Essays on the Effects of Information on Incentives and on People's Awareness and Assessment of Biases

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Introduction

Strategically interacting individuals often lack knowledge about each others' actions or attributes. This may introduce incentive problems into the interaction. Economists already realized for quite some time that it is important to analyze how incentive problems are dealt with in reality and how one can solve or mitigate incentive problems. Incentive problems are present in many different contexts in real life like insurance contracts, taxation, the provision of public goods, auctions, compensation schemes, and incentive schemes for employees.

Incentive problems induce, for instance, shirking on the part of employees when their actions are not observable or verifiable. Implications are, for example, that contracts can no longer condition on actions if these are not observable (or not verifiable). Instead, the contract may condition on (observable) outcomes (Chapter 2). Another alternative can be to change the compensation scheme to some relative compensation scheme like a tournament (Chapter 1). Relative compensation schemes can in particular be useful if individuals have to fear not to be paid as agreed upon since indicators of performance are not verifiable. The tournament structure then commits the organizer (principal) to pay the prize to one of the contestants. Relative compensation schemes can be useful if the production technologies of individuals are dependent

In the first two chapters of this thesis, we consider relative compensation schemes and incentive schemes when information asymmetries are present. In the first chapter, individuals have private information about their abilities. In the second chapter, the actions of individuals cannot be observed (moral hazard). In both chapters, we analyze whether and how intermediate information changes incentives. Here, intermediate information arises endogenously in the game when we change the timing of the game such that individuals act sequentially instead of simultaneously. If they act sequentially, the second mover can observe the action (Chapter 1) or the outcome (Chapter 2) of the first mover.

In Chapter 1, we consider contests, in which two players compete for an exogenously given prize by exerting efforts. The player, who exerts an higher effort, wins the contest with a higher probability. We do not derive the optimal prize scheme here, but focus on incentive effects caused by intermediate information, when contestants act sequentially instead of simultaneously. We compare contests, in which players either move simultaneously or sequentially, under different information settings: Contestants' types are either publicly known or private information. Players are ex ante symmetric, but realized types may be heterogeneous. The joint distribution of types allows for correlation of the players' types.

The combination of private information and sequential moves (as defined above) in contests as well as the comparisons of sequential contests under private and complete information are novel to the literature.

We find that the expected effort sum in sequential contests is higher than in simultaneous contests irrespective of the information setting. Hence, incentives increase (from an ex ante perspective), when intermediate information is available, not only under complete information but also under private information. It is known that sequential contests Pareto dominate simultaneous contests under complete information. We can extend this result to private information, when contestants' types are sufficiently negatively correlated. Then, also the second mover prefers sequential contests. This is due to an *efficiency gain effect* together with an *ability effect*. The latter effect implies higher bids by "better" contestants, who value the prize more or are of higher ability. The more negatively types are correlated, the higher is the probability that types differ. Given the same probability for high and low types, it thus becomes more likely that the player, who values the prize more, wins the contest (*ability effect*). In addition, the *efficiency gain effect* increases expected payoffs in this situation. When this effect becomes sufficiently strong, the second mover's expected payoff increases enough so that he prefers sequential contests. The first mover, however, still prefers sequential contests (ex ante) irrespective of the information setting.

It has already been shown that for simultaneous contests the information setting does not matter from an ex ante perspective: The ex ante expected effort sum as well as ex ante expected payoffs for the contestants do not change. Comparing the two information settings given sequential contests, we can show, however, that the ex ante expected effort sum is higher under private information, whereas contestants' expected payoffs are higher under complete information. Hence, incentives under complete and private information change when intermediate information is available. Incentives rise (from an ex ante perspective), when there is private information. This result is novel to the literature on contests and is due to a *commitment effect* for the first mover that offsets the usual *competition intensity effect*. The latter effect implies that contestants exert more effort, the more similar their types are.

In Chapter 2, we consider a model of team production under moral hazard. Again, we analyze incentive effects of intermediate information. We investigate whether the principal prefers his agents to work simultaneously or sequentially. In case agents act sequentially the second mover can observe the quality of the first mover's contribution to the joint (team) project. This means that the second mover receives intermediate information, on which he can condition his action. Irrespective of the timing structure, the principal only observes the value of the joint project in the end.

In contrast to the first chapter, agents no longer compete against each other, but jointly work on a project. We derive the optimal wage scheme for both structures of the game, in which wages condition on the value of the joint project. The optimal structure for the principal depends on whether the agents' contributions are complements or substitutes in his production function: A sequential structure is optimal when the agents' contributions are perfect complements, whereas a simultaneous structure is optimal when they are perfect substitutes.

So far, the literature on teams only considered effort complementarities and two possible values of the joint project: The project either fails or succeeds. We introduce output complementarities and an intermediate value of the project. In contrast to findings in the literature on teams, we find that intermediate information does not necessarily increase incentives and is thus not necessarily favorable to the principal.

While we deal with an exogenously fixed wage scheme in the first chapter, we derive optimal wage schemes in the second chapter. We find that results in the second chapter hinge on a change in the feasible wage scheme between both timing structures and as well between contributions being either substitutes or complements. In the first chapter, however, results depend on four "strategic effects" that we identify.

Both Chapters 1 and 2 aim at studying the interaction between intermediate information and incentives. We believe that this is an important issue in many economic situations, and we hope to provide some new insights into the interplay of intermediate information and incentives when information asymmetries are present.

In the third chapter of this thesis, we deal with a different topic. Nevertheless, the topic is related to the first two chapters as we explain below. In the third chapter, we experimentally investigate what people know about the bias of other people. We focus on people's self-assessment of their abilities. We consider a situation, in which people do not know the ability (or type) of others and in addition, they do not even know their own ability (type) for sure. We can say that people have private information on their *belief* about their own type. If people interact with each other in such a situation, people have to form beliefs about other people's self-assessment. We do not consider the interaction itself, but we elicit beliefs people hold about the own ability. This issue is, for example, considered in experiments on overconfidence.[1] Moreover, we go one step

[1]For an overview on overconfidence see Englmaier (forthcoming).

further, which is novel to the literature in this context: We elicit beliefs people have about the beliefs of others.

If we think of the issues considered in the first two chapters, this new issue seems to be an important extension. If we do not know the attributes or actions of people with whom we interact, we form beliefs about it. People's actions usually depend on their type (or attributes). If people do not perfectly know their type, actions will depend on their perceived type. Hence, if we interact with them, we have to form beliefs about their perceived type to be able to infer their behavior and act optimally. We do, however, not consider how optimal behavior or contracts change when such higher order beliefs are important, but analyze whether people are aware that other people's self-assessment might be biased and what they think about the others' true ability. Thus, we investigate how accurate beliefs about other people's attributes and about their beliefs are.

More precisely, we consider people's self-assessment about their number of correct answers when answering a set of multiple choice questions. While different types of individuals exist (they either underestimate, correctly estimate or overestimate their ability), our results confirm that people tend to overestimate their ability, i.e. the population – on average – is biased. We then test whether individuals are aware of other people's bias and what they think about the relation between their own and other people's bias. We find that most individuals do not think that other people have a bias. The more information subjects receive about the task, which the other group handles, the more subjects realize that others are on average biased. Moreover, people tend to think that their own self-assessment is better (they make less likely a mistake) than the self-assessment of others. They believe this, even if "the others" are a group of people such that (random) mistakes cancel out. We observe, however, that subjects partly revise this judgement if they receive more information about the task or are incited to reason about mistakes or biases in people's self-assessment.

Hence, in the third chapter, we again observe effects of information: Information helps subjects to make better judgements.

Chapter 1

Sequential versus Simultaneous Contests

1.1 Introduction

Contests are situations in which agents spend resources (they make a bid or exert an effort) in order to win a prize. This expenditure influences an agent's probability of winning the prize. The bid is irreversible and contestants bear the costs of their action independent of whether they win a prize or not. A contestant's reward in this competitive scheme depends on his relative performance. The contestant with the highest performance, however, does not necessarily win the contest. Only in a perfectly discriminating contest the contestant with the highest performance wins for sure. A perfectly discriminating contest is known as an all-pay *auction*.

In this paper, we consider imperfectly discriminating contests, which means that the winner of the contest is determined probabilistically and needs not be the one with the highest performance.

These contest models capture essential features of rent-seeking competition, patent races, job promotion or sports contests. They are also used to model incentive schemes in organizations.[1]

Tullock (1980) proposed the traditional contest-framework: He considered imperfectly discriminating contests with symmetric contestants who move simultaneously and in which no agent is able to commit to an expenditure level. This standard framework has been extended in many ways. For an overview on rent-seeking competition see, for example, Nitzan (1994) or Nti (1999). For a review of the literature on sports contests see Szymanski (2002). Even if there are various extensions of the standard framework, the main focus of the literature is on simultaneous (i.e. Cournot-type) contests and on symmetric contestants or at least the agents' types are publicly known. In the following, we refer to the case in which the realizations of types are publicly known

[1] For tournaments as incentive schemes in organizations see, e.g., Lazear and Rosen (1981) or – for a review of the literature – see Prendergast (1999).

as the "public information" setting. This means that not only the contestants but also the designer of the contests know the types.

Compared to contests in reality, the assumptions of simultaneity and symmetry seem very strict. A first observation is that in most sports contests (e.g. skiing championships), contestants move one after the other such that contestants who move later can observe the action(s) of their predecessor(s) before it is their own turn. In other contests – for instance, the Tour de France or triathlon competitions – each contestant can observe his opponent(s) during the ongoing contest and react to his (their) action(s). Purely simultaneous contests are rare in sports. In weight lifting or high jump championships, for example, contestants act "in turns". Here, one can observe that actions are often adjusted to the action of the predecessor or that athletes try to preempt their competitors. Other examples for sequential structures include rent-seeking payments in case institutions announce the contributions publicly during the ongoing process. Similarly, in tendering procedures, it is often the case that one company has some "priority"[2]: Knowing the offers of its competitors, this company is asked to make a (final) offer. Also in court proceedings it is common that plaintiff and defense act sequentially: In general, the plaintiff first submits evidence and then the defense reacts.

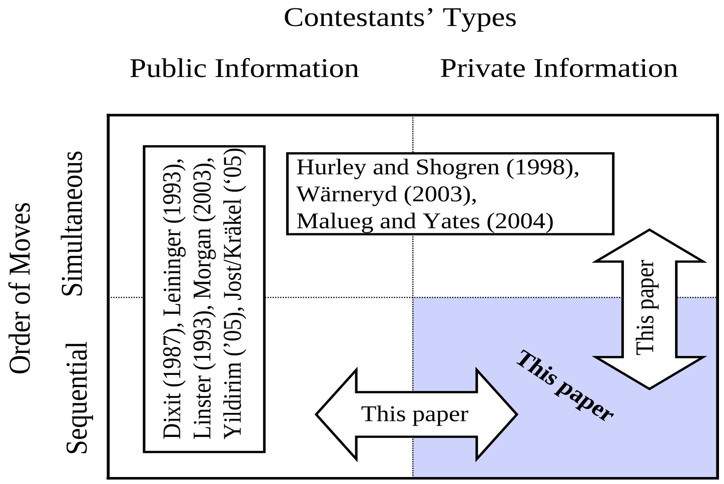
The question we are interested in, is whether the sequential order of moves in contests arises just because of outside restrictions (e.g. a ski jump can only be used by one athlete at a time) or whether agents can be better incentivized in this setting compared to simultaneous contests.

A second important observation is that in real life contestants not only tend to be asymmetric, but – more importantly – they do not necessarily know their rivals' abilities or valuations of the prize. Like in auctions, it is realistic to assume that valuations are the agents' private information. Regarding an agent's ability – which can be interpreted as the effectiveness of an agent's effort or lower effort costs – there are many examples showing that the relevant (actual) ability may not be publicly known: Even in professional sports, where abilities of athletes are *in principle* publicly known, athletes only have some vague belief about their opponents' current form as temporary fluctuations are common. One often observes that a designated favorite (e.g. because of his pre-championship performance) in a championship does not win because he performs worse than expected by others. Of course, there are also random factors (changing weather conditions during a contest, pressure of being the favorite, physical injuries etc.), which influence an athlete's performance. The actual ability, however, is not random in the sense that it may well be known to the athlete himself but not publicly.

Furthermore, the agents' actual abilities may be correlated: In many disciplines athletes prefer

[2]In general, such prior claims may be due, for example, to close collaboration with one company.

Figure 1.1: The Four Institutional Settings



some venues, techniques or equipment, which is more or less helpful under certain conditions (e.g. weather conditions). Hence, given these conditions, one contestant's ability might be positively or negatively correlated with his opponent's ability.

In this paper, we take into account these two aspects, private information (where we allow for correlation of the agents' types) *and* a sequential order of moves. The combination of both aspects is novel to the literature on contests in this form.
The sequential setting that we consider is a Stackelberg game: The second mover can perfectly observe the first mover's action and thus reacts to this action.[3] When, in addition, contestants' types are private knowledge (and correlated), the second mover cannot only react to the first mover's action, but also update his prior about the first mover's type. Besides the combination of these two aspects itself and the analysis of this institutional setting, we compare the four possible institutional settings by two approaches: We compare the two timing structures, i.e. the order of moves is sequential or simultaneous, from an ex ante perspective given the information setting, i.e. agents' types are either public or private information. Then, given the timing structure, we compare the two information settings from an ex ante perspective. The four institutional settings and the aforementioned comparisons (indicating related literature) are illustrated in Figure 1.1.

Our main results are that the risk neutral designer of the contest – when he wants to max-

[3]Fudenberg et al. (1983) deal with the case of imperfect observability of the rival's action in an R&D race where the favorite makes the first move.

imize the expected effort sum – and the first mover prefer the sequential setting irrespective of the information setting. For the second mover, this only holds true for the case of public information. Whether he prefers the sequential or simultaneous contest under private information depends on the distribution of types. When agents' types are sufficiently negatively correlated, the second mover prefers the sequential structure, otherwise he prefers the simultaneous one. The intuition for this result is an *efficiency gain effect* together with an *ability effect*: The more negatively contestants' types are correlated, the higher is the probability that they are heterogeneous. Moreover, the agent with the higher *ability* bids more than his rival (*ability effect*). In our model the agent with the higher ability is the one who values the prize more.[4] Hence, the probability that the agent with the higher valuation wins the prize increases. This *efficiency gain effect* increases the surplus of both agents in the sequential contest. If the effect is sufficiently strong, expected payoffs of the second mover become larger under the sequential contest than under the simultaneous contest. As these results suggest, we find that in the sequential contest with private information a first-mover advantage exists.[5]

When comparing the information settings for simultaneous contests, an effort maximizing designer as well as the contestants are indifferent between both informational settings. This result has been shown by Malueg and Yates (2004). The result for sequential contests is new and different: An effort maximizing designer prefers the private information setting, whereas contestants prefer public information from an ex ante perspective. This result is driven by a *commitment effect* for the first mover that is only present if contestants act sequentially. The high type of the first mover commits to a higher effort level when the second mover is a low type than if the second mover is a high type. This *commitment effect* thus offsets the usual *competition intensity effect* for the high type of the first mover. The latter effect implies that the more similar agents are, the higher is their effort as competition becomes more intense. Moreover, the *commitment effect* drives up expected efforts and thus lowers the agents' expected payoffs and enhances the designer's payoff.

In case the designer only cares about a close race and not the effort sum, we find that he prefers simultaneous to sequential contests irrespective of the information setting. Given a sequential order of moves, he prefers private information to public information and vice versa if the order of moves is simultaneous.

The structure of the paper is as follows. Before we compare the different settings, we deal with all four settings separately. We begin with reviewing the two public information settings

[4]We show later that we can equivalently model this with agents who differ in their effort costs.

[5]Interestingly, in an experimental analysis of sequential tournaments by Weimann et al. (2000) with symmetric agents – where the first mover advantage is even more pronounced – a second mover advantage is observed.

where we make in particular use of results by Morgan (2003) and Leininger (1993). Next, we consider private information. First, we summarize results on simultaneous contests with private information where we rely on results by Malueg and Yates (2004). Afterwards, we introduce sequential contests with private information. Using these results, we compare the institutional settings from an ex ante perspective as mentioned before. The comparison for the case of sequential contests, however, is novel to the literature. We also extend the previous literature by analysing the effects of the timing assumption given private information on equilibrium actions and on which structure the designer – who maximizes the ex ante expected effort sum – and the contestants, respectively, prefer. Finally, we consider the case that the principal wants to have a "close race" and does not maximize the expected effort sum.

Dixit (1987) was the first who analyzed the impact on incentives, when contestants can precommit to their effort levels. He shows that the favorite (i.e. the player whose winning probability in equilibrium is larger than one half) – when given the chance to move first – overcommits to effort compared to the case without commitment, whereas the underdog (the one with the lower probability of winning) undercommits to effort. While in Dixit the order of moves is exogenously given, Baik and Shogren (1992) determine the order of moves endogenously. They show that the underdog chooses to move first and the favorite to move second. Moreover, they find that both players' equilibrium efforts are lower in the sequential setting than in the simultaneous move game. An extension to n identical players who move sequentially is numerically investigated by Glazer and Hassin (2000). They find that the first mover makes higher profits than later movers – independent of whether these move sequentially or simultaneously. The profit of the first mover, however, needs not be higher than in a simultaneous n player contest. Aggregate efforts of the contestants are at least as high as in a simultaneous n player contest, and hence their aggregate payoffs are smaller than in the simultaneous contest (or at best equally high).
Most closely related to our paper regarding the timing structure are the studies by Yildirim (2005), Morgan (2003), Linster (1993) and Leininger (1993). In contrast to our paper, all four studies restrict to public information.[6] Linster (1993) as well as Leininger (1993) contrast Cournot and Stackelberg contests with two players, who can be either homogeneous or heterogeneous, which is publicly known. In addition, Leininger endogenizes the order of moves by considering a game in which two players can choose whether to move first or second knowing their own and the rival's valuation. He finds that if players are asymmetric, then equilibrium play will be in a particular order. When players are symmetric, both a simultaneous and a sequential order of moves form

[6]Linster (1993) briefly addresses the case of one-sided asymmetric information but does not analyze implications or compare different settings at all.

an equilibrium. Morgan (2003) extends this analysis by allowing for ex ante uncertainty about the players' types. His main result is that the designer of the contest – who maximizes the effort sum – and the contestants (ex ante) prefer a sequential order of moves under public information. Morgan also endogenizes the order of moves: He shows that sequential play arises in equilibrium when the timing decision is endogenously determined prior to the realization of valuations (when agents decide on effort, however, types are publicly known). For the comparison under public information, we rely on results by Morgan. We, however, extend the analysis of Morgan by allowing for two-sided asymmetric information during the contest. We find that the result that the principal and the first mover prefer the sequential order of moves is still true for the case of private information. For the second mover it depends on the distribution of types, whether he prefers the sequential or simultaneous setting. As aforementioned, he only receives a higher ex ante expected payoff if the correlation between types is sufficiently negative.

Yildirim (2005) considers contests with multiple rounds and public information about the players' types. Both players choose an effort in each round. His main result stands in contrast to Baik and Shogren (1992) and Leininger (1993). They examine a game, in which agents simultaneously commit to the period in which they want to exert an effort. Given the resulting order of moves, either a Cournot-Nash or one of two possible Stackelberg contests is played. Baik and Shogren as well as Leininger find that the unique equilibrium implies that first the underdog (i.e. the player who wins the contest with a lower probability) and then the favorite moves. In Yildirim (2005), equilibrium outcomes for heterogeneous contestants contain a notion of "leadership" of the favorite in the sense that his effort lies on the other player's reaction function. Hence, in settings where contestants exert effort in multiple rounds, it cannot happen that the underdog moves first in equilibrium. Furthermore, Yildirim finds that the total equilibrium effort is – in general – weakly higher than in equilibrium of the simultaneous game. The latter finding is perfectly in line with the results by Morgan (2004) for public information and our results for private information.

Jost (2001) and Jost and Kräkel (2005) consider sequential tournaments with symmetric and also with asymmetric agents and public information.[7] Jost (2001) studies risk neutral as well as risk averse agents. With risk neutral contestants, equilibrium outcomes do not change when intermediate information is available or not. When agents are risk averse, however, the designer rather prefers a sequential order of moves. This means that risk aversion – similar to private information in our model – makes the setting with intermediate information more favorable compared to the setting without from the point of view of the designer. Another study that analyzes the optimality of intermediate information within a tournament model and symmetric

[7] We stick here to the usual terminology that uses "tournaments" to refer to models that use a probit form contest success function, in contrast to "contests" with a logit form contest success function.

agents is Aoyagi (2003). Here, both agents act in each period. Aoyagi shows that the optimality of providing intermediate information depends on the shape of the marginal cost function for effort in the second period. If the marginal cost function is concave (convex), then it is optimal for the principal to choose a setting where (no) intermediate information is provided.

Romano and Yildirim (2005) derive equilibrium outcomes for a very general class of games – so-called games of accumulation – under public information about the players types. Each player has a fixed amount of a strategic variable. After observing first period choices, a player can adjust his strategic variable in a second period, but he can only add on the initial amount. These games are related to this paper since contests are one possible application and accumulation games have both sequential and simultaneous features.

All the papers mentioned so far consider either symmetric players or assume complete information about each contestant's type. The only exception, we are aware of, is Münster (2004). He examines signaling issues in repeated contest, in which agents act simultaneously in both rounds. Before the second round starts, first round effort choices are revealed. This means that in contrast to our setting, *both* agents receive intermediate information and both can react in the second period. Münster finds that high ability contestants might put in little effort in the first round to make their opponents believe that they are of low ability. He compares expected efforts in the repeated game to twice the expected effort in the one-shot game with asymmetric information. This corresponds to a comparison of settings with and without intermediate information. In contrast to our result, intermediate information leads to lower expected equilibrium efforts.

Other papers, which compare simultaneous and sequential contests, are Moldovanu and Sela (2006) (who study an all-pay auction), Gradstein (1999) and Rosen (1986). The difference of these papers to ours is that they focus on a different notion of 'sequentiality': In these papers an elimination tournament is played where the losers of each round are eliminated. Our notion of sequentiality, in contrast, is 'moving one after the other'.[8]

Sequential games have also been studied in the industrial organization literature, examining the behavior of oligopolists. See, for example, Saloner (1987) who solves a Cournot duopoly with two production periods and a homogeneous product. Pal (1991) extends Saloner's work by allowing for cost changes over time.

Regarding asymmetric information, there is only few related work. Hurley and Shogren (1998a) consider asymmetric information in contests. They study a simultaneous two-player contest with one-sided asymmetric information. More precisely, one contestant is uninformed about his rival's valuation, whereas the other one has complete information. Results for total effort are mostly ambiguous. If contestants' efforts are strategic complements, unambiguous results are possible in

[8]Also related are the studies on racing by Harris and Vickers (1985) and (1987) that offer dynamic features.

case the uninformed players' beliefs change (in the sense that the variance increases or decreases). If contestants' efforts are strategic substitutes, unambiguous results are possible in case relative abilities change (and induce a change in the uninformed players' beliefs). The case of one- as well as two-sided asymmetric information in simultaneous contests when valuations are independently drawn is, in general, analytically intractable, but Hurley and Shogren (1998b) approach this case numerically. Similar to the one-sided asymmetric information case, general conclusions are mostly ambiguous. The driving forces of behavior are again perceptions of relative abilities and risk. These forces can be related to the *competition intensity effect* that we identify as driving effect. Most closely related to our analysis is the study by Malueg and Yates (2004). They derive a Bayesian equilibrium in a simultaneous contest with private information solving the aforementioned problem by the assumption of a distribution that allows for correlation of the valuations. We adopt this distributional assumption for our analysis of sequential and simultaneous contests in order to be able to derive explicit solutions. Malueg and Yates main result is that the ex ante expected effort sum under private and public information is the same. We review their results in detail when analyzing the simultaneous contest with private information.

As aforementioned, Münster (2004) studies repeated contests (two rounds) with asymmetric information about the agents' types. Agents simultaneously choose an effort each round and both receive intermediate information before they enter the second round. In a separating equilibrium, second round efforts are identical to efforts in the one-shot game under complete information, since there is complete information in this case. He does, however, not compare the expected effort sum to the complete information case for any kind of equilibrium.

Closely related is also the paper by Wärneryd (2003). He deals asymmetric information in the sense that one player knows the common value of the prize whereas the other player is uncertain about it (he knows the prior distribution). In contrast to the current paper, he only considers simultaneous contests. For contests with a Tullock contest success function, the effort sum in equilibrium is lower when one agent is informed and the other one uninformed than when either both are informed or both uninformed. Like in our simultaneous contest, the expected effort sum is identical under public and private information (if both agents are uninformed).

The remainder of this paper is organized as follows: In Section 1.2, we present the basic model. In Section 1.3, we review results concerning the public information settings. In Section 1.4, we turn to private information first considering simultaneous contests. Then, we introduce the new setting of sequential contests under private information and derive results for this setting. In Sections 1.5 and 1.6, we compare the four institutional settings. In Section 1.7, we briefly discuss other aims of the designer than maximizing the expected effort sum and finally, we conclude in Section 1.8.

Table 1.1: Probability distribution of valuations

		$\mathcal{V}_1$: V_L	$\mathcal{V}_1$: V_H
$\mathcal{V}_2$	V_L	$\frac{1}{2}r$	$\frac{1}{2}(1-r)$
	V_H	$\frac{1}{2}(1-r)$	$\frac{1}{2}r$

1.2 Setting

We consider two risk neutral agents $i = 1, 2$ who are competing for one prize by making irreversible, non-negative bids x_i. These bids can be viewed as effort levels or amounts of money. Henceforth, we interchangeably use the terms bids or effort. We consider two different settings: Contestants either make their bids simultaneously or sequentially.[9] In the latter case, the second mover can observe the bid of the first mover. Agent i's valuation of the prize is $\mathcal{V}_i$. We model $\mathcal{V}_1$ and $\mathcal{V}_2$ as random variables, which take on either a low value, V_L, or a high value, V_H, where $0 < V_L \leq V_H$. We refer to the agents' valuations as the agents' *types*. The prior probability distribution of valuations $(\mathcal{V}_1, \mathcal{V}_2)$ is common knowledge and is given in Table 1.1. As this distribution is symmetric, agents are identical from an ex ante perspective. The distribution, however, allows for heterogeneity in valuations and correlation between the valuations.[10] We consider two different settings in the following: Either agents' types are public information or they are private information. The parameter r (see Table 1.1) is monotonically related to the correlation coefficient ρ of the valuations as $\rho = 2(r - \frac{1}{2})$. Therefore, we can use both r and ρ as a measure of correlation of the valuations: When valuations are perfectly negative correlated (i.e. $\rho = -1$) $r = 0$, and when valuations are perfectly positive correlated (i.e. $\rho = 1$) we have $r = 1$. $r = \frac{1}{2}$ corresponds to independence (i.e. $\rho = 0$) of the valuations. When addressing the degree of correlation of the valuations, we refer to ρ in the following if not stated otherwise.

[9] When agents act sequentially, we abstract from discounting.

[10] This distribution allows us to calculate equilibrium bids explicitly. It does not allow, however, for more general cases of independently drawn valuations since simultaneous contests with private valuations become – in general – analytically intractable in the sense that we cannot derive explicit solutions. This is attributable to the fact that equilibrium efforts of each type of agent depend on the best reply of each type of his opponent, and moreover, they (indirectly) depend on the best reply of the other type of the first agent (as the opponent's best reply depends on it). A numerical analysis of simultaneous contests with two-sided incomplete information that also allows for more general cases of independence is conducted by Hurley and Shogren (1998b).

The probability that agent i wins the prize is given by Tullock's logit form contest success function, where x denotes an agent's bid (or effort outlay)[11]

$$\pi_i(x_1, x_2) = \begin{cases} \frac{x_i}{x_1+x_2} & \text{if } (x_1, x_2) \neq (0,0) \\ \\ \frac{1}{2} & \text{otherwise.} \end{cases} \tag{1.1}$$

The contest is imperfectly discriminating. This means the winner is determined probabilistically: A higher effort compared to the opponent makes winning more likely but does not guarantee success. An axiomatic foundation for the contest success function having this specific form is given by Skaperdas (1996) for symmetric contests and by Clark and Riis (1998) for asymmetric contests. If an agent does not win the prize, he earns zero but has to pay his bid. The expected payoff of agent i given both agents' bids and agent i's valuation $\mathcal{V}_i$ is then

$$\Psi_i(x_1, x_2, \mathcal{V}_i) = \frac{x_i}{x_1 + x_2}\mathcal{V}_i - x_i. \tag{1.2}$$

Note that the contest success function given in (1.1) is discontinuous at $(x_1, x_2) = (0, 0)$. This implies that the best reply of an agent to zero effort of the other agent is not well-defined (as the first agent can win for sure by exerting any strictly positive effort $\epsilon > 0$ and – by continuity – his payoff is larger: $V_i - \epsilon > \frac{1}{2}V_i - 0$). We solve this problem by assuming that there is some smallest unit of effort (e.g. a smallest unit of money if we think of monetary investments). This means that a contestant has to exert at least some strictly positive amount of effort, $\epsilon > 0$ – if he wants to exert a positive amount of effort – where ϵ can be arbitrarily small.[12]

The timing of the game is as follows. First, the risk neutral designer of the contest fixes whether agents act simultaneously or sequentially. Then, the agents' types are drawn according to the distribution described above and each agent learns his own type. In the public information setting, agents learn the opponent's type, too. Next, given a simultaneous structure, contestants simultaneously choose their bid. In a sequential setting, first, agent 1 chooses his bid. Agent 2 can observe the first mover's bid before he decides on his own bid. Finally, payoffs realize.

Instead of modelling heterogeneous types by different valuations of the prize, we can also model different abilities of the agents in terms of different effort costs. It is possible to do this

[11] For simplicity, we consider a discriminatory power equal to one. Regarding simultaneous contests, one could easily generalize the analysis. Malueg and Yates (2004) consider more general levels of discriminatory power in simultaneous contests with private information.

[12] An alternative approach to solve the discontinuity problem are endogenous tie-breaking (or sharing) rules in the sense as proposed by Simon and Zame (1990) – for the case of complete information – and by Jackson et al. (2002) for the case of incomplete information.

in a way such that both notions are strategically equivalent: The reason is that preferences are invariant with respect to affine transformations of the expected utility (Expected Utility Theorem). Instead of using the expected payoff function as given in (1.2), we could consider the following function

$$\widetilde{\Psi_i}(x_1, x_2, \tau_i) = \frac{x_i}{x_1 + x_2}\mathcal{V} - \tau_i x_i,$$

where heterogeneity is captured by different effort costs, with $\tau_i \in \mathbb{R}^+$. Let $\mathcal{V}_i = \frac{\mathcal{V}}{\tau_i}$. Then

$$\Psi_i(x_1, x_2, \mathcal{V}_i) = \frac{1}{\tau_i}\widetilde{\Psi_i}(x_1, x_2, \tau_i). \tag{1.3}$$

Hence, whether we model heterogeneity by different valuations or by different effort costs does not matter for equilibrium outcomes, as long as we assume $\mathcal{V}_i = \frac{\mathcal{V}}{\tau_i}$. We only scale expected payoffs by $\frac{1}{\tau_i}$. For ease of presentation we restrict to the case of different valuations as described above but call it also different *abilities*.

In the following, we analyze the four institutional settings, beginning with the two public information settings.

1.3 Contests with Public Information

In this section, we review the standard contest where agents' valuations are common knowledge and agents move simultaneously. Equilibrium bids of this *Cournot-Nash* contest are well-known and given as follows.

Proposition 1 *[Morgan (2003)] In a two-player contest with public information about the contestants' types* $(\mathcal{V}_1, \mathcal{V}_2)$ *and simultaneous moves the unique Nash-equilibrium in pure strategies is*

$$x_1^* = \mathcal{V}_1 \frac{\mathcal{V}_1\mathcal{V}_2}{(\mathcal{V}_1 + \mathcal{V}_2)^2} =: \omega\mathcal{V}_1 \quad \textit{and} \quad x_2^* = \mathcal{V}_2 \frac{\mathcal{V}_1\mathcal{V}_2}{(\mathcal{V}_1 + \mathcal{V}_2)^2} =: \omega\mathcal{V}_2. \tag{1.4}$$

Obviously, both agents bid the same fraction $0 < \omega = \frac{\mathcal{V}_2\mathcal{V}_1}{(\mathcal{V}_1+\mathcal{V}_2)^2} < 1$ of their own valuation. Hence, equilibrium bids are proportional in that $\frac{x_1^*}{\mathcal{V}_1} = \frac{x_2^*}{\mathcal{V}_2}$; meaning that identical agents make the same bid in equilibrium and the agent with the higher valuation spends more than the one with the lower valuation. Moreover, an agent's bid is rising in his own valuation (as $\frac{\partial x_i^*}{\partial \mathcal{V}_i} = \frac{2\mathcal{V}_i\mathcal{V}_j^2}{(\mathcal{V}_i+\mathcal{V}_j)^2} > 0$ where $i, j = 1, 2$ and $i \neq j$). We refer to the well-known effect that an agent bids more when he is a high type than when he is a low type as *ability effect*. In the simultaneous contest with public information, the only reason why one agent bids more than the other one is the *ability effect*: the one who has the higher valuation bids more. Equilibrium bids are strictly increasing in the opponent's valuation if the opponent has a 'lower' type, otherwise bids decrease in the rival's valuation (strictly if the other's type is strictly 'higher') as $\frac{\partial x_i^*}{\partial \mathcal{V}_j} > (\leq)\ 0$ is equivalent to $\mathcal{V}_i > (\leq)\ \mathcal{V}_j$ where $i, j = 1, 2$ and $i \neq j$. This means, equilibrium effort increases in the opponent's valuation when the contest evens out (valuations become closer) and thus, there is more competition. We call this *competition intensity effect* in the following. This also explains that in a contest with homogeneous contestants each contestant bids more than when the contest is heterogeneous.

In the second setting, the Stackelberg-contest, agents' types are common knowledge, but contestants make their bids sequentially. Let agent 1 denote the first mover and agent 2 the second mover. The second mover can perfectly observe the bid of the first mover before he decides on his bid. The following subgame perfect equilibrium bids are derived, for example, in Leininger (2003) and also in Romano and Yildirim (2005).

Proposition 2 *In the two-player contest with public information about the contestants' types* $(\mathcal{V}_1, \mathcal{V}_2)$ *and sequential order of moves – agent 1 is the first mover and agent 2 the second mover – the unique pure strategy subgame perfect equilibrium outcome is*

$$x_1^* = \frac{\mathcal{V}_1}{2}\left(\frac{\mathcal{V}_1}{2\mathcal{V}_2}\right) \text{ and } x_2^* = \frac{\mathcal{V}_1}{2}\left(1 - \frac{\mathcal{V}_1}{2\mathcal{V}_2}\right) \text{ if } \mathcal{V}_1 \leq 2\mathcal{V}_2, \tag{1.5}$$

$$x_1^* = \mathcal{V}_2 \quad \text{and } x_2^* = 0 \quad \text{otherwise.} \tag{1.6}$$

In case valuations are rather "close" (i.e. $\mathcal{V}_1 \leq 2\mathcal{V}_2$) all types of players spend a positive amount of effort, i.e. we have an interior solution, whereas in case valuations are not "close" (i.e. $\mathcal{V}_1 > 2\mathcal{V}_2$) and the first mover has the higher valuation, he can preempt the second mover such that the latter spends zero, i.e. we have a boundary solution.

It can easily be seen that the more the first mover bids, the less bids the second. The follower only responds with a positive bid if the leader bids less than the follower's valuation. The maximum bid of the leader is $\mathcal{V}_2$ (even if his own valuation is higher): by bidding $\mathcal{V}_2$, the leader already wins with certainty.[13] Because of this strategic behavior, contestants no longer bid the same fraction of their valuation as we have seen in the Cournot-Nash contest.

Comparing the agents' bids, we see that as in the simultaneous contest, symmetric agents exert the same effort in equilibrium and this effort is, moreover, the same as in the simultaneous contest.[14] Given an equally strong opponent, no contestant is able to make a strategic gain by moving first, which implies that Cournot-Nash and Stackelberg contests lead to identical outcomes in such a case.[15] This can be intuitively explained by considering the Nash-equilibrium of the simultaneous contest: Departing from the Nash-equilibrium, a small change in an agent's bid does not induce

[13]Since the first mover bids $\mathcal{V}_2$ in the boundary solution, this implies that his maximum bid cannot be higher than V_L.

[14]As agents are symmetric, the case $\mathcal{V}_1 \leq 2\mathcal{V}_2$ is the relevant one. Hence, we only have to analyze the *interior* solution.

[15]See also Romano and Yildirim (2005) and Morgan (2003).

the other agent to change his bid. Therefore, although the leader can commit to a bid in the sequential contest, this does not change the follower's behavior.

For asymmetric contestants – like in the simultaneous contest – the first mover exerts more (less) effort than the second mover when he has the higher (lower) valuation as this is equivalent to $\frac{\mathcal{V}_1}{\mathcal{V}_2} \geq (\leq)\ 1$.[16] This means that the first mover does not preempt the follower when the latter has the higher valuation as this becomes too costly for him given the follower's high incentives to win the prize. Hence, although agents are no longer completely homogeneous in the sequential contests as they differ in the their role of being the first or second mover, we see that it is still the *ability effect* that determines which agent bids more (like in the simultaneous contest): equilibrium efforts rise in an agent's valuation such that the agent with the higher valuation bids more aggressively than his rival (independent of the agent's role).[17]

Moreover, we can verify that an agent receives a higher expected payoff than his opponent if and only if he has the higher valuation and, therefore, bids more aggressively: Comparing the first mover's expected payoff – which is given by $\frac{\mathcal{V}_1^2}{4\mathcal{V}_2}$ – with the follower's expected payoff – which is given by $\mathcal{V}_2 - \mathcal{V}_1 + \frac{\mathcal{V}_1^2}{4\mathcal{V}_2}$ – immediately gives the result. Like Morgan (2003) finds for independently drawn valuations, it also holds true for our distributional assumption that from an ex ante perspective, expected payoffs of the first and second mover are identical for $\mathcal{V}_1 \leq 2\mathcal{V}_2$ (otherwise the expected payoff of the first mover is higher). Committing to a publicly observable bid does not lead to a first mover advantage for the interior solution (from an ex ante perspective). This observation can easily be verified: Symmetric agents make the same expected payoff as they make equal bids. Given asymmetric contestants, expected payoffs of the first mover are given by $\frac{\mathcal{V}_1}{4\mathcal{V}_2}$ and for the follower by $\mathcal{V}_2 - \mathcal{V}_1 + \frac{\mathcal{V}_1}{4\mathcal{V}_2}$ where $\mathcal{V}_1 \neq \mathcal{V}_2$. Obviously, it is – in general – not the case that the high (low) type receives the same expected payoff independent of whether he moves first or second. The second mover, however, looses exactly as much compared to the first mover when he is the low type as he wins when he is the high type. Since each contestant is a high or a low type with equal probabilities, their ex ante expected payoffs are identical.

The crucial effect of the sequential order of moves is that it offsets the *competition intensity effect* for the high type of the first mover: When the first mover is a high type, he invests more when the follower is a low type than when he is a high type as well. This means that the first mover profits from increasing the distance between himself and the follower, such that this increase even outweighs the *competition intensity effect*. More precisely, for the interior solution, we see

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[16] Regarding the boundary solution (in which the first mover necessarily has the higher valuation), the first mover obviously bids more than the second mover.

[17] For the boundary solution, bids do not change with the own valuation as long as $\mathcal{V}_1 > 2\mathcal{V}_2$ still holds. Nevertheless, it is the agent with the higher valuation who bids more.

that the first mover's effort falls in the second mover's valuation as $\frac{\partial x_1^*}{\partial \mathcal{V}_2} = -\frac{\mathcal{V}_1^2}{4\mathcal{V}_2^2} < 0$. As long as $V_2 \geq V_1$, this can be explained by less competition. For $V_2 < V_1$, however, competition rises and nevertheless the leader's effort decreases, which contradicts the *competition intensity effect*. For the boundary solution, $\mathcal{V}_1 > 2\mathcal{V}_2$, the effect that the first mover's effort *rises* in the second mover's valuation is again due to intensified competition. By the commitment of the first mover to a publicly observable bid he can realize a first-mover advantage when he is a high type and the *competition intensity effect* is offset by this *commitment effect*.[18] The low type of the first mover, however, cannot make use of the chance to commit to an effort level. As aforementioned, his expected payoff is (weakly) lower than the follower's expected payoff. For the second mover, observations are quite similar to the simultaneous contest: His effort rises in the first mover's valuation if the first mover has the lower valuation[19] – thus, if the contest evens out – since $\frac{\partial x_2^*}{\partial \mathcal{V}_1} = \frac{1}{2}(1 - \frac{\mathcal{V}_1}{\mathcal{V}_2})$. Otherwise, the second mover's effort decreases in $\mathcal{V}_1$ (or remains equal to zero iff $\mathcal{V}_1 > 2\mathcal{V}_2$).

Similar to our observation for the first mover, Jost and Kräkel (2005) who analyze a sequential rank-order tournament with heterogeneous agents find a first mover advantage, too. In their setting, the first mover can have a higher expected payoff than the second mover even if he has the lower ability (given that the prize spread between winner and loser prize is not too large).

Next, we consider contests with private information starting with the simultaneous setting and then introducing the sequential setting. Afterwards, in Section 1.5, we analyze whether simultaneous or sequential contests are preferred by the designer and the contestants from an ex ante perspective conditional on the information setting and finally, in Section 1.6, we analyze which information setting is preferred.

1.4 Contests with Private Information

Now, we vary the information regime and consider the settings, in which valuations are private information. The aforementioned prior distribution of valuations is publicly known. At first, we consider simultaneous contests again. We restrict attention to the analysis of *symmetric* pure strategy Bayesian equilibria. The following result for a discriminatory power of one follows from Malueg and Yates (2004) who consider more general cases of the discriminatory power. We denote

[18] For the boundary solution it is still the *competition intensity effect* that dominates as preempting the second mover is profitable.

[19] For $\mathcal{V}_1 \geq 2\mathcal{V}_2$, i.e. in particular $\mathcal{V}_1 > \mathcal{V}_2$, the second mover exerts zero effort and at $\mathcal{V}_1 = 2\mathcal{V}_2$ the second mover's bid decreases in the first mover's valuation. The latter observation is intuitively plausible as competition gets less intense.

by x^*_{it} (where $i = 1, 2$ and $t = L, H$) the equilibrium bids of agent i when he has type t.

Proposition 3 *There exists a unique symmetric pure strategy Bayesian equilibrium of the simultaneous contest with private information. In this equilibrium*

$$x^*_{iL} = \left(\frac{r}{4} + (1-r)\frac{V_H V_L}{(V_H + V_L)^2}\right) V_L =: \lambda V_L \quad (1.7)$$

$$x^*_{iH} = \left(\frac{r}{4} + (1-r)\frac{V_H V_L}{(V_H + V_L)^2}\right) V_H =: \lambda V_H \quad (1.8)$$

for $i = 1, 2$.

Proof.

See Malueg and Yates (2004). As existence for general levels of discriminatory power is not guaranteed, existence for the special case considered here (discriminatory power equal to one) is shown in the appendix. ■

Remember that for $r = 1$ and $r = 0$ we are back to public information[20] as types are perfectly positive or negative, respectively, correlated. $r = 1$ implies that agents 1 and 2 have identical valuations ($\mathcal{V}_1 = \mathcal{V}_2$) and $r = 0$ implies that they have different valuations ($\mathcal{V}_1 \neq \mathcal{V}_2$). Additionally, if high and low valuation are identical ($V_L = V_H$), valuations are common knowledge, too. Note that for these three cases, equilibrium bids coincide with the public information setting.

Obviously, as in the case with public information, equilibrium bids are proportional in the sense that $\frac{x^*_{iL}}{V_L} = \frac{x^*_{iH}}{V_H}$. Like in the contest with public information, the bid of a high type is larger than the bid of a low type (*ability effect*), and agents invest a fraction, here denoted by λ, of their own valuation, which is the same for a high and a low type. It can easily be verified that $0 < \lambda < 1$. Thus, like in the public information setting, agents never bid zero or exactly their valuation. If we compare λ to the fraction ω which is invested in the public information case, we receive the following result.[21]

Proposition 4 *Consider simultaneous contests and let* $\rho \in (-1, 1)$ *and* $V_H > V_L$. *Under public information, the fraction invested in equilibrium (according to Propositions 1 and 3) is smaller than under private information if the contest under public information is asymmetric (i.e.* $\mathcal{V}_1 \neq \mathcal{V}_2$*) and it is larger if the contest under public information is symmetric (i.e.* $\mathcal{V}_1 = \mathcal{V}_2$*).*

[20] To be precise, it is not exactly public information for $r = 0$: The agents know each other's valuation. An outsider, however, only knows the realization of valuations but does not know which agent is the high or low type. Nevertheless, we make no further differentiation here as this is not the point of the paper.

[21] We restrict here to $r \in (0, 1)$ and $V_H > V_L$ as $r = 0$, $r = 1$ and $V_H = V_L$ are equivalent to public information. As aforementioned, for these cases bids and hence the fraction λ that is invested coincide in the contests under public and private information.

Proof.

Suppose the contest under public information is asymmetric (i.e. $\mathcal{V}_1 \neq \mathcal{V}_2$). We have to verify that $\lambda > \frac{V_H V_L}{(V_H+V_L)^2}$. This is equivalent to $r(V_H - V_L)^2 > 0$ which always holds as $V_H > V_L$ and $\rho \in (-1, 1)$ (which is equivalent to $r \in (0, 1)$).

Suppose now the contest under public information is symmetric (i.e. $\mathcal{V}_1 = \mathcal{V}_2$). We have to verify that $\lambda < \frac{1}{4}$. This is equivalent to $-(1-r)(V_H - V_L)^2 < 0$ which is always fulfilled as $V_H > V_L$ and $\rho \in (-1, 1)$ (which is equivalent to $r \in (0, 1)$). ■

Trivially, this result implies that when the fraction invested is higher (lower) under public information, then also the bid – conditional on an agent's type – is higher (lower) under public information. The intuition for the result is that contestants bid more aggressively, the more 'similar' their types are. This means a *competition intensity effect* drives the result: The "closer" the contest, the higher the bids. To see this, we consider first the fraction $\omega = \frac{\mathcal{V}_2\mathcal{V}_1}{(\mathcal{V}_1+\mathcal{V}_2)^2}$, which is invested of the own valuation in the public information case. When types are symmetric (i.e. $\mathcal{V}_1 = \mathcal{V}_2$), then $\omega = \frac{1}{4}$. For asymmetric types (i.e. $\mathcal{V}_1 \neq \mathcal{V}_2$) the invested fraction ω is smaller than for symmetric types (as $\frac{\mathcal{V}_2\mathcal{V}_1}{(\mathcal{V}_1+\mathcal{V}_2)^2} \leq \frac{1}{4}$ is equivalent to $0 \leq (\mathcal{V}_1 - \mathcal{V}_2)^2$ which holds with strict inequality for $\mathcal{V}_1 \neq \mathcal{V}_2$). Note that this holds *independent* of whether two low or two high types compete against each other. Hence, an even contest implies higher competition, which is reflected by higher bids.

Under public information contestants know for sure whether they have the same type or different ones. Under private information, however, agents do not know for sure the type of their opponent (as long as $\rho \in (-1, 1)$ and $V_L < V_H$). Therefore, competition intensity increases relative to a public information contest with asymmetric types and decreases relative to one with symmetric types. It follows that the fraction that is invested under *private* information is higher (lower) compared to the fraction under a public information contest with asymmetric (symmetric) types.

In order to see the effects of 'more similar' types on competition and bids more clearly, consider now how the agents' bids vary with the degree of correlation of their valuations. The invested fraction λ in a private information contest becomes larger (smaller) the more positively (negatively) the agents' types are correlated. Conditional on an agent's valuation, these results hold true for the bids as well. A stronger positive correlation between the contestants' types (i.e. an increase in ρ) can be interpreted as a more even contest since – given that an agent knows his own valuation – this corresponds to a higher probability of the opponent having the same valuation. Hence, there is more aggressive bidding if the contest gets closer.

In the limit – for perfect positive or negative correlation, respectively – the fraction invested under private information coincides with the corresponding bid under public information. This means that if under private information uncertainty about the opponent's type diminishes, bids under private information approach the corresponding bids under public information.

Proposition 5 *Bids in the symmetric Bayesian equilibrium in pure strategies of simultaneous contests with private information are weakly larger (smaller) than the Nash-equilibrium bids of asymmetric (symmetric) simultaneous contests with public information. They approach the bids of asymmetric (symmetric) contests with public information the more negatively (positively) types are correlated and coincide for perfect negative (positive) correlation.*

Proof.

As already shown, equilibrium bids in an asymmetric simultaneous contest with public information are smaller than in the symmetric one. Moreover, from Proposition 4 we know that for $\rho \in (-1, 1)$ and $V_H > V_L$ the fraction invested under public information is larger (smaller) than under private information if the contest under public information is symmetric (asymmetric). Conditional on an agent's type this also holds for equilibrium bids. For $\rho = 1$, $\rho = -1$ or $V_H = V_L$ invested fractions coincide under both settings. Using $\rho = 2(r - \frac{1}{2})$, we have $\frac{\partial \lambda}{\partial \rho} = \frac{1}{8} - \frac{V_H V_L}{2(V_H + V_L)^2}$. It follows $\frac{\partial \lambda}{\partial \rho} \geq 0$ if and only if $(V_H - V_L)^2 \geq 0$ (with strict inequality if $V_L > V_H$) and the result follows. ■

An interesting result that is intuitively appealing is that the bid of a high (low) type under private information is the same as the expected bid of a high (low) type under public information – i.e. the expected bid of an agent under public information *conditional* on this agent being a high (low) type – as Malueg and Yates (2004) show. Therefore, also the ex ante expected effort sum of both agents is the same under public and private information. This immediately implies that a designer who aims at maximizing the ex ante expected effort sum is indifferent between the two settings.

