^2 - \frac{f(x)x((1+G(x)\gamma)-\sigma^2)}{2} + \frac{\sqrt{f(x)^2(-8G(x)\gamma\sigma^4 + (x+G(x)x\gamma-(x+2f(x)\gamma)\sigma^2)^2)}}{2}} = \tilde{w}.
$$

We begin by bringing all terms to the left side and multiplying by $wf(x)$:

$$
w^2 2\gamma f(x)^2 G(x) + wf(x)(x(\frac{1}{\sigma^2} - 1) - \gamma(2f(x) - \frac{x}{\sigma^2}G(x))) + 1 > 0
$$

Collecting all x together and multiplying by σ^2 we obtain:

$$
w^2 2\gamma f(x)^2 G(x)\sigma^2 + wf(x)(x((1+\gamma G(x)) - \sigma^2) - 2\gamma f(x)\sigma^2) + \sigma^2 > 0
$$

To solve for w we now use the quadratic formula for the equality $w = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ where $a = 2\gamma f(x)^2G(x)\sigma^2$, $b = f(x)(x((1+\gamma G(x)) - \sigma^2) - 2\gamma f(x)\sigma^2)$ and $c = \sigma^2$. Plugging in a, b and c yields:

$$
\begin{aligned}
w =& \frac{f(x)(x(\sigma^2 - (1+\gamma G(x))) + 2\gamma f(x)\sigma^2)}{4\gamma\sigma^2 f(x)^2 G(x)} \\
& \frac{\pm\sqrt{(f(x)(x((1+\gamma G(x)) - \sigma^2) - 2\gamma f(x)\sigma^2))^2 - 8\gamma\sigma^4 f(x)^2 G(x)}}{4\gamma\sigma^2 f(x)^2 G(x)}
\end{aligned}
$$

To deal with this big term we now temporarily express it as $\frac{s-\sqrt{t}}{4\gamma\sigma^2 f(x)^2 G(x)}$. We neglect the postive root, as we are looking for a conservative upper bound. The crucial step to obain (1.14) is to realise that the denominator $4\gamma\sigma^2 f(x)^2 G(x)$ can be rewritten as $\frac{s^2-t}{2\sigma^2}$, which we now show:

$$
\begin{aligned}
s^2 - t =& (f(x)(x((1+\gamma G(x)) - \sigma^2) - 2\gamma f(x)\sigma^2))^2 \\
& - (f(x)(x((1+\gamma G(x)) - \sigma^2) - 2\gamma f(x)\sigma^2))^2 + 8\gamma\sigma^4 f(x)^2 G(x) \\
=& 8\gamma\sigma^4 f(x)^2 G(x)
\end{aligned}
$$

Thus $\frac{s^2-t}{2\sigma^2} = 4\gamma\sigma^2 f(x)^2 G(x)$. We can now say $\frac{2\sigma^2(s-\sqrt{t})}{s^2-t} = \frac{2\sigma^2(s-\sqrt{t})}{(s-\sqrt{t})(s+\sqrt{t})} = \frac{2\sigma^2}{s+\sqrt{t}}$ which is equal to:

$$
\frac{\sigma^2}{f(x)^2\gamma\sigma^2 - \frac{f(x)x((1+G(x)\gamma)-\sigma^2)}{2} + \frac{\sqrt{f(x)^2(-8G(x)\gamma\sigma^4 + (x+G(x)x\gamma - (x+2f(x)\gamma)\sigma^2)^2)}}{2}}
$$

Chapter 2

Competing Firms on New Media

In this chapter, I model a reviewing process on new media as contest situation between two firms. The complexity of the model increases when taking the risk of losing potential buyers into account and when considering that hosts of blogs and video channels might evaluate using objective as well as subjective criteria. It can be seen, that especially unknown firms with low costs which produce decent quality products which are not that pricy have an incentive to participate in such contests, using it as inexpensive advertisement to win new customers. By contrast, firms with high market share that might be often forced into these contests due to their popularity are compelled to rise their quality. To gain further insights, a one stage Tullock contest with the two decisive variables quality and taste is developed and analyzed. As a result, an evaluation including quality and taste at the same time is most likely to increase the consumer's utility. Furthermore, it can be shown that the second variable is more likely to be used by the weaker opponent.

1. Introduction

In 2001 the company "Blogger" started to provide a platform for people to publish their own journals online for the first time.[1] A few years later the first youtube video went online in 2005.[2] Nowadays, many people write blogs - private blogs about topics like traveling or commercial blogs about their business. Most people are online every day and use the internet as an additional source for information and entertainment next to television, radio or print. Youtube has become a ginormous platform and community where people publish their channels and follow other hosts for entertainment and information. Another modern way of being entertained and informed is listening to podcasts. These broadcasts have been made accessible for the mass in 2005 by apple implementing a podcast service into their music application iTunes.[3] All these media channels were intended for privat use at first and were discovered by companies as important way to address a different class of customers later on.

[1] For further information see Baker (2009).

[2] This is the first youtube video posted by one of the founders: Karim (2005).

[3] Further information can be found e.g. in this article BBCNews (2005).

To promote launches of new products or product lines, companies might sent samples to known blogs of a fitting topic. E.g. the beauty company Sally Hansen sent her new nail polish collection to one of the biggest German beauty blogs Grüttner (2016) to promote the introduction of their products in the drug store "dm" in Germany. The blogger was able to test the products and wrote a review about the experience mentioning the upcoming introduction in the stores. However, blogger do not only review products separately but they explicitly look for similarities between comparable products. E.g. beauty bloggers look for cheaper duplicates of hyped highly priced products called "dupes" like in the show by Kusak and Engelshowe (2015) called "beautea time" where a limited edition from a drugstore label, that has bean provided by the brand and is reviewed with focus on possible "dupes". Especially, bloggers or hosts in general are a good source of information concerning comparisons between products as they have a lot of experience in their field and have seen or tried many products. For this reason they often relate to other products during a review and compare them. To have a product being revied or compared to other products by a host it is beneficial for a company to provide the product to the host even if it is costly. In his report, Olhoeft (2016) explains, why a sponsored mobile phone should not be seen as bribe as you usually do not take it and sell it on eBay, for example, but it is needed to compare it to other mobile phones with comparable specifications. E.g. the brand Huawei wants to reach a status on the same level with apple and samsung in the mobile phone market. For this reason, they do not only organize big key notes to signal their importance but they also provide their phones to the tech bloggers and channel owners invited to become part of the comparison.

Hence, firms can use new media like blogs, video channels or podcasts as possibilities for advertising and winning new customers. Hosts have alternative ways to evaluate products as they do not claim to be objective but the characteristic about their format is their personal opinion and their experience. Furthermore, it is possible for firms to enter this reviewing process on their own by providing their product to the hosts as mentioned above. The recommendations of the hosts can be assumed to motivate some of their viewers, readers or listeners to follow their advice and buy the product favored in the article or episode.

In this chapter, I capture the main characteristics of these reviewing processes with comparison component in order to formalize this new way of evaluation process and analyses influences on the product specifications. The focus is set on the optimal behavior of firms in this context but it will be payed attention to the factors that motivate a firm to voluntarily enter this contest situation as well. Furthermore, possible benefits and drawbacks for consumers and firms resulting from this new way of competing for customers will be discussed. I focus on the way the host can evaluate the products in objective as well as subjective ways, by first developing a one dimensional model taking into account either the objective or the subjective approach and finally combining both in a two dimensional model. As the new media, especially blogs, video platforms and podcasts are channels that can be used in very individual ways there may be a variety of different manifestations that are more or less related to the models presented here.

In this chapter, I do not claim to model the general process of reviewing in social media, but I want to elucidate these specific occasions where products are compared to each other in a review by a host influencing his followers buying decisions.

This chapter is one of the first essays that model a certain process or aspect developed by using new media. The important insight provided, is that these processes can be captured by using wellknown models and when introducing specific new characteristics. This is an important step in order to learn more about mechanisms in online communities.

A multitude of guidebooks for social media marketing is available like Hettler (2012). There are several introductions to new media as Lister (2009) examining the different aspects of new media also including “on demand TV services”, fandoms and other communities as well. Besides these very general overviews there are more specific paper focusing on certain aspects like Johnson and Kaye (2004) who investigate the perceived credibility of webblogs or Mangold and Faulds (2009) analyzing new media as part of the promotion mix of firms. Liu et al. (2007) investigating the influence of blog references on product sales and develops a model for predictive usage.

Chen and Xie (2005) investigated Third-Party reviews but focus on evaluations by official institutions and professionals only. In their subsequent paper, Chen and Xie (2008) looked for the best reaction on product reviews e.g. providing the content on the firms own web page. In their model, firms only react after receiving a review but are neither able to optimize their product in beforehand nor to enter the decision process actively. Dean and Biswas (2001) investigated third-party organization endorsements and their effect on the prepurchase valuation on consumers.

Furthermore, his chapter adds to the advertisement literature in certain aspects as the reviewing process can be seen as informative advertisement to some extend following the classifications of Armstrong and Porter (2007). Also related is Friedman (1958) as they introduce competitive behavior in advertising.

The models developed in this chapter base on a contest setting. There are two main modeling approaches for rank-order tournaments commonly used in economics. The first model was introduced by Tullock (1980) while the second model came up one year later introduced by Lazear and Rosen (1981) using additive noise to model uncertainty in the contest as variation to a first prize all-pay auction. In this context a model following Tullock (1980) will be used. This typical rent seeking contest is characterized by full information on all players with no probability distribution adding uncertainty needed. In contrast to Lazear and Rosen (1981) the modeling structure of Tullock (1980) ensures a pure strategy solution in most cases.[4] Nti (1999) enhanced Tullock’s model with asymmetric valuations. His nomenclature is used to evaluate a simplified notation in this chapter which becomes especially useful in the more complex model versions. The use of different valuation is significantly different to the modeling of different abilities

[4]The last model variation will show a case where this solution is not always guaranteed.

like in Baik (1994) or Cornes and Hartley (2005). The asymmetry in valuations is part of the objective function, while the asymmetry in abilities is influencing the structure of the contest success function directly.

In this chapter, I show that joining this type of contest and using it as advertisement possibility is attractive to firms with several characteristics. The contest is more attractive to firms with low costs but producing decent quality products. Additionally, it can be expected more beneficial for firms that are new in the market as these have no consumers to lose in the contest but the contest gives the possibility to win lots of new customers. The price plays two roles in this context. First, it determines the profits from selling a product to a consumer. Second, it determines the hight of the entrance fee, as the company has to provide his product to the host to enter the reviewing process. For this reason, it may be attractive to firms with high price products in case they can gain lots of money from winning the contest and from finding new customers but most likely this competition is more attractive to firms with lower price products that are maybe even bought frequently. Interestingly especially those firms who would like to not participate in these contests as they have a high reputation going along with a high market share, are forced to enter these contests as the reviews are highly requested. In consequence, these contests force these firms to keep their quality high as they are not willing to risk losing customers.

Besides the possibility of using only quality as decisive variable, an alternative approach is presented where the taste of the host is crucial. It is shown how the new approach can be converted to be analyzed according to the model derived in beforehand. Following the idea, that hosts can use both an objective criterion as quality and a subjective criterion as taste in their evaluation, the last model presents a version, where both approaches are combined. It can be seen, that allowing for two variables will lead most likely to more beneficial products for consumers as the rent dissipation meaning the amount invested in both instruments overall rises. Furthermore, it becomes evident, that all results for the two-dimensional model hold only under a certain condition following from the optimization process.[5] It can be shown that the possibility of using quality and taste in the contest is not always used by both players. There are even cases, where nobody is using the second instrument. However, the option to aim at the taste of the host is more likely to be used by the weaker opponent.

The situation of interest includes several aspects differing from a standard Tullock model. For this reason the investigation and the modeling process evolve step by step. At first, the situation is captured in a basic model (Section 2). This allows to draw first intuitions from the basic setting before adding further aspects to address more specific characteristics. In section 3, the possibility of losing customers by a bad review is taken into account. In section 4, I present a way of modeling the blogger deciding on their personal taste instead of the objective criterion quality. Furthermore, this section provides a general interpretation of the expanded model and a discussion of possible

[5]This restriction has not been taken into account by Epstein and Hefeker (2003), who used a related model. Considering the restriction some of their results would change or not hold in general.

benefits for customers and firms. In section 5, I introduce a model that combines both criteria, investigate the influence on the firms' optimal decision and examine possible benefits for consumers. Concluding remarks are given in section 6.

2. Basic Model

This basic modeling approach captures first characteristics of the setting being analyzed. Firms offering a product can enter a contest, staged by a host of a social media channel. The host evaluates the products and gives a review including a ranking of the reviewed products. This assessment influences the purchasing decision of the followers attending the channel. To prevent introducing too much complexity this chapter is constrained to a contest situation between two players. In case of more than two players there would certainly appear other phenomenas as shown by Cornes and Hartley (2005) for example.

The basic contest is only considering the quality x of a product i or j. This leads to a straightforward contest success function (csf):

$$\varrho_i = \frac{x_i}{x_i + x_j} \tag{2.1}$$

As profit function, a very simple version is used to gain a first impression of the mechanisms. The host has a specific number of people he reaches with his review and a partial of this is willing to buy the specific type of product in the near future. This amount of people willing to buy is called h. Therefore, in case of winning, the firm earns hp_i, where p_i declares the price of the specific product, with $p_i \geq 0$. It can be assumed, that the price is given for each firm, as it can be seen realistic that the pricing process has already taken place for the whole market while this specific contest is not influencing the pricing decision. The products being compared in the reviewing contests can be in the same price range but differ in their quality. However, it can also occur that a more expensive product is compared to a cheeper model. This can be caused by different reasons. On the one hand there can be a firm producing a product with similar performance but more efficiently in productions or with less margin while for example the more expensive product is produced by a well known firm that prices in higher levels. On the other hand two products can be compared for one use but have different functions that are not all needed. In this case the price does not need to reflect the actual value of the product for the specific use.

$$\Pi_i = \varrho_i h p_i - c(x_i) - p_i \tag{2.2}$$

The cost function will depend on the quality choice x_i. The higher the quality of the product is, the higher the costs will be. To describe this effect and allow for additional analysis regarding differences in costs, the cost function is defined as $c(x_i) = c_i x_i$, with $c_i \geq 0$. As already mentioned for the prices, the costs may differ due to different production processes of the firms but also for products with different ranges of functions being compared for one specific use. It can be expected, that there are no fix costs of production caused by the contest but the product has to be provided to the host to enter the contest so he is able to make a review.[6] In the model, p_i can be seen as entry costs in order to take part in the contest.

The next step is to derive the optimal decisions on quality for each firm. For this purpose the first order conditions of both players are considered.

$$\begin{aligned} FOC\ 1: \ \tfrac{\partial \Pi_i}{\partial x_i} &= hp_i \tfrac{x_j}{(x_i+x_j)^2} - c_i = 0 \\ FOC\ 2: \ \tfrac{\partial \Pi_j}{\partial x_j} &= hp_j \tfrac{x_i}{(x_i+x_j)^2} - c_j = 0 \end{aligned}$$

$$FOC\ 1\ \&\ FOC\ 2 \qquad \Rightarrow \qquad \frac{p_i}{c_i} x_j = \frac{p_j}{c_j} x_i \tag{2.3}$$

To ensure calculating an optimal decision for both firms both second derivatives need to be smaller than zero. Which is actually the case. The terms below can at most become zero in case no quality is chosen which is not really a choice in this setting as quality can only be bad but not nothing. However, even the zero case would be included.

$$\begin{aligned} SOC\ 1: \ \tfrac{\partial^2 \Pi_i}{\partial x_i^2} &= -2hp_i \tfrac{x_j}{(x_i+x_j)^3} < 0 \\ SOC\ 2: \ \tfrac{\partial^2 \Pi_j}{\partial x_j^2} &= -2hp_j \tfrac{x_i}{(x_i+x_j)^3} < 0 \end{aligned}$$

Inserting equation (2.3) in FOC 1 and FOC 2 leeds to the optimal solutions:

$$x_i^* = \frac{\left(\frac{hp_i}{c_i}\right)^2 \frac{hp_j}{c_j}}{\left(\frac{hp_i}{c_i} + \frac{hp_j}{c_j}\right)^2} \tag{2.4}$$

This solution goes along with Nti (1999).[7] He investigates asymmetric valuation in rent-seeking contests. Instead, the factors h and p_i can be seen as valuation following Nti (1999). The asymmetry is introduced by the different prices. But this is not the only factor influencing the valuation as can be seen in equations 2.4 and 2.5. This model is using a more complex cost function as Nti decided to choose $c_i = 1$. The optimal result from Nti (1999) can be expressed as equations:

$$x_i^* = \frac{(V_i)^2 V_j}{(V_i + V_j)^2} \tag{2.5}$$

Following the notation introduced by Nti, the overall valuations of the firms can be expressed as $V_i = \frac{hp_i}{c_i}$ and $V_j = \frac{hp_j}{c_j}$. This way the differences in costs can be easily taken into account talking about the valuation of a firm in this contest. A firm's valuation increases in the price it can demand and in the potential number of buyers they can win as well. In contrast, the higher quality costs are, the lower is the valuation of winning the contest.

[6] This is a common procedure, however there are e.g. cases where products are only provided for a certain time frame. This point will also be addressed during the analysis.

[7] The factor r used to declare the degree of homogeneity in Nti (1999) is set to one in this model.

