Circumplex Analysis: Addressing Methodological Challenges of Data With Circular Structure

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Abstract

Many psychological traits, such as interpersonal behavior, human values, and affect, can be described by a circular structure (e.g. Gurtman, 1993, 2009; Kiesler, 1983; Leary, 1957; Russell et al., 1989; Schwartz & Boehnke, 2004; Schwartz et al., 2012; Stanisławski et al., 2021; Wiggins, 1979). Circumplex models make assumptions about the geometric distribution of variables, which translate into correlational patterns (L. Guttman, 1954; Rounds et al., 1992; Tracey, 1997) or factor loadings on two orthogonal axes (Acton & Revelle, 2004). Factor analysis and principal component analysis (PCA) are commonly used in circumplex analysis (Fabrigar et al., 1997; Tracey, 2000). For example, the interpersonal circumplex is mapped on the axes Dominance and Love (Gurtman, 1993, 1995; Gurtman & Pincus, 2000), providing the theoretical basis for widely used measures like the Interpersonal Adjective Scales (IAS; Jacobs & Scholl, 2005; Wiggins et al., 1988) and the Inventory of Interpersonal Problems (IIP; Horowitz et al., 1988, 2017). Many models also incorporate an overarching general factor, such as overall interpersonal distress in the IIP. The IAS, IIP, and other circumplex instruments comprise eight subscales, ideally evenly spaced, creating a configuration with two competing simple structures (double simple structure), in which two rotational positions align the circumplex axes with four subscales each. A challenge in developing circumplex instruments is the conceptual and geometrical proximity of subscales, which complicates generating items that comprehensively represent each subscale without excessive overlap. Moreover, most circumplex techniques are only suited for the subscale level. However, circumplexity should hold for items as well because scale-level circumplexity can be an artifact resulting from intentional item aggregation. Researchers often assign items to subscales by visually dividing item plots from smallest space analysis (e.g., Perrinjaquet et al., 2007; Redeker et al., 2014; Schwartz et al., 2012) or PCA (e.g., Hatcher & Rogers, 2009; Horowitz et al., 2017; Locke, 2000, 2019; Wiggins et al., 1988), which lacks objectivity and does not ensure optimal spacing. Traditional cluster analysis provides an objective criterion by minimizing within-cluster distances but does not consider the required circular arrangement. In light of the popularity of the circumplex factor model, this thesis examined factor analytic and PCA approaches in circumplex settings. Moreover, given the challenges associated with assigning items to circumplex subscales, a new clustering method tailored to circumplex data was developed, designed to accommodate the circular structure in the idenfication of subscales. For these purposes, three studies were conducted.

Study 1 addressed local optima in factor rotation, a well-known issue for various rotation procedures (Weide & Beauducel, 2019). Although perfect circumplexity implies rotational invariance, there are preferred rotational positions of the circumplex axes when the variables are not uniformly distributed on the circumplex. For example, the two rotational positions in double simple structure represent two optimal solutions according to the rotation criterion. A simulation study compared Varimax rotation based on the gradient projection algorithm (GPA) as a newer method (Bernaards & Jennrich, 2005) to the traditional Kaiser algorithm (Kaiser, 1958) in PCA. The design included orthogonal simple structure models and more complex double simple structure (circumplex) models featuring global and local optima. Results showed that GPA-Varimax generally matched the Kaiser algorithm's performance—also in an empirical example—but was vulnerable to local optima in some circumplex conditions. Using multiple random starts and selecting the solution with the highest Varimax criterion successfully overcame this problem, resulting in better performance than the Kaiser algorithm. These findings indicate that GPA-Varimax, combined with multiple starts, is a valuable alternative, particularly in the presence of local optima as in circumplex structures.

Study 2 modeled the higher-level structure of the IIP with the two circumplex factors (Dominance and Love) and the general factor of interpersonal difficulties (Distress) on the eight subscales in empirical data from 822 participants (Weide et al., 2021). Compared with confirmatory factor analysis based on traditional maximum-likelihood estimation with fixed circumplex loadings, Bayesian confirmatory factor analysis (BCFA) showed greater model fit, more robust parameter estimates, and fewer correlations between error terms. Exploratory factor analysis with target rotation was also examined but showed greater deviations from the ideal circumplex than BCFA without clear advantages in model fit. The study further evaluated higher-level scores for Dominance, Love, and Distress: Regression factor scores, BCFA posterior means, and weighted sum scores were examined, all of which produced highly reliable scores that closely matched the underlying factors and retained the circumplex structure. External validity was supported by expected associations with Big Five traits—Agreeableness, Extraversion, and Neuroticism—as well as with narcissism. The findings highlight the benefits of Bayesian modeling for circumplex structures and support using higher-level scores for the IIP.

The third and largest research project addressed the lack of a dedicated, objective method for clustering items into circumplex subscales (Weide et al., 2025). To this end, I developed ClusterCirc, a clustering method that optimizes circumplexity at both the item and subscale level using an objective spacing criterion. ClusterCirc minimizes within-cluster distances of items and yields clusters that approximate an even distribution of subscales in the circle. Simulation results demonstrated that ClusterCirc consistently outperformed traditional Ward's cluster analysis and k-means clustering in recovering circumplex clusters. This advantage was most prominent for larger within-cluster distances but was also found under conditions with unequal cluster spacing and an additional general factor alongside the circumplex axes. To illustrate its practical value, ClusterCirc was applied to unused data of a German IAS version from Study 2, clustering 52 out of 64 items according to the original subscales and producing subscales with improved circumplex fit and sound psychometric properties. I developed an R package (https://github.com/ancleo/ClusterCirc) and SPSS syntax (https://github.com/ancleo/ClusterCirc_SPSS) for ClusterCirc with three core functions: ClusterCirc-Data for empirical circumplex clustering, ClusterCirc-Simu for a tailored simulation study to evaluate circumplex fit of the data, and ClusterCirc-Fix to compute circumplex indices for user-defined item clusters (e.g., in the case of pre-existing subscales).

Despite these advances, the thesis‘ research has limitations, including the exclusive use of nonrandom and nonclinical samples and the short length of some measures. Future studies could extend the research to other circular constructs such as values or affect, examine more complex scenarios and violations of circumplex assumptions, and test additional analytic techniques and models. Moreover, future work could extend factor extraction methods to accommodate circumplex assumptions and investigate whether a combination of rotational procedures and ClusterCirc could optimize circumplexity more effectively than using either method alone. Furthermore, ClusterCirc could be expanded to incorporate varying theoretical assumptions, for example, regarding specific violations of circumplex spacing and expected item heterogeneity within clusters.

Zusammenfassung

Viele psychologische Merkmale, wie interpersonelles Verhalten, menschliche Werte und Affekt, lassen sich durch eine Kreisstruktur beschreiben (z. B. Gurtman, 1993, 2009; Kiesler, 1983; Leary, 1957; Russell et al., 1989; Schwartz & Boehnke, 2004; Schwartz et al., 2012; Stanisławski et al., 2021; Wiggins, 1979). Solche Circumplex-Modelle treffen Annahmen über die geometrische Anordnung von Variablen, die sich in charakteristischen Korrelationsmustern (L. Guttman, 1954; Rounds et al., 1992; Tracey, 1997) oder Faktorladungen auf zwei orthogonalen Achsen niederschlagen (Acton & Revelle, 2004). Häufig in der Analyse von Circumplex-Modellen eingesetzte Verfahren sind die Faktorenanalyse und die Hauptkomponentenanalyse (HKA; Fabrigar et al., 1997; Tracey, 2000). So wird etwa der interpersonelle Circumplex auf den Achsen Dominanz und Liebe abgebildet (Gurtman, 1993, 1995; Gurtman & Pincus, 2000) und bildet die theoretische Grundlage für weit verbreitete Instrumente wie die Interpersonellen Adjektivskalen (IAS; Jacobs & Scholl, 2005; Wiggins et al., 1988) und das Inventar zur Erfassung Interpersoneller Probleme (IIP; Horowitz et al., 1988, 2017). Viele Modelle integrieren außerdem einen übergeordneten Generalfaktor, beispielsweise das übergeordnete Maß an interpersonellen Schwierigkeiten im IIP. Die IAS, das IIP und andere Circumplex-Instrumente bestehen jeweils aus acht Subskalen, die idealerweise gleichmäßig über den Kreis verteilt sind. So ergibt sich eine doppelte Einfachstruktur, bei der zwei Rotationspositionen die Circumplex-Achsen jeweils an vier Subskalen ausrichten. Eine zentrale Herausforderung bei der Entwicklung von Circumplex-Instrumenten besteht in der inhaltlichen und geometrischen Nähe der Subskalen. Diese erschwert die Itemgenese, da Items ihre Subskala möglichst umfassend, aber ohne übermäßige Überlappung mit benachbarten Subskalen repräsentieren sollen. Zudem sind die meisten Circumplex-Analysetechniken nur auf Subskalenebene anwendbar. Allerdings sollte Circumplexität auch auf Itemebene vorliegen, da Circumplexität auf Skalenebene ein Artefakt intentionaler Itemaggregation sein kann. Forschende sortieren Items oft durch eine visuelle Unterteilung von Itemplots aus der Smallest Space Analyse (z. B. Perrinjaquet et al., 2007; Redeker et al., 2014; Schwartz et al., 2012) oder HKA (z. B. Hatcher & Rogers, 2009; Horowitz et al., 2017; Locke, 2000, 2019; Wiggins et al., 1988) in Subskalen – ein Vorgehen, dem es an Objektivität mangelt und welches keine optimalen Circumplex-Abstände garantiert. Traditionelle Clusteranalysen bieten zwar ein objektives Kriterium durch die Minimierung von Itemdistanzen innerhalb

von Clustern, berücksichtigen jedoch nicht die Kreisstruktur. In Anbetracht der Popularität des Circumplex-Faktormodells untersuchte diese Dissertation faktorenanalytische und HKA- Ansätze im Circumplex-Kontext. Zusätzlich wurde angesichts der Schwierigkeiten bei der Zuordnung von Items zu Circumplex-Subskalen eine neue, auf Circumplex-Daten zugeschnittene Clustermethode entwickelt, die darauf ausgelegt ist, die Kreisstruktur bei der Identifikation von Subskalen zu berücksichtigen. Zu diesen Zwecken wurden drei Studien durchgeführt.

Studie 1 befasste sich mit dem Problem lokaler Optima in der Rotation von Faktoren, einem bekannten Phänomen verschiedener Rotationsverfahren (Weide & Beauducel, 2019). Obwohl perfekte Circumplexität rotationale Invarianz impliziert, können bevorzugte Ausrichtungen der Circumplex-Achsen bestehen, wenn Variablen nicht perfekt gleichmäßig über den Kreis verteilt sind. Beispielsweise stellen die beiden Rotationspositionen der doppelten Einfachstruktur zwei optimale Lösungen nach dem Rotationskriterium dar. In einer Simulationsstudie wurde die Varimax-Rotation auf Basis des Gradientenprojektionsalgorithmus (GPA) als relativ neue Methode (Bernaards & Jennrich, 2005) mit dem traditionellen Kaiser-Algorithmus (Kaiser, 1958) in der HKA verglichen. Das Design umfasste orthogonale Einfachstrukturen und komplexere doppelte Einfachstrukturen (Circumplexe) mit globalen und lokalen Optima. GPA-Varimax erzielte im Allgemeinen ähnliche Lösungen wie der Kaiser-Algorithmus – auch in einem empirischen Beispiel –, war jedoch in bestimmten Circumplex-Bedingungen anfällig für lokale Optima. Durch die Verwendung mehrerer Zufalls-Startwerte und die Auswahl der Lösung mit dem höchsten Varimax-Kriterium konnte dieses Problem überwunden werden, so dass auch die Rotationsleistung des Kaiser-Algorithmus übertroffen wurde. Diese Befunde zeigen, dass GPA-Varimax mit mehreren Zufalls-Startwerten eine nützliche Alternative darstellt, insbesondere im Falle von lokalen Optima wie bei Circumplex-Strukturen.

Studie 2 modellierte die Struktur höherer Ordnung des IIP mit den beiden Circumplex-Faktoren (Dominanz und Liebe) sowie dem Generalfaktor (interpersoneller) Stress auf den acht Subskalen anhand der Daten von 822 Teilnehmenden (Weide et al., 2021). Im Vergleich zur traditionellen konfirmatorischen Faktorenanalyse basierend auf Maximum-Likelihood-Schätzung mit fixierten Circumplex-Ladungen zeigte die Bayes'sche konfirmatorische Faktorenanalyse (BCFA) einen besseren Modellfit, robustere Parameterschätzungen und weniger Korrelationen zwischen Fehlertermen. Auch eine explorative Faktorenanalyse mit Zielrotation wurde untersucht, wich jedoch stärker als die BCFA vom idealen Circumplex ab, ohne den Modellfit substantiell zu verbessern. Weiterhin wurden für die drei Faktoren Scores höherer Ordnung – Regressions-Faktorscores, BCFA-Posterior-Mittelwerte und gewichtete Summenscores – evaluiert. Alle Varianten produzierten hoch reliable Scores, die eng mit den zugrundeliegenden Faktoren übereinstimmten und die Circumplex-Struktur gut abbildeten. Die externe Validität der Faktorenmodelle und der Scores wurde durch erwartete Zusammenhänge mit den Big-Five-Dimensionen Verträglichkeit, Extraversion und Neurotizismus sowie mit Narzissmus gestützt. Die Ergebnisse unterstreichen die Vorteile Bayes'scher Modellierung für Circumplex-Strukturen und befürworten die Nutzung von Scores höherer Ordnung für das IIP.

Das dritte und umfangreichste Forschungsprojekt adressierte das Fehlen einer objektiven Methode zur Clusterbildung von Items für Circumplex-Subskalen (Weide et al., 2025). Zu diesem Zweck wurde ClusterCirc entwickelt, eine Clustermethode, die Circumplexität auf Item- und Subskalenebene unter Verwendung eines objektiven Abstandskriteriums optimiert. ClusterCirc minimiert die Distanzen innerhalb von Clustern und erzeugt Cluster, die eine gleichmäßige Verteilung der Subskalen im Kreis annähern. Simulationsergebnisse zeigten, dass Circumplex-Cluster konsistent besser durch ClusterCirc als durch die traditionelle Ward-Clusteranalyse oder k-means-Clustering identifiziert werden konnten. Dieser Vorteil war besonders bei größeren Itemdistanzen innerhalb von Clustern ausgeprägt, zeigte sich aber auch bei ungleichen Clusterabständen und bei einem zusätzlichen Generalfaktor neben den Cicumplex-Achsen. Zur Demonstration des praktischen Nutzens wurde ClusterCirc auf eine deutsche IAS-Version von ungenutzten Daten aus Studie 2 angewendet. Dabei gruppierte ClusterCirc 52 von 64 Items entsprechend den Original-Subskalen und produzierte Subskalen mit verbessertem Circumplex-Fit und guten psychometrischen Eigenschaften. ClusterCirc ist als R-Paket (https://github.com/ancleo/ClusterCirc) und als SPSS-Code (https://github.com/ancleo/ClusterCirc_SPSS) mit drei Hauptfunktionen verfügbar: ClusterCirc-Data für empirische Clusteranalysen, ClusterCirc-Simu für eine simulationsbasierte Evaluation des Circumplex-Fit der Daten und ClusterCirc-Fix zur Berechnung von Circumplex-Indizes für benutzerdefinierte Itemcluster.

Trotz dieser Beiträge weist die Forschung dieser Dissertation Limitationen auf, darunter die ausschließliche Verwendung nichtrandomisierter und nichtklinischer

Stichproben sowie die geringe Itemzahl einiger Instrumente. Künftige Studien könnten die Forschung auf andere zirkuläre Konstrukte (z. B. Werte oder Affekt) ausweiten, komplexere Szenarien und Verletzungen der Circumplex-Annahmen untersuchen sowie weitere analytische Verfahren und Modelle testen. Zudem könnten Faktorextraktionsmethoden so erweitert werden, dass sie Circumplex-Annahmen berücksichtigen, und es könnte geprüft werden, ob eine Kombination von Rotationsverfahren und ClusterCirc Circumplexität effektiver optimieren kann als der Einsatz jeder einzelnen Methode. Außerdem könnte ClusterCirc erweitert werden, um verschiedene theoretische Annahmen zu integrieren, etwa bezüglich spezifischer Abweichungen von idealen Abständen im Kreis oder erwarteter Itemheterogenität innerhalb von Clustern.

1. Introduction

1.1. The Circle and Circular Models in Science

The circle is one of the most fundamental and enduring forms in human culture and understanding. Objects like coins, plates, wheels, and clocks all come in circular shapes, having played a major role throughout history and continuing to shape everyday life. Mathematically, the circle is defined as the set of all points equidistant from a central point in a plane—a definition both simple and profound (Stewart, 2016). The properties of circles have fascinated scholars since antiquity. Babylonian clay tablets dating back to around 1700 B.C.E. include approximations of the area of a circle using a value close to π (Robson, 2008). By the time of Euclid's *Elements* (original work ca. 300 B.C.E.; Euclid, 1956), circular geometry had become central to classical mathematics. Building on this foundation, the study of the circle has also given rise to trigonometry, which remains an essential framework for understanding the relationships between angles and side lengths of triangles (Stewart, 2016). Beyond pure mathematics, the concept of circularity has found applications across diverse scientific disciplines—from the orbits of celestial bodies in physics (Feynman et al., 2011) and recursive loops in computer science (Cormen et al., 2013) to metabolic cycles in biology (Urry, 2019) and rhythmic brain activity in neuroscience (Buzsáki, 2006), to name just a few.

When a circular structure is applied in psychometrics, the resulting model is typically referred to as a *circumplex*. Circumplex models have been employed across a wide range of psychological domains. For example, circumplex models have been used to conceptualize cognitive abilities (L. Guttman, 1954, 1957; Marshalek et al., 1983), interpersonal traits (Fournier et al., 2010; Gurtman, 1993, 2009; Kiesler, 1983; Leary, 1957; Wiggins, 1979), human values (Schwartz & Boehnke, 2004; Schwartz et al., 2012), vocational interests (Etzel et al., 2021; Etzel & Nagy, 2019; Holland, 1997; Tracey & Rounds, 1996), emotional states (Posner et al., 2005; Russell, 1980; Russell et al., 1989; Stanisławski et al., 2021), stress, and coping (Stanisławski, 2019, 2025). Major personality frameworks such as the Big Five and the HEXACO model have also been situated within circumplex models (Barford et al., 2015; DeGeest & Schmidt, 2015; DeYoung et al., 2013; Gurtman, 1995; McCrae & Costa, 1989; Nysæter et al., 2009; Trapnell & Wiggins, 1990). Furthermore, a general circumplex model of personality has been proposed that integrates a broad range of personality dimensions under two higher-level traits (Strus & Cieciuch, 2021; Strus et al., 2014). In clinical psychology, circumplex models have been used to differentiate between patient groups (Alden & Phillips, 1990; Horowitz et al., 1988; Pincus & Wiggins, 1990), predict and evaluate the outcome of therapeutic interventions (Alden & Capreol, 1993; Horowitz et al., 1988; M. A. Ruiz et al., 2004), and conceptualize personality disorders (Pincus & Hopwood, 2012; Williams & Simms, 2016; Wilson et al., 2017; Wright et al., 2012). Beyond personality and clinical research, circumplex models have been applied to leadership styles (Redeker et al., 2014), organizational culture (Locke, 2019), family systems (Olson, 2000; Olson et al., 2019), and parenting styles (Meisel et al., 2025).

1.2. Focus and Structure of the Thesis

The prevalence of circular models within and beyond psychology demonstrates that they are useful for organizing complex phenomena. Given their relevance across a wide range of empirical sciences, it is crucial to develop and apply appropriate methods to analyze circularly structured data. Accordingly, the present thesis focuses on the evaluation and advancement of methods for analyzing circumplex structure. If a domain is supposed to display circumplex structure, there are theoretical assumptions about the geometrical arrangement of the variables, and thus, their conceptual and statistical relationships: Quite simply, they should form a circle in a two-dimensional space. Numerous models and techniques have been employed in the conceptualization and analysis of circumplex structure (Browne, 1992; Grassi et al., 2010; Gurtman, 2009; Gurtman & Pincus, 2000; L. Guttman, 1954, 1968; Rounds et al., 1992; Shepard, 1974; Tracey, 1997; Tracey, 2000). These approaches vary in their assumptions and rigor—ranging from loose and subjective interpretations of how well a circular pattern can be discerned in a variable plot to the most rigorous ideal circumplex, in which variables are precisely aligned on the circumference and evenly distributed along the perimeter to form an almost perfectly circular shape. Factor analysis and principal component analysis (PCA) are among the most widely used tools for circumplex analysis (Acton & Revelle, 2004; Fabrigar et al., 1997; Tracey, 2000). They provide a flexible and well-established framework, but they come with challenges such as local optima in factor rotation, specifying appropriate factor models, choosing a suitable analysis technique, or the estimation of reliable and valid factor scores (Browne, 2001; Gorsuch, 2015; Mulaik, 2010).

Another important aspect refers to the measurement of traits that are supposed to fulfill circumplex structure. Circumplex instruments are typically built from subscales that are composed of multiple items. While circumplex assumptions should ideally hold both at the item and subscale level, most statistical models focus only the latter, and there is no circumplex-specific technique for item clustering as a basis for circumplex subscales. As a result, item–subscale assignment in circumplex scale development is often guided by subjective judgment and visual inspection of circumplex plots (e.g., Horowitz et al., 2017; Locke, 2000; Perrinjaquet et al., 2007; Redeker et al., 2014; Schwartz et al., 2012; Wiggins et al., 1988), which may compromise the reproducibility of item selection and the fidelity of circumplex measurement (Fabrigar et al., 1997).

The present thesis addresses these methodological challenges through three research projects:

1. The first project investigates local optima in rotation methods as a well-known issue in factor and component analysis, using complex circumplex structures in simulation designs (Weide & Beauducel, 2019).
2. The second project examines the higher-level factor structure of an established circumplex instrument, comparing different confirmatory modeling approaches and scoring methods (Weide et al., 2021).
3. The third and largest project introduces ClusterCirc, a new clustering procedure developed to optimize and evaluate assignment of items to subscales in circumplex instruments (Weide et al., 2025).

The thesis is structured as follows: Chapter 2 provides the theoretical background, including an overview of different circumplex models, the circumplex factor model, typical approaches in scale development for circumplex instruments, and the description of two circumplex instruments measuring the interpersonal circumplex that were used in the thesis' studies. Chapter 3 describes the rationale of the thesis by addressing methodological challenges in the analysis of circumplex structure and introduces the research projects, including the development of ClusterCirc. Chapter 4 presents the three studies: a simulation study on local optima in component rotation, an empirical study comparing confirmatory approaches to higher-level circumplex factor analysis, and the evaluation of the new clustering procedure ClusterCirc based on simulated and empirical data. Chapter 5 discusses the findings in light of existing research, addresses limitations of the thesis' research, and presents ideas for future directions.

2. Background

2.1. The Circumplex Model

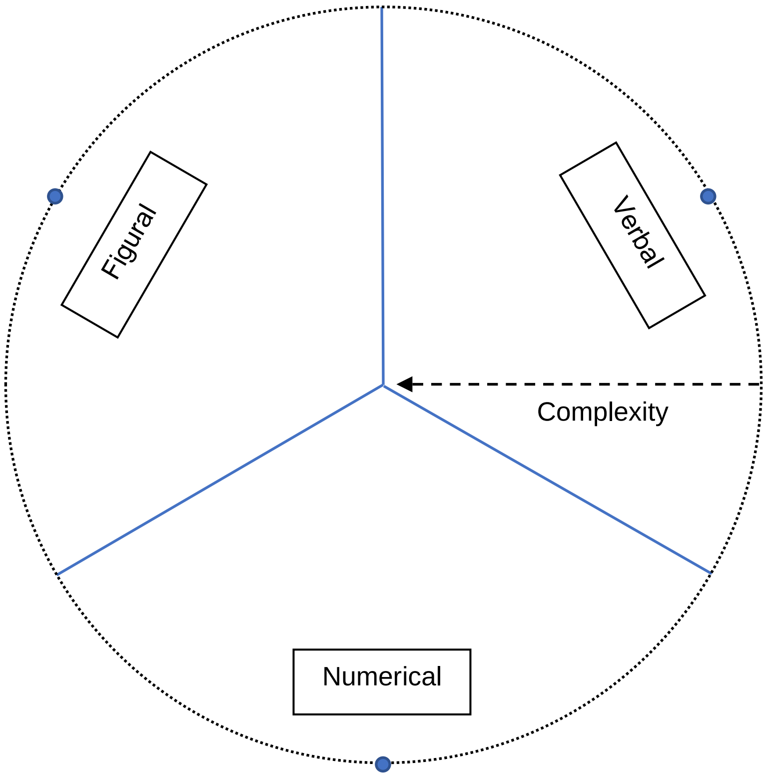
2.1.1. First Conceptualization

First notions of the circumplex in psychology can be traced back to L. Guttman's radex model of personality and mental abilities within the discussion of conceptualizing and analyzing intelligence (L. Guttman, 1954, 1957; Horn & Cattell, 1966; Marshalek et al., 1983; Schlesinger & Guttman, 1969; Spearman, 1930; Thurstone, 1934, 1943). According to the radex model, the structure of mental abilities can be projected onto a two-dimensional, circular surface (see Figure 1 for the version by Marshalek et al., 1983). In the center of the circle are tasks of high complexity, and complexity decreases as one moves to the outer edges of the circle. The different content areas of mental abilities (e.g., verbal, numerical, figural) are positioned along the circumference of the circle, with the disk divided into same-sized segments to represent each content area. The decreasing complexity level within each content area can be represented by a straight line from the center to the outer edge of the circle, forming a simple order of complexity or a *simplex*. If the complexity level is held constant, tasks of the different content areas form a circle—a circumplex—of the same complexity level. If both complexity and content area are varied, the resulting structure of the whole disk illustrates the general radex model as a radial expansion of complexity within different content areas.

L. Guttman (1954, 1957) states that the simplex and circumplex are associated with certain correlational patterns. Hence, the geometric pattern can be translated into statistical coefficients of similarity. If there is a simplex structure and variables are ordered accordingly in the correlation matrix, the following pattern is expected: The largest correlations appear next to the main diagonal, and correlation coefficients decrease monotonically with greater distance from the diagonal, while all correlations remain positive (see Table 1). If there is a circumplex structure, correlations also decrease at first when moving away from the main diagonal but then increase again, indicating a circular structure of variables (see also Browne, 1992). Hence, variables that are close to each other on the circle correlate more strongly with each other than variables that are farther apart (Table 1, Table 2; see also Gurtman, 1993, 2009; Gurtman & Pincus, 2000; Tracey & Rounds, 1993).

Figure 1

The Radex Model of Mental Abilities



Note. The radex model consists of a circumplex structure spanned by the different content areas (figural, verbal, numeric) and a simplex structure represented by increasing complexity from the outer arc to the center of the circle. Adapted from a refined version of L. Guttman's (1957) model in "The Complexity Continuum in the Radex and Hierarchical Models of Intelligence" by B. Marshalek, D. F. Lohman, and R. E. Snow, 1983, *Intelligence*, 7(2), p. 110 (https://doi.org/10.1016/0160-2896(83)90023-5).

Mathematically, the relationship between variables can be expressed using their angular positions in the circle, where $0° \leq \Theta \leq 360°$ (angles in degrees) or $0 \leq \Theta \leq 2\pi$ (angles in radians). The position of 0° can be chosen arbitrarily, for example by fixing it to the position of any one of the variables. Then, the correlation—or similarity as a more general concept—between two variables in the circle can be expressed by the equation

$$r_{x,y} = f^{-1}(\Theta_x - \Theta_y) \quad , \tag{1}$$

where Θ_x and Θ_y are the angular positions of the variables in the circle. Hence, the angular distance between the variables is negatively related to their correlation (Gurtman & Pincus, 2000). This fundamental equation can be satisfied by a large number of functions, depending on the constraints specified for how the variables should be distributed across the circle. Since L. Guttman's (1954, 1957) first conceptualization, various circumplex models have followed, which will be explored in the next chapter.

Table 1

Correlational Pattern of the Circumplex (and the Simplex) for an Example of Eight Variables

Variable	V1	V2	V3	V4	V5	V6	V7	V8
V1	1							
V2	ρ_1	1						
V3	ρ_2	ρ_1	1					
V4	ρ_3	ρ_2	ρ_1	1				
V5	ρ_4	ρ_3	ρ_2	ρ_1	1			
V6	ρ_3 (ρ_5)	ρ_4	ρ_3	ρ_2	ρ_1	1		
V7	ρ_2 (ρ_6)	ρ_3 (ρ_5)	ρ_4	ρ_3	ρ_2	ρ_1	1	
V8	ρ_1 (ρ_7)	ρ_2 (ρ_6)	ρ_3 (ρ_5)	ρ_4	ρ_3	ρ_2	ρ_1	1

Note. $\rho_1 > \rho_2 > \rho_3 > \rho_4 > \rho_5 > \rho_6$. The circumplex and simplex patterns are identical from V1 to V5. In the lower part of the table, the circumplex pattern is shown as the main entries, and the entries in parentheses represent the continuation for the simplex pattern. In the circular order model, correlations denoted by the same index (e.g. ρ_1) do not need to have the same value. In the geometric correlation model (ideal circumplex model), the same index denotes the same value. V = variable; ρ = correlation coefficient. The circumplex pattern is adapted from "Exploring Personality With the Interpersonal Circumplex" by M. B. Gurtman, 2009, *Social and Personality Psychology Compass, 3*(4), p. 608 (https://doi.org/10.1111/j.1751-9004.2009.00172.x).

2.1.2. Overview of different circumplex models

Given the geometrical shape of the circle, assumptions can be specified for the spacing and the radius of the variables. Different models regarding spacing and radius of the variables are presented in Table 1, Table 2, and Figure 2. Spacing refers to the angular distances between variables. In other words: Are the variables evenly distributed along the circle's circumference (equal spacing), or do some neighboring variables have larger or smaller distances than others? Radius refers to the vector lengths of variables, defined as the straight lines drawn from the circle's center to each variable's position[1]. This raises the question: Do the variables form a circular arc with comparable distances to the center (equal radius), or are some closer to and others farther away from the center?

[1] Strictly speaking, an individual variable does not have a radius because it does not span the whole circular surface. The term *vector length* is more precise, but *radius* is more common and will be used as a synonym for a variable's vector length in this thesis.

Table 2

Two Examples of Correlations for an Ideal Circumplex of Eight Variables

Angle/ Variable	0°	45°	90°	135°	180°	225°	270°	315°
V1	1	.85	.50	.15	0	.15	.50	.85
V2	.71	1	.85	.50	.15	0	.15	.50
V3	0	.71	1	.85	.50	.15	0	.15
V4	-.71	0	.71	1	.85	.50	.15	0
V5	-1	-.71	0	.71	1	.85	.50	.15
V6	-.71	-1	-.71	0	.71	1	.85	.50
V7	0	-.71	-1	-.71	0	.71	1	.85
V8	.71	0	-.71	-1	-.71	0	.71	1

Note. V = variable. The depicted correlations are in line with the geometric model of correlations (ideal circumplex). The upper triangle represents expected correlations in the presence of an overarching common dimension causing all correlations to be nonnegative (e.g., a third factor in addition to the two circumplex factors). The lower triangle represents circumplex correlations in the absence of a common dimension beyond the variables. Adapted from "Constructing Personality Tests to Meet a Structural Criterion: Application of the Interpersonal Circumplex" by M. B. Gurtman, 1993, *Journal of Personality, 61*(2), p. 241 https://doi.org/10.1111/j.1467-6494.1993.tb01033.x).

The least restrictive circumplex model is the spatial representation model (Shepard, 1974; Figure 2, all panels). It suggests that variables are projected onto a circular surface in Euclidean space without explicit constraints on spacing or radius. The spatial representation model is typically assessed informally by visual inspection and subjective interpretation. For example, nonmetric multidimensional scaling, specifically smallest space analysis (SSA; L. Guttman, 1968; L. Guttman & Levy, 1991; Schlesinger & Guttman, 1969), can be applied to the data. Similarly, loading patterns from exploratory factor analysis (EFA) or PCA can be plotted and inspected (Acton & Revelle, 2004; Tracey, 2000).

Following L. Guttman's (1954, 1957) example, several circumplex models have been proposed regarding the correlations of variables forming the circular structure (Rounds et al., 1992; Tracey, 1997; Tracey, 2000; Tracey & Schneider, 1995). These correlation models are mainly concerned with spacing rather than radius (see Table 1; Table 2; Figure 2). Mathematically, bivariate correlations can be directly translated into angular distances between variables, and vice versa, by the law of cosines (Dunham, 1924). However, they cannot be easily transformed into radial information—the distance to the center or circumference of an overarching circle. To this end, further analytical steps are required (Browne, 1992; Nagy et al., 2019). Nonetheless, if a variable has a particularly small radius in the context of factor analysis and PCA, it often indicates a lesser degree of shared variance with the other variables (e.g., communality), which in turn reduces its correlations with them.

Already L. Guttman (1954, 1957) distinguished between an ideal circumplex with equal spacing and a quasi-circumplex with less restricted spacing between variables. In the quasi-circumplex, only order relations between correlations are specified, representing the circular order model of correlations (see Table 1; Figure 2C and 2D; Rounds et al., 1992; Tracey, 1997; Tracey, 2000; Tracey & Schneider, 1995). The circular order model imposes ordinal constraints on variable spacing. As in L. Guttman's (1954) original theory, it requires that adjacent variables have higher correlations than variables two steps apart, which in turn have higher correlations than variables three steps apart, and so on. The circular order model of the quasi-circumplex can be tested with programs specifically designed to evaluate the order of correlations (RANDALL; Tracey, 1997).

