Three Essays in Empirical and Theoretical Macroeconomics: Implications for Monetary and Fiscal Policy

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Introduction

Mephistopheles

Wo fehlt's nicht irgendwo auf dieser Welt?
Dem dies, dem das, hier aber fehlt das Geld.
Vom Estrich zwar ist es nicht aufzuraffen;
Doch Weisheit weiß das Tiefste herzuschaffen.
In Bergesadern, Mauergründen,
Ist Gold gemünzt und ungemünzt zu finden,
Und fragt ihr mich, wer es zutage schafft:
Begabten Manns Natur- und Geisteskraft...

Ein solch Papier, an Gold und Perlen Statt,
Ist so bequem, man weiß doch was man hat,
Man braucht nicht erst zu markten noch zu tauschen,
Kann sich nach Lust in Lieb' und Wein berauschen.
Will man Metall, ein Wechsler ist bereit,
Und fehlt es da, so gräbt man eine Zeit.
Pokal und Kette wird verauktioniert,
Und das Papier, sogleich amortisiert,...

— Johann Wolfgang von Goethe,
Faust. Der Tragödie zweiter Teil

"Die Entstehung des Papiergelds - Lustgarten", part of a series of paintings by Siegfried Rischar (1987) for Deutsche Bundesbank Regional Office in Hesse, illustrates the role of money in Goethe's epic *Faust II*. The throne room scene, in parts quoted above, tells us a lot about the two unifying themes of this dissertation - the role of interest rates and debt for the economy. Interest rates and debt are key policy instruments of monetary and fiscal policy and relevant not only for individual decision making but also for the economy as a whole. In Goethe's *Faust II*, the emperor is in grave need of money to finance current expenditures and service the Reich's out-

Figure 1. Die Entstehung des Papiergeldes (Detail) – Siegfried Rischar, 1987. ©Deutsche Bundesbank, Foto: Waltraud Krase

standing debt. Mephisto, his fool, elaborates on that the emperor could pay all his obligations using prospective future revenues from yet unmined natural resources. An endless stream of noninterest-bearing debt obligations, or paper money, no longer backed by solid metal, seems to be the sweet promise that solves all financial worries. Nevertheless, as the painting illustrates likewise, bankruptcy looms in the distance as creditors want to get repaid at some point in time. In modern societies, central banks have the *Mephisto-esque* power to create money from nothing. However, for the story to end differently than in Goethe's epic drama, they depend on the treasury to honour outstanding obligations and the central bank to keep prices stable. Hence, successful macroeconomic policy must walk the tightrope between minimizing welfare losses by stabilizing shocks and ensuring that debt remains sustainable in the long run.

The present thesis takes a glance at both these aims. The FIRST CHAPTER contributes to the literature on the equity premium and risk-free rate puzzle that discuss and analyze why consumption behavior (at the macro level) is so much at odds with model predictions. Its main result is that discrepancies between model predictions and empirical observations can be explained by monetary policy surprises, and the way the central bank conducts its policy.

The SECOND CHAPTER analyzes the transmission of monetary policy in an incomplete markets Dynamic Stochastic General Equilibrium (DSGE) Model with heterogeneous agents and a life-cycle motive in the form of stochastic aging. Its main result is that implications from demographic change matter for the conduct of monetary policy - not only in the far future but already today in understanding competing saving motives of young and old households. The chapter is joint work with Christian Bayer and Ralph Lütticke.

The THIRD CHAPTER contributes to the literature on fiscal policy in the Euro Area in light of debates on fiscal rules, sustainable public finances and European economic integration lasting for years. Using a Kalman-Filter approach it assesses the fiscal policy stance of European governments before and after the European Debt Crisis. The following paragraphs summarize the chapters in more detail.

CHAPTER 1: Empirical evidence on the movement of US consumption growth rates and real interest rates does not square with predictions from theoretical models. In these models, the real Federal Funds works as main transmission device of monetary policy. Following recessions or an intervention by the central bank, the ex-post observed Federal Funds Rate and a model-implied Euler equation rate move into opposite directions. In theory however, they should correlate perfectly.

In the work *Data, Model-implied Policy Rates and Consumption Euler Equations*, I extend research by Canzoneri, Cumby, and Diba (2007) who were the first to document this result. I provide new evidence and find a possible way to resolve this problem. In a first step, I consider the importance of the choice of the sample period. The original paper regards predictions from quarterly Data from 1966 to 2005 which I update until 2017Q1. The first result is that the sign of the correlation is not stable over a long time horizon. From the 1960's to the early 1980's this chapter confirms the results of Canzoneri, Cumby, and Diba (2007), however this relation breaks down during the so-called *Great Moderation*, from the 1980's onwards. Moreover, various specifications of the consumption function adequately capture the long-term decrease in real interest rates and consumption growth. From that time on the Federal Funds Rate and the model-implied Euler rate correlate positively, as theory predicts. In a second step, I test whether the Federal Funds rate and the Euler rate still move into opposite directions following a recession or a monetary policy intervention. Chapter 1 finds that regardless of the time period, both rates move into opposite directions following a change in the Federal Funds rate, i.e. the Euler rate systematically reacts to monetary policy. In a last step, I investigate what could be a potential driver of this reaction. Macroeconomic theory under rational expectations tells us that the variables of a model should not react to anticipated shocks. Here, this includes everything that is in the information set of the central bank. Romer and Romer (2004) provided us event study-based evidence on monetary policy surprises outside the scope of the central bank, called narrative monetary policy shocks. I use them to cleanse the Federal Funds rate from its unexpected component. Using only the anticipated component of the Federal Funds rate, there is no longer a significant reaction of the Euler rate to the stance of monetary policy.

CHAPTER 2: Consumption decisions of households have been reformulated as problems of intertemporal choice since the days of Fisher (1930). However, they also depend on the current stage of the household's life-cycle as Modigliani and Brumberg (1954) show. Moreover, adding income and wealth heterogeneity as described by models in the tradition of Hansen and Imrohoroglu (1992), Huggett (1993), or Aiyagari (1994), allows to discuss inter- and intratemporal effects of monetary or fiscal policy on consumption decisions. In the work *Monetary Policy in a Heterogeneous Agent Model with Aging* we extend variants of the models in Bayer et al. (2015) and Lütticke (2017) using a stochastic aging framework. The chapter builds on the premise discussed e.g. in Miles (2002) who argues that the transmission of monetary policy might be different in a greyer world. In the presence of nominal rigidities and a life-cycle motive of consumption, effects of monetary policy work among other things through intertemporal substitution, wealth effects and the individual (in)ability to borrow against expected lifetime income. Changes in the age composition of the economy - or demographic change - may alter the relative importance of each channel for the respective household. Obviously, young households will react differently to shocks to the policy rate as old households, given the fact that intertemporal substitution plays a greater role for them, still. On the contrary, both young and old households face wealth effects of monetary policy. Asset-rich, old households tend to benefit from positive rate hikes whereas young households find themselves faster close to their borrowing limit. Nevertheless, as the amount of asset-rich households is generally small, an increase of the central bank's policy rate still depresses aggregate demand. Thus, monetary policy that considers overall welfare, should take possible effects on inter-age group inequality into account, as well. This chapter builds a dynamic stochastic general equilibrium (DSGE) model with heterogeneous agents and stochastic aging. First-order perturbations around the stationary steady-steady are used to answer the questions formulated above. From a methodological point of view this allows to model a parsimonious life-cycle structure without having to track every single generation as in the literature on overlapping generation (OLG) models. This enables us to study the general equilibrium effects of monetary policy both between age groups and between different steady-states, i.e. comparing very young and very old economies. Furthermore, including agents that differ *ex-post* through different realizations of idiosyncratic labor productivity creates heterogeneity in the accumulation of assets. Using our model we are able to confirm results found in the literature. Along the transition to an older society, the real return to capital will fall by approximately 0.5 percentage points. Moreover, the consumption response to a monetary policy shock is different relative to the response of an incomplete markets economy without age structure. Furthermore, this response differs significantly depending on the age composition of the economy. Finally, including an age-structure into the HANK framework further reduces the effect of forward guidance shocks, here measured as the difference between a persistent and a transitory monetary policy innovation.

Chapter 3: The Maastricht Treaty, the Stability and Growth Pact (SGP) and its successor, the Fiscal Compact have been praised as the cornerstones of successful European economic integration. Nevertheless, the last 20-25 years have seen endless discussions, failures to reform and to commit credibly, and finally, the European debt crisis from 2010 onwards. The popular debate in Germany favors the hypothesis of profligate southern Europeans bailed out using German tax payers money. However, others correctly point out that the 2003 violation without consequences of Germany and France has been the original Fall of Man. During times of distress - as it is the case in every family - the discussion circles around arguments of solidarity versus responsibility. It seems that abiding by European rules in fiscal policy (and other areas) has not worked too well over the last 25 years. This includes Germany who usually takes a moral high ground on that issue. For some countries even, the SGP has never been complied with seriously, once they had been admitted to joining the European Monetary Union (EMU) in 1999. Especially, questions of enforcement remain a pending issue. Nevertheless, rule-based fiscal policy evaluation is still seen by many as a necessary condition for the well-working of the

European Monetary Union. Among other indicators, this evaluation is based on the structural deficit, i.e. the discretionary part of the budget balance that does not fluctuate over the business cycle. This figure or its components are unobservable and need to be estimated. Moreover, data on variables relevant for fiscal policy as e.g. tax revenue statistics are usually only available at an annual frequency. Thus, causal implications from fiscal policy measures, as can be derived from DSGE models calibrated to a quarterly frequency, are difficult to reconcile with the data.

In the work *Revisiting the Stability and Growth Pact: A 20 Years Empirical Perspective*, I estimate quarterly structural deficit series of 31 European countries on the basis of an Unobserved Components Model (UCM) and the Kalman Filter. This enables an empirical assessment of European fiscal policy in the light of the Maastricht Treaty, the Stability and Growth Pact and their successor treaties. In comparison to related approaches to the question at hand, the chosen econometric approach employs a direct, one-step estimation technique. This allows to evaluate the fiscal policy stance within clearly defined statistical boundaries. The chapter documents that the fiscal policy stance is countercyclical in most countries and that fiscal policy is active in the sense of the Fiscal Theory of the Price Level. From a policy perspective, Chapter 3 finds that after admittance to the monetary union in the late 1990's, the Maastricht criteria, especially the 3% deficit ceiling were no longer taken seriously. Using safety-margins, i.e. values the structural deficit can assume to stay within the 3% boundary, reveals that discretionary and not business cycle-driven spending has been the driver of deficits in many European countries.

1

Data, Model-Implied Policy Rates and Consumption Euler Equations

1.1 Introduction

There exists a consensus in macroeconomic literature towards the modeling of conventional monetary policy within Dynamic Stochastic General Equilibrium (DSGE) models. The monetary authority is assumed to set its operating instrument[1] following a feedback rule towards policy goals such as an inflation target or a certain level of unemployment. Additionally, the policy instrument responds to exogenous, monetary disturbances which are not in the information set of the central bank and affect policy goals only with a time lag - the monetary policy shock.

In monetary models with price rigidities, the transmission of a monetary policy shock to the real economy is engineered via the Consumption Euler equation. In DSGE models, variants of this equation represent the demand side of the economy and are at the core of the linearized dynamic system that constitutes a rational expectations equilibrium, as e.g. the 3-equation basic New Keynesian Model in Galí (2008). In contrast to the old Keynesian consumption function that related consumption to disposable, aggregate income, the Consumption Euler equation describes optimal decisions of an individual, representative household over the current and future level of his consumption.[2] A variant of it, linearized around the non-stochastic steady-state, describes the relation between expected consumption growth and the real interest rate. Modern consumption theory suggests that both should be perfectly and positively correlated. Additionally, differences in each series' volatility only stem from the degree of risk aversion as

[1] As operating or policy instrument central banks usually use an overnight interest rate charged on reserves borrowed from the central bank or use the reserve requirement ratio, i.e. the ratio of bank reserves to bank deposit liabilities.

[2] The gain in utility from consuming one unit less of the consumption good today, has to equal the discounted gain in utility from consuming one more unit tomorrow times the return received on the previous' period saving.

captured by the household's preferences. Thus, models that analyze monetary policy typically view the Euler equation-implied interest rate as a market rate - the rate that clears the capital market - and equate it to the policy instrument of the central bank.

Empirically however, tests based on predictions made by Consumption Euler equation models perform poorly. Following the famous rejection of the Consumption Capital Asset Pricing model (CCPAM) by Hansen and Singleton (1983), Hall (1988) or Campbell and Mankiw (1989) among others, there exists repeatedly documented evidence that data on returns and consumption expenditures are not consistent with model implications. A at that time *standard* model[3] has not been capable of generating plausible real interest rate behavior in comparison to what can be observed in the data. This failure has generated a lot of effort to improving the standard model such that it matches empirical evidence from structural Vector Autoregressive Models (VAR) as e.g. Christiano, Eichenbaum, and Evans (1999).

Fuhrer (2000) documents that preferences that include habit formation are key to generate a more realistic, humped-shaped response of e.g. consumption or inflation to a monetary policy shock. Moreover, Carroll (1992), Kimball (1990) or Aiyagari (1994) have documented the importance of precautionary savings motives in an adequate description of consumption behavior. However, Canzoneri, Cumby, and Diba (2007) among others, challenge the modeling practice that equates the nominal interest rate targeted by the central bank with an interest rate implied out of the Consumption Euler equation. For a variety of preference specifications - including habits - they find that interest rates calculated out of a Consumption Euler equation cannot be reconciled with observed market rates. Furthermore, the two rates are uncorrelated or even negatively correlated. Their approach follows Fuhrer (2000) and estimates a reduced-form VAR of consumption and inflation. They make use of the VAR's dynamics and the first and second conditional moments to derive the implied paths of the nominal or real interest rate. These rates are then compared with ex-post observed rates. Additionally, the authors show that the spread between both rates reacts systematically to the stance of monetary policy. In this case, the model-implied interest rate responds negatively to contractionary monetary policy.

Ahmad (2005) extends this analysis towards six of the G7 countries and finds similar results - an increase in the nominal interest rate leads to a decline in the implied model rate. He confirms this result by comparing the responses of consumption and output following a money market interest rate shock. Using the Christiano, Eichenbaum, and Evans (1999) approach to VAR-identification, he shows that the response of the model-implied interest rate to an increase of the money market interest rate is negative. Both Canzoneri, Cumby, and Diba (2007) and Ahmad (2005) conclude that movements in implied interest rates can not be reconciled with the theoretical foundation of the Consumption Euler equation. More recent work on that topic by Collard and Dellas (2012) argues that the non-separability of consumption and leisure helps to bring the data closer to theoretical predictions. In their paper, the model-implied interest rate is positively correlated with the data and the average discrepancy between model and data is lower. Additionally, the response of the model-rate towards an increase in the federal funds

[3] The term *standard* refers to investors with logarithmic, or constant-relative risk aversion (CRRA), time-separable preferences and the absence of habits, borrowing constraints or other financial frictions.

rate is now slightly positive nevertheless, insignificant. Furthermore, Gareis and Mayer (2013) argue that the negative correlation can be explained by risk premium disturbances. Finally, Florio (2013) shows that accounting for financial frictions within the Euler equation helps to bring the model closer to the data.

Using US data from 1960 to 2017, this chapter applies the methodology of Canzoneri, Cumby, and Diba (2007) and highlights that the qualitative statement and the quantitative magnitude of their results is highly conditional on the regarded time period. The correlation between model and data is neither negative nor negligible for all subsamples. Two time periods are outstanding: From the 1960's to the early 1980's there exists a strong negative correlation between model and data and the spread between both rates reacts severely and highly significantly to an increase in the federal funds rate. On the contrary, over the entire sample or a second subsample from the 1980's until today, model and data correlate strongly positive. A partition of data and model-implied rates into a trend and a cyclical component shows that the correlation is biased towards negative values by the cyclical components. In contrast to that, the model is able to adequately capture the trend behavior of consumption and interest rates over the sample. The reaction of the spread to the stance of monetary policy is four to five times smaller in the later subsample but still highly significant. Using narrative Romer and Romer (2004) shocks as instruments, this chapter documents that the significant reaction to monetary policy has been due to surprise actions by the monetary authority. Accounting for them, the reaction of the spread between model and data to a change in the Federal Funds rate becomes insignificant in both economic and statistic terms.

The remaining sections of the chapter proceed as follows: Section 1.2 discusses the properties of the dataset and section 1.3 discusses the model, inherent assumptions and states first estimation results. Section 1.4 analyzes the influence of the sample horizon and section 1.5 explains how to reconcile these findings with the model's predictions. Section 1.6 concludes.

1.2 Data

I use quarterly data from 1960 - 2017 constructed using the Federal Reserve St. Luis database. Variables are: Real per capita consumption expenditures of nondurable goods and services, real per capita disposable income, a gross measure of inflation using nominal and real consumption expenditures, the Federal Funds rate, producer prices, a measure of real GDP per capita without consumption and the monetary aggregate M1. To account for the effects of unconventional monetary policy, a Federal Funds shadow rate following Wu and Xia (2016) is used instead of the Federal Funds rate from the years 2009 onwards. For the most recent years, both rates coincide again. All data except the Federal Funds rate are in logs and the VAR is estimated with four lags, the lag order suggested by both BIC and Hannan-Quinn information criterion. The dataset corresponds to the specification of Canzoneri, Cumby, and Diba (2007). However, the sample length and the price index used for producer prices, differ.[4] Figure 1.1 visualizes the

[4] Canzoneri, Cumby, and Diba (2007) use the Journal of Commerce industrial materials commodity. My robustness checks contain the Thomson Reuters/CoreCommodity CRB Index (TR/CC CRB) obtained from Thomson/Reuters

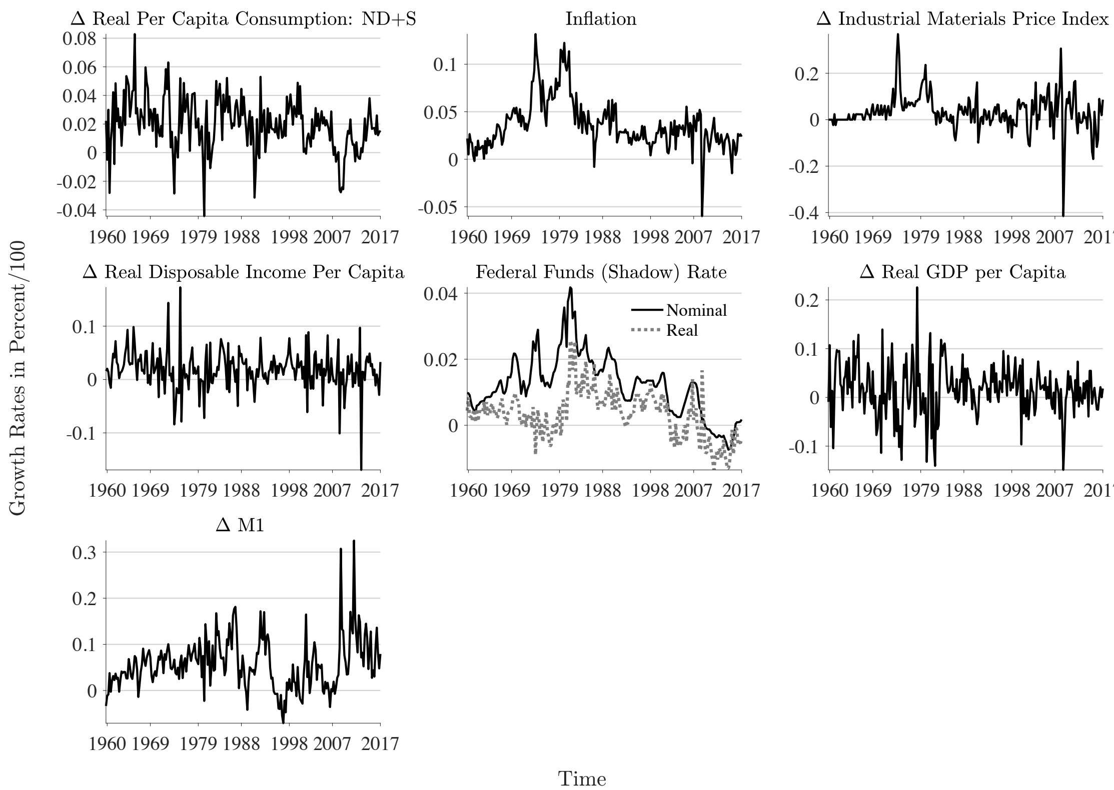

Figure 1.1. Key U.S. Macroeconomic Variables, 1960-2017, Annualized at Quarterly Frequency

Notes: The figure shows annual growth rates (log-differences) at quarterly frequency for all variables except the Federal Funds rate. The subplot for the Federal Funds rate contains the inflation-adjusted real rate (dashed), as well. From the year 2009 onwards, the Federal Funds shadow rate provided by Wu and Xia (2016) substitutes the effective Federal Funds rate.

Table 1.1. Summary Statistics of Model Variables Over Different Samples

	Δ Real Per Capita Consumption: ND+S	Inflation	Δ Industrial Materials Price Index	Δ Real Disposable Income Per Capita	Federal Funds (Shadow) Rate	Δ Real GDP per Capita	Δ M1
Sample Period: 1960-2017							
Mean	0.0198	0.0365	0.035	0.0183	0.05	0.0148	0.0586
St. deviation	0.018	0.0263	0.0766	0.0364	0.0394	0.0528	0.0544
Min	-0.0444	-0.0599	-0.416	-0.169	-0.0292	-0.149	-0.0708
Max	0.0828	0.132	0.369	0.173	0.178	0.226	0.325
Sample Period: 1969-1982							
Mean	0.0193	0.0699	0.0881	0.0122	0.0853	-0.00219	0.0631
St. deviation	0.0208	0.0262	0.0739	0.0461	0.0381	0.0757	0.0298
Min	-0.0444	0.03	0	-0.0842	0.0354	-0.14	-0.0225
Max	0.0631	0.132	0.369	0.173	0.178	0.226	0.144
Sample Period: 1983-2017							
Mean	0.0169	0.0272	0.0215	0.0174	0.0378	0.017	0.06
St. deviation	0.0158	0.0162	0.0786	0.0342	0.0349	0.0396	0.065
Min	-0.0315	-0.0599	-0.416	-0.169	-0.0292	-0.149	-0.0708
Max	0.053	0.0625	0.306	0.0971	0.114	0.118	0.325

growth rates of the sample's variables and Table 1.1 depicts summary statistics over the full sample and subsamples which become important in later parts of the chapter. Appendix A.1 provides details and the exact sources of the employed data.

1.3 Economic Model and Preference Specifications

The following section considers the household part of a very stylized general equilibrium model in which a representative consumer maximizes his life-time utility subject to an intertemporal budget constraint. The resulting Consumption Euler equation builds - under different preference specifications - the basis for the calculation of interest rates implied by the first and second moments of a Vector Autoregressive Model (VAR).

1.3.1 The Model

The model assumes complete markets, no borrowing constraints and decisions under full information rational expectations. This is important to stress e.g. given the fact that precautionary savings motives and imperfect or limited information could be an alternative explanation for the spread between empirically-observed and theoretically-implied interest rates, as argued in Kaplan, Moll, and Violante (2018).

The specification of the utility function considers the cases of constant, relative risk-aversion (CRRA), internal habit formation in the tradition of Fuhrer (2000) and external habit formation as in Abel (1990) or Abel (1999). The later two are described in the following subsection. Every period, the representative agent divides his interest income from existing financial wealth b_{t-1} between consumption expenditures and the holding of two riskless one-period bonds. The first one pays one unit of the consumption good in the next period and the second one pays one dollar. The life-time utility maximization problem of the representative agent takes the following form:

$$U_t = \sum_{j=0}^{\infty} \beta^t E_t u(C_{t+j}, Z_{t+j}) \tag{1.1}$$

subject to the following period budget constraint:

$$P_t C_t + B_t = (1 + i_{t-1}) B_{t-1}, \tag{1.2}$$

where β is the consumer's discount factor and Z_{t+j} the reference or habit level of consumption evolving as:

$$Z_t = \rho Z_{t-1} + (1 - \rho) C_{t-1}. \tag{1.3}$$

Datastream. How the variable selection influences the replication of the original results in Canzoneri, Cumby, and Diba (2007) is discussed in A.2.2.

Pricing both bonds, using a stochastic discount factor looks as follows:

$$\frac{1}{1+r_t} = \beta E_t\left(\frac{U_c'(C_{t+1},Z_{t+1})}{U_c'(C_t,Z_t)}\right)$$
$$\frac{1}{1+i_t} = \beta E_t\left(\frac{U_c'(C_{t+1},Z_{t+1})}{U_c'(C_t,Z_t)}\frac{1}{1+\pi_{t+1}}\right),$$

where r_t is the real and i_t is the nominal interest rate. P_t is the price of one unit of the consumption good such that $\frac{P_{t+1}}{P_t} = (1+\pi_{t+1})$, the gross inflation rate. The ex-post real interest rate r_t is defined as $(1+i_t)/(1+\pi_{t+1})$. The different preference specifications discussed below thus, affect the respective interest rate via different marginal rates of substitution. Out of convenience I abstract from a consumption-leisure trade-off, which is unproblematic as long as preferences are additively-separable.[5] The generality of the model makes such extensions however, straightforward.

1.3.2 CRRA Preferences

If the representative agent has preferences under constant, relative risk-aversion (CRRA), his period utility is described by the following function:

$$u(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma},$$

where $\sigma > 0$ is the coefficient of relative risk-aversion. This is nested in equation (1.1) and (1.3) for $\rho = 0$, i.e. there exist no habits in consumption. For $\sigma \rightarrow 1$ this specification converges towards log-utility preferences. The associated Euler equation looks as follows:

$$\frac{1}{1+i_t} = \beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{1+\pi_{t+1}}\right]. \tag{1.4}$$

The corresponding expression for the real return looks identical except for the terms corresponding to the inflation rate in the denominator. Under the additional assumption that variables are conditionally lognormal distributed this implies for the Euler equation under CRRA preferences that:

$$\frac{1}{1+i_t} = \beta \exp\left\{-\sigma(E_t c_{t+1} - c_t) - E_t\pi_{t+1} + \frac{\sigma^2}{2}V_t c_{t+1} + \frac{1}{2}V_t\pi_{t+1} + \sigma cov_t(c_{t+1},\pi_{t+1})\right\}. \tag{1.5}$$

Conditional lognormality is a strong assumption and Carroll (1997) points out the weaknesses of Euler equation approximations. However, as DSGE models are often cast into a linearized form it

[5] For the consequences of non-separability compare the already mentioned paper by Collard and Dellas (2012).

is a comprehensive way to test their performance. Assuming that the dynamics of consumption and inflation can be represented by a VAR model in companion form:

$$Y_t = A_0 + A_1 Y_{t-1} + e_t, \qquad e_t \overset{iid}{\sim} N(0, \Sigma) \tag{1.6}$$

where $Y_t = [c_t, \Pi_t, ppi_t, y_t^{disp}, R_t, y_t, m_t]$. Variables denote the log of real consumption expenditures and services per capita, the log of gross inflation. Additional control variables are the log of a producer price index, the log of real disposable income per capita, the federal funds rate, nonconsumption real GDP per capita and the log of the monetary aggregate M1. If necessary, data has been seasonally adjusted.[6] Conditional on the observed data the VAR specification allows to form expectations one- or t-steps ahead. The VAR thus works as a predictor function of the representative agent's expectations. Using his expectations about consumption and inflation, it is possible to derive implied nominal and real interest rates out of the Euler equation specification derived above.

In the CRRA preferences case, I set $\sigma = 2$ and $\beta = 0.9967$ which corresponds to the average real interest rate over the whole sample. I obtain first and second moments of the model by using the following specification:

$$E_t Y_{t+1} = A_0 + A_1 Y_t \tag{1.7}$$

$$E_t Y_{t+2} = A_0 + A_1 E_t Y_{t+1} \tag{1.8}$$

$$V_t(Y_{t+1}) = \Sigma \tag{1.9}$$

$$V_t(Y_{t+2}) = A_1 \Sigma A_1' + \Sigma \tag{1.10}$$

$$Cov_t(Y_{t+1}, Y_{t+2}) = \Sigma A_1', \tag{1.11}$$

where it is important to note that in this case, the conditional second moments are time constant. In contrast to Canzoneri, Cumby, and Diba (2007), I do not use a linear-segmented trend. Their proposition of a trend break around 1974 seems arbitrary with respect to other potential breaks we can observe in the data, especially in the 1980's. The following paragraph compares implied nominal and real rates with the ex-post observed rates from the data. Alternative preference specifications used to obtain the remaining results are described further below. Table 1.2 illustrates estimation results and Figure 1.2 plots ex-post observed real market and real implied model rates for the CRRA preference specification. In addition, it shows the respective spread between model and data.

In contrast to the results of Canzoneri, Cumby, and Diba (2007)[7], the estimation results depicted in Table 1.2 suggest that the respective model matches the data better on average.

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[6] For further details on the data used in the estimation of the VAR see the appendix.

[7] For results of the original paper, the interested reader is referred to Table 1 on p. 1867 in Canzoneri, Cumby, and Diba (2007). Appendix A.2 has a paragraph on estimation results for the original time period and comparability issues.

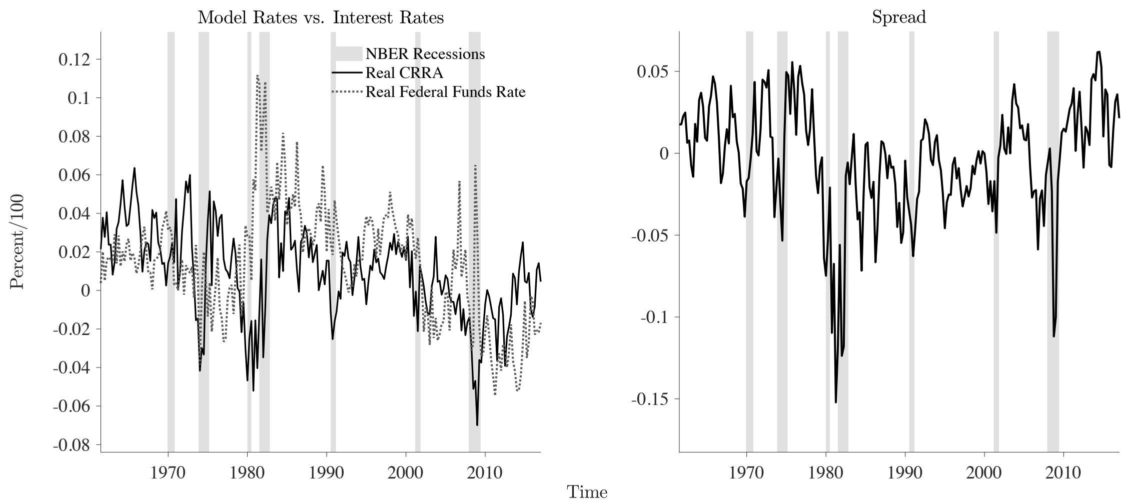

Figure 1.2. Model-Implied Rates vs. Ex-Post Observed Rates (annualized)

Notes: The figure's first column shows the development of ex-post observed real interest rates (dashed) compared to rates predicted from the VAR and the DSGE model (solid) for the CRRA preference specification. The second column calculates the spread, i.e. the difference between model rate and the data and sets and highlights period when the US has been in a recession as classified by the NBER.

Table 1.2. Ex-post Observed and Model-Implied Nominal Interest Rates, 1960-2017

Time period: 1960-07-01 to 2017-01-01				
Rates computed from models	Data	CRRA	Fuhrer (2000)	Abel (1999)
Real Rates				
Mean	0.013	0.0095	0.0032	0.0014
St. deviation	0.029	0.024	0.026	0.022
Min	-0.053	-0.07	-0.13	-0.071
Max	0.11	0.064	0.087	0.045
corr(Data,Model)	1	0.15	0.032	0.15
Nominal Rates				
Mean	0.051	0.046	0.04	0.038
St. deviation	0.04	0.031	0.033	0.03
Min	-0.029	-0.072	-0.05	-0.073
Max	0.18	0.13	0.14	0.11
corr(Data,Model)	1	0.51	0.44	0.54

The estimated model fits the mean interest rate quite well. Furthermore, the standard deviation as well as the extrema come relatively close to the sample counterparts. Overall, observed and model-implied rates seem to follow a similar long term or trend pattern: Higher rates in the early phase of the sample and lower rates for the most recent years. This is also the case for nominal rates which are not plotted here. Surprisingly, the correlation coefficient describes a strong, positive relationship between data and model for nominal interest rates. In addition, implied real and observed real rates are weakly positively correlated. Figure 1.2 shows that the model is able to replicate the trend found in interest rates but fails with respect to the cyclical component. Cochrane (2017) argues that risk premiums have a clear business-cycle correlation such that the above observation could be an indication of preference or risk premium shocks driving model and data apart. Nevertheless, key findings of the work of Canzoneri, Cumby, and Diba (2007) and Ahmad (2005) remains visible - it is not possible to reconcile the divergence of data and model-implied rates following monetary policy (MP) interventions of the central bank. With respect to the modeling of the transmission mechanism of monetary policy in DSGE models described above this constitutes a severe problem. Figure 1.2 and Table 1.3, show five periods of

Table 1.3. Implied- vs. Observed Interest Rates During Recessions or MP Interventions

Rates computed from models	Data	CRRA	Fuhrer (2000)	Abel (1999)
Real Rates				
1967Q3 - 1968Q2	0.022	0.025	0.0098	0.015
1973Q4 - 1974Q3	-0.0064	0.026	0.029	0.018
1978Q3 - 1979Q2	0.021	-0.027	-0.012	-0.031
1979Q4 - 1980Q3	0.082	-0.03	-0.048	-0.036
2007Q4 - 2009Q2	-0.0037	-0.031	-0.021	-0.035

the Federal Reserves' reaction to a monetary policy shock as identified in Ahmad (2005) plus the Great Recession. The first period is 1967Q3 - 1968Q2 has rates moving in opposite directions, the second 1973Q4 - 1974Q3 as well has model-implied rates which are high whereas market rates are low, followed by 1978Q3-1979Q2 and 1979Q4 - 1980Q3 where model-implied rates are low and market rates high.

Interest rates moving into opposite directions imply that the spread between model and data widens. Using grey-shaded NBER recession dates in the right panel of Figure 1.2, this suggests that model and data are more at odds in times of recessions or presumably, times of monetary policy intervention. Finally, the model predicts strongly negative real interest rates following the onset of the recent financial crisis, whereas as the observed nominal interest rate is of course subject to the zero-lower bound (ZLB). A way to combine the shortcomings of the ZLB and the negative model-implied rates is the use of a Federal Funds shadow rate as e.g. recently proposed in Wu and Xia (2016). The shadow rate accounts for the effects of unconventional monetary policy and thus the overall stance of monetary policy at the ZLB which is otherwise not present in the effective Federal Funds rate. The results suggest that both the selection of the sample horizon and monetary policy surprises seem to drive results apart.

1.3.3 Preference Shocks

The significant difference in the behavior of the two interest rate series depicted in Figure 1.2 may stem from shocks to consumer's preferences. A positive preference shock, incorporated in an otherwise standard CRRA utility function looks as follows:

$$u(C_t) = \left(\frac{\left(\frac{C_t}{\Delta_t}\right)^{1-\sigma}}{1-\sigma} \right), \tag{1.12}$$

where a preference shock is a (positive) innovation to $E_t(\Delta_{t+1} - \Delta_t)$. Incorporated into equations (1.4) and (1.5) this implies

$$\frac{1}{1+i_t} = \left(\underbrace{\beta E_t \left(\frac{\Delta_t}{\Delta_{t+1}} \right)^{\sigma}}_{\tilde{\beta}_t} \right) E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{1+\pi_{t+1}} \right], \tag{1.13}$$

such that we can think of the preference- or taste shock as a time-varying component of the discount factor. The conditional log-normal approximation is then given by:

$$\begin{aligned} \frac{1}{1+i_t} &= \beta \exp\{-\sigma(E_t c_{t+1} - c_t)) + \sigma(E_t(\log(\Delta_{t+1}) - \log(\Delta_t)) - E_t \pi_{t+1} \\ &\quad + \frac{\sigma^2}{2} V_t c_{t+1} + \frac{1}{2} V_t \pi_{t+1} + \sigma cov_t(c_{t+1}, \pi_{t+1})\}. \end{aligned} \tag{1.14}$$

Other things equal, the positive preference shock pushes down the interest rate and agents become less patient to clear the market. Suppose now that the model-implied rate is equalized with the observed Federal Funds rate - however, the model does not account for movements in the data due to innovations in preferences. Then the model-implied rate as shown in Figure 1.2 is too high compared to the data as the influence of the preference shock which depresses the interest rate is falsely neglected. As already mentioned above, monetary policy uncertainty until the Volcker disinflation regime switch plays a huge role. Incorporating the influence of taste shocks in specifications that include habit formation as those described below yields the same qualitative result. Before further investigating these findings for subsample periods, the following section outlines and applies preferences with habit formation to the model.

