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Introduction

This dissertation consists of three self-contained essays in microeconomic theory. The first chapter examines how pre-election polls affect the election outcome through the voters' participation decision and studies the resulting incentives that arise for the poll participants. The second chapter, which is joint work with Tobias Rachidi, compares two search technologies—evaluating one or multiple candidates at a time in each time period—in a committee search framework in terms of welfare and acceptance standards. The third chapter analyzes the equilibria of three-candidate plurality-rule elections in a costly voting framework, revisiting Duverger's law.

In chapter 1, I study the effect of pre-election polls on the participation decision of citizens in a large, two-candidate election and the resulting incentives for the poll participants. Citizens have private values, and voting is costly and instrumental. The environment is ex ante symmetric and features aggregate uncertainty about the distribution of preferences. An opinion poll conducted prior to the election publishes its results of preferences for each candidate as expressed by those participating in the poll. Citizens base their participation decision in the election on their own preferences and on the information provided in the poll.

Starting with the voting equilibrium, I show that there exists an equilibrium for any posterior belief induced by the polling outcome. As the population size grows large, the limit of the ratio of participation rates of supporters of either candidate is unique. This limit ratio reflects the underdog effect—the expected minority participates at higher rates than the expected majority—and it is monotonic in the posterior belief. However, the underdog effect is only partial, meaning that the increased turnout of the expected minority does not fully offset the majority's initial advantage. Thus, in the limit, the majority candidate wins the election almost surely, regardless of voters' posterior beliefs. If all participants answer the poll truthfully, the results imply that the supporters of the trailing candidate turn out at higher rates than the supporters of the leader of the poll, and that this effect is monotonic in the poll's margin. This yields incentives for the poll participants to misrepresent their preferences to encourage the voters who have the same preferences to turn out. If poll participants are strategic, however, there does not exist an equilibrium in which the poll conveys any information.

In chapter 2, we study committee search where members either assess candidates "one at a time", i.e., on a rolling basis, or they simultaneously review a set of candidates of fixed size in each time period. The former search procedure has been studied before, whereas the exploration of the latter search technology is novel in the committee search literature. We compare both search procedures in terms of acceptance standards and welfare. There is a trade-off between the expected value of a candidate conditional on stopping and the expected search costs. The resolution of this trade-off depends on the voting rule and the specification of search costs associated with the simultaneous evaluation of multiple candidates. The adoption of a qualified majority rule changes the evaluation of search procedures compared to the unanimity rule, revealing that the presence of a search committee alters the search design problem in comparison with the single decision-maker case, which is a special case of a committee operating under unanimity voting rule. This is the main qualitative insight and we discuss its implications for committee search in practice.

In chapter 3, I consider a large election with three candidates under plurality voting rule. I examine whether Duverger's law—which stipulates that plurality-rule elections favor a two-party system (cf. Duverger (1954))—holds in a framework in which participation is endogenous, i.e., in which voting is costly and voluntary. Citizens have private values. In the limit as the population size grows large, the existence of an equilibrium in which only two candidates receive a positive vote share, a *Duvergerian* equilibrium, follows from the analysis of Xefteris (2019). I study the question whether any non-Duvergerian equilibria exist in the limit. I show first that there cannot exist an equilibrium in which only one candidate receives any votes. Next, I prove that an equilibrium in which all three candidates are expected to receive positive vote shares in the limit does not exist if one candidate is expected to be behind the others. The only potential non-Duvergerian equilibria are thus the ones in which either all candidates tie, or two candidates are expected to tie for second and third place behind a front-runner. I give necessary and sufficient conditions on the distribution of voter types for the former equilibrium to exist for any finite population size. These conditions are knife-edge. Finally, I discuss the latter type of equilibrium.

References

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Chapter 1

Polls and Elections: Strategic Respondents and Turnout Implications

1.1 Introduction

Polls matter. They receive widespread attention in the media, and they are perceived to have an influence on voting behavior and turnout. As an example, on 23 June 2016, the United Kingdom decided to leave the European Union with a 52% majority. By contrast, opinion polls released on the eve of the referendum predicted that 51% of voters would support remaining (cf. Wells (22 June 2016)). Later, several sources, such as Low (24 October 2016), voiced concern that many citizens might not have voted because they had believed that "Brexit" would be defeated. Government regulations restricting the timing of the publication of polls in the run-up to elections reflect the perceived influence of polls. Several countries prohibit the publication of opinion polls in specific circumstances, usually quite close to Election Day itself. In a study of policies that address the publication of voter polls in 133 countries, Frankovic, Johnson, and Stavrakantonaki (2018) find that 60% of these countries ban the publication of polls before elections for a certain period of time, called the blackout period. In France, for example, the blackout period is currently set at two days.

Generally, turnout decisions are strategic and depend on the citizens' beliefs about the preferences and turnout decisions of other voters.[1] Polls inform citizens about these preferences and allow rational voters to update their beliefs. Therefore, polls can influence elections by updating beliefs, which, in turn, affect the incentives of poll participants. The aim of this paper is to analyze the effect of the information provided through polls on voter turnout, the incentives of the poll participants, and

[1] For empirical evidence, see Agranov, Goeree, Romero, and Yariv (2018), Klor and Winter (2018), Morton, Muller, Page, and Torgler (2015), Blais, Gidengil, and Nevitte (2006), and Cantoni, Yang, Yuchtman, and Zhang (2019).

the implications for regulations. To this end, I study and build upon the canonical model of costly voting as introduced by Palfrey and Rosenthal (1983). I add the feature that there is aggregate uncertainty about the distribution of preferences. This captures the observation that voters rarely know with certainty whether they are part of the majority. Finally, I introduce polls that inform citizens about the preferences of others.

In more detail, I consider a large election taking place between candidates A and B. Citizens have private values, and there is aggregate uncertainty about the preference distribution, which is governed by the state of the world. In state α, the probability that a randomly drawn voter supports candidate A is $q > \frac{1}{2}$, and in state β, the probability that a randomly drawn voter supports candidate A is $1-q$, where each state is equally likely. Thus, ex ante, it is equally likely that a random voter prefers candidate A or B. Each voter knows his or her own preferences, but does not know whether he or she is part of the majority. An opinion poll conducted prior to the election publishes its results of preferences for each candidate as expressed by those participating in the poll. Voters in the electorate use the information from the poll as well as their own preference type to update their beliefs about the state of the world. It is assumed that a known and finite number of citizens, randomly drawn from the population, participate in the opinion poll and indicate their preferred candidate. So, the sample of citizens surveyed in the pre-election poll is representative.[2]
Voting is costly and voluntary, and costs are drawn from a smooth distribution that has bounded support $[0, \bar{c}]$ and a strictly positive density everywhere on the support. The election is decided by majority voting. The candidate with the most votes wins the election, and ties are broken randomly.

I solve the game backwards, starting with the voting equilibrium. For any given polling outcome, and for any strategy the poll participants pursue, voters hold some posterior belief about the state of the world. I show that there always exists a voting equilibrium with strictly positive participation rates by both groups. Focusing on large elections, i.e., taking the limit as the size of the population goes to infinity, I show that there exists a unique limit of the ratio of participation rates. This limit ratio reflects the *underdog effect*: the expected minority participates at higher rates.[3]
Mentioned informally decades ago in Palfrey and Rosenthal (1983), the underdog effect has been observed repeatedly in the experimental literature, e.g. Levine and Palfrey (2007). As formalized in Ledyard (1984), a vote for the expected minority candidate is pivotal with higher probability because it pushes the election closer to a tie; by contrast, a vote for the majority candidate pushes the likely outcome

[2] To isolate effects, the citizens participating in the poll are excluded from voting in the election, but it is assumed that they derive the same utility from the election outcome as eligible voters who share their preferences.

[3] Note that if there are no polls, supporters of both candidate A and B have the same probability of turning out.

further away from a tie. Consequently, because a voter's perceived benefit of voting increases with his or her belief in the likelihood that his or her vote will be pivotal, those supporting the perceived underdog in the race have a higher incentive to vote, implying higher participation rates.[4] Importantly, I show that this underdog effect is monotonic in the sense that the limit ratio of participation rates is monotonic in the posterior beliefs. However, the underdog effect is only *partial*: the limit ratio of participation rates is closer to one than the ratio of the respective population shares of A and B supporters. Consequently, in the limit, the majority candidate almost surely wins the election. Notably, this holds for any posterior belief voters might hold, including the case in which any aggregate uncertainty about the distribution of preferences is resolved and, thus, the state of the world is known.

The solution of the voting stage has interesting implications if polls are answered truthfully. Due to the partial underdog effect, the margin of victory decreases, but the majority candidate still wins the election in the limit almost surely. Yet, prior research has found that poll participants do not necessarily answer truthfully[5]. So, I next consider the strategic behavior of participants in the polling stage of the election by initially assuming that all participants behave in strategic ways. By contrast, suppose that all poll participants answer truthfully and that the electorate believe this to be the case. In such a situation, posterior beliefs are monotonic in the poll's margin, and, thus, so is the underdog effect. However, given this monotonicity, there cannot exist an equilibrium with any information transmission. In equilibrium, the poll is uninformative. Intuitively, misrepresenting preferences is profitable because it simultaneously stimulates the participation of voters who have the same preferences and discourages the participation of opponent voters. In Appendix 1.C, I consider an extension in which a fixed share of poll participants is prescribed to answer the poll truthfully and the other poll participants are strategic. Then, it holds again that the majority candidate almost surely wins the election in the limit. The direction of the underdog effect will depend on the share of exogenously truthful poll participants. If this share is larger than one-half, the poll is informative. Else, the poll is not informative. In any case, the strategic poll participants have an incentive to misrepresent their preferences.

In conclusion, in the limit, polls do not prevent the almost sure election of the majority candidate. If the poll result is informative, it stimulates participation of voters who support the minority candidate. However, the higher participation rate is not sufficient to overturn the election outcome. In contrast, if poll participants are

[4] Note that the underdog effect is related to the free-riding problem in public goods games because voting is comparable to contributing to the public good.

[5] See, for example, Clarke and Whiteley (2016) who are concerned with false answers regarding voting intention, or Keeter and Samaranayake (2007) and Hopkins (2009) who consider the Bradley effect.

strategic, the poll is uninformative and voters behave as if there were no poll in the first place.

Goeree and Großer (2007) study the effect of exogenously truthful information on the distribution of preferences. They find that this information (for example, as provided by truthful polls) is detrimental for welfare because, as a result, both candidates are equally likely to win. In their paper, all voters face homogenous voting costs, which are set such that turnout is incomplete and positive. For voters to be willing to employ mixed strategies and, thus, to be indifferent between abstaining and voting, given the homogenous cost, the expected benefit of voting needs to coincide for all voter types, implying equal pivot probabilities for votes for either candidate. Since a vote for the trailing candidate has a higher probability of being pivotal, pivot probabilities (and, thus, expected voting benefits) can only be equal if the expected vote shares coincide. As a result, models with homogenous voting costs (such as the model of Goeree and Großer (2007)) observe a *full underdog effect* because the minority's heightened participation has to completely offset the majority's advantage in equilibrium. By contrast, if one assumes, as I do, that the distribution of costs is smooth, the underdog effect must be partial. Intuitively, if the underdog effect were to fully compensate for the majority's advantage such that expected vote shares would be equal, the pivot probabilities would be equal for the two groups. However, in my model, if the probability of being pivotal would indeed be the same for both groups, the participation rates would be equal, and, thus, vote shares would necessarily be strictly different. This contrasts with Goeree and Großer (2007), where the vote shares can be the same if the pivot probabilities are the same. Overall, while they show how drastic the effects of polls can be, I demonstrate that the negative welfare effects do not carry over when considering a slightly different model framework. Coming back to the "Brexit" example, my results can be interpreted to demonstrate that in equilibrium, and in a large election, polls should not have been of concern for the election outcome. However, polls generally do matter in the sense that they reduce the margin of victory in elections (or referenda) compared to the actual advantage of the majority candidate (or alternative) through the partial underdog effect. This effect has consequences if the margin of victory or vote shares themselves matter.

The remainder of this paper is organized as follows: Section 1.2 gives an overview of the related literature. Section 1.3 introduces the model, and section 1.4 analyzes the voting equilibrium. Section 1.5 analyzes the polling equilibrium, and section 1.6 concludes. All omitted proofs appear in Appendix 1.A. Appendix 1.B contains the properties of posterior beliefs. Appendix 1.C considers a poll with a share of exogenously truthful participants.

1.2 Related Literature

The theory of costly participation in a two-candidate election was introduced by Palfrey and Rosenthal (1983), Ledyard (1984), and Palfrey and Rosenthal (1985), who explore the paradox of not voting and give conditions for equilibria with positive turnout for given candidate platforms. Palfrey and Rosenthal (1983) characterize multiple equilibria in a setting in which voting costs are identical for all voters. Nöldeke and Peña (2016) provide missing proofs. Ledyard (1984) considers spatial preferences of voters and a smooth cost distribution. He characterizes the voting equilibrium, showing that if candidates can freely set their platforms, the welfare maximizing platform is chosen by all candidates, and there is no turnout in equilibrium. Palfrey and Rosenthal (1985) also consider fixed platforms and allow for different distributions of costs across groups. They show that in large elections, only voters with negative or zero costs of voting turn out.

The partial underdog effect has been identified in the literature studying idiosyncratic uncertainty about voters' preferences, assuming, as I do, a smooth distribution of costs. Herrera, Morelli, and Palfrey (2014) contrast the voting systems of majority and proportional representation in a setting with population uncertainty; they characterize the differences in turnout. Krishna and Morgan (2015) give conditions under which simple majority rule selects the utilitarian candidate.[6] In a model of ethical voting, Evren (2012) assumes that a fraction of agents is altruistic, and that there is aggregate uncertainty about the expected share of altruists for supporters of either candidate. While selfish agents abstain from voting, altruistic agents turn out if their private voting cost is outweighed by their vote's contribution to the welfare of society.

Myatt (2017) studies protest voting in a setting in which voting is not costly, but the possibility to protest bears opportunity costs, and voting for the opponent potentially influences policy. He observes an "offset" effect which is directly related to the underdog effect. The anticipation of a larger protest reduces the motivation of like-minded agents to join the protest—thus reducing the size of the protest, but not fully compensating for the increase in enthusiasm. I show that the partial underdog effect exists for any posterior belief that can be induced by a pre-election poll if there is aggregate uncertainty about the distribution of preferences and voters only expect to be part of the majority or the minority, but do not know this with certainty.

Aggregate uncertainty about the distribution of preferences in costly voting models has been studied by Goeree and Großer (2007), Taylor and Yildirim (2010b), and Myatt (2015). In similar papers, Goeree and Großer (2007) and Taylor and Yildirim (2010b) both contrast the effects that occur in two different environments. The first is an environment in which voters are informed about the distribution of

[6] Grüner and Tröger (2019) study utilitarian-optimal voting rules if voting is costly.

preferences; the second is an environment in which there is aggregate uncertainty about this distribution and the prior over the state distribution is symmetric. Voting costs are homogenous for all voters. Their papers differ in that Goeree and Großer (2007) consider two states of the world and focus on small elections, whereas Taylor and Yildirim (2010b) allow for finitely many states of the world and consider small and large elections. Under their common assumptions on costs, the underdog effect implies that expected vote shares are equal. In the informed case, this results in a toss-up election in expectation. In the case with aggregate uncertainty about the preference distribution, the symmetric prior yields identical participation rates for both types of voters, such that the majority candidate is more likely to win the election. Goeree and Großer (2007) and Taylor and Yildirim (2010b) thus conclude that information provision that resolves the aggregate uncertainty is unambiguously detrimental to voters' welfare since it decreases the probability of the majority candidate winning the election. My modeling of aggregate uncertainty about the preference distribution follows Goeree and Großer (2007). I show that their conclusion about the welfare implications of information is sensitive to assumptions on the distribution of costs. My work is distinct in two other aspects: I consider the incentives of poll participants, and I do not require the poll to perfectly reveal the state of the world.

In Myatt (2015), the probability that candidate A is preferred by a randomly drawn voter is given by p, which is itself a random variable with mean $\bar{p}$ and density $f(p)$. He studies the response of turnout and the election outcome to, amongst others, varying assumptions on costs, the importance of the election, the preference intensities, or the perceived popularity of candidates. He also finds that there exists an underdog effect, which is complete if costs are the same for all voters, and partial if the cost distribution is smooth. Our two models are not nested. First, Myatt (2015) assumes full support on $[0, 1]$ for the density f. Further, in Myatt (2015), reducing the aggregate uncertainty about the distribution of preferences corresponds to decreasing the variance; if the uncertainty is resolved, voters' beliefs coincide with the mean $\bar{p}$. By contrast, in my model, if the state of the world is known, the probability of preferring A is either q or $1 - q$, but never their mean.[7] Further, Myatt (2015) does not consider the incentives faced by poll participants but only studies the impact of different beliefs about preferences on voting.[8]

This paper is also related to the literature on information provision in elections and on signaling in elections.

Burke and Taylor (2008) study polls with signaling incentives, assuming that the same voters participate in the poll and in the election. There is only idiosyncratic uncertainty about voters' preferences, and voting costs are the same for all voters. They find that truthful reporting is an equilibrium in the pre-election poll

7 A similar argument is made in Agranov et al. (2018).
8 Cvijanović, Groen-Xu, and Zachariadis (2020) study corporate voting, building on Myatt (2015).

for a three-person electorate and low voting costs. This holds because in the case of a two-citizen majority, if preferences are known, there is no underdog effect for sufficiently low voting costs, and therefore, the majority is more likely to win. The incentive to truthfully reveal preferences in the case of a two-citizen majority dominates the incentive to misrepresent in all other cases. For general n, Burke and Taylor (2008) derive sufficient conditions for the non-existence of a truthful reporting equilibrium. Finally, they show that if a truthful equilibrium exists, it is welfare enhancing because the minority is discouraged from participating in the election. My model marries the incentive considerations in a poll that is intended to inform the electorate about the prevailing preferences with the assumption that the distribution of these preferences is initially unknown. Then, truthtelling cannot be an equilibrium of the polling stage.

Another subject of study of the roles played by polls concerns their ability to serve as signaling and coordination devices, or a means to inform politicians about the desired policy.

Hummel (2011) proposes a model of polling in sequential elections, in which the winner of the first election faces a third candidate, and finds incentives to misrepresent preferences to increase the winning probability of one's favorite candidate in the second election. Piketty (2000) analyzes a similar sequential election, in which there are no polls, but voters use their votes in the first election round to communicate their preferences—thereby trading off sincere with strategic voting. Hummel (2014) considers a three-candidate election, in which the third candidate is supported by a minor party; he explains why third party candidates achieve better results in pre-election polls than in elections.

Meirowitz (2005) and Morgan and Stocken (2008) analyze the incentives of poll participants if candidates use the information revealed in the poll to select policy platforms. To be more precise, Morgan and Stocken analyze a setting in which a policy maker polls the constituents, who differ in terms of information they have and ideology they hold, about their preferred policy, and provide conditions for full information aggregation. Relatedly, Battaglini (2017) and Ekmekci and Lauermann (2019) study information aggregation through informal elections, such as public protests.

Communication in committees prior to a binding vote has also been modeled through straw votes (a full poll). Coughlan (2000) and Austen-Smith and Feddersen (2006) give conditions for full information revelation in a non-binding straw vote for a Condorcet jury setting. Gerardi and Yariv (2007) allow for general communication protocols; they show that the set of equilibrium outcomes is invariant to the voting institution, as long as it is non-unanimous.

The incentives of poll participants and the effects of polls, or exogenous information release, on voters' beliefs have been studied in the experimental literature as well. Agranov et al. (2018) study the effect of the release of exogenous information by testing the model proposed by Goeree and Großer (2007). They do not observe an underdog effect and find that information about the distribution of preferences does not reduce welfare. Agranov et al. (2018) argue that the data can be explained by assuming that voters have preferences to vote for the winner.[9] Klor and Winter (2018) also consider exogenous information; they find that close polls stimulate turnout, and that the effect is greater for majority voters because of false beliefs about the probability of casting a pivotal vote. Morton et al. (2015) employ a natural experiment featuring exit polls in France; their findings show that the publication of exit polls while the election was ongoing led to a decrease in turnout by 11%, and an increase in bandwagon voting, i.e., voting for the expected winner of the election. Großer and Schram (2010) find that polls stimulate turnout, and that this is driven by undecided voters. Blais, Gidengil, and Nevitte (2006) examine the impact of polls in the 1988 Canadian election. They find that the polls affected the beliefs about the outcome of the election and voting itself by discouraging turnout of supporters of a party that was not considered likely to win. They do not observe a bandwagon effect. Because I abstract from voter preferences that prescribe that participants want to vote for the winner, I avoid the effect described by Blais, Gidengil, and Nevitte (2006) that would counteract some of my results. Cantoni et al. (2019) conduct a field experiment to elicit the beliefs of individuals about others' planned participation in a public protest and the effects on turnout. The authors find that there is strategic substitutability related to the underdog effect in the sense that turnout is stimulated if and only if others are believed not to participate.

Methodically, this paper is related to the seminal work of Myerson (1998a), Myerson (1998b), and Myerson (2000) on population uncertainty. It is also related to Krishna and Morgan (2012), who study welfare properties of majority voting in a two-candidate election with common values and population uncertainty, in which the state of the world indicates which candidate is more competent.

1.3 The Model

Two candidates, A and B, vie for election. Citizens have independent private values. An A supporter receives a utility of $v > 0$ if and only if A is elected, and zero otherwise, and a B supporter receives a utility of $v > 0$ if and only if B is elected, and zero otherwise. There is aggregate uncertainty about the distribution of preferences that is governed by two states, $\omega \in \Omega = \{\alpha, \beta\}$. In state α, the probability that a randomly drawn citizen prefers A is $\Pr(A|\alpha) = q > \frac{1}{2}$, while, in state β, the probability that a randomly drawn citizen prefers A is $\Pr(A|\beta) = 1 - q$. The number of eligible voters is finite but uncertain, and it is Poisson distributed with mean n. Hence, the probability that the electorate consists of k citizens is $e^{-n}\frac{n^k}{k!}$. This induces an extended Poisson game as introduced by Myerson (1998a).

[9] Callander (2007) shows in a theoretical model that preferences to vote for the winner can result in the so-called *bandwagon effect*.

Voting is costly and voluntary. Each citizen can decide between the actions "vote for A", "vote for B", and "abstain". If a citizen chooses to vote for one of the candidates, he or she incurs a voting cost c. The voting cost is distributed according to the cumulative distribution function F with density f that is strictly positive on its support $[0, \bar{c}]$, with $\bar{c} \geq v$.[10] Further, F is assumed to be differentiable. Costs are drawn independently for each individual citizen and, thus, do not depend on preferences. The candidate who obtains the majority of votes wins, and ties are broken by the toss of a fair coin. Prior to the election, but after the state and preferences have been realized, an opinion poll is conducted. To this end, m independently drawn citizens are asked which candidate they prefer. Then, the poll result is published in the form of the pair $\tau = (\tau_A, \tau_B)$, where τ_i denotes the number of poll participants who indicated a preference for candidate i, for $i \in \{A, B\}$. For tractability, the m participants of the poll are assumed not to take part in the main election. That is, they will not belong to the electorate. They will, however, have the same preferences over the election outcome as the members of the electorate, and so, have the same stakes in the election, absent the cost of voting.

Thus, the overall timing is as follows: Nature draws the number of voters and the state of the world, preferences are determined by independent draws from the state-dependent Bernoulli distribution, the pre-election poll is conducted and published, and, finally, the election is held.

I will consider symmetric Perfect Bayesian equilibria, in which all supporters of the same candidate employ the same strategy.[11]

1.4 Voting Equilibrium

This section addresses the equilibrium of the election stage and collects its properties. I take as given the citizens' posterior beliefs about the state of the world. First, I derive the existence of voting equilibria and show that for all n, participation rates are equal if and only if the posterior beliefs coincide with the prior beliefs. Then, I turn to the analysis of large elections. I show that the limit ratio of participation rates is unique and that it reflects the underdog effect, which is monotonic in the posterior beliefs. Finally, I show that the majority candidate almost surely wins the election in the limit, independently of posterior beliefs.

1.4.1 Equilibrium Existence

Observe first that given the assumption that voting is costly, for every supporter of candidate i, voting for candidate j is strictly dominated by abstention. Thus, if a

[10] This condition is sufficient, but not necessary to guarantee positive and incomplete turnout.

[11] Given the Poisson setting, the assumption of symmetric strategies is without loss of generality. For details, see Myerson (1998a).

citizen chooses to vote, he or she will vote for his or her preferred candidate. Voting is always sincere.

So, a citizen trades off voting for his or her favorite candidate against abstaining. To that end, a citizen will contrast the expected benefit of his or her vote with the associated costs. His or her vote will directly benefit him or her only if the vote changes the outcome of the election, i.e., only if his or her vote is *pivotal*. A vote for candidate A is pivotal in two cases: 1) if both candidates are tied, that is, if there are $2k$ other voters, where k are voting for A and k are voting for B, and 2), if candidate A is exactly one vote behind, that is, if there are $2k+1$ other voters, where k are voting for A and $k+1$ are voting for B. Piv_A denotes the event that a vote for candidate A is pivotal, analogously, Piv_B denotes the event that a vote for B is pivotal.

Upon observing their own preference type, citizens do not hold uniform priors. That is, a citizen of type i, for $i \in \{A, B\}$, holds the prior $\Pr(\omega|i)$ for $\omega \in \{\alpha, \beta\}$. After observing a poll result τ, the posterior belief of a citizen of type i that the state is ω is denoted by $\Pr(\omega|i, \tau)$. The properties of these beliefs are derived in Appendix 1.B.

The expected benefit of voting for an i supporter is thus given by

$$\Pr(\alpha|i,\tau) \cdot \Pr(Piv_i|\alpha) \cdot v + \Pr(\beta|i,\tau) \cdot \Pr(Piv_i|\beta) \cdot v.$$

A citizen will vote for his or her preferred candidate if and only if the expected benefit of voting is weakly larger than his or her voting cost c. Since the expected benefit of voting is independent of c, there exist cost cutoffs c_A^*, c_B^*,[12] satisfying

$$\Pr(\alpha|A,\tau) \cdot \Pr(Piv_A|\alpha) \cdot v + \Pr(\beta|A,\tau) \cdot \Pr(Piv_A|\beta) \cdot v \;=\; c_A^* \tag{1.1}$$
$$\Pr(\alpha|B,\tau) \cdot \Pr(Piv_B|\alpha) \cdot v + \Pr(\beta|B,\tau) \cdot \Pr(Piv_B|\beta) \cdot v \;=\; c_B^*.\text{[13]} \tag{1.2}$$

The cost cutoffs determine the voters' participation decision. Thus, the equilibrium strategy for a voter of type i, for $i \in \{A, B\}$, will now be identified by the cutoff cost c_i^*. A voting equilibrium is a pair of cutoff costs (c_A^*, c_B^*) such that it is optimal for a citizen of type i with cost $c \le c_i^*$ to turn out and vote for candidate i if all other citizens in the electorate pursue this strategy.

Let p_A denote the probability that an A supporter chooses to vote for A, and analogously, let p_B denote the probability that a B supporter chooses to vote for B. Then,

[12] The cost cutoffs may depend on τ. The dependence is omitted in the notation for the sake of readability.

[13] The probability that a vote is pivotal is larger if the election is expected to be close. The above equations imply that close elections induce higher turnout because higher pivot probabilities increase the cost cutoffs.

the probability that an i supporter abstains is $1 - p_i$. I will call p_A and p_B the participation rates of A and B supporters, respectively.
Given the pair of cost cutoffs, these participation rates are

$$p_A := F(c_A^*),\ p_B := F(c_B^*).$$

Since the size of the electorate follows a Poisson distribution with mean n, the number of votes for candidate i conditional on the state of the world ω is distributed according to a Poisson distribution with mean denoted by $\lambda(i|\omega), i \in \{A, B\}, \omega \in \{\alpha, \beta\}$. Note that the number of votes for candidate i is independent of the number of votes for candidate j conditional on the state, cf. Myerson (2000).
These means, which I also call expected conditional votes, are given by

$$\begin{aligned}
\lambda(A|\alpha) &:= n \cdot q \cdot p_A, \\
\lambda(A|\beta) &:= n \cdot (1-q) \cdot p_A, \\
\lambda(B|\alpha) &:= n \cdot (1-q) \cdot p_B, \\
\lambda(B|\beta) &:= n \cdot q \cdot p_B.
\end{aligned}$$

Since in an extended Poisson game, the pivot probabilities depend only on the expected conditional votes for either candidate, I can now calculate these probabilities. As mentioned above, a vote is pivotal if it either creates a tie or breaks a tie. Let T be the event of a tie, let T_{-1}^A be the event that candidate A is one vote behind, and T_{-1}^B be the event that candidate B is one vote behind. The probabilities of these events are given by

$$\begin{aligned}
\Pr(T|\omega) &= e^{-\lambda(A|\omega)-\lambda(B|\omega)} \cdot \sum_{k=0}^{\infty} \frac{\lambda(A|\omega)^k}{k!} \cdot \frac{\lambda(B|\omega)^k}{k!}, \\
\Pr(T_{-1}^A|\omega) &= e^{-\lambda(A|\omega)-\lambda(B|\omega)} \cdot \sum_{k=1}^{\infty} \frac{\lambda(A|\omega)^{k-1}}{(k-1)!} \cdot \frac{\lambda(B|\omega)^k}{k!}, \\
\Pr(T_{-1}^B|\omega) &= e^{-\lambda(A|\omega)-\lambda(B|\omega)} \cdot \sum_{k=1}^{\infty} \frac{\lambda(A|\omega)^k}{k!} \cdot \frac{\lambda(B|\omega)^{k-1}}{(k-1)!}.
\end{aligned}$$

Therefore, the probability that an A supporter's vote for A is pivotal in state ω is

$$\Pr(Piv_A|\omega) = \frac{1}{2}\Pr(T|\omega) + \frac{1}{2}\Pr(T_{-1}^A|\omega),$$

and the probability that a B supporter's vote for B is pivotal for B in state ω is

$$\Pr(Piv_B|\omega) = \frac{1}{2}\Pr(T|\omega) + \frac{1}{2}\Pr(T_{-1}^B|\omega).$$

The existence of a voting equilibrium is now established in the following proposition.

Proposition 1.1 (Existence). *There exists an equilibrium of the election stage; that is, there exist equilibrium cutoff costs* (c_A^*, c_B^*) *for every pair* $\tau = (\tau_A, \tau_B)$. *All voting equilibria involve interior participation rates, i.e.,* $p_A, p_B \in (0,1)$.

Proof. Recall that $p_A := F(c_A^*)$ and $p_B := F(c_B^*)$. Further, $\lambda(i|\omega)$, for $i \in \{A,B\}$ and $\omega \in \{\alpha, \beta\}$, are functions of p_A, p_B, and the pivot probabilities are functions of $\lambda(i|\omega)$. This defines the pivot probabilities as functions of the cost cutoffs. Note that all above functions are continuous, and so are the cost cutoffs as functions of the pivot probabilities.

Define $h,g : [0,\bar{c}] \times [0,\bar{c}] \to [0,\bar{c}] \times [0,\bar{c}]$, with $c_A^* =: h(c_A, c_B)$, $c_B^* =: g(c_A, c_B)$. Since $[0,\bar{c}] \times [0,\bar{c}]$ is a compact convex subset of $\mathbb{R}^2$, Brouwer's fixed-point theorem guarantees the existence of cutoff costs c_A^*, c_B^* that simultaneously satisfy $c_A^* = h(c_A^*, c_B^*)$ and $c_B^* = g(c_A^*, c_B^*)$. This establishes the existence of the voting equilibrium.

To see that the equilibrium cutoffs must be interior, i.e., $c_A^*, c_B^* \in (0,\bar{c})$, assume first, by contradiction and without loss of generality, that $c_A^* = 0$. Then, $p_A = 0$. In this case, an A supporter is pivotal if either no B supporter or if exactly one B supporter shows up at the ballot. But then, the probability of a vote being pivotal for candidate A is strictly positive in both states, implying a positive cost cutoff c_A^*, which is the desired contradiction. Secondly, by equation (1.1), and since $\bar{c} \geq v$, $c_A^* < \bar{c}$, so $p_A = F(c_A^*) = 1$ is impossible. □

For the equilibrium analysis, it is crucial to understand the influence of the voters' beliefs about the state of the world on their participation rates. Consider a sequence of equilibria with the corresponding sequence of equilibrium participation rates $(p_A(n), p_B(n))_n$. Lemma 1.1 establishes that along all equilibrium sequences, participation rates coincide if and only if the voters hold their prior beliefs. Intuitively, since the prior beliefs are symmetric, both A and B supporters have the same incentives to participate in the election.[14] Therefore, the participation rates must coincide.

Lemma 1.1. *Along all equilibrium sequences, the participation rates of A and B supporters coincide for all n if and only if voters hold their prior beliefs. That is, for all n,* $p_A(n) = p_B(n)$ *if and only if* $\Pr(\alpha|A,\tau) = \Pr(\alpha|A) = q$ *and* $\Pr(\beta|B,\tau) = \Pr(\beta|B) = q$.

1.4.2 Pivot Probabilities

Having established the common properties of all voting equilibria, let me now turn to the limiting case of a large election. That is, for the remainder of the paper, I assume that n goes to infinity. I start with rather technical results which will allow

[14] Note that upon learning his or her own preferences, any voter believes to be in the majority. For this, see also the derivations in Appendix 1.B.

the application of an approximation of the pivot probabilities, making the model more tractable. All subsequent results will rely on this approximation.

Lemma 1.2 establishes that in the limit, as n goes to infinity, the participation rates must go to zero. To see why this is the case, suppose that, to the contrary, participation rates are strictly positive in the limit. Then, the probability of being pivotal goes to zero in the limit; with this, the gross benefit of voting also disappears. But this means that any randomly drawn voter would be better off by abstaining than by voting, contradicting positive participation rates.

Lemma 1.2. *As $n \to \infty$, the participation rates p_A, p_B go to zero along every sequence of equilibria, that is,* $\limsup_{n\to\infty} p_A(n) = \limsup_{n\to\infty} p_B(n) = 0$.

Lemma 1.3 reveals that the participation rates converge slowly enough to zero such that expected conditional votes nevertheless go to infinity as n grows large. For this result, it is essential that zero is in the support of the cost distribution. To see this, suppose total turnout were finite in the limit. Then, the pivot probabilities would have strictly positive limits, yielding a strictly positive gross benefit of voting. If costs can arbitrarily become close to zero, there will be voters who are better off voting than abstaining—implying strictly positive participation rates and contradicting finite turnout as n goes to infinity. However, if costs are bounded away from zero, even with a strictly positive gross benefit of voting, abstaining might be more profitable than voting, yielding finite turnout.

Lemma 1.3. *As $n \to \infty$, the expected conditional votes $\lambda(A|\alpha)$, $\lambda(B|\alpha)$, $\lambda(A|\beta)$, $\lambda(B|\beta)$ go to infinity. That is,* $\liminf_{n\to\infty} \lambda(i|\omega) = \infty$ *for $i \in \{A,B\}$ and $\omega \in \{\alpha,\beta\}$. Further, the participation rates are of the same order of magnitude, that is,* $\liminf_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 0$ *and* $\limsup_{n\to\infty} \frac{p_A(n)}{p_B(n)} < \infty$.

Given these properties in large elections, the pivot probabilities can be approximated by employing modified Bessel functions (cf. Abramowitz and Stegun (1965)), as suggested by Myerson (2000).

Lemma 1.4. *As $n \to \infty$,*

$$\Pr(Piv_A|\omega) \approx \frac{1}{2}\frac{e^{-\left(\sqrt{\lambda(A|\omega)}-\sqrt{\lambda(B|\omega)}\right)^2}}{\sqrt{4\pi\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}\left(1+\sqrt{\frac{\lambda(B|\omega)}{\lambda(A|\omega)}}\right), \tag{1.3}$$

$$\Pr(Piv_B|\omega) \approx \frac{1}{2}\frac{e^{-\left(\sqrt{\lambda(A|\omega)}-\sqrt{\lambda(B|\omega)}\right)^2}}{\sqrt{4\pi\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}\left(1+\sqrt{\frac{\lambda(A|\omega)}{\lambda(B|\omega)}}\right). \tag{1.4}$$

1.4.3 Underdog Effect

Based on these approximations, I can prove three important results: (i) the limit ratio of participation rates reflects the underdog effect; (ii) this limit ratio is unique in large elections; and (iii) the underdog effect is monotonic in beliefs, meaning that the limit ratio of participation rates is monotonic in the posteriors.

The underdog effect captures that the supporters of the expected underdog (i.e., the voters who are expected to be in the minority) participate with higher probability than the supporters of the expected leader. This is already a well-known result given the assumption that there is only idiosyncratic uncertainty about voters' preferences. Evren (2012), Myatt (2015), and Myatt (2017) also observe this result. Consider the posterior probabilities $\Pr(\alpha|\tau), \Pr(\beta|\tau)$. Supporters of candidate A are the expected minority if and only if $\Pr(\alpha|\tau) < \Pr(\beta|\tau)$.[15]

Proposition 1.2 (Underdog effect). *Fix some posterior probabilities* $\Pr(\alpha|\tau)$, $\Pr(\beta|\tau)$. *Along all equilibrium sequences,*

(1) if $\Pr(\alpha|\tau) > \Pr(\beta|\tau)$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$, *and*

(2) if $\Pr(\alpha|\tau) < \Pr(\beta|\tau)$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$.

For an intuition, consider the effect of a vote for the expected underdog compared to the expected leader. A vote for the leader increases the expected margin between the candidates and pushes the election further away from a tie. In contrast, a vote for the expected underdog decreases the margin and increases the probability of an election toss-up. Thus, a vote for the underdog is pivotal with higher probability, yielding a higher expected benefit of voting for the supporters of the underdog. Consequently, the supporters of the expected minority candidate turn out at higher rates. Further note that the turnout decision is related to a public goods problem, since (costly) voting is comparable to contributing to the public good. Therefore, just as in the public goods problem, there is an incentive to free ride on the participation of like-minded voters. The underdog effect can be interpreted as a situation in which the free-riding problem is less pronounced among the minority.
Proposition 1.3 establishes the uniqueness of the limit ratio of participation rates.

Proposition 1.3 (Uniqueness). *The limit of the ratio of participation rates,* $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$, *is unique.*

I can now prove that in a large election, the underdog effect that is reflected by the unique limit ratio of participation rates is actually monotonic in the posterior beliefs.

Proposition 1.4 (Monotonicity). *In a large election, the limit of the ratio of participation rates of the A supporters relative to B supporters,* $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$, *is strictly decreasing in* $\Pr(\alpha|\tau)$.

15 If $\Pr(\alpha|\tau) = \Pr(\beta|\tau)$, then Lemma 1.1 has bite.

Consider two poll results τ and τ', and suppose that the state is more likely to be α upon observing τ, than when observing τ'. Then, the limit of the ratio of participation rates of A supporters over those of B supporters is lower under posterior beliefs induced by τ than under posterior beliefs induced by τ'. Intuitively, as posterior beliefs shift toward state α, supporters of candidate A become increasingly optimistic of their victory, leading them to adopt relatively ever lower cost thresholds compared to supporters of candidate B. This monotonicity will be the main driver of poll participants' incentives.

1.4.4 Election Outcome

Because the underdog effect favors the expected minority—attenuating the expected majority candidate's advantage through relatively lower turnout probabilities of the expected majority—one might worry about the implications for election outcomes. That is, one might now worry that the underdog will be more likely to win the election because of this effect. Indeed, in Goeree and Großer (2007), the underdog effect yields a toss-up election, in which both candidates are equally likely to win.

By contrast, in my model, the *partial underdog compensation* result by Herrera, Morelli, and Palfrey (2014) carries over to the present setting with aggregate uncertainty about the distribution of preferences. The advantage of the majority candidate is only partially attenuated by the increased turnout of the minority. Therefore, in both states of the world, the majority candidate wins the election almost surely. The result holds for any fixed posterior belief induced by any poll. In particular, the result is independent of the poll size, the polling outcome, and the poll participants' strategies.

Proposition 1.5. *As $n \to \infty$, in each state of the world, the majority candidate will win the election almost surely, regardless of the poll result. That is, the probability that, in the limit, A will win in state α, and candidate B will win in state β, is 1.*

For an intuition, assume that the state of the world is α, and hence, candidate A is the majority candidate. If posterior beliefs (mistakenly) indicate that the state of the world is more likely to be β, i.e., $\Pr(\beta|\tau) > \frac{1}{2}$, it holds that $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$. So, candidate A will win the election with probability one because A is preferred by the majority and, at the same time, A supporters turn out at higher rates.

