Essays on Financial and Labor Market Institutions

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Introduction

Risk plays a key role in the economy. The capacity to assume risk has driven economic prosperity, yet risk-taking must not compromise system stability. The concept of limited liability exemplifies this. By limiting investors' liability to the amount of their investment, limited liability companies can attract funds from numerous small investors, collectively enabling the financing of major technological advances, such as the construction of railroad networks, which required large amounts of funds and involved substantial risks. However, this same privilege of limited liability for investment banks was at the heart of the Global Financial Crisis (GFC) in 2008, which led to a near collapse of the financial system.

Economies rely on institutions, encompassing the legal system and the rules governing markets, that incentivize individuals to engage in entrepreneurial activity by mitigating downside risks through insurance mechanisms. Against this background, the design of institutions is pivotal for ensuring that risk-taking is neither too excessive, jeopardizing the system's stability, nor subdued, hindering innovation and sectoral transformation. This thesis consists of three self-contained papers on financial and labor market institutions in the context of risk and insurance.

The chapters in this thesis, each corresponding to a paper, draw on recent developments. The first paper is motivated by the regulatory push in the aftermath of the GFC towards institutions that mitigate counterparty default risk in over-the-counter derivatives markets. Their effect on competition and on market-based incentives for prudent risk management is little understood. The starting point of the second paper was the unprecedented usage during the COVID-19 pandemic of short-time work (STW), a labor market policy with the goal of keeping jobs at firms facing temporary economic difficulties. While the scheme may preserve jobs, it could also hinder productive reallocation of workers across firms and sectors in the longer term. The third paper addresses the fundamental question of whether and how firms balance different types of risks. Bridging the labor and the financial side, I focus, at the firm level, on the interaction of risks stemming from human capital accumulation and operations in foreign currency.

Methodologically, the thesis relies on the interplay between theoretical and empirical analysis. Collecting and using new – potentially very large – data and employing the state-of-the-art toolkit of econometrics for rigorous empirical analysis is indispensable for understanding the functioning of modern economies. However, any empirical analysis must start with a hypothesis. Mapping relevant real-world features for the question at hand to a mathematical model framework and formally testing the rigor of an argument sharpens our understanding of the mechanisms at play. In this vein, the first chapter is purely theoretical, while the second and third are empirical, using models to guide and inform the empirical analysis.

Chapter 1 studies incentives for insurance sellers to take precautionary measures to ensure solvency when insurer default risk is a dimension of the competition. Specifically, I introduce insurer default risk as a quality dimension of the insurance product. In the model, two insurers sequentially choose their default risk before competing for clients that differ in risk aversion, resulting in vertical product differentiation. As a first step, I demonstrate that more risk-averse clients self-select to purchase from the insurer with the lower default risk, leading to market segmentation. I show that a unique price equilibrium exists for any pair of default risks, with the insurer offering the lower default risk earning larger profits. Unlike standard results of vertical product differentiation, I find that market discipline in the choice of default risk emerges: the first mover chooses a low default risk, and the second mover follows with a (potentially small) default risk gap.

The model reflects essential features of over-the-counter derivatives markets, where derivatives are akin to insurance products offered by dealers, typically large banks. These markets are highly concentrated, with a few dealers selling to diverse clients, matching the model structure. The model highlights a market force driven by competition in price and default risk that incentivizes prudent risk management beyond regulatory requirements. This market force may be absent for market structures where dealers are shielded from competition in default risks, such as when derivatives are cleared via a central counterparty.

Chapter 2, which is joint work with Simon Jäger, Moritz Kuhn, Farzad Saidi, and Stefanie Wolter, studies the take-up of STW and its effects on worker outcomes and firm behavior, using novel German administrative data from 2009 to the present. We focus on extensions of STW along two dimensions: first, with respect to extending individual eligibility, and second, with respect to the potential duration of benefits. While estimates comparing individuals who receive STW benefits to co-workers at the same firm who do not suggest a positive employment effect of STW benefits of up to eight percentage points, we uncover that individual STW take-up highly correlates with predicted retention probability. This indicates that firms target STW towards workers who are more likely to stay and that these estimates constitute an upper bound. For identification, we zoom in on cohorts reaching the statutory retirement age, at which one automatically loses access to potential STW benefits. Workers above the retirement age, ineligible for STW, have identical employment trajectories compared to their slightly younger, eligible peers when their establishment takes up STW, suggesting no employment effect of an extension along this dimension.

Second, we investigate a policy lever widely used by governments: extension of the potential benefit duration (PBD) of STW. Exploiting a 2012 reform doubling PBD from 6 to 12 months through a regression discontinuity design, we find that STW extensions did not secure employment at treated firms 12 months after take-up, with little heterogeneity across worker characteristics. We find substantial and persistent positive wage effects in treated firms, consistent with control firms, which do not get an extension, lowering wages relative to treated firms. Across industry-region-size cells, larger wage effects are associated with smaller employment effects, indicating that downward wage flexibility prevents layoffs. Our results point to the crucial role of labor market institutions, sectoral bargaining, and local works councils in shaping the effects of STW, with the decentralized German wage-setting institutions substituting for STW extensions by enabling negotiations that prevent layoffs.

Chapter 3 starts with the observation that hiring is a risky investment decision: Higher employment levels reduce the likelihood of production being limited by personnel shortages, but firms face more idle labor and a larger wage bill during periods of low demand. The chapter proposes a trade-off between cash flow volatility from fixed labor and cash flow volatility from other sources. Using novel administrative data on short-time work in Germany, matched with employer-employee data and firm financials, I construct a firm-level measure for temporarily unused labor (*surplus labor*). As a second source of risk, I focus on cash flow risk from selling or sourcing in foreign exchange (FX) and document that firms with more surplus labor exhibit lower exchange rate-induced cash flow volatility.

I build a model that formalizes this trade-off and introduces firm-specific human capital as a dimension of firm heterogeneity to explain surplus labor choices. Guided by the model, I instrument surplus labor with proxies for firm-specific human capital and find evidence of a causal effect of surplus labor on firms' FX-induced cash flow risks. Hand-collected data on firms' use of FX derivatives reveal that the effect exists for both derivatives users and non-users, indicating that financial and operational hedges matter. The results suggest that firm-specific human capital plays a role in firms' hedging decisions.

In sum, this thesis demonstrates the scope of modern economic research, combining theoretical and empirical analysis, for better understanding institutions in financial markets, institutions in labor markets, and risk management of firms operating under both.

Chapter 1

Differences in Default Risks and Competition in Insurance Markets

1.1 Introduction

One characteristic of an insurance product is the default risk of its seller. Sellers can influence their own default risk through precautionary measures to ensure solvency. At the same time, sellers compete for clients, and the competition may create incentives to maintain low default risks – a relevant consideration for market stability. Consider, as a large market for risk transfer, the over-the-counter (OTC) derivatives market. Counterparty default risk is a major concern therein, especially given its role in the instabilities during the Global Financial Crisis (Duffie, 2019). This market is highly concentrated, with few large banks at the core selling derivatives to numerous heterogeneous clients. However, little is known about how oligopolistic competition in insurance markets affects insurers' choices of default risk.

This paper introduces insurer default risk as a quality dimension of the insurance product in a basic insurance model. Insurers sequentially choose their default risks while competing for risk-averse clients. Although all clients prefer lower default risk, their willingness to pay varies due to different levels of risk aversion. I investigate whether market discipline in the choice of default risk emerges in the resulting model of vertical product differentiation.

I find that this is the case when risk aversion is sufficiently relevant. The insurer with the lower default risk has larger profits, incentivizing the first mover in the choice of default risk to choose a low default risk. The second mover then follows with a (potentially small) default risk gap. I discuss implications of endogenous market discipline in the model for introducing central clearing in derivatives markets whereby sellers are shielded from competition in default risk.

In the model, two insurers offer insurance contracts to clients seeking to hedge against a common macro risk. The insurance contracts feature full coverage, except when the insurer defaults. Insurers choose their default risk by deciding on measures to ensure their solvency, e.g., setting aside capital or having balanced trading books. As a result, an insurance product is characterized by the price and its seller's default risk. Clients have CARA utility with varying levels of absolute risk aversion. Competition occurs in two stages: insurers sequentially choose their publicly observable default risks before engaging in simultaneous price competition.

The main results of the model are as follows, presented following backward induction through the stages of the model.

First, in stage three, when clients make their purchase decisions, there is self-selection. All clients prefer a low default risk to a high default risk, but their willingness to pay for low default risks varies due to differences in risk aversion. As a result, there is an indifferent client that segments the market with more risk-averse clients self-selecting to buy from the insurer with the lower default risk. This market segmentation hinges on differentiated default risks. Insurers make positive profits.

Second, in stage two, when prices are set, for every pair of default risks, a unique pair of prices exists that forms a subgame-perfect Nash equilibrium. The price equilibrium is such that the insurer with the higher default risk chooses a lower price.

Third, price equilibria and, subsequently, profits depend only on a function in default risks that is close to a function in the difference in default risks. I call the difference in default risks the *default risk gap*.

Fourth, in equilibrium, the insurer with the lower default risk (i.e., that offers the insurance product of higher quality) has larger profits than the other insurer. This renders the leadership position in quality more attractive.

This has two key implications for the first stage of the game, when default risks are sequentially chosen. First, the first mover is under pressure to choose a low default risk: Since the insurer with the lower default risk has larger profits, the first mover aims to occupy this position vis-à-vis the second mover. As a result, he chooses a sufficiently low default risk to exclude the possibility that the second mover reverses roles. In particular, the smallest optimal default risk gap approximately determines an upper bound for the default risk of the first mover. In general, the default risk of the first mover may not exceed roughly half of the worst (externally given) admissible default risk since this is an upper bound for the default risk gap.

Second, there are push-and-pull factors on the second-mover's choice of default risk. That is, if there is an optimal default risk gap for the second mover, the second mover will keep this gap (relative to the first mover). Under two conditions that are simple but probably more restrictive than necessary, the competitive situation can be summed up based on the default risk gap. If the profit of the second mover as a function of the second-mover's default risk has a unique interior maximum, and if the profit of the first mover as a function of the second-mover's default risk is increasing, one can fully characterize the equilibrium default risks. Broadly speaking, the first mover will choose a default risk below a threshold lower than the optimal default risk gap, and the default risk of the second mover will be that of the first mover plus the optimal default risk gap.

Lastly, in a numerical example, I demonstrate that the two above conditions hold for a plausible set of parameter values, and that the first-mover's default risk and the default risk gap can be small – much smaller than the admissible default risks in the model. Thus, competitive forces alone can lead to low default risks. Since the overall outcome varies somewhat smoothly with the parameter values, the conclusions drawn from the numerical example extend to a neighborhood of parameter values and are, therefore, "locally generic" for the parameters of the numerical example.

In sum, pressure to choose a low default risk for the first mover and a push-and-pull effect on the second-mover's choice of default risk can be seen as market discipline in the choice of default risks.

The model captures essential features of derivatives markets and may serve as a framework for exploring open questions about market structure. Derivatives can be seen as insurance products offered by dealers, typically large banks. Derivatives markets exhibit a hub-and-spoke structure (Abad, Aldasoro, Aymanns, D'Errico, Rousová, et al., 2016), with few dealers at the core and numerous heterogeneous clients in the periphery, which aligns with the model setup. This structure persists even when the market is centrally cleared through a central counterparty (CCP)[1], as typically only dealers are members of the CCP, and most market participants access central clearing as clients of these members (*client clearing*)[2]. However, interposing a CCP at the core of a highly concentrated market raises the question of the effect of central clearing on competition.

The model provides a framework to conceptualize the effects of central clearing on competition. A salient feature of a centrally cleared market is that members of the CCP do not differ in their default risks from the client's perspective, primarily due to mechanisms that port clients' portfolios from one member to another in case of a default (Braithwaite and Murphy, 2020). This reduces competition in price and default risk to competition in prices alone. However, the model demonstrates that market discipline in choosing default risk emerges *as a result of* two-dimensional competition (price and default risk). Thus, the model highlights a market force that may be absent in centrally cleared markets where dealers are shielded from competition in default risks.

1. A CCP replaces a contract between two of its members with two contracts that each have the CCP on one end. It thereby insulates the contracting parties from the risk that the counterparty defaults.

2. See, e.g., Financial Stability Board (2018) and CPMI, IOSCO (2022).

Related Literature. This paper contributes to three strands of the literature. First, I extend the literature on insurance markets following the seminal work by Rothschild and Stiglitz (1976). I introduce the seller's default risk as a quality dimension of the insurance product. Consequently, the focus of client heterogeneity shifts: clients differ in their risk aversion but are identical in the underlying endowment risk. A model introducing differences in insurer *service* quality has been developed by Schlesinger and Von der Schulenburg (1991), but their model centers around horizontal product differentiation and search costs.

Second, I add to the body of work on vertical product differentiation from the industrial organization literature (Gabszewicz and Thisse, 1979; Gabszewicz and Thisse, 1980; Shaked and Sutton, 1982; Shaked and Sutton, 1983). In the standard model in Tirole (1988), which closely follows Shaked and Sutton (1982), two firms compete in product quality and price. Maximal differentiation in quality choices emerges because differentiation in quality softens price competition. I adapt this class of models to the insurance context by mapping insurers' default risks to (inverse) qualities and introducing risk aversion into consumer utility. As a result, consumer utility becomes non-linear, reversing the result of maximal differentiation: push-and-pull factors and upward pressure on both qualities emerge, which I interpret as market discipline in quality choices.[3]

Third, I contribute to a growing literature on the market structure of OTC derivatives markets, initiated by Duffie, Gârleanu, and Pedersen (2005) and Atkeson, Eisfeldt, and Weill (2015).[4] Seminal papers on central clearing in derivatives markets have examined netting benefits (Duffie and Zhu, 2011), transparency (Acharya and Bisin, 2014), and the role of margins (Biais, Heider, and Hoerova, 2012; Biais, Heider, and Hoerova, 2016; Biais, Heider, and Hoerova, 2021). I add to this literature by focusing on the nature of competition between the dealers at the core of the market and by introducing the notion of differentiation in default risks. Competition between dealers is also studied in Carapella and Monnet (2020), who investigate the effect of central clearing in derivatives markets on dealers' entry decisions. The idea is that if more dealers enter as a result of the regulation, more intense competition and a resulting lower level of spreads may alter incentives to invest in efficient technologies ex-ante. Unlike their model, where all agents are risk-neutral, and the focus is on search frictions for dealers intermediating derivatives, the model in this paper emphasizes clients' risk aversion as the driving force behind dealer competition and default risk differentiation.

3. The Online Appendix of Brinkmann (2023) revisits the standard model to clarify which assumptions need to be removed from the standard linear model to produce analogous results. Differently from Moorthy (1988) and Moorthy (1991), who lifts the same assumptions and numerically computes and compares outcomes, I use a general convex cost function and derive the push factor directly from profit-maximizing incentives.

4. See Dugast, Üslü, and Weill (2022) for recent work on the coexistence of OTC and centralized markets.

The rest of the paper is organized as follows. Section 1.2 introduces the model framework. Section 1.3 derives key results on self-selection and illustrates the setup. Section 1.4 shows uniqueness and existence of price equilibria for any pair of default risks. Section 1.5 analyses the choices in default risks. Section 1.6 presents a numerical example. Section 1.7 discusses an application of the model setup to derivatives markets, and Section 1.8 concludes.

1.2 Model

1.2.1 Setup

There are a continuum of risk-averse *clients* with a hedging need and two risk-neutral *insurers*.

Clients. Each client has an asset $\tilde{x}$ which takes the value $\underline{\theta}$ with probability p and $\overline{\theta}$ with probability $(1-p)$. Let the expected value of the asset be zero, and the bad endowment state a loss.[5] The endowment risk is the *same* across all clients, and p is commonly known. Clients are risk-averse with CARA utility[6]

$$u_a(x) \;=\; -\exp(-ax). \tag{1.2.1}$$

Insurance Contract and Default Risk. Each insurer offers a full-coverage insurance contract for a fixed payment of γ. However, insurers default with some probability in the bad endowment state, in which case they do not honor the contractual obligations of the insurance contract. Insurers choose their default risk $b_i, i \in \{1,2\}$, i.e., the probability that they default in the bad endowment state.

A client with risk aversion parameter a derives the following utility from a contract (b,γ), sold by insurer with default risk b at price γ,

$$U_a(b,\gamma) \;:=\; (1-bp)u_a(-\gamma) + bpu_a(\underline{\theta}). \tag{1.2.2}$$

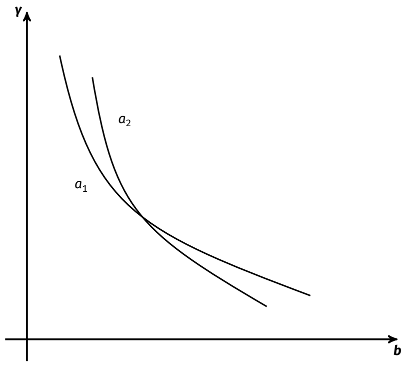
As illustrated in Figure 1.2.1, the marginal rate of substitution, i.e. the necessary reduction in the price γ for an increase in default risk b to keep a client indifferent, is increasing in a.[7] More risk-averse clients have a larger willingness to pay for an increase in quality.

5. Otherwise $E[\tilde{x}]$ is a certain payment and consider the random variable $\tilde{x}-E[\tilde{x}]$ instead of $\tilde{x}$.

6. The model remains unchanged with the cardinally equivalent utility $v_a(x) = 1/a(1-\exp(-ax))$.

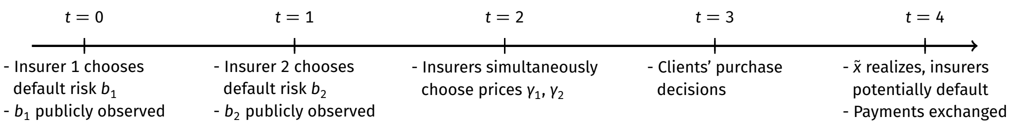
7. One can verify that $\partial MRS(a)/\partial a = -p/((1-bp)a)\big(\exp(-a(\underline{\theta}+\gamma))\left[1/a+\underline{\theta}+\gamma\right]-1/a\big)$. To see that this expression is positive, note that for $(\underline{\theta}+\gamma) < -1/a$ it follows directly. For $0 > (\underline{\theta}+\gamma) > -1/a$ it follows, since for all $x \neq 0$ $\exp(x) > 1+x$.

Figure 1.2.1. Illustration of Indifference Curves for Two Clients with $a_1 < a_2$



Timing. There are five points in time, $t \in \{0,1,2,3,4\}$. In $t = 0$, insurer 1 chooses default risk b_1. In $t = 1$, upon observing insurer 1's default risk, insurer 2 chooses his default risk b_2. In $t = 2$ they simultaneously choose prices $\gamma_i, i \in \{1,2\}$ for a full-coverage insurance contract.[8] Upon observing the insurers' choices (b_1, γ_1) and (b_2, γ_2), clients decide from whom to buy in $t = 3$. Lastly, clients' endowments are realized, and payments are exchanged in $t = 4$ unless there is a default. Figure 1.2.2 summarizes the timing of events.

Figure 1.2.2. Timeline



We study subgame-perfect Nash equilibria in the resulting game.

1.2.2 Discussion

The assumption that default risks are chosen before other actions embeds a commitment assumption. Once chosen, a default risk cannot be modified at later points in time. This excludes a situation in which an insurer abandons precautionary measures after they signaled a low default risk. In the simple timing structure of the present model, commitment seems a reasonable assumption. First, precautionary measures that insurers undertake to reduce the probability of their own default, such as setting aside capital, sufficient liquidity buffers, or balancedness of

8. Specifically, it is assumed that γ is the upfront premium for establishing the client-insurer relationship. Afterward, the insurer offers the actuarily fair price, and clients subsequently pick trade volumes that result in full insurance. Hence, the insurer's profit per client is γ. See Appendix 1.B.1 for details.

the trading books, are relatively long-term strategic decisions. They are here seen as investments that are sunk costs during later phases of competition, not continued period-per-period expenses. Second, default risks need to become public information before clients' purchase decisions. The disclosure of such information and the associated build-up of reputation also takes time.

I assume that default risks are chosen sequentially, which is a simplifying assumption. With simultaneous choice, any pure-strategy equilibrium in qualities cannot be symmetric, as equal qualities yield zero profits for insurers. Thus, with simultaneous quality choices, there are multiple equilibria (with reversed roles). In Shaked and Sutton (1982), roles are thus assigned upfront. In this model, roles are instead assigned via sequential quality choice (as, e.g., in Aoki and Prusa (1997) and Lehmann-Grube (1997)).

The setup implies somewhat restrictive assumptions regarding the contract terms: Insurers are limited to offering full-coverage insurance (unless they default) and only have discretion over the premia. The situation maps to a situation in which clients pay a fixed premium to establish the client-insurer relationship, after which the insurer provides insurance at fair prices and clients subsequently choose full insurance. The setup rules out a situation in which insurers offer a menu of contracts that differ in coverage and thus additionally compete in coverage. I make two points in defense of this assumption. First, the novel aspect of the model is competition in default probabilities seen as a quality dimension of the insurance products. To keep this analysis tractable, I keep other dimensions of the competition as simple as possible. Second, in the context of derivatives markets, which will be discussed later, full coverage is a typical feature. For example, a plain-vanilla interest rate swap specifies the exchange of a fixed interest rate for a floating rate without variation in coverage.

Similarly, it is assumed that insurers are unable to discriminate among clients based on their risk aversion. In other words, I assume that risk aversion is private information to clients. One may debate how much information insurers are able to acquire about the risk attitudes of their clients. In the context of derivatives markets as an over-the-counter market, clients may additionally have a hard time comparing prices. Assuming that risk aversion is private information, nonetheless, seems a natural starting point and one that facilitates an analysis with respect to vertical product differentiation. In related work on vertical product differentiation, first-degree price discrimination is ruled out.

1.3 Stage 3: Clients' Purchase Decisions

The model is solved by backward induction, starting with clients' purchase decisions in $t = 3$. This section establishes that the market is segmented with more risk-averse clients buying from the insurer with the lower default risk (Proposition 2) and derives some properties to graphically illustrate the setup (Figure 1.3.2).

In $t = 3$, insurers' default risks are given. To fix roles, insurer 1 defaults with a lower probability or, in other words, offers a product of higher quality. That is, let $\Delta b := b_2 - b_1 > 0$. Let $\vec{b} := (b_1, b_2)$ and $\vec{\gamma} := (\gamma_1, \gamma_2)$ denote the pairs of default risks and prices.

Lemma 1 (Characterization of the Indifferent Client). *A client with degree of risk aversion a is indifferent between two contracts (b_1, γ_1) and (b_2, γ_2) with $\Delta b > 0$ if*

$$g(a, \vec{\gamma}) := \frac{\exp(-a\Delta\gamma) - 1}{\exp(-a(\underline{\theta} + \gamma_2)) - 1} = \frac{p\Delta b}{1 - b_1 p} =: \tilde{g}(\vec{b}). \tag{1.3.1}$$

Proof. See Appendix 1.A.1. □

For any two contracts with $b_2 > b_1$, if there is a solution to (1.3.1), then $\gamma_1 > \gamma_2$.[9] That is, the insurer that offers the product of higher quality sets the higher price.

The main result of this section (Proposition 2) establishes that there is at most one client characterized by some a^* who is indifferent between contracts (b_1, γ_1) and (b_2, γ_2) and segments the market. For the existence of a unique indifferent client, the utility loss due to the payment of the price relative to the utility loss due to the bad endowment needs to diminish as clients become more risk-averse. In other words, the function g needs to decrease in the risk aversion parameter – akin to a single crossing condition. A lower bound on $-a(\underline{\theta} + \gamma_i)$ is sufficient for this, which is ensured by the following set of assumptions.[10]

Assumption A1.

$$p < \frac{1}{3}.$$

Assumption A2.

$$\text{For } i \in \{1, 2\} : b_i \in [0, b^{max}] \text{ with } b^{max} \leq \frac{1}{3}.$$

9. To see this, note that with $\Delta b > 0$, the RHS of (1.3.1) is positive. The denominator of the LHS of (1.3.1) is positive, which necessitates $\Delta\gamma < 0$.

10. From assumptions A3 and A4 we get $-a(\underline{\theta} + \gamma_i) > 2$. To see this, note that $-a(\underline{\theta} + \gamma_i) > 2 \Leftrightarrow \gamma^{max} < 2/(-\underline{a}) - \underline{\theta}$. The RHS holds, since by assumption A3 $\gamma^{max} < (-\underline{\theta})/3$ and $2/(\underline{a}) - \underline{\theta} > (-\underline{\theta})/3 \Leftrightarrow (-\underline{a})\underline{\theta} > 3$, which is ensured by assumption A4.

Assumption A3.

$$\text{For } i \in \{1,2\} : \gamma_i \in [0, \gamma^{max}] \text{ with } \gamma^{max} \leq \frac{1}{3}(-\underline{\theta}).$$

Assumption A4.

$$\underline{a}(-\underline{\theta}) > \log\left[\frac{1-\frac{1}{8}}{\exp(-2)-\frac{1}{8}}\right] \approx 4.4.$$

The assumptions bound the probability of the bad endowment state (assumption A1) and the default risks (assumption A2). They, thus, focus attention on a setup of insurance against an infrequent large event as well as a setup with the default of an insurer being the exception rather than the norm.

Regarding assumption A3, note that $\gamma^{max} \leq (-\underline{\theta})$ by construction, since otherwise the price exceeds the bad endowment. A priori, there is no market for prices exceeding the price above which the most risk-averse client is unwilling to buy insurance even if offered with the lowest default risk (see Appendix 1.B.2 for details). In the numerical example, assumption A3 is non-binding in equilibrium.

Assumption A4 imposes a lower bound on the degree of risk aversion times the absolute value of the bad endowment, $a(-\underline{\theta})$ for all $a \in [\underline{a}, \overline{a}]$. It is a condition on both the range of a and $\underline{\theta}$: For any large $\underline{\theta}$, one can find a small a such that assumption A4 is violated. Intuitively, for any large payment without limitations on a, one can find clients whose utility is sufficiently close to a risk-neutral one (i.e., a close to 0) such that risk aversion barely kicks in. Assumption A4 rules out such almost risk-neutral clients – relative to the bad endowment. Hence, it demands that risk aversion is relevant for all clients.

Proposition 2 (Self-Selection). *Suppose assumptions A1 - A4. For given contracts* (b_1, γ_1) *and* (b_2, γ_2) *with* $\Delta b > 0$*, there is at most one indifferent client* $a^*(\vec{\gamma})$ *satisfying*

$$g(a^*(\vec{\gamma}), \vec{\gamma}) = \tilde{g}(\vec{b}) = \frac{p\Delta b}{1-b_1 p}. \tag{1.3.2}$$

Such an indifferent client $a^*(\vec{\gamma}) \in [\underline{a}, \overline{a}]$ *indeed exists, if*

$$g(\overline{a}, \vec{\gamma}) \leq \frac{p\Delta b}{1-b_1 p} \leq g(\underline{a}, \vec{\gamma}). \tag{1.3.3}$$

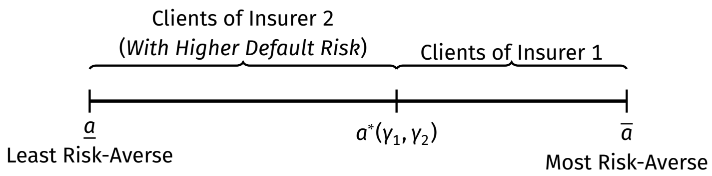
In this case, client a will choose insurer 1 iff

$$a \geq a^*(\vec{\gamma}). \tag{1.3.4}$$

Proof. See Appendix 1.A.2. □

The result implies market segmentation. Clients with a risk aversion larger than $a^*(\vec{\gamma})$ buy from insurer 1, while insurer 2 receives clients with a level of risk aversion below the threshold $a^*(\vec{\gamma})$, as depicted in Figure 1.3.1.

Figure 1.3.1. Market Segmentation



Notes: The figure illustrates market segmentation for two contracts (b_1, γ_1) and (b_2, γ_2) with $b_2 > b_1$.

Formally, for given default risks $\vec{b}$, a^* is defined via $g(a^*(\vec{b}, \vec{\gamma}), \vec{\gamma}) = \tilde{g}(\vec{b})$ on the set

$$\mathscr{G}_{[\underline{a}, \bar{a}]} := \{\vec{\gamma} \,|\, 0 \leq \gamma_2 < \gamma_1 \leq \gamma^{max} \text{ and } g(\underline{a}, \vec{\gamma}) \leq \tilde{g}(\vec{b}) \leq g(\bar{a}, \vec{\gamma})\}. \quad (1.3.5)$$

Let $\mathscr{G}_0 := \{0 \leq \gamma_2 < \gamma_1 \leq \gamma^{max}\}$. Then the insurers' profits are

$$\Pi_1(\gamma_1, \gamma_2) = \begin{cases} \big(\bar{a} - a^*(\gamma_1, \gamma_2)\big)\gamma_1 & \text{on } \mathscr{G}_{[\underline{a}, \bar{a}]} \\ \big(\bar{a} - \underline{a}\big)\gamma_1 & \text{on } \mathscr{G}_0 \setminus \mathscr{G}_{[\underline{a}, \bar{a}]} \text{ if } \tilde{g}(\vec{b}) \leq g(\underline{a}, \vec{\gamma}) \\ 0 & \text{on } \mathscr{G}_0 \setminus \mathscr{G}_{[\underline{a}, \bar{a}]} \text{ if } g(\bar{a}, \vec{\gamma}) \leq \tilde{g}(\vec{b}) \end{cases} \quad (1.3.6)$$

$$\Pi_2(\gamma_1, \gamma_2) = \begin{cases} \big(a^*(\gamma_1, \gamma_2) - \underline{a}\big)\gamma_2 & \text{on } \mathscr{G}_{[\underline{a}, \bar{a}]} \\ 0 & \text{on } \mathscr{G}_0 \setminus \mathscr{G}_{[\underline{a}, \bar{a}]} \text{ if } \tilde{g}(\vec{b}) \leq g(\underline{a}, \vec{\gamma}) \\ \big(\bar{a} - \underline{a}\big)\gamma_2 & \text{on } \mathscr{G}_0 \setminus \mathscr{G}_{[\underline{a}, \bar{a}]} \text{ if } g(\bar{a}, \vec{\gamma}) \leq \tilde{g}(\vec{b}) \end{cases} \quad (1.3.7)$$

In the following we restrict attention to the set $\mathscr{G}_{[\underline{a}, \bar{a}]}$.

The setup admits no closed-form solutions. Instead, in the remainder of this section, we characterize market share elasticities and graphically illustrate the setup. The following notation is introduced for an explicit characterization in the next Lemma of how the indifferent client changes as either insurer increases prices, but not needed for the subsequent text. Define

$$\tilde{A} : [\underline{a}, \bar{a}] \times [0, -\underline{\theta})^2 \to \mathbb{R}, \quad (a, \vec{\gamma}) \mapsto \exp(-a\Delta\gamma) \quad (1.3.8)$$

$$\text{and } \tilde{B}_i : [\underline{a}, \bar{a}] \times [0, -\underline{\theta}) \to \mathbb{R}, \quad (a, \gamma_i) \mapsto \exp(-a(\underline{\theta} + \gamma_i)) \quad (1.3.9)$$

and let

$$A(\vec{\gamma}) := \tilde{A}(a^*(\vec{\gamma}), \vec{\gamma}), \quad \text{and} \quad B_i(\vec{\gamma}) := \tilde{B}_i(a^*(\vec{\gamma}), \gamma_i) \quad (1.3.10)$$

be the two functions, defined on $[0,-\underline{\theta})^2$, one obtains when inserting the indifferent client $a^*(\vec{\gamma})$ into (1.3.8) and (1.3.9). Since, for given $\vec{b}$, the RHSs of (1.3.1) and (1.A.30) are constant, we infer that the respective LHSs, i.e.

$$g(a^*(\vec{\gamma}),\vec{\gamma}) = \frac{A(\vec{\gamma})-1}{B_2(\vec{\gamma})-1} \quad \text{and} \quad h(a^*(\vec{\gamma}),\vec{\gamma}) = \frac{1-\frac{1}{A(\vec{\gamma})}}{B_1(\vec{\gamma})-1}, \tag{1.3.11}$$

are constants, and call them g and h, respectively. Finally, define

$$\xi_2 := (\underline{\theta}+\gamma_2), \quad \varphi_1 := \xi_2 B_1 \quad \text{and} \quad \tau_1 := \big(\Delta\gamma - g\varphi_1\big), \tag{1.3.12}$$

$$\text{as well as} \quad \xi_1 := (\underline{\theta}+\gamma_1), \quad \varphi_2 := \xi_1 B_2, \quad \text{and} \quad \tau_2 := \big(\Delta\gamma - h\varphi_2\big). \tag{1.3.13}$$

The following Lemma shows that both insurers indeed lose market share when increasing prices.

Lemma 3 (Market Shares). *Suppose assumptions A1 - A4. The indifferent client is increasing in* γ_1 *and decreasing in* γ_2*, namely*

$$\partial_1 a^* = \frac{a^*}{\tau_1} > 0 \tag{1.3.14}$$

$$\partial_2 a^* = \frac{-a^*}{\tau_2} < 0. \tag{1.3.15}$$

For the slope of a contour line $\{(\gamma_1,\gamma_2)|\,a^*(\gamma_1,\gamma_2)$ *constant*$\}$ *we have*

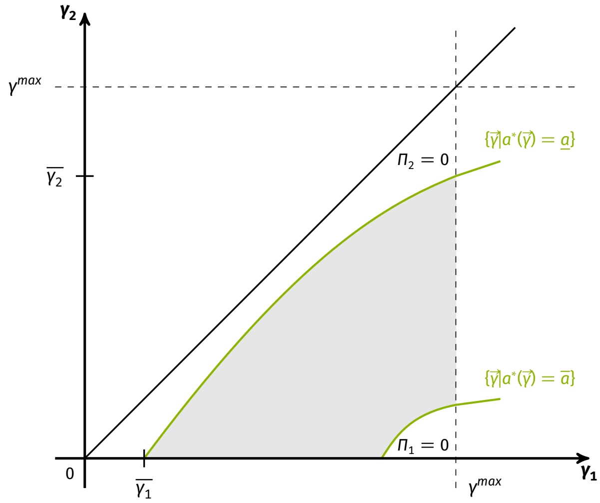
$$\frac{-\partial_2 a^*}{\partial_1 a^*} =: \alpha < 1. \tag{1.3.16}$$

Proof. See Appendix 1.A.4. □

Figure 1.3.2 visualizes the setup with prices set by insurers 1 and 2 on the x- and y-axis, respectively. With insurer 1 the insurer with the lower default risk offering insurance at a higher price, pairs of prices lie below the diagonal. The green line just below the diagonal depicts the pairs of prices above which insurer 2 has no market share and, subsequently, no profits. For $\gamma_2 \in [0,\gamma^{max}]$, we parameterize these pairs by defining $\gamma_1^{\underline{a}}(\gamma_2)$ such that $a^*(\gamma_1^{\underline{a}}(\gamma_2),\gamma_2) = \underline{a}$. From Lemma 3 we know that contour lines of a^* have a slope below one.

The visualization in Figure 1.3.2 offers an alternative justification for assumption A4. Denote by $\overline{\gamma_1}$ and $\overline{\gamma_2}$ the intercepts of upper green line, $\gamma_1^{\underline{a}}(\cdot)$, with the x- and y-axis, respectively. Assumption A4 is equivalent to demanding $\overline{\gamma_2} > 0$ (see Appendix 1.B.3 for details). In other words, assumption A4 demands that insurer

Figure 1.3.2. Illustration of the Setup



Notes: The figure depicts, for given default risks, insurer 1's and 2's prices on the x- and y-axis, respectively. See the text for a detailed explanation.

1 does not a priori get the entire market – making the setup interesting to begin with.

1.4 Stage 2: Price Setting

This section establishes existence and uniqueness of a Nash equilibrium in prices in $t = 2$ for a given pair of default risks under the following two additional assumptions.

Assumption A5.

$$\overline{a} \;\leq\; \frac{3}{2}\underline{a}.$$

Assumption A6.

$$\partial_1 \Pi_1 \left(\gamma_1^{\underline{a}}(\gamma_2^*(\gamma^{max})), \gamma_2^*(\gamma^{max})\right) \;\geq\; 0.$$

Assumption A6 is a technical assumption involving insurer 2's reaction function γ_2^*. It demands that at a point at which insurer 1 "owns" the entire market, insurer 1 has no incentive to decrease prices. The assumption is required because a negative market share at negative prices also leads to positive turnover – a case certainly not of interest.

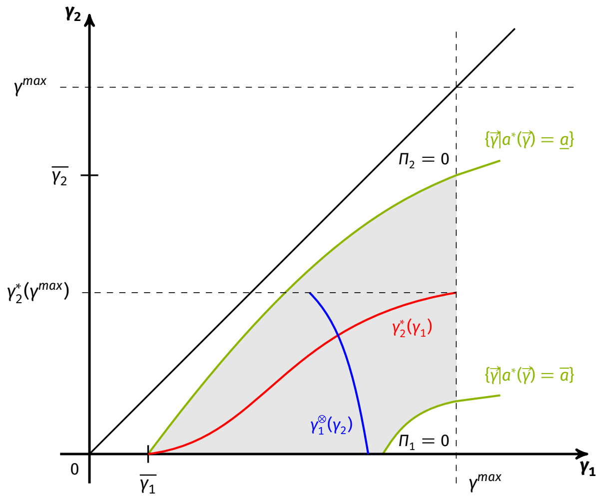
Proposition 4 (Existence and Uniqueness). *Suppose assumptions A1 - A6. Consider a pair of default risks* (b_1, b_2) *with* $\Delta b \geq 0$. *Then,*

i) *If* $b_1 < b_2$, *there exists a unique Nash equilibrium in prices* (γ_1, γ_2).

ii) *If* $b_1 = b_2$, *a client can be indifferent only if* $\gamma_2 = \gamma_1$. *That is, if insurers' default risks coincide, pure price competition drives prices to marginal costs (which are set to zero here).*

Proof. See Appendix 1.A.11. The proof builds on the existence of insurer 1's and insurer 2's reaction functions (Propositions 5 and 6). □

Figure 1.4.1. Price Equilibrium



Notes: The figure depicts, for given default risks, insurer 1's and 2's prices on the x- and y-axis, respectively. See the text for a detailed explanation.

The intuition of the proof is as follows: Insurer 2's reaction function, $\gamma_2^*(\gamma_1)$ in red, is strictly increasing. Thus, there exists an inverse function. From the boundary values of the inverse function, there must be an intersection with insurer 1's reaction function, $\gamma_1^{\otimes}$ (as depicted in Figure 1.4.1). Formally, we apply Brouwer's Fixed Point Theorem for existence. From the bounds on $\partial_2 \gamma_1^{\otimes}$ and $\partial_1 \gamma_2^*$ in Propositions 6 and 5, respectively, it follows that there can be at most one intersection.

We now formally show existence and properties of insurer 1's and insurer 2's reaction functions.

Proposition 5 (Insurer 2's Reaction Function). *Suppose assumptions A1 - A4. Suppose some fixed default risks* (b_1, b_2) *with* $\Delta b > 0$. *Then,*

i) *for any* $\gamma_1 \in [0, \gamma^{max}]$, *there is a unique best response in prices for insurer 2,* $\gamma_2^*(\gamma_1)$. *For* $\gamma_1 \in (\overline{\gamma}_1, \gamma^{max})$, γ_2^* *is in the interior of* $\mathcal{G}_{[\underline{a},\overline{a}]}$ *and uniquely characterized via* $\partial_2 \Pi_2 = 0$.

ii) for $\gamma_1 \in [\overline{\gamma}_1, \gamma^{max}]$, γ_2^* *is a smooth function and strictly increasing in* γ_1.

iii) $\partial_1 \gamma_2^* < 1/\alpha^*$ *with* $\alpha^* := \alpha(\gamma_1, \gamma_2^*(\gamma_1))$, *i.e.,* α *evaluated on insurer 2's reaction function.*

Proof. See Appendix 1.A.6. □

The strategy of the proof is standard: Uniqueness follows from $\partial_2^2 \Pi_2 < 0$, and existence follows since profits are a continuous function that is zero at the boundaries of the interval.

For the other insurer, existence of a reaction function is not straightforward since insurer 1's profit function is not necessarily concave. In fact, parameter restrictions ensuring concavity are not compatible with the existing set of assumptions that require risk aversion to have enough bite. Without concavity of insurer 1's profit function, points that satisfy the first-order condition need not correspond to best responses. Instead, we prove an auxiliary Lemma (Lemma 13 in the Appendix) for a smooth real-valued function f on some interval $[a, b]$ with $\partial f(a) > 0$: If there exists a point in the interval below which local extrema may only be local minima and above which local extrema may only be local maxima, then f has a global maximum. Assumptions A5 and A6 ensure that we can use this Lemma to obtain insurer 1's best responses for the relevant interval, that is, for $\gamma_2 \in [0, \gamma_2^*(\gamma^{max})]$.

Proposition 6 (Insurer 1's Reaction Function). *Suppose assumptions A1 - A6. Suppose some fixed default risks* (b_1, b_2) *with* $\Delta b > 0$. *Then,*

i) for any $\gamma_2 \in [0, \gamma_2^*(\gamma^{max})]$, *there is a unique best response in prices for insurer 1,* $\gamma_1^{\otimes}(\gamma_2)$. $\gamma_1^{\otimes}$ *is uniquely characterized via*

$$\partial_1 \Pi_1(\gamma_1^{\otimes}(\gamma_2), \gamma_2) = 0 \qquad \text{or}$$
$$\left(\gamma_1^{\otimes}(\gamma_2) = \gamma^{max} \text{and } \forall \mu \geq \gamma_1^{a}(\gamma_2) : \partial_1 \Pi_1(\mu, \gamma_2) > 0\right).$$

ii) $\gamma_1^{\otimes}$ *is a continuous function, smooth except at finitely many points.*

iii) $\partial_2 \gamma_1^{\otimes} < \alpha^{\otimes}$ *with* $\alpha^{\otimes} := \alpha(\gamma_1^{\otimes}(\gamma_2), \gamma_2)$, *i.e.,* α *evaluated on insurer 1's reaction function.*

Proof. See Appendix 1.A.7. □

1.5 Stage 1: Choices of Default Risks

This section analyses subgame-perfect equilibria in default risks and the competitive mechanism at play.

The model features vertical product differentiation: Ceteris paribus, all clients prefer the insurer with the lower default risk, but clients differ in their valuation for low default risks. Compared to the standard model of vertical product differentiation as in Shaked and Sutton (1982) or Tirole (1988), the inclusion of risk aversion leads to non-linear utility which does not admit closed-form solutions. Yet, an analog to the result in Shaked and Sutton (1982) that the high-quality firm has larger profits still holds.

Proposition 7 (Lower Default Risk More Attractive). *Suppose assumptions A1 - A6. At any Nash equilibrium in prices,*

i) the insurer with the lower default risk (quality leader) has larger profits, $\Pi_1 > \Pi_2$,

ii) the insurer with the lower default risk has a larger market share, $(\overline{a} - a^*) > (a^* - \underline{a})$.

Proof. See Appendix 1.A.12. □

There is a simple characterization of default risks that lead to the same price equilibrium.

Proposition 8 (Price Equilibrium Depends on the Default Risk Gap). *Suppose assumptions A1 - A6.*

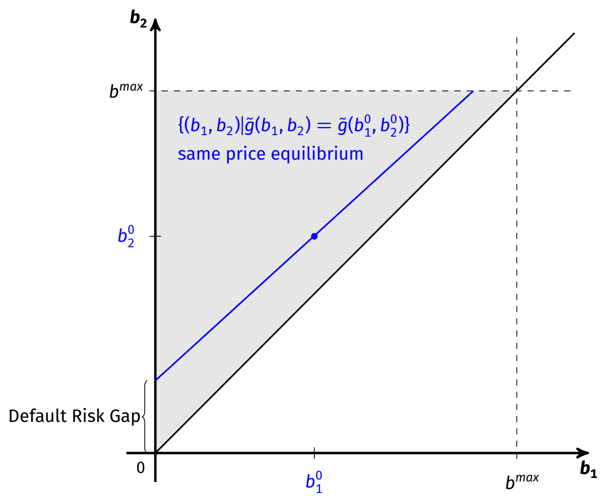
i) Default risks (b_1^0, b_2^0) *and* (b_1, b_2) *with* $\tilde{g}(b_1, b_2) = \tilde{g}(b_1^0, b_2^0)$ *lead to the same price equilibrium.*

ii) On the set of default risks $\{b \in [0, b^{max}]^2 | b_1 < b_2\}$*, price equilibria (and subsequently profits) are constant on straight lines with slope* $(1 - \tilde{g})$.

Proof. See Appendix 1.A.13. □

Figure 1.5.1 illustrates the result, with default risks of insurers 1 and 2 on the x- and y-axis, respectively. Since insurer 1 has the lower default risk, the default risks lie above the diagonal (shaded). For (b_1^0, b_2^0), the blue line depicts all pairs of default risks that lead to the same value of $\tilde{g}$ and, consequently, the same price equilibrium. $\tilde{g}$ is small, so the slope of the blue line is nearly parallel to the 45-degree line. Therefore, the gap between pairs of default risks that lead to the same price equilibrium (*default risk gap*) changes only slightly as b_1 changes.

Figure 1.5.1. Default Risks Leading to the Same Price Equilibrium Lie on Straight Lines



Notes: The figure depicts insurer 1's and 2's default risks on the x- and y-axis, respectively. See the text for a detailed explanation.

For default risks $\vec{b} = (b_1, b_2)$, let $\vec{\gamma}^{\square}(\vec{b})$ be the corresponding price equilibrium. As shown in Proposition 4, the price equilibrium exists and is unique; hence $\vec{\gamma}^{\square}(\vec{b})$ is well-defined. In Appendix 1.B.4, we show that price equilibria are smooth functions in qualities. By $\Pi_i^{\square}(\vec{b})$, we denote profits associated with a pair of default risks under optimal price setting in the subsequent period, $\Pi_i^{\square}(\vec{b}) := \Pi_i\left(\vec{\gamma}^{\square}(\vec{b}), \vec{b}\right)$.

Proposition 9 (First Mover Chooses a Low Default Risk, Second Mover Follows). *Suppose assumptions A1 - A6. Let Π_2^* be the global maximum of $\Pi_2^{\square}(0, b_2)$ as a function of b_2. Let b_2^s and b_2^l be the smallest and largest b_2 for which this maximum is assumed, i.e., b_2^s is the smallest default risk gap of a subgame-perfect equilibrium of the form $(0, b_2)$. Let (b_1^*, b_2^*) be a subgame-perfect Nash equilibrium in default risks. Then*

i) $b_1^* < b_2^s$.

ii) $b_2^* \leq \left(2 - \tilde{g}(0, b_2^l)\right) b_2^l$.

A general upper bound for b_1^* is

$$b_1^* < \left(\frac{8}{15}\right) b^{max}. \tag{1.5.1}$$

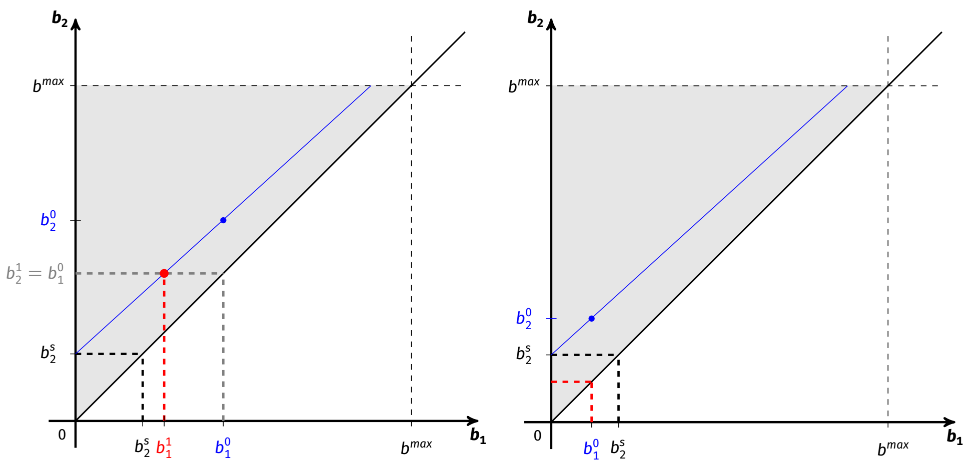
Proof. See Appendix 1.A.14. □

The intuition is illustrated in Figure 1.5.2, where Panel (a) shows a feasible and profitable deviation for the second mover, which is infeasible in Panel (b).

The default risk of the lower-risk and higher-risk insurer is depicted on the x- and y-axis, respectively. In Panel (a), consider the default risk pair $b_1^0 < b_2^0$ as indicated by the blue dot. Other default risk pairs leading to the same price equilibrium lie on straight lines, as shown by Proposition 8 and indicated by the blue line. A profitable deviation for the second mover is to choose a default risk that leads to the same price equilibrium but with reversed roles (the reversal of roles is indicated in Panel (a) by the grey dotted lines, while the profitable deviation for the second mover is indicated by the red dotted line). Such profitable deviations are infeasible for default risk choices of the first mover below b_2^s — as illustrated in Panel (b). The risk level that rules out this profitable deviation is characterized by i) or (1.5.1) in Proposition 9.

Figure 1.5.2. Competitive Mechanism

(a) (b_1^1, b_2^1) Leads to Reversed Roles **(b)** Profitable Deviation in (a) Infeasible



Notes: Each panel depicts insurer 1's and 2's default risks on the x- and y-axis, respectively. See the text for a detailed explanation.

Proposition 9 implies that the maximal default risk to ensure the position of quality leader is smaller than the smallest optimal default risk gap (at $b_1 = 0$.) This is approximately equal to the smallest optimal default risk gap at other b_1, since the blue line is almost parallel to the 45-degree line. Hence, the smallest optimal default risk gap is approximately an upper bound for the default risk of the first mover, and therefore, twice the default risk gap is approximately an upper bound for the second-mover's default risk.

Under two conditions that are simple but probably more restrictive than necessary, the competitive situation can be summed up based on the default risk gap.

Proposition 10 (Push-and-Pull Effect for Second Mover). *Suppose assumptions A1 - A6 and that the following two conditions hold*

$$\Pi_2^{\square}(0,b_2) \text{ as a function of } b_2 \text{ has a unique maximum for a } b_2 < b^{max}, \quad \text{(N1)}$$
$$\Pi_1^{\square}(0,b_2) \text{ as a function of } b_2 \text{ is increasing in } b_2. \quad \text{(N2)}$$

Let Π_2^* be the global maximum of $\Pi_2^{\square}(0,b_2)$, assumed at b_2^s. Let $\bar{b_1}$ be the minimum of all b_1 such that $\Pi_1^{\square}(0,b_1) = \Pi_2^*$. Then, (b_1^*, b_2^*) is a subgame-perfect equilibrium iff

$$b_1^* \in [0, \bar{b}_1] \quad (1.5.2)$$
$$b_2^* = (1 - \tilde{g}(0, b_2^s))b_1^* + b_2^s. \quad (1.5.3)$$

The second-mover's choice of default risk is pinned down by the first-mover's choice plus the optimal default risk gap. The first-mover's choice of default risk thus exerts a push-and-pull effect on the second-mover's choice of default risk.

Proof. See Appendix 1.A.15. □

Proposition 10 suggests that the first mover chooses a default risk pinned down by the default risk gap, and the second mover follows at an optimal distance. This is in contrast to the result of maximal product differentiation as in the standard model in Tirole (1988), which closely follows Shaked and Sutton (1982). In Tirole (1988), two firms compete in quality (chosen first) and price (chosen second) for clients that differ in their valuation of quality. The key mechanism is that for any two pairs of quality choices, firms choose prices in such a way that the resulting market shares remain unchanged. This eliminates a quantity effect, and with only a price effect left, firms soften price competition as much as possible by choosing maximally differentiated qualities. The result of maximal differentiation in qualities in the standard model hinges on three assumptions: first, clients' utility is linear; second, it is assumed that the market is always fully covered; and third, costs are quality-invariant. In the present model, the main departure from the standard model is the non-linearity of the utility function stemming from risk aversion in the insurance context. As a result, market shares are no longer invariant for varying quality pairs, and we obtain market discipline in quality choices. The Online Appendix of Brinkmann (2023) revisits the standard model and shows that one can obtain a similar result in the standard model with linear utility when removing the assumptions of full market coverage and introducing (general) convex costs for quality provision.

While conditions (N1) and (N2) cannot hold in general, e.g., a parameter value of b^{max} sufficiently small may violate (N1), I conjecture that they hold for a wide range of parameters. They hold in a numerical example for plausible parameter values, as shown in the following section.

1.6 Numerical Example

In a numerical example with plausible parameter values, I explicitly characterize the subgame-perfect Nash equilibria and demonstrate that the default risk gap can indeed be small.

Parameter Values. Consider the model for a specific set of parameters, namely

$$\underline{\theta} = -100 \cdot 10^6 \tag{1.6.1}$$

$$p = 0.03 \tag{1.6.2}$$

$$\underline{a}(-\underline{\theta}) = 4.5 \tag{1.6.3}$$

$$\bar{a} = \frac{3}{2}\underline{a} \tag{1.6.4}$$

$$\gamma^{max} = 33 \cdot 10^6 \tag{1.6.5}$$

$$b^{max} = \frac{1}{3} \tag{1.6.6}$$

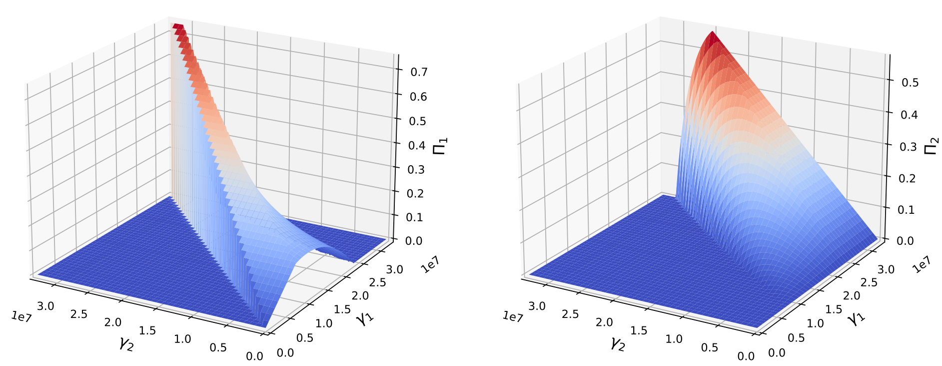
(1.6.1) and (1.6.2) correspond to a scenario with a large rare loss, e.g., a 100 million loss from a sudden movement in exchange rates that occurs every 33 years on average. (1.6.3), (1.6.4), (1.6.5) and (1.6.6) are chosen in the simplest way such that assumptions A4, A5, A3 and A2, respectively, are satisfied.

Based on Proposition 8, we first consider $b_1 = 0$.

Solving for the price equilibrium for $(0, b_2)$ *for some fixed* b_2. For $(0, b_2)$, we numerically solve for the indifferent client as a function of prices (γ_1, γ_2). As an illustration, for $b_2 = 0.15$, Figure 1.6.1 shows the resulting profit functions for both insurers.

Figure 1.6.1. Illustration of Insurers' Profit Functions at $b_2 = 0.15$

(a) Insurer 1 **(b)** Insurer 2

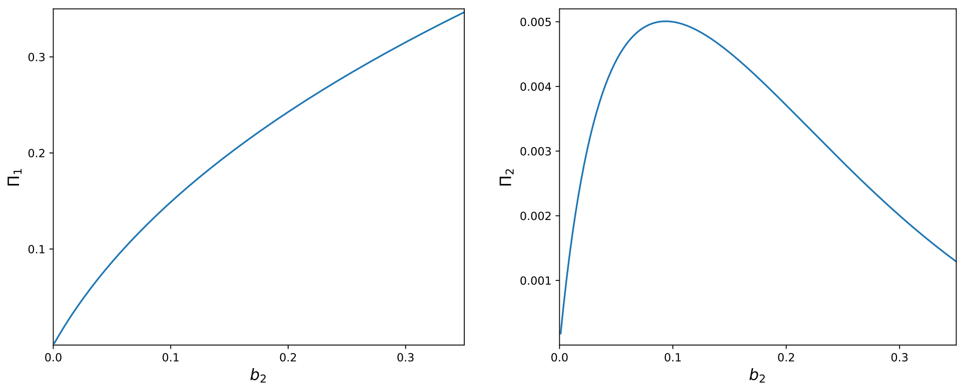


Notes: The figure shows insurer 1's (Panel (a)) and insurer 2's (Panel (b)) profits (z-axis) as functions of prices (γ_1 on the x-axis, γ_2 on the y-axis) in a numerical example. Profit functions are drawn for the following vector of default risks: $b_0 = 0, b_1 = 0.15$. The parameter values used in the numerical example are listed in the text.

Equilibrium profits for $(0, b_2)$ *as a function of* b_2. We then solve for price equilibria (and subsequently profits) for a range of b_2. Figure 1.6.2 shows the resulting equilibrium profits for both insurers as a function of b_2. In particular, the second

Figure 1.6.2. Equilibrium Profits as a Function of b_2

(a) Insurer 1 **(b)** Insurer 2



Notes: The figure shows insurer 1's (Panel (a)) and insurer 2's (Panel (b)) profits (y-axis) as functions of the default risk gap in a numerical example. As motivated in the text, b_1 is fixed at zero and insurer 2's default risk, b_2, is depicted on the x-axis. The parameter values used in the numerical example are listed in the text.

mover's profit as a function of b_2 has a unique interior maximum, while the first mover's profit as a function of b_2 is increasing. That is, conditions (N1) and (N2) hold. Additionally, insurer 1's are an order of magnitude larger than insurer 2's.

Equilibrium qualities. We then calculate $\bar{b}_1 \approx 0.0023$, hence the resulting equilibrium default risks are

$$b_1^* \in [0, 0.0023] \tag{1.6.7}$$

$$b_2^* = 0.9972 b_1^* + 0.0937. \tag{1.6.8}$$

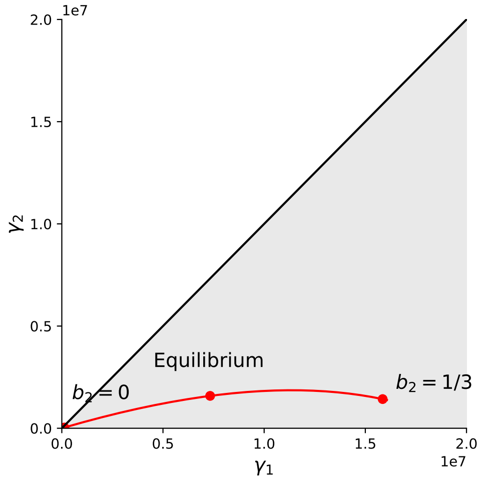
In particular, the first mover chooses a default risk close zero, and the second mover follows at a distance that equals the optimal default risk gap. This default risk gap is lower than a third of the largest admissible default risk in the model setup.

Equilibrium prices are depicted in Figure 1.6.3.

1.7 Application to Derivatives Markets

An application of the model setup is the derivatives market, as it naturally maps key features of these markets. First, the stylized insurance contract considered in the model is typical for derivatives markets: the contract features full coverage against a macro risk (as does, e.g., a plain-vanilla interest rate swap) but comes

Figure 1.6.3. Equilibrium Prices



Notes: The figure shows optimal prices at varying default risk gaps in a numerical example. Insurer 1's and insurer 2's prices are depicted on the x- and y-axis, respectively. The red line depicts pairs of optimal prices for $(0, b_2)$ with b_2 ranging in $[0, 1/3]$, with the prices at the equilibrium in default risks marked. The parameter values used in the numerical example are listed in the text.

with the risk of counterparty default. Thus, both the price of the derivative and the counterparty risk may influence the purchase decision. Second, derivatives markets have a hub-and-spoke structure with numerous clients with differing risk attitudes seeking insurance from a small set of large banks (called *dealers*) – which aligns with the model structure. Third, dealers choose their own default risk, e.g., by setting aside capital, choosing liquidity buffers, or maintaining balanced trading books. Gregory (2014, p.135), for example, details how an institution's creditworthiness as assessed by ratings plays a role, as well as its capital base, liquidity, and operational requirements for processing trades.

Introducing a central counterparty (CCP) in derivatives markets raises questions about its impact on competition among dealers. In a centrally cleared market, a CCP interposes itself between a buyer and a seller, replacing the existing contract between them with two contracts that each have the CCP on one end. It thereby insulates the contracting parties from the risk that the counterparty defaults. CCPs can support financial stability through netting, enforcing margining and improving transparency for better regulatory oversight. However, the effects on competition in a highly concentrated market are little understood.

The starting point of this project was the observation that in a centrally cleared market unless there is a default, the client-dealer relationship remains largely unchanged because clients do not directly interact with the CCP. Only members of the CCP can directly clear with the CCP, while most market participants access clearing services through members (CPMI, IOSCO, 2022). Consider a trade where a market participant buys a derivative from a dealer. Suppose the common situation in which the dealer is a member of the CCP and not only the executing broker of the trade but also the client's clearing service provider. Then, the resulting flow is: client - client account at the dealer - CCP - house account dealer. Thus, from the dealer's perspective, client clearing changes little as the CCP protects the dealer from its own default while the client still buys the derivative from the dealer.

However, central clearing alters the nature of competition between dealers, and my model is able to clarify a market force that may be absent in a centrally cleared market. A CCP facilitates porting arrangements, meaning that if a clearing member defaults, clients' portfolios are transferred to another solvent member (Braithwaite, 2016; Braithwaite and Murphy, 2020). The success of the London Clearing House (LCH) during the Global Financial Crisis can largely be attributed to such porting arrangements. From the client's perspective, there is no longer differentiation in contract continuity between dealers, eliminating this quality dimension of competition. Viewed through the lens of the model, the market force that incentivizes dealers to choose a low default risk – beyond requirements mandated by regulation – may then be absent: This paper shows that with two-dimensional competition in price and default risk, market discipline in the choice of default risks emerges. Without perceived differences in default risks, pure price competition prevails.

1.8 Conclusion

I study precautionary measures insurance sellers undertake to ensure their solvency within a model of vertical product differentiation. To that end, I introduce the seller's default risk as a quality dimension of the insurance product. Analogous to standard analyses of vertical product differentiation, I show that more risk-averse clients self-select to buy from the dealer with the lower default risk, leading to market segmentation and higher profits for the dealer with the lower default risk. The key insight from the model is that competition in two dimensions (price, default risk) gives rise to market discipline in insurers' default risk choices: the first over in the choice of default risk chooses a low default risk, and the second mover follows suit.

I discuss the model implications for competition in derivatives markets. The result highlights a market force that may be absent in a centrally cleared market where dealers compete for clients but are insulated from competition in default risk.

A central counterparty in the model framework is conceptualized ad-hoc and not formally introduced, leaving many aspects of central clearing (e.g., loss-sharing mechanisms, margins, CCP's default probability) beyond the scope of the current model. Retaining the simple framework that maps the market structure with client clearing and incorporates risk aversion while modeling a CCP in more detail is left for future research.

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Appendix 1.A Proofs

Remark. I use the following notation. For functions $G : \mathbb{R}^3 \to \mathbb{R}$ and $H : \mathbb{R}^2 \to \mathbb{R}^3$,

$$D(G \circ H) \;=\; \big(\partial_1(G \circ H), \partial_2(G \circ H)\big) \;=:\; \big(d_1 G, d_2 G\big).$$

This is to indicate that the chain rule on the composite function is considered although we write G instead of the composite function $(G \circ H)$ for brevity. Here, typically $H : (\gamma_1, \gamma_2) \mapsto (a^*(\gamma_1, \gamma_2), \gamma_1, \gamma_2)$.

1.A.1 Proof of Lemma 1

For the indifferent client we have

$$U_a(b_1, \gamma_1) \;=\; U_a(b_2, \gamma_2) \tag{1.A.1}$$

$$\Leftrightarrow\; (1 - b_1 p) u_a(-\gamma_1) + b_1 p u_a(\underline{\theta}) \;=\; (1 - b_2 p) u_a(-\gamma_2) + b_2 p u_a(\underline{\theta}) \tag{1.A.2}$$

$$\Leftrightarrow\; u_a(-\gamma_1) - u_a(-\gamma_2) + p\,[b_2 u_a(-\gamma_2) - b_1 u_a(-\gamma_1)] \;=\; p \Delta b u_a(\underline{\theta}) \tag{1.A.3}$$

$$\Leftrightarrow\; [u_a(-\gamma_1) - u_a(-\gamma_2)]\,(1 - b_1 p) \;=\; p \Delta b \big[u_a(\underline{\theta}) - u_a(-\gamma_2)\big] \tag{1.A.4}$$

$$\Leftrightarrow\; \frac{u_a(-\gamma_1) - u_a(-\gamma_2)}{u_a(\underline{\theta}) - u_a(-\gamma_2)} \;=\; \frac{p \Delta b}{1 - b_1 p} \tag{1.A.5}$$

$$\Leftrightarrow\; \frac{\exp(-a \Delta \gamma) - 1}{\exp(-a(\underline{\theta} + \gamma_2)) - 1} \;=\; \frac{p \Delta b}{1 - b_1 p}. \tag{1.A.6}$$

□

1.A.2 Proof of Proposition 2

ad i). The proof proceeds by showing that $\partial_a g < 0$. Suppose this was true. Then the LHS of (1.3.1) is monotonically decreasing, while the RHS of (1.3.1) is fixed, yielding at most one solution.

Claim. $\partial_a g < 0$.

Proof of claim. For the derivative of the function g with respect to a we get

$$\frac{\partial g(a)}{\partial a} = \frac{-\Delta\gamma \exp(-a\Delta\gamma)\left(\exp(-a(\underline{\theta}+\gamma_2))-1\right)}{\left(\exp(-a(\underline{\theta}+\gamma_2))-1\right)^2} + \frac{\left(\exp(-a\Delta\gamma)-1\right)(\underline{\theta}+\gamma_2)\exp(-a(\underline{\theta}+\gamma_2))}{\left(\exp(-a(\underline{\theta}+\gamma_2))-1\right)^2} \tag{1.A.7}$$

$$= \frac{1}{\left(\exp(-a(\underline{\theta}+\gamma_2))-1\right)^2}\Big[\exp(-a\Delta\gamma)\Big(-\Delta\gamma\left(\exp(-a(\underline{\theta}+\gamma_2))-1\right) + (\underline{\theta}+\gamma_2)\exp(-a(\underline{\theta}+\gamma_2))\Big) - (\underline{\theta}+\gamma_2)\exp\left(-a(\underline{\theta}+\gamma_2)\right)\Big] \tag{1.A.8}$$

$$= \frac{1}{\left(\exp(-a(\underline{\theta}+\gamma_2))-1\right)^2}\Big[\exp(-a\Delta\gamma)\Big(\exp\left(-a(\underline{\theta}+\gamma_2)\right)(\underline{\theta}+\gamma_1)+\Delta\gamma\Big) - (\underline{\theta}+\gamma_2)\exp(-a(\underline{\theta}+\gamma_2))\Big] \tag{1.A.9}$$

$$= \underbrace{\frac{\exp(-a\Delta\gamma)}{\left(\exp(-a(\underline{\theta}+\gamma_2))-1\right)^2}}_{>0}\Big[\underbrace{\Delta\gamma}_{<0} + \underbrace{\exp\left(-a(\underline{\theta}+\gamma_1)\right)}_{>0}\underbrace{\Big(\exp(-a\Delta\gamma)(\underline{\theta}+\gamma_1)-(\underline{\theta}+\gamma_2)\Big)}_{:=f(a)}\Big] \tag{1.A.10}$$

using that

$$\exp(-a(\underline{\theta}+\gamma_2)) = \exp(-a(\underline{\theta}+\gamma_1))\exp(-a\Delta\gamma). \tag{1.A.11}$$

Then

$$f(a) < 0 \Rightarrow \frac{\partial g(a)}{\partial a} < 0. \tag{1.A.12}$$

We have

$$f(a) = \exp(-a\Delta\gamma)(\underline{\theta}+\gamma_1) - (\underline{\theta}+\gamma_2) < 0 \tag{1.A.13}$$

$$\Leftrightarrow \exp(-a\Delta\gamma)(\underline{\theta}+\gamma_1) < (\underline{\theta}+\gamma_2) \tag{1.A.14}$$

$$\Leftrightarrow \frac{\exp(-a(\underline{\theta}+\gamma_2))}{\exp(-a(\underline{\theta}+\gamma_1))}(\underline{\theta}+\gamma_1) < (\underline{\theta}+\gamma_2) \tag{1.A.15}$$

$$\Leftrightarrow \frac{\exp(-a(\underline{\theta}+\gamma_2))}{(\underline{\theta}+\gamma_2)} < \frac{\exp(-a(\underline{\theta}+\gamma_1))}{(\underline{\theta}+\gamma_1)}. \tag{1.A.16}$$

For $x < 0$ the function

$$h(x) := \frac{\exp(-ax)}{x} \tag{1.A.17}$$

is negative and

$$h'(x) \;=\; h(x)\left[-a-\frac{1}{x}\right] \;>\; 0 \quad\Leftrightarrow\quad a+\frac{1}{x} \;>\; 0 \quad\Leftrightarrow\quad a(-x) \;>\; 1. \tag{1.A.18}$$

For $x = \underline{\theta} + \gamma$ this is true from assumption A3. Since $\underline{\theta} + \gamma_2 < \underline{\theta} + \gamma_1$, (1.A.16) indeed holds and proves the claim.

ad ii). With $g(\cdot, \vec{\gamma})$ strictly decreasing, existence under (1.3.3) follows immediately.

ad iii). A client with risk aversion parameter a chooses insurer 1 if

$$U_a(b_1,\gamma_1) \;>\; U_a(b_2,\gamma_2) \tag{1.A.19}$$

$$\Leftrightarrow\; (1-b_1p)u_a(-\gamma_1) + b_1pu_a(\underline{\theta}) \;>\; (1-b_2p)u_a(-\gamma_2) + b_2pu_a(\underline{\theta}) \tag{1.A.20}$$

$$\Leftrightarrow\; [u_a(-\gamma_1) - u_a(-\gamma_2)](1-b_1p) \;>\; p\Delta b\underbrace{\big(u_a(\underline{\theta}) - u_a(-\gamma_2)\big)}_{<0} \tag{1.A.21}$$

$$\Leftrightarrow\; \frac{u_a(-\gamma_1) - u_a(-\gamma_2)}{u_a(\underline{\theta}) - u_a(-\gamma_2)} \;<\; \frac{p\Delta b}{1-b_1p} \tag{1.A.22}$$

$$\Leftrightarrow\; g(a) \;<\; g(a^*) \tag{1.A.23}$$

$$\Leftrightarrow\; a \;>\; a^*(\gamma_1,\gamma_2). \tag{1.A.24}$$

□

1.A.3 Proof of Lemma 11

The idea is to proceed analogously to the proof of Proposition 1, but add and subtract $b_2u_a(-\gamma_1)$ instead of $b_1u_a(-\gamma_2)$. Namely, for the indifferent client we have

$$U_a(b_1,\gamma_1) \;=\; U_a(b_2,\gamma_2) \tag{1.A.25}$$

$$\Leftrightarrow\; u_a(-\gamma_1) - u_a(-\gamma_2) + p\,[b_2u_a(-\gamma_2) - b_1u_a(-\gamma_1)] \;=\; p\Delta bu_a(\underline{\theta}) \tag{1.A.26}$$

$$\Leftrightarrow\; [u_a(-\gamma_1) - u_a(-\gamma_2)](1-b_2p) \;=\; p\Delta b\big[u_a(\underline{\theta}) - u_a(-\gamma_1)\big] \tag{1.A.27}$$

$$\Leftrightarrow\; \frac{u_a(-\gamma_1) - u_a(-\gamma_2)}{u_a(\underline{\theta}) - u_a(-\gamma_1)} \;=\; \frac{p\Delta b}{1-b_2p} \tag{1.A.28}$$

$$\Leftrightarrow\; \frac{1-\exp(-(-a\Delta\gamma))}{\exp(-a(\underline{\theta}+\gamma_1))-1} \;=\; \frac{p\Delta b}{1-b_2p}. \tag{1.A.29}$$

□

1.A.4 Proof of Lemma 3

An auxiliary lemma offers a second characterization of the indifferent client, symmetric to the one in Lemma 1, and exploiting this symmetry will be key in the sequel.

Lemma 11. *The client who is indifferent between two contracts* (b_1, γ_1) *and* (b_2, γ_2) *with* $\Delta b > 0$*, has a second characterization*

$$h(a, \vec{\gamma}) := \frac{1 - \exp(-(-a\Delta\gamma))}{\exp(-a(\underline{\theta} + \gamma_1)) - 1} = \frac{p\Delta b}{1 - b_2 p}. \tag{1.A.30}$$

Proof.
The idea is to proceed analogously to the proof of Proposition 1, but add and subtract $b_2 u_a(-\gamma_1)$ instead of $b_1 u_a(-\gamma_2)$. Namely, for the indifferent client we have

$$U_a(b_1, \gamma_1) = U_a(b_2, \gamma_2) \tag{1.A.31}$$

$$\Leftrightarrow u_a(-\gamma_1) - u_a(-\gamma_2) + p\left[b_2 u_a(-\gamma_2) - b_1 u_a(-\gamma_1)\right] = p\Delta b u_a(\underline{\theta}) \tag{1.A.32}$$

$$\Leftrightarrow [u_a(-\gamma_1) - u_a(-\gamma_2)](1 - b_2 p) = p\Delta b\left[u_a(\underline{\theta}) - u_a(-\gamma_1)\right] \tag{1.A.33}$$

$$\Leftrightarrow \frac{u_a(-\gamma_1) - u_a(-\gamma_2)}{u_a(\underline{\theta}) - u_a(-\gamma_1)} = \frac{p\Delta b}{1 - b_2 p} \tag{1.A.34}$$

$$\Leftrightarrow \frac{1 - \exp(-(-a\Delta\gamma))}{\exp(-a(\underline{\theta} + \gamma_1)) - 1} = \frac{p\Delta b}{1 - b_2 p}. \tag{1.A.35}$$

□

Following the notation as noted in the remark at the beginning of the appendix, $\partial_i a^* = d_i a^*$. We show three claims from which Lemma 3 directly follows.

Claim 1. $a^* = \tau_1(d_1 a^*)$ with $\tau_1, (d_1 a^*) > 0$.

Claim 2. $a^* = \tau_2(-d_2 a^*)$ with $\tau_2, (-d_2 a^*) > 0$.

Claim 3. $(-d_2 a^*)/(d_1 a^*) =: \alpha = (1 - gB_1) = 1/(1 + hB_2) = \tau_1/\tau_2 < 1$.

Proof of claim 1. For the function $g(a^*(\vec{\gamma}), \vec{\gamma})$, as defined in (1.3.1), we have from Proposition 1

$$0 = d_1 g = \partial_1 g|_{a=a^*} + \partial_a g|_{a=a^*} \cdot d_1 a^* \tag{1.A.36}$$

$$\Leftrightarrow d_1 a^* = \frac{-\partial_1 g|_{a=a^*}}{\partial_a g|_{a=a^*}}. \tag{1.A.37}$$

In the following write $\partial_i g$ shorthand for $\partial_i g|_{a=a^*}$. We have

$$\partial_1 g = a^* \frac{A}{B_2 - 1} > 0. \tag{1.A.38}$$

and from Proposition 2 we know that $\partial_a g < 0$. Hence, in light of (1.A.37), we have $d_1 a^* > 0$.

Further, note that the expression for $\partial_a g$, derived in the proof of Proposition 2, can be written in short-hand notation as follows

$$\partial_a g \;=\; \frac{A}{(B_2-1)}\Big[-\Delta\gamma + (\underline{\theta}+\gamma_2)\underbrace{\frac{(A-1)}{(B_2-1)}}_{=g}\underbrace{\frac{B_2}{A}}_{=B_1}\Big] \overset{(1.A.38)}{=} \frac{\partial_1 g}{a^*}\left[-\Delta\gamma + g\varphi_1\right]. \tag{1.A.39}$$

Inserted into (1.A.36) this yields

$$0 \;=\; \partial_1 g + \frac{\partial_1 g}{a^*}(-\Delta\gamma + g\varphi_1)d_1 a^* \tag{1.A.40}$$

$$\;=\; \underbrace{\frac{\partial_1 g}{a^*}}_{>0}\left[a^* + (-\Delta\gamma + g\varphi_1)d_1 a^*\right]. \tag{1.A.41}$$

Hence

$$a^* \;=\; (\Delta\gamma - g\varphi_1)\underbrace{d_1 a^*}_{>0}, \tag{1.A.42}$$

and subsequently

$$\tau_1 \;=\; (\Delta\gamma - g\varphi_1) \;>\; 0. \tag{1.A.43}$$

Proof of claim 2. Analogously, for the function $h(a^*(\vec{\gamma}),\vec{\gamma})$, as defined in (1.A.30), we have

$$0 \;=\; d_2 h \;=\; \partial_2 h|_{a=a^*} + \partial_a h|_{a=a^*}\cdot d_2 a^* \tag{1.A.44}$$

$$\Leftrightarrow\; d_2 a^* \;=\; \frac{-\partial_2 h|_{a=a^*}}{\partial_a h|_{a=a^*}}. \tag{1.A.45}$$

Similar to before we write $\partial_i h$ shorthand for $\partial_i h|_{a=a^*}$. Then we have

$$\partial_2 h \;=\; (-a^*)\frac{1}{A(B_1-1)} \;<\; 0, \tag{1.A.46}$$

and

$$\begin{aligned}
\partial_a h &= (-\Delta\gamma)\frac{1}{A(B_1-1)} + (\underline{\theta}+\gamma_1)\frac{(1-\frac{1}{A})B_1}{(B_1-1)^2} && (1.A.47)\\
&= \frac{1}{A(B_1-1)^2}\left[\Delta\gamma - \Delta\gamma B_1 - (\underline{\theta}+\gamma_1)B_1 + (\underline{\theta}+\gamma_1)AB_1\right] && (1.A.48)\\
&= \underbrace{\frac{1}{A(B_1-1)^2}}_{>0}\Big[\underbrace{\Delta\gamma}_{<0} + \underbrace{B_1}_{>0}\big(A(\underline{\theta}+\gamma_1) - (\underline{\theta}+\gamma_2)\big)\Big]. && (1.A.49)
\end{aligned}$$

From the proof of Proposition 2 we know that $A(\underline{\theta}+\gamma_1)-(\underline{\theta}+\gamma_2)$ is negative, hence $\partial_a h < 0$. Then from (1.A.45) we get $d_2 a^* < 0$.

For the remaining part, note that $AB_1 = B_2$ and hence (1.A.47) can also be written as

$$\begin{aligned}
\partial_a h &= \frac{1}{A(B_1-1)}\left[-\Delta\gamma + (\underline{\theta}+\gamma_1)AB_1\frac{(1-\frac{1}{A})B_1}{(B_1-1)}\right] && (1.A.50)\\
&= \frac{\partial_a h}{a^*}\left[\Delta\gamma - \varphi_2 h\right]. && (1.A.51)
\end{aligned}$$

Inserted into (1.A.44) this yields

$$0 = \underbrace{\frac{\partial_2 h}{a^*}}_{<0}\left[a^* + (\Delta\gamma - \varphi_2 h)d_2 a^*\right]. \tag{1.A.52}$$

Hence,

$$a^* = -(\Delta\gamma - \varphi_2 h)\underbrace{d_2 a^*}_{<0}, \tag{1.A.53}$$

and subsequently

$$\tau_2 = (\Delta\gamma - \varphi_2 h) > 0. \tag{1.A.54}$$

Proof of claim 3. We first establish that

$$(1-gB_1) = \frac{B_1-1}{B_2-1} = \frac{1}{(1+hB_2)}. \tag{1.A.55}$$

This follows, since from the definition

$$\begin{aligned}
1-gB_1 &= 1-\frac{A-1}{B_2-1}\frac{B_2}{A} = \frac{B_2-A}{A(B_2-1)} = \frac{B_1-1}{B_2-1} && (1.A.56)\\
1+hB_2 &= 1+\frac{1-\frac{1}{A}}{B_1-1}B_2 = \frac{B_1-1+B_2-\frac{B_2}{A}}{B_1-1} = \frac{B_2-1}{B_1-1}. && (1.A.57)
\end{aligned}$$

In light of (1.3.14) and (1.3.15) we have

$$\begin{aligned}
\alpha &= \frac{\Delta\gamma - g\varphi_1}{\Delta\gamma - h\varphi_2} && \text{(1.A.58)}\\
&= \frac{(\underline{\theta}+\gamma_2)-(\underline{\theta}+\gamma_1)-g\varphi_1}{(\underline{\theta}+\gamma_2)-(\underline{\theta}+\gamma_1)-h\varphi_2} && \text{(1.A.59)}\\
&= \frac{(\underline{\theta}+\gamma_2)(1-gB_1)-(\underline{\theta}+\gamma_1)}{-(\underline{\theta}+\gamma_1)(1+hB_2)+(\underline{\theta}+\gamma_2)} && \text{(1.A.60)}\\
&= \frac{(1-gB_1)\Big((\underline{\theta}+\gamma_2)-\frac{1}{(1-gB_1)}(\underline{\theta}+\gamma_1)\Big)}{(\underline{\theta}+\gamma_2)-(1+hB_2)(\underline{\theta}+\gamma_1)} && \text{(1.A.61)}\\
&= (1-gB_1), && \text{(1.A.62)}
\end{aligned}$$

which concludes the proof.

□

1.A.5 Auxiliary Properties

Proposition 12. *As always, we consider the set $\mathscr{G}_{[\underline{a},\overline{a}]}$. Then the following properties hold*

$$\begin{aligned}
d_2^2 a^* &= \frac{(-d_2a^*)}{\tau_2}\Big[2 + h\varphi_2 a^*\big(1-\frac{\xi_2}{\tau_2}\big)\Big] < 0 && \text{(1.A.63)}\\
d_1d_2a^* &= (d_1a^*)^2\frac{\alpha}{a^*}\Big[a^*\xi_2\frac{h\varphi_2}{\tau_2} - 2\Big] > 0 && \text{(1.A.64)}\\
d_1^2a^* &= \Big[\frac{2}{a^*} - g\varphi_1\xi_2\frac{\alpha}{\tau_1}\Big] && \text{(1.A.65)}\\
d_2^2\Pi_2 &= (d_2a^*)\Big[2 + \frac{\gamma_2}{\tau_2}\Big(\frac{(a^*\xi_1)h\varphi_2}{\tau_1} - 2\Big)\Big] < 0 && \text{(1.A.66)}\\
d_1d_2\Pi_2 &= (d_1a^*)\Big[1 + \frac{\gamma_2}{\tau_2}\Big(\frac{(a^*\xi_2)h\varphi_2}{\tau_2} - 2\Big)\Big] > 0 && \text{(1.A.67)}\\
d_1^2\Pi_1 &= (-d_1a^*)\Big[2 + \frac{\gamma_1}{\tau_1}\Big(2 - \frac{a^*\xi_2 g\varphi_1}{\tau_2}\Big)\Big] && \text{(1.A.68)}\\
d_1d_2\Pi_1 &= (-d_2a^*)\Big[1 + \frac{\gamma_1}{\tau_1}\Big(2 - \frac{a^*\xi_2 h\varphi_2}{\tau_2}\Big)\Big] && \text{(1.A.69)}\\
d_1^2\Pi_1 + \frac{1}{\alpha}d_1d_2\Pi_1 &< 0, \textit{ hence } d_1^2\Pi_1 \neq 0 \vee d_1d_2\Pi_1 \neq 0. && \text{(1.A.70)}
\end{aligned}$$

Proof.
ad $d_2^2a^*$. Note that

$$\frac{\xi_1}{\tau_1} - \frac{\xi_2}{\tau_2} = \frac{1}{\tau_1}\underbrace{[\xi_1 - \alpha\xi_2]}_{=-\tau_1} = -1 \qquad \text{(1.A.71)}$$

We have,

$$
\begin{aligned}
d_2^2 a^* &= d_2\left(-\frac{a^*}{\tau_2}\right) && (1.A.72)\\
&= -\frac{d_2 a^*}{\tau_2} + a^*\frac{1}{\tau_2^2} d_2\tau_2 && (1.A.73)\\
&= -\frac{d_2 a^*}{\tau_2}[1 + d_2\tau_2] && (1.A.74)\\
&= -\frac{d_2 a^*}{\tau_2}[1 + 1 + h\xi_1 B_2(a^* + (d_2 a^*)\xi_2)] && (1.A.75)\\
&= -\frac{d_2 a^*}{\tau_2}\left[2 + h\varphi_2 a^*\left(1 - \frac{\xi_1}{\tau_2}\right)\right] && (1.A.76)\\
&= -\frac{d_2 a^*}{\tau_2}\left[2 + h\varphi_2 a^*\frac{\xi_1}{\tau_1}\right] && (1.A.77)\\
&= \underbrace{(d_2 a^*)}_{<0}\frac{1}{\tau_2^2}\left[-2\tau_2 + h\varphi_2(-a^*\xi_1)\frac{1}{\alpha}\right] && (1.A.78)
\end{aligned}
$$

$d_2^2 a^*$ is negative iff

$$
-a^*\xi_1 > 2\underbrace{\frac{\tau_2}{-h\varphi_2}}_{\in(0,1)}\underbrace{\alpha}_{<1}, \qquad (1.A.79)
$$

which is ensured by $-\underline{a}\xi_1 > 2$ from assumption A3.

ad $d_1 d_2 a^*$. We have

$$
\begin{aligned}
d_1\varphi_2 &= B_2 + \xi_1 d_1 B_2 && (1.A.80)\\
&= B_2 + \xi_1 B_2(-d_2 a^*)\xi_2 && (1.A.81)\\
&= B_2\big((1 - \xi_1\xi_2 d_1 a^*\big) && (1.A.82)
\end{aligned}
$$

Then

$$
\begin{aligned}
d_1 d_2 a^* &= -d_1\left(\frac{a^*}{\tau_2}\right) && (1.A.83)\\
&= -\frac{d_1 a^*\tau_2 - a^*(-1 - hB_2(1 - \xi_1\xi_2 d_1 a^*)}{\tau_2^2} && (1.A.84)\\
&= \frac{-d_1 a^*[\tau_2 - a^*\xi_1\xi_2 hB_2] + a^*(1 + hB_2)}{\tau_2^2} && (1.A.85)\\
&= \underbrace{\frac{-d_1 a^*}{\tau_2^2}}_{<0}\underbrace{[\tau_2 - a^*\xi_1\xi_2 hB_2 + \tau_1(1 + hB_2)]}_{=:W} && (1.A.86)
\end{aligned}
$$

Hence, $d_1 d_2 a^* > 0$ if the expression in brackets is negative. This is indeed the case, since

$$\begin{aligned} W &= 2\Delta\gamma + hB_2[\tau_1 - \xi_1 - a^*\xi_1\xi_2] - g\varphi_1 && (1.A.87) \\ &= \Delta\gamma(2 + hB_2) - \xi_1 hB_2 \underbrace{(1 + a^*\xi_2)}_{=-1+(2+a^*(\underline{\theta}+\gamma_2))} - g\varphi_1(1 + hB_2) && (1.A.88) \\ &= hB_2[\xi_1 + \Delta\gamma] - g\xi_2 B_1(1 + hB_2) - \xi_1 hB_2(2 + a^*\xi_2) + 2\Delta\gamma && (1.A.89) \\ &= \xi_2 \underbrace{\big(hB_2 - gB_1(1 + hB_2)\big)}_{=\frac{\Delta B}{B_1-1}-\frac{\Delta B}{B_2-1}\frac{B_1-1}{B_2-1}=0} - \xi_1 hB_2(2 + a^*\xi_2) + 2\Delta\gamma && (1.A.90) \\ &= \underbrace{-\xi_1}_{>0} hB_2 \underbrace{(2 + a^*\xi_2)}_{<0} + \underbrace{2\Delta\gamma}_{<0} && (1.A.91) \\ &< 0, && (1.A.92) \end{aligned}$$

which together yields

$$\begin{aligned} d_1 d_2 a^* &= \frac{(d_1 a^*) h\varphi_2}{\tau_2^2} \left(a^*\xi_2 + 2\frac{\tau_2}{(-h\varphi_2)} \right) && (1.A.93) \\ &= \frac{(d_1 a^*) h\varphi_2}{\tau_2^2} \left[a^*\xi_2 + 2\frac{\tau_2}{(-h\varphi_2)} \right] && (1.A.94) \\ &= (d_1 a^*)^2 \frac{\alpha}{a^*} \left[a^*\xi_2 \frac{h\varphi_2}{\tau_2} - 2 \right]. && (1.A.95) \end{aligned}$$

ad $d_1^2 a^*$. We know

$$d_1\varphi_1 = \varphi_1\xi_2\alpha(d_1 a^*), \qquad (1.A.96)$$

hence

$$\begin{aligned} d_1^2 a^* &= d_1\left(\frac{a^*}{\tau_1}\right) && (1.A.97) \\ &= \frac{d_1 a^*}{\tau_1} + a^* d_1\left(\frac{1}{\tau_1}\right) && (1.A.98) \\ &\overset{(1.A.96)}{=} \frac{d_1 a^*}{\tau_1} - a^* \frac{1}{(\tau_1)^2}[-1 + g\varphi_1\xi_2\alpha(d_1 a^*)] && (1.A.99) \\ &= \frac{d_1 a^*}{\tau_1}[2 - g\varphi_1\xi_2\alpha(d_1 a^*)] && (1.A.100) \\ &= (d_1 a^*)\frac{a^*}{\tau_1}\left[\frac{2}{a^*} - g\varphi_1\xi_2\alpha\frac{d_1 a^*}{a^*}\right] && (1.A.101) \\ &= (d_1 a^*)^2\left[\frac{2}{a^*} - g\varphi_1\xi_2\frac{\alpha}{\tau_1}\right]. && (1.A.102) \end{aligned}$$

ad $d_2^2 \Pi_2$. Using (1.A.64) and (1.A.63),

$$d_2^2\Pi_2 = 2d_2a^* + \gamma_2 d_2^2 a^* \tag{1.A.103}$$

$$= (d_2a^*)\left[2 + \frac{\gamma_2}{\tau_2^2}\Big((-h\varphi_2)a^*(\tau_2 - \xi_2) - 2\tau_2\Big)\right] < 0 \tag{1.A.104}$$

We use $\xi_1/\tau_1 - \xi_2/\tau_2 = (-1)$ to simplify to

$$d_2^2\Pi_2 = (d_2a^*)\left[2 + \frac{\gamma_2}{\tau_2}\Big(\frac{(a^*\xi_1)h\varphi_2}{\tau_1} - 2\Big)\right]. \tag{1.A.105}$$

ad $d_1d_2\Pi_2$. Using (1.A.64) and (1.A.63),

$$d_1d_2\Pi_2 = (d_1a^*) + (d_1d_2a^*)\gamma_2 \tag{1.A.106}$$

$$= (d_1a^*)\Big[1 + \frac{\gamma_2}{\tau_2^2}\underbrace{\Big((-a^*\xi_2)(-h\varphi_2) - 2\tau_2\Big)}_{:=E}\Big] > 0, \tag{1.A.107}$$

since $E > 0$ by assumption (A4). Again this further simplifies to

$$d_1d_2\Pi_2 = (d_1a^*)\Big[1 + \frac{\gamma_2}{\tau_2}\Big(\frac{(a^*\xi_2)h\varphi_2}{\tau_2} - 2\Big)\Big]. \tag{1.A.108}$$

ad $d_1^2\Pi_1$. Using (1.A.65),

$$d_1^2\Pi_1 = d_1\left[(\overline{a} - a^*) - (d_1a^*)\gamma_1\right] \tag{1.A.109}$$

$$= -2(d_1a^*) - \gamma_1(d_1^2a^*) \tag{1.A.110}$$

$$= -d_1a^*\left[2 + \gamma_1(d_1a^*)\left(\frac{2}{a^*} - g\varphi_1\xi_2\frac{\alpha}{\tau_1}\right)\right] \tag{1.A.111}$$

$$= -d_1a^*\left[2 + \frac{\gamma_1}{\tau_1}\left(2 - \frac{a^*\xi_2 g\varphi_1}{\tau_2}\right)\right]. \tag{1.A.112}$$

ad $d_1d_2\Pi_1$. Using (1.A.64),

$$d_2d_1\Pi_1 = d_2\left[(\overline{a} - a^*) - (d_1a^*)\gamma_1\right] \tag{1.A.113}$$

$$= (-d_2a^*) - \gamma_1(d_1a^*)^2\frac{\alpha}{a^*}\left[a^*\xi_2\frac{h\varphi_2}{\tau_2} - 2\right] \tag{1.A.114}$$

$$= (-d_2a^*)\left[1 + \frac{\gamma_1}{\tau_1}\left(2 - \frac{a^*\xi_2 h\varphi_2}{\tau_2}\right)\right]. \tag{1.A.115}$$

ad $d_1^2\Pi_1 + \frac{1}{\alpha}d_1 d_2\Pi_1$. Using (1.A.64) and (1.A.66),

$$d_1^2\Pi_1 + \frac{1}{\alpha}(d_1 d_2\Pi_1) = (-d_1 a^*)\left[2 + \frac{\gamma_1}{\tau_1}\left(2 - \frac{(a^*\xi_2)g\varphi_1}{\tau_2}\right)\right] + (d_1 a^*)\left[1 + \frac{\gamma_1}{\tau_1}\left(2 - \frac{(a^*\xi_2)h\varphi_2}{\tau_2}\right)\right] \tag{1.A.116}$$

$$= (-d_1 a^*) + (d_1 a^*)\frac{\gamma_1}{\tau_1}\left[\frac{(a^*\xi_2)g\varphi_1}{\tau_2} - \frac{(a^*\xi_2)h\varphi_2}{\tau_2}\right] \tag{1.A.117}$$

$$= \underbrace{(-d_1 a^*)}_{<0} + (d_1 a^*)\frac{\gamma_1}{\tau_1}\underbrace{\frac{(-a^*\xi_2)}{\tau_2}}_{2/\tau_2>0}\underbrace{(h\varphi_2 - g\varphi_1)}_{<0} \tag{1.A.118}$$

$$< 0, \tag{1.A.119}$$

with $(h\varphi_2 - g\varphi_1) < 0$, since

$$g\varphi_1 - h\varphi_2 = hB_2\Big[\underbrace{\frac{gB_1}{hB_2}}_{=\alpha}\xi_2 - \xi_1\Big] = hB_2\tau_2 > 0 \tag{1.A.120}$$

where the last equality follows, since

$$\tau_2 - \xi_2 = \Delta\gamma - h\varphi_2 - (\xi_1 + \Delta\gamma) = (-\xi_1)(1 + hB_2) = \frac{-\xi_1}{\alpha}. \tag{1.A.121}$$

□

1.A.6 Proof of Proposition 5

Following the notation as noted in the remark at the beginning of the appendix, $\partial_1\gamma_2^* = d_1\gamma_2^*$ and $\partial_i\Pi_2 = d_i\Pi_2$. Auxiliary properties are proven in Appendix 1.A.5. We first prove the following central claim.

Claim. The following notation is used: For a function $f(\vec{\gamma})$ let $f^*(\gamma_1) := f(\gamma_1, \gamma_2^*(\gamma_1))$. Then,

$$d_1\gamma_2^* = \frac{(d_1 d_2\Pi_2)^*}{(-d_2^2\Pi_2)^*}. \tag{1.A.122}$$

Proof of claim. By definition, $0 \equiv (d_2\Pi_2)^*$ and thus

$$0 = d_1((d_2\Pi_2)^*) = (d_1 d_2\Pi_2)^* + (d_2^2\Pi_2)^* d_1\gamma_2^* \tag{1.A.123}$$

$$\Leftrightarrow d_1\gamma_2^* = \frac{(d_1 d_2\Pi_2)^*}{(-d_2^2\Pi_2)^*}. \tag{1.A.124}$$

ad i). From (1.A.66) we have concavity of Π_2, which ensures uniqueness of a solution. For existence, note that $\gamma_2 \mapsto \Pi_2(\gamma_1, \gamma_2)$ as continuous function on a compact interval, assumes its maximum. But $\Pi_2(\gamma_1, 0) = \Pi_2(\gamma_1, \gamma^{max}) = 0$, hence the maximum is assumed in the interior.

ad ii). For $\gamma_2^* \in \mathscr{C}^\infty$, we make use of the implicit function theorem. We know $d_2\Pi_2 \in \mathscr{C}^\infty$ and $d_2^2\Pi_2 < 0$. Hence, from the implicit function theorem the mapping

$$\gamma_1 \;\mapsto\; \gamma_2^*(\gamma_1) \;=\; \arg_{\gamma_2}\{d_2\Pi_2(\gamma_1, \gamma_2) \;=\; 0\} \tag{1.A.125}$$

is smooth. Monotonicity of γ_2^* follows from the claim together with (1.A.67) and (1.A.66).

ad iii). Using (1.A.66) and (1.A.67),

$$\alpha^* d_1\gamma_2^* \;=\; \alpha^* \frac{(d_1 d_2 \Pi_2)^*}{(-d_2^2\Pi_2)^*} \tag{1.A.126}$$

$$= \alpha^* \frac{(d_1 a^*)\left[1 + \frac{\gamma_2}{\tau_2}\left(\frac{(a^*\xi_2)h\varphi_2}{\tau_2} - 2\right)\right]}{(-d_2 a^*)\left[2 + \frac{\gamma_2}{\tau_2}\left(\frac{(a^*\xi_1)h\varphi_2}{\tau_1} - 2\right)\right]} \tag{1.A.127}$$

$$= \frac{1 + \frac{\gamma_2}{\tau_2}\left(\frac{(a^*\xi_2)h\varphi_2}{\tau_2} - 2\right)}{2 + \frac{\gamma_2}{\tau_2}\left(\frac{(a^*\xi_1)h\varphi_2}{\tau_1} - 2\right)}. \tag{1.A.128}$$

In the numerator

$$\frac{(a^*\xi_2)h\varphi_2}{\tau_2} - 2 \;=\; (-a^*\xi_2)\frac{(-h\varphi_2)}{\tau_2} - 2 \;>\; 0, \tag{1.A.129}$$

since $a^*(-\xi_2) > 2$ from assumption A3, and in the denominator

$$\frac{(a^*\xi_1)h\varphi_2}{\tau_1} - 2 \;=\; (a^*h\varphi_2)\left(1 - \frac{\xi_2}{\tau_2}\right) - 2 \;=\; \underbrace{\frac{(a^*\xi_2)h\varphi_2}{\tau_2} - 2}_{>0} + \underbrace{(-h\varphi_2)a^*}_{>0} \;>\; 0, \tag{1.A.130}$$

hence, we get $\alpha^* d_1\gamma_2^* < 1$.

□

1.A.7 Proof of Proposition 6

Following the notation as noted in the remark at the beginning of the appendix, $\partial_2\gamma_1^{\otimes} = d_2\gamma_1^{\otimes}$ and $\partial_i\Pi_1 = d_i\Pi_1$. The proof proceeds by showing a basic lemma from real analysis (Lemma 13) and then proving its applicability in the present context (Lemma 14 and Lemma 15). The lemmata are presented upfront and proven in the subsequent appendices.

Notation. $\gamma_1^{\overline{a}}(\gamma_2)$ is defined similar to $\gamma_1^{\underline{a}}(\gamma_2)$. In particular, $\gamma_1^{\overline{a}}(\gamma_2)$ is defined as $a^*(\gamma_1^{\overline{a}}(\gamma_2), \gamma_2) = \overline{a}$ if there is a solution in $\mathcal{G}_{[\underline{a},\overline{a}]}$, and as $\gamma_1^{\overline{a}}(\gamma_2) = \gamma^{max}$ otherwise.

Lemma 13. *Let f be a smooth function on some interval $[a,b] \subset \mathbb{R}$. If there exists a $\mu \in [a,b]$ such that*

$$\forall x < \mu : \quad df(x) = 0 \Rightarrow d^2f(x) > 0 \tag{S1}$$

$$\forall x > \mu : \quad df(x) = 0 \Rightarrow d^2f(x) < 0 \tag{S2}$$

$$df(a) > 0, \tag{S3}$$

then f has a global maximum τ and $\forall x < \tau : df(x) > 0$ and $\forall x > \tau : df(x) < 0$.

Lemma 14. *Consider a fixed γ_2 for which*

$$d_1\Pi_1(\gamma_1^{\underline{a}}(\gamma_2), \gamma_2) > 0. \tag{T3}$$

If assumption A5 holds, there exists a $\mu \in [\gamma_1^{\underline{a}}(\gamma_2), \gamma_1^{\overline{a}}(\gamma_2)]$ such that for all $\gamma_1 \in [\gamma_1^{\underline{a}}(\gamma_2), \gamma_1^{\overline{a}}(\gamma_2)]$

$$\gamma_1 < \mu \Rightarrow \big(d_1\Pi_1 = 0 \Rightarrow d_1^2\Pi_1(\gamma_1) > 0\big) \tag{T1}$$

$$\gamma_1 > \mu \Rightarrow \big(d_1\Pi_1 = 0 \Rightarrow d_1^2\Pi_1(\gamma_1) < 0\big). \tag{T2}$$

If $\mu = \gamma_1^{\overline{a}}(\gamma_2)$, then $\gamma_1^{\overline{a}}(\gamma_2) = \gamma^{max}$.

Lemma 15. *Assumption A6 implies that, for all $\gamma_2 \in [0, \gamma_2^*(\gamma^{max})]$,*

$$d_1\Pi_1(\gamma_1^{\underline{a}}(\gamma_2), \gamma_2) > 0. \tag{1.A.131}$$

ad i). Lemma 14 and Lemma 15 show that for any $\gamma_2 \in [0, \gamma_2^*(\gamma^{max})]$ we can make use of Lemma 13. Then we know from Lemma 13 that for all $\gamma_2 \in [0, \gamma_2^*(\gamma^{max})]$, a) $\Pi_1(\cdot, \gamma_2)$ has a unique global maximum $\tau \in [\gamma_1^{\underline{a}}(\gamma_2), \gamma_1^{\overline{a}}(\gamma_2)]$, b) $\tau = \text{argmin}_\mu\{d_1\Pi_1 = 0 \vee \mu = \gamma^{max}\}$, i.e., τ is either the unique solution to $d_1\Pi_1(\tau) = 0$, or $\tau = \gamma^{max}$, and, c) for all $\gamma_1 < \tau$: $d_1\Pi_1(\gamma_1) > 0$ and for all $\gamma_1 > \tau$: $d_1\Pi_1(\gamma_1) < 0$.

ad ii). We consider $\gamma_1^{\otimes}$ separately on

$$\mathcal{N}_1 := \{\gamma_2 \in [0, \gamma_2^*(\gamma^{max})] | \gamma_1^{\otimes} < \gamma^{max}\} \tag{1.A.132}$$
$$\mathcal{N}_2 := \{\gamma_2 \in [0, \gamma_2^*(\gamma^{max})] | \gamma_1^{\otimes} = \gamma^{max}\} \tag{1.A.133}$$

and first show continuity on $[0, \gamma_2^*(\gamma^{max})] = \mathcal{N}_1 \dot{\cup} \mathcal{N}_2$.

On $\mathcal{N}_1$, we already know from (1.A.69) that $d_1^2\Pi_1 \neq 0 \vee d_1 d_2 \Pi_1 \neq 0$. Hence $\{d_1\Pi_1 = 0\}$ is a smooth curve. Then we make a case distinction.

i) If $d_1^2\Pi_1 \neq 0$, we know from the implicit function theorem that one can parameterize $\{d_1\Pi_1 = 0\}$ via $\gamma_1^{\otimes}(\gamma_2)$. In particular, such a parameterization is smooth.

ii) At a point $q = (\gamma_1^{\otimes}(\gamma_2), \gamma_2)$ with $d_1^2\Pi_1(q) = 0$, we have $d_1 d_2 \Pi_1(q) \neq 0$, hence one can parameterize $\{d_1\Pi_1 = 0\}$ locally via $\gamma_2^{\otimes}(\gamma_1)$. $\gamma_2^{\otimes}(\gamma_1)$ has to strictly increase in some neighborhood around q or strictly decrease in some neighborhood around q, since otherwise the inverse couldn't exist. Hence, $\gamma_2^{\otimes}$ is bijective on some neighborhood U of $\gamma_1^{\otimes}(\gamma_2)$ and V of $\gamma_2^{\otimes}$. Then $\gamma_1^{\otimes}$ is monotone on U and continuous.

Hence, $\gamma_1^{\otimes}$ is continuous on $\mathcal{N}_1$, $\mathcal{N}_1$ is open and $\gamma_1^{\otimes}$ is also continuous on the closure of $\mathcal{N}_1$, $\overline{\mathcal{N}}_1$. Since the complement of $\mathcal{N}_1$ is $\mathcal{N}_2$, $\mathcal{N}_2$ is closed. On $\mathcal{N}_2$, $\gamma_1^{\otimes}$ is a constant function and as such continuous on $\mathcal{N}_2$. Thus, since $\gamma_1^{\otimes}$ is continuous on $\overline{\mathcal{N}}_1$ and on $\mathcal{N}_2$, it is continuous on $[0, \gamma_2^*(\gamma^{max})]$.

It remains to show smoothness except at isolated points.

Again, we consider $\mathcal{N}_1$ first. If $d_1^2\Pi_1 \neq 0$, the above argument has already shown smoothness. At a point $q = (\gamma_1^{\otimes}(\gamma_2), \gamma_2)$ with $d_1^2\Pi_1(q) = 0$, we have $d_1 d_2 \Pi_1(q) \neq 0$, hence one can parameterize $\{d_1\Pi_1 = 0\}$ locally via $\gamma_2^{\otimes}(\gamma_1)$. $d_1^2\Pi_1(\gamma_1, \gamma_2^{\otimes}(\gamma_1))$ is an analytic function, i.e., the Taylor expansion converges at every point with positive radius of convergence. From complex analysis (see e.g. Theorem 4.8 in Shakarchi and Stein (2003)) we know that, if the zeros of the function accumulate, then $d_1^2\Pi_1(\gamma_1, \gamma_2^{\otimes}(\gamma_1)) \equiv 0$ on some open neighborhood U of γ_1. But this is a contradiction: Consider the image $V := \{(\gamma_1, \gamma_2^{\otimes}(\gamma_1)) | \gamma_1 \in U\} \subset \{d_1\Pi_1 = 0\}$. There, $d_1^2\Pi_1 = 0 \wedge d_1 d_2 \Pi_1 \neq 0$ everywhere. So the tangent to $\{d_1\Pi_1 = 0\}$ may not have a component in d_2-direction. But this means that $\gamma_2^{\otimes}(\gamma_1)$ is constant on U – a contradiction to the argument in the proof of i). This proves the claim on an open neighborhood of $\overline{\mathcal{N}}_1$.

On $\mathcal{N}_2$, $\gamma_1^{\otimes}$ is constant and thus smooth on all interior points of $\mathcal{N}_2$, i.e., except possibly on points on $\mathcal{N}_2 \cap \overline{\mathcal{N}}_1$. But these points are isolated by the proof for $\mathcal{N}_1$.

In addition, exception points are well-behaved:

Claim. Let γ_2^0 be a point at which $\gamma_1^{\otimes}$ is non-differentiable, i.e. $d_1^2\Pi_1(\gamma_1^{\otimes}(\gamma_2^0), \gamma_2^0) = 0$. Then,

i) $d_2\gamma_1^{\otimes}$ converges to minus infinity in γ_2^0.

ii) $\gamma_1^{\otimes}$ decreases in a neighborhood of γ_2^0.

Proof of claim.
Consider a point $q = (\gamma_1^{\otimes}(\gamma_2^0), \gamma_2^0)$ with $d_1^2 \Pi_1(q) = 0$, and a locally inverse function $\gamma_2^{\otimes}$ of the parameterization $\gamma_1^{\otimes}$ in a neighborhood V. Since $d_1 \Pi_1(q) = 0$, $d_1 \gamma_2^{\otimes}(\gamma_1^{\otimes}) = 0$ and $\lim_{\gamma_1 \to \gamma_1^{\otimes}} d_1 \gamma_2^{\otimes}(\gamma_1) = 0$. From monotonicity of $\gamma_2^{\otimes}(\gamma_1)$, either for all $\gamma_2^0 \neq \gamma_2 \in V$, $d_1 \gamma_2^{\otimes} > 0$ or for all $\gamma_2^0 \neq \gamma_2 \in V$, $d_1 \gamma_2^{\otimes} < 0$. Since $d_2 \gamma_1^{\otimes} = 1/d_1 \gamma_2^{\otimes}$, either $\lim_{\gamma_2 \to \gamma_2^0} d_2 \gamma_1^{\otimes} = \infty$ or $\lim_{\gamma_2 \to \gamma_2^0} d_2 \gamma_1^{\otimes} = -\infty$, but the first case is ruled out by part iii).

ad iii). $d_2 \gamma_1^{\otimes}$, if defined, is either equal to 0 or $\gamma_1^{\otimes} < \gamma^{max}$. and $d_1^2 \Pi_1 \neq 0$. But $0 < \alpha$, so we may assume $\gamma_1^{\otimes} < \gamma^{max}$. We first prove a preliminary claim.

Claim. Consider a continuously differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ and vectors $(1, a)$ and $(1, b)$ with $0 < a < b \in \mathbb{R}$. Consider a point $p \in \mathbb{R}^2$ with $Df(p) \neq 0$. If in p the directional derivatives $D_{(1,a)}f$ and $D_{(1,b)}f$ have the same sign, then all directional derivatives $D_{(1,x)}f$ with $a \leq x \leq b$ have the same sign in p. If in p one of the directional derivatives, $D_{(1,a)}f, D_{(1,b)}f$, is equal and the other unequal to zero, then for all $x \in (a, b)$ $D_{(1,x)}f \neq 0$ and has the same sign.

Proof of claim. We have $Df(p) \neq 0$, hence the gradient $\text{grad}(f) = (d_1 f, d_2 f)$ does not vanish at p. Hence,

$$D_{(1,x)}f \;=\; \langle (1, x), \text{grad}(f) \rangle \;=\; d_1 f + x\, d_2 f \tag{1.A.134}$$

is a linear function in x. Subsequently, if $d_1 f + x d_2 f$ (as a function in x) has the same sign for $x = a$ and $x = b$, it has the same sign for all $x \in (a, b)$. This also holds in case one of the two directional derivatives $D_{(1,a)}f, D_{(1,b)}f$ are zero. This proves the claim.

$d_2 \gamma_1^{\otimes}$ is defined by $D_{(d_2 \gamma_1^{\otimes}, 1)}(d_1 \Pi_1) = 0$. Thus, it is to show that for $\kappa > \alpha$, $D_{(\kappa,1)}(d_1 \Pi_1) \neq 0$. Since $D_{(\kappa,1)}f = \kappa D_{(1,1/\kappa)}f$, this is equivalent to showing that for $1/\kappa \in (0, 1/\alpha)$, $D_{(1,1/\kappa)}(d_1 \Pi_1) \neq 0$. With the above claim it thus remains to show that

$$d_1^2 \Pi_1(\gamma_1^{\otimes}(\gamma_2), \gamma_2) \;=\; D_{(1,0)}(d_1 \Pi_1) \;\leq\; 0 \tag{1.A.135}$$

$$\text{and} \qquad D_{(1,1/\alpha)}(d_1 \Pi_1) \;<\; 0. \tag{1.A.136}$$

(1.A.136) holds, since $D_{(1,1/\alpha)}(d_1 \Pi_1) = d_1^2 \Pi_1 + \frac{1}{\alpha}(d_1 d_2 \Pi_1) < 0$ from (1.A.70). (1.A.135) holds since $\gamma_1^{\otimes}(\gamma_2)$ is a local maximum..

□

1.A.8 Proof of Lemma 13

Condition (S1) requires that for $x < \mu$, f only has local minima. Condition (S2), on the other hand, requires that for $x > \mu$, f only has local maxima. Hence, f is increasing on the interval $[a, \mu]$, since otherwise from condition (S3) there was a local maxima below μ. On the interval $[b,\mu]$ there can be at most one local maximum τ, since otherwise there would be another local minima in between - contradiction.

Subsequently, f is increasing on $[a,\mu]$ and decreasing on $[\mu,b]$. Hence, τ is a global maximum and from monotonicity we have $\forall x<\tau : df(x)\geq 0$ and $\forall x>\tau : df(x)\leq 0$. But $df(x)$ must not be zero for $x\neq\tau$, since otherwise from (S1) and (S2) at that point there would be another local extremum, which would entail another extremum in between - contradiction.

□

1.A.9 Proof of Lemma 14

If $d_1\Pi_1=0$,

$$
\begin{aligned}
d_1^2\Pi_1 &\geq 0 && (1.A.137)\\
\Leftrightarrow -(2d_1a^*+\gamma_1 d_1^2a^*) &\geq 0 && (1.A.138)\\
\Leftrightarrow 2d_1a^* &\leq -\gamma_1(d_1a^*)^2\left[\frac{2}{a^*}-g\varphi_1\frac{\xi_2\alpha}{\tau_1}\right] && (1.A.139)\\
\overset{d_1\Pi_1=0}{\Leftrightarrow} 2 &\leq -\gamma_1\frac{(\overline{a}-a^*)}{\gamma_1}\left[\frac{2}{a^*}-g\varphi_1\frac{\xi_2\alpha}{\tau_1}\right] && (1.A.140)\\
\Leftrightarrow 2a^* &\leq (\overline{a}-a^*)\Big[-2+g\varphi_1\xi_2\underbrace{a^*\frac{\alpha}{\tau_1}}_{=(-d_2a^*)}\Big] && (1.A.141)\\
\Leftrightarrow 2\overline{a} &\leq (\overline{a}-a^*)g\varphi_1\xi_2(-d_2a^*), && (1.A.142)
\end{aligned}
$$

where we used $d_1\Pi_1=0\Leftrightarrow d_1a^*=\frac{(\overline{a}-a^*)}{\gamma_1}$ as well as (1.A.65).

Define the RHS of (1.A.142) as

$$R(\gamma_1,\gamma_2) := (\overline{a}-a^*)g\varphi_1\xi_2(-d_2a^*). \tag{1.A.143}$$

Claim. It suffices to show $d_1R<0$.

Proof of claim. If $d_1R<0$, there can be at most one μ with $2\overline{a}=R(\mu,\gamma_2)$ and for this μ (T1) and (T2) hold. In case there is no μ with $2\overline{a}=R(\mu,\gamma_2)$, we distinguish the following cases:

i) If there is an interior local maximum, at this interior local maximum we must have $d_1\Pi_1^2<0$. Hence $2\overline{a}>R$ on the entire interval and $\mu=\gamma_1^{a}(\gamma_2)$ satisfies the condition.

ii) If there is no interior local maximum, Π_1 increases on the entire interval by Assumption A6 and Lemma 15, and $\mu=\gamma^{max}$ satisfies the condition. In that case also $\gamma_1^{\overline{a}}(\gamma_2)=\gamma^{max}$, because otherwise $\Pi_1(\gamma_1^{\overline{a}}(\gamma_2),\gamma_2)=0$ would contradict monotonicity of Π_1.

iii) By Assumption A6 and Lemma 15 there can be no interior local minima.

Claim. $d_1 R < 0$.

Proof of claim. Using (1.A.68) in Proposition 6,

$$\begin{aligned} d_1 R &= (-d_1 a^*)\frac{R}{(\overline{a}-a^*)} + d_1\varphi_1 \frac{R}{\varphi_1} - (d_1 d_2 a^*)\frac{R}{(-d_2 a^*)} &(1.A.144)\\ &= R(d_1 a^*)\cdot\left[-\frac{1}{(\overline{a}-a^*)} + \alpha\xi_2 - \frac{1}{\alpha}\frac{(d_1 d_2 a^*)}{(d_1 a^*)^2}\right] &(1.A.145)\\ &= R(d_1 a^*)\cdot\left[-\frac{1}{(\overline{a}-a^*)} + \alpha\xi_2 - \frac{1}{\alpha}\frac{\alpha}{a^*}\left(a^*\xi_2\frac{h\varphi_2}{\tau_2} - 2\right)\right] &(1.A.146)\\ &= \underbrace{R(d_1 a^*)}_{>0}\cdot\left[-\frac{1}{(\overline{a}-a^*)} + \frac{2}{a^*} - \xi_2(\alpha + \frac{h\varphi_2}{\tau_2})\right] &(1.A.147)\end{aligned}$$

Subsequently

$$\begin{aligned} d_1 R < 0 &\Leftrightarrow \frac{2\overline{a}-3a^*}{(\overline{a}-a^*)a^*} < \xi_2\left(\alpha + \frac{h\varphi_2}{\tau_2}\right) &(1.A.148)\\ &\Leftrightarrow \underbrace{\frac{2\overline{a}-3a^*}{(\overline{a}-a^*)}}_{<2} < \underbrace{a^*\xi_2}_{<(-2)}\Big(\underbrace{\alpha}_{\in(0,1)} + \underbrace{\frac{h\varphi_2}{\tau_2}}_{<(-1)}\Big). &(1.A.149)\end{aligned}$$

Assumption A5 ensures that the LHS of (1.A.149) is negative, and, thus, under assumption A5 (1.A.149) holds.

□

1.A.10 Proof of Lemma 15

From assumption A6 we have $d_1\Pi_1(\gamma_1^{\underline{a}}(\gamma_2),\gamma_2) > 0$ for $\gamma_2 = \gamma_2^*(\gamma^{max})$. From (1.A.69), we know

$$D_{(1,1/\alpha)}(d_1\Pi_1) = d_1^2\Pi_1 + \frac{1}{\alpha}(d_1 d_2\Pi_1) < 0. \qquad (1.A.150)$$

Hence, $d_1\Pi_1(\gamma_1^{\underline{a}}(\gamma_2),\gamma_2)$ increases along $\{a^* = \underline{a}\}$ as γ_2 decreases, and, thus, $d_1\Pi_1(\gamma_1^{\underline{a}}(\gamma_2),\gamma_2) > 0$ for all $\gamma_2 \in [0, \gamma_2^*(\gamma^{max})]$.

□

1.A.11 Proof of Proposition 4

ad Existence. We consider insurer 2's reaction function

$$\gamma_2^* \; : \; [\gamma_1^a(0), \gamma^{max}] \; \to \; [0, \gamma_2^*(\gamma^{max})] \tag{1.A.151}$$

$$\gamma_1 \; \mapsto \; \gamma_2^*(\gamma_1) \tag{1.A.152}$$

and insurer 1's reaction function

$$\gamma_1^\otimes \; : \; [0, \gamma_2^*(\gamma^{max})] \; \to \; [\gamma_1^a(0), \gamma^{max}] \tag{1.A.153}$$

$$\gamma_2 \; \mapsto \; \gamma_1^\otimes(\gamma_2). \tag{1.A.154}$$

From Propositions 6 and 5 we know that γ_2^* and $\gamma_1^\otimes$ are continuous functions. Subsequently,

$$(\gamma_2^* \circ \gamma_1^\otimes) \; : \; [0, \gamma_2^*(\gamma^{max})] \; \to \; [0, \gamma_2^*(\gamma^{max})] \tag{1.A.155}$$

is a continuous self-mapping on a nonempty, compact and convex set and, hence, by Brouwer's fixed point theorem (rf Mas-Colell, Whinston, and Green (1995, p. 952)) there exists a fixed point. By construction a fixed point either satisfies both FOCs or lies at the boundary.

ad Uniqueness. Since insurer 2's reaction function γ_2^* is strictly increasing, there exists an inverse function, denoted by γ_1^{*-1}. From part iii) of Proposition 6 we have for insurer 2's reaction function $d_1\gamma_2^* < 1/\alpha^*$, hence, for its inverse function

$$d_2\gamma_1^{*-1} \; > \; \alpha^*. \tag{1.A.156}$$

At the same time, we know from Proposition 5 that for insurer 1's reaction function

$$d_2\gamma_1^\otimes \; < \; \alpha^\otimes. \tag{1.A.157}$$

Consider the mapping

$$\gamma_2 \; \mapsto \; \gamma_1^{*-1}(\gamma_2) \; \mapsto \; a^*(\gamma_1^{*-1}(\gamma_2), \gamma_2). \tag{1.A.158}$$

Then $a^*(\gamma_1^{*-1}(\cdot), \cdot)$ as a function of γ_2 is increasing in γ_2, since

$$0 \; < \; d_2 a^*(\gamma_1^{*-1}(\gamma_2), \gamma_2) \; = \; (d_1 a^*)(d_2\gamma_1^*) + d_2 a^* \; \Leftrightarrow \; d_2(\gamma_1^*) \; > \; \frac{(-d_2 a^*)}{(d_1 a^*)} \; = \; \alpha, \tag{1.A.159}$$

$$\Leftrightarrow \; \frac{1}{d_1(\gamma_2^*)} \; > \; \alpha, \tag{1.A.160}$$

which holds by Proposition 5 part iii).

Likewise, one can consider the analogous mapping using insurer 1's reaction function

$$\gamma_2 \mapsto \gamma_1^{\otimes}(\gamma_2) \mapsto a^*(\gamma_1^{\otimes}(\gamma_2), \gamma_2). \tag{1.A.161}$$

Then $a^*(\gamma_1^{\otimes}(\cdot), \cdot)$ as a function of γ_2 is decreasing in γ_2, since

$$0 > d_2 a^*(\gamma_1^{\otimes}(\gamma_2), \gamma_2) = (d_1 a^*)(d_2 \gamma_1^{\otimes}) + d_2 a^* \iff d_2(\gamma_1^{\otimes}) < \frac{(-d_2 a^*)}{(d_1 a^*)} = \alpha, \tag{1.A.162}$$

which holds by Proposition 6 part iii).

Since a^* values at a point must coincide at a point at which the two function intersect, there can be at most one intersection.

□

1.A.12 Proof of Proposition 7

First, at a Nash equilibrium $\vec{\gamma}$ one has $d_1 \Pi_1(\vec{\gamma}) \geq 0 = d_2 \Pi_2(\vec{\gamma})$ with $d_1 \Pi_1(\vec{\gamma}) > 0$ only if $\gamma_1 = \gamma^{max}$. Note furthermore that

$$d_1 \Pi_1 \geq 0 \iff (\overline{a} - a^*) - \gamma_1 d_1 a^* \geq 0 \tag{1.A.163}$$

$$d_2 \Pi_2 = 0 \iff (a^* - \underline{a}) + \gamma_2 d_2 a^* = 0. \tag{1.A.164}$$

Using Lemma 3 part iii), it thus follows that at a point $\vec{\gamma}$ with $d_1 \Pi_1(\vec{\gamma}) \geq 0 = d_2 \Pi_2(\vec{\gamma})$ we have

$$1 > \alpha = \frac{-d_2 a^*}{d_1 a^*} \geq \frac{-d_2 a^*}{(\overline{a} - a^*)} \gamma_1 = \frac{(a^* - \underline{a})}{(\overline{a} - a^*)} \frac{\gamma_1}{\gamma_2} = \frac{\Pi_2}{\Pi_1} \frac{\gamma_1^2}{\gamma_2^2} > \frac{\Pi_2}{\Pi_1}, \tag{1.A.165}$$

where the last inequality follows since $\Delta\gamma < 0 \iff \gamma_1/\gamma_2 > 1$. Hence, (1.A.165) yields $(a^* - \underline{a}) < (\overline{a} - a^*)$ and $\Pi_2 < \Pi_1$.

□

1.A.13 Proof of Proposition 8

The optimization problem for a given vector of default risks $\vec{b}^0$ depends only on $\tilde{g}(\vec{b}^0) = p(b_2^0 - b_1^0)/(1 - b_1^0 p)$. Hence, vectors of default risks with the same $\tilde{g}$ yield the same Nash equilibria.

Claim. For a given pair of default risks $(b_1^0, b_2^0) = \vec{b}^0$ with $\tilde{g}(\vec{b}^0)$, the set of default risks $\vec{b}$ with the same $\tilde{g}$ is

$$\left\{ \left(b_1^0 - \alpha, b_2^0 - (1 - \tilde{g}(b_1^0, b_2^0))\alpha \right) | \alpha \in \left[b_1^0 - \frac{1}{3}, b_1^0 \right] \right\}. \tag{1.A.166}$$

Proof of claim. We have

$$\partial_{b_2}\tilde{g}|_{\vec{b}^0} = \frac{p}{1-b_1^0 p} \tag{1.A.167}$$

$$\partial_{b_1}\tilde{g}\Big|_{\vec{b}^0} = \frac{-p(1-b_1^0 p)+p(b_2^0-b_1^0)p}{(1-b_1^0 p)^2} \tag{1.A.168}$$

$$= -p\frac{(1-b_2^0 p)}{(1-b_1^0 p)^2} \tag{1.A.169}$$

$$= -\frac{p}{(1-b_1^0 p)}[1-\underbrace{\frac{p\Delta b}{(1-b_1^0 p)}}_{=\tilde{g}(\vec{b}^0)}] \tag{1.A.170}$$

$$-\frac{\partial_{b_1}\tilde{g}}{\partial_{b_2}\tilde{g}}\Big|_{\vec{b}^0} = (1-\tilde{g}(\vec{b}^0)) \in (0,1) \tag{1.A.171}$$

Hence, from the implicit function theorem we know that sets $\{\vec{\gamma}|\tilde{g}(\vec{\gamma})=c\}$ are submanifolds that have (for a given c) the same slope $(1-\tilde{g})$ at each point. Hence they are straight lines.

□

1.A.14 Proof of Proposition 9

ad i). Let (b_1^*, b_2^*) be a subgame-perfect equilibrium for prescribed roles. If Π_2^* has multiple maxima, b_2^s and b_2^l are defined as the smallest and largest b_2 at which the maximum is assumed. The following goes through for b_2^s and b_2^l, in particular for b_2^s. From the definition of b_2^s

$$b_2^s = b_2^* - \left(1-\tilde{g}(b_1^*, b_2^*)\right)b_1^*. \tag{1.A.172}$$

For all $\lambda \in [0, b_1^*]$, all $b^\lambda = (\lambda, b_2^s + (1-\tilde{g}(b_1^*, b_2^*))\lambda)$ are also subgame-perfect equilibria with

$$\Pi_i^\square(b_1^*, b_2^*) = \Pi_i^\square(b_1^\lambda, b_2^\lambda) \tag{1.A.173}$$

for $i \in \{1,2\}$ from from Proposition 8 ii), since

$$\frac{b_2^\lambda - b_2^*}{b_1^\lambda - b_1^*} = \frac{-(1-\tilde{g}(b_1^*, b_2^*))(b_1^*-\lambda)}{(\lambda - b_1^*)} = 1-\tilde{g}(b_1^*, b_2^*). \tag{1.A.174}$$

Now, suppose $b_1^* > b_2^s$. Then, for $\lambda = (b_1^* - b_2^s)/(1-\tilde{g}(b_1^*, b_2^*)) > 0$, $b_2^\lambda = b_1^*$ and

$$\lambda - b_1^* = \frac{1}{1-\tilde{g}(b_1^*, b_2^*)}\Big(b_1^* - \big(b_2^s + (1-\tilde{g}(b_1^*, b_2^*))b_1^*\big)\Big) \tag{1.A.175}$$

$$= \frac{1}{1-\tilde{g}(b_1^*, b_2^*)}(b_1^* - b_2^*) < 0, \tag{1.A.176}$$

hence $\lambda < b_1^*$. So in this case, $(b_1^\lambda, b_2^\lambda)$ offers a profitable deviation for insurer 2 by choosing λ and thereby reserving roles and capturing profit $\Pi_1^\square(b_1^\lambda, b_2^\lambda) > \Pi_2^\square(b_1^\lambda, b_2^\lambda) = \Pi_2^\square(b_1^*, b_2^*)$. Contradiction.

ad ii). In any subgame-perfect equilibrium, we have

$$b_2^* \le b_2^l + \big(1-\tilde{g}(0, b_2^l)\big)b_1^* \le \big(2-\tilde{g}(0, b_2^l)\big)b_2^l. \tag{1.A.177}$$

ad Upper Bound. In general, from Proposition 8 we know that pairs of default risks (b_1, b_2) with

$$(b_1, b_2) = (b_1^* - z, b_2^* - (1-\tilde{g}(b_1^*, b_2^*))z), \tag{1.A.178}$$

$z \in [b_1^* - \frac{1}{3}, b_1^*]$, lead to the same Nash equilibria in prices. Hence, the second mover has the option to choose a quality $b_2^1 < b_1^*$ with

$$(b_2^1, b_1^*) = \left(\frac{1}{(1-\tilde{g})}\big[(1-\tilde{g})b_1^* - (b_2^* - b_1^*)\big], b_1^*\right) \tag{1.A.179}$$

that leads to the same Nash equilibrium in prices, but with reversed roles. By Proposition 7 this is a profitable deviation. This deviation is infeasible if

$$b_2^* - b_1^* > (1-\tilde{g}(b_1^*, b_2^*))b_1^* \tag{1.A.180}$$

$$\Leftrightarrow b_2^* > (2-\tilde{g}(b_1^*, b_2^*))b_1^* \tag{1.A.181}$$

$$\Leftrightarrow b_1^* < \underbrace{\frac{1}{(2-\tilde{g})}}_{<2-1/8 \text{ from Lemma 19}} \underbrace{b_2^*}_{<b^{max}}. \tag{1.A.182}$$

□

1.A.15 Proof of Proposition 10

"⇐" Similar to the proof of Proposition 9, insurer 1 must choose b_1 in such a way that it is not profitable for insurer 2 to become quality leader. This is the case if Π_2^* exceeds any profit insurer 2 can capture with reversed roles. Since from condition (N2) the profit of the quality leader is increasing in the lower quality, consider the smallest $\bar{b}_1$ such that $\Pi_1^\square(0, \bar{b}_1) = \Pi_2^*$. Then, $b_1 < \bar{b}_1$ leaves no profitable deviation for the follower.

"⇒" Let (b_1^*, b_2^*) be a subgame-perfect equilibrium with prescribed roles. Then for $i \in \{1,2\}$ and $\lambda \leq b_1^*$

$$\Pi_i^{\square}(b_1^* - \lambda, b_1^*) = \Pi_i^{\square}\big(0, b_1^* - (1 - \tilde{g}(b_1^* - \lambda, b_1^*))(b_1^* - \lambda)\big) \quad (1.A.183)$$

$$= \Pi_i^{\square}\big(0, \lambda + \tilde{g}(b_1^* - \lambda, b_1^*)(b_1^* - \lambda)\big). \quad (1.A.184)$$

Suppose $b_1^* > \bar{b}_1$. Then let $\lambda = \bar{b}_1$. Hence

$$\Pi_1^{\square}(b_1^* - \bar{b}_1, b_1^*) = \Pi_1^{\square}\big(0, \bar{b}_1 + \tilde{g}(b_1^* - \bar{b}_1, b_1^*)(b_1^* - \bar{b}_1)\big) > \Pi_2^* \quad (1.A.185)$$

by definition of $\bar{b}_1$. Since $\tilde{g}(b_1^* - \bar{b}_1, b_1^*)(b_1^* - \bar{b}_1) > 0$, this is a feasible profitable deviation. Contradiction.

□

Appendix 1.B Additional Results

1.B.1 Optimal Choice of State-Contingent Payments

This section clarifies the contracting problem that has the specified insurance contract as outcome. Suppose an insurer with default risk b offers a contract that involves a fixed rate γ for establishing the client-insurer relationship, after which the insurer offers the actuarially fair price and the coverage is determined endogenously. All payments are due in $t=4$. This includes γ, which, although set ex-ante, is also exchanged in $t=4$ and hence only due if the insurer survives.

Clients chooses payments (y,z) with

$$y \text{ due if } \tilde{x} = \overline{\theta} \text{ and the insurer survives} \tag{1.B.1}$$

$$z \text{ due if } \tilde{x} = \underline{\theta} \text{ and the insurer survives} \tag{1.B.2}$$

to maximize expected utility

$$(1-p)u\big(\overline{\theta}-y\big) + p(1-b)u\big(\underline{\theta}-z\big) + bpu(\underline{\theta}) \tag{1.B.3}$$

subject to the constraint

$$(1-p)y + p(1-b)z - \left[\gamma - \frac{bp\underline{\theta}}{(1-bp)}\right](1-bp) \;\geq\; 0 \tag{1.B.4}$$

$$\Leftrightarrow\; (1-p)y + p(1-b)z \;\geq\; \gamma(1-bp) - bp\underline{\theta}. \tag{1.B.5}$$

(1.B.4) and (1.B.5) offer two views on the constraint. (1.B.5) demands that the expected cash flows to the insurer (LHS) must be at least as high as the expected fee already agreed upon minus the expected endowment if the insurer survives. To see the latter part note that

$$E[\tilde{x}|\text{insurer survives}]P[\text{insurer survives}] \;=\; (1-p)\overline{\theta} + p(1-b)\underline{\theta} \;\overset{E[\tilde{x}]=0}{=}\; -bp\underline{\theta} \tag{1.B.6}$$

$$\Leftrightarrow\; E[\tilde{x}|\text{insurer survives}] \;=\; \frac{-bp\underline{\theta}}{(1-bp)} \;>\; 0. \tag{1.B.7}$$

The risk-averse clients passes the risky endowment to the insurer unless the insurer defaults.

(1.B.4) offers an alternative explanation. Let γ^{nom} be the expression in brackets, i.e.

$$\gamma^{nom} \;:=\; \gamma - \frac{bp\underline{\theta}}{(1-bp)}. \tag{1.B.8}$$

Then the third term on the LHS of (1.B.4) is the "nominal" fee per client-insurer relationship, γ^{nom}, times the survival probability of the insurer, since only in that case the payment is actually exchanged. It is subtracted because this fee for establishing the client-insurer relationship has already been agreed upon, so the insurer already "mentally set it aside" and subsequently wants to break even in $t = 3$. Compared to γ, from the definition we have $\gamma = \gamma^{nom} + bp\underline{\theta}/(1-bp) < \gamma^{nom}$. In view of (1.B.7) the adjustment term, $bp\underline{\theta}/(1-bp)$, is precisely the expected endowment conditional on the survival of the insurer. Since it is positive, the client claims this extra revenue for himself, rendering γ the "true" fees for the insurer. In the formulation of the insurer's constraint in (1.B.4) one assumes that the insurer chooses "true" fees γ instead of "nominal" ones γ^{nom}. This reparametrization will make subsequent calculations tractable as we will see, while simplifying the intuition.

The following proposition then is a direct result from solving a client's optimization problem

$$\max_{y,z}\Big\{(1-p)u(\overline{\theta}-y) + p(1-b)u(\underline{\theta}-z) + bpu(\underline{\theta}) \;\Big|\; (1-p)y + p(1-b)z \;=\; \gamma(1-bp) - bp\underline{\theta}\Big\}$$

Proposition 16. *For a given (b,γ), the client optimally chooses*

$$y^*(b,\gamma) \;=\; \gamma + \frac{p(1-b)\overline{\theta} - p\underline{\theta}}{(1-bp)} \tag{1.B.9}$$

$$z^*(b,\gamma) \;=\; \gamma - \frac{(1-p)}{(1-bp)}\overline{\theta} - \frac{b(1-bp)-(1-p)}{(1-b)(1-bp)}\underline{\theta}. \tag{1.B.10}$$

Let $r^*(b,\gamma)$ be the payoff a client is left with in an optimal insurance contract unless the counterparty defaults (residual endowment), i.e. $r^*(b,\gamma) := \overline{\theta} - y^*(b,\gamma) = \underline{\theta} + z^*(b,\gamma)$. Then, as one would expect from risk aversion, $r^*(b,\gamma)$ does not depend on the endowment state, namely

$$r^*(b,\gamma) \;=\; -\gamma. \tag{1.B.11}$$

1.B.2 Market Coverage

An insurance contract (b,γ) is called *feasible for a* if client a prefers the contract to none. This translates into the following condition

$$pu_a(\underline{\theta}) + (1-p)u_a(\overline{\theta}) \;\leq\; (1-bp)u_a(-\gamma) + bpu_a(\underline{\theta}) \tag{1.B.12}$$

$$\Leftrightarrow\; bp\left[u_a(-\gamma) - u_a(\underline{\theta})\right] + u_a(\overline{\theta}) - u_a(-\gamma) \;\leq\; p\left[u_a(\overline{\theta}) - u_a(-\gamma) + u_a(-\gamma) - u_a(\underline{\theta})\right] \tag{1.B.13}$$

$$\Leftrightarrow\; (1-p)\left[u_a(\overline{\theta}) - u_a(-\gamma)\right] \;\leq\; p(1-b)\left[u_a(-\gamma) - u_a(\underline{\theta})\right] \tag{1.B.14}$$

(1.B.14) admits an intuitive interpretation: Client a prefers the contract to no insurance, if the expected utility gain from avoiding the bad endowment in case the seller does not default (RHS) outweighs the expected utility loss from the fee if the good endowment materializes (LHS).[11]

The following proposition characterizes the client that is indifferent between insurance contract (b,γ) and no insurance.

Proposition 17. *Client a is indifferent between (b,γ) and no insurance, if*

$$\gamma \;=\; \gamma_a^{exit}(b) \;:=\; (-\underline{\theta}) - \frac{1}{a}\ln\left(\frac{K(b)+1}{K(b)+\exp(-a(\overline{\theta}-\underline{\theta}))}\right) \tag{1.B.15}$$

with $K(b) = (1-b)p/(1-p)$. $\gamma_a^{exit}(b)$ is strictly increasing in a and decreasing in b.

Proof. In light of (1.B.14), a client a is indifferent between buying contract (b,γ) and no insurance, if

$$\frac{u_a(\overline{\theta}) - u_a(-\gamma)}{u_a(-\gamma) - u_a(\underline{\theta})} \;=\; \frac{p}{1-p}(1-b) \tag{1.B.16}$$

$$\Leftrightarrow\; \frac{\exp(-a\overline{\theta}) - \exp(a\gamma)}{\exp(a\gamma) - \exp(-a\underline{\theta})} \;=\; K(b) \tag{1.B.17}$$

$$\Leftrightarrow\; \frac{\exp(-a(\overline{\theta}+\gamma)) - 1}{1 - \exp(-a(\underline{\theta}+\gamma))} \;=\; K(b) \tag{1.B.18}$$

$$\Leftrightarrow\; \frac{\exp(-a\Delta\theta)\exp(-a(\underline{\theta}+\gamma)) - 1}{1 - \exp(-a(\underline{\theta}+\gamma))} \;=\; K(b) \tag{1.B.19}$$

$$\Leftrightarrow\; \exp(-a(\underline{\theta}+\gamma)) \;=\; \frac{K(b)+1}{K(b)+\exp(-a\Delta\theta)} \tag{1.B.20}$$

$$\Leftrightarrow\; \gamma \;=\; \gamma_a^{exit}(b) \;:=\; (-\underline{\theta}) - \frac{1}{a}\ln\left(\frac{K(b)+1}{K(b)+\exp(-a\Delta\theta)}\right), \tag{1.B.21}$$

11. Note that from (1.B.14) we also know that for any feasible contract $(\underline{\theta}+\gamma) < 0$. (Since $-\gamma < 0 < \overline{\theta}$, the LHS of (1.B.14) is positive, hence, the RHS needs to be positive as well.) Indeed, we already restricted attention to $\gamma < (-\underline{\theta})$ by assumption A3.

with $K(b) := (1-b)p/(1-p)$ and $\Delta\theta := (\overline{\theta} - \underline{\theta})$.

ad $\gamma_a^{exit}(b)$ *increasing in a.* We have

$$\frac{\partial \gamma_a^{exit}}{\partial a} = \frac{1}{a}\left[\frac{1}{a}\log\left(\frac{K(b)+1}{K(b)+\exp(-a\Delta\theta)}\right) - \frac{\exp(-a\Delta\theta)}{K(b)+\exp(-a\Delta\theta)}\Delta\theta\right]. \quad (1.B.22)$$

With

$$y := \frac{1-\exp(-a\Delta\theta)}{K(b)+\exp(-a\Delta\theta)} \quad (1.B.23)$$

this reads

$$\frac{\partial \gamma_a^{exit}}{\partial a} = \frac{1}{a}\left[\frac{1}{a}\ln(1+y) + \left(y - \frac{1}{K(b)+\exp(-a\Delta\theta)}\right)\Delta\theta\right] \quad (1.B.24)$$

$$= \frac{1}{a}\left[\frac{1}{a}y\left(\frac{\log(1+y)}{y} + a\Delta\theta\right) - \frac{1}{K(b)+\exp(-a\Delta\theta)}\Delta\theta\right] \quad (1.B.25)$$

$$= \left(\frac{1}{a}\right)^2 \frac{1}{K(b)+\exp(-a\Delta\theta)}\left[(1-\exp(-a\Delta\theta))\left(\frac{\log(1+y)}{y} + a\Delta\theta\right) - a\Delta\theta\right] \quad (1.B.26)$$

$$= \left(\frac{1}{a}\right)^2 \frac{1}{K(b)+\exp(-a\Delta\theta)}\left[(1-\exp(-a\Delta\theta))\frac{\log(1+y)}{y} - \exp(-a\Delta\theta)a\Delta\theta\right] \quad (1.B.27)$$

With $x := a\Delta\theta$ this expression is positive if and only if

$$\frac{\exp(x)-1}{x} > \frac{y}{\log(1+y)} \quad (1.B.28)$$

$$\Leftrightarrow \log(1+y) > y\frac{x}{\exp(x)-1} \quad (1.B.29)$$

$$\Leftrightarrow \log\left(\frac{K(b)+1}{K(b)+\exp(-x)}\right) > \frac{x}{\exp(x)}\frac{1}{K(b)+exp(-x)}. \quad (1.B.30)$$

For $x = 0$ the LHS and RHS are 0. For $x > 0$ the derivative w.r.t. x of the LHS reads

$$\frac{\partial LHS}{\partial x} = \frac{\exp(-x)}{K(b)+\exp(-x))}, \quad (1.B.31)$$

while the derivative of the RHS reads

$$\frac{\partial RHS}{\partial x} = \frac{\exp(-x)}{K(b)+\exp(-x)}\underbrace{\left[(1-x) + \frac{1}{K(b)+\exp(-x)}\frac{x}{\exp(x)}\right]}_{<1}. \quad (1.B.32)$$

To see why the expression in brackets is smaller one, note that

$$(1-x)+\frac{1}{K(b)+\exp(-x)}\frac{x}{\exp(x)} < 1 \Leftrightarrow \frac{1}{K(b)+\exp(-x)} < \exp(x) \Leftrightarrow 0 < K(b)\exp(x),$$

which always holds and proves the claim.

ad $\gamma_a^{exit}(b)$ increasing in b. Follows directly, since

$$\frac{\partial \gamma_{\underline{a}}^{exit}}{\partial K(b)} = \frac{1-\exp(-a\Delta\theta)}{(1+K(b))(K(b)+\exp(-a\Delta\theta))} > 0 \quad (1.B.33)$$

$$\text{and } \partial_b K(b) < 0. \quad (1.B.34)$$

□

The result is intuitive: the fee at which a client is indifferent between the contract and no insurance is higher the more risk-averse he is. The next corollary follows as a direct consequence.

Corollary 18. *i) For fixed default probability b_i, an insurance contract (b_i, γ_i) is feasible for client a if $\gamma_i < \gamma_a^{exit}(b_i)$.*

ii) Let $a^{exit}(b_i, \gamma_i)$ be the client that is indifferent between contract (b_i, γ_i) and no insurance. For γ_i outside of $[\gamma_{\underline{a}}^{exit}(b_i), \gamma_{\overline{a}}^{exit}(b_i)]$, a^{exit} lies outside of the interval $[\underline{a}, \overline{a}]$ and is set to the respective boundary. Then clients with $a < a^{exit}(b_i, \gamma_i)$ prefer no insurance.

iii) If the fee set by the unsafer insurer, γ_2, is smaller than $\gamma_{\underline{a}}^{exit}(b_2)$, then $a^{exit} < \underline{a}$ and there is full market coverage.

In the analysis, I restrict attention to the case in which the market is fully covered.[12]

1.B.3 Formal Results on the Illustration

Lemma 19. *i) For $\gamma_2 \in [0, \gamma^{max}]$ define*

$$\gamma_1^{\underline{a}}(\gamma_2) \text{ such that } a^*(\gamma_1^{\underline{a}}(\gamma_2), \gamma_2) = \underline{a} \quad (1.B.35)$$

$$\gamma_1^{\overline{a}}(\gamma_2) \text{ such that } a^*(\gamma_1^{\overline{a}}(\gamma_2), \gamma_2) = \overline{a}. \quad (1.B.36)$$

12. Later we will introduce $\gamma_2^*(\gamma^{max})$, that is, insurer 2's best response to the largest possible fee set by insurer 1. insurer 2's reaction function is increasing. Hence $\gamma_2^*(\gamma^{max})$ is the largest fee possibly set by insurer in equilibrium, and if $\gamma_2^*(\gamma^{max}) \le \gamma_{\underline{a}}^{exit}(b_2)$ there is full market coverage anyways. Otherwise, insurer 2's reaction function remains unaltered until $\gamma_{\underline{a}}^{exit}(b_2)$. Above that point, insurer 2 potentially looses market share "from below" when increasing fees, which may induce him to set fees as best responses. Hence, we expect the reaction function to change above $\gamma_{\underline{a}}^{exit}(b_2)$, but it should leave the core of the analysis unchanged.

Then $\gamma_1^{\underline{a}} < \gamma_1^{\overline{a}}$ *and*

$$\gamma_1^{\underline{a}} \leq \gamma^{max} \quad \Leftrightarrow \quad \gamma_2 \leq \overline{\gamma_2} \tag{1.B.37}$$

$$\text{with } \overline{\gamma_2} := \arg_\gamma\{a^*(\gamma^{max},\gamma) = \underline{a}\} = (-\underline{\theta}) - \frac{1}{\underline{a}}\log\left[\frac{1-\tilde{g}(\vec{b})}{\exp(-2)-\tilde{g}(\vec{b})}\right]. \tag{1.B.38}$$

ii) *Analogously, for* $\gamma_1 \in [0,\gamma^{max}]$ *define*

$$\gamma_2^{\underline{a}}(\gamma_1) \text{ such that } a^*(\gamma_1,\gamma_2^{\underline{a}}(\gamma_1)) = \underline{a} \tag{1.B.39}$$

$$\gamma_2^{\overline{a}}(\gamma_1) \text{ such that } a^*(\gamma_1,\gamma_2^{\overline{a}}(\gamma_1)) = \overline{a} \tag{1.B.40}$$

Then $\gamma_2^{\underline{a}} > \gamma_2^{\overline{a}}$ *and*

$$\gamma_2^{\underline{a}} \geq 0 \quad \Leftrightarrow \quad \gamma_1 \geq \overline{\gamma_1} \tag{1.B.41}$$

$$\text{with } \overline{\gamma_1} := \arg_\gamma\{a^*(\gamma,0) = \underline{a}\} = \frac{1}{\underline{a}}\log\left[1+\tilde{g}(\vec{b})\left(\exp(\underline{a}(-\underline{\theta}))-1\right)\right]. \tag{1.B.42}$$

iii) *As one would expect from the picture* $\gamma_2 \leq \overline{\gamma_2}$ *iff* $\gamma_1 \geq \overline{\gamma_1}$.

iv) *insurer 1 gets the entire market if*

$$\overline{\gamma_2} \leq 0 \quad \Leftrightarrow \quad \underline{a}(-\underline{\theta}) \leq \log\left[\frac{1-\tilde{g}(\vec{b})}{\exp(-2)-\tilde{g}(\vec{b})}\right]. \tag{1.B.43}$$

With $\tilde{g}(\vec{b}) < 1/8$ *from assumption A1 and A2,*

$$\underline{a}(-\underline{\theta}) > \log\left[\frac{1-\frac{1}{8}}{\exp(-2)-\frac{1}{8}}\right] \approx 4.4, \tag{1.B.44}$$

ensures that the setup is interesting. This is exactly assumption A4.

Proof. ad i). First of all, we show that for a fixed $\gamma_2 \in [0,\gamma^{max}]$ such $\gamma_1^{\underline{a}}(\gamma_2), \gamma_1^{\overline{a}}(\gamma_2)$ indeed exist. Whenever clear form the context we suppress the dependence on γ_2. Note that for $a \in [\underline{a},\overline{a}]$ $g(a,\gamma_2,\gamma_2) = 0$, while $\lim_{\gamma\to\infty} g(a,\gamma,\gamma_2) = \lim_{\gamma\to\infty} \frac{1}{c_1}\left(\exp(a\gamma)c_2 - 1\right) = \infty$ with $c_1 := \exp(-a(\underline{\theta}+\gamma_2))$ and $c_2 := \exp(-a\gamma_2)$ independent of γ. Hence, from continuity such $\gamma_1^{\underline{a}}, \gamma_1^{\overline{a}}$ exist and, since $\partial_1 g > 0$, they are also unique.

Claim. $\gamma_1^{\underline{a}} < \gamma_1^{\overline{a}}$

Proof of claim. Since $\partial_a g < 0$ we have $\tilde{g}(\vec{b}) = g(\overline{a}, \gamma_1^{\overline{a}}, \gamma_2) = g(\underline{a}, \gamma_1^{\underline{a}}, \gamma_2) > g(\overline{a}, \gamma_1^{\underline{a}}, \gamma_2)$. With $\partial_1 g > 0$ this implies $\gamma_1^{\underline{a}} < \gamma_1^{\overline{a}}$.

For the last part of the statement we have

$$\begin{aligned} \gamma_1^{\underline{a}} &\leq \gamma^{max} && (1.B.45)\\ \Leftrightarrow g(\underline{a}, \gamma^{max}, \gamma_2) &\geq \tilde{g}(\vec{b}) && (1.B.46)\\ \frac{\exp(-\underline{a}(\underline{\theta}+\gamma_2))\exp(-2)-1}{\exp(-\underline{a}(\underline{\theta}+\gamma_2))-1} &\geq \tilde{g}(\vec{b}) && (1.B.47)\\ \exp(-\underline{a}(\underline{\theta}+\gamma_2))\left(\exp(-2)-\tilde{g}(\vec{b})\right) &\geq 1-\tilde{g}(\vec{b}) && (1.B.48)\\ -\underline{a}(\underline{\theta}+\gamma_2) &\geq \log\left[\frac{1-\tilde{g}(\vec{b})}{\exp(-2)-\tilde{g}(\vec{b})}\right] && (1.B.49)\\ \gamma_2 &\leq (-\underline{\theta}) - \frac{1}{\underline{a}}\log\left[\frac{1-\tilde{g}(\vec{b})}{\exp(-2)-\tilde{g}(\vec{b})}\right]. && (1.B.50) \end{aligned}$$

Note that we use $\tilde{g}(\vec{b}) < \exp(-2)$ here, which is ensured by assumptions A1 and A2.

ad ii). The argument for existence is analogous to before, so is the argument for $\gamma_2^{\underline{a}} > \gamma_2^{\overline{a}}$ except that now $\partial_2 g < 0$. For the last part we have

$$\begin{aligned} \gamma_2^{\underline{a}} &\geq 0 && (1.B.51)\\ \Leftrightarrow g(\underline{a}, \gamma_1, 0) &\geq \tilde{g}(\vec{b}) && (1.B.52)\\ \Leftrightarrow \frac{\exp(\underline{a}\gamma_1)-1}{\exp(\underline{a}(-\underline{\theta}))-1} &\geq \tilde{g}(\vec{b}) && (1.B.53)\\ \Leftrightarrow \exp(\underline{a}\gamma_1) &\geq 1+\tilde{g}(\vec{b})\left(\exp(\underline{a}(-\underline{\theta}))-1\right) && (1.B.54)\\ \Leftrightarrow \gamma_1 &\geq \overline{\gamma_1} =: \frac{1}{\underline{a}}\log\left[1+\tilde{g}(\vec{b})\left(\exp(\underline{a}(-\underline{\theta})-1\right)\right]. && (1.B.55) \end{aligned}$$

ad iii). We have

$$\begin{aligned} \overline{\gamma_2} &\geq 0 && (1.B.56)\\ \Leftrightarrow (-\underline{\theta}) - \frac{1}{\underline{a}}\log\left[\frac{1-\tilde{g}(\vec{b})}{\exp(-2)-\tilde{g}(\vec{b})}\right] &\geq 0 && (1.B.57)\\ \Leftrightarrow \log\left[\frac{1-\tilde{g}(\vec{b})}{\exp(-2)-\tilde{g}(\vec{b})}\right] &\leq \underline{a}(-\underline{\theta}). && (1.B.58) \end{aligned}$$

At the same time

$$
\begin{aligned}
\overline{\gamma_1} &\leq \gamma^{max} && (1.B.59)\\
\Leftrightarrow\ 2 + \log\left[1 + \tilde{g}(\vec{b})\left(\exp(\underline{a}(-\underline{\theta})) - 1\right)\right] &\leq \underline{a}(-\underline{\theta}) && (1.B.60)\\
\Leftrightarrow\ \log\left[\exp(2)\left(1 + \tilde{g}(\vec{b})\left(\exp(\underline{a}(-\underline{\theta})) - 1\right)\right)\right] &\leq \underline{a}(-\underline{\theta}) && (1.B.61)\\
\Leftrightarrow\ \exp(2)\left(1 + \tilde{g}(\vec{b})\left(\exp(\underline{a}(-\underline{\theta})) - 1\right)\right) &\leq \exp(\underline{a}(-\underline{\theta})) && (1.B.62)\\
\Leftrightarrow\ \exp(2)\left(1 - \tilde{g}(\vec{b})\right) &\leq \exp(\underline{a}(-\underline{\theta}))\left(1 - \exp(2)\tilde{g}(\vec{b})\right) && (1.B.63)\\
\Leftrightarrow\ \frac{1 - \tilde{g}(\vec{b})}{\exp(-2) - \tilde{g}(\vec{b})} &\leq \exp(\underline{a}(-\underline{\theta})) && (1.B.64)\\
\Leftrightarrow\ \overline{\gamma_2} &\geq 0. && (1.B.65)
\end{aligned}
$$

ad iv). Since the LHS of (1.B.58) is increasing in $\tilde{g}(\vec{b})$ and under Assumption A2 $\tilde{g}(\vec{b}) < 1/8$,

$$\underline{a}(-\underline{\theta}) > \log\left[\frac{1 - \frac{1}{8}}{\exp(-2) - \frac{1}{8}}\right] \approx 4.4 \qquad (1.B.66)$$

ensures $\overline{\gamma_2} \geq 0$ for all admissible parameters and hence renders the setup interesting. □

1.B.4 Price Equilibria are Smooth Functions of Qualities

Proposition 20 (Price Equilibrium Smooth Function in Qualities). *Without loss of generality let $b_1 = 0$. Let*

$$\mathscr{D}_1 := \left\{b_2 | \gamma_1^{\square}(b_2) \lneq \gamma^{max}\right\} \qquad (1.B.67)$$

be the set of b_2 that lead to price equilibria in the interior. Let

$$\mathscr{D}_2 := \left\{b_2 | \gamma_1^{\square}(b_2) = \gamma^{max}, (d_1\Pi_1)^{\square} \gneq 0\right\} \qquad (1.B.68)$$

be the set of b_2 that lead to price equilibria in which insurer 1 chooses the highest admissible price. The price equilibrium $\vec{\gamma}^{\square}(\vec{b})$ as a function of quality choices $\vec{b}$ is a smooth function on $\mathscr{D}_1$ and $\mathscr{D}_2$.

Proof. Let

$$\mathscr{M} := \{(b_2, \vec{\gamma}) | b_2 \in (0, b^{max}], 0 \leq \gamma_2 < \gamma_1 \leq \gamma^{max}\} \qquad (1.B.69)$$

and

$$
\begin{aligned}
\mathscr{L}_1 &:= \{d_2\Pi_2 = 0\} \cap \{d_1\Pi_1 = 0\} \subset \mathscr{M} && (1.B.70)\\
\mathscr{L}_2 &:= \{d_2\Pi_2 = 0\} \cap \{\gamma_1 = \gamma^{max}\} \subset \mathscr{M}. && (1.B.71)
\end{aligned}
$$

Then we know that price equilibria are a subset of $\mathscr{L} := \mathscr{L}_1 \cup \mathscr{L}_2$, and, that $\mathscr{L}_1$ consists of price equilibria.

Claim 1. $\mathscr{L}_1$ is a smooth submanifold of $\mathscr{M}$ and $\vec{\gamma}^{\square}$ is smooth on $\mathscr{D}_1$.

Proof of claim 1. $\mathscr{L}_1$ is the intersection of nullsets of smooth functions

$$\vec{f} := \begin{pmatrix} d_2\Pi_2 \\ d_1\Pi_1 \end{pmatrix}. \tag{1.B.72}$$

The intersection of two nullsets $\{\vec{f} = 0\}$ is smooth if $rank(Df) = 2$. If $d_1^2\Pi_1 \neq 0$,

$$\begin{aligned} \det(D_{\vec{\gamma}}f) &= \det\begin{pmatrix} d_1d_2\Pi_2 & d_2^2\Pi_2 \\ d_1^2\Pi_1 & d_2d_1\Pi_1 \end{pmatrix} && (1.B.73)\\ &= -(d_2^2\Pi_2)(d_1^2\Pi_1)\left[1 - \frac{(d_1d_2\Pi_2)}{(d_2^2\Pi_2)}\frac{(d_2d_1\Pi_1)}{(d_1^2\Pi_1)}\right] && (1.B.74)\\ &= -(d_2^2\Pi_2)(d_1^2\Pi_1)\Big[1 - \underbrace{(d_2\gamma_1^{\otimes})}_{<1/\alpha}\underbrace{(d_1\gamma_2^{*})}_{<\alpha}\Big] && (1.B.75)\\ &\neq 0. && (1.B.76) \end{aligned}$$

If $d_1^2\Pi_1 = 0$, then $d_1d_2\Pi_1 \neq 0$, and thus

$$\det(D_{\vec{\gamma}}f) = (d_1d_2\Pi_2)(d_2d_1\Pi_1) \neq 0. \tag{1.B.77}$$

As shown in Proposition 4, for any $\vec{b}$ there is exactly one price equilibrium $\vec{\gamma}^{\square}(\vec{b})$ such that $(\vec{b}, \vec{\gamma}^{\square}(\vec{b})) \in \mathscr{L}$. This defines a function

$$\begin{aligned} \vec{\gamma}^{\square} : (0, b^{max}) &\to \mathscr{L} && (1.B.78)\\ b_2 &\mapsto \vec{\gamma}^{\square}(0, b_2)) && (1.B.79) \end{aligned}$$

with $\vec{\gamma}^{\square} : \mathscr{D}_i \to \mathscr{L}_i$ for $i \in \{1, 2\}$. Hence, from the Implicit Function Theorem, $\vec{\gamma}^{\square}|_{\mathscr{D}_1}$ is the smooth parameterization of the submanifold $\mathscr{L}_1$.

Claim 2. $\mathscr{L}_2$ is a smooth submanifold of $\mathscr{M}$ and $\vec{\gamma}^{\square}$ is smooth on $\mathscr{D}_2$.

Proof of claim 2. The proof proceeds analogously, but now $\mathscr{L}_2$ is the intersection of nullsets of

$$\vec{g} := \begin{pmatrix} d_2\Pi_2 \\ \gamma_1 - \gamma^{max} \end{pmatrix}, \tag{1.B.80}$$

with

$$\det(D_{\vec{\gamma}}g) = \det\begin{pmatrix} d_1d_2\Pi_2 & 1 \\ d_2^2\Pi_2 & 0 \end{pmatrix} = d_2^2\Pi_2 < 0. \tag{1.B.81}$$

□

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Chapter 2

Short-Time Work Extensions

Joint with Simon Jäger, Moritz Kuhn, Farzad Saidi, and Stefanie Wolter

2.1 Introduction

Short-time work (STW) schemes are a widely used policy tool by governments to preserve jobs during economic downturns (Giupponi, Landais, and Lapeyre, 2022). These schemes provide subsidies to firms to reduce employee hours instead of laying off workers. The efficiency of STW schemes hinges, first and foremost, on its ability to save jobs in the short and long run. However, the extent of such employment effects also depends on the tightness of firms' liquidity constraints regarding their wage bill. The latter, in turn, interact with pre-existing (wage) bargaining institutions.

In this paper, we provide evidence on the effect of STW extensions—along both the extensive and intensive margin—on employment, wages, and across firms. We do so by leveraging novel administrative data on short-time work receipt in Germany, allowing us to characterize the take-up of extended STW benefits from 2009 to 2021 at the firm level and individual STW eligibility of workers during the COVID-19 crisis. This allows us to estimate effects on employment at the individual level.

While endogenous estimates suggest a positive employment effect of STW benefits of up to eight percentage points, we uncover that individual STW take-up correlates highly with predicted retention probabilities. This strongly suggests that these estimates constitute an upper bound. For identification, we zoom in on cohorts reaching their statutory retirement age, at which one automatically loses access to potential STW benefits, before and after firms took up STW during the first year of the COVID-19 pandemic, which is when it was made widely available. That is, a cohort is affected when it reaches the statutory retirement age after

April 2020. We find no difference in the probability of remaining employed at the same firm when reaching the statutory retirement age for affected vs. unaffected cohorts, suggesting no employment effects from the extensive margin of STW benefits.

A key policy lever during recessions is the potential benefit duration (PBD), which sets the maximum time period that firms can receive STW subsidies for during a given spell. During the COVID-19 pandemic, for example, France introduced a new STW scheme with up to 48 months PBD, Switzerland extended PBD from 12 to 24 months, and Germany from 12 to 28 months, with the costs of short-time work in Germany in 2020 alone estimated at about 22.1 billion EUR (Bundesrechnungshof, 2022). By extending the PBD, policymakers aim to give firms more flexibility to temporarily reduce labor costs while retaining employees, under the assumption that firms will use this opportunity to hoard labor during temporary downturns. Even though existing work has studied the extensive-margin effects of receiving STW on workers and firms in different labor markets outside of Germany (see, e.g., Cahuc, Kramarz, and Nevoux, 2021; Kopp and Siegenthaler, 2021; Giupponi and Landais, 2023), the effects of extending PBD — the intensive margin of STW—remain an open empirical question.

To shed light on the employment and wage effects of extending PBD, we exploit a unique policy reform in Germany that unexpectedly doubled the PBD from 6 to 12 months in December 2012. The backward-binding nature of this reform generated quasi-experimental variation in STW PBD across firms that had already started using short-time work earlier in 2012. For firms that had started spells after July 2012, the December 2012 reform extended their benefit duration, while for those starting just before, it did not.

This sharp policy change allows us to use a regression discontinuity design (RDD) comparing firms on either side of the reform's timing cutoff. Our analysis draws on administrative data on monthly STW receipt for the universe of establishments in Germany starting in 2009 — the first representative administrative dataset on individual firms' STW receipt for the German labor market. We enrich this data by matching it to longitudinal employer-employee data from German social security records and firm financial information. This integrated dataset allows us to comprehensively study the reform's impacts on worker outcomes over several years.

In line with our extensive-margin evidence, extending the PBD did not increase employment retention at treated firms compared to control firms in the 12-48 month period after starting STW. We find a point estimate of 0.028 (SE 0.03) 12 months after take-up when we would expect retention effects to be biggest. Our tight confidence intervals allow us to reject moderate positive effects on the probability of remaining employed at treated firms, and we similarly find no employment effects at longer horizons. In the first 12 months, we find an even smaller effect of 0.003 (SE 0.01) on the probability of workers initially employed by treated firms to be employed anywhere in the labor market (including in the original firm). Taken together, our estimates imply that in the short run even the positive (though small) point estimates on employment at treated firms is partly due to a reallocation of employment from other firms rather than from reductions in non-employment. We further investigate heterogeneity in several dimensions—namely tenure, age, education, and the position in the wage distribution—and find at most small effects across all categories considered.

To rationalize the absence of employment effects, we develop a stylized model with mutual-consent bargaining that allows matches to prevent layoffs by adjusting wages instead of employment. In the model, firms have access to STW benefits in case of negative productivity shocks and renegotiate wages to prevent layoffs. In such an environment, we find that flexible wage bargaining can through downward wage adjustments substitute for a longer PBD. The model predicts that firms facing shocks can utilize STW to avoid wage cuts if benefit durations are extended, or they need to negotiate wage concessions to prevent layoffs if benefit durations are short.

To test to what extent flexible wage setting and (efficient) bargaining may have prevented layoffs in control group firms with a shorter PBD, we investigate wage growth. We find substantial and positive wage effects of STW extensions, with treated firms' wage growth exceeding that at control firms by up to 5.9 percentage points. This difference persists over several years after the treatment. To shed further light on the role of wage flexibility, we split our sample into cells based on sector, region and size, and calculate cell-specific treatment effects on wages and employment. Providing support for wage flexibility preventing layoffs, we find a negative relationship between cell-specific treatment effects on wages and employment: firms that can reduce wage growth in response to negative shocks lay off fewer workers.

This implies that control firms with a shorter PBD insure their employees at the expense of the latter's wage growth. To bolster our evidence for this mechanism, we explore firm-level heterogeneity in their insurance responses to exogenous variation in PBD. The insurance premium (incurred by employees) is larger for firms in regions with above-average local unemployment, and for those with worse access to liquidity. We further investigate the role of sectoral bargaining agreements and works councils in mediating wage and employment effects, and find wage effects driven by firms without works councils. Works councils allow to stabilize employment without cutting wages, thereby effectively substituting for firms' insurance response under shorter PBD.

Overall, our evidence points to the crucial role of the institutional environment in shaping the response to labor market policies. In the German context, decentralization of wage setting appears to be sufficiently high such that firms and workers can efficiently negotiate over wages and thereby prevent layoffs (Jäger, Schoefer, Young, and Zweimüller, 2020; Jäger, Schoefer, and Zweimüller, 2023).

The institutional environment thus appears to substitute for the policy response of STW extensions. Our evidence on wage rigidity or flexibility as a key mediator of the effect of STW extensions can also help to account for why the employment effects we estimate qualitatively differ from the ones in other contexts. For example, comparing Italy and Germany, the settings of Giupponi and Landais (2023) and our study, respectively, Boeri, Ichino, Moretti, and Posch (2021) document large differences in wage rigidity and decentralization of bargaining between the two countries. The intra-German heterogeneity in employment and wage effects of STW extensions that we document thus helps to understand the overall small employment effects as a consequence of more wage flexibility.

Our paper contributes to several strands of literature. A recent set of design-based research has used policy reforms to estimate the employment effects of STW programs (Cahuc, Kramarz, and Nevoux, 2021; Kopp and Siegenthaler, 2021; Giupponi and Landais, 2023). The existing literature has focused on the extensive margin of STW program introduction or eligibility. In contrast, our paper focuses on a key policy lever that governments use in crises—adjustments of potential benefit duration. Our work relates to the macroeconomic literature evaluating the aggregate effects of STW policies, including Boeri et al. (2021), Cahuc and Carcillo (2011), Hijzen and Martin (2013), and Balleer, Gehrke, Lechthaler, and Merkl (2016). We provide the first quasi-experimental estimate of how changes in the PBD—a primary policy tool for regulating the generosity of STW schemes—affects employment and wage outcomes. While an extensive literature studies the effects of adjusting PBD for unemployment insurance Schmieder and Von Wachter (see 2016, for an overview), including more recent evidence on heterogeneous effects of UI extensions across different initial durations (Acosta, Mueller, Nakamura, and Steinsson, 2024), ours is the first design-based estimate for the understudied yet quantitatively important policy level of STW extensions.

Our work also contributes to the literature by providing the first comprehensive analysis of Germany's STW scheme combining novel administrative data on the universe of firms participating in STW matched with employer-employee data and firm financials. Despite Germany being the largest OECD economy with a significant STW scheme and, in fact, the birthplace of STW schemes (Cahuc, 2024), previous work on the German STW largely relied on surveys to measure STW take-up, with the exception of one innovative study drawing on administrative data on STW take-up from the city of Nuremberg (Tilly and Niedermayer, 2016).

Our results suggest that wage negotiations can undo the direct effects of job retention policies when labor markets have flexible decentralized bargaining, a force overlooked in the previous macroeconomic literature on STW program impacts.

The remainder of the paper is organized as follows. Section 2.2 describes the institutional context and the reform we study. Section 2.3 introduces the datasets we use for the analysis, and Section 2.4 discusses descriptive evidence on take-up and selection into short-time work both at the individual and at the firm level. We present employment effects of individual STW eligibility in Section 2.5. Section 3.2 discusses a model of STW in the presence of wage rigidity. Section 2.7 presents evidence on employment and wage effects of varying the PBD of short-time work benefits, and the role of decentralized bargaining and wage flexibility. The last section concludes.

2.2 Short-Time Work in Germany: Institutional Context

We start by providing institutional background information on the short-time work (STW) policy scheme in Germany. The short-time work scheme allows firms to temporarily reduce working hours, while the employment agency replaces a significant share of the gap in wages for affected employees. The regular replacement rate is 60% of net wages (67% for employees with children). Once admitted to the program, firms decide every month on the reduction of working hours per employee and pay wages for hours worked as well as STW benefits to employees. After handing in detailed documentation (*Abrechnungslisten*), firms are reimbursed for the STW benefits by the employment agency.

Firms file an application for admission (*Anzeige*) to the STW scheme and need to meet certain eligibility criteria. First, the reduction in working hours must be temporary and due to economic reasons or an unavoidable event. Second, other accommodating measures such as reducing working time accounts must have already been exhausted. Third, the shock must be sizeable enough such that at least one third of the employees must each face a reduction in working hours of at least 10%. Even after successful initial admission to the program, benefit claims are preliminary until a final examination at the end that determines whether all criteria were met (*Abschlussprüfung*).

The top panel of Figure 2.8.1 illustrates the take-up of STW, measured by both the total share of establishments in STW and the share of employees in STW within establishments (all employment-weighted). At the height of the COVID-19 crisis, approximately one-third of establishments made use of STW for a grand total of one-sixth of the workforce. This suggests an average use for half the workforce within firms, conditional on firms using short-time work. This is in line with the bottom panel of Figure 2.8.1, which depicts the intensity of STW use, as measured by the share of employees in STW in the starting month of a given establishment’s STW stint. The respective distribution is almost uniform, indicating that one-quarter of all establishments used STW for at least 80% of their workforce.

Variation in the potential benefit duration. Firms may receive STW benefits for up until the potential benefit duration (PBD) as part of one successful admission to the program (STW spell). Changes in the PBD have been a key policy lever that governments use during economic downturns.

Since 2009, the PBD has been adjusted multiple times. Notably, the government increased PBD during the financial crisis as well as during the COVID-19 pandemic. Formally, a law sets the default PBD (§104 SGB III); the federal government can temporarily increase PBD by federal ordinance "in case of exceptional circumstances in the labor market" (§109 (4) SGB III). Until the end of 2015 the default PBD set by law was 6 months. The government has temporarily increased PBD by executive ordinance multiple times (18m decided on November 26, 2008 (BGBl. I. S. 2332); 24m decided on May 29, 2009 (BGBl. I. S. 1223); 18m decided on December 8, 2009 (BGBl. I. S. 3855); 12m decided on December 1, 2010 (BGBl. I. S. 1823); prolongation extended on December 7, 2012 (BGBl. I. S. 2570); October 31, 2013 (BGBl. I. S. 3905) and November 13, 2014 (BGBl. I. S. 1749)). Since a change in the law in 2016, the default PBD has been 12 months. During the COVID-19 pandemic PBD has also been temporarily extended multiple times (final extension to 28m).

2.3 Data

Our main data source is novel data on STW receipt at the establishment level starting in 2009, and on STW receipt at the individual level starting in 2020. We match the STW data to matched employer-employee data based on German Social Security Records and supplement it with firm-level financial information from Bureau van Dijk (BvD) (see Jäger, Schoefer, and Heining, 2021; Moser, Saidi, Wirth, and Wolter, 2022, for recent work with BvD data matched with German administrative data). Below, we describe our four main data sources in detail.

Establishment-level information on monthly STW receipt. We use novel data on monthly STW receipt at the establishment level starting in 2009. An establishment that has successfully been admitted to the STW program submits a detailed application every month to get reimbursed by the employment agency. The data we use is compiled for statistical purposes by the Statistics of the Federal Employment Agency (*Statistik der Bundesagentur für Arbeit: Tabellen, Realisierte Kurzarbeit, Nürnberg, Oktober 2021, Daten mit einer Wartezeit von bis zu 5 Monaten (ohne Hochrechnung)*). The close link to the operational system upon which actual payment of benefits is based ensures high data reliability. The data includes monthly information on whether an establishment receives STW benefits, the number of short-time workers, and the wage bill gap.

We match this data with the Establishment History Panel (BHP, (Ganzer, Schmucker, Stegmaier, and Wolter, 2022)) which contains information on all establishments in Germany with at least one employee liable to social security as of June 30 each year. The match allows us to add information on the establishment's location, industry, and age. Details on the matching procedure are provided in Appendix 2.A.2.1.

A STW spell is defined as the period of consecutive STW usage under the same application. A pause in STW receipt for one or two months is allowed and disregarded in the calculation of the spell's benefit duration. Throughout our analyses, we restrict attention to establishments that had not started another STW spell in the previous twelve months.

Individual-level information on monthly STW receipt. We additionally use novel data on individual-level STW receipt (*PKUG Personen in Kurzarbeit*). Since the employment agency reimburses employers for STW benefits paid to employees, the data compiled during the payment of benefits is at the establishment level, as described above. In their monthly applications (*Abrechnungslisten*), however, establishments list employees in STW and calculate their STW benefits step-by-step, documenting the wage gap and reduction in hours. In a unique data collection effort, these typically manual applications were digitalized for the period between March 2020 and December 2021, aiming to link individuals in the applications to their employment biographies. To address challenges in the digitalization process, a thorough validation procedure cross-checked information with both establishment-level data and individual employment biographies for each month. The final dataset contains, for all individuals working at establishments using STW between March 2020 and April 2021, a monthly likelihood of being in STW after various cross-checks. The likelihood is categorized as 0%, small (0-20%), medium (20-50%), high (above 50%) and 100%. Details are provided in Appendix 2.A.1. For our analysis, we consider an individual to be in STW if the likelihood is at least high.

Matched employer-employee data. We combine the information on STW receipt with employee data based on German Social Security Records since 2008. The data stem from the Integrated Employment Biographies (IEB) database of the Institute for Employment Research. Specifically, the data is based on employers' reports to the German social insurance system and includes the start and end date of each job, employees' earnings up to the censoring limit at the social security maximum earnings limit, an indicator for part-/full-time employment, and data on education levels, occupation as well as demographic information. We use standard procedures to create cross sections of the data originally stored in spell format (Stüber, Dauth, and Eppelsheimer, 2023), transforming it into a monthly panel at the individual level (see details in Appendix 2.A.2.2).

Firm-level financial information. We enrich our dataset on the policy variation of PBD with firm-level financial information from the commercial database Dafne, provided by Creditreform and Bureau von Dijk (BvD). Dafne contains financial information of German firms since 2008 and is the underlying source for data on German firms in BvD's Orbis dataset. Appendix 2.A.2.4 summarizes how we assemble and clean the firm-level financial data. We link establishments to firms using the record linkage key Orbis-ADIAB (Antoni, Koller, Laible, and Zimmermann, 2018) and focus on establishments that can successfully be matched. Table 2.C.4 in the Online Appendix shows characteristics of matched and unmatched establishments. Restricting attention to establishments that can be linked to the firm level primarily excludes very small establishments with less than five employees for which average wages are inherently volatile by construction. We aggregate the data to the firm level and conduct our main analysis at the firm level, focusing on firms with more than five full-time employees who are fully liable to social security. We provide details on the aggregation procedure in Appendix 2.A.2.3.

2.4 Take-Up and Selection into STW

We study individuals who work at establishments in Germany that started STW in 2020. Specifically, we consider the universe of establishments with more than five employees in Germany that started STW in some month (which we call start month) since April 2020, and consider individuals who work at these establishments in the start month. We add information on individual-level STW receipt in the start month and follow individual employment trajectories over time. In the following, we furthermore focus on establishments with high-quality information on individual STW receipt (see Appendix 2.A.1 for details).

Establishment-level evidence on take-up and selection. Table 2.8.1 presents summary statistics for users and non-users of short-time work at the establishment level and over different time periods. In particular, we consider the total time period with available data, 2009-2021, and dissect it into subperiods of interest, specifically the aftermath of the Global Financial Crisis (2009-2010), the European sovereign debt crisis (2011-2012), the COVID-19 crisis (2020-2021, which matches the time period for which we have individual-level data), and the remaining years in between (2013-2019).

The analysis of establishment-level data from 2009 to 2021 reveals distinct patterns in STW take-up over time. Averaged over the entire sample, STW users tended to be larger (42.41 vs. 33.65 employees) and slightly older (19.62 vs. 18.61 years) compared to non-users. Average daily wages were marginally lower for STW users (€89.35 vs. €89.44). STW users consistently exhibited negative employment growth in the year *preceding* STW take-up (-2.76 vs. 1.23 percentage points), indicating that STW was often implemented in response to ongoing employment declines.

Key differences in take-up and selection emerge between the COVID-19 pandemic and earlier time periods. While the size gap between STW users and non-users was substantial in earlier periods (e.g., 62.70 vs. 31.89 employees in 2009/2010), it narrowed significantly during the pandemic (36.19 vs. 34.55 in 2020/2021). The wage pattern also dramatically reversed: in pre-pandemic periods, STW users generally had higher average daily wages, but in 2020/2021, non-users users had significantly higher wages compared to users (€107.92 vs. €90.16).

The education composition of STW-using establishments also shifted. In earlier periods, STW users had higher shares of middle-educated workers and lower shares of low-educated workers. However, this pattern inverted in 2020/2021, with STW users showing higher shares of low-educated workers (25% vs. 20%) compared to non-users. The age-distribution differences that were prominent in earlier periods (with STW users having smaller shares of young workers) largely disappeared in 2020/2021.

Notably, the scale of STW usage increased dramatically during the pandemic. The number of STW-using establishments rose from 30,415 in 2013-2019 to 402,008 in 2020/2021. This substantial increase, combined with the changes in establishment characteristics, speaks to the much broader adoption of STW during the pandemic across various establishment types, likely reflecting the widespread economic impact of COVID-19 rather than the more selective use seen in previous economic downturns. Despite these significant changes in STW take-up patterns during the COVID-19 period, one feature remained consistent with earlier periods: STW users continued to exhibit lower establishment growth in the preceding year compared to non-users (-2.23 vs. 1.88 percentage points in 2020/2021), mirroring the pattern observed in previous years and suggesting that STW continued to be taken up in response to longer-running employment declines regardless of the broader economic context.

Individual-level evidence on take-up and selection. In Table 2.8.2, we turn to individual-level data for the COVID-19 period and focus on establishments with STW take-up and differentiate between workers on STW vs. workers with no take-up.[1] We consider establishments with short-time work in April 2020 vs. any start month from April to December 2020, capturing heterogeneity in how the crisis unfolded.

STW workers earned higher daily wages (€103.96 vs. €88.44), were more likely to have mid-level education (69% vs. 61%), and were less likely in the low-education group (12% vs. 22%). They were overrepresented in production (34% vs. 29%) and commercial service (31% vs. 28%) occupations, and more frequently engaged in complex specialist tasks (17% vs. 14%). STW take-up was more com-

1. For this analysis, we focus on establishments with a high quality of individual STW data. Appendix Table 2.C.1 reports summary statistics for all establishments, without conditioning on the quality of individual STW receipt data.

mon among middle-aged workers (35-54 years old, 49% vs. 40%) and less likely among older and younger workers. There is also a slight skew towards those with longer job tenure. Our findings suggest that firms targeted STW programs primarily at a more established workforce segment, characterized by higher wages, mid-level education, and more complex job roles, potentially reflecting a strategy to retain skilled labor during economic uncertainty.

The establishment-level and individual-level data reveal some contrasting patterns of STW take-up during the COVID-19 period. While at the establishment level, STW users had lower average daily wages compared to non-users (€90.16 vs. €107.92), within STW-using establishments, workers on STW earned higher daily wages than their non-STW counterparts (€103.96 vs. €88.44). Similarly, although STW-using establishments had higher shares of low-educated workers (25% vs. 20%) compared to non-users, within these establishments, STW workers were less likely to be in the low-education group (12% vs. 22%) and more likely to have mid-level education (69% vs. 61%). This suggests that while the COVID-19 crisis led to broader STW adoption across various establishment types, including those with lower average wages and education levels, within these establishments, STW was still predominantly used for retaining higher-wage, more educated workers in more complex job roles.

Firms target STW towards workers with high retention probabilities. We next investigate more formally whether establishments target STW towards workers with high retention probabilities (even in the absence of STW). To evaluate this possibility, we estimate a logistic regression model of retention at the same employer 12 months later on rich individual and establishment characteristics in a training sample in the pre-COVID-19 pandemic period, and use the coefficients to predict the retention probability for individuals in the sample (for details see Appendix 2.A.1). The respective summary statistics are reported in the last row of Table 2.8.2.

Panel (a) of Figure 2.8.2 zooms into establishments with STW take-up between April and December 2020, and shows that predicted retention strongly predicts individual STW take-up in a binned scatter plot.[2] A ten percentage point increase in the predicted retention probability increases STW take-up by 4.9 percentage points (with a standard deviation of 0.2 percentage points). This implies that short-time work was targeted towards individuals that were very likely to be retained even in the absence of a STW-triggering event or STW take-up itself.

We additionally validate the prediction model in Figure 2.8.2, Panel (b), which demonstrates a remarkably linear relationship between predicted retention probability and actual retention in a binned scatter plot. A ten percentage point increase in the predicted retention probability corresponds to a 9.3 percentage point in-

2. We focus on the first STW spell in case of multiple.

crease in actual retention (with a standard deviation of 0.1 percentage points). The strength of this relationship is particularly noteworthy given that our prediction model was trained on pre-pandemic data, yet maintains its predictive power when applied to the pandemic period—a dramatically different economic context—and specifically in firms utilizing STW. This robust performance suggests that the underlying factors influencing employee retention remained relatively stable despite the unprecedented economic disruptions caused by the pandemic.

Panel (c) of Figure 2.8.2 further dissects this relationship by comparing individuals with and without STW take-up. We find a slope of close to one (0.98) between actual and predicted retention for non-STW individuals. For STW recipients, the slope is substantially lower at 0.77. At lower levels of predicted retention, STW recipients demonstrate a notably higher likelihood of remaining with their firm. This disparity diminishes as the predicted retention probability increases. This pattern suggests that STW is associated with a higher probability of actual retention, driven by individuals who would otherwise have been at higher risk of separation (i.e., those with lower predicted retention probabilities). The evidence, while purely correlational, leaves room for the possibility that STW, while targeted towards individuals with higher predicted retention, may be most impactful in retaining employees who, based on pre-pandemic patterns, would have been more likely to leave or be dismissed.

2.5 Effects of Individual Eligibility

In this section, we turn to formally estimating the effects of individual STW eligibility on employment—a primary policy concern.

Based on the set of establishments, with high-quality information, that took up STW between April and December 2020, we start out with individual-level regressions, using as dependent variable whether or not a given worker is still with the same employer 12 months later. The results are in Table 2.8.3.[3] After controlling for time-invariant unobserved heterogeneity at the firm level in column 2, the employment effect remains not only positive and statistically significant, but increases further to eight percentage points. This attests to the endogeneity in individual take-up, as shown in Figure 2.8.2: firms are more likely to take up STW for individuals with a higher ex-ante retention probability. The resulting positive employment effect is robust to including a rich set of individual-level controls in column 3.

In the remaining columns of Table 2.8.3, we consider as dependent variable an indicator variable for being employed *anywhere* one year after the start month. All

3. Tables 2.C.2 and 2.C.3 in the Online Appendix present the respective estimates for the total sample and full-time employees only.

estimates drop in size compared to their counterparts in the first three columns. This suggests that the effect on employment at the initial employer is partly due to hindered reallocation due to short-time work.

To address potential endogeneity issues of individual STW take-up, we exploit sharp policy variation in individual-level eligibility around the statutory retirement age. In particular, we use the fact that even though many continue to work, workers generally lose potential access to STW after reaching the statutory retirement age (SRA). Based on the set of establishments from before which take up STW in April 2020, we differentiate different cohorts of individuals by the month in which they reach the SRA: three months before up until three months after their respective employers take up STW. Among these cohorts, workers that reach the SRA before (or in) April 2020 are never eligible for STW and, thus, experience no change in their potential access to STW at their statutory retirement age. Unlike those control cohorts, the remaining treated cohorts comprise individuals that reach the SRA after STW take-up in April 2020, meaning that they are eligible for short-time work before they reach the statutory retirement age and lose access to it thereafter.

Panel (a) of Figure 2.8.3 plots event studies around STW take-up for cohorts reaching the SRA around the time their establishment takes up STW in April 2020. Across all cohorts, there is no discernible difference in the drop in employment at the initial employer at the statutory retirement age. To visualize this differently, Panel (b) sorts by calendar time around the establishments' STW take-up in April 2020. In particular, this makes it evident that employment levels are not any higher before reaching the SRA before and after April 2020, despite the fact that the post-April 2020 cohorts have increasing shares of individual STW take-up over time (cf. Panel (c)). Panel (d) concludes this analysis by showing that the drop in employment rates is approximately 30 percentage points (two months before vs. two months after reaching the SRA), irrespective of whether a cohort loses potential STW benefits or not upon reaching the SRA.

While the focus on individuals still working very few months before reaching the statutory retirement age comes at the cost of limited external validity of our quasi-experimental design, the absence of any effect of individual STW eligibility on employment outcomes does suggest that our endogenous estimates in Table 2.8.3 are, at best, an upper bound. This casts severe doubt on the existence of any employment effects along the extensive margin of extending short-time work.

2.6 Conceptualizing the Absence of Employment Effects

We now turn to a simple model of STW that can rationalize the absence of employment effects. In particular, we argue that firms with a shorter PBD can retain employees to the same extent as firms with prolonged PBD do by adjusting wages.

2.6.1 Model Setup

In this section, we develop a stylized three-period model of STW that allows for persistent wage effects between firms with different maximum potential benefit duration. The model focuses on firms that experience a negative productivity shock in the first period and abstracts from firms with "normal" productivity that will not use STW benefits in the first period.[4] We assume that working hours can be set flexibly each period and that wages are negotiated with mutual consent (Postel-Vinay and Turon, 2010). Mutual-consent bargaining is relevant in periods 1 and 2 of the model when productivity is low and wages might need to be reduced to guarantee positive continuation values of firms. As we only consider negative productivity shocks, the only relevant wage adjustments are wage reductions. The wage adjustment after a negative shock makes the firm indifferent between continuing the match and a layoff.[5]

We model STW as in Cahuc, Kramarz, and Nevoux (2021). Firms receive a wage subsidy σ per reduced hour whenever the current hours worked are below a threshold value $\hat{h}$. Importantly, our model differs from Cahuc, Kramarz, and Nevoux (2021) in two respects. First, to speak to our empirical evidence, we allow for firms with short and long maximum potential benefit duration. Differences in maximum benefit duration will be relevant in the second period of the model, when firms with long maximum duration are allowed to still use STW whereas firms with short maximum benefit duration cannot use STW. Second, we assume mutual-consent bargaining for wages whereas Cahuc, Kramarz, and Nevoux (2021) assume Nash bargaining. The mutual-consent bargaining implies that we have the last bargained wage as an additional state variable to the problem and that wages become rigid.

The state variables of the problem are the negotiated wage w and the current level of productivity A. We denote by $\bar{A}$ the "normal" productivity level and A is a random productivity level with support $[0,\bar{A})$. We assume that firms start into period 1 with some negotiated wages w and a productivity draw $A < \bar{A}$, i.e., all firms start with a negative productivity shock. We consider persistent but mean-reverting productivity shocks. In the third period, productivity will have always recovered so that $A = \bar{A}$. In period 2, there is a probability π that productivity recovers, and with probability $1 - \pi$ productivity remains persistently low at $A < \bar{A}$. We assume that workers and firms have linear utility and discount the future at discount rate $\beta \in (0, 1)$. In unemployment, the worker receives flow utility b

4. Firms can be arbitrarily close to their normal productivity level and the model also features distortions in the hours choice. Cahuc, Kramarz, and Nevoux (2021) provide a detailed analysis of this problem.

5. We consider a model in partial equilibrium but the underlying assumption is that free entry leads to a continuation value of zero for the firm with a vacancy.

and we assume that unemployment is an absorbing state. We solve the model by backward iteration.

Period 3. For the third period, we solve a fixed point problem with a firm having its "normal" productivity state $\bar{A}$. Initially, the match enters the third period with a wage w from period 2 that might be below its "normal" long-run level $\bar{w}$. The wage will, however, recover in the third period. We assume a reduced form where the wage recovers to $\bar{w}$ with probability λ that we interpret as the probability of an outside offer from another firm. We abstract from firm heterogeneity, so wages in this case will be set by mutual consent to $\bar{w}$.[6]. The only decision for the match in period 3 is to set hours. We assume that the hours choice is made to maximize the joint surplus of the match $S(A)$.[7]

The value functions of the worker in employment $V_3(w,A)$ and unemployment U in period 3 are:

$$\begin{aligned} U &= b + \beta U \\ V_3(\bar{A},w) &= wh^* - \psi(h^*) + \beta\Big((1-\rho)\Big(\lambda V_3(\bar{w},\bar{A}) + (1-\lambda)V_3(w,\bar{A})\Big) + \rho U\Big), \end{aligned}$$

where h^* denotes the optimal hours choice and $\psi(h)$ is the disutility from work that is increasing and convex. We also allow for an exogenous separation rate ρ. The worker surplus Δ in period 3 is:

$$\begin{aligned} \Delta_3(\bar{A},w) &= V_3(\bar{A},w) - U \\ &= wh^* - \psi(h^*) - b + \beta\Big((1-\rho)\Big(\lambda\Delta_3(\bar{A},\bar{w}) + (1-\lambda)\Delta_3(\bar{A},w)\Big)\Big). \end{aligned}$$

The value function of the firm is:

$$J_3(\bar{A},w) = \bar{A}h^* - wh^* + \beta(1-\rho)\Big(\lambda J_3(\bar{A},\bar{w}) + (1-\lambda)J_3(\bar{A},w)\Big),$$

where we exploit that the continuation value of the firm in case of separation is zero. The total surplus of the match in period 3 is then given by:

$$\begin{aligned} S_3(\bar{A}) &= \Delta_3(\bar{A},w) + J_3(w,\bar{A}) \\ &= \bar{A}h^* - \psi(h^*) - b + \beta\Big((1-\rho)\Big(\lambda S_3(\bar{A},\bar{w}) + (1-\lambda)S_3(\bar{A},w)\Big)\Big). \end{aligned}$$

6. Postel-Vinay and Turon (2010) provide a model with on-the-job search.

7. Note that the total surplus of the match depends only on productivity A but not on the wage w that splits the total surplus.

To maximize the joint surplus, hours worked are set to satisfy the first-order condition, i.e., the hours choice in period 3 solves

$$h^* = \psi'^{-1}(\bar{A}).$$

Period 2. Productivity shocks are persistent and all firms start the first period with a below-normal productivity A. In period 2, productivity recovers with probability π, so some firms have $A = \bar{A}$ and some firms have persistent realizations with $A < \bar{A}$ from the first period. We distinguish between two types of firms regarding their eligibility status $i \in \{S, L\}$. Long-eligibility firms ($i = L$) have access to STW benefits in period 2, whereas short-eligibility firms ($i = S$) cannot rely on STW benefits in period 2. The value functions for firms of each type in the second period are:

$$\begin{aligned} J_2^L(A,w) &= Ah^{L,*} - wh^{L,*} + \sigma \max\{\hat{h} - h^{L,*}, 0\} + \beta(1-\rho)J_3(\bar{A},w) \\ J_2^S(A,w) &= Ah^{S,*} - wh^{S,*} + \beta(1-\rho)J_3(\bar{A},w), \end{aligned}$$

where $\hat{h}$ denotes the hours threshold to be eligible for STW benefits σ. We follow Cahuc, Kramarz, and Nevoux (2021) for the specification of STW benefits with subsidy σ for hours below the eligibility threshold $\hat{h}$. The value functions of the worker and the resulting worker surplus are:

$$\begin{aligned} V_2^i(w,A) &= wh^{i,*} - \psi(h^{i,*}) + \beta\Big((1-\rho)V_3(\bar{A},w) + \rho U\Big) \\ U &= b + \beta U \\ \Delta_2^i(w,A) &= h^{i,*} - \psi(h^{i,*}) - b + \beta(1-\rho)\Delta_3(\bar{A},w). \end{aligned}$$

The joint match surplus is:

$$\begin{aligned} S_2^L(A) &= J_2^S(A,w) + \Delta_2^L(A,w) \\ &= Ah^{L,*} - \psi(h^{L,*}) - b + \sigma \max\{\hat{h} - h^{L,*}, 0\} + \beta(1-\rho)S_3(\bar{A}) \\ S_2^S(A) &= J_2^S(A,w) + \Delta_2^S(A,w) \\ &= Ah^{S,*} - \psi(h^{S,*}) - b + \beta(1-\rho)S_3(\bar{A}). \end{aligned}$$

First-order conditions for hours to maximize the join surplus imply:

$$h^{i,*} = \begin{cases} \psi'^{-1}(A-\sigma) & \text{if } h^{i,*} < \hat{h} \text{ and } i = L \\ \psi'^{-1}(A) & \text{otherwise,} \end{cases}$$

with the boundary condition $h^* = 0$ if $A - \sigma \leq 0$ in case the firm is eligible and chooses to use STW. For wages, mutual-consent bargaining implies that if A is so low relative to the currently bargained wage w that the value of the firm turns negative, then the renegotiated wage $\hat{w}$ is set such that the continuation value of the firm is zero. The implied wage setting rule is:

$$w = \begin{cases} w & \text{if } J_2^i(A,w) > 0 \\ \hat{w} : J_2^i(A,\hat{w}) = 0 & \text{otherwise.} \end{cases}$$

The decision to use STW follows directly from the hours choice. We therefore assume that the decision to use STW is also taken so as to maximize the joint surplus of the match. Given that the decision to implement STW is negotiated between the firm and the works council in Germany, we argue that surplus maximization is a reasonable approximation. Hence, the worker and the firm compare the joint surplus with and without using STW, if eligibile, and choose the maximum of the two, i.e.,

$$S^L(A) = \max\{S^L_{stw}(A), S^L_{no}(A)\},$$

where $S^L_{stw}(A)$ denotes the joint match surplus when using STW, i.e., when $h^* < \hat{h}$, and $S^L_{no}(A)$ denotes the joint surplus when not using STW, i.e., when $h^* \geq \hat{h}$.

Period 1. The first period is similar to the second period except that now all firms are eligible to use STW and productivity has not yet recovered, so that all firms have a productivity level $A < \bar{A}$. We still need to distinguish between firms with short and long eligibility because of their continuation value. Firms with long eligibility that are still eligible in the second period will have a higher expected firm surplus and, therefore, have less often a binding participation constraint for the firm. Firms with short eligibility face the risk that in the second period their productivity has not recovered, so they are experiencing a persistent shock but they are no longer eligible for STW benefits. The value functions of the two firm types are:

$$\begin{aligned} J_1^L(A,w) &= Ah^* - wh^* + \sigma \max\{\hat{h} - h^*, 0\} + \beta(1-\rho)\Big(\pi J_2^L(\bar{A},w) + (1-\pi)J_2^L(A,w)\Big) \\ J_1^S(A,w) &= Ah^* - wh^* + \sigma \max\{\hat{h} - h^*, 0\} + \beta(1-\rho)\Big(\pi J_2^S(\bar{A},w) + (1-\pi)J_2^S(A,w)\Big). \end{aligned}$$

As hours are set intratemporally to maximize the joint surplus, they only depend on the current productivity level A but not the eligibility status. The value functions of the worker and the corresponding worker surplus are for $i \in \{L, S\}$:

$$\begin{aligned} V_1^i(w,A) &= wh^* - \psi(h^*) + \beta(1-\rho)\Big(\pi V_2^i(w,\bar{A}) + (1-\pi)V_2^i(w,A)\Big) + \beta\rho U, \\ U &= b + \beta U \\ \Delta_1(A,w) &= wh^* - \psi(h^*) - b + \beta(1-\rho)\Big(\pi \Delta_2^i(w,\bar{A}) + (1-\pi)\Delta_2^i(w,A)\Big). \end{aligned}$$

The resulting joint surplus is:

$$
\begin{aligned}
S_1^L(A) &= J_1^L(A,w) + \Delta_1^L(A,w) \\
&= Ah^* - \psi(h^*) - b + \sigma \max\{\hat{h} - h^*, 0\} + \beta\Big(\pi S_2^L(\bar{A}) + (1-\pi) S_2^L(A)\Big) \\
S_1^S(A) &= J_1^S(A,w) + \Delta_1^S(A,w) \\
&= Ah^* - \psi(h^*) + \sigma \max\{\hat{h} - h^*, 0\} + \beta\Big(\pi S_2^S(\bar{A}) + (1-\pi) S_2^S(A)\Big),
\end{aligned}
$$

and first-order conditions for hours imply

$$
h^* = \begin{cases} \psi'^{-1}(A-\sigma) & \text{if } h^* < \hat{h} \\ \psi'^{-1}(A) & \text{otherwise,} \end{cases}
$$

with the boundary condition $h^* = 0$ if $A - \sigma \leq 0$ in case the firm is using STW. For wages, it might be the case that because of a low productivity realization the firm has a binding participation constraint. In this case, there is mutual consent to rebargain to $\hat{w}$. The wage $\hat{w}$ is characterized by the condition that the firm has a continuation value of zero. We can derive the wage setting rule:

$$
w = \begin{cases} w & \text{if } J_1^i(A,w) > 0 \\ \hat{w} : J_1^i(A,\hat{w}) = 0 & \text{otherwise} \end{cases} \tag{2.6.1}
$$

for $i \in \{L, S\}$. Finally, the decision to enter STW, i.e., to choose $h^* < \hat{h}$, is again a direct consequence of the hours choice and made to maximize the joint surplus of the match.

Parameterization. The model is highly stylized and we therefore abstain from calibrating it directly to the data. Instead, speaking to our empirical design, we parameterize the model to demonstrate the differential wage adjustments absorb employment adjustments for firms with differences in PBD of STW. For the (dis)utility function from work, we assume a standard functional form

$$
\psi(h) = \frac{h^{1+\phi}}{1+\phi}.
$$

We set the remaining parameters as shown in Table 2.8.4. The low discount factor β and the high wage relative to productivity $\bar{w} = 0.9\bar{A}$ imply that the stable employment situation starting in period 3 does not dominate the surplus in the two initial periods. Effectively, the discount factor is a stand-in for the expected duration of the match that is affected by future job-to-job mobility, retirement, or quits of workers. We set the threshold for STW access to 70 percent of the normal hours choice $\bar{h}$, i.e., the hours choice when productivity is $\bar{A}$. To use STW benefits, firms during the time period considered in our empirical analysis had to have an hours reduction for 30 percent of total employment.

2.6.2 Model Results

To illustrate the effect of differences in the PBD on wages, we start all firms at $\bar{w}$ and consider an initial productivity shock of 50 percent, i.e., we set $A = 0.5\bar{A}$. We assume that productivity recovers in period 2 and stays at its normal level from then on. Hence, we only consider relative to expectations a transitory productivity shock.

Figure 2.8.10 shows the wage difference between a firm with long and a firm with short PBD. The firm with prolonged PBD lowers wages by less than the firm with the shorter PBD. The reason is that the option for the longer access to STW benefits increases the value of the firm in period 1. Therefore, the firm does not have to enter into wage negotiations to keep its surplus positive. The value of the firm that only has access to STW benefits in the first period turns negative in the first period because of the risk that STW benefits will not be available when productivity is still low in period 2. Importantly, wage negotiations happen by mutual consent, and the wages of workers in the firm with short access to STW benefits will be cut. The wage effect in the model will always be transitory as all firms recover from the initial shock and workers will eventually receive outside offers that will lead to mean reversion of wages in the future.

The wage effect in the model is the result of efficient negotiation between the worker and the firm to lower wages in an attempt to avoid layoffs. This flexible wage setting in case of productivity shocks provides insurance and employment stability to the worker. Period-by-period Nash bargaining of wages would also provide a mechanism to trade off wages and job stability, but only the infrequent wage adjustment with mutual-consent bargaining yields persistent wage dynamics from transitory shocks and differences in future eligibility to STW benefits.

2.7 Extending the Potential Benefit Duration of Short-Time Work

Having empirically considered the possibility of employment effects along the extensive margin of short-time work, we next turn to intensive-margin variation. In particular, a key policy lever that governments use is the potential benefit duration (PBD) of STW benefits.

The top panel of Figure 2.8.4 shows the PBD (left y-axis) for firms that started STW in the respective months since 2005. The figure also shows the unemployment rate in Germany (right y-axis) to illustrate the countercyclical nature of extensions, alongside the PBD of unemployment insurance (UI). Unlike in the US where UI PBD is, by design, countercyclical (see, e.g., Schmieder and Von Wachter, 2016), Germany has historically not changed UI PBD in response to crises.

For the purpose of identifying the effect of PBD on employment and wages, we focus on a sharp and unexpected reform in 2012 that doubled the STW PBD from 6 to 12 months, which we describe next.

2.7.1 The 2012 Reform: STW Extension

After executive ordinances had continuously extended the default PBD of 12 months in the aftermath of the global financial crisis, no further extension was planned beyond the end of 2011. For firms starting STW in January of 2012, the PBD was set back to the default length of 6 months.

Even though the German economy was entering an economic slowdown around the time of the European debt crisis of 2012, the labor minister, Ursula von der Leyen, publicly denounced plans to change the PBD as late as November 25, 2012. Yet, on December 7, 2012, she reversed course and decided to double the PBD from 6 to 12 months. The timing of the reform and the policy reversal was sudden and sharp; we include an overview of newspaper coverage from November and December of 2012 in Appendix Figure 2.B.1, highlighting the unexpected and sharp nature of the policy reversal.

The extension also applied to firms with benefit receipt at the time and was backward-binding: firms that started STW in 2012 could not anticipate the subsequent change in PBD, but depending on when they started are either be covered by the extension or not. If their benefits had run out by December, they were not eligible for the extension. If they were still on benefits in December, they could claim benefits for another 6 months.

In the bottom panel of Figure 2.8.4, we illustrate the reform for starters in May and July of 2012. Firms that started STW receipt in July and still received STW benefits in December (last month of a 6 month spell under the old PBD regime) could benefit from the extension and continue using STW in 2013. For firms that had started in May 2012, the PBD for uninterrupted usage ended in October, hence before the reform.

When a firm's STW spell reaches the PBD limit, the firm has to pause STW receipt. In principle, it can apply for benefits again in the future. However, this necessitates a new STW application and can only occur after a mandatory moratorium of at least three months. In principle, gaps in STW receipt of up to two months are allowed within one STW spell and prolong the PBD accordingly. We address this concern in two ways. First, we document that among all STW spells that start in 2011 or 2012 84% do not have interruptions. Second, we ignore starters in June of 2012 whose PBD expired in November, but who may still receive STW benefits if they had a gap of one month.

2.7.2 Research Design: Regression Discontinuity Based on STW Start Date

Our design exploits the 2012 reform in the sense that firms that started STW in the second half of 2012 and were still using STW in December were ex post eligible for the prolonged benefit duration.

We estimate the following linear regression discontinuity model for outcome $y_{i,h}$ for horizon h for firm i that starts STW in start month $m(i) \in \{2011m1, 2011m2\ldots, 2012m12\}$:

$$y_{i,h} = \beta_{1,h}D_{m(i)} + \beta_{2,h}D_{m(i)}\mathbf{1}(D_{m(i)} > 0) + \tau_h\mathbf{1}(D_{m(i)} > 0) + \alpha_m + \beta_{3,h}X_{m(i)} + \epsilon_{i,h}, \tag{2.7.1}$$

with running variable D_m and controls X_m defined as follows:

$$\begin{aligned} D_m &:= (m - 2012m6) \cdot X_m \\ X_m &:= \mathbf{1}(m \in \{2012m1, \ldots, 2012m5, 2012m7, \ldots, 2012m12\}). \end{aligned}$$

α_m denotes calendar-month fixed effects.

The specification is a regression discontinuity design with distance to the cutoff 2012m6 (D_m) as running variable. The coefficient of interest for horizon h is τ_h, which we interpret as the treatment effect of the STW extension.

The design is estimated for firms that start in 2012 (X_m); we also include firms that start STW in 2011 so as to be able to include calendar-month fixed effects, allowing us to account for seasonality in the usage pattern. We exclude establishments that start STW in the cutoff month itself as we only have start date information at the monthly level and firms starting in June 2012 may or may not be eligible for the extension depending on whether they started before or after June 7, 2012.

Our baseline specification includes industry-by-region fixed effects. Industries are defined at the 1-digit level as sections based on the Classification of Economic Activities (WZ 2008) and regions as states (*Bundesländer*). The outcome variables of interest are employment (share of initially employed that are employed anywhere), employment at initial employer (share of initially employed that are still employed at the firm) as well wage growth in average daily wages relative to the start month of STW.

Summary statistics and descriptive evidence. For our analysis on the PBD as an important policy lever, we focus on firms that start STW in 2011 and 2012, and investigate worker and firm outcomes in terms of employment and wages over time. Specifically, we define a firm based on its employees in the start month of STW, and follow their employment status as well as wages in the months following the start of STW. To reduce noise when studying the evolution of wages, we restrict attention to individuals that work full-time and are fully liable to social security (*Sozialversicherungspflichtig Beschäftigte ohne besondere Merkmale, Personengruppenschlüssel 101*).

Table 2.8.5 shows key summary statistics for firms that start STW in 2012, the year of the policy variation. The median firm has 20 employees. The difference to the size of the average firm (67 employees) implies a skewed size distribution. While financial information based on balance sheets (assets, cash) is widely available, the availability of financial information based on income statements is substantially worse. This is due to German reporting requirements: small firms (*Kleinst-Kapitalgesellschaften* and *kleine Kapitalgesellschaften*), defined based on a combination of thresholds for revenue, assets and employees, are not required to publish information beyond their balance sheet.

To better interpret the magnitude of subsequent effects on wage growth, Table 2.8.5 also includes summary statistics of the growth rates of average wages for different horizons. On average, wages increase by 3 (6, 9, 11) percent one (two, three, four) years after the start of STW and relative to the level at the start of STW. For at least 75% of firms, wage growth is non-negative in the first year since the start of STW.

Figure 2.8.5 shows the differences in consecutive use of STW and employment outcomes for our treatment and control groups non-parametrically. Treated firms are more likely to use STW for more than six but fewer than twelve months, and it is during the same short time period that any potential employment differences emerge.

2.7.3 RDD Balancedness and First Stage

In the following, we consider statistics relevant for the validity of our regression discontinuity design. First, consistent with the fact that we leverage an unexpected and backward-binding reform, we find that characteristics of firms are smooth around the cutoff date. In Figure 2.8.6, we show that firms are similar around the cutoff date in terms of (i) their total number of employees, (ii) average daily wage paid, (iii) the number of observations available, which reflects the number of firms starting STW in a given month, and (iv) the share of manufacturing firms, which faced particularly severe economic conditions.

We next turn to the first stage of our regression discontinuity, and confirm that firms in the treatment group indeed had substantially longer STW benefit receipt compared to firms in the control group. This is the case irrespective of whether we consider firms' consecutive (Panel (e) in Figure 2.8.6) or nonconsecutive (Figure 2.B.2 in the Online Appendix) use of short-time work.

Finally, we consider firm-level determinants of using more than six months of short-time work when it becomes available for firms (starting STW in the second half of 2012) in Table 2.C.5 of the Online Appendix. We include as covariates in the cross-sectional regressions firms' total number of employees, their average daily wage, and their age as we have shown previously that firms are balanced in terms of those characteristics around the cutoff date. As such, from this exercise one learns what types of firms would have desired a longer PBD than was available in the first half of 2012. We find that older firms, those with higher average wages, and smaller firms are more likely to take up short-time work benefits for more than six months when it is possible to do so, while the wage growth compared to the year prior to the start of STW bears no statistically significant effect. Note that by controlling for industry by region fixed effects, we also account for any potential differences in the severity of economic conditions across local sectors.

2.7.4 (No) Employment Effects of Short-Time Work Extensions

Using our RDD, Figure 2.8.7 tests whether firms that for plausibly exogenous reasons have a longer PBD relative to otherwise equivalent firms retain a larger fraction of their workforce.[8] At least 24 months later, the propensity to retain employees is indistinguishable between the two groups.

Zooming in on the first 18 months after the start of STW, the top panel of Figure 2.8.8 shows that the employment effect in Panel A of Figure 2.8.7 is statistically insignificant. When considering not only employment at the firm at which individuals were employed at the time said firm started STW but employment anywhere in the bottom panel of Figure 2.8.8, the short-term effect becomes even smaller, in line with our results for the extensive margin in Table 2.8.3. This implies that at least in the short run, any positive, albeit small, point estimates on employment at treated firms are due to a reallocation of employment from other firms rather than from reductions in non-employment.

Table 2.8.6 tests these differences more formally, and presents the RDD estimates for different horizons using as the dependent variable employment at initial employer in the top panel. Neither are all point estimates positive nor is any one of them statistically significant. Thus, firms with prolonged PBD (twelve months) are no more or less likely to keep their employees than firms with shorter PBD (six months). This holds also for employment anywhere (including the initial employer) in the bottom panel, where the point estimates become somewhat larger after 18 months.

We next investigate heterogeneity in several dimensions of worker-level characteristics. For the sake of compactness, we summarize our results graphically, and present the point estimates alongside confidence bands for the baseline effects and the respective interaction effects.[9]

8. Starters in June are excluded from the regression, but are included (in gray) in Figure 2.8.7 as well.

9. We include the full tables for both dependent variables in the Online Appendix, in Tables 2.C.6 to 2.C.13.

Regardless of whether we consider employment at the initial employer (Figure 2.8.9) or employment anywhere (Figure 2.B.3 in the Online Appendix), we find only small and never any statistically significant effects across all worker characteristics that we consider, ranging from tenure, age, education to the position in the wage distribution.

2.7.5 Wage Effects and the Role of Wage Flexibility

In line with our estimates from the extensive-margin variation in individual STW eligibility, we uncover precisely estimated zero employment effects from a longer PBD, i.e., variation in the intensive margin of short-time work. At first glance, this is at odds with other design-based work that has found positive employment effects of STW, be it in France (Cahuc, Kramarz, and Nevoux, 2021), Switzerland (Kopp and Siegenthaler, 2021), or—at least in the short run—also in Italy (Giupponi and Landais, 2023).

In striking contrast to these countries, Germany has substantially more decentralized (wage) bargaining institutions. Wage rigidity, on the other hand, is a key friction that inhibits efficient renegotiation. In particular, wage flexibility can preserve jobs where firm surplus would have been negative, leading to layoffs when wages are fixed, but joint surplus remains positive (Jäger, Schoefer, and Zweimüller, 2023). This opens up the possibility that decentralized bargaining and wage flexibility are potential remedies.

2.7.5.1 Effect of STW Extensions on Wage Growth

To test the role of wage flexibility, as also highlighted in the model, we return to our empirical setting and study effects on wages. We focus on workers initially employed at firms that use short-time work in 2012. In particular, we use as dependent variable the growth in average daily wages relative to a given firm's short-time work start month. In doing so, we consider workers' wages in the post-period, measured one to four years later, earned anywhere, possibly at another firm.[10] Due to the fact that wages in the first year upon receipt of short-time work are potentially mismeasured, we focus on longer horizons starting 24 months.[11]

Figure 2.8.11 shows that firms with shorter PBD adjust their wages downward relative to otherwise equivalent firms that are treated with extended benefits. We test this more formally in Figure 2.8.12 and Table 2.8.7 where the effect size is long-lasting and increasing in the horizon, leading to treated firms' wage growth exceeding that at control firms by up to 5.9 percentage points. Our empirical findings are broadly consistent with the model-implied paths of wage effects.

10. We will separately focus on wage effects among stayers and switchers below.

11. STW, albeit to a small extent, affects social security contributions, and, thus, during STW receipt, contaminates wages as reported to the German social insurance system.

As treated and control firms do not vary in employment outcomes—i.e., firms with a shorter PBD offer the same level of employment protection—our evidence is consistent with an insurance premium incurred by employees at firms with a shorter PBD.

Unless there are adverse effects on individual workers' matching in the labor market subsequent to working at a firm with a shorter PBD, it should be primarily employees remaining with the same firm that see relative wage cuts in exchange for employment protection in spite of shorter PBD. Across panels in Table 2.8.8, we consider heterogeneous treatment effects for workers that are no longer with the same firm—i.e., switchers—one to four years upon said firm starting to use STW.

Switching almost eradicates the treatment effect on wages. Especially workers that switch within the first two years see no wage adjustments. Naturally, our estimates for the coefficient on the respective interaction effect become weaker for longer horizons when we consider switchers within three or four years, as the ex-post probability of having already switched by the time wage growth is measured decreases in the horizon length.

2.7.5.2 Interdependency of Employment and Wage Effects

In this section, we seek to characterize the circumstances under which firms trade off employment versus wage effects. Our findings suggest that, on average, firms with a shorter PBD retain their employees just like treated firms with a longer PBD, but they do so at an insurance premium in the form of lower wage growth. This begs the question as to whether there exists underlying heterogeneity in firms' responses. To shed light on this, we split the sample into cells based on sector (manufacturing, wholesale and retail trade, other), region (East/West), and size (up to 5, 6-15,16-50, more than 50). We then calculate cell-specific treatment effects on employment and wages.

If the absence of an effect on employment across treated and control firms is symptomatic of control firms with a shorter PBD insuring their employees at the expense of the latter's wage growth, then one should detect an employment effect, but no wage effect, for control firms that do not insure their employees, even if this is not their average response in our data. Using all available establishments—i.e., without requiring firm-level data—Figure 2.8.13 reveals for both employment-related outcomes and the shortest valid horizon (24 months) that positive wage effects go hand in hand with zero or negative employment effects, while positive employment effects are associated with zero or negative wage effects. Firms that lower wages more in response to shorter PBD (in comparison to the treatment group) preserve more employment. The elasticity is -0.86, i.e., a ten percent decrease in wages is associated with an 8.6 percent increase in employment.

As our baseline sample is conditional on available firm-level data from Orbis, this also implies an admittedly modest sample selection in terms of firm size, although even small and medium-sized companies are covered by Orbis. However, the sample is fairly representative as it covers 77% of all employees at establishments that made use of short-time work in 2012. To establish whether firms with balance-sheet data that populate our baseline sample are indeed focused on a different quadrant of the cell-level analysis, we split up the previous figure into the latter group and the remaining group without firm-level balance-sheet data coverage, comprising arguably smaller firms. Figure 2.B.4 in the Online Appendix shows that in contrast to firms with balance-sheet data, those that do not make part of our baseline sample are indeed more likely to exhibit employment effects, but no (positive) wage effects. These results also hold for a longer horizon of 36 months (Figures 2.B.5 and 2.B.6 in the Online Appendix).

2.7.5.3 Heterogeneity by Local Labor Market Conditions

We next return to our baseline sample with firm-level balance-sheet data, and discuss concrete channels underlying firm-level heterogeneity in their insurance responses to exogenous variation in PBD. We first consider to what extent the treatment effect varies by local labor market conditions. For this purpose, we generate an indicator variable that equals one for above-average local unemployment.

As can be seen in the top panel of Table 2.8.9, although often borderline insignificant (with the exception of the last column), worse labor market conditions tend to be problematic for the remaining employed (at one's initial employer). Prolonged PBD undoes any such effect, however, as the sum of the coefficients on our indicator variable for above-average local unemployment and the latter's interaction with the treatment dummy is insignificantly different from zero (the lowest p-value across all horizons/columns is 0.34).

In such labor markets, a larger reduction in wage growth is required in order to stabilize employment (bottom panel): the treatment effect (i.e., the difference in wage growth between firms with long and short PBD) is significantly higher in such labor markets.

2.7.5.4 Role of Decentralized Wage Bargaining

We next investigate the role of wage flexibility in preventing layoffs, and probe our mechanism by testing how an alternative insurance scheme— decentralized wage bargaining—interacts with prolonged PBD. For this purpose, we consider the existence of works councils and sectoral bargaining agreements.

Drawing on survey data, we predict the probability that an establishment in our sample has a works council, and compare employment and wage effects across establishments with high or low (predicted) works council presence. We predict the presence of works councils based on survey data. Specifically, we draw on the

IAB Establishment Panel (2012 wave), a representative employer survey based on more than 15,000 establishments from all branches and sizes. We fit a logistic regression model for the presence of a works council using information on the establishment's size, region, industry as well as age, and use this model to predict the probability that an establishment in our sample has a works council. We present details in Appendix 2.A.2.5. We align our data as closely as possible to the IAB Establishment Panel by considering establishments instead of firms for our analysis at this point. Figure 2.B.7 in the Online Appendix shows the receiver operating characteristic curve (ROC curve) for the prediction exercise based on a random 15% subsample of the IAB Establishment Panel. For the prediction in our sample, we pick the threshold that maximizes the Area Under The Curve (AUC).

As can be seen by comparing the respective intercept effects across the top and bottom panels of Table 2.8.10, works councils are associated with greater employment protection and smaller treatment effects on employment. As such, they substitute for firms' response to shorter PBD where firms use relatively lower wage growth to provide the same level of employment protection as those with prolonged PBD.

As a result, there is no difference in the treatment effect regardless of whether or not there is a works council in place, as the sum of the coefficients on our works-council indicator and the respective interaction is not significantly different from zero in the top panel (the lowest p-value across all horizons/columns is 0.37). By substituting for firms' insurance response under shorter PBD, works councils allow to stabilize employment without cutting wages by as much as would be necessary in the absence of works councils. This is reflected in the fact that while at least for the two longest horizons the sum of the three coefficients (adding the coefficient on our main treatment to the previous two coefficients) is still statistically significant at the 2% (36 months) and 5% level (48 months), the sum of the two coefficients is negative in the bottom panel.

Our results lend support to the idea that decentralized wage setting can substitute for STW policies in preventing layoffs during economic downturns.

2.7.5.5 Heterogeneity by Liquidity

Just as the existence of a works council independently enables a firm to offer insurance to its employees, so might access to internal funds, especially if the event that triggers the use of short-time work extensions is related to financial distress. To test this, we consider firms in the top and bottom terciles of the distribution of their cash-to-assets ratio. Due to the resulting considerable drop in sample size (also because the respective variable is not available for all firms), we omit industry by region fixed effects in Table 2.8.11, but our findings are qualitatively similar when not doing so (Table 2.C.18 in the Online Appendix).

Similar to the employment protection due to works councils, more liquid firms are more likely to retain their employees, as the respective intercept effect is positive and statistically significant for all horizons starting 12 months in the top panel. Analogously to works councils, firms' liquidity can substitute for their response to shorter PBD. As a consequence, there is no difference in the treatment effect of prolonged PBD on employment irrespective of firms' cash-to-asset ratio. While the sum of the coefficients on firms' cash-to-asset ratio (indicator) and the respective interaction is borderline significant at the 10% level for only one horizon (36 months), the sum of the three coefficients (adding the coefficient on our main treatment to the previous two coefficients) is insignificantly different from zero throughout (the lowest p-value across all horizons/columns is 0.35).

More liquid firms can offer employment protection without sacrificing wages. This holds irrespective of the potential benefit duration for more liquid firms. Similarly to firms with works councils, firms with a higher cash-to-assets ratio stabilize employment under shorter PBD without cutting wages altogether: the sum of the above-mentioned three coefficients is always insignificantly different from zero in the bottom panel (the lowest p-value across all horizons/columns is 0.14). Unlike works councils, however, corporate liquidity reduces only the need to adjust wages to retain employees in spite of shorter PBD (cf. negative coefficient on the interaction between the treatment and firms' cash-to-assets ratio), but not generally so (cf. positive, albeit insignificant, intercept effect of firms' cash-to-assets ratio).

2.8 Conclusion

Our paper provides new evidence on the employment and wage impacts of extending the availability and the PBD of short-time work schemes, using novel individual-level administrative data from the COVID-19 crisis and a reform in Germany that sharply increased the PBD from 6 to 12 months for some firms but not others. In a puzzle for the conventional view of STW as a labor hoarding device, we find no employment effects of both individual STW eligibility and extending the PBD—firms that received the longer duration were no more likely to retain employees 12 to 48 months later compared to those with the standard 6-month duration.

There are two potential reasons why our extensive-margin and intensive-margin results differ from other extensive-margin analyses that found STW preserved employment, at least in the short run (Kopp and Siegenthaler, 2021; Giupponi and Landais, 2023). First, the institutional setting we study, the German labor market, may have greater wage flexibility that facilitated alternative channels to prevent layoffs. Second, adjusting the intensive PBD margin of an established program may operate through different mechanisms than introducing or changing eligibility for STW to begin with.

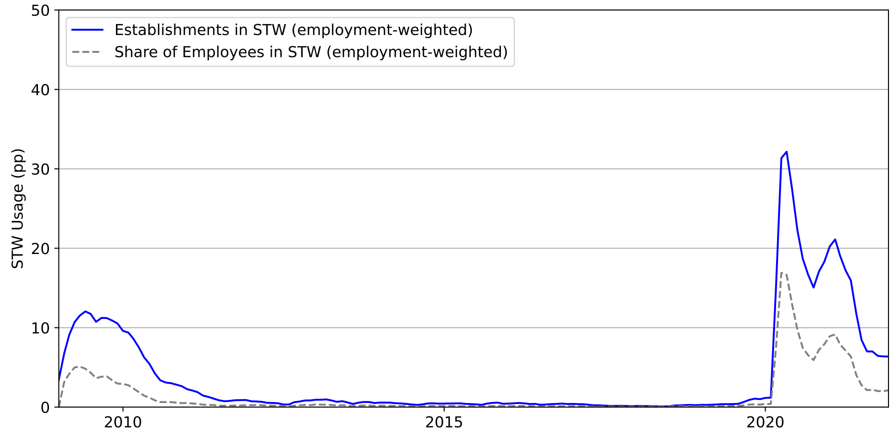
We find that the lack of employment effects of STW extensions masks important wage dynamics. Firms that did not receive the PBD extension negotiated significantly lower wage growth compared to those treated with the longer duration. This difference in wage trajectories suggests control firms relied on wage flexibility rather than STW subsidies to prevent layoffs when facing binding duration constraints. Indeed, labor market cells in which the STW scheme led to greater wage reductions also saw more employment protection.

Overall, our findings suggest that the impacts of STW policies depend critically on the underlying wage-setting institutions and the bargaining environment. While STW extensions did not directly preserve job matches on average in our setting, the reform enabled certain firms to sustain employment by relaxing binding constraints on wage bargaining imposed by limited PBD. More broadly, policies aimed at promoting labor hoarding during downturns may be substituted or complemented by wage dynamics when firms and workers can engage in decentralized negotiations over employment preservation.

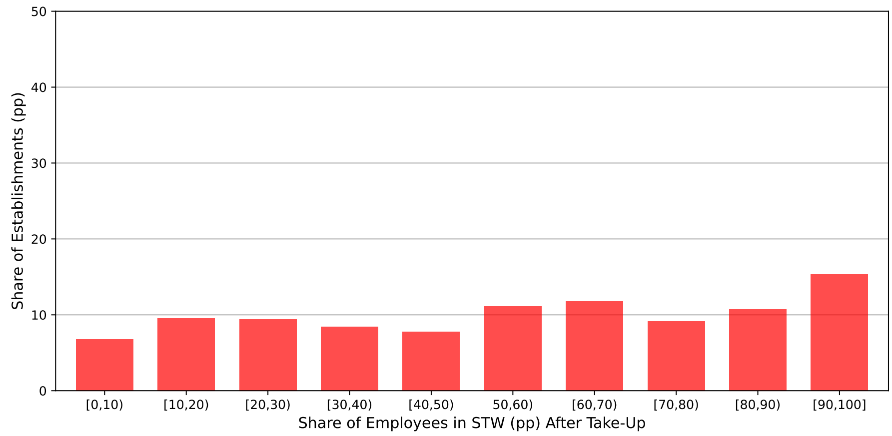
Figures

Figure 2.8.1. STW Take-Up Over Time and Within Firm

(a) STW Take-Up Over Time



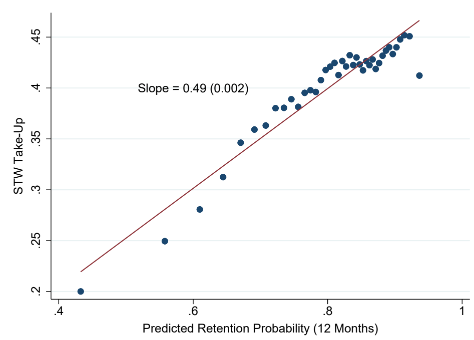
(b) Within-Firm Distribution of STW Take-Up



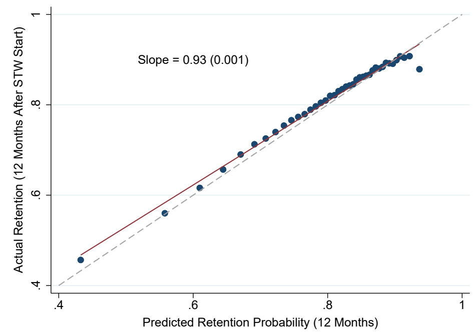
Notes: Panel (a) shows monthly STW usage since 2009. The solid line in depicts the employment-weighted share of establishments in STW, the dashed line depicts the establishment-level share of employees in STW—again employment-weighted. We use the Establishment History Panel since 2009 as universe, and add information on STW receipt. Establishments with five employees or less as well as establishments that are eligible for seasonal STW (*Baugewerbetarif*) are excluded (see Appendix 2.A.2.1 for details). Panel (b) shows the share of employees in STW per establishment in the start month of a STW spell. We consider all STW spells in Germany since 2009, with the same sample restrictions as in Panel (a). In a small number of cases of multi-establishment firms (3,254 of 481,137), the reported number of employees in STW exceeds establishment-level employment and we set the share to 100%.

Figure 2.8.2. Within-Firm STW Take-Up and Employment Outcomes By Predicted Retention Probability

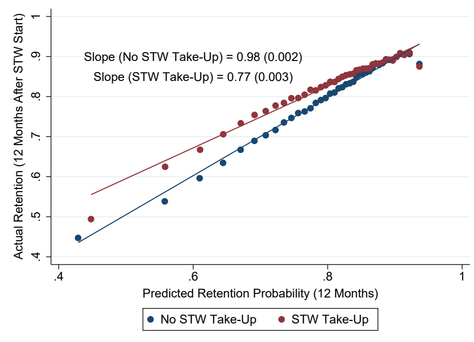
(a) Within-Firm STW Take-Up Probability By Predicted Retention Probability



(b) Actual Retention After STW Start By Predicted Retention Probability



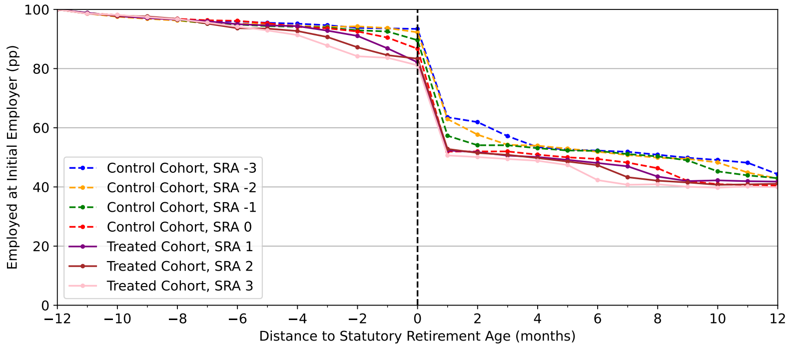
(c) Actual Retention After STW Start By Predicted Retention Probability (By Individual STW Take-Up)



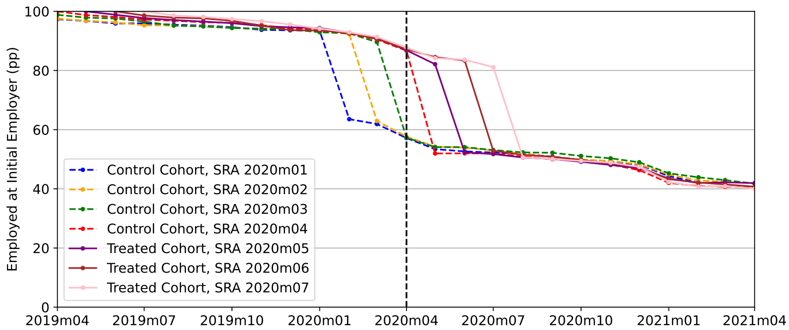
Notes: The figures focus on establishments with STW take-up between April and December 2020 (focusing on the first STW spell in case of multiple). Panel (a) plots individual STW take-up against the predicted retention probability. To estimate the predicted retention probability, we estimate a logit regression model of retention at the same employer 12 months later on rich individual and establishment characteristics in a training sample in the pre-COVID-19 pandemic time period, and use the coefficients to predict the retention probability for individuals in the sample (for details see Appendix 2.A.1). Panels (b) and (c) plot actual retention at the initial employer 12 months after the start of STW (Panel b and c) against the predicted retention probability. In Panel (c), we split the sample ex post by actual individual-level take-up of STW.

Figure 2.8.3. Cohort-Specific Event Studies Around STW Take-Up in April 2020

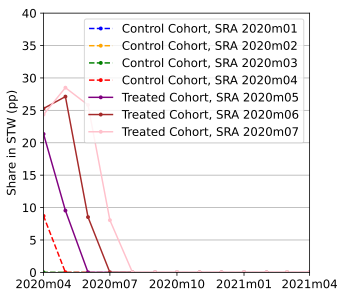
(a) Cohort-Specific Employment Trajectories Around Statutory Retirement Age (SRA)



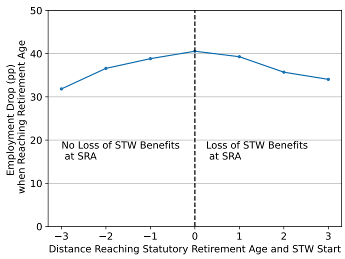
(b) Cohort-Specific Employment Trajectories Around Establishment STW Take-Up



(c) Cohort-Specific STW Take-Up Trajectories Around Establishment STW Take-Up



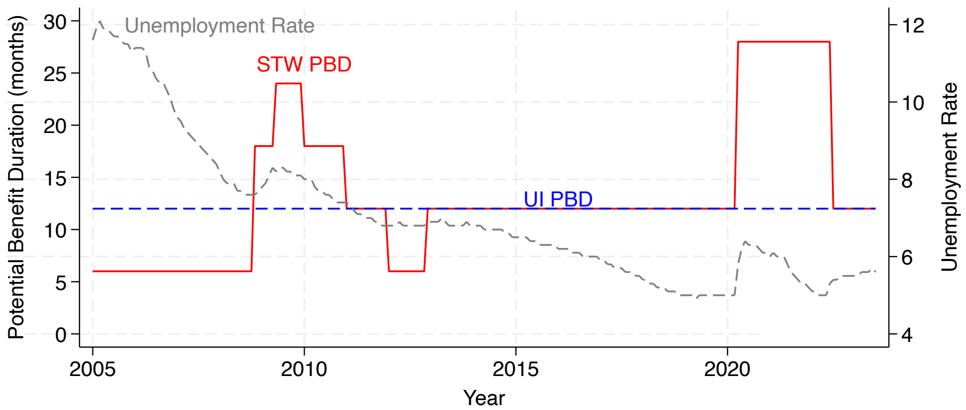
(d) Cohort-Specific Drop in Employment at SRA



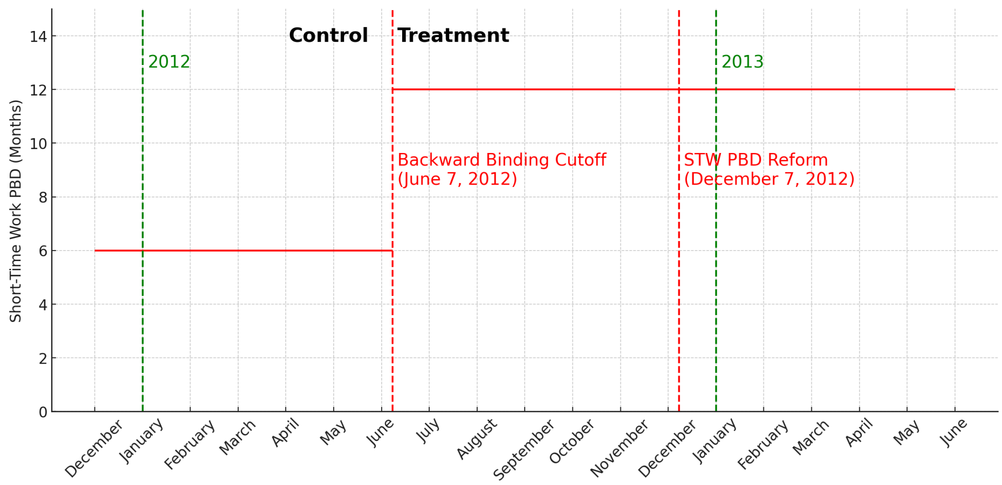
Notes: This figure plots event studies around STW take-up for cohorts reaching the Statutory Retirement Age (SRA) around the time their establishment takes up STW. We leverage the institutional feature that individuals above the SRA are ineligible for STW benefits and zoom in on establishments taking up STW in April 2020. Panel (a) plots the cohort-specific share of initially employed workers who are still employed at their initial employer over time around the SRA. The dashed lines depict cohorts who reach SRA before their establishment takes up STW and thus never receive STW benefits, the solid lines depict cohorts who reach SRA after STW take-up and thus are initially eligible for STW benefits. Panel (b) plots the same outcome for the same cohort but sorted by calendar time around the establishment's STW take-up in April 2020. Panel (c) plots cohort-specific, individual-level STW take-up by calendar time. Panel (d) reports the cohort-specific drop in employment when reaching the SRA (defined as the employment drop from two months before SRA to two months after SRA). Cohorts to the left of the dashed line reach SRA before STW take-up, cohorts to the right of the dashed line reach SRA after STW take-up and thus lose STW benefits when reaching SRA.

Figure 2.8.4. Short-Time Work Potential Benefit Duration (PBD) Over Time and 2012 Reform Research Design

(a) Short-Time Work PBD, UI PBD, and the German Unemployment Rate Over Time

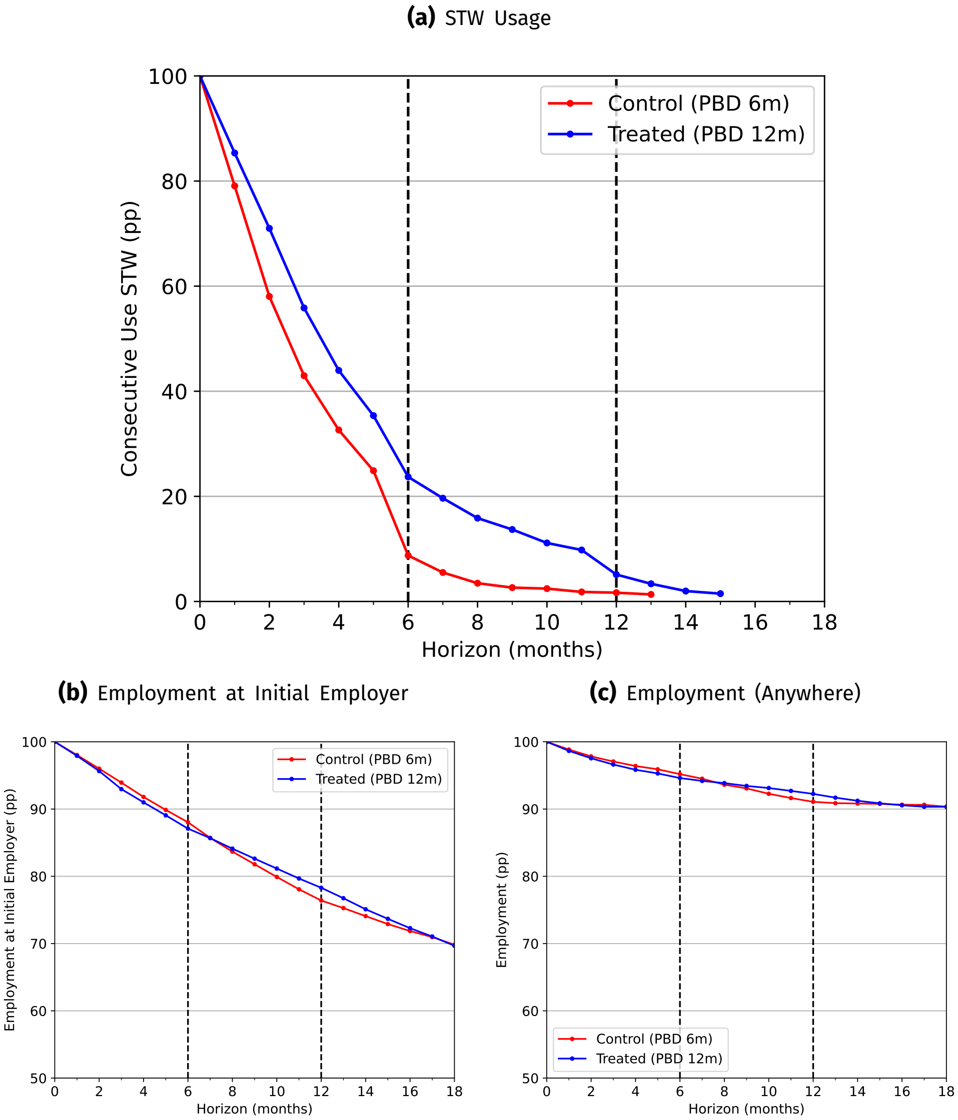


(b) 2012 Reform Research Design: Illustration



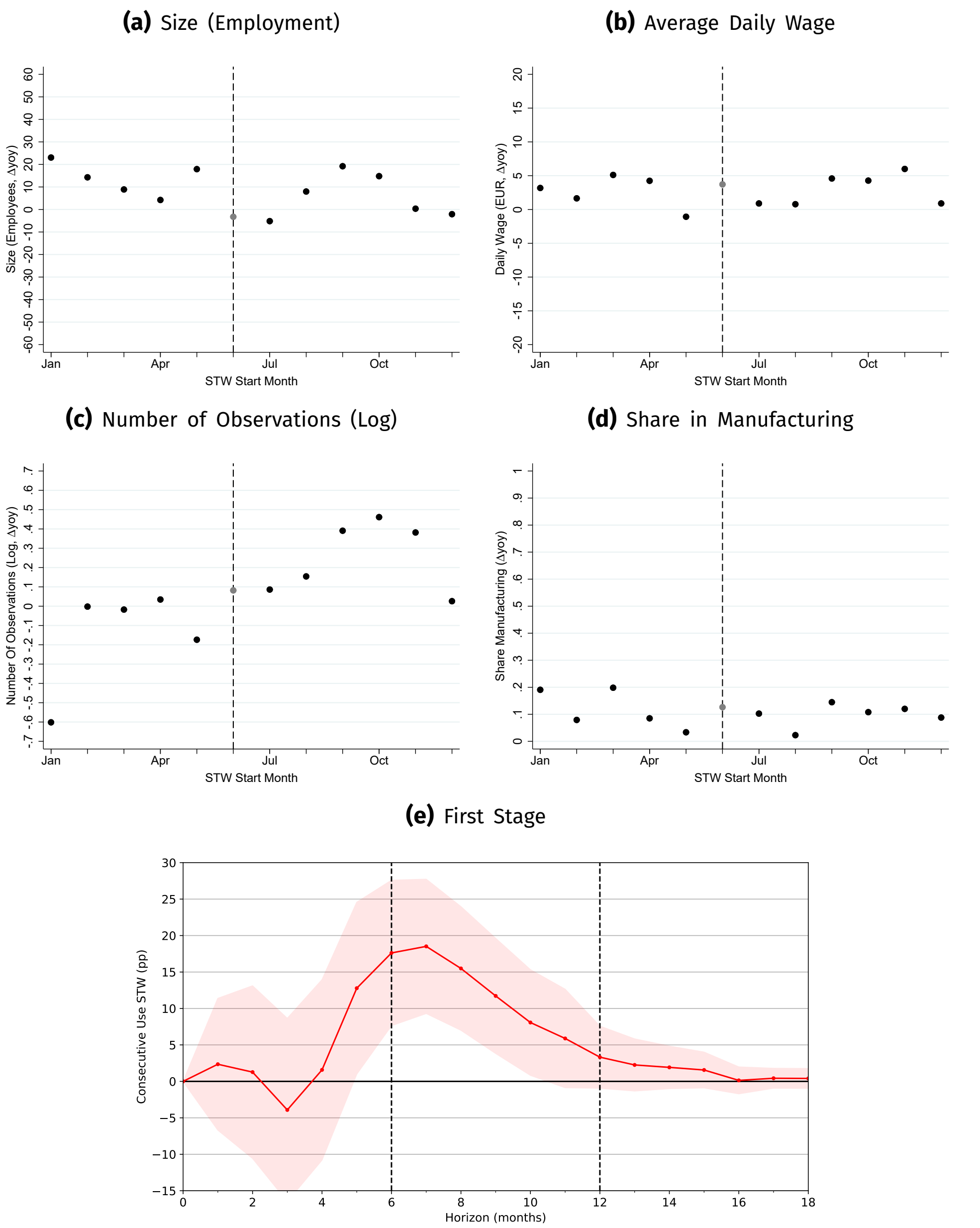
Notes: Panel (a) plots STW potential benefit duration (PBD) (solid red line, LHS scale). For comparison, we also plot the PBD for unemployment insurance (UI) (dashed blue, LHS scale) as well the monthly unemployment rate in Germany (dashed gray, RHS scale). Panel (b) illustrates the 2012 STW PBD reform that was announced by executive ordinance on December 7, 2012 and extended STW PBD from 6 to 12 months. It was backward-binding as it also applied to firms that had already been admitted to the program and were still receiving benefits (under the STW PBD of 6 months applicable until then). This splits firms that start STW in 2012 ex post into a treatment (PBD of 12 months) and control (PBD of 6 months) group as indicated by the red dotted lines.

Figure 2.8.5. 2012 Reform: STW Usage and Employment Outcomes By STW Start Date (Before/After June 2012)



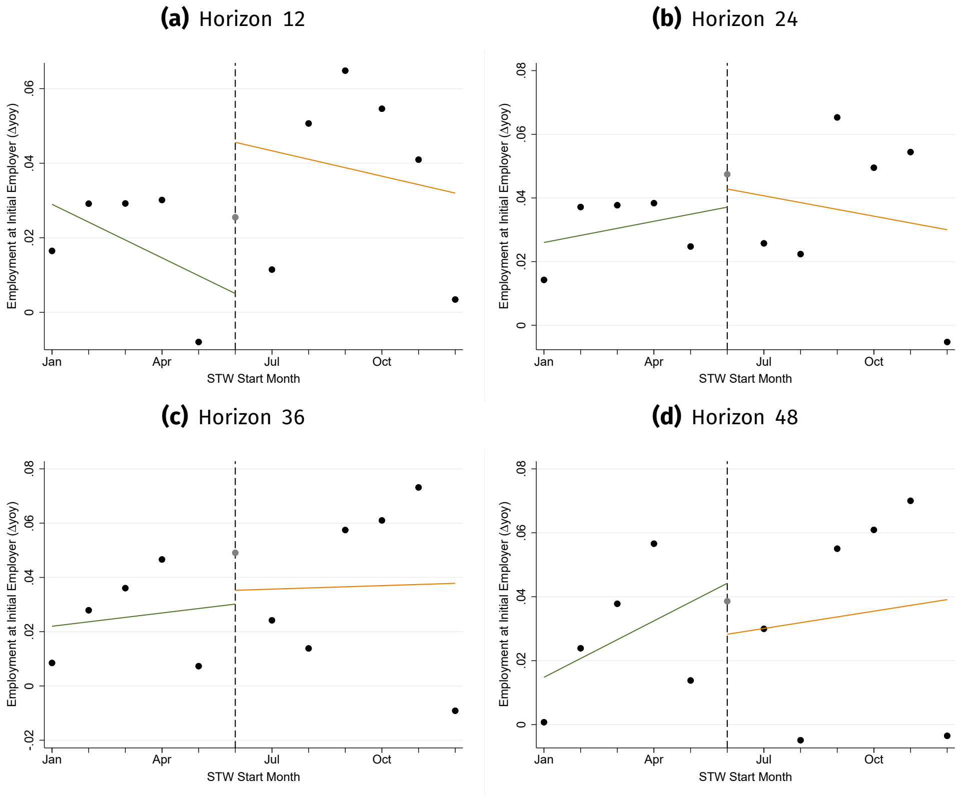
Notes: The figures focus on establishments that start STW within 3 months of June 2012 (omitting June 2012). In red, we plot outcomes for establishments that take up STW in March, April, or May 2012 and are thus eligible for 6 months of STW. In blue, we plot outcomes for establishments that take up STW in July, August, or September 2012 and are thus eligible for 12 months of STW. In Panel (a) we consider as outcome variable an indicator variable that is equal to one if the firm still receives STW benefits. The outcome variable in Panel (b) plots the share of initially employed workers (i.e., employed at the start of the establishment's STW spell) who are still employed at the same firm. In panel (d), we plot employment at any employer for the same cohorts of workers (initially employed by the firm taking up STW).

Figure 2.8.6. Regression Discontinuity Analysis for 2012 Reform: Balancedness and First Stage



Notes: The figure plots firm characteristics by timing of the start of STW (x-axis). We compute the number of firms in each cohort of the same start month, as well as cohort means of employment in the start month, average daily wage in the start month, and a dummy whether the firm is in the manufacturing sector. Employment and wages are winsorized at the 1% level. The figure plots the effect of the reform at different horizons after the start of STW using as outcome variable an indicator variable that is equal to one if the firm still receives STW benefits (*Consecutive Use STW*). We report the treatment effects using the regression discontinuity design specified in (2.7.1) including industry by region fixed effects. The data is at the firm-horizon level; a separate regression is run for each horizon. Only STW receipt as part of the initial application is considered. Figure 2.B.2 in the Online Appendix shows the analogous result with an indicator variable that is equal to one regardless of the STW spell as outcome variable. 95% confidence intervals based on robust standard errors are depicted.

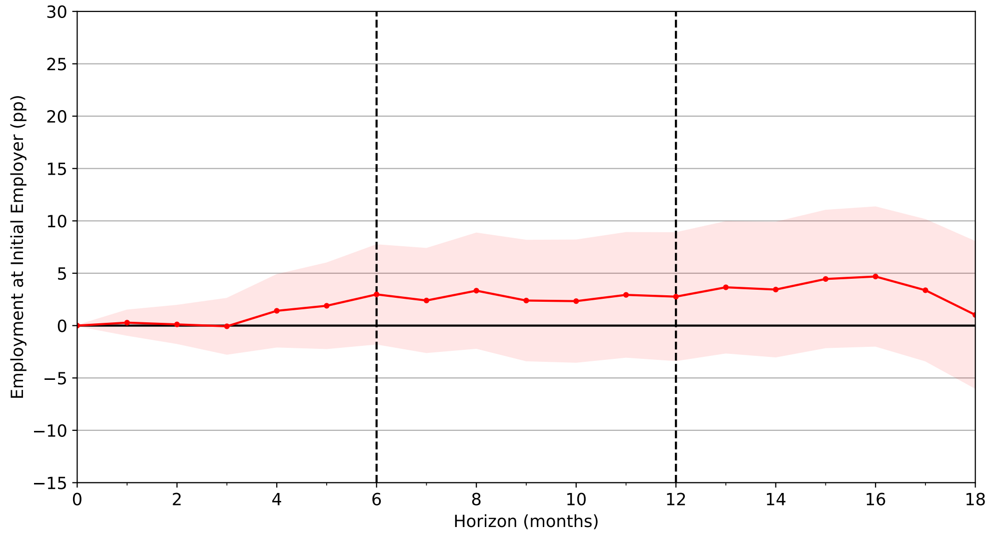
Figure 2.8.7. RDD Design: Employment at Initial Employer



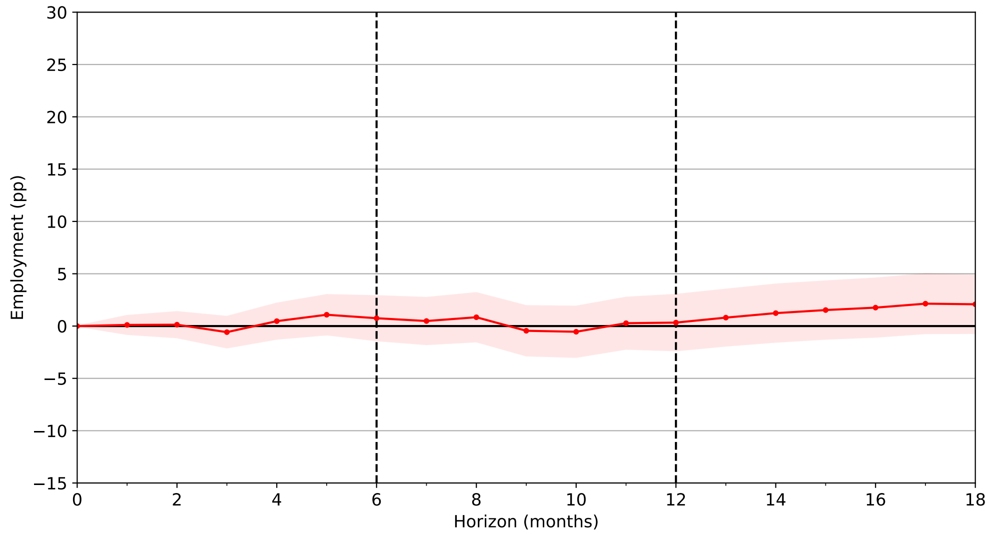
Notes: The figure plots the regression discontinuity design for the outcome variable considered 12, 24, 36 and 48 months after the start of STW. As outcome variable, we use for each firm the share of initially employed (i.e. employed at the start of STW) who are still employed at the firm after the respective time horizon. Potential re-entries after an exit are ignored. To account for seasonality, we use the difference in cohort means per calendar month between 2012 and 2011. The cohort that starts STW in the cutoff month which we exclude from the analysis is shown in gray. The sample is restricted to firms that in the start month have more than five employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*).

Figure 2.8.8. Effects on Employment at Initial Employer and Anywhere

(a) Employment at Initial Employer

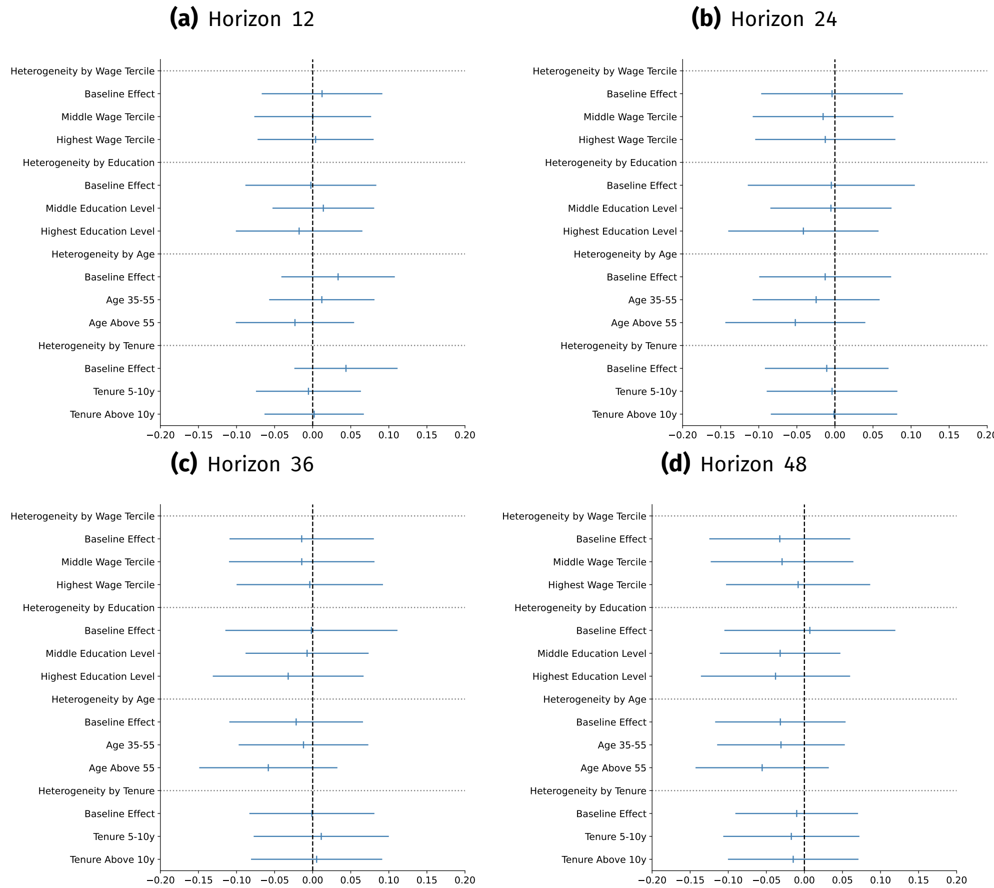


(b) Employment Anywhere



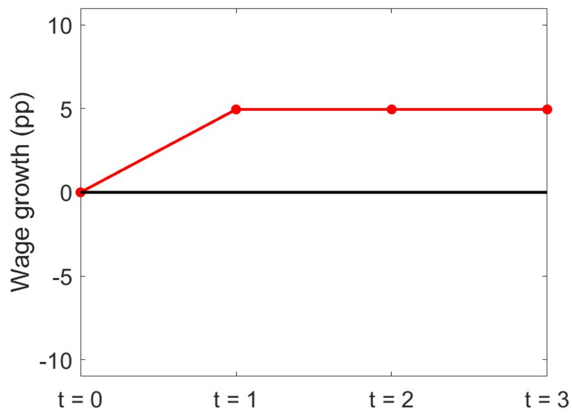
Notes: The figure plots the effect of the reform on employment at different horizons after the start of STW. The outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are still employed at the same firm (Panel (a)) or employed anywhere (Panel (b)). Potential re-entries after an exit are ignored. We report treatment effects using the regression discontinuity design specified in (2.7.1) including industry by region fixed effects. The data is at the firm-horizon level; a separate regression is run for each horizon. 95% confidence intervals based on robust standard errors are depicted. The sample is restricted to firms that in the start month have more than five employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*).

Figure 2.8.9. Heterogeneity by Demographics: Employment at Initial Employer



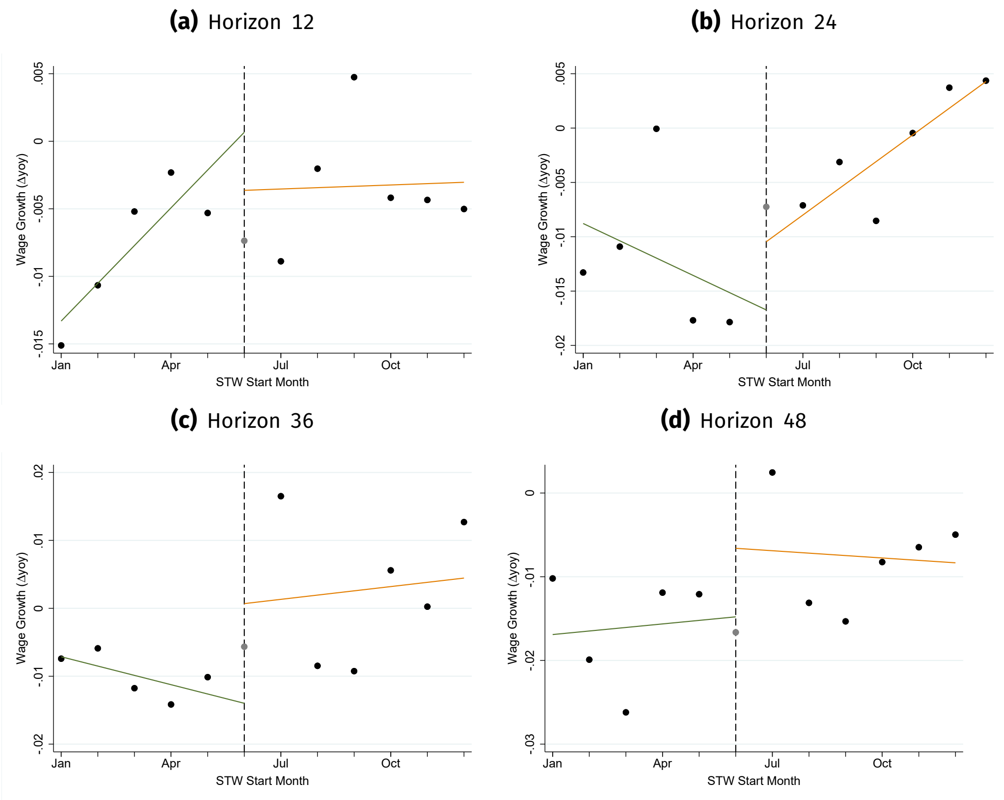
Notes: The figure plots heterogeneous employment effects by demographics at different horizons after the start of STW. We define groups within firms based on demographic characteristics at the start of STW (age, tenure at the firm, education level, wage tercile within the firm). The data is at the group-firm-horizon level. The coefficients shown are heterogeneous treatment effects of a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. As outcome variable, we use for each group-firm cell the share of initially employed (i.e. employed at the start of STW) who are still employed at the firm after the respective time horizon. Potential re-entries after an exit are ignored. The baseline education level is defined as no training or missing information, individuals with a middle (high) education level have a vocational training (hold a degree from an university of university of applied sciences). The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*).

Figure 2.8.10. Wage Growth in Model



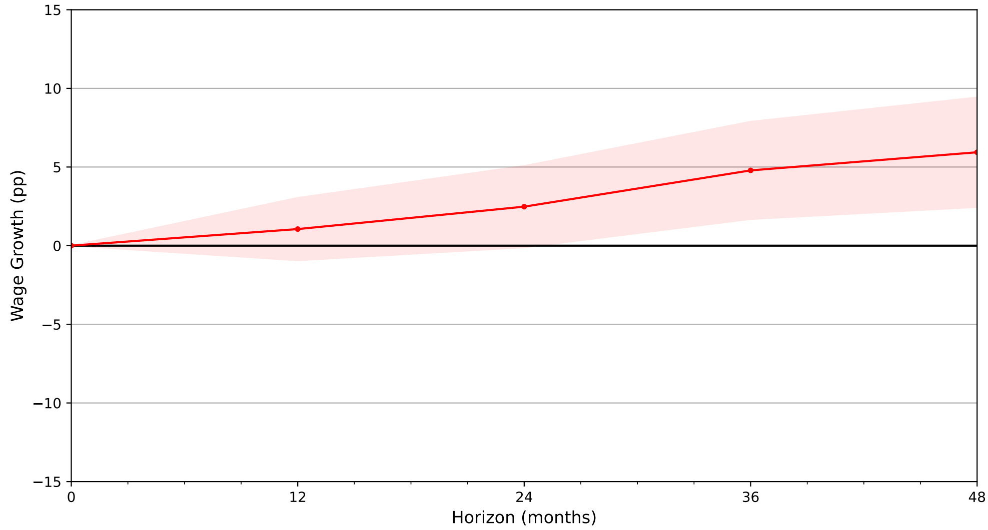
Notes: The figure shows the model-implied difference in wage growth (in %) between long-eligibility and short-eligibility firms. The two firms have the same realization of productivity with a transitory productivity shock of 50 percent in period 1 after which they recover to their normal productivity level.

Figure 2.8.11. RDD Design: Wage Growth



Notes: The figure plots the regression discontinuity design for the outcome variable considered 12, 24, 36 and 48 months after the start of STW. As outcome variable, we use the growth rate of average daily wages relative to the start of STW. To account for seasonality, we use the difference in cohort means per calendar month between 2012 and 2011. The cohort that starts STW in the cutoff month which we exclude from the analysis is shown in gray. The sample is restricted to firms that in the start month have more than five employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*).

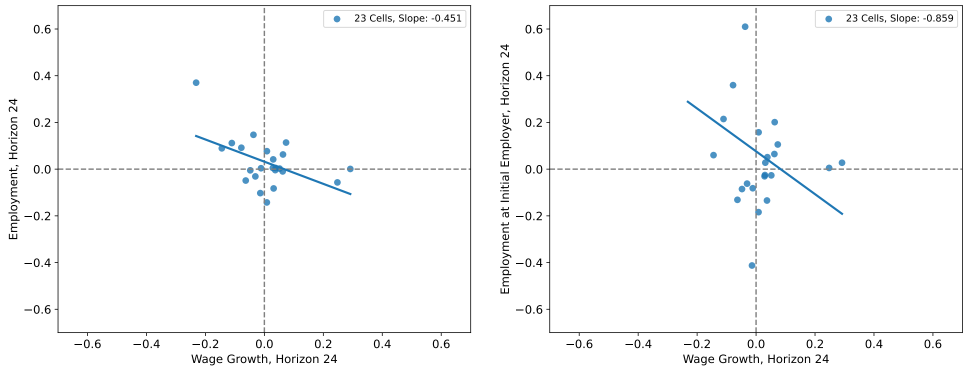
Figure 2.8.12. Effects on Wage Growth over Four Years



Notes: The figure plots the effect of the reform on wages at different horizons after the start of STW. The outcome variable considered is the growth rate of average daily wages relative to the start of STW. We report treatment effects using the regression discontinuity design specified in (2.7.1) including industry by region fixed effects. The data is at the firm-horizon level; a separate regression is run for each horizon. Since in the majority of cases the administrative information on wages is based end-of-year reports, we consider coefficients at annual frequency. 95% confidence intervals based on robust standard errors are depicted. The sample is restricted to firms that in the start month have more than five employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*).

Figure 2.8.13. Cell-Level Analysis after 24 months

(a) Employment **(b)** Employment at Initial Employer



Notes: The figure plots the treatment effect on employment (y-axis) against the treatment effect on wage growth (x-axis) in different cells 24 months after the start of STW at the establishment level. Establishments are assigned to cells based on their sector (manufacturing (43%), wholesale and retail trade (14%), rest (43%)), region (East (28%), West (72%)), and size (up to 5 (51%), 6-15 (23%), 16-50 (15%), more than 50 employees (11%)). One cell (wholesale and retail trade, east, more than 50 employees) is excluded because there are too few observations. In Panel (a), the outcome variable for employment is for each firm the share of initially employed (i.e., employed at the start of STW) who are employed anywhere. In Panel (b), the outcome variable for employment is for each firm the share of initially employed who are still employed at the same firm. Potential re-entries after an exit are ignored. Wage growth is the growth rate of average daily wages relative to the start of STW. We report treatment effects using the regression discontinuity design specified in (2.7.1) at the establishment level without industry by region fixed effects for a horizon of 24 months. Attention is restricted to employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*).

Tables

Table 2.8.1. STW Take-Up Across Establishments

	Time Periods									
	2009-2021		2009/2010		2011/2012		2013-2019		2020/2021	
	Nonuser	User	Nonuser	User	Nonuser	User	Nonuser	User	Nonuser	User
Number of Employees	33.65	42.41	31.89	62.70	33.42	56.71	33.95	64.53	34.55	36.19
	(59.17)	(71.78)	(56.66)	(91.11)	(59.12)	(85.97)	(59.57)	(93.57)	(60.05)	(63.33)
Average Daily Wage (Imp.)	89.44	89.35	75.99	84.24	81.03	83.64	91.26	96.66	107.92	90.16
	(36.00)	(32.77)	(32.33)	(29.18)	(33.16)	(28.56)	(35.48)	(30.98)	(37.12)	(33.62)
Establishment Age	18.61	19.62	16.51	18.45	17.30	19.34	19.00	22.32	20.90	19.67
	(12.64)	(13.40)	(11.23)	(11.40)	(11.71)	(11.91)	(12.79)	(13.29)	(14.09)	(13.80)
Employment Growth Previous Year (pp.)	1.23	-2.76	1.85	-4.88	-0.78	-3.11	1.49	-3.59	1.88	-2.23
	(43.00)	(39.99)	(41.19)	(27.56)	(43.30)	(26.69)	(43.30)	(24.40)	(43.16)	(43.49)
Education (Establishment-Level Shares)										
Low (Neither or Missing)	0.19	0.22	0.19	0.15	0.17	0.14	0.19	0.14	0.20	0.25
	(0.19)	(0.21)	(0.18)	(0.13)	(0.18)	(0.13)	(0.19)	(0.13)	(0.20)	(0.22)
Middle (Vocational Training)	0.68	0.65	0.71	0.74	0.71	0.75	0.68	0.73	0.64	0.63
	(0.23)	(0.23)	(0.21)	(0.18)	(0.21)	(0.19)	(0.23)	(0.19)	(0.24)	(0.24)
High (Degree from University/FH)	0.13	0.12	0.10	0.11	0.12	0.12	0.13	0.13	0.16	0.13
	(0.19)	(0.17)	(0.16)	(0.15)	(0.17)	(0.16)	(0.19)	(0.16)	(0.21)	(0.17)
Age (Establishment-Level Shares)										
Younger Than 35	0.36	0.35	0.37	0.31	0.36	0.28	0.35	0.28	0.34	0.36
	(0.21)	(0.21)	(0.22)	(0.18)	(0.22)	(0.18)	(0.21)	(0.17)	(0.20)	(0.21)
35-54	0.45	0.44	0.48	0.52	0.47	0.52	0.45	0.49	0.43	0.42
	(0.18)	(0.17)	(0.19)	(0.15)	(0.18)	(0.15)	(0.17)	(0.14)	(0.16)	(0.17)
55 and older	0.19	0.21	0.15	0.16	0.16	0.20	0.20	0.24	0.23	0.22
	(0.15)	(0.16)	(0.14)	(0.12)	(0.14)	(0.13)	(0.16)	(0.14)	(0.17)	(0.16)
Minimum Number of Observations	7,833,554	536,920	1,105,604	84,187	1,226,735	20,310	4,588,408	30,415	912,807	402,008

Notes: The table reports establishment-level summary statistics. Standard deviations are reported below the means in parentheses. We use the Establishment History Panel since 2009 as universe, and add information on STW receipt. Establishments with five employees or less as well as establishments that are eligible for seasonal STW (*Baugewerbetarif*) are excluded (see Appendix 2.A.2.1 for details). We pool observations in the establishment-year panel for the time periods considered. An establishment is defined as a user in some year if it receives STW benefits at some point during that year. Number of employees, average daily wages (imputed) and employment growth are winsorized at the 1% level. We use the symmetric growth rate for calculation of the employment growth.

Table 2.8.2. Individual-Level Summary Statistics

	Start Months			
	2020m4		2020m4-2020m12	
	No STW	STW	No STW	STW
Wage				
Daily Wage	88.44	103.96	93.67	106.21
	(64.08)	(50.39)	(64.45)	(50.22)
Education Level				
Low (Neither or Missing)	0.22	0.12	0.20	0.12
	(0.41)	(0.33)	(0.40)	(0.32)
Middle (Vocational Training)	0.61	0.69	0.61	0.69
	(0.49)	(0.46)	(0.49)	(0.46)
High (Degree from University/FH)	0.18	0.19	0.19	0.19
	(0.38)	(0.39)	(0.39)	(0.39)
Occupation (Horizontal)				
Production	0.29	0.34	0.32	0.37
	(0.46)	(0.47)	(0.47)	(0.48)
Personal Service	0.16	0.16	0.15	0.15
	(0.37)	(0.37)	(0.36)	(0.35)
Commercial Service	0.28	0.31	0.28	0.30
	(0.45)	(0.46)	(0.45)	(0.46)
IT Service	0.05	0.04	0.05	0.04
	(0.21)	(0.19)	(0.22)	(0.20)
Other Service	0.22	0.15	0.20	0.14
	(0.41)	(0.35)	(0.40)	(0.35)
Occupation (Vertical)				
Unskilled/ Semiskilled Tasks	0.20	0.16	0.19	0.16
	(0.40)	(0.36)	(0.39)	(0.36)
Skilled Tasks	0.54	0.55	0.54	0.55
	(0.50)	(0.50)	(0.50)	(0.50)
Complex Specialist Tasks	0.14	0.17	0.14	0.17
	(0.34)	(0.38)	(0.35)	(0.38)
Highly Complex Tasks	0.12	0.12	0.13	0.12
	(0.33)	(0.32)	(0.33)	(0.32)
Age				
Younger 35	0.33	0.29	0.33	0.28
	(0.47)	(0.45)	(0.47)	(0.45)
35-54	0.40	0.49	0.41	0.49
	(0.49)	(0.50)	(0.49)	(0.50)
Older 55	0.27	0.22	0.26	0.22
	(0.44)	(0.41)	(0.44)	(0.42)
Tenure				
Less Than 5y	0.55	0.50	0.53	0.49
	(0.50)	(0.50)	(0.50)	(0.50)
5-10y	0.17	0.19	0.17	0.20
	(0.37)	(0.40)	(0.38)	(0.40)
Above 10y	0.28	0.30	0.30	0.32
	(0.45)	(0.46)	(0.46)	(0.47)
Predicted Retention Probability	0.77	0.80	0.78	0.81
	(0.12)	(0.10)	(0.12)	(0.09)
Observations	1386877	1021931	2087262	1384087

Notes: The table reports individual-level summary statistics for workers at establishments that used short-time work in 2020. We restrict attention to establishments with high-quality information (see Appendix 2.A.1 for details). We differentiate between workers on short-time work vs. all other workers. STW Take-up is defined as high or 100% probability of STW receipt in the start month (see Appendix 2.A.1 for details). Columns 1 and 2 restrict attention to establishments with April 2020 as start month of STW, and consider the universe of individuals who work there in the start month. Columns 3 and 4 pool across start months in 2020 Q2-Q4. Standard deviations are reported below the means in parentheses.

Table 2.8.3. Effect of Individual STW Eligibility on Employment

	Employment (12 Months)					
	At Initial Employer			Anywhere		
	(1)	(2)	(3)	(4)	(5)	(6)
STW in Start Month	0.063***	0.081***	0.076***	0.042***	0.055***	0.052***
	(0.0004)	(0.0011)	(0.0010)	(0.0003)	(0.0007)	(0.0007)
Start Month FEs	Yes	Yes	Yes	Yes	Yes	Yes
Employer FEs	No	Yes	Yes	No	Yes	Yes
Control for Age	No	No	Yes	No	No	Yes
Education Group FEs	No	No	Yes	No	No	Yes
Control for Tenure	No	No	Yes	No	No	Yes
Control for Gender	No	No	Yes	No	No	Yes
Occupation Group FEs	No	No	Yes	No	No	Yes
N Individuals	3,471,349	3,471,127	3,471,058	3,471,349	3,471,127	3,471,058
R Squared	0.007	0.205	0.215	0.006	0.075	0.080
Adj. R Squared	0.007	0.184	0.195	0.006	0.051	0.056
N Establishments	88,047	87,825	87,825	88,047	87,825	87,825
Mean Outcome (No STW)	0.77	0.77	0.77	0.89	0.89	0.89

Notes: The level of observation is a worker i that is initially employed at an establishment that took up short-time work between April and December 2020. We focus on the first STW spell in case of multiple. The sample is limited to establishments with high-quality information (see Appendix 2.A.1 for details). In the first three columns, the dependent variable is an indicator variable for whether a given worker is still employed at the initial employer 12 months after the start of STW. In the last three columns, the dependent variable is an indicator variable for whether a given worker is employed anywhere 12 months after the start of STW at the initial employer. *STW in Start Month*$_i$ is an indicator variable for individual STW receipt in the start month. STW Take-up is defined as high or 100% probability of STW receipt in the start month (see Appendix 2.A.1 for details). Individual-level control variables are included where indicated. The education groups are no training or missing information, vocational training, and (any) university degree. We include five occupation groups (horizontal): production, personal service, commercial service, IT service, and other service. Robust standard errors clustered at the establishment level are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.8.4. Model Parameterization

β	0.8	π	0.1
ϕ	2	σ	0.3
λ	0.25	$\hat{h}$	0.7 $\bar{h}$
ρ	0.1	$\bar{w}$	0.9 $\bar{A}$

Notes: Model parameters of three-period model. See text for parameter description.

Table 2.8.5. Firm-Level Summary Statistics

	Firms that Start STW in 2012						
	Mean	p10	p25	p50	p75	p90	N
Number of Employees (Start Month)	67.27	7.00	10.00	20.00	53.00	142.00	3683
Average Daily Wage (Start Month)	87.38	57.24	69.46	86.14	103.06	119.81	3683
Age	20.86	5.00	10.00	20.00	37.00	37.00	3683
Employment Growth Previous Year (pp)	-1.47	-20.69	-9.52	0.00	3.77	14.33	3682
Financial Information							
Assets (Mio EUR)	8.38	0.37	0.67	1.50	4.37	15.07	3125
Revenue (Mio EUR)	52.86	1.00	2.17	7.43	34.55	105.62	917
Cash-to-Asset Ratio (pp)	12.28	0.05	0.45	4.23	18.58	38.18	3078
Value Added per Employee (Mio EUR)	0.06	0.03	0.04	0.05	0.07	0.08	424
Wagebill-to-Value-Added Ratio (pp)	82.50	59.52	72.78	83.26	91.82	104.51	657
Wagebill-to-Revenue Ratio (pp)	31.97	13.32	20.55	30.32	39.64	53.78	517
Education (Firm-Level Shares)							
Low (Neither or Missing)	0.08	0.00	0.00	0.03	0.12	0.23	3683
Middle (Vocational Training)	0.79	0.55	0.72	0.83	0.92	1.00	3683
High (Degree from University/FH)	0.13	0.00	0.00	0.08	0.16	0.33	3683
Age (Firm-Level Shares)							
Younger Than 35	0.23	0.05	0.13	0.21	0.32	0.45	3683
35-55	0.56	0.38	0.47	0.57	0.66	0.74	3683
Above 55	0.20	0.00	0.11	0.19	0.29	0.39	3683
Tenure (Firm-Level Shares)							
Less Than 5y	0.38	0.07	0.15	0.29	0.54	1.00	3683
5-10y	0.22	0.00	0.08	0.17	0.29	0.50	3683
Above 10y	0.40	0.00	0.00	0.43	0.67	0.81	3683
Average Wage Growth							
Wage Growth Previous Year (pp)	0.94	-5.42	-1.75	1.23	3.72	7.00	3656
Wage Growth Within 1y (pp)	3.07	-4.10	0.15	3.14	6.11	9.96	3683
Wage Growth Within 2y (pp)	5.92	-3.76	1.82	6.13	10.14	15.25	3682
Wage Growth Within 3y (pp)	8.81	-2.97	3.29	8.68	13.91	20.75	3682
Wage Growth Within 4y (pp)	10.89	-3.54	4.58	10.64	16.89	25.19	3683

Notes: The table reports firm-level summary statistics. Firms that start in 2011 (3,559) which we include in the analysis to facilitate the use of calendar month fixed effects are not included. Number of employees, average daily wages, employment growth (symmetric growth rate), financial information as well as wage growth variables are winsorized as the 1% level. Age refers to the age of the largest establishment in case of multi-establishment firms (for details on the aggregation to the firm-level see Appendix 2.A.2.3). The sample is restricted to firms that in the start month have more than five employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*). Wage Growth Within 1y (2y, 3y, 4y) is the growth rate in average wages relative to the firm's start of STW after 1y (2y, 3y, 4y) based on employees that were initially employed at the respective firm – regardless of their future employer. Wage Growth Previous Year is the 1y-growth rate in average wages based on employees that were employed at the respective firm 12 months prior to the start of STW. Financial information is based on 2012 information from the Dafne database by Creditreform/ BvD. Details on the cleaning procedures applied can be found in Appendix 2.A.2.4. Availability of financial information drops for items in income statements (revenue, value added, wagebill) rather than balance-sheet-items (cash, assets) since small firms in Germany need not publish information beyond their balance sheet.

Table 2.8.6. Effects on Employment over Four Years

(a) Employment at at Initial Employer

	Employment at Initial Employer, Horizon (months)					
	6	12	18	24	36	48
Running Variable	-0.005	-0.000	0.002	0.003	0.002	0.007
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable	0.003	-0.003	-0.004	-0.002	-0.000	-0.003
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD)	0.030	0.028	0.010	-0.011	0.004	-0.023
	(0.02)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	6,969	6,969	6,969	6,969	6,969	6,969
N Individuals	664,634	664,634	664,634	664,634	664,634	664,634

(b) Employment Anywhere

	Employment, Horizon (months)					
	6	12	18	24	36	48
Running Variable	-0.002	0.001	-0.004*	-0.004	-0.003	0.002
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	0.002	0.000	0.004	0.005	0.004	0.001
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD)	0.007	0.003	0.021	0.011	0.019	-0.011
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	6,969	6,969	6,969	6,969	6,969	6,969
N Individuals	664,634	664,634	664,634	664,634	664,634	664,634

Notes: The table reports the effect of the reform on employment at different horizons after the start of STW. We report the results of the regression discontinuity design specified in (2.7.1) including industry by region fixed effects. The outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are still employed at the same firm (Panel A) or employed anywhere (Panel B). Potential re-entries after an exit are ignored. The data is at the firm-horizon level; a separate regression is run for each horizon. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown includes firms that start in 2011, which are included to facilitate calendar month fixed effects in order to account for seasonality. The data is a balanced panel, the number of individuals refers to the number of individuals the calculation is based upon. The sample is restricted to firms that in the start month have more than five employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.8.7. Effect on Wage Growth

	Wage Growth Since Start, Horizon (months)			
	12	24	36	48
Running Variable	-0.000	-0.005**	-0.006*	-0.007**
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	0.000	0.008**	0.004	0.004
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD)	0.010	0.025*	0.047***	0.059***
	(0.01)	(0.01)	(0.02)	(0.02)
Industry x Region FE	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes
N Firms	6,969	6,968	6,968	6,969
N Individuals	623,638	605,768	592,361	579,913

Notes: The table reports the effect of the reform on wage growth at different horizons after the start of STW. We report the results of the regression discontinuity design specified in (2.7.1) including industry by region fixed effects. The outcome variable is the growth rate of average daily wages relative to the start of STW. The data is at the firm-horizon level; a separate regression is run for each horizon. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown includes firms that start in 2011, which are included to facilitate calendar month fixed effects in order to account for seasonality. The number of individuals per horizon refers to the number of individuals among all initially employed who are still in the labor market at this horizon and, thus, for whom wage growth can be calculated. A drop and subsequent increase in the number of firms can occur if, at some firm, all initially employed have gaps in their employment history (e.g., due to parental leave or sickness). The sample is restricted to firms that in the start month have more than five employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.8.8. Role of Job Switches in Explaining the Effect on Wage Growth

	Wage Growth Since Start, Horizon (months)			
	12	24	36	48
Switch Within 1y				
Treatment (12m PBD)	-0.001	0.035**	0.055***	0.060***
	(0.01)	(0.01)	(0.02)	(0.02)
Treatment (12m PBD) × Switch Within 1y	-0.037*	-0.037*	-0.027	-0.030
	(0.02)	(0.02)	(0.03)	(0.03)
Switch Within 2y				
Treatment (12m PBD)	0.011	0.033**	0.053***	0.074***
	(0.01)	(0.01)	(0.02)	(0.02)
Treatment (12m PBD) × Switch Within 2y	-0.032**	-0.036*	-0.041*	-0.057**
	(0.01)	(0.02)	(0.02)	(0.02)
Switch Within 3y				
Treatment (12m PBD)	0.016	0.023	0.034**	0.052**
	(0.01)	(0.01)	(0.02)	(0.02)
Treatment (12m PBD) × Switch Within 3y	-0.027**	-0.022	0.008	-0.019
	(0.01)	(0.01)	(0.02)	(0.02)
Switch Within 4y				
Treatment (12m PBD)	0.012	0.012	0.047***	0.042**
	(0.01)	(0.01)	(0.02)	(0.02)
Treatment (12m PBD) × Switch Within 4y	-0.020**	-0.027**	-0.022	-0.019
	(0.01)	(0.01)	(0.02)	(0.02)

Notes: The table reports heterogeneous treatment effects by job switching status (defined in four different ways) of the reform on wage growth at different horizons after the start of STW. For the specification *Switch Within 1y (2y, 3y, 4y)* we define groups per firm based on whether an invidual has switched employer within 1y (2y, 3y, 4y) after the start of STW. The data is at the group-firm-horizon level. The coefficients shown are heterogeneous treatment effects of a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. As outcome variable, we use for each group-firm cell the growth rate of average daily wages relative to the start of STW. The table presented is a condensed version of the four specifications; the full tables can be found in the Appendix (Tables 2.C.14, 2.C.15, 2.C.16, 2.C.17). Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.8.9. Heterogeneity by Local Labor Market Conditions

(a) Effect on Employment at Initial Employer

	Employment at Initial Employer, Horizon (months)					
	6	12	18	24	36	48
Running Variable	-0.006	-0.001	-0.001	0.000	0.000	0.007
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Running Variable × Local Unemployment Above Mean	0.001	0.002	0.001	0.002	0.001	-0.000
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	0.001	-0.004	0.005	0.006	0.007	-0.001
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Local Unemployment Above Mean	0.004	0.003	-0.009	-0.009	-0.006	-0.000
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD)	0.033	0.022	-0.017	-0.044	-0.028	-0.044
	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Treatment (12m PBD) × Local Unemployment Above Mean	-0.018	0.001	0.047	0.050	0.036	0.021
	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)
Local Unemployment Above Mean	0.002	-0.009	-0.011	-0.013	-0.018	-0.025**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	6,431	6,431	6,431	6,431	6,431	6,431
N Individuals	609,872	609,872	609,872	609,872	609,872	609,872

(b) Effect on Wage Growth

	Wage Growth Since Start, Horizon (months)			
	12	24	36	48
Running Variable	0.002	-0.005*	-0.005*	-0.007*
	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Local Unemployment Above Mean	-0.002	-0.002	-0.002	-0.003
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	-0.001	0.009**	0.008*	0.009*
	(0.00)	(0.00)	(0.00)	(0.01)
Treatment (12m PBD) x Running Variable × Local Unemployment Above Mean	0.000	-0.004	-0.009**	-0.009*
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD)	0.000	0.016	0.032*	0.050**
	(0.01)	(0.01)	(0.02)	(0.02)
Treatment (12m PBD) × Local Unemployment Above Mean	0.011	0.023*	0.041**	0.034*
	(0.01)	(0.01)	(0.02)	(0.02)
Local Unemployment Above Mean	0.001	0.007*	0.004	0.005
	(0.00)	(0.00)	(0.01)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes
N Firms	6,431	6,430	6,430	6,431
N Individuals	573,371	557,709	545,171	533,636

Notes: The table reports heterogeneous effects by local labor market conditions of the reform on employment and wage growth at different horizons after the start of STW. We report the results of the regression discontinuity design specified in (2.7.1) including industry by region fixed effects. In Panel (a), the outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are still employed at the same firm. Potential re-entries after an exit are ignored. In Panel (b), the growth rate of average daily wages relative to the start of STW is considered as outcome variable. The variable *Local Unemployment Above Mean* takes the value one if the unemployment rate in the month of the start of STW in the area (*Kreis*) is above the mean unemployment rate across all areas in Germany that month. Definitions are based on the 2017 data-version (*Kreisschlüssel 2017*, SIAB 1975-2017). We assign the area of the largest establishment to a multi-establishment firm. The data is at the firm-horizon level; a separate regression is run for each horizon. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown includes firms that start in 2011, which are included to facilitate calendar month fixed effects in order to account for seasonality. In Panel (a), the data is a balanced panel with the number of individuals referring to the number of individuals the calculation is based upon. In Panel (b), the number of individuals per horizon refers to the number of individuals among all initially employed who are still in the labor market at this horizon and, thus, for whom wage growth can be calculated. A drop and subsequent increase in the number of firms can occur if, at some firm, all initially employed have gaps in their employment history (e.g., due to parental leave or sickness). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.8.10. Heterogeneity by Existence of a Works Council

(a) Effect on Employment at Initial Employer

	Employment at Initial Employer, Horizon (months)					
	6	12	18	24	36	48
Running Variable	-0.005	-0.000	0.002	0.006	0.006	0.012*
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Running Variable × Works Council	0.005**	0.010**	0.010**	0.007	0.005	0.004
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	0.001	-0.005	-0.008	-0.006	-0.009	-0.013
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Works Council	0.008	0.005	0.004	0.002	0.013	0.014
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD)	0.035	0.036	0.026	-0.012	0.010	-0.016
	(0.02)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)
Treatment (12m PBD) × Works Council	-0.064**	-0.064**	-0.064*	-0.039	-0.072*	-0.072*
	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)
Works Council	0.041***	0.063***	0.071***	0.073***	0.072***	0.072***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Establishments	7,378	7,378	7,378	7,378	7,378	7,378
N Individuals	461,120	461,120	461,120	461,120	461,120	461,120

(b) Effect on Wage Growth

	Wage Growth Since Start, Horizon (months)			
	12	24	36	48
Running Variable	0.000	-0.004	-0.004	-0.006
	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Works Council	-0.001	-0.002	-0.003*	-0.002
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	-0.002	0.004	0.002	0.002
	(0.00)	(0.00)	(0.00)	(0.01)
Treatment (12m PBD) x Running Variable × Works Council	0.007***	0.006**	0.004	0.006
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD)	0.012	0.025*	0.043**	0.055***
	(0.01)	(0.01)	(0.02)	(0.02)
Treatment (12m PBD) × Works Council	-0.010	-0.001	0.006	-0.006
	(0.01)	(0.01)	(0.02)	(0.02)
Works Council	-0.006***	-0.008***	-0.007**	-0.012***
	(0.00)	(0.00)	(0.00)	(0.00)
Industry x Region FE	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes
N Establishments	7,378	7,377	7,377	7,378
N Individuals	431,060	418,216	407,898	398,740

Notes: The table reports heterogeneous effects by existence of a works council of the reform on employment and wage growth at different horizons after the start of STW. We report the results of the regression discontinuity design specified in (2.7.1) including industry by region fixed effects at the establishment level. In Panel (a), the outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are still employed at the same firm. Potential re-entries after an exit are ignored. In Panel (b), the growth rate of average daily wages relative to the start of STW is considered as outcome variable. We predict the existence of a works council drawing on the *IAB Establishment Panel* for the prediction (for details see Appendix 2.A.2.5). To match the level of observation of the *IAB Establishment Panel*, we run this analysis at the establishment level. The variable *Works Council* takes the value one if the predicted probability of the existence of a works council exceeds the threshold chosen to maximize the AUC in the prediction. The sample consists of establishments that can be matched to the firm-level using Orbis-ADIAB and, analogous to before, restricting to those establishments that in the start month have more than five employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*). The data is at the estblishment-horizon level; a separate regression is run for each horizon. The running variable is distance to the cutoff 2012m6. Treated establishments are those that start STW after the cutoff. The number of establishments shown includes establishments that start in 2011, which are included to facilitate calendar month fixed effects in order to account for seasonality. Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.8.11. Heterogeneity by Firm-Level Characteristics: Liquidity

(a) Effect on Employment at Initial Employer

	Employment at Initial Employer, Horizon (months)					
	6	12	18	24	36	48
Running Variable	0.003	-0.004	-0.003	-0.001	-0.003	0.004
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Running Variable × Cash-to-Asset Ratio Above p66	-0.009***	0.000	0.001	-0.002	-0.003	-0.003
	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable	0.004	0.010	0.014	0.015	0.016	0.001
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Cash-to-Asset Ratio Above p66	0.009	0.000	-0.008	-0.006	-0.001	0.009
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD)	-0.021	-0.017	-0.048	-0.055	-0.031	-0.020
	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
Treatment (12m PBD) × Cash-to-Asset Ratio Above p66	0.003	0.008	0.045	0.046	0.020	-0.043
	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)
Cash-to-Asset Ratio Above p66	-0.010	0.004	0.017	0.027**	0.051***	0.063***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	3,924	3,924	3,924	3,924	3,924	3,924
N Individuals	299,701	299,701	299,701	299,701	299,701	299,701

(b) Effect on Wage Growth

	Wage Growth Since Start, Horizon (months)			
	12	24	36	48
Running Variable	0.001	-0.007**	-0.005	-0.009**
	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Cash-to-Asset Ratio Above p66	0.002	0.003	0.004	0.004
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	-0.001	0.007	0.000	0.003
	(0.00)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Cash-to-Asset Ratio Above p66	0.002	0.007	0.006	0.005
	(0.00)	(0.00)	(0.00)	(0.01)
Treatment (12m PBD)	0.004	0.028	0.050**	0.065**
	(0.01)	(0.02)	(0.02)	(0.03)
Treatment (12m PBD) × Cash-to-Asset Ratio Above p66	-0.017	-0.030*	-0.038**	-0.038*
	(0.01)	(0.02)	(0.02)	(0.02)
Cash-to-Asset Ratio Above p66	0.003	0.004	0.009	0.011*
	(0.00)	(0.00)	(0.01)	(0.01)
Calendar Month FE	Yes	Yes	Yes	Yes
N Firms	3,924	3,923	3,923	3,924
N Individuals	280,869	272,334	265,805	260,033

Notes: The table reports heterogeneous effects by liquidity of the reform on employment and wage growth at different horizons after the start of STW. We report the results of the regression discontinuity design specified in (2.7.1).s In Panel (a), the outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are still employed at the same firm. Potential re-entries after an exit are ignored. In Panel (b), the growth rate of average daily wages relative to the start of STW is considered as outcome variable. The cash-to-asset ratio is based on BvD data in 2012 (2011) for firms that start in 2012 (2011). Details on the cleaning procedures data can be found in appendix 2.A.2.4. The variable *Cash-to-Asset Ratio Above p66* takes the value one if the firm's cash-to-asset ratio is above the p66 among firms that start in the same year. The sample includes the bottom and top tercile. Due to the resulting drop in the number of observations we report the specification excluding industry by region fixed effects here (the results of the specification including industry by region fixed effects can be found in the Appendix, Table 2.C.18). The data is at the firm-horizon level; a separate regression is run for each horizon. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown includes firms that start in 2011. In Panel (a), the data is a balanced panel with the number of individuals referring to the number of individuals the calculation is based upon. In Panel (b), the number of individuals per horizon refers to the number of individuals among all initially employed who are still in the labor market at this horizon and, thus, for whom wage growth can be calculated. A drop and subsequent increase in the number of firms can occur if, at some firm, all initially employed have gaps in their employment history (e.g., due to parental leave or sickness). The sample is restricted to firms that in the start month have more than five employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

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Appendix 2.A Data Appendix

2.A.1 Data on Individual STW Receipt

Information on individual STW benefits is extracted from establishments' monthly applications (*Abrechnungslisten*) using an automated optical character recognition (OCR) procedure. The procedure reads out the social security number, reduction in hours, regular remuneration, actual remuneration, and STW benefits per individual.

The OCR procedure faced several challenges, such as illegible handwriting and the discontinuation of information extraction for long applications after a certain number of pages. Additionally, for multi-establishment firms and temporary employment agencies, the establishment applying for STW may not coincide with the individual's employer in the Social Security Records.

Mapping individual STW benefits to employment biographies requires thorough cross-checks with both establishment-level data and Social Security Records. The key variable, indicating an individual's STW risk, is constructed as follows: an individual eligible for STW based on cross-checks with Social Security Records and found in the digitalized lists is assigned a 100% STW risk. Employees at an establishment are eligible for STW if they are below the statutory retirement age, not on parental leave, and either fully liable to social security or in vocational training (beyond the second month). If in a month the number of employees with a 100% STW risk coincides with the number of employees in STW from the establishment-level data, the remaining employees are assigned a STW risk of 0%. If there is a discrepancy, the remaining individuals are assigned a positive STW risk based on the share of eligible employees in STW per gender per establishment.

The upper panel of Table 2.C.19 shows the results of the cross-checks at the establishment level for establishments starting STW in April 2020 (columns 1 and 2) and pooled across all establishments starting between April and December 2020 (columns 3 and 4). We define individual-level data as high quality if the individual works at an establishment for which the aggregated individual-level information on STW receipt coincides with the establishment-level data.

The bottom panel of Table 2.C.19 shows the STW risk for individuals working at establishments under the same restrictions as in the upper panel.

We drop individuals with incalculable STW risk. This is often due to the fact that there is no 1:1 or 1:n mapping between the establishment that applies for STW and the employer from Social Security Records (often the case when a temporary employment agency is involved).

2.A.2 Details on Data Construction for the Dataset on PBD Extensions

2.A.2.1 Matching BTR KUG and BHP

This section describes the procedure for combining the administrative data on STW receipt (BTR KUG) with the Establishment History Panel (BHP).

1) We create STW spells from BTR KUG, defining them as periods of STW usage with a maximum gap of two months, and transform the data into a monthly panel.

2) This unbalanced monthly panel is matched to the Establishment History Panel (BHP), which was expanded to the monthly level.

3) We drop all establishments that qualify for the seasonal STW scheme (*Baugewerbetarif*) at any point. This STW scheme targets establishments in the construction sector that are dependent on weather conditions and, thus, regularly face fluctuations in working hours in the winter.

4) We exclude establishments that only appear in BTR KUG and never in BHP.

5) We also exclude establishments that cannot be successfully matched to BHP for the years of interest (2011 and 2012).

2.A.2.2 Processing IEB

This section provides details on how we create a monthly panel with information on employment status and daily wage from excerpts of the Integrated Employment Biographies (IEB).

1) As a first step, we create two monthly panels: one based on reports with positive daily wages and another based on reports for periods with zero daily wages when an individual was still employed but received compensation from other sources, e.g., because of parental leave or longer illness (*Unterbrechungsmeldung wg Entgeltersatzleistung (151), Erziehungsurlaub (152), gesetzliche Dienstpflicht (153)*). We exclude these periods in the calculation of wages but include them for analyzing employment status.

2) For the first panel, we use standard procedures (see Dauth and Eppelsheimer (2020)) to transform the data originally stored in spell format into a monthly panel. Specifically, for multiple simultaneous employments, we focus on the employment liable to social security, and if there are multiple, the one with the highest wage. One-time payments are converted into daily payments for the reported period and added to the daily wage. We create cross-sections at the end of each month to create a monthly panel at the individual level.

3) For the second panel, we use the same cutoff dates to create monthly cross-sections.

4) In a second step, we enrich the first panel with periods of temporary interruptions in employment from the second panel.

2.A.2.3 Aggregation to the Firm Level

This section describes the aggregation of establishment-level information to the firm level.

1) We drop firms with establishments that started STW multiple times in the 12 months prior to a start of STW in 2012 (excludes 15% of the 6,416 firms that started in 2012).

2) In case a firm has multiple establishments that started STW, we keep the firm only if the starts happen either in the same month or one month apart. In the latter case, we define the earlier start months of the two to as the start month of the firm (fewer than 20 firms dropped).

3) If the remaining firms have an establishment that starts STW in 2012 and another establishment that starts in 2011, we exclude the firm in the reference group of firms that start STW in 2011 (78 firms dropped in the reference year 2011).

4) We assign each firm the industry, region and age of its largest establishment.

2.A.2.4 Preparing Dafne

This section explains how we assemble and clean the firm-level financial data from the Dafne database.

1) We start with the universe of firms in Dafne (as of May 2022) and use financial information from the lowest level of consolidation available.

2) To identify the lowest level of consolidation available we follow the following procedure. We use financial information at the unconsolidated level whenever possible. Some firms only report financial information at the group level (i.e., they are exempt by HGB 264 to report at both levels). If we can identify such a firm as the group head and thus identify other subsidiaries of the group, we use the consolidated information and drop other subsidiaries of the group. If we cannot identify the firm as the group head, the firm is dropped. If a firm reports both consolidated and unconsolidated information, we use the unconsolidated information of the group head as long as its revenues exceed 5% of the group revenue. Below this threshold, we assume that the group head is merely a financial holding and should not be treated as an individual firm (within the group).

3) We add balance sheet information and income statement data from 2008 until 2020.

4) We follow standard cleaning procedures but focus on balance sheet variables, since many firms in the sample are so small that they are not required to publish their income statement:

   a) We drop firms that have negative or zero total assets in any year.
   b) We drop firms that have larger equity than total assets in any year.

2.A.2.5 Predicting the Existence of a Works Council

This section contains details on the prediction exercise for the existence of a works council based on the IAB Establishment Panel (*IAB Establishment Panel 9319, DOI: 10.5164/IAB.IABBP9319.de.en.v1*).

We split the 2012 wave of the IAB Establishment Panel into a random test sample (15%) and a training sample (remaining 85%). We fit a logit model using information on industry (as in the IAB Establishment Panel), region (*Bundesland*), wages (average monthly wage per employee), size (1-4,5-10,11-19,20-49,50-99,100-199,200-499,500 employeed and more) and age (founded before/after 1990). Panel B of Figure 2.B.7 shows the number of establishments per bin of length 0.1 of the predicted probabilities on the LHS and the actual share of establishments with a works council per bin on the RHS. This indicates that the predicted probabilities are of the right order of magnitude. The ROC curve is shown in Panel B of Figure 2.B.7.

As a robustness check, we run a Lasso version of the logit model described above and an alternative specification of the logit model with also includes the share of employees with high and middle education level as well the share of female employees. The prediction quality remains similar in all cases.

We use the estimated coefficients to predict the existence of a works council for establishments that start STW in 2012.

2.A.3 Details on Data Construction for the Dataset on Individual STW Eligibility

For the match of BTR KUG and BHP we proceed as described in Appendix 2.A.2.1, focusing on 2020 in step 5. We create a monthly panel with information on employment for individuals working at establishments in STW in 2020 or 2021, as described in Appendix 2.A.2.2. Since wages are not considered in this analysis, we only use one panel based on all periods including those of zero wages.

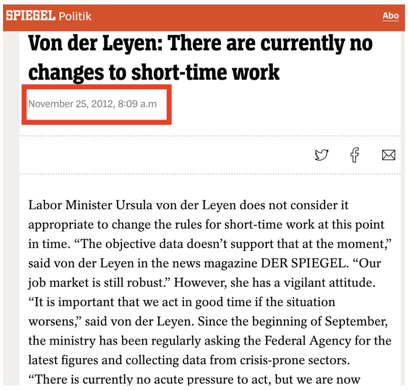
2.A.3.1 Predicting the Retention Probability based on Individual Characteristics

We predict the probability of an employee remaining with the same employer 12 months later based on individual characteristics. For this prediction, we use the universe of employees in Germany who were working on June 30, 2018, at establishments that can be linked to the firm level.

We fit a logit model using the following information: industry of the employer (1-digit), size of the employer (1-4,5-10,11-19,20-49,50-99,100-199,200-499,500 employeed and more), wage tercile at employer, occupation (*Berufssegment*, *Anforderungsniveau*), education (low, middle, high), full-time dummy, gender, tenure (year bins capped at 40) as well as age (5 year bins).

Appendix 2.B Supplementary Figures

Figure 2.B.1. Newspaper Coverage in November and December 2012 (translated)

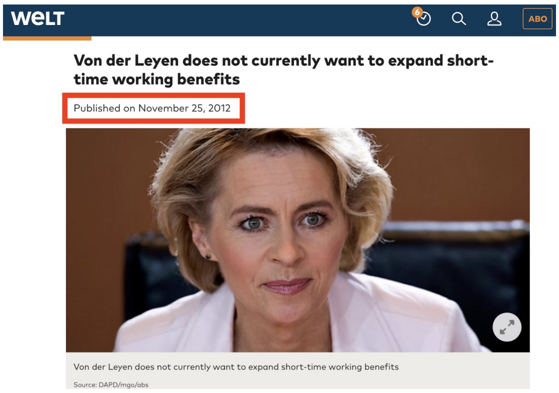


SPIEGEL Politik Abo

Von der Leyen: There are currently no changes to short-time work

November 25, 2012, 8:09 a.m

Labor Minister Ursula von der Leyen does not consider it appropriate to change the rules for short-time work at this point in time. "The objective data doesn't support that at the moment," said von der Leyen in the news magazine DER SPIEGEL. "Our job market is still robust." However, she has a vigilant attitude. "It is important that we act in good time if the situation worsens," said von der Leyen. Since the beginning of September, the ministry has been regularly asking the Federal Agency for the latest figures and collecting data from crisis-prone sectors. "There is currently no acute pressure to act, but we are now



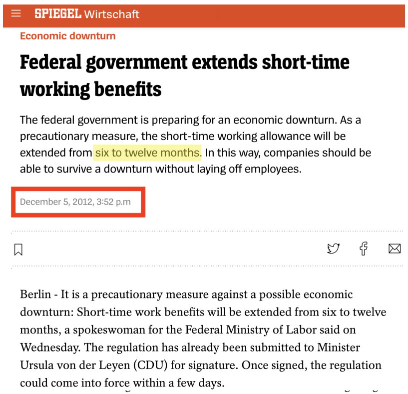
WELT ABO

Von der Leyen does not currently want to expand short-time working benefits

Published on November 25, 2012

Von der Leyen does not currently want to expand short-time working benefits

Source: DAPD/mgo/abs



SPIEGEL Wirtschaft

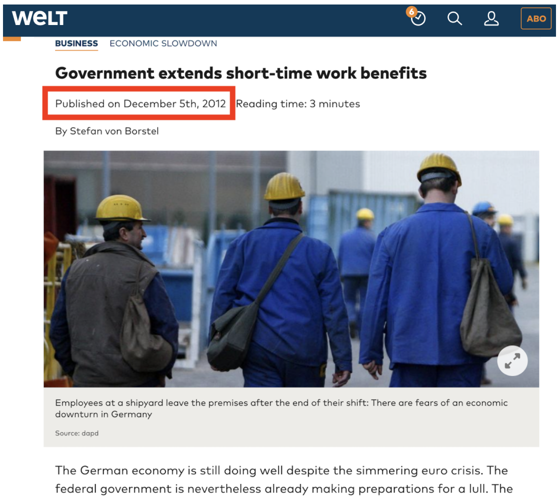
Economic downturn

Federal government extends short-time working benefits

The federal government is preparing for an economic downturn. As a precautionary measure, the short-time working allowance will be extended from six to twelve months. In this way, companies should be able to survive a downturn without laying off employees.

December 5, 2012, 3:52 p.m

Berlin - It is a precautionary measure against a possible economic downturn: Short-time work benefits will be extended from six to twelve months, a spokeswoman for the Federal Ministry of Labor said on Wednesday. The regulation has already been submitted to Minister Ursula von der Leyen (CDU) for signature. Once signed, the regulation could come into force within a few days.



WELT ABO

BUSINESS ECONOMIC SLOWDOWN

Government extends short-time work benefits

Published on December 5th, 2012 Reading time: 3 minutes

By Stefan von Borstel

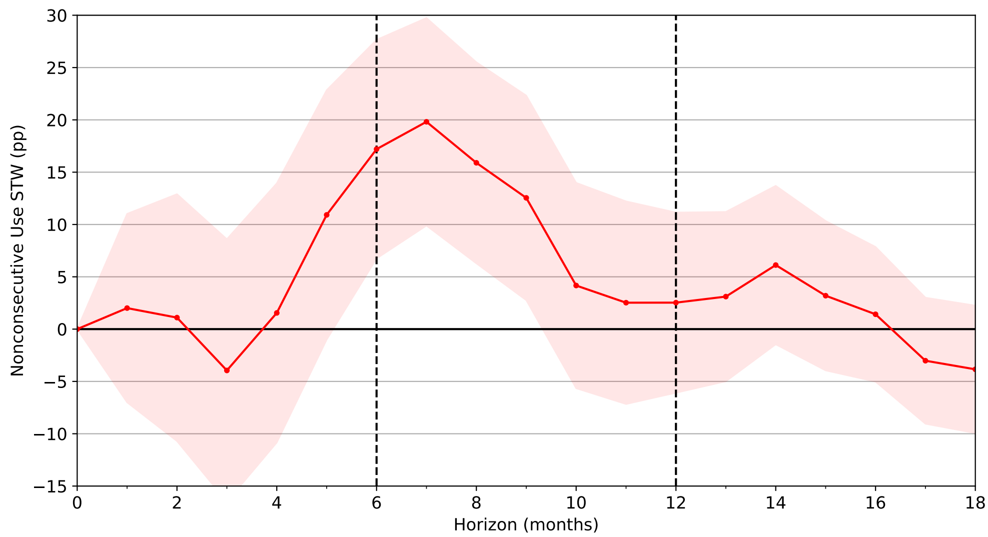
Employees at a shipyard leave the premises after the end of their shift: There are fears of an economic downturn in Germany

Source: dapd

The German economy is still doing well despite the simmering euro crisis. The federal government is nevertheless already making preparations for a lull. The

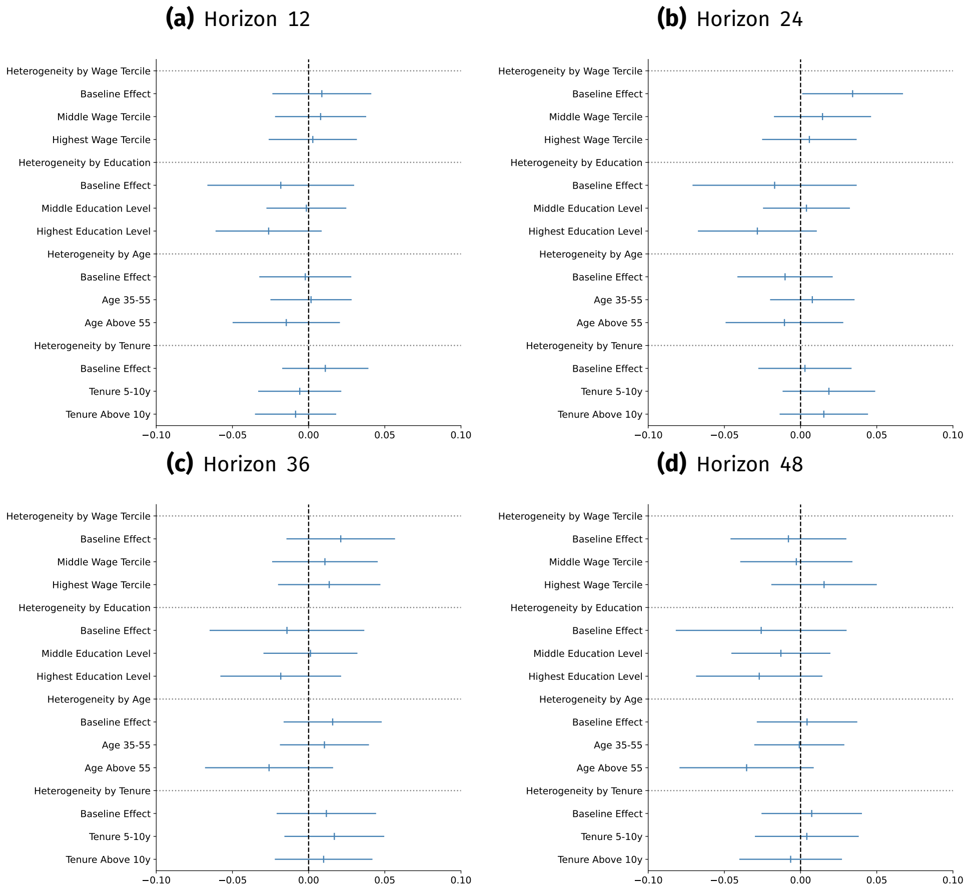
Notes: We include screenshots of newspaper coverage in two highly visible news outlets, the magazine *Der Spiegel* and the newspaper *Die Welt*, respectively. The top row shows news articles published on November 25, 2012, and highlights the stance of the Federal Labor Minister, Ursula von der Leyen, opposing STW extensions. The bottom row shows news articles from December 5, 2012, by which time the government had sharply reversed coures and announced a doubling of STW PBD. We translated the screenshots using Google Translate and added highlights in red around the dates as well as in yellow marking the policy change from 6 to 12 months of PBD.

Figure 2.B.2. First Stage (Use of Short-Time Work)



Notes: The figure plots the effect of the reform at different horizons after the start of STW using as outcome variable an indicator variable that is equal to one if the firm still receives STW benefits regardless of the STW spell (*Nonconsecutive Use STW*). We report the treatment effects using the regression discontinuity design specified in (2.7.1) including industry by region fixed effects. The data is at the firm-horizon level; a separate regression is run for each horizon. 95% confidence intervals based on robust standard errors are depicted. The sample is restricted to firms that in the start month have more than five employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*).

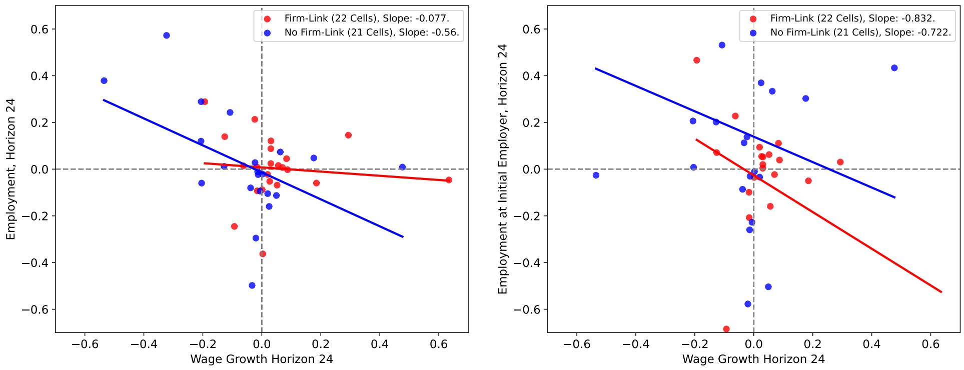
Figure 2.B.3. Heterogeneity by Demographics: Employment



Notes: The figure plots heterogeneous employment effects by demographics at different horizons after the start of STW. We define groups within firms based on demographic characteristics at the start of STW (age, tenure at the firm, education level, wage tercile within the firm). The data is at the group-firm-horizon level. The coefficients shown are heterogeneous treatment effects of a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. As outcome variable, we use for each group-firm cell the share of initially employed (i.e. employed at the start of STW) who are employed anywhere. The baseline education level is defined as no training or missing information, individuals with a middle (high) education level have a vocational training (hold a degree from an university of university of applied sciences). The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*).

Figure 2.B.4. Cell-Level Analysis after 24 months: Establishments w/ and w/o Firm Link

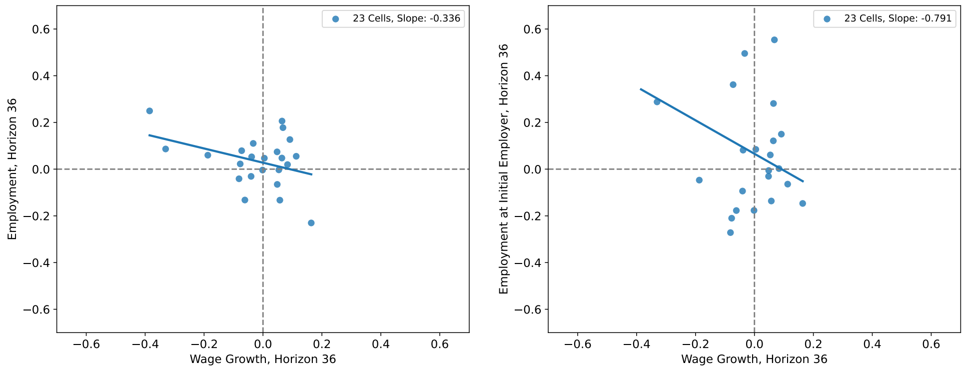
(a) Employment **(b)** Employment at Initial Employer



Notes: The figure plots the treatment effect on employment (y-axis) against the treatment effect on wage growth (x-axis) in different cells 24 months after the start of STW, separately for establishments that can be linked to the firm level (red) or not (blue). Orbis-ADIAB (see Antoni, Koller, Laible, and Zimmermann, 2018, for details) is used for linking establishments to firms. Establishments are assigned to cells based on sector (manufacturing, wholesale and retail trade, rest), region (East/West), and size (up to 5, 6-15, 16-50, more than 50 employees). In Panel A, the outcome variable for employment is for each firm the share of initially emplyoed (i.e., employed at the start of STW) who are employed anywhere. In Panel B, the outcome variable for employment is for each firm the share of initially employed who are still employed at the same firm. Potential re-entries after an exit are ignored. Wage growth is the growth rate of average daily wages relative to the start of STW. We report treatment effects using the regression discontinuity design specified in (2.7.1) at the establishment level without industry by region fixed effects for a horizon of 24 months. Attention is restricted to employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*).

Figure 2.B.5. Cell-Level Analysis after 36 months

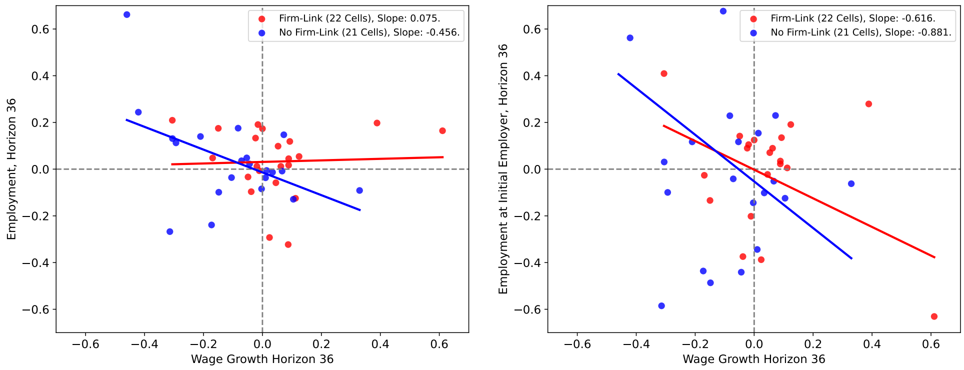
(a) Employment **(b)** Employment at Initial Employer



Notes: The figure plots the treatment effect on employment (y-axis) against the treatment effect on wage growth (x-axis) in different cells 36 months after the start of STW at the establishment level. Establishments are assigned to cells based on their sector (manufacturing (43%), wholesale and retail trade (14%), rest (43%)), region (East (28%), West (72%)) and size (up to 5 (51%), 6-15 (23%), 16-50 (15%), more than 50 employees (11%)). One cell (wholesale and retail trade, east, more than 50 employees) is excluded because there are too few observations. In Panel A, the outcome variable for employment is for each firm the share of initially employed (i.e., employed at the start of STW) who are employed anywhere. In Panel B, the outcome variable for employment is for each firm the share of initially employed who are still employed at the same firm. Potential re-entries after an exit are ignored. Wage growth is the growth rate of average daily wages relative to the start of STW. We report treatment effects using the regression discontinuity design specified in (2.7.1) at the establishment level without industry by region fixed effects for a horizon of 36 months. Attention is restricted to employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*).

Figure 2.B.6. Cell-Level Analysis after 36 months: Establishments w/ and w/o Firm Link

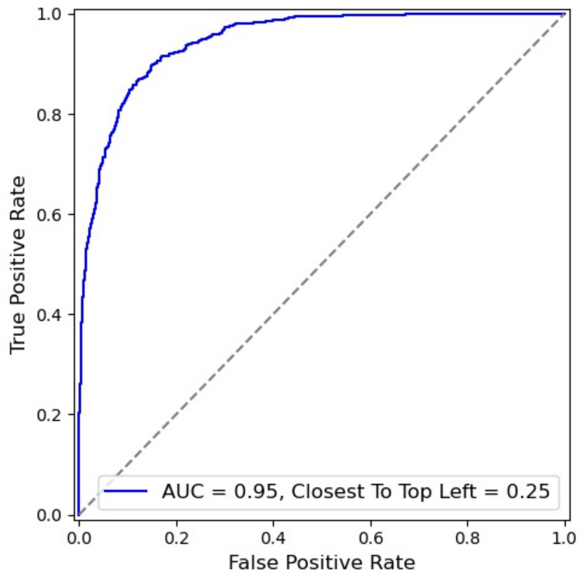
(a) Employment **(b)** Employment at Initial Employer



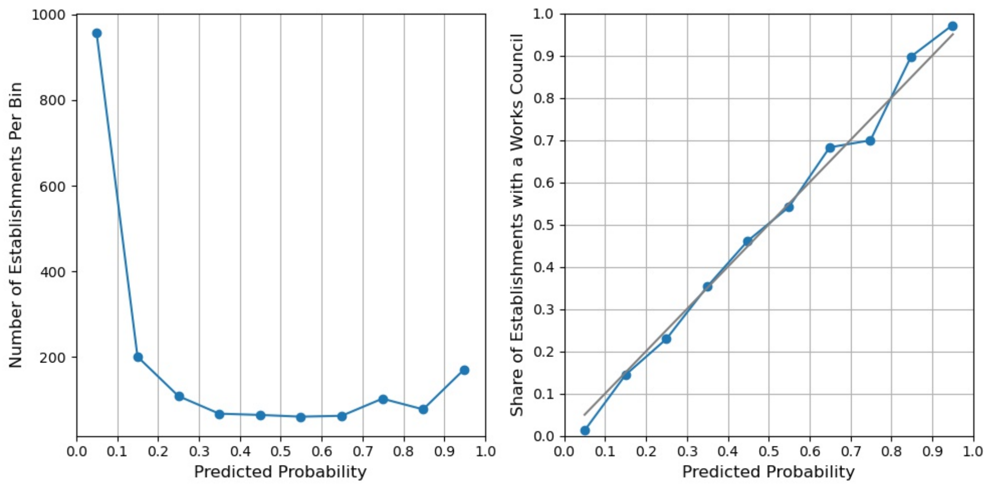
Notes: The figure plots the treatment effect on employment (y-axis) against the treatment effect on wage growth (x-axis) in different cells 36 months after the start of STW, separately for establishments that can be linked to the firm level (red) or not (blue). Orbis-ADIAB (see Antoni, Koller, Laible, and Zimmermann, 2018, for details) is used for linking establishments to firms. Establishments are assigned to cells based on sector (manufacturing, wholesale and retail trade, rest), region (East/West), and size (up to 5, 6-15, 16-50, more than 50 employees). In Panel A, the outcome variable for employment is for each firm the share of initially emplyoed (i.e., employed at the start of STW) who are employed anywhere. In Panel B, the outcome variable for employment is for each firm the share of initially employed who are still employed at the same firm. Potential re-entries after an exit are ignored. Wage growth is the growth rate of average daily wages relative to the start of STW. We report treatment effects using the regression discontinuity design specified in (2.7.1) at the establishment level without industry by region fixed effects for a horizon of 36 months. Attention is restricted to employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*).

Figure 2.B.7. Evaluation of Prediction (based on IAB Establishment Panel)

(a) ROC Curve



(b) Distribution of Predicted Probabilities



Notes: The figure shows an evaluation of the prediction of existence of a works council based on the IAB Establishment Panel. We split the IAB Establishment panel into a random test sample (15%) and training sample, and present the results on the test sample. Panel A shows the receiver operating characteristic curve (ROC curve) for a logit specification described in 2.A.2.5. Panel B shows the results of a simple evaluation whether the predicted probabilities are of the right order of magnitude. The LHS of Panel B shows the distribution of predicted probabilities. The chosen bin size is 0.1 and midpoints of bins are shown. The RHS of Panel B shows for each bin (x-axis) the true share of establishments with a works council (y-axis).

Appendix 2.C Supplementary Tables

Table 2.C.1. Individual-Level Summary Statistics—All Establishments

	Start Months			
	2020m4		2020m4-2020m12	
	No STW	STW	No STW	STW
Wage				
Daily Wage	91.95	110.86	98.55	111.91
	(63.84)	(52.70)	(64.22)	(52.01)
Education Level				
Low (Neither or Missing)	0.21	0.11	0.19	0.11
	(0.41)	(0.32)	(0.40)	(0.32)
Middle (Vocational Training)	0.61	0.69	0.61	0.70
	(0.49)	(0.46)	(0.49)	(0.46)
High (Degree from University/FH)	0.18	0.19	0.20	0.19
	(0.38)	(0.39)	(0.40)	(0.39)
Occupation (Horizontal)				
Production	0.30	0.36	0.32	0.38
	(0.46)	(0.48)	(0.47)	(0.49)
Personal Service	0.16	0.14	0.16	0.13
	(0.36)	(0.35)	(0.36)	(0.34)
Commercial Service	0.28	0.31	0.28	0.30
	(0.45)	(0.46)	(0.45)	(0.46)
IT Service	0.05	0.04	0.05	0.04
	(0.21)	(0.19)	(0.22)	(0.20)
Other Service	0.22	0.15	0.19	0.14
	(0.41)	(0.36)	(0.39)	(0.35)
Occupation (Vertical)				
Unskilled/ Semiskilled Tasks	0.20	0.15	0.18	0.15
	(0.40)	(0.36)	(0.39)	(0.36)
Skilled Tasks	0.54	0.55	0.54	0.55
	(0.50)	(0.50)	(0.50)	(0.50)
Complex Specialist Tasks	0.14	0.18	0.14	0.17
	(0.34)	(0.38)	(0.35)	(0.38)
Highly Complex Tasks	0.12	0.12	0.13	0.12
	(0.33)	(0.33)	(0.34)	(0.33)
Age				
Younger 35	0.33	0.28	0.32	0.28
	(0.47)	(0.45)	(0.47)	(0.45)
35-54	0.40	0.49	0.42	0.50
	(0.49)	(0.50)	(0.49)	(0.50)
Older 55	0.26	0.23	0.26	0.23
	(0.44)	(0.42)	(0.44)	(0.42)
Tenure				
Less Than 5y	0.54	0.50	0.52	0.48
	(0.50)	(0.50)	(0.50)	(0.50)
5-10y	0.17	0.19	0.17	0.19
	(0.37)	(0.39)	(0.38)	(0.39)
Above 10y	0.29	0.31	0.31	0.33
	(0.45)	(0.46)	(0.46)	(0.47)
Predicted Retention Probability	0.78	0.81	0.79	0.81
	(0.12)	(0.09)	(0.11)	(0.09)
Observations	2450192	1872371	3815372	2501467

Notes: The table reports individual-level summary statistics for workers at establishments that used short-time work in 2020. We differentiate between workers on short-time work vs. all other workers. STW Take-up is defined as high or 100% probability of STW receipt in the start month (see Appendix 2.A.1 for details). Columns 1 and 2 restrict attention to establishments with April 2020 as start month of STW, and consider the universe of individuals who work there in the start month. Columns 3 and 4 pool across start months in 2020 Q2-Q4. Standard deviations are reported below the means in parentheses.

Table 2.C.2. Effect of Individual STW Eligibility on Employment—All Establishments

	Employment (12 Months)					
	At Initial Employer			Anywhere		
	(1)	(2)	(3)	(4)	(5)	(6)
STW in Start Month	0.059***	0.089***	0.082***	0.038***	0.059***	0.055***
	(0.0003)	(0.0009)	(0.0009)	(0.0002)	(0.0006)	(0.0006)
Start Month FEs	Yes	Yes	Yes	Yes	Yes	Yes
Employer FEs	No	Yes	Yes	No	Yes	Yes
Control for Age	No	No	Yes	No	No	Yes
Education Group FEs	No	No	Yes	No	No	Yes
Control for Tenure	No	No	Yes	No	No	Yes
Control for Gender	No	No	Yes	No	No	Yes
Occupation Group FEs	No	No	Yes	No	No	Yes
N Individuals	6,316,839	6,316,460	6,316,360	6,316,839	6,316,460	6,316,360
R Squared	0.007	0.202	0.211	0.005	0.064	0.069
Adj. R Squared	0.007	0.187	0.196	0.005	0.046	0.051
N Establishments	119,846	119,467	119,467	119,846	119,467	119,467
Mean Outcome (No STW)	0.78	0.78	0.78	0.90	0.90	0.90

Notes: The level of observation is a worker i that is initially employed at an establishment that took up short-time work between April and December 2020. We focus on the first STW spell in case of multiple. In the first three columns, the dependent variable is an indicator variable for whether a given worker is still employed at the initial employer 12 months after the start of STW. In the last three columns, the dependent variable is an indicator variable for whether a given worker is employed anywhere 12 months after the start of STW at the initial employer. $\textit{STW in Start Month}_i$ is an indicator variable for individual STW receipt in the start month. STW Take-up is defined as high or 100% probability of STW receipt in the start month (see Appendix 2.A.1 for details). Individual-level control variables are included where indicated. The education groups are no training or missing information, vocational training, and (any) university degree. We include five occupation groups (horizontal): production, personal service, commercial service, IT service, and other service. Robust standard errors clustered at the establishment level are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.3. Effect of Individual STW Eligibility on Employment—Full-Time Employees Only

	Employment (12 Months)					
	At Initial Employer			Anywhere		
	(1)	(2)	(3)	(4)	(5)	(6)
STW in Start Month	0.026***	0.051***	0.055***	0.015***	0.031***	0.032***
	(0.0004)	(0.0010)	(0.0010)	(0.0002)	(0.0006)	(0.0006)
Start Month FEs	Yes	Yes	Yes	Yes	Yes	Yes
Employer FEs	No	Yes	Yes	No	Yes	Yes
Control for Age	No	No	Yes	No	No	Yes
Education Group FEs	No	No	Yes	No	No	Yes
Control for Tenure	No	No	Yes	No	No	Yes
Control for Gender	No	No	Yes	No	No	Yes
Occupation Group FEs	No	No	Yes	No	No	Yes
N Individuals	4,368,844	4,362,643	4,362,611	4,368,844	4,362,643	4,362,611
R Squared	0.003	0.229	0.237	0.001	0.059	0.064
Adj. R Squared	0.003	0.210	0.218	0.001	0.035	0.040
N Establishments	115,067	108,866	108,866	115,067	108,866	108,866
Mean Outcome (No STW)	0.82	0.82	0.82	0.92	0.92	0.92

Notes: The level of observation is a worker i that is initially employed at an establishment that took up short-time work between April and December 2020. We focus on the first STW spell in case of multiple. In the first three columns, the dependent variable is an indicator variable for whether a given worker is still employed at the initial employer 12 months after the start of STW. In the last three columns, the dependent variable is an indicator variable for whether a given worker is employed anywhere 12 months after the start of STW at the initial employer. $\textit{STW in Start Month}_i$ is an indicator variable for individual STW receipt in the start month. STW Take-up is defined as high or 100% probability of STW receipt in the start month (see Appendix 2.A.1 for details). Individual-level control variables are included where indicated. The education groups are no training or missing information, vocational training, and (any) university degree. We include five occupation groups (horizontal): production, personal service, commercial service, IT service, and other service. Robust standard errors clustered at the establishment level are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.4. Summary Statistics by Match Outcome

	Starter in 2012		w/ Firm-Link		w/o Firm-Link	
	N	Mean	N	Mean	N	Mean
Share in East Germany	9813	0.26	5425	0.24	4388	0.28
Age	9813	18.13	5425	19.28	4388	16.71
Average Daily Wage (Start Month)	9813	78.18	5425	84.80	4388	70.01
Share in Manufacturing	9813	0.49	5425	0.58	4388	0.38
Size (Start Month)						
1-4 Employees	9813	0.42	5425	0.26	4388	0.63
5-9 Employees	9813	0.19	5425	0.20	4388	0.17
10-19 Employees	9813	0.14	5425	0.19	4388	0.08
20-49 Employees	9813	0.12	5425	0.17	4388	0.06
50-99 Employees	9813	0.06	5425	0.08	4388	0.03
100-199 Employees	9813	0.04	5425	0.05	4388	0.02
200-499 Employees	9813	0.02	5425	0.03	4388	0.01
More Than 500 Employees	9813	0.01	5425	0.01	4388	0.00

Notes: The table shows summary statistics of establishments that start STW in 2012. The first two columns include all establishments (no size restrictions), the middle two columns the subset thereof that can be linked to the firm level using Orbis-ADIAB and the last two the subset thereof for which no such link can be established. Size refers to employment in the start month of STW including only employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*).

Table 2.C.5. Complier Analysis

	Complier (Benefit Duration Exceeds 6m)			
	(1)	(2)	(3)	(4)
Log Employees (Start Month)	-0.004	-0.011	-0.015*	-0.016*
	(0.01)	(0.01)	(0.01)	(0.01)
Log Avrg Daily Wage (Start Month)		0.090*	0.088*	0.083*
		(0.05)	(0.05)	(0.05)
Age			0.004***	0.004***
			(0.00)	(0.00)
Pre-Period Wage Growth				0.038
				(0.19)
Start Month FE	Yes	Yes	Yes	Yes
Industry x Region FE	Yes	Yes	Yes	Yes
N Firms	1,762	1,762	1,762	1,750
R Squared	0.071	0.073	0.082	0.081
R Squared Adj.	0.024	0.025	0.035	0.033

Notes: The table shows the results of a regression of a dummy indicating a STW benefit duration that exceeds 6 months (*Complier*) on firm characteristics. The sample consists of firms that start STW between 2012m7 and 2012m12. *Pre-Period Wage Growth* is the 1y-growth rate in average wages based on employees that were employed at the respective firm 12 months prior to the start of STW. The sample is restricted to firms that in the start month have more than five employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.6. Heterogeneity in Individual Characteristics: Wage Tercile

	Employment at Initial Employer, Horizon (months)					
	6	12	18	24	36	48
Running Variable	-0.003	0.009	0.009	0.011	0.012	0.013
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Running Variable × Middle Wage Tercile	-0.002	-0.004*	-0.006**	-0.006***	-0.007***	-0.005**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Highest Wage Tercile	-0.002	-0.005**	-0.005**	-0.006**	-0.007***	-0.006**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	0.008	-0.010	-0.008	-0.013	-0.011	-0.011
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Middle Wage Tercile	0.001	0.005	0.004	0.007	0.005	0.004
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable × Highest Wage Tercile	-0.003	0.006	0.003	0.005	0.003	-0.000
	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD)	0.012	0.012	-0.002	-0.004	-0.014	-0.032
	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
Treatment (12m PBD) × Middle Wage Tercile	-0.013	-0.012	-0.005	-0.012	0.000	0.003
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Treatment (12m PBD) × Highest Wage Tercile	0.005	-0.008	-0.003	-0.009	0.011	0.024
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Middle Wage Tercile	0.053***	0.066***	0.069***	0.070***	0.069***	0.064***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Highest Wage Tercile	0.062***	0.083***	0.097***	0.102***	0.102***	0.100***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	4,239	4,239	4,239	4,239	4,239	4,239
N Individuals	639,228	639,228	639,228	639,228	639,228	639,228

Notes: The table reports heterogeneous treatment effects of the reform by within-firm wage tercile on employment at different horizons after the start of STW. We define groups within firms based on demographic characteristics at the start of STW. The data is a balanced panel at the group-firm-horizon level. The results are from a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. The outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are still employed at the same firm. Potential re-entries after an exit are ignored. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown is the number of clusters including firms that start in 2011. The number of individuals refers to the number of individuals the calculation is based upon. Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.7. Heterogeneity in Individual Characteristics: Education

	Employment at Initial Employer, Horizon (months)					
	6	12	18	24	36	48
Running Variable	-0.005	0.006	0.006	0.005	0.007	0.012
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Running Variable × Middle Education Level	-0.001	-0.004	-0.001	0.001	0.002	0.000
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Running Variable × Highest Education Level	0.001	-0.000	0.005	0.004	0.003	-0.000
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable	0.008	0.002	0.001	-0.001	-0.003	-0.013
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Middle Education Level	-0.001	-0.005	-0.011	-0.008	-0.008	0.002
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Highest Education Level	0.000	-0.001	-0.010	-0.004	-0.004	0.004
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD)	0.025	-0.002	-0.004	-0.005	-0.002	0.007
	(0.04)	(0.04)	(0.05)	(0.06)	(0.06)	(0.06)
Treatment (12m PBD) × Middle Education Level	-0.014	0.016	0.010	-0.001	-0.006	-0.039
	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)
Treatment (12m PBD) × Highest Education Level	-0.017	-0.015	-0.022	-0.037	-0.030	-0.045
	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
Middle Education Level	0.013*	0.013	0.018*	0.013	0.021**	0.019*
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Highest Education Level	0.024***	0.034***	0.041***	0.030**	0.038***	0.039***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	6,411	6,411	6,411	6,411	6,411	6,411
N Individuals	652,717	652,717	652,717	652,717	652,717	652,717

Notes: The table reports heterogeneous treatment effects of the reform by education on employment at different horizons after the start of STW. The baseline education level is defined as no training or missing information, individuals with a middle (high) education level have vocational training (hold a degree from an university of university of applied sciences). We define groups within firms based on demographic characteristics at the start of STW. The data is a balanced panel at the group-firm-horizon level. The results are from a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. The outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are still employed at the same firm. Potential re-entries after an exit are ignored. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown is the number of clusters including firms that start in 2011. The number of individuals refers to the number of individuals the calculation is based upon. Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.8. Heterogeneity in Individual Characteristics: Age

	Employment at Initial Employer, Horizon (months)					
	6	12	18	24	36	48
Running Variable	-0.007	0.001	0.006	0.008	0.007	0.009
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Running Variable × Age 35-55	0.003	0.004	0.001	0.000	-0.000	-0.001
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Age above 55	0.003	0.005*	-0.001	-0.000	0.001	-0.000
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	0.011	-0.001	-0.004	-0.002	0.003	-0.000
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Age 35-55	-0.008*	-0.006	-0.003	-0.003	-0.006	-0.003
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Age above 55	-0.003	-0.001	0.003	-0.001	-0.004	-0.005
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD)	0.022	0.033	0.012	-0.013	-0.022	-0.032
	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Treatment (12m PBD) × Age 35-55	-0.003	-0.021	-0.026	-0.012	0.010	0.001
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Treatment (12m PBD) × Age above 55	-0.041*	-0.057**	-0.049	-0.039	-0.037	-0.024
	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Age 35-55	0.070***	0.100***	0.115***	0.125***	0.134***	0.137***
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Age above 55	0.071***	0.087***	0.082***	0.075***	0.044***	0.007
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	5,567	5,567	5,567	5,567	5,567	5,567
N Individuals	645,138	645,138	645,138	645,138	645,138	645,138

Notes: The table reports heterogeneous treatment effects of the reform by age on employment at different horizons after the start of STW. We define groups within firms based on demographic characteristics at the start of STW. The data is a balanced panel at the group-firm-horizon level. The results are from a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. The outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are still employed at the same firm. Potential re-entries after an exit are ignored. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown is the number of clusters including firms that start in 2011. The number of individuals refers to the number of individuals the calculation is based upon. Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.9. Heterogeneity in Individual Characteristics: Tenure

	Employment at Initial Employer, Horizon (months)					
	6	12	18	24	36	48
Running Variable	-0.006	0.001	0.003	0.006	0.005	0.007
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Running Variable × Tenure 5-10y	0.003	0.007**	0.006*	0.004	0.007**	0.005
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Tenure above 10y	0.003	0.006*	0.004	0.001	0.001	0.002
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	0.012*	-0.004	0.000	0.000	0.000	-0.002
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Tenure 5-10y	-0.005	0.000	-0.010	-0.008	-0.012*	-0.006
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Tenure above 10y	-0.013**	-0.006	-0.014*	-0.011	-0.011	-0.008
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD)	0.010	0.044	0.009	-0.011	-0.001	-0.010
	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)
Treatment (12m PBD) × Tenure 5-10y	0.001	-0.049**	-0.003	0.007	0.012	-0.007
	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)
Treatment (12m PBD) × Tenure above 10y	0.015	-0.042	0.003	0.010	0.006	-0.005
	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Tenure 5-10y	0.087***	0.138***	0.152***	0.156***	0.156***	0.150***
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Tenure above 10y	0.117***	0.183***	0.201***	0.207***	0.207***	0.199***
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	5,926	5,926	5,926	5,926	5,926	5,926
N Individuals	649,531	649,531	649,531	649,531	649,531	649,531

Notes: The table reports heterogeneous treatment effects of the reform by tenure on employment at different horizons after the start of STW. We define groups within firms based on demographic characteristics at the start of STW. The data is a balanced panel at the group-firm-horizon level. The results are from a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. The outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are still employed at the same firm. Potential re-entries after an exit are ignored. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown is the number of clusters including firms that start in 2011. The number of individuals refers to the number of individuals the calculation is based upon. Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.10. Heterogeneity in Individual Characteristics: Wage Tercile

	Employment, Horizon (months)					
	6	12	18	24	36	48
Running Variable	0.002	0.004	-0.002	-0.003	-0.000	-0.000
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Middle Wage Tercile	-0.001	-0.004***	-0.001	-0.000	-0.002	0.001
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Highest Wage Tercile	-0.001	-0.003**	0.000	-0.000	-0.001	0.001
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	0.003	-0.005	-0.000	-0.003	-0.002	0.002
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable × Middle Wage Tercile	-0.002	0.003	0.001	0.003	0.003	-0.004
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable × Highest Wage Tercile	-0.004*	0.005*	0.002	0.006*	0.002	-0.007*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD)	-0.024	0.009	0.022	0.034**	0.021	-0.008
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Treatment (12m PBD) × Middle Wage Tercile	0.007	-0.001	-0.003	-0.020	-0.010	0.005
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) × Highest Wage Tercile	0.023**	-0.006	-0.015	-0.028**	-0.008	0.023
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Middle Wage Tercile	0.027***	0.033***	0.038***	0.041***	0.039***	0.040***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Highest Wage Tercile	0.031***	0.040***	0.048***	0.050***	0.048***	0.044***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	4,239	4,239	4,239	4,239	4,239	4,239
N Individuals	639,228	639,228	639,228	639,228	639,228	639,228

Notes: The table reports heterogeneous treatment effects of the reform by within-firm wage tercile on employment at different horizons after the start of STW. We define groups within firms based on demographic characteristics at the start of STW. The data is a balanced panel at the group-firm-horizon level. The results are from a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. The outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are employed anywhere. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown is the number of clusters including firms that start in 2011. The number of individuals refers to the number of individuals the calculation is based upon. Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.11. Heterogeneity in Individual Characteristics: Education

	Employment, Horizon (months)					
	6	12	18	24	36	48
Running Variable	0.002	0.001	-0.004	-0.003	-0.000	0.003
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Middle Education Level	-0.002	-0.001	0.004	0.003	0.003	0.002
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Highest Education Level	-0.001	-0.000	0.006**	0.000	-0.000	-0.001
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	0.006	0.006	0.012**	0.009	0.008	0.003
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Middle Education Level	-0.002	-0.005	-0.013**	-0.009	-0.011*	-0.009
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Highest Education Level	-0.007	-0.002	-0.014**	-0.003	-0.004	-0.002
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD)	-0.027	-0.018	-0.016	-0.017	-0.014	-0.026
	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)
Treatment (12m PBD) × Middle Education Level	0.017	0.017	0.018	0.021	0.015	0.013
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Treatment (12m PBD) × Highest Education Level	0.033	-0.008	0.001	-0.011	-0.004	-0.001
	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)
Middle Education Level	0.018***	0.030***	0.043***	0.039***	0.056***	0.059***
	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
Highest Education Level	0.028***	0.048***	0.057***	0.055***	0.066***	0.068***
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	6,411	6,411	6,411	6,411	6,411	6,411
N Individuals	652,717	652,717	652,717	652,717	652,717	652,717

Notes: The table reports heterogeneous treatment effects of the reform by education on employment at different horizons after the start of STW. The baseline education level is defined as no training or missing information, individuals with a middle (high) education level have vocational training (hold a degree from an university of university of applied sciences). We define groups within firms based on demographic characteristics at the start of STW. The data is a balanced panel at the group-firm-horizon level. The results are from a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. The outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are employed anywhere. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown is the number of clusters including firms that start in 2011. The number of individuals refers to the number of individuals the calculation is based upon. Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.12. Heterogeneity in Individual Characteristics: Age

	Employment, Horizon (months)					
	6	12	18	24	36	48
Running Variable	0.001	0.002	0.001	0.001	-0.001	0.001
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Age 35-55	-0.001	0.000	-0.000	-0.001	0.001	0.002
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Age above 55	0.000	0.001	-0.002	-0.003	0.000	-0.002
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	0.004	-0.001	0.003	0.003	0.001	-0.001
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable × Age 35-55	-0.001	-0.003	-0.003	-0.004	-0.001	-0.003
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable × Age above 55	-0.002	-0.001	0.000	-0.001	0.004	0.006
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)
Treatment (12m PBD)	-0.016	-0.002	-0.006	-0.010	0.016	0.004
	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Treatment (12m PBD) × Age 35-55	0.009	0.004	0.011	0.018	-0.005	-0.005
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) × Age above 55	-0.008	-0.012	-0.002	-0.000	-0.042**	-0.040*
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Age 35-55	0.017***	0.024***	0.024***	0.025***	0.024***	0.019***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Age above 55	-0.008***	-0.033***	-0.065***	-0.093***	-0.166***	-0.246***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	5,567	5,567	5,567	5,567	5,567	5,567
N Individuals	645,138	645,138	645,138	645,138	645,138	645,138

Notes: The table reports heterogeneous treatment effects of the reform by age on employment at different horizons after the start of STW. We define groups within firms based on demographic characteristics at the start of STW. The data is a balanced panel at the group-firm-horizon level. The results are from a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. The outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are employed anywhere. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown is the number of clusters including firms that start in 2011. The number of individuals refers to the number of individuals the calculation is based upon. Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.13. Heterogeneity in Individual Characteristics: Tenure

	Employment, Horizon (months)					
	6	12	18	24	36	48
Running Variable	-0.001	0.002	-0.002	-0.002	-0.002	-0.002
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Tenure 5-10y	0.001	0.001	0.002	-0.000	0.002	0.003*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Tenure above 10y	0.000	-0.002	-0.001	-0.002	0.001	0.003
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	0.005	-0.003	0.004	0.005	0.004	0.002
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable × Tenure 5-10y	-0.004	0.003	-0.007**	-0.004	-0.005	-0.003
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable × Tenure above 10y	-0.007**	0.004	-0.004	-0.003	-0.003	-0.003
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD)	-0.015	0.011	0.005	0.003	0.012	0.007
	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Treatment (12m PBD) × Tenure 5-10y	0.014	-0.017	0.020	0.016	0.005	-0.003
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) × Tenure above 10y	0.027**	-0.019*	0.015	0.012	-0.002	-0.014
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
Tenure 5-10y	0.032***	0.046***	0.042***	0.043***	0.041***	0.033***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Tenure above 10y	0.031***	0.045***	0.033***	0.029***	0.013***	-0.004
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	5,926	5,926	5,926	5,926	5,926	5,926
N Individuals	649,531	649,531	649,531	649,531	649,531	649,531

Notes: The table reports heterogeneous treatment effects of the reform by tenure on employment at different horizons after the start of STW. We define groups within firms based on demographic characteristics at the start of STW. The data is a balanced panel at the group-firm-horizon level. The results are from a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. The outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are employed anywhere. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown is the number of clusters including firms that start in 2011. The number of individuals refers to the number of individuals the calculation is based upon. Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.14. Role of Job Switches Within 1y in Explaining the Effect on Wage Growth

	Wage Growth Since Start, Horizon (months)			
	12	24	36	48
Running Variable	-0.001	-0.006**	-0.006*	-0.007**
	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Switch Within 1y	0.005*	0.005*	0.007**	0.006*
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) × Running Variable	0.005*	0.008**	0.003	0.005
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) × Running Variable × Switch Within 1y	-0.013**	-0.012**	-0.015**	-0.013*
	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD)	-0.001	0.035**	0.055***	0.060***
	(0.01)	(0.01)	(0.02)	(0.02)
Treatment (12m PBD) × Switch Within 1y	-0.037*	-0.037*	-0.027	-0.030
	(0.02)	(0.02)	(0.03)	(0.03)
Switch Within 1y	0.008	0.076***	0.099***	0.122***
	(0.01)	(0.01)	(0.01)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes
N Firms	6,573	6,574	6,576	6,576
N Individuals	617,349	599,940	586,636	574,209

Notes: The table reports heterogeneous treatment effects of the reform by job switches within 1y on wage growth at different horizons after the start of STW. We define groups within firms based on whether an individual has switched employer within the respective horizon after the start of STW. The data is at the group-firm-horizon level. The results are from a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. As outcome variable, we use for each group-firm cell the growth rate of average daily wages relative to the start of STW. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown is the number of clusters including firms that start in 2011. The number of individuals per horizon refers to the number of individuals among all initially employed who are still in the labor market at this horizon and, thus, for whom wage growth can be calculated. A drop and subsequent increase in the number of firms can occur if, at some firm, all initially employed have gaps in their employment history (e.g., due to parental leave or sickness). Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.15. Role of Job Switches Within 2y in Explaining the Effect on Wage Growth

	Wage Growth Since Start, Horizon (months)			
	12	24	36	48
Running Variable	-0.002	-0.005**	-0.008**	-0.011***
	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Switch Within 2y	0.004**	0.003	0.009***	0.010***
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) × Running Variable	0.004	0.008**	0.008*	0.009*
	(0.00)	(0.00)	(0.00)	(0.01)
Treatment (12m PBD) × Running Variable × Switch Within 2y	-0.005	-0.006	-0.012**	-0.009
	(0.00)	(0.00)	(0.01)	(0.01)
Treatment (12m PBD)	0.011	0.033**	0.053***	0.074***
	(0.01)	(0.01)	(0.02)	(0.02)
Treatment (12m PBD) × Switch Within 2y	-0.032**	-0.036*	-0.041*	-0.057**
	(0.01)	(0.02)	(0.02)	(0.02)
Switch Within 2y	0.026***	0.005	0.084***	0.113***
	(0.00)	(0.00)	(0.01)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes
N Firms	6,397	6,379	6,399	6,399
N Individuals	616,800	598,364	585,425	573,025

Notes: The table reports heterogeneous treatment effects of the reform by job switches within 2y on wage growth at different horizons after the start of STW. We define groups within firms based on whether an individual has switched employer within the respective horizon after the start of STW. The data is at the group-firm-horizon level. The results are from a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. As outcome variable, we use for each group-firm cell the growth rate of average daily wages relative to the start of STW. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown is the number of clusters including firms that start in 2011. The number of individuals per horizon refers to the number of individuals among all initially employed who are still in the labor market at this horizon and, thus, for whom wage growth can be calculated. A drop and subsequent increase in the number of firms can occur if, at some firm, all initially employed have gaps in their employment history (e.g., due to parental leave or sickness). Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.16. Role of Job Switches Within 3y in Explaining the Effect on Wage Growth

	Wage Growth Since Start, Horizon (months)			
	12	24	36	48
Running Variable	-0.002	-0.002	-0.005	-0.007**
	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Switch Within 3y	0.001	-0.002	-0.002	0.003
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) × Running Variable	0.004	0.006	0.008*	0.010*
	(0.00)	(0.00)	(0.00)	(0.01)
Treatment (12m PBD) × Running Variable × Switch Within 3y	0.001	-0.001	-0.009*	-0.011**
	(0.00)	(0.00)	(0.01)	(0.01)
Treatment (12m PBD)	0.016	0.023	0.034**	0.052**
	(0.01)	(0.01)	(0.02)	(0.02)
Treatment (12m PBD) × Switch Within 3y	-0.027**	-0.022	0.008	-0.019
	(0.01)	(0.01)	(0.02)	(0.02)
Switch Within 3y	0.018***	0.026***	0.002	0.088***
	(0.00)	(0.00)	(0.00)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes
N Firms	6,308	6,309	6,296	6,307
N Individuals	616,459	598,928	584,909	572,862

Notes: The table reports heterogeneous treatment effects of the reform by job switches within 3y on wage growth at different horizons after the start of STW. We define groups within firms based on whether an individual has switched employer within the respective horizon after the start of STW. The data is at the group-firm-horizon level. The results are from a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. As outcome variable, we use for each group-firm cell the growth rate of average daily wages relative to the start of STW. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown is the number of clusters including firms that start in 2011. The number of individuals per horizon refers to the number of individuals among all initially employed who are still in the labor market at this horizon and, thus, for whom wage growth can be calculated. A drop and subsequent increase in the number of firms can occur if, at some firm, all initially employed have gaps in their employment history (e.g., due to parental leave or sickness). Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.17. Role of Job Switches Within 4y in Explaining the Effect on Wage Growth

	Wage Growth Since Start, Horizon (months)			
	12	24	36	48
Running Variable	-0.000	-0.003	-0.008**	-0.008**
	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Switch Within 4y	0.002	0.003	0.003	0.003
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) × Running Variable	0.001	0.008**	0.011**	0.013***
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) × Running Variable × Switch Within 4y	-0.002	-0.004	-0.008*	-0.010*
	(0.00)	(0.00)	(0.00)	(0.01)
Treatment (12m PBD)	0.012	0.012	0.047***	0.042**
	(0.01)	(0.01)	(0.02)	(0.02)
Treatment (12m PBD) × Switch Within 4y	-0.020**	-0.027**	-0.022	-0.019
	(0.01)	(0.01)	(0.02)	(0.02)
Switch Within 4y	0.019***	0.034***	0.044***	0.013***
	(0.00)	(0.00)	(0.00)	(0.00)
Industry x Region FE	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes
N Firms	6,225	6,225	6,224	6,207
N Individuals	616,241	598,825	585,460	572,571

Notes: The table reports heterogeneous treatment effects of the reform by job switches within 4y on wage growth at different horizons after the start of STW. We define groups within firms based on whether an individual has switched employer within the respective horizon after the start of STW. The data is at the group-firm-horizon level. The results are from a regression discontinuity design analogous to the one specified in (2.7.1) at the group-firm level, including industry by region fixed effects. As outcome variable, we use for each group-firm cell the growth rate of average daily wages relative to the start of STW. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown is the number of clusters including firms that start in 2011. The number of individuals per horizon refers to the number of individuals among all initially employed who are still in the labor market at this horizon and, thus, for whom wage growth can be calculated. A drop and subsequent increase in the number of firms can occur if, at some firm, all initially employed have gaps in their employment history (e.g., due to parental leave or sickness). Robust standard errors clustered at the firm level are reported in parentheses. The sample is restricted to group-firm cells that in the start month contain more than five employees in full-time that are fully liable to social security (*Personengruppenschlüssel 101*). Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.18. Heterogeneity by Firm-Level Characteristics: Liquidity

(a) Effect on Employment at Initial Employer

	Employment at Initial Employer, Horizon (months)					
	6	12	18	24	36	48
Running Variable	0.005	0.000	0.001	0.002	-0.000	0.007
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Running Variable × Cash-to-Asset Ratio Above p66	-0.005*	0.004	0.005	0.002	0.001	0.001
	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable	0.001	0.005	0.010	0.011	0.012	-0.002
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Cash-to-Asset Ratio Above p66	0.007	-0.004	-0.011	-0.011	-0.006	0.005
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD)	-0.027	-0.030	-0.062	-0.067	-0.042	-0.033
	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)
Treatment (12m PBD) × Cash-to-Asset Ratio Above p66	-0.007	0.006	0.042	0.049	0.024	-0.039
	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)
Cash-to-Asset Ratio Above p66	-0.001	0.015*	0.026**	0.037***	0.063***	0.075***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes	Yes	Yes
N Firms	3,879	3,879	3,879	3,879	3,879	3,879
N Individuals	298,195	298,195	298,195	298,195	298,195	298,195

(b) Effect on Wage Growth

	Wage Growth Since Start, Horizon (months)			
	12	24	36	48
Running Variable	0.002	-0.006*	-0.005	-0.010**
	(0.00)	(0.00)	(0.00)	(0.00)
Running Variable × Cash-to-Asset Ratio Above p66	0.001	0.002	0.003	0.002
	(0.00)	(0.00)	(0.00)	(0.00)
Treatment (12m PBD) x Running Variable	-0.002	0.006	-0.001	0.004
	(0.00)	(0.01)	(0.01)	(0.01)
Treatment (12m PBD) x Running Variable × Cash-to-Asset Ratio Above p66	0.001	0.006	0.006	0.006
	(0.00)	(0.00)	(0.01)	(0.01)
Treatment (12m PBD)	0.005	0.031	0.056**	0.070***
	(0.01)	(0.02)	(0.02)	(0.03)
Treatment (12m PBD) × Cash-to-Asset Ratio Above p66	-0.011	-0.022	-0.033*	-0.032
	(0.01)	(0.02)	(0.02)	(0.02)
Cash-to-Asset Ratio Above p66	-0.001	-0.001	0.002	0.002
	(0.00)	(0.00)	(0.01)	(0.01)
Industry x Region FE	Yes	Yes	Yes	Yes
Calendar Month FE	Yes	Yes	Yes	Yes
N Firms	3,879	3,878	3,878	3,879
N Individuals	279,496	271,052	264,537	258,808

Notes: The table reports heterogeneous effects by liquidity of the reform on employment and wage growth at different horizons after the start of STW. We report the results of the regression discontinuity design specified in (2.7.1). In Panel (a), the outcome variable is for each firm the share of initially employed (i.e., employed at the start of STW) who are still employed at the same firm. Potential re-entries after an exit are ignored. In Panel (b), the growth rate of average daily wages relative to the start of STW is considered as outcome variable. The cash-to-asset ratio is based on BvD data in 2012 (2011) for firms that start in 2012 (2011). Details on the cleaning procedures behind the BvD data can be found in appendix 2.A.2.4. The variable *Cash-to-Asset Ratio Above p66* takes the value one if the firm's cash-to-asset ratio is above the p66 among firms that start in the same year. The sample includes the bottom and top tercile. The data is at the firm-horizon level; a separate regression is run for each horizon. The running variable is distance to the cutoff 2012m6. Treated firms are those that start STW after the cutoff. The number of firms shown includes firms that start in 2011, which are included to facilitate calendar month fixed effects in order to account for seasonality. In Panel (a), the data is a balanced panel with the number of individuals referring to the number of individuals the calculation is based upon. In Panel (b), the number of individuals per horizon refers to the number of individuals among all initially employed who are still in the labor market at this horizon and, thus, for whom wage growth can be calculated. A drop and subsequent increase in the number of firms can occur if, at some firm, all initially employed have gaps in their employment history (e.g., due to parental leave or sickness). The sample is restricted to firms that in the start month have more than five employees in full-time who are fully liable to social security (*Personengruppenschlüssel 101*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.C.19. Details on Data on Individual-Level STW Receipt

	Start Month 2020m4		Start Months 2020m4-2020m12	
	All	High Quality	All	High Quality
	Mean	Mean	Mean	Mean
Establishment-Level				
Cross-Check: Aggregated Individual-Level With Establishment-Level				
Coincide	0.74	1.00	0.73	1.00
Divergence (Number of Employees in STW)	0.08	0.00	0.08	0.00
Divergence (Month of STW Receipt)	0.10	0.00	0.10	0.00
Divergence (Both)	0.00	0.00	0.00	0.00
Incalculable	0.09	0.00	0.09	0.00
Observations	86513	11598	119923	88059
Individual-Level				
STW Risk (pp)				
Ineligible	0.13	0.15	0.12	0.14
Confirmed 0	0.26	0.35	0.29	0.38
Estabishment-Level Gender-Specific Share in STW 0-20	0.05	0.01	0.07	0.02
Estabishment-Level Gender-Specific Share in STW 20-50	0.07	0.02	0.07	0.02
Estabishment-Level Gender-Specific Share in STW 50-100	0.14	0.04	0.12	0.04
Confirmed 100	0.26	0.36	0.25	0.34
Incalculable	0.08	0.05	0.07	0.05
Observations	4689821	2528057	6775140	3636864

Notes: The table shows the results of various cross-checks of the individual level data on STW receipt. The grand total of the individual-level data includes individuals working at an establishment in STW in the month of STW receipt, for April 2020 (columns 1 and 2) and pooled across the months April to December 2020 (columns 3 and 4). Columns 1 and 3 consider all establishments, while columns 2 and 4 restrict attention to establishments with high quality data, defined as coinciding numbers of short-time workers between aggregated individual-level and establishment-level data. The top panel shows the results of cross-checking the individual-level data, aggregated to the establishment level, with the establishment-level information on monthly STW receipt (maximum of the variable *Qualitätsklasse* per establishment). The number of individuals in STW can either match (first row) or diverge for a given month. Divergence can occur if the number of individuals differs (second row) or if the establishment is not found in both datasets for that month (third row). Cross-checks may be infeasible (last row) if there is no 1:1 or 1:n mapping between the establishment applying for STW and the employer in the Social Security Records, often due to the involvement of a temporary employment agency. The bottom panel shows the individual-level risk of being in STW after cross-checks with the establishment-level data and Social Security Records (variable *Kug-Status*). An individual can be classified as ineligible (e.g., above the statutory retirement age, first row), confirmed not in STW (second row, e.g., the establishment-level number of individuals matches the aggregated individual-level information, and the individual is not in the digitized list), or confirmed in STW. If there is a discrepancy between the establishment-level data and individual-level data, the individual is assigned the gender-specific share of eligible employees at the establishment in buckets (third to fifth rows).

Chapter 3

Do Firms Hedge Human Capital?

3.1 Introduction

Hiring is a long-term investment decision (Oi, 1962). This is, for example, due to the time required to build firm-specific knowledge, employment protection legislation, and potential difficulties in hiring employees with specialized skills. Decisions on employment levels are risky as they are decided upon under uncertainty around future demand. For example, the US Quarterly Survey of Plant Capacity Utilization lists insufficient orders and insufficient supply of labor as the most cited reasons for production being below full production capabilities (Figure 3.8.1). Higher employment levels reduce the likelihood of production being limited by an insufficient supply of labor, but firms have more idle labor and a larger wage bill during periods of insufficient orders. This raises the question of how firms' inherently risky employment decisions interact with their risk management.

It is well-known that the fixed nature of labor costs creates a form of operating leverage (Donangelo, Gourio, Kehrig, and Palacios, 2019; Schoefer, 2021). The literature has relied on the labor share as a measure thereof, suggesting a positive association between the labor share and cash flow fluctuations. However, when focusing on a single source of cash flow fluctuations that firms can actively manage, such as those from exchange rate movements, a negative correlation with labor share emerges (RHS of Figure 3.8.2). In this paper, I argue that firms face a trade-off between cash flow volatility from fixed labor and cash flow volatility from other sources and establish firm-specific human capital as a driver in this trade-off.

I use short-time work (STW), a German labor market policy that facilitates flexible adjustments in work hours, to construct an empirical firm-level measure of temporarily unused fixed labor, which I call *surplus labor*. By leveraging novel administrative data on STW combined with matched employer-employee data and firm financial information, including hand-collected data on foreign exchange (FX) hedging, I show that firms with more surplus labor experience lower cash flow volatility from unhedged exchange rate movements. Using proxies for firm-specific human capital as instruments for surplus labor, I find evidence that choices of surplus labor influence firms' willingness to assume FX-induced cash flow risk.

I begin by presenting a model that formalizes the trade-off between surplus labor and another source of cash flow risk. The model serves two purposes: it defines surplus labor and formalizes the mechanism, and derives predictions that I empirically test in the remainder of the paper.

The model has similarities to the example in Arellano, Bai, and Kehoe (2019) but innovates along three dimensions. First, it explicitly models expected unused fixed labor (*surplus labor*). Second, it introduces an unrelated price risk as another source of cash flow risk, which can be hedged at a cost. Both innovations allow a better mapping to the data and the empirical setup. Third, the model adds a dimension of firm heterogeneity (*firm-specific human capital*) to explain why firms choose different levels of surplus labor.

In the model, firms vary in their reliance on workers with specialized knowledge who cannot be hired on demand (*fixed labor*). Firms choose their fixed labor under demand uncertainty, and the level of fixed labor puts a cap on their future production. They maximize expected profit but have limited risk-bearing capabilities. The wage bill for fixed labor is a fixed cost, and, thus, more fixed labor and less hedging of an unrelated price risk compete for scarce risk-bearing capabilities. I analytically characterize the unique labor and hedging choices and derive empirical predictions from a numerical solution of the model. All else equal, firms with more firm-specific human capital (a) opt for a lower capacity but still hold more fixed labor, (b) have more surplus labor, and (c) hedge more.

To test the model predictions empirically, I overcome the challenge that surplus labor is hard to measure since employees' temporary idleness is private information to firms. I use STW, a scheme that allows firms facing economic difficulties to adjust work hours flexibly while affected workers receive benefits from the employment agency to cover a large part of the wage gap. Typically, access to the scheme is very restrictive, but in the wake of the Covid-19 pandemic, these restrictions were temporarily lifted. Take-up reached unprecedented levels across a broad range of firms, even in the second half of 2020 and in industries where economic activity had largely resumed. Assuming that firms with similar output in 2020 as in 2019 had similar labor inputs and temporarily underutilized labor in both years, I use their average STW usage intensity in 2020 under eased access to proxy their typical level of temporarily idle fixed labor.

A salient concern is that while surplus labor is an ex-ante concept, I empirically measure the ex-post underutilization of labor due to the Covid-19 shock. Through the lens of the model, surplus labor is the expected level of underutilized labor associated with a chosen level of employment. I empirically map this expectation to averages over months since June 2020, excluding the lockdown period but nevertheless potentially overlapping with firms' labor hoarding in anticipation of economic recovery. To validate that STW usage in 2020 reflects more than just exposure to the Covid-19 shock, I link STW usage to the year-on-year change in revenue in 2020. For firms with revenue declines, STW usage is strongly correlated with exposure to the Covid-19 shock, but this correlation disappears for firms with increased revenue. For firms with revenue growth, there is no reason to expect labor hoarding motives, yet usage levels remain at around 30%.

To further alleviate concerns around using data in 2020 to back out firm-specific levels of temporarily idle fixed labor, I take the following steps. First, I focus on firms with not too-unusual revenue in 2020 compared to 2019 and exclude sectors heavily impacted by the pandemic, such as restaurants. As a result, 62% of the firms in the sample are in the manufacturing sector. Second, I control for the size of the Covid-19 shock by including the revenue drop from 2019 to 2020 as a control variable. I include industry-by-region fixed effects to compare firms within the same industry and region. Third, I replicate the analysis using average STW usage in 2009, when access restrictions were also eased.

I use novel data from the Research Institute of the Federal Employment Agency (IAB) on STW usage, enriched with matched employer-employee data and firm financial information. At the IAB, the administrative employment data is matched with firm financial information and information on firms' financial hedging, which is extracted from annual reports using text analysis. The baseline sample - anonymized and following the strict data protection rules at the IAB - consists of approximately 2,300 firms. It covers roughly a quarter of all German firms that are required to report an income statement and can be reasonably well matched to the confidential employment data at the IAB.

Exchange rate-induced cash flow risk is studied as a second source of cash flow risk, and I follow Adams and Verdelhan (2022) in constructing a measure thereof. In Germany, small to medium-sized firms (the so-called *Mittelstand*) export a lot outside of the eurozone, making exchange rate movements a significant and relevant risk for them. These firms are of particular interest for testing my hypothesis, since it is likely that their financial and labor decisions are taken at the same point within the firm and thus interact. I construct a measure of exchange rate-induced cash flow risk using the accounting variable *FX Transaction Gains/Losses*. Broadly speaking, it captures accumulated revaluations due to movements in exchange rates between invoicing and the settlement of a transaction, net of hedging.

As a first step, I show that surplus labor and exchange rate-induced cash flow volatility are negatively correlated. This finding aligns with the model's prediction that firms with higher cash flow volatility from surplus labor have lower exchange rate-induced cash flow risk. However, this correlation does not imply a causal effect, and concerns around reverse causality are salient: Firms that choose a less risky FX portfolio may have more room to hold surplus labor.

Theoretically motivated by the model, I argue that firm-specific human capital is a suitable instrument for surplus labor, and I proxy for it by using the share of employees with vocational training. In the model, firms differ in their level of firm-specific human capital in the production process. When firms solely maximize expected profits, firm-specific human capital shapes the capacity decision and, subsequently, the decision on how much surplus labor to hold but does not affect the hedging decision. Firm-specific human capital impacts the capacity *and* hedging decision only through the trade-off induced by risk aversion. Empirically, I use the share of employees with vocational training as a proxy for firm-specific human capital. Vocational training schemes in Germany are successful partly because they are firm-based, providing on-the-job training (Dustmann and Schönberg, 2012). Firms have an incentive to invest in building firm-specific knowledge, as they often hire apprentices after training.

Using this instrumental variable approach, I find evidence for a causal effect of surplus labor on exchange rate-induced cash flow volatility. In the first stage, I show that a higher share of employees with vocational training is associated with more surplus labor. The resulting 2SLS estimates are negative and statistically significant, indicating that a one standard deviation increase in surplus labor decreases exchange rate-induced cash flow volatility by 1.5 standard deviations. The coefficients are an order of magnitude larger than the OLS estimates, suggesting that omitted variable bias is a greater concern than reverse causality. I conduct several robustness checks: First, I validate the result using surplus labor based on STW usage in 2009, a period with similarly eased access to the scheme. Second, I find qualitatively similar but smaller effects using an alternative instrument based on the share of employees in shortage occupations, as classified by the Federal Employment Agency. This approach rests on the idea that the effect of firm-specific human capital on surplus labor choices depends on firms' inability to hire specialized employees at short notice.

I leverage hand-collected data on the usage of financial hedging instruments from annual reports to explore how firms manage their FX portfolio. The FX-induced cash flow volatility considered thus far is net of hedging, aligning with the idea that firms manage their FX volatility through different hedge intensities. However, this does not indicate whether firms use FX derivatives or operational hedges. A quarter of firms in the sample use FX derivatives. I find positive but statistically insignificant estimates for the effect of surplus labor on the extensive margin of FX derivatives usage. Additionally, there is no heterogeneity in the effect between FX derivatives users and non-users, suggesting that financial and operational hedges play a role. While I find no effect of surplus labor on mean net FX gains, I find effects on mean FX gains and mean FX losses separately. This finding suggests that firms reduce exposure rather than use natural hedging strategies that offset FX gains with FX losses.

The results presented in this paper offer a new perspective on the link between rigidities in labor markets and financial markets. In a labor market with increasing labor shortages and a trend towards specialization, firms' dependence on firm-specific human capital may grow. This paper suggests that in such cases, firms hold more surplus labor and reduce cash flow volatility from price risks. The channel studied in this paper in principle applies to various other sources of cash flow volatility, e.g. new product launches, and measures to reduce them may differ in their implications for innovativeness or growth. For example, using FX derivatives may not matter for innovation activity, while reducing R&D projects does. Therefore, the findings of this paper highlight the importance of non-financial firms' access to financial hedging tools, such as FX derivatives.

Related Literature. This paper contributes to several strands of the literature. First, it adds to a seminal body of work that recognizes labor as a quasi-fixed factor (Oi, 1962) and attempts to explain why labor productivity is procyclical (see Biddle (2014) for a literature review). The explanation that emerged was that firms have time-varying labor utilization rates because they retain workers through downturns to economize on hiring and firing costs, i.e., they engage in labor hoarding (e.g., Clark (1973), Fair (1985), Rotemberg and Summers (1990), and Bertola and Caballero (1994)). My framework, however, does not focus on business cycle fluctuations or hiring/firing costs explicitly. Instead, it views labor as an ex-ante investment decision under demand and price uncertainty. Therefore, I refer to unused fixed labor as *surplus labor* rather than labor hoarding. This paper innovates by providing a novel firm-level measure for unused fixed labor.

Second, I contribute to the literature on the role of labor in the context of firm-level volatility. Previous studies have shown that rigid wages for incumbent workers (Schoefer, 2021) and the inflexibility of labor expenses (Donangelo et al., 2019; Acabbi and Alati, 2021) amplify fluctuations in firms' cash flows. The model in Bentolila and Bertola (1990) stresses the role of demand uncertainty in shaping firms' employment policies. More recently, Arellano, Bai, and Kehoe (2019) proposed a mechanism explaining why output and labor declined while volatility, observed as dispersion in firm growth, increased during the Great Recession. They argue that demand volatility makes hiring labor a risky endeavor and firms react to increased volatility by reducing their labor inputs. I introduce unused fixed labor and hedging of a second source of unrelated price uncertainty into a model framework similar to their example. Thereby, I connect the core mechanism to the firm-level and corporate financial decisions.

Third, my findings complement the literature on the connection between labor and corporate financial policies. Workers' exposure to unemployment risk (Agrawal and Matsa, 2013), firing costs (Serfling, 2016), and dependence on talent (Baghai, Silva, Thell, and Vig, 2021) have been shown to impact corporate financial policies. Giroud and Mueller (2017) suggest a link between financial constraints and labor hoarding. The study by Ghaly, Anh Dang, and Stathopoulos (2017) is most closely related to this paper, providing evidence that firms with less labor flexibility due to a higher share of skilled workers hold more precautionary cash. Compared to their study, I sharpen the theoretical connection between labor flexibility and corporate financial policies by conceptually introducing surplus labor and by empirically providing measures at the firm level.

Fourth, I add to the literature on the role of firm-specific human capital (Becker (1962) and Lazear (2009)). While the worker's perspective in this context has extensively been studied, the firm's perspective has received less attention with the exception of Jäger and Heining (2022), who exploit exogenous worker exits to study how firms respond to find replacement for firm-specific human capital. I use firm's reliance on workers with firm-based vocational training and those in hard-to-replace occupations to proxy for firms' level of firm-specific human capital. This is consistent with Jäger and Heining (2022) whose findings suggest larger replacement costs in thin labor markets. In this paper, I connect firms' firm-specific human capital to their decisions to hold surplus labor, a link first suggested by Hart and Malley (1996).

3.2 Mechanism in a Stylized Model

In this section, I present a model about the choice of surplus labor and hedging of an unrelated price risk. Section 3.2.1 presents the model setup and formally defines surplus labor. It introduces a dimension of heterogeneity (*firm-specific human capital*) as an explanation for differential choices of surplus labor. Section 3.2.2 solves the model analytically as a function of the level of firm-specific human capital and discusses the key mechanism. Building on the analytical solution, I numerically solve the model for a fixed set of parameters and derive testable predictions in Section 3.2.3.

3.2.1 Setup and Definition of Surplus Labor

Consider a firm that produces a good or service sold at a price normalized to one. It operates in the following two-period environment.

Demand Uncertainty. The firm employs two types of workers: workers with specialized knowledge or training who need to be hired in advance (*fixed labor*) and workers who can be employed flexibly depending on demand (*variable labor*). A firm is characterized by a level of *firm-specific human capital* $\gamma \in [\gamma_{min}, \gamma_{max}]$, fixed by their technology, which determines the relative importance of fixed labor in the production process. Specifically, a firm with γ requires γc fixed labor and $(1-\gamma)c$ variable labor to produce output c.

In $t = 0$, firm γ chooses its fixed labor γc, and consequently *capacity* c, under demand uncertainty. In $t = 1$, the firm receives *incoming orders* $X \sim \mathcal{N}(\mu, \sigma^2)$. The firm serves incoming orders up to its chosen capacity c, producing $min(X, c)$. Expectation μ and variance σ^2 of the normally distributed random variable X with cdf F are known to the firm. There is no capital, and the wage per unit of labor is $w \in [0, 1]$.

Unrelated Price Uncertainty. The firm faces a second type of uncertainty: unrelated price risk, which materializes in $t = 1$. To fix ideas, suppose that the firm exports at a price denominated in foreign currency. Let Y be the value in the firm's home currency, a discrete random variable which is one in expectation and takes three values: for some fixed $a \in (0,1)$, $P[Y = (1-a)] = P[Y = (1+a)] = p$ and $P[Y = 1] = 1-2p$, for $p \in [0,1/2]$. Thus, $Var[Y] = 2pa^2$. X and Y are independent.

The firm has access to a hedging tool against exchange rate fluctuations. In $t = 0$, the firm chooses a *hedge level* $h \in [0, h_{max}]$, $h_{max} \leq a$, and is subsequently not exposed to Y, but to a *partially hedged exchange rate* $\tilde{Y}$ with $P[\tilde{Y} = 1-(a-h)] = P[\tilde{Y} = 1+(a-h)] = p$ and $P[\tilde{Y} = 1] = 1-2p$. Let $K(h)$ be the per-unit costs associated with hedge level h such that no hedging is costless, $K(0) = 0$, and higher levels of hedging are associated with higher costs, $K' > 0$. Specifically, let $K(h) = kh$ with $k \in (0,1)$.

With some fixed obligations, e.g., debt payments, $b \geq 0$ due in $t = 1$, cash flows in $t = 1$ thus are

$$CF_\gamma(c,h) \ := \ \min(X,c)\left[\tilde{Y} - K(h) - (1-\gamma)w\right] - \gamma wc - b. \tag{3.2.1}$$

Risk Aversion. The firm has limited risk-bearing capabilities and operates under the constraint that the probability of default in the bad realization of the exchange rate remains below some threshold α. Hence, a firm γ solves the following optimization problem[1]

$$\max_{c,h} E[CF_\gamma] \quad \text{s.t.} \quad P[CF_\gamma \ < \ 0 | Y \ = \ (1-a)] \ \leq \ \alpha. \tag{3.2.2}$$

Definition. Surplus labor is defined as expected unused fixed labor. That is, for a firm with firm-specific human capital γ that chooses capacity c

$$sl(c,\gamma) \ := \ \gamma\left(c - E[\min(X,c)]\right). \tag{3.2.3}$$

A firm that hired γc fixed labor expects to need $\gamma E[\min(X,c)]$ fixed labor for production. The difference, as in (3.2.3), represents expected unused fixed labor. Therefore, the sum of labor used in production and surplus labor equals the total

1. I derive the model solution analytically for the constraint in (3.2.2). In the numerical simulation, I also consider the alternative constraint $P[CF_\gamma < 0] \leq \alpha$ (see Figure 3.C.2) with little change. The alternative constraint is stricter as it demands that the overall probability of default may not exceed α.

workforce size:

$$\underbrace{\gamma\left(c - E[\min(X,c)]\right)}_{\text{surplus labor}} + \underbrace{E[\min(X,c)]}_{\text{labor used in production}} = \underbrace{E[\min(X,c)](1-\gamma)}_{\text{variable labor}} + \underbrace{c\gamma}_{\text{fixed labor}}.$$

This definition aligns with two intuitive features of surplus labor. First, a firm with no firm-specific human capital ($\gamma = 0$) can flexibly choose employment depending on demand and thus has no surplus labor. Second, a firm entirely dependent on firm-specific human capital ($\gamma = 1$) cannot hire employees based on demand, so surplus labor corresponds to unused capacity.

Discussion. The two-period model is highly stylized and makes a series of assumptions which I briefly discuss in the following.

The model assumes a fixed price in the foreign currency and that demand is independent of the exchange rate, reflecting a short-run perspective. With the price denominated in foreign currency, exchange rate changes do not affect foreign demand. Over longer horizons, however, a correlation between demand and the exchange rate is expected: for an exporter, an appreciation of the home currency (higher Y) should reduce foreign demand.

Fixing prices while allowing demand fluctuations may seem like a stark assumption. However, for an individual firm, the correlation between incoming orders and price is ambiguous. In a non-capacity-constrained industry, competitive pressures dampen demand at higher prices. However, in an industry operating near capacity, a positive demand shock can lead to a non-linear price increase, reflecting the convexity of industry supply curves, as shown theoretically and empirically in Boehm and Pandalai-Nayar (2022). In the current model, if the industry is close to capacity constraints, the comovement between incoming orders and price would intensify the model's mechanism: limited production due to insufficient fixed labor becomes even more costly as the firm forgoes higher prices.

The structure of hedge costs implies that while the firm sets a hedge level in $t = 0$, it can adjust the hedged volume depending on realized demand in $t = 1$. For example, with an FX forward contract, this flexibility might come from a baseline agreement with a relationship bank that allows adjustments to the notional amount. In the case of an operational hedge, this could involve the firm invoicing a portion of its goods in its home currency as a first step. This impacts the firm's bargaining position with customers and may result in a reduced margin on the output, which can be seen as a hedge cost.

3.2.2 Analytical Model Solution

As a starting point, consider the firm's unconstrained problem

$$\max_{c,h} E[CF]. \tag{3.2.4}$$

Lemma 21 (Trade-Off Behind Capacity Choice). *Consider a firm with firm-specific human capital γ. Then, the firm's unconstrained problem (3.2.4) has a unique solution $\left(c^*(\gamma), h^*(\gamma)\right)$ with*

$$h^*(\gamma) = 0 \tag{3.2.5}$$

$$c^*(\gamma) \text{ s.t. } \Big[1-(1-\gamma)w\Big]\Big[1-F\big(c^*(\gamma)\big)\Big] = \gamma w. \tag{3.2.6}$$

Proof. See Section 3.B.1. □

In the absence of the constraint, the firm does not hedge, and the trade-off around capacity choice is intuitive. Hedging has no benefits in expectation since it does not change the expected exchange rate, but is costly. Hence, the firm chooses not to hedge when solely maximizing expected cash flow. Regarding capacity choice, (3.2.6) states that, at the optimum, the expected marginal cost of increasing capacity equals the expected marginal benefit. The marginal cost of increasing capacity is the wage for fixed labor (RHS). The marginal expected benefit (LHS) is the expected price net of variable costs, $(1-(1-\gamma)w)$, times the probability that the firm benefits from the increased capacity, i.e., that incoming orders exceed the current capacity, $(1-F(c^*(\gamma)))$.

Now I turn to the constrained problem (3.2.2), which requires the following set of parameter assumptions.

$$\mu \geq 5\sigma \tag{A1}$$

$$\gamma_{max} < \bar{\gamma}_{max} = (1-w-kh_{max})/w, \gamma_{max} \leq 1 \tag{A2}$$

$$\gamma_{min} > \bar{\gamma}_{min} = (1-w)/(9w) \tag{A3}$$

$$a \leq (4/9)(1-w)-(1/3)kh_{max} \tag{A4}$$

$$k \leq F^{-1}(\alpha)/\sigma(\sqrt{2/\pi}-3/4)/(3+F^{-1}(\alpha)/\sigma(\sqrt{2/\pi}-3/4)) \tag{A5}$$

$$(c-\mu/\mu)\gamma_{max}w < (1-a-w-b/\mu)-(2/5)(1-a-w). \tag{A6}$$

I briefly discuss the parameter assumptions. Assumption A1 limits demand volatility by requiring that the standard deviation of the demand distribution does not exceed one-fifth of its expectation. For a normal distribution, this implies that a drop in demand by 20% relative to the expected level has a likelihood of less than 16% - still a lot by industry standards. Assumptions A2 and A3 restrict attention to optimal capacity choices above the expected level but below such a high level that demand exceeds capacity in less than 10% of the cases. More formally, they restrict capacity choices to the range $[\mu, \mu+(5/4)\sigma]$. Assumptions A4 and A5 restrict the amplitude of exchange rate fluctuations and the per-unit costs for hedging. Assumption A6 demands that the fixed costs relative to the profit margin are bounded from above. Specifically, the first term on the RHS of (A6) represents the profit margin when capacity and demand match expectations. The assumption then ensures that the profit margin can accommodate some additional costs per unit of production resulting from fixed labor choices that differ from expected demand.

Proposition 22 (Solution for Fixed γ). *Suppose assumptions A1 - A6. Consider a firm with firm-specific human capital γ. Then there is a unique solution $\left(c^{opt}(\gamma), h^{opt}(\gamma)\right)$ to (3.2.2).*

a) Either the contraint does not bind, and we get the unconstrained solution from Lemma 21.

b) Or the constraint binds with no hedging, $h^{opt}(\gamma) = 0$.

c) Or the constraint binds in an interior solution with

$$\frac{\partial_c E[CF]}{\partial_c P[CF < 0 | Y = (1-a)]} = \frac{\partial_h E[CF]}{\partial_h P[CF < 0 | Y = (1-a)]}.$$

d) Or the constraint binds with full hedging, $h^{opt}(\gamma) = h_{max}$.

Proof. See Section 3.B.2. □

The interior solution in Proposition 22 states that capacity and hedging are complements. Increasing capacity and decreasing hedging are both profitable in expectation, but they come with costs as they raise the probability of default. Hence, at the optimum, the shadow costs of increasing capacity equal the shadow costs of decreasing hedging. In other words, more capacity and *less* hedging (both profitable in expectation) compete for scarce risk-bearing capability.

Which case occurs depends on the level of γ. Figure 3.C.1 illustrates the model solution for three increasing levels of γ, from Panel (a) to (c). In each panel, points that satisfy the relevant conditions (constraint, unconstrained optimality, Lagrange optimality) are depicted in red, yellow, and blue. As γ increases, the constraint becomes stricter, foreshadowing the next proposition, which characterizes the model solution as a function of γ.

Proposition 23 (Full Model Solution). *Suppose assumptions A1 - A6. Consider a continuum of firms $\gamma \in [\gamma_{min}, \gamma_{max}]$. Then there exist thresholds $\gamma_1 < \gamma_2 < \gamma_3$ such that firms' optimal capacity and hedging choices $(c^{opt}(\gamma), q^{opt}(\gamma))$ are*

$$\begin{cases} \text{the unconstrained optimum a) in Proposition 22} & \text{if } \gamma \leq \gamma_1 \\ \text{the corner solution with no hedging b) in Proposition 22} & \text{if } \gamma_1 < \gamma \leq \gamma_2 \\ \text{the interior optimum c) in Proposition 22} & \text{if } \gamma_2 < \gamma \leq \gamma_3 \\ \text{the corner solution with full hedging d) in Proposition 22} & \text{if } \gamma_3 < \gamma. \end{cases} \tag{3.2.7}$$

Not all four cases need to occur, e.g., if $\gamma_{max} < \gamma_3$.

Proof. See Section 3.B.3. □

The intuition behind the effect of an increase in γ is as follows. An increase in γ means that, all else equal, a larger fraction of the wage bill is borne as fixed rather than variable costs. Higher fixed costs increase the probability of default, making the constraint more binding. Therefore, as γ increases, the solution transitions from unconstrained to constrained.

3.2.3 Empirical Predictions

Equipped with a characterization of the model solution as a function of γ, I numerically solve the model for a fixed set of parameters and derive testable predictions.

As a first step, Panels (a) and (b) of Figure 3.8.3 show that as γ increases, optimal capacity decreases while the optimal choice of fixed labor increases. The intuition behind the decrease in optimal capacity is similar to the key mechanism in Arellano, Bai, and Kehoe (2019), where firms reduce their labor input as demand volatility rises to counteract the increase in default probability associated with the increase in demand volatility. Here, an increase in γ is associated with a higher default probability. Consequently, under a binding constraint, the firm chooses lower capacity. However, as the level of firm-specific human capital increases, the fraction of the workforce that is fixed also rises. In the simulation, this effect outweighs the reduction in capacity, leading to an overall increase in fixed labor.

Next, I study optimal choices of surplus labor and the variance of hedged exchange rates change as a function of γ. Panels C and D of Figure 3.8.3 show that as γ increases, optimal surplus labor also increases while the chosen exchange rate variance decreases. The intuition is simple: At the interior optimum (γ in the range $[\gamma_2, \gamma_3]$ as characterized in Proposition 23), more capacity and less hedging compete for scarce risk-bearing capabilities. At higher levels of γ, the default probability rises, increasing the shadow costs of capacity expansion and leading to higher levels of hedging. In summary, this leads to the following two empirical predictions:

Testable Prediction 1. All else equal, firms with higher γ hold more surplus labor.

Testable Prediction 2. All else equal, firms with higher γ hedge more and experience a lower exchange rate volatility.

Figure 3.8.4 shows comparative statics at the considered parameters along four dimensions. The parameter a in the model maps to the export share: a higher fraction of output invoiced in foreign currency increases the pass-through of exchange rate movements. Combing the two bottom panels of Figure 3.8.3, optimal choices of surplus labor are depicted on the x-axis and the variance of hedged exchange rate on the y-axis in each panel for firms with different γ. The relationship between surplus labor and variance of the exchange rate is prevalent for more firms at a higher export share (higher a in Panel (a)), higher labor share (higher w in Panel (b)), higher demand volatility (higher σ in Panel (c)) and higher hedge costs (higher k in Panel (d)). In each case, the unconstrained optimum with no hedging is feasible for fewer firms.

3.3 Institutional Context for Measuring Surplus Labor

The key empirical challenge for testing the model predictions is a lack of data on surplus labor since the extent to which employees are temporarily idle is private information to firms. I overcome this challenge by using novel data on short-time work. In this section, I provide institutional details on short-time work in Germany and argue how periods with eased access to the scheme lend themselves to the construction of a firm-level measure for surplus labor.

3.3.1 Institutional Setting: Short-Time Work in Germany

Short-time work (STW) is a policy scheme that allows firms to temporarily reduce work hours, with affected workers receiving benefits from the employment agency to replace most of the wage gap. The replacement rate is 60% (67% for employees with children). For example, a childless employee whose hours are reduced by 50% still receives 80% of their regular wage (50% from the employer and 30% from STW benefits). This policy targets firms facing temporary economic difficulties to preserve employment (Cahuc, 2024). Firms initially pay the STW benefits to employees and are later reimbursed by the employment agency.

Firms apply for access (*Anzeige*) to the short-time work scheme and, if approved, can choose monthly whether and to what extent to use STW. Typically, the maximum duration of STW is 12 months. Each month, firms submit detailed documentation (*Abrechnungslisten*) on STW usage per employee to receive benefit reimbursements. Payments from the employment agency are preliminary until the end of the STW period when a final examination (*Abschlussprüfung*) verifies whether eligibility criteria were met throughout the scheme's duration.

Access to short-time work is typically very restrictive, requiring firms to meet several eligibility criteria. First, the economic difficulties must be temporary and beyond the firm's control. Second, the firm must have exhausted all other measures, such as working-time accounts, and justify the necessity of short-time work for each job. Third, the shock must be sizeable, with at least a third of employees facing a reduction in hours of at least 10%.

3.3.2 Short-Time Work with Eased Access

Access restrictions to short-time work have been a policy lever and have been temporarily eased during crises. During the global financial crisis, the requirement that at least one-third of employees be affected was dropped (March 2, 2009, BGBl I. S. 430f), with this change extended until the end of 2011 (October 27, 2010, BGBl I. S. 1420f; December 20, 2011). During the Covid-19 pandemic, only 10% of employees needed to be affected, and working time accounts did not need to be exhausted first (March 13, 2020, BGBl I. S. 493f).

At the beginning of the Covid-19 pandemic, access restrictions to STW were minimal due to the unprecedented number of applications and the need to handle them operationally. An internal anonymized survey by proIAB among eight local employment agency branches in 2020 confirms this. In the first month after March 2020, mentioning "Covid" sufficed for admission to the STW scheme. By the summer of 2020, procedures had become slightly stricter, following a general directive. However, until the second lockdown starting in mid-December 2020, a brief reference to Covid typically sufficed without additional documentation (also see Bossler, Osiander, Schmidtke, and Trappmann (2023) and Bossler, Fitzenberger, Osiander, Schmidtke, and Trappmann (2024) for studies on free-riding on STW). In 2021, pre-pandemic requirements for proof of eligibility were reinstated.

Usage of STW reached unprecedented levels in 2020 – even in the second half of the year and in industries that quickly recovered. Figure 3.8.5 shows the share of firms using STW since 2009 among those with revenue data in 2019 and 2020, and that can be matched to administrative employment data. Usage levels were high following the global financial crisis but reached unprecedented levels in spring 2020, with nearly 40% of firms using STW. The dotted lines indicate periods of eased access (2009-2011 and after March 2020). Strict lockdown measures in Germany ended in May 2020 and were not reimposed until mid-December 2020. Figure 3.8.6 shows monthly industry-wide revenue (blue, LHS scale) and the share of firms using STW (red, RHS scale) for the four largest industries in the sample. Despite revenue returning to pre-crisis levels, STW usage remained high. This indicates that usage levels were not only driven by economic circumstances but also influenced by relaxed access requirements.

A key idea in this paper is to back out a measure of surplus labor from detailed information on STW usage in 2020, a period when a wide range of firms, not just those with severe economic difficulties, used the scheme.

3.4 Data

This section describes the data sources and sample selection. I compile the dataset from four main sources: novel establishment-level information on the monthly usage of short-time work at the Research Institute of the Federal Employment Agency (IAB), matched employer-employee data, firms' financial information from *Dafne*, provided by Creditreform/ Bureau van Dijk (BvD), as well as hand-collected information on firms' usage of financial hedging tools extracted from annual reports using text analysis.

The dataset starts with the universe of German establishments that can be linked to a firm in Dafne (henceforth referred to as the *universe of linked establishments*). The confidential matching procedure used to link establishments to firms is detailed in Antoni, Koller, Laible, and Zimmermann (2018) (see Jäger, Schoefer, and Heining (2021) and Moser, Saidi, Wirth, and Wolter (2022) for recent work with BvD data matched with German administrative data).

For the universe of linked establishments, the data on short-time work contain information on STW usage from 2009 to 2020 (*Statistik der Bundesagentur für Arbeit: Tabellen, Realisierte Kurzarbeit, Nürnberg, Oktober 2021, Daten mit einer Wartezeit von bis zu 5 Monaten (ohne Hochrechnung)*). For each STW episode, I have the number of employees in STW, the shortfall in wages (in buckets), and the shortfall in hours in worker equivalents (in buckets). I transform the data into a monthly panel and merge it with the annual panel of German establishments (*Establishment History Panel*, Ganzer, Schmucker, Stegmaier, and Wolter (2022)). This merge allows me to ensure basic consistency (details in Appendix 3.A.1) and add location and industry information. I then aggregate the establishment-level data to the firm level, using the information from the largest establishment for location and industry.

For the universe of linked establishments, I have employment histories since 2008 for all individuals employed at these establishments at any point since then (*excerpt of Integrierte Erwerbsbiografien (IEB)*, IAB). Using standard procedures described in Dauth and Eppelsheimer (2020), I take monthly snapshots to (a) obtain monthly employment information and (b) calculate the share of employees per occupation per firm as of December 2019. These occupation shares are later used to construct an instrument similar to a shift-share instrument based on occupation characteristics. While I could proceed analogously to calculate the share of employees with vocational training per firm, I take a shortcut by using the existing data on the number of employees with vocational training (as of June 30) per establishment available in the Establishment History Panel.

The dataset on firms' financials contains annual information from firms' balance sheets and income statements at the unconsolidated level. I enhance this data with information on FX derivative hedging, extracted from manually downloaded annual reports using text analysis. Details on the text analysis and classification procedure are provided in Appendix 3.A.4.

I use the combined data to select the following sample of firms (details in Appendix 3.A.2). The resulting number of firms after each step is provided in parentheses. The starting point includes firms that report an income statement, specifically revenues, at the unconsolidated level in 2019 and 2020 (approximately 21,000 firms).[2] I exclude firms that are likely just holdings or fail basic data consistency requirements similar to Kalemli-Ozcan, Sorensen, Villegas-Sanchez, Volosovych, and Yesiltas (2015), reducing the sample to 16,300 firms. These firms are then matched with the confidential data at the IAB, with a successful match rate of 71%. I restrict attention to firms with at most 20 establishments (11,500 firms) and further to those firms where employment information from annual reports roughly coincides with the aggregated establishment-level employment information at the IAB (within a tolerance of -20% to +100%, 10,000 firms). This ensures that firms in the sample primarily have employment in Germany. Following standard data cleaning methodology from the literature, I exclude regulated utilities (sections D and E of the Classification of Economic Activities (WZ 2008)), financial firms (section K), and firms in public service (section O), resulting in 9,100 firms. Of these, 4,200 firms report exchange rate transaction income in at least two years between 2010 and 2019. As will be discussed in the next section, I am not interested in firms for which 2020 was a very exceptional year in terms of revenue development and thus exclude firms with a relative change in revenue from 2019 to 2020 below −20% or above 20% (3,000 firms). Finally, I demand that firms have non-missing information on their export share.

Table 3.8.1 reports the summary statistics of the final dataset (*baseline sample*) consisting of a cross-section of 2,352 firms. It contains firms' financial information in 2019, information on whether the firm exports and uses financial hedging tools in 2019 as well as employment information (employment, share of employees per occupation, share of employees with vocational training) in 2019. I explain how I construct a measure for surplus and two measures of exchange rate-induced cash flow volatility in the next section.

Additional Data. To gauge service provision around FX derivatives, I use information on firms' relationship banks and hand-collected information on whether banks continued selling FX derivatives in-house. For 95% of firms, I have information on their relationship banks provided by Creditreform, which I match to banks in SNL Fundamentals banks by name. Based on banks' annual reports from 2010 until 2019, I extract annual information on whether banks had outstanding FX derivatives in their client business and I identify banks that continued offering

2. The number is not larger since firms that exceed not more than one of three size thresholds (12 mio revenues, 6 mio assets, and 50 employees) need not publish an income statement.

FX derivatives in-house under the new regulation on derivatives markets (EMIR). Details on the data construction are provided in Appendix 3.A.5.

For a robustness check, I use novel data on the usage of short-time work at the individual level (*Personen in Kurzarbeit (PRS KUG), Betaversion*, IAB), which contains for a subset of employees information on benefit receipt at the individual level.

3.5 Measuring Surplus Labor and FX-Induced Cash Flow Risk

This section details the construction of empirical measures for surplus labor and exchange rate-induced cash flow volatility. In principle, the hypothesized mechanism linking cash flow risks from surplus labor to other sources of cash flow risk applies to any unrelated cash flow risk. I focus on exchange rate risk for two reasons. First, relevant data is available from firms' annual reports. Second, in Germany, many small and medium-sized firms export extensively, making exchange rate fluctuations a significant concern for them.

3.5.1 Empirical Construction of Surplus Labor

I map the notion of surplus labor as defined in the model to the data for periods of eased access to STW. I focus on 2020, but in a robustness exercise, replicate the analysis for 2009.

The building block for an empirical measure of surplus is information at the firm-month level on the intensity of short-time work usage. I define *Unused Fixed Labor* of firm i in month m as

$$\text{Unused Fixed Labor}_{im} := \frac{\text{Short-Time Work in Employee Equivalents}_{im}}{\text{Number of Employees}_{im}}. \quad (3.5.1)$$

Short-Time Work in Employee Equivalents$_{im}$ is obtained by multiplying the number of short-time workers and the relative wage bill gap among short-time workers (for details on the relative wage bill gap see Appendix 3.A.3).

In the model, surplus labor is defined from an ex-ante perspective, which I empirically map to averages. Firms form expectations about the level of unused fixed labor associated with their choice on the level of employment, knowing the distribution of incoming orders but not their actual values. One can think of the known distribution as stable annual levels of incoming orders but uncertain monthly levels. Empirically, I map the expectation to averages of realized underutilization over some time window revealed through STW usage.

Building on this, I define surplus labor for firm i as

$$\text{Surplus Labor}_i := mean\left(\text{Unused Fixed Labor}_{im}, m \in \mathscr{M}\right), \quad (3.5.2)$$

with $\mathscr{M}$ a set of months with eased access to STW. For the measure based on 2020, I take June until December as time window to exclude the first lockdown period (March until May). For the measure based on 2009, I take the average over the full year.

I interpret the average level of temporarily unused labor in the second half of 2020 after controlling for the size of the Covid-19 shock, as firm-specific level of surplus labor. The starting point for such an interpretation is the assumption that firms with similar output in 2020 and 2019 had similar overall labor inputs and levels of temporarily underutilized labor in both years. Due to eased access, however, these firms used STW for temporarily unused labor in 2020 but not in 2019.

To validate that STW usage in 2020 reflects more than just the exposure to the Covid-19 shock, Panel (a) of Figure 3.8.7 shows a binned scatterplot of STW usage against the relative change in revenue from 2019 to 2020. While STW usage is strongly associated with the year-to-year revenue change, this link disappears for firms with increased revenue. Usage levels, however, are stable around 30% even for these firms. A regression analog of this result is shown in Table 3.C.1 in the Appendix. Panel (b) displays a similar pattern for the intensity of STW usage (defined as mean since June 2020). To the contrary, a replication exercise of the same figure (see Figure 3.C.3) pooling information for the years 2012-2019 shows that there is barely any usage in normal times when regular access restriction are in place.

To alleviate further concerns, I restrict attention to firms with not too unusual revenue in 2020. Specifically, I focus on firms with a relative change in revenue from 2019 to 2020 in the range of $[-20\%, 20\%]$. A priori, requiring available data on exchange rate transaction income already shifts focus away from sectors that were heavily hit by the pandemic like gastronomy towards sectors with international exposure such as manufacturing, trade as well as the information and communication sector. Figure 3.C.6 shows the revenue distribution in the sample of firms with available information on FX transaction income with the part of the distribution included in the sample colored in green.

Throughout the rest of the paper, I control for the relative change in revenue between 2019 and 2020 to proxy for the drop in output due to the Covid-19 shock. Revenue changes are a composite effect of price and quantity changes. If the year-on-year revenue changes in 2020 primarily reflect price changes, they would be a poor proxy for exposure to the Covid-19 shock. To test this, I examine the correlation between year-on-year changes in revenue and material expenses, using the latter as a proxy for input quantities, assuming that broad-based price changes affecting all prices did not occur until 2021. I find a positive correlation between year-on-year revenue changes and changes in material expenses (correlation coefficient 0.64), supporting the use of revenue change as a proxy for the drop in output.

3.5.2 Exchange Rate-Induced Cash Flow Risk

I follow Adams and Verdelhan (2022) in using the accounting variables FX Losses and FX Gains from firms' annual reports to construct a measure for FX-induced cashflow risk. Firms are required to report these gains and losses in the appendices of their annual reports.

The following example illustrates what the variables FX Losses and FX Gains capture. Consider a German firm that exports to the US. It invoices and ships the good on March 1 at a price of $1 mil, with payment due three months later on June 1. At invoicing, $1 is worth 1.05 EUR, so the firm records 1/1.05 mil EUR on March 1. Suppose the exchange rate moved to 1.15 EUR per USD by the settlement date, such that, at the time of settlement, the firm receives 1/1.15 mil EUR. The change in value is recorded as an FX loss of (1/1.15 - 1/1.05) mil EUR = 80,000 EUR. Throughout the year, the firm conducts multiple such transactions and collects revaluations due to movements in exchange rates in the variables FX losses and FX gains.[3]

From a data standpoint, this paper innovates compared to Adams and Verdelhan (2022) by showing that the accounting variable FX transaction income can also be used to study private firms. While they consider public firms, only 3% (81 firms) of firms in my sample are publicly traded. In Germany, reporting FX gains and FX losses is mandatory for all firms required to publish an income statement.

I consider net FX gains scaled by revenue in year t,

$$\text{Net FX Gains}_t = (\text{FX Gains}_t - \text{FX Losses}_t) \,/\, \text{Revenue}_t. \tag{3.5.3}$$

Figures 3.C.5 and 3.8.8 illustrate the magnitude of the variable both cross-sectionally and over time. Figure 3.C.5 shows the distribution of Net FX Gains in 2019, revealing that more than a quarter of firms have net FX gains exceeding five basis points of revenue. Figure 3.8.8 focuses on FX net losses (negative FX Net Gains) per firm over time, showing the distribution of the largest FX net loss relative to revenue between 2010 and 2019. Due to data protection, bins with fewer than 20 firms are not shown, but 13% of firms have net FX losses exceeding 1% of their annual revenue at some point. For a firm with a return on sales of 5%, this loss represents more than one-fifth of their annual profits.

The accounting variables on FX Transaction Income (FX Gains and FX Losses) serve as a proxy for FX risks but likely underestimate the true exchange rate risks.

3. In practice, a German firm exporting to the US usually has a US subsidiary, but FX gains and losses typically still accrue to the parent company if the subsidiary only distributes, rather than produces. In this case, the subsidiary buys goods from the parent company at arm's length prices denominated in USD, transferring the FX risk to the parent company. Given that the firms in the sample have most of their employees in Germany, it is reasonable to assume their foreign subsidiaries are only involved in distribution, not production.

This variable primarily captures movements in the exchange rate between invoicing and payment. Invoicing usually occurs when the good is shipped, thus, price movements between the point of sale and invoicing are ignored. Likewise, long-term contracts, such as those for large machinery, often involve interim payments, meaning that a significant portion of the payment is already completed when the good is shipped.

FX transaction income may also be confounded by FX risks from investing or borrowing in foreign currency, as discussed in Adams and Verdelhan (2022). However, foreign currency-denominated debt is unlikely to be a significant confounding factor in this context for two reasons. First, the firms in the sample are relatively small and privately held, and given Europe's bank-based financing structure, they are unlikely to finance through the bond market. Second, according to BIS location banking statistics, only 1.5% of total bank claims or liabilities in Germany are denominated in currencies other than EUR.

In the presence of financial hedging, the variable *Net FX Gains* captures the effect net of hedging. To illustrate, suppose the firm in the previous example purchases a forward contract with a notional of $1 mil at a forward rate equal to the spot rate on March 1. In this case, the firm is perfectly hedged, and no revaluation effect is expected. When the forward contract matures on June 1, it has the same value as the spot rate. Hence, the change in value of the hedged item, (1/1.05-1/1.15) mil EUR, is exactly offset by the change in value of the hedge, (1/1.15-1/1.05) mil EUR. Under the German Commercial Code, a firm using hedge accounting (specifically fair value hedges) can choose between two accounting methods. With the freezing method (*Einfrierungsmethode*), the hedge eliminates the FX transaction risk. With the pass-through method (*Durchbuchungsmethode*), the FX loss from the value change of the hedged item is offset by an FX gain of the same amount from the value change of the hedge. Although these methods imply different interpretations for the FX gains and FX losses variables separately, both result in the same value (net of hedging) for FX net gains.[4]

Building on this, I construct two measures of unhedged exchange rate-induced cash flow volatility. I define for firm i

$$\begin{aligned}\text{sd net gains}_i &= sd\left\{\text{FX Net Gains/Revenue}_{2010},\ldots,\text{FX Net Gains/Revenue}_{2019}\right\}\cdot 100\\ \text{max net loss}_i &= -min\left\{\text{FX Net Gains/Revenue}_{2010},\ldots,\text{FX Net Gains/Revenue}_{2019}\right\}\cdot 100\end{aligned}$$

Both measures are winsorized at the 1% and 99% level to remove outliers. The first measure provides an intuitive starting point for measuring FX-induced cash

4. For further discussion, including on cash flow hedges and the case where the forward rate differs from the spot rate at the point of sale, see Adams and Verdelhan (2022). In the latter case, there is an economic loss equal to the difference between the spot rate at the point of sale and the forward rate.

flow volatility. The largest loss induced by net FX positions, as captured by the second measure, aligns more closely with heightened default risk from exchange rate movements – the ultimate concern for risk-averse firms. Thus, the second measure more closely maps to the constraint in the model.

3.6 Negative Correlation

Equipped with empirical measures for surplus labor and exchange rate-induced cash flow volatility, I empirically test the model prediction that these two variables are negatively correlated.

As a starting point, Figure 3.8.9 shows a binned scatterplot of exchange rate-induced cash flow volatility against surplus labor, controlling for size, industry and the exposure to the Covid-19 shock. The binned scatterplot supports the model prediction that firms with more surplus labor have less exchange rate-induced cash flow risk. I test the correlation more formally by running the following regression:

$$\text{FX-Induced CF Volatility}_i \;=\; \beta\,\text{Surplus Labor}_i + \theta'\mathbf{X}_i + \varepsilon_i, \tag{R1}$$

where *Surplus Labor*$_i$ and *FX-Induced CF Volatility*$_i$ are defined as in the previous section, and $\mathbf{X}_i$ is a vector of control variables and fixed effect dummies.

β correctly captures a negative correlation between surplus labor and FX-induced cash flow risk if there are no unobserved firm characteristics that affect the ranking of firms in terms of surplus labor differentially from the ranking of firms in terms of FX-induced cash flow volatility. Since the regression uses cross-sectional data, firm fixed effects are not feasible. Instead, I control for firm characteristics as well as possible by comparing firms in the same industry (section based on the Classification of Economic Activities (WZ 2008)) in the same region (*Bundesland*) and including controls for size (*log(assets)*), export activity (*export share*) and exposure to the Covid-19 shock (*revenue change 19-20*).

Table 3.8.2 shows the estimation results for (R1). The baseline specifications in columns 3 and 4 confirm a negative correlation between surplus labor and FX-induced CF volatility for both measures. The point estimate for *sd net gains* suggests a small magnitude, however. It suggests that when comparing one firm to a similar firm with a one standard deviation (5.3 pp) higher level of surplus labor, the latter exhibits exchange rate-induced cash flow volatility that is 1/25 standard deviations lower. The coefficients' magnitudes are consistent across both measures.

In a robustness check, I control for further firm characteristics. Columns 5 and 6 of Table 3.8.2 include the cash-to-asset ratio as a proxy for liquidity preferences, leaving the coefficient of interest unchanged. Another concern is that more productive firms are less likely to use STW in general but more likely to export outside of Europe (with transaction less likely invoiced in Euro), affecting both variables of interest. Since the export share does not differentiate between exports within Europe and overseas, it may not be a sufficient control. Columns 7 and 8 of Table 3.8.2 control for value added per employee in 2019 as a proxy for productivity for firms with available data. The coefficient increases by 50% compared to the baseline specification but does not change the overall picture.

To address concerns about disentangling ex-ante surplus labor choices from labor hoarding motives in observed STW usage, even when controlling for exposure to the Covid-19 shock, I rerun the regressions using only firms that did not experience a substantial drop in revenue in 2020. Specifically, I focus on firms with a year-on-year revenue change in 2020 within the range of $[-5\%, 20\%]$, reducing the sample by half. The signs of all coefficients remain unchanged. The estimates become statistically insignificant unless value added per employee, a proxy for productivity, is included as a control. Productivity may be a particularly important confounding factor among these firms, as their potential STW use occurred despite revenue growth during the Covid-19 shock. When controlling for productivity, the coefficients are similar in significance and magnitude to those in the full sample.

Guided by the comparative statics of the model, I investigate heterogeneity across three dimensions. Columns 1 and 2 of Table 3.8.3 show that the correlation is driven by firms with a high (above-median) export share. In columns 3 and 4, I sort granular industries by their average labor share and validate that the effect comes from firms with a high (above-median) labor share. For the subset of manufacturing firms, I draw on data per granular industry on the order volatility from the Federal Statistical Office of Germany (tables 42151-0002) to proxy for demand volatility. Specifically, I calculate the volatility of a value index of monthly incoming orders between 2010 and 2020. The results, estimated on less than half the sample, are inconclusive when comparing firms with high (above-median) and low order volatility.

Discussion of Potential Biases. The negative OLS coefficient may not capture a causal effect of surplus labor on exchange rate-induced cash flow volatility due to different sources of endogeneity. One concern is that the design suffers from reserve causality. Firms' decision on how much exchange rate-induced cash flow volatility to assume could leave more or less room for surplus labor. This concern is heightened because the measures for exchange rate-induced cash flow volatility are based on earlier years (2010-2019) than the measure for surplus labor.

Another concern is omitted variable bias: any unobserved firm characteristic that increases a firms' propensity to hold surplus labor and its activity on international markets may create an omitted variable bias. A potential omitted variable is managerial risk aversion or managerial sophistication. For example, firms with more sophisticated risk management systems may have job positions dedicated to so-called staff level optimization as well as divisions for financial FX hedging.

Staff level optimization could involve assigning employees to skillsets within divisions, allowing them to rotate between divisions within their skillset as needed to minimize periods of idleness.

While reverse causality would result in OLS estimates that overstate the effect, unobserved differences in managerial risk aversion suggest a negative bias of the OLS estimates. More risk-averse firms or those with more sophisticated risk management practices may choose lower levels of surplus labor as well as lower levels of exchange rate-induced cash flow risk.

3.7 Firms' Response to Surplus Labor

This section presents evidence of a causal negative effect of increased surplus labor on exchange rate-induced cash flow volatility, using an instrumental variable approach. Guided by the model, I use a proxy for firm-specific human capital as an instrument for surplus labor. The results suggest that firm-specific human capital influences firms' hedging decisions. Leveraging hand-collected data on the use of FX derivatives, I further explore the mechanism.

3.7.1 Identification Strategy Through Firm-Specific Human Capital

The model suggests using firm-specific human capital as instrument for surplus labor. In the model, firms require two complementary types of workers for production: fixed and variable labor, with firms differing in their dependence on fixed labor. As shown in Lemma 21, when firms only maximize expected profits, the level of firm-specific human capital shapes their decision on how much fixed labor and subsequently surplus labor to hold, but has no bearing on the decision of how much exchange rate risk to assume. The firm characteristic impacts surplus labor *and* hedging only when the constraint binds. Subsequently, from a theoretical perspective, while firm-specific human capital drives the labor decision, it influences the hedging decision only through the trade-off induced by risk aversion.

I use the firm-level share of employees with vocational training as a proxy for firm-specific human capital. Employees with vocational training have completed firm-based on-the-job training as part of apprenticeship schemes, which are supplemented by classes at vocational schools once or twice a week. An apprenticeship typically lasts two to three years and concludes with a final examination.

During firm-based vocational training, firms have an incentive to invest in the development of firm-specific knowledge and skills. Dustmann and Schönberg (2012) argue that Germany's vocational training is successful because it occurs within firms, not just at vocational schools. Firms know the skills necessary in the workplace and invest in them, motivated by the likelihood of hiring their apprentices post-training. Survey evidence shows that firms are willing to offer employment contracts to apprentices in about 90% of cases (Mohr, 2015).

I estimate the following 2SLS specification:

$$\text{Surplus Labor}_i = \alpha \text{ Share Vocational Training}_i + \theta' \mathbf{X}_i + \eta_i \qquad \text{(R2)}$$

$$\text{FX-Induced CF Volatility}_i = \beta \widehat{\text{Surplus Labor}}_i + \theta' \mathbf{X}_i + \varepsilon_i, \qquad \text{(R3)}$$

with $\mathbf{X}_i$ a vector of control variables and fixed effect dummies.

Instrument validity hinges on the standard relevance and exclusion restriction. Regarding relevance, I expect that a higher share of employees with vocational training is associated with a higher level of surplus labor. In support of this, the bottom of Table 3.8.5 reports the estimated coefficient of α, which is positive as hypothesized. The magnitude of the coefficient implies that a 100 basis point increase in the share of employees with vocational training raises the fraction of the workforce fully idle during low demand by 26 basis points. The resulting first stage F statistic (Kleibergen-Paap Wald statistic, see Andrews, Sock, and Sun (2023)) is 15.150, and passes the Stock and Yogo (2005) threshold for weak instruments.

The exclusion restriction demands η_i and ε_i are uncorrelated. In other words, it demands that after conditioning on controls the instrument is uncorrelated with unobserved variables that are relevant for the relationship of interest in (R1). The exclusion restriction would be violated, for example, if firms' exposure to global markets, and thus their cash flow volatility due to exchange rate movements, shapes their technology and, as a result, their demand for employees with vocational training. To alleviate this concern, as before, I compare firms in the same industry and region, and control for size, export share and the exposure to the Covid-19 shock.

I provide two additional pieces in support of the exclusion restriction. Table 3.8.4 shows that firms with an above-median share of employees with vocational training are indistinguishable in means by size and export activity to firms with a below-median share. While this is not the case for all characteristics, in an alternative specification including industry-by-region fixed effects (Table 3.C.2), I find that the share of employees with vocational training does neither significantly correlate with the return on assets nor with the propensity to export to destinations outside Europe. The share with vocational training still correlates with the cash-to-asset ratio as well value added per employee and I control for these characteristics in a robustness.

Second, for the subset of firms for which I have annual information on foreign revenue, I exploit the panel dimension of the information on employees with vocational training. Table 3.C.3 shows that the share with vocational training is not correlated with the export share when including firm and time fixed effects. Importantly, the inclusion of firm fixed effects explains almost all variation in the share with vocational training, suggesting that the latter is stable over time. This supports the idea that firms fixed technology drives their employment composition across employees with or without vocational training.

3.7.2 Impact of More Surplus Labor

Panel (a) of Table 3.8.5 presents regression estimates of the 2SLS specification in (R2) and (R3). The coefficients for both measures of FX-induced cash flow volatility are statistically significant, supporting the hypothesis. Columns 5 and 6 report the reduced-form estimates and the strong statistical significance corroborates the existence of an effect.

The size of the 2SLS coefficient estimates suggests that a one standard-deviation increase in surplus labor reduces FX-induced cash flow volatility by 1.5 standard deviations ($= (18.432 \times 0.05)/0.62$). Compared to the OLS estimates, the IV estimates thus increase by an order of magnitude. Following the practice suggested by Jiang (2017), I discuss the result and robustness checks in the following.

The relative magnitudes of the OLS and IV estimates suggest that an omitted variable bias outweighs reverse causality concerns, and I support this by replicating the analysis for 2009 where surplus labor is measured before FX-induced cash flow volatility. In the replication, surplus labor is defined as mean STW usage in 2009, a period when access restrictions to the STW scheme were substantially eased. Analogous to before, I control for the exposure to the Global Financial Crisis via the year-on-year change in revenue in 2009, size and export share. Table 3.8.6 shows negative OLS estimates of a similar magnitude to the original analysis, consistent with previous findings. Although an analogous IV design using the share of employees with vocational training in 2008 results in a weak first-stage F statistic (bottom of Table 3.8.6), the weak-instrument-robust Anderson-Rubin Chi-Squared test still passes at the 1% level.

I report the partial R^2 of the excluded instrumental variables for explaining variation in surplus labor. The partial R^2 of the first stage is 0.05, which is not very large. Therefore, if the instrument has a direct effect on the outcome, the IV estimate would combine the true effect and an exaggerated version of the direct effect. In the robustness exercise in Panel (b) of Table 3.8.5, the IV coefficient indeed decreases for specifications with a larger partial R^2 in the first stage (columns 7 and 8).

The inclusion of additional control variables in Panel (b) of Table 3.8.5, however, leaves the magnitude of the IV estimate largely unchanged. In columns 2 and 3, I include cash-to-assets as proxy for liquidity preferences as control, while in columns 7 and 8 I include value added per employee as proxy for productivity as control. In all specifications, the first-stage F statistic exceeds 10, rejecting the hypothesis that the instrument is week.

3.7.3 Alternative Instrument

The effect of firm-specific human capital on surplus labor theoretically hinges on firms inability to hire employees with specialized skills at short notice, which motivates using the share of employees in shortage occupations as alternative instrument, similar to a shift-share instrument. Suppose firms' needs for employees across occupations as determined by their fixed technologies. Also suppose that occupations differ in how long it takes to find, hire, and train a suitable candidate for specialized tasks. If this time exceeds the forecast horizon for demand, the firm must hire employees in that occupation in advance. Empirically, I calculate the share of employees in firm i in occupation j as of December 2019 ($\textit{Share Occupation}_{ij}$) using the most granular occupation information (5-digit occupations). The Federal Employment Agency in Germany analyses and classifies whether occupations are shortage occupations and I use their classification from December 2019.[5] Then I define $\textit{Shortage Share}_i$ as

$$\text{Shortage Share}_i = \sum_j \text{Share Occupation}_{ij} \cdot \mathbf{1}(\text{Occupation j is Shortage Occupation}),$$

where $\mathbf{1}(\cdot)$ is an indicator function that equals 1 if occupation j is classified as a shortage occupation.

The definition of shortage occupations by the Federal Employment Agency (version until 2019) is based on three indicators. First, the vacancy duration in this occupation is at least 30% above the average. Second, the ratio of unemployed to job postings in this occupation is smaller than 2:1 for experts and specialists, and 4:1 for experts. Third, the unemployment rate in this occupation is lower than 3%. If all three criteria are met and the classification passes the evaluation of an expert, an occupation is classified as a shortage occupation. This definition aims to identify structural problems in filling positions within an occupation, rather than hiring challenges faced by individual firms, such as unattractive working conditions or limited mobility among the unemployed. I enrich the information on shortage occupation at the federal level with information on regional (*Bundesland*) shortages.

Similar to the previous instrument, the relevance condition requires that a higher share of employees in shortage occupations induces firms to hold more surplus labor, as they cannot hire in these occupations on demand. The bottom of Panel (a) in Table 3.8.7 reports the coefficients from the first-stage regression of surplus labor on the shortage share, showing a positive effect. The associated F statistic for the first-stage regression in the baseline specification is 12.290 and thus passes the threshold for weak instruments.

5. https://statistik.arbeitsagentur.de/SiteGlobals/Forms/Suche/Einzelheftsuche_Formular.html?nn=20626&topic_f=fk-engpassanalyse

Regarding the exclusion restriction, firms with an above-mean shortage share have similar means for size measures like assets and revenue compared to firms with a below-median shortage share (Table 3.C.4). The two groups do not differ in characteristics like leverage and return on assets, but in others (cash/assets, value added per employee). I control for the cash-to-asset ratio and value added per employee in a robustness check. Firms with a high shortage share also have higher average export shares. However, a separate analysis (Table 3.C.5) confirms that when including industry-by-region fixed effects, the shortage share is not correlated with the propensity to export outside of Europe.

Panel (a) of Table 3.8.7 shows the results when using the shortage share as instrument for surplus labor, corroborating the existence of an effect of surplus labor on FX-induced cash flow volatility. The magnitude of the 2SLS estimates is reduced to one-third. This reduction in magnitude remains robust after including additional controls (Panel B).

In a robustness, I rerun the analysis for the subsample of firms with available data on export destinations that have export destinations outside of Europe. Firms that export only with the Euro as invoicing currency should not experience currency fluctuations for their exports. I classify firms based on text information about their export destinations. Following Gopinath and Itskhoki (2022), I assume that exports within Europe (excluding the UK) are denominated in Euros, while the dollar functions as dominant currency elsewhere. In Table 3.C.7, I restrict the analysis to firms that name at least one export destination outside of Europe and confirm the prior results for both instruments.

3.7.4 Mechanism

The variables considered so far are net of hedging, and I explore how firms adjust their hedging intensity. Firms in principle have several options to manage their FX risks: purchasing FX derivatives, replicating FX derivatives through foreign currency borrowing or investing, using natural hedges, managing invoicing currencies, or altering their export destinations. For firms using FX derivatives, hedge intensity corresponds to hedge coverage levels. For firms not using FX derivatives, adjusting the hedge intensity may involve decisions such as how much to invoice in the home currency despite reduced margins or changing the mix of export destinations.

3.7.4.1 FX Derivatives Usage

I use hand-collected data on financial FX hedging instruments, extracted from appendices of firms' annual reports through text analysis. Table 3.8.8 presents summary statistics for firms using FX derivatives in 2019 compared to those that do not. A quarter of firms in the sample use FX derivatives. Derivatives users are larger by all size measures, with the median derivatives user being twice as large as the median non-user. Non-users hold more liquidity and are slightly more profitable, and their average export share is 10 percentage points smaller.

Table 3.8.9 investigates the link between hedging with derivatives and FX-induced cash flow volatility. Export share is a major driver of FX-induced cash flow volatility (Panel (a)), but less so for derivatives users, as shown by the negative interacted coefficient in column 1. The sample substantially shrinks in size when including the import share due to data availability (column 2). The result persists when focusing on firms with available export destination data that export outside Europe (columns 6 and 7). Interestingly, the link between FX-induced cash flow volatility and derivatives usage is only present for the export share and disappears when considering the import share. Panel (b) of Table 3.8.9 presents the results with imports and exports reversed. Notably, derivatives users and non-users do not differ in how the import share correlates with FX-induced cash flow volatility. Overall, these findings indicate that financial hedging with FX derivatives is primarily targeted towards exports rather than imports.

The starting point for understanding the role of financial hedging in the trade-off between labor and financial risks is to examine whether the choice of surplus labor is directly linked to FX derivative usage. I estimate regressions of the form

$$\mathbf{1}(\text{Uses FX Derivatives in 2019})_i \;=\; \beta\,\text{Surplus Labor}_i + \theta'\mathbf{X}_i + \varepsilon_i. \qquad \text{(R4)}$$

Table 3.8.10 presents results for OLS, 2SLS using the share of employees with vocational training as instrument and the reduced form. The point estimates are positive but statistically insignificant.

While I find no effect of surplus labor on the decision to use FX derivatives, there could be differences in the trade-off between derivatives users and non-users. Due to the instrument's weakness for interactions, I focus on the OLS. However, columns 1 and 2 of Table 3.8.11 show no such heterogeneity, suggesting that financial and operational hedges play a role.

3.7.4.2 Service Provision Around FX Hedging via Relationship Banks

I study the role of service provision around FX hedging through firms' relationship banks by examining banks' offerings of FX derivatives. Following the introduction of the European Market Infrastructure Regulation (EMIR) in 2014, the costs for using FX derivatives increased due to the need for costly identification numbers and additional back-office capacity for new reporting requirements. Firms could delegate these reporting requirements to banks, which also faced increased infrastructure demands. For many local banks (savings and commercial banks), FX derivatives were previously the most important types of derivatives sold to customers and a core part of their business. I hand-collected information from around annual reports since 2010 for approximately 800 banks on outstanding FX derivatives for their clients and classify a bank as having stopped offering FX derivatives if the outstanding amounts dropped to zero and remained at zero (for details, see Appendix 3.A.5). Figure 3.C.7 shows a substantial consolidation in the number of local banks offering FX derivatives to their clients. The following analysis assumes that all banks reevaluated their derivative offerings in response to the regulatory requirements, and for those that continued offering these services, FX was a focus of their business.

In columns 3 and 4 of Table 3.8.11, I explore heterogeneity between firms that are connected to a local that continued offering FX derivatives in-house proxying for better service quality around FX hedging. I find weak evidence that the effect of interest is less pronounced for firms connected to local banks with presumably better service provision, controlling for whether the firm has any local bank among its relationship banks. The proxy is likely confounded by banks that started commissioned trading or delegated their customers to other banks within the banking groups (mirrored in Figure 3.C.8 where there is no drop in outstanding amounts in 2014 per banking group).

3.7.4.3 Natural Hedging Through Matching Exposures

To shed light on the role of operational hedging, I examine gross FX positions. As a starting point, I consider mean FX net gains in column 1 of Table 3.8.12 and find no effect, consistent with the notion that the volatility, not the levels, of net FX positions matter. However, I do find an effect of surplus labor on both mean FX losses and mean FX gains. This contradicts neutral hedging strategies that offset FX losses with FX gains. If firms primarily used such strategies, we would expect no effect on gross positions. The result thus suggests that firms primarily directly reduce exposure rather than offsetting it.

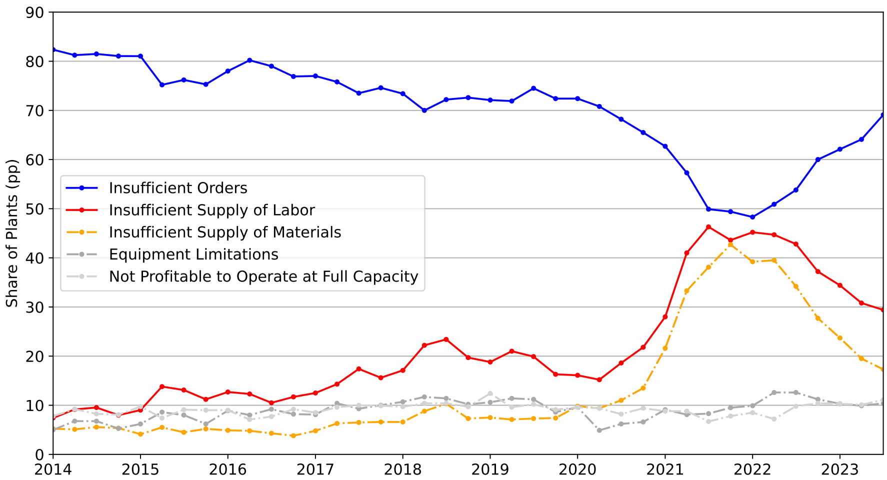
3.8 Conclusion

This paper establishes a link between firms' choice of expected temporarily unused labor (surplus labor) and their hedging decisions for other cash flow risks. Using novel data on short-time work in Germany, matched with employer-employee data and firm financials, I construct a firm-level measure for temporarily underutilized labor. I show a negative correlation between this measure and firms' exchange rate-induced cash flow volatility, the latter presenting a significant source of risk for exporting firms. This finding is consistent with firms trading off cash flow volatility from fixed labor against other cash flow volatility sources. I formalize this trade-off in a model that introduces firm-specific human capital as a driver of surplus labor choices. By instrumenting surplus labor with proxies for firm-specific human capital, I find empirical evidence of a causal effect of surplus labor on firms' willingness to assume exchange rate-induced cash flow risks.

The results indicate that the level of firm-specific human capital, through surplus labor choices, influences firms' hedging decisions for other cash flow risks. The framework presented in this paper may serve as starting point for discussions about firms' willingness to assume risks, particularly if their dependence on firm-specific human capital increased due to labor market shortages and higher levels of specialization.

Figures

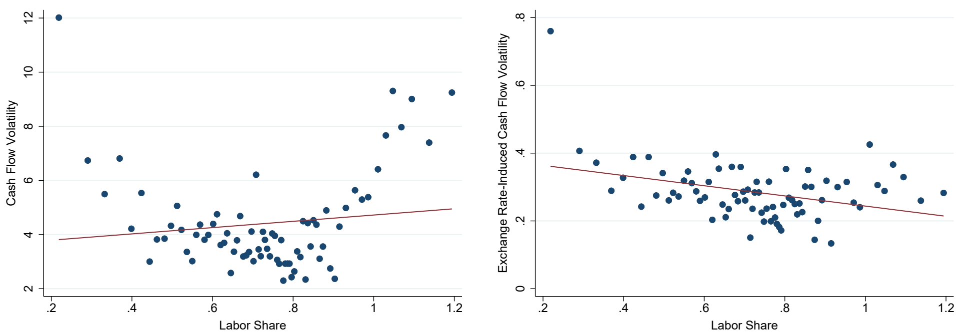
Figure 3.8.1. Most Cited Reasons Why Production Is Below Full Production Capabilities



Notes: This figure uses quarterly data from the Quarterly Survey of Plant Capacity Utilization (QSPC) of the US Census Bureau. The figure shows the share of plants among those with reduced production indicating each reason as a primary reason why actual production was less than full production capability. Multiple answers are possible.

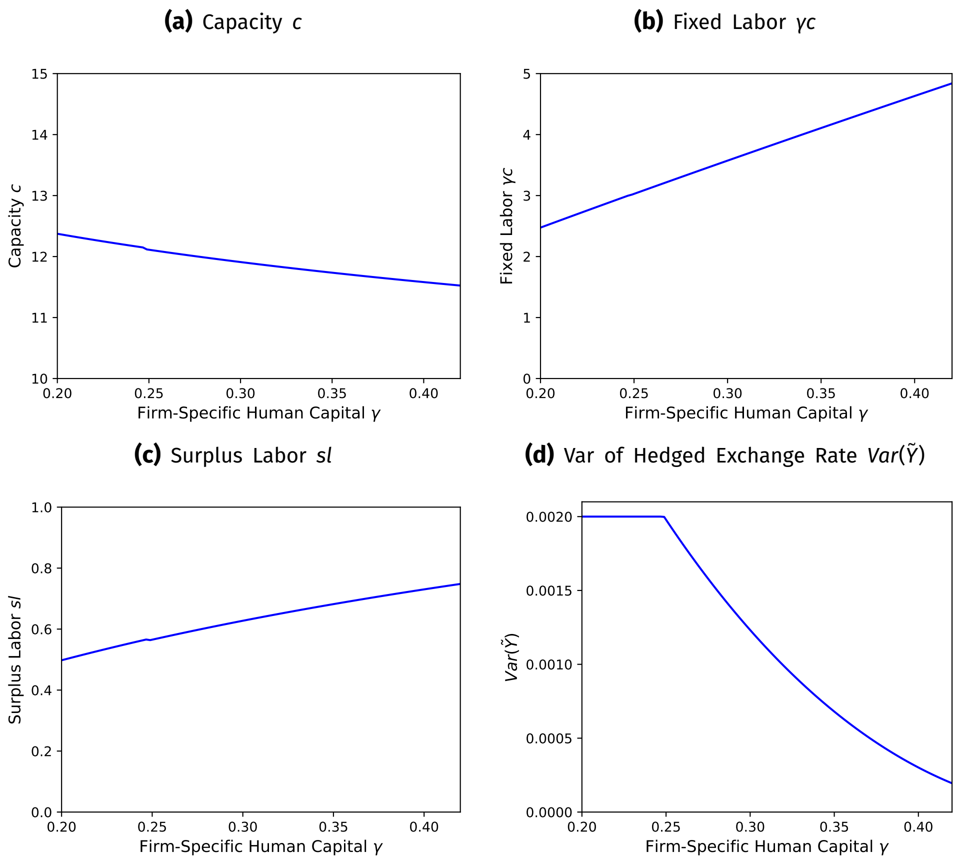
Figure 3.8.2. Connection to Literature on Labor Leverage: Labor Share vs Cash Flow Volatility

(a) Total CF Volatility **(b)** Exchange Rate-Induced CF Volatility



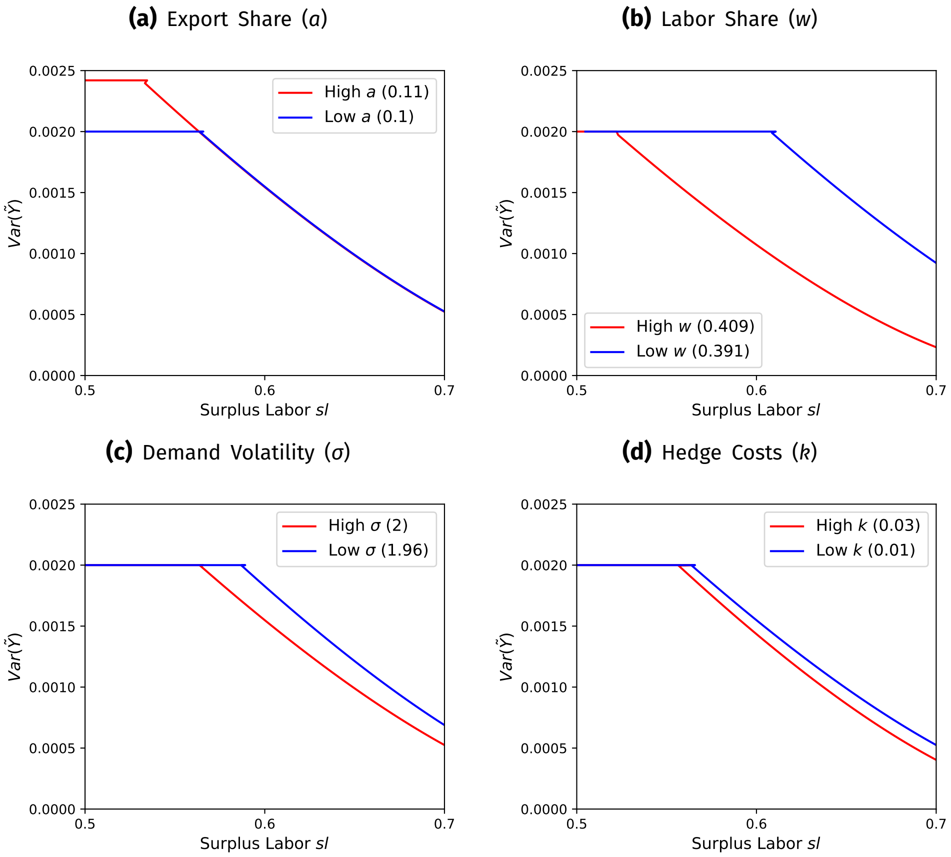
Notes: The figure shows a binned scatterplot of cash flow (CF) volatility and the labor share. *Total CF Volatility* in Panel (a) is defined as the standard deviation of CF/revenue between 2010 and 2019. *Exchange Rate-Induced CF Volatility* in Panel (b) is defined as the standard deviation of net FX gains/ revenue between 2010 and 2019. *Labor Share* is defined as wage bill/ value added in 2019. The variables are winsorized at the 5% level. Estimates are based on the sample without matching with the IAB data, following otherwise analogous sample restriction procedures. Controls include size, export share as well as industry fixed effects.

Figure 3.8.3. Model: Firm Outcomes As Functions of Firm-Specific Human Capital



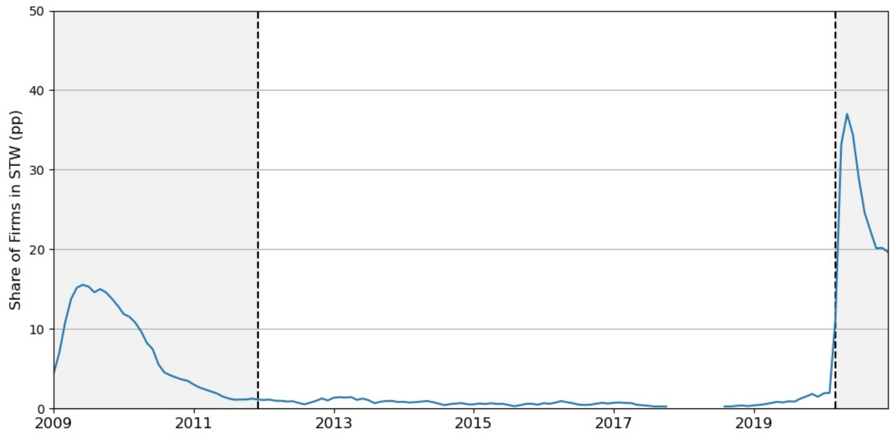
Notes: The figure shows how optimal *capacity*, c, *fixed labor*, γc, *surplus labor*, $sl = \gamma(c - E[\min(X, c)])$, and the *variance of the hedged exchange rate*, $Var(\tilde{Y}) = 2p(a - h)^2$, change as a function of firm-specific human capital γ. The constraint considered is $P[CF < 0|Y = (1 - a)] < \alpha$. The model is numerically solved for the following set of parameters: $\mu = 10, \sigma = 2, b = 2, a = 0.1, p = 0.1, w = 0.4, \alpha = 0.01, k = 0.01, q_{min} = 0.02$.

Figure 3.8.4. Model: Comparative Statics



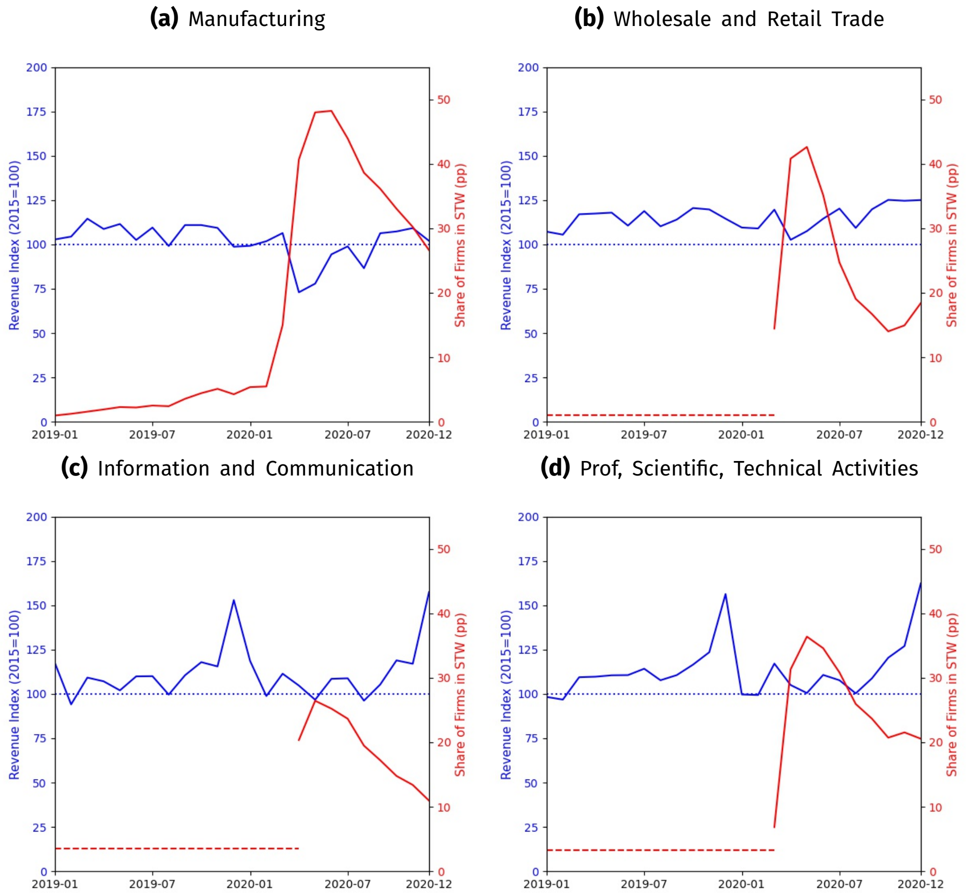
Notes: The figure shows comparative statics in a, w, σ and k of the optimal choice of surplus labor (x-axis) and the variance of the hedges exchange rate ($Var(\tilde{Y})$, y-axis). The baseline parameter specification is as before: $\mu = 10, \sigma = 2, b = 2, a = 0.1, p = 0.1, w = 0.4, \alpha = 0.01, k = 0.01, q_{min} = 0.02$.

Figure 3.8.5. Short-Time Work (STW) Usage and Eased Access Over Time



Notes: The figure shows the monthly share of firms in STW from 2009 until 2020. The sample consists of all German firms with available revenue information in 2019 and 2020 that can be reasonably-well matched to the administrative employment data at the IAB (9,145 in 2020) (see section 3.4 for details). Availability of information on FX transaction income or export shares is not required, nor are limits on the revenue change imposed yet. The shaded areas indicate episodes of eased access to STW (2009-2011, since March 2020).

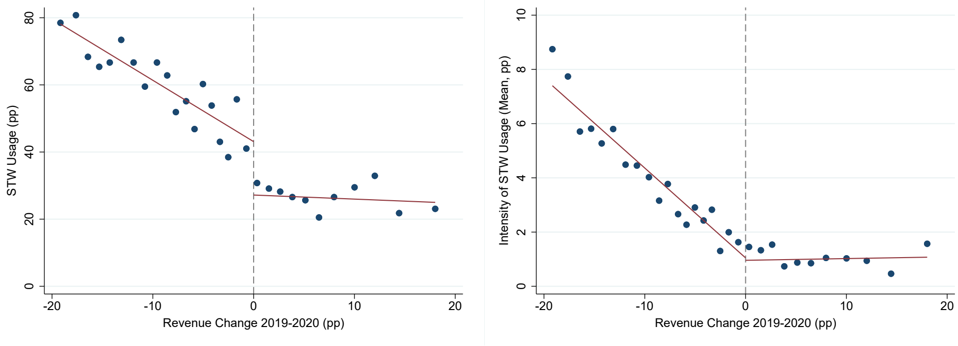
Figure 3.8.6. Industry-Level: STW Usage and the Exposure to the Covid-19 Shock



Notes: This figure plots industry-wide revenue (blue, LHS scale) against the share of firms in STW in the sample (red, RHS scale) for four large industries (largest industries in the baseline sample). The sample consists of all German firms with available revenue information in 2019 and 2020 that can be reasonably-well matched to the administrative employment data at the IAB (9,145 in 2020) (see section 3.4 for details). Availability of information on FX transaction income or export shares is not required, nor are limits on the revenue change imposed yet. The frequency of the data is monthly. Revenue is a value index which is normalized to 100 in 2015 (raw series). The revenue data is from the Federal Statistical Office of Germany (tables 42152-0001, 45212-0005 and 47414-0005). For the series of STW usage, no data is available below the dotted red lines per industry due to data protection.

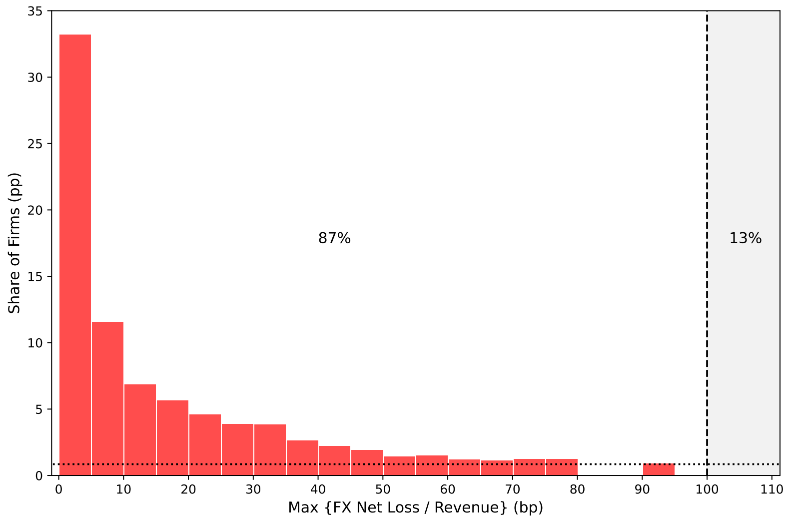
Figure 3.8.7. Firm-Level: STW Usage Beyond the Covid-19 Shock

(a) STW Usage **(b)** STW Usage Intensity



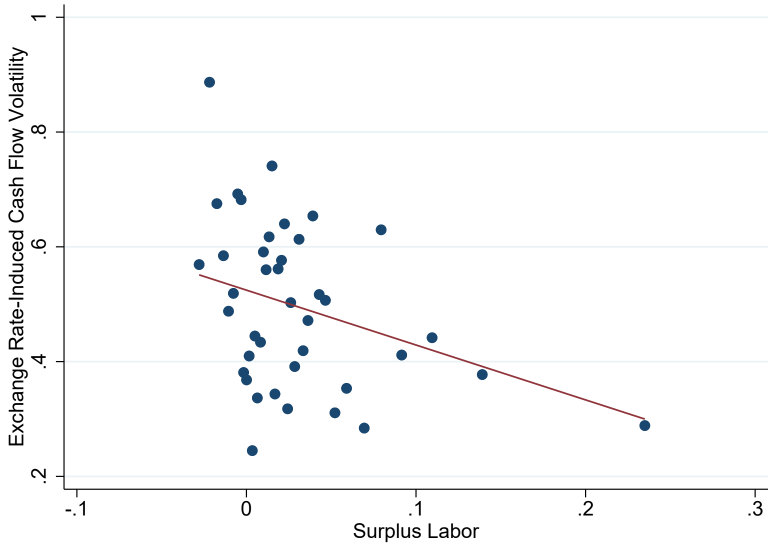
Notes: The figure shows a binned scatterplot of STW Usage (Panel A) and Intensity of STW Usage (Panel B) against the relative change in revenue from 2019-2020. Intensity of STW Usage is defined as *Surplus Labor*, for details see Section 3.5.

Figure 3.8.8. Relevance of FX-Induced Cash Flow Volatility



Notes: The figure shows the distribution of the maximum net FX loss relative to revenue between 2010 and 2019 in basis points (corresponding to the variable *max net loss* as defined in Section 3.5.2 divided by 100). No data is shown below the dotted line, due to data protection. The shaded area indicates maximum net losses exceeding 1% of revenue in some year. 13% of firms in the sample experience losses beyond this magnitude at some point.

Figure 3.8.9. Negative Correlation Between Surplus Labor and FX-Induced CF Volatility



Notes: The figure shows the binned scatterplot *FX-Induced CF Volatility* (*max net loss*) and *Surplus Labor*. For details of the definitions of the variables see Section 3.5. For details on the definition of the variables *Surplus Labor* and measures for *FX-Induced CF Volatility* (*sd net gains*, *max net loss*) see Section 3.5. Included controls are size, relative revenue change from 2019 to 2020, export share as well as industry fixed effects.

Tables

Table 3.8.1. Summary Statistics

	Mean	SD	p5	p50	p95	N Firms
Core Financial Information (2019)						
Assets (mil EUR)	305.75	4118.04	9.14	46.01	505.91	2352
Revenue (mil EUR)	236.75	1823.66	15.34	72.97	647.06	2352
Employees	450.73	2510.38	34.00	221.00	1182.00	2352
Equity/Assets (pp)	40.73	31.56	2.23	41.16	84.00	2352
Cash/Assets (pp)	9.53	13.02	0.00	4.10	37.91	2352
ROA (pp)	7.45	13.68	-10.77	6.13	28.75	2352
Value Added per Employee (mil EUR)	0.17	1.50	0.05	0.09	0.27	1661
Information on Exports and FX-Volatility						
Export Share	0.44	0.28	0.02	0.45	0.90	2352
sd net gains	0.32	0.62	0.00	0.12	1.31	2352
max net losses	0.50	1.03	0.00	0.14	2.16	2352
1(Exports to Outside Europe)	0.82	0.38	0.00	1.00	1.00	1192
Financial Hedging 2019	0.26	0.44	0.00	0.00	1.00	2352
Information on Employment						
Surplus Labor (based on 2020)	0.03	0.05	0.00	0.00	0.14	2352
Surplus Labor (based on 2009)	0.03	0.21	0.00	0.00	0.12	2276
Firm-Level Shares per Education Level (2019)						
Neither/ Missing	0.11	0.08	0.01	0.10	0.27	2352
Vocational Training	0.62	0.19	0.21	0.67	0.84	2352
Degree from University/FH	0.27	0.20	0.05	0.20	0.71	2352
Firm-Level Shares per Occupation (2019)						
Production	0.41	0.29	0.00	0.42	0.82	2352
Service (IT, Scientific)	0.10	0.18	0.00	0.03	0.56	2352
Service (Rest)	0.33	0.24	0.06	0.25	0.82	2352

Notes: The table reports firm-level summary statistics for the baseline sample. For details on the definition of the variables *Surplus Labor* and measures for *FX-Induced CF Volatility* (*sd net gains*, *max net loss*) see Section 3.5. *Financial Hedging 2019* is an indicator that takes the value 1 if the firm uses FX derivatives in 2019.

Table 3.8.2. Negative Correlation Between Surplus Labor and FX-Induced CF Volatility

	Dependent Variable							
	OLS		OLS		OLS		OLS	
	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss
Surplus Labor	-0.426**	-0.742**	-0.451**	-0.766**	-0.382**	-0.693**	-0.708***	-1.187***
	(0.18)	(0.32)	(0.20)	(0.37)	(0.18)	(0.33)	(0.22)	(0.32)
Log Assets	0.065***	0.099***	0.065***	0.099***	0.070***	0.104***	0.070***	0.102***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)
Export Share	0.457***	0.692***	0.456***	0.691***	0.445***	0.679***	0.478***	0.743***
	(0.06)	(0.10)	(0.06)	(0.10)	(0.06)	(0.10)	(0.08)	(0.13)
Revenue Change 19-20			-0.039	-0.038				
			(0.16)	(0.26)				
Cash/Assets					0.313**	0.349*		
					(0.12)	(0.19)		
Value Added per Employee							0.014	-0.004
							(0.01)	(0.01)
Industry x Region FEs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R squared	0.112	0.092	0.112	0.092	0.116	0.094	0.117	0.099
R squared adj	0.082	0.062	0.082	0.061	0.086	0.063	0.080	0.061
N Firms	2,319	2,319	2,319	2,319	2,319	2,319	1,640	1,640

Notes: This table reports the estimated OLS coefficients from specification (R1). Two versions of the dependent variable *FX-Induced CF Volatility* are considered: *sd net gains* and *max net loss*. For details on the definition of the variables *Surplus Labor* and measures for *FX-Induced CF Volatility* (*sd net gains*, *max net loss*) see Section 3.5. Control variables are as of 2019 (or available information in Dafne as of May 2022 for *Export Share*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.8.3. Heterogeneity Guided by Comparative Statics of the Model

	Dimensions of Heterogeneity					
	Low Export Share		Low Labor Share		Low Order Volatility	
	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss
Surplus Labor	-0.788***	-1.532***	-0.992***	-1.540***	-0.725**	-0.659
	(0.28)	(0.45)	(0.25)	(0.39)	(0.36)	(0.56)
Dimension of Heterogeneity × Surplus Labor	0.781**	1.701***	1.120***	1.639***	0.126	-0.935
	(0.33)	(0.57)	(0.35)	(0.57)	(0.45)	(0.76)
Dimension of Heterogeneity	-0.225***	-0.361***	-0.053*	-0.075	-0.081*	-0.032
	(0.03)	(0.05)	(0.03)	(0.06)	(0.05)	(0.08)
Log Assets	0.067***	0.101***	0.064***	0.098***	0.073***	0.109***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)
Export Share			0.458***	0.692***	0.381***	0.538***
			(0.06)	(0.10)	(0.09)	(0.15)
Revenue Change 19-20	-0.057	-0.070	-0.047	-0.039	0.139	0.016
	(0.16)	(0.26)	(0.16)	(0.25)	(0.23)	(0.38)
Industry x Region FEs	Yes	Yes	Yes	Yes	Yes	Yes
R squared	0.101	0.085	0.114	0.094	0.089	0.059
N Firms	2,319	2,319	2,316	2,316	1,054	1,054

Notes: This table reports the estimated OLS coefficients from specification (R1) allowing for heterogeneity of the effect in three different dimensions. Two versions of the dependent variable *FX-Induced CF Volatility* are considered: *sd net gains* and *max net loss*. For details on the definition of the variables *Surplus Labor* and measures for *FX-Induced CF Volatility* (*sd net gains*, *max net loss*) see Section 3.5. *Low Export Share* is defined as below-median export share. A granular (3-digit) industry has a *Low Labor Share* if its average labor share (wagebill to value added) is below median. A granular (3-digit) industry has a *Low Order Volatility* if the standard deviation of monthly industry-level incoming orders between 2010 and 2020 is below median (data only available for the manufacturing sector). Control variables are as of 2019 (or available information in Dafne as of May 2022 for *Export Share*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.8.4. Summary Statistics for Firms With a High/ Low Share With Vocational Training

	Low Share Vocational					High Share Vocational					t-test Means
	Mean	p10	p50	p90	N	Mean	p10	p50	p90	N	
Core Financial Information (2019)											
Assets (mil EUR)	354.58	14.12	46.59	343.56	1176	256.93	14.33	45.49	206.08	1176	0.57
Revenue (mil EUR)	260.58	21.23	72.08	375.21	1176	212.92	24.13	74.47	279.14	1176	0.53
Employees	431.23	41.00	189.50	752.00	1176	470.24	67.00	250.00	709.00	1176	0.71
Equity/Assets (pp)	38.73	7.10	38.79	76.09	1176	42.73	9.70	42.97	77.28	1176	0.00
Cash/Assets (pp)	11.35	0.06	5.21	31.54	1176	7.72	0.02	3.28	21.75	1176	0.00
ROA (pp)	8.38	-4.11	6.68	24.06	1176	6.52	-5.16	5.64	19.34	1176	0.00
Value Added per Employee (mil EUR)	0.24	0.06	0.11	0.25	837	0.09	0.05	0.08	0.14	824	0.05
Information on Exports and FX-Volatility											
Export Share	0.45	0.05	0.46	0.85	1176	0.43	0.08	0.42	0.79	1176	0.14
1(Export Outside Europe)	0.83	0.00	1.00	1.00	555	0.81	0.00	1.00	1.00	637	0.39

Notes: The table reports firm-level summary statistics separately for firms with a high (above-median) and low share with vocational training.

Table 3.8.5. Impact of Surplus Labor on FX-Induced CF Volatility: 2SLS

(a) Panel A: Core

	Dependent Variable					
	OLS		2SLS		Reduced Form	
	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss
Surplus Labor	-0.451**	-0.766**	-18.432***	-29.022***		
	(0.20)	(0.37)	(6.41)	(9.73)		
Log Assets	0.065***	0.099***	-0.027	-0.045	0.062***	0.095***
	(0.02)	(0.02)	(0.04)	(0.06)	(0.02)	(0.02)
Export Share	0.456***	0.691***	0.606***	0.927***	0.407***	0.613***
	(0.06)	(0.10)	(0.10)	(0.17)	(0.06)	(0.09)
Revenue Change 19-20	-0.039	-0.038	-3.423***	-5.356***	0.013	0.055
	(0.16)	(0.26)	(1.21)	(1.85)	(0.14)	(0.23)
Share Vocational Training					-0.485***	-0.764***
					(0.12)	(0.18)
Industry x Region FEs	Yes	Yes	Yes	Yes	Yes	Yes
Instrument 1st Stage			.026	.026		
Partial R Squared 1st Stage			.005	.005		
Kleibergen-Paap F statistic			15.150	15.150		
Anderson-Rubin Chi-Squared p-value			0.000	0.000		
N Firms	2,319	2,319	2,319	2,319	2,319	2,319

(b) Panel B: Robustness

	Dependent Variable							
	2SLS		2SLS		2SLS		2SLS	
	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss
Surplus Labor	-15.144***	-23.877***	-18.432***	-29.022***	-15.318***	-24.593***	-13.772***	-24.557***
	(4.90)	(7.41)	(6.41)	(9.73)	(5.27)	(8.04)	(4.90)	(7.67)
Log Assets	-0.014	-0.025	-0.027	-0.045	-0.016	-0.032	0.003	-0.018
	(0.03)	(0.05)	(0.04)	(0.06)	(0.04)	(0.05)	(0.04)	(0.05)
Export Share	0.630***	0.965***	0.606***	0.927***	0.635***	0.982***	0.708***	1.155***
	(0.10)	(0.16)	(0.10)	(0.17)	(0.10)	(0.17)	(0.13)	(0.22)
Revenue Change 19-20			-3.423***	-5.356***				
			(1.21)	(1.85)				
Cash/Assets					-0.060	-0.248		
					(0.22)	(0.34)		
Value Added per Employee							0.007	-0.015
							(0.01)	(0.01)
Industry x Region FEs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Instrument 1st Stage	.032	.032	.026	.026	.03	.03	.04	.04
Partial R Squared 1st Stage	.007	.007	.005	.005	0	0	.011	.011
Kleibergen-Paap F statistic	20.304	20.304	15.150	15.150	17.741	17.741	19.601	19.601
Anderson-Rubin Chi-Squared p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
N Firms	2,319	2,319	2,319	2,319	2,319	2,319	1,640	1,640

Notes: This table reports the estimated coefficients from specification (R2), (R3) instrumenting *Surplus Labor* with *Share Vocational Training*, the share of employees with vocational training. Two versions of the dependent variable *FX-Induced CF Volatility* are considered: *sd net gains* and *max net loss*. For details on the definition of the variables *Surplus Labor* and measures for *FX-Induced CF Volatility* (*sd net gains*, *max net loss*) see Section 3.5. Control variables are as of 2019 (or available information in Dafne as of May 2022 for *Export Share*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.8.6. Surplus Labor Based on 2009

	Dependent Variable					
	OLS		2SLS		Reduced Form	
	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss
mean STW usage 2009 (rel mthly)	-0.572**	-0.763*	-23.414*	-40.997*		
	(0.24)	(0.40)	(13.30)	(22.87)		
Log Assets 2008	0.056***	0.099***	-0.013	-0.020	0.054***	0.098***
	(0.02)	(0.03)	(0.04)	(0.08)	(0.02)	(0.03)
Export Share	0.462***	0.779***	0.749***	1.276***	0.435***	0.725***
	(0.07)	(0.12)	(0.18)	(0.31)	(0.07)	(0.12)
Revenue Change 08-09	0.087	0.216**	-1.569*	-2.701*	0.105	0.233**
	(0.08)	(0.11)	(0.94)	(1.62)	(0.07)	(0.11)
Share Vocational Training					-0.366***	-0.633***
					(0.12)	(0.19)
Industry x Region FEs	Yes	Yes	Yes	Yes	Yes	Yes
Instrument 1st Stage			.016	.016		
Partial R Squared 1st Stage			.006	.006		
Kleibergen-Paap F statistic			3.993	3.993		
Anderson-Rubin Chi-Squared p-value			0.003	0.001		
N Firms	1,558	1,558	1,554	1,554	1,560	1,560

Notes: This table reports the estimated coefficients from the specification (R2), (R3) focussing on 2009 with *Surplus Labor* defined based on STW usage in 2009. Two versions of the dependent variable *FX-Induced CF Volatility* are considered: *sd net gains* and *max net loss*. For details on the definition of the variables *Surplus Labor* and measures for *FX-Induced CF Volatility* (*sd net gains*, *max net loss*) see Section 3.5. Control variables as well as *Share Vocational Training* are as of 2008 (or available information in Dafne as of May 2022 for *Export Share*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.8.7. Shortage Share as Alternative Instrument

(a) Panel A: Core

	Dependent Variable					
	OLS		2SLS		Reduced Form	
	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss
Surplus Labor	-0.451**	-0.766**	-6.071*	-10.827**		
	(0.20)	(0.37)	(3.18)	(5.22)		
Log Assets	0.065***	0.099***	0.036	0.047	0.066***	0.101***
	(0.02)	(0.02)	(0.02)	(0.04)	(0.02)	(0.02)
Export Share	0.456***	0.691***	0.503***	0.775***	0.453***	0.685***
	(0.06)	(0.10)	(0.07)	(0.11)	(0.06)	(0.10)
Revenue Change 19-20	-0.039	-0.038	-1.097*	-1.932*	0.031	0.080
	(0.16)	(0.26)	(0.61)	(1.03)	(0.15)	(0.23)
Shortage Share					-0.158**	-0.282**
					(0.07)	(0.11)
Industry x Region FEs	Yes	Yes	Yes	Yes	Yes	Yes
Instrument 1st Stage			.026	.026		
Partial R Squared 1st Stage			.005	.005		
Kleibergen-Paap F statistic			12.290	12.290		
Anderson-Rubin Chi-Squared p-value			0.024	0.012		
N Firms	2,319	2,319	2,319	2,319	2,319	2,319

(b) Panel B: Robustness

	Dependent Variable							
	2SLS		2SLS		2SLS		2SLS	
	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss
Surplus Labor	-4.756**	-8.511**	-6.071*	-10.827**	-4.883**	-8.606**	-5.650*	-9.411**
	(2.36)	(3.82)	(3.18)	(5.22)	(2.33)	(3.77)	(2.99)	(4.60)
Log Assets	0.042**	0.057*	0.036	0.047	0.044**	0.059*	0.045*	0.060
	(0.02)	(0.03)	(0.02)	(0.04)	(0.02)	(0.03)	(0.03)	(0.04)
Export Share	0.508***	0.784***	0.503***	0.775***	0.502***	0.779***	0.565***	0.888***
	(0.07)	(0.11)	(0.07)	(0.11)	(0.07)	(0.11)	(0.10)	(0.15)
Revenue Change 19-20			-1.097*	-1.932*				
			(0.61)	(1.03)				
Cash/Assets					0.201	0.152		
					(0.14)	(0.21)		
Value Added per Employee							0.011	-0.008
							(0.01)	(0.01)
Industry x Region FEs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Instrument 1st Stage	.034	.034	.026	.026	.034	.034	.035	.035
Partial R Squared 1st Stage	.008	.008	.005	.005	.002	.002	.008	.008
Kleibergen-Paap F statistic	18.163	18.163	12.290	12.290	18.739	18.739	14.026	14.026
Anderson-Rubin Chi-Squared p-value	0.023	0.010	0.024	0.012	0.017	0.008	0.033	0.018
N Firms	2,319	2,319	2,319	2,319	2,319	2,319	1,640	1,640

Notes: This table reports the estimated coefficients from a specification analogous to (R2), (R3) now instrumenting *Surplus Labor* with *Shortage Share*, the share of employees in shortage occupations as of the end of 2019. Two versions of the dependent variable *FX-Induced CF Volatility* are considered: *sd net gains* and *max net loss*. For details on the definition of the variables *Surplus Labor* and measures for *FX-Induced CF Volatility* (*sd net gains*, *max net loss*) see Section 3.5. Control variables are as of 2019 (or available information in Dafne as of May 2022 for *Export Share*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.8.8. Stylized Facts on FX Derivatives Usage: Summary Statistics

	Non-User					Derivatives User 2019					t-test Means
	Mean	p10	p50	p90	N	Mean	p10	p50	p90	N	
Core Financial Information (2019)											
Assets (mil EUR)	111.25	12.38	40.99	171.82	1729	845.55	20.76	80.06	535.91	623	0.00
Revenue (mil EUR)	128.73	19.85	63.53	227.49	1729	536.53	35.02	114.71	772.64	623	0.00
Employees	307.04	55.00	203.00	588.00	1729	849.52	52.00	283.00	1286.00	623	0.00
Equity/Assets (pp)	40.56	6.63	41.80	77.10	1729	41.19	10.46	39.73	75.21	623	0.67
Cash/Assets (pp)	9.86	0.03	4.52	28.11	1729	8.62	0.04	3.43	24.31	623	0.04
ROA (pp)	7.82	-4.73	6.44	22.68	1729	6.42	-3.97	5.43	19.04	623	0.03
Value Added per Employee (mil EUR)	0.16	0.05	0.09	0.19	1214	0.19	0.06	0.10	0.21	447	0.72
Information on Exports and FX-Volatility											
Export Share	0.42	0.06	0.40	0.80	1729	0.51	0.11	0.55	0.85	623	0.00
sd net gains	0.28	0.00	0.09	0.72	1729	0.44	0.02	0.21	1.03	623	0.00
max net loss	0.44	0.00	0.10	1.16	1729	0.64	0.01	0.24	1.48	623	0.00
1(Export Outside Europe)	0.80	0.00	1.00	1.00	877	0.89	0.00	1.00	1.00	315	0.00

Notes: This table shows summary statistics for firms in the baseline sample, separately for derivatives users (RHS) and non-users (LHS). *Export Share* (*Import Share*) is the information availabe from Creditreform (as of May 2022). For details on the definition *sd net gains* and *max net loss* see Section 3.5.

Table 3.8.9. Stylized Facts on FX Derivatives Usage: Determinants of FX-Induced CF Volatility

(a) Relevance of Exports

	FX-Induced CF Volatility (sd net gains)			
	Baseline		Exports Outside EA	
	(1)	(2)	(3)	(4)
Export Share	0.489***	0.477***	0.652***	0.542***
	(0.06)	(0.07)	(0.10)	(0.10)
Export Share × Derivatives Usage	-0.210*	-0.091	-0.429**	-0.225
	(0.12)	(0.18)	(0.19)	(0.24)
Derivatives Usage	0.203***	0.160**	0.317***	0.200
	(0.07)	(0.08)	(0.11)	(0.12)
Log Assets	0.059***	0.022	0.048**	0.031
	(0.02)	(0.01)	(0.02)	(0.02)
Import Share		0.271***		0.431***
		(0.07)		(0.09)
Industry x Region FEs	Yes	Yes	Yes	Yes
R Squared	0.117	0.138	0.153	0.170
R Squared adj	0.087	0.093	0.108	0.108
N Firms	2,319	936	957	555

(b) Relevance of Imports

	FX-Induced CF Volatility (sd net gains)			
	Baseline		Exports Outside EA	
	(1)	(2)	(3)	(4)
Import Share	0.254***	0.224***	0.418***	0.366***
	(0.08)	(0.07)	(0.11)	(0.10)
Import Share × Derivatives Usage	0.141	0.185	0.173	0.252
	(0.14)	(0.14)	(0.18)	(0.17)
Derivatives Usage	0.088	0.042	0.040	-0.012
	(0.06)	(0.06)	(0.08)	(0.07)
Log Assets	0.038**	0.023*	0.050***	0.032*
	(0.01)	(0.01)	(0.02)	(0.02)
Export Share		0.457***		0.482***
		(0.08)		(0.12)
Industry x Region FEs	Yes	Yes	Yes	Yes
N Firms	936	936	555	555
R Squared	0.089	0.140	0.123	0.172
R Squared adj	0.042	0.095	0.059	0.110

Notes: This table reports the estimated coefficients from a regression of FX-Induced CF Volatility (*sd net gains*) on the export share (Panel A) and on the import share (Panel B), allowing for heterogeneity between derivatives users and non-users. *Derivatives Usage* is equal to one if the firm uses FX derivatives in 2019. Control variables are as of 2019 (or available information in Dafne as of May 2022 for *Export Share*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.8.10. Mechanism: FX Derivatives Usage as Outcome

	Dependent Variable: Derivatives Usage		
	OLS	2SLS	Reduced
Surplus Labor	0.287	0.955	
	(0.19)	(2.36)	
Log Assets	0.091***	0.094***	0.090***
	(0.01)	(0.01)	(0.01)
Export Share	0.182***	0.176***	0.187***
	(0.04)	(0.04)	(0.04)
Revenue Change 19-20	0.282***	0.408	0.230**
	(0.10)	(0.45)	(0.09)
Share Vocational Training			0.025
			(0.06)
Industry x Region FEs	Yes	Yes	Yes
Instrument 1st Stage		.026	
Partial R Squared 1st Stage		.005	
Kleibergen-Paap F statistic		15.150	
Anderson-Rubin Chi-Squared p-value		0.679	
N Firms	2,319	2,319	2,319

Notes: This table reports the estimated coefficients from the specification (R4) with *Surplus Labor* instrumented with *Share Vocational Training*, the share of employees with vocational training. *Derivatives Usage* is equal to one if the firm uses FX derivatives in 2019. Control variables are as of 2019 (or available information in Dafne as of May 2022 for *Export Share*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.8.11. Mechanism: Heterogeneity by FX Derivatives Usage and Service Provision

	Dimension of Heterogeneity			
	Derivatives Usage		Local Bank Continued	
	sd net gains	max net loss	sd net gains	max net loss
Surplus Labor	-0.477**	-0.990***	-0.692***	-0.954**
	(0.23)	(0.37)	(0.24)	(0.42)
Dimension of Heterogeneity × Surplus Labor	-0.009	0.565	0.623*	0.774
	(0.41)	(0.72)	(0.33)	(0.61)
Dimension of Heterogeneity	0.102***	0.090	-0.008	0.016
	(0.04)	(0.06)	(0.04)	(0.06)
Log Assets	0.055***	0.089***	0.064***	0.098***
	(0.02)	(0.02)	(0.02)	(0.02)
Export Share	0.437***	0.675***	0.443***	0.666***
	(0.06)	(0.10)	(0.06)	(0.10)
Revenue Change 19-20	-0.068	-0.081	-0.001	0.049
	(0.16)	(0.26)	(0.16)	(0.27)
Industry x Region FEs	Yes	Yes	Yes	Yes
Local Bank FEs	No	No	Yes	Yes
R squared	0.117	0.094	0.116	0.098
N Firms	2,319	2,319	2,193	2,193

Notes: This table reports the estimated OLS coefficients from specification (R1) allowing for heterogeneity of the effect in two dimensions. Two versions of the dependent variable *FX-Induced CF Volatility* are considered: *sd net gains* and *max net loss*. For details on the definition of the variables *Surplus Labor* and measures for *FX-Induced CF Volatility* (*sd net gains*, *max net loss*) see Section 3.5. *Derivatives Usage* is equal to one if the firm uses FX derivatives in 2019. *Local Bank Continued* is one if the firm has a relationship bank that continued offering FX derivatives in-house in 2014 (for details see Section 3.A.5). In columns 3 and 4 fixed effect dummies based on whether the firm has a local bank among their relationship bank are included. Control variables are as of 2019 (or available information in Dafne as of May 2022 for *Export Share*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.8.12. Mechanism: Average Gross FX Exposures

	Dependent Variable: Means								
	OLS			2SLS			Reduced Form		
	Net Gains	Gains	Losses	Net Gains	Gains	Losses	Net Gains	Gains	Losses
Surplus Labor	0.014	-0.623**	-0.561**	-2.002	-38.084***	-36.283***			
	(0.08)	(0.26)	(0.23)	(1.62)	(11.19)	(10.47)			
Log Assets	0.006	0.135***	0.138***	-0.005	-0.056	-0.044	0.005	0.129***	0.132***
	(0.01)	(0.02)	(0.02)	(0.01)	(0.07)	(0.06)	(0.01)	(0.02)	(0.02)
Export Share	0.022	0.795***	0.768***	0.039	1.108***	1.065***	0.017	0.696***	0.673***
	(0.02)	(0.08)	(0.07)	(0.03)	(0.19)	(0.18)	(0.02)	(0.07)	(0.07)
Revenue Change 19-20	-0.018	-0.076	-0.060	-0.397	-7.126***	-6.782***	-0.024	-0.026	-0.018
	(0.05)	(0.21)	(0.18)	(0.31)	(2.13)	(2.00)	(0.05)	(0.19)	(0.17)
Share Vocational Training							-0.053	-1.003***	-0.955***
							(0.04)	(0.16)	(0.14)
Industry x Region FEs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Instrument 1st Stage				.026	.026	.026			
Partial R Squared 1st Stage				.005	.005	.005			
Kleibergen-Paap F statistic				15.150	15.150	15.150			
Anderson-Rubin Chi-Squared p-value				0.188	0.000	0.000			
N Firms	2,319	2,319	2,319	2,319	2,319	2,319	2,319	2,319	2,319

Notes: This table reports the estimated coefficients from a specification analogous to (R2), (R3), but with different outcomes. *Net Gains* are mean net FX gains, *Gains* are mean FX gains, and *Losses* are mean FX losses constructed based on data from 2010 until 2019, multiplied by 100. I instrument *Surplus Labor* with *Share Vocational Training*, the share of employees with vocational training. Control variables are as of 2019 (or available information in Dafne as of May 2022 for *Export Share*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

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Appendix 3.A Data Appendix

3.A.1 Cleaning BTR KUG

In BTR KUG, I create STW spells, i.e., periods of STW usage with a maximal gap of two months and transform the data into a monthly panel. I match this unbalanced panel at the establishment-month level to the annual panel of all German establishments (BHP) which I have previously expanded to the monthly level.

I drop all establishments that are in a special construction scheme *(Baugewerbetarif)* at any point in time (around 5% of observation in the initial BTR KUG). I also drop establishments that in some year appear in BTR KUG, but not in BHP, except when this happens in the year that marks the establishment's last (first) appearance in BHP. Since BHP is based on establishments with at least one employee subject to social insurance contributions on June 30 of each year, such cases can occur if an establishment closes before June 30, but used STW in earlier months that year.

3.A.2 Cleaning Dafne

Before merging firms' financials from Dafne to the employment data at the IAB, I clean Dafne as follows. Starting point are firms that report an income statement in 2019 *(48,000)* at the unconsolidated level. I further restrict attention to firms that report revenues in 2019 and 2020 *(21,000)*. Among the firms that report at the consolidated and unconsolidated level (i.e. group heads) I restrict attention to firms that are likely not just holdings. In particular, I demand that a) firms have more than 10 employees at the unconsolidated level in 2019 and 2020 (if reported) and b) firms' unconsolidated revenues are at least 10% of consolidated revenues between 2016 and 2020 (if consolidated revenues are available) *(17,800)*.

Similar to the standard data cleaning methodology for ORBIS, I discard firms that do not pass basic data consistency checks on their key financials (whenever assets are available they are positive, equity exceeds assets in 2019 and 2020, fixed assets are never negative, revenues are never negative, sales-to-asset ratio is below the 99.9 percentile (pooled across all years), assets to not exceed those of VW, fixed asset-to-asset ratio below 1) *(17,200)*. I demand that information on cashflow, cash and equity is available in 2019 *(16,400)*.

I consolidate information on FX gains and FX losses across two accounting formats (*Umsatzkostenverfahren* and *Gesamtkostenverfahren*) and two FX reporting schemes (*Aufwendungen/ Ertraege aus Währungsumrechnung*, *Währungsgewinne/ Währungsverluste*). I identify which of the two FX schemes is the predominant one at the firm level (i.e. which one appears more often than the other). I consolidate information on currency gains and on currency losses across the two FX schemes, as the same information in annual reports is collected inconsistently across both schemes. Here, I take the predominant FX scheme. If information on gains is missing in the predominant FX scheme, but available in the other format, I add the information from the other (analogously for losses). If only gains or only losses are reported, I set the other to zero.

3.A.3 Details on the Relative Wage Bill Gap

The data contains the monthly number of short-time workers and information on the relative wage bill gap among them. The gap is defined as the gap in wages among short-time workers divided by the regular wage bill of short-time workers. Is it available in buckets: for values below 0.25 it takes value 0.175, for values in $(0.25, 0.5]$ 0.375, for values in $(0.5, 0.75]$ 0.625, for values in $(0.75, 0.99]$ 0.87 and it takes value 1 for values above 0.99.

For a subsample of establishments that use STW in 2020, I have individual-level information on the wage gap. I aggregate this individual-level information to the establishment level and confirm that it aligns well with the described bucketed variable (Figure 3.C.4).

3.A.4 Text Analysis of Annual Reports on the Use of FX Hedging Instruments

I have annual reports for 28,495 firm-year observations. Firms are required to include information on their risk management in the appendix of annual reports, and I conduct a text analysis to identify mentions of FX hedging instruments. The reports are in German.

1) I extract the name of the company and year from the report.

2) I search for explicit mentions of words indicating FX hedging. Specifically, as first pattern, I search for the word FX Forward or FX Option (*Devisentermin, Devisenoption, Devisenswap*), and, as second pattern, for other words related to FX hedging (*Währungssicherung, Währungsabsicherung, Kurrsicherung, Devisenabsicherung, kursgesichert*).

3) I count raw occurrences of each pattern. Aditionally, I check if each pattern occur in combination with words suggesting negation (*keine, nicht durch, bestehen nicht, bestanden nicht, verzichtet*), or in combination with words that suggest a conditional sentence structure like "If foreign exchange hedges exist, we use xyz accounting ..." (*sofern, soweit, falls*).

4) For each pattern, I classify for each year the occurrence structure as "No mention" (assigned value 0, pattern not found), "Only negated mentions" (assigned value 1, pattern only occurs in combination with words that suggest negation),

"Sentences with mentions all conditional" (assigned value 2, pattern only occurs in combination with words that suggest a conditional sentence), "Partially negated mentions" (assigned value 3, not all mentions occur in a combination with a word that suggests negation) and "Hedges" (assigned value 4, none of the above).

	Pattern 1		Pattern 2	
	Percent	N	Percent	N
No mention	77.15%	21,983	86.40%	24,621
Only negated mentions	0.71%	203	0.73%	207
Sentences with mentions all conditional	0.34%	97	0.96%	274
Partially negated mentions	0.96%	274	0.33%	94
Hedges	20.84%	5,939	11.58%	3,299
Sum	100%	28,495	100%	28,495

5) I use the highest classification across the two patterns (combined classification value), except when one pattern has only negated mentions in which case I set the combined classification value to 1.

6) I classify a firm as using FX derivatives in a year if the combined classification value is at least two.

3.A.5 Data on FX Derivatives Offered by Banks

I compile information on outstanding FX derivatives from 7,360 bank-year observations, as banks are required to include this information in the appendices of their annual reports.

1) The starting point is relationship banks of firms in Dafne with FX transaction income data and a revenue change from 2019 to 2020 in the range of $[-20\%, 20\%]$. These banks are matched by name to institutions in SNL Fundamentals (accessed via WRDS). The matched sample consists of 745 banks, including 321 savings banks (*Sparkassen*), 345 cooperative banks (*Volksbanken*), three major banks (*Deutsche Bank, Commerzbank, Unicredit*) and 65 other.

2) Their annual reports from 2010 to 2019 were manually downloaded. I extract annual information on outstanding FX derivatives from tables of varying format within pdfs.

3) A bank is classified as having stopped offering FX derivatives in-house if it reported positive amounts outstanding at any point since 2010 but none thereafter (until 2018).

4) Anecdotal evidence suggests that some banks delegated their FX business to other banks within their banking group (Savings Banks Financial Group for savings banks, German Cooperative Financial Group for cooperative banks). I check whether commissioned trading in connection to FX derivatives or membership in S-International (part of the Savings Banks Financial Group) is mentioned in annual reports; this is the case for 144 institutions.

5) I classify a firm as being connected to a bank that continued offering derivatives if the firm is connected to a bank that offered FX derivatives at some point and neither stopped nor delegated.

Appendix 3.B Model Proofs

3.B.1 Proof of Lemma 21

We consider the amplitude of the partially hedged exchange rate $q := a - h$, instead of h.

For the density $f(\cdot)$ of a normal distribution with mean μ and variance σ^2 the following property holds:

$$f'(x) = -\frac{(x-\mu)}{\sigma^2} f(x). \tag{3.B.1}$$

Hence,

$$E[\min(X,c)] = \int_{-\infty}^{c} x f(x) dx + \int_{c}^{\infty} c f(x) dx \tag{3.B.2}$$

$$= -\sigma^2 \int_{-\infty}^{c} -\frac{(x-\mu)}{\sigma^2} f(x) dx + \mu \int_{-\infty}^{c} f(x) dx + \int_{c}^{\infty} c f(x) dx \tag{3.B.3}$$

$$= -\sigma^2 f(c) + \mu F(c) + c(1 - F(c)), \tag{3.B.4}$$

and

$$\partial_c E[\min(X,c)] = -\sigma^2 f'(c) + (\mu - c) f(c) + (1 - F(c)) \tag{3.B.5}$$

$$= (1 - F(c)). \tag{3.B.6}$$

With E short-hand for the expected cashflow,

$$E := E[CF_\gamma(c,q)] = E[\min(X,c)][1 - k(a-q) - (1-\gamma)w] - (\gamma wc + b), \tag{3.B.7}$$

it follows that

$$\partial_c E = [1 - k(a-q) - (1-\gamma)w](1 - F(c)) - \gamma w \tag{3.B.8}$$

$$\partial_c^2 E = -[1 - k(a-q) - (1-\gamma)w] f(c) \quad < 0. \tag{3.B.9}$$

For a fixed q, from (3.B.9) and $\lim_{c\to\infty} \partial_c E < 0$, $\partial_c E = 0$ is a necessary and sufficient condition for a unique local maximum, which is also a global one here. Since the optimal solution c^* is larger 0 (otherwise the setup is not interesting), from (3.B.9) we also know that $\partial_c E > 0$ for $c < c^*$. Since $\partial_q E = kE[\min(X,c)] > 0$, the firm chooses the highest possible q.

□

3.B.2 Proof of Proposition 22

We consider the amplitude of the partially hedged exchange rate $q := a - h$, instead of h.

Step 1: Preliminary properties.

For ease of notation we define the following objects and show some preliminary properties first. Denote by $i \in \{o, m, u\}$ the good, neutral and bad realization of the exchange rate. Then the fixed costs, β_2, and marginal return in the different states read

$$\beta_2 := \gamma wc + b \tag{3.B.10}$$

$$\beta_{1i} := \begin{cases} (1+q) - k(a-q) - (1-\gamma)w & \text{for } i = o, Y = (1+a) \\ 1 - k(a-q) - (1-\gamma)w & \text{for } i = m, Y = 1 \\ (1-q) - k(a-q) - (1-\gamma)w & \text{for } i = u, Y = (1-a) \end{cases} \tag{3.B.11}$$

Further, for $i \in \{o, m, u\}$

$$\lambda_i := \frac{\beta_2}{\beta_{1i}}. \tag{3.B.12}$$

Then the derivatives of E read

$$\partial_c E = \beta_{1m}(1 - F(c)) - \gamma w \tag{3.B.13}$$

$$\partial_q E = kE[\min(X, c)] \qquad > 0 \tag{3.B.14}$$

$$\partial_c^2 E = -\beta_{1m} f(c) \qquad < 0 \tag{3.B.15}$$

$$\partial_c \partial_q E = k(1 - F(c)) \qquad > 0 \tag{3.B.16}$$

$$\partial_q^2 E = 0. \tag{3.B.17}$$

Note that

$$P[\min(X, c) < \Omega] = \begin{cases} F[\Omega] & \text{for } \Omega < c \\ 1 & \text{for } \Omega \geq c. \end{cases} \tag{3.B.18}$$

The unconstrained optimum for a fixed level of hedging, c^*, is in the interval $[\mu, \mu + (5/4)\sigma]$, since

$$\frac{1}{10} < 1 - F(c^*) = \frac{\gamma w}{1 - k(a-q) - (1-\gamma)w} < \frac{1}{2}. \tag{3.B.19}$$

from assumptions A2 and A3. We know

$$\lambda_u < \frac{3}{5}\mu, \tag{3.B.20}$$

since from assumption A6, we have for all $c > \mu$

$$\frac{(c-\mu)}{\mu}\gamma w \;<\; (1-a-w-\frac{b}{\mu}) - \frac{2}{5}(1-a-w) \tag{3.B.21}$$

$$\Rightarrow\; \frac{b}{\mu} + \frac{(c-\mu)}{\mu}\gamma w \;<\; \frac{3}{5}\big(1-k(a-q)-q-(1-\gamma)w\big) - \gamma w \tag{3.B.22}$$

$$\Leftrightarrow\; \frac{b+\gamma wc}{\beta_{1u}} \;<\; \frac{3}{5}\frac{\beta_{1u}}{\beta_{1u}}\mu \tag{3.B.23}$$

$$\Leftrightarrow\; \lambda_u \;<\; \frac{3}{5}\mu. \tag{3.B.24}$$

Hence the default probability takes the form

$$P \;:=\; P[CF_\gamma(c,q) \;<\; 0|Y \;=\; (1-a)] \tag{3.B.25}$$

$$= P\left[\min(X,c) \;<\; \frac{\beta_2}{\beta_{1u}}\middle| Y \;=\; (1-a)\right] \;=\; F[\lambda_u]. \tag{3.B.26}$$

Let

$$Q \;:=\; f(\lambda_u)\lambda_u \tag{3.B.27}$$

With

$$\partial_c\lambda_i \;=\; \lambda_i\frac{\gamma w}{\beta_2} \tag{3.B.28}$$

$$\partial_q\lambda_i \;=\; (-\lambda_i^2)\frac{(k+\delta_i)}{\beta_2} \text{ with } \delta_i \;=\; \begin{cases} 1 & \text{for } i \;=\; o \\ 0 & \text{for } i \;=\; m \\ -1 & \text{for } i \;=\; u, \end{cases} \tag{3.B.29}$$

then

$$\partial_q Q \;=\; \big[f'(\lambda_u)\lambda_u + f(\lambda_u)\big](\partial_q\lambda_u) \qquad > \; 0 \tag{3.B.30}$$

$$\partial_c Q \;=\; \big[f'(\lambda_u)\lambda_u + f(\lambda_u)\big](\partial_c\lambda_u) \qquad > \; 0. \tag{3.B.31}$$

Subsequently

$$\partial_c P \;=\; f(\lambda_u)(\partial_c\lambda_u) \;=\; \frac{\gamma w}{\beta_2}Q \qquad > \; 0 \tag{3.B.32}$$

$$\partial_q P \;=\; f(\lambda_u)(\partial_c\lambda_u) \;=\; \frac{(1-k)}{\beta_2}\lambda_u Q \qquad > \; 0. \tag{3.B.33}$$

Note that

$$\partial_q Q \;=\; (\partial_q P)\Big[1 + \underbrace{\frac{\mu-\lambda_u}{\sigma}\frac{\lambda_u}{\sigma}}_{=:\tau}\Big] \;=\; (\partial_q P)(1+\tau) \tag{3.B.34}$$

$$\partial_c Q \;=\; (\partial_c P)(1+\tau). \tag{3.B.35}$$

With $\lambda_u < \mu$ from (3.B.20),

$$\partial_c^2 P = \frac{\gamma w}{\beta_2}\left(\partial_c Q - \frac{\gamma w}{\beta_2} Q\right) = \left(\frac{\gamma w}{\beta_2}\right)^2 f'(\lambda_u)\lambda_u^2 > 0 \quad (3.B.36)$$

$$\partial_c \partial_q P = \frac{\gamma w}{\beta_2}(\partial_q Q) > 0 \quad (3.B.37)$$

$$\partial_q^2 P = \frac{(1-k)}{\beta_2}\lambda_u\left(2f(\lambda_u) + f'(\lambda_u)\lambda_u\right)(\partial_q \lambda_u) > 0. \quad (3.B.38)$$

Step 2: There is a smooth function $c^E(q)$ that parameterizes $\{\partial_c E = 0\}$ with $\partial_q c^E > 0$.

From the proof of Lemma 21, we know that for any q there exists a unique solution to $\partial_c E = 0$. Since $\partial_q \partial_c E \neq 0$ by (3.B.16) there is a smooth function, $c^E(q)$ that parameterizes $\{\partial_c E = 0\}$ and is uniquely characterized by

$$(\partial_q \partial_c E)(\partial_q c^E) + \partial_c^2 E = 0 \iff \partial_q c^E = -\frac{\partial_c^2 E}{\partial_q \partial_c E} > 0, \quad (3.B.39)$$

where the inequality follows from (3.B.15) and (3.B.17).

Step 3: There is a smooth function $c^P(q)$ that parameterizes $\{P = \alpha\}$ with $\partial_q c^P < 0$.

Since $\partial_q P \neq 0$ by (3.B.33), there is a smooth function, $c^P(q)$, that parameterizes $\{P = \alpha\}$. As above and using (3.B.32) and (3.B.33) for the inequality, it follows that

$$(\partial_q P)(\partial_q c^P) + \partial_c P = 0 \iff \partial_q c^P = -\frac{\partial_c P}{\partial_q P} < 0. \quad (3.B.40)$$

Step 4: There is a smooth function $c^L(q)$ that parameterizes $\{(\partial_c E)(\partial_q P) - (\partial_q E)(\partial_c P) = 0\}$ with $\partial_q c^L > 0$.

The first order conditions for the Lagrangian associated with the value-at-risk constraint,

$$\mathscr{L} = E[CF] + \lambda\left(P[CF < 0] - t - \alpha\right), \quad (3.B.41)$$

read for non-negative t

$$\partial_c E + \lambda \partial_c P = 0 \quad (3.B.42)$$

$$\partial_q E + \lambda \partial_q P = 0 \quad (3.B.43)$$

$$P[CF < 0] + t = \alpha \quad (3.B.44)$$

$$t\lambda = 0. \quad (3.B.45)$$

For a binding constraint the optimality condition thus reads

$$\frac{\partial_c E}{\partial_q E} = \frac{\partial_c P}{\partial_q P}. \tag{3.B.46}$$

Let

$$L := (\partial_c E)(\partial_q P) - (\partial_q E)(\partial_c P). \tag{3.B.47}$$

Then we have $\partial_c E > 0$ on $\{L = 0\}$, since otherwise $L = (\partial_c E)(\partial_q P) - (\partial_q E)(\partial_c P) < 0$, contradiction. Hence, together with (3.B.15), (3.B.33), (3.B.13), (3.B.37), (3.B.16), (3.B.32), (3.B.14) and (3.B.36) we have

$$\partial_c L = \underbrace{(\partial_c^2 E)}_{<0}\underbrace{(\partial_q P)}_{>0} + \underbrace{(\partial_c E)}_{>0}\underbrace{(\partial_c \partial_q P)}_{>0} - \underbrace{(\partial_c \partial_q E)}_{>0}\underbrace{(\partial_c P)}_{>0} - \underbrace{(\partial_q E)}_{>0}\underbrace{(\partial_c^2 P)}_{>0} \tag{3.B.48}$$

and, additionally with (3.B.38) and (3.B.17),

$$\partial_q L = \underbrace{(\partial_q \partial_c E)}_{>0}\underbrace{(\partial_q P)}_{>0} + \underbrace{(\partial_c E)}_{>0}\underbrace{(\partial_q^2 P)}_{>0} - \underbrace{(\partial_q^2 E)}_{=0}\underbrace{(\partial_c P)}_{>0} - \underbrace{(\partial_q E)}_{>0}\underbrace{(\partial_c \partial_q P)}_{>0}. \tag{3.B.49}$$

We first show

$$\partial_q L > 0 \quad \text{on} \quad \{L = 0\}. \tag{3.B.50}$$

From (3.B.49), it suffices to show

$$(\partial_q E)(\partial_c \partial_q P) < (\partial_c E)(\partial_q^2 P) \tag{3.B.51}$$

$$\overset{L=0}{\Leftrightarrow} (\partial_c E)\frac{\partial_q P}{\partial_c P}(\partial_c \partial_q P) < (\partial_c E)(\partial_q^2 P) \tag{3.B.52}$$

$$\Leftrightarrow (\partial_q P)(\partial_c \partial_q P) < (\partial_c P)(\partial_q^2 P) \tag{3.B.53}$$

$$\Leftrightarrow \frac{\gamma w}{\beta_w} Q \frac{1-k}{\beta_2} \lambda_u \left[\lambda_u f'(\lambda_u) + f(\lambda_u)\right](\partial_q \lambda_u) < \frac{\gamma w}{\beta_2} Q \frac{1-k}{\beta_2} \lambda_u \left[\lambda_u f'(\lambda_u) + 2f(\lambda_u)\right](\partial_q \lambda_u) \tag{3.B.54}$$

$$\Leftrightarrow 0 < f(\lambda_u), \tag{3.B.55}$$

which is true.

We now show

$$\partial_c L < 0 \quad \text{on} \quad \{L = 0\}. \tag{3.B.56}$$

From (3.B.48) is suffices to show

$$\left[(\partial_c E)(\partial_c \partial_q P) - (\partial_q E)(\partial_c^2 P) + (\partial_c^2 E)(\partial_q P)\right]\frac{(\partial_c P)}{(\partial_c E)} < 0. \tag{3.B.57}$$

Using $L = 0$, i.e., (3.B.46), we have

$$(\partial_c E)(\partial_c \partial_q P) - (\partial_q E)(\partial_c^2 P) = \frac{(\partial_c E)}{(\partial_c P)} \left[(\partial_c P)(\partial_c \partial_q P) - (\partial_q P)(\partial_c^2 P) \right] \tag{3.B.58}$$

$$= \frac{(\partial_c E)}{(\partial_c P)} \left(\frac{\gamma w}{\beta_2} \right)^2 \frac{1-k}{\beta_2} f(\lambda_u)^2 \lambda_u^3 \tag{3.B.59}$$

and

$$(\partial_c^2 E)(\partial_q P) = \frac{(\partial_c E)}{(\partial_c P)} \frac{(\partial_c^2 E)}{\partial_c E} \left[(\partial_q P)(\partial_c P) \right] \tag{3.B.60}$$

$$= \frac{(\partial_c E)}{(\partial_c P)} \frac{(-\beta_{1m}) f(c)}{\beta_{1m}(1-F(c)) - \gamma w} f(\lambda_u)\lambda_u^2 f(\lambda_u)\lambda_u \left(\frac{\gamma w}{\beta_2} \right) \frac{1-k}{\beta_2} \tag{3.B.61}$$

$$\leq (-1) \frac{(\partial_c E)}{(\partial_c P)} \frac{\gamma w}{\beta_2} \frac{1-k}{\beta_2} f(\lambda_u)^2 \lambda_u^3 \frac{f(c)}{(1-F(c))}. \tag{3.B.62}$$

Hence

$$\left[(\partial_c E)(\partial_c \partial_q P) - (\partial_q E)(\partial_c^2 P) + (\partial_c^2 E)(\partial_q P) \right] \frac{(\partial_c P)}{(\partial_c E)} \leq \frac{1-k}{\beta_2} \frac{\gamma w}{\beta_2} f(\lambda_u)^2 \lambda_u^3 \left[\frac{\gamma w}{\beta_2} - \frac{f(c)}{1-F(c)} \right] < 0, \tag{3.B.63}$$

where the RHS is negative, since the hazard rate $f(c)/(1-F(c))$ of the normal distribution is increasing on $[\mu, \mu + (5/4)\sigma]$, thus

$$\left[\frac{\gamma w}{\beta_2} - \frac{f(c)}{1-F(c)} \right] < 0 \Leftarrow \frac{f(\mu)}{1-F(\mu)} \geq \frac{\gamma w}{\gamma w \mu + b} \tag{3.B.64}$$

$$\Leftrightarrow \sqrt{\frac{2}{\pi}} \frac{1}{\sigma} \geq \frac{\gamma w}{\gamma w \mu + b} \tag{3.B.65}$$

$$\Leftrightarrow b \geq \gamma w \left[\sqrt{\frac{\pi}{2}} \sigma - \mu \right], \tag{3.B.66}$$

which holds since the expression in brackets is negative from assumption A1.

Since $\partial_q L \neq 0$, there is a smooth function, $c^L(q)$, that parameterizes $\{(\partial_c E)(\partial_q P) - (\partial_q E)(\partial_c P) = 0\}$. Using (3.B.50) and (3.B.56), we have

$$(\partial_q L)(\partial_q c^L) + \partial_c L = 0 \Leftrightarrow \partial_q c^L = -\frac{\partial_c L}{\partial_q L} > 0. \tag{3.B.67}$$

Step 5: Unique solution which is one of four cases.

Since $\partial_q c^E > 0$ and $\partial_q c^P < 0$, as shown in step 2 and 4, there is at most one intersection between $\{P = \alpha\}$ and $\{\partial_c E = 0\}$. Likewise, since $\partial_q c^L > 0$ and $\partial_q c^P < 0$, as shown in step 3 and 4, there is at most one intersection between $\{P = \alpha\}$ and $\{L = 0\}$. Also, as we have shown in the proof that $\{L = 0\} \subset \{\partial_c E > 0\}$, so we have $c^L < c^E$. Hence, there are four cases

a) There is no intersection between c^E and c^P and $\{\partial_c E = 0\} \subset \{P < \alpha\}$. Then the unconstrained optimal solution is feasible and therefore chosen.

b) There exists an intersection between c^E and c^P, but none between c^L and c^P. Then $\{L = 0\} \subset \{P < \alpha\}$, since otherwise $\{L = 0\} \subset \{P > \alpha\}$. But since $c^L < c^E$ this would imply $\{\partial_c E = 0\} \subset \{P > \alpha\}$, contradiction. Hence, since there is no intersection between c^L and c^P, there is no internal optimum on the range of optimization $\{\partial_c E \geq 0\} \cap \{P \leq \alpha\}$. But then, since $\partial_c E > 0$ and $\partial_q E > 0$, the firm chooses the point on the constraint with no hedging. The same is true if there is neither an intersection between c^L and c^P nor an intersection between c^E and c^P, and $\{\partial_c E = 0\} \subset \{P > \alpha\}$.

c) There is an intersection between c^L and c^P. Then the solution is the constrained solution, since it is the (internal) optimum.

d) There is neither an intersection between c^L and c^P nor an intersection between c^E and c^P and $\{L = 0\} \subset \{P > \alpha\}$. Then the firm chooses the point on the constraint with most hedging (if such a point still yields positive profits - otherwise the case is not of interest, since there is no feasible profitable solution at all).

□

3.B.3 Proof of Proposition 23

For ease of notation, we omit the subscript for λ and take λ to be λ_u, and omit the subscript for β_1 and take $\beta_1 = \beta_{1m}$. As before in the proofs, we consider the amplitude of the partially hedged exchange rate $q := a - h$, instead of h.

Step 1: Further preliminary properties.

We have

$$\partial_\gamma \partial_c E = (\partial_\gamma \beta_1)(1 - F(c)) - w = -wF(c) < 0 \quad (3.B.68)$$

$$\partial_\gamma \partial_q E = k(\partial_\gamma E[\min(X, c)]) = 0. \quad (3.B.69)$$

With

$$\partial_\gamma \lambda = \frac{w}{\beta_2} \lambda (c - \lambda). \quad (3.B.70)$$

also

$$\partial_\gamma P = f(\lambda)(\partial_\gamma \lambda) = \frac{w}{\beta_2}(c-\lambda)Q \qquad > 0 \tag{3.B.71}$$

$$\begin{aligned}
\partial_\gamma \partial_q P &= \frac{1-k}{\beta_2}\left[\partial_\gamma(\lambda Q) - \frac{wc}{\beta_2}(\lambda Q)\right] && (3.B.72)\\
&= \frac{1-k}{\beta_2}\left[\lambda(\partial_\gamma Q) - \frac{\lambda w}{\beta_2}(\lambda Q)\right] && (3.B.73)\\
&= \frac{1-k}{\beta_2}\Big[\lambda(1+\tau)(\partial_\gamma P) - \underbrace{\frac{w}{\beta_2}(c-\lambda)Q}_{\partial_\gamma P}\lambda\frac{\lambda}{(c-\lambda)}\Big] && (3.B.74)\\
&= \frac{1-k}{\beta_2}\lambda\left[(1+\tau) - \frac{\lambda}{(c-\lambda)}\right](\partial_\gamma P) && (3.B.75)\\
\partial_\gamma \partial_c P &= \frac{w}{\beta_2}\left[\gamma\partial_\gamma Q + \left(1 - \frac{\gamma wc}{\beta_2}\right)Q\right] && (3.B.76)\\
&= \frac{w}{\beta_2}\left[\gamma(1+\tau) + \frac{b}{w}\frac{1}{(c-\lambda)}\right](\partial_\gamma P). && (3.B.77)
\end{aligned}$$

Rearranging (3.B.36) yields

$$\begin{aligned}
\partial_c^2 P &= \frac{\gamma w}{\beta_2}\left(\partial_c Q - \frac{\gamma w}{\beta_2}Q\right) && (3.B.78)\\
&= \frac{\gamma w}{\beta_2}(\partial_c P)(1+\tau) - \left(\frac{\gamma w}{\beta_2}\right)^2 Q && (3.B.79)\\
&= \frac{\gamma w}{\beta_2}(1+\tau)(\partial_q P)\frac{(\partial_c P)}{(\partial_q P)} - \frac{\gamma w}{\beta_2}(\partial_c P) && (3.B.80)\\
&= \frac{\gamma w}{\beta_2}(1+\tau)\frac{\gamma w}{1-k}\frac{1}{\lambda}(\partial_q P) - \frac{\gamma w}{\beta_2}(\partial_c P). && (3.B.81)
\end{aligned}$$

We have

$$\begin{aligned}
\partial_\gamma L &= \underbrace{(\partial_\gamma \partial_c E)}_{<0}\underbrace{(\partial_q P)}_{>0} + \underbrace{(\partial_c E)}_{>0}(\partial_\gamma \partial_q P) - \underbrace{(\partial_\gamma \partial_q E)}_{=0}\underbrace{(\partial_c P)}_{>0} - \underbrace{(\partial_q E)}_{>0}\underbrace{(\partial_\gamma \partial_c P)}_{>0} && (3.B.82)\\
&= (\partial_\gamma \partial_c E)(\partial_q P) + Z && (3.B.83)
\end{aligned}$$

with

$$
\begin{aligned}
Z &:= (\partial_c E)(\partial_\gamma \partial_q P) - (\partial_q E)(\partial_\gamma \partial_c P) \\
&= (\partial_c E)\frac{1-k}{\beta_2}\lambda\left[(1+\tau) - \frac{\lambda}{(c-\lambda)}\right](\partial_\gamma P) - (\partial_q E)\frac{\gamma w}{\beta_2}\left[(1+\tau) + \frac{b}{\gamma w}\frac{1}{(c-\lambda)}\right](\partial_\gamma P) \\
&= (\partial_\gamma P)\frac{(1-k)}{\gamma w}\left[(\partial_c E)(1+\tau)\frac{\gamma w}{\beta_2} - (\partial_q E)(1+\tau)\frac{\gamma w}{\beta_2}\frac{\gamma w}{(1-k)}\right] \\
&\qquad\qquad + (\partial_\gamma P)\left[-\frac{\lambda^2}{(c-\lambda)}\frac{(1-k)}{\beta_2}(\partial_c E) + (\partial_q E)\frac{b}{\beta_2}\frac{1}{(c-\lambda)}\right] \\
&= (\partial_\gamma P)\frac{(1-k)}{\gamma w}\lambda G \\
&\quad + (\partial_\gamma P)\left[-(\partial_q E)\frac{\gamma w}{\beta_2} + k(1-F_c) - \frac{\lambda^2}{(c-\lambda)}\frac{1-k}{\beta_2}(\partial_c E) + (\partial_q E)\frac{b}{\beta_2}\frac{1}{(c-\lambda)}\right] \\
&= \left[G\lambda\frac{(1-k)}{\gamma w}\right](\partial_\gamma P) + H(\partial_\gamma P)
\end{aligned}
$$

with

$$
G := \frac{1}{\lambda}\left[(\partial_c E)(1+\tau)\frac{\gamma w}{\beta_2}\lambda - k(1-F_c)\frac{\gamma w}{1-k} - (\partial_q E)\tau\frac{\gamma w}{\beta_2}\frac{\gamma w}{1-k}\right] \tag{3.B.84}
$$

and

$$
H := k(1-F_c) - (\partial_c E)\frac{\lambda^2}{(c-\lambda)}\frac{(1-k)}{\beta_2} - (\partial_q E)\frac{\gamma w}{\beta_2}\left(1 + \frac{b}{\gamma w(c-\lambda)}\right). \tag{3.B.85}
$$

At the same time for $\partial_c L$, we have with (3.B.48)

$$
\partial_c L = (\partial_c^2 E)(\partial_q P) + N \tag{3.B.86}
$$

with

$$
\begin{aligned}
N &:= (\partial_c E)(\partial_c \partial_q P) - (\partial_q E)(\partial_c^2 P) - (\partial_c \partial_q E)(\partial_c P) \\
&= (\partial_c E)\frac{\gamma w}{\beta_2}(1+\tau)(\partial_q P) - (\partial_q E)\left[(\partial_q P)\frac{\gamma w}{\beta_2}\frac{\gamma w}{1-k}(1+\tau)\frac{1}{\lambda} - \frac{\gamma w}{\beta_2}(\partial_c P)\right] - k(1-F_c)(\partial_q P)\frac{\partial_c P}{\partial_q P} \\
&= \left[(\partial_c E)(1+\tau)\frac{\gamma w}{\beta_2}\lambda - (\partial_q E)(1+\tau)\frac{\gamma w}{\beta_w}\frac{\gamma w}{(1-k)} + (\partial_q E)\frac{\gamma w}{\beta_2}\frac{\gamma w}{(1-k)} - k(1-F_c)\frac{\gamma w}{(1-k)}\right]\frac{\partial_q P}{\lambda} \\
&= \frac{1}{\lambda}\left[(\partial_c E)(1+\tau)\frac{\gamma w}{\beta_2}\lambda - (\partial_q E)\tau\frac{\gamma w}{\beta_w}\frac{\gamma w}{(1-k)} - k(1-F_c)\frac{\gamma w}{(1-k)}\right](\partial_q P) \\
&= G(\partial_q P).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\partial_c L &= \left[(\partial_c^2 E) + G\right](\partial_q P) &\quad (3.B.87)\\
&=: \tilde{G}(\partial_q P). &\quad (3.B.88)
\end{aligned}
$$

Step 2: $\partial_\gamma c^L < 0$ on $\{L = 0\}$, $\partial_\gamma c^E < 0$ and $\partial_\gamma c^P < 0$.

By definition of c^E, we have $\partial_c E(\gamma, c^E(\gamma)) = 0$, hence

$$\partial_\gamma \partial_c E + (\partial_c^2 E)(\partial_\gamma c^E) \;=\; 0 \quad \overset{(3.B.68),(3.B.15)}{\Rightarrow} \quad \partial_\gamma c^E \;=\; -\frac{\partial_\gamma \partial_c E}{\partial_c^2 E} \;<\; 0. \tag{3.B.89}$$

Likewise,

$$\partial_\gamma P + (\partial_c P)(\partial_\gamma c^P) \;=\; 0 \quad \overset{(3.B.71),(3.B.32)}{\Rightarrow} \quad \partial_\gamma c^P \;=\; -\frac{\partial_\gamma P}{\partial_c P} \;<\; 0. \tag{3.B.90}$$

Likewise, from (3.B.56) and (3.B.50) we have

$$\partial_\gamma L + (\partial_c L)(\partial_\gamma c^L) \;=\; 0 \;\Leftrightarrow\; \partial_\gamma c^L \;=\; -\frac{\partial_\gamma L}{\partial_c L} \;<\; 0 \quad \text{on} \quad \{L \;=\; 0\}. \tag{3.B.91}$$

Step 3: $|\partial_\gamma c^L| < |\partial_\gamma c^P|$ and $|\partial_\gamma c^E| < |\partial_\gamma c^P|$.

From (3.B.48) and (3.B.83), we have

$$\begin{aligned}
\partial_\gamma L \;&=\; (\partial_\gamma \partial_c E)(\partial_q P) + Z && (3.B.92)\\
&=\; (\partial_\gamma \partial_c E)(\partial_q P) + \left[\tilde{G}\lambda \frac{(1-k)}{\gamma w}\right](\partial_\gamma P) + H(\partial_\gamma P) - (\partial_c^2 E)\lambda \frac{(1-k)}{\gamma w}(\partial_\gamma P) && (3.B.93)\\
&=\; \tilde{G}\frac{(\partial_q P)}{(\partial_c P)}(\partial_\gamma P) + H(\partial_\gamma P) + \Big[\underbrace{(\partial_\gamma \partial_c E)\frac{\partial_q P}{\partial_\gamma P} - (\partial_c^2 E)\lambda \frac{(1-k)}{\gamma w}}_{:=R}\Big](\partial_\gamma P) && (3.B.94)\\
&=\; \partial_c L \frac{(\partial_\gamma P)}{(\partial_c P)} + (H+R)(\partial_\gamma P), && (3.B.95)
\end{aligned}$$

with

$$\begin{aligned}
R \;&=\; (\partial_\gamma \partial_c E)\frac{\partial_q P}{\partial_\gamma P} - (\partial_c^2 E)\lambda \frac{(1-k)}{\gamma w} && (3.B.96)\\
&=\; (\partial_\gamma \partial_c E)\frac{\partial_q P}{\partial_\gamma P} - (\partial_c^2 E)\frac{\partial_q P}{\partial_c P}. && (3.B.97)
\end{aligned}$$

Hence,

$$-\partial_\gamma c^L \;=\; \frac{\partial_\gamma L}{\partial_c L} \;=\; \frac{\partial_\gamma P}{\partial_c P} + \frac{(H+R)(\partial_\gamma P)}{(\partial_c L)} \;=\; -\partial_\gamma c^P + (H+R)\underbrace{\frac{(\partial_\gamma P)}{(\partial_c L)}}_{<0 \text{ on } \{L=0\}}, \tag{3.B.98}$$

and from step 2 and on $\{L = 0\}$

$$|\partial_\gamma c^L| \;<\; |\partial_\gamma c^P| \;\Leftrightarrow\; -\partial_\gamma c^L \;<\; -\partial_\gamma c^P \;\Leftrightarrow\; (H+R) \;>\; 0. \tag{3.B.99}$$

Similarly,

$$-\partial_\gamma c^E = \frac{(\partial_\gamma \partial_c E)}{(\partial_c^2 E)} = \frac{\partial_\gamma P}{\partial_c P} + \frac{(\partial_c \partial_\gamma E) - (\partial_\gamma P)/(\partial_c P)(\partial_c^2 E)}{(\partial_c^2 E)} = -\partial_\gamma c^P + R \underbrace{\frac{(\partial_\gamma P)}{(\partial_c^2 E)(\partial_q P)}}_{<0}, \tag{3.B.100}$$

and from step 2

$$|\partial_\gamma c^E| < |\partial_\gamma c^P| \Leftrightarrow -\partial_\gamma c^E < -\partial_\gamma c^P \Leftrightarrow R > 0. \tag{3.B.101}$$

It remains to show $R > 0$ and $(H + R) > 0$.

Claim: $R > 0$.

Proof of claim.

$$R = (\partial_\gamma \partial_c E)\frac{\partial_q P}{\partial_\gamma P} - (\partial_c^2 E)\frac{\partial_q P}{\partial_c P} \tag{3.B.102}$$

$$= -F_c(1-k)\frac{\lambda}{(c-\lambda)} + \beta_1 f_c \frac{(1-k)}{\gamma w}\lambda \tag{3.B.103}$$

$$= \frac{(1-k)\lambda}{(1-F_c)}\Big[-\frac{F_c(1-F_c)}{(c-\lambda)} + \underbrace{\frac{\beta_1}{\gamma w}(1-F_c)}_{>1 \text{ since } \partial_c E>0} f_c\Big] \tag{3.B.104}$$

$$\geq \frac{(1-k)\lambda}{(1-F_c)}\Big[-\frac{1}{4(c-\lambda)} + f_c\Big] \tag{3.B.105}$$

From (3.B.20), assumption A2 and A1, we have

$$(c-\lambda) > c - \frac{3}{5}\mu > \frac{2}{5}\mu > 2\sigma. \tag{3.B.106}$$

From assumption A3, we know $c < \mu + (5/4)\sigma$, hence $f_c > 1/(8\sigma)$. Plugged into (3.B.105), this yields $R > 0$.

Claim: $(H + R) > 0$.

Proof of claim. From (3.B.20) we have $(c - \lambda) > (1/3)c$, hence

$$1 + \frac{b}{\gamma w(c-\lambda)} \leq \frac{\gamma w + 3b/c}{\gamma w} \leq \frac{3\beta_2/c}{\gamma w}. \tag{3.B.107}$$

Thus, we have

$$\begin{aligned} H+R \ &\geq \ k(1-F_c) - \left[(\partial_c E)\frac{\lambda^2(1-k)}{(c-\lambda)\beta_2} + (\partial_q E)\frac{3}{c}\right] + \frac{(1-k)\lambda}{(1-F_c)}\left[-\frac{F_c(1-F_c)}{(c-\lambda)} + \frac{\beta_1}{\gamma w}(1-F_c)f_c\right] \\ &\geq \ k\left[(1-F_c) - 3\frac{E[\min(X,c)]}{c}\right] + \frac{(1-k)\lambda}{(1-F_c)}\left[-\frac{F_c(1-F_c)}{(c-\lambda)} + f_c - \frac{\lambda(1-F_c)}{(c-\lambda)\beta_2}(\partial_c E)\right] \\ &\geq \ -3k + \frac{(1-k)\lambda}{(1-F_c)}\Big[-\frac{(1-F_c)}{(c-\lambda)}\underbrace{[F_c + \frac{\lambda}{\beta_2}(\partial_c E)]}_{\leq \lambda/\beta_2(\beta_1-\gamma w)} + f_c\Big] \\ &\geq \ -3k + (1-k)\lambda\left[\frac{f_c}{(1-F_c)} - \frac{\lambda}{(c-\lambda)}\frac{1}{\beta_2}(\beta_1-\gamma w)\right] \end{aligned}$$

Since the hazard rate is increasing for $c \geq \mu$ and

$$\frac{f_\mu}{(1-F_\mu)} = \sqrt{\frac{2}{\pi}}\frac{1}{\sigma} \approx 0.79\frac{1}{\sigma} \geq \frac{3}{4}\frac{1}{\sigma}, \tag{3.B.108}$$

the expression in brackets is positive if

$$\frac{\lambda}{(c-\lambda)}\frac{1}{\beta_2}(\beta_1-\gamma w) \leq \frac{3}{4}\frac{1}{\sigma} \tag{3.B.109}$$

$$\Leftrightarrow \frac{(\beta_1-\gamma w)}{(\beta_1-q)}\frac{3}{4}\sigma \leq (c-\lambda). \tag{3.B.110}$$

But $(c-\lambda) \geq 2\sigma$, hence,

$$\frac{(\beta_1-\gamma w)}{(\beta_1-q)}\frac{3}{4} \leq 2 \Leftrightarrow \frac{q}{w} - \gamma \leq \frac{(\beta_1-\gamma w)}{3w}, \tag{3.B.111}$$

is sufficient for the expression in brackets to be positive. This is ensured by assumptions A4 and A3, since then

$$a \leq \frac{4}{9}(1-w) - \frac{1}{3}kh_{max} \tag{3.B.112}$$

$$\Leftrightarrow a \leq \frac{1}{9}(1-w) + \frac{1}{3}(1-w-kh_{max}) \tag{3.B.113}$$

$$\Rightarrow \bar{\gamma}_{\min} \geq \frac{a}{w} - \frac{(\beta_1-\gamma w)}{3w}. \tag{3.B.114}$$

From (3.B.108), the expression in brackets can be bounded from below by $(\sqrt{2/\pi} - 3/4)(1/\sigma)$. With $\lambda \geq 1$, assumption A5 then ensures $H+R > 0$.

Step 4: The values of γ that lead to case c) are one interval in $[\gamma_{min}, \gamma_{max}]$.

Let

$$\mathbb{D} := \left\{(\gamma,q,c)|\gamma \in [\gamma_{min},\gamma_{max}], q \in [q_{min},a], c \in \mathbb{R}^+\right\} =: \mathbb{D}_1 \times \mathbb{D}_2 \times \mathbb{D}_3 \tag{3.B.115}$$

and consider E and P as functions on $\mathbb{D}$, subsequently also $L = (\partial_c E)(\partial_q E) - (\partial_q E)(\partial_c P)$. Define

$$\mathbb{C}^{LP} := \{L = 0\} \cap \{P = \alpha\}. \tag{3.B.116}$$

$\mathbb{C}^{LP}$ is a smooth submanifold of dimension 1 of $\mathbb{D}$ if everywhere on $\mathbb{C}^{LP}$

$$rank\begin{pmatrix} DL \\ DP \end{pmatrix} = 2. \tag{3.B.117}$$

This is indeed the case since on $\{L = 0\}$

$$\det\begin{pmatrix} \partial_c L & \partial_q L \\ \partial_c P & \partial_q P \end{pmatrix} = (\partial_c L)(\partial_q P) - (\partial_q L)(\partial_c L) < 0. \tag{3.B.118}$$

Hence, for all $x \in \mathbb{C}^{LP}$ one can locally parameterize $\mathbb{C}^{LP}$ via γ. Since from Proposition 22, for each γ, there is at most one (q, c) such that $(\gamma, q, c) \in \mathbb{C}^{LP}$, there is an open subset $I^{LP} \subset [\gamma_{min}, \gamma_{max}]$ such that some $g^{LP} : I^{LP} \to \mathbb{D}^0$ (interior of $\mathbb{D}$) globally parameterizes $\mathbb{C}^{LP} \cap \mathbb{D}^0$ with $g^{LP}(\gamma) = (q^{LP}(\gamma), c^{LP}(\gamma))$.

$\mathbb{C}^{LP}$ is closed in $\mathbb{D}$ and for some large c also bounded on $\mathbb{D}_1 \times \mathbb{D}_2 \times [0, c]$, hence compact. Thus, the boundary of $\mathbb{C}^{LP}$ needs to lie on the boundary of $\mathbb{D}$, hence in

$$\{\gamma_{min}, \gamma_{max}\} \times \mathbb{D}_2 \times \mathbb{D}_3 \quad \cup \quad \mathbb{D}_1 \times \{q_{min}, a\} \times \mathbb{D}_3. \tag{3.B.119}$$

It remains to show that I^{LP} consists of only one interval. For this it suffices to show that $\partial_\gamma q^{LP} < 0$. If I^{LP} consisted of multiple intervals, there were $x_1, x_2 \in \mathbb{C}^{LP}$ with $\partial_\gamma q^{LP}(x_1) < 0 < \partial_\gamma q^{LP}(x_2)$. (Loosely speaking, if there was a gap in I^{LP}, i.e. $\gamma_1 < \gamma_2 < \gamma_3$ such that $\gamma_1, \gamma_3 \in I^{LP}$, but $\gamma_2 \notin I^{LP}$, then $q^{LP}(\gamma_2) \in \{q_{min}, a\}$, hence either bigger or smaller than both $q^{LP}(\gamma_1), q^{LP}(\gamma_3) \in (q_{min}, a)$. Hence, in the first case, $\partial_\gamma q^{LP} < 0$ for some $\gamma > \gamma_1$ and $\partial_\gamma q^{LP} > 0$ for some $\gamma < \gamma_3$.)

Claim: $\partial_\gamma q^{LP} < 0$.

Proof of claim. For some γ_1, consider the plane $\{\gamma_1\} \times \mathbb{D}_1 \times \mathbb{D}_2$ and the corresponding point therein in $\mathbb{C}^{LP}$, namely $(q^{LP}(\gamma_1), c^L(\gamma_1, q^{LP}(\gamma_1)))$. By definition, $c^L(\gamma_1, q^{LP}(\gamma_1)) = c^P(\gamma_1, q^{LP}(\gamma_1))$. For some small $\varepsilon > 0$ consider the plane $\{\gamma_2 = \gamma_1 + \varepsilon\} \times \mathbb{D}_1 \times \mathbb{D}_2$ at the previous level of q, $q^{LP}(\gamma_1)$. Then,

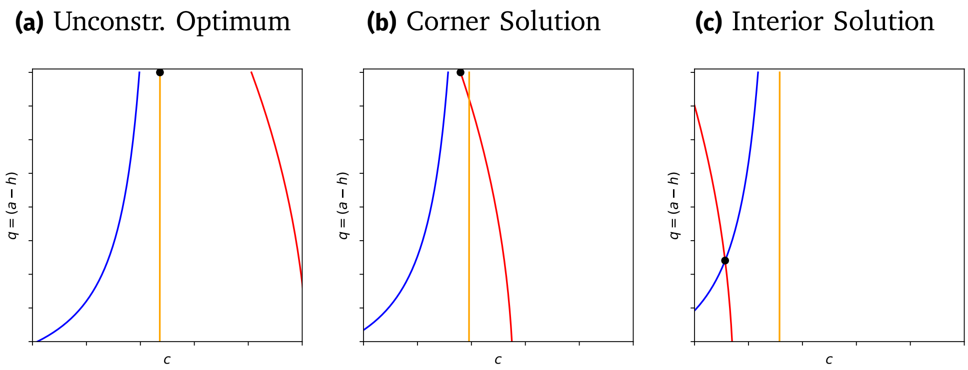
$$\begin{aligned} c^P(\gamma_2, q^{LP}(\gamma_1)) &\approx c^P(\gamma_1, q^{LP}(\gamma_1)) + \varepsilon\partial_\gamma c^P = c^L(\gamma_1, q^{LP}(\gamma_1)) + \varepsilon\partial_\gamma c^P \\ &< c^L(\gamma_1, q^{LP}(\gamma_1)) + \varepsilon\partial_\gamma c^L \approx c^L(\gamma_2, q^{LP}(\gamma_1)), \end{aligned} \tag{3.B.120}$$

since by step 3, $\partial_\gamma c^L > \partial_\gamma c^P$. Since $\partial_q c^P < 0$ and $\partial_q c^L > 0$, the point in $\mathbb{C}^{LP}$ in $\{\gamma_2\} \times \mathbb{D}_1 \times \mathbb{D}_2$ needs to have $q^{LP}(\gamma_2) < q^{LP}(\gamma_1)$. Hence, $\partial_\gamma q^{LP} < 0$.

Step 5: The values of γ that lead to case b) are one interval in $[\gamma_{min}, \gamma_{max}]$. For γ in case b) we already know that $q = a$ and that $c^P(\gamma, a) < c^E(\gamma, a)$. From step 3 we have $\partial_\gamma c^E > \partial_\gamma c^P$. Hence, c^E can cross c^P at most once.

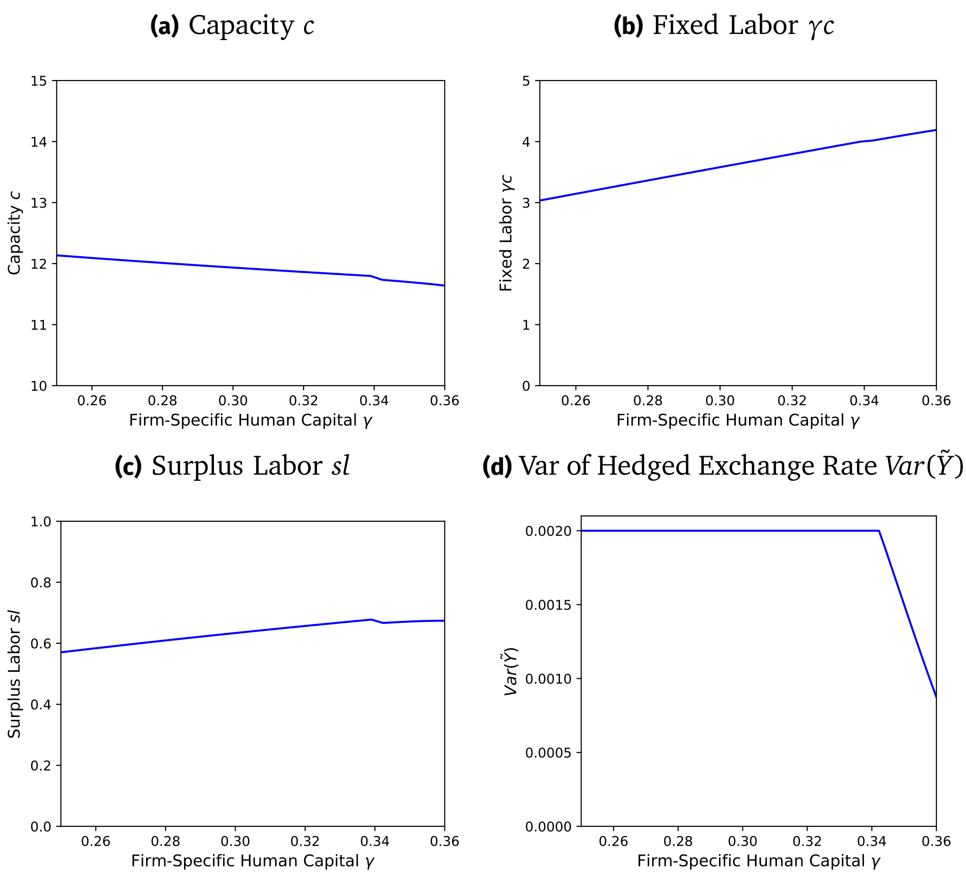
Appendix 3.C Additional Figures and Tables

Figure 3.C.1. Illustration of the Model Solution for Increasing Levels of γ



Notes: This figure illustrates the model solution (black dot) for increasing levels of γ from Panel A to C. Panels A, B and C correspond to cases a), b) and c) in Proposition 22, respectively. On the x-axes the capacity choice, c, and on the y-axis amplitude of the partially hedged exchange rate, $q = (a - h)$ is depicted. In each panel, the blue line corresponds to points on which the Lagrange optimality is satisfied, the yellow line to unconstrained optimal capacity choices for given levels of q and the red line to the points on which the constraint binds.

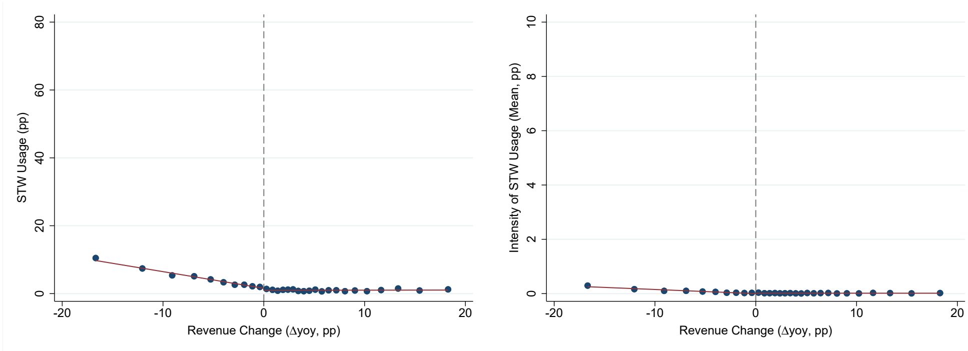
Figure 3.C.2. Model Solution: Alternative Constraint



Notes: This figure shows how optimal *capacity*, c, optimal *fixed labor*, γc, optimal *surplus labor*, $sl = \gamma(c - E[\min(X, c)])$, and, the optimal *variance of the hedged exchange rate*, $Var(\tilde{Y}) = 2p(a - h)^2$ change as a function of firm-specific human capital γ. The constraint considered is $P[CF < 0] < \alpha$. The model is numerically solved for the following set of parameters: $\mu = 10, \sigma = 2, b = 2, a = 0.1, p = 0.1, w = 0.4, \alpha = 0.006, k = 0.005, q_{min} = 0.02$.

Figure 3.C.3. Firm-Level: STW Usage by Revenue Change 2012-2019

(a) STW Usage **(b)** STW Usage Intensity



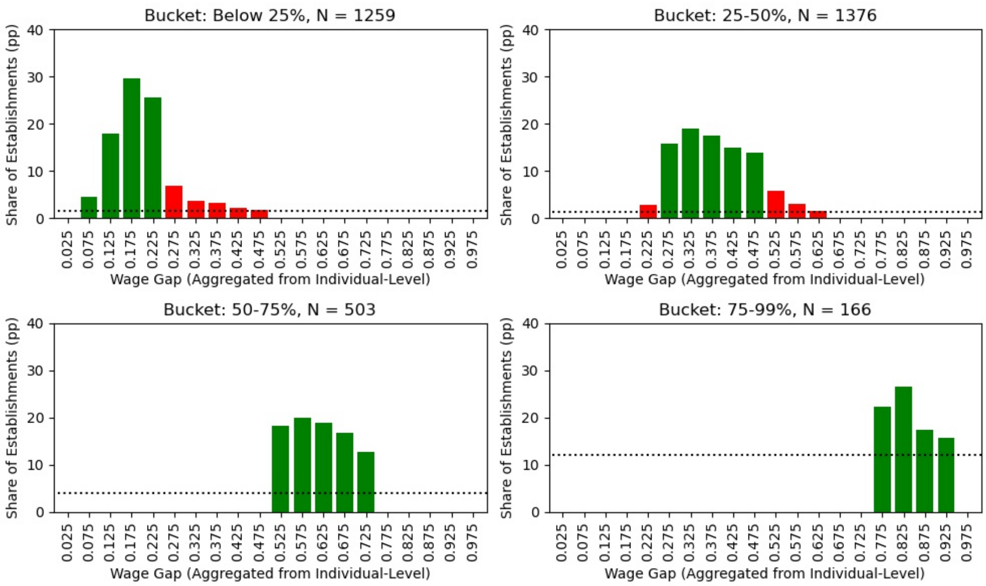
Notes: The figure shows a binned scatterplot of *STW Usage* (Panel A) and *Intensity of STW Usage* (Panel B) against the year-on-year change in revenue. The sample consists of pooled firm-year observations for the years 2012-2019. *Intensity of STW Usage* is defined analogously to the variable *Surplus Labor (2009)*, see Section 3.5 for details.

Table 3.C.1. Firm-Level: STW Usage Beyond the Covid-19 Shock

	STW Usage		Intensity of STW Usage	
	Δ Revenue ≤ 0	Δ Revenue > 0	Δ Revenue ≤ 0	Δ Revenue > 0
	(1)	(2)	(3)	(4)
Revenue Change 19-20	-2.249***	0.184	-0.401***	0.068
	(0.24)	(0.41)	(0.03)	(0.05)
Log Assets	-0.006	-0.005	-0.006***	-0.002*
	(0.01)	(0.02)	(0.00)	(0.00)
Constant	0.561***	0.363	0.111***	0.039**
	(0.21)	(0.35)	(0.02)	(0.02)
R Squared	0.073	0.001	0.177	0.016
R Squared adj	0.071	-0.004	0.175	0.011
N Firms	1,009	452	1,009	452

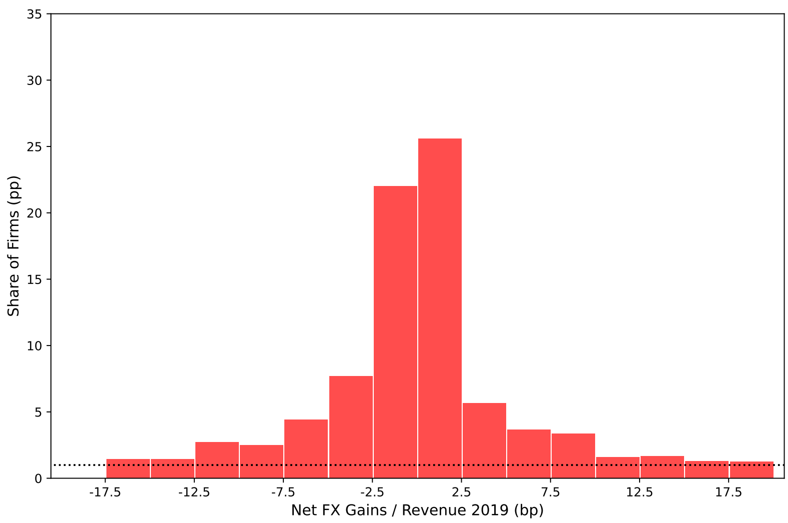
Notes: The table shows the results of a regression of STW Usage (columns 1 and 2) or Intensity of STW Usage (columns 3 and 4) on the relative change in revenue 2019-2020. In each case, firms with a decline in revenue are considered separately from firms with revenue growth. Intensity of STW Usage is defined as *Surplus Labor*, for details see Section 3.5. The sample consists of the manufacturing in order to omit fixed effects, because the constant is of interest.

Figure 3.C.4. Validation of Establishment-Level Data with Individual-Level Data



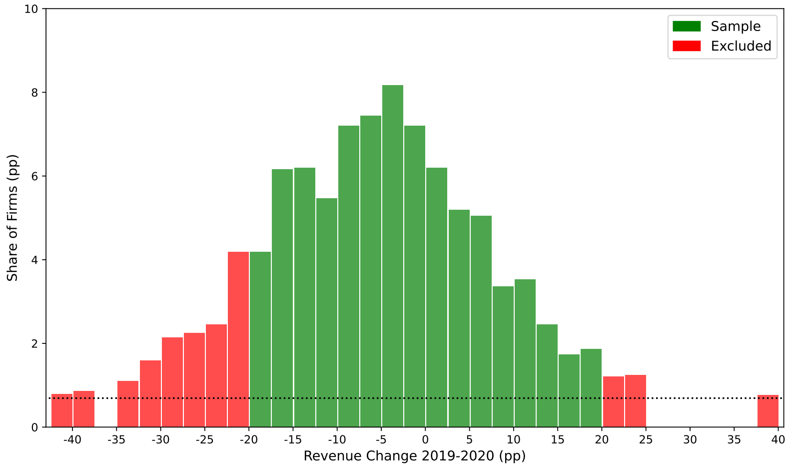
Notes: This figure shows for establishments for which individual-level information is available in 2020 per bucket of the establishment-level variable wage gap (panels), the distribution of the relative wage gap aggregated from individual-level information. Green bars indicate that the variable from aggregated individual-level data falls in the same bucket as the establishment-level variable. No information below the dotted line is available due to data protection (less than 20 establishments).

Figure 3.C.5. Distribution of Net FX Gains to Revenue in 2019



Notes: This figure plots the distribution of *Net FX Gains* relative to revenue in 2019. *Net FX Gains* are FX gains minus FX losses as reported in firms' annual reports. No information below the dotted line is available due to data protection (less than 20 establishments). 8% of firms have zero net FX gains.

Figure 3.C.6. Distribution of the Revenue Change 2019-2020



Notes: This figure shows the distribution of the relative change in revenue from 2019 to 2020 in the sample before the restriction on the drop in revenue is imposed. No information below the dotted line is available due to data protection (less than 20 establishments).

Table 3.C.2. Share with Vocational Training and Other Firm Characteristics

	Share Vocational Training			
	(1)	(2)	(3)	(4)
ROA (pp)	-0.000			
	(0.00)			
Cash/Assets		-0.114***		
		(0.03)		
Value Added per Employee			-0.008***	
			(0.00)	
1(Exports to Outside Europe)				-0.002
				(0.01)
Industry x Region FEs	Yes	Yes	Yes	Yes
R Squared	0.433	0.438	0.443	0.360
R Squared adj	0.414	0.419	0.420	0.331
N Firms	2,319	2,319	1,640	1,163

Notes: The table shows the results of a cross-sectional regression of the firm-level share with vocational training on various other firm characteristics. Variables are defined as of 2019. Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.C.3. Share with Vocational Training and the Export Share

	Share Vocational Training		
	(1)	(2)	(3)
Export Share	0.000**	-0.001	0.000
	(0.00)	(0.00)	(0.00)
Firm FEs	Yes	No	Yes
Year FEs	No	Yes	Yes
R Squared	0.965	0.003	0.967
R Squared adj	0.959	0.002	0.962
N Observations	10,991	10,991	10,991
N Firms	1,678	1,678	1,678

Notes: The table shows the results of a panel regression of the firm-level share with vocational training on the export share, defined as foreign revenue relative to revenue. The regression is based on the subset of the baseline sample for which panel information is available between 2010 and 2019. Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.C.4. Summary Statistics for Firms With a High/ Low Shortage Share

	Low Shortage Share					High Shortage Share					t-test Means
	Mean	p10	p50	p90	N	Mean	p10	p50	p90	N	
Core Financial Information (2019)											
Assets (mil EUR)	265.80	13.73	44.99	253.40	1176	345.71	14.62	47.08	276.55	1176	0.64
Revenue (mil EUR)	217.29	23.09	78.21	333.95	1176	256.20	21.54	68.36	332.99	1176	0.61
Employees	323.50	37.00	172.00	605.00	1176	577.97	87.00	277.00	866.00	1176	0.01
Equity/Assets (pp)	40.52	6.63	41.15	76.33	1176	40.94	9.72	41.17	77.02	1176	0.75
Cash/Assets (pp)	9.07	0.02	3.73	25.51	1176	9.99	0.04	4.70	27.64	1176	0.09
ROA (pp)	7.75	-3.66	6.11	22.31	1176	7.14	-5.78	6.18	21.14	1176	0.28
Value Added per Employee (mil EUR)	0.24	0.05	0.10	0.25	818	0.10	0.06	0.09	0.15	843	0.06
Information on Exports and FX-Volatility											
Export Share	0.42	0.06	0.40	0.80	1176	0.47	0.07	0.48	0.82	1176	0.00
1(Export Outside Europe)	0.76	0.00	1.00	1.00	557	0.87	0.00	1.00	1.00	635	0.00

Notes: The table reports firm-level summary statistics separately for firms with a high (above-median) and low shortage share.

Table 3.C.5. Shortage Share and Other Firm Characteristics

	Shortage Share			
	(1)	(2)	(3)	(4)
ROA (pp)	-0.000			
	(0.00)			
Cash/Assets		0.033		
		(0.02)		
Value Added per Employee			-0.005***	
			(0.00)	
1(Exports to Outside Europe)				0.014
				(0.01)
Industry x Region FEs	Yes	Yes	Yes	Yes
R Squared	0.182	0.182	0.193	0.191
R Squared adj	0.156	0.156	0.160	0.155
N Firms	2,319	2,319	1,640	1,163

Notes: The table shows the results of a cross-sectional regression of the firm-level shortage share on various other firm characteristics. Variables are defined as of 2019. Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.C.6. Robustness: Subset of Firms With (Almost) No Drop in Revenue

	Dependent Variable							
	OLS		OLS		OLS		OLS	
	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss
Surplus Labor	-0.254	-0.188	-0.162	-0.058	-0.228	-0.170	-0.776*	-1.321**
	(0.34)	(0.70)	(0.32)	(0.68)	(0.33)	(0.70)	(0.41)	(0.58)
Log Assets	0.059***	0.112***	0.059***	0.112***	0.062***	0.114***	0.055**	0.111**
	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)	(0.03)	(0.04)
Export Share	0.480***	0.737***	0.480***	0.736***	0.473***	0.732***	0.505***	0.776***
	(0.08)	(0.13)	(0.08)	(0.14)	(0.08)	(0.14)	(0.10)	(0.17)
Revenue Change 19-20			0.563*	0.799*				
			(0.30)	(0.45)				
Cash/Assets					0.185	0.131		
					(0.18)	(0.23)		
Value Added per Employee							-0.013	-0.028
							(0.02)	(0.03)
Industry x Region FEs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R squared	0.135	0.115	0.138	0.118	0.136	0.116	0.154	0.137
R squared adj	0.087	0.067	0.090	0.069	0.088	0.066	0.097	0.079
N Firms	1,265	1,265	1,265	1,265	1,265	1,265	890	890

Notes: This table reports the estimated OLS coefficients from specification (R1) on the sample restricted to firms with a year-on-year change in revenue in 2020 in the range of $[-5\%, 20\%]$. Two versions of the dependent variable *FX-Induced CF Volatility* are considered: *sd net gains* and *max net loss*. For details on the definition of the variables *Surplus Labor* and measures for *FX-Induced CF Volatility* (*sd net gains*, *max net loss*) see Section 3.5. Control variables are as of 2019 (or available information in Dafne as of May 2022 for *Export Share*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3.C.7. Robustness: Subset of Firms with Export Destinations Information Exporting to Outside of Europe

(a) Instrument: Share with Vocational Training

	Dependent Variable							
	2SLS		2SLS		2SLS		2SLS	
	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss
Surplus Labor	-6.772	-16.487*	-8.218	-19.703*	-6.465	-16.966*	-7.984	-20.237**
	(4.90)	(8.44)	(6.25)	(11.09)	(5.27)	(9.11)	(5.67)	(10.12)
Log Assets	0.027	0.019	0.020	0.004	0.030	0.015	0.008	-0.026
	(0.03)	(0.06)	(0.04)	(0.07)	(0.04)	(0.06)	(0.05)	(0.09)
Export Share	0.618***	0.942***	0.622***	0.952***	0.605***	0.962***	0.697***	1.223***
	(0.13)	(0.24)	(0.14)	(0.25)	(0.14)	(0.25)	(0.22)	(0.40)
Revenue Change 19-20			-1.475	-3.281				
			(1.26)	(2.29)				
Cash/Assets					0.156	-0.243		
					(0.30)	(0.51)		
Value Added per Employee							0.081	-0.041
							(0.10)	(0.03)
Industry x Region FEs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Instrument 1st Stage	.051	.051	.041	.041	.048	.048	.054	.054
Partial R Squared 1st Stage	.01	.01	.007	.007	.001	.001	.014	.014
Kleibergen-Paap F statistic	11.294	11.294	8.224	8.224	9.957	9.957	9.730	9.730
Anderson-Rubin Chi-Squared p-value	0.135	0.018	0.141	0.022	0.189	0.023	0.124	0.012
N Firms	957	957	957	957	957	957	706	706

(b) Instrument: Shortage Share

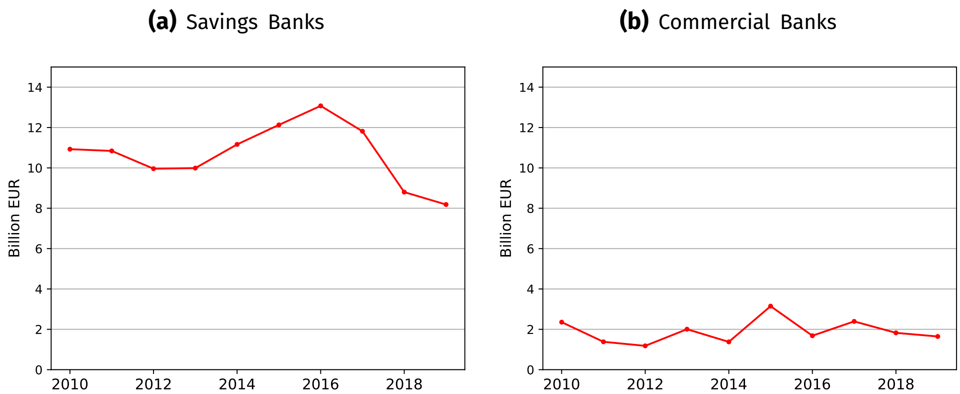
	Dependent Variable							
	2SLS		2SLS		2SLS		2SLS	
	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss	sd net gains	max net loss
Surplus Labor	-7.106**	-10.507**	-7.745**	-11.235**	-7.074**	-10.509**	-5.524*	-10.200**
	(3.19)	(4.87)	(3.60)	(5.50)	(3.21)	(4.91)	(3.10)	(5.19)
Log Assets	0.026	0.046	0.022	0.042	0.027	0.046	0.024	0.037
	(0.03)	(0.04)	(0.03)	(0.05)	(0.03)	(0.05)	(0.04)	(0.06)
Export Share	0.622***	0.866***	0.617***	0.860***	0.614***	0.867***	0.631***	0.951***
	(0.12)	(0.19)	(0.12)	(0.19)	(0.12)	(0.19)	(0.16)	(0.26)
Revenue Change 19-20			-1.380*	-1.574				
			(0.75)	(1.20)				
Cash/Assets					0.134	-0.011		
					(0.24)	(0.35)		
Value Added per Employee							0.087	-0.018
							(0.10)	(0.02)
Industry x Region FEs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Instrument 1st Stage	.05	.05	.046	.046	.05	.05	.052	.052
Partial R Squared 1st Stage	.012	.012	.01	.01	.003	.003	.014	.014
Kleibergen-Paap F statistic	11.626	11.626	10.627	10.627	11.480	11.480	11.029	11.029
Anderson-Rubin Chi-Squared p-value	0.003	0.006	0.004	0.008	0.004	0.006	0.039	0.017
N Firms	957	957	957	957	957	957	706	706

Notes: This table reports the estimated coefficients from specification (R1) instrumenting *Surplus Labor* with *Share Vocational Training*, the share of employees in vocational training, in Panel A and with *Shortage Share*, the share of employees in shortage occupations, in Panel B. The sample is restricted to firms with export destinations information that export to outside of Europe. Two versions of the dependent variable *FX-Induced CF Volatility* are considered: *sd net gains* and *max net loss*. For details on the definition of the variables *Surplus Labor* and measures for *FX-Induced CF Volatility* (*sd net gains*, *max net loss*) see Section 3.5. Control variables are as of 2019 (or available information in Dafne as of May 2022 for *Export Share*). Robust standard errors are reported in parentheses. Stars denote statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Figure 3.C.7. Banks that Stopped Selling FX Derivatives by Type of Bank

Notes: The figure shows the number of banks that stopped offering FX derivatives over time per type of bank (Savings Banks (*Sparkassen*), Commercial Banks (*Volksbanken*), major German banks (*Deutsche Bank, Commerzbank, Unicredit*) and other). The depicted year corresponds to the last year a bank reported outstanding FX derivatives on behalf of clients in their annual report (for details on the data construction see Appendix 3.A.5).

Figure 3.C.8. Outstanding Amounts of FX Derivatives by Type of Bank



Notes: The figure shows the outstanding amounts of FX derivatives aggregated per banking group: savings banks (*Sparkassen*) in Panel A and commercial banks (*Volksbanken*) in Panel B. For details on the construction of the dataset see Appendix 3.A.5.