1.3.4 Habit Formation

Preferences that incorporate habits are important for an adequate description of a consumer's consumption-saving problem. This paragraph provides a brief introduction to the problem at hand. For a more thorough discussion the interested reader is referred to Schmitt-Grohe and Uribe (2008).

The literature on empirical asset pricing identifies the fact that agents develop consumption habits over time as one approach to explain the equity premium puzzle, as first identified in the work of Mehra and Prescott (1985). The equity premium puzzle states that observed excess returns of shares over less risky assets are too high in order to be consistent with observed consumption behavior unless risk aversion is tremendously high. Empirically, fluctuations in consumption growth are small (around 1.8 percentage points in the chapter's sample from 1960 to 2017, compare Table 1.1), which implies that high returns on risky assets can only be explained if already small fluctuations in consumption have a strong negative impact on consumer's utility. A model, where consumers have CRRA or even log-preferences and do not form any habits, as discussed, is not able to generate volatile enough consumption growth. In contrast to that, habit formation specifies the period utility function in dependence of current consumption and a reference level of consumption. As the term *habit* suggests, consuming more today makes consumers want to consume at least as much or even more tomorrow. In other words, habits break up the separability of preferences over time. If the stock of habits is of autoregressive nature as depicted in (1.3), the parameter ρ captures persistence and sensitivity towards past consumption levels. Under habits, consumers care more about variations of consumption relative to their habit level, rather than variations in the level of consumption itself. The following paragraphs describe two distinct variants of habit formation and apply them to the model.

Fuhrer et al. (2000)

For preferences incorporating the results on habit formation as in Fuhrer (2000), the representative agent has the following utility function:

$$u(C_t, Z_t) \;=\; \frac{1}{1-\sigma}\left(\frac{C_t}{C_{t-1}}C_{t-1}^{1-\gamma}\right)^{1-\sigma}, \quad 0 \;\leq\; \gamma \;\leq\; 1 \tag{1.15}$$

Fuhrer (2000) estimated the autoregressive parameter of the habit formation process as insignificant and close to zero, such that equation 1.3 reduces to $Z_t = C_{t-1}$. Unlike under CRRA preferences, period utility is no longer additively-separable over time. The current level of period utility depends on today's and yesterday's choice of consumption. As a consequence, the Consumption Euler equation including habits looks as follows:

$$\frac{1}{\beta(1+i_t)} \;=\; E_t\left(\frac{\left[C_{t-1}^{-\gamma(1-\sigma)}C_t^{-\sigma} - \beta\gamma C_{t+1}^{1-\sigma}C_t^{-\gamma(1-\sigma)-1}\right]}{\left[C_t^{-\gamma(1-\sigma)}C_{t+1}^{-\sigma} - \beta\gamma C_{t+2}^{1-\sigma}C_{t+1}^{-\gamma(1-\sigma)-1}\right](P_t/P_{t+1})}\right), \tag{1.16}$$

which nests the standard CRRA case for $\gamma = 0$. In contrast to the standard case, the representative agent wants to smooth both - consumption and its growth rate over time. Using the assumption of conditional log-normality again, the approximation of (1.16) becomes:

$$[\beta(1+i_t)]^{-1} \;=\; \frac{\exp(d_t) - \beta\gamma\exp(e_t)}{\exp(a_t) - \beta\gamma\exp(b_t)}, \tag{1.17}$$

where coefficients are given by:

$$a_t = -\gamma(1-\sigma)c_{t-1} - \sigma c_t \tag{1.18}$$

$$b_t = (-\gamma(1-\sigma)-1)c_t + (1-\sigma)E_t c_{t+1} + \frac{(1-\sigma)^2}{2}V_t c_{t+1} \tag{1.19}$$

$$d_t = -\gamma(1-\sigma)c_t - \sigma E_t c_{t+1} - E_t\pi_{t+1} + \frac{\sigma^2}{2}V_t c_{t+1} + \frac{1}{2}V_t\pi_{t+1} + \sigma cov_t(c_{t+1},\pi_{t+1}) \tag{1.20}$$

$$\begin{aligned} e_t = {} & (-\gamma(1-\sigma)-1)E_t c_{t+1} + (1-\sigma)E_t c_{t+2} - E_t\pi_{t+1} + \frac{(\gamma(\sigma-1)-1)^2}{2}V_t c_{t+1} \\ & + \frac{(1-\sigma)^2}{2}V_t c_{t+2} + \frac{1}{2}V_t\pi_{t+1} + (1-\sigma)(\gamma(\sigma-1)-1)cov_t(c_{t+1},c_{t+2}) \\ & - (1-\sigma)(cov_t(\pi_{t+1},c_{t+2}) - (\gamma(\sigma-1)-1)cov_t(\pi_{t+1},c_{t+1})). \end{aligned} \tag{1.21}$$

Again, a reduced-form VAR gives the first and second moments which are then used to calculate the implied nominal and real rates out of the model. Following Canzoneri, Cumby, and Diba (2007), I set the discount factor $\beta = 0.986$, risk aversion $\sigma = 2$ and the degree of habit persistence $\gamma = 0.6$. Results are again shown in Table 1.2 and Figure 1.2. Similar to the case of CRRA preferences, the correlation between model-implied and ex-post observed rates is moderately positive for nominal rates and close to zero for real rates. Additionally, all summary statistics come very close to the data - the Fuhrer (2000) specification even outperforms the CRRA case for a few summary statistics. Nevertheless, as for the CRRA case, both rates diverge during the identified periods of monetary policy action.

Abel (1990, 1999)

In the specification of Abel (1990, 1999), period utility depends on the ratio of current consumption to a reference level, as specified in (1.1). In contrast to Fuhrer (2000) however, habits are external and not internal. Here, the habit level is a function of the lagged aggregate instead of the lagged individual level of consumption. This assumption simplifies the first-order condition of the household and is feasible as long as in equilibrium, aggregate and individual growth rates of consumption will be the same. Parameter values of the log-linear approximated Consumption Euler equation are picked following an algorithm that matches unconditional first and second moments of the model with respective sample moments. In Abel (1999)'s model, the representative agent i maximizes the following utility function:

$$U_t(i) = E_t\left\{\sum_{j=0}^{\infty}\beta^j\left(\frac{1}{1-\sigma}\right)\left(\frac{C_t(i)}{Z_t}\right)^{1-\sigma}\right\}, \tag{1.22}$$

where σ is the degree of risk aversion, β the discount factor and $C_t(i)$ the individual level of consumption. The habit level of consumption is given by $Z_t = C_t^{\gamma_0}C_{t-1}^{\gamma_1}G^{\gamma_2}$. It refers to the current and past aggregate level of consumption and its average trend growth rate, G. Using that in equilibrium both growth rates coincide, the implied nominal interest rate out of the Consumption Euler equation looks as follows:

$$\frac{1}{1+i_t} = \tilde{\beta} E_t \left[\left(\frac{C_t}{C_{t-1}} \right)^{\gamma_1(\sigma-1)} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right], \tag{1.23}$$

where $\tilde{\beta} = \beta G^{\gamma_2(\sigma-1)}$. The equation results from imposing three restrictions that guarantee unique parameter values: $\gamma_0 = 0$, $0 \leq \gamma_i \leq 1$, and $G = 1 + \mu$, where μ refers to the mean growth rate of aggregate consumption. As above, the implied real rate is the same except for the terms involving the price level. Under the restrictions imposed above, Abel (1999)'s methodology only applies to the case where consumption growth is iid. lognormal. However, following the results of Fuhrer (2000), the dynamics of consumption and inflation are well represented using a Vector Autoregressive Model (VAR), where variables are *jointly* lognormal. Abel (1999)'s calibration method leads to biased parameters estimates. Nevertheless, as Ahmad (2005) argues, Abel's methodology is a good reference point even under a joint lognormal distribution in order to generate a series whose mean and variance are close to the observed ones in the sample. Taking a lognormal approximation around (1.23) and imposing the restrictions on the γ's gives:

$$\begin{aligned} \frac{1}{1+i_t} = \ & \tilde{\beta} \exp\{-\gamma_1(\sigma-1)c_{t-1} + (\sigma + \gamma_1(\sigma-1))c_t - \sigma E_t c_{t+1} - E_t \pi_{t+1} \\ & + \frac{\sigma^2}{2} Var_t c_{t+1} + 0.5 Var_t \pi_{t+1} + \sigma Cov(c_{t+1}, \pi_{t+1})\}. \end{aligned} \tag{1.24}$$

Results are virtually identical to the cases considered above however, the first to specifications match the data more closely.

1.4 The Importance of the Sample Horizon

Canzoneri, Cumby, and Diba (2007) argue that model-generated and ex-post observed rates are generally uncorrelated and move into opposite directions in response to a change in the monetary policy instrument. Assuming that the way the household forms expectations does not change over time (i.e. estimate one VAR over the entire sample), the size and the sign of the correlation coefficient should not change between subsamples.[8] As this section shows, this is not the case. The moderate correlation for nominal rates and the low correlation for real rates estimated over the full sample horizon could be due to averaging. For this reason, the following step splits the sample into two subsamples. The first comprises an episode called the *Great Inflation* whereas the second subsample looks at the last 30 years. It includes the so-called *Great Moderation*, the recent financial crisis and beyond. Differences over the sample horizon could

[8] Looking at smaller subsamples means that the series, estimated using the full-sample VAR is split at several points and compared with observed rates within this period.

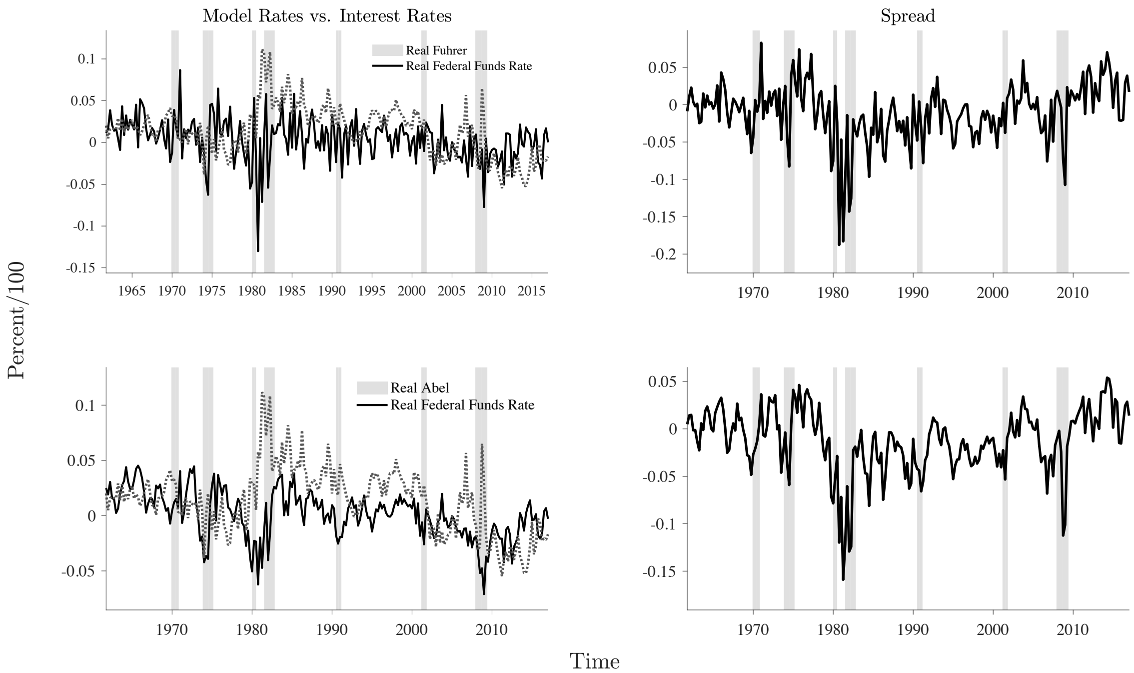

Figure 1.3. Model-Implied Rates vs. Ex-Post Observed Rates (annualized)

Notes: The figure's first column shows the development of ex-post observed real interest rates (dashed) compared to rates predicted from the VAR and the DSGE model (solid) for the Fuhrer (2000) and Abel(1990,1999) preference specification. The second column calculates the spread, i.e. the difference between model rate and the data and sets and highlights period when the US has been in a recession as classified by the NBER.

be either due to a change in the way agents form expectations or because the macroeconomic environment has changed tremendously. This chapter focuses on the later and thus assumes that the estimated coefficients of the VAR are time-constant. The following subsections motivate and discuss the choice of these subsamples.

1.4.1 Great Inflation

Meltzer (2005) describes the high inflation period from the mid-1960's to the mid-1980's with inflation rates varying between two and 15 percent as the []..."the climactic monetary event of the last part of the 20th century." Siegel (1994) even calls it "[]...the greatest failure of American macroeconomic policy in the postwar period". During this period, the Bretton-Woods system which had been established during World War II collapsed, two oil-price shocks caused severe damage and the economy suffered from four recessions. In addition, macroeconomic theory and the way policy makers thought about monetary policy started to change as a consequence of outcomes of this episode. Finally, this period provides the biggest spread between model and data, as the right half of Figure 1.2 shows. Furthermore, the spread seems to be positive on average, whereas it appears to be negative on average for the later part of the sample.

The following Table 1.4 displays sample moments and correlation from 1969 to 1982. Model

Table 1.4. Ex-post Observed and Model-Implied Nominal Interest Rates, 1969-1982

Time period: 1969-01-01 to 1982-10-01

Rates computed from models	Data	CRRA	Fuhrer (2000)	Abel (1999)
Real Rates				
Mean	0.016	0.0097	0.0015	0.0013
St. deviation	0.032	0.031	0.037	0.028
Min	-0.035	-0.052	-0.13	-0.062
Max	0.11	0.06	0.065	0.045
corr(Data,Model)	1	0.0061	-0.05	0.0072
Nominal Rates				
Mean	0.087	0.078	0.07	0.069
St. deviation	0.039	0.024	0.034	0.021
Min	0.035	0.0076	-0.035	0.0019
Max	0.18	0.13	0.14	0.11
corr(Data,Model)	1	-0.41	-0.24	-0.41

and data are far from being positively correlated as the correlation coefficients are close to zero for real and negative, in the range of -0.30 to -0.13 for nominal rates. Upon other explanations as e.g. the high inflation period, the Federal Reserve's behavior in response to shocks could be an explanation for the switch of the sign of the correlation coefficient. If agents had been surprised a lot by the Fed's policy but their expectations had been based on a version of the models revisited above, the huge spread between model and data appears to be quite natural. The next section will cast a closer look upon that hypothesis using narrative policy evidence provided by Romer and Romer (2004).

1.4.2 Great Moderation and Beyond

The second subsample includes the years from 1983 to 2017. It comprises the years of the *Great Moderation*, a term coined by Stock and Watson (2002) and former Federal Reserve chairman Ben Bernanke.[9] This period has seen a reduction in the volatility of Real GDP and inflation relative to the 1970's as Table 1.1 showed. Moreover, both nominal interest and real rates and inflation exhibited some trend decline. Monetary policy fought high inflation rates by responding aggressively to inflationary shocks (as argued in Clarida, Galí, and Gertler (2000), Boivin

[9] Bernanke referred to this in a speech at the meeting of the Eastern Economic Association, Washington, DC on February 20, 2004.

and Giannoni (2006) or Lubik and Schorfheide (2004) among others). The years of the financial crisis or *Great Recession* are included into the subsample on purpose, as both interest rates and consumption follow a similar trend decrease since the 1980's and are thus rather positively correlated. Between 2008 to 2015, when the effective Federal Funds rate hit the ZLB, i.e. is lower than 0.25%, the shadow rate calculated from the model by Wu and Xia (2016) has been used instead of the Federal Funds rate. Furthermore, Figure 1.2 supports the argument to distinguish these two episodes as mean and variance of the spread are smaller on average over the last 30 years. In contrast to the first subsample, the second one shows a strong positive

Table 1.5. Ex-post Observed and Model-Implied Nominal Interest Rates, 1983-2017

Time period: 1983-01-01 to 2017-01-01

Rates computed from models	Data	CRRA	Fuhrer (2000)	Abel (1999)
Real Rates				
Mean	0.0092	0.0035	-0.00056	-0.0041
St. deviation	0.031	0.019	0.022	0.017
Min	-0.053	-0.07	-0.077	-0.071
Max	0.08	0.048	0.058	0.038
corr(Data,Model)	1	0.38	0.2	0.38
Nominal Rates				
Mean	0.036	0.03	0.026	0.022
St. deviation	0.034	0.025	0.025	0.023
Min	-0.029	-0.072	-0.05	-0.073
Max	0.11	0.091	0.1	0.081
corr(Data,Model)	1	0.62	0.49	0.63

correlation that ranges from 0.5 to 0.6 for nominal and a moderate one between 0.2 and 0.35 for real rates. Table 1.5 displays the respective results.

1.4.3 What's driving the correlation

I conclude this section with an alternative way to think about the surprisingly distinct results for correlation coefficients between model and data at different points in time. For a given sample or subsample period, the calculated correlations coefficients report the ratio of the model's and the data's covariance over the product of the sample variances. In any case, this involves some sort of averaging over the *entire* (sub)sample. Using a correlation coefficient, augmented with a weighted, two-sided rolling window, reveals that the correlation centered around a specific date or point is far from being always negative in earlier and always positive in later years.[10] Figure 1.4 depicts this sequence of correlations within an eight quarter window, and illustrates

[10] Appendix A.3 provides the formula used to obtain smoothed correlation coefficients.

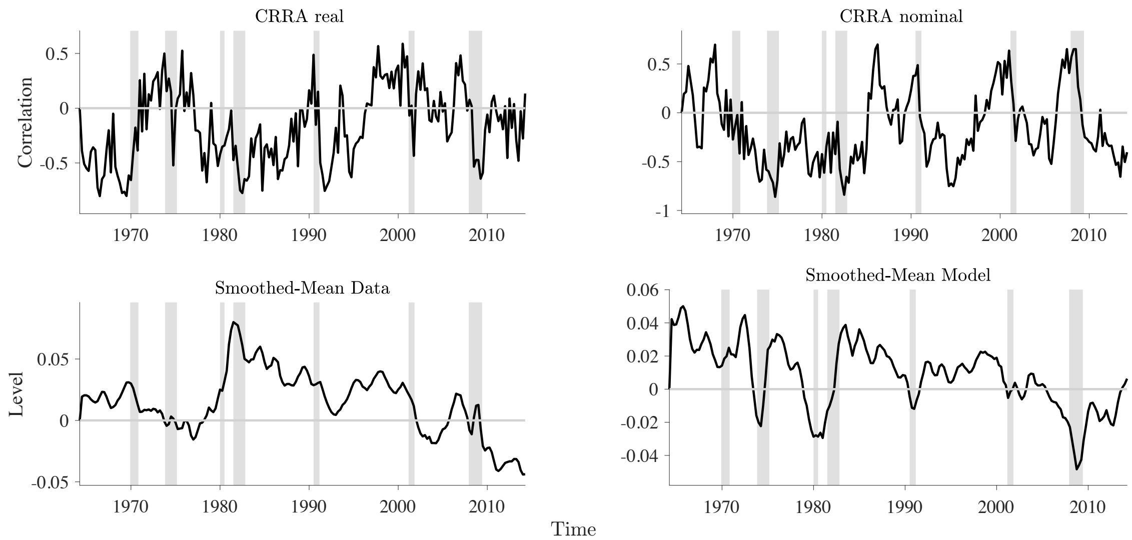

Figure 1.4. Smooth Correlation and Smooth Mean of Interest Rate and Model Rate

Notes: The figure shows the smoothed correlation and smoothed mean of observed interest- and model-implied rates using a eight-year rolling window. Appendix A.3 provides the exact formulas.

that the rolling-window correlation is much more volatile than expected. Furthermore, big reversals of the series of correlation coefficients seems to take place around NBER recession dates which underlines the hypothesis that the business cycle plays an important role in explaining the divergence of model and data. In addition, the sample average over the rolling-window correlation coefficients does not coincide with the coefficients calculated in Tables 1.2, 1.4 or 1.5. To understand, where the volatility, inherent in Figure 1.4 comes from, I cast a closer look on the smoothed, weighted means, used to calculate the rolling correlation. A similar pattern as in Figure 1.2 emerges. The model is not too bad in capturing the long-term trend movements, but it fails at predicting business cycle movements.

Using a standard HP-filtering approach to separate trend and cyclical component of both model-implied and observed rates, reveals that the cyclical components of the model and the data are negatively correlated. From the Consumption Euler equation we know that cyclical components should be highly positively correlated. In contrast to that, the size of the correlation coefficient for the trend components is close to the one calculated in the replication of Canzoneri, Cumby, and Diba (2007)'s results. Moreover, the cyclical component of the spread between model and data is negatively correlated with the cyclical component of both real and nominal interest rates. This confirms the result of Canzoneri, Cumby, and Diba (2007) that monetary expansions over the business cycle are associated with a wider discrepancy between model and data. The following table visualizes this result using the HP-Filter and the Baxter-King-Filter. Results for the later are robust to findings using the HP-Filter. The filtered data comprises the

Table 1.6. Cyclical and Trend Correlation of Data with Model Rates

Time period: 1963 to 2013			
HP-Filter	CRRA	Fuhrer et. al. (2000)	Abel (1999)
Trend			
real	0.319	0.0841	0.304
nominal	0.643	0.489	0.652
Cycle			
real	-0.276	-0.299	-0.281
nominal	-0.257	-0.11	-0.228
corr(Spread^C, Interest Rate^C)			
real	-0.777	-0.661	-0.802
nominal	-0.757	-0.605	-0.771

Time period: 1963 to 2013			
Baxter-King-Filter	CRRA	Fuhrer et. al. (2000)	Abel (1999)
Trend			
real	0.504	0.443	0.501
nominal	0.785	0.795	0.793
Cycle			
real	-0.23	-0.189	-0.238
nominal	-0.216	-0.101	-0.186
corr(Spread^C, Interest Rate^C)			
real	-0.759	-0.8	-0.788
nominal	-0.752	-0.727	-0.77

Notes: The Table displays the correlation coefficient between model and data rate when the interest rate is seperated into a trend and a cyclical component. This is done on the full sample, the data loss on both sides is due to the Baxter-King filter.

full sample, however, use of the Baxter-King filter leads to a loss of 8 quarters at both sides of the sample. To ensure comparability of results between both filters I truncate the HP-filtered data, respectively.

1.5 Explaining the Spread between Model and Data

Section 1.3 confirmed the result of Canzoneri, Cumby, and Diba (2007) and Ahmad (2005) who link the stance of monetary policy to the wedge between model and data. Also, section 1.4

showed that the selection of the sample horizon seems to matter in so far as the model-implied rates track the data much closer from the 1980's on. There are several possibilities to bring this observation to the data. In a first step, the following subsections show that the wedge between model and data indeed moves into opposite directions following a monetary policy intervention. In the second step, this chapter provides a way to resolve this issue.

1.5.1 Canzoneri et al. approach

Table 1.7. Response of Interest Rate Spreads to Monetary Policy (Quarterly Data)

Time period: 1969-01-01 to 1982-10-01

Monetary Policy Indicator	CRRA	Fuhrer (2000)	Abel (1999)
Real Rates			
FFR	-0.843	-0.92	-0.915
	(0.2151)	(0.3272)	(0.3247)
R^2_{ad}	0.6077	0.3399	0.3393
Nominal Rates			
FFR	-1.45	-1.6	-1.45
	(0.1665)	(0.2408)	(0.1635)
R^2_{ad}	0.8486	0.6903	0.8616

Time period: 1983-01-01 to 2008-10-01

Monetary Policy Indicator	CRRA	Fuhrer (2000)	Abel (1999)
Real Rates			
FFR	-0.225	-0.572	-0.447
	(0.1243)	(0.1486)	(0.1305)
R^2_{ad}	0.4247	0.3527	0.2501
Nominal Rates			
FFR	-0.253	-0.511	-0.267
	(0.06008)	(0.1235)	(0.06112)
R^2_{ad}	0.7183	0.3392	0.7307

Notes: HAC robust standard errors in parentheses. In each regression, the spread (defined as the interest rate computed from each models Euler equation less the federal funds rate) is regressed on four lags of the spread and an indicators of monetary policy. Only the coefficients and corresponding standard errors for the monetary policy variables are reported.

Canzoneri, Cumby, and Diba (2007) regress the spread between model and data on measures of monetary policy and investigate the impact of a movement in the Federal Funds rate on the

Consumption Euler Rate. In contrast to that, Ahmad (2005) conducts a structural VAR analysis in the tradition of Christiano, Eichenbaum, and Evans (1999). The difference between both approaches is that Canzoneri, Cumby, and Diba (2007) considers any movement in the policy instrument of the central bank, i.e. expected and unexpected whereas Ahmad (2005) only analyzes surprise ones.

Following Canzoneri, Cumby, and Diba (2007), I regress the spread (defined as the model-generated interest rate minus the data) on four own lags and the contemporaneous Federal Funds rate.

$$spread_t = \beta_0 + \sum_{i=1}^{4} \beta_i spread_{t-i} + FFR_t + \nu_t \tag{1.25}$$

The included lags capture the autoregressive nature of the series (as can be seen in Figure 1.2) and the Federal Funds rate represents the instrument of monetary policy. If monetary policy drives the difference between data and model, an increase in the monetary policy instrument should lead to a significant and negative coefficient. Furthermore, the original paper uses a second measure (the ratio of non-borrowed reserves plus extended credit to total reserves) which I will not employ here. A reason for that is the explosion of non-borrowed reserves from 2008 on which would bias any result towards that period. The following Table 1.7 reports regression results. Monetary contractions, i.e. a higher federal funds rate cause the spread to narrow. All estimated coefficients are highly significant. This is surprising as Canzoneri, Cumby, and Diba (2007) report insignificant results for rates calculated from Fuhrer (2000)'s model. Again, there is a substantial difference between the two periods from 1969 to 1982 and 1983 to 2017. In addition, there is no qualitative difference in the reaction of nominal and real model-implied interest rates. However, the coefficient for nominal rates approaches the coefficient for real rates in the latter half of the sample, whereas there is a enormous difference between both in the first half. As a robustness check, I calculate the spread between data and model using the 3-month T-Bill rate as money market rate. The Federal Funds rate remains the instrument of monetary policy. This ensures that the sign of the monetary policy indicator in Table 1.7 is not negative by construction. Results are qualitatively identical and quantitatively very close to the original setting. Appendix A.2.2 provides the respective results. Overall, these results confirm and extend the finding of Canzoneri, Cumby, and Diba (2007). A possible shortcoming of this approach is the role of unconventional monetary policy from 2008 onwards which is present in the top of Table 1.7. The zero-lower bound for nominal interest rates induced a shift from the conventional policy instrument as the implied federal funds rate to more unconventional instruments. Using the a Federal Funds shadow rate as proposed in Wu and Xia (2016) is one way to account for these effects. Nevertheless, the main statement of this section, that the impact of a change in the federal funds rate on the spread is tremendously higher from 1969 to 1982 does not change when the regarded second sample already ends in 2008.

1.5.2 Narrative Identification

In what follows I will cast a closer look upon the early sample, i.e. the late 1960's until the early 1980's. The above results suggest that changes in the policy rate often surprised agents which drives the spread between model and data apart. If this was the case, the above regression is misspecified as ex-post observed data on the Federal Funds rate is correlated with these policy surprises. Hence, results are biased. The standard way to heal this bias is to find a suitable instrument for the Federal Funds Rate. Assume that the Federal Reserves follows some sort of Taylor (1991) rule with an endogenous reaction to the state of the economy and a surprise component. Using the residual of the regression of changes in the implied Federal Funds Rate on Greenbook forecasts, i.e. the Romer and Romer (2004) measure of monetary policy shocks, it is possible to control for these surprise moments. In order to test this hypothesis I will use narratively identified monetary policy shocks[11] until 2008 to instrument the Federal Funds rate at the first-stage of an IV regression. Analogously to Romer and Romer (2004), I include the cumulated sum of the narrative shock series provided that the federal funds rate is usually measured in levels. Table 1.8 shows the results. In comparison to Table 1.7 results change tremendously. Monetary policy action does no longer have a highly significant influence on the spread between model and data in case of CRRA preferences. In comparison to the results in Table 1.7, the reaction to the FFR becomes larger (less negative) as well. Puzzling however is that Fuhrer (2000)'s and Abel (1999)'s specification still react significantly - and even stronger - to the stance of monetary policy.

Nevertheless, this puzzle vanishes with regards to the second subsample (due to Greenbook forecast lag only until 2008). All coefficients lose their economic and statistical significance which is precisely the result this exercise was looking for. Controlling for the unpredictable component of monetary policy implies that movements in the FFR no longer widen the spread between model and data. This result is in line with the literature on announcement effects of monetary policy, e.g. Gürkaynak, Sack, and Swanson (2005).

1.6 Conclusion

When DSGE models with are very standard approach to towards the consumption-savings problem are put to the data, a fundamental problem concerning the transmission mechanism of monetary policy arises. Theoretical predictions that the real interest rate was a sufficient statistic for consumption growth are not reconcilable with the data. Especially, interest rates implied out of a Consumption Euler equation - are weakly or even negatively correlated with their em-

[11] This measure of monetary policy shocks has been introduced by Romer and Romer (2004) and has been updated by Coibion et al. (2012) and can be downloaded from Yuri Gorodnichenko's homepage. For the time period 1969 (due to availability of the shock series) until 1982 I will employ the original series. For the period 1983 until 2008 (due to the publication lag of the Greenbook forecasts) I will make use of the extended series. Figure A.1 in the appendix shows that the extended and revised series comes very close to the Romer and Romer (2004) series during the original time period.

Table 1.8. Response of Interest Rate Spreads to Monetary Policy (Narrative Approach)

Time period: 1969-01-01 to 1982-10-01

Monetary Policy Indicator	CRRA	Fuhrer (2000)	Abel (1999)
Real Rates			
FFR	-0.384	-1.03	-0.886
	(0.3667)	(0.3656)	(0.3914)
R^2_{ad}	0.5532	0.5696	0.6135
Nominal Rates			
FFR	-0.473	-1.87	-0.833
	(0.543)	(0.6097)	(0.4371)
R^2_{ad}	0.5899	0.49	0.7453

Time period: 1983-01-01 to 2008-10-01

Monetary Policy Indicator	CRRA	Fuhrer (2000)	Abel (1999)
Real Rates			
FFR	0.0257	0.118	0.0579
	(0.1341)	(0.2389)	(0.1557)
R^2_{ad}	0.375	0.09589	0.268
Nominal Rates			
FFR	0.0486	-0.129	0.045
	(0.1148)	(0.2066)	(0.1079)
R^2_{ad}	0.5436	0.2123	0.6106

Notes: Standard errors in parentheses. In each regression, the Federal Funds Rate is regressed in a first stage on the narratively identified Romer and Romer rates plus four lags of the spread. The second stage regresses the fit of the first stage plus 4 lags of the spread on the spread (defined as the interest rate computed from each models Euler equation less the Federal Funds rate). Only the coefficients and corresponding standard errors for the monetary policy variables are reported.

pirical counterparts. A lot of recent work in the tradition of Canzoneri, Cumby, and Diba (2007) showed that this poses a severe problem for the modeling of the transmission mechanism of monetary policy. In this class of models the nominal interest rate is set by the central bank via a Taylor rule. Changes in the policy instrument affect expectations over the price level and the growth rate of consumption. The model predicts that the correlation between a money market rate as e.g. the Federal Funds rate and a rate implied from changes in the expectations over consumption growth has to be unity. If empirical evidence is not able to confirm this prediction, this suggests that we should further augment the modeling of the transmission mechanism of monetary policy. In order to solve this problem, previous work focused on alternative household preferences as e.g. habit formation or non-separability between consumption and leisure. Both approaches have been able to improve the fit of the model, however none plausibly solved this surprising result.

The present chapter of my thesis contributes to the existing literature on the empirical validity of Consumption Euler equation-implied behavior with respect to three points:

First, the existing lack of a clear-cut explanation for the puzzling finding that the correlation between model-implied and observed rates has the wrong sign is not only due to using a bad model. A closer look to different periods of a long data sample (1960-2017) as this chapter did, reveals that the sample horizon plays a crucial role for the determination of the sign of the correlation coefficient between model and data. This correlation has been zero or even negative during a subsample from 1969 until 1982 when the Volcker disinflation heralded a shift towards a new monetary policy regime. The strong positive correlation between model and data observed during the last 30 years confirms this notion. The last ten years amplify this result as consumption growth and nominal / real interest rates moved into the same direction during the recession.

Second, the model does surprisingly well in capturing the long-term trends observed in the data: consumption growth and real rates have been declining for 35 years, a behavior model-implied interest rates can account for excellently. Digging deeper into that observation reveals that following a separation of model-implied and observed rates into a trend and a cyclical component, the trend components of model and data are strongly correlated over the full sample. In contrast to that, the cyclical behavior seems to drive a lot of the observed negative correlation from 1969 to 1982 and works as a downward bias of the correlation coefficient during from 1983 until today. Alternative approaches towards the calculation of the correlation using a rolling-window support this result. Furthermore, the graphical results revealed that variation or discrepancy between model and data is strongly pronounced around recession dates.

This brings the chapter to the final and third contribution, which picks up a point made in Canzoneri, Cumby, and Diba (2007): The spread between model-implied and observed interest rate reacts systematically to the stance of monetary policy. This fact can be replicated plus the observation that this reaction has been much stronger pronounced for the 1969 to 1982 subsample. However, if one controls for surprise movements in the Federal Funds rate using the (extended) Romer and Romer (2004) shock series, this significant reaction disappears in both economic and statistical terms. Concluding, the chapter suggests that it is worth investigating more into what affects the business cycle components of variables inside the Consumption Euler equation. Risk-premium or preference shocks and a departure from the representative agent framework seem to be plausible candidates.

A.1 Data Appendix

VAR Data:

All data has been downloaded from `https://research.stlouisfed.org/`, the FRED database of the Federal Reserve Bank of St. Luis. The following overview provides the exact classification of the data series employed and how all variables used in the estimation process are constructed. This will help to ensure replicability and transparency of the empirical work done. The maximum data length is 1959 to 2017. One year was lost due to constructing an inflation rate out of consumer prices. Data series used in the VAR are:

- Real Personal Consumption Expenditures ND+Services
  `A796RX0Q048SBEA + A797RX0Q048SBEA`
- Nominal Personal Consumption Expenditures ND+Services
  `A796RC0Q052SBEA + A797RC0Q052SBEA`
- Real Disposable Income, `DPIC96`
- Civilian Population, `CNP16OV`
- Federal Funds Rate 7Day Average (quarterly data as monthly average), `DFF`
- Inflation Rate, calculated as $\log(\frac{P_t}{P_{t-1}})$ out of nominal and real consumption expenditures.
- Real GDP, `GDPC96`
- Production Price Index, `PPIIDC`
- Monetary Aggregate M1, `M1SL`

which have been used to construct the following variables described in the text:

$$
\begin{aligned}
c_t &= \log\left(\frac{\texttt{A796RX0Q048SBEA + A797RX0Q048SBEA}}{\texttt{CNP16OV}}\right)\\
y_t^{disp} &= \log\left(\frac{\texttt{DPIC96}}{\texttt{CNP16OV}}\right)\\
R_t &= \sqrt[4]{\left(1+\frac{\texttt{DFF}}{100}\right)}-1\\
P_t &= \left(\frac{\texttt{A796RC0Q052SBEA + A797RC0Q052SBEA}}{\texttt{A796RX0Q048SBEA + A797RX0Q048SBEA}}\right)\\
y_t &= \log\left(\frac{\texttt{GDPC96} - \texttt{A796RX0Q048SBEA + A797RX0Q048SBEA}}{\texttt{CNP16OV}}\right)\\
ppi_t &= \log(\texttt{PPIIDC})\\
m_t &= \log(\texttt{M1SL})
\end{aligned}
$$

Other Data:

The 3-Month Treasury Bill: Secondary Market Rate can be downloaded from the FRED database using the following series identifier: TB3MS The Federal Funds shadow rate has been provided by Wu and Xia (2016) and can be downloaded for monthly updates on: https://sites.google.com/site/jingcynthiawu/home/wu-xia-shadow-rates As producer prices might have been an issue in the VAR specification, I run a robustness-check using data including the Thomson Reuters/CoreCommodity CRB Index (TR/CC CRB) obtained from Thomson/Reuters Datastream, as the Journal of Commerce JOC-ECRI Industrial Price Index used in Canzoneri, Cumby, and Diba (2007) has not been publically available for me.