Inserting the optimal quality choices in the contest success function (2.1) simplifies to:

$$\varrho_i^* = \frac{V_i}{V_i + V_j} = \frac{\frac{hp_i}{c_i}}{\frac{hp_i}{c_i} + \frac{hp_j}{c_j}} \tag{2.6}$$

In the following, comparative statics are performed to get a better understanding in the mechanics between the different factors of the firm's valuation on the optimal choices, the resulting probabilities.

$$\frac{\partial x_i^*}{\partial h} = \frac{c_i c_j p_i^2}{(c_j p_i + c_i p_j)^2} > 0$$

As h declares the number of buyers descending to buy the product because of the contest won, it is intuitive that an increase in this number increases the expected gain and in consequence leads to a willingness to improve quality to perform better. As the number of influenceable people correlated to the reach of the host channel it is obvious that larger channels get products offered for reviews more often as the expected revenue is higher than for smaller channels. Furthermore, channels with larger audience can be expected to reflect the opinion of the customers much better. This means, basing the quality decision on opinions of hosts with large followership can be seen as potential good estimates for the consumers preferences. As h is influencing both firms equally, the winning probabilities ϱ_i and ϱ_j are not influenced by changes in h.

$$\frac{\partial x_i^*}{\partial p_i} = \frac{2hc_i c_j p_i p_j^2}{(c_j p_i + c_i p_j)^3} > 0$$

In case of rising prices earnings grow as well while the number of followers potentially buying is assumed to stay the same. For this reason each product sold becomes more valuable which increases the motivation to increase the quality provided.

$$\frac{\partial \varrho_i^*}{\partial p_i} = \frac{c_i c_j p_j}{(c_j p_i + c_i p_j)^2} > 0$$

In consequence, higher prices lead to higher quality which increases the probability to get the better review and thereby win the contest.

$$\frac{\partial x_i^*}{\partial c_i} = -\frac{2hc_j p_i^2 p_j^2}{(c_j p_i + c_i p_j)^2} < 0$$

Increasing costs shrink the profits made by selling a product. For this reason, the motivation to sell pieces is lower i.e. to reduce quality becomes more attractive.

$$\frac{\partial \varrho_i^*}{\partial c_i} = -\frac{c_j p_i p_j}{(c_j p_i + c_i p_j)^2} < 0$$

In consequence, the lower quality caused by the higher costs leads to lower probability to get a positive review especially better than the opponent and to win the contest.

$$\frac{\partial x_i^*}{\partial p_j} = \frac{p_i^2 c_j h(c_j p_i - c_i p_j)}{(c_j p_i + c_i p_j)^3} > 0 \qquad if\ c_j p_i > c_i p_j \Leftrightarrow \frac{p_i}{c_i} > \frac{p_j}{c_j}$$

The influence of changes in the opponent's prices depends on a condition that can be interpreted in several different ways. Regarding Nti (1999) the condition $\frac{p_i}{c_i} > \frac{p_j}{c_j}$ is equivalent to $V_i > V_j$ as V_i was equivalent to $\frac{hp_i}{c_i}$ in this model. Considering the optimal csf given in equation (2.1) $V_i > V_j$ means that firm i is stronger while firm j is the underdog in the contest. It can be seen as well that the valuations V_i can be reduced to the price to cost ratio of each firm in this condition i.e. the firm having the better price cost ratio is the stronger firm. In other words, margin compared in relation to the costs changes the way a firm reacts to a price change of his opponent.

$$\frac{\partial \varrho_i^*}{\partial p_j} = -\frac{c_i c_j p_i}{(c_j p_i + c_i p_j)^2} < 0$$

If the firm i is the underdog and firm j rises her prices the rising quality firm j is offering demotivates firm i resulting in even lower quality produced by the underdog. In case of firm i dominating the contest an increase in p_j also motivates firm i to increase the quality of her product. The reaction of ϱ_i reveals that this increase in quality in case of i being the stronger firm never evens out the increase in x_j as the influence on the csf is always negative.

$$\frac{\partial x_i^*}{\partial c_j} = \frac{p_i^2 p_j h(c_i p_j - c_j p_i)}{(c_j p_i + c_i p_j)^3} < 0 \qquad if\ c_j p_i > c_i p_j \Leftrightarrow \frac{p_i}{c_i} > \frac{p_j}{c_j}$$

A similar condition decides about the reaction on changes in the opponent's costs. In case of firm i being stronger the rising costs of j lead to shrinking quality of j allowing also i to reduce quality slightly as the competition becomes tight. However, in case of being the underdog, firm i would use the weakness of her opponent to increase her chances by increasing the quality produced.

$$\frac{\partial \varrho_i^*}{\partial c_j} = \frac{c_i p_i p_j}{(c_j p_i + c_i p_j)^2} > 0$$

In both cases an increase in the opponents costs leads to an advantage for firm i whether it raises or lowers quality depends on the valuations.

Using the profit function (2.2) and inserting the optimal solution into the csf leads to the expected profits in equilibrium.

$$\Pi_i^* = \frac{V_i^3 c_i}{(V_i + V_j)^2} - p_i = \frac{\left(\frac{hp_i}{c_i}\right)^3 c_i}{\left(\frac{hp_i}{c_i} + \frac{hp_j}{c_j}\right)^2} - p_i$$

As the contest is only profitable in case the expected profits of participation are larger or equal zero. This means the profit function being positive can be seen as participation constraint. For this reason there are factors increasing the attractiveness of entering the reviewing process while others are impedimental.

$$\frac{\partial \Pi_i^*}{\partial h} = \frac{c_j^2 p_i^3}{(c_j p_i + c_i p_j)^2} > 0$$

As h is a positive factor in each revenue function, the expected optimal profit rises with increasing h. The degree a firm can profit from this increase in potential buyers depends on the price it is able to get out of the sells of her product and also on the costs the other firm has producing their quality.

Changes in price p_i influence the profit in two different ways. First, an increase in price increases the revenue. Second, a product has to be provided to the host for the review. These costs are fix and necessary to enter this type of contest. For this reason, the influence of rising prices is positive in case the marginal revenue is larger than one.

$$\frac{\partial \Pi_i^*}{\partial p_i} = \frac{c_j^2 h p_i^2 (c_j p_i + 3 c_i p_j)}{(c_j p_i + c_i p_j)^3} - 1 > 0 \qquad if \; \frac{c_j^2 h p_i^2 (c_j p_i + 3 c_i p_j)}{(c_j p_i + c_i p_j)^3} > 1$$

In consequence, firms with more pricy products have a higher threshold to even enter this contest. In the field, there are also arrangements where products are only lent for a limited period. However, this is not only advantages for the firm. The time limit can prevent the product from being compared to several products as there may be more than one comparable product in the market. Sometimes it can even lead to the instance that the host is not able to directly compare it to any product at all.[8] Additionally, a product provided to the host is more likely to become his favorite product, as he does not have to buy it as it is already his property. Usually the personally preferred product is used as kind of anchor for quality and mentioned in many following contest situation most likely in a positive way. This increases buying incentives for the followers in a strong way especially as these hosts often produce some kind of peer effects as e.g. investigated by Entorf and Lauk (2008) investigating social multipliers or Moretti (2011) looking at consumption decisions.

$$\frac{\partial \Pi_i^*}{\partial c_i} = -\frac{2 h c_j^2 p_i^3 p_j}{(c_j p_i + c_i p_j)^3} < 0$$

Increases in cost reduce profit directly but also reduce the probability of winning, because the incentives to produce high quality shrink. Inefficiently high cost are not only

[8]Multiple contests are not modeled in this chapter but possible extensions are practiced in reality.

debilitating in the market but making the contest less attractive.

$$\frac{\partial \Pi_i^*}{\partial p_j} = -\frac{2c_j^2 h p_i^3 p_j}{(c_i c_j p_i + c_i)^3} < 0$$

An opponent rising prices is always decreasing the probability to win meaning gaining some profit from the contest. E.g. a firm with strong reputation that is able to demand high prices is a strong opponent that is maybe even able to prevent the firm from entering the contest.

$$\frac{\partial \Pi_i^*}{\partial c_j} = \frac{2c_i c_j h p_i^3 p_j}{(c_j p_i + c_i p_j)^3} > 0$$

Same applies for firms being able to produce very cost efficient. A high quality product produced in Europe with high costs can find a dangerous competitor in a product produced for lower costs e.g. in Asia while providing decent quality.

These investigations provided an application of Nti (1999) as well as a more detailed idea of valuations represented by asymmetric prices, costs and a symmetric parameter h. However, choosing $r = 1$ and $c_i = 1$ reduce the equation to the results from Nti (1999) with $V_i = p_i$.[9] Especially the new interpretation of the valuation as price to cost ratio influencing decisions is important as it can display profitability of the firm. In contrast to Nti (1999) in this chapter, the influence of the quality choice is set in relation to the changes in winning probability. It has been explained above that the opponent's valuation causes different quality adjustments in case of being the stronger player or the underdog in the contest. However, the resulting csf reflecting the relation between both quality choices is always influenced in the same way, independent of the leadership.

3. Extended Model

The first modeling part aimed to provide a good intuition for the basic mechanisms. However, there are further aspects important when looking at reviewing processes and their results on sales. Getting a positive review is definitely increasing the overall sales but getting a negative review will not leave the sales unaffected, too. Every firm has a certain market share and a negative publicity can reduce this. E.g. a high end Dell laptop is one of the most powerful laptops on the market. If this laptop is not performing well and consistent for a host and has diverse errors, this can drastically reduce trust in the brand and result in decreasing sales.

The new revenue function will include this possibility of losses in case of losing the competition. As the winning probability is given by ϱ_i, $(1 - \varrho_i)$ it declares the probability that player i loses the competition. In consequence, losses are encountered with the factor $(1 - \varrho_i)$.

[9] In general, propositions 4 and 5 apply to the presented model as well, the cost parameter simply causes the inverse reactions as the price parameter.

$$\Pi_i = \varrho_i h p_i - (1 - \varrho_i)\eta_i h p_i - c_i(x_i) - p_i \tag{2.7}$$

The parameter η_i, with $\eta_i \in [0, 1]$, declares the market share without the contest. In case of losing, firm i will lose the part of the followers that would have bought their products without seeing the contest outcome or the bad review, respectively. This means the sales will reduce by $\eta_i h$ in case of being inferior.

This possibility of losing money entering the contest is adding riskiness to the outcome and will have influence on the optimization.

$$FOC: \ \frac{\partial \Pi_i}{\partial x_i} = hp_i \frac{x_j}{(x_i + x_j)^2}(1 + \eta_i) - c_i = 0$$

Again, the second order conditions ensure that the result is indeed an optimal solution for both firms.

$$SOC: \ \frac{\partial^2 \Pi_i}{\partial x_i^2} = -2hp_i(1 + \eta_i)\frac{x_j}{(x_i + x_j)^3} < 0$$

As it can be easily seen looking at the first order condition, this new variation introduced can be reduced to an additional factor following the notation of Nti (1999):

$$V_i = \frac{hp_i(1 + \eta_i)}{c_i}$$

The additional factor $(1+\eta_i)$ implies that a higher market share goes along with a higher valuation in this contest. This means, a high market share makes it more important to win the competition. This can be seen in the new optimal choices, as well:

$$x_i^* = \frac{\left(\frac{hp_i(1+\eta_i)}{c_i}\right)^2 \frac{hp_j(1+\eta_j)}{c_j}}{\left(\frac{hp_i(1+\eta_i)}{c_i} + \frac{hp_j(1+\eta_j)}{c_j}\right)^2}$$

The new optimal choice for j is equivalent. Together, these two terms yield the new corresponding winning probabilities where ϱ_j^* is again equivalent to ϱ_i^*.

$$\varrho_i^* = \frac{\frac{hp_i(1+\eta_i)}{c_i}}{\frac{hp_i(1+\eta_i)}{c_i} + \frac{hp_j(1+\eta_j)}{c_j}}$$

The comparative statics will be very similar for the old parameters included. The only new aspect would be the additional factor $(1+\eta_i)$. For this reason the upcoming analysis is narrowed down to the new parameter η_i and how this factor influences the optimal decision.[10]

$$\frac{\partial x_i^*}{\partial \eta_i} = \frac{2c_i c_j h p_i^2 p_j^2 (1 + \eta_i)(1 + \eta_j)^2}{(c_j p_i(1 + \eta_i) + c_i p_j(1 + \eta_j))^3} > 0$$

[10] In the end of the next section there is a table providing a summary of al comparative statics for the extended model.

The optimal quality choice increases in the market share. As the firm has more to lose, the willingness to improve quality rises. E.g. being the brand leader means the firm has the highest market share. This firm would highly suffer from losing this contest in quality as clients expect a constant high quality so they would lose lots of customers.

$$\frac{\partial \varrho_i^*}{\partial \eta_i} = \frac{c_i c_j p_i p_j (1+\eta_j)}{(c_j p_i (1+\eta_i) + c_i p_j (1+\eta_j))^2} > 0$$

In reality this may even spill over to other products from the same brand as the brand itself stands for high quality. In case this is proven wrong the brand itself can lose its status to some extend. The reason why the firm wants to increase the quality is to prevent failure as rising the quality increases the probability to win.

Facing an opponent with a high market share may again lead to different reactions depending on the leadership as already seen in the section before.

$$\frac{\partial x_i^*}{\partial \eta_j} = \frac{c_j h p_i^2 p_j (1+\eta_i)^2 [c_j p_i (1+\eta_i) - c_i p_j (1+\eta_j)]}{(c_j p_i (1+\eta_i) + c_i p_j (1+\eta_j))^3} > 0 \qquad if \ \frac{p_i(1+\eta_i)}{c_i} > \frac{p_j(1+\eta_j)}{c_j}$$

A stronger firm i tries to keep up with the opponent who in return will increase the quality along with a higher market share. However, being the underdog firm i will resonate and reduce quality because of the strength of the opponent.

$$\frac{\partial \varrho_i^*}{\partial \eta_j} = -\frac{c_i c_j p_i p_j (1+\eta_i)}{(c_j p_i (1+\eta_i) + c_i p_j (1+\eta_j))^2} < 0$$

Even in case of firm i trying to catch up, the stronger position of the opponent will always lead to a decrease in the probability to win independent of firm i being the stronger firm or the underdog.

$$\Pi_i^* = \frac{V_i^3 c_i}{(V_i + V_j)^2} - V_i c_i - p_i = \frac{\left(\frac{hp_i(1+\eta_i)}{c_i}\right)^3 c_i}{\left(\frac{hp_i(1+\eta_i)}{c_i} + \frac{hp_j(1+\eta_j)}{c_j}\right)^2} - (1 + h(1+\eta_i))p_i$$

Having a look at the expected profits, it can be seen that the extension of the model leads to an additional term entering the profit function negatively.

For this reason firms with a higher share would realize higher losses. This goes along with the observation that wellknown firms with high reputation and a strong position in the market are very cautious to make no mistakes.

$$\frac{\partial \Pi_i^*}{\partial \eta_i} = -\frac{c_i^2 h p_i p_j^2 (1+\eta_j)^2 (3 c_j p_i (1+\eta_i) + c_i p_j (1+\eta_j))}{(c_j p_i (1+\eta_i) + c_i p_j (1+\eta_j))^3} < 0$$

But, a strong opponent causes a reduction in the expected revenue as well. The high quality realized by the competitor leads to a higher probability to actually lose the tournament and realize the losses.

$$\frac{\partial \Pi_i^*}{\partial \eta_j} = -\frac{2c_i c_j^2 h p_i^3 p_j (1+\eta_i)^3}{(c_j p_i (1+\eta_i) + c_i p_j (1+\eta_j))^3} < 0$$

As it can be seen above, a higher η_i can make it very unattractive to enter the contest. On the one hand, a firm facing a very strong opponent may not be willing to enter the contest as chances of losing would be too high. On the other hand, a firm with a high market share has much at stake, that even with a very low probability of failing, losses would be rather painful.

Unfortunately, especially these well known firms are often big players who are very requested by the followers to be reviewed. For this reason, these firms are mostly not able to choose wether or not to enter the contest. The hosts can either buy these products on their own or lent it somewhere. An other possibility is, that a retailer sponsors the product or lent it for review in exchange for being mentioned by the host. In this case, the entrance fee has not to be paid by the known firm but is payed by someone else. Unknown firms are still able to choose whether or not to enter the contest. This will be profitable if they are able to compete with the high quality standards of the known firm in any means. In this case of an unknown firm with a product of solid quality it might be beneficial to enter the contest as it might be inexpensive but effective advertising.

4. Taste Model

Product tests performed by institutions try to be as objective as possible. However, this is different for hosts making reviews. Followers want to know the personal opinion of the blogger. How they think about quality, usefulness and handiness. The personality of the host is a very important characteristic in Blogs and Channels. Many bloggers or vloggers (video bloggers) publish content on their private life, what they are doing at the moment, what experiences they are making and what they are thinking about. A blog is forming a community where the followers are not only interested in the information they are receiving but in the person as well. For this reason many hosts make FAQ videos, where they answer many questions about themselves not only regarding the topic their channel is dealing with, but about their personal opinion and life as well. For this reason, they produce so called "follow me arounds" to introduce them to their life, their work, their background like Slimani (2013). The relation between a host and his followers is always much more private than between an editor and the readers, for example. For this reason, especially subjective opinions can be evaluated very accurately by the consumers as they have lots of background information on the reviewer.