We now turn to *sequential* contests with private information. Afterwards, in Section 1.5, we compare the four different institutional settings from the point of view of the designer and the contestants.

In sequential contests with private information, the second mover perfectly observes the first mover's action before he decides on his bid – exactly like under public information. The agents' types, however, are private information. We consider symmetric perfect Bayesian equilibria in pure strategies. In a weak perfect Bayesian equilibrium, the bid of each type of the first mover, x^*_{1t} $(t = L, H)$, maximizes this type's expected payoff given his beliefs about the second mover's type and the best response of each type of the second mover. The conditional probabilities, which are obtained from the prior probability distribution of the contestants' types, determine the first mover's equilibrium beliefs about the second mover's type[22]: With probability r the second mover

[22]Observe that both information sets of the first mover (and as well those of the second mover) are always reached with positive probability.

is of the same type as the first mover and with probability $1 - r$ he is of a different type.

Equilibrium bids of each type of the second mover maximize this type's expected payoff contingent on the *observed action* of the first mover. We denote equilibrium bids of type $k = L, H$ of the second mover by x_{2k}^{t*}. $t = H, L$ indicates the type (and therefore the bid) of the first mover.[23]

It is important to note that the beliefs of the second mover about the type of the first mover do not influence equilibrium outcomes. The perfectly observable action of the first mover completely determines the optimal reaction of the second mover (irrespective of the first mover's true type): The second mover responds to the first mover's action in a way that maximizes his expected payoffs. His payoffs only depend on the first mover's action that he observes but not on the first mover's true type (except that the first mover's action may depend on the first mover's type in equilibrium). As the optimal action of the second mover is independent of his beliefs, they can be arbitrary, and we do not need to specify his beliefs in the following. Moreover, because of this independence, we can solve the game backward, starting with the second mover. In the following derivation of equilibrium bids, we do not provide a complete proof. All missing steps are in the appendix (see proof of Proposition 6).

The second mover maximizes his expected payoff given the action of the first mover, which he observes, and given his own type $k = H, L$. First note that we do not need to consider the case that nobody exerts a positive amount of effort as this cannot happen in equilibrium: Suppose the first mover exerts zero effort. Then, the second mover can ensure winning the prize with an arbitrarily small effort $\epsilon > 0$. This dominates exerting no effort at all and winning only with probability one half. Furthermore, in equilibrium, as we show in the appendix, the first mover's bid is strictly positive for each type $t = L, H$. Hence, the relevant best response of the second mover (i.e. given $x_{1t} > 0$) can be derived from the following maximization problem

$$\max_{x_{2k} \geq \epsilon} \frac{x_{2k}}{x_{2k} + x_{1t}} V_k - x_{2k}.$$

It can easily be verified that the maximization problem is concave in the second mover's bid given the first mover's bid. Thus, the best response function of the second mover is given by the first order condition, as long as the first order condition results in a non-negative bid of the second mover, otherwise the optimal bid is zero:

$$x_{2k}(x_{1t}) = \max\{\sqrt{x_{1t}}\sqrt{V_k} - x_{1t}, 0\} \text{ for } x_{1t} > 0. \tag{1.9}$$

We can see from the best response function in (1.9) that the second mover completely drops out of the contest – i.e. both types bid zero – when the first mover bids more than the high valuation.

[23] Of course, the second mover actually observes the leader's action but not his type. In equilibrium, however, the two types of the first mover "separate" in their actions – as we show in the appendix. Hence, the second mover can in principle infer the first mover's type from the observed action. For ease of presentation, we therefore denote the observed action by the first mover's type.

When $V_H > x_{1t} \geq V_L$ only the low type of the second mover drops out.
Regarding the first mover, this implies two different cases: Either he faces an active second mover (i.e. both types of the second mover make a positive bid) or he faces an (partly) inactive second mover (i.e. at least the low type bids zero).
Knowing his own type, the first mover updates his beliefs about his rival's type and hence his rival's bid. As aforementioned, the conditional probability that the second mover is a low (high) type given that the first mover is a low (high) type – i.e. agents are homogeneous – is given by r. $(1-r)$ denotes the conditional probability that the second mover is a low (high) type given that the first mover is a high (low) type – i.e. agents are heterogeneous. The first mover maximizes his expected payoff conditional on his own type $t = L, H$ and on each type of the second mover reacting according to his best response function given in (1.9).

Suppose first that both types of the second mover are active. We know from (1.9) that this is not possible if the first mover bids the low valuation or more. Hence, both types of the first mover have to bid less than V_L, which implies that the best response of the second mover is given by $x_{2k}(x_{1t}) = \sqrt{x_{1t}}\sqrt{V_k} - x_{1t}$. Moreover, we disregard here the case that the first mover in inactive. As argued above, in this case, the second mover's best response is to bid an arbitrarily small amount $\epsilon > 0$, which ensures that he wins the prize and hence the first mover's payoff from being inactive is zero. In the appendix we show, that the first mover is never inactive in equilibrium as he can insure a positive expected payoff by actively taking part in the contest. The maximization problem of type $t = L, H$ of the first mover for the case that both types of the follower are active is as follows where $n = L, H$:

$$\max_{V_L > x_{1t} > 0} \left(\frac{x_{1t}}{x_{1t} + x_{2t}(x_{1t})} r + \frac{x_{1t}}{x_{1t} + x_{2n}(x_{1t})} (1-r) \right) V_t - x_{1t} \quad \text{with} \quad t \neq n.$$

Plugging in the relevant best response of the second mover yields

$$\max_{V_L > x_{1t} > 0} \left(\frac{1}{\sqrt{V_t}} r + \frac{1}{\sqrt{V_n}} (1-r) \right) \sqrt{x_{1t}} V_t - x_{1t} \quad \text{with } t \neq n. \tag{1.10}$$

This maximization problem is concave in x_{1t}. Therefore, the first order condition is sufficient if we consider the unconstrained maximization problem (neglecting the constraints on x_{1t} for the moment). The optimal bid, x_{1t}^*, is given by

$$x_{1t}^* = \frac{V_t^2}{4} \left(r \left(\frac{1}{\sqrt{V_t}} - \frac{1}{\sqrt{V_n}} \right) + \frac{1}{\sqrt{V_n}} \right)^2 =: \alpha_t^2 \quad \text{with } t \neq n. \tag{1.11}$$

Obviously, x_{1t}^* is always strictly positive. Hence, the only constraint that remains to check is whether $x_{1t}^* < V_L$. Otherwise, at least the low type of the second mover exerts zero effort and – for $\alpha_t^2 > V_L$ – (1.10) is no longer the corresponding maximization problem of the first mover and α_t^2 not the optimal bid. To check this, we consider both types of the leader separately.

For the low type of the first mover, it can never be optimal to bid more than V_L. The reason is that if he invests more than V_L his payoff is negative for sure and, for example, investing zero – yielding zero payoff – is better. In fact, the first mover can ensure a positive expected payoff (as is shown in the appendix) by bidding α_L^2, which is strictly smaller than the low valuation ($\alpha_L^2 < V_L \Leftrightarrow (1-r)V_L^{\frac{1}{2}} < (2-r)V_H^{\frac{1}{2}}$). Thus, the low type of the first mover bids strictly less than the low valuation in equilibrium, implying that we always have an *interior* solution – i.e. all contestants spend a positive amount of effort – when the first mover is a low type. The corresponding best responses of type $k = L, H$ of the second mover are determined according to (1.9). Hence, his equilibrium bids given that the first mover bids $x_{iL}^* = \alpha_L^2$ are given by $x_{2k}^{L*} = \alpha_L(\sqrt{V_k} - \alpha_L)$.
When the first mover is a high type, however, he may bid more than the low valuation. Hence, we have to verify for an interior solution under which condition $\alpha_H^2 < V_L$. This holds if and only if

$$V_H\left(r\left(\frac{1}{\sqrt{V_H}} - \frac{1}{\sqrt{V_L}}\right) + \frac{1}{\sqrt{V_L}}\right) < 2\sqrt{V_L}. \tag{1.12}$$

Intuitively, when the high and low valuation coincide (i.e. $V_H = V_L$) condition (1.12) holds, which means that both types of the first mover bid less than the low valuation. For $V_H > V_L$ we can rewrite (1.12) as

$$r > \frac{2V_L - V_H}{\sqrt{V_H}\left(\sqrt{V_L} - \sqrt{V_H}\right)} =: \widetilde{r}. \tag{1.13}$$

Since the denominator of $\widetilde{r}$ is negative, $r > \widetilde{r}$ is fulfilled in case $2V_L > V_H$ as then $\widetilde{r} < 0$ but $r \in [0, 1]$. For $4V_L < V_H$, we get $\widetilde{r} > 1$, implying that there cannot be an interior solution in this case as $r > \widetilde{r}$ is impossible. Hence, we always have an interior solution if $2V_L > V_H$. When $2V_L < V_H \leq 4V_L$, we have an interior solution if $r > \widetilde{r}$. Otherwise, there is a boundary solution in which at least one type of the second mover bids zero. The bid of type $k = L, H$ of the follower in an interior solution when the leader is a high type is determined according to (1.9), i.e. $x_{2k}^{H*} = \alpha_H(\sqrt{V_k} - \alpha_H)$.

Up to now, we considered the case that both types of the second mover are active. Suppose now that the second mover is (partly) inactive, i.e. we have a boundary solution. We know from (1.9) that this happens if and only if the leader bids at least the low valuation (i.e. if $V_L < V_H$ and $r \leq \widetilde{r}$).
Since the low type of the follower drops out first, this implies that $x_{2L}^{t*} = 0$ (for $x_{1t} \geq V_L$) where t denotes the type of the leader . Therefore, we only consider the high type of the follower, who may still make a positive bid. With a similar argument as given above for the low type of the first mover, we obtain that the leader invests less than V_H when he is a high type. This means that the second mover is never completely inactive and the relevant best response of the high type of the second mover is given by $x_{2H}(x_{1t}) = \sqrt{x_{1t}}\sqrt{V_H} - x_{1H}$. The reduced maximization problem of the

first mover, when the low type of the follower is inactive, is

$$\max_{V_H \geq x_{1H} \geq V_L} \left(\frac{x_{1H}}{x_{1H} + x_{2H}(x_{1H})} r + (1-r) \right) V_H - x_{1H}. \tag{1.14}$$

Plugging in the relevant best response of the high type of the second mover, we can rewrite the maximization problem as follows

$$\max_{V_H \geq x_{1H} \geq V_L} r\sqrt{x_{1H}}\sqrt{V_H} + (1-r)V_H - x_{1H}. \tag{1.15}$$

Consider for the moment only the unconstrained maximization problem (neglecting the restrictions $V_H \geq x^*_{1H} \geq V_L$). The unconstrained problem is concave in x_{1H} and thus the first order condition,

$$\frac{1}{2} r\sqrt{V_H} - \sqrt{x_{1H}} = 0, \tag{1.16}$$

is also sufficient. Solving for x_{1H} yields $x_{1H} = \frac{r^2 V_H}{4}$. As argued above, the restriction limiting x_{1H} from above is always fulfilled ($\frac{r^2 V_H}{4} < V_H$). Hence, the equilibrium bid of the high type of the first mover, x^*_{1H}, is given by (1.16) if $\frac{r^2 V_H}{4} \geq V_L$. This condition is equivalent to

$$r > 2\frac{\sqrt{V_L}}{\sqrt{V_H}} =: \widetilde{\widetilde{r}}. \tag{1.17}$$

If (1.17) is not satisfied (as well as if it is satisfied with equality), we have $x^*_{1H} = V_L$. This follows from the concavity of the objective function, and since the first mover's payoff from bidding V_H is zero whereas the payoff from bidding V_L is strictly positive as is shown in the appendix.

As aforementioned, equilibrium bids of the high type of the second mover are determined according to the follower's relevant best response function. The bid is $x^{H*}_{2H} = \sqrt{V_L}(\sqrt{V_H} - \sqrt{V_L})$ if the high type of the first mover bids exactly the low valuation and is $x^{H*}_{2H} = \frac{rV_H}{2}\left(1 - \frac{r}{2}\right)$ if the first mover bids more than the low valuation. Thus, for a boundary solution the conditions $V_L < V_H$ and $r \leq \widetilde{r}$ have to be satisfied and the bids of the high types depend on whether $r \gtrless \widetilde{\widetilde{r}}$. These results are summarized in the following proposition.

Proposition 6 *The unique symmetric weak perfect Bayesian equilibrium outcome in pure strategies of sequential contests with private information with belief r of the high (low) type of the first mover that the second mover is high (low) and belief $1-r$ that he is low (high) and arbitrary beliefs of the second mover is as follows ($t = H, L$ denotes the type of the first mover and $k = H, L$ the type of the second mover):*

Interior solution if either (i) $V_L = V_H$ or if (ii) $r > \widetilde{r}$ and $V_L < V_H$:

$$x^*_{1L} = \alpha_L^2$$
$$x^{t*}_{2L} = \alpha_t\left(\sqrt{V_k} - \alpha_t\right)$$

Boundary solutions if $r \leq \widetilde{r}$ and $V_L < V_H$:

Boundary case (i) if $r \leq \min\{\widetilde{r}, \widetilde{\widetilde{r}}\}$:

$$\begin{aligned} x^*_{1L} &= \alpha_L^2, & x^*_{1H} &= V_L, \\ x^{*L}_{2L} &= \alpha_L\left(\sqrt{V_L} - \alpha_L\right), & x^{*H}_{2L} &= 0, \\ x^{*L}_{2H} &= \alpha_L\left(\sqrt{V_H} - \alpha_L\right), & x^{*H}_{2H} &= \sqrt{V_L}\left(\sqrt{V_H} - \sqrt{V_L}\right) \end{aligned}$$

Boundary case (ii) if $\widetilde{\widetilde{r}} < r \leq \widetilde{r}$:

$$\begin{aligned} x^*_{1L} &= \alpha_L^2, & x^*_{1H} &= \tfrac{r^2 V_H}{4}, \\ x^{*L}_{2L} &= \alpha_L\left(\sqrt{V_L} - \alpha_L\right), & x^{*H}_{2L} &= 0, \\ x^{*L}_{2H} &= \alpha_L\left(\sqrt{V_H} - \alpha_L\right), & x^{*H}_{2H} &= \tfrac{rV_H}{2}\left(1 - \tfrac{r}{2}\right) \end{aligned}$$

with $\alpha_L = \frac{V_L}{2}\left(r(\frac{1}{\sqrt{V_L}} - \frac{1}{\sqrt{V_H}}) + \frac{1}{\sqrt{V_H}}\right)$, $\alpha_H = \frac{V_H}{2}\left(r(\frac{1}{\sqrt{V_H}} - \frac{1}{\sqrt{V_L}}) + \frac{1}{\sqrt{V_L}}\right)$,
$\widetilde{r} = \frac{2V_L - V_H}{\sqrt{V_H}\left(\sqrt{V_L} - \sqrt{V_H}\right)}$, *and* $\widetilde{\widetilde{r}} = 2\frac{\sqrt{V_L}}{\sqrt{V_H}}$.

Proof.

See appendix. ■

Notice that we can easily rewrite the conditions on r for the interior and boundary solutions in terms of the correlation coefficient ρ using the relation $\rho = 2r - 1$. The condition $r > \widetilde{r}$ for an interior solution if $V_L < V_H$ becomes then

$$\rho > \frac{4V_L - V_H - \sqrt{V_L}\sqrt{V_H} + V_H}{\sqrt{V_H}(\sqrt{V_L} - \sqrt{V_H})} =: \widetilde{\rho}. \tag{1.18}$$

This implies that for an interior solution valuations have to have a sufficiently "strong tendency to positive correlation", which does not mean that $\widetilde{\rho}$ necessarily has to be positive. It follows that expected valuations of the contestants have to be sufficiently close to each other for an interior solution: Given an agent's valuation, the expected valuation of the opponent must be sufficiently close, which is just an extension of the condition under public information to private information. Remember that in the sequential contest with public information the condition for an interior solution is that valuations are rather close ($\mathcal{V}_1 \leq 2\mathcal{V}_2$).

Similar to the results under public information, we know from the second mover's best response function that the more the first mover bids, the less bids the second. Again, the follower only responds with a positive bid if the leader bids less than the follower's valuation. Like before, only the low type of the second mover can be preempted such that he drops out of the contest in equilibrium. In contrast to the setting under public information, the maximum bid of the leader under private information can exceed V_L when he is a high type and tries to preempt the second mover. The reason is that the first mover faces uncertainty about the second mover's type. With bidding V_L he can in general not be sure to win. Because of the strategic effect when contestants move sequentially, they no longer bid the same fraction of their valuation, like we have seen for the case of public information.

Furthermore, it is again the *ability effect* that drives behavior in equilibrium:

Proposition 7 *Let $V_L < V_H$ and $r \in (0,1)$. In the interior solution of a weak perfect Bayesian equilibrium of the sequential contest with private information (see Proposition 6)*

(i) a contestant bids strictly more when he has the high valuation than when he has the low valuation and

(ii) the first mover bids strictly more (less) than the second mover when he is a high (low) type.

Proof.

See appendix. ■

We show within the proof that these results hold true for the boundary solutions as well with one exception: It need not be true that the high type of the first mover bids more than the high type of the second mover.

In the limit, when there is either perfect positive or negative correlation or the high and low valuation is identical (i.e. $r = 1$, $r = 0$ or $V_L = V_H$), i.e. there is actually complete information, equilibrium bids coincide with the corresponding bids of the sequential contest under public information. As long as valuations are not perfectly correlated, bids of identical types, however, differ, which follows from Proposition 7 part (ii).

It follows from Proposition 7 that in the sequential contest with private information, the dominant force for bidding behavior is still the *ability effect* independent of whether an agent is the leader of the follower. In addition, bidding more than the rival (having the higher type) translates again into higher expected payoffs for the interior solution.[24]

Moreover, the more positively valuations are correlated, the more the second mover bids in equilibrium. The more positively valuations are correlated, the more (less) the first mover bids when he is a low (high) type. For the second mover, the intuition for this result is the *competition intensity effect*. As we have already seen, a closer contest – in the sense that the contestants have "more similar" valuations – leads to more aggressive bidding. A stronger positive correlation between the contestants' types (i.e. an increase in ρ) implies as well that the contest under private information evens out from a contestant's perspective: given the own type, a stronger positive correlation corresponds to a higher probability that the opponent has the same type. For the first mover,

[24]See Proof E in the appendix.

however, the result can only partly be explained by the *competition intensity effect*. Like in the sequential contest with public information, only the behavior of the low type can be explained by higher competition. The high type, in contrast, bids less when there is higher competition. The *competition intensity effect* is again offset by the *commitment effect*. These results are summarized in the following proposition.

Proposition 8 *In a weak perfect Bayesian equilibrium of the sequential contest with private information (see Proposition 6)*

(i) the bid of the first mover is increasing (decreasing) in the correlation coefficient ρ when the first mover is a low (high) type and

(ii) bids of the second mover are increasing in ρ.

Proof.

See appendix. ■

In the following section, we compare the four different institutional settings for the designer and the contestants from an ex ante perspective.

1.5 Sequential versus Simultaneous Contests

We begin the comparison of the different institutional settings by comparing simultaneous and sequential contests given the information setting and afterwards contrast private and public information given the order of moves. Here, and in the remainder of the paper we assume for the sequential settings that the conditions for interior solutions are satisfied if not stated otherwise. Moreover, we assume that in every setting, contestants make the corresponding equilibrium bids as given in Propositions 1, 2, 3 and 6. In the following comparison of sequential and simultaneous contests, we first refer to public information, then we turn to private information.

1.5.1 Sequential versus Simultaneous Contests given Public Information

The comparison of the oder of moves given public information leads to the following insights. We have seen in Section 1.3 that bids of homogeneous types in the sequential and simultaneous contest with public information are identical as the first mover's commitment to a bid does not affect equilibrium behavior in this case. Regarding the Stackelberg and Cournot-Nash equilibrium outcomes when types are heterogeneous, the outcomes no longer coincide: Morgan (2003) shows that a first mover who has a high valuation commits to a higher bid in the sequential contest than in the simultaneous contest, whereas a first mover who has a low valuation commits to a lower bid.

Intuitively, this can be explained as follows. The high type of the first mover anticipates that the low type of the follower reduces his bid when he faces a higher bid of the first mover. Hence, the leader can profit from increasing his bid as his probability of winning rises. In contrast, the low type of the leader knows that a reduction in his bid induces the high type of the follower to reduce his bid as well as the marginal gain in his probability of winning decreases for higher bids.[25]
Hence, in a heterogeneous contest, the sum of efforts is lower (in equilibrium) in the sequential contest, when the low type moves first as both agents reduce their efforts compared to the simultaneous contest. When the high type moves first, the sum of efforts is higher in the sequential contest as the first mover's increase in his effort outweighs the decrease of the follower (see Morgan (2003)). As aforementioned, for homogeneous contests equilibrium efforts under both timing are identical. Therefore, to conclude under which timing structure the ex ante expected effort sum is higher, we only need to compare the expected effort sum when the first mover is a high type and the second mover a low type and vice versa. Ex ante, both situations are equally likely as each contestant is with provability one half a high or a low type, respectively. This property is also satisfied for the symmetric distribution with independently drawn valuations that is considered by Morgan. Thus, his result that the ex ante expected effort sum is higher in the sequential contest can be extended to our distributional assumption. A designer who wants to maximize the expected effort sum, hence prefers the sequential contest to the simultaneous one under public information.
Regarding the contestants' payoffs, we already know that in simultaneous contests ex ante expected payoffs of both agents are identical as well as in sequential contests (for the interior solution). Although there is no difference between the contestant's expected payoff given a timing structure, ex ante expected payoffs are higher in sequential contests than in simultaneous contests.[26] Taken together, this implies that sequential contests Pareto dominate simultaneous contests from an ex ante perspective. This may be a bit surprising since agents spend more effort from an ex ante perspective. The intuition for the result is an *efficiency gain effect*: the agent with the higher valuation wins "more often" in the sequential setting since the gap between the bid of heterogeneous types becomes larger[27]. This *efficiency gain effect* increases the surplus of both agents and outweighs the effect of higher expected efforts.

[25]It is straightforward to verify the follower's reactions by differentiating the best response function of the follower (that is identical to (1.9)) w.r.t. the leader's bid and evaluating the derivative at the equilibrium bid of agent 1 in the simultaneous contest (as given in Proposition 1).

[26]The proof is omitted since the result can straightforwardly be derived from Morgan (2003) by changing the distributional assumption.

[27]It immediately follows from above that equilibrium bids in a heterogeneous contest are more unequal in the sequential contest than in the simultaneous contest when the high type moves first. For the low type see the proof of Proposition 16.

Baik and Shogren (1992) and Leininger (1993) show that the sequential contest also arises endogenously in the following two-stage game. In stage 1, knowing their own and the opponent's type, agents decide simultaneously whether to move "early" or "late". Timing decisions are publicly announced and afterwards, knowing the decision of the opponent, contestants choose their bid in the period they committed to. In this game, agents move sequentially in equilibrium if types are heterogeneous with the low type being first mover, otherwise sequential and simultaneous order of moves form an equilibrium.[28]

Morgan (2003) finds that the sequential contest also arises endogenously when the timing decision is taken *before* contestants know their valuations. In stage 1, agents decide simultaneously whether to move "early" or "late" *before* knowing their valuations. Then valuations realize, are publicly announced, and according to the timing decision in stage 1, effort is exerted in the period agents committed to before. In this game, the only possible outcome of a subgame perfect equilibrium in pure strategies is that agents act sequentially. This result also holds for the joint distribution of valuations considered in this paper.[29] Although a sequential order of moves is the only equilibrium outcome, a drawback of this game is that it involves a coordination problem: one agent has to move first and the other one second.

1.5.2 Sequential versus Simultaneous Contests given Private Information

In this section, we compare simultaneous and sequential contests when valuations are private information. We find that with private information – like in the public information setting – the ex ante expected effort sum is (weakly) higher in the sequential contest than in the simultaneous one, which implies the following result.

Proposition 9 *Suppose contestants bid in each setting according to the corresponding equilibrium outcomes in Propositions 1, 2, 3 and 6. Then the designer of the contest (weakly) prefers a sequential setting when he aims at maximizing the expected effort sum (strictly so if correlation is not perfectly positive or* $V_H > V_L$*).*

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[28] Baik and Shogren (1992) consider "favorites" and "underdogs" where the underdog (favorite) wins with the lower (higher) probability in equilibrium. This is equivalent to a heterogeneous contest in our setting, in which the high type bids more than the low type and therefore, wins with a higher probability. Leininger (1993) considers both cases, heterogeneous and homogeneous contestants.

[29] The proof is omitted as the result can straightforwardly derived from Morgan (2003) by replacing the distribution with the one used here.

Proof.

See appendix. ■

Moreover, if we think of the prize agents win as money the designer spends, the sequential contest is "cheaper" for the designer. He can extract the same effort sum that he extracts under the simultaneous contest but at a lower price. This may be crucial for the institutional choice if the sequential contest is – in some other sense – more expensive than the simultaneous contest. For example, it might take longer to carry out a sequential contest, which may cause higher costs. Hence, even if one includes discounting, there will be parameter combinations in which sequential contests dominate simultaneous ones.

Our result thus differs from Münster (2004). He considers repeated contests, in which contestants twice choose an effort simultaneously and both receive intermediate information before they enter the second stage. In contrast to our result that the sequential contest (in which intermediate information is available to the second mover) leads to a higher expected effort sum, intermediate information leads to lower expected equilibrium efforts in his setting. The main reason for this difference is that in Münster high ability contestants might put in little effort in the first round to make their opponents believe that they are of low ability.

The result that sequential contests Pareto dominate simultaneous ones can only partly be extended to the case of private information. Higher expected efforts lead to higher expected effort costs for the contestants. For the first mover, the effect of an increased probability of winning prevails such that he nevertheless prefers the sequential contest. The second mover prefers the sequential contest only when ρ is sufficiently "negative". We call the correlation sufficiently "negative" although this does not mean, that correlation needs to be actually negative. By sufficiently negative, we mean that there is a sufficiently strong tendency to negative correlation between both agents' valuations, the threshold, however, can be strictly positive.

The intuition for this result is an *efficiency gain effect*. Suppose the first mover is a high type. We know that he bids more than his rival because of the *ability effect*. Moreover, his bid is increasing the smaller ρ (*commitment effect*). The bid of the second mover is decreasing the smaller ρ (*competition intensity effect*) and the higher the bid of the first mover. Hence, the first mover, who is a high type, wins with a higher probability the smaller ρ. Suppose the leader is a low type. Then his bid is decreasing the smaller ρ and also the bid of the follower is decreasing (as ρ decreases and as the first mover bids less). When the second mover is a high type, his bid decreases less than the first mover's bid and he bids more than the first mover (*ability effect*). Hence, the probability that the second mover – who is the high type – wins the prize increases. Taken together, this implies that the agent with the higher valuation is expected to win more often.

In addition, the smaller the correlation, the higher the probability that valuations actually differ. Note that in case contestants have the same type, it does not matter in terms of allocative efficiency which one of them wins. Thus, the smaller the correlation ("the more negative"), the more important the *efficiency gain effect* becomes. The second mover needs to be compensated for increased effort outlays under the sequential structure to prefer the sequential structure. When the correlation is sufficiently "negative", and thus the *efficiency gain effect* is sufficiently strong, the second mover can be compensated.

Hence, the result that the sequential contest Pareto dominates the simultaneous one under public information still holds under private information if valuations are sufficiently "negatively" correlated. These observations are summarized in the following proposition.

Proposition 10 *Consider equilibrium outcomes of simultaneous and sequential contests as given in Proposition 3 and 6. In the interior solution of sequential contests with private information*

(i) the first mover always receives a higher ex ante expected payoff than in simultaneous contests,

(ii) the second mover receives a higher ex ante expected payoff than in simultaneous contests for $V_L < V_H$ *and* $\rho < 1$ *if the correlation is sufficiently "negative", i.e. if*

$$\rho \leq \widetilde{\rho}^c := \frac{\left(V_L^{\frac{3}{2}} - V_H^{\frac{3}{2}}\right)^2 - V_L V_H (V_H + V_L) - 10 V_H^{\frac{3}{2}} V_L^{\frac{3}{2}}}{(V_L + V_H)\left(V_H^2 + 4 V_H^{\frac{1}{2}} V_L^{\frac{1}{2}} (V_H + V_L) + V_L^2\right)}$$

given the condition for an interior solution is satisfied, i.e. $\rho > \widetilde{\rho} = \frac{4V_L - V_H - \sqrt{V_L}\sqrt{V_H} + V_H}{\sqrt{V_H}(\sqrt{V_L} - \sqrt{V_H})}$. *For* $V_H = V_L$ *or* $\rho = 1$ *his ex ante expected payoffs in simultaneous and sequential contests are identical.*

Proof.

See appendix. ■

Comparing the numerator and denominator of $\widetilde{\rho}^c$, it is straightforward to verify $|\widetilde{\rho}^c| < 1$ (see Proof of Proposition 10).

Note that the critical value of ρ is derived from the interior solution of equilibrium bids. Therefore, we have to make sure that the condition for an interior solution ($r \geq \widetilde{r}$) is satisfied as well. The result is illustrated in Figure 1.2 for the case that the low valuation equals 1.5. The solid line describes the critical value of the correlation coefficient. Below this line the sequential contest leads to higher expected payoffs for the second mover and above the simultaneous contest. The dashed line marks the threshold for the interior solution. Unlike Münster (2004), the setting with intermediate information (i.e. a sequential contest) is not necessarily favorable to both contestants. This difference is due to the fact that in Münster, agents act simultaneously. They play two stages and

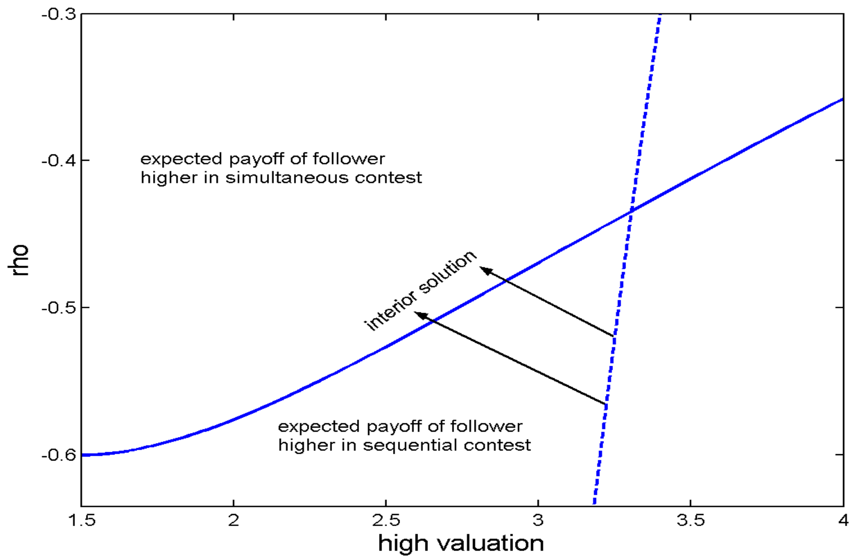

Figure 1.2: Critical Value $\widetilde{\rho}^c$ for $V_L = 1.5$

both receive intermediate information after the first stage. This means no first mover advantage arises – which reduces the second mover's payoff – as in our case.

Another criterion for the institutional choice is the overall (ex ante) expected payoff of the contestants. Suppose contestants have the choice whether they want to play a simultaneous or sequential contest. Moreover, they have to decide on it *before* they know their valuation and whether they move first or second – because of ex ante symmetry they are assigned to the role of being the first mover and the opponent the second mover (and vice versa) with probability one half. In this setting, whether contestants prefer a simultaneous or sequential setting depends on where the overall expected payoff is larger. Although the first mover always prefers the sequential setting, this cannot outweigh the fact that the second mover prefers the simultaneous contest for sufficiently "positive" correlation.

Proposition 11 *Consider equilibrium outcomes of simultaneous and sequential contests under private information (interior solution) as given in Propositions 3 and 6. The overall expected payoff of the contestants is higher in sequential contests for $V_L < V_H$ and $\rho < 1$ if the correlation is sufficiently "negative", i.e. if*

$$\rho \leq \rho^c := \frac{V_L^3+V_H^3+2V_H^{\frac{1}{2}}V_L^{\frac{1}{2}}(V_H-V_L)^2-V_LV_H(V_L+V_H)-4V_L^{\frac{3}{2}}V_H^{\frac{3}{2}}}{V_L^3+V_H^3+V_H^{\frac{1}{2}}V_L^{\frac{1}{2}}(V_L+V_H)\left(2(V_H+V_L)+V_H^{\frac{1}{2}}V_L^{\frac{1}{2}}\right)}$$

given the condition for an interior solution is satisfied, i.e. $\rho > \widetilde{\rho} = \frac{4V_L-V_H-\sqrt{V_L}\sqrt{V_H}+V_H}{\sqrt{V_H}(\sqrt{V_L}-\sqrt{V_H})}$.

For $V_H = V_L$ of $\rho = 1$ the overall expected payoff is identical in simultaneous and sequential contests.

Proof.

See appendix. ■

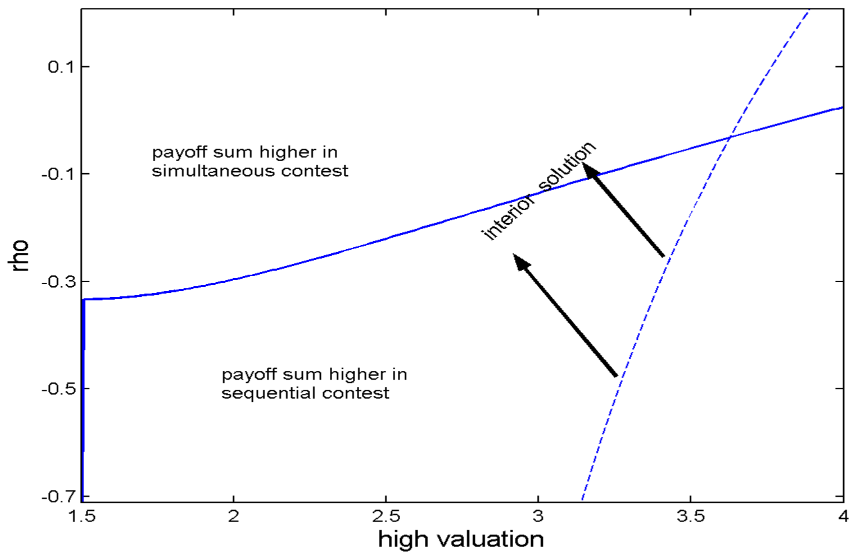

Figure 1.3: Critical Value ρ^c for $V_L = 1.5$

Comparing the numerator and denominator of ρ^c, it can be seen that $|\rho^c| < 1$.

Again, the intuition for the result is an *efficiency gain effect* like for the comparison of individual expected payoffs in Proposition 10. Evidently, the critical value for the overall expected payoff being higher in the sequential contest is larger than the critical value for the expected payoff of the second mover being higher (because of the influence of the expected payoff of the first mover).

Note that the critical value of ρ is derived from the interior solution of equilibrium bids. Therefore, we have to make sure that the condition for an interior solution ($r \geq \widetilde{r}$) is satisfied as well. The result is illustrated in Figure 1.3 (for the case that the low valuation equals 1.5), where we plot the critical value of the correlation coefficient (solid line). Below this line the sequential contest leads to higher overall expected payoffs for the contestants and above the simultaneous contest. The dashed line marks the threshold for the interior solution.

Under public information, we have seen that sequential contests arise endogenously. For the case of private information, we consider again the aforementioned two-stage game in which agents 1 and 2 simultaneously decide whether they want to move 'early' or 'late' in stage 1, these choices are publicly announced and valuations realize. In stage 2, knowing the own valuation – but not the opponent's type – and the period choice of the opponent, contestants make their bid in the period they committed to. This means that if both agents announce the same period, the subsequent subgame is the simultaneous contest with private information and if they announce different periods, it is the corresponding sequential contest with private information.

Assuming that contestants bid in each subgame according to the corresponding equilibria in Propo-

Table 1.2: Ex ante expected payoffs

		1: early	1: late
2	early	Ψ^{sim}, Ψ^{sim}	Ψ^{1st}, Ψ^{2nd}
	late	Ψ^{2nd}, Ψ^{1st}	Ψ^{sim}, Ψ^{sim}

sitions 1, 2, 3 and 6, we have already analyzed all possible outcomes in stage 2 and know the ex ante expected payoffs for each timing choice, it remains to investigate the simultaneous move game in stage 1. Table 1.2 summarizes the ex ante expected payoffs of agent 1 and 2 given their choices whether to make their bid 'early' or 'late'. Ψ^{sim} denotes the ex ante expected payoff in the simultaneous contest with private information, Ψ^{1st} and Ψ^{2nd} denote the ex ante expected payoff in the sequential contest with private information of the first mover and the second mover, respectively. Regarding these payoffs, the only relationship, we did not analyze up to now is whether the first or second mover has a higher ex ante expected payoff. Intuitively appealing, it turns out that the first mover has a higher (ex ante) expected payoff than the second mover.

Lemma 1 *In the interior solution of a weak perfect Bayesian equilibrium of the sequential contest with private information (see Proposition 6) a first mover advantage exists from an ex ante perspective.*

Proof.

See appendix. ■

Using this result, we find that in the private information case the sequential contest does not necessarily arise endogenously in the aforementioned two-stage game:

Proposition 12 *Suppose contestants bid in each subgame of the proposed two-stage game according to the corresponding equilibrium outcomes in Propositions 3 and 6, respectively. In the subgame perfect equilibria in pure strategies of the proposed two-stage game, contestants choose a sequential order of moves when the correlation between types is "sufficiently negative", i.e. $\rho < \widetilde{\rho}^c$. If $\rho > \widetilde{\rho}^c$, both agents choose to move "early" in equilibrium. If $\rho = \widetilde{\rho}^c$, both simultaneous moves at the early stage and sequential order of moves arise in subgame perfect equilibria.*

Proof.

The only step that is left to show is to solve for the (pure strategy) Nash equilibria of the simultaneous-move game in the first stage. Table 1.2 shows the payoff matrix of this game. To derive the Nash equilibria, the relationship between Ψ^{sim} and Ψ^{2nd} is crucial. If $\Psi^{2nd} < \Psi^{sim}$, which holds when $\rho < \widetilde{\rho}^c$ (as we have shown for Proposition 10), the only equilibrium is that both contestants choose to move "early". If $\Psi^{2nd} > \Psi^{sim}$, which holds when $\rho > \widetilde{\rho}^c$ (as we have shown for Proposition 10), sequential play forms an equilibrium. If $\Psi^{2nd} = \Psi^{sim}$, which holds for $\rho = \widetilde{\rho}^c$, sequential order of moves as well as both contestants choosing to move early forms an equilibrium. ∎

1.6 Private versus Public Information

Finally, we compare the different information regimes given the order of moves. In the simultaneous contest a high (low) type bids more (less) if he knows that the opponent is also a high type, than when the opponent is a low type (i.e. there is public information) compared to the bid of a high (low) type under private information. Conditional on an agent's type, the expected bid of an agent is the same under public and private information as we have seen in Section 1.4. Therefore, a risk neutral designer, who wants to maximize the ex ante expected sum of efforts, is indifferent between both information regimes. If we refer to the effort sum as the revenue of the designer, we can say that revenue equivalence holds for the designer. Moreover, the contestants' conditional probability of winning the prize is equal under both regimes in the simultaneous contest. Malueg and Yates (2004) derive these results, which we summarize in the following proposition.

Proposition 13 *[Malueg and Yates (2004)] Given the corresponding equilibrium outcomes of simultaneous contests (see Propositions 1 and 3), contestants' ex ante expected payoffs are identical under public and private information and revenue equivalence holds for a risk neutral designer, who maximizes the expected effort sum.*

For simultaneous contests with a Tullock contest success function in which agents are either both uninformed or informed about the common value of the prize, Wärneryd (2003) finds as well that the expected effort sum is identical under both settings. If, however, only one player is informed and the other one not, expected efforts decrease. Proposition 13 implies that – from an ex ante point of view – there is no incentive for the agents to share their private information in the simultaneous contest.

When we compare sequential contests given private or public information, respectively, results change: Here, the different information setting matters from an ex ante perspective. Intuitively, private information dampens the differences between ex ante expected efforts of the first and second mover. As under public information, the first mover exerts a higher expected effort than the second mover. The expected effort of the first mover is, however, lower than under public information. The second mover's expected effort is higher than under public information. The effect on the second mover's expected effort compensates for the first mover's effort reduction given private information.[30] These results are driven by the *commitment effect* for the high type of the first mover, who exerts more effort when the second mover is a low type than when he is a high type under public information. This deters the second mover when the first mover is a high type such that the distance in expected efforts between public and private information is larger for the second mover. Thus, we have the following proposition.

Proposition 14 *Given the corresponding equilibrium outcomes of sequential contests (see Propositions 2 and 6), a risk neutral designer, who maximizes the expected effort sum, prefers a private information setting to a public information setting from an ex ante perspective.*

Proof.

See appendix. ■

This means that revenue equivalence across information regimes does not hold for sequential contests. When agents move simultaneously, no *commitment effect* – which drives up the expected effort sum – is present.

Suppose that the designer knows the realization of types when the contest is played. We can conclude from Proposition 14 that the designer has no incentives to reveal his information from an ex ante perspective (i.e. when he can commit to an order of moves and an information policy *before* he knows the realization of types).

Regarding the contestants' overall expected payoff, we can immediately conclude that it is smaller under private information. Remarkably, this result also holds true for individually expected payoffs of the first as well as of the second mover. This is due to the dampening effect of private information: Ex ante expected bids of the contestants are more similar, which we can interpret as higher competition, which reduces the agents' payoffs. Although the first mover saves effort costs since his expected effort is lower, his "overall expected" probability of winning decreases disproportionately as the second mover's expected effort is higher under private information. For the second mover, effort costs drive up because his expected bis is higher.

Proposition 15 *Given the corresponding equilibrium outcomes of sequential contests (see Propositions 2 and 6), contestants prefer public information to private information.*

Proof.

See appendix. ■

[30] It is straightforward to verify these results but not necessary for the proof of Proposition 14. Therefore, we omit the proofs.

Thus, there is an incentive for the contestants to share the information about their types in the sequential contest. If they could commit to information disclosure, contestants would like to do so (ex ante). Whether contestants would like to disclose their private information at the time they know the realization is an interesting question but we do not want to address it in this paper.

1.7 Other Aims of the Designer

1.7.1 Minimizing the Expected Effort Sum

Up to now, we assumed that the principal wants to maximize the expected effort sum. There are of course settings in which this is not true. If we think of wasteful expenditures, like in the rent-seeking literature, then the principal in contrast aims at minimizing the expected effort sum. From our analysis, we can immediately conclude that in this case the principal prefers simultaneous contests (compare Proposition 9). Regarding the contestants nothing changes to before. Given public information, they still prefer the sequential setting as well. Given private information, the first mover still prefers the sequential setting and for the second mover the result depends on the correlation of valuations. This implies that it is never possible to align preferences of all three.
Regarding the information setting, nothing changes when agents move simultaneously. All parties are indifferent between public and private information from an ex ante perspective. When agents act sequentially, however, all three would then prefer public information (compare Propositions 14 and 15).

1.7.2 Close Race

There may be different aims of the designer of a contest than maximizing or minimizing the effort sum. In particular, when we think of sports contests, it may be that a designer aims at having a close race between athletes in order to attract the audience rather than to maximize the sum of efforts.[31] The gap between high and low types is larger in sequential contests under public information than in simultaneous contests (compare Section 1.5.1). Similarly, the expected gap in the sequential contest is also higher than in the simultaneous contest in case of private information.[32]

[31] Another aim in sports contests would be that athletes break records. In this case the designer would like to maximize "the highest effort" in the contest. See e.g. Moldovanu and Sela (2006). Given public information, the sequential contest would then be preferred since for homogeneous types efforts are identical but for heterogeneous types either the high type in the sequential contest makes a higher bid than in the simultaneous contest. This result will at least partly hold true for private information since the expected gap between low and high types is larger in the sequential contest and also the expected effort sum can be larger.

[32] We assume that agents are randomly assigned to be the first or second mover. Also under public information, the designer fixes the order of moves before he knows the realization of types. This means that when types are

The reason is that in the sequential contest with private information there is not only a gap between the efforts of heterogeneous types, but also between the efforts of homogeneous types as the first and second mover do no longer exert the same effort.

For simultaneous contest, the ex ante expected gap between the efforts of the contestants is higher under private information than under public information. Irrespective of the information setting, homogeneous types exert the same effort in equilibrium in simultaneous contests. Thus, no gap arises with homogeneous types. When types are heterogeneous, then the gap under private information is larger. This is due to increased competition under private information when types are actually heterogeneous: Under private information contestants are not sure that their types differ. Thus, the bid of a low (high) type under private information is larger than the bid of a low (high) type under public information who knows that his opponent is a high (low) type. This effect is larger for the high type than for the low type. We can say that the *ability effect* increases the effect of more competition when agents do not know that their types actually differ.[33]

For sequential contests it is the other way around. The ex ante expected gap is larger under public information. For the sequential setting private information dampens the differences, although no gap arises for homogeneous types under public information but a gap arises under private information. The reason is the additional *commitment effect* that offsets the *competition intensity effect* when the first mover is a high type. Under public information this effect is stronger than under private information: The high type of the first mover makes a very large bid, when the second mover is a low type and correspondingly the second mover makes a low bid in this case. Private information dampens this result as contestants do not know the type of the opponent with certainty. This yields the following proposition.

Proposition 16 *Suppose contestants bid according to the corresponding equilibrium outcomes of simultaneous and sequential contests (see Propositions 1, 2, 3 and 6). Then a risk neutral designer who aims at having a close race, prefers simultaneous contests to sequential contests from an ex ante perspective. Given a simultaneous setting, he prefers public information to private information and given a sequential setting he prefers private information to public information from an ex ante perspective.*

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heterogeneous, the 1^{st} mover in the sequential contest is with probability one half the high and the low type, respectively. This assumption does not change the result for public information. With private information, if the probability that the 1^{st} mover is the high type is larger than 1/2, the result still holds, too. We can show that – given heterogeneous types – the expected effort difference in the sequential contest is larger than in the simultaneous contest, when the 1^{st} mover is the high type. If the probability is lower than 1/2, however, it is possible that the expected gap in the simultaneous contest is larger.

[33]These results follow from the observations in Proposition 4.

Proof.

See appendix. ■

1.8 Conclusion

In this paper we compare a two-player contest under four different institutional settings: We consider simultaneous and sequential orders of moves and public as well as private information about the player's types. Ex ante, sequential contests are preferred to simultaneous contests by an effort-maximizing designer and also by the first mover irrespective of the information setting. The second mover, however, does not necessarily prefer the sequential contest in the private information setting. Whether he prefers the simultaneous or sequential setting depends on the distribution of types. When correlation between types is sufficiently negative, he prefers a sequential order of moves (ex ante), too. In the setting with public information sequential contests arise endogenously. This result can only partly be extended to private information since there exists a first mover advantage and therefore a strong incentive to move first.

Furthermore, an effort-maximizing designer ex ante prefers private information given a sequential contest, whereas contestants prefer public information when the order of moves is sequential. Given simultaneous contests, the designer as well as the contestants are indifferent between public and private information from an ex ante perspective.

This result may no longer hold when we include considerations of time. A sequential order of moves prolongs the contest. Depending on the type of contest, this may be a crucial feature reducing or even offsetting the aforementioned advantage of sequential contests. It may be expensive to organize a contest that lasts longer. Sequential contests,however, are also "cheaper" for the effort-maximizing designer in the sense, that he can reduce the prize up to some point and his expected revenue in the sequential contest still equals his expected revenue in the simultaneous one. This advantage may outweigh the costs of the extra time.

Hence, we can conclude that it is not only outside restrictions that determine the timing structure of contests we observe in reality, but the combination of both these restrictions and the differing incentive effects of the structures.

Chapter 2

Teams and Intermediate Information[1]

2.1 Introduction

This paper studies the issue of how to organize the information structure between agents who jointly work on a project. Should the principal instruct his agents to work simultaneously (i.e. no intermediate information is available to the agents) or is it better when they work sequentially (i.e. intermediate information is available)? When analyzing this question, we take the team structure as given and focus on the effect the informational change has on incentives.

In real life, we frequently observe team production or problem-solving teams. Osterman (1994, 2000) estimates in a survey that in 1992 and 1997 about 40 percent of the manufacturing establishments (that have more than 50 employees) in the U.S. have more than half of their employees working in teams. Hence, teams became important work practices. A recent trend is the assignment of software development teams instead of single agents. Teasley et al. (2002) conduct an empirical study on the performance of software development teams working either in open space offices or private offices.

Team members are, in general, paid according to some joint performance evaluation scheme. Therefore, team performance suffers from free-riding problems. Nevertheless, teams may be performance enhancing in reality for other reasons like synergy effects or increased employee satisfaction. The free-riding problem, however, may still remain. There are studies that examine how team production can be improved by circumventing the free-riding problem. One argument is that a sequential order of moves – where the second mover receives non-verifiable information about the first mover's action – reduces shirking (see Winter (2005)) and may attain efficient production under budget-balancing (see Strausz (1999)). In line with these approaches, we compare two settings: one in which intermediate information is available as agents act sequentially and one in which it is not. In contrast to the literature, we allow that the joint project takes

[1] This chapter is based on joint work with Julia Nafziger.

an additional intermediate value. This means that the project cannot only succeed or fail, but can also take an intermediate value. We show that in this case it need no longer be true that the sequential setting with intermediate information is necessarily better: The principal prefers a simultaneous setting (i.e. no intermediate information is available) if individual contributions are perfect substitutes rather than perfect complements regarding the joint output. If we depart from the case of perfect complements, it is possible that the simultaneous structure becomes optimal for complementary contributions.