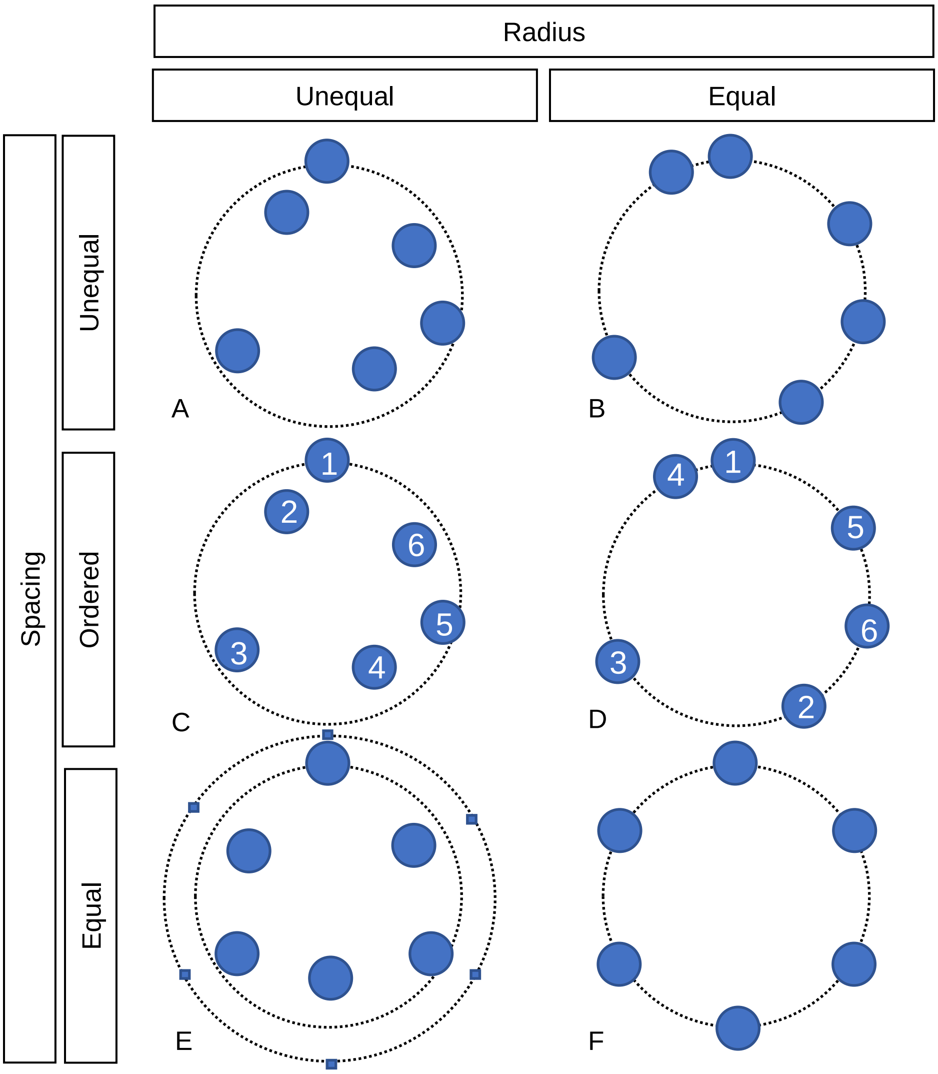
In contrast to the quasi-circumplex, the ideal circumplex requires equal spacing with angular distances of 360/m (m = number of variables) between all adjacent variables (L. Guttman, 1954, 1957). The corresponding correlation matrix is therefore more restricted in the geometric model of correlations (Table 1; Table 2; Figure 2E and 2F; Gurtman, 2009; Rounds et al., 1992). This model requires that correlations are perfectly related to the (equally spaced) angular distances in the circle and that all correlations are proportionally and linearly related to each other. The exact size of the correlations depends on the existence and strength of a common dimension underlying all variables (L. Guttman, 1954, 1957; Rounds et al., 1992; Tracey, 2000). For example, in L. Guttman's (1954) radex model of mental abilities, all circumplex correlations are nonnegative, implying the presence of a common dimension underlying the variables, such as the g-factor of intelligence (Spearman, 1930). However, the assumption of nonnegativity is specific to the intelligence domain, and negative correlations can be conceived in circumplex structures, particularly in the personality domain (Table 2, lower triangle; Browne, 1992; Gurtman, 1993; Gurtman & Pincus, 2000; Wiggins et al., 1981). If there is no underlying common dimension and the correlation at a 180° distance is −1, circumplex correlations of the geometric model of correlations are given by the simple formula

$$r_{x,y} = \cos(\theta_x - \theta_y) \quad , \qquad (2)$$

where θ is a variable's angular position in radians within the circle (Dunham, 1924; Gurtman, 1993; Gurtman & Pincus, 2000). The geometric model of correlations for the ideal circumplex is extremely parsimonious because all correlations are based on one single parameter.

Figure 2

Overview of Different Circumplex Models



Note. All depicted structures conform to the spatial representation model. Panels C and D illustrate the circular order model of correlations; the ordered variables could also display equal spacing, which is not shown for simplicity. Panels E and F conform to the geometric model of correlations. In Panel E, the outer arc displays circular distances between the variables to illustrate equality of angular spacing despite unequal radii. Panel F displays an ideal circumplex. The stochastic processing model of the circumplex (SPMC) allows for the modeling of Panels A, B, E and F.

Nowadays, the most prominent circumplex model is Browne's (1992) stochastic process model of the circumplex (SPMC; see also Grassi et al., 2010; Nagy et al., 2019). The SPMC states that each manifest variable consists of a common part and a unique part. Circumplex constraints are formulated for the latent, common parts of the variables since measurement error is unavoidable and manifest variables will never form an ideal circumplex. The SPMC is the most flexible circumplex model because equality constraints for both spacing and radius can be specified separately (see Figure 2, Panels A, B, E, and F). This allows for different variants of the quasi-circumplex to be modeled, up to the ideal circumplex with equal spacing and equal radius of the latent variables. The SPMC is based on a stochastic function that describes the distribution of the variables in a circle (Anderson, 1960). The respective correlations are expressed as a Fourier series with trigonometric functions based on the arc length between all pairs of variables in the circle. The minimal correlation at a 180° distance is ≥ –1, allowing for both positive and negative correlations. If the correlation at 180° is greater than –1, variables that are expected to be opposite to each other in the two-dimensional circumplex are still slightly related. This would indicate a common general dimension underlying all variables, which is implicitly accounted for by the SPMC.

In contrast to the previously introduced correlation models, the SPMC does not require an a priori defined order of variables. Instead, it determines the appropriate position of each variable (polar angle) in the circle based on its correlations with all other variables. Because measurement error is allowed in the SPMC, it allows for a confirmatory test of the circumplex model. For this purpose, Browne (1992) developed CIRCUM, an implementation of structural equation modeling for circumplex structure (see also CircE by Grassi et al., 2010; Nagy et al., 2019, for adaptations and newer developments). In CIRCUM (Browne, 1992)/CircE (Grassi et al., 2010), model fit of the SPMC can be evaluated in a flexible manner for all combinations of equality constraints on spacing and radius of the variables. For example, if spacing is fixed to be equal but radius is freely estimated, CIRCUM/CircE evaluates deviations of estimated angular distances from equal spacing. If both spacing and radius are constrained to equality, model fit will decrease because deviations from perfect alignment of variables on the circle's circumference are also assessed.

2.1.3. The Circumplex Factor Model

In addition to the previously described models, the circumplex can be defined by a factor analytic model, which offers major advantages. Factor analysis is a well-established method for modeling and investigating latent structures (Fabrigar & Wegener, 2012; Gorsuch, 2015; Mulaik, 2010; Thurstone, 1965). Framing the circumplex within this broader family of factor models makes it accessible to a wide audience and allows for a thorough examination of the circular structure using powerful and widely applied analytic tools. Accordingly, the research in this thesis is primarily based on the factor analytic circumplex model.

The circumplex factor model typically consists of two orthogonal factors, which can be viewed as the axes of a coordinate system surrounded by a circle (see Figure 3; Acton & Revelle, 2004; Gurtman, 1993; Gurtman & Pincus, 2000). The Cartesian coordinates of the variables are given by

$$x = u \times \cos(\Theta) \quad \text{and} \tag{3}$$

$$y = u \times \sin(\Theta) \quad , \tag{4}$$

where u is the radial coordinate[2] (the radius) of the variable, and Θ is the angular coordinate or polar angle (in radians) of the variable. The radius of the variable can be determined by forming a right triangle with the x- and y-coordinates as the short sides (see Figure 3A). The radius u is the hypotenuse, and its length is given by the Pythagorean theorem:

$$u = \sqrt{x^2 + y^2} \quad . \tag{5}$$

Expressed in terms of factor analysis, the coordinates x and y represent the loadings of a variable on two orthogonal factors a_1 and a_2. Therefore, in the case of a purely two-dimensional circumplex model, the radius of a variable is the square root of its communality[3] h^2, and the formula can be written as

$$u = \sqrt{h^2} = \sqrt{a_1^2 + a_2^2} \quad . \tag{6}$$

The loadings of the two-dimensional circumplex factor model are thus given by

$$a_1 = h \times \cos(\Theta) \quad , \tag{7}$$

for loadings on the factor representing the x-axis of the coordinate system, and

$$a_2 = h \times \sin(\Theta) \quad , \tag{8}$$

[2] The radius of a variable is usually denoted by *r*. In this thesis, it is denoted by *u* to distinguish it from correlation coefficients.

[3] This is only true for a two-dimensional model. In the case of additional factors alongside the circumplex axes, the communality also comprises the loadings on the other factors, whereas the radius is only concerned with the two circumplex axes. In this case, *h* needs to be replaced by *u* in Equations 6-10.

for loadings on the factor representing the y-axis. Conversely, the polar angle of a variable can be recovered by

$$\Theta = \arccos\left(\frac{a_1}{h}\right) \quad \text{or} \tag{9}$$

$$\Theta = \arcsin\left(\frac{a_2}{h}\right) \quad . \tag{10}$$

According to Acton and Revelle (2004), the circumplex factor model should meet the following criteria:

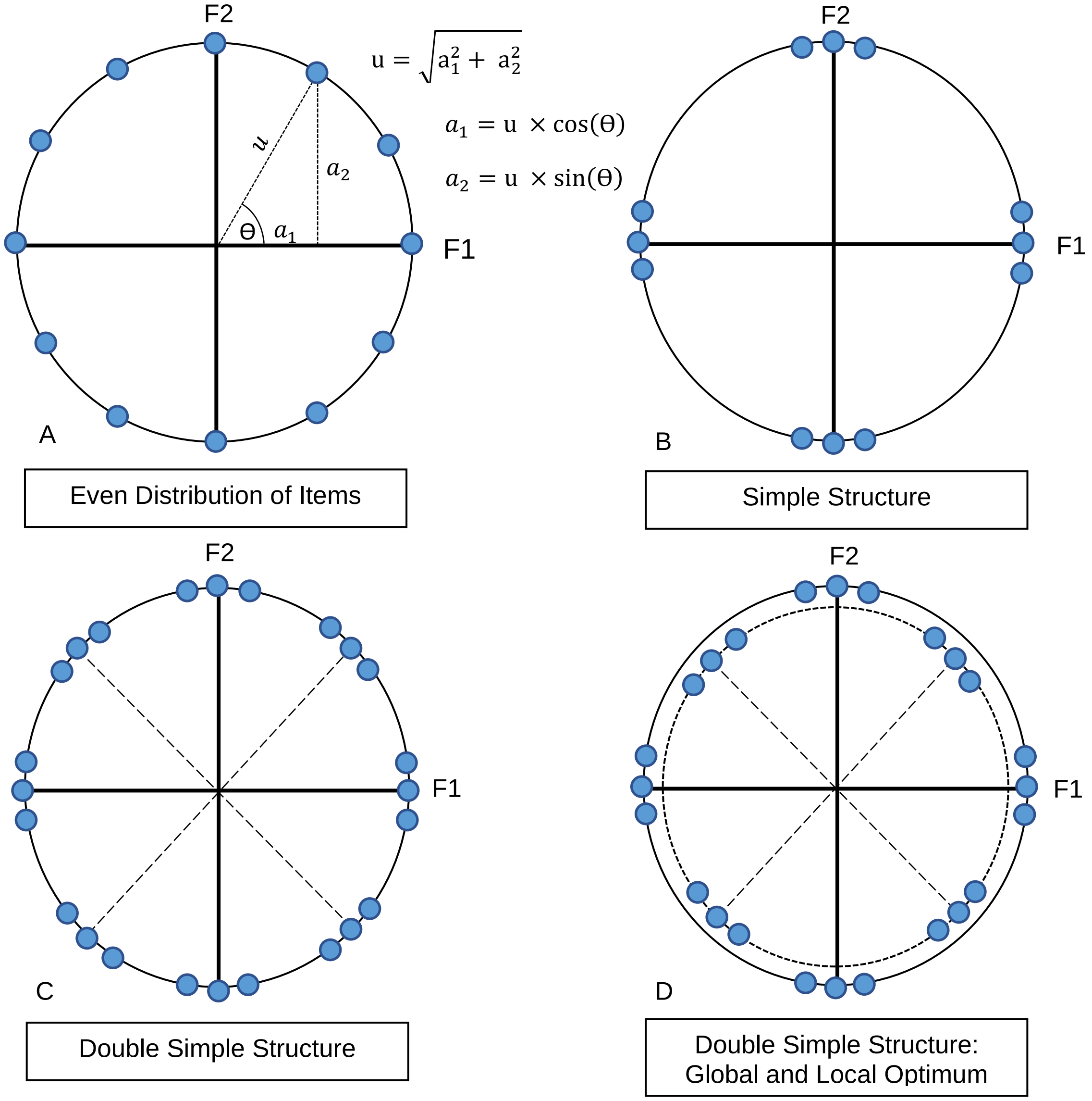
1. Systematic interrelations: Variables are interrelated in a systematic manner; the circular structure is not based on random noise.
2. Two axes: Exactly two orthogonal factors span the circular structure (see Figure 3, axes of the coordinate system).
3. No clumping/interstitiality: Variables should not cluster along the main axes, as would be expected for simple structure. Instead, interstitial variables are distributed in a circular pattern. Ideally, variables are equally spaced to form an ideal circumplex (Gurtman, 2009; Wiggins, 1979).
4. Equal radius: The radius of the variables should ideally be constant. In factor analytic terms, variables should have equal communalities in the two-dimensional circumplex (see Equation 6; Fisher, 1997).
5. Rotational invariance: There is no preferred rotation of the two main axes; any orthogonal rotation can reveal the circular structure. The specific loading pattern changes, but it remains consistent with the circumplex model (Conte & Plutchik, 1981).

These criteria overlap substantially with those of earlier circumplex models (e.g., Browne, 1992; L. Guttman, 1957; Tracey, 2000) but are adapted to the factor analytic framework. Strictly speaking, the SPMC is also a type of factor analytic model (Grassi et al., 2010; Nagy et al., 2019).

Although the second criterion specifies that exactly two factors define the circumplex, it is possible to embed the two circumplex factors within a model with more factors. For example, many circumplex models include a third, general factor in addition to the two circumplex factors (Acton & Revelle, 2002; Browne, 1992; Hatcher & Rogers, 2009; Hopwood & Good, 2019; Tracey et al., 1996; Wendt et al., 2019; Wilson et al., 2013). In circumplex correlation models, as stated earlier, this is represented in models with only positive (and zero) correlations, since a general factor would yield nonnegative correlations between all variables (Anderson, 1960; Browne, 1992; L. Guttman, 1954, 1957; Rounds et al., 1992). The SPMC also embeds the possibility of a third, general factor because the number of common factors can vary within the Fourier series.

Figure 3

The Circumplex Factor Model



Note. The circumplex factor model is defined by exactly two orthogonal axes. Panel A shows the geometrical relationship between and items' radius (u), angular position (Θ), and loadings on Factor 1 (a_1) and Factor 2 (a_2) based on the Pythagorean theorem and trigonometric functions. In the two-dimensional model, u^2 is the item's communality. Any rotational position of the axes is possible, particularly in cases with evenly distributed items, as shown in Panel A. Panels B, C, and E depict preferred rotational positions in the case of item clustering. In Panels C and D, the dashed lines illustrate alternative positions of the axes that reveal a competing simple structure. The alternative position in Panel D is modeled with smaller item loadings (and, thus, radius), representing a locally optimal solution as opposed to the global optimum.

The circumplex factor model can be examined using PCA, EFA, or confirmatory factor analysis (CFA; Gorsuch, 2015; Jöreskog et al., 2016; Mulaik, 2010). Because of rotational invariance, any orthogonal rotation should reveal the circular structure, although the factors can be fixed to a theoretically expected alignment. Spacing and radius of the variables are assessed through the loading pattern on the two circumplex factors (Equations 6, 9, and 10). Depending on the method—exploratory or confirmatory—and the expected degree of conformity with the ideal circumplex (equal spacing and equal radius), model suitability can be evaluated in various ways. For example, PCA or EFA loading plots can be visually inspected ("eyeball tests") to assess whether variables form a circular arrangement consistent with the spatial representation model (Shepard, 1974; Tracey, 2000).

More formal approaches are also available. Radius can be examined by evaluating the equality of vector lengths, which correspond to communalities in a two-dimensional circumplex (Equation 6). Radius of the whole variable pattern can be assessed through the Fisher test (Fisher, 1997), which divides the standard deviation of vector lengths by their mean—a proxy of the overall circle's radius. If all variables have the identical vector lengths, the result is zero, indicating a perfectly equal radius for the circle.

Circumplex spacing can formally be assessed by transforming loadings into polar angles (Equations 9 and 10) and comparing observed positions—or variable distances— with theoretical expectations. This can be done for PCA/EFA loadings or within CFA, with loadings fixed to the expected circumplex pattern given a defined axis alignment (see Equations 7 and 8; Figure 3A; e.g., Cieciuch et al., 2014; Schwartz & Boehnke, 2004; Wilson et al., 2013). Circumplex-specific spacing measures also exist. The squared loadings index (Saucier, 1992), for instance, captures interstitiality—that is, whether variables are spread between the two axes or clustered along them. Additionally, different gap tests use angular distances between variables or the distances between actual and ideal angular positions to assess deviations from equal spacing (Acton & Revelle, 2004; Gurtman, 1993).

Finally, rotational invariance can also be examined. When a sufficiently large number of variables are evenly distributed around the circle without clustering or clumping, the rotational position of the two circumplex axes should be arbitrary. Rotation criteria should then yield identical values regardless of the chosen rotational position. Acton and Revelle (2004) proposed tests of rotational invariance, which, among other criteria, examine whether variables display circumplex-specific interstitiality or, instead, cluster along the axes. Such clustering along the axes would be more consistent with the well-established simple structure pattern, which Acton and Revelle explicitly distinguish from circumplex structure (see also Gurtman, 2009; Russell, 1980). The following section, however, will examine whether there are cases in which—despite their apparent differences—simple structure and circumplex structure may in fact be connected.

2.1.4. The Circumplex and (Double) Simple Structure

In most common factor analytic approaches, researchers aim to achieve simple structure in the loading pattern (Carroll, 1953; Fabrigar & Wegener, 2012; Hendrickson & White, 1964; Kaiser, 1958; Mulaik, 2010). According to Thurstone's (1935) definition of simple structure, each row must contain at least one zero loading; that is, each variable should have a (close to) zero loading on at least one factor. The number of zeros should also be maximized, such that many (close to) zero loadings are present across the entire loading pattern. The idea is that each variable should be explained ideally by fewer than the total number of factors, while each factor should represent only a subset of variables (see Figure 3B). Although the criteria for simple structure have been refined (Thurstone, 1954, 1965), the basic idea remains the same. Simple structure is widely considered desirable because it helps assign variables to common factors, reduces redundancy, and enhances interpretability (Gorsuch, 2015; Mulaik, 2010).

By contrast, in the case of circumplex structure, a clear-cut simple structure along the two main axes is per definition not to be found (Acton & Revelle, 2004). Instead, interstitial variables are placed between the main axes to form the circular shape (see Figure 3A). This leads to a loading pattern in which many variables have neither zero nor main loadings. Rather, interstitial variables typically have divided loadings across both circumplex factors, and both factors are needed to explain their variance. Furthermore, rotation to simple structure is usually intended to resolve rotational indeterminacy of factor solutions by applying a set of reasonable criteria (Kaiser, 1958; Mulaik, 2010). By contrast, rotational invariance of the circumplex factor model implies that rotational indeterminacy is not to be resolved but accepted as a feature of the model (Acton & Revelle, 2004).

Nevertheless, simple structure and circumplex structure can be conceptually linked. Thurstone (1934, 1935) illustrated his theory on the vectors of mind using circles and spheres (also including higher dimensions). He explained that correlations between traits are related to the cosine of their angular positions on a sphere and demonstrated the extraction of two factors in a circular arrangement (e.g., for radicalism or mental ability tests). These illustrations are consistent with the least restrictive spatial representation model of the circumplex (Shepard, 1974). More restrictive quasi-circumplex and ideal circumplex models with constraints on radius and spacing of variables—as well as rotational invariance of the two factors—are less compatible with these demonstrations of simple structure (Acton & Revelle, 2004; Gibson, 1963; L. Guttman, 1954).

However, consider a scenario with two orthogonal factors. If variables display perfect simple structure—zero loadings on one factor and high loadings on the other, half positive and half negative—the circumplex condition of (quasi-)equal radius is met (see Figure 3B). However, interstitiality with equal spacing between variables, as well as rotational invariance, would not be satisfied. Alternatively, if only one variable is positioned at the extreme ends of each factor, one might argue that such a simple structure also constitutes a circumplex. Although this scenario is highly unlikely and factor extraction with only two variables per factor poses serious identification problems (Anderson & Rubin, 1956; Marsh et al., 1998; Mulaik, 2010), equal spacing (without interstitiality) and equal radius would technically be satisfied.

A more realistic and common structure that connects simple structure and circumplex structure is double simple structure. This pattern consists of two orthogonal axes, along with an alternative position of the axes rotated by 45°, producing an octant division of the circle (see Figure 3C). Many circumplex theories and inventories are based on this model (e.g., Alden et al., 1990; Gurtman, 1993; Gurtman & Pincus, 2000; Hopwood & Good, 2019; Horner et al., 2025; Horowitz et al., 2017; Stanisławski, 2025). When double simple structure is present and variables lie on the circumference of the circle, the circumplex conditions of equal radius and interstitiality with equal spacing are met[4]. Regarding rotational invariance (Acton & Revelle, 2004), the exact position of the two axes is arbitrary from a circumplex perspective. However, if the typical loading pattern of double simple structure is desired, one of the two rotational positions is usually chosen. Simple structure emerges for half of the variables in one of the rotational positions and for the other half in the alternative position. Thus, double simple structure exemplifies a case where both simple structure and circumplex structure can be regarded as satisfied.

2.2. Development and Validity of Circumplex Instruments

Factor-analytic approaches are frequently applied in the development of psychometric instruments (Briggs & Cheek, 1986; Clark & Watson, 1995; Reise et al., 2000). Psychometric

[4] Perfect equal spacing is met when there is only one variable per extreme end of each axis position.

instruments designed to adhere to circumplex structure are typically based on the circumplex factor model described above, and many of them comprise a double simple structure. Examples include the Interpersonal Adjective Scales (IAS; Adams & Tracey, 2004), the Inventory of Interpersonal Problems (IIP; Horowitz et al., 1988, 2017), the Inventory of Interpersonal Strengths (Hatcher & Rogers, 2009), the Interpersonal Emotion Inventory (Horner et al., 2025), the Circumplex Scales of Interpersonal Values (Locke, 2000), the Parenting Styles Circumplex Inventory (Meisel et al., 2025), and the Personal Globe Inventory, which measures vocational interests (Etzel et al., 2021; Etzel & Nagy, 2019).

2.2.1. Three Levels in Circumplex Instruments: Factors, Subscales, and Items

These inventories comprise several subscales, each composed of multiple items. Thus, circumplex structure can be considered at different levels. Up to this point, the circumplex has been described at only two levels: the two overarching factors that span the circular structure and the variables distributed along the circumference of the circle. However, many psychological theories include facets (Beauducel & Kersting, 2020; R. Guttman & Greenbaum, 1998; Shye, 1998). In psychometric measurement, facets are typically represented by subscales, which constitute a third, intermediate level between the two circumplex factors and individual items. For example, in L. Guttman's (1954) radex model of mental abilities, the content areas (verbal, numerical, and figural) represent facets or subscales, whereas the specific tasks within each area correspond to the item level.

Frequently, the circumplex is modeled at the subscale level, such that the subscale centers should ideally be equally spaced and have equal radii (Adams & Tracey, 2004; Gurtman, 2009; Hatcher & Rogers, 2009; Horner et al., 2025; Horowitz et al., 1988). Here, *subscale center* refers to the angular centroid of the subscale when projected onto the circle's perimeter rather than the geometric center of its segment within the disk (e.g., the straight lines in Figure 3C point to the subscale centers). If the circumplex were modeled exclusively at the item level with equal item spacing across the circle, dividing the circular structure into subscales would be arbitrary because no clear-cut subscale boundaries could be drawn. In such a case, sorting items into subscales would be impossible. Nevertheless, circumplexity at the item level is theoretically compelling because items comprise the core of each psychological instrument. For this reason, item wording should be chosen so that the items themselves conceptually form a circular pattern. With respect to their position in the circular structure, items should lie along the circumference within the boundaries of their assigned subscale, ideally close to the subscale center to avoid subscale indeterminacy.

The procedures specifically tailored to analyze circumplex structure are not designed to account for the multilevel structure in psychometric instruments. Researchers must therefore decide whether to examine circumplexity at the subscale level or at the item level, with the latter being more difficult to assess. Although Fisher's (1997) test for equal radius can be applied to either items or subscales, most tests concerned with circumplex spacing are not suitable for the item level when items are clustered within subscales. For example, if the SPMC is analyzed with CIRCUM/CircE (Browne, 1992; Grassi et al., 2010) under equal-spacing constraints at the item level, the method presumes that items are evenly distributed around the circle—an unrealistic and undesirable expectation in the context of subscales. The same applies to the squared loadings index (Saucier, 1992), which would treat item clustering as a problematic deviation from circumplexity. Similarly, gap tests (Acton & Revelle, 2004; Gurtman, 1993) are prone to yield unfavorable spacing results at the item level since adjacent items from different subscales are typically further apart than adjacent items within the same subscale. This makes the development of psychometric instruments that conform to circumplex structure at both the subscale and item levels particularly challenging.

2.2.2. Typical Approaches in Developing and Evaluating Circumplex Instruments

When developing psychometric instruments with multiple subscales, researchers usually generate items based on theoretical considerations to match the intended meaning of each subscale. Subsequently, they conduct empirical studies and analyze their data to select, rephrase, or replace items, and sometimes adjust the conceptual scope or even the number of subscales. This process is typically repeated in multiple cycles of theorization and empirical analysis (Clark & Watson, 1995; Coulacoglou & Saklofske, 2017; Nunnally & Bernstein, 1994).

Many instruments are based on the assumption that each item measures unequivocally one subscale and that subscales are distinct from each other or only weakly correlated. However, in the case of circumplex structure, there is an intended statistical and conceptual overlap between neighboring subscales (Gurtman, 1993; Gurtman & Pincus, 2000; L. Guttman, 1957). Hence, items in circumplex instruments must simultaneously distinguish between neighboring subscales and represent their theoretical proximity. For this reason, it is often desirable that items within a subscale show a certain level of heterogeneity to cover the theoretical breadth of the segment, while still distinctively belonging to a particular subscale rather than the neighboring one. This makes item generation and item–subscale assignment more challenging than in instruments based on orthogonal constructs, such as in the Big Five (Costa & McCrae, 1995, 2006) or HEXACO model of personality (Ashton & Lee, 2007).

It is important to note that in circumplex structure, item heterogeneity does not necessarily correspond to item intercorrelations. Heterogeneity is determined by angular distances along the circle that can be drawn around the item plot. Items may occupy the same angular position but have low communalities; that is, they may possess substantial unique variance not shared with other items in the instrument. In such a case, they would display low intercorrelations despite being circularly homogeneous. Conversely, items with high communalities but different angular positions may correlate strongly, yet are more heterogeneous from a circumplex perspective.

To assess circumplex structure at different stages of scale development, researchers rely on the procedures described above. Spacing and radius have often been evaluated simultaneously by eyeball tests of SSA plots (Adams & Tracey, 2004; Bardi & Schwartz, 2003; Fegg et al., 2016; Fisher et al., 1985; Gurtman & Pincus, 2000; Perrinjaquet et al., 2007; Redeker et al., 2014; Russell, 1980; Russell et al., 1989; Schwartz et al., 2012; Stanisławski et al., 2021). Similarly, visualization and examination of PCA loadings and Browne's (1992) procedure CIRCUM (R package: CircE by Grassi et al., 2010) are popular for assessing spacing and radius within the same procedure. They have often been performed in conjunction with RANDALL, which assesses the circumplex order of spacing (PCA + RANDALL: Boudreaux et al., 2018; Hopwood et al., 2011; PCA + RANDALL + CIRCUM/CircE: Horner et al., 2025; Horowitz et al., 2017; Locke, 2000, 2019; Richardson et al., 2020).

Radius of circumplex instruments has been frequently assessed by the Fisher test (e.g., Acton & Revelle, 2002; Fisher et al., 1985; Horowitz et al., 2017; Jacobs & Scholl, 2005). In these investigations, spacing is studied more often at the subscale level than at the item level because the circumplex theory is often more elaborated at the facet/subscale level, the procedures can be applied at only one level at a time, and many are unfit for the item level. Item radius is typically assessed in a straightforward manner by computing communalities, sometimes with an overall test such as the Fisher test (Hopwood et al., 2011; Horner et al., 2025; Locke, 2019). Item circumplexity within subscales has, for example, been evaluated by correlating items with their respective subscales, which can be interpreted as an indirect indicator of item radius and spacing. For instance, when only items with relatively high correlations to their subscale are retained, they are supposedly positioned close to the subscale's central angle and also close to the circle's circumference (Alden et al., 1990; Horner et al., 2025).

Item spacing has further been examined through PCA with two orthogonal (circumplex) components on the items. In early stages of scale development, new circumplex subscales have been identified by the following procedure: Item loadings on the two orthogonal components are plotted on a circular plane. Subsequently, the circle is divided into (same-sized) segments based on visual inspection and theoretical considerations regarding the intended meaning of the desired subscales. All items within the angular range of a segment are assembled to form a subscale. This method was, for example, used in developing the Inventory of Interpersonal Strengths (Hatcher & Rogers, 2009), the Circumplex Scales of Interpersonal Values (Locke, 2000), and the Circumplex Measure of the Interpersonal Culture in Work Teams and Organizations (Locke, 2019). The IIP was not originally designed as a circumplex instrument (Horowitz et al., 1988), but circumplex structure was later applied to its items, leading to the identification of new subscales through this method of slicing PCA plots.

At later stages of development, the suitability of items for a subscale can be evaluated similarly. The subscale's central angle serves as the reference point, and items may deviate from it within a limited range, typically corresponding to the subscale's angular segment (Gurtman, 1995). This permits moderate item heterogeneity within subscales while preserving distinctiveness between subscales. For example, in validation studies of the IAS (Wiggins et al., 1988) and the IIP (Horowitz et al., 2017), an item's acceptable deviation from its subscale angle was restricted to ±22.5°, resulting in 45° subscale segments. The IAS and IIP are two key measures of the interpersonal circumplex, an influential model for describing interpersonal behavior (Fournier et al., 2010; Gurtman, 1993, 2009; Kiesler, 1983; Leary, 1957; Wiggins, 1979). The process of developing and evaluating circumplex instruments will therefore be illustrated in more detail for these two inventories, given their central role in the circumplex tradition.

2.3. The Interpersonal Circumplex: Theory and Instruments

The interpersonal circumplex is one of the most widely studied circumplex models. It is defined by a factor model with double simple structure, and all three levels of analysis—factors, subscales, and items—are relevant in measures of the interpersonal circumplex (Gurtman, 1995; Gurtman & Pincus, 2000; Horowitz et al., 2017; Horowitz et al., 1988;

Wiggins et al., 1988; Wiggins, 1979). It has been applied to describe a variety of interpersonal constructs, including interpersonal motives and goals (Horowitz et al., 2006; Locke, 2014; Ojanen et al., 2005), values (Locke, 2000), behavior (Adams & Tracey, 2004), strengths (Hatcher & Rogers, 2009), problems (Horowitz et al., 1988, 2017), and sensitivities (Hopwood et al., 2011), as well as social support (Trobst, 2000).

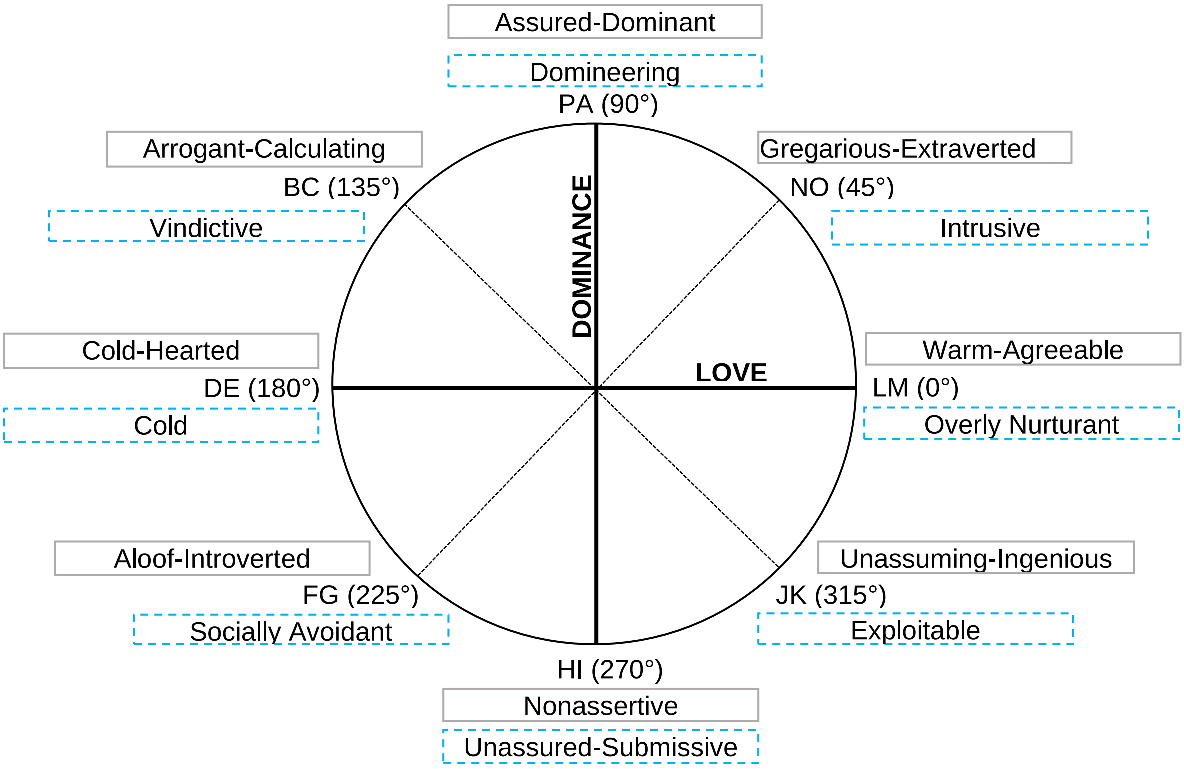
The interpersonal circumplex is typically modeled with two orthogonal axes, which will be called by their most prominent labels, *Dominance* and *Love* (Gurtman, 1993, 1995; Gurtman & Pincus, 2000) throughout this thesis (Figure 4). The Dominance axis captures the extent to which individuals behave in an assured, dominant versus submissive, unassured manner toward others. The Love axis, assumed to be independent of Dominance, represents warmth and agreeableness versus coldness and hostility. Together, these axes define four facets at their poles, and an additional four facets between them, yielding an octant structure of the circle. This structure corresponds to a double simple structure with 45° distances between octant centers.

The interpersonal circumplex has been linked to the Big Five and HEXACO models of personality (Du et al., 2021; Nysæter et al., 2009; Pincus, 1992; Pincus et al., 1998). Particular attention has been paid to the alignment of Extraversion and Agreeableness within the interpersonal circumplex due to their conceptual resemblance to Dominance and Love (see Figure 5; Barford et al., 2015; DeYoung et al., 2013; Gurtman, 1995; McCrae & Costa, 1989). Extraversion has been aligned close to Dominance, tilted toward Love (between PA/Assured-Dominant and NO/Gregarious-Extraverted). Agreeableness has been mapped close to the Love axis toward the more submissive end of the Dominance axis (between LM/Warm-Agreeable and JK/Unassuming-Ingenious).

Although numerous instruments have been developed for the interpersonal circumplex, two are especially influential: the IAS (Adams & Tracey, 2004; Jacobs & Scholl, 2005; Wiggins et al., 1988), which measures interpersonal behavior in general, and the IIP (Alden et al., 1990; Horowitz et al., 2017, 1988), which measures difficulties in interacting with others. Both instruments play a central role in research and practice and were employed and analyzed in two of the thesis' studies (Weide et al., 2021; Weide et al., 2025).

Figure 4

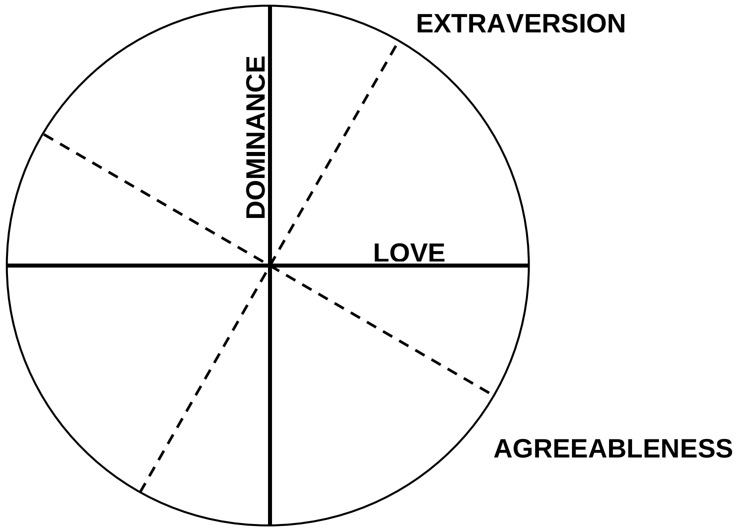
The Interpersonal Circumplex: General Interpersonal Behavior and Interpersonal Problems



Note. The expressions framed by (gray) solid lines represent general interpersonal behavior; the expressions framed by (blue) dashed lines represent interpersonal problems.

Figure 5

Big Five Extraversion and Agreeableness Within the Interpersonal Circumplex



Note. Adapted from "Unifying the Aspects of the Big Five, the Interpersonal Circumplex, and Trait Affiliation" by C. G. DeYoung, Y. J. Weisberg, L. C. Quilty, & J. B. Peterson, 2013, *Journal of Personality*, *81*(5), p. 446 (https://doi.org/10.1111/jopy.12020).

2.3.1. The Interpersonal Adjective Scales (IAS)

The IAS is, as its name suggests, a list of adjectives intended to capture stable interpersonal traits. It was originally developed by Wiggins (1979) with explicit reference to circumplex theory (Leary, 1957; Sullivan, 2013) and a lexical approach to item generation. Drawing on Leary's interpersonal theory, Wiggins proposed eight bipolar interpersonal categories spanning the two main axes Love and Status (Dominance). These categories, and their labels, were revised over time, with items added or discarded in an iterative process of theorization and statistical evaluation.

The initial item pool consisted of 1,710 trait items. Preliminary subscales were created through rational item selection (i.e., do the items match the meaning of the category), PCA plots of the resulting subscales on the two circumplex components with heuristic evaluation of spacing, and correlations of items with subscales. An item was retained if it correlated highly positively with its designated subscale, highly negatively with the opposite subscale, and close to zero with orthogonal subscales. Various item clusters were tested, and the loading pattern of the resulting subscales on the circumplex components was compared to the ideal circumplex configuration based on visual inspection. The final version of the IAS contained 128 items (16 per subscale) that achieved the most consistent circumplex loading pattern at the subscale level. The focus of this development process lay primarily on optimizing circumplex spacing for subscales, while item spacing was treated more heuristically. No formal indices of item spacing were computed. Instead, the above-described correlation patterns (positive, zero, negative) used in item selection can be considered a loose ordinal criterion for item spacing. Moreover, by favoring items with highly positive correlations with its designated subscale correlations, the circumplex criterion of equal radius was implicitly addressed, as such items tend to lie near the circumference of the circle rather than closer to the center of the disk.

The IAS was refined (Wiggins et al., 1988) using a large student sample (N = 1,161). PCA with two orthogonal components was performed on the subscale sum scores of the original IAS. The components were rotated by orthogonal Procrustes rotation toward the expected octant positions as target matrix. Correlations of the items with the component scores were then transformed trigonometrically to determine their polar angles, while the squared sum of these correlations served as a proxy for item communalities. From the original 128 items, those with the highest communality-proxies were retained, resulting in a 64-item scale with eight items per subscale. This strategy optimized item radius, whereas spacing was largely disregarded at the item level. Spacing was assessed only for the resulting subscales by plotting and visually inspecting their loadings on the two circumplex components in a second PCA in the same sample, consistent with the spatial representation model (Gurtman & Pincus, 2000; Shepard, 1974; Tracey, 2000).

The validity of the IAS has since been evaluated in numerous studies. Different circumplex models were investigated: the spatial representation model through evaluation of PCA and SSA plots, the circular order model (software RANDALL; Tracey, 1997), the SPMC with different equality constraints on spacing and radius (software CIRCUM; Browne, 1992; or CircE; Grassi et al., 2010) and the circumplex factor model with various tests for its criteria (Acton & Revelle, 2002; Adams & Tracey, 2004; Gaines et al., 1997; Gurtman & Pincus, 2000; Tracey & Schneider, 1995). In these studies, spacing and radius were typically assessed at the subscale rather than item level.

For German versions of the IAS, two options are available: the IAS-revidiert-deutsch (IAS-R-dt; Ostendorf, 2001) or the Interpersonal Adjective List (IAL; Jacobs & Scholl, 2005). The structural validity of the IAL and IAS-R-dt is supported by a battery of studies using both exploratory and confirmatory approaches (Jacobs & Scholl, 2005). As with the English versions, circumplex criteria were primarily evaluated at the subscale level. Both instruments converge strongly, but the IAL demonstrates superior internal consistency and better fit to the circumplex model (e.g., two-dimensionality, circumplex order of correlations, equal radius, and equal spacing of subscales). Accordingly, the IAL was selected as the measure of general interpersonal tendencies for the thesis' research. Like the English IAS, the IAL consists of eight subscales: PA/Assured-Dominant, BC/Arrogant-Calculating, DE/Cold-Hearted, FG/Aloof-Introverted, HI/Unassured-Submissive, JK/Unassuming-Ingenious, LM/Warm-Agreeable, and NO/Gregarious-Extraverted (see Figure 4). Each subscale comprises eight adjectives describing interpersonal attributes, for example, "confident" (PA), "cynical" (BC), "hostile" (FG), "modest" (JK), or "sociable" (NO). Respondents rate each adjective on a 5-point Likert scale indicating the degree to which it describes their typical interpersonal behavior (*not at all* to *very much*).

2.3.2. The Inventory of Interpersonal Problems (IIP)

In addition to general interpersonal tendencies, people may also experience difficulties in their interactions with others. Such interpersonal problems are relevant in many areas of research and practice. For example, they play a major role in clinical psychology, particularly in personality disorders, in which interpersonal impairment is a principal characteristic

(Alden & Capreol, 1993; Pincus & Hopwood, 2012; Pincus & Wiggins, 1990; Williams & Simms, 2016; Wilson et al., 2017; Wright et al., 2012). Moreover, interpersonal problems concern a wide range of human relationships, including couples and families, friendships, acquaintances, and workplace settings (Horowitz et al., 2017).

Interpersonal problems are routinely measured with the IIP (Alden et al., 1990; Horowitz et al., 1988, 2017). As stated earlier, the IIP was originally not designed as a circumplex instrument, but the circumplex model was superimposed in later versions (Alden et al., 1990) in accordance with the interpersonal theory (Kiesler, 1983; Leary, 1957; Sullivan, 2013). The first version of the IIP (Horowitz et al., 1988) was developed using videos of interviews with patients seeking outpatient psychotherapy. Independent observers recorded all problem statements, which were subsequently categorized as interpersonal or not. This resulted in 127 items, which were subjected to SSA, where three major dimensions were identified: the degree of psychological involvement, the nature of involvement (friendly or hostile), and the intention to influence, change, or control others. In a follow-up study, the original IIP items were entered in a PCA. A general factor was recovered on which all items had positive loadings, indicating a person's overall level of interpersonal distress. In addition, Horowitz et al. (1988) retained six further components describing different areas of interpersonal problems, like (too much or too little) assertiveness, intimacy, responsibility, or exerting control. Items were assigned to a single subscale based on their highest component loading. Horowitz et al. also conducted a PCA on ipsatized sum scores of the subscales to eliminate the effect of the general distress factor. They found two relevant components for the ipsatized subscales: dominance versus submissiveness and hostility versus friendliness, which are closely related to the later recovered circumplex factors Dominance and Love (Alden et al., 1990). Ipsatized scores represent how problematic a certain area is for an individual relative to other interpersonal domains.

Recognizing the similarity between the IIP's higher-level structure and interpersonal circumplex theory (Kiesler, 1983; Leary, 1957; Sullivan, 2013), Alden et al. (1990) created a revised circumplex version of the IIP. They postulated eight dimensions along the two main axes Dominance and Love and applied PCA with two orthogonal components on the ipsatized scores of the original 127 items. Item loadings were converted into polar angles, and the circular structure was divided into eight sectors of 45° each (double simple structure). Within each sector, the eight items with the highest communalities, the strongest item–subscale correlations, and the best theoretical fit to the intended subscale were retained. Selecting items with high communalities supported the equal radius criterion for circumplex structure. Item spacing was addressed only implicitly, as items with high item–scale correlations tend to cluster around the subscale's central angle, thereby improving subscale spacing. In a follow-up study, Alden et al. demonstrated that PCA projections of the revised eight subscales produced better circumplex spacing and radius than the original six subscales. The IIP octants also closely aligned with the IAS octants, further supporting their structural validity.

The German version of the IIP (Horowitz et al., 2017) is based on this revised circumplex version (Alden et al., 1990). It was translated and validated in several studies, most notably in a large norm sample by Brähler et al. (1999). The circumplex structure of the German IIP was supported by analyses of ipsatized item and subscale scores (Horowitz et al., 2017). Several statistical tests confirmed the circumplexity of the IIP octants, including RANDALL for correlational order (Tracey, 1997), the Fisher test for constant radius (Fisher, 1997), the gap test for equal spacing (Acton & Revelle, 2004), and confirmatory tests of the SPMC (Browne, 1992; Grassi et al., 2010). Item–subscale assignment was evaluated by computing polar angles of ipsatized item scores from their PCA loadings on two Varimax-rotated circumplex components (Dominance and Love). Items were considered suitable if they fell within their subscale's ideal angle ±22.5° in the norm sample. Overall, the circumplex structure of both the English and German IIP appears to be well supported (Horowitz et al., 2017).

Notably, most studies examining the structural validity of the IIP circumplex have used ipsatization to recover the Dominance and Love axes (Alden et al., 1990; Horowitz et al., 1988, 2017). However, ipsatization may be problematic because the psychometric properties of ipsatized IIP scores have been shown to be worse than their non-ipsatized counterpart (Becker & Mohr, 2005). Without ipsatization, a third factor typically emerges, with positive loadings for all items and subscales. This pattern has been observed in both exploratory analyses (Horowitz et al., 1988, 2017) and confirmatory tests (Acton & Revelle, 2002; Hopwood & Good, 2019; Wendt et al., 2019; Wilson et al., 2013). While some have interpreted this factor as a response artifact (Alden et al., 1990; Horowitz et al., 1988), others argue it reflects a meaningful construct: an individual's general level of interpersonal difficulties (Tracey et al., 1996). Moreover, it accounts for substantial variance of the IIP subscales and should thus not be dismissed. Instead, the IIP's higher-level structure seems best captured by a three-factor solution comprising Dominance, Love, and a general factor of interpersonal difficulties, which—in accordance with Wilson et al. (2013) and Wendt et al. (2019)—will be labeled *Distress* throughout the thesis.

Regarding external validity, the IIP has been linked to the Big Five as broader personality frameworks (Du et al., 2021; Gurtman, 1995; Nysæter et al., 2009). It has also been associated with more specific traits, such as assertiveness in role plays (Leising et al., 2007) and the ability to identify and distinguish emotions (Weinryb et al., 1996). Given the central role of interpersonal problems in psychological disorders, especially personality disorders, the IIP has been extensively investigated in clinical contexts. Distinct patterns of interpersonal problems measured by the IIP have been associated with personality disorders (Hilsenroth et al., 2007; Monsen et al., 2006), anxiety, and depression (Alden & Phillips, 1990; Wongpakaran et al., 2012). Moreover, IIP scales have been shown to predict therapeutic outcomes (Davies-Osterkamp et al., 1996; M. A. Ruiz et al., 2004). Finally, associations between narcissism and the IIP subscales have been examined in both clinical and nonclinical samples (Dickinson & Pincus, 2003; Ogrodniczuk et al., 2009).

The IIP's subscales are PA/Domineering, BC/Vindictive, DE/Cold, FG/Socially Avoidant, HI/Nonassertive, JK/Exploitable, LM/Overly Nurturant, and NO/Intrusive (see Figure 4). Despite identical positions in the circumplex, the IIP subscale labels differ from the IAL subscale labels because the IIP subscales capture interpersonal difficulties in the specific domains rather than general interpersonal tendencies. IIP items include statements such as "I try to please other people too much" (LM/Overly Nurturant) or "It is hard for me to trust other people" (BC/Vindictive), which are rated on a 5-point Likert scale (not at all to very much).

3. Rationale and Research Projects of the Thesis

As alluded to in the previous sections, circumplex structure poses methodological challenges for data analysis. For example, the circumplex factor model assumes rotational invariance of the two circumplex axes, which implies that rotational indeterminacy of the factor solution is, by definition, not to be resolved. Moreover, within the conventional simple structure framework, maximizing similarity between items (item homogeneity) is a reasonable strategy for forming subscales. Circumplex theory, however, extends beyond mere similarity-based classification and acknowledges that semantic distinctions between items can occur at a more fine-grained level. It is also important to consider the different levels involved in developing circumplex instruments, as circumplex structure at the subscale level may result from strategic aggregation of non-circumplex items. Although many research questions can be formulated regarding methodological challenges in analyzing circumplex structure, this thesis focuses on two aspects in particular: factor-analytic approaches to the circumplex (Studies 1 and 2) and the challenge of assigning items to subscales in the development of circumplex instruments (Study 3).

3.1. Factor and Component Analysis of the Circumplex

The first two projects focus on traditional factor analytic procedures[5] since factor analysis is widely recognized both within and beyond the circumplex domain (Fabrigar & Wegener, 2012; Gorsuch, 2015; Mulaik, 2010; Thurstone, 1965). Factor analysis requires large samples and certain distributional assumptions, but when applied appropriately, it has high statistical power and helps answer numerous questions in psychological research.

In factor and component analysis, several decisions need to be made, for example: Should one adopt an exploratory or confirmatory approach? And in the case of exploratory analysis, should PCA or EFA be used? Moreover, how many factors or components are necessary to describe the psychological construct? In circumplex models, this often refers to the question of whether the two axes are sufficient, or whether a third overarching factor is needed to provide an adequate description (Acton & Revelle, 2002; Hatcher & Rogers, 2009; Hopwood & Good, 2019; Hopwood et al., 2011; Tracey, 1996). Furthermore, a major question in factor and component analysis concerns the loading pattern of variables. In exploratory analysis, this involves factor rotation, which should reveal a meaningful pattern among the variables. In confirmatory analysis, it involves the allocation of variables to factors and the extent to which loadings are fixed versus freely estimated, such as in the expected circumplex pattern. Finally, with the growing interest in Bayesian inference and its probabilistic treatment of unknown quantities (Bolstad & Curran, 2016; van de Schoot et al., 2017), one may also consider whether a traditional frequentist or a Bayesian approach is more suitable for analyzing the circumplex factor model (Conti et al., 2014; Hoofs et al., 2018; Lopes, 2014).

Regarding the first two questions, the circumplex factor model is often analyzed by an exploratory approach, and more commonly via PCA rather than EFA (Boudreaux et al.,

[5] Although the results from factor analysis and PCA may be very different in some contexts (Fokkema & Greif, 2017), this is not an issue with respect to the present thesis. Therefore, both approaches will be referred to as factor analytic approaches in the following.

2018; Hopwood et al., 2011; Horowitz et al., 2017, 1988; Locke, 2000, 2014, 2019; Richardson et al., 2020; Stanisławski et al., 2021). In accordance with the PCA model, it has been argued that the circumplex axes summarize the segments of the circle rather than representing underlying factors that cause or explain the segments (Locke, 2019). In many circumplex models, the two axes are discovered post hoc and later superimposed onto the structure (Alden et al., 1990; Russell, 1980; Russell et al., 1989; Schaefer, 1959; Schwartz et al., 2012), which aligns more closely with the PCA approach than with EFA modeling. Furthermore, the PCA model is more parsimonious, which can lead to more robust results than EFA because the latter requires computation of error factors in addition to common factors (Harman, 1967).

With respect to rotation, the circumplex model implies rotational invariance: The circumplex is satisfied regardless of axis orientation, provided the axes remain orthogonal (Acton & Revelle, 2004). However, some rotational positions may be theoretically and statistically more compelling. For example, in circumplex instruments with eight (roughly equally spaced) subscales, the two rotational positions that reveal double simple structure would constitute a meaningful orientation of the axes (see Figure 3C; Adams & Tracey, 2004; Etzel et al., 2021; Gurtman & Pincus, 2000; Hatcher & Rogers, 2009; Hopwood & Good, 2019; Horner et al., 2025; Horowitz et al., 2017; Locke, 2000; Meisel et al., 2025; Stanisławski, 2025). If there are theoretical expectations about axis alignment, Procrustes rotation toward a target matrix is an obvious solution (Browne, 2001). Target rotation has been used to reveal circular arrangements in circumplex contexts (Horner et al., 2025; Jacobs & Scholl, 2005; Locke, 2014; Stanisławski et al., 2021; Wiggins et al., 1988).

However, many researchers rely on standard orthogonal rotations, especially when aiming for double simple structure. In these cases, procedures that optimize a simple structure criterion are often performed, typically Varimax rotation (Kaiser, 1958). Varimax is also the most widely used rotation method toward orthogonal simple structure outside of the circumplex realm (Browne, 2001; Fabrigar et al., 1999). In circumplex settings, Varimax is often applied to components from a PCA (Boudreaux et al., 2018; Hopwood et al., 2011; Horowitz et al., 1988, 2017; Richardson et al., 2020). In double simple structure, Varimax and related methods would yield two local optima of axis rotation, each producing simple structure for half the variables. Ideally, both rotational positions yield identical results for the rotation criterion. In empirical settings, however, one solution typically prevails, for example, due to differences in measurement reliability or deviations from equal octant spacing, which may pull the axes toward denser variable clusters.

3.2. First Project: Local Optima in (Circumplex) Double Simple Structure as a Challenge for Rotation Procedures

Given the two local optima in double simple structure as a popular circumplex model, it is well suited to test whether rotation procedures stop at a local optimum or discover the global optimum of a data structure. For this purpose, the double simple structure needs to be modeled with higher main loadings for one of the rotational positions of the circumplex axes. This rotational position would reveal a stronger simple structure for half of the variables, as opposed to the weaker simple structure for the other half in the alternative rotational position (see Figure 3D). The position with higher loadings thus represents the global optimum of the rotational criterion, whereas the alternative rotation constitutes the local optimum.

The issue of local optima is highly relevant in analytic factor rotation because the loss functions on which the algorithms operate are complex and curvilinear, with multiple minima and maxima (Rozeboom, 1992). To overcome local optima and ensure that the globally optimal solution is found, researchers can employ multiple random starts for the search algorithm and examine whether results differ depending on the start matrix (Browne, 2001; Hattori et al., 2017; Kiers, 1994; Trendafilov & Jolliffe, 2006).

A relatively new procedure to analytic factor rotation was proposed by Bernaards and Jennrich (2005). They developed a method for factor rotation that can approximate most established rotation criteria toward simple structure based on a single algorithm (Jennrich, 2001, 2002), the gradient projection algorithm (GPA). GPA rotation is promising because it subsumes different rotation criteria within a single-algorithm approach, making it both universal and parsimonious. Furthermore, it is accessible via an openly available R package, thereby supporting open science standards (https://cran.r-project.org/web/packages/GPArotation/index.html).

The basic approach of GPA rotation is as follows (Bernaards & Jennrich, 2005; Jennrich, 2001, 2002; Mulaik, 2010): Any rotation toward simple structure is based on an optimization criterion (either maximization or minimization), with certain components of the optimization function subject to constraints. Constrained optimization problems of this type can be solved by GPA. An initial loading matrix is rotated by

$$\mathbf{\Lambda} = \mathbf{AT} \quad , \tag{11}$$

where $\mathbf{\Lambda}$ is the rotated loading matrix, $\mathbf{A}$ is the initial, typically unrotated $m \times l$ loading matrix (with m as the number of variables and l as the number of factors), and $\mathbf{T}$ is the $l \times l$ transformation matrix. The columns of $\mathbf{T}$ are unit length, such that the sums of squares across the columns equal 1. In circumplex structure, the two factors are required to be orthogonal (Acton & Revelle, 2004), which puts constraints on the correlation matrix between rotated factors or components. Orthogonality constrains the off-diagonal entries to zero, yielding

$$\mathbf{\Phi} = \mathbf{T}'\mathbf{T} = \mathbf{I} \quad , \tag{12}$$

where $\mathbf{\Phi}$ is the $l \times l$ correlation matrix between rotated factors or components and $\mathbf{I}$ is the identity matrix. The optimization criterion in analytic factor rotation Q is computed based on the rotated loadings, such that it is a function of the transformation matrix, given by

$$Q(\mathbf{\Lambda}) = f(\mathbf{T}) \quad . \tag{13}$$

In Varimax rotation, for instance, the variance of the squared loadings is supposed to be maximized (Kaiser, 1958), which makes the Varimax criterion a function of the transformation matrix. In GPA, the algorithm aims to find a minimum of $f(\mathbf{T})$. Accordingly, the Varimax criterion is defined as the negative of $Q(\mathbf{\Lambda})$ within the GPA approach.

The algorithm is based on the negative gradient $\mathbf{G}$ of $Q(\mathbf{\Lambda}) = f(\mathbf{T})$. In orthogonal rotation, the negative gradient is based on the initial loadings, partial differentiation of the optimization criterion, and the rotated loadings:

$$\mathbf{G} = \mathbf{A}' \frac{\partial Q}{\partial \mathbf{\Lambda}} \tag{14}$$

The exact expression for ∂Q depends on the rotation criterion. The algorithm searches for a solution within the search space of possible transformation matrices $\mathbf{T}$ that satisfy the constraints (such as uncorrelated factors in the case of orthogonal rotation). It begins with an initial $\mathbf{T}$. Next, $\mathbf{T}$ is displaced by its negative gradient with step length b, which gives

$$\mathbf{M} = \mathbf{T} - b\mathbf{G} \quad . \tag{15}$$

The columns of $\mathbf{M}$ are then normalized to unit length to project $\mathbf{M}$ back onto $\mathbf{T}$, which is denoted by $\sim\mathbf{T}$. The rotation criterion $f(\sim\mathbf{T})$ is computed after each iteration and compared with that of the previous transformation matrix. The algorithm is strictly descending if b is sufficiently small, such that

$$f(\sim\mathbf{T}) < f(\mathbf{T}) \quad . \tag{16}$$

If $f(\sim\mathbf{T}) \geq f(\mathbf{T})$, the step length b should be reduced until the optimization criterion is smaller for $\sim\mathbf{T}$ than for T. After each iteration, $\mathbf{T}$ is replaced with $\sim\mathbf{T}$, and the same steps are repeated until the algorithm converges, that is, until the rotation criterion reaches its minimum or a previously defined maximum number of iterations is reached.

However, the algorithm does not explore the entire search space of possible transformation matrices. Instead, GPA is prone to stop at a solution close to the initial rotation matrix (Jennrich, 2004, 2006). It is therefore possible that GPA rotation yields a local rather than the global optimum for the rotation criterion, which will be examined in the thesis' first project. The investigation will be conducted for Varimax rotation in PCA, in line with its popularity within and beyond the circumplex realm. For this purpose, a population model based on a modified double simple structure with a local and a global optimum will be used, as described above (see Figure 3D). If local optima are relevant in GPA, results should differ when multiple random start matrices are applied in such a double-optimum scenario. It should also be contrasted against a data structure, in which no local optima of the Varimax criterion are expected. For example, this is the case in perfect orthogonal simple structure, where variables have main loadings on only one factor and no cross loadings on other factors (Figure 3B), unlike in double simple structure. In such cases, there is only a single optimum for the Varimax criterion. If GPA rotation indeed yields local optima, multiple random starts should result in different solutions for the modified double simple structure, whereas no differences should emerge in the case of perfect simple structure.

Given that GPA rotation is a relatively new method serving the same purpose as other rotation procedures, it should be evaluated against established approaches to support its use. Therefore, GPA rotation toward the Varimax criterion will be compared to the Kaiser algorithm (Kaiser, 1958), which is the built-in procedure for Varimax rotation in most statistical software such as SPSS or SAS. Hence, in the first project of this thesis, the circumplex model of double simple structure will serve as a challenging data structure to investigate local optima in GPA rotation toward the Varimax criterion in PCA, and this will be compared with rotation using the conventional Kaiser algorithm.

3.3. Second Project: Bayesian Modeling in Circumplex Confirmatory Factor Analysis and Higher-Level Scores of Interpersonal Problems

Although exploratory analysis—particularly PCA with Varimax rotation—is very popular in examining the circumplex factor model, confirmatory analysis of the factor structure is also important. CFA is especially suitable when the circumplex structure has already been demonstrated and established as a model for a given domain. Because the expected angular positions of variables can be translated into factor loadings on the two circumplex axes via trigonometric transformation (Equations 7 and 8; Figure 3A), assumptions about loadings can be incorporated into CFA. In this way, CFA can corroborate the circumplex structure of psychometric instruments.

Traditionally, CFA is conducted with a frequentist approach and maximum-likelihood estimation of parameters (MCFA; Li, 2016). In frequentist MCFA, loadings are typically either fixed to zero or freely estimated to reveal a simple structure pattern. Loadings entered into the analysis are treated as fixed, “true” parameters. Model tests then evaluate how likely the observed loadings are under the assumption that the model is true. This frequentist rationale has been criticized both philosophically and methodologically, and it may lead to false inferences if not applied with caution (J. Cohen, 1994; Gigerenzer, 2004; R. D. Morey et al., 2016; Wagenmakers, 2007).

Circumplex structures add further complications. It must be decided how assumptions about angular positions should be translated into model specifications. A straightforward strategy is to fix all loadings to exact circumplex values (Equations 7 and 8; Figure 3A; Wilson et al., 2013), which assumes perfect equal spacing. However, the ideal circumplex model might be too strict, and loosening some assumptions might be more appropriate. In frequentist MCFA, this can be addressed in a post-hoc manner by inspecting modification indices. Loadings with large modification indices can be investigated by misspecification analysis (Saris et al., 1987, 2009) and freed in a follow-up CFA to test whether model fit improves. However, this can lead to a large number of inferential tests and thus a higher chance of erroneous inferences (MacCallum et al., 1992).

A further complication arises because many circumplex models and instruments also include a third, general factor in addition to the two circumplex factors (Acton & Revelle, 2002; Browne, 1992; Hatcher & Rogers, 2009; Hopwood & Good, 2019; Tracey et al., 1996; Wendt et al., 2019; Wilson et al., 2013). Such hierarchical circumplex models pose particular challenges for frequentist MCFA because a general factor with high loadings on all items can strongly affect model fit of the overall solution. It is therefore difficult to differentiate between the adequacy of the circumplex pattern for the two other factors and the suitability of the general factor. Researchers are left either to speculate on the extent to which model fit reflects the circumplex factors or to rely on modification indices and misspecification analyses (Saris et al., 1987, 2009), which involve the shortcomings of multiple comparisons.

Moreover, evaluating model fit at the level of individual loadings does not capture the overall circumplex pattern. As a result, in hierarchical circumplex models, MCFA cannot test the circumplex pattern simultaneously and independently of the general factor.

In summary, frequentist MCFA in circumplex models faces four major problems: First, the assumption of fixed, true parameters for circumplex loadings may be philosophically flawed—a general criticism of the frequentist framework. Second, circumplex models might involve greater uncertainty in specifying loadings than simple structure models because they imply very specific values for the loading pattern rather than freely estimated and zero loadings. Third, addressing uncertainty post-hoc through misspecification analysis of individual loadings and/or a sequence of modified CFA models might capitalize on chance. Fourth, if a hierarchical model is most appropriate, the general factor will have a large impact on model fit, and it is not possible to disentangle the two circumplex factors from the third factor in assessing model adequacy.

These problems can be addressed by adopting a Bayesian approach to modeling and testing circumplex factor models (Hoofs et al., 2018; Muthén & Asparouhov, 2012). Bayesian CFA (BCFA) offers several advantages. First, BCFA assumes a priori that the circumplex pattern is not a fixed entity, but one possible model with a given probability. Each model parameter is treated as a random variable, which may better reflect the uncertainty about variable positions around the circle. Second, BCFA does not require a strict distinction between fixed and freely estimated loadings. Instead, uncertainty can be incorporated directly into the model by entering the circumplex loadings as priors. The posterior estimates for the loadings are then based on a combination of the circumplex priors and the observed data. The posterior loadings can be compared to the (prior) ideal circumplex loadings to evaluate for which variables the circumplex arrangement is more suitable and which variables might deviate to a greater extent from their expected position in the circle. Third, BCFA priors allow to investigate the whole circumplex pattern within a single analysis, without reliance on multiple post hoc tests to address uncertainty about the circumplex loadings. If BCFA with circumplex priors is performed instead of a sequence of post-hoc MCFA tests, BCFA results might be more robust with a reduced risk of reaching statistical significance by random chance. Fourth, BCFA may also be advantageous over frequentist MCFA for hierarchical circumplex models. Instead of modeling completely free loadings on the general factor and completely fixed loadings on the two circumplex factors, priors allow for a more balanced model with greater flexibility in the circumplex loadings.

With increased flexibility in estimating the circumplex pattern, model fit will likely improve, and the circumplex factors will exert greater influence on the fit of the three-factor model in BCFA than in MCFA. The suitability of the circumplex structure for the data can then be assessed both globally and at the level of individual loadings by comparing (prior) ideal circumplex loadings with posterior loadings. For global indices of circumplex fit, the similarity between the ideal circumplex factors and posterior factors can, for example, be evaluated using Tucker's congruence coefficients (Lorenzo-Seva & ten Berge, 2006).

It is also possible to conduct EFA or PCA with Procrustes rotation toward a target matrix that specifies the expected circumplex pattern (Browne, 2001). In target rotation, the rotated loading pattern may deviate from the a priori specified ideal circumplex pattern, similar to BCFA. Model fit can also be evaluated using structural equation modeling techniques (Asparouhov & Muthén, 2009). Exploratory analysis with target rotation thus provides another alternative to MCFA for a more flexible investigation of the circumplex model, in addition to BCFA. In previous studies on the circumplex, target rotation was performed on components from PCA rather than factors from EFA (Horner et al., 2025; Jacobs & Scholl, 2005; Locke, 2014; Stanisławski et al., 2021; Wiggins et al., 1988). However, the factor model from EFA is more comparable to the CFA model than the PCA model and can be tested in a confirmatory fashion, like BCFA and MCFA. Therefore, target rotation within EFA (TEFA) is more suitable as a competing strategy to BCFA and MCFA in the investigation of the circumplex model.

Based on these arguments, the second research project will compare MCFA, BCFA, and TEFA to test the structural validity of the IIP as an important measure of interpersonal problems (Alden et al., 1990; Horowitz et al., 1988, 2017). However, since target rotation toward the circumplex pattern has been used in previous investigations, whereas BCFA has not yet been established for circumplex applications, the focus will be on the comparison between MCFA and BCFA. As pointed out previously, the IIP comprises eight subscales that are best described by a three-factor model with the two circumplex factors Dominance and Love and Distress as the general factor of interpersonal difficulties (Acton & Revelle, 2002; Hopwood & Good, 2019; Horowitz et al., 1988, 2017; Wendt et al., 2019; Wilson et al., 2013). In a previous MCFA on the IIP subscales, Wilson et al. (2013) specified a bi-factor model with the general Distress factor and two orthogonal circumplex factors for the IIP subscales. They fixed the circumplex loadings to the exact pattern based on the angular positions of the IIP scales (see Equations 7 and 8, MCFA loadings on Dominance and Love in Table 3; see below;

adapted from Weide et al., 2021, p. 6) and left the loadings on the general factor to be freely estimated. This MCFA bi-factor model will be the reference model for the second research project. The MCFA model will be compared with a BCFA model with circumplex priors and freely-estimated loadings on the general factor as well as with TEFA toward the ideal circumplex. Like the previous studies on CFA for the IIP, the study will focus on the higher-level model—that is, the structure of the IIP subscales within the three-factor model—and leave the item level aside to reduce complexity and ensure comparability with previous findings. Item level circumplexity will, however, be addressed by the third research project (see next section).

In view of the well-fitting three-factor model for the IIP scales, Wendt et al. (2019) suggest using higher-level scores for Dominance, Love, and Distress. These could be useful for practical applications of the IIP, such as in diagnostics, as well as for further scientific endeavors (Devlieger & Rosseel, 2017; Zitzmann & Helm, 2021). A good model fit of the three-factor model supports the use of higher-level scores for the IIP. However, fit of the factor model cannot be automatically transferred to higher-level scores because different higher-level scores can be calculated based on the same CFA model. Moreover, higher-level scores might carry a different theoretical meaning than the underlying factors (Beauducel, 2005). Therefore, both the investigation of the factor model itself as well as the evaluation of higher-level scores are important and will be addressed in the research project on the IIP.

In addition to the internal validity of the CFA models for the IIP and corresponding higher-level scores, it is also of interest whether external criteria can be associated with the models and scores. As pointed out earlier, the Big Five personality traits have been linked to the interpersonal circumplex (Barford et al., 2015; DeYoung et al., 2013; Gurtman, 1995; McCrae & Costa, 1989; Nysæter et al., 2009), with Extraversion and Agreeableness slightly displaced to the Love and Dominance axes (Figure 5). Moreover, Neuroticisim has been associated with the global score of interpersonal impairment of the IIP (Nysæter et al., 2009). It would therefore be most compelling if Extraversion, Agreeableness, and Neuroticism could be integrated in the CFA models of the IIP and linked to the higher-level scores in line with previous findings. According to their theoretical alignment within the interpersonal circumplex, it is hypothesized that Extraversion is positively associated with the Love and Dominance factors and the corresponding scores of the IIP, with the latter relationship being stronger than the former. Agreeableness is expected to be positively linked to Love and—to a lesser degree—negatively linked to Dominance. Furthermore, Neuroticism is expected to show a positive relationship with Distress as a general factor of interpersonal difficulties (see Table 4, upper part, for hypotheses).

In addition to the Big Five as broader concepts of personality, numerous studies have linked grandiose narcissism to the axes of the interpersonal circumplex (Dickinson & Pincus, 2003; Dowgwillo & Pincus, 2017; Miller et al., 2012; Ogrodniczuk et al., 2009; Rauthmann & Kolar, 2013; J. M. Ruiz et al., 2001) and, in particular, to interpersonal problems. Clinical narcissism has been associated with greater interpersonal distress in general (Ogrodniczuk et al., 2009). However, this relationship has not been confirmed for subclinical narcissism, possibly due to a relatively positive and affirmative self-image in individuals who score high on narcissism without the impairments experienced by individuals at the extreme, clinical end of the spectrum (Dickinson & Pincus, 2003). With respect to the circumplex axes, narcissism has been consistently found to be positively related to Dominance, whereas findings on Love have been mixed (Dowgwillo & Pincus, 2017; Miller et al., 2012; Rauthmann & Kolar, 2013; J. M. Ruiz et al., 2001). Based on these findings, it is hypothesized that subclinical narcissism is positively associated with Dominance, whereas no substantial associations are expected between narcissism and Love or Distress (see Table 4, upper part).

In line with this argumentation, the second research project will compare BCFA modeling with MCFA and TEFA and evaluate higher-level scores for the IIP (Horowitz et al., 1988, 2017). This approach will address both the internal validity of the models and scores and their external validity, including the Big Five factors Extraversion, Agreeableness, and Neuroticism, as well as subclinical narcissism.

3.4. Third Project: Item–Subscale Assignment in the Development of Circumplex Instruments

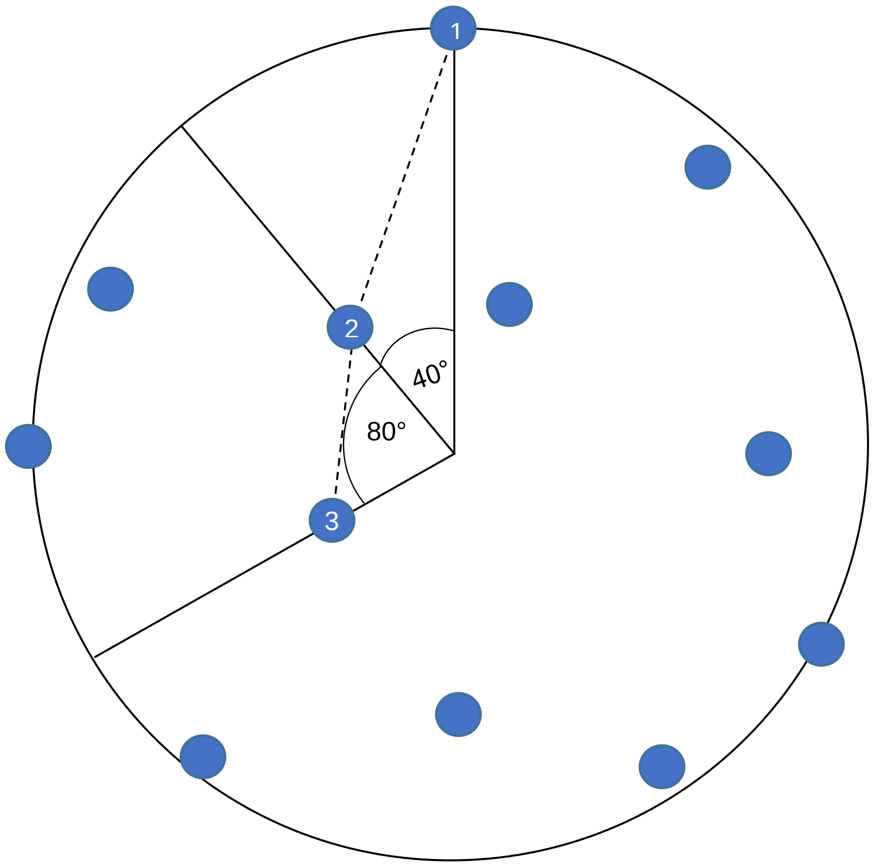
While the first two projects of the thesis address the factor structure of the circumplex model, the third and largest project examines how items can be sorted into subscales when developing circumplex instruments. As demonstrated for the IAS/IAL (Adams & Tracey, 2004; Jacobs & Scholl, 2005; Wiggins et al., 1988) and the IIP (Horowitz et al., 1988, 2017), many circumplex instruments are based on a factor model and comprise subscales, each consisting of several items distributed across the circular structure. In line with previous research (Wendt et al., 2019; Wilson et al., 2013), the second research project investigates the higher-level structure of the IIP, focusing exclusively on the subscale level. However, the item level is also of major relevance, as items form the foundation of each psychometric instrument, and subscale-level circumplexity may simply be an artifact of item aggregation rather than a meaningful psychological structure. Ideally, circumplex subscales are composed of items that themselves conform to circumplexity, so that both the item and subscale levels reveal the circular pattern. Suggestions for a combined analysis of item- and subscale-level circumplexity will be detailed later (see Section 3.4.1., and in particular, Section 3.4.1.3).

Besides theoretical considerations and classical item analysis, items have often been evaluated and circumplex subscales have been identified by slicing item plots from SSA (e.g., Perrinjaquet et al., 2007; Redeker et al., 2014; Schwartz et al., 2012) or PCA (e.g., Hatcher & Rogers, 2009; Horowitz et al., 2017; Locke, 2000, 2019; Wiggins et al., 1988) into segments representing subscales. As mentioned above, this approach has a major flaw: The segmentation of the circle heavily depends on subjective decisions (Fabrigar et al., 1997). No formal index currently exists to evaluate item–subscale assignment from a circumplex perspective. However, item–subscale assignment is not completely arbitrary because theoretical considerations regarding the content of the items and the meaning of the desired subscales typically guide decisions. Nonetheless, this can be particularly challenging in the case of circumplex structure due to the geometric and theoretical proximity of subscales. For instance, distinguishing between items belonging to the subscale Social Facilitating versus the neighboring subscale Helping in vocational interests might be difficult (Etzel et al., 2021). Similarly, determining which items are more suited for Tradition as opposed to Conformity in human values can be challenging (Schwartz & Boehnke, 2004; Schwartz et al., 2012).

To address this issue, it would be helpful to perform a statistical analysis based on a formal criterion that sorts items into subscales according to their position in the circle. By analogy with other psychometric instruments, items within a circumplex subscale are expected to be more similar—and thus closer—to each other than to those of adjacent subscales. This raises the question of whether cluster analysis might offer a solution because it seeks to form clusters with maximum item proximity or minimum within-cluster variance based on a formal search criterion (Bridges, 1966). If highly homogeneous subscales are desired, item clusters from traditional cluster analysis could indeed serve as a basis for circumplex subscales. However, subscales are generally expected to cover a broader theoretical range, meaning that items should show moderate heterogeneity even within subscales. Moreover, traditional cluster analysis relies on Euclidean distances, which are the straight-line distances between items in two-dimensional space. In contrast, the circumplex model is based on the angular positions of variables. Angular distances—measured as the angle between vectors drawn from the center of the coordinate system to the items—are therefore more appropriate (see Figure 6). For example, two items with short radii might have a large angular distance. The short radii could produce a small Euclidean distance, and the items could be clustered together by cluster analysis, even though this would be inappropriate from a circumplex perspective. Conversely, items with small angular distances but very different radii might not be clustered together by cluster analysis despite fitting well together within the circumplex framework.

Figure 6

Euclidean Versus Angular Distances in a Circumplex Model



Note. Euclidean distances are indicated by the dashed lines; angular distances are indicated by the solid lines. Conventional cluster analysis is based on (squared) Euclidean distances, whereas ClusterCirc is based on angular distances. The Euclidean distance between Item 1 and Item 2 is larger than that between Item 2 and Item 3, whereas the opposite holds for angular distances. This discrepancy can lead to different clustering solutions, particularly when items differ greatly in radius.

Beyond item proximity, it is also crucial to consider how overall circumplexity can be optimized, taking both the item level and the subscale/cluster level into account. Maximizing item proximity alone, as in traditional cluster analysis, may yield clusters that do not display any circumplex structure at all. Only when cluster centroids also conform to the circumplex pattern does maximizing the proximity of items to their cluster centroids enhance the overall circumplexity of the instrument. Thus, although cluster analysis is well established in psychometrics, its usefulness may be limited in the case of circumplex structure. In light of the shortcomings of SSA and PCA plot slicing, as well as traditional cluster analysis, in finding circumplex subscales, a new method is needed—one that assigns items to subscales based on an objective criterion optimizing circumplexity simultaneously at both the item and subscale levels. This challenge will be addressed in the third research project of the thesis.

3.4.1. ClusterCirc: A New Method to Find Item Clusters With Optimal Circumplexity

The third research project is concerned with the development of ClusterCirc, a new method for identifying item clusters and evaluating item–subscale assignment in circumplex instruments. In the assignment of items to subscales, spacing is more important than radius because circumplex subscales are supposed to cover slices from the center of the circle to the outer arc, where items can be located at different radii. However, radius is also relevant in item selection because equal radius is an important characteristic of circumplex structure. Furthermore, in a two-dimensional model, item radius translates into communalities, which constitute a relevant standard item statistic. Nonetheless, for item–subscale assignment, spacing is more meaningful. Therefore, ClusterCirc focuses on the spacing of items and subscales while leaving radius aside.

Like SSA and PCA plot slicing, ClusterCirc makes use of data topology to divide the circle into same-sized segments for preliminary clusters. However, instead of selecting a particular segmentation based on subjective interpretation, an objective search criterion is applied to find optimal circumplex clusters. ClusterCirc iterates across different segmentations of the circle and selects the one with the best results for item and cluster circumplexity simultaneously. Like cluster analysis, ClusterCirc aims to find item clusters with minimal distances between items within each cluster. However, as noted above, in addition to having relatively small within-cluster variance, as is typical in psychometrics, circumplex clusters are usually preferred to be (nearly) evenly distributed across the circle.

Therefore, ClusterCirc takes into account within-cluster distances as well as between-cluster distances, considering the item and cluster level simultaneously. Furthermore, in contrast to cluster analysis, ClusterCirc is based on angular distances instead of Euclidean distances to align with the expected circular structure. Moreover, ClusterCirc is designed to be more flexible and to offer more applications than traditional cluster analysis, which will be elaborated on in the following sections.

3.4.1.1. Goals and Intended Use

The primary goal of ClusterCirc is to cluster items in a way that circumplex spacing of the resulting subscales is optimal at both the item and subscale levels. Hence, ClusterCirc can be used at an early stage of scale development, after item generation and preselection, when it is still unclear how items should be clustered to form circumplex subscales. In the development of non-circumplex instruments, one might opt for traditional cluster analysis in these cases. ClusterCirc offers an alternative to traditional cluster analysis and produces item clusters as candidate subscales based on an objective circumplex criterion. Within the procedure, it is possible to manually adjust the relative importance of within-cluster proximity versus between-cluster spacing, with the option to entirely disregard either criterion. For instance, if researchers believe that equal spacing between clusters does not reflect the structure of their data and prefer not to optimize it, they can configure ClusterCirc to solely minimize within-cluster distances, similar to traditional cluster analysis.

In addition to finding completely new subscales, ClusterCirc can also be used when researchers have a priori hypotheses about the assignment of items to subscales. These can be addressed in two ways. First, if researchers are more confident about the allocation of some items than others, they can assign different weights to items in ClusterCirc. Greater weights reflect stronger confidence in item–subscale assignment, whereas smaller weights represent more uncertainty. Items with greater weights exert greater influence on the clustering outcome. In contrast, items with smaller weights have a smaller impact on the solution and may be reallocated into different subscales by ClusterCirc. Thus, this strategy allows for (partial) reallocation of items based on the degree of confidence about item–subscale assignment in a highly flexible and adjustable manner.

The second option for including a priori hypotheses on item–subscale assignment is to omit the ClusterCirc search and compute ClusterCirc indices only for predefined subscales. It may also be useful to compare user-defined sorting of items into subscales with item clusters from the ClusterCirc search to evaluate whether reallocations are warranted. Both the assignment itself and the spacing indices can be inspected and compared. ClusterCirc spacing indices can be evaluated at the global level as well as for individual items and clusters. Researchers can also manually reallocate some items and compare ClusterCirc indices for different user-defined solutions to identify the configuration with the most favorable indices, such as the one closest to the optimal ClusterCirc solution.

In addition to item sorting, ClusterCirc indices can inform item selection. The item-level indices can guide decisions about which items to retain, revise, or exclude from the instrument—regardless of whether the subscales are newly identified or pre-established. Nonetheless, such decisions should be integrated into a broader psychometric evaluation framework, including standard indices such as communalities and item–total correlations. In summary, ClusterCirc can assist in finding new circumplex subscales, assessing and improving the circumplexity of existing subscales through (partial) reallocation of items, and guiding item selection and modification. The methodological approach for achieving these objectives will be detailed in the subsequent section.

3.4.1.2. Settings in ClusterCirc

The ClusterCirc search for optimal circumplex clusters is based on the angular positions of items in a two-dimensional space. Item angles can be inserted directly, or they can be derived from a trigonometric transformation of item loadings on two orthogonal components or factors from PCA, EFA, or CFA. If raw scores are inserted, the procedure performs PCA and extracts two orthogonal components without rotation by default, in line with the popularity of PCA in circumplex research (Boudreaux et al., 2018; Horowitz et al., 2017; Locke, 2000, 2019; Richardson et al., 2020; Tracey, 2000). However, allows for the option to obtain item angles by conducting CircE (Grassi et al., 2010) on the raw scores before the ClusterCirc algorithm searches for an optimal item clustering. The Fourier series in CircE implies the presence of a possible third factor. CircE angles might therefore be better suited than PCA angles if a third factor is relevant, as in hierarchical circumplex models with a general factor in addition to the two circumplex axes. If loadings from PCA or EFA are used in ClusterCirc and the theoretical model consists of more than two axes, ClusterCirc should be performed on those factors or components that best represent the circular structure on theoretical grounds. It is furthermore advised to restrict ClusterCirc analyses to factors or components whose eigenvalues exceed the mean eigenvalues derived from parallel analysis (Crawford et al., 2010). This should mitigate the effects of sampling error and reduce the risk of overfitting.

Before conducting the ClusterCirc search on item angles, the desired number of clusters must be specified. Researchers can test different numbers of clusters and select an appropriate solution, such as the one that yields the best ClusterCirc indices or the one that best aligns with theory. Item clusters are identified by dividing the circle into segments of the same size and iterating across different segmentations. This approach is consistent with many psychological theories, in which same-sized segments are a typical configuration of circumplex models (Adams & Tracey, 2004; Gurtman & Pincus, 2000; Horowitz et al., 1988, 2017; Locke, 2000; Trobst, 2000). By default, ClusterCirc assumes that within-cluster distances should be minimized and that equal cluster spacing should be approximated, though users can adjust the relative importance of these criteria. Assumptions regarding item–subscale assignment can also be adjusted by assigning item weights. These parameters are incorporated into the ClusterCirc spacing index, which the algorithm seeks to optimize.

3.4.1.3. The ClusterCirc Spacing Index

The ClusterCirc spacing index is based on angular distances between each item and each cluster. It is defined as an error term that describes how much the empirical item–cluster distances deviate from those expected in a perfect circumplex structure. In this context, perfect circumplex structure is defined to consist of clusters that that are equally spaced around the circle and items that are perfectly aligned on their respective cluster centroids. The ClusterCirc spacing index can be computed for any segmentation of the circle into clusters. It is given by

$$spc_w = \sqrt{\frac{1}{m}\frac{1}{\overline{w_i}}\sum_{i=1}^{m} w_i \sum_{c=1}^{k} w_c \left(\frac{d_{i,c} - \delta_{i,c}}{360/k}\right)^2}, \qquad 0 \leq spc_w \leq 1. \tag{17}$$

Here, the term in brackets describes deviations from ideal spacing, with $d_{i,c}$ denoting the empirical angular distance of item i from cluster c and $\delta_{i,c}$ the ideal distance of item i from cluster c. The number of items is m, the number of clusters is k, the item weight is w_i (with mean item weight $\overline{w_i}$), and each cluster has a weight of w_c.

For the empirical distances $d_{i,c}$, angular positions of all items and all clusters are required. The angular position of each item is computed from factor or component loadings or obtained by CircE, as described above. The angular position of each temporary cluster is defined as the angular cluster centroid, which differs from the centroid in traditional cluster analysis. In ClusterCirc, it is computed as the central angular position between the two outermost items of the temporary cluster rather than the mean Euclidean coordinate of all items that belong to the cluster, as in cluster analysis. This approach allows for maximum item heterogeneity within clusters while maintaining the centroid at the cluster's central angle. The ideal distance $\delta_{i,c}$ between each item and cluster is based on the equal spacing criterion and the requirement of minimum within-cluster distances. The ideal distance between an item and the cluster to which it temporarily belongs is zero, given that, ideally, all items are aligned on their angular cluster centroid. Ideal distances between an item and other clusters depend on the number of clusters. Specifically, the distance of an item to the two neighboring clusters is ideally $360/k$ (e.g., 45° for eight clusters or 120° for three clusters), to the next-to-neighbor clusters $2*360/k$, and so forth.

It is possible to adjust the relative importance of within- versus between-cluster distances by the parameter e, which is embedded in the cluster weight w_c. Specifically, $w_c = e$ for the cluster to which an item (temporarily) belongs, and $w_c = (1 - e)/(k - 1)$ for all other clusters, with $0 \leq e \leq 1$. The default value is $e = 1/k$, which assigns equal weight to all clusters. Thus, by default, the distances of items to their own cluster centroids (within-cluster distances) have the same impact on spc_w as their distances to any other cluster. If $e = 0$, within-cluster proximity is disregarded, and ClusterCirc only enhances equality of spacing between clusters. If $e = 1$, between-cluster distances are ignored, and ClusterCirc maximizes only within-cluster proximity. If $e = 0.5$, within-cluster proximity and between-cluster spacing are equally weighted.

In addition to cluster weights, item weights w_i, can also be specified to adjust the relative impact of each item on the clustering solution. By default, ClusterCirc uses item communalities as weights to ensure that items with greater reliability exert more influence on the final solution. However, item weights can also be varied to reflect theoretical considerations. Items with a clearer expected assignment to a subscale may be given greater weights, so that they have a stronger impact on clustering, whereas items with uncertain subscale assignment may be weighted less. It is also possible to calibrate the relative importance of empirical results (communalities) versus a priori expectations by adjusting the extent to which weights deviate from item communalities.

The spacing index spc_w can range from 0 (ideal spacing) to 1 (worst possible spacing). If $0 < e < 1$, spc_w becomes zero when there is no within-cluster heterogeneity (i.e., all items are placed exactly on their angular cluster centroid) and when all clusters display perfect equal spacing. Otherwise, spc_w yields the proportional deviation from this ideal scenario. In the worst case, the spacing index spc_w becomes 1. For example, this would occur if items were positioned on the centroids of neighboring clusters. Spacing indices of $spc_w > .90$ indicate extreme model misfit and require a complete revision of the theory.

3.4.1.4. Additional ClusterCirc Indices

The weighted spacing index spc_w is used in the ClusterCirc search. However, it may not be easily interpretable as an index of circumplexity due to the inclusion of item and cluster weights. Therefore, ClusterCirc also provides additional indices that can be interpreted more directly. For example, spacing is also computed without item and cluster weights:

$$spc = \sqrt{\frac{1}{m}\frac{1}{k}\sum_{i=1}^{m}\sum_{c=1}^{k}\left(\frac{d_{i,c} - \delta_{i,c}}{360/k}\right)^2}, \qquad 0 \leq spc \leq 1. \tag{18}$$

This unweighted spacing index is reported at the overall level and for each item individually as item spacing, which can be useful in item selection procedures:

$$spc_i = \sqrt{\frac{1}{k}\sum_{c=1}^{k}\left(\frac{d_{i,c} - \delta_{i,c}}{360/k}\right)^2}, \qquad 0 \leq spc_i \leq 1. \tag{19}$$

Like spc_w, the unweighted spacing indices spc and spc_i equal zero in the case of perfect equal between-cluster spacing and zero within-cluster distances of items.

ClusterCirc furthermore yields an index of spacing solely at the cluster level with between-cluster spacing:

$$bcs = \sqrt{\frac{2}{k\,(k-1)}\sum_{c=1}^{k}\sum_{c'=c+1}^{k}\left(\frac{d_{c,c'} - \delta_{c,c'}}{360/k}\right)^2}, \qquad 0 \leq bcs \leq 1, \tag{20}$$

where $d_{c,c'}$ is the empirical distance between cluster c and cluster c'. Only the upper (or lower) triangle of the distance matrix is included in bcs to avoid redundancies, yielding $2/(k(k\text{-}1))$ comparisons. Between-cluster spacing bcs is zero in the case of perfect equal distances between clusters. Otherwise, it represents the departure of such perfect circumplex spacing at the cluster level.

Homogeneity of items within clusters is captured by within-cluster proximity:

$$wcp = \sqrt{\frac{1}{k}\sum_{c=1}^{k}\frac{1}{m_c}\sum_{i=1}^{m_c}\left(\frac{d_{i,c}}{360/2k}\right)^2}, \qquad 0 \leq wcp \leq 1, \tag{21}$$

with m_c denoting the number of items within each cluster and k the number of clusters. Unlike spc_w, spc, and spc_i, the distance $d_{i,c}$ here describes the distance of item i from only its own cluster centroid. Thus, *wcp* becomes zero when all items are placed exactly on their cluster centroid, regardless of distances to the other clusters. Values greater than zero indicate deviations from such maximum homogeneity of items within clusters. The maximum value of $wcp = 1$ would, for instance, occur if all items were aligned on the boundary to neighboring clusters while clusters were equally spaced. Within-cluster proximity is conceptually similar to (the inverse of) within-cluster variance from cluster analysis. However, as mentioned before, it is based on angular rather than Euclidean distances, and the cluster centroid is defined as the central position rather than the mean coordinates of the items within the cluster. At the item level, ClusterCirc reports $d_{i,c}/(360/2k)$, the proportional distance of an item from its cluster centroid. At the cluster level, wcp_c indicates how much items deviate from their cluster centroid within cluster c. All ClusterCirc indices can be inspected at the overall level (Equations 15, 16, 18, and 19) to assess circumplex spacing for the instrument as a whole, at the cluster level to evaluate individual clusters, and at the item level to assist in item analysis and item selection (Supplement A of the published article, p. 2, Weide et al., 2025).

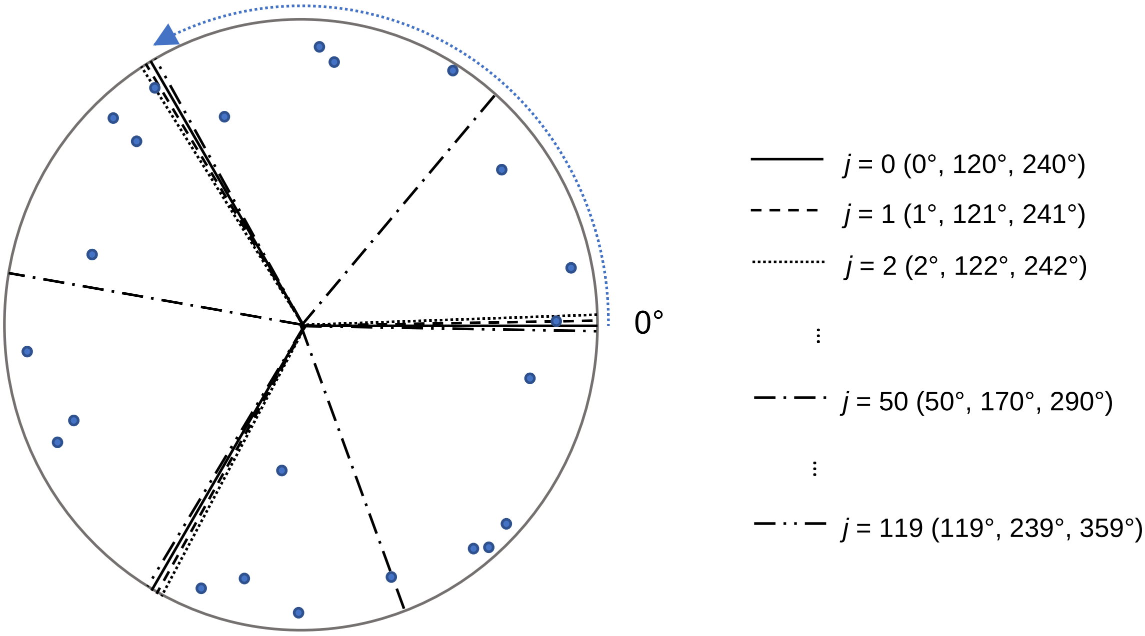
3.4.1.5. ClusterCirc-Data: The ClusterCirc Search for Optimal Circumplex Clusters

The main purpose of ClusterCirc is to find item clusters with optimal circumplex spacing. For this purpose, the main function—ClusterCirc-Data—allows users to search for optimal circumplex clusters based on their data (raw scores, loadings on two orthogonal axes, or item angles directly as input). ClusterCirc-Data searches for a segmentation of the circle into same-sized clusters that yields the best circumplex spacing. The algorithm is a simple brute-force search, exhaustively exploring all possible solutions to find the best one among them. It is based on the weighted ClusterCirc spacing index spc_w—initially set to a large value—which is minimized by the ClusterCirc algorithm. ClusterCirc begins by dividing the circle into same-sized segments, with boundaries depending on the number of clusters k. The first cluster boundaries are set to 0°, 360°/k, 2×360°/k, … , (k-1)×360°/k in the given coordinate system. For example, initial cluster boundaries are 0°, 120°, 240° in the case of three clusters, 0°, 90°, 180°, 270° in the case of four clusters, 0°, 72°, 144°, 216°, 288° in the case of five clusters, and so forth. All items that lie within the range of a segment are automatically assigned to that cluster. Hence, the number of items per cluster can vary between clusters. For this initial division, spc_w is calculated and stored to be compared with the subsequent solution.

Next, cluster boundaries are shifted by a step length of $1/q°$. The parameter q controls precision of the algorithm, with a default of $q = 10$, balancing computational speed and accuracy. On high-performance devices, higher q values are possible but generally have minimal impact on the results. The spc_w value of the new solution is compared with that of the previous solution. If it is smaller than the previous spc_w, the earlier solution and its spc_w value are dismissed, and the new solution and corresponding spc_w value are stored for subsequent comparisons. This process iterates until all possible boundary shifts with a step length of $1/q°$ are tested, and the configuration with the smallest spc_w is selected for the final solution. The ClusterCirc algorithm for an example of three clusters and a precision index of $q = 1$ is illustrated in Figure 7 (adapted from Weide et al., 2025; p.11).

Figure 7

Illustration of the ClusterCirc Algorithm for an Example of Three Clusters



Note. Each dot in the circle represents an item. Each dot in the outer (blue) arc represents an iteration *j* with a possible division of items (*j* ranges from 0 to $360 \times q/k$ in steps of $1/q$; here illustrated for $q = 1$). In every iteration, the circle is divided into same-sized segments with a range of $360°/k$ (here: 3 clusters with 120°). Cluster limits are represented by the straight lines within the circle. All items that fall within a cluster's range are automatically assigned to it. ClusterCirc then computes the spacing index spc_w for each division *j*. If spc_w is smaller in a subsequent division, the previous value is replaced and the respective item clusters are stored, representing better circumplexity. From all $360 \times q/k$ possible divisions, the one with the smallest spacing index is selected, and items are assigned to the corresponding clusters for the final solution. *j* = iteration with suggested division of items; *k* = number of clusters; *q* = precision index of the algorithm. Adapted from "ClusterCirc: Finding Item Clusters for Circumplex Instruments" by A. C. Weide, T. Kuhl, & A. Beauducel, 2025, *Journal of Educational and Behavioral Statistics*, Advance online publication, p. 11. (https://doi.org/10.3102/10769986251323017).

3.4.1.6. ClusterCirc-Simu: Model Fit of the ClusterCirc Solution

To test whether the circumplex model fits the data, ClusterCirc includes a simulation module, ClusterCirc-Simu. This generates reference values for the indices under ideal circumplexity. The population model is based on empirical parameters (sample size, number of clusters, number of items, angular within-cluster range, mean communality), but it assumes perfect spacing of clusters and even item distribution within clusters. ClusterCirc-Simu draws the desired number of samples (default: 500 samples) from the population with perfect circumplexity and runs ClusterCirc on population data as well as on the simulated samples. ClusterCirc indices are reported for the population and for the simulated samples (mean and standard deviation). The empirical spc_w*-data* value is compared against the mean spc_w*-simu* value from the simulation study in a one-tailed significance test based on a normalized distribution of spc_w*-simu* values. The circumplex model is considered acceptable for the data if spc_w*-data* does not exceed the critical value of the spc_w*-simu* distribution given a desired significance level α. Hence, the null hypothesis assumes that the empirical data conform to a circumplex structure, which is typically the desired outcome. In contrast, the alternative hypothesis posits that the data do not agree with the circumplex model. Therefore, increasing the significance level α (e.g., to 25%) enhances the power to detect deviations from circumplex structure by reducing the Type II error rate, that is, the risk of incorrectly accepting circumplexity. However, perfect circumplexity is rarely observed in empirical data due to factors such as item unreliability and sample heterogeneity, even when the underlying structure can be considered sufficiently circular. As a result, the test is inherently conservative. To avoid excessive stringency, the default α level is set at the rather small value of 1% to acknowledge the difficulty of achieving perfect circumplexity in real-life data. For comparability, ClusterCirc-Simu should be run with the same estimation method for item angles (based on PCA loadings or CircE estimation) and with the same relative importance of within-cluster proximity versus between-cluster spacing (parameter *e*) as ClusterCirc-Data. The current version of ClusterCirc-Simu weights items by their communalities; therefore, ClusterCirc-Data should also be run with communalities to ensure a valid comparison.

3.4.1.7. ClusterCirc-Fix: ClusterCirc Indices for Fixed a Priori Clusters

Researchers who wish to evaluate predefined item clusters (e.g., preliminary subscales) without running the full ClusterCirc search for optimal clustering can use ClusterCirc-Fix. ClusterCirc-Fix does not alter item allocation but instead reports ClusterCirc indices at all levels: overall circumplexity, cluster indices, and item indices. Users specify which items belong to which subscale and how item angles should be computed (e.g., PCA loadings or CircE), unless angles are provided directly. They can also decide whether to weight items by communalities or user-defined values and whether to adjust the balance between within- and between-cluster spacing in the computation of spc_w for the fixed item clusters. Fixed clusters can be compared with the suggested clustering by ClusterCirc-Data, ideally run with the same specifications as in ClusterCirc-Fix for an adequate comparison. If multiple user-defined versions of item clusters are compared, the one with spacing indices closest to the optimal values from the ClusterCirc-Data solution might be preferred. This comparison can inform decisions about whether the initial subscales should be retained or revised to improve circumplex structure—for instance, by revising, reallocating, or completely removing items from the instrument.

3.4.1.8. ClusterCirc R Package and SPSS Codes

The ClusterCirc procedures (ClusterCirc-Data, ClusterCirc-Simu, and ClusterCirc-Fix) can be performed by installing and loading the ClusterCirc R package from the github repository (https://github.com/ancleo/ClusterCirc) or by downloading and running the corresponding code in SPSS/SPSS Matrix syntax (https://github.com/ancleo/ClusterCirc_SPSS). Technical information on how to insert parameters, how to apply the ClusterCirc functions, and how to interpret the results can be found on the github websites, in Supplement B of the published article (Weide et al., 2025), in the vignette, description, and function documentation of the R package, and in the comments of the SPSS syntax files.

3.4.2. Research Questions for the Evaluation of ClusterCirc

As ClusterCirc is a novel method for item clustering in circumplex instruments, it is essential to examine the spc_w value and the effectiveness of ClusterCirc in identifying circumplex clusters for different scenarios created in simulation studies. To establish the added value of ClusterCirc over existing procedures, its performance should be directly compared with alternative item clustering techniques. While widely used methods such as visual inspection of SSA and PCA plots are popular in practice, they are inherently subjective and thus unsuitable for controlled evaluation in simulation studies. In contrast, traditional cluster analysis methods are based on objective distance metrics (typically decreasing distances of items within clusters) and can thus be employed as a benchmark for comparison. Accordingly, the third research project of this thesis focuses on a systematic comparison between ClusterCirc and conventional cluster analysis techniques. This includes both hierarchical clustering using Ward's method (Bridges, 1966; L. C. Morey et al., 1983) and nonhierarchical k-means clustering (MacQueen, 1967).

For this comparison, the influence of different levels of item heterogeneity within clusters is of interest. Both ClusterCirc and traditional cluster analysis aim to minimize within-cluster item distances; however, ClusterCirc additionally incorporates spacing between clusters, which may offer an advantage in scenarios characterized by relatively large item heterogeneity within clusters. Given the preference for equally spaced clusters in ClusterCirc, simulation conditions should include both equally spaced and unequally spaced cluster configurations in the population model to assess the method's robustness. Another factor likely to affect the performance of the clustering methods is the dimensionality of the underlying model. In particular, differences may emerge between purely two-dimensional circumplex structures and hierarchical circumplex models that include a general factor alongside the circumplex axes. In such cases, hierarchical methods like Ward's clustering may have an advantage over nonhierarchical techniques such as k-means and ClusterCirc. Moreover, the method used to estimate item angular positions may influence clustering outcomes. CircE estimation of item angles inherently accommodates a possible third factor, which could improve ClusterCirc's performance in hierarchical circumplex models compared with angle estimates based on PCA loadings.

Beyond simulation-based evaluations, it is also critical to assess how ClusterCirc performs when applied to empirical data. This will involve analyzing data from a well-established circumplex instrument to determine how ClusterCirc assigns items to clusters and to what extent these assignments deviate from the instrument's original subscales. This empirical evaluation will be conducted using data from the German IAS version, the IAL (Adams & Tracey, 2004; Jacobs & Scholl, 2005), within the third research project of the thesis.

4. Studies

The three research projects were associated with three studies that investigated the research questions of the thesis (Weide & Beauducel, 2019; Weide et al., 2021; Weide et al., 2025). For this purpose, two large simulation studies (Research project 1 and 3) and one empirical study were conducted (Research Project 2, with an additional empirical evaluation in Project 3). The articles of the peer-reviewed publications can be found by the links provided in the

Appendix of the thesis. This chapter provides an overview of the research questions, study designs, and main findings. A more detailed description can be found in the original articles.

4.1. Study 1: Varimax Rotation Based on Gradient Projection and Local Optima in Double Simple Structure

The first study examined the performance of GPA rotation (Bernaards & Jennrich, 2005; Jennrich, 2001, 2002; Mulaik, 2010) toward the Varimax criterion in PCA (Weide & Beauducel, 2019). Given that the algorithm is susceptible to converge on a rotation matrix close to the start matrix, a modified double simple structure was used as a difficult circumplex model to test for local optima of the rotation procedure (see Section 3.2; Figure 3D). To evaluate its validity as a relatively new rotational method, GPA rotation was compared to the built-in Varimax rotation in SPSS (Kaiser algorithm; Kaiser, 1958) to assess whether it performs comparably, worse, or better than established methods. The research questions were as follows:

1. Can GPA-Varimax rotation be used as an alternative to SPSS-Varimax? Does it offer any advantages over the established and widely-used Kaiser algorithm?
2. Does GPA-Varimax rotation find the global optimum of the Varimax criterion, or does it stop at a local optimum? If local optima are of concern, how can the procedure be modified to find the global optimum?

4.1.1. Conditions of the Simulation Study

The research questions were addressed in a large simulation study. Two population models were created, comprising a single-optimum condition with perfect orthogonal simple structure and a double-optimum condition with a modified (circumplex) double simple structure. The double simple structure was altered so that one of the simple structures received greater loadings than the alternative one. Hence, one of the rotational positions of the two axes constituted the global optimum of the Varimax criterion, whereas the other constituted the local optimum. Loading patterns are shown in Table P1.1 of the article for the single-optimum condition and Table P1.2 for the double-optimum condition. Figure 3D displays a schematic illustration of modified double simple structure, and Figure P1.1 in the article depicts the loading plot with exact population loadings (Weide & Beauducel, 2019, pp. 4-6).

The number of components was varied with l = 3, 6, 9, and 12 for both the single-optimum and the double-optimum condition. In the double-optimum condition, two thirds of the components were modeled as circumplex pairs, whereas the remaining components displayed simple structure (e.g., 3 circumplex pairs and 3 simple structure components in $l = 9$), with orthogonality maintained for all components. The number of variables was varied with $m/l = 4$, 6, and 8 variables per component for the single-optimum condition. For the double-optimum condition, 16 variables per circumplex plus 8 variables for each additional component were created (e.g., 24 variables in a 3-component double-optimum condition). Population data consisted of 1,000 samples per condition. Samples were drawn with $n = 100$, the minimum sample size recommended for factor analysis (Gorsuch, 2015), and $n = 300$, which has been considered *good* for factor analytic procedures (MacCallum et al., 1999). The simulated data were subjected to PCA with Varimax rotation based on GPA versus the built-in Kaiser algorithm in SPSS. GPA rotation was performed with an unrotated start loading matrix as well as random start matrices (1, 10, 50, and 100 random start loading matrices and selection of the best solution) to investigate possible local optima. For both SPSS-Varimax and GPA-Varimax, the number of iterations per rotation was varied with 25 and 250, and rotation was performed on both Kaiser-normalized loadings and nonnormalized loadings.

4.1.2. Results of the Simulation Study

Population data and sample data were analyzed, and congruence coefficients (Lorenzo-Seva & ten Berge, 2006) were computed between each sample solution and the corresponding population solution. Congruence coefficients were averaged across the 1,000 samples of each condition to assess the performance of the rotation procedure in identifying the respective population structure. Additionally, the Varimax criterion was retained for each solution and averaged across samples for each condition to assess the effectiveness of the rotation procedures in optimizing the Varimax criterion. For equality of results, cut-offs were defined that were scaled according to the size of the congruence coefficients (mean values ranging from $g = .729$ to $g = .984$) and the Varimax criterion (mean values ranging from $v = .0066$ to $v = .0417$) of the simulated data. Accordingly, results were considered equal if both the absolute difference between mean congruence coefficients was $|\Delta| < .001$ and the absolute difference in the mean Varimax criterion was $|\Delta| < .0001$.

In the single-optimum condition, SPSS-Varimax and GPA-Varimax showed fairly similar performance with equal results in over 90% of the comparisons. Very few and small differences (close to the cut-offs) were found in favor of SPSS-Varimax. These differences diminished when loadings were Kaiser-normalized before rotation and 250 (instead of 25) iterations were used per rotation. As expected, inserting random start matrices in GPA rotation and selecting the best solution among them did not improve rotational performance over an unrotated start loading matrix in the single-optimum condition, where no additional local optima were modeled in the population data structure (Tables P1.3 and P1.4 in the article; Weide & Beauducel, 2019, pp.8–9).

In the double-optimum circumplex condition, however, more relevant differences emerged between SPSS and GPA rotation and between different random starts in GPA, indicating the relevance of local optima. Rotation performance was comparable between SPSS and GPA rotation in 50 out of 96 conditions, better for GPA rotation in 30 conditions, and slightly better for SPSS rotation in 16 conditions (Tables P1.5 and P1.6 in the article; Weide & Beauducel, 2019, p. 10). Differences in favor of SPSS were again marginal, whereas differences in favor of GPA were more substantial, with more than half of the congruence differences exceeding .020. These substantial differences in favor of GPA occurred in conditions with large samples (n = 300), when loadings were Kaiser-normalized, and when the best solution from multiple random starts was selected in GPA rotation. Hence, as expected, GPA rotation required multiple random starts to overcome local optima in some conditions with the double-optimum circumplex model. GPA with random starts thereby outperformed the built-in Kaiser algorithm in SPSS for Varimax rotation. However, results indicated that 10 random starts were sufficient for optimal results in GPA-Varimax rotation, as rotational performance did not further improve with 50 or 100 random starts.

4.1.3. Empirical Example

To complement the simulation study, empirical data were analyzed to examine the suitability of GPA-Varimax rotation in practice. PCA was conducted on 17 newly developed single-choice knowledge test items completed by 397 German high school students (55 females, 342 males, no nonbinary participants; M = 19.53 years, SD = 2.49). The items covered three knowledge domains—geography/history, science, and culture/arts—that were expected to be uncorrelated and comply with orthogonal simple structure. Three components explaining 30.52% of the variance were retained for Varimax rotation. Rotation was carried out using SPSS-Varimax and GPA-Varimax with up to 10 random start matrices, as the simulation study showed no benefit beyond 10 random starts. Despite some secondary loadings, the component structure aligned well with the three domains. GPA results closely matched SPSS loadings, with congruence coefficients of g > .999 (with Kaiser normalization) and g > .990 (without). The Varimax criterion was approximately v = .1900 across all procedures (SPSS and GPA with and without Kaiser normalization, GPA with unrotated start loadings or random starts) with only small differences at the third decimal place. Hence, results were highly similar in the empirical example, and component interpretation was not affected by differences in the rotation procedure.

4.1.4. Conclusion and Recommendations

The results of Study 1 suggest that Varimax rotation based on GPA can be used as an alternative to the well-established Kaiser algorithm implemented in SPSS and other statistical software. With appropriate adjustments in GPA-Varimax rotation, the small advantages of SPSS in some conditions were eliminated, and GPA even yielded solutions that were superior to the SPSS Kaiser algorithm. This was mainly the case in the more difficult conditions comprising circumplex structures with a global and a local optimum of the Varimax criterion. The results in these double-optimum conditions—as opposed to the single-optimum conditions—support the notion that local optima are of concern in factor and component rotation. Consistent with previous research on other rotation procedures (Browne, 2001; Hattori et al., 2017; Kiers, 1994; Trendafilov & Jolliffe, 2006), the results suggest that local optima can be overcome and the global optimum can be identified by using multiple random starts and selecting the best solution among them in GPA-Varimax rotation. For optimal results of GPA-Varimax, loadings should be Kaiser-normalized before rotation, rotation should be based on 250 iterations, and the best solution from at least 10 random start matrices should be selected. R code and SPSS syntax for these modifications are provided in the Supplement of the paper: https://www.frontiersin.org/articles/10.3389/fpsyg.2019.00645/full#supplementary-material

4.2. Study 2: Bayesian Confirmatory Factor Analysis and Higher-Level Scores of the IIP

In contrast to the exploratory approach of Study 1, Study 2 focused on confirmatory testing of the circumplex factor model. In particular, the higher-level structure of the IIP comprising the three factors Dominance, Love (circumplex factors) and Distress (general factor), as well as respective higher-level scores, was investigated (Weide et al., 2021). The research questions were:

1. Is BCFA more suitable than MCFA in examining the higher-level structure of the IIP given its greater flexibility for modeling the circumplex via Bayesian priors? Could TEFA also be an alternative?

2. Can the use of higher-level scores be supported for the IIP? Are there differences between weighted sum scores, BCFA scores, and MCFA regression scores?
3. Can external criteria be related to the three MCFA and BCFA factors and the higher-level scores in line with previous research (see Table 4, upper part)?

4.2.1. Method

The research questions were addressed by an online questionnaire study conducted in compliance with the Declaration of Helsinki and approved by the Ethics Board of the Psychology Department from the University of Bonn. The final data set comprised 822 participants (62.8% female, 37.2% male, no nonbinary participants), aged between 16 and 89 years, with a mean of M = 36.68 years (SD = 14.79). Participation was voluntary, lasted approximately 45–60 minutes, and participants received course credit if needed for their degree. The empirical study formed part of a larger research project, in which a variety of psychometric instruments were completed. The measures used in the present study were the IIP (Horowitz et al., 2017) with 64 items measuring interpersonal difficulties, a 40-item measure for the Big Five from the International Personality Item Pool (Hartig et al., 2003), and a 17-item version of the Narcissistic Personality Inventory (NPI-17; con Collani, 2014), all administered in German.

4.2.2. Analysis and Results

Before examining the higher-level structure of the IIP, sum scores of the eight IIP subscales were entered in CircE (Grassi et al., 2010) to test if their overall correlational pattern conformed to the expected circular arrangement. CircE analysis showed that model fit improved substantially when spacing of the IIP octants was estimated freely rather than fixed to equal distances. This suggests that the strict equal spacing assumption as also implied by MCFA might not be appropriate for the IIP octants. CircE estimated the minimum correlation between scores at a 180° distance as r = .001, supporting the three-factor model rather than a simple two-factor circumplex, for which r(180°) = –1 would have been expected.

4.2.2.1. Factor Analyses

The MCFA bi-factor model for the IIP octants consisted of freely-estimated loadings on the general factor Distress and fixed loadings on Dominance and Love, in line with the ideal circumplex (Equations 7 and 8, Table 3; adapted from Weide et al., 2021, p. 6). The three factors were specified as uncorrelated. The BCFA and TEFA models comprised the same specifications as the MCFA model. However, prior variances were defined for the circumplex loadings in BCFA, allowing posterior loadings to deviate from the ideal circumplex. The prior variance was initially set to a smaller value of $\sigma^2 = 0.01$ and raised to $\sigma^2 = 0.1$ in a second step, following recommendations (Asparouhov et al., 2015). In TEFA, circumplex loadings were entered as target loadings for factor rotation. Loadings entered in MCFA, posterior loadings from BCFA, and the rotated loading pattern from TEFA can be inspected in Table 3 (adapted from Weide et al., 2021; p. 6). The fit of the MCFA model was $\chi^2(18) = 207.13$, $p < .001$, *CFI* = .94, *SRMR* = .09, *RMSEA* = .11, 90% CI [.10, .13]. The fit of the BCFA model with a prior variance of $\sigma^2 = 0.01$ was 95% CI [34.10, 86.48] for $\chi^2(40)$, the χ^2-based posterior predictive *p*-value was $p < .001$, *CFI* = .98, *RMSEA* = .09, 90% CI [.08, .10]. A prior variance of $\sigma^2 = 0.1$ in BCFA yielded 95% CI [29.59, 80.05] for $\chi^2(40)$, the χ^2-based posterior predictive *p*-value was $p < .001$, *CFI* = .98, *RMSEA* = .09, 90% CI [.08, .11]. TEFA model fit was $\chi^2(5) = 62.03$, $p < .001$, *CFI* = .99, *SRMR* = .01, *RMSEA* = .12, 90% CI [.09, .15].

The similarity between the BCFA and TEFA circumplex factors with the ideal circumplex as entered in the MCFA model was assessed by Tucker's congruence coefficients (Lorenzo-Seva & ten Berge, 2006). As expected, BCFA factors with smaller prior variances showed higher congruences (*g* = .984 for Dominance and *g* = .984 for Love) than BCFA factors with larger prior variances (*g* = .976 for Dominance and *g* = .977 for Love). Therefore, and since the fit indices of the two BCFA models were comparable, all follow-up analyses were based on the smaller prior variance $\sigma^2 = 0.01$. TEFA congruence coefficients with the ideal circumplex were much lower, with *g* = .735 for Love and *g* = .798 for Dominance (*g* = .714 for Love and *g* = .812 for Dominance after Kaiser-normalization). As TEFA circumplex factors showed greater deviations from the ideal circumplex while not resulting in greater model fit than BCFA, no further TEFA analyses were conducted.

Across all BCFA loadings, deviations from the perfect MCFA circumplex were most prominent for the LM/Overly Nurturant scale on Dominance and the PA/Domineering scale on Love, both of which were set to zero in the MCFA model. An additional misspecification analysis of these loadings in MCFA (Saris et al., 1987, 2009) indicated large expected parameter changes, suggesting that zero loadings might be overly restrictive. To further test model validity, follow-up analyses allowed for correlated error terms. Twelve correlations with modification indices > 10 in the initial MCFA were freely estimated. MCFA resulted in seven significant correlated errors ($p < .05$), whereas BCFA yielded only two, suggesting that circumplex priors offer a solution that better satisfies the CFA assumption of uncorrelated

Table 3

Loading Patterns of the Three-Factor Models for the Subscales of the Inventory of Interpersonal Problems in Study 2

Factor	Subscale	MCFA	BCFA	TEFA
	PA	1	1	.36
	BC	.71	.62***	.13
	DE	0	.06	-.20
Dominance	FG	-.71	-.53***	-.57
(circumplex)	HI	-1	-.89***	-.78
	JK	-.71	-.74***	-.74
	LM	0	-.26***	-.51
	NO	.71	.64***	.09
	PA	0	-.23***	-.53
	BC	-.71	-.79***	-.78
	DE	-1	-.88***	-.76
Love	FG	-.71	-.70***	-.66
(circumplex)	HI	0	.19***	-.12
	JK	.71	.84***	.15
	LM	1	1	.14
	NO	.71	.83***	.05
	PA	.66***	.51***	.53
	BC	.49***	.61***	.33
	DE	.47***	.65***	.21
Distress	FG	.59***	.74***	.18
(general)	HI	.64***	.62***	.30
	JK	.73***	.62***	.52
	LM	.64***	.63***	.68
	NO	.65***	.49***	.78

Note. Dominance, Love, and Distress were uncorrelated in all factor models. MCFA loadings were fixed according to the ideal circumplex model and inserted as priors for BCFA and as target loadings for TEFA. For scaling adjustments, the loadings of PA on Dominance and of LM on Love were fixed to 1 in the BCFA. MCFA = Confirmatory factor analysis; BCFA = Bayesian confirmatory factor analysis; TEFA = Exploratory factor analysis with target rotation. Adapted from "Bayesian and Maximum-Likelihood Modeling and Higher-Level Scores of Interpersonal Problems With Circumplex

Structure" by A. C. Weide, V. Scheuble, & A. Beauducel, 2021, *Frontiers in Psychology, 12*, Article 761378, p. 6 (https://doi.org/10.3389/fpsyg.2021.761378).

*** $p < .001$.

errors. Furthermore, robustness checks were conducted via an odd-even split of the sample.Based on the Bayesian Information Criterion, model fit differences between samples were smaller for BCFA than MCFA, indicating greater stability of BCFA results across samples.

4.2.2.2. Higher-level Scores of the IIP

In view of the well-fitting three-factor model of the IIP, higher-level scores for the three IIP factors Love, Dominance, and Distress were examined to test whether they could serve as feasible and economic measures of interpersonal problems. For this purpose, MCFA regression factor scores, mean plausible values from BCFA, and weighted sum scores based on the ideal circumplex (weights like MCFA loadings, Table 3) were computed. To evaluate the reliability and validity of these scoring methods, regression component loadings implied by the scoring models were computed (Beauducel, 2005; Schönemann & Steiger, 1976). Reliability estimates of the scoring-implied loadings exceeded .90 for all scores across methods. Internal validity was examined via congruence coefficients between MCFA and BCFA factors with the respective scoring-implied loadings. All congruence coefficients were $g \geq .99$, indicating excellent agreement. Further, CircE (Grassi et al., 2010) was performed on reproduced intercorrelations of the IIP octants based on the scoring-implied loadings. Compared to the original intercorrelations, weighted sum scores improved CircE model fit for all types of constraints (equal spacing and radius). MCFA and BCFA scores also improved CircE model fit under some conditions. The gap difference test (Acton & Revelle, 2004) showed that reproduced IIP octants based on the scoring models were closer to equal spacing than the original IIP octants. Values were $\sqrt{GDIFF} = 18.61$ for MCFA scores, $\sqrt{GDIFF} = 23.26$ for weighted sum scores, $\sqrt{GDIFF} = 23.67$ for BCFA scores, and $\sqrt{GDIFF} = 29.26$ for original octants.

4.2.2.3. External Validity of the Factor Models and Higher-Level Scores

To assess external validity, the external measures were included in the BCFA and MCFA models. For MCFA, including Extraversion, Agreeableness, and Neuroticism yielded $\chi^2(36) = 335.00$, $p < .001$, $CFI = .94$, $SRMR = .09$, $RMSEA = .10$, 90% CI [.09, .11]. The inclusion of NPI-17 scores (narcissism) in the MCFA model resulted in $\chi^2(23) = 248.41$, $p < .001$, $CFI = .94$, $SRMR = .09$, $RMSEA = .11$, 90% CI [.10, .12]. For BCFA, inclusion of the Big Five measures resulted in 95% CI [113.73, 179.30] for $\chi^2(55)$, χ^2-based posterior predictive $p < .001$, *CFI* =.97, *RMSEA* = .08, 90% CI [.08, .09]. The BCFA model with NPI-17 scores yielded 95% CI [67.24, 124.26] for $\chi^2(45)$, χ^2-based posterior predictive $p < .001$, *CFI* =.98, *RMSEA* = .09, 90% CI [.08, .10]. The loading pattern of the external measures on the three IIP factors was largely comparable across MCFA and BCFA and aligned with hypotheses (Table 4; adapted from Weide et al., 2021, p. 9). One deviation occurred: The negative loading of Agreeableness on Dominance was higher than its positive loading on Love, contrary to expectations. The correlational pattern of higher-level scores was consistent with the factor model loadings across all scoring methods (Table 4; adapted from Weide et al., 2021, p. 9). However, in contrast to prior expectations, NPI-17 scores measuring subclinical narcissism showed a slight negative correlation with Love scores.

4.2.3. Conclusion and Recommendations

In summary, the results of Study 2 support the use of BCFA for evaluating circumplex structure, particularly in hierarchical models. Bayesian priors provide greater flexibility in modeling the circular structure, leading to improved model fit, fewer correlations between error terms, and greater robustness across samples compared with traditional MCFA. Although TEFA also offers more flexibility than MCFA, the findings suggest that target-rotated loadings may deviate more strongly from the ideal circumplex without yielding substantial improvements in model fit relative to BCFA. Results further support the use of higher-level scores for the IIP. All three types—weighted sum scores, MCFA regression factor scores, and BCFA mean plausible values—proved to be reliable and valid indicators of the IIP factors Dominance, Love, and Distress. Beyond internal structural validity, the inclusion of external criteria in MCFA and BCFA and their correlations with the higher-level scores largely aligned with hypotheses based on previous research, which provides evidence for the external validity of the IIP's higher-level model.

Table 4

Hypotheses and Empirical Associations of Higher-Level Factors and Higher-Level Scores of the Inventory of Interpersonal Problems With External Measures from Study 2

		Extraversion	Agreeableness	Neuroticism	Grandiose narcissism
Hypotheses	DOM	++	-		+
	LOV	+	++		
	GEN			+	
MCFA factor loadings	DOM	**1.00*****	**-.96*****	-.14*	**1.05*****
	LOV	**.81*****	**.58*****	-.19**	.04
	GEN	-.36***	-.20***	**.60*****	.01
BCFA factor loadings	DOM	**.87*****	**-.94*****	-.14*	**.94*****
	LOV	**.77*****	**.68*****	-.16**	-.05
	GEN	-.35***	-.23***	**.61*****	.04
MCFA regression factor scores	DOM	**.40*****	**-.59*****	-.07	**.53*****
	LOV	**.28*****	**.45*****	-.07*	-.09*
	GEN	-.35***	-.16***	**.57*****	-.01
BCFA factor scores	DOM	**.45****	**-.56****	-.09*	**.53****
	LOV	**.34****	**.40****	-.08*	-.10**
	GEN	-.35**	-.18**	**.57****	.02
Weighted sum scores	DOM	**.45*****	**-.56*****	-.14***	**.53*****
	LOV	**.31*****	**.43*****	-.08*	-.13***
	GEN	-.32***	-.19***	**.57*****	.02

Note. Double versus single signs (e.g. ++ vs. +) indicate the relative sizes of the expected associations within the respective column; they do not represent hypotheses across the different external constructs. Boldface entries show results for which hypotheses on the correlational pattern were formulated. MCFA = Frequentist confirmatory factor

analysis based on maximum-likelihood estimation; BCFA = Bayesian confirmatory factor analysis; DOM = Dominance; LOV= Love; GEN = General interpersonal distress. BCFA factor scores were Bayesian mean plausible values. Adapted from "Bayesian and Maximum-Likelihood Modeling and Higher-Level Scores of Interpersonal Problems With Circumplex Structure" by A. C. Weide, V. Scheuble, & A. Beauducel, 2021, *Frontiers in Psychology, 12*, Article 761378, p. 9 (https://doi.org/10.3389/fpsyg.2021.761378).
* $p < .01$ (two-tailed). ** $p < .01$ (two-tailed). *** $p < .001$ (two-tailed).

4.3. Study 3: Evaluation of ClusterCirc

The third study of the thesis evaluated ClusterCirc as a new method for item clustering in circumplex instruments (Weide et al., 2025). The performance of ClusterCirc was investigated in a large simulation and compared with hierarchical Ward clustering and nonhierarchical k-means clustering as alternative techniques. ClusterCirc was also applied to the IAL, a German IAS version (Adams & Tracey, 2004; Jacobs & Scholl, 2005), as an empirical example. The research questions and hypotheses were as follows:

1. How does ClusterCirc perform in detecting circumplex clusters across different levels of item heterogeneity within clusters, under equal- versus unequal-spacing scenarios, and in two- versus three-dimensional (hierarchical) models? How do the number of items, clusters, and sample size impact clustering performance?
2. How does clustering by ClusterCirc compare with traditional hierarchical cluster analysis based on Ward's method and with k-means clustering?

   2.a. With greater item heterogeneity within clusters, ClusterCirc could be advantageous because it optimizes both within- and between-cluster distances, whereas cluster analysis only minimizes within-cluster distances.

   2.b. Unequal cluster spacing may be particularly challenging for ClusterCirc due to its preference for equal spacing between clusters, whereas cluster analysis does not impose equal spacing constraints.

   2.c. In hierarchical data structures with a third general factor alongside the two circumplex axes, hierarchical Ward clustering might have an advantage over ClusterCirc and k-means clustering as nonhierarchical techniques.
3. For item angles as input in ClusterCirc: In hierarchical data structures, CircE angles may be better suited than angles based on PCA loadings because CircE automatically accounts for the possibility of a third factor.
4. To what extent do item clusters suggested by ClusterCirc align with, or deviate from, the original subscales of the IAL and from Ward clusters in an empirical example?

4.3.1. Conditions of the Simulation Study

The simulation study comprised five two-dimensional, nonhierarchical population models and four hierarchical population models with an additional general factor alongside the two circumplex factors (Table 5; adapted from Weide et al., 2025, p. 21). For the nonhierarchical models, four levels of item heterogeneity within clusters were created: zero (all items perfectly aligned on their cluster centroid), small, medium, and large, with progressively greater distances between items within clusters. In these models, clusters were equally spaced across the circumplex. Additionally, one nonhierarchical model was created with unequal cluster spacing and medium within-cluster item heterogeneity. The hierarchical model consisted of two conditions with equal cluster spacing and two with unequal cluster spacing. Loadings on the general factor were varied with a_3 = .30 and a_3 = .40, all modeled with medium item heterogeneity.

For all nine population models, the number of clusters was varied with k = 3, 4, 5, and 6, as was the number of items per cluster with m_c = 3, 4, 5, and 6. Sample sizes were n = 100, 300, 500, and 1,000. Item radius on the two circumplex axes was kept constant at u = .80; except in two conditions modeled with k = 3, m_c = 6, n = 100 and k = 6, m_c = 3, n = 100, both with medium item heterogeneity and equal cluster spacing. These were modeled with an item radius of u = .90 in two clusters and u = .70 in the others. This resulted in 9 (population model) × 4 (number of clusters) × 4 (number of items per cluster) × 4 (sample size) +2 (different radii) = 578 conditions in the simulation study. For each condition, 500 samples were drawn.

4.3.2. Results of the Simulation Study

Both population and sample data were analyzed by ClusterCirc (with default settings), Ward cluster analysis (with squared Euclidean and Minkowski-2 distances), and k-means clustering. Item input for all clustering techniques was based on CircE results and PCA loadings. The ClusterCirc spacing index spc_w ranged from 0 (population data, two-dimensional model, no item heterogeneity within clusters) to .30 (sample data, two-dimensional model, unequal cluster spacing with medium item heterogeneity) in the simulation study. Increasing data complexity raised spc_w values, with item heterogeneity, unequal spacing, and number of items per cluster having the largest effects; number of clusters and sample size had minimal effects on spc_w. Hierarchical models showed similar spc_w to nonhierarchical models when item heterogeneity and spacing were matched (for an overview of spc_w values see Table P3.1 in the article; Weide et al., 2025, p. 19).

Clustering performance was assessed by defining intended population clusters as those in which items were closer to their within-cluster neighbors than to adjacent-cluster items. Table 5 gives an overview of sorting accuracy of the different clustering procedures across conditions (adapted from Weide et al., 2025, p. 21). In population data, ClusterCirc recovered the intended clusters in all 578 conditions. In contrast, Ward cluster analysis and k-means clustering often failed, even in the absence of sampling error. In sample data, all techniques achieved almost perfect detection rates (>99%) when clusters had no item heterogeneity. However, such conditions—all items perfectly aligned on their cluster centroids—are neither realistic nor desirable, as subscales should cover a certain thematic range. With increasing item heterogeneity, sorting accuracy decreased for all clustering techniques in the samples. Nonetheless, ClusterCirc (with the appropriate input) consistently outperformed Ward and k-means clustering in all conditions with nonzero item heterogeneity. As expected, greater item heterogeneity within clusters was associated with a more pronounced advantage of ClusterCirc.

Clustering performance also decreased with other aspects of data complexity, such as unequal cluster spacing, hierarchical models, more clusters, and more items (Table 5; adapted from Weide et al., 2025, p. 21). For ClusterCirc, larger samples reliably counteracted these effects, whereas Ward and k-means did not consistently benefit from increasing sample size. Overall, data complexity and sample size had monotonic effects on ClusterCirc's performance, whereas the cluster analytic techniques showed greater irregularities, for example, an odd-even pattern for the number of items per cluster in Ward's performance (see Figure 8, adapted from Weide et al., 2025, p. 23).

Ward and k-means showed no advantage over ClusterCirc in unequal spacing or hierarchical models, either (Table 5; adapted from Weide et al., 2025, p. 21). In contrast, ClusterCirc remained superior under these challenging conditions as well. Within Ward clustering, Minkowski-2 distances yielded slightly higher sorting accuracy than squared Euclidean distances (default in Ward), although this effect was minor compared with ClusterCirc's overall advantage. Regarding ClusterCirc input, CircE and PCA angles performed similarly in most conditions. As expected, in hierarchical three-dimensional models, CircE angles—implicitly accounting for the third factor—achieved greater accuracy, particularly in conditions with unequal cluster spacing (see Figure 9 for the most difficult conditions; adapted from Weide et al., p. 28; more simulation results are depicted in Figures P3.5–7 on pp. 25–27).

Table 5

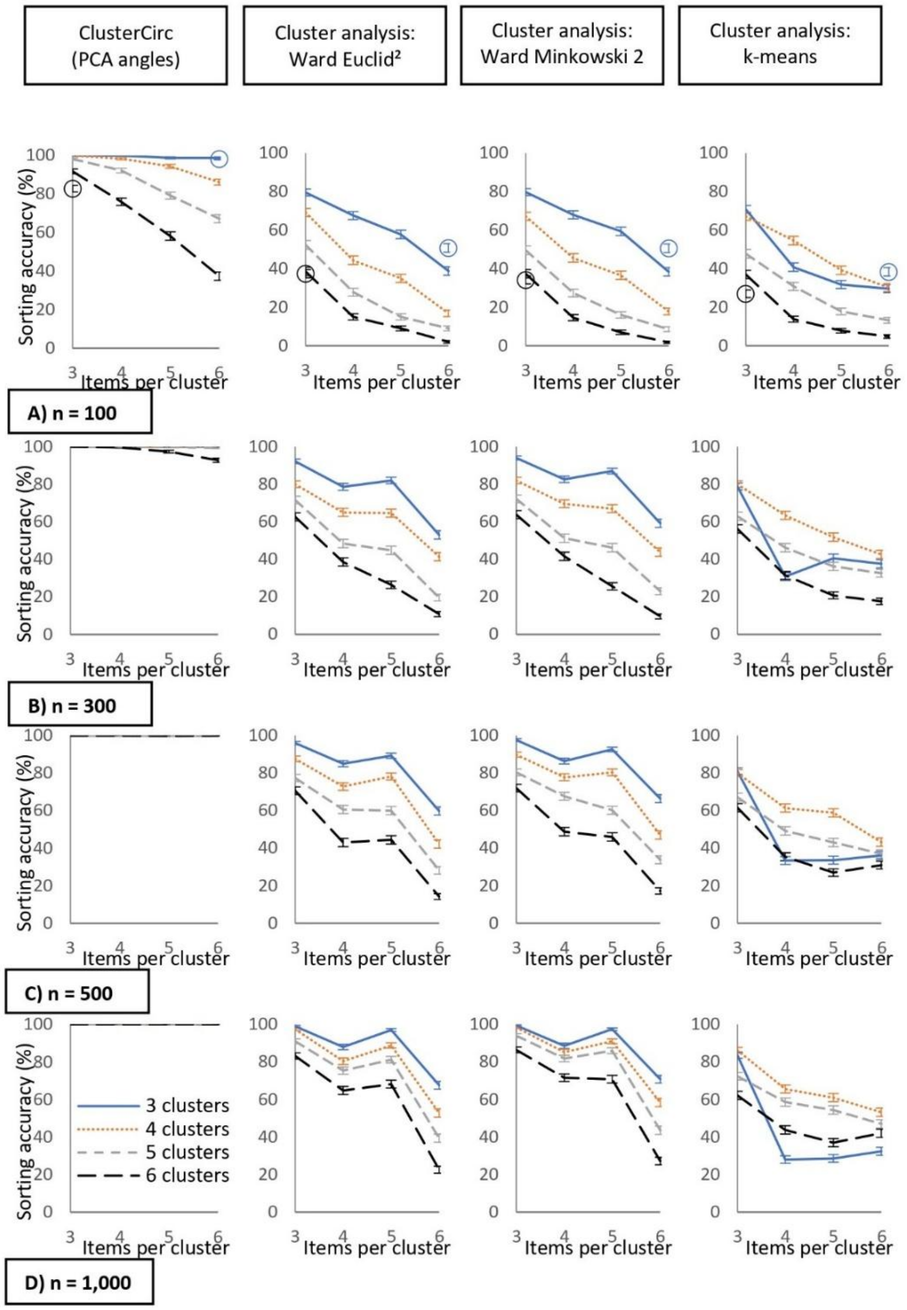
Sorting Accuracy of ClusterCirc Versus Cluster Analysis in the Simulation of Study 3

Simulation model		Population				Samples			
		Cluster Circ	Cluster analysis			Cluster Circ	Cluster analysis		
Factors	Item distribution		Ward E^2	Ward M	k-means		Ward E^2	Ward M	k-means
Non-hierarchical model: two circumplex factors	No item heterogeneity	100% (94%)	100% (94%)	100% (94%)	94%	100%	>99%	>99%	>99%
	Small item heterogeneity	100%	100%	100%	97%	>99%	97%	97%	94%
	Medium item heterogeneity	100%	91% (75%)	95% (75%)	69%	96%	57%	60%	45%
	Large item heterogeneity	100%	23% (13%)	23% (13%)	6%	86%	8%	8%	11%
	Unequal cluster spacing	100%	25% **(44%)**	25% **(44%)**	81%	82%	36%	37%	47% (46%)
Hierarchical model: two circumplex factors + general factor	Equal cluster spacing, a_3 = .30	100%	92% (88%)	91% (88%)	63%	91% **(97%)**	67% (66%)	69%	48% **(49%)**
	Equal cluster spacing, a_3 = .40	100%	89% (81%)	89% (81%)	56%	95% **(97%)**	62%	64%	47%
	Unequal cluster spacing, a_3 = .30	100%	56% (44%)	56% (44%)	75%	50% **(85%)**	42%	43%	51% **(53%)**
	Unequal cluster spacing, a_3 = .40	100%	31% **(50%)**	31% **(44%)**	75%	72% **(83%)**	39%	40%	49% **(50%)**

Note. All values were computed as means across subconditions of the simulation study (sample sizes n = 100, 300, 500, 1,000; number of clusters k = 3, 4, 5, 6; number of items per cluster m_c = 3, 4, 5, 6). In conditions with unequal cluster spacing and in conditions with a hierarchical model, there was medium item heterogeneity within clusters. In the hierarchical model, a third, general factor was modeled in addition to the two circumplex factors. Item angles as input for all methods were obtained by PCA and by CircE (Browne's method). Values in parentheses display sorting accuracy for CircE angles in all conditions where sorting accuracy differed between PCA and CircE angles. Boldface indicates cases in which CircE angles improved sorting accuracy. E^2 = squared Euclidean distances; M = Minkowski-2 distances; a_3 = item loadings on the general factor. Adapted from "ClusterCirc: Finding Item Clusters for Circumplex Instruments" by A. C. Weide, T. Kuhl, & A. Beauducel, 2025, *Journal of Educational and Behavioral Statistics*, Advance online publication, p. 21 (https://doi.org/10.3102/10769986251323017).

Figure 8

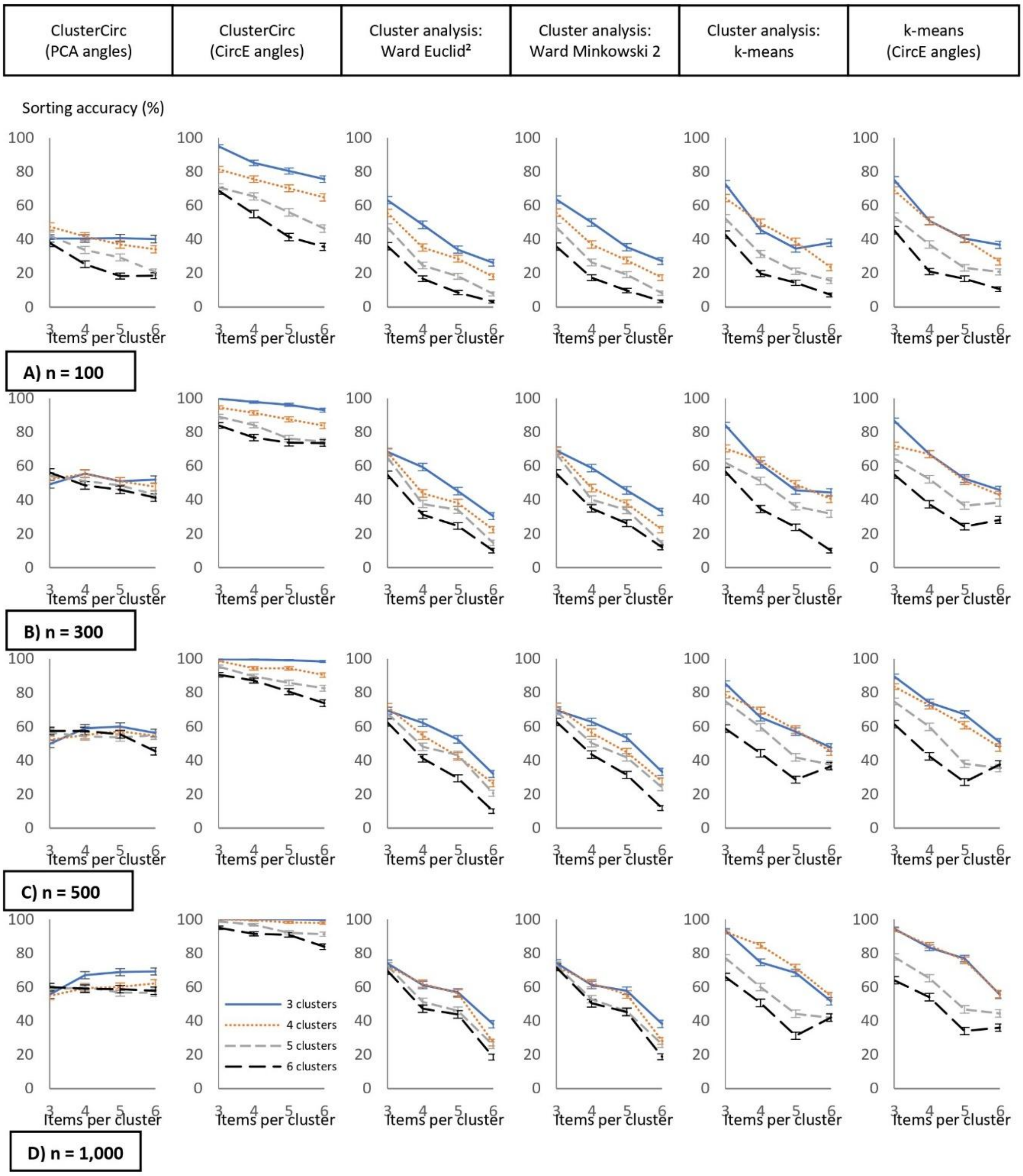
Sorting Accuracy of ClusterCirc Versus Cluster Analysis in Nonhierarchical Data With Equal Spacing of Clusters and Medium Item Heterogeneity Within Clusters (Study 3)



Note. The circled data points below/above the lines in Panel A show results from two subconditions with unequal communalities of items. Results are depicted for input based on PCA for all clustering techniques; results were identical for CircE input. PCA = principal component analysis; Euclid² = squared Euclidean distances; Minkowski-2 = Euclidean distances. Copied from "ClusterCirc: Finding Item Clusters for Circumplex Instruments" by A. C. Weide, T. Kuhl, & A. Beauducel, 2025, *Journal of Educational and Behavioral Statistics*, Advance online publication, p. 23 (https://doi.org/10.3102/10769986251323017).

Figure 9

Sorting Accuracy of ClusterCirc Versus Cluster Analysis in Hierarchical Data With Unequal Spacing of Clusters and Medium Item Heterogeneity Within Clusters (Study 3)



Note. Hierarchical structure refers to two circumplex factors and a general factor with population loadings of a_3 = .40 for all items. The figure illustrates results for the most difficult conditions of the simulation study. Results are depicted for input based on PCA for all clustering techniques and additionally for input based on CircE when results differed from PCA input. PCA = principal component analysis; Euclid² = squared Euclidean distances; Minkowski-2: Euclidean distances. Copied from "ClusterCirc: Finding Item Clusters for Circumplex Instruments" by A. C. Weide, T. Kuhl, & A. Beauducel, 2025, *Journal of Educational and Behavioral Statistics*, Advance online publication, p. 28 (https://doi.org/10.3102/10769986251323017).

4.3.3. Empirical Example

ClusterCirc and Ward clustering were applied to the IAL (Jacobs & Scholl, 2005). The data were collected within the same online study as Study 2, with 823 complete IAL forms from 517 women and 306 men (no nonbinary participants), aged 16–89 (M = 32.68 years, SD = 14.79). ClusterCirc-Data and ClusterCirc-Simu were conducted with the R package's default settings. The precision index was varied with q = 1 and q = 10, yielding identical results. Of the 64 items, ClusterCirc sorted 52 into their original subscales and 12 into different subscales. Most deviations involved items from the JK/Unassuming-Ingenuous subscale, which were reassigned into the neighboring subscale HI/Unassured-Submissive. Ward clustering also diverged from the original subscales (see Figure 10, adapted from Weide et al., 2025, p. 30). ClusterCirc-Data indices were spc_w-$data$= .26, spc = .26, bcs = .09, and wcp = .52. ClusterCirc-Simu resulted in $M(spc_w$-$simu)$ = .24 and SD = .01 for simulated samples with perfect population circumplexity. It yielded acceptable circumplex fit of the IAL clusters as suggested by ClusterCirc-Data: The spc_w-$data$ value did not exceed the cut-off for deviation from circumplex structure (mean spc_w-$simu$ + 2.33 standard deviations, α = 1%). Cronbach's alpha and item–scale correlations for ClusterCirc subscales were comparable to those for the original subscales, although ClusterCirc item–scale correlations showed less variation. CircE analysis on the original subscales, subscales based on ClusterCirc and on Ward clustering showed that ClusterCirc subscales yielded the best circumplex model fit for all combinations of spacing and radius constraints.

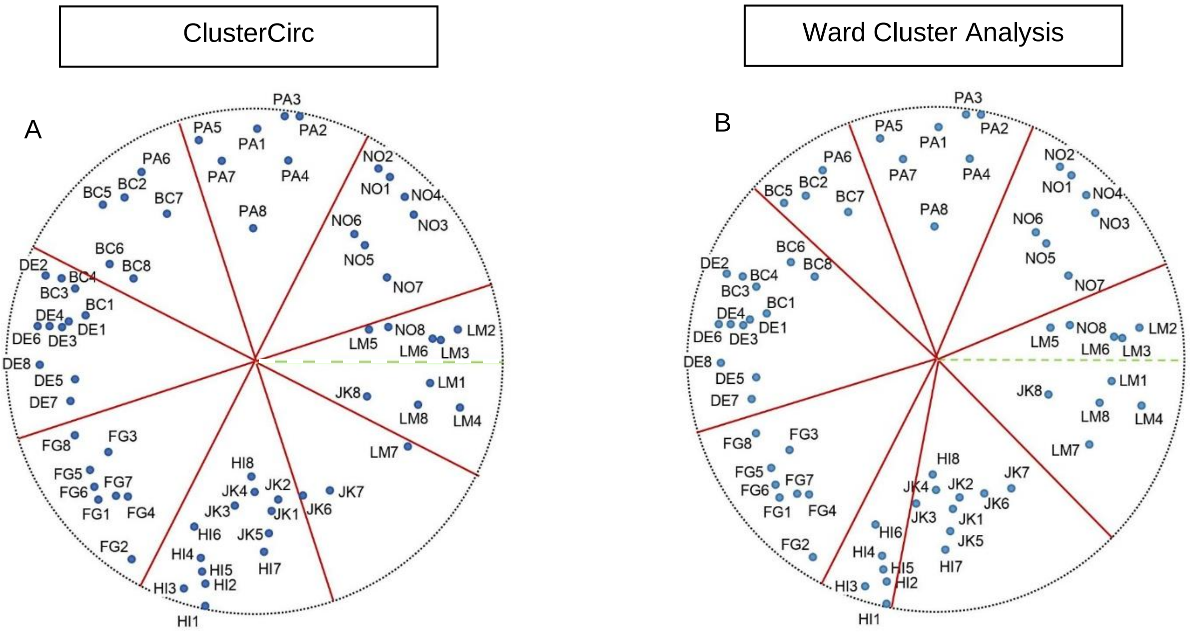
4.3.4. Conclusion and Recommendations

The simulation results suggest that ClusterCirc is a feasible method to identify circumplex clusters. It generally outperformed Ward and k-means clustering, particularly under challenging conditions such as large item heterogeneity within clusters, but also in unequal spacing scenarios and three-dimensional structures. In hierarchical, three-dimensional data, ClusterCirc should be performed with CircE input instead of PCA input to take into account the third factor. For highly complex data (many clusters and items, large item heterogeneity, possible unequal spacing), ClusterCirc should be applied only in large samples (n ≥ 500) to ensure optimal sorting accuracy. The empirical example indicates that ClusterCirc produces item clusters largely consistent with the original IAL subscales (Jacobs & Scholl, 2005), while improving circumplex fit compared with both the original and Ward-based subscales. It also maintained good psychometric properties for the resulting subscales.

Hence, ClusterCirc appears to be a suitable clustering technique for assigning items to circumplex subscales, offering added value over traditional cluster analysis methods.

Figure 10

Item Clusters Found by ClusterCirc Versus Ward Cluster Analysis for the Interpersonal Adjective Scales in the Empirical Example of Study 3



Note. The straight lines within the circle mark the angular separation of the clusters; the dashed line marks the reference angle of 0°. Adapted from "ClusterCirc: Finding Item Clusters for Circumplex Instruments" by A. C. Weide, T. Kuhl, & A. Beauducel, 2025, *Journal of Educational and Behavioral Statistics*, Advance online publication, p. 30 (https://doi.org/10.3102/10769986251323017).

5. Discussion

The projects of this thesis addressed methodological challenges in analyzing data with circular structure. After reviewing different approaches to modeling circumplex data, a particular attention was given to the factor analytic model of circumplex structure (Acton & Revelle, 2004; Boudreaux et al., 2018; Gurtman, 1995, 2009; Richardson et al., 2020; Tracey, 2000), which was examined in both exploratory and confirmatory analyses in the first two research projects. Factor analytic procedures are often employed in the development of psychometric instruments, for example, by slicing PCA plots of items on two circumplex axes to find circumplex subscales (Fabrigar et al., 1997; Tracey, 2000). However, assigning items to distinct subscales is particularly challenging in circumplex structure because of the theoretical and statistical proximity of subscales implied by their circular arrangement. This was addressed by the third research project through the development of ClusterCirc, a clustering technique that can help find new subscales, optimize, or evaluate existing ones according to circular spacing criteria. The findings of the studies support the use of GPA-Varimax with multiple random starts in PCA as an alternative to SPSS-Varimax (Weide & Beauducel, 2019), Bayesian modeling in CFA instead of frequentist MCFA (Weide et al., 2021), and ClusterCirc as a more suitable alternative to SSA- or PCA-plot slicing and traditional cluster analysis in circumplex applications (Weide et al., 2025). A summary of the main findings of the research projects can be found in the conclusion sections of the studies chapter.

5.1. Key Discussion Points and Interpretation of Results

The following sections elaborate on selected aspects of circumplex analysis, with the main findings from the thesis's studies integrated and interpreted in relation to them.

5.1.1. Preferred Alignment and Local Optima in Circumplex Structure

Although local optima were explicitly examined only in the first research project (Weide & Beauducel, 2019), the topic can be extended to the other projects within the context of alignment of the two circumplex axes and clustering of variables in the circular structure. In general, local optima in circumplex analysis can arise for different reasons. One source lies in the clustering of subscales or items, which corresponds to meaningful aspects of the latent structure. In these cases, two situations can be distinguished: (a) All locally optimal solutions may be equally acceptable, or (b) one solution may be preferable—either because it is more meaningful in terms of theoretical considerations or because it represents the statistical global optimum. A second source of local optima, by contrast, arises from unwanted variance—such as measurement error or other empirical impurities—which does not reflect the latent structure and should therefore be regarded as artifactual.

If variables were to form a perfect circle with a large number of variables being evenly spread along the circumference, rotational invariance of the circumplex axes would hold (Acton & Revelle, 2004). In practice, however, this is typically not the case, and most circumplex models comprise larger gaps between variables, mostly at the subscale level (Alden et al., 1990; Etzel & Nagy, 2019; Etzel et al., 2021; Gurtman, 1993; Gurtman & Pincus, 2000; Hopwood & Good, 2019; Horner et al., 2025; Horowitz et al., 2017; Meisel et al., 2025;

Stanisławski, 2025). Under such conditions, a rotational position of the circumplex axes might be more appealing with at least one of the axes aligned on one of the subscales. In the interpersonal circumplex with double simple structure, for instance, the Dominance axis is usually spanned from the PA/Assured-Dominant subscale to the HI/Unassured-Submissive subscale, and the Love axis is spanned from the LM/Warm-Agreeable subscale to the DE/Cold-Hearted subscale (see Figure 4; Adams & Tracey, 2004; Alden et al., 1990; Gurtman, 1993, 2009). These preferences were represented in the factor models in Study 2 (Weide et al., 2021), in which circumplex loadings were entered as fixed parameters (MCFA), priors (BCFA), or targets (TEFA) according to the expected axis alignment. Such model specifications either eliminate (MCFA) or reduce (BCFA, TEFA) the risk of (undesired) local optima for circumplex estimation due to empirical impurities.

Although MCFA (Wendt et al., 2019; Wilson et al., 2013) and target rotation (Horner et al., 2025; Jacobs & Scholl, 2005; Locke, 2014; Stanisławski et al., 2021; Wiggins et al., 1988) have been applied in the context of circumplex analysis, researchers frequently rely on more exploratory procedures without explicit constraints on axis positioning, particularly Varimax rotation in PCA, as investigated in Study 1 (Boudreaux et al., 2018; Hopwood et al., 2011; Horowitz et al., 2017; Richardson et al., 2020). The exploratory approach of PCA with Varimax rotation aligns well with the idea of perfect circumplexity, in which any position of the two circumplex axes is possible. However, it can also be criticized because researchers often apply it even when they have theoretical expectations about axis rotation, although it is generally more appropriate in the absence of a priori assumptions about axis rotation (Fabrigar et al., 1999; Fokkema & Greiff, 2017; Gorsuch, 2015). However, such preferences are typically mirrored by statistical parameters, since rotation toward simple structure often reaches a maximum for the rotation criterion for an axis position where the axes align with some of the subscales. Even though there are interstitial variables in circumplex structure, rotation toward simple structure still facilitates interpretation (Gorsuch, 2015; Mulaik, 2010; Thurstone, 1965) because the axes can be understood in relation to the aligning subscales.

Nonetheless, the two alternative positions of the two circumplex axes in double simple structure yield two local optima of the Varimax criterion as thoroughly examined for GPA rotation in Study 1 (Weide & Beauducel, 2019). The number of local optima of rotation criteria depends on the number of subscales. Simple structure will have more pronounced optima if the number of—evenly spread—subscales is a multiple of four than for different numbers of subscales because it implies an alignment of the two axes on four subscales with high main loadings. If the number of subscales is not a multiple of four, the alignment of the subscales is less clear. For instance, in the case of three subscales like in L. Guttman's (1954) radex model of intelligence, only one of the two orthogonal circumplex axes can be aligned with only one of the subscales, with three local optima of a simple structure criterion. However, even in such cases—or in structures with many variables that have small distances and no clear clustering into potential subscales—it might be more compelling to align at least one axis with one of the variables. Hence, in circumplex structure, there may be several local optima of the rotation criteria, and the corresponding factor solutions are generally more interpretable than any other solutions outside these optima. Accordingly, statistical tests of rotational invariance like the variance tests, rotation tests, or Minkowski test (Acton & Revelle, 2004) are rarely applied in the context of circumplex analysis.

The occurrence of (expected) local optima of rotation criteria in circumplex structure allowed to test whether the relatively new GPA rotation (Bernaards & Jennrich, 2005) would find the global optimum of the Varimax criterion in Study 1. The results showed that rotational performance of GPA-Varimax improved by selecting the best solution from at least 10 random starts in the double-optimum circumplex conditions. In contrast, rotational performance remained unaffected by the start transformation matrix in the single-optimum conditions with perfect simple structure. This difference between the double-optimum and the single-optimum conditions indicates that local optima are relevant in GPA rotation and can be overcome by using random starts, in line with previous research on local optima in analytic factor rotation (Browne, 2001; Hattori et al., 2017; Kiers, 1994; Trendafilov & Jolliffe, 2006).

Following our research, other authors have also implemented random starts to examine and improve factor rotation. For example, Beauducel and Kersting (2020) used random starts to investigate orthogonal rotation methods in the identification of two-facet simple structure, and Scharf et al. (2022) suggested and provided code to run Geomin rotation in GPA based on multiple random starts. Nguyen and Waller (2023) conducted an extensive simulation study on local solutions in factor rotation for different rotation methods. Similar to our results in the single-optimum conditions, they found that Varimax was relatively robust and rarely yielded local solutions in ideal, well-defined data structures. In contrast, oblique Geomin rotation was more likely to yield local solutions depending on the start transformation matrix, also for GPA rotation, in line with the research by Hattori et al. (2017) and Scharf et al. (2022). Thus, while the results from Study 1 suggest that random starts can be beneficial for finding the global optimum in Varimax rotation by GPA, they might be even more relevant in oblique Geomin rotation, whether performed by GPA or other rotational algorithms.

Moreover, Nguyen and Waller (2023) recommend using at least 15,000 iterations per rotation and 2,000 random starts, which far exceeds the number of iterations (250) and recommended random starts (10) from Study 1 of the thesis. This is likely due to their examination of a larger number of procedures and data structures to ensure stable and globally optimal results across a wide variety of applications. However, such high numbers of iterations and random starts require substantial computing power and might not be necessary for all rotation procedures (e.g., Varimax rotation). They also found that the solution with the maximum hyperplane count—that is, the number of near-zero loadings in the total loading matrix—was more likely to represent the modeled population structure. Therefore, in addition to selecting the solution with the optimal value of the rotation criterion, as recommended based on the findings from Study 1, it could also be beneficial to take into account the number of near-zero loadings when selecting a factor solution from multiple random starts.

The discussion on local optima and preferred alignment of factor solutions typically refers to only one level of variables, which can be either the subscales or the items of a psychometric measure (unless otherwise specified, for instance, in a multilevel CFA). However, both the item and subscale (or facet) level are important in psychometric instruments (Beauducel & Kersting, 2020; R. Guttman & Greenbaum, 1998; Shye, 1998). Therefore, local optima and eventual preferences for localizations within the circular structure apply not only to the alignment of the two circumplex axes but also to the separation of the circle into segments representing circumplex subscales. If items were not arranged within clusters but evenly distributed along the circle's circumference, any division of the circle into subscale segments would be equally suitable to represent the structure, analogous to rotational invariance of the circumplex axes (Acton & Revelle, 2004).

However, given that subscales are often preferred that can be meaningfully distinguished from each other, circumplex items usually form clusters along the circle, which represent the subscales (e.g., Adams & Tracey, 2004; Hatcher & Rogers, 2009; Horowitz et al., 1988, 2017). Nonetheless, it should be noted that, in the absence of clustering, one may also perform a weighted aggregation of items according to the circumplex at each position of the circle. Such aggregates may be evaluated, for instance, using factor score reliability methods to assess the reliability of the aggregates at any point on the circle. Thus, subscale homogeneity is not strictly necessary from a psychometric point of view.

However, when items cluster to form subscales that are somewhat distinguishable from one another, this typically implies that the distances between items within each cluster are smaller than the distances between items from different clusters. This principle is implemented both in cluster analysis, which aims to minimize such within-cluster distances (Bridges, 1966; MacQueen, 1967; L. C. Morey et al., 1983), as well as in ClusterCirc. However, ClusterCirc also optimizes distances between clusters to optimize the equal spacing criterion and enhance circumplexity at the subscale level. When clusters are well defined by small within-cluster distances and equal between-cluster spacing, the data structure likely contains a single global optimum for item clustering, which can reliably be detected by clustering procedures. This is supported by the results from Study 3, where clustering procedures identified the correct solution in all conditions with zero and most conditions with small item heterogeneity and equal cluster spacing (Weide et al., 2025).

However, in more complex scenarios with less clearly defined clusters, characterized by larger item heterogeneity and uneven cluster spacing, local optima might be an issue, especially when sampling error is added. In such complex conditions, sorting accuracy was substantially better for ClusterCirc than for Ward cluster analysis and k-means clustering (Weide et al., 2025). ClusterCirc does not stop if the spacing index for the algorithm, spc_w, converges to a local optimum. Instead, it iterates across all possible solutions of dividing the circle into same-sized segments to find the minimum for spc_w across this extensive search space. The brute-force approach increases the chance of finding item clusters with a global optimum for spc_w. However, local optima may occur if the precision index of the algorithm q, which defines the step length, is too large. With the default $q = 10$, local optima can appear when an alternative solution in which the segmentation is displaced by steps smaller than 1/10° (with greater precision) would produce a smaller spc_w value. Nonetheless, the results for the empirical example in Study 3 suggest that even a smaller precision index of $q = 1°$ (iterating in 1° steps for cluster boundaries) can produce the same results as the ClusterCirc default of $q = 10$ (iterating in 1/10° steps for cluster boundaries). Therefore, it may be assumed that local optima are not a major concern in the ClusterCirc search for optimal circumplex clusters.

In contrast, the occurrence of local optima has been discussed for k-means clustering (Bradley et al., 2000; Steinley, 2003). This might partly explain the relatively poor performance of k-means clustering in revealing the population clusters in Study 3. All population clusters in the simulation study were modeled so that item distances were smaller within clusters than between clusters. This is in line with the k-means criterion (and Ward criterion) of minimizing within-cluster distances and, accordingly, maximizing between-cluster distances. Therefore, one might have expected that k-means clustering would have had a fair chance at detecting population clusters. However, k-means clustering (and also Ward clustering) failed to find the intended population clusters in many conditions, surprisingly even for population data in the absence of sampling error (Table 5; adapted from Weide et al., 2025; p. 21). Besides performing worse than ClusterCirc, the sorting accuracy of k-means clustering was also inferior to Ward cluster analysis in most conditions. In addition to differences in the constraints and algorithms of the procedures, local optima in k-means clustering may have partially contributed to these results.

5.1.2. Constraints and Flexibility in the Analysis of Circumplex Structure

As thoroughly elaborated in the Introduction, the circumplex model imposes certain constraints and assumptions on the data. Depending on theoretical expectations, the model can be formulated with stricter or more flexible assumptions regarding the spacing and radius of variables. For example, the spatial representation model (Gurtman & Pincus, 2000; Shepard, 1974) is generally considered the most lenient as it does not impose formal constraints on spacing or radius. At the opposite extreme, the ideal circumplex requires equal radius and equal spacing of variables around the circle (Gurtman, 2009; Gurtman & Pincus, 2000). Other models fall between these poles. The circular order model of correlations, for example, imposes only ordinal constraints on variable spacing (Rounds et al., 1992; Tracey, 2000; Tracey & Schneider, 1995), and the SPMC (Browne, 1992; Grassi et al., 2010; Nagy et al., 2019) allows flexible combination of constraints on spacing and radius. From a factor analytic perspective, circumplex models can also be specified with varying degrees of rigidity. The procedures applied in the thesis' research can be compared in terms of the constraints they impose. Do their assumptions and criteria align with the circumplex model? To what extent can flexibility in the constraints be beneficial in examining the circumplex? Many of the results reported here can likely be attributed to the criteria approximated by the procedures and the constraints placed on the data.

With respect to the rotation procedures investigated in Study 1, both GPA rotation as well as the SPSS-implemented Kaiser algorithm applied the same constraint by optimizing the Varimax criterion. The Varimax criterion states that the sum of the variances of the squared loadings should be maximal for the solution that is selected from all possible rotational positions (Kaiser, 1958). It thereby helps recover simple structure with a maximum of high main loadings and (close to) zero loadings. Typically, circumplex structure is distinguished from simple structure as the interstitial loadings of the variables positioned between the main axes are a key feature of the model (Acton & Revelle, 2004; Tracey, 2000). Nonetheless, Varimax and related criteria can detect the typical loading pattern of double simple structure as an important circumplex model. Therefore, the Varimax criterion as investigated in Study 1 can be considered suitable in this particular case, which is supported by its frequent use in circumplex measures with double simple structure in PCA (Boudreaux et al., 2018; Hopwood et al., 2011; Horowitz et al., 2017, 1988; Richardson et al., 2020). Results showed that SPSS-Varimax and GPA-Varimax produced highly similar solutions in almost all conditions with perfect orthogonal simple structure and also in the majority of (modified) double simple structure scenarios. This equality in results may be largely attributed to the fact that the procedures approximated the same (Varimax) criterion.

Study 2 examined differences between conventional MCFA and BCFA in modeling the three-factor structure of the IIP octants (Horowitz et al., 2017; Weide et al., 2021). In MCFA, one must decide whether loadings should be freely estimated or fixed to a particular value. The MCFA model in Study 2 was, in line with previous (Wilson et al., 2013) and recent studies (Israel & Langeveld, 2021), defined as a bi-factor model with free loadings on the general factor Distress and loadings fixed to the ideal circumplex on the two circumplex factors Dominance and Love. Thus, the MCFA model in Study 2 imposed highly rigorous constraints on the circumplex factors, whereas loadings on the general factor were entirely data-driven. BCFA, in contrast, allows for a more balanced and flexible approach, as the expected circumplex pattern can be inserted as priors. Posterior loadings for the final solution are then found by combining information from the prior distribution and the empirical data. Given that perfect equal spacing is rarely met in empirical data, and yet spacing is unlikely to be entirely random as might be implied by freely estimated loadings, BCFA with circumplex priors seems more suitable than MCFA to model circumplex structure. The findings from Study 2 support this conclusion: Compared to MCFA, BCFA resulted in improved fit indices, fewer correlated errors, and more robust results across samples. In particular, the results from follow-up misspecification analyses suggest that strict zero loadings in MCFA might be too rigorous for the IIP.

With respect to the fit indices in Study 2, *RMSEA* values for the MCFA and BCFA models were relatively high (Weide et al., 2021). However, *RMSEA* is known to potentially misclassify well-fitting models as inadequate when strong main loadings occur, especially in circumplex structure (Rogoza et al., 2021; Saris et al., 1987, 2009). In line with this, Gurtman and Pincus (2003) considered RMSEA values below .13 acceptable for circumples models. According to this cut-off, both the MCFA model and the BCFA model from Study 2 would be deemed acceptable. Moreover, Tucker's congruence coefficients between BCFA posterior loadings and the MCFA-defined ideal circumplex were exceptionally high. According to the cut-offs proposed by Lorenzo-Seva and ten Berge (2006), these results suggest that the factors can be considered equivalent. Thus, BCFA seems to produce circumplex factors that closely approximate an ideal circumplex, while also relaxing unrealistic constraints to improve model fit.

TEFA also imposes fewer constraints than MCFA, as the ideal circumplex is included as a target rather than fixed loading matrix. However, TEFA resulted in smaller congruences with the ideal circumplex factors and did not substantially improve model fit over BCFA (Weide et al., 2021). This suggests that BCFA offers more advantages than TEFA when a priori hypotheses about variable positions in the circle exist. Although Study 2 supports the use of BCFA on circumplex structure, the choice of methods and constraints should—of course—also be based on theoretical considerations and the greater context of the investigation. For instance, one might prefer a more rigorous MCFA model if the latent structure of a circumplex measure is well established and should be tested by a more conservative analysis. On the other hand, during earlier stages of scale development, when variable spacing might be less certain, TEFA might be preferred over BCFA and MCFA.

Scale development is also the focus of ClusterCirc, which was introduced and investigated in the third research project as a clustering technique for item–subscale assignment in circumplex instruments (Weide et al., 2025). Previous research has often relied on slicing PCA or SSA item plots into subscale segments based on visual inspection and subjective decisions (e.g., Hatcher & Rogers, 2009; Horowitz et al., 2017; Locke, 2000; Locke, 2019; Perrinjaquet et al., 2007; Redeker et al., 2014; Schwartz et al., 2012; Wiggins et al., 1988). This approach is in line with the most lenient circumplex model, the spatial representation model (Shepard, 1974) because it does not apply formal constraints on the data. Nonetheless, it might be argued that researchers are also guided by theoretical assumptions and employ certain constraints when assessing spacing and radius of the items and deciding how to segment the circle. Despite this, it is difficult to find item subscales that would also be optimal when evaluated by formal circumplex criteria using this procedure. ClusterCirc addresses this issue by introducing spc_w, a formal circumplex index optimized by the search procedure for circumplex clusters.

The spc_w index balances two assumptions: maximizing within-cluster proximity of items and minimizing deviations from equal spacing between clusters. Thus, ClusterCirc combines within-cluster proximity as the primary criterion of traditional cluster analysis with the equal spacing criterion of circumplex structure. By default, ClusterCirc assumes that both criteria should be considered for optimal circumplex clusters. However, users can flexibly adjust the relative importance of either criterion depending on individual preferences. For instance, users can decide to only maximize within-cluster proximity and completely ignore equal cluster spacing. In this case, ClusterCirc puts fewer circumplex constraints on the data. Nonetheless, even though within-cluster proximity is not inherently part of the circumplex model, ClusterCirc's use of angular rather than Euclidean distances—also within clusters—still promotes a circular arrangement of item clusters. If between-cluster spacing is prioritized, the ClusterCirc search puts more emphasis on the circular structure. Yet equal spacing is not rigidly imposed on the data, as in the ideal circumplex; ClusterCirc simply identifies a clustering solution with the smallest deviation from an even distribution of clusters around the circle.

Traditional cluster analysis can also be used to find item clusters; however, it is not circumplex-specific. Cluster analysis typically minimizes within-cluster variance of item distances, a reasonable criterion for many psychometric instruments (Revelle, 1979; Sireci & Geisinger, 1992). Equal spacing between clusters is not explicitly addressed. Moreover, even if ClusterCirc is adjusted to only maximize within-cluster proximity, it might lead to different results than cluster analysis because the latter is usually based on (squared) Euclidean distances (straight lines in the Euclidean coordinate system). As outlined previously, items might be close with regard to their angular positions in the circle but could be further apart by their Euclidean distance if they have different radii. They would likely be sorted into the same circumplex cluster by ClusterCirc, whereas they might be sorted into different clusters by traditional cluster analytical procedures (see Figure 6). Although traditional cluster analysis is not primarily designed for circumplex structure, it might thus put more emphasis on radius of the variables than ClusterCirc, which largely ignores the equal radius criterion.

Although radius is also an important characteristic of circumplex structure, it is less relevant in the assignment of items to circumplex subscales because the circle is typically divided into subscale segments that extend from the circle's center to circumference (see Figure 7; adapted from Weide et al., 2025, p. 11). Radius, however, is relevant in item selection because items with a particularly small radius necessarily have low communalities in two-dimensional data (see Equation 6) and likely in higher-dimensional data as well. ClusterCirc addresses this implicitly by weighting items according to their communalities, unless user-defined item weights are specified. Thus, the default assumption in ClusterCirc is that items with high communalities should impact the final clustering solution to a greater extent than those with low communalities. However, item weights can also be altered in ClusterCirc, such that individual preferences for item–subscale assignment can be included. Hence, in addition to being explicitly developed for circumplex applications, ClusterCirc provides more flexibility in the criteria that are optimized by the procedure than cluster analysis. Beyond the criteria implied by minimizing spc_w, ClusterCirc imposes constraints on the data by dividing the circle into segments of the same size, which is not required by cluster analysis. Although this approach can be criticized, dividing the circle into same-sized segments is in line with the frequently performed slicing of PCA and SSA plots and conforms to numerous psychological circumplex theories (Adams & Tracey, 2004; Etzel et al., 2021; Gurtman & Pincus, 2000; Hatcher & Rogers, 2009; Horowitz et al., 1988, 2017; Locke, 2000; Meisel et al., 2025; Stanisławski, 2025; Trobst, 2000).

The findings of Study 3 suggest that ClusterCirc is beneficial over Ward cluster analysis and k-means clustering when circumplex structure is present (Weide et al., 2025). The superiority of ClusterCirc in the simulation study might largely be attributed to differences in the search criterion: Cluster analysis minimizes (squared) Euclidean within-cluster distances, whereas ClusterCirc optimizes both within-cluster as well as between-cluster spacing based on angular distances. This difference in the search criterion might be responsible for the advantage of ClusterCirc over Ward and k-means cluster analysis when item heterogeneity within clusters increased. Given that cluster analysis only minimized within-cluster distances, sorting accuracy of Ward and k-means clustering declined more sharply when item heterogeneity within the modeled clusters increased. In contrast, ClusterCirc also relied on spacing between clusters, such that ClusterCirc's sorting accuracy was less affected by increasing within-cluster distances. Thus, ClusterCirc can be particularly useful if subscales with a broader conceptual scope are desired while maintaining optimal circumplex spacing. However, as mentioned earlier, all modeled population clusters consisted of items that were closer to each other than to adjacent items of neighboring clusters to ensure fair chances of detecting population clusters for cluster analysis as well. Yet, Ward and k-means clustering revealed different clusters than the intended ones in many conditions, even in population data (Table 5; adapted from Weide et al., 2025, p. 21). To allow for a more challenging test of clustering performance for ClusterCirc, population clusters were modeled that were unevenly spaced, which contradicts the equal spacing criterion implied by spc_w. In these conditions, sorting accuracy of ClusterCirc decreased by approximately 14% compared to similar conditions with equal cluster spacing. Nonetheless, Ward and k-means cluster analysis still performed worse than ClusterCirc in revealing the population clusters. Hence, ClusterCirc can also be considered more suitable than traditional clustering techniques to find circumplex clusters in the case of unequal cluster spacing.

Results for the empirical example of Study 3 also illustrate differences in the criteria that are approximated and, moreover, the same-sized segmentation of the circle imposed by ClusterCirc (Weide et al., 2025). In contrast to ClusterCirc, Ward cluster analysis resulted in clusters for the IAL (Jacobs & Scholl, 2005) that largely differed in angular range, although with greater proximity of items within some clusters. For example, one of the ClusterCirc clusters consisted of only three items with relatively large distances between them (JK6, JK7, LM7), because the area of the circle was sparsely covered in the data (see Figure 10, adapted from Weide et al., 2025, p. 30). Nonetheless, same-sized segmentation agrees with the interpersonal circumplex model (Alden et al., 1990; Gurtman, 1993, 2009; Gurtman & Pincus, 2000; Kiesler, 1983) and has previously been performed for PCA plots of the IAS (Wiggins et al., 1988). Therefore, comparing the same-sized IAL clusters based on an objective circumplex criterion by ClusterCirc with the original IAL subscales can provide insights into the circumplexity of the IAL. The majority of the IAL items were sorted into the same clusters by ClusterCirc as the original IAL subscales, which corroborates the validity of the IAL as a circumplex measure. However, some discrepancies were found, mostly for JK items (Unassuming–Ingenuous), many of which were sorted into the neighboring subscale HI (Unassured–Submissive). Angular inspection confirmed that JK items overlapped strongly with HI items, making it difficult to statistically distinguish them. Content analysis also revealed semantic overlap. For instance, “compliant” (JK1) and “obedient” (JK2) closely resemble “submissive” (HI6) in meaning. Similarly, “susceptible” (JK3) can be considered a synonym for “influenceable” (HI8), and “cautious” (JK5) closely aligns with “hesitant” (HI4). Moreover, someone who is “conflict-averse” (HI7) is likely to also be “indulgent” (JK4). This pattern is consistent with previous research, which showed that the HI and JK subscales often appear closer together than the theoretically expected 45° (Alden et al., 1990; Jacobs & Scholl, 2005; Weide et al., 2021). To better distinguish between the subscales, it may be useful to develop new items that capture the domain of JK/Unassuming-Ingenuous more precisely and reduce overlap with the HI/Unassured-Submissive subscale. As could be expected, subscales based on ClusterCirc also showed improved overall circumplexity when tested with CircE (Grassi et al., 2010), compared with both the original and Ward-based subscales. Moreover, psychometric properties of items and subscales were comparable to those of the original subscales. This suggests that ClusterCirc can yield subscales with enhanced circumplex structure for the IAS while preserving validity in other psychometric aspects.

5.1.3. Whole-Pattern Analysis Versus Separating Elements From the Data Structure

In addition to differences in the criteria and constraints imposed on the data, the procedures also differ in whether they analyze the entire data structure or split it into smaller parts. The circumplex model implies an overarching two-dimensional structure for a set of variables. Therefore, it may be more suitable to examine circumplex structure using procedures that analyze the whole pattern of the circumplex rather than separating elements from the circular structure. The results of the thesis’ studies support the benefits of whole-pattern analysis in circumplex structure.

In Study 1, relevant differences between GPA-Varimax rotation (Bernaards & Jennrich, 2005) and the SPSS-implemented Kaiser algorithm (Kaiser, 1958) were found only in conditions comprising circumplex models of (modified) double simple structure, in which GPA-Varimax outperformed SPSS-Varimax (Weide & Beauducel, 2019). GPA rotation optimizes the rotation criterion simultaneously for all factors (or components), whereas the Kaiser algorithm maximizes the sum of the variances of the squared loadings for all combinations of factor or component pairs. The simulation study comprised population structures with a minimum of three components, whereby two thirds of the variables (and thus components) formed a circumplex structure. Hence, the population models consisted of one or more separate circumplex(es), each with two components, and one or more non-circumplex components. The Kaiser algorithm optimized the Varimax criterion for all combinations of components pairs: Some pairs represented the modeled circumplexes, whereas others combined components from different circumplexes or a circumplex component with a non-circumplex component. In contrast, GPA rotation optimized the Varimax criterion for the whole pattern of components—circumplexes and non-circumplex components—simultaneously. The advantage of GPA rotation in conditions with double simple structure suggest that maximizing the Varimax criterion for the entire loading pattern may be less sensitive to sampling error than pairwise algorithms when the data contain circumplex structure. However, there was no condition with only two circumplex components in Study 1. In such cases, results could be more similar for GPA-Varimax and SPSS-Varimax because there would be only a single (circumplex) component pair that constitutes the entire loading matrix.

While Study 1 examined data structures with a minimum of three components, Studies 2 and 3 investigated circumplex structures with only two or three factors (Weide et al., 2021, 2025). Thereby, it can be differentiated whether the procedures analyzed the two-dimensional circumplex structure as a whole or whether some variables were separated from the overarching structure. MCFA, TEFA, and BCFA, which were applied to the IIP (Horowitz et al., 1988, 2017) in Study 2, can be considered whole-pattern analyses because they examine the entire loading matrix within a single analysis. However, if the circumplex is to be analyzed separately from the general factor of the IIP, frequentist MCFA only allows for misspecification analysis of individual loadings rather than of the entire circumplex pattern (Saris et al., 1987, 2009). These individual loadings can then be freed in a follow-up MCFA. Although this was not explicitly investigated, it is well known that model modifications in structural equation modeling may capitalize on chance (MacCallum et al., 1992), especially given the typically large number of circumplex loadings that could be analyzed individually. Hence, results for individual circumplex loadings in MCFA may reflect sample-specific artifacts and may not generalize to other samples or the population. The two significant misspecification results for MCFA circumplex loadings found in Study 2 should therefore be interpreted with caution. However, these follow-up analyses were based on BCFA results, which support the notion that strict zero loadings, as defined in the MCFA model, might not be suitable in circumplex analysis. In contrast to frequentist MCFA, BCFA allows circumplex estimates to be analyzed and modified directly within a single analysis, without requiring model modifications and follow-up analyses. Instead, the posterior loading pattern may differ as a whole from the initial model specifications as defined by the Bayesian priors. When the BCFA converges according to the potential scale reduction, this makes the analysis more robust and reduces the risk of results based on sample-specific anomalies. TEFA also allows for estimation of loadings according to a circumplex target matrix within a single analysis of factor rotation. However, as stated previously, the higher congruence of the BCFA posterior loadings with the ideal circumplex found in Study 2, together with the lack of substantial improvement in model fit when using TEFA, suggests an advantage of BCFA over TEFA in the analysis of circumplex structure.

Regarding item clustering as a basis for circumplex subscales, it also seems suitable to take a whole-pattern approach in order to adequately address the desired overall circular structure. Within the frequently applied procedure of slicing PCA or SSA plots and dividing the circle into item clusters based on visual inspection, it can be assumed that the entire circular structure is taken into account (e.g., Hatcher & Rogers, 2009; Horowitz et al., 2017; Locke, 2000; Locke, 2019; Perrinjaquet et al., 2007; Redeker et al., 2014; Schwartz et al., 2012; Wiggins et al., 1988). As stated earlier, SSA and PCA plot slicing heavily rely on subjective decisions by individual researchers, which is a major pitfall of the approach (Fabrigar et al., 1997). In contrast, ClusterCirc objectifies the spatial information of the items while maintaining the whole-pattern approach. ClusterCirc optimizes circumplex spacing of the entire instrument by dividing the entire circle into same-sized segments and computing spc_w based on all items and (temporary) clusters in each iteration.

K-means clustering (MacQueen, 1967) can also be considered a whole-pattern approach. This method begins with an initial set of *k* centroids and assigns the items to the cluster with the nearest mean based on squared Euclidean distances. The centroids are updated with the current items, and items are re-assigned to the clusters based on the updated centroids in each iteration. In this way, it splits the entire pattern into *k* clusters, including all items simultaneously in each step of the algorithm. In contrast, Ward cluster analysis performs a hierarchical procedure, beginning with individual item pairs as initial clusters and subsequently enlarging the clusters until the algorithm converges (Breckenridge, 2000; Bridges, 1966; L. C. Morey et al., 1983). Hence, Ward's cluster analysis does not examine the entire variable pattern simultaneously but rather performs successive analyses of smaller units that are gradually combined.

It can be discussed whether this pairwise approach of the Ward algorithm contributed to the results of the simulation study. As can be seen in Figure 8 (Figure P3.4 of the article; see also Figure P3.7; Weide et al., p. 23 and p. 27), sorting accuracy of Ward cluster analysis did not decrease monotonically when the number of variables per cluster increased. Instead, Ward cluster analysis might have better chances to reveal circumplex clusters with an odd number of items than clusters with an even number of items. K-means clustering did not yield monotonic results either. However, unlike Ward cluster analysis, there was no odd/even pattern, and the effects of data complexity were even more irregular. For example, in two-dimensional models with equal spacing and medium item heterogeneity, sorting accuracy of k-means was worse for configurations with three clusters than in configurations with more than three clusters. Similarly, sorting accuracy generally decreased with an increase in the number of items per cluster but was better for six items per cluster than for five items per cluster in configurations with six clusters (see Figure 8; Weide et al., 2025, p. 23). Since k-means clustering relies on a whole-pattern algorithm, these effects cannot be attributed to a whole-pattern versus individual-elements distinction that might explain some of the Ward effects. Instead, k-means effects are more likely due to other characteristics of the k-means algorithm, such as its sensitivity to local optima.

In contrast, data complexity had more regular (and monotonic) effects on sorting accuracy for ClusterCirc. The only exception was hierarchical population models with unequal cluster spacing, in which performance of ClusterCirc with PCA angles as input did not decrease strictly monotonically with an increase in the number of clusters or items (Figure 9, adapted from Weide et al., 2025; p. 28). However, when CircE angles were used as input in ClusterCirc—which automatically accounts for the third factor in hierarchical models—the pattern of results showed similar regularity as in the other conditions. Moreover, the adverse effects of data complexity were reliably compensated by increasing sample size in ClusterCirc, which was not the case for Ward or k-means cluster analysis in many conditions of the simulation study. Hence, in addition to its overall better performance in detecting circumplex population clusters than Ward and k-means cluster analysis, another advantage of ClusterCirc could be a better predictability of the impact of data complexity and sample size on clustering performance.

5.1.4. The Number of Factors in Circumplex Models (and Personality)

By definition, the circumplex model consists of two and only two orthogonal factors (Acton & Revelle, 2004). It has been useful in modeling cognitive abilities (L. Guttman, 1954, 1957), affective states (Posner et al., 2005; Russell, 1980; Russell et al., 1989; Stanisławski et al., 2021), vocational interests (Tracey & Rounds, 1993), human values (Locke, 2000; Ponikiewska et al., 2020), and interpersonal behavior, difficulties, and strengths (Adams & Tracey, 2004; Hatcher & Rogers, 2009; Horowitz et al., 1988, 2017; Wiggins et al., 1988), to repeat some of the major applications of circumplex theory. However, the two-dimensional structure is often revealed only when variables are ipsatized, due to the presence of a third, overarching factor (Alden et al., 1990; Hopwood et al., 2011; Horowitz et al., 1988, 2017; Richardson et al., 2020). This third factor has been modeled explicitly as a meaningful construct, like the Distress factor in interpersonal problems (Acton and Revelle, 2002; Hopwood & Good, 2019; Israel & Langeveld, 2021; Wendt et al., 2019; Wilson et al., 2013), a general resource factor in interpersonal strength (Hatcher & Rogers, 2009), or a severity factor in interpersonal sensitivities (Hopwood et al., 2011). This suggests that two factors may sometimes not be sufficient to describe some psychological domains with circumplex structure.

In addition, more comprehensive models of personality typically comprise more than two factors. The central debate concerns whether personality is best conceptualized by three (Eysenck, 1991; Eysenck et al., 1992), five (Costa & McCrae, 1995, 2006), or six (Ashton & Lee, 2007; Lee & Ashton, 2004) distinct dimensions. Importantly, both the Big Five and HEXACO traits have been incorporated into circumplex models (Barford et al., 2015; DeGeest & Schmidt, 2015; DeYoung et al., 2013; Gurtman, 1995; McCrae & Costa, 1989; Nysæter et al., 2009; Trapnell & Wiggins, 1990). Moreover, a general circumplex model of metatraits has been proposed (Strus & Cieciuch, 2021; Strus et al., 2014), which subsumes diverse psychological constructs—the Big Five and HEXACO traits, values, temperament, mental health, and affect—under two orthogonal metatraits that account for intercorrelations among personality constructs. Thus, although the circumplex model is defined by only two factors, more factors often need to be considered to capture relevant aspects of personality, either within the circular structure or in addition to it.

In line with the most popular personality models, Study 1 examined data structures with a minimum of three components (Weide & Beauducel, 2019). The standard two-factor circumplex model or a hierarchical circumplex model with a general factor in addition to the circumplex factors was not investigated. Instead, three-component models with circumplex structure consisted of two circumplex components plus an additional component with zero loadings on the circumplex components. Circumplex models with more than three components consisted of separate circumplexes covering two thirds of the variables and independent components with main loadings on the remaining one third of the variables. Such models with several circumplexes align with the circumplex model of metatraits, which proposes a second set of metatraits in addition to the main personality circumplex (Strus & Cieciuch, 2021; Strus et al., 2014). Moreover, the scope of the first research project extended beyond a mere focus on circumplex structure. It was concerned with creating scenarios suitable for testing Varimax rotation by GPA. For this reason, different simple structure models of varying difficulty in recovering the population structure through Varimax rotation were created. Three- and higher-component circumplex models (with double simple structures) therefore included additional components with zero and main loadings, consistent with the Varimax criterion. Hierarchical models would have been less suitable for investigating Varimax because it maximizes the variance of loadings—resulting in a pattern of both low and high loadings—rather than producing a general factor defined by uniformly high loadings across all variables. Nonetheless, these modeling choices limit the interpretation of the results, which may have been different had two-factor circumplexes or hierarchical circumplex models been included.

Study 2 investigated MCFA and BCFA for the higher-level structure of the IIP (Horowitz et al., 1988, 2017; Weide et al., 2021), consisting of a hierarchical model with the two circumplex factors Dominance and Love and the general factor (interpersonal) Distress. The three-factor model of the IIP is well established (Acton & Revelle, 2002; Hopwood & Good, 2019; Israel & Langeveld, 2021; Wendt et al., 2019; Wilson et al., 2013) and is supported by the present findings. CircE results of the SPMC for the IIP octants showed positive intercorrelations between the subscales, with an estimated correlation of r = .001 at 180°, suggesting the presence of a third, overarching factor. Accordingly, MCFA, BCFA, and TEFA showed acceptable model fit for the hierarchical three-factor model (see Gurtman & Pincus, 2003; Rogoza et al., 2021; Saris et al., 1987, 2009). Beyond providing more flexible constraints and improved model fit, BCFA is especially well suited for hierarchical circumplex models compared with MCFA. Bayesian priors for the circumplex loadings provide a more balanced model, in which uncertainty is allowed for both the circumplex factors as well as the general factor. By contrast, by tightly constraining circumplex loadings but freely estimating the general factor, MCFA with fixed circumplex loadings produces an asymmetrically specified model. Thus, in addition to its general advantage of greater flexibility in modeling circumplex structure, BCFA offers the added benefit of more proportionate specification for hierarchical three-factor circumplex models.

The simulation study of the third research project on ClusterCirc compared with cluster analysis comprised conditions with two-factor circumplex models and conditions based on hierarchical three-factor circumplex models, thereby covering the two most popular versions of circumplex structure (Weide et al., 2025). As mentioned before, ClusterCirc and k-means clustering simultaneously consider the entire data structure, whereas Ward's method follows a hierarchical algorithm, beginning with item pairs and successively enlarging clusters according to the algorithm. Accordingly, differences in sorting accuracy were expected between ClusterCirc and k-means, on the one hand, and Ward's method, on the other, depending on whether the data structure was hierarchical or not. The results showed that ClusterCirc outperformed both Ward and k-means cluster analysis, both in two-dimensional and hierarchical three-dimensional conditions (Table 5; adapted from Weide et al., 2025; p. 21). However, the advantage of ClusterCirc over Ward cluster analysis was smaller in hierarchical conditions than in nonhierarchical conditions. Moreover, Ward cluster analysis performed better in hierarchical conditions than in the nonhierarchical counterparts (Table 5; see also Figure 8/P.4 vs. Figure P.7 in the article; Weide et al., 2025; p. 23 and p. 27). These findings suggests that Ward's method is more suitable for hierarchical as opposed to nonhierarchical circumplex models. Contrary to expectations, k-means did not yield higher sorting accuracy than Ward's method in nonhierarchical conditions, although it is a nonhierarchical procedure.

Overall, ClusterCirc was superior to the other clustering procedures. Yet in hierarchical data, CircE input had an advantage over PCA input in ClusterCirc. This can be attributed to the estimation procedure in CircE, which automatically accounts for the possibility of a third factor in the Fourier series. Especially when cluster spacing was not equal, CircE input substantially improved sorting accuracy of ClusterCirc. CircE input should therefore be preferred for ClusterCirc analyses of hierarchical circumplex models.

5.1.5. Convergence of Results and Methodological Pluralism

Alongside relevant differences between the examined methods that were found in the thesis' studies, there were also considerable similarities in results that are worth highlighting. In Study 1, GPA-Varimax (Bernaards & Jennrich, 2005,) and SPSS-Varimax (Kaiser algorithm; Kaiser, 1958) yielded highly similar results in most conditions of the simulation study (Weide & Beauducel, 2019). In the single-optimum simple structure conditions, identical results were found in more than 90% of comparisons. Remaining differences between SPSS and GPA were marginal and disappeared with Kaiser normalization and 250 iterations per rotation. For double-simple structure circumplex models, rotational performance differed to a greater extent between GPA and SPSS—mostly in favor of GPA—in some conditions. Yet, in more than half of the double-optimum conditions, rotational results did not exceed the cut-offs either. Furthermore, GPA-Varimax and SPSS-Varimax yielded components with congruence coefficients indicating equivalent solutions for the knowledge test in the empirical example. These null results are not trivial. Given that GPA is a relatively new rotation method, it was important to demonstrate that its performance is comparable to established procedures. Thus, the similarity in results support GPA as a valid alternative to the Kaiser algorithm in Varimax rotation, while the differences suggest that GPA may even offer added benefit in certain cases. However, the cut-offs for determining equality versus differences in results were based on the scaling of the simulation results. This can be criticized as a data-driven rather than a statistically principled approach. Yet, because of the specific focus of the investigation, a priori benchmarks for the comparison from prior research were not available. Defining cut-offs from the simulation results therefore appeared most appropriate.

In Study 2, differences between MCFA and BCFA affected internal validity more strongly, whereas external validity results were highly similar (Weide et al., 2021). Although model fit indices differed between MCFA and BCFA when external measures were included, this was likely due to the strict constraints on circumplex loadings in MCFA versus the more flexible priors in BCFA, like in the models without external criteria. More relevant for external validity was the pattern of loadings of the external measures on the IIP factors: They closely matched the hypotheses and were highly similar for MCFA and BCFA, thus supporting the general three-factor model of the IIP (Israel & Langeveld, 2021; Wendt et al., 2019; Wilson et al., 2013). Previous research has mapped Extraversion and Agreeableness onto the interpersonal circumplex for general behavior (Barford et al., 2015; DeYoung et al., 2013; McCrae & Costa, 1989). The present research suggests that their positioning could be extended to interpersonal difficulties when the third factor is taken into account. The findings also align with research by Nysæter et al. (2009). When ipsatized scores were used (i.e., with the general factor controlled for), they found correlations of the IIP quadrants with Extraversion and Agreeableness similar to our results, and slightly different results for non-ipsatized scores. The positive loading of Neuroticism on Distress in both MCFA and BCFA further highlights the importance of the third factor as a meaningful indicator of general interpersonal impairment (Acton & Revelle, 2002; Israel & Langeveld, 2021; Tracey et al., 1996; Wilson et al., 2013).

External validity results for MCFA and BCFA were consistent with correlational patterns for the higher-level IIP scores. Because factor-model results cannot automatically be transferred to corresponding factor scores (Beauducel, 2005), these scores were examined separately. The findings indicate that the three scoring methods—MCFA regression scores, mean plausible values from BCFA, and weighted sum scores—all yield valid, reliable, and efficient measures of the IIP's higher-level factors. Compared to the original IIP octants, scoring-implied octants showed improved circumplex properties. Although expected, given the circumplex constraints imposed by the scoring methods, this nonetheless supports the validity of the higher-level scores for both research and diagnostic applications. Correlations with external criteria were also highly convergent across scoring methods and largely in line with MCFA and BCFA results. A minor deviation emerged with subclinical grandiose narcissism. Narcissism was associated exclusively with Dominance in MCFA and BCFA, as hypothesized; however, NPI-17 scores also correlated slightly with higher-level Love scores. This suggests that narcissism may fall within the 90°–135° segment of the interpersonal circumplex, consistent with findings by Nagy et al. (2019), who located the admiration facet of narcissism in the same range using the SPMC model.

It should be noted that BCFA scores were estimated via mean plausible values, for which potential biases have been reported (Lüdtke & Robitzsch, 2017; Zitzmann & Helm, 2021). However, in this study, BCFA scores were strongly correlated with other scoring types and yielded no substantial differences in results (see also Beauducel & Hilger, 2022). Thus, there was no evidence of relevant bias for BCFA scores[6]. Hence, all three types of higher-level scores can be recommended for the IIP, with the choice depending on theoretical and practical preferences. For example, weighted sum scores may be most appropriate in diagnostic contexts that might require simple computation, whereas MCFA regression scores or BCFA scores may be preferable for research applications with a stronger emphasis on theoretical depth.

[6] The final published article excluded BCFA scores at the editor's request, despite reviewer approval. In light of their convergence with other scoring methods (Beauducel & Hilger, 2022), the favorable results for BCFA, and in the interest of transparency and methodological pluralism, BCFA scores are included in the thesis.

In Study 3, considerable differences in performance were observed between ClusterCirc and traditional clustering methods. Yet, in data structures with zero or small item heterogeneity (two-dimensional, equal cluster spacing), all procedures recovered the intended circumplex clusters in over 94% of conditions (Table 5; adapted from Weide et al., 2025, p. 21). With medium heterogeneity (hierarchical and nonhierarchical models), Ward and k-means also recovered many intended clusters, although ClusterCirc consistently performed better. Thus, while ClusterCirc may generally be better suited in circumplex settings, Ward and k-means could also be effective if items show high homogeneity within clusters. With respect to the distance measure in Ward cluster analysis, only minor differences were observed between squared Euclidean distances (the default) and Euclidean distances (Minkowski-2), suggesting comparable outcomes across the investigated distance measures. Regarding item angles and communalities as input for the clustering procedures, PCA and CircE produced highly similar results for all clustering methods in two-dimensional circumplex structures. Differences emerged only in hierarchical, three-dimensional models, in which CircE outperformed PCA. Thus, in nonhierarchical models, researchers can select PCA or CircE based on practical and theoretical considerations. For instance, PCA angles can typically be computed faster, but CircE angles may be advantageous if a third factor cannot be entirely ruled out.

The convergence between different methodological approaches described here is not irrelevant. Instead, it promotes methodological pluralism in psychological research (Zitzmann & Loreth, 2021). Although psychological methods should be selected wisely based upon statistical guidelines backed up by simulation studies and theoretical reasoning, it is important to recognize that different approaches might produce similar—or even identical—results. Hence, some of the heated debates among statistician (e.g., J. Cohen, 1994; Gigerenzer, 2004; R. D. Morey et al., 2016) may reflect research traditions and stylistic preferences more than substantial differences in outcomes. For example, frequentist confidence intervals and Bayesian credible intervals can have the same numerical boundaries in some cases (Albers et al., 2018). Likewise, BCFA and frequentist MCFA produce similar results in normally distributed data and large samples when the same factor model is specified (Hoofs et al., 2018; Scheines et al., 1999). Nevertheless, genuine methodological differences should not be overlooked when they are indeed relevant. For instance, potential advantages of GPA-Varimax with random starts over SPSS-Varimax in double-optimum structures, the increased flexibility and improved model fit of BCFA with circumplex priors compared to frequentist MCFA, and the benefits of ClusterCirc over traditional cluster analysis in circumplex settings should be acknowledged. At the same time, overidentification with particular methods should be avoided. Recommendations should rest on empirical evidence and methodological coherence rather than individual (or peer-pressured) attachment to specific approaches. Furthermore, it is important to maintain a tolerant stance toward other researchers' methodological preferences, provided that the chosen methods are theoretically sound (e.g., Nalborczyk et al., 2019) and do not inherently produce bias in results.

5.2. Limitations and Future Research

In addition to interpreting the results and discussing broader implications, it is essential to elaborate on the major limitations of the thesis' research and to propose directions for future research to address these gaps.

5.2.1. Conditions of the Simulation Studies

For the first and third research project, simulation studies were conducted to investigate the performance of different methodological approaches. In Study 1, which compared GPA versus Kaiser rotation (SPSS) toward the Varimax criterion, the population models comprised perfect simple structures with a single optimum for the Varimax criterion and modified double simple structures (circumplexes) with a global and a local optimum for the Varimax criterion (Weide & Beauducel, 2019). The population models were created as ideal models without any violations of perfect simple or double simple structure, which limits the conclusions that can be drawn. For example, simple structure conditions included only main and zero loadings, with all main loadings set to the same value (a = .50). Double simple structures were modeled as eight clusters with two variables per cluster and perfect equal cluster spacing. Moreover, all circumplex conditions included at least three components (two circumplex components plus an additional third component); the basic two-dimensional circumplex was not examined. Future research could address these limitations by extending the population models. For instance, factor saturation (Guadagnoli & Velicer, 1988) could be systematically varied by adjusting the magnitude of the main loadings. In addition, secondary loadings could be introduced alongside the main loadings to further increase structural complexity. These could either preserve orthogonality—e.g., by alternating loading signs—or introduce obliqueness into the model. The latter is particularly relevant, as oblique structures are generally considered more realistic than orthogonal ones, despite the widespread use of Varimax rotation (Browne, 2001; Fabrigar et al., 1999). Oblique models would allow researchers to examine GPA-Varimax rotation under violations of the orthogonality assumption. Moreover, circumplex models could be created that comprise the typical two-dimensional data structure in addition to the three- or higher-dimensional structures in Study 1. For double simple structure models, deviations from equal spacing of clusters could be introduced by shifting one or more clusters by a set number of degrees and adjusting the corresponding population loadings accordingly. Moreover, the number of variables per cluster in the circumplex could also be varied systematically to test the robustness of rotation procedures when cluster sizes are uneven. Beyond standard double simple structures, more complex circumplex configurations could also be modeled. For instance, the radex model with six facets per circumplex (L. Guttman, 1954; 1957) poses challenges for rotation toward orthogonal simple structure, as only one of the two axes can be aligned with only one of the six facets. It could be modeled with a hierarchy of local optima by varying loading sizes across the six clusters, providing a challenging test for local optima of rotation procedures.

Limitations of the simulation study in the third research project on the comparison between ClusterCirc and traditional cluster analysis also need to be addressed. Sorting accuracy was defined as the detection rate of the intended population clusters (Weide et al., 2025). As stated previously, in all conditions, item distances were smaller within clusters than distances between items from adjacent clusters, consistent with the search criteria of traditional cluster analysis (Bridges, 1966; MacQueen, 1967). While this definition was appropriate for the present research, alternative definitions of cluster membership may be more suitable depending on the theoretical model and purpose of research. Consequently, the results are limited to population structures in which clusters are defined by minimal within-cluster distances among items. In many conditions, population clusters were evenly spaced around the circle, and items were evenly spaced within each cluster, resulting in segments of the same size. Such symmetric segmentation is generally preferred in circumplex models (e.g., Hatcher & Rogers, 2009; Horowitz et al. 1988, 2017; Locke, 2000; Locke, 2019; Meisel et al., 2025), as asymmetrical arrangements may not be considered consistent with circumplex structure. For instance, if most items are located on one side of the circle, this may suggest a rotational position of the axes resembling an unrotated solution, with many items loading strongly on the first axis and few on the second. As a result, unless the axes' rotational position is fixed a priori, only minor deviations from equal spacing and symmetric segmentation are compatible with the circumplex model. Nonetheless, violations of assumptions are of particular interest, which is why conditions with unequal cluster spacing, and thus asymmetric segmentation of the circle, were included. Future studies could systematically extend this approach by modeling more pronounced deviations—such as further shifting of cluster centroids or introducing uneven item distributions within clusters. Such modifications would likely present greater challenges for ClusterCirc, potentially reducing its accuracy while increasing the relative utility of conventional cluster analysis. Additionally, incorporating broader variability in communalities could further enrich the complexity of the simulated data. Exploring how such deviations affect the magnitude of ClusterCirc indices may also contribute to a more nuanced interpretation of empirical results, particularly in studies where ideal circumplex structure cannot be assumed.

5.2.2. Empirical Data

While simulation studies are well suited to examining methodological aspects of statistical procedures, assessing their practical relevance requires applying them to real-world data. Accordingly, empirical data were included in the present studies. However, these data bring their own limitations concerning both the samples and the psychometric measures.

5.2.2.1. Samples

The thesis' studies relied on nonrandom convenience samples. As a result, the findings may not generalize to the broader population, for example, due to selection bias and samples that are more homogeneous in both the investigated and other traits than the population. The majority of the participants were recruited by members of the Department of Psychology of the University of Bonn, likely resulting in WEIRD[7] (Henrich et al., 2010) and otherwise biased samples. Parameter estimates may be biased due to factors such as higher educational levels, younger age, or deviations in gender distribution and ethnic background compared with the general population. For example, the mean age was 19.53 years in Study 1 (Weide & Beauducel, 2019) and 32.68 years in Study 2 and 3 (Weide et al., 2021), compared with a median age of 45.49 years in Germany (World Health Organization, 2025). However, populations in other regions of the world are younger and closer in age to the study samples (World Health Organization, 2025). Gender distribution also differed, with 14% female participants in Study 1 and 63% in Studies 2 and 3, diverging from the near-equal distribution

[7] WEIRD = White, educated, industrialized, rich, and democratic.

in the general population (World Population Review, 2025). Such sampling characteristics may have biased parameter estimates. In Study 1, for instance, differences between SPSS and GPA-Varimax solutions in the short knowledge test may have been more pronounced in a more diverse sample. In Study 2, factor loadings and correlations with external variables may have varied with broader sampling. Similarly, item loadings and angles of the IAS in Study 3 may have differed in other samples, potentially leading to different clustering outcomes for the IAS with ClusterCirc.

Nonetheless, the results generally align with a priori expectations, previous empirical findings, and simulation results, which supports the robustness of the findings. For instance, the occurrence of local optima in GPA rotation found in Study 1 is consistent with other research on the topic (Browne, 2001; Hattori et al., 2017; Kiers, 1994; Nguyen & Waller, 2023; Trendafilov & Jolliffe, 2006). Findings on the IIP in Study 2 remained stable across odd-even splits of the sample, converged across different methodological approaches, and were in line with the literature. In the empirical example of Study 3, ClusterCirc yielded item clusters that were largely consistent with the original IAL subscales, further supporting robustness of the findings (Weide et al., 2025). Importantly, this thesis primarily sought to investigate methodological aspects and structural patterns within the available samples rather than to estimate population-level effect sizes. For basic research on structural patterns, convenience sampling of sufficiently large samples for multivariate analyses is both feasible and economic (Sherman, 2024). Nevertheless, future studies using random or more diverse samples are necessary to confirm generalizability of the findings.

Moreover, the research of the thesis relied on nonclinical samples. The presence of a psychological or psychiatric diagnosis was not an exclusion criterion, so as not to artificially create bias, given the high prevalence of mental health conditions in the general population (point estimate at approximately 40% in Germany in 2023; Robert Koch-Institut, 2024). While studying nonclinical samples is valuable for establishing baseline patterns in the general population, including clinical samples would be particularly interesting for research on the IIP (Study 2; Weide et al., 2021), which is designed to assess problematic interpersonal behavior in clinical contexts as well (Horowitz et al., 2017). Thus, future studies should investigate whether the findings reported in this thesis can be replicated in clinical samples.

5.2.2.2. Measures and Investigated Traits

In addition to sampling characteristics, it is also important to critically discuss the psychometric instruments administered in the studies and the traits that they measure.

Given the broader research purpose of Study 1—beyond a narrow focus on circumplex structure—the empirical example was based on a knowledge test with simple structure (Weide & Beauducel, 2019). However, it would be worthwhile to explore whether the findings from the simulation study on local optima in double simple structure can be extended to empirical settings. For instance, if GPA rotation based on multiple random starts yields a solution with a smaller Varimax criterion than GPA rotation without multiple random starts or SPSS-Varimax in circumplex questionnaires, this would corroborate the validity of the simulation findings.

Study 2 and Study 3 examined the circumplex structure of the IAS (Adams & Tracey, 2004; Jacobs & Scholl, 2005; Wiggins et al., 1988) and the IIP (Horowitz et al., 1988, 2017). Like many psychological instruments, the items of the IAS and the IIP might reach only ordinal scale level rather than interval scale level. However, procedures such as conventional PCA, EFA, CFA, and the Fourier transformation by CircE (e.g., for item input in ClusterCirc) assume interval measurement. Therefore, it could be suitable to use polychoric correlations instead of Pearson correlations as input for the procedures in future research. Polychoric correlations, especially when combined with estimation of factor loadings through weighted least squares or robust variants thereof, are better suited to ordinal data and may yield more robust loading patterns (Beauducel & Herzberg, 2006; A. Cohen & Migliorati, 2017; Li, 2016). Notably, ClusterCirc does not require metric variables, and polychoric-based loadings can serve as valid input for the algorithm when the interval level of variables is in question.

Moreover, in the empirical example on ClusterCirc, subscales for the IAL—derived using ClusterCirc, Ward cluster analysis, and the original IAL subscales—were computed using simple sum scores (Weide et al., 2025). This decision was made to keep the example straightforward and accessible, particularly for practitioners who commonly rely on sum scoring in diagnostic practice. While this enhances applicability, it does not fully align with the assumptions of the circumplex model. A more accurate representation of the underlying structure could be achieved through model-based scoring methods, which should be considered in future, more comprehensive investigations of subscale validity. For example, factor scores estimated through CFA using model-implied item weights may provide a closer fit to the theoretical model. Alternatively, subscale scores derived from ClusterCirc could be calculated by weighting items inversely to their distance from the cluster centroid, potentially with additional weighting based on item communalities, offering a more nuanced and theoretically grounded scoring method.

It would also be valuable to extend the research from Study 2, which examined the three-dimensional IIP, to two-dimensional circumplex measures such as the IAS—and conversely, to apply ClusterCirc to a three-dimensional measure like the IIP in addition to the two-dimensional IAS. Relevant research questions include the following: Is BCFA with Bayesian circumplex priors also more suitable than MCFA for two-dimensional circumplex structures? How do BCFA model fit and congruence with the ideal circumplex differ between two-dimensional circumplex measures and the three-dimensional IIP? For ClusterCirc, it would be informative to investigate whether CircE angles yield different clustering solutions with lower spc_w values than PCA angles when applied to three-dimensional circumplex measures, as observed in the simulation study using hierarchical population models.

Another limitation of Studies 2 and 3 concerns the exclusive use of interpersonal circumplex instruments based on three levels (two factors, eight subscales, and 64 items) with same-sized and equally spaced segments for the subscales (Weide et al., 2021, 2025). While this design allowed for coherent and theoretically grounded modeling, it limits generalizability to other circular constructs. For example, models of human values (Cieciuch et al., 2014; Hinz et al., 2005; Schwartz & Boehnke, 2004; Schwartz et al., 2012) involve hierarchical structures with multiple levels, varying segment sizes, and unequal spacing of subscales. Future studies could extend the present research to such measures, which would present a greater challenge for ClusterCirc and could decrease model fit for MCFA, BCFA, and TEFA if an equally spaced circumplex is specified. However, deviations from equal cluster spacing can be translated into an expected loading pattern in CFA and TEFA, whereas this is not yet available in ClusterCirc.

Beyond internal validity, external validity was also addressed by including external measures in Study 2. However, the International-Personality-Item-Pool (Hartig et al., 2003), containing 40 Big Five items, and the NPI-17 (con Collani, 2014), measuring narcissism, are relatively short scales with limited conceptual coverage. This may reduce the reliability and validity of the observed associations with the IIP's higher-level domains. Future research would benefit from employing more comprehensive personality assessments, such as the Revised NEO Personality Inventory (Costa & McCrae, 2006) for the Big Five, as well as more elaborate measures of narcissism. Moreover, the current study focused exclusively on subclinical grandiose narcissism and neglected vulnerable narcissism, which may yield different patterns. Vulnerable narcissism is characterized by heightened sensitivity to rejection and a stronger focus on one's own distress and has been shown to relate differently to interpersonal functioning than grandiose narcissism (Dickinson & Pincus, 2003; Şen & Barişkin, 2024). For instance, it may be more strongly associated with emotional withdrawal and general interpersonal distress than grandiose narcissism. Beyond narcissism, the other components of the dark triad—psychopathy and machiavellianism—are also of interest, as they have been linked to distinct profiles within the interpersonal circumplex in previous research (Dowgwillo & Pincus, 2017; Rauthmann & Kolar, 2013). Including them in future studies could offer a more thorough investigation of the external validity of the IIP's higher-level domains and a richer understanding of maladaptive interpersonal dynamics.

In line with the suggestion to use a clinical sample, it would also be worthwhile to examine whether the IIP's higher-level scores can predict treatment outcomes or distinguish between diagnostic groups, as has been previously demonstrated for its subscales (Alden & Phillips, 1990; Pincus & Wiggins, 1990; M. A. Ruiz et al., 2004; Tilden et al., 2010). In this context, future research could examine which of the three higher-level scores—Dominance, Love, and Distress—shows the greatest predictive validity for specific psychological disorders. The IIP's higher-level scores may be particularly informative for personality disorders, which are closely linked to impairments in interpersonal functioning (Alden & Capreol, 1993; Hilsenroth et al., 2007; Monsen et al., 2006; Pincus & Hopwood, 2012; Pincus & Wiggins, 1990; Williams & Simms, 2016; Wilson et al., 2017; Wright et al., 2012). Given that impairments in personality disorders typically extend across multiple interpersonal domains, the overall Distress score may be particularly relevant. Additionally, it would be valuable to investigate whether different scoring methods for the IIP's higher level dimensions (e.g., MCFA, BCFA, and weighted sum scores as in Study 2) differ in effectiveness for distinguishing psychological disorders. Beyond the three-factor model of the IIP (Israel & Langeveld, 2021; Wendt et al., 2019; Wilson et al., 2013), Stern et al. (2000) proposed a competing five-factor model for an IIP version specifically designed to assess personality disorders. This modified IIP version could be used in future studies, and it could be examined if the original three-factor model or the alternative five-factor model is more appropriate for clinical versus nonclinical samples. To increase the validity of such clinical investigations, it would be essential to incorporate clinician-administered interviews and observer ratings in addition to self-report measures, as individuals with personality disorders may evaluate themselves differently than external evaluators.

In contrast to Study 2, the empirical example on the IAL in Study 3 examined only the internal validity of the ClusterCirc solution (Weide et al., 2025). To further investigate the validity of ClusterCirc in empirical settings, future research could incorporate external constructs such as the Big Five (as in Study 2) or the HEXACO model of personality. As they have been previously linked to the interpersonal circumplex (Barford et al., 2015; DeYoung et al., 2013; Nysæter et al., 2009), they would offer a meaningful framework for assessing the convergent validity of subscales derived from ClusterCirc solutions.

5.2.3. Investigated Methods

Another limitation of the present research concerns the statistical procedures that were employed and investigated. In Study 1, GPA-Varimax was conducted both without and with random starts for the transformation matrix. In contrast, SPSS-Varimax was conducted only with a single (identity) start transformation matrix (Weide & Beauducel, 2019). The only substantial differences found in the simulation study favored GPA-Varimax over SPSS-Varimax in double-optimum conditions, but only when random starts were used in GPA. Although this suggests that random starts are the primary cause of the observed effect, the findings do not fully support this conclusion, as the effect was confounded with differences in the algorithm (Kaiser vs. GPA). Therefore, it would be interesting to disentangle the effect of random starts from differences in the algorithm by also applying random starts to the Kaiser algorithm in future research. Moreover, it would be worthwhile to investigate whether the findings on GPA rotation remain stable when extraction methods other than PCA are used. For example, methods that incorporate unique variances—such as conventional EFA with maximum-likelihood estimation or principal-axis factoring (Harman, 1967), and even Bayesian EFA (Conti et al., 2014)—could be examined. Additionally, different algorithms and rotation procedures could be investigated. For example, if oblique structures are modeled as well, analysis could be extended to oblique rotation methods, such as Promax, Oblimin, Quartimin, or Geomin, the latter being applicable to both orthogonal and oblique rotations (Browne, 2001; Hattori et al., 2017). Findings by Nguyen and Waller (2023) suggest that local optima are also relevant in EFA and for various rotation methods, particularly Geomin. However, they did not use circumplex models with multiple optima in the population models, which would present a more challenging test of local optima.

Regarding factor rotation in circumplex structures, rotation toward simple structure can help recover double simple structure and related models (e.g., other models with a number of facets that is a multiple of four). However, rotation toward simple structure primarily aims to reduce the number of interstitial variables between the two axes (Gorsuch, 2015; Mulaik, 2010; Thurstone, 1954, 1965), which contradicts the circumplex factor model (Acton & Revelle, 2004). A potential extension of the present approach could involve a combined procedure integrating factor rotation and ClusterCirc. Conceptually, two procedures may be interesting. Both would begin with (1) extracting factors or components and apply Varimax rotation, followed by (2) a ClusterCirc analysis. Next, one could (3a) target-rotate the Varimax loadings toward the ClusterCirc-based distances of items to their cluster centroid, and (4a) perform ClusterCirc again to evaluate whether the overall circumplex structure is optimized. Alternatively, starting from step (3), one could (3b) multiply the factor loadings by the distance of each item to the centroid of its nearest cluster, (4b) perform a Varimax rotation on these proximity-weighted loadings, (5b) apply the resulting transformation matrix to the original Varimax loadings from step (1), and finally (6b) conduct a new ClusterCirc analysis to examine whether the classification improves relative to the initial ClusterCirc solution from step (2). Such iterative procedures may help to integrate the strengths of orthogonal rotation and circumplex-based clustering, potentially leading to a more precise alignment between factor-analytic and circumplex structures.

Moreover, the concept can be extended beyond factor rotation to the level of factor extraction itself. Specifically, one could develop a new extraction method that allocates variance according to the alignment of variables along the circumplex. As such a method is not yet available, a provisional approach could involve a two-dimensional analysis (e.g., PCA) to derive circumplex weights for each variable. These weights could then be used to reweight the variable intercorrelation matrix, which in turn would serve as input for a subsequent circumplex-oriented PCA extraction. In this way, the extracted components would more directly reflect the circumplex-aligned variance structure of the variables.

With respect to the methods and factor models investigated in Study 2, a greater variety of circumplex models could be modeled. In TEFA, the loadings were freely estimated, with the circumplex pattern entered only as target loadings. In BCFA, the circumplex loadings were partially constrained, whereas in MCFA, the circumplex loadings were strictly fixed to the ideal circumplex. This design ensured comparability with previous (Wilson et al., 2013) and follow-up (Israel & Langeveld, 2021) research that used the same MCFA bi-factor model. Future research could incorporate more nuances, for instance, by fixing only the loadings of variables aligned with the circumplex axes while leaving the rest to be freely estimated, or vice versa. Additionally, a BCFA model with identical specifications to the MCFA model but without Bayesian priors could be examined to determine whether it yields results comparable to MCFA, as would be expected in large samples with normally distributed data (Hoofs et al., 2018; Scheines et al., 1999). Moreover, only the higher-level structure of the IIP was investigated, in accordance with the model by Wilson et al. (2013). Nevertheless, as pointed out in the previous section and highlighted by the development of ClusterCirc, examining both the item and subscale levels is highly relevant for circumplex structure. Therefore, future studies on the IIP should examine the item level as well, for example, by a multilevel CFA. Beyond comparing MCFA and BCFA, Study 2 also included CircE analysis to test the SPMC with various combinations of equality constraints for spacing and radius of the IIP octants. Future research could extend this comparison for CFA to the SPMC by comparing the default frequentist approach with maximum-likelihood estimation in CircE with Bayesian estimation of the SPMC (Lenk et al., 2006). Furthermore, external criteria could be incorporated in the SPMC (Nagy et al., 2019) to examine, for instance, whether narcissism is positioned between PA/Assured-Dominant and BC/Arrogant-Calculating, as suggested by the results of Study 2 (Weide et al., 2021).

Study 3 comprised an extensive simulation study comparing ClusterCirc with hierarchical Ward cluster analysis and k-means clustering (Weide et al., 2025). However, cluster analysis is rarely applied in circumplex analysis (e.g., Fegg et al., 2016). Commonly applied slicing of PCA or SSA plots (Hatcher & Rogers, 2009; Horowitz et al., 2017; Locke, 2000; Locke, 2019; Perrinjaquet et al., 2007; Redeker et al., 2014; Schwartz et al., 2012; Wiggins et al., 1988) could not serve as a reference for clustering performance of ClusterCirc because subjective decisions cannot be modeled in simulation studies. Therefore, because cluster analysis is an alternative strategy for item clustering, it was used as a benchmark for clustering performance of ClusterCirc in circumplex structures. ClusterCirc was performed using default weighting of between-cluster distances and within-cluster proximity. To better compare it with traditional cluster analysis, it would be informative to set the parameter e to 1, so that only within-cluster proximity is maximized in future studies. This would reduce differences in the search criterion between ClusterCirc and cluster analysis and emphasize differences in the algorithmic procedures. Moreover, a greater variety of q values, as the precision index in ClusterCirc, could be examined to rule out local optima of the search procedure. To ensure that differences in item input would not confound the results, the same item input (based on PCA and CircE) was used for ClusterCirc, Ward cluster analysis, and k-means clustering. However, ClusterCirc relied on angular distances of items, whereas traditional cluster analyses were performed with Euclidean (Minkowski-2 and default in k-means) and squared Euclidean distances (default in Ward). Future simulation studies could enter angular distances directly into cluster analytical procedures to disentangle effects attributable to the algorithm (ClusterCirc vs. cluster analysis) from effects of the distance measure. In addition, it could be worthwhile to examine more clustering techniques in circumplex structures, such as centroid clustering, single linkage, or nearest neighbor (Jaeger & Banks, 2023), and compare them with ClusterCirc.

5.2.4. Limitations and Further Development of ClusterCirc

Beyond discussing the research design of the studies, it is also necessary to reflect on limitations of ClusterCirc and outline directions for further development. A central concern relates to model error, particularly regarding the assumptions of ClusterCirc. Although perfect circumplex spacing is not imposed on the data, the search criterion spc_w assumes that ideal circumplex clustering is defined by equal spacing between clusters and high item homogeneity within clusters. The search algorithm divides the circle into same-sized segments in an iterative process and finds a segmentation that minimizes spc_w. The final item clusters can then be used as subscales for the instrument. However, such ideal structures are rarely observed in empirical data.

Numerous circumplex models assume equal spacing between subscales (e.g., Adams & Tracey, 2004; Alden et al., 1990; Etzel et al., 2021; Gurtman & Pincus, 2000; Meisel et al., 2025), although empirical deviations are common. Already, L. Guttman (1957) proposed larger angular distances between verbal ability and other mental domains, which he modeled closer together in his early theory, before later developments suggested an equally spaced model of mental abilities (Marshalek et al., 1983). Other circumplex theories, such as models of interpersonal strengths (Hatcher & Rogers, 2009) or human values (Hinz et al., 2005; Schwartz & Boehnke, 2004), have also implemented unequal spacing to reflect empirical findings and theoretical considerations. In its current implementation, ClusterCirc allows researchers to decrease or even discard the importance of equal cluster spacing if it does not fit the theoretical model of the instrument. However, if cluster spacing is taken into account and the clusters deviate from equal distances, this increases ClusterCirc's spacing indices and may result in lower model fit when tested via ClusterCirc-Simu. To enhance flexibility, future versions of ClusterCirc could include user-defined angular distances between clusters. For example, rather than assuming uniform distances (e.g., 120° in a three-cluster model), researchers could specify asymmetric angles (e.g., 100°, 120°, and 140°) to align the analysis more closely with theoretical expectations.

Regarding item spacing within clusters, ClusterCirc assumes that within-cluster proximity should be maximized, consistent with the logic of traditional cluster analysis (Bridges, 1966; Jaeger & Banks, 2023). While this assumption is often useful, it is not universally appropriate. Similar to equal cluster spacing, ClusterCirc allows researchers to reduce or disregard this constraint. However, by default, greater item heterogeneity increases spacing indices in ClusterCirc-Data. Importantly, ClusterCirc-Simu incorporates actual item heterogeneity within clusters when assessing model fit, allowing some heterogeneity without automatically penalizing the solution. Nonetheless, a potential improvement would be to incorporate a user-defined tolerance for item heterogeneity directly into the search procedure, rather than optimizing for minimal heterogeneity by default.

Another limitation of the current version is its reliance on same-sized segments. Although same-sized segmentation is frequently employed when finding circumplex subscales (Adams & Tracey, 2004; Gurtman & Pincus, 2000; Horowitz et al., 2017, 1988; Locke, 2000; Trobst, 2000), and it simplifies analysis, it may not reflect the structure of all instruments. Following the same rationale as with unequal spacing, a more flexible segmentation—tailored to theoretical or empirical requirements—could improve the usefulness and precision of ClusterCirc.

Currently, ClusterCirc is primarily concerned with item and cluster spacing, with item radius addressed only indirectly by including communalities as default item weights in spc_w. Yet, radius is a core feature of circumplex structure (Acton & Revelle, 2004; Gurtman & Pincus, 2000). Future developments could explicitly incorporate item radius, possibly in combination with item selection strategies. Although item selection is not the primary goal of the current version, researchers can already use ClusterCirc item indices (e.g., the item spacing index or distance to its cluster centroid) to evaluate item suitability. A combined index of communality and spacing could be developed to aid in item selection, including the assessment of how removing individual items would affect the overall spacing index. This would extend ClusterCirc's utility in item selection beyond item–subscale assignment.

Moreover, ClusterCirc in its current form is rather exploratory in nature. The method is primarily designed for early stages of scale development, helping identify item clusters that approximate optimal circumplex spacing for a specified number of clusters and items (ClusterCirc-Data). The exploratory approach is, for example, valuable when there is uncertainty about which two among multiple factors (or components) are supposed to exhibit circumplex structure. In this case, ClusterCirc can assist in identifying the optimal pair of circumplex factors by comparing spc_w values across all two-factor combinations. Model fit can be assessed through ClusterCirc-Simu, but this is done without reference to pre-specified subscales. However, some researchers may prefer to retain existing subscales due to strong theoretical justifications. In such cases, reallocating items—for example, moving items from Power to Security in Schwartz's value model (Cieciuch et al., 2014; Hinz et al., 2005; Schwartz & Boehnke, 2004; Schwartz et al., 2012)—might not be appropriate. For these scenarios, ClusterCirc-Fix offers a suitable alternative. It does not perform a cluster search but computes circumplex indices for user-defined item clusters based on pre-defined subscales. These indices can be used to evaluate the alignment of individual items with their assigned subscales, for example, by assessing item-level spacing indices. Furthermore, researchers may compare ClusterCirc-Fix results to those generated by ClusterCirc-Data to assess the size of spacing indices and the extent of agreement in item allocations. However, ClusterCirc does not yet provide a confirmatory test for fixed subscales entered in ClusterCirc-Fix. Such an extension could further enhance the tool's applicability.

At the subscale level, confirmatory analysis of circumplexity is already implemented in CIRCUM/CircE (Browne, 1992; Grassi et al., 2010), which tests the SPMC for user-defined combinations of equal spacing and radius. If CircE is applied to items, it does not account for potential clustering. As a result, CircE model fit for spacing decreases if items are arranged within clusters rather than evenly spread around the circle. Thus, CircE is suitable for testing equal radius but not equal spacing of items when (desired) item clustering is present. Conversely, when analysis is restricted to the subscale level, as is feasible in CircE, subscales may show good fit for equal spacing even if item-level spacing is suboptimal—a limitation that CircE may overlook. A major advantage of ClusterCirc is its combined analysis of both items and clusters (as a basis for subscales), such that circumplex spacing is optimized at both levels. However, its utility could be further enhanced if more confirmatory model tests, like those implemented in CircE, were integrated. Future research could therefore explore combining ClusterCirc and CircE for confirmatory testing of circumplex structure across different combinations of equality constraints at both the item and subscale levels.

6. Conclusion

To conclude, this thesis examined circumplex structure and methodological challenges inherent in its analysis. Traditionally, psychometric theory has been grounded in the principle of simple structure, which has shaped the field since Thurstone's introduction of primary factors and his definition of simple structure for factor rotation (Gorsuch, 2015; Mulaik, 2010; Thurstone, 1934, 1935, 1943, 1954, 1965). This paradigm is deeply embedded in the discipline's thinking. It seems that many researchers try to find names for dimensions and to subsume items under these umbrella terms. However, quantitative science may be a matter of degree rather than of linguistic similarity. For example, while one may label a substance an "acid" or a "base", chemistry relies on the pH-value to represent this property numerically rather than by word classifications. Similarly, psychometrics may eventually reach a stage where gradual similarities among items are quantified directly, replacing classifications based on subscale names that may inadvertently constrain theoretical development.

To date, simple structure has essentially only ever been challenged by three prominent alternatives: hierarchical models with a general factor (G. Cohen, 2000; Deary, 2012; Spearman, 1930), facet theory (R. Guttman & Greenbaum, 1998; Shye, 1998), and circumplex models (Gurtman, 1993, 2009; L. Guttman, 1954, 1957). Closer examination of circumplex models and the refinement of their analysis therefore hold promise for expanding theoretical perspectives and advancing psychological science. Accordingly, this thesis systematically investigated, refined, and developed statistical approaches for analyzing circumplex structure and offered recommendations based on both simulation studies and empirical data.

Key findings include the identification of local optima in GPA-Varimax when applied to circumplex double simple structure, and the demonstration that these can be overcome by using random starts for rotation, thereby offering advantages over the well-established Kaiser algorithm (Weide & Beauducel, 2019). Moreover, BCFA with Bayesian circumplex priors appears better suited than frequentist MCFA with fixed circumplex loadings or TEFA for modeling circumplex structure, as illustrated using the IIP (Weide et al., 2021). Finally, the development of ClusterCirc introduces an alternative to conventional clustering techniques for circumplex applications, which appears more effective in recovering circumplex clusters as a basis for psychometric subscales (Weide et al., 2025).

While these notable differences emerged across methods, considerable similarities also warrant attention. For example, GPA-Varimax and SPSS-Varimax produced highly congruent solutions in most conditions of Study 1, and the three scoring methods examined in Study 2 yielded nearly identical outcomes. Furthermore, applying ClusterCirc to empirical IAL data resulted in item clusters that largely resembled the original IAL subscales—however, with changes leading to improved circumplexity—even though the IAL was developed prior to the introduction of ClusterCirc. This finding supports both the circumplex structure of the IAL as well as the validity of ClusterCirc. Such convergence supports a tolerant stance toward methodological choices of researchers, provided that the approaches are theoretically grounded and yield empirically robust and unbiased results. Thus, beyond evaluating existing methods and proposing new ones for analyzing circumplex structure, the present research also advocates for methodological pluralism in psychological science, so long as the validity and reliability of outcomes can be reasonably assured.

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List of Mathematical and Statistical Notations

Matrices

Symbol	Definition
$\mathbf{A}$	Unrotated loading matrix
$\mathbf{A}'$	Transpose of **A**
$\mathbf{I}$	Identity matrix
$\mathbf{G}$	Negative gradient (of a transformation matrix **T** in factor rotation)
$\mathbf{\Lambda}$	Rotated loading matrix
$\partial\mathbf{\Lambda}$	Partial derivative of $\mathbf{\Lambda}$
$\mathbf{M}$	A matrix given by a displacement of the transformation matrix **T** by its negative gradient with step length *b*
$\mathbf{T}$	Transformation matrix for factor rotation
$\sim\mathbf{T}$	Projection of **M** onto the possible **T** after each iteration
$\mathbf{T}'$	Transpose of **T**
$f(\mathbf{T})$	Rotation criterion for a given transformation matrix
$f(\sim\mathbf{T})$	Rotation criterion for a subsequent transformation matrix in the gradient projection algorithm
$\mathbf{\Phi}$	Correlation matrix between factors or components

Symbols denoted by Greek letters

Symbol	Definition
α	Type-I error in statistical significance testing
β	Type-II error in statistical significance testing
δ	Ideal angular distance given perfect circumplex spacing
$\lvert\Delta\rvert$	Absolute difference
Θ	Angular position of a variable (subscale or item) in a circular configuration
σ^2	Variance (population or model value – used for Bayesian prior variances)

Symbols denoted by Latin letters

Symbol	Definition
a	Factor or component loading
b	Step length in gradient projection
bcs	Between-cluster spacing, indicating deviations from equal cluster spacing
c	Cluster
CI	Confidence interval
CFI	Comparative fit index
d	Observed angular distance
e	Relative importance of within-cluster proximity vs. equal spacing between clusters in ClusterCirc. $0 < e < 1$, and $e = 1/k$ is default; $e = 1$ makes equal distances irrelevant, $e = 0$ makes within-cluster distances irrelevant.
g	Tucker's congruence coefficient
$\sqrt{GDIFF}$	Results for the gap difference test
h^2	Communality of a variable (item or subscale) in factor analysis
$\overline{h^2}$	Mean communality across all variables (typically, items)
i	Item/variable
j	Iteration of the ClusterCirc algorithm
k	Number of clusters (in ClusterCirc, k-means, Ward cluster analysis)
l	Number of factors or components
M	Mean (empirical value)
m	Number of items/variables
m_c	Number of items in cluster c
m/l	Number of variables per factor/component
N, n	Sample size
p	Level of statistical significance
q	Precision index for the ClusterCirc algorithm
Q	Optimization criterion in factor rotation
∂Q	Partial derivative of Q in gradient projection
r	Correlation coefficient
$RMSEA$	Root-mean square error of approximation
SD	Standard deviation (empirical value)
spc	Unweighted spacing index in ClusterCirc
spc_i	Spacing index for an individual item in ClusterCirc
spc_w	Spacing index for the ClusterCirc search algorithm, with item and cluster weights
spc_w-$data$	The spc_w value of an empirical data set
spc_w-$simu$	The mean spc_w value of simulated samples in ClusterCirc-Simu. Is used as a benchmark for the evaluation of circumplex fit of a data set.
$SRMR$	Standardized root mean squared residual
u	Radius (vector length) of a variable in a circular arrangement
v	Varimax criterion (observed value)
w_i	Weight for item i in ClusterCirc
$\overline{w_i}$	Mean weight across all items in ClusterCirc
w_c	Weight for cluster c in ClusterCirc, with $w_c = e$ for clusters to which the item belongs, $w_c = (1- e)/(1-k)$ for the other clusters.
wcp	Within-cluster proximity, overall index of item homogeneity within clusters.

List of Abbreviations

BCFA	Bayesian confirmatory factor analysis
CFA	Confirmatory factor analysis
EFA	Exploratory factor analysis
GPA	Gradient projection algorithm
IAL	Interpersonal Adjective List, Interpersonelle Adjektivliste
IAS	Interpersonal Adjective Scales
IIP	Inventory of Interpersonal Problems
MCFA	Maximum-likelihood confirmatory factor analysis (frequentist)
NPI	Narcissistic Personality Inventory
PCA	Principal component analysis
SPMC	Stochastic process model of the circumplex
SSA	Smallest space analysis
TEFA	Exploratory factor analysis with target rotation
WEIRD	White, educated, industrialized, rich, democratic

List of Figures and Tables

Appendix

The present thesis is based on three publications. To avoid copyright violations, the articles are not included in this thesis. They can be found online via the following references:

Weide, A. C., & Beauducel, A. (2019). Varimax rotation based on gradient projection is a feasible alternative to SPSS. *Frontiers in Psychology*, *10*, Article 645. https://doi.org/10.3389/fpsyg.2019.00645

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