The more intricate case is one in which beliefs accurately reflect that the state of the world is more likely to be α, i.e., $\Pr(\alpha|\tau) > \frac{1}{2}$. Here, in the limit, the underdog effect leads B supporters to turn out at higher rates. Proposition 1.4 implies that the limit of the ratio of participation rates is monotonic in the beliefs. Therefore, p_A is lowest relative to p_B if the beliefs are such that citizens are convinced that the state of the world is α—that is, if the aggregate uncertainty about the state of the world is completely resolved. Yet, this is equivalent to a model setup in which the state of the world is known from the outset. For this setup it has already been shown that the underdog effect is only partial (e.g., Herrera, Morelli, and Palfrey (2014)). Intuitively, if the underdog effect were fully compensating such that expected vote shares would be equal, both groups of voters would have the same cost cutoffs because of equal pivot probabilities. Given that the share of A supporters is strictly larger than one-half, this then contradicts equal expected vote shares. Finally, if the majority candidate almost surely wins at the relatively lowest participation rates of the majority, he or she will win for all intermediate cases as well.

1.5 Polling Equilibrium

Having analyzed the equilibrium of the voting subgame for any induced posterior belief, let me now focus on the polling stage. To isolate effects, I first assume that all citizens participating in the opinion poll answer truthfully such that τ represents the participants' true underlying preferences. Understanding the way voters react to exogenously truthful information about the state of the world is necessary to understand poll participants' incentives.

1.5.1 Truthful Reporting

Assuming truthful reporting in the poll, how do different poll results translate into posterior beliefs and, eventually, into participation rates? Consider first the posterior beliefs that are induced if a poll of fixed size is assumed to be answered truthfully, its result being given by (τ_A, τ_B). The derivations in Appendix 1.B reveal that a lead for candidate A in the poll induces posteriors according to which state α is more likely than state β, and that these beliefs are monotonic in the poll's margin. More formally, $\Pr(\alpha|\tau) > \Pr(\beta|\tau)$ if and only if $\tau_A > \tau_B$; and $\Pr(\alpha|\tau)$ is increasing in the poll's margin $\tau_A - \tau_B$ (Claim 1.3). Lastly, if the poll is balanced such that $\tau_A = \tau_B$, posterior beliefs are the same as if no poll had been released. Applying now Lemma 1.1 and Proposition 1.2 yields that if $\tau_A = \tau_B$, A and B supporters will turn out at the same rates, and if the poll favors candidate A, A supporters will turn out at lower rates than B supporters, and vice versa. This is summarized in Corollary 1.1.

Corollary 1.1. *Fix a poll of size m. Assume that polled agents state their preferences truthfully and that the poll result is given by $\tau = (\tau_A, \tau_B)$. Then,*

(1) If no poll is conducted ($m = 0$), or if $\tau_A = \tau_B$, then $p_A(n) = p_B(n)$ for all n.
(2) If $\tau_A > \tau_B$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$.
(3) If $\tau_A < \tau_B$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$.

To summarize, the supporters of the trailing candidate turn out at higher rates. Finally, applying Proposition 1.4 yields that in a large election, this relation is monotonic in the poll result: As the perceived support of candidate A in comparison to candidate B increases—which is measured by an increasing margin $\tau_A - \tau_B$—the participation rate of A supporters relative to the participation rate of B supporters decreases. The result is summarized in Corollary 1.2.

Corollary 1.2. *Fix the poll size $m = \tau_A + \tau_B$, and assume that poll participants state their preferences truthfully. Then, in a large election, the limit of the ratio of participation rates of the A supporters relative to B supporters is strictly increasing in the margin of the poll $\tau_B - \tau_A$, i.e., $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$ is a strictly increasing function of $\tau_B - \tau_A$.*

Importantly, these results imply that each vote in the poll affects the participation rates unambiguously. A vote for candidate A in the poll decreases the participation rate of A supporters relative to the participation rate of B supporters in the limit.

1.5.2 Incentives

In Section 1.5.1, the analysis was based on the premise that citizens participating in the pre-election poll state their preferences truthfully. Suppose now that all poll participants are strategic. Consider the incentives of an A supporter who is questioned by a pollster. Even though poll participants are excluded from the main election, they still have the same stakes concerning the election winner. Therefore, the A supporter seeks to maximize the probability that A wins, which is increasing in $\frac{p_A}{p_B}$. If the A supporter assumes that all other poll participants answer truthfully, Corollary 1.1 and Corollary 1.2 together imply that an additional vote for A in the poll decreases the limit of $\frac{p_A}{p_B}$ in the election. This reveals an incentive to misrepresent the preferences in the poll and yields that truthtelling cannot be an equilibrium. Intuitively, if an A supporter claims to prefer candidate B in the poll, he or she increases free-riding among the B supporters, and simultaneously decreases free-riding among the A supporters, shifting $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$ in a favorable direction.

Likewise, there cannot exist an equilibrium in which everybody misrepresents their preferences because this would be understood and the true preferences could be worked out by the electorate, giving an incentive to deviate to revealing preferences truthfully.

Say that there is information transmission if the voters update their beliefs about the state of the world after observing the poll's publication such that $\Pr(\alpha|\tau) \neq \Pr(\beta|\tau)$, and call the corresponding equilibrium of the polling stage informative. Proposition 1.6 states that there does not exist an informative equilibrium of the polling stage. Consequently, there can only exist the babbling equilibrium and the poll is discarded by the electorate.

Proposition 1.6. *The babbling equilibrium is the unique equilibrium of the polling stage.*

The implications of Proposition 1.6 are as follows: If it is reasonable to assume that all citizens participating in a poll are strategic, the poll result does not convey any information. Thus, eligible voters can ignore it. As a result, the beliefs of the electorate coincide with their prior beliefs. Lemma 1.1 implies that in this case, for all n and along all equilibrium sequences, the probability of turning out to vote is the same for both A and B supporters. Therefore, by the law of large numbers, in each state, the probability that the majority candidate wins goes to one as n grows large. With this in mind, if the poll participants behave strategically—behavior which undermines the purpose of the poll—the results do not reduce the probability that the majority candidate will be elected.

Appendix 1.C extends this result by assuming that a fixed share of poll participants is exogenously truthful. Again, strategic poll participants never have an incentive to truthfully reveal their preferences. Yet, the poll is informative if and only if the share of truthful agents is strictly larger than one-half. In any case, Proposition 1.5 applies, and, in the limit, the majority candidate almost surely wins the election.

1.6 Conclusion

This paper analyzes the effect of pre-election polls on election outcomes. I analyze how information revealed through polls affects the participation decision of citizens in large elections, and the incentives this yields for poll participants. My analysis relies on a framework with aggregate uncertainty about the distribution of preferences, in which the poll is conducted to resolve the aggregate uncertainty; voting is voluntary and costly, and the cost is drawn from a smooth cost distribution.

My main findings are that for any posterior belief about the state of the world induced by the poll (i) there exists a unique limit of the ratio of participation rates. This limit ratio (ii) reflects the underdog effect, and (iii) it is monotonic in the posterior belief. However, the limit ratio of participation rates is (iv) closer to one than the ratio of the respective population shares of supporters of candidates A and B, such that, in the limit, the majority candidate almost surely wins for any given belief. Given the underdog effect and its monotonicity in beliefs, (v) citizens participating in the poll always have an incentive to avoid truthfully reporting their preferences. There does not exist an equilibrium in which the poll provides any information transmission.

My findings contrast with those of Goeree and Großer (2007), who study the effect of exogenously truthful information about the state of the world in a framework in which voting costs are homogenous. This assumption on the cost of voting implies that the authors obtain a *full underdog effect*, where the increased turnout by the minority completely offsets the majority's initial advantage. Because the cost of voting is set such that turnout is incomplete and positive, voters are employing mixed strategies in equilibrium. To be willing to mix, the expected benefit of voting needs to equal the voting cost. Therefore, given homogenous costs for all voters, expected benefits of voting coincide for all voters. These can only be the same if the

expected vote shares coincide, yielding the full underdog effect.[16]
However, if the cost distribution is smooth, and its support is bounded below by zero, cost cutoffs are interior. If expected vote shares would coincide, the participation rates would also be the same, contradicting equal vote shares, because there is a strict majority. The underdog effect can therefore only be *partial*. This has already been observed for idiosyncratic preference uncertainty, e.g. Herrera, Morelli, and Palfrey (2014), and under different forms of aggregate uncertainty, e.g. Evren (2012) or Myatt (2015). Note that these assumptions on the voting costs allow the inclusion of voters who vote because of a sense of duty or ethical reasons. Thus, the assumptions capture the potential for voting costs to differ across voters.
The full underdog effect implies that both candidates are equally likely to win the election, supporting the conclusion that polls are detrimental to welfare. Goeree and Großer (2007) conclude that their results may explain why several countries impose a black-out period prior to elections. By contrast, I show that the partial underdog effect does not result in such a toss-up election. Rather, for any posterior belief induced by the poll, the majority candidate almost surely wins in the limit. This includes the extreme cases where the poll is either uninformative—and the game is as if no poll were conducted in the first place—or perfectly reveals the state of the world. Consequently, my work demonstrates that the conclusions of Goeree and Großer (2007) on the possibly drastic effect of polls do not hold if the model framework is slightly altered.

While, in my model, polls do not overturn election outcomes, polls still matter because the partial underdog effect has real implications. Affecting the turnout margin and vote shares, the partial underdog effect implies that referenda or elections will be closer than they truly are. If vote shares themselves have policy implications, this effect of polls might be concerning. This is an interesting topic which will be left for future research.

[16] The same result is obtained if costs are continuously distributed but bounded away from zero. Then, only those voters with a cost realization at the lower bound turn out to vote, and the argument boils down to the one with fixed costs. This is the case in Taylor and Yildirim (2010a).

Appendix 1.A Proofs

1.A.1 Preliminaries

For the subsequent analysis, it is useful to express the pivot probabilities in terms of modified Bessel functions (cf. Abramowitz and Stegun (1965) and Krishna and Morgan (2012)), which are defined as

$$I_0(z) = \sum_{k=0}^{\infty} \frac{(\frac{z}{2})^k (\frac{z}{2})^k}{k! \cdot k!}, \; I_1(z) = \sum_{k=1}^{\infty} \frac{(\frac{z}{2})^{k-1} (\frac{z}{2})^k}{(k-1)! \cdot k!}.$$

Reformulating the pivot probabilities yields for all $\omega \in \{\alpha, \beta\}$:

$$\begin{aligned}
\Pr(T|\omega) &= e^{-\lambda(A|\omega)-\lambda(B|\omega)} \cdot I_0\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right), \\
\Pr(T_{-1}^{A}|\omega) &= e^{-\lambda(A|\omega)-\lambda(B|\omega)} \cdot \sqrt{\frac{\lambda(B|\omega)}{\lambda(A|\omega)}} \cdot I_1\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right), \\
\Pr(T_{-1}^{B}|\omega) &= e^{-\lambda(A|\omega)-\lambda(B|\omega)} \cdot \sqrt{\frac{\lambda(A|\omega)}{\lambda(B|\omega)}} \cdot I_1\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right).
\end{aligned}$$

So,

$$\begin{aligned}
\Pr(Piv_A|\omega) &= \frac{1}{2}e^{-\lambda(A|\omega)-\lambda(B|\omega)}\left[I_0\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right) + \sqrt{\frac{\lambda(B|\omega)}{\lambda(A|\omega)}} \cdot I_1\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right)\right], \\
\Pr(Piv_B|\omega) &= \frac{1}{2}e^{-\lambda(A|\omega)-\lambda(B|\omega)}\left[I_0\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right) + \sqrt{\frac{\lambda(A|\omega)}{\lambda(B|\omega)}} \cdot I_1\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right)\right].
\end{aligned}$$

For $z \to \infty$, Abramowitz and Stegun (1965) show that $I_0(z) \approx \frac{e^z}{\sqrt{2\pi z}} \approx I_1(z)$.[17]

1.A.2 Proof for Section 1.4.1 (Existence)

Proof of Lemma 1.1.
"If"
Recall that citizens update their beliefs about the state of the world upon observing their own type. It is derived in Appendix 1.B that $\Pr(\alpha|A) = \Pr(\beta|B) = q$.
Assume that the voters' posterior beliefs coincide with these prior beliefs, that is, $\Pr(\alpha|A,\tau) = q = \Pr(\beta|B,\tau)$.
Recall that the cost cutoffs are defined as follows

$$\begin{aligned}
\Pr(\alpha|A,\tau) \cdot \Pr(Piv_A|\alpha) \cdot v + \Pr(\beta|A,\tau) \cdot \Pr(Piv_A|\beta) \cdot v &= c_A^*(n), \\
\Pr(\alpha|B,\tau) \cdot \Pr(Piv_B|\alpha) \cdot v + \Pr(\beta|B,\tau) \cdot \Pr(Piv_B|\beta) \cdot v &= c_B^*(n).
\end{aligned}$$

[17] Suppose x_n and y_n are functions of n. Then, $x_n \approx y_n$ indicates that $\lim_{n\to\infty} \frac{x_n}{y_n} = 1$

Suppose without loss of generality that along some subsequence, there exists n s.t. $p_A(n) > p_B(n) \Leftrightarrow c_A^*(n) > c_B^*(n)$. Suppressing the dependence on n,

$$\begin{aligned}
c_A^* - c_B^* &= q \cdot v \cdot [\Pr(Piv_A|\alpha) - \Pr(Piv_B|\beta)] + (1-q) \cdot v \cdot [\Pr(Piv_A|\beta) - \Pr(Piv_B|\alpha)] \\
&= \frac{1}{2} \cdot q \cdot v \cdot \Big\{ I_0\Big(2\sqrt{n^2 q(1-q) p_A p_B}\Big) \cdot \big(e^{-n[qp_A+(1-q)p_B]} - e^{-n[(1-q)p_A+qp_B]}\big) \\
&+ I_1\Big(2\sqrt{n^2 q(1-q) p_A p_B}\Big) \cdot \sqrt{\frac{1-q}{q}} \cdot \left(\sqrt{\frac{p_B}{p_A}} e^{-n[qp_A+(1-q)p_B]} - \sqrt{\frac{p_A}{p_B}} e^{-n[(1-q)p_A+qp_B]}\right)\Big\} \\
&+ \frac{1}{2}(1-q) \cdot v \cdot \Big\{ I_0\Big(2\sqrt{n^2 q(1-q) p_A p_B}\Big) \cdot \big(e^{-n[(1-q)p_A+qp_B]} - e^{-n[pqp_A+(1-q)p_B]}\big) \\
&+ I_1\Big(2\sqrt{n^2 q(1-q) p_A p_B}\Big) \cdot \sqrt{\frac{q}{1-q}} \cdot \left(\sqrt{\frac{p_B}{p_A}} e^{-n[(1-q)p_A+qp_B]} - \sqrt{\frac{p_A}{p_B}} e^{-n[qp_A+(1-q)p_B]}\right)\Big\}.
\end{aligned}$$

Rearranging yields

$$\begin{aligned}
& c_A^* - c_B^* \\
=& \frac{1}{2} \cdot v \Big\{ (2q-1) \cdot I_0\Big(2\sqrt{n^2 q(1-q) p_A p_B}\Big) \cdot \big(e^{-n[qp_A+(1-q)p_B]} - e^{-n[(1-q)p_A+qp_B]}\big) \\
&+ I_1\Big(2\sqrt{n^2 q(1-q) p_A p_B}\Big) \cdot \sqrt{q(1-q)} \cdot \left[\left(\sqrt{\frac{p_B}{p_A}} - \sqrt{\frac{p_A}{p_B}}\right) \cdot \big(e^{-n[qp_A+(1-q)p_B]} + e^{-n[(1-q)p_A+qp_B]}\big)\right]\Big\} \\
<& 0,
\end{aligned}$$

contradicting the assumption that $c_A^*(n) > c_B^*(n)$.
The inequality holds, since $e^{-n[qp_A+(1-q)p_B]} < e^{-n[(1-q)p_A+qp_B]}$, $q > \frac{1}{2}$, and $\sqrt{\frac{p_B}{p_A}} < \sqrt{\frac{p_A}{p_B}}$ because of $p_A > p_B$.
$c_B^*(n) > c_A^*(n)$ is analogous. Thus, for all equilibrium sequences and for all n, $p_A(n) = p_B(n)$ if $\Pr(\alpha|A,\tau) = \Pr(\alpha|A) = \Pr(\beta|B) = \Pr(\beta|B,\tau)$.

"Only if"
Assume now that for all n, $p_A(n) = p_B(n) =: p$.

$$\begin{aligned}
0 &= c_A^*(n) - c_B^*(n) \\
&= \Pr(\alpha|A,\tau) \cdot \frac{1}{2} e^{-np} \cdot \left[I_0\big(2np\sqrt{q(1-q)}\big) + \sqrt{\frac{1-q}{q}} \cdot I_1\big(2np\sqrt{q(1-q)}\big)\right] \\
&+ \Pr(\beta|A,\tau) \cdot \frac{1}{2} e^{-np} \cdot \left[I_0\big(2np\sqrt{q(1-q)}\big) + \sqrt{\frac{q}{1-q}} \cdot I_1\big(2np\sqrt{q(1-q)}\big)\right] \\
&- \Pr(\alpha|B,\tau) \cdot \frac{1}{2} e^{-np} \cdot \left[I_0\big(2np\sqrt{q(1-q)}\big) + \sqrt{\frac{q}{1-q}} \cdot I_1\big(2np\sqrt{q(1-q)}\big)\right] \\
&- \Pr(\beta|B,\tau) \cdot \frac{1}{2} e^{-np} \cdot \left[I_0\big(2np\sqrt{q(1-q)}\big) + \sqrt{\frac{1-q}{q}} \cdot I_1\big(2np\sqrt{q(1-q)}\big)\right] \\
&= \frac{1}{2} e^{-np} \cdot \Big[I_0\big(2np\sqrt{q(1-q)}\big) \cdot \big(\Pr(\alpha|A,\tau) + \Pr(\beta|A,\tau) - \Pr(\alpha|B,\tau) - \Pr(\beta|B,\tau)\big) \\
&+ I_1\big(2np\sqrt{q(1-q)}\big) \cdot \left(\sqrt{\frac{1-q}{q}}[\Pr(\alpha|A,\tau) - \Pr(\beta|B,\tau)] + \sqrt{\frac{q}{1-q}}[\Pr(\beta|A,\tau) - \Pr(\alpha|B,\tau)]\right)\Big] \\
&= \frac{1}{2} e^{-np} \cdot I_1\big(2np\sqrt{q(1-q)}\big) \\
&\cdot \left(\sqrt{\frac{1-q}{q}}[\Pr(\alpha|A,\tau) - \Pr(\beta|B,\tau)] + \sqrt{\frac{q}{1-q}}[\Pr(\beta|A,\tau) - \Pr(\alpha|B,\tau)]\right),
\end{aligned}$$

where the last step follows from

$$\Pr(\alpha|A,\tau) + \Pr(\beta|A,\tau) - \Pr(\alpha|B,\tau) - \Pr(\beta|B,\tau) \;=\; 0.$$

Now,

$$\begin{aligned}
&\sqrt{\frac{1-q}{q}} \cdot [\Pr(\alpha|A,\tau) - \Pr(\beta|B,\tau)] + \sqrt{\frac{q}{1-q}} \cdot [\Pr(\beta|A,\tau) - \Pr(\alpha|B,\tau)] \\
=&\sqrt{\frac{1-q}{q}} \cdot [\Pr(\alpha|A,\tau) - \Pr(\beta|B,\tau)] + \sqrt{\frac{q}{1-q}} \cdot [(1-\Pr(\alpha|A,\tau)) - (1-\Pr(\beta|B,\tau))] \\
=&\sqrt{\frac{1-q}{q}} \cdot [\Pr(\alpha|A,\tau) - \Pr(\beta|B,\tau)] + \sqrt{\frac{q}{1-q}} \cdot [\Pr(\beta|B,\tau) - \Pr(\alpha|A,\tau)] \\
=&[\Pr(\alpha|A,\tau) - \Pr(\beta|B,\tau)] \cdot \left(\sqrt{\frac{1-q}{q}} + \sqrt{\frac{q}{1-q}}\right).
\end{aligned}$$

This term is zero if and only if $\Pr(\alpha|A,\tau) = \Pr(\beta|B,\tau)$. Since $I_1(\cdot) > 0$, it follows that $p_A(n) = p_B(n) = 0$ holds if and only if $\Pr(\alpha|A,\tau) = \Pr(\beta|B,\tau)$. In Appendix 1.B, I show that this is equivalent to $\Pr(\tau|\alpha) = \Pr(\tau|\beta)$. Thus, the participation rates coincide only if the poll is uninformative and the posterior beliefs are equal to the prior beliefs about the state of the world. □

1.A.3 Proofs for Section 1.4.2 (Pivot Probabilities)

Proof of Lemma 1.2.
Assume, by contradiction, that, along some subsequence, $\lim_{n\to\infty} c_A^*(n) > 0$, implying $\lim_{n\to\infty} p_A(n) > 0$. Then,

$$\lim_{n\to\infty} \Pr(\alpha|A,\tau) \cdot \Pr(Piv_A|\alpha) \cdot v + \Pr(\beta|A,\tau) \cdot \Pr(Piv_A|\beta) \cdot v \;>\; 0,$$

suppressing the dependence on n for notational simplicity.

Since $\lim_{n\to\infty} p_A(n) > 0$, $\lambda(A|\alpha), \lambda(A|\beta) \to \infty$.

If $\lim_{n\to\infty} \sqrt{\lambda(A|\omega)\cdot\lambda(B|\omega)} < \infty$, $\Pr(Piv_A|\omega) \to 0$, since $e^{-\lambda(A|\omega)-\lambda(B|\omega)} \to 0$, $I_0\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right)$ is finite and $e^{-\lambda(A|\omega)-\lambda(B|\omega)}\sqrt{\frac{\lambda(B|\omega)}{\lambda(A|\omega)}} \to 0$.

If $\lim_{n\to\infty} \sqrt{\lambda(A|\omega)\cdot\lambda(B|\omega)} = \infty$, the modified Bessel functions can be approximated by

$$I_0\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right) \;\approx\; \frac{e^{2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}{\sqrt{2\pi\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right)}} \;\approx\; I_1\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right),$$

yielding

$$\Pr(Piv_A|\omega) \;\rightarrow\; \frac{1}{2}\frac{e^{-\left(\sqrt{\lambda(A|\omega)}-\sqrt{\lambda(B|\omega)}\right)^2}}{\sqrt{4\pi\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}\left(1+\sqrt{\frac{\lambda(B|\omega)}{\lambda(A|\omega)}}\right).$$

This probability converges to 0 in both states of the world, since the denominator is unbounded, whereas the numerator is bounded.
But then, in any case,

$$\lim_{n\to\infty} \Pr(\alpha|A,\tau)\cdot\Pr(Piv_A|\alpha)\cdot v + \Pr(\beta|A,\tau)\cdot\Pr(Piv_A|\beta)\cdot v \;=\; 0,$$

contradicting $\lim_{n\to\infty} p_A(n) > 0$. □

Proof of Lemma 1.3.
Recall that

$$\begin{aligned}
\lambda(A|\alpha) &= n\cdot q\cdot p_A,\\
\lambda(A|\beta) &= n\cdot(1-q)\cdot p_A,\\
\lambda(B|\alpha) &= n\cdot(1-q)\cdot p_B,\\
\lambda(B|\beta) &= n\cdot q\cdot p_B.
\end{aligned}$$

Assume, by contradiction, that it is not true that $\forall i \in \{A,B\}\ \forall\omega\in\{\alpha,\beta\}$, $\liminf_{n\to\infty}\lambda(i|\omega)=\infty$.
Suppose first that along some subsequence, $\lambda(A|\alpha),\lambda(A|\beta),\lambda(B|\alpha),\lambda(B|\beta)<\infty$ as $n\to\infty$, i.e., the expected number of votes for each candidate is finite in each state. Then, along this subsequence, the pivot probabilities are strictly positive in every state: $\Pr(Piv_A|\omega)>0, \Pr(Piv_B|\omega)>0\ \forall\omega$.

This implies that $\lim_{n\to\infty} c_i^*(n) > 0$ for $i\in\{A,B\}$, and, given the assumptions on the cdf F and the corresponding density f, the participation rates p_A, p_B must remain strictly positive in the limit as $n\to\infty$: $\lim_{n\to\infty} p_i(n)>0$ for $i\in\{A,B\}$. But then, expected turnout must go to infinity for every candidate in every state as $n\to\infty$—a contradiction.

Suppose now that along some subsequence, $\lambda(A|\alpha)<\infty$ and $\lambda(B|\alpha)\to\infty$ as $n\to\infty$.
Given the definitions of $\lambda(\cdot|\omega)$, this implies that $\lambda(A|\beta)<\infty$ and $\lambda(B|\beta)\to\infty$ as $n\to\infty$.
Consider $\lim_{n\to\infty}\frac{\lambda(B|\beta)}{\lambda(A|\alpha)}$:

$$\lim_{n\to\infty}\frac{\lambda(B|\beta)}{\lambda(A|\alpha)} \;=\; \lim_{n\to\infty}\frac{n\cdot q\cdot p_B(n)}{n\cdot q\cdot p_A(n)} \;=\; \lim_{n\to\infty}\frac{p_B(n)}{p_A(n)}.$$

Since $\lim_{n\to\infty} p_i(n) = 0$, $i \in \{A, B\}$, a Taylor expansion of F around zero yields

$$\begin{aligned}
\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} &= \lim_{n\to\infty} \frac{F(c_A^*(n))}{F(c_B^*(n))} \\
&\approx \lim_{n\to\infty} \frac{F(0) + f(0)\cdot(c_A^*(n)-0) + \frac{1}{2}f'(0)(c_A^*(n)-0)^2 + \dots}{F(0) + f(0)\cdot(c_B^*(n)-0) + \frac{1}{2}f'(0)(c_B^*(n)-0)^2 + \dots} \\
&\approx \lim_{n\to\infty} \frac{f(0)\cdot c_A^*(n)}{f(0)\cdot c_B^*(n)} \\
&= \lim_{n\to\infty} \frac{c_A^*(n)}{c_B^*(n)}.
\end{aligned}$$

By the assumption above, $\lim_{n\to\infty} \frac{\lambda(B|\beta)}{\lambda(A|\alpha)} = \infty$ and this implies

$$\lim_{n\to\infty} \frac{c_B^*(n)}{c_A^*(n)} = \infty.$$

For ease of exposition, define for the following step $z := \lambda(A|\alpha)$ and $y := \lambda(B|\beta)$ and observe that $\lambda(A|\beta) = \frac{1-q}{q}z$ and $\lambda(B|\beta) = \frac{q}{1-q}y$. Since $\lambda(B|\beta)/\lambda(A|\alpha) \to \infty$, $y/z \to \infty$.
Working towards a contradiction, derive an expression for $\lim_{n\to\infty} \frac{c_B^*(n)}{c_A^*(n)}$.

$$\begin{aligned}
&\lim_{n\to\infty} \frac{c_B^*(n)}{c_A^*(n)} \\
=&\lim_{n\to\infty} \frac{\Pr(\alpha|B,\tau)\cdot\Pr(Piv_B|\alpha) + \Pr(\beta|B,\tau)\cdot\Pr(Piv_B|\beta)}{\Pr(\alpha|A,\tau)\cdot\Pr(Piv_A|\alpha) + \Pr(\beta|A,\tau)\cdot\Pr(Piv_A|\beta)} \\
=&\lim_{\frac{y}{z}\to\infty} \frac{\Pr(\alpha|B,\tau)\frac{1}{2}\frac{e^{-(\sqrt{z}-\sqrt{y})^2}}{\sqrt{4\pi\sqrt{zy}}}\left(1+\sqrt{\frac{z}{y}}\right) + \Pr(\beta|B,\tau)\frac{1}{2}\frac{e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2}}{\sqrt{4\pi\sqrt{zy}}}\left(1+\frac{1-q}{q}\sqrt{\frac{z}{y}}\right)}{\Pr(\alpha|A,\tau)\frac{1}{2}\frac{e^{-(\sqrt{z}-\sqrt{y})^2}}{\sqrt{4\pi\sqrt{zy}}}\left(1+\sqrt{\frac{y}{z}}\right) + \Pr(\beta|A,\tau)\frac{1}{2}\frac{e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2}}{\sqrt{4\pi\sqrt{zy}}}\left(1+\frac{q}{1-q}\sqrt{\frac{y}{z}}\right)} \\
=&\lim_{\frac{y}{z}\to\infty} \frac{\Pr(\alpha|B,\tau)e^{-(\sqrt{z}-\sqrt{y})^2}\left(1+\sqrt{\frac{z}{y}}\right) + \Pr(\beta|B,\tau)e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2}\left(1+\frac{1-q}{q}\sqrt{\frac{z}{y}}\right)}{\Pr(\alpha|A,\tau)e^{-(\sqrt{z}-\sqrt{y})^2}\left(1+\sqrt{\frac{y}{z}}\right) + \Pr(\beta|A,\tau)e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2}\left(1+\frac{q}{1-q}\sqrt{\frac{y}{z}}\right)} \\
=&\lim_{\frac{y}{z}\to\infty} \frac{\Pr(\alpha|B,\tau)\left(1+\sqrt{\frac{z}{y}}\right) + \Pr(\beta|B,\tau)e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2+(\sqrt{z}-\sqrt{y})^2}\left(1+\frac{1-q}{q}\sqrt{\frac{z}{y}}\right)}{\Pr(\alpha|A,\tau)\left(1+\sqrt{\frac{y}{z}}\right) + \Pr(\beta|A,\tau)e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2+(\sqrt{z}-\sqrt{y})^2}\left(1+\frac{q}{1-q}\sqrt{\frac{y}{z}}\right)} \\
\leq& 1.
\end{aligned}$$

The last step follows from $\lim_{\frac{y}{z}\to\infty} e^{-(\sqrt{\frac{1-q}{q}z}-\sqrt{\frac{q}{1-q}y})^2+(\sqrt{z}-\sqrt{y})^2} = 0$ and $Pr(\alpha|A,\tau) > Pr(\alpha|B,\tau)$. The second step follows from $y \to \infty$, allowing to apply the approximation of the modified Bessel functions.

Overall, $\lim_{n\to\infty}\frac{c_B^*(n)}{c_A^*(n)} \leq 1$ contradicts $\lim_{n\to\infty}\frac{c_B^*(n)}{c_A^*(n)} = \infty$.
The case in which $\lambda(B|\omega) < \infty$ and $\lambda(A|\omega) \to \infty$ is analogous.
Therefore, it must be the case that expected turnout goes to infinity for each candidate and in each state when n goes to infinity.
From the last part of the proof it follows immediately that it is not possible that along some subsequence either $\liminf_{n\to\infty}\frac{p_A(n)}{p_B(n)} = 0$ or that $\limsup_{n\to\infty}\frac{p_A(n)}{p_B(n)} = \infty$, proving the lemma. □

Proof of Lemma 1.4.
Since Lemma 1.3 implies that $2\sqrt{\lambda(A|\omega)\lambda(B|\omega)} \to \infty$ as $n \to \infty$, by Abramowitz and Stegun (1965), it holds that

$$I_0\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right) \approx \frac{e^{2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}{\sqrt{2\pi\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right)}} \approx I_1\left(2\sqrt{\lambda(A|\omega)\lambda(B|\omega)}\right).$$

Therefore, the pivot probabilities can indeed be approximated by

$$\Pr(Piv_A|\omega) \approx \frac{1}{2}\frac{e^{-\left(\sqrt{\lambda(A|\omega)}-\sqrt{\lambda(B|\omega)}\right)^2}}{\sqrt{4\pi\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}\left(1+\sqrt{\frac{\lambda(B|\omega)}{\lambda(A|\omega)}}\right),$$
$$\Pr(Piv_B|\omega) \approx \frac{1}{2}\frac{e^{-\left(\sqrt{\lambda(A|\omega)}-\sqrt{\lambda(B|\omega)}\right)^2}}{\sqrt{4\pi\sqrt{\lambda(A|\omega)\lambda(B|\omega)}}}\left(1+\sqrt{\frac{\lambda(A|\omega)}{\lambda(B|\omega)}}\right).$$

□

1.A.4 Proofs for Section 1.4.3 (Underdog Effect)

Proof of Proposition 1.2.
Before commencing the proof, let me state some preliminary claims.
Preliminaries:
By the Taylor expansion from the proof of Lemma 1.3,

$$\lim_{n\to\infty}\frac{p_A(n)}{p_B(n)} = \lim_{n\to\infty}\frac{c_A^*(n)}{c_B^*(n)}.$$

Thus, suppressing the dependence of p_i on n, for $n \to \infty$,

$$\frac{p_A}{p_B} \approx \frac{Pr(\alpha|A,\tau)e^{-(\sqrt{nqp_A}-\sqrt{n(1-q)p_B})^2}\left(1+\sqrt{\frac{1-q}{q}}\sqrt{\frac{p_B}{p_A}}\right)+Pr(\beta|A,\tau)e^{-(\sqrt{n(1-q)p_A}-\sqrt{nqp_B})^2}\left(1+\sqrt{\frac{q}{1-q}}\sqrt{\frac{p_B}{p_A}}\right)}{Pr(\alpha|B,\tau)e^{-(\sqrt{nqp_A}-\sqrt{n(1-q)p_B})^2}\left(1+\sqrt{\frac{q}{1-q}}\sqrt{\frac{p_A}{p_B}}\right)+Pr(\beta|B,\tau)e^{-(\sqrt{n(1-q)p_A}-\sqrt{nqp_B})^2}\left(1+\sqrt{\frac{1-q}{q}}\sqrt{\frac{p_A}{p_B}}\right)}.$$

If $qp_A \neq (1-q)p_B$, $\left(\sqrt{nqp_A}-\sqrt{n(1-q)p_B}\right)^2$ diverges for $n \to \infty$, and
if $(1-q)p_B \neq qp_B$, $\left(\sqrt{n(1-q)p_A}-\sqrt{nqp_B}\right)^2$ diverges for $n \to \infty$, given that $q \neq \frac{1}{2}$.
This implies that for $n \to \infty$

$$p_A > p_B \Rightarrow e^{-\left(\sqrt{nqp_A}-\sqrt{n(1-q)p_B}\right)^2+\left(\sqrt{n(1-q)p_A}-\sqrt{nqp_B}\right)^2} \to 0, \tag{1.A.1}$$

$$p_A < p_B \Rightarrow e^{-\left(\sqrt{n(1-q)p_A}-\sqrt{nqp_B}\right)^2+\left(\sqrt{nqp_A}-\sqrt{n(1-q)p_B}\right)^2} \to 0. \tag{1.A.2}$$

I will now prove the proposition.
Assume without loss that candidate A is the underdog, that is, $\Pr(\alpha|\tau) < \Pr(\beta|\tau)$ and suppose, by contradiction, that $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} \le 1$.

The assumption that $Pr(\alpha|\tau) < Pr(\beta|\tau)$ together with Lemma 1.1 imply that for all n, $p_A(n) \neq p_B(n)$. Therefore, it must hold that for n sufficiently large, $p_A(n) < p_B(n)$. By (1.A.2), as $n \to \infty$,

$$\begin{aligned}\frac{p_A(n)}{p_B(n)} &\to \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \frac{\frac{\sqrt{qp_A(n)}+\sqrt{(1-q)p_B(n)}}{\sqrt{qp_A(n)}}}{\frac{\sqrt{qp_A(n)}+\sqrt{(1-q)p_B(n)}}{\sqrt{(1-q)p_B(n)}}} \\ &= \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \sqrt{\frac{1-q}{q}} \cdot \sqrt{\frac{p_B(n)}{p_A(n)}}.\end{aligned}$$

However,

$$\begin{aligned}\frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} &= \frac{Pr(A|\alpha)}{Pr(B|\alpha)} \cdot \frac{Pr(B|\alpha)Pr(\tau|\alpha) + Pr(B|\beta)Pr(\tau|\beta)}{Pr(A|\alpha)Pr(\tau|\alpha) + Pr(A|\beta)Pr(\tau|\beta)} \\ &= \frac{q}{1-q} \cdot \frac{(1-q)Pr(\tau|\alpha) + q \cdot Pr(\tau|\beta)}{q \cdot Pr(\tau|\alpha) + (1-q)Pr(\tau|\beta)} \\ &> \frac{q}{1-q},\end{aligned}$$

where the last inequality holds, since given Claim 1.1,

$$\begin{aligned}&Pr(\alpha|\tau) < Pr(\beta|\tau) \iff Pr(\tau|\alpha) < Pr(\tau|\beta) \\ \Rightarrow\ & \frac{(1-q)Pr(\tau|\alpha) + q \cdot Pr(\tau|\beta)}{q \cdot Pr(\tau|\alpha) + (1-q)Pr(\tau|\beta)} > 1.\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{p_A(n)}{p_B(n)} &\to \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \sqrt{\frac{1-q}{q}} \cdot \sqrt{\frac{p_B(n)}{p_A(n)}} \\ &> \sqrt{\frac{q}{1-q}} \cdot \sqrt{\frac{p_B(n)}{p_A(n)}} \\ &> 1,\end{aligned}$$

a contradiction to $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} \le 1$!
The proof for the case in which B is the underdog is analogous and therefore omitted.

□

Proof of Proposition 1.3.
From Proposition 1.2 and Lemma 1.1, along all equilibrium sequences, either $p_A(n) = p_B(n)$ for all n, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$ or $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$.

If $p_A(n) = p_B(n)$, the claim obviously holds.

Next, assume that in equilibrium, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$.
Then, for n sufficiently large, $p_A(n) < p_B(n)$ and by (1.A.2), as $n \to \infty$,

$$\begin{aligned}
\frac{p_A(n)}{p_B(n)} &\approx \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \frac{\frac{\sqrt{qp_A(n)}+\sqrt{(1-q)p_B(n)}}{\sqrt{qp_A(n)}}}{\frac{\sqrt{(1-q)p_B(n)}+\sqrt{qp_A(n)}}{\sqrt{(1-q)p_B(n)}}} \\
&= \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \sqrt{\frac{1-q}{q}}\sqrt{\frac{p_B(n)}{p_A(n)}} \\
\Rightarrow \lim_{n\to\infty} \left(\frac{p_A(n)}{p_B(n)}\right)^{\frac{3}{2}} &= \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \sqrt{\frac{1-q}{q}}.
\end{aligned}$$

Since the left-hand side is strictly increasing in $\frac{p_A(n)}{p_B(n)}$ and the right-hand side is independent of $\frac{p_A(n)}{p_B(n)}$, the limit of the ratio of participation rates is unique for any equilibrium sequence that satisfies $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$.

Analogously, the limit of the ratio of participation rates is unique for any equilibrium sequence that satisfies $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$. □

Proof of Proposition 1.4.
Consider two poll results τ, τ' such that $\Pr(\alpha|\tau') < \Pr(\alpha|\tau)$. Abusing notation, denote by p'_A, p'_B the equilibrium participation rates if the poll result is τ' and by p_A, p_B the participation rates if the poll result is τ. Recall that given some beliefs, the limit of the ratio of participation rates is unique. Thus, the limit of the ratio of participation rates is monotonic in the beliefs if and only if $\Pr(\alpha|\tau') < \Pr(\alpha|\tau)$ implies $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < \lim_{n\to\infty} \frac{p'_A(n)}{p'_B(n)}$.

Assume, by contradiction, that there exist poll results τ, τ' with $\Pr(\alpha|\tau') < \Pr(\alpha|\tau)$ such that $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} \geq \lim_{n\to\infty} \frac{p'_A(n)}{p'_B(n)}$.

<u>Case 1</u>: $\Pr(\alpha|\tau') < \Pr(\alpha|\tau) < \frac{1}{2}$.
Then, by Proposition 1.2, $\lim_{n\to\infty} \frac{p'_A(n)}{p'_B(n)} > 1$. Thus, by (1.A.1) and by the Taylor approximation around zero,

$$
\begin{aligned}
\lim_{n\to\infty}\frac{p'_A(n)}{p'_B(n)} &\le \lim_{n\to\infty}\frac{p_A(n)}{p_B(n)}\\
\Leftrightarrow \frac{\Pr(\beta|A,\tau')}{\Pr(\beta|B,\tau')}\cdot\sqrt{\lim_{n\to\infty}\frac{p'_B(n)}{p'_A(n)}} &\le \frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)}\cdot\sqrt{\lim_{n\to\infty}\frac{p_B(n)}{p_A(n)}}\\
\Leftrightarrow \frac{\frac{\Pr(\beta|A,\tau')}{\Pr(\beta|B,\tau')}}{\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)}} &\le \sqrt{\frac{\lim_{n\to\infty}\frac{p_B(n)}{p_A(n)}(x)}{\lim_{n\to\infty}\frac{p'_B(n)}{p'_A(n)}}} \le 1.
\end{aligned}
$$

By Claim 1.2, $\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)}$ is strictly decreasing in $\Pr(\alpha|\tau)$, meaning that the left hand side of the above inequality is strictly larger than 1—a contradiction!