Why is this difference so important? One could expect a higher quality product to be always better respectively preferred to a lower quality product. However, as a product can be used for different purposes the demands of the consumers can differ. A professional photographer looking for a new camera may prefer a very reliable solid camera. In case he is working in a studio resting the camera on a tripod most of the time, he will not care about the weight as much. In consequence, he would claim a big sturdy camera of good quality as fitting to its taste. A travel photographer on the other hand is looking for a solid and reliable camera as well but it also has to be lightweight and got to be carried around the whole day. Both are looking for a good camera but have very different ideas what that means.

These tastes need to be met by the product to be bought by the customers. It can be assumed, that the taste of the followers resemble the taste of the host as they would otherwise not visit this channel for informational reasons. Certainly, there are some more entertaining channels on the internet, providing more comedy like content. However, they may be good for publicity of the product and for creating a certain image, this is not modeled in this chapter.

This aspect of taste can be introduced in the model above by some minor adjustments: instead of using the quality x_i as variable of choice for the csf, an indicator of the fit of the product to the taste of the host can be used. The new interpretation will be called d_i[11]. As a higher quality is indicated by a higher number, the distance is wished to be smaller to be better. To transform the distance indicator into a variable behaving as the quality the taste variable t_i can be defined as follows:

$$t_i = \frac{1}{d_i} \tag{2.8}$$

Because t_i is increasing when the distance decreases, a higher number in t_i is preferred. Thus, the new variable t_i can be introduced in the model replacing x_i. The new csf is given by:

$$\varrho_i = \frac{t_i}{t_i + t_j} = \frac{\frac{1}{d_i}}{\frac{1}{d_i} + \frac{1}{d_j}} \tag{2.9}$$

The new profit function the firm is confronted with, by following the model presented above, will be given by:

$$\Pi_i = \varrho_i h p_i - (1 - \varrho_i)\eta_i h p_i - s_i(d_i) - p_i \tag{2.10}$$

It can be assumed that the cost function will differ to the cost of quality. The new cost function $s_i(d_i)$ can be interpreted as a representation of search costs for the firm analyzing the taste of the hosts and the followership they are representing. In reality some hosts even stay in contact with the firms giving feedback on the products and convey the desires of their followers. According to the cost function used in the model on quality, the cost function for search costs is defined as $s_i(d_i) = s_i \frac{1}{d_i} = s_i t_i = s_i(t_i)$.

[11]The underlying variable d_i could be calculated using the distance measured between the preference of the host θ_h and the preferences the product is meeting θ_i: $d_i = |\theta_i - \theta_h|$.

Table 2.1: Summary

Valuation	Situation	Variable	Consumer	Firm
price		$p_i \uparrow$	$x_i \uparrow$	$\Pi_i \uparrow$*
	i stronger	$p_j \uparrow$	$x_i \uparrow$	$\Pi_i \downarrow$
	i weaker	$p_j \uparrow$	$x_i \downarrow$	$\Pi_i \downarrow$
share		$\eta_i \uparrow$	$x_i \uparrow$	$\Pi_i \downarrow$
	i stronger	$\eta_j \uparrow$	$x_i \uparrow$	$\Pi_i \downarrow$
	i weaker	$\eta_j \uparrow$	$x_i \downarrow$	$\Pi_i \downarrow$
costs		$c_i \uparrow$	$x_i \downarrow$	$\Pi_i \downarrow$
	i stronger	$c_j \uparrow$	$x_i \downarrow$	$\Pi_i \uparrow$
	i weaker	$c_j \uparrow$	$x_i \uparrow$	$\Pi_i \uparrow$
		$s_i \uparrow$	$d_i \uparrow$	$\Pi_i \downarrow$
	i stronger	$s_j \uparrow$	$d_i \uparrow$	$\Pi_i \uparrow$
	i weaker	$s_j \uparrow$	$d_i \downarrow$	$\Pi_i \uparrow$

*Only under the condition shown in part 2.

As the relationship between d_i and t_i is negative, the derivative of $s_i(d_i)$ in d_i is negative as well, in other words: to decrease the difference between the preferences the search costs increase.

$$\frac{\partial s_i(d_i)}{\partial d_i} < 0 \qquad \text{while} \qquad \frac{\partial s_i(t_i)}{\partial t_i} > 0$$

Regarding the interpretation of the model results, there are two perspectives that of the customers and of the firms. Customers want to maximize their utility and it can be assumed that a higher quality product or a product fitting their taste better, respectively, is increasing their utility. For this reason a higher investment in the contest is positive for customers. This is different to the consideration in the standard rent seeking literature. In rent seeking contests the aim is to have a preferably low effort as this effort can be seen as wasted. Again, in this model the pricing decision for the product is assumed to be made in beforehand.

In this model, three types of valuations can be differentiated: the price, share and costs. All can be reduced to the valuation term following Nti (1997) during the optimization process but they influence the decision differently as they enter the profit function in different ways (See Table 2.1).

The first row of each category depicts the reaction on variations in the factors of the own firm. A decrease in quality costs is positive for customers as well as for the firm as the product will be better as well as the expected profit will increase. An increase in the market share ensures good products for the customers but leads to higher expected losses and therefore to lower expected profits. The market share is often linked to high prestige for firms and even lower profits are approved to increase the market share (e.g. Simon (2015)). However, the results presented in this chapter indicat that a lower market share would be more desirable in some cases. The higher potential losses caused by the higher marked share should not be neglected in the overall calculation.

An increase in prices leads to better products and higher profits. This establishes a positive relation between increasing prices and quality. Certainly, the price can be expected to influence the utility of the consumer negatively at the same time. For this reason a higher market share and therefore higher losses are likely to be more preferable for customers than an increase in price leading to better products. Lower costs can be considered as positive for the firm as well as for the customers in general. In contrast, the effects of market share and price could depend on the actual preferences of the firm and the customer which may not only be influenced by better products and higher profits as described above.

Regarding changes in the opponent's valuation it can be seen that a reduction of the distance between the two firms is always resulting in a better product as the competition gets closer. In this context the distinction between the reaction as stronger and weaker firm becomes comprehensible. The perspective changes at the point of being even and therefore the reaction differs for the two cases. The expected profit always suffers in case of an increase in valuation on the opponent's side (meaning an increase in price or share as well as a decrease in costs).

5. Combined Model

The previous chapters considered either quality or taste as relevant variable for the valuation. However, these are two extremes: one being very objective, only looking at the quality and one being totally subjective, considering just the individual taste. It can be expected that a more realistic evaluation will take both aspects into account. Most reviewers will consider the quality as well as the individual taste to a certain amount. For this reason, a csf that combines both aspects would be of interest. It can be seen as most reasonable that quality and taste will work together as complements. A higher quality product that fits the taste will be valued much higher.

$$\varrho_i = \frac{x_i(1+\alpha t_i)}{x_i(1+\alpha t_i) + x_j(1+\alpha t_j)} \tag{2.11}$$

This new csf includes taste as additional aspect. The influence it has on the overall decision is reflected by $\alpha \geq 0$. With an α of 0 the model reduces to the objective model regarding quality only. This csf follows the model introduced by Epstein and Hefeker (2003). As there are now two aspects entering the decision process, the firm's profit function needs to be adjusted as well:

$$\Pi_i = \varrho_i h p_i(1+\eta_i) - \eta_i h p_i - c_i x_i - s_i t_i - p_i \tag{2.12}$$

The profit function developed in Section 3 is now enlarged by an additional cost function. Now both costs for quality x_i as well as taste t_i are considered. In contrast to Epstein and Hefeker (2003), I use a distinct cost function for each instrument as it is very unlikely that both aspects will cause the same costs.

The new first order conditions are:

$$FOC\ 1: \ \frac{\partial \Pi_i}{\partial x_i} = 0 \ \Leftrightarrow \ \frac{x_j(1+\alpha t_j)(1+\alpha t_i)}{(x_i(1+\alpha t_i)+x_j(1+\alpha t_j))^2} \underbrace{\frac{h(1+\eta_i)p_i}{c_i}}_{V_i^c} = 1$$

$$FOC\ 2: \ \frac{\partial \Pi_j}{\partial x_j} = 0 \ \Leftrightarrow \ \frac{x_i(1+\alpha t_i)(1+\alpha t_j)}{(x_i(1+\alpha t_i)+x_j(1+\alpha t_j))^2} \underbrace{\frac{h(1+\eta_j)p_j}{c_j}}_{V_j^c} = 1$$

$$FOC\ 3: \ \frac{\partial \Pi_i}{\partial t_i} = 0 \ \Leftrightarrow \ \frac{x_j(1+\alpha t_j)\alpha x_i}{(x_i(1+\alpha t_i)+x_j(1+\alpha t_j))^2} \underbrace{\frac{h(1+\eta_i)p_i}{s_i}}_{V_i^s} = 1$$

$$FOC\ 4: \ \frac{\partial \Pi_j}{\partial t_j} = 0 \ \Leftrightarrow \ \frac{x_i(1+\alpha t_i)\alpha x_j}{(x_i(1+\alpha t_i)+x_j(1+\alpha t_j))^2} \underbrace{\frac{h(1+\eta_j)p_j}{s_j}}_{V_j^s} = 1$$

Again, a valuation type term can be defined that differs in the costs for each instrument. For this reason, the valuation including quality costs will be called V_i^c while the valuation depending on the costs for meeting the taste will be named V_i^s.

Using the first order conditions, several relations between the choice variables can be derived:

$$\begin{array}{ll} R\ 1 : (FOC\ 1\ \&\ FOC\ 2) & x_j = x_i \frac{V_j^c}{V_i^c} \\ R\ 2 : (FOC\ 3\ \&\ FOC\ 4) & t_j = (\frac{1}{\alpha}+t_i)\frac{V_j^s}{V_i^s} - \frac{1}{\alpha} \\ R\ 3 : (FOC\ 1\ \&\ FOC\ 3) & x_i = (\frac{1}{\alpha}+t_i)\frac{V_i^c}{V_i^s} \\ R\ 4 : (FOC\ 2\ \&\ FOC\ 4) & x_j = (\frac{1}{\alpha}+t_j)\frac{V_j^c}{V_j^s} \end{array}$$

The optimal choices having two instruments are calculated in the following.

$$x_i^* = \frac{(V_i^c)^2(V_j^c V_i^s V_j^s)}{(V_i^c V_i^s + V_j^c V_j^s)^2}$$

The optimal choice from the one dimensional model was $x_i^* = \frac{(V_i)^2 V_j}{(V_i+V_j)^2}$. It can be seen, even better than in Epstein and Hefeker (2003), that the characteristics stay the same. The corresponding valuation V_i^c has a quadratic influence on the optimal decision while all remaining valuations, for the other instrument as well as both of the opponent, enter single only.

$$t_i^* = \max\left\{\frac{(V_i^s)^2(V_i^c V_j^c V_j^s)}{(V_i^c V_i^s + V_j^c V_j^s)^2} - \frac{1}{\alpha}, 0\right\} \tag{2.13}$$

The optimal choice for the second instrument is equivalent to the first one but includes the negative term $\frac{1}{\alpha}$ reflecting the influence of the weighting factor α in the reviewing process. While x_i^* is always larger or equal to 0, as the valuations can only be larger or equal 0, t_i^* could become negative for small α. As the taste variable should not become smaller 0 it yields $t_i^* = 0$ for those cases.

$$\varrho_i^* = \frac{V_i^c V_i^s}{V_i^c V_i^s + V_j^c V_j^s} \tag{2.14}$$

Again the result from the one-dimensional model was very similar, $\varrho_i^* = \frac{V_i}{V_i+V_j}$, but now the winning probability depends on the valuations of each instrument.

In order to ensure investigating optima, the second derivative has to be considered. As the optimization problem has enlarged to two instruments, the Hessian matrix has to be negative semidefinite. Unfortunately this important step is missing in Epstein and Hefeker (2003).

In standard Tullock contests the second order condition is easily met but in the two dimensional case the Hessian has to be negative semidefinite i.e. condition 1 and 3 need to be fulfilled, ensuring the first leading principal minor to be negative and the second to be positive.

$$Condition\ 1: \ H_{11} = \frac{\partial^2 \Pi_i}{\partial x_i^2} = -\frac{2(1+\alpha_i t_i)^2 h p_i (1+\eta_i)(x_j + \alpha t_j x_j)}{(x_i + \alpha t_i x_i + x_j + \alpha t_j x_j)^3} \leq 0$$

$$Condition\ 2: \ H_{22} = \frac{\partial^2 \Pi_i}{\partial t_i^2} = -\frac{2\alpha^2 (1+\alpha t_j) h p_i (1+\eta_i) x_i^2 x_j}{(x_i + \alpha t_i x_i + x_j + \alpha t_j x_j)^3} \leq 0$$

$$H_{12} = \frac{\partial \Pi_i}{\partial x_i \partial t_i} = \frac{\alpha(1+\alpha t_j) h p_i (1+\eta_i) x_j (-(1+\alpha t_i) x_i + x_j + \alpha t_j x_j)}{(x_i + \alpha t_i x_i + x_j + \alpha t_j x_j)^3}$$

$$Condition\ 3: H_{11} * H_{22} - H_{12}^2 \geq 0 \qquad if\ x_j(1+\alpha t_j) \leq 3[x_i(1+\alpha t_i)]$$

As shown above, conditions 1 and 2 are the same as the second order conditions in standard Tullock contests. The derivative is negative in both cases. The more delicate part is contidion 3. This equation holds under certain conditions only. [12]

In the following it will be derived under which circumstances the optimization is valid: For this purpose two relationships $x_i = x_j \frac{V_i^c}{V_j^c}$ $(R1)$ and $t_i = (\frac{1}{\alpha} + t_j)\frac{V_i^s}{V_j^s} - \frac{1}{\alpha}$ $(R2)$ are used.

$$x_i \frac{V_j^c}{V_i^c}\left(1 + \alpha\left(\left(\frac{1}{\alpha} + t_i\right)\frac{V_j^s}{V_i^s} - \frac{1}{\alpha}\right)\right) \leq 3[x_i(1+\alpha t_i)]$$

$$\Leftrightarrow \qquad x_i \frac{V_j^c}{V_i^c}\left((1+\alpha t_i)\frac{V_j^s}{V_i^s}\right) \leq 3[x_i(1+\alpha t_i)]$$

$$\Leftrightarrow \qquad \frac{V_j^c}{V_i^c}\left((1+\alpha t_i)\frac{V_j^s}{V_i^s}\right) \leq 3(1+\alpha t_i) \tag{2.15}$$

[12] The condition would be equivalent for Epstein and Hefeker (2003).

As it can be seen above, the ratio between the valuations of both firms is important. If the ratios are 1, the equation holds but if the differences are too large the condition will not be met any longer. This means, for competitors with valuations in the same range the optimization can be done.

In order to derive an exogenous condition the optimal solution t_i^* from equation (2.13) is inserted in the inequality (2.15).

$$\frac{V_j^c}{V_i^c} \leq \frac{(6+V_i^s)V_j^cV_j^s + 3V_i^cV_i^s(2+\alpha V_i^sV_j^cV_j^s)}{V_i^cV_i^s + V_j^cV_j^s} \tag{2.16}$$

The same must hold for the opponent at the same time.

$$\frac{V_i^c}{V_j^c} \leq \frac{(6+V_j^s)V_i^cV_i^s + 3V_j^cV_j^s(2+\alpha V_j^sV_i^cV_i^s)}{V_j^cV_j^s + V_i^cV_i^s} \tag{2.17}$$

As both instruments will not become smaller 0 we have an interior solution in case the conditions (2.16) and (2.17) are fulfilled. In this case, the extremum is the only extremum and therefore is a maximum. This is the optimal point. We have an interior extremum in all cases except those where the second instrument is zero. For further calculations, the condition is assumed to be fulfilled.

In order to gain intuition in which cases the condition (2.16) may hold, I investigate the limits for the different valuations going towards 0.

$$V_j^c \to 0: \qquad 0 \leq 6$$

If their own valuation for quality of the opponent goes towards 0 the condition will be always fulfilled. This is intuitive as in this case the opponent becomes weaker. However, this condition has to hold for both players at the dame time. The next limit will show, that in the above case the opponents condition would not hold and therefore no equilibrium would exist.

$$V_i^c \to 0: \qquad V_j^c \leq 0 \qquad \lightning$$

Letting the own valuation for quality go toward 0 would need the opponents evaluation to be negative. This means, the condition cannot be fulfilled in the limit. This goes along with the intuition presented above that the valuations should not be too different to have the condition been fulfilled.

$$V_j^s \to 0: \qquad \frac{V_j^c}{V_i^c} \leq 6$$

Similar results can be found looking at the valuations for taste. If this valuation goes towards 0 the ratio between the valuation for quality has to be in a certain range. The same holds for their own valuation for taste.

$$V_i^s \to 0: \qquad \frac{V_j^c}{V_i^c} \leq 6$$

As mentioned above, the second instrument is not always used. This has been shown by Epstein and Hefeker (2003) for their model in beforehand. In this specific model the condition slightly changes. To derive the condition, the optimal solution given in equation (2.13) is used.

$$t_i \geq 0 \qquad if\ \frac{(V_i^s)^2(V_i^c V_j^c V_j^s)}{(V_i^c V_i^s + V_j^c V_j^s)^2} \geq \frac{1}{\alpha} \tag{2.18}$$

Now the valuations are expressed in terms of each other following the idea of Epstein and Hefeker (2003) as $V_j^c = k^c V_i^c$ and $V_j^s = k^s V_i^s$. Inserting this in the equation above leads to the condition given in equation (2.19).

$$\frac{(V_i^s)^2(V_i^c k^c V_i^c k^s V_i^s)}{(V_i^c V_i^s + k^c k^s V_i^c V_i^s)^2} \geq \frac{1}{\alpha}$$

$$\Leftrightarrow \qquad V_i^s \geq \frac{(1+k^c k^s)^2}{\alpha k^c k^s} \tag{2.19}$$

$$\Rightarrow\ t_j \geq 0 \qquad if\ \ V_i^s \geq \frac{(1+k^c k^s)^2}{\alpha k^c (k^s)^2} \tag{2.20}$$

Using $V_j^s = k^s V_i^s$ and otherwise performing the same transformations for firm j as for firm i leads to equation (2.20). Inserting the comparative way of expressing the valuations using k^c and k^s in equation (2.14) reveals that firm i is leading if $k^c k^s > 1$.
For i being stronger it follows, that if $V_i^s \geq \frac{(1+k^c k^s)^2}{\alpha k^c k^s}$ both use their second instrument t_i and if $\frac{(1+k^c k^s)^2}{\alpha k^c k^s} \geq V_i^s \geq \frac{(1+k^c k^s)^2}{\alpha k^c (k^s)^2}$ only j uses the second instrument. In consequence depending on the difference in valuations both firms, no firm or just the weaker firm will use the second instrument.