A.2 Robustness Checks

A.2.1 Replicating the Canzoneri et al. (2007) Setup

The following Table A.1 compares results estimated for the period 1966Q1 until 2004Q4 as Table 1 in the original Canzoneri, Cumby, and Diba (2007) paper on page 1867. To bring the specification as close as possible to the original one, I use a commodity price index instead of a standard producer price index and furthermore, no monetary aggregate M1. Two observations are striking. Comparing my results to the respective original model formulation shows that they are quite distinct. Whereas the deviation is still tolerable in the CRRA case, results differ tremendously for the cases with habits. Moreover, my results fit the data much better although the same calibration of parameters (e.g. discount factor, risk aversion etc.) has been chosen. Moreover, the correlation between model and data is only close to the original paper for nominal rates in the CRRA case. For real rates the sign of the correlation coefficient is identical and negative.

Table A.1. Replication of Canzoneri et al. (2007)

Time period: 1966-01-01 to 2004-10-01

Rates computed from models	Data	Orig. CRRA	CRRA	Orig. Fuhrer (2000)	Fuhrer (2000)	Orig. Abel (1999)	Abel (1999)
Real Rates							
Mean	0.0209	0.0708	0.015	0.0566	-0.0711	0.0834	-0.00475
St. deviation	0.0253	0.0255	0.0199	0.313	0.0377	0.266	0.0176
Min	-0.0349	0.0164	-0.0511	-0.757	-0.277	-0.704	-0.072
Max	0.107	0.106	0.0542	0.952	0.0323	0.703	0.0332
corr(Data,Model)	1	-0.37	-0.121	-0.07	-0.191	-0.36	-0.132
Nominal Rates							
Mean	0.0649	0.116	0.06	0.101	-0.0289	0.128	0.0396
St. deviation	0.0327	0.0198	0.0333	0.315	0.0395	0.259	0.0217
Min	0.01	0.0746	0.0144	0.0144	-0.191	-0.632	-0.00357
Max	0.178	0.163	0.128	1.05	0.099	0.731	0.101
corr(Data,Model)	1	0.2	0.224	-0.1	0.0799	-0.61	0.264

Reasons for this difference may lie in the use of another commodity price index - Thomson Reuters CRB index instead of Journal of Commerce JOC index - and a questionable trend break assumption. The trend break assumption is questionable in so far as the long-term trend reversal does not start in the early 1970's but rather after the monetary regime change in the early 1980's. Because deviations from specific break dates produced highly volatile and distinct results concerning summary statistics and correlation coefficients I decided to stick with a formulation without an explicitly modeled time trend. The model's ability to capture the long-term behavior of consumption and interest rates well confirms this decision. Finally, a complete set of summary statistics of Canzoneri, Cumby, and Diba (2007)'s sample has neither been available in an appendix nor in their paper. Hence, data issues might explain the differences. Nonetheless, the main message their paper conveys for this period also holds in my results. The correlation between model and data is extremely low or even negative in parts of the sample which remains in stark contrast to all theoretical predictions. This shows that the core message is robust even over distinct model formulations or data variations and needs other explanations - some that my work tries to add.

A.2.2 Spread Regressions using T-Bill Rate

Time period: 1969-01-01 to 1982-10-01

Monetary Policy Indicator	CRRA	Fuhrer et. al. (2000)	Abel, 1999
Real Rates			
T-Bill	-0.508	-0.575	-0.551
	(0.1855)	(0.2881)	(0.2643)
R^2_{ad}	0.5505	0.2283	0.25
Nominal Rates			
T-Bill	-0.985	-1.21	-0.961
	(0.2191)	(0.3254)	(0.2417)
R^2_{ad}	0.7672	0.5768	0.7781

Time period: 1983-01-01 to 2008-10-01

Monetary Policy Indicator	CRRA	Fuhrer et. al. (2000)	Abel, 1999
Real Rates			
T-Bill	-0.173	-0.49	-0.362
	(0.1189)	(0.1368)	(0.1234)
R^2_{ad}	0.3614	0.3097	0.1967
Nominal Rates			
T-Bill	-0.208	-0.428	-0.219
	(0.05977)	(0.1106)	(0.05935)
R^2_{ad}	0.6901	0.2722	0.6998

Notes: Standard errors in parentheses. In each regression, the spread (defined as the interest rate computed from each models Euler equation less the 3M-TBill rate) is regressed on four lags of the spread and an indicators of monetary policy. Only the coefficients and corresponding standard errors for the monetary policy variables are reported.

A.3 Rolling Correlations

Figure 1.4 plots a rolling window correlation between model and observed rates. Let Y_t and M_t describe the respective data or model-implied interest rate. The chosen bandwidth is 8 quarters in each direction such that the rolling correlation window looks as follows:

$$Cov_t(Y_t, M_t, w) = \frac{\sum_{i=-8}^{8} w_i (Y_{t+i} - E_t(Y \mid Y_{t-8} \dots Y_{t+8}, w))(M_{t+i} - E_t(M \mid M_{t-8} \dots M_{t+8}, w))}{\sum_{i=-8}^{8} w_i} \tag{A.1}$$

where

$$w_i = \begin{cases} \frac{1}{(i)^2} & \forall i \neq 0 \\ 1 & \text{if } i = 0, \end{cases} \tag{A.2}$$

and the weighted mean for any $Z_t = \{Y_t, M_t\}$ is calculated as:

$$E_t(Z_t) = \frac{\sum_{i=-8}^{8} w_{t+i} Z_{t+i}}{\sum_{i=-8}^{8} w_{t+i}} \tag{A.3}$$

i.e. the weights increase with the distance i from Y_t or M_t. The rolling window correlation coefficient is thus given by the following formula:

$$\rho_t(Y, M, w) = \frac{Cov_t(Y_t, M_t, w)}{\sqrt{Cov_t(Y_t, Y_t, w) Cov_t(M_t, M_t, w)}} \tag{A.4}$$

A.4 Narrative Monetary Policy Shocks

The following Figure A.1 shows the original Romer and Romer (2004) monetary policy shock series which has been extended by Coibion et al. (2012).

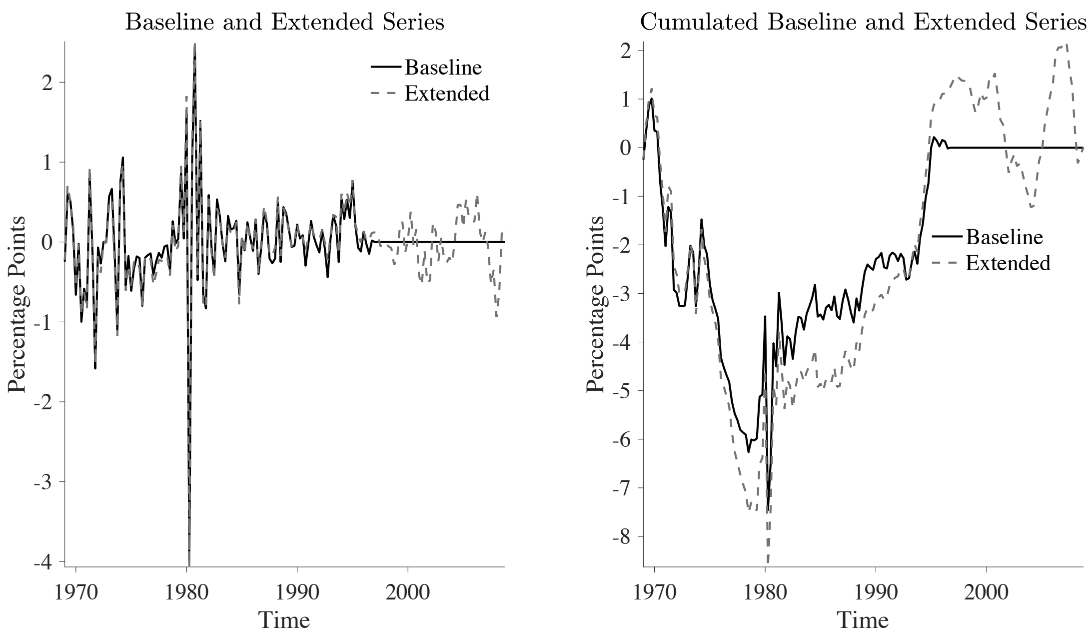

Figure A.1. Baseline (1996) and Extended (2008) Narrative Monetary Policy Shock Series

2

Monetary Policy in a Heterogeneous Agent Model With Aging

2.1 Introduction

Declining fertility rates and an increasing life-expectancy have lead to greying societies in many countries of the developed world. While the U.S. old-age dependency ratio, i.e. the share of retirees relative to the population at working age, has been 15% during the 1960s, recent projections suggest that this coefficient will increase to 50% in 2065.[1] In other words, people aged 65 or older will account for half of the population. At least since Modigliani and Brumberg (1954) we know that the consumption-savings decision of a household depends on his current stage within the life-cycle. Hence, aging could, among many other things, have an impact on the economy's aggregate savings and thus the real return of capital. Following the argument in Bean (2004), the real interest rate falls along the transition towards the new steady state of the relatively older society. Thus, the dynamic reaction of macroeconomic variables may differ. Moreover, the transmission mechanism of monetary policy, in particular how interest rate changes affect aggregate consumption, might change tremendously over the course of these developments.

In the presence of nominal rigidities, the liquidity effect of monetary policy works via three channels: Intertemporal substitution, wealth effects and the individual (in)ability to borrow against expected lifetime income. In an aging society, the relative importance of these channels will change. Monetary policy generates wealth effects as households differ with respect to their idiosyncratic productivity. Moreover, the propensity to substitute consumption today against consumption tomorrow is a function of their age. Consequently, the transmission of monetary policy innovations to the economy will be a function of the demographic structure. Young house-

[1] Estimates are provided by the UN World population projection database. The case of 60 persons older than 65 per 100 persons between 15 and 64 is the worst case scenario of low fertility and low immigration.

holds typically hold less assets than old ones. Hence, intertemporal substitution of consumption over the life-cycle is their preferred response to movements of the policy rate. On the contrary, wealth effects in response to interest rate changes matter more for older households. In an economy, that is on a transition path between *young* and *old*, the importance of wealth effects relative to substitution effects will increase. This implies that the direct impact of a policy rate move will decline and indirect effects through labor and profit income will increase. Monetary policy that considers overall welfare, should take possible effects on inter-age group inequality into account, as well.

This chapter builds a dynamic stochastic general equilibrium (DSGE) model with heterogeneous agents and stochastic aging. First-order perturbations around the stationary steady-steady are used to quantify the line of argumentation formulated above. From a methodological point of view this allows to model a parsimonious life-cycle structure without having to track every single generation as in the literature on overlapping generation (OLG) models. This enables us to study the general equilibrium effects of monetary policy both between age groups and between different steady states, i.e. comparing very young and very old economies. The model is able to replicate the decline of the real return to capital as a function of the old-age dependency ratio. Moreover, our results show that there is a significant difference in the consumption response of young and old agents to a monetary policy shock. The size of this effect depends on the fiscal stance towards debt and fluctuations in aggregate output. In addition, the way how the government stabilizes the economy, i.e. using either monetary policy, fiscal policy or a combination of both has a significant effect on results. Finally, including an age-structure into the HANK framework further reduces the effect of forward guidance shocks, here measured as the difference between a persistent and a transitory monetary policy innovation.

The chapter proceeds as follows: Section 2.2 discusses related literature. Sections 2.3 and 2.4 present the theoretical model and its numerical implementation. Section 2.5 explains how parameters are chosen to match the data. Section 2.6 describes results and section 2.7 concludes.

2.2 Literature

The present chapter builds on three important strands of macroeconomic literature: First, New Keynesian DSGE models in the tradition of Clarida, Galí, and Gertler (1999), Woodford (2003) or Galí (2008); second, incomplete markets models in which heterogeneous agents face idiosyncratic income risk in the tradition of Hansen and Imrohoroglu (1992), Huggett (1993), Aiyagari (1994) or Krusell and Smith (1997, 1998); and - finally, literature on aging as overlapping generation (OLG) or stochastic life-cycle models in the spirit of Auerbach and Kotlikoff (1987) or Gertler (1999).

DSGE models in which nominal rigidities and the non-neutrality of monetary policy meet heterogeneous agents, have emerged in recent years. Using a so called Heterogeneous Agent New Keynesian (HANK) model, Kaplan, Moll, and Violante (2018) show that one of the central theoretical predictions of classical New Keynesian models - monetary policy works mainly through direct effects of intertemporal substitution on consumption - no longer holds. The authors show instead that general equilibrium effects of monetary policy on disposable income matter much more for consumption, once uninsurable income risk and the presence of liquid and illiquid assets in the household portfolio have been taken into account. In particular, the amount of liquid, low-interest earning assets in household portfolios determines how the household reacts to a monetary stimulus.[2] Instead of consuming more, households tend to rebalance their portfolio away from liquid towards illiquid assets with a higher return. On the contrary, Bayer et al. (2015), show that in response to higher uncertainty about future idiosyncratic income, households rebalance their portfolio towards the more liquid asset. A further feature of Hank models is, that monetary policy is not independent from fiscal policy, as Ricardian equivalence fails to hold. Interest rate cuts that affect liquid assets in form of e.g. short-term government debt have an effect on the intertemporal budget constraint of the government and thus, household income. In addition, Lütticke (2017) shows that heterogeneity in the composition of household portfolios increases the responsiveness of consumption to monetary policy relative to the classical New Keynesian model.

There is a rich literature on the the “glacial” nature of demographic change and its long-run implications for monetary policy. Miles (2002) points out that within an OLG setup, aging could increase or decrease the effectiveness of monetary policy.[3] Imam (2015) estimates negative long-run effects of aging on the effectiveness of monetary policy in five of the G7 countries. In particular, for a one percentage point increase in the old-age dependency ratio, the cumulative impact of an expansionary stimulus to inflation and unemployment falls by 0.1 and 0.35 percentage points, respectively. However, Kara and von Thadden (2010) argue that the negative effect of the decrease in the population size and the increase in life-expectancy on the real interest rate does not exceed 50 basis points over a 20-year horizon. Moreover, this order of magnitude would not require a monetary policy reaction as the effect is too small to be visible within the horizon regarded for traditional monetary stabilization. Carvalho, Ferrero, and Nechio (2016) identify the increase of life-expectancy as the main driver of the decline of the real interest rate. In their calibrated model it accounts for one third of the overall decline since 1990. Moreover, they discuss challenges for monetary policy in light of the zero lower bound on nominal interest rates.

With post-crisis unconventional monetary policy the literature on monetary policy shifted towards aspects of distribution and the effect of inequality on monetary policy transmission (cf. Coibion et al. (2017); Doepke, Selezneva, and Schneider (2015); Sterk and Tenreyro (2015)).

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[2] This result in line with the growing empirical evidence that the consumption response to a cut in the central bank’s policy rate is heterogeneous across households (Broda and Parker (2014); Misra and Surico (2014)); Cloyne and Surico (2017)). Moreover, as Auclert (2017) shows, redistribution itself can be a channel trough which monetary policy affects the economy.

[3] The transmission of monetary policy could become more effective in an economy with a higher share of richer and older households that rely on savings instead of labor income (wealth channel). However, as borrowing constraints become less binding, the role of credit for consumption smoothing might decline (intertemporal substitution channel).

Only a small part of the literature however, focuses on inter-age-group heterogeneity and monetary policy:

Fujiwara and Teranishi (2008) use a New-Keynesian life-cycle model with workers and retirees to show that impulse responses to a monetary policy shock in fact depend on the age structure of society. They argue that, if consumption responses to interest rate innovations diverge, there exists a severe policy trade-off for the central bank between lower interest rate (young households) and higher interest rate (retirees) levels. Kantur (2013) confirms this result using an analytically tractable two-period overlapping generation New Keynesian model. Cwik, Lakdawala, and Peterman (2015) examine the effects of monetary policy in a life-cycle model with heterogeneous agents and show that monetary policy has quantitatively different effects on households both between and within age cohorts. In a recent contribution using the U.S. Consumer Expenditure Survey (CEX), Wong (2016) analyses the contribution of population aging to the transmission of monetary policy. Her main empirical results are: First, expansionary shocks to the monetary policy instrument are large and have a persistent effect on consumption. Second, the consumption response of young households is significantly larger than that of old ones and drives the response of aggregate consumption. Third, homeowners who adjust their loans in response to a monetary policy shock display a stronger consumption response than those of retirees or owners that do not adjust their loans. Finally, younger households tend to adjust their loans more often which explains their higher consumption elasticity. Furthermore, using a theoretical model, Wong (2016) shows that the fixed-rate mortgage structure is essential to generate the heterogeneous consumption response between young and old found in the data.

2.3 The Model

The model adds the dimension of aging to a model of the heterogeneous-agent New Keynesian (HANK) class. It is related closely to previous work in Bayer et al. (2015) and Lütticke (2017) however, with a focus on demographic aspects of monetary policy.

The economy consists of heterogeneous households, a firm sector, a treasury and a central bank. Markets are incomplete and households use physical capital or government bonds to self-insure against idiosyncratic income risk. There are two types of households - workers and entrepreneurs. When households are workers, they supply labor and physical capital and own all final goods producing firms. Otherwise, as entrepreneurs they receive an equal share of economy-wide profits. The firm sector features perfect competition among intermediate goods producers, however, final goods producers operate under monopolistic competition and set prices subject to a friction as in Rotemberg (1982). Monopolistic competition arises due to the product variety of differentiated intermediate goods. The fiscal and monetary authorities close the model by setting the nominal interest rate on bonds, levying taxes and issuing bonds to match desired expenditures and ensure the stability of government debt and inflation. The model is hit by aggregate disturbances to total factor productivity (TFP) and the Taylor rule. In what follows I will discuss the model parts in more detail.

2.3.1 Stochastic Aging

The economy consists of a continuum of households of measure one, where i describes the idiosyncratic productivity- and j the corresponding age-cohort-state. Each household is born at age 20 and ages from that time onwards. Households maximize their current and future felicity over an infinite horizon. As each household belongs to a certain age cohort j at any point in time t, aging can be described by a J-state Markov chain with transition probability matrix P_J. For a given annual life-expectancy X, there exists a transition probability from one age cohort into the next given by:[4]

$$P_J(j+1|j) \;:=\; \xi_{j+1|j} \;=\; \frac{J-1}{X}, \quad 0 \,\le\, \xi \,\le\, 1. \tag{2.1}$$

The probability to remain in an age-group j is given by $1-\xi_{j+1|j}$. Aging works only in one direction - once the household has transitioned into the next cohort, he remains there for a while or ages even further. In the limit, he dies, and is *reborn* with probability 1 in the next period.[5]

To visualize this, assume aging comprises only four different states, an amount high enough to capture the life-cycle motive of young, middle-aged, old and dying households, sufficiently. The transition probability matrix for aging described by $J = 4$ states, looks as follows:

$$P_J \;=\; \begin{bmatrix} 1-\xi_{j+1|j} & \xi_{j+1|j} & 0 & 0 \\ 0 & 1-\xi_{j+2|j+1} & \xi_{j+2|j+1} & 0 \\ 0 & 0 & 1-\xi_{j+3|j+2} & \xi_{j+3|j+2} \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{2.2}$$

The size of each age-cohort determines the position of the economy. The transition probability matrix for aging allows to calculate the stationary age distribution and thus, old age-dependency ratios. There are several ways to change the age structure of the economy in order to compare different steady state outcomes. One possibility is to decrease fertility and let population growth decline, the other is an increase in life-expectancy. Average life-expectancy (over both sexes) in the U.S. has increased from 70.4 years (1960-1965) to 78.9 years (2010-2015) and UN (2018) projects it to increase to 86.5 years in the year 2065. Put otherwise, each *new* old cohort has aged a bit slower than the previous one. To translate this into the model framework, some age cohorts age faster or slower, depending on the desired outcome. Life-expectancy X is the sum

[4] The implicit assumption behind this formulation is that the size of each age-cohort is identical in the baseline scenario.

[5] In this state the household eats up all his remaining wealth. This allows to abstract from tracing bequests from one generation to the other. This renders the model numerically much simpler. Nevertheless, bequests are important to explain the concentration of wealth in the top five- and one- percentile of the income distribution. A simple life-cycle framework fails to account for that (compare e.g. De Nardi (2015), De Nardi and Yang (2014).

of the time spent in each age cohort. If time spent is now allowed to vary, this implies that the transition probabilities are calculated as:

$$\xi_{j+1|j} = \frac{1}{X_j}, \text{using that } X = \sum_{j=1}^{J} X_j. \tag{2.3}$$

This allows to study the *effect of greying* as discussed in Miles (2002), Kantur (2013) or Imam (2015).

2.3.2 Idiosyncratic Productivity

Agents not only differ with respect to their age cohort but with respect to two more dimensions. For a given stochastic transition probability, households become either workers ($s_{ijt} = 1$) or entrepreneurs ($s_{ijt} = 0$).[6] This approach follows Castañeda, Díaz-Giménez, and Ríos-Rull (1998) and generates the entrepreneur state as high income state. Furthermore, conditional on being in age cohort j, workers are *ex-ante* identical, but are *ex-post* heterogeneous with respect to their idiosyncratic labor productivity realization, h_{ijt}. We assume that productivity evolves according to a log-AR(1)-process using Tauchen (1986)'s algorithm such that:

$$\log(h_{ijt}) = \rho_h \log(h_{ijt-1}) + \varepsilon_{ht}, \quad \varepsilon_{ht} \sim \mathbf{N}(0, \sigma_h^2). \tag{2.4}$$

To make work experience matter, the productivity process is scaled over the j age cohort, approximating a quadratic scaling. This implies that regardless of the idiosyncratic productivity realization, the middle-age household cohort will always be more productive than his younger and older counterparts. Individual labor productivity is given by

$$h_{ijt} = \frac{\tilde{h}_{ijt}}{\int h_{ijt} dji}. \tag{2.5}$$

To ensure that average worker productivity is constant, we scale $\tilde{h}_{ijt}$ by its cross-sectional average over idiosyncratic productivity and age-cohort. Entrepreneurs have zero labor productivity but receive a constant share of the economy-wide profits from imperfect competition as income. Profit incomes are subject to the same rate of taxation as labor income. This allows to neglect considerations about the optimal fiscal mix between labor taxes and taxes on pure profits.

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[6] Note that although the selection between type *worker* and type *entrepreneur* is stochastic, the economy-wide share of entrepreneurs is constant over time and small.

2.3.3 Preferences and Labor Supply

Households have time-separable preferences and discount future utility with the time-discount factor β. The discount factor does not change when agents get older. This is on purpose, as *ad-hoc* aggregate differences in subjective discounting could overlay idiosyncratic differences in capital accumulation that stem from the diverse age-productivity endowment. Households derive utility from consuming c_{ijt} and, when working, disutility from supplying labor n_{ijt}. The consumption-labor-trade-off is captured by Greenwood, Hercowitz, and Huffman (1988) (GHH) preferences such that households maximize the discounted sum of felicity:

$$E_0 \max_{\{c_{ijt},n_{ijt}\}} \sum_{t=0}^{\infty} \beta^t u\left[c_{ijt} - G(n_{ijt}, h_{ijt})\right]. \tag{2.6}$$

All worker-type households supply labor and pay taxes on their wage income.[7] Using GHH preferences makes the household problem numerically much more tractable. Households do not alter their labor supply decision in response to aggregate income uncertainty, as e.g. a TFP shock. Effectively, i.e. taking idiosyncratic labor productivity into account, all households supply the same amount of labor in terms of hours.[8] Assumptions on the functional forms of the felicity function, on the aggregate price level and the consumption bundle of differentiated goods are standard in the literature. Utility $u(\cdot)$ is obtained under constant relative risk-aversion, where $\sigma > 0$ is the degree of risk aversion such that:

$$u(x_{ijt}) = \frac{x_{ijt}^{1-\sigma}}{1-\sigma} = \frac{\left[c_{ijt} - G(n_{ijt}, h_{ijt})\right]^{1-\sigma}}{1-\sigma}, \tag{2.7}$$

where x_{ijt} is the household's demand for consumption c_{ijt} minus the disutility from labor supply, described in $G(\cdot)$. There exists are variety k of differentiated goods, bundled into a composite consumption good using the Dixit-Stiglitz aggregator:

$$c_{ijt} = \int \left(c_{ijkt}^{\frac{\eta-1}{\eta}} dk\right)^{\frac{\eta}{1-\eta}}. \tag{2.8}$$

Variety justifies price differences such that each of this differentiated goods is offered at price p_{kt}. Aggregate price level P_t and the demand for each single variety are thus given by:

$$P_t = \left(\int p_{tk}^{1-\eta} dk\right)^{\frac{1}{1-\eta}} \quad \text{and} \quad c_{ijkt} = \left(\frac{p_{kt}}{P_t}\right)^{-\eta} c_{ijt}. \tag{2.9}$$

[7] Modeling an additional "retiree problem" implied that retirees no longer work but received tax-financed lump-sum transfers. Its implementation would be straightforward and is a possible future extension. The scaling of idiosyncratic productivity realizations over the life-cycle already fulfills a similar purpose.

[8] This translates into the functional form of $G(\cdot)$ which is a function of h_{ijt}, as well. Hence, there exists a functional form such that the household's effective labor supply does only depend on the nominal wage and no longer on idiosyncratic productivity.

For hours n_{ijt} worked at wage rate w_t and effective labor tax rate τ, the first-order condition of $G(n_{ijt}, h_{ijt})$ with respect to n_{ijt} leads to the following expression:

$$\frac{\partial G(n_{ijt}, h_{ijt})}{\partial n_{ijt}} = (1-\tau) w_t h_{ijt}. \tag{2.10}$$

Under a constant Frisch-elasticity of labor supply and the income distribution as calibration target we can choose a functional form

$$G(n_{ijt}, h_{ijt}) = h_{ijt} \frac{n_{ijt}^{1+\gamma}}{1+\gamma}, \tag{2.11}$$

where $\gamma > 0$ is the inverse Frisch-elasticity. Idiosyncratic productivity h_{ijt} drops then from the FOC with respect to n_{ijt}. This result eliminates Hartmann-Abel effects of uncertainty on labor supply discussed above and implies that we can drop the household-specific indices i, j and substitute N_t for n_{ijt}, as $n_{ijt} = N_t(w_t)$.[9] This allows to simplify the expression for x_{ijt} and $G(\cdot)$, which now read:

$$x_{ijt} = c_{ijt} - \frac{(1-\tau) w_t h_{ijt} N_t}{1+\gamma} \tag{2.12}$$

$$G(h_{ijt}, N_t) = \frac{N_t^{1+\gamma}}{1+\gamma}. \tag{2.13}$$

Asset markets are incomplete. The household derives his portfolio decision subject to the following budget constraint:

$$c_{ijt} + b_{t+1} + q_t k_{ijt+1} = (q_t + r_t) k_{ijt} + (1-\tau^w) w_t h_{ijt} N_t + b_t \frac{R_t^b}{\pi_t}, \tag{2.14}$$
$$k_{ijt+1} > 0, \quad b_{t+1} > 0,$$

where households can either save into capital k_{ijt} or into government bonds b_t. Capital k_{ijt} rents out at at price q_t and pays r_t as dividend, which has to equal the marginal product of capital net of depreciation. Government bonds b_t are a linear combination of total assets and the capital stock and pay nominal return R_t^b.[10] The return on bonds has to be adjusted by realized inflation, $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$.

[9] This assumption does not alter the portfolio-choice problem of the household. However, higher realized uncertainty translates into an increase of the dispersion of h_{ijt}. Without this assumption individual labor supply n_{ijt} increased in h_{ijt} and thus, total labor supply increased in the dispersion of h_{ijt}. This complicates aggregation.

[10] The assumption to abstract from including bonds into the household's decision problem is strong and neglects important life-cycle considerations, as e.g. portfolio rebalancing in response to income uncertainty (cf.. Bayer et al. (2015)). Nonetheless, bonds as aggregate asset suffice to fulfill the purpose of having a second asset in the model.

Substituting $x_{ijt} = c_{ijt} - \frac{(1-\tau)w_t h_{ijt} N_t}{1+\gamma}$ into (2.14) results in the following expression:

$$x_{ijt} + b_{t+1} + q_t k_{ijt+1} \;=\; (q_t + r_t)k_{ijt} + (1 - \tau^w)\left(\frac{\gamma}{1+\gamma} w_t h_{ijt} N_t\right) + b_t \frac{R_t^b}{\pi_t}, \qquad (2.15)$$

$$k_{ijt+1} \;>\; 0, \quad b_{t+1} \;>\; 0,$$

The dynamic planning problem of the household's optimization problem is characterized by the following Bellman equation:

$$V(k,h,J,\Theta;K,R^b,s) \;=\; \max_{\Delta k'} u\Big[x(k,k',h,J) + \beta EV(k',h',J',\Theta';K',R^{b'},s')\Big]. \qquad (2.16)$$

The distribution function Θ is the joint distribution of (k,h,J). Aggregate bond holdings and inflation, described in the paragraph on the government sector further below, depend on Θ_t which thus becomes a state variable of the planning problem.

The First-order necessary conditions for the full maximization problem can be summarized as follows:

$$c_{ijt} \;:\; u'(x_{ijt}) - \lambda_t \;=\; 0 \qquad (2.17)$$

$$N_t \;:\; u'(x_{ijt})h_{ijt}N^{\gamma} \cdot (-1) + \lambda_t(1-\tau)w_t h_{ijt} \;=\; 0 \qquad (2.18)$$

$$b_{t+1} \;:\; -\lambda_t + \beta E_t \frac{R_{t+1}^b}{\pi_{t+1}} \lambda_{t+1} \;=\; 0 \qquad (2.19)$$

$$k_{ijt+1} \;:\; -\lambda_t q_t + \beta E_t(q_{t+1} + r_{t+1})\lambda_{t+1} \;=\; 0, \qquad (2.20)$$

where λ_t is the Lagrange multiplier on the budget constraint. The last two equations describe the sequence of consumption Euler equations and the no-arbitrage condition between the net marginal return on capital r and the nominal interest rate R^b, corrected for inflation. Adjusting capital is subject to a friction given by:

$$\Delta k_{ijt+1} \;=\; \frac{1}{q_t}\left[I_{ijt} - \frac{\theta}{2}\Big(\frac{\Delta k_{ijt+1}}{k_{ijt}}\Big)^2 k_{ijt}\right], \qquad (2.21)$$

where I_{ijt} is investment. An equilibrium condition for the price of capital q_t can be derived by maximizing (2.16) subject to (2.14) and (2.21) with respect to Δk_{ijt+1}. This includes the assumption that $q_t = \frac{\lambda_t}{\mu_t}$, where μ_t is the Lagrange multiplier on (2.21) such that:

$$-\lambda_t q_t + \mu_t q_t + \mu_t q_t \theta\Big(\frac{\Delta k_{ijt+1}}{k_{ijt}}\Big) \;=\; 0, \qquad (2.22)$$

which can be rearranged to:

$$q_t \;=\; 1 + \theta\Big(\frac{\Delta k_{ijt+1}}{k_{ijt}}\Big). \qquad (2.23)$$

2.3.4 Firm Sector

The model breaks up production into two parts. Intermediate goods and final goods producers. Whereas the first work under perfect competition, the later differentiate the intermediate good and exercises a degree of market power via its price setting mechanism.

Firm Problem: Intermediate Goods Producers

The intermediate goods producing firm operates under perfect competition and employs labor and capital as factor inputs. The production function has constant returns to scale:

$$Y_t = A_t K_t^{\alpha} N_t^{(1-\alpha)}, \tag{2.24}$$

where N_t is total labor supply and K_t the amount of total assets invested into the capital stock. Total factor productivity (A_t) is an AR(1)-process that moves according to an exogenous innovation such that:

$$\log(A_{t+1}) = \rho_A \log(A_t) + \varepsilon_{t+1}^A \quad \varepsilon_t^A \sim \mathbf{N}(0, \sigma_A^2). \tag{2.25}$$

Define MC_t as the price at which the intermediate good is sold to final good producers. The profit-maximization problem then reads as follows:

$$MC_t Y_t = MC_t A_t K_t^{\alpha} N_t^{(1-\alpha)} - (\delta + r_t)K_t - w_t N_t. \tag{2.26}$$

Intermediate goods firms make zero profits and factor inputs earn their (net) marginal product. The real wage rate and the real interest rate are given by:

$$r_t = \alpha A_t MC_t \left(\frac{N_t}{K_t}\right)^{1-\alpha} - \delta \tag{2.27}$$

$$w_t = (1-\alpha) A_t MC_t \left(\frac{K_t}{N_t}\right)^{\alpha}, \tag{2.28}$$

where MC_t are real marginal cost and δ the depreciation rate of capital.

Firm Problem: Final Good Producers

Final good producers differentiate the intermediate good and set prices under monopolistic competition. Following Rotemberg (1982), the monopolistic firm faces quadratic cost of adjusting nominal prices. Risk-neutral managers, which are a group of households with mass zero, set prices. They have the same time-constant discount factor as households. They are compensated by a share of profits and do not participate in the asset market. The profit-maximization problem over real profits Π_t subject to the demand of good k is:

$$\Pi_t \;=\; \max_{a} E_0 \sum_{t=0}^{\infty} \beta^t Y_t \left\{ \left(\frac{p_{kt}}{P_t} - MC_t \right) \left(\frac{p_{kt}}{P_t} \right)^{-\eta} - \frac{\eta}{2\kappa} \left(\log \left(\frac{p_{kt}}{p_{kt-1}} \right) \right)^2 \right\}, \tag{2.29}$$

taking demand of good k

$$y_{kt} \;=\; \left(\frac{p_{kt}}{P_t} \right)^{-\eta} Y_t, \tag{2.30}$$

as given. The first-order necessary condition w.r.t. allows to derive a variant of the New Keynesian Phillips curve:

$$\log(\pi_t) \;=\; \beta E_t \left[\log(\pi_{t+1}) \frac{Y_{t+1}}{Y_t} \right] + \kappa \left(MC_t - \frac{\eta - 1}{\eta} \right). \tag{2.31}$$

The nominal rigidity inherent in the price setting problem leads to an inefficiency of output. This holds regardless of the type of nominal rigidity employed. Under the Calvo (1983) price setting mechanism, price dispersion creates a wedge between aggregate employment and aggregate output. Under Rotemberg (1982) adjustment cost however, the nominal rigidity creates a wedge between aggregate production and aggregate consumption as a part of production goes into the price adjustment cost, making output less efficient. Nevertheless, up to a second-order approximation those approaches yield an identical Phillips curve.

2.3.5 Fiscal and Monetary Policy

Two policy rules close the model. First, the central bank sets the nominal interest rate on bonds. Thus, the bank implicitly targets an inflation rate that equates the net real return on capital and the real return on bonds. The treasury supplies bonds such that total expenditures equal the sum of total tax revenue and bonds rolled over in the previous period.

Monetary policy follows a modified Taylor (1993)-type feedback rule with interest rate smoothing:

$$\frac{R^b_{t+1}}{\bar{R}^b} \;=\; \left(\frac{R^b_t}{\bar{R}^b} \right)^{\rho_{RB}} \left(\frac{\pi_t}{\bar{\pi}} \right)^{(1-\rho_R)\theta_\pi} v^R_{t+1}, \tag{2.32}$$

where $\bar{R}^b$ is the steady state interest rate and ε a monetary policy shock. The coefficient, $0 < \rho_R < 1$ is usually estimated to be close to one.[11] We assume that, like TFP, the monetary policy innovation is described by an AR(1) process given by:

$$\log(v^R_{t+1}) \;=\; \rho_R \log(v^R_t) + \varepsilon^R_{t+1}, \quad \varepsilon^R_t \;\sim\; \mathbf{N}(0, \sigma^2_R). \tag{2.33}$$

[11] Clarida, Galí, and Gertler (1999) estimate the degree of interest rate smoothing to be around 0.8.

The specification of this rule suggests that the Taylor rate is adjusted gradually in response to a shock. Furthermore, $\theta_\pi \geq 0$ determines the monetary policy stance towards inflation and is usually larger than one (*Taylor principle*). The higher θ_π, the stronger the reaction of the central bank to a deviation of inflation from its steady state, which is modeled to be zero.

Following Woodford (1995), the fiscal policy rule takes fiscal theory of the price-level (FTPL) considerations into account and looks as follows:

$$\frac{B_{t+1}}{\bar{B}} = \left(\frac{B_t R_t^b/\pi_t}{\bar{B}\bar{R}^b/\bar{\pi}}\right)^{\rho_B} \left(\frac{\pi_t}{\bar{\pi}}\right)^{-\gamma_\pi} \left(\frac{T_t}{\bar{T}}\right)^{-\gamma_\tau}, \tag{2.34}$$

where tax revenues T_t are equal to $\tau(w_t N_t + \Pi_t)$. The parameter ρ_B captures the fiscal policy stance towards debt. A coefficient smaller than one actively stabilizes debt, whereas for $\rho_B = 1$, the treasury rolls over all outstanding debt including interest rate payments. Moreover, a degree of cyclicality is inherent in the fiscal policy rule. Setting $\gamma_\pi = \gamma_\tau = 0$ implied that debt does not automatically respond price movements and the business cycle. However, for $\gamma_\pi > 0 > \gamma_\tau$ the treasury takes a countercyclical, for $\gamma_\pi < 0 < \gamma_\tau$ a procyclical stance on debt.

