

Book Review

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Willi Freeden, M. Zuhair Nashed: Recovery methodologies: Regularization and sampling

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Willi Freeden is a professor of Geomathematics at the University of Kaiserslautern and has published many books as an author or an editor. He is an editor-in-chief of the International Journal of Geomathematics, the Handbook of Geomathematics, the Handbook of (Deep and Near Surface) Geothermal Energy, the Encyclopedia Frontiers In Entropy (with M.Z. Nashed), the Handbook of Geodesy (with R. Rummel), the book series Geosystems Mathematics, besides other book series. M. Zuhair Nashed is a professor of Mathematics at the University of Central Florida, Orlando, and has held distinguished visiting scholar positions at many universities around the world. He is an editor-in-chief of the journals Numerical Functional Analysis and Optimization and Sampling Theory in Signal Processing, the Handbook of Mathematical Geodesy (with W. Freeden), the Birkhäuser book series Geomathematics and Computing, the World Scientific book series Contemporary Mathematics, besides other book series. The basis of geoscience and geodesy are measurements and in this way the bridge to this book. Wherever there are data and observations it becomes mathematically. Mathematics serves here as a key for modelling and interpretation of geomagnetic or gravity measurements. The decisive question “Why should a geodesist or geoscientist study this book?” can shortly be answered: “The recovery of objects from a set of data demands key solvers of inverse and sampling problems.” Advances in several areas of applied mathematics have revealed common threads between them. A unifying description of the underlying mathematical ideas and concepts for their application e.g. in gravimetry is therefore necessary. Although essential features of practical relevance may be based on arbitrary data systems and conventional regularization procedures such as least-squares method and pseudoinverse,

regularization in function spaces (e.g., Tikhonov regularization), regularization by control-dimensionality, iterative regularization, filter methods, Backus-Gilbert methods, stochastic regularization, non-linear methods, etc. Besides this material under a synoptic recovery scope, an essential decision by the authors of the book additionally was to look at simply structured input data systems. In fact, the price the book is willing to accept for establishing progress in methodological efficiency and economy is the restriction to observational and/or measurement data systems showing a uniform pattern. As a consequence, the far-reaching constraint to use uniform data systems opens manifold perspectives, namely to develop affordable mollifier regularization strategies in inverse problem theory of, e.g., gravimetry and magnetometry, to point out new wavelet Shannon-type sampling for reproduction techniques by means of exponentials using so-called generalized measurements and, last but not least, to circumvent any kind of numerical calamities with large linear systems of equations, in particular instabilities with the solution. As a significant background, new sampling methods are presented within the recovery context to achieve better results in comparison to the methods used so far. All in all, the book represents more than a consistent overview of developments in the field of recovery problems in the form of a monograph. It may be understood as a compendium to handle physics-driven problems for potential fields, e.g., in gravitation and magnetics, particularly restricted to uniformly given input data. The work as such is meant as a textbook for a broad spectrum of scientists and engineers interested in progressive recovery facets. This circumstance undoubtedly is the reason why the book, besides the research related fields to geodesy and geo-magnetics, can be used in teaching a graduate course about recovery strategies for students and scientists from geosciences, all branches of engineering and science. It is demonstrated that advances in theory lead to new developments in other mathematical as well as scientific branches. In order to keep the book manageable, proofs are omitted. An extensive bibliography also including broad-ranging facets of geodetic application can be found at the end of the book. Everyone who wants to apply or is using the aforementioned

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methods in order to make full use of the possibilities these new methods offer in getting better results for the exploitation of his data respectively his measurements is invited, to look into the mathematical background for solving

inverse and sampling problems. From the reviewer's standpoint of view the book can be recommended for all scientists, who deal with the various questions of recovery problems. *Bertold.witte@uni-bonn.de.