Having a second instrument at choice in this reviewing process, it is straightforward, as result 2 from Epstein and Hefeker (2003) shows, that the chosen amount of the first instrument tends to be smaller than with one alone. This means the quality will be at least not higher than in the one dimensional case. On the other hand consumers now get a product with possibly slightly lower quality but more fitting to specific needs which may increase their overall utility. Epstein and Hefeker (2003) proved that rent dissipation, $RD = x_i + x_j + t_i + t_j$, increases for small differences in valuation. Hence, a close enough contest can be expected to provide more utility for the customers in case of having two instruments at choice.[13]

This last step modeled in this chapter showed a possibility to combine the two as-

[13] Epstein and Hefeker (2003) argue in Result 3 that for larger differences in valuation the rent dissociation will decrease. However, in case the difference in valuation is large enough an equilibrium may not be ensured as shown above. In consequence, it would be very important for Epstein and Hefeker (2003) to consider the limitations in optimization occurring by introducing a second instrument in a Tullock contest.

pects taste and quality in a reviewing process using a Tullock type contest. As shown above, this is possible for contests with opponents having a small enough difference in valuation, only. Otherwise the results are no longer guaranteed to provide an optimal result. Adapting to the results of Epstein and Hefeker (2003) the second instrument is more likely to be used by the underdog. Furthermore, the possibility of having a second instrument at hand increases the rent dissipation compared to the one instrument model. This means that especially younger firms being the underdog in the competition will use the possibility to adapt to fit the taste of the host. This can be seen in reality as these upcoming firms are usually more active in social media and therefore doing more search regarding what taste a host has. This might not be able to make the underdog becoming leading in probability but increases the probability to win after all. However, including the taste provides the consumers with a product increasing their utility most likely as the performance of both products will be raised. At the same time, the negative influence on profits might be small, as the modern firms are more familiar with social media which reduces their costs for using the second instrument most likely.

6. Conclusion

Blogs and channels are playing a more and more important role in advertising and marketing. For this reason, new media with its rules and new processes are becoming more interesting for firms and scientists. Modern firms are almost forced to engage in social media and other online channels in order to decide how to position their company in this environment.

In this chapter, I modeled the evaluation process of a host reviewing and comparing two products. The level of complexity increases taking losses and different evaluation criteria into account. The models reveal that prices, costs and market share can be converted in a valuationlike variable. It has been shown above that, nevertheless the influence of these three types of valuations enter the optimization process similarly influence the objective function of the firm in totally different ways.

The comparative statics of the one dimensional model permitted several insights. On the one hand, especially unknown firms with lower costs and lower prices that are producing decent quality products might be able to benefit from this type of contests. These firms can use the contest as inexpensive advertisement to promote their product and to win new customers. On the other hand, firms that own a high market share are those who suffer the most from the possibility of losing customers due to the contest outcome. However, these firms are often not able to avoid the contest situation as they are requested to be compared in the reviewing process. In consequence, the firms do not need to invest the entrance fee of sending in a product but are forced to compete because the host buys or lends the product itself or the product is provided by a retailer for being named as sponsor. This mechanism of holding a known and successful firm in the contest forces the firm to increase the quality of the product even further to minimize the probability of experiencing the potentially large losses in case of being beaten by another firm.

After providing possibilities how to model an objective competition in quality as well as looking at the possibility of subjective evaluations where the firms want to match the host's taste best, both, the objective and the subjective criteria, should be combined in one contest where the amount of subjective evaluation may vary. This setting is investigated in the last model presented. This two-dimensional approach models taste as additional instrument to quality entering as complement in the evaluation process. This chapter reveals that the optimal solution is only guaranteed to be existing under certain conditions where the difference in valuations is sufficiently small. This restriction has not been taken into account by Epstein and Hefeker (2003). For the optimal solutions, it can be seen that the winning probability is determined by costs of the first and second instrument equally. The weight of the second instrument enters as threshold for the second instrument being used only.

It has been shown that the possibility to meet the taste of a host is especially attractive for the weaker firm and is used by these firms more likely. Considering the possible benefits for the customers, the evaluation process including subjective and objective criteria is most likely to increase the utility of the consumers as the rent dissipation rises for firms being sufficiently similar in valuations. In consequence, this model underlines a reason why the new rules in new media provide the consumers with more benefits not only because they are informed better but because their utility might be increased even more, because of products with better quality respectively a better fit.

In this chapter, I gave an example on how new media implement new processes that include new characteristics. The most important insight given is, that even in this new setup, mechanisms can still be led back to wellknown models like the Tullock tournament model used in this case. The crucial point is to isolate new driving characteristics and to include these into the old models. In my case, I introduced the aspect of a subjective criterion that became reliable due to the personal contact between host and followers. Furthermore, I presented a tournament model combining two criteria, the subjective criterion, taste, and the objective criterion, quality.

In the upcoming years new media will even increase their importance in our everyday life. Furthermore, they will become crucial for marketing and sales to some extend. For this reason, research like it is presented in this chapter will become important in order to understand the new conditions that are determining modern life. The online world might seem very complex and vast, but this should not discourage researchers from isolating and modeling particular aspects in order to gain important insights on the world we are living in today.

Chapter 3

Women in Academia Empirical Findings from Business Administration in Germany

In this chapter, I analyse a data set of German PhD students and professors in Business Administration (BA). It provides information on the trends in prospects of women in a male dominated profession. The collected data reveal that since 1998 the transition rate from PhD students to professors for women has become equal to that of men. While the increase in the percentage of female PhD students is quite linear, the trend in the percentage of female professors shows a significant increase in the beginning of the 90th. This is a remarkable increase especially when compared to the development in the US. The change in trend coincides with the establishment of laws to ensure gender equality in hiring new professors in Germany during the nineties. In this chapter, I addresse the mentioned incidence via an event study providing insights in the German gender equality laws introduced in several federal states. Further analyses regarding the faculties' composition and the decision of PhD students to become professors are featured. Also, this chapter presents significant influences of fellow students' gender composition as well as the gender composition of full professors at a faculty.

1. Introduction

Life of women has changed drastically, especially during the last century. Whereas merely a few decades ago, it was considered appropriate for a woman to raise children and to do the housework, nowadays, double income households are likely to become the new standard. In consequence, women are entering the labour market which leads to changes in family structures, labour policy etc., though, not all women do choose to work in typical female work fields like education, nursing or administrative assistance. Interestingly, they started to enter male dominated sectors like finance, technology and science to name just a few. These days, it is common that girls attend university like their male age-mates. As this development is already ongoing for a while, one should expect women to be represented in leading positions by now. However, even today women are rarely found in leading positions of big enterprises, banks, in politics or science. Especially, the amount of female professors in academic fields like science or economics is still relatively small.

Often, ad hoc examples and agglomerated data are used as arguments when discussing gender equality in careers. In this chapter, I use a data set on a specific male dominated field to provide a stronger foundation than ad hoc examples would do while providing more detailed insights than a more agglomerated data set. The first aim of this chapter is to depict the current situation as well as the progress in the last decades. In this chapter, I focus on the academic field of Business Administration (BA) using data on full professors and doctoral students. BA is still male dominated in the academic world as well as in the world of business. Including professors, one of the highest positions in an academic career, as well as PhD students, allows to compare the changes in both career levels and to draw conclusions on the differences in gender equality in both stages. To increase the assessability of the progresses, I will put the results in an international context by using data from several academic fields in the US. The fact that writing a doctoral thesis can be seen as a preliminary stage for a possible professorship combined with the circumstance that students are taught by professors allows to investigate interactions between two career levels. It is possible that the gender distribution in an academic environment has an influence on decisions affecting women's careers. This thought will be addressed in this chapter focusing on the following two aspects: First, in which way is the gender distribution correlated with the allocation of female professors at the faculties – second, what is influencing the decision of women for or against an academic career as a professor after making their doctorate.

The analysis shows that over the last decades, the amount of female PhD students and professors in BA has raised to approximately one quarter respectively which is low for PhD students but high for professors comparing to similar data from the US. Additionally, the data show that since 1998 the transition rate from PhD students to professors for women has become equal to that of men. While the increase in the percentage of female PhD students is quite linear, the trend in the percentage of female professors features a significant increase in the beginning of the 90th. The growth of the percentage of female German BA professors found in this chapter is impressive compared to the development in the US. This change in trend coincides with the establishment of laws to ensure gender equality in hiring new professors in Germany during the nineties.

This chapter addresses these changes in law via an event study having a deeper look at the German gender equality laws introduced in the federal states. Additionally, the dataset is analyzed regarding two different types of new laws, one with a priority rule concerning the hiring of women and one without. Furthermore, this chapter features an analysis of the faculties' compositions. The investigation shows changes in gender composition of full professors at the faculties regrading samples before and after 1994. After 1994, the gender composition follows a ratio of approximately one quarter with increasing tendency. There was found no influence of the gender composition of faculties on hiring decisions. Concerning the decision of PhD students to become professors, this chapter shows significant influences of fellow students' gender composition as well as the gender composition of full professors at a faculty, whereas, the supervisor's gender does not seem to be related to the decision at all.

The next section gives an overview on literature of various research areas, related to the questions discussed in this chapter. Before starting the investigation of the data set, the collected data are characterized in detail in Section 3. Section 4 provides the whole analysis. Finally, I will sum up the drawn insights and provide a couple of conclusive thoughts.

2. Literature

To start from scratch: Men and women are different in certain aspects. This is an insight conveyed by everyday experience as well as scientific findings. Numerous publications reveal a variety of aspects in which the behavior of both sexes differ. One of the most widespread differences is the amount of risk aversion. Byrnes et al. (1999) analyzed five different age groups in different tasks and found significant differences between the two genders, especially for intellectual as well as for physical risk taking. However, these differences are shrinking as the authors observed a smaller gender gap for younger generations compared to older ones. Jianakoplos and Bernasek (1998) look at financial decision making of single women compared to single men and found women to be more risk averse.[1] They state, this could be part of the explanation why women are less financially wealthy then men.

Another well known characteristic of females is their different attitude towards competition. Already in early life, this pattern is intrinsic as described by Gneezy and Rustichini (2004). In their experiment they observe 140 fourth grade children in their running performance measuring their time needed single and in pair wise competition. They found that the running times in single runs are not significantly different between girls and boys. However, in the competitive scenario girls ran significantly slower while boys ran faster. This finding goes along with Gneezy et al. (2003), analyzing the influence of a competitive environment on performance. They pointed out that the difference in competitiveness also has an influence on the labour supply of women. Additionally, their findings are not just that women perform better in piece rate paid tasks than in tournaments, but, that they perform even worse competing against men then in single sex tournaments. Ivanova-Stenzel and Kübler (2005) made the same observation by analyzing team competition between male and female teams. They confirmed that men spend more effort in presence of women even when they are in the same team – so it seems not to be an incentive of beating the 'weak sex' but showing off in their presence.

As described by Finucane et al. (2000), the above mentioned differences in gender do not seem to be the result of pure genetics – who observed an impact of culture on

[1] They also found that risk aversion depends on other factors as age, ethnic background or number of children.

competitiveness. In their paper the authors describe a gender difference in risk attitudes among 'whites' but not among other ethnic groups. They call this the 'white male' effect. Thus, the cultural background can be crucial for the performance of the sexes. An even more drastic example is provided by Gneezy et al. (2008). They investigated the competitiveness of the Maasai, a very patriarchal society, and the Khasi, a matrilineal Indian society. Finding Khasi women more competitive as Khasi men and even slightly more competitive than Maasai men who are themselves twice as competitive as Maasai women.

As some differences between men and women seem to be not intrinsic but culturally rooted, they might be prone to changes in society. This thought has created several movements during the last centuries and also found its way into sociological research. Epstein (1970) describes how the culture of profession can limit women's career options as a profession may be considered as not being appropriate. He discusses the influence of social changes on traditional structures. Correspondingly, there is a whole field of literature looking at discrimination of women and minorities. In his book Becker (2010) addresses this topic in a theoretical way while e.g. Blau and Ferber (1987) provide empirical evidence of discrimination.

The economic perspective of this inequality builds the root of the enormous field of research on the gender gap. Even theoretical models where formed to explain the gender gap to some extend as in Lazear and Rosen (1990). However, most of the gender gap literature is data based research Ferrer and Azmat (2013) who investigated the gender gap in performance among associate lawyers in the United States. Interestingly, they also found evidence, that career aspiration of women and men differ significantly. A gender gap can be found in many situations like Goldin et al. (2006) who looked at the gender gap among US college students from 1957 to 1992. Their work provides evidence that gender gaps tend to narrow down over time accompanied by a controversy and resulting in changes in society.

These changes in society, the behavior towards women and their appropriate behavior, are just one aspect. Women's identity – the way they see themselves – is changing in this process as well, as described in Bertrand et al. (2010). In consequence, society enables women to behave differently, but, the intrinsic aspiration of women changes as well. This is crucial as it comes to affirmative actions to provide equal chances for both sexes. The effects of affirmative actions and equal opportunity laws in an experimental environment are shown by Schotter and Weigelt (1992). They found, starting from an uneven tournament, affirmative actions lead to more effort over all subjects and, in consequence, to more profit for the game administrator or principal.

However, it is not as simple as that. Probably the most popular and most discussed affirmative action is a quota. Ellingsen et al. (2013) showed – also in an experimental environment – that the introduction of a quota might induce undesirable effects. They found that introducing a quota based selection rule causes uncooperative behavior. This effect appeared for an initial performance based selection rule as the subjects selected via quota are seen as being there without deserving it. Even providing a fairness argument as explanation is not changing the uncooperative behavior. This effect totally disappears by replacing the performance based selection by random choice. Several paper investigate the consequences of introducing a quota in the field like Ahern and Dittmar (2012) who analyzed the quota introduction in Norwegian boards of directors. They recognized a significant drop in firm performance caused by the lack of qualified women needed to fulfill the quota. Here, we see another difficulty in generating equality by changing society. Not only the women have to be willing to fill in the positions they are offered – as explained above, but, we need enough women being qualified to do so as well.

An alternative device to induce changes in society and in female self-concept is the introduction of role models. This is a much less intrusive tool then a politically enforced quota. Marx and Roman (2002) found qualified female role models have significant influence on performance of women in examination situation. Gibson (2004) underlined the influence of role models on our career especially in contrast to mentoring. According to one of the research questions investigated in this chapter concerning the influence of female faculty members on future decisions of doctoral students, Bettinger and Long (2005) already hinted on a positive influence of female role models rising further interest for their topic.

As I analyze the selection of female doctoral students into jobs, likewise, it contributes to the class of selection literature like Bacolod (2007) who investigated the selection of female workers into the profession of teaching. While he aims for the selection mechanism, this chapter intends to mainly quantify the status quo as well as the developments over time.

There are related data sets like the data used by Bosquet et al. (2013) looking at French concours data for promotions of professors. They focus on the competitiveness of women finding that women entered the competition less often whereas the selected sample that entered performed as good as their male competitors. Ülkü-Steiner et al. (2000) use survey data to analyze differences in graduate programs that are male-dominated towards gender-balanced programs. They found evidence, that faculty's gender composition influences career commitment and self-concept of female students in the direction that both tend to be lower in male dominated environments.

3. Data

The data used in this chapter consists of two individual data sets. The first set comprises mile stones in career of German professors in Business Administration (BA) from 1971 to 2006. The data was collected by Rainer Haselmann (Goethe University Frankfurt) and Matthias Kräkel (University Bonn). It includes variables like the place and year of the professor's graduation and habilitation as well as their supervisors' names. Furthermore, the data provides information on subsequent career stages with beginning and end dates. The second data set was collected for this chapter, specifically. It is a data set of BA students who have finished their doctorate in BA in Germany. The data was collected from two sources. The first source used were dissertation registers from German universities. Over 20 % of the German universities provide information from their dissertation registers. The first data point is from 1971 and the last observations are from 2013. To fill in the gaps, data from the German national library (DNB) was used. Every dissertation in Germany has to be transmitted to the DNB. For this reason, we have a full record of all dissertations from 1990 till 2009. The search criteria used were 'dissertation' in the category of academic records and 'Economics (330)' and 'Management (650)' as subject categories. Additionally, only students having a supervisor who is included in the professors' data set were concidered. This step guarantees that the definition of BA applied is congruent within both data sets.