Price-Level Determinacy

The life-cycle motive, distortionary taxes and idiosyncratic income shocks present in the model's setup make the economy non-Ricardian. Thus, monetary policy alone does not uniquely determine the price-level. Price-level determinacy depends on fiscal policy as well given that households also save into government debt to smooth consumption. Hence, for any given interest rate path there exists a uniquely pinned down path of inflation, such that the government budget constraint holds in every period. The fiscal-monetary interaction allows to discuss the effects of monetary vs. fiscal stabilization polices in response to shocks as depicted in Appendix A.4.

2.3.6 Market Clearing and Recursive Equilibrium

In equilibrium, the goods, bond, labor and capital market have to clear. The following two paragraphs state the market clearing conditions and define a recursive equilibrium.

Market Clearing

The labor market clears at the competitive wage rate as defined in (2.28). The household demand for bonds is an exogenously determined fraction of total assets and thus a function of the capital stock. The implicit assumption is that all households hold the same bond-to-capital ratio. In equilibrium, this is share determined by the supply of government bonds as stated in equation (2.34).

$$B_{t+1}^d := TA_{t+1} - q_t K_{t+1} = B_{t+1}^s(\Delta Y_{t+1}, B_t, \pi_t), \tag{2.35}$$

where TA_{t+1} stands for total assets. The market for capital clears under the following conditions:

$$q_t = 1 + \phi\left(\frac{\Delta k_{ijt+1}}{k_{ijt}}\right) \tag{2.36}$$

$$E_t\left(\frac{q_{t+1} + r_{t+1}}{q_t}\right) = E_t\left(\frac{R^b_{t+1}}{\pi_{t+1}}\right), \tag{2.37}$$

where (2.36) implies that under competitive markets the price of capital has to equal its replacement cost.[12] Equation (2.37) is the no-arbitrage condition derived from the FOCs of the household. The return on capital has to equal the real return on bonds. Under Walras' law, the goods market clears as long as the market clearing conditions hold in all other markets.

Recursive Equilibrium

The *recursive equilibrium* is a set of policy functions $\{x^*, k^*\}$, a value function V, pricing functions $\{w, r, q, \pi, R^b\}$, aggregate capital and labor supply functions $\{K, N\}$, distributions Θ_t over individual capital holdings, productivity, aging and perceived laws of motion Γ_A, Γ_R, such that:

1. Given V, $\{\Gamma_A, \Gamma_R\}$, prices and distributions, the policy functions $\{x^*, k^*\}$ solve the household's decision problem, and given the policy functions $\{x^*, k^*\}$, prices and distributions, the value function V is a solution to the Bellman equation.

2. All markets clear and interest rates on bonds are set according to the Taylor rule of the central bank, as stated in (2.32).

3. The perceived laws of motion Γ_A, Γ_R coincide with the actual laws of motion, i.e. $\Theta'_i = \Gamma_i(\Theta, s'_i), \quad i \in \{A, R\}$.

2.4 Numerical Implementation

The household's Bellman equation and the corresponding recursive equilibrium is not computable, because it involves the infinite-dimensional object Θ_t.[13] Discretization and representation of Θ_t by its histogram makes the distribution a finite-dimensional object and thus computable.

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[12] Adjustment cost on capital only have to be paid when capital is actively destructed and replaced or the capital stock increases. There are no adjustment costs for replacing the existing capital destroyed by depreciation.

[13] The state vector contains the entire cross-sectional distribution of idiosyncratic states. If there is a continuum of agents, as assumed in this model, the state vector becomes an infinite-dimensional object.

2.4.1 Household Problem

A discrete Markov chain describes both the (stochastic-)aging and the idiosyncratic productivity process. The number of age states is set to four and the number of productivity states to five. Four age states are sufficient to capture the life-cycle dynamics, dividing the model into young, middle-aged, old and dying households. Five productivity states allow a quasi-quadratic scaling over the age-distribution which again supports life-cycle dynamics found in the data. The approximation of the AR-(1) process of productivity as described in (2.4) relies on the standard method proposed in Tauchen (1986).

We solve the household problem using 80 grid points on the asset, eight grid points on the productivity and four grid points on the age grid. The stationary equilibrium in the absence of aggregate shocks is found by iterating over first-order conditions of the household problem. The employed endogenous grid-point method (EGM) has been originally developed by Carroll (2006) and extended by Hintermaier and Koeniger (2010).[14]

2.4.2 Aggregate Fluctuations

After calculating the stationary equilibrium, it remains to compute aggregate dynamics. The dynamic system can be represented as a set of non-linear difference equations, such that:

$$E_t F(S_t, S_{t+1}, C_t, C_{t+1}) = 0, \tag{2.38}$$

where $\{S, C\}$ are the sets of state- and control-variables. Basically any dynamic, non-linear system of this form can be solved using the Schmitt-Grohé and Uribe (2004) perturbation approach as suggested by Reiter (2009). However, full grid-size is $n^F_{grid} = n_{age} \times n_h \times n_{TA} = 2560$. Perturbation around the full consumption policy, the value function and the distribution, plus aggregate states and controls contains over 7600 points, what makes solutions without state- and control-space reduction at least time consuming if not also numerically unstable. Full second-order approximations will be infeasible for sure.

We employ the method as proposed in Bayer et al. (2015) that extends the Reiter (2002, 2009) method. To reduce the number of idiosyncratic states, we use the fact that the three-dimensional distribution Θ can be approximated with a distribution that has a fixed copula and (possibly) time-varying marginal distributions.[15] This approach builds on a notion in Krusell and Smith (1998) who showed that not all moments of the cross-sectional distribution have the same impact on the distribution of e.g. wages and interest rates that households need to forecast.

We fix the Copula to the stationary distribution of the model and obtain its representation by fitting a cubic spline to the marginal distributions of productivity and assets over all age states.

[14] Changing the number of grid points on the grid for total assets had only minor and negligible impact on results.
[15] Following Skar's theorem, every multivariate cumulative distribution function of a random vector can be expressed in terms of its marginal distributions and a Copula.

This technique reduces the number of endogenous states tremendously. As shocks to idiosyncratic productivity or the age distribution are neglected in this baseline version, we assume their marginal distributions to be time-constant as well. Hence, instead of perturbating the full distribution with $n^{F}_{S-grid} = 2560$ points, we only have to perturbate around the marginal distribution of assets with $n^{R}_{S-grid} = n_{TA} = 80$ points. In comparison to Reiter (2009), the full distribution function maintains its shape.

Reduction of the control space relies on (inverse) discrete cosine transformations (DCT) of stationary equilibrium value function and consumption policies. Simply put, the idea behind DCT transformation is to identify pieces of information in e.g. the consumption policy that can be effectively "discarded" without seriously compromising its informative content (akin to image compression as e.g. *jpeg*). This approach reduces the size of the control-space substantially.[16]

2.5 Calibration

We calibrate the economic and demographic structure of the model to the US economy. Aggregate data used for calibration spans the post-Volcker disinflation time until 2008. One model period equals a quarter of a year, given that many macroeconomic variable that are relevant for stabilization policy run at this frequency. The following tables summarize the choice of parameters and their corresponding target in the data.

Table 2.1. Calibrated Household Parameters

Parameter	Value	Description	Source
Households			
β	0.993	Discount factor	K/Y = 285%
γ	0.75	Inv. Frisch elasticity	Chetty et al. (2011)
σ	4	Coeff. Relative Risk Aversion	Kaplan, Moll, and Violante (2018)
Idiosyncratic Income			
ρ_H	0.979	Persistence of Income Process	Standard Value
σ_H	0.08	STD of Income Process	Standard Value

[16] However, as the number of coefficients retained from the compression step is implicitly chosen by the researcher, robustness of the results should be checked.

2.5.1 Household Preferences

An overview over the parameters relevant for the household problem is provided in Table 2.1. Households' felicity over the composite good x is of constant-relative risk aversion (CRRA) form. Their degree of risk aversion is set to $\sigma = 4$, as in Kaplan and Violante (2014). The inverse Frisch elasticity of labor supply is $\gamma = 0.75$, and builds on the analysis in Chetty et al. (2011) that finds values that typically range between 0.5 and 1. For a discount factor of $\beta = 0.993$ that matches the annual real interest of the regarded time period, the annual capital output ratio produced by the model is too low. As Table 2.2 shows, the sum of both liquid assets, capital and bonds, relative to output only amounts to 1.98 instead of 2.86 as in the data. The quarterly standard deviation of shocks to idiosyncratic labor productivity is set to 0.08 and the degree of quarterly persistence to 0.979 which follows Storesletten, Telmer, and Yaron (2004). The probability to leave the high-income entrepreneur state follows Guvenen, Kaplan, and Song (2014) and is set to 6.25%. This corresponds to their annual probability of 25% to drop out of the top 1% income group. The share of household's in the high-income state is set to 0.5% of all households. However, as Table 2.2 shows, the model does not perform well in matching US wealth inequality as the implied-Gini coefficient of the model is only 0.58 instead of 0.78.[17]

Table 2.2. Model-Implied Moments and Data

Targets	Model	Data	Source	Parameter
Asset-to-Output Ratio (K+B)/Y	1.98	2.86	NIPA	Discount factor
Mean government expenditures (G/Y)	0.19	0.2	NIPA	Discount Factor
Mean government revenue (T/Y)	0.21	0.2	NIPA	Tax Rate
Gini total wealth	0.59	0.78	SCF	Fraction Entrepreneurs

2.5.2 Aging

The number of households in each age group is not necessarily equal. To capture differences between young and old societies, the mass of older relative to younger households has to vary. When younger persons age faster or older persons age slower the average lifetime increases. Hence, the transition probability $\xi_j = P_J(j+1|j)$ from one to another age-group will vary in size.

17 The too-low wealth inequality is driven by the assumption that only old, dying households enter the high income state and then consume all assets. Hence, there is no high-income inequality in earlier stages that accumulates over the life-cycle. This has been done for numerical reasons such that the dynamic response of consumption to a monetary policy shock remains plausible.

Table 2.3. Parameters for Economies with Different Age Composition

Parameter	Value	Description	Source
Young Economy			
X_y	70.4	Life expectancy, years	UN Data: 1965-1970
$\xi_{1,2}$, $\xi_{2,3}$	1.25%	Age prob. states (1,2) to (2,3)	own calculations
$\xi_{3,4}$	4.25%	Age prob. state 3 to 4	own calculations
φ_y	15%	Old-age dep. ratio	UN Data: 15.7% (1965)
Middle-aged Economy			
X_m	78.9	Life expectancy, years	UN Data: 2010-2015
$\xi_{1,2}$, $\xi_{2,3}$	1.25%	Age prob. states (1,2) to (2,3)	own calculations
$\xi_{3,4}$	2.5%	Age prob. state 3 to 4	own calculations
φ_m	25%	Old-age dep. ratio	UN Data: 22.1% (2015)
Old-aged Economy			
X_o	86.5	Life expectancy, years	UN Data: 2065-2070
$\xi_{1,2}$, $\xi_{2,3}$	1.25%	Age prob. states (1,2) to (2,3)	own calculations
$\xi_{3,4}$	0.96%	Age prob. state 3 to 4	own calculations
φ_o	51%	Old-age dep. ratio	UN Data: 49.3% (2065)

The number of age states is set to $J = 4$ to keep the model specification parsimonious. The transition probability matrix will thus correspond to (2.2). Agents are born at age 20 and start to work immediately. In the young society, average life expectancy is around 70 years, while in the grey society people get 86 years old on average. I assume that the increase in life expectancy only matters for the old. Using equation (2.3), the probabilities $\{\xi_{1,2}, \xi_{2,3}\}$ as in equation (2.2) are thus set to 1.25%, which corresponds to 20 years of young and middle-aged life, respectively. On the contrary, agents spend between 10 and 26 years in their last phase of life. Thus, $\xi_{3,4}$ should be set to values between 4.25% and 0.96%.[18]This corresponds to an old age-dependency ratio between 0.15 and 0.7 which is in line with projected US population statistics. The mass of dying households, i.e. agents that are in their final life phase varies between 0.5% and 0.1% of total population.

[18] Age transition probabilities correspond to the quarterly frequency of the model, as in equation (2.1).

2.5.3 Firm Sector

Table 2.4. Calibrated Firm Parameters

Parameter	Value	Description	Source
Intermediate Goods			
δ	0.0135	Depreciation Rate	NIPA: Fixed Assets and Consumer Durables
α	0.7018	Labor Share	Standard Value
ρ_A	0.9	Persistence of TFP Shock	Standard Value
σ_A	0.026	STD of TFP Shock	STD(Y) = 1
θ	11.4	Capital Adj. Cost	Investment-Volatiliy Ratio = 4.5
Vendors			
κ	0.085	Degree of Nominal Rigidity	Avg. price duration 4 quarters
μ	0.95	Mark-up on Marginal Cost	Standard Value

The labor share adjusted for profit income is set to $1 - \alpha = 0.702$, and hence, the capital share equals $\alpha = 0.298$, which are standard, long-run averages for the U.S. economy. The rate of depreciation is set to $\delta = 0.0135$, which corresponds to an annual consumption of fixed capital of 5.4%, which has been the long-run average over the regarded time period in the data. Resellers sell intermediate goods as final goods at a markup of 5% percent, which implies marginal cost of $\mu = \frac{\eta-1}{\eta} = 0.95$ in steady state. The degree of price stickiness, $\kappa = 0.085$ is set to target an average stickiness of four quarters. Both parameters, markup and price stickiness assume values which are standard in the literature on New Keynesian Models (compare e.g.Christiano, Eichenbaum, and Evans (1999), Galí (2008)). We choose capital adjustment cost of $\phi = 11.4$. This comes close to an investment volatility of 4.5 in response to TFP shocks, as found in U.S. data. The TFP shock is highly persistent. The degree of persistence is set to $\rho_A = 0.9$ and its standard deviation $\sigma_A = 0.026$.

2.5.4 Government Sector

Treasury

The treasury issues debt according to a rule that is similar as in Woodford (1995) or Bi, Leeper, and Campbell (2013). The tax rate and government expenditures are jointly calibrated to match the 20% expenditure share of output what implies an effective tax rate of 30%. US government debt is highly persistent and the autocorrelation parameter ρ_B assumes a value of 0.86 which can be estimated from US data.

Central Bank

The nominal interest rate on bonds R_t^b is set according to a Taylor rule. We linearize the model around the zero-inflation steady state, such that in absence of shocks, the nominal interest rate equals the real return on capital, set to 2.6% (annually). The degree of persistence or interest-rate smoothing captured in ρ_{RB} is set to 0.9. The parameter θ_π, which describes the reaction of the nominal interest rate towards deviations of inflation from its steady state value is set to $1.5 > 1$ and satisfies the *Taylor principle* in the baseline scenario. The scenarios of full monetary vs. full fiscal stabilization of inflation and its influence on parameter choice are discussed below. The persistence of the monetary policy shock captured in ρ_R is set to 0.839. The volatility of the monetary policy shock is set according to results in Christiano, Eichenbaum, and Evans (1999).

Table 2.5. Calibrated Government Parameters

Parameter	Value	Description	Source
Treasury			
τ	0.3	Tax Rate	G/Y = 20%
γ_π	1.5	Reaction to Infl.	Standard Value
γ_τ	0.75	Reaction to Revenue	Standard Value
ρ_B	0.86	Autocorrelation of Gov't Debt	
Central Bank			
θ_π	1.5	Inflation Stance of CB	Standard Value
ρ_{RB}	0.9	Policy Persistence Parameter	Clarida, Galí, and Gertler (1999)
ρ_R	0.839	Persistence of MP Shock	Standard Value
σ_R	0.18	STD of MP Shock	Christiano, Eichenbaum, and Evans (1999)

2.6 Quantitative Results

This section discusses the impacts of aging from two perspectives: The first one assumes a long-run perspective and illustrates the effect of greying on monetary policy as e.g. discussed in Miles (2002) or Imam (2015). Thereafter, the perspective changes towards one of short-run stabilization policy. The model allows to discuss how age heterogeneity and the demographic composition matter for the transmission of monetary policy. This is done in two steps. First, we look at the dynamic response of the model to a monetary policy shock, as stated in (2.32). Second, we compare this outcome to a version of the model without aging. To isolate the effect of age heterogeneity, all other parameters will be kept constant. Finally, we will vary the age composition resulting in a very young, an intermediate and an old age society in steady state.

2.6.1 Long-run Effect of Greying: Impact on the Real Return to Capital

Following the argument made in Bean (2004), Miles (2002) or Miles (1999), the natural rate of interest is a decreasing function of the demographic structure of the economy. Hence, the effectiveness of monetary policy decreases as Imam (2015) argues. Here, aging and the real return to capital are connected via the policy function of capital. Thus, it is not possible to state a functional form that allows to derive the real interest rate for a variation of the old-age dependency ratio. However, using equations (2.2) and (2.3) and the population statistics targets as discussed in Table 2.3, we come up with transition probabilities that imply a young, a medium-aged and an old-aged economy. In our case, the steady state of the economy is solved over a grid of old-age dependency ratios ranging from 0.15 to 0.5. Figure 2.1 displays the steady state real interest rate as a function of the age structure and the evolution of the old-age. Starting in the 1960's, where the old-age dependency ratio has been 0.15, the real return to capital will decreases by 0.5 percentage points in the year 2065, where the share of old has risen to 0.5. Results are in line with Miles (1999) who found similar results, however using European data.

2.6.2 Effects of Monetary Policy: Aging vs. No-aging

In section 2.2 we discussed how taking life-cycle considerations into account could change the transmission channel of a monetary policy innovation.[19] For this reason we compare our model with an incomplete market economy that neglects aging, i.e. has no built-in OLG-structure but is otherwise identically calibrated. The latter variant without aging corresponds to a special case of the model in Bayer et al. (2015) where all assets are liquid. All parameters are set to coincide with the calibration in 2.5. Figure 2.2 displays the dynamic response of the system to a monetary policy shock that, all else equal, increased the nominal interest rate by 18 basis points in the first quarter.

Three observations immediately attract attention. First, the model with OLG-structure seems to be more persistent as the shock takes a relatively longer time to fade out. A similar point has been made in Wong (2016), though. Second, for some variables the response on impact is substantially stronger than in the model without aging. Third, except for the response of aggregate consumption, the shock amplification follows similar patterns.

Moreover, Figure 2.2 shows that the size of intertemporal substitution in response to a shift in the policy rate depends on your current age state. On impact, aggregate consumption, defined as

[19] A variant that compares the case of a TFP shock is displayed in Figure A.5 in Appendix A.2.

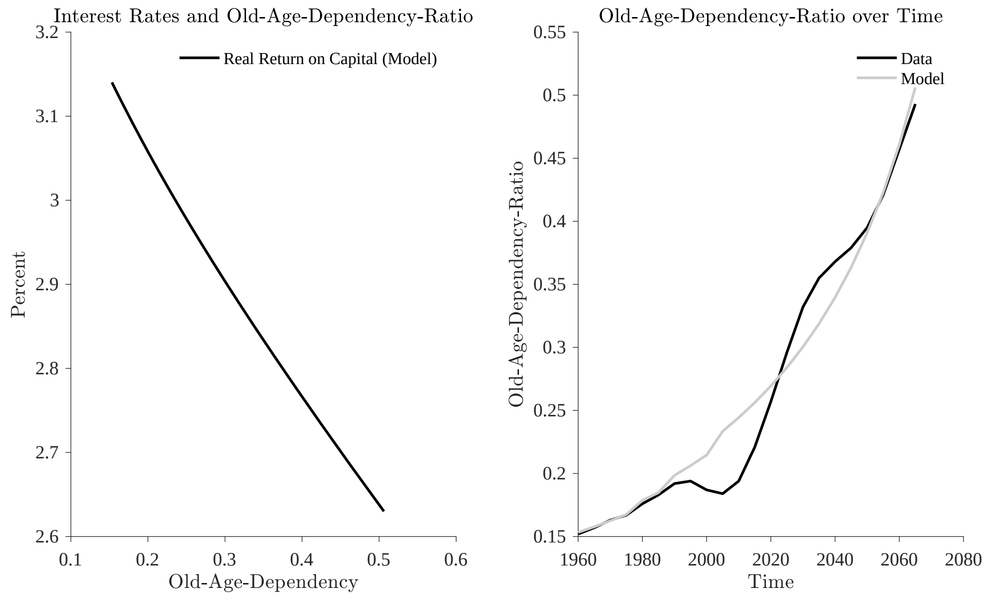

Figure 2.1. Real Return to Capital and Demographic Change

Notes: The figures displays the evolution of the real return to capital over different old-age dependencies in the left panel. The variation over the old-age share roughly corresponds with UN population statistics data on the (projected) demographic development between 1960 and 2065 as the right panel shows. Deviations from the data should stem from (one-time-)effects like higher fertility, migration or the retirement of the baby-boomer generation.

the sum over the consumption policies in every age-state, drops by more than half a percentage point. However, it drops only half as much as aggregate consumption in the world without aging. Given that the drop of old-age consumption is only slightly negative, the consumption response of young households will be much more negative in order to match aggregate consumption. This finding is in line with results in Wong (2016).

Furthermore, the response of dividend, profit and wage income is less prominent than in the model without age-structure. One reason could be the difficulty of the model to generate comparable, plausible income inequality.

2.6.3 Effects of Monetary Policy: Changing the Age Distribution

In a next step, we vary the age composition of the economy. While in the baseline scenario, every age group was populated by an equal amount of households, adjusting survival probabilities allows to change that into a young, medium-old and old economy. The respective old-age-

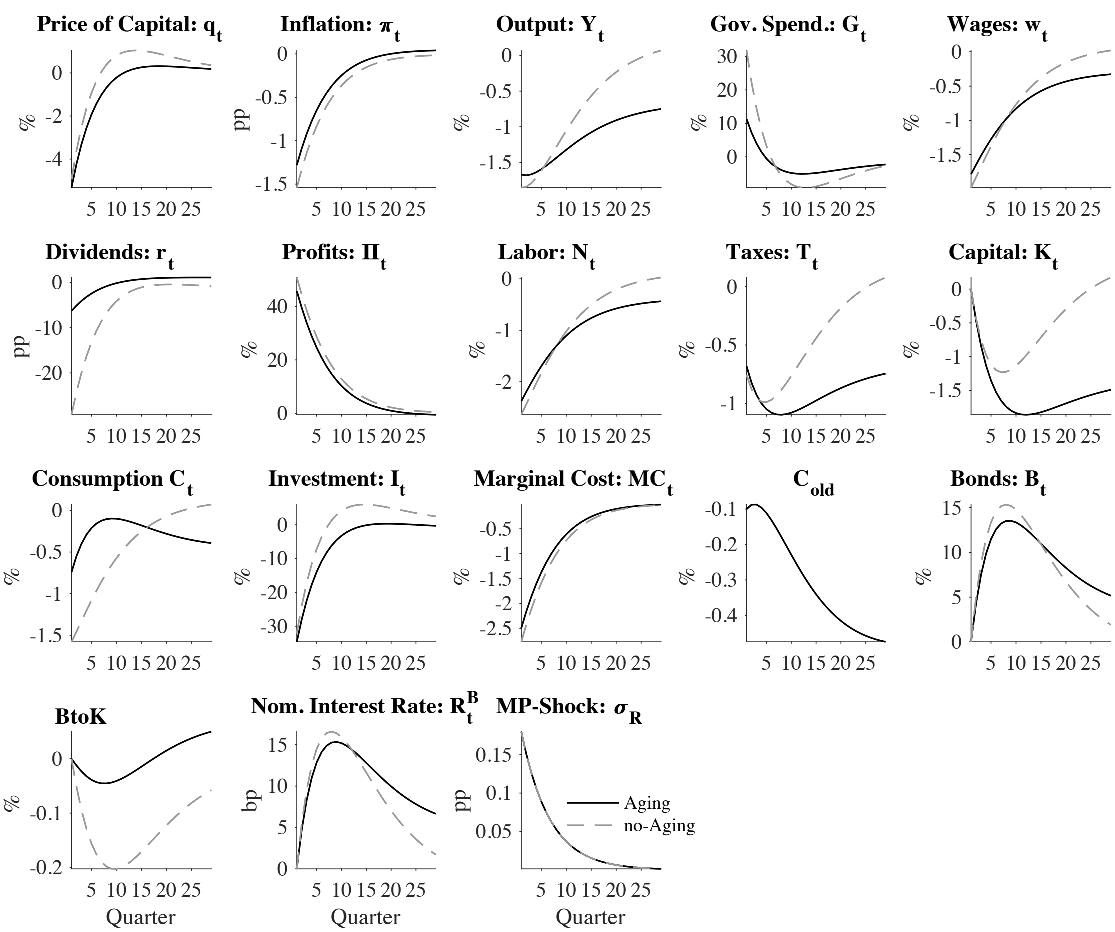

Figure 2.2. Aggregate Response to a MP Shock in a Model with and w/o OLG-structure

Notes: Impulse Responses to a one standard deviation monetary policy shock, $\nu^R = 0.18\ pp$ (annualized). Solid line: The Model with OLG-structure (aging). Dashed line: Identically calibrated model but w/o OLG-structure (no-aging).

dependency ratios correspond to the ones discussed in Table 2.3.[20] The aggregate effects of changing the age distribution are small for most variables. It mainly affects the on-impact level of the response but not its shape. Hence, for reasons of space, not all IRFs are reported again. However, changing the age composition has a pronounced effect on consumption aggregates. The difference between the response of aggregate consumption in the young relative to the old

[20] The baseline scenario where every age group is populated by an equal amount of households corresponds to an old-age dependency ratio of 50% and thus, coincides with the old-economy setup. This would change however, if the model was solved with a higher number of age states, as we define *old* only as the (J-1)th and Jth age state.

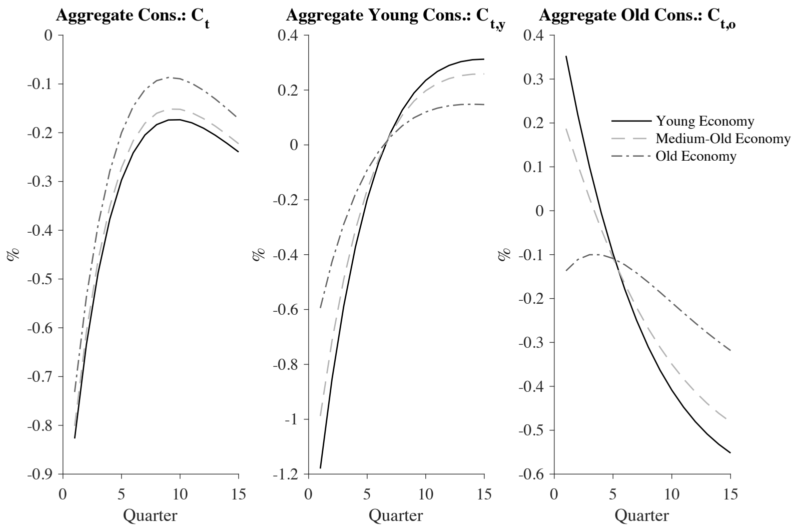

Figure 2.3. Consumption Response to a MP Shock Over Different Economies

Notes: Impulse Responses to a one standard deviation monetary policy shock, $\nu^R = 0.18\ pp$ (annualized). Solid line: Young economy corresponding to and old-age dependency ratio of 0.15. Dashed line: Medium-old economy corresponding to and old-age dependency ratio of 0.4. Dashed-dotted line: Old economy corresponding to and old-age dependency ratio of 0.6.

economy is 0.1 percentage points on impact. This pattern is even more pronounced in the case of consumption of young households. When there are many of them, the responds to a contractionary monetary policy shock is 0.6 percentage points larger on impact compared to a world with only a few young households. The consumption response of old households holds some important insights for monetary policy, as the response of old households to a contractionary monetary policy shock could turn positive. Here, wealth effects of monetary policy seem to exceed the effects of intertemporal substitution as for the later is probably not much time left. A bit puzzling are sign and magnitude of the response of old households when living in an old economy. Here, the response on impact displays the usual, negative response but is rather small. A possible explanation lies in the model setup. Old households do not retiree in the sense that they receive pension transfers instead of labor income but they work until they die. This implies that old households do not rely on a declining number of young households at working age to finance their pensions. As the economy gets older, the aggregates of composite good x_{ijt} as defined in equation (2.12) become almost identical between young and old households. In the case of a young or a medium-old economy instead, composite consumption of the old is only half of the amount of composite consumption of the young. Furthermore, the Gini coefficients of consumption and capital drop from 0.41 to 0.38 and 0.63 to 0.58 respectively, as the economy gets greyer. As wealth accumulated over the life-cycle to distributed on more shoulders, the individual motive to consume more in reaction to an increase of interest rates might decline and even turn around again. A further possible explanation can be numerical reasons given the parsimonious number of age states chosen and the specification of the entrepreneur state as last state of the joint distribution.

2.6.4 Effects of Monetary Policy: Forward Guidance

During the last decade, central banks relied more and more on unconventional monetary policy tools as e.g. forward guidance. Forward guidance is a communication device of central banks that conveys information on future changes of the policy rate. Its impact on the transmission of monetary policy can be twofold as Campbell et al. (2012) argue. First, households could interpret the announcement of such a strategy as *more* monetary stimulus and expect a lower future path of the policy rate which stimulated economic activity (*Odyssean interpretation*). Second, households could read this strategy as *bad news* about the future economic outlook assuming that the central bank had superior information about the state of the economy (*Delphic interpretation)*. Hence, the effect of forward guidance depends very much on how markets read central bank communication (Campbell et al. (2012)).

So far, quantifying the effects of forward guidance on the macroeconomy has been difficult. Medium-scale DSGE models tend to overestimate the effects of unconventional monetary policy on key macroeconomic variables dramatically. McKay and Reis (2016) analyze a situation in which the monetary authority announces to lower the real interest rate by 0.5 percentage points for a single quarter at some point in the future. Within their three-equation New Keynesian model such a transitory shock generates an 18 times larger impact on inflation than a persistent shock to the current real rate. In some specifications a forward guidance shock even generates explosive paths for inflation and output (Carlstrom and Paustian (2012)). This phenomenon has been called *forward guidance puzzle* by e.g. Giannoni, Patterson, and Del Negro (2015). As resolutions for this puzzle, McKay and Reis (2016) and Giannoni, Patterson, and Del Negro (2015) propose to add either an incomplete markets economy and borrowing constraints or a life-cycle structure. This dramatically reduced the effect identified in standard RANK models. Their finding builds on the fact that in a incomplete market economy the possibility to hit the borrowing limit within the next years is positive and thus, households become wary to increase their current consumption through dissaving. This reduces the power of forward guidance as intertemporal substitution plays a smaller role for consumption. This is a general result of the effects of monetary policy on consumption in HANK economies (compare Kaplan, Moll, and Violante (2018)). In an economy which includes a life-cycle structure, new born generations cannot increase their current consumption already today as they need time to accumulate assets, first.

In the model discussed in Section 2.3, the interest rate does not follow an exogenous path set by the central bank as in McKay and Reis (2016). Instead, monetary policy sets the interest rate according to a feedback rule that responds to fluctuations in inflation and deviations of tax revenues from their steady state. Thus, one way to think about forward guidance in the present framework would be the difference between the effect of a transitory and a highly persistent monetary policy shock on aggregate variables. This follows the argumentation in Nakamura

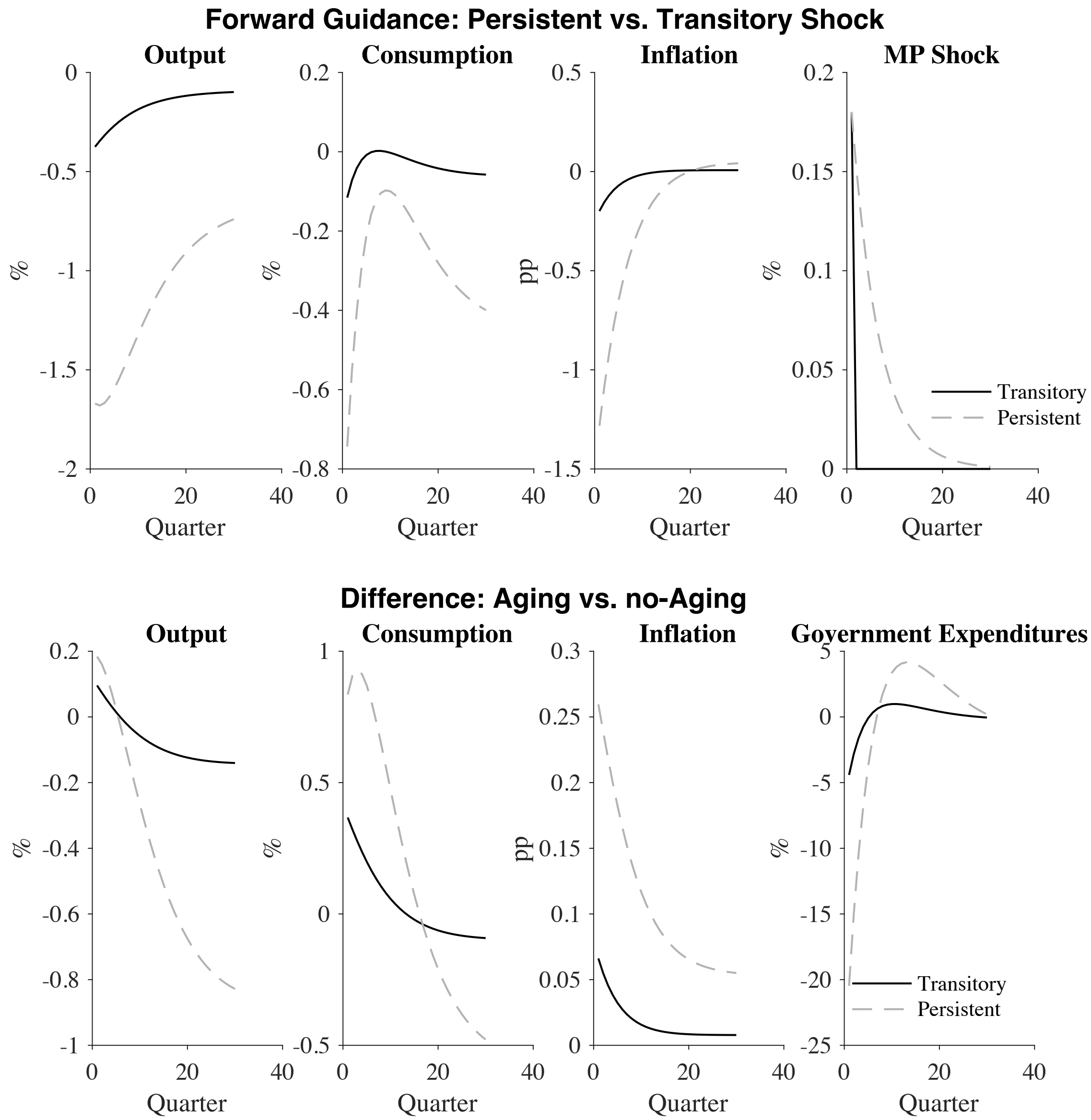

Figure 2.4. The Effect of Forward Guidance: Persistent vs. Transitory Shocks

Notes: The top panel depicts the Impulse Responses to a one standard deviation monetary policy shock, $\nu^R = 0.18\ pp$ (annualized) for the baseline specification that does not change the age distribution. The shock process is either transitory ($\rho^R = 0$) or persistent as in the baseline specification. The lower panel depicts the difference in the response to a persistent and a transitory shock between a model with OLG-structure and a model w/o OLG-structure.

and Steinsson (2014). Their shock estimates have little effect on short-term interest rates but a much larger impact on rates eight to twelve quarters ahead. They argue that the shock high persistence can be mainly interpreted as forward guidance shocks about future monetary policy. More precisely, the difference in the effect on aggregate variables between a persistent and a transitory monetary policy shock could be interpreted as the forward guidance component of the shock.

Figure 2.4 illustrates the results. Forward guidance, here interpreted as the difference between a persistent and a transitory disturbance has an effect. As the top panel shows, the transmission of a shock has a four to five times larger effect on impact and decays much slower. Nevertheless, this is, although of course difficult to compare, in no range of the tremendously big effects found in the representative agent model of e.g. McKay and Reis (2016). What drives the difference with the complete markets economy becomes visible in the response of consumption in the top panel. The dynamic responses to both the transitory and the persistent shock peak after eight quarters and fall again afterwards. This occurs as a monetary tightening redistributes wealth away from wage-earning to profit-earning households. Moreover, borrowers lose relative to lenders and thus, inequality increases. Redistribution of wealth away from households with high marginal propensities to consume towards households with low marginal propensities to consume increases output (through aggregate demand) until it converges back to steady state. Such effects are generally muted in a representative agent model.