Case 2: $\Pr(\alpha|\tau') < \frac{1}{2} \le \Pr(\alpha|\tau)$
Then, by Proposition 1.2, $\lim_{n\to\infty}\frac{p_A(n)}{p_B(n)} \le 1$ for all n and $\lim_{n\to\infty}\frac{p'_A(n)}{p'_B(n)} > 1$, a contradiction.

Case 3: $\Pr(\alpha|\tau') \le \frac{1}{2} < \Pr(\alpha|\tau)$
Then, $\lim_{n\to\infty}\frac{p'_A(n)}{p'_B(n)} \ge 1$ for all n and $\lim_{n\to\infty}\frac{p_A(n)}{p_B(n)} < 1$, a contradiction.

Case 4: $\frac{1}{2} < \Pr(\alpha|\tau') < \Pr(\alpha|\tau)$
By Proposition 1.2 and by (1.A.2),

$$
\begin{aligned}
\lim_{n\to\infty}\frac{p'_A(n)}{p'_B(n)} &\le \lim_{n\to\infty}\frac{p_A(n)}{p_B(n)}\\
\Leftrightarrow \frac{\Pr(\alpha|A,\tau')}{\Pr(\alpha|B,\tau')}\cdot\sqrt{\lim_{n\to\infty}\frac{p'_B(n)}{p'_A(n)}} &\le \frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)}\cdot\sqrt{\lim_{n\to\infty}\frac{p_B(n)}{p_A(n)}}\\
\Leftrightarrow \frac{\frac{\Pr(\alpha|A,\tau')}{\Pr(\alpha|B,\tau')}}{\frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)}} &\le \sqrt{\frac{\lim_{n\to\infty}\frac{p_B(n)}{p_A(n)}(x)}{\lim_{n\to\infty}\frac{p'_B(n)}{p'_A(n)}}} \le 1.
\end{aligned}
$$

However, by Claim 1.2, $\frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)}$ is strictly decreasing in $\Pr(\alpha|\tau)$, implying that $\frac{\frac{\Pr(\alpha|A,\tau')}{\Pr(\alpha|B,\tau')}}{\frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)}} > 1$—a contradiction!

Thus, the limit of the ratio of participation rates is monotonic in the beliefs. □

1.A.5 Proof for Section 1.4.4 (Election Outcome)

Proof of Proposition 1.5.
Candidate A is the majority candidate in state α and candidate B is the majority

candidate in state β. As $n \to \infty$, by the law of large numbers, the majority candidate wins the election in each state if and only if

$$\frac{1-q}{q} < \lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < \frac{q}{1-q}.$$

If $\tau_A = \tau_B$ or if no poll is considered, for all n, $p_A(n) = p_B(n) =: \hat{p}(n)$. Then, for all n, $\frac{p_A(n)}{p_B(n)} = 1$ and the result holds.
If $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$, by (1.A.2), as $n \to \infty$,

$$\begin{aligned}
\frac{q}{1-q} \cdot \frac{p_A(n)}{p_B(n)} &\approx \frac{q}{1-q} \cdot \frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} \cdot \sqrt{\frac{1-q}{q}} \sqrt{\frac{p_B(n)}{p_A(n)}} \\
&> \frac{q}{1-q} \cdot \sqrt{\frac{1-q}{q}} \sqrt{\frac{p_B(n)}{p_A(n)}} \\
&= \sqrt{\frac{q}{1-q}} \sqrt{\frac{p_B(n)}{p_A(n)}} \\
&> 1,
\end{aligned}$$

where the third to last step follows from $\frac{Pr(\alpha|A,\tau)}{Pr(\alpha|B,\tau)} > 1$, which is derived in Appendix B, and the last step follows because $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$ and $q > \frac{1}{2}$ by assumption.
Finally, if $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$, by (1.A.1), as $n \to \infty$,

$$\begin{aligned}
\frac{q}{1-q} \cdot \frac{p_B(n)}{p_A(n)} &\to \frac{q}{1-q} \cdot \frac{Pr(\beta|B,\tau)}{Pr(\beta|A,\tau)} \cdot \sqrt{\frac{1-q}{q}} \sqrt{\frac{p_A(n)}{p_B(n)}} \\
&> \frac{q}{1-q} \cdot \sqrt{\frac{1-q}{q}} \sqrt{\frac{p_A(n)}{p_B(n)}} \\
&= \sqrt{\frac{q}{1-q}} \sqrt{\frac{p_A(n)}{p_B(n)}} \\
&> 1,
\end{aligned}$$

since $\frac{Pr(\beta|B,\tau)}{Pr(\beta|A,\tau)} > 1$.
Therefore, as $n \to \infty$, candidate A wins in state α and candidate B wins in state β with probability 1, so the majority candidate is elected almost surely. □

1.A.6 Proof for Section 1.5.2 (Incentives)

Proof of Proposition 1.6.
Following the derivations in Appendix 1.C, in particular Proposition 1.7, and setting $\gamma = 0$ immediately yields that the unique equilibrium strategy prescribes poll participants to reveal their preferences truthfully with probability $\frac{1}{2}$ ($\mu = \frac{1}{2}$). Consequently, $\Pr(\alpha|\tau) = \Pr(\beta|\tau)$, and babbling is the unique equilibrium of the polling stage.

□

Appendix 1.B Posterior Beliefs

This section is concerned with the derivation of posterior beliefs about the state of the world, firstly after observing one's own preference type, and secondly after additionally observing the result of the pre-election poll. Further, useful properties of related conditional probabilities will be derived.

Priors upon observing the preference type

Recall that it is assumed that $\Pr(\alpha) = \Pr(\beta) = \frac{1}{2}$ and that $\Pr(A|\alpha) = q = \Pr(B|\beta)$, where the $\Pr(A|\alpha)$ indicates the probability that a randomly drawn citizen prefers candidate A over B given that the state is α. Learning about his or her own preferences, a citizen updates his or her beliefs about the state of the world as follows:

$$\begin{aligned}
\Pr(\omega = \alpha|A) &= \frac{\Pr(\omega = \alpha, A)}{\Pr(A)} = \frac{q}{q \cdot \frac{1}{2} + (1-q)\frac{1}{2}} = q, \\
\Pr(\beta|A) &= 1 - q, \\
\Pr(\alpha|B) &= 1 - q, \\
\Pr(\beta|B) &= q.
\end{aligned}$$

Posteriors after observing the poll

Additionally observing the pre-election poll result $\tau = (\tau_A, \tau_B)$ yields the posterior beliefs

$$\begin{aligned}
\Pr(\alpha|A, \tau) &= \frac{\Pr(\alpha, \tau, A)}{\Pr(\tau, A)} \\
&= \frac{\Pr(A, \tau|\alpha) \cdot \Pr(\alpha)}{\Pr(A, \tau|\alpha) \cdot \Pr(\alpha) + \Pr(A, \tau|\beta) \cdot \Pr(\beta)} \\
&= \frac{\Pr(\tau|\alpha) \cdot \Pr(A|\alpha) \cdot \Pr(\alpha)}{\Pr(\tau|\alpha) \cdot \Pr(A|\alpha) \cdot \Pr(\alpha) + \Pr(\tau|\beta) \cdot \Pr(A|\beta) \cdot \Pr(\beta)},
\end{aligned}$$

where $\Pr(\tau|\omega)$ denotes the posterior probability that the state is ω if the poll result is τ.
For $q > \frac{1}{2}$,

$$\Pr(\alpha|A, \tau) > \Pr(\alpha|B, \tau), \ \Pr(\beta|B, \tau) > \Pr(\beta|A, \tau),$$

and

$$\Pr(\alpha|A, \tau) - \Pr(\alpha|B, \tau) = \Pr(\beta|B, \tau) - \Pr(\beta|A, \tau).$$

The following relation will be prove useful:

Claim 1.1.

$$\Pr(\alpha|A, \tau) < \Pr(\beta|B, \tau) \iff \Pr(\tau|\alpha) < \Pr(\tau|\beta) \iff \Pr(\alpha|\tau) < \Pr(\beta|\tau)$$

Proof.

$$\frac{\Pr(\alpha|A,\tau)}{\Pr(\beta|B,\tau)} = \frac{\Pr(\tau|\alpha)}{\Pr(\tau|\beta)} \cdot \frac{(1-q)\cdot\Pr(\tau|\alpha) + q\cdot\Pr(\tau|\beta)}{q\cdot\Pr(\tau|\alpha) + (1-q)\cdot\Pr(\tau|\beta)}$$

$$\Rightarrow \frac{\Pr(\alpha|A,\tau)}{\Pr(\beta|B,\tau)} \begin{cases} = 1 & \Leftrightarrow \Pr(\tau|\alpha) = \Pr(\tau|\beta) \\ > 1 & \Leftrightarrow \Pr(\tau|\alpha) > \Pr(\tau|\beta) \\ < 1 & \Leftrightarrow \Pr(\tau|\alpha) < \Pr(\tau|\beta), \end{cases}$$

and further,

$$\Pr(\alpha|\tau) = \frac{\Pr(\tau|\alpha)\cdot\Pr(\alpha)}{\Pr(\tau)} = \frac{\Pr(\tau|\alpha)}{2\cdot\Pr(\tau)},$$

$$\Rightarrow \Pr(\alpha|\tau) < \Pr(\beta|\tau) \Leftrightarrow \Pr(\tau|\alpha) < \Pr(\tau|\beta).$$

□

The following claim is used in the proof of Proposition 1.4.

Claim 1.2. $\frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)}$ *and* $\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)}$ *are strictly decreasing in* $\Pr(\alpha|\tau)$.

Proof. By the derivations above and since $\Pr(\alpha) = \Pr(\beta)$,

$$\begin{aligned} \frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)} &= \frac{q}{1-q}\frac{\Pr(\tau|\alpha)(1-q) + \Pr(\tau|\beta)q}{\Pr(\tau|\alpha)q + \Pr(\tau|\beta)(1-q)} \\ \frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} &= \frac{1-q}{q}\frac{\Pr(\tau|\alpha)(1-q) + \Pr(\tau|\beta)q}{\Pr(\tau|\alpha)q + \Pr(\tau|\beta)(1-q)}. \end{aligned}$$

Since $\Pr(\tau|\omega) = 2\Pr(\omega|\tau)\Pr(\tau)$ for $\omega \in \{\alpha,\beta\}$, and $\Pr(\beta|\tau) = 1 - \Pr(\alpha|\tau)$,

$$\begin{aligned} \frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)} &= \frac{q}{1-q}\frac{\Pr(\alpha|\tau)(1-2q) + q}{\Pr(\alpha|\tau)(2q-1) + (1-q)} \\ \frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} &= \frac{1-q}{q}\frac{\Pr(\alpha|\tau)(1-2q) + q}{\Pr(\alpha|\tau)(2q-1) + (1-q)}. \end{aligned}$$

Since

$$\frac{d}{d\Pr(\alpha|\tau)}\frac{\Pr(\alpha|\tau)(1-2q) + q}{\Pr(\alpha|\tau)(2q-1) + (1-q)} = \frac{1-2q}{(\Pr(\alpha|\tau)(2q-1) + 1 - q)^2} < 0,$$

the claim follows, since $q > \frac{1}{2}$. □

Truthfully answered polls

Assuming now that the poll was answered truthfully, the posterior beliefs become

$$\begin{aligned}
\Pr(\alpha|\tau) &= \frac{\binom{\tau_A+\tau_B}{\tau_A}\cdot q^{\tau_A}\cdot(1-q)^{\tau_B}}{\binom{\tau_A+\tau_B}{\tau_A}\cdot\left(q^{\tau_A}\cdot(1-q)^{\tau_B}+(1-q)^{\tau_A}\cdot q^{\tau_B}\right)},\\
&= \frac{1}{1+(\frac{q}{1-q})^{\tau_B-\tau_A}},\\
\Pr(\beta|\tau) &= \frac{1}{1+(\frac{q}{1-q})^{\tau_A-\tau_B}}.
\end{aligned}$$

Thus, $\tau_A > \tau_B \iff \Pr(\alpha|\tau) > \Pr(\beta|\tau)$ and $\Pr(\alpha|\tau) = \Pr(\beta|\tau)$ if and only if $\tau_A = \tau_B$.

Claim 1.3. *The posterior probability* $\Pr(\alpha|\tau)$ *is increasing in* $\tau_A - \tau_B$.

Proof.

$$\frac{d}{d(\tau_A-\tau_B)}\Pr(\alpha|\tau) = \frac{(\frac{q}{1-q})^{(\tau_B-\tau_A)}\log(\frac{q}{1-q})}{\left((\frac{q}{1-q})^{(\tau_B-\tau_A)}+1\right)^2} > 0.$$

□

$$\begin{aligned}
\Pr(\alpha|A,\tau) &= \frac{\binom{\tau_A+\tau_B}{\tau_A}\cdot q^{\tau_A}\cdot(1-q)^{\tau_B}\cdot q\cdot\frac{1}{2}}{\binom{\tau_A+\tau_B}{\tau_A}\cdot\frac{1}{2}\cdot(q^{\tau_A}\cdot(1-q)^{\tau_B}\cdot q+(1-q)^{\tau_A}\cdot q^{\tau_B}\cdot(1-q))},\\
&= \frac{1}{1+(\frac{q}{1-q})^{-\tau_A+\tau_B-1}},\\
\Pr(\beta|A,\tau) &= \frac{1}{1+(\frac{q}{1-q})^{\tau_A-\tau_B+1}},\\
\Pr(\alpha|B,\tau) &= \frac{1}{1+(\frac{q}{1-q})^{-\tau_A+\tau_B+1}},\\
\Pr(\beta|B,\tau) &= \frac{1}{1+(\frac{q}{1-q})^{\tau_A-\tau_B-1}}.
\end{aligned}$$

Note that $\Pr(\omega|A) = \Pr(\omega|A,\tau)$ if and only if either $\tau_A = \tau_B$ or $q = \frac{1}{2}$. This implies that a balanced poll where $\tau_A = \tau_B$ has the same effect as if no poll was published at all.

Appendix 1.C Polls with Exogenously Truthful Participants

In this extension, I prescribe a share $\gamma > 0$ of poll participants to always state their preferences truthfully.[18] The share $1-\gamma$ is strategic, and states preferences truthfully with probability $\mu \in [0,1]$. I show that the majority candidate is elected with probability one in the limit. Further, I demonstrate how the underdog effect extends, and how this affects the incentives of the strategic share of poll participants.

Fix the strategy $\mu \in [0,1]$, and denote the probability that a poll participant states a preference for candidate i in state ω by $\Pr(\text{“}i\text{”}|\omega)$. Define $\kappa := \Pr(\text{“}A''|\alpha)$. Then,

$$\begin{aligned}
\Pr(\text{“}A\text{”}|\alpha) &= q \cdot (\gamma \cdot 1 + (1-\gamma)\cdot\mu) + (1-q)\cdot(1-\gamma)\cdot(1-\mu), \\
&=: \kappa, \\
&= \Pr(\text{“}B\text{”}|\beta), \\
\Pr(\text{“}B\text{”}|\alpha) &= q\cdot(1-\gamma)\cdot(1-\mu) + (1-q)\cdot(\gamma\cdot 1 + (1-\gamma)\cdot\mu), \\
&= 1-\kappa, \\
&= \Pr(\text{“}A\text{”}|\beta).
\end{aligned}$$

Note that $\kappa = q$ if and only if $\gamma = 1$ or $\mu = 1$, and $\kappa < q$ else.
The posterior beliefs after observing the poll become

$$\begin{aligned}
\Pr(\alpha|A,\tau) &= \frac{\binom{\tau_A+\tau_B}{\tau_A}\cdot\kappa^{\tau_A}\cdot(1-\kappa)^{\tau_B}\cdot q\cdot\frac{1}{2}}{\binom{\tau_A+\tau_B}{\tau_A}\cdot\frac{1}{2}\cdot[\kappa^{\tau_A}\cdot(1-\kappa)^{\tau_B}\cdot q + (1-\kappa)^{\tau_A}\cdot\kappa^{\tau_B}\cdot(1-q)]}, \\
&= \frac{1}{1+(\frac{\kappa}{1-\kappa})^{\tau_B-\tau_A}\cdot\frac{1-q}{q}}, \\
\Pr(\beta|A,\tau) &= \frac{1}{1+(\frac{\kappa}{1-\kappa})^{\tau_A-\tau_B}\cdot\frac{q}{1-q}}, \\
\Pr(\alpha|B,\tau) &= \frac{1}{1+(\frac{\kappa}{1-\kappa})^{\tau_B-\tau_A}\cdot\frac{q}{1-q}}, \\
\Pr(\beta|B,\tau) &= \frac{1}{1+(\frac{\kappa}{1-\kappa})^{\tau_A-\tau_B}\cdot\frac{1-q}{q}}.
\end{aligned}$$

Lemma 1.5 reveals that the underdog effect depends on κ. For any fixed μ, if $\kappa > \frac{1}{2}$, as before, supporters of the candidate obtaining the higher vote count in the poll turn out at lower rates. Intuitively, as $\kappa > \frac{1}{2}$, a vote for A is more likely to occur if the state is α, and, thus, leads voters to update their beliefs toward α. However, if $\kappa < \frac{1}{2}$, the effect is reversed. Supporters of the candidate with the higher vote count in the poll turn out at higher rates because voters understand that $\tau_A > \tau_B$

[18] If $\gamma = 0$, all poll participants are strategic. Plugging in $\gamma = 0$ in the analysis below yields the proof for Proposition 1.6.

actually implies that the state is more likely to be β. If $\kappa = \frac{1}{2}$ or $\tau_A = \tau_B$, the poll is not informative, and both groups turn out at equal rates.

Lemma 1.5.

(1) Let $\kappa > \frac{1}{2}$. Then,

a. if $\tau_A > \tau_B$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$,

b. if $\tau_A = \tau_B$, $p_A(n) = p_B(n)\ \forall n$,

c. and if $\tau_A < \tau_B$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$.

(2) Let $\kappa = \frac{1}{2}$. Then, $p_A(n) = p_B(n)\ \forall n$.

(3) Let $\kappa < \frac{1}{2}$. Then,

a. if $\tau_A > \tau_B$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} > 1$,

b. if $\tau_A = \tau_B$, $p_A(n) = p_B(n)\ \forall n$,

c. and if $\tau_A < \tau_B$, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)} < 1$.

Proof. Suppose $\kappa > \frac{1}{2}$ and $\tau_A > \tau_B$.
Assume, by contradiction, that as $n \to \infty$, $p_A(n) > p_B(n)$.[19] By (1.A.1), as $n \to \infty$,

$$\frac{p_A(n)}{p_B(n)} \to \frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} \cdot \sqrt{\frac{q}{1-q}} \cdot \sqrt{\frac{p_B(n)}{p_A(n)}}.$$

Claim 1.4. $\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} \cdot \sqrt{\frac{q}{1-q}} < 1.$

Proof.

$$\begin{aligned} &\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} \cdot \sqrt{\frac{q}{1-q}} < 1 \\ \Leftrightarrow\ &\frac{1 + (\frac{\kappa}{1-\kappa})^{\tau_A-\tau_B} \cdot \frac{1-q}{q}}{1 + (\frac{\kappa}{1-\kappa})^{\tau_A-\tau_B} \cdot \frac{q}{1-q}} < 1 \\ \Leftrightarrow\ &\sqrt{\frac{q}{1-q}} - 1 < \left(\frac{\kappa}{1-\kappa}\right)^{\tau_A-\tau_B} \cdot \left(\frac{q}{1-q} - \sqrt{\frac{1-q}{q}}\right). \end{aligned}$$

The last statement is true because $\left(\frac{\kappa}{1-\kappa}\right)^{\tau_A-\tau_B} > 1$ given that $\kappa > \frac{1}{2}$ and $\tau_A > \tau_B$ by assumption, and because $\frac{q}{1-q} - \sqrt{\frac{1-q}{q}} > \sqrt{\frac{q}{1-q}} - 1$.

□

[19] By Lemma 1.1, $p_A(n) = p_B(n)$ can be excluded.

This yields a contradiction because $\frac{p_A(n)}{p_B(n)} > 1$ by assumption, but
$\frac{\Pr(\beta|A,\tau)}{\Pr(\beta|B,\tau)} \cdot \sqrt{\frac{q}{1-q}} \cdot \sqrt{\frac{p_B(n)}{p_A(n)}} < 1$.
The proofs of parts 1 c), 3 a) and 3 c) are analogous.
For the proofs of parts 1 b), 2 and 3 b), observe that if either $\tau_A = \tau_B$ or $\kappa = \frac{1}{2}$, $\Pr(\omega|i,\tau) = \Pr(\omega|i)$. Hence, by Lemma 1.1, the result obtains. □

Similarly, the direction of the monotonicity of the underdog effect in the poll margin depends on κ.

Corollary 1.3.

(1) *If $\kappa > \frac{1}{2}$, in a large election, the limit of the ratio of participation rates of the A supporters relative to B supporters is strictly increasing in the margin of the poll $\tau_B - \tau_A$, i.e., $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$ is a strictly increasing function of $\tau_B - \tau_A$.*
(2) *If $\kappa < \frac{1}{2}$, in a large election, the limit of the ratio of participation rates of the A supporters relative to B supporters is strictly decreasing in the margin of the poll $\tau_B - \tau_A$, i.e., $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$ is a strictly decreasing function of $\tau_B - \tau_A$.*

The result immediately follows by observing that given $\tau_A > \tau_B$, $\Pr(\alpha|\tau) > \frac{1}{2}$ if and only if $\kappa > \frac{1}{2}$, and given $\tau_A < \tau_B$, $\Pr(\alpha|\tau) < \frac{1}{2}$ if and only if $\kappa > \frac{1}{2}$.

How does this affect the incentives of the share $1-\gamma$ of poll participants who are strategic? As it turns out, the optimal strategy μ^* depends on the share of exogenously truthful poll participants, γ.

Proposition 1.7. *Fix the share of exogenously truthful poll participants, γ.*

(1) *If $\gamma > \frac{1}{2}$, $\mu^* = 0$. That is, all strategic poll participants misrepresent their preferences to be exactly the opposite of their true preferences. Since $\kappa > \frac{1}{2}$, the poll is informative.*
(2) *If $\gamma = \frac{1}{2}$, $\mu^* = 0$. Since $\kappa = \frac{1}{2}$, the poll is not informative.*
(3) *If $\gamma < \frac{1}{2}$, $\mu^* = \frac{1-2\gamma}{2(1-\gamma)}$. Since $\kappa = \frac{1}{2}$, the poll is not informative.*

Proof. Note first that $\frac{d\kappa}{d\mu} > 0$, $\frac{d\kappa}{d\gamma} > 0$ and that $\kappa = \frac{1}{2}$ if $\gamma = \frac{1}{2}$ and $\mu = 0$.

Case 1: $\gamma > \frac{1}{2}$.
Then, $\kappa > \frac{1}{2}$ for all $\mu \in [0,1]$. By Lemma 1.5 and Corollary 1.3, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$ is a strictly increasing function of $\tau_B - \tau_A$. Therefore, it is optimal for all strategic poll participants to play $\mu^* = 0$ and claim to have the exact opposed preferences. Since $\kappa > \frac{1}{2}$, the poll is informative.

Case 2: $\gamma = \frac{1}{2}$.
Then, $\kappa \geq \frac{1}{2}$ and the inequality is strict if $\mu = 0$. Suppose, by contradiction, that all

other poll participants play according to $\mu > 0$. Since $\kappa > \frac{1}{2}$, by Lemma 1.5 and Corollary 1.3, it is optimal for an individual poll participant to deviate to $\mu = 0$, thereby increasing the relative participation rate of like-minded voters. So, $\mu > 0$ cannot be part of an equilibrium. In contrast, if all other poll participants play according to $\mu = 0$, $\kappa = \frac{1}{2}$. Then, the poll is not informative and in particular, voters do not take the poll into account. Thus, $\mu^* = 0$ is the equilibrium best response.

Case 3: $\gamma < \frac{1}{2}$.
Then, there exists a unique μ^* such that $\kappa = \frac{1}{2}$ if and only if $\mu = \mu^* = \frac{1-2\gamma}{2(1-\gamma)}$. Suppose that $\mu < \mu^*$. Then, $\kappa < \frac{1}{2}$ and by Lemma 1.5 and Corollary 1.3, $\lim_{n\to\infty} \frac{p_A(n)}{p_B(n)}$ is a strictly decreasing function of $\tau_B - \tau_A$. Thus, $\mu = 1$ is an optimal deviation. If $\mu > \mu^*$, then, $\kappa > \frac{1}{2}$ and $\mu = 0$ is an optimal deviation. Finally, if $\mu = \mu^*$, the poll is uninformative and does not affect the voters' beliefs. Then, $\mu = \mu^*$ is the equilibrium best response. □

Proposition 1.7 reveals that the poll is informative if and only if the share of exogenously truthful poll participants is strictly larger than one-half. While the strategic poll participants will again misrepresent their preferences, the truthful response of the majority of poll participants allows the electorate to derive some information from the poll.

Finally, Corollary 1.4 states that utilitarian efficiency also holds in a large election if poll participants play mixed behavioral strategies.

Corollary 1.4. *In the limit, the majority candidate wins the election with probability 1.*

Proof. The result follows by repeating the same steps as in the proof of Proposition 1.5, and observing that

$$\frac{\Pr(\alpha|A,\tau)}{\Pr(\alpha|B,\tau)} = \frac{1 + (\frac{\kappa}{1-\kappa})^{\tau_B-\tau_A} \cdot \frac{q}{1-q}}{1 + (\frac{\kappa}{1-\kappa})^{\tau_B-\tau_A} \cdot \frac{1-q}{q}} > 1.$$

□

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Chapter 2

Committee Search: Evaluating One or Multiple Candidates at a Time?

Joint with Tobias Rachidi

2.1 Introduction

Academic hiring is mostly conducted by search committees. Often several candidates are reviewed simultaneously after the application deadline has been reached, and the committee either selects one suitable candidate or the hiring process starts over if neither of the candidates satisfied the committee's acceptance standards. So far, the literature on committee search has mainly focused on a search process where candidates are reviewed "one at a time", i.e., hiring is conducted on a rolling basis. Define this search procedure as *single-option sequential search*.[1] In this paper, we consider a search technology in which committees evaluate several candidates simultaneously in each time period, which we denote by *multi-option sequential search*. Our aim is to explore whether single- or multi-option sequential search yields higher acceptance standards and a higher ex ante utilitarian welfare for the search committee.[2] Under multi-option sequential search, committee members can directly compare candidates. This has two implications: On the one hand, the expected value of a candidate conditional on hiring increases; on the other hand, the probability of hiring a particular candidate decreases, and, thus, the expected search costs are altered. Generally, there is a trade-off between these two objects that determine the committee's welfare. The resolution of this trade-off depends on the voting rule and the specification of search costs associated with the simultaneous evaluation of multiple candidates.

1. In the search literature, this is mostly denoted as *sequential search*.

2. A second application is a family searching for a house. The decision to purchase can be made after each showing or after a certain number of houses have been seen. Compte and Jehiel (2010) suggest an application to project selection: Suppose that a firm has scarce resources and needs to decide on a new project to fund, where project ideas arrive randomly.

We find that under unanimity voting, the ranking of the two search procedures depends on how the search costs vary with the number of candidates simultaneously evaluated in each period. In contrast, under qualified majority voting distinct from unanimity, this sensitivity to the shape of the cost function partly disappears. Consequently, the problem of the design of search technologies for committees is different from the search design problem for a single decision-maker, noting that the latter is a special case of a search committee operating under the unanimity voting rule. This insight constitutes the main contribution of this paper.

In our model, a search committee consisting of at least one member seeks to hire one candidate, and we consider two search technologies: Under single-option sequential search, exactly one candidate arrives per period, and under multi-option sequential search, the committee reviews in each time period a fixed number $K > 1$ of candidates simultaneously. The time horizon is infinite, and rejected candidates cannot be recalled. Note that multi-option sequential search can also be interpreted as delayed voting: Suppose that one candidate per period arrives. Then, simultaneously evaluating K candidates in some round of the dynamic search procedure can be viewed as taking voting decisions only every K periods instead of every single period. In other words, choosing the number of candidates to be evaluated simultaneously can be viewed as selecting voting times.[3]

The committee members' preferences feature independent private values. For every member, the value of a candidate is a random variable, which is distributed independently and identically across time, members, and candidates. Each committee member observes his or her own value realization for every candidate and has distributional knowledge about the other members' values.

We consider a class of voting rules where each member may either vote for one of the available candidates or may opt to continue search. A candidate is then hired if and only if the number of votes he or she receives exceeds a qualified majority threshold ranging from simple majority to unanimity. This class of voting rules is frequently used in practice, for example in certain resolutions of the German Caritas (cf. Deutscher Caritasverband e.V. (2018)), in the election of the President of Germany (cf. Bundesrepublik Deutschland (2019)),[4] or in the papal conclave (cf. Benedict XVI (2013)).[5] In the latter circumstances, it is common that more than two candidates are in contention.

If a candidate is hired, search stops; otherwise, search continues, and each committee member bears an additive search cost $c \cdot h(K) > 0$. We restrict the committee members' voting strategies to symmetric and neutral[6] stationary Markov strategies.

3. We thank Olivier Compte for suggesting this interpretation.

4. In these cases, abstaining is equivalent to voting in favor of continuing search

5. Abstention is not allowed in the papal conclave. However, since any male Catholic is a potential candidate for papacy, voting for a person without a chance is equivalent to voting to continue search.

6. A strategy is *neutral* if it does not condition on the identity of the candidate.

Then, a member votes in favor of some candidate if and only if the candidate's value is the highest among all observed K values and it exceeds some cutoff representing the member's acceptance standard. Acceptance standards coincide with welfare because values are private.[7]

We first prove the existence and uniqueness of a symmetric and neutral stationary Markov equilibrium in the single- and multi-option sequential search setting for all qualified majority voting rules including unanimity voting. The uniqueness of equilibrium is shown for value distributions that admit a log-concave density. In the subsequent comparison of the two search procedures, we maintain this distributional assumption.

Then, we study the case of unanimity voting in detail. We find that if the cost function h is superadditive or linear in the number of evaluated candidates, single-option sequential search yields higher acceptance standards and higher welfare than multi-option sequential search. Intuitively, given some acceptance standard, the expected value of a candidate conditional on stopping is higher if $K > 1$ than if $K = 1$. However, at the same time, expected search costs are also higher because the probability of hiring a particular candidate is lower and costs are superadditive or linear. We show that the increase in the expected value conditional on stopping is limited and that the overall trade-off is resolved in favor of single-option sequential search. In contrast, if the cost function h is strictly subadditive in the number of candidates, multi-option sequential search yields higher welfare if the magnitude of search costs quantified by the parameter c is sufficiently small. Here, if c is small, acceptance standards are close to the upper bound of the support of the value distribution. Hence, while the probability of hiring a particular candidate is higher under single-option sequential search, it is low for both search protocols. Therefore, if c is sufficiently small, expected search costs are actually lower under multi-option sequential search because h is assumed to be subadditive. In addition, as before, the expected value conditional on stopping for $K > 1$ is not lower than the respective value for $K = 1$. Extensions to interdependent values and correlated values show the robustness of these results.

Next, we consider qualified majority voting rules that do not require full unanimity. We find that multi-option sequential search yields a higher welfare than single-option sequential search for all cost functions h as long as c is sufficiently small. Thus, the sensitivity to the shape of the cost function h that we find for the unanimity rule partly disappears. To prove this result, we first establish that the ranking of the expected values conditional on stopping from the unanimity voting case carries over to qualified majority, meaning, the respective expected value is higher if $K > 1$ compared to $K = 1$. Then, we show that if c is sufficiently small, this increase in the expected value conditional on stopping outweighs the potential rise

7. To be precise, this holds if and only if the equilibrium cutoff is interior.

in expected costs.[8] Consequently, as alluded to above, the comparison of single- to multi-option sequential search differs considerably if the search committee operates under qualified majority voting instead of unanimity voting. Thus, our results imply in particular that the conclusions for the single decision-maker case do not carry over to committee search with qualified majority voting.

We are aware of only one other paper analyzing the differences between the committee search setting and the single-searcher case with respect to the search technology and the implications for its design. In independent work, Cao and Zhu (2019) compare single-option sequential search to fixed-sample-size search, where the committee first determines the total number of candidates to be reviewed, then reviews the candidates sequentially, and finally selects one of these candidates. The latter is conceptually different to multi-option sequential search. Cao and Zhu (2019) show that previous results from the single-searcher setup do not carry over to the committee search setting. We will discuss further details and differences in the next section.

The paper is organized as follows: Section 2.2 reviews the related literature, Section 2.3 introduces the model, and Section 2.4 proves the existence and uniqueness of the equilibrium. Section 2.5 treats the unanimity voting case, Section 2.6 contains the results for qualified majority voting rules, and Section 2.7 concludes while discussing the implications of our results for committee search in practice. Appendix 2.A contains the proofs, and Appendix 2.B derives expressions for the probability of approving a particular candidate and the expected value conditional on stopping.

2.2 Related Literature

Our paper contributes to the growing literature on committee search where a committee conducts search dynamically over time.[9] Albrecht, Anderson, and Vroman (2010), Compte and Jehiel (2010), and Moldovanu and Shi (2013) assume that the committee employs single-option sequential search, where, in each time period, the committee draws exactly one alternative from a known distribution and decides by voting whether to accept the current alternative or to continue costly search by drawing a new alternative. The time horizon is infinite, and there is no recall.

In Albrecht, Anderson, and Vroman (2010) and Compte and Jehiel (2010), preferences feature private values, and search is costly because of multiplicative discounting. Albrecht, Anderson, and Vroman (2010) show that there exists a unique

8. Depending on the shape of the cost function h, expected search costs might also decrease. Of course, this only reinforces our reasoning.

9. The static case of committee decision-making has also been analyzed in depth, cf. the survey by Li and Suen (2009).

equilibrium if the density of the value distribution is log-concave.[10] Further, they compare the committee search problem and the single-agent search problem in terms of acceptance standards and expected search duration. More generally, they study the effect of increasing either the committee size or the majority requirement on acceptance standards and the expected search duration. Finally, they show that the welfare-maximizing majority requirement increases in the members' patience. Our paper focuses on the effect of different search procedures on acceptance standards and welfare given the voting rule, whereas Albrecht, Anderson, and Vroman (2010) study the implications of different voting rules or committee sizes while fixing the search technology, i.e., single-option sequential search.

In Compte and Jehiel (2010), in each period, one candidate or proposal is drawn from a potentially multi-dimensional proposal space. The members' values of some proposal are given by utility functions mapping proposals into values. Compte and Jehiel (2010) show how different aspects shape the final outcome under single-option sequential search. If the proposal space is multi-dimensional, they find a systematic difference between unanimity voting and qualified majority voting. In the former case, the size of the agreement set, meaning, the set of proposals that are approved by at least as many members as the majority threshold requires for acceptance, becomes small, while, in the latter case, the size of the agreement set does not vanish as members become arbitrarily patient. When restricting attention to single-option sequential search, in the framework of Compte and Jehiel (2010), our model essentially corresponds to the case of a multi-dimensional proposal space with linear utility. The proposal space is a hypercube whose dimension is equal to the committee size, each member is associated with one distinct dimension of the proposal space, and the members' values are given by the characteristic of the proposal they are linked to. Consequently, this systematic difference between unanimity and qualified majority voting arises in our setting as well, and it turns out to be important for our results concerning the comparison of different search technologies.

Moldovanu and Shi (2013) assume that committee members have interdependent preferences, and they focus on the unanimity voting rule. Committee members face additive search costs. Each member is linked to a signal about the candidate arriving in some period, and a member's value of this candidate amounts to a weighted average of this member's own signal and the signals associated with the other members. The weight attached to a member's own signal is interpreted as the level of partisanship in the committee. If members only observe their own signals, they are called specialists, whereas, if members observe all signals, they are termed generalists. Moldovanu and Shi (2013) analyze how acceptance standards and welfare react to varying degrees of partisanship and compare the decisions of committees

10. Conceptually, the proof strategy of our uniqueness result follows Albrecht, Anderson, and Vroman (2010), but, as outlined below, the presence of more than one candidate per period requires a substantial amount of supplementary arguments.

composed of either generalists or specialists. For the case of unanimity voting, our results concerning acceptance standards and partly welfare extend when committee members have interdependent preferences.

We know of only one other contribution that is concerned with the comparison of different search technologies in the committee search environment.[11] In independent work, Cao and Zhu (2019) compare single-option sequential search with simple majority voting to a fixed-sample-size search technology that can be described as follows: First, the committee determines the total sample size via the random proposer mechanism. Then, in each period, one alternative is drawn until the predetermined sample size is reached. Finally, the committee selects an alternative according to plurality voting. There is no discounting, but members bear additive search costs that are linear in the number of alternatives. Moreover, their main model focuses on committees with two members and uniformly distributed values.[12] Cao and Zhu (2019)'s main insight is that the finding from the single decision-maker setting that single-option sequential search always dominates fixed-sample-size search, as for example established in Rothschild (1974), does not extend to the committee search setting. In contrast, fixed-sample-size search dominates sequential search if the per-observation search cost is very small or large enough. These results are driven by the trade-off of the flexibility advantage of single-option sequential search, which captures that search can be stopped as soon as an appropriate alternative is drawn, against the commitment advantage of fixed-sample-size search, which captures that the number of observations is ex ante optimal. While Cao and Zhu (2019) independently ask a similar research question to ours, they study a conceptually different search technology, inducing different results driven by different effects. Therefore, we view our paper to be complementary to their work.

In the literature on search conducted by a single decision-maker, not only single-option sequential search due to McCall (1970), but also other search technologies such as fixed-sample-size search have been discussed and contrasted, see for instance Stigler (1961), Rothschild (1974), and Burdett and Judd (1983). In Morgan (1983) as well as Manning and Morgan (1985), search is conducted by a single decision-maker, and they consider general classes of search procedures, in which, in each period, the single agent decides how many alternatives to draw in the following period if search continues, and whether to stop search in the current period. Therefore, multi-option sequential search conducted by a single decision-maker is

11. In the literature on auctions, the comparison between different selling technologies has been studied before. Wang (1993) compares auctions to posted-price selling in terms of revenue and prices, and finds that the ranking of the two technologies depends on the seller's auctioning costs and on the steepness of the marginal revenue curve.

12. Cao and Zhu (2019) show that their results extend to values that are drawn from the exponential distribution. Further, they establish via numerical simulations that their findings also partly carry over to larger committees.

part of the search technologies studied in Morgan (1983) as well as Manning and Morgan (1985).

Morgan (1983) derives properties of the optimal sample size in each time period depending on the searcher's recall, time horizon, and outside option, but he does not analytically identify conditions on the primitives of the model under which single-option sequential search is optimal. However, he mentions numerical simulations indicating in particular that single-option sequential search might not be optimal if there is no recall and there are intraperiodic economies of scale in the simultaneous evaluation of multiple alternatives. To some extent, our analytical result for committee search with unanimity voting and subadditive costs specialized to the single-agent case addresses this point.

Manning and Morgan (1985) show analytically that single-option sequential search conducted by a single agent is optimal if the time horizon is infinite, there is full recall, and the searcher bears additive search costs that are non-decreasing and strictly convex in the number of alternatives per period. This result resembles our finding for committee search with unanimity voting and superadditive or linear search costs when specializing to the single-searcher case. Note that Manning and Morgan (1985) assume full recall, whereas we assume that rejected alternatives cannot be recalled. Yet, as long as the sample size per period does not depend on calendar time (as it is the case under single-option as well as multi-option sequential search), in the single-agent case, the no recall assumption is without loss.[13] Therefore, our finding for committee search with unanimity voting and superadditive or linear search costs specialized to the single-agent case can be derived from Manning and Morgan (1985)'s result.[14]

2.3 The Model

A committee consisting of members $\mathcal{N} := \{1,...,N\}$ with $N \geq 1$, who are indexed by i, seeks to hire one candidate. In each discrete period of time t, a set of candidates $\mathcal{K} := \{1,...,K\}$ with $1 \leq K < \infty$ arrives. If $K = 1$, we call the resulting search procedure *single-option sequential search*, whereas, if $K > 1$, the search technology is termed *multi-option sequential search*.

Preferences feature private values. For each committee member $i \in \mathcal{N}$, the value of hiring candidate $k \in \mathcal{K}$ is governed by the random variable X_i^k, where X_i^k is distributed independently and identically across time periods, candidates, and members according to the cumulative distribution function F with density f. We assume that the distribution of X_i^k has full support on the bounded interval $[0,\bar{x}]$ with $\bar{x} > 0$.

13. For single-option sequential search and a single decision-maker, this point has been made previously by Albrecht, Anderson, and Vroman (2010).

14. However, note that our assumption on the shape of the cost function is slightly more general because non-decreasing and strictly convex costs are also superadditive or linear.