The overall data set contains 23018 observations from Eastern and Western Germany. To match gender data, forenames were used. For this purpose, 4879 names were categorized, resulting in 22801 observations with matched gender.[2] The data set includes 961 professors. The register as well as the electronic doctoral theses from the DNB contain information about the supervisors – using again the names to determine the supervisor's gender – providing 2289 observations with information about the supervisor. In case there were multiple supervisors, always the first supervisor was chosen as that person is expected to have the strongest impact. The professors' data set also includes the duration of dissertation, habilitation and each professorship. In the data set on PhD students only the moment of finishing the doctoral thesis is documented. The time needed to finish the thesis, was estimated with four years and is used accordingly in the PhD students' data set.[3]

There are several surroundings within you make a doctorate in Germany: at a chair – working for the supervisor, at a graduate school, at a scientific institution or at a company. As in Germany a supervisor with a full professorship at a German university is needed to do a doctorate, all these tracks are included in the data set. However, the used datasources have one drawback: Registers and the DNB catalog used to generate the underlying data set do not contain many additional observed characteristics than the ones used in this investigation. Of course it would be advantageous to have information on e.g. cultural background, family status of students as well as professors. For this reason, further causal analysis would require supplementary data. Nevertheless, this extensive data set provides innovative information on the academic field of BA in Germany over the last decades. It includes the full population of full professors in Eastern and Western Germany for the observed period and the same for PhD students after 1989.[4]

[2] It was not possible to translate all names into gender information as some names are not gender specific.

[3] In Germany, universities award a doctoral degree. Strictly speaking, this is not the same as a PhD. However, as it is used synonymously in normal parlance, PhD and doctorate are used synonymously in this thesis.

[4] As described above, the sample is reduced to all observations having a gender specific name for the following analysis.

4. Analysis

This section is divided into four parts. In Section 4.1, we will have a look at the general composition of the data set leading into deeper analysis regarding the development of gender composition over the last decades. In Subsection 4.2, this part is followed by a comparison within an international context using data from the US. Section 4.3 addresses the changes in law regarding hiring decisions in public service during the nineties. Subsection 4.4 provides a deeper look at the faculties and their gender compositions. Finally, we will explore factors that might influence the doctoral students in their career decisions (Subsection 4.5).

4.1 Women in Business Administration

When looking at the data set it becomes obvious that only a small proportion of PhD students stays in academia and obtains a professorship in BA. The data comprises 23,801 PhD students (with known gender) but only 961 of these became BA professors, which implies that 21,840 completed PhDs deciding to choose a profession outside academia or at least choose not to become a professor in their field. In stakes, only 4 % of PhDs remain in academic research and train new generations of scientists in their field (Table 3.1).

Table 3.1: Data Overview

Observation	Total of Observations	Females	Males
Students	23040	-	-
Names	1990	1263	3071
Students with known Gender	22801	5204	17597
Supervisors' Gender	5534	1233	4238
Professors	961	132	829
Other Jobs	21840	5072	16768
Start year of dissertation	956	130	823
End year of dissertation	23018	5200	17513
Percentage of Professors	4 %	3 %	5 %
Percentage of Female Supervisors	7 %	11 %	6 %

The first panel quantifies variables of interest. All characteristics are also splitted into observation numbers for the female and male subgroup. The variable "Names" contains all names found in the data set that could be matched to gender. This data base was used to determine students' as well as supervisors' gender. The second panel provides some resulting proportions giving first hints on gender differences. The first line displays the percentage of professorships on all job choices while the second line describes the amount of female supervisors given the different subgroups of PhD students.

Looking at the amount of females in the data set, we see that women are in distinct minority with 5,204 women. In percentage terms this translates into 77.18 % men and

22.82 % women resulting in a ratio below 1:3. Table 3.1 reveals the selection of women into the different career paths. Of the 961 professors only 132 are female (13.74 %) while we have 5,072 women choosing other professions (23.68 %). In consequence, only 2.54 % of the women holding a PhD decided to become a professor. In contrast, nearly twice as many of men (4.71 %) did so. Running a probit regression we get a coefficient of -0.3*** (0.000) by regressing being a professor on the characteristic of being female. (The p-value is given in parentheses.) In other words, the probability of females to become professors is nearly 30 % lower than that of their male fellows.

Knowing that only a small fraction of PhDs in BA become professors the interesting question is, how this fraction has developed over time. Figure 3.1 shows the total number of completed PhDs in each year since 1990 splittet in those who became a professor in BA (mid grey) and all others (light grey).[5] The total number of dissertations shows a clearly increasing trend. From the beginning of the timeline in 1990 till the end in 2006 the number of students doing a doctorate in BA has more then doubled. However, the amount of PhDs staying in academia as professors has decreased over time as the percentages provided in Figure 3.1 illustrate.

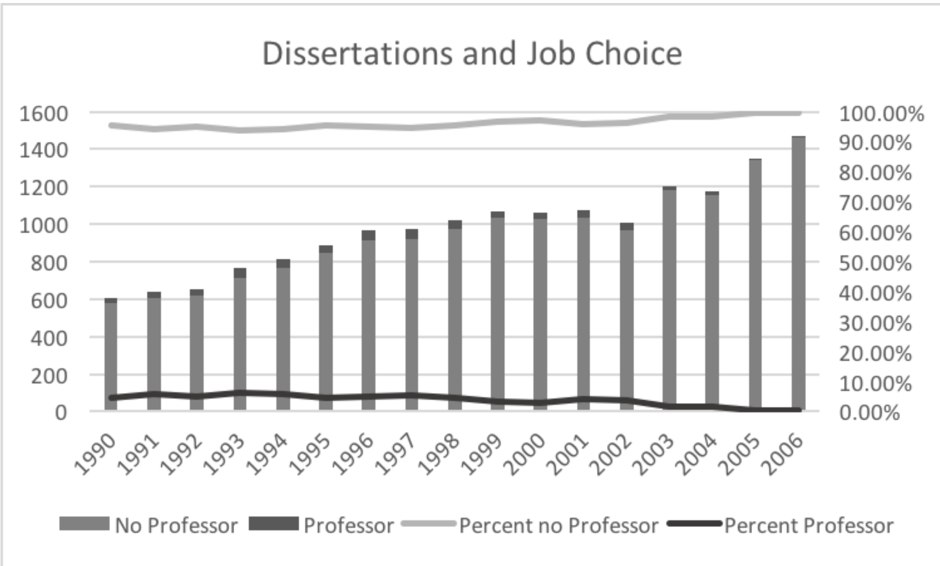


Figure 3.1: Development of Job Decisions over Years

This diagram displayes the job decisions of PhDs over the years. The bar chart provides the total numbers indicated on the left axes while the percentages are given in the graphs with the scale on the right.

Plotting the percentage of PhDs in BA becoming a professor over the same period emphasizes this observation (Figure 3.2). The overall tendency is clearly downward trending. Starting at a value of around 5 %, it goes down to less than 1 % in 2005 and

[5]Here, the data set is displayed from the moment in time, where all sources are included – professors' data, doctorate registers and DNB data.

2006. This shows a change in the use of a doctoral degree. It has developed from the entry stair to an academic career and a career in science to a qualification certificate needed to enter a large amount of professions outside the academic and scientific field.

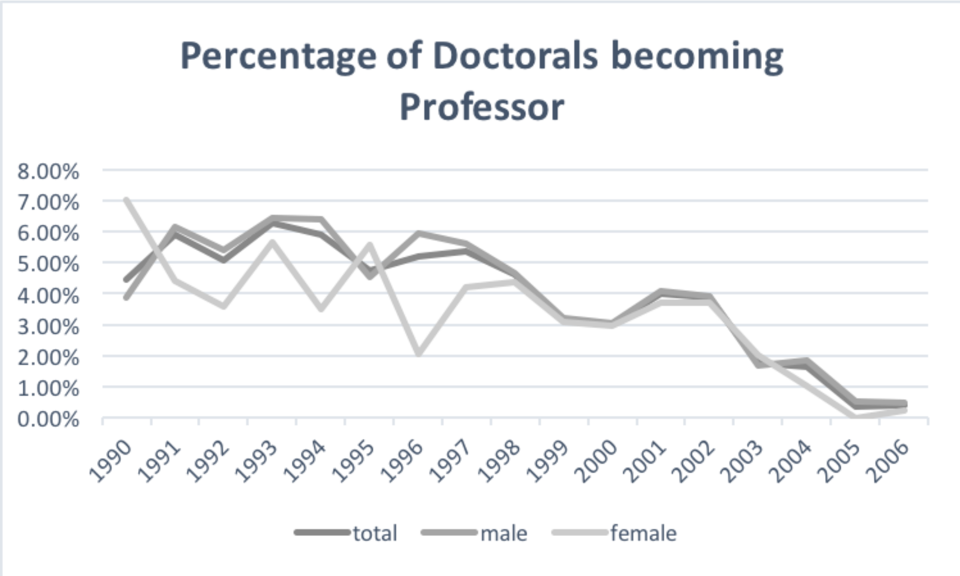


Figure 3.2: Percentage of Doctorals becoming BA Professor, Development over Years

This diagram shows how the percentages of PhDs becoming professor evolved over the years. The development is also provided for the male and female subgroup.

Splitting the sample in male and female PhDs and observing the persistence rate again, two different patterns become apparent. The percentage of females is lower then the male one most of the time, in the first half of the sample. The male graph is alternating at around 6 % while the female evolves at about 4 %. From 1998 onwards, both develop surprisingly similar. This means, taking into account that less females decide to make a doctoral degree, the amount of PhDs becoming professors in their field has already balanced out regarding both sexes. In other words, the women who selected themselves into the group of doctoral students behave as their male colleges, regarding the decision to stay in academia.[6] This is interesting, as – in the nineties – affirmative actions were introduced by the governments of several German federal states in order to give female PhD students equal opportunities to become a professor.[7]

The number of students making their doctoral degree is shown in Figure 3.3. The positive trend can be seen for both, male and female doctoral students. Although, the male doctoral students are still in majority, we can recognize a stronger increase in female PhD students than in male. This is becoming more apparent in Figure 3.4 showing the percentage of female dissertations already starting in 1970. As we now look

[6]The finding goes along with Bosquet et al. (2013) showing that less women opt into a competition while the selected sample in the competition behaves like their male opponents.

[7]This will be discussed in more depth in subsection 4.3.

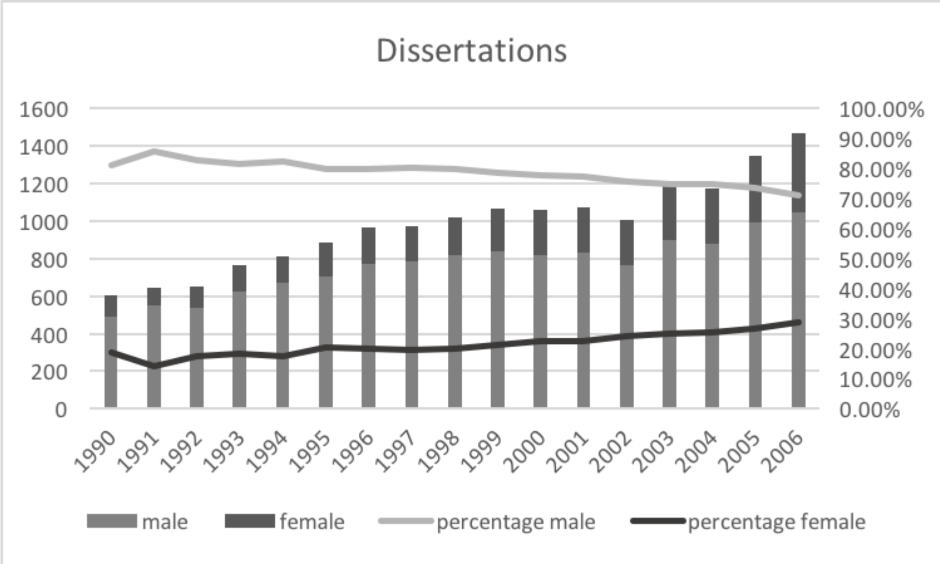


Figure 3.3: Dissertations over Years and Gender

This diagram displays the number of dissertations regarding the male and female subgroups. The bar chart is again providing the total numbers indicated on the left while the percentages are given in graphs with scales on the right.

at proportions instead of absolute numbers, there is more data at disposal. From 1970 onwards, there is data on dissertations from the data set of BA professors and some observations from the registers. Starting in 1990 the DNB data set provides the full coverage of all PhD students in BA in Germany.[8] As assumed before, the percentage of female students is strongly increasing over time starting from nearly 0 % in 1970 and increasing to nearly 30 % in 2006. Due to the lower number of observation, the fluctuation in the graph is quite strong in the first part of the sample (1970-1989). This makes the results more sensitive to outliers. In this first part, the volatility is noticeably higher than in the second period with very low deviations from a common trend. Estimating the increase in percentage of women making their PhD leads to a coefficient of 0.71 (0.000) for the whole sample from 1970 till 2006.[9] Reducing the sample to the core period (1990-2006) leads to nearly the same result (0.72*** with p-value = 0.000). Both estimations are highly significant and fit the data very well, as Figure 3.4 demonstrates.

Next, the same analysis is shown in Figure 3.5 for professors beginning with the number of professors and the gender composition. Starting up from nearly no professors in BA in 1971, the number increases till the end of the eighties where it plateaus fluctuating around a value slightly below 60. We see that the number of female professors started rising in the early eighties staying at a comparatively low level till the beginning of the nineties. This can be seen most easily in Figure 3.6. Looking at percentages we see an enormous peak in the beginning of the time line between 1973 and

[8]The criteria for being a BA student in this data set are described in section 3.

[9]The p-value indicating the level of significance is always given in parentheses unless otherwise noted.

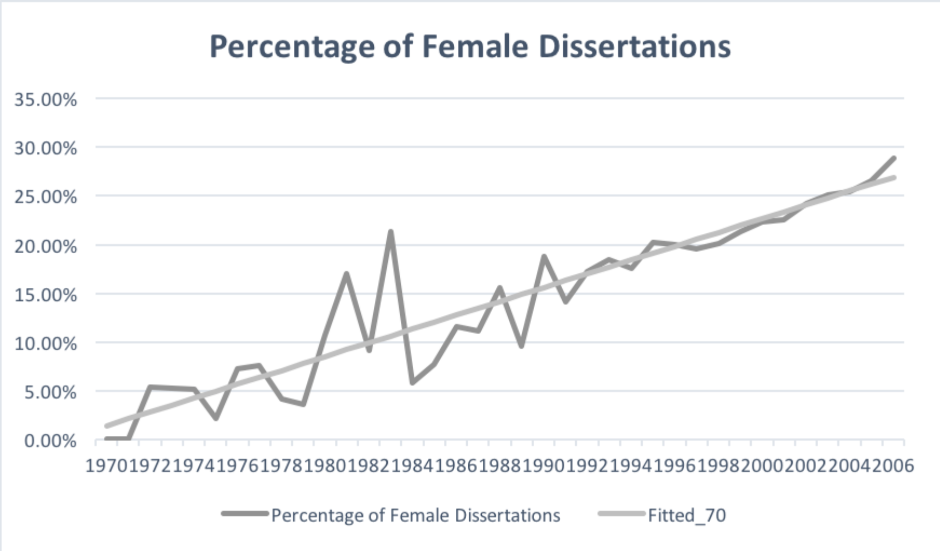


Figure 3.4: Increase in Female Dissertations over Years

The graph displays the percentage of females among new PhDs in BA. The data set includes a smaller sample till 1983 only. A fitted graph is also included in the diagram.

1977. This is an outlier of one woman in interaction with the low overall number of BA professors creating the peak.

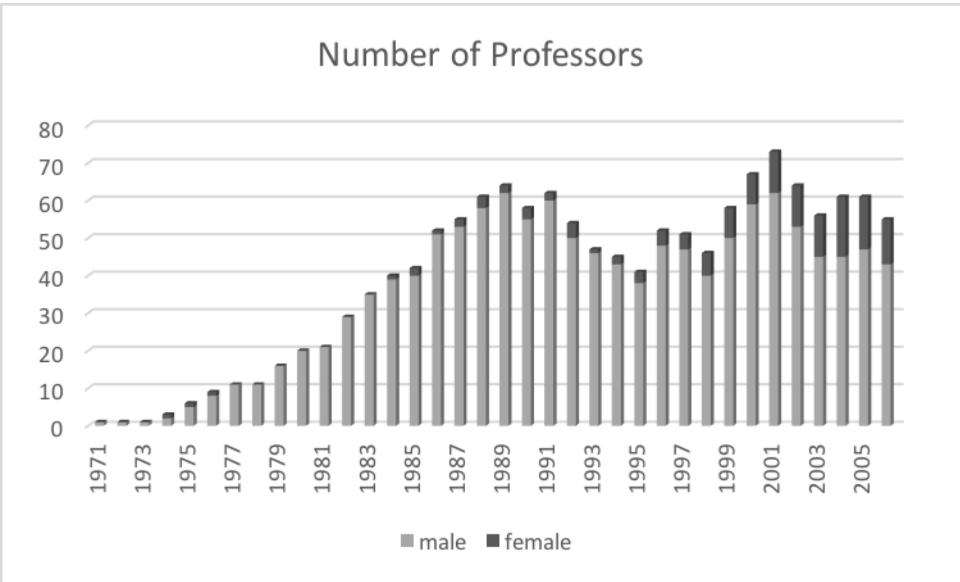


Figure 3.5: Distribution of Professors over Years, conditioned on gender

This bar chart provides the number of professors in BA in Germany and its development over the years.

Starting in 1977 the graph reflects the observations from the bar chart. Figure 3.7

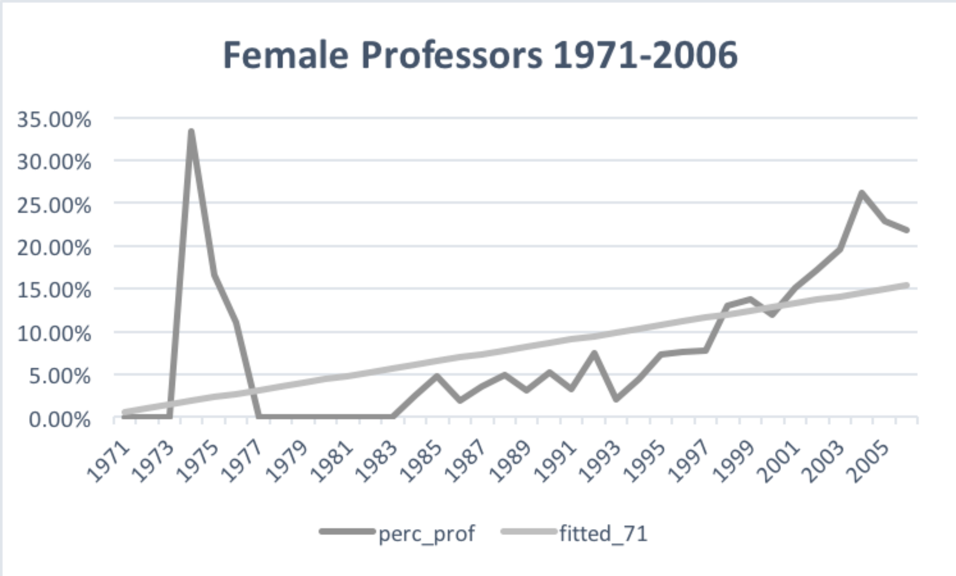


Figure 3.6: Increase in Percentage of Female Professors over Years (1971 to 2006)

The graph displays the percentage of female professors in BA. The data set includes a smaller sample till 1983 only. A fitted graph is also included in the diagram.

also includes a fitted trend. Unlike the approximation for the dissertations (Figure 3.6), most of the points are below the fitted graph between 1984 and 2000 and all later values are exceeding it. Apparently, this linear estimation does not seem to be a good fit. Hence, this could be a hint for a structural break. For this reason the following breakpoint regression is estimated: $fem\_perc = \alpha + \beta\ t + \gamma\ t \times I_b + \delta\ I_b + \varepsilon_t$. I_b is a dummy variable becoming one for breakpoint period b and later.

To evaluate an appropriate year for the breakpoint, several regressions were performed (1994, 1991 and 1989) and are summarized in Table 3.2.[10] A change is very likely as the coefficient incorporating the change in trend is highly significant in all cases. Due to the lowest p-values and the strongest change in trend, Figure 3.8 shows the new fitted values allowing for a break in 1994.

The regressions above were performed for the displayed period from 1977 to 2006. To check the robustness of this breakpoint regression, it is repeated for two additional starting years 1971 and 1977, each with breakpoint in 1994. As Table 3.3 reveals, the significance of the breakpoint can be found independently of the starting year. However, we see changes in the slope for the first half of the regression when including more years. Including years till 1977 leads to a less pronounced increase as there were no female professors before 1984 and after 1977. Starting in 1971 makes the slope even negative because of the peak induced by the one pioneer woman holding a professorship in BA from 1974 to 1976.

[10]The regressions also allow for a change in the constant. The constant is adjusting as well, which is not displayed in the tables as the focus is on the change in trends.

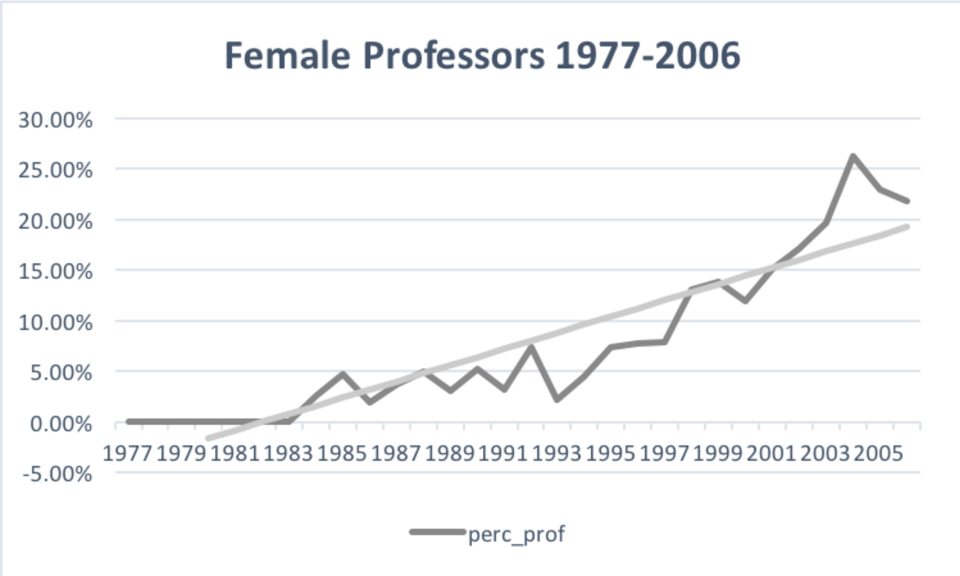


Figure 3.7: Increase in Percentage of Female Professors over Years (1977 to 2006)

The graph displays the percentage of female professors in BA for a sample starting in 1977. A fitted graph is also included in the diagram.

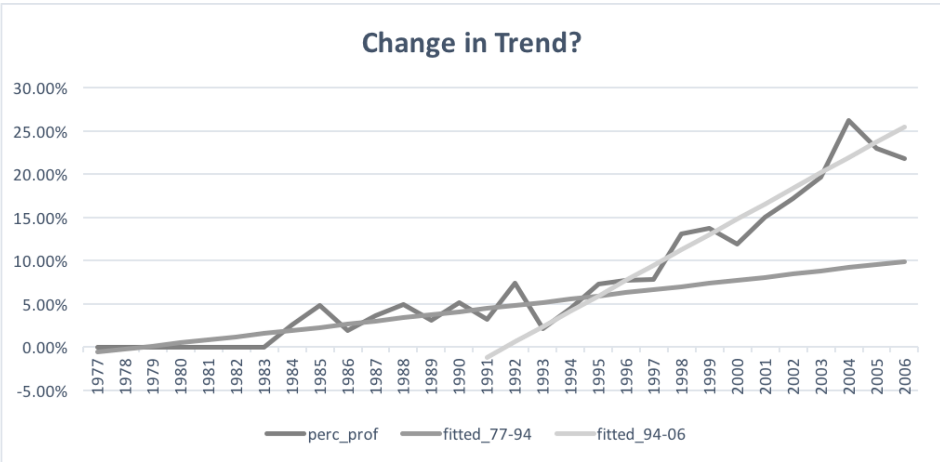


Figure 3.8: Percentage of Female Professors fitted with breakpoint in 1994

The graph displays the percentage of female professors in BA including a graph resulting from a breakpoint regression with breakpoint in 1994.

Table 3.2: Different Breakpoints in Slope

Dependent Variable: fem_perc_t	Regression I (b=1994)	Regression II (b=1991)	Regression III (b=1989)
$time$	0.004***	0.004***	0.005**
	[0.000]	[0.002]	[0.019]
$time \times I_b$	0.0130***	0.010***	0.009***
	[0.000]	[0.000]	[0.000]
I_b	-25.809***	-20.717***	-17.051***
	[0.000]	[0.000]	[0.000]
$const.$	-7.180***	-8.601***	-9.107**
	[0.000]	[0.000]	[0.020]
n	30	30	30
R^2	0.952	0.943	0.9272

This table displays the statistics for the breakpoint regression on the number of female professors over a period from 1977 till 2006. I_b is a dummy variable becoming one for breakpoint period b and later. The breakpoint b is varied between 1994, 1991 and 1989. P-Values are provided in parentheses.

Table 3.3: Different Periods with Breakpoint in 1994

Dependent Variable: fem_perc_t	Regression I ($t_0 = 1971$)	Regression II ($t_0 = 1977$)	Regression III ($t_0 = 1984$)
$time$	0.002	0.004***	0.001
	[0.361]	[0.000]	[0.501]
$time \times I_{1994}$	0.018***	0.013***	0.0015***
	[0.001]	[0.000]	[0.000]
I_{1994}	-36.667***	-25.809***	-30.076***
	[0.001]	[0.000]	[0.000]
$const.$	3.678	-7.18***	-2.913
	[0.356]	[0.000]	[0.507]
n	36	30	23
R^2	0.5243	0.943	0.940

As an additional check for robustness of the results, this table displays the statistics for the breakpoint regression on the number of female professors with different starting periods t_0. I_{1994} is a dummy variable becoming one for the breakpoint period 1994 and later. P-Values are provided in parentheses.

Finally, Figure 3.9 presents growth rates for three periods for the percentage of women on all students completing their PhD on the one hand and the percentage of female professors on the other hand. The first section is from 1985 to 1990 taking 1985 as base year. Over this period, the increase in dissertations is pretty huge, as the percentage more than doubles, while the growth in professorships is only slightly different from zero. Regarding the next two sections the picture becomes a totally different one.[11] Now, the percentage of female professors increases drastically with nearly the same rate as dissertations did beforehand and only becoming a little bit less pronounced in the last period, while the percentage of females making their doctoral degree is staying fairly constant at around 0.4.

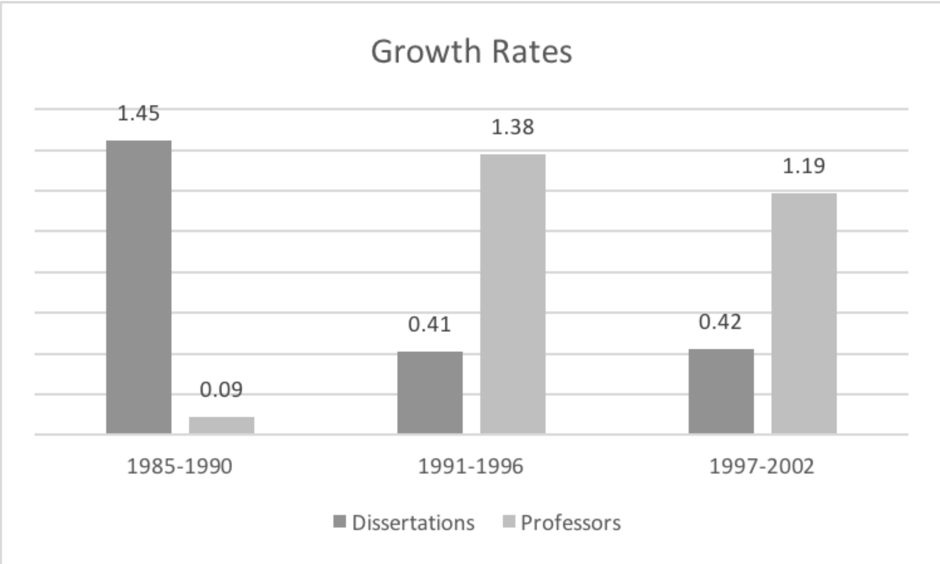


Figure 3.9: Growth Rates of the Percentage of Females

This bar chart provides growth rates for female dissertations and professorships. The base year is always the first year of each period.

Initially, observing the small number of female professors could lead to the presumption of a problem often called 'glass ceiling' meaning that women are able to reach a certain stage of career as well as their male colleges but are mostly unable to reach any higher position in hierarchy. There are several papers about women in science as by Monroe et al. (2008) and Rossi (1965) giving different explanations for this observed effect. Possible reasons are said to be early family influence, a higher interest in social interaction on the women's side, career interruption or a priority of marriage for females, saying that it is more important for females to whom they are married to and what the husband is working as their own stage of career, leading to negligence or harshly spoken sabotage of their own career in favor of a good catch. Bain and Cummings (2000) summarize these reasons for distinctions in careers as result of the cultural background meaning unstated norms in society. Empirically, Ferrer and Azmat (2013) verified different career aspiration between men and women which underlines the arguments above.

[11] The base year is again the first year of each period.

The above analysis shows that the transition rates from PhD to professorship converged for male and female students around 1998. Before 1990 the increase in the percentage of female professors was close to zero but then it increased drastically as the growth rates reveal. This development could be pinned down in a breakpoint regression where the breakpoint seems to be somewhere in the beginning of the nineties. Interestingly, in the nineties there were introduced several changes in law to give females a better opportunity to become a professor as their male colleagues.[12] Starting in 1991 these changes where implemented in several German federal states. These legal changes will be subject of Section 4.3. Despite the fact, that these changes coincide with the German reunion, a corresponding analysis does not yield further insights due to the small number of professors in Eastern Germany.

Overall, the situation of female professors does not look like a glass ceiling problem from the stage of PhD students to professors. However, the low number of females (PhD students as well as professors) is still immanent and probably culturally founded. It would be up for discussion, whether affirmative actions for female PhD students would keep the high transition rate to professorship upright or if it would just lead to an increase of females making their doctoral degree but then opting out before the professor's stage.

4.2 Comparison to Data from the US

In order to bring the data from Germany into an international context, we will use US data for comparison. Burrelli (2008) investigated the percentage of women among US PhD students as well as professors using National Science Foundation (NSF) Data. For this comparison the same data set is used. The NSF provides data from the National Center for Science and Engineering Statistics (NCSES) which is one of 13 federal statistical agencies in the US. They make periodical national surveys and collect other national data in the context of research and development, paying special attention to STEM education (Science, Technology, Engineering, Mathematics).

First, we will have a look at the percentage of females making their PhD, displayed in Figure 3.10. The US data set reaches from 1958 to 2006. The German data set starts in 1970 on the same low level as engineering in the US. From there on, the female percentage in engineering is increasing in both countries, but, the percentage in BA in Germany is rising even more, reaching the fraction of women in mathematics in the last years observed. However, the percentage of female PhDs in BA locates between two fields that are fairly male dominated as the average percentage over all fields shows and fare away from the US percentage in social science.

Performing the same comparison regarding professorships we recognize a surprising pattern (Figure 3.11). The German graph starts out on one level with the percentage of US mathematicians data set in 1984. The data set for engineering starts in 1985 at a certainly lower level. Here, we have to take into account that the number of professors in Germany is just increasing and not very large meaning a small amount of females

[12] A list of the legislative texts with the corresponding year of publication is provided in the appendix.

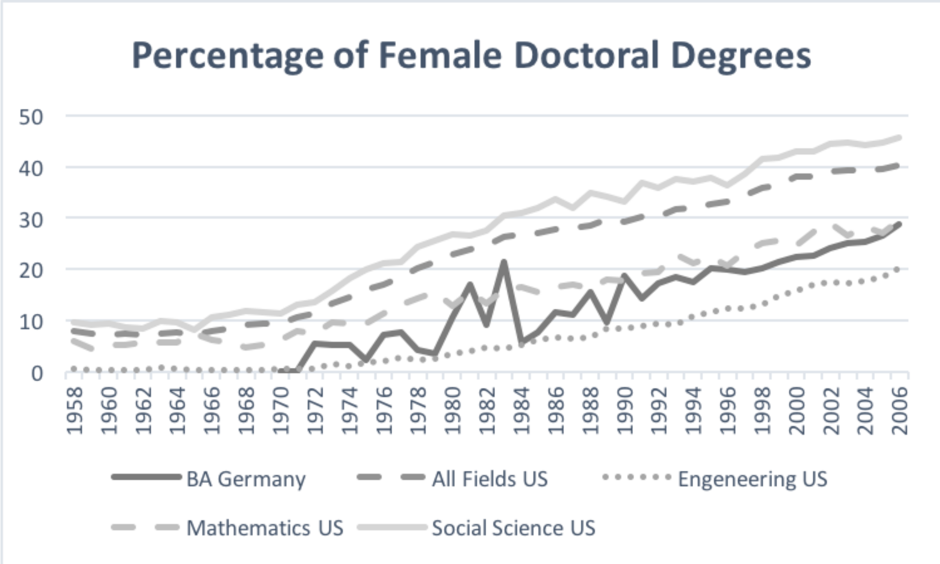


Figure 3.10: Development of German Dissertations in Comparison to Data from the US

This diagram compares the development of the percentage of females making a doctoral degree in BA in Germany to the US in various fields using data from the National Science Foundation (NSF).

is having a stronger impact on a small total of BA professors. Till 1993, the amount of females in BA has shrunk towards the level of engineers in the US. From there on, it overshoots the percentage in mathematics and even the overall fraction leveling with the representation of females in social sciences in the US in the last years observed.

The change in trend for female BA professors can not be found in the US for any kind of studies, as there is a non-kinked smooth increase in percentages in the US. This, again, hints on an external impact on the percentage of female professors in Germany. Furthermore, it could be seen that the percentage of female PhDs in Germany in the field of BA is far below the comparable field of social sciences in the US.

4.3 German Equality Laws

This section will have a deeper look at German gender equality laws. First, a short overview on the historical main steps in gender equality concerning the legal system will be given[13] : The first equality law was commenced in 1958. Prior to that, the husband could decide if his wife is allowed to work and could quit her job without notice. In 1980, a new law was introduced to facilitate equal treatment at work and in 1994 the second equality law followed, which focuses mostly on equal job opportunities for women. The last one was accompanied by new laws in federal states concerning gender

[13] The historical classification is abstracted from LpB (1996) "Dokumentation: Unsere Stadt braucht Frauen - Unser Kreis braucht Frauen".

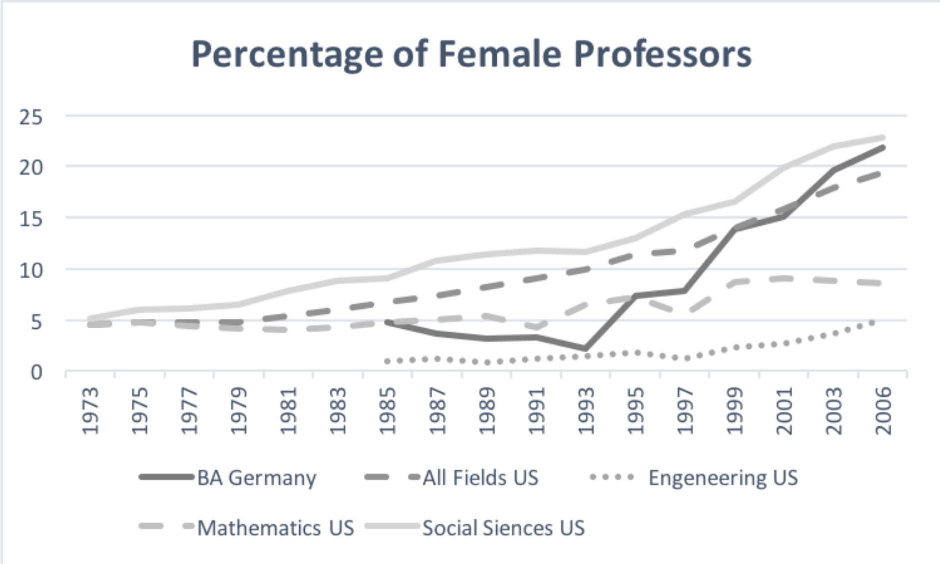


Figure 3.11: Development of German Professorships in Comparison to US Data

This diagram compares the development of the percentage of female professors in BA in Germany to the US in various flies using data from the National Science Foundation.

equality in public service. All these laws were commenced during the nineties.

As these federal laws for public service apply for universities, in particular for the hiring process of new professors, an event study will be performed to align the effects of the changes in law in all federal states. For this purpose the year of the commencement is normalized to period zero. This way the analysis remains its statistical power instead of splitting the dataset federal state wise by introducing controls for the change in law.

Recalling the idea of the breakpoint regression performed in Section 4.1, the breakpoint itself is now intrinsically given by the event period zero. The corresponding regression is performed as follows: $fem\_perc = \alpha + \beta\ t + \gamma\ t \times I_e + \delta\ I_e + \varepsilon_t$. The dependent variable fem_perc_i is the percentage of female professors in period i and I_e is a dummy variable switching to one for all periods after the event. It can be expected that before and after the commencement, behavior could already have been changed earlier or needed a while to adapt. To exclude this noise there are some periods excluded around the event. The index e indicates how many years have been left out before and after the event.

Table 3.4 shows that, already without leaving out any transition period, regression I indicates significant changes when compared to the increase in the percentage of female professors before and after the commencement of the new laws. By increasing the transition period the explanatory power of the performed regressions II to IV becomes even better.

Interestingly, two types of equality laws can be distinguished. Some federal states

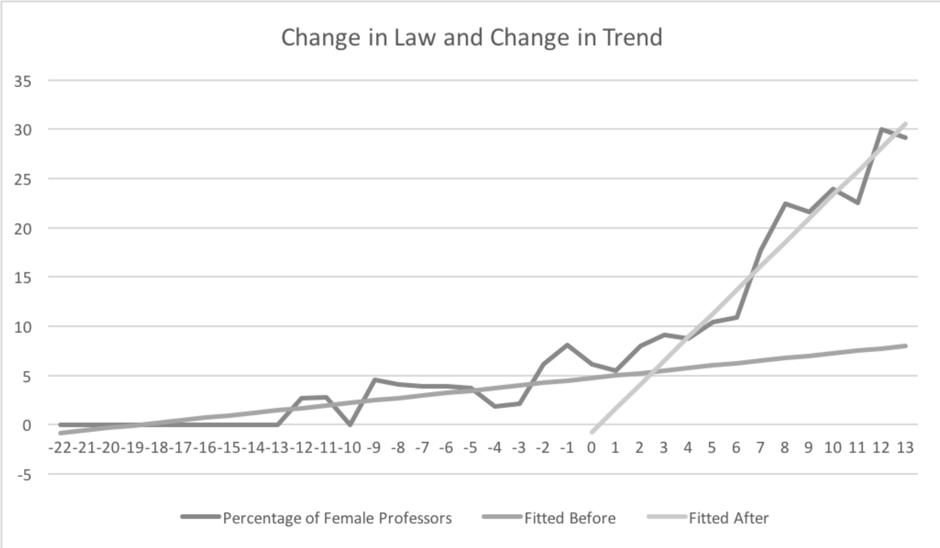


Figure 3.12: Change in Public Service Gender Equality Laws

This diagram shows the increase in the percentage of female professors in the dataset. The year of introduction of the federal gender equality law in public service is set to zero. The fitted graphs display the performed breakpoint regression leaving out a transition period of three years before and after the event.

Table 3.4: Change in Laws

Dep.Var.: fem_perc	Regression I $e = 0$	Regression II $e = 1$	Regression III $e = 2$	Regression IV $e = 3$
$time$	0.003***	0.003***	0.002***	0.003***
	[0.000]	[0.000]	[0.002]	[0.001]
I_e	-0.033**	-0.032**	-0.039**	-0.055***
	[0.016]	[0.034]	[0.025]	[0.008]
$time \times I_e$	0.018***	0.019***	0.021***	0.022***
	[0.000]	[0.000]	[0.000]	[0.000]
$const.$	0.054***	0.048***	0.043***	0.047***
	[0.000]	[0.000]	[0.000]	[0.000]
n	35	33	31	29
R^2	0.961	0.97	0.973	0.976

This table displays the influence of the introduction of the new equality law on the percentage of female professors fem_perc_i *using the normalized data set with the year the new law has been introduced in each federal state set to period zero.* I_e *is a dummy variable switching to one for all periods after the event and the index e indicates how many years have been left out before and after the event. P-Values are provided in parentheses.*

included a priority rule, meaning in case of equal qualification the female applicant has to become favor. 12 federal states chose to include this priority rule into the law: Brandenburg, Bremen, Hamburg, Mecklenburg-Hither Pomerania, Lower Saxony, Rhineland-Palatinate, Saxony-Anhalt, Schleswig-Holstein, Berlin, Saarland, Thuringia and Northrhein-Westphalia. Four federal states chose not to do so, all of them being located in the south of Germany. Only Saxony is a previously eastern federal state while Bavaria, Hesse and Baden-Wuerttemberg are previously western federals states.

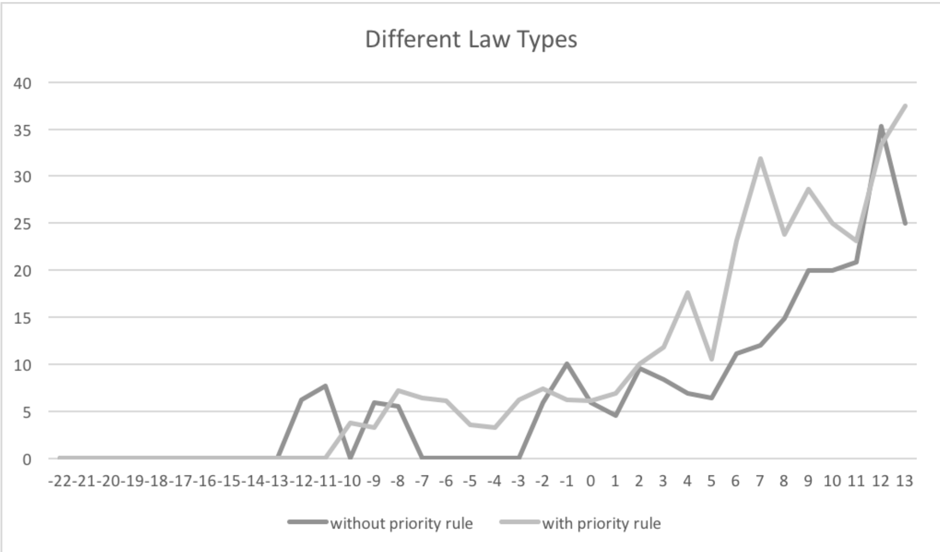


Figure 3.13: Development with and without Priority Rule

This diagram shows the increase in the percentage of female professors in the dataset distinguishing between laws with and without a priority rule. The year of introduction of the federal gender equality law in public service is set to zero.

To evaluate if there is a difference in the percentage of female professors regarding the two law types the following regression was performed: $fem\_perc = \alpha + \beta\ t + \gamma\ t \times I_p + \delta\ I_p + \varepsilon_t$. I_p is a dummy variable indicating that the priority rule is introduced in these federal states. Again, the dataset with the event period set to zero was used.

Regression I (Table 3.5) shows significant difference in the percentage of female professors regarding the period after the event. However, it looks like the values of the federal states without the priority rule are mostly below those of the federal states with priority rule, especially in the first years after the commencement. To address this, regression 2 includes data only till 10 periods after the event. Now, weak significancy appears regarding a difference in slopes.

Testing for significant differences in federal states before the event can be rejected as regression III reveals. For this reason, it can be assumed that the constant for the regression regarding the post event period will be the same. In consequence, the assumption $\delta = 0$ is introduced which allows to bring more statistical power to find a potential difference in slopes as the observations after the event are limited and may be too few to pin down a smaller difference. Indeed, regression IV resembling regression II with the same constant for both types of laws, leads to highly significant differences in the first 10 years after the event. Regression V, repeating the first regression with the new assumption, now even shows significant differences in the whole post event period.

This event study showed significant changes comparing the period before and after the commencement of the new gender equality laws in the nineties. It seems that a priority rule leads at least to a faster effect on gender equality. Whether a priority rule is more powerful in the long run than a law without has to be investigated at a later point in time.

4.4 Faculty Composition

As the data set provides information on the university of each professorship, this section tests the hypothesis that gender composition is correlated with upcoming hiring decisions. Consequently, the relationship between faculty size and the share of women is investigated.[14]

The percentage of female professors at a faculty might influence the hiring decision in different ways. The first hypothesis is, that female professors will be allocated at faculties where the percentage of females is higher. The paper by Rivera (2012) describes this effect as cultural matching. Konrad and Pfeffer (1991) observe a similar pattern calling it demographic group power using data of hiring process for higher university administrative positions. In this case, the effect could be caused by different actions: On the one hand, female professors at a faculty could be influencing the hiring decision towards new female professors. On the other hand, female professors could tend to apply to faculties having a higher number of females.[15] The second hypothesis implies that the fact of having a low percentage of female professors produces social pressure to increase the number of females more strongly to achieve more gender equality. The corresponding probit regression yields $fem\_hire_i = \alpha + \beta \ fem_i + \varepsilon_i$ with fem_i as percentage of females at a faculty at the time of hiring and fem_hire_i as probability to be hired as woman.

We get a strong and significant positive correlation (Table 3.6) supporting the first hypothesis. Including the year of hiring as $year_i$ results in the regression $fem\_hire_i = \alpha + \beta \ fem_i + \gamma \ year_i + \varepsilon_i$. In consequence the significancy disappears. Thus, there is a positive correlation between the percentage of female professors and new females being hired. This might be due to the fact that in times when we had a lower percentage of female professors there were also less potential female professors available to hire over the years both the number of female professors and the number of female PhDs increased as seen in Section 4.1. This result is robust against changes in the examined

[14] Faculty size is measured by the total number of full professors, here.

[15] Unfortunately, we do not observe who applied for a professorship so we are not able to potentially disentangle both effects.

period. Regression 3 provides the result for the corresponding analysis to regression 2 for years after 1994.[16]

To achieve a better understanding of the distribution of female professors over the different faculties, Figure 3.14 displays the relationship between the number of full professors and the number of female professors at a faculty in a scatterplot. Most faculties do not exceed a number of 2 female professors. Considering faculties with more than 12 full professors in total, only, a positive relation between the two observed features becomes apparent. As already described in Section 4.1, there is a structural break in the data set. In consequence, it appears to be conclusive to split the data in two periods.

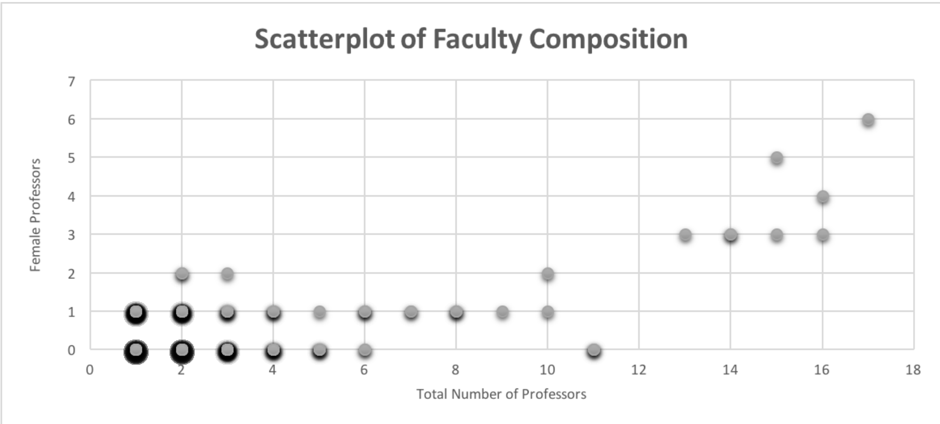
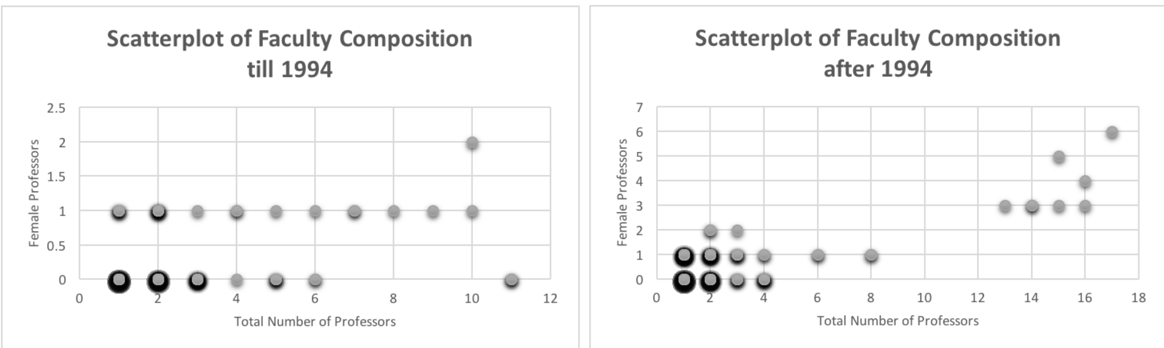


Figure 3.14: The Number of Female Professors vs. the Total Number of Professors

This scatter plot displays the number of female professors at a faculty against the total number of professors working there for the whole data set.



(a) Before the Breakpoint

(b) After the Breakpoint

Figure 3.15: Scatterplots for Splitted Sample before and after the Breakpoint in 1994

These scatter plots display the number of female professors at a faculty against the total number of professors working there for two different periods in time.

[16] This starting point is chosen because 1994 is the breakpoint detected in Section 4.1.

Table 3.5: Effects of Priority Rule

Dep.Var.: *dem_perc*	Reg. I $1 < t < 13$	Reg. II $1 < t < 10$	Reg. III $-22 < t < -1$	Reg. IV $1 < t < 10$	Reg. V $1 < t < 13$
time	0.02***	0.016***	0.002**	0.015***	0.017***
	[0.000]	[0.001]	[0.020]	[0.000]	[0.000]
$I_p \times time$	0.001	0.012**	0.001	0.011***	0.005**
	[0.843]	[0.049]	[0.320]	[0.000]	[0.026]
I_p	0.037	-0.008	0.018	-	-
	[0.321]	[0.827]	[0.269]		
const.	0.007	0.025	0.045***	-0.034**	0.035**
	[0.793]	[0.305]	[0.000]	[0.021]	[0.035]
n	26	20	42	22	28
R^2	0.773	0.836	0.394	0.835	0.804

This table displays regressions investigating differences in the federal states having introduced the new equality law with and with a priority rule on the percentage of female professors fem_perc_i using the normalized data set with the year the new law has been introduced in each federal state set to period zero. I_p is the dummy variable indicating that a priority rule has been introduced. P-Values are provided in parentheses.

Table 3.6: Probability for Hiring a Female Professor

Dependent Variable: fem_hire_i	Regression I	Regression II	Regression III
fem_i	1.042**	0.583	0.508
	[0.011]	[0.207]	[0.308]
$year_i$	-	0.042**	0.046
		[0.019]	[0.278]
const.	-1.382***	-86.028**	-94.075
	[0.000]	[0.017]	[0.265]
n	211	211	93
pseudo-R^2	0.043	0.085	0.044

Regression I displays the correlation between the percentage of females at a faculty (fem_i) and the probability for a female Professor to be hired (fem_hire_i). In Regression II and III it is controlled for the year of hiring and Regression III is restricted to data after the breakpoint in 1994. P-Values are provided in parentheses.

Figure 3.15 displays the same scatterplot for two particular periods: The first part includes all data till 1994, which resembles the breakpoint derived in Section 4.1, while the second part contains all observations after 1994. The decomposition of the data set reveals two different patterns. In the first period the majority of faculties tend to have at most one female professor and small faculties even non. In contrast, the second part of the sample is subdivided into small and large faculties. While small faculties with up to 8 full professors have up to 2 female colleagues, the larger faculties exhibit the positive relationship already visible in Figure 3.14.

In order to pin down the relationships described above in a quantitative way the probit regression $numb\_fem_j = \alpha + \beta \; size_j + \varepsilon_j$ was performed. The variable $numb\_fem_i$ represents the number of female professors at a faculty while the total number of professors at the faculties is given by $size_i$. Table 3.7 displays the corresponding results. Regarding the whole sample, the coefficient yields 0.1825, meaning barely 20 % of the faculty members (full professors) in the whole sample are female. Performing this regression for the sample before 1994 reveals a ratio of less than 10 %. In contrast, the sample after 1994 exhibits a proportion of even more than 23 %. Thus, for the second part of the sample there is a 1 to 3 ratio on average. Increasing the starting year of the second sample to 1998 leads to a coefficient of 25 %, while setting it to 2002 of about 30 %. Meanwhile, a decrease in significance is noticeable which is expectable as the sample size is reducing correspondingly. The negative sign of the constant appearing in all regressions might depict a tendency to adjust the share of females downwards rather than upwards. Summing up, the analysis features changes in gender composition of full professors at the faculties regarding samples before and after 1994. After 1994, the gender composition seems to follow a ratio of approximately one quarter with increasing tendency.

Table 3.7: Influences on the Number of Female Professors

Dep.Var.: $numb\_fem_j$	Reg. I *total*	Reg. II ≤ 1994	Reg. III > 1994	Reg. IV > 1998	Reg. V > 2002
size	0.182***	0.072***	0.238***	0.250***	0.303***
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
const.	-0.137***	-0.060***	-0.120***	-0.101***	-0.108***
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
n	901	456	445	316	171
R^2	0.401	0.172	0.544	0.601	0.689

These regressions show the correlation between the number of professors working at a faculty ($size_i$) and the number of female professors working there ($numb\_fem_i$) for different time periods. P-Values are provided in parentheses.

4.5 Influences on Doctoral Students

The last section of the analysis focuses on factors correlating with the decision of PhD students to become a professor. There are three parameters I am going to investigate: the influence of the percentage of female fellow students, the influence of the percentage of female professors at the PhD student's faculty and the supervisors' gender.[17]

The average percentage of female students is 23 %, varying between 0 % and 66 %. To investigate which influences the decision to stay in academia ($job\_choiche_i$) the following probit regression is performed with different measures of the composition of cohorts: $job\_choice_i = \alpha + \beta\ comp_i + \gamma\ fem_i + \delta\ comp_i \times fem_i + \varepsilon_i$.

The first row of Table 3.8 shows the corresponding probit regression on the percentage of female PhD students.[18] Both effects are negative, but, the additional effect on women is significantly less negative than the general effect. Interestingly, the same regressions performed with other related parameters such as the number of female students, the number of male students and the total number of students provided in Table 3.8 have negative general effects that are a lot less pronounced. Additionally, we do not see the significantly different effect on females as found for the percentages. The relative variable gives more impact to the smaller universities with a high percentage of females than the absolute numbers do, meaning the effect is even stronger at small universities with a high percentage of women.

Since the effect is generally negative it could be caused by other values in life of women such as wanting to contribute something to society, founding a family (Rossi, 1965; Monroe et al., 2008). Maybe with a higher percentage of females they are becoming more powerful as a peer group strengthening their values and even transferring it to the other sex.[19] As the effect is probably driven by smaller universities with a high fraction of females it could also be that men do not feel comfortable being surrounded by a larger amount of women.[20]

It has already been pointed out in the literature section that role models can have a strong influence on behavior. Female professors could be seen as role models for PhD students at a faculty encouraging them to become professor, as well. This leads to the hypothesis that women have an additional positive effect on the probability of women becoming professors compared to their influence on men becoming professors. The corresponding probit regression indicating the influence of the presence of a female professor ($influence_i$) on the decision to stay in academia (job_choice_i) appears as follows: $job\_choice_i = \alpha + \beta\ influence_i + \gamma\ fem_i + \delta\ influence_i \times fem_i + \varepsilon_i$. It is

[17] In this section, effects are assumed to be constant over time.

[18] As there could be variations in the percentage of females during the PhD time, the average percentage of females over the studying time is used.

[19] In her analysis Sax (2001) recognized a strong influence of peer mean regarding scientific orientation.

[20] Heterogeneous groups can have impact on the evaluation of the own performance. Baugh and Graen (1997) found out that the evaluation of performance decreases when the team becomes heterogeneous.

Table 3.8: Influence of other PhD Students on Job Choice

Dep. Var.: job_choice_i	Regression I fem_perc	Regression II numb_fem	Regression III numb_male	Regression IV total_numb
$comp_i$	-3.909***	-0.024***	-0.007***	-0.012***
	[0.000]	[0.000]	[0.000]	[0.000]
$tcomp_i \times fem_i$	1.611***	-0.002	0.001	-0.001
	[0.005]	[0.590]	[0.636]	[0.805]
fem_i	-0.465***	-0.150**	-0.282***	-0.251***
	[0.001]	[0.042]	[0.000]	[0.001]
$const.$	-0.92 ***	-1.284***	-1.256***	-1.232***
	[0.000]	[0.000]	[0.000]	[0.000]
n	21494	21494	21494	21494
pseudo-R^2	0.058	0.068	0.053	0.060

These regressions display the influence on the composition of the cohort ($comp_i$) on the job decision (job_choice_i). Regression I uses the female percentage, Regression II The total number of females, Regression III the total number of males and Regression IV the total number of fellow students making their PhD at the same faculty. P-Values are provided in parentheses.

distinguished between having at least one year spend with at lest one female professor around and at least two years. There does not seem to be a strong change in an increasing number of female professors, hence, the investigation only longs for the existence of at least one female professor. The results are given in Table 3.9.

There is an additional significant effect for females, meaning the effect on male is 0.119 while the effect on females is 0.219. Including students experiencing at least two years with a female professor at the same time, the effect gets stronger but more noisy.[21] This could be caused by the even smaller sample resulting in a less powerful regression. Surprisingly, we find a positive effect on male students, too. Interestingly even this effect is higher when having spent longer time at a gender mixed faculty. There is no obvious reason why women are also encouraging men to stay in academia. Possibly, the presence of female professors changes the atmosphere or the way of teaching.

The above investigation has shown that the gender composition of the faculty is indeed correlated with the decision of becoming a professor. The supervisor can be expected to have an even more pronounced influence on a PhD student. To examine this hypothesis the following probit regression was performed to investigate the influence of students' gender (fem_i) and supervisors' gender (sup_i)[22] on the job decision (job_choice_i): $job\_choice_i = \alpha + \beta\ fem_i + \gamma\ sup_i + \delta\ fem_i \times sup_i + \varepsilon_i$.The first regression in Table 3.10 shows the probability to become a professor for those students

[21] It is significant to a 10 % level instead of a 5 % level.

[22] Dummy variables always indicate female as 1 and male as 0.

Table 3.9: Influence of Female Percentages at Faculties

Dependent Variable: job_choice_i	Regression I min 1 year	Regression II min 2 years
$influence_i$	0.119***	0.468***
	[0.000]	[0.000]
$influence_i \times fem_i$	0.100**	0.319*
	[0.033]	[0.065]
fem_i	-0.447***	-0.429***
	[0.000]	[0.000]
$const.$	-1.488***	-1.494***
	[0.000]	[0.000]
n	4575	4575
pseudo-R^2	0.039	0.032

Theres probit regressions display the influence of the presence of a female professor ($influence_i$) on the decision to stay in academia (job_choice_i). It is distinguished between having at least one year spend with at lest one female professor around (Regression I) and at least two years (Regression II). P-Values are provided in parentheses.

who have a male supervisor. The next column provides the same regression for those having a female supervisor. We see a lower probability for female students to become professor in general, but, the reduction seems to be slightly less pronounced in case of a female supervisor. However, the coefficient in the second regression is not significant – which could also be caused by the extremely small number of 198 female supervisors.

Regression 3 shows the influence of the supervisor's gender on the probability of becoming a professor independent of the student's gender. Here, we see no significant influence. Finally, regression 4 gives neither a significant correlation for the supervisor's gender nor the interaction between the supervisor's and the student's gender. This explains that the difference between the parameters in regression 1 and 2 are not displaying a significant disparity. Only the lower probability of females to become professor stays significant, which is driven by the lower percentage of women becoming professor before 1998 (Figure 3.2).

Summing up, although the gender composition of the faculty shows a strong correlation with the career decision, no comparable relation can be found regarding the supervisor's gender. The analysis shows that the gender of a supervisor does not have an influence on the probability of becoming a professor and, in consequence, no different influence on female than on male students. There is one interesting observation left, regarding the supervisors: Regressing the supervisor's gender on the student's gender gives a correlation of 0.34*** (0.000) that is significantly different from zero. This means, female students tend to be supervised by female professors more often.

Table 3.10: Probability of Becoming a Professor Depending on the Supervisor

Dep. Var.: job_choice_i	Reg. I male sup	Reg. II fem sup	Reg. III	Reg. IV
fem_i	-0.325***	-0.210	-	-0.326***
	[0.000]	[0.293]		[0.000]
sup_i	-	-	-0.184*	-0.181
			[0.052]	[0.110]
$fem_i \times sup_i$	-	-	-	0.116
				[0.581]
$const.$	-1.098	-1.279	-1.165	-1.098
	[0.000]	[0.000]	[0.000]	[0.000]
n	5100	371	5534	5471
pseudo-R^2	0.008	0.005	0.001	0.009

These regressions indicate the influence of students gender (fem_i) and supervisors gender (sup_i) on the job decision (job_choice_i). The first regression restricts the investigation on the student's gender and was performed on the subgroup with male supervisors only, while the second one was performed on the subgroup with female supervisors. Regression III limits the influence of the supervisor's gender while regression IV incorporates all factors. Dummy variables always indicate female as 1 and male as 0. P-Values are provided in parentheses.

5. Conclusion

The analysis has shown that there have been distinct changes over the last decades. On the one hand, the amount of dissertations has doubled from 1990 to 2009 while the percentage of PhDs deciding to become a professor has decreased to a number below 1 %. On the other hand, the number of professors has increased from below 10 in the seventies to over 60 in 1988. However, only 5 % of the overall number of professors in 1988 were women. In 2005 the share of female students as well as professors reached a level of around one quarter. While this is a huge increase, one quarter could be seen as still fairly small. Interestingly, the transition rate from doctoral students to professors has become equal for men and women in 1998. Thus, the number of females relative to all female PhDs who choose to become professors is the same as the number of males becoming professor. Due to the smaller fraction of females completing a PhD the fraction of women can be seen as adequate.

We have also seen a stronger increase in the female percentage of professors than for students over the years and even more, a change in trend for female professors. This change, taking place around 1994, seems to be related to affirmative action policies by some German federal states regarding the hiring decisions for female professors. When seen in this international context, the fraction of female students is below the US average. The strong increase in female professors, probably caused by political and social changes in Germany, has pushed the amount of female professors in BA in Germany above the US average and reached the same level as social sciences in the US. During the nineties, German federal states have introduced laws to enforce gender equality in hiring decisions. The corresponding event study showed significant changes comparing the period before and after the commencement of the new gender equality laws in the nineties. It seams that a priority rule leads at least to a faster implementation of an increase in female professors. Whether a priority rule is more powerful in the long run then a law without cannot be finally seen within this data set.

Despite the changes in gender equality laws the implementation of a quota for professors was always renounced. However, the composition of full professors at the faculties follows a ratio of approximately one quarter with increasing tendency. This resembles the fraction of females completing a PhD. The question arises if either the transition rate, which appears to be the same for men and women since 1998, is resulting in this even distribution of females, or the unwritten quota – maybe formed by social leverage on the faculties hiring decisions – is resulting in the aligned transition rates. The direction of causality cannot be pinned down with this data set. However, the analysis revealed that the faculty composition itself does not seem to have any influence on the gender of the next professor being hired.

Finally, testing for influences on PhD students' decisions to become professors showed that both the percentage of female fellow students as well as the percentage of female professors at a faculty, are related with this career decision. Surprisingly, both effects point in the opposite direction. Female professors have a positive influence on the probability of becoming professor on both male and female students. However, the influence is stronger on females. So, the female professors could act as role models for female students. However, this effect seems to be existent even for male students. This means, a higher percentage of female professors at a faculty increases the attractiveness of becoming a professor in general. Possible reasons would be an improved professional environment as well as different teaching methods. Looking at female fellow students, their presence seems to have a negative impact on the decision in general as well. In this case, the effect is less pronounced for females, though. Although, supervisors are playing an important role in the career of doctoral students, the data show no significant influence for them acting as role models. In consequence, the faculty, meaning their scientific environment, seems to be more defining for PhD students than their supervisors. Interestingly, the supervisor's gender does not seem to play a role in career decisions, but the decision for a supervisor is connected to his or her gender. Female students tend to have a female supervisor significantly more often. This does not strictly imply that female students prefer to choose female supervisors. It could also be possible that female students are rejected by male supervisors. Hence, they would be forced to choose a female supervisor.

This raises another question: If there were more female supervisors, would this increase the number of female PhD students? We have seen that they do not influence the decision to become a professor. However, we also found the rate of becoming professor to be equal for men and women. If the transition rate stayed unaffected, an increase in the percentage of female PhD students would result in a proportional increase of female professors. Nevertheless, it is possible that the transition rate would not stay constant as it is likely that the fraction of women being able to become a PhD as well as a professor and willing to decide for this career is still low in society. Women who already choosing a career in academia seem to be able to handle the present situation. In order to increase their number, there are two possible approaches: first, by making a professorship more attractive for women – for themselves as well as society. Second, by enabling them to become a professor by providing the appropriate education. The second point itself has again a lot to do with the women's self-concept as well as the social environment. Science is still seen as unfeminine. If female students continue to avoid choosing sciences as subjects, they will not have the chance to decide whether they want to become a science professor or not as they will not be qualified.

Summing up, a lot has changed in the field of BA regarding gender equality. Those women deciding to do a doctorate are behaving like their male colleagues and the institutional changes allow them to behave equally in the decision of becoming a professor. There does not seem to be an invisible barrier left from PhD to a professorship as the percentage of female PhDs and professors in BA is nearly the same. Comparing to data from the US, this is outstanding as the fraction of females is distinctively smaller for professors than for PhD students in the US. In this regard, the field of BA in Germany performs reasonably well with respect to gender equality. While the development for female BA professors in Germany can be seen as positive and encouraging, the percentage of female PhD students in BA in Germany is rather disillusioning and demands for action if it is desired to adjust to the US in this concern.

6. Appendix

During the nineties, there have been several legislation amendments in order to increase gender equality in the recruitment process of professors.

- Bremen (1990): Gesetz zur Gleichstellung von Frau und Mann im öffentlichen Dienst des Landes Bremen (Landesgleichstellungsgesetz)
- Mecklenburg-Vorpommern (1998): Gesetz zur Gleichstellung von Frau und Mann im öffentlichen Dienst des Landes Mecklenburg-Vorpommern (Gleichstellungsgesetz - GlG M-V)
- Niedersachsen (1994): Niedersächsisches Gleichberechtigungsgesetz
- Brandenburg (1994): Gesetz zur Gleichstellung von Frauen und Männern im öffentlichen Dienst im Land Brandenburg (Landesgleichstellungsgesetz – LGG)
- Hamburg (1991): Gesetz zur Gleichstellung von Frauen und Männern im hamburgischen öffentlichen Dienst (Gleichstellungsgesetz)
- Sachsen-Anhalt (1997): Frauenfördergesetz (FrFG) Sachsen-Anhalt
- Schleswig-Holstein (1994): Gesetz zur Gleichstellung der Frauen im öffentlichen Dienst (GstG)
- Berlin (1991): Landesgleichstellungsgesetz zur Gleichstellung von Frauen und Männern im Berliner Landesdienst (LGG)
- Saarland (1996): Landesgleichstellungsgesetz - LGG Saarland
- Thüringen (1998): Thüringer Gleichstellungsgesetz (ThürGleichG)
- Baden-Württemberg (1995): Landesgleichberechtigungsgesetz
- Sachsen (1994): Gesetz zur Förderung von Frauen und der Vereinbarkeit von Familie und Beruf im öffentlichen Dienst im Freistaat Sachsen (Sächsisches Frauenförderungsgesetz – SächsFFG)
- Bayern (1996): Bayerisches Gesetz zur Gleichstellung von Frauen und Männern
- Hessen (1993): Hessisches Gesetz über die Gleichberechtigung von Frauen und Männern und zum Abbau von Diskriminierungen von Frauen in der öffentlichen Verwaltung (Hessisches Gleichberechtigungsgesetz - HGlG)
- NRW (1999): Gesetz zur Gleichstellung von Frauen und Männern für das Land Nordrhein-Westfalen (Landesgleichstellungsgesetz - LGG)
- Rheinland-Pfalz (1995): Landesgleichstellungsgesetz (LGG) Rheinland-Pfalz

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