The problem of how to design team production is, for instance, known from the automobile production. On the one hand, there is the assembly line approach, where all agents act sequentially but can observe their predecessors' performance. On the other hand, there is the possibility that several agents work simultaneously on the product before it passes to the next production stage. BMW, for example, uses the latter structure, where several teams work sequentially.

Another example of team production, mentioned above, are software development teams. It is possible that agents work simultaneously on the project but perform different tasks. Explicit agreements on different working hours may induce a sequential setting. In the latter setting, it might well be that the second mover can evaluate the performance of his colleague (since he is involved in the same project), but for a third person it might be very difficult to evaluate the performance on some program that is not yet finished. Similar issues arise with other kinds of joint research projects.

Our setting corresponds to such projects where it is difficult or prohibitively costly (at least compared to the projects value) to evaluate intermediate results for a person that is not directly involved, but easy to evaluate the whole project.

We consider the question of the optimal information structure in a moral hazard model with a risk neutral principal and two agents, who are risk neutral but protected by limited liability. The agents jointly work on a project. Each agent's contribution to the joint project can be either of high or low quality. The quality of an agent's contribution depends on this agent's (unobservable) effort. In the following we denote the probability that a contribution is of high quality by *success* probability of an agent. As agents work in a team, the principal can only observe the (verifiable) joint output – i.e. the value of the project – and not the quality of each agent's contribution. The principal can, however, instruct his agents to make their contributions simultaneously or sequentially. This means he can specify ex ante which agent is in control. In case agents work sequentially, the second mover (agent 2) can perfectly observe the quality of the first mover's (agent 1) contribution. Thus, the second mover can condition his effort on the quality the first mover provided. Since effort is unobservable, the principal offers an incentive scheme to each agent to implement the desired effort. We derive the optimal wage scheme for both information scenarios – either agents move simultaneously or they move sequentially. When analyzing the effects of the information structure on incentives and on the principal's expected payoff, we find that the optimal structure depends on whether the individual contributions enter the principal's production function in a complementary or substitutable way. Our main result is that the sequential structure is superior if contributions are perfect complements, while the simultaneous structure is superior when contributions are perfect substitutes.

When contributions are complements, an agent's contribution is more effective regarding the value of the whole project, the higher the quality of the other agent's contribution. We begin by analyzing the special case of perfect complements, in which a low quality contribution of one agent leads to a failure of the whole project. This means, the principal only observes whether both agents' contributions are of high quality (hence the project succeeded) or whether the project failed, i.e. at most one agent's contribution is of high quality. In this special case, the project is of no value if at least one agent's contribution is of low quality. Under the sequential structure, the principal cannot incentivize the second mover when the first mover provided low quality. Hence, the second mover exerts no effort when the first agent provided low quality and the principal pays a wage of zero. The second mover only needs to be paid in case the first mover's contribution is of high quality. This enables the principal to better incentivize him: The principal saves implementation costs. Moreover, his revenues do not decrease since a high quality contribution from both agents is needed to generate revenues. This implies that expected revenues are not influenced by the effort the second mover exerts in case the first mover provides low quality. Thus, if contributions are perfect complements, the principal prefers the sequential structure. In an extension, we also consider more general cases of complements. Here, it may happen that the simultaneous structure becomes optimal.

For the case of substitutability, we assume that one agent's contribution is equally (or less) effective irrespective of the quality of the other agent's contribution. When contributions are substitutes, the principal observes three different values of the project. Either it is of high value (i.e. both contributions are of high quality), low value (both are of low quality) or of intermediate value (one contribution is of high, one of low quality). In the latter case, the principal does not know, whose contribution has which quality (as agents work in a team). We find that the principal prefers the simultaneous structure when contributions are perfect substitutes (which we define as an agent's contribution is *equally* effective irrespective of the quality of the other agent's contribution). There are two driving forces for this result: On the one hand, the feasible wage scheme changes between the structures as soon as the principal implements a positive effort for the second mover in case the first mover provided low quality. This change drives up implementation costs. On the other hand, a negative effect on expected revenues arises if the principal conditions the second mover's effort to the state of the world. When the effort of the second mover differs contingent on the quality of the first mover's contribution, the value of the concave expected revenue function decreases. This implies that irrespective of whether the principal implements zero effort for the second mover after a low quality contribution of the first mover or not, the simultaneous structure leads to higher expected profits. The negative revenue effect of the sequential structure that is crucial for our result has not been considered in the literature by now to the best of our knowledge. This effect only arises when different values of the project are taken into account, which has been ignored so far. In an extension, we also consider more general cases of substitutability.

The literature on multi-agent moral hazard problems started with Holmström (1982). In Holmström's partnership model, agents jointly produce and commonly share an output. The output deterministically depends on the agents' efforts, which are unobservable. Holmström shows that such form of organization is inefficient if budget-balancing is required for the sharing rule. The reason is that a team member shirks as he has to bear the cost for his effort himself, while the marginal benefits of his effort are shared.[2] For such partnerships, Rasmusen (1987) finds that with risk-averse agents efficiency can be reached by allowing for random punishments. Legros and Matthews (1993) show that with Leontief production functions (i.e. effort are complements) the inefficiency resolves as well.

The literature on team production under moral hazard discusses various mechanisms to alleviate the problem of free-riding. As a means to solve the free-riding problem in one-shot interactions, Kandel and Lazear (1992) consider how teams generate social pressure. They analyze how the disutility from such social pressure can be optimally exploited. This peer pressure may, for example, arise from the possibility that team members can monitor each other. If they can do so, the principal could also implement a mechanism that induces agents to report on their colleague's effort to reduce incentives to free-ride (see Marx and Squintani (2002)). Miller (1997) analyzes such a message game in case that a single agent can observe his colleagues' efforts and can report his observation to the group. He shows that efficiency can be sustained by a budget-balancing sharing rule. When teams are subject to moral hazard and adverse selection, McAfee and McMillan (1991) show that – under some conditions – it does not matter whether the principal observes the final output or individual contributions to the output. Hence, monitoring is not necessary to prevent shirking in this case. In this paper, we abstract from the possibility of monitoring and reporting but instead change the production process such that the second mover can observe the first mover's contribution. Che and Yoo (2001) consider repeated interactions and find that if an agent can observe after each period whether the colleague shirked or not, free-riding decreases.

Most closely related to our paper is the paper by Winter (2005). He compares different information

[2] The basic model is extended by Battaglini (2004) to a multi-dimensional strategy and output space.

structures when a sequence of agents collectively works on a project and moral hazard is present. Winter consideres any information structure in between (and including) the following two extreme cases. One extreme case is that agents are not able to observe effort decisions of their predecessors (corresponding to simultaneous effort choices). The other extreme case is that each agent knows the effort decision of all his predecessors (corresponding to sequential effort choices). More (less) transparency then corresponds to more (less) agents being able to observe the effort of more (less) of their predecessors. There is no mutual knowledge about effort decisions in the sense that an agent who acts earlier does not know the effort of an agent who acts later. In contrast to our model, Winter analyzes the case that agents are only rewarded in case of a success of the joint project. When agents have a binary effort choice – either they exert effort or not – more transparency is always favorable to the principal if he wants to induce all agents to exert effort. This result holds irrespective of whether the agents' efforts are complements or substitutes (i.e. whether the production technology possesses decreasing or increasing returns to scale). When more agents make their effort choice contingent on the decision of (more of) the other agents, shirking is more harmful to the probability that the joint project succeeds. Hence, incentives increase and implementation costs decrease. Winter's result stands in sharp contrast to our result that a sequential structure needs not be optimal when different values of the project are considered (and can be rewarded). The reasons for these different results are the aforementioned change in the feasible wage scheme and the revenue effect.

For the case that agents move sequentially and each agent can observe the effort decision of all his predecessors, Winter (forthcoming) shows that agents who move later (i.e. their effort is observed by less agents) should receive (weakly) higher rewards. Agents who move later, face a minor threat that their own shirking induces their followers to shirk as well.[3]

Similar to our approach, Gould and Winter (2005) consider a model, in which an agent's wage depends on the vector of the agents' performances. They show that positive as well as negative peer effects may arise under team production without behavioral effects as peer pressure. A positive peer effect means that a high (low) performance of one agent increases (decreases) the other agent's effort. Depending on whether agents deal with complementary or substitutable tasks, respectively, positive or negative peer effects arise. Task complementarities are reflected by the wage scheme. These properties of the wage scheme are imposed and are not part of the optimal wage scheme (which they do not derive).[4] In contrast to our paper, Gould and Winter do not compare different information structures and do not include complementarities of the agents' performances. They present empirical evidence for their theory using data on the performance of

[3]Winter (forthcoming) also shows that agents with a lower effort costs (i.e. a higher ability) should move later.

[4]The optimal wage scheme that we derive, satisfies their definition of complementary tasks.

professional baseball players.

We do not analyze whether it is optimal to employ a team, but assume that this is an existing relationship that cannot be changed for the project. The question of how to optimally structure agents' interaction, is, for instance, considered by Goldfain (2006) and Hemmer (1995). Goldfain (2006) analyzes in an R&D setting when it is optimal to employ a team (where synergy effects are present), two competing agents or only one agent. Regarding the team, Goldfain (2006) distinguishes between simultaneous and sequential effort choices. She presents numerical results, which suggest that when efforts are strategic substitutes, the performance of the team is not increased under the sequential structure compared to the simultaneous one. This is in line with our result when contributions are substitutes. Hemmer (1995) approaches the question how to assign agents to a sequence of tasks. He finds that it is optimal to organize agents in a team to deal with these subsequent tasks instead of assigning different agents to single tasks, when there exist synergies from dealing with several tasks.

The question whether to provide intermediate information is optimal is also considered by Lizzeri et al. (2002) and Ludwig and Nafziger (2006). Lizzeri et al. (2002) analyze the question whether the principal should provide a single agent (who works for two periods) with intermediate information about his output or not. In contrast to Winter (2005), they find that it is always optimal for the principal *not* to provide feedback. The driving force that turns intermediate information unfavorable, is a change in the wage scheme as it also arises in our setting: The agent has to be rewarded after a failure in the first period, too, otherwise he does not exert any effort when he gets the "negative" feedback.

Ludwig and Nafziger (2006) consider two agents who do not work on a joint project, but separately produce an output. The agents' success probabilities are allowed to be dependent. Like in the current paper, the principal can instruct his agents to work simultaneously (i.e. no intermediate information is available) or sequentially (i.e. intermediate information is available to the second mover). In the latter case, the second mover can observe the output of the first mover. Which information structure is optimal, depends on whether the colleague's output is informative about an agent's effort or not. If it is uninformative, a sequential structure is optimal. If the colleague's output is informative, the simultaneous structure can be optimal as well. Independent success probabilities – like we consider in the current paper – imply that the colleagues output is uninformative and thus that a sequential structure is optimal. Similarly to the present paper, one driving force is a (possible) change in the feasible wage scheme.

Another field that is related to our issue, is the literature on how to design jobs or allocate tasks when moral hazard is present (see e.g Prescott and Townsend (2006), Schmitz (2005) or Itoh (1994)). Furthermore, the literature on sequential tournaments is related to this paper. In tournaments, agents generally compete against each other and do not form a team. Contrary to the current paper, in the tournament literature, the wage scheme is often exogenous (e.g. Aoyagi (2003), Ederer (2004) or Morgan (2003)) or it is solely a strategic effect that drives the result (e.g. Jost (2001) or Jost and Kräkel (2000)), which means that the first mover tries to influence the action of the second mover.

The paper is structured as follows. In Section 2.2, we present the basic model. In Section 2.3, we first derive the optimal wage scheme for the simultaneous and sequential structure given that contributions are perfect complements. Then, in Section 2.4, we analyze which of the two structures the principal prefers. In Section 2.5, we consider the case that contributions are perfect substitutes. Again, we derive the optimal wage schemes and, in Section 2.6, we analyze which structure the principal prefers. Afterwards, in Section 2.7, we extend the analysis to more general cases of complementarities. In Section 2.8, we discuss our results and relate them to the literature. We conclude in Section 2.9.

2.2 The Model

There are two agents, $i \in \{1, 2\}$, and a risk neutral principal. Both agents jointly work on a project. They are risk neutral but protected by limited liability. The value of their outside option is zero. If both agents accept to work for the principal production takes place, otherwise the relationship terminates and the payoff of every player is zero. In the former case, each agent makes a contribution $Y_i \in \{L, H\}$ to the joint project. The contribution is either of high quality (H) or of low quality (L). We refer to a specific quality realization as "state of the world". The probability that an agent's contribution is of high quality depends on his effort $e_i \in [0, \bar{e}] \equiv \mathcal{I}$ where $p(e_i) := \Pr(H|e_i)$, $p : \mathcal{I} \to (0, 1)$. Moreover, $p \in C^3$, concave and strictly increasing in effort. The probability that the contribution has low quality is $1 - p(e_i)$. We refer to the probability that an agent's contribution has high quality as this agent's *success* probability.

The quality combination Y_iY_{-i} realizes with probability $p^{Y_iY_{-i}}(e_i, e_{-i}) := \Pr(Y_iY_{-i}|e_i, e_{-i})$, where $p^{Y_iY_{-i}} : \mathcal{I} \times \mathcal{I} \to (0, 1)$ and $p^{Y_iY_{-i}} \in C^3$. We consider independent individual success probabilities. This implies that $p^{Y_iY_{-i}}(e_i, e_{-i})$ is equal to the product of the respective individual probabilities. Hence, an agent's individual success probability $p(e_i) = p^{HH}(e_i, e_{-i}) + p^{HL}(e_i, e_{-i})$ depends only on his own effort and not on the other agent's effort.

Denoting by $p^{HH}_{e_i}(e_i, e_{-i})$ the derivative of $p^{HH}(e_i, e_{-i})$ with respect to agent i's effort, it follows that $p^{HY_{-i}}_{e_i}(e_i, e_{-i}) > 0$, $p^{LY_{-i}}_{e_i}(e_i, e_{-i}) < 0$, $p^{HY_{-i}}_{e_ie_i}(e_i, e_{-i}) \leq 0$, and $p^{LY_{-i}}_{e_ie_i}(e_i, e_{-i}) \geq 0$.

Providing effort is costly for the agents. The cost to provide effort e is $c(e)$, where $c : \mathbb{R}^+_0 \to \mathbb{R}^+_0$, $c \in C^3$ with $c'(\cdot) > 0\ \forall e_i > 0$, $c''(\cdot) > 0$ and $c(0) = c'(0) = 0$.

The revenue of the principal is $\pi\mathcal{Y}(Y_i, Y_{-i})$, where π is some strictly positive constant and $\mathcal{Y}(Y_i, Y_{-i})$ is the value of the project (or the agents' joint output), which depends on both agents' contributions

Y_i and Y_{-i}. We distinguish between two cases: Contributions are either complements or substitutes. When contributions are complements, the quality of an agent's contribution is more effective, the higher the quality of the other agent's contribution, i.e. $\mathcal{Y}(H,H) - \mathcal{Y}(H,L) > \mathcal{Y}(L,H) - \mathcal{Y}(L,L)$. When contributions are substitutes, $\mathcal{Y}(H,H) - \mathcal{Y}(H,L) \leq \mathcal{Y}(L,H) - \mathcal{Y}(L,L)$. In the main part of the paper, we consider perfect complements and perfect substitutes as introduced below. In an extension, we refer to more general cases of complements and substitutes.

First, we analyze the case that agents' contributions are perfect complements. This means $\mathcal{Y}_c(Y_i, Y_{-i}) := \mathcal{Y}(Y_i, Y_{-i}) = \min\{Y_i, Y_{-i}\}$. For simplicity, we assume $H = 1$ and $L = 0$. In case agents' contributions are perfect complements, a high quality contribution of both agents is needed for the project to be of high quality (which we denote by $\mathbf{H}$), otherwise the project is of low quality (which we denote by $\mathbf{L}$). Therefore, $\mathcal{Y}_c(Y_i, Y_{-i}) \in \{0,1\} \equiv \{\mathbf{L}, \mathbf{H}\}$. Secondly, we consider that the agents' contributions are perfect substitutes. Again, we assume $H = 1$ and $L = 0$. Substitues imply that the value of the project can take three values: It can be of high value when both agents contribute high quality, which we denote by $\mathcal{H}$. If only one agent contributes high quality, it takes an intermediate value (which we denote by $\mathcal{M}$) and if both agents contribute low quality, the project is of low value (which we denote by $\mathcal{L}$). The value of the joint project when individual contributions are perfect substitutes is $\mathcal{Y}_s(Y_i, Y_{-i}) := Y_i + Y_{-i}$. Thus, $\mathcal{Y}_s(Y_i, Y_{-i}) \in \{0,1,2\} \equiv \{\mathcal{L}, \mathcal{M}, \mathcal{H}\}$. We change the notation here only to not confuse both cases later on. Hence, the set of possible values of the project differs for the case of perfect complements and substitutes.

The timing and the information structure of the game are as follows. At date 0, the principal decides whether agents move sequentially (we denote the first mover by agent 1 and the second mover by agent 2) or simultaneously. At date 1, the principal offers to each agent i a wage scheme $\mathbf{w}(\mathcal{O})$. This scheme conditions on all observable and verifiable variables ($\mathcal{O}$). If agents move simultaneously, both agents provide unobservable effort at date 2. If they move sequentially, only agent 1 provides unobservable effort at date 2. Agent 2 can observe the realization of the quality of the first mover's contribution[5] and then provides effort as well. The principal, however, can neither observe the quality of the first mover's contribution nor the agents' efforts, but only the value of the project $\mathcal{Y}(Y_i, Y_{-i})$ (which is also verifiable). Finally, the project's value and payoffs realize: The principal receives his revenues and pays the wages to the agents as specified in the wage scheme. Each agent receives his wage minus his effort costs.

[5]We show later that it does not matter for our results whether the second mover also observes the effort of the first mover or not.

2.3 Contributions are Complements

In this section, we consider the case that agents' contributions are perfect complements in the principal's production function. This means that the value of the project is $\mathcal{Y}_c = \min\{Y_i, Y_{-i}\}$. Hence, the principal observes whether both agents perform well or whether at least one of them performs poorly. Note that we stick to the case of perfect complements in the whole section. Whenever we refer to complements, we indeed mean perfect complements in this section.

The principal maximizes his expected profit (i.e. expected revenues minus implementation costs) with respect to wages and efforts, subject to the incentive, participation and limited liability constraints of the agents.[6] Since agents are protected by limited liability, their expected payoff in equilibrium is at least zero. Hence, by entering the relationship, they cannot be worse off than by not entering it (the outside option is zero). We assume that agents accept to work for the principal when they are indifferent between accepting or not accepting.

As usual in a moral hazard model, we can decompose the principal's maximization problem into two parts. In the first part, we take the efforts the principal wants to implement as given and maximize profits with respect to wages subject to the three constraints. As expected revenues (which are π times the probability that both agents provide high quality) do not depend on wages, this is equivalent to minimizing the expected wage payment. The solution to the problem is the optimal wage scheme. In the second part of the problem, the principal maximizes his expected revenues minus the wages (which depend on effort) with respect to effort. We do not investigate whether it is indeed optimal for the principal to employ both agents (by implementing zero effort/paying a wage of zero he can ensure a payoff of zero).

2.3.1 The Wage Scheme for the Simultaneous Structure

In this section, we consider the first part of the principal's problem and derive the optimal wage scheme for the simultaneous structure to implement any desired effort level $(\hat{e}_i, \hat{e}_{-i})$. Under the simultaneous structure (as well as under the sequential one), the principal offers a wage scheme to each agent that depends on the value of the project, which is the only observable and verifiable variable: $\mathbf{w}(\mathcal{Y}_c) = \{w_i^{\mathbf{L}},\ w_i^{\mathbf{H}}\}$. In the appendix we argue that the participation constraint of each agent is satisfied if the limited liability constraint (which requires $w_i^{\mathcal{Y}_c} \geq 0\ \forall i,\ \forall \mathcal{Y}_c(Y_i, Y_{-i})$) and the agent's incentive constraint are satisfied. Hence, we can drop the participation constraint in the following and consider only the incentive and limited liability constraint. Using this result, the problem of the principal (under the simultaneous structure) is to minimize expected wage payments

[6] For the sequential structure we require that the ex ante participation constraint is satisfied.

subject to the incentive and limited liability constraints.

$$\max_{\mathbf{w_i^H},\mathbf{w_i^L}} \quad -\sum_i p^{HH}(\hat{e}_i,\hat{e}_{-i})w_i^{\mathbf{H}} - \sum_i [1-p^{HH}(\hat{e}_i,\hat{e}_{-i})]w_i^{\mathbf{L}}$$

$$\text{s.t. } \hat{e}_i \in \arg\max_{e_i \in \mathcal{I}} \; p^{HH}(e_i,\hat{e}_{-i})w_i^{\mathbf{H}} + (1-p^{HH}(e_i,\hat{e}_{-i}))w_i^{\mathbf{L}} - c(e_i), \; \forall i \qquad (IC_i^{sim_c})$$

$$\text{s.t. } w_i^{\mathcal{Y}_c} \geq 0 \; \forall i, \; \forall \mathcal{Y}_c,$$

where $\hat{e}_i$ and $\hat{e}_{-i}$ denote the Nash-equilibrium effort levels.[7]

From the incentive constraint $(IC_i^{sim_c})$, we see that setting $w_i^{\mathbf{L}} > 0$ decreases incentives as $(1 - p^{HH}(e_i,\hat{e}_{-i}))$ is decreasing in e_i. Moreover, it lowers the principal's profits. Hence, setting $w_i^{\mathbf{L}} = 0$ is optimal. Using this result, we can derive the optimal wage when both agents provide high quality from the agent's incentive constraint. The solutions to the principal's problem are summarized in the following lemma.

Lemma 2 *Suppose $\mathcal{Y}(Y_i,Y_{-i}) = \mathcal{Y}_c(Y_i,Y_{-i})$ and agents move simultaneously. Then the wage for agent i to implement effort $\hat{e}_i$ is zero when at least one agent fails to provide high quality, i.e. $w_i^{\mathbf{L}} = 0$. If both agents perform well, the wage is non-negative: $w_i^{\mathbf{H}} = \frac{c'(\hat{e}_i)}{p_{e_i}^{HH}(\hat{e}_i,\hat{e}_{-i})}$.*

It straightforwardly follows from Lemma 2 that expected costs to implement effort $\hat{e}_i$ for agent i – given an effort $\hat{e}_{-i}$ of the other agent – are

$$\mathcal{W}_i^{sim_c} := \frac{p^{HH}(\hat{e}_i,\hat{e}_{-i})}{p_{e_i}^{HH}(\hat{e}_i,\hat{e}_{-i})}c'(\hat{e}_i) = \frac{p(\hat{e}_i)}{p'(\hat{e}_i)}c'(\hat{e}_i). \tag{2.1}$$

2.3.2 The Wage Scheme for the Sequential Structure

In this section, we derive the wage scheme for the sequential structure, starting with the second mover. Note that in the sequential structure, the second mover only observes the first mover's performance and *not* his effort before he decides on his own effort.[8] Hence, he forms beliefs about the first mover's effort that have to be correct in equilibrium. The principal does neither observe the first mover's effort nor his performance. As the second mover learns the quality $Y_1 \in \{H, L\}$ of the first mover's contribution, he can update his success probabilities:

$$\Pr[Y_2|H,\hat{e}_1,e_2] = \frac{p^{Hy_2}(\hat{e}_1,e_2)}{p(\hat{e}_1)} \text{ and } \Pr[Y_2|L,\hat{e}_1,e_2] = \frac{p^{Ly_2}(\hat{e}_1,e_2)}{1-p(\hat{e}_1)}. \tag{2.2}$$

[7] Note that the chosen effort needs not only be the maximizer of the agent's profit, but also that an agent's beliefs have to be correct in equilibrium, i.e. the agent correctly predicts the effort level $\hat{e}_{-i}$ of his colleague. A Nash-equilibrium in pure strategies of this game exists as the strategy space is is compact and convex and the objective function is quasi-concave. Concerning uniqueness, we follow Mookherjee (1984) and assume that Nash-equilibrium implementation is possible.

[8] We can also allow the second mover to additionally observe the effort of the first mover without changing our results as we show in the appendix (See Proof $\mathcal{A}$.).

Moreover, the second mover can condition his effort on the observed quality. Thus, we have to consider two incentive constraints – one for the case that the first mover's contribution is of high quality and one when it is of low quality:

$$\text{After seeing } H: \; e_2^H \in \arg\max_{e_2 \in \mathcal{I}} \; \Pr[H|H, \hat{e}_1, e_2] w_2^{\mathbf{H}} + \Pr[L|H, \hat{e}_1, e_2] w_2^{\mathbf{L}} - c(e_2),$$
$$\text{After seeing } L: \; e_2^L \in \arg\max_{e_2 \in \mathcal{I}} \; w_2^{\mathbf{L}} - c(e_2).$$

After observing a low quality contribution of the first mover, the second mover receives the failure wage $w_2^{\mathbf{L}}$ for sure. This implies that it is not possible to set incentives for the second mover to exert any positive amount of effort in state L. Thus, $e_2^L = 0$ irrespective of $w_2^{\mathbf{L}}$. The failure wage also enters the incentive constraint of the second mover after having observed a high quality contribution of the first mover. Here, we can see that setting $w_2^{\mathbf{L}}$ larger than zero, decreases incentives of the second mover in state H since $\Pr[L|H, \hat{e}_1] = 1 - p(e_2)$ decreases in e_2. Taken both observations together, we have that $w_2^{\mathbf{L}} = 0$ is optimal for the principal. Using $w_2^{\mathbf{L}} = 0$, the second order conditions of the maximization problems are obviously satisfied as the cost function is strictly convex and p^{HH} is concave in e_2. Therefore, we can derive the optimal wage for the second mover when both agents perform well, i.e. $w_2^{\mathbf{H}}$, from the incentive constraint in state H. This yields that $w_2^{\mathbf{H}} = \frac{p(\hat{e}_1)}{p_{e_2}^{HH}(e_2^H, \hat{e}_1)} c'(e_2^H)$ is optimal to implement effort levels (e_2^L, e_2^H), given the effort $\hat{e}_1$ of the first mover. From this we obtain the following lemma.

Lemma 3 *Suppose $\mathcal{Y}(Y_1, Y_2) = \mathcal{Y}_c(Y_1, Y_2)$ and agents move sequentially. Then the wages for the second mover satisfy $w_2^{\mathbf{L}} = 0$ and $w_2^{\mathbf{H}} = \frac{c'(e_2^H)}{p'(e_2^H)}$. This yields expected implementation costs for the second mover – given an effort $\hat{e}_1$ of the first mover – of $\mathcal{W}_2^{seq_c} := p(\hat{e}_1) \frac{p(e_2^H)}{p'(e_2^H)} c'(e_2^H)$.*

As intuition suggests, the second mover only receives a positive wage if the joint project is of high value (like under the simultaneous structure). Compared to expected implementation costs under the simultaneous structure (see (2.1)), however, the second agent can be easier incentivized under the sequential structure. In order to implement the same effort for the second agent, the principal has to pay (weakly) less under the sequential structure. This stems from the fact that the second mover has to be incentivized only in case the first mover provided high quality. Hence, when the second mover observes the first mover performing well, he knows that the success of the project and thus his payment only depends on his own contribution. This means that free-riding is not profitable for the second mover. This result is similar to the result of Winter (2005), when he compares structures with different degrees of transparency. A more transparent structure in his model, is a structure that allows more agents to condition their effort decision on the decision of (more of the) other agents. He shows that it is cheaper to incentivize agents, whose decisions are observed by (more of) the others, as it makes shirking less attractive to them. The reason for his result is that shirking induces the later moving agents to shirk as well. Therefore, the probability that the project succeeds decreases even more.

The derivation of the wage scheme for the first mover is analogue to the simultaneous case. What changes is the first mover's belief about the second agent's effort: He anticipates correctly that the second mover provides effort e_2^H after observing a high performance and $e_2^L = 0$ after observing a poor performance. Hence, his incentive constraint becomes

$$\hat{e}_1 \in \arg\max_{e_1 \in \mathcal{I}} \; p^{HH}(e_1, e_2^H) w_1^{\mathbf{H}} + (1 - p^{HH}(e_1, e_2^H)) w_1^{\mathbf{L}} - c(e_1). \tag{2.3}$$

It follows (analogue to the proof of Lemma 2) that the first mover's wage if at least one agent performs poorly is zero and if both agents perform well it is $w_1^{\mathbf{H}} = \frac{c'(\hat{e}_1)}{p_{e_1}^{HH}(\hat{e}_1, e_2^H)}$. Hence, expected implementation costs for the first mover are $\mathcal{W}_1^{seq_c} = \frac{p(\hat{e}_1)}{p_{e_1}(\hat{e}_1)} c'(\hat{e}_1)$. Comparing the expected wage payment for the first mover with the one under the simultaneous structure (see (2.1)), we see that it is identical under both structures if the first agent's Nash-equilibrium effort levels are identical. Consider the first agent's incentive constraints under both structures for fixed and equal wages: The only difference stems from the second agent's Nash-equilibrium effort levels $\hat{e}_2$ and e_2^H, respectively. If these effort levels differ, the first agent's effort levels differ under both structures, otherwise they are identical. For fixed and equal wages for the second agent under both structures, $\hat{e}_2$ and e_2^H differ.[9]

2.4 Comparison of the Structures for the Case of Complements

After having derived the optimal wage scheme, the next step in solving the principal's problem would be to maximize the principal's profits (expected revenues minus implementation costs) with respect to effort – given expected implementation costs as derived in Lemma 2 and 3, respectively. We do not derive these optimal effort levels, but compare the problems for the sequential and simultaneous structure to make statements about the optimal information structure.

The principal's maximization problem for the sequential structure is

$$\max_{\mathbf{e_1} \in \mathcal{I}, \mathbf{e_2^H} \in \mathcal{I}} \quad 2p^{HH}(e_1, e_2^H)\pi - \frac{p(e_1)}{p'(e_1)} c'(e_1) - p(e_1) \frac{p(e_2^H)}{p'(e_2^H)} c'(e_2^H),$$

and for the simultaneous structure it is

$$\max_{\mathbf{e_1} \in \mathcal{I}, \mathbf{e_2} \in \mathcal{I}} \quad 2p^{HH}(e_1, e_2)\pi - \frac{p(e_1)}{p'(e_1)} c'(e_1) - \frac{p(e_2)}{p'(e_2)} c'(e_2).$$

Comparing these two problems, we see that for all effort pairs (e_1, e_2) the expected profit function for the sequential structure cannot lie below the one for the simultaneous structure since $p(e_1) \leq 1$. Hence, the two functions never (strictly) cross and the maximum of the expected profit function

[9] In principle, it is possible that the principal implements $\hat{e}_2 = e_2^H$ (and thus wages differ for the second agent under both structures), but in general, as we will see in Section **??**, the implemented effort levels differ.

for the sequential structure has to lie above the one for the simultaneous structure. It follows that – in general – the principal implements different effort levels under both structures (at least for one of the two agents). Thus, we have the following proposition.

Proposition 17 *Suppose $\mathcal{Y}(Y_1, Y_2) = \mathcal{Y}_c(Y_1, Y_2)$. Then the sequential structure leads to higher expected profits than the simultaneous structure given the optimal wage schemes and efforts. Expected profits are strictly higher if $(e_2^H, e_2) \neq (0, 0)$.*

Intuitively, this can be explained as follows. Under the sequential structure, the principal pays a wage of zero and the second mover exerts no effort when the first agent provided low quality. This saves implementation costs, but does not decrease revenues since high quality contributions from both agents are needed to generate the revenue π. As already mentioned in Section 2.3.2, the fact that the second mover only needs to be incentivized in case the first mover's contribution is of high quality, enables the principal to better incentivize him.

The only possibility that both structures do equally good is that it is optimal to implement zero effort for the second agent under *both* structures, i.e. $e_2^H = e_2^L = e_2 = 0$. This means that for the sequential structure to do strictly better than the simultaneous one, the principal necessarily *tailors* the second mover's efforts to the quality states under the sequential structure (i.e. he sets $e_2^H \neq e_2^L = 0$) as otherwise $e_2^H = e_2 = 0$.

This result confirms the result by Winter (2005) in a model with a binary effort decision: Winter finds that the principal always (weakly) gains from a more transparent structure (a structure that allows more agents to observe more of their predecessors). In Winter's model, however, the agents' efforts are complementary and not the quality of the individual contributions to the joint project. Hence, Winter does not consider different "values" of the joint project. When contributions are complements in our model, our results are thus similar to Winter as the project is of no value if at least one agent fails to contribute high quality. For a model of team production with synergy effects, Goldfain (2006) provides numerical results that are similar: If efforts are strategic complements, a sequential structure is beneficial for the principal. For the case of independent success probabilities (which we consider in the current paper), the result that the sequential structure cannot be worse than the simultaneous structure is also present in Ludwig and Nafziger (2006) with two agents, who do not form a team. Independent success probabilities imply that the output is uninformative in Ludwig and Nafziger.[10]

Moreover, we can relate this result to the literature on contests, where the reward for the winning agent is exogenously given. Morgan (2004) compares simultaneous and sequential moves of contestants in a model with complete information about the players' types. He shows that from an ex

[10] In Ludwig and Nafziger (2006), we consider also dependent success probabilities. For dependent success probabilites, it is possible (when output is informative) that a simultaneous structure is optimal.

ante perspective a sequential structure even Pareto dominates a simultaneous one. With private information about the players' types this result can be extended if types are sufficiently negatively correlated. The principal, however, always prefers the sequential structre (see Ludwig (2006)). In a tournament model with risk averse agents, Jost (2001) finds that for optimal prizes, the principal prefers a sequential structure.

Hence, the finding that a sequential structure is (weakly) beneficial for the principal seems to be quite robust across different types of models. In the following section, we show that this is no longer the case, when the agents' contributions are substitutes.

2.5 Contributions are Substitutes

We now turn to the analysis for the case that agents' contributions are perfect substitutes in the principal's production function. The difference to perfect complements is that a contribution of low quality of a single agent does not necessarily lead to a value of zero of the team's project. If the other agent performs well, the project is still of some strictly positive value. This means that the project can generate revenues for the principal although one agent performed poorly. This implies that the principal can distinguish a complete failure of the team (i.e. both agents provide low quality) from the case that only one agent fails to provide high quality. In the latter case he cannot, however, observe which agent provides high and which one low quality, respectively: The principal does not observe individual performances, but the joint output $\mathcal{Y}_s(Y_i, Y_{-i}) = \sum_i Y_i$. This change implies that the wage scheme offered differs from the case of complements as it now specifies three wages $\mathbf{w}(\mathcal{Y}_s) = \{w_i^{\mathcal{L}}, w_i^{\mathcal{M}}, w_i^{\mathcal{H}}\}$ for each agent i.

2.5.1 The Wage Scheme for the Simultaneous Structure

Like for the case of complements, we begin by considering the first part of the principal's maximization problem. In the following section, we derive the optimal wage scheme for the simultaneous structure to implement any desired effort level $(\hat{e}_i, \hat{e}_{-i})$. As before, the problem of the principal is to minimizing the wage payments to the agents, subject to the incentive, participation and limited liability constraints. Analogue to the analysis with complements, we can drop the participation constraints and restrict to the limited liability and incentive constraints. Again, we can set the failure wages equal to zero ($w_i^{\mathcal{L}} = 0$ for $i = 1, 2$). Hence, the problem of the principal is:

$$\begin{aligned}
\max_{\mathbf{w_i^{\mathcal{H}}}, \mathbf{w_i^{\mathcal{M}}}} \quad & -\sum_i p^{HH}(\hat{e}_i, \hat{e}_{-i}) w_i^{\mathcal{H}} - \sum_i [p^{HL}(\hat{e}_i, \hat{e}_{-i}) + p^{LH}(\hat{e}_i, \hat{e}_{-i})] w_i^{\mathcal{M}} \\
& \text{s.t. } \hat{e}_i \in \arg\max_{e_i \in \mathcal{I}} \; p^{HH}(e_i, \hat{e}_{-i},) w_i^{\mathcal{H}} + \left[p^{HL}(e_i, \hat{e}_{-i}) + p^{LH}(e_i, \hat{e}_{-i})\right] w_i^{\mathcal{M}} - c(e_i) \; \forall i \\
& \text{s.t. } w_i^{\mathcal{Y}_s} \geq 0 \; \forall i, \; \forall \mathcal{Y}_s(Y_i, Y_{-i}).
\end{aligned}$$

In the appendix, we show that the solution to the principal's problem is as follows.

Lemma 4 *Suppose $\mathcal{Y}(Y_i, Y_{-i}) = \mathcal{Y}_s(Y_i, Y_{-i})$ and agents move simultaneously. To implement effort level $\hat{e}_i$, the wage for agent i if both agents or only one of them performs poorly is equal to zero, i.e. $w_i^{\mathcal{L}} = 0$ and $w_i^{\mathcal{M}} = 0$. The optimal wage if both agents provide high quality is $w_i^{\mathcal{H}} = \frac{c'(\hat{e}_i)}{p_{e_i}^{HH}(\hat{e}_i, \hat{e}_{-i})}$.*

In line with our result for complementary contributions in Lemma 2, only the wage if the joint project has a high value can be positive in the simultaneous setting with substitutes. The intuition for this result is that if agents are paid a positive wage in case only one of them performs well, each agent will try to free-ride on the other agent's effort as agents work in a team and the principal cannot observe which agent performs well. Therefore, it cannot be optimal to implement some kind of relative performance scheme by setting $w_i^{\mathcal{M}} > 0$. The literature on tournaments uses relative performance schemes to create incentives but, in general, the wage scheme is exogenously given in this literature. In Ludwig and Nafziger (2006), with two agents, who do not work in a team, the optimal wage scheme can be one of relative performance payment contingent upon the informativeness of the colleague's output about an agent's effort.[11] With two agents, who do not form a team, the principal can observe which agent contributes high quality and hence only pay *this* agent. This counteracts free-riding and can make relative performance schemes profitable.
It follows from Lemma 4 that the expected wage payment for agent i is

$$\mathcal{W}_i^{sim_s} = \frac{p(\hat{e}_i)}{p'(\hat{e}_i)} c'(\hat{e}_i). \tag{2.4}$$

The expected wage equals the expected wage $\mathcal{W}_i^{sim_c}$ when contributions are complements and agents move simultaneously. Hence, contribution complementarities make no difference for the wage scheme when agents move simultaneously. Winter (2005), in contrast, finds that with effort instead of performance complementarities, the optimal wage scheme differs. This is due to the fact that with complementary efforts implicit incentives are more effective in his model compared to the case of substitutes. This change in incentives is not present in our model. Irrespective of whether contributions are substitutes or complements, the agents' efforts are complements in Winter's sense. In Winter's model, efforts are complements when an agent's effort is more effective, the more of his colleagues exert effort.[12] In our model this means that the cross derivative of $p^{HH}(e_i, e_{-i})$ is positive, which is satisfied by assumption.

[11] As mentioned earlier, the case of independent success probabilities, we consider in the current paper, implies that output is uninformative in Ludwig and Nafziger (2006). For this case, the optimal wage scheme can be one of relative performance payment.

[12] Winter (2005) considers a binary effort choice and multiple agents.

2.5.2 The Wage Scheme for the Sequential Structure

In the sequential setting, the second mover again learns the quality $Y_1 \in \{H, L\}$ of the first mover (like in Section 2.3.2).[13] He updates his success probabilities as given in (2.2) and can condition his effort on the observed quality. By an analogue argument as for complements, it is optimal to set an agent's wage equal to zero in case both agents fail (i.e. $w_i^{\mathcal{L}} = 0$). Thus, the incentive constraints for the second mover are as follows (where we already used $w_i^{\mathcal{L}} = 0$):

$$\begin{aligned}\text{After seeing } H: \; e_2^H &\in \arg\max_{e_2 \in \mathcal{I}} \; \Pr[H|H, \hat{e}_1, e_2] w_2^{\mathcal{H}} + \Pr[L|H, \hat{e}_1, e_2] w_2^{\mathcal{M}} - c(e_2), \\ \text{After seeing } L: \; e_2^L &\in \arg\max_{e_2 \in \mathcal{I}} \; \Pr[H|L, \hat{e}_1, e_2] w_2^{\mathcal{M}} - c(e_2).\end{aligned}$$

The crucial change compared to the case of complements is that the principal can set incentives for the second mover in state L. By setting $w_2^{\mathcal{M}} > 0$, he can implement a positive effort for the second mover. Note that this decreases incentives in state H: The probability that the second mover fails to provide high quality given that the first mover succeeded decreases in the second mover's effort. Later, we also ask when it is optimal to implement $e_2^L > 0$.

As the second mover's maximization problem in state L is strictly concave in e_2, the first order condition – $w_2^{\mathcal{M}} = \frac{c'(e_2^L)}{p'(e_2^L)}$ – yields a global maximum. One can easily check (by plugging in the conditional probabilities) that the first order condition of the agent's maximization problem in state H is also sufficient when $w_2^{\mathcal{H}} \geq w_2^{\mathcal{M}}$. We show below that this holds in equilibrium. Plugging in the optimal wage that is derived from state L into the latter first order condition yields the optimal wage when the value of the project is high.

Regarding the wage scheme of the first mover, the same argument applies as for the sequential structure in Section 2.3.2. Hence, the first mover's wages are as for the simultaneous structure (see Lemma 4) but with the Nash-equilibrium effort level $\hat{e}_2$ of the second mover replaced by e_2^L and e_2^H, respectively. In the appendix, we derive the optimal wages as follows.

Lemma 5 *Suppose $\mathcal{Y}(Y_1, Y_2) = \mathcal{Y}_s(Y_1, Y_2)$ and agents move sequentially.*

(i) For the first mover, wages if at least one agent contributes low quality are zero: $w_1^{\mathcal{M}} = w_1^{\mathcal{L}} = 0$. When the value of the joint project is high, the wage satisfies $w_1^{\mathcal{H}} = \frac{c'(\hat{e}_1)}{p_{e_1}^{HH}(\hat{e}_1, e_2^H)}$.

(ii) For the second mover, the wage if the joint project fails is equal to zero $w_2^{\mathcal{L}} = 0$. If only one agent contributes low quality, his wage is positive if the principal implements $e_2^L > 0$ and is then $w_2^{\mathcal{M}} = \frac{c'(e_2^L)}{p'(e_2^L)}$. The wage when both agents provide high quality is $w_2^{\mathcal{H}} = \frac{c'(e_2^H)}{p'(e_2^H)} + \frac{c'(e_2^L)}{p'(e_2^L)}$.

Obviously, the second mover's wage when the project has a high value is at least as large than the wage when the project has an intermediate value, as we claimed above. This result is also intuitive

[13] Like for the case of complements in Section 2.3.2, it does not matter for our results if the second mover also observes the first mover's effort besides his performance. See Proof $\mathcal{B}$ in the appendix.

since two high quality contributions are more valuable for the principal than only one (at least if we abstract from implementation costs).

Note that only the second mover's effort in state H, e_2^H, influences the first mover's incentives since the principal sets $w_1^{\mathcal{M}} = w_1^{\mathcal{L}} = 0$. If also the effort in state L entered, we would have a strategic effect in the sense of the first mover attempting to influence the action of the follower: When deciding on his effort, the first mover takes into account that if he exerted more effort, it would be more likely that he produces high quality and therefore, the second mover provides effort e_2^H (instead of e_2^L if he performs poorly).[14] The optimal wage scheme, however, is such that no strategic effect arises. This stands in contrast to the literature on sequential tournaments – for example, Aoyagi (2003) or Ederer (2004) – where agents compete against each other (and do not form a team). In this literature the relative performance scheme (which is comparable to $w_i^{\mathcal{M}} > 0$ in this model) is mostly exogenously given and (only) a strategic effect drives the results.

Comparing the wage scheme for the second mover under the sequential structure to the one under the simultaneous structure, we find that the wage scheme changes: Firstly, under the simultaneous structure (compare Lemma 4), the principal does not want to set incentives for the case that only one agent performs well. Under the sequential structure with substitutes, however, if he wants to implement a positive effort for the second mover in state L, he can (and has to) set incentives in this state. Hence, the question arises whether it is optimal for the principal to implement a strictly positive effort after a poor performance by setting $w_2^{\mathcal{M}} > 0$. In the appendix[15], we show that if the revenue parameter π is sufficiently large (Condition $\mathcal{Z}$), it is indeed optimal to implement a strictly positive effort for agent 2 after a low quality contribution. The second change arises irrespective of whether $e_2^L = 0$ or not. Suppose that $e_2^L = 0$, then under the simultaneous structure, the wage for the second agent is $w_2^{\mathcal{H}} = \frac{c'(\hat{e}_2)}{p_{e_2}^{HH}(\hat{e}_1,\hat{e}_2)}$ and under the sequential structure it is $w_2^{\mathcal{H}} = \frac{c'(e_2^H)}{p_{e_2}(e_2^H)}$. The wage under the simultaneous structure is larger, when the same effort for the second agent is implemented (i.e. $\hat{e}_2 = e_2^H$). Intuitively, agent 2 can be easier incentivized under the sequential structure given the first mover provided high quality since his action is conditional on the high performance of agent 1. This ignores, however, the low state of the world. To implement the same *expected* effort for agent 2 under both structures needs not be less expensive under the sequential structure. We show this in the following section, when we compare both information structures in more detail.

Note the difference to Winter (2005): When efforts instead of contributions are substitutes, there is no change in the optimal wage scheme between the different information structures. We analyze

[14]Of course, this strategic effect does only play a role if $e_2^H \neq e_2^L$.

[15]See Condition $\mathcal{Z}$ in the appendix.

the implications of this change in the wage scheme on the optimal information structure in Section 2.6.

Moreover, we can compare the optimal wage scheme for the sequential structure with substitutes to the setting with complements. In the latter case, we have seen that the principal cannot set incentives for the second mover when the second mover sees the first mover failing (compare Section 2.3.2). Consequently, the second mover exerts zero effort in this state. The optimal wage scheme for the case of complements is $w_2^{\mathcal{H}} = \frac{c'(e_2^H)}{p_{e_2}(e_2^H)}$ if the project is of high value and otherwise zero. This scheme is identical to the one with substitutes (see Lemma 5) if and only if the effort for the second mover after a poor performance of agent 1 is zero. Thus, when agents move sequentially and Condition $\mathcal{Z}$ is satisfied, the optimal wage schemes for complements and substitutes differ as well. In the following, we will also see how this change in the optimal wage scheme drives the optimality of the information structures.[16]

2.6 Comparison of the Structures for the Case of Substitutes

As the wage schemes differ between the sequential and simultaneous structure, deriving the optimal structure is more complicated for the case of substitutes than for the case where contributions are complements. In particular, we cannot immediately see from the two maximization problems which structure does better.

One way to determine the optimal structure would be to proceed with the next step of the principal's problem and derive the optimal effort levels. Given these values we could then compare total profits. This comparison would yield necessary and sufficient conditions for the optimal structure. As usual in a moral hazard problem, we skip this step and take the effort vector the principal wants to implement – denoted by $(e_1^\star, e_2^\star)$ for the simultaneous structure and by the triple $(e_1^\star, e_2^{L\star}, e_2^{H\star})$ for the sequential structure – as given. As long as we do not restrict the effort choice, these effort levels can also involve the optimal ones.

To compare the two information scenarios, we restrict, however, the choice for one structure. More specifically, we assume that the principal implements the same *expected* effort for both structures. This means, we assume that he implements $e_1^\star$ for the first mover and an expected effort of $e_2^\star = p(e_1^\star)e_2^{H\star} + (1 - p(e_1^\star))e_2^{L\star}$ for the second mover (i.e. under the sequential structure, he implements $e_2^{H\star}$ after a high quality contribution and $e_2^{L\star}$ after a low quality one). Thus, for example, $(e_1^\star, e_2^\star)$ can be any effort vector under the simultaneous structure (which includes the optimal one). This effort vector determines the effort triple under the sequential structure. Since efforts under the sequential structure are restricted by this assumption, the optimal efforts may be excluded. Therefore, the

[16]Remember that the wage scheme for the first agent does not change from the case of complements to substitutes.

following analysis gives us sufficient conditions for one structure to be optimal.

When $e_2^{H\star} = e_2^{L\star}$, then not only expected efforts for the second mover, but also the effort in each state is identical to the one under the simultaneous structure. Some of our results will depend on whether efforts of the second mover are state-contingent or not. Therefore, we first consider whether the principal tailors the effort of the second mover to the state of the world under the sequential structure.

Then, we proceed as follows. We analyze for the "fixed" effort levels under which structure expected revenues of the principal are larger. Afterwards, we examine under which structure expected wages are lower for the first and second agent. Finally, we calculate overall profits to make statements about the optimal structure.

2.6.1 Tailoring of Efforts

Before we compare revenues and implementation costs of the two information structures, we consider an important issue regarding the sequential structure: Does the principal tailor the effort of the second mover to the state of the world? Tailoring implies that e_2^H and e_2^L are set differently.

Concerning the case of complements, we have already seen that the principal cannot set incentives for the second mover after a poor performance of the first mover. Hence, the second mover exerts zero effort and the principal necessarily tailors the efforts to the state of the world as soon as he implements any positive level of effort for the second mover. When contributions are substitutes, the result is not obvious. For the analysis of this question when contributions are substitutes, we make the following assumption, which is sufficient for the principal's maximization problem to be concave in effort:

Assumption $\mathcal{C}$ $\frac{c'(e)}{p'(e)}$ *is convex in* e.

Using this assumption, we show the subsequent result in the appendix.

