

Production of $\eta\pi^-$ and $\eta'\pi^-$ final states in high-energy $\pi^- p$ scattering at COMPASS

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Abstract

Understanding the mechanisms of hadron production at high energies provides crucial insight into the strong interaction and the non-perturbative regime of Quantum Chromodynamics (QCD). This thesis focuses on the production of the $\eta^{(\prime)}\pi^-$ system in high-energy π^-p scattering at the COMPASS experiment, with particular emphasis on the regime dominated by double-Regge exchange.

We analyze the full dataset collected during two dedicated years of data taking at COMPASS, selecting the exclusive $\eta^{(\prime)}\pi^- + p$ final state and reconstructing the $\eta^{(\prime)}\pi^-$ system via the decay chain $\eta^{(\prime)} \rightarrow \pi^-\pi^+\pi^0(\eta)$, $\pi^0(\eta) \rightarrow \gamma\gamma$. Of key importance for the description of the data is an accurate modeling of the detector acceptance. We describe the implementation of previously missing detector components in the Monte Carlo simulation frame work of COMPASS. Furthermore, we have significantly improved the calorimeter description. Previously, an effective shower simulation in the COMPASS calorimeters was needed to process events. We have enhanced the implementation such that this effective simulation is no longer required, resulting in a more detailed and precise shower simulation.

For the first time, the double-Regge formalism is applied to fit the data in the high-mass region, with resonance and double-Regge contributions separated using kinematic cuts. Six double-Regge amplitudes are considered: two for forward $\eta^{(\prime)}$ production and four for backward $\eta^{(\prime)}$ production. The fit, performed with only 13 free parameters, determines the relative strengths of the contributing amplitudes and accurately models the t -dependence at each vertex. The forward region is dominated by $a_2\mathbb{P}$ exchange, with a minor a_2f_2 contribution, while the backward region is dominated by $\mathbb{P}\mathbb{P}$ exchange, with significant interference effects involving f_2 and Pomeron exchange.

This thesis demonstrates the first event-based extraction of double-Regge amplitudes for the $\eta^{(\prime)}\pi^-$ system. The methods and results establish a foundation for future studies, including the extraction of coupling strengths at individual vertices and the connection of the double-Regge and resonance regions using theoretical approaches such as finite energy sum rules.

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Introduction

We know about four fundamental interactions: gravity, electromagnetism, the weak interaction, and the strong interaction. Throughout this thesis, we focus on the interactions of the smallest particles, with particular emphasis on the strong interaction. Quantum Chromodynamics (QCD) is the standard theoretical framework for describing strong interactions. Its key features are asymptotic freedom, where the strong force weakens at extremely short distances [1, 2], and confinement, where quarks cannot exist freely at large distances [3].

Particles composed of quarks are categorized as mesons (integer spin) or baryons (half-integer spin). The constituent quark model describes mesons as quark-antiquark pairs and baryons as three-quark objects. The mediator of the strong interaction is the gluon, which, like quarks, carries color charge. Consequently, the strong interaction exhibits both quark-gluon interactions and self-interacting gluons.

Even before the introduction of QCD, Tullio Regge observed that hadronic resonances can be organized according to spin and squared mass [4]. This relationship, known as a Regge trajectory, plays a central role in this thesis and will be discussed in detail.

Diffractive scattering, which involves t -channel exchanges, has been studied extensively at the COmmon Muon Proton Apparatus for Structure and Spectroscopy (COMPASS) [5–10]. In this process, the exchanged particle is typically a Pomeron, which can be interpreted as a multi-gluon exchange with vacuum quantum numbers at high energies. The reaction $\pi^- + p \rightarrow \eta^{(\prime)} + \pi^- + p$ was previously analyzed in Ref. [5], using half of the COMPASS dataset. The diffractive regime up to $m_{\eta^{(\prime)}\pi^-} = 3 \text{ GeV}/c^2$ was considered and a comprehensive partial-wave decomposition was performed.

More recently, the JPac collaboration used the partial waves extracted in Ref. [5] to construct a model describing the high-mass regime ($m_{\eta^{(\prime)}\pi^-}$ from 2.4 to $3 \text{ GeV } c^{-2}$) using double-Regge exchange [11]. Due to the detour via partial waves, which were extracted without any t -dependence, it was impossible to fit the t -dependence accurately. This led to the conclusion that the $f_2 f_2$ exchange was dominant, challenging the common assumption at COMPASS that exchanges are entirely Pomeron dominated.

In this work, we focus on the high-mass regime ($m_{\eta\pi^-}$ from 2.9 to $3 \text{ GeV } c^{-2}$ and $m_{\eta'\pi^-}$ from 2.6 to $3 \text{ GeV } c^{-2}$) and describe it using the double-Regge formalism, derived in Chapter 2 and following the model in Ref. [11]. An updated description of diffractive data in the resonance region is provided in Ref. [12]. We briefly introduce the COMPASS experiment in Chapter 3 and analyze the full dataset from two dedicated years of data taking. The event selection process is described in detail in Chapter 4.

To fit the data reliably, an accurate detector acceptance description is essential. The COMPASS

Monte Carlo (MC) simulation framework [13] lacked several detectors present during the diffractive campaign and used a computationally slow description of the calorimeters, which forced the use of GFlash that essentially replaced the detailed calorimeter description with an approximation. In Chapter 5, we discuss the detector implementation in the framework and present improvements to the calorimeter simulation, resulting in a complete and precise simulation.

The double-Regge fit is presented in Chapter 6, where the strengths of the contributing amplitudes are extracted. We close with a summary and outlook in Chapter 7.

Motivation

Throughout this thesis, we will study the reaction $\pi^- + p \rightarrow \eta^{(\prime)} + \pi^- + p$. In order to delve into that reaction, we will first discuss some basic particle properties in Section 2.1 and scattering theory in Section 2.2. Lastly, we will discuss the application to the reaction under investigation in Section 2.3.

2.1 The Constituent-Quark Model

In the constituent-quark model, a meson is made up of a quark (q) and anti quark ($\bar{q}$). For the sake of this thesis, we will concern ourselves only with light, non-strange mesons, i.e. mesons that are made up of (anti-)up ($\bar{u}/u$) and (anti-)down ($\bar{d}/d$) quarks¹.

Up and down quarks have spin and isospin of 1/2 and can therefore form states with total spin and isospin of 0 or 1. The other characteristic quantum numbers of a meson are given in Eq. (2.1).

$$|L - S| \leq J \leq L + S, \quad (2.1a)$$

$$P = P_q \cdot P_{\bar{q}} \cdot (-1)^L = (-1)^{L+1}, \quad (2.1b)$$

$$C = (-1)^{L+S}, \quad (2.1c)$$

$$G = (-1)^{L+S+I}, \quad (2.1d)$$

$$\eta = P \cdot (-1)^J, \quad (2.1e)$$

where J is the total spin, S is the intrinsic spin and L is the orbital angular momentum between the quark and anti quark. The parity is given by P , the behavior under charge conjugation² by C , the extension of C -parity to charged particles made of light non strange quarks (G) and the Isospin I . Lastly we have the naturality η .

¹ Strictly speaking, we consider mesons with total strangeness of 0. The $\eta^{(\prime)}$ for example has a strange quark contribution. We will not consider open strangeness, like the Kaon.

² Note that C is only a good quantum number for eigenstates of the charge conjugation operator, i.e. neutral light non-strange mesons. It is however convention that the charged partner states, e.g. $\pi^\pm$ for the π^0 , are given the same C -Parity as their neutral partners.

With Eq. (2.1), only a few combinations of quantum numbers are allowed for traditional $q\bar{q}$ mesons. Every unique combination is given a name, so that one can clearly identify them. Note that J is not included in the naming convention. In Table 2.1, one can find the naming convention.

Table 2.1: Possible quantum numbers for light non-strange $q\bar{q}$ mesons and their naming convention. It is convention, due to historic reasons to omit the J label for the ground state $\eta^{(\prime)}$, π , ω , ϕ and ρ . Mixture states of light non-strange quarks and strange quarks are denoted by a pair, e.g. ω/ϕ or η/η' [14].

S	L	I	I^G	J^{PC}	Naturality (η)	Name
0	odd	0	0^-	odd^{+-}	negative	h_J, h'_J
0	odd	1	1^+	odd^{+-}	negative	b_J
0	even	0	0^+	even^{-+}	negative	$\eta_{(J)}/\eta'_{(J)}$
0	even	1	1^-	even^{-+}	negative	$\pi_{(J)}$
1	odd	0	0^+	even^{++}	positive	f_J
1	odd	0	0^+	odd^{++}	negative	f_J, f'_J
1	odd	1	1^-	even^{++}	positive	a_J
1	odd	1	1^-	odd^{++}	negative	a_J
1	even	0	0^-	even^{--}	negative	$\omega_{(J)}/\phi_{(J)}$
1	even	0	0^-	odd^{--}	positive	$\omega_{(J)}/\phi_{(J)}$
1	even	1	1^+	even^{--}	negative	$\rho_{(J)}$
1	even	1	1^+	odd^{--}	positive	$\rho_{(J)}$

2.2 Scattering Theory

In this thesis, we will cover scattering of negatively charged pions off a fixed proton target. In the course of the event selection (see Chapter 4) we will show that we only select events, where the proton stays in tact and is recoiled with little energy, leaving the beam pion to be excited and then produce the $\eta\pi^-$ or $\eta'\pi^-$ system.

2.2.1 Resonances Formation and Single-Regge Exchange

The differential cross section for 2 to 2 scattering is given by Ref. [15]

$$d\sigma = \frac{1}{F} |A_{fi}(s, \cos \theta)|^2 d\Phi_2, \quad (2.2)$$

where F is the flux factor, A_{fi} is the amplitude that encodes the transition from the initial state i to the final state f and $d\Phi_2$ is the two body phase space. The construction of A_{fi} depends on the scattering process under investigation. If we consider $1 + 2 \rightarrow 3 + 4$ scattering in the s -channel (Fig. 2.1(a)) for two spinless particles, $A_{fi}(s, \cos \theta)$ is given by a series of partial waves

$$A_{fi}(s, \cos \theta) = \sum_{l=0}^{\infty} (2l+1) T_{l,fi}(s) P_l(\cos \theta), \quad (2.3)$$

where l is the orbital angular momentum between particles 3 and 4 and $P_l(\cos \theta)$ are the Legendre Polynomials, which encode the angular distribution as a function of the scattering angle θ in the center-of-momentum frame, i.e. $\theta(p_3^{\text{CM}})$. The dynamics of the scattering process in a given partial wave l is given by $T_{l,fi}$.

In the low energy regime, i.e. $\sqrt{s} < 3 \text{ GeV}$, we observe resonances that follow the angular distribution of the Legendre Polynomials in the two pseudoscalar final state, e.g. $\eta^{(\prime)}\pi^-$. In the total cross section of π^- proton scattering (see Fig. 2.2(b)), we see peaks in that energy regime.

Regge theory [4] predicts that there is a functional behavior between the mass of a resonance of a given family and the spin of the same. A family is defined to have the same I, I_3, P, S and naturality (η) but different J . Experimentally, we observe a straight line behavior, which is demonstrated in the form of the Chew-Frautschi Plot (see Fig. 2.2(a))³. In the t -channel, we parameterize this so called Regge Trajectory as

$$\text{Re}\{\alpha(t)\} = \alpha_0 + \alpha_1 t. \quad (2.4)$$

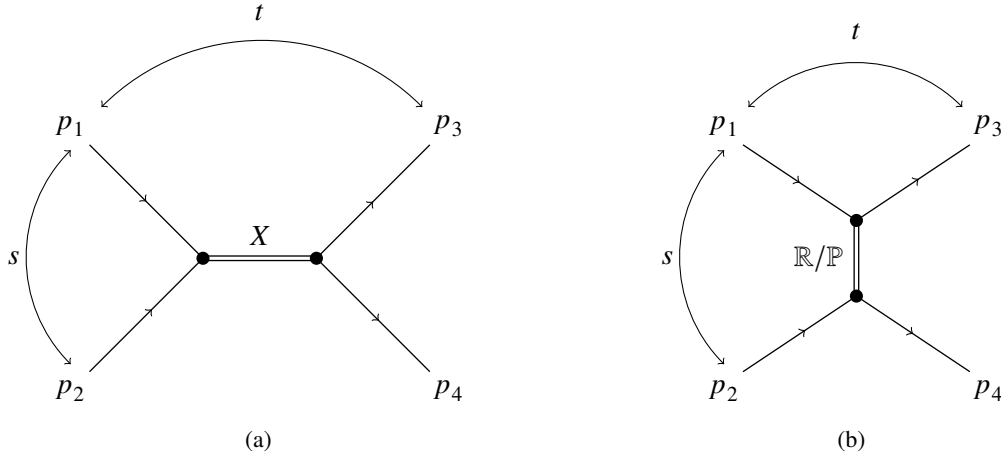


Figure 2.1: s -channel diagram (a), where the final state is produced via the formation of a resonance state X and t -channel diagram, where a Pomeron or Reggeon is exchanged to produce the final state.

Energy, momentum and mass of a particle i are related via

$$m_i^2 = E_i^2 - |\vec{p}_i|^2. \quad (2.5)$$

The Mandelstam variables s and t are given by

$$s = (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2(E_1 E_2 - |\vec{p}_1| |\vec{p}_2| \cos \theta_{12}), \quad (2.6a)$$

$$t = (p_3 - p_1)^2 = m_1^2 + m_3^2 - 2(E_1 E_3 - |\vec{p}_1| |\vec{p}_3| \cos \theta_{13}). \quad (2.6b)$$

³ A linear trajectory seems to fit best and is preferred in the literature. In principle, nothing forbids a behavior of a higher order polynomial.

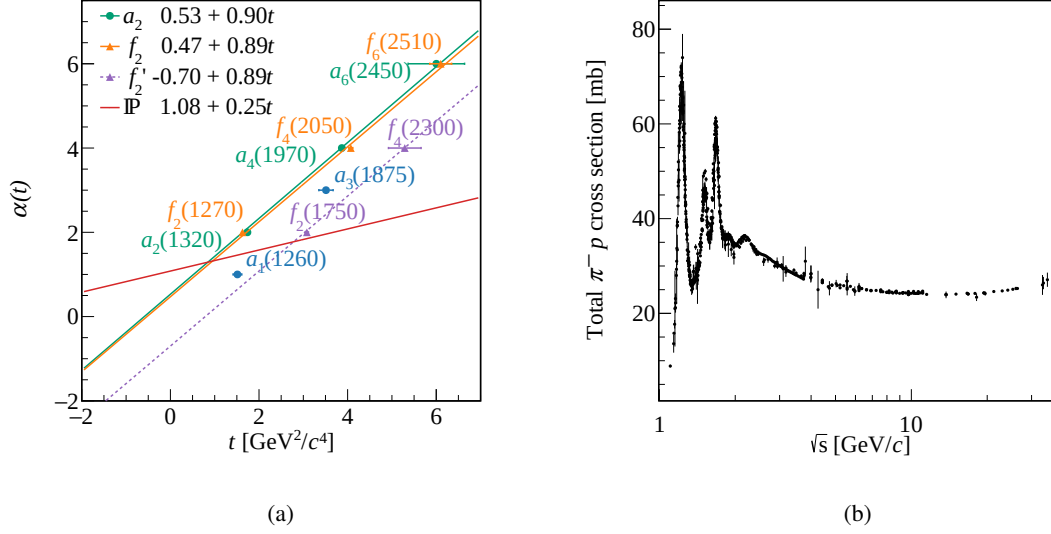


Figure 2.2: In (a) we draw the Chew-Frautschi plot with data from Ref. [15]. We draw the Regge trajectories, as they are used throughout this thesis. They are taken from Ref. [11] and explicitly given in Eq. (2.55). An exception is the first daughter trajectory of the f_2 , the f_2' , where we use the same slope as the one for f_2 but a different intercept to showcase that daughter trajectories have indeed the same slope but different intercepts. Note that the slope of the Regge trajectories has the unit c^2/GeV^2 , which is omitted for readability. In (b) we show the total $\pi^- p$ cross section with data from Ref. [15].

In the center-of-momentum frame ($\theta_{12} = 180$ deg) and the high energy limit, i.e. $s \rightarrow \infty$, one can neglect all particle masses and identify $E_i = |\vec{p}_i| = E$. Equation (2.6) simplifies to

$$s = 4E^2, \quad (2.7a)$$

$$t = -2E^2(1 - \cos \theta_{13}) = -\frac{s}{2}(1 - \cos \theta_{13}), \quad (2.7b)$$

$$\cos \theta_{13} = 1 + \frac{2t}{s}, \quad (2.7c)$$

where $\theta_{13} = \theta$ is the same scattering angle as in Eq. (2.3). Due to crossing symmetry, s and t are exchanged if we consider t -channel exchange (Fig. 2.1(b)), instead of s -channel exchange (Fig. 2.1(a)). We get

$$\cos \theta = 1 + \frac{2s}{t}. \quad (2.8)$$

In the Regge limit, i.e. $s \rightarrow \infty$ and t fixed, one therefore arrives at

$$\cos \theta \propto s. \quad (2.9)$$

Another way of expressing the high-energy limit is to say that $|\cos \theta|$ will approach 1. In order to describe this angular behavior an infinite number of partial waves is needed, due to the oscillating

behavior of the Legendre polynomials. Resonances with the same name but different signature⁴ $\tau = \pm 1$, lie on different trajectories. Therefore, it is convenient to split the amplitude (Eq. (2.3)) in two parts, one that represents odd integer l and one that represents even integer l resonances.

$$A_{fi}(t, s) = A_{fi}^{\tau=+1}(t, s) + A_{fi}^{\tau=-1}(t, s). \quad (2.10)$$

Note that we have now switched into the t -channel representation and, hence, replaced s by t in the argument of the amplitude. Additionally, we used Eq. (2.9) to replace $\cos \theta$ with s . Now, we use the Sommerfeld-Watson transform [16] to express the amplitude in terms of a contour integral. In order to do so, we analytically continue l to the complex plane. For any complex function, that has only simple poles, we can use Cauchy's theorem to write

$$\oint_C f(l) dl = 2\pi i \sum_{k=1}^N \text{Res}_{a_k} f(l) = 2\pi i \sum_{k=1}^N \lim_{l \rightarrow a_k} (l - a_k) f(l). \quad (2.11)$$

Here, a_k denote the simple poles of $f(l)$, and C is a contour enclosing all such poles. We use

$$f^\tau(l) = \frac{1 + \tau e^{-i\pi l}}{\sin \pi l}, \quad (2.12)$$

where the numerator is constructed such, that poles with odd l are canceled by zeros for positive signature. Even l poles are canceled for negative signature. Therefore, $f^+(l)$ only has poles for even l and $f^-(l)$ has poles for odd l . Now, we expand l near integer values n :

$$\sin \pi l \sim \sin(\pi n + \pi(l - n)) = (-1)^n \pi(l - n) + O((l - n)^3) = \begin{cases} -\pi(l - n) & \text{for odd } n, \\ +\pi(l - n) & \text{for even } n, \end{cases} \quad (2.13)$$

and get

$$\frac{1}{4i} \oint_C \frac{1 + \tau e^{-i\pi l}}{\sin \pi l} dl = \tau \left(1 + \tau e^{-i\pi l} \right) = \tau. \quad (2.14)$$

Using this mathematical trick, we can use Eq. (2.14) in Eq. (2.3) to write

$$A_{fi}^\tau(t, s) = \frac{1}{4i} \oint_C \frac{1 + \tau e^{-i\pi l}}{\sin \pi l} \frac{(2l + 1)}{\tau} T_{l,fi}(t) P_l(s) dl. \quad (2.15)$$

Next, we can replace l by the Regge trajectory $\alpha(t)$ (see Eq. (2.4)) and use the large s limit for Legendre Polynomials with a complex argument

$$P_{\alpha(t)}(\cos \theta) \sim \frac{a(\alpha(t))}{\Gamma(\alpha(t) + 1)} s^{\alpha(t)}. \quad (2.16)$$

Together with the mathematical identity

$$\Gamma(-\alpha(t)) = \frac{\pi}{\Gamma(\alpha(t) + 1) \cdot \sin \pi \alpha(t)}, \quad (2.17)$$

⁴ negative signature corresponds to odd J and positive signature to even J resonances. In the two pseudoscalar case throughout this work, $J = l$.

we write for the signature amplitude

$$A_{fi}^\tau(t, s) = \frac{1}{2i} \oint_C \Gamma(-\alpha(t)) \cdot \left(1 + \tau e^{-i\pi\alpha(t)}\right) \tilde{T}_l^\tau(t, \alpha(t)) s^{\alpha(t)} d\alpha(t), \quad (2.18)$$

where we have absorbed numerical constants in $\tilde{T}_l^\tau(t, l)$. Now, we can change the integration contour, to include the integer poles in the contour and pick up non integer Regge poles instead. Using Cauchy's theorem, the integral is solved.

$$A^\tau(t, s, \alpha(t)) \sim \Gamma(-\alpha(t)) \beta^\tau(t, \alpha(t)) \left(1 + \tau e^{-i\pi\alpha(t)}\right) \tilde{s}^{\alpha(t)}, \quad (2.19)$$

where $\beta^\tau(t, \alpha(t))$ is the residuum for one Regge pole. Additionally, we have introduced

$$\tilde{s} = s \cdot 0.8 c^2 \text{ GeV}^{-2}, \quad (2.20)$$

such that $\tilde{s}$ is dimensionless. The actual value of this normalization is chosen here in accordance to Ref. [11].

2.2.2 The Pomeron

The optical theorem connects the imaginary part of the forward elastic scattering amplitude A_{ii} and the total cross section. In the high energy limit it is given by Ref. [14]

$$\sigma_{\text{tot}} \sim \frac{1}{s} \text{Im}(A_{ii}(0, s)) \sim (\tilde{s})^{\alpha_0 - 1}. \quad (2.21)$$

In other words, the total cross section should fall for any trajectory with $\alpha_0 < 1$. This is contradictory to Fig. 2.2(b), where the cross section is shown to be constant or better slightly rising for large $\sqrt{s}$. In order to account for this, a new trajectory called the Pomeron trajectory⁵ was invented [17], which has $\alpha_0 \gtrsim 1$. The Pomeron corresponds to a ladder of quarks and gluon exchanges and we use throughout this work the common parameterization [11, 14]

$$\alpha_{\mathbb{P}} = 1.08 + 0.25 \frac{c^2}{\text{GeV}^2} t. \quad (2.22)$$

The Pomeron has vacuum quantum numbers [18]

$$I^G J^{PC} = 0^+ \text{even}^{++}. \quad (2.23)$$

Note that the f_{even} mesons carry the same quantum numbers. Since the Pomeron is postulated for exchanges at high $\sqrt{s}$, we expect mainly Pomeron exchange in Fig. 2.3(a). The beam energy used during the 2008 and 2009 data taking was around 190 GeV, which leads to $\sqrt{s} \sim 19 \text{ GeV } c^{-2}$, which is well above the resonance region in Fig. 2.2(b). Single Pomeron exchange (Fig. 2.3(a)) is often called diffractive scattering. Ref. [19] draws the Pomeron contribution next to the $\pi^- p$ cross section, showing a complete saturation of the cross section at $\sqrt{s} \approx 100 \text{ GeV } c^{-2}$ and a contribution of more than 95% at $\sqrt{s} \approx 19 \text{ GeV } c^{-2}$ (COMPASS center of mass energy). Therefore, we expect Pomeron exchange to dominate.

⁵ We will refer to it throughout this work simply as the Pomeron ($\mathbb{P}$).

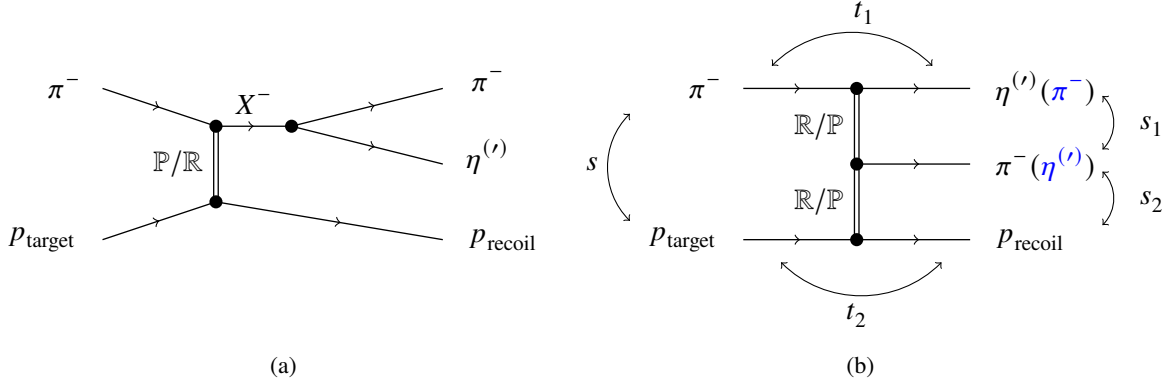


Figure 2.3: Production of the $\eta^{(\prime)}\pi^-$ system via an intermediate resonance X^- (a) and via double-Regge exchange (b). For double-Regge exchange there are two possibilities. Forward $\eta^{(\prime)}$ production, where the $\eta^{(\prime)}$ is produced at the top vertex, marked in black, and backward production, where the $\eta^{(\prime)}$ is produced at the middle vertex, marked in blue.

2.2.3 Double-Regge Exchange

The reaction under interest throughout this thesis is $\pi^- + p \rightarrow \eta^{(\prime)} + \pi^- + p$. Therefore, we need to transition from $1 + 2 \rightarrow 3 + 4$ scattering to $1 + 2 \rightarrow 3 + 4 + 5$ scattering. Here, we have two possibilities to produce the $\eta^{(\prime)}\pi^- + p$ final state. At low $s_1 = s_{\eta^{(\prime)}\pi^-}$, single Pomeron exchange with resonance production dominates (see Fig. 2.3(a)). This is basically s -channel exchange where particle 1 and 2 are the beam pion and the P/R , respectively, while the $\eta^{(\prime)}$ and π^- are the particles 3 and 4. Compare Fig. 2.1(a) and Fig. 2.3(a). In the high energy limit, i.e. $s_1 \rightarrow \infty$, t -channel Regge exchange will be dominant and therefore produce the $\eta^{(\prime)}\pi^-$ system directly, without an intermediate resonance (see Fig. 2.3(b)).

Our s and t notation of the previous section considered $1 + 2 \rightarrow 3 + 4$ scattering. Now, we introduce more variables. In particular

$$s = (p_{\eta^{(\prime)}} + p_{\pi^-} + p_{\text{recoil}})^2, \quad (2.24a)$$

$$s_1 = s_{\eta^{(\prime)}\pi^-} = (p_{\eta^{(\prime)}} + p_{\pi^-})^2, \quad (2.24b)$$

$$s_2 = s_{\eta^{(\prime)}p} = (p_{\eta^{(\prime)}} + p_{\text{recoil}})^2, \quad (2.24c)$$

$$s_2 = s_{\pi p} = (p_{\pi^-} + p_{\text{recoil}})^2, \quad (2.24d)$$

$$t_1 = t_{\eta^{(\prime)}} = (p_{\eta^{(\prime)}} - p_{\text{beam}})^2, \quad (2.24e)$$

$$t_1 = t_{\pi} = (p_{\pi^-} - p_{\text{beam}})^2 \quad (2.24f)$$

$$t_2 = t_p = (p_{\eta^{(\prime)}} + p_{\pi^-} - p_{\text{beam}})^2. \quad (2.24g)$$

Note that the choice of s_2 and t_1 depends on the fact if a π^- or $\eta^{(\prime)}$ is produced at the top vertex in Fig. 2.3(b). In double-Regge exchange, we have two variables s_1 and s_2 that are tied to the exchange

particles. In addition, there are now two different t variables. We will now derive the amplitude for double-Regge exchange in $\pi^- + p \rightarrow \eta^{(\prime)} + \pi^- + p$. Note that much more details on the general five body double-Regge amplitude are given in Refs. [18, 20]. In the double-Regge limit, effectively double t -channel exchange, we build on the single-Regge limit of Eq. (2.9). Therefore, the asymptotic behavior is given as

$$\cos \theta_1 \propto s_1 \rightarrow \infty \text{ and } \cos \theta_2 \propto s_2 \rightarrow \infty, \text{ with } t_1, t_2 \text{ fixed.} \quad (2.25)$$

In addition, one can show [20], that

$$s \propto \cos \theta_1 \cos \theta_2. \quad (2.26)$$

Therefore, the double-Regge limit is defined as [20]

$$s \rightarrow \infty, s_1 \rightarrow \infty, s_2 \rightarrow \infty, t_1, t_2, \kappa \text{ fixed,} \quad (2.27)$$

with

$$\kappa = \frac{\tilde{s}_1 \tilde{s}_2}{\tilde{s}}. \quad (2.28)$$

Expanding Eq. (2.19) for two exchange Reggeons, i.e. multiplying Eq. (2.19) with itself for different Regge trajectories and signatures, leads to

$$\begin{aligned} A^{\tau_1 \tau_2}(t_1, t_2, s, s_1, s_2) &\sim \Gamma(-\alpha_1) \Gamma(-\alpha_2) (1 + \tau_1 (-1)^{\alpha_1} + \tau_2 (-1)^{\alpha_2} + \tau_1 \tau_2 (-1)^{\alpha_1 + \alpha_2}) \\ &\quad \cdot \tilde{\beta}^{\tau_1}(t_1, \alpha_1) \tilde{\beta}^{\tau_2}(t_2, \alpha_2) \tilde{s}_1^{\alpha_1} \tilde{s}_2^{\alpha_2} \gamma^{\tau_1}(t_1, \alpha_1) \gamma^{\tau_2}(t_2, \alpha_2) \\ &= \Gamma(-\alpha_1) \Gamma(-\alpha_2) \tilde{\beta}^{\tau_1}(t_1, \alpha_1) \tilde{\beta}^{\tau_2}(t_2, \alpha_2) \cdot R^{\tau_1 \tau_2}(t_1, t_2, s, s_1, s_2), \end{aligned} \quad (2.29)$$

with $\alpha_1 \equiv \alpha_1(t_1)$ and $\alpha_2 \equiv \alpha_2(t_2)$. We have separated the vertex dependence $\tilde{\beta}^{\tau_1}(t_1, \alpha_1)$ and $\tilde{\beta}^{\tau_2}(t_2, \alpha_2)$ from the rest of the residuum, which represents the Reggeon-Reggeon coupling ($\gamma^{\tau_1}(t_1, \alpha_1) \gamma^{\tau_2}(t_2, \alpha_2)$). The residuum functions need to be modeled and we choose the representation in [20], where

$$\tilde{s}_1^{\alpha_1} \tilde{s}_2^{\alpha_2} \beta^{\tau_1}(t, \alpha_1) \beta^{\tau_2}(t, \alpha_2) = (-\tilde{s}_2)^{\alpha_2 - \alpha_1} (-\tilde{s})^{\alpha_1} V(t_1, t_2, \kappa) + (-\tilde{s}_1)^{\alpha_1 - \alpha_2} (-\tilde{s})^{\alpha_2} V(t_2, t_1, \kappa). \quad (2.30)$$

Therefore,

$$\begin{aligned} R^{\tau_1 \tau_2}(t_1, t_2, s, s_1, s_2) &= (-\tilde{s}_2)^{\alpha_2 - \alpha_1} (-\tilde{s})^{\alpha_1} V(t_1, t_2, \kappa) + (-\tilde{s}_1)^{\alpha_1 - \alpha_2} (-\tilde{s})^{\alpha_2} V(t_2, t_1, \kappa) \\ &\quad + \tau_1 ((-\tilde{s}_2)^{\alpha_2 - \alpha_1} (\tilde{s})^{\alpha_1} V(t_1, t_2, \kappa) + (\tilde{s}_1)^{\alpha_1 - \alpha_2} (\tilde{s})^{\alpha_2} V(t_2, t_1, \kappa)) \\ &\quad + \tau_2 ((\tilde{s}_2)^{\alpha_2 - \alpha_1} (\tilde{s})^{\alpha_1} V(t_1, t_2, \kappa) + (-\tilde{s}_1)^{\alpha_1 - \alpha_2} (\tilde{s})^{\alpha_2} V(t_2, t_1, \kappa)) \\ &\quad + \tau_1 \tau_2 ((\tilde{s}_2)^{\alpha_2 - \alpha_1} (-\tilde{s})^{\alpha_1} V(t_1, t_2, \kappa) + (\tilde{s}_1)^{\alpha_1 - \alpha_2} (-\tilde{s})^{\alpha_2} V(t_2, t_1, \kappa)), \end{aligned} \quad (2.31a)$$

where the so called vertex function is modeled as

$$V(t_1, t_2, \kappa) = \frac{1}{\Gamma(-\alpha_1)\Gamma(-\alpha_2)} \sum_{n=0}^{\infty} \frac{1}{n!} \Gamma(-\alpha_1 + n) \Gamma(-\alpha_2 + \alpha_1 - n) \kappa^n, \quad (2.32a)$$

$$V(t_2, t_1, \kappa) = \frac{1}{\Gamma(-\alpha_2)\Gamma(-\alpha_1)} \sum_{n=0}^{\infty} \frac{1}{n!} \Gamma(-\alpha_2 + n) \Gamma(-\alpha_1 + \alpha_2 - n) \kappa^n. \quad (2.32b)$$

We can write

$$\sin(x - n)\pi = \frac{e^{i\pi(x-n)} - e^{-i\pi(x-n)}}{2i} = \frac{e^{i\pi(x-n)} - e^{-i\pi(x-n)}}{2i} = \frac{e^{-i\pi n} e^{i\pi x} - e^{i\pi n} e^{-i\pi x}}{2i} \quad (2.33a)$$

$$= (-1)^n \sin x\pi, \quad (2.33b)$$

where we used that for integer n , $e^{\pm i\pi n} = (-1)^n$. Together with the identity

$$\Gamma(1 - x) = \frac{\pi}{\Gamma(x) \cdot \sin \pi x}, \quad (2.34)$$

we get

$$\Gamma(x - n)\Gamma(1 - (x - n)) = \frac{\pi}{\sin(x - n)\pi} = (-1)^{-n} \frac{\pi}{\sin x\pi} = (-1)^{-n} \Gamma(x)\Gamma(1 - x). \quad (2.35)$$

Reordering Eq. (2.35) gives

$$\Gamma(x - n) = (-1)^{-n} \frac{\Gamma(x)\Gamma(1 - x)}{\Gamma(1 - x + n)}. \quad (2.36)$$

Now, we can plug Eq. (2.36) into Eq. (2.32a), where we use $x = -\alpha_2 + \alpha_1$ and get

$$V(t_1, t_2, \kappa) \quad (2.37)$$

$$= \frac{\kappa^n}{\Gamma(-\alpha_1)\Gamma(-\alpha_2)} \sum_{n=0}^{\infty} \frac{1}{n!} \Gamma(-\alpha_1 + n) (-1)^{-n} \frac{\Gamma(-\alpha_2 + \alpha_1)\Gamma(1 + \alpha_2 - \alpha_1)}{\Gamma(1 + \alpha_2 - \alpha_1 + n)} \quad (2.38)$$

$$= \frac{\Gamma(\alpha_1 - \alpha_2)}{\Gamma(-\alpha_2)} \sum_{n=0}^{\infty} \frac{(-\kappa)^n}{n!} \frac{\Gamma(-\alpha_1 + n)}{\Gamma(-\alpha_1)} \frac{\Gamma(1 + \alpha_2 - \alpha_1)}{\Gamma(1 + \alpha_2 - \alpha_1 + n)} \quad (2.39)$$

$$= \frac{\Gamma(\alpha_1 - \alpha_2)}{\Gamma(-\alpha_2)} {}_1F_1(-\alpha_1, 1 - \alpha_1 + \alpha_2, -\kappa), \quad (2.40)$$

where ${}_1F_1(-\alpha_1, 1 - \alpha_1 + \alpha_2, -\kappa)$ is the confluent hypergeometric function of the first kind. We analyze double-Regge exchange in the so called Gottfried-Jackson (GJ) frame [21], where the $\eta^{(\prime)}\pi^-$ system is at rest and the beam π^- defines the z -axis. In addition, the production plane is parallel to $\vec{p}_{\text{target}}^{\text{GJ}} \times \vec{p}_{\text{beam}}^{\text{GJ}}$. A sketch of the GJ frame is shown in Fig. 2.4. In literature, one finds sometimes that the y -axis is defined as $\vec{p}_{\text{recoil}}^{\text{GJ}} \times \vec{p}_{\text{beam}}^{\text{GJ}}$ [14], which is equivalent in the GJ frame (see Eq. (2.41)).

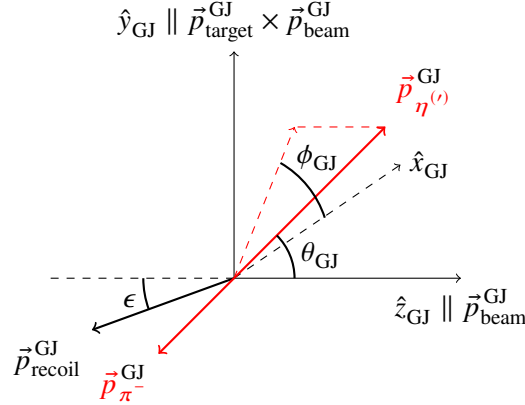


Figure 2.4: Sketch of the GJ frame.

$$\vec{p}_{\text{beam}}^{\text{GJ}} + \vec{p}_{\text{target}}^{\text{GJ}} = \underbrace{\vec{p}_{\eta^{(\prime)}}^{\text{GJ}} + \vec{p}_{\pi^-}^{\text{GJ}} + \vec{p}_{\text{recoil}}^{\text{GJ}}}_{\vec{0}}, \quad (2.41a)$$

$$\Leftrightarrow \left(\vec{p}_{\text{beam}}^{\text{GJ}} + \vec{p}_{\text{target}}^{\text{GJ}} \right) \times \vec{p}_{\text{beam}}^{\text{GJ}} = \vec{p}_{\text{recoil}}^{\text{GJ}} \times \vec{p}_{\text{beam}}^{\text{GJ}}, \quad (2.41b)$$

$$\Leftrightarrow \vec{p}_{\text{target}}^{\text{GJ}} \times \vec{p}_{\text{beam}}^{\text{GJ}} = \vec{p}_{\text{recoil}}^{\text{GJ}} \times \vec{p}_{\text{beam}}^{\text{GJ}}. \quad (2.41c)$$

In order to account for the correct angular behaviour, we include the K factor [11] (see Eq. (2.42)) in the amplitude. This is especially important if one describes an energy regime where there might be overlap between the resonance region and the double-Regge region. In our case, considering the 2008 and 2009 COMPASS data this is always the case.

$$K = \epsilon_{\alpha\beta\gamma\delta} (p_{\text{beam}}^{\text{GJ}} + p_{\text{recoil}}^{\text{GJ}})^{\alpha} (p_{\text{beam}}^{\text{GJ}} - p_{\text{recoil}}^{\text{GJ}})^{\beta} (p_{\eta^{(\prime)}}^{\text{GJ}} + p_{\pi^-}^{\text{GJ}})^{\gamma} (p_{\eta^{(\prime)}}^{\text{GJ}} - p_{\pi^-}^{\text{GJ}})^{\delta} \quad (2.42a)$$

$$= 4m_{\eta^{(\prime)}\pi^-} \left| \vec{p}_{\text{beam}}^{\text{GJ}} \right| \left| \vec{p}_{\eta^{(\prime)}}^{\text{GJ}} \right| \left| \vec{p}_{\text{recoil}}^{\text{GJ}} \right| \sin \epsilon \sin \theta_{\text{GJ}} \sin \phi_{\text{GJ}} \quad (2.42b)$$

In the double-Regge limit, $K \rightarrow s$ [22]. This dependency needs to be canceled in the overall amplitude. We do that by introducing a shift $\alpha_i \rightarrow \alpha_i - 1$. The final form of the vertex function reads

$$V(t_1, t_2, \kappa) = \frac{\Gamma(\alpha_1 - \alpha_2)}{\Gamma(1 - \alpha_2)} {}_1F_1(1 - \alpha_1, 1 - \alpha_1 + \alpha_2, -\kappa). \quad (2.43)$$

Using the following rearrangements

$$(\tilde{s})^{\alpha_1}(\tilde{s}_2)^{\alpha_2-\alpha_1} \stackrel{\alpha_i \rightarrow \alpha_i-1}{=} (\tilde{s})^{\alpha_1-1}(\tilde{s}_2)^{\alpha_2-\alpha_1} = \frac{(\tilde{s}_1)^{\alpha_1}(\tilde{s}_2)^{\alpha_2}}{\tilde{s}} \frac{(\tilde{s})^{\alpha_1}}{(\tilde{s}_1)^{\alpha_1}(\tilde{s}_2)^{\alpha_1}} = \frac{(\tilde{s}_1)^{\alpha_1}(\tilde{s}_2)^{\alpha_2}}{\tilde{s}} \frac{1}{\kappa^{\alpha_1}}, \quad (2.44a)$$

$$(\tilde{s})^{\alpha_2}(\tilde{s}_1)^{\alpha_1-\alpha_2} \stackrel{\alpha_i \rightarrow \alpha_i-1}{=} (\tilde{s})^{\alpha_2-1}(\tilde{s}_1)^{\alpha_1-\alpha_2} = \frac{(\tilde{s}_1)^{\alpha_1}(\tilde{s}_2)^{\alpha_2}}{\tilde{s}} \frac{(\tilde{s})^{\alpha_2}}{(\tilde{s}_1)^{\alpha_2}(\tilde{s}_2)^{\alpha_2}} = \frac{(\tilde{s}_1)^{\alpha_1}(\tilde{s}_2)^{\alpha_2}}{\tilde{s}} \frac{1}{\kappa^{\alpha_2}}, \quad (2.44b)$$

we can rewrite Eq. (2.31) as

$$R^{\tau_1 \tau_2}(t_1, t_2, s, s_1, s_2) = \frac{(\tilde{s}_1)^{\alpha_1}(\tilde{s}_2)^{\alpha_2}}{\tilde{s}} \left(\frac{V(t_1, t_2, \kappa)}{\kappa^{\alpha_1}} ((-1)^{\alpha_2-\alpha_1}(-1)^{\alpha_1} + \tau_1(-1)^{\alpha_2-\alpha_1} + \tau_2 + \tau_1 \tau_2(-1)^{\alpha_1}) + \frac{V(t_2, t_1, \kappa)}{\kappa^{\alpha_2}} ((-1)^{\alpha_1-\alpha_2}(-1)^{\alpha_1} + \tau_1 + \tau_2(-1)^{\alpha_1-\alpha_2} + \tau_1 \tau_2(-1)^{\alpha_2}) \right). \quad (2.45)$$

We define the signature factors as

$$\xi_i = e^{-i\pi\alpha_i(t_i)} + \tau_i, \quad (2.46a)$$

$$\xi_{ij} = e^{-i\pi(\alpha_i(t_i) - \alpha_j(t_j))} + \tau_i \tau_j, \quad (2.46b)$$

where $\alpha_i(t_i)$ and $\alpha_j(t_j)$ are either α_1 or α_2 . Multiplying them leads to

$$\xi_1 \xi_{21} = \left(\tau_1 + e^{-i\pi\alpha_1} \right) \left(\tau_1 \tau_2 + e^{-i\pi(\alpha_2-\alpha_1)} \right) \quad (2.47a)$$

$$= \tau_1 \tau_1 \tau_2 + \tau_1 e^{-i\pi(\alpha_2-\alpha_1)} + \tau_1 \tau_2 e^{-i\pi\alpha_1} + e^{-i\pi\alpha_1} e^{-i\pi(\alpha_2-\alpha_1)} \quad (2.47b)$$

$$= (-1)^{\alpha_1}(-1)^{\alpha_2-\alpha_1} + \tau_1(-1)^{\alpha_2-\alpha_1} + \tau_2 + \tau_1 \tau_2(-1)^{\alpha_1}, \quad (2.47c)$$

$$\xi_2 \xi_{12} = \left(\tau_2 + e^{-i\pi\alpha_2} \right) \left(\tau_1 \tau_2 + e^{-i\pi(\alpha_1-\alpha_2)} \right) \quad (2.47d)$$

$$= \tau_2 \tau_1 \tau_2 + \tau_2 e^{-i\pi(\alpha_1-\alpha_2)} + \tau_1 \tau_2 e^{-i\pi\alpha_2} + e^{-i\pi\alpha_2} e^{-i\pi(\alpha_1-\alpha_2)} \quad (2.47e)$$

$$= (-1)^{\alpha_2}(-1)^{\alpha_1-\alpha_2} + \tau_1 + \tau_2(-1)^{\alpha_1-\alpha_2} + \tau_1 \tau_2(-1)^{\alpha_2}, \quad (2.47f)$$

where we used $\tau_i = \pm 1 \Rightarrow \tau_i \tau_i = 1$. Now we can apply Eq. (2.47c) and Eq. (2.47f) in Eq. (2.45) and arrive at

$$R^{\tau_1 \tau_2}(t_1, t_2, s, s_1, s_2) = \frac{(\tilde{s}_1)^{\alpha_1}(\tilde{s}_2)^{\alpha_2}}{\tilde{s}} \left(\frac{V(t_1, t_2, \kappa)}{\kappa^{\alpha_1}} \xi_1 \xi_{21} + \frac{V(t_2, t_1, \kappa)}{\kappa^{\alpha_2}} \xi_2 \xi_{12} \right). \quad (2.48)$$

Keeping in mind the shift from $\alpha_i \rightarrow \alpha_i - 1$, as well as Eqs. (2.42) and (2.48), we write the double-Regge amplitude from Eq. (2.29) as

$$A^{\tau_1 \tau_2}(t_1, t_2, s, s_1, s_2) = K \tilde{\beta}^{\tau_1}(t_1, \alpha_1) \tilde{\beta}^{\tau_2}(t_2, \alpha_2) \Gamma(1 - \alpha_1) \Gamma(1 - \alpha_2) \frac{(\tilde{s}_1)^{\alpha_1} (\tilde{s}_2)^{\alpha_2}}{\tilde{s}} \cdot \left(\frac{V(t_1, t_2, \kappa)}{\kappa^{\alpha_1}} \xi_1 \xi_{21} + \frac{V(t_2, t_1, \kappa)}{\kappa^{\alpha_2}} \xi_2 \xi_{12} \right). \quad (2.49)$$

Left is to figure out the t dependence at the vertices ($\tilde{\beta}_i^{\tau_i}(t_i, \alpha_i)$). Experimentally, we know that the differential cross section follows an exponential fall with $|t|$ at small $|t|$ [14]. Therefore, we substitute

$$\tilde{\beta}_i^{\tau_i}(t_i, \alpha_i) = e^{-b_{i, \alpha_i} t_i}. \quad (2.50)$$

This is called the formfactor and b_{i, α_i} is called the slope parameter. We will use one unique slope parameter per unique vertex.

2.3 Production of the $\eta^{(\prime)} \pi^- + p$ Final State at COMPASS

In order to analyze $\pi^- + p \rightarrow \eta^{(\prime)} + \pi^- + p$ in the double-Regge limit, we need to figure out what Reggeons / if the Pomeron can be exchanged at what vertices. From the general behavior of the Regge amplitude (Eq. (2.19)), i.e. $\sim s^{\alpha_0 + \alpha_1 t}$, we see that for high s , trajectories with large α_0 dominate. We know that unnatural trajectories have a lower α_0 than natural trajectories [23]. Since COMPASS operates at high $s \sim 380 \text{ GeV}^2 c^{-2}$, we will completely ignore unnatural trajectories in our considerations of the final model. Note that the Pomeron has the same quantum numbers as the f_{even} mesons. Therefore, quantum number argument that allow f_{even} exchange, automatically allow Pomeron exchange. Due to Isospin symmetry, we consider C parity to be a good quantum number and conserved during our considerations, i.e. the process $\pi^0 + p \rightarrow \eta^{(\prime)} \pi^0 + p$ should make use of the same exchange Reggeons / Pomeron as the charged partner which we consider throughout this work. We use the quantum numbers of the particles from Table 2.1 and Eq. (2.23), as well as basic coupling rules for $a + b \rightarrow c$, given in Eq. (2.51).

$$|I_a - I_b| \leq I_c \leq I_a + I_b, \quad (2.51a)$$

$$G_c = G_a \cdot G_b \quad (2.51b)$$

$$||S_a - S_b| - L| \leq J_c \leq S_a + S_b + L, \quad (2.51c)$$

$$P_c = P_a \cdot P_b \cdot (-1)^L, \quad (2.51d)$$

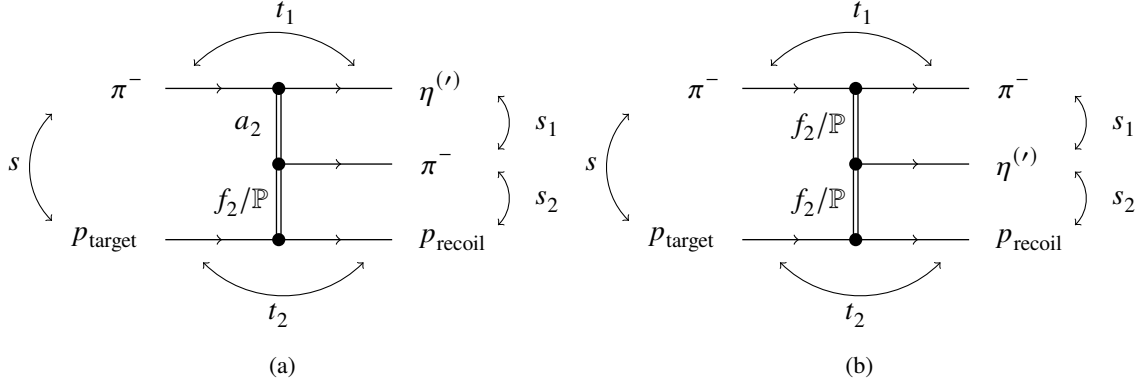
$$C_c = C_a \cdot C_b \cdot (-1)^L. \quad (2.51e)$$

Lets start with the process, where $\eta^{(\prime)}$ is produced at the top vertex (black in Fig. 2.3(b)). If we consider the top vertex, i.e. $\pi^- + \eta \rightarrow R$, we arrive at possible quantum numbers

$$I^G J^{PC} = \begin{cases} 1^- \text{even}^{++} \rightarrow a_{\text{even}}, \\ 1^- \text{odd}^{-+} \rightarrow \pi_{\text{odd}}^6. \end{cases} \quad (2.52)$$

For the Regge exchange, we therefore consider the a_2 trajectory. Now we can couple the a_{even} and the

⁶ A $q\bar{q}$ meson can not form these quantum numbers with odd J . We nevertheless know about the $\pi_1(1600)$ coupling to


 Figure 2.5: Forward (a) and backward (b) production of the $\eta^{(\prime)}\pi^-$ system with the leading order exchanges.

Pion and get

$$I^G J^{PC} = \begin{cases} 0^+ J^{++} \rightarrow f/\mathbb{P}, \\ 0^+ J^{-+} \rightarrow \eta^{(\prime)}, \end{cases} \quad (2.53)$$

where we made use of the fact that the neutral a_2 and π^0 can not couple to Isospin 1 particles due to the Clebsch-Gordan Coefficient being 0 and the fact that a $q\bar{q}$ meson can only have Isospin 0 or 1. The $\eta^{(\prime)}$ does not lie on any Regge trajectory and would anyways be an unnatural exchange. Therefore, we consider the $f_2/\mathbb{P}$ trajectory at the bottom vertex and arrive at Fig. 2.5(a).

Now, we consider the process where the $\eta^{(\prime)}$ is produced at the middle vertex (blue in Fig. 2.3(b)). If we look at the top vertex there, i.e. the $\pi^- + \pi^+ \rightarrow R$, we get the same quantum numbers for R as for the a_{even} Pion vertex, discussed above. Therefore, we can either exchange a f_2 trajectory or the Pomeron. Lastly, we have the $f_{\text{even}}\eta$ vertex, where we get the quantum numbers

$$I^G J^{PC} = \begin{cases} 0^+ J^{++} \rightarrow f/\mathbb{P}, \\ 0^+ J^{-+} \rightarrow \eta^{(\prime)}, \end{cases} \quad (2.54)$$

which again leads to either an f_2 trajectory or the Pomeron. The final diagram is given in Fig. 2.5(b).

Of course, we only considered the leading order here. In principle, also daughter trajectories, like the f'_2 or unnatural exchanges like the a_1 could contribute. Due to their considerably lower α_0 , we will safely ignore them throughout this work. This also means that all our trajectories have positive signature and we can identify $\tau_1 = \tau_2 = +1$ in Eq. (2.49). For the parameterizations of the trajectories we use (in agreement with Ref. [11])

$\eta^{(\prime)}\pi^-$ [24]. It is considered a hybrid particle where a gluon contributes to the quantum numbers.

$$\alpha_{a_2}(t) = 0.53 + 0.90 \frac{c^2}{\text{GeV}^2} t, \quad (2.55a)$$

$$\alpha_{f_2}(t) = 0.47 + 0.89 \frac{c^2}{\text{GeV}^2} t, \quad (2.55b)$$

$$\alpha_{\mathbb{P}}(t) = 1.08 + 0.25 \frac{c^2}{\text{GeV}^2} t. \quad (2.55c)$$

2.4 Asymmetries

The low $\eta^{(\prime)}\pi^-$ mass regime is described by partial waves. In Eq. (2.3), we have noted down the partial-wave expansion, while considering independence in the azimuthal angle ϕ . We can extend the partial-wave expansion to include an azimuthal component, which arises from the non-zero spin projection m of the produced states [25]. If we consider the two pseudoscalar final state, the amplitude becomes [14, 26]

$$A_{fi}(\theta, \phi) \propto \sum_{l=0}^{\infty} P_l^m(\cos \theta) \sin m\phi, \quad (2.56)$$

where m is the spin projection and $P_l^m(\cos \theta)$ are the associated Legendre functions. Therefore, the intensity function becomes

$$I(\theta, \phi) \propto \left| \sum_{l=0}^{\infty} P_l^m(\cos \theta) \sin m\phi \right|^2. \quad (2.57)$$

2.4.1 Forward-Backward Asymmetry

A key feature of the associated Legendre functions are their symmetry. Those with even l are symmetric around $\cos \theta_{\text{GJ}} = 0$, while those with odd l are anti-symmetric around $\cos \theta_{\text{GJ}} = 0$. The intensity will therefore only inhibit non-symmetric behavior when partial-waves with odd and even l are involved. We define

$$F \equiv \int_0^1 I \, d \cos \theta_{\text{GJ}}, \quad (2.58a)$$

$$B \equiv \int_{-1}^0 I \, d \cos \theta_{\text{GJ}}, \quad (2.58b)$$

$$A \equiv \frac{F - B}{F + B}, \quad (2.58c)$$

where F is the forward intensity and B is the backward intensity with respect to $\cos \theta_{\text{GJ}}$. The forward-backward asymmetry is defined as A with I being the intensity within our model. In the partial-wave interpretation, we would use Eq. (2.57) and in the double-Regge exchange model, we use

Eq. (6.1). We can further split these quantities in bins of $m_{\eta^{(\prime)}\pi^-}$, to see how they evolve over mass. Since we are dealing with an event based analysis, we write for a single bin in $m_{\eta^{(\prime)}\pi^-}$

$$F(m_1; m_2) = \sum_{i=1}^N r(m_1; m_2) \Theta(\cos \theta_{\text{GJ}}), \quad (2.59\text{a})$$

$$B(m_1; m_2) = \sum_{i=1}^N r(m_1; m_2) \Theta(-\cos \theta_{\text{GJ}}), \quad (2.59\text{b})$$

$$A(m_1; m_2) \equiv \frac{F(m_1; m_2) - B(m_1; m_2)}{F(m_1; m_2) + B(m_1; m_2)}, \quad (2.59\text{c})$$

where Θ is the Heaviside step function and

$$r(m_1; m_2) = \begin{cases} 1 & \text{if } m_1 < m_{\eta^{(\prime)}\pi^-} \leq m_2, \\ 0 & \text{otherwise.} \end{cases} \quad (2.60)$$

N represents all events in the given data set, e.g. all 2008 / 2009 selected COMPASS events. With this, we can test in which mass regime a forward-backward asymmetry manifests itself and if so, we can directly conclude that odd and even partial-waves need to be present in the data. Moreover, odd partial waves are spin exotic, i.e. can not be formed by simple $q\bar{q}$ states, which makes them especially interesting. We can also investigate the asymmetry in the high mass region and look how well the double-Regge model can reproduce it. This will give us a clue on how well we should be able to unite the double-Regge and partial-wave model with the approach discussed in Section 2.5.

2.4.2 Right-Left Asymmetry

Similar to the forward-backward asymmetry, described in Eq. (2.58), we can define a right-left asymmetry in the following way

$$R \equiv \int_{-\pi/2}^{\pi/2} I d\phi, \quad (2.61\text{a})$$

$$L \equiv \int_{\pi/2}^{3\pi/2} I d\phi, \quad (2.61\text{b})$$

$$A_{\text{RL}} \equiv \frac{R - L}{R + L}. \quad (2.61\text{c})$$

With this asymmetry, we can test the involvement of $m = 1$ and $m = 2$ partial waves⁷. If we only consider partial-waves up to $m = 2$, we can write for Eq. (2.57)

$$I = |w_1 \sin \phi_{\text{GJ}} + w_2 \sin 2\phi_{\text{GJ}}|^2, \quad (2.62)$$

⁷ Note that $m = 0$ is forbidden for natural exchange in this final state [5, 26].

where w_1 and w_2 are weights that absorb the parts of the partial-wave amplitudes that do not depend on ϕ_{GJ} . Now, we can integrate Eq. (2.61), while using Eq. (2.62) and arrive at

$$L = \int_{\pi/2}^{3\pi/2} \text{Id}\phi_{\text{GJ}} = \frac{\pi}{2} \left[\left(w_1 - \frac{8}{3\pi} w_2 \right) w_1^* + \left(w_2 - \frac{8}{3\pi} w_1 \right) w_2^* \right], \quad (2.63a)$$

$$R = \int_{-\pi/2}^{\pi/2} \text{Id}\phi_{\text{GJ}} = \frac{\pi}{2} \left[\left(w_1 + \frac{8}{3\pi} w_2 \right) w_1^* + \left(w_2 + \frac{8}{3\pi} w_1 \right) w_2^* \right], \quad (2.63b)$$

$$R - L = \frac{8}{3} (w_2 w_1^* + w_1 w_2^*), \quad (2.63c)$$

$$R + L = \pi \left(|w_1|^2 + |w_2|^2 \right), \quad (2.63d)$$

$$A_{\text{RL}} = \frac{R - L}{R + L} = \frac{8}{3\pi} \frac{w_2 w_1^* + w_1 w_2^*}{|w_1|^2 + |w_2|^2}. \quad (2.63e)$$

The right-left asymmetry (Eq. (2.63e)) vanishes if w_1 or w_2 vanish. Therefore, if we measure an asymmetry, it is a direct indication of the presence of $m = 2$ partial waves. In addition, we can split the data to a forward- and backward component and investigate the right-left asymmetry therein. Similar to Eq. (2.59), we write

$$R(m_1; m_2) \equiv \sum_{i=1}^N r(m_1; m_2) \Theta \left(-|\phi_{\text{GJ}}| + \frac{\pi}{2} \right), \quad (2.64a)$$

$$L(m_1; m_2) \equiv \sum_{i=1}^N r(m_1; m_2) \Theta \left(+|\phi_{\text{GJ}}| - \frac{\pi}{2} \right), \quad (2.64b)$$

$$A_{\text{RL}}(m_1; m_2) \equiv \frac{R(m_1; m_2) - L(m_1; m_2)}{R(m_1; m_2) + L(m_1; m_2)}, \quad (2.64c)$$

$$R^{\text{F}}(m_1; m_2) \equiv \sum_{i=1}^N r(m_1; m_2) \Theta(\cos \theta_{\text{GJ}}) \Theta \left(-|\phi_{\text{GJ}}| + \frac{\pi}{2} \right), \quad (2.64d)$$

$$L^{\text{F}}(m_1; m_2) \equiv \sum_{i=1}^N r(m_1; m_2) \Theta(\cos \theta_{\text{GJ}}) \Theta \left(+|\phi_{\text{GJ}}| - \frac{\pi}{2} \right), \quad (2.64e)$$

$$A_{\text{RL}}^{\text{F}}(m_1; m_2) \equiv \frac{R^{\text{F}}(m_1; m_2) - L^{\text{F}}(m_1; m_2)}{R^{\text{F}}(m_1; m_2) + L^{\text{F}}(m_1; m_2)}, \quad (2.64f)$$

$$R^{\text{B}}(m_1; m_2) \equiv \sum_{i=1}^N r(m_1; m_2) \Theta(-\cos \theta_{\text{GJ}}) \Theta \left(-|\phi_{\text{GJ}}| + \frac{\pi}{2} \right), \quad (2.64g)$$

$$L^{\text{B}}(m_1; m_2) \equiv \sum_{i=1}^N r(m_1; m_2) \Theta(-\cos \theta_{\text{GJ}}) \Theta \left(+|\phi_{\text{GJ}}| - \frac{\pi}{2} \right), \quad (2.64h)$$

$$A_{\text{RL}}^{\text{B}}(m_1; m_2) \equiv \frac{R^{\text{B}}(m_1; m_2) - L^{\text{B}}(m_1; m_2)}{R^{\text{B}}(m_1; m_2) + L^{\text{B}}(m_1; m_2)}, \quad (2.64i)$$

for our event based analysis. Note that we define $\phi_{GJ} \in (-\pi, \pi]$ as usual.

2.5 Finite Energy Sum Rules

In this chapter, we have discussed the low energy and high energy region of $\pi^- + p \rightarrow \eta^{(\prime)} + \pi^- + p$ separately, while focusing on the derivation of the asymptotic behavior at high $\sqrt{s_1}$. At COMPASS, we focus most of our studies in the low mass regime (see, for instance, [6–8, 10, 27]). In the low mass regime, we perform a partial wave decomposition to split the data in different partial waves, from where we try to extract resonance parameters. Naturally, the double-Regge process is considered as a background process in that kind of studies. We usually model it by some phenomenological background parameterizations. Since these models violate unitarity, it would be of great interest to infer the background of double-Regge exchange in the low mass region. Since quite some time, there exists the concept of finite energy sum rules [28], which try to predict the behavior of one region by knowing precisely the behavior in the other. While these models are quite advanced and applicable in $1 + 2 \rightarrow 3 + 4$ scattering [29], there is no sound model for $1 + 2 \rightarrow 3 + 4 + 5$ scattering, yet.

The COMPASS Experiment

The COMPASS experiment was located at Conseil Européen pour la Recherche Nucléaire (CERN) and took data from 2002 to 2022 with a large number of physics programs, using the high intensity muon or hadron beam, provided via the M2 beam line at the Super Proton Synchrotron (SPS). At the SPS, protons are accelerated to 400 GeV/c, which are then extracted and scattered on a production target. From there, the secondary beam is guided through a complex magnet and collimator setup to the COMPASS target. One extraction period is called spill. The COMPASS data acquisition groups data of a maximum of 200 spills into one run.

3.1 The 2008 and 2009 Experimental Setup

Throughout this work, we focus on the data taking period in 2008 and 2009, where we used a negative hadron beam, composed of π^- (96.9 %), K^- (2.4 %) and $\bar{p}$ (0.8 %), at an energy around 190 GeV. In Ref. [30], one can find a detailed explanation of the setup and the detectors used at COMPASS. In the following, we will briefly describe the spectrometer setup.

In Fig. 3.1 we show the COMPASS spectrometer, as it was in 2008 and 2009. It can be divided into four parts. First, we have the beam line, where two dedicated particle identification detectors (Cherenkov Detector with Achromatic Ring focus (CEDAR)) are put. Their working principle is based on Cherenkov radiation and they are tuned to differentiate between Kaons and Pions in the beam. The second region is the target region, which is composed of silicon and scintillating fiber detectors for trajectory measurements and at its center, we have the LH_2 target. In 2009, also different targets were used. However, in this work only data with the LH_2 target is analyzed (see Section 3.1.1 for more details). The target is surrounded by the Recoil Proton Detector (RPD), which is a time-of-flight detector, built to measure the trajectory of recoiling protons from the target. More details on the RPD are given in Section 3.1.1. With the first spectrometer magnet, the Large Angle Spectrometer (LAS) region starts. Here, all detectors have a large hole in their middle to ensure that particles that are created with a large forward boost can pass through the detectors and can be detected in the Small Angle Spectrometer (SAS), which starts at the second spectrometer magnet. Detectors in the SAS have only a small hole in the middle to ensure that non-interacting beam particles can pass through the detectors. The dipole magnets bend charged particles and together with various tracking detectors, e.g. Gas Electron Multipliers (GEMs), Drift Chambers (DCs), Micromesh Gaseous Structure (MicroMegas)

and Rich Wall (DR), one can reconstruct the momenta of charged particles. The LAS features also the Ring Imaging CHerenkov detector (RICH) for final state particle identification, i.e. it can distinguish charged kaons, pions and (anti-)protons. In order to reconstruct neutral particles and to measure the energy of all particles, we have one electromagnetic calorimeter and one hadronic calorimeter in the LAS and SAS, respectively. More information about the calorimeters can be found in Section 3.1.4. Downstream of the calorimeters we find a dedicated muon system, consistent of Muon Wall 1 (MW1) in LAS and Muon Wall 2 (MW2) in SAS. These are drift tube detectors. In addition to them, a large block of concrete ensures that only muons can survive until this point of the spectrometer. Tracks detected by MW1 or MW2 can therefore safely be identified as muons. For this work, muon identification plays no role. Nevertheless, it is important to simulate the correct material budget and we discuss in Section 5.3.6, why some significant changes needed to be implemented in Total GEometry ANd Tracking (TGEANT), regarding these detectors.

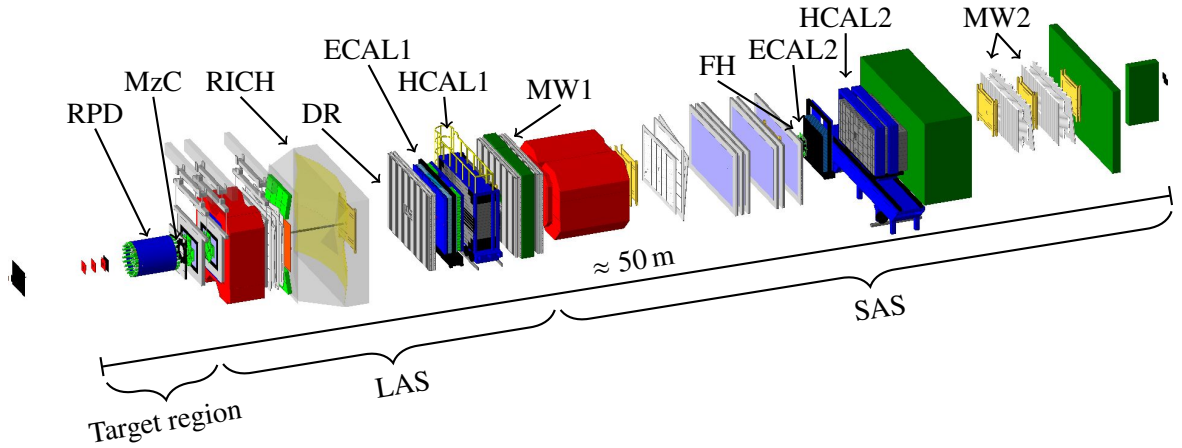


Figure 3.1: The COMPASS spectrometer for the 2008 / 2009 diffractive setup. Marked are the detectors, which are discussed in more detail throughout this thesis. In red, one can see the spectrometer magnets, which mark the start of the LAS and SAS, respectively. Absorber material is displayed in dark green.

3.1.1 The LH_2 target

The liquid hydrogen target is a cylindrical vessel with a radius of 1.75 cm and a length of 40 cm [30]. In order to hold the liquid hydrogen in place, a complex holding structure with as little material as possible was built. A schematic view can be found in Fig. 3.2 and a detailed technical drawing in Fig. 3.3. The holding structure is partly made of aluminum or stainless steel.

3.1.2 The Recoil-Proton detector

The RPD is a central detector for diffractive events at COMPASS. It is a time-of-flight detector, made up of two layers of scintillators and positioned around the LH_2 target. In the $\eta^{(\prime)}\pi^-$ final state, very little energy, $\mathcal{O}(500 \text{ MeV})$, is transferred to the recoil proton. Therefore, it is emitted with an angle of around 80° , relative to the beam axis. In Fig. 3.4(d), the two scintillator layers are depicted. The layer close to the beam axis is called Ring A and the other one Ring B. Ring A consists of 12 scintillators

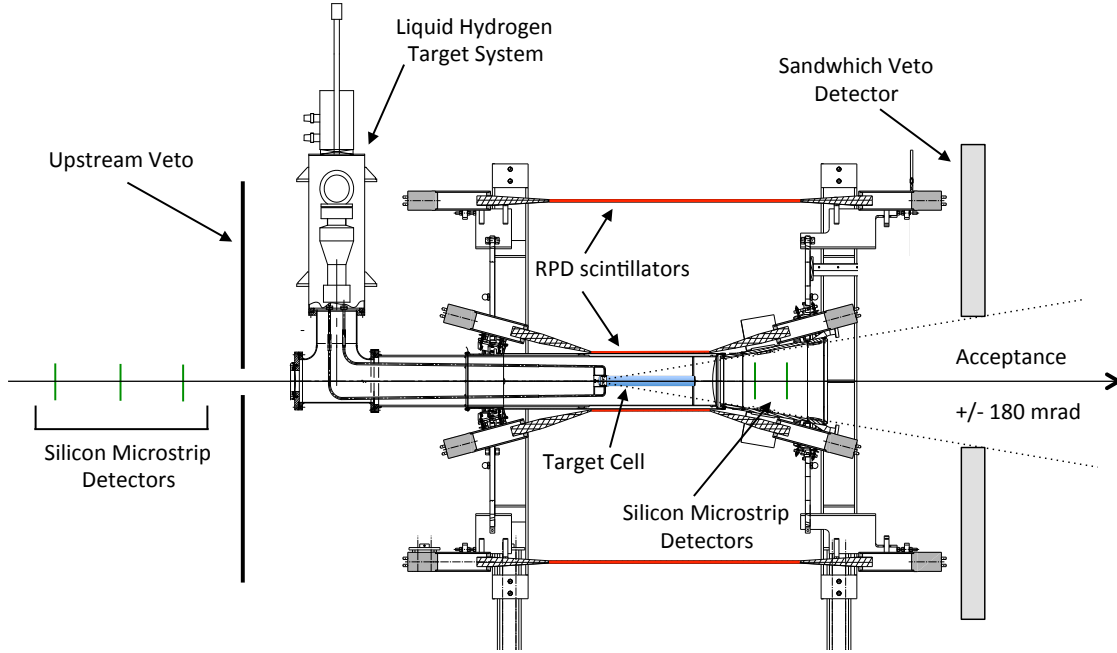


Figure 3.2: Side view of the target region for the 2008 / 2009 hadron setup from Ref. [30] fig. 4.

and Ring B of 24. The ones in Ring B are a little longer, so that recoil protons with smaller opening angles, relative to the beam axis, can still be detected. A front view of the RPD can be seen in Fig. 3.4(a) and a side view in Fig. 3.4(c). The scintillators in Ring A are shown in Fig. 3.4(b). Each scintillator is read out by one photomultiplier on each end. Throughout this work, a complete software recalibration of the RPD was done [32]. In its core, we have replaced the hard-coded calibration values of each scintillator of the RPD by a run-dependent t_0 calibration, in combination with a global z alignment of the two rings. The final values and the code is documented in Refs. [32, 33]. More information on the RPD can be found in Ref. [34].

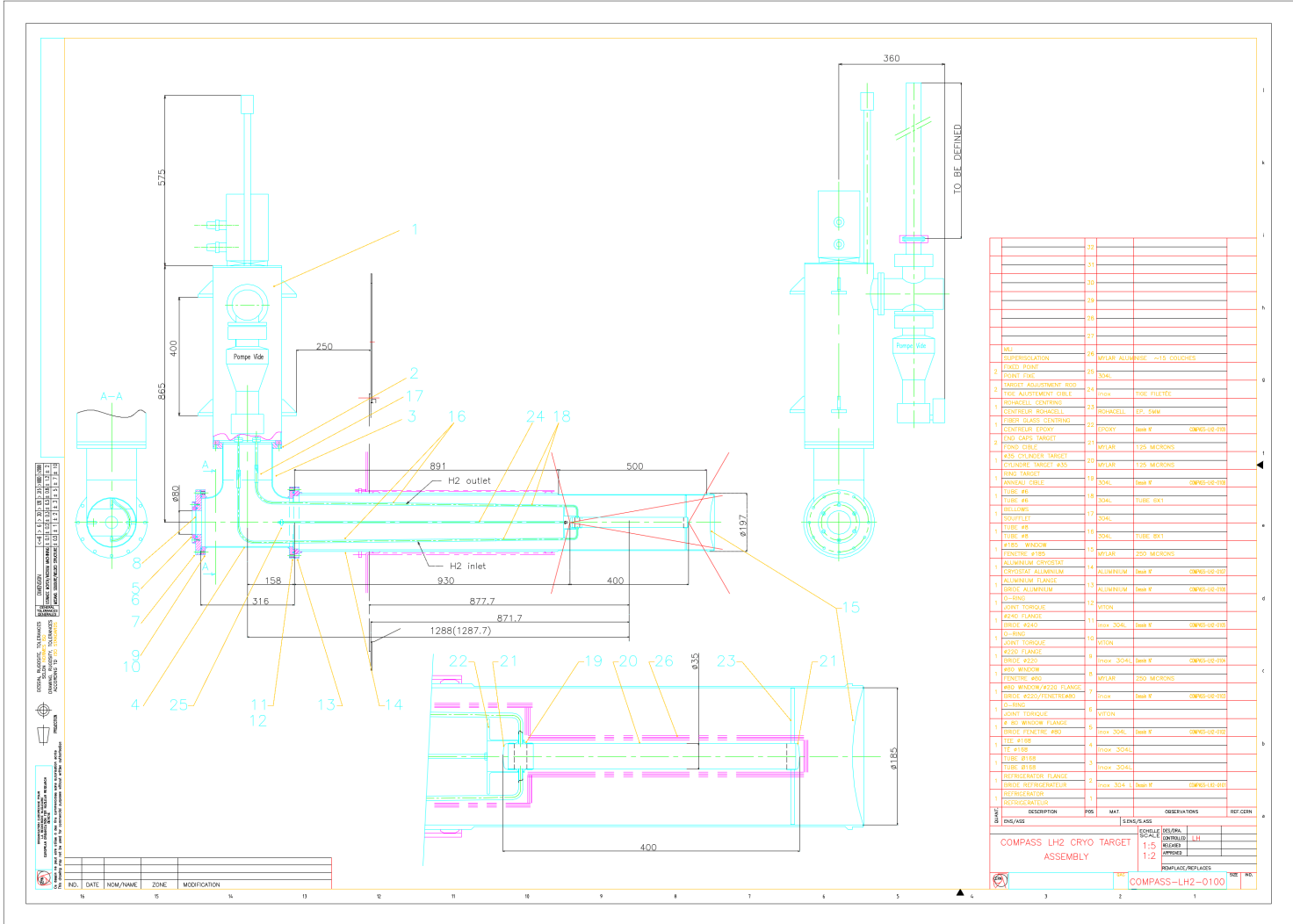


Figure 3.3: Technical drawing of the target region for the 2008 / 2009 hadron setup from Ref. [31].

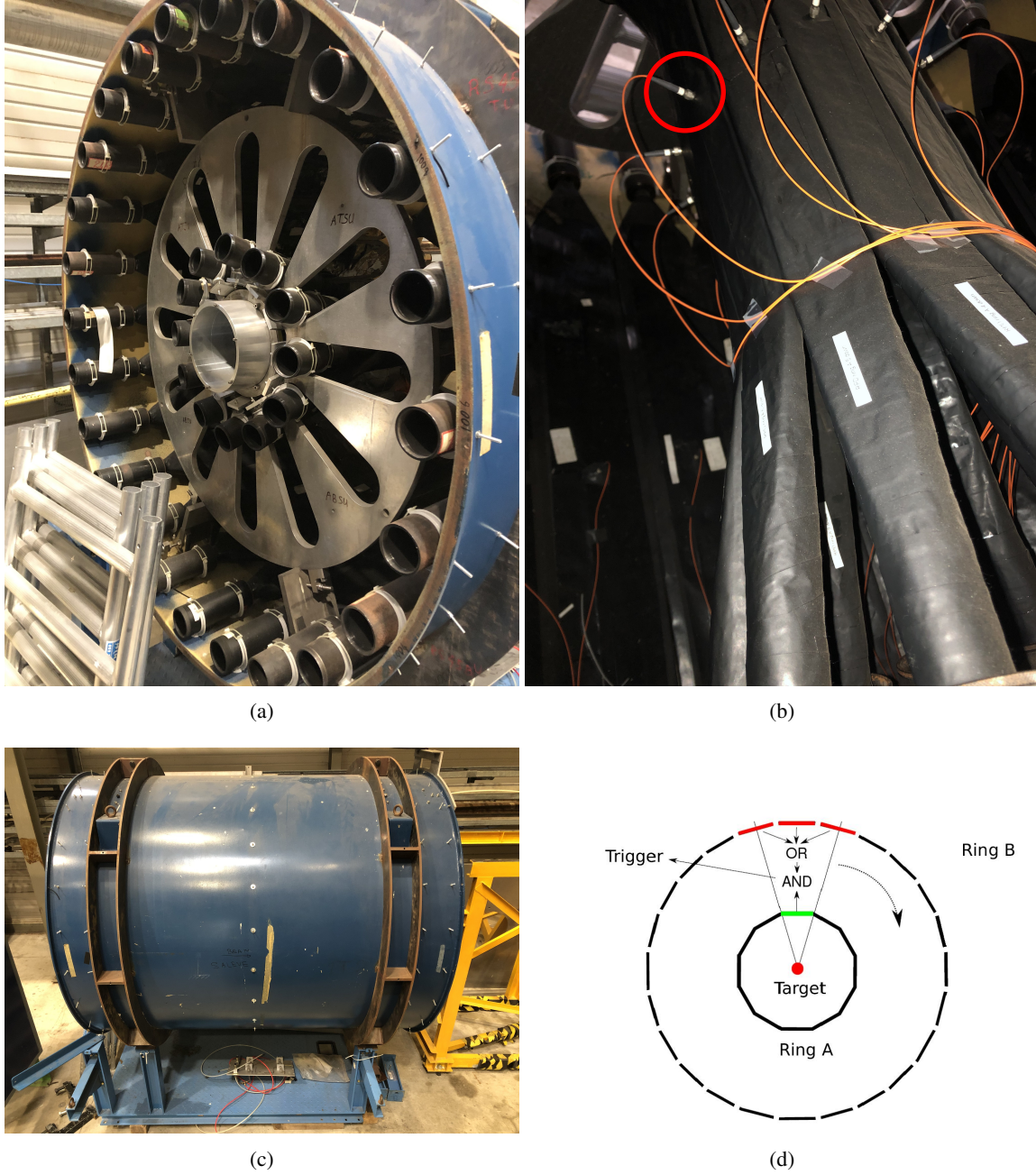


Figure 3.4: Front (a) and side (c) view of the RPD. In (b), one LED connector is marked with a red circle on a scintillator of the inner Ring (Ring A) of the RPD. The trigger logic of the RPD is shown in (d). Diameter and length of the RPD are approximately 2 m.

3.1.3 The Trigger

COMPASS is a triggered spectrometer, meaning that there are certain detectors in place that shall determine if an interesting event occurred. Usually, from hits in all the different trigger detectors, a trigger mask is constructed, based on which one can decide during data taking, or during analysis, if a particular event is of interest. During the 2008 and 2009 diffractive data taking, the trigger logic from Table 3.1 and Table 3.2 respectively was in place.

Explanation of the different trigger components The veto detectors are scintillators, located around the spectrometer, which cover areas outside of the reach of the other detectors, i.e. they trigger on particles that leave the spectrometer and are not seen by the spectrometer. By definition, events with those kind of particles should be rejected for the main physics analysis. Part of the veto system are the Veto Inner1, Veto Inner2, Veto Outer, HOuter4 and Beam Killer (BK) detectors. The latter is placed far downstream of the target, directly on the beam axis. It serves as an identification for non interacting beam particles. Additionally, there is the Sandwich Veto detector, placed directly downstream of the RPD to cover the angular range between the acceptance of the RPD and that of the rest of the spectrometer [35].

The trigger logic of the RPD is shown in Fig. 3.4(d). It only requires that at least one scintillator in Ring A (example shown in green) and one of the three corresponding scintillators in Ring B (shown in red) fired, without any restriction on the hit positions in the slabs.

The alternative beam trigger (aBT) is a coincidence of a hit in the fiber detectors upstream of the target and the beam counter (a scintillating disc upstream of the target that covers the full transverse target area), to ensure that a beam particle was present and hit the target cell.

The Mainz Counter (MzC) and Munich Counter (MuC) (see Fig. 3.5(a) for a technical drawing) cover the hole in the central part of the Sandwich Veto. More information about these two detectors may also be found in Ref. [36]. The Forward Hodoscope (FH) (see Fig. 3.5(b) for a technical drawing) is located upstream of the Electromagnetic CALorimeter 2 (ECAL2).

For this work, we only consider the DT0 trigger, which requires that no veto detector saw a signal, a recoil proton was present and the beam interacted with the target protons. Other trigger detectors act only as dead material in the spectrometer.

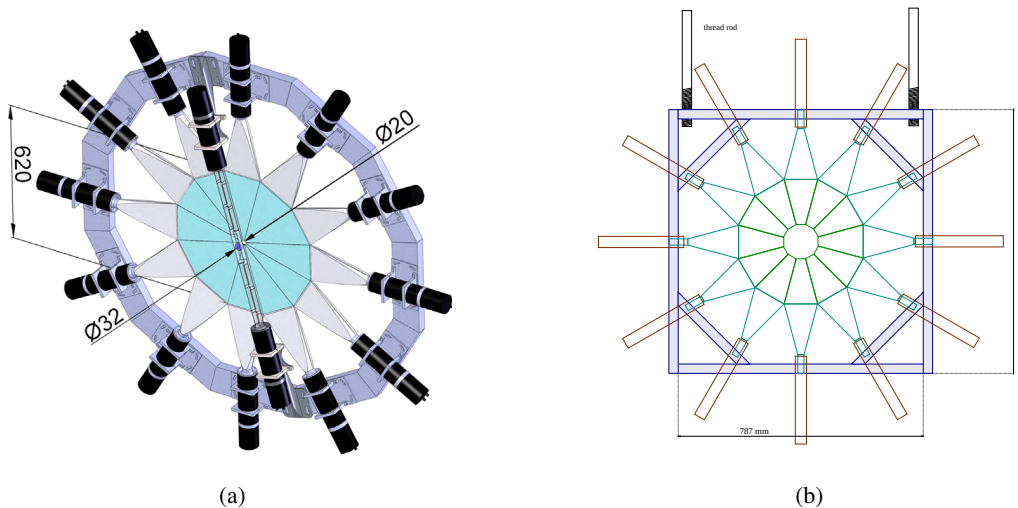


Figure 3.5: Technical drawing of the MzC and MuC (a), with dimensions in mm, taken from Ref. [30]. Design of the FH (b), taken from Ref. [34].

Table 3.1: Trigger logic for the 2008 diffractive hadron run [37].

Bit	Name	Logic
01	DT0	aBT && RPD && !(Veto)
02	DT1	DT0 && (MzC or HCAL2)
03	KaonT	aBT && CEDAR1* && CEDAR2* && !(Veto)
04	CT1	DT0 && FH
05	NIT	aBT && BK
06	VI	Veto Inner1 && Veto Inner2
07	Halo	Veto Outer && HOuter4
08	BT	Beamtrigger
09	aBT	altern. Beamtrigger
10	RPD	Proton Trigger
11	accT	accepted synchronous triggers
12	random	Noise generator

Table 3.2: Trigger logic for the 2009 diffractive hadron run [37].

Bit	Name	Logic
01	DT0	aBT && RPD && !(Veto)
02	LOWT2	aBT && !(Veto) (minimum bias)
03	LOWT3	aBT && MuC (2.5MIPS threshold = 465mV) && !(Veto)
04	LOWT1	aBT && MzC (low threshold 50mV) && !(Veto)
05	NIT	aBT && BK
06	VI	Veto Inner1 && Veto Inner2
07	Halo	Veto Outer && HOuter4
08	BT	Beamtrigger
09	aBT	altern. Beamtrigger
10	RPD	Proton Trigger
11	Laser	ECAL1 Laser Trigger
12	TRand	True Random Trigger

3.1.4 The Calorimeters

COMPASS features the Electromagnetic CALorimeter 1 (ECAL1) and ECAL2. The former is located in the first half of the spectrometer, aiming to measure the energy and position of photons and electrons/positrons with a large opening angle, relative to the z -axis. The latter is located more downstream in the spectrometer and aims to measure the energy and position of photons and electrons/positrons with a small opening angle, relative to the z -axis. ECAL1 is a homogeneous calorimeter that consists of 1500 lead glass modules. It is split in three different parts with modules of different sizes. The largest outer modules are the OLGA modules, the most inner once the GAMS modules and the third module type are the Mainz modules. The layout of ECAL1 is depicted in Fig. 3.6(a). ECAL2 (see Fig. 3.6(b)) consists of 1332 lead glass modules (GAMS) which are identical to the GAMS modules of ECAL1. Additionally, there are 848 radiation-hardened modules (GAMS-R) towards the more central part of the calorimeter. The part close to the beam axis is covered with 860 Shashlik type modules (see Fig. 3.7).

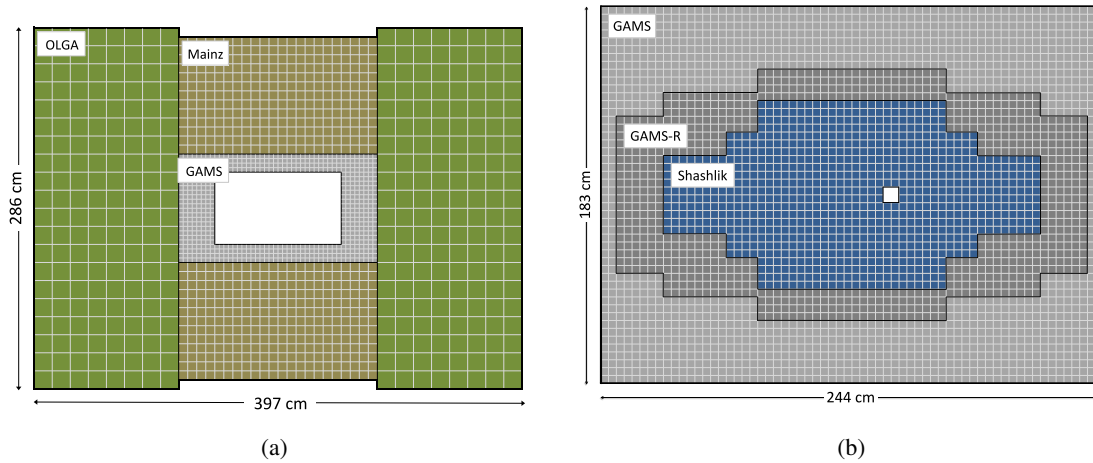


Figure 3.6: Overview of ECAL1 (a) and ECAL2 (b), taken from Ref. [14]. Denoted are the different calorimeter regions in different colors. The names refer to the experiments, from where the crystals originate, before they were moved to the COMPASS spectrometer. An exception are the shashlik modules, which were newly built for COMPASS. The white spots denote the different beam holes.

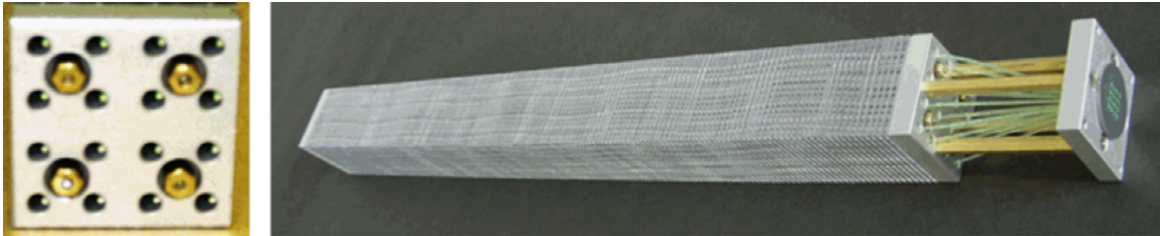


Figure 3.7: Photo of one Shashlik cell at COMPASS [30]. The dimensions are $3.82 \text{ cm} \times 3.82 \text{ cm} \times 38.75 \text{ cm}$.

One shashlik module¹ is composed of alternating layers of scintillator and absorber material, in

¹ We sometimes call one module “cell”.

order to withstand the radiation and provide enough radiation lengths to assure that a large amount of energy is deposited in ECAL2 even for high energy (> 50 GeV) photons or electrons/positrons. These layers are fixed in space by four holding rods and the light, created in the scintillators, is guided by wave-length shifting (WLS) fibers to the photomultipliers. Details on how the rods and fibers are implemented are given in Fig. 3.8.

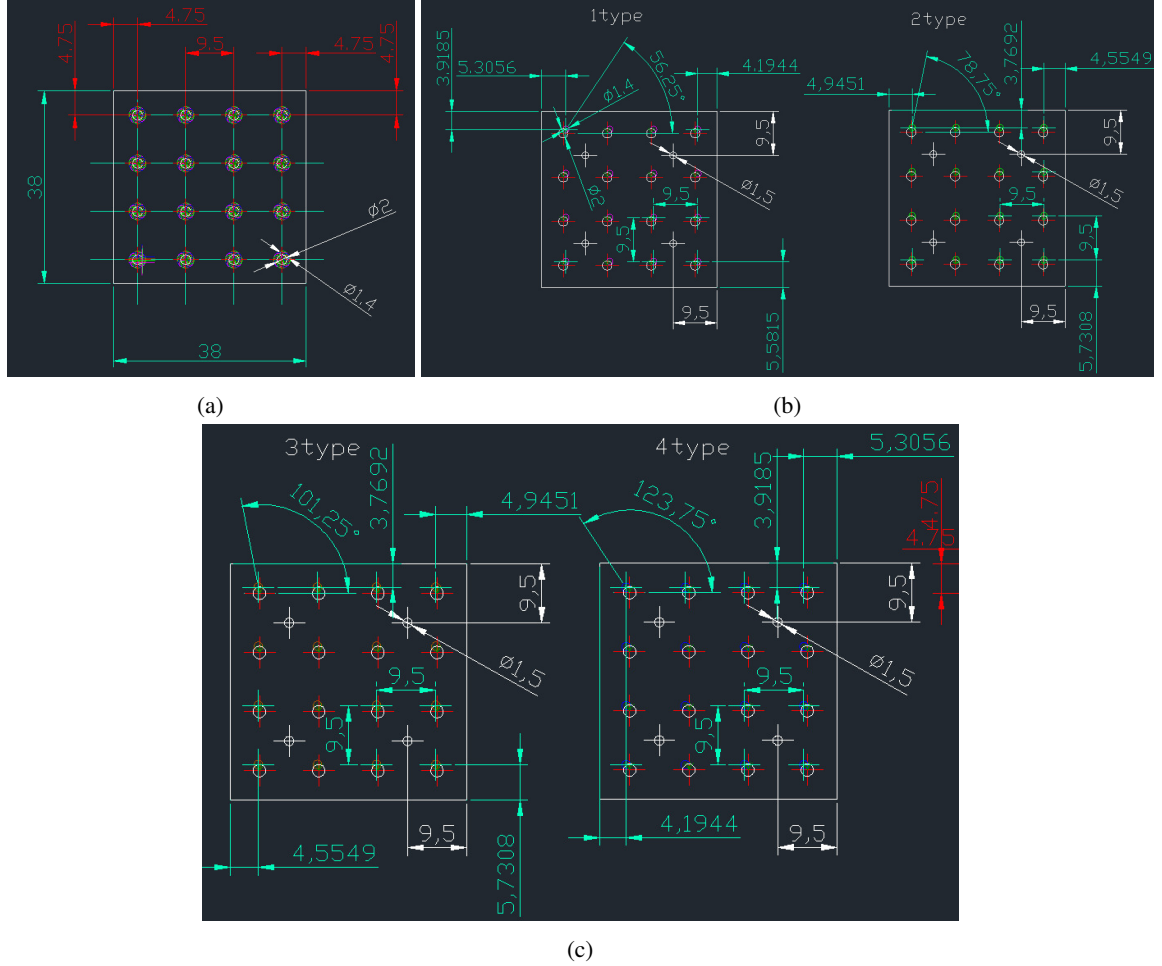


Figure 3.8: Technical drawings for the location of the rods and fibers in one shashlik cell. Shown is a projection overview for the full cell in (a) and the four different rotations, i.e. four different types of shashlik slices in (b) and (c). Technical drawings from Ref. [38].

One can see that four types of slices were manufactured. For each slice, holes were drilled to leave space for the rods and fibers. The rods are positioned in a straight line, while the fibers are spiraling around the rods. The reason being that incident photons or electrons/positrons, hitting the fibers would have the possibility to not interact with the absorber material, if the fibers were positioned in a straight line.

Electrons, positrons, or photons entering the calorimeter initiate an electromagnetic shower. Electrons and positrons primarily interact with the detector material via bremsstrahlung, emitting secondary photons and losing energy in the process. Photons, on the other hand, interact through

pair production, generating electron–positron pairs. These secondary particles can further interact with the material, producing a cascade of particles known as an electromagnetic shower. The depth and development of the shower depend on the energy of the initial particle and the properties of the detector material.

A photomultiplier is used at the downstream end of each calorimeter module to collect the created light and with it measure the deposited energy. More details on the working principle of calorimeters can be found in Ref. [39] and a more detailed description of the COMPASS calorimeters in Ref. [30].

3.2 General Purpose COMPASS Software

Apart from the software developed specifically for this thesis, we will be using three frameworks developed by COMPASS. The first one covers the simulation of the spectrometer, the second one covers the reconstruction of real COMPASS events and the MC events and the last one is used to analyze the output of the event reconstruction and perform event selections. All of these programs are written in C++.

Monte Carlo simulation framework (TGEANT) MC simulations will be covered in detail in Chapter 5. In the course of this work, we have made considerable improvements to the MC simulation framework TGEANT, such as massive speed improvements (see Sections 5.3.5 and 5.3.6) or implementation of missing detectors / detector parts (see Sections 5.3.1 to 5.3.3). In TGEANT, we simulate the detector geometry and do particle tracking with GEometry ANd Tracking 4 (GEANT4). The usual workflow, analyzed in this work, is the tracking of a beam particle to the COMPASS target, followed by an event generation, i.e. a forced interaction with a target particle, where a given final state is created. These final state particles are then tracked through the detector geometry. The output of TGEANT is a compressed file that includes generator (MC truth) information as well as simulated detector responses in form of energy deposits in the calorimeter cells or hits in the tracking detectors. In addition, one can produce geometry files in the form of ROOT geometries and detector tables. The former encapsulates the geometry as ROOT objects with extent and positioning. The latter is a custom COMPASS format, called detector.dat, where additional information such as unique names, calibration constants, wire positions and more is stored. It is used in Compass Reconstruction ALgorithm (CORAL) and TGEANT as an alignment file.

Event reconstruction with CORAL CORAL is used to reconstruct events, measured with the COMPASS spectrometer and MC events simulated with TGEANT. It uses a large database of calibration files and is operated by an option file, where calibrations for all different detector types, procedures for track fitting and much more can be customized. In addition, it needs the information about detector positioning from the ROOT geometry and detector.dat, which is produced by TGEANT. CORALs output is a large ROOT tree with event based information about trajectories, calorimeter clusters, detector hits and more, which can easily be read by PHysics Analysis Software Tools (PHAST). After CORAL, the only way to differentiate between COMPASS events and MC events is the MC flag per event in the ROOT tree. We call the produced files mini Data Storage Tree (mDST) files.

Event based analysis with PHAST PHAST provides an interface to implement user routines that intervene on an event level, where one can perform event selections or simply histogram certain

event based quantities. It also allows for refitting of trajectories or extrapolating of tracks, since the geometry is also saved in the mDST files, produced by CORAL. In Chapter 4, we will elaborate on the event selection performed with PHAST and have written a dedicated routine to automatically select final states of the form $3\pi\gamma\gamma + p$ [40]. We chose this particular signature, since the reaction under investigation is

$$\pi^- + p \rightarrow \eta^{(\prime)} + \pi^- + p, \quad (3.1a)$$

$$\eta^{(\prime)} \rightarrow \pi^- + \pi^+ + \eta(\pi^0), \quad (3.1b)$$

$$\eta(\pi^0) \rightarrow \gamma\gamma, \quad (3.1c)$$

and therefore ends up in the $3\pi\gamma\gamma + p$ final state. See Chapter 4 for more details.

Data Set and Event Selection

Throughout this work, we will analyze the reaction $\pi^- + p \rightarrow \eta^{(\prime)} + \pi^- + p$ with COMPASS data, taken in 2008 and 2009. This is the first analysis, where we use both data taking periods. Recently, we have improved on the calibration of the Electromagnetic CALorimeters 1 and 2 (ECALs), which we describe in Ref. [41]. In the now outdated Slot4 production of the data, the π^0 mass position was sometimes off by more than 10 MeV. For the new Slot5 production, we additionally corrected a trigger time shift in the early runs of 2008 in form of a new SciFi calibration and performed run-dependent calorimeter-cluster timing corrections, as well as a new calibration of the RPD [32]. More details about the effect of these corrections can be found in Refs. [27, 41]. All data, used in this work originates from the Slot5 production.

4.1 Preselection

COMPASS collected over 30×10^9 events in the data taking period under investigation. This is about 138 TB of tape data. Obviously, not every recorded event is a candidate for the reaction $\pi^- p \rightarrow \eta^{(\prime)} \pi^- p \rightarrow \pi^- \pi^- \pi^+ \gamma \gamma p$. There are many different interesting final states currently analyzed at COMPASS that have a similar signature as the one analyzed in this work [6, 8, 9, 27]. Therefore, COMPASS has performed some preselections that cover different final state signatures.

The preselection that is the starting point for this analysis is defined by the following cuts (see Ref. [42]):

- A best Primary Vertex (PV)¹ was found.
- The z position of the PV is between -200 cm and 160 cm (roughly the target region).
- Exactly three charged tracks are leaving the PV (not counting the recoil proton!).
- Charge sum of these tracks is -1 (ensures charge conservation).

This reduces the amount of tape data from about 138 TB to 4.7 TB, i.e. roughly 1×10^9 events, which is very useful in order to reduce computing time when performing the full event selection in

¹ A Primary Vertex is an interaction point with the beam particle and at least one other particle. Best refers to the PV with the lowest χ^2 , determined by the vertex fitting.

different final states. In addition, it allows us to copy the data to our local cluster in Bonn [43] and perform a second event selection, tailored to the $\eta^{(\prime)}\pi^-$ final state, there.

4.2 Selection of the $\eta\pi^-$ and $\eta'\pi^-$ Final States

We have released the event selection on Slot4 data in Ref. [44]. In this section, we will describe the selection procedure shortly and especially point to the changes made for the Slot5 production.

The selection is displayed in two different styles. The first one is the Waterfall-Style, where we apply one cut after the other in a fixed order. The second style we call Antok-Style since it is inspired by the program of the same name, developed in the hadron subgroup of COMPASS [45]. Here, for every cut, we perform all other cuts and then look at the impact of only the given cut. This gives a good hint on the actual importance of the given cut.

The selection is performed with PHAST [46]. PHAST is a C++ library that provides an interface to the reconstructed COMPASS events, where one may implement a so called `UserEvent` function that is processed for each event. We developed a set of classes and functions that perform the event selection and automatically save plots for each cut in Waterfall-Style and Antok-Style [40]. The same code is also used for the study of the full $3\pi\eta$ final state in Ref. [27]. In this chapter, we will show most of the plots for the $\eta'\pi^-$ final state only and in Antok-Style. In Appendix A, we document all plots for both final states in Antok-Style and Waterfall-Style.

4.2.1 Removing Problematic Data-Taking Periods

There was a detailed study by Simon Havemann that is documented in Ref. [44], where we made a run-by-run and even spill-by-spill scan on the Slot4 data and investigated if there are anomalies in certain runs compared to the full data sample, e.g. errors on specific detectors, trigger changes, missing sectors in the calorimeters, etc. Runs or spills that are identified as bad are completely cut from the data sample.

4.2.2 Selecting a Good Beam-Target Interaction

We require that one and exactly one PV was found (see Figs. 4.1(a) and A.3). In the preselection, at least one PV was required.

A valid beam time For each track, the relative time to the trigger is recorded (see Figs. 4.1(b) and A.4). We require this track time for the beam particle to be

$$-2.641 \text{ ns} < t_{\text{Beam}}(\eta\pi^-) < 2.601 \text{ ns} , \quad (4.1a)$$

$$-2.491 \text{ ns} < t_{\text{Beam}}(\eta'\pi^-) < 2.461 \text{ ns} , \quad (4.1b)$$

such that we do not include tracks from earlier or later events. More details on the determination of these values are given in Appendix A, specifically in Figs. A.1(a) and A.2(a).

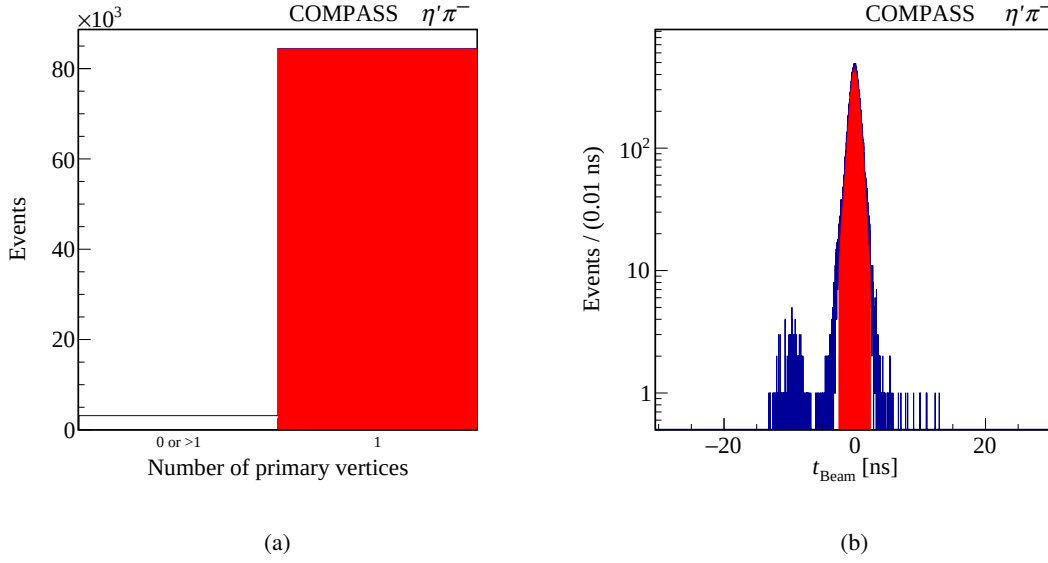


Figure 4.1: Requirement of exactly one PV (a) and the beam time distribution (b) for the $\eta'\pi^-$ final state in Antok-Style. The remaining events after the cut are shown in red.

Trigger The trigger logic is described in Section 3.1.3 in great detail. For the selection of the $\eta^{(\prime)}\pi^- + p$ final state, we require a recoil proton to be detected and all particles to be inside the COMPASS acceptance. Additionally, we require that the beam was properly identified upstream of the target, by the beam counter and SciFi detectors, and no hit in the beam killers downstream of the target was measured. This is reflected by the DT0 trigger. Therefore we only allow events where the DT0 trigger bit is set.

Primary Vertex position We require that the PV was found inside the target region, i.e. an interaction with a proton from the LH_2 target took place. We define $(\text{PV}_x, \text{PV}_y, \text{PV}_z)$ to be the space point of the PV and with that the following cut (see Eq. (4.2))

$$\sqrt{\text{PV}_x^2 + \text{PV}_y^2} < 1.55 \text{ cm (see Figs. 4.2(a) and A.5)}, \quad (4.2a)$$

$$-66 \text{ cm} \leq \text{PV}_z \leq -29 \text{ cm (see Figs. 4.2(b) and A.6)}. \quad (4.2b)$$

Charge conservation We have three charged particles in the final state, namely two π^- and one π^+ . Therefore, we need exactly three tracks leaving the PV² and their charge sum needs to be minus one. In the reconstruction, the charge of a particle can be extracted by two different methods.

1. Curvature of the track

² Note that the recoil proton is reconstructed in PHAST, while the vertexing is done in CORAL. Therefore, the recoil-proton track is not considered for any vertex.

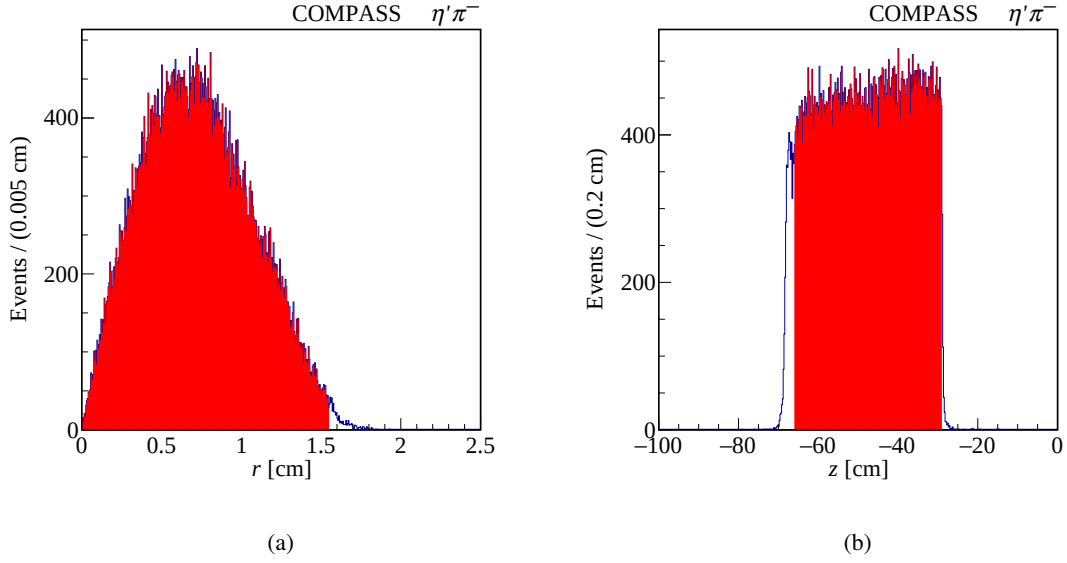


Figure 4.2: The radial distance of the PV to the z axis (a) and the z position of the PV (b) for the $\eta'\pi^-$ final state in Antok-Style. The remaining events after the cut are shown in red.

2. Curvature of the refitted track, where the PV is included as a track point

In cases where the track is almost straight, both methods might lead to different results. Such events are cut from the final event sample.

4.2.3 Selecting a π^0 or η from Two Photons

We measure the η and π^0 in the $\eta'\pi^-$ and $\eta\pi^-$ final state, respectively, in the decay to two photons. Consequently, we need to reconstruct events with exactly 2 photons in the final state. The main challenge is that photons do not leave tracks in the tracking detectors, due to them not being charged. We have to solely rely on the information of the ECALs. Here, major challenges apply, since there might be photon clusters measured in the ECALs that did not originate from the PV and are therefore not candidates for decay products of either the π^0 or η , such as conversion photons, bremsstrahlung photons, etc.

We test every reconstructed particle per event and accept those as good photon candidates that have a charge of 0. Furthermore, the associated cluster needs to be found in one of the ECALs and a valid cluster time needs to be set. In addition to these basic requirements, the cluster time (see Figs. 4.3(a) and A.7 for ECAL1 and Figs. 4.3(b) and A.8 for ECAL2) needs to be

$$-3.790 \text{ ns} < t_{\text{Cluster}}^{\text{ECAL1}} - t_{\text{Beam}} < 3.900 \text{ ns}, \quad (4.3a)$$

$$-2.593 \text{ ns} < t_{\text{Cluster}}^{\text{ECAL2}} - t_{\text{Beam}} < 2.595 \text{ ns}, \quad (4.3b)$$

for the $\eta\pi^-$ final state and

$$-2.821 \text{ ns} < t_{\text{Cluster}}^{\text{ECAL1}} - t_{\text{Beam}} < 2.880 \text{ ns}, \quad (4.4a)$$

$$-2.616 \text{ ns} < t_{\text{Cluster}}^{\text{ECAL2}} - t_{\text{Beam}} < 2.517 \text{ ns}, \quad (4.4b)$$

for the $\eta'\pi^-$ final state. The fits that were done to extract these ranges are shown in Figs. A.1(b), A.1(c), A.2(b) and A.2(c).

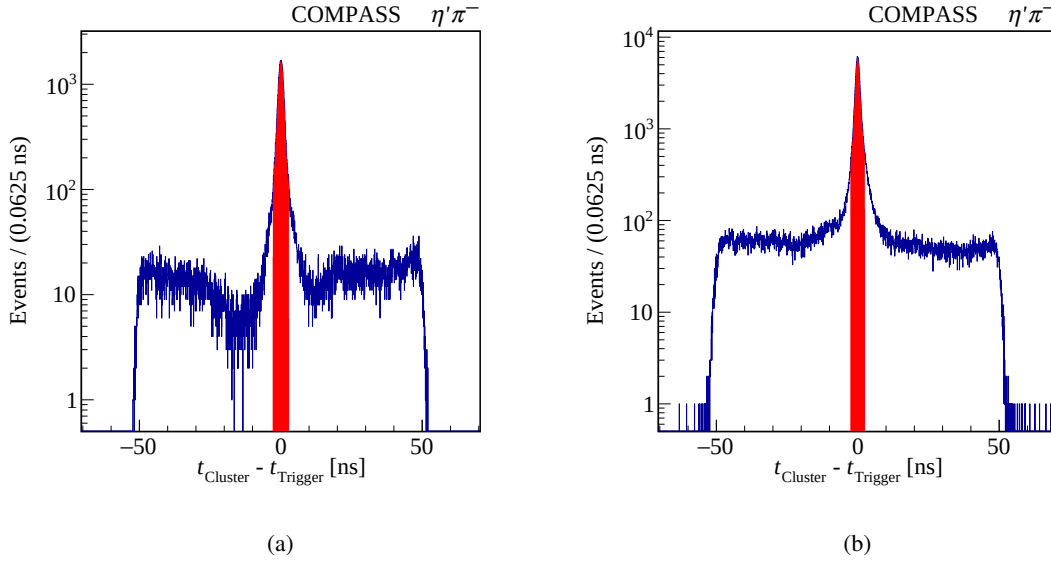


Figure 4.3: The time difference for clusters measured in ECAL1 (a) and ECAL2 (b) for the $\eta'\pi^-$ final state in Antok-Style. The remaining events after the cut are shown in red.

Also, the cluster position must not be in the Hadronic CALorimeter 1 (HCAL1) shadow region of the ECAL2. The shadow region is defined as all points in ECAL2 whose straight-line connection to any point in the target passes through the material of HCAL1, i.e. misses the hole in its center. Clusters in this area are removed from the final event sample. This cut is depicted in Figs. 4.4(a) and A.9.

There are still a lot of photons remaining per event, where most of them are low energetic. We performed a significance study in Ref. [41], which showed that the best cut-off values for photon energies are 1 GeV for ECAL1 photons and 4.2 GeV for ECAL2 photons. All photons below that threshold are considered secondary photons, i.e. not originating from the PV, or noise. After this, only events with exactly two good-photon candidates are kept. Additionally, we show in Figs. 4.4(b) and A.10 the different combinations of photon clusters per event, i.e. how many photons were found in which calorimeter per event. Due to the larger mass of the η , compared to the π^0 , we expect photons from the η decay to be produced with a larger opening angle to the z axis. Consequently, we should reconstruct more events with photons in ECAL1 for the $\eta'\pi^-$ final state. This is exactly what we see in Fig. A.10.

Now, we assume these two photons to originate from the PV, i.e. we assign four-momentum such that the three-momentum direction points from the PV to the cluster center:

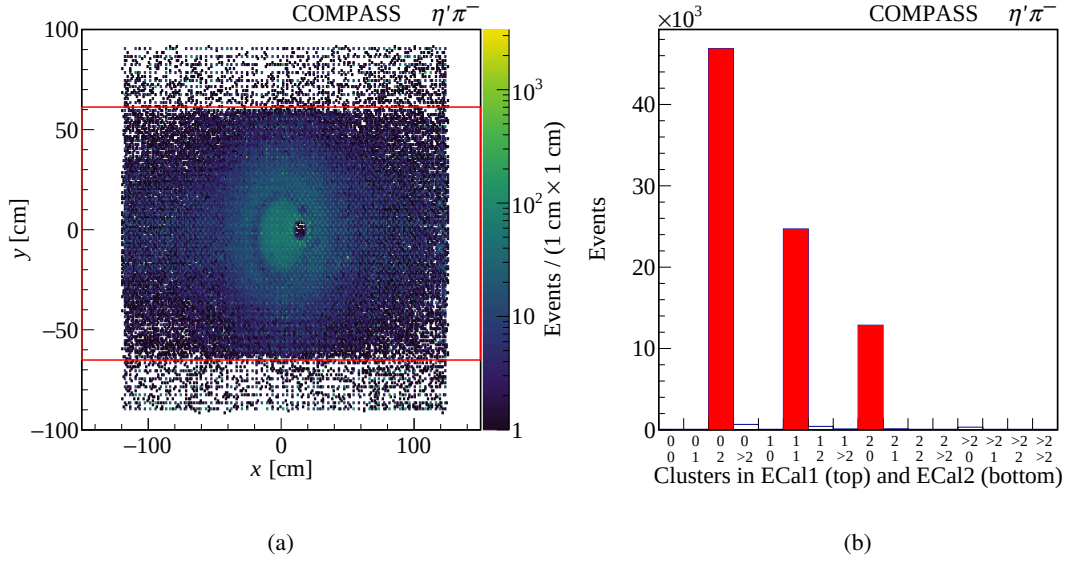


Figure 4.4: The ECAL2 cluster position (a), with cut ranges indicated in red lines and the distribution of number of photon clusters per event, split into the number of ECAL1 and ECAL2 clusters (b) for the $\eta'\pi^-$ final state in Antok-Style. In (b) the remaining events after the cut are shown in red.

$$E_\gamma = E_{\text{measured}}, \quad (4.5a)$$

$$\hat{p}_\gamma = \frac{\vec{p}_{\text{cluster}} - \vec{p}_{\text{PV}}}{|\vec{p}_{\text{cluster}} - \vec{p}_{\text{PV}}|}, \quad (4.5b)$$

$$\vec{p}_\gamma = E_\gamma \cdot \hat{p}_\gamma, \quad (4.5c)$$

where E_{measured} is the energy contained in the calorimeter cluster and $\vec{p}_{\text{cluster}}$ points to the center of gravity of the cluster at the most upstream point of the calorimeter cell. The two resulting photon four-momenta are then added up and become a π^0 or η candidate. Due to the magnitudes smaller decay width of π^0 or η , with respect to the invariant-mass peak width in Figs. 4.5 and A.11, we can conclude that the width of the invariant-mass peak originates from the detector resolution, i.e. the uncertainty of the energy measurement by the calorimeters. Therefore, we have one range per possible calorimeter combination and final state, since ECAL1 and ECAL2 have a different resolution. We then apply the cuts, tabulated in Table 4.1.

The fits, which were used to extract these cut ranges, are shown in Figs. A.1(d) to A.1(f) and A.2(d) to A.2(f). In Figs. 4.5 and A.11, we show the two-photon invariant mass spectrum for events, where we reconstruct both photons in ECAL1 (Figs. 4.5(a) and A.11(a)), both photons in ECAL2 (Figs. 4.5(b) and A.11(b)), one photon in ECAL1 and one in ECAL2 (Figs. 4.5(c) and A.11(c)) and all combinations (Figs. 4.5(d) and A.11(d)). Note that these plots are produced in Antok-Style, where no kinematic fit of the two-photon system and no cut on the $\eta^{(\prime)}\pi^-$ mass was applied. Exclusivity and coplanarity (see Section 4.2.5) however are applied on the final state with raw, not yet kinematically-fitted, photons. With this we get peaks with very little background, where the detector resolution is clearly visible.

Table 4.1: Cut ranges for the two-photon invariant mass.

Final state	Photons reconstructed in	Cut range
$\eta\pi^-$	ECAL1	$0.105 \text{ GeV} < m_{\gamma\gamma} < 0.163 \text{ GeV}$
$\eta\pi^-$	ECAL1 + ECAL2	$0.103 \text{ GeV} < m_{\gamma\gamma} < 0.167 \text{ GeV}$
$\eta\pi^-$	ECAL2	$0.121 \text{ GeV} < m_{\gamma\gamma} < 0.150 \text{ GeV}$
$\eta'\pi^-$	ECAL1	$0.477 \text{ GeV} < m_{\gamma\gamma} < 0.623 \text{ GeV}$
$\eta'\pi^-$	ECAL1 + ECAL2	$0.483 \text{ GeV} < m_{\gamma\gamma} < 0.615 \text{ GeV}$
$\eta'\pi^-$	ECAL2	$0.502 \text{ GeV} < m_{\gamma\gamma} < 0.589 \text{ GeV}$

We use the method of kinematic fitting [35, 44] to vary the energy of the reconstructed photons such that the resulting mass of the two-photon system matches the PDG values of the π^0 or η [15]. If not specified otherwise, the kinematically-fitted π^0 or η is used for all further tasks.

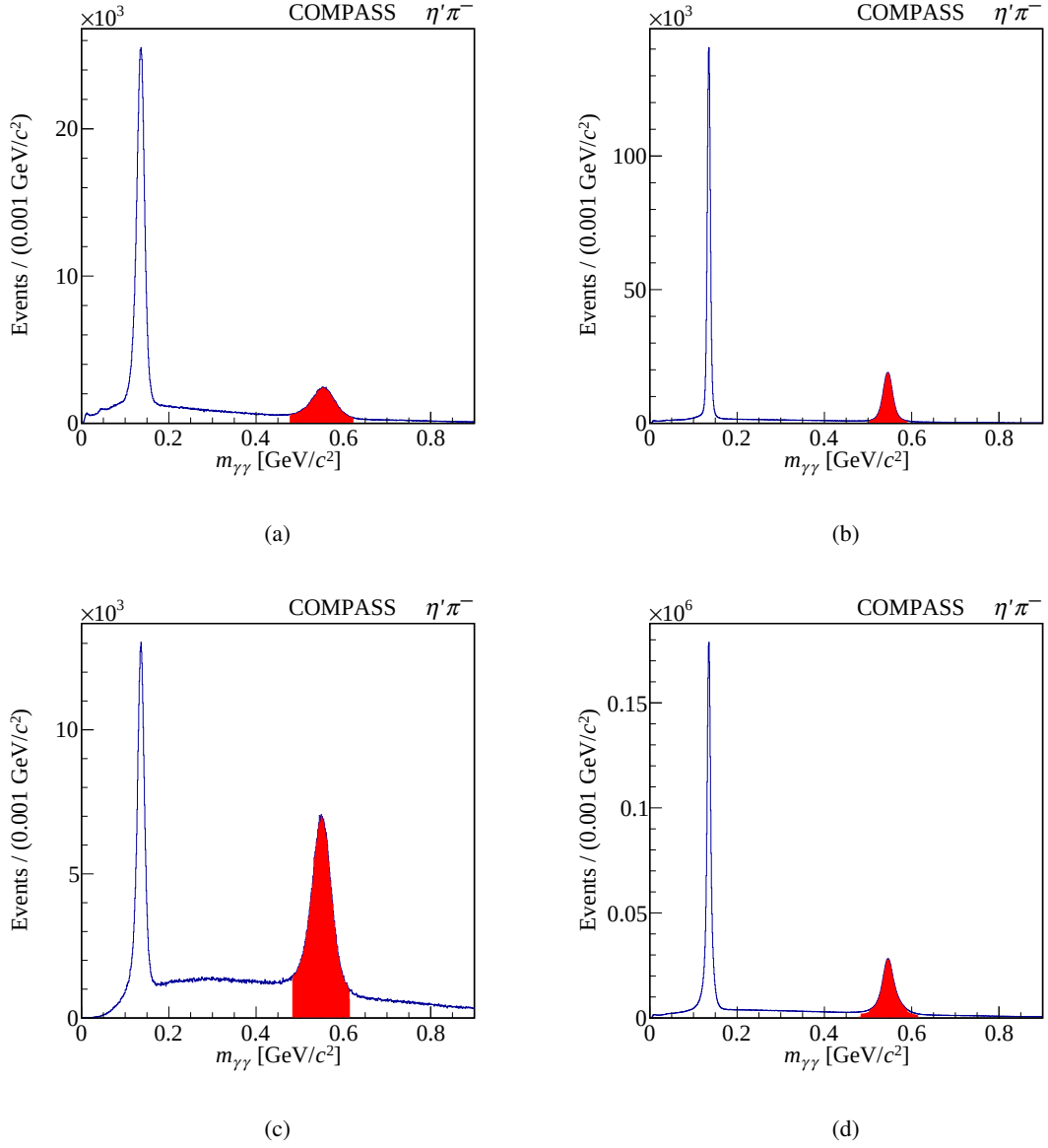


Figure 4.5: Two-photon invariant mass for the $\eta'\pi^-$ final state. In (a), both photons are reconstructed in ECAL1. In (b), both photons are reconstructed in ECAL2. In (c), one photon is reconstructed in ECAL1 and one in ECAL2. In (d), all combinations are shown together. The histograms are created in Antok-Style, where no kinematic fit of the two-photon system and no cut on the $\eta'\pi^-$ mass was applied. Exclusivity and coplanarity cuts are applied on the final state with raw, not yet kinematically-fitted, photons. The remaining events after the cut are shown in red.

4.2.4 The Recoil Proton

The layout of the RPD is described in Section 3.1.2. Throughout this work, we noticed that, often, events occur where one up- or downstream photomultiplier in Ring A or B of the RPD measured 2 signals per event, while the corresponding down- or upstream one did only measure one. This is most likely attributed to pile up, but results in more than one reconstructed recoil-proton track. These two tracks will therefore differ in the measured z position on the slab and the reconstructed z position of the interaction vertex. In Waterfall-Style (Figs. A.12(a), A.12(c), A.14(a) and A.14(c)), as well as in Antok-Style (Figs. 4.6(a), 4.6(c), A.13(a) and A.13(c)), we see a large number of events that have 2 or more recoil-proton tracks, where the track of the reconstructed recoil protons lies outside of the physical extension of the slabs, depicted by black lines.

Usual cut criteria are that we require exactly one good recoil-proton track reconstructed by the RPD. However, we need to define “good recoil-proton track” well. Originally, only a rough requirement on the recoil-proton reconstruction was imposed, where we required the reconstructed z position of the PV, seen by the RPD, to lie roughly inside the target. If there was more than one reconstructed recoil-proton track, the event was deselected. We introduce now a new procedure, where we require a good recoil-proton track to hit Ring A and B within its geometrical margins. After this we of course still require one good recoil-proton track per event. With this, the distributions change drastically (see Figs. A.12(b), A.12(d), A.14(b) and A.14(d) for the distributions in Waterfall-Style and Figs. 4.6(b), 4.6(d), A.13(b) and A.13(d) for the distributions in Antok-Style). In these figures, it becomes evident that events with 2 or more reconstructed recoil-proton tracks in reality only have one “good recoil-proton track”, i.e. some reconstructed recoil-proton tracks have hit positions outside of the geometrical extends of the rings. The cases where events with 2 or more recoil-proton tracks become events with one “good recoil-proton track” are directly gained as good $\eta^{(\prime)}\pi^-$ events in the event selection. Using this cut procedure, we select roughly 10 % more events. The new cut ranges are

$$-120 \text{ cm} < z_A < 20 \text{ cm} , \quad (4.6a)$$

$$-140 \text{ cm} < z_B < 70 \text{ cm} . \quad (4.6b)$$

4.2.5 Energy and Momentum Conservation

We require our $\eta^{(\prime)}\pi^-$ final state to be exclusively selected, i.e. the final state consists of exactly $\eta^{(\prime)}\pi^-$ and the recoil proton. To ensure this, we check that no energy and no momentum is missing from the final state. First, we check the conservation of momentum direction. For that, we define the coplanarity angle as

$$\Delta\phi_{\text{cop.}} = \angle(\vec{p}_{\eta^{(\prime)}\pi^-}^\perp, \vec{p}_{\text{Recoil}}^\perp) , \quad (4.7)$$

where $\perp$ means perpendicular to the beam axis. The beam axis deviates from event to event slightly, but is roughly given by the z -axis. If momentum direction is perfectly conserved, the recoil proton and the $\eta^{(\prime)}\pi^-$ system would be produced back to back in the plane perpendicular to the beam axis. The RPD has a limited resolution due to multiple scattering ($\sigma_{\text{MS}} = 0.06 \text{ rad}$) and due to the geometrical extent of the slabs (intrinsic resolution). This can best be seen at Fig. 3.4(d), where the 12 scintillators

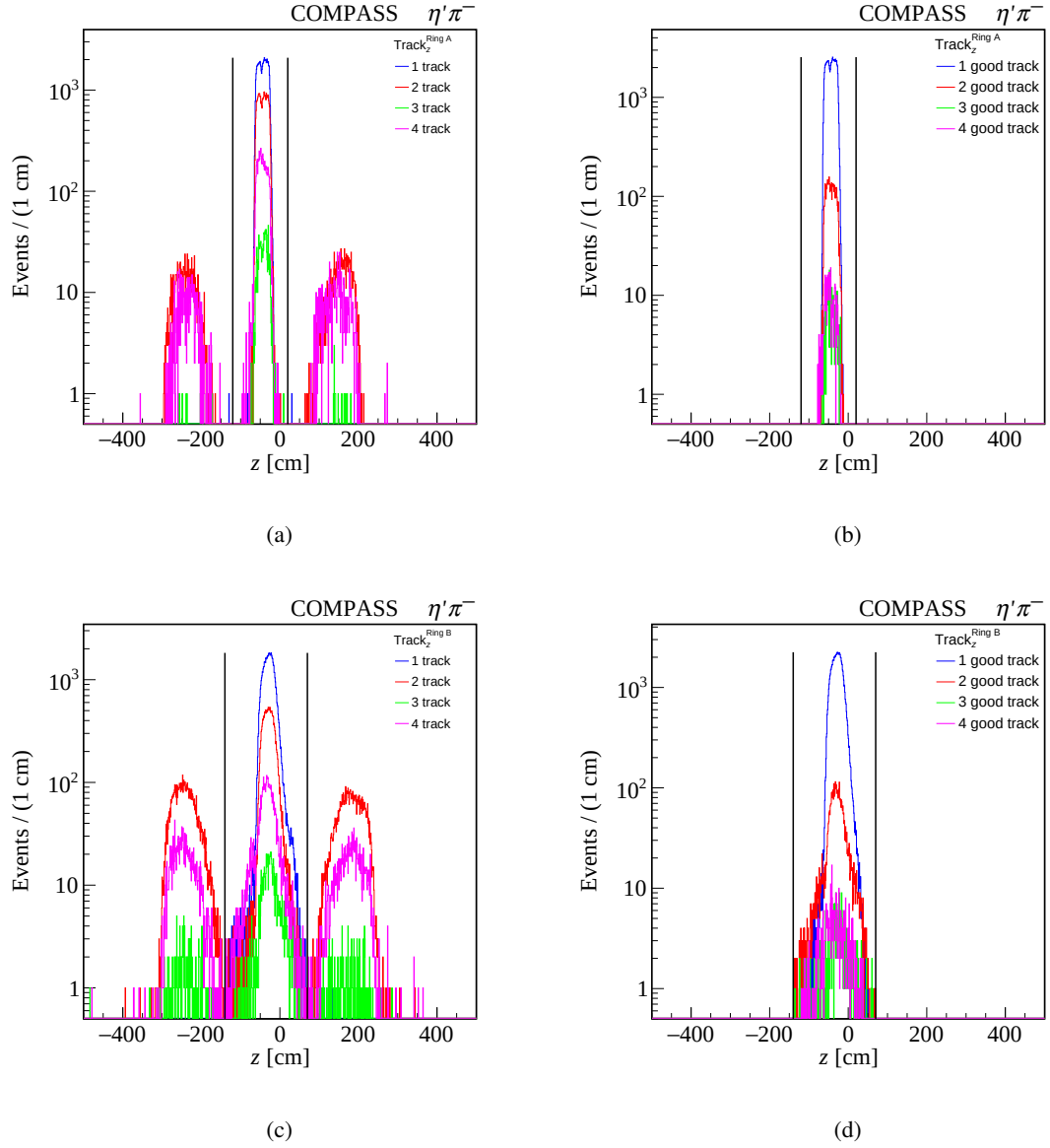


Figure 4.6: z position of the recoil-proton tracks for the $\eta'\pi^-$ final state at Ring A (a) and Ring B (c). In (b) and (d) the z position of good recoil-proton tracks, as defined in Eq. (4.6), is shown at Ring A and B, respectively. Cut ranges are indicated with black lines and all plots shown are in Antok-Style.

in Ring A and 24 scintillators in Ring B are clearly visible. In one scintillator, we cannot distinguish hits at the same z position, i.e. the transverse position of any hit is only as precise as the extent of the scintillator in that plane. Due to the trigger condition of the RPD, there are only two possibilities for any reconstructed track. Either, the recoil proton hits a scintillator in Ring B that is in line of that one in Ring A³, or one that is next to the one that is inline. From simple geometry, one can derive that the intrinsic resolution in the former case (we call this an even combination⁴) is $\sigma_{\text{int}} = 7.5^\circ$ and in the latter case (odd) is $\sigma_{\text{int}} = 3.5^\circ$. Combining this uncertainty yields

$$\sigma_{\phi_{\text{cop}}} = \sqrt{\sigma_{\text{MS}}^2 + \sigma_{\text{int}}^2} = \begin{cases} 143 \text{ mrad for even,} \\ 96 \text{ mrad for odd.} \end{cases} \quad (4.8)$$

We define

$$\phi_{\text{cop}} = (\Delta\phi_{\text{cop}} - \pi) \cdot \begin{cases} -1 & \text{for } \left(\vec{p}_{\eta^{(\prime)}\pi^-}^\perp \times \vec{p}_{\text{Recoil}}^\perp \right)_z > 0, \\ 1 & \text{for } \left(\vec{p}_{\eta^{(\prime)}\pi^-}^\perp \times \vec{p}_{\text{Recoil}}^\perp \right)_z < 0, \end{cases} \quad (4.9)$$

such that ϕ_{cop} is centered and symmetric around 0. From this resolution alone, it is clear that the four-momentum of the recoil proton is not reconstructed to a high precision with the RPD. At the analysis stage, we will therefore omit the measured value of the recoil proton four-momentum, and instead calculate it from the other final state particles. In Figs. 4.7(a) and A.15 we show the coplanarity angle for even and in Figs. 4.7(b) and A.16 for odd combinations.

The momentum direction of the beam is measured with the COMPASS spectrometer. However, we only roughly know the energy of the beam to be around 190 GeV. In order to fully reconstruct the beam four-momentum from measured quantities, we start from the squared four-momentum transfer

$$t = (p_{\text{Beam}} - p_{\eta^{(\prime)}\pi^-})^2 \quad (4.10a)$$

$$= m_{\text{Beam}}^2 + m_{\eta^{(\prime)}\pi^-}^2 - 2 \left(E_{\text{Beam}} E_{\eta^{(\prime)}\pi^-} - \left| \vec{p}_{\text{Beam}} \right| \left| \vec{p}_{\eta^{(\prime)}\pi^-} \right| \cos \theta_1 \right), \quad (4.10b)$$

$$t = (p_{\text{Target}} - p_{\text{Recoil}})^2 \quad (4.10c)$$

$$= m_{\text{Target}}^2 + m_{\text{Recoil}}^2 - 2 \left(E_{\text{Target}} E_{\text{Recoil}} - \left| \vec{p}_{\text{Target}} \right| \left| \vec{p}_{\text{Recoil}} \right| \cos \theta_2 \right). \quad (4.10d)$$

With a target at rest and energy conservation, we obtain

³ If we look at Fig. 3.4(d), this would be a track from the target through the green scintillator and the middle scintillator of the three red ones in Ring B.

⁴ Due to the numbering scheme of the RPD, in this case even-numbered slabs in Ring B are combined with one slab in Ring A for a recoil-proton track.

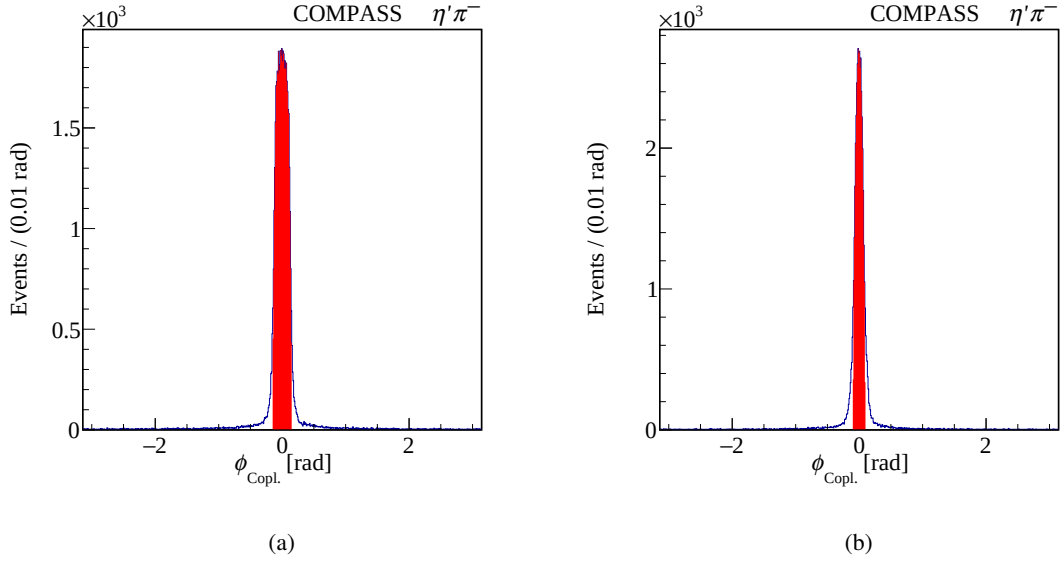


Figure 4.7: The coplanarity angle for an even slab combination in the RPD (a) and an odd one (b) for the $\eta'\pi^-$ final state in Antok-Style. The remaining events after the cut are shown in red.

$$\left| \vec{p}_{\text{Target}} \right| = 0, E_{\text{Target}} = m_{\text{Target}}, \quad (4.11a)$$

$$m_{\text{Beam}} = m_{\pi^-}, \quad (4.11b)$$

$$m_{\text{Target}} = m_{\text{Recoil}} = m_{\text{proton}} \equiv m_p, \quad (4.11c)$$

$$E_{\text{Beam}} + E_{\text{Target}} = E_{\eta^{(\prime)}\pi^-} + E_{\text{Recoil}} \quad (Eq. (4.11a)) \Leftrightarrow E_{\text{Recoil}} = m_p + E_{\text{Beam}} - E_{\eta^{(\prime)}\pi^-} \text{ and} \quad (4.11d)$$

$$\theta \equiv \theta_1, \quad (4.11e)$$

Inserting Eq. (4.11d) into Eq. (4.10d) and setting it equal to Eq. (4.10b) gives us

$$m_{\pi^-}^2 + m_{\eta^{(\prime)}\pi^-}^2 - 2E_{\text{Beam}}E_{\eta^{(\prime)}\pi^-} + 2\left|\vec{p}_{\text{Beam}}\right|\left|\vec{p}_{\eta^{(\prime)}\pi^-}\right|\cos\theta = 2m_pE_{\eta^{(\prime)}\pi^-} - 2m_pE_{\text{Beam}} \quad (4.12a)$$

$$\Leftrightarrow \underbrace{\left(m_{\pi^-}^2 + m_{\eta^{(\prime)}\pi^-}^2 - 2m_pE_{\eta^{(\prime)}\pi^-}\right)}_{\equiv a} + \underbrace{\left(2\left|\vec{p}_{\eta^{(\prime)}\pi^-}\right|\cos\theta\right)\left|\vec{p}_{\text{Beam}}\right|}_{\equiv b} = \underbrace{\left(2E_{\eta^{(\prime)}\pi^-} - 2m_p\right)E_{\text{Beam}}}_{\equiv c}. \quad (4.12b)$$

Now we can square that equation and insert

$$E_{\text{Beam}}^2 = m_{\pi^-}^2 + \left|\vec{p}_{\text{Beam}}\right|^2, \quad (4.13)$$

to get

$$a^2 + 2ab|\vec{p}_{\text{Beam}}| + b^2|\vec{p}_{\text{Beam}}|^2 = c^2 \left(m_{\pi^-}^2 + |\vec{p}_{\text{Beam}}|^2 \right) \quad (4.14a)$$

$$\Leftrightarrow 0 = \left(a^2 - c^2 m_{\pi^-}^2 \right) + 2ab|\vec{p}_{\text{Beam}}| + \left(b^2 - c^2 \right) |\vec{p}_{\text{Beam}}|^2 \quad (4.14b)$$

$$= |\vec{p}_{\text{Beam}}|^2 + \frac{2ab}{b^2 - c^2} |\vec{p}_{\text{Beam}}| + \frac{a^2 - c^2 m_{\pi^-}^2}{b^2 - c^2} \quad (4.14c)$$

$$\Leftrightarrow |\vec{p}_{\text{Beam}}| = -\frac{ab}{b^2 - c^2} \pm \sqrt{\left(\frac{ab}{b^2 - c^2} \right)^2 - \frac{a^2 - c^2 m_{\pi^-}^2}{b^2 - c^2}} \quad (4.14d)$$

$$= -\frac{ab}{b^2 - c^2} \pm \sqrt{\frac{a^2 b^2 - (b^2 - c^2)(a^2 - c^2 m_{\pi^-}^2)}{(b^2 - c^2)^2}} \quad (4.14e)$$

$$= \frac{1}{c^2 - b^2} \left(ab \mp \sqrt{a^2 b^2 - (b^2 - c^2)(a^2 - c^2 m_{\pi^-}^2)} \right) \quad (4.14f)$$

$$= \frac{1}{c^2 - b^2} \left(ab \mp c \sqrt{a^2 - m_{\pi^-}^2 (c^2 - b^2)} \right). \quad (4.14g)$$

In order to figure out, if the + or – solution is the physical one, we can approximate $m_{\pi^-}^2 (c^2 - b^2) \ll a^2$ and $E_{\text{Beam}} = |\vec{p}_{\text{Beam}}|$. In this approximation we get from Eq. (4.12b)

$$a + b|\vec{p}_{\text{Beam}}| = c|\vec{p}_{\text{Beam}}| \Leftrightarrow |\vec{p}_{\text{Beam}}| = \frac{a}{c - b}, \quad (4.15)$$

and from Eq. (4.14g)

$$|\vec{p}_{\text{Beam}}| = a \frac{b \mp c}{(c + b)(c - b)}, \quad (4.16a)$$

$$|\vec{p}_{\text{Beam}}|_- = \frac{a}{c + b}, \quad (4.16b)$$

$$|\vec{p}_{\text{Beam}}|_+ = \frac{a}{c - b}. \quad (4.16c)$$

Therefore,

$$|\vec{p}_{\text{Beam}}| = |\vec{p}_{\text{Beam}}|_+ = \frac{1}{c^2 - b^2} \left(ab + c \sqrt{a^2 - m_{\pi^-}^2 (c^2 - b^2)} \right), \quad (4.17)$$

is the physical solution. Finally, the four-momentum of the beam only depends on particle masses and the four-momentum of the $\eta^{(\prime)}\pi^-$ system, which is reconstructed with high accuracy.

Now, we can histogram the reconstructed beam energy (see Figs. 4.8(a) and A.17) and use a phenomenological fit, commonly used at COMPASS [8], to describe the distribution (see Eq. (4.18)).

$$f(E_{\text{Beam}}) = \frac{A}{\sqrt{2\pi}\sigma^2} \exp\left\{-\frac{1}{2} \frac{(E_{\text{Beam}} - \mu)^2}{\sigma^2}\right\} + \left[\frac{\pi}{2} - \arctan(p_2(E_{\text{Beam}} - \mu))\right] \cdot \left(\sum_{i=0}^1 p_i E_{\text{Beam}}^i\right), \quad (4.18)$$

with fit parameters A , σ , μ , p_0 , p_1 and p_2 . The final cut ranges are given in Eq. (4.19).

$$187.005 \text{ GeV} < E_{\text{Beam}}(\eta\pi^-) < 196.352 \text{ GeV}, \quad (4.19a)$$

$$186.674 \text{ GeV} < E_{\text{Beam}}(\eta'\pi^-) < 196.436 \text{ GeV}. \quad (4.19b)$$

The fit can be found in Fig. A.1(g) for $\eta\pi^-$ and Fig. A.2(g) for $\eta'\pi^-$.

In addition to the traditional exclusivity cut, we introduce an anti-exclusivity cut, where we check that the combination of all charged final-state particles, i.e. the final state without the two photons, does not survive the coplanarity and exclusivity cuts described in this section. Fortunately, this cut does not remove many events by its own (see Figs. 4.8(b) and A.18).

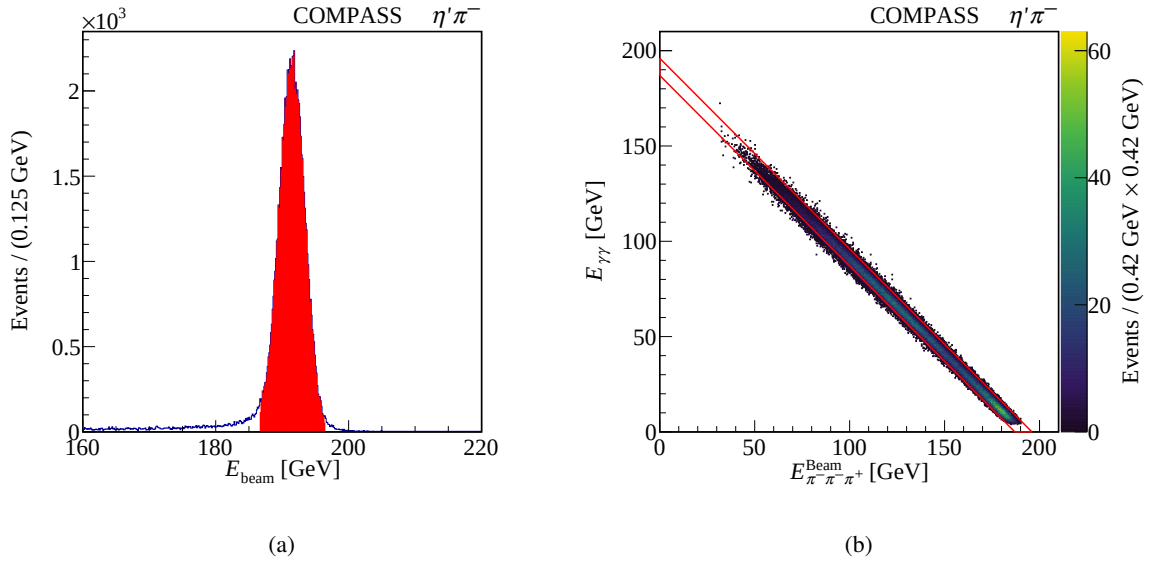


Figure 4.8: The exclusivity in form of the reconstructed beam energy in (a) and the anti-exclusivity in a two dimensional representation of the beam energy without the two photons in (b) for the $\eta'\pi^-$ final state in Antok-Style. In (a) the remaining events after the cut are shown in red. In (b) the two dimensional cut ranges are given with red lines.

4.2.6 Particle Identification

The 190 GeV negative hadron beam, delivered by the M2 beamline for the data analyzed throughout this work, consists of pions (96.8 %), kaons (2.4 %) and antiprotons (0.8 %) [30]. We are only interested in events with a negative pion beam. The particle species of the beam can be determined with two CEDAR detectors. They operate on the Cherenkov principle to distinguish pions, kaons and antiprotons. In Ref. [8], a detailed likelihood fit method was developed to ensure large confidence of identifying beam kaons. The method determines one likelihood value per CEDAR detector and particle species. We accept an event if

$$R_{\text{CEDAR}} = \log_{10} \frac{L_1(K) \cdot L_2(K)}{L_1(\pi) \cdot L_2(\pi)} < 0, \quad (4.20)$$

where L_1 and L_2 are likelihoods for CEDAR 1 and 2, respectively. A detailed explanation of the reconstructed likelihoods and the exact cut-off values is discussed in Refs. [8, 47].

Similar to the initial-state particle identification, we perform a final-state particle identification in addition. The RICH detector operates on the Cherenkov principle, with a limited acceptance. In this work, we use the RICH only as a cross check that no final-state kaon is created. For each charged final-state particle (except the recoil proton), we check the likelihood ratio

$$R_{\text{RICH}} = \frac{\mathcal{L}(K)}{\max \mathcal{L}(S)} \quad \text{with} \quad S = \pi, p \text{ and background.} \quad (4.21)$$

We do not accept events where at least one particle has $R_{\text{RICH}} > 1.15$ [8]. Due to the limited acceptance of the RICH, we can only check particles with $10 \text{ GeV} < |\vec{p}| < 60 \text{ GeV}$, $\theta < 0.25$ and $\sqrt{p_x^2 + p_y^2} > 5 \text{ cm}$. More details on these numbers can be found in Refs. [8, 44]. In Fig. A.19 and Fig. A.20, we see that the Kaons are nicely cut out in the Waterfall-Style. In Antok-Style, the Kaon band is not visible even before the cut (see Fig. 4.9).

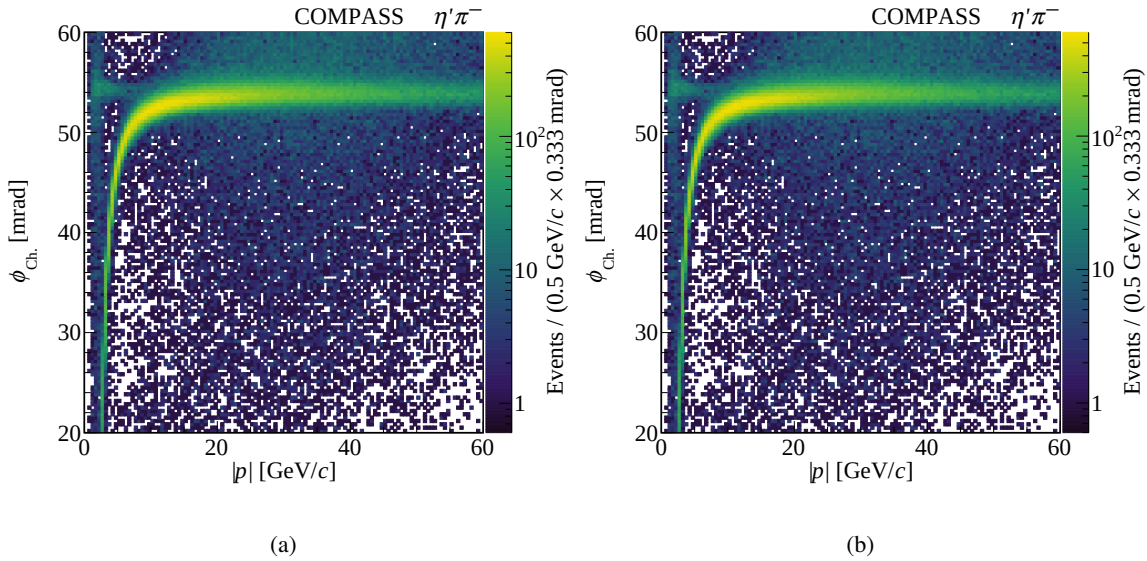


Figure 4.9: The Cherenkov angle of all charged particles in the $\eta'\pi^-$ final state, measured by the RICH vs. the absolute particle momentum before the likelihood cut (a) and after (b) in Antok-Style.

4.2.7 Final-State Mass

Lastly, we cut on the $\pi^+\pi^-\gamma\gamma$ mass, selecting exactly one $\eta^{(\prime)}$. There are two π^- in the final state and we have to determine, which combination is used for the $\eta^{(\prime)}$ in the final state. If both combinations are valid, we randomly select one of the combinations. The $\pi^+\pi^-\gamma\gamma$ mass is shown in Fig. 4.10, where we clearly see the $\eta^{(\prime)}$ peak. The cut ranges given in Eq. (4.22)

$$0.537346 \text{ GeV} < E_{\text{Beam}}(\eta\pi^-) < 0.559198 \text{ GeV}, \quad (4.22a)$$

$$0.943406 \text{ GeV} < E_{\text{Beam}}(\eta'\pi^-) < 0.97291 \text{ GeV}. \quad (4.22b)$$

and the fits to determine this cut ranges are given in Figs. A.1(h) and A.2(h). In addition, we see an ω peak in the $\eta\pi^-$ final state and an $f_1(1285)$ peak in the $\eta'\pi^-$ final state. Both peaks lie on top of the combinatorial background, i.e. the $\pi^+\pi^-\gamma\gamma$ mass with the wrong π^- . Note that the reaction $\pi^-p \rightarrow \pi^-\eta$ conserves G-Parity. However, $\pi^-p \rightarrow \pi^-\omega$ does not. The quantum numbers of all involved particles are given in Table 4.2. In order to conserve G-Parity, we would need an additional π^0 in the final state. This reaction is comprehensively studied in Ref. [9]. The fact that we still see an $\omega(782)$ in the $\pi^+\pi^-\gamma\gamma$ spectrum tells us that an additional π^0 need to be present for the given events that was not seen by the spectrometer. This however only effects events inside the $\omega(782)$ peak and is of no concern for the work in this thesis.

Table 4.2: Quantum numbers for the particles that are visible in Fig. 4.10(a).

	p	$\pi^\pm$	π^0	η	$\omega(782)$
$I^{(G)}$	$\frac{1}{2}$	1^-	1^-	0^+	0^-
$J^{P(C)}$	$\frac{1}{2}^+$	0^-	0^{-+}	0^{-+}	1^{--}

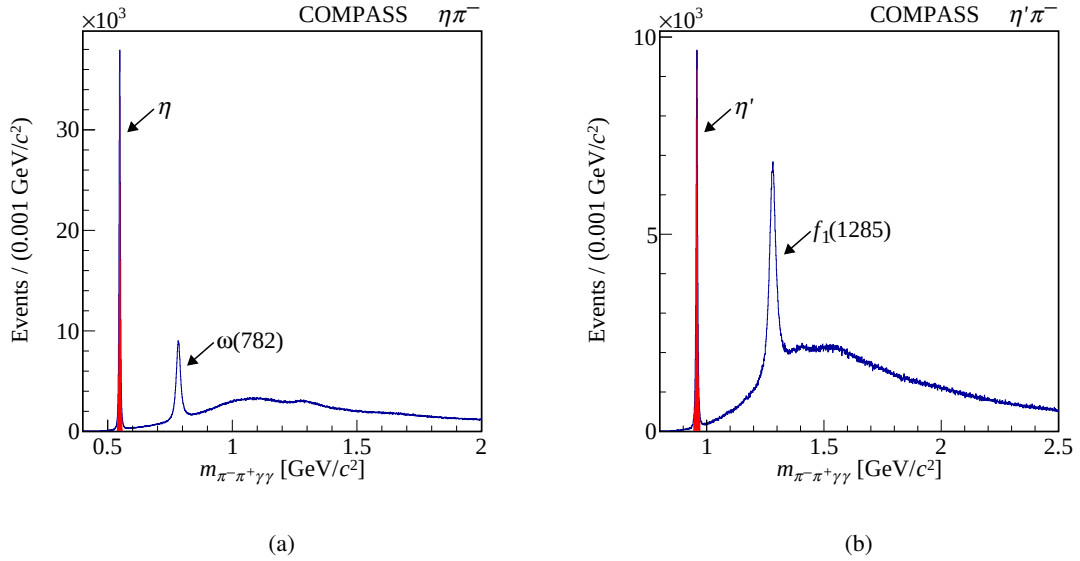


Figure 4.10: The mass of the $\pi^-\pi^+\gamma\gamma$ system for the $\eta\pi^-$ (a) and $\eta'\pi^-$ (b) final state, in Antok-Style. The remaining events after the cut are shown in red. The histograms are filled twice, since there are two π^- in the final state and it is a priori unknown which one contributed to the $\eta^{(\prime)}$ system.

4.2.8 Impact of the Different Selection Stages

In Table 4.3 we document the impact of the different cuts in Waterfall-Style for both final states. The event selection was cross checked for the full data set by Philipp Haas [44] and is documented in Appendix A.

Table 4.3: The number of events in the $\eta^{(\prime)}\pi^-$ final state left after each cut in Waterfall-Style.

Cut name	$\eta\pi^-$	$\eta'\pi^-$
After preselection	971 996 615	971 996 615
Bad run	914 723 154	914 723 154
Exactly one PV	852 766 288	852 766 288
Trigger	834 817 477	834 817 477
PV radius	824 193 368	824 193 368
PV z position	764 245 776	764 245 776
Three charged particles	764 245 776	764 245 776
Charge conservation	764 088 739	764 088 739
Beam time	748 087 212	745 174 038
Two photons	114 094 784	114 238 497
Recoil proton	97 522 305	97 635 187
Anti exclusivity	88 461 165	88 594 283
RICH	82 124 460	82 246 338
CEDAR	78 905 881	79 027 754
Two-photon invariant mass	20 163 587	6 288 601
Kinematic fit	20 150 848	6 283 048
Coplanarity	4 516 343	2 049 211
Exclusivity	2 290 913	1 294 792
$\pi^+\pi^-\gamma\gamma$ invariant mass	268 009	84 429

Monte Carlo Simulations at COMPASS

No detector is perfect. It has limitations due to size, holding structures, electronics or inefficient areas. Nevertheless, we try to build the detectors with as little inaccuracies as possible. Still, there are some challenges that are hard to overcome. Particles with a very small opening angle, i.e. flying very close to the beam axis, are hard to differentiate from beam particles that have not interacted with the target at all. In contrast to that, particles with large opening angles might not hit the spectrometer detectors at all. Then there is of course always the possibility of a particle hitting some holding structure or similar of a detector, preventing the detector to get a signal for that particle or worse, introducing secondary particles that need to be detected and might introduce structures that are not or wrongly understood. At COMPASS, we try to cover the small and large angle problem with so called veto detectors. These are usually scintillators that are placed such that particles that fly outside of the acceptance of the other spectrometer detectors leave a signal in them. Events, where we collect hits in one or more veto detectors are then discarded on the trigger level.

The concept of what the spectrometer can detect and what not is a convolution of the detector acceptance and their efficiency. In order to perform any analysis with data taken from the spectrometer, this needs to be understood first. Since the acceptance can not be determined analytically, it is common practice to tackle it with MC simulations. We simulate the COMPASS spectrometer and randomly sample the phase space until we have enough statistics to ensure that we cover the full realm of possibilities. It is of course not clear when this point is reached and we will show later how we determine that we collected enough MC statistics.

5.1 Software Used for the Simulations

This work is not the first that is performing MC simulations for the COMPASS spectrometer. There are two established MC simulation frameworks that are both in use to date and maintained by the COMPASS collaboration. We will describe both of them shortly and discuss what framework shall be used in this work.

The original MC simulation framework is COMPASS GEometry AND Tracking (COMGEANT) [48]. It is based on GEometry AND Tracking 3 (GEANT3) [49], which was developed by CERN and its purpose is to simulate particle movement and interaction in matter. It is written in FORTRAN. Researchers all over the world move to C++, Python or other languages. Consequently, COMGEANT

is no longer maintained. It served as the MC simulation framework for, for example, the analyses of the $\pi^- \pi^- \pi^+ + p$ final state [6, 7], the $K^- \pi^- \pi^+ + p$ final state [8], and the original $\eta^{(\prime)} \pi^-$ partial-wave decomposition [5]. A disadvantage of COMGEANT is that the COMPASS calorimeters are only described by an effective material. For this work, it is essential to simulate the interaction of the final state photons accurately.

All the points above lead us to the decision to use the new COMPASS simulation framework TGEANT. It is based on GEANT4 [50], which is the successor of GEANT3 and completely based on C++. GEANT4 is maintained and developed to this date and ideally suited for detailed detector descriptions and particle simulations. In addition, it is commonly used by almost all CERN experiments as backend for MC simulations. The detector implementation of TGEANT aims to be more detailed as that of COMGEANT. Across this chapter, we will discuss major differences and advantages of TGEANT. It was developed originally for a different physics program of COMPASS and is the standard simulation framework for all data taking periods of COMPASS after 2012. Prior to this work, most of the detectors, present in the 2008 and 2009 setup, were already implemented in TGEANT. However, the setup was never used for an analysis of the 2008 and 2009 data. In addition, some key detectors were described inaccurately or were even missing in TGEANT for the 2008 and 2009 setup. Throughout this work, TGEANT was updated to describe the full 2008 and 2009 setup.

5.2 Working Principle of GEANT4

GEANT4, similar to GEANT3, is a library of physics process, materials, fundamental particles and more. It is the foundation for simulating the COMPASS spectrometer and particle interaction within it. It also provides the infrastructure for an event-by-event simulation, storage of the detector response, MC truth information and user interaction.

First, we will discuss how to implement detectors in GEANT4. Here, the concept of logical volumes and solids is used. A logical volume defines material properties, magnetic fields and sensitivity, meaning if deposited energy should be saved or not. This mechanism is for example used in calorimeters, where we explicitly collect energy deposited in the scintillators. On the other hand, one might have absorber materials where we are not interested in the deposited energy. These detectors are then not sensitive. The logical volume is connected to a solid which defines the physical space that the logical volume should take up. There are a lot of different predefined solids in GEANT4, e.g. tubes, boxes or trapezoids, and in addition, there is the possibility to define your own solids. One can make unions of solids, intersections or define polygon borders that form complicated shapes. There are many possibilities to construct the physical extent of the detector, which comes with certain disadvantages or advantages. Describing the detector as detailed as possible might slow down the simulation, while an effective description might not be accurate enough. Usually, we want to be fast enough so that we can simulate enough statistics in a reasonable time frame while being as accurate as possible in describing the different detectors. Building detectors as a huge number of intersections and unions has the disadvantage that the GEANT4 kernel may need to calculate many border effects. It might be beneficial to describe the detectors as a polygon instead. In addition, there are certain tricks to describe repetitive structures of the detectors. We will report on this more in Section 5.3.5.

Once, the logical volume is fully described, it may be placed in a physical volume. This is the step, where the volume becomes active for the detector simulation. During this step, it might also be rotated and translated in space. Usually, one starts with a big volume, the so called world volume. Here, we

place all our different detectors. With this logic, one defines smaller simple volumes that will easily hold single detectors. In them, we build the detectors themselves with their complicated structures. This means that one can layer logical volumes as much as one wants.

As an example, one can create a box like world volume and fill it with air. Then we construct four smaller boxes, where we have a tracking detector in the first three boxes and a complicated calorimeter structure in the last one. Now, we place the boxes after each other with some distance in between them in the world volume. With the tracking detectors, we are able to measure a charged particle track and with the calorimeter the energy deposition of a photon, electron or positron. This setup would be nicely suited to simulate a single electron traversing through space, measuring its trajectory and energy. If we add a magnet to the setup, we could extract the momentum of the particle via the curvature of the track and the energy with the calorimeter. This strategy can be explored for all different detectors, holding structures, magnets, etc., with the advantage that one can develop every detector individually and care about rotation and placement in the end.

GEANT4 comes with an overlap checker that may be executed. It finds geometrical overlaps between logical volumes. It is important to avoid overlaps since the simulation does not know what volume to take if there is no hierarchy between them. If logical volumes are layered, the most internal layer defines the material properties of course.

Now that the basic idea of how to construct a detector in GEANT4 was explained, we address how particle interaction and event-by-event simulation is performed.

GEANT4 works with a so called run manager as its main operating force. It is a global object that coordinates the simulation. One has to connect at least four different features to the run manager in order for GEANT4 to work.

1. The detector construction must inherit from `G4UserDetectorConstruction` and override the `Construct` function. In the `Construct` function, the full spectrometer geometry will be constructed.
2. In the physics list, we define all processes, particle properties and production cuts. A production cut defines the threshold for the creation of secondary particles, for example during an electromagnetic shower. If a secondary's expected range is shorter than this threshold, the particle is not generated explicitly, and its energy is deposited locally in the medium. Every particle type, e.g. photon, electron, muon etc., has its own processes attached. It is clear that a photon interacts differently with matter, compared to an electron. The GEANT4 philosophy is that each particle is stepped through matter. The step length usually depends on current volume conditions, i.e. the GEANT4 kernel may use larger steps when there is a large block of material and smaller steps when there are many material borders. In reality it is more complicated than that but this is the general idea. There are three different kinds of processes: A process before the step starts, one along the step and one after the step.
3. The event manager, which inherits from `G4UserEventAction`, defines actions that may be executed before the event starts and after the event is done. This is useful for initialising properties or saving all event information in the end.
4. The primary generator that inherits from `G4VUserPrimaryGeneratorAction`, determines how the event should be initialized, i.e. how it is generated, what particles will be used, etc.

Secondary interactions are treated by GEANT4 itself with respect to the chosen physics list.

5.3 Simulating the 2008 and 2009 Setup

In this section, we will describe the working principle and philosophy of TGEANT. After that, we move to specifics for the 2008 / 2009 hadron setup and how they were implemented throughout this work.

TGEANT is completely written in C++ and comes with a handy Graphical User Interface (GUI) where one can control the main aspects of the simulation. Fig. 5.1 shows the **General** tab of the GUI, where one can choose the main layout of the simulation. Most of the configurations are set once and stay constant for basically any simulation done in this thesis.

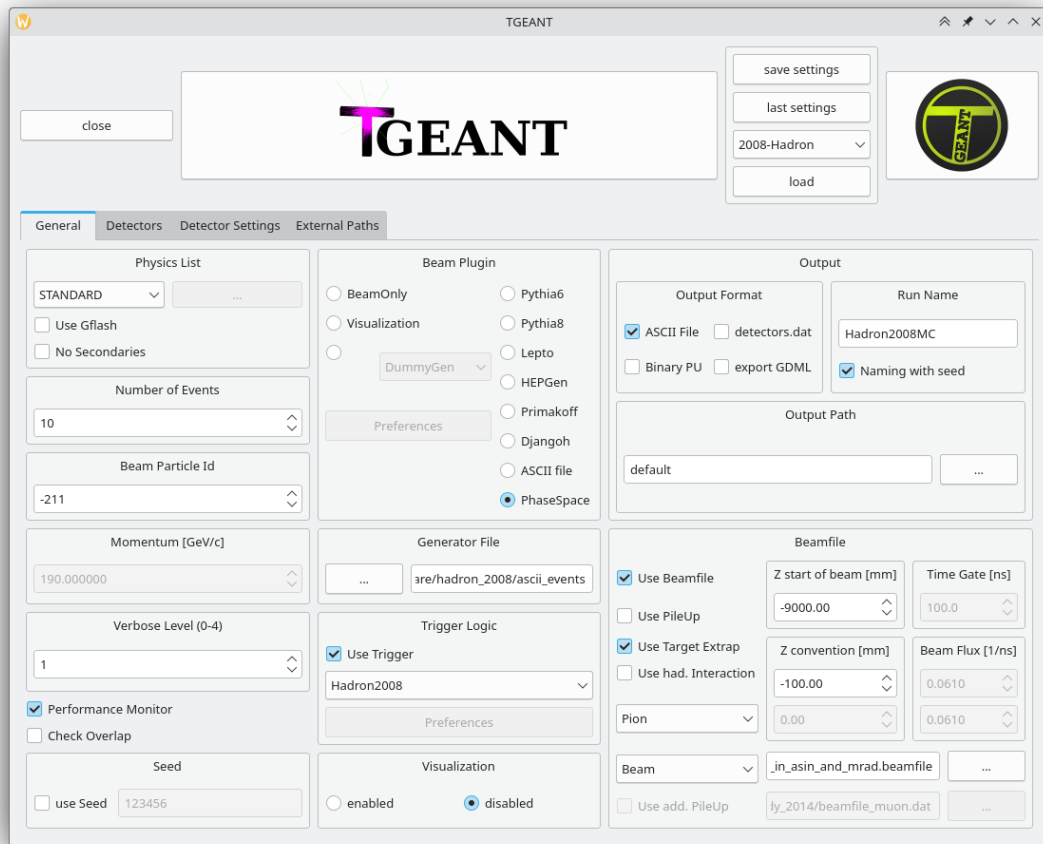


Figure 5.1: Snapshot of the TGEANT GUI. Displayed is the **General** tab, where the layout of the simulation is defined.

The GUI has a box-like structure, where aspects of the simulations can be modified. In the following, we will describe the boxes that are used in the simulations, conducted for this work.

Physics List

One may choose from three different predefined physics lists. **EM_ONLY**, **STANDARD** and **FULL**.

Usually we choose the `STANDARD` physics list, which describes all relevant processes and interactions of the particles involved in the simulation. In edge cases, where we only want to simulate electromagnetic interaction, we can choose the `EM_ONLY` physics list. The `FULL` physics list also includes propagation of optical photons, which is only relevant for an accurate `RICH` simulation and which is not yet fully implemented. In addition, one can switch on / off `GFlash` and the production of secondary particles, i.e. no particle created by the `GEANT4` kernel is simulated. This option should never be checked for our simulations. `GFlash` will be discussed in more detail in Section 5.3.5.

Beam Particle Id

One may choose the beam particle according to the PDG MC numbering scheme [15]. Negatively charged particles get a minus in front of the number. -211 from Fig. 5.1 translates to a π^- .

Seed

Setting a specific seed for the random engines at the start of the simulations might be useful to get reproducible results and for debugging. For the final simulation, a random seed is chosen.

Beam Plugin

The beam plugin is the `TGEANT` way of choosing the event generator of the simulation. Throughout this work and together with [10, 51] we developed our own generator, called `phasespace` in the GUI, which is based on `TGenPhaseSpace` from `ROOT`. This is described in more detail in Section 5.4.2.

Generator File

Some event generators are steered by a settings file. This may be chosen here. The `phasespace` generator needs one such file. We explain the `phasespace` generator in detail in Section 5.4.2.

Trigger Logic

The trigger logic reads detector information of all trigger detectors after each event and stores them in a trigger mask, which is represented in the same way as it is for `COMPASS` taken data. The trigger logic, used in 2008 and 2009 was implemented in the course of this work. See Section 5.3.1.

Output Format

Here, the output of the simulation is defined. In an event simulation, hits and trajectories are produced and stored in a format compatible with `CORAL`, enabling event reconstruction in the same manner as for `COMPASS` data. For this purpose, the `ASCII File` output format is used. `TGEANT` ships its own output format for this, which is based on the `gzstream` library [52]. In addition, one can output the `detector.dat` file. `export GDML` stores the full detector geometry as a Geometry Description Markup Language (GDML) file. Throughout this work, we found inaccuracies in the GDML file and also unsupported geometry types. Furthermore, conversion from GDML to `ROOT` is broken for certain geometries. The default way, how the geometry is used in `CORAL` is an import of the GDML file into `ROOT`, which will fail with certain geometries. We will describe in Section 5.3.7 that we solved this problem, by using the third party program Virtual Geometry Model (VGM) [53], where the detector geometry is directly exported to a `ROOT` geometry in `TGEANT`, without the detour via the GDML file. Consequently, that fix only covers the geometry stored in the `ROOT` file. It is therefore mandatory to use the `ROOT` geometry and not the GDML file as an input for `CORAL`.

Beamfile

The used beamfile format, and how to create a beamfile is covered in more detail in Section 5.4.1. In this box you may chose if a beamfile should be used and how we interact with it.

Others

The other fields should be self explanatory and are not described in detail here.

Fig. 5.2 shows the detector tab of the GUI. COMPASS is a composition of many different detectors, which may be present in different data-taking periods. This configuration page makes it easy to toggle detectors which were present or not in the given period that you are simulating for. For the 2008 / 2009 setup, the shown detectors were in the spectrometer.

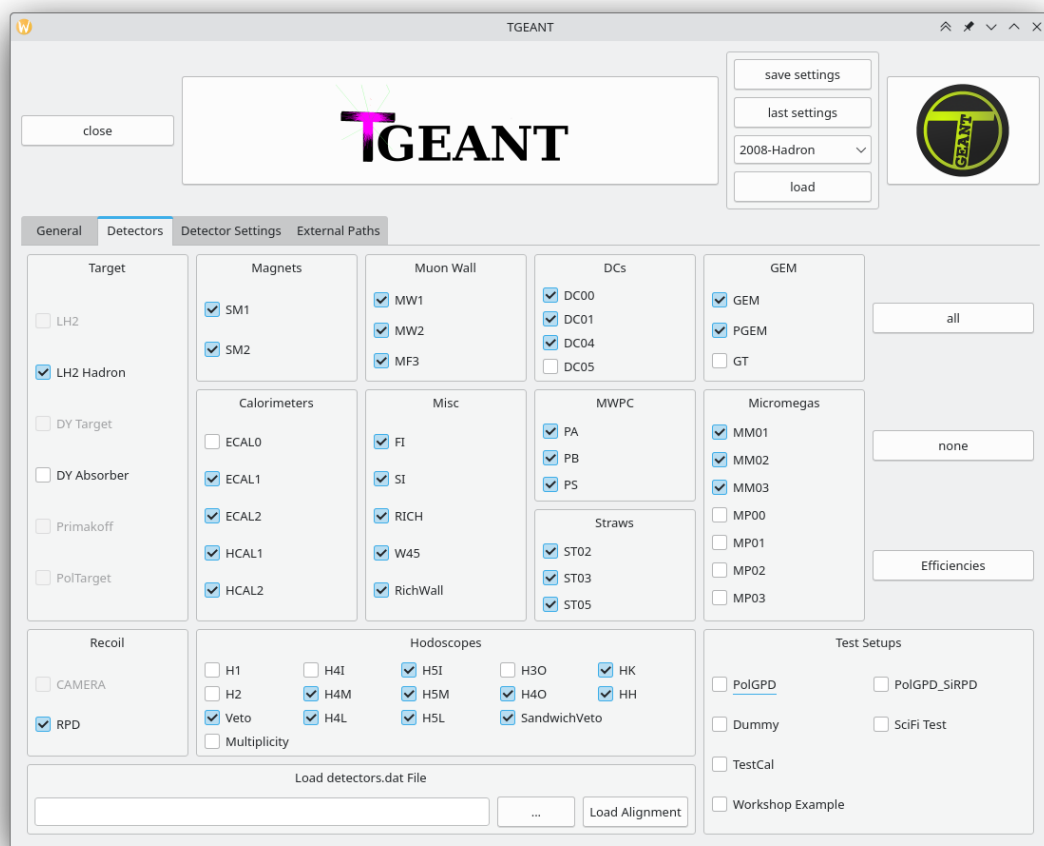


Figure 5.2: Snapshot of the TGEANT GUI. Displayed is the Detectors tab, where one can include or remove detectors for the simulation.

Now that we have selected all detectors that are important for the simulation, we need to place them correctly in space. For this, we have the Detector Settings tab, shown in Fig. 5.3.

Detectors in TGEANT are implemented in a standalone way and placed, aligned at their center

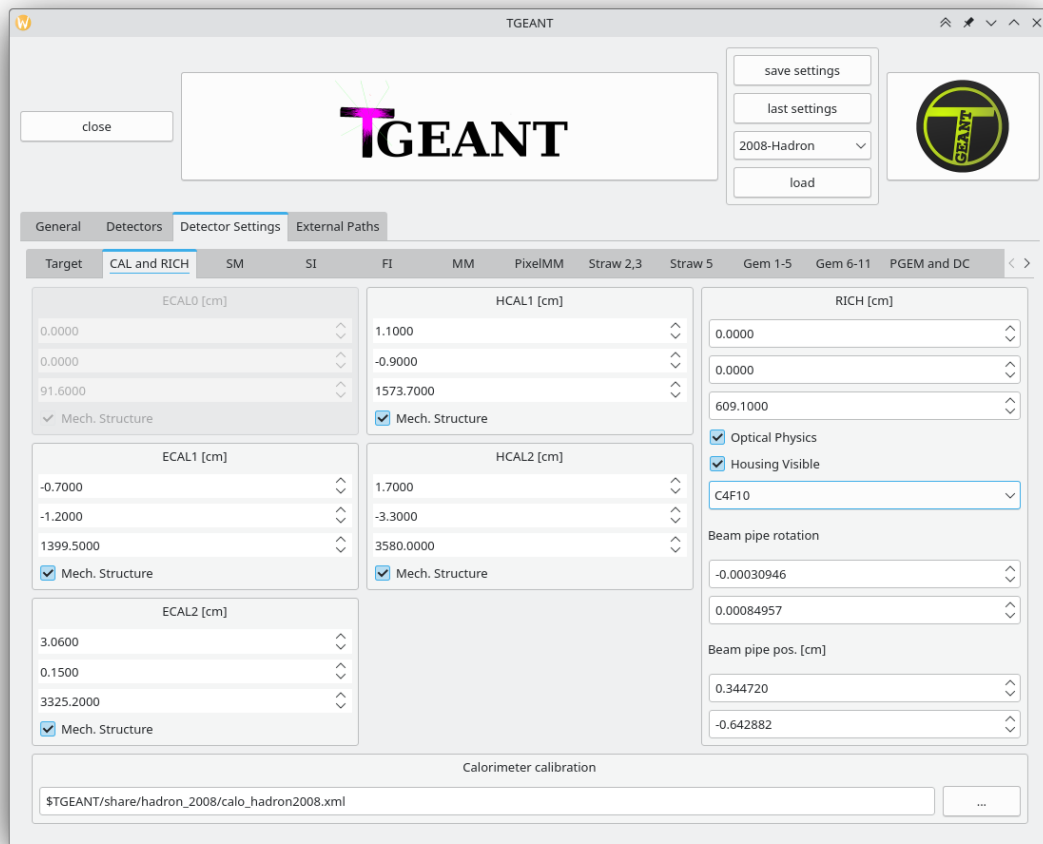


Figure 5.3: Snapshot of the TGEANT GUI. Displayed is the **Detector Settings - CAL and RICH** tab, where placement specifications of the calorimeters and the RICH detector are defined.

coordinates, anywhere in the setup¹. Therefore, we can align them the same way, as they were aligned during data taking. We use the detector.dat file for this. It can be loaded in the **Detectors** tab (see Fig. 5.2). Layout of one detector setting:

First field

Center position in x-direction

Second field

Center position in y-direction

Third field

Center position in z-direction

¹ Care must be taken to avoid placing two detectors such that they overlap in space, as having multiple detectors at the same geometrical position results in undefined behavior.

Mech. Structure

Enable or disable the inclusion of the mechanical structure of the detector, e.g holding material of certain detectors. We always include the mechanical structure in the simulation.

Other fields

Other fields may switch off certain planes of the detector or switch gas mixture, etc.

Lastly, we see the **External Paths** tab in Fig. 5.4, where some miscellaneous features may be defined, which are largely irrelevant to the 2008 / 2009 simulation. The only relevant settings are the production cuts, which will be discussed in more detail in Section 5.3.5.

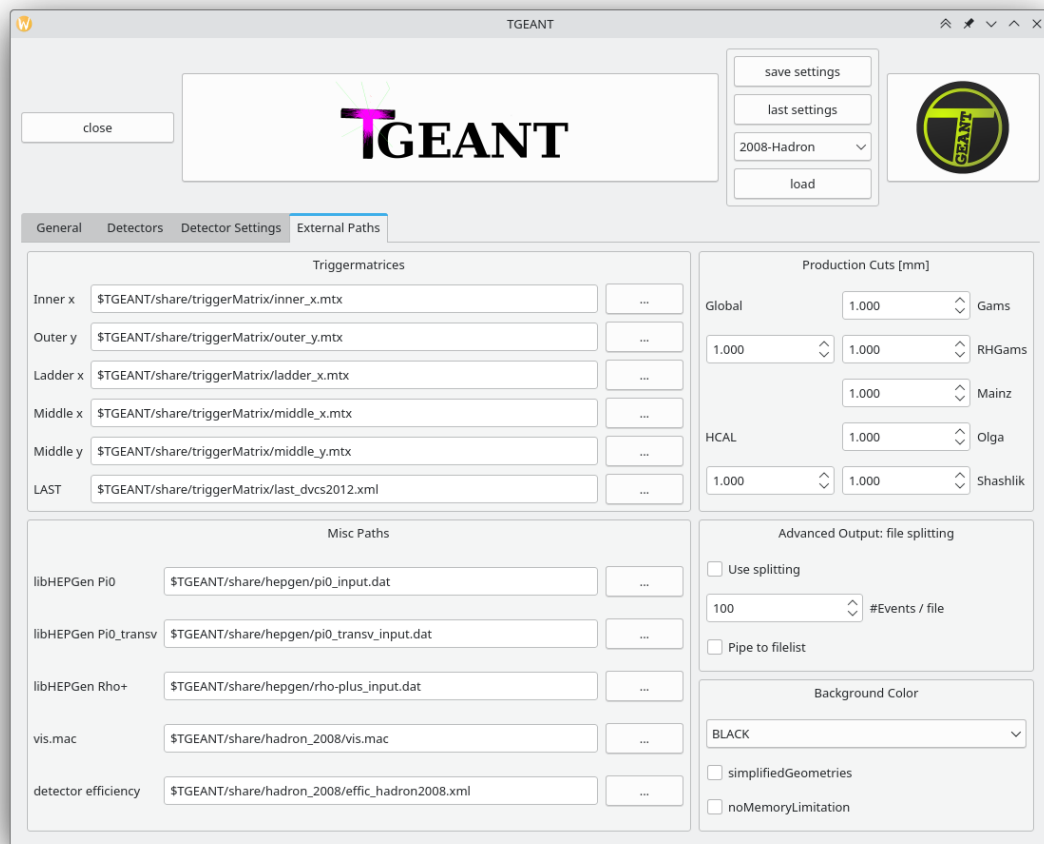


Figure 5.4: Snapshot of the TGEANT GUI. Displayed is the **External Paths** tab, where miscellaneous parameters are defined.

5.3.1 Implementation of the Trigger Logic

In TGEANT, we have all the hits in the different trigger detectors available after each event. In the end-of-event routine, these hits are scanned and the trigger mask is constructed in accordance to Tables 3.1 and 3.2.

Apart from the trigger detectors that are important to the DT0 trigger, there are other trigger detectors which were missing in the TGEANT implementation and were implemented throughout this work. These detectors are the MzC (Fig. 5.5(a)), MuC (Fig. 5.5(c)) and the FH (Fig. 5.5(b)). The implementation of these detectors follows the technical drawings in Fig. 3.5. The MzC was installed for the 2008 and 2009 hadron program. In 2009, the Munich counter was mounted on top of the MzC to better cover the central area. Note however that this upgrade was installed after the diffractive data-taking [54]. For the simulations in this work, the MuC is not present. It can be switched on by activating the trigger plugin `hadron-2009` in TGEANT. The implementation is documented in Ref. [55].

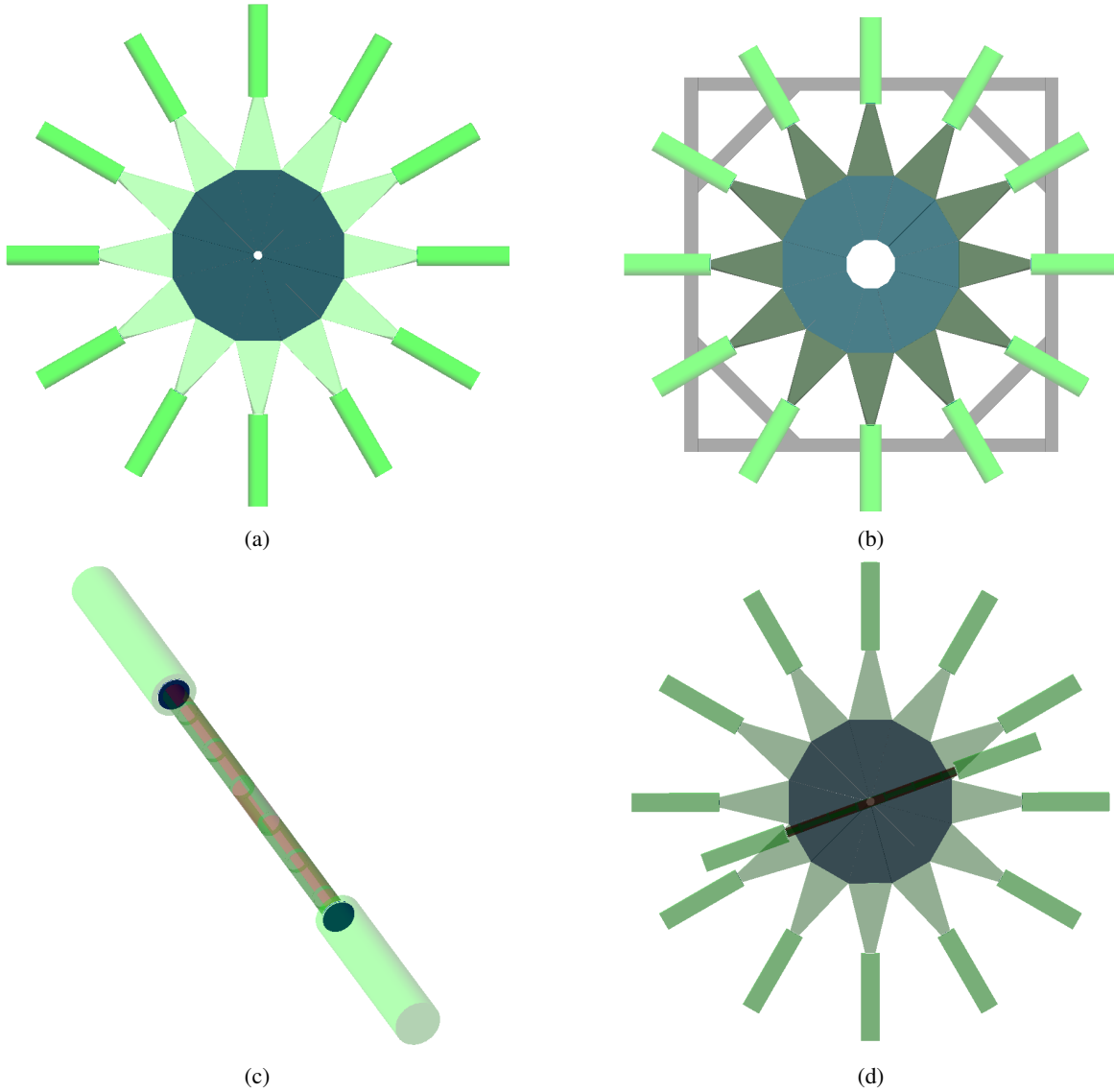


Figure 5.5: Implementation of the MzC (a), the FH (b), the MuC (c) and the combination of the MzC and MuC (d) in TGEANT.

5.3.2 Implementation of the LH₂ Target

Since the holding structure of the LH₂ target is partly made of aluminum or stainless steel, it is to be assumed that recoiling protons will hit these structures from time to time. In reality, this can mean that protons with certain recoil angles are more likely to not interact with the RPD. Of course, this has to be understood and covered in the MC simulation. Unfortunately, the LH₂ target was not implemented to great precision in TGEANT. We have now implemented the key components of the target holding (see Fig. 5.6). The most accurate representation of the target holding structure can be found in Fig. 3.3. Figure A.6 shows the z position of the PV in the 2008 and 2009 data. This can be compared to the simulation in Fig. 5.20(b). The main features are covered well.

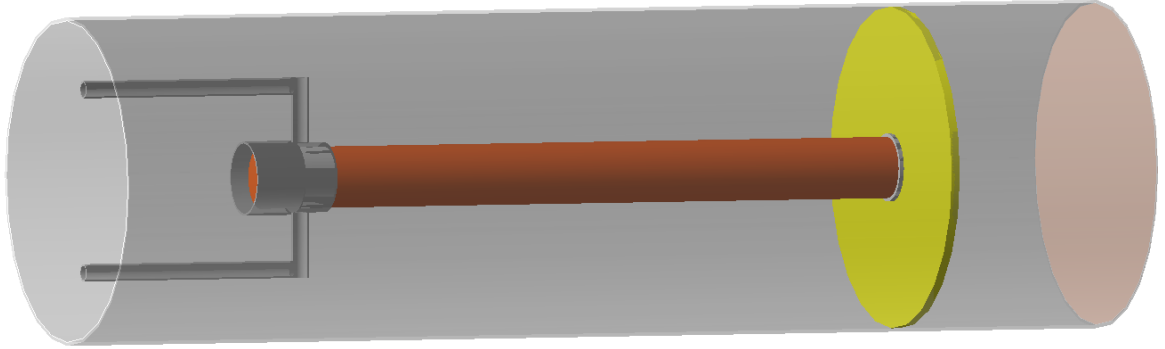


Figure 5.6: Implementation of the LH₂ target region in TGEANT. In orange, we see the LH₂ target. In yellow, we see the Rohacell spacer to hold the target in the light gray aluminum vessel. In dark gray, we see the pipe that carry the liquid hydrogen to the target and the one that carries it away to the cooling interface.

5.3.3 Improvements of the Recoil Proton Detector

In Fig. 3.4(a), we show a front view of the RPD as it is built inside the COMPASS spectrometer. Note that the picture was taken after the data taking, where the RPD was put to parking position. In Fig. 5.7, one can see the implementation of the RPD in TGEANT. The most important feature of the implementation are the black LED connectors that are mounted on top of the scintillators in the inner and outer ring. They were used to calibrate the scintillators of the RPD. One may also see them in Fig. 3.4(b). Due to the recoil protons having only a couple of hundred MeV of kinetic energy [34], their energy loss in the connectors is actually significant. One can see the impact in Fig. A.6 (the dip around -50 cm). We modified the RPD to feature the connectors in TGEANT. The z -Dip is not described the same way as in the data (see Fig. 5.20(a)). This is either due to the fact that the material budget is not correctly implemented or we might have missed some other material in the RPD. However, tests with a largely overestimated material budget of the connectors showed that a dip can be reproduced. It might also be possible that in the digitization step of the MC hits of the RPD something is treated differently than for the data digitization. The dip is only visible in the data, when looking at the early stages of the event selection. In the future, one could investigate this issue further, but it will not make a large difference in the acceptance description, since the dip is not visible in the PV z -position distribution after all cuts (see Fig. 4.2(b)).

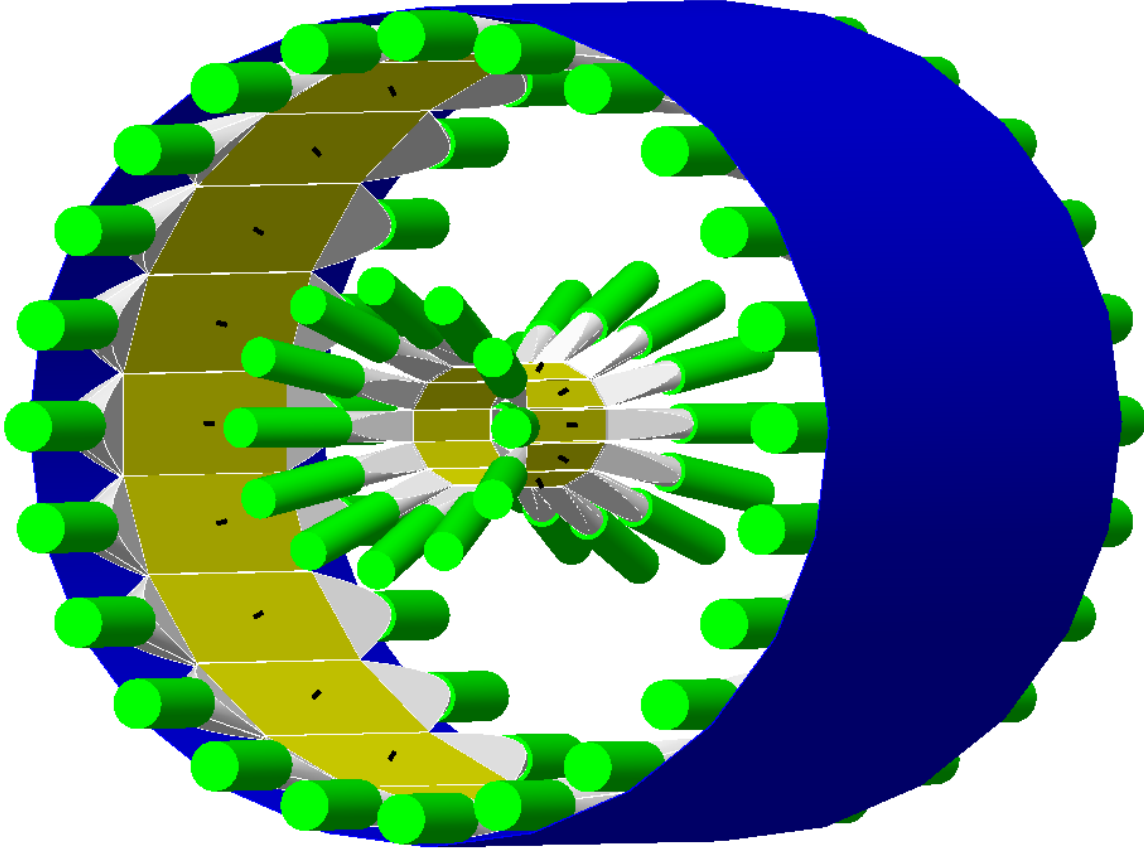


Figure 5.7: Implementation of the RPD in TGEANT.

5.3.4 Updates of the RICH Detector

One key feature of the RICH detector is the beam pipe, where beam particles that did not interact with the target can just travel through the detector without interacting with the gas (C_4F_{10}) inside the RICH. This is needed, since these particles shall not disturb particle identification, performed by the RICH. In 2012, the beam pipe of the RICH was exchanged. The thick stainless steel pipe ($0.85\%X_0$ for orthogonal crossing) was changed to a lighter one ($0.044\%X_0$ for orthogonal crossing). Nevertheless, for this thesis, we need the implementation of the heavy pipe, which was newly implemented by Andrii Maltsev [56]. The RICH pipe considerably changes our photon reconstruction acceptance, as one can see in Fig. A.9. Due to the heavy material, many photons convert there and the acceptance is lowered by this. Additionally, we found out that the mirror structure of the RICH is described poorly [57]. This should not impact this simulation, since we do not use the actual response of the RICH, but only the material budget. Nevertheless, this is still an open issue in TGEANT. For the event selection of the MC data, we use the knowledge about kinematic acceptance from real COMPASS events. More details are in Section 5.4.4.

5.3.5 Speedup of the Simulation - Calorimeters

The two calorimeters (ECAL1 and ECAL2) are implemented in TGEANT. The chosen implementation at the point where TGEANT was released [13] was to construct every cell individually and place it at the correct position in space. This of course is a large amount of volumes that in principle only vary in their placement position. Moreover, each cell is read out individually - as it is also in the COMPASS spectrometer - which produces a large amount of sensitive detector objects as well. Furthermore, the implementation of one shashlik cell is not as straight forward as a homogeneous module. The alternating structure of scintillator and absorber material of the shashlik cell was realized by individual placements in TGEANT. The holding rods and WLS fibers were not implemented. Especially the rods contribute to a considerable difference in radiation lengths along the calorimeter cells and it is important for a good description of the cell in MC that they are implemented properly.

Considering the original implementation, the time to simulate a singular photon of 100 GeV was around 400 s. To put this into perspective, photon simulation in COMGEANT was of the order of 10 s. One has to mention that COMGEANT did not implement the shashlik cells in great detail, i.e. with the different slices, but rather used an effective material description.

Using the original TGEANT implementation of the calorimeters, we were not able to perform simulations with event times² going up to 500 s. The original approach of TGEANT used a fast shower simulation algorithm called GFlash. In this method, the calorimeter material is approximated as an effective homogeneous block. The electromagnetic shower is then simulated within this effective description. The resulting output from GFlash, i.e. the spatial distribution of energy depositions, is subsequently mapped back onto the full TGEANT geometry. This ensures that energy deposited in inactive material is not collected. For more details on the GFlash algorithm, see Ref. [58]. GFlash was tuned to reflect the calorimeter properties at COMPASS to the best possible degree by the TGEANT developers [13] and during a bachelor thesis [59], where in addition to the scintillator and absorber material, the holding rods and WLS fibers were taken into account. Still, even with GFlash, event times were around 60 s on average and one had to rely on approximations of the material. It is evident that the original TGEANT implementation had some major flaws. Due to the nature of the implementation, each cell (and slice in the shashlik case) was built individually, i.e. there was one object per slice and cell. For GEANT4, this meant that navigation of the particles through these volumes is not clear a priori. To be more precise, every few cm there may be a volume border, where the GEANT4 kernel needs to check in what volume the particle is and what may change from volume to volume. This comes with a huge memory and computation overhead. GEANT4 provides a direct solution for repetitive structures, so-called parameterized volumes.

The main idea is that there is only one object and it is reused in space, i.e. one provides a look up table of positions, where the same volume shall be. This is ideally suited for the calorimeter problem. There is one obstacle that comes along with the parameterization. Each volume can only hold one parameterized volume or many normal placements, i.e. one needs to create a vessel volume first that shall hold the full parameterization. But nothing prevents the user from implementing parameterized volumes inside a parameterized volume. We chose to create one vessel volume per calorimeter type, i.e. one for all shashlik cells, GAMS cells etc. Then we construct one single cell and reuse it in form of parameterizations across the whole vessel. Ultimately, this results in only one volume that is shared across the full shashlik vessel. The original way of attaching one readout per cell is therefore not

² In this context, one event is the full simulation of the reaction under investigation ($\pi^- + p \rightarrow \eta^{(\prime)} + \pi^- + p$).

usable anymore. We introduce a new parametrized readout, where we determine the cell that is hit by the copy number of the parameterized volume. The copy number is assigned in chronological order from the first placement of the parameterization to the last by GEANT4. With this, we can compute the unique cell id during the readout of the cell volume and save the deposited energy accordingly. In total, this formalism reduces the number of volumes and readout objects for each calorimeter type from N_{cells} to 1.

In the same manner, we use parameterizations to build the complicated shashlik cells. We know from Fig. 3.8 that there are four unique slice layouts that can be used via rotation of 90 deg to form the full spiral of the WLS fibers. Therefore, we need to construct one object per slice and type for the scintillator and absorber material, which has 16 fiber volumes and 4 rod volumes inside. These 20 volumes are again parameterized, where we precompute the position of the rod and fiber volumes and parameterize according to the copy number (first 4 for rods and last 16 for fibers) the material and shape of the volume. Now, we place all 12 slices (4 types and 3 materials) in one vessel and reuse it across one shashlik cell. Note that 155 shashlik slices is not dividable by 4. The leftover slices need to be placed normally. Therefore, we create a vessel for the first 152 slices, and use a parameterized volume there. Then we place this vessel together with the 3 leftover slices in the shashlik cell volume as normal placements. Lastly, there is an aluminum plate at the front of the cells, where the rods are fixed to. All in all, the complete shashlik cell is depicted in Fig. 5.8.

One can see that the rods and fibers are implemented correctly and in the same way as in the spectrometer (see Fig. 3.7). The readout of the shashlik cell is a little more complicated as for the simple crystals of the other calorimeter types. Here, all energy deposited in the scintillator material as well as the fibers is collected. Energy deposited in the rods or absorber material is ignored.

Therefore, we expect that most of the energy of the particle shower is lost but then later recovered via the reserv method, described in Section 5.3.9. In total, these improvements lead to a major computational gain and to the fact that we do not need to use GFlash anymore, i.e. we can use the true GEANT4 simulation of the shower and make use of the detailed material description. Simulating a 100 GeV photon on the shashlik calorimeter takes around 4 s now. This is an effective gain in computing time of roughly a factor 25 to the photon simulation without GFlash and the old implementation. Of course this factor varies for different photon energies. Switching on GFlash for the new calorimeter implementation, still speeds up the simulation. The average event time for an $\eta^{(\prime)}\pi^-$ event (see Section 5.4.4) is 18 s without GFlash and 6 s with GFlash. Still, we used GFlash in the first place because the simulation was not feasible without it. With event times of 18 s, a simulation for the $\eta^{(\prime)}\pi^-$ final state is easily feasible with a computer cluster like BAF2 [43]. Therefore, there is no need to use GFlash anymore at COMPASS.

In order to test the implementation, we shoot 500 000 photons, with an energy of 100 GeV, randomly from one fixed point at (0, 0, -500 mm), which is in the middle of the target region, onto one shashlik cell, with a center position of (0, 0, 30 020 mm). Thereby, we preserve the relative distance from the PV to ECAL2 and may compare the result to the study in chapter 9.7.6 from Ref. [30]. The calorimeter cell is divided into small bins where each photon contributes with average energy / measured energy. Finally, this plot is normalized by the number of hits per bin. The result is a ratio plot, where a ratio of 1 (with statistical fluctuations) would be expected for a homogeneous material. Since we have a substructure in the shashlik cells, i.e. rods and fibers, we expect to lose more energy when we hit a rod and very slightly less energy when we hit a fiber since more energy might escape the calorimeter in that case. In fact that is exactly what we see in Fig. 5.9. The study from Ref. [30] was conducted with electrons, since the position measurement could then be extracted by the tracking detectors. We use

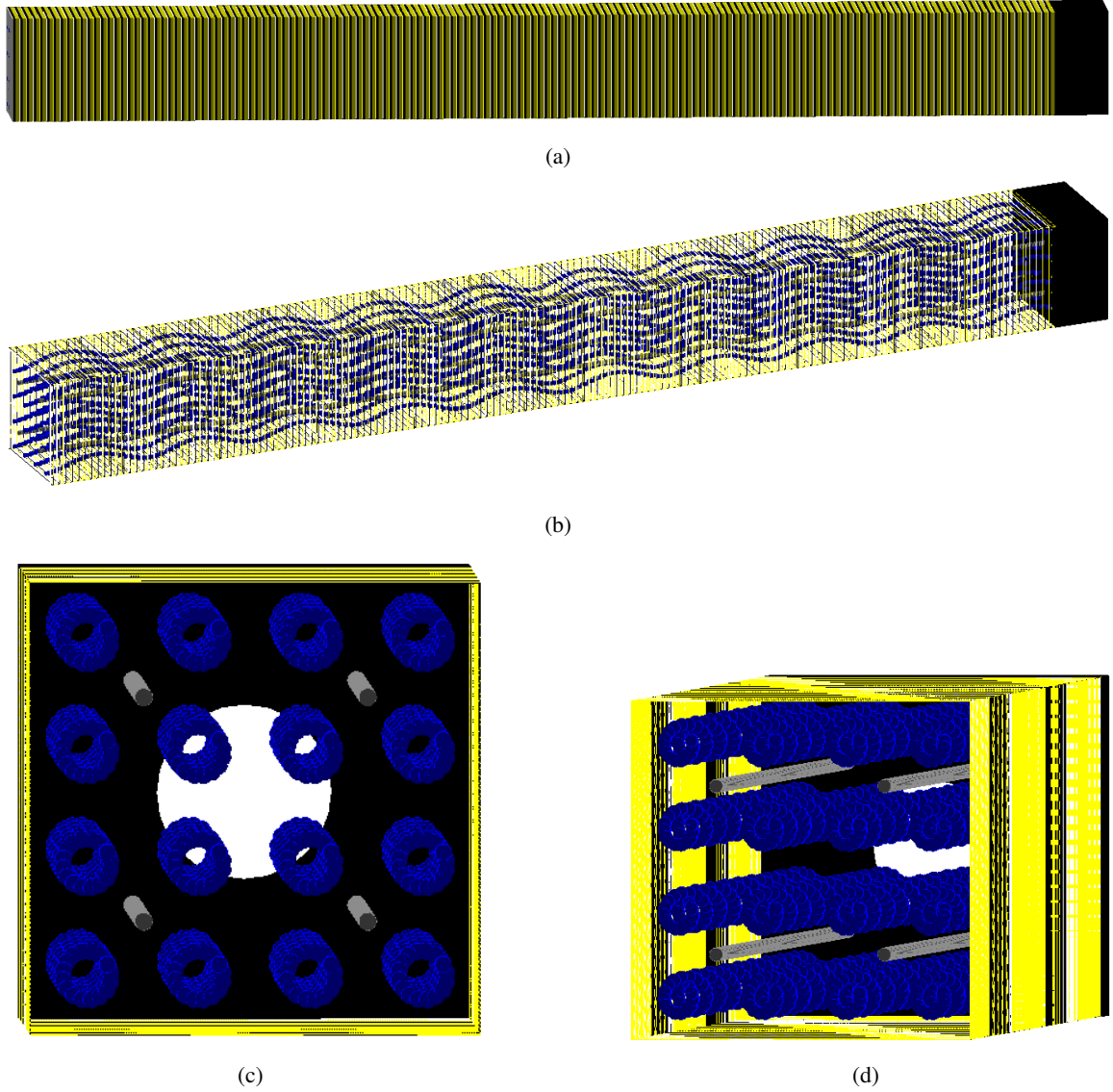


Figure 5.8: Side view (a), transparent side view to see the spiraling WLS fibers (b), backwards view (c) and sideways backward view (d) of the TGEANT implementation of one shashlik cell. The spiraling WLS fibers are shown in blue, the rods in gray and the alternating structure of the scintillator and absorber plates in yellow and black. The black plate at the front of the module is an aluminum plate for thermal shielding.

photons for the sake of simplicity and the MC truth information for the hit position at the calorimeter. Therefore, we do not take inaccuracies of the reconstruction into account and expect a much cleaner picture in our simulation, compared to the study in Ref. [30].

When we compare this to the measured plot in Fig. 5.9 we see that the spots are not as clear and some structures that are visible in MC are not visible in the measured plot. This can have a multitude of reasons.

- We are using the MC truth values and have no reconstruction inefficiency

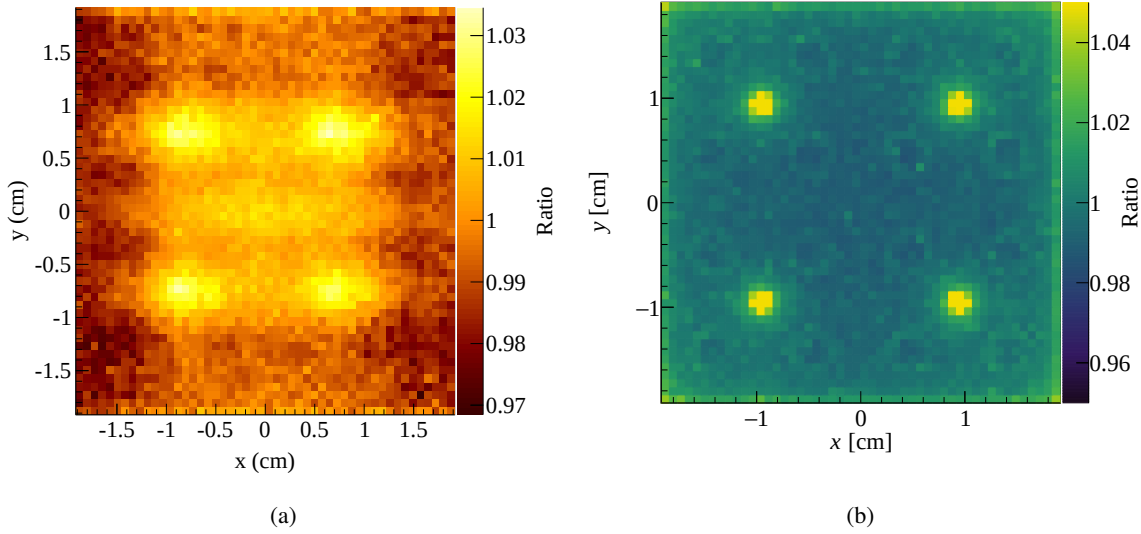


Figure 5.9: Ratio of track momentum over calorimeter energy for electrons (a), measured at COMPASS (taken from Ref. [30]) and photons simulated in TGEANT (b).

- We use a fixed high photon energy of 100 GeV, while a range of energies was used in Ref. [30]
- The inclination angle of the photon is fixed, while it was fluctuating in the study, documented in Ref. [30]
- We did not implement the holding nuts of the rods, which are slightly larger than the rods itself.

The use of MC truth energies and hit positions explains the better visibility of the rods and fibers in Fig. 5.9(b), compared to Fig. 5.9(a). In the future, it might be useful to implement the holding nuts and investigate their impact on the present study.

Production cuts in GEANT4 describe the flight path of a photon or electron/positron from which on it will not be tracked anymore and will deposit its energy directly. Ideally, one wants to make this path much smaller than the geometrical extents of the detