Three Essays on Markets with Constrained Price Mechanisms

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Introduction

In the current times of turmoil, we see that it is not always supply and demand that determine the price. Often, the price mechanism of the market is constrained. Sometimes, the constraints are intentional: At the time of writing, European governments fiercely debate setting price caps for energy and the German minimum wage is about to jump by almost 15%. Sometimes, the constraints stem from the individual behavior of market participants: Although they sold out within minutes after a delivery, supermarkets did not increase the price of toilet paper during the pandemic, and because the sellers of gas in Germany had agreed on fixed prices before the war in Ukraine, the German government considered a new tax on gas to help those sellers increase their prices.

My dissertation consists of three independent research projects that deal with different aspects of markets in which the price mechanism is constrained.

The first chapter investigates price caps. Price caps are a tool to fight market power. A monopolist restricts its supply to drive up the market price. This restriction of the supply is inefficient because the monopolist would be able to produce more goods at a cost that consumers would be willing to pay. If a regulator steps in and caps the price, it is not the market anymore that determines the price. Because the monopolist cannot ramp up the prices, it supplies more. The price cap improves the efficiency of the market.

While the case of monopoly regulation is well researched and, in fact, part of every introductory micro course, the case of oligopoly regulation is not: Especially the situation in which duopolists get different price caps has not been researched before. Yet, this situation is relevant. Governments wanting to set price caps face legislative burdens. For example, the German state may only intervene if a firm dominates the market, and even then, the state may only put a price cap on the dominant firm, which has happened to the former state monopoly for postal services. Furthermore, even if the price caps are seemingly symmetric, they affect the firms asymmetrically if one firm offers a better product: This firm's effective price cap is tighter.

Thus, **Chapter 1 "Cournot Competition with Asymmetric Price Caps"** deals with the question: Do price caps still help against market power even if we can regulate only one firm? To answer this question, I add asymmetric price caps to the well-known Cournot quantity competition duopoly model. "Asymmetric" includes both different price caps and the special case in which only one firm has a price cap.

I find that the asymmetry of the price caps distorts the production incentives. In the relevant range of price caps, the firm with the tighter price cap produces more. So, if the marginal costs are identical and increasing, the firm produces at a higher cost. Production gets inefficient. That is, asymmetric price caps have the downside of reducing production efficiency.

Nevertheless, there always exist asymmetric price caps that are better than no price caps. So, the take-away for the regulator is that price cap regulation can still be beneficial if it is only possible to regulate a dominant firm (even though the regulation further increases the dominant firm's market share), but that the asymmetry has to be taken into account when setting the price cap.

The second chapter deals with the firms' reaction to price regulation. In the absence of market power, the state might regulate prices to redistribute wealth.[1] A common example for this use of price caps are minimum wages. Because the firms employing minimum wage workers lose from the redistribution, they have an incentive to devise methods undermining the regulator's intent.

Fabian Schmitz and I explore in **Chapter 2 "Do Non-Compete Clauses Undermine Minimum Wages?"** why adding non-compete clauses to employment contracts might be such a method. Non-compete clauses forbid an employee to be employed at or to found a competitor to the current employer for some time after the employment ended.

How can firms prevent the redistribution through minimum wages? A minimum wage above the market-clearing price for labor implies that the employee gets an economic rent. That is, the employee would be willing to pay the firm for the opportunity to work a minimum wage job—and the firm wants the employee to pay. This payment could be in money, meaning kickbacks, but the minimum wage law forbids that. So, to make the redistribution fail, the firms need a substitute for letting their employees pay with money. We investigate whether non-compete clauses allow the employees to pay with effort instead.

The mechanism that lets effort substitute for money builds on the classic moral hazard problem: The firm wants the employee to exert effort, but cannot directly

1. Using price caps to redistribute is controversial because it leads to efficiency losses. A price cap without fighting market power discourages production, but encourages consumption, so there is overdemand and undersupply. In contrast to the case of market power, the price cap makes the traded quantity inefficiently small. Moreover, the market's price mechanism allocates goods to those with the highest willingness to pay; a property that is lost when the price mechanism gets constrained. Further, excess demand encourages using less efficient methods to allocate the goods than transferring money: Time spent waiting in a queue is ultimately lost.

Bulow and Klemperer (2012) show that the negative effects might outweigh the redistributive effect even in "normal" situations. Thus, the orthodox stance on redistribution is that the state should not meddle with prices, but use transfers.

contract on this. Instead, the firm has to rely on rewarding signals for high effort ("successes") and on punishing signals for low effort ("failures").

A non-compete clause can be used to transfer economic rents to the firm, undermining redistribution, because it offers an opportunity to punish failures: If an employee has a non-compete clause, being laid off is worse. To avoid being laid off, the employee exerts more effort. The additional effort that the employee exerts due to the non-compete clause benefits the firm, while the labor cost is borne by the employee.

The effort incentives from non-compete clauses, however, have an advantage, too. Minimum wages lead to inefficiently little effort because they make rewarding successes relatively more expensive. A non-compete clause counteracts this inefficiency by inducing more effort. The efficiency gain from more effort might outweigh the efficiency loss from restricted possibilities after a lay-off. This additional efficiency compared to minimum wages alone can also benefit the employee: Putting a suitable bound on non-compete clauses keeps redistribution possible by limiting the effort incentives and, thus, how much rent the firm can extract from the employee.

Our research contributes to the American debate about whether non-compete clauses should be banned for low wage workers. We explain why these low wage workers have non-compete clauses in the first place and argue that non-compete clauses undermine the redistribution using minimum wages. Nevertheless, the best option is not necessarily to ban non-compete clauses, as they might mitigate problems resulting from the minimum wages if they are bounded accordingly. The takeaway is that effort incentives from non-compete clauses and other substitutes for monetary transfers interact with minimum wages. A regulator has to take these interactions into account and to adjust legislation accordingly to make minimum wages effective.

The last chapter is concerned with price stickiness. Even in unregulated markets, the prices sometimes behave as if the market's price mechanism was constrained: Changes in the environment might fail to change prices. An important reason for this price stickiness are fairness considerations. To avoid alienating their customers, firms might keep their prices constant when, for example, the demand for toilet paper suddenly increases. Thus, the firms run out of stock.

To answer the question what kind of changes in the environment lead to price changes, Maximilian Weiß and I conducted a survey, whose results are in **Chapter 3 "Surveying Price Stickiness and Fair Price Increases."** We surveyed German hairdressers because they had suffered several shocks shortly before due to the second lockdown during the pandemic. Following Blinder, Canetti, Lebow, and Rudd (1998), we confronted the hairdressers with hypotheses verbalizing economic theories about their sticky prices and asked them to grade these hypotheses.

Our results reflect that it is important for hairdressers that their regular customers perceive their price increases as fair. What is considered fair is consistent with the existing research. Kahneman, Knetsch, and Thaler (1986) and follow-up studies survey the society's fairness notions concerning price increases and find that passing on cost increases is generally considered fair, whereas increasing the price due to an increased demand is not.

A novel result is suggestive evidence for how trust affects the customers' fairness perceptions. As the customers do not observe cost shocks directly, they can only find price increases fair if they believe the hairdresser that, for example, the cost has increased and is being passed on. Having built a trusting relationship with the regular customers and being transparent about the pricing seems to help the hairdressers convince their customers. Thus, it gets easier for the hairdressers to pass on cost increases.

Our results explain why keeping the inflation down is difficult for the Western states and central banks at the time of writing. The problem is that the inflation is driven by (energy) cost increases, which it is considered fair to pass on. Thus, the firms that have a good relationship with their customers will eventually increase their prices. Here, it could help to increase the interest rates to slow down the economy. On the other hand, firms that do not have a good relationship with their customers will struggle to increase their prices, which leads to the risk of default; especially if the economy is slowed down. To keep the inflation down, the decision makers have to balance these opposite forces.

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Chapter 1

Cournot Competition with Asymmetric Price Caps*

1.1 Introduction

In imperfectly competitive markets, price caps are supposed to improve the welfare by increasing the traded quantity. But if the price cap regulation affects the firms in a duopoly asymmetrically, even if only marginally so, judging price caps by the total quantity alone is shortsighted: The regulation distorts the allocation of production across firms—it causes inefficient production.

Asymmetric price caps arise if a regulator can legally put a price cap on only one firm in a market; either because the state owns the firm or because the firm dominates the market. An example is the German market for postal services: Because of its dominant market position, the former German state monopoly is subject to price caps for the delivery of letters, whereas its competitors are not.[1] Another source of asymmetric price caps are symmetric price caps on differentiated goods: The real price cap is then tighter for firms with better products.[2] Lastly, asymmetric price caps might arise from freezing current prices: Some laws against price gouging re-

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1. See the German law, Postgesetz (PostG) § 19 "Genehmigungsbedürftige Entgelte."
2. See Subsection 1.4.8.

strict price increases of incumbent sellers during crises differently than the prices of entering sellers,[3] and even without such laws prices often fail to adjust upwards.[4]

This chapter is concerned with the questions: How does asymmetric price regulation distort the production in a Cournot duopoly? Can asymmetric price caps still improve the welfare?

To answer these questions, I add asymmetric price caps to the canonical Cournot quantity competition model with increasing marginal cost. The price that the firm anticipates to receive depends on its inverse residual demand and on its price cap. In the canonical model, the inverse residual demand function follows from the prices' rationing the (total) demand efficiently. Because price caps impede price rationing, I assume efficient non-price rationing. This rationing rule yields the canonical inverse residual demand function. Given an inverse residual demand function, the price cap applies: A firm gets paid the minimum of its inverse residual demand and its price cap. Intuitively, a price cap makes the inverse residual demand function flat at the price cap.

I find that with asymmetric price caps, the firms produce unequal quantities. Even an arbitrarily small asymmetry in the price caps may change the allocation of production discontinuously. Moreover, there is a trade-off between total quantity and production efficiency when only one price cap binds. Changing the binding price cap to increase the total quantity makes the production more unequal—less efficient. Thus, increasing the traded quantity is not equivalent to increasing the welfare anymore.

The good news is that asymmetric price caps that increase the welfare (compared to no price caps) generally exist: In the Cournot-Nash equilibrium without price caps, the firms' quantities are identical, so production is efficient. When one firm gets a price cap marginally below the Cournot-Nash equilibrium price, the total quantity increases, but, as the firms have the same marginal cost, the marginal distortion of production has no effect.

These results are driven by two assumptions. *Efficient rationing* means that the allocation maximizes the consumer surplus, even if the price does not clear the market. Intuitively, there might be an unregulated resale market that allocates the goods to those with the highest willingness to pay. This assumption pins down the firms' inverse residual demand functions.

Strategic substitutability is a standard regularity assumption. It means that, in the absence of price caps, if one firm increases its quantity, it is optimal for the

3. See, for example, California. Executive Order N-44-20 (Executive Department, State of California, 2020) completes the Penal Code Section 396. Because an emergency has been declared, sellers of essential goods were forbidden to increase their prices by more than 10%, unless they were only passing on additional costs. Sellers that only started selling the goods after the emergency had been declared, were not allowed to charge prices exceeding their cost by more than 50%.

4. See Nakamura and Steinsson (2011) for anecdotal evidence of firms' committing themselves to not increase their prices.

other firm to decrease its quantity. A log-concave inverse demand function is an assumption on the model primitives that guarantees strategic substitutability.

Strategic substitutability provides the basis for the trade-off between quantity and production efficiency that changing the price cap entails: Let a binding price cap be set to relieve a firm of its effect on its price, such that it is no longer optimal for the firm to withhold production and it produces more than without the price cap. If the firm's binding price cap is changed to incentivize it to expand its quantity and the other firm's price cap does not bind, the other firm's production is crowded out because of strategic substitutability. Because the firm with the binding price cap produces a larger quantity, this means that expensive production crowds out cheap production.

In a similar setting with symmetric price caps, there is a continuum of pure-strategy Nash equilibria (Okumura, 2017). In each of these equilibria, the total quantity in the market is the same, only the split between the duopolists differs. In particular, the continuum contains a symmetric equilibrium with efficient production.

My contribution to Okumura (2017) is showing that small asymmetries in the price caps can have big effects. The continuum of equilibria exists only because the price caps are symmetric: In each equilibrium of the continuum, both firms' price caps just bind. Therefore, both firms are at a discontinuous drop of their marginal profit: For any higher quantity, the price cap does not bind anymore and the firm depresses its price when increasing its quantity, so increasing the quantity causes a loss for all inframarginal units. For any lower quantity, the firm does not affect its price because the price cap strictly binds. The appearance of the inframarginal loss makes the marginal profit drop. As both firms are at the drop in their marginal profit, there is some leeway in the firms' optimality conditions, so a continuum of quantities satisfies them.

With the slightest asymmetry, however, the continuum of equilibria collapses to a single equilibrium. The reason is that asymmetric price caps cannot just bind at the same total quantity. Thus, the firms are not both at the drops in their marginal profit. The firm whose price cap is not just binding is not at the drop and has a (unique) optimal quantity; so there is no leeway, but a unique equilibrium. This equilibrium lies at one of the continuum's borders—so it has the most inefficient production.

The rest of the chapter is structured as follows. In Section 1.2, I explain the model and the price caps. In Section 1.3, I derive the unique pure-strategy Nash equilibria and their welfare implications. Subsection 1.3.5 deals with the special case of linear demand and quadratic cost, in which observable data might help to evaluate the price caps. In Section 1.4, I discuss extensions, generalizations, and applications. Extensions are the consumer surplus as an alternative objective (Subsection 1.4.1), the special case of constant marginal cost (Subsection 1.4.2), and the relationship with the sequential Stackelberg competition, which offers the novel interpretation that the Stackelberg leader commits itself to a price cap instead of a quantity (Subsection 1.4.3). Generalizations are the case in which both firms have a price cap (Subsection 1.4.4), heterogeneous cost functions (Subsection 1.4.5), mixed-strategy Nash equilibria as another solution concept (Subsection 1.4.6), and proportional rationing as another rationing rule (Subsection 1.4.7). Applications are symmetric nominal price caps with vertically differentiated goods (Subsection 1.4.8), and research strands that might pick up the model (Subsection 1.4.9). In Section 1.5, I summarize and conclude. In the Appendix are the proofs (Appendix 1.A) and the benchmark of price caps in a monopoly (Appendix 1.B).

1.2 Model

There are two firms, 1 and 2. The firms engage in quantity competition: They simultaneously choose their quantities, non-negative real numbers, to maximize their profits. The quantity that firm 1 chooses is named q_1 and the quantity that firm 2 chooses is named q_2. Both firms produce the same good and have the same cost function $c(q_i)$. The cost function is two times continuously differentiable, and the marginal cost is weakly positive, $c'(\cdot) \geq 0$, and strictly increasing, $c''(\cdot) > 0$.

The market-clearing price for each total quantity is given by the inverse demand function $p(q)$. It is two times continuously differentiable and strictly decreasing wherever it is positive, $\forall q: \ p(q) > 0 \implies p'(q) < 0$. The market exists, that is, at least one firm wants to produce, $p(0) > c'(0)$, and the market is finite, $\exists x: \ p(x) < c'(x)$.

Another standard assumption is that the firms' quantities are strategic substitutes. An assumption on the inverse demand function that is sufficient for the quantities to be strategic substitutes is log-concavity (Amir, 1996). Furthermore, log-concavity is almost necessary for the existence of a pure-strategy Nash equilibrium in Cournot competition (Amir, 2005).[5] Therefore, I will refer to Assumption 1.1 directly as "strategic substitutability."

Assumption 1.1 (Strategic substitutability)**.** The inverse demand function is strictly log-concave wherever it is positive,

$$\forall q \ : \ p(q) \ > \ 0 \quad \Longrightarrow \quad \frac{\partial^2(\ln(p(q)))}{\partial q^2} \ < \ 0. \tag{1.1}$$

5. "Almost" because the set of quantities for which the inverse demand function has to be log-concave can be restricted. Also, if the marginal cost is strictly increasing, this adds some wiggle room for the inverse demand function.

Benchmark without Price Caps. A firm's problem is to maximize its profit by choosing q_i, taking as given the q_j that the other firm chooses,

$$\max_{q_i} \ \pi_i(q_i, q_j) \ = \ \max_{q_i} \ q_i \cdot p(q_i + q_j) - c(q_i). \tag{1.2}$$

A firm's marginal profit is the price it gets for the marginal unit less the inframarginal loss from depressing the price it gets for the inframarginal units and less the marginal cost,

$$\frac{\partial \pi_i(q_i, q_j)}{\partial q_i} \ = p(q_i + q_j) + q_i \cdot p'(q_i + q_j) - c'(q_i). \tag{1.3}$$

The strategic substitutability assumption implies that the firms' profit functions are strictly quasi-concave,

$$\forall q_j \ : \ \frac{\partial \pi_i(q_i, q_j)}{\partial q_i} = 0 \ \implies \ \frac{\partial^2 \pi_i(q_i, q_j)}{\partial q_i^2} \ < \ 0. \tag{1.4}$$

Thus, the best response is unique.

As mentioned above, strategic substitutability also implies that each firm's best response function is decreasing in the quantity of the other firm as long as the best response is positive, so

$$\forall q_j \ : \ \frac{\partial \pi_i(q_i, q_j)}{\partial q_i} = 0 \ \implies \ \frac{\partial^2 \pi_i(q_i, q_j)}{\partial q_i \, \partial q_j} \ < \ 0. \tag{1.5}$$

Together with the other standard assumptions, strategic substitutability implies the existence and uniqueness of the pure-strategy Cournot-Nash equilibrium.[6]

The unique pure-strategy Cournot-Nash equilibrium is symmetric. Both firms choose q^C such that the first-order condition $p(2q^C) + q^C \cdot p'(2q^C) - c'(q^C) \stackrel{!}{=} 0$ is satisfied. In the following, I will refer to this equilibrium as the *Cournot-Nash equilibrium*, to the equilibrium quantity q^C as the *Cournot-Nash quantity*, and to the equilibrium price $p^C \equiv p(2q^C)$ as the *Cournot-Nash price*.

6. Existence is proven in Amir (1996). The uniqueness is a corollary of Proposition 1.1: The best response functions are continuous, and the slopes are strictly between -1 and 0. Thus, the best response functions can intersect at most once. Furthermore, the symmetry of the best response functions implies that the intersection is in positive quantities. There are no further equilibria involving corner solutions. When firm 1 plays a quantity of 0, then firm 2 plays the monopoly quantity, but because the monopoly price is larger than the marginal cost of the first unit, firm 1's best response to the monopoly quantity is positive. Figure 1.12 is a sketch of the best response functions in the benchmark (among other things).

Price Caps. The novel element that I introduce to the Cournot setting are asymmetric price caps. In the main part, only firm 1 has a price cap, $\bar{p}$. The case in which firm 2 also has a price cap is deferred to Subsection 1.4.4. The difference that firm 1's price cap makes, can be split into how the inverse residual demand functions (the price that a firm expects when producing a certain quantity while taking the other firm's quantity as given) are determined and, given its inverse residual demand function, at which price firm 1 can sell its good.

Before considering the price cap's effect, it is helpful to reconsider the standard Cournot case to better understand the meaning and derivation of the inverse residual demand function without price caps. When firm 1 makes its quantity choice, it expects its inverse residual demand to depend on its own choice and on the choice of its opponent, $p_1(q_1, q_2)$. Moreover, it rightly expects that $p_1(q_1, q_2) \equiv p(q_1 + q_2)$. The reason for this equivalence is that price rationing is efficient: After the firms have chosen their quantities, as the price is free to adjust, the price that clears the market will realize. Firm 1 anticipates this when choosing its quantity. Thus, firm 1's price when producing its first marginal unit is $p(q_2)$. Analogously, the price when producing q_1 units is $p(q_1 + q_2)$. That is, firm 1's inverse residual demand curve is the inverse demand curve shifted to the left by q_2 units. Figure 1.1 illustrates this concept.

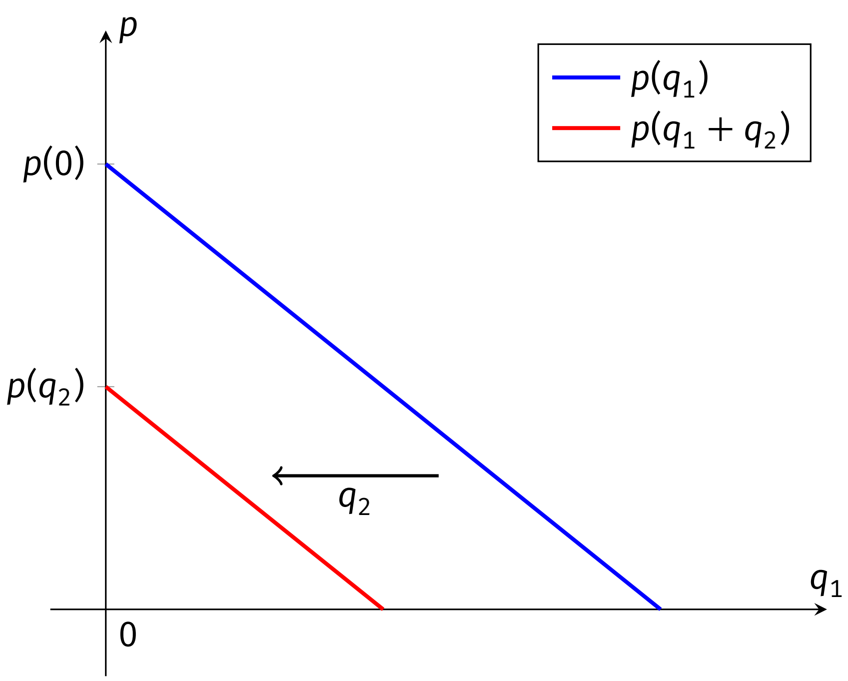

Figure 1.1. Firm 1's inverse residual demand curve is the inverse demand curve shifted to the left by q_2 units.

Given an inverse residual demand function, the only difference that a firm's price cap makes, is to flatten the inverse residual demand function at the price cap.

Denote firm 1's inverse residual demand function if it had no price cap by $p_1(q_1, q_2)$. If firm 1 chooses a quantity that is so small such that $p_1(q_1, q_2)$ exceeds the price cap, firm 1 still receives only the price cap for its good. If firm 1 chooses a quantity that is so large such that $p_1(q_1, q_2)$ falls below the price cap, the price cap does not bind and has no effect. Thus, the inverse residual demand function is $p_1(\bar{p}, q_1, q_2) = \min\{\bar{p},\ p_1(q_1, q_2)\}$. Figure 1.2 illustrates this concept.

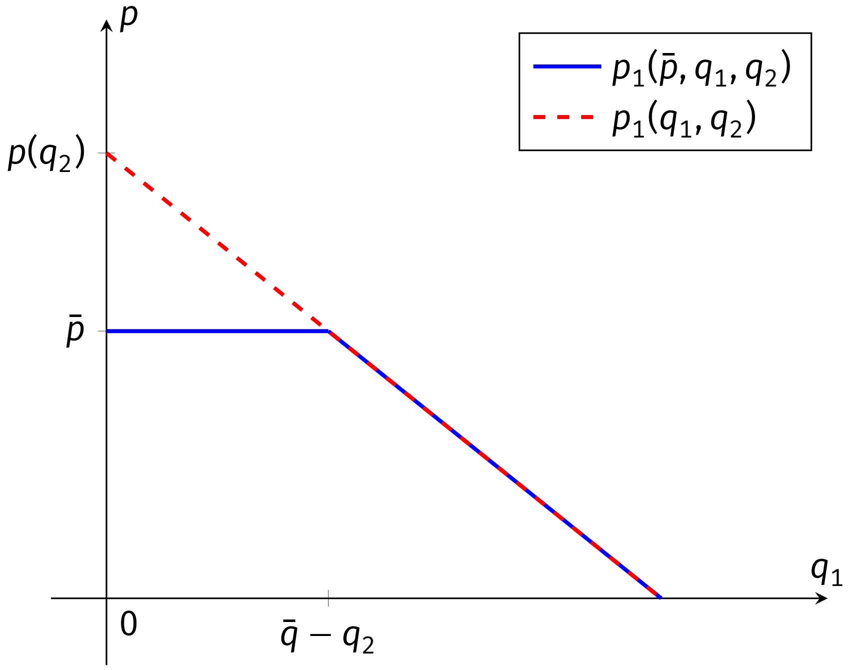

Figure 1.2. Given an inverse residual demand curve without a price cap, the price cap flattens it at the top. If the inverse demand would exceed the price cap, the firm still only gets paid the price cap.

To close the model, it only remains to determine how the asymmetric price cap affects the original inverse residual demand functions. For firm 1, there is no difference to the standard Cournot case. Because firm 2 has no price cap, its quantity is rationed efficiently as in the standard Cournot case. To simplify notation, I will denote the total quantity that makes firm 1's price cap just bind by $\bar{q} \equiv p^{-1}(\bar{p})$. Thus,

$$p_1(\bar{p}, q_1, q_2) = \begin{cases} \bar{p} & \text{if } q_1 < \bar{q} - q_2 \\ p(q_1 + q_2) & \text{if } q_1 \geq \bar{q} - q_2. \end{cases} \tag{1.6}$$

The inverse residual demand function of firm 2 is not immediate. As firm 1 has a price cap, its quantity will be sold below the market-clearing price if firm 2 chooses a low enough quantity. Thus, it depends on a (non-price) rationing rule, which parts of the demand function get satisfied and which remain for firm 2.[7]

In the literature, there are two different rationing rules, the first of which is efficient rationing. I assume this rationing rule in the main part. It says that the quantities always get rationed to maximize the consumer surplus. Thus, the inverse residual demand is the left-shifted inverse demand; the same as in the standard Cournot case in Figure 1.1. Intuitively, when microfounding the demand function as a continuum of consumers with different valuations, this means that the consumers get served in the decreasing order of their valuations; for example because there is an unregulated secondary market. Expressed in terms of firm 2's problem, efficient rationing is summarized in Assumption 1.2.

Assumption 1.2 (Efficient rationing)**.** Firm 2's inverse residual demand function is $p(q_1 + q_2)$.

The other usual rationing rule in the literature is proportional rationing (see Subsection 1.4.7).

1.3 Equilibria and Welfare Analysis

In this section, I solve for the unique pure-strategy Nash equilibrium for each price cap of firm 1. See Subsection 1.4.6 for a condition under which no mixed-strategy Nash equilibria exists. For readability, I will refer to the pure-strategy Nash equilibrium simply as "equilibrium."

I restrict the range of permissible price caps from below (exclusively) by the marginal cost of the first unit and from above (inclusively) by the Cournot-Nash price, $\bar{p} \in (c'(0), p^C]$. A lower price cap would exclude firm 1 from the market, and a higher price cap would have no effect.

1.3.1 The Firms' Maximization Problems

Due to efficient rationing, firm 2's profit maximization problem is the same as in the benchmark. Whenever firm 2's best response is positive, it is indirectly defined by the solution to its first-order condition,

7. When the price caps are symmetric (as in Okumura, 2017), the inverse residual demand functions are independent of the rationing rule. To see this, consider the interpretation of the demand function as mapping prices into measures of consumers whose willingness to pay exceeds the price. As both firms have the same price cap, even consumers with a larger willingness to pay never pay more than the symmetric price cap. Thus, the inverse demand function could be replaced with the same function, but flattened at the price cap. When the price cap binds, only consumers whose willingness to pay equals the price cap are served, so it does not matter who exactly gets served; the firms' inverse residual demand functions remain the same.

$$BR_2(q_1) \;=\; q_2^*(q_1) \;: \quad p(q_1+q_2^*(q_1)) + q_2^*(q_1)\cdot p'(q_1+q_2^*(q_1)) - c'(q_2^*(q_1)) \;=\; 0. \tag{1.7}$$

To improve the readability, I suppress the (irrelevant) corner solution $q_2 = 0$, which is only optimal if $p(q_1) - c'(0) \leq 0$, that is, if firm 1 supplies enough to serve the whole market at the marginal cost of the first unit or less.

Firm 1 maximizes its profit using the residual demand function from above,

$$\max_{q_1} \pi_1(q_1,q_2) \;=\; \max_{q_1} \begin{cases} q_1 \cdot \bar{p} - c(q_1) & \text{if } q_1 \;<\; \bar{q} - q_2 \\ q_1 \cdot p(q_1+q_2) - c(q_1) & \text{if } q_1 \;\geq\; \bar{q} - q_2. \end{cases} \tag{1.8}$$

The marginal profit is, expressed as the right-derivative at the drop[8] at $q_1 = \bar{q} - q_2$,

$$\frac{\partial_+ \pi_1(q_1,q_2)}{\partial q_1} \;=\; \begin{cases} \bar{p} - c'(q_1) & \text{if } q_1 \;<\; \bar{q} - q_2 \\ p(q_1+q_2) + q_1 \cdot p'(q_1+q_2) - c'(q_1) & \text{if } q_1 \;\geq\; \bar{q} - q_2. \end{cases} \tag{1.9}$$

The composition of the marginal profit is illustrated in Figure 1.3. Firm 1's marginal profit consists of two parts. For $q_1 < \bar{q} - q_2$, the price cap strictly binds, so firm 1 is a price-taker. Because of the price cap, the marginal revenue is constantly $\bar{p}$ and the marginal profit is strictly decreasing as the marginal cost is strictly increasing. The root of this part is $(c')^{-1}(\bar{p})$. For $q_1 \geq \bar{q} - q_2$, the price cap just binds or does not bind, so firm 1's marginal profit is that of a standard Cournot duopolist. The own effect on the price—the inframarginal loss—from increasing the quantity is $q_1 \cdot p'(\bar{q})$. It is the inframarginal loss that makes imperfectly competitive firms withhold quantity (and eliminating the inframarginal loss is the reason why price caps work). The root of this part is $BR_2(q_2)$. As the price and the marginal cost are continuous at $q_1 = \bar{q} - q_2$, firm 1's marginal profit drops by the inframarginal loss, $q_1 \cdot p'(\bar{q})$.[9]

Firm 1's profit function is strictly quasi-concave in q_1: The profit function is continuous and the marginal profit functions of both the price-taker and the standard Cournot duopolist intersect zero exactly once and from above. The price-taker's because it is strictly decreasing and the standard Cournot duopolist's because it is the benchmark. Because firm 1's marginal profit drops downwards at $q_1 + q_2 = \bar{q}$, the combined marginal profit does not contain both intersections with zero but exactly one (possibly at the drop).

As firm 1's profit function is strictly quasi-concave in q_1, the intersection of the marginal profit and zero determines firm 1's best response if it is positive. The corner

8. There is a strict drop for all price caps except for $\bar{p} = p^C$. In this case, there is only a kink. Nevertheless, I will refer to the "drop."

9. In fact, for all $q_1 > \bar{q} - q_2$, the marginal profit of the standard Cournot duopolist (the second part of the marginal profit function) is strictly less than the marginal profit of the price-taker (the first part of the marginal profit function): The price is lower, the inframarginal loss is negative, and the marginal cost is the same.

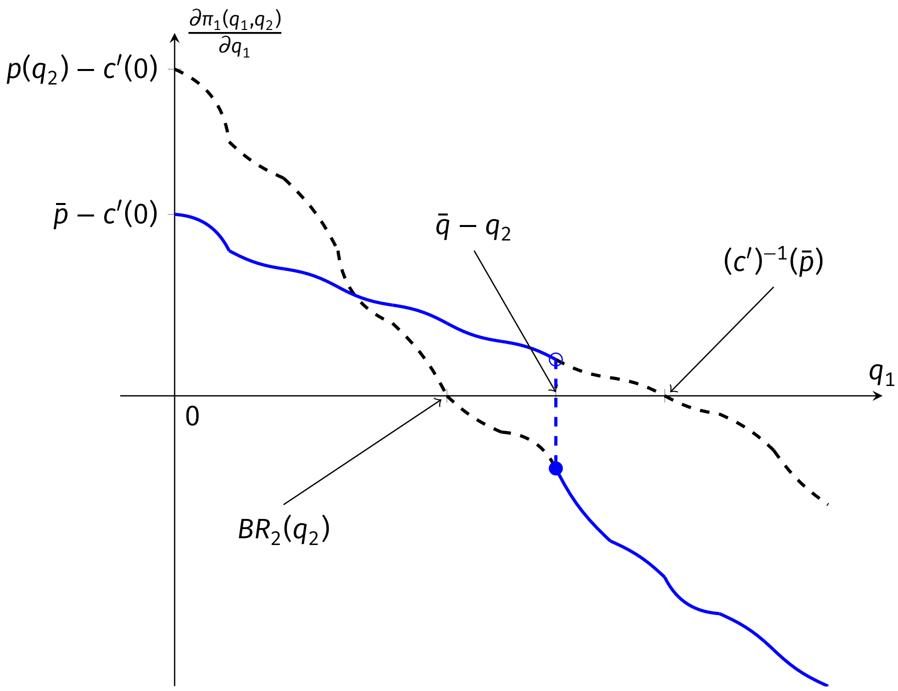

Figure 1.3. The marginal profit of the price-taker starts at $\bar{p} - c'(0)$ and its root is $(c')^{-1}(\bar{p})$. The marginal profit of the standard Cournot duopolist starts at $p(q_2) - c'(0)$ and its root is $BR_2(q_2)$. The marginal profit of firm 1 is in blue.

solution is optimal in the same case as for firm 2 and is, as well, ignored. The interior best responses of firm 1 depend both on the opponent's choice and the price cap, and they can be grouped in three cases, which also occur when applying a price cap to a monopoly (see Appendix 1.B).

1) The intersection can be in the part where $q_1 > \bar{q} - q_2$. This case is illustrated in Figure 1.4. Even with the inframarginal loss after the price cap stops binding, the firm profits from expanding its quantity. As the price cap does not bind, it has no effect. Thus, the best response of a standard Cournot duopolist, $BR_2(q_2)$, is optimal if $BR_2(q_2) \geq \bar{q} - q_2$. In the monopoly, this case corresponds to high, ineffective price caps.

2) The intersection can be at the drop at $q_1 = \bar{q} - q_2$. This case is illustrated in Figure 1.5. As argued above, when the price cap stops binding, the inframarginal loss from depressing the price appears. In this case, the marginal profit from expanding its quantity is positive as long as the firm does not influence its price. But, the inframarginal loss is so large that the firm would lose from producing more when the price cap stops binding. Thus, $q_1 = \bar{q} - q_2$ is optimal whenever the marginal profit's drop starts in the weakly positive and ends in the weakly negative, that is, if $BR_2(q_2) \leq \bar{q} - q_2 \leq (c')^{-1}(\bar{p})$. In the monopoly, this case corresponds to intermediate price caps for which the inverse demand curve determines the quantity.

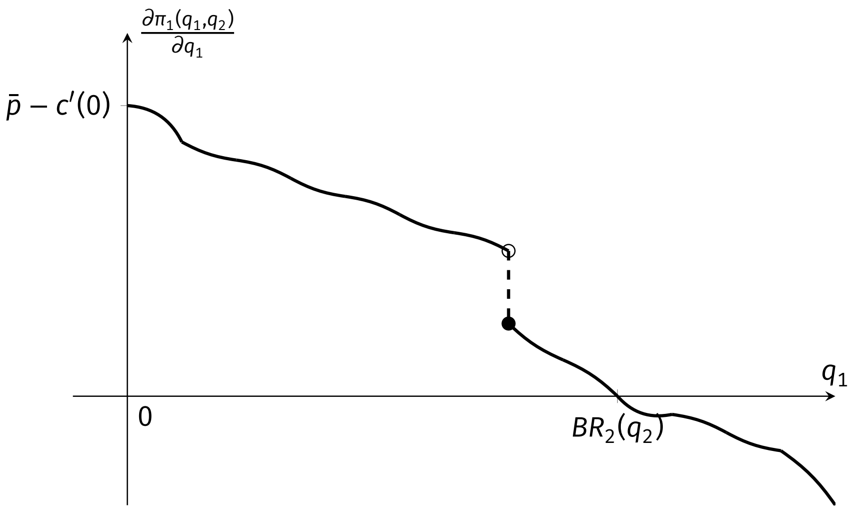

Figure 1.4. If the marginal profit is weakly positive after the drop, it is optimal for firm 1 to produce $BR_2(q_2)$.

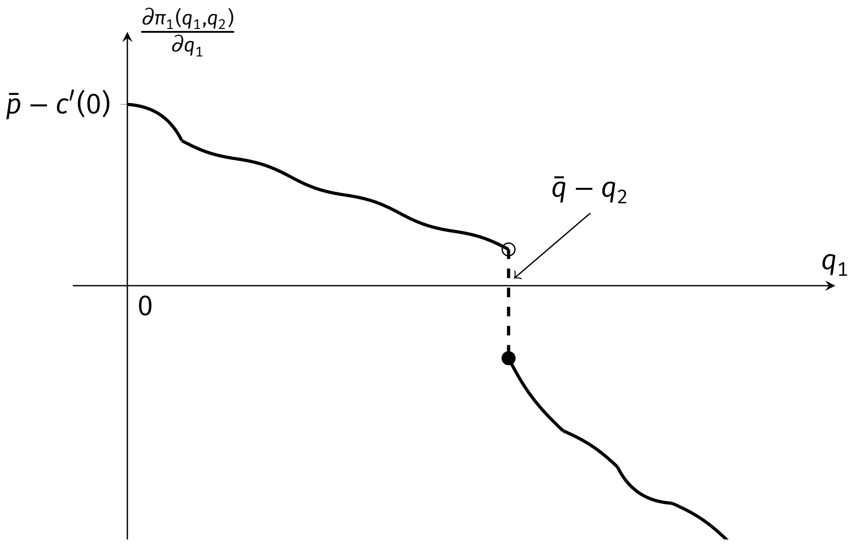

Figure 1.5. If the marginal profit is weakly positive above and weakly negative below the drop, it is optimal for firm 1 to produce $\bar{q} - q_2$.

3) The intersection can be in the part where $q_1 < \bar{q} - q_2$. This case is illustrated in Figure 1.6. The price cap and the other firm's quantity are so low that firm 1's marginal cost reaches the price cap while it still binds: Firm 1 is a price-taker. Thus, the root of the marginal profit of the price-taker, $(c')^{-1}(\bar{p})$, is optimal whenever it is smaller than the quantity at which the price cap stops binding, that is, if $(c')^{-1}(\bar{p}) < \bar{q} - q_2$. In the monopoly, this case corresponds to low price caps for which the marginal cost curve determines the quantity.

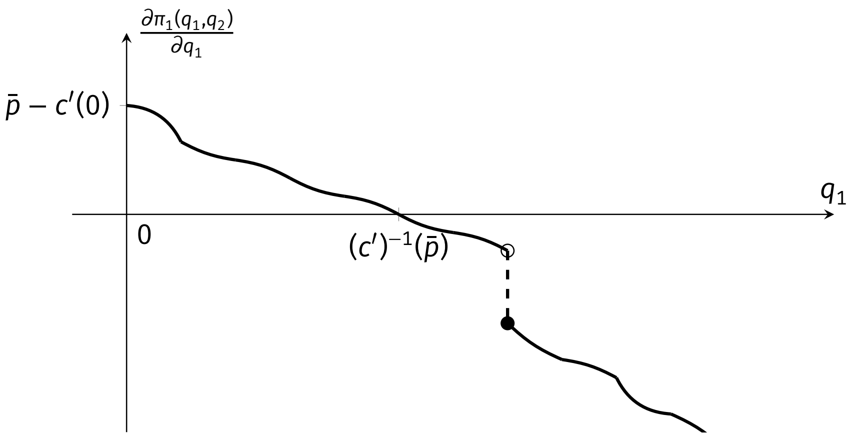

Figure 1.6. If the marginal profit is weakly negative above the drop, it is optimal for firm 1 to produce $(c')^{-1}(\bar{p})$.

These cases can be summarized in firm 1's best response function—ignoring corner solutions at 0—[10]

$$BR_1(q_2;\bar{p}) = \min\left\{\max\left\{BR_2(q_2),\ \bar{q}-q_2\right\},\ (c')^{-1}(\bar{p})\right\}. \tag{1.10}$$

You can see the best response function in Figure 1.7.

The intersections of the two firms' best response functions are the equilibria of the game. The properties of the equilibria depend on the part of firm 1's best response function in which the intersection is.

The intersection cannot be in the interior of the first part of firm 1's best response function, $BR_2(q_2)$, because the total quantity would exceed $\bar{q}$. As Proposition 1.1 shows, then, at least one firm could profitably deviate. The idea is that both firms are standard Cournot duopolists if the total quantity exceeds $\bar{q}$. The slope of their best response functions is, then, strictly between −1 and 0 due to strategic substitutability. So if a firm produces more than in the Cournot-Nash equilibrium, its marginal profit gets negative unless the other firm reduces its quantity by even more; which means that the total quantity falls short of $\bar{q}$. This idea is illustrated in Figure 1.8.

Proposition 1.1. *If $q_1 + q_2 > \bar{q}$, then the marginal profit of at least one firm is strictly negative.*

Proof. The proof is in Appendix A, Subsection 1.A.1 □

10. If you have kept track of the permutations, you might wonder whether this best response function yields the wrong value if $\bar{q} - q_2 < (c')^{-1}(\bar{p}) < BR_2(q_2)$ (it yields $(c')^{-1}(\bar{p})$, but $BR_2(q_2)$ maximizes the profit). The solution is that the above inequality cannot occur. Remember the fact that the standard Cournot duopolist part of the marginal profit is strictly smaller than the price-taker part for all $q_1 > \bar{q} - q_2$. Thus, if $\bar{q} - q_2 < BR_2(q_2)$, the root of the price-taker part has to be at an even higher quantity, so $\bar{q} - q_2 < BR_2(q_2) < (c')^{-1}(\bar{p})$. In this case, the best response function yields the right value: $BR_2(q_2)$.

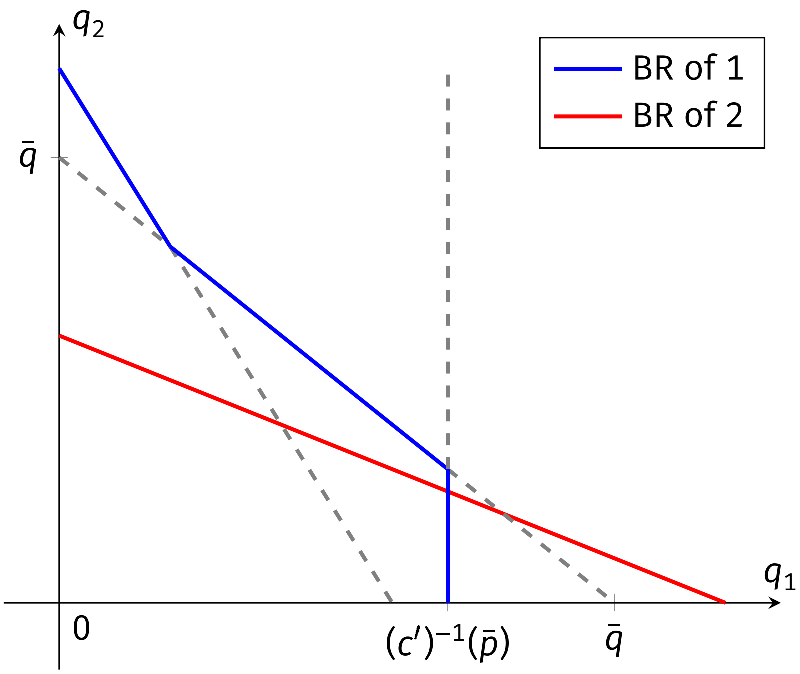

Figure 1.7. On the x-axis is the quantity of firm 1 and on the y-axis is the quantity of firm 2. The best response function of firm 2 (in red) is that of the standard Cournot duopolist. The best response function of firm 1 (in blue) consists of three parts. The left part is case 1), where the standard Cournot duopolist's best response is optimal. The middle part is case 2), where choosing the quantity at which the price cap just binds, $\bar{q} - q_2$, is optimal. The right part is case 3), where firm 1 is a price-taker and stops producing when the marginal cost reaches the price cap. For the comparative statics in the price cap, remember that the first part is independent of the price cap and that $\bar{q}$ and $(c')^{-1}(\bar{p})$ move in opposite directions. You can try out the comparative statics for a linear demand and quadratic cost in this Desmos Graphing Calculator graph: https://www.desmos.com/calculator/ritredhsbu (last accessed September 27, 2022).

In the other two parts, there are equilibria. If the intersection is in the second part of $BR_1(q_2; \bar{p})$, then $q_1 + q_2 = \bar{q}$ and the price cap just binds in equilibrium. These are "clearing equilibria." If the intersection is in the third part of $BR_1(q_2; \bar{p})$, then $q_1 + q_2 < \bar{q}$ and the price cap strictly binds in equilibrium. These are "rationing equilibria." In the following, I explain the names and show that if the price cap is above a cutoff, κ, the unique equilibrium is a clearing equilibrium and that if the price cap is below the cutoff κ, the unique equilibrium is a rationing equilibrium.

1.3.2 Clearing Equilibria

For each price cap, there is a unique candidate for a clearing equilibrium. The reason is that the equilibrium condition $q_1 + q_2 = \bar{q}$ leaves only one split of $\bar{q}$ into $q_1^*(\bar{q})$ and $q_2^*(\bar{q})$ such that the first-order condition of firm 2 is satisfied. This candidate is an equilibrium if its suggested quantity is optimal for firm 1; that is, if the marginal

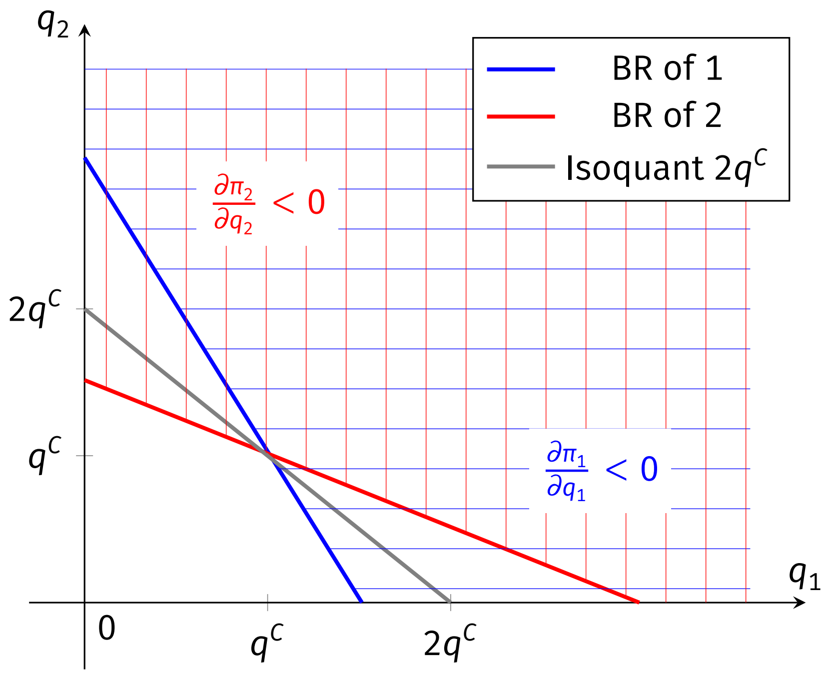

Figure 1.8. On the x-axis is the quantity of firm 1 and on the y-axis is the quantity of firm 2. The gray isoquant shows the Cournot-Nash total quantity and has a slope of -1. The best response function of firm 2 (red) is everywhere flatter, and the best response function of firm 1 (blue) is everywhere steeper (because the axes are inverted). Above the best response function, firm 2's marginal profit is negative and to the right of the best response function, firm 1's marginal profit is negative because the profit functions are strictly quasi-concave.

profit is weakly positive above and weakly negative below the drop (as in Figure 1.5). Theorem 1.1 proves that this candidate is the unique equilibrium if the price cap is above the cutoff κ.

Theorem 1.1 (Clearing Equilibria). *The quantities*

$$q_1^*(\bar{q}) \;=\; \bar{q} - q_2^*(\bar{q}) \quad \textit{and} \quad q_2^*(\bar{q}) \;:\; p(\bar{q}) + q_2^*(\bar{q}) \cdot p'(\bar{q}) - c'(q_2^*(\bar{q})) \;=\; 0 \tag{1.11}$$

have the properties that

(i) $q_1^*(\bar{q}) \geq q^C \geq q_2^*(\bar{q})$, *with strict inequalities for* $\bar{q} > 2q^C$.
(ii) $q_1^*(\bar{q})$ *is strictly increasing in* $\bar{q}$ *and* $q_2^*(\bar{q})$ *is strictly decreasing in* $\bar{q}$.

There is a cutoff $\kappa \in (c'(0), p^C)$ *that is indirectly defined by*

$$\kappa \;:\; \kappa - c'\left(q_1^*(p^{-1}(\kappa))\right) \;=\; 0. \tag{1.12}$$

It has the properties that

(iii) $q_1^*(\bar{q})$ *and* $q_2^*(\bar{q})$ *are the unique equilibrium for all* $\bar{p} \in [\kappa, p^C]$.

(iv) $q_1^*(\bar{q})$ *and* $q_2^*(\bar{q})$ *are no equilibrium for all* $\bar{p} \in (c'(0), \kappa)$.

Proof. The proof is in Appendix A, Subsection 1.A.2. □

I call these equilibria "clearing" because there is no excess demand, as the price cap just binds; both firms receive the same price. Theorem 1.1 contains the message of this chapter: The asymmetric price cap makes the production asymmetric and, thus, inefficient, and the more so the more it increases the total quantity.

To analyze the impact of the inefficient production on the usefulness of asymmetric price caps, consider the (utilitarian) *welfare*. It is the sum of the consumer surplus and the firms' profits. Formally, the welfare is the area between the inverse demand curve and the marginal cost curves,

$$W(\bar{q}) = \int_0^{\bar{q}} p(x)\, dx - c(q_1^*(\bar{q})) - c(q_2^*(\bar{q})). \tag{1.13}$$

The derivative of the welfare with respect to the price cap shows the trade-off between the total quantity and the production efficiency. When the price cap is decreased to increase the total quantity, firm 1 increases its quantity, which crowds out some of firm 2's quantity. Thus, increasing the total quantity affects the welfare in two ways: The net increase in the total quantity is the (weakly) positive quantity effect, and the crowding out of firm 2's quantity is the (weakly) negative production efficiency effect. Formally, the derivative is[11]

$$\frac{\partial W(\bar{q})}{\partial \bar{q}} = \underbrace{\bar{p} - c'(q_1^*(\bar{q}))}_{\text{quantity}} \underbrace{- \left(-\frac{\partial q_2^*(\bar{q})}{\partial \bar{q}}\right) \cdot \Big(c'(q_1^*(\bar{q})) - c'(q_2^*(\bar{q}))\Big)}_{\text{production efficiency}}, \tag{1.14}$$

where I adjusted the minuses to make all brackets weakly positive and used that $q_1^*(\bar{q}) = \bar{q} - q_2^*(\bar{q})$.

The quantity effect is (weakly) positive because firm 1's increasing the total quantity increases the welfare: The marginal contribution to the welfare is $\bar{p}$ and the social marginal cost is that firm 1 incurs additional costs of $c'(q_1^*(\bar{q}))$. If the price cap is strictly above the cutoff, the difference is strictly positive.

The production efficiency effect is (weakly) negative because firm 1 produces weakly more and has, thus, a weakly larger marginal cost. Therefore, crowding out cheaper quantity from firm 2 is socially wasteful. The production efficiency effect is the product of two terms: By how much firm 2 reduces its quantity—the stronger firm 2 reacts to a change in the price cap, the more firm 1 has to compensate—and

11. I take the derivative with respect to the total equilibrium quantity $\bar{q}$ to improve readability. To get the marginal effect in terms of a one unit decrease of the price cap, multiply with the negative term $\frac{\partial \bar{q}}{\partial \bar{p}}$.

how unequal the marginal cost is already—the larger the difference is, the more socially costly is the compensation.

The lower the price cap is, the more tends the social loss from a more inefficient production to outweigh the social gain from more production. When the price cap is lower, the positive marginal effect of a larger total quantity gets smaller as the social benefit and the social cost of a larger quantity converge. The negative marginal effect from a less efficient production tends to get larger, as the marginal cost are more unequal. This tendency might be locally overturned because the other factor—how much firm 2 reacts to a marginal change in the price cap—might be smaller for some price caps.[12] Thus, the welfare effect of a marginal change in the price cap is generally ambiguous—except for the extreme price caps.

When the price cap equals the Cournot-Nash price, the production efficiency effect vanishes because both firms produce the same quantity and have the same marginal cost. Thus, shifting production marginally from firm 2 to firm 1 does not change the social cost. Evaluating the derivative at the Cournot-Nash price yields

$$\left.\frac{\partial_+ W(\bar{q})}{\partial \bar{q}}\right|_{\bar{q}=2q^C} = p^C - c'(q^C) \, > \, 0. \tag{1.15}$$

The inequality follows from the Cournot-Nash equilibrium condition. This result means that introducing asymmetric price caps just below the Cournot-Nash price always increases the welfare.

When the price cap equals the cutoff between clearing and rationing equilibria, κ, the quantity effect vanishes: At the cutoff, the marginal cost of firm 1 equals the social marginal benefit. Thus, only the negative production efficiency effect remains,

$$\left.\frac{\partial_- W(\bar{q})}{\partial \bar{q}}\right|_{\bar{q}=p^{-1}(\kappa)} = -\left(-\frac{\partial_- q_2^*(p^{-1}(\kappa))}{\partial \bar{q}}\right) \cdot \left(c'(q_1^*(p^{-1}(\kappa))) - c'(q_2^*(p^{-1}(\kappa)))\right) \, < \, 0. \tag{1.16}$$

This result means that it is not innocuous that the price cap distorts the production: At the cutoff, a marginally higher price cap would increase the welfare although the total quantity would decrease. So, the regulator should not rely on the total quantity alone to evaluate asymmetric price caps.

1.3.3 The Cutoff

Theorem 1.1 has only shown that the cutoff, κ, lies in the range of permissible price caps, $(c'(0), p^C]$. In this subsection, I will explore the nature of the cutoff and present bounds.

12. With linear demand and quadratic cost, firm 2's reaction is constant in the price cap. Then, the negative effect from a more inefficient production is monotone in the price cap (see Subsection 1.3.5).

The reason for the existence of the cutoff is the monotonicity of the comparative statics: The lower the price cap is, the higher is the clearing equilibrium candidate quantity of firm 1, which in turn means that the marginal profit above the drop is lower. As long as the price cap is strictly above the cutoff, the marginal profit above the drop is positive. When the price cap is at the cutoff, κ, the marginal profit above the drop is exactly zero. For all lower price caps, firm 1's marginal profit above the drop is negative, so firm 1 wants to deviate to $(c')^{-1}(\bar{p})$ (see Figure 1.6).

As mentioned above, price regulation in a Cournot duopoly is closely related to price regulation in a monopoly (see Appendix 1.B). The cutoff, κ, is analogous to the perfectly competitive price in the monopoly: As long as the price cap is above the cutoff, the quantity of firm 1 is determined by the intersection of the marginal revenue and the inverse residual demand curve. When the price cap is below the cutoff, its quantity is determined by the intersection of the marginal revenue and the marginal cost curve.

Because there is another firm in the market, the cutoff, κ, lies strictly above the perfectly competitive price as Proposition 1.2 shows.[13] There cannot be a clearing equilibrium with the perfectly competitive price as the price cap. The reason is that both firms would have to supply half of the perfectly competitive quantity, but firm 2 profits from deviating to a lower quantity. This result means that it is impossible to achieve full efficiency with asymmetric price caps.

Concerns that the cutoff could be (arbitrarily) close to the Cournot-Nash equilibrium and that clearing equilibria could be, thus, not particularly interesting, are unnecessary. Proposition 1.2 also shows that the cutoff lies strictly below the Stackelberg equilibrium price, if the Stackelberg leader's profit function is strictly quasi-concave.[14]

Proposition 1.2. *Define the competitive price, p^W, as the price at which the inverse demand curve, $p(q)$, and the social marginal cost curve, $c'(\frac{q}{2})$, intersect.*

Define p^S as the unique Stackelberg equilibrium price if the Stackelberg leader's profit function is strictly quasi-concave.

It is true that $p^W < \kappa < p^S$.

Proof. The proof is in Appendix A, Subsection 1.A.3. □

13. In a duopoly with symmetric cost, the perfectly competitive price is given by the intersection of the inverse demand curve and the social marginal cost curve. The social marginal cost is each firm's marginal cost when splitting the total quantity equally—when producing efficiently.

14. The qualification assures that the Stackelberg equilibrium price is unique and determined by the Stackelberg leader's first-order condition, which is used to prove the inequality. A sufficient condition for strict quasi-concavity is, for example, a weakly concave inverse demand function. For a short definition of the sequential Stackelberg quantity competition model, see Subsection 1.4.3.

1.3.4 Rationing Equilibria

When the price cap is below the cutoff, κ, the total quantity has to fall short of $\bar{q}$ in all equilibria: Theorem 1.1 shows that the unique candidate for an equilibrium with total quantity $\bar{q}$ is no equilibrium and Proposition 1.1 shows that no equilibrium has a total quantity exceeding $\bar{q}$.

Because the total quantity is below $\bar{q}$ in equilibrium, firm 1's price cap binds strictly, so it acts as a price-taker: Firm 1's equilibrium strategy is to produce until the marginal cost equals the price cap. Firm 2's equilibrium strategy is the solution to the monopolist's problem in the market for residual demand. Theorem 1.2 formally summarizes the rationing equilibrium strategies. Because $\bar{q}$ has no particular meaning in the rationing equilibria and to distinguish them from the clearing equilibria, I denote the equilibrium strategies as functions of $\bar{p}$.

Theorem 1.2 (Rationing Equilibria). *If $\bar{p} \in (c'(0), \kappa)$, the only pure-strategy Nash equilibrium is*

$$q_1^*(\bar{p}) \;=\; (c')^{-1}(\bar{p}) \quad \textit{and} \tag{1.17}$$

$$q_2^*(\bar{p}) \;:\; p\Big(q_1^*(\bar{p}) + q_2^*(\bar{p})\Big) + q_2^*(\bar{p}) \cdot p'\Big(q_1^*(\bar{p}) + q_2^*(\bar{p})\Big) - c'(q_2^*(\bar{p})) \stackrel{!}{=} 0. \tag{1.18}$$

In the limit of $\bar{p} \to \kappa$, the equilibrium converges to the clearing equilibrium presented in Theorem 1.1.

$q_1^*(\bar{p})$ is strictly increasing in $\bar{p}$ and $q_2^*(\bar{p})$ is strictly decreasing in $\bar{p}$. The total quantity $q_1^*(\bar{p}) + q_2^*(\bar{p})$ is strictly increasing in $\bar{p}$.

Proof. The proof is in Appendix A, Subsection 1.A.4. □

In the rationing equilibria, there is rationing in the equilibrium. Firm 1 sells its quantity at the price cap, and firm 2 does not want to serve all the excess demand at that price. Thus, firm 2 sells its quantity at a higher price that clears the market for the residual demand.

The comparative statics are reversed compared to the clearing equilibria. When the price cap decreases, firm 1, being a price-taker, produces a smaller quantity in equilibrium. Firm 2 produces a larger quantity as its market gets larger because of strategic substitutability. The total quantity, however, decreases.

Table 1.1 summarizes the equilibria and their comparative statics that go in opposite directions.

The reversal of the comparative statics is no coincidence: Clearing and rationing equilibria are symmetric to each other around the cutoff. Whenever the total quantity is the same in a clearing and in a rationing equilibrium, both firms' quantities are identical. The reason is that firm 2's best response is unique for each total quantity. Furthermore, the total quantity is monotone and continuous in the price cap both within the clearing and the rationing equilibria. Lemma 1.1 proves the symmetry.

Table 1.1. This table summarizes the equilibrium quantities and the respective comparative statics in the two different types of equilibria.

	Price caps	Firm 1	Firm 2	Total quantity
Clearing equilibria (Theorem 1.1)	$p^C > \bar{p} \geq \kappa$	$q_1^* = \bar{q} - q_2^*$ $\frac{\partial q_1^*}{\partial \bar{p}} < 0$	q_2^* solves FOC $\frac{\partial q_2^*}{\partial \bar{p}} > 0$	$\frac{\partial(q_1^*+q_2^*)}{\partial \bar{p}} < 0$
Rationing equilibria (Theorem 1.2)	$\kappa > \bar{p} > c'(0)$	$q_1^* = (c')^{-1}(\bar{p})$ $\frac{\partial q_1^*}{\partial \bar{p}} > 0$	q_2^* solves FOC $\frac{\partial q_2^*}{\partial \bar{p}} < 0$	$\frac{\partial(q_1^*+q_2^*)}{\partial \bar{p}} > 0$

Lemma 1.1. *Define $\bar{p}_B$ as the price cap for which the total quantity in the rationing equilibrium is equal to the Cournot-Nash quantity, $\bar{p}_B : q_1^*(\bar{p}_B) + q_2^*(\bar{p}_B) = 2q^C$. It holds that $c'(0) < \bar{p}_B < \kappa$.*

There is a monotone bijection between the clearing equilibria and the rationing equilibria. For each price cap $\bar{p}_c \in (\kappa, p^C]$, there is exactly one price cap $\bar{p}_r \in [\bar{p}_B, \kappa)$ such that the equilibrium quantities of the firms are the same:

$$q_1^*(\bar{q}_c) = q_1^*(\bar{p}_r) \qquad \textit{and} \qquad q_2^*(\bar{q}_c) = q_2^*(\bar{p}_r). \tag{1.19}$$

Proof. The proof is in Appendix A, Subsection 1.A.5. □

Because equal equilibrium quantities imply equal welfare, the symmetry result extends to the welfare: The welfare effects of a decreasing price cap are reversed for the rationing equilibria. As long as the total quantity exceeds the Cournot-Nash quantity, a lower price cap reduces the total quantity but makes the production more efficient.

If the price cap is below $\bar{p}_B$, there is no trade-off between total quantity and production efficiency anymore. A lower price cap means that the total quantity decreases and that the production gets less efficient because firm 2 produces already initially more than firm 1 and its quantity increases further. When the price cap approaches $c'(0)$, the welfare goes to the welfare in a monopoly. Anyway, the welfare with any asymmetric price cap below $\bar{p}_B$ is lower than the welfare in the Cournot-Nash equilibrium without a price cap.

Due to the symmetry, the welfare takes a local minimum at the cutoff. As a decreasing price cap decreases the welfare in the clearing equilibria just above the cutoff, a decreasing price cap increases the welfare in the rationing equilibria just below the cutoff.

Even if the regulator that sets the price cap knows nothing about the functional forms, she might be able to observe whether a price cap is at the cutoff: The market is at the brink of segmenting into a regulated low-price part and an unregulated high-price part. If the regulator observes this beginning segmentation, she should either decrease or increase the price cap.

1.3.5 Special Case of Linear Demand and Quadratic Cost

A special case often analyzed in the literature is a linear inverse demand, $p(q) = a - b \cdot q$, and quadratic cost, $c(q_i) = \frac{c}{2} \cdot q_i^2$. These functional forms eliminate many of the higher derivatives; so the best response functions are (piece-wise) linear. Below, there are plots of the equilibrium quantities (Figure 1.9) and of the consumer surplus and the welfare (Figure 1.10).

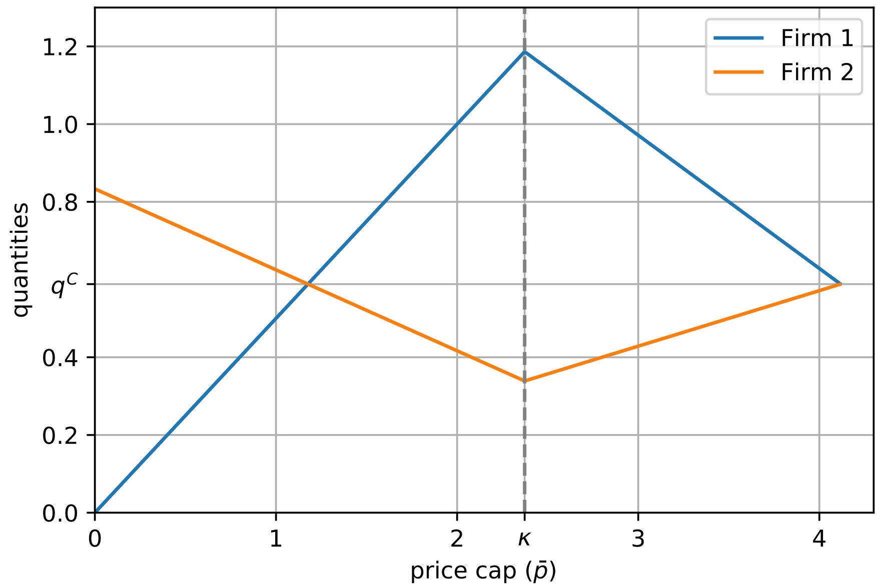

Figure 1.9. The equilibrium quantities of the firms when the cost functions are $c(q_i) = q_i^2$ and the inverse demand is $p(q) = 10 - 5 \cdot q$.

Figure 1.9, depicting the equilibrium quantities, shows that the comparative statics in clearing and in rationing equilibria are reversed. It also illustrates the symmetry result: Whenever firm 1's quantity is the same, so is firm 2's quantity.

Figure 1.10, depicting the consumer surplus and the welfare, shows some additional welfare results beyond the general case. The welfare is concave both within the clearing and in the rationing equilibria. The reason is that the functional forms make the production efficiency effect monotone in the price cap because the slope of firm 2's best response function is constant.

For all linear demand and quadratic cost functions, the welfare at the cutoff is larger than the welfare both in the Cournot-Nash equilibrium and in the unique Stackelberg equilibrium. This fact might help to evaluate the welfare effect of an asymmetric price cap. It implies that all clearing equilibria have a higher welfare than the Cournot-Nash equilibrium. The symmetry then implies that the asymmetric price cap improves the welfare if and only if it increases the total quantity, which is

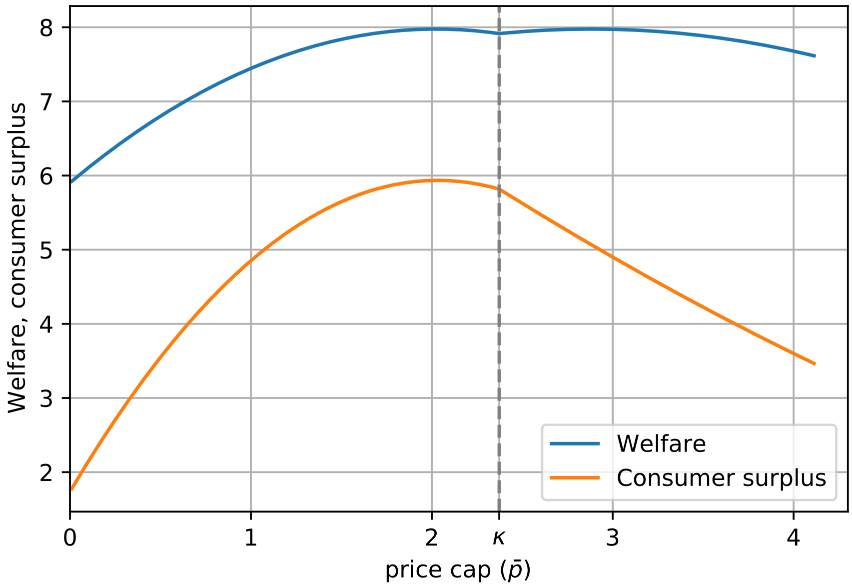

Figure 1.10. The welfare and consumer surplus in equilibrium when the cost functions are $c(q_i) = q_i^2$ and the inverse demand is $p(q) = 10 - 5 \cdot q$.

the same as saying that it improves the welfare if and only if firm 1 produces a larger quantity than firm 2. Therefore, a regulator that chooses a price cap to maximize the total quantity, on the one hand, ends up in a local minimum of the welfare, but, on the other hand, still improves the welfare compared to the benchmark without a price cap.

1.4 Extensions, Generalizations, and Applications

In this section, I explore extensions and additional results, discuss generalizations and alternative assumptions, and present applications.

1.4.1 Consumer Surplus

This subsection deals with a different objective that the regulator might have: maximizing the *consumer surplus*. The consumer surplus—with efficient rationing—is defined as the area between the inverse demand curve and the prices up to the total quantity.

In clearing equilibria, the consumer surplus is

$$CS(\bar{q}) = \int_0^{\bar{q}} p(x) - \bar{p} \; dx. \tag{1.20}$$

It is unambiguously increasing in a decreasing price cap because the total quantity increases and the price decreases.

In contrast to the welfare, the consumer surplus is not symmetric between the rationing and the clearing equilibria. The reason is that while the firms' quantities are symmetric, the prices are not: The price of firm 1 is lower in the corresponding rationing equilibrium. Thus, compared to the consumer surplus in the corresponding clearing equilibrium, the consumers receive the price differential as an additional transfer from firm 1. Figure 1.11 illustrates the consumer surplus. The additional transfer is the rectangle $q_1^*(\bar{p}) \cdot (p(Q(\bar{p})) - \bar{p})$.

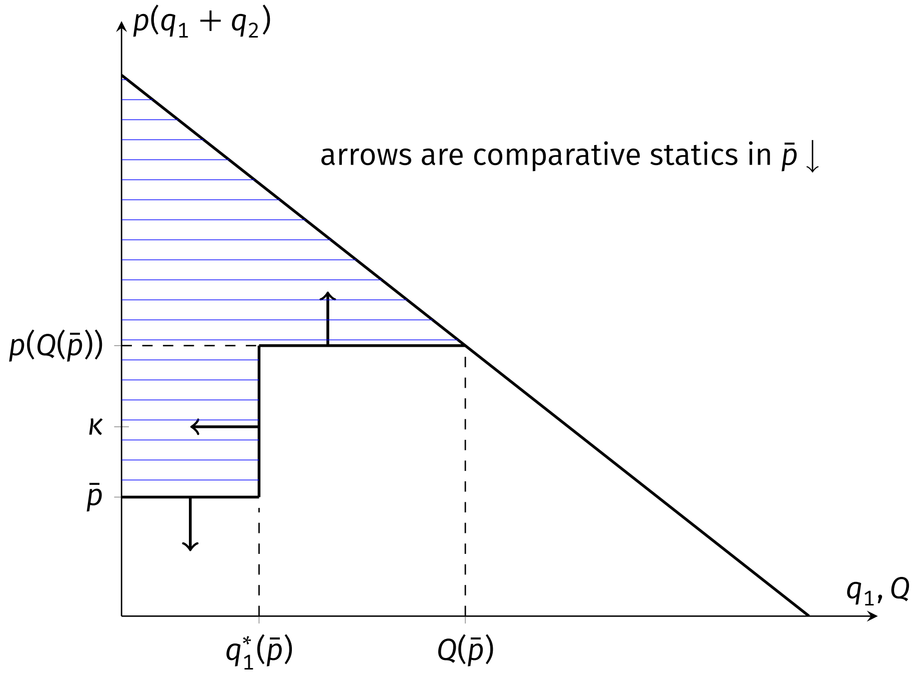

Figure 1.11. The consumer surplus is the area below the inverse demand curve less the consumers' expenditure. I replaced the equilibrium total quantity, $q_1^*(\bar{p}) + q_2^*(\bar{p})$, by $Q(\bar{p})$. Marginally decreasing the price cap has three effects on the consumer surplus. Each of them is illustrated with an arrow.

Formally, the consumer surplus in a rationing equilibrium is

$$CS(\bar{p}) = \int_0^{q_1^*(\bar{p})+q_2^*(\bar{p})} p(x)\,dx - q_1^*(\bar{p}) \cdot \bar{p} - q_2^*(\bar{p}) \cdot p(q_1^*(\bar{p}) + q_2^*(\bar{p})); \tag{1.21}$$

the area under the inverse demand curve up to the total quantity less the expenditure for the quantity of firm 1 less the expenditure for the quantity of firm 2.

A marginal decrease in the price cap has three effects on the consumer surplus:

$$-\frac{\partial CS(\bar{p})}{\partial \bar{p}} = q_1^*(\bar{p}) - \frac{\partial q_1^*(\bar{p})}{\partial \bar{p}} \cdot \left(p(Q(\bar{p})) - \bar{p}\right) - q_2^*(\bar{p}) \cdot (-p'(Q(\bar{p}))) \cdot \frac{\partial Q(\bar{p})}{\partial \bar{p}}, \tag{1.22}$$

where I replaced the equilibrium total quantity, $q_1^*(\bar{p}) + q_2^*(\bar{p})$, by $Q(\bar{p})$ to fit the expression in one line.

The first term in equation (1.22) is the gain of those consumers that buy from firm 1 before and after the marginal decrease of the price cap and save one marginal unit. In Figure 1.11, this is the blue rectangle's expanding down. The second term in equation (1.22) is the loss of those consumers that buy from firm 1 before but have to buy from firm 2 at a higher price after the marginal decrease of the price cap because firm 1 reduces its quantity. In Figure 1.11, this is the blue rectangle's shrinking to the left. The third term in equation (1.22) is the loss of those consumers that buy from firm 2 before and after the marginal change because the price they have to pay increases as the total quantity decreases. In Figure 1.11, this is the blue triangle's shrinking.

The total effect of a marginal decrease in the price cap on the consumer surplus is ambiguous. The positive effect is large when $q_1^*(\bar{p})$ is large because this increases the transfer from firm 1 to the consumers. This is the case close to the cutoff, κ. The negative effects are small when the firms' quantities adjust only little because few consumers switch from firm 1 to firm 2 and the price of firm 2 rises only little. This is the case when the marginal cost is very steep at firm 1's optimal quantity.

The levels of the consumer surplus at the extreme price caps can be compared to the benchmark level. At the cutoff, the consumer surplus is larger than in the Cournot-Nash equilibrium because it has been increasing through all clearing equilibria. If the price cap goes to the marginal cost of the first unit, the consumer surplus is lower than in the Cournot-Nash equilibrium because it goes to the consumer surplus in a monopoly as firm 1 leaves the market. For the marginal change in the consumer surplus, these results imply that it has to be decreasing for at least some price caps.

In the case of a linear demand and quadratic cost (see Figure 1.10.), the consumer surplus increases when the price cap is decreased beginning at the cutoff. For all such functional forms, the transfer from firm 1 to the consumers outweighs the quantity reduction and price increase. Therefore, the price cap that uniquely maximizes the consumer surplus corresponds to a rationing equilibrium.

Furthermore, the consumer surplus is strictly concave within the rationing equilibria. When the price cap decreases, the positive effect gets monotonically smaller (because the quantity of firm 1 is decreasing), whereas the negative effects get monotonically larger (because the firms' reactions to a change in the price cap are constant and the price differential between the firms increases).

Whether the price cap that maximizes the consumer surplus is greater or less than the price cap that maximizes the welfare within the rationing equilibria depends on the cost parameter and on the slope of the inverse demand function. The reason is that the cost directly enters into the welfare, whereas it only enters indirectly through the price into the consumer surplus. If the cost parameter is sufficiently large compared to the slope of the inverse demand function, the consumer surplus gets maximized at a lower price cap.[15]

1.4.2 Constant Marginal Cost

When the marginal cost is a constant c, there is no cutoff and all equilibria are clearing equilibria. Figure 1.12 depicts the best response functions of both firms. Whenever firm 2's best response is positive, it is indirectly defined by the solution to its first-order condition,

$$BR_2(q_1) = q_2^*(q_1) : \quad p(q_1 + q_2^*) + q_2^* \cdot p'(q_1 + q_2^*) - c = 0. \tag{1.23}$$

Whenever firm 1's best response is positive, it is

$$BR_1(q_2; \bar{p}) = \max\{BR_2(q_2), \bar{q} - q_2\} \tag{1.24}$$

for reasons analogous to the case of increasing marginal cost (just let the inverse of the marginal cost function be infinity everywhere because it does not exist).

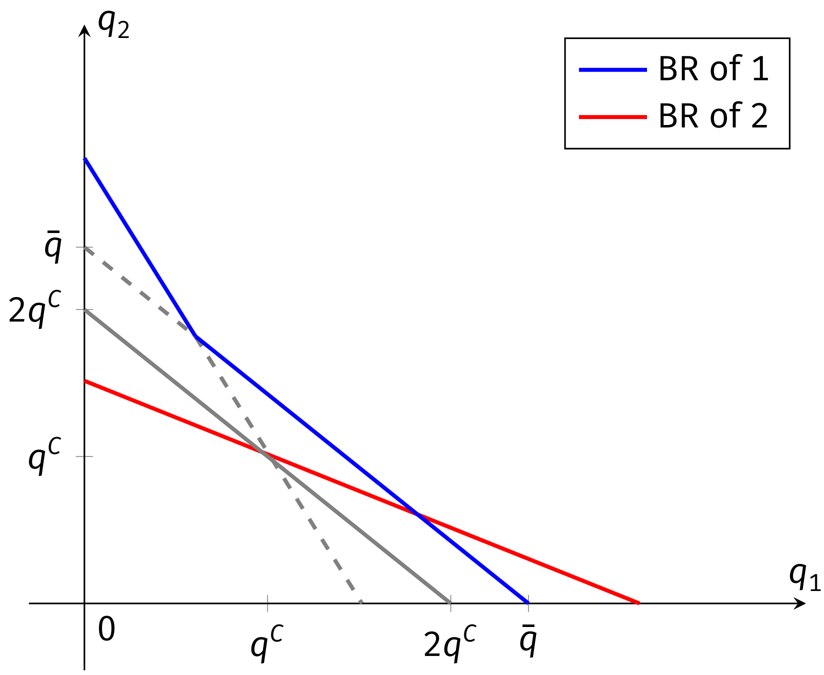

Figure 1.12. On the x-axis is the quantity of firm 1 and on the y-axis is the quantity of firm 2. The best response function of firm 2 is in red. The best response function of firm 1 is in blue. The dashed gray lines are the two parts of firm 1's best response function where they are not optimal.

15. The exact condition is $c > (\sqrt{2} - 1) \cdot b$.

Proposition 1.3. *When the marginal cost is constant at c, the unique equilibrium is*

$$q_1^*(\bar{q}) = \bar{q} - q_2^*(\bar{q}) \quad \text{and} \quad q_2^*(\bar{q}) = \frac{p(\bar{q}) - c}{-p'(\bar{q})}. \tag{1.25}$$

Proof. The proof is in Appendix A, Subsection 1.A.6. □

1.4.3 Stackelberg Competition

This subsection explains and exploits the relationship between asymmetric price caps and the sequential Stackelberg competition. In the Stackelberg game, the Stackelberg leader chooses its quantity, s_1, first. The Stackelberg follower, then observes the choice of the leader and optimally chooses its own quantity, s_2. This allows the Stackelberg leader to choose its optimal point on the best response function of the Stackelberg follower, which is that of a standard Cournot duopolist.

Commitment power is equivalent to sequential choice: If firm 1 has the possibility to commit itself to a quantity choice, firm 1 chooses the same quantity as a Stackelberg leader does in the Stackelberg equilibrium. Firm 2 anticipates this and replies as a Stackelberg follower does in the Stackelberg equilibrium.

Because of strategic substitutability, the Stackelberg leader can profitably deviate from the Cournot-Nash equilibrium of the simultaneous game. If the Stackelberg leader increases its quantity, the Stackelberg follower reacts by reducing its quantity, which counteracts the price depressing effect of increasing the quantity in the first place: The Stackelberg follower's reaction increases the Stackelberg follower's marginal profit.

Formally, the Stackelberg leader's marginal profit, anticipating the Stackelberg follower's reaction, is

$$\frac{\partial \pi(s_1, s_2(s_1))}{\partial s_1} = p(s_1 + s_2(s_1)) - c'(s_1) + s_1 \cdot p'(s_1 + s_2(s_1)) \cdot \left(1 + \frac{\partial s_2(s_1)}{\partial s_1}\right). \tag{1.26}$$

Evaluated at the Cournot-Nash equilibrium quantities, it is strictly positive,

$$\begin{aligned} \left.\frac{\partial \pi(s_1, s_2(s_1))}{\partial s_1}\right|_{s_1=q^C=s_2} &= p(2q^C) - c'(q^C) + q^C \cdot p'(2q^C) \cdot \left(1 + \left.\frac{\partial s_2(s_1)}{\partial s_1}\right|_{s_1=q^C}\right) \\ &= \underbrace{q^C}_{>0} \cdot \underbrace{p'(2q^C)}_{<0} \cdot \underbrace{\left(\left.\frac{\partial s_2(s_1)}{\partial s_1}\right|_{s_1=q^C}\right)}_{<0} > 0, \end{aligned} \tag{1.27}$$

where the last equality follows from the Cournot-Nash equilibrium condition. Strategic substitutability implies that $\frac{\partial s_2(s_1)}{\partial s_1} < 0$.

There is a relationship between the well-known profit function of the Stackelberg leader and firm 1's equilibrium profit for different price caps. If the Stackelberg leader and firm 1 choose the same quantity, so do the Stackelberg follower and firm 2 because their best response functions are the same. So, firm 1's equilibrium profit in a clearing equilibrium is the same as the Stackelberg leader's profit when it chooses firm 1's equilibrium quantity. To get firm 1's equilibrium profit in a rationing equilibrium, the additional transfer from firm 1 to the consumers (see Subsection 1.4.1) has to be subtracted from the Stackelberg leader's profit. The reason is that in rationing equilibria, firm 1 has a different price—the price cap—than the Stackelberg leader when it chooses firm 1's equilibrium quantity—the price that firm 2 gets in the rationing equilibrium.

Although firm 1's price is capped, it can actually profit from having a price cap. This result follows from the relationship with the Stackelberg leader's profit function and from the monotonicity of firm 1's equilibrium quantity in the price cap. When the price cap is the Cournot-Nash price, firm 1's equilibrium quantity is the Cournot-Nash quantity. Marginally decreasing the price cap increases firm 1's equilibrium quantity. Because the Stackelberg leader's profit increases when marginally increasing its quantity starting at the Cournot-Nash quantity, so does firm 1's equilibrium profit. Thus, firm 1's equilibrium profit with a price cap marginally below the Cournot-Nash price exceeds the benchmark profit without a price cap. The reason is the strategic effect—the crowding out—that firm 1's price cap exerts on the other firm.

Assuming additionally that the Stackelberg leader's profit is strictly quasi-concave in its quantity makes firm 1's equilibrium profit strictly quasi-concave in the price cap for the clearing equilibria. As is generally true, if the price cap decreases starting from the Cournot-Nash price, the profit initially increases. Because of strict quasi-concavity, the profit keeps increasing up to its maximum at the unique Stackelberg equilibrium price. As Proposition 1.2 has shown, the Stackelberg equilibrium price lies within the range of clearing equilibria. If the price cap decreases further, firm 1's equilibrium profit decreases again.

In the rationing equilibria, firm 1's equilibrium profit decreases monotonically in a decreasing price cap: Both firm 1's quantity and the price it receives decrease.

Figure 1.13 illustrates firm 1's equilibrium profits for different price caps in the special case of linear demand and quadratic cost. For these functional forms, the Stackelberg leader's profit function is strictly quasi-concave in its quantity, so the equilibrium profit of firm 1 increases until the price cap is the Stackelberg equilibrium price and then decreases. Below the cutoff, it always decreases.

The relationship to the game with asymmetric price caps offers a novel interpretation for the Stackelberg game: In the first stage, the Stackelberg leader commits to an individual price cap.[16] In the second stage, both firms choose quantities simultaneously.

16. It is not necessary to set a price cap for the Stackelberg follower, too. In fact, if the Stackelberg leader could only choose a symmetric price cap for both firms, it might hurt itself. With symmetric price caps, there is a continuum of equilibria, that includes the reversed quantities (Okumura, 2017). So if

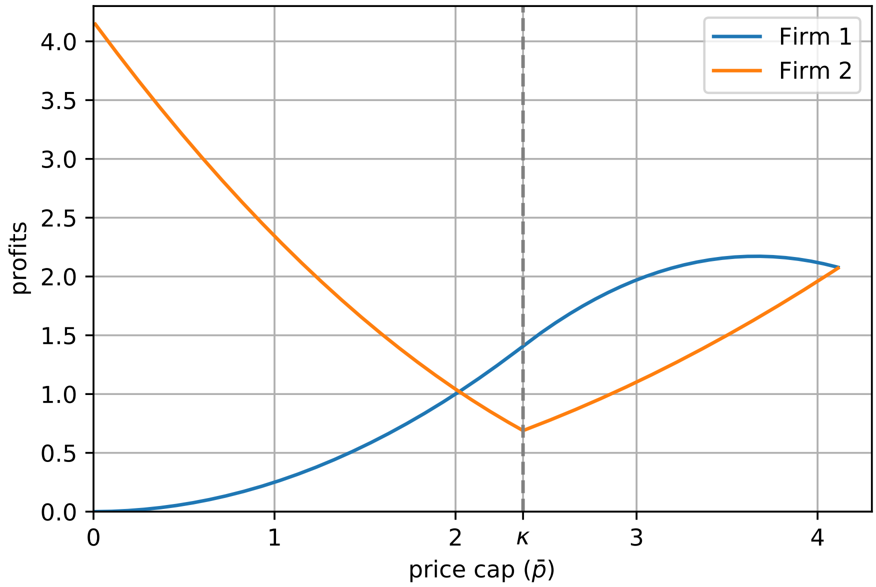

Figure 1.13. The equilibrium profits of the firms when the cost functions are $c(q_i) = q_i^2$ and the inverse demand is $p(q) = 10 - 5 \cdot q$.

The subgame perfect equilibrium in this alternative interpretation can be found by backward induction. In the second stage, given a price cap for the Stackelberg leader, the firms play the unique equilibria analyzed in the main part. In the first stage, the Stackelberg leader optimally chooses a Stackelberg equilibrium price as its price cap. The equilibrium outcome is identical to the outcome in the traditional Stackelberg game. Committing to a price cap can replace the commitment to a quantity because it eliminates the inframarginal loss in the second stage, so choosing a larger quantity becomes optimal for firm 1.

1.4.4 Both Firms Have Price Caps

This subsection deals with the case in which both firms have price caps. Without loss of generality, assume that firm 2 has a strictly higher price cap.

Firm 2's price cap might affect its own equilibrium quantity, but never firm 1's equilibrium quantity. If firm 1's price cap is above the cutoff, κ, there is a clearing equilibrium and firm 2's price cap does not bind, so it neither affects firm 1's nor its own equilibrium quantity. If firm 1's price cap is below the cutoff, κ, there is a

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the Stackelberg leader chooses the Stackelberg equilibrium price as a price cap, there is an equilibrium in which it makes the Stackelberg follower's profit.

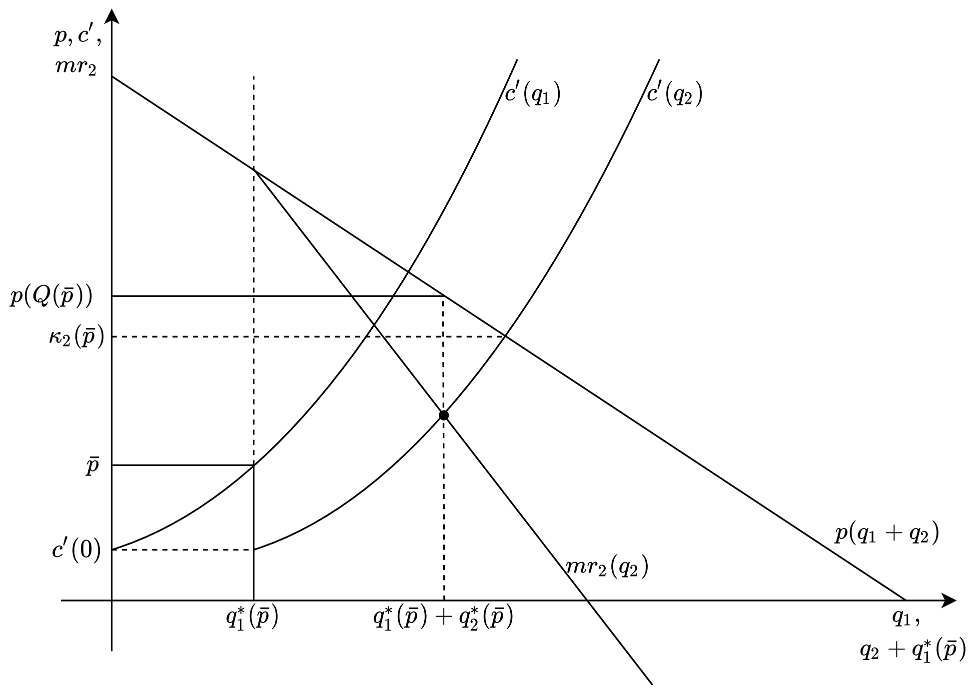

Figure 1.14. Beginning on the left, firm 1 produces the quantity at which its price cap $\bar{p}$ and its marginal cost intersect, $q_1^*(\bar{p})$. At this quantity, q_2 starts at 0. Firm 2's inverse residual demand curve is the original inverse demand curve from hereon (starting to count q_2 at $q_1^*(\bar{p})$ is the same as shifting the original inverse demand curve to the left by $q_1^*(\bar{p})$). Firm 2 produces the quantity at which its marginal revenue and its marginal cost intersect, $q_2^*(\bar{q})$. The corresponding market-clearing price, $p(q_1^*(\bar{p}) + q_2^*(\bar{p}))$, is abbreviated with $p(Q(\bar{p}))$. The price at which firm 2's marginal cost and the inverse residual demand curve intersect is $\kappa_2(\bar{p})$. If firm 2's price cap, $\bar{p}_2$, is above $p(Q(\bar{p}))$, it does not bind and is ineffective. If $p(Q(\bar{p})) > \bar{p}_2 \geq \kappa_2(\bar{p})$, there is a partially rationing equilibrium. If $\bar{p} < \kappa_2(\bar{p})$, there is a doubly rationing equilibrium.

rationing equilibrium. Because firm 1's price cap strictly binds, it plays the price-takers quantity, $(c')^{-1}(\bar{p})$ in any equilibrium.

Whether firm 2's price cap affects its own quantity in a rationing equilibrium depends on its level. As firm 2 is the monopolist on the market for the residual demand, there are the same three cases as for the monopolist in Appendix 1.B: If the price cap is above the monopoly price, $p(q_1^*(\bar{p}) + q_2^*(\bar{p}))$, it has no effect; if the price cap is between the monopoly price and the competitive price on the market for residual demand, $\kappa_2(\bar{p})$, it binds and the price clears the market; if the price cap is between the marginal cost for the first unit and the competitive price, the price cap binds and the price does not clear the market.

Proposition 1.4. *Assume that firm 1 has the price cap $\bar{p}$ and firm 2 has a price cap $\bar{p}_2$. Without loss of generality, assume that $\bar{p} < \bar{p}_2$.*

Define $\kappa_2(\bar{p})$ as the value at which $p(q_1^*(\bar{p}) + q_2)$ and $c'(q_2)$—both functions of q_2—intersect; illustrated in Figure 1.14. $\kappa_2(\bar{p})$ is decreasing in $\bar{p}$.

(i) If $\bar{p} \geq \kappa$, the only pure-strategy Nash equilibrium is the clearing equilibrium described in Theorem 1.1. It does not depend on $\bar{p}_2$.

(ii) If $\bar{p} < \kappa$ and $\bar{p}_2 \geq p(q_1^*(\bar{p}) + q_2^*(\bar{p}))$ (as defined in Theorem 1.2), the only pure-strategy Nash equilibrium is the rationing equilibrium described in Theorem 1.2. It does not depend on $\bar{p}_2$.

(iii) If $\bar{p} < \kappa$ and $p(q_1^*(\bar{p}) + q_2^*(\bar{p})) > \bar{p}_2 \geq \kappa_2(\bar{p})$, the only pure-strategy Nash equilibrium is a partially rationing equilibrium. In this equilibrium, firm 1 produces $q_1^*(\bar{p}) = (c')^{-1}(\bar{p})$ and firm 2 produces the quantity that brings the market-clearing price to $\bar{p}_2$, which is $q_2^*(\bar{p}, \bar{p}_2) = p^{-1}(\bar{p}_2) - q_1^*(\bar{p})$.

(iv) If $\bar{p} < \kappa$ and $\kappa_2(\bar{p}) > \bar{p}_2$, the only pure-strategy Nash equilibrium is a doubly rationing equilibrium. In this equilibrium, firm 1 produces $q_1^*(\bar{p}) = (c')^{-1}(\bar{p})$ and firm 2 produces $q_2^*(\bar{p}_2) = (c')^{-1}(\bar{p}_2)$, which depends only on the own price cap.

Proof. The proof is in Appendix A, Subsection 1.A.7. □

When the marginal cost is constant, neither the existence nor the level of the higher price cap influence the equilibrium quantities, as Proposition 1.5 shows. The reason is that there are only clearing equilibria in which firm 2's price cap cannot bind.

Proposition 1.5. *Assume that the marginal cost is constant. Assume that firm 1 has the price cap $\bar{p}$ and firm 2 has a price cap $\bar{p}_2$ with $\bar{p} < \bar{p}_2$. Then, for all $\bar{p} \in (c, p^C)$, the only pure-strategy Nash equilibrium is the clearing equilibrium described in Proposition 1.3.*

Proof. The arguments in the proof of Proposition 1.3 are still true: There can be no equilibrium with $q_1 + q_2 < \bar{q}$ because firm 1 could profitably deviate, and there can be no equilibrium with $q_1 + q_2 > \bar{q}$ because at least one firm could profitably deviate.

So, if there is an equilibrium, it has to be that $q_1 + q_2 = \bar{q}$. But if the total quantity is $\bar{q}$, the price cap of firm 2 does not bind and it has, thus, no effect. □

These results are extreme, especially for clearing equilibria. The continuum of equilibria in the case of symmetric price caps described in Okumura (2017) collapses to one of its boundaries if there is the slightest asymmetry in the price caps. The welfare implication of this result is that the inefficiency on the production side arises discontinuously when one of two perfectly symmetric price caps is changed marginally.

The existence of the continuum of equilibria hinges on the fact that both firms' price caps bind simultaneously, so both firms are at the drop in their marginal profits. This creates some leeway in satisfying the optimality conditions. Asymmetric price caps cannot, however, bind simultaneously. So, the firm with the non-binding price cap has a unique best response, pinning down the best response of the other firm.

1.4.5 Heterogeneous Cost Functions

In the main part, I assume that the firms have the same cost function. This assumption makes the interpretation of the price cap's welfare effects simpler, but it is, for example, not necessary to determine the signs of the comparative statics in the price cap.

Heterogeneous cost functions do not change the comparative statics of the equilibrium quantities in the price cap. As mentioned above, the comparative statics follow from the assumption that the inverse demand function is strictly log-concave, implying strategic substitutability. More accurately, the comparative statics depend on the slope of firm 2's best response function's being strictly between -1 and 0, which is implied by the strict log-concavity of the residual inverse demand function and the marginal cost's being strictly increasing (see the proof of Proposition 1.1). Neither the strict log-concavity of firm 2's residual inverse demand function nor the monotonicity of its marginal cost needs homogeneous costs. In fact, the slope of firm 2's best response function does not depend on firm 1's cost function at all.

To see how heterogeneous cost functions change the equilibrium quantities, it is helpful to look at the best response functions in a simple example. Let firm 1 have higher marginal cost for each quantity: The marginal cost of firm 1 is $\alpha \cdot c'(q_1)$, with $\alpha > 1$, whereas the marginal cost of firm 2 is $c'(q_2)$. Figure 1.15 illustrates how the best response function of firm 1 differs from the case with homogeneous cost. The first of the three parts is the best response of a standard Cournot duopolist, so higher marginal costs lead to smaller best responses. Because there can still be no equilibrium in the first part, this does not matter.[17] The second part, $\bar{q} - q_2$, is the quantity at which the price cap just binds, so it is independent of the cost function. The third, vertical part, $(c')^{-1}(\bar{p})$ is smaller when the marginal cost is higher.

As the second part of firm 1's best response function is unchanged, so are the equilibrium quantities in clearing equilibria. The cutoff between clearing and rationing equilibria, however, is a larger price cap: The marginal profit above the drop hits zero at a higher price cap because the higher marginal cost reduces the marginal profit. In rationing equilibria, firm 1's equilibrium quantity is smaller and so firm 2's equilibrium quantity, its best response, is larger due to strategic substitutability.

The trade-off between total quantity and production efficiency remains. Although firm 1 produces less than firm 2 in the benchmark equilibrium without price caps, its marginal cost is larger.[18] If a price cap above the cutoff is introduced

17. Proposition 1.1 follows from the fact that the slopes of both best response functions are strictly between -1 and 0.

18. To see this, look at the optimality conditions

$$\begin{aligned} p(q_1^* + q_2^*) + q_1^* \cdot p'(q_1^* + q_2^*) - \alpha \cdot c'(q_1^*) &= 0 \\ p(q_1^* + q_2^*) + q_2^* \cdot p'(q_1^* + q_2^*) - \quad c'(q_2^*) &= 0. \end{aligned} \tag{1.28}$$

Because the marginal cost is strictly increasing and $\alpha > 1$, it follows that $q_1^* < q_2^*$. Then,

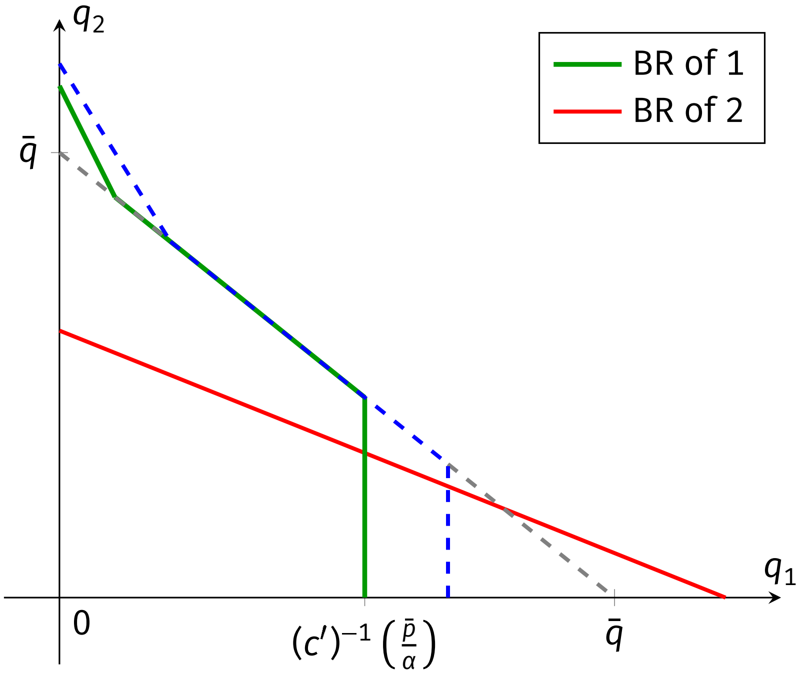

Figure 1.15. The red curve is the best response function of firm 2. The blue, dashed curve would be the best response function of firm 1 if it had the same cost function as firm 2. The green curve is the actual best response function of firm 1 when its marginal cost is $\alpha \cdot c'(q_1)$, with $\alpha > 1$, whereas the marginal cost of firm 2 is $c'(q_2)$.

and marginally decreased, firm 1's equilibrium quantity increases—the comparative statics having the same sign as with homogeneous cost. Thus, firm 1 produces more and its expensive production crowds out the socially cheaper production of firm 2.

If, on the other hand, firm 2 has higher marginal costs than firm 1, the trade-off vanishes for at least some price caps. If firm 2's marginal cost function is $\alpha \cdot c'(q_2)$, firm 2's best response is lower than in the symmetric case for all q_1. In the benchmark equilibrium without price caps, it is now firm 2 that has the higher marginal cost. If firm 1 gets a price cap marginally below the Cournot-Nash price, firm 1 produces more, crowding out some of firm 2's expensive production. Thus, starting in any clearing equilibrium in which firm 2 has the higher marginal cost, marginally decreasing the price cap not only increases the total quantity but also makes the production more efficient. So, lower price caps unambiguously improve the welfare. Due to the symmetry between clearing and rationing equilibria, this is also true for higher price caps in the corresponding rationing equilibria.

$$p(q_1^* + q_2^*) + q_1^* \cdot p'(q_1^* + q_2^*) \; > \; p(q_1^* + q_2^*) + q_2^* \cdot p'(q_1^* + q_2^*). \tag{1.29}$$

Thus, $\alpha \cdot c'(q_1^*) > c'(q_2^*)$.

1.4.6 Mixed-Strategy Nash Equilibria

In the main part, I restrict the analysis to pure-strategy Nash equilibria. I cannot rule out that mixed-strategy Nash equilibria exist. There is, however, an alternative assumption on the primitives implying strategic substitutability under which all results remain true and under which no mixed-strategy Nash equilibrium exists.

Keep all assumptions in this chapter except for Assumption 1.1 that the inverse demand function is strictly log-concave. Replace Assumption 1.1 with Assumptions 2 and 3 of Theorem 3 in Novshek (1985, p. 90):[19]

$$\exists Z \; : \; p(Z) \; = \; 0 \tag{1.30}$$

$$\forall q \in [0, Z) \; : \; p'(q) + q \cdot p''(q) \; < \; 0. \tag{1.31}$$

These assumptions imply that, without price caps, each firm's marginal revenue is weakly decreasing in the other firm's quantity and, thus, also in the own quantity. Combined with strictly increasing marginal cost, the assumptions imply that each firms' profit function is strictly concave in the own quantity.

In the game with price caps, firm 2's profit function is the same as that of a standard Cournot duopolist, so it is still strictly concave in q_2 for all q_1. Then, firm 2's expected profit, facing a mixed strategy of firm 1, is also strictly concave. Thus, its best response is unique. So, firm 2 has to play a pure strategy in each Nash equilibrium. The best response of firm 1 to a pure strategy is also a pure strategy, as shown above. Thus, in any Nash equilibrium, both firms play pure strategies.

1.4.7 Proportional Rationing

As explained in Section 1.2, asymmetric price caps make it necessary to assume a rationing rule to determine the inverse residual demand function. In the main part, I have considered the efficient rationing rule, meaning that the consumer surplus is always maximized. The other common assumption in the literature is proportional rationing. With proportional rationing, the firms serve all consumers that want to buy at a price with equal probability.

To understand what this means, it is helpful to consider a microfoundation of a demand function: There is a continuum of consumers with unit demand and each consumer's valuation (their willingness to pay) for the good is drawn according to

19. Curiously, the alternative assumptions from Novshek (1985) are neither stronger nor weaker than the original assumption (which is the assumption from Amir, 1996). I have chosen the original assumption because the range on which the assumption holds could be constrained such that it is weaker than the alternative assumption while all of my results remain true (I have not actually constrained the range in Assumption 1.1 to keep the exposition as simple as possible). For a discussion about the relationship between the two alternative assumptions and how to constrain the range of Assumption 1.1, see Amir (2005).

a distribution.[20] The demand function states for each price the measure of people with valuations exceeding this price—who are willing to buy at that price.

Proportional rationing includes an implicit assumption about the timing of purchasing actions: If there is excess demand at the price cap, everyone tries to buy at the cheaper price from firm 1 first.[21] The quantity of firm 1 is then allocated by proportional rationing. Consumers that get served leave the market.

Firm 2's residual demand consists of those consumers that did not get served by firm 1 and remain on the market. If there is no excess demand for the quantity of firm 1, $q_1 \geq \bar{q}$, there is no rationing and all consumers with valuations above $p(q_1)$ get served and leave the market. Firm 2's residual demand is then described by $p(q_1+q_2)$ as in the standard Cournot case. If there is excess demand for the quantity of firm 1, $q_1 < \bar{q}$, each consumer with a valuation of at least $\bar{p}$ has the same probability of getting served by firm 1 and leaving the market, $\frac{q_1}{\bar{q}} < 1$. The counter probability of not getting served and remaining on the market is $\frac{\bar{q}-q_1}{\bar{q}}$. The densities of the remaining consumers' valuations consist of two parts. Up to the price cap of firm 1, the measures of consumers in firm 2's residual demand are given by the initial density—as none of these consumers was served. For the prices above the price cap, the initial density is multiplied with the probability of remaining on the market. The construction of the density of the remaining consumers' valuations is illustrated in Figure 1.16.

The inverse residual demand function of firm 2 depends on whether firm 1's price cap binds or not. If $q_1+q_2 < \bar{q}$, the price cap binds and firm 2 sells only to those who participated in the lottery for the quantity of firm 1 but lost. When deriving the inverse demand function from the distribution of valuations of the consumers, the axis are inverted, so firm 2's inverse residual demand function is $p\left(\frac{\bar{q}}{\bar{q}-q_1} \cdot q_2\right)$. If $q_1+q_2 \geq \bar{q}$, the price cap does not bind. As the price clears the market, the inverse residual demand function of firm 2 is given by $p(q_1+q_2)$. To sum up, the inverse residual demand function of firm 2 is

$$p_2(q_1,\bar{p},q_2) = \begin{cases} p\left(\frac{\bar{q}}{\bar{q}-q_1} \cdot q_2\right) & \text{if } q_2 < \bar{q}-q_1 \\ p(q_1+q_2) & \text{if } q_2 \geq \bar{q}-q_1. \end{cases} \tag{1.32}$$

Intuitively, the inverse residual demand function is a compression in the quantity direction of the original inverse demand function for $q_2 < \bar{q}-q_1$ as some consumers with a high valuation have already been served by firm 1. The inverse residual demand and how it can be constructed graphically from the original inverse demand, firm 1's quantity, and the price cap are illustrated in Figure 1.17.

20. The continuum prevents market power problems, justifies considering goods to be divisible, and implies that the demand is not stochastic.

21. With efficient rationing, the timing does not matter. Firm 2 knows that in the end, all production will end up with the measure q_1+q_2 of consumers with the highest willingness to pay, so the last served consumer pins down its price at $p(q_1+q_2)$.

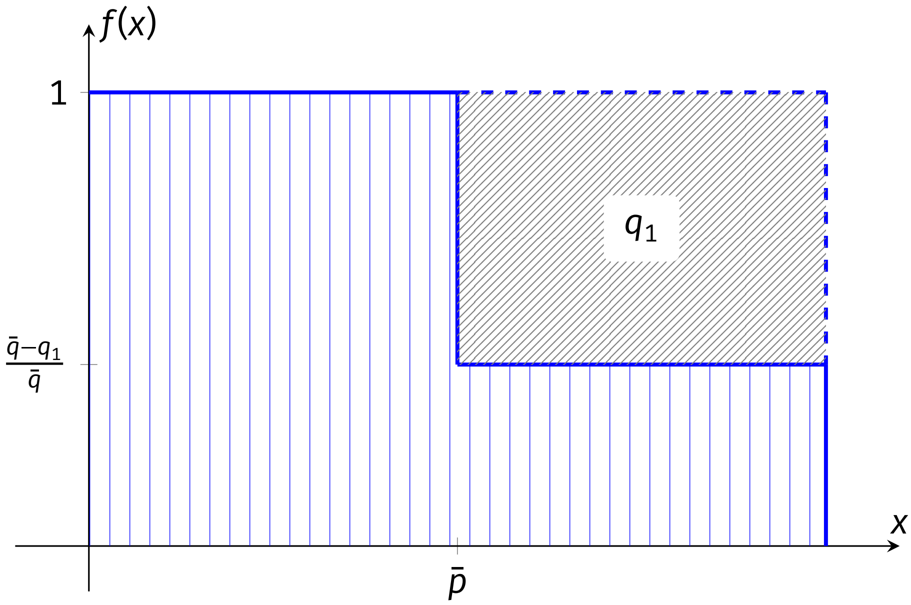

Figure 1.16. On the x-axis is the valuation, on the y-axis is the density. For simplicity, I chose a uniform distribution, which yields a linear demand function. Above $\bar{p}$, some consumers got served and left the market. The measure of consumers that got served is the gray area of size q_1. The blue line is the density of the remaining consumers' valuations, from which the residual demand for firm 2 is derived.

Firm 2's profit function consists of two functions,

$$\pi_2(q_1,\bar{p},q_2) = \begin{cases} q_2 \cdot p\left(\frac{\bar{q}}{\bar{q}-q_1} \cdot q_2\right) - c(q_2) & \text{if } q_2 < \bar{q}-q_1 \\ q_2 \cdot p(q_1+q_2) - c(q_2) & \text{if } q_2 \geq \bar{q}-q_1. \end{cases} \tag{1.33}$$

Both functions are individually strictly quasi-concave in q_2 because the corresponding inverse residual demand functions are strictly log-concave in q_2. Thus, the profit-maximizing quantity of each individual function is determined by the intersection of the marginal profit and zero.

Determining the profit-maximizing quantity of the actual profit function is, however, complicated. The marginal profit jumps upwards at $q_2 = \bar{q} - q_1$. The left-derivative and at the right-derivative at $q_2 = \bar{q} - q_1$ are

$$\begin{aligned} \left.\frac{\partial_-\pi_2(q_1,\bar{p},q_2)}{\partial q_2}\right|_{q_2=\bar{q}-q_1} &= \bar{p} + (\bar{q}-q_1) \cdot \frac{\bar{q}}{\bar{q}-q_1} \cdot p'(\bar{q}) - c'(\bar{q}-q_1) \quad < \\ \left.\frac{\partial_+\pi_2(q_1,\bar{p},q_2)}{\partial q_2}\right|_{q_2=\bar{q}-q_1} &= \bar{p} + (\bar{q}-q_1) \cdot p'(\bar{q}) - c'(\bar{q}-q_1). \end{aligned} \tag{1.34}$$

The inequality follows from $\frac{\bar{q}}{\bar{q}-q_1} > 1$. As some consumers with valuations above $\bar{p}$ have left the market, firm 2 depresses its price more by increasing its quantity, so the inframarginal loss is larger.

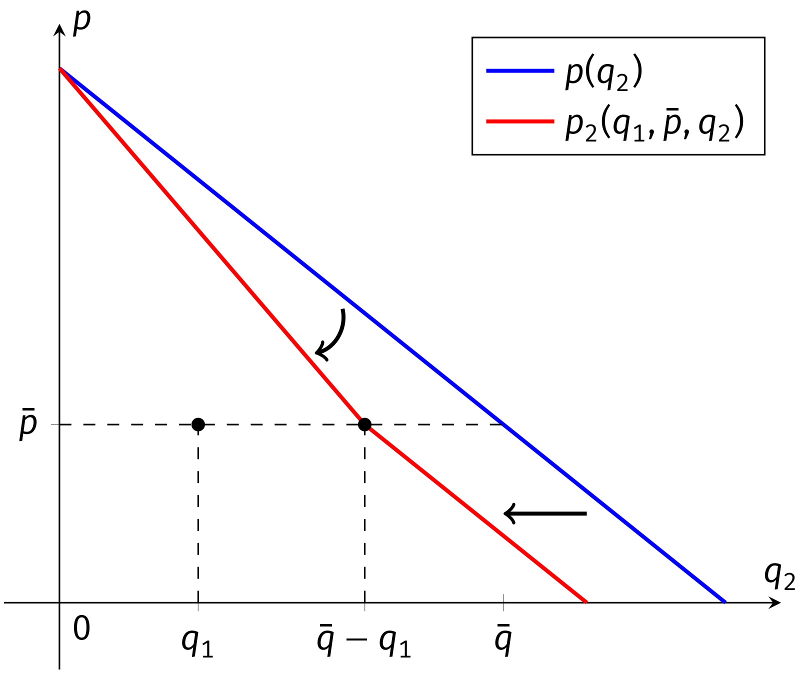

Figure 1.17. The inverse demand function is in blue. Firm 1 has the price cap $\bar{p}$ and chooses the quantity $q_1 < \bar{q}$. Firm 2's inverse residual demand function is in red. Consumers with a valuation larger than $\bar{p}$ are served with probability $\frac{q_1}{\bar{q}}$, so firm 2's inverse residual demand curve is a compression in the x-direction of the inverse demand curve for $q_2 < \bar{q} - q_1$. For a linear inverse demand curve, compressing is the same as tilting inwards. For $q_2 \geq \bar{q} - q_1$, the inverse demand curve is shifted to the left as in the standard Cournot case to get the inverse residual demand curve.

Thus, there are three cases for the profit-maximizing quantity of firm 2, depending on the sign of its marginal profit before and after the jump at $\bar{q} - q_1$. If the marginal profit begins in the negative and is still negative after the jump, the profit-maximizing quantity is given by the profit-maximizing quantity of the function that is the first part of firm 2's profit function. If the marginal profit begins in the positive, then it is still positive after the jump, and the profit-maximizing quantity is given by the profit-maximizing quantity of the function that is the second part of firm 2's profit function. If, however, the marginal profit begins in the negative and is positive after the jump, then either of the profit-maximizing quantities in the two parts may be the profit-maximizing quantity: Firm 2's profit function is not strictly quasi-concave in q_2.

As a consequence of firm 2's marginal profit's jumping up, clearing equilibria do not exist (see Proposition 1.6). The marginal profit jumps at exactly the quantity that makes firm 1's price cap just bind. If firm 2 provides less quantity, there is proportional rationing, so firm 2 has a stronger effect on its price. If firm 2 provides more quantity, there is price rationing, so firm 2 has a weaker effect on its price. For these reasons, it is optimal for firm 2 to either provide less quantity and to profit from a higher price or to expand its quantity further and to profit from depressing its price less. Thus, it is never a best response of firm 2 to bring the total quantity to $\bar{q}$ and to make firm 1's price cap just bind.

Moreover, as the fact that firm 2's best response function jumps over the middle part of firm 1's best response function suggests, there is no equilibrium at all for some price caps below the Cournot-Nash price. Proposition 1.6 shows that firm 1's price cap has to bind in all equilibria; otherwise at least one firm could profitably deviate as both firms became standard Cournot duopolists. Thus, in any equilibrium, the total quantity is either $\bar{q}$ (so the equilibrium would be clearing) or less (so the equilibrium would be rationing). Therefore, to show that there is no equilibrium, it is sufficient to verify that there is no rationing equilibrium for a given price cap. Intuitively, firm 2 must discretely decide whether to stay on its small isolated market and to produce a small quantity such that there is rationing or to produce a large quantity such that both firms sell on the same market without a price cap or rationing. Due to the jump in its marginal profit, the latter would mean a total quantity exceeding $\bar{q}$. Numerical exercises verify that for high price caps, given the rationing equilibrium quantity of firm 1, it is optimal for firm 2 to produce a large quantity: There is no equilibrium.

Rationing equilibria exist when it is optimal for firm 2 to stay on its isolated small market, as this means that firm 1's price cap strictly binds and firm 1 acts as a price-taker. With proportional rationing, the cutoff below which there are rationing equilibria is larger as Proposition 1.6 shows. The comparative statics, have the same signs: If the price cap falls, firm 1 produces less and firm 2 produces more.

The trade-off between a larger total quantity and the production efficiency, however, might vanish. With proportional rationing, the slope of firm 2's best response function is not necessarily bounded between 0 and -1. Thus, when the price cap falls and firm 1 produces less, it might be that firm 2 expands it quantity sufficiently to increase the total quantity. The reason is an additional positive effect on firm 2's quantity when the price cap decreases: More consumers with high valuations remain on the market as firm 1 reduces its supply and more consumers with lower valuations enter the lottery for the quantity of firm 1. Thus, firm 2's residual demand gets less compressed and firm 2's depressing effect on its price gets attenuated.

There is, however, an additional welfare-reducing effect from misallocation on the consumers' side of the market. For fixed quantities, a lower price cap leads to more misallocation because more consumers with a low valuation enter and win the lottery for the quantity of firm 1. For adjusting quantities in rationing equilibria, the effect is ambiguous: While more consumers with low valuations enter the lottery, firm 2 replaces some of the quantity of firm 1, and the quantity of firm 2 is allocated efficiently.

Proposition 1.6.

(i) There are no equilibria with $q_1 + q_2 > \bar{q}$ *(Proposition 1.1 remains true with proportional rationing).*

(ii) There are no clearing equilibria with $q_1 + q_2 = \bar{q}$.

(iii) There are rationing equilibria with $q_1 + q_2 < \bar{q}$. *There is an* $\epsilon > 0$, *such that the unique equilibrium for each price cap in* $(c'(0), \kappa + \epsilon)$ *is a rationing equilibrium. When the price cap decreases, firm 1 decreases and firm 2 increases its quantity.*

Proof. The proof is in Appendix A, Subsection 1.A.8. □

1.4.8 Symmetric Regulation of Vertically Differentiated Products

Often, price cap regulation is symmetric for all firms. If the products of the firms are, however, vertically differentiated, symmetric regulation is in fact asymmetric, as the following example shows.

Example 1.1. In the initial situation, there is no price cap. The goods are vertically differentiated, with firm 1 offering the superior product. The inverse residual demand function for the worse good of firm 2 is $p_2(q_1 + q_2)$. The inverse residual demand function for the superior good of firm 1 is $p_1(q_1 + q_2) \equiv p_2(q_1 + q_2) + x$, with $x > 0$. Thus, the price for the good of firm 1 is always by x units larger than the price for the good of firm 2. A possible microfoundation is that each consumer's marginal willingness to pay is larger by x for good 1 than for good 2, reflecting the superiority of good 1. This form of vertical differentiation is proposed in Ritz (2018).

Now, both firms get the same price cap $\bar{p}$.

The price caps stop binding at different total quantities because the goods are differentiated. For firm 2, the price cap stops binding if the total quantity exceeds $\bar{q}_2 \equiv p_2^{-1}(\bar{p})$. Because the price for the good of firm 1 is larger for each total quantity, price cap stops binding for firm 1 at a larger quantity, $\bar{q}_1 \equiv p_2^{-1}(\bar{p} - x)$. Effectively, firm 1's price cap is tighter than firm 2's price cap.

Because the firms' price caps cannot just bind simultaneously, the situation with vertically differentiated goods is comparable to the situation with asymmetric price caps (Subsection 1.4.4). Symmetric price caps on vertically differentiated goods cause the same misallocation of quantities on the producers' side of the market as asymmetric price caps. There is, however, an additional benefit to the welfare because the production is distorted in favor of the superior good.

1.4.9 Applications

One application of my model is concerned with the modeling of mixed oligopolies (Cremer, Marchand, and Thisse, 1989, Fraja and Delbono, 1989, and Fraja and Delbono, 1990). In mixed oligopolies, the oligopolists have different objectives. Private firms maximize their profits and public firms maximize the welfare. A typical finding is that oligopolists trying to maximize the welfare sometimes make larger profits than profit-maximizing firms because their expanding their quantity makes other firms reduce their quantities.[22]

I propose a new approach to modeling mixed oligopolies: Letting the regulator choose a price cap for the public firm instead of choosing a quantity. Price capping public firms seems more realistic, is equally tractable, and has slightly different implications that could be used to empirically evaluate the different modeling approaches.

A first result follows from the analysis of heterogeneous cost functions (see Subsection 1.4.5). A common assumption in the literature on mixed oligopolies is that the public firm is less efficient than the private firm. In such settings, applying a price cap to the public firm leads to a trade-off between production efficiency and total quantity. It would be better if the regulator could apply price caps to the private firms.

My analysis, however, also shows that even if the public firm is less efficient, price-capping the public firm can improve the welfare. Future research could focus on how to identify the optimal price cap by using only observable data, such as market shares or whether a market is segmented into low-price and high-price parts.

Another application is price stickiness. In some situations, shocks leave prices unchanged, although standard arguments would predict price changes. My model is applicable to price stickiness in two ways.

On the one hand, as mentioned in the introduction, asymmetric price stickiness across firms are a microfoundation of asymmetric price caps. For example, the fairness considerations of the “invisible handshake” (Okun, 1981): Consumers become regular customers to save on search costs and the firm, in return, forgoes “unfair” price increases. Thus, when a shock happens that passing on would be considered unfair, a firm with a large share of regular customers would have a high cost of adjusting its price—a price cap. A firm with few regular customers would not have a high cost of adjusting its price. My model offers a framework to think about and to estimate the welfare consequences of such price stickiness.

On the other hand, my model offers a novel explanation for why firms might want to make their prices stickier to increase their profits.[23] As Subsection 1.4.3 has illustrated, the firm with a price cap can make larger profits than in the benchmark. By making price adjustments costly, firms can make the current Cournot-Nash price

22. For the benevolent firm, it is of course bad that the private firms reduce their quantities as the objective is to maximize the welfare, not the profit.

23. One existing explanation is habit-forming goods (Nakamura and Steinsson, 2011): Consumers do not consume habit-forming goods because they anticipate being exploited afterwards—unless the firm makes its prices sticky to commit to not exploiting the consumers.

their price cap. If a small cost shock happens, increasing the Cournot-Nash price marginally, a firm that has capped its price at the old Cournot-Nash price crowds out the other firm's production and makes larger profits.

1.5 Conclusion

My innovation is adding asymmetric price caps to the canonical Cournot quantity competition model. Besides standard regularity assumptions, the asymmetric price caps make it necessary to assume a rationing rule to determine the firms' residual demands. I consider the efficient and the proportional rationing rule.

My main result is that asymmetric price caps distort the production efficiency. Moreover, in many settings, there is a trade-off between the total quantity in the market and the production efficiency. Hence, the welfare is not necessarily improved when a price cap is changed and a larger quantity is traded. Nevertheless, there are always asymmetric price caps that increase the welfare compared to the benchmark without any price caps.

My contribution to the literature is showing that with asymmetric price caps, distorted production is not a possibility, but an inevitability. In the existing literature with symmetric price caps, there is a continuum of equilibria with the same total quantities, which also contains a symmetric equilibrium with efficient production (Okumura, 2017). A tiny asymmetry in the price caps is sufficient for the continuum to collapse to a unique equilibrium at one of its boundaries, which potentially causes substantial waste in the production. An example suggests that an asymmetry in the goods might have the same effect: A setting with symmetric price caps on vertically differentiated goods can be reformulated into a setting with asymmetric price caps on identical goods.

Further, while clearing equilibria have been known, I introduce rationing equilibria. If the lower price cap is above a cutoff, the unique pure-strategy Nash equilibrium is a clearing equilibrium. Both firms' equilibrium price is the lower price cap and there is no rationing in equilibrium. Because the firm with the binding price cap does not depress its own price when expanding its quantity as long as the price cap binds, it produces a larger quantity in equilibrium. A larger production of the firm with the binding price cap crowds out production of the other firm. Whenever this means that socially expensive production crowds out socially cheap production, there is the trade-off between quantity and production efficiency. For the firms, the crowding out implies that a firm can induce the Stackelberg equilibrium by committing to a price cap instead of a quantity in a first stage.

If the lower price cap is below a cutoff, the unique pure-strategy Nash equilibrium is a rationing equilibrium. The firms have different equilibrium prices and there is non-price rationing in equilibrium. The firm with the lower price cap behaves as a price-taker and produces until its marginal cost equals its price cap, at which it sells. The other firm sells at a higher price. Only in this case, it makes a difference whether the other firm has a price cap, too. Because the other firm is the monopolist on the market for residual demand, its price cap has the same effect as a price cap in a monopoly: It might be non-binding, it might bind while the price clears the market for residual demand, or it might bind while there is also non-price rationing on the market for residual demand.

Although seemingly different, both types of equilibria are symmetric to each other if the higher price cap is non-binding. Whenever the total equilibrium quantity is the same, both firms' equilibrium quantities are the same. Thus, also the welfare and the trade-off between quantity and production efficiency are the same.

In the special case with linear demand and quadratic cost, observable equilibrium outcomes predict whether price-capping only one firm is better than no regulation: Whenever the price-capped firm produces a larger quantity, the welfare is strictly larger than without a price cap, although the production is distorted. In particular, at the price cap at which the market starts to segment into a low-price and a high-price part, which might be observable, the welfare is higher than without a price cap, although it attains a local minimum.

A possible avenue for further research is adjusting the model to evaluate real-world regulation. As the example of linear demand and quadratic cost has shown, making structural assumptions can allow identifying welfare implications from observable data. Another avenue for future research is to tackle open questions. The subsection on proportional rationing has shown that it is difficult to determine equilibria with this rationing rule. As a consequence, it is also difficult to evaluate the welfare effects of asymmetric price caps, taking into account the misallocation on the consumers' side of the market. Future research could extend the results for proportional and alternative rationing rules, maybe combining results with the general approach in (Bulow and Klemperer, 2012), which takes industry supply functions as given.

Appendix 1.A Proofs

1.A.1 Proof of Proposition 1.1

If $q_1 + q_2 > \bar{q}$, then the marginal profit of at least one firm is strictly negative.

Figure 1.8 illustrates the idea of the proof.

Proof. Because the price cap does not bind, $q_1 + q_2 > \bar{q}$, both firms' marginal profits are as in the standard Cournot model. Applying the implicit function theorem to the first-order condition yields a bound for the slope of the best response function.

Firm i's first-order condition is

$$\frac{\partial \pi_i(q_i^*, q_j)}{\partial q_i} = p(q_i^* + q_j) + q_i^* \cdot p'(q_i^* + q_j) - c'(q_i^*) \stackrel{!}{=} 0. \tag{1.A.1}$$

Note that

$$\frac{\partial^2 \pi_i(q_i^*, q_j)}{\partial q_i^2} = 2 \cdot p'(q_i^* + q_j) + q_i^* \cdot p''(q_i^* + q_j) - c''(q_i^*) \tag{1.A.2}$$

$$= \frac{\partial^2 \pi_i(q_i^*, q_j)}{\partial q_i \, \partial q_j} + p'(q_i^* + q_j) - c''(q_i^*), \tag{1.A.3}$$

which is strictly negative because of strategic substitutability.

Applying the implicit function theorem to (1.A.1) yields

$$\frac{\partial q_i^*(q_j)}{\partial q_j} = -\frac{p'(q_i^* + q_j) + q_i^* \cdot p''(q_i^* + q_j)}{2 \cdot p'(q_i^* + q_j) + q_i^* \cdot p''(q_i^* + q_j) - c''(q_i^*)} \tag{1.A.4}$$

$$= -\frac{\frac{\partial^2 \pi_i(q_i^*, q_j)}{\partial q_i \, \partial q_j}}{\frac{\partial^2 \pi_i(q_i^*, q_j)}{\partial q_i^2}} = -\frac{\frac{\partial^2 \pi_i(q_i^*, q_j)}{\partial q_i \, \partial q_j}}{\frac{\partial^2 \pi_i(q_i^*, q_j)}{\partial q_i \, \partial q_j} + p'(q_i^* + q_j) - c''(q_i^*)} > -1. \tag{1.A.5}$$

The inequality follows because all terms are negative (the cross derivative because of strategic substitutability, and p' and $-c''$ by assumption).

Because the slope of the best response function is always between 0 and -1, the total quantity cannot exceed $2q^C$ without making the marginal profit of at least one firm strictly negative. The marginal profits of both firms are 0 if both firms produce q^C. If now one firm produces one marginal unit more, this firm's marginal profit becomes negative unless the other firm reduces its quantity by the reciprocal of the slope of the best response function of the other firm, which means by more than one marginal unit. This holds for all marginal units. □

1.A.2 Proof of Theorem 1.1

The quantities

$$q_1^*(\bar{q}) \;=\; \bar{q} - q_2^*(\bar{q}) \quad \textit{and} \quad q_2^*(\bar{q}) \;:\; p(\bar{q}) + q_2^*(\bar{q}) \cdot p'(\bar{q}) - c'(q_2^*(\bar{q})) \;=\; 0 \tag{1.11}$$

have the properties that

(i) $q_1^*(\bar{q}) \geq q^C \geq q_2^*(\bar{q})$*, with strict inequalities for* $\bar{q} > 2q^C$.
(ii) $q_1^*(\bar{q})$ *is strictly increasing in* $\bar{q}$ *and* $q_2^*(\bar{q})$ *is strictly decreasing in* $\bar{q}$.

There is a cutoff $\kappa \in (c'(0), p^C)$ *that is indirectly defined by*

$$\kappa \;:\; \kappa - c'\left(q_1^*(p^{-1}(\kappa))\right) \;=\; 0. \tag{1.12}$$

It has the properties that

(iii) $q_1^*(\bar{q})$ *and* $q_2^*(\bar{q})$ *are the unique equilibrium for all* $\bar{p} \in [\kappa, p^C]$.
(iv) $q_1^*(\bar{q})$ *and* $q_2^*(\bar{q})$ *are no equilibrium for all* $\bar{p} \in (c'(0), \kappa)$.

Proof. I begin by proving part (ii).

$q_1^*(\bar{q})$ *is strictly increasing in* $\bar{q}$ *and* $q_2^*(\bar{q})$ *is strictly decreasing in* $\bar{q}$.

The implicit function theorem yields the derivative of $q_2^*(\bar{q})$:

$$\frac{\partial q_2^*(\bar{q})}{\partial \bar{q}} \;=\; -\frac{p'(\bar{q}) + q_2^*(\bar{q}) \cdot p''(\bar{q})}{p'(\bar{q}) - c''(q_2^*(\bar{q}))}. \tag{1.A.6}$$

The denominator of this expression is negative by assumption. It remains to show that the numerator is negative, too.

Rearranging firm 2' first-order condition yields

$$q_2^*(\bar{q}) \;=\; \frac{p(\bar{q}) - c'(q_2^*(\bar{q}))}{p'(\bar{q})}. \tag{1.A.7}$$

Plugging this into the numerator of the derivative yields

$$p'(\bar{q}) + \frac{p(\bar{q}) - c'(q_2^*(\bar{q}))}{p'(\bar{q})} \cdot p''(\bar{q}) \;\overset{?}{<}\; 0 \tag{1.A.8}$$

$$\iff \quad (p(\bar{q}) - c'(q_2^*(\bar{q}))) \cdot p''(\bar{q}) - [p'(\bar{q})]^2 \;\overset{?}{<}\; 0, \tag{1.A.9}$$

where $\overset{?}{<}$ means that the inequality remains to be shown.

The strict log-concavity of $p(q)$ implies that

$$\forall q \;:\; p(q) \;>\; 0 \quad \Longrightarrow \quad p(q) \cdot p''(q) - [p'(q)]^2 \;<\; 0. \tag{1.A.10}$$

Since $p(\bar{q}) > 0$, this implies the above inequality because $c'(\cdot)$ is positive.

The derivative of $q_1^*(\bar{q})$ is

$$\frac{\partial q_1^*(\bar{q})}{\partial \bar{q}} = 1 - \frac{\partial q_2^*(\bar{q})}{\partial \bar{q}} > 0. \tag{1.A.11}$$

This proves part (ii).

I proceed by proving part (i).

$q_1^*(\bar{q}) \geq q^C \geq q_2^*(\bar{q})$, with strict inequalities for $\bar{p} < p^C$.

By definition, $\bar{q} \geq 2q^C$. In the corner case of $\bar{q} = 2q^C$, the equilibria in the game with a price cap and in the standard Cournot game coincide (because the price cap just binds): $q_1^*(2q^C) = q^C$ and $q_2^*(2q^C) = q^C$.

Because $q_1^*(\bar{q})$ is increasing, it follows that $q_1^*(\bar{q}) \geq q^C$.

Because $q_2^*(\bar{q})$ is decreasing, it follows that $q_2^*(\bar{q}) \leq q^C$.

This proves part (i).

Begin by assuming that $\kappa \in (c'(0), p^C)$ and verify at the end.

As $q_2^*(\bar{q})$ solves firm 2's first-order condition (and firm 2's profit function is strictly quasi-concave because of strategic substitutability), firm 2 has no profitable deviation. Whether $q_1^*(\bar{q})$ and $q_2^*(\bar{q})$ are an equilibrium, thus, depends only on whether firm 1 has a profitable deviation or not.

I proceed by proving part (iii).

I will show that $q_1^*(\bar{q})$ and $q_2^*(\bar{q})$ are an equilibrium if $\bar{p} \in [\kappa, p^C]$. Firm 1's marginal profit drops at $q_1^*(\bar{q})$ because the price cap stops binding, which introduces an inframarginal loss. Firm 1 has no profitable deviation if its marginal profit is weakly positive everywhere to the left of the drop at $q_1 + q_2 = \bar{q}$ and weakly negative everywhere to the right of the drop.

The marginal profit before the drop is given by the left derivative with respect to the own quantity, evaluated at $q_1 = q_1^*(\bar{q})$ and $q_2 = q_2^*(\bar{q})$,

$$\left. \frac{\partial_- \pi_1(q_1, q_2)}{\partial q_1} \right|_{q_1 = q_1^*(\bar{q}),\ q_2 = q_2^*(\bar{q})} = \bar{p} - c'(q_1^*(\bar{q})). \tag{1.A.12}$$

This expression is positive if the price cap is sufficiently large: In particular, if the price cap is the Cournot-Nash price, then $q_1^*(\bar{q}) = q_2^*(\bar{q}) = q^C$ and the left-derivative of the marginal profit is $p^C - c'(q^C) > 0$. This inequality follows from the Cournot-Nash equilibrium condition

$$p^C + \underbrace{q^C \cdot p'(2q^C)}_{<0} - c'(q^C) = 0.$$

If the price cap decreases, $q_1^*(\bar{q})$ increases continuously, as shown above. Thus, the marginal profit just above the drop decreases continuously. By definition of κ, the marginal profit is weakly positive for all $\bar{p} \in [\kappa, p^C]$.

The marginal profit after the drop is given by the right-derivative

$$\frac{\partial_+ \pi_1(q_1, q_2)}{\partial q_1}\bigg|_{q_1 = q_1^*(\bar{q}),\ q_2 = q_2^*(\bar{q})} = p(\bar{q}) + q_1^*(\bar{q}) \cdot p'(\bar{q}) - c'(q_1^*(\bar{q})). \tag{1.A.13}$$

This is the same marginal profit as in the standard Cournot game without price caps. That this expression is weakly negative for all $\bar{p} \in (c'(0), p^C]$ can be shown by decomposing the change in the total quantity compared to the Cournot-Nash equilibrium. Specifically, keep the quantity of firm 2 at q^C in the first step, while firm 1 brings the total quantity to $\bar{q}$. For this purpose, define $\hat{q_1}(\bar{q}) \equiv \bar{q} - q^C$. Because $\bar{q} \geq 2q^C$, it is true that $\hat{q_1}(\bar{q}) \geq q^C$. To show that the right-derivative of the marginal profit is negative, I use a set of inequalities (slightly abusing the notation, the marginal profits are those of the standard Cournot model; i.e. without a price cap):

$$0 = \frac{\partial \pi_1(q_1^C, q_2^C)}{\partial q_1} \geq \frac{\partial \pi_1\left(\hat{q_1}(\bar{q}), q_2^C\right)}{\partial q_1} \geq \frac{\partial \pi_1\left(q_1^*(\bar{q}), q_2^*(\bar{q})\right)}{\partial q_1}. \tag{1.A.14}$$

The equality follows from the Cournot-Nash equilibrium. The first inequality is true because the marginal profit is strictly quasi-concave in the own quantity. Moreover, if $\hat{q_1}(\bar{q}) > q^C$, the inequality is strict. To see that the second inequality is true, look at it written out (note that $\hat{q_1}(\bar{q}) + q^C = \bar{q} = q_1^*(\bar{q}) + q_2^*(\bar{q})$):

$$p(\bar{q}) + \hat{q_1}(\bar{q}) \cdot p'(\bar{q}) - c'(\hat{q_1}(\bar{q})) \geq p(\bar{q}) + q_1^*(\bar{q}) \cdot p'(\bar{q}) - c'(q_1^*(\bar{q})). \tag{1.A.15}$$

The inequality is true because $p'(\bar{q})$ is negative and because $q_1^*(\bar{q}) \geq \hat{q_1}(\bar{q})$. Moreover, if $q_1^*(\bar{q}) > \hat{q_1}(\bar{q})$, the inequality is strict. This implies that the marginal profit below the drop is weakly negative for all price caps.

Therefore, $q_1^*(\bar{q})$ and $q_2^*(\bar{q})$ are an equilibrium if $\bar{p} \in [\kappa, p^C]$.

Furthermore, this equilibrium is unique in this range:

For $q_1 + q_2 = \bar{q}$, there can be no other equilibrium because the solution to the first-order condition of firm 2 is unique.

For $q_1 + q_2 > \bar{q}$, Proposition 1.1 shows that there is no equilibrium.

For $q_1 + q_2 < \bar{q}$, there exists no equilibrium in this range of price caps: The first-order condition of firm 2 implies that, due to strategic substitutability, the optimal $q_2 > q_2^*(\bar{q})$. Therefore, to not violate $q_1 + q_2 < \bar{q}$, it has to be that $q_1 < q_1^*(\bar{q})$. This, however, implies that the marginal profit of firm 1 is strictly positive as $\bar{p} - c'(q_1) > \bar{p} - c'(q_1^*(\bar{q})) \geq 0$. The first inequality is true because $c'(\cdot)$ is strictly increasing. The second inequality is shown to be true above. Therefore, firm 1 could profitably deviate by expanding its quantity.

This proves part (iii).

I proceed by proving part (iv).

I will show that $q_1^*(\bar{q})$ and $q_2^*(\bar{q})$ are no equilibrium if $\bar{p} \in (c'(0), \kappa)$. As shown above, the left-derivative of firm 1's marginal profit at $q_1^*(\bar{q})$ and $q_2^*(\bar{q})$ is strictly decreasing in $\bar{q}$; so it is strictly increasing in $\bar{p}$. At $\bar{p} = \kappa$, the left-derivative of the marginal profit is 0. If $\bar{p} < \kappa$, it is negative. Therefore, firm 1 could profitably deviate by reducing its quantity.

This proves part (iv).

I conclude by proving that $\kappa \in (c'(0), p^C)$.

By definition, κ is the price cap for which the left-derivative of firm 1's marginal profit evaluated at $q_1^*(\bar{q})$ and $q_2^*(\bar{q})$ is 0. As shown above, the marginal profit given these quantities is strictly increasing in $\bar{p}$ and it is strictly positive at p^C. Thus, $\kappa < p^C$.

To show that $\kappa > c'(0)$, I show that when the price cap is $c'(0)$, the suggested quantities are no equilibrium because firm 1 could profitably deviate by reducing its quantity: Its marginal profit above the drop given the suggested equilibrium quantities is strictly negative. The idea is that the equilibrium price is the marginal cost of the first unit, but firm 1's suggested equilibrium quantity is positive. Plugging the price cap into the suggested equilibrium quantities yields $q_2^*(p^{-1}(c'(0))) = 0$ and, hence, $q_1^*(p^{-1}(c'(0))) = p^{-1}(c'(0)) > 0$. Thus, firm 1's marginal profit above the drop is $c'(0) - c'(p^{-1}(c'(0)))$, which is negative because $c'(\cdot)$ is strictly increasing.

□

1.A.3 Proof of Proposition 1.2

Define the competitive price, p^W, as the price at which the inverse demand curve, $p(q)$, and the social marginal cost curve, $c'(\frac{q}{2})$, intersect.

Define p^S as the unique Stackelberg equilibrium price if the Stackelberg leader's profit function is strictly quasi-concave.

It is true that $p^W < \kappa < p^S$.

Proof. I first prove that $p^W < \kappa$ by contradiction.

Define $p^{-1}(p^W) \equiv q^W$.

Assume that there is a clearing equilibrium with $\bar{p} = p^W$. One of the optimality conditions of firm 1 says that the marginal profit just above the drop has to be weakly positive,

$$p^W - c'(q_1^*) \geq 0. \tag{1.A.16}$$

This expression is strictly decreasing in q_1^*. If $q_1^* = \frac{q^W}{2}$, then the inequality has to bind because of the definition of the competitive price, p^W. Thus, firm 1 can never produce more than $\frac{q^W}{2}$ in any clearing equilibrium.

Because the in the clearing equilibrium the total quantity has to be q^W, firm 2 has to produce at least $\frac{q^W}{2}$. This, however, cannot be optimal for firm 2, as $\frac{q^W}{2} > q^C$, but Theorem 1.1 has shown that firm 2's suggested equilibrium quantity for such a total quantity has to be less than q^C. Firm 2's optimality condition fails because of the inframarginal loss,

$$p^W - c'(q_2^*) + q_2^* \cdot p'(q^W) \;=\; 0. \tag{1.A.17}$$

The first two terms together are at most 0 (if $q_2^* = \frac{q^W}{2}$). The third term, the inframarginal loss, is strictly negative. Thus, there can be no clearing equilibrium if the price cap is the competitive price.

I now prove that $\kappa < p^S$.

The Stackelberg equilibrium $(s_1^*, s_2(s_1^*))$ is defined by

$$p(s_1^* + s_2(s_1^*)) - c'(s_1^*) + \underbrace{s_1^*}_{>0} \cdot \underbrace{p'(s_1^* + s_2(s_1^*))}_{<0} \cdot \underbrace{\left(1 + \frac{\partial s_2(s_1^*)}{\partial s_1}\right)}_{>0} \stackrel{!}{=} 0. \tag{1.A.18}$$

The cutoff κ is defined by

$$p(q_1^*(p^{-1}(\kappa)) + q_2^*(p^{-1}(\kappa))) - c'(q_1^*(p^{-1}(\kappa))) \;=\; 0. \tag{1.A.19}$$

Combining these equations yields

$$p(s_1^* + s_2(s_1^*)) - c'(s_1^*) \;>\; p(q_1^*(p^{-1}(\kappa)) + q_2^*(p^{-1}(\kappa))) - c'(q_1^*(p^{-1}(\kappa))). \tag{1.A.20}$$

Now use two facts:

(i) On both sides are values of the same function of q_1: Because the Stackelberg follower and firm 2 both have the best response function of a standard Cournot duopolist as their best response function, they respond with the same quantities to the same q_1.

(ii) This function of q_1 is strictly decreasing in q_1: The price falls in q_1 because the total quantity increases in q_1 (the best response function's slope is larger than -1, see Proposition 1.1) and the marginal cost is increasing in q_1.

Therefore, the inequality implies that $s_1^* < q_1^*(p^{-1}(\kappa))$, so the total quantity in the unique Stackelberg equilibrium is strictly less than in the clearing equilibrium at the cutoff. So, the Stackelberg equilibrium price is strictly larger than the cutoff, $p^S > \kappa$.

□

1.A.4 Proof of Theorem 1.2

If $\bar{p} \in (c'(0), \kappa)$, the only pure-strategy Nash equilibrium is

$$q_1^*(\bar{p}) = (c')^{-1}(\bar{p}) \quad \text{and} \tag{1.17}$$

$$q_2^*(\bar{p}) : p\Big(q_1^*(\bar{p}) + q_2^*(\bar{p})\Big) + q_2^*(\bar{p}) \cdot p'\Big(q_1^*(\bar{p}) + q_2^*(\bar{p})\Big) - c'(q_2^*(\bar{p})) \stackrel{!}{=} 0. \tag{1.18}$$

In the limit of $\bar{p} \to \kappa$, the equilibrium converges to the clearing equilibrium presented in Theorem 1.1.

$q_1^*(\bar{p})$ is strictly increasing in $\bar{p}$ and $q_2^*(\bar{p})$ is strictly decreasing in $\bar{p}$. The total quantity $q_1^*(\bar{p}) + q_2^*(\bar{p})$ is strictly increasing in $\bar{p}$.

Proof. As $\bar{p} < \kappa$, a clearing equilibrium with $q_1 + q_2 = \bar{q}$ does not exist (see Theorem 1.1). Proposition 1.1 shows that no equilibrium exists with $q_1 + q_2 > \bar{q}$. So, if an equilibrium exists, it has to be that $q_1 + q_2 < \bar{q}$.

Firm 1's optimality condition then prescribes that its marginal profit has to get negative before the drop. Its profit-maximization condition is $\bar{p} = c'(q_1)$. So, the unique candidate for an equilibrium strategy is $q_1 = (c')^{-1}(\bar{p})$. Note that this candidate strategy converges to the equilibrium strategy in the clearing equilibria at the cutoff, $\lim_{\bar{p}\to\kappa}(c')^{-1}(\bar{p}) = q_1^*(p^{-1}(\kappa))$, as both solve $\kappa = c'(q_1)$.

Plugging firm 1's optimal choice into the first-order condition of firm 2 yields

$$p(q_1^*(\bar{p}) + q_2) + q_2 \cdot p'(q_1^*(\bar{p}) + q_2) - c'(q_2) \stackrel{!}{=} 0. \tag{1.A.21}$$

This equation has a solution because the left-hand side is continuous and it is positive at $q_2 = 0$ (because $p(q_1^*(\bar{p})) > c'(0)$) and negative for large q_2, for example at $q_2 = p^{-1}(c'(0))$. The solution is unique because strategic substitutability makes firm 2's profit function strictly quasi-concave.

The comparative statics of firm 1's quantity follow from the inverse function theorem:

$$\frac{\partial q_1^*(\bar{p})}{\partial \bar{p}} = \frac{\partial (c')^{-1}(\bar{p})}{\partial \bar{p}} = \frac{1}{c''\big((c')^{-1}(\bar{p})\big)} > 0. \tag{1.A.22}$$

The comparative statics of firm 2's quantity follow from a decomposition: The choice of firm 2 only depends on the price cap via the choice of firm 1. Thus, the derivative is given by the product of the slope of firm 2's best response function (which is strictly between −1 and 0) and the change in firm 1's choice,

$$\frac{\partial q_2^*(\bar{p})}{\partial \bar{p}} = \underbrace{\frac{\partial q_2^*(\bar{p})}{\partial q_1}}_{<0} \cdot \underbrace{\frac{\partial q_1^*(\bar{p})}{\partial \bar{p}}}_{>0} < 0. \tag{1.A.23}$$

The same decomposition can be used to determine the comparative statics of the total equilibrium quantity,

$$\frac{\partial (q_1^*(\bar{p}) + q_2^*(\bar{p}))}{\partial \bar{p}} = \underbrace{\frac{\partial q_1^*(\bar{p})}{\partial \bar{p}}}_{>0} \cdot \underbrace{\left(1 + \frac{\partial q_2^*(\bar{p})}{\partial q_1}\right)}_{>0} > 0. \tag{1.A.24}$$

□

1.A.5 Proof of Lemma 1.1

Define $\bar{p}_B$ as the price cap for which the total quantity in the rationing equilibrium is equal to the Cournot-Nash quantity, $\bar{p}_B : q_1^*(\bar{p}_B) + q_2^*(\bar{p}_B) = 2q^C$. It is true that $c'(0) < \bar{p}_B < \kappa$.

There is a monotone bijection between the clearing equilibria and the rationing equilibria. For each price cap $\bar{p}_c \in (\kappa, p^C]$, there is exactly one price cap $\bar{p}_r \in [\bar{p}_B, \kappa)$ such that the equilibrium quantities of the firms are the same:

$$q_1^*(\bar{q}_c) = q_1^*(\bar{p}_r) \quad \text{and} \quad q_2^*(\bar{q}_c) = q_2^*(\bar{p}_r). \tag{1.19}$$

Proof. Because the slope of firm 2's best response function lies strictly between 0 and -1, for any fixed total quantity, $q_1 + q_2$, there is at most one possible split between q_1 and q_2 such that q_2 is the best response to q_1. Because firm 2's profit-maximizing problem is the same, this fact is true for both clearing and rationing equilibria. So, if the total quantity in a clearing equilibrium and in a rationing equilibrium are equal, it has to be that also the firms' individual quantities are equal.

The price cap $\bar{p}_B$ exists and is unique because the total quantity in equilibrium in the rationing equilibria is continuous and monotone. When the price cap goes to $c'(0)$, the total quantity goes to the monopoly quantity q^M. When the price cap goes to κ, the total quantity exceeds $2q^C$, as the clearing and the rationing equilibrium converge at κ (see Theorem 1.2). Because in the clearing equilibria the total quantity is $\bar{q} = p^{-1}(\bar{p})$ and $\bar{p} = \kappa < p^C$, the total quantity exceeds $2q^C$.

The range of total quantities in the equilibria is the same for price caps in $(\kappa, p^C]$ and for price caps in $[\bar{p}_B, \kappa)$ because the total quantities are the same for the extreme points and in-between the total quantity is monotone and continuous. In the extreme points, the total quantities are the same by construction of the intervals: Theorem 1.2 has shown that the equilibrium quantities converge for the price cap's going to κ from above in clearing and from below in rationing equilibria. The price cap $\bar{p}_B$ is defined such that the total quantity in the rationing equilibrium is equal to the total quantity in the clearing equilibrium with price cap p^C. The monotonicity and continuity of the equilibrium quantities follows in the clearing equilibria from the fact that the total quantity is $\bar{q}$, which is monotone and continuous in $\bar{p}$, and in the rationing equilibria, it has been shown in Theorem 1.2.

The monotonicity also implies that the bijection between $\bar{p}_c$ and $\bar{p}_r$ is monotone. Thus, if there is a pair of price caps that induce the same equilibrium quantities $\bar{p}_c^1$ and $\bar{p}_r^1$ and another such pair $\bar{p}_c^2$ and $\bar{p}_r^2$ with $\bar{p}_c^1 > \bar{p}_c^2$, then it has to be that $\bar{p}_r^1 < \bar{p}_r^2$. □

1.A.6 Proof of Proposition 1.3

When the marginal cost is constant at c, the unique equilibrium is

$$q_1^*(\bar{q}) \;=\; \bar{q} - q_2^*(\bar{q}) \quad \text{and} \quad q_2^*(\bar{q}) \;=\; \frac{p(\bar{q}) - c}{-p'(\bar{q})}. \tag{1.25}$$

Proof. First, note that with constant marginal cost, the range of price caps becomes $(c, p^C]$, where, re-purposing notation, p^C is the Cournot-Nash price with constant marginal cost.

With constant marginal cost, firm 1's marginal profit becomes (expressed as the right-derivative at the drop $q_1 + q_2 = \bar{q}$)

$$\frac{\partial_+ \pi_1(q_1, q_2)}{\partial q_1} \;=\; \begin{cases} \bar{p} - c & \text{if } q_1 + q_2 \;<\; \bar{q} \\ p(q_1 + q_2) + q_1 \cdot p'(q_1 + q_2) - c & \text{if } q_1 + q_2 \;\geq\; \bar{q}. \end{cases} \tag{1.A.25}$$

The first step is to prove that there can be no equilibrium that is not clearing.

If $q_1 + q_2 < \bar{q}$, firm 1's marginal profit is strictly positive, so it could profitably deviate by increasing q_1.

If $q_1 + q_2 > \bar{q}$, Proposition 1.1 (at least one firm has a strictly negative marginal profit if $q_1 + q_2 > \bar{q}$) can be adjusted to the case of constant marginal cost: Simply drop $c''(q_i^*)$ from Equation (1.A.5).

Hence, in all equilibria it has to be that $q_1 + q_2 = \bar{q}$, which means, that they are clearing equilibria. The equilibrium strategies follow analogously to Theorem 1.1:

Plugging the equilibrium condition $q_1 + q_2 = \bar{q}$ into firm 2's first-order condition yields

$$p(\bar{q}) + q_2 \cdot p'(\bar{q}) - c \;=\; 0 \quad\Longleftrightarrow\quad q_2^*(\bar{q}) \;=\; \frac{p(\bar{q}) - c}{-p'(\bar{q})}, \tag{1.A.26}$$

which has a unique solution because firm 2's profit function is strictly quasi-concave.

Rearranging the equilibrium condition $q_1 + q_2 = \bar{q}$ yields a unique candidate for the equilibrium quantity of firm 1,

$$q_1^*(\bar{q}) \;=\; \bar{q} - q_2^*(\bar{q}). \tag{1.A.27}$$

Showing that this candidate is actually an equilibrium is, again, analogous to Theorem 1.1: Firm 1's marginal profit is weakly positive above and weakly negative below the drop. The marginal profit above the drop is $\bar{p} - c > 0$. That the marginal profit below the drop is weakly negative follows from the same decomposition as in Theorem 1.1.

By the same argument as in Theorem 1.1, it is true that $\frac{\partial q_2^*(\bar{q})}{\partial \bar{q}} < 0$, so $q_2^*(\bar{q}) \leq q^C$. Define $\hat{q_1}(\bar{q}) \equiv \bar{q} - q^C$, for which it is true that $\hat{q_1}(\bar{q}) \geq q^C$. Then,

$$0 = \frac{\partial \pi_1(q_1^C, q_2^C)}{\partial q_1} \geq \frac{\partial \pi_1\left(\hat{q_1}(\bar{q}), q_2^C\right)}{\partial q_1} \geq \frac{\partial \pi_1\left(q_1^*(\bar{q}), q_2^*(\bar{q})\right)}{\partial q_1}. \tag{1.A.28}$$

Again, slightly abusing the notation, the marginal profits denote the marginal profits of standard Cournot duopolists. The equality follows from the Cournot-Nash equilibrium. The first inequality is true because the marginal profit is strictly quasi-concave in the own quantity. To see that the second inequality is true, look at it written out (note that $\hat{q_1}(\bar{q}) + q^C = \bar{q} = q_1^*(\bar{q}) + q_2^*(\bar{q})$):

$$p(\bar{q}) + \hat{q_1}p'(\bar{q}) - c \geq p(\bar{q}) + q_1^* p'(\bar{q}) - c. \tag{1.A.29}$$

The inequality is true because $p'(\bar{q})$ is negative and because $q_1^*(\bar{q}) \geq \hat{q_1}(\bar{q})$. Moreover, if $q_1^*(\bar{q}) > \hat{q_1}(\bar{q})$, the inequality is strict.

□

1.A.7 Proof of Proposition 1.4

Assume that firm 1 has the price cap $\bar{p}$ and firm 2 has a price cap $\bar{p}_2$. Without loss of generality, assume that $\bar{p} < \bar{p}_2$.

Define $\kappa_2(\bar{p})$ as the value at which $p(q_1^*(\bar{p}) + q_2)$ and $c'(q_2)$—both functions of q_2—intersect; illustrated in Figure 1.14. $\kappa_2(\bar{p})$ is decreasing in $\bar{p}$.

(i) If $\bar{p} \geq \kappa$, the only pure-strategy Nash equilibrium is the clearing equilibrium described in Theorem 1.1. It does not depend on $\bar{p}_2$.

(ii) If $\bar{p} < \kappa$ and $\bar{p}_2 \geq p(q_1^*(\bar{p}) + q_2^*(\bar{p}))$ (as defined in Theorem 1.2), the only pure-strategy Nash equilibrium is the rationing equilibrium described in Theorem 1.2. It does not depend on $\bar{p}_2$.

(iii) If $\bar{p} < \kappa$ and $p(q_1^*(\bar{p}) + q_2^*(\bar{p})) > \bar{p}_2 \geq \kappa_2(\bar{p})$, the only pure-strategy Nash equilibrium is a partially rationing equilibrium. In this equilibrium, firm 1 produces $q_1^*(\bar{p}) = (c')^{-1}(\bar{p})$ and firm 2 produces the quantity that brings the market-clearing price to $\bar{p}_2$, which is $q_2^*(\bar{p}, \bar{p}_2) = p^{-1}(\bar{p}_2) - q_1^*(\bar{p})$.

(iv) If $\bar{p} < \kappa$ and $\kappa_2(\bar{p}) > \bar{p}_2$, the only pure-strategy Nash equilibrium is a doubly rationing equilibrium. In this equilibrium, firm 1 produces $q_1^*(\bar{p}) = (c')^{-1}(\bar{p})$ and firm 2 produces $q_2^*(\bar{p}_2) = (c')^{-1}(\bar{p}_2)$, which depends only on the own price cap.

Proof. Define $\bar{q}_2 \equiv p^{-1}(\bar{p}_2)$.

The price cap of firm 2 has the same effect as for firm 1 as long as it binds, so the marginal profit becomes

$$\frac{\partial_+ \pi_2(q_1, q_2)}{\partial q_2} = \begin{cases} \bar{p}_2 - c'(q_2) & \text{if } q_1 + q_2 < \bar{q}_2 \\ p(q_1 + q_2) + q_2 \cdot p'(q_1 + q_2) - c'(q_2) & \text{if } q_1 + q_2 \geq \bar{q}_2. \end{cases} \tag{1.A.30}$$

Case (i): At the equilibrium quantities, the price cap of firm 2 does not bind because the price is $\bar{p}$. Firm 1 does not want to deviate because nothing has changed in comparison to Theorem 1.1. Firm 2 does not want to deviate because for all smaller quantities, the marginal profit is strictly positive, and for all larger quantities, the marginal profit is strictly negative.

The uniqueness follows from Proposition 1.1 (no equilibrium with $q_1 + q_2 > \bar{q}$) and Theorem 1.1 (no other equilibrium with $\bar{q}_2 < q_1 + q_2 \leq \bar{q}$) for all total quantities, for which the price cap of firm 2 does not bind.

For all quantities for which the price cap of firm 2 does bind, there is a decomposition argument similar to the one used in Theorem 1.1 that shows that firm 2 always has a profitable deviation. To derive a contradiction, assume that q_1' and q_2' constitute an equilibrium, such that $q_1' + q_2' \leq \bar{q}_2$.

Firm 1's optimality condition, because its price cap strictly binds, is that

$$\bar{p} - c'(q_1') = 0. \tag{1.A.31}$$

In the equilibrium with $q_1^* + q_2^* = \bar{q}$, firm 1's marginal profit above the drop is weakly positive,

$$\bar{p} - c'(q_1^*(\bar{q})) \geq 0. \tag{1.A.32}$$

As the marginal cost is strictly increasing, both conditions combined imply that $q_1' \geq q_1^*(\bar{q})$.

Since the total quantity has to be smaller in the fictitious equilibrium, it has to hold that $q_2' < q_2^*(\bar{q})$.

Define $\hat{q_2}(\bar{q}) \equiv \bar{q}_2 - q_1^*(\bar{q})$. It is true that $q_2' \leq \hat{q_2}(\bar{q}) < q_2^*(\bar{q})$. The first inequality follows from plugging in for $\bar{q}_2 \geq q_1' + q_2'$ in the definition of $\hat{q_2}(\bar{q})$ and then applying the inequality $q_1' \geq q_1^*(\bar{q})$. The second inequality follows from $\hat{q_2}(\bar{q}) = \bar{q}_2 - q_1^*(\bar{q}) < \bar{q} - q_1^*(\bar{q}) = q_2^*(\bar{q})$.

Applying this decomposition of the difference between q_2' and $q_2^*(\bar{q})$ to the marginal profit of a standard Cournot duopolist, that is, including the inframarginal loss, yields

$$0 = \frac{\partial \pi_2\left(q_1^*(\bar{q}), q_2^*(\bar{q})\right)}{\partial q_2} < \frac{\partial \pi_2\left(q_1^*(\bar{q}), \hat{q_2}(\bar{q})\right)}{\partial q_2} \leq \frac{\partial \pi_2\left(q_1', q_2'\right)}{\partial q_2}. \tag{1.A.33}$$

The equality follows from the optimality condition in the confirmed equilibrium. The first inequality follows from strict quasi-concavity of the profit function because the quantity of firm 2 is strictly reduced. In the step to the right-hand side, the total quantity is kept fixed, while the quantity of firm 1 is weakly increased and the quantity of firm 2 weakly decreased. The marginal profit for a fixed total quantity is decreasing in the own quantity because $p'(\bar{q}_2)$ is strictly negative and $c'(\cdot)$ is strictly increasing. So if $q_1' > q_1^*(\bar{q})$, the inequality is strict.

The fictitious equilibrium, however, is only an equilibrium if the right-hand side is weakly negative: It is firm 2's marginal profit below the drop. As it is strictly positive, firm 2 profits from deviating to a larger quantity. Thus, (q_1', q_2') are no equilibrium.

The other three cases correspond to rationing equilibria in the case when only firm 1 has a price cap. In the rationing equilibria, the game is essentially nonstrategic and firm 2 is a monopolist on the market for residual demand. Thus, the three cases directly correspond to the three cases in monopoly regulation (see Appendix 1.B). The equilibrium price when firm 2 has no price cap, $p(q_1^*(\bar{p}) + q_2^*(\bar{p}))$, corresponds to the monopoly price, p^M. A cutoff that depends on the quantity of firm 1, $\kappa_2(\bar{p})$, corresponds to the competitive price.

For the following three cases, the uniqueness argument is the same: Proposition 1.1 and Theorem 1.1 show that there can be no pure-strategy Nash equilibria in which $q_1 + q_2 \geq \bar{q}$, so in any equilibrium, it has to be that firm 1's price cap strictly binds (that is, firm 1 is not at the drop of its marginal revenue). Because firm 1's price cap strictly binds in any equilibrium, the only optimal strategy of firm 1 is to play $q_1^*(\bar{p}) = (c'^{-1})(\bar{p})$. Given that firm 1 plays $q_1^*(\bar{q})$ in any equilibrium, the uniqueness of the equilibria follows from the uniqueness of firm 2's best response.

Case (ii): This corresponds to the case in the monopoly in which the price cap is so high that it makes no difference. The monopolist's marginal profit intersects zero at a quantity beyond the drop.

The equilibrium price is $p(q_1^*(\bar{p}) + q_2^*(\bar{p})) \leq \bar{p}_2$, so the additional constraint of firm 2's price cap is not violated in the solution, in which firm 2 ignores its price cap. Thus, this solution remains optimal and the price cap $\bar{p}_2$ makes no difference.

For the following two cases, the competitive price on the market for the residual demand, $\kappa_2(\bar{p})$, is needed. Given that firm 1 produces the quantity $q_1^*(\bar{p}) = (c')^{-1}(\bar{p})$, firm 2's inverse residual demand curve and its marginal cost curve intersect at the price $\kappa_2(\bar{p})$. Figure 1.14 illustrates the principle. Formally,

$$\kappa_2(\bar{p}) \equiv p(q_1^*(\bar{p}) + q_2^*(\bar{p})), \tag{1.A.34}$$

$$\text{where } q_2^*(\bar{p}) \text{ solves} \quad p(q_1^*(\bar{p}) + q_2^*(\bar{p})) - c'(q_2^*(\bar{p})) \stackrel{!}{=} 0. \tag{1.A.35}$$

The comparative statics follow from the comparative statics of firm 1's equilibrium quantity. If $\bar{p}$ increases, $q_1^*(\bar{p})$ increases, so firm 2's inverse residual demand function is shifted to the left (decreases). Therefore, the inverse residual demand curve and the marginal cost curve intersect at a lower price, $\kappa_2(\bar{p})$.[24]

Case (iii): This corresponds to the case in the monopoly in which the price cap is between the monopoly price and the competitive price, so it binds, and the price clears the market. The monopolist's marginal profit intersects zero at the drop.

The total quantity for the proposed strategies is $\bar{q}_2$, that is, firm 2's price cap just binds, and firm 2 is at the drop in its marginal profit. Firm 1 does not want to deviate because nothing has changed in comparison to Theorem 1.2. Firm 2 has no incentive to deviate either.

Its marginal profit above the drop is weakly positive. By definition of $\kappa_2(\bar{p})$, given $q_1^*(\bar{p})$, the quantity q_2' that equates the inverse demand and the competitive price on the market for residual demand, $p(q_1^*(\bar{p}) + q_2') = \kappa_2(\bar{p})$ solves also $p(q_1^*(\bar{p}) + q_2') - c'(q_2') = 0$. This latter function is strictly decreasing in q_2. By the case condition, $p(q_1^*(\bar{p}) + q_2^*(\bar{p},\bar{p}_2)) = \bar{p}_2 \geq \kappa_2(\bar{p})$, hence $q_2^*(\bar{p},\bar{p}_2) \leq q_2'$. The marginal profit above the drop is $\bar{p}_2 - c'(q_2^*(\bar{p},\bar{p}_2)) \geq \kappa_2(\bar{p}) - c'(q_2') \equiv 0$ because $\bar{p}_2 \geq \kappa_2(\bar{p})$ and $c'(q_2^*(\bar{p},\bar{p}_2)) \leq c'(q_2')$.

After the drop and for all larger quantities, firm 2's marginal profit is strictly negative. This follows from three previously established and one new fact. First, after the drop, the price cap does not bind, so firm 2's marginal profit is the same as that of a standard Cournot duopolist. Second, the profit function of a standard Cournot duopolist is strictly quasi-concave in the own quantity. Thirdly, given $q_1^*(\bar{p})$, the root of the standard Cournot duopolist's marginal profit is given by $q_2^*(\bar{p})$, as shown in Theorem 1.2. Lastly, $q_2^*(\bar{p},\bar{p}_2) > q_2^*(\bar{p})$ because $p(q_1^*(\bar{p}) + q_2^*(\bar{p},\bar{p}_2)) = \bar{p}_2 < p(q_1^*(\bar{p}) + q_2^*(\bar{p}))$, where the inequality follows from the case condition.

Case (iv): This corresponds to the case in the monopoly in which the price cap is between the competitive price and the marginal cost of the first unit, so the price cap binds, and the price does not clear the market. The monopolist's marginal profit intersects zero at a quantity before the drop.

At the proposed quantities, both price caps bind. Firm 1 does not want to deviate because nothing has changed in comparison to Theorem 1.2. Firm 2 has no incentive to deviate because its marginal profit intersects zero already before the drop. As shown above, the marginal profit above the drop is $\bar{p}_2 - c'(q_2^*(\bar{p},\bar{p}_2)) < \kappa_2(\bar{p}) - c'(q_2') \equiv 0$, where q_2' follows from the definition of the competitive price on the market for residual demand, $p(q_1^*(\bar{p}) + q_2') = \kappa_2(\bar{p})$. The inequality follows from the case condition $\bar{p}_2 < \kappa_2(\bar{p})$, which also implies that $q_2^*(\bar{p},\bar{p}_2) > q_2'$. Thus, firm 2's marginal profit intersects zero at $q_2^*(\bar{p}_2) = (c')^{-1}(\bar{p}_2)$ and the marginal profit is

24. In the monopoly analogy, the competitive price gets lower when the inverse demand function is shifted to the left.

strictly positive for all smaller quantities and strictly negative for all larger quantities.

□

1.A.8 Proof of Proposition 1.6

(i) There are no equilibria with $q_1 + q_2 > \bar{q}$ (Proposition 1.1 remains true with proportional rationing).

(ii) There are no clearing equilibria with $q_1 + q_2 = \bar{q}$.

(iii) There are rationing equilibria with $q_1 + q_2 < \bar{q}$. There is an $\epsilon > 0$, such that the unique equilibrium for each price cap in $(c'(0), \kappa + \epsilon)$ is a rationing equilibrium. When the price cap decreases, firm 1 decreases and firm 2 increases its quantity.

Proof. Firm 1's best response function is the same as in the case of efficient rationing because it does not depend on the rationing rule (the quantity of firm 2 is efficiently rationed because the price is free to adjust).

(i) When $q_1 + q_2 > \bar{q}$, both firms are standard Cournot duopolists as the price cap does not bind. Thus, Proposition 1.1 remains true.

(ii) Because firm 2's marginal profit jumps at $q_2 = \bar{q} - q_1$, this quantity is never optimal for firm 2.

There are four sub-cases. Remember that firm 2's profit function is strictly quasi-concave in q_2 within both parts and that the marginal profit crosses zero at least once.

(ii.i) If the marginal profit jumps from the strictly negative into the weakly negative, a strictly lower quantity is optimal.

(ii.ii) If the marginal profit jumps from the weakly positive into the strictly positive, a strictly larger quantity is optimal.

(ii.iii) If the marginal profit jumps from the weakly negative into the strictly positive, it is both better to choose a slightly smaller or larger quantity.

(ii.iv) If the marginal profit has no jump, which means that $q_1 = 0$, the monopoly quantity, which is strictly less than $\bar{q}$, is optimal.

(iii) As before, in any rationing equilibrium, firm 1 plays $q_1^*(\bar{p}) = (c')^{-1}(\bar{p})$.

There is a rationing equilibrium if the best response functions intersect in the vertical part of firm 1's best response function. That is, firm 2's best response to $q_1^*(\bar{p})$ is such that $q_2 \leq \bar{q} - q_1^*(\bar{p})$. The global maximizer of firm 2's two-part profit function is in the first part. I now show that a rationing equilibrium exists for all price caps $\bar{p} < \kappa$.

The second part of firm 2's profit function is the profit function of a standard Cournot duopolist. If $\bar{p} < \kappa$, the best response of a standard Cournot duopolist is $BR_2(q_1^*(\bar{p})) < \bar{q} - q_1^*(\bar{p})$. This inequality is necessarily true because, with efficient rationing, a rationing equilibrium exists for these price caps (see Theorem 1.2).

Furthermore, as the standard Cournot duopolist's profit function is strictly quasi-concave in q_2, its marginal profit is strictly negative for all $q_2 > BR_2(q_1^*(\bar{p}))$. So, in particular, the marginal profit is strictly negative for $q_2 = \bar{q} - q_1^*(\bar{p})$. Or, expressed in terms of firm 2 with proportional rationing and its two-part profit function: The right-derivative of firm 2's profit with respect to q_2 is strictly negative in the second part at the transition point at $\bar{q} - q_1^*(\bar{p})$.

The above fact and the fact that the profit function of firm 2 is strictly quasi-concave in q_2 within each of the two parts, implies that the global maximizer of firm 2's profit is in the first part of its two-part profit function: As firm 2's marginal profit is strictly negative above the jump at $q_2 = \bar{q} - q_1^*(\bar{p})$, it is also strictly negative below the jump. Thus, the only intersection of firm 2's marginal profit and zero is in the first part of the profit function.

Therefore, $q_1^*(\bar{p})$ and the q_2 that solves

$$p\left(\frac{\bar{q}}{\bar{q} - q_1^*(\bar{p})} \cdot q_2\right) + q_2 \cdot \frac{\bar{q}}{\bar{q} - q_1^*(\bar{p})} \cdot p'\left(\frac{\bar{q}}{\bar{q} - q_1^*(\bar{p})} \cdot q_2\right) - c'(q_2) \overset{!}{=} 0 \quad (1.A.36)$$

constitute a rationing equilibrium.

Moreover, continuity implies that there are also rationing equilibria for some price caps above the cutoff κ.

For the price cap κ, there is a rationing equilibrium: By definition of κ, it is true that $BR_2(q_1^*(\kappa)) = p^{-1}(\kappa) - q_1^*(\kappa)$. So, firm 2's marginal profit is 0 above the jump at the transition point at $p^{-1}(\kappa) - q_1^*(\kappa)$. Below the jump, the marginal profit is strictly negative. Thus, the maximum in the first part of the profit function is strictly larger than the corner maximum in the second part of the profit function.

When marginally increasing the price cap, the global maximum remains in the first part of firm 2's profit function because $q_1(\bar{p})$ changes continuously and the two candidates for firm 2's best response change continuously, so the corresponding profits change continuously. As the profit in the maximum of the first part was strictly larger than the profit in the maximum of the second part, it remains strictly larger for some larger price caps. Then, $q_1^*(\bar{p})$ and the q_2 described by (1.A.36) still are a rationing equilibrium.

The comparative statics of the equilibrium quantity of firm 1, $q_1^*(\bar{p})$, with respect to the price cap are the same as with efficient rationing. The comparative statics of $q_2^*(\bar{p})$ follow from applying the implicit function theorem on firm 2's first-order condition (1.A.36). I take the derivative with respect to $\bar{q} = p^{-1}(\bar{p})$. The derivative with respect to $\bar{p}$ has the opposite sign. The derivative is, slightly abusing the $\bar{p}$ and $\bar{q}$ notation,

$$\frac{\partial q_2^*(\bar{p})}{\partial \bar{q}} = -q_2^*(\bar{p}) \cdot \alpha \cdot \frac{\gamma}{\beta \cdot \gamma - c''(q_2^*)}, \quad (1.A.37)$$

where I substituted to improve readability:

$$\alpha \equiv \frac{\bar{q} \cdot \frac{\partial q_1^*(\bar{p})}{\partial \bar{q}} - q_1^*(\bar{p})}{(\bar{q} - q_1)^2} < 0, \tag{1.A.38}$$

$$\beta \equiv \frac{\bar{q}}{\bar{q} - q_1^*(\bar{p})} > 1, \quad \text{and} \tag{1.A.39}$$

$$\gamma \equiv 2p'(\beta \cdot q_2^*) + q_2^* \cdot \beta \cdot p''(\beta \cdot q_2^*) < 0. \tag{1.A.40}$$

The bound for α follows from $\frac{\partial q_1^*(\bar{p})}{\partial \bar{q}} < 0$. The bound for γ follows from the strict log-concavity of firm 2's inverse residual demand function in q_2. Thus, whenever $q_2^*(\bar{p}) > 0$, then

$$\frac{\partial q_2^*(\bar{p})}{\partial \bar{q}} > 0. \tag{1.A.41}$$

□

Appendix 1.B Price Regulation in Monopoly

This section summarizes the textbook case of price regulation in a monopoly.

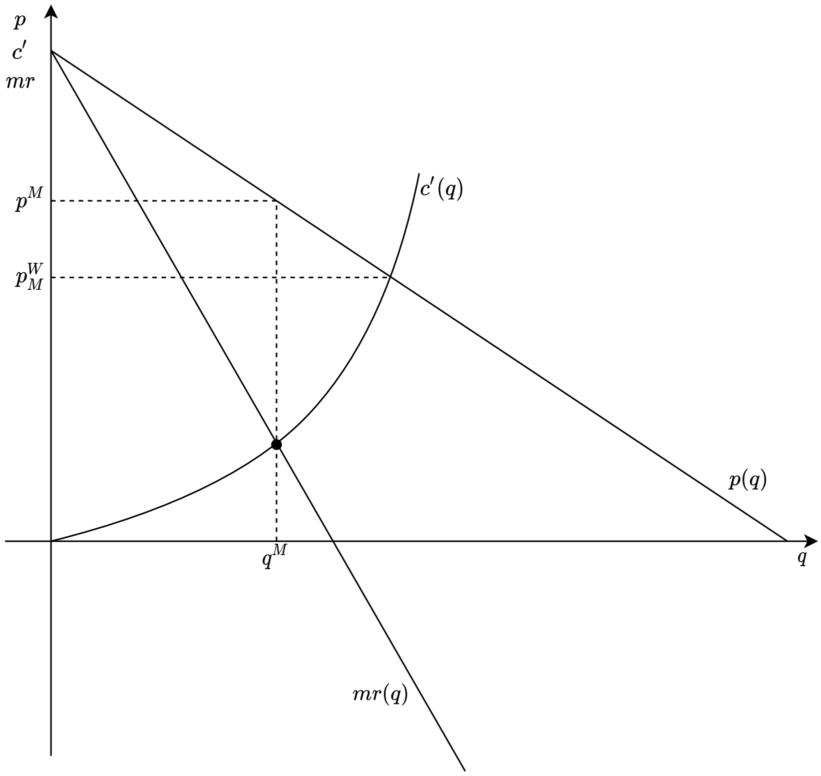

Figure 1.B.1. The monopolist's profit maximization problem without price caps. The profit is maximized by the quantity at which the marginal revenue and the marginal cost intersect. The welfare is maximized at the quantity at which the inverse demand curve and the marginal cost curve intersect.

Figure 1.B.1 illustrates the monopolist's maximization problem. As in the main part, I assume that the inverse demand curve is falling and strictly log-concave and that the marginal cost is strictly increasing. The monopolist maximizes its profit, $\pi = q \cdot p(q) - c(q)$. The marginal revenue is $p(q) + q \cdot p'(q)$, which includes the inframarginal loss from depressing the price when increasing the quantity. Log-concavity of the inverse demand function implies that the marginal revenue is decreasing whenever it is positive.[25]

In the absence of a price cap, the monopolist maximizes its profit at the intersection of the marginal revenue and the marginal cost curve: It produces the quantity q^M, which leads to a price of p^M. This quantity falls short of the welfare-maximizing

25. See https://economics.stackexchange.com/questions/24833/can-marginal-revenue-be-increasing (last accessed September 27, 2022).

quantity, q_M^W, at which the marginal willingness to pay of the market is equal to the marginal cost of the monopolist (which are equal to p_M^W).

A price cap, $\bar{p}$, changes the marginal revenue curve as the monopolist is relieved of its effect on the price—as long as the price cap binds. As a price-taker, the marginal revenue is equal to the price as long as the price does not change. When the quantity is so large that the price cap stops binding, the monopolist has again a price effect and the marginal revenue drops to the normal marginal revenue.

There are now three cases, depending on the level of the price cap.

First Case, $\bar{p} \geq p^M$. The price cap stops binding before the marginal revenue intersects the marginal cost. Thus, the price cap has no effect and the monopolist produces q^M.

This is sketched in Figure 1.B.2.

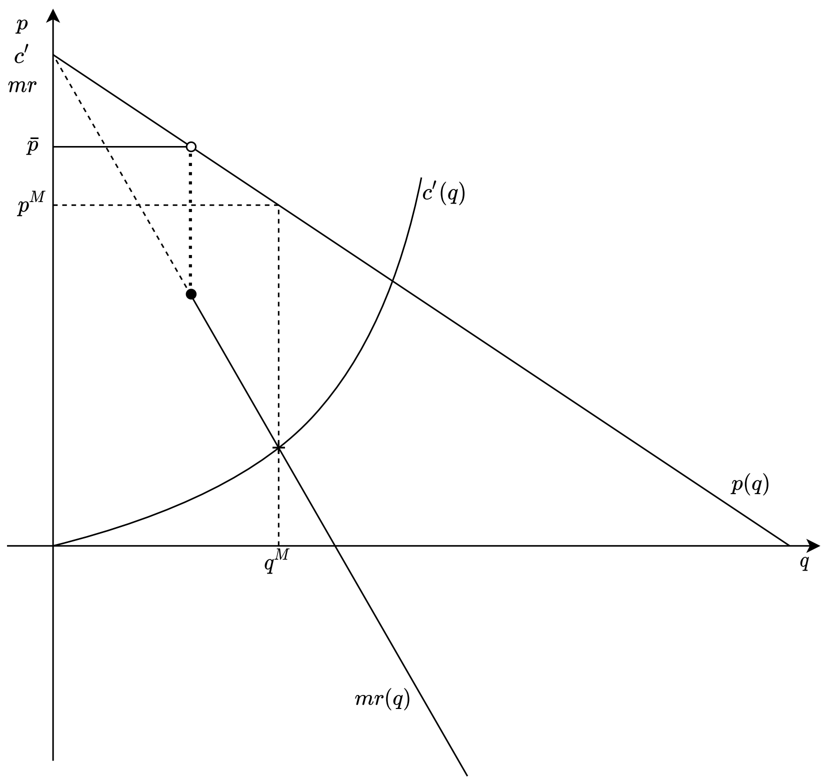

Figure 1.B.2. If the price cap is above the monopoly price, the marginal revenue becomes the standard monopolist's marginal revenue before intersecting the marginal cost curve. Thus, the price cap has no effect and the profit-maximizing quantity is still the monopoly quantity.

Second Case, $p^M > \bar{p} \geq p_M^W$. The equilibrium quantity is determined by the inverse demand curve. The marginal revenue is constant until the price cap stops binding. At the corresponding quantity, the marginal revenue drops. Because the price cap is below the monopoly price, the marginal revenue after the drop is below the marginal cost. Thus, the quantity that the monopolist optimally produces is determined by $p(q^*) = \bar{p}$.

This is sketched in Figure 1.B.3.

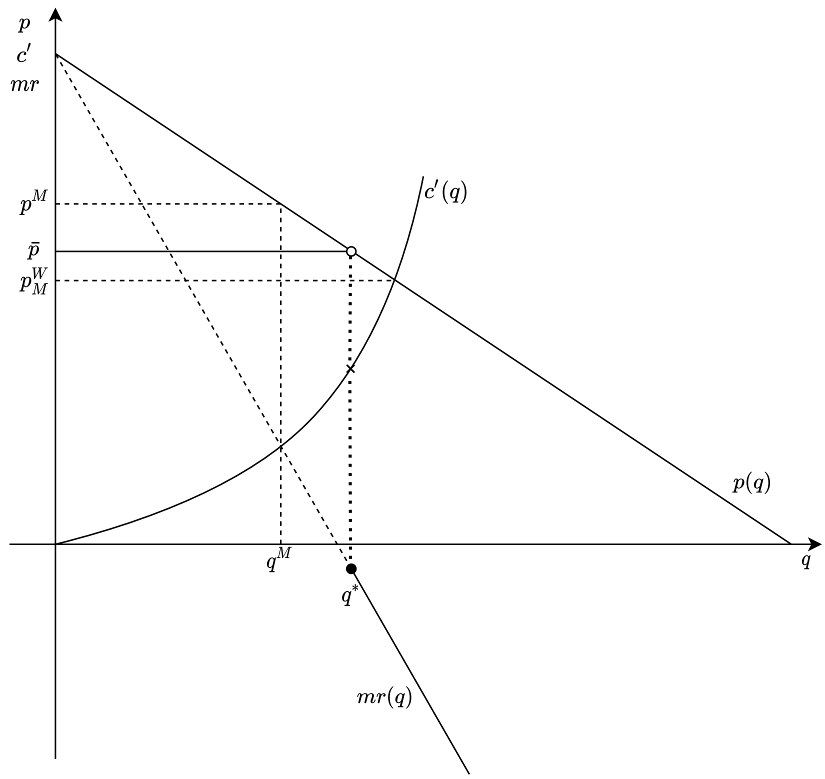

Figure 1.B.3. Price caps between the monopoly price and the welfare-maximizing price increase the welfare.

Third Case, $p^W_M > \bar{p} > c'(0)$. The equilibrium quantity is determined by the marginal cost. In this case, the marginal revenue intersects the marginal cost before the price cap stops binding. Thus, the quantity that the monopolist optimally produces is determined by $c'(q^*) = \bar{p}$. If the price cap is very low, the monopolist produces even less than it would without a price cap, albeit that quantity is sold at a much lower price. The lower price causes a rationing problem: At the price cap, the demand exceeds the supply; the price cannot (efficiently) allocate the good.

This is sketched in Figure 1.B.4.

Constant Marginal Cost. When the monopolist has constant marginal cost, most of the above remains true. The difference is that the third case does not exist as $p^W_M = c'(0)$.

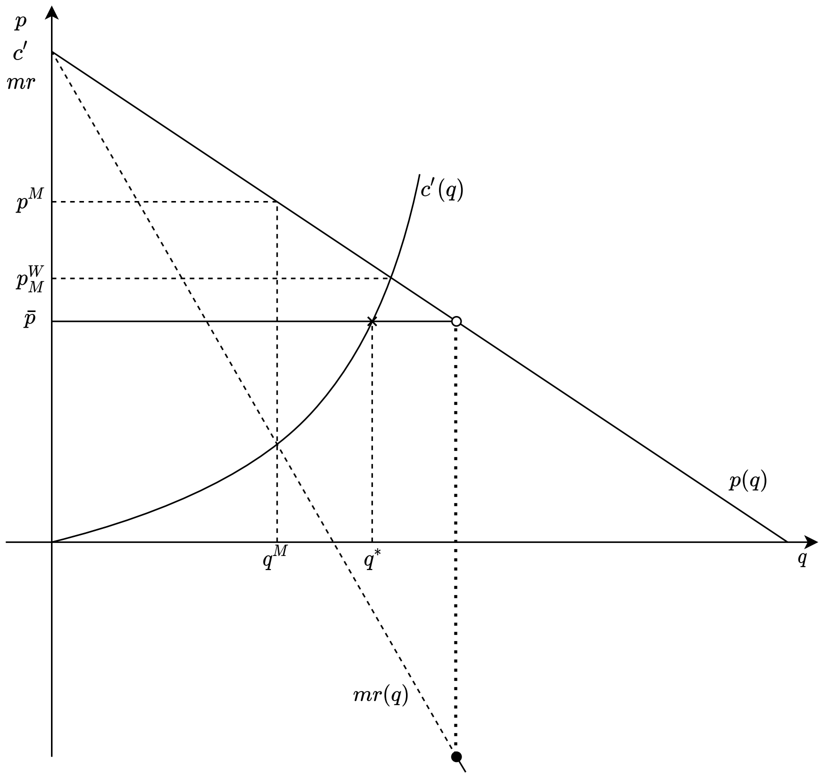

Figure 1.B.4. If the price cap is below the welfare-maximizing price, the marginal revenue intersects the marginal cost in the range in which it is constant.

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Chapter 2

Do Non-Compete Clauses Undermine Minimum Wages?*

Joint with Fabian Schmitz

2.1 Introduction

A non-compete clause (*NCC*) is part of an employment contract that prohibits employees from working for a competitor or from starting their own business within specific geographic or temporal boundaries. A significant fraction of the US labor force is currently bound by a non-compete clause: 20% of the labor force were restricted by such a clause in 2014 and 40% had signed one in the past (Starr, Prescott, and Bishara, 2021). Moreover, many low-wage workers are bound by NCCs. 29% of the sampled workplaces that pay an average hourly salary of less than 13 dollars and 20% of the workplaces in which the typical employee has not graduated from high school have each employee sign an NCC (Colvin and Shierholz, 2019).

While the public seems to accept NCCs in the contracts of CEOs, media reports about NCCs in the contracts of low-wage workers caused a public outrage.[1] Some politicians, too, believe that NCCs exploit low-wage workers. As a result, there have

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1. The fast-food firm *Jimmy John's* made its employees sign that they were not allowed to work for "any business which derives more than ten percent (10%) of its revenue from selling submarine, hero-type, deli-style, pita and/or wrapped or rolled sandwiches and which is located with three (3) miles of either [the Jimmy John's location in question] or any such other Jimmy John's Sandwich Shop." (Jamieson, 2014, for *Huffington Post*: "`Jimmy John's Makes Low-Wage Workers Sign 'Oppressive' Noncompete Agreements.`") Jimmy John's has settled with the Attorney General in New York State and has stopped using non-compete clauses for sandwich workers in 2016. For more

been several attempts at restricting the use of NCCs in the last years, particularly concerning low-wage workers.[2] The exact mechanism by which NCCs make low-wage workers worse off has, however, remained unclear.

Our main contribution is showing that effort incentives through NCCs can be such a mechanism. We use the canonical partial-market moral hazard model and add the possibility to costlessly reduce the agent's payoff with an NCC. This feature captures the employer's opportunity to terminate the agent after a bad performance, which activates the agent's NCC, restricting his future employment possibilities. To avoid this, the agent exerts more effort. Thus, both a bonus wage and an NCC provide incentives—they are substitutes in the incentive constraint.

The property that makes providing incentives via an NCC interesting to the principal is that a bonus wage and an NCC are opposites in the participation constraint: The bonus wage makes the participation constraint slack, as it increases the agent's payoff after a good outcome, whereas the NCC makes the participation constraint tight, as it decreases the payoff after a bad outcome. If a sufficiently large minimum wage prevents the principal from using wages to extract the surplus, she resorts to an NCC. While the minimum wage leaves the agent a rent—slackens the participation constraint—adding an NCC extracts the agent's rent—tightens the participation constraint. If the agent gets a rent, adding an NCC provides incentives at no cost to the principal; the additional effort cost is borne by the agent's rent.

Thus, we find that if NCCs may arbitrarily reduce the agent's payoff after a failure, the profit maximizing contract never leaves the agent a rent, irrespective of the minimum wage's level. While NCCs lead to weak Pareto improvements if minimum wages are so low that they do not redistribute but merely lead to inefficiency, they make the principal better off and the agent worse off whenever the minimum wage redistributes. Surprisingly, bounded NCCs (see Appendix 2.A) might lead to strict Pareto improvements over minimum wages alone. NCCs with a suitably chosen bound can reduce the inefficiency from minimum wages, while the bound prevents the principal from extracting the agent's rent completely. Considering an extensive margin, NCCs reduce the employment effect of minimum wages because they counteract the inefficiency (and the redistribution).

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details, see Whitten (2016) for *CNBC*: "`Jimmy John's drops noncompete clauses following settlement.`"

2. On the federal level, President Biden issued an executive order to "curtail the unfair use of non-compete clauses and other clauses or agreements that may unfairly limit worker mobility" (The White House, 2021) on July 09, 2021. Furthermore, the "Mobility and Opportunity for Vulnerable Employees Act," the "Workforce Mobility Act," and the "Freedom to Compete Act" have been introduced, but neither has been passed. There has also been progress on the state level: Some states now make NCCs unenforceable if the employee's salary lies below a threshold.

Further welfare results are concerned with the utilitarian welfare.[3] We decompose the total effect into an idleness effect and an incentive effect. The *idleness effect* is the direct reduction in the social surplus from reducing the agent's payoff after a failure. It always reduces the utilitarian welfare. The *incentive effect* works through the increase of the equilibrium effort, which is how an NCC transfers utility from the agent to the principal. If the minimum wage is binding but low, the incentive effect is positive, as the equilibrium effort in the benchmark without NCCs is inefficiently low. The NCC brings the equilibrium effort closer to the first-best level (Proposition 2.3). If the minimum wage is large, however, the incentive effect becomes negative because the equilibrium effort with NCCs gets inefficiently large (Proposition 2.4). The principal induces an inefficiently large effort, because the only way of extracting the agent's rent is through higher equilibrium efforts. Bounding NCCs may prevent the incentive effect from turning bad and keep the total effect positive.

The effort incentives from an NCC provide a convincing reason for why a rational minimum wage worker is asked to sign an NCC and does so. Due to her market power, the principal can extract rents from the agent. Because of the minimum wage laws (or limited liability), this cannot be done via money, but only through effort incentives.

The transfer motive complements the usual four reasons for the use of NCCs in the literature; most of which are not particularly appealing for the case of low-wage workers: Firstly, employers can use NCCs to improve their bargaining power in future wage bargaining.[4] Yet, minimum wage workers rarely bargain for wage increases.[5] Secondly, like non-disclosure agreements and non-solicitation agreements, NCCs protect proprietary information and client lists. Yet, many low-wage workers do not possess sensitive information. Thirdly, NCCs increase the job tenure, which reduces the turnover.[6] This reduces training and hiring costs. Yet, these costs are rather low for most low-wage jobs.[7] Fourthly, NCCs mitigate the hold-up problem of

3. We define the utilitarian welfare as the unweighted sum of the agent's payoff and the principal's profit.

4. The verbal argument is developed in Arnow-Richman (2006). Empirical findings from the ban of NCCs in the high-tech sector in Hawaii (Balasubramanian et al., 2020) are consistent with the argument. Moreover, for 30% of the surveyed employees with NCCs, the NCC was not mentioned during their negotiation, but they were asked to sign an NCC on their first day at work after having declined all other offers (Starr, Prescott, and Bishara, 2021, p. 69).

5. Cahuc, Postel-Vinay, and Robin (2006) find that low-wage workers possess no significant bargaining power. Instead, the wage growth for low-wage workers often comes from changing jobs, which is also shut down by NCCs (Colvin and Shierholz, 2019).

6. A positive correlation of (the enforceability of) NCCs and the average length of job tenure has been found by Balasubramanian et al. (2020) and by Starr, Frake, and Agarwal (2019).

7. A meta-study (Boushey and Glynn, 2012) finds that the turnover costs average around 20% of the annual salary and are rather lower for low-skilled jobs. For the fast-food industry, reports range between $600 and $2000 while the turnover rate is around 150% (Rosenbaum, 2019). Yet, many firms do not even know their turnover cost and seem to ignore them as they are not salient (Altman, 2017).

investments in human capital. If workers are liquidity constrained and cannot invest in their industry-specific or general human capital, an NCC allows the employer to recoup her investment (Rubin and Shedd, 1981).[8] Yet, it is debatable how much employers actually invest in their minimum wage workers' industry-specific or general human capital.

Our model also refines empirical predictions. Hair salon owners are more likely to make their employees sign NCCs when the minimum wage increases (Johnson and Lipsitz, 2020). Johnson and Lipsitz (2020) show that this can be explained if NCCs can be used to transfer utility. We complement their study by showing that effort provision is a possible microfoundation for the utility transfer. Our model also implies the monotonicity of NCCs in the minimum wage, both on the extensive and on the intensive margin (Proposition 2.2). Furthermore, we derive additional empirically testable predictions, for example, employees with a, ceteris paribus, worse outside option should have more severe NCCs or should be more likely to have an NCC at all (see Section 2.6).

This chapter is organized as follows. In Section 2.2, we provide background information on the use of non-compete clauses and when they are enforceable, and we discuss the related literature. We introduce the model in Section 2.3, and characterize the profit maximizing contracts in the benchmark and with NCCs in Section 2.4. In Section 2.5, we analyze the welfare implications of these contracts. In Section 2.6, we discuss the simplifying assumptions and summarize empirical predictions of our model. We conclude in Section 2.7.

2.2 Background and Related Literature

In this section, we summarize the relevant legislation on NCCs and the related research.

2.2.1 Background of Non-Compete Clauses

As the legislation on non-compete clauses is very different across the United States, we focus on the aspects that are relevant for our model. The principal uses an NCC to threaten the agent into exerting more effort. For the threat to be credible, courts have to be willing to enforce such an NCC.

There are attempts by Bishara (2011) and Garmaise (2011) to compare whether a state's courts tend to rule in favor of the employees or the employers. Both use a comprehensive survey of courts' decisions (Malsberger, 2019) and questionnaires to calculate one-dimensional measures of NCCs' enforceability for all states. This

8. Long (2005) proposes repayment agreements as a better alternative to NCCs in this case. The disadvantage of NCCs is that they usually remain in the contract even after the employer has recouped his investment, whereas repayment agreements expire.

allows them to order the states on a spectrum, going from states that do not enforce NCCs at all—California, North Dakota, and Oklahoma—to states in which courts are ordered to ignore hardships that NCCs cause for employees—Florida. In many states, employers can use NCCs in the way they want to.

That NCCs might be used to provide incentives is also reflected in the enforceability questionnaire of Bishara (2011): “Question 8: If the employer terminates the employment relationship, is the covenant enforceable?” (Bishara, 2011, p. 777). The states are awarded scores on a scale from 0 to 10, where 0 means that a termination makes an NCC unenforceable and 10 means that a termination makes no difference whatsoever. Only five states score less than 6. Moreover, 15 jurisdictions score 10. That is, NCCs stay active when being dismissed for poor job performance in most states.

Even if the NCC became unenforceable after dismissal for bad performance, having signed an NCC might negatively affect the search for a new job. The cost of litigating an unenforceable NCC is high for low-wage workers (Colvin and Shierholz, 2019, p. 5-6), so former employees might rather adhere to an unenforceable NCC. Empirical evidence shows that unenforceable NCCs affect the employees’ behavior (Starr, Prescott, and Bishara, 2020). Moreover, although California and North Dakota do not enforce NCCs, the prevalence is the same as in states that enforce NCCs (Starr, Prescott, and Bishara, 2021). Lastly, some NCCs specify that trials are not to be held by official courts but by mandatory arbitration. Since mandatory arbitrators’ rulings are usually confidential, the enforceability of an NCC might differ from the expected enforceability in a given state.

Summing up, in many states, NCCs are unaffected by a dismissal due to bad performance on the job. Even if a states’ law renders NCCs unenforceable after a dismissal due to bad performance, there are reasons to believe that the existence of an NCC affects the employee’s job searching behavior, and, thus, also the employee’s outcome.

2.2.2 Related Literature

This chapter is related to multiple strands of literature. We first summarize the small literatures on the incentive effects of NCCs and on utility transfers using NCCs.[9] Then, we summarize two related concepts: efficiency wages and collateralized debt. Lastly, we explain our methodological contribution to the literature on moral hazard.

In Kräkel and Sliwka (2009), contrasting our model, an NCC reduces the agent’s incentives. In their model, exerting more effort increases the probability of outside offers. Outside offers lead to a wage increases. If the agent has an NCC, however, he may not accept an outside offer, reducing the expected payoff from exerting effort.

9. We refer the reader interested in other theoretical and empirical articles on NCCs to the survey McAdams (2019).

Cici, Hendriock, and Kempf (2021) empirically test the incentive effect of NCCs. Their identification strategy is using exogenous legislative changes in the enforceability of NCCs. The hypotheses are derived without a formal model. They find that mutual fund managers perform better when NCCs get more enforceable. This evidence suggests that the mechanism in our model exists in the real world.

NCCs have been argued before to redistribute rent from the agent to the principal. Wickelgren (2018) proposes a hold-up model with investments in human capital. A minimum wage prevents the principal from extracting all rents without an NCC. By making the agent sign an NCC, the principal can prevent the agent from leaving without increasing the wage. The optimal contract does not leave a rent to the agent. In contrast to our work, this model relies on human capital investments for minimum wage workers.

Johnson and Lipsitz (2020) find in the data that higher minimum wages are associated with more NCCs. They also provide a model in which NCCs are used to transfer utility if a minimum wage restricts the transfer of utility via money. If the terms of trade favor the employers, the employees have to sign NCCs to (inefficiently) transfer utility to the employers in equilibrium. When signing an NCC, employees incur an exogenous cost while employers receive an exogenous benefit. Whether NCCs are used or not is determined by the participation constraint of the least productive firm according to a "law of one price." We complement their work by providing a microfoundation for NCCs' transferring utility.

Non-compete clauses as a means to provide incentives reminds of two similar concepts. Firstly, there are efficiency wages. In the literature started by Shapiro and Stiglitz (1984), an agent is also retained after a good outcome and dismissed after a bad outcome. The differential of the corresponding payoffs provides incentives to exert effort. The difference to our model is that efficiency wages—wages above the market-clearing level—increase the payoff in the good state. Thus, with limited liability, efficiency wages make the agent's participation constraint slack and grant him a rent. NCCs, in contrast, reduce the payoff in the bad state after a dismissal. Thus, even with limited liability, they make the agent's participation constraint tight and extract his rent.

Secondly, there is collateralized debt (e.g. Stiglitz and Weiss, 1981, Bernanke and Gertler, 1989, Chan and Thakor, 1987, Bester, 1987, Boot, Thakor, and Udell, 1991, and Tirole, 2006). An agent that is cash constrained might pledge an asset in order to improve his access to a credit line. After a signal for low effort (default), the asset is transferred to the bank. Collateralized debt both incentivizes the agent and reduces the bank's loss after the bad outcome.

Non-compete clauses in our model are similar to collaterals in lending agreements: The agent pledges his labor. After a bad signal, the NCC is activated, and the agent is not allowed to sell his labor to anyone else.

One difference in collateralized debt is the efficiency loss from transferring the asset. In the one extreme, pledging a perfectly resalable asset is a perfect substitute to monetary payments because the asset has the same value to the principal as to the agent. Thus, the friction from limited liability vanishes. In the other extreme, the principal has a negative value for the asset she has to seize (in Chwe, 1990, the asset is bodily integrity, and whipping the agent also hurts the principal). Our model is in-between these extremes: Activating an NCC costs the agent, but neither costs nor benefits the principal.

Methodologically, we contribute to the literature on agency models with moral hazard in continuous effort and with limited liability (e.g. Schmitz, 2005, Kräkel and Schöttner, 2010, Ohlendorf and Schmitz, 2012, and Englmaier, Muehlheusser, and Roider, 2014). Especially, we contribute to the agency literature with multidimensional (monetary and non-monetary) payoffs. In our model, the payoff's dimensions are present and future payoff. Minimum wages affect only present payoffs. NCCs can reduce only future payoffs via unemployment. As in our model, Kräkel and Schöttner (2010) show that controlling the access to future rents can be used to incentivize current effort.

There are articles with similar models that interpret the second argument of the agent's payoffs as pain or unfriendliness. It is pain in the coerced labor settings of Chwe (1990) and Acemoglu and Wolitzky (2011). Chwe (1990) provides a model in which the principal can inflict costly pain to the agent. As in our model, inflicting pain maximizes the profit if monetary transfers are limited due to wealth constraints. Another variant of this model is used in Acemoglu and Wolitzky (2011): The principal can pay to reduce the agent's reservation utility. In Dur, Kvaløy, and Schöttner (2022), the reduction of the agent's payoff is interpreted as an unfriendly leadership style.

2.3 Model

We consider a moral hazard model with continuous effort, binary output, and limited liability. There is a risk-neutral principal P (*she*) who owns a project. The project can be either a success and pay off V or a failure and pay off nothing. P wants to hire a risk-neutral agent A (*he*) to work on the project for one period. The principal offers the agent a contract that consists of three items: a base wage w, which is paid unconditionally, a bonus wage b, which is paid conditionally on a success, and a non-compete clause (*NCC*).[10] The wages are subject to a minimum wage that limits the agent's liability.

The agent's expected utility accrues in two stages: the effort provision stage and a continuation in which an NCC might come into play. For simplicity, we present a

10. Various forms of incentive pay are common in minimum wage jobs. We refer the interested reader to Section 2.6. In our model, we use explicit bonus payments as a stand-in for more complicated methods of incentive pay. The qualitative results of our model remain the same if the bonus wage is exogenously set to 0. The model is then closer to the efficiency wage literature.

partial market model. That is, we do not microfound the continuation payoff. Instead, we directly assume that having an NCC when losing a job reduces the expected discounted future payoff. In Section 2.6, we justify this assumption and present details about how to think about the outside option.

We now consider the effort provision stage in more detail. The agent chooses his effort $e \in [0,1]$ at a strictly convex cost of $c(e)$, where $c(0) = 0$. We assume the standard Inada conditions that $c'(0) = 0$ and $\lim_{e \to 1} c'(e) = \infty$. We also assume that $\frac{c'''(e)}{c''(e)} > \frac{1}{1-e}$ $\forall e \in [0,1)$ to get a concave objective function (see Lemma 2.3 in Appendix 2.B.2).[11] Two examples are $c(e) = -\ln(1-e) - e$ and $c(e) = \frac{e^2}{1-e}$.[12] The effort level that A chooses is private information and, thus, creates a moral hazard problem. The chosen effort is the probability that the project is successful, that is, a success payoff V accrues to the principal with probability e, $\text{Prob}(\text{success}|\ e) = e$. Successes are verifiable and serve as a signal for the agent's effort. In the case of a success, the agent gets the bonus wage b.

We now consider the simplified continuation (as mentioned above, see Section 2.6 for details). After the project is completed, the agent's continuation payoff is determined. The continuation payoff can take two values. If the agent is retained, we set the continuation payoff to zero. If the agent is fired at the end of the effort provision stage, the NCC gets activated and reduces the continuation payoff. The contract's NCC directly specifies the agent's continuation payoff, $\bar{v} \leq 0$. Concerning the principal, we assume that dismissing the agent has no effect on her continuation profit. That is, hiring a replacement is costless. As we show in Section 2.6, under this condition, it is optimal for the principal to fire the agent after a failure and to retain the agent after a success.

To sum up, a contract between the principal and the agent is defined by the tuple $(w, b, \bar{v})$. These items are constrained. The minimum wage law demands that the agent is paid at least the minimum wage $\underline{w}$ for the effort-provision stage.[13] After a failure, the principal pays the agent $w \geq \underline{w}$, and after a success, she pays him $w + b \geq \underline{w}$. The level of the minimum wage is relative to the agent's outside option

11. Compared to the canonical principal agent model, the principal has an additional choice variable, the NCC. Therefore, to get a well-behaved problem, we need a stronger assumption on the cost function than the standard assumption that $c'''(e) > 0$. Chwe (1990) and Acemoglu and Wolitzky (2011) use the same assumption in their models. In the proofs in Appendix 2.B, we will state which assumptions on the cost function we need in the respective steps. The concavity assumption is simpler and implies all of them.

12. These cost functions are only defined for $e \in [0,1)$ and $\lim_{e \to 1} c(e) = \infty$.

13. The use of NCCs to extract rents is not restricted to minimum wages. For example, the downward-rigidity of nominal wages might prevent the principal from reducing the wages but not from adding an NCC. Cici, Hendriock, and Kempf (2021) have shown that NCCs incentivize fund managers, but the rent that gets extracted in this case hardly comes from a minimum wage. Note that not all rents can be extracted, for example, information rents cannot, as they are needed to incentivize truth-telling: Anticipating that he would be asked to sign an NCC, an agent will not reveal his private information.

that we have normalized to zero.[14] The NCC is constrained, $\bar{v} \leq 0$, because it can only reduce the agent's continuation payoff. We say that a contract does have *no non-compete clause* if $\bar{v} = 0$. The lower $\bar{v}$, the lower is the agent's continuation payoff after being dismissed. We refer to a lower $\bar{v}$ as a *more severe non-compete clause*.

If he signs the contract, the agent's expected utility is given by

$$\mathbb{E}U \;=\; w + e \cdot b + (1 - e) \cdot \bar{v} - c(e). \tag{2.1}$$

The base wage is paid unconditionally, the bonus wage only in the case of success, and if the agent fails, the NCC is activated. Given his contract, the agent maximizes his expected utility by choosing his effort.

The principal's expected profit is given by

$$\pi \;=\; -w + e \cdot (V - b). \tag{2.2}$$

The principal anticipates whether the agent will sign the offered contract and, if so, which effort the agent will exert. The principal then maximizes her expected profit by choosing the contract. We assume that the success payoff, V, is large enough such that the principal makes a profit that exceeds her outside option, and therefore ignore the extensive margin (except for one paragraph in Section 2.5).

The timing of the game is as follows. The principal offers a contract to the agent. The agent can reject or accept the offer. If he rejects, the game ends and he gets his outside option. If he accepts, the game continues. The agent then chooses his effort from the unit interval. The payoffs to the agent and the principal are determined according to the accepted contract, including that the principal dismisses the agent and activates the NCC after a failure. The solution concept is subgame perfect Nash equilibrium. We find it by backward induction. The timeline in Figure 2.1 summarizes the game.

First-best welfare analysis. First, consider the benchmark without any frictions. A social planner maximizes the expected welfare

$$W^{FB} \;=\; \max_{e \in [0,1],\; \bar{v} \leq 0} \; e \cdot V - c(e) + (1 - e) \cdot \bar{v}. \tag{2.3}$$

The first-order condition shows that in the social optimum there is no NCC because, due to the Inada conditions, the effort will be interior. As a result, any NCC comes into action with positive probability, which inefficiently burns surplus. The social surplus is maximized by $\bar{v} = 0$.

Given that $\bar{v} = 0$, the first-best effort equates the marginal benefit and the marginal cost, $V = c'(e^{FB})$. This is optimal due to the welfare function's concavity.

14. Other models with limited liability often normalize, on the contrary, the minimum wage (or the limited liability) to zero. The agents are then heterogeneous in their outside options. As we assume that human capital plays no role and the agents are homogeneous, normalizing the outside option to zero fits better to our interpretation: We are interested in the effects of (an increase in) the minimum wage. For the interpretation of heterogeneous agents, keep in mind that a better outside option is equivalent to a lower minimum wage.

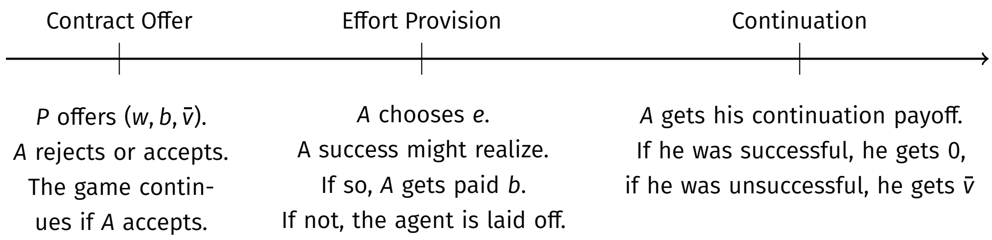

Figure 2.1. The timing of the game.

2.4 The Profit Maximizing Contract

In this section, we characterize the profit maximizing contracts for different minimum wages.

To build intuition, we begin by analyzing how an NCC changes the incentive compatibility and the participation constraint. Given the contract $(w, b, \bar{v})$, the agent chooses the effort level e^* that maximizes his expected utility,

$$e^* = \underset{e \in [0,1]}{\arg\max} \; w + e \cdot b + (1-e) \cdot \bar{v} - c(e). \tag{IC}$$

This is the agent's incentive compatibility constraint. If $b - \bar{v}$ is non-negative, the agent's optimal effort choice is characterized by the first-order condition

$$b - \bar{v} = c'(e^*). \tag{2.4}$$

The equilibrium effort is unique because the marginal cost is strictly increasing. The first-order condition shows that the bonus wage and the NCC are perfect substitutes for giving incentives. Therefore, the NCC has an *incentive effect*. P must decide to what extent to provide incentives through an NCC and to what extent through a bonus wage.

The agent only accepts the contract if his participation constraint

$$w + e^* \cdot b + (1-e^*) \cdot \bar{v} - c(e^*) \geq 0. \tag{PC}$$

is satisfied. The bonus wage and the severity of the NCC go into opposite directions in the participation constraint. A higher bonus wage makes the participation constraint slack. A more severe NCC makes the participation constraint tight. This already hints at the use and the distributional effects of NCCs: Whenever the agent would get a rent without an NCC, the principal will add an NCC to the contract and convert the rent into more incentives. The participation constraint will always bind. In the participation constraint, the NCC enters twice. Firstly, it enters indirectly via the equilibrium effort, through the incentive effect. Secondly, the *idleness effect* can be seen in the participation constraint: The NCC enters directly as $(1-e^*) \cdot \bar{v} \leq 0$. This expression is the agent's utility that gets burned in case of a failure—the labor force that the NCC forces to lie idle.

One could also decompose the effect of an NCC differently. Rearranging the agent's expected utility yields $(w+\bar{v})+e^*\cdot(b-\bar{v})-c(e^*)$.[15] This means that the NCC reduces the base and increases the bonus wage as perceived by the agent. Because the minimum wage is supposed to increase the base wage, in that sense, NCCs undermine minimum wage laws.

The principal does not profit from the reduction in the perceived base wage as the activation of the NCC, the idleness effect, burns surplus instead of transferring it. The benefit of the NCC for the principal comes from the increase in the perceived bonus wage, which increases the equilibrium effort without the principal's having to pay for it. In Section 2.5, we will take a closer look at the welfare effects of the incentive and the idleness effect.

With the possibility of imposing an NCC, the principal's problem becomes

$$\max_{w,b,\bar{v}} \quad -w+e^*\cdot(V-b) \tag{2.5}$$

$$\text{subject to} \quad e^* = \arg\max_{e\in[0,1]} \; w+e\cdot b+(1-e)\cdot\bar{v}-c(e) \tag{IC}$$

$$w+e^*\cdot b+(1-e^*)\cdot\bar{v}-c(e^*) \geq 0 \tag{PC}$$

$$\bar{v} \leq 0 \tag{NCC}$$

$$w \geq \underline{w} \qquad w+b \geq \underline{w}. \tag{MWC1) and (MWC2}$$

The principal maximizes her expected profit subject to the incentive-compatibility constraint, the participation constraint, the NCC feasibility constraint, and the minimum wage constraints.

The benchmark without non-compete clauses. Before we proceed and analyze the optimal contract with NCCs, we briefly consider the benchmark without NCCs. Formally, this means that $\bar{v}=0$ is set exogenously and P can only choose the base and the bonus wage. The optimal contracts under limited liability with those two tools are well known (see for example Laffont and Martimort, 2002, and Schmitz, 2005). Proposition 2.1 derives the optimal contract that the principal offers to the agent in the benchmark.

Proposition 2.1. *Consider the problem without NCCs. There exist threshold values in the minimum wage κ_1 and κ_3 such that*

(i) if $\underline{w}\leq\kappa_1$, then P offers the contract $(w,b)=(\kappa_1,V)$.

(ii) if $\kappa_1<\underline{w}\leq\kappa_3$, then P offers the contract $(w,b)=(\underline{w},c'(e_2^{BM}))$. Where $e_2^{BM}(\underline{w})$ is implicitly defined by $c(e_2^{BM})-e_2^{BM}\cdot c'(e_2^{BM})=\underline{w}$.

15. In this reformulation, the incentive effect is hidden in the equilibrium effort and the idleness effect is the two $\bar{v}$.

(iii) *if* $\kappa_3 < \underline{w}$, *then P offers the contract* $(w, b) = (\underline{w}, c'(e_3^{BM}))$.
Where $e_3^{BM}(\underline{w})$ *is implicitly defined by* $c'(e_3^{BM}) + e_3^{BM} \cdot c''(e_3^{BM}) = V$.

Proof. The proof is in Appendix B, Subsection 2.B.1. □

Note that the subscripts are 1 and 3. There is a specific κ_2 that lies in-between κ_1 and κ_3, but it is irrelevant in the benchmark.

The three parts of Proposition 2.1 correspond to the three cases of binding and non-binding constraints; depending on the level of the minimum wage.

Case 1. The minimum wage is lower than the base wage the principal wants to set when she ignores the minimum wage constraints. Therefore, the optimal contract is the same as with unlimited liability. The principal leaves the success payoff to the agent and uses the base wage to extract the complete surplus from the agent. Therefore, this case is commonly referred to as "selling the firm."

Case 2. If the minimum wage is above κ_1, selling the firm violates the minimum wage condition; the principal cannot extract the full social surplus anymore. To provide incentives, the base wage is chosen as low as possible: the minimum wage. The optimal bonus wage makes the agent's participation constraint binding. As the bonus wage is below the success payoff, the effort is inefficiently small. Furthermore, if the minimum wage increases, so does the base wage. Therefore, a lower bonus wage makes the participation constraint bind, implying a lower equilibrium effort. The binding participation constraint means that the minimum wage does not redistribute from the principal to the agent; it solely induces inefficiency.

Case 3. For minimum wages above κ_3, the principal does not want to lower the bonus wage further to keep the participation constraint binding. Because the participation constraint is slack, the agent gets a rent. The optimal bonus wage is constant in the minimum (and base) wage. The social surplus is, thus, constant. A minimum wage now becomes a tool of perfect redistribution: An increase of the minimum wage by one unit translates into an increase of the agent's rent by one unit.

The equilibrium analysis with non-compete clauses. Proposition 2.2 summarizes the optimal contracts with NCCs.

Proposition 2.2. *Consider the problem with NCCs. There exist threshold values in the minimum wage* κ_1 *and* κ_2 *and, if* $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2} \cdot V < 1$, *another threshold* κ_4 *such that*

(i) *if* $\underline{w} < \kappa_1$, *then P offers the contract* $(w,\ b,\ \bar{v}) = (\kappa_1,\ V,\ 0)$.

(ii) *if* $\kappa_1 \le \underline{w} \le \kappa_2$, *then P offers the contract* $(w,\ b,\ \bar{v}) = (\underline{w},\ c'(e_2^{BM}),\ 0)$.
$e_2^{BM}(\underline{w})$ *is defined by* $c(e_2^{BM}) - e_2^{BM} \cdot c'(e_2^{BM}) = \underline{w}$.

(iii) *if* $\kappa_2 < \underline{w} < \kappa_4$, *then P offers the contract*
$(w,\ b,\ \bar{v}) = \big(\underline{w},\ (1-e_3^{NCC})c'(e_3^{NCC})\ + c(e_3^{NCC}) - \underline{w},\ c(e_3^{NCC}) - \underline{w} - e_3^{NCC}c'(e_3^{NCC})\big)$.
$e_3^{NCC}(\underline{w})$ *is defined by* $c(e_3^{NCC}) + (1-e_3^{NCC}) \cdot (c'(e_3^{NCC}) + e_3^{NCC} \cdot c''(e_3^{NCC})) = V + \underline{w}$.

(iv) if $\kappa_4 \le \underline{w}$*, then P offers the contract* $(w,\ b,\ \bar{v}) = \left(\underline{w},\ 0,\ -\frac{\underline{w}-c(e_4^{NCC})}{1-e_4^{NCC}}\right)$.
$e_4^{NCC}(\underline{w})$ *is defined by* $(1-e_4^{NCC}) \cdot c'(e_4^{NCC}) + c(e_4^{NCC}) = \underline{w}$.

Proof. The proof is in Appendix B, Subsection 2.B.2. □

The four parts of Proposition 2.2 correspond to the four combinations of binding and non-binding constraints for different levels of the minimum wage. Figure 2.2 illustrates which constraints are binding in the optimum, depending on the level of the minimum wage. If $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2} \cdot V \ge 1$, Combination 4 is never optimal. Importantly, the participation constraint binds in all combinations; the agent never gets a rent. If the participation constraint were slack, there would be a profitable deviation: making the NCC more severe. The equilibrium effort increases and, because the agent gets less than the success payoff, the principal profits.

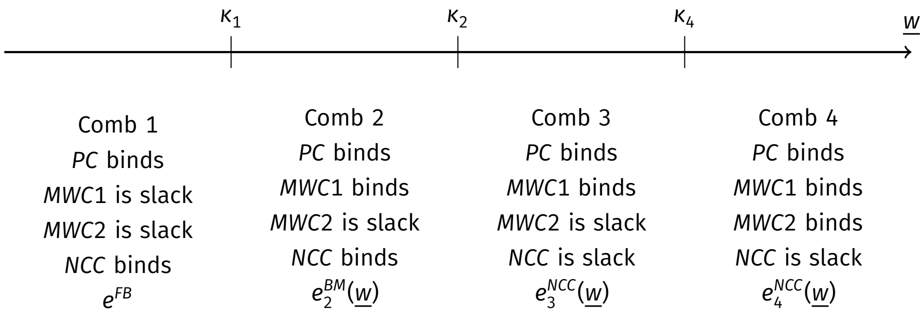

Figure 2.2. The combinations of binding and non-binding constraints that characterize the optimal contract when NCCs are allowed. The combinations are from Table 2.B.1. When there are NCCs, the cutoff κ_3 is meaningless.

Figure 2.3 illustrates and compares the optimal contracts with NCCs and without NCCs for a specific effort cost function.

We will now consider each combination in more detail.

Combination 1. This combination is identical to Case 1 in the benchmark. As the principal's profit is already equal to the first-best social surplus, she cannot do any better by introducing an NCC.

Combination 2. For minimum wages between κ_1 and κ_2, it is optimal for the principal not to use an NCC. The optimal contract is the same as in the benchmark in Case 2, although it stops at a lower minimum wage, $\kappa_2 < \kappa_3$. As the bonus wage alone makes the participation constraint binding, the principal would have to increase the wages to compensate the agent for an NCC's idleness effect after a failure. An NCC's incentive effect would be, however, small because the equilibrium effort is already high. Using an NCC is too costly.

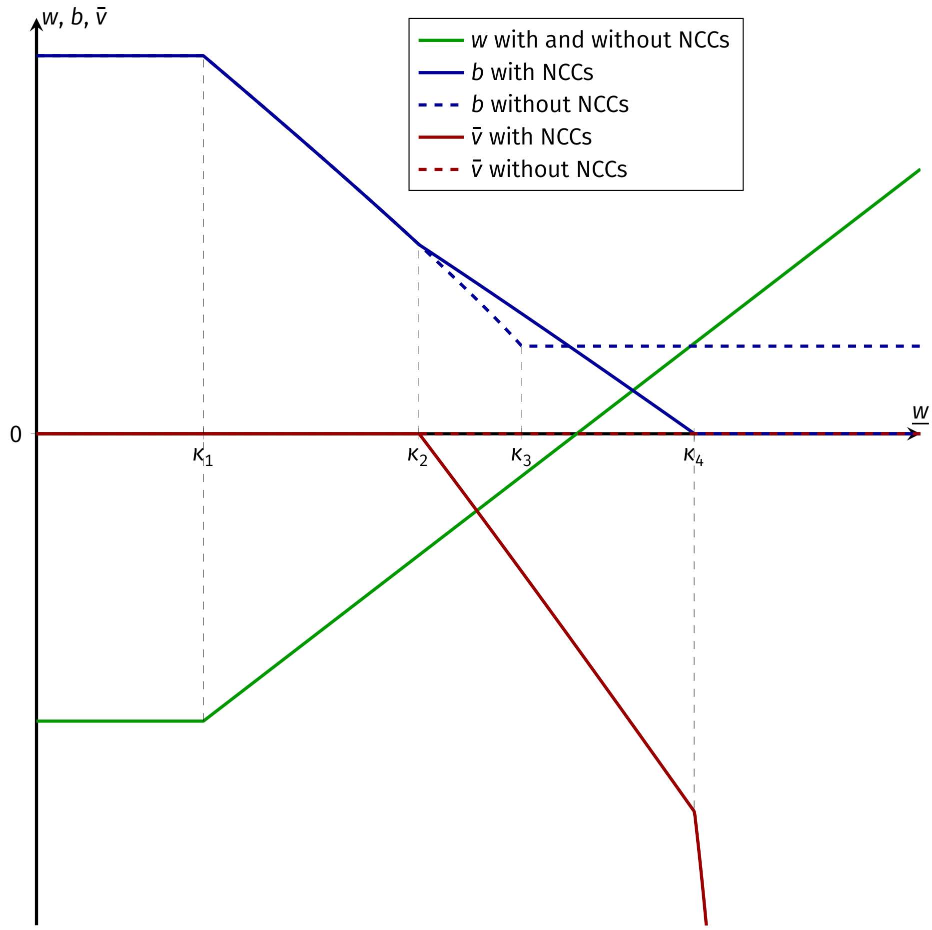

Figure 2.3. Illustration of the optimal contract for different minimum wages for $c(e) = -\ln(1-e) - e$ and $V = 10$.

Combination 3. When the minimum wage increases, the bonus wage and, hence, the equilibrium effort without NCCs decrease. Due to the lower equilibrium effort, both the incentive and the idleness effect become larger. The incentive effect because one unit of NCC affects the equilibrium effort more, and the idleness effect because the probability of a failure increases. Because the effort cost is convex, the incentive effect grows faster. At a minimum wage of κ_2, both effects are equally strong. If the minimum wage is above κ_2, the incentive effect prevails and the principal uses an NCC. Moreover, Proposition 2.3 shows that the optimal NCC gets monotonically more severe in the minimum wage.

The equilibrium effort is non-monotone in the minimum wage (Proposition 2.3). If the principal does not use an NCC, the equilibrium effort is strictly decreasing in the minimum wage. If the minimum wage is sufficiently large such that the principal uses an NCC, the NCC gets more severe when the minimum wage increases. We show that the increase in the NCC's severity overcompensates the decrease in the bonus wage: The sum of incentives increases in the minimum wage. This non-monotonicity of the equilibrium effort in the minimum wage is a novel result in the moral hazard literature.

Proposition 2.3 (Non-Monotonicity of Optimal Effort). *The equilibrium effort is non-monotone in the minimum wage.*

(i) If $\underline{w} < \kappa_1$, the equilibrium effort is constant in the minimum wage.
(ii) If $\kappa_1 \leq \underline{w} \leq \kappa_2$, the equilibrium effort is strictly decreasing in the minimum wage.
(iii) If $\kappa_2 < \underline{w}$, the equilibrium effort is strictly increasing in the minimum wage.

Proof. The proof is in Appendix B, Subsection 2.B.3. □

Not only is the equilibrium eventually increasing in the minimum wage, but it also gets inefficiently large (Proposition 2.4). If the minimum wage goes to infinity, the equilibrium effort goes to one. The principal keeps on making the NCC more severe when the minimum wage increases, as the NCC is the principal's only way of extracting the agent's rent from a higher minimum wage. The social loss from the NCC affects only the agent.

Proposition 2.4 (Inefficiently Large Optimal Effort). *As the minimum wage goes to infinity, the equilibrium effort goes to 1. Hence, the equilibrium effort level exceeds the first-best effort if the minimum wage is sufficiently large.*

Proof. The proof is in Appendix B, Subsection 2.B.4. □

Close to κ_2, for a fixed minimum wage, the bonus wage is larger when the principal may use an NCC. The reason is that the participation constraint binds already without an NCC and the NCC's idleness effect harms the agent. With NCCs, the principal can provide "double incentives" by increasing the bonus wage: The higher bonus wage makes the participation constraint slack. This allows for a more severe NCC, which makes the participation constraint binding again. Both the increase in the bonus wage and in the NCC's severity provide incentives.

At some minimum wage above κ_3, the optimal bonus wage with NCCs falls below the optimal, constant bonus wage without NCCs. It is unnecessary for the principal to pay higher bonus wages to allow herself to use a more severe NCC because the high minimum wage would make the agent's participation constraint slack anyway. As the equilibrium effort is again quite high, the benefit from providing more incentives is diminishing.

Combination 4. This combination is characterized by the lack of bonus wages: All incentives come from an NCC. Using no bonus wage is only ever optimal if the equilibrium effort reacts weakly enough to an increase in the incentives.[16] Otherwise, it is more profitable to use a bonus wage and double incentives.

16. This might answer the empirical question why some employers do not use explicit bonus wages, although they have verifiable performance measures: Other forms of implicit incentives might

As the equilibrium effort gets inefficiently large for high minimum wages, the principal would even want to pay negative bonus wages—to charge the agent for successes. The minimum wage condition for the case of a success, however, prevents negative bonus wages: The base wage would have to be above the minimum wage. As we show in the proof of Proposition 2.2, increasing the base wage above the minimum wage is too expensive to be profitable.

2.5 Welfare Analysis

Having characterized the profit-maximizing contracts, we can now look at the welfare effects of NCCs.

2.5.1 Utilitarian Welfare

The first welfare criterion that we consider is utilitarian welfare—the sum of the agent's rent and the principal's profit. From the previous section, we already know that with NCCs, the agent never gets a rent. Hence, the utilitarian welfare is equal to the principal's profit.

An NCC affects the utilitarian welfare through two channels: the incentive and the idleness effect. The NCC's total effect on the utilitarian welfare is the sum of the two effects.

The incentive effect works indirectly through the increasing equilibrium effort due to an NCC. Formally, the incentive effect is

$$\int_{e^{\text{No NCC}}}^{e^{NCC}} \left(V - c'(x)\right)\, dx, \tag{2.6}$$

where $e^{\text{No NCC}}$ denotes the equilibrium effort without NCCs and e^{NCC} denotes the equilibrium effort with NCCs, both of which depend on the minimum wage.

The incentive effect first increases in the minimum wage and finally decreases again: For minimum wages slightly above κ_2, without NCCs, the equilibrium effort is inefficiently low. An NCC moves the equilibrium effort closer to the first-best; the incentive effect is positive. When the minimum wage increases, the equilibrium effort without NCCs (weakly) decreases, while the equilibrium effort with NCCs increases. As long as the equilibrium effort with NCCs lies below the first-best, the incentive effect is, thus, increasing in the minimum wage. For large minimum wages, however, the equilibrium effort with NCCs gets wastefully large as it increases above the first-best level (Proposition 2.4). Because the equilibrium effort without NCCs is constant,

be more profitable than using bonus wages. An NCC allows the principal to increase the incentives without having to pay for it since the agent pays for the additional incentives with his rent.

the incentive effect then decreases in the minimum wage. Finally, from some minimum wage on, the equilibrium effort is large enough to make the incentive effect negative.

The idleness effect directly reduces the utilitarian welfare by reducing the agent's payoff: In the case of a failure, the NCC gets activated and burns $\bar{v}$ of the social surplus. Thus, this effect is unambiguously negative. It formally is

$$(1 - e^{\text{NCC}}) \cdot \bar{v}, \tag{2.7}$$

where e^{NCC} again denotes the equilibrium effort with NCCs, which depends on the minimum wage.

We split the evaluation of an NCC's total effect on the utilitarian welfare into two parts: minimum wages below κ_3 and minimum wages above κ_3. For minimum wages below κ_3, the agent gets no rent in the benchmark without NCCs. Thus, both in the benchmark and with NCCs, the utilitarian welfare equals the principal's profit. For minimum wages between κ_2 and κ_3, NCCs increase the principal's profit and, hence, the utilitarian welfare. NCCs mitigate the inefficiency that accompanies minimum wages.

For minimum wages above κ_3, the agent gets a rent in the benchmark without NCCs. Moreover, as the equilibrium effort is constant in the minimum wage in the benchmark, so is the utilitarian welfare. The total effect of an NCC on the utilitarian welfare is ambiguous. For minimum wages slightly above κ_3, an NCC improves the utilitarian welfare: It does so for the minimum wage of κ_3 and the incentive and the idleness effect are continuous in the minimum wage. If the minimum wage increases, however, the incentive effect begins to decrease and the idleness effect becomes more negative. For the extreme minimum wage, the total effect is negative. Therefore, there is a minimum wage above which the utilitarian welfare is smaller with an NCC. The position of this minimum wage depends on the functional form of the effort cost.

2.5.2 Pareto Dominance

As the utilitarian welfare does not consider the distribution of the social surplus, it is maximized without a minimum wage. Thus, the existence of a minimum wage hints at the policymaker's putting weight on the distribution. Therefore, we also compare equilibrium outcomes using Pareto dominance. This welfare criterion is relatively uncontroversial, as it remains agnostic about how the policymaker aggregates profits and rents in her welfare measure. An equilibrium outcome strictly Pareto dominates another if both the agent's rent and the principal's profit are strictly larger; it weakly Pareto dominates another if either rent or profit is strictly larger and the other one is equal. An equilibrium outcome that weakly Pareto dominates another also has a strictly larger utilitarian welfare.

For minimum wages between κ_2 and κ_3, the outcome with NCCs weakly Pareto dominates the benchmark. For minimum wages above κ_3, Pareto dominance has no bite, as the principal is better, but the agent is worse off with an NCC.

Extensive margin. There might be a weak Pareto improvement and, thus, efficiency gain on the extensive margin. In all of the above, we have assumed that the principal wants to offer a contract to the agent irrespective of the minimum wage. That is, the success payoff is large enough such that the profit exceeds the principal's outside option for all minimum wages, with or without NCCs. For this paragraph, we drop this simplifying assumption.

Whenever the optimal contract includes an NCC, the principal's profit is strictly larger than in the benchmark. Both without and with NCCs, the principal's profit is strictly decreasing in the minimum wage. Hence, if the principal's profit at a minimum wage of κ_2 is larger than her outside option, she participates for more minimum wages when NCCs are allowed. For all minimum wages for which the principal does not participate in the benchmark but does participate with NCCs, the NCC leads to a weak Pareto improvement: The agent gets his outside utility in both cases, whereas the principal makes a profit that exceeds her outside option.

Furthermore, the extensive margin corresponds to the employment effect of minimum wages: If the minimum wage drives a principal out of the game, there is one fewer job in the economy. Since the principal might participate for more minimum wages when NCCs are allowed, NCCs reduce the employment effect of minimum wages (more on this empirical prediction in Section 2.6).

Bounded non-compete clauses. In Appendix 2.A, we also consider bounded NCCs. The bound limits how much of the agent's rent the principal can extract. Thus, a sufficiently large minimum wage redistributes from the principal to the agent. Therefore, Pareto dominance gets back some of its bite. Unfortunately, it is prohibitively difficult to characterize the optimal contracts analytically with bounded NCCs. Nevertheless, we provide an example, in which a combination of suitably chosen minimum wages and bounds on NCCs strictly Pareto dominates any outcome that can be achieved by minimum wages alone.

2.6 Discussion

In this section, we derive empirical predictions from our model that future work could take to the data. Moreover, we defend the assumptions that we made. These entail the use of incentive pay with minimum wage jobs, the partial market setting including the outside option and the continuation payoff, and the firing rule that the principal uses.

Empirical predictions. In our model, the minimum wage is defined as "minimum wage minus the outside option," because we normalized the outside option. With heterogeneous agents, thus, the same minimum wage is "low" for those with good outside options, and "high" for those with bad outside options. Therefore, our model predicts that an agent with a worse outside option, everything else equal, should be more likely to sign an NCC or have a more severe NCC. Agents might have worse outside options if they are less educated, older, less mobile, or less healthy. Surprisingly, those who would have trouble finding a new job anyway are predicted to be bound by NCCs.[17]

The same mechanism may explain why NCCs have become so frequent. There is some evidence that NCCs reduce the payoff of those who have not signed one (Starr, Frake, and Agarwal, 2019). Furthermore, the FTC considers banning NCCs on the grounds that they negatively affect parties other than the signers of the NCC, for example because they help employers collude to weaken the competition for employees. If an employee's NCC reduces the outside options of other employees (without NCCs), our model suggests that those employees become more likely to be offered NCCs. Thus, NCCs might be a self-reinforcing phenomenon.

Concerning the wages, our model makes ambiguous predictions about the effect of NCCs. Whenever the agent's participation constraint binds, the expected wage is equal to the effort cost plus the expected damage from the NCC (the idleness effect). In the benchmark, NCCs are not allowed, so this reduces to the effort cost. Thus, whenever the agent gets no rent and the equilibrium effort is larger with NCCs, the expected wage is larger with NCCs (because the idleness effect is always negative).

If the minimum wage is above κ_3, the participation constraint gets slack in the benchmark without NCCs. The expected wage is the minimum wage plus the (constant) equilibrium effort times the (constant) bonus wage, which increases linearly in the minimum wage. If NCCs are allowed, the participation constraint binds, so the expected wage is still the effort cost plus the expected damage from the NCC. Thus, at κ_3, the expected wage with NCCs is larger than in the benchmark. Above some threshold minimum wage, however, the expected wage with NCCs is lower than in the benchmark: The bonus wage goes to zero, so the expected wage goes to the minimum wage. This implies that the realized wages stop varying for high minimum wages if NCCs are allowed. Empirical research could also test whether with NCCs there is more incentive pay for low minimum wages and less incentive pay for high minimum wages.

Furthermore, our model predicts that the wages are not that informative for the well-being of employees. Although they might receive higher wages, the agent

17. We abstract from the literal clauses of an NCC and define "severity of an NCC" directly on by how much the agent's payoff is reduced after a dismissal. This reduction hides that agents with a worse outside option probably have more severe literal clauses in their NCCs anyway. If the same literal NCC affects the job market outcome of an agent with a worse outside option less (e.g. because the most likely outcome is unemployment anyway), this agent has to be offered an NCC with more severe literal clauses to achieve the same $\bar{v}$.

loses his rent due to an NCC because he has to exert more effort. As measured by his rent, an agent is strictly worse off when NCCs are allowed compared to the benchmark, whenever the minimum wage lies above κ_3. Our model predicts that for such minimum wages, minimum wage workers should be happier in states in which NCCs are unenforceable compared to states in which NCCs are enforceable.[18]

On the macro level, the extensive margin analysis of our model predicts that the effect of minimum wages on the employment is lower when NCCs can be used. When NCCs are allowed and used (that is, if the minimum wage is above κ_2), the principal makes strictly larger profits. Therefore, when NCCs can be used, there should be fewer market exits due to the minimum wage. Johnson and Lipsitz (2020) derive the same hypothesis and test it in their Section V. They interact the enforceability measure of Bishara (2011) with the minimum wage to check whether access to NCCs moderates the employment effects of a minimum wage. They find a significant and robust effect that supports the hypothesis. This might help explain the empirical puzzle on why minimum wage increases have so little of an impact on employment.

Incentive pay. The central problem in our model is that the principal has to incentivize the agent to exert effort, which she does by using an NCC.[19] Thus, for our model to be a valid explanation for why minimum wage workers sign NCCs, it has to be the case that effort is important in minimum wage jobs. We argue that this is the case by the revealed preferences of real-world employers: There are many examples of explicit and implicit incentives in minimum wage jobs. This can only be optimal if effort is important, but not contractible.

In many jobs, employees get a bonus for reaching a quota. Examples include salesforce agents—telemarketers often get paid the minimum wage as a base wage—shelf stackers in supermarkets, or pickers in the storehouses of e-commerce firms. Some fast food firms use explicit bonus payments.[20] A large German bakery retailer uses team bonuses (Friebel et al., 2017).

Commissions are common bonuses in sales jobs (Joseph and Kalwani, 1998, p. 149). An example of minimum wage workers that receive commissions are taxi drivers. Furthermore, tips (in restaurants, at the hairdresser's, for food deliveries, and again for taxi drivers) are a kind of (stochastic) commissions.

Another kind of incentive pay are promised promotions and pay increases, which can be a form of efficiency wages. Skimming job-search websites for low-wage jobs

18. Measures other than happiness that are interesting might be (self-reported) effort at work or stress-related health issues. In the fast food industry, work effort of minimum wage workers could be measured by cleanliness, customer satisfaction with the service (or amount of complaints), or customer waiting time (or number of sales during peak hours).

19. The existence of bonus wages is not crucial.

20. "Chipotle Mexican Grill implemented a bonus program that gives hourly employees the opportunity to earn up to an extra month's pay each year. To qualify for the quarterly bonus program, restaurant teams must meet certain criteria such as predetermined sales as well as cashflow and throughput goals." (Chipotle Mexican Grill, 2019).

shows that many firms advertise their jobs with advancement options.[21] While promotions are often not the direct consequence of meeting a verifiable success, the literature on relational contracts shows that employers can build a reputation for rewarding high effort, which allows them to use unverifiable measures to incentivize effort.

A last set of examples concerns non-monetary "bonuses." One example is the personal interaction between the employer and the employee: Praise can be a bonus (Dur, Kvaløy, and Schöttner, 2022). Another example are work-related perks (Marino and Zábojník, 2008). A third example are tournament incentives: Some firms let their best employees choose their favorite shifts.[22]

Partial market. Here, we describe how we think about the agent's outside option and continuation payoff.

The outside option is the expected payoff from searching a job on the labor market. On the labor market, firms are grouped into several sectors. A sector consists of those firms to which an NCC applies. So, if an employee of a fast-food firm has an NCC that forbids him to work for any other fast-food firm, these fast-food firms are one sector. As the NCC does not rule out performing janitorial services, this is another sector.

A friction in the labor market causes involuntary unemployment. Being unemployed yields an exogenously fixed payoff. If an agent is unemployed, he searches for matches with any firm. If the agent has signed an NCC, he is not allowed to match with firms in the barred sector.[23] The more firms the agent is allowed to work for, the more probable he is to find a match.

Working for some firms in a sector yields the agent a rent over the fixed unemployment payoff. Not all minimum wage workers are asked to sign NCCs. As mentioned in the introduction, Colvin and Shierholz (2019) find that around a quarter of firms make all their low-wage workers sign an NCC. Due to the minimum wage, finding a job at a firm that uses no NCCs leaves the agent a rent.[24]

21. "Chipotle's career trajectory begins with a path from crew member to general manager to the elite level of Restaurateur. Chipotle's focus on development shows as 80% of general managers have been promoted from within, often starting as line level crew members" (Chipotle Mexican Grill, 2019).

22. Anecdotal evidence suggests that in a supermarket in New Jersey in the late 1970s, the best shifts were on weekend afternoons (Cowen, 2021). In the fast food industry, night shifts are popular because they are usually calm.

23. The severity of an NCC can be interpreted as the duration for which the agent is barred from matching with the firms in that sector or as how widely a sector is defined.

24. As our simplified model implies that the principal profits from extracting the agent's rent using an NCC, we cannot answer why not all firms make their employees sign NCCs. We have, however, three suspicions. First, real NCCs are bounded, so it might be impossible for some principals to extract the whole rent. Second, it might be that some firms have other motives than maximizing their profits: In the FTC workshop on NCCs, many comments criticized NCCs for restricting the liberty of workers,

To sum up, the parts needed to give an NCC its incentive effect are: First, there are some firms that leave their employees a rent. Second, an NCC reduces the agent's probability of finding a match with a firm that leaves a rent, so the NCC has a threat potential. Third, there is involuntary unemployment, so agents are better off working for firms with an NCC than not working at all.

Our model is a snapshot of this larger model. The partial market is an agent that matches with a firm that uses NCCs. The principal, then, offers a contract that makes the agent indifferent between continuing to search and accepting, which means losing access to the firms that do not use NCCs in the same sector after being terminated.

Another simplification in the main part is that we set the continuation payoff of an agent without an NCC to zero, which is not a normalization, as we already normalized the outside option to zero. On the one hand, this simplifies the model substantially. On the other hand, this distorts which contracts are optimal. We decided for the simplification because the qualitative results are the same, as we will now argue.

Without the simplification, in the benchmark without an NCC, the agent gets a rent in each period if he is retained. Existing work has shown that future rents can be used to provide incentives in the same vein as NCCs are used in our model: When retention is conditioned on good performance, the agent exerts more effort Kräkel and Schöttner (2010).

If the principal can additionally use NCCs, however, she can still do better. When using future rents to incentivize the agent, the agent gets a rent in each period with a success. With NCCs, the principal can turn the rents into even more incentives by reducing the payoff after a bad performance. So, the principal can extract the future rents. Setting the continuation payoff to zero, thus, merely shifts the level of efforts.

Firing rule. We have assumed that the principal can commit herself to a specific firing rule. We now argue that while it is important that we assume the commitment power, the firing rule we use (retain after success, fire after failure) is optimal, assuming that the principal can replace the agent at no cost.

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curtailing the "American dream" and being "un-American" (see for example `Comment 15`, `Comment 96`, `Comment 196`, `Comment 271`, and `Comment 297`). Third, it might be that there are losses associated with using NCCs: Jimmy John's experienced a public outrage after the media reported about its use of NCCs. A firm that wants to protect its image from such a disaster might prefer to leave its employees a rent.

Note that our simplified model would cause a paradox if all firms used unbounded NCCs to extract all rents: Then, no job would yield the agent a rent above the exogenous payoff from being unemployed. But then, the NCCs cannot reduce the agent's continuation payoff, the agent does not exert more effort, and his rent is not extracted. While the above three reasons solve this problem, we leave the exploration of the paradox in a general equilibrium model for future research.

Without commitment power, renegotiations would lead to a spiral of ever more severe NCCs in a dynamic version of the model.[25] An agent who has signed an NCC has a different outside option than an agent who has not signed an NCC: It is $\bar{v}$ instead of 0 because the principal can activate the NCC by firing the agent. Thus, the principal can offer another contract to the agent that includes a more severe NCC, such that the agent is (almost) indifferent between the new contract and $\bar{v}$. This spiral would continue until the NCC is infinitely severe. Anticipating this, a rational agent would never sign an NCC. A principal that commits herself to a firing rule breaks the spiral, as she cannot activate the NCC at will. Thus, there is no reason for the agent to accept a contract with a more severe NCC.

Reputation might be an alternative for commitment power. The principal might be infinitely-lived and embedded into a larger, infinitely repeated game with multiple short-lived agents that play one after another. If there is a small, yet strictly positive probability for the principal's being a commitment type, Proposition 2 of Fudenberg, Kreps, and Maskin (1990, p. 560) applies: If the discount factor is sufficiently large, there is a subgame perfect equilibrium in which the principal without commitment power gets almost the same payoff as the commitment type.

Given that the principal can commit herself or build a reputation for following some firing rule, it is optimal to choose to retain the agent with certainty after a success and to fire the agent with certainty after a failure if the principal can replace the agent at no cost. Lemma 2.1 proves this by showing that the extreme firing rule maximizes the agent's expected utility for a fixed bonus wage and fixed incentives from the NCC. As the principal can extract all surplus by increasing the incentives from the NCC until the participation constraint binds, she wants to choose the firing rule that maximizes the agent's expected utility for a given equilibrium effort.

Lemma 2.1. *Let f_f be the probability that the agent gets fired after a failure and f_s the probability that the agent gets fired after a success. In equilibrium, the principal chooses $f_s = 0$ and $f_f = 1$ if she can replace the agent at no cost.*

Proof. The proof is in Appendix B, Subsection 2.B.5. □ □

There are two effects at play: The more succeeding increases the probability of retention, the more incentive the agent has to exert effort. Thus, the extreme firing rule provides the most incentives. On the other hand, the extreme firing rule leads to the NCC's being activated more often, which reduces the agent's expected utility. However, to still provide the same incentives, less extreme firing rules have to be paired with more severe NCCs. Lemma 2.1 shows that the negative effect from more severe NCCs outweighs the positive effect from a reduced probability of activating the NCC.

25. We thank an anonymous referee for pointing this out.

2.7 Conclusion

We introduce the effort incentives that non-compete clauses have as a new effect to the public discussion and to research. Our simple model shows that a single premise is sufficient to endow non-compete clauses with an incentive effect: A non-compete clause has to worsen the employee's prospects after a dismissal.

Our model shows that non-compete clauses can transfer utility from the agent to the principal. Without a minimum wage, the principal can extract all of the agent's surplus using money and does not use a non-compete clause. With a minimum wage—a purposefully created friction to transfer utility via money to the agent—the principal uses a non-compete clause to extract the agent's rent again. It is the agent's rent that pays for the additional incentives from the non-compete clause. Thus, non-compete clauses undermine the policymakers' attempt to transfer utility from the principal to the agent with a minimum wage.

This new effect can enrich the public discussion and research. The public discussion has assumed that non-compete clauses in their employment contracts harm minimum wage workers, but it has not voiced a channel. In fact, it has been an open question why rational employees sign non-compete clauses at all in the absence of reasons such as human capital, protection of proprietary information, bargaining, or reduction of turnovers. We argue that the effort incentives from non-compete clauses explain both. Importantly, there is first evidence that our proposed mechanism exists, although in a different setting: Non-compete clauses increase the effort of mutual fund managers as measured by their performance (Cici, Hendriock, and Kempf, 2021).

Effort provision can explain some observed patterns: If the minimum wage is increased, the prevalence of non-compete clauses increases (Johnson and Lipsitz, 2020). As the non-compete clauses return the minimum wage to the employers, minimum wages have little effect on employment. Effort provision may also explain why a change in the enforceability of non-compete clauses does not imply that wages change in a certain direction. After banning non-compete clauses, the wages increased in Oregon (Lipsitz and Starr, 2022), but not in Austria (Young, 2021). Our model predicts that the direction of the change in wages depends on the existence and the level of a minimum wage.

Our model is too simplified and the mechanism too complicated to derive a recommendation for whether non-compete clauses for minimum wage workers should be banned: Even when ignoring all other mechanisms, whether banning non-compete clauses is beneficial, depends on the several parameters and functional forms. What we can say for sure is that introducing a minimum wage without taking into account the possible interactions with non-compete clauses is a mistake.

While our model makes empirically testable predictions, we lack suitable data to test them. Counter-intuitively, our model predicts that, ceteris paribus, those agents with the worst outside options are the most likely to being offered a non-compete clauses (because the minimum wage is effectively larger for these agents). Also, if non-compete clauses are, ceteris paribus, more enforceable in a state, this should lead to lower rents for minimum wage workers, which might be reflected in a lower job satisfaction. We leave testing these predictions for future research.

Appendix 2.A Bounded Non-Compete Clauses

As we have seen in Section 2.2, the legislation on NCCs varies across the United States. No state, however, would enforce an NCC that, say, forbade the employee to ever work in the same field again: Real NCCs cannot be arbitrarily severe. In the main part, we have abstracted from that to keep the intuition simple. In this section, we assume that the severity of NCCs has an exogenous bound. The differences in the legislation across states can be interpreted as different bounds.

In the following, we will formally define a bound on NCCs and solve for the optimal contracts with this additional constraint. We find that whenever the optimal NCC without a bound would be more severe than the bound, then the optimal NCC is equal to the bound. Moreover, there is a (large) minimum wage, for which the optimal NCC has reached the bound, and from which on the bonus wage is constant. In contrast to the case with unbounded NCCs, there is a range with constant bonus wages (i) irrespective of the cost function and (ii) the constant bonus wage might be positive.

Having characterized the optimal contracts with bonus wages, we revisit the welfare analysis using Pareto dominance. The bound limits the principal's power to extract the agent's rent: From some minimum wage on, the agent is left a rent. While we cannot derive general results, we show with an example that a combination of minimum wages and bounded NCCs might in some cases Pareto dominate minimum wages alone. The intuition is that redistribution with minimum wages alone causes a welfare loss due to inefficiently low effort. Redistribution with minimum wages and bounded NCCs causes a welfare loss due to the idleness effect and possibly inefficient effort (either too low or too high). Depending on the cost function and the parameters, either of those two scenarios might cause less of a loss.

2.A.1 Optimal Contracts with Bounded Non-Compete Clauses

We define $\underline{\bar{v}} < 0$ as the most severe NCC that the principal may use. The additional constraint takes the form $\bar{v} \geq \underline{\bar{v}}$.

We formalize our findings as Proposition 2.5.

Proposition 2.5 (Bounded Non-Compete Clauses). *Let $\underline{\bar{v}} < 0$ be a lower bound on the NCC.*

(i) Let, without a bound on NCCs, the optimal NCC be $\bar{v} \geq \underline{\bar{v}}$. Then, the optimal contract remains the same with a bound on NCCs.

(ii) Let, without a bound on NCCs, the optimal NCC be $\bar{v} < \underline{\bar{v}}$. Then, the optimal contract with a bound on NCCs has $\bar{v} = \underline{\bar{v}}$. If the optimal bonus wage is positive, when the bound on the NCC starts binding, the bonus wage decreases more steeply than without a bound. At some larger minimum wage, the optimal bonus wage becomes constant, either at a positive level or at zero. If the optimal bonus wage is

zero when the bound on the NCC starts binding, the bonus wage remains at zero for all larger minimum wages.

Proof. The proof is in Appendix B, Subsection 2.B.6. □

As the profit-maximizing NCC gets infinitely severe if the minimum wage goes to infinity, it will eventually reach the bound.

After the bound is reached, positive bonus wages decrease faster than without a bound because there are no more double incentives: Increasing the bonus wage means that the agent gets a rent that cannot be converted into more incentives, as the NCC cannot be made more severe. This reduces the benefit of bonus wages.

As soon as the NCC has reached the bound and the bonus wage remains constant, there is redistribution as in the benchmark. The minimum wage at which the bonus wage becomes constant is larger than that in the benchmark, κ_3.

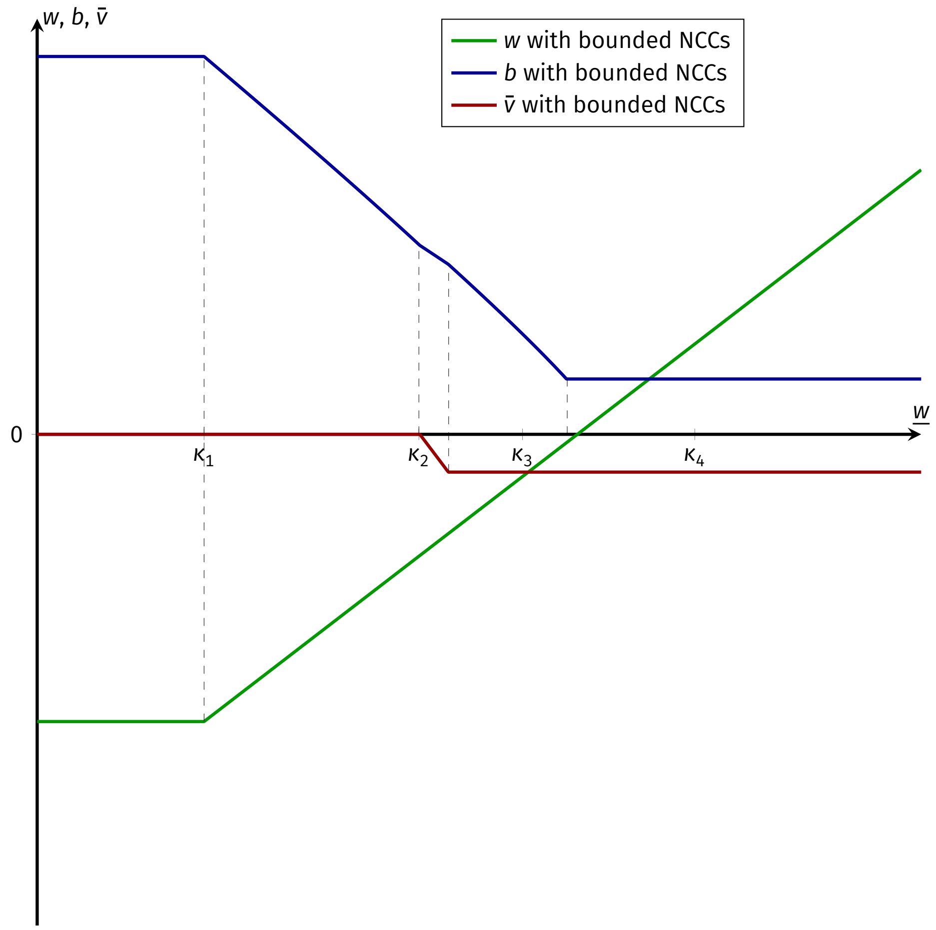

Figure 2.A.1. Illustration of the optimal contract for different minimum wages for $c(e) = -\ln(1-e) - e$, $V = 10$ and a bound on the NCC of $\underline{v} = -1$.

Figure 2.A.1 illustrates the optimal contract with bounded NCCs for a specific effort cost function and a specific bound. In the depicted case, the optimal constant

bonus wage is positive. The optimal contract is the same as without a bound up to a minimum wage slightly above κ_2. Then, the bound on the NCC starts to bind and the optimal bonus wage has a kink. Somewhere to the right of κ_3, the optimal bonus wage gets constant and the participation constraint gets slack. If the bound on the NCC were looser, the optimal constant bonus wage might be zero.

2.A.2 Welfare Effects of Bounded Non-Compete Clauses

When NCCs are bounded, minimum wages can again redistribute from the principal to the agent. If the minimum wage increases, the profit maximizing contract eventually has a constant bonus wage and an NCC that lies at the bound (Proposition 2.5). If the minimum wage increases further, the utilitarian welfare remains constant as in the benchmark for minimum wages above κ_3. In this area, a one unit increase of the minimum wage reduces the principal's profit by one unit and increases the agent's rent by one unit. Because of the NCC, this particular minimum wage is larger than in the benchmark.

For an exemplary effort cost function, we show that the constant utilitarian welfare with bounded NCCs exceeds that in the benchmark, if the bound is suitably chosen. This implies that, setting the minimum wage correspondingly, bounded NCCs can lead to outcomes that strictly Pareto dominate any benchmark outcome.

We reconsider the functional form of the cost function and the parameters that we have plotted above: $c(e) = -\ln(1-e) - e$ and $V = 10$. Our simple example relies on the peculiar fact that the principal coincidentally induces first-best effort at κ_4; that is, without a bonus wage, using only an NCC of $-V$. We choose this NCC as the bound, $\underline{\bar{v}} = -V$. Thus, the equilibrium effort remains constantly at the first-best level for all higher minimum wages and the redistribution begins at κ_4. As the effort is at the first-best level, the incentive effect is maximized and exactly cancels out the inefficiency due to the minimum wage in the benchmark without NCCs. The inefficiency from the idleness effect is also constant in the minimum wage because the equilibrium effort and the NCC are constant. Thus, with the logarithmic cost function and the bound $\underline{\bar{v}} = -V$ for $\underline{w} \geq \kappa_4$, the utilitarian welfare is $(1-e^{FB}) \cdot V$ below the first-best.

Consider now the constant utilitarian welfare in the benchmark for minimum wages above κ_3; the minimum wages for which there is redistribution from the principal to the agent. There is one source of inefficiency: too little effort. The utilitarian welfare is $\int_{e_3^{BM}}^{e^{FB}} V - c'(x)\, dx$ below the first-best.

We can now compare the constant levels of the utilitarian welfare with the bounded NCC and without NCCs. With a bounded NCC, the idleness effect reduces the utilitarian welfare by $(1-e^{FB}) \cdot V$ compared to the first-best. In the benchmark, the inefficiently low equilibrium effort reduces the utilitarian welfare by

$\int_{e_3^{BM}}^{e^{FB}} V - c'(x)\ dx$ compared to the first-best. That is, if V is large enough, the outcome with a bounded NCC is more efficient.[26]

The example is illustrated in Figure 2.A.2. On the x-axis is the minimum wage and on the y-axis are the principal's expected profits for the benchmark, with unbounded NCCs, and with the bounded NCC. In the benchmark, the utilitarian welfare equals the expected profit up to κ_3 and is constant for higher minimum wages, whereas the expected profit is decreasing with slope -1. With the bounded NCC, the utilitarian welfare equals the expected profit up to κ_4 and is constant for higher minimum wages, whereas the expected profit is decreasing with slope -1. The respective constant level is marked by a dotted line. With unbounded NCCs, the utilitarian welfare always equals the expected profit and is never constant. As is illustrated, the utilitarian welfare decreases fast when the equilibrium effort approaches one because the marginal cost of effort increases fast.

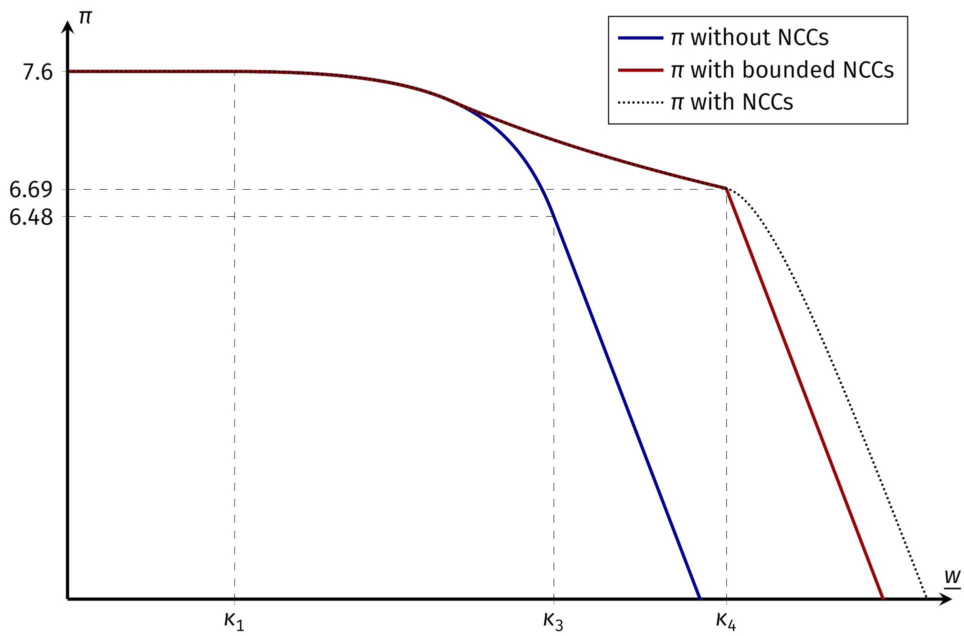

Figure 2.A.2. Bounded non-compete clauses potentially allow for strict Pareto improvements. We choose $c(e) = -\ln(1-e) - e$, $V = 10$ and $\underline{v} = -10$.

Whenever the constant utilitarian welfare, which can be distributed between the principal and the agent, is larger with a bounded NCC than in the benchmark, one

26. When $V \to 0$, both social losses go to zero. The loss with a bounded NCC is coincidentally equal to e^{FB}; it is concave in V. The loss in the benchmark is a more complicated expression, $\sqrt{1+V} - 1 - \frac{1}{2} \cdot \ln(1+V)$. It is the area between V and the marginal cost in the range from e_3^{BM} to e^{FB}; it is convex in V. When increasing V, the loss with a bounded NCC increases initially faster than the loss in the benchmark. For larger V, the loss in the benchmark increases faster. Numerically, they intersect at $V \approx 7.873$.

can construct an equilibrium that Pareto dominates any benchmark equilibrium. In the benchmark, for minimum wages above κ_3, the agent's rent is $\underline{w} - \kappa_3$. With the exemplary bounded NCC, for minimum wages above κ_4, the agent's rent is $\underline{w} - \kappa_4$. To give the agent the same rent as in the benchmark, the minimum wage has to be increased; in this example by $\kappa_4 - \kappa_3$. This procedure can be exported to all other effort cost functions, success payoffs, and bounds by replacing κ_4 by the minimum wage at which the utilitarian welfare becomes constant.

Whether bounded NCCs can lead to Pareto improvements over minimum wages alone hinges on two constraints.

A technological constraint: Whether the constant utilitarian welfare with a bounded NCC can be larger than that without NCCs depends on the effort cost function. Without NCCs, there is inefficiently little effort. With a bounded NCC, there is a welfare loss from the idleness effect and a different equilibrium effort because of the incentive effect. The relative sizes of the effects depend on the effort cost function (and the bound on the NCC).

An informational constraint: The policymaker must have sufficient information to choose the right bound on NCCs and the suitable minimum wage to attain a Pareto improvement. The bound on NCCs has to be chosen optimally to increase the utilitarian welfare. If the bound on NCCs is either too small or too large, the utilitarian welfare might be smaller than with minimum wages alone. The minimum wage has to be chosen such that the agent receives a certain rent, which also depends on the bound on the NCC. The looser the bound on NCCs, the larger minimum wages have to be to redistribute at all. Additionally, all of this depends on the effort cost function. Heterogeneity in agents could make it impossible to find a minimum wage that suits all.

Appendix 2.B Proofs

2.B.1 Proof of Proposition 2.1

Consider the problem without NCCs. There exist threshold values in the minimum wage κ_1 and κ_3 such that

(i) *if $\underline{w} \leq \kappa_1$, then P offers the contract $(w,b) = (\kappa_1, V)$.*

(ii) *if $\kappa_1 < \underline{w} \leq \kappa_3$, then P offers the contract $(w,b) = (\underline{w}, c'(e_2^{BM}))$. Where $e_2^{BM}(\underline{w})$ is implicitly defined by $c(e_2^{BM}) - e_2^{BM} \cdot c'(e_2^{BM}) = \underline{w}$.*

(iii) *if $\kappa_3 < \underline{w}$, then P offers the contract $(w,b) = (\underline{w}, c'(e_3^{BM}))$. Where $e_3^{BM}(\underline{w})$ is implicitly defined by $c'(e_3^{BM}) + e_3^{BM} \cdot c''(e_3^{BM}) = V$.*

Proof. First, we show that the objective function is strictly concave in the bonus wage. Let $E(b)$ be the maximizer of the agent's utility, that is, the equilibrium effort,

$$E(b) = \begin{cases} (c')^{-1}(b) & \text{if } b \geq 0 \\ 0 & \text{if } b < 0. \end{cases} \tag{2.B.1}$$

If the bonus wage is non-negative, the equilibrium effort is determined by the solution of the agent's first-order condition. Furthermore, $E(b)$ is strictly increasing in this range. If the bonus wage is negative, a corner solution, $E(b) = 0$, is optimal. We will use this function with a different argument again, when NCCs are allowed. The first and second derivative of $E(b)$ with respect to its positive argument are $E'(b) = \frac{1}{c''(E(b))}$ and $E''(b) = -\frac{c'''(E(b))}{(c''(E(b)))^3}$.

Remember that the expected profit is $\pi = -w + E(b) \cdot (V - b)$. The first and second derivatives with respect to the bonus wage are then given by

$$\frac{\partial \pi}{\partial b} = E'(b) \cdot (V - b) - E(b) \quad \text{and} \tag{2.B.2}$$

$$\frac{\partial^2 \pi}{\partial b^2} = E''(b) \cdot (V - b) - 2E'(b). \tag{2.B.3}$$

Since $E''(\cdot) < 0$ and $E'(\cdot) > 0$, the second derivative is negative. This implies that P's objective function is strictly concave in the bonus wage.

Next, we look at the constraints of P's problem. We now show that $MWC2$ is always slack. Assume to the contrary that $MWC2$ binds. Rearranging $MWC2$ yields $b = \underline{w} - w$. By $MWC1$ we know that $w \geq \underline{w}$, which then implies that $b \leq 0$. A non-positive bonus wage, however, implies that the equilibrium effort is zero, which cannot be optimal.[27] Hence, $MWC2$ is always slack.

27. The maximum profit is zero for negative minimum wages and $-\underline{w}$ for positive minimum wages. As we assume that the success payoff is sufficiently large for the principal to be able to achieve a positive profit, a non-positive bonus wage cannot be optimal.

This leaves two constraints that can either bind or be slack, the *PC* and *MWC*1. We now show that it cannot be the case that both *PC* and *MWC*1 are slack. Assume to the contrary that both *PC* and *MWC*1 are slack. This means that there is a profitable deviation: Decreasing w by ϵ still leaves *PC* and *MWC*1 slack, but increases *P*'s expected profit. Therefore, in the optimum, either *PC* or *MWC*1 or both bind.

This leaves us with the following three possible cases:
Case 1: *PC* binds and *MWC*1 is slack.
Case 2: *PC* binds and *MWC*1 binds.
Case 3: *PC* is slack and *MWC*1 binds.

Next, we focus on each case in more detail.

Case 1. *P*'s problem is given by

$$\max_{w,b} \quad -w + E(b) \cdot (V - b) \tag{2.B.4}$$

$$\text{subject to} \quad w + E(b) \cdot b - c(E(b)) \;=\; 0 \tag{PC}$$

$$w \;>\; \underline{w} \quad \text{and} \quad w + b \;>\; \underline{w}. \qquad \text{(MWC1) and (MWC2)}$$

We will ignore the slack constraints for the moment and later check for which minimum wages they are not violated. The *PC* can be rewritten as $E(b) \cdot b = c(E(b)) - w$. We plug this into *P*'s objective function and maximize over the equilibrium effort instead of the bonus wage. The first-order condition is $V = c'(E(b)) = b$. Since the objective function is concave, we know that the first-order condition yields the global maximum. Therefore, $b = V$, $E(V) = e^{FB}$, and $w = c(e^{FB}) - e^{FB} \cdot c(e^{FB})$. Now, we check the constraints. Because $V > 0$, *MWC*2 is slack. *MWC*1 is slack if $\underline{w} < c(e^{FB}) - e^{FB}c'(e^{FB}) \equiv \kappa_1$.

Case 2. *P*'s problem is given by

$$\max_{w,b} \quad -w + E(b) \cdot (V - b) \tag{2.B.5}$$

$$\text{subject to} \quad w + E(b) \cdot b - c(E(b)) \;=\; 0 \tag{PC}$$

$$w \;=\; \underline{w} \quad \text{and} \quad w + b \;>\; \underline{w}. \qquad \text{(MWC1) and (MWC2)}$$

There are two unknowns and two binding constraints. Plugging *MWC*1 into *PC* implicitly characterizes $E(b)$ and the bonus wage. There are three subcases: negative minimum wages, $\underline{w} = 0$, and positive minimum wages.

For each negative $\underline{w}$, there are exactly one b and one $E(b)$ such that the participation constraint binds. The reason is the following: Rearrange the binding participation constraint to get

$$E(b) \cdot b - c(E(b)) \;=\; -\underline{w}. \tag{2.B.6}$$

The left-hand side is the part of the agent's utility that is generated by exerting effort. Graphically, it is the area above an increasing function ($c'(e)$), between 0 and $E(b)$, another increasing function. It is zero for a bonus wage of zero, and is strictly increasing in the bonus wage because $c''(e) > 0$. Therefore, there can be at most one bonus wage for each negative minimum wage such that this holds. Furthermore, for negative minimum wages, there is a bijection between b and $E(b)$. Since the right-hand side is strictly positive, so is the bonus wage, which implies $MWC2$.

Consider the minimum wage $\underline{w} = 0$. Since the right-hand side of equation (2.B.6) is zero, so is the equilibrium effort, which means that the bonus wage has to be non-positive. $MWC2$ is only slack if the bonus wage is positive. Thus, there is no bonus wage such that PC binds and $MWC2$ is slack.

Consider positive minimum wages. The participation constraint is always slack. That is, there are no bonus wage and no equilibrium effort that satisfy equation (2.B.6).

Summing up the optimal contract in Case 2: For negative minimum wages, let $e_2^{BM}(\underline{w})$ denote the effort that makes the participation constraint (2.B.6) binding. Then, $e_2^{BM}(\underline{w})$ is implicitly defined by $e_2^{BM}(\underline{w}) \cdot c'(e_2^{BM}(\underline{w})) - c(e_2^{BM}(\underline{w})) = -\underline{w}$. We also get that $b = c'(e_2^{BM}(\underline{w}))$ and from $MWC1$ we get $w = \underline{w}$.

Case 3. P's problem is given by

$$\max_{w,b} \quad -w + E(b) \cdot (V - b) \tag{2.B.7}$$

$$\text{subject to} \quad w + E(b) \cdot b - c(E(b)) \ > \ 0 \tag{PC}$$

$$w \ = \ \underline{w} \quad \text{and} \quad w + b \ > \ \underline{w}. \tag{MWC1) and (MWC2}$$

We will ignore the slack constraints for the moment and later check for which minimum wages they are not violated. We plug $MWC1$ into the objective function and take the derivative. The optimal bonus wage is characterized by the marginal profit's being 0. The solution to the first-order condition implicitly defines the optimal effort in Case 3, e_3^{BM}: $c'(e_3^{BM}) + e_3^{BM} \cdot c''(e_3^{BM}) = V$. Hence, $e_3^{BM} < e^{FB}$. We also get that $w = \underline{w}$ and $b = c'(e_3^{BM})$. Next, we check the constraints. As $e_3^{BM} > 0$, $MWC2$ is slack. PC is slack if $\underline{w} > c(e_3^{BM}) - e_3^{BM} c'(e_3^{BM}) \equiv \kappa_3$.

The optimal contract. We have verified that the optimal contract from Case 1 is feasible if $\underline{w} < \kappa_1$, the optimal contract from Case 2 is feasible if $\underline{w} < 0$, and the optimal contract from Case 3 is feasible if $\underline{w} > \kappa_3$. These thresholds are $\kappa_1 = c(e^{FB}) - e^{FB} c'(e^{FB}) < 0$ and $\kappa_3 = c(e_3^{BM}) - e_3^{BM} c'(e_3^{BM}) < 0$. Because $e_3^{BM} < e^{FB}$, it follows that $\kappa_1 < \kappa_3$.

Thus, for $\underline{w} < \kappa_1$, we have two candidates: Case 1 and Case 2. The maximization problem in Case 2 has two binding constraints, while the maximization problem in Case 1 has none. As a result, the profit from the optimal contract in Case 1 is weakly larger. The concavity of the objective function and the fact that the bonus wages from Case 1 and Case 2 are different for all $\underline{w} < \kappa_1$ imply that the profit is strictly larger. For $\kappa_1 \leq \underline{w} \leq \kappa_3$, the only candidate is Case 2; thus, this contract is optimal. For $\kappa_3 < \underline{w}$, we have again two candidates: Case 2 and Case 3. Since the maximization problem in Case 3 has only one binding constraint, the profit from the optimal contract in Case 3 is weakly larger. Again, concavity and different solutions imply strictly larger profits.

□

2.B.2 Proof of Proposition 2.2

Consider the problem with NCCs. There exist threshold values in the minimum wage κ_1 *and* κ_2 *and, if* $\lim_{e\to 1} \frac{c'''(e)}{[c''(e)]^2} \cdot V < 1$, *another threshold* κ_4 *such that*

(i) if $\underline{w} < \kappa_1$, *then P offers the contract* $(w,\ b,\ \bar{v}) = (\kappa_1,\ V,\ 0)$.

(ii) if $\kappa_1 \leq \underline{w} \leq \kappa_2$, *then P offers the contract* $(w,\ b,\ \bar{v}) = (\underline{w},\ c'(e_2^{BM}),\ 0)$. $e_2^{BM}(\underline{w})$ *is defined by* $c(e_2^{BM}) - e_2^{BM} \cdot c'(e_2^{BM}) = \underline{w}$.

(iii) if $\kappa_2 < \underline{w} < \kappa_4$, *then P offers the contract* $(w,\ b,\ \bar{v}) = \big(\underline{w},\ (1-e_3^{NCC})c'(e_3^{NCC})\ + c(e_3^{NCC}) - \underline{w},\ c(e_3^{NCC}) - \underline{w} - e_3^{NCC}c'(e_3^{NCC})\big)$. $e_3^{NCC}(\underline{w})$ *is defined by* $c(e_3^{NCC}) + (1-e_3^{NCC}) \cdot (c'(e_3^{NCC}) + e_3^{NCC} \cdot c''(e_3^{NCC})) = V + \underline{w}$.

(iv) if $\kappa_4 \leq \underline{w}$, *then P offers the contract* $(w,\ b,\ \bar{v}) = \left(\underline{w},\ 0,\ -\frac{\underline{w}-c(e_4^{NCC})}{1-e_4^{NCC}}\right)$. $e_4^{NCC}(\underline{w})$ *is defined by* $(1-e_4^{NCC}) \cdot c'(e_4^{NCC}) + c(e_4^{NCC}) = \underline{w}$.

Proof. The proof proceeds in two main parts. The first part is about simplifying the problem. Since there are four inequality constraints, there are 16 possible combinations of slack and binding constraints. First, we identify those four combinations that can be optimal. In all of those combinations, the participation constraint is binding; the agent does not get a rent. We use this fact to reduce the problem's dimensionality by using the participation constraint to express the optimal NCC in terms of the minimum wage and the bonus wage. The first combination is the same as Case 1 in the benchmark, which means that this contract is profit maximizing for $\underline{w} < \kappa_1$. For all $\underline{w} \geq \kappa_1$, the base wage has to be the minimum wage. This fact and an additional piece of notation simplify the problem further. This yields a strictly quasi-concave objective function of only the bonus wage with one inequality constraint. The optimal bonus wage and whether the inequality constraint binds show into which combination the contract falls. In the second part, we solve this rewritten problem.

The possibly optimal combinations. The agent's first-order condition for the optimal effort with NCCs is

$$b - \bar{v} \ = \ c'(e). \tag{2.B.8}$$

Whenever the left-hand side is non-negative, the first-order condition yields the optimal equilibrium effort, which we, express as $E(b-\bar{v}) \equiv (c')^{-1}(b-\bar{v})$. As above, a negative left-hand side implies that the corner solution $E(b-\bar{v}) = 0$ is optimal.

The principal's problem is

$$\max_{w,b,\bar{v}} \quad -w + E(b-\bar{v}) \cdot (V-b) \tag{2.B.9}$$

$$\text{subject to} \quad w + E(b-\bar{v}) \cdot b + (1 - E(b-\bar{v})) \cdot \bar{v} - c(E(b-\bar{v})) \geq 0 \tag{PC}$$

$$\bar{v} \leq 0 \tag{NCC}$$

$$w \geq \underline{w} \qquad w + b \geq \underline{w}. \qquad \text{(MWC1) and (MWC2)}$$

To solve the principal's problem, one has to know which constraints bind and which are slack for different minimum wages. In total, there are 16 combinations. They are summarized in Table 2.B.1. The combinations' order in Figure 2.2 reflects their

Table 2.B.1. The 16 combinations of binding constraints.

No.	PC	NCC	MWC1	MWC2	Relevant?
1	binds	binds	slack	slack	$\underline{w} \leq \kappa_1$
2	binds	binds	binds	slack	$\kappa_1 < \underline{w} \leq \kappa_2$
3	binds	slack	binds	slack	$\kappa_2 < \underline{w} \leq \kappa_4$
4	binds	slack	binds	binds	$\kappa_4 < \underline{w}$
5	slack	binds	binds	binds	no, *PC*
6	slack	binds	binds	slack	no, *PC*
7	slack	binds	slack	binds	no, *PC*
8	slack	binds	slack	slack	no, *PC*
9	slack	slack	binds	binds	no, *PC*
10	slack	slack	binds	slack	no, *PC*
11	slack	slack	slack	binds	no, *PC*
12	slack	slack	slack	slack	no, *PC*
13	binds	binds	binds	binds	no, no effort
14	binds	binds	slack	binds	no, no effort
15	binds	slack	slack	binds	no, deviation
16	binds	slack	slack	slack	no, deviation

occurrence when the minimum wage increases. We will now prove that the optimal contract always falls into the Combinations 1 to 4 and never into the Combinations 5 to 16 for three distinct reasons (see column six of Table 2.B.1).

Firstly, the participation constraint has to bind. Otherwise, there is a profitable deviation: Make the NCC more severe, keeping everything else fixed. Note that the bonus wage is optimally never larger than the success. Then, the agent exerts more effort, which leads to more successes and more profit.

Secondly, it cannot be that *MWC*2 and *NCC* bind simultaneously. If they did, the agent would exert no effort. Then, the principal has no revenue. This cannot be optimal by our assumption that the success payoff is sufficiently large to allow for positive profits.

Thirdly, *MWC*1 can only be slack when the NCC feasibility constraint binds. Otherwise, there is a profitable deviation. In these combinations, the principal uses an

NCC and pays a larger than necessary base wage. This cannot be optimal because there is a profitable deviation: Decrease the base wage by one unit and increase the bonus wage and make the NCC less severe by one unit. Because bonus wage and the NCC's severity are perfect substitutes, the equilibrium effort stays the same. Furthermore, the participation constraint remains satisfied: The agent loses one unit on the base wage but gains one unit both if there is a success and if there is a failure. The principal's profit increases because he saves on the base wage one unit with certainty and loses on the bonus wage one unit with the success probability (less than one by the Inada conditions). The principal can repeat this deviation until either $MWC1$ or NCC binds.

When is the first combination optimal? In the benchmark, we have seen that in the first combination, the optimal contract implements the first-best effort. Additionally, the principal extracts the whole surplus. Therefore, this contract is profit-maximizing whenever it is feasible.

As we have seen in the benchmark, the contract in the first combination is only feasible if $\underline{w} < \kappa_1 = c(e^{FB}) - e^{FB}c'(e^{FB}) < 0$. This implies that for all $\underline{w} \geq \kappa_1$, the optimal contract is from either the second, the third, or the fourth combination. In all of these combinations, the base wage optimally is the minimum wage; $MWC1$ binds.

From now on, $\underline{w} \geq \kappa_1$, which eliminates w from the problem. Thus, b and $\bar{v}$ remain. Furthermore, the participation constraint PC has to bind. This lets us express $\bar{v}$ as an implicit function of $\underline{w}$ and b,

$$\bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))}. \tag{2.B.10}$$

Note that $(b - \bar{v})$ is non-negative because $MWC1$ binds, which simplifies $MWC2$ to $b \geq 0$, and because $\bar{v} \leq 0$ (NCC). Thus, the agent's first-order condition yields the equilibrium effort.

$\bar{v}(\underline{w}, b)$ is the most severe NCC that the agent is willing to accept given a base wage $\underline{w}$ and a bonus wage b. Lemma 2.2 shows that the higher the minimum wage is, the more severe is this NCC for a given bonus wage. The higher the bonus wage is, the more severe is this NCC for a given minimum wage. Furthermore, due to monotonicity, the values of $\bar{v}(\underline{w}, b)$ are unique in b for a fixed $\underline{w}$ and the other way around.

Therefore, the principal's problem can also be expressed as

$$\max_{b} \quad -\underline{w} + E(b - \bar{v}(\underline{w}, b))\,(V - b) \tag{2.B.11}$$

$$\text{subject to} \quad \bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))} \tag{PC'}$$

$$\bar{v} \leq 0 \tag{NCC}$$

$$b \geq 0. \tag{MWC2}$$

Whenever $\underline{w} \geq \kappa_1$, a contract is optimal if and only if it solves the simplified problem. In the second combination, *MWC*2 is slack and *NCC* binds. In the third combination, *MWC*2 and *NCC* are both slack. In the fourth combination, *MWC*2 binds and *NCC* is slack.

Lemma 2.2. *i) Fix a minimum wage. The NCC that makes the participation constraint bind $\bar{v}(\underline{w}, b)$ is strictly decreasing in the bonus wage: $\frac{\partial \bar{v}(\underline{w},b)}{\partial b} < 0$.*
ii) Fix a bonus wage. The NCC that makes the participation constraint bind $\bar{v}(\underline{w}, b)$ is strictly decreasing in the minimum wage: $\frac{\partial \bar{v}(\underline{w},b)}{\partial \underline{w}} < 0$.

Proof. Rearrange the binding participation constraint to

$$Z \;\equiv\; \underline{w} + E(b-\bar{v}) \cdot (b-\bar{v}) + \bar{v} - c(E(b-\bar{v})) \;=\; 0. \tag{2.B.12}$$

Because this is continuously differentiable, the implicit function theorem can be used to get the derivatives of $\bar{v}$ with respect to $\underline{w}$ and b,

$$\begin{aligned}\frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} &= -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial \bar{v}}} = -\frac{1}{-E'(b-\bar{v}) \cdot (b-\bar{v}) - E(b-\bar{v}) + 1 + c'(E(b-\bar{v})) \cdot E'(b-\bar{v})} \\ &= -\frac{1}{1-E(b-\bar{v})} \end{aligned} \tag{2.B.13}$$

and

$$\begin{aligned}\frac{\partial \bar{v}(\underline{w}, b)}{\partial b} &= -\frac{\frac{\partial Z}{\partial b}}{\frac{\partial Z}{\partial \bar{v}}} = -\frac{E'(b-\bar{v}) \cdot (b-\bar{v}) + E(b-\bar{v}) - c'(E(b-\bar{v})) \cdot E'(b-\bar{v})}{-E'(b-\bar{v}) \cdot (b-\bar{v}) + 1 - E(b-\bar{v}) + c'(E(b-\bar{v})) \cdot E'(b-\bar{v})} \\ &= -\frac{E(b-\bar{v})}{1-E(b-\bar{v})}. \end{aligned} \tag{2.B.14}$$

Where we use the agent's first-order condition, $(b-\bar{v}-c'(E)) = 0$, to simplify. □

We will now define a useful term to simplify the maximization problem further. Let $b_2^{**}(\underline{w})$ denote the optimal bonus wage in Case 2 of the benchmark (binding *PC*, binding *MWC*1, slack *MWC*2). The case conditions imply a property of $b_2^{**}(\underline{w})$: It makes the participation constraint binding in the absence of an NCC.

To use this particular bonus wage to simplify the problem, we have to extend the definition of $b_2^{**}(\underline{w})$ to minimum wages above κ_3 for which it is not the optimal bonus wage. Let $b_2^{**}(\underline{w})$ denote the *minimal non-negative* bonus wage that keeps the participation constraint *satisfied* in the absence of an NCC,

$$\forall \underline{w} \;\geq\; \kappa_1 \qquad b_2^{**}(\underline{w}) \;\equiv\; \min\left\{b \;\in\; \mathbb{R}_0^+ \;\mid\; \underline{w} + E(b) \cdot b - c(E(b)) \;\geq\; 0\right\}. \tag{2.B.15}$$

For non-positive minimum wages, $b_2^{**}(\underline{w})$ is determined by the minimum wage that makes the participation constraint binding. For positive minimum wages the participation constraint is always slack without NCCs; there is no bonus wage that makes the participation constraint binding. Thus, if $\underline{w} \geq 0$, then $b_2^{**}(\underline{w}) = 0$. Furthermore, $b_2^{**}(\underline{w})$ has the nice property that it exists and it is strictly decreasing in the minimum wage between κ_1 and 0.

To simplify the problem, we now replace the inequality constraints using $b_2^{**}(\underline{w})$: As long as PC' holds, the conditions NCC and $MWC2$ are equivalent to another condition, $b \geq b_2^{**}(\underline{w})$.

Consider $\underline{w} < 0$. In this case, PC' and NCC imply $MWC2$. The bonus wage has to be at least $b_2^{**}(\underline{w})$, even without an NCC, to satisfy the participation constraint. If $\underline{w} < 0$, then $b_2^{**}(\underline{w}) > 0$, implying $MWC2$. In this case, the new constraint $b \geq b_2^{**}(\underline{w})$ is binding if and only if NCC is binding.

Consider $\underline{w} \geq 0$. In this case, PC' and $MWC2$ imply NCC. If $\underline{w} \geq 0$, then $b_2^{**}(\underline{w}) = 0$; for $\underline{w} = 0$ the participation constraint is binding without an NCC, for $\underline{w} > 0$, the participation constraint is slack without an NCC. In both cases, the binding PC means that $\bar{v} \leq 0$, implying NCC. In this case, the new constraint is binding if and only if $MWC2$ is binding.

The problem is, thus, equivalent to

$$\max_b \quad -\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot (V - b) \tag{2.B.16}$$

$$\text{subject to} \quad \bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))} \tag{PC'}$$

$$b \geq b_2^{**}(\underline{w}). \tag{2.B.17}$$

The problem (2.B.16) is simpler because it has only one inequality constraint, which constrains the only argument of the objective function. Under the assumptions made in Section 2.3, moreover, the objective function is strictly concave, as Lemma 2.3 shows. We introduced this assumption because it implies all assumptions that we need in this proof. To make the proof tighter, however, we make weaker assumptions wherever possible. Thus, for determining whether the second or the third combination is optimal, we will use a weaker assumption and the notion of strict quasi-concavity that is sufficient to derive the results. In Lemma 2.4, we determine the necessary and sufficient condition that makes the objective function strictly quasi-concave in the bonus wage.

Lemma 2.3. *(2.B.16) is strictly concave in b if for all bonus wages*

$$\frac{c'''(E(b, \bar{v}(\underline{w}, b)))}{c''(E(b, \bar{v}(\underline{w}, b)))} > \frac{1}{1 - E(b, \bar{v}(\underline{w}, b))}. \tag{2.B.18}$$

Proof. The objective function's first and second derivatives with respect to the bonus wage are

$$\frac{\partial \pi(b)}{\partial b} = \frac{E'(b, \bar{v}(\underline{w}, b))}{1 - E(b, \bar{v}(\underline{w}, b))} \cdot (V - b) - E(b, \bar{v}(\underline{w}, b)) \tag{2.B.19}$$

and (omitting the argument of $E(b, \bar{v}(\underline{w}, b))$ for readability)

$$\frac{\partial^2 \pi(b)}{\partial b^2} = \left[\frac{E''}{(1-E)^2} + \frac{(E')^2}{(1-E)^3} \right] \cdot (V-b) - \frac{2E'}{1-E}. \tag{2.B.20}$$

Because $E'(b, \bar{v}(\underline{w}, b)) > 0$, a sufficient condition for the concavity of the objective function is that $\frac{E''}{(1-E)^2} + \frac{E'E'}{(1-E)^3} < 0$. Rearranging and simplifying shows that this is true under our assumption on the cost function,

$$E''(b, \bar{v}(\underline{w}, b)) + \frac{(E'(b, \bar{v}(\underline{w}, b)))^2}{1 - E(b, \bar{v}(\underline{w}, b))} < 0 \quad \Longrightarrow \quad \frac{\partial^2 \pi(b)}{\partial b^2} < 0. \tag{2.B.21}$$

Plugging in for $E'(\cdot) \equiv \frac{1}{c''(E(\cdot))}$ and $E''(\cdot) \equiv -\frac{c'''(E(\cdot))}{(c''(E(\cdot)))^3}$ yields

$$\frac{c'''(E(b, \bar{v}(\underline{w}, b)))}{c''(E(b, \bar{v}(\underline{w}, b)))} > \frac{1}{1 - E(b, \bar{v}(\underline{w}, b))}. \tag{2.B.22}$$

□

Lemma 2.4. *(2.B.16) is strictly quasi-concave in b if for all bonus wages*

$$\frac{c'''(E(b - \bar{v}(\underline{w}, b)))}{c''(E(b - \bar{v}(\underline{w}, b)))} > \frac{1}{1 - E(b - \bar{v}(\underline{w}, b))} - \frac{2}{E(b - \bar{v}(\underline{w}, b))}. \tag{2.B.23}$$

Proof. The objective function, $\pi(b)$, is twice continuously differentiable. It is strictly quasi-concave in b if the second derivative is negative at each critical point.

For readability, we will omit the argument of $E(b - \bar{v}(\underline{w}, b))$ and its derivatives, and instead write $E(\cdot)$. The objective function's first derivative with respect to b is

$$\begin{aligned} \frac{\partial \pi(b)}{\partial b} &= E'(\cdot) \cdot \left(1 - \frac{\partial \bar{v}(\underline{w}, b)}{\partial b}\right) \cdot (V-b) - E(\cdot) \\ &= \frac{E'(\cdot)}{1 - E(\cdot)} \cdot (V-b) - E(\cdot). \end{aligned} \tag{2.B.24}$$

Since $1 - E(\cdot)$ is the equilibrium probability of a failure, it is positive due to the Inada conditions. Critical points are characterized by

$$V - b = \frac{E(\cdot) \cdot (1 - E(\cdot))}{E'(\cdot)}. \tag{2.B.25}$$

The objective function is strictly quasi-concave in b if and only if the derivative of equation (2.B.24) is negative at every critical point. After some calculus, the sign of the derivative of equation (2.B.24) is seen equal to the sign of "expression 1":

$$E'(\cdot) \cdot (V-b) - E(\cdot) \cdot (1 - E(\cdot)). \tag{Expression 1}$$

Expression 1's derivative is

$$\begin{aligned}
&E''(\cdot)\cdot\left(1-\frac{\partial\bar{v}(\underline{w},b)}{\partial b}\right)\cdot(V-b)-E'(\cdot)-E'(\cdot)\cdot(1-2E(\cdot))\cdot\left(1-\frac{\partial\bar{v}(\underline{w},b)}{\partial b}\right)\\
&=\frac{E''(\cdot)}{1-E(\cdot)}\cdot(V-b)-E'(\cdot)-\frac{E'(\cdot)\cdot(1-2E(\cdot))}{1-E(\cdot)}\\
&=\frac{E''(\cdot)}{1-E(\cdot)}\cdot(V-b)-\frac{E'(\cdot)\cdot(2-3E(\cdot))}{1-E(\cdot)}. \qquad (2.B.26)
\end{aligned}$$

Since we only care about the sign at the critical points, we can now plug in the solution of the first-order condition (2.B.25) for $(V-b)$. This yields an expression that we want to show is negative.

$$\frac{E''(\cdot)}{1-E(\cdot)}\cdot\frac{E(\cdot)\cdot(1-E(\cdot))}{E'(\cdot)}-\frac{E'(\cdot)\cdot(2-3E(\cdot))}{1-E(\cdot)}\overset{?}{<}0, \qquad (2.B.27)$$

where $\overset{?}{<}$ means that the inequality remains to be shown. Rearranging yields

$$E''(\cdot)\overset{?}{<}\frac{(E'(\cdot))^2\cdot(2-3E(\cdot))}{E(\cdot)\cdot(1-E(\cdot))}. \qquad (2.B.28)$$

Using the definition of $E(\cdot)$, this can be simplified with

$$E(\cdot)=(c')^{-1}(\cdot),\quad E'(\cdot)=\frac{1}{c''(E(\cdot))},\quad E''(\cdot)=-\frac{c'''(E(\cdot))}{(c''(E(\cdot)))^3}. \qquad (2.B.29)$$

Therefore, (2.B.28) is equivalent to our assumption

$$\frac{c'''(E(\cdot))}{c''(E(\cdot))}>\frac{1}{1-E(\cdot)}-\frac{2}{E(\cdot)} \qquad (2.B.30)$$

□

For equilibrium efforts below $\frac{2}{3}$, the assumption is always satisfied. For equilibrium efforts above $\frac{2}{3}$, the assumption says that the marginal cost has to be convex enough. As a result, the equilibrium effort reacts not too strongly to increased incentives and the strict quasi-concavity is preserved when introducing NCCs.

Strict quasi-concavity in the bonus wage implies that the maximum is unique if it exists. To see that the maximum exists, note that the maximum is equivalent to the maximum of the problem constraining $b_2^{**}(\underline{w})\le b\le V$, since the optimal bonus wage cannot be above V. Because of the extreme value theorem, we know that the latter problem has a solution ($b_2^{**}(\underline{w})\le b\le V$ is a compact set and the objective function is continuous).

This last simplification concludes the first part of the proof. In the second part of the proof, we look at the three remaining combinations and determine for which minimum wages they are optimal. We first characterize the different combinations in the simplified problem. Then, we use the monotonicity of the marginal profit in the bonus wage evaluated at the bonus wage $b_2^{**}(\underline{w})$ to find the minimum wages for which the second combination is optimal. Lastly, we derive a condition under which the fourth combination is optimal for some minimum wages.

Negative minimum wages. Consider negative minimum wages first. For $\kappa_1 \leq \underline{w} < 0$, only the second or the third combination can be optimal. The sign of the derivative of the objective function with respect to the bonus wage at the lower bound $b_2^{**}(\underline{w})$ shows whether there is an interior solution or not. If the derivative is non-positive, there is a corner solution and, thus, no NCC. The second combination is optimal. If the derivative is positive, there is an interior solution and, thus, an NCC. The third combination is optimal. The monotonicity of the derivative evaluated at $b_2^{**}(\underline{w})$ in the minimum wage yields uniqueness of the minimum wage at which a switch happens.

Lemma 2.5. *Assume that* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{2}{E(b-\bar{v}(\underline{w},b))}$ *for all bonus wages. There is a unique cutoff* $\kappa_2 < 0$ *in the minimum wage such that for all* $\kappa_1 \leq \underline{w} \leq \kappa_2$*, the optimal contract has* $b = b_2^{**}(\underline{w})$*, and for all* $\kappa_2 < \underline{w} < 0$*, the optimal contract has* $b > b_2^{**}(\underline{w})$.

Proof. The derivative of the profit with respect to the bonus wage evaluated at the lower bound is

$$\begin{aligned} \left.\frac{\partial \pi(\underline{w},b)}{\partial b}\right|_{b=b_2^{**}(\underline{w})} &= \left.\frac{\partial E(b-\bar{v}(\underline{w},b))}{\partial b}\right|_{b=b_2^{**}(\underline{w})} \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w})) \\ &= \frac{E'(b_2^{**}(\underline{w}))}{1-E(b_2^{**}(\underline{w}))} \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}). \end{aligned} \tag{2.B.31}$$

We will now look at different minimum wages and show that there is exactly one minimum wage at which the optimum switches from a corner to an interior solution. The corresponding minimum wage is the minimum wage from which on NCCs are used, κ_2. Technically, at κ_2, the objective function's first derivative evaluated at the lowest possible bonus wage $b_2^{**}(\underline{w})$ switches the sign from negative (corner solution) to positive (interior solution).

We use the same strategy as when proving quasi-concavity: We show that in all candidates for κ_2, the derivative goes from negative to positive. By continuity, there can be only one candidate.

A candidate for κ_2 is a minimum wage such that the derivative is zero:

$$\begin{aligned} \left.\frac{\partial \pi(\underline{w},b)}{\partial b}\right|_{b=b_2^{**}(\underline{w})} &= \frac{E'(b_2^{**}(\underline{w}))}{1-E(b_2^{**}(\underline{w}))} \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w})) \stackrel{!}{=} 0 \\ \iff \qquad (V - b_2^{**}(\underline{w})) &= \frac{E(b_2^{**}(\underline{w})) \cdot (1-E(b_2^{**}(\underline{w})))}{E'(b_2^{**}(\underline{w}))}. \end{aligned} \tag{2.B.32}$$

To see how the derivative of the profit with respect to the bonus wage at the lower bound changes, take the derivative with respect to the minimum wage. Note that although $\bar{v}(b,\underline{w})$ is a function of both the bonus and the minimum wage, it will not change: At $b_2^{**}(\underline{w})$, the participation constraint binds without an NCC. Thus, $\bar{v}(b_2^{**}(\underline{w}),\underline{w}) = 0$ for all negative minimum wages.

Again, we work with another expression that has the same sign as the first derivative but which is easier to work with. “Expression 2” is

$$E'(b_2^{**}(\underline{w})) \cdot (V - b_2^{**}(\underline{w})) - E(b_2^{**}(\underline{w})) \cdot (1 - E(b_2^{**}(\underline{w}))). \qquad \text{(Expression 2)}$$

The derivative of expression 2 with respect to the minimum wage (where we express $E(b_2^{**}(\underline{w}))$ and its derivatives as E to improve readability) is

$$\begin{aligned} \frac{\partial\left(\frac{\partial\pi}{\partial b}|_{b=b_2^{**}(\underline{w})}\right)}{\partial\underline{w}} &= E'' \cdot (V - b_2^{**}(\underline{w})) \cdot \frac{\partial b_2^{**}(\underline{w})}{\partial\underline{w}} - E' \cdot \frac{\partial b_2^{**}(\underline{w})}{\partial\underline{w}} \\ &\quad - (1-E) \cdot E' \cdot \frac{\partial b_2^{**}(\underline{w})}{\partial\underline{w}} + E' \cdot E \cdot \frac{\partial b_2^{**}(\underline{w})}{\partial\underline{w}} \\ &= \frac{\partial b_2^{**}(\underline{w})}{\partial\underline{w}}\left(E'' \cdot \frac{E(1-E)}{E'} - 2E' \cdot (1-E)\right) > 0. \end{aligned} \qquad (2.B.33)$$

The second line follows from plugging (2.B.32) in. At the critical point, the derivative of the profit with respect to the bonus wage evaluated at the lower bound is increasing because $\frac{\partial b_2^{**}}{\partial\underline{w}} < 0$; the lowest bonus wage to satisfy the participation constraint is decreasing in the minimum wage because a higher minimum wage makes the participation constraint already slack. Moreover, it is globally true that $E' > 0$, and $E'' < 0$.

We have shown that any switches between corner and interior solutions have to be from corner to interior solutions. Moreover, there can be at most one switching point. That is, conditional on existence, κ_2 is unique.

To show that there is at least one critical point, we use that the derivative of the profit with respect to the bonus wage is continuous in the minimum wage. There is a minimum wage for which the derivative is negative and there is a minimum wage for which the derivative is positive. Thus, there is also a minimum wage for which the derivative is zero.

The derivative is negative for the minimum wage κ_1. The principal implements first-best effort and extracts all surplus by selling the firm. Because all of the success payoff goes to the agent, increasing the bonus wage further reduces the profit. Plugging κ_1 in, yields $b_2^{**}(\kappa_1) = V$. The derivative is

$$\left.\frac{\partial\pi}{\partial b}\right|_{b=b_2^{**}(\kappa_1)} = -E(V) < 0. \qquad (2.B.34)$$

The derivative is positive for the minimum wage κ_3. Following a similar argument as above, we know from the benchmark that the derivative of the profit with respect to the bonus wage without access to NCCs at the minimum wage κ_3 is zero: Left of κ_3, the optimal bonus wage just satisfies the participation constraint, right of

κ_3, the optimal bonus wage makes the participation constraint slack. The derivative of the profit with respect to the bonus wage without NCCs is

$$\left.\frac{\partial \pi^{\text{No NCC}}}{\partial b}\right|_{b=b_2^{**}(\kappa_3),\bar{v}=0} = E'(b_2^{**}(\kappa_3)) \cdot (V - b_2^{**}(\kappa_3)) - E(b_2^{**}(\kappa_3)) = 0. \quad (2.B.35)$$

With NCCs, there are double incentives. Thus, the derivative with NCCs is strictly larger: The marginal benefit gets multiplied with $\frac{1}{1-E} > 1$. Therefore, the positive term is larger. The negative term is the same. Since at κ_3 the derivative without NCCs is zero, the derivative with NCCs is positive,

$$\left.\frac{\partial \pi(\underline{w}, b)}{\partial b}\right|_{b=b_2^{**}(\kappa_3),\bar{v}=0} = \frac{E'(b_2^{**}(\kappa_3))}{1 - E(b_2^{**}(\kappa_3))} \cdot (V - b_2^{**}(\kappa_3)) - E(b_2^{**}(\kappa_3)) > 0. \quad (2.B.36)$$

To sum up: The profit's first derivative evaluated at the bonus wage $b_2^{**}(\underline{w})$ is continuous and monotonically increasing. It is strictly negative at κ_1 and strictly positive at κ_3. Thus, its root, κ_2, exists and lies strictly in-between, $\kappa_1 < \kappa_2 < \kappa_3 < 0$.

□

For all minimum wages below κ_2, the optimal contract and, thus, the profit is the same as in the benchmark. For minimum wages above κ_2, an NCC is used and the principal's profits are strictly larger than in the benchmark: Strict quasi-concavity of the profit in the bonus wage means that the maximum is unique. The principal could mimic the world without NCCs. He does, however, not want to. Uniqueness of the maximum means that the optimal contract with an NCC is strictly better than the optimal contract without an NCC.

A minimum wage of zero. For $\underline{w} = 0$, the second combination is not feasible. The binding participation constraint with no NCC implies that the bonus wage has to be zero. In the second combination, the bonus wage has to be strictly positive. Furthermore, the fourth combination is not feasible. The binding participation constraint with no bonus wage implies that the most severe NCC is no NCC. In the fourth combination, the NCC has to be strictly negative. Thus, the optimal contract has to have both a bonus wage and an NCC.

Having established that the first, the second, and then the third combination are optimal in an increasing minimum wage, we now turn to positive minimum wages.

Positive minimum wages. For positive minimum wages, contracts from the second combination are not feasible: It is not possible to make the participation constraint binding without an NCC. In this range, only the third or the fourth combination can be optimal. We show that starting at a minimum wage of 0, the third combination is optimal. We derive one condition on the effort cost function for the existence and one condition for the uniqueness of there being a minimum wage $\kappa_4 > 0$ such that for all $\underline{w} < \kappa_4$, the third combination is optimal and for all $\underline{w} \geq \kappa_4$, the fourth combination is optimal. At κ_4, the principal stops using a bonus wage. Instead, all incentives follow from an NCC. If the condition is not met, the third combination is optimal for all positive minimum wages.

To get uniqueness of κ_4, we need an assumption on the cost function. For all bonus wages, it has to hold that $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$. While this assumption is stronger than the assumption to get strict quasi-concavity, it is also implied by our assumptions in Section 2.3 that imply strict concavity of the objective function. With this assumption, we can show that there is at most one minimum wage at which the principal switches between the third and the fourth combination. Furthermore, this assumption implies that the switch is such that for lower minimum wages there is a positive bonus wage, while for higher minimum wages, the optimal bonus wage is zero.

The strategy of the proof is to determine the sign of the marginal profit of the bonus wage, evaluated at a bonus wage of 0. If it is positive, there is an interior solution and the optimal bonus wage is positive. To make the participation constraint binding, an NCC is needed. The optimal contract is, thus, from the third combination. Using no bonus wage is optimal if the marginal profit is negative. Then, the first unit of the bonus wage is not worth the marginal cost. The optimal contract is, thus, from the fourth combination. The assumption on the uniqueness implies that every switch of the sign goes from the positive to the negative.

To prove existence, we show that the sign of the marginal profit of the bonus wage, evaluated at a bonus wage of 0, is initially positive. We assume that the condition for uniqueness is met. The marginal profit of the first unit of bonus wage is continuous in the minimum wage. Because its sign is initially positive, can switch its sign at most once, and the marginal profit's continuity, the sign in the limit is negative if and only if the switch happened for a finite minimum wage. We then derive the (necessary and sufficient) condition under which the sign is negative in the limit. This is the condition for the existence of κ_4. To determine the sign in the limit, we use L'Hôpital's rule.

Lemma 2.6. *If for all bonus wages* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$, *then there is at most one minimum wage for which* $\frac{\partial \pi}{\partial b}|_{b=0} = 0$.

Proof. Again, we will employ the same strategy of proof as above to show the uniqueness of a critical point. The critical point in the minimum wage is characterized by

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=0} = \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V - E(-\bar{v}(\underline{w},0)) \overset{!}{=} 0. \tag{2.B.37}$$

The equation defines the critical points in the minimum wage for which the marginal profit from using a bonus wage is zero. Since $\underline{w} > 0$, the principal will use an NCC to provide incentives. The optimal contract falls into the fourth combination.

Thus, a critical point is defined by

$$V = \frac{E(-\bar{v}(\underline{w},0)) \cdot (1 - E(-\bar{v}(\underline{w},0)))}{E'(-\bar{v}(\underline{w},0))}. \tag{2.B.38}$$

As above, we show that this critical point is unique if it implies that the marginal profit from the first unit of bonus wage hits zero from above. Then, to the left of the critical point, it is optimal to use positive bonus wages; to the right of the critical point, it is optimal to use no bonus wages. We want to show that

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=0} \overset{!}{=} 0 \quad \Longrightarrow \quad \frac{\partial \left(\left.\frac{\partial \pi}{\partial b}\right|_{b=0} \right)}{\partial \underline{w}} < 0. \tag{2.B.39}$$

To do so, we compute this derivative (we again omit the arguments and express $E(-\bar{v}(\underline{w},0))$ as E to improve readability)

$$\frac{\partial \left(\left.\frac{\partial \pi}{\partial b}\right|_{b=0} \right)}{\partial \underline{w}} = \frac{(1-E)E'' + E' \cdot E'}{(1-E)^3} \cdot V - \frac{E'}{1-E}. \tag{2.B.40}$$

Plugging in the characterization of a critical point ($V = \frac{E\cdot(1-E)}{E'}$) and simplifying yields

$$\frac{c'''(E)}{c''(E)} > \frac{1}{1-E} - \frac{1}{E}, \tag{2.B.41}$$

which holds by assumption. □

Lemma 2.7. *Assume that for all bonus wages* $\frac{c'''(E(b-\bar{v}(\underline{w},b)))}{c''(E(b-\bar{v}(\underline{w},b)))} > \frac{1}{1-E(b-\bar{v}(\underline{w},b))} - \frac{1}{E(b-\bar{v}(\underline{w},b))}$. *If*

$$\lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} \cdot V < 1, \tag{2.B.42}$$

then there is a minimum wage $\kappa_4 > 0$ such that the optimal contract uses a bonus wage for all lower minimum wages and the optimal contract uses no bonus wage for all larger minimum wages.

Proof. κ_4 exists if there is a positive minimum wage that equates the marginal benefit and the marginal cost of the first unit of bonus wage.

$$\left.\frac{\partial \pi}{\partial b}\right|_{b=0} = \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V - E(-\bar{v}(\underline{w},0)) = 0. \tag{2.B.43}$$

We have shown above that there is at most one such minimum wage. Furthermore, we have shown that the intersection has to be such that the marginal benefit intersects the marginal cost from above. Now we show under which conditions there is at least one such intersection.

Initially, the marginal benefit is larger than the marginal cost. Consider the minimum wage $\underline{w} = 0$. Together with $b = 0$, this implies that $\bar{v} = 0$ to make the PC binding and that the equilibrium effort is 0. The marginal benefit is $\frac{E'(0)}{1} \cdot V$. Since $E'(\cdot) \equiv \frac{1}{c'(E(\cdot))}$, this is strictly positive for a minimum wage of 0. The marginal cost is $E(0) = 0$ at a minimum wage of 0. Hence, we showed that for $\underline{w} = 0$, the bonus wage's marginal benefit is higher than the marginal cost. By continuity, this also holds for some positive minimum wages.

Since the marginal benefit is initially larger, can intersect the marginal cost only from above, and both are continuous, it is sufficient to look at the limits of the minimum wage's going to infinity. Without a bonus wage, the non-compete clause will then become ever more severe, which implies that the equilibrium effort will go to 1.

First, consider the marginal cost of increasing the bonus wage, starting at $b = 0$. When the minimum wage goes to infinity, the equilibrium effort goes to 1 and the marginal cost goes to 1. Second, consider the marginal benefit of increasing the bonus wage starting at $b = 0$. When the minimum wage goes to infinity, the equilibrium effort goes to 1 and the marginal benefit goes to $\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V$. Let us consider numerator and denominator separately. The numerator goes to zero because $\lim_{\underline{w}\to\infty} E'(-\bar{v}(\underline{w},0)) = \lim_{\underline{w}\to\infty} \frac{1}{c''(E(-\bar{v}(\underline{w},0)))}$ and $\lim_{\underline{w}\to\infty} c''(E(-\bar{v}(\underline{w},0))) = \infty$. This follows because $\underline{w} \to \infty$ implies that $E(-\bar{v}(\underline{w},0)) \to 1$ which implies that $c'(e) \to \infty$. For the same reason, the denominator also goes to zero.

Thus, we use L'Hôpital's rule to evaluate $\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V$. In order to use L'Hôpital's rule, we need to check two conditions:

First, we must check that for all (positive) finite minimum wages $\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial\underline{w}} \neq 0$. This condition is fulfilled because $\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial\underline{w}} = -\frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))}$. By assumption, the numerator is positive.

Second, we must check that the limit of the ratio of the derivatives exists. This condition is fulfilled because

$$\lim_{\underline{w}\to\infty} \frac{\frac{\partial E'(-\bar{v}(\underline{w},0))}{\partial\underline{w}}}{\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial\underline{w}}} \cdot V = \lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} \cdot V < 1. \tag{2.B.44}$$

by assumption and it is continuous on (0,1).

All in all, L'Hôpital's rule yields

$$\begin{aligned}\lim_{\underline{w}\to\infty} \frac{E'(-\bar{v}(\underline{w},0))}{1-E(-\bar{v}(\underline{w},0))} \cdot V &= \lim_{\underline{w}\to\infty} \frac{\frac{\partial E'(-\bar{v}(\underline{w},0))}{\partial\underline{w}}}{\frac{\partial(1-E(-\bar{v}(\underline{w},0)))}{\partial\underline{w}}} \cdot V \\ &= \lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} \cdot V. \end{aligned} \tag{2.B.45}$$

Therefore, there is a critical minimum wage κ_4 if and only if

$$\lim_{\underline{w}\to\infty} \frac{c'''(E(-\bar{v}(\underline{w},0)))}{[c''(E(-\bar{v}(\underline{w},0)))]^2} \cdot V \; < \; 1. \tag{2.B.46}$$

The assumption can also be expressed in properties of the effort cost function. It is an assumption on the convergence speeds of the second and the third derivative. Note that both $c''(\cdot)$ and $c'''(\cdot)$ go to infinity when the minimum wage goes to infinity because the equilibrium effort goes to 1 and then $c'(\cdot)$ goes to infinity. Therefore, if $(c''(\cdot))^2$ goes to infinity strictly faster than $c'''(\cdot)$, the marginal benefit converges to zero. If the convergence of $(c''(\cdot))^2$ and $c'''(\cdot)$ has the same speed, the limit is some number. If this number times V is less than 1, the assumption is also satisfied. Whenever the convergence of $c'''(\cdot)$ is faster than that of $(c''(\cdot))^2$, the assumption does not hold.

□

Having characterized which constraints bind in which combination, we can now characterize the optimal contract in each combination. Note that the contract in the first (second) combination mirrors the one in Case 1 (2). The base and bonus wages are equal and the principal does not want to use an NCC. The derivations of base and bonus wage are therefore identical to the derivations in Case 1 and 2 in Proposition 2.1 and therefore are skipped here. We now characterize the optimal bonus wage and the optimal non-compete clause, depending on the effort level that will be chosen in each combination.

Next, we consider the third combination.

Third combination. Let E be the effort level that the agent chooses given the contract. $MWC1$ binds, which implies that $w = \underline{w}$. PC binds as well. We substitute IC and $MWC1$ into PC and rewrite to get

$$\bar{v} \; = \; c(E) - E \cdot c'(E) - \underline{w}, \tag{2.B.47}$$

where we suppress the arguments of E and $\bar{v}$ for readability.

Combining $MWC1$, PC and IC by substituting for $\bar{v}$ yields

$$b \; = \; (1-E) \cdot c'(E) + c(E) - \underline{w}. \tag{2.B.48}$$

Now, we substitute for w and b in P's objective function to get

$$\pi \; = \; E \cdot V - (1-E) \cdot \underline{w} - E \cdot (1-E) \cdot c'(E) - E \cdot c(E). \tag{2.B.49}$$

P maximizes over the incentive-compatible effort level and hence $E = e_3^{NCC}$ is chosen such that

$$c(e_3^{NCC}) + (1-e_3^{NCC}) \cdot c'(e_3^{NCC}) + e_3^{NCC} \cdot (1-e_3^{NCC}) \cdot c''(e_3^{NCC}) \; = \; V + \underline{w}. \tag{2.B.50}$$

Next, we consider the fourth combination.

Fourth combination. Let E be the effort level that the agent chooses given the contract. $MWC1$ binds, which implies that $w = \underline{w}$. $MWC2$ binds, which together with the binding $MWC1$ implies that $b = 0$. $\bar{v}$ is then determined by the binding participation constraint

$$\bar{v} \;=\; -\frac{\underline{w} - c(E)}{1 - E}. \tag{2.B.51}$$

The optimal effort choice is then determined by the IC and hence $E = e_4^{NCC}$ is characterized by

$$\underline{w} + e_4^{NCC} \cdot c'(e_4^{NCC}) - c(e_4^{NCC}) \;=\; c'(e_4^{NCC}). \tag{2.B.52}$$

□

2.B.3 Proof of Proposition 2.3

The equilibrium effort is non-monotone in the minimum wage.

(i) *If $\underline{w} < \kappa_1$, the equilibrium effort is constant in the minimum wage.*
(ii) *If $\kappa_1 \leq \underline{w} \leq \kappa_2$, the equilibrium effort is strictly decreasing in the minimum wage.*
(iii) *If $\kappa_2 < \underline{w}$, the equilibrium effort is strictly increasing in the minimum wage.*

Proof. We show that the equilibrium effort is constant in the minimum wage in the first combination, decreasing in the minimum wage in the second combination and increasing in the minimum wage if P uses an NCC, that is, in the third and fourth combination.

We start with the first combination. Note that we showed in Proposition 2.1 and Proposition 2.2 that P does not use an NCC and induces the first-best effort level in the first combination. First-best effort level is constant at e^{FB} and hence does not change in the minimum wage.

We proceed with the second combination. Note that we showed in Proposition 2.2 that P does not use an NCC. The equilibrium effort is hence defined by $c'(E) = b(\underline{w})$. Since the marginal cost is increasing, the equilibrium effort gets smaller if the right-hand side gets smaller. Thus, we have to show that the right-hand side is decreasing in the minimum wage. The binding participation constraint gives us

$$G(\underline{w}, b) \;\equiv\; E(b) \cdot b - c(E(b)) + \underline{w} \;=\; 0. \tag{2.B.53}$$

We use the implicit function theorem on the binding participation constraint. From now on, we will skip the argument of E for readability. Since G is continuously differentiable, the implicit function theorem can be used to calculate the derivative of b with respect to $\underline{w}$,

$$\frac{\partial b(\underline{w})}{\partial \underline{w}} \;=\; -\frac{\frac{\partial G(\underline{w},b)}{\partial \underline{w}}}{\frac{\partial G(\underline{w},b)}{\partial b}} \;=\; -\frac{1}{E}. \tag{2.B.54}$$

Hence, we get that $\frac{\partial b(\underline{w})}{\partial \underline{w}} < 0$ which then implies that the equilibrium effort decreases in the minimum wage.

We continue with the third combination, in which the optimal contract has both a bonus wage and an NCC. We, therefore, need to evaluate their combined effect on the effort. The equilibrium effort is defined by $c'(E) = b(\underline{w}) - \bar{v}(\underline{w}, b(\underline{w}))$. Since the marginal cost is increasing, the equilibrium effort gets larger if the right-hand side gets larger. Thus, we need to show that the right-hand side is increasing in the minimum wage. Taking the derivative with respect to the minimum wage of the right-hand side yields

$$\frac{\partial b(\underline{w})}{\partial \underline{w}} - \left(\frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial \underline{w}} + \frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial b(\underline{w})} \cdot \frac{\partial b(\underline{w})}{\partial \underline{w}} \right). \tag{2.B.55}$$

To show that this expression is positive, we look at its parts in turn. We already calculated the effect of a change in the minimum wage and in the bonus wage on the NCC that makes the participation constraint bind in Lemma 2.2. For convenience, we reproduce the result here:

$$\frac{\partial \bar{v}(\underline{w}, b)}{\partial \underline{w}} = -\frac{1}{1 - E(b - \bar{v})}, \quad \text{and} \quad \frac{\partial \bar{v}(\underline{w}, b)}{\partial b} = -\frac{E(b - \bar{v})}{1 - E(b - \bar{v})}. \tag{2.B.56}$$

It remains to characterize how the optimal bonus wage changes in the minimum wage. We use the implicit function theorem on the first-order condition of the expected profit maximization problem. Again, we will from now on skip the argument of E for readability. The FOC of P's expected profit with respect to the bonus wage is

$$Z(\underline{w}, b) \equiv E'(b - \bar{v}) \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) \cdot (V - b) - E(b - \bar{v}) = 0. \tag{2.B.57}$$

Before we apply the implicit function theorem to this equation to see how b changes in $\underline{w}$, we need two intermediary derivatives: $\frac{\partial^2 \bar{v}(\underline{w}, b)}{\partial b \partial \underline{w}}$ and $\frac{\partial^2 \bar{v}(\underline{w}, b)}{\partial b^2}$. And again, we can use Lemma 2.2, which shows that $\frac{\partial \bar{v}(\underline{w}, b)}{\partial b} = -\frac{E}{1-E}$.

Thus,

$$\frac{\partial^2 \bar{v}(\underline{w}, b)}{\partial b \partial \underline{w}} = -\frac{E' \cdot (1 - E) \cdot \frac{\partial \bar{v}}{\partial \underline{w}} + E' \cdot E \cdot \frac{\partial \bar{v}}{\partial \underline{w}}}{(1 - E)^2} = -\frac{E'}{(1 - E)^3} \tag{2.B.58}$$

and

$$\frac{\partial^2 \bar{v}(\underline{w}, b)}{\partial b^2} = \frac{-E' \cdot (1 - E) \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right) - E' \cdot E \cdot \left(1 - \frac{\partial \bar{v}}{\partial b}\right)}{(1 - E)^2} = -\frac{E'}{(1 - E)^3}. \tag{2.B.59}$$

Since $Z(\underline{w}, b)$ is continuously differentiable, the implicit function theorem can be used to get the derivative of b with respect to $\underline{w}$,

$$\begin{aligned}\frac{\partial b(\underline{w})}{\partial \underline{w}} &= -\frac{\frac{\partial Z(\underline{w},b)}{\partial \underline{w}}}{\frac{\partial Z(\underline{w},b)}{\partial b}} \\ &= -\frac{-E''\cdot\frac{\partial \bar{v}(\underline{w},b)}{\partial \underline{w}}\cdot\left(1-\frac{\partial \bar{v}(\underline{w},b)}{\partial b}\right)\cdot(V-b)-E'\cdot\frac{\partial^2 \bar{v}(\underline{w},b)}{\partial b\partial \underline{w}}\cdot(V-b)+E'\cdot\frac{\partial \bar{v}(\underline{w},b)}{\partial \underline{w}}}{E''\cdot\left(1-\frac{\partial \bar{v}(\underline{w},b)}{\partial b}\right)^2\cdot(V-b)-E'\cdot\frac{\partial^2 \bar{v}(\underline{w},b)}{\partial b^2}\cdot(V-b)-2E'\cdot\left(1-\frac{\partial \bar{v}(\underline{w},b)}{\partial b}\right)} \\ &= -\frac{\left(\frac{1}{1-E}-\frac{c'''(E)}{c''(E)}\right)\cdot\frac{V-b}{(1-E)^2\cdot(c''(E))^2}-\frac{1}{(1-E)\cdot c''(E)}}{\left(\frac{1}{1-E}-\frac{c'''(E)}{c''(E)}\right)\cdot\frac{V-b}{(1-E)^2\cdot(c''(E))^2}-\frac{2}{(1-E)\cdot c''(E)}}. \end{aligned} \tag{2.B.60}$$

Since $E(\cdot) < 1$, $c''(\cdot) > 0$, $c'''(\cdot) > 0$, $b \leq V$ and strict concavity $\left(\frac{c'''(E)}{c''(E)} > \frac{1}{1-E}\right)$, we get that $\frac{\partial b(\underline{w})}{\partial \underline{w}} < 0$. Hence, a higher minimum wage implies a lower bonus wage.

On the one hand, we found that a higher minimum wage leads to a lower bonus wage, which provides fewer incentives. On the other hand, we found that a higher minimum wage implies a more severe NCC, which provides more incentives. It remains to show that the effect on the NCC is stronger than on the bonus wage. Rearranging the marginal change of the incentives in the minimum wage (2.B.55) and plugging in yields

$$-\frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial \underline{w}} + \frac{\partial b(\underline{w})}{\partial \underline{w}}\cdot\left(1-\frac{\partial \bar{v}(\underline{w}, b(\underline{w}))}{\partial b}\right) \tag{2.B.61}$$

$$= \frac{1}{1-E} + \frac{\partial b(\underline{w})}{\partial \underline{w}}\cdot\left(1+\frac{E}{1-E}\right) \tag{2.B.62}$$

$$= \frac{1}{1-E}\cdot\left(1+\frac{\partial b(\underline{w})}{\partial \underline{w}}\right). \tag{2.B.63}$$

To show that this is positive, it now suffices to show that the bracket is positive. That is, $\frac{\partial b(\underline{w})}{\partial \underline{w}} > -1$.

Consider $-\frac{\partial b(\underline{w})}{\partial \underline{w}}$ as it is characterized in equation (2.B.60). For simplicity, let

$$x \equiv \left(\frac{1}{1-E}-\frac{c'''(E)}{c''(E)}\right)\frac{V-b}{(1-E)^2(c''(E))^2} \quad \text{and} \quad y \equiv \frac{1}{(1-E)c''(E)}. \tag{2.B.64}$$

We have that $x < 0$ and $y > 0$. It is then easy to check that $-\frac{\partial b(\underline{w})}{\partial \underline{w}} = \frac{x-y}{x-2y} < 1$. Which was to be shown. Therefore, the equilibrium effort is increasing in the minimum wage in the third combination.

We now show that in the fourth combination, the equilibrium effort is also increasing in the minimum wage. The principal does not use a bonus wage anymore. Lemma 2.2 shows that $\frac{\partial \bar{v}(\underline{w})}{\partial \underline{w}} = -\frac{1}{1-E(-\bar{v}(\underline{w}))}$ where $E(-\bar{v}(\underline{w}))$ is the solution to the agent's incentive problem. This shows that higher minimum wages lead to more severe NCCs, which then leads to higher effort through the incentive constraint.

To sum up, if $\underline{w} > \kappa_2$, then higher minimum wages lead to more effort incentives, and, thus, a non-monotonicity of the equilibrium effort.

□

2.B.4 Proof of Proposition 2.4

As the minimum wage goes to infinity, the equilibrium effort goes to 1. Hence, the equilibrium effort level exceeds the first-best effort if the minimum wage is sufficiently large.

Proof. We show that the principal induces a higher effort level than first-best effort if the minimum wage is sufficiently large. Due to the Inada conditions, the first-best effort level will be strictly smaller than 1. We show that the equilibrium effort in the third (which is relevant in case the fourth combination is never optimal) and in the fourth combination must go to 1. This directly implies that the equilibrium effort level will be higher than the first-best effort level if the minimum wage is large enough. We start with the third combination. Formally, we want to show that

$$\lim_{\underline{w}\to\infty} E(b(\underline{w}) - \bar{v}(\big(\underline{w}), b(\underline{w})\big) \;=\; 1, \tag{2.B.65}$$

where E is continuous and monotonically increasing in the bonus wage, in the severity of the NCC, and in the minimum wage (Proposition 2.3). Therefore, we can rewrite the limit such that

$$\begin{aligned}
&\lim_{\underline{w}\to\infty} E(b(\underline{w}) - \bar{v}(\big(\underline{w}), b(\underline{w})\big)) && (2.B.66)\\
=&\lim_{\underline{w}\to\infty} (c')^{-1}(b(\underline{w}) - \bar{v}(\big(\underline{w}), b(\underline{w})\big)) && (2.B.67)\\
=&(c')^{-1}\left(\lim_{\underline{w}\to\infty} b(\underline{w}) - \lim_{\underline{w}\to\infty} \bar{v}(\big(\underline{w}), b(\underline{w})\big)\right) && (2.B.68)\\
=&(c')^{-1}(\infty) && (2.B.69)\\
=&1, && (2.B.70)
\end{aligned}$$

due to the Inada conditions.

We proceed with the fourth combination. Formally, we want to show that

$$\lim_{\underline{w}\to\infty} E(-\bar{v}\left(\underline{w}\right)) \;=\; 1, \tag{2.B.71}$$

where E is continuous and monotonically increasing in the severity of the NCC, and in the minimum wage (Proposition 2.3). Therefore, we can rewrite the limit such that

$$\lim_{\underline{w}\to\infty} E(-\bar{v}\left(\underline{w}\right)) \tag{2.B.72}$$

$$= \lim_{\underline{w}\to\infty} (c')^{-1}(-\bar{v}\left(\underline{w}\right)) \tag{2.B.73}$$

$$=(c')^{-1}\left(-\lim_{\underline{w}\to\infty} \bar{v}\left(\underline{w}\right)\right) \tag{2.B.74}$$

$$=(c')^{-1}(\infty) \tag{2.B.75}$$

$$=1, \tag{2.B.76}$$

due to the Inada conditions.

□

2.B.5 Proof of Lemma 2.1

Let f_f be the probability that the agent gets fired after a failure and f_s the probability that the agent gets fired after a success. In equilibrium, the principal chooses $f_s = 0$ and $f_f = 1$ if she can replace the agent at no cost.

Proof. Let f_f be the probability that the agent gets fired after a failure and f_s the probability that the agent gets fired after a success. Plugging this general firing rule into the agent's problem changes his incentive constraint:

$$e^* \;=\; \underset{e\in[0,1]}{\arg\max}\;\; w + e\cdot b + e\cdot f_s\cdot\bar{v} + (1-e)\cdot f_f\cdot\bar{v} - c(e). \tag{IC'}$$

The agent's first-order condition (the Inada conditions ensure an interior solution) is then

$$b - (f_f - f_s)\bar{v} - c'(e) \;=\; 0. \tag{2.B.77}$$

The agent's incentives from the NCC are now given by $-(f_f - f_s)\cdot\bar{v}$ instead of $-\bar{v}$. Since the principal has to make the agent willing to participate, she chooses the firing rule and the NCC that reduce the agent's expected utility as little as possible. To implement a fixed effort, given a fixed base and bonus wage, the principal, thus, chooses the firing rule that solves

$$\max_{f_f, f_s, \bar{v}} \quad w + e\cdot b + e\cdot f_s\cdot\bar{v} + (1-e)\cdot f_f\cdot\bar{v} - c(e) \tag{2.B.78}$$

$$\text{subject to} \quad 0 \;\le\; f_f \;\le\; 1 \tag{2.B.79}$$

$$0 \;\le\; f_s \;\le\; 1 \tag{2.B.80}$$

$$\bar{v} \;\le\; 0 \tag{2.B.81}$$

$$-(f_f - f_s)\bar{v} \;=\; K \;\ge\; 0. \tag{2.B.82}$$

K is the "amount of incentives" from the NCC. The principal only wants to use NCCs at all if she wants to provide more incentives than with the bonus wage alone. That is why K is positive. Ignoring constants and rearranging the constraints, this problem simplifies to $\bar{v} = -\frac{K}{f_f - f_s}$ and

$$\max_{f_f, f_s} \quad -\frac{f_f}{f_f - f_s} \quad \text{subject to} \quad 0 \leq f_s < f_f \leq 1. \tag{2.B.83}$$

The derivative with respect to f_f is globally positive, given the constraint. Therefore, it is optimal to set $f_f = 1$. The derivative with respect to f_s is globally negative, given the constraint. Therefore, it is optimal to set $f_s = 0$. The optimal $\bar{v}$ is then given by $-K$. The firing rule that makes the agent's participation constraint as slack as possible is to fire him if and only if he fails. □

2.B.6 Proof of Proposition 2.5

Let $\underline{\bar{v}} < 0$ be a lower bound on the NCC.

(i) Let, without a bound on NCCs, the optimal NCC be $\bar{v} \geq \underline{\bar{v}}$. Then, the optimal contract remains the same with a bound on NCCs.

(ii) Let, without a bound on NCCs, the optimal NCC be $\bar{v} < \underline{\bar{v}}$. Then, the optimal contract with a bound on NCCs has $\bar{v} = \underline{\bar{v}}$. If the optimal bonus wage is positive, when the bound on the NCC starts binding, the bonus wage decreases more steeply than without a bound. At some larger minimum wage, the optimal bonus wage becomes constant, either at a positive level or at zero. If the optimal bonus wage is zero when the bound on the NCC starts binding, the bonus wage remains at zero for all larger minimum wages.

Proof. Let the principal's expected profit be strictly quasi-concave in the bonus wage, that is,

$$\frac{c'''(E(b - \bar{v}(\underline{w}, b)))}{c''(E(b - \bar{v}(\underline{w}, b)))} > \frac{1}{1 - E(b - \bar{v}(\underline{w}, b))} - \frac{2}{E(b - \bar{v}(\underline{w}, b))} \tag{2.B.84}$$

holds for all minimum wages.

With a bounded NCC we have the additional constraint that $\bar{v} \geq \underline{\bar{v}}$. This changes P's maximization problem to

$$\max_{b} \quad -\underline{w} + E(b - \bar{v}) \cdot (V - b) \tag{2.B.85}$$

$$\text{subject to} \quad \bar{v} = \max\left\{\bar{v}(\underline{w}, b), \underline{\bar{v}}\right\} \tag{NCC}$$

$$b \geq b_2^{**}(\underline{w}), \tag{2.B.86}$$

where again $\bar{v}(\underline{w}, b) = -\frac{\underline{w} + E(b - \bar{v}(\underline{w}, b)) \cdot b - c(E(b - \bar{v}(\underline{w}, b)))}{1 - E(b - \bar{v}(\underline{w}, b))}$. The *NCC* condition already uses that the profit is increasing in more severe non-compete clauses (because the optimal bonus wage is smaller than the success payoff). Thus, the principal would never use an NCC that is less severe than the specific NCC that makes the PC binding ($\bar{v}(\underline{w}, b)$), except if this would violate the bound on NCCs ($\underline{\bar{v}}$). As a result, the optimal NCC is determined by which constraint binds first: the participation constraint ($\bar{v}(\underline{w}, b)$) or the bound on NCCs ($\underline{\bar{v}}$).

We now split the minimum wages into two ranges. One for which the bound on NCCs is insubstantial and one for which the bound on NCCs makes the formerly optimal contracts infeasible. This is possible because the optimal $\bar{v}$ without a bound decreases continuously and strictly monotonically in the minimum wage above κ_2. Moreover, $\bar{v}$ lies between zero and minus infinity such that any bound binds for some minimum wages. We define $\underline{w}_{bound}$ as the minimum wage for which the optimal contract without a bound on NCCs uses an NCC that is exactly the bound. That is, the optimal contract is $(\underline{w}_{bound}, b(\underline{w}_{bound}), \underline{\bar{v}})$. As argued above, $\underline{w}_{bound}$ exists and is unique for each bound $\underline{\bar{v}}$.

Case i) $\underline{w} < \underline{w}_{bound}$. For these minimum wages, the optimal contract without a bound on NCCs does not violate the bound on NCCs. Since the bound only introduces another constraint, these contracts remain optimal. The bound on NCCs can be ignored.

Case ii) $\underline{w} \geq \underline{w}_{bound}$. For all minimum wages above $\underline{w}_{bound}$, the optimal contracts without a bound on NCCs are not feasible anymore: they violate the bound on NCCs. In the simplified problem, the only choice variable of the principal is the bonus wage. Thus, the optimal NCC is implicitly defined by the optimal bonus wage.

For $\underline{w} \geq \underline{w}_{bound}$, the constraint $b \geq b_2^{**}(\underline{w})$ can be ignored. The constraint said that, first, the participation constraint must not be violated if $\bar{v} = 0$, and, second, that the bonus wage must be non-negative. Since the optimal NCC at $\underline{w}_{bound}$ is strictly negative (and because of its comparative statics), we know that the participation constraint without an NCC would be satisfied. Furthermore, the optimal bonus wage can never be negative because there is a profitable deviation, as argued in the proof of Proposition 2.2; this deviation exists independently of a bound on NCCs.

For minimum wages $\underline{w} \geq \underline{w}_{bound}$, the optimal contract without a bound is either the one from the third combination or the one from the fourth combination. We can distinguish these as different cases. For each case, we show that once the binding NCC is optimal, it will remain optimal for all larger minimum wages, and we characterize the optimal bonus wage.

a) The optimal contract for the minimum wage $\underline{w}_{bound}$ is from the third combination. That is, the optimal bonus wage without a bound is strictly positive. Thus, the optimal bonus wage is determined by the first-order condition; the bonus wage for which the marginal profit gets zero. It is unique because the objective function is quasi-concave by assumption. For $\underline{w} = \underline{w}_{bound}$, the optimal contract remains optimal and just makes the bound on the NCC binding. Thus, the marginal profit at the bonus wage $b(\underline{w}_{bound})$ is 0. We will reconsider this particular minimum wage after describing the marginal profit in the bonus wage in general.

How does the marginal profit with respect to the bonus wage behave for a fixed minimum wage $\underline{w} > \underline{w}_{bound}$? For a sketch of the marginal profit, see Figure 2.B.1.

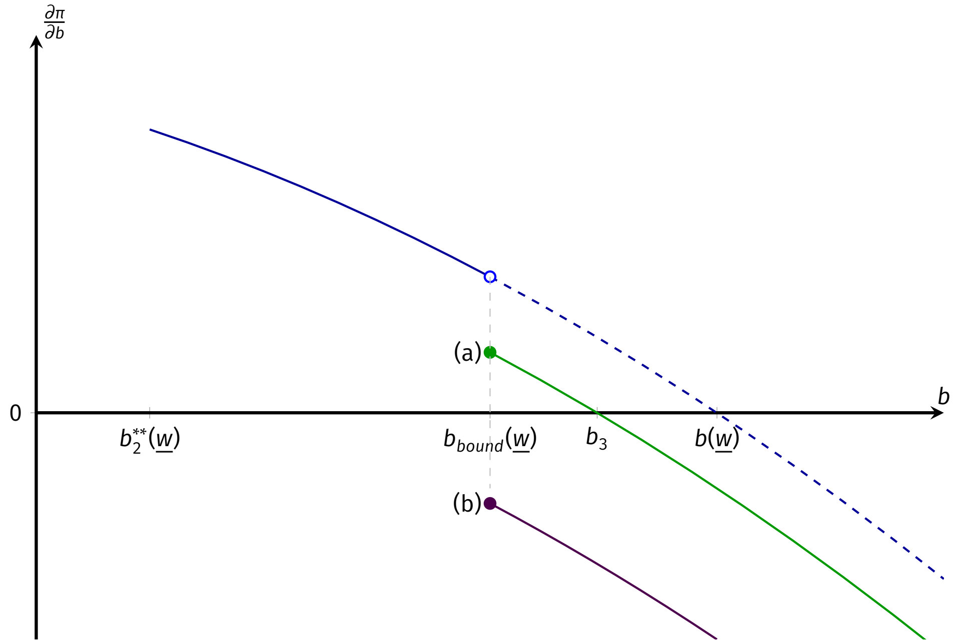

Figure 2.B.1. The derivative of the profit with respect to the bonus wage drops as soon as the bound on the NCC binds. If (a) the drop ends above zero, the agent gets a rent and the optimal bonus wage is the same for higher minimum wages. If (b) the drop ends below zero, the agent gets no rent. Drawn for a concave objective function.

As mentioned above, starting at $b_2^{**}(\underline{w})$, the marginal profit is positive. When increasing the bonus wage, it keeps being positive. Then, the bonus wage, $b_{bound}(\underline{w})$, is reached that allows the principal to reach the bound $\bar{v}(\underline{w}, b_{bound}(\underline{w}) = \underline{\bar{v}}$. Importantly, at this minimum wage, the derivative is still positive: The optimal bonus wage is $b(\underline{w})$ and by the case assumption it is true that $\bar{v}(\underline{w}, b(\underline{w})) < \underline{\bar{v}}$. Because $\bar{v}(\underline{w}, b)$ is decreasing in the bonus wage, and because the root of the first-order condition is unique, we know that $b_{bound}(\underline{w}) < b(\underline{w})$. From $b_{bound}(\underline{w})$ on, the principal cannot make the NCC more severe when increasing the bonus wage. Therefore, there are no double incentives anymore. The marginal profit, thus, drops downwards.

Formally, the marginal profit for bonus wages up to (exclusively) $b_{bound}(\underline{w})$ is given by the derivative of the profit function $\pi(\underline{w}, b, \bar{v}(\underline{w}, b(\underline{w})))$ because a larger bonus wage means a more severe NCC. The marginal profit for bonus wages above $b_{bound}(\underline{w})$ is given by the derivative of the profit function $\pi(\underline{w}, b, \underline{\bar{v}})$ because the NCC's severity is constrained by its bound. The marginal profits just above and just below the drop are

$$\left.\frac{\partial \pi(\underline{w}, b, \bar{v}(\underline{w}, b(\underline{w})))}{\partial b}\right|_{b=b_{bound}(\underline{w})}$$
$$= \frac{E'(b_{bound}(\underline{w}) - \underline{\bar{v}})}{1 - E(b_{bound}(\underline{w}) - \underline{\bar{v}})} \cdot (V - b_{bound}(\underline{w})) - E(b_{bound}(\underline{w}) - \underline{\bar{v}}) > 0 \qquad (2.B.87)$$

and

$$\left.\frac{\partial \pi(\underline{w}, b, \underline{\bar{v}})}{\partial b}\right|_{b=b_{bound}(\underline{w})}$$
$$= E'(b_{bound}(\underline{w}) - \underline{\bar{v}}) \cdot (V - b_{bound}(\underline{w})) - E(b_{bound}(\underline{w}) - \underline{\bar{v}}). \qquad (2.B.88)$$

For bonus wages after the drop, the profit function is strictly concave in the bonus wage.[28] The marginal profit is, thus, strictly decreasing.

The optimal bonus wage is now either $b_{bound}(\underline{w})$, if the marginal profit drops (weakly) below zero, or a higher bonus wage if the marginal profit remains positive after the drop. In any case, this implies that $\bar{v}(\underline{w}, b) \leq \underline{\bar{v}}$ in the optimum. Therefore, the bound on the NCC is the binding constraint; thus $\underline{\bar{v}}$ is the optimal NCC.

To find the optimal bonus wage, we have to find out which constraints will bind. This depends on whether the optimal bonus wage is at the drop point or not. If it is at the drop point, the participation constraint binds ($\bar{v}(\underline{w}, b_{bound}(\underline{w})) = \underline{\bar{v}}$); which implies that the agent gets no rent. If it is to the right of the drop point, the participation constraint is slack because the NCC that would make the participation constraint binding lies outside the bound. Therefore, it is slack; which implies that the agent gets a rent.

For the other constraints (MWC_1, MWC_2, NCC), the same reasoning as above, in the proof of Proposition 2.2, applies. The minimum wage condition on the base wage binds. Otherwise, there is a profitable deviation. Due to the case assumption, the optimal bonus wage without a bound is positive, thus MWC_2 is slack. With a bound, it might also be that MWC_2 binds if ignoring the constraint leads to a violation. Due to the case assumption, an NCC is used, which means that the *NCC* feasibility constraint is slack. As a result, the optimal base wage always is the minimum wage and, as shown above, the optimal NCC is the binding NCC.

First, we now determine the optimal bonus wage depending on where the drop ends and then, second, we show that there always is a range of minimum wages for which the drop ends in the negative.

We start with the case in which the marginal profit's drop ends in the non-positive. In this case, the optimal bonus wage is at the drop point and makes the participation constraint binding. Thus, the participation constraint pins down the optimal bonus wage. How does the optimal bonus wage change in the minimum

28. The second derivative is $E''(\cdot) \cdot (V - b) - 2E'(\cdot) < 0$. $E''(\cdot)$ is globally negative and $E'(\cdot)$ is globally positive.

wage? We use the implicit function theorem to show that the bonus wage that makes the participation constraint binding is strictly decreasing in the minimum wage. Rearrange the binding participation constraint to

$$Z \equiv \underline{w} + E(b_{bound} - \underline{v}) \cdot (b_{bound} - \underline{v}) + \underline{v} - c(E(b_{bound} - \underline{v})) = 0. \qquad (2.B.89)$$

Because this is continuously differentiable, the implicit function theorem can be used to get the derivatives of b_{bound} with respect to $\underline{w}$,

$$\frac{\partial b_{bound}(\underline{w})}{\partial \underline{w}} = -\frac{\frac{\partial Z}{\partial \underline{w}}}{\frac{\partial Z}{\partial b_{bound}}} = -\frac{1}{E'(\cdot) \cdot (b_{bound} - \underline{v}) + E(\cdot) - c'(E(\cdot)) \cdot E'(\cdot)}$$
$$= -\frac{1}{E(\cdot)}. \qquad (2.B.90)$$

where we suppress the argument of E for readability. The simplification is due to the agent's first-order condition, $b_{bound} - \underline{v} - c'(E) = 0$. Since $E(b_{bound} - \underline{v}) > 0$, the bonus wage that makes the participation constraint binding is strictly decreasing in the minimum wage.

Further, we can say that the optimal bonus wage with a bound lies below the optimal bonus wage without a bound on the NCC. In both cases, the participation constraint is binding and the bonus wage is positive (due to the case assumption). Without a bound on the NCC, the optimal NCC is weakly more severe than the bound because $\underline{w} \geq \underline{w}_{bound}$; strictly more severe if $\underline{w} > \underline{w}_{bound}$. For a fixed minimum wage, a strictly more severe NCC needs a strictly larger bonus wage to keep the participation constraint satisfied. Thus, with a bound on the NCC, the optimal bonus wage is smaller.

When the optimal bonus wage hits zero, it stays at zero for all larger minimum wages. It can never become negative because of the profitable deviation. Note that when the bonus wage hits zero, for all larger minimum wages, the participation constraint is slack and the agent gets a rent.

We now look at the optimal bonus wage if the drop in the marginal profit ends in the positive and the participation constraint can be ignored. The optimal bonus wage is constant because the minimum wage does not enter the problem anymore. The optimal bonus wage is determined by the marginal profit's being zero or the minimum wage condition on the bonus wage. We define b_3 as the root,

$$\frac{\partial \pi}{\partial b} \stackrel{!}{=} 0 \iff b_3 : \quad E'(b_3 - \underline{v}) \cdot (V - b_3) - E(b_3 - \underline{v}) = 0. \qquad (2.B.91)$$

Note that $E'(\cdot)$ is decreasing in its arguments because $E''(\cdot) < 0$. Furthermore, $E(\cdot)$ is increasing in its arguments. Therefore, compared to the third case in the benchmark, the marginal benefit of the bonus wage is smaller and the marginal cost is larger for all bonus wages. We shift $E'(\cdot)$ to the left and $E(\cdot)$ to the right. Thus, $b_3 < b^{***}$. If the marginal profit is zero for a negative bonus wage, the optimal bonus wage is zero because of the minimum wage condition. Thus, the optimal bonus wage is $b_3^+ \equiv \max\{0, b_3\}$.

What is the relation between the solution when the drop ends in the negative and when it ends in the positive? The maximization problem when ignoring the participation constraint yields a weakly larger maximum than taking into account the participation constraint. Therefore, the profit with b_3^+ is weakly larger than the profit with $b_{bound}(\underline{w})$. b_3^+ is optimal whenever it does not violate the participation constraint.

We now show that there are some minimum wages for which b_3^+ does violate the participation constraint, such that $b_{bound}(\underline{w})$ is the optimal solution. Reconsider the minimum wage $\underline{w}_{bound}$. The optimal contract is $(\underline{w}_{bound}, b(\underline{w}_{bound}), \underline{\bar{v}})$. By the case assumption, $b(\underline{w}_{bound}) > 0$. Thus, without a bound on NCCs, the marginal profit of an additional unit of bonus wage is 0 at $b(\underline{w}_{bound})$. With a bound on NCCs, this is the bonus wage at which the drop from double incentives to incentives (only through bonus wage) happens. The drop, thus, has to end in the negative. Thus, this is one minimum wage for which the participation constraint would be violated for b_3^+. Furthermore, the point at which the drop ends, moves continuously in the minimum wage: The marginal profit is a continuous function of the bonus wage and the bonus wage at which the drop happens is a continuous function of the minimum wage. Thus, the drop also ends in the negative for some larger minimum wages.

b) The optimal contract for the minimum wage $\underline{w}_{bound}$ is from the fourth combination, that is, the optimal bonus wage is 0. With a bound on NCCs, the optimal contract now is $(\underline{w}, 0, \underline{\bar{v}})$. A positive bonus wage cannot increase the profits. The optimal contract only falls into the fourth combination if the marginal profit from the first unit of bonus wage is negative. Because the binding NCC does not violate the participation constraint even without a bonus wage, there never are double incentives. Thus, the marginal profit is smaller than without a bound on NCCs (intuitively, the drop happened for a negative bonus wage). Since the marginal profit was negative with double incentives, the marginal profit is still negative. It is optimal not to use a positive bonus wage.

A negative bonus wage cannot increase the profits because this means increasing the base wage above the minimum wage (otherwise the minimum wage constraint on the bonus would be violated). Then, there is a profitable deviation (making the NCC less severe, the bonus wage larger and the base wage lower by one marginal unit).

□

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Chapter 3

Surveying Price Stickiness and Fair Price Increases*

Joint with Maximilian Weiß

3.1 Introduction

Firms face costs when they attempt to adjust their prices. The nature of these price-setting costs is of interest to policy makers for explaining movements in the aggregate price level. The survey approach by Blinder et al. (1998) tries to select the "right" theory of price-setting costs by directly asking the managers: The managers are presented a list of hypotheses. Each expresses an academic theory of "price stickiness" in layperson's terms. Then, the managers grade how well these hypotheses describe what they are thinking while not adjusting their firms' prices. Unfortunately, unclear rankings of hypotheses are common: Often, several theories with similar and only intermediate scores are at the top (Blanchard, 1994). The survey approach fails to select a single best theory.

We think that there are two main forces that produce more similar scores in most of the surveys conducted. First, many of the existing surveys average the hypotheses' scores both over different markets and over heterogeneous firms within a market.[1] The tendency for scores to equalize arises if the main reason for price stickiness actually differs across different industries and market environments. Second,

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1. Many existing surveys construct their sample to resemble the overall economy, as they are looking for the one main cause of price stickiness in the economy. If, after averaging the hypotheses' scores over different sectors, one hypothesis clearly outscores all others, the theory behind it should be used to explain changes in the aggregate price level. It is unclear, however, whether such a dominant cause of price stickiness exists.

most studies ask the respondents for their behavior in hypothetical situations. This adds a source of noise to the respondents' answers: respondents may interpret the hypothetical situation differently, and the mental cost of the exercise may introduce a recency bias to their answers. The added noise tends to attenuate the difference between the scores.

We design our survey in order to mitigate these two forces: We survey German hairdressers. The German hairdressing market consists of relatively homogeneous firms, which are small and managed by the owner, who is a hairdresser. By asking only those firms, we avoid averaging over different markets and organizational forms. Second, we mitigate the noise from the respondents' interpretations of hypotheticals by asking the hairdressers about their actual responses to recent shocks caused by the COVID-19 pandemic. The spread of the virus, as well as public policy measures that were adopted in response, have hit German hairdressers with several shocks that, theoretically, should induce higher prices. We ask the hairdressers about their actual behavior in response to these shocks.

We find that the design of our survey sufficiently reduces the tendency to similar scores. Our results imply that one theory is most important in explaining price stickiness in the German hairdressing market: Firms abstain from increasing their prices in order to retain their regular customers. Behind this winning theory, many theories receive similar, intermediate scores.[2]

Having identified the main source of price stickiness in this market, we use our data to investigate it. When and why does the desire to retain regular customers prevent German hairdressers from increasing their prices? We find that firms are more likely to increase their prices—to have less sticky prices—when they have a closer and more trusting relationship with their regular customers.

To explain this finding, we verbally extend theories of fair pricing under asymmetric information with customer relationship. Customers want to save on search costs and become regulars of a firm that (implicitly) promises to increase its price only for "fair" reasons (Okun, 1981).[3] The firm experiences shocks, which are pri-

2. The only other hypothesis that scores as highly as wanting to retain regular customers is having already increased the price during the previous year. This finding hints at the importance of *time-dependent* (or *interval-dependent*) price setting. We have excluded the hypothesis from our analysis because the hypothesis does not correspond definitely to a single underlying theory (besides time-dependent price setting, it might be that one method to retain the regular customers is to increase the price rarely or that the previous price increase has put the current price close to the optimal price, so time-invariant pricing cost deter price increases). Furthermore, we have few observations for this hypothesis as we added it later (after several participants voiced that they were missing that hypothesis).

3. The dual entitlement theory (Kahneman, Knetsch, and Thaler, 1986) derives rules to determine which price increases are generally considered fair. The initial price sets a reference point that determines how the surplus from trade is split between the customer and the firm. Both the customer and the firm are entitled to these reference surpluses. If a price increase leaves the customer less than her reference surplus, the customer finds this unfair, unless the shock decreases the total surplus from

vate information, and decides whether to increase its price. The customers only observe the price and try to infer whether a price increase is fair. If they infer that it is not fair, they stop buying from that firm (Okun, 1981, p. 153f). We add to this theory the aspect of a relationship between the firm and the regular customers. We suppose that, when customers have a good relationship with the firm, they trust more in the firm's commitment to fair pricing. Then, they are more easily convinced that a particular price increase is fair.[4] For the firm, fostering a trusting relationship, thus, reduces obstacles to price increases in the presence of asymmetric information.

Besides the finding that a good relationship is associated with price increases, we establish other stylized facts about the German hairdresser market: A hairdresser that has a good relationship with her customers is on average less likely to be constrained by the risk of losing her regular customers when increasing prices. Furthermore, the average hairdresser who has a good relationship with her regular customers (i) was more able to restore her profit margin from before the pandemic, (ii) is less pessimistic, and (iii) is more satisfied with her pricing. These observations are consistent with our theory: The theory states that a hairdresser who has a good relationship with her regular customers has more leeway to increase prices, as customers are more willing to accept the price increase to be fair. Thus, she can restore her profit margin more easily, and has generally more options in pricing when responding to shocks, which improves the business outlook.

An interview with a practitioner supports the theory of fair pricing: Regular customers generally demanded justifications for price increases. Referring to shocks that lower the total surplus—like a cost shock—was the best argument to convince the customer of the fairness of the price increase.

We contribute to the literature using the survey approach to research price stickiness. To our knowledge, so far, 27 price stickiness surveys have been conducted in 26 countries. We list the rankings and short descriptions of the presented hypotheses in Appendix 3.A.

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trade—for example, a cost shock. In this case, it is considered fair that the firm restores its reference profit by increasing its price.

4. Xia, Monroe, and Cox (2004, p. 6) propose a similar mechanism. A trusting relationship between a buyer and a seller might make it less likely that the buyer perceives a price as unfair. To do so, the relationship would need to reach the highest possible trust level in business transactions, identification-based trust (Shapiro, Sheppard, and Cheraskin, 1992), which only few business relationships do. The relationship between a customer and her hairdresser might be one of these instances. A hairdresser told us that her relationship with her customers are especially trusting as it usually lasts for many years and she was the only "stranger" that was allowed to touch her customer's head and face, besides medical practitioners.

Another similar mechanism is the extension of the dual entitlement theory to include the seller's motive for a price increase (Campbell, 1999). Even if a price increase only restores a firm's profit, the customers might still find it unfair if the firm's inferred motive for the price increase is bad, as for example, greed. A firm with a good reputation might get the benefit of a doubt that its motives are good (Campbell, 1999, p. 190).

In particular, we respond to the critique of the survey approach by Blanchard (1994). The critique refers to the somewhat unclear ranking (see Figure 3.1) that resulted from the original survey by Blinder (1994). While Blanchard also criticizes some ill-posed questions in the specific survey, the methodological critique is directed at the whole literature. Blanchard suspects that mono-causal hypotheses were often too abstract, so that respondents had to think that many hypotheses contained some, but not all, of the truth. This would lead to many hypotheses' scoring similarly (Blanchard, 1994, p. 150). We respond to this critique by showing that the forces equalizing the hypotheses' scores can be mitigated sufficiently to get a clear ranking by adjusting our survey design—in particular, by narrowing the field of survey participants and by asking about recently experienced shocks. Therefore, we argue that the weakness discussed in Blanchard (1994) does not necessarily disqualify the survey approach as a whole.

We also contribute to the literature on fair pricing in connection with asymmetric information (for example Okun, 1981, Rotemberg, 2005, Rotemberg, 2011, Nakamura and Steinsson, 2011, and Eyster, Madarász, and Michaillat, 2021). Asymmetric information complicates fairness evaluations by adding a signaling component to price increases, as firms want their customers to infer that a price increase is fair. We provide suggestive evidence that transparent pricing and a good relationship between the firm and its regular customers reduces the firms' price-stickiness. This is in line with the predictions of the literature (for example in Xia, Monroe, and Cox, 2004).

In Section 3.2, we outline the design of our survey, describe our sample, and explain how the firms in our sample were affected by the pandemic and the ensuing public policy measures in Germany at the time. In Section 3.3, we present the results of our survey. In Section 3.4, we explore how the relationship between the firms and their customers interacts with fair pricing and price stickiness. In Section 3.5, we conclude.

3.2 Market Description and Summary Statistics

We conducted our online survey on the platform *SoSci Survey* from Monday, March 08, 2021, to Friday, April 16, 2021, shortly after the end of the second lockdown in Germany.[5] The questionnaire we used is in Appendix 3.B. It was necessary to possess the URL to participate. We recruited participants in two ways. First, on March 08, 2021, we contacted all local Chambers of Handicrafts (*Handwerkskammern*) be-

5. There were two lockdowns during which selling any hairdressing services was forbidden. The first lockdown went from March 22 to May 4, 2020. Afterwards, hygiene rules were introduced, such as distancing rules, mandatory masking, and mandatory hair washing before any hairdressing service. The second lockdown, after which we conducted our survey, went from December 13, 2020, to February 28, 2021.

cause membership is mandatory for German hairdressing firms. However, only few Chambers of Handicrafts reacted at all, and most refused to forward our e-mail to the hairdressers. Thus, second, we contacted the heads (*Obermeister*) of all local hairdressing guilds (*Friseur-Innungen*)[6], on March 15, 2021, and asked them to participate and to forward our e-mail to the other members. On April 1, we sent a reminder to the heads of the local hairdressing guilds.

After deleting answers with mostly missing or contradictory answers, 281 responses remained. In 2020, 77,166 hairdressing firms were registered in Germany (Zentralverband des Deutschen Friseurhandwerks, 2021, p. 12). That is, we reached around 0.4% of all German hairdressing firms.

We suppose that almost all participants come from the local guilds, which is helpful, as our goal was to sample similar firms. A head of a guild characterized the average guild member as somewhat larger than regular and predominantly in the middle price segment. The data presented below support our supposition.

Our participants are somewhat larger than regular. Table 3.1 summarizes the respondents' answers to our question measuring their firms' sizes by employees. For comparison with the distribution in the general market, we use data from 2018 from Zentralverband des Deutschen Friseurhandwerks (2021, p. 12). 53,484 firms (71% of the registered firms) reported their revenues to the authorities. A firm whose revenue is below the cutoff for exemption to report cannot have a single employee (paid at minimum wage) without making a loss.[7] So, in the market, almost a third of the firms has most likely no employees. Of the firms that reported their revenue, around half of the firms has less than 5 employees, around a fifth has between 5 and 9 employees, and around a twentieth has between 10 and 19 employees.

Our participants have somewhat higher prices than regular. Our measure for a hairdresser's prices is a standard men's haircut ("short back and sides, wash, cut, blow dry, 25 minutes"). We asked whether the price contained a "hygiene surcharge" and if so, what amount it is, and we asked whether the firm has passed on the temporary VAT reduction to the customers. The answers are summarized in Table 3.2. For a comparison with the general market, we use the data from a data-sharing project: A business consultant and a large supplier offer hairdressing firms the opportunity

6. Guilds are lobby groups with voluntary membership. The local hairdressing guilds are organized on a county-level or slightly larger, and there are in total 247 of them in Germany.

7. A firm is exempt from paying VAT if it has had a revenue of less than €17,500 in the previous year and expects a revenue of less than €50,000 in the current year. The federal minimum wage in 2018 was €8.84 per hour. This wage times 40 hours per week times 4.34 weeks per month times 12 months yields more than €18,000, additional to which the employer has to pay social security contributions. It is possible that a firm is exempt from reporting its revenue because it is newly founded and expects a revenue of less than €50,000, so it might have employees without making a loss. There are, however, few entrants to the market. The federal guild reports that 5,867 salons—not firms—were newly registered in 2020, so this number includes existing firms' moving or opening branches (Zentralverband des Deutschen Friseurhandwerks, 2021, p. 11).

Table 3.1. Size distribution of the participants. When using the size of a firm in regressions, we compromise between continuous variables and dummies. We treat the first three options as a continuous variable that takes the middle value of the interval, so 0, or 2, or 4.5. The answer "more than 6 employees" is captured by a dummy.

Number of employees	*n*	Frequency	continuous size variable	dummy size variable
no employees	22	8%	0	0
1 to 3 employees	105	37%	2	0
3 to 6 employees	88	31%	4.5	0
more than 6 employees	66	23%	0	1
Total	281	100%		

to submit their data to get access to the other firms' (aggregated) data. We suppose that the average firm in this self-selected sample is, as well, larger than a regular firm. In the sample from 2018, the average price of our reference haircut is €22.90 (Zöllner, 2019). Considering that the sample is self-selected and that the inflation rates were rather low between 2018 and 2021, we conclude that our participants are rather from the middle price segment.

Table 3.2. Summary of the participants' prices and related variables. The prices and surcharges are in Euros. 1 means the VAT reduction has been passed on.

Variable	*n*	Mean	SD	Min	Max
Price before the lockdown	281	25.93	6.22	14	58
Price after the lockdown	281	27.35	6.48	14	59
Planned price in April	263	27.58	6.57	14	59
Hygiene surcharge	96	2.38	0.98	0.5	5
Passed on the VAT reduction?	277	0.14	0.35	0	1

In our sample, the average price has increased by 5.5% during the lockdown, and the spread between prices got larger. Around a third of the firms charge hygiene surcharges that are on average around 9% of the average total price.

Table 3.3 shows the price information split by whether the firm increased its price. Almost two thirds of the sampled firms (64%) have increased their prices. Increasers had on average slightly lower prices before and slightly higher prices after their price increase compared to the non-increasers. Conditional on having increased the price, the average price increase is 12.6%. Hygiene surcharges seem to be independent of whether a firm increased its price or not.

We use that the COVID-19 pandemic and the associated lockdowns have hit all hairdressers with similar shocks, so that we can ask the hairdressers about their actual behavior instead of presenting hypothetical situations.

First, the firms have lost months worth of profits, paid bills from their reserves, and some had to borrow money to keep their businesses.

Table 3.3. Comparison of the prices and related variables between increasers and non-increasers, using a *t*-test.

	Non-increasers			Increasers			Difference	
Variable	*n*	Mean	SD	*n*	Mean	SD	Δ	*t* statistics
Price before the lockdown	102	26.55	6.80	179	25.58	5.85	0.97	(1.20)
Price after the lockdown	102	26.55	6.80	179	27.80	6.26	-1.25	(-1.53)
Price in April 2021	99	26.84	6.79	164	28.03	6.42	-1.19	(-1.40)
Hygiene surcharge	32	2.27	0.77	64	2.43	1.07	-0.16	(-0.85)

Second, the hygiene rules became slightly stricter: It got prohibited to serve walk-in customers (they needed to book appointments beforehand), the hairdresser had to wear a medical face mask and to replace it after each customer, there had to be a continuous stream of fresh air in the salon, although it was winter, and in some regions with many infections, customers had to be tested negatively. In some states, the hairdresser was allowed to conduct the test.

Third, the federal VAT was increased back to the normal rate: From June to December, the VAT rate had been reduced from 19% to 16%. Legally, price tags in Germany have to display the price including the VAT. So, firms that have passed on (some of) the tax reduction had changed their price lists. In our survey, few firms indicated having passed on the VAT reduction, which weakens the upward price shock from the end of the VAT reduction.

Fourth, many employees in hairdressing received pay rises: January is a common month for discretionary wage increases, and the legal minimum wage for hairdressing increased in several German states on January 1, 2021.[8]

Fifth, the demand directly after the second lockdown was huge. Several hairdressers auctioned off their first appointments for three-digit prices (and donated the revenue to charity).[9] The average waiting time for an appointment in the online booking tool Treatwell was more than two weeks.[10] Somewhat surprisingly, the demand later decreased to a constant level below the demand before the lockdown.[11]

8. In some states, there are binding collective agreements determining the minimum wage in hairdressing. This is the case in Hesse, where the collectively agreed minimum wage increased on January 1, 2021. In some states, there are collective agreements, but employers decide themselves whether to opt in. And in the other states, mostly in Eastern Germany, there are no collective agreements on the minimum wage in hairdressing. In this case, the federal minimum wage applies, which increased on January 1, 2021.

9. See Freund (2021) for *handwerksblatt.de*: “Friseure versteigern Termine für den guten Zweck.”

10. See Top Hair (2021): “Buchungsrekord zum Re-Start.”

11. See Roßberger (2021) for *BR24*: “Bayerische Friseure leiden weiter unter der Pandemie.” The hairdressers in the article conjecture that the hygiene and testing rules are the reason. Other reasons might be the fear of getting infected at the hairdresser’s, the diminished importance of having a fresh or professional haircut, and the customers’ smaller budgets because of the recession.

3.3 Ranking of Hypotheses

The goal of the manager survey by Blinder et al. (1998) is to evaluate theories explaining price stickiness. Because many theories received similar, intermediate scores and acceptance rates (see Figure 3.1),[12] Blanchard (1994) doubts the fruitfulness of this attempt: The survey approach did not help to determine the most important theories for price stickiness.

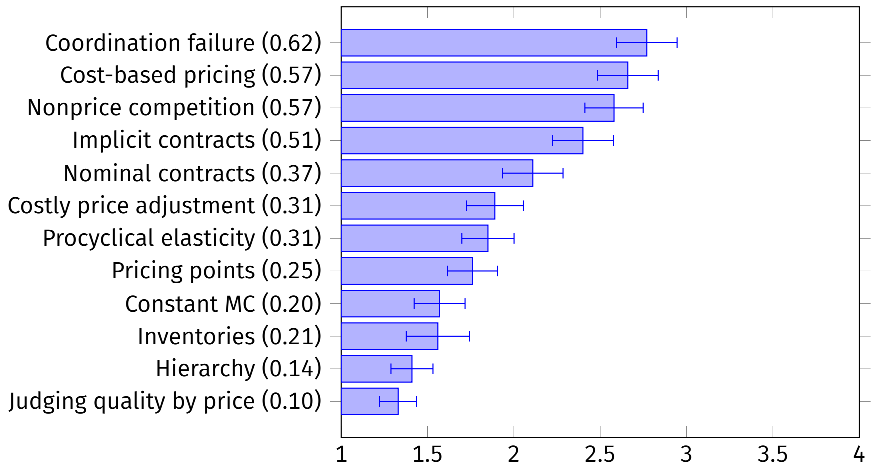


Figure 3.1. The original survey results by Blinder et al. (1998, p. 110). The grades go from 1 to 4. In the brackets behind the name of the hypothesis is the acceptance rate, which is defined as the share of managers that grade the respective hypothesis 3 or 4. If a manager rejected the premise of the hypothesis (e.g. the manager stated that the firm does not have inventories), Blinder et al. directly assigned the grade 1. On the x-axis is the average score of the hypothesis. The error bands are the 95% confidence intervals. For short summaries of the theories that underlie the hypotheses, see Appendix 3.A.

Whether the survey approach is generally unhelpful, however, depends on the reasons for the similar scores. Only reasons that are innate in the survey approach such that adjusting the survey design cannot mitigate the tendency to similar scores make surveying less useful. To get more specific, consider the reasons for the similar scores.

Two reasons for similar scores are unavoidable: First, following Blanchard (1994), as the world is complicated, no single theory will ever be able to cover all the reasons for which managers fail to adjust their prices. Therefore, there is a tendency to assign lots of intermediate grades. Second, there is noise from translation. The survey translates academic theories into terms that managers can relate to and the managers' evaluations into a grade. Each translation and each reduction of the information's dimensions causes noise.

12. The data are from Table 5.2 in Blinder et al. (1998, p. 110). It is the finalized version of the table to which Blanchard's critique referred (Table 4.4 in Blinder, 1994, p. 124).

We design our survey to avoid two other reasons for similar scores: The first is averaging within and across markets. As different theories are differently important in different markets, averaging leads to more intermediate scores. The same might be true for averaging within a market over small shops and multinational firms. To make the ranking clearer, we survey a single market which consists of rather homogeneous firms—made even more similar due to our sampling bias favoring guild members. The second is the noise from asking about pricing decisions in hypothetical situations. The availability bias makes the managers overweight recent, potentially contrarian experiences when imagining hypothetical situations. The projection bias makes them misjudge what they would do if the hypothetical situation became real. To reduce this noise, we ask the hairdressers about their actual behavior in the wake of recent shocks.

With this suitably adjusted survey design, we present the German hairdressers hypotheses based on those in Blinder et al. (1998) and get a clearly winning hypothesis at the top of the ranking (see Figure 3.2).[13] The clearly winning hypothesis is that firms did not increase their prices to retain their regular customers. That is, for the German hairdressing market, the survey approach identifies the main reason for price stickiness when the survey design mitigates averaging and the noise from hypothetical questions.

Next, we make the firms even more homogeneous, to see whether the similar scores in the middle of the ranking are due to averaging about slightly heterogeneous firms. We follow two approaches to determine subgroups of firms. Top-down, we select firms based on their characteristics, and bottom-up, we select firms based on their grades.

In the top-down approach, we use the descriptive variables from the survey to form subgroups of even more similar firms and compute the rankings within these subgroups. However, this approach does not yield clearer rankings.[14]

Nevertheless, there might be subgroups of firms whose owners evaluate the hypotheses more similarly, but the subgroups are not determined by different values

13. We asked about versions of coordination failure (competitors' prices not up), cost-based pricing (cost not up, not passed on VAT reduction), implicit contracts (retain regular customers), nominal contracts (prices contracted), costly price adjustments (unsure about increasing, avoid temporary increase), procyclical elasticity (customers' budgets smaller), pricing points, and hierarchy (could not agree on increase). We excluded menu costs (in their literal meaning), non-price competition, inventories, constant marginal cost, and judging quality by price for different reasons. Mostly, we expected those theories to not apply or to be rated as unimportant. Because participants asked for it, we added the option "I did not increase the price because I have increased the price already last spring" later. As explained above, we exclude it from the results. See Appendix 3.A for short descriptions of the theories underlying the hypotheses.

14. We formed subgroups based on the responses to (i) the size of the firm, (ii) the share of regular customers, (iii) the price level, (iv) different pricing methods, and three variables described in the next section (measuring customer relationship, pessimism, and satisfaction with the own pricing).

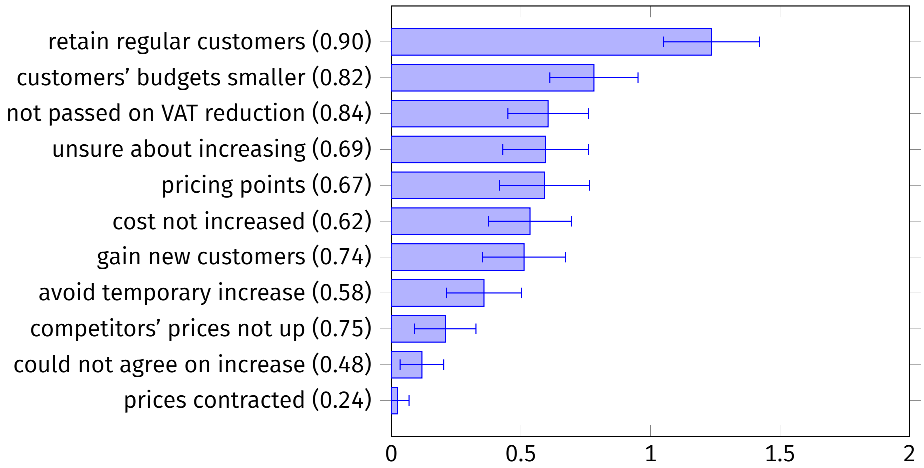
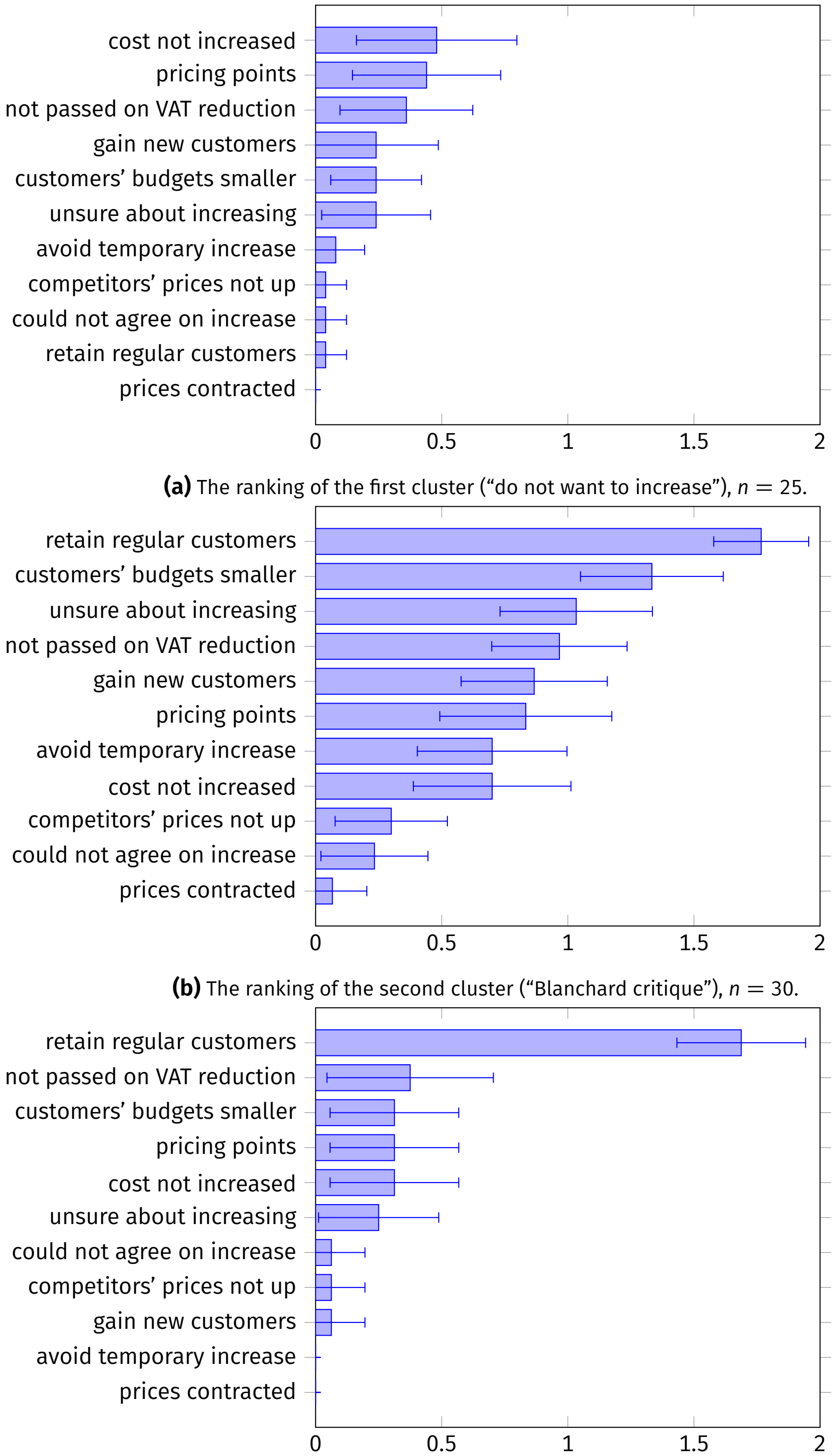


Figure 3.2. The ranking of our survey. The grading scale has four items: "does not apply," "has played no role" (0), "has played some role" (1), and "has played an important role" (2). Following Blinder et al. (1998), if the respondent marked a hypothesis as not applicable and assigned no grade, we assign the lowest grade, 0. In contrast to Blinder et al. (1998), we define the acceptance rate in the bracket as the share of respondents that did not say the hypothesis does not apply, but assigned a score. On the *x*-axis is the average score with error bars showing the 95% confidence interval.

of a single variable. To identify these subgroups, we use a clustering algorithm. We apply Ward's method to yield 3 clusters with as similar scores as possible. As we have the scores from 71 firms, we chose to form 3 clusters to avoid overfitting. The clustering algorithm begins with 71 singleton clusters and then iteratively merges clusters until there are only 3 left. To determine which clusters are merged, the algorithm computes the variance of each hypothesis' score within each cluster and sums up the variances both across hypotheses and clusters. The algorithm merges clusters to minimize this total variance. The ranking within each cluster is illustrated in Figure 3.3.

We conduct a multinomial logit regression of cluster membership on several variables to explain to which cluster a firm belongs (see Table 3.C.1 in Appendix 3.C). The explaining variables are the size of a firm, its price, dummies for different important factors when setting prices, the satisfaction with the own pricing, how the owner evaluates the mandate to hair washing, pessimism, and the quality of the relationship with the customers (the last four variables are explained below). To evaluate the coefficients, we calculate the marginal effects of variables at the means (Table 3.C.2 in Appendix 3.C).

In the first cluster ("do not want to increase"), all hypotheses receive low grades. It might be that these firms did not want to raise their prices at all, rather than suffering from sticky prices. The regression shows that the owners in this cluster are more likely to base their pricing on factors that did not change during the lockdown: the quality of the offered service and the customer satisfaction. Still, the firms in this



(a) The ranking of the first cluster ("do not want to increase"), $n = 25$.

(b) The ranking of the second cluster ("Blanchard critique"), $n = 30$.

(c) The ranking of the third cluster ("only retain regulars"), $n = 16$.

Figure 3.3. The rankings for three clusters grouped using Ward's method.

cluster were affected by the shocks, which is reflected by their owners' being more pessimistic and reporting lower profit margins compared to before the lockdown.

In the second cluster ("Blanchard critique"), the ranking is less clear as more hypotheses have high scores. The ranking is reminiscent of the criticized raking in the original study. The regression shows that owners in this cluster are less likely to adjust their prices to inflation. Maybe, this means that they are not used to increasing their prices and are, thus, somewhat insecure about price increases, which they attribute to different hypotheses.

In the third cluster ("only retain regulars"), the ranking is even clearer than in the total sample: All owners agree that retaining the regular customers is the only important reason for not increasing their prices. The regression shows that these owners are also more likely to say that the cost is important for setting their prices. They face the problem discussed in the next section: Even if the firm only passes on cost increases, which customers usually accept as fair (Kahneman, Knetsch, and Thaler, 1986), it still has to convince the regular customers that the price increase is fair to retain them.

3.4 Customer Relationship and Asymmetric Information

In some markets, firms gain and retain regular customers by (implicitly) promising to only increase their prices if doing so is "fair" (Okun, 1981, p. 153). For example, consumers consider passing on cost increases fair, but exploiting demand shocks unfair (Kahneman, Knetsch, and Thaler, 1986). When confronted with an unfair price increase, regular customers get angry and stop buying from that firm (Okun, 1981, p. 153f).

In practice, consumers only see the prices, but not the shocks. How can they find out whether a price increase is fair, so they remain regulars, or whether it is unfair, so they go back to shopping around?

We theorize that the quality of the regular customers' relationship with their firm is decisive. If the relationship with the firm is bad, the customers are less inclined to believe that a price increase is fair. If the relationship with the firm is good, on the other hand, the customers give the firm the benefit of a doubt and believe that the price increase is fair. Thus, in bad times, when shocks happen that threaten the firm's profits, a good relationship with the regular customers allows the firm to increase the prices to restore its profit margin without losing regular customers, protecting the firm's profits.

We derive stylized facts from our survey's data to support our theory. To do so, we construct four variables measuring underlying constructs.

The first of these variables measures how good the relationship between the hairdressers and their customers is. We construct this variable as the sum of four Likert-item scale answers. Because we are especially interested in how the customers perceive the fairness of a price increase, our measure for the quality of the relationship contains questions about the prices' transparency—about whether the customers understand the prices and the pricing. In the survey, we inverted some statements, so that the answers from inattentive respondents cancel out. Table 3.4 lists the statements and shows with which sign they enter into the sum.

Table 3.4. Construction of a variable measuring the quality of the relationship between the hairdresser and the regular customers. The respondents were asked to express their agreement with the statements on a scale from 1 (totally disagree) to 3 (undecided) to 5 (totally agree).

Sign	Statement
+	The customers express understanding for my/our prices.
−	The customers complain to me about their own financial situation.
−	Some customers accuse me of profiteering.
+	The reasons for price increases are understandable for customers.

The second variable measures how satisfied the owners are with their own pricing. Table 3.5 lists the statements and shows which enter positively and which negatively into the sum.

Table 3.5. Construction of a variable measuring how satisfied the owners are with their own pricing. The respondents were asked to express their agreement with the statements on a scale from 1 (totally disagree) to 3 (undecided) to 5 (totally agree).

Sign	Statement
+	I am satisfied with my pricing method.
+	My prices are optimal for the firm.
−	Actually, my prices should be higher.

The third variable measures to what extent the owners see the hygiene rule mandating a hair wash before any other procedure as a pricing tool. The mandate could be interpreted as a price increase and, thus, deter owners from an additional price increase. Indeed, the respondents slightly agree that mandatory hair washing is like a price increase, but they slightly disagree to profiting from it. Table 3.6 summarizes the construction of the variable.

Table 3.6. Construction of a variable measuring how the owners view the mandatory hair washing. The respondents were asked to express their agreement with the statements on a scale from 1 (totally disagree) to 3 (undecided) to 5 (totally agree).

Sign	Statement
+	The mandatory hair washing is like a price increase.
+	I profit from the mandatory hair washing.

The last variable measures how pessimistic the owners are. Table 3.7 summarizes the construction of the variable.

Table 3.7. Construction of a variable measuring the owners' expectations and professional uncertainty, expressed as pessimism. The respondents were asked to express their agreement with the statements on a scale from 1 (totally disagree) to 3 (undecided) to 5 (totally agree).

Sign	Statement
+	There will be another lockdown this year.
−	We will be back to normal in one year.
+	The hygiene measures will stay for years.
+	Fear will deter customers for a long time.
+	Customers' WTP will lastingly decrease.
−	My personal financial situation will improve.

Our data give rise to five stylized facts that are consistent with our theory. To improve readability, all regression tables are in Appendix 3.C.

Stylized Fact 1. Among the non-increasers, the better a firm's relationship with its regular customers, the less important for price stickiness is the motive of retaining its regular customers.

This stylized fact follows from a logit and an ordered logit regression. In the logit regression, the dependent variable is whether the respondent marked the hypothesis "I did not increase my prices to retain my regular customers" as applicable. In the ordered logit regression, the dependent variable is the grade that the respondent assigns to the hypothesis, where we assign the lowest score when the respondent chose "not applicable." In both regressions, the explaining variables are the size of the firm, the price level, the satisfaction with the own pricing, the mandatory hair washing variable, the pessimism variable, and the quality of the relationship with the regular customers. The stylized fact follows from the coefficient of the relationship with the customers' being negative and significant on the 10%-level in the first regression and the coefficient's being negative, although not significant, in the second regression (see Table 3.C.3 and Table 3.C.4). So, the main reason for price stickiness in the market applies less for firms that have a good relationship with their regular customers.

Stylized Fact 2. Firms that have a better relationship with their customers are more likely to increase their prices.

This stylized fact follows from a logit regression. The dependent variable is a dummy for whether the respondent increased her price. The explaining variables are the size of the firm, the price level, the satisfaction with the own pricing, the mandatory hair washing variable, the pessimism variable, and the quality of the relationship with the regular customers. The stylized fact follows from the coefficient of the relationship with the customers' being positive and significant on the 1%-level (see Table 3.C.5). As they do not fear to lose their regular customers, firms with a better relationship with their customers are more likely to increase their prices.

Stylized Fact 3. Firms that have a good relationship with their regular customers are able to restore their profit margin from before the pandemic.

We asked the owners how their profit margins after the lockdown compare to their profit margins, first, before the pandemic and, second, before the lockdown. The possible answers to both questions are “smaller” (-1), “equal” (0), or “larger” (1). To see that the firms that have a good relationship with their regular customers could on average restore their profit margin, we compare the average answers for the lowest and the highest tertile of the variable customer relationship (Table 3.C.6). The average answer of the firms with a good relationship are slightly larger than 0 and significantly larger than the average answer from firms with a bad relationship. A two-sample *t*-test shows that firms with a better relationship answer rather that they could restore or even increase their profit margin. This result establishes the stylized fact.

Stylized Fact 4. Owners whose firms have a better relationship with their regular customers are more satisfied with their own pricing.

This stylized fact follows from an OLS regression. The dependent variable is the owners’ satisfaction with the own pricing. The explaining variables are the size of the firm, the price level, the mandatory hair washing variable, the pessimism variable, and the quality of the relationship with the regular customers. The stylized fact follows from the coefficient of the relationship with the customers’ being positive and significant on the 1%-level (see Table 3.C.7). Owners seem to feel less constrained in their pricing when their firm has a good relationship with its customers.

Stylized Fact 5. Owners whose firms have a better relationship with their customers are less pessimistic.

This stylized fact follows from an OLS regression. The dependent variable is the owners’ pessimism. The explaining variables are the size of the firm, the price level, the mandatory hair washing variable, the satisfaction with the own pricing variable, and the quality of the relationship with the regular customers. The stylized fact follows from the coefficient of the relationship with the customers’ being negative and significant on the 1%-level (see Table 3.C.8). The owners might be less pessimistic both because they could increase the prices to restore their profit margin and because they know that they could also pass on further shocks.

Note that the alternative theory of relationship marketing does not explain the stylized facts. Firms using relationship marketing foster a good relationship with their regular customers to increase the customers’ valuations for (buying) the firm’s good. While relationship marketing could explain a positive correlation between the quality of the relationship and the firm’s prices, there should be no interaction with price stickiness in the wake of shocks. Furthermore, our variable measuring the customer relationship includes the transparency of the pricing, which is not how relationship marketers would measure the quality of the relationship.

As a good relationship with the customers seems to be valuable to a hairdresser, why do not all hairdressers have a good relationship with their customers? Because our survey was not intended to answer this question, we do not have much data on this. Hence, we spoke with a hairdresser, who is the head of a local guild.

The conversation reflects fair pricing and the problem of asymmetric information. We were told that customers confronted their hairdressers in the weeks after a price increase, asking for the reasons for the price increase. The hairdresser, then, had to justify the price increase. In their explanations, the hairdressers ought to focus on reasons that were evident (e.g. when the news reported increased energy costs) or directly attributable to an individual treatment (e.g. increased wage costs or increased cost of dye). On the other hand, the customers would not accept, for example, that the hygiene rule decreased a hairdresser's capacity or that the competitors had increased their prices. Because the customers would be able to tell, the hairdressers should not lie. Our conclusion is that building a good relationship takes effort and honesty, and it limits the hairdresser's opportunities for price increases.

That building up a good relationship with their customers restricts hairdressers in their pricing is somewhat supported by our data. In some specifications, there is a statistically significant negative relationship between the quality of a hairdresser's relationship with their customers and the initial price level before the lockdown (see Table 3.C.9 for an OLS regression of the price before the lockdown on the tertile of customer relationship). Adding other variables, such as the firm's size, however, shrinks the coefficient slightly, so it is not statistically significant anymore.

3.5 Conclusion

We ask German hairdressers about price stickiness following Blinder et al. (1998). To achieve a clear result, we design our survey to mitigate the criticized tendency to similar scores: We concentrate on a single market with homogeneous firms and manager-owners instead of averaging across and within markets, and we ask about actual reactions to recent shocks instead of presenting hypothetical situations.

We find a clearly winning hypothesis at the top and several theories with similar scores in the middle part of the ranking. Our demonstration that the survey approach can identify the single best theory predicting price stickiness in at least one situation might open different avenues for further research.

Future research could improve the survey approach methodologically. We avoid averaging and ask about real behavior in the hope that this would mitigate the tendency to similar scores. A random control trial could dissect the tendency to similar scores more fundamentally. Varying the survey design over a large sample could show which channels lead to more similar scores and how strong their respective effects are.

The improved survey approach can then be used to determine the main cause of price stickiness in single markets and in sectors. On the macro-level, these findings could be used to build prediction models in which sectors with different causes for price stickiness interact to evaluate monetary policy.

On the micro-level, our survey results suggest that a better relationship between a firm and its regular customers and transparent pricing mitigate price stickiness. A possible channel is that a better relationship enables the firm to convince its regular customers that a price increase is fair, although the reason for the price increase is private information. On the other hand, a better relationship with the customers might be costly as it is associated with fewer opportunities to increase the prices. Future work could test these hypotheses.

Appendix 3.A Other Surveys on Price Stickiness

After Blinder et al. (1998), the Inflation Persistence Network of the European Central Bank has conducted similar surveys in many European countries: Austria (Kwapil, Baumgartner, and Scharler, 2005), Belgium (Aucremanne and Druant, 2005), France (Loupias and Ricart, 2004), Germany (Stahl, 2005), Italy (Fabiani, Gattulli, and Sabbatini, 2004), Luxembourg (Lünnemann and Mathä, 2006), the Netherlands (Hoeberichts and Stokman, 2006), Portugal (Martins, 2005), and Spain (Álvarez and Hernando, 2005). Their results are summarized in the meta study by Fabiani, Druant, et al. (2006). Independent researchers have also conducted similar studies in other countries: the United Kingdom (Hall, Walsh, and Yates, 2000 and Greenslade and Parker, 2012), Japan (Nakagawa, Hattori, and Takagawa, 2000), Canada (Amirault, Kwan, and Wilkinson, 2006), Sweden (Apel, Friberg, and Hallsten, 2005), Norway (Langbraaten, Nordbø, and Wulfsberg, 2008), Romania (Copaciu, Neagu, and Braun-Erdei, 2010), Estonia (Dabušinskas and Randveer, 2006), Turkey (Sahinoz and Saraçoğlu, 2008), Pakistan (Malik, Satti, and Saghir, 2008), Poland (Jankiewicz and Kolodziejczyk, 2008), Iceland (Ólafsson, Pétursdóttir, and Vignisdóttir, 2011), Lithuania (Virbickas, 2011), New Zealand (Parker, 2014), Brazil (Correa, Petrassi, and Santos, 2018), Tanzania (Kimolo, 2018), and Vietnam (Pham, Nguyen, and Nguyen, 2019).

The following tables summarize the results of these studies. The first table presents the hypotheses. Because both the selection of hypotheses and their number differ across the studies, we categorize the hypotheses in 8 categories and color-code the rankings to make them more comparable. We also add short descriptions of the theories underlying the hypotheses. The second table lists the authors of a study, where the results are published, when and where the survey was conducted, how many respondents there are, the scale on which hypotheses are rated, and their ranking. We interpret all hypotheses that are referred to as “kinked-demand curve” as coordination failure, and we exclude the hypotheses for why prices are increased (instead of why prices are sticky) in Loupias and Ricart (2004).

List of the Categories and the Hypotheses

There is no reason to change the prices	
Constant MC	The supply is perfectly elastic in the relevant range.
Factor stability	Nothing changes, so there is no reason to change the prices.
Low inflation	The price level does not change, so there is no reason to change the prices.
Rules (of thumb) how prices are set	
Regular date	Prices are only changed on specific dates.
Regular interval	Prices are only changed after specific (potentially stochastic intervals.
Cost-based pricing	Price = Piece cost + mark-up
Pricing points	Exploit the consumers' leading digit bias by letting prices end in .99.
Markup (keep markup in a specific range)	Change the price only if the (real) mark-up falls out of a pre-specified range.
Explicit contracts	Long-time contracts with customers that fix prices (might be pegged to inflation measures).
Customer goodwill	
Implicit contracts	Invisible handshake: Customers want to be regular customers and stable prices reducing uncertainty.
Customer relations	Don't want to lose the customers' goodwill.
Market environment changes in cycles	
Countercyclical cost of finance	In recessions, financing costs are larger, so prices cannot be reduced.
Liquidity constraints	Firms have to recoup their fixed costs, so they cannot reduce mark-ups too much in recessions.
Procyclical elasticity	The mark-up changes over the cycle because the price elasticity of demand changes (e.g. in recessions only loyal customers buy)
Thick market (demand side)	In booms, the demand is more price elastic because customers are more likely to search for cheaper prices as they buy more.
Thick market (supply side)	In booms, firms can reach customers easier and get more demand by not increasing their prices.
Competition	
Coordination failure (upwards)	The first firm to increase the price gets punished by the customers that leave.
Coordination failure (downwards)	Decreasing prices starts a price war.
Deviation from collusion	Decreasing the price breaks the collusion and leads to punishment from other firms.

Adjustment costs	
(Physical) Menu costs	Changing the price involves costs directly.
Costly information	Gathering information and making decisions is costly.
Hierarchy	Within the firm, consensus has to be reached.
Formal and legal difficulties	Price changes might have to be justified.
Temporary shocks	To save adjustment costs, the price might not be changed if the optimal price is expected to revert soon.
Adjust other things than price	
Nonprice competition	Change other things than the price. E.g. increase delivery lags instead of increasing prices.
Inventories	Keep a stock to satisfy excess demand and build up a stock if demand is low.
Asymmetric information	
Judging quality by price	If the price goes down, people think that the quality went down.

Authors (Source)	Blinder, Canetti, Lebow, Rudd (1998 Monography)	Hall, Walsh, Yates (2000 Oxford Economic Papers)	Nakagawa, Hattori, Takagawa (2000 Bank of Japan Working Paper)
Country (Year of Survey)	United States (1990-1992)	United Kingdom (1995)	Japan (2000)
Sample Size (Responses)	200	654	630
Hypotheses and Ranking	Scale: 1 to 4	Scale: 7 to 1	Scale: 5 to 1
1	Coordination failure (2.77)	Explicit contracts (2.2)	Coordination failure (2.86)
2	Cost-based pricing (2.66)	Cost-based pricing (2.3)	Implicit contracts (2.86)
3	Nonprice competition (2.58)	Coordination failure (2.5)	Explicit contracts (3.10)
4	Implicit contracts (2.40)	Pricing points (2.8)	Pricing points (3.60)
5	Nominal contracts (2.11)	Implicit contracts (2.9)	Nonprice competition (3.61)
6	Costly price adjustment (1.89)	Constant MC (3.1)	Procyclical elasticity (3.99)
7	Procyclical elasticity (1.85)	Inventories (3.1)	Menu costs (4.18)
8	Pricing points (1.76)	Nonprice competition (3.3)	Judging quality by price (4.23)
9	Constant MC (1.57)	Procyclical elasticity (3.3)	Delivery lags/service (4.35)
10	Inventories (1.56)	Judging quality by price (3.6)	
11	Hierarchy (1.41)	Physical menu costs (3.8)	
12	Judging quality by price (1.33)		
13			
14			
15			

Authors (Source)	Amirault, Kwan, Wilkinson (2006 Bank of Canada Working Paper)	Apel, Friberg, Hallsten (2005 Journal of Money, Credit and Banking)	Langbraaten, Nordbø, Wulfsberg (2008 Norges Bank Economic Bulletin)
Country (Year of Survey)	Canada (2002-2003)	2000	2007
Sample Size (Responses)	170	48.7% of 1,285	725
Hypotheses and Ranking	Scale: 0 or 1	Scale: 1 to 4	Scale: 1 to 4 (scores not reported)
1	Cost-based pricing (67.1%)	Implicit contracts (3.00)	Explicit contracts
2	Customer relations (55.3%)	Sluggish costs (cost-based pricing and constant MC) (2.45)	Coordination failure
3	Explicit contracts (45.3%)	Explicit contracts (2.27)	Customer relationship
4	Nonprice competition (44.1%)	Kinked demand curve (coordination failure) (2.17)	Pricing points
5	Coordination failure upwards (41.2%)	Countercyclical cost of finance (2.08)	Costly information Menu costs
6	Low inflation (33.5%)	Liquidity constraints (1.85)	
7	Implicit contracts (31.8%)	Pricing points (1.85)	
8	Coordination failure downwards (31.2%)	Procyclical elasticity (1.75)	
9	Factor stability (= no reason to change prices) (31.2%)	Deviation from collusion (1.68)	
10	Menu costs (21.2%)	Thick market (supply side) (1.60)	
11	Sticky information (13.5%)	Physical menu costs (1.54)	
12		Thick market (demand side) (1.50)	
13		Information costs (1.40)	
14			
15			

Authors (Source)	Inflation Persistence Network Meta-Study (2006 International Journal of Central Banking)	Kwapil, Scharler, Baumgartner (2005 European Central Bank Working Paper)	
Country (Year of Survey)	Eurpean Union (2003-2004)	Austria (2004)	
Sample Size (Responses)	more than 11,000	873	
Hypotheses and Ranking	Scale: 1 to 4 (Unweighted averages of country scores)	Scale: 1 to 4 Price increases only	Scale: 1 to 4 Price decreases only
1	Implicit contracts (2.7)	Implicit contracts (3.04)	Implicit contracts (3.04)
2	Explicit contracts (2.6)	Explicit contracts (3.02)	Explicit contracts (2.94)
3	Cost-based pricing (2.6)	Cost-based pricing (2.72)	Kinked demand curve (coordination failure) (2.69)
4	Coordination failure (2.4)	Kinked demand curve (coordination failure) (2.69)	Cost-based pricing (2.49)
5	Judging quality by price (2.1)	Coordination failure (2.47)	Coordination failure (2.13)
6	Temporary shocks (2.0)	Information costs (1.61)	Nonprice competition (1.98)
7	Nonprice competition (1.7)	Menu costs (1.52)	Judging quality by price (1.88)
8	Menu costs (1.6)	Nonprice competition (1.49)	Temporary shocks (1.470
9	Costly information (1.6)	Temporary shocks (1.42)	Information costs (1.61)
10	Pricing points (1.6)	Pricing points (1.32)	Menu costs (1.52)
11		Judging quality by price (not applicable)	Pricing points (1.24)
12			
13			
14			
15			

Authors (Source)	Aucremanne, Druant (2005 European Central Bank Working Paper)	Loupias, Ricart (2004 European Central Bank Working Paper)	
Country (Year of Survey)	Belgium (2004)	France (2003-2004)	
Sample Size (Responses)	1,979	1,662	
Hypotheses and Ranking	Scale: 1 to 4	Scale: 1 to 4 Price increases only	Scale: 1 to 4 Price decreases only
1	Implicit contracts (2.5)	Cost-based pricing (3.0 commodity prices) (2.5 labor cost) (1.8 productivity)	coordination failure (2.8 match others' prices) (2.1 others don't change)
2	Explicit contracts (2.4)	coordination failure (3.0 others don't change) (2.3 match others' prices)	Cost-based pricing (2.6 commodity prices) (1.9 labor cost) (2.2 productivity)
3	Sluggish costs (cost-based pricing and constant MC) (2.4)	Nominal contracts (2.7)	Nominal contracts (2.5)
4	Liquidity constraints (2.2)	Implicit contracts (2.2)	Negative demand shock (2.3)
5	Kinked demand curve (coordination failure) (2.2)	Temporary shocks (2.1)	Temporary shocks (2.1)
6	Procyclical elasticity (2.1)	Positive demand shock (2.0)	Implicit contracts (2.0)
7	Thick market (demand side) (2.0)	Fewer competitors (1.8)	More competitors (2.0)
8	Judging quality by price (1.9)	Pricing points (1.7)	Pricing points (1.6)
9	Thick market (supply side) (1.8)	Inventory + delay (1.4)	Inventory + delay (1.6)
10	Temporary shock (1.8)	Physical menu costs (1.4)	Physical menu costs (1.4)
11	Nonprice competition (1.7)		
12	Countercyclical cost of finance (1.7)		
13	Pricing points (1.7)		
14	Information costs + bureaucracy (1.6)		
15	Menu costs (1.5)		

Authors (Source)	Stahl (2005 European Central Bank Working Paper)		Fabiani, Gattulli, Sabbatini (2004 European Central Bank Working Paper)
Country (Year of Survey)	Germany (2004)		Italy (2003)
Sample Size (Responses)	1,200		333
Hypotheses and Ranking	Scale: 1 to 4 Price increases only	Scale: 1 to 4 Price decreases only	Scale: 1 to 4
1	Coordination failure (2.6)	Nominal contracts (2.4)	Nominal contracts (2.64)
2	Nominal contracts (2.4)	Coordination failure (2.2)	Coordination failure (2.59)
3	High elasticity for increases (kinked demand?) (2.1)	Low elasticity for decreases (kinked demand?) (2.1)	Temporary shocks (1.97)
4	Regular date (2.0)	Temporary shock (2.0)	Menu costs (1.58)
5	Regular interval (1.9)	Regular date (2.0)	Pricing points (1.43)
6	Temporary shock (1.8)	Regular interval (1.9)	Bureaucratic costs 1.30
7	Sluggish costs (1.8)	Sluggish costs (1.8)	
8	Menu costs (1.4)	Menu costs (1.4)	
9	„Other“ (1.1)	„Other“ (1.1)	
10			
11			
12			
13			
14			
15			

Authors (Source)	Lünnemann, Mathä (2006 European Central Bank Working Paper)	Hoeberichts, Stokman (2006 European Central Bank Working Paper)	Martins (2005 European Central Bank Working Paper)
Country (Year of Survey)	Luxemburg (2004)	Netherlands (2004)	Portugal (2004)
Sample Size (Responses)	367	1,246	1,370
Hypotheses and Ranking	Scale: 1 to 4 (scores not reported)	Scale: 1 to 4	Scale: 1 to 4
1	Implicit contracts	Implicit contracts (2.66)	Implicit contracts (3.14)
2	Constant MC	Explicit contracts (2.57)	Coordination failure (2.84)
3	Explicit contracts	Judging quality by price (2.34)	High fixed costs (=liquidity constraints) (2.80)
4	Procyclical elasticity	Temporary shocks (2.34)	Constant MC (2.70)
5	Thick markets (demand)	Coordination failure (2.22)	Explicit contracts (2.63)
6	Liquidity constraints	Nonprice competition (2.07)	Procyclical elasticity (2.61)
7	Judging quality by price	Pricing points (1.80)	Temporary shock (2.46)
8	Thick markets (supply)	Menu costs (1.71)	Bureaucratic delays (2.45)
9	Coordination failure		Judging quality by price (2.28)
10	Pricing points		Menu costs (1.89)
11	Temporary shock		Pricing points (1.78)
12	Countercyclical cost of finance		Costly information (1.70)
13	Menu cost		
14	Nonprice competition		
15	Costly information		

Authors (Source)	Alvarez, Hernando (2005 European Central Bank Working Paper)		Copaciu, Neagu, Braun-Erdei (2010 Managerial and Decision Economics)
Country (Year of Survey)	Spain (2004)		Romania (2006)
Sample Size (Responses)	2,008		377
Hypotheses and Ranking	Scale: 1 to 4 Price increases only	Scale: 1 to 4 Price decreases only	Scale: 1 to 4
1	Implicit contracts (2.56)	Coordination failure (2.21)	Implicit contracts (3.12)
2	Coordination failure (2.42)	Explicit contracts (2.09)	Explicit contracts (3.10)
3	Explicit contracts (2.25)	Temporary shocks (1.82)	Judging quality by price (2.19)
4	Temporary shocks (1.82)	Judging quality by price (1.82)	Price readjustments (2.15)
5	Pricing points (1.49)	Pricing points (1.42)	Coordination failure (1.97)
6	Menu costs (1.43)	Menu costs (1.39)	Costly information (1.74)
7	Nonprice competition (1.34)	Nonprice competition (1.34)	Menu costs (1.62)
8	Costly information (1.33)	Costly information (1.30)	
9	Judging quality by price (not applicable)	Implicit contracts (not asked)	
10			
11			
12			
13			
14			
15			

Authors (Source)	Ólafsson, Pétursdóttir, Vignisdóttir (2011 Central Bank of Iceland Working Paper)	Dabušinskas, Randveer (2006 Bank of Estonia Working Paper)	
Country (Year of Survey)	Iceland (2008)	Estonia (2005)	
Sample Size (Responses)	580	208	
Hypotheses and Ranking	(100 * #most important + 50 *#second most important) / #most important	Scale: 1 to 4 Price increases only (scores not reported)	Scale: 1 to 4 Price decreases only (scores not reported)
1	Implicit contracts (34.1)	Implicit contracts	Cost-based pricing
2	Explicit contracts (31.0)	Explicit contracts	Implicit contracts
3	Temporary shocks (28.8)	Cost-based pricing	Judging quality by price
4	Coordination failure (26.1)	Coordination failure	Coordination failure
5	Pricing points (15.0)	Pricing points	Explicit contracts
6	Menu costs	Nonprice competition	Nonprice competition
7	Nonprice competition	Costly information	Pricing points
8	Judging quality by price	Menu costs	Menu costs
9			Costly information
10			
11			
12			
13			
14			
15			

Authors (Source)	Sahinoz, Saraçoğlu (2008 Developing Economies)	Greenslade, Parker (2012 Economic Journal)	
Country (Year of Survey)	Turkey (2005)	United Kingdom (unclear)	
Sample Size (Responses)	999	693	
Hypotheses and Ranking	Scale: 0 to 3 and *100	Scale: 0 or 1 Price increases only	Scale: 0 or 1 Price decreases only
1	Markup (keep markup in a specific range) (44.8)	Coordination failure (60%)	Coordination failure (35%)
2	Temporary shocks (40.6)	It would anger customers (56%)	Explicit contracts (35%)
3	Explicit contracts (37.1)	Explicit contracts (47%)	Implicit contracts (29%)
4	Implicit contracts (36.9)	Implicit contracts (38%)	Temporary shocks (28%)
5	Coordination failure (30.8)	Temporary shocks (32%)	Constant MC (26%)
6	Constant MC (22.6)	Constant MC (31%)	It would anger customers (25%)
7		Pricing points (24%)	Pricing points (15%)
8		Menu costs (10%)	Menu costs (9%)
9			
10			
11			
12			
13			
14			
15			

Authors (Source)	Malik, Satti, Saghir (2008 Pakistan Development Review)	Jankiewicz, Kolodziejczyk (2008 Bank i Kredyt)	
Country (Year of Survey)	Pakistan (2008)	Poland (2006)	
Sample Size (Responses)	343	752	
Hypotheses and Ranking	Scale: 1 to 4	Frequency of being among top two answers Price increases only	Frequency of being among top two answers Price decreases only
1	Implicit contracts (framed as „customers prefer stable prices") (2.66)	Coordination failure (53.4%)	Temporary shocks (33.5%)
2	Explicit contracts (2.41)	Explicit contracts (40.5%)	Explicit contracts (30.8%)
3	Coordination failure (2.35)	Temporary shocks (22.0%)	None (29.0%)
4	Temporary shocks (1.84)	None (17.1%)	Other (22.1%)
5	Judging quality by price (1.84)	Other (15.7%)	Judging quality by price (19.1%)
6	Costly information (1.62)	Formal and legal difficulties (7.3%)	Pricing points (5.3%)
7	Menu costs (1.59)	Pricing points (misreported) 3.8??	Formal and legal difficulties (3.2%)
8		Menu costs (1.4%)	Menu costs (2.0%)
9			
10			
11			
12			
13			
14			
15			

Authors (Source)	Virbickas (2011 Bank of Lithuania Working Paper)		Parker (2014 Reserve Bank of New Zealand Discussion Paper)
Country (Year of Survey)	Lithuania (2008)		New Zealand (2010)
Sample Size (Responses)	343		5,369
Hypotheses and Ranking	Scale: 1 to 4 Frequency of 3 and 4 Price increases only	Scale: 1 to 4 Frequency of 3 and 4 Price decreases only	Scale: 0 or 1 (scores not reported)
1	Cost-based pricing (74.2%)	Cost-based pricing (61.7%)	Explicit contracts
2	Explicit contracts (63.2%)	Explicit contracts (51.1%)	Implicit contracts
3	Implicit contracts (50.9%)	Temporary shocks (50.9%)	Coordination failure
4	Coordination failure (41.1%)	Judging quality by price (48.1%)	Temporary shocks
5	Costly information (40.5%)	Coordination failure (37.8%)	Pricing points
6	Temporary shocks (33.4%)	Costly information (30.2%)	Nonprice competition
7	Pricing points (21.5%)	Nonprice competition (27.4%)	Menu costs
8	Nonprice competition (18.3%)	Pricing points (19.6%)	
9	Menu costs (17.0%)	Menu costs (16.4%)	
10			
11			
12			
13			
14			
15			

Authors (Source)	Kimolo (2018 Journal of Economics and Sustainable Development)	Correa, Petrassi, Santos (2018 Journal of Business Cycle Research)	Pham, Nguyen, Nguyen (2019 working paper)
Country (Year of Survey)	Tanzania (2014)	Brazil (2011-2012)	Vietnam (2014)
Sample Size (Responses)	79	7,002	1,560
Hypotheses and Ranking	Scale: 5 to 1	unclear (authors' ranking)	not reported
1	Implicit contracts (2.00)	Menu cost and costly information (46.7%)	
2	Explicit contracts (2.70)	Cost-based pricing (79.4%)	
3	Pricing points (2.94)	Explicit contracts (20%)	
4	Judging quality by price (3.14)	Implicit contracts and not antagonizing customers (79%)	
5	Coordination failure (3.28)	Coordination failure (67%)	
6	Nonprice competition (3.33)	Non-price competition (75% - 54%)	
7	Menu Costs (3.53)	Judging quality by price (38.3%)	
8	Temporary shocks (3.68)		
9			
10			
11			
12			
13			
14			
15			

Appendix 3.B Questionnaire

The following is the translation of our survey into English. “()” indicates single choice and “[]” indicates multiple choice. The German original is below.

English Translation of Our Questionnaire

Page 1
Dear Sir or Madam,

on March 1, you were finally allowed to open up again. For our dissertations in economics at the University of Bonn, we investigate how the pandemic and the lockdown in Germany affect the hairdressers and the prices for haircuts.

We kindly ask you to take 10 to 15 minutes to fill out our survey. If you have less time at your proposal, we would also be happy for partially filled out forms (all answers are optional). You can also save your progress and continue the survey later; to do so, please click on “save progress” on the bottom of the page.

The survey is anonymous. We do not ask for or save any personal data. Your answers will be treated confidentially and only used for scientific purposes.

Thank you very much for your support!

Thomas Kohler and Maximilian Weiß

Page 2

First, we would like to get to know you and your firm better.

1. What is your role in the firm?
   () I am the owner.
   () I am a franchisee.
   () I am an employed manager.
   () I am an employee.
   () Other: [free text field]
   () not applicable

2. Are you involved in the pricing in your firm?
   () Yes, I set the prices.
   () Yes, I suggest prices to my superior.
   () Yes, I set the prices in accordance with my franchising contract.
   () Yes, my associates and I set the prices together.
   () No
   () Other: [free text field]

3. How many branches does your firm have? (In case of franchises, please for the franchisee)
   () no branch (mobile hairdresser)
   () one branch
   () two branches
   () three to five branches
   () more than five branches
   () can't or won't say

4. How many employees does your firm have? (In case of franchises, please for the franchisee)
   () none
   () one to three
   () three to six
   () more than six
   () can't or won't say
   Comment: [free text field]

5. Which share of your customers are regulars?
   () 0 % to 19 %
   () 20 % to 39 %
   () 40 % to 59 %
   () 60 % to 79 %
   () 80 % to 100 %
   () can't or won't say

Page 3

On this page, we'll ask you some questions about the price of a man's haircut in your firm. If you do not offer this haircut, please indicate so (You will then receive questions about the price of a woman's haircut).

6. What is the price of the following man's haircut in your firm?
short back and sides, wash, cut, blow dry, 25 minutes

Please fill in the price including a possible hygiene surcharge.
Please fill in the base price if you charge other surcharges (e.g. for Mondays, late appointments, new customers, or other).

Before this lockdown (until December 16, 2020): [free text field] Euros
() can't or won't say

First week of March 2021: [free text field] Euros
() can't or won't say

[Planned] April 2021: [free text field] Euros
() can't or won't say

() I don't offer this kind of haircut (in this case, please indicate "can't or won't say" everywhere in this question, ignore the rest of the page, and click on "Continue").

7. Had you lowered your prices because of the VAT reduction in the second half-year of 2020?
() yes
() no
() can't or won't say

8. Pricing parts (beginning of March 2021)
If the price you filled in (for the beginning of March 2021) contains a hygiene surcharge, please indicate what it is. If you charge different hygiene surcharges for different services, please indicate the hygiene surcharge for the haircut described above.
If new customers pay more than regular customers, please indicate the price difference.
If you charge a surcharge for late appointments, Monday appointments or weekend appointments, please indicate the surcharge.

hygiene surcharge: [free text field] Euros
new customer surcharge: [free text field] Euros
surcharge for late appointments: [free text field] Euros
surcharge for Monday appointments: [free text field] Euros
surcharge for weekend appointments: [free text field] Euros
() can't or won't say

9. Do you make more or less profit per customer with the haircut described above compared to before the pandemic (February 2020)?
() today less
() same
() today more
() can't or won't say

10. Do you make more or less profit per customer with the haircut described above compared to before the last lockdown (December 2020)?
() today less
() same
() today more
() can't or won't say

Page 4 [only if indicated that the reference man's haircut is not offered]

On this page, we'll ask you some questions about the price of a woman's haircut in your firm.

11. What is the price of the following woman's haircut in your firm?
Length is to the shoulders; wash, cut, brush, blow dry. Total time around 45 minutes. No dying or highlights or similar.

Please fill in the price including a possible hygiene surcharge.
Please fill in the base price if you charge other surcharges (e.g. for Mondays, late appointments, new customers, or other).

Before this lockdown (until December 16, 2020): [free text field] Euros
() can't or won't say

First week of March 2021: [free text field] Euros
() can't or won't say

[Planned] April 2021: [free text field] Euros
() can’t or won’t say

12. Had you lowered your prices because of the VAT reduction in the second half-year of 2020?
() yes
() no
() can’t or won’t say

13. Pricing parts (beginning of March 2021)
If the price you filled in (for the beginning of March 2021) contains a hygiene surcharge, please indicate what it is.
If you charge different hygiene surcharges for different services, please indicate the hygiene surcharge for the haircut described above.
If new customers pay more than regular customers, please indicate the price difference.
If you charge a surcharge for late appointments, Monday appointments or weekend appointments, please indicate the surcharge.

hygiene surcharge: [free text field] Euros
new customer surcharge: [free text field] Euros
surcharge for late appointments: [free text field] Euros
surcharge for Monday appointments: [free text field] Euros
surcharge for weekend appointments: [free text field] Euros
() can’t or won’t say

14. Do you make more or less profit per customer with the haircut described above compared to before the pandemic (February 2020)?
() today less
() same
() today more
() can’t or won’t say

15. Do you make more or less profit per customer with the haircut described above compared to before the last lockdown (December 2020)?
() today less
() same
() today more
() can’t or won’t say

Page 5 [only if the indicated price for March strictly larger than the price for December]

16. Why have you increased your prices since December?
You have indicated that at least one of your prices was larger in March 2021 than in December 2020. Which role did the following factors play in your increasing the prices?
Reduced capacity due to distancing rules
() no role
() a small role
() a big role
() does not apply
() can't or won't say

Recoup lost revenue / reduced reserves due to lockdown
() no role
() a small role
() a big role
() does not apply
() can't or won't say

Increased demand
() no role
() a small role
() a big role
() does not apply
() can't or won't say

Increased financing cost (for example because of new loans)
() no role
() a small role
() a big role
() does not apply
() can't or won't say

Adjustment to the general price level
() no role
() a small role
() a big role
() does not apply
() can't or won't say

Increased wage cost

() no role
() a small role
() a big role
() does not apply
() can't or won't say

The price increase is only temporary

() no role
() a small role
() a big role
() does not apply
() can't or won't say

Increased incidental cost

() no role
() a small role
() a big role
() does not apply
() can't or won't say

Increased hygiene cost (masks, disinfection, time)

() no role
() a small role
() a big role
() does not apply
() can't or won't say

Expectation that the customers understand the price increases

() no role
() a small role
() a big role
() does not apply
() can't or won't say

Competitors have increased their prices

() no role
() a small role
() a big role
() does not apply
() can't or won't say

End of the VAT reduction
() no role
() a small role
() a big role
() does not apply
() can’t or won’t say

Other important factors:
[free text field]
[free text field]
[free text field]

17. To what extent do you agree with these statements about your experiences with your customers?

The customers express understanding for my/our prices.
() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can’t or won’t say

The customers complain to me about their own financial situation.
() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can’t or won’t say

Some customers accuse me of profiteering.
() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can’t or won’t say

The customers tip more.

() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can’t or won’t say

The customers tip less.

() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can’t or won’t say

Page 6 [only if the indicated price for March is not larger than the price for December]

18. Why have you not increased your prices since last December?

You have indicated that at least one of your prices is not larger in March 2021 than in December 2020.
Which role did the following factors play in your decision to not increase the price?
The prices are contracted [in the ranking table: prices contracted]

() no role
() a small role
() a big role
() does not apply
() can’t or won’t say

Within the firm, we could not agree on a price increase [in the ranking table: could not agree on increase]

() no role
() a small role
() a big role
() does not apply
() can’t or won’t say

I am not sure whether increased prices would be better for the firm [in the ranking table: unsure about increasing]

() no role
() a small role
() a big role
() does not apply
() can't or won't say

A price increase would seem larger than it actually is [in the ranking table: pricing points]

() no role
() a small role
() a big role
() does not apply
() can't or won't say

Increase the market share / gain new customers [in the ranking table: gain new customers]

() no role
() a small role
() a big role
() does not apply
() can't or won't say

The prices were already increased after the first lockdown (spring 2020) [not in the ranking table]

() no role
() a small role
() a big role
() does not apply
() can't or won't say

The customers' budgets are smaller during the pandemic [in the ranking table: customers' budgets smaller]

() no role
() a small role
() a big role
() does not apply
() can't or won't say

VAT reduction was not passed on in the second half-year of 2020 [in the ranking table: not passed on VAT reduction]
() no role
() a small role
() a big role
() does not apply
() can't or won't say

The competitors have not increased their prices [in the ranking table: competitors' prices not up]
() no role
() a small role
() a big role
() does not apply
() can't or won't say

The prices were not increased, so they don't have to be decreased again soon [in the ranking table: avoid temporary increase]
() no role
() a small role
() a big role
() does not apply
() can't or won't say

The costs have not increased [in the ranking table: cost not increased]
() no role
() a small role
() a big role
() does not apply
() can't or won't say

Retaining regular customers [in the ranking table: retain regular customers]
() no role
() a small role
() a big role
() does not apply
() can't or won't say

Other important factors:
[free text field]
[free text field]
[free text field]

19. To what extent do you agree with these statements about your experiences with your customers?

The customers express understanding for my/our prices.
() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

The customers complain to me about their own financial situation.
() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

Some customers accuse me of profiteering.
() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

The customers tip more.
() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

The customers tip less.

() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

Page 7

On this page we ask you questions about how your company is dealing with the political measures and how you assess future developments.

20. If you received more requests for appointments for the beginning of March than you could satisfy: How did you deal with it?

Multiple answers are possible.

[ ] preferential treatment of new customers
[ ] hire more employees to offer more appointments
[ ] preferential treatment of customers that had appointments canceled in the past months
[ ] preferential treatment of regular customers
[ ] first come, first served
[ ] extend the opening hours to offer more appointments
[ ] charge a surcharge for new customers
() does not apply
() can't or won't say

21. To what extent do you agree with these statements about the mandate to wash the customers' hair?

I feel safer when I wash the customers' hair before the treatment.

() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

The mandatory hair washing is like a price increase.

() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

The customers find the mandatory hair washing acceptable.

() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

I profit from the mandatory hair washing.

() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

22. How accurate do you think the following predictions are?

We will be back to normal in one year.

() not at all
() rather not
() unclear
() rather
() very
() can't or won't say

The hygiene measures will stay for years.

() not at all
() rather not
() unclear
() rather
() very
() can't or won't say

Fear will deter customers for a long time.
() not at all
() rather not
() unclear
() rather
() very
() can’t or won’t say

My personal financial situation will improve (compared to today).
() not at all
() rather not
() unclear
() rather
() very
() can’t or won’t say

Due to (fighting) the pandemic, the customers’ willingness to pay will lastingly decrease.
() not at all
() rather not
() unclear
() rather
() very
() can’t or won’t say

There will be another lockdown this year.
() not at all
() rather not
() unclear
() rather
() very
() can’t or won’t say

23. How unsure are you about your own professional future?
() not at all
() barely
() somewhat
() a lot
() can’t or won’t say

Page 8
On this page, we ask general questions about pricing in your firm.

24. In general, what do you pay most attention to when setting prices?
Multiple answers are possible.

[ ] Costs
[ ] The competitors' prices
[ ] The quality of my offer
[ ] Customer satisfaction
[ ] Adjustment to the general price level
[ ] Something else: [free text field]
() can't or won't say

25. To what extent do you agree with these statements about your pricing?

I am satisfied with my pricing method.
() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

My prices are optimal for the firm.
() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

Actually, my prices should be higher.
() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

The reasons for price increases are understandable for customers.

() totally disagree
() somewhat disagree
() undecided
() somewhat agree
() totally agree
() can't or won't say

Page 9

Thank you very much for participating in our study!

26. If you want to tell us anything, you can do so anonymously here (note: this answer will be saved together with the other answers, but without any personal information).
If you have a question that you would like an answer to, please feel free to email us.

[free text field]

Last page

Thank you again for participating!
Your answers have been saved, you may close the browser window now.

soSci
oFb - der onlineFragebogen

oeffnung_der_friseure → base 19.04.2021, 11:38

Seite 01

Startseite

Sehr geehrte Damen und Herren,

am 01. März durften Sie endlich wieder öffnen. Im Rahmen unserer Doktorarbeiten in VWL an der Universität Bonn untersuchen wir, wie sich die Pandemie und der Lockdown in Deutschland auf die Friseur/innen und die Preise für Haarschnitte auswirken.

Wir bitten Sie, sich 10 bis 15 Minuten Zeit zu nehmen, um unseren Fragebogen auszufüllen. Sollten Sie weniger Zeit zur Verfügung haben, würden wir uns auch über teilweise ausgefüllte Bögen freuen (alle Antworten sind optional). Sie können auch Ihren zwischenzeitlichen Fortschritt abspeichern und die Befragung zu einem späteren Zeitpunkt an der Stelle fortsetzen; dazu klicken Sie bitte auf "Fortschritt speichern" am unteren Rand der Seite.

Die Befragung ist anonym. Es werden keinerlei personenbezogene Daten erhoben oder gespeichert. Ihre Angaben werden vertraulich behandelt und nur für wissenschaftliche Zwecke verwendet.

Herzlichen Dank für Ihre Unterstützung!
Thomas Kohler und Maximilian Weiß

PHP-Code

```
$pageNr = 1;
replace('%ownPageNumber%',$pageNr);
option('progress',round(100*$pageNr/7));
option('progress.last','KO');
```

Seite 02

PHP-Code

```
$pageNr = 2;
replace('%ownPageNumber%',$pageNr);
option('progress',round(100*$pageNr/7));
```

Teil 1 Allgemein

Zunächst möchten wir etwas über Sie und Ihr Unternehmen erfahren.

AI03

1. Was ist Ihre Rolle in Ihrem Unternehmen?

- ○ Ich bin der/die Besitzer/in
- ○ Ich bin Franchise- oder Lizenznehmer/in
- ○ Ich bin angestelle/r Betriebsleiter/in
- ○ Ich bin Angestellte/r
- ○ Anderes: ________

- ○ Nicht zutreffend

AI02

2. Sind Sie an der Preissetzung in Ihrem Unternehmen beteiligt?

- ○ Ja, ich bestimme die Preise selbst
- ○ Ja, ich schlage meiner/m Vorgesetzten Preise vor
- ○ Ja, ich wähle die Preise im Rahmen meines Franchise-Vertrags
- ○ Ja, mein/e Geschäftspartner/in und ich wählen die Preise gemeinsam
- ○ Nein
- ○ Anderes: ________

AI04

3. Wie viele Filialen hat Ihr Unternehmen? (Bei Franchises bitte für das Franchise-nehmende Unternehmen)

- ○ keine Filiale (mobiler Friseur)
- ○ eine Filiale
- ○ zwei Filialen
- ○ drei bis fünf Filialen
- ○ mehr als fünf Filialen

- ○ Kann / Möchte ich nicht sagen

AI05

4. Wie viele Angestellte hat Ihr Unternehmen? (Bei Franchises bitte für das Franchise-nehmende Unternehmen)

- ○ keine
- ○ eine/n bis drei
- ○ drei bis sechs
- ○ mehr als sechs

- ○ Kann / Möchte ich nicht sagen

AI08

Anmerkung:

AI01

5. Welcher Anteil Ihrer Kunden sind Stammkunden?

- ○ 0 % bis 19 %
- ○ 20 % bis 39 %
- ○ 40 % bis 59 %
- ○ 60 % bis 79 %
- ○ 80 % bis 100 %

- ○ Kann / Möchte ich nicht sagen

Seite 03

PHP-Code

```
$pageNr = 3;
replace('%ownPageNumber%',$pageNr);
option('progress',round(100*$pageNr/7));
```

Teil 2 Preise Haarschnitt 1

Auf dieser Seite stellen wir Ihnen einige Fragen zum Preis eines Herren-Haarschnitts in Ihrem Unternehmen. Falls Sie diesen Haarschnitt nicht anbieten, markieren Sie dies bitte (Sie erhalten dann Fragen zum Preis eines Damen-Haarschnitts).

PL01

6. Wie viel kostet der folgende Herren-Haarschnitt in Ihrem Unternehmen?

Klassischer Fassonschnitt. Waschen, Schneiden, Föhnen. Gesamtdauer etwa 25 Minuten.

Bitte geben Sie den Preis inklusive einer eventuellen Hygienepauschale an.

Bitte geben Sie den Grundpreis an, falls Sie andere Zuschläge (z.B. montags, späte Termine, für Neukunden oder ähnliches) erheben.

Vor diesem Lockdown (bis zum 16. Dezember 2020)		Euro	☐ Kann / Möchte ich nicht sagen
Erste Märzwoche 2021		Euro	☐ Kann / Möchte ich nicht sagen
April 2021		Euro	☐ Kann / Möchte ich nicht sagen

PL14

☐ Ich biete diese Art Haarschnitt nicht an (Bitte kreuzen Sie in diesem Fall bei dieser Frage überall „Kann ich nicht sagen“ an und ignorieren Sie bitte den Rest dieser Seite und klicken auf „Weiter“.)

PL16

7. Hatten Sie aufgrund der Mehrwertsteuersenkung im zweiten Halbjahr 2020 Ihre Preise gesenkt?

○	○	○
ja	nein	Kann / Möchte ich nicht sagen

PL05

8. Preisbestandteile (Anfang März 2021)

Falls der angegebene Preis (Anfang März 2021) eine Hygienepauschale beinhaltet, geben Sie bitte an, wie hoch diese ist. Falls Sie eine unterschiedlich hohe Hygienezuschläge für unterschiedliche Dienstleistungen erheben, geben Sie bitte den Hygienezuschlag für den oben beschriebenen Haarschnitt an.

Falls Neukunden mehr zahlen als Stammkunden, geben Sie bitte den Preisunterschied an.

Falls Sie einen Zuschlag für späte Termine, für Termine am Montag oder für Termine am Wochenende erheben, geben Sie bitte die Höhe des Zuschlags an.

☐ Hygienepauschale: ______ Euro

☐ Neukunden-Zuschlag: ______ Euro

☐ Zuschlag für späte Termine: ______ Euro

☐ Zuschlag für Termine am Montag: ______ Euro

☐ Zuschlag für Termine am Wochenende: ______ Euro

☐ Kann / Möchte ich nicht sagen

9. Machen Sie mit dem oben beschriebenen Haarschnitt pro Kunde heute mehr oder weniger Gewinn als vor der Pandemie (Februar 2020)?

heute weniger	gleich viel	heute mehr	Kann / Möchte ich nicht sagen
○	○	○	○

PL10

10. Machen Sie mit dem oben beschriebenen Haarschnitt pro Kunde heute mehr oder weniger Gewinn als vor dem letzten Lockdown (Dezember 2020)?

heute weniger	gleich viel	heute mehr	Kann / Möchte ich nicht sagen
○	○	○	○

Seite 04

PHP-Code

```
if (value('PL14_01')==1){
goToPage('PH');
}
$pageNr = 3;
replace('%ownPageNumber%',$pageNr);
option('progress',round(100*$pageNr/7));
```

Teil 2 Preise Haarschnitt 2

Auf dieser Seite stellen wir Ihnen einige Fragen zum Preis eines Damen-Haarschnitts in Ihrem Unternehmen.

PL02

11. Wie viel kostet der folgende Damen-Haarschnitt in Ihrem Unternehmen?

Haarlänge: etwa schulterlang

Waschen, Schneiden, Kämmen, Föhnen. Gesamtdauer etwa 45 Minuten.

Keine Farbe, Strähnchen oder ähnliches.

Bitte geben Sie den Preis inklusive einer eventuellen Hygienepauschale an.

Bitte geben Sie den Grundpreis an, falls Sie andere Zuschläge (z.B. montags, späte Termine, für Neukunden oder ähnliches) erheben.

Vor diesem Lockdown (bis zum 16. Dezember 2020)		Euro	☐ Kann / Möchte ich nicht sagen
Erste Märzwoche 2021		Euro	☐ Kann / Möchte ich nicht sagen
April 2021		Euro	☐ Kann / Möchte ich nicht sagen

PL17

12. Hatten Sie aufgrund der Mehrwertsteuersenkung im zweiten Halbjahr 2020 Ihre Preise gesenkt?

○ ja ○ nein ○ Kann / Möchte ich nicht sagen

PL13

13. Preisbestandteile (Anfang März 2021)

Falls der angegebene Preis (Anfang März 2021) eine Hygienepauschale beinhaltet, geben Sie bitte an, wie hoch diese ist. Falls Sie eine unterschiedlich hohe Hygienezuschläge für unterschiedliche Dienstleistungen erheben, geben Sie bitte den Hygienezuschlag für für den oben beschriebenen Haarschnitt an.

Falls Neukunden mehr zahlen als Stammkunden, geben Sie bitte den Preisunterschied an.

Falls Sie einen Zuschlag für späte Termine, für Termine am Montag oder für Termine am Wochenende erheben, geben Sie bitte die Höhe des Zuschlags an.

☐ Hygienepauschale: Euro

☐ Neukunden-Zuschlag: Euro

☐ Zuschlag für späte Termine: Euro

☐ Zuschlag für Termine am Montag: Euro

☐ Zuschlag für Termine am Wochenende: Euro

☐ Kann / Möchte ich nicht sagen

14. Machen Sie mit dem oben beschriebenen Haarschnitt pro Kunde heute mehr oder weniger Gewinn als vor der Pandemie (Februar 2020)?

heute weniger	gleich viel	heute mehr	Kann / Möchte ich nicht sagen
○	○	○	○

PL12

15. Machen Sie mit dem oben beschriebenen Haarschnitt pro Kunde heute mehr oder weniger Gewinn als vor dem letzten Lockdown (Dezember 2020)?

heute weniger	gleich viel	heute mehr	Kann / Möchte ich nicht sagen
○	○	○	○

Seite 05
PH

PHP-Code

```
if (
(
(value('PL14_01') == 1) and (value('PL01_02') <= value('PL01_01'))
)
or
(
(value('PL14_01') == 2) and (value('PL02_02') <= value('PL02_01'))
)
){
goToPage('PG');
}
$pageNr = 4;
replace('%ownPageNumber%',$pageNr);
option('progress',round(100*$pageNr/7));
```

PL03

16. Weshalb haben sich Ihre Preise seit letztem Dezember erhöht?

Sie haben angegeben, dass mindestens einer Ihrer Preise im März 2021 höher ist als er im Dezember 2020 war. Welche Rolle haben die folgenden Faktoren bei der Preiserhöhung gespielt?

	Keine Rolle	Eine kleine Rolle	Eine große Rolle	Trifft nicht zu	Kann / Möchte ich nicht sagen
verringerte Kapazität durch Abstandsregelungen	○	○	○	○	○
Ausgleich des entgangenen Umsatzes / des Rücklagenabbaus durch den Lockdown	○	○	○	○	○
höhere Nachfrage	○	○	○	○	○
gestiegene Finanzierungskosten (zum Beispiel wegen Kreditaufnahme)	○	○	○	○	○
Anpassung an das allgemeine Preisniveau	○	○	○	○	○
gestiegene Lohnkosten	○	○	○	○	○
Die Preiserhöhung ist nur kurzfristig.	○	○	○	○	○
gestiegene Nebenkosten	○	○	○	○	○
gestiegener Hygieneaufwand (Masken, Desinfektionsmittel und Zeit)	○	○	○	○	○
Erwartung, dass Kunden für Preiserhöhung Verständnis haben	○	○	○	○	○
gestiegene Preise der Konkurrenz	○	○	○	○	○
Ende der Mehrwertsteuersenkung	○	○	○	○	○

PL07

Sonstige wichtige Faktoren:

17. Inwiefern stimmen Sie diesen Aussagen über Ihre Erfahrungen mit Ihren Kunden zu? PL15

	stimme gar nicht zu	stimme eher nicht zu	unent-schieden	stimme eher zu	stimme voll zu	Kann / Möchte ich nicht sagen
Die Kunden äußern Verständnis für meine/unsere Preise.	○	○	○	○	○	○
Die Kunden beklagen sich aufgrund ihrer eigenen finanziellen Situation über die Preise.	○	○	○	○	○	○
Einzelne Kunden haben mir vorgeworfen von der Krise profitieren zu wollen.	○	○	○	○	○	○
Die Kunden geben mehr Trinkgeld.	○	○	○	○	○	○
Die Kunden geben weniger Trinkgeld.	○	○	○	○	○	○

Seite 06
PG

PHP-Code

```
if ((value('PL01_02') > value('PL01_01')) or (value('PL02_02') > value('PL02_01'))) {
goToPage('RA');
}
$pageNr = 4;
replace('%ownPageNumber%',$pageNr);
option('progress',round(100*$pageNr/7));
```

PL04

18. Weshalb haben sich Ihre Preise seit letztem Dezember nicht erhöht?

Sie haben angegeben, dass mindestens einer Ihrer Preise im März 2021 nicht höher ist als er im Dezember 2020 war. Welche Rolle haben die folgenden Faktoren bei der Entscheidung, den Preis nicht zu erhöhen, für Sie gespielt?

	keine Rolle	eine kleine Rolle	eine große Rolle	Trifft nicht zu	Kann / Möchte ich nicht sagen
Die Preise sind vertraglich festgelegt.	○	○	○	○	○
Innerhalb des Unternehmens konnten wir uns nicht auf Preissteigerungen einigen.	○	○	○	○	○
Ich weiß nicht, ob höhere Preise besser für das Unternehmen wären.	○	○	○	○	○
Eine Preiserhöhung würde größer scheinen als sie wirklich ist.	○	○	○	○	○
Erhöhung des Marktanteils / neue Kunden gewinnen	○	○	○	○	○
Die Preise wurden bereits nach dem 1. Lockdown (Frühjahr 2020) erhöht.	○	○	○	○	○
Zahlungskraft der Kunden ist in der Pandemie geringer	○	○	○	○	○
Mehrwertsteuersenkung im zweiten Halbjahr 2020 wurde nicht weitergegeben	○	○	○	○	○
Die Konkurrenz hat ihre Preise nicht erhöht.	○	○	○	○	○
Die Preise wurden nicht erhöht, um sie nicht in absehbarer Zeit wieder senken zu müssen.	○	○	○	○	○
Die Kosten sind nicht gestiegen.	○	○	○	○	○
Erhalt der Stammkunden	○	○	○	○	○

PL08

Sonstige wichtige Faktoren:

19. Inwiefern stimmen Sie diesen Aussagen über Ihre Erfahrungen mit Ihren Kunden zu? PL15

	stimme gar nicht zu	stimme eher nicht zu	unent-schieden	stimme eher zu	stimme voll zu	Kann / Möchte ich nicht sagen
Die Kunden äußern Verständnis für meine/unsere Preise.	○	○	○	○	○	○
Die Kunden beklagen sich aufgrund ihrer eigenen finanziellen Situation über die Preise.	○	○	○	○	○	○
Einzelne Kunden haben mir vorgeworfen von der Krise profitieren zu wollen.	○	○	○	○	○	○
Die Kunden geben mehr Trinkgeld.	○	○	○	○	○	○
Die Kunden geben weniger Trinkgeld.	○	○	○	○	○	○

Seite 07

RA

PHP-Code

```
$pageNr = 5;
replace('%ownPageNumber%',$pageNr);
option('progress',round(100*$pageNr/7));
```

Teil 3 Zustand nach Lockdown

Auf dieser Seite stellen wir Ihnen Fragen dazu, wie Ihr Unternehmen mit den politischen Maßnahmen umgeht, und wie Sie die zukünftige Entwicklung einschätzen.

LO04

20. Falls Sie für Anfang März mehr Terminanfragen erhalten haben, als Sie Termine zu vergeben hatten. Wie sind Sie damit umgegangen?

Mehrfachantworten sind möglich

- ☐ Bevorzugung von Neukunden
- ☐ Anstellung von Mitarbeitern, um mehr Termine anbieten zu können
- ☐ Bevorzugung von Kunden, deren Termine in den letzten Monaten abgesagt werden mussten
- ☐ Bevorzugung von Stammkunden
- ☐ Wer zuerst angefragt hat, hat Termine bekommen
- ☐ Ausweitung der Öffnungszeiten, um mehr Termine anbieten zu können
- ☐ Erhebung eines Zuschlags für Neukunden

- ☐ Trifft nicht zu
- ☐ Kann / Möchte ich nicht sagen

LO01

21. Inwiefern stimmen Sie diesen Aussagen über die Pflicht zum Haarewaschen zu?

	Stimme gar nicht zu	Stimme eher nicht zu	Unent-schieden	Stimme eher zu	Stimme voll zu	Kann / Möchte ich nicht sagen
Ich fühle mich sicherer, wenn die Haare der Kunden vor der Behandlung gewaschen werden.	○	○	○	○	○	○
Die Pflicht zum Haarewaschen ist wie eine Preiserhöhung.	○	○	○	○	○	○
Die Kunden finden die Pflicht zum Haarewaschen akzeptabel.	○	○	○	○	○	○
Ich profitiere finanziell von der Pflicht zum Haarewaschen.	○	○	○	○	○	○

LO07

22. Für wie zutreffend halten Sie die folgenden Vorhersagen?

	gar nicht	eher nicht	unklar	eher ja	sehr	Kann / Möchte ich nicht sagen
In einem Jahr werden wir wieder den Zustand von vor der Pandemie haben.	○	○	○	○	○	○
Infektionsschutzmaßnahmen werden noch für Jahre vorgeschrieben bleiben.	○	○	○	○	○	○
Die Angst vor dem Virus wird manche Menschen noch lange Zeit von einem Friseurbesuch abhalten.	○	○	○	○	○	○
Meine persönliche finanzielle Situation wird sich längerfristig verbessern (verglichen zu heute).	○	○	○	○	○	○
Infolge der Pandemie(bekämpfung) wird die Zahlungsbereitschaft meiner/unserer Kunden nachhaltig sinken.	○	○	○	○	○	○
Es wird dieses Jahr einen weiteren Lockdown geben, in dem Friseurläden wieder schließen müssen.	○	○	○	○	○	○

LO08

23. Wie unsicher sind Sie sich über Ihre berufliche Zukunft?

gar nicht	kaum	etwas	sehr	Kann / Möchte ich nicht sagen
○	○	○	○	○

Seite 08

PHP-Code

```
$pageNr = 6;
replace('%ownPageNumber%',$pageNr);
option('progress',round(100*$pageNr/7));
```

Teil 4 Preissetzung allgemein

Auf dieser Seite stellen wir Ihnen allgemeine Fragen zur Preissetzung in Ihrem Unternehmen.

PA01

24. Im Allgemeinen, worauf achten Sie am meisten bei der Preissetzung?

Mehrfachantworten sind möglich

- ☐ Kosten
- ☐ Preise der Konkurrenz
- ☐ Qualität meines Angebots
- ☐ Kundenzufriedenheit
- ☐ Anpassung an das allgemeine Preislevel
- ☐ Anderes: ______

- ☐ Kann / Möchte ich nicht sagen

PA02

25. Inwiefern stimmen Sie diesen Aussagen über Ihre Preissetzung zu?

	stimme gar nicht zu	stimme eher nicht zu	unent-schieden	stimme eher zu	stimme voll zu	Kann / Möchte ich nicht sagen
Ich bin zufrieden mit der Art wie ich/wir Preise setze/n.	○	○	○	○	○	○
Die Preise sind optimal für das Unternehmen gewählt.	○	○	○	○	○	○
Eigentlich sollten die Preise höher sein.	○	○	○	○	○	○
Die Gründe für Preiserhöhungen sind für die Kunden nachvollziehbar.	○	○	○	○	○	○

Seite 09
KO

PHP-Code

```
$pageNr = 7;
replace('%ownPageNumber%',$pageNr);
option('progress',round(100*$pageNr/7));
```

Danke

Vielen Dank für Ihre Teilnahme an unserer Studie!

S001

26. Wenn Sie uns etwas mitteilen möchten, können Sie dies hier anonym tun

Anmerkung: Diese Antwort wird zusammen mit Ihren anderen Antworten, aber ohne personenbezogene Informationen gespeichert.

Sollten Sie eine Frage haben, auf die Sie eine Antwort wünschen, können Sie uns gerne eine E-Mail schreiben.

Letzte Seite

Nochmals vielen Dank für Ihre Teilnahme!

Ihre Antworten wurden gespeichert, Sie können das Browser-Fenster nun schließen.

Thomas Kohler und Maximilian Weiß, Bonn Graduate School of Economics

Rheinische Friedrich-Wilhelms Universität Bonn – 2021

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Appendix 3.C Regression Tables

3.C.1 Membership in Clusters

Table 3.C.1. Multinomial logit regression. The dependent variable is the membership in a cluster. Membership in Cluster 1 (“do not want to increase”) is the baseline. The price setting variables are dummies. We offered five factors and asked the owners to indicate which of them are the most important in their pricing. Because no owner in the third cluster and few overall chose “the competitors’ prices,” we dropped this variable. Multiple answers were possible.

Membership in	Cluster 2 (Blanchard critique)	Cluster 3 (only retain regulars)
Employees (linear part)	0.503	0.325
	(1.31)	(1.09)
Dummy for many employees	1.218	0.262
	(0.76)	(0.18)
Share of regular customers	2.419**	0.400
	(2.32)	(0.40)
Price before the lockdown	0.0242	0.0339
	(0.51)	(0.75)
Price setting: Cost	1.946*	3.672***
	(1.77)	(2.75)
Price setting: Quality of my service	-3.206***	-2.969***
	(-3.09)	(-2.94)
Price setting: Customer satisfaction	-1.189	-1.856*
	(-1.25)	(-1.73)
Price setting: Inflation adjustment	-2.679***	-1.018
	(-2.89)	(-1.18)
Satisfaction with pricing	-0.0936	-0.114
	(-0.42)	(-0.52)
Hairwashing	0.0434	0.187
	(0.27)	(0.99)
Pessimism	-0.561***	-0.228*
	(-3.32)	(-1.87)
Customer relationship	-0.163	0.107
	(-1.02)	(0.70)
Constant	-7.391	-3.232
	(-1.36)	(-0.64)
Observations	62	

z statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3.C.2. To interpret the coefficients in the above multinomial logit regression, we calculate the marginal effect of each variable on the probability of being in a certain cluster, evaluated at the means. For the dummy variables (the size and the price setting factors), the coefficients indicate the change in the probability when the dummy is 1 compared to its being 0.

Marginal effect evaluated at the means on membership in	Cluster 1 (do not want to increase)	Cluster 2 (Blanchard critique)	Cluster 3 (only retain regulars)
Employees (linear part)	-0.061	0.043	0.019
	(-1.44)	(1.02)	(0.47)
Dummy for many employees=1	-0.101	0.147	-0.047
	(-0.55)	(0.75)	(-0.26)
Share of regular customers	-0.186	0.281^{**}	-0.095
	(-1.30)	(2.53)	(-0.72)
Price before the lockdown	-0.005	0.001	0.004
	(-0.80)	(0.15)	(0.56)
Price setting: Cost	-0.408^{***}	0.072	0.336^{***}
	(-4.06)	(0.75)	(4.19)
Price setting: Quality of my service	0.430^{***}	-0.189^{*}	-0.241^{**}
	(5.12)	(-1.81)	(-2.22)
Price setting: Customer satisfaction	0.245^{**}	-0.034	-0.211^{*}
	(2.06)	(-0.37)	(-1.88)
Price setting: Inflation adjustment	0.224^{**}	-0.201^{***}	-0.024
	(2.23)	(-2.98)	(-0.21)
Satisfaction with pricing	0.016	-0.004	-0.012
	(0.54)	(-0.19)	(-0.43)
Hairwashing	-0.020	-0.0068	0.027
	(-0.83)	(-0.40)	(1.11)
Pessimism	0.056^{***}	-0.056^{***}	0.001
	(3.25)	(-3.99)	(0.04)
Customer relationship	0	-0.028	0.028
	(0)	(-1.50)	(1.34)
Observations	25	30	16

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.C.2 Retaining Regulars Applies Less Often

Stylized Fact 1. Among the non-increasers, the better a firm's relationship with its regular customers, the less important for price stickiness is the motive of retaining its regular customers.

Table 3.C.3. Logit regression. The dependent variable is whether the respondent marked the hypothesis "Retain regular customers" as applicable or not.

	Dummy for retain regulars applies
Dummy for retain regulars applies	
Employees (linear part)	-7.697***
	(-8.87)
Dummy for many employees	-33.47***
	(-10.70)
Price before the lockdown	-0.146**
	(-2.12)
Customer relationship	-1.126*
	(-1.88)
Satisfaction with pricing	0.315
	(1.63)
Hairwashing	-0.106
	(-0.33)
Pessimism	-0.295
	(-0.94)
Constant	47.48***
	(6.06)
Observations	74

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3.C.4. Ordered logit regression. The dependent variable is the grade that the respondent assigned to the hypothesis “Retain regular customers.” For this regression, we assigned the lowest grade if the respondent marked the hypothesis as not applicable (instead of assigning a grade).

	Grade for retain regulars
Employees (linear part)	-0.160
	(-0.89)
Dummy for many employees	-0.790
	(-1.10)
Price before the lockdown	-0.00574
	(-0.18)
Customer relationship	-0.186
	(-1.58)
Satisfaction with pricing	-0.00961
	(-0.07)
Hairwashing	0.121
	(1.19)
Pessimism	-0.137*
	(-1.83)
cut1	
Constant	-2.832**
	(-1.98)
cut2	
Constant	-1.997
	(-1.41)
Observations	74

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.C.3 More Likely to Increase Prices

Stylized Fact 2. Firms that have a better relationship with their customers are more likely to increase their prices.

Table 3.C.5. Logit regression. The dependent variable is a dummy indicating whether the respondent increased the price during the lockdown or not.

	Dummy for increaser
Employees (linear part)	0.107
	(0.88)
Dummy for many employees	0.521
	(1.07)
Price before the lockdown	-0.0496*
	(-1.95)
Customer relationship	0.224***
	(3.10)
Satisfaction with pricing	-0.128
	(-1.64)
Hairwashing	0.0283
	(0.46)
Pessimism	-0.00437
	(-0.08)
Constant	0.913
	(0.82)
Observations	210

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.C.4 Rather Restored Profit Margin

Stylized Fact 3. Firms that have a good relationship with their regular customers are able to restore their profit margin from before the pandemic.

Table 3.C.6. Means of the answers to the question about the profit margin compared to before the pandemic and before the lockdown in the tertile with the worst customer relationship (first column) and the tertile with the best customer relationship (second column). The third column is the *t* statistic of a two-sample *t*-test, testing whether the means are equal.

customer relationship	lowest tertile	highest tertile	
	mean	mean	*t* statistic
Profit margin before pandemic	-0.20	0.12	-2.74***
Profit margin before lockdown	-0.28	0.14	-4.08***
Observations	86	66	

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.C.5 More Satisfied with Own Pricing

Stylized Fact 4. Owners whose firms have a better relationship with their regular customers are more satisfied with their own pricing.

Table 3.C.7. OLS regression. The dependent variable is the summary variable of the respondent's satisfaction with the own pricing method.

	Satisfaction with own pricing
Employees (linear part)	-0.0260
	(-0.23)
Dummy for many employees	0.322
	(0.71)
Price before the lockdown	0.00791
	(0.40)
Customer relationship	0.357^{***}
	(5.65)
Hairwashing	0.0838
	(1.38)
Pessimism	-0.0381
	(-0.82)
Constant	2.428^{**}
	(2.26)
Observations	210

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.C.6 Are Less Pessimistic

Stylized Fact 5. Owners whose firms have a better relationship with their customers are less pessimistic.

Table 3.C.8. OLS regression. The dependent variable is the summary variable of the respondent's pessimism concerning the firm's and the own professional future.

	Pessimism
Employees (linear part)	-0.177
	(-1.21)
Dummy for many employees	-0.316
	(-0.49)
Price before the lockdown	-0.0349
	(-1.31)
Customer relationship	-0.270***
	(-3.22)
Satisfaction with pricing	-0.0787
	(-0.81)
Hairwashing	0.0433
	(0.54)
Constant	10.53***
	(10.04)
Observations	210

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.C.7 Lower Initial Prices

Table 3.C.9. OLS regression. The dependent variable is the initial price before the lockdown. The explaining variable indicates whether the firm is in the first, second, or third tertile of the summary variable for the relationship with the customers.

	Price before the lockdown
Tertile Customer Relationship	-0.999**
	(-1.99)
Constant	27.99***
	(24.66)
Observations	235

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

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