Lemma 6 *Suppose the principal wants to implement an effort of* $e_1^\star$ *for the first mover and an expected effort of* $e_2^\star = p(e_1^\star)e_2^{H\star} + (1-p(e_1^\star))e_2^{L\star}$ *for the second mover. Let in addition Assumption* $\mathcal{C}$ *be satisfied. Then the optimal wage scheme induces the second mover to exert more effort after having observed a high performance of the first mover than after having observed a poor one.*

Hence, the principal optimally tailors the efforts of the second mover to the state of the world and does not set them equally given the assumptions of Lemma 6. More precisely, he induces the second mover to work harder after a high quality contribution in the first period than after a low quality contribution. If additionally Condition $\mathcal{Z}$ holds, the ranking of efforts of the second mover is $e_2^H > e_2^L > 0$.

2.6.2 Expected Revenue

When agents' contributions are substitutes, the principal's expected revenue under the sequential structure – if he implements $(e_1^\star, e_2^{L\star}, e_2^{H\star})$ – is $\pi(2p^{HH}(e_1^\star, e_2^{H\star}) + p^{HL}(e_1^\star, e_2^{H\star}) + p^{LH}(e_1^\star, e_2^{L\star}))$. For the simultaneous structure, we only have to replace $e_2^{L\star}$ and $e_2^{H\star}$ by $e_2^\star = p(e_1^\star)e_2^{H\star} + (1 - p(e_1^\star))e_2^{L\star}$. Using that the effort that is implemented for the first agent is the same under both structures ($e_1^\star$), the difference in expected revenues (denoted by ΔR) between the simultaneous and the sequential structure becomes

$$\Delta R = \left[p(e_2^\star) - \left(p(e_1^\star)p(e_2^{H\star}) + (1 - p(e_1^\star))p(e_2^{L\star})\right)\right]\pi.$$

The sign of this difference depends on the curvature of the success probability p as the following lemma summarizes. The proof is in the appendix.

Lemma 7 *Suppose $\mathcal{Y}(Y_1, Y_2) = \mathcal{Y}_s(Y_1, Y_2)$ and the principal wants to implement effort levels $e_1^\star$ and $e_2^\star = p(e_1^\star)e_2^{H\star} + (1 - p(e_1^\star))e_2^{L\star}$. Then expected revenues for the simultaneous structure are higher, strictly so if p is strictly concave and $e_2^{H\star} \neq e_2^{L\star}$.*

The first agent's contribution is with probability $p(e_1^\star) = p(e_1^\star)p(\tilde{e}_2) + p(e_1^\star)(1 - p(\tilde{e}_2))$ of high quality, where $\tilde{e}_2$ equals $e_2^{H\star}$ for the sequential structure and equals $e_2^\star$ for the simultaneous structure. Hence, we can say that agent 1 generates revenue π for the principal when performing high. Since the effort that is implemented for the first agent is the same under both structures ($e_1^\star$), he performs well with the same probability. Thus, expected revenues "from the first agent" are the same under both structures. The difference in expected revenues is, therefore, exactly the difference in expected revenues generated by the second agent. He contributes high quality with probability $p(e_2^\star)$ under the simultaneous structure and with probability $p(e_1^\star)p(e_2^{H\star}) + (1 - p(e_1^\star))p(e_2^{L\star})$ under the sequential structure.

The main difference between the two information settings is that under the sequential structure efforts can be made state-contingent (by allowing $e_2^{H\star}$ to differ from $e_2^{L\star}$). There is no difference in expected revenues if efforts are not tailored to the state of the world. Tailoring is, however, optimal for the principal under Assumption $\mathcal{C}$ (compare Lemma 6). Lemma 7 shows that for state-contingent efforts ($e_2^{H\star} \neq e_2^{L\star}$) under the sequential structure, the simultaneous structure leads to higher expected revenues since the probability of success function is concave (strictly higher for strict concavity and $e_2^H \neq e_2^L$). Why is this the case? Conditioning effort on the state of the world implies that expected revenue is uncertain in effort from an ex ante perspective . Since the individual probability of success function is concave in an agent's effort, Jensen's Inequality implies that the value of the convex combination of expected revenues (having with probability $p(e_1^\star)$ revenues arising from e_2^H and with probability $1 - p(e_1^\star)$ from e_2^L) is smaller than the value expected revenue that arises from the convex combination of efforts – i.e. e_2^*.[17]

When the probability of success function is linear in effort, then revenues for both structures are the same as linearity implies that the convex combination of the values is equal to the value of the convex effort combination. Hence, conditioning effort on the state of the world does not influence expected revenues in this case.

2.6.3 Implementation Costs for Agent 1

We now consider the implementation costs for agent 1. As we have shown in Sections 2.5.1 and 2.5.2, the expected wage payment for each structure ($e_1^\star$ is implemented for the first agent under both structures) is

$$W_1 := p^{HH}(e_1^\star, e_2^\star)w_1^{\mathcal{H}} = \frac{p(e_1^\star)}{p'(e_1^\star)}c'(e_1^\star). \tag{2.5}$$

This means that for agent 1 implementation costs are identical under both information structures as the same effort is implemented. This result is in line with Winter (2005), who finds no difference in wages under both structures when efforts are substitutes. Furthermore, we see from (2.5) that the expected wage is independent of the second agent's effort. This establishes the following lemma.

Lemma 8 *Suppose $\mathcal{Y}(Y_1, Y_2) = \mathcal{Y}_s(Y_1, Y_2)$ and the principal implements for both structures the same effort $e_1^\star$ for the first agent. Then expected implementation costs for the first agent are equal under both structures.*

2.6.4 Implementation Costs for Agent 2

Next, we turn to the comparison of expected implementation costs for agent 2. These are under the simultaneous structure

$$W_2^{sim_s} := p^{HH}(e_2^\star, e_1^\star)w_2 = \frac{p(e_2^\star)}{p'(e_2^\star)}c'(e_2^\star). \tag{2.6}$$

The expected wage payment for agent 2 under the sequential structure is[18]

$$W_2^{seq_s} := p(e_1^\star)\frac{p(e_2^{H\star})}{p'(e_2^{H\star})}c'(e_2^{H\star}) + (1-p(e_1^\star))\left(\frac{p(e_2^{L\star})}{p'(e_2^{L\star})} + \frac{p(e_1^\star)}{p'(e_2^{L\star})(1-p(e_1^\star))}\right)c'(e_2^{L\star}). \tag{2.7}$$

For the comparison of expected implementation costs for agent 2, we again use Assumption $\mathcal{C}$. Under this assumption, expected implementation costs for agent 1 and 2 are convex as we show in

[17] For dependent success probabilities this "uncertainty effect" may be outweighed by an informational effect the sequential structure provides as is shown in Ludwig and Nafziger (2006): It might pay in terms of expected revenues to condition the second agent's effort on the state of the world (the quality of the first agent) since this state of the world contains information about the second agent's success probability conditional on this state.

[18] Note that $w_2^{\mathcal{M}}$ is paid if exactly one agent provides high quality and the other one low quality.

the appendix (see Proof C).[19]

Comparing expected implementation costs for agent 2, we can derive the following lemma. The proof is in the appendix.

Lemma 9 *Suppose* $\mathcal{Y}(Y_1, Y_2) = \mathcal{Y}_s(Y_1, Y_2)$ *and the principal implements* $e_1^\star$ *and* $e_2^\star = p(e_1^\star)e_2^{H\star} + (1 - p(e_1^\star))e_2^{L\star}$. *Let Assumption* $\mathcal{C}$ *be satisfied, then the simultaneous structure has lower expected implementation costs for agent 2.* [20] *They are strictly lower if*

(i) efforts for the second mover are state-dependent: $e_2^{L\star} \neq e_2^{H\star}$,

(ii) the effort implemented in state L *is strictly positive:* $e_2^{L\star} > 0$.

Regarding part (i), recall that we have shown (Lemma 6) that state dependent efforts are optimal when agents move sequentially, Assumption $\mathcal{C}$ holds, and the principal implements the same expected effort for agent 2 under both information structures. Concerning part (ii), we have seen before, that the principal optimally implements a strictly positive effort for the second mover in the low state of the world if condition $\mathcal{Z}$ is satisfied.

Furthermore, note that *any* effort vector $(e_1^\star, e_2^{L\star}, e_2^{H\star})$ implemented under the sequential structure (including the optimal one), can be implemented *in expectation* ($e_1^\star$ and $e_2^\star = p(e_1^\star)e_2^{L\star} + (1 - p(e_1^\star))e_2^{L\star}$) at (strictly) lower expected implementation costs under the simultaneous structure. These efforts need, however, not be optimal for the simultaneous structure. Regarding optimal effort levels (i.e. profit-maximizing efforts), we cannot conclude here, which structure leads to lower implementation costs without knowing the optimal efforts.

There are two driving forces behind the result that the simultaneous structure leads to strictly lower expected implementation costs: state-dependent efforts for the second mover (part (i) of Lemma 9) and – more importantly – a change in the feasible wage scheme when a strictly positive effort is implemented for the second mover in state L (part (ii) of Lemma 9). The latter effect is due to the change in incentives for the second agent under the sequential structure. The principal has to pay agent 2 a positive wage in both states of the world (to induce a positive effort in both states), whereas under the simultaneous structure he only pays a positive wage if the value of the joint project is high.

Under the simultaneous structure, expected implementation costs are the costs to implement the convex effort combination $e_2^\star = p(e_1^\star)e_2^{H\star} + (1 - p(e_1^\star))e_2^{L\star}$. In order to compare these costs to the ones under the sequential structure, let first $e_2^{L\star} = e_2^{H\star} = e_2^\star$, i.e. efforts are not state-contingent.

[19] We could rewrite our results for concave expected implementation costs. We believe, however, that convex implementation costs are the most relevant case and restrict to this case in the following.

[20] In the appendix, we show that Assumption $\mathcal{C}$ implies that $\frac{p(e)}{p'(e)}c'(e)$ is strictly convex (see Proof C).

Comparing (2.6) and (2.7) for this case, we see that costs under the sequential structure equal the costs under the simultaneous structure *plus* an additional non-negative cost term $\frac{p(e_1^\star)}{p'(e_2^{L\star})}c'(e_2^{L\star})$. This additional term results from the change in the wage scheme: As soon as the principal implements a positive effort for the second mover after a poor performance of the first mover, implementation costs for agent 2 are strictly higher under the sequential structure if the principal does not tailor the efforts.

Suppose now, the principal tailors the second mover's effort to the (ex ante uncertain) state of the world, i.e. $e_2^{L\star} \neq e_2^{H\star}$ (compare Lemma 6). The expected effort of agent 2 is, however, the same as under the simultaneous structure. Can tailoring reduce expected implementation costs? With tailoring, expected implementation costs under the sequential structure (compare (2.7)) consist of two parts again: One part is again the additional cost term $\frac{p(e_1^\star)}{p'(e_2^{L\star})}c'(e_2^{L\star})$. The other part is the convex combination of the costs that would arise if $e_2^{H\star}$ and $e_2^{L\star}$ had been implemented under the simultaneous structure. Dropping the additional term – but noting that this additional term is non-negative and therefore, cannot decrease costs – Jensen's Inequality implies that expected implementation costs under the simultaneous structure are strictly lower (given strict convexity of $\frac{p(e)}{p'(e)}c'(e)$, which follows from Assumption $\mathcal{C}$). Hence, convex implementation costs cannot be reduced by tailoring the effort to the state of the world, but are even further increased.

It is important to note that our result of lower implementation costs for the simultaneous structure does not only hinge on this "convexity effect". The crucial difference between the simultaneous and sequential structure that drives our result, stems from the change in the feasible wage scheme (paying a positive wage in state L) as we have seen above: Irrespective of whether the principal tailors or does not tailor the second mover's effort to the state of the world, expected implementation costs are higher under the sequential structure because of the change in the feasible wage scheme (as long as $e_2^{L\star} > 0$).

Ludwig and Nafziger (2006) find for two agents, who do not work in a team, that even if success probabilities are dependent – which leads to an informational gain under the sequential structure – the negative effect of the change in the feasible wage scheme (paying a positive wage after a low quality contribution) under the sequential structure prevails.

2.6.5 Overall Effect

From the analysis in the previous sections, we can derive the following general result for the overall effect of the information setting on expected profits by combining Lemma 7, 8 and 9.

Proposition 18 *Suppose* $\mathcal{Y}(Y_1, Y_2) = \mathcal{Y}_s(Y_1, Y_2)$ *and* $\frac{p(e)}{p'(e)}c'(e)$ *is convex. Then any effort vector* (e_1, e_2^L, e_2^H) *implemented under the sequential structure (including the optimal effort vector) yields higher expected profits when implemented in expectation under the simultaneous structure. Expected* *profits are strictly higher if*

(i) $\frac{p(e)}{p'(e)}c'(e)$ *is strictly convex and* $e_2^L \neq e_2^H$

(ii) p *is strictly concave and* $e_2^L \neq e_2^H$

(iii) $e_2^L > 0$.

This implies that *any* effort vector (e_1, e_2^L, e_2^H) implemented under the sequential structure (including the optimal effort vector) leads to higher expected profits when implemented in expectation (i.e. $e_1, e_2 = p(e_1)e_2^H + (1 - p(e_1))e_2^L$) under the simultaneous structure (if Assumption $\mathcal{C}$ is satisfied). The latter efforts need, however, not be optimal under the simultaneous structure. With optimal efforts, the simultaneous structure can only do better but not worse. Therefore, the result also holds true for optimal efforts.

Using the results of the previous sections, the finding can be explained as follows. On the one hand, if the principal implements $e_2^L = 0$, then there is no difference in the feasible wage schemes of both information structures. The principal pays each agent only a positive wage if the value of the team project is high. Hence, it is not the wage scheme effect that drives the result but the revenue effect. If $e_2^L = 0$, and a positive effort is implemented for the second mover after a high performance of the first mover, the principal tailors the efforts to the state of the world. This reduces revenues compared to the simultaneous structure since the probability of success function is concave. On the other hand, if the principal implements $e_2^L > 0$, then the change in the wage scheme makes the sequential structure less attractive. The principal has to reward the second agent also after a low quality contribution in the first stage, whereas under the simultaneous structure he only pays him if both agents perform well.[21] Compared to the case when $e_2^L = 0$, tailoring is less "strong" now: The same expected effort for the second mover can be implemented with a smaller gap between e_2^L and e_2^H if e_2^L is positive. Regarding expected revenues, this leads to a less pronounced negative effect. The driving force for the sequential structure performing worse than the simultaneous structure is, therefore, the wage scheme. Hence, irrespective of whether the principal implements a positive effort for the second mover or not, one of the two effects reduces expected profits when agents move sequentially compared to the simultaneous structure.

In contrast, when contributions are complements, the sequential structure does (strictly) better than the simultaneous one when contributions are substitutes. The intuition is the following: When agents move sequentially and contributions are complements, the principal still rewards the

[21] Note that as soon as the principal implements $e_2^L > 0$, the wage scheme for the sequential structure also differs compared to the one for the sequential structure with complements. This is similar to Winter (forthcoming), who finds a change in the wage scheme when the production technology possesses either increasing or decreasing returns to scale given a sequential structure.

second agent only if both agents perform well. He cannot set incentives after a low quality contribution in the first period. Hence, there is no change in the feasible wage scheme between both information structures. Zero effort of the second mover after a poor performance of the first mover here decreases implementation costs. Moreover, the aforementioned negative revenue effect is not present here as revenue is only generated if both agents contribute high quality. The fact that the second agent exerts zero effort after a low quality contribution in the first period does not matter for revenues. Revenues may even be positively influenced if the effort of the second agent after a high performance in the first period under the sequential structure is higher than the second agent's effort under the simultaneous structure depending on the effort of the first agent ($2p(e_1)p(e_2^H)\pi$ versus $2p(\hat{e}_1)p(\hat{e}_2)\pi$). Overall, the sequential structure outperforms, therefore, the simultaneous structure if contributions are complements.

2.7 Extension

Up to now, we considered special examples for contributions being perfect substitutes or perfect complements. We now extend the preceding analysis to more general cases.

The value of the joint project is $\mathcal{Y}(Y_1, Y_2)$. If the quality of an agent's contribution is more effective, the higher the quality of the other agent's contribution, i.e. $\mathcal{Y}(H,H) - \mathcal{Y}(H,L) > \mathcal{Y}(L,H) - \mathcal{Y}(L,L)$, the qualities of the agents' contributions are complements. Accordingly, if $\mathcal{Y}(H,H) - \mathcal{Y}(H,L) \leq \mathcal{Y}(L,H) - \mathcal{Y}(L,L)$ the qualities of the agents' contributions are substitutes.

We assume that it does not matter for the value of the project, which agent provides low or high quality. Hence, there is exactly one intermediate value of the project $\mathcal{Y}(H,L) = \mathcal{Y}(L,H) = \mathcal{M}$.[22]

Moreover, we assume that the values of the project satisfy $\mathcal{H} = \mathcal{Y}(H,H) \geq \mathcal{M} \geq \mathcal{Y}(L,L)$, and we normalize the value of the project if both agents provide low quality to zero ($\mathcal{L} = \mathcal{Y}(L,L) = 0$). These assumptions imply that qualities of the individual contributions are complements if $\mathcal{H} > 2\mathcal{M}$ and otherwise substitutes.

Note that in Section 2.3, we analyzed perfect complements with $\mathcal{H} = 1$ and $\mathcal{M} = \mathcal{L} = 0$ and in Section 2.5 perfect substitutes with $\mathcal{H} = 2$, $\mathcal{M} = 1$, $\mathcal{L} = 0$.

We begin with the analysis of the general case of substitutes, i.e. $\mathcal{H} \leq 2\mathcal{M}$, where $\mathcal{H} \geq \mathcal{M} \geq 0$. It is important to note that there is no change regarding the feasible wage scheme for both information structures compared to the special case in Section 2.5. Only expected revenues are influenced. For the analysis of expected revenues, we proceed exactly like in Section 2.6.2. This means, the principal implements the same expected effort for agent 2 under both information structures and the same effort for agent 1. When the principal implements $(e_1^\star, e_2^{L\star}, e_2^{H\star})$ under the sequential structure,

[22]Note that otherwise, it would be possible for the principal to infer from the projects value, which agent provided low or high quality. Thus, the team production problem would resolve.

expected revenues are $\pi[p^{HH}(e_1^\star, e_2^{H\star})\mathcal{H} + (p^{HL}(e_1^\star, e_2^{H\star}) + p^{LH}(e_1^\star, e_2^{L\star}))\mathcal{M}]$. For the simultaneous structure, we only have to replace $e_2^{L\star}$ and $e_2^{H\star}$ by $e_2^\star = p(e_1^\star)e_2^{H\star} + (1 - p(e_1^\star))e_2^{L\star}$. Hence, we can write the difference in expected revenues between the simultaneous and the sequential structure as follows:

$$\Delta R = \pi\left[p(e_1^\star)(\mathcal{H} - \mathcal{M})\left(p(e_2^\star) - p(e_2^{H\star})\right) + (1 - p(e_1^\star))\mathcal{M}\left(p(e_2^\star) - p(e_2^{L\star})\right)\right]. \tag{2.8}$$

By Jensen's Inequality, we know that $p(e_2^\star) \geq p(e_1^\star)p(e_2^{H\star}) + (1 - p(e_1^\star))p(e_2^{L\star})$ (with strict inequality for strict concavity of p). Using this and simplifying, we get

$$\Delta R \geq \pi[p(e_1^\star)(1 - p(e_1^\star))(2\mathcal{M} - \mathcal{H})\left(p(e_2^{H\star}) - p(e_2^{L\star})\right)]. \tag{2.9}$$

Since $2\mathcal{M} - \mathcal{H} \geq 0$, the difference in expected revenues is non-negative (strictly positive for p strictly concave and for $2\mathcal{M} - \mathcal{H} > 0$), as long as $e_2^{H\star} \geq e_2^{L\star}$. Hence, if it is optimal for the principal to implement a (weakly) larger effort in state H, results are the same as for the special case analyzed in Section 2.5: Any effort vector implemented under the sequential structure yields higher expected profits when implemented in expectation under the simultaneous structure (compare Proposition 18). As we show in the appendix (see Proof $\mathcal{G}$), it need, however, no longer be optimal to implement a higher effort in state H. In case it is optimal to implement a higher effort in state L, it can be that the sequential structure does better than the simultaneous one. The revenue effect works in favor of the sequential structure, but the wage scheme effect (which is like in Section 2.5) works still in favor of the simultaneous structure.

Consider now the general case of complements, i.e. $\mathcal{H} > 2\mathcal{M}$ with $\mathcal{M} > 0$. Here, not only the revenue effect might change, but the optimal wage scheme changes for the sequential structure changes as well (compared to the special case in Section 2.3). Since there are now three possible values of the project, the optimal wage scheme looks exactly like for the case of substitutes. Remember that the only difference between the case of complements and the one of substitutes when deriving the optimal wage scheme was that the project could take two or three different values, respectively. This wage scheme effect works against the sequential structure (as we have seen already when analyzing substitutes in Section 2.5). Regarding expected revenues, there is also a crucial change compared to the special case. In Section 2.3, we have seen that the difference in expected revenues does not depend on the effort of the second mover in state L. This is no longer true for the general case. The difference in expected revenues is now given by (2.8). We can again infer from inequality (2.9) in which direction the revenue effect works. Since for complements $\mathcal{H} > 2\mathcal{M}$, the difference in expected revenues becomes negative for $e_2^{H\star} > e_2^{L\star}$. Thus, if it is still optimal to implement a higher effort for the second mover in the high state of the world, the revenue effect works in favor of the sequential structure.[23] We show in the appendix (see Proof $\mathcal{G}$) that this still holds given Assumption $\mathcal{C}$ (i.e. Lemma 6 still applies for the case of complements). If the negative effect of the change in the wage scheme is outweighed by the positive revenue effect, the sequential structure can, therefore, lead to higher profits for the principal. But otherwise the simultaneous structure becomes optimal even for the case of complements.

2.8 Discussion

Contrary to our result that intermediate information is rather disadvantageous when contributions are substitutes, Winter (2005) finds that observability (i.e. the sequential structure) *always* yields higher expected profits: when *efforts* are complements as well as when they are substitutes.[24] What drives the different result? Intuitively, if we take intermediate values of the project into account, the second agent's incentives decrease if the principal pays a positive wage for the intermediate outcome. Since the principal does not know which agent provided high or low quality, he always pays the wage if exactly one agent performed poorly. Therefore, the second mover gets for sure a positive wage when the first agent provided high quality. In Winter, there is no difference in the wage scheme between the different information structures when efforts are substitutes and the project cannot take intermediate values. Hence, the negative effect on the sequential setting stemming from the change in the wage scheme – that we observe – is not present. Furthermore, also the revenue effect does not arise (only a successful project generates a payoff, there is no intermediate value). Concerning the free-riding problem within teams, our results thus imply that intermediate observation (or more transparency) does not necessarily work in favor of the principal as it has been shown by Winter. This result is similar to Goldfain (2006). She presents numerical results for an R&D model, which suggest that when agents' efforts are strategic substitutes, the performance of the team does not increase under the sequential structure.

Lizzeri et al. (2002) derive in a related setting that it is not optimal to provide an agent with intermediate information. They consider a moral hazard model with a single agent, who works for two periods, and either receives intermediate information about his performance (corresponding to the sequential structure considered here) or does not receive it (corresponding to the simultaneous structure). In contrast to our result, providing intermediate information can *never* be optimal in their setting as there are no informational gains from it. Like in our paper, a comparable change in the wage scheme is the driving force that turns intermediate information disadvantageous. The

[23] Remember that for the special case considered in Section 2.3, the second mover always exerted zero effort in the low state of the world.

[24] Winter finds that when efforts are substitutes, the effect is less strong.

same expected effort can be implemented at lower cost if no intermediate information is provided.[25] A change in the feasible wage scheme between the settings, in which either intermediate information is available or not, is also crucial for the results in Ludwig and Nafziger (2006). When the feasible wage scheme does not necessarily change (i.e. the same wage scheme can be optimal as well), a sequential structure is optimal (i.e. intermediate information is available), otherwise a simultaneous structure can also be optimal.

Moreover, Schmitz (2005) finds similar effects than we do. He analyzes whether integration (i.e. one agent is in charge for two production stages) or separation (i.e. two different agents are responsible for stage one and two) is optimal in sequential production processes. Consider first the case that the principal wants to implement zero effort after a poor performance in the first period (like in the case of complementary contributions) and high effort after a high performance in the first period. Under separation, each of the two agents receives a positive wage if performance in both stages is high. Integration, however, is cheaper. As Schmitz shows, under integration it suffices to pay the agent a wage to induce effort in the second stage. The agent works hard in the first period since by not working hard he loses the chance to receive the positive wage. Hence, it is a cost-saving argument that drives the result like we have seen for the case of complements. If the principal, however, wants to implement high effort after a poor performance in the first period as well, the result in Schmitz changes as it does with substitutes in our case: It becomes more expensive to induce a single agent to work hard irrespective of the first-period outcome than it is under separation.

2.9 Conclusion

In this paper, we analyze whether it is profitable for the principal if he instructs his two agents – who are responsible for a joint project – to work simultaneously or sequentially. If agents work sequentially, the second mover (but not the principal) can observe the quality of the contribution of the first mover. Hence, the second mover can condition his effort on the first mover's outcome. The value of the joint project depends on the qualities of the two agents' contributions. These contributions can be either complements or substitutes. When the agents' contributions are perfect complements, the project is of no value as soon as one agent provides low quality. Only if both agents perform well, the joint project succeeds. In case the agents' contributions are substitutes, however, the joint project takes a strictly positive value also if only one agent provides high quality. We find that when contributions are perfect substitutes, it is optimal to instruct the agents to work simultaneously. When contributions are perfect complements, however, the sequential structure is optimal. The reason for this change is that between the two information settings a change in the feasible wage scheme can arise when contributions are substitutes: If agents act sequentially, the

[25] Lizzeri et al. (2002) abstract from revenue effects.

principal has to pay a positive wage to the agent 2 not only if the realized value of the project is high but also if it has an intermediate value (if he wants to implement a positive effort for the agent 2 in the low state of the world). When contributions are perfect complements, however, the principal cannot set incentives in the low state of the world. Therefore, the second mover exerts no effort, when the first mover performed poorly. This reduces wage payments for the principal, but does not reduce his expected revenues. A zero effort after observing low quality of the first mover does not reduce revenues when the project is only of some positive value if both agents perform well. In this case, expected revenues are not influenced by the effort the second mover exerts in the low state of the world.

The change in the feasible wage scheme when contributions are substitutes, however, only matters if the principal implements a strictly positive effort for the second mover under the sequential structure. It can be optimal for the principal to implement zero effort. Although there is no effect on the feasible wage scheme in this case, there is a negative revenue effect. In contrast to the case of complements, the effort the second mover exerts in the low state of the world influences expected revenues: The joint project is of some positive value also if only one agent provides high quality. Thus, allowing for different values of the project works against the principal under the sequential structure. The finding that providing intermediate information is disadvantageous is new to the literature in this context. Of course, if one adds informational gains that arise in the sequential structure (e.g. dependent success probabilities), the sequential structure might gain an advantage over the simultaneous structure. Then, depending upon which of the opposing effects is larger, either the sequential or the simultaneous structure will be optimal.

Chapter 3

My and Your Bias – What Do You Know About Them?[1]

3.1 Introduction

Knowledge about other people's attributes is important in many economic situations. Imagine you hire a manager, give him a perfectly designed incentive contract, and after some time you wonder why things go wrong in your firm. The manager may have invested in too risky projects, made insensible acquisitions or hired wrong people. What went wrong – according to your incentive contract he shouldn't have done all these things! Well, maybe you did not know that your manager is overconfident. We are interested in whether people know that biases like overconfidence exist in the population.

Overconfidence can be defined and measured in different ways. On the one hand, one can define overconfidence in own knowledge or ability or one can define it as being too optimistic regarding the own performance ("optimistic overconfidence"), which does not necessarily depend on own knowledge. An example of "optimistic overconfidence" is that people assess the likelihood that they get divorced too optimistically. On the other hand, overconfidence can refer to absolute abilities as well as to relative abilities, i.e. people make assessments either regarding their own ability or regarding their ability compared to other people's ability (like estimating their rank or percentile in a distribution). Much of the evidence for overconfidence comes from calibration studies by psychologists, in which subjects make probability judgements, e.g., that their answer to a question is correct. People's confidence often exceeds their actual accuracy (for a review of this literature see Yates (1990)). Besides being poorly calibrated, people also state confidence intervals that are too narrow.

The fact that individuals are overconfident – in the sense that they overestimate their absolute or

[1] This chapter is based on joint work with Julia Nafziger.

relative abilities – is confirmed by economists (see e.g. Camerer and Lovallo (1999) or Hoelzl and Rustichini (2005)), who also point out that the presence of overconfident individuals in economic settings has far going implications. For example, if you know that your opponent or employee is overconfident you should adjust your behavior in contests accordingly (Ando (2004) or Santos-Pinto (2005b)), write different incentive contracts (Santos-Pinto (2005a) or De La Rosa (2005)), or choose different strategies in Bertrand and Cournot Competition (Englmaier (2004) or Eichberger et al. (2005)). Malmendier and Tate (2005a, 2005b) observe that managers are indeed overconfident and that this characteristic is a disadvantage to the firm, whereas in Kyle and Wang (1997), for example, overconfidence is unilaterally beneficial.
The cornerstone of all these models is that people know that other people have a bias. In some models it is also important that people know whether they have a bias themselves and that they know about the relation of these biases (and that all this is common knowledge). Suppose you do not know that others are overconfident. Why should you behave differently in a contest if you have no idea that your opponent is overconfident? Why would you write a non-standard incentive contract if you have no idea that your manager is overconfident? Why do you hire overconfident managers in case this is a disadvantage for your firm – don't you know that they have a bias?

The aim of our experiment is to examine what people know about such biases: Are individuals aware that others have a bias in assessing their (absolute) abilities? What do they think about the relation of their and other people's biases? Are there some hints that they know about their own bias and correct for it?

Since overconfidence is a common phenomenon, we consider a bias that is one possible interpretation of overconfidence: over-or underestimation of one's absolute ability. Subjects assess their number of correct answers to a set of questions, which means they assess their absolute ability in this task. In contrast to calibration studies, we cannot claim that a subject is biased when her self-assessment is wrong, she might just make a mistake. If a group of subjects, however, tends to either over- or underestimate their abilities (i.e. their mistakes do not cancel out), we can say that the group is biased.
In order to avoid to influence people in their reasoning (and thus, their choice) by asking them the questions what they think about biases explicitly, we construct simple decision problems to elicit beliefs. Moreover, we conduct another treatment to see whether choices differ, when subjects face either these decision problems or the explicit questions. To analyze the effects of such framed instructions (i.e. asking the subjects explicitly) compared to the neutral way (i.e. the decision problems) is – besides the two questions above – another topic of our paper. So far, relatively little research in the overconfidence field considers whether asking subjects directly (as psychologist do

it) changes behavior. For an overview on framing effects in other fields see Rabin (1998). Asking people directly whether others over- or underestimate or correctly estimate their abilities might cause that people become aware of problems like over- or underestimation. Therefore, subjects may adjust their beliefs or people may start to overrate the relevance of wrong estimates. This may lead to "over-adjustment" of beliefs.

The design of our basic experiment is as follows. At first, subjects in the reference treatment (R) answer seven general knowledge questions (multiple choice) – we refer to these subjects as *Rs* in the following. Then, the *Rs* choose an action, where the optimal choice depends on *R's* belief about her number of correctly answered questions. Subjects in *another* treatment (T) are informed about the questions (not the correct answers) the *Rs* had to answer and the 'average action' the *Rs* have chosen. This average action reflects the *Rs'* average assessment of their number of correctly answered questions. Given this information, subjects in T have a choice between three actions. The chosen action reveals whether subjects in T think the *Rs* are either underconfident, rational or overconfident. Further, subjects in T choose a number reflecting their belief about the true average number of correct answers of the *Rs*.

Besides this baseline treatment we explore several extensions. In the first one, subjects in T answer the questions themselves and assess their own number of correct answers before evaluating the *Rs*. This does not only give subjects a better feeling for the plausibility of the estimate of others, but also enables us to compare the own bias of a subject and the belief about the bias of others (the *Rs*): Do people, who are more biased themselves, also think that others are more likely to be biased or is it just the other way round? In another extension we test (as mentioned above) the impact of using a non-neutral language in the instructions. Furthermore, we consider whether subjects could be forced to recognize that the *Rs* are biased. To analyze this issue, we let the *Rs* answer very tricky questions instead of the hard ones, and subjects in T also see the correct answers to the tricky questions before they judge the *Rs*. These tricky questions are designed in a way to increase subject's confidence that they answered correctly but are in fact wrong with their answer (i.e. the correct answer is rather surprising). Lastly, we confront subjects in T not with the average guess of the *Rs*, but with single *Rs*. By doing so, we can infer whether subjects know that others make mistakes (these mistakes need not be systematic as they need to be to form a bias), even though they do not know that others are biased. In this treatment, we apply the strategy method to elicit the beliefs of subjects given any possible belief R can have.

Concerning relative biases, we add in several of the above treatments an additional decision problem. Here, subjects evaluate the relation of their own bias or mistakes and the average bias of the subjects in the reference treatment.

We observe that there are different types of subjects: Subjects who overestimate their number of correct answers as well as subjects who underestimate or correctly estimate it. The largest group is – with more than 50 and up to 90 percent – the group that overestimates the own ability. Our first result is that even if overestimation frequently occurs in the population (like previous studies have shown), a majority of subjects does not know that others have (on average) a bias. This result is striking as overestimation of one's own ability seems to be such a prevalent phenomenon in our experiment (and in the real world) that it should be self-evident that people are also aware of it. The more familiar subjects are with a task, however, the more subjects learn that others are on average biased. We cause this familiarity in our experiment by letting the subjects answer the questions themselves, by framed instructions (asking the subjects explicitly as explained above) or by letting them evaluate Rs who answered tricky questions and showing them the correct answers to these questions.

We observe that asking subjects explicitly whether they think that others estimate their ability correctly gives subjects a hint about the existence of erroneous self-assessments: in contrast to the setting where subjects are confronted with the neutrally framed decision problem, more of them recognize that others are biased. Moreover, subjects in the framed session are less biased – indicating that the wording does not only make them recognize that others are biased, but also that they are biased themselves (for which they then correct). Finally, when confronted with single Rs, subjects recognize that those might make mistakes. Combining our observations indicates that subjects think that Rs make unsystematic mistakes (which cancel out on average), but not that these mistakes are systematic (implying that the Rs are really biased).

An important question is how subjects make their judgement of the Rs (or a single one). In those treatments, where subjects answer the questions themselves, we see that they think that others are similar to them: if subjects think, for example, 2 is a good guess for their own ability, they also guess that 2 is the (average) number of correct answers of a single R (the group of Rs). This result can be interpreted in the way that subjects show a "false consensus bias" (see Mullen et al. (1985)): subjects' estimates of others are biased in the direction of their own belief about themselves. Even more interestingly, subjects think that similar[2] Rs are very likely to be correct with their choice. One possible interpretation of this finding is that a similar R is just a projection of the own self, i.e. subjects think about themselves that they are correct.

The largest group of subjects thinks that they are themselves more likely to judge their ability correctly than is the average population.[3] This assessment of relative biases is consistent with

[2] "Similar" subjects in the sense that R has the same belief about the number of correct answers as the subject in T has about herself. This can be seen in the treatment, where subjects are faced with single Rs and where we applied the strategy method.

[3] Some might be surprised that the largest groups thinks that others are biased, while oneself is not, did we state before that the majority of subjects does not know that others are biased. One should be careful here with "largest"

observations that people are overconfident in the sense that they think they are better than the average, where "better" in our case means to be less biased. Although this finding can be explained by the "better-than-average" effect – or more precisely by a self-serving bias[4] – it is surprising, since "the others" represent an average here. For this average, mistakes should cancel out (in case mistakes were just random), while for a single subject they do not.[5] Furthermore, we relate this "better-than-average" bias with the bias when assessing the own number of correct answers. The result is that subjects, who are biased in the question task, also have a "better-than-average" bias.

The main question of our paper – what people know about about themselves and others – is also prominent in other fields in economics and psychology like the hyperbolic discounting model, game theory (where we especially mention beauty-contest experiments) and divorce statistics.

The hyperbolic discounting model was developed to explain time-inconsistent preferences (see, e.g., Laibson (1997) and O'Donoghue and Rabin (1999)). It is usually distinguished between people who are sophisticated, which means that they know that they have a bias[6], and people who are (partially) naive, i.e. they are (partially) not aware of their bias. Empirical evidence suggests that people are (partially) naive and not sophisticated (see, e.g., Della Vigna and Malmendier (2005)). It seems to be an open topic for future research to examine further the degree of "partiality". Although, the overconfidence bias and the hyperbolic discounting bias have many conceptual differences, we think that we contribute to this debate with our observations. Our results show that people do not even partially know that others are biased and suggest that people are not aware of their own bias.

The assumption that rationality of players is common knowledge is crucial for game theory and has been tested, for example, in so called beauty-contest experiments (see, e.g., Nagel (1995), Bosch-Domenech et al. (2002) or Ho et al. (1998)). In these experiments subjects play a game that is solvable by iterated deletion of strictly dominated strategies. Here it is interesting to observe how many iteration steps subjects are typically perform. The number of steps depends

and "majority". A minority (35 percent) states that they are more likely to be correct than others, but this minority still forms the largest group compared to those subjects who think that others are rational and they are biased themselves (33 percent) or who think that others and they themselves are rational (32 percent).

[4]For a general discussion on self-serving biases see Rabin (1995).

[5]Svenson (1981) conducted the well-known study showing that people think they are better drivers than is an average person. Here, it is not clear what the reference group and reference ability of agents is. In contrast to our study, the average in Svenson is not likely to be better as there are no mistakes that could cancel out.

[6]They could not perfectly correct for their bias in these models because the player today and tomorrow are typically modelled as two different players. With biases like overconfidence one typically assumes in case people "know their bias" that they are uncertain about the exact size and direction of their bias and could hence not always perfectly correct for it.

on a subject's own depth of thinking, what she knows about the depth of her opponents ("their bias"), the relation between the two ("relative bias") and that all of this is common knowledge. From observing the choice of a subject, however, it cannot be fully disentangled for which reasons this choice is made in beauty-contests: Is it her own limited depth of reasoning or that she thinks the others do – on average – not think as many steps ahead as she does or that the others do not know that she thinks so many steps ahead (and she either knows this or not)? Thus, we cannot unambiguously conclude from beauty-contests what people think about other people's reasoning (or "bias") or about the relation between one's own and the others' reasoning. As aforementioned, we consider a much more simple decision problem without strategic interaction. We are able to certainly identify individuals who are aware of other people's bias and those who are not and what people think about the relation between the own and other people's bias.
A study by the psychologists Baker and Emery (1993) suggests that people may be better at detecting "biases" of other people than biases of themselves. While individuals know quite accurately the likelihood of divorces (about 50 percent of U.S. couples who marry), they have extremely optimistic expectations assessing the likelihood that they get divorced themselves. People think that a divorce is rather unlikely to happen to them. Although our subjects have to go one step further in their reasoning, i.e. we ask subjects whether they think that others know their likelihood of divorce correctly (translated to the divorce example), some of our results are related. The finding by Baker and Emery indicates that many people think that they are "better" than – or different from – the average. This is related to our result that subjects say that similar subjects are unbiased, while other subjects might be biased. The phenomenon that people think they are better than others also arises in the second part of our study, where we examine relative biases. Here, people think that they are "better than the average" in the sense that they are less biased. Again, one can explain this result by a self-serving bias.

In a study by Frederick (2005), subjects face questions that induce "intuitive mistakes". This means that the answer that comes first to one's mind is wrong. Frederick does not aim at analyzing what subjects think about others, but on the influence of cognitive ability on decision making. Nevertheless, there is one similarity to our experiment. Subjects judge the difficulty of the questions by estimating the proportion of others who answered them correctly. Those who correctly answered the questions state that they are more difficult (as they are aware of the possible "intuitive mistakes") than do those who failed to answer correctly. This result is in line with our result that more information helps subjects to realize that others are wrong. The information in Frederick is, however, endogenous: it is only available to subjects who solved the questions correctly. Concerning our tricky questions, subjects cannot realize the trickiness (or the find correct answer) just by thinking a bit longer about the question.

The issue that the type of questions that subjects answer matters for overconfidence has been intensively investigated in the literature with different results. A well-known result is the hard-easy effect. Lichtenstein and Fischhoff (1977), for instance, show that with easy questions overconfidence vanishes and even turns into underconfidence. Gigerenzer (1993) claims that the type of questions does not matter, but that it matters whether questions are randomly selected or not. If they were selected randomly, overconfidence would vanish. Among others, Brenner et al. (1996) show that this is not true. We do not want to add to this discussion. The tricky questions that we use are just a means to be able to provide subjects with a strong signal (by showing them the correct answers) that others might be wrong with their assessment.

The paper is structured as follows. In Section 3.2, we describe the experimental design for the treatments that deal with the question whether people know that others have a bias (or make mistakes). We first present the basic and the reference treatment, before we explain the extensions. In Section 3.3, we derive and discuss the theoretical predictions and present the results in Section 3.4. Afterwards, in Section 3.5, we analyze the question whether subjects are aware that others make mistakes. In Section 3.6, we consider the question what people think about their relative bias - first presenting the design, then the predictions and finally the results. In the last section, we conclude.

3.2 Experimental Design

The experiment was conducted at the University of Bonn. A total of 116 subjects participated in six sessions (with 18 to 22 participants each) - one session for each treatment (*T Average, T AveragePlus, T Frame, T Individual*) and one for each reference treatment (*R Hard, R Tricky*). Each subject participated in only one of the treatments. Note that we refer to a subject, who participated in one of the four T treatments, as "he" in the following and to one in the R treatments as "she". The experiment was programmed with the software z-Tree (Fischbacher (1999)). Subjects have been recruited via the internet by using the software ORSEE (Online Recruitment System for Economic Experiments) developed by Greiner (2004). The instructions[7] have been read out loudly before the experiment started and the subjects answered clarifying questions to make sure that they understand the experimental procedure. The wording of all but one instructions (see later) was kept neutral to avoid framing effects. We did not use terms like self-assessment, type, overconfidence, etc. which we use in the following to describe the design. Subjects could earn Tokens during the experiment, where 210 Tokens = 1 Euro. Average hourly earnings were 8 Euros.

[7]Instructions are in the appendix (translated from German).

3.2.1 Treatment Design - The Basics (*T Average* and *R Hard*)

In our baseline treatment, *T Average*, 20 subjects have to state whether they think "others" are on average overconfident, underconfident or rational. The "others" are 20 subjects (we call them *Rs* or she in the following) from the reference treatment *R Hard*, who answered seven very hard multiple choice questions from different fields of general knowledge. They were paid 190 Tokens for each correct answer.[8] After having answered these questions, *R* had to estimate her number of correctly answered questions – we denote this estimate by $q \in \{0, 1 \ldots 7\}$ – without knowing her true number of correctly answered questions $t \in \{0, 1 \ldots 7\}$. The resulting payoff, $\pi(t, q)$, from her estimate q depends on whether her guess is correct, i.e. equal to her true number of correctly answered questions questions, or not correct:

$$\pi(t, q) = 525 - 495\ \mathbf{1(t \neq q)}$$

where $\mathbf{1}(\cdot) = 1$ if and only if $t \neq q$ and 0 otherwise. This means that *R* is punished if she over- or underestimates her number of correctly answered questions t.[9] As we will show later, her estimate q should be equal to her belief about t (i.e. the t she considers as most probable). When answering the questions, she knows that she has to make a decision later on, where her payoff depends on her number of correctly answered questions. She does not know yet, however, the task and the relevant payoff table. With this procedure, we avoid that *Rs* try to game the experiment by deliberately giving wrong answers (e.g. by giving no answer at all) to be able to make the correct guess.[10]

We did not ask the *Rs* explicitly what they think how many questions they have answered correctly for not influencing their choice. Instead we let them choose between eight actions and show them the corresponding payoffs in a payoff table (see Table 3.1). From this payoff table one can easily infer that it is optimal, for example, to choose "Action 3" if one thinks it is most probable that one answered three questions correctly.

In the instructions for a subject in *T Average* (he), we explained him what the *Rs* had to do, how they were paid for this and we also showed him the multiple choice questions (without indicating the correct answers). In order to elicit whether he thinks she is over-, underconfident or rational on average, we told him the average q (average estimate of number of correct answers) of the *Rs* rounded to one decimal place, which is denoted by $\bar{q}$ in the following. Similarly, we denote by $\bar{t}$ the average t (average true number of correct answers), however, he is not told $\bar{t}$. Then, he has to state

[8] Subjects are free not to give an answer at all, which leads to a payoff of zero for this question.

[9] One might wonder why we did not punish more, the larger the deviation of an estimate q from t is. The answer is that with risk averse subjects, one can then no longer be sure whether they choose the number q that equals t if and only if they think their true number is t.

[10] As we pay subjects for each correctly answered question and for their estimate, this problem should be alleviated, but we wanted to avoid such motivations completely. In fact, all subjects gave an answer to all questions.

	Correct Answers (t)							
	0	1	2	3	4	5	6	7
Action 0	525	30	30	30	30	30	30	30
Action 1	30	525	30	30	30	30	30	30
Action 2	30	30	525	30	30	30	30	30
Action 3	30	30	30	525	30	30	30	30
Action 4	30	30	30	30	525	30	30	30
Action 5	30	30	30	30	30	525	30	30
Action6	30	30	30	30	30	30	525	30
Action 7	30	30	30	30	30	30	30	525

Table 3.1: Payoffs - How Many Questions Do You Think You Have Correct?

whether he thinks that $\bar{t}$ is smaller than $\bar{q} - 0.5$ (which means thinking the *Rs* are overconfident), that $\bar{t}$ is between $\bar{q} - 0.5$ and $\bar{q} + 0.5$ (*Rs* are rational) or that $\bar{t}$ is larger than $\bar{q} + 0.5$ (*Rs* are underconfident). By adding/subtracting 0.5 we capture rounding effects and small mistakes which remain on, even though the *Rs* are rational on average. A subject in *T Average* receives 1680 Tokens if he is correct, which means he states that the *Rs* are overconfident (underconfident or rational, resp.) when they are indeed, otherwise he earns 315 Tokens. Note that we did not ask a subject in *T Average* explicitly whether he thinks that the *Rs* are underconfident, overconfident or rational, but gave him the choice between three actions $left$, *middle* and *right*. It could be inferred from the payoff table (see Table 3.2) that it is optimal to choose, for example, action *middle* if one thinks that the *Rs* estimated the number of questions correctly.

Moreover, a subject in *T Average* states how many questions he thinks the *Rs* answered on average correctly, i.e. how large he thinks $\bar{t}$ roughly is. For this statement, he chooses a number z out of the set $\{0, 0.1, 0.2, \ldots, 6.9, 7\}$. Of course, he could only choose a number smaller than $\bar{q} - 0.5$ if he stated before that the *Rs* are underconfident and correspondingly if he stated that they are overconfident or rational, respectively. He receives 105 Tokens in case his guess z of the average number of correct answers $\bar{t}$ is almost perfect – which means that the distance between his guess z and the true average $\bar{t}$ is smaller than 0.5 – and 20 Tokens otherwise.

For the estimation of $\bar{t}$ we implemented a similar procedure than before: We did not ask "How large do you think is $\bar{t}$?", but let subjects choose a number and let them infer from the payoffs what this choice means.

In the following sections, we describe all other treatments that extend the baseline treatment *T Average* in various ways.

	Action		
	left	middle	right
$\bar{t} < \bar{q} - 0.5$	315	315	1680
$\bar{q} - 0.5 \geq \bar{t} \leq \bar{q} + 0.5$	315	1680	315
$\bar{t} > \bar{q} + 0.5$	1680	315	315

Table 3.2: Payoffs - Are The Others Biased?

3.2.2 Extension – Impact of Answering The Questions Oneself (*T AveragePlus*)

We are interested in the question whether a subject's belief about the *Rs* being underconfident, overconfident or rational is influenced, when he answers the questions himself and estimates his own true number of correct answers. By completing these tasks he might get a better feeling for the difficulty of the questions and whether the average guess of the *Rs* is realistic. Therefore, in the treatment *T AveragePlus*, 17 subjects answered the same multiple choice questions, estimated their number of correct answers and stated whether they think that others are under-, overconfident or rational.[11]

3.2.3 Extension – Hard versus Tricky Questions (*R Tricky* and *T Frame*)

With another treatment, we want to test whether some form of feedback helps subjects to recognize that others are biased. To test this, we first conducted the treatment *R Tricky*, which is identical to *R Hard*, except that these new 20 *Rs* answered different multiple choice questions. Instead of the hard ones, we selected "tricky" ones, i.e. questions that look very simple, but are in fact very difficult: subjects are quite certain that they choose the right answer, but actually select the wrong one.

Subjects in the treatment *T Frame* (where we use non-neutral wording in the instructions; see the next subsection) answered both the hard and the tricky questions and performed all the tasks as subjects in *T AveragePlus*. They had, however, not only to judge the *Rs* in *R Hard* but as well those in *R Tricky*. In addition, they state how many questions they think the *Rs* (in *R Hard* and *R Tricky*, respectively) have on average correct by choosing a number z for subjects in both R treatments. Note that we want to highlight the trickiness of the questions to see whether subjects are forced by this kind of information to recognize problems like overestimation. Therefore, we showed subjects in *T Frame* the correct answers to the tricky questions before they assessed whether the *Rs* are under-, overconfident or rational (but of course *after* they answered the questions). In order to avoid hedging effects, we randomly selected for the payment one block of questions (either the hard

[11] These 17 subjects did, however, not state z, i.e. how many questions they think R answered on average correctly.

or the tricky ones) and one block of decisions (corresponding to the hard or tricky questions) after subjects finished all decisions.