The lower panel isolates the effect of adding a life-cycle structure on the effect of forward guidance. As expected from the results in Giannoni, Patterson, and Del Negro (2015), adding stochastic aging reduces the effect of forward guidance even further. The positive difference in case of output, consumption and inflation indicates that the effect of both a transitory and a persistent shock as been more pronounced in the economy without aging. The reason for that lies in the finiteness of life in our model setup. When households enter the last age state they know that they will be reborn with probability one. However, even if they knew they would be hit by a shock in the next period, they could not respond to it *ex ante* as they have to consume all wealth in their final period and restart "naked". Hence, they do not suffer from the future interest rate increase but are reborn with a lower level of consumption.

2.7 Conclusion and Outlook

The chapter has contributed to the incomplete markets literature with nominal rigidities in the following respect: Aging and the demographic composition of the economy matter for monetary policy. Not only in the distant future as demographic change is "glacial", but also for means of short-run shock stabilization by a central bank or fiscal policy.

The present chapter built a stochastic aging OLG-structure around a model, similar to those discussed in Bayer et al. (2015) and Lütticke (2017). Demographic change is one explanation for the constant decline in real interest rates observed in the data. The model is able to replicate results found in the literature on the long-run macroeconomic impacts of demographic change as in Miles (2002) or Imam (2015). By calculating the transition path of the real return to capital using UN population projections from 1965 to 2065, the model shows that, ceteris paribus (all else being equal) the real return to capital falls by approximately half a percentage point.

Moreover, the OLG-structure introduced a further degree of heterogeneity to a HANK model: In addition to idiosyncratic labor productivity, households behave differently conditional on their current position in the life-cycle. This effect is mainly visible in different consumption responses towards monetary policy shocks. By comparing our model to a similar model without aging structure, we can show that the impact on aggregate consumption is muted given that the consumption motives of young and old households not necessarily coincide. As would be expected, intertemporal substitution plays a much stronger role for young households relative to old ones. This has also been documented in recent literature as e.g. in Wong (2016). Furthermore, the effect of a monetary policy shock aggregate consumption gets smaller the greyer the economy becomes.

Our model contributed to the discussion on the effects of forward guidance. Following arguments made in Giannoni, Patterson, and Del Negro (2015) and McKay and Reis (2016) we document that the inclusion of a stochastic aging framework reduces the difference between persistent and purely transitory monetary policy shocks, relative to an economy which only has an incomplete markets setting. Moreover, although not directly comparable, the magnitude of the forward guidance effect is much smaller than in literature that uses the standard three-equation New Keynesian or a medium-scale DSGE model.

To our best knowledge, this chapter has been the first to integrate a stochastic aging framework into HANK models in the tradition of Kaplan, Moll, and Violante (2018) or Bayer et al. (2015). However, not only leave results room for further research - the easily-computable solution method as proposed in Bayer et al. (2015) calls for the integration of richer model features. Especially, the following three points would be worth to look at: First, the present model lacks a *full* second asset, i.e. there is no distinction between a liquid and an illiquid asset. This mutes the entire portfolio-choice problem. Bonds, here introduced to integrate debt-financed government deficits and inflation into the model, have the same equilibrium return as capital. This drawback has been due to the idea to keep the model specification parsimonious at first and focus on the age structure instead. Bayer et al. (2015) and Lütticke (2017) discuss how a departure from that, by having a liquid and an illiquid asset, influences the transmission of shocks.

Second, model (a more complex) pension system and split the household maximization problem into a worker- and a retiree problem, where the later receive tax-financed transfers. This guarantees that at a certain point of the life-cycle, the household has to rely more on savings than tax-financed transfers. This would especially be the case if we thought about a public pension system that guarantees only a minimum amount to everybody (lump-sum transfers). Households that anticipated this would then save much stronger into physical capital and dissave when they become old. Furthermore, this allowed to study more traditional questions related to demographic change as e.g. the sustainability of public finances. Finally, a full, second-order welfare analysis would allow deeper, optimal insights into the redistributionary effects of monetary and fiscal policy.

A.1 Unconditional Business Cycle Statistics

Table A.6. Unconditional Business Cycle Statistics

	GDP	C	I
Data			
STD	1.03	0.783	4.5
AC(1)	0.882	0.835	0.805
Corr GDP	1	0.879	0.887
Model (MP)			
STD	1	1.09	0.562
AC(1)	0.45	0.445	0.451
Corr GDP	1	0.787	0.526
Model (TFP)			
STD	1	0.668	4.27
AC(1)	0.998	0.998	0.801
Corr GDP	1	0.984	0.772

Notes: Real GDP, Consumption (C), Investment (I) in logs. All data are HP-filtered with $\lambda = 1600$. Model refers to simulation following a first-order approximation in response to a monetary policy and TFP shock. Standard deviations are multiplied with 100.

A.2 Model Response to TFP Shocks

The following section reports the model response to shocks to total factor productivity (TFP) and compares it with the setup without an OLG-structure for robustness. Figure A.5 displays the results. As in the case of a monetary policy shock, introducing an OLG-structure to the model changes the initial shock impact, tremendously. Deviations might stem foremost from differences in the age-productivity/productivity distribution that arise by construction. However, in most cases, the transition path follows similar patterns, regardless of the age structure.

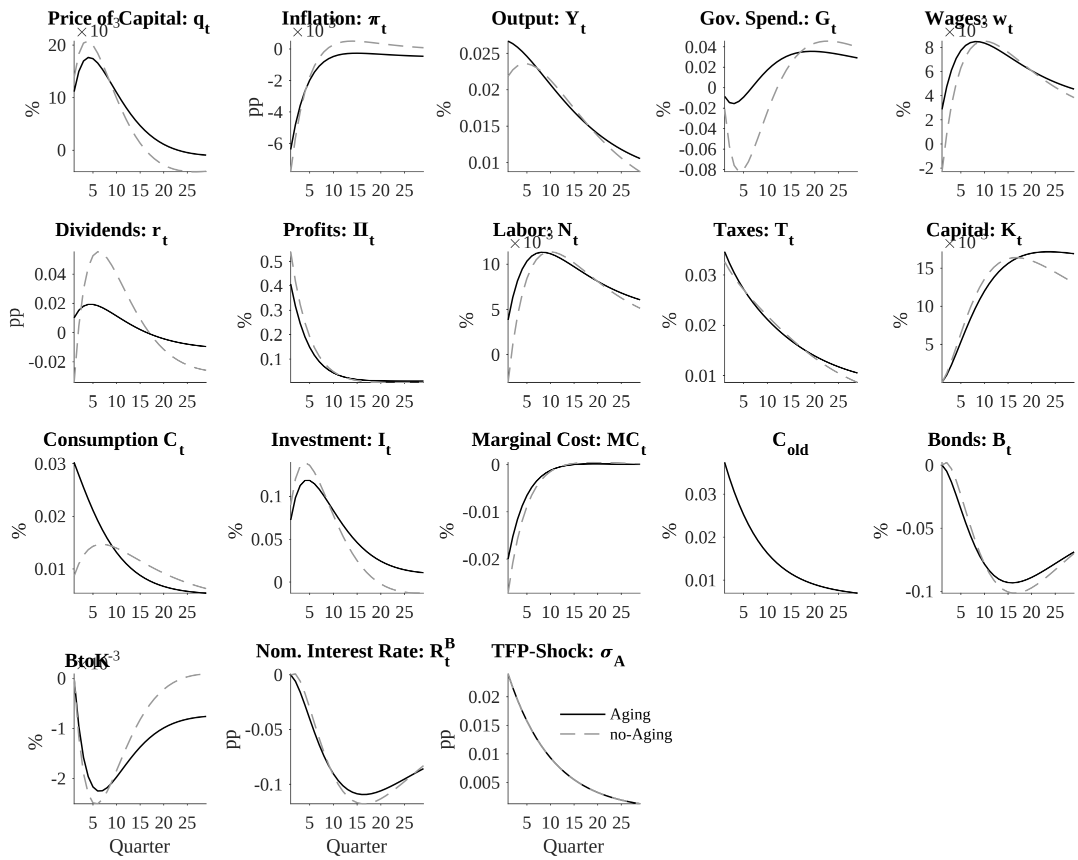

Figure A.5. Aggregate Response to a TFP Shock in a Model with and w/o OLG-structure

Notes: Impulse Responses to a one standard deviation TFP shock, $\nu^R = 0.09pp$ (annualized). Solid line: The Model with OLG-structure (aging). Dashed line: Identically calibrated model but w/o OLG-structure (no-aging).

A.3 Numerical Performance

The following sections discuss performance, speed and accuracy of the method proposed in Bayer et al. (2015) and used in this chapter by comparing it to versions w/o state-space reduction via the copula.

A.3.1 Quality of the Approximation

To show that the state-space reduction via the Copula does not change the first-order perturbation around the non-stochastic steady state, we solve both models using the same parameter values and compare the simulation for the aggregate stock of capital. Both Figure A.6 and Table

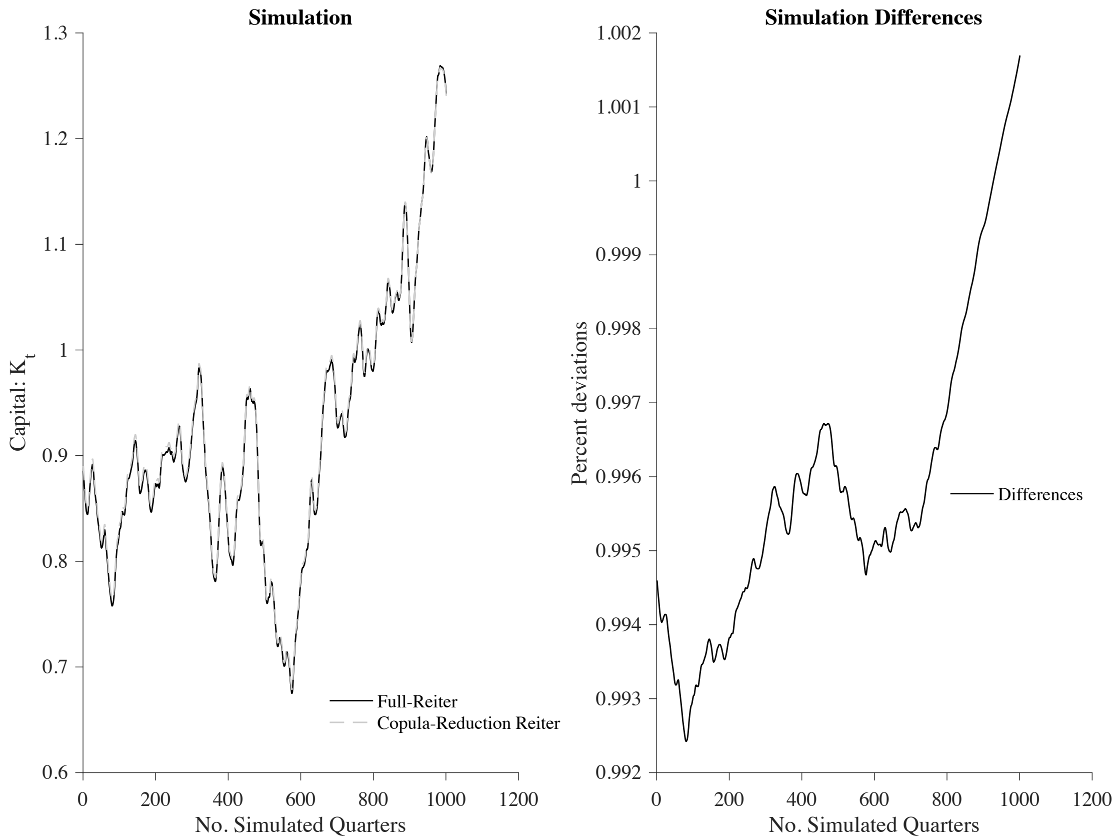

Figure A.6. Quality of Approximation

Notes: Both panels show simulations of the model discussed in Section 2.3 with TFP shocks solved with (1) the Reiter method with state-space reduction via Copula function. (2) the Reiter method with full histogram.

A.7 show that the response of capital to a shock to total factor productivity is virtually identical in both simulations. Furthermore, the advantage of the state-space reduction as proposed in Bayer et al. (2015) and further explained in Bayer and Lütticke (2018) becomes visible with respect to the time-needed to perturbate around the non-stochastic steady state.

Table A.7. Simulation Errors and Run-Time relative to Full-Reiter Method

Model	Full-Reiter	Copula-Reiter
Mean Value (%):	0.000	0.005
Run-Time (seconds)	634.450	11.203

Notes: Differences in percent between simulations of aggregate capital for the model discussed in Section 2.3 solved with state-space reduction via Copula function (1) and the Reiter method on the full joint histogram (2). The first column shows the mean percent deviation from the full histogram and the second column compares run time in seconds on a MacBook Pro (2016) with an 2.7 GHz Intel Core i5 processor. Code in Matlab.

A.4 Effects of Monetary Policy: Stabilization Policy

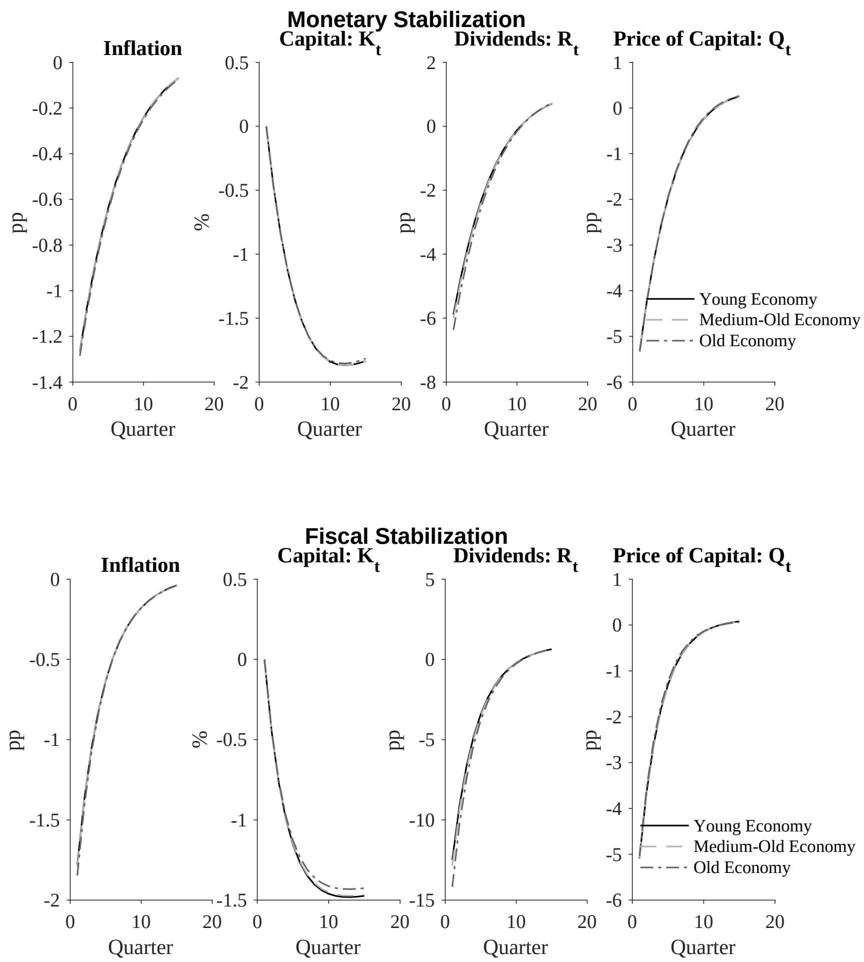

Figure A.7. Aggregate Response to MP Shock with Stabilization Policy

Notes: Impulse Responses to a one standard deviation monetary policy shock, $\nu^R = 0.18\ pp$ (annualized). The parameters of the above scenarios discussed are as follows:

(a) $\gamma_\pi = 1.5$, γ_T = 0, $\theta_\pi = 0$, $\rho_R = 0.83$

(b) $\gamma_\pi = 0$, $\gamma_T = 0.75$, $\theta_\pi = 1.5$, $\rho_R = 0.83$

3

Revisiting the Stability and Growth Pact: A 20 Years Empirical Perspective

3.1 Introduction

The Stability and Growth Pact (SGP) and its successor, the Fiscal Compact have been praised as the cornerstones of successful European economic integration. Nevertheless, the last 20 years have seen endless discussions, failures to reform and to commit credibly, and finally the European debt crisis from 2010 onwards. While popular debate in Germany favors the hypothesis of profligate southern Europeans bailed out using German tax payers money - others blame the 2003 violation without consequences of Germany and France as the original Fall of Man. The present chapter argues into another direction. For some countries the SGP has never been complied with seriously once they had been admitted to joining the EMU in 1999. Huge rescue-, bail-out and recovery packages in the aftermath of the Great Recession further contributed to the distress of public finances in Europe. The recent handling of Portugal's and Spain's excessive deficits in 2015 by the EU and national governments provide additional arguments in support of this hypothesis.

By constructing a broad data set of 31 countries, i.e. 28 EU countries plus Iceland, Norway and Switzerland, this chapter estimates quarterly structural deficit series out of government budget balances on the basis of an Unobserved Components Model (UCM) and the Kalman Filter. Using a one-step estimation approach, employing a variant of a model first proposed by Camba-Mendez and Lamo (2004), the estimation of the unobserved components does not rely on several datasets and independent regressions as e.g. the current method chosen by the European Commission.[1] The advantage is that a one-step approach allows to calculate standard errors and thus a confidence interval for e.g. the structural deficit. In addition, a statement

[1] For details on the econometric approach chosen by the European Commission compare Havik et al. (2014); Mourre et al. (2013); Mourre, Astarita, and Princen (2014).

about the fiscal policy stance, i.e. whether fiscal policy tends to accommodate or enhance business cycle fluctuations can be put on solid, statistical ground. Given that different approaches towards the calculation of the latter lead to huge variations in predicted results (cf. German Council of Economic Experts (2016), box 6 pp. 92-94), a direct approach helps to mitigate estimation uncertainty. I will provide an overview on other frequently used methodologies and relate them to my findings.

From an institutional perspective, the chapter allows an empirical assessment of European fiscal policy in the light of the Maastricht Treaty, the Stability and Growth Pact (SGP) and their successor treaties. Moreover, a partition into several country blocks that share some common characteristics (Euro Area, 2004 EU entry countries, EU Med or Northern Europe) shows that until now there has not been a convergence towards a permanent and more sound fiscal policy. With respect to the requirements agreed upon in the various treaties mentioned above, the chapter documents that for most EU countries the Maastricht criteria, and especially the 3% deficit ceiling might have worked as a benchmark to policy makers but were no longer taken seriously after the run-up phase to the European Monetary Union (EMU). Using safety-margins, i.e. values the structural deficit can assume to stay within the 3% boundary, reveals that discretionary and not business cycle-related spending has been the driver of deficits in many European countries.

The chapter proceeds as follows: Section 3.2 gives a brief literature overview on the European fiscal governance framework of the last 20 years. Section 3.3 discusses the data and gives a brief exposition on the institutional details of the treaties on fiscal policy in Europe. Section 3.4 introduces the UCM in the spirit of Harvey (1990) and Camba-Mendez and Lamo (2004) and deviations from it related to the research question. Section 3.5 briefly describes related methods to calculate the structural budget balance. Section 3.6 discusses estimation results and compares them with the results from applying related methods. Section 3.7 concludes.

3.2 Literature

Theoretical and empirical literature on the SGP and the surveillance of budgetary positions in a monetary union is numerous and a huge amount has by now been outdated by institutional changes and reform debates following the European Debt crisis. This section provides a brief overview over 20 years of academic literature, whereas the next section reviews the institutional details of the numerous (changes to) treaties on European fiscal policies.

3.2.1 Early Years Of The SGP

In addition to a politically independent central bank, the rules laid down in the SGP were thought to prevent coordination failures between the single monetary policy authority and multiple, national fiscal policies. Prior to the start of the EMU, Buti, Franco, and Ongena (1998) had already warned that problems with the implementation of the pact may arise in early years in the event of a slow-down in economic activity. Early literature on the Fiscal Theory of the Price-Level (FTPL, compare Leeper (1991), Sims (1994) and Woodford (1995)) provides the theoretical foundation for this: Monetary policy is *active*, i.e. the European Central Bank (ECB) ensures price stability, defined as an inflation rate of below, but close to two percent. The fiscal policy side is *passive*, i.e. European governments credibly commit not to run excessive deficits (max. three percent of GDP, but balanced in the long-run) and not to accumulate too high stocks of debt (60 percent of GDP). These constraints to fiscal policy have been criticized as inflexible by taking governments the capability to adequately respond to economic shocks during a crisis. Calmfors and Corsetti (2003) discuss how to reform the SGP while walking the tightrope between short-run flexibility and long-run sustainability. Eichengreen and Wyplosz (1998) and Brunila, Buti, and in 't Veld (2002) among others discuss how to recover space for the working of automatic stabilizers, despite having the fiscal policy rule of the SGP in place. Galí and Perotti (2003) find that implementing the SGP did not harm the flexibility of EMU countries but rather supporting the countercyclical nature of automatic stabilizers. Focusing more on the political process behind, Beetsma and Uhlig (1999) argue that the election-oriented, short-term perspective policy makers usually take leads to a temptation to raise additional debt.

3.2.2 Violations And First Reform

The SGP has not been enforceable against big countries as France and Germany (first in 2003) and fines have not been applied in the case of Portugal (2002) and Greece (2005). In 2005, the European Council then *reformed* or rather relaxed the requirements of the treaty.[2] Annett (2006) analyses how the reformed SGP can be made more enforceable with respect to politically induced deficit biases and documents why some countries fared better under the fiscal framework than others. Buti (2006) discusses the implementation of the 2005 reform. They conclude that the reform went into the right direction concerning "asymmetric incentives and lack of a long-term view" however, question whether the key weakness - its lack of enforceability - has been eliminated. Filipek and Schreiber (2010) review the pact in the light of the 2005 reform and the Greek debt crisis and propose to adapt a European debt brake and medium-term objectives (MTOs) that also consider an consideration of external imbalances.

3.2.3 Recent Reforms

Lane (2012) argues that implementation problems of the new Fiscal Compact based on the evaluation of structural deficits require a quasi real-time monitoring of government expenditures. Furthermore, the German Council of Economic Experts (2016), box 6 pp. 92-94 discusses that real-time measurement of the output gap, which is a key variable to calculate the structural deficit is challenging and different approaches lead to huge variations in the value the output gap assumes. Andrle et al. (2015) outline ways to simplify the European fiscal governance

[2] For a broader overview on this discussion and the 2005 reform see De Grauwe (2007) and Senior Nello (2009), p.250 and Annett (2006)

framework after the recent reforms (Six Pact, Fiscal Compact, Two Pact). They propose to use the public debt-to-GDP ratio as single anchor in a common fiscal rule for all countries. Furthermore, they emphasize the necessity of greater automaticity in enforcing violations of the constraints to fiscal policy. Since then, several supranational institutions (IMF, European Commission or the European Fiscal Board) bring up the need to reform the existing framework using e.g. expenditure rules. Their main aim is to increase transparency and to reduce complexity and exceptions and escape clauses (compare German Council of Economic Experts (2017)).

3.3 Fiscal Policy Treaties and Data Description

The following two subsections give a brief overview of the development of fiscal policy treaties in the EMU. A possible conclusion that could be drawn from this discussion is that reforms of the various treaties have not enhanced but rather complicated the adequate surveillance of government budgets. Furthermore, loop-holes and other exceptions that justify a violation of the deficit and debt criteria have become bigger with every reform undertaken. The section concludes with a description of key summary statistics over the regarded countries in the sample.

3.3.1 A History of Fiscal Policy Treaties in Europe

Maastricht Treaty

Signed and established in 1992 by the member states of the European Community, the Maastricht Treaty created the European Union and led to the creation of the Euro as common currency. Among other things, the Maastricht Treaty imposes constraints on the conduct of fiscal policy. Limitations on the size of debt and budget deficits become part of the first pillar within the pillar structure of the European Union. The imposed criteria state that countries “shall avoid excessive government spending”, i.e. the ratio of government net lending to nominal GDP must not exceed three percent. In case of “exceptional” circumstances, as which structural reforms with positive medium-run effects as well as severe economic downturns are considered, the deficit may range in between three and three and a half percent. Furthermore, a restriction on the amount of debt relative to GDP set to 60 percent, is introduced. In case a country fails to meet this criterion, it has to prove efforts of a significant reduction over the last years and aspired convergence to the reference value within “satisfactory” time. In addition, long-run interest rates on sovereign bonds have to converge, such that “nominal long-term interest rate[s are not] more than two percentage points higher than in the three lowest inflation member states”. Whereas these criteria are still binding for all EU members, restrictions on annual inflation and the exchange rate (devaluation of currency etc.) have been more important in the run-up to EMU. Nevertheless, every prospective member of the Euro Zone has to fulfill these restrictions before being allowed to enter and introducing the Euro as its currency. The next subsections discuss refinements and reforms of the legal limitations on the conduct of fiscal policy.

Stability and Growth Pact

To guarantee and preserve the sustainability of public finances, the SGP is an agreement among the member states of the European Union which had initially been ratified in 1998. All EU members are obliged to be in compliance with the SGP not only at the end of the preceding fiscal year but also in the medium-run. Its initial purpose has been to ensure that the Maastricht criteria are not only met before entering the currency union, but are maintained subsequently. The pact especially implies fiscal monitoring by the European Commission and the Council of Ministers, including recommendations through its preventive, and in utmost cases excessive deficit procedures (EDP), through its dissuasive arm. The preventive arm complements the Maastricht criteria by demanding convergence to a common medium-term objective (MTO). They entered into force in 1998. The original version of the SGP recommends this common budgetary position, to be either balanced or in surplus. In contrast, the dissuasive arm is in charge of enforcing the limitations agreed upon in the Maastricht Treaty. It defines corrective policy measures, e.g. increases in the VAT rate, a non-compliant country has to undertake within a certain time frame. Breaches of the restrictions on deficits and debt intensify surveillance and can lead to economic sanctions as last resort. The SGP's corrective rules entered into force in 1999. The 2005 SGP revision deals with criticism from politicians and academics, complaining about the low flexibility and weak enforceability of the original treaty version. Annett (2006) even identifies enforceability as the "Achilles' heel" of the treaty, compared to other core characteristics of good fiscal governance. Major complaints have been that the pact jeopardized growth if governments do not possess full spending flexibility during economic downturns. In addition, Greece and Portugal have used "creative accounting" - techniques to meet the three percent deficit ceiling at any cost. Finally, the European Commission has been unable to implement sanctions against France and Germany, which at that time had repeatedly violated the deficit constraint. The 2005 reform does not alter the three percent deficit and 60 percent debt ceiling, however, the settlement whether a country runs an excessive deficit or not does now depend on additional factors. Deficit procedures now regard the structural or cyclically adjusted deficit, the duration of low growth periods plus the possibility that the violation stems from reforms increasing medium-run competitiveness. Important changes in the preventive arm include the deviation from a *common* MTO. Instead, MTOs for each member country are calculated depending on the respective debt-to-GDP ratio and potential GDP growth. MTOs include country-wise safety margins, i.e. the maximum structural deficit which is still in line with the Maastricht figure of three percent. Moreover, the maximum structural deficit to be set as MTO objective is one percent of potential GDP in case the country is not highly indebted and has a strongly growing economy. Nevertheless, in case a country is prone to demographic long run risks, the upper MTO limit should be set to be zero or in surplus. Furthermore, fiscal consolidation has become part of the preventive arm. Under "favorable" economic conditions, i.e. if the output gap is positive, countries that do not meet the above criteria agree to improve their structural deficit by half a percent of potential GDP per annum. If reforms undertaken during the consolidation process are aimed to improve overall competitiveness, a short-run deviation from the MTO is possible and not prosecuted. As 3.5 shows, different methodologies to calculate the output gap lead to huge variations in the structural deficit figures. The focus on the structural component thus enables space for a political interpretation and a less stricter enforcement of the self-given rules. In 2011, a collection of governance rules concerning the monitoring of economic policies, known as the "Six Pack" became mandatory in 2011. In 2013, further economic coordination between members states has been laid down in the "Two Pack".

Fiscal Compact

The Fiscal Compact is part (Title III) of the Treaty on Stability, Coordination and Governance in the Economic and Monetary Union, signed on 2 March 2012.[3] The purpose has been to foster discipline of public spending and strengthen the SGP following the insights of the European debt crisis. Its main elements build on the reformed SGP and include a balanced budget rule which needs to become part of national law plus an automatic correction mechanism. Furthermore, the excessive deficit procedure has been strengthened towards higher automatization of the decision, whether a country runs an excessive deficit or not. Finally, high debt countries need to ex-ante report strategies for the issuance of new debt and reduce their debt per year by 1/20th of each percentage point above the 60 percent debt ceiling.

The balanced budget rule is respected if annual, structural expenditures do not exceed the country-specific MTO, as formulated above. As this has been based on the SGP however, it contains more flexible upper limits for structural deficits depending on the country-specific debt-to-GDP ratio. Countries whose level of debt does not exceed the 60% threshold are allowed to run a structural deficit of at most 1% of nominal GDP in contrast to the otherwise allowed 0.5%. According to the SGP, the Fiscal Compact demands a "rapid convergence" to the MTO under consideration of country-specific risks. The fiscal policy rule needs to be mandatory, preferably at constitutional level, as does e.g. the German "debt brake". Moreover, an automatic correction mechanism whose guidelines were delineated by the European Commission but is individually implemented by each country shall work against the fiscal reality of significant deviations from the respective MTO.

3.3.2 Overview How EU Regulation Affects Countries in Sample

The data sample contains 31 countries. 28 of them are members of the European Union, which implies that the Maastricht Treaty and the Stability and Growth Pact applies to them. Rules imposed by successive agreements, such as the Fiscal Compact apply only to 20 countries in the sample, as countries outside the Eurozone as Sweden, Poland or Hungary are not bound by any economic or fiscal policy coordination mechanisms. So far only the United Kingdom has not ratified the treaty however, the upcoming Brexit makes a ratification more than illusional. Denmark and Romania are outside the Eurozone and may accede to the treaty only after negotiations with representatives of the Eurozone. Iceland, Norway and Switzerland are not affected by any of the above treaties. However, both countries are closely associated with the EU and

[3] A comprehensive, more detailed overview on the Fiscal Compact has been provided in ECB (2012).

the Eurozone and have access among other things to the European Single Market. Moreover, Switzerland is interesting as they agreed upon maintaining sustainable public finances using a debt brake.

For a more comprehensive analysis, it is useful to sort countries into political, historical or regional subgroups. This might even allow to generalize the vast amount of information the results will convey.[4] I base my classifications on the "EU supranational bodies", discussed in an Wikipedia article on the "Regions of Europe" - which is no official classification and not to confuse with official European regions which are represented in the Assembly of European Regions. A lot of European governments coordinate their EU politics and especially their voting behavior on the European Council with neighbors they have strong historical or geographical ties with in inter-governmental or -parliamentary bodies: Examples for that are the:

- Nordic Council (**Denmark**, Sweden, Finland, Iceland, and Norway among others)
- Baltic Assembly (**Estonia**, Latvia and Lithuania)
- Benelux States (**The Netherlands**, Belgium and Luxembourg)
- Craiova Group (**Romania** and Bulgaria among others)
- Visegrád Group (**Poland**, Hungary, Czech Republic, Slovakia, Austria)
- EU MED Group (**Spain**, Cyprus, France, Greece, Italy, Malta and Portugal),

among others. Based on that, I will select a country that is the most representative of the supranational body it belongs to (here marked in **bold**). Southern Europe is a special case: France and Italy belong to the (economically) most-important countries and Greece has been hit the hardest by the European Debt Crisis and the Great Recession, such that I will identify these countries on their own. Finally, as Germany is no member of a smaller, very homogeneous supranational body of countries, Germany is added to the list of countries for which I will discuss results in detail.

3.3.3 Data Description

All quarterly data stems from the Eurostat National Accounts Database. The dataset contains the log of seasonally-adjusted real GDP, seasonally-adjusted general government net lending in percent of seasonally-adjusted nominal GDP and general government gross debt in percent of nominal GDP. In cases when data was not available for the entire time span, e.g. only at annual frequency during some periods, Section 3.4 describes a method how to still obtain the structural components of the model. For some countries, the annual budget balances had to be obtained directly from the IMF's World Economic Outlook Database as they were not covered by the Eurostat National Accounts Database. Hence, the data is either of quarterly or annual

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[4] If not in the main text, full results on individual countries will be provided in the respective appendix.

frequency and covers a time span from usually 1995 to 2015 for most countries, for some even longer.[5] Variables used for estimation are quarterly, seasonally-adjusted real GDP and the quarterly government budget balance in percent of nominal GDP. Consolidated, general government gross debt in percent of GDP is used as additional exogenous regressor.

Table 3.1 and Figure 3.1 provide an overview over the development of GDP growth, deficits and debt for all countries. I divide the sample into four subperiods and calculate the respective subsample averages. The summary statistics show that during the years 1995 - 2000, all countries (except for Norway, Luxembourg and Ireland) have run substantial deficits. Substantial refers to the fact that the across-country average deficit is close to three percent, in particular after excluding the three countries named above. The budgetary position improved on average from 2000 to 2005. However, the group of countries which joined the European Union in 2004 give a mixed picture. Some of them consolidate or even run surpluses, whereas others as e.g. Hungary or Poland have a deteriorated budget balance. With the exception of Bulgaria, Luxembourg and the Scandinavian countries, all countries are hugely affected by the recession in 2008-2010 and the post- bank-bailout costs following the financial crisis. For the most recent period from 2010 to 2015, more than half of the countries in the sample face even higher deficits on average than before. Despite austerity programs and expenditure cuts, many countries in the other half still run substantial deficits highly above three percent of GDP. It seems that in the short run, deficits below the three percent boundary are hard to achieve for many countries following the European debt crisis. This effect is supported by the fact, that the sanctioning mechanism did never fully work and has been weakened or politically adjusted over and over gain.

The European economies have been growing strongly from 1996 to 2000. Exceptions were Germany, France or Italy - economies with a lot of problems concerning structural growth during that time. Data on GDP growth of the Eastern European transition economies exhibits catch-up growth and the Scandinavian economies have successfully recovered from the Scandinavian banking crisis in the early 1990's. Growth increases even more during the early 2000's but becomes slow and moderate on average for the years 2005-2010, as many countries have been hit strongly by the Great Recession in 2008-2009. Post 2010 GDP growth does not give a clear picture. Some countries quickly recovered, whereas especially the southern European Economies shrank on average.

With the exception of Italy and Greece, the level of debt in percent of GDP has been centered around 60% for major European economies in the late 1990's. For other European countries debt has been even much lower. During the early 2000's almost all countries converged below the now binding threshold of 60% debt to GDP ratio. Prominent exceptions among others are again Germany and France who have also been the first countries to receive an official warning by the European Commission in 2003 for breaking the Stability and Growth Pact. A sanctioning process which has been voted down by the council of European ministers of finance later. Deficits increased almost everywhere from 2005 until 2010 as the financial crisis and the subsequent debt crisis included debt-financed bank-bailouts and recovery programs. This trend continued

[5] For some Eastern European countries data is only available from 2000 onwards.

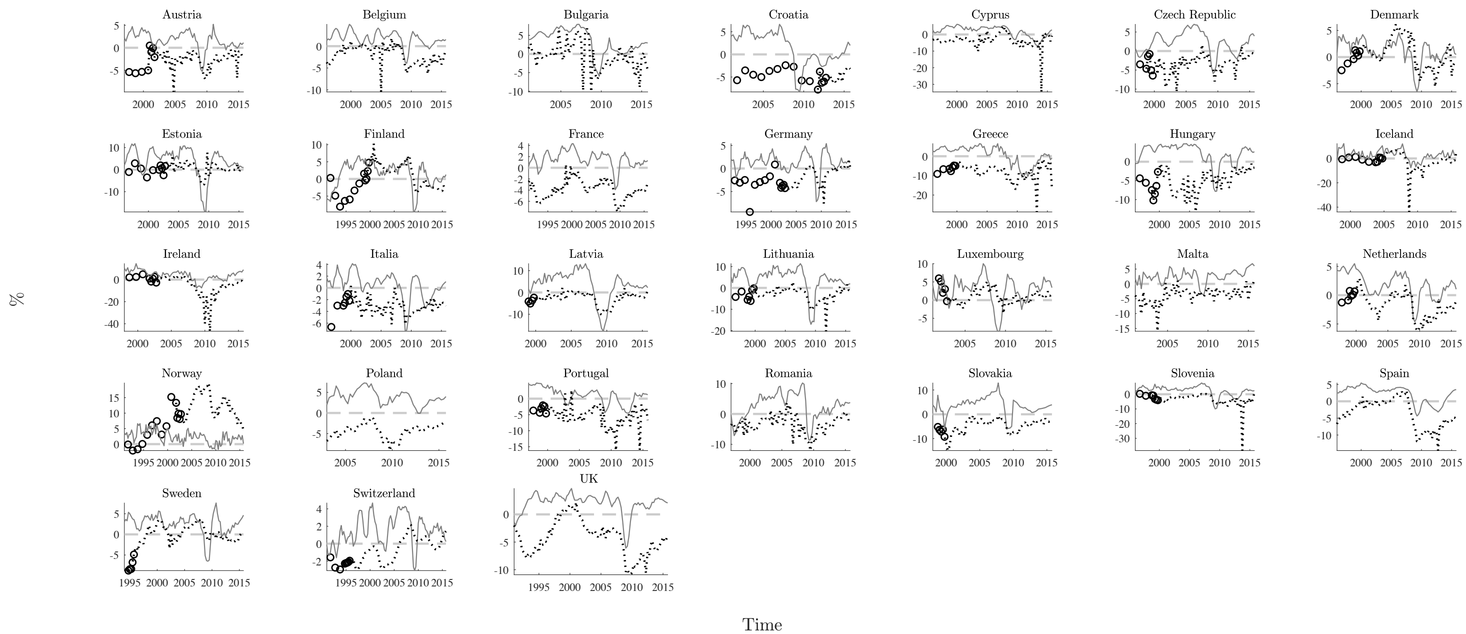

Figure 3.1. Quarterly GDP Growth (grey-solid) and Budget Balances (black-dashed) in % of GDP Until 2015

Notes: The figure shows quarterly, seasonally-adjusted real GDP growth rates and budget balance ratios in percent of nominal GDP. In case data has only been available every 4 quarters, corresponding value is reported as circle. All data is measured in Euro and has been provided by the European System of National and Regional Accounts (ESA 2010) and in some cases the IMF World Economic Outlook Database

Table 3.1. Fiscal Developments in Europe, 1995 - 2015

Countries	Balance in Percent of GDP				Growth Rate of GDP				Debt in Percent of GDP			
	1995 - 2000	2000 - 2005	2005 - 2010	2010 - 2015	1996 - 2000	2000 - 2005	2005 - 2010	2010 - 2015	1995 - 2000	2000 - 2005	2005 - 2010	2010 - 2015
Austria	-5.8	-2.3	-3.1	-2	3.2	1.7	1.3	1	70.4	69.7	72.8	83.2
Belgium	-2.4	-0.41	-1.8	-3.5	2.7	2.1	1.3	1.2	124	105	95.4	106
Bulgaria	0.27	0.49	-2.3	-1.7	5.3	4.4	0.9	2.9	76.3	50.4	17	19
Croatia	-4.9	-3.6	-5.6	-3.3	4.5	1.7	-1.1	1.6	NaN	38.6	42.3	75.4
Cyprus	-3.2	-3.6	-0.93	-5.3	3.4	3.9	2.9	-1.3	53.7	61.4	56	87.7
Czech Republic	-3.5	-4.8	-3	-2.1	1.2	3.8	2.4	1.3	15.6	25.7	29.9	42.1
Denmark	-0.53	0.98	3.1	-1.6	2.8	1.6	0.2	0.81	56.4	48.5	34.2	44.9
Estonia	-0.34	1.3	0.33	0.28	5	7	1	3.3	5.67	5.14	4.84	8.95
Finland	-0.31	4.1	2.7	-2.4	4.8	3.1	0.78	0.55	44.7	40.8	37	54.8
France	-3.4	-2.6	-3.7	-4.7	2.6	2.1	0.73	1.1	59.1	62.2	70.4	91.2
Germany	-4	-3.5	-1.6	-0.74	1.6	0.99	0.62	1.9	59.6	61.6	67.7	77.3
Greece	-7	-6.5	-8.9	-9.1	3.5	4.5	0.95	-4.3	NaN	104	113	166
Hungary	-6.7	-5.9	-6.2	-3.1	2.6	4.2	0.53	1.5	65.4	56.8	70.7	79.1
Iceland	0.076	2.4	-6.2	-1.7	4.7	4.2	0.81	2.8	NaN	NaN	NaN	NaN
Ireland	2.3	1.1	-13	-5	8.6	5.2	0.05	3.5	42.6	31.3	41.1	107
Italy	-3.4	-2.9	-3.4	-3.2	1.5	1.5	-0.44	-0.33	109	105	106	126
Latvia	-2.5	-0.65	-5.1	-1.3	5.1	9.5	-2.7	2.8	12.6	13.4	19.1	41
Lithuania	-3.4	-2.3	-2.7	-3.8	4.7	6.6	2.4	3.2	24.2	21.7	19.8	38.7
Luxembourg	1.2	1.8	0.53	1.5	2.9	2	3.1	4.8	NaN	6.73	11.5	22.1
Malta	-6.2	-3.1	-2.9	-1.5	1.6	2	3.2	6.2	NaN	67	65	69.1
Netherlands	0.46	-1.5	-1.9	-2.9	4.5	1.3	1.3	0.76	57	50.3	50.3	65.2
Norway	5.1	9.8	16	11	3.7	2.3	1.4	1.5	23.4	35.7	44.3	31.3
Poland	NaN	-5.2	-4	-4.3	NaN	4.3	4.5	3	NaN	NaN	44.3	46.6
Portugal	-3.5	-5	-6.4	-5.9	4.1	0.86	0.61	-0.95	51.2	57	73.9	123
Romania	-4	-1.7	-5.3	-2.6	-1.3	5.5	2.9	2.3	22.3	21.8	16.2	36.3
Slovakia	-8.1	-3.9	-4.8	-3.1	2	5.8	3.6	2.2	49.6	43.4	32.4	51.4
Slovenia	-1.7	-2.9	-2	-6.4	4.1	3.5	2.3	0.63	24.8	26.9	27.8	62.4
Spain	-4.1	-0.51	-1.9	-7.9	3.7	3.6	1.8	-0.14	62.7	50.1	42.9	86.1
Sweden	-2	0.48	1.7	-0.66	3.2	3	1	2.6	64.1	49.7	40	40.2
Switzerland	-2	-1.7	0.57	0.27	1.8	1.7	2.2	1.7	NaN	NaN	NaN	NaN
UK	-2.2	-1.4	-4.9	-6.9	3	2.9	0.68	2	38	36	49.9	84.3

Notes: The above table shows average summary statistics over five-year intervals. For some countries the first interval is not exactly five years (e.g. the sample only starts in 1996 or 2002) - however, to keep notation simple the corresponding value is then just the average of the remaining years. If the years until the year 2000 are missing completely, the corresponding first column is set to NaN.

on average for the most recent period. Similar to the summary statistics of the budget balances, the early criteria on a debt limit seem to be no longer binding and not achievable anymore in the medium term.

To conclude, it appears that fiscal discipline and the sustainability of public finance had already ended after the post-Euro or post-European Union convergence phase. The ratification of treaties such as the Maastricht Treaty or the Stability and Growth Pact should have exactly prevented that. Also, from a 2016 perspective - automatic sanctions as imposed following the ratification of the Fiscal Compact do not make sense if they are not credibly imposed. How large has the scope of discretionary fiscal policy really been? How large was the structural deficit in reality? The descriptive statistics suggest that there has already been a constant decline in fiscal discipline prior to the recent crisis which we should be able to detect in the data, as well.

The next section puts the data to use in an empirical model. Unobserved Component Models (UCM) enable to decompose observed variables such as GDP or government figures into their unobservable parts. Taking a time series perspective, where all seasonal influences have been eliminated, it is possible to discuss and evaluate the influence of cyclical and structural components, separately. Following Taylor (2000) or Galí and Perotti (2003), the decomposition of the observed deficit can be regarded as a fiscal rule, analogously to the ones used to conduct monetary policy. Fiscal rules help to monitor fiscal discipline as agreed upon in the Stability and Growth Pact. The precise rule is discussed in section 3.4.

3.4 An Unobserved Components Model

The econometric model used for the estimation of cyclically-adjusted government budget balances belongs to the class of Unobserved Component Models (UCM) in tradition of e.g. Harvey (1990) or Harvey (1991). The basic idea is a signal-extraction problem. The econometrician is interested in unobservable components (the states) of an observable time series. Economic theory is used to pin down the underlying structure and Kalman Filtering- and Fixed Interval Smoothing-techniques are employed to extract unobservable states out of the data. The following section delineates the empirical model and the chosen estimation approach.

3.4.1 The Model

The UCM decomposes time series into a trend, a cyclical and an idiosyncratic component. The statistical treatment of these unobserved components depends on knowledge or assumptions about the states' underlying law of motion. Individual components, as the structural or cyclical component are not directly observable but we do have an idea about how they evolve over time. In combination with observables, this constitutes a state-space system. Observable data will be quarterly real GDP and the budget balance ratio defined as quarterly government net lending over quarterly nominal GDP. Gross debt in percent of nominal GDP is an additional regressor - but not part of the underlying law of motion of the unobservable states and added as an exogenous regressor.

The European System of National and Regional Accounts (ESA 2010) contains quarterly data for both GDP and the government deficit for at least from 2000 onwards for most countries. This allows straightforward estimation and interpretation of quarterly, cyclically-adjusted deficit series. However, prior to the year 2000 there is a shortage in the availability of quarterly government spending data for a variety of countries. Using a model first proposed by Camba-Mendez and Lamo (2004)[6], the signal-extraction problem is augmented with an update step from annual to quarterly frequency. I will show below that quarterly states can be extracted even when new information is only available every fourth quarter, i.e. at annual frequency. This allows a longer-term evaluation of the fiscal policy stance in the European Union.

Real GDP is modeled as the sum of a stochastic trend $\mu^y_{t,q}$, a cyclical component $\psi_{t,q}$ and a measurement error $\zeta_{1t,q}$. The budget balance ratio follows a stochastic trend $\mu^d_{t,q}$, a cyclical component $\Phi_d(L)\psi_{t,q}$ proportional to the business cycle, and a presumably negative level effect of the European Debt crisis between 2008Q4 and 2012Q4 $\eta^d_{t,q}$. In addition, the budget balance ratio is affected by gross debt over nominal GDP plus a component $\zeta_{2t,q}$ accounting for measurement errors. The model assumes that trend processes are independent, as economic theory tells us that potential or long-run output is usually determined by supply-side factors that concern the structure of the economy and are not altered by government spending or monetary policy. On the contrary, the structural component of the budget balance is at the discretion of policy makers and depends mainly on legislative and other political factors. The nature of the proportionality between the business cycle and the cyclical budget balance lies in automatic stabilizers. Common examples are tax revenues and unemployment benefits which fluctuate proportional to the business cycle. During a recession, tax revenues decrease which supports disposable income and dampens the pass through to aggregate demand. In times of boom, higher revenues are collected which diminishes the effect on aggregate demand. The more progressive the tax code, the stronger the stabilizing nature of taxation. In addition, higher unemployment benefits during recessions absorb shocks to aggregate demand, whereas the converse holds during booms. Stated otherwise, these variables work as automatic stabilizers on the business cycle. A very recent contribution to this otherwise extensively researched field has been made by McKay and Reis (2016).

In order to account for measurement errors, $\zeta_{ti,q}$ is an *iid*, normally-distributed process with standard deviation σ_{ζ_i}, $\forall i \in \{1,2\}$. A measurement error of zero implied that the interpolated series is exactly the sum of its unobserved components.

The observation equation of the state-space system can be described as a two-dimensional vector-autoregression where:

$$\begin{aligned} y_{t,q} &= \mu^y_{t,q} + \alpha\psi_{t,q} + \zeta_{1t,q} \\ d_{t,q} &= \mu^d_{t,q} + (\alpha_1 + \alpha_2 L)\psi_{t,q} + \lambda_f \eta^f_{t,q} + \lambda_d \eta^d_{t,q} + \zeta_{2t,q}, \end{aligned} \tag{3.1}$$

[6] Whereas the original paper documented quarterly, structural deficit series of Germany and Italy from 1970 to 2000, the present chapter supplies the decomposition of the government budget balance for a set of 30 countries for the years 1995 to 2015 or even earlier.

where t, q indicate the specific year and quarter. Output $y_{t,q}$ is described by the log of quarterly real GDP, whereas $d_{t,q}$ is nominal government net lending over nominal GDP. The joint, cyclical component is modeled as a second-order autoregressive process:

$$\psi_{t,q} = \rho_1 \psi_{t,q-1} + \rho_2 \psi_{t,q-2} + \zeta_{3t,q}, \qquad \zeta_{3t,q} \sim N(0,1). \tag{3.2}$$

The polynomial lag operator $\Phi_d(L) = (\alpha_1 + \alpha_2 L)$ on $\psi_{t,q}$ accounts for the proportionality with the business cycle. The stochastic trend components evolve according to the following two equations, where $i = \{y, d\}$:

$$\begin{aligned} \mu^i_{t,q} &= \beta^i_{t,q} + \mu^i_{t,q-1} + \varepsilon^i_{t,q}, & \varepsilon^i_{t,q} &\overset{iid}{\sim} N(0, \sigma^2_{\varepsilon i}) \\ \beta^i_{t,q} &= \beta^i_{t,q-1} + \nu^i_{t,q}. & \nu^y_{t,q} &\overset{iid}{\sim} N(0, \sigma^2_{\nu y}) \end{aligned} \tag{3.3}$$

The trend component of both GDP and budget balance ratio are assumed to follow independent random walks. In addition, Camba-Mendez and Lamo (2004) model the slope of this trend, $\beta_{t,q}$ as a second random walk. If however, the variance affecting this slope-random walk is near zero, the entire trend component converges against a random walk with drift. Moreover, I include exogenous variables $\eta^f_{t,q}$ and $\eta^d_{t,q}$ to model changes in the level of the budget balance ratio after 2008Q4 until the end of 2012 (indicator variable) and the reaction of the budget balance to the debt-to-GDP balance.

3.4.2 State-Space System

In what follows I describe the state-space system for the fully observable model. In an additional step I explain how it is possible to interpolate the missing 3 quarters in case annual budget balance data has been used to extend the sample to a longer time horizon. To keep things comparable, the notation still closely follows Camba-Mendez and Lamo (2004). A more detailed and comprehensive overview of all the steps and equations can be found in Appendix B.2.

Fully-observable Model

The two-dimensional vector-autoregressive process in equation (3.1) is summarized by the following equation, representing the measurement equation of the state-space system:

$$x_{t,q} = \mathbf{A} s_{t,q} + \mathbf{B} z_{t,q} + \varepsilon_{t,q}, \tag{3.4}$$

where $\mathbf{A}$ is a selection matrix on the state-vector $s_{t,q}$, $\mathbf{B}$ is a vector capturing the additional level effect on the budget balance ratio between 2008Q4-2012Q4 and the effect of the gross debt over GDP ratio and $\varepsilon_{t,q}$ allows to control for measurement errors. The vector $x_{t,q}$ stacks the two observables, $(y_{t,q}, d_{t,q})'$. The state-vector $s_{t,q}$ collects the unobservable components discussed above. The underlying transition equation is given by:

$$s_{t,q} = \mathbf{C} s_{t,q-1} + e_{t,q}. \tag{3.5}$$

If quarterly data is available for both real GDP and the budget balance ratio at all times no partial-updating steps are needed. The state vector is augmented such that the new state vector contains both the unknown states $s_{t,q}$ and the known observables, $x_{t,q}$ represented in $q_{t,q} = (x_{t,q}, s_{t,q})'$. This results in the following state-space system at quarterly frequency:

$$x_{t,q} = \mathbf{Z}' q_{t,q} \tag{3.6}$$

$$q_{t,q} = \mathbf{N} q_{t,q-1} + \mathbf{M} z_{t,q} + \mathbf{R} v_{t,q}. \tag{3.7}$$

In eight out of 31 cases all data is fully available at quarterly frequency and the structural components of the model can be recovered directly without any further interpolation steps. In case annual budget balance data is used to help obtaining the desired sample length, an interpolation based on the full sample requires the aggregation to an annual frequency. A subsequent partial updating step then recovers the structural components also at quarterly frequency. Exploiting the recursive structure of equation 3.7, the annualized representation at quarterly frequency looks as follows:

$$x_{t,4} = \mathbf{Z}' q_{t,4} \tag{3.8}$$

$$q_{t,4} = \mathbf{N_t}^4 q_{t-1,4} + \mathbf{M}_t^4 (z_{t1} \ldots z_{t4}) + \xi_{t,4}^4 \tag{3.9}$$

$$\mathbf{L_t^4} = Var(\xi_{t,4}^4). \tag{3.10}$$

The annualized representation is used to estimate the unknown parameters of the model conditional on the full sample length. The model's equations are now time-dependend due to the construction of an auxiliary budget balance-ratio series for the missing quarters. Whenever quarterly data is available, the later will be used during the updating procedure. The state-space system allows to construct the joint log-likelihood function of the model via the prediction error decomposition. Prediction errors and the prediction error variance stem from a Kalman Filtering algorithm robust to outliers which evaluates the log-likelihood function at every point in time, t. Maximum Likelihood estimates are obtained through numerical minimization of the negative of the joint log-likelihood function. In addition, the standard errors of the estimated parameters are obtained from the inverse Hessian matrix. Finally, performing Ljung-Box Tests on the correlation of standardized residuals ensures that the model is well-specified.

Interpolation Step

In case quarterly data on government deficits are only partially available, annual deficit series are used to interpolate the unobservable states to a quarterly frequency. Camba-Mendez and Lamo (2004) overcome this problem by introducing auxiliary variables represented in a vector $x_{t,q}^a$ which is fully observable even at quarterly frequency.

$$x_{t,j}^a = \sum_{i=1}^{j} \mathbf{W}_{t,i} x_{t,i},$$

where

$$\mathbf{W}_{t,i} = \begin{bmatrix} 1 & 0 \\ 0 & w_{t,i} \end{bmatrix} \qquad \text{and} \qquad w_{t,j} = \frac{Y^n_{t,j}}{\sum_{i=1}^{4} Y^n_{t,i}}.$$

The vector $x^a_{t,q}$ transforms quarterly real GDP at quarterly level into quarterly real GDP at annual level, i.e. it contains the cumulated sum of quarterly GDP figures for the regarded year. Additionally, annual budget balance ratios are weighted and cumulated afterwards. The weight is constructed from quarterly nominal GDP, $Y^n_{t,j}$ over annual nominal GDP in the respective year.[7] This implies that $x^a_{t,4}$ contains just the end-of-period value for GDP and the corresponding annual budget balance ratio.

The state-space system for the model using annual data basically looks like the system presented in 3.7 and 3.9. However, as $w_{t,q}$ is time-varying, all system matrices except the selection matrix **Z** are time-varying as well. Furthermore, the annualized system which is used to estimate the unknown model parameters only uses every fourth observation, given that at this frequency all data is fully-observable. See the appendix for a detailed derivation of all equations and estimation steps.

Once the parameters are estimated, Kalman filtering with partial updating finds the unobserved parts of the state vector $q_{t,q}$ at quarterly frequency. Partial updating implies updating for real GDP every quarter and updating for the budget balance ratio only every fourth quarter. Since this chapter reconstructs the unobservable states until at least the first quarter of 1995, the filtered outcome is passed to a Kalman Smoothing algorithm. All estimation and specification test results will be discussed thoroughly in the succeeding section.

3.5 Further Approaches towards Structural Budget Balances

Before comparing the approach delineated above with other possible approaches employed to estimate cyclically-adjusted budget balances, this section provides a brief overview on the European Commission's methodology and estimation based on HP-filtering and Hamilton (2017)-Filtering.

3.5.1 The European Commission's Approach

The European Commission follows its own approach to monitor the fiscal policy rules delineated in the EU treaties above. Their approach views the budget balance as the sum of a component proportional to the business cycle and a discretionary part corrected for one-time measures and

[7] For standard flow variables, the annual value is just the sum of its quarterly components. This does not hold for ratios, i.e. the weighting has to account for differences between quarterly nominal GDP values.

the influence of automatic stabilizers:

$$BB_t = \frac{R_t - G_t}{Y_t} = CAB_t + \varepsilon OG_t, \tag{3.11}$$

where BB_t denotes the budget balance or net government spending, $R_t - G_t$ stands for revenue minus expenditures, Y_t for GDP, CAB_t for the cyclically-adjusted budget balance, and OG_t for the output gap. The parameter ε denotes the joint reaction parameter of revenue and expenditures towards the cycle. The division into these parts is not uniquely pinned down by theory and discussions about the quantitative magnitude and the qualitative reliability of these figures are always related to the estimation of the output gap and the sensitivity parameter. The German Council of Economic Experts (2016) pointed out that different methodologies lead to severe deviations between the values the structural deficit assumes and makes qualitative judgment less reliable. Two major revisions in the EU's methodological approach recently took place. Havik et al. (2014) describes a revised approach to estimate potential GDP and the output gap, also employing Kalman Filtering techniques. In addition, Mourre et al. (2013) show and compare two approaches and the impact on results, if, instead of estimating a budget sensitivity, ε is estimated as semi-elasticity.

Output Gap Calculation

Calculating, forecasting and evaluating the output gap, i.e. the difference between actual, observed output and output when the economy was running at full potential belongs to the central questions of (empirical) macroeconomics during the past 60 years. Giorno et al. (1995) compare the use of a production function-based methodology with the HP-filter to find a smooth GDP trend and calculate structural budget balances. While later parts of this section discuss the HP-filter, I will briefly describe the production function approach and its recent elaborations (Havik et al. (2014)) which are now part of the ECOFIN approved methodology to calculate potential output.

Using a simple two-factor type Cobb-Douglas production function of the form

$$y = c + \alpha \ln(N) + (1 - \alpha)\ln(K) + e, \tag{3.12}$$

as regression equation, the Solow-residual e or the log of total factor productivity can be calculated. The residual series is then smoothed using the HP-filter to provide a measure of trend factor productivity. Actual (log-)capital input k, the smoothed residuals e^p and a measure for potential employment n^p are then re-inserted into the equation to obtain potential output as:

$$y^p = c + \alpha n^p + (1 - \alpha)k + e^p, \tag{3.13}$$

where potential employment has been calculated as the log of $TLF(1 - NAWRU) - EG$, i.e. trend labor force times one minus the estimated non-accelerating wage-rate of unemployment minus employment in the government sector. The new propositions in Havik et al. (2014) concern the estimation of the $NAWRU$ and the estimation of trend total factor productivity e^p using more appropriate statistical tools.

Instead of HP-filtering, a Bayesian bivariate Kalman-filtering approach following Koopman and Durbin (2003) has been chosen. Here, the trend-cycle decomposition is based on a structural recursive model which in case of the NAWRU is augmented with information that stems from a Phillips Curve trade-off. The overall benefit of this revised approach is to give economic theory more space in contrast to a purely statistical approach that might generate arbitrary outcomes.

Reaction Parameter ε

The estimation of ε is based on a two-step methodology, following van den Noord (2000) and has been updated by Girourard and André (2005). In a nutshell they comprise: Revenue elasticities of corporate, personal income and indirect taxes and social security contributions with respect to the output gap. On the expenditure side the elasticity of unemployment benefits to the output gap is regarded. The output gap elasticity of those budget items can be obtained by multiplication of a weighting parameter (elasticity of e.g. individual revenue item to base revenue) times the elasticity of the base to the output gap. This allows to keep track of country-specific tax codes or the significance of unemployment benefits.

The revised reaction parameter ε accounts for a second order approximation error which does not matter as long as both output gap and budget balance are small. Until 2012, ε had been defined as time-constant *budget-sensitivity*, i.e. measuring the ratio of incremental changes in the budget to incremental changes in output. However, as Mourre et al. (2013) show on pp. 11-13, using the definition of the budget sensitivity does not exactly lead to the desired result, i.e. the cyclically-adjusted budget balance. Instead they propose to use a time-constant *budget semi-elasticity*, i.e. measuring the ratio of incremental changes in the budget-balance *ratio* to the percentage change in GDP.[8] Mourre et al. (2013) point out that during the recent financial crisis this approximation error made up to half a percentage point of GDP. Furthermore, using (3.11) and the results and definitions of footnote (8) allows to break down the semi-elasticity parameter ε into its components.

$$\varepsilon = \frac{dB}{dY} - \frac{B}{Y} = \left(\frac{(dR/R)}{(dY/Y)} - 1\right)\frac{R}{Y} - \left(\frac{(dG/G)}{(dY/Y)} - 1\right)\frac{G}{Y} = (\eta_R - 1)\frac{R}{Y} - (\eta_G - 1)\frac{G}{Y}. \tag{3.14}$$

Mourre et al. (2013) propose to use this approach instead of the sensitivity-approach because cyclical adjustment is much stronger on the expenditure side relative to the revenue side. Here, a main driver of adjustment are unemployment expenses and transfers. This feature is not adequately captured by using revenue and expenditure sensitivities instead. Given that revenues are sensitive to the business cycle, a change in output is more or less off-set by a change in

[8] The approximation error which is usually of second-order magnitude can be derived using the following approximation of the cyclically-adjusted balance plus the output gap and the corresponding definition of the reaction parameter ε.

$$CAB = \frac{B - dB}{Y^p} = \frac{B}{Y} - (\varepsilon_R - \varepsilon_G) \cdot OG = \frac{B}{Y} - \varepsilon \frac{Y - Y^p}{Y^p},$$

The budget-sensitivity is defined as :

$$\frac{dB}{dY} = \frac{dR - dG}{dY} = \frac{(dR/R)}{(dY/Y)} \cdot \frac{R}{Y} - \frac{(dG/G)}{(dY/Y)} \cdot \frac{G}{Y},$$

and substitution for ε in the definition of CAB leads to the following cyclically-adjusted balance:

$$CAB_e = \left(\frac{B}{Y} - \frac{dB}{Y^p}\right),$$

whereas the budget-elasticity has been defined as

$$\frac{dB}{dY} - \frac{B}{Y} = \frac{dB/Y}{dY/Y} = \frac{(dB/dY) \cdot Y - (dY/Y) \cdot B}{Y} = \frac{dB}{dY} - \frac{B}{Y},$$

and leads to the following CAB_e which corresponds to the definition above.:

$$CAB_e = \left(\frac{B}{Y^p} - \frac{dB}{Y^p}\right).$$

The substitution for ε in both cases shows that the budget sensitivity only adjusts the budget balance but not current output, which is of course affected by the current cyclical movement, whereas the the semi-elasticity correctly describes the reaction of the budget-balance to GDP -ratio to cyclical change in GDP. As Mourre et al. (2013) show, the difference between the *correct* CAB and the one based on the budget sensitivity amounts to:

$$CAB_e - CAB_s = \left(\frac{B}{Y^p} - \frac{dB}{Y^p}\right) - \left(\frac{B}{Y} - \frac{dB}{Y^p}\right) = \cdots = OG \cdot \frac{B}{Y},$$

which only matters in case both factors are large as e.g. during the recent financial crisis.

revenues. The revenue elasticity η_R close to 1 and thus, the revenue semi-elasticity ε_R is close to 0. In contrast to that, a huge part of total expenditures is of non-cyclical nature, which implies that the expenditure elasticity η_G is close to zero. Hence, the expenditure-to-GDP-ratio strongly responds to changes in the business cycle if the expenditure-to-GDP-ratio is approximated by 0.5, roughly the EU average. Nevertheless, Mourre et al. (2013) conclude that in absence of the recent crisis years, the revision had a minor impact on quantitative results.

3.5.2 Alternatives

Another popular and easy to implement alternative towards this trend-cycle-separation problem is the HP-filter, after Hodrick and Prescott (1981, 1997). Let a time series Y_t be described as $Y_t = T_t C_t \varepsilon_t$ such that in logs, the cyclical component c_t of the series is approximately (abstracting from noise) equal to the log of the original series y_t minus the trend component τ_t. The basic idea behind that is to simultaneously minimize the squared deviation of the log time series from its trend plus to penalize excessive curvature in the trend component, using λ. The optimization problem which needs to be solved looks as follows:

$$\min_{\tau}\left(\sum_{t=1}^{T}(y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1}[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2\right). \tag{3.15}$$

Using the ratio of government net lending over nominal GDP, this series can be split into a trend or structural and a cyclical part. This very simple approach can be good approximation provided the fact that the cycle of the budget balance and GDP are proportional and trend excess expenditures are related to long-term growth prospects of the economy.

However, the HP-Filter comes with enormous drawbacks such that the European Commission no longer uses it (see Havik et al. (2014) p.6 for further details) to calculate potential output. The filter is biased at the start- and endpoints of the sample because estimates are needed to calculate initial differences. It is not causal as it is forward- and backward-looking at the same time. This may lead to problems when HP-filtered data is used to estimate recursively-ordered structural models. Moreover, especially the endpoint problems imply that the filter should not be used for real-time policy recommendations as results might be biased. Furthermore, the choice of λ which is related to the length of the cycle is somehow arbitrary. Longer deviations from trend cannot be detected by construction and would be counted as a trend reversal. This has become obvious during or right after the recent financial crisis. Whereas Cogley and Nason (1995) illustrate how some of these shortcomings differently affect the detrending of trend- and difference stationary time series, Ravn and Uhlig (2002) provide further insights on issues concerning the choice of λ. Stock and Watson (1999) define a 1-sided, or purely-backward looking Kalman-Filter equivalent version of the filter that mitigates some of the criticism delineated above.

Finally, Hamilton (2017) proposes to use a simple but (in his opinion) statistically more superior regression-based method of fourth differences instead, which works surprisingly well. For quarterly data, the OLS regression equation looks as follows:

$$y_{t+h} = c + y_{t-p+4} + y_{t-p+3} + y_{t-p+2} + y_{t-p+1} + \varepsilon_t, \tag{3.16}$$

where I follow Hamilton (2017) and set h to 8 and p to 4. In absence of other influences (e.g. seasonality), the residuals $\hat{e} = (y_{t+h} - \hat{y}_{t+h})$ are the cyclical component of the series. In the following section I will compare my results with results based on the methods described in this section.

3.6 Empirical Results

In what follows I am going to discuss estimation results and their implication for the stance of fiscal policy in the respective country. Furthermore, I analyze the estimated structural budget balances with respect to medium-term objectives and safety-margins over different time periods. Finally, the results are compared with outcomes of the alternative approaches discussed further above.

3.6.1 Estimation Results

Figure 3.2 depicts the interpolated, quarterly budget balance series and the respective cyclically-adjusted balance for each country. The black, solid line is the actual budget balance observed and for some early years extended with annual values in form of bubbles. The grey-dotted line is the cyclically-adjusted budget balance. Depending on individual data availability, the time horizon covered is shorter for some countries. The respective sample length becomes visible from the summary statistics in Table 3.1. For a subset of countries[9], chosen to make the fiscal policy stance comparable across countries with similar characteristics, the results in Table 3.2 display a subset of the estimated coefficients of the model. The full vector of estimated coefficients looks as follows:

$$\theta = [\alpha\ \alpha_1\ \alpha_2\ \rho_1\ \rho_2\ \sigma_{\varepsilon y}\ \sigma_{\eta y}\ \sigma_{\varepsilon d}\ \sigma_{\eta d}\ \sigma_{\xi_1}\ \sigma_{\xi_2}\ \lambda_f\ \lambda_d] \tag{3.17}$$

The estimation coefficients can be broadly separated into three groups: Business cycle related, potential GDP or structural government balance and exogenous influences. I follow Fatás and Mihov (2010) who think of fiscal policy as a combination of automatic stabilizers and endogenous and exogenous discretionary fiscal policy. My classification looks as follows:

- *Business Cycle related*: The reaction of fiscal policy to cyclical fluctuations as a result of the tax code and social security benefits that link the budget to the business cycle. Here α_1, α_2 represent the cyclical reaction of the budget balance ratio. The cycle generating coefficients are ρ_1 and ρ_2. The coefficient α links GDP to the cycle.

[9] The choice of the subset and the selection criteria have been discussed in section 3.3, based on the "Regions of Europe" article on Wikipedia.

Table 3.2. Estimation Results for Subset of Countries

	Coefficients related to the Business Cycle			Cycle-generating AR(2)-Coefficients		Reaction to Crisis	Reaction to Lagged Debt
	GDP	Budget Balance					
Countries	α	α_1	α_2	ρ_1	ρ_2	λ_f	λ_d
Nordic Council	0.0457	0.00727	0.0149	-0.6	-0.564	-0.00154	-0.000742
	(0.000)	(0.000)	(0.000)	(0.0004)	(0.0003)	(0.0001)	(0.0001)
Baltic Assembly	0.0504	0.0146	0.0112	-0.396	-1.14	-0.00112	-0.0012
	(0.000)	(0.000)	(0.000)	(0.0002)	(0.0005)	(0.0001)	(0.0001)
France	0.0316	0.0267	0.000201	-0.467	-0.617	-0.000986	-0.000981
	(0.000)	(0.000)	(0.000)	(0.0002)	(0.0002)	(0.0162)	(0.0000)
Germany	0.0518	0.0232	0.0141	-0.21	-0.931	-0.000794	-0.000865
	(0.000)	(0.000)	(0.000)	(0.0001)	(0.0004)	(0.0001)	(0.0000)
Greece	0.105	0.00369	0.0146	0.0447	-0.816	0.00133	0.000601
	(0.000)	(0.000)	(0.000)	(0.0000)	(0.0004)	(0.0173)	(0.0001)
Italy	0.0514	0.00764	0.00519	-0.432	-0.925	-0.000806	-0.0016
	(0.000)	(0.000)	(0.000)	(0.0002)	(0.0004)	(0.0000)	(0.0000)
Benelux	0.0544	0.0157	-0.000261	-0.386	-0.574	-0.000933	-0.00145
	(0.000)	(0.000)	(0.000)	(0.0786)	(0.0006)	(0.0002)	(0.0000)
Visegrad Group	0.0672	0.0193	0.00551	-0.864	-0.991	-0.00202	-0.00018
	(0.000)	(0.000)	(0.000)	(0.0003)	(0.0003)	(0.0001)	(0.0001)
Craiova Group	0.0983	0.0109	0.00044	-0.689	0.355	-0.00115	-0.000974
	(0.000)	(0.000)	(0.000)	(0.0006)	(0.0002)	(0.0777)	(0.1812)

Notes: Standard Errors in parentheses. The table shows 7 out of 13 estimated coefficients. The remaining 6 are the standard deviations of the trend processes and the measurement errors. Representative countries chosen for the respective supranational body are in order of appearance: Denmark, Estonia, Netherlands, Poland and Romania.

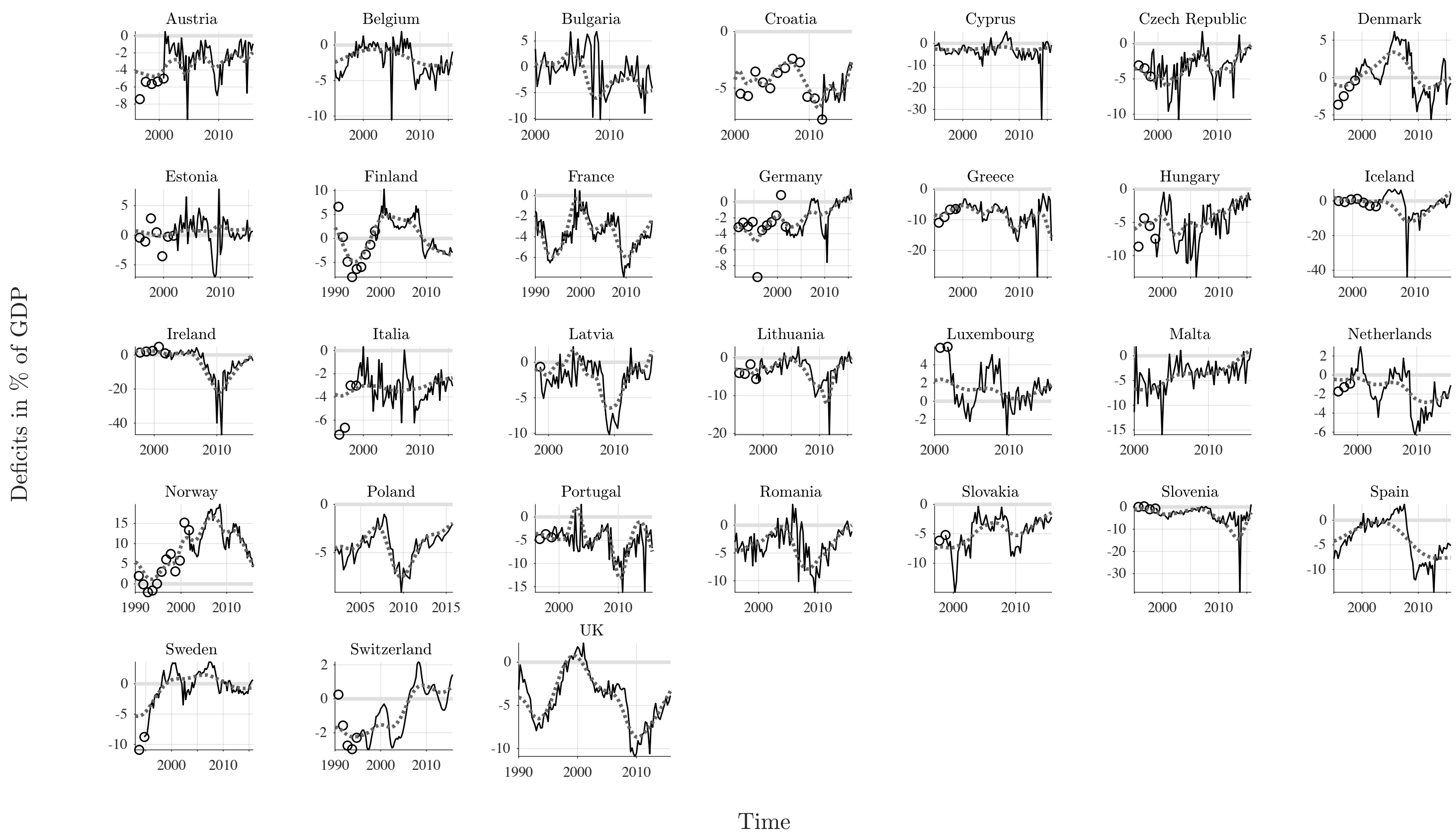

Figure 3.2. Structural Deficits (dashed) and Budget Balance (solid) in Europe until 2015 (Quarterly Data)

Notes: The figure shows quarterly, government budget balances in percent of seasonally-adjusted nominal GDP and its corresponding trend component, i.e. the structural budget balance. In case some parts of the respective country budget balance are only available at annual frequency, circles depict the annual budget balance value which by construction is equivalent with the value the budget balance would assume in the fourth quarter provided all data was available.

- *Potential or trend components*: An adjustment of fiscal policy in response to changing (long-run) economic conditions. Standard deviations of the budget balance's trend process is estimated using $(\sigma_{\varepsilon d},\ \sigma_{\eta d})$. The trend component of GDP or potential GDP is estimated using the following standard deviations: $(\sigma_{\varepsilon y},\ \sigma_{\eta y})$
- *Exogenous influences*: Policies unrelated to (direct) economic conditions such as political considerations, one-time events as crisis or the level of debt that affect the budget only via the amount of interests that need to be paid. The exogenous influence of the financial crisis and gross debt, (λ_f, λ_d) on the level of the budget balance ratio. Standard deviations of the measurement error $(\sigma_{\xi_1}\ \sigma_{\xi_2})$.

A table with full estimation results on all countries is provided in the Appendix in Table B.1.

The cycle-generating coefficients ρ_1, ρ_2 are consistently negative and highly significant for all countries. This is in line with the commonly accepted view of a stationary cycle, fluctuating around zero.[10] Positive coefficients for α and α_1 on the common underlying cycle suggest a countercyclical fiscal policy stance. In this representative table of results this is the case for all countries / supranational body representatives. The t-statistics are very significant in all cases. In combination with moderate deficits, a countercyclical fiscal policy stance smoothes business cycle fluctuations by supporting the effectiveness of automatic stabilizers (see Melitz (2000)). The coefficient α_2, relating the budget balance ratio to the lag of the business cycle is usually positive and smaller than α_1 however, for some countries it is also negative.

Following e.g. Fatás and Mihov (2010) or Alesina, Campante, and Tabellini (2008), the change in the fiscal stance can be highlighted through the correlation between the growth rate of the cyclically-adjusted budget balance ratio and the output gap. A positive correlation expresses a countercyclical and a negative a pro-cyclical policy stance. Figure 3.3 shows this for the subset of countries and the years 2005-2015. This approach generally confirms the countercyclical stance of fiscal policy in Europe. However, for that time period, the results for Estonia (Baltic Assembly) and Romania (Craiova Group) are puzzling, given that the estimated coefficients on the cycle (α_1, α_2) are both positive and thus suggest a countercyclical stance.[11] An explanation might be the state of development of both countries. Kumar and Ter-Minassian (2007) argue that the main causes for pro-cyclical fiscal policy stem from weak institutions, corruption, financial constraints and low confidence of foreign investors. Apart from that, the findings are in line with results of Fatás and Mihov (2010) who find that fiscal policy in Europe has become more countercyclical after 1999.

The cyclically-adjusted balance is displayed in Figure 3.2 - here for all countries. In most cases it is smooth, a feature of the general linear exponential smoothing model (ARIMA(0,1,2)) - assumption, underlying the trend components. Furthermore, especially in the case when only quarterly data was used, accounting for outliers further smooths results a bit.

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[10] Employing an augmented Dickey-Fuller test revealed that the null hypothesis of no unit root in the cycle cannot be rejected for 27 out of 31 countries, testing at a 5% level of significance.

[11] Boiciuc (2015) finds that fiscal policy in Romania has been pro-cyclical from the year 2000 onwards for all years but 2013.

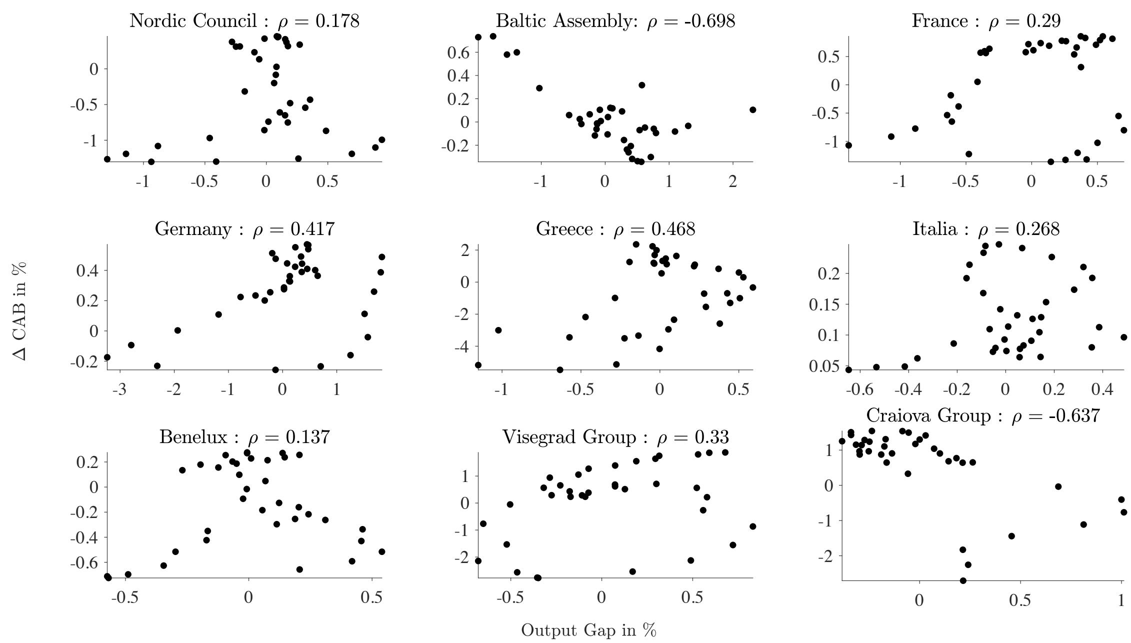

Figure 3.3. Fiscal Policy Stance and the Output Gap (2005-2015)

Notes: The figure shows the annual change of the quarterly, cyclically-adjusted budget balance and the Output Gap for the years 2005 to 2015. The correlation coefficient is an indicator for the stance of fiscal policy. Positive indicates a countercyclical and negative a pro-cyclical stance. Representative countries chosen for the respective supranational body are in order of appearance: Denmark, Estonia, Netherlands, Poland and Romania.

Two exogenous influences on the government budget balance are described via the coefficients λ_f which captures the influence of the European Debt Crisis (2008Q4-2012Q4) and λ_d which measures the impact of high states of (lagged) debt over GDP. The influence of a higher debt-to GDP ratio on the budget is negative and highly significant in 25 out of 29 countries where quarterly data on gross debt has been available in Euros. However, in Table 3.2 the reaction of Romania's (Craiova Group) budget balance to lagged debt has the same sign but is insignificant. In the case of Romania this makes sense, given that in Bulgaria, the second Craiova Group country, the reaction to lagged debt even displays a positive reaction. Furthermore, as non-Euro country, Romania is not affected by the ECB's policy. A negative sign of the reaction to debt implies that higher debt yesterday leads to a less positive balance or a higher deficits today and governments do not stabilize debt by cutting expenditures today. Hence, fiscal policy could be *active* in the definition of Leeper (1991)'s *Fiscal Theory of the Price Level*. A *passive* monetary policy which is needed to ensure a stable equilibrium is also in line with the behavior of the ECB in recent years. Furthermore, the magnitude of the reaction to debt does not differ tremendously and is rather small. This might stem from “debt brakes” anchored in the respective constitutions, especially with respect to the Fiscal Compact.

On the contrary, the additional influence the financial crisis had on the countries’ spending behavior is not so clear-cut. For 19 out of 31 countries I find an additional, significant impact of the crisis on the level of the budget balance ratio. Nevertheless, this additional effect has to be taken with a grain of salt. The economic significance of both coefficients - the reaction to debt and the financial crisis - is not too big which could be explained by the fact that sudden higher spending due to e.g. bank bail-out programs or economic recovery programs might be already captured by an increase in the structural deficit, although from a conceptual point of view this was not correct.

The next subsection compares the my findings with results that can be derived using alternative approaches as discussed in section 3.5.

3.6.2 The Fiscal Policy Stance In Comparison With Other Methods

As previous sections already pointed out, there is more than one unique road towards the “Rome” of monitoring public finances, the calculation of the cyclically-adjusted budget balance. Section 3.5 discussed two main approaches: 1. Cyclically-adjusted budget balances based on potential GDP derived from Kalman Filtering supply-side variables of the economy and estimating the tax code and social security benefit semi-elasticities to the output gap and 2. purely statistical approaches as the HP-Filter or Hamilton’s eight quarter detrending.

First, I propose to combine the estimated output gap (based on Kalman Filtering of Real GDP using the UCM discussed in section 3.4) with the 2005 and 2014 version of the budget sensitivities / semi-elasticities as calculated in Mourre, Astarita, and Princen (2014). This is an approximation, given that potential GDP in Mourre et al. (2013); Mourre, Astarita, and Princen (2014) is based on various supply-side variables. Second, I suggest to detrend quarterly real GDP using both the HP- and the Hamilton Filter and calculate the cyclically-adjusted budget balance as the difference between the observed or smoothed balance (in case of data limitations) and the product of the output gap and the budget elasticities. I will display my results graphically and in a table, summarizing the fiscal stance over different methodologies. Figure 3.4 displays the graphical approach towards this comparison. For the majority of countries / regional representatives the cyclically-adjusted budget balances calculated using alternative methodologies resemble each other, and are similar to the version calculated using the model in section 3.4 (black, dashed). However, in some cases and for single years, bigger derivations of course underline the difficulty in finding a plausible approach towards this trend-separation problem. For the case of the Baltic Assembly (Estonia), the huge derivation stems from a mean reversion between 2008 and 2010 as Figure 3.2 illustrates. Here, the outlier-robust version of the chapter’s Kalman filtering approach steps in and “produces” a smoother trend component. The relatively “bad” performance of Hamilton’s filter in some cases depends on the choice of the length of the underlying cycle.

Table 3.3 provides more quantitative evidence on the performance of other approaches relative to the UCM: The first six columns show the correlations of the respective alternative method

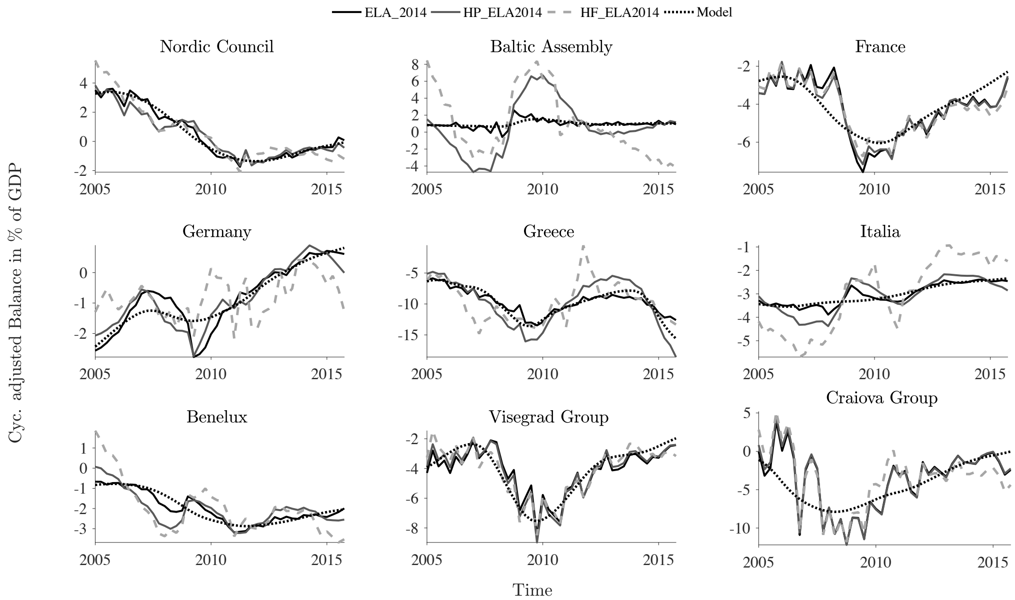

Figure 3.4. Comparing the CABs Using Different Methodologies, (2005-2015)

Notes: The graph depicts three versions of cyclically-adjusted budget (CAB) balances based on the 2014 budget semi-elasticity of Mourre, Astarita, and Princen (2014) and the CAB balance derived from the UC model for the years 2005-2015. Representative countries chosen for the respective supranational body are in order of appearance: Denmark, Estonia, Netherlands, Poland and Romania

with the approach chosen in this chapter. Correlations are very high and positive in all cases, though it is fair enough to admit that in case of the first to columns results might be biased towards the chapter's result by using the cyclical component of GDP calculated using the UCM. Especially when the ratio of α_1, α_2 and α comes close to the semi-elasticities calculated by Mourre, Astarita, and Princen (2014), results resemble each other strongly.

The degree of pro- or countercyclicality, i.e. the fiscal stance of the respective government is displayed in the last three columns of the table. Ω_α measures the percentage point change of the budget balance ratio for a 1 percent change in the GDP. Because the numerator of Ω_α is in levels and the denominator in logs, this ratio can also be interpreted as a semi-elasticity. This allows to compare Ω_α to the semi-elasticities presented in Mourre et al. (2013); Mourre, Astarita, and Princen (2014). A bit puzzling is the result that for some countries the semi-elasticities almost coincide, for other however, there is a big discrepancy. On the one hand, given the fact that both elasticities have been obtained using totally different approaches the results might not be perfectly comparable in all cases. On the other hand, as Table B.1 and Appendix B.1.2 show, the fiscal stance calculated using an UCM directly provides standard errors for the semi-elasticities. Nevertheless, the sign clearly indicates a countercyclical stance in all cases which is in line with the literature and earlier results found above.

Table 3.3. Comparing Methodologies across Subset of Countries, (2005-2015)

	Correlation of CAB with CAB of UC Model						Fiscal Stance of EU Approach		UC Model
Countries	(1) ELA_{2005}	(2) ELA_{2014}	(3) HP_{2005}	(4) HP_{2014}	(5) HF_{2005}	(6) HF_{2014}	Budget Sensitivity (2005)	Semi-Elasticity (2014)	Ω_α $[(\alpha_1 + \alpha_2)/(\alpha)]$
Nordic Council	0.99	0.98	0.97	0.94	0.94	0.91	0.49	0.58	0.48
Baltic Assembly	0.61	0.62	0.94	0.94	0.5	0.5	0.55	0.61	0.51
France	0.81	0.81	0.84	0.84	0.78	0.79	0.32	0.31	0.85
Germany	0.91	0.93	0.9	0.91	0.53	0.44	0.43	0.47	0.72
Greece	0.96	0.94	0.95	0.94	0.66	0.58	0.45	0.52	0.17
Italy	0.93	0.9	0.79	0.75	0.87	0.86	0.39	0.43	0.25
Benelux	0.93	0.93	0.67	0.66	0.53	0.52	0.61	0.62	0.28
Visegrad Group	0.86	0.85	0.89	0.89	0.9	0.88	0.3	0.44	0.37
Craiova Group	0.66	0.66	0.66	0.66	0.51	0.5	0.53	0.57	0.12

Notes: The table reports correlation coefficients of six cyclically-adjusted budget balances calculated using different approaches towards obtaining potential GDP and the reaction of the budget to the cycle. Furthermore, indicators of the fiscal policy stance are reported using the budget sensitivity / semi-elasticity of Mourre et al. (2013); Mourre, Astarita, and Princen (2014) and coefficients estimated from the UCM. For details see section 3.6.2. Representative countries chosen for the respective supranational body are in order of appearance: Sweden, Estonia, Netherlands, Poland and Romania.

3.6.3 Safety Margins

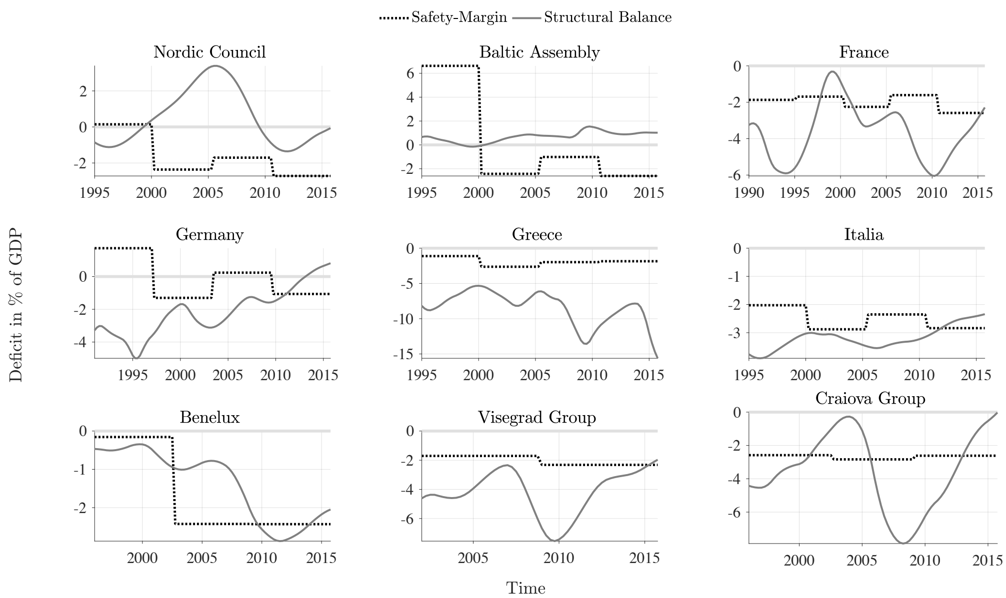

Figure 3.5. Structural Deficits (solid) and Medium-Term Objective (dashed) in Europe until 2015 (Quarterly Data)

Notes: The figure shows quarterly, structural budget balances in percent of seasonally-adjusted nominal GDP (solid) and a medium-term-objective (MTO) of fiscal policy (dashed) which changes around a five-year window. The MTO is defined as the maximum value discretionary deficits can assume such that the 3% Maastricht deficit threshold is never exceeded over the cycle. Representative countries chosen for the respective supranational body are in order of appearance: Denmark, Estonia, Netherlands, Poland and Romania

The last subsection of this chapter analyses a fiscal policy evaluation tool which is also part of the EU treaties. Starting from the reformed version of the Stability and Growth Pact in 2005, so-called "safety-margins" and "medium-term objectives" (MTO) play an important role in the evaluation whether a country runs an excessive deficit or not. According to the treaties countries may have a maximum structural deficit of 1% (revised 2005 version of the SGP) and a maximum structural deficit of 0.5% of GDP according to the Fiscal Compact which constitutes the MTO in order to guarantee sustainable debt.[12] Furthermore, all countries that have not yet reached their MTO should consolidate discretionary spending by a minimum of 0.5% each year until the MTO is reached.

Table 3.4 regards so-called safety margins, i.e. the maximum value discretionary spending can assume such that the 3% Maastricht deficit threshold is never exceeded over the cycle. It

[12] Structural Deficits higher than 0.5% but below 1% are possible if the respective country's debt is below 40% of GDP.

is calculated as the three percent threshold deficit minus the minimum cyclical component of the budget balance ratio, i.e. it is a worst-recession margin. Calculation over the entire sample does not really make sense as the financial crisis is a huge outlier and provides a too negative picture of the state of the economy. The idea is to calculate the safety margins for a five-year window, such that it is possible to evaluate structural spending within this episode. This allows for variation due to huge outliers as e.g. the recent crisis. The regarded periods correspond to the periods regarded in the summary statistics. Furthermore, Table 3.4 depicts average figures for the structural balance in the respective time period. Figure 3.5 plots the the safety margins within a five-year window and the corresponding structural deficit.

For a majority of countries, structural expenses are not within or above the safety margins that guarantee a compliance to the Maastricht criteria. Furthermore, especially for the periods 2005-2008 and 2008 until 2015, when the revised SGP and later the Fiscal Compact where signed and ratified, almost all countries (except Germany and the Sweden) are far away from their MTO. This can be regarded as a consequence of the recent crisis which has been the worst recession since the Great Depression. However, the figure shows more than that. Structural deficits declined over the last years and many countries are on a convergence path towards the respective safety margin. This is a result of the austerity programs that included harsh consolidation and structural reform obligations following the bailout by the rescue funds ESFS or ESM. Nonetheless, the country-specific safety margin are lower than the maximum structural deficit in the MTO agreed upon in the SGP for most countries, i.e. more consolidation is needed. This is especially important as public finances of most European countries are challenged by a high stock of debt and an aging population. Moreover, the figure shows that many countries became fatigue to reform their budgets at least after joining the EMU, as we see structural deficits increase already around the year 2000.

What is the conclusion of these results? These results highlight that fiscal discipline, which should have been imposed by the Maastricht Treaty or Stability and Growth Pact (and the numerous successor treaties) did not exist. Average figures, overall (as in the summary statistics) and structural failed to be in compliance with the Maastricht criteria over almost the entire sample. It has not been the Franco-German complicity with assistance of the Italian presidency that undermined the Stability and Growth Pact and provided bad incentives. Compared to annual or even quarterly figures, using averages here allows to assess the performance of the structural deficit over years of boom and recession. Structural spending, especially in good times has been too high and has left little room for fiscal policy when the tide turned. This confirms the result of Buti, Franco, and Ongena (1998), mentioned in the introduction. The frequent violation of the Stability and Growth Pact and the political "necessity" to weaken its criteria and its enforceability in 2005 are the sad consequence of this story and pose a big threat to the sustainability of public finances in Europe.

Table 3.4. Safety Margins and the Structural Deficit for a Subset of Countries

Countries	Average Safety Margin				Average Struct. Balance			
	1995-2000	2000-2005	2005-2008	2008-2015	1995-2000	2000-2005	2005-2008	2008-2015
Nordic Council	0.144	-2.37	-1.71	-2.72	-0.59	1.95	1.49	-0.855
Baltic Assembly	-2.43	-2.43	-1.01	-2.6	0.238	0.539	0.982	1.01
France	-1.69	-1.69	-2.25	-1.61	-4.83	-2.32	-2.75	-4.39
Germany	-1.3	-1.3	0.237	-1.06	-3.86	-2.53	-1.9	-0.268
Greece	-2.63	-2.63	-1.98	-1.85	-7.05	-6.83	-9.8	-10
Italy	-2.88	-2.88	-2.35	-2.84	-3.53	-3.19	-3.37	-2.64
Benelux	-2.43	-2.43	-2.43	-2.43	-0.539	-1.11	-2.54	-2.54
Visegrad Group	-	-1.71	-2.32	-2.32	-	-3.99	-4.5	-4.5
Craiova Group	-2.84	-2.84	-2.62	-2.62	-3.14	-4.01	-3.39	-3.39

Notes: Safety Margins are defined as the three percent threshold deficit minus the minimum cyclical component of the budget balance ratio. Representative countries chosen for the respective supranational body are in order of appearance: Denmark, Estonia, Netherlands, Poland and Romania.

3.7 Conclusion

This chapter provided new quantitative evidence on almost 20 years of European government spending under the Stability and Growth Pact and its successor treaties. Using an unobserved components model (UCM) and the Kalman Filter, the present chapter estimated structural deficit series for 31 European countries between 1995 and 2015. Comparing these values against the 3% Maastricht benchmark reveals that the Stability and Growth Pact (SGP) has not really been taken seriously by many European governments. Political decisions to vote down a pending excessive deficit procedure as in the case of France and Germany in 2003 or Spain and Portugal last year, do not improve the popularity of these treaties in the population. Furthermore, the chapter assessed the stance of European fiscal policy. In accordance with earlier research, this chapter documents that European fiscal policy is in general strongly countercyclical. Moreover, the degree of countercyclicality varies between the countries. This is less pronounced with respect to the reaction of fiscal policy to gross debt. European fiscal policy is very active in the definition of the FTPL. Surprisingly, this does not differ too much among European countries. The economic significance of that effect is small however, given that a lot of European countries have anchored so-called debt brakes in their constitutions in line with the Fiscal Compact. Using safety-margins, i.e. values the structural deficit can assume to stay within the 3% boundary, reveals that discretionary and not business cycle-driven spending has been the driver of deficits in many European countries. Finally, the chapter provides quarterly series of the structural and the primary structural deficit which can be used for policy analysis or can be set into perspective with other methods to obtain these figures.

Table B.1. Full Estimation Results

Countries	α	α_1	α_2	Ω_α	ρ_1	ρ_2	λ_f	λ_d
Austria	0.14	0.041	-0.0109	0.216	0.489	-0.0261	-0.000911	-0.00188
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0003)	(0.0001)	(0.0016)	(0.0001)
Belgium	0.139	0.0311	0.0118	0.3089	-0.723	0.0958	-0.000393	-0.000532
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0008)	(0.0001)	(0.0060)	(0.0000)
Bulgaria	0.122	0.0217	0.0295	0.4219	0.416	-1.01	0.000594	0.00192
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0000)	(0.0000)	(0.0003)	(0.0000)
Croatia	0.0307	0.0201	-0.0143	0.1876	-0.464	-0.645	-0.003	-0.00332
	(0.000)	(0.000)	(0.000)	(0.001)	(0.0003)	(0.0004)	(0.0001)	(0.0001)
Cyprus	0.163	0.0171	-0.00661	0.0641	-0.0803	-1	-0.0041	-0.000381
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Czech Republic	0.0342	0.000452	-0.0111	-0.3099	-0.651	-0.43	-0.0029	-0.00189
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0005)	(0.0003)	(0.0001)	(0.2793)
Denmark	0.0457	0.00727	0.0149	0.485	-0.6	-0.564	-0.00154	-0.000742
	(0.000)	(0.000)	(0.000)	(0.001)	(0.0004)	(0.0003)	(0.0001)	(0.0001)
Estonia	0.0504	0.0146	0.0112	0.5119	-0.396	-1.14	-0.00112	-0.0012
	(0.000)	(0.000)	(0.000)	(0.001)	(0.0002)	(0.0005)	(0.0001)	(0.0001)
Finland	0.0619	0.0211	0.0145	0.5756	-0.279	-0.763	-0.000126	-0.000923
	(0.000)	(0.000)	(0.000)	(0.001)	(0.0001)	(0.0003)	(0.0204)	(0.0002)
France	0.0316	0.0267	0.000201	0.8486	-0.467	-0.617	-0.000986	-0.000981
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0002)	(0.0002)	(0.0162)	(0.0000)
Germany	0.0518	0.0232	0.0141	0.7197	-0.21	-0.931	-0.000794	-0.000865
	(0.000)	(0.000)	(0.000)	(0.001)	(0.0001)	(0.0004)	(0.0001)	(0.0000)
Greece	0.105	0.00369	0.0146	0.1738	0.0447	-0.816	0.00133	0.000601
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0000)	(0.0004)	(0.0173)	(0.0001)
Hungary	0.0562	0.0172	-0.00956	0.1355	-0.401	0.0241	-0.00239	-0.00476
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0001)	(0.0001)	(0.0001)	(0.0000)
Iceland	0.0767	0.00992	-0.00236	0.0986	-0.294	-0.94	-0.000616	-
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0001)	(0.0005)	(0.0001)	-

Countries	α	α_1	α_2	Ω_α	ρ_1	ρ_2	λ_f	λ_d
Ireland	0.0339	0.021	0.0085	0.8702	-0.502	-0.709	-0.000772	-0.00105
	(0.000)	(0.000)	(0.000)	(0.002)	(0.0003)	(0.0003)	(0.0001)	(0.0001)
Italy	0.0514	0.00764	0.00519	0.2497	-0.432	-0.925	-0.000806	-0.0016
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0002)	(0.0004)	(0.0000)	(0.0000)
Latvia	0.0438	0.0152	-0.0133	0.04392	-0.444	-0.745	-0.00292	-0.00388
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0004)	(0.0006)	(0.1161)	(0.0001)
Lithuania	0.0878	0.0157	0.0019	0.2005	-0.567	-0.921	-0.000983	-0.00161
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0003)	(0.0005)	(0.0001)	(0.0001)
Luxembourg	0.0379	0.0231	-0.00264	0.539	-0.331	-0.983	-0.000658	-0.000866
	(0.000)	(0.000)	(0.000)	(0.002)	(0.0003)	(0.0009)	(0.0031)	(0.0001)
Malta	0.0433	0.0176	0.0126	0.699	-0.383	-0.722	-0.00105	-0.000834
	(0.000)	(0.000)	(0.000)	(0.001)	(0.0003)	(0.0004)	(0.0000)	(0.0001)
Netherlands	0.0544	0.0157	-0.000261	0.284	-0.386	-0.574	-0.000933	-0.00145
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0786)	(0.0006)	(0.0002)	(0.0000)
Norway	0.122	0.0177	0.0239	0.341	0.0978	0.0385	0.0015	-0.00141
	(0.000)	(0.000)	(0.000)	(0.001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Poland	0.0672	0.0193	0.00551	0.369	-0.864	-0.991	-0.00202	-0.00018
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0003)	(0.0003)	(0.0001)	(0.0001)
Portugal	0.0421	0.0191	-0.0176	0.0356	-0.507	-0.301	-0.00311	-0.00345
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0005)	(0.0003)	(0.0001)	(0.0001)
Romania	0.0983	0.0109	0.00044	0.116	-0.689	0.355	-0.00115	-0.000974
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0006)	(0.0002)	(0.0777)	(0.1812)
Slovakia	0.0284	0.0215	0.00948	1.09	-0.822	-0.729	-0.00105	-0.000629
	(0.000)	(0.000)	(0.000)	(0.003)	(0.0005)	(0.0004)	(0.0109)	(0.0001)
Slovenia	0.0399	0.0083	0.02	0.71	-1.03	-0.267	-0.00105	-0.00186
	(0.000)	(0.000)	(0.000)	(0.001)	(0.0004)	(0.0001)	(0.0065)	(0.0001)
Spain	0.115	0.00704	-0.0103	-0.0282	-0.0485	-0.541	-0.00429	-0.00265
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0001)	(0.0003)	(0.0004)	(0.0002)
Sweden	0.0357	0.0292	0.00894	1.07	-0.119	-0.339	-0.000846	-0.00128
	(0.000)	(0.000)	(0.000)	(0.001)	(0.0000)	(0.0001)	(0.0059)	(0.0068)
Switzerland	0.0823	0.0136	0.0141	0.337	-0.387	-0.592	-0.000845	-
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0002)	(0.0004)	(0.0038)	-
UK	0.0466	0.0286	0.0108	0.845	-0.389	-1	-0.000679	-0.000614
	(0.000)	(0.000)	(0.000)	(0.000)	(0.0000)	(0.0002)	(0.0276)	(0.0275)

Notes: Standard Erros in parentheses. The table shows 7 out of 13 estimated coefficeints. The remaining 6 are the standard deviations of the trend processes and the measurement errors.

B.1 Specification Results

B.1.1 Ljung-Box-Statistics

The model's specification is discussed via Ljung-Box statistics. In a correctly specified model the correlation structure is rich enough to absorb all serial correlation in standardized prediction error residuals, computed from the Kalman Filter. Stated otherwise, a model is well specified when it is not possible to reject the null hypothesis of no autocorrelation in the residuals. Following Camba-Mendez and Lamo (2004), statistics are reported for the first 4 and 6 autocorrelations in Table B.2. In the overall evaluation, the model does fairly well and seems a good approximation towards quarterly figures in European public finance.

B.1.2 Variance of Ratio of Parameters

Derivation of the standard error of the ratio of three estimated coefficients: $\frac{\alpha_1+\alpha_2}{\alpha}$ For any known variance-covariance matrix of θ which contains among others the coefficients $\alpha, \alpha_1, \alpha_2$ it is straightforward to calculate individual standard errors following an maximum likelihood estimation of the parameters. However, from an economic perspective I am interested in the combination of those coefficients as given by $\frac{\alpha_1+\alpha_2}{\alpha}$. Hence, the calculation of the standard errors needs to account for potential covariation between the estimated coefficients. This can be done using a first- and second order Taylor-series expansion evaluated at the expected value of the random variables.

The first step combines $x = \alpha_1 + \alpha_2$ to a new random variable with expected value:

$$E(x) = E(\alpha_1 + \alpha_2) = \alpha_1 + \alpha_2 \tag{B.1}$$

$$VAR(x) = VAR(\alpha_1 + \alpha_2) = VAR(\alpha_1) + VAR(\alpha_2) + 2COV(\alpha_1 + \alpha_2) \tag{B.2}$$

Hence, we end up with a function $G = g(x, \alpha) = \frac{x}{\alpha}$. Let $c = (E(\alpha), E(x))$. The first-order approximation is then given by:

$$g(x, \alpha) \approx g(c) + g'_\alpha(c)(\alpha - E(\alpha)) + g'_x(c)(x - E(x)) + O(n^{-1}) \tag{B.3}$$

Taking expectations on both sides gives:

$$\begin{aligned}
E(g(x,\alpha)) &= E\left[g(c) + g'_\alpha(c)(\alpha - E(\alpha)) + g'_x(c)(x - E(x)) + O(n^{-1})\right] && \text{(B.4)}\\
&\approx E[g(c)] + g'_\alpha(c)E[\alpha - E(\alpha)] + g'_x(c)E[x - E(x)] && \text{(B.5)}\\
&= E[g(c)] + 0 + 0 && \text{(B.6)}\\
&\approx g(E(x), E(\alpha)) = \frac{E(x)}{E(\alpha)} && \text{(B.7)}
\end{aligned}$$

Using this result, the second-order expansion is:

$$\begin{aligned} g(x,\alpha) &= g(c) + g'_\alpha(c)(\alpha - E(\alpha)) + g'_x(c)(x - E(x)) \\ &\quad + \frac{1}{2}\Big\{g''_{\alpha\alpha}(c)(\alpha - E(\alpha))^2 + 2g''_{\alpha x}(c)(\alpha - E(\alpha))(x - E(x)) + g''_{xx}(c)(x - E(x))^2)\Big\} + O(n^{-2}) \end{aligned} \tag{B.8}$$

such that by taking expectations on both sides again we obtain:

$$E(g(x,\alpha)) = g(c) + \frac{1}{2}\Big\{g''_{\alpha\alpha}(c)VAR(\alpha) + 2g''_{\alpha x}(c)COV(\alpha,x) + g''_{xx}(c)VAR(x)\Big\} \tag{B.9}$$

Using that g is defined as $g = x/\alpha$, $g''_{xx} = 0$, and $g''_{x\alpha} = -\alpha^{-2}$ and $g''_{\alpha\alpha} = 2x/\alpha^3$. Thus,

$$E(x/\alpha) \equiv E(g(x,\alpha)) \approx \frac{E(x)}{E(\alpha)} - \frac{COV(\alpha,x)}{E(\alpha)^2} + \frac{2E(x)VAR(\alpha)}{E(\alpha)^3} \tag{B.10}$$

Using the definition of the variance and that $E(g(x,\alpha)) \approx g(c)$, the variance of $g(x,\alpha)$ is given by:

$$VAR(g(\alpha,x)) \approx E\Big\{[g(\alpha,x) - g(c)]^2\Big\} \tag{B.11}$$

Now, using the first-order Taylor expansion for $g(x,\alpha)$ expanded at c gives:

$$VAR(g(x,\alpha)) \approx E\Big\{\big[g(c) + g'_\alpha(c)(\alpha - E(\alpha)) + g'_x(c)(x - E(x)) - g(c)\big]^2\Big\} \tag{B.12}$$

$$= g'^2_\alpha(c)VAR(\alpha) + 2g'_\alpha(c)g'_x(c)COV(\alpha,x) + g'^2_x(c)VAR(x) \tag{B.13}$$

Now returning to our ratio $g(x,\alpha) = x/\alpha$ we end up with

$$VAR(g(x,\alpha)) \equiv VAR\Big(\frac{x}{\alpha}\Big) \approx \frac{1}{E(\alpha)^2}VAR(x) + 2\frac{E(x)}{E(\alpha)^3}COV(\alpha,x) + \frac{E(x)^2}{E(\alpha)^4}VAR(\alpha) \tag{B.14}$$

$$= \frac{E(x)^2}{E(\alpha)^2}\left[\frac{VAR(x)}{E(x)^2} - 2\frac{COV(\alpha,x)}{E(x)E(\alpha)} + \frac{VAR(\alpha)}{E(\alpha)^2}\right] \tag{B.15}$$

Under the assumption that the estimated coefficient and its expected value coincide (how should this be different when not working from a Bayesian perspective) and assuming that the covariance between α, x is equal to the $COV(\alpha,\alpha_1) + COV(\alpha,\alpha_2)$ we end up with:

$$VAR\Big(\frac{\alpha_1 + \alpha_2}{\alpha}\Big) = \frac{(\alpha_1 + \alpha_2)^2}{\alpha^2}\left[\frac{\sigma^2_\alpha}{\alpha^2} - 2 \times \frac{COV(\alpha,(\alpha_1,\alpha_2))}{\alpha(\alpha_1 + \alpha_2)} + \frac{\sigma^2_{\alpha_1+\alpha_2}}{(\alpha_1 + \alpha_2)^2}\right] \tag{B.16}$$

The standard error is the respective square root of this expression.

Table B.2. Ljung-Box Test-Statistics

Countries	GDP		Balance	
	Q(4)	Q(6)	Q(4)	Q(6)
Belgium	0.598	0.736	4.34	13.8
	(0.897)	(0.981)	(0.227)	(0.0169)
Bulgaria	3.42	3.68	1.79	2.14
	(0.331)	(0.596)	(0.618)	(0.829)
Czech Republic	0.336	0.55	0.336	0.55
	(0.953)	(0.99)	(0.953)	(0.99)
Denmark	5.78	9.89	1.56	2.01
	(0.123)	(0.0785)	(0.67)	(0.848)
Germany	0.238	0.379	0.238	0.379
	(0.971)	(0.996)	(0.971)	(0.996)
Estonia	8.56	12.8	3.56	7.81
	(0.0357)	(0.0251)	(0.314)	(0.167)
Ireland	2.65	2.66	1.65	1.69
	(0.448)	(0.752)	(0.649)	(0.89)
Greece	7.97	8.73	4.98	5.58
	(0.0467)	(0.12)	(0.173)	(0.349)
Spain	11.6	12.4	6.46	7.06
	(0.00286)	(0.0154)	(0.0913)	(0.216)
France	3.99	4.96	2.91	3.41
	(0.262)	(0.421)	(0.405)	(0.637)
Croatia	6.95	12	7.11	13.7
	(0.0736)	(0.0354)	(0.0685)	(0.0175)
Italy	13.4	14.6	5.24	6.19
	(0.00391)	(0.0121)	(0.155)	(0.288)
Cyprus	3.09	3.19	1.87	2.29
	(0.378)	(0.671)	(0.6)	(0.807)
Latvia	5.52	6.07	1.02	1.58
	(0.137)	(0.3)	(0.797)	(0.903)
Lithuania	1.9	2.52	3.5	4.55
	(0.594)	(0.773)	(0.32)	(0.473)
Luxembourg	4.52	4.56	10.4	10.4
	(0.21)	(0.471)	(0.0157)	(0.0641)
Hungary	3.06	5.28	2.61	3.64
	(0.382)	(0.383)	(0.455)	(0.602)
Malta	7.6	7.72	8.55	8.66
	(0.055)	(0.172)	(0.036)	(0.124)

Countries	GDP		Balance	
	Q(4)	Q(6)	Q(4)	Q(6)
Netherlands	4.44	12.6	0.686	0.946
	(0.218)	(0.027)	(0.877)	(0.967)
Austria	1.58	2.45	2.68	3.23
	(0.664)	(0.783)	(0.443)	(0.665)
Poland	2.22	2.56	1.76	1.92
	(0.528)	(0.767)	(0.623)	(0.86)
Portugal	5.03	5.41	0.417	2.36
	(0.17)	(0.368)	(0.937)	(0.797)
Romania	1.23	5.59	2.35	2.88
	(0.745)	(0.349)	(0.502)	(0.718)
Slovenia	7.37	9.19	2.62	3.68
	(0.0611)	(0.102)	(0.453)	(0.596)
Slovakia	12.5	13.9	3.77	4.09
	(0.00578)	(0.0165)	(0.288)	(0.536)
Finland	1.93	2.14	5.23	5.38
	(0.586)	(0.829)	(0.156)	(0.372)
Sweden	5.02	5.1	0.256	0.409
	(0.17)	(0.403)	(0.968)	(0.995)
UK	11.4	12.2	4.81	9.84
	(0.00988)	(0.032)	(0.186)	(0.0799)
Iceland	0.31	4.5	2.06	4.4
	(0.958)	(0.48)	(0.56)	(0.493)
Norway	7.03	7.77	2.24	4.86
	(0.0709)	(0.17)	(0.525)	(0.433)
Switzerland	2.27	2.42	3.69	3.77
	(0.517)	(0.788)	(0.296)	(0.583)

Notes: P-Values in parentheses. The table displays Ljung-Box Test statistics using 4 and 6 lags. An insignificant test statistic implies that the null hypothesis of no autocorrelation in the residuals cannot be rejected and the model is well-specified.

B.1.3 Critical Repraisal of the Estimation Methodology

The appendix section on specification issues concludes with a critical acknowledgment of the employed method, the available data and the question at hand. The estimation strategy this chapter uses builds on widely accepted and state-of the art time series methods. Compared to ad-hoc or two-step approaches as e.g. the HP-Filter or the estimation of semi-elasticities, the structural vector autoregression (SVAR) approach is a direct one. Of advantage is that the SVAR approach does not need to deal with additional uncertainty from measurement errors and estimation errors of the fiscal elasticities. It accounts for them directly. Furthermore, the semi-elasticities are not necessarily time-constant and the two-step approach does not take variations into account. The direct approach chosen here does that by assuming that the cycle itself is a stochastic process. Nevertheless, some drawbacks of the underlying model are worth mentioning. For the countries where quarterly government data is still relatively scarce, estimation at an annual frequency covering 15 years for 13 coefficients can be problematic. The short horizon and the enormous state-space may lead to likelihood functions which have no unique (global) maximum and are not very well-defined. Additionally, the Kalman Filter Gain might not converge against its steady state value. The formulation of prior beliefs about the distribution of the coefficients and estimation within a Bayesian framework should improve results, especially with respect to probability boundaries obtained from Monte Carlo simulation. In addition, severe structural breaks affect the budget balances. Although the present model accounts for country-specific outlier observations via the Mahalanobis distance, it would be preferable to model them directly. However, that increased the number of coefficients to be estimated only further. For big trend reversals that affected almost all countries equally as e.g. the European Debt crisis an additional indicator variable solves this issue.

B.2 Deriving the Model

B.2.1 Basic Setup

This variant is based on the model by Camba-Mendez and Lamo (2004). The initial state-space setup looks as follows: The measurement equation is given by

$$\underbrace{\begin{pmatrix} y_{t,q} \\ d_{t,q} \end{pmatrix}}_{x_{t,q}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 1 & 0 & \alpha_1 & \alpha_2 \end{bmatrix}}_{A} \underbrace{\begin{pmatrix} \mu^y_{t,q} \\ \beta^y_{t,q} \\ \mu^d_{t,q} \\ \beta^d_{t,q} \\ \psi_{t,q} \\ \psi_{t,q-1} \end{pmatrix}}_{s_{t,q}} + \underbrace{\begin{bmatrix} 0 & 0 \\ \lambda_f & \lambda_d \end{bmatrix}}_{B} \underbrace{\begin{pmatrix} \eta^f_{t,q} \\ \eta^d_{t,q} \end{pmatrix}}_{z_{t,q}} + \underbrace{\begin{pmatrix} \zeta_{1t,q} \\ \zeta_{2t,q} \end{pmatrix}}_{\varepsilon_{t,q}}, \tag{B.17}$$

with

$$\Sigma_{\varepsilon\varepsilon} = \begin{bmatrix} \sigma^2_{\zeta_1} & 0 \\ 0 & \sigma^2_{\zeta_2} \end{bmatrix}$$

and the state equation is given by:

$$\underbrace{\begin{pmatrix} \mu^y_{t,q} \\ \beta^y_{t,q} \\ \mu^d_{t,q} \\ \beta^d_{t,q} \\ \psi_{t,q} \\ \psi_{t,q-1} \end{pmatrix}}_{s_{t,q}} = \underbrace{\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_1 & \rho_2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{C} \underbrace{\begin{pmatrix} \mu^y_{t,q-1} \\ \beta^y_{t,q-1} \\ \mu^d_{t,q-1} \\ \beta^d_{t,q-1} \\ \psi_{t,q-1} \\ \psi_{t,q-2} \end{pmatrix}}_{s_{t,q-1}} + \underbrace{\begin{pmatrix} \varepsilon^y_{t,q} \\ \nu^y_{t,q} \\ \varepsilon^d_{t,q} \\ \nu^d_{t,q} \\ \zeta_{3t,q} \\ 0 \end{pmatrix}}_{e_{t,q}} \tag{B.18}$$

with

$$\Sigma_{ee} = \begin{bmatrix} \sigma^2_{\varepsilon y} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{\nu y} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\varepsilon d} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{\nu d} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Both matrices, Σ_{ee} and $\Sigma_{\epsilon\epsilon}$ are positive-definite and bounded. If the estimated value of $\sigma_{\nu i}$ is very small, the ARIMA(0,2,1) trend components reduce to random walks with drift.

B.2.1.1 Building Quarterly Series out of Annual Data

For convenience, the state-space representation derived above is stated again:

$$x_{t,q} = \mathbf{A}s_{t,q} + \mathbf{B}\eta^d_{t,q} + \varepsilon_{t,q} \tag{B.19}$$

$$s_{t,q} = \mathbf{C}s_{t,q-1} + e_{t,q} \tag{B.20}$$

Remember that for some countries $y_{t,q}$ is observed every whereas $d_{t,q}$ is unobserved only every fourth quarter. You can account for these missing observations by applying a variable transformation described in what follows. For this purpose an additional variable is defined, which cumulates quarterly observed GDP values and weights the annually observed deficit numbers:

$$x^a_{t,j} = \sum_{i=1}^{j} \mathbf{W}_{t,i} x_{t,i},$$

where

$$\mathbf{W}_{t,i} = \begin{bmatrix} 1 & 0 \\ 0 & w_{t,i} \end{bmatrix} \qquad \text{and} \qquad w_{t,j} = \frac{Y^n_{t,j}}{\sum_{i=1}^{4} Y^n_{t,i}}$$

For the respective fourth quarter, we now observe (cumulated quarterly) values for GDP and the annual budget balance. It remains to integrate this auxiliary variable in the state-space setup described above. By using that $x^a_{t,j+1} = \Psi_u I x^a_{t,j} + \mathbf{W}_{t,j+1} x_{t,j+1}$ it is possible to rewrite the state-space system in the following way:

$$\begin{aligned} x^a_{t,q} &= \Psi_u I x^a_{t,q-1} + \mathbf{W}_{t,q}\left(\mathbf{AC}s_{t,q-1} + \mathbf{B}\eta^d_{t,q} + \mathbf{A}e_{t,q} + \varepsilon_{t,q}\right) \\ s_{t,q} &= \mathbf{C}s_{t,q-1} + e_{t,q}, \end{aligned}$$

where Ψ_q is an indicator equal to 0 if we are in the first quarter and equal to 1 else. Stacking both equations into one system and defining a new selection matrix for the augmented observation equation gives:

$$x^a_{t,q} = \mathbf{Z}' q_{t,q},$$

with $\mathbf{Z}' = [\mathbf{I} \quad \mathbf{0}]$ and as augmented transition equation

$$\underbrace{\begin{bmatrix} x^a_{t,q} \\ s_{t,q} \end{bmatrix}}_{q_{t,q}} = \underbrace{\begin{bmatrix} \Psi_u I & \mathbf{W}_{t,q}\mathbf{AC} \\ \mathbf{0} & \mathbf{C} \end{bmatrix}}_{\mathbf{N}_{t,q}} \underbrace{\begin{bmatrix} x^a_{t,q-1} \\ s_{t,q-1} \end{bmatrix}}_{q_{t,q-1}} + \underbrace{\begin{bmatrix} \mathbf{W}_{t,q}\mathbf{B} \\ \mathbf{0} \end{bmatrix}}_{\mathbf{M}_{t,q}} \underbrace{\begin{bmatrix} (\eta^f_{t,q}, \eta^d_{t,q})' \\ \mathbf{0} \end{bmatrix}}_{z_{t,q}} + \underbrace{\begin{bmatrix} \mathbf{W}_{t,q} & \mathbf{W}_{t,q}\mathbf{A} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}}_{\mathbf{R}_{t,q}} \underbrace{\begin{bmatrix} \varepsilon_{t,q} \\ e_{t,q} \end{bmatrix}}_{v_{t,q}} \tag{B.21}$$

where the combined shock process has zero mean and covariance matrix:

$$\Sigma = \begin{bmatrix} \Sigma_{\varepsilon\varepsilon} & \mathbf{0} \\ \mathbf{0} & \Sigma_{ee} \end{bmatrix}$$

The new state-space system at quarterly frequency reads as follows:

$$\begin{aligned} x^a_{t,q} &= \mathbf{Z}' q_{t,q} \\ q_{t,q} &= \mathbf{N}_{t,q} q_{t,q-1} + \mathbf{M}_{t,q} z_{t,q} + \mathbf{R}_{t,q} v_{t,q} \\ \mathbf{Q} &= \mathbf{R}_{t,q} v_{t,q} v'_{t,q} \mathbf{R}'_{t,q} = \mathbf{R}_{t,q} \Sigma \mathbf{R}'_{t,q} \end{aligned} \tag{B.22}$$

B.2.2 State Space Transformation into Annual Frequency

Finally, the joint quarterly state-space system is transformed in a way such that the model parameters can be estimated at an fully observed annual frequency. By construction, the auxiliary variable, $x^a_{t,4}$ displays the observed (annual) deficit value and cumulated real GDP, every fourth quarter of a year. By exploiting the recursive structure of the system, it is possible to construct the respective annual system matrices out of the quarterly frequency representation:

$$\begin{aligned} q_{t,4} &= \mathbf{N}_{t,4} q_{t,3} + \mathbf{M}_{t,4} z_{t,4} + \mathbf{R}_{t,4} v_{t,4} \iff \\ q_{t,4} &= \mathbf{N}_{t,4}\mathbf{N}_{t,3} q_{t,2} + \mathbf{N}_{t,4}\mathbf{M}_{t,3} z_{t,3} + \mathbf{N}_{t,4}\mathbf{R}_{t,3} v_{t,3} + \mathbf{M}_{t,4} z_{t,4} + \mathbf{R}_{t,4} v_{t,4} \iff \\ &\ldots \\ q_{t,4} &= \mathbf{N}_{t,4}\mathbf{N}_{t,3}\mathbf{N}_{t,2}\mathbf{N}_{t,1} q_{t-1,4} + \mathbf{N}_{t,4}\mathbf{N}_{t,3}\mathbf{N}_{t,2}\mathbf{M}_{t,1} z_{t,1} + \mathbf{N}_{t,4}\mathbf{N}_{t,3}\mathbf{N}_{t,2}\mathbf{R}_{t,1} v_{t,1} + \mathbf{N}_{t,4}\mathbf{N}_{t,3}\mathbf{M}_{t,2} z_{t,2} + \\ &\quad \mathbf{N}_{t,4}\mathbf{N}_{t,3}\mathbf{R}_{t,2} v_{t,2} + \mathbf{N}_{t,4}\mathbf{M}_{t,3} z_{t,3} + \mathbf{N}_{t,4}\mathbf{R}_{t,3} v_{t,3} + \mathbf{M}_{t,4} z_{t,4} + \mathbf{R}_{t,4} v_{t,4} \iff \end{aligned}$$

$$q_{t,4} = \mathbf{N}^4_t q_{t-1,4} + \underbrace{\mathbf{M}_{t,4} z_{t,4} + \sum_{i=1}^{3} \left(\prod_{j=0}^{i-1} \mathbf{N}_{t,4-j} \right) \mathbf{M}_{t,4-i} z_{t,4-i}}_{\mathbf{M}^4_t (z_{t,1},\ldots,z_{t,4})} + \underbrace{\mathbf{R}_{t,4} v_{t,4} + \sum_{i=1}^{3} \left(\prod_{j=0}^{i-1} \mathbf{N}_{t,4-j} \right) \mathbf{R}_{t,4-1} v_{t,4-i}}_{\xi_{t,4}}$$

with

$$\mathbf{N}^k_t = \left(\prod_{j=0}^{k-1} \mathbf{N}_{t,4-j} \right)$$

and

$$\mathbf{E}\left(\xi_{t,4}\xi'_{t,4}\right) = \mathbf{R}_{t,4}\Sigma\mathbf{R}'_{t,4} + \sum_{i=1}^{3}\left(\prod_{j=0}^{i-1}\mathbf{N}_{t,4-j}\right)\mathbf{R}_{t,4-i}\Sigma\mathbf{R}'_{t,4-i}\left(\prod_{j=0}^{i-1}\mathbf{N}_{t,4-j}\right)' \equiv \mathbf{L}_{t,4}$$

by assuming that $\mathbf{E}\left(v_{t,q}v'_{u-i}\right) = 0$ for $i \neq 0$. The complete state-space system at annual frequency reads as follows:

$$\begin{aligned} x^a_{t,4} &= \mathbf{Z}' q_{t,4} \\ q_{t,4} &= \mathbf{N_t^4} q_{t-1,4} + \mathbf{M_t^4}(z_{t1}, \ldots z_{t4}) + \xi_{t,4} \\ \mathbf{L}_{t,4} &= \mathbf{R}_{t,4}\Sigma\mathbf{R}'_{t,4} + \sum_{i=1}^{3}\mathbf{N}^i_t\mathbf{R}_{t,4-i}\Sigma\mathbf{R}'_{t,4-i}\mathbf{N}^{i\prime}_t \end{aligned}$$

B.3 Kalman Filter Recursions

B.3.1 Recursive Estimation of Model Parameters Using Annual Series

The following state-space representation displays the system of equations at an annual frequency, with details on the transformation steps discussed above.

$$x^a_{t,4} = \mathbf{Z}' q_{t,4} \tag{B.23}$$
$$q_{t,4} = \mathbf{N_t^4} q_{t-1,4} + \mathbf{M_t^4}(z_{t1}, \ldots z_{t4}) + \xi_{t,4}, \tag{B.24}$$

where equation (B.23) represents the measurement and (B.24) the transition equation.

Assumptions:

- $x^a_{t,4}$ is a (2×1) vector. It contains the cumulated sum of the logs of quarterly real GDP and the annual budget balance ratio, both observable at point t.

- $q_{t,4}$ is an $[(2+m) \times 1]$ state vector, containing the models data input $x_{t,4}$ and the vector of unobservable state variables $s_{t,q}$ with length m.

- $\xi_{t,4}$ is a zero mean process formed as a linear combination of two independent zero mean processes, with variance $\mathbf{L}_{t,4} = \left\{\xi_{t,4}\xi'_{t,4}\right\}$

- $\mathbf{Z}' = [\mathbf{I}\ \ \mathbf{0}]$ is a $[2 \times (2+m)]$ selection matrix, ensuring the equality sign in the measurement equation.

- $\mathbf{N_t^4}$ is a $[(2+m) \times (2+m)]$ transition or system matrix

- $\mathbf{M_t^4}(z_{t1}, ...z_{t4})$ is a vector, an exogenous variable combining the influence of the financial crisis and gross debt.

The following page presents outlier-robust Kalman Filter recursions in order to numerically evaluate the log-Likelihood function of the annual unobserved components model.[1] The log-Likelihood function is then maximized in order to estimate the model's unknown parameters.

The filtering method, initially developed by Kalman (1960) is a method for the recursive estimation of (unobserveable) states, given measured observations of the output variable. This setup follows Lütkepohl (2007), Hamilton (1994) and Chang (2014).

Initialization phase:

Kalman filter recursions are initialized with the unconditional mean and variance of the state vector at $t = 1$. If all eigenvalues of the transition matrix are inside the unit circle, the process describing the state variables is covariance-stationary. In this case, $q_{1,4|0,4} = \mathbf{0}$ and $vec(P_{1,4|0,4}) = \left[I_{n^2} - \left(\mathbf{N_1^4} \otimes \mathbf{N_1^4}\right)\right]^{-1} \cdot vec(\Lambda_{1,4})$ provide the initial conditions for starting the recursion.
In our case, however, unity is an eigenvalue of the system matrix which implies singularity of $\left(I_n - \mathbf{N_t^4}\right)$ and a non-unique solution.[2] A proper way to initialize the unstable parts of the variance-covariance matrix of the state vector is to set large numbers (like e.g. 10^7) on parts of the main diagonal. This procedure is called diffuse state initialization. A more exact version, accounting for the accumulation of numerical inaccuracies is presented in Koopman and Durbin (2003).

Prediction Phase:

Given starting values $\hat{q}_{1,4|0,4}$ and $P_{1,4|0,4}$, the idea is to find estimates of $\hat{q}_{t,4|t-1,4}$ and

[1] In the case when all data is observable at quarterly frequency, the same recursions are applied however no partial updating steps are necessary. In this case the state-space aggregated to annual frequency but estimated using every quarter.

[2] This due to unit root behavior of $\beta^i_{t,q-1}$ and $\mu^i_{t,q-1}$ for $i = \{y, d\}$ in the independent trend component of our UC model, which are part of the state vector, $s_{t,q}$.

$P_{t,4|t-1,4}$ for $t = 2, 3, \dots T$. Since $z_{t,q}$ is deterministic, its entire path is known. For the prediction of $q_{t,4}$ this implies by transition equation (B.24):

$$\hat{E}\left(q_{t,4}|z_{t,4}, x^a_{t-1,4}\dots, x^a_{1,4}, \ , z_{t-1,4}, \dots, z_{1,4}\right) = \hat{q}_{t,4|t-1,4} + \mathbf{M_t^4}(z_{t1}, \dots z_{t4}) + \mathbf{0} \qquad \text{(B.25)}$$

Forecasting for the observed variables gives:

$$\begin{aligned}\hat{x}^a_{t,4|t-1,4} &= \hat{E}\left(x^a_{t,4}|z_{t,4}, q_{t,4}\right) \\ &= Z'\hat{E}\left(q_{t,4}|z_{t,4}, x^a_{t-1,4}\dots, x^a_{1,4}, \ z_{t-1,4}, \dots, z_{1,4}\right) = Z'\hat{q}_{t,4|t-1,4} \qquad \text{(B.26)}\end{aligned}$$

such that the forecast error and its associated variance-covariance matrix become:

$$x^a_{t,4} - \hat{x}^a_{t,4|t-1,4} = Z'\left(q_{t,4} - \hat{q}_{t,4|t-1,4}\right) \qquad \text{(B.27)}$$

$$E\left[(x^a_{t,4} - \hat{x}^a_{t,4|t-1,4})(x^a_{t,4} - \hat{x}^a_{t,4|t-1,4})'\right] = Z'P_{t,4|t-1,4}Z \qquad \text{(B.28)}$$

Evaluation phase:

Let θ be the set of hyperparameters to be estimated. It comprises the following parameters:

$$\theta = \left[\alpha \ \alpha_1 \ \alpha_2 \ \rho_1 \ \rho_2 \ \sigma_{\varepsilon y} \ \sigma_{vy} \ \sigma_{\varepsilon d} \ \sigma_{vd} \ \sigma_{\xi_1} \ \sigma_{\xi_2} \ \lambda_f \ \lambda_d\right]$$

By assuming that the initial state $\hat{q}_{1,4|0,4}$ and the innovation in the transition equation $\{\xi_{t,4}\}_{t=1}^T$ are multivariate Gaussian, the distribution of our data conditional on the past is normal with mean $\hat{x}^a_{t,4|t-1,4} = Z'\hat{q}_{t,4|t-1,4}$ and variance $Z'P_{t,4|t-1,4}Z$ such that:

$$x^a_{t,4}|z_{t,4}, x^a_{t-1,4}\dots, x^a_{1,4}, \ z_{t-1,4}, \dots, z_{1,4} \sim N\left((Z'\hat{q}_{t,4|t-1,4}), (Z'P_{t,4|t-1,4}Z)\right) \qquad \text{(B.29)}$$

For the (log-)Likelihood function follows:

$$\begin{aligned}&f_{X^a_{t,4}|Z_{t,4}, x^a_{t,4}\dots, x^a_{1,4}, z_{t,4}\dots z_{1,4}}\left(x^a_{t,4}|z_{t,4}, x^a_{t-1,4}\dots, x^a_{1,4}, z_{t-1,4}\dots z_{1,4}; \ \theta\right) \\ &= (2\pi)^{n/2}\,|Z'P_{t,4|t-1,4}Z|^{-1/2} \times exp\{-\frac{1}{2}\left(x^a_{t,4} - Z'\hat{q}_{t,4|t-1,4}\right)'\left(Z'P_{t,4|t-1,4}Z\right)^{-1}\left(x^a_{t,4} - Z'\hat{q}_{t,4|t-1,4}\right)\}\end{aligned} \qquad \text{(B.30)}$$

$$\begin{aligned}&L_{t,4}(\theta) = \log f_{X^a_{t,4}|Z_{t,4}, x^a_{t,4}\dots, x^a_{1,4}, z_{t,4}\dots z_{1,4}}\left(x^a_{t,4}|z_{t,4}, x^a_{t-1,4}\dots, x^a_{1,4}, z_{t-1,4}\dots z_{1,4}; \ \theta\right) = \\ &-\left(\frac{n}{2}\right)\log(2\pi) - \frac{1}{2}\log|Z'P_{t,4|t-1,4}Z|^{-1} - \frac{1}{2}\left(x^a_{t,4} - Z'\hat{q}_{t,4|t-1,4}\right)'\left(Z'P_{t,4|t-1,4}Z\right)^{-1}\left(x^a_{t,4} - Z'\hat{q}_{t,4|t-1,4}\right)\end{aligned} \qquad \text{(B.31)}$$

which is evaluated for $t = 1, 2, \ldots, T$ by the prediction error decomposition. Variable n is the length of the vector of observed variables at point in time t. The joint (log-)Likelihood of the model is just the sum of the conditional likelihoods.

Outlier Correction

At this point the Kalman-Filter Algorithm is extended for an outlier-correction mechanism. This extension is based on Chang (2014). Sometimes outliers violate the assumptions made on the conditional distribution of the data - or stated otherwise - the distribution is contaminated by other distributions. Chang (2014) thinks of that as a modeling error and constructs a hypothesis test in order to check if the actual observation is compatible with the model, i.e. the null hypothesis of the test cannot be rejected. Using a judging criterion based on the square of the Mahalanobis distance, the test determines whether the Chi-squared, with m degrees of freedom distributed distance between the true and the predicted observations is larger than the distance between the observed and the predicted observations. The test statistic is given by:

$$\bar{\gamma}_k = \left(\sqrt{\left(x^a_{t,4} - Z'\hat{q}_{t,4|t-1,4}\right)' \left(Z'P_{t,4|t-1,4}Z\right)^{-1} \left(x^a_{t,4} - Z'\hat{q}_{t,4|t-1,4}\right)}\right)^2 \tag{B.32}$$

If the test statistic exceeds a critical value (e.g. for a 1% significance) this indicates an outlier detection. The following algorithm shows how to correct the prediction error variance for outliers:

- Calculate initial distance $\bar{\gamma}_k(0)$ and set the initial scaling factor $\lambda_k(0) = 1$.

- If and as long as $\bar{\gamma}_k > \chi_\alpha$: Inflate $R_k(t) = Z'\Lambda^4_t Z$, which is part of the prediction error variance with the scaling factor λ_k and calculate a new prediction error variance given by:

$$Q_k(t) = Z'\left(N^4_t P_{t-1,4|t-1,4} N^{4'}_t\right) Z + R_k(t) \tag{B.33}$$

- Calculate $\lambda_k(t+1)$ as follows:

$$\lambda_k(t+1) = \lambda_k(t) + \frac{\bar{\gamma}_k(t) - \chi_\alpha}{\left(x^a_{t,4} - Z'\hat{q}_{t,4|t-1,4}\right)' \left(Q_k(t)\right)^{-1} \left(R_k\right) \left(Q_k(t)^{-1}\right) \left(x^a_{t,4} - Z'\hat{q}_{t,4|t-1,4}\right)} \tag{B.34}$$

- Update $\bar{\gamma}_k$.

The scaled version of the prediction error variance leads to the fact that in the updating phase, outliers don't play too big a role anymore when calculating the Kalman Filter Gain.

Updating phase:

Updating the inference about the state vector $q_{t,4}$:

$$\begin{aligned}\hat{q}_{t,4|t,4} &= \hat{E}\left(q_{t,4}|x^a_{t,4},\ z_{t,4}, x^a_{t-1,4}\dots, x^a_{1,4},\ z_{t-1,4},\dots, z_{1,4}\right)\\ &= \hat{q}_{t,4|t-1,4} - P_{t,4|t-1,4}Z(Z'P_{t,4|t-1,4}Z')^{-1}\left(x^a_{t,4} - Z'\hat{q}_{t,4|t-1,4}\right) \end{aligned} \tag{B.35}$$

given by the fact that: $\hat{E}(\xi_{t,4}|x^a_{t,4}, x^a_{t-1,4}\dots, x^a_{1,4},\ z_{t,4}, z_{t-1,4},\dots, z_{1,4}) = \mathbf{0}$,

The associated MSE follows:

$$\begin{aligned}P_{t,4|t,4} &= E\left[(q_{t,4} - \hat{q}_{t,4|t-1,4})[(q_{t,4} - \hat{q}_{t,4|t-1,4})'\right]\\ &= P_{t,4|t-1,4} - P_{t,4|t-1,4}Z\left(Z'P_{t,4|t-1,4}Z\right)^{-1}Z'P_{t,4|t-1,4}\end{aligned} \tag{B.36}$$

Forecasting phase:

Producing a forecast of $q_{t+1,4|t,4}$:

$$\begin{aligned}\hat{q}_{t+1,4|t,4} &= \hat{E}\left(q_{t+1,4}|z_{t+1,4}, x^a_{t,4}, x^a_{t-1,4}\dots, x^a_{1,4},\ z_{t,4}, z_{t-1,4},\dots, z_{1,4}\right)\\ &= \mathbf{N^4_{t+1}}\hat{E}\left(q_{t,4}|x^a_{t,4}, x^a_{t-1,4}\dots, x^a_{1,4},\ z_{t,4}, z_{t-1,4},\dots, z_{1,4}\right) + \hat{E}\left(\mathbf{M^4_{t+1}}(z_{t+1,1},\dots z_{t+1,4})|x^a_{t,4}, z_{t,4},\dots\right)\\ &+ \hat{E}\left(\xi_{t+1,4}|x^a_{t,4}, x^a_{t-1,4}\dots, x^a_{1,4},\ z_{t,4}, z_{t-1,4},\dots, z_{1,4}\right)\\ &= \mathbf{N^4_{t+1}}\ \hat{q}_{t,4|t,4} + \mathbf{M^4_{t+1}}(z_{t+1,1},\dots z_{t+1,4}) + \mathbf{0}\end{aligned}$$

By substitution of (B.35) follows:

$$\hat{q}_{t+1,4|t,4} = \mathbf{N^4_{t+1}}\ \hat{q}_{t,4|t-1,4} + \mathbf{N^4_{t+1}}P_{t,4|t-1,4}Z\left(Z'P_{t,4|t-1,4}\right)^{-1}\left(x^a_{t,4} - Z'\hat{q}_{t,4|t-1,4}\right) + \mathbf{M^4_{t+1}}(z_{t+1,1},\dots z_{t+1,4}) \tag{B.37}$$

The associated variance-covariance matrix can be found by substituting from (B.37) and (B.24):

$$P_{t+1,4|t,4} = \mathbf{N^4_{t+1}}\left[P_{t,4|t-1,4} - P_{t,4|t-1,4}Z\left(Z'P_{t,4|t-1,4}Z\right)^{-1}Z'P_{t,4|t-1,4}\right]\mathbf{N^4_{t+1}}' + \mathbf{0} + \Lambda_{t+1,4}, \tag{B.38}$$

where $\Lambda_{t+1,4} = E\left[\xi_{t+1,4}\xi'_{t+1,4}\right]$ and $var(\mathbf{M^4_{t+1}}(z_{t+1,1},\dots z_{t+1,4})) = \mathbf{0}$ for all $t = 0, 1, 2, \dots, T$

[0.5em] Repeating this algorithm until T and implementing a numerical maximization (or minimization for the negative of the (log-)Likelihood function) search routine estimates the models hyperparameters, given the data.

B.4 Partial Updating and Fixed Interval Smoothing

After estimating the models unknown parameters at a fully observed annual frequency[3] it remains to find estimates of the unknown state $q_{t,q}$ using full sample information. This is done via Kalman Filter recursions where new information on real GDP and inflation is received every quarter, whereas updates on the budget balance ratio are only available every fourth quarter. Camba-Mendez and Lamo (2004) alter the standard updating equations such that:

$$\hat{q}_{t,q|t,q} = \hat{q}_{t,q|t,q-1} + P_{t,q|t,q-1} Z J_q' F_{t,q}^{-1} J_q \left(x_{t,q}^a - Z' \hat{q}_{t,q|t,q-1} \right) \tag{B.39}$$

$$P_{t,q|t,q} = P_{t,q|t,q-1} - P_{t,q|t,q-1} Z J_q' F_{t,q}^{-1} J_q Z' P_{t,q|t,q-1}, \tag{B.40}$$

where $F_{t,q} = J_q Z' P_{t,q|t,q-1} Z J_q'$ represents the MSE of the prediction error, transformed in such a way that full information is only obtained when $q = 4$. This is done by defining J_q as matrix equal to $[1 \;\; 0]$ if $q < 4$ and equal to an (2×2) identity matrix if $q = 4$.

The prediction equations are similar to those at annual frequency, for sake of completeness they are provided again:

$$\hat{q}_{t,q|t,q-1} = \mathbf{N}_{t,q} \hat{q}_{t,q-1|t,q-1} + \mathbf{M}_{t,q} z_{t,q}$$

$$P_{t,q|t,q} = \mathbf{N}_{t,q} P_{t,q|t,q-1} \mathbf{N}_{t,q}' + \mathbf{R}_{t,q} \Sigma \mathbf{R}_{t,q}'$$

Using the outcome of the Kalman Filter recursions, the filtered states are smoothed in order to recover the exact states, given all information on $x_{1,1}^a, \dots, x_{4,T}^a$. Sequences of filtered state vector and variance plus the forecast MSE are stored. The smoothed state vector and variance at point in time T are initialized with the last filtered values obtained from the Kalman Filter. For all $t < T$ the Fixed Interval Smoothing Algorithm of Ansley and Kohn (1982) is given by:

$$\hat{q}_{t,q|T,4} = \hat{q}_{t,q|t,q} + H_{t,q} \left(\hat{q}_{t,q+1|T,4} - \mathbf{N}_{t,q+1} \hat{q}_{t,q|t,q} - \mathbf{M}_{t,q+1} z_{t,q} \right) \tag{B.41}$$

$$P_{t,q|T,4} = P_{t,q|t,q} - H_{t,q} \left(P_{t,q+1|t,q} - P_{t,q+1|T,4} \right) H_{t,q}', \tag{B.42}$$

where $H_{t,q} = P_{t,q|t,q} \, \mathbf{N}_{t,q+1|t,q}' P_{t,q+1|t,q}^{-1}$ is the Kalman Smoothing matrix.[4]

[3] This step is only required when the sample length of the data for country j is extended using annual data.

[4] In case that $P_{t,q|t+1,q}$ is not invertible, Ansley and Kohn (1982) propose to use an generalized inverse, instead.

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