Let μ denote the mean of the random variable X_i^k. For all candidates $k \in \mathscr{K}$, committee member $i \in \mathscr{N}$ observes the realization of X_i^k perfectly and has only distributional knowledge about the value X_j^k that any committee member j other than i assigns to candidate k.

The timing is as follows: In every time period, member i observes a realization of the vector of random variables $(X_i^1, \dots, X_i^K)$, that is, K values. Then, members simultaneously cast a vote, voting either for one candidate k (action k) or for the option to continue search (action 0). Candidate k is hired and search is stopped if and only if the number of votes in favor of k is larger than or equal to the (qualified) majority threshold $M \in \{1, \dots, N\}$, with $M > \frac{N}{2}$.[15] This class of voting rules encompasses, for instance, unanimity voting corresponding to the case where $M = N$ or simple majority voting with an odd number of members, that is, $M = \frac{N+1}{2}$. If search is continued, each committee member incurs a per period cost of $c \cdot h(K) > 0$, where $h(K)$ is the value of some function $h : \mathbb{N}_+ \to \mathbb{R}_{>0}$ evaluated at K, and $c > 0$ represents a scaling parameter. Finally, we assume that the search horizon is infinite, and that rejected candidates cannot be recalled.

2.4 Equilibrium Analysis

Committee member i's strategy is a sequence of functions $\sigma_i = \{\sigma_i(H_t)\}_t$, mapping from any history H_t until period t to $\Delta(\{0\} \cup \mathscr{K})$, i.e., all probability distributions over the set of actions $\{0\} \cup \mathscr{K}$ that are available in each period. As is common in the literature on committee search, we restrict strategies to be (1) Markovian, meaning, the action that member i's strategy prescribes in period t does not depend on the entire history up to period t, but only on the evaluation of the most recent K candidates, and we focus on (2) stationary and (3) symmetric equilibria, that is, the equilibrium strategies are neither sensitive to calendar time nor to the identity of the committee member. In addition, we assume strategies to be (4) neutral, that is, they have to be invariant with respect to permutations of the candidates' labels.[16] Essentially, neutrality rules out stationary and symmetric equilibria in Markov strategies in which voters coordinate on ignoring one or more candidates. Apart from conditions (1) - (4), we also impose that search terminates in finite time, excluding dominated equilibria in which all members always vote to continue search, independently of the value realizations. Subsequently, we simply write equilibrium when referring to a stationary and symmetric Markov equilibrium in neutral strategies.
Strategies that satisfy these refinements are characterized by cutoffs $z \in [0, \bar{x})$. More

15. The assumption $M > \frac{N}{2}$ ensures that no two distinct candidates meet the (qualified) majority requirement at the same time.

16. Any stationary Markov strategy can be described by a mapping $s : [0, \bar{x}]^K \to \Delta(\{0\} \cup \mathscr{K})$. A strategy s satisfies neutrality if, for all $(x^1, \dots, x^K) \in [0, \bar{x}]^K$, it holds that $s(x^{\rho(1)}, \dots, x^{\rho(K)}) = (s^0(x^1, \dots, x^K), s^{\rho(1)}(x^1, \dots, x^K), \dots, s^{\rho(K)}(x^1, \dots, x^K))$ for any permutation ρ of the set $\mathscr{K}$.

specifically, in any time period, upon observing the value realizations $(x_i^1, ..., x_i^K) \in [0, \bar{x}]^K$, member $i \in \mathcal{N}$ votes in favor of candidate $k \in \mathcal{K}$ if and only if

$$x_i^k \;\geq\; \max_{l \neq k} x_i^l \text{ and } x_i^k \;\geq\; z.$$

We call these strategies *maximum-strategies with cutoff*. In words, every member chooses the best among the K available candidates and approves this candidate if and only if the respective value exceeds the cutoff, or acceptance standard, z. Intuitively, since candidates are identical ex ante and because members treat candidates in a neutral way, all candidates have the same chance to be elected from the perspective of an individual member. Consequently, no member has an incentive to vote in favor of any candidate but the best.[17]

Interior equilibrium cutoffs $z \in (0, \bar{x})$ solve $z = v$, where v is the continuation value implied by this strategy profile.[18] The continuation value which coincides with the ex ante utilitarian welfare per committee member is given by

$$v \;=\; -\frac{c \cdot h(K)}{K \cdot \Pr(\text{candidate } k \text{ hired})} + \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}].$$

The continuation value amounts to the difference between the expected value conditional on stopping $\mathbb{E}[X_i^k | \text{candidate } k \text{ hired}]$ and the expected search costs $\frac{c \cdot h(K)}{K \cdot \Pr(\text{candidate } k \text{ hired})}$.

Let $Q^K(z, N, M)$ be the cumulative distribution function of the Binomial distribution with parameters N and $\Pr(X_i^k \geq z \text{ and } X_i^k \geq \max_{l \neq k} X_i^l)$ evaluated at $M - 1$. Also, for any $b \in \mathbb{N}_0$ with $b \leq N$, $q^K(z, N, b)$ denotes the corresponding probability mass function evaluated at b. Further, we argue in Appendix 2.B.2 that

$$\Pr(X_i^k \;\geq\; z \text{ and } X_i^k \;\geq\; \max_{l \neq k} X_i^l) \;=\; \frac{1}{K}[1 - F(z)^K].$$

Then, the equilibrium equation can be written as

$$z \;=\; -\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]} + \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}]. \tag{2.1}$$

Intuitively, acceptance standards z arising in equilibrium are calibrated in a way such that a member is indifferent between stopping and continuing search whenever the value of some candidate coincides with the cutoff z. A derivation of the equilibrium strategies and the equilibrium equation (2.1) can be found in Appendix 2.A.1.

17. Note that mixed strategies do not arise in equilibrium.

18. Boundary solutions, i.e., equilibria involving some maximum strategy with cutoff $z = 0$, may arise if the search costs $c \cdot h(K)$ are large. Subsequently, we take care of this issue.

2.4.1 Equilibrium Existence

We claim that there exists an equilibrium. The reasoning in the previous part implies that there exists an equilibrium if and only if there either exists $0 \leq z < \bar{x}$ that solves equation (2.1), or there is a boundary equilibrium in which the maximum-strategy with cutoff $z = 0$ forms an equilibrium.

Proposition 2.1. *There exists an equilibrium.*

We prove the existence of an equilibrium while making use of the intermediate value theorem. Similar existence arguments appear in Albrecht, Anderson, and Vroman (2010), Compte and Jehiel (2010), and Moldovanu and Shi (2013).[19]

2.4.2 Equilibrium Uniqueness

We turn to the problem of equilibrium uniqueness. Apart from being of interest in itself, the uniqueness of equilibrium is important for a transparent comparison between single-option sequential search and multi-option sequential search. It turns out that the equilibrium is unique if we impose the assumption that the density f is log-concave.[20]

Proposition 2.2. *If the density f is log-concave, the equilibrium is unique.*

Many well-known distributions including, for instance, the uniform distribution or the truncated normal distribution meet this requirement.[21]
Conceptually, the proof strategy follows Albrecht, Anderson, and Vroman (2010), but, as discussed below, the presence of more than one candidate per period requires a substantial amount of supplementary steps that are not needed if $K = 1$. The arguments from the previous parts imply that there is a unique equilibrium if and only if either equation (2.1) admits exactly one solution and there is no supplementary boundary equilibrium, or there is a boundary equilibrium and the equilibrium equation has no solution. Rearrange equation (2.1):

$$\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]} = \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}] - z.$$

The essential part of the proof is to establish that the left-hand side of this equation is increasing in z, whereas the right-hand side is decreasing in z. Then, the uniqueness result follows from the opposite monotonicities of the discussed functions.

19. In particular, Moldovanu and Shi (2013) show the existence of an equilibrium for the case of single-option sequential search with unanimity voting, i.e., $K = 1$ and $M = N$.

20. For the case of single-option sequential search with unanimity voting, i.e. $K = 1$ and $M = N$, the uniqueness of equilibrium has been established in Moldovanu and Shi (2013).

21. For a comprehensive list of distributions that admit a log-concave density, we refer to Bagnoli and Bergstrom (2005).

First, it is straightforward to derive that the left-hand side is increasing in z. Intuitively, if the acceptance standard z increases, the probability of voting in favor of some candidate k decreases, and, hence, the probability of hiring this candidate k and the overall probability of stopping decrease as well. Thus, the expected search costs increase. Consequently, it remains to show that $\mathbb{E}[X_i^k|\text{candidate } k \text{ hired}] - z$ is decreasing in z. This claim is stated as Lemma 2.1.[22] Define $S^K(z, N, M) := \mathbb{E}[X_i^k|\text{candidate } k \text{ hired}]$ to emphasize that the expected value conditional on hiring depends on K and M.

Lemma 2.1. *Consider any $K \geq 1$. If the density f is log-concave, the function*

$$S^K(z, N, M) - z$$

is decreasing in z.

Subsequently, we discuss the proof of Lemma 2.1. Define

$$\begin{aligned} \mu_a^K(z) &:= \mathbb{E}[X_i^k|X_i^k \geq z \text{ and } X_i^k \geq \max_{l \neq k} X_i^l], \text{ and} \\ \mu_r^K(z) &:= \mathbb{E}[X_i^k|X_i^k < z \text{ or } X_i^k < \max_{l \neq k} X_i^l]. \end{aligned}$$

These conditional expectations capture the expected value of an arbitrary candidate $k \in \mathscr{K}$ for an arbitrary member $i \in \mathscr{N}$ conditional on approving or rejecting this candidate, respectively. We argue in Appendix 2.B.1 that

$$\mathbb{E}[X_i^k|\text{candidate } k \text{ hired}] = w^K(z)\mu_a^K(z) + [1 - w^K(z)]\mu_r^K(z), \tag{2.2}$$

with $w^K(z)$ being defined as

$$w^K(z) := \sum_{l=M}^{N} \frac{q^K(z, N, l)}{1 - Q^K(z, N, M)} \frac{l}{N}.$$[23]

Intuitively, conditional on stopping, the accepted candidate k might be supported or rejected by an arbitrary member. Therefore, the expected value of k conditional on stopping amounts to an average of the expected values conditional on supporting as well as rejecting candidate k. The weight $w^K(z)$ represents the expected share of members supporting k conditional on k meeting the majority requirement. Note that under unanimity voting, hired candidates must be accepted by every member. Thus, in this case, the expected value conditional on hiring simplifies to $\mathbb{E}[X_i^k|\text{candidate } k \text{ hired}] = \mu_a^K(z)$.

22. For single-option sequential search, i.e., $K = 1$, this property has been shown in Albrecht, Anderson, and Vroman (2010).

23. This kind of representation of the expected value conditional on stopping is due to Albrecht, Anderson, and Vroman (2010).

After some intermediate steps that are similar to those in the proof of Albrecht, Anderson, and Vroman (2010) we obtain that, for $z \in (0,\bar{x})$,

$$\frac{d\mathbb{E}[X_i^k|\text{candidate } k \text{ hired}]}{dz} < w^K(z)\frac{d\mu_a^K(z)}{dz} + [1 - w^K(z)]\frac{d\mu_r^K(z)}{dz}.$$

Hence, the key proof step is to show that $\frac{d\mu_a^K(z)}{dz} \leq 1$ and $\frac{d\mu_r^K(z)}{dz} \leq 1$. Notice that if $K = 1$, these conditional expected values are truncated means:

$$\mu_a^1(z) = \mathbb{E}[X_i^k|X_i^k \geq z], \text{ and } \mu_r^1(z) = \mathbb{E}[X_i^k|X_i^k < z].$$

It is well-known that log-concavity of f implies the desired Lipschitz conditions on the truncated means, i.e., $\frac{d\mu_a^1(z)}{dz} \leq 1$ and $\frac{d\mu_r^1(z)}{dz} \leq 1$ (see e.g. Bagnoli and Bergstrom (2005)). However, for $K > 1$, the discussed implications are not standard because the involved expected values conditional on rejecting or supporting a candidate do no longer constitute truncated means. To obtain that $\frac{d\mu_a^K(z)}{dz} \leq 1$, we establish that the conditional density $\Pr(X_i^k = x|X_i^k \geq \max_{l \neq k} X_i^l)$ is log-concave by employing the fact that log-concavity is preserved under integration, which has been shown in Prékopa (1973). Then, like in the case of $K = 1$, log-concavity implies the desired Lipschitz condition on $\mu_a^K(z)$. Next, we show that $\frac{d\mu_r^K(z)}{dz} \leq 1$ by directly invoking the log-concavity of f as well as its implications. Again, the preservation of log-concavity under integration due to Prékopa (1973) is important. Taking both aspects together, Lemma 2.1 follows, and we obtain that the right-hand side of the equation above is decreasing in z. When comparing the welfare induced by single-option sequential search and multi-option sequential search, we repeatedly make use of Lemma 2.1. We believe that the technical property established in Lemma 2.1 might be useful beyond its application in this paper.

2.5 Unanimity Voting

Having established equilibrium existence and uniqueness, in this section we assume that the committee employs unanimity voting, i.e., we set $M = N$. We contrast the unique equilibria under single-option and multi-option sequential search in terms of acceptance standards and welfare and show how the superiority of one or the other search technology depends on the structure of the search costs.

2.5.1 Superadditive or Linear Costs

In this part, we study cost functions h that satisfy

$$\frac{h(K)}{K} \geq h(1) \tag{2.3}$$

for all $K > 1$. In a slight abuse of wording, we say that condition (2.3) gives rise to *superadditive or linear* costs.[24] Intuitively, the restriction on the function h means that the search costs per candidate under multi-option sequential search are at least as high as under single-option sequential search. For instance, if $h(K) = K^\alpha$ for some $\alpha \geq 1$, costs are superadditive or linear.

Denote the ex ante utilitarian welfare per committee member in the game with $K \geq 1$ candidates per period by v_K. Proposition 2.3 establishes that the welfare under multi-option sequential search with arbitrarily many candidates $K > 1$ is strictly lower than the welfare under single-option sequential search.

Proposition 2.3. *Suppose that the voting rule is unanimity, i.e., $M = N$, and assume that the density f is log-concave. If the search costs are superadditive or linear in the number of candidates, i.e., satisfy (2.3), the committee's ex ante utilitarian welfare is higher under single-option sequential search relative to multi-option sequential search, i.e., $v_1 > v_K$ for all $K > 1$.*

The basic trade-off when moving from $K = 1$ to $K > 1$ is that, on the one hand, the expected value conditional on stopping rises, but on the other hand, expected search costs rise, too. The former effect arises because unanimity voting means that, conditional on stopping, all members vote in favor of the hired candidate, and when there are multiple candidates, members only approve some candidate if the associated value is the maximum out of the K values they observe. The latter effect is due to two aspects: First, the probability of hiring an arbitrary candidate k is smaller if $K > 1$, and, second, since costs are superadditive or linear, the search costs per candidate are weakly higher if $K > 1$ compared to $K = 1$. Thus, a priori, the ranking of the two search procedures in terms of welfare is ambiguous. The key proof step is to show that the increase in the expected value conditional on stopping is limited when moving from single-option sequential search to multi-option sequential search. Concretely, for any $K > 1$ and any fixed cutoff z, we derive an upper bound for the ratio

$$\frac{\mu_a^K(z) - z}{\mu_a^1(z) - z} = \frac{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z] - z}{\mathbb{E}[X_i^k | X_i^k \geq z] - z}.$$

Lemma 2.2. *Consider any $K > 1$. For all $z \in [0, \bar{x})$,*

$$\frac{\mu_a^K(z) - z}{\mu_a^1(z) - z} < \frac{1 - F(z)}{\frac{1}{K}[1 - F(z)^K]}.$$

24. Note that the condition is actually weaker than (strict) superadditivity or linearity.

Note that it is easy to see that a lower bound of this ratio is 1 because in the numerator, the maximum over $K > 1$ values is considered. Lemma 2.2 reveals that an upper bound of the ratio is given by the ratio of the probability that an individual member votes in favor of candidate k if there is only one candidate to this probability if there are $K > 1$ candidates. We believe that this technical property might be useful beyond its application in this paper.

Now, let us sketch the proof of Proposition 2.3 for interior cutoffs. In this case, acceptance standards coincide with welfare.[25] Consider the ratio of the expected value conditional on stopping net of the cutoff when $K > 1$ compared to the net value when $K = 1$, that is,

$$\frac{\mathbb{E}[X^k | X^k \ \geq \ \max_{l \neq k} X^l, X^k \ \geq \ z_K] - z_K}{\mathbb{E}[X^k | X^k \ \geq \ z_1] - z_1},$$

where z_K denotes the cutoff when there are $K \geq 1$ candidates. Towards a contradiction, assume that $z_1 \leq z_K$. By the equilibrium equation, i.e., equation (2.1), the considered ratio is equal to the ratio of the expected search costs when $K > 1$ versus when $K = 1$. Then, the superadditive or linear cost assumption yields a lower bound on this ratio of expected search costs. Moreover, recall that Lemma 2.1 reveals that the log-concavity of f implies the Lipschitz condition $\frac{d\mu_a^K(z)}{dz} \leq 1$. While invoking $\frac{d\mu_a^K(z)}{dz} \leq 1$ and making use of Lemma 2.2, we obtain an upper bound on the discussed ratio of expected values conditional on stopping. It turns out that the derived lower bound is larger than the upper bound, which constitutes the desired contradiction.

2.5.2 Subadditive Costs

Next, we consider cost functions h that satisfy

$$\frac{h(K)}{K} \ < \ h(1) \tag{2.4}$$

for all $K > 1$. We say, again in a slight abuse of wording, that condition (2.4) gives rise to *subadditive* costs.[26] This assumption is reasonable if there are fixed costs associated with the hiring process, or if there are cost savings when multiple candidates can be considered. For example, if $h(K) = K^\beta$ for some $\beta < 1$, costs are subadditive. Proposition 2.4 reveals that under the assumption of subadditive costs, the conclusion of the previous part of this section is partly reversed: If the magnitude of the search costs as quantified by the parameter c is sufficiently small, evaluating multiple candidates at a time improves welfare.

25. We emphasize that the result also holds if some equilibria constitute boundary solutions.

26. Note that the distinction between superadditive or linear costs and subadditive costs is not exhaustive.

Proposition 2.4. *Suppose that the voting rule is unanimity, i.e., $M = N$, assume that the density f is log-concave, and consider any function h giving rise to subadditive costs, i.e., satisfying (2.4). Then, for all $K > 1$, there exists $\bar{c}_K > 0$ such that for all $c < \bar{c}_K$, the committee's ex ante utilitarian welfare is higher under multi-option sequential search with K candidates per period relative to single-option sequential search, i.e., $v_K > v_1$.*

Intuitively, again, the expected value conditional on stopping is not lower for $K > 1$ relative to $K = 1$. However, in contrast to the previous cost regime, for subadditive costs and sufficiently small magnitudes of search costs c, the expected search costs are actually lower if $K > 1$ compared to $K = 1$, yielding a higher welfare for the committee if multi-option sequential search is employed.

Let us sketch the proof of Proposition 2.4 in more detail. Assume, by contradiction, that there exists $K > 1$, such that for all $\bar{c}_K > 0$, there exists $c < \bar{c}_K$ such that $v_1 \geq v_K$. Without loss of generality, suppose that both cutoffs are interior. Then, they coincide with welfare and, thus, we have that $z_1 \geq z_K$. First, we show that given $z_1 \geq z_K$, the expected value conditional on stopping is increasing when moving from $K = 1$ to $K > 1$. This is a consequence of the log-concavity of f and, more precisely, the Lipschitz condition $\frac{d\mu_a^K(z)}{dz} \leq 1$ we derived in Lemma 2.1. The equilibrium condition (2.1) then implies that the expected search costs should also be higher if $K > 1$ compared to $K = 1$. However, if c becomes small, under both search procedures, the equilibrium acceptance standards are close to the upper bound of the value distribution, $\bar{x}$. This conclusion crucially relies on the fact that the voting rule is unanimity and fails in the case of qualified majority rules distinct from unanimity. Then, even though the probability of hiring an arbitrary candidate k is higher for $K = 1$, this probability is small for $K = 1$ as well as for $K > 1$. In fact, if c is small enough, the difference is low enough such that, given subadditive costs, the expected search costs are overall actually smaller for $K > 1$ than for $K = 1$. This is the desired contradiction.

2.5.3 Extensions

For the unanimity voting rule, we explore the robustness of our results via two extensions: Allowing for interdependent values instead of private values, and allowing for correlated values instead of independent values.[27]

For the case of interdependent values, we follow the approach in Moldovanu and Shi (2013), assuming that the value a member derives from hiring some candidate is a weighted average of his own observed signal and the signals of all other members. We find that our results regarding acceptance standards carry over from the analysis under private values. As far as welfare is concerned, note that under the assumption of interdependent values, acceptance standards and welfare no longer coincide even

27. The arguments for these extensions are available on request.

if the equilibrium cutoff is interior (cf. Moldovanu and Shi (2013)). If costs are subadditive, the ranking of single-option and multi-option sequential search in terms of welfare from the private-values case extends to interdependent values. Overall, this suggests that our results concerning unanimity voting are not driven by the private-values assumption on preferences.

To relax the assumption that candidates' values are distributed independently across committee members, we introduce an unknown state of the world s_k for each candidate $k \in \mathscr{K}$, which we assume to be independently and identically distributed across time and candidates. Conditional on the state realization s_k, the values associated with candidate k are then independently and identically distributed across committee members. The state-dependent value distributions are assumed to be stochastically ranked according to the likelihood-ratio ordering. While relaxing the independence of values across members, we maintain the assumption that committee members have private values. Thus, acceptance standards and welfare again coincide whenever the equilibrium is interior. We find that both results for the unanimity voting rule carry over from the private-values case to correlated values. Therefore, we conclude that while the assumption of independently distributed values is admittedly strong, it does not drive our results for the unanimity voting rule.

2.6 Qualified Majority Voting

Having studied the case of unanimity voting, in this section, we turn to qualified majority voting, considering a majority requirement M such that $M < N$. As before, we compare the unique equilibria of multi-option sequential search and single-option sequential search in terms of acceptance standards and welfare. Again, let v_K be the ex ante utilitarian welfare per committee member if there are $K \geq 1$ candidates per period. As already stated, the welfare induced by a search procedure is determined by two ingredients: the expected value conditional on hiring and the expected search costs. To start, in Lemma 2.3 we compare the expected values conditional on stopping when there are $K > 1$ versus $K = 1$ candidates per period. Recall that $S^K(z,N,M) = \mathbb{E}[X_i^k|\text{candidate } k \text{ hired}]$.

Lemma 2.3. *Consider any $K > 1$. For all $z \in [0, \bar{x})$,*

$$S^1(z,N,M) \; < \; S^K(z,N,M).$$

Lemma 2.3 reveals that, when fixing a cutoff value z, the expected value conditional on stopping when $K > 1$ is higher than the corresponding expected value when $K = 1$. If the voting rule is unanimity, this conclusion is immediate because, in this case, for any $K > 1$,

$$S^1(z,N,N) \; = \; \mathbb{E}[X_i^k|X_i^k \; \geq \; z] \; < \; \mathbb{E}[X_i^k|X_i^k \; \geq \; z \text{ and } X_i^k \; \geq \; \max_{l \neq k} X_i^l] \; = \; S^K(z,N,N).$$

Yet, if the voting rule is qualified majority, the conclusion is not obvious because there are two forces pulling in opposite directions.
Consider the average representation of the expected value conditional on hiring as introduced in equation (2.2):

$$S^K(z,N,N) \;=\; w^K(z)\mu_a^K(z) + [1 - w^K(z)]\mu_r^K(z).$$

Note that for $M < N$, in contrast to unanimity, it does not hold that $w^K(z) = 1$ for all $K \geq 1$, but $w^K(z)$ depends non-trivially on K. Now, take any $K > 1$, and fix a cutoff value z. Observe that $\mu_a^K(z) > \mu_a^1(z)$ as well as $\mu_r^K(z) > \mu_r^1(z)$, that is, both the expected value conditional on approving as well as conditional on rejecting an arbitrary candidate are higher under multi-option sequential search compared to single-option sequential search. Similar to the case of unanimity voting, $\mu_a^K(z)$ increases since a member approves a candidate only if the candidate's value is the highest among the K values that this member observes. The reason why $\mu_r^K(z)$ increases is, in intuitive terms, as follows: If $K = 1$, rejecting some candidate means that this candidate's value is below the cutoff z. In contrast, if $K > 1$, a member might also reject some candidate with a value above the cutoff z in case another candidate has an even higher value.

However, at the same time, we have that $w^K(z) < w^1(z)$. Conditional on stopping, the expected share of members who approve some candidate k decreases when moving from single-option to multi-option sequential search. This holds because the probability that a single member approves some candidate k decreases, since the candidate's value has to be the maximum out of K values in addition to being above the cutoff z.

Finally, since $\mu_a^K(z) > \mu_r^K(z)$ as well as $\mu_a^1(z) > \mu_r^1(z)$, the overall effect on the expected value conditional on stopping is a priori ambiguous. We prove Lemma 2.3 by employing a technical result from Albrecht, Anderson, and Vroman (2010) related to the expected share of members who approve some candidate k conditional on stopping.

In Proposition 2.5, we claim that multi-option sequential search dominates single-option sequential search independently of the shape of the cost function (superadditive or linear, subadditive, or none of the two) as long as the magnitude of the search costs is sufficiently small.

Proposition 2.5. *Suppose that the density f is log-concave, take any qualified majority voting rule distinct from unanimity, i.e., $M < N$, and consider any function h. Then, for all $K > 1$, there exists $\bar{c}_K > 0$ such that for all $c < \bar{c}_K$, the committee's ex ante utilitarian welfare is higher under multi-option sequential search with K candidates per period relative to single-option sequential search, i.e., $v_K > v_1$.*

Intuitively, the increase in the expected value conditional on hiring when moving from single-option sequential search to multi-option sequential search as revealed by

Lemma 2.3 outweighs the potential rise of expected search costs[28] if the magnitude of costs c is sufficiently small. We emphasize once again that this result does not depend on the type of the cost function. For any function h, and for any $K > 1$, there are cost levels c such that single-option sequential search is dominated by multi-option sequential search.[29]

Let us discuss the proof of Proposition 2.5. To the contrary, suppose that there exists $K > 1$, such that for all $\bar{c}_K > 0$, there exists $c < \bar{c}_K$ such that $v_1 \geq v_K$. Again, without loss of generality, focus on interior cutoffs. Thus, we have that $z_1 \geq z_K$ where, again, z_K denotes the equilibrium cutoff if there are $K \geq 1$ candidates per period. Recall that Lemma 2.1 implies that the log-concavity of f is sufficient for $\frac{dS^1(z,N,M)}{dz} \leq 1$. When employing this Lipschitz condition, we obtain that the difference $S^K(z_K, N, M) - S^1(z_K, N, M)$ is bounded above by the difference in expected search costs between $K > 1$ and $K = 1$. Now, in contrast to unanimity voting, if $M < N$, the equilibrium cutoffs arising under single-option as well as under multi-option sequential search do not converge to the upper bound of the value distribution as the magnitude of search costs c becomes small, but they remain bounded away from $\bar{x}$.[30] The intuition for this result is as follows: Under qualified majority voting, conditional on stopping, a candidate might be hired even though some particular member has rejected this candidate. Taking that scenario—which does not arise under unanimity voting—into account, members do not become arbitrarily picky if search costs become small. Consequently, if c goes to 0, the difference in expected search costs discussed above vanishes. However, due to Lemma 2.3, the difference $S^K(z_K, N, M) - S^1(z_K, N, M)$ remains strictly positive.[31] This is the desired contradiction.

Our analysis reveals that the ranking of the two types of search technologies for the single-searcher case does not generally extend to the committee search case. Again, note that the single decision-maker case is equivalent to the case of a committee with size $N = 1$ operating under the unanimity voting rule. Thus, our results from section 2.5 apply. For the committee search case, we have shown that if costs are superadditive or linear and the magnitude of search costs c is small, single-option sequential search is superior if the voting rule is unanimity, whereas multi-option sequential search yields a higher welfare under qualified majority voting. What drives this difference? If the voting rule is unanimity, there is a race between the difference

28. We write potential rise of expected search costs because depending on the shape of the function h the expected search costs might also be lower under multi-option sequential search compared to single-option sequential search. Of course, this only reinforces our reasoning.

29. However, as emphasized in Proposition 2.5, the threshold $\bar{c}_K$ depends on the number of candidates $K > 1$ that are simultaneously evaluated in each time period.

30. For the case of single-option sequential search, this observation has been made previously in Albrecht, Anderson, and Vroman (2010) as well as Compte and Jehiel (2010).

31. This step fails if the voting rule is unanimity because, in this case, if c goes to 0, z_K converges to $\bar{x}$ and, thus, $S^K(z_K, N, N) - S^1(z_K, N, N)$ would vanish as well.

in the expected value conditional on stopping and the difference in the expected search costs between $K > 1$ and $K = 1$: if c becomes small, the difference in expected search costs between $K > 1$ and $K = 1$ vanishes, and, in addition, the difference in the expected value conditional on hiring also goes to 0. In contrast, under qualified majority voting, if c becomes small, as in the unanimity voting case, the difference in the expected search costs goes to 0. However, in contrast to the unanimity voting case, the difference in the expected value conditional on stopping does not vanish because equilibrium cutoffs do not converge to $\bar{x}$, but they stay bounded away from it. This discrepancy explains why the ranking of the two types of search procedures is different when the voting rule is qualified majority instead of unanimity. Therefore, when comparing the single-searcher case with the committee search case, the choice of the voting rule crucially matters.

2.7 Conclusion

In this paper, we contrast two committee search procedures: the well-known sequential search procedure, in which candidates are evaluated "one at a time", and multi-option sequential search, where, in each period, committees simultaneously evaluate a set of candidates of fixed size. We study the equilibrium behavior under these search procedures and show equilibrium existence as well as uniqueness within some reasonably restricted class of equilibria. Based on the equilibrium analysis, we compare single-option and multi-option sequential search in terms of acceptance standards and welfare. We identify circumstances under which the "one at a time"-policy commonly studied in the committee search literature is not optimal. Generally, the superiority of one or the other search technology depends on two important ingredients of the search problem: the voting rule and the specification of the search costs associated with the simultaneous evaluation of multiple candidates.

If the committee operates under the unanimity rule, single-option sequential search outperforms any multi-option sequential search procedure if the search costs increase at least linearly in the number of candidates evaluated in each period. In contrast, if the search costs are strictly below the linear benchmark, even if they are only slightly below it, multi-option sequential search improves welfare if the magnitude of costs is sufficiently small. Therefore, in the case of unanimity voting, the conclusion is sensitive to the shape of the cost function. Allowing for correlation among the committee members' values does not alter these findings. Similarly, our results concerning acceptance standards and partly welfare also carry over to the case of interdependent-value preferences. Thus, these findings appear to be robust.

This dependence on the form of the cost function partly vanishes when committees employ a qualified majority rule different from unanimity. In this case, evaluating multiple candidates in each time period improves welfare compared to single-option sequential search for any type of cost function as long as the magnitude of the search costs is sufficiently small. Consequently, the assessment of single-option and multi-option sequential search considerably changes when moving from the unanimity rule to qualified majority rules. This is the main qualitative insight of this paper. Again, note that the search conducted by a single agent is a special case of committee search with unanimity voting. Consequently, our analysis reveals that the results for the single decision-maker case (cf. section 2.2 for references) do not carry over to the committee setting, but the presence of a committee alters the search design problem and implies different rankings of search procedures.

Finally, let us discuss the implications of our results for committee search in practice. First, consider the application to academic hiring, and suppose that a university seeks to hire a full professor. It seems reasonable to assume that search costs are rather negligible in view of the importance of the hiring decision. Therefore, in this case, if the hiring committee employs a qualified majority rule distinct from unanimity, our results suggest that the committee should not hire on a rolling basis, but rather evaluate multiple candidates at a time. In reality, we indeed observe that hiring committees often employ some kind of multi-option sequential search procedure, making their choice of the search procedure consistent with our results.[32] Second, go back to the example of a family searching for a house. Here, it seems natural that the voting rule is unanimity. Now, the family might search for a house in their current area of residence or they might be planning to move to an ulterior region. The former situation might correspond to the case of linear costs whereas the latter circumstances give rise to subadditive costs because the family has to travel to the region where they search for a house. Thus, our findings suggest that the family should employ single-option sequential search in the first case and multi-option sequential search in the second case. Third, consider the application of project search conducted by a committee in a firm. If the project choice is of high importance for the firm, the situation appears to be similar to academic hiring, and, hence, as long as the voting rule is not unanimity, our results suggest relying on multi-option sequential search instead of single-option sequential search. In contrast, if the required search effort is substantial relative to the importance of the value of the projects, search should rather be conducted according to the “one at at time”-policy. The above discussion demonstrates that our findings have practically relevant implications which appear to be intuitive.

32. See for example the descriptions of the hiring processes of the Columbia University in the City of New York (2016), The University of Arizona (2019) or The University of California, Berkeley (2019).

Appendix 2.A Proofs

2.A.1 Characterization

To begin with, we claim that the best response of any member $i \in \mathcal{N}$ against an arbitrary neutral stationary Markov strategy that is symmetric across all other members amounts to a maximum-strategy with cutoff, that is, member i votes in favor of candidate $k \in \mathcal{K}$ if and only if

$$x_i^k \;\geq\; \max_{l \neq k} x_i^l \text{ and } x_i^k \;\geq\; z$$

with $z \in [0, \bar{x})$ being some cutoff.

Assume that all members except for member $i \in \mathcal{N}$ in some period t behave according to a common Markovian strategy that is stationary and neutral. First of all, let v be the continuation value member i obtains when search continues. Note that v does not depend on past or current actions, or value realizations since the continuation strategy adopted by all members in periods following t is Markovian. Also, it is neither sensitive to the identity i of the member nor to calendar time because continuation strategies are symmetric across members and stationary. Now, suppose that member i observes the value realizations $(x_i^1, \ldots, x_i^K)$ in period t. Member i is pivotal for candidate k if and only if exactly $M-1$ out of the other $N-1$ members choose action k in the given period, that is, approve candidate k.

Let $p_k(a,b) > 0$ with $a \in \mathbb{N}$, $b \in \mathbb{N}_0$ and $b \leq a$ denote the probability that exactly b out of a members choose action k in the given period. Similarly, $P_k(a,b) > 0$ with $a, b \in \mathbb{N}$ and $b \leq a$ describes the probability that at most $b-1$ out of a members select action k. Then, the probability that member i is pivotal in favor of candidate k is given by $p_k(N-1, M-1)$.

The expected utility that member i obtains when approving candidate k can be expressed as follows:

$$\begin{aligned} &[(1 - P_k(N-1, M)) + p_k(N-1, M-1)]x_i^k + \sum_{l \in \{1,\ldots,K\}: l \neq k} [1 - P_l(N-1, m)]x_i^l \\ &\quad + [P_k(N-1, M) - p_k(N-1, M-1) - \sum_{l \in \{1,\ldots,K\}: l \neq k} (1 - P_l(N-1, M))]v. \end{aligned}$$

The expected payoff of member i when voting in favor of continuing search, i.e., selecting action 0, amounts to

$$\sum_{l \in \{1,\ldots,K\}} [1 - P_l(N-1, M)]x_i^l + [1 - \sum_{l \in \{1,\ldots,K\}} (1 - P_l(N-1, M))]v.$$

Since the stationary Markov strategy that is commonly adopted by members distinct from i is neutral, it holds that $P_d(a,b) = P_e(a,b)$ as well as $p_d(a,b) = p_e(a,b)$ for all $d, e \in \mathcal{K}$. For simplicity, write $P(a,b)$ and $p(a,b)$ to denote these probabilities.

Consequently, the expected utility of choosing action k can be reformulated in the following way:

$$p(N-1,M-1)x_i^k + [1-P(N-1,M)] \sum_{l\in\{1,\ldots,K\}} x_i^l \\ + [1-K(1-P(N-1,M)) - p(N-1,M-1)]v.$$

Similarly, the expected payoff of action 0 simplifies to the expression

$$[1-P(N-1,M)] \sum_{l\in\{1,\ldots,K\}} x_i^l + [1-K(1-P(N-1,M))]v.$$

Thus, voting in favor of candidate k is optimal for member i if and only if, for all $m \in \mathcal{K}$ with $m \neq k$,

$$\begin{aligned} & p(N-1,M-1)x_i^k + [1-P(N-1,M)] \sum_{l\in\{1,\ldots,K\}} x_i^l \\ & \qquad + [1-K(1-P(N-1,M)) - p(N-1,M-1)]v \\ \geq\ & p(N-1,M-1)x_i^m + [1-P(N-1,M)] \sum_{l\in\{1,\ldots,K\}} x_i^l \\ & \qquad + [1-K(1-P(N-1,M)) - p(N-1,M-1)]v, \end{aligned}$$

and, at the same time,

$$\begin{aligned} & p(N-1,M-1)x_i^k + [1-P(N-1,M)] \sum_{l\in\{1,\ldots,K\}} x_i^l \\ & \qquad + [1-K(1-P(N-1,M)) - p(N-1,M-1)]v \\ \geq\ & [1-P(N-1,M)] \sum_{l\in\{1,\ldots,K\}} x_i^l + [1-K(1-P(N-1,M))]v. \end{aligned}$$

The former condition is equivalent to requiring that $x_i^k \geq \max_{l\neq j} x_i^l$. The latter condition reduces to $x_i^k \geq v$. This means that there exists a cutoff value $z_i(t) \in [0,\overline{x})$ such that this condition is met if and only if $x_i^j \geq z_i(t)$. Moreover, the cutoff value solves $z_i(t) = v$ whenever it is interior. Hence, given an arbitrary neutral stationary Markov strategy commonly adopted by all members except for member i in period t, it is optimal for member i to employ a maximum-strategy with cutoff $z_i(t)$ in this period. In the following, we make use of this claim, and we establish the sufficiency and the necessity part separately.

Regarding necessity, it is immediate from the previous claim that any symmetric stationary Markov equilibrium in neutral strategies must involve a maximum-strategy with cutoff $z \in [0,\bar{x})$ solving $z = v$ whenever being interior, and that this strategy is commonly adopted by all members since, otherwise, at least one member has a profitable deviation. In particular, the cutoffs are neither sensitive to the members' identities nor to calendar time because, by assumption, equilibria are symmetric and stationary. Moreover, the consistency of continuation values and equilibrium strategies implies that v must satisfy

$$v = -c \cdot h(K) + [1 - K(1 - P(N,M))]v + K \cdot [1 - P(N,M)]\mathbb{E}[X_i^k|\text{candidate } k \text{ hired}].$$

Rearranging this equation yields

$$v = -\frac{c \cdot h(K)}{K \cdot [1 - P(N,M)]} + \mathbb{E}[X_i^k|\text{candidate } k \text{ hired}].$$

Therefore, equilibrium cutoffs solve the equation

$$z = -\frac{c \cdot h(K)}{K \cdot [1 - P(N,M)]} + \mathbb{E}[X_i^k|\text{candidate } k \text{ hired}]$$

whenever they are interior. Finally, recall that $P(N,M)$ denotes the probability that at most $M-1$ out of N members approve some candidate k. Thus, when using the notation introduced in the main text, we have that $P(N,M) = Q^K(z,N,M)$. This concludes the proof of the necessity part.

Next, we turn to sufficiency. First of all, observe that strategy profiles in which all members adopt the same maximum-strategy with cutoff $z \in [0,\bar{x})$ are symmetric, neutral, and stationary Markov. Furthermore, as argued in the necessity part of this proof, these strategy profiles give rise to continuation values satisfying

$$v = -\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z,N,M)]} + \mathbb{E}[X_i^k|\text{candidate } k \text{ hired}].$$

Consequently, it remains to verify that these strategy profiles constitute equilibria. To this end, consider any strategy with cutoff $z \in [0,\bar{x})$ solving

$$z = v = -\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z,N,M)]} + \mathbb{E}[X_i^k|\text{candidate } k \text{ hired}]$$

whenever the cutoff z is interior. First, by construction, the consistency of continuation values and strategies is fulfilled. Second, if all members apart from member $i \in \mathcal{N}$ in period t adopt the discussed strategy, the claim above implies that it is optimal for member i to follow the same strategy in period t, that is, the maximum-strategy with cutoff $z_i(t)$ solving $z_i(t) = v = z$ whenever it is interior. Now, the one-shot deviation principle implies that no member has a profitable deviation. Thus, the maximum-strategy with cutoff z solving

$$z = v = -\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z,N,M)]} + \mathbb{E}[X_i^k|\text{candidate } k \text{ hired}]$$

whenever being interior constitutes an equilibrium. This completes the sufficiency part.

2.A.2 Existence and Uniqueness

Proof of Proposition 2.1.
Recall that $S^K(z,N,M) = \mathbb{E}[X_i^k|\text{candidate } k \text{ accepted}]$. Rewriting equation (2.1) which characterizes equilibrium cutoff values yields

$$\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z,N,M)]} \;=\; S^K(z,N,M) - z.$$

Suppose that $z = 0$. In this case, the left-hand side amounts to

$$\frac{c \cdot h(K)}{K \cdot [1 - Q^K(0,N,M)]} \;=\; \frac{c\frac{h(K)}{K}}{1 - Q^K(0,N,M)},$$

and the right-hand side reduces to $S^K(0,N,M)$.
In contrast, if $z \to \bar{x}$, the left-hand side goes to ∞ whereas the right-hand side amounts to $S^K(\bar{x},N,M) - \bar{x} \leq 0$.

Depending on the magnitude of the search costs c, we perform a case distinction:

1) $\frac{c\frac{h(K)}{K}}{1-Q^K(0,N,M)} < S^K(0,N,M)$

In this case, we observe that the left-hand side is strictly smaller than the right-hand side of the equilibrium equation when evaluating both sides at $z = 0$. In contrast, if z is sufficiently close to $\bar{x}$, the left-hand side is strictly larger than the right-hand side. Moreover, note that both sides of the equation involve functions that are continuous in z. Hence, the intermediate value theorem yields the existence of a cutoff z that solves equation (2.1).

2) $\frac{c\frac{h(K)}{K}}{1-Q^Kk(0,N,M)} = S^K(0,N,M)$

Here, the cutoff $z = 0$ solves the equilibrium equation which means that the maximum-strategy with cutoff $z = 0$ constitutes an equilibrium.

3) $\frac{c\frac{h(K)}{K}}{1-Q^K(0,N,M)} > S^K(0,N,M)$

In this case, suppose that all members apart from member $i \in \mathcal{N}$ in period t adopt the maximum-strategy with cutoff $z = 0$. In this case, the arguments in Appendix 2.A.1 still apply, and, thus, it is optimal for member i to follow some maximum-strategy with cutoff. However, since

$$v \;=\; -\frac{c\frac{h(K)}{K}}{1 - Q^K(0,N,M)} + S^K(0,N,M) \;<\; 0$$

by assumption, the optimal cutoff for member i in the given period is $z = 0$. The reason is that member i wants to stop search as quickly as possible, and the probability of voting in favor of some candidate k is maximized at $z = 0$. Alluding to the one-deviation-principle, this shows that there exists a boundary equilibrium such that the maximum-strategy with cutoff amounting to $z = 0$ forms an equilibrium. □

Proof of Lemma 2.1.
We establish that $S^K_z(z,N,M) \leq 1$ which implies that the function $S^K(z,N,M) - z$ is non-increasing in z. Subsequently, again, we make use of the notation

$$\begin{aligned} \mu^K_a(z) &= \mathbb{E}[X^k_i | X^k_i \geq z \text{ and } X^k_i \geq \max_{l \neq k} X^l_i] \text{ and} \\ \mu^K_r(z) &= \mathbb{E}[X^k_i | X^k_i < z \text{ or } X^k_i < \max_{l \neq k} X^l_i]. \end{aligned}$$

Then, as shown in Appendix 2.B.1, $S^K(z,N,M)$ can be expressed as

$$S^K(z,N,M) = w^K(z)\mu^K_a(z) + (1 - w^K(z))\mu^K_r(z),$$

where $w^K(z)$ is given by

$$w^K(z) = \sum_{l=M}^{N} \frac{q^K(z,N,l)}{1 - Q^K(z,N,M)} \frac{l}{N}.$$

Further, to simplify the notation, define

$$1 - R^K(z) := \Pr(X^k_i \geq \max_{l \neq k} X^l_i, X^k_i \geq z).$$

First, we obtain that $\frac{dw^K(z)}{dz} \leq 0$.[33] Observe that $w^K(z)$ constitutes the average of terms of form $\frac{l}{N}$ with weights

$$w^K_l(z) := \frac{q^K(z,N,l)}{1 - Q^K(z,N,M)}.$$

We claim that, for all $l < l'$, $\frac{w^K_l(z)}{w^K_{l'}(z)}$ is non-decreasing in z. This means that increasing z yields a stochastic decrease according to the likelihood-ratio ordering which, as is well-known, implies a stochastic decrease in terms of first-order stochastic dominance. Hence, exploiting the average structure of $w^K(z)$, when increasing z, the average $w^K(z)$ decreases. In other words, we have $\frac{dw^K(z)}{dz} \leq 0$. In order to see that $\frac{w^K_l(z)}{w^K_{l'}(z)}$ is increasing in z, note that

$$\frac{w^K_l(z)}{w^K_{l'}(z)} = \frac{\binom{N}{l}}{\binom{N}{l'}} R^K(z)^{l'-l}(1 - R^K(z))^{l-l'},$$

33. The argument yielding $\frac{dw^K(z)}{dz} \leq 0$ is analogous to step 2 in the proof of Lemma 1 in Albrecht, Anderson, and Vroman (2010).

and, therefore, straightforward differentiation yields

$$\frac{d\frac{w_l^K(z)}{w_{l'}^K(z)}}{dz} = \frac{\binom{N}{l}}{\binom{N}{l'}}\frac{dR^K(z)}{dz}(l'-l)R^K(z)^{l'-l-1}(1-R^K(z))^{l-l'-1}.$$

The derivation in Appendix 2.B.2 reveals that

$$1-R^K(z) = \frac{1}{K}[1-F(z)^K].$$

Thus, $\frac{dR^K(z)}{dz} = F(z)^{K-1}f(z) \geq 0$ and we obtain that $\frac{d\frac{w_l^K(z)}{w_{l'}^K(z)}}{dz} \geq 0$ which is the desired claim. Therefore, we conclude that $\frac{dw^K(z)}{dz} \leq 0$.

Second, we show that $\mu_a^K(z) - z$ is non-increasing or, in other words, $\frac{d\mu_a^K(z)}{dz} \leq 1$. Consider the density

$$\begin{aligned} g^K(x) \; := \; & \Pr(X_i^k = x | X^k \geq \max_{l \neq k} X_i^l) \\ = \; & \frac{\Pr(X_i^k = x, X_i^k \geq \max_{l\neq k} X_i^l)}{\Pr(X_i^k \geq \max_{l\neq k} X_i^l)} \\ = \; & \frac{\Pr(X_i^k = x, x \geq \max_{l\neq k} X_i^l)}{\Pr(X_i^k \geq \max_{l\neq k} X_i^l)} \\ = \; & \frac{\Pr(X_i^k = x)\Pr(x \geq \max_{l\neq k} X_i^l)}{\Pr(X_i^k \geq \max_{l\neq k} X_i^l)} \\ = \; & Kf(x)[F(x)]^{K-1}. \end{aligned}$$

We know from Prékopa (1973) that the log-concavity of the density f implies that the cdf F is also log-concave. Moreover, since the product of log-concave functions must be again log-concave, we obtain that the density g^K is log-concave as well. Therefore, as is well-known, the log-concavity of g^K implies that the random variable $X_i^k | X_i^k \geq \max_{l\neq k} X_i^l$ has the decreasing mean residual life property which means that $\mu_a^K(z) - z$ is non-increasing.[34] Thus, we conclude that $\frac{d\mu_a^K(z)}{dz} \leq 1$.

Third, we establish that $\frac{d\mu_r^K(z)}{dz} \leq 1$. By the law of total expectation, we obtain

$$\mu = \mathbb{E}[X_i^k] = \mu_a^K(z)[1-R(z)] + \mu_r^K(z)R(z).$$

Again, in Appendix 2.B.2, we derive that

$$1-R^K(z) = \frac{1}{K}[1-F(z)^K].$$

34. Bagnoli and Bergstrom (2005) discuss the relationship between log-concave densities and concepts from reliability theory.

Thus,

$$\mu = \mu_a^K(z)[\frac{1}{K}(1-F(z)^K)] + \mu_r^K(z)[1-\frac{1}{K}(1-F(z)^K)].$$

Let G^K be the cdf of the random variable $X_i^k | X_i^k \geq \max_{l \neq k} X_i^l$. Hence, rearranging yields

$$\begin{aligned}
\mu_r^K(z) &= \frac{\mu - \mu_a^K(z)[\frac{1}{K}(1-F(z)^K)]}{1-\frac{1}{K}(1-F(z)^K)} \\
&= \frac{\int_0^{\bar{x}} sf(s)ds - [\frac{1}{K}(1-F(z)^K)]\int_z^{\bar{x}} s\frac{g^K(s)}{1-G^K(z)}ds}{1-\frac{1}{K}(1-F(z)^K)} \\
&= \frac{\int_0^{\bar{x}} sf(s)ds - \int_z^{\bar{x}} sf(s)F(s)^{K-1}ds}{1-\frac{1}{K}(1-F(z)^K)}.
\end{aligned}$$

Taking the derivative of $\mu_r^K(z)$ with respect to z yields

$$\begin{aligned}
&\frac{d\mu_r^K(z)}{dz} \\
&= \frac{(zf(z)F(z)^{K-1}))\cdot(1-\frac{1}{K}(1-F(z)^K)) - \left(\int_0^{\bar{x}} sf(s)ds - \int_z^{\bar{x}} sf(s)F(s)^{K-1}ds\right)\cdot f(z)F(z)^{K-1}}{[1-\frac{1}{K}(1-F(z)^K)]^2} \\
&= \frac{f(z)F(z)^{K-1}\left[z(1-\frac{1}{K}) + z\frac{1}{K}F(z)^K - sF(s)\Big|_0^{\bar{x}} + \int_0^{\bar{x}} F(s)ds + s\frac{1}{K}F(s)^K\Big|_z^{\bar{x}} - \int_z^{\bar{x}} \frac{1}{K}F(s)^K ds\right]}{[1-\frac{1}{K}(1-F(z)^K)]^2} \\
&= \frac{f(z)F(z)^{K-1}\left[z(1-\frac{1}{K}) + z\frac{1}{K}F(z)^K - \bar{x}(1-\frac{1}{K}) - z\frac{1}{K}F(z)^K + \int_0^{\bar{x}} F(s)ds - \int_z^{\bar{x}} \frac{1}{K}F(s)^K ds\right]}{[1-\frac{1}{K}(1-F(z)^K)]^2} \\
&= \frac{f(z)F(z)^{K-1}\left[(z-\bar{x})(1-\frac{1}{K}) + \int_0^{\bar{x}} F(s)ds - \int_z^{\bar{x}} \frac{1}{K}F(s)^K ds\right]}{[1-\frac{1}{K}(1-F(z)^K)]^2} \\
&= \frac{f(z)F(z)^{K-1}\left[\int_0^{\bar{x}} F(s)ds - \int_z^{\bar{x}} [1-\frac{1}{K}(1-F(s)^K)]ds\right]}{[1-\frac{1}{K}(1-F(z)^K)]^2} \\
&= \frac{f(z)F(z)^{K-1}\left[\int_0^{z} F(s)ds + \int_z^{\bar{x}} F(s)ds - \int_z^{\bar{x}} [1-\frac{1}{K}(1-F(s)^K)]ds\right]}{[1-\frac{1}{K}(1-F(z)^K)]^2} \\
&= \frac{f(z)F(z)^{K-1}[\int_0^{z} F(s)ds + \int_z^{\bar{x}} F(s) - [1-\frac{1}{K}(1-F(s)^K)]ds]}{[1-\frac{1}{K}(1-F(z)^K)]^2}.
\end{aligned}$$

Since we have $\frac{d\mu_r^K(z)}{dz}\Big|_{z=0} = 0 \leq 1$, for the remainder of the proof of $\frac{d\mu_r^K(z)}{dz} \leq 1$, suppose that $z \neq 0$.

Again, due to Prékopa (1973), log-concavity is preserved under integration. Hence, since the density f is log-concave, the cdf $F(z) = \int_0^z f(s)ds$ is also log-concave and, consequently, the left-hand integral $\int_0^z F(s)ds$ must be log-concave as well. By definition of log-concavity, this means that $\int_0^z F(s)ds \leq \frac{F(z)^2}{f(z)}$.[35]

Moreover, note that, for all $s \in [0, \bar{x}]$,

$$\begin{aligned}\frac{1}{K}(1 - F(s)^K) \; = \; 1 - R^K(s) \; &= \; \Pr(X^k \; \geq \; \max_{l \neq k} X^l \text{ and } X^k \; \geq \; s) \\ &\leq \; \Pr(X^k \; \geq \; s) \\ &= \; 1 - F(s).\end{aligned}$$

Thus, we obtain, for all $s \in [0, \bar{x}]$, that

$$F(s) - [1 - \frac{1}{K}(1 - F(s)^K)] \; \leq \; 0,$$

and, in particular, it holds that

$$\int_z^{\bar{x}} F(s) - [1 - \frac{1}{K}(1 - F(s)^K)]ds \; \leq \; 0.$$

Also, observe that $F(z) - [1 - \frac{1}{K}(1 - F(z)^K)] \leq 0$ is equivalent to

$$\frac{1}{1 - \frac{1}{K}(1 - F(z)^K)} \; \leq \; \frac{1}{F(z)}.$$

Employing the derived inequalities yields

$$\begin{aligned}\frac{d\mu_r^K(z)}{dz} \; &= \; \frac{f(z)F(z)^{K-1}\left[\int_0^z F(s)ds + \int_z^{\bar{x}} F(s) - [1 - \frac{1}{K}(1 - F(s)^K)]ds\right]}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\ &\leq \; \frac{f(z)F(z)^{K-1}\int_0^z F(s)ds}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\ &\leq \; \frac{f(z)F(z)^{K-1}\frac{F(z)^2}{f(z)}}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\ &= \; \frac{F(z)^{K+1}}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\ &\leq \; \frac{F(z)^{K+1}}{F(z)^2} \\ &= \; F(z)^{K-1} \\ &\leq \; 1.\end{aligned}$$

35. Again, for a discussion of these kinds of implications, we refer to Bagnoli and Bergstrom (2005).

Therefore, we conclude that $\frac{d\mu_r^K(z)}{dz} \leq 1$.
Further, note that $\mu_a^K(z) > \mu_r^K(z)$ or, equivalently, $\mu_a^K(z) - \mu_r^K(z) > 0$. Taking together the three ingredients $\frac{dw^K(z)}{dz} \leq 0$, $\frac{d\mu_a^K(z)}{dz} \leq 1$ and $\frac{d\mu_r^K(z)}{dz} \leq 1$, we have

$$\begin{aligned}
S_z^K(z,N,M) &= \frac{d\left[w^K(z)\mu_a^K(z) + (1-w^K(z))\mu_r^K(z)\right]}{dz} \\
&= \frac{d\left[w^K(z)[\mu_a^K(z) - \mu_r^K(z)] + \mu_r^K(z)\right]}{dz} \\
&= \frac{dw^K(z)}{dz}\left[\mu_a^K(z) - \mu_r^K(z)\right] + w^K(z)\left[\frac{d\mu_a^K(z)}{dz} - \frac{d\mu_r^K(z)}{dz}\right] + \frac{d\mu_r^K(z)}{dz} \\
&= \frac{dw^K(z)}{dz}\left[\mu_a^K(z) - \mu_r^K(z)\right] + w^K(z)\frac{d\mu_a^K(z)}{dz} + [1-w^K(z)]\frac{d\mu_r^K(z)}{dz} \\
&\leq w^K(z)\frac{d\mu_a^K(z)}{dz} + [1-w^K(z)]\frac{d\mu_r^K(z)}{dz} \\
&\leq w^K(z) + [1-w^K(z)] \\
&= 1.
\end{aligned}$$

In conclusion, as desired, we infer that $S_z^K(z,N,M) \leq 1$ which, implies that the function $S^K(z,N,M) - z$ is non-increasing in z. Additionally, the argument reveals that $S_z^K(z,N,M) < 1$ whenever $z \neq 0$ and, thus, $S^K(z,N,M) - z$ is strictly decreasing in z. □

Proof of Proposition 2.2.
To begin with, by Proposition 2.1, there exists an equilibrium. Moreover, we know from Lemma 2.1 that the function $S^K(z,N,M) - z$ is decreasing in z. Next, we show that the function

$$\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z,N,M)]}$$

is increasing in z.
Again, to simplify the notation, define

$$1 - R^K(z) \;:=\; \Pr(X_i^k \;\geq\; \max_{l \neq k} X_i^l, X_i^k \;\geq\; z).$$

Taking the derivative of the discussed function with respect to z yields

$$\frac{d}{dz}\left[\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z,N,M)]}\right] \;=\; \frac{c \cdot h(K) \cdot Q_z^K(z,N,M)}{K \cdot [1 - Q^K(z,N,M)]^2}.$$

Further, using the relationship between the Binomial and the Beta distribution,[36] we have

$$\begin{aligned} Q^K(z,N,M) &= \sum_{l=0}^{M-1} \binom{N}{l} (1-R^K(z))^l \cdot R^K(z)^{N-l} \\ &= \frac{N!}{(N-M)! \cdot (M-1)!} \int_0^{R^K(z)} s^{N-M}(1-s)^{M-1} ds. \end{aligned}$$

Taking the derivative of $Q^K(z,N,M)$ with respect to z yields

$$Q_z^K(z,N,M) = \frac{N!}{(N-M)! \cdot (M-1)!} \frac{dR^K(z)}{dz} R^K(z)^{N-M}(1-R^K(z))^{M-1}.$$

Again, the derivation in Appendix 2.B.2 reveals that

$$1-R^K(z) = \frac{1}{K}[1-F(z)^K].$$

Thus, we have that $\frac{dR^K(z)}{dz} = F(z)^{K-1}f(z) \geq 0$. Hence, we obtain that $Q_z^K(z,N,M) \geq 0$, yielding the desired inference that

$$\frac{d}{dz}\left[\frac{c \cdot h(K)}{K \cdot [1-Q^K(z,N,M)]}\right] = \frac{c \cdot h(K) \cdot Q_z^K(z,N,M)}{K \cdot [1-Q^K(z,N,M)]^2} \geq 0.$$

Additionally, the argument shows that this derivative is strictly larger than 0 whenever $z \neq 0$ and, hence, $\frac{c \cdot h(K)}{K \cdot [1-Q^K(z,N,M)]}$ is strictly increasing in z.

Consider the equation characterizing equilibrium cutoff values

$$S^K(z,N,M) - z = \frac{c \cdot h(K)}{K \cdot [1-Q^K(z,N,M)]} = \frac{c\frac{h(K)}{K}}{1-Q^K(z,N,M)}.$$

Depending on the magnitude of the search costs, we perform a case distinction:

1) $\frac{c\frac{h(K)}{K}}{1-Q^K(0,N,M)} < S^K(0,N,M)$

In this case, all cutoffs associated with equilibrium strategies are interior, satisfying $z \neq 0$. In particular, these cutoffs must solve the equilibrium equation. However, due to Lemma 2.1, the left-hand side of the discussed equation is strictly decreasing, and the right-hand side is strictly increasing. Therefore, both sides of the equation have at most one intersection which establishes uniqueness of equilibrium.

36. Cf. Casella and Berger (2002)

2) $\frac{c\frac{h(K)}{K}}{1-Q^K(0,N,M)} \geq S^K(0,N,M)$

Here, the cutoff $z = 0$ is part of an equilibrium. Either $z = 0$ solves the equilibrium equation, or there is a boundary equilibrium involving the cutoff $z = 0$. To the contrary, suppose that there is another equilibrium with some cutoff $z' > 0$. This cutoff must solve the equilibrium equation because it is interior. However, employing the monotonicity properties of the functions involved in the equilibrium equation that are partly derived in Lemma 2.1, we have

$$\frac{c\frac{h(K)}{K}}{1-Q^K(z',N,M)} > \frac{c\frac{h(K)}{K}}{1-Q^K(0,N,M)} \geq S^K(0,N,M) > S^K(z',N,M) - z'.$$

Hence, the cutoff $z' > 0$ cannot be part of an equilibrium which constitutes the desired contradiction. □

2.A.3 Unanimity Voting

Proof of Lemma 2.2.
Consider any $K > 1$. Suppose, by contradiction, that there exists some $z \in [0, \bar{x})$ such that

$$\frac{\mu_a^K(z) - z}{\mu_a^1(z) - z} = \frac{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z] - z}{\mathbb{E}[X_i^k | X_i^k \geq z] - z} \geq \frac{K[1-F(z)]}{1-F(z)^K}.$$

Rewriting the left-hand side of the inequality yields

$$\begin{aligned}
\frac{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z] - z}{\mathbb{E}[X_i^k | X_i^k \geq z] - z} &= \frac{\frac{\int_z^{\bar{x}} f(s)F(s)^{k-1} s ds}{\frac{1}{K}[1-F(z)^K]} - z}{\frac{\int_z^{\bar{x}} f(s) s ds}{1-F(z)} - z} \\
&= \frac{\frac{\int_z^{\bar{x}} f(s)F(s)^{k-1} s ds}{\frac{1}{K}[1-F(z)^K]}[1-F(z)] - z[1-F(z)]}{\int_z^{\bar{x}} f(s) s ds - z[1-F(z)]} \\
&= \frac{K[1-F(z)]}{1-F(z)^K} \frac{\int_z^{\bar{x}} f(s)F(s)^{K-1} s ds - z[\frac{1-F(z)^K}{K}]}{\int_z^{\bar{x}} f(s) s ds - z[1-F(z)]},
\end{aligned}$$

where the first step uses the fact that

$$\Pr(X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z) = \frac{1}{K}[1-F(z)^K],$$

which is derived in Appendix 2.B.2.

Thus, we get

$$\frac{\int_z^{\bar{x}} f(s)F(s)^{K-1}sds - z[\frac{1-F(z)^K}{K}]}{\int_z^{\bar{x}} f(s)sds - z[1-F(z)]} \geq 1.$$

Since $\int_z^{\bar{x}} f(s)sds - z[1-F(z)] > 0$, we have

$$\int_z^{\bar{x}} f(s)F(s)^{K-1}sds - z[\frac{1-F(z)^K}{K}] \geq \int_z^{\bar{x}} f(s)sds - z[1-F(z)],$$

or, equivalently,

$$\int_z^{\bar{x}} f(s)[F(s)^{K-1}-1]sds \geq z\left[\frac{1-F(z)^K}{K} - (1-F(z))\right].$$

Moreover, since $z < \bar{x}$,

$$\begin{aligned}\int_z^{\bar{x}} f(s)[F(s)^{K-1}-1]sds &< z\int_z^{\bar{x}} f(s)[F(s)^{K-1}-1]ds \\ &= z\left[\frac{1}{K}(1-F(z)^K) - (1-F(z))\right].\end{aligned}$$

Hence,

$$\begin{aligned}z\left[\frac{1-F(z)^K}{K} - (1-F(z))\right] &> \int_z^{\bar{x}} f(s)[F(s)^{K-1}-1]sds \\ &\geq z\left[\frac{1-F(z)^K}{K} - (1-F(z))\right],\end{aligned}$$

which is the desired contradiction.

□

Proof of Proposition 2.3.

We begin by deriving conditions for when boundary solutions of either of the search procedures arise.

First of all, note that the proofs of Propositions 2.1 and 2.2 reveal that for $K = 1$, the unique equilibrium is a corner solution if and only if $c \geq \frac{\mu}{h(1)} =: c^1$. Similarly, if $K > 1$, a boundary equilibrium arises if and only if

$$c \geq \frac{S^K(0,N,N)[1-Q^K(0,N,N)]}{\frac{h(K)}{K}} = \frac{\mu_a^K(0)[\frac{1}{K}]^N}{\frac{h(K)}{K}} =: c^K.$$

We claim that $c^K < c^1$.

Suppose not, i.e., assume that $c^K \geq c^1$. By definition, this means that

$$\frac{\mu_a^K(0)[\frac{1}{K}]^N}{\frac{h(K)}{K}} \geq \frac{\mu}{h(1)}.$$

Applying Lemma 2.2 by setting $z = 0$ yields

$$\frac{\mu_a^K(0)}{\mu} < K.$$

Combining the two inequalities, we obtain

$$\frac{h(1)\mu_a^K(0)[\frac{1}{K}]^N}{\frac{h(K)}{K}} \geq \mu > \frac{\mu_a^K(0)}{K}.$$

Hence, since, by assumption, $\frac{h(K)}{K} \geq h(1)$,

$$[\frac{1}{K}]^{N-1} > 1.$$

If $N = 1$, we obtain that $1 = [\frac{1}{K}]^0 > 1$ and, in the case that $N \geq 2$, we must have that $K < 1$. Thus, in both cases, we derived the desired contradiction.

We are now ready to perform a case distinction depending on the magnitude of the scaling parameter c:

1) $c \geq c^1 > c^K$

In this case, single-option sequential search ($K = 1$) as well as multi-option sequential search ($K > 1$) give both rise to a boundary equilibrium with equilibrium cutoffs $z_1 = 0$ and $z_K = 0$, respectively. The respective welfare levels of the two search procedures amount to

$$\begin{aligned} v_1 &= -c \cdot h(1) + \mu, \text{ and} \\ v_k &= \mu_a^K(0) - \frac{c \cdot h(K)}{K[\frac{1}{K}]^N} = \mu_a^K(0) - K^N c \frac{h(K)}{K}. \end{aligned}$$

Towards a contradiction, suppose $v_K \geq v_1$. Applying Lemma 2.2 and using $\frac{h(K)}{K} \geq h(1)$,

$$\begin{aligned} -c \cdot h(1) + \mu = v_1 \leq v_K &= \mu_a^K(0) - K^N c \frac{h(K)}{K} \\ &< K\mu - K^N c \frac{h(K)}{K} \leq K\mu - K^N c \cdot h(1). \end{aligned}$$

Thus, we conclude that

$$[K^N - 1]c \cdot h(1) < [K - 1]\mu.$$

Since $c \geq c^1 = \frac{\mu}{h(1)}$, we have $K^N - 1 < K - 1$ or, equivalently, $K^N < K$.
In the case of $N = 1$, there is a contradiction.

If $N \geq 2$, we must have that $K < 1$ which also constitutes a contradiction.

2) $c^1 > c \geq c^K$

Here, single-option sequential search ($K = 1$) admits an interior equilibrium described by the cutoff value $z_1 > 0$, whereas multi-option sequential search ($K > 1$) has a boundary equilibrium with cutoff $z_K = 0$. Therefore, the resulting welfare levels of both search procedures are given by

$$\begin{aligned} v_1 &= z_1, \text{ and} \\ v_K &= \mu_a^K(0) - \frac{c \cdot h(K)}{K[\frac{1}{K}]^N} = \mu_a^K(0) - K^N c \frac{h(K)}{K}. \end{aligned}$$

By definition of c^K, and because of $c \geq c^K$, we directly obtain that $v_K \leq 0$. In contrast, it holds that $v_1 = z_1 > 0$, directly implying $v_K < v_1$.

3) $c^1 > c^K > c$

In this case, single-option sequential search ($K = 1$) as well as multi-option sequential search ($K > 1$) give both rise to an interior equilibrium. Denote the unique equilibrium cutoff values in the game with $K > 1$ candidates per period by z_K and the unique equilibrium cutoff value in the search game with $K = 1$ candidate per period by z_1. Given private value preferences, cutoff values, or acceptance standards, coincide with welfare, i.e., $v_1 = z_1$ and $v_K = z_K$.

Assume, by contradiction, that that $v_1 = z_1 \leq z_K = v_K$. The equilibrium cutoff values satisfy the following equations:

$$\begin{aligned} S^K(z_K, N, N) - z_K &= \frac{c \cdot h(K)}{K \cdot [1 - Q^K(z_K, N, N)]} \\ S^1(z_1, N, N) - z_1 &= \frac{c \cdot h(1)}{1 - Q^1(z_1, N, N)}. \end{aligned}$$

In the following, we derive bounds on the ratio

$$\frac{S^K(z_K, N, N) - z_K}{S^1(z_1, N, N) - z_1} = \frac{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z_K] - z_K}{\mathbb{E}[X_i^k | X_i^k \geq z_1] - z_1}.$$

First, Lemma 2.1 yields that the log-concavity of f and the assumption $z_1 \leq z_K$ imply the inequality

$$\frac{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z_K] - z_K}{\mathbb{E}[X_i^k | X_i^k \geq z_1] - z_1} \leq \frac{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z_1] - z_1}{\mathbb{E}[X_i^k | X_i^k \geq z_1] - z_1}.$$

Lemma 2.2 then yields

$$\frac{\mathbb{E}[X^k|X^k \geq \max_{l\neq k} X^l, X^k \geq z_K] - z_K}{\mathbb{E}[X^k|X^k \geq z_1] - z_1} < \frac{K(1-F(z_1))}{1-F(z_1)^K}.$$

Second, by the equilibrium conditions,

$$\begin{aligned}\frac{\mathbb{E}[X^k|X^k \geq \max_{l\neq k} X^l, X^k \geq z_K] - z_K}{\mathbb{E}[X^k|X^k \geq z_1] - z_1} &= \frac{\frac{c\frac{h(K)}{K}}{1-Q^K(z_K,N,N)}}{\frac{c\cdot h(1)}{1-Q^1(z_1,N,N)}} \\ &= \frac{\frac{c\frac{h(K)}{K}}{[\Pr(X^k\geq \max_{l\neq k} X^l, X^k\geq z_K)]^N}}{\frac{c\cdot h(1)}{[\Pr(X^k\geq z_1)]^N}} \\ &= \frac{h(K)}{K}\frac{1}{h(1)}\frac{[\Pr(X^k \geq z_1)]^N}{[\Pr(X^k \geq \max_{l\neq k} X^l, X^k \geq z_K)]^N}.\end{aligned}$$

Since $\frac{h(K)}{K} \geq h(1)$ and, by assumption, $z_1 \leq z_K$, we obtain

$$\begin{aligned}\frac{h(K)}{K}\frac{1}{h(1)}\frac{[\Pr(X^k \geq z_1)]^N}{[\Pr(X^k \geq \max_{l\neq k} X^l, X^k \geq z_K)]^N} &\geq \frac{[\Pr(X^k \geq z_1)]^N}{[\Pr(X^k \geq \max_{l\neq k} X^l, X^k \geq z_1)]^N} \\ &= \frac{[1-F(z_1)]^N}{[\frac{1}{K}(1-F(z_1)^K)]^N} \\ &= \left[\frac{K(1-F(z_1))}{1-F(z_1)^K}\right]^N.\end{aligned}$$

Therefore, we get

$$\frac{\mathbb{E}[X^k|X^k \geq \max_{l\neq k} X^l, X^k \geq z_K] - z_K}{\mathbb{E}[X^k|X^k \geq z_1] - z_1} \geq [\frac{K(1-F(z_1))}{1-F(z_1)^K}]^N.$$

Putting both bounds on $\frac{S^K(z_K,N,N)-z_K}{S^1(z_1,N,N)-z_1}$ together, we conclude

$$\frac{K(1-F(z_1))}{1-F(z_1)^K} > [\frac{K(1-F(z_1))}{1-F(z_1)^K}]^N. \tag{2.A.1}$$

If $N = 1$, inequality (2.A.1) cannot be met. If $N \geq 2$, observe that

$$\frac{K(1-F(z_1))}{1-F(z_1)^K} = \frac{1-F(z_1)}{\frac{1}{K}(1-F(z_1)^K)} = \frac{1-R^1(z_1)}{1-R^K(z_1)}$$

is the ratio of the acceptance probabilities for $K = 1$ candidate compared to $K > 1$ candidates for a fixed cutoff z_1. Since the acceptance probability is smaller for $K > 1$ than for $K = 1$, this ratio must be strictly larger than 1. Hence, in the case of $N \geq 2$, inequality (2.A.1) yields also the desired contradiction.
Thus, it must be true that $v_1 = z_1 > z_K = v_K$. □

Proof of Proposition 2.4.
To begin with, denote the unique equilibrium cutoff value in the game with $K > 1$ candidates per period by z_K, and the unique equilibrium cutoff value in the search game with $K = 1$ candidate per period by z_1. Suppose to the contrary that there exists some $K > 1$ such that for all $\bar{c}_K > 0$ there exists $c < \bar{c}_K$ such that $v_1 \geq v_K$. Without loss of generality, restrict attention to sufficiently small values of c such that the equilibria under both procedures are interior. Then, cutoff values coincide with welfare, i.e., $v_1 = z_1$ and $v_K = z_K$.

The respective equilibrium thresholds satisfy the following equations:

$$\begin{aligned} S^K(z_K,N,N) - z_K &= \frac{c \cdot h(K)}{K \cdot [1 - Q^K(z_K,N,N)]} \\ S^1(z_1,N,N) - z_1 &= \frac{c \cdot h(1)}{1 - Q^1(z_1,N,N)}. \end{aligned}$$

Lemma 2.1 implies that

$$S^K(z_K,N,N) - z_K = \mathbb{E}[X_i^k | X_i^k \geq \max_{l \neq k} X^l, X_i^k \geq z_K] - z_K$$

is non-increasing.
Therefore, the assumption $z_1 \geq z_K$ yields the inequality

$$\mathbb{E}[X_i^k | X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z_1] - z_1 \leq \mathbb{E}[X_i^k | X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z_K] - z_K.$$

Moreover, since

$$\mathbb{E}[X_i^k | X_i^k \geq z_1] \leq \mathbb{E}[X_i^k | X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z_1],$$

we obtain that

$$\begin{aligned} & \mathbb{E}[X_i^k | X_i^k \geq z_1] - z_1 \leq \mathbb{E}[X_i^k | X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z_K] - z_K \\ \Leftrightarrow \quad & S^1(z_1,N,N) - z_1 \leq S^K(z_K,N,N) - z_K. \end{aligned}$$

Exploiting the equilibrium equations, we get

$$\begin{aligned} & \frac{c \cdot h(1)}{1 - Q^1(z_1,N,N)} \leq \frac{c\frac{h(K)}{K}}{1 - Q^K(z_K,N,N)} \\ \Leftrightarrow \quad & \frac{c \cdot h(1)}{[\Pr(X_i^k \geq z_1)]^N} \leq \frac{c\frac{h(K)}{K}}{[\Pr(X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z_K)]^N} \\ \Leftrightarrow \quad & [\Pr(X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z_K)]^N \leq \frac{h(K)}{K} \frac{1}{h(1)} [\Pr(X_i^k \geq z_1)]^N. \end{aligned}$$

Furthermore, again because of $z_1 \geq z_K$, we have $[\Pr(X_i^k \geq z_1)] \leq [\Pr(X_i^k \geq z_K)]$. Thus, we obtain

$$[\Pr(X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z_K)]^N \leq \frac{h(K)}{K}\frac{1}{h(1)}[\Pr(X_i^k \geq z_K)]^N.$$

Using the expressions for these probabilities derived in Appendix 2.B.2, we obtain

$$\begin{aligned} \Pr(X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z_K) &= \frac{1}{K}[1 - F(z_K)^K] \\ \Pr(X_i^k \geq z_K) &= 1 - F(z_K). \end{aligned}$$

Finally, plugging these terms into the inequality yields

$$[\frac{1}{K} \cdot \frac{1 - F(z_K)^K}{1 - F(z_K)}]^N \leq \frac{h(K)}{K}\frac{1}{h(1)}.$$

Since, by assumption, $\frac{h(K)}{K} < h(1)$, we have that the right-hand side of this inequality is strictly smaller than 1. We claim that, no matter the fixed value of

$$0 < \frac{h(K)}{K}\frac{1}{h(1)} < 1,$$

as long as the cost parameter c is sufficiently small, the left-hand side of the inequality is below 1, but arbitrarily close to it. The first part of this statement follows from the proof of Proposition 2.3. To see the second part, note that as $c \to 0$, $z \to \bar{x}$, implying that $F(z_K) \to 1$. Using L'Hôpital's rule then yields that as $c \to 0$, the left-hand side of the inequality tends to 1. Therefore, eventually, for small c, the left-hand side of the inequality exceeds the right-hand side because $\frac{h(K)}{K}\frac{1}{h(1)} < 1$. This is the desired contradiction. □

2.A.4 Qualified Majority Voting

Proof of Lemma 2.3.

To begin with, take any $K > 1$, and fix any value $z \in (0, \bar{x})$. In order to improve readability, we often drop the dependence of the involved functions on z. We tackle the case of $z = 0$ at the end of this proof.

First, we derive an expression for $S^1(z, N, M)$ in terms of $w^1(z)$, $\mu_a^1(z)$, $F(z)$ and μ. By the law of total expectation, we have

$$\mu_r^1 = \frac{\mu - (1 - F)\mu_a^1}{F},$$

and, consequently, we obtain

$$\mu_a^1 - \mu_r^1 = \mu_a^1 - \frac{\mu - (1 - F)\mu_a^1}{F} = \frac{\mu_a^1 - \mu}{F}.$$

Therefore, $S^1(z,N,M)$ can be written as

$$\begin{aligned} S^1(z,N,M) &= w^1\mu_a^1 + [1-w^1]\mu_r^1 \\ &= \mu_r^1 + w^1[\mu_a^1 - \mu_r^1] \\ &= \frac{\mu-(1-F)\mu_a^1}{F} + w^1\frac{\mu_a^1-\mu}{F} \\ &= \mu\left[\frac{1-w^1}{F}\right] + \mu_a^1\left[\frac{w^1-1+F}{F}\right] \\ &= \mu + \left[\frac{w^1-1+F}{F}\right][\mu_a^1-\mu]. \end{aligned}$$

Further, the law of total expectation yields

$$S^1(z,N,M) = \left[\frac{w^1-1+F}{F}\right][\mu_a^1-\mu] + \left[\frac{1}{K}(1-F^K)\right]\mu_a^K + \left[1-\frac{1}{K}(1-F^K)\right]\mu_r^K.$$

Second, we develop an expression for $\mu_a^K - \mu_r^K$ as well as a lower bound on this term. The law of total expectation implies

$$\mu_r^K = \frac{\mu - \frac{1}{K}(1-F^K)\mu_a^K}{1-\frac{1}{K}(1-F^K)}.$$

Thus, we obtain

$$\begin{aligned} \mu_a^K - \mu_r^K &= \mu_a^K - \frac{\mu - \frac{1}{K}(1-F^K)\mu_a^K}{1-\frac{1}{K}(1-F^K)} \\ &= \frac{\mu_a^K-\mu}{1-\frac{1}{K}(1-F^K)} \\ &\geq \frac{\mu_a^1-\mu}{1-\frac{1}{K}(1-F^K)}, \end{aligned}$$

where the inequality follows from $\mu_a^K \geq \mu_a^1$.
Now, suppose to the contrary that $S^1(z,N,M) \geq S^K(z,N,M)$. This means that

$$\begin{aligned} S^1(z,N,M) &= \left[\frac{w^1-1+F}{F}\right][\mu_a^1-\mu] + \left[\frac{1}{K}(1-F^K)\right]\mu_a^K + \left[1-\frac{1}{K}(1-F^K)\right]\mu_r^K \\ &\geq \mu_a^K w^K + \mu_r^K[1-w^K] = S^K(z,N,M). \end{aligned}$$

Rearranging this inequality yields

$$\begin{aligned} &\left[\frac{w^1-1+F}{F}\right][\mu_a^1-\mu] + \mu_r^K\left[1-\frac{1}{K}(1-F^K)-1+w^K\right] \geq \mu_a^K[w^K - \frac{1}{K}(1-F^K)] \\ &\Leftrightarrow \left[\frac{w^1-1+F}{F}\right][\mu_a^1-\mu] \geq [\mu_a^K-\mu_r^K]\left[w^K-\frac{1}{K}(1-F^K)\right]. \end{aligned}$$

Employing the lower bound on $\mu_a^K - \mu_r^K$, we have

$$\left[\frac{w^1 - 1 + F}{F}\right][\mu_a^1 - \mu] \;\geq\; \left[\frac{\mu_a^1 - \mu}{1 - \frac{1}{K}(1 - F^K)}\right]\left[w^K - \frac{1}{K}(1 - F^K)\right],$$

because $w^K - \frac{1}{K}(1 - F^K) > 0$. To see the latter point, observe that

$$\begin{aligned}
w^K &= \sum_{l=M}^{N} \frac{q^K(z,N,l)}{1 - Q^K(z,N,M)} \frac{l}{N} \\
&\geq \frac{M}{N} \sum_{l=M}^{N} \frac{q^K(z,N,l)}{1 - Q^K(z,N,M)} \\
&= \frac{M}{N} \\
&> \frac{1}{2}.
\end{aligned}$$

Moreover, since $K > 1$, we have

$$\frac{1}{K}(1 - F^K) \;\leq\; \frac{1}{2}(1 - F^K) \;\leq\; \frac{1}{2}.$$

Hence, it holds that $w^K - \frac{1}{K}(1 - F^K) > 0$.
Next, we note that $[\mu_a^1 - \mu] > 0$ because F has full support and, by assumption, $z > 0$. Thus, we arrive at the following expression:

$$\frac{w^1 - 1 + F}{F} \;\geq\; \frac{w^K - \frac{1}{K}(1 - F^K)}{1 - \frac{1}{K}(1 - F^K)}.$$

Rewriting this inequality yields

$$1 - w^1 \;\leq\; \frac{F}{1 - \frac{1}{K}(1 - F^K)}[1 - w^K].$$

Now, Albrecht, Anderson, and Vroman (2010) provide an alternative expression for the weights as a function of the probability that some member approves some candidate; they rely on the Gaussian hypergeometric function as well as the Euler integral.[37] We apply those expressions to the weights w^1 and w^K. In order to simplify the notation, let A^1 and A^K be the probability of approving some candidate k if there are one or K candidates respectively. In other words, define

$$\begin{aligned}
A^1(z) &:= 1 - F, \text{ as well as} \\
A^K(z) &:= \frac{1}{K}(1 - F^K).
\end{aligned}$$

37. See, for example, Abramowitz and Stegun (1965).

Making use of this notation, the expressions in Albrecht, Anderson, and Vroman (2010) read as follows:[38]

$$\begin{aligned} w^1 &= A^1 + \frac{M}{N}(1-A^1)\left\{\int_0^1 \left[1 + \frac{A^1}{1-A^1}(1-y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}, \text{ and} \\ w^K &= A^K + \frac{M}{N}(1-A^K)\left\{\int_0^1 \left[1 + \frac{A^K}{1-A^K}(1-y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}. \end{aligned}$$

Therefore, we obtain

$$\begin{aligned} 1-w^1 &= 1-A^1 - \frac{M}{N}(1-A^1)\left\{\int_0^1 \left[1 + \frac{A^1}{1-A^1}(1-y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1} \\ &= [1-A^1]\cdot\left[1 - \frac{M}{N}\left\{\int_0^1 \left[1 + \frac{A^1}{1-A^1}(1-y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}\right] \\ &= F\cdot\left[1 - \frac{M}{N}\left\{\int_0^1 \left[1 + \frac{A^1}{1-A^1}(1-y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}\right], \end{aligned}$$

as well as

$$\begin{aligned} 1-w^K &= 1-A^K - \frac{M}{N}(1-A^K)\left\{\int_0^1 \left[1 + \frac{A^K}{1-A^K}(1-y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1} \\ &= [1-A^K]\cdot\left[1 - \frac{M}{N}\left\{\int_0^1 \left[1 + \frac{A^K}{1-A^K}(1-y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}\right] \\ &= [1-\frac{1}{K}(1-F^K)]\cdot\left[1 - \frac{M}{N}\left\{\int_0^1 \left[1 + \frac{A^K}{1-A^K}(1-y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}\right]. \end{aligned}$$

Then, the inequality $1-w^1 \leq \frac{F}{1-\frac{1}{K}(1-F^K)}[1-w^K]$ becomes

$$\begin{aligned} &F\cdot\left[1 - \frac{M}{N}\left\{\int_0^1 \left[1 + \frac{A^1}{1-A^1}(1-y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}\right] \\ \leq &\frac{F}{1-\frac{1}{K}(1-F^K)}\left[1-\frac{1}{K}(1-F^K)\right]\left[1 - \frac{M}{N}\left\{\int_0^1 \left[1 + \frac{A^K}{1-A^K}(1-y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}\right]. \end{aligned}$$

38. The derivation can be found on pages 1403 f. in Albrecht, Anderson, and Vroman (2010).

Simplifying and rearranging this inequality yields

$$\int_0^1 \left[1+\frac{A^1}{1-A^1}(1-y^{\frac{1}{M}})\right]^{N-M} dy \;\leq\; \int_0^1 \left[1+\frac{A^K}{1-A^K}(1-y^{\frac{1}{M}})\right]^{N-M} dy.$$

In the following, we claim that, for all $y \in [0,1)$,

$$\left[1+\frac{A^1}{1-A^1}(1-y^{\frac{1}{M}})\right]^{N-M} \;>\; \left[1+\frac{A^K}{1-A^K}(1-y^{\frac{1}{M}})\right]^{N-M},$$

which implies that the former inequality cannot be true.
To begin with, note that $A^1 = A^1(z) > A^K(z) = A^K$ since $z \neq \bar{x}$. Now, take any $y \in [0,1)$ and observe that

$$\begin{aligned}
& & A_1 &> A_K \\
&\Leftrightarrow & \frac{A^1}{1-A^1} &> \frac{A^K}{1-A^K} \\
&\Leftrightarrow & 1+\frac{A^1}{1-A^1}(1-y^{\frac{1}{M}}) &> 1+\frac{A^K}{1-A^K}(1-y^{\frac{1}{M}}) \\
&\Leftrightarrow & \left[1+\frac{A^1}{1-A^1}(1-y^{\frac{1}{M}})\right]^{N-M} &> \left[1+\frac{A^K}{1-A^K}(1-y^{\frac{1}{M}})\right]^{N-M}.
\end{aligned}$$

This establishes the claim yielding the desired contradiction. Therefore, overall, we conclude that $S^1(z,N,M) < S^K(z,N,M)$ for all $z \in (0,\bar{x})$.

Finally, it remains to tackle the case of $z = 0$. Here, observe that $S^1(0,N,M) = \mu$. Suppose, towards a contradiction, that $\mu = S^1(0,N,M) \geq S^k(0,N,M)$. By the law of total expectation, we obtain

$$\begin{aligned}
\left[\frac{1}{K}(1-[F(0)]^K)\right]\mu_a^K + \left[1-\frac{1}{K}(1-[F(0)]^K)\right]\mu_r^K \;=\; \mu \;\geq\; & S^k(0,N,M) \\
=\; & \mu_a^K w^K + \mu_r^K[1-w^K].
\end{aligned}$$

Rearranging this inequality yields

$$0 \;\geq\; [\mu_a^K - \mu_r^K][w^K - \frac{1}{K}].$$

However, we have that

$$0 \;\geq\; [\mu_a^K - \mu_r^K][w^K - \frac{1}{K}] \;>\; 0,$$

because $\mu_a^K - \mu_r^K > 0$ as well as $w^K - \frac{1}{K}(1-F^K) > 0$, as established in the first part of this proof. Hence, we arrive at the desired contradiction. □

Proof of Proposition 2.5.
Suppose, by contradiction, that there exists $K > 1$ such that for all $\bar{c}_K > 0$ there exists $c < \bar{c}_K$ such that $v_1 \geq v_K$. Without loss of generality, restrict attention to sufficiently small values of c such that the equilibria under both procedures are interior. Let z_1 and z_K denote the equilibrium cutoffs corresponding to single-option as well as multi-option sequential search with K candidates, respectively. These cutoffs solve the respective equilibrium equations

$$\begin{aligned} S^1(z_1,N,M) - z_1 &= \frac{c \cdot h(1)}{1 - Q^1(z_1,N,M)} \\ S^K(z_K,N,M) - z_K &= \frac{c \cdot h(k)}{K[1 - Q^K(z_K,N,M)]} = \frac{c\frac{h(K)}{K}}{1 - Q^K(z_K,N,M)}, \end{aligned}$$

and they coincide with welfare: $z_1 = v_1$ as well as $z_K = v_K$. Thus, by assumption, $z_1 \geq z_K$.
Lemma 2.1 implies that $\frac{d[S^1(z,N,M)-z]}{dz} \leq 0$ for all $z \in [0,\bar{x})$. Making use of this property and employing the equilibrium equations as well as $z_1 \geq z_K$, we obtain

$$\begin{aligned} \frac{c \cdot h(1)}{1 - Q^1(z_1,N,M)} &= S^1(z_1,N,M) - z_1 \\ &\leq S^1(z_K,N,M) - z_K \\ &= S^1(z_K,N,M) + \frac{c\frac{h(K)}{K}}{1 - Q^K(z_K,N,M)} - S^K(z_K,N,M). \end{aligned}$$

Rearranging this inequality yields

$$S^K(z_K,N,M) - S^1(z_K,N,M) \leq \frac{c\frac{h(K)}{K}}{1 - Q^K(z_K,N,M)} - \frac{c \cdot h(1)}{1 - Q^1(z_1,N,M)}.$$

Now, we claim that there exists $B < \bar{x}$ such that for all $c > 0$, it holds $z_K < B$ and $z_1 < B$.

First, towards a contradiction, suppose that for all $B^1 < \bar{x}$ there exist $c > 0$ such that $z_1 \geq B^1$. By the equilibrium equation and the monotonicity properties of the involved functions established in the proofs of Lemma 2.1 and Proposition 2.2, we have that z_1 is weakly decreasing in c. Thus, the previous assumption requires that $z_1 \to \bar{x}$ as $c \to 0$. Consider the following rearranged version of the equilibrium equation:

$$z_1 = S^1(z_1,N,M) - \frac{c \cdot h(1)}{1 - Q^1(z_1,N,M)}.$$

If we take the limit on both sides of the equation as $c \to 0$, we obtain

$$\begin{aligned}
\bar{x} &= \lim_{c\to 0}[z_1] \\
&= \lim_{c\to 0}\left[S^1(z_1,N,M) - \frac{c\cdot h(1)}{1-Q^1(z_1,N,M)}\right] \\
&\leq \lim_{c\to 0}[S^1(z_1,N,M)] \\
&= S^1(\bar{x},N,M) \\
&< \bar{x},
\end{aligned}$$

which constitutes the desired contradiction. Recalling the average representation of $S^1(\bar{x},N,M)$, the final inequality holds since $w^1(\bar{x}) < 1$ and $\mu_r^1(\bar{x}) = \mu < \bar{x}$ due to $M < N$. Therefore, there exists $B^1 < \bar{x}$ such that for all $c > 0$, it holds that $z_1 < B^1$.

Second, applying the same argument in an analogous way to multi-option sequential search, we infer that there exists $B^K < \bar{x}$ such that for all $c > 0$, $z_K < B^K$. Consequently, setting $B := \max\{B^1, B^K\}$, we conclude that $z_K < B$ and $z_1 < B$ for all $c > 0$.

Making use of this feature, we obtain the following upper bound on the difference of expected search costs:

$$\begin{aligned}
&\frac{c\frac{h(K)}{K}}{1-Q^K(z_K,N,M)} - \frac{c\cdot h(1)}{1-Q^1(z_1,N,M)} \\
<\ &\frac{c\frac{h(K)}{K}}{1-Q^K(B,N,M)} - \frac{c\cdot h(1)}{1-Q^1(0,N,M)} \\
=\ &\frac{c\frac{h(K)}{K}}{1-Q^K(B,N,M)} - c\cdot h(1) \\
=\ &c\left[\frac{\frac{h(K)}{K}}{1-Q^K(B,N,M)} - h(1)\right].
\end{aligned}$$

Note that this upper bound does not depend on z_1 or z_K.

Let us perform a case distinction:

1) $\frac{\frac{h(K)}{K}}{[1-Q^K(B,N,M)]} - h(1) \leq 0$

In this case, we obtain

$$S^K(z_K,N,M) - S^1(z_K,N,M) \;\leq\; 0,$$

which contradicts Lemma 2.3. Let $\bar{c}_K$ be the cost value such that for all $c < \bar{c}_K$, the unique equilibrium under both search procedures is interior. That is, set

$$\bar{c}_K \;:=\; \min\left\{\frac{S^K(0,N,M)[1-Q^K(0,N,M)]}{\frac{h(K)}{K}}, \frac{\mu}{h(1)}\right\} \;>\; 0,$$

recalling the proofs of Propositions 2.1 and 2.2. Then, the established contradiction implies that, for all these levels of c, we have $v_1 < v_K$.

2) $\frac{\frac{h(K)}{K}}{[1-Q^K(B,N,M)]} - h(1) > 0$

To begin with, define

$$r \ := \ \min_{s \in [0,B]} [S^K(s,N,M) - S^1(s,N,M)].$$

Observe that r is well-defined because the involved minimum exists due to the extreme value theorem. Also, it does not depend on z_1, z_K or c. Lemma 2.3 implies that $r > 0$ and, moreover, we have

$$S^K(z_K,N,M) - S^1(z_K,N,M) \ \geq \ r.$$

Taking the upper bound on the cost difference together with this lower bound on the difference in terms of expected quality, we arrive at the following inequality:

$$r \ < \ c\left[\frac{\frac{h(K)}{K}}{1 - Q^K(B,N,M)} - h(1)\right].$$

Now, set

$$\bar{c}_K \ := \ \frac{r}{\frac{\frac{h(K)}{K}}{[1-Q^K(B,N,M)]} - h(1)}.$$

Note that $\bar{c}_K > 0$ since $\frac{\frac{h(K)}{K}}{[1-Q^K(B,N,M)]} - h(1) > 0$ by assumption and, again, $r > 0$ because of Lemma 2.3. Then, for all $c < \bar{c}_K$, we have that

$$\begin{aligned} r \ &< \ c\left[\frac{\frac{h(K)}{K}}{[1 - Q^K(B,N,M)]} - h(1)\right] \\ &< \ \frac{r}{\frac{\frac{h(K)}{K}}{[1-Q^K(B,N,M)]} - h(1)} \cdot \left[\frac{\frac{h(K)}{K}}{[1 - Q^K(B,N,M)]} - h(1)\right] \\ &= \ r. \end{aligned}$$

This constitutes the desired contradiction. □

Appendix 2.B Derivations

2.B.1 Expected Value Conditional on Stopping

First, we derive the expression for the value quality of some candidate $k \in \mathscr{K}$ for some member $i \in \mathscr{N}$ conditional on stopping:

$$
\begin{aligned}
& S^K(z,N,M) \\
&= \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}] \\
&= \sum_{l=M}^{N} \Pr(\#k\ supporters\ =\ l | k\ hired) \mathbb{E}[X_i^k | k\ hired \text{ and } \#k\ supporters\ =\ l] \\
&= \sum_{l=M}^{N} \frac{q^K(z,N,l)}{1 - Q^K(z,N,M)} \mathbb{E}[X_i^k | \#k\ supporters\ =\ l] \\
&= \sum_{l=M}^{N} \frac{q^K(z,N,l)}{1 - Q^K(z,N,M)} \\
&\qquad \big\{ \Pr(voter\ i\ supports\ k | \#k\ supporters\ =\ l) \mathbb{E}[X_i^k | voter\ i\ supports\ k] \\
&\qquad + \Pr(voter\ i\ rejects\ k | \#k\ supporters\ =\ l) \mathbb{E}[X_i^k | voter\ i\ rejects\ k] \big\} \\
&= \sum_{l=M}^{N} \frac{q^K(z,N,l)}{1 - Q^K(z,N,M)} \left[\frac{l}{N} \mu_a^K(z) + \frac{N-l}{N} \mu_r^K(z) \right] \\
&= w^K(z) \mu_a^K(z) + [1 - w^K(z)] \mu_r^K(z),
\end{aligned}
$$

where $w^K(z)$ is defined as

$$
w^K(z) \ := \ \sum_{l=M}^{N} \frac{q^K(z,N,l)}{1 - Q^K(z,N,M)} \frac{l}{N}.
$$

2.B.2 Probability of Acceptance

Second, we derive the expression for the probability that some member $i \in \mathscr{N}$ votes in favor of some candidate $k \in \mathscr{K}$ as a function of K, F, and the employed cutoff z:

$$
\begin{aligned}
&\Pr(X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z) \\
&= \int_0^{\bar{x}} \Pr(X_i^k \geq s, X_i^k \geq z) \Pr(\max_{l \neq k} X_i^l = s) ds \\
&= \int_0^{\bar{x}} \Pr(X_i^k \geq \max\{s, z\}) \Pr(\max_{l \neq k} X_i^l = s) ds \\
&= \int_0^{z} \Pr(X_i^k \geq z) \Pr(\max_{l \neq k} X_i^l = s) ds + \int_z^{\bar{x}} \Pr(X_i^k \geq s) \Pr(\max_{l \neq k} X_i^l = s) ds \\
&= [1 - F(z)] \int_0^{z} \frac{dF(s)^{K-1}}{ds} ds + \int_z^{\bar{x}} [1 - F(s)](K-1)F(s)^{K-2} f(s) ds \\
&= [1 - F(z)]F(z)^{K-1} + \int_z^{\bar{x}} (K-1)F(s)^{K-2} f(s) ds - \int_z^{\bar{x}} (K-1)F(s)^{K-1} f(s) ds \\
&= [1 - F(z)]F(z)^{K-1} + \int_z^{\bar{x}} \frac{dF(s)^{K-1}}{ds} ds - \int_z^{\bar{x}} \frac{d[\frac{K-1}{K} F(s)^K]}{ds} ds \\
&= [1 - F(z)]F(z)^{K-1} + [1 - F(z)^{K-1}] - \frac{K-1}{K} + \frac{K-1}{K} F(z)^K \\
&= F(z)^{K-1} - F(z)^K + 1 - F(z)^{K-1} - 1 + \frac{1}{K} + F(z)^K - \frac{1}{K} F(z)^K \\
&= \frac{1}{K}[1 - F(z)^K].
\end{aligned}
$$

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Chapter 3

Costly Voting: Duverger's Law Revisited

3.1 Introduction

Plurality voting is a wide-spread electoral system; it is prevalent in the United Kingdom, the United States of America, Canada, and India. A common observation is that countries operating under plurality voting usually have two major parties. This was formalized by Duverger (1954), who states that "the simple majority, single ballot system favors the two-party system", and is known as *Duverger's law*. The law is driven by two factors: (i) over time, small parties tend to dissipate because they are unlikely to win an election, and (ii) voters tend to abandon small parties because they do not want to waste their votes. Instead, small party supporters vote strategically for one of the two leading candidates. This paper focuses on the second aspect—the voting behavior of citizens.

In the analysis of plurality voting with more than two alternatives, the literature has so far mostly focused on costless or compulsory voting. However, participation is generally voluntary and, thus, endogenous. It is therefore imperative to examine plurality voting in a model in which voting is costly and participation is voluntary. To the best of my knowledge, only two papers have approached this task so far. Arzumanyan and Polborn (2017) study plurality voting with homogenous voting costs and homogenous cardinal utilities. They find that at least two candidates receive votes in equilibrium, and that all candidates who receive votes are equally likely to win.[1] Voters who do turn out vote sincerely for their preferred candidate. By contrast, Xefteris (2019) assumes that costs are smoothly distributed and proves the existence of *Duvergerian equilibria*, i.e., equilibria, in which exactly two candi-

1. Note that already in costly voting models of two-candidate elections, toss-up elections, i.e., elections in which all candidates are equally likely to win, are a consequence of homogenous voting costs. For a discussion, see for example Herrera, Morelli, and Palfrey (2014) or chapter 1 of this dissertation.

dates receive votes. Strategic voting by some voters may emerge in equilibrium.[2] The question whether and under which conditions *non-Duvergerian equilibria* exist when costs are smoothly distributed remains open—a partial answer to this question is the main contribution of this paper.

I study plurality voting in a large, three-candidate election. Voting is costly and voluntary, and costs are smoothly distributed, capturing that voting costs might be different across voters. I show that Duverger's law applies whenever one candidate is expected to be trailing, i.e., has the lowest expected vote share. Further, I give necessary and sufficient conditions on the distribution of preferences for the existence of non-Duvergerian equilibria in which all three candidates receive the same vote share for any finite electorate. These conditions are knife-edge. Finally, I discuss the case where exactly two candidates have the lowest expected vote share.

In more detail, three candidates, A, B, and C, vie for election. All six ordinal preference rankings are feasible and occur with positive probability. Voters have private values, and cardinal utilities are homogenous, meaning that the first-, second-, and third-ranked candidate yield cardinal utilities of $1, v \in (0,1)$, and 0, respectively. Voting is voluntary and costly, and costs are drawn from a continuous distribution that has a bounded support $[0, \bar{c}]$, with $\bar{c} > 0$, and a strictly positive density everywhere on the support. A voter's preference type and his or her voting costs are private information. The election is decided by plurality voting, and ties are broken randomly.

I show first that there does not exist an equilibrium in which only one candidate receives any votes. The existence of Duvergerian equilibria in large elections, i.e., as the size of the electorate goes to infinity, is an immediate consequence of the analysis in Xefteris (2019). Next, I show that these Duvergerian equilibria are the only possible equilibria in a large election whenever one candidate is expected to be trailing. This has also been established in the context of costless voting by Palfrey (1989). Intuitively, in a large election, the probability that a single vote for the trailing candidate affects the outcome of the election, i.e., is pivotal, is infinitesimally small compared to the probability that a vote for one of the leading candidates is pivotal. Therefore, avoiding wasting their vote, supporters of the trailing candidate abandon this candidate to vote strategically for whichever leading candidate they prefer. Thus, in the limit, the only potential non-Duvergerian equilibria are (i) equilibria in which all candidates receive equal vote shares, and (ii) equilibria in which two candidates are tied behind the front-runner. I show that the first type exists for any given population size if and only if either candidate is the first choice of exactly one-third of the electorate. This necessary and sufficient condition is clearly knife-edge: the equilibrium ceases to exist if the type shares deviate only slightly from one-third, contrasting the results of Arzumanyan and Polborn (2017). The result implies that an equilibrium sequence that converges to equal vote shares in the

2. To be precise, voting strategically means not voting for the most preferred candidate in this context.

limit also has to satisfy the necessary condition if vote shares are equal at least once along the equilibrium sequence for some finite population size. Finally, consider the second non-Duvergerian equilibrium type. All voters who rank the front-runner first have an incentive to vote sincerely. If the two trailing candidates are already tied for a sufficiently large electorate, in the limit, the election is most likely to be decided between one of the trailing candidates and the front-runner, and both events are equally likely. This yields incentives for voters who rank the front-runner last to vote sincerely. Depending on the incentives of voters who rank the front-runner second, I derive necessary conditions on the distribution of voter preferences for such equilibria to exist. These conditions are again knife-edge. I conclude that Duverger's law applies whenever one candidate is expected to be trailing and that the non-Duvergerian equilibria specified above are knife-edge. Finally, I discuss what types of non-Duvergerian equilibria still might arise and how my findings relate to the results of Arzumanyan and Polborn (2017) and the results in the literature on costless voting.

The remainder of this paper is organized as follows: Section 3.2 reviews the related literature, and section 3.3 introduces the model. Section 3.4 defines the equilibrium strategies and treats one-candidate and two-candidate equilibria, that is, equilibria in which exactly one or exactly two candidates receive positive vote shares, respectively. Section 3.5 considers three-candidate equilibria, and section 3.6 concludes. All omitted proofs appear in the appendix.

3.2 Related Literature

This paper contributes to two bodies of work. One is the literature on costly voting. Palfrey and Rosenthal (1983), Ledyard (1984) and Palfrey and Rosenthal (1985) introduced the canonical model of a two-candidate election. The theory on endogenous participation has been continually growing. To name but a few strands of this literature, Campbell (1999), Börgers (2004), Krasa and Polborn (2009), Krishna and Morgan (2012), Krishna and Morgan (2015), Kartal (2015), and Grüner and Tröger (2019) are concerned with the welfare properties of different election systems, Herrera, Morelli, and Palfrey (2014) compare turnout under different election systems, and Evren (2012) and Feddersen and Sandroni (2006) study ethical voting. Goeree and Großer (2007), Taylor and Yildirim (2010), Myatt (2015), and chapter 1 of this dissertation introduce aggregate uncertainty about the distribution of preferences and the effect of information.

To the best of my knowledge, there are only three other papers that study costly voting in plurality-rule elections with more than two candidates: Feddersen (1992), Arzumanyan and Polborn (2017), and Xefteris (2019). Feddersen (1992) assumes that the policy space is a ball in the Euclidean n-space. Voters have quadratic utility over policy positions and incur a homogenous and positive cost conditional on voting. He finds that, generally, an equilibrium exists and exactly two positions receive votes, and thus confirms Duverger's law. Arzumanyan and Polborn (2017) assume that multiple candidates vie for election under plurality rule. Voters face homogenous and positive voting costs, and homogenous utility (1, v and 0 for the first-, second-, and third-ranked candidate, respectively). They find that in equilibrium, for a sufficiently large electorate, a set of relevant candidates arises. All relevant candidates receive votes and are equally likely to win. Voters who do turn out vote sincerely because voters who rank one of the non-relevant candidates first have lower stakes in the election and prefer to abstain. For the case of a three-candidate election, Arzumanyan and Polborn (2017) identify an open set of voting costs c for which the set of relevant candidates contains exactly two candidates ($0 < c \leq \frac{1}{4}$) or all three candidates ($0 < c \leq \frac{2}{3}(1 - \frac{v}{2})$). Thus, Duverger's law does not hold. Xefteris (2019) studies plurality-rule elections with multiple candidates and assumes a smooth distribution of costs. He proves that Duvergerian equilibria, i.e., equilibria in which exactly two candidates receive votes, generally exist in the limit. Further, he shows that, by contrast to Arzumanyan and Polborn (2017), strategic voting emerges in equilibrium. This is because cost cutoffs arise endogenously, so even voters with lower stakes turn out to vote if their cost realization is sufficiently low. In a related setting with three candidates, I derive that there cannot exist an equilibrium where exactly one candidate is expected to be trailing in the limit. Further, I show that the three-candidate equilibria derived in Arzumanyan and Polborn (2017) are knife-edge when the distribution of voting costs is smooth.

The second body of related literature is concerned with multicandidate elections when voting is costless or compulsory. One strand of this literature has studied Duverger's law and its applicability.[3] Palfrey (1989) considers a large three-candidate election and shows that Duvergerian equilibria arise if one candidate is expected to be trailing. He argues verbally that all potential three-candidate equilibria must be knife-edge. However, this claim is rebutted by Myerson and Weber (1993), who contrast the equilibria arising in multicandidate elections under plurality rule, approval voting, and the Borda count. They provide an example of a non-Duvergerian, quasi-Bayesian equilibrium in which two candidates are expected to tie behind the front-runner and show that this equilibrium persists under small parameter changes. Fey (1997) builds on Palfrey (1989) and shows that non-Duvergerian equilibria as described by Myerson and Weber (1993) indeed exist in a Bayesian equilibrium model. However, Fey (1997) shows that these equilibria are not stable, employing the stability notion introduced in Palfrey and Rosenthal (1991). He argues that while the equilibrium itself is not knife-edge, it requires a "type of knife-edge coordination of beliefs" that makes it unstable.

3. See Fey (1997) for a review of the earlier literature approaching Duverger's law.

Plurality-rule elections with multiple candidates have since been studied under many different aspects. Among others, robustness of equilibria has, for example, also been studied by Messner and Polborn (2007) and Messner and Polborn (2011), aggregate uncertainty about the distribution of preferences has been examined by Clough (2007), Myatt (2007), and Fisher and Myatt (2017), ethical voting has been studied by Bouton and Ogden (2017), and information aggregation has been studied by Hummel (2011). Forsythe, Myerson, Rietz, and Weber (1993) and Hermann (2012) take an empirical approach.

Methodologically, this paper builds on the analysis of large Poisson games as introduced by Myerson (2000). Further, it is related to the analysis of approval voting in multicandidate elections, in particular to Myerson (2002), Núñez (2010), Bouton and Castanheira (2012), and Durand, Macé, and Núñez (2019).

3.3 The Model

There is a set of three candidates, $L := \{A, B, C\}$, who vie for election. A citizen's ranking over the three candidates is given by his or her type $t \in T$, where $T := \{AB, AC, BA, BC, CA, CB\}$. The prior distribution over types is given by the vector $r = (r(t))_{t \in T}$, where $r(t) > 0$ for all types $t \in T$, and $\sum_{t \in T} r(t) = 1$. Citizens have independent private values. Type $t = ij$ derives utility 1 if candidate i is elected, utility $v \in (0,1)$ if candidate j is elected, and 0 else. The number of citizens follows a Poisson distribution with mean n, meaning that the probability that there are n citizens in the electorate is $e^{-n}\frac{n^k}{k!}$.

Voting is voluntary and costly. The set of available actions is denoted by $Z = \{A, B, C, \emptyset\}$; the actions $z \in Z$ being voting for candidate A, B, C, or abstaining, respectively. If a citizen decides to turn out and casts a vote for either of the candidates, the citizen incurs the voting cost c. The voting cost is identically and independently distributed according to the cumulative distribution function F, which has a strictly positive density f on $[0, \bar{c}]$, where $\bar{c} \geq 1$.[4] Further, F is assumed to be differentiable. While the distribution of preferences is commonly known, an individual citizen's preference type and voting cost realization are private information.

The voting rule is plurality, and ties are broken randomly. The equilibrium concept is symmetric Bayesian equilibrium.

3.4 Equilibrium

3.4.1 Strategies

The above specifies a Poisson game as introduced by Myerson (1998b). A pure strategy $\sigma : T \times [0, \bar{c}] \to Z$ assigns for each combination of preference type and voting

4. This is a sufficient condition for turnout to be incomplete.

costs in $T \times [0, \bar{c}]$ an action $z \in Z$. I consider strategies that are symmetric across citizens.[5]

Note that for any preference type, voting for the least preferred candidate is strictly dominated by abstention. Hence, in equilibrium, a citizen will either vote for his or her first-best or second-best candidate, or abstain altogether. For most ballots, a single vote does not affect the outcome of the election. In these circumstances, an individual citizen would rather abstain and save the cost of voting, given that his or her own vote does not change the outcome of the election. Therefore, when deciding whether to turn out to vote or to abstain, any voter conditions on the event of being *pivotal*, i.e., affecting the election outcome with one single vote. A vote is pivotal if it either breaks a tie between two or three candidates, or creates such a tie.

Denote by X the set of vote profiles $x = (x(l))_{l \in \{A,B,C\}}$. The event that candidates i and j are tied is denoted by

$$tie_{ij} \;:=\; \{x \;\in\; X \,|\, x(i) \;=\; x(j)\}.$$

Similarly, the event that all three candidates are tied is denoted by

$$tie_{ABC} \;:=\; \{x \;\in\; X \,|\, x(A) \;=\; x(B) \;=\; x(C)\}.$$

The pivotal event that candidates i and j are ahead of k and tied for first place is defined by

$$piv_{ij} \;:=\; \{x \;\in\; X \,|\, x(i) \;=\; x(j) \;>\; x(k)\}.$$

Finally, for any set of events $E \in X$, the set E^{-l} contains all events such that one additional vote for l yields an event in E. For example, piv_{AB}^{-A} is the set of ballots such that casting one more vote for A yields a tie between A and B while C is trailing. Equipped with this notation, the expected benefit of voting for the first-best candidate i for type $t = ij$ can be written as

$$\begin{aligned} U(i, ij) \;=\; & \frac{2-v}{3}\Pr(tie_{ABC}) + \frac{2-v}{6}\Pr(tie_{ABC}^{-i}) + \frac{1-v}{2}\Pr(piv_{ij}) + \frac{1-v}{2}\Pr(piv_{ij}^{-i}) \\ & + \frac{1}{2}\Pr(piv_{ik}) + \frac{1}{2}\Pr(piv_{ik}^{-i}). \end{aligned} \tag{3.1}$$

To derive the expected benefit of voting for i in the event that a vote for i breaks a tie between all three candidates, tie_{ABC}, note that i will win the election because of the additional vote, yielding a utility of 1. However, without this vote, i would have won with probability $\frac{1}{3}$, and j would have won with probability $\frac{1}{3}$, which would have yielded utilities of 1 and v, respectively. Thus, the expected benefit is $1 - \frac{1}{3}(1 + v) =$

5. The symmetry assumption is without loss of generality given the Poisson setting. For details, see Myerson (1998a).

$\frac{2-\nu}{3}$. The other terms are derived accordingly.
Similarly, the expected benefit of voting for the second-best candidate j is given by

$$U(j,ij) = \frac{2\nu-1}{3}\Pr(tie_{ABC}) + \frac{2\nu-1}{6}\Pr(tie_{ABC}^{-j}) + \frac{\nu-1}{2}\Pr(piv_{ij}) + \frac{\nu-1}{2}\Pr(piv_{ij}^{-j}) \quad (3.2)$$
$$+\frac{\nu}{2}\Pr(piv_{jk}) + \frac{\nu}{2}\Pr(piv_{jk}^{-j}).$$

Viewed in isolation, turning out and voting for either candidate i or j is beneficial if and only if the expected benefit of voting exceeds the citizen's realized cost of voting c. Since the expected benefit of voting is independent of c, for every type ij, there is a pair of cost cutoffs $(c(i,ij), c(j,ij))$ such that

$$c(i,ij) = U(i,ij), \text{ and} \quad (3.3)$$
$$c(j,ij) = U(j,ij). \quad (3.4)$$

Since only voting for the least preferred candidate is dominated, conditional on voting, the citizen of type $t = ij$ has to trade off voting for i against voting for j. Here, two effects come into play: First, whenever j is behind i, or the election is a close race between i and j, i.e., candidate k is expected to be trailing, voting for j harms type $t = ij$ because it reduces the winning probability of the preferred candidate i. However, whenever i is trailing behind both j and k, i.e., the election is a close race between j and k, a vote for i would be wasted, but a vote for j might contribute to defeating the worst candidate. The resolution of this trade-off depends on which event has the higher probability. Overall, conditional on voting, a citizen will vote for the candidate who yields the largest expected benefit. Consequently, the equilibrium strategy of a voter of type ij can be summarized as follows:

$$\sigma(ij,c) = \begin{cases} i & \text{if } c \leq c(i,ij) \text{ and } c(i,ij) \geq c(j,ij),^{6} \\ j & \text{if } c \leq c(j,ij) \text{ and } c(j,ij) > c(i,ij), \\ \emptyset & \text{else.} \end{cases}$$

An equilibrium is a vector of cutoff costs $(c(i,ij), c(j,ij))_{t=ij\in T}$ such that it is optimal for a citizen of type $t = ij$ and costs $c \leq \max\{c(i,ij), c(j,ij)\}$ to participate in the election and vote for candidate $\sigma(ij,c)$ if all other citizens in the electorate follow the same strategy. Say that a voter of type ij votes *sincerely* if $c(i,ij) > c(j,ij)$, that is, if he or she votes for his or her first-best candidate. Say that a three-candidate equilibrium is *sincere* if all voters vote sincerely.

6. Thus, the tie-breaking assumption is that voters vote for their preferred candidate when indifferent. I do not consider mixed equilibria because I conjecture that indifference does not occur in large elections.

It is useful to define participation rates $p(l,t)$ as the probability that type $t \in T$ turns out to vote for candidate $l \in L = \{A,B,C\}$. The equilibrium strategies yield

$$\begin{aligned} p(i,ij) &= \begin{cases} F(c(i,ij)) & \text{if } c(i,ij) \geq c(j,ij), \\ 0 & \text{else}, \end{cases} \\ p(j,ij) &= \begin{cases} F(c(j,ij)) & \text{if } c(j,ij) > c(i,ij), \\ 0 & \text{else}, \end{cases} \\ p(k,ij) &= 0. \end{aligned}$$

Define the voting probability of type t, $p(t)$, as the probability that type t turns out to vote for any candidate instead of abstaining:

$$p(t) = \max_{l \in L}\{p(l,t)\}.$$

Note that the pivot probabilities, and, thus, cost cutoffs and participation rates, generally depend on the expected number of citizens, n. This dependence is sometimes omitted to save notation and is otherwise denoted by a subscript.[7]

Given the citizens' strategies and given the expected number of citizens n, the expected share of citizens voting for candidate l is $\tau_n(l) = \sum_{t \in T} r(t) p_n(l,t)$, and the expected number of votes candidate l receives is given by $n\tau_n(l)$. Since the size of the electorate is Poisson distributed with mean n, the number of votes for candidate l is also Poisson-distributed with mean $n\tau_n(l)$, and, further, the number of votes for candidate l is independent of the number of votes for candidate $l' \neq l$. Since in a Poisson game the probability of any event depends only on the expected number of votes cast for each of the candidates, the probabilities of the pivotal events are:

$$\Pr(tie_{ABC}) = e^{-n\tau_n(A)-n\tau_n(B)-n\tau_n(C)} \sum_{k=0}^{\infty} \frac{(n\tau_n(A))^k}{k!} \frac{(n\tau_n(B))^k}{k!} \frac{(n\tau_n(C))^k}{k!}, \tag{3.5}$$

$$\Pr(tie_{ABC}^{-A}) = e^{-n\tau_n(A)-n\tau_n(B)-n\tau_n(C)} \sum_{k=1}^{\infty} \frac{(n\tau_n(A))^{k-1}}{(k-1)!} \frac{(n\tau_n(B))^k}{k!} \frac{(n\tau_n(C))^k}{k!}, \tag{3.6}$$

$$\Pr(piv_{AB}) = e^{-n\tau_n(A)-n\tau_n(B)-n\tau_n(C)} \sum_{k=1}^{\infty} \frac{(n\tau_n(A))^k}{k!} \frac{(n\tau_n(B))^k}{k!} \sum_{j=0}^{k-1} \frac{(n\tau_n(C))^j}{j!}, \tag{3.7}$$

$$\Pr(piv_{AB}^{-A}) = e^{-n\tau_n(A)-n\tau_n(B)-n\tau_n(C)} \sum_{k=1}^{\infty} \frac{(n\tau_n(A))^{k-1}}{(k-1)!} \frac{(n\tau_n(B))^k}{k!} \sum_{j=0}^{k-1} \frac{(n\tau_n(C))^j}{j!}, \tag{3.8}$$

where the probabilities of the other pivotal events are computed analogously.

7. Of course, the strategy of a citizen also depends on the strategies of the other citizens, which is, too, omitted.

3.4.2 Equilibrium Types

There are three conceivable types of equilibria: (i) one-candidate equilibria, in which only one candidate receives votes, (ii) two-candidate equilibria, or *Duvergerian* equilibria, in which two candidates receive votes, and (iii) three-candidate equilibria, in which all three candidates receive votes.

Proposition 3.1 states that one-candidate equilibria can never exist. Thus, the only potential *non-Duvergerian* equilibria are three-candidate equilibria.

Proposition 3.1. *For any n, there does not exist an equilibrium in which exactly one candidate receives votes.*

Proof. Take any n and suppose, by contradiction, that only one candidate receives any votes. Assume without loss of generality that this candidate is A. Since $r(t) > 0$ for all $t \in T$, a positive share of the population, $r(BC) + r(CB) > 0$, ranks candidate A last. Because voting for A is dominated by abstention for types BC and CB, only A receiving any votes implies that $c_n(B,BC) = c_n(C,BC) = c_n(B,CB) = c_n(C,CB) = 0$. Recall the definition of $c_n(B,BC)$:

$$\begin{aligned} c_n(B,BC) \;&=\; \frac{2-v}{3}\Pr_n(tie_{ABC}) + \frac{2-v}{6}\Pr_n(tie_{ABC}^{-B}) + \frac{1-v}{2}\left(\Pr_n(piv_{BC}) + \Pr_n(piv_{BC}^{-B})\right) \\ &\quad + \frac{1}{2}\left(\Pr_n(piv_{AB}) + \Pr_n(piv_{AB}^{-A})\right). \end{aligned}$$

Given the prescribed equilibrium strategies, a vote for B can be pivotal against A if and only if either no voter turns out to vote for A or exactly one voter turns out to vote for A. But then,

$$c_n(B,BC) \;=\; \frac{2-v}{3}e^{-n\tau_n(A)} + \frac{1}{2}e^{-n\tau_n(A)}n\tau_n(A) \;>\; 0$$

—a contradiction! Consequently, given the assumption that F has full support on $[0,\bar{c}]$, there is a strict incentive for citizens of types BC and CB to turn out to vote for either B or C. □

Next, Proposition 3.2 reveals that two-candidate equilibria always exist in large elections.

Proposition 3.2. *As $n \to \infty$, all three Duvergerian equilibria exist.*

The result is due to Xefteris (2019). Note that Xefteris (2019) allows for $|L| \geq 3$ and allows for cardinal utilities to be smoothly distributed, however ruling out indifference. Yet, his results also go through when assuming, as I do, that cardinal utilities are homogenous. The proof idea is to first restrict the strategy space to the options of voting for two of the three candidates—say, without loss of generality, either for A or for B—or to abstain, and then showing that the equilibrium of the restricted game continues to be an equilibrium for sufficiently large n in the unrestricted game, in which voters might also vote for the third candidate, say C. Intuitively, if all other voters are expected to vote for either A or B, the probability that a single vote is pivotal for C is infinitesimally small compared to the probability that a vote for A or B is pivotal. Since voters who rank C first still derive utility from having their second-ranked candidate elected, it is more beneficial for them to vote strategically for their second choice. Clearly, this argument requires voters who prefer C not to be indifferent between A and B. The argument holds for all three types of Duvergerian equilibria: one, where A does not receive any votes, one where B does not receive any votes, and one where C does not receive any votes.[8]

Now, the remaining question is whether three-candidate equilibria exist in the limit, which will be in the focus of the next section.

3.5 Three-Candidate Equilibria

3.5.1 Large Elections

The subsequent analysis is based on the analysis of large elections, that is, as n goes to infinity. Define $\tau(l) := \lim_{n\to\infty} \tau_n(l)$, and write $\tau = (\tau(A), \tau(B), \tau(C))$. Let me begin with some preliminary results as $n \to \infty$.

Lemma 3.1. *Along all equilibrium sequences, participation rates are interior, i.e., for all $t \in T$ and for all n, $p_n(t) \in (0, 1)$.*

This result is a consequence of the assumptions that the support of the cost distribution F is bounded below by zero and that the upper bound, $\bar{c}$, is larger than 1, which is the utility a voter derives from the election of his or her preferred candidate. The proof of this and subsequent results are displayed in the appendix.

Define the share of citizens who turn out to vote for any candidate by $\phi_n := \tau_n(A) + \tau_n(B) + \tau_n(C)$ and define the total turnout by $\Phi_n := n\phi_n$.

Lemma 3.2. *Along all equilibrium sequences, total turnout goes to infinity as n goes to infinity. That is,* $\liminf_{n\to\infty} \Phi_n = \infty$.

Suppose that total turnout were finite in the limit. Then, the pivot probabilities would have strictly positive limits, and so would the equilibrium cost cutoffs. Since F has full support on $[0, \bar{c}]$, there would be a strictly positive fraction of voters who would turn out to vote, implying infinite turnout in the limit, since $r(t) > 0$ for all $t \in T$.

In the costly voting game with three candidates, one is interested in the vote shares of the candidates, while the share of abstaining voters is not of great interest. Define the vote shares of the candidates by $\xi_n(l) := \frac{\tau_n(l)}{\phi_n}$, and $\lim_{n\to\infty} \xi_n(l) =:$

8. Note that all three Duvergerian equilibria require a significant amount of coordination between the voters. The question of how voters coordinate on candidates is outside of the scope of this model.

$\xi(l) = \frac{\tau(l)}{\phi}$. Note that, by definition, $\xi(A) + \xi(B) + \xi(C) = 1$. Since, by Lemma 3.2, the total turnout Φ_n goes to infinity as $n \to \infty$, the game with abstention can be interpreted as one without abstention by taking Φ_n as the population mean of the reduced game.[9] Then, the analysis in Myerson (2000) applies. Note that a three-candidate equilibrium exists in the limit if and only if $\xi(A), \xi(B), \xi(C) > 0$.

3.5.2 Pivot Probabilities and Their Magnitudes

The incentives of citizens to vote are pinned down by the pivot probabilities. These, however, are difficult to compute, since they generally tend to zero as n grows large. Myerson (2000) introduces the concept of the *magnitude* of a sequence of events, which measures the rate at which the probability of the event sequence goes to zero as the population mean n goes to infinity. This section collects results on magnitudes and asymptotic pivot probability ratios that will be useful for proving the ensuing results.

First, I define the *magnitude* of a sequence of events in the reduced game without abstention which, in this context, measures the rate at which the probability of the event sequence goes to zero as the total turnout Φ_n goes to infinity.

Definition 3.1. Given the sequence of vote shares $(\xi_n)_{n=1}^{\infty}$, the *magnitude* μ of the sequence of events $E = \{E_n\}_{n=1}^{\infty}$ with $E_n \subset X$ is

$$\mu(E|\Phi_n \xi_n) \ = \ \lim_{n\to\infty} \frac{1}{\Phi_n} \log \Pr(E_n|\Phi_n \xi_n).$$

Observe that the magnitude is always smaller than or equal to zero because the logarithm of a probability can never be positive. If the magnitude $\mu(E|\Phi_n \xi_n)$ is negative, the probability of the sequence of events E converges to zero at the rate of $e^{\Phi_n \mu(E|\Phi_n \xi_n)}$ as Φ_n, or equivalently, n, grows large.[10]

The notion of the magnitude is useful to compare pivot events with different magnitudes. The following lemma is due to Myerson (2000) and Bouton and Castanheira (2012).

Lemma 3.3. *Consider two sequences of events, $\{E_1\}_{n=1}^{\infty}, \{E_2\}_{n=1}^{\infty}$, $E_{1,n}, E_{2,n} \subset X$, and suppose that $\mu(E_1) < \mu(E_2)$. Then, the probability ratio of the two events is approximately given by $e^{\Phi_n(\mu(E_1)-\mu(E_2))}$ and goes to zero as n tends to infinity:*

$$\lim_{n\to\infty} \frac{\Pr(E_1)}{\Pr(E_2)} \ = \ 0.$$

9. This step was already suggested in Myerson (2000). Qualitatively, it does not change the analysis since $\xi_n(i) = \tau_n(i)/\phi_n$.

10. This means that $\Pr_n(E) \sim \mathcal{O}(e^{\Phi_n \mu(E|\Phi_n \xi_n)})$ as $n \to \infty$.

For an intuition, recall that the magnitude measures the rate at which a probability converges to zero as n grows large. If $\mu(E_1) < \mu(E_2)$, then the probability of the sequence $\{E_1\}_{n=1}^{\infty}$ converges to zero faster than the probability of $\{E_2\}_{n=1}^{\infty}$. The result implies that if there is a strict ordering of the pivot probabilities, as n goes to infinity, the incentives of the voters are determined by the pivot event with the largest magnitude, and the relative probabilities of all other pivot events vanish.

There are three useful results that can be employed to compute the magnitude of a given event: the Magnitude Theorem according to Myerson (2000), the Magnitude Equivalence Theorem according to Núñez (2010), and a result from Durand, Macé, and Núñez (2019) that has been derived in the context of approval voting.

Lemma 3.4 (Magnitude Theorem, Myerson (2000)). *Let $E = \{E_n\}_{n=1}^{\infty}$ be any sequence of events in X, and let $(\xi_n)_{n=1}^{\infty}$ be any sequence of vote shares. Then,*

$$\begin{aligned}
\mu(E|\Phi_n\xi_n) &= \lim_{n\to\infty} \frac{1}{\Phi_n} \log(\Pr(E_n|\Phi_n\xi_n)) \\
&= \lim_{n\to\infty} \max_{x_n\in E_n} \frac{1}{\Phi_n} \log(\Pr(x_n|\Phi_n\xi_n)) \\
&= \lim_{n\to\infty} \max_{x_n\in E_n} \sum_{l\in\{A,B,C\}} \xi_n(l)\psi\left(\frac{x_n(l)}{\Phi_n\xi_n(l)}\right),
\end{aligned}$$

where $\psi(\theta) := \theta(1-\log\theta)-1$.

The sequence $\{x_n\}_{n=1}^{\infty} \in \{E_n\}_{n=1}^{\infty}$ that maximizes the sum on the right-hand side above is called a *major sequence of points* in the sequence of events $\{E_n\}_{n=1}^{\infty}$. The Magnitude Theorem states that the magnitude of any sequence of events in X is equal to the magnitude of a major sequence of points in these events, where almost all the probability of the sequence of events is concentrated.
Applying the theorem yields the following expressions:

Lemma 3.5.

$$\begin{aligned}
\mu(tie_{ABC}|\Phi_n\xi_n) &= 3(\xi(A)\xi(B)\xi(C))^{\frac{1}{3}} - 1, \\
\mu(tie_{AB}|\Phi_n\xi_n) &= 2\sqrt{\xi(A)\xi(B)} - \xi(A) - \xi(B).
\end{aligned}$$

The magnitudes of the events tie_{AC}, tie_{BC} are computed analogously. Observe that $\mu(tie_{ABC}|\Phi_n\xi_n) \leq \mu(tie_{ij}|\Phi_n\xi_n)$ for all pairs ij because the event that three candidates tie is a subset of the event that two candidates tie. The expressions above imply that the magnitude of any tie event is zero if all expected vote shares are equal in the limit, and that the magnitude of a tie between two candidates is zero if the expected vote shares of these two candidates are equal in the limit.

Next, the Magnitude Equivalence Theorem allows me to derive the magnitudes of events that differ by a single translation from events with known magnitude.

Lemma 3.6 (Magnitude Equivalence Theorem, Núñez (2010)). *Let $Y \subset L$ and $pivot(Y) := \{x \in X | \forall y \in Y,\ x(y) \geq \max_{l \in L} x(l) - 1, \forall l \notin Y,\ x(l) \leq \max_{l \in L} x(l) - 2\}$. Given a sequence of vote shares $\{\xi_n\}_{n=1}^{\infty}$, it holds that*

$$\mu(pivot(Y)|\Phi_n \xi_n) \ = \ \mu(D|\Phi_n \xi_n),$$

for some outcome $D \subset X$ defined by

$$D \ := \ \{x(i) \ = \ x(j) \forall i,j \ \in \ Y\} \cup \{x(i) \ \geq \ x(j) \ \forall i \ \in \ Y \text{ and } j \ \in \ L \backslash Y.\}$$

Applying this theorem directly yields

Claim 3.1.

$$\begin{aligned} \mu(tie_{ij}|\Phi_n \xi_n) \ &= \ \mu(tie_{ij}^{-i}|\Phi_n \xi_n) \ = \ \mu(tie_{ij}^{-j}|\Phi_n \xi_n), \\ \mu(tie_{ABC}|\Phi_n \xi_n) \ &= \ \mu(tie_{ABC}^{-A}|\Phi_n \xi_n) \ = \ \mu(tie_{ABC}^{-B}|\Phi_n \xi_n) \ = \ \mu(tie_{ABC}^{-C}|\Phi_n \xi_n), \text{ and} \\ \mu(piv_{ij}|\Phi_n \xi_n) \ &= \ \mu(piv_{ij}^{-i}|\Phi_n \xi_n) \ = \ \mu(piv_{ij}^{-j}|\Phi_n \xi_n). \end{aligned}$$

Thus, the event in which casting one single vote yields a tie has the same magnitude as the event of the tie itself.

Finally, Durand, Macé, and Núñez (2019) have derived a result that allows to compute the magnitude of the event piv_{ij}. Their proof directly applies to this model and is therefore omitted.

Lemma 3.7 (Durand, Macé, and Núñez (2019)). *For any vote share profile $(\xi(l))_{l \in L}$, $L = \{i,j,k\}$, and for any pair of candidates ij,*

$$\mu(piv_{ij}|\Phi_n \xi_n) \ = \ \begin{cases} \mu(tie_{ij}|\Phi_n \xi_n), & \text{if } \delta_{ij}(\xi) \ > \ 0 \\ \mu(tie_{ABC}|\Phi_n \xi_n), & \text{if } \delta_{ij}(\xi) \ \leq \ 0, \end{cases}$$

where $\delta_{ij}(\xi) := \sqrt{\xi(i)\xi(j)} - \xi(k)$.

To understand the result, consider the probability of the pivotal event piv_{AB}, which is the event that A and B are tied and C has less votes than A. This event is equivalent to the event tie_{AB} with the additional constraint that C has less votes than A. As already stated in Durand, Macé, and Núñez (2019), $\delta_{AB}(\xi) > 0$ can be interpreted as the expected vote share difference between A and C conditional on the event that A and B are tied. If this difference is positive, the constraint is slack and, conditional on A and B being tied, the probability of piv_{AB} must be the same as the probability of tie_{AB}. However, if $\delta_{AB}(\xi) \leq 0$, the constraint is binding and, conditional on A and B being tied, the probability of piv_{AB} must be infinitesimally smaller than that of tie_{AB}. In this case, piv_{AB} has the same magnitude as tie_{ABC}—the event that all three candidates tie.

Lemma 3.7 relates the magnitudes of the events piv_{ij} and tie_{ij}, but does not directly allow to draw precise conclusions about the ratio of the respective probabilities. The next result reveals that if the threshold $\delta_{ij}(\xi)$ is positive, not only do tie_{ij} and piv_{ij} have the same magnitude, but the limit of their probability ratio must be one.

Lemma 3.8. *For any vote share profile* $(\xi(l))_{l\in L}$, $L = \{i,j,k\}$, *and for any pair of candidates ij,*

$$\lim_{n\to\infty} \frac{\Pr(piv_{ij})}{\Pr(tie_{ij})} \;=\; 1 \textit{ if } \delta_{ij}(\xi) \;>\; 0,$$

with $\delta_{ij}(\xi) := \sqrt{\xi(i)\xi(j)} - \xi(k)$.

The result is derived by decomposing the event tie_{ij} into three disjunct sets, including piv_{ij} and tie_{ABC}, and showing that all sets of events but piv_{ij} must have a smaller magnitude than tie_{ij} if the stated condition is satisfied. Thus, all the probability mass of tie_{ij} is concentrated on the subset piv_{ij} in the limit. I believe that this lemma is useful beyond the application in this paper.

3.5.3 Duverger's Law

The main result of this section states that whenever one candidate has a strictly lower limiting expected vote share than the other two candidates, there cannot exist a three-candidate equilibrium. This establishes Duverger's law in costly voting, provided that exactly one candidate is expected to be behind the other two candidates. Suppose, without loss of generality, that the trailing candidate is C.

Proposition 3.3. *There does not exist a sequence of equilibria in which* $\xi(A) \geq \xi(B) > \xi(C) > 0$.

Let me sketch the proof. I show first that if candidate C is expected to be trailing, the probability of the pivot event that candidates A and B are tied (or nearly tied) while C is behind goes to zero at a slower rate than the probability of any pivotal event involving C. Intuitively, conditional on C having the lowest expected vote share, it must be more likely that A and B are tied (or nearly tied) while C is trailing, than that C is actually tied for first place with either A, B, or both. By Lemma 3.3, this implies that, as $n \to \infty$, the probability of any pivot event involving C vanishes relatively to the probabilities of the pivotal events involving A and B. Take any voter who ranks C first, for example type CA. The previous step yields that the ratio of cost cutoffs, $\frac{c_n(C,CA)}{c_n(A,CA)}$, tends to zero as n goes to infinity. In particular there must exists some $\bar{n}$ such that for all $n > \bar{n}$, $\frac{c_n(C,CA)}{c_n(A,CA)} < 1$. Intuitively, voting strategically for A yields utility in the pivotal event involving A and B, while voting for C yields utility only in the pivotal events involving C, whose probabilities vanish. Thus, types CA and CB have an incentive to vote strategically. Similarly, all voter types who rank C

second have an incentive to vote sincerely because the relative expected benefit of voting for the second-ranked candidate goes to zero. Consequently, all voters who turn out either vote for A or B, but never C, yielding a zero vote share for candidate C for sufficiently large n, and in particular in the limit.

The result relies on the assumption that voters derive strictly positive utility from their second choice being elected. If there would be a type of voter who only derives utility from C being elected and receives a utility of zero otherwise, the result would not hold for the following reason: Suppose that the equilibrium strategies prescribe that nobody should vote for C. Then, even if the probability of being pivotal is much smaller for a vote for C than for a vote for either A or B, it is still positive. Given that F has full support on $[0, \bar{c}]$, the C supporter will prefer to vote for C instead of abstaining.

3.5.4 Non-Duvergerian Equilibria

The previous section established that three-candidate equilibria fail to exist if one candidate is expected to be trailing behind the other two candidates in the limit. So, there are two potential scenarios in which a three-candidate equilibrium might still exist as n grows large: (i) the case where all candidates are expected to tie, i.e., where $\xi(A) = \xi(B) = \xi(C) > 0$, and (ii) the case where two candidates are tied for second and third place, i.e., where $\xi(A) > \xi(B) = \xi(C) > 0$ (up to a relabeling of candidates).
Proposition 3.4 gives a necessary and sufficient condition for the existence of equilibria in which vote shares are exactly equal for finite n.

Proposition 3.4. *[Equal vote shares]*

(1) If $r(AB) + r(AC) = r(BA) + r(BC) = r(CA) + r(CB) = \frac{1}{3}$, there exists an equilibrium sequence with $\xi_n(A) = \xi_n(B) = \xi_n(C) = \frac{1}{3}$ for all n.

(2) For any n, an equilibrium sequence satisfying $\xi_n(A) = \xi_n(B) = \xi_n(C) = \frac{1}{3}$ exists only if $r(AB) + r(AC) = r(BA) + r(BC) = r(CA) + r(CB) = \frac{1}{3}$.

For an intuition for the first part note that if each candidate is preferred by one-third of the population, by symmetry, there exists a sequence of cost cutoffs $(\hat{c}_n)_{n\geq 1}$ such that for each n, every citizen turns out if his or her cost realization is smaller than this cutoff, and voting is sincere. Then, expected vote shares are equal for all n. To derive the existence of such a cutoff, I first show that if all other voters employ the same cost cutoff and vote sincerely, it is a best response for some individual voter to do the same. Then, this common cost cutoff is a continuous functions of the pivot probabilities, and vice versa. The existence of the equilibrium cutoff follows from Brouwer's fixed point theorem.

The second part of Proposition 3.4 reveals that the sufficient condition is also necessary for a three-candidate equilibrium sequence that satisfies equal vote shares for all candidates to exist for some n. Intuitively, if the expected vote shares are equal, all two-way ties are equally likely, and so are the corresponding pivot events. This implies first that all voters have an incentive to vote sincerely. Second, the cost cutoffs coincide for all voter types, and so do the participation rates. Thus, the expected vote share of a candidate is given by the participation rate multiplied with the share of citizens who rank this candidate first, divided by the total turnout rate. Consequently, vote shares can only be equal if indeed each candidate is expected to be ranked first by an equal share of citizens. This condition is clearly knife-edge—the equilibrium will cease to exist if the distribution of voter preference types is slightly different because, fixing the strategies induced by equal expected vote shares, the candidate who is the first choice of a plurality of voters will receive a higher vote share than the other candidates.

The second part of Proposition 3.4 implies that the stated necessary condition is also a necessary condition for the existence of an equilibrium with equal vote shares in the limit—i.e., satisfying $0 < \xi(A) = \xi(B) = \xi(C)$—provided that the corresponding equilibrium sequence satisfies $0 < \xi_n(A) = \xi_n(B) = \xi_n(C)$ for at least one value of n. It remains an open question what conditions are necessary for an equilibrium sequence to exist that satisfies equal vote shares in the limit, but does not satisfy equal vote shares for any finite n.

The last potential limit three-candidate equilibrium is the one in which two candidates are tied behind the front-runner in the limit as the population size grows large. For example, let A and B be tied behind C as $n \to \infty$, meaning that $0 < \xi(A) = \xi(B) < \xi(C)$. In such an equilibrium, in the limit, the pivotal events between A and C and between B and C have the same magnitude and are much more likely than the pivotal events between A and B. Therefore, all voters who rank C first have a strict incentive to vote for C. By contrast, the incentives of the types who rank C second or last depend on the shape of the equilibrium sequence $(\xi_n(A), \xi_n(B), \xi_n(C))$ and on the parameter v.

The next proposition treats the case where the equilibrium sequence is constant for sufficiently large n in the sense that $0 < \xi_n(A) = \xi_n(B) < \xi_n(C)$ for all n larger than some $\bar{n}$. In this case, for $n > \bar{n}$, the probability of the pivotal events between A and C and between B and C are equally likely from the perspective of an individual voter (yielding symmetric strategies for types AC and BC), and, again, are much more likely than the pivotal events between A and B. Then, all voters who rank C last have a strict incentive to vote for their favorite candidate. Since $r(t) > 0$ for all $t \in T$ and $f > 0$ on $[0, \bar{c}]$, $\xi(A)$ and $\xi(B)$ are strictly positive.[11] Depending on v, types AC and BC will either vote sincerely or strategically. Thus, two types of equilibria

11. This argument fails if A is expected to be behind B in the limit as n grows large, albeit slightly. Then, the event that the election is decided between B and C is much more likely than the event that the election is decided between A and C, yielding the incentive for types AB to abandon A.

are conceivable. Proposition 3.5 states the two respectively necessary conditions on the distribution of voter types for such equilibria to exist in the limit as $n \to \infty$.

Proposition 3.5. *An equilibrium sequence satisfying* $0 < \xi(A) = \xi(B) < \xi(C)$ *in the limit and* $0 < \xi_n(A) = \xi_n(B) < \xi_n(C)$ *for all n larger than some* $\bar{n}$

(1) *exists only if* $r(AB) + (1-v)r(AC) = r(BA) + (1-v)r(BC)$, *provided that all types vote sincerely.*

(2) *exists only if* $r(AB) = r(BA)$, *provided that types AB, BA, CA, CB vote sincerely and types AC, BC vote strategically for C.*

The two stated conditions are necessary for limiting expected vote shares of candidates A and B to be equal in the respective type of equilibrium. These imply that both equilibrium types are knife-edge because even a slight deviation from the respective condition upsets the respective equilibrium. For an intuition for the first condition, $r(AB) + (1-v)r(AC) = r(BA) + (1-v)r(BC)$, observe that the event piv_{AB} has a smaller magnitude than the event piv_{AC} (or, equivalently, of piv_{BC}). Thus, as $n \to \infty$, the cost cutoffs are shaped by the events piv_{AC} and piv_{BC}, while piv_{AB} becomes negligible. For type AB (or BA), the expected benefit of creating or breaking a tie against C by voting for A (or B) yields utility one-half, while for type AC (or BC), this only yields utility $\frac{1-v}{2}$. This explains the factor $(1-v)$. For the second equilibrium type, if only types AB and BA vote for A and B, respectively, by symmetry, both types have the same cost cutoff. Therefore, expected vote shares can only be equal if the type shares are equal, yielding the second condition, $r(AB) = r(BA)$.

The previous proposition only covers the case where the equilibrium sequence is constant for sufficiently large n. Thus, the case where the equilibrium sequence converges in any other way to its limit $0 < \xi(A) = \xi(B) < \xi(C)$ remains open and is left for future research. The necessary conditions stated in Proposition 3.5 remain valid if it can be shown that expected vote shares of A and B being equal implies that, along the equilibrium sequence, $\frac{\Pr(piv_{AC})}{\Pr(piv_{BC})}$ converges to 1 in the limit. This, in turn, would imply that types AB and BA, and AC and BC, respectively, face the same incentives, and the arguments from Proposition 3.5 apply. Without further assumptions, however, this is not immediate because the sequence of vote shares converges to its limit at a sublinear rate.[12,13]

3.5.5 Discussion

In the previous section, I have derived necessary conditions for certain kinds of three-candidate equilibria which turned out to be knife-edge. Thus, for these equilibria

12. This follows from the fact that participation rates converge to zero, but total turnout goes to infinity as n grows large.

13. What does follow from the previous analysis is that $\frac{\Pr(piv_{AC})}{\Pr(piv_{BC})}$ is bounded above zero and finite. Further, by Lemma 3.8, and using the closed-form approximations in Myerson (2000),

Duverger's law holds and the intuition developed in Palfrey (1989) applies. Yet, it is still an open question what are necessary conditions for the remaining types of three-candidate equilibria and whether they, too, are knife-edge or can be satisfied for an open set of parameters. In a costly voting framework with homogenous costs, Arzumanyan and Polborn (2017) have identified a continuum of non-Duvergerian equilibria, and Fey (1997) has derived a non-Duvergerian equilibrium when voting is costless. Let me now discuss their findings in relation to my model.

In Arzumanyan and Polborn (2017), three-candidate equilibria have the feature that all candidates are expected to tie. To be precise, Arzumanyan and Polborn (2017) state that for voting costs $c \leq \frac{2}{3}(1 - \frac{v}{2})$, and a fixed preference distribution $(r(t))_{t \in T}$, there exists $\bar{N}(c)$ such that for all $n > \bar{N}(c)$, there exist three-candidate equilibria satisfying $0 < \xi_n(A) = \xi_n(B) = \xi_n(C)$. Thus, these equilibria exist for a whole range of parameter constellations. This stands in contrast to my findings captured in Proposition 3.4 which state that such equilibria exist if and only if $r(ij) + r(ik) = \frac{1}{3}$ for all $ij, ik \in T$. The difference in these results is driven by the different assumptions on the cost distribution. Since voting costs are homogenous in Arzumanyan and Polborn (2017), in equilibrium, voters need to be exactly indifferent between voting and abstaining. Thus, the expected benefit of voting needs to be the same for every voter, meaning that expected vote shares need to coincide. Given a fixed distribution of preferences, the voting probabilities of different types adjust accordingly to equalize the expected vote shares. By contrast, if, as in my model, the cost distribution is smooth, equal expected vote shares and equal expected voting benefits imply equal cost cutoffs, and thus, equal participation rates. Therefore, the candidates' vote shares can be equal only if each candidate is preferred by an equal share of the population.[14]

The second type of three-candidate equilibria in which two candidates tie behind a front-runner corresponds to the ones first described in Myerson and Weber (1993) and analyzed in Fey (1997). In Fey (1997), voting is compulsory, and the utility a voter derives from his or her second choice, v, is smoothly distributed. There are only three types of voters: types AB (share $0.3 + \varepsilon$), BA (share $0.3 - \varepsilon$), and C (share

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$$\lim_{n\to\infty} \frac{\Pr(piv_{AC})}{\Pr(piv_{BC})} = \lim_{n\to\infty} \frac{\Pr(tie_{AC})}{\Pr(tie_{BC})} = \lim_{n\to\infty} \frac{e^{-n\phi_n(\sqrt{\xi_n(A)-\xi_n(C)})^2}}{e^{-n\phi_n(\sqrt{\xi_n(B)-\xi_n(C)})^2}} \frac{\sqrt{4\pi n\phi_n \sqrt{\xi_n(B)\xi_n(C)}}}{\sqrt{4\pi n\phi_n \sqrt{\xi_n(A)\xi_n(C)}}}$$
$$= \lim_{n\to\infty} \frac{e^{-n\phi_n(\sqrt{\xi_n(A)-\xi_n(C)})^2}}{e^{-n\phi_n(\sqrt{\xi_n(B)-\xi_n(C)})^2}}.$$

This, however, need not imply that the limit ratio is 1 because $\lim_{n\to\infty} \frac{-n\phi_n(\sqrt{\xi_n(A)-\xi_n(C)})^2}{-n\phi_n(\sqrt{\xi_n(B)-\xi_n(C)})^2} = 1$ need not imply that $\lim_{n\to\infty} \frac{e^{-n\phi_n(\sqrt{\xi_n(A)-\xi_n(C)})^2}}{e^{-n\phi_n(\sqrt{\xi_n(B)-\xi_n(C)})^2}} = 1$.

14. As already stated, models with homogenous voting costs generally yield toss-up elections. In two-candidate elections, the difference between the assumptions of homogenous versus smooth costs has already been discussed; in particular, with respect to the underdog effect. For more details, see Herrera, Morelli, and Palfrey (2014) or chapter 1 of this dissertation.

0.4), where C types are indifferent between A and B. A strategy of types AB and BA is described by a cutoff $\bar{v}$ such that a voter votes for his or her first-ranked candidate if and only if $v < \bar{v}$. Fey (1997) shows that there exists a sequence of equilibria satisfying $0 < \xi(A) = \xi(B) < \xi(C)$. In such an equilibrium, types BA and C vote sincerely and type AB-voters with a large v are prescribed to vote for B. In a finite electorate, this means that A receives fewer votes than B, reinforcing the strategic voting of the fraction of AB types with high v's. In the limit, A and B receive equal vote shares. Since this equilibrium exists for a range of $\varepsilon > 0$, the conditions are not knife-edge.

In my model, this equilibrium construction cannot hold if the limit of the ratio $\frac{\Pr(piv_{AC})}{\Pr(piv_{BC})}$ goes to one as n grows large because $\xi(A) = \xi(B)$ would imply equal voting benefits for types AB and BA, yielding equal participation rates. Then, vote shares could not be equal unless the respective population shares coincide. For the case that $\lim_{n\to\infty} \frac{\Pr(piv_{AC})}{\Pr(piv_{BC})} \neq 1$, it is an open question whether a similar construction as employed by Fey (1997) will yield an equilibrium in the present model of costly voting. Since the parameter v is homogenous in my model, there is no mixing and all voters of the same type will vote for the same candidate, conditional on turning out. However, cost cutoffs might differ between voters of types AB and BA, potentially enabling a similar equilibrium construction as in Fey (1997), where candidate A receives fewer votes than B for sufficiently large, but finite n.

3.6 Conclusion

In this paper, I analyze plurality voting with three candidates in an environment in which voting is costly and voluntary, and costs are drawn from a smooth distribution. I examine the question whether three-candidate equilibria can exist. My main finding is that such equilibria do not exist in the limit as the expected number of citizens grow large if exactly one candidate is expected to have the lowest vote share. Equilibria in which all candidates are expected to tie exist for finite populations only under knife-edge conditions. Finally, equilibria in which two candidates are expected to tie behind the front-runner for sufficiently large n and in the limit can exist only under knife-edge conditions. The general existence of equilibria in which the second-and third-placed candidate are tied in expectation still remains an open question.

I show first that, as $n \to \infty$, equilibria must be Duvergerian whenever one candidate is expected to be trailing. Thus, there are two remaining potential types of three-candidate equilibria: Either (i) all candidates are expected to tie, or (ii) two candidates tie behind the front-runner. For given n, type (i) can be supported as an equilibrium if and only if each candidate is ranked first by exactly one-third of the electorate. If, along an equilibrium sequence, condition (ii) is satisfied already for sufficiently large n, type (ii) has two subtypes—depending on the incentives of the voters who rank the front-runner second. For both subtypes, I find necessary conditions on the distribution of preferences for existence in a large election. These necessary conditions are again knife-edge. If condition (ii) is satisfied only in the limit, the analysis does not yield general necessary or sufficient conditions for the existence of a type (ii) .

My results extend the analysis of Xefteris (2019), who proves the existence of Duvergerian equilibria in a slightly more general model, and contrast with the analysis of Arzumanyan and Polborn (2017), who assume that voting costs are homogenous. Homogenous voting costs imply that all voters need to be indifferent between voting and abstaining in equilibrium. As a result, the expected benefit of voting must be equal for all preference types, implying a toss-up election. Thus, the fundamental difference between costly voting models with smooth versus homogenous costs that has already been discussed for two-candidate elections extends to models with multiple candidates.

Further, my results reflect those of Palfrey (1989) who proves that non-Duvergerian equilibria cannot exist if one candidate is expected to be trailing in the limit in a model of costless voting. The question whether the type of non-Duvergerian equilibria derived in Fey (1997) can also exist under costly voting, and, thus, endogenous participation, remains open.

Finally, the literature on costly voting in multi-candidate elections has yet to answer the question of how voters coordinate. Duvergerian equilibria require voters to agree on which candidate is expected to receive the lowest vote share and will, thus, be abandoned. This does not need to be the candidate with the lowest ex ante support. One possible equilibrium refinement is the focal point-approach by Schelling (1980), which claims that agents are likely to coordinate on the salient equilibrium and would likely eliminate the candidate who is ranked first by the lowest fraction of citizens. In this context, polls could be an important coordination device. Fixing how voters coordinate given a certain poll result, it will be interesting to analyze the arising incentives for poll participants. In two-candidate elections, the underdog effect yields incentives for poll participants to misrepresent their preferences to stimulate turnout of like-minded voters. Yet, in multicandidate elections, the trailing candidate in the poll might be abandoned in the election, yielding incentives for poll participants to support their favorite candidate in the poll. It is not obvious which incentive dominates, and the answer to this question is left for future research.

Appendix 3.A Proofs

3.A.1 Proofs for Section 3.5.1 (Large Elections)

Proof of Lemma 3.1.
Recall that the probability of type t voting for any candidate is $\max_l p_n(l,t)$. First, this probability cannot be one for any type. Suppose, without loss of generality, that the probability of voting is one for type AB. Then, either $p_n(A,AB) = 1$ or $p_n(B,AB) = 1$, meaning that either $c_n(A,AB) = \bar{c}$ or $c_n(B,AB) = \bar{c}$. Inspecting equations (3.1) and (3.2) and recalling that $\bar{c} > 1$ yields the desired contradiction.

Second, the probability of voting cannot be zero for any type. Suppose, without loss of generality, that the probability of voting is zero for type AB. Then, both $p_n(A,AB) = 0$ and $p_n(B,AB) = 0$, meaning that both $c_n(A,AB) = 0$ and $c_n(B,AB) = 0$. Recall the definition of $c_n(A,AB)$, and consider the cost cutoffs for types BA and CA.

$$\begin{aligned}
c_n(A,AB) =& \frac{2-v}{3}\Pr_n(tie_{ABC}) + \frac{2-v}{6}\Pr_n(tie_{ABC}^{-A}) \\
&+ \frac{1-v}{2}\big(\Pr_n(piv_{AB}) + \Pr_n(piv_{AB}^{-A})\big) + \frac{1}{2}\big(\Pr_n(piv_{AC}) + \Pr_n(piv_{AC}^{-A})\big), \\
c_n(A,BA) =& \frac{2v-1}{3}\Pr_n(tie_{ABC}) + \frac{2v-1}{6}\Pr_n(tie_{ABC}^{-A}) \\
&+ \frac{v-1}{2}\big(\Pr_n(piv_{AB}) + \Pr_n(piv_{AB}^{-A})\big) + \frac{v}{2}\big(\Pr_n(piv_{AC}) + \Pr_n(piv_{AC}^{-A})\big), \\
c_n(A,CA) =& \frac{2v-1}{3}\Pr_n(tie_{ABC}) + \frac{2v-1}{6}\Pr_n(tie_{ABC}^{-A}) \\
&+ \frac{v-1}{2}\big(\Pr_n(piv_{AC}) + \Pr_n(piv_{AC}^{-A})\big) + \frac{v}{2}\big(\Pr_n(piv_{AB}) + \Pr_n(piv_{AB}^{-A})\big).
\end{aligned}$$

Since $c_n(A,AB) = 0$ is possible only if all pivot events involving A have zero probability, i.e., only if, for all n, $\Pr_n(piv_{ABC}) = \Pr_n(piv_{ABC}^{-A}) = \Pr_n(piv_{AB}) = \Pr_n(piv_{AB}^{-A}) = \Pr_n(piv_{AC}) = \Pr_n(piv_{AC}^{-A}) = 0$, it must be true that $c_n(A,BA) = c_n(A,CA) = 0$. Thus, no citizen's strategy prescribes to vote for A. Similarly, all pivot events involving B must have zero probability. This means that $c_n(A,t) = c_n(B,t) = 0$ for all $t \in T$, and no voter is prescribed to vote for either A or B. Consider now the incentives of any type who ranks candidate C third. A vote for either A or B is pivotal against C if nobody turns out to vote for C, which occurs with probability $e^{-n(\tau_n(A)+\tau_n(B)+\tau_n(C))} > 0$, or if exactly one voter turns out for C, which occurs with probability $e^{-n(\tau_n(A)+\tau_n(B)+\tau_n(C))} n\tau_n(C)$. However, this implies $c_n(A,t), c_n(B,t) > 0$ for all types who rank C last, and in particular, that $c_n(A,AB) > 0$—a contradiction. The other cases are entirely analogous. □

Proof of Lemma 3.2.
Suppose, by contradiction, that along some subsequence, $n(\tau_n(A) + \tau_n(B) + \tau_n(C))$

has a finite limit. Thus, the probability that nobody turns out to vote is given by $e^{-n(\tau_n(A)+\tau_n(B)+\tau_n(C))} > 0$. Thus, inspecting equations (3.5)-(3.8) yields that $\lim_{n\to\infty} U(i, ij) > 0$ for all types $ij \in T$. Hence, for all types $t \in T$, the cost cutoffs converge to strictly positive limits. In particular, the maximum of the two cost cutoffs has to be strictly positive in the limit for all preference types. Then, given that F is assumed to have full support on $[0, \bar{c}]$, for each type $t \in T$, $\lim_{n\to\infty} p_n(t) > 0$. Now, $\tau_n(A) + \tau_n(B) + \tau_n(C) = \sum_{t\in T} r(t)p_n(t)$, and thus, $\lim_{n\to\infty} n(\tau_n(A) + \tau_n(B) + \tau_n(C)) = \lim_{n\to\infty} n \sum_{t\in T} r(t)p_n(t) = \infty$—a contradiction! □

3.A.2 Proof for Section 3.5.2 (Magnitudes)

Proof of Lemma 3.5.
Magnitude of *tie*$_{ABC}$

$$\mu(tie_{ABC}|\Phi_n\xi_n) \;=\; \lim_{n\to\infty} \max_{\gamma\geq 0} \sum_{l\in\{A,B,C\}} \xi_n(l)\psi\left(\frac{\gamma}{\Phi_n\xi_n(l)}\right).$$

The first order condition yields

$$\begin{aligned} -\sum_{l\in\{A,B,C\}} \xi_n(l)\frac{1}{\Phi_n\xi_n(l)}\log\frac{\gamma}{\Phi_n\xi_n(l)} &= 0 \\ \Leftrightarrow \frac{1}{\Phi_n}\left[\sum_{l\in\{A,B,C\}}\log\frac{\gamma}{\Phi_n\xi_n(l)}\right] &= 0 \\ \Leftrightarrow \frac{\gamma}{\Phi_n\xi_n(A)}\cdot\frac{\gamma}{\Phi_n\xi_n(B)}\cdot\frac{\gamma}{\Phi_n\xi_n(C)} &= 1 \end{aligned}$$

$$\Leftrightarrow \frac{\gamma^3}{\Phi_n^3} \;=\; \xi_n(A)\xi_n(B)\xi_n(C).$$

The second order condition yields $\frac{\partial^2}{(\partial\gamma)^2} = -\frac{3}{\Phi_n\gamma} < 0$. Thus, the objective function is strictly concave.
Ignore the integer restriction on γ for now. The unique optimal solution is given by

$$\frac{\gamma}{\Phi_n} \;=\; \prod_{l\in\{A,B,C\}} (\xi_n(l))^{\frac{1}{3}}.$$

Define $g(n) := \frac{\gamma}{\Phi_n}$, and let $g := \lim_{n\to\infty} g(n)$.

As noted in Myerson (2000), when $\gamma(n)$ is the integer rounding of $\Phi_n g(n)$, the sequence $\{\gamma(n)\}_{n\geq1}$ is a *major sequence* in *tie*$_{ABC}$. This means that $\{\gamma(n)\}_{n\geq1}$ has a magnitude that is equal to the greatest magnitude of any sequence in tie_{ABC}. This magnitude is given by

$$\begin{aligned}
\mu(tie_{ABC}|\Phi_n\xi_n) &= \lim_{n\to\infty} \sum_{l\in\{A,B,C\}} \xi_n(l)\psi\left(\frac{\Phi_n g(n)}{\Phi_n\xi_n(l)}\right) \\
&= \lim_{n\to\infty} \sum_{l\in\{A,B,C\}} \xi_n(l)\left(\frac{g(n)}{\xi_n(l)}\left(1-\log\frac{g(n)}{\xi_n(l)}\right)-1\right) \\
&= \lim_{n\to\infty} 3g(n) - \sum_{l\in\{A,B,C\}} \xi_n(l) \\
&= \lim_{n\to\infty} 3\prod_{l\in\{A,B,C\}} [\xi_n(l)]^{\frac{1}{3}} - \sum_{l\in\{A,B,C\}} \xi_n(l) \\
&= 3\prod_{l\in\{A,B,C\}} [\xi(l)]^{\frac{1}{3}} - \sum_{l\in\{A,B,C\}} \xi(l). \\
&= 3[\xi(A)\xi(B)\xi(C)]^{\frac{1}{3}} - 1.
\end{aligned}$$

Magnitude of tie_{AB}

$$\mu(tie_{AB}|\Phi_n\xi_n) = \lim_{n\to\infty} \max_{\gamma\geq 0,\delta\geq 0} \left[\sum_{l\in\{A,B\}} \xi_n(l)\psi\left(\frac{\gamma}{\Phi_n\xi_n(l)}\right) + \xi_n(C)\psi\left(\frac{\delta}{\Phi_n\xi_n(C)}\right)\right].$$

The first order conditions yield

$$\begin{aligned}
\frac{\partial}{\partial\gamma} &: -\sum_{l\in\{A,B\}} \xi_n(l)\frac{1}{\Phi_n\xi_n(l)}\log\frac{\gamma}{\Phi_n\xi_n(l)} = 0 \\
&\Leftrightarrow \frac{1}{\Phi_n}\left[\sum_{l\in\{A,B\}} \log\frac{\gamma}{\Phi_n\xi_n(l)}\right] = 0 \\
&\Leftrightarrow \frac{\gamma}{\Phi_n\xi_n(A)}\cdot\frac{\gamma}{\Phi_n\xi_n(B)} = 1 \\
&\Leftrightarrow \frac{\gamma^2}{\Phi_n^2} = \xi_n(A)\xi_n(B).
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial\delta} &: -\xi_n(C)\frac{1}{\Phi_n\xi_n(C)}\log\frac{\delta}{\Phi_n\xi_n(C)} = 0 \\
&\Leftrightarrow \frac{\delta}{\Phi_n} = \xi_n(C).
\end{aligned}$$

The Hessian is given by

$$H = \begin{bmatrix} \frac{-2}{\Phi_n\gamma} & 0 \\ 0 & \frac{-1}{\Phi_n\delta} \end{bmatrix}.$$

Since H is negative definite, the objective function is strictly concave. Ignoring the integer restrictions on γ and δ for now, the unique optimal solution is given by

$$\frac{\gamma}{\Phi_n} = \prod_{l\in\{A,B\}} [\xi_n(l)]^{\frac{1}{2}}, \quad \frac{\delta}{\Phi_n} = \xi_n(C).$$

Define $g(n) := \frac{\gamma}{\Phi_n}$ and $d(n) := \frac{\delta}{\Phi_n}$, and let $g := \lim_{n\to\infty} g(n)$ and $d := \lim_{n\to\infty} d(n)$. Then,

$$\begin{aligned}
\mu(tie_{AB}|\Phi_n\xi_n) &= \lim_{n\to\infty} \sum_{l\in\{A,B\}} \xi_n(l)\psi\left(\frac{\Phi_n g(n)}{\Phi_n\xi_n(l)}\right) + \xi_n(C)\psi\left(\frac{\Phi_n d(n)}{\Phi_n\xi_n(C)}\right)\\
&= \lim_{n\to\infty} \sum_{l\in\{A,B\}} \xi_n(l)\left(\frac{g(n)}{\xi_n(l)}\left(1-\log\frac{g(n)}{\xi_n(l)}\right)-1\right)\\
&\quad + \xi_n(C)\left(\frac{d(n)}{\xi_n(C)}\left(1-\log\frac{d(n)}{\xi_n(C)}\right)-1\right)\\
&= \lim_{n\to\infty} g(n)\left(2-\log\left(\frac{g(n)^2}{\xi_n(A)\xi_n(B)}\right)\right)\\
&\quad + d(n)\left(1-\log\left(\frac{d(n)}{\xi_n(C)}\right)\right) - \xi_n(A)-\xi_n(B)-\xi_n(C)\\
&= \lim_{n\to\infty} 2\sqrt{\xi_n(A)\xi_n(B)} - \xi_n(A) - \xi_n(B)\\
&= 2\sqrt{\xi(A)\xi(B)} - \xi(A) - \xi(B).
\end{aligned}$$

□

Proof of Lemma 3.8.
Consider some sequence of vote shares $(\xi_n)_{n\in\mathbb{N}}$ with $\delta_{ij}(\xi) = \sqrt{\xi(i)\xi(j)} - \xi(k) > 0$. Recall that $tie_{ij} := \{x \in X : x(i) = x(j)\}$, and define $notpiv_{ij} := \{x \in X : x(i) = x(j) < x(k)\}$. Then,

$$\begin{aligned}
tie_{ij} =& \{x \in X : x(i) = x(j) > x(k)\} \cup \{x \in X : x(i) = x(j) = x(k)\}\\
&\cup \{x \in X : x(i) = x(j) < x(k)\}\\
=& piv_{ij} \cup tie_{ABC} \cup notpiv_{ij},
\end{aligned}$$

and, for all n,

$$\Pr_n(tie_{ij}) = \Pr_n(piv_{ij}) + \Pr_n(tie_{ABC}) + \Pr_n(notpiv_{ij}).$$

Since $\delta_{ij}(\xi) > 0$, by Lemma 3.7,

$$\mu(piv_{ij}|\Phi_n\xi_n) > \mu(tie_{ABC}|\Phi_n\xi_n).$$

This implies that $\frac{\Pr_n(tie_{ABC})}{\Pr_n(piv_{ij})} \to 0$ as $n \to \infty$.
Let me now derive $\mu(notpiv_{ij}|\Phi_n\xi_n)$.

Following Myerson (2002) and Bouton and Castanheira (2012), the magnitude of the event *notpiv*$_{ij}$ is defined as

$$\begin{aligned}
\mu(notpiv_{ij}|\Phi_n\xi_n) &= \lim_{n\to\infty} \max_x \sum_{l\in L} \frac{x(l)}{n\phi_n}\left(1-\log\left(\frac{x(l)}{n\phi_n\xi_n}\right)\right)-1\\
s.t.\, x(i) &= x(j), \text{ and}\\
x(k) &> x(j).
\end{aligned}$$

If I ignore the second constraint, or if the constraint is not binding at the optimum, this magnitude is exactly $\mu(tie_{ij}|\Phi_n\xi_n)$. So, let me calculate the optimum.
Set $x(i)=\gamma=x(j)$, $x(k)=\kappa$, and maximize ignoring the second constraint for now:

$$\max_{\gamma\geq 0,\kappa\geq 0} \frac{\gamma}{n\phi_n}\left(2-\log\left(\frac{\gamma^2}{n^2\phi_n^2\xi_n(i)\xi_n(j)}\right)\right)+\frac{\kappa}{n\phi_n}\left(1-\log\left(\frac{\kappa}{n\phi_n\xi_n(k)}\right)\right)-1$$

The first order conditions yield

$$\begin{aligned}
\frac{\partial}{\partial\gamma}\ :\ \frac{1}{n\phi_n}\left(2-\log\left(\frac{\gamma^2}{n^2\phi_n^2\xi_n(i)\xi_n(j)}\right)\right)-\frac{\gamma}{n\phi_n}\frac{2}{\gamma} &= 0\\
\Leftrightarrow\ \frac{1}{n\phi_n}\log\left(\frac{\gamma^2}{n^2\phi_n^2\xi_n(i)\xi_n(j)}\right) &= 0\\
\Rightarrow\ \gamma^* &= n\phi_n\sqrt{\xi_n(i)\xi_n(j)}.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial\kappa}\ :\ \frac{1}{n\phi_n}\left(1-\log\left(\frac{\kappa}{n\phi_n\xi_n(k)}\right)\right)-\frac{\kappa}{n\phi_n}\frac{1}{\kappa} &= 0\\
\Leftrightarrow\ \frac{1}{n\phi_n}\log\left(\frac{\kappa}{n\phi_n\xi_n(k)}\right) &= 0\\
\Rightarrow\ \kappa^* &= n\phi_n\xi_n(k).
\end{aligned}$$

The Hessian is given by

$$H = \begin{bmatrix} \frac{-2}{n\phi_n\gamma} & 0\\ 0 & \frac{-1}{n\phi_n\kappa}\end{bmatrix}.$$

Since the Hessian is negative definite, the objective function is strictly concave. Thus, the first order conditions are sufficient for an optimum. It remains to check whether these values imply $\gamma^*<\kappa^*$, which would yield that the second constraint is not binding at the optimum.
Now, by assumption, $\delta_{ij}(\xi)=\sqrt{\xi(i)\xi(j)}-\xi(k)>0$. This implies that there exists

some $\bar{n}$ such that for all $n > \bar{n}$, $\sqrt{\xi_n(i)\xi_n(j)} - \xi_n(k) > 0$. Consequently, for all $n > \bar{n}$, given that $n\phi_n > 0$,

$$\begin{aligned} n\phi_n \left(\sqrt{\xi_n(i)\xi_n(j)} - \xi_n(k) \right) &> 0 \\ \Leftrightarrow \gamma^* - \kappa^* &> 0. \end{aligned}$$

Thus, for all $n > \bar{n}$, the unique unconstrained maximizer (γ^*, κ^*) does not satisfy the second constraint, and, hence, cannot do so in the limit as $n \to \infty$. Consequently, the value of constrained maximization problem, $\mu(notpiv_{ij}|\Phi_n\xi_n)$, has to be strictly smaller than the value of the unconstrained maximization problem, $\mu(tie_{ij}|\Phi_n\xi_n)$. As a result, $\lim_{n\to\infty} \frac{\Pr_n(notpiv_{ij})}{\Pr_n(piv_{ij})} = 0$.
But then,

$$\begin{aligned} \lim_{n\to\infty} \frac{\Pr_n(tie_{ij})}{\Pr_n(piv_{ij})} &= \lim_{n\to\infty} \frac{\Pr_n(piv_{ij}) + \Pr_n(tie_{ABC}) + \Pr_n(notpiv_{ij})}{\Pr_n(piv_{ij})} \\ &= 1 + \lim_{n\to\infty} \left(\frac{\Pr_n(tie_{ABC})}{\Pr_n(piv_{ij})} + \frac{\Pr_n(notpiv_{ij})}{\Pr_n(piv_{ij})} \right) \\ &= 1. \end{aligned}$$

Thus, employing Myerson (2000), equation (5.3), as $n \to \infty$,

$$\Pr_n(piv_{ij}) \approx \frac{e^{-n\phi_n\left(\sqrt{\xi_n(i)}-\sqrt{\xi_n(j)}\right)^2}}{\sqrt{4\pi n\phi_n\sqrt{\xi_n(i)\xi_n(j)}}}.^{15}$$

□

3.A.3 Proof for Section 3.5.3 (Duverger's Law)

Proof of Proposition 3.3.
Suppose, by contradiction, that $\xi(A) \geq \xi(B) > \xi(C) > 0$.
Then, because of $r(t) > 0$ for all $t \in T$, there must exist a (convergent) subsequence of cost cutoffs along which at least one of the following inequalities is satisfied:

$$\begin{aligned} \lim_{n\to\infty} \frac{c_n(C, CA)}{c_n(A, CA)} &> 1, \lim_{n\to\infty} \frac{c_n(C, CB)}{c_n(B, CB)} > 1, \\ \lim_{n\to\infty} \frac{c_n(C, AC)}{c_n(A, AC)} &> 1, \lim_{n\to\infty} \frac{c_n(C, BC)}{c_n(B, BC)} > 1. \end{aligned}$$

Note first that, $\delta_{AB}(\xi) = \sqrt{\xi(A)\xi(B)} - \xi(C) > \xi(C) - \xi(C) = 0$. Thus, by Lemma 3.7, $\mu(piv_{AB}|\Phi_n\xi_n) > \mu(tie_{ABC}|\Phi_n\xi_n)$.

15. The expression $x_n \approx y_n$ indicates that $\lim_{n\to\infty} \frac{x_n}{y_n} = 1$.

Claim 3.2. *$\xi(A), \xi(B) > \xi(C)$ implies that $\mu(piv_{AB}|\Phi_n \xi_n) > \mu(piv_{AC}|\Phi_n \xi_n)$ and $\mu(piv_{AB}|\Phi_n \xi_n) > \mu(piv_{BC}|\Phi_n \xi_n)$.*

Proof. There are three different possibilities: (i) $\xi(A) = \xi(B)$, (ii) $\xi(A) > \xi(B)$, or (iii) $\xi(B) > \xi(A)$.

Case (i): Then, $\delta_{AC}(\xi) = \sqrt{\xi(A)\xi(C)} - \xi(B) < \xi(A) - \xi(B) = 0$, so, by Lemma 3.7, $\mu(piv_{AC}|\Phi_n \xi_n) = \mu(tie_{ABC}|\Phi_n \xi_n)$.
By the same argument, $\mu(piv_{BC}|\Phi_n \xi_n) = \mu(tie_{ABC}|\Phi_n \xi_n)$.

Case (ii): Then, $\delta_{BC}(\xi) = \sqrt{\xi(B)\xi(C)} - \xi(A) < \xi(A) - \xi(A) = 0$, and, thus, by Lemma 3.7, $\mu(piv_{AB}|\Phi_n \xi_n) > \mu(piv_{BC}|\Phi_n \xi_n)$. Thus, it is left to show that $\mu(piv_{AB}|\Phi_n \xi_n) > \mu(piv_{AC}|\Phi_n \xi_n)$.
By Lemma 3.7, either, $\mu(piv_{AC}|\Phi_n \xi_n) = \mu(tie_{AC}|\Phi_n \xi_n) = 2\sqrt{\xi(A)\xi(C)} - \xi(A) - \xi(C)$ or $\mu(piv_{AC}|\Phi_n \xi_n) = \mu(tie_{ABC}|\Phi_n \xi_n)$. If the latter holds, the conclusion is again immediate. If the former holds, by Lemma 3.5, observe that

$$
\begin{aligned}
\mu(piv_{AB}|\Phi_n \xi_n) &> \mu(piv_{AC}|\Phi_n \xi_n) \\
\Leftrightarrow\ 2\sqrt{\xi(A)\xi(B)} - \xi(A) - \xi(B) &> 2\sqrt{\xi(A)\xi(C)} - \xi(A) - \xi(C) \\
\Leftrightarrow\ 2\sqrt{\xi(A)}\left(\sqrt{\xi(B)} - \sqrt{\xi(C)}\right) &> \xi(B) - \xi(C) \\
\Leftrightarrow\ 2\sqrt{\xi(A)} &> \sqrt{\xi(B)} + \sqrt{\xi(C)},
\end{aligned}
$$

which is true because of the assumption that $\xi(A) > \xi(B) > \xi(C) > 0$.

Case (iii) is analogous to case (ii). □

By the Magnitude Equivalence Theorem, $\mu(tie^{-C}_{ABC}) = \mu(tie^{-A}_{ABC}) = \mu(tie_{ABC})$, $\mu(piv^{-C}_{AC}) = \mu(piv^{-A}_{AC}) = \mu(piv_{AC})$, and $\mu(piv^{-C}_{BC}) = \mu(piv_{BC})$. Now, by Lemma 3.3, as $n \to \infty$, $\frac{\Pr_n(piv_{BC})}{\Pr_n(piv_{AB})} \to 0$, $\frac{\Pr_n(piv_{AC})}{\Pr_n(piv_{AB})} \to 0$, $\frac{\Pr_n(tie_{ABC})}{\Pr_n(piv_{AB})} \to 0$, and so on. Consequently,

$$
\begin{aligned}
&\lim_{n\to\infty} \frac{c_n(C, CA)}{c_n(A, CA)} \\
&= \lim_{n\to\infty} \frac{\frac{2-\nu}{3}\Pr_n(tie_{ABC}) + \frac{2-\nu}{6}\Pr_n(tie^{-C}_{ABC}) + \frac{1-\nu}{2}[\Pr_n(piv_{AC}) + \Pr_n(piv^{-C}_{AC})] + \frac{1}{2}[\Pr_n(piv_{BC}) + \Pr_n(piv^{-C}_{BC})]}{\frac{2\nu-1}{3}\Pr_n(tie_{ABC}) + \frac{2\nu-1}{6}\Pr_n(tie^{-A}_{ABC}) + \frac{\nu-1}{2}[\Pr_n(piv_{AC}) + \Pr_n(piv^{-A}_{AC})] + \frac{\nu}{2}[\Pr_n(piv_{AB}) + \Pr_n(piv^{-A}_{AB})]} \\
&= \lim_{n\to\infty} \frac{\Pr_n(piv_{AB})}{\Pr_n(piv_{AB})} \frac{\frac{2-\nu}{3}\frac{\Pr_n(tie_{ABC})}{\Pr_n(piv_{AB})} + \frac{2-\nu}{6}\frac{\Pr_n(tie^{-C}_{ABC})}{\Pr_n(piv_{AB})} + \frac{1-\nu}{2}\left[\frac{\Pr_n(piv_{AC})}{\Pr_n(piv_{AB})} + \frac{\Pr_n(piv^{-C}_{AC})}{\Pr_n(piv_{AB})}\right] + \frac{1}{2}\left[\frac{\Pr_n(piv_{BC})}{\Pr_n(piv_{AB})} + \frac{\Pr_n(piv^{-C}_{BC})}{\Pr_n(piv_{AB})}\right]}{\frac{2\nu-1}{3}\frac{\Pr_n(tie_{ABC})}{\Pr_n(piv_{AB})} + \frac{2\nu-1}{6}\frac{\Pr_n(tie^{-A}_{ABC})}{\Pr_n(piv_{AB})} + \frac{\nu-1}{2}\left[\frac{\Pr_n(piv_{AC})}{\Pr_n(piv_{AB})} + \frac{\Pr_n(piv^{-A}_{AC})}{\Pr_n(piv_{AB})}\right] + \frac{\nu}{2}\left[1 + \frac{\Pr_n(piv^{-A}_{AB})}{\Pr_n(piv_{AB})}\right]} \\
&= 0.
\end{aligned}
$$

Similarly, $\lim_{n\to\infty} \frac{c_n(C,CB)}{c_n(B,CB)} = 0$. As a result, in the limit as n grows large, citizens who prefer candidate C over candidates A and B have a strict incentive to vote for their second-preferred candidate or to abstain and will never vote sincerely. By the same line of argument, $\lim_{n\to\infty} \frac{c_n(C,AC)}{c_n(A,AC)} = 0$ and $\lim_{n\to\infty} \frac{c_n(C,BC)}{c_n(B,BC)} = 0$.

Thus, $\xi(C) = 0$—a contradiction!

Moreoever, since

$$\frac{c_n(C,CA)}{c_n(A,CA)} \rightarrow 0, \frac{c_n(C,CB)}{c_n(B,CB)} \rightarrow 0, \frac{c_n(C,AC)}{c_n(A,AC)} \rightarrow 0, \text{ and } \frac{c_n(C,BC)}{c_n(B,BC)} \rightarrow 0,$$

there exists $\bar{n}$ such that for all $n > \bar{n}$,

$$\frac{c_n(C,CA)}{c_n(A,CA)} < 1, \frac{c_n(C,CB)}{c_n(B,CB)} < 1, \frac{c_n(C,AC)}{c_n(A,AC)} < 1, \text{ and } \frac{c_n(C,BC)}{c_n(B,BC)} < 1,$$

yielding the contradiction also for finite, but sufficiently large n. □

3.A.4 Proofs for Section 3.5.4 (Non-Duvergerian Equilibria)

Proof of Proposition 3.4.

Part 1: "If": Suppose that $r(AB) + r(AC) = r(BA) + r(BC) = r(CA) + r(CB) = \frac{1}{3}$.

Fix any n, and take some voter q of type ij. Note first that if all voters other than voter q vote sincerely and employ the cost cutoff $\hat{c}_n$, it is a best response for q to vote sincerely and employ the cutoff $\hat{c}_n$, too, independent of his or her type. That is, $\hat{c}_n = c_n(i,ij) \geq c_n(j,ij)$. To see this, consider $c_n(i,ij) - c_n(j,ij)$:

$$\begin{aligned} c_n(i,ij) - c_n(j,ij) =& (1-v)\Pr_n(tie_{ABC}) + \left(\frac{2-v}{6}\Pr_n(tie_{ABC}^{-i}) - \frac{2v-1}{6}\Pr_n(tie_{ABC}^{-j})\right) \\ &+ (1-v)\Pr_n(piv_{ij}) + \left(\frac{1-v}{2}\Pr_n(piv_{ij}^{-i}) - \frac{v-1}{2}\Pr_n(piv_{ij}^{-j})\right) \\ &+ \frac{1}{2}\Pr_n(piv_{ik}) + \frac{1}{2}\Pr_n(piv_{ik}^{-i}) - \frac{v}{2}\Pr_n(piv_{jk}) - \frac{v}{2}\Pr_n(piv_{jk}^{-j}). \end{aligned}$$

If all voters other than voter q vote sincerely with cutoff $\hat{c}_n$, it holds that $\tau_n(A) = r(AB)F(\hat{c}_n) + r(AC)F(\hat{c}_n) = \frac{1}{3}F(\hat{c}_n) = \tau_n(B) = \tau_n(C)$ because $r(ij) + r(ik) = \frac{1}{3}$ for all $ij, ik \in T$ by assumption. Thus, the difference simplifies to

$$\begin{aligned} c_n(i,ij) - c_n(j,ij) =& (1-v)\Pr_n(tie_{ABC}) + \frac{1-v}{2}\Pr_n(tie_{ABC}^{-i}) \\ &+ \frac{3(1-v)}{2}\Pr_n(piv_{ij}) + \frac{3(1-v)}{2}\Pr_n(piv_{ij}^{-i}), \end{aligned}$$

which is obviously positive because of $v < 1$. Moreover, if $\hat{c}_n$ is the equilibrium cutoff for all voters other than q, $c_n(i,ij)$ must be equal to $\hat{c}_n$ by symmetry.

Such an equilibrium exists if the cutoff $\hat{c}_n$ solves equation (3.3) for all types $t \in T$. Thus, $\hat{c}_n$ must be a fixed point of (3.3). Since the cutoff $\hat{c}_n$ does not depend on voter types and because vote shares are identical for all candidates, equation (3.3) is identical for all voter types. Apply now Brouwer's fixed-point theorem to just one equation, that is, consider the mapping from $[0, \bar{c}]$ into itself. Note that

$[0, \bar{c}]$ is non-empty, compact and convex because F has full support on $[0, \bar{c}]$ with $\bar{c} > 1$. Further, $\tau_n(l)$, $l \in L$, are continuous functions of the cost cutoff and so are the pivot probabilities. Thus, Brouwer's fixed-point theorem guarantees the existence of a fixed point for each isolated equation (3.3). But then, this fixed point must solve the equilibrium equation (3.3) for all types $t \in T$.

I conclude that the sincere equilibrium with equal expected vote shares indeed exists if $r(ij) + r(ik) = \frac{1}{3}$ for all $ij, ik \in T$.

Part 2: "Only if": Take any n and let $\xi_n(A) = \xi_n(B) = \xi_n(C) > 0$.

First, $\Pr_n(piv_{AB}) = \Pr_n(piv_{AC}) = \Pr_n(piv_{BC})$, and $\Pr_n(piv_{ij}^{-i}) = \Pr_n(piv_{ik}^{-i})$ for all $ij, ik \in T$. Further, $\Pr_n(tie_{ABC}^{-A}) = \Pr_n(tie_{ABC}^{-B}) = \Pr_n(tie_{ABC}^{-C})$, and $\Pr_n(piv_{ij}^{-i}) = \Pr_n(piv_{ij}^{-j})$ for all $ij \in T$. This implies that

$$
\begin{aligned}
& c_n(i, ij) - c_n(j, ij) \\
=& (1 - v) \cdot \left[\Pr_n(tie_{ABC}) + \frac{1}{2} \Pr_n(tie_{ABC}^{-i}) + \frac{3}{2} \left(\Pr_n(piv_{ij}) + \Pr_n(piv_{ij}^{-i}) \right) \right] \\
>& 0,
\end{aligned}
$$

since $v < 1$, meaning that every voter will vote sincerely for his most preferred candidate.

Further, this implies that $c_n(i, ij) = c_n(i, ik) \equiv c_n(i)$ and furthermore that $c_n(i) = c_n(j)$ for all i, j, k. Thus, all voters participate with the same probability $p_n(i) \equiv p_n = F(c_n)$. Then,

$$\tau_n(A) \; = \; p_n \cdot [r(AB) + r(AC)].$$

So, $\tau_n(A) = \tau_n(B) = \tau_n(C)$ can hold only if indeed,

$$r(AB) + r(AC) \; = \; r(BA) + r(BC) \; = \; r(CA) + r(CB) \; = \; \frac{1}{3}.$$

□

Let me introduce an intermediate result. Lemma 3.9 will allow me to apply a Taylor expansion in the proof of Proposition 3.5.

Lemma 3.9. *Suppose* $0 < \xi(A) = \xi(B) < \xi(C)$. *As* $n \to \infty$, *the participation rates* $p(t)$, $t \in T$, *go to zero along every equilibrium sequence. That is,* $\limsup_{n\to\infty} p_n(t) = 0$ *for all* $t \in T$.

Proof. Suppose, by contradiction, that, along some convergent subsequence, $\lim_{n\to\infty} p_n(AB) > 0$, meaning that $\lim_{n\to\infty} \max\{p_n(A, AB), p_n(B, AB)\} > 0$.

Suppose first that $\lim_{n\to\infty}\max\{p_n(A,AB),p_n(B,AB)\}=\lim_{n\to\infty}p_n(A,AB)$. This implies that

$$\begin{aligned}\lim_{n\to\infty}c_n(A,AB) &= \lim_{n\to\infty}\frac{2-v}{3}\Pr_n(tie_{ABC})+\frac{2-v}{6}\Pr_n(tie_{ABC}^{-A})\\ &+\frac{1-v}{2}\left(\Pr_n(piv_{AB})+\Pr_n(piv_{AB}^{-A})\right)\\ &+\frac{1}{2}\left(\Pr_n(piv_{AC})+\Pr_n(piv_{AC}^{-A})\right)\\ &>0.\end{aligned}$$

Now, since $\mu(tie_{ABC})=\mu(piv_{AB})<\mu(piv_{AC})$, it holds that

$$\lim_{n\to\infty}c_n(A,AB) < \lim_{n\to\infty}\left(\frac{2-v}{3}+\frac{1-v}{2}+\frac{1}{2}\right)\left(\Pr_n(piv_{AC})+\Pr_n(piv_{AC}^{-A})\right).$$ [16]

Further, since $piv_{AC}\subset tie_{AC}$, it holds that $\Pr_n(piv_{AC})\le\Pr_n(tie_{AC})$.[17] Thus,

$$\lim_{n\to\infty}c_n(A,AB) < \lim_{n\to\infty}\frac{10-5v}{6}\left(\Pr_n(tie_{AC})+\Pr_n(tie_{AC}^{-A})\right).$$

From the Offset Theorem in Myerson (2000) [Theorem 2],

$$\lim_{n\to\infty}\frac{\Pr_n(tie_{AC}^{-A})}{\Pr_n(tie_{AC})} = \lim_{n\to\infty}\sqrt{\frac{\xi_n(C)}{\xi_n(A)}}.$$

Further, from Myerson (2000), equation (5.3), as $n\to\infty$,

$$\Pr_n(tie_{AC}) \approx \frac{e^{-n\phi_n\left(\sqrt{\xi_n(A)-\xi_n(C)}\right)^2}}{\sqrt{4\pi n\phi_n\sqrt{\xi_n(A)\xi_n(C)}}}.$$

Thus, as $n\to\infty$,

$$\lim_{n\to\infty}c_n(A,AB) < \lim_{n\to\infty}\frac{10-5v}{6}\left(1+\sqrt{\frac{\xi_n(C)}{\xi_n(A)}}\right)\frac{e^{-n\phi_n\left(\sqrt{\xi_n(A)-\xi_n(C)}\right)^2}}{\sqrt{4\pi n\phi_n\sqrt{\xi_n(A)\xi_n(C)}}}.$$

The right-hand side converges to 0 as $n\to\infty$, since the denominator is unbounded, whereas the numerator is bounded—a contradiction. The other cases are analogous. □

16. The bound can be found by using the fact that $\Pr_n(tie_{ABC}),\Pr_n(piv_{AB})<\Pr_n(piv_{AC})$.
17. Actually, by Lemma 3.8, it even holds that $\lim_{n\to\infty}\frac{\Pr_n(tie_{AC})}{\Pr_n(piv_{AC})}=1$.

Proof of Proposition 3.5.
Let $0 < \xi(A) = \xi(B) < \xi(C)$ and suppose that there exists some $\bar{n}$ such that for all $n > \bar{n}$, $0 < \xi_n(A) = \xi_n(B) < \xi_n(C)$.

Asymptotic pivot ratios
Recalling equation (3.7), this implies that for all $n > \bar{n}$, $\Pr_n(piv_{AC}) = \Pr_n(piv_{BC})$. Consequently,

$$\lim_{n\to\infty} \frac{\Pr_n(piv_{AC})}{\Pr_n(piv_{BC})} = 1.$$

Similarly, by equation (3.8), for all $n > \bar{n}$, $\Pr_n(piv_{AC}^{-A}) = \Pr_n(piv_{BC}^{-B})$. This implies that

$$\lim_{n\to\infty} \frac{\Pr_n(piv_{AC}) + \Pr_n(piv_{AC}^{-A})}{\Pr_n(piv_{BC}) + \Pr_n(piv_{BC}^{-B})} = 1.^{18} \tag{3.A.1}$$

Next, note that

$$\begin{aligned}
\delta_{AB}(\xi) &= \sqrt{\xi(A)\xi(B)} - \xi(C) < \xi(C) - \xi(C) = 0,\\
\delta_{AC}(\xi) &= \sqrt{\xi(A)\xi(C)} - \xi(B) > \xi(A) - \xi(B) = 0,\\
\delta_{BC}(\xi) &= \sqrt{\xi(B)\xi(C)} - \xi(A) > \xi(B) - \xi(A) = 0.
\end{aligned}$$

Therefore, by Lemma 3.7,

$$\mu(piv_{AC}|\Phi_n\xi_n) = \mu(piv_{BC}|\Phi_n\xi_n) > \mu(piv_{AB}|\Phi_n\xi_n) = \mu(tie_{ABC}|\Phi_n\xi_n).$$

Thus,

$$\begin{aligned}
\lim_{n\to\infty} \frac{\Pr_n(piv_{AB})}{\Pr_n(piv_{AC})} &= \lim_{n\to\infty} \frac{\Pr_n(piv_{AB})}{\Pr_n(piv_{BC})} = 0, \text{ and}\\
\lim_{n\to\infty} \frac{\Pr_n(tie_{ABC})}{\Pr_n(piv_{AC})} &= \lim_{n\to\infty} \frac{\Pr_n(tie_{ABC})}{\Pr_n(piv_{BC})} = 0.
\end{aligned}$$

18. This can also be derived by using Lemma 3.8. Then,

$$\lim_{n\to\infty} \frac{\Pr_n(piv_{AC}) + \Pr_n(piv_{AC}^{-A})}{\Pr_n(piv_{BC}) + \Pr_n(piv_{BC}^{-B})} = \lim_{n\to\infty} \frac{\Pr_n(tie_{AC})}{\Pr_n(tie_{BC})} \frac{\frac{\Pr_n(piv_{AC})}{\Pr_n(tie_{AC})} + \frac{\Pr_n(piv_{AC}^{-A})}{\Pr_n(piv_{AC})}\frac{\Pr_n(piv_{AC})}{\Pr_n(tie_{AC})}}{\frac{\Pr_n(piv_{BC})}{\Pr_n(tie_{BC})} + \frac{\Pr_n(piv_{BC}^{-B})}{\Pr_n(piv_{BC})}\frac{\Pr_n(piv_{BC})}{\Pr_n(tie_{BC})}} = \frac{1+\sqrt{\frac{\xi(C)}{\xi(A)}}}{1+\sqrt{\frac{\xi(C)}{\xi(A)}}} = 1.$$

Types *AB*, *BA*, *CA*, and *CB* vote sincerely

As $n \to \infty$, types AB and BA vote sincerely:

$$
\begin{aligned}
&\lim_{n\to\infty} \frac{c_n(A,AB)}{c_n(B,AB)} \\
=&\lim_{n\to\infty} \frac{\frac{2-v}{3}\Pr_n(tie_{ABC}) + \frac{2-v}{6}\Pr_n(tie_{ABC}^{-A}) + \frac{1-v}{2}[\Pr_n(piv_{AB}) + \Pr_n(piv_{AB}^{-A})] + \frac{1}{2}[\Pr_n(piv_{AC}) + \Pr_n(piv_{AC}^{-A})]}{\frac{2v-1}{3}\Pr_n(tie_{ABC}) + \frac{2v-1}{6}\Pr_n(tie_{ABC}^{-B}) + \frac{v-1}{2}[\Pr_n(piv_{AB}) + \Pr_n(piv_{AB}^{-B})] + \frac{v}{2}[\Pr_n(piv_{BC}) + \Pr_n(piv_{BC}^{-B})]} \\
=&\lim_{n\to\infty} \frac{\Pr_n(piv_{AC})}{\Pr_n(piv_{BC})} \cdot \frac{\frac{2-v}{3}\frac{\Pr_n(tie_{ABC})}{\Pr_n(piv_{AC})} + \frac{2-v}{6}\frac{\Pr_n(tie_{ABC}^{-A})}{\Pr_n(piv_{AC})} + \frac{1-v}{2}\left[\frac{\Pr_n(piv_{AB})}{\Pr_n(piv_{AC})} + \frac{\Pr_n(piv_{AB}^{-A})}{\Pr_n(piv_{AC})}\right] + \frac{1}{2}\left[1 + \frac{\Pr_n(piv_{AC}^{-A})}{\Pr_n(piv_{AC})}\right]}{\frac{2v-1}{3}\frac{\Pr_n(tie_{ABC})}{\Pr_n(piv_{BC})} + \frac{2v-1}{6}\frac{\Pr_n(tie_{ABC}^{-B})}{\Pr_n(piv_{BC})} + \frac{v-1}{2}\left[\frac{\Pr_n(piv_{AB})}{\Pr_n(piv_{BC})} + \frac{\Pr_n(piv_{AB}^{-B})}{\Pr_n(piv_{BC})}\right] + \frac{v}{2}\left[1 + \frac{\Pr_n(piv_{BC}^{-B})}{\Pr_n(piv_{BC})}\right]} \\
=&\lim_{n\to\infty} \frac{\frac{1}{2}[\Pr_n(piv_{AC}) + \Pr_n(piv_{AC}^{-A})]}{\frac{v}{2}[\Pr_n(piv_{BC}) + \Pr_n(piv_{BC}^{-B})]} \\
=&\frac{1}{v} \\
>&1.
\end{aligned}
$$

The second to last step uses Lemma 3.3 and the fact that $\mu(piv_{AC}|\Phi_n\xi_n) = \mu(piv_{BC}|\Phi_n\xi_n) > \mu(piv_{AB}|\Phi_n\xi_n) = \mu(tie_{ABC}|\Phi_n\xi_n)$. The last step uses equation (3.A.1). The same argument applies to type BA.

Similarly, as $n \to \infty$, types CA and CB vote sincerely:

$$
\begin{aligned}
&\lim_{n\to\infty} c_n(A, CA) \\
=&\lim_{n\to\infty} \Pr_n(piv_{AC}) \Bigg[\frac{2v-1}{3}\frac{\Pr_n(tie_{ABC})}{\Pr_n(piv_{AC})} + \frac{2v-1}{6}\frac{\Pr_n(tie_{ABC}^{-A})}{\Pr_n(piv_{AC})} \\
&\qquad + \frac{v-1}{2}\left(1 + \frac{\Pr_n(piv_{AC}^{-A})}{\Pr_n(piv_{AC})}\right) + \frac{v}{2}\left(\frac{\Pr_n(piv_{AB})}{\Pr_n(piv_{AC})} + \frac{\Pr_n(piv_{AB}^{-A})}{\Pr_n(piv_{AC})}\right)\Bigg] \\
=&\lim_{n\to\infty} \frac{v-1}{2}\left[\Pr_n(piv_{AC}) + \Pr_n(piv_{AC}^{-A})\right].
\end{aligned}
$$

By contrast,

$$
\begin{aligned}
&\lim_{n\to\infty} c_n(C, CA) \\
=&\lim_{n\to\infty} \Pr_n(piv_{AC}) \Bigg[\frac{2-v}{3}\frac{\Pr_n(tie_{ABC})}{\Pr_n(piv_{AC})} + \frac{2-v}{6}\frac{\Pr_n(tie_{ABC}^{-C})}{\Pr_n(piv_{AC})} \\
&\qquad + \frac{1-v}{2}\left(1 + \frac{\Pr_n(piv_{AC}^{-C})}{\Pr_n(piv_{AC})}\right) + \frac{1}{2}\left(\frac{\Pr_n(piv_{BC})}{\Pr_n(piv_{AC})} + \frac{\Pr_n(piv_{BC}^{-C})}{\Pr_n(piv_{AC})}\right)\Bigg] \\
=&\lim_{n\to\infty} \frac{1-v}{2}\left[\Pr_n(piv_{AC}) + \Pr_n(piv_{AC}^{-C})\right] + \frac{1}{2}\left[\Pr_n(piv_{BC}) + \Pr_n(piv_{BC}^{-C})\right].
\end{aligned}
$$

Thus, there exists $\bar{n}$ such that for all $n > \bar{n}$, $c_n(A,AC) > c_n(C,AC)$.

Thus, types AB, BA, CA, and CB vote sincerely. By symmetry, if voters of type AC vote sincerely, then so do voters of type BC. It is therefore sufficient to consider two equilibrium subtypes: the sincere equilibrium and the equilibrium in which types

AC and BC vote strategically for C.

Sincere equilibria

$$
\begin{aligned}
\frac{\xi(A)}{\xi(B)} &= \lim_{n\to\infty} \frac{\xi_n(A)}{\xi_n(B)} \\
&= \lim_{n\to\infty} \frac{\tau_n(A)}{\tau_n(B)} \\
&= \lim_{n\to\infty} \frac{r(AB)p_n(A,AB) + r(AC)p_n(A,AC)}{r(BA)p_n(B,BA) + r(BC)p_n(B,BC)} \\
&= \lim_{n\to\infty} \frac{r(AB)F(c_n(A,AB)) + r(AC)F(c_n(A,AC))}{r(BA)F(c_n(B,BA)) + r(BC)F(c_n(B,BC))}.
\end{aligned}
$$

Since, by Lemma 3.9, $\lim_{n\to\infty} p_n(t) = 0$ for all $t \in T$, a Taylor expansion of F around 0 yields

$$
\begin{aligned}
&\lim_{n\to\infty} \frac{\xi_n(A)}{\xi_n(B)} \\
&\approx \lim_{n\to\infty} \frac{r(AB)\big(F(0) + f(0)(c_n(A,AB) - 0) + \frac{1}{2}f'(0)\cdot\ldots\big) + r(AC)\big(F(0) + f(0)(c_n(A,AC) - 0) + \ldots\big)}{r(BA)\big(F(0) + f(0)(c_n(B,BA) - 0) + \frac{1}{2}f'(0)\cdot\ldots\big) + r(BC)\big(F(0) + f(0)(c_n(B,BC) - 0) + \ldots\big)} \\
&\approx \lim_{n\to\infty} \frac{f(0)}{f(0)} \frac{r(AB)c_n(A,AB) + r(AC)c_n(A,AC)}{r(BA)c_n(B,BA) + r(BC)c_n(B,BC)}
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\lim_{n\to\infty} \frac{\xi_n(A)}{\xi_n(B)} &\approx \lim_{n\to\infty} \frac{r(AB)c_n(A,AB) + r(AC)c_n(A,AC)}{r(BA)c_n(B,BA) + r(BC)c_n(B,BC)} \\
&= \lim_{n\to\infty} \frac{\Pr_n(piv_{AC})}{\Pr_n(piv_{BC})} \cdot \frac{r(AB)\frac{c_n(A,AB)}{\Pr_n(piv_{AC})} + r(AC)\frac{c_n(A,AC)}{\Pr_n(piv_{AC})}}{r(BA)\frac{c_n(B,BA)}{\Pr_n(piv_{BC})} + r(BC)\frac{c_n(B,BC)}{\Pr_n(piv_{BC})}} \\
&= \lim_{n\to\infty} \frac{\Pr_n(piv_{AC})}{\Pr_n(piv_{BC})} \cdot \lim_{n\to\infty} \frac{r(AB)\frac{c_n(A,AB)}{\Pr_n(piv_{AC})} + r(AC)\frac{c_n(A,AC)}{\Pr_n(piv_{AC})}}{r(BA)\frac{c_n(B,BA)}{\Pr_n(piv_{BC})} + r(BC)\frac{c_n(B,BC)}{\Pr_n(piv_{BC})}}.
\end{aligned}
$$

Using now Lemma 3.3 and the fact that $\mu(piv_{AC}|\Phi_n\xi_n) = \mu(piv_{BC}|\Phi_n\xi_n) > \mu(piv_{AB}|\Phi_n\xi_n) = \mu(tie_{ABC}|\Phi_n\xi_n)$,

$$
\begin{aligned}
\frac{\xi(A)}{\xi(B)} &\approx \lim_{n\to\infty} \frac{r(AB) + (1-\nu)r(AC)}{r(BA) + (1-\nu)r(BC)} \cdot \frac{\Pr_n(piv_{AC}) + \Pr_n(piv_{AC}^{-A})}{\Pr_n(piv_{BC}) + \Pr_n(Piv_{BC}^{-B})} \\
&= \lim_{n\to\infty} \frac{r(AB) + (1-\nu)r(AC)}{r(BA) + (1-\nu)r(BC)},
\end{aligned}
$$

where the last step holds because of equation (3.A.1). Thus, $\xi(A) = \xi(B)$ only if $r(AB) + (1-\nu)r(AC) = r(BA) + (1-\nu)r(BC)$.

Equilibria in which types *AC* and *BC* do not vote sincerely

Employing again the Taylor approximation from the previous step,

$$\begin{aligned}
\lim_{n\to\infty} \frac{\xi_n(A)}{\xi_n(B)} &= \lim_{n\to\infty} \frac{\tau_n(A)}{\tau_n(B)} \\
&\approx \lim_{n\to\infty} \frac{r(AB)c_n(A,AB)}{r(BA)c_n(B,BA)} \\
&= \lim_{n\to\infty} \frac{\Pr_n(piv_{AC})}{\Pr_n(piv_{BC})} \frac{r(AB)\,\frac{c_n(A,AB)}{\Pr_n(piv_{AC})}}{r(BA)\,\frac{c_n(B,BA)}{\Pr_n(piv_{BC})}} \\
&= \frac{r(AB)}{r(BA)} \cdot \lim_{n\to\infty} \frac{\Pr_n(piv_{AC}) + \Pr_n(piv_{AC}^{-A})}{\Pr_n(piv_{BC}) + \Pr_n(piv_{BC}^{-B})} \\
&= \frac{r(AB)}{r(BA)},
\end{aligned}$$

given equation (3.A.1).

Thus, $\xi(A) = \xi(B)$ only if $r(AB) = r(BA)$.

$\square$

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