3.2.4 Extension – Impact of Framing (*T Frame*)

Psychologists generally use a non-neutral language in their experiments. We want to see whether such framing[12] has some impact on our results. Subjects might think differently about a problem when they read the word "overestimate" instead of "action *right*". By reading the word "overestimate" a subject might get an idea that overestimation is a problem (why else should he read this word in the instructions?). To analyse the impact of the wording, we "framed" the instructions in *T Frame*. The main differences are as follows: in *T Frame* we explicitly asked subjects "How many questions do you think you have correct?", while in all other treatments we let them choose between eight actions.[13] Furthermore, subjects had to state whether they think that the *Rs* under- or overestimate the true number of correct answers or estimate it correctly. In the treatments with neutral instructions, however, subjects choose between three corresponding actions $left$, $right$ and *middle*. Similarly, for the statement of the belief about the *Rs'* average number of correct answers, we explicitly asked in *T Frame* "How many questions do you think the others answered on average correctly?", while in the neutral treatments we let subjects choose a number z and they have to infer the meaning from the payoffs.

3.2.5 Extension – Single Subject versus The Average (*T Individual*)

Does it make a difference whether the "others" represent the average of the *Rs* or a single *R*? As we explain more precisely in the next section, in theory it does: For a single subject one cannot distinguish by observing the guess q and the true number t of correct answers whether she makes just an unsystematic mistake or is really biased if the numbers differ. By observing the average numbers $\bar{q}$ and $\bar{t}$ of a group of subjects, however, one can conclude that these subjects are on average biased if the averages differ.

From the treatments described above, in which subjects face the average of the *Rs*, we can infer whether subjects think others are on average biased or not, but we cannot infer whether they think others make unsystematic mistakes. Yet, we are also interested in whether subjects think that *Rs* just make mistakes but are not biased, do not even make mistakes, or that they are biased

[12] The term framing should not be misleading here. We just mean by it that we use a non-neutral language which hints at problems like a wrong self-assessment.

[13] Note that subjects in the *R Treatments* faced a different decision problem when estimating their number of correct answers as they face a payoff table and have to infer the meaning. This decision problem was explained to the subjects in *T Frame* and we made clear that it meant the same as the question "How many questions do you think you have correct?".

	R Hard	*R Tricky*	*T Average*	*T AveragePlus*	*T Individual*	*T Frame*
Hard Questions	√			√	√	√
Tricky Questions		√				√
Estimate Own t	√	√		√	√	√
Others Biased?			√	√	√	√
Relative Bias?				√	√	√
Guess z of $\bar{t}$ (t)			√		√	√
Info about $\bar{q}$			√	√		√

Table 3.3: Overview of the Tasks in the Treatments and in which Treatment Subjects Receive Information about the *Rs'* Average Belief $\bar{q}$. "Hard/Tricky Questions" means that subjects answer the hard/tricky questions.

(i.e. mistakes are systematic). Regarding the first case, for example, subjects would be aware that a single subject might make mistakes, but that these cancel out for a group of subjects. Thus, the group is unbiased.

To analyze the issue, we additionally conducted the treatment *T Individual*, in which subjects state beliefs about single *Rs* and not the complete group. In *T Individual*, 20 subjects have to perform all the tasks subjects in *T AveragePlus* have to. A difference to *T AveragePlus* is that we implemented the strategy method in *T Individual*. Thus, subjects do not receive specific information about a single R, but they state for every possible estimate $q \in \{0, 1, \ldots, 7\}$ of R whether they think she is under-, overconfident or rational (strictly speaking: makes mistakes or not – see Section 3.3.1). For the numbers 0 and 7 on the boundary, subjects only choose between the two appropriate possibilities. In case he thinks R is under- or overconfident, he has to choose a number $z \in \{0, 1, \ldots 7\}$, $z \neq q$, that mirrors his belief about her true number of correct answers. A subject in *T Individual* was not paid for all his decisions, but for his decision when facing a particular estimate q of an R. For his payment, one R was randomly selected. Her q and t – together with his decision when facing her estimate q – determined his payment. Again, we did not ask all these questions directly, but confronted subjects with simple decision problems to infer their beliefs.

In Table 3.3, we provide an overview of all the tasks subjects have to complete in each treatment. We also indicate, in which treatment we inform subjects about the average estimate $\bar{q}$ of the *Rs*. In Table 3.4, we list the timing of the single tasks and when we inform subjects about $\bar{q}$. Since not all stages are present in all treatments, the corresponding timing of a treatment follows by skipping the missing stages. In these tables, we show already the "relative bias" task that is explained in more detail in Section 3.6.

$T = 1$	$T = 2$	$T = 3$	$T = 4$	$T = 5$
Questions	Estimate Own t	Relative Bias?	Info about $\bar{q}$	Others Biased? & Guess z

Table 3.4: Timing

3.3 Predictions

3.3.1 Definitions and Assumptions

For the theoretical predictions of our experiment, we need some weak assumptions and definitions about the players' behavior. We assume that individuals are subjective expected utility maximizers, with a strictly increasing utility function, i.e. they prefer more money compared to less.[14]

Next, we define what we mean by an under- or overconfident individual – i.e. a *biased* individual – or by a rational individual. Biased means that an agent's self-assessment is wrong – he *systematically* under- or overestimates his number of correct answers and is thus under- or overconfident, respectively. By systematically we mean that the mistakes an individual makes when estimating her ability are not random, in the sense that they do not cancel out on average. A rational agent in contrast makes on average no mistakes.[15] Thus, we can identify whether a population of individuals is rational – if they were rational, then $\bar{t} = \bar{q}$ (roughly) holds.[16] If they were not rational, then $\bar{t} \neq \bar{q}$. In the latter case, we define the bias as $b := \bar{t} - \bar{q}$ and say that a population with $b < 0$ is overconfident (or overestimates its ability, i.e. the true number of correct answers) and one with $b > 0$ is underconfident (or underestimates its ability). For a single individual, however, we cannot infer from observing her t and q that she is biased or not, since she could have made only an unsystematic mistake ($b < 0$: negative mistake, $b > 0$: positive mistake).

Note that in *T Individual*, we ask subjects whether they think that a single subject R is right with her self-assessment. Thus, we can in general not conclude from *T Individual* whether subjects think

[14]This seems reasonable since here are no concerns for concepts like fairness. Note that we make no assumptions regarding the curvature or differentiability of the utility function. Thus, we could – by an appropriate definition of the reference point – also think of the utility function as a value function in the spirit of Kahneman and Tversky (1979) to capture concepts like gain-loss utility.

[15]Statistical: A *rational* individual estimates that her type is $\mathrm{E}[t|\xi]$, when her true type is t with $\mathrm{E}[t|\xi] = t - \varepsilon$ and ξ is the available information of the individual. Hence, $\mathrm{E}(\varepsilon) = 0$, i.e. a rational individual makes on average no mistakes. Assuming that across individuals ($i \in I$) the ε_i's are uncorrelated random variables, one could apply the weak law of large numbers to see that $\lim_{I \to \infty} \frac{1}{I} \sum_i \varepsilon_i = E(\varepsilon) = 0$. For a biased individual $E(\varepsilon) = b \neq 0$, i.e. on average it makes mistakes.

[16]In the experiment, we allow for small deviations from $\bar{t} = \bar{q}$ for a rational group.

that a single R is biased or not, but only whether it makes (systematic or unsystematic) mistakes or not.[17]

Our main interest is whether subjects think others are biased or not. Hence, in most treatments, we consider averages over the beliefs q. Nevertheless, we want to know whether subjects think others make mistakes and thus discuss most results of *T Individual* separately in Section 3.5.

3.3.2 Eliciting Beliefs

In the following, we look at individuals' choices and explain how these mirror a subject's beliefs. In our experiment, all decision problems the individuals face have the same structure: A subject has the choice between several alternatives ($J = \{2, 3, 4, 8, 70\}$). For example, a subject has eight alternatives for the statement how many questions she thinks she answered correctly. If a subject makes the "right" choice (e.g., she states the right number of correctly answered questions), she receives a high payoff and if her choice is not correct, she receives a low payoff. Of course, an individual might be uncertain which alternative is true, and hence forms beliefs about the probabilities of the different alternatives being true. We show in the appendix that *an individual chooses the alternative on which she puts the largest probability to be the correct one* (Proposition 19 in the appendix).[18]

This proposition implies that subjects should state the number of questions they think they answered most likely correctly. As explained above, the *average* values of stated and true number should not differ much in case individuals are roughly rational and only make random mistakes. When individuals tend to be biased in a certain direction (i.e. either over- or underconfidence), however, these numbers differ even on average.

3.3.3 Hypotheses on the Beliefs About the Bias of the Average

Based on previous studies by psychologists and economists (see introduction), we predict the following.

Hypothesis 1 *Subjects overestimate their abilities on average – more so with the tricky questions.*

The fact that subjects overestimate their abilities and that the degree of overestimation depends on the type of questions is a well known result from psychology. In the psychological literature on

[17] In Section 3.5 we also try to infer whether subjects think mistakes are systematic or not.

[18] So called "probability matching" (see e.g. Shanks et al. (2002)) could occur in our decision problem. Suppose the majority of subjects chooses the action "the others are biased". Then also if probability matching happens the results should imply that subjects put the largest probability on this action (similar for the other tasks). Shanks et al. show that this anomaly occurs less often in case financial incentives are provided. Thus, "probability matching" should not be a severe problem for our experiment.

overconfidence the so-called “hard-easy” effect arises: People have been found to be underconfident for “easy” questions and overconfident for “hard” ones (see, e.g., Juslin (1994)). Our tricky questions are designed in a way that provokes more negative mistakes: Subjects are more sure that they selected the right answer but, in fact, this answer turns out to be wrong. This means that confidence rises and the number of correct answers decreases compared to the hard questions. This effect of such “surprising” questions is also addressed in Juslin (1994).
Whether one can say that subjects are more overconfident with the tricky questions depends on the way one defines overconfidence. On the one hand, one can simply say that a population is more biased (here: overconfident) if and only if the absolute value of their bias is larger – i.e. $\bar{t} - \bar{b}$ is smaller – the statement is correct. On the other hand, one can argue that subjects are not more biased because they really *are* more biased, but because the tricky questions *make* them more biased, i.e. subjects only seem more biased (see Brenner et al. (1996)). We do not deepen this discussion as our main point is not the influence of the tricky questions on the degree of overconfidence – instead, we want to see whether subjects can be induced by these questions to recognize that *Rs* are overconfident. Whenever we say in this context that overestimation is more pronounced with the tricky questions, we do not want to claim that these subjects have a stronger bias.
Under the assumption of a symmetric distribution of mistakes, one could for instance use a Wilcoxon Test (to test the hypothesis that the difference between q and t has median value zero given the pairs (q_i, t_i) of individuals i) for testing whether subjects are rational or biased.

Proposition 19 (see appendix) also implies that whenever a subject believes that the *Rs* are more likely to be either over-, underconfident or unbiased, he also states this when asked for his assessment. Further, his guess of $\bar{t}$ (the average of the true number of correct answers) should be the number that he thinks mirrors $\bar{t}$ most likely. Therefore, for agents who are uncertain between positive and negative biases or between different sizes of biases (including positive, negative and zero biases), we interpret their choice as reflecting what they think to be most likely true (and say sometimes for simplicity “they think”, without the most likely). Obviously, we cannot distinguish between agents who are certain or uncertain about their statement being true.
We predict that at least some subjects know that the population is biased. A priori, it is not clear whether more subjects think that *Rs* are biased or more of them think that they are rational. From experiments and field evidence about hyperbolic discounting we know that some individuals are only “partially naive” and not fully naive (see e.g. Della Vigna and Malmendier (2006)). Partially naive means that they know their *own* bias to some extent. In case people know that they are biased themselves, there is some chance that they also know that others are biased. Note that in our experiment a wrong guess could just be a mistake and not a bias, while hyperbolic

discounters are always biased. Thus, subjects in our experiment might be aware that people make mistakes (un-, or systematic), but many might expect mistakes to cancel out on average.
Concerning our different treatments we make the following prediction:

Hypothesis 2 *The more information subjects receive about the problem (no information in T Average, answering questions themselves in T AveragePlus, seeing the correct answers and framed instructions in T Frame), the more subjects state that others are biased.*

This hypothesis seems evident in the (theoretical) sense that subjects, who can use more information, can update their beliefs and thus, make better decisions. Experimental studies on whether subjects update information according to Bayes' rule, however, rather provide evidence that subjects are not "perfect Bayesians" (e.g. Kahneman and Tversky (1972) or Zizzo et al. (2000)). Nevertheless, we think that in our experiment, more information works in the stated direction. Subjects are forced to reason better how realistic it is that the *Rs* have on average $\bar{q}$ questions correct, once they answered the questions themselves and recognize that it is indeed very hard to give so many correct answers. This effect is reinforced when they see the correct answers of the tricky questions – here they could recognize that these tricky questions induce overestimation.
Effects of better reasoning on decisions are for instance explored by Croson (2000). She finds that the frequency of equilibrium play in prisoner's dilemma and public good games increases when first subject's beliefs about the actions of others are elicited before the game is played. It is a priori not clear whether the effect of framing is stronger or weaker than the one of answering the questions oneself. Nevertheless, we think that there is an effect – reading words like "overestimation" gives subjects a hint that such things could occur. Hence, we predict that more subjects state that the *Rs* are biased.

When thinking about others, individuals often tend to conclude from their own behavior or own beliefs on others. This is the so-called false consensus effect, see e.g. Mullen et al. (1985).[19] We expect this effect to be crucial, when subjects judge the others. Thus, we have the following hypothesis.

Hypothesis 3 *When making statements about the Rs (about their bias or about their average number $\bar{t}$ of correct answers), this statement tends in the direction of the own behavior (own bias or guess of own number of correct answers).*

---

[19]Note that doing this need not be suboptimal, especially when subjects have no further information on the identity or characteristics of others. Therefore, the term "false" can be misleading.

	$\bar{t}$	$\bar{q}$	Bias	p
R Hard	2.3	3.4	-1.1	0.006
R Tricky	1.2	4.6	-3.4	0.000
T AveragePlus	2.1	3.5	-1.4	0.005
T Individual	1.7	2.7	-1.0	0.068
T Frame Hard	2.6	3.1	-0.5	0.207
T Frame Tricky	1.6	3.2	-1.6	0.001

Table 3.5: Reported and True Number of Correct Answers

3.4 Results

In the following, we first discuss the results on the own bias of subjects. This refers to the first part of the experiment, the question task and the self-assessment, which is present in all treatments but *T Average*. Note that in *T Frame* we pose both types of questions and subjects have to evaluate both reference groups *R Hard* and *R Tricky*, respectively. When presenting the results, we therefore split this treatment into *T Frame Hard* and *T Frame Tricky*, where each part refers to either the hard or tricky questions and the corresponding decisions. Since overconfidence has already been extensively investigated in the psychological literature, our discussion is very brief.

We then turn to our results on the new issues – the knowledge about other people's bias and the belief about the relation between own and other people's biases.

3.4.1 The Own Bias (Hypothesis 1)

Table 3.5 shows the average type $\bar{t}$ and the average estimate $\bar{q}$ for each treatment, the difference between the two (bias) and the p-values from a Wilcoxon Test. With the hard questions the bias ranges from -0.5 to -1.4, with the tricky ones from -1.6 to -3.4. The p-values indicate significant differences between $\bar{t}$ and $\bar{q}$ for all treatments except for *T Individual* and *T Frame Hard*. Although not significant in *T Individual*, the average bias is -1 which is quite large.

Even if the average bias of the whole group indicates overestimation, different types of individuals exist. Figure 3.1 illustrates the percentage of subjects that are under-, overconfident or rational (make positive, negative or none mistakes) in the different treatments. The fraction of subjects, who are overconfident (make negative mistakes), ranges from 53 to 90 percent in the different treatments. As intended, with the tricky questions the percentage of those, who overestimate their number of correct answers, is higher. Moreover, it is interesting to see that on average 75 percent of subjects in *R Tricky* focused on one and the same answer for each question. With the hard questions, in contrast, the answer that has been chosen most often for a question, has on average

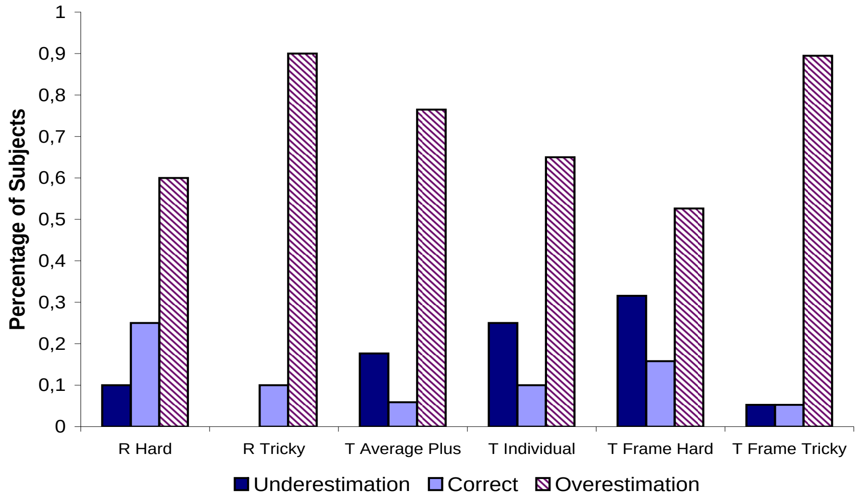

Figure 3.1: Percentage of Subjects Being (Not) Correct in the Treatments

only chosen by 44 percent of the subjects in *R Hard*. One could take this as a vague hint that subject's confidence in an answer also increased with the tricky questions. How does the type of questions influence the true number of correct answers and the belief about it? As discussed in Section 3.3.3, the size of the bias cannot necessarily be interpreted as stronger overconfidence. Nevertheless, the effect of the type of questions on the true and believed number of correct answers is important. It might be that subjects recognized that these questions are tricky and adjusted their beliefs accordingly. As argued in Section 3.3.3, we find that in *R Tricky* the true numbers of correct answers t are significantly smaller than in *R Hard*, whereas the estimated numbers q and thus the mistakes are significantly larger in *R Tricky* (Mann-Whitney U test: $p = 0.009$, $p = 0.001$ and $p = 0.000$, respectively). This indicates that people in *R Tricky* seem *not* to recognize the trickiness of the questions. Similarly, within the two parts in *T Frame* (hard and tricky questions), true numbers of correct answers $t's$ are significantly larger and mistakes are significantly smaller for the hard questions (Mann-Whitney U test: $p = 0.026$ and $p = 0.03$, respectively).

Moreover, we are interested in the effects of framing. We observe that the estimates q are larger and that overestimation is much more pronounced in *R Tricky* compared to the framed treatment *T Frame Tricky*. The $q's$ and also the mistakes of subjects are significantly different across these treatments (Mann-Whitney U test: $p = 0.001$ and $p = 0.002$, respectively). This result is (to our knowledge) new in this context. A possible explanation for it is what psychologists call self-impression management: "[This concept] suggests that a person acts to show himself in a positive light, even when he is the only observer of his own behavior." (Murnighana et al. (2001)). Comparing the neutral with the framed treatment, subjects in the neutral treatment do not have as strong emotions when their decision turns out *not* to be optimal as subjects in the framed treatment who are forced to think of terms like self-assessment. The latter subjects feel ashamed or more stupid when they are wrong or they even do not want to appear themselves arrogant. Therefore, in the framed treatment, subjects are reluctant to make overly optimistic guesses – instead they make more realistic guesses such that overestimation is reduced.

3.4.2 What Do You Think about the Bias of Others? (Hypothesis 2)

In this subsection we analyze the subjects' perception of the *Rs'* bias.

Result 1 *Without further information, a majority of the subjects thinks that others estimate their ability correctly. The more familiar subjects are with the task or the more information they receive (answering the questions themselves, framed instructions, seeing the correct answers of the questions), the less subjects think that others estimate their ability correctly.*

This result is illustrated in Figure 3.2. The figure shows the percentage of subjects in the different treatments believing that the *Rs* are underconfident, rational or overconfident. Except for the second part in the framed treatment, where subjects saw the correct answers before evaluating the *Rs'* average estimate, a majority of subjects states that the *Rs* are rational. Being asked for their choice in a questionnaire after the experiment, subjects say that they made this choice because they either think the mistakes the *Rs* make cancel out on average or that the *Rs* have better information about their own number of correct answers, or that the *Rs* are simply able to make the correct choices.

Next, we explore the impact of a single piece of information. First, we ask about the effect of answering the questions oneself. Does this induce more subjects to recognize that others are biased? As aforementioned, answering the questions oneself gives subjects a better feeling for the difficulty of the task. Hence, subjects get a better impression how realistic it is that the *Rs* indeed answered $\bar{q}$ questions correctly on average as these estimate. In Figure 3.2, we see that the percentage of subjects, who think that the *Rs* are biased, is slightly higher in *T AveragePlus* compared to *T Average*. We cannot reject, however, the hypothesis that there is no relation between the number of subjects in the two treatments, who think that the *Rs* are rational or biased, according to a Fisher's exact test ($p = 0.234$ one-sided).

Framing and answering the questions together, in contrast, (i.e. comparing *T Frame Hard* and *T Average*) has a significant effect (Fisher's exact test: $p = 0.038$ one-sided). It increases (decreases) the percentage of subjects who think the others are biased (unbiased). Nevertheless, no significant difference arises, when only framing is added given that subjects answer the questions themselves: Comparing *T AveragePlus* and *T Frame* yields no significant effect ($p = 0.252$). In Figure 3.2, we

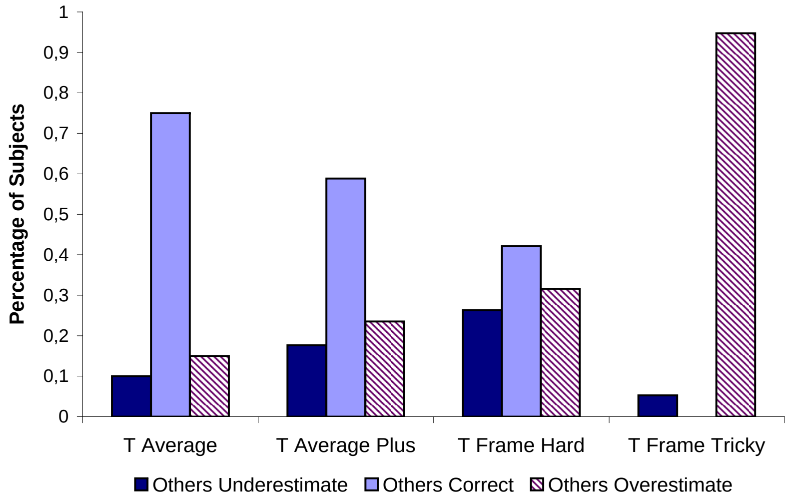

Figure 3.2: Beliefs About Others' Self-Assessment

can see, however, that the percentage of subjects thinking that the *Rs* are biased is larger in *T Frame Hard* than in *T Average*/*T AveragePlus*. This increase in the percentage of people thinking that the *Rs* are biased might be caused by the frame – by reading words like "overestimate" and "underestimate" subjects get forced to recognize that people's self-assessment might be wrong.
Does it have an effect when subjects see the correct answers to the tricky questions? This kind of feedback has a significant effect – provided with this information, almost all subjects believed that the *Rs* are overconfident.[20] Psychologists have shown (for an overview see Pulford and Colman (1997)) that feedback in form of giving the correct answers has the greatest impact on a subject's *own* bias when feedback contradicts a subject's belief most. Our result indicates that this also holds for giving feedback when evaluating others and not oneself. This is interesting since here the adjustment has to proceed in two steps as subjects conclude from their own bias on the bias of others: At first, subjects recognize that it is impossible for themselves to have as many questions correct as the *Rs* think they have on average in *R Tricky*, i.e. $\bar{q}$. In a second step, subjects conclude from their own ability that it must also be impossible for the *Rs* to answer that many questions correctly.

What do the subjects guess is the average true number of correct answers $\bar{t}$ of the *Rs* given the feedback $\bar{q}$ they receive about the others' average belief about $\bar{t}$? This means, we consider the

[20]Comparing *T Frame Tricky* to *T Frame Hard*/*T AveragePlus*/*T Average*, there are more (less) subjects who think that the *Rs* are biased (rational) according to a Fisher's exact test ($p = 0.0015/0.0001/0.00$ one-sided).

	Feedback $\bar{q}$	Guess z	p-value (Mann-Whitney U test)
T Average	3.4	3.2	0.002
T Frame Hard	3.4	3.4	0.138
T Frame Tricky	4.6	2.9	0.000

Table 3.6: Belief z about the Others' Average Number of Correct Answers ($\bar{t}$) versus the Others' Average Belief $\bar{q}$ about Their Own Number of Correct Answers (t)

subjects' estimate z given information $\bar{q}$ of the reference treatments (this information thus differs whether the hard or tricky questions are considered). The result is summarized in Table 3.6. The figure shows the average estimate $\bar{q}$ chosen by the *Rs* for the tricky and hard questions, respectively, the estimates z and the p-values from a Mann-Whitney U test – testing whether z and $\bar{q}$ are different from each other.

For *T Average*, we see that although subjects think that the *Rs* are roughly correct when evaluating their abilities, they think that the *Rs* are a little bit overconfident ($\bar{q} > z$). With the tricky questions, where subjects recognized after seeing the correct answers that the *Rs* are overconfident, they adjust their estimate z of $\bar{t}$ downward to 2.9. Although this is significantly smaller than $\bar{q} = 4.6$, the estimate is still higher than the true average $\bar{t} = 1.2$. Interestingly, the estimate z is not that much smaller than 3.4, which was the subject's guess for the hard questions in *T Frame*. Thus, subjects recognize that the *Rs* overestimate their abilities, but are still not aware that overestimation is such a severe problem.

3.4.3 Why Do You Think What You Think About the Bias of Others? (Hypothesis 3)

In the last section, we already got some hints that subjects conclude from their own behavior on the behavior of others. Now, we want to investigate the reasons for the subjects' choices in more detail.

First, we have a closer look at the relationship between the bias a subject in *T AveragePlus* has himself and the belief he has about the bias of the *Rs*. The cumulative distribution functions of the average value of the bias of subjects who either think that the *Rs* are correct, overestimate or underestimate their ability is shown in Figure 3.3. The cumulative distribution function of those subjects who think that the *Rs* are on average correct is always below the other two functions. Hence, those subjects have less extreme (negative) biases. From the average biases, we see that in *T AveragePlus* those who think the *Rs* are overconfident have on average a bias of -2, those who think that the *Rs* are roughly unbiased have a bias of -0.7 and the rest has a bias of -3. The difference

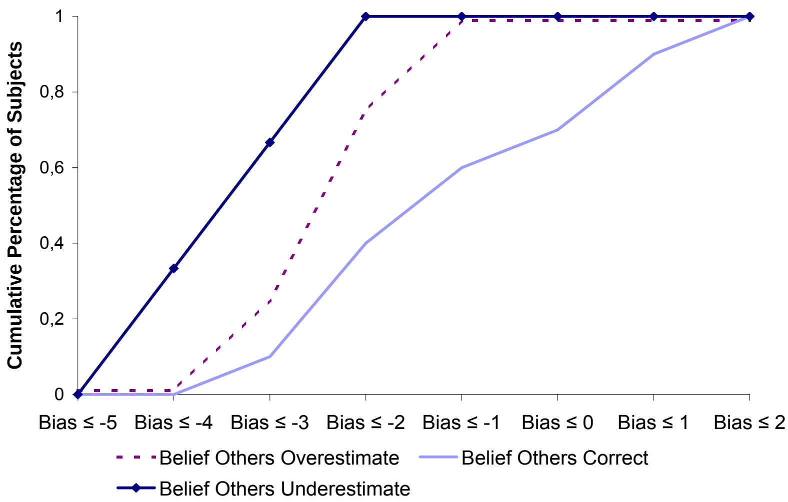

Figure 3.3: Own Bias given Belief About Others' Self-Assessment

between the biases of subjects who say that others are biased and those who say they are rational are according to a Mann-Whitney U test significant (p=0.033). Moreover, in *T AveragePlus* 85 percent of the subjects having a "small" bias (larger or equal to -1) think that others are rational, while 60 percent of those who have a bias smaller than -1 (i.e. who overestimate more heavily) say that others are biased. This result is striking since we cannot directly explain it by a false consensus effect. Recall that subjects do not know how good their own self-assessment is. The result can, however, be taken as evidence that subjects have some[21] knowledge about the degree of their own bias. Subjects may conclude from their bias onto the bias of others. For instance, a subject may reason as follows: "I am rational and I know this, so the others are rational, too". These findings are summarized more generally in the following:

Result 2 *Those who think that others make on average the correct choice, make on average better choices themselves; while those who think that others are biased, make on average more biased choices. Moreover, the other way around, most subjects that are unbiased also think that others are unbiased.*

This result is also striking in another aspect. It gives us some hint that the choice of subjects is not driven by a "better than the average effect" (or self-serving bias), but by their implicit

[21]If we sometimes say a subjects "knows about his bias", we mean the following: The subject knows that he is, e.g., overconfident to some extent, but he does not know the exact magnitude of this bias. Would he know the magnitude, he could perfectly correct for the bias.

self-knowledge as described above. What do we mean by "better than the average effect" in this context? After subjects estimated their own number of correct answers to be q, they learn the estimate $\bar{q}$ of the *Rs*. Thus, subjects can see whether – according to their own and the *Rs'* beliefs – they are better or worse than the average. If they think that they are better than the average but $q \leq \bar{q}$, they can simply state that the others are overconfident in order to sustain their self-image of being better than the average, as this means the others are actually worse than $\bar{q}$ and thus maybe even worse than q. There is not a large difference, however, between the percentage of those stating that others are overconfident or rational: $q \leq \bar{q}$ holds for 40 percent of the subjects stating that others are rational and for 50 (33) percent of those, who say that others are overconfident (underconfident). Hence, we find no clear evidence that subjects try to fool themselves to make them better than the average by stating others are overconfident.

In *T Frame Hard* Result 2 is slightly different. Those saying that the *Rs* underestimate their ability, have on average a bias of -1.2. Those saying that the *Rs* are roughly correct, have a bias of -0.75. And those, who say that the *Rs* are overconfident, are in fact (on average) underconfident with a bias of 0.5.[22] In *T Frame Hard* subjects, who say that the *Rs* are overconfident, could be aware (and this awareness could be caused by the frame) that overconfidence not only exists in the population but also for themselves. Hence, they might adjust their choice accordingly, which leads to a less severe bias (and even underconfidence). With those, who say that the *Rs* tend to be underconfident, it is exactly the other way around (as well as in *T AveragePlus*). These subjects have the most severe bias. Thinking that they are underconfident themselves might induce them to choose an estimate q that is too high such that overconfidence arises.

How does the belief z about the (average) true number of correct answers t ($\bar{t}$) of the *Rs* relate to a subject's own stated number of correct answers q? We can analyze this issue in the treatments *T Average*, *T Frame* and *T Individual*. We find that subjects think that others have a similar (average) number of correct answers than they have themselves and – assuming some implicit self-knowledge when subjects make their choice – they think that the others are rational (or do not make mistakes). Hence, we can also see the following result as a strengthening of our interpretation of Result 2: Subjects may not only think that others have a similar bias, but also that they have a similar (average) number of correct answers.

Result 3 *Subjects think that others are similar to them, i.e. they have a belief z about the others' average ability $\bar{t}$ that is close to the belief q about their own ability. Moreover, they think that*

[22]The estimate q of those subjects, who say that others are over- or underconfident, is significantly smaller than the estimate q of those, who say that others are rational (according to a Mann-Whitney U test $p = 0.004$ and $p = 0.011$, respectively).

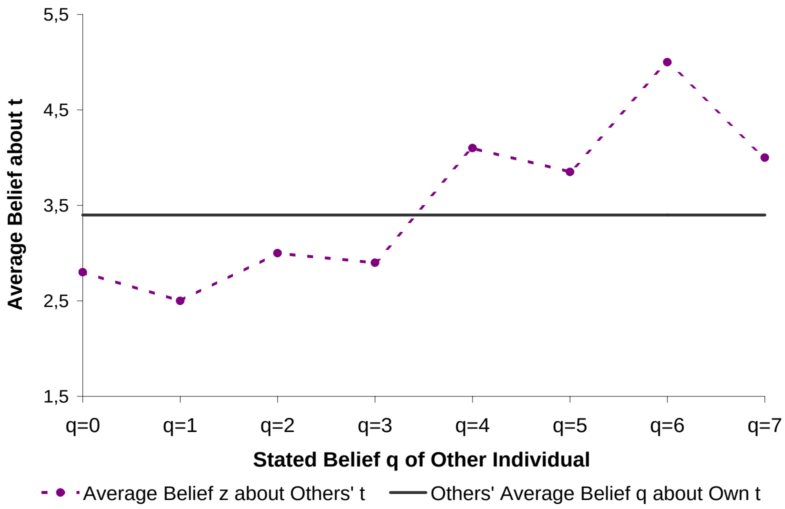

Figure 3.4: Average Belief z about the Others' Ability given the Belief q about the Own Ability

similar subjects are likely to be correct when estimating their ability.

The first part of this result can be explained by the false consensus effect, which says that people tend to overestimate the degree to which, for example, their own behavior or beliefs are shared by other people (compare our prediction). Hence, by the false consensus effect people overestimate the frequency with which their own estimate q is present in the population. Therefore, it is likely that subjects in our experiment adjust their estimate z of $\bar{t}$ in the direction of their own estimate q – under the restriction that they think that the *Rs* are roughly rational. This is illustrated in Figure 3.4, which shows the average estimate z chosen by the subjects in *T Frame* given their belief q of the own number of correct answers and the information the subjects receive (i.e. the *Rs'* average belief $\bar{q}$ which is 3.4). It can be seen that subjects with lower q's (up to 3) choose on average an estimate z that is lower than 3.4, whereas subjects with higher beliefs about the own ability t (from 4 on) choose on average a higher z.

The first part of Result 3 is further supported by the following observations. The average difference between a subject's q and the chosen z is only -0.09 in *T Frame Hard* (and for the tricky questions it is still only 0.41). According to a Wilcoxon Test there is no significant difference in the median of the chosen number and the chosen action ($p = 0.647$ and $p = 0.155$). Furthermore, in *T Frame Hard* the estimates q and z are correlated: The Spearman rank order correlation coefficient is 0.737 (with $p = 0.0002$).

The second part of the result can be derived from *T Individual.* Recall that a subject in *T*

Individual states a belief z about an *R's* true number of correct answers t for each possible belief R can have about her t. This means that a subject in *T Individual*, which states a belief z that equals *R's* belief q, believes that this R is correct.

We compare a subject's estimate z of *R's* true number of correct answers t with this *R's* estimate q of the own number of correct answers given that R is *similar*. A similar R has exactly the same belief q about her number of correct answers than the subject in *T Individual* has about his number of correct answers. For example, an R who thinks she has three questions correct is similar to a subject in *T Individual* that thinks he has three questions correct himself. Given such a similar R, we consider the estimate z a subject in *T Individual* has about this *R's* number of correct answers. The average difference between the estimate z for a similar subject and this similar subject's belief q about his own number of correct answers is -0.05. The medians of these numbers do not differ significantly ($p = 0.476$, Wilcoxon test). Moreover, the belief about a similar individual and the *Rs* own belief are correlated (Spearman rank order correlation coefficient is 0.59 with $p = 0.003$[23]). This implies that a subject thinks that the similar subject is correct with her self-assessment. Subjects even think that similar *Rs* are likely to be correct if these *Rs* hold "extreme" beliefs, for which most other subjects say that this extreme belief must be mistaken.[24]

What can we learn from this? Baker and Emery (1993) showed that individuals know that the average married person in their country gets divorced, but state at the same time that they themselves will not get divorced. If we replace "getting divorced" by "being biased", we get a similar result in our experiment. In case subjects make their statement because they think similar subjects are like them – not only with respect to their ability, but also with respect to their bias (see Result 2 and the first part of Result 3) – we can conclude that subjects also think about themselves that they are unbiased or do not make mistakes as they think that others are unbiased. Note again that a similar R has the same belief about her number of correct answers than has the subject in *T Individual* about himself. If this is true, it also implies that we can explain the second part of Result 3 by a false consensus bias: Subjects conclude from their own beliefs on others. If they think they are correct, then also a similar individual is correct.

As the estimates z are significantly different from the beliefs $q's$ of a single R for all her estimates q besides 2,3 and 4 ($p \leq 0.002$, Mann-Whitney U test), subjects in *T Individual* know that the *Rs* make mistakes.

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[23]If we exclude one subject (that always chose 0 for high q's of the other person), the numbers are even more similar to *T Frame Hard*.

[24]For *Rs* that are *not* similar and who have belief $q \in \{0, 4, 5, 6, 7\}$, the absolute values of the differences between q and z are significantly larger than than they are for similar *Rs* ($p \leq 0.02$). Dissimilar *Rs* with belief $q \in \{1, 2, 3\}$, however, are most often considered to be correct, too.

Furthermore, we are interested in the question whether subjects make the same mistakes (or have the same bias) when evaluating themselves and when evaluating the *Rs*. On the one hand, subjects have better information about themselves than about the *Rs* and this should make it easier to judge themselves. On the other hand, individuals often reject information about themselves, such that they could see themselves in a good light (see, e.g., Bénabou and Tirole (2002)). This should make evaluating the *Rs* easier since a subject does not care about the implications of his choice (which the *Rs* will never get to know) on *R's* self-image.

In *T Frame*, we find no significant difference between the own bias and the bias in assessing the average number of correct answers $\bar{t}$ of the *Rs* by choosing z. On average, the own bias in *T Frame Hard* is about 0.26 larger in absolute terms (it is more negative) than the bias in assessing $\bar{t}$ and in *T Frame Tricky* the own bias is about 0.24 larger in absolute terms (it is more negative again). The latter finding is surprising as with the tricky questions the subjects see the correct answers to the questions after having assessed the own ability t, but *before* assessing $\bar{t}$ of the *Rs*. This additional information seems not to improve the subjects' assessment about the others.

Finally, we compare the own bias (mistake) and the bias in guessing the ability of a similar R in *T Individual*.[25] We find that the own bias is significantly larger – in the sense that overestimation is more pronounced – according to a Wilcoxon test ($p = 0.024$). The average own bias is -1.8, whereas the average bias when assessing a single R is -0.87.

3.5 *T Individual*

For *T Individual* one should be aware that one should replace "over-, underconfident or unbiased", by "negative, none or positive mistake", as we explained in Section 3.3.1. Even for a rational subject the true and stated number of correct answers can differ – the subject might simply make (unsystematic) mistakes. In the following, we try to disentangle what subjects think about the bias or mistake of single subjects.

3.5.1 Beliefs About the Bias/Mistake of Single Subjects

Consider a subject that assesses the *Rs*. He might think that "each R is rational" (and only makes unsystematic mistakes). If he thinks each R is rational, this is consistent with the belief that the *Rs* are unbiased on average. Yet, if he thinks (some) single *Rs* are biased this can still be consistent with a belief that the whole population of *Rs* is on average unbiased: He might think that the

[25] As in *R Hard* no one has belief zero or one, we have to skip those subjects in *T Individual* who have a belief of zero or one.

biases cancel out for the population, i.e. "the population is rational".[26] Under some assumptions, we can calculate, which choices of subjects in *T Individual* are consistent with a belief that *Rs* are rational on average. We present two alternative ways to do this. The first alternative corresponds to the possibility "each subject is rational", the second one to "the population is rational" as just explained.

Regarding the first alternative, we assume that if an individual is rational the distribution of the mistake is uniform and symmetric around zero. This implies that the precision of *Rs* that state extreme q's is higher - for example, someone who says "I answered zero questions correctly" is always right (since he e.g. did not mark any answer). Denote the possible beliefs of R about her number of correct answers by $\{zero, \ldots, seven\}$. The choices of the subject in *T Individual* are $left$, *middle* and $right$, which mean that a subject thinks the other subject overestimates, correctly estimates or underestimates her correct answers. Given a specific belief of R, which (rough) choices of a subject in *T Individual* are consistent if he believes R is rational? We can infer that the following (rough) choices[27] are consistent given a belief $\{zero, \ldots, seven\}$: for belief *zero* and *one* action $right$ (i.e. underestimation), for *two*, *three*, $four$ and $five$ action *middle* (i.e. correct estimation) and for *six* and *seven* action $left$ (i.e. overestimation). For example, an R, who states she has one question correct, how many questions could it actually have correct? Under the assumption that mistakes are uniformly and symmetrically distributed around zero, a subject that has actually one, two, three or four questions correct could state that it has one correct (i.e. have the belief *one*). Then the average of actually correct questions is 2.5 (remember that we assume different mistakes are equally likely). This is by 1.5 larger than 1 what the subject guessed herself. Hence, one should choose for such an R $right$ (i.e. underestimation).

Concerning the second alternative, we assume that single subjects can be biased, but that their (systematic) mistakes cancel out in the population ("the population is rational"). Thus, a subject that states $q = 0$ can make a mistake. We assume that the absolute value of mistakes is at most three and that all mistakes have the same probability. We can derive (similar to above) that the prediction, when a subject in *T Individual* should choose $left$, *middle* or $right$, is exactly the same as for the first alternative.[28]

Similarly, one can see, which choice of subjects in *T Individual* is consistent with his belief that *Rs* are unbiased on average, when they are asked to guess the number of correct answers of R (making

[26] Note that a subject in *T Individual* does not know the distribution of types of the *Rs*, i.e. he does not know how many *Rs* think they have one question correct etc. Therefore, we make assumptions on a subject's belief about this distribution in the following.

[27] These are "rough" choices in the sense that we calculate averages and round these to receive the choice.

[28] To be precise, only rough choices are the same, i.e. when we consider the rounded values. The unrounded values differ.

the same assumptions as above). Namely, if a subject believes that each R is rational but makes mistakes, he should choose the following numbers for each of *R's* beliefs $\{zero, \dots, seven\}$: *zero*: 2, *one*: 2.5, *two*: 2.5, *three*: 3.5, $four$: 3.5, $five$: 4.5, *six*: 4.5, and *seven*: 5.5. If a subject believes that the population of *Rs* is rational but single *Rs* are biased, he should choose the following numbers for each of *R's* beliefs $\{zero, \dots, seven\}$: *zero*: 1.5, *one*: 2, *two*: 2.5, *three*: 3, $four$: 3, $five$: 4.5, *six*: 5, and *seven*: 5.5.

Hence, given the assumption that subjects think the others are rational on average[29], we can compare how the two alternative approaches (assumptions see above) fit the experimental data: Given subjects think that others are rational on average, is it rather the case that they think that each individual is rational or that individuals might be biased but the population is, nevertheless, rational?

3.5.2 Results on Knowledge About Mistakes

When analyzing *T Individual* in more detail, we are interested in the question whether subjects are aware that others make mistakes and whether (and when) they think such mistakes are unsystematic or systematic. In order to investigate these questions, we compare the two different approaches explained in the preceding section: We assume that subjects either think that each R is rational or that the population of *Rs* is rational. Regarding the belief in *T Individual* about the goodness of the *Rs'* guess, we can infer the following from the solid line in Figure 3.5[30]: For low stated q's of R, subjects think that she has more likely a higher q than she stated, while *Rs* with high q's are expected to have more likely a lower q than stated. This means that especially *Rs* with extreme beliefs are considered to be wrong. Remarkably, *no* subject states for all possible estimates q's of R that she makes the correct choice. For each value of the belief q, $50 - 95$ percent of subjects state that R is wrong. Thus, subjects know that *Rs* make mistakes when these assess their q. Furthermore, we compare the average choice of subjects in *T Individual* with the predicted choice that would be consistent with subjects in *T Individual* thinking that the *Rs* are rational. In Figure 3.5, we derive the predicted choice under the assumption that the population is rational and in Figure 3.6, we derive it under the assumption that each single individual is rational and just make mistakes. We see in both figures that the curves for the predicted and actual choice are close. The second prediction fits, however, better for small qs.

Figures 3.7 and 3.8 again differ only in the predicted choices, but the actual considered choice is the same (i.e. the solid line). We see from the solid line that for more extreme beliefs of R, the

[29] We only consider this case here as it turned out that a majority of subjects thinks that the *Rs* are rational on average.

[30] The solid line is the same in Figure 3.6

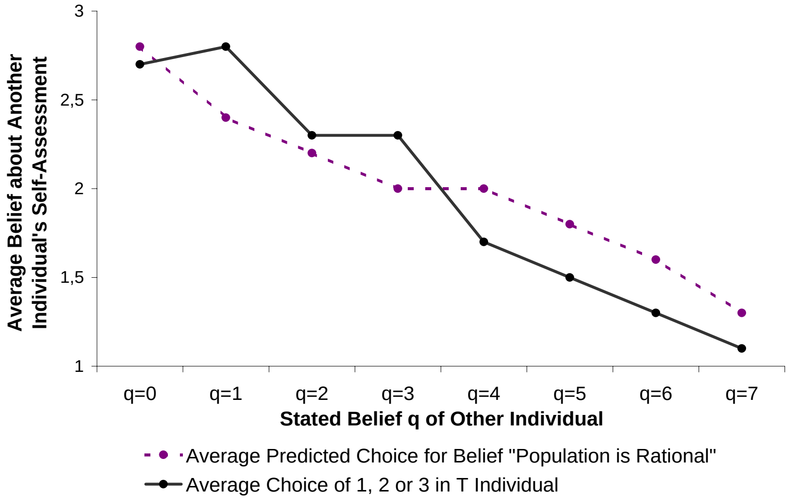

Figure 3.5: Average Assessment of *R's* Self-Assessment for Each of Her Possible Beliefs q about Her Ability t: 1=Overestimate, 2=Right, 3=Underestimate. Assumption: "Population is Rational"

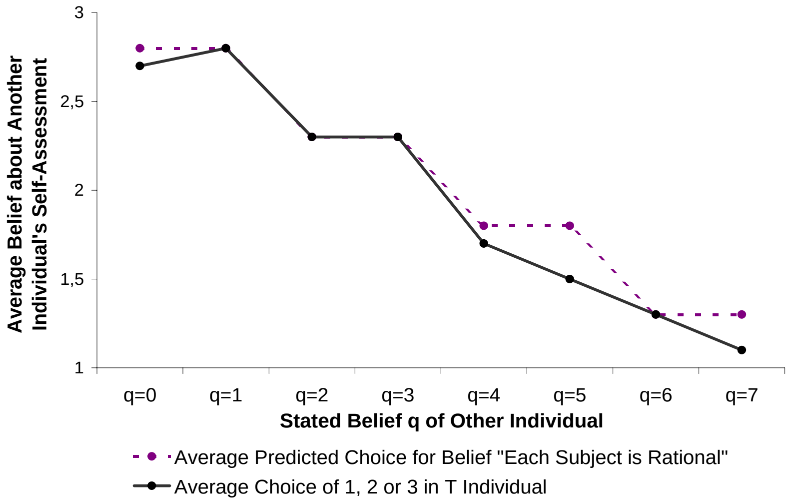

Figure 3.6: Average Assessment of *R's* Self-Assessment for Each of Her Possible Beliefs q about Her Ability t: 1=Overestimate, 2=Right, 3=Underestimate. Assumption: "Each Subject is Rational"

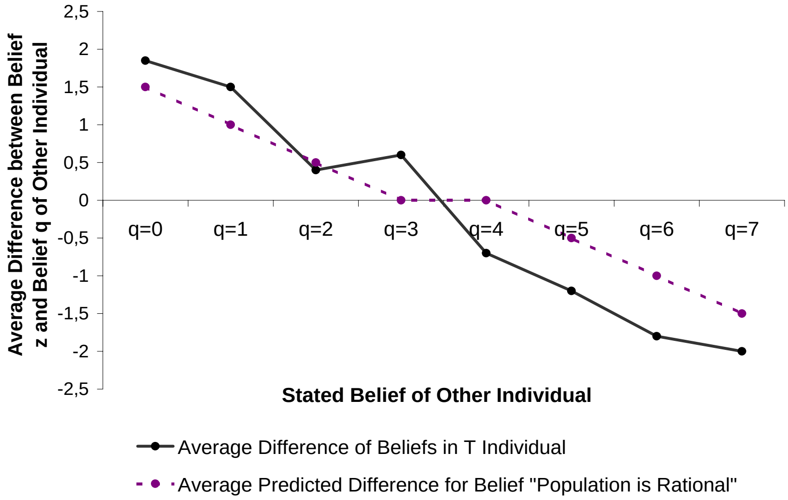

Figure 3.7: Average Value of Absolute Distance Between a Subject's Belief z about R's Ability t for Each of R's Possible Beliefs q about Her Own Ability. Assumption: "Population is Rational"

average distance between the estimate z about R and the stated belief q of this R is increasing. Consistently with the choice above, subjects think that mistakes are more severe for Rs with extreme beliefs: The higher q, the more overestimation is pronounced and similarly, the lower q, the more underestimation is pronounced. We compare these actual choices to the predicted choice that would be consistent with subjects thinking that the Rs are rational but make mistakes. We see that the predicted curve in Figure 3.8 – where the assumption is that a subject thinks "each subject is rational but makes mistakes" – and the true curve are quite close for stated beliefs smaller than 4. In Figure 3.7 – where the assumption is that subjects think the population is rational – this is not true. For higher beliefs, the true curve and the predicted one diverge (under both assumptions). One possible interpretation of this is that subjects rather think that "pessimists" (those with low qs) are rational and just making mistakes instead of being biased. Whereas for "optimists" (those with high qs), we cannot infer which assumption fits observed behavior better.

3.6 Relative Bias

Finally, we want to analyze what subjects think about the relation of their own possible bias or mistake when assessing the number of correct questions and the bias or mistake of the Rs.

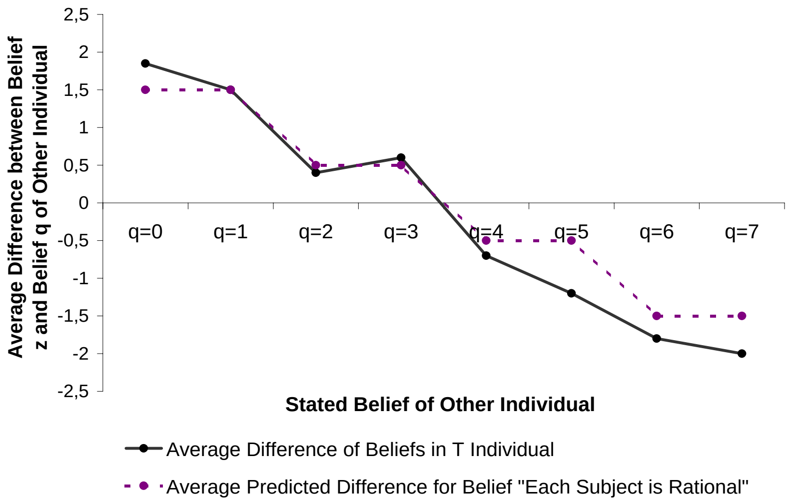

Figure 3.8: Average Value of Absolute Distance Between a Subject's Belief z about *R's* Ability t for Each of *R's* Possible Beliefs q about Her Own Ability. Assumption: "Each Subject is Rational"

3.6.1 Experimental Design

An additional task is included in the following treatments: *T AveragePlus, T Frame, T Individual*. In *T Frame*, we explicitly asked subjects whether they think that "I and others made the correct choice" (or both are wrong/ others right/ I am right). If subjects are right with their statement, they receive 400 Tokens, otherwise they receive 50 Tokens. In *T AveragePlus* and *T Individual*, subjects choose between two alternatives (in *T Individual*, with the strategy method, they choose for each of the eight possible estimates q of R between the two alternatives). Subjects in *T AveragePlus* made this decision based on a payoff table (see Table 3.7). This payoff table shows that the payoffs of the alternatives I and II for the four possible events, where $q = t$ ($q \neq t$) refers to the self-assessment of a subject in *T AveragePlus* and $|\bar{q} - \bar{t}| \gtrless 0.5$ refers to the average self-assessment of the *Rs*. We see in Table 3.7 that payoffs of the two alternatives only differ for the second and third case, i.e. for the event that the subject is correct himself while the *Rs* are wrong on average or when the subject is wrong but the *Rs* are correct on average. In case that both are correct or both are wrong the alternatives lead to identical payoffs. Combining this with the statement whether he thinks that *Rs* are under-, overconfident or rational, one can see whether he thinks that both make the right/wrong decision or only one is wrong while the other is right (we will explain this in more detail below).

The decision of subjects in *T Individual* between the two alternatives is exactly the same as in *T AveragePlus* besides a difference in the payoff table. In *T Individual*, $|\bar{q} - \bar{t}| \gtrless 0.5$ is replaced by

	Alternative I	Alternative II
$q = t$ and $\|\bar{q} - \bar{t}\| < 0.5$	800	800
$q = t$ and $\|\bar{q} - \bar{t}\| > 0.5$	500	300
$q \neq t$ and $\|\bar{q} - \bar{t}\| < 0.5$	300	500
$q \neq t$ and $\|\bar{q} - \bar{t}\| > 0.5$	210	210

Table 3.7: Payoffs - Relative Biases

$q = t$ and $q \neq t$, i.e. whether the self-assessment of single R is correct or wrong instead of the average self-assessment.

3.6.2 Predictions

In this context, Proposition 19 (see appendix) is to be interpreted as follows. Define two states of the world: state 1 is the state in which a subject guesses his number of right answers correctly and the *Rs* are biased ($|\bar{q} - \bar{t}| > 0.5$). State 2 is the state in which the subject guesses his number of right answers not correctly and the *Rs* are (roughly) unbiased ($|\bar{q} - \bar{t}| < 0.5$). According to our Proposition, this subject should choose alternative I if he believes that state 1 occurs with a strictly larger probability than state 2 and otherwise alternative II. Combining the choice of alternative I or II with the statement that others are over-, underconfident or rational, we can deduce what individuals in *T AveragePlus* think about their mistakes or biases and others' biases. If, for example, a subject says that others are biased and chooses alternative I, this means that he thinks that he makes more likely the right decision himself, while the *Rs* do not, i.e. he is rational (or does not make a mistake) but the *Rs* are biased. If he says others are rational and chooses alternative I, this can be translated into the statement that the subject thinks that it is more likely that both are unbiased (or he makes no mistake). Saying that others are biased and choosing alternative II implies that a subject thinks that both, himself and the *Rs* are biased (he may only make a mistake). And finally, saying that others are rational and choosing alternative II suggests that the *Rs* are unbiased, while oneself might be more likely biased (or make more likely a mistake).

In *T Frame* the inference about relative biases is easier as subjects immediately choose between the four alternatives "both are right/wrong", "only oneself is right", "only the others are right". Given these four alternatives, our proposition implies that a subject chooses the alternative with the statement, he believes most likely to occur.

As it is known from psychologists (see, e.g., Svenson (1981)), people tend to say that they are better than the average in ability tasks. As mentioned earlier, one can explain this observation by a self-serving bias. The tasks in our experiment do not require to say who is better in answering questions, but who is better in estimating the own ability or who is more rational. This is not the same but similar. Hence, we expect that subjects tend to indicate that they are rational (or do not make mistakes), while others are biased or that they at least state (by choosing alternative I) that this is more likely than the converse. Even though a subject may think that he makes better choices than a single R, this belief, however, seems surprising when he faces the complete group of Rs and their average estimate. The reason is that for him – even though he is rational – mistakes do not cancel out while for a rational average mistakes should (roughly) cancel out. Thus, we predict the following:

Hypothesis 4 *When facing a single R a majority of subjects tends to say that she is biased, while oneself is not (does more likely (not) make a mistake). When facing the average of the Rs it is the other way round.*

Applying again the assumption that "each subject is rational", it can further be seen that if a subject believes that an R is rational, then for the Rs that indicate that they have $q \in \{2, 3, 4, 5\}$ questions correct, the average deviation between stated and true number of correct answers is 0.5, while for the remaining q's it is strictly larger than 1. Thus, if one thinks that oneself and R are rational but make/s mistakes, then it could be plausible[31] to state that it is more likely that oneself is wrong and R is correct than is the converse (i.e. choosing alternative II) for $q \in \{2, 3, 4, 5\}$ and to state that it is more likely that oneself is correct and she is wrong than is the converse (i.e. alternative I) for the remaining actions q.

3.6.3 Results (Hypothesis 4)

In this section, we present the results on the question what individuals think about the relation between their own bias and others' biases. Who is more likely to be biased or make mistakes?

Figure 3.9 shows the percentages of subjects in *T AveragePlus* and *T Frame* (*Hard* and *Tricky*) thinking that oneself does not make a mistake and the Rs are biased, that oneself and the Rs are correct, that oneself makes a mistake while the Rs are rational or that both are wrong (make a mistake/have a bias).

In *T AveragePlus*, we observe that 65 percent of the subjects choose alternative I – i.e. they rather think that they are correct themselves and the Rs are wrong – and 35 percent choose alternative II. Combining this choice with their statement about the rationality of Rs, we get the percentages in Figure 3.9. In the part of *T Frame* with the hard questions the percentage of those, who think that their self-assessment is better, is lower. In both treatments roughly the same percentage of subjects

[31] One could not say what is implied here, since it depends on the belief about the size of the own mistake relative to the other one.

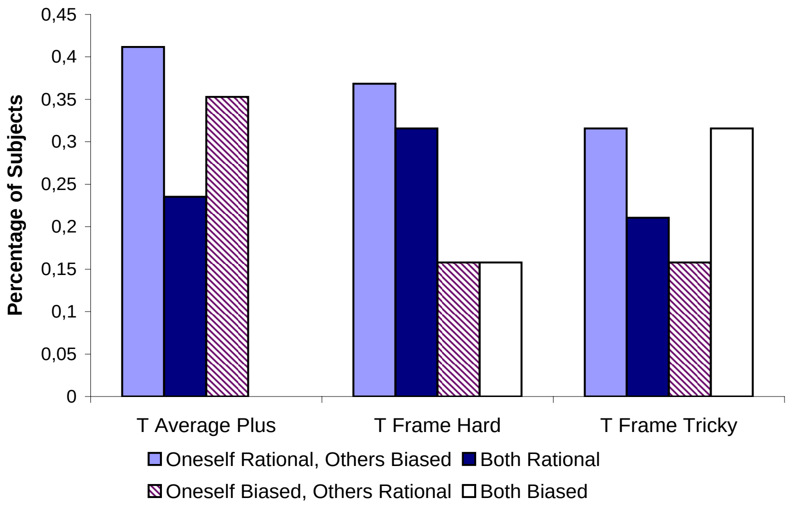

Figure 3.9: Relative Beliefs

thinks that they are rather wrong themselves. In *T AveragePlus*, however, 35 percent think that the *Rs* are rather better, while in *T Frame Hard* 16 percent think that the *Rs* are better or that both are biased/make mistakes, respectively. With the tricky questions, less subjects think that both are correct (21%), slightly less think that they are better themselves (32%), and many more think that both are wrong (32%). Thus, with the tricky questions, we find that the percentage of those, who think that oneself (not neccessarily the *Rs*) is right, decreases.[32] We summarize these findings in the following result:

Result 4 *The majority of subjects thinks that it is more likely that they do not make a mistake, while the others are biased. This percentage decreases as subjects receive more information (i.e. framing and seeing the correct answers).*

This result is somehow surprising. If one thinks that all subjects (oneself and the others) are rational, one should tend to choose alternative II since a single rational individual makes mistakes, while for the average they cancel out. The choice of alternative I is only consistent with the beliefs "I do not make mistakes at all" or "the average is very likely biased and I am unlikely to make a mistake". The first belief is surprising as it implies that subjects are not aware that they might make

[32]We can, however, not reject the hypothesis that there is no relation between the number of subjects who think that oneself is correct or wrong according to Fisher's exact tests ($p > 0.05$ one-sided). If we consider only those, who think that the *Rs* are wrong, there are significantly less (more) who think that they are correct (wrong) themselves than in *T AveragePlus* (Fisher's exact test, p=0.034 one-sided).

mistakes. Also the second belief is surprising as subjects seem to be aware that there is something like a bias in the population, but are not aware that they are biased (or make a mistake).[33] From the analysis of *T Individual* we already know, that subjects are aware that others make mistakes. Although subjects are aware of it, they do not think that they make mistakes themselves. We can explain this again by a self-serving bias. Subjects think that they are different from the others or better than these are.

Next, we consider how the own bias is related to the belief about the relative bias. When evaluating the relative bias, subjects may be reluctant to say that they are better than others in their self-assessment. This behavior would reveal overconfidence of the type "I think I am better than the average". It is hence interesting to see whether subjects, who have a bias when evaluating absolute abilities (here: answering questions), also have a bias when assessing relative abilities (here: evaluation of relative bias). In *T AveragePlus*, for those, who choose alternative I (i.e. they rather think that they are correct themselves and the *Rs* are wrong), the difference between true type and believed type is on average -2.18, while for those, who choose alternative II it is 0 (meaning that they are unbiased).[34] According to a Mann-Whitney U test, subjects who think that they are more likely to be correct, are significantly more biased than those who think that rather the *Rs* are correct whilst oneself is wrong ($p = 0.007$). Remarkably, *all* subjects who choose alternative I – i.e. who rather think that they are correct themselves and the *Rs* are wrong – are overconfident themselves.

The pattern in *T Frame* is similar. Those subjects, who think that they are right, while the *Rs* have a bias, have the largest bias (average bias is -1 with the hard questions, -2 with the tricky ones). Those, who say that they as well as the *Rs* are likely to be wrong, are either underconfident (with the hard questions their bias is 1) or have the smallest bias with the tricky questions. This may be due to the fact that subjects, who know that individuals are biased, try to behave accordingly and adjust their guess of the number of correct answers downward. Those, who say that both are correct, have roughly the same bias as those, who say that the *Rs* may make better guesses (-0.5 (-1.75) versus -0.67 (-1.7) for hard (tricky) questions).

Finally, we turn to the analysis of *T Individual.* The average choice of alternatives I or II of the subjects for every single belief of an *R* is shown in Figure 3.10. Here we see again, that subjects

[33]They might think that the only possible state of the world is that both are wrong. Hence, they are indifferent between the two alternatives. We think, however, that subjects put some (small) positive probability on the other states of the world such that the choice of alternative I reveals that they think they are correct, while the others are not.

[34]Since positive and negative biases cancel out, also the average *absolute values* of the biases are interesting. These are 2.18 given alternative I and 1.33 given alternative II, i.e. they are still lower for alternative II.

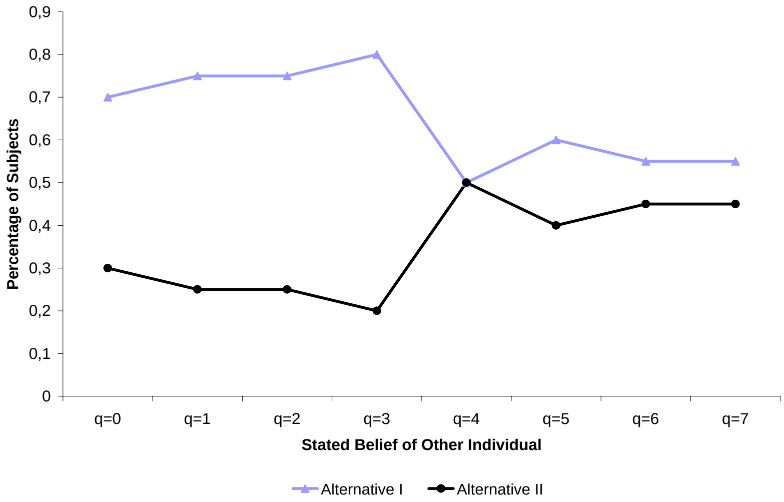

Figure 3.10: Average Choice of Alternatives in *T Individual*

tend to think that they are more likely to make the correct self-assessment themselves, i.e. they tend to choose more often alternative I (confirming our Hypothesis). Moreover, for low beliefs of an R about her type (i.e. pessimistic Rs), subjects in *T Individual* tend even more to alternative I than for high beliefs of R (i.e. optimistic Rs). Thus, it seems that they trust an optimist more to make the right decision than a pessimist. This observation is interesting since subjects think that both, optimists as well as pessimists make mistakes as we have also seen in Section 3.5.2. This means that although subjects realize that Rs with high qs might just appear to be a good type (since they might be actually worse then they think), subjects still seem to believe that these "high types" are somehow better than others.[35]

As we have mentioned before, if one thinks that oneself and R are rational (but make/s mistakes), one should choose for $q \in \{2, 3, 4, 5\}$ alternative II. We see in the figure that the proportion of subjects that chooses alternative I is always larger than the proportion choosing alternative II. For $q \in \{4, 5, 6, 7\}$ the proportions are close. Thus, there is again a tendency that subjects think that they are better (more rational) than others (here: better than single individuals in *R Hard*).

[35]A According to a Wilcoxon test the medians of the number of subjects choosing alternative I/alternative II when q is lower than 4 or at least 4 significantly differ ($p = 0.05$, two-sided).

3.7 Conclusion

Empirical studies show that overconfidence occurs in various settings: People overestimate their driving abilities, students their scores in exams or their rank in the distribution, couples the likelihood of not getting divorced, and portfolio managers their prediction abilities. In our experiment, subjects estimated how many out of seven multiple choice questions they answered correctly. Our observations confirm that overestimation of the own ability is a prominent phenomenon in the population. As overconfidence is such a common characteristic of people's behavior and is observed so frequently in real life, it seems obvious that people are also aware of this bias. Remarkably, we find that a majority of subjects does not think or know that others have a bias.

What are the consequences of this ignorance? If we think of economic interactions, it is often important that agents are aware that others are biased in order to make optimal decisions. For instance, an agent who is not aware that his opponent in a contest is biased or who does not know at least, which belief the opponent has about his type, cannot adjust his effort optimally, as is assumed in Ando (2004). Malmendier and Tate (2005a, 2005b) observe that managers are overconfident and that this is disadvantageous for the firm. Thus, principals should be aware of overconfident managers in order to be able to counteract possible decision defects. Santos-Pintos (2005a) show that incentive contracts should be designed in a special way for overconfident agents. Given that a majority of subjects tends to be overconfident, ignorance of this bias leads to suboptimal contracts for a majority of agents.

Our results indicate that the more information subjects receive on the task the others have to complete – i.e. the more familiar they are with the task – the more subjects learn that others are on average biased regarding this task. Hence, more familiarity with the task others have to complete might help subjects to recognize that others are biased. Therefore, it helps to make better decisions when subjects face biased individuals. This highlights the importance of information and feedback to make better economic decisions.

Moreover, we observe that when subjects are confronted with the question what they think about the relation between their own and others' biases, a majority states that they are more likely able to estimate their ability correctly than is the population. This has, for example, implications for decision making in firms: Suppose a principal has to decide whether to delegate to subordinates or not. In real life, it is often observed that people do not want to delegate even though there is no incentive or verifiability problem. Our results indicate that one explanation for this phenomenon is that either the principal believes that other agents are not able to make the right decision as they have a bias while he has no bias himself or that his bias is smaller.

We have also seen that individuals think that similar subjects are very likely not to make mistakes, while they, nevertheless, know that mistakes occur. This indicates that subjects are not only unaware of the bias in the population, but also of their own bias (or mistakes). Although we think that this can be taken as some evidence, we think it is a topic for future research to investigate the knowledge about own biases (like hyperbolic discounting or overconfidence) further.

Appendix

Proof of Proposition 3 (Existence of equilibrium).

For existence we have to verify that the expected payoffs of a low and a high type are non-negative when bidding x_{iL}^* and x_{iH}^*, respectively. Otherwise it would be better to bid zero and make an expected payoff of zero. The expected payoff of a low type is

$$\begin{aligned} & \left(\frac{1}{2}r + \frac{x_{iL}^*}{x_{iL}^* + x_{iH}^*}(1-r)\right) V_L - x_{iL}^* \\ = & \left(\frac{1}{2}r + \frac{V_L}{V_L + V_H}(1-r) - \lambda\right) V_L \end{aligned} \tag{A.1}$$

and of a high type

$$\begin{aligned} & \left(\frac{1}{2}r + \frac{x_{iH}^*}{x_{iL}^* + x_{iH}^*}(1-r)\right) V_H - x_{iH}^* \\ = & \left(\frac{1}{2}r + \frac{V_H}{V_L + V_H}(1-r) - \lambda\right) V_H. \end{aligned} \tag{A.2}$$

If the expected payoff of the low type is non-negative, then also the expected payoff of a high type is non-negative. Therefore, it is sufficient to check whether (A.1) is non-negative. Plugging in λ and rearranging yields

$$\left(\frac{1}{4}r + \frac{V_L^2}{(V_L + V_H)^2}(1-r)\right) V_L$$

which is strictly larger than zero. ■

Proof of Proposition 6.

In order to prove existence, we have to verify that expected payoffs of both contestants are non-negative for each type. Starting with the first mover, we calculate his expected payoff, Ψ_t^{1st}, in the interior solution when his type is $t = L, H$.

$$\begin{aligned} \Psi_t^{1st} &= V_t \left(r\frac{\alpha_t}{\sqrt{V_t}} + (1-r)\frac{\alpha_k}{V_k} \right) - \alpha_t^2 \text{ with } t \neq k \\ &= \alpha_t^2 = \frac{V_t^2}{4} \left(r(V_t^{-\frac{1}{2}} - V_k^{-\frac{1}{2}}) + V_k^{-\frac{1}{2}} \right)^2 \text{ with } t \neq k \end{aligned} \tag{A.3}$$

Hence, Ψ_t^{1st} is strictly larger than zero.

The expected payoff of the first mover in the two boundary cases, when he is the low type, is exactly the same as in (A.3). It remains to check whether the high type makes non-negative expected payoff in these cases. We first consider the case, in which the first mover bids exactly V_L.

$$\begin{aligned}\Psi_H^{1st} &= V_H\left(r\frac{\sqrt{V_L}}{\sqrt{V_H}} + (1-r)\right) - V_L \\ &= \sqrt{V_L} + \sqrt{V_H} + (1-r)\left(\sqrt{V_H} - \sqrt{V_L}\right).\end{aligned}$$

Obviously, $\Psi_H^{1st} > 0$ as $r \in [0,1]$ and $V_H \geq V_L$.

In the second boundary case, the expected payoff of the first mover is

$$\begin{aligned}\Psi_H^{1st} &= V_H\left(r\frac{r^2}{2} + (1-r)\right) - \frac{r^2 V_H}{4} \\ &= \left(\frac{r^2}{4} + (1-r)\right)V_H.\end{aligned}$$

As $r \in [0,1]$, the first mover's payoff is strictly positive.

Turning to the second mover, we first check his expected payoff Ψ_k^{2nd}, in the interior solution, when his type is $k = H, L$ and the first mover has with probability r the same type, $t = k$, and with probability $1-r$ a different type, $t \neq k$.

$$\begin{aligned}\Psi_k^{2nd} &= V_k\left(r\left(\frac{\sqrt{V_k}-\alpha_k}{\sqrt{V_k}}\right) + (1-r)\left(\frac{\sqrt{V_k}-\alpha_t}{\sqrt{V_k}}\right)\right) - r\left(\sqrt{V_k}\alpha_k - \alpha_k^2\right) - (1-r)\left(\sqrt{V_k}\alpha_t - \alpha_t^2\right) \\ &\quad \text{where } t \neq k.\end{aligned}$$

This can be rewritten as

$$\Psi_k^{2nd} = r\left(\sqrt{V_k} - \alpha_k\right)^2 + (1-r)\left(\sqrt{V_k} - \alpha_t\right)^2, \tag{A.4}$$

which is non-negative as $r \in [0,1]$.

In the boundary solutions, the expected payoff of the low type of the second mover is zero, as he exerts zero effort. For the high type of the second mover, there are two cases:

In case the high type of the first mover bids V_L, the high type of the second mover expects

$$\Psi_H^{2nd} = V_H\left(r\frac{\sqrt{V_H}-\sqrt{V_L}}{\sqrt{V_H}} + (1-r)\frac{\sqrt{V_H}-\alpha_L}{\sqrt{V_H}}\right) - r\sqrt{V_H}\left(\sqrt{V_H}-\sqrt{V_L}\right) - (1-r)\alpha_L\left(\sqrt{V_H}-\alpha_L\right).$$

This can be rewritten as

$$\Psi_H^{2nd} = r\left(\sqrt{V_H} - \sqrt{V_L}\right)^2 + (1-r)\left(\sqrt{V_H} - \alpha_L\right)^2,$$

which is non-negative as $r \in [0,1]$.

In case the high type of the first mover bids $\frac{r^2 V_H}{4}$, the high type of the second mover expects

$$\Psi_H^{2nd} = V_H\left(r(1-\frac{r}{2}) + (1-r)(1-\alpha_L V_H^{-\frac{1}{2}})\right) - r\left(\frac{r_H^V}{2}(1-\frac{r}{2})\right) - (1-r)(\alpha_L(V_H^{\frac{1}{2}} - \alpha_L)).$$

Plugging in α_L and rearranging yields

$$\begin{aligned}\Psi_H^{2nd} = & -\frac{V_H}{4}\left((1-r)\left(-4V_H^{\frac{3}{2}}(V_H^{\frac{1}{2}} - rV_L^{\frac{1}{2}}) - 2rV_L V_H^{\frac{1}{2}}(2V_H^{\frac{1}{2}} - V_L^{\frac{1}{2}}(1+r)) - rV_L(V_H + rV_L)\right)\right. \\ & \left. -V_H(V_L + r^3(V_H - V_L))\right).\end{aligned}$$

This is non-negative as $V_H \geq V_L$ and $r \in [0,1]$.

Proof that the first mover makes a strictly positive bid in equilibrium:
Suppose type $t = H, L$ of the first mover bids zero. Then his payoff is zero as the second mover bids $\epsilon > 0$ and wins the prize. If type t of the first mover, however, bids α_t^2, his expected payoff is strictly larger than zero as shown before.

Proof that equilibrium bids of the high and low type of the first mover differ:
In order to show that the first mover's bid signals his type, we have to show for $V_L \neq V_H$ (for $V_L = V_H$ there is only one type and therefore the first mover's bids coincide for "both" types, L and H) that $\alpha_H \neq \alpha_L$ for the interior solution. Suppose $\alpha_H = \alpha_L$. This is equivalent to

$$r\left(V_H^{\frac{1}{2}} - V_L^{-\frac{1}{2}}V_H\right) + V_L^{-\frac{1}{2}}V_H = r\left(V_L^{\frac{1}{2}} - V_L^{-\frac{1}{2}}V_H\right) + V_H^{-\frac{1}{2}}V_L.$$

This can be rewritten as

$$V_L^{-\frac{1}{2}}V_H^{-\frac{1}{2}}\left(V_H^{\frac{3}{2}} - V_L^{\frac{3}{2}}\right) + V_H^{\frac{1}{2}} - V_L^{\frac{1}{2}} = 0.$$

The left hand side is obviously larger than zero for $V_L \neq V_H$ which is a contradiction, hence $\alpha_H \neq \alpha_L$. Regarding the two boundary cases, we know that the low first mover bids strictly less than V_L and the high type bids at least V_L. It follows immediately, that low and high type never bid the same amount. Hence, the bid of the first mover reveals his type.
In case that $r \leq \widetilde{r}$ and $r < 2\frac{\sqrt{V_L}}{\sqrt{V_H}}$, the first order condition (1.16) does not yield the optimal bid of the constrained maximization problem. Because of concavity of the objective function in the first mover's bid, the optimal bid is a corner solution. Hence, it is either V_L or V_H. If the high type of the first mover bids V_H, his payoff is zero as the second mover bids zero, and the first mover wins the prize. In contrast, if the high type of the first mover bids V_L, his expected payoff is strictly larger than zero as shown above. Hence, we have $x_{1H}^* = V_L$ in case $r \leq \widetilde{r}$ and $r < 2\frac{\sqrt{V_L}}{\sqrt{V_H}}$. ■

Proof of Proposition 7.

(i) For the second mover we have to verify $\alpha_t(\sqrt{V_L} - \alpha_t) \leq \alpha_t(\sqrt{V_H} - \alpha_t)$ (where $t = L, H$) for the interior solution. This obviously holds as $V_H \geq V_L$ (with equality for $V_H = V_L$). For the boundary solutions the result is trivial as the second mover exerts zero effort, when he is a low type, and a non-negative effort, when he is a high type, which follows from the derivation of equilibrium bids.

For the first mover, we have to verify $\alpha_L \leq \alpha_H$. This is equivalent to

$$r\left(\frac{1}{\sqrt{V_L}} - \frac{1}{\sqrt{V_H}}\right)(V_L + V_H) \leq \frac{V_H}{\sqrt{V_L}} - \frac{V_L}{\sqrt{V_H}}.$$

Rearranging yields

$$r\frac{1}{\sqrt{V_H}\sqrt{V_L}}(\sqrt{V_H} - \sqrt{V_L})(V_L + V_H) \leq \frac{1}{\sqrt{V_H}\sqrt{V_L}}(V_H^{\frac{3}{2}} - V_L^{\frac{3}{2}}).$$

This holds with equality if $V_L = V_H$. For $V_L < V_H$, we can simplify this to

$$r\left(V_L + V_H\right) \leq \left(V_H + \sqrt{V_H}\sqrt{V_L} + V_L\right)$$

which always holds.

For the boundary solutions we have to check $V_L \geq \alpha_L^2$, which is equivalent to

$$2 \geq \sqrt{V_L}\left(r\left(\frac{1}{\sqrt{V_L}} - \frac{1}{\sqrt{V_H}}\right) + \frac{1}{\sqrt{V_H}}\right).$$

This can be simplified to

$$1 \geq \left(\frac{\sqrt{V_L}}{\sqrt{V_H}} - 1\right)(1-r),$$

which is always fulfilled (for $r = 1$ with equality).

Since $\frac{r^2 V_H}{4} \geq V_L$ for the relevant range (as $r > \frac{2\sqrt{V_L}}{\sqrt{V_H}}$), it holds that $\frac{r^2 V_H}{4} > \alpha_L^2$ (as $\alpha_L^2 < V_L$). Hence, the first mover exerts more effort when he is a high type.

(ii) To show that the first mover exerts (a) more effort than the second mover when he is a high type and (b) less when he is a low type it suffices to check that he makes more (less) effort than the high (low) type of the second mover (as we know from part (i) that the high type makes a higher effort than the low type).

(a) $x_{1H}^* \geq x_{2H}^{*H}$ is equivalent to $2\alpha_H \geq \sqrt{V_H}$ for the interior solution. This is equivalent to $(1-r)(\frac{\sqrt{V_H}}{\sqrt{V_L}}-1) \geq 0$ which is fulfilled (for $V_L = V_H$ and $r = 1$ with equality). As $x_{2H}^{*H} \geq x_{2L}^{*H}$, it follows $x_{1H}^* \geq x_{2L}^{*H}$.

For the first boundary case, we have to verify $V_L \geq \sqrt{V}_L(\sqrt{V}_H - \sqrt{V}_L)$, which is equivalent to $2\sqrt{V}_L \geq \sqrt{V}_H$. This need not hold since it is possible that $2\sqrt{V}_L \leq \sqrt{V}_H$ for the first boundary case. Note that the high type of the first mover exerts a positive amount of effort, hence more than the low type of the second mover.

For the second boundary case, we have to verify $\frac{r^2 V_H}{4} \geq \frac{r V_H}{2}(1 - \frac{r}{2})$, which is equivalent to $r^2 \geq r$. As $r \in (0, 1)$, this is never fulfilled. Thus, the high type of the first mover exerts less effort than the high type of the second mover in this case. Nevertheless, the high type of the first mover exerts a positive amount of effort, hence more than the low type of the second mover.

(b) $x_{1L}^* \leq x_{2L}^{*L}$ is equivalent to $2\alpha_L \leq \sqrt{V_L}$ for the interior solution. This is equivalent to $(1-r)V_L^{\frac{1}{2}} \leq (1-r)V_H^{\frac{1}{2}}$, which holds with equality for $V_L = V_H$ and for $r = 1$. For $V_L < V_H$ and $r < 1$ the left hand side is strictly smaller than the right hand side. As $x_{2H}^{*L} \geq x_{2L}^{*L}$, it follows $x_{1L}^* \leq x_{2H}^{*L}$.

Note that we do not have to check the boundary cases when the first mover is a low type as there is no difference to the interior solution for the low type and hence, there is also no difference for the second mover who observes this action.

■

Proof E.

For the interior solution, the first mover's expected payoff when he has type $t = L, H$ is given by α_t^2 and the follower's expected payoff when the follower has type $k = H, L$ is given by $(\sqrt{V}_k - \alpha_t)^2$ when the first mover has type $t = L, H$ (compare (A.4). In order to show that the first mover receives a higher expected payoff than the second mover when he is a high type, we have to verify that $2\alpha_H > \sqrt{V}_k$ for $k = H, L$. As $V_H \geq V_L$, it suffices to check $2\alpha_H > \sqrt{V}_H$. substituting α and rearranging yields $\sqrt{V}_H\left((1-r)(1-\frac{\sqrt{V}_H}{\sqrt{V}_L})\right) \leq 0$, which holds (with strict inequality for $V_L < V_H$ and $r < 1$). In order to show that the first mover receives a lower expected payoff than the second mover when he is a low type, we have to verify that $2\alpha_L < \sqrt{V}_k$ for $k = H, L$. As $V_H \geq V_L$, it suffices to check $2\alpha_L > \sqrt{V}_L$. substituting α and rearranging yields $\sqrt{V}_L\left((1-r)(1-\frac{\sqrt{V}_L}{\sqrt{V}_H}\right) \geq 0$, which holds (with strict inequality for $V_L < V_H$). ■

Proof of Proposition 8.

In order to show that equilibrium bids are non-decreasing in the correlation coefficient ρ, we first show that bids are non-decreasing in r.

Low type of the first mover, interior (hence, also boundary) solution:
$\frac{\partial x_{1L}^*}{\partial r} = \alpha_L V_L\left(\frac{1}{\sqrt{V_L}} - \frac{1}{\sqrt{V_H}}\right) \geq 0$ since $V_L \leq V_H$ and $\alpha_L \geq 0$.

High type of the first mover, interior solution: $\frac{\partial x_{1H}^*}{\partial r} = \alpha_H V_H\left(\frac{1}{\sqrt{V_H}} - \frac{1}{\sqrt{V_L}}\right) \leq 0$ since $V_L \leq V_H$ and $\alpha_H \geq 0$.

High type of the first mover, boundary solution:
(i) $\frac{\partial x_{1H}^*}{\partial r} = \frac{rV_H}{2} \geq 0$ and (ii) $\frac{\partial x_{1H}^*}{\partial r} = 0$.

Low type of the second mover, interior solution if the first mover is a low type:
$\frac{\partial x_{2L}^{*L}}{\partial r} = \frac{\partial \alpha_L}{\partial r}(\sqrt{V}_L - 2\alpha_L) = \frac{V_L^{\frac{3}{2}}}{2}(V_L^{-\frac{1}{2}} - V_H^{-\frac{1}{2}})(1-r)(1 - V_L^{\frac{1}{2}}V_H^{-\frac{1}{2}}) \geq 0.$
Low type of the second mover, interior solution if the first mover is a high type:
$\frac{\partial x_{2L}^{*H}}{\partial r} = \frac{\partial \alpha_H}{\partial r}(\sqrt{V}_L - 2\alpha_H) = \frac{V_H V_L^{-\frac{1}{2}}}{2}(V_H^{-\frac{1}{2}} - V_L^{-\frac{1}{2}})(V_L^{\frac{1}{2}} - V_H^{\frac{1}{2}})((1-r)V_H^{\frac{1}{2}} + V_L^{\frac{1}{2}}) \geq 0.$
In the boundary solutions, the low type of the second mover exerts zero effort, hence his bid does not change when ρ rises.

High type of the second mover, interior solution if the first mover is a low type:
$\frac{\partial x_{2H}^{*L}}{\partial r} = \frac{\partial \alpha_L}{\partial r}(\sqrt{V}_H - 2\alpha_L) = \frac{V_L V_H^{-\frac{1}{2}}}{2}(V_L^{-\frac{1}{2}} - V_H^{-\frac{1}{2}})(V_H^{\frac{1}{2}} - V_L^{\frac{1}{2}})((1-r)V_L^{\frac{1}{2}} + V_H^{\frac{1}{2}}) > 0.$
High type of the second mover, interior solution if the first mover is a high type:
$\frac{\partial x_{2H}^{*H}}{\partial r} = \frac{\partial \alpha_H}{\partial r}(\sqrt{V}_H - 2\alpha_H) = \frac{V_H^{\frac{3}{2}}}{2}(V_H^{-\frac{1}{2}} - V_L^{-\frac{1}{2}})(1-r)(1 - V_H^{\frac{1}{2}}V_L^{-\frac{1}{2}}) \geq 0.$
High type of the second mover, boundary solution:
(i) $\frac{\partial x_{2H}^{*H}}{\partial r} = \frac{V_H}{2}(1-r) \geq 0$ and (ii) $\frac{\partial x_{2H}^{*H}}{\partial r} = 0$.

Note that we can immediately conclude from the sign of the derivatives of the contestants' bids with respect to r whether the bids are increasing or decreasing in ρ as $r = \frac{1}{2}(\rho + 1)$: The signs of the derivatives with respect to r and to ρ are identical. Hence, we have established the proposition. ■

Proof of Proposition 9.

For the case of public information see Morgan (2003). Thus, it remains to verify the case of private information.

The ex ante expected effort sum in the sequential contest with private information (restricting to the interior solution) is

$$\begin{aligned}\xi_{seq}^{priv} := & \frac{1}{2}\left[\alpha_L^2 + \alpha_H^2 + r\alpha_L\left(\sqrt{V_L} - \alpha_L\right) + (1-r)\alpha_H\left(\sqrt{V_L} - \alpha_H\right)\right. \\ & \left. + r\alpha_H\left(\sqrt{V_H} - \alpha_H\right) + (1-r)\alpha_L\left(\sqrt{V_H} - \alpha_L\right)\right].\end{aligned}$$

Simplifying yields

$$\xi_{seq}^{priv} = \frac{1}{2}\left[r(\alpha_H - \alpha_L)\left(\sqrt{V_H} - \sqrt{V_H}\right) + \alpha_L\sqrt{V_H} + \alpha_H\sqrt{V_L}\right].$$

Substituting $\alpha_L = \frac{V_L}{2}\left(r(\frac{1}{\sqrt{V_L}} - \frac{1}{\sqrt{V_H}}) + \frac{1}{\sqrt{V_H}}\right)$ and $\alpha_H = \frac{V_H}{2}\left(r(\frac{1}{\sqrt{V_H}} - \frac{1}{\sqrt{V_L}}) + \frac{1}{\sqrt{V_L}}\right)$ and rearranging yields

$$\xi_{seq}^{priv} = \frac{1}{4}\left[V_L + V_H + r(1-r)\left(2\left(V_L^{\frac{1}{2}}V_H^{\frac{1}{2}} - (V_L + V_H)\right) + V_L^{-\frac{1}{2}}V_H^{-\frac{1}{2}}\left(V_L^2 + V_H^2\right)\right)\right]. \quad \text{(A.5)}$$

The ex ante expected effort sum in the simultaneous contest with private information is

$$\begin{aligned}\xi_{sim}^{priv} := & \lambda(V_L + V_H) \\ = & \frac{1}{4(V_L + V_H)}\left[r(V_L - V_H)^2 + 4V_H V_L\right].\end{aligned}$$

Routine transformations yield that $\xi_{seq}^{priv} \geq \xi_{sim}^{priv}$ is equivalent to

$$(1-r)\left[(V_L - V_H)^2 + r(V_L + V_H)(2\sqrt{V_L V_H} - 2V_L - 2V_H + \frac{1}{\sqrt{V_L V_H}}(V_L^2 + V_H^2))\right] \geq 0.$$

This holds with equality for $r = 1$. The term

$$2\sqrt{V_L V_H} - 2V_L - 2V_H + \frac{1}{\sqrt{V_L V_H}}(V_L^2 + V_H^2)$$

can be written as

$$\frac{1}{\sqrt{V_L V_H}}(V_L + V_H)\left(\sqrt{V_H} - \sqrt{V_L}\right)^2,$$

which is non-negative. Thus, we have for $r \neq 1$ that $\xi_{seq}^{priv} \geq \xi_{sim}^{priv}$ (with equality for $V_L = V_H$). ■

Proof of Proposition 10.

The ex ante expected payoff, Ψ, of a contestant in the simultaneous contest with private information is the sum of his expected payoff when he is a low and a high type respectively, each with probability one half:

$$\Psi = \frac{1}{2}\left(V_L\left(\frac{x_{iL}}{x_{iL} + x_{jL}}r + \frac{x_{iL}}{x_{iL} + x_{jH}}(1-r)\right) - x_{iL} + V_H\left(\frac{x_{iH}}{x_{iH} + x_{jH}}r + \frac{x_{iH}}{x_{iH} + x_{jL}}(1-r)\right) - x_{iH}\right).$$

Plugging in equilibrium bids and simplifying yields

$$\Psi = \frac{1}{2}\left[\frac{V_L^2 - V_H V_L + V_H^2}{V_L + V_H} - \frac{3}{4} r \frac{(V_L - V_H)^2}{V_L + V_H}\right]. \tag{A.6}$$

The ex ante expected payoff of the first mover in the sequential contest with private information is the sum of his expected payoff when he is a low and a high type respectively, i.e. Ψ_H^{1st} and Ψ_L^{1st} as given in (A.3), each with probability one half:

$$\Psi^{1st} = \frac{1}{2}(\alpha_L^2 + \alpha_H^2).$$

Plugging in α_L and α_H yields

$$\frac{1}{8V_L V_H}\left[(V_l^2 + V_H^2)(V_L^{\frac{1}{2}} - V_H^{\frac{1}{2}})^2 \left(r^2 - 2r(V_L^2 + V_L V_H + V_H^2 + V_L^{\frac{1}{2}} V_H^{\frac{1}{2}}(V_H + V_L))\right) + V_L^3 + V_H^3\right] \tag{A.7}$$

The ex ante expected payoff of the second mover is the sum of his expected payoff when he is a low and a high type respectively, i.e. Ψ_L^{2nd} and Ψ_H^{2nd} as given in (A.4), each with probability one half:

$$\begin{aligned}
\Psi^{2nd} &= \frac{1}{2}\Bigg[\left(\sqrt{V_L} - \alpha_H\right)^2 + \left(\sqrt{V_H} - \alpha_L\right)^2 + \\
&\quad r\left(\left(\sqrt{V_L} - \alpha_L\right)^2 - \left(\sqrt{V_L} - \alpha_H\right)^2 + \left(\sqrt{V_H} - \alpha_H\right)^2 - \left(\sqrt{V_H} - \alpha_L\right)^2\right)\Bigg] \\
&= \frac{1}{2}\left[\left(\sqrt{V_L} - \alpha_H\right)^2 + \left(\sqrt{V_H} - \alpha_L\right)^2 + 2r\left(\alpha_H - \alpha_L\right)^2\left(\sqrt{V_L} - \sqrt{V_H}\right)^2\right].
\end{aligned} \tag{A.8}$$

(i) Routine transformations yield that

$$\Psi^{1st} \geq \Psi \;\text{ is equivalent to }\; z + ry + r^2 x \geq 0$$

where

$$\begin{aligned}
z &= \left(V_L^{\frac{1}{2}} - V_H^{\frac{1}{2}}\right)^2 \left(V_L^{\frac{1}{2}} + V_H^{\frac{1}{2}}\right)^2 \left(V_H^2 - V_H V_L + V_L^2\right) \\
y &= -\left(V_L^{\frac{1}{2}} - V_H^{\frac{1}{2}}\right)^2 \left[V_L V_H (V_L + V_H) + \left(V_H^{\frac{3}{2}} - V_L^{\frac{3}{2}}\right)^2 + 2V_L^{\frac{1}{2}} V_H^{\frac{1}{2}}\left(V_L^2 + V_H^2\right) + V_H^3 + V_L^3\right] \\
x &= \left(V_L^{\frac{1}{2}} - V_H^{\frac{1}{2}}\right)^2 \left(V_H^2 + V_L^2\right)\left(V_H + V_L\right).
\end{aligned}$$

Obviously, $x \geq 0$ and $z \geq 0$ both with strict inequality for $V_L < V_H$. Moreover, $y \leq 0$ (with strict inequality for $V_L < V_H$). Note that for $V_L = V_H$ we have $z = w = x = 0$ and hence expected payoffs of the first mover are identical to his payoffs under the simultaneous structure. For the rest of the proof, we restrict to $V_H > V_L$, which implies $x > 0$, $z > 0$ and $y < 0$.

Consider the function $\hat{f}(r) = z + ry + r^2 x$ for $r \in [0, 1]$. $\hat{f}(r) \geq 0$ is equivalent to $\Psi^{1st} \geq \Psi$. As $x > 0$, and hence, $\hat{f}(0) > 0$, we have (by continuity) for "sufficiently small"r that $\Psi^{1st} \geq \Psi$. $\hat{f}$ is decreasing in r for $r \leq \frac{-y}{2x} =: \hat{r}^*$ and increasing for $r > \hat{r}^*$. Depending on the sign of $\hat{f}(1)$, whether $\hat{r}^*$ is larger or smaller than one, and the roots of $\hat{f}$, we can determine the regions of r for which $\Psi^{1st} \geq \Psi$.

First, we check whether $\hat{r}^* > 1$:

This is equivalent to $-y > 2x$. Plugging in y and x and simplifying yields (for $V_H > V_L$)

$$2V_L^{\frac{5}{2}} V_H^{\frac{1}{2}} + 2V_L^{\frac{1}{2}} V_H^{\frac{5}{2}} - 2V_L^{\frac{3}{2}} V_H^{\frac{3}{2}} - V_L^2 V_H - V_L V_H^2 > 0.$$

We can write this as

$$V_L^{\frac{1}{2}} V_H^{\frac{1}{2}} (V_L^{\frac{1}{2}} - V_H^{\frac{1}{2}})^2 \left[(V_L + V_L^{\frac{1}{2}} V_H^{\frac{1}{2}} + V_H) + \left(V_L^{\frac{1}{2}} + V_H^{\frac{1}{2}}\right)^2\right] > 0,$$

which is obviously satisfied as $V_H > V_L$. Hence, $\hat{r}^* > 1$.

Next, we check whether $\hat{f}(1) = x + y + z$:

It is straightforward to verify $x + y + z = 0$, hence $\hat{f}(1) = 0$. This means that $r = 1$ is one of the roots of $\hat{f}$. As $\hat{r}^* > 1$, $\hat{f}$ decreasing in r for $r \leq \hat{r}^*$, and $\hat{f}(1) = 0$, it must be that the second root ($\hat{r}_2$) of $\hat{f}$ is larger than $\hat{r}^* > 1$. It follows that for all $r \in [0,1]$ we have $r^2 x + ry + z \geq 0$ and hence, $\Psi^{1st} \geq \Psi$ (with equality for $r = 1$ and $V_L = V_H$ as shown before).

(ii) For the second mover, we receive (by plugging in α_L and α_H) that

$$\Psi^{2nd} \geq \Psi \text{ is equivalent to } a + rb + r^2 c \geq 0$$

where

$$\begin{aligned}
a: &= \frac{1}{4V_H V_L}\left[4V_H^2 V_L^2 - 3\left(V_H^3 V_L + V_L^3 V_H\right) + V_H^4 + V_L^4\right],\\
b: &= -\frac{(\sqrt{V_L} - \sqrt{V_H})^2}{4V_H V_L}\left[2\left(V_H^3 + V_L^3\right) + V_H V_L\left(V_H + V_L\right) + 6\sqrt{V_L}\sqrt{V_H}\left(V_H V_L + V_H^2 + V_L^2\right)\right]\\
c: &= \left(\frac{1}{\sqrt{V_L}} - \frac{1}{\sqrt{V_H}}\right)^2 \frac{(V_H + V_L)}{4}\left[V_H^{\frac{3}{2}}\left(\sqrt{V_H} + 4\sqrt{V_L}\right) + V_L^{\frac{3}{2}}\left(\sqrt{V_L} + 4\sqrt{V_H}\right)\right].
\end{aligned}$$

Obviously, $b \leq 0$ and $c \geq 0$ (with equality if $V_L = V_H$). We can rewrite a as follows:

$$\begin{aligned}
a &= \frac{1}{4V_H V_L}\left(V_H - V_L\right)\left[V_H^3 - V_L^3 - 2V_H V_L\left(V_H - V_L\right)\right]\\
&= \frac{1}{4V_H V_L}\left(V_H - V_L\right)^2\left(V_H^2 - V_H V_L + V_L^2\right).
\end{aligned}$$

Hence, a is larger than or equal to $\frac{1}{4V_H V_L}\left(V_H - V_L\right)^4$, which is non-negative (and strictly positive if $V_H > V_L$). Thus, $a \geq 0$.

Note that for $V_L = V_H$ we have $a = b = c = 0$ and hence $\Psi^{2nd} = \Psi$. For the rest of the proof, we restrict to $V_H > V_L$.

Consider $\widetilde{f}(r) = a + rb + r^2 c$ for $r \in [0,1]$. Note that $\widetilde{f}(r) \geq 0$ ($\leq$) is equivalent to $\Psi^{2nd} \geq \Psi$ ($\leq$). Hence, since a is positive, we have (by continuity) for "sufficiently small" r that $\widetilde{f}(r) \geq 0$ and therefore $\Psi^{2nd} \geq \Psi$. $\widetilde{f}$ is decreasing in r for $r \leq -\frac{b}{2c} =: \widetilde{r}^*$ and increasing for $r > \widetilde{r}^*$.

Depending on the sign of $\widetilde{f}(1)$, whether $\widetilde{r}^* > 1$ or $\widetilde{r}^* \leq 1$, and the roots of $\widetilde{f}$, we can determine the regions of r for which $\Psi^{2nd} \geq \Psi$.

First, we check whether $\widetilde{r}^* \leq 1$:

This is equivalent to $-b \leq 2c$. Plugging in b and c and simplifying yields (for $V_H > V_L$)

$$V_L V_H \left(V_L^{\frac{1}{2}} + V_H^{\frac{1}{2}}\right)^2 + 2V_L^{\frac{1}{2}} V_H^{\frac{1}{2}}(V_L^2 + V_H^2) \geq 0.$$

Note that this is always fulfilled with strict inequality. Hence, $\widetilde{r}^* < 1$.

Next, we check whether $\widetilde{f}(1) = a + b + c \geq 0$:

It is straightforward to verify $a + b + c = 0$. Hence, $\widetilde{f}(1) = 0$. This means that $r = 1$ is one of the roots of $\widetilde{f}$. As $\widetilde{r}^* < 1$, $\widetilde{f}$ decreasing in r for $r \leq \widetilde{r}^*$, and $\widetilde{f}(1) = 0$, it must be that the second root ($\widetilde{r}_2$) of $\widetilde{f}$ is smaller than $\widetilde{r}^* \leq 1$. Moreover, it follows that $\widetilde{r}_2$ is the critical value of r, i.e. $\widetilde{r}^c$, such that we have $\Psi^{2nd} \geq \Psi$ for $r \leq \widetilde{r}^c = r_2$ (as $\widetilde{f}(r) \geq 0$ for $r \leq \widetilde{r}^c = \widetilde{r}_2$).

The roots of $\widetilde{f}(r)$ are given by $\widetilde{r}_{1,2} = \frac{1}{2c}\left[-b \pm \sqrt{b^2 - 4ac}\right]$.

We know that the first root is equal to one. As $\frac{1}{2c}\left[-b+\sqrt{b^2-4ac}\right] > \frac{1}{2c}\left[b-\sqrt{b^2-4ac}\right]$, we have that $\widetilde{r}_1 = \frac{1}{2c}\left[b+\sqrt{b^2-4as}\right] = 1$, and hence $\widetilde{r}^c = \widetilde{r}_2 = \frac{1}{2c}\left[b-\sqrt{b^2-4ac}\right]$. By substitution of a, b and c, and simplifying we can derive

$$\widetilde{r}^c = \widetilde{r}_2 = \frac{\left(V_L^{\frac{3}{2}} - V_H^{\frac{3}{2}}\right)^2 + 2V_H^{\frac{1}{2}}V_L^{\frac{1}{2}}(V_H^2+V_L^2)}{(V_L+V_H)\left(V_H^2+4V_H^{\frac{1}{2}}V_L^{\frac{1}{2}}(V_H+V_L)+V_L^2\right)}. \tag{A.9}$$

Summarizing the results, it follows that for $V_L = V_H$ and for perfect positive correlation (i.e. $r=1$ or equivalently $\rho = 1$) expected payoffs of the second mover are identical under the simultaneous and sequential structure. For $V_L < V_H$ and for $r < 1$ the sequential structure leads to higher expected payoffs if and only if $r \leq \widetilde{r}^c$.

Since $r = \frac{1}{2}(\rho+1)$, we can rewrite the results in terms of ρ. For $\rho < 1$ and $V_L < V_H$ we receive from condition (A.9) that expected payoffs are higher under the sequential structure if

$$\rho \leq \frac{2\left(\left(V_L^{\frac{3}{2}} - V_H^{\frac{3}{2}}\right)^2 + 2V_H^{\frac{1}{2}}V_L^{\frac{1}{2}}(V_H^2+V_L^2)\right)}{(V_L+V_H)\left(V_H^2+4V_H^{\frac{1}{2}}V_L^{\frac{1}{2}}(V_H+V_L)+V_L^2\right)} - 1$$

This is equivalent to

$$\rho \leq \frac{\left(V_L^{\frac{3}{2}} - V_H^{\frac{3}{2}}\right)^2 - V_HV_L(V_H+V_L) - 10V_H^{\frac{3}{2}}V_L^{\frac{3}{2}}}{(V_L+V_H)\left(V_H^2+4V_H^{\frac{1}{2}}V_L^{\frac{1}{2}}(V_H+V_L)+V_L^2\right)} =: \widetilde{\rho}^c.$$

Comparing the denominator and numerator of $\widetilde{\rho}^c$, it is easy to see that $\widetilde{\rho}^c$ is strictly smaller than one. Moreover, it can be easily verified that $\widetilde{\rho}^c > -1$ is equivalent to $2(V_H^{\frac{3}{2}} - V_L^{\frac{3}{2}})^2 + 4V_L^{\frac{1}{2}}V_H^{\frac{1}{2}}(V_H^2+V_L^2) > 0$, which is satisfied as $V_L < V_H$. ∎

Proof of Proposition 11.

The ex ante expected payoff sum of the contestants in the simultaneous contest with private information is

$$W^{sim} := \frac{V_L^2 - V_HV_L + V_H^2}{V_L+V_H} - \frac{3}{4}r\frac{(V_L-V_H)^2}{V_L+V_H}.$$

The ex ante expected payoff sum in the sequential contest with private information is given by the sum of payoffs of the first and second mover, Ψ^{1st} and Ψ^{2nd}, respectively:

$$W^{seq} := \Psi^{1st} + \Psi^{2nd},$$

where the ex ante expected payoff of the first mover is (as given in (A.7))

$$\begin{aligned}\Psi^{1st} &= \frac{1}{2}(\alpha_L^2+\alpha_H^2)\\ &= \frac{1}{8V_LV_H}\left[(V_l^2+V_H^2)(V_L^{\frac{1}{2}}-V_H^{\frac{1}{2}})^2\left(r^2-2r(V_L^2+V_LV_H+V_H^2+V_L^{\frac{3}{2}}V_H^{\frac{1}{2}}+V_H^{\frac{3}{2}}V_L^{\frac{1}{2}})\right)+V_L^3+V_H^3\right].\end{aligned}$$

The ex ante expected payoff of the second mover is (as given in (A.8))

$$\begin{aligned}\Psi^{2nd} &= \frac{1}{2}\Bigg[\left(\sqrt{V_L}-\alpha_H\right)^2 + \left(\sqrt{V_H}-\alpha_L\right)^2 + \\ &\quad r\left(\left(\sqrt{V_L}-\alpha_L\right)^2 - \left(\sqrt{V_L}-\alpha_H\right)^2 + \left(\sqrt{V_H}-\alpha_H\right)^2 - \left(\sqrt{V_H}-\alpha_L\right)^2\right)\Bigg]\\ &= \frac{1}{2}\left[\left(\sqrt{V_L}-\alpha_H\right)^2 + \left(\sqrt{V_H}-\alpha_L\right)^2 + 2r\left(\alpha_H-\alpha_L\right)^2\left(\sqrt{V_L}-\sqrt{V_H}\right)^2\right].\end{aligned}$$

The rest of the proof works like the proof of Proposition 10.

By standard transformations we can show that

$$W^{seq} \geq W^{sim}$$

is equivalent to

$$u - rv + r^2 w \geq 0$$

where

$$\begin{aligned} u: &= \frac{1}{V_L V_H}(V_L^2 - V_L V_H + V_H^2)(V_L - V_H)^2, \\ v: &= \frac{1}{V_L V_H}\left[\left(V_L^2 - V_H^2\right)^2 + \left(V_H^3 - V_L^3\right)(V_H - V_L) + 4V_L V_H\left(V_L^{\frac{3}{2}} - V_H^{\frac{3}{2}}\right)\left(V_H^{\frac{1}{2}} - V_L^{\frac{1}{2}}\right)\right], \\ w: &= 2(V_L + V_H)\frac{1}{V_L V_H}\left(\sqrt{V_L} - \sqrt{V_H}\right)^2\left(\frac{V_L^2}{2} + \frac{V_H^2}{2} + \sqrt{V_L}\sqrt{V_H}(V_L + V_H)\right). \end{aligned}$$

Obviously, $u \geq 0$ and $w \geq 0$ (strictly for $V_L < V_H$). Moreover, we can write v as

$$v = \frac{1}{V_L V_H}\left[\left(V_L^2 - V_H^2\right)^2 + \left(V_H^{\frac{3}{2}} - V_L^{\frac{3}{2}}\right)\left(V_H^{\frac{1}{2}} - V_L^{\frac{1}{2}}\right)^3\left(V_H + 3V_L^{\frac{1}{2}}V_H^{\frac{1}{2}} + V_L\right)\right],$$

which is larger than or equal to zero as $V_H \geq V_L > 0$ (with equality for $V_H = V_L$).

Note that for $V_L = V_H$ the ex ante expected payoffs under the simultaneous and sequential structure are identical as $u = v = w = 0$. We restrict to $V_H > V_L$ for the rest of the proof.

Consider the function $f(r) = u - rv + r^2 w$ for $r \in [0, 1]$. $f(r) \geq 0$ is equivalent to $W^{seq} \geq W^{sim}$. As $u > 0$ for $V_H > V_L$ and hence, $f(0) > 0$, we have (by continuity) $W^{seq} \geq W^{sim}$ for "sufficiently small" r. f is decreasing in r for $r \leq \frac{v}{2w} =: r^*$ and increasing for $r > r^*$. We can determine the regions of r for which $W^{seq} \geq W^{sim}$ if we know the roots of f, the sign of $f(1)$, and whether $r^* \leq 1$ or $r^* > 1$.

First, we check whether $r^* \leq 1$:

This is equivalent to $v \leq 2w$. Plugging in v and w and simplifying yields

$$10V_L^2 V_H^2 - V_L^3 V_H - V_L V_H^3 - 4V_L^{\frac{5}{2}}V_H^{\frac{3}{2}} - 4V_L^{\frac{3}{2}}V_H^{\frac{5}{2}} \leq 0.$$

We can write this as

$$V_L V_H\left[-(V_L - V_H)^2 - 4V_L^{\frac{1}{2}}V_H^{\frac{1}{2}}\left(V_L^{\frac{1}{2}} - V_H^{\frac{1}{2}}\right)^2\right] \leq 0,$$

which is obviously fulfilled with strict inequality for $V_H > V_L$. Hence, $r^* < 1$.

Next, we check whether $f(1) = u - v + w \geq 0$:

It is straightforward to verify $u - v + w = 0$. Hence, $f(1) = 0$ and $r = 1$ is one of the roots of f. This means that for perfect positive correlation overall expected payoffs are identical under the simultaneous and sequential contest.

As $r^* \leq 1$ and f is decreasing in r for $r \leq r^*$ and $f(1) = 0$, it must be that the second root (r_2) of f is smaller than $r^* \leq 1$. Moreover, it follows that r_2 is the critical value of r, i.e. r^c, such that we have $W^{seq} \geq W^{sim}$ for $r \leq r^c = r_2$ (as $f(r) \geq 0$ for $r \leq r^c = r_2$).

The roots of $f(r)$ are given by $r_{1,2} = \frac{1}{2w}\left[v \pm \sqrt{v^2 - 4uw}\right]$. We know that the first root is equal to one. As $\frac{1}{2w}\left[v + \sqrt{v^2 - 4uw}\right] > \frac{1}{2w}\left[v - \sqrt{v^2 - 4uw}\right]$, we have that $r_1 = \frac{1}{2w}\left[v + \sqrt{v^2 - 4uw}\right] = 1$, and hence $r^c = r_2 = \frac{1}{2w}\left[v - \sqrt{v^2 - 4uw}\right]$. By substitution of w, u and v, and simplifying we can derive

$$r^c = r_2 = \frac{V_L^3 + V_H^3 + 2V_H^{\frac{1}{2}}V_L^{\frac{1}{2}}(V_H^2 - V_L V_H + V_L^2)}{V_L^3 + V_H^3 + V_H^{\frac{1}{2}}V_L^{\frac{1}{2}}(V_L + V_H)\left(2(V_H + V_L) + V_H^{\frac{1}{2}}V_L^{\frac{1}{2}}\right)}. \tag{A.10}$$

Summarizing the results, it follows that for $V_L = V_H$ and for perfect positive correlation (i.e. $r = 1$ or equivalently $\rho = 1$) overall expected payoffs are identical under the simultaneous and sequential structure. For $V_L < V_H$ and for $r < 1$ the sequential structure leads to higher expected payoffs if and only if $r \leq r^c$. Since $r = \frac{1}{2}(\rho + 1)$, we can rewrite the results in terms of ρ. For $\rho < 1$ and $V_L < V_H$, we receive from condition (A.10) that overall expected payoffs are higher under the sequential structure if

$$\rho \leq \frac{2(V_L^3 + V_H^3 + 2V_H^{\frac{1}{2}} V_L^{\frac{1}{2}} (V_H^2 - V_L V_H + V_L^2))}{V_L^3 + V_H^3 + V_H^{\frac{1}{2}} V_L^{\frac{1}{2}} (V_L + V_H) \left(2(V_H + V_L) + V_H^{\frac{1}{2}} V_L^{\frac{1}{2}}\right)} - 1$$

This is equivalent to

$$\rho \leq \frac{V_L^3 + V_H^3 + 2V_H^{\frac{1}{2}} V_L^{\frac{1}{2}} (V_H - V_L)^2 - V_L V_H (V_L + V_H) - 4V_L^{\frac{3}{2}} V_H^{\frac{3}{2}}}{V_L^3 + V_H^3 + V_H^{\frac{1}{2}} V_L^{\frac{1}{2}} (V_L + V_H) \left(2(V_H + V_L) + V_H^{\frac{1}{2}} V_L^{\frac{1}{2}}\right)} =: \rho^c.$$

Comparing the denominator and numerator of ρ^c, it is easy to see that ρ^c is strictly smaller than one and larger than minus one. ■

Proof of Lemma 1.

The ex ante expected payoff of the first mover in the interior solution of the equilibrium of the sequential contest with private information is

$$\Psi^{1st} = \frac{1}{2}(\alpha_L^2 + \alpha_H^2)$$

as given in equation (A.7). The ex ante expected payoff of the second mover is (as given in equation (A.8))

$$\begin{aligned}
\Psi^{2nd} &= \frac{1}{2}\Big[\left(\sqrt{V_L} - \alpha_H\right)^2 + \left(\sqrt{V_H} - \alpha_L\right)^2 + \\
&\quad r\left(\left(\sqrt{V_L} - \alpha_L\right)^2 - \left(\sqrt{V_L} - \alpha_H\right)^2 + \left(\sqrt{V_H} - \alpha_H\right)^2 - \left(\sqrt{V_H} - \alpha_L\right)^2\right)\Big] \\
&= \frac{1}{2}\left[\left(\sqrt{V_L} - \alpha_H\right)^2 + \left(\sqrt{V_H} - \alpha_L\right)^2 + 2r\left(\alpha_H - \alpha_L\right)^2 \left(\sqrt{V_L} - \sqrt{V_H}\right)^2\right].
\end{aligned}$$

Routine transformations yield that

$$\Psi^{1st} \geq \Psi^{2nd}$$

is equivalent to

$$\alpha_L^2 + \alpha_H^2 \geq \left(\sqrt{V_L} - \alpha_H\right)^2 + \left(\sqrt{V_H} - \alpha_L\right) + 2r\left(\alpha_H - \alpha_L\right)^2 \left(\sqrt{V_L} - \sqrt{V_H}\right)^2.$$

Rearranging leads to

$$r\left(\sqrt{V_L}\alpha_L + \sqrt{V_H}\alpha_H\right) + (1 - r)\left(\sqrt{V_L}\alpha_H + \sqrt{V_H}\alpha_L\right) \geq \frac{V_H + V_L}{2}.$$

Substituting α_L and α_H and simplifying, it is straightforward to verify that for $V_L = V_H$ or $r = 0$ or $r = 1$ (i.e. each other's types are known to the contestants) ex ante expected payoffs of the first and second mover are identical. For $r \in (0, 1)$ and $V_L > V_H$ we have that $\Psi^{1st} \geq \Psi^{2nd}$ is equivalent to

$$V_H^{\frac{3}{2}} - V_L^{\frac{3}{2}} + V_H^{\frac{1}{2}} V_L - V_L^{\frac{1}{2}} V_H \quad \geq \quad 0.$$

This can also be written as

$$\left(V_H^{\frac{1}{2}} - V_L^{\frac{1}{2}}\right)(V_H + V_L) \quad \geq \quad 0,$$

which is satisfied with strict inequality as $V_L > V_H$. Hence, for $V_L > V_H$ and $r \in (0,1)$, the ex ante expected payoff of the first mover is strictly higher than of the second mover in the interior solution. ■

Proof of Proposition 14.

The ex ante expected effort sum in the sequential contest with public information is

$$\begin{aligned}\xi_{seq}^{pub} : &= \frac{1}{2}r\left(\frac{V_L}{2} + \frac{V_H}{2}\right) + \frac{1}{2}(1-r)\left(\frac{V_L}{2} + \frac{V_H}{2}\right) \\ &= \frac{V_L}{4} + \frac{V_H}{4},\end{aligned}$$

and in the sequential contest with private information it is (for the interior solution) as given in (A.5)

$$\xi_{seq}^{priv} \quad = \quad \frac{1}{4}\left[V_H + V_L + r(1-r)\left(2\left(V_H^{\frac{1}{2}}V_L^{\frac{1}{2}} - V_L - V_H\right) + V_L^{-\frac{1}{2}}V_H^{-\frac{1}{2}}\left(V_H^2 + V_L^2\right)\right)\right].$$

It follows that $\xi_{seq}^{priv} \geq \xi_{seq}^{pub}$ is equivalent to

$$r(1-r)\left[2\left(V_H^{\frac{1}{2}}V_L^{\frac{1}{2}} - V_L - V_H\right) + V_L^{-\frac{1}{2}}V_H^{-\frac{1}{2}}\left(V_H^2 + V_L^2\right)\right] \geq 0 \qquad \text{(A.11)}$$

It can be seen from (A.11) that for $r = 0$ and $r = 1$ the ex ante expected effort sum is identical under both information settings. This is intuitive since in these cases there is public information about the valuations. For $r \in (0,1)$ we can simplify (A.11) to

$$V_L^{-\frac{1}{2}}V_H^{-\frac{1}{2}}\left(V_H + V_L\right)^2 - 2\left(V_L + V_H\right) \geq 0$$

and further to

$$\left(V_H^{\frac{1}{2}} - V_L^{\frac{1}{2}}\right)^2 \geq 0,$$

which is fulfilled (with equality for $V_L = V_H$). Hence, we have shown $\xi_{seq}^{priv} \geq \xi_{seq}^{pub}$. ■

Proof of Proposition 15.

As we have already seen in Section 1.3, the ex ante expected payoff for the first mover, $\Psi^{1st/pub}$, and the second mover, $\Psi^{2nd/pub}$, in the sequential contest with public information is identical. The expected payoffs are given by

$$\Psi^{1st/pub} = \Psi^{2nd/pub} = \frac{1}{2}\left[r\left(\frac{V_L}{4} + \frac{V_H}{4}\right) + (1-r)\left(\frac{V_L^2}{4V_H} + \frac{V_H^2}{4V_L}\right)\right].$$

The ex ante expected payoff of the first mover in the sequential contest with private information is Ψ^{1st} as in (A.7) and for the second mover it is Ψ^{2nd} as in (A.8). By rearranging and simplifying, we can show for $r \in (0,1)$ that $\Psi^{1st/pub} \geq \Psi^{1st}$ is equivalent to

$$\left(V_H^{\frac{1}{2}} - V_L^{\frac{1}{2}}\right)^2\left(V_H^2 + V_L^2\right) \geq 0, \quad \text{which always holds.}$$

For $r=1$ and $r=0$, we have $\Psi^{1st/pub}=\Psi^{1st}$ – which is intuitive as in these cases agents know each other's valuations. Thus, the first mover prefers public information from an ex ante point of view given that a sequential contest is played.

$\Psi^{2nd/pub}\geq\Psi^{2nd}$ is equivalent to

$$\begin{aligned}0 \quad \leq \quad & r\left[-\frac{7}{4}\left(V_H+V_L\right)+\frac{3}{4}\left(V_H^2V_L^{-1}+V_L^2V_H^{-1}\right)+\frac{1}{2}\left(V_H^{\frac{3}{2}}V_L^{-\frac{1}{2}}+V_L^{\frac{3}{2}}V_H^{-\frac{1}{2}}\right)+2V_L^{\frac{1}{2}}V_H^{\frac{1}{2}}\right] \\ & -r^2\left(V_L^{-\frac{1}{2}}-V_H^{-\frac{1}{2}}\right)^2\left[V_L^{\frac{1}{2}}V_H^{\frac{1}{2}}\left(V_H+V_L\right)+\frac{1}{4}\left(V_H^2+V_L^2\right)\right]. \qquad \text{(A.12)}\end{aligned}$$

For $r=0$ we have $\Psi^{2nd/pub}=\Psi^{2nd}$.

For $r\neq 0$ we can divide (A.12) by r. Then, the right-hand side of the inequality is decreasing in r as $-\left(V_L^{-\frac{1}{2}}-V_H^{-\frac{1}{2}}\right)^2\left[V_L^{\frac{1}{2}}V_H^{\frac{1}{2}}\left(V_H+V_L\right)+\frac{1}{4}\left(V_H^2+V_L^2\right)\right]$ is non-positive. This implies that if (A.12) holds for $r=1$, we have $\Psi^{2nd/pub}\geq\Psi^{2nd}$. For $r=1$ and $V_H>V_L$ (for $V_H=V_L$ we get $\Psi^{2nd/pub}=\Psi^{2nd}$), we can simplify (A.12) to

$$\begin{aligned}&\frac{1}{2}\left[\left(V_H^{\frac{3}{2}}+V_L^{\frac{3}{2}}\right)+\left(V_HV_L^{\frac{1}{2}}+V_LV_H^{\frac{1}{2}}\right)+V_H^2V_L^{-\frac{1}{2}}+\frac{1}{2}V_L^2V_H^{-\frac{1}{2}}\right]\left(V_L^{-\frac{1}{2}}-V_H^{-\frac{1}{2}}\right) \\ &+\frac{1}{4}V_H^{-1}V_L^{\frac{3}{2}}\left(V_L^{\frac{1}{2}}+V_H^{\frac{1}{2}}\right)\geq 0,\end{aligned}$$

which is always satisfied. Hence, $\Psi^{2nd/pub}\geq\Psi^{2nd}$. ■

Proof of Proposition 16.

In the simultaneous contest with public information, homogeneous types exert the same effort. Thus, the effort difference in this case is equal to zero. When types are heterogeneous, the high type exerts a higher effort and the effort difference is

$$\Delta x_{sim}^{pub}:=\frac{V_HV_L\left(V_H-V_L\right)}{\left(V_H+V_L\right)^2}.$$

In the sequential contest with public information, homogeneous types exert identical efforts as well, i.e. the effort difference is zero again. Hence, for homogeneous types there is no gap in both settings.

When types are heterogeneous in the sequential contest under public information, the effort difference is

$$\Delta x_{seq}^{pub}:=|x_1^*-x_2^*|=\begin{cases}x_{1H}^*-x_{2L}^*=\frac{V_H}{2}\left(\frac{V_H-V_L}{V_L}\right) & \text{when } 1^{st} \text{ mover is a high type}\\[2ex] x_{2L}^*-x_{1H}^*=\frac{V_L}{2}\left(\frac{V_H-V_L}{V_H}\right) & \text{when } 1^{st} \text{ mover is a low type.}\end{cases}$$

It can easily be seen that the gap is larger, when the 1^{st} mover is a high type as $\frac{V_H}{V_L}\geq\frac{V_L}{V_H}$.[36] Therefore, if $\Delta x_{seq}^{pub}\geq\Delta x_{sim}^{pub}$ when the 1^{st} mover is a low type, it also holds when the 1^{st} mover is a high type.

It remains to show that $\Delta x_{seq}^{pub}\geq\Delta x_{sim}^{pub}$ holds when the 1^{st} mover is a low type. Trivially, for $V_H=V_L$ we obtain $\Delta x_{seq}^{pub}=\Delta x_{sim}^{pub}$ since then types are homogeneous. For $V_H>V_L$, $\Delta x_{seq}^{pub}\geq\Delta x_{sim}^{pub}$ is equivalent to

$$\left(V_H+V_L\right)^2\geq 2V_H^2.$$

[36] Compare Section 1.5.1.

This can be written as

$$V_L^2 + V_H\left(2V_L - V_H\right) \geq 0,$$

which holds as $(2V_L - V_H) \geq 0$ for the interior solution. Hence, we have shown that a risk neutral designer, who aims at a close race, prefers a simultaneous contests given public information.

In the simultaneous contest with private information, homogeneous types exert the same effort. Thus, the effort difference is equal to zero. Given heterogeneous types, the effort difference is

$$\begin{aligned}
\Delta x_{sim}^{priv} : &= x_{iH}^* - x_{iL}^* \\
&= (V_H - V_L)\left(\frac{r}{4} + (1-r)\frac{V_H V_L}{(V_H + V_L)^2}\right) \\
&= r\frac{(V_H - V_L)^3}{4(V_H + V_L)^2} + \frac{V_H V_L (V_H - V_L)}{(V_H + V_L)^2}.
\end{aligned}$$

Using (3.7), it follows that

$$\Delta x_{sim}^{priv} = \Delta x_{sim}^{pub} + r\frac{(V_H - V_L)^3}{4(V_H + V_L)^2}.$$

Hence, $\Delta x_{sim}^{priv} \geq \Delta x_{sim}^{pub}$ (with strict inequality if $V_H > V_L$ and $r > 0$). This gives us the result that a risk neutral designer, who aims at a close race, prefers public information to a private information given that a simultaneous contest is played.

In the sequential contest with private information, homogeneous types do not exert the same effort, unless $V_H = V_L$ or $r = 0$. This means that there is a gap between efforts of the first and second mover. Thus, the effort difference for homogeneous types is larger than in the simultaneous contest with private information and in the sequential contest with public information. Using the results from Proposition 7, where we show that the first mover makes a higher (lower) bid than the second mover when he is the high (low) type, we can derive the following gaps in the contestants' bids:

$$\Delta x_{seq}^{\text{hom}} := \begin{cases} x_{1H}^* - x_{2H}^{*H} = \alpha_H\left(2\alpha_H - \sqrt{V_H}\right) & \text{when contestants are high types} \\ \\ x_{2L}^{*L} - x_{1L}^* = \alpha_L\left(\sqrt{V_L} - 2\alpha_L\right) & \text{when contestants are low types.} \end{cases}$$

For heterogeneous types, the effort difference is

$$\Delta x_{seq}^{het} := \begin{cases} x_{1H}^* - x_{2L}^{*H} = \alpha_H\left(2\alpha_H - \sqrt{V_L}\right) & \text{when the } 1^{st} \text{ mover is a high type} \\ \\ x_{2H}^{*L} - x_{1L}^* = \alpha_L\left(\sqrt{V_H} - 2\alpha_L\right) & \text{when the } 1^{st} \text{ mover is a low type.} \end{cases}$$

Hence, the ex ante expected gap in case of private information is given by

$$\begin{aligned}
\widetilde{\Delta x_{seq}^{priv}} : &= \frac{1}{2}r\left(x_{1H}^* - x_{2H}^{*H} + x_{2L}^{*L} - x_{1L}^*\right) + \frac{1}{2}(1-r)\left(x_{1H}^* - x_{2L}^{*H} + x_{2H}^{*L} - x_{1L}^*\right) \\
&= \frac{1}{2}\left[2(\alpha_H^2 - \alpha_L^2) - \alpha_H V_L^{\frac{1}{2}} + \alpha_L V_H^{\frac{1}{2}} + r(\alpha_H + \alpha_L)\left(V_L^{\frac{1}{2}} - V_H^{\frac{1}{2}}\right)\right].
\end{aligned}$$

Plugging in α and rearranging yields

$$\widetilde{\Delta x_{seq}^{priv}} = (1-r)\frac{V_H - V_L}{4V_H V_L}\left[(1-r)(V_H^2 + V_L^2) + rV_H^{\frac{1}{2}}V_L^{\frac{1}{2}}(V_H + V_L)\right].$$

The ex ante expected gap in case of public information is

$$\begin{aligned}\widetilde{\Delta x^{pub}_{seq}} := & \frac{1}{2} r \cdot 0 + \frac{1}{2}(1-r)\left(\frac{V_H}{2}\left(\frac{V_H - V_L}{V_L}\right) + \frac{V_L}{2}\left(\frac{V_H - V_L}{V_H}\right)\right) \\ = & \frac{1}{4V_H V_L}(1-r)(V_H - V_L)\left(V_H^2 + V_L^2\right).\end{aligned}$$

Comparing the ex ante expected gaps under public and private information, we see that for $V_L = V_H$ as well as for $r = 0$ and $r = 1$ they are equally large. This is intuitive as in these cases there is in fact public information. Let now $V_< V_H$ and $r \in (0,1)$. It is straightforward to verify that $\widetilde{\Delta x^{priv}_{seq}} < \widetilde{\Delta x^{pub}_{seq}}$ is equivalent to

$$r^2\left(V_H^{-\frac{1}{2}} - V_L^{-\frac{1}{2}}\right)^2\left[V_H^{\frac{1}{2}} V_L^{\frac{1}{2}}(V_H - V_L) + V_H^2 - V_L^2\right] + r\left(V_H^{-\frac{1}{2}} - V_L^{-\frac{1}{2}}\right)\left[2V_L^{\frac{3}{2}} + V_H V_L^{\frac{1}{2}} + V_L V_H^{\frac{1}{2}}\right] < 0.$$

This can be rewritten as

$$\left(V_H^{\frac{1}{2}} - V_L^{\frac{1}{2}}\right)\left(V_L^{\frac{3}{2}} - V_H^{\frac{3}{2}}\right) < 0,$$

which is obviously true. Thus, $\widetilde{\Delta x^{priv}_{seq}} \leq \widetilde{\Delta x^{pub}_{seq}}$ is always satisfied (with equality for $r = 0$, $r = 1$ and $V_H = V_L$). This implies that given a sequential contest is played, the designer prefers private information from an ex ante perspective if he wants to have a close race.

The last step of the proof is to show that in the private information setting for heterogeneous types, the designer prefers the simultaneous contest, when he aims at a close race. In order to show this, we consider the expected effort difference $\widetilde{\Delta x^{priv}_{seq}}$ in the sequential contest and show that this difference is larger than in the simultaneous contest, i.e.

$$\widetilde{\Delta x^{priv}_{seq}} \geq \frac{1}{2}(1-r)2\Delta x^{priv}_{sim} + \frac{1}{2} r \cdot 0 =: \widetilde{\Delta x^{priv}_{sim}} \tag{A.13}$$

using the result that the effort difference for homogeneous types is zero in the simultaneous contest. Evidently, for $V_L = V_H$ and also for $r = 1$ the expected gaps are identical. Suppose now $V_L < V_H$ and $r < 1$. Then, (A.13) is equivalent to

$$(V_H + V_L)^2\left[(1-r)(V_H^2 + V_L^2) + rV_H^{\frac{1}{2}} V_L^{\frac{1}{2}}(V_H + V_L)\right] \geq rV_H V_L (V_H - V_L)^2 + 4V_L^2 V_H^2.$$

We can rewrite this as

$$\begin{aligned}&2V_H V_L(V_H - V_L) + (1-r)(V_H^4 + V_L^4 + 2V_L^3 V_H) + r\left(V_H^2(V_H^{\frac{1}{2}} - V_L^{\frac{1}{2}})^2 + V_H^2 V_L^{\frac{1}{2}}(2V_L^{\frac{1}{2}} - V_H^{\frac{1}{2}})\right. \\ &\left. + V_L^2 V_H^{\frac{1}{2}}(V_H^{\frac{1}{2}} - V_L^{\frac{1}{2}}) + V_L^3 + 2V_L^2 V_H\right) \geq 0.\end{aligned} \tag{A.14}$$

Note that for the interior solution $r \geq \tilde{r} = \frac{2V_L - V_H}{\sqrt{V_H}(\sqrt{V_L} - \sqrt{V_H})}$ has to be satisfied. This can only be satisfied if $\tilde{r} \leq 1$, which is equivalent to $4V_L \geq V_H$. Hence, for $4V_L < V_H$ we cannot have an interior solution. This implies that $V_H^{\frac{1}{2}} \leq 2V_L^{\frac{1}{2}}$ holds for the interior solution. Using this observation and $V_L \leq V_H$, it can be seen that (A.14) is satisfied and thus, $\widetilde{\Delta x^{priv}_{seq}} \geq \widetilde{\Delta x^{priv}_{sim}}$. Hence, given private information, the designer prefers simultaneous contests from an ex ante perspective when his aim is a close race. ■

Proof of Lemma 2.

The participation constraint for agent i if the principal wants to implement the effort vector $(\hat{e}_i, \hat{e}_{-i})$ is

$$p^{HH}(\hat{e}_i, \hat{e}_{-i})w_i^{\mathbf{H}} + (1 - p^{HH}(\hat{e}_i, \hat{e}_{-i}))w_i^{\mathbf{L}} - c(\hat{e}_i) \geq 0.$$

If the agent provides zero effort, he can ensure an expected payoff of zero. Moreover, the incentive constraint is the maximizer of the agent's expected payoff, hence, the agent cannot earn a negative expected payoff. Note that a sufficient condition for the agent's problem (the incentive constraint) to be concave in e_i is $w_i^{\mathbf{H}} \geq w_i^{\mathbf{L}}$ as the probability of success function is concave and the cost function convex in e_i. For the optimal wage scheme, $w_i^{\mathbf{H}} \geq w_i^{\mathbf{L}}$ – as we show below. Thus, a global maximum of the agent's problem exists. The principal's problem is a linear optimization problem, where the feasible set is convex and the objective function is continuous and linear in $w_i^{\mathcal{Y}_c}$.

As already mentioned in the text, setting $w_i^{\mathbf{L}} > 0$ decreases incentives compared to setting $w_i^{\mathbf{L}} = 0$ as $\frac{\partial p^{HL}(\hat{e}_i, \hat{e}_{-i})}{\partial e_i} < 0$ (which can be seen from the incentive constraints). Thus, $\hat{e}_i$ has to be smaller, too. This cannot be profit maximizing for the principal since effort is smaller and the principal pays the agent more by setting $w_i^{\mathbf{L}} > 0$, which reduces his profit. Therefore, $w_i^{\mathbf{L}} = 0$.

It remains to solve for the optimal wage if the project succeeds, $w_i^{\mathbf{H}}$, to implement efforts $(\hat{e}_1, \hat{e}_2)$, which we can derive from the incentive constraints. Using $w_i^{\mathbf{L}} = 0$, we can write the incentive constraints $(IC_i^{sim_c})$ as

$$\frac{\partial p^{HH}(\hat{e}_i, \hat{e}_{-i})}{\partial e_i} w_i^{\mathbf{H}} = c'(\hat{e}_i) \ \ \forall i. \tag{A.15}$$

This first order condition yields a global maximum since the second order conditions

$$\frac{\partial^2 p^{HH}(\hat{e}_i, \hat{e}_{-i})}{\partial e_i^2} w_i^{\mathbf{H}} - c''(\hat{e}_i) < 0 \ \ \forall i \tag{A.16}$$

are satisfied for the optimal wage scheme as the cost function is strictly convex, $p^{HH}(e_i, e_{-i})$ is concave in e_i by assumption, and $w_i^{\mathbf{H}} \geq 0$ must hold true by the limited liability constraint. Assuming for the moment that the limited liability constraints are met, the optimal wage for agent i to implement an effort level of $\hat{e}_i$ when both agents perform well (given an effort level $\hat{e}_{-i}$) has to satisfy $w_i^{\mathbf{H}} = \frac{c'(\hat{e}_i)}{p_{e_i}^{HH}(\hat{e}_i, \hat{e}_{-i})}$ as stated in Lemma 2. Note that this wage is non-negative and strictly larger than zero for $e_i > 0$. Hence, the limited liability constraints are satisfied and $w_i^{\mathbf{H}} \geq w_i^{\mathbf{L}} = 0$.

Note that the principal pays the wage for a project with high value with probability $p^{HH}(\hat{e}_i, \hat{e}_{-i})$ to agent i. Thus, expected implementation costs for effort $\hat{e}_i$ are $\mathcal{W}_i^{sim_c} = \frac{p^{HH}(\hat{e}_i, \hat{e}_{-i})}{p_{e_i}^{HH}(\hat{e}_i, \hat{e}_{-i})} c'(\hat{e}_i) = \frac{p(\hat{e}_i)}{p'(\hat{e}_i)} c'(\hat{e}_i)$. ■

Proof $\mathcal{A}$.

In order to show that our results do not change if the second mover can also observe the effort of the first mover, it suffices to consider the incentive constraints of the second mover (which we give later in Section 2.3.2):

$$\begin{aligned}
&\text{After seeing } H: \ e_2^H \in \arg\max_{e_2 \in \mathcal{I}} \ \Pr[H|H, \hat{e}_1, e_2]w_2^{\mathbf{H}} + \Pr[L|H, \hat{e}_1, e_2]w_2^{\mathbf{L}} - c(e_2), \\
&\text{After seeing } L: \ e_2^L \in \arg\max_{e_2 \in \mathcal{I}} \ w_2^{\mathbf{L}} - c(e_2).
\end{aligned}$$

Obviously, the effort of the first mover does not enter the incentive constraint of the second mover when he observes low quality. Hence, only the incentive constraint after having observed a high quality contribution can be affected when effort is observable. Plugging in the conditional probabilities according to (2.2), we can write the incentive constraint as follows:

$$\text{After seeing } H: \; e_2^H \in \arg\max_{e_2 \in \mathcal{I}} \; p(e_2)w_2^{\mathbf{H}} + (1-p(e_2))w_2^{\mathbf{L}} - c(e_2).$$

Thus, this incentive constraint is also independent of the effort of the first mover. This implies immediately that our results do not change when the second mover can also observe the effort of the first mover. ■

Proof of Lemma 4.

Like for the case of complements, the principal's problem is a linear optimization problem, where the feasible set is convex and the objective function is continuous and linear in $w_i^{\mathcal{Y}_s}$. By a similar argument than for the case of complements, we can drop the participation constraints and focus only on the incentive and limited liability constraints. The incentive constraint ($IC_i^{sim_s}$) for agent i is:

$$\begin{aligned} \hat{e}_i \in \text{argmax}_{e_i \in \mathcal{I}} \quad & p^{HH}(e_i, \hat{e}_{-i},)w_i^{\mathcal{H}} + p^{HL}(e_i, \hat{e}_{-i})w_i^{\mathcal{M}} + p^{LH}(e_i, \hat{e}_{-i})w_i^{\mathcal{M}} \\ & + p^{LL}(e_i, \hat{e}_{-i})w_i^{\mathcal{L}} - c(e_i), \end{aligned}$$

where we denote Nash equilibrium effort levels by $\hat{e}_i$. We can rewrite this problem as

$$\sum_{Y_{-i}} \sum_{Y_i} \frac{\partial p^{Y_i Y_{-i}}(\hat{e}_i, \hat{e}_{-i})}{\partial e_i} w_i^{\mathcal{Y}_s} - c'(\hat{e}_i) = 0. \tag{A.17}$$

Like we have seen in the proof of Lemma 2 for the case that contributions are complements, $w_i^{\mathcal{L}} > 0$ cannot be optimal by the same argument: it decreases incentives and reduces profits. Thus, $w_i^{\mathcal{L}} = 0$. By the same argument, it cannot be optimal to set $w_i^{\mathcal{M}} > 0$ if $\frac{\partial(p_i^{LH}(\hat{e}_i, \hat{e}_{-i}) + p_i^{HL}(\hat{e}_i, \hat{e}_{-i}))}{\partial e_i} \leq 0$ (condition $\mathcal{A}$). Thus, $w_i^{\mathcal{M}} = 0$ under condition $\mathcal{A}$.

If the wages in case that at least one agent contributes low quality are zero, we can derive the wage when both agents perform well from the incentive constraint in (A.17). If condition $\mathcal{A}$ is, however, not satisfied, i.e. $\frac{\partial(p_i^{LH}(\hat{e}_i, \hat{e}_{-i}) + p_i^{HL}(\hat{e}_i, \hat{e}_{-i}))}{\partial e_i} > 0$, we have to consider the problem of the principal.

Let first condition $\mathcal{A}$ be satisfied. Using $w_i^{\mathcal{M}} = w_i^{\mathcal{L}} = 0$, the problem of the agent simplifies to

$$\frac{\partial p^{HH}(\hat{e}_i, \hat{e}_{-i})}{\partial e_i} w_i^{\mathcal{H}} - c'(\hat{e}_i) = 0. \tag{A.18}$$

The second order conditions

$$\frac{\partial^2 p^{HH}(e_i, e_{-i})}{\partial e_i^2} w_i^{\mathcal{H}} - c''(e_i) < 0 \tag{A.19}$$

are satisfied as the cost function is strictly convex, $p^{HH}(e_i, e_{-i})$ is concave in e_i, and $w_i^{\mathcal{H}} \geq 0$ by the limited liability constraint. Hence, (A.18) yields a global maximum: The wage for agent i to implement effort $\hat{e}_i$ (given the other agent's effort e_{-i}) has to be $w_i^{\mathcal{H}} = \frac{c'(\hat{e}_i)}{p_{e_i}^{HH}(\hat{e}_i, \hat{e}_{-i})}$. Note that $w_i^{\mathcal{H}}$ is non-negative (strictly positive for $\hat{e}_i > 0$). Thus, the limited liability constraints are met.

If condition $\mathcal{A}$ does not hold, i.e. $\frac{\partial(p_i^{LH}(\hat{e}_i, \hat{e}_{-i}) + p_i^{HL}(\hat{e}_i, \hat{e}_{-i}))}{\partial e_i} > 0$ (condition $\mathcal{B}$), we cannot (immediately) conclude that $w_i^{\mathcal{M}} = 0$, but have to consider the problem of the principal. The problem of the principal under condition $\mathcal{B}$ is

$$\max_{\mathbf{w_i^{\mathcal{H}}},\mathbf{w_i^{\mathcal{M}}}} \quad -\sum_i p^{HH}(\hat{e}_i,\hat{e}_{-i})w_i^{\mathcal{H}} - \sum_i [p^{HL}(\hat{e}_i,\hat{e}_{-i}) + p^{LH}(\hat{e}_i,\hat{e}_{-i})]w_i^{\mathcal{M}}$$
$$\text{s.t } \hat{e}_i \in \text{argmax}_{e_i\in\mathcal{I}}\ p^{HH}(e_i,\hat{e}_{-i},)w_i^{\mathcal{H}} + (p^{HL}(e_i,\hat{e}_{-i}) + p^{LH}(e_i,\hat{e}_{-i}))w_i^{\mathcal{M}} - c(e_i)\ \forall i,$$
$$\text{s.t } w_i^{\mathcal{Y}_s} \geq 0\ \forall \mathcal{Y}_s(Y_i,Y_{-i}),\ \forall i$$

Denoting by δ the Lagrange multiplier, the first order conditions with respect to wages are

$$w_i^{\mathcal{H}}: \quad -p^{HH}(e_i,\hat{e}_{-i}) + \delta\left(\frac{\partial p^{HH}(e_i,\hat{e}_{-i})}{\partial e_i}\right) \leq 0,\ w_i^{\mathcal{H}}\frac{\partial \mathcal{L}}{\partial w_i^{\mathcal{H}}} = 0,$$
$$w_i^{\mathcal{M}}: \quad -[p^{HL}(e_i,\hat{e}_{-i}) + p^{LH}(\hat{e}_i,\hat{e}_{-i})] + \delta\left(\frac{\partial(p^{HL}(e_i,\hat{e}_{-i}) + p^{LH}(\hat{e}_i,\hat{e}_{-i}))}{\partial e_i}\right) \leq 0,\ w_i^{\mathcal{M}}\frac{\partial \mathcal{L}}{\partial w_i^{\mathcal{M}}} = 0.$$

It follows from these first order conditions that if

$$\frac{p^{HH}(e_i,\hat{e}_{-i})}{p_{e_i}^{HH}(e_i,\hat{e}_{-i})} < \frac{p^{HL}(e_i,\hat{e}_{-i}) + p^{LH}(e_i,\hat{e}_{-i})}{p_{e_i}^{HL}(e_i,\hat{e}_{-i}) + p_{e_i}^{LH}(e_i,\hat{e}_{-i})}, \tag{A.20}$$

then $w_i^{\mathcal{H}} > 0$ and $w_i^{\mathcal{M}} = 0$.[37] We refer to condition (A.20) as condition $\mathcal{W}$.

If condition $\mathcal{W}$ is not satisfied, but (A.20) holds instead with equality, then all $w_i^{\mathcal{H}} - w_i^{\mathcal{M}}$ combinations that satisfy the incentive constraint (A.17) are optimal. If the inequality in (A.20) is reversed, then $w_i^{\mathcal{H}} = 0$ and $w_i^{\mathcal{M}} > 0$.

Using independence of individual success probabilities, condition $\mathcal{B}$ simplifies to $p'(e_i)(1-2p(\hat{e}_{-i})) > 0$ and condition $\mathcal{W}$ to $\frac{p(e_i)}{p'(e_i)} < \frac{p(e_i)(1-2p(\hat{e}_{-i}))+p(\hat{e}_{-i})}{p'(e_i)(1-2p(\hat{e}_{-i}))}$.[38] Since $p'(e_i)(1-2p(\hat{e}_{-i})) > 0$, we can further simplify condition $\mathcal{W}$ to $p(\hat{e}_{-i}) > 0$.

Hence, for $p(\hat{e}_{-i}) > 0$, we have that $\frac{p^{HH}(e_i,\hat{e}_{-i})}{p_{e_i}^{HH}(e_i,\hat{e}_{-i})} \geq \frac{p^{HL}(e_i,\hat{e}_{-i})+p^{LH}(e_i,\hat{e}_{-i})}{p_{e_i}^{HL}(e_i,\hat{e}_{-i})+p_{e_i}^{LH}(e_i,\hat{e}_{-i})}$ cannot be satisfied. Thus, we have $w_i^{\mathcal{H}} > 0$ and $w_i^{\mathcal{M}} = 0$ (like under condition $\mathcal{A}$). Moreover, we can derive $w_i^{\mathcal{H}}$ – exactly like under condition $\mathcal{A}$ – from the agent's incentive constraint (A.18) and hence, $w_i^{\mathcal{H}} = \frac{c'(\hat{e}_i)}{p_{e_i}^{HH}(\hat{e}_i,\hat{e}_{-i})}$.

■

Proof $\mathcal{B}$.

Exactly like for the case of complements, we can show that the incentive constraints of the second mover (see Section 2.5.2) are independent of the first mover's effort by plugging in the conditional probabilities:

$$\text{After seeing } H:\ e_2^H \in \arg\max_{e_2\in\mathcal{I}}\ p(e_2)w_2^{\mathcal{H}} + (1-p(e_2))w_2^{\mathcal{M}} - c(e_2),$$
$$\text{After seeing } L:\ e_2^L \in \arg\max_{e_2\in\mathcal{I}}\ (1-p(e_2))w_2^{\mathcal{M}} - c(e_2).$$

Again, it immediately follows, that observability of the first mover's effort does not change our results. ■

Proof of Lemma 5.

As argued in the text, the wage for the second mover when both agents fail is equal to zero (compare the

[37]We assume here that $p_{e_i}^{HH}(e_i,\hat{e}_{-i}) > 0$ and $p_{e_i}^{HL}(e_i,\hat{e}_{-i}) + p_{e_i}^{LH}(e_i,\hat{e}_{-i}) > 0$ such that (A.20) is always defined. We see below that this holds under condition $\mathcal{B}$.

[38]Note that condition $\mathcal{B}$ implies $p'(e_i) > 0$ and hence condition $\mathcal{W}$ is always defined.

argument in Section 2.3.2.). We can rewrite the incentive constraint of the second mover after observing a poor performance of the first mover as

$$\frac{p_{e_2}^{LH}(\hat{e}_1, e_2^L)}{1-p(\hat{e}_1)} w_2^{\mathcal{M}} - c'(e_2^L) = 0. \tag{A.21}$$

As mentioned in the text, the second order condition is satisfied for the optimal wage scheme since the cost function is strictly convex, $p^{LH}(\hat{e}_1, e_2)$ is concave in e_2, and given that the limited liability constraint is satisfied (what we assume for the moment and show in the following that it is indeed true). Hence, the first oder condition in (A.21) yields the optimal wage

$$w_2^{\mathcal{M}} = \frac{c'(e_2^L)}{p_{e_2}^{LH}(\hat{e}_1, e_2^L)}(1-p(\hat{e}_1)) = \frac{c'(e_2^L)}{p'(e_2^L)}.$$

This wage is non-negative and strictly positive for $e_2^H > 0$. Thus, the limited liability constraint is satisfied. We can write the incentive constraint of the second mover after observing a high performance of the first mover as follows:

$$p'(e_2^L)\left[w_2^{\mathcal{H}} - w_2^{\mathcal{M}}\right] - c'(e_2^L).$$

Note that the second order conditions are satisfied if $w_2^{\mathcal{H}} \geq w_2^{\mathcal{M}}$, which is satisfied in equilibrium as we show below. Hence, we can derive from the first order condition the optimal wage for the case that the project has a high value. Plugging in $w_2^{\mathcal{M}}$ into the other first order condition in state H and solving for $w_2^{\mathcal{H}}$ yields

$$w_2^{\mathcal{H}} = \frac{c'(e_2^H)}{p'(e_2^H)} + \frac{c'(e_2^L)}{p'(e_2^L)}.$$

Obviously, $w_2^{\mathcal{H}} \geq w_2^{\mathcal{M}}$ as $w_2^{\mathcal{H}} = \frac{c'(e_2^H)}{p'(e_2^H)} + w_2^{\mathcal{M}}$. Since $w_2^{\mathcal{M}} \geq 0$, also $w_2^{\mathcal{H}} \geq 0$, which satisfies the limited liability constraint.

By an analogue argument as given for the simultaneous structure (see Proof of Lemma 5) and for the sequential structure when contributions are complements (see Section 2.3.2), the first mover's wages are $w_1^{\mathcal{H}} = \frac{c'(\hat{e}_1)}{p_{e_1}^{HH}(\hat{e}_1, e_2^H)}$, $w_1^{\mathcal{M}} = w_1^{\mathcal{L}} = 0$. Similar to the observation when contributions are complements, however, as mentioned earlier in Section 2.3.2, equilibrium efforts of the first mover need not be identical under both structures when e_2^H differs from $\hat{e}_2$. ∎

Condition $\mathcal{Z}$.

Consider the sequential structure when contributions are substitutes. Given the optimal wage scheme, the problem of the principal is to maximize his expected profits with respect to efforts:[39]

$$\max_{\mathbf{e_1}\in\mathcal{I}, \mathbf{e_2^H}\in\mathcal{I}, \mathbf{e_2^L}\in\mathcal{I}} \quad \Bigg[\left(2p(e_1)p(e_2^H) + p(e_1)(1-p(e_2^H)) + p(e_2^L)(1-p(e_1))\right)\pi - p(e_1)p(e_2^H)\frac{c'(e_1)}{p'(e_1)p(e_2^H)}$$
$$-p(e_1)p(e_2^H)\left(\frac{c'(e_2^H)}{p'(e_2^H)} + \frac{c'(e_2^L)}{p'(e_2^L)}\right) - \left[(1-p(e_1))p(e_2^L) + p(e_1)(1-p(e_2^L))\right]\frac{c'(e_2^L)}{p'(e_2^L)}\Bigg]$$

Summarizing yields

$$\max_{\mathbf{e_1}\in\mathcal{I}, \mathbf{e_2^H}\in\mathcal{I}, \mathbf{e_2^L}\in\mathcal{I}} \quad \Bigg[\left(p(e_1)p(e_2^H) + p(e_1) + p(e_2^L)(1-p(e_1))\right)\pi - p(e_1)\frac{c'(e_1)}{p'(e_1)}$$
$$-p(e_1)p(e_2^H)\frac{c'(e_2^H)}{p'(e_2^H)} - \left[(1-p(e_1))p(e_2^L) + p(e_1)\right]\frac{c'(e_2^L)}{p'(e_2^L)}\Bigg] \tag{A.22}$$

[39]Note that $w_2^{\mathcal{M}}$ is paid in case the first mover provides high quality and the second mover low quality as well as when the first one performs poorly and the second one performs well.

Suppose now that the principal implements $e_2^L = 0$. Denote by e_1° and $e_2^{H\circ}$ the profit-maximizing efforts for this case. Note that e_1° and $e_2^{H\circ}$ need not be profit-maximizing for the case that $e_2^L > 0$. We calculate the difference in expected profits when either $e_2^L > 0$ or $e_2^L = 0$. For both cases, we evaluate expected profits at e_1° and $e_2^{H\circ}$, which are optimal for $e_2^L = 0$, but not necessarily for $e_2^L > 0$. If this difference in expected profits is, nevertheless, positive, it is also positive for the optimal efforts e_1 and e_2^H given $e_2^L > 0$. Hence, if the difference is positive, $e_2^L > 0$ is optimal. The difference in expected profits is[40]

$$(1 - p(e_1^\circ))\left(p(e_2^L) - p(0)\right)\pi - \left[(1 - p(e_1^\circ))p(e_2^L) + p(e_1^\circ)\right]\frac{c'(e_2^L)}{p'(e_2^L)}.$$

Thus, the difference in expected profits is larger than zero under the following condition (Condition $\mathcal{Z}$):

$$\pi > \frac{c'(e_2^L)}{p'(e_2^L)}\left[\frac{p(e_1^\circ) + (1 - p(e_1^\circ))p(e_2^L)}{(1 - p(e_1^\circ))\left(p(e_2^L) - p(0)\right)}\right].$$

Hence, if π is sufficiently large it is optimal to implement a strictly positive effort for the second mover after a poor performance of the first mover. ■

Proof of Lemma 6.

Suppose the principal wants to implement an effort level of $e_1^\star$ for the first mover and an expected effort of $e_2^\star = p(e_1^\star)e_2^{H\star} + (1 - p(e_1^\star))e_2^{L\star}$ for the second mover. Given the optimal wage scheme, the problem of the principal becomes (compare (A.22))

$$\begin{aligned}
\max_{\mathbf{e_2^{L\star}} \in \mathcal{I}} \quad & \Bigg[\left(p(e_1^\star)p(e_2^{H\star}) + p(e_1^\star) + p(e_2^{L\star})(1 - p(e_1^\star))\right)\pi - p(e_1^\star)\frac{c'(e_1^\star)}{p'(e_1^\star)} \\
& -p(e_1^\star)p(e_2^{H\star})\frac{c'(e_2^{H\star})}{p'(e_2^H)} - \left[(1 - p(e_1^\star))p(e_2^{L\star}) + p(e_1^\star)\right]\frac{c'(e_2^{L\star})}{p'(e_2^L)}\Bigg] \\
\text{s.t.} \quad & e_2^\star = p(e_1^\star)e_2^{H\star} + (1 - p(e_1^\star))e_2^{L\star}.
\end{aligned}$$

The first order condition to this problem is

$$\begin{aligned}
&\left[(1 - p(e_1^\star))\left(p'(e_2^{L\star}) - p'(e_2^{H\star})\right)\right]\pi + (1 - p(e_1^\star))\left[c'(e_2^{H\star}) - c'(e_2^{L\star})\right] \\
&-(1 - p(e_1^\star))p(e_2^{H\star})\left[\frac{-p'(e_2^{H\star})c''(e_2^{H\star}) + c'(e_2^{H\star})p''(e_2^{H\star})}{p'(e_2^{H\star})^2}\right] - p(e_1^\star)\left[\frac{p'(e_2^{L\star})c''(e_2^{L\star}) - c'(e_2^{L\star})p''(e_2^{L\star})}{p'(e_2^{L\star})^2}\right] \\
&-(1 - p(e_1^\star))p(e_2^{L\star})\left[\frac{p'(e_2^{L\star})c''(e_2^{L\star}) - c'(e_2^{L\star})p''(e_2^{L\star})}{p'(e_2^{L\star})^2}\right] = 0.
\end{aligned}$$

The first order condition is also sufficient (since p is strictly increasing and concave and c is strictly convex) if Assumption $\mathcal{C}$ is satisfied. This is evident from the maximization problem if we note that under our assumptions on p and c it follows from Assumption $\mathcal{C}$ that $p(e)\frac{c'(e)}{p'(e)}$ is strictly convex. To see this implication of Assumption $\mathcal{C}$, let $f(e) := \frac{c'(e)}{p'(e)}$ and $g(e) := p(e)\frac{c'(e)}{p'(e)} = p(e)f(e)$. Hence, $g'(e) = p'(e)f(e) + p(e)f'(e)$ is strictly positive by the assumptions on p and c. These imply, in particular, $f(e) \geq 0$ and $f'(e) = \frac{p'(e)c''(e) - p''(e)c'(e)}{p'(e)^2} > 0$. Consider now the second derivative of g:

$$g''(e) = p''(e)f(e) + 2p'(e)f'(e) + p(e)f''(e).$$

[40] Note that $c'(0) = 0$.

Under Assumption $\mathcal{C}$ (i.e. $f''(e) \geq 0$)), for this derivative being positive, it suffices to show that $p''(e)f(e) + 2p'(e)f'(e) > 0$. $p''(e)f(e) + 2p'(e)f'(e)$ is equal to $\frac{2p'(e)c''(e)-p''(e)c'(e)}{p'(e)}$, which is strictly larger than zero. Hence, we have by Assumption $\mathcal{C}$ that $g''(e) > 0$. This establishes that the first order condition is also sufficient.

Rewriting the first order condition by using $k(e) := p(e)\left[\frac{p'(e)c''(e)-c'(e)p''(e)}{p'(e)^2}\right]$ yields

$$(1 - p(e_1^\star))\left[\left(p'(e_2^{L\star}) - p'(e_2^{H\star})\right)\pi + \left(c'(e_2^{H\star}) - c'(e_2^{L\star})\right) - \left(k(e_2^{L\star}) - k(e_2^{H\star})\right)\right] = p(e_1^\star)k(e_2^{L\star}). \quad \text{(A.23)}$$

Note that $k(e) = p(e)f'(e) > 0$, which implies that the right hand side of (A.23) is strictly positive. Moreover, under Assumption $\mathcal{C}$ it follows that $k'(e) = p'(e)f'(e) + p(e)f''(e) > 0$.

Suppose now that $e_2^{L\star} = e_2^{H\star}$. The first order condition then simplifies to

$$0 = p(e_1^\star)k(e_2^{L\star}).$$

This condition can, however, never be satisfied as the right hand side is strictly larger than zero. Hence, it cannot be optimal to set $e_2^{L\star} = e_2^{H\star}$.

In order to show that it is optimal to set $e_2^{L\star} < e_2^{H\star}$, suppose first $e_2^{L\star} > e_2^{H\star}$. $e_2^{L\star} > e_2^{H\star}$ implies that the left hand side of (A.23) is strictly negative as $p'(e_2^{L\star}) - p'(e_2^{H\star})$ becomes non-negative, $c'(e_2^{H\star}) - c'(e_2^{L\star})$ becomes strictly negative, and $k(e_2^{L\star}) - k(e_2^{H\star})$ becomes strictly positive. Since the right hand side of (A.23) is strictly positive, the first order condition cannot be satisfied for $e_2^{L\star} > e_2^{H\star}$. Hence, $e_2^{L\star} < e_2^{H\star}$ must hold true. ■

Proof of Lemma 7.

The difference in expected revenues between the simultaneous and the sequential structure is

$$\Delta R \quad = \quad \left[p(e_2^\star) - \left(p(e_1^\star)p(e_2^{H\star})) + (1 - p(e_1^\star))p(e_2^{L\star})\right)\right]\pi$$

since $e_1^\star$ is implemented for agent 1 under both structures. Suppose now that p is concave and the principal implements $e_2^\star = p(e_1^\star)e_2^{H\star} + (1 - p(e_1^\star))e_2^{L\star}$ under the simultaneous structure. Then, by Jensen's Inequality $p(e_2^\star) = p(p(e_1^\star)e_2^{H\star} + (1 - p(e_1^\star)e_2^{L\star}) \leq p(e_1^\star)p(e_2^{H\star}) + (1 - p(e_1^\star))p(e_2^{L\star})$ (with strict inequality for strict concavity and $e_2^{L\star} \neq e_2^{H\star}$).

■

Proof C.

Expected implementation costs for both agents under the simultaneous structure and for the first mover under the sequential structure (compare (2.5) and (2.6)) are convex if and only if $\frac{p(e)}{p'(e)}c'(e)$ is convex in effort. Moreover, the convexity of $\frac{p(e)}{p'(e)}c'(e)$ is a sufficient condition for expected implementation costs for the second mover under the sequential structure (compare (2.7)) to be convex.

Within the Proof of Lemma 6 we show that Assumption $\mathcal{C}$) implies that $\frac{p(e)}{p'(e)}c'(e)$ is strictly convex. Hence, under Assumption $\mathcal{C}$, expected implementation costs are convex in effort. ■

Proof of Lemma 9.

The difference in expected implementation costs for the second agent between the simultaneous and sequential structure is

$$\begin{aligned}\Delta SM: &= W_2^{sim_s} - W_2^{seq_s} \\ &= \frac{p(e_2^\star)}{p'(e_2^\star)}c'(e_2^\star) - p(e_1^\star)\frac{p(e_2^{H\star})}{p'(e_2^{H\star})}c'(e_2^{H\star}) - (1-p(e_1^\star))\frac{p(e_2^{L\star})}{p'(e_2^{L\star})}c'(e_2^{L\star}) - \frac{p(e_1^\star)}{p'(e_2^{L\star})}c'(e_2^{L\star}).\end{aligned}$$

Note that the term $\frac{p(e_1^\star)}{p'(e_2^{L\star})}c'(e_2^{L\star})$ in ΔSM increases expected implementation costs of the sequential structure relative to the simultaneous one. Dropping this term, we have

$$\Delta SM^+ := \frac{p(e_2^\star)}{p'(e_2^\star)}c'(e_2^\star) - \left[p(e_1^\star)\frac{p(e_2^{H\star})}{p'(e_2^{H\star})}c'(e_2^{H\star}) + (1-p(e_1^\star))\frac{p(e_2^{L\star})}{p'(e_2^{L\star})}c'(e_2^{L\star})\right].$$

Let Assumption $\mathcal{C}$ be satisfied. Under Assumption $\mathcal{C}$, $\frac{p(e_2)}{p'(e_2)}c'(e_2) =: f(e_2)$ is strictly convex (see Proof C). Suppose the principal implements for the first agent the same effort under both structures (i.e. $e_1^\star$) and for the second agent, he implements the same effort (in expectation), i.e. $e_2^\star = p(e_1^\star)e_2^{H\star} + (1-p(e_1^\star))e_2^{L\star}$. Then, by Jensen's Inequality, $f(e_2^\star) = f(p(e_1^\star)e_2^{H\star} + (1-p(e_1^\star))e_2^{L\star}) \leq p(e_1^\star)f(e_2^{H\star}) + (1-p(e_1^\star))f(e_2^{L\star})$.[41] Thus, $\Delta SM^+ \leq 0$ and therefore, $\Delta SM \leq 0$.

Further on assuming that Assumption $\mathcal{C}$ is satisfied, the above result holds with strict inequality if $e_2^{H\star} \neq e_2^{L\star}$, which establishes part (i) of the lemma.

For part (ii) of the lemma, note that $\Delta SM^+ < \Delta SM$ if $e_2^{L\star} >$. Since $\Delta SM^+ \leq 0$ when f is convex, it follows that $\Delta SM > 0$. ■

Proof $\mathcal{G}$.

We want to check whether Lemma 6 still holds true for the general case of substitutes and complements. This means whether it is optimal for the principal to implement a higher for the second mover in the high state of the world than in the low state. We should first note what changes for the general case compared to the special case considered in the proof of Lemma 6. As the optimal wage scheme does not change, the only difference is captured by expected revenues. This implies that we just need to plug in the new expected revenue function and otherwise can stick to the procedure of the proof of Lemma 6. Expected revenues under the sequential structure are now

$$\pi[p^{HH}(e_1^\star, e_2^{H\star})\mathcal{H} + (p^{HL}(e_1^\star, e_2^{H\star}) + p^{LH}(e_1^\star, e_2^{L\star}))\mathcal{M}].$$

We can rewrite expected revenues as follows

$$\pi[p(e_1^\star)p(e_2^{H\star})(\mathcal{H} - \mathcal{M}) + (p(e_1^\star) + p(e_2^{L\star}) + p(e_1^\star)p(e_2^{L\star}))\mathcal{M}].$$

[41] Note that for this result weak convexity of f is sufficient.

Hence, the first order condition of the principals maximization problem, which we derived in the proof of Lemma 6 (see equation (A.23)), becomes

$$(1-p(e_1^\star))\left[\pi[\left(p'(e_2^{H\star})+p'(e_2^{L\star})\right)\mathcal{M}-\mathcal{H}p'(e_2^{H\star})]\right]+ \\ (1-p(e_1^\star))\left[\left(c'(e_2^{H\star})-c'(e_2^{L\star})\right)-\left(k(e_2^{L\star})-k(e_2^{H\star})\right)\right]=p(e_1^\star)k(e_2^{L\star}), \tag{A.24}$$

where $k(e)=p(e)\left[\frac{p'(e)c''(e)-c'(e)p''(e)}{p'(e)^2}\right]$.

Similar to the proof of Lemma 6, we can again conclude that the first order condition is sufficient given Assumption $\mathcal{C}$: Since only expected revenues differ here, we just check whether it is still true that the second derivative of expected revenues with respect to $e_2^{L\star}$ subject to $e_2^\star = p(e_1^\star)e_2^{H\star}+(1-p(e_1^\star))e_2^{L\star}$ is negative. This derivative is

$$\pi(1-p(e_1^\star))[\mathcal{M}p''(e_2^{L\star})+\frac{1-p(e_1^\star)}{p(e_1^\star)}(\mathcal{H}-\mathcal{M})p''(e_2^{H\star})],$$

which is smaller than zero since p is concave and $\mathcal{H}\geq\mathcal{M}$. Hence, given Assumption $\mathcal{C}$, the first order condition is again sufficient.

Can it be optimal to implement $e_2^{L\star}=e_2^{H\star}$? If we plug in $e_2^{L\star}=e_2^{H\star}$ into equation (A.24), we have

$$\pi p(e_1^\star)p(e_2^{L\star})(2\mathcal{M}-\mathcal{H}))=p(e_1^\star)k(e_2^{L\star}).$$

Since $k(e)>0$, the right hand side is strictly positive. $2\mathcal{M}-\mathcal{H}$ is negative for complements, thus the left hand side is negative. Hence, the first order condition can never be satisfied. This means that for complements $e_2^{L\star}=e_2^{H\star}$ cannot be optimal. For substitutes, however, $2\mathcal{M}-\mathcal{H}\geq 0$. Thus, it can be optimal to set $e_2^{L\star}=e_2^{H\star}$ if $2\mathcal{M}-\mathcal{H}>0$, otherwise it cannot be optimal.

Is it optimal to implement $e_2^{L\star}<e_2^{H\star}$? Consider the first order condition (A.24) and suppose first that the principal implements $e_2^{L\star}>e_2^{H\star}$. Given $e_2^{L\star}>e_2^{H\star}$, we have

$$\left[\left(c'(e_2^{H\star})-c'(e_2^{L\star})\right)-\left(k(e_2^{L\star})-k(e_2^{H\star})\right)\right]<0$$

(compare proof of Lemma 6). Moreover, $p'(e_2^{L\star})\leq p'(e_2^{H\star})$. This implies that

$$\pi[\left(p'(e_2^{H\star})+p'(e_2^{L\star})\right)\mathcal{M}-\mathcal{H}p'(e_2^{H\star})]\leq\pi[\left(p'(e_2^{H\star})\right)(2\mathcal{M}-\mathcal{H})].$$

For complements, $\pi[\left(p'(e_2^{H\star})\right)(2\mathcal{M}-\mathcal{H})]<0$. Hence, the left hand side of (A.24) is negative for complements. Thus, for complements Lemma 6 still applies, i.e. $e_2^{L\star}<e_2^{H\star}$ is optimal given that Assumption $\mathcal{C}$ holds. Similarly, we can conclude for substitutes that $e_2^{L\star}<e_2^{H\star}$ is optimal given that Assumption $\mathcal{C}$ holds if $2\mathcal{M}-\mathcal{H}=0$. For $2\mathcal{M}-\mathcal{H}>0$, however, we cannot exclude any case as $\pi[\left(p'(e_2^{H\star})+p'(e_2^{L\star})\right)\mathcal{M}-\mathcal{H}p'(e_2^{H\star})]>0$ holds true for $e_2^{L\star}<e_2^{H\star}$ but can also hold true for $e_2^{L\star}>e_2^{H\star}$. Thus, for the general case of substitutes $e_2^{L\star}\geq e_2^{H\star}$ as well as $e_2^{L\star}<e_2^{H\star}$ can be optimal. This means Lemma 6 only applies for substitutes if $2\mathcal{M}-\mathcal{H}=0$ (which we earlier denoted by perfect substitutes).

■

Proof: Subjects Play a Pure Strategy

Before we prove that subjects play (in general) a pure strategy, we first want to define beliefs and strategies of a subject. We let μ_j be the individual's belief that alternative j is true, where

$j \in \{0, \dots J\}$, $J \in \{2, 3, 4, 8, 70\}$ with $\sum_j^J \mu_j = 1$. Next, we define a strategy of an individual. A pure strategy of an individual is an action or alternative a subject can choose, i.e. a pure strategy is $j \in \mathcal{J} = \{0, \dots J\}$. Given an individual's pure strategy set $\mathcal{J}$, an individual's mixed strategy, $\sigma : \mathcal{J} \to [0, 1]$, assigns to each pure strategy j a probability $\sigma_j \geq 0$ with which j will be played, where $\sum_j \sigma_j = 1$. Further, we denote by c the high payoff (i.e. 525, 1680, 105, 400 or 500 Tokens) and by $c - \kappa$ the low one (i.e. 20, 30, 315, 50 or 300 Tokens). We assume, without loss of generality, that $\mu_1 \geq \mu_2 \geq \dots \geq \mu_J$.

Then one can show the following:

Proposition 19 *Unless $\mu_1 = \mu_2$, an individual plays a pure strategy. More precisely, the individual sets $\sigma_1 = 1$ if $\mu_1 > \mu_j \ \forall j \neq 1$. If $\mu_1 = \dots = \mu_n > \mu_{n+1} \geq \dots \geq \mu_J$ with $J \geq n \geq 2$, then any σ with $\sigma_1 + \dots + \sigma_n = 1$ can be optimal.*

This result implies that a mixed strategy is not optimal as long as an individual that is uncertain about the right action attaches a higher probability to one possible action than to all other actions. Since all the decision problems in our experiment have this structure – subjects have the choice between $\{1, \dots, J\}$ alternatives with $J \in \{2, 4, 3, 8\}$, we can apply this proposition to all of them. If subjects make the "right" choice (the right guess for the interval, the right guess for the relative bias or the right guess for the number of correctly answered questions), they receive a high payoff, say c, and if the choice is not correct, they receive $c - \kappa$.

Proof of Proposition 19.

The subjectively expected utility of an individual from strategy σ is

$$\mu_1[\sigma_1 u(c) + \sigma_2 u(c-\kappa) + \dots + \sigma_J u(c-\kappa)] + \mu_2[\sigma_1 u(c-\kappa) + \sigma_2 u(c) + \dots + \sigma_J u(c-\kappa)] \\ + \dots + \mu_J[\sigma_1 u(c-\kappa) + \sigma_2 u(c-\kappa) \dots \sigma_J u(c)].$$

Rearranging yields

$$u(c)\sum_j \mu_j\sigma_j + u(c-\kappa)\underbrace{[\sigma_1(\sum_{j\neq 1}\mu_j) + \sigma_2(\sum_{j\neq 2}\mu_j) + \dots + \sigma_J(\sum_{j\neq J}\mu_j)]}_{=\sum_j \sigma_j(\sum_{i\neq j}\mu_i)}. \tag{A.25}$$

Suppose now that subjects never put the same probability on alternatives. Without loss of generality $\mu_1 > \mu_2 > \dots > \mu_J$. The expected utility under a strategy that sets $\sigma_1 = 1$ would be

$$u(c)\mu_1 + u(c-\kappa)\sum_{j\neq 1}\mu_j. \tag{A.26}$$

Compare this to a strategy σ' that puts some positive weight on other alternatives (i.e. $\sigma_1' < 1$). This means, we subtract (A.25) from (A.26), where we, however, replace all σ_j by σ_j' in the latter. This yields

$$u(c)\underbrace{[(1-\sigma_1')\mu_1 - \sum_{j\neq 1}\mu_j\sigma_j']}_{(A)} + u(c-\kappa)\underbrace{[\sum_{j\neq 1}\mu_j - \sum_j \sigma_j'(\sum_{i\neq j}\mu_i)]}_{(B)}$$

As long as this difference is positive, the strategy that sets $\sigma_1 = 1$ is optimal. Consider term (A) using that $\sigma'_2 = 1 - \sum_{j\neq 2} \sigma'_j$:

$$(1-\sigma'_1)\mu_1 - \sum_{j\neq 1} \mu_j \sigma'_j = (1-\sigma'_1)(\mu_1 - \mu_2) + \mu_2(\sum_{j>2} \sigma'_j) - \sum_{j>2} \mu_j \sigma'_j = (1-\sigma'_1)(\mu_1-\mu_2) + \sum_{j>2}(\mu_2 - \mu_j)\sigma'_j.$$

This is strictly larger than zero since $\mu_1 > \mu_2 > \dots > \mu_J$. The smallest value it can take is zero if and only if $\mu_1 = \mu_2 = \dots = \mu_J$. Consider now term (B):

$$\sum_{j\neq 1} \mu_j - \sum_j \sigma'_j(\sum_{i\neq j} \mu_i) = (1-\sigma'_1)\sum_{j\neq 1} \mu_j - \sum_{j\neq 1} \sigma'_j \underbrace{(\sum_{i\neq j} \mu_i)}_{1-\mu_j} = (1-\sigma'_1)(\mu_2 - \mu_1) + \sum_{j>2} \sigma'_j(\mu_j - \mu_2).$$

This term is (strictly) negative (the term equals zero if $\mu_1 = \mu_2 = \dots = \mu_J$), but the absolute value is the same for the term (A) and (B). Since the first is weighted by $u(c) > u(c-\kappa)$, subjectively expected utility from the strategy setting $\sigma_1 = 1$ is larger than from σ' and hence, this is the optimal strategy.

It is easy to see that this result also holds true for $\mu_1 > \mu_2 \geq \dots \geq \mu_J$, since $\sigma'_1 < 1$. If, however, $\mu_1 = \dots = \mu_n > \mu_{n+1} \geq \dots \geq \mu_J$ with $J \geq n \geq 2$, then any σ with $\sigma_1 + \dots + \sigma_n = 1$ can be optimal. To see this, note that term (A) simplifies to

$$\sum_{j>n}(\mu_2 - \mu_j)\sigma'_j$$

and (B) to

$$\sum_{j>n} \sigma'_j(\mu_j - \mu_2)$$

as $\mu_1 = \mu_n$. Consider a strategy σ' that sets $\sigma'_j = 0$ for all $j > n$ (i.e. all j for which $\mu_2 - \mu_j > 0$) and $\sum_{j\leq n} \sigma'_j = 1$. Then term (A) and term (B) would be both equal to zero under this strategy σ'. Hence, the strategy setting $\sigma_1 = 1$ yields the same expected payoff than σ'. Thus, any strategy that sets $\sigma_1 + \dots + \sigma_n = 1$ can be optimal. ■

Instructions (translated from German)

Instructions *R Hard* – Part 1

In this scientific experiment you can earn money with your decisions. During the experiment your payoffs are given in tokens.
After the experiment this amount of tokens will be converted into euros according to the exchange rate of **1 euro for 210 tokens** and paid cash to you.

Course of the Experiment:

The experiment consists of two stages. In stage 1 you answer 7 multiple-choice questions. In stage 2 you make a decision. **The payoff for this decision depends among other things on the number of multiple-choice questions you answered correctly**. You get the instructions for stage 2 after having answered the 7 questions.

Stage 1:

- **7 multiple-choice questions** are posed. For each question you get **4 possible answers** to choose from. At a time, only **one** of these possible answers is correct.
  You choose your answer to a question by clicking on the circle in front of the corresponding answer and then clicking “OK”. As soon as you click OK, you cannot change your answer any more and the next question appears.
- You have at most 45 seconds to give your answer to each of the questions. During these 45 seconds you can give your answer at any time. The time that is left for a question is shown on the screen. When time has run out, the computer automatically shows the following question.
- **Please note:** If you do **not** click on one answer or **not** click OK before the time has run out, this means the same as if you give a wrong answer.
- Once you have answered all questions, the computer determines how many questions you have answered correctly. **You receive the information how many correct answers you have after the experiment, i.e. after stage 2.**

Payoff for stage 1:

For each **correct** answer you receive **190 tokens** and for each **wrong one you receive 10 tokens**.

Instructions *R Hard* – Part 2

Stage 2:

In stage 2 you choose one out of eight possible **actions 0, 1, 2, 3, 4, 5, 6, 7**. This is done by entering one of these numbers in the corresponding cell on the computer screen and you confirm your choice by clicking on "OK".

Payoff stage 2:

The following table shows the payoffs in tokens, which you receive depending on you choice and how many questions you answered correctly in stage 1. **You are not told until after the experiment how many questions you answered correctly.**

		Number of correct questions							
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Action	**Action 0**	525	30	30	30	30	30	30	30
	Action 1	30	525	30	30	30	30	30	30
	Action 2	30	30	525	30	30	30	30	30
	Action 3	30	30	30	525	30	30	30	30
	Action 4	30	30	30	30	525	30	30	30
	Action 5	30	30	30	30	30	525	30	30
	Action 6	30	30	30	30	30	30	525	30
	Action 7	30	30	30	30	30	30	30	525

Calculation of your total payoff:

- Your total payoff from the experiment is given by the **number of all your correctly answered questions** multiplied by 190 tokens and the **number of wrong answers** multiplied by 10 tokens (your payoff in stage 1) and the payoff from your chosen action (your payoff in stage 2). In addition you receive a payment of 525 tokens.
- This total payoff is converted into euros according to the exchange rate 1 Euro = 210 tokens.

Instructions *R Tricky*

The instructions for *R Tricky* are identical to *R Hard*. The difference is that subjects answer the tricky instead of the hard questions.

Hard Questions:

When did the Holy Roman Empire of the German Nation stop existing?

- 1618
- 1918
- 1815
- 1806 (+)

Which frequency has home power in middle Europe?

- 220 volt
- 110 volt
- 60 hertz
- 50 hertz (+)

Who wrote „Iphigenie auf Aulis“?

- Goethe
- Euripides (+)
- Schiller
- Sophokles

How many symphonies wrote Joseph Haydn?

- 104 (+)
- 41
- 21
- 9

Which is no chemical element?

- selenium
- calcium
- arsenic
- americium

How do you call the dark spots of the moon?

- Mare (+)
- Mire
- Mure
- More

Which boxers fought against each other at „Rumble in the Jungle“?

- Joe Frazier and George Foreman
- George Foreman and Muhammed Ali (+)
- Evander Holyfield and Mike Tyson
- Muhammed Ali and Joe Frazier

Tricky questions:

The most expensive picture, which was bought at a German auction is from:

- Gerhardt Richter
- Ottmar Alt
- Pablo Picasso
- Max Beckmann (+)

Which metropolis region has the most inhabitants?

- Ranstad (Netherlands) (+)
- Johannesburg (South Africa)
- Dallas (USA)
- Zurich (Switzerland)

Which of these mountains is the highest?

- Olymp
- Sinai
- Zugspitze
- Etna (+)

Which sportsman earns the most money (sum of prize money, sponsoring, promotion, fan articles etc.)

- Michael Schumacher
- Tiger Woods (+)
- Davis Beckham
- Lance Armstrong

Which mature animal (male) weighs most on average?

- tiger
- domestic pig
- polar bear
- giraffe (+)

Who has had his title for the longest period?

- Helmut Kohl: chancellor
- Johannes Paul II: pope
- Bill Gates: Microsoft founder
- Franz Beckenbauer: „emperor“ (+)

Which food has the most kilocalories per 100g?

- crispbread (+)
- apple
- camembert with 45% fat
- cured eel

Instructions *T Average*

Course of the experiment:

You make a decision between three actions and and a decision about a number. In order to make this decisions, you receive some information on another experiment *(Experiment I)*, which has been conducted a week before.

Description of *Experiment I*

Experiment I had **20 participants**. The experiment consisted of two stages.

Stage 1

- In the first stage, the participants answered 7 multiple-choice questions. For each of the questions there have been 4 possible answers. At a time, only **one** of these possible answers was correct. For each of the questions, the participants had at most 45 seconds to give their answer. When time had run out, the computer automatically showed the next question. In case no answer had been clicked on during this time, this was equivalent to giving a wrong answer.
- The questions are attached to these instructions and you can look at them later on.

Stage 2

- In the second stage the participants have chosen one out of eight possible **actions 0, 1, 2, 3, 4, 5, 6, 7**.
- The payoffs (in tokens) for every possible combination of the "number of correctly answered questions" and the "chosen action" have been determined according to the payoff table below. The participants of *Experiment I* had this table in stage 2 in order to make their decision.

		Number of correct questions							
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Action	**Action 0**	525	30	30	30	30	30	30	30
	Action 1	30	525	30	30	30	30	30	30
	Action 2	30	30	525	30	30	30	30	30
	Action 3	30	30	30	525	30	30	30	30
	Action 4	30	30	30	30	525	30	30	30
	Action 5	30	30	30	30	30	525	30	30
	Action 6	30	30	30	30	30	30	525	30
	Action 7	30	30	30	30	30	30	30	525

<u>Further relevant information</u>

- The participants knew in stage 1 (when answering the 7 questions) that they make a decision in stage 2 and that the payoff depends on the number of correctly answered questions. The payoff table and detailed instructions for stage 2 have not been handed to the participants until the beginning of stage 2.
- The number of correct questions has been determined for each participant by the computer. At the end of the experiment, the participants received 190 tokens for each correct answer and 10 tokens for each wrong one. **Each participant has not been told the number of correctly answered questions and the payoff until he/she has chosen his/her action in stage 2.**
- At the end of the experiment, the payoff of the participants from answering the questions and from their decision, as well as an additional payment of 525 tokens has been converted into euros according to the exchange rate **210 tokens = 1 euro** and paid cash to the participants.

Description of today's Experiment

<u>Relevant results from *Experiment I*:</u>

Based on the answers and the decisions of the participants of *Experiment I*, two averages have been calculated **after** the experiment:

1. The **average number of correct answers "R"** of all participants:

   The average is calculated as follows: the number of correct answers of all participants is added and then divided by the number of participants (20). The resulting value is rounded on one **decimal place**. Thus, the average can take values from 0 to 7 in steps of 0.1.

2. The **average action "A"** chosen by the participants:

   The average is calculated as follows: each participant chooses an action whereat the actions are assigned numbers from 0 to 7 (see table). The **numbers** of the chosen action of each participant are added and then divided by the number of participants (20). The resulting value is rounded on one **decimal place**. Thus, the average action can also take values from 0 to 7 in steps of 0.1.

<u>Your decision:</u>

Before you make your decision, you are **told the value of the average action** (**A**) chosen by the participants of *Experiment I*.

- You choose between three actions: **action 1**, **action 2 und action 3**. You select action 1, 2 or 3 by clicking on the corresponding action on the computer screen. In the following the actions are explained more detailed.
- After you have chosen one of the actions, you choose a number as described in the following:
  - If you have chosen **action 1**, you can choose a **number which is larger than or equal to A - 0.5 and smaller than A + 0.5**.
  - If you have chosen **action 2**, you can choose a **number which is larger than A + 0.5 and smaller than or equal to 7.**

- If you have chosen **action 3**, you can choose a **number which is larger than or equal to 0 and smaller than A - 0.5**.

A= average action of the participants of Experiment I

- You can give the number in **steps of 0.1**. You chose a number by entering the number you want to choose in the corresponding cell on the screen.
- When you have made all decisions, please confirm your choice by clicking on “OK”.

Your payoff consists of the following two components:

<u>Payoff component 1:</u>

		Actions		
		Action 1	**Action 2**	**Action 3**
Value of the Average number of correct questions (R)	**R is smaller than *A-0.5***	315	315	1680
	R is larger/equal *A - 0.5 and* smaller/equal *A+0.5*	1680	315	315
	R is larger than *A+0.5*	315	1680	315

A= Value of the average action of the participants of *Experiment I*

R= Value of the average number of correct questions in *Experiment I*

<u>Payoff component 2:</u>

If the distance (explanation see below) between the number you have chosen and the average number of correct questions **R** is smaller than or equal to 0.5 and your payoff from component 1 is *1680* tokens, then you receive in addition *105* tokens, if the distance is larger than 0.5 or your payoff from component 1 is *315* tokens, you receive *20* tokens.

<u>Explanation „Distance“:</u>

Consider two numbers X and Y. The distance between these two numbers is

X-Y if X is larger than Y and Y-X if X is smaller than Y.

<u>Total payoff:</u>

Your total payoff ist he sum of your payoffs from component 1 and 2 and an additional payment of 625 tokens.

Instructions *T AveragePlus* – Part 1

Course of the experiment:

The experiment consists of 4 stages. In stage 1 you answer 7 multiple-choice questions. In stage 2 you make a decision. **The payoff for this decision depends among other things on the number of multiple-choice questions you answered correctly**. After stage 2 you receive some information on another experiment (*Experiment I*). In *Experiment I* stage 1 and 2 have been played as well. Having received this information, you make a decision between two alternatives in stage 3. The payoff you get from the choice of an alternative depends on *Experiment I* and your decision in stage 2. In stage 4 you make a decision between three alternatives, whereat your payoff from the choice of an alternative depends on *Experiment I*. You get the instructions for stage 2, 3 and 4 after having answered the 7 questions.

Stage 1:

Exactly like in *R Hard.*

Instructions *T Average+* – Part 2

Stage 2:

Exactly like in *R Hard.*

Relevant Information on *Experiment I:*

In ***Experiment I*** there have been **20 participants**. The experiment consisted of exactly the same 2 stages as just described: Answering 7 multiple-choice questions in stage 1 and choice between actions 0, 1, 2, 3, 4, 5, 6, 7 in stage 2.

At the end of *Experiment I*, payoffs of the participants from answering the questions and from the decisions as well as an additional payment of 525 tokens have been converted into euros according to the exchange rate **1 Euro per 210** tokens and paid cash to the participants.

Based on the answers and the decisions of the participants of *Experiment I*, two averages have been calculated **after** the experiment:

The **average number of correct answers "R"** of all participants:

The average is calculated as follows: the number of correct answers of all participants is added and then divided by the number of participants (20). The resulting value is rounded on one **decimal place**. Thus, the average can take values from 0 to 7 in steps of 0.1.

The **average action “A” chosen** by the participants:

The average is calculated as follows: each participant chooses an action whereat the actions are assigned numbers from 0 to 7 (see table). The **numbers** of the action of each participant are added and then divided by the number of participants (20). The resulting value is rounded on one **decimal place**. Thus, the average action can take values from 0 to 7 in steps of 0.1.

Before you make your decision in stage 3 and 4, you are **told the value of the average action** (**A**) chosen by the participants of *Experiment I*.

Stage 3:

Decision stage 3:

In stage 3 you can choose between the following two alternatives. The choice is done by clicking on the alternative on the screen and confirming the choice with “OK”.

Payoff stage 3:

Your payoff in stage 3 depending on the distance between **R** and **A**, your payoff in stage 2 (which you are not told until the end of the experiment) and your choice between the two alternatives is:

	Alternative 1	**Alternative 2**
Your payoff in stage 2 is 525 and the distance between R and A is smaller than or equal to 0.5	800	800
Your payoff in stage 2 is 525 and the distance between R and A is larger than 0.5	500	300
Your payoff in stage 2 is 30 and the distance between R and A is smaller than or equal to 0.5	300	500
Your payoff in stage 2 is 30 and the distance between R and A is larger than 0.5	210	210

Explanation „Distance“:

Consider the two numbers R and A. The distance between these two numbers is

R-A if R is larger than A and A-R if R is smaller than A.

Stage 4:

Decision stage 4:

You choose between three alternatives: **Left**, **Middle and Right** by clicking on the corresponding alternative on the computer screen. Please confirm your choice by clicking on “OK”.

Payoff stage 4:

		Alternatives		
		Middle	**Right**	**Left**
Value of the Average number of correct questions (R)	**R is smaller than *A-0.5***	315	315	1680
	R is larger/equal *A - 0.5 and* smaller/equal *A+0.5*	1680	315	315
	R is larger than *A+0.5*	315	1680	315

A= Value of the average action of the participants of *Experiment I*

R= Value of the average number of correct questions in *Experiment I*

Calulation of your total payoff:

Your total payoff in the experiment is given by the sum of:

- The number of all your correctly answered questions multiplied by 190 tokens and the number of wrong answers multiplied by 10 tokens (your payoff in stage 1).
- Your payoff in stage 2.
- Your payoff in stage 3.
- Your payoff in stage 4.
- In addition you receive a payment of 725 tokens.

This total payoff is converted into euros according to the exchange rate 1 Euro = 210 tokens.

Instructions *T Frame* – Part I

Course of the experiment:

The experiment consists of two parts: In part I you answer **two blocks of questions A and B** each with 7 multiple-choice questions. In part II you make 8 decisions. The first four decisions **(1A-4A)** refer to question block **A**, the next four decisions **(1B-4B) to** question block **B**.

The payoff from decision 1A (1B) depends among other things on your number of correctly answered multiple-choice questions in block A (B). Afterwards you receive some information on another experiment (*Experiment I, II resp.*). In *Experiment I (II)* question block A (B) have been answered and decision 1A (1B) have been made, too. Having received the information, you make decision 2A (2B). The payoff for decision 2A (2B) depends on *Experiment I (II)* and on your decision 1A (1B). Subsequently, you make decision 3A (3B) and 4A (4B), whereat your payoffs depend on *Experiment I (II)*.

Stage 1:

Exactly like in *R Hard* except that subjects answer two different blocks of 7 multiple-choice questions (the hard and the tricky questions). Subjects are paid like in *R Hard* but only for one block of questions that is randomly selected.

Instructions *T Frame* – Part II

Decision 1A:

Decision 1A:

You state how many of the 7 questions in question block A you think you have answered correctly. For this, you enter a whole number between 0 and 7 in the corresponding cell and then click on „OK“.

Payoff decision 1A:

If your statement coincides with the actual number of correctly answered questions in block A („Your estimation is correct“), you receive 525 tokens, i fit does not coincide (“Your estimation is not correct”), you receive 30 tokens.

Relevant information on *Experiment I and II:*

In ***Experiment I and II*** respectively there have been **20 participants**. These experiments consisted of answering the questions of block A in *Experiment I* and block B in *Experiment II* and each time a statement, how many questions have been answered correctly. For this statement, the participants have chosen between eight actions 0, 1, 2, 3, 4, 5, 6, 7. In case the actual number of correctly answered questions coincides with the number of the action, a participant received 525 tokens, if there was no coincidence he/she received 30 tokens.

At the end of *Experiment I (II)* payoffs of the participants from answering the questions and from the decisions as well as an additional payment of 525 tokens have been converted into euros according to the exchange rate **1 Euro per 210** tokens and paid cash to the participants.

Based on the answers and the decisions of the participants of *Experiment I* and *II* respectively, two averages for each experiment have been calculated **after** the experiment:

The **average number of correct answers "R"** of all participants:

The average is calculated as follows: the number of correct answers of all participants is added and then divided by the number of participants (20).

The **average estimation "E"** of the participants:

The average is calculated as follows: the chosen statements about the number of correctly answered questions of each participant are added and then divided by the number of participants (20).

Both averages **E** and **R** of *Experiment I* and *II* are rounded on **one** decimal place. Thus, the averages can take values from 0 up to 7 in steps of 0.1.

Decisions 2A, 3A and 4A:

Before you make decisions 2A, 3A and 4A, you are told the **value of the average estimation (E)** of the participants of *Experiment I*.

Decision 2A:

You decide how good your estimation of the number of correct questions is and how good the average estimation (E) of the participants of *Experiment I* is. There are four alternatives:

- **"Both estimations are good**": your estimation is correct (see above) and the distance (explanation see below) between the average estimation (E) and the average number of correct questions (R) in *Experiment I* is smaller than or equal to 0.5.
- **"My own estimation is better"**: your estimation is correct and the distance between E and R in *Experiment I* is larger than 0.5.
- **„Average estimation is better"**: your estimation is not correct and the distance between E and R in *Experiment I* is smaller than or equal to 0.5.
- **"Both estimations are bad"**: your estimation is not correct and the distance between E and R in *Experiment I* is larger than 0.5.

Payoff decision 2A:

If you select the alternative that is actually true, you receive 400 tokens, otherwise you receive 50 tokens.

Explanation „distance":

Consider the two numbers R and E. The distance between these two numbers is R-E if R is larger than E and is E-R if R iss maller than E.

Decision 3A:

You state how well you think the participants in *Experiment I* assess themselves:

- The participants **overestimate** their actual number of correctly answered questions on average. This means that the average number of correct (R) in *Experiment I* is by more than **0.5 smaller** than the average estimation (E).
- The participants estimate their actual number of correctly answered questions on average almost **correct**. This means that the average number of correct (R) in *Experiment I* is larger than or equal to **E-0.5** and smaller than or equal to **E+0.5**.

- The participants **underestimate** their actual number of correctly answered questions on average. This means that the average number of correct (R) in *Experiment I* is by more than **0.5 larger** than the average estimation (E).

You choose between the three alternatives **(overestimate, correct, underestimate)** by clicking on the corresponding alternative and confirming with OK.

<u>Payoff decision 3A:</u>

When the alternative you have chosen is actually true, then you receive **1680 tokens**, when it is **not** true, you receive **315 tokens**.

<u>Decision 4A:</u>

You state, what you think **how large the average number of correctly answered questions (R)** of the participants in *Experiment I* is. This is done by entering a number between 0 and 7 in steps of 0.1 in the corresponding cell.

Take notice of the following conditions:

- If you have chosen "**correct**" in decision 3A, you can choose a **Number** that is **larger than or equal to E - 0.5 and smaller than or equal to E + 0.5**.
- If you have chosen **"underestimate"** in decision 3A, you can choose a **Number** that is **larger than E + 0.5 and smaller than or equal to 7.**
- If you have chosen **"overestimate"** in decision 3A, you can choose a **Number** that is **larger than or equal to 0 and smaller than E - 0.5**.

<u>Payoff decision 4A:</u>

If the distance between the number you have chosen and the average number of correct questions **(R)** is smaller than or equal to 0.5 and you selected in decision 3A the alternative that is actually true, then receive **105 tokens**, otherwise you receive **20 tokens**.

Decision 1B-4B

After decisions 1A-4A decisions 1B-4B regarding block B follow.

Here, the following decisions are equivalent 1A-1B, 2A-2B, 3A-3B, 4A-4B besides that they refer now to **block B** and ***Experiment II***.

After decision 2B you are told the correct answers to the questions of block B. Afterwards you make decision 3B and 4B.

Calculation of your total payoff:

Your total payoff from the experiment is the sum of:

- The number of your correctly answered questions in the block of questions randomly selected by the computer multiplied 190 tokens and the number of wrong answers in this block multiplied by 10 tokens.
- Your payoff from decisions 1A-4A or 1B-4B: For the payment the computer again randomly selects whether decisions 1A-4A or 1B-4B are paid.
- In addition you receive a payment of 420 tokens.

This total payoff is converted into euros according to the exchange rate 1 Euro = 210 tokens.

Instructions *T Individual* – Part 1

Course of the Experiment:

The experiment consists of 4 stages. In stage 1 you answer 7 multiple-choice questions. In stage 2 you make a decision. **The payoff for this decision depends among other things on the number of multiple-choice questions you answered correctly**. After stage 2 you receive some information on another experiment (*Experiment I*). In *Experiment I* stage 1 and 2 have been played as well. Having received this information, you make eight times a decision between two alternatives in stage 3. The payoff you get from the choice of an alternative depends on *Experiment I* and your decisions in stage 2. In stage 4 you make eight times a decision between three alternatives, whereat your payoff from the choice of an alternative depends on *Experiment I*. You get the instructions for stage 2, 3 and 4 after having answered the 7 questions.

Stage 1:

Exactly like stage 1 in *R Hard.*

Instructions *T Individual* – Part 2

Stage 2:

Decision stage 2:

In stage 2 you choose one out of eight possible **actions 0, 1, 2, 3, 4, 5, 6, 7**. This is done by entering one of these numbers in the corresponding cell on the computer screen and you confirm your choice by clicking on "OK".

Payoff stage 2:

The following table shows the payoffs in tokens, which you receive depending on you choice and how many questions you answered correctly in stage 1. **You are not told until after the experiment how many questions you answered correctly.**

		Number of correct questions							
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Action	**Action 0**	525	30	30	30	30	30	30	30
	Action 1	30	525	30	30	30	30	30	30
	Action 2	30	30	525	30	30	30	30	30
	Action 3	30	30	30	525	30	30	30	30
	Action 4	30	30	30	30	525	30	30	30
	Action 5	30	30	30	30	30	525	30	30
	Action 6	30	30	30	30	30	30	525	30
	Action 7	30	30	30	30	30	30	30	525

Relevant Information on *Experiment I:*

In ***Experiment I*** there have been **20 participants**. The experiment consisted of exactly the same 2 stages as just described: Answering 7 multiple-choice questions in stage 1 and choice between actions 0, 1, 2, 3, 4, 5, 6, 7 in stage 2.

At the end of *Experiment I*, payoffs of the participants from answering the questions and from the decisions as well as an additional payment of 525 tokens have been converted into euros according to the exchange rate **1 Euro per 210** tokens and paid cash to the participants.

Based on the answers and the decisions of the participants of *Experiment I*, two values have been identified after the experiment:

1. The **number of correct answers "R"** of a participant.
2. The **action "A" chosen** by a participant. The value A of an action is a number between 0 and 7 next to an action (see table).

You are randomly assigned to one participant of *Experiment I*. When you make your decisions, you do not know which participant it is. Therefore, you make your decisions in stage 3 and 4 for all possible values of A, i.e. 0,1,2,3,4,5,6,7. For none of these values of **A** you get to know the value of **R**.

Stage 3:

Decision stage 3:

In stage 3 you choose for every possible **A** (0,1,2,3,4,5,6,7) between two alternatives – i.e. you make **eight** times a decision between the two alternatives. The choice is done by clicking on the alternative on the screen and confirming the choice with "OK" when you finished **all eight** decisions.

Payoff stage 3:

The following table shows your payoff in stage 3 depending on the values of R **and** A of the participant of ***Experiment I*** that is assigned to you**, your payoff in stage 2 (which you are not told until the end of the experiment) and your choice between the two alternatives:**

	Alternative 1	**Alternative 2**
Your payoff in stage 2 is 525 and R equals A	800	800
Your payoff in stage 2 is 525 and R is larger or smaller than A but not equal to A.	500	300
Your payoff in stage 2 is 30 and R equals A.	300	500
Your payoff in stage 2 is 30 and R is larger or smaller than A but not equal to A.	210	210

Stage 4:

Your decision:

- You make for each possible **A** (0,1,2,3,4,5,6,7) a decision between three alternatives – i.e. you make eight times a decision between three alternatives: **Left**, **Middle and Right** by clicking on the corresponding alternative on the computer screen. Please confirm your choice by clicking on "OK".

- Then, you choose a number for each possible **A** (i.e. you choose eight times a number):
  - When you chose **Left**, you choose a whole number between **0** and **A-1**
  - When you chose **Right**, you choose a whole number between **A+1** and **7**
  - When you chose **Middle,** you choose exactly the number **A**

  A **table** that shows you all possible numbers you can choose in stage 4 for each possible choice of alternatives in stage 4 and all values of A is attached to the instructions.
- After you have made **all sixteen** decisions, please confirm your choice by clicking on "OK".

<u>Payoff stage 4:</u>

1. You receive 105 tokens, when the number you have chosen coincides with the value **R** of the participant that is assigned to you. If there is no coincidence, you receive 20 tokens.

2. Based on your decision and the values R and A **of the participant that is assigned to you**, you receive the following payoff:

		Alternatives		
		Middle	**Right**	**Left**
Number of correct questions (R) of the selected participant	**R is smaller than *A***	315	315	1680
	R equal to *A*	1680	315	315
	R is larger than A	315	1680	315

A= Action chosen by a participant of *Experiment I*

R= Number of correct questions of a participant in *Experiment I*

Calulation of your total payoff:

Your total payoff in the experiment is given by the sum of:

- The number of all your correctly answered questions multiplied by 190 tokens and the number of wrong answers multiplied by 10 tokens (your payoff in stage 1).
- Your payoff in stage 2.
- Your payoff in stage 3.
- Your payoff in stage 4.
- In addition you receive a payment of 400 tokens.

This total payoff is converted into euros according to the exchange rate 1 Euro = 210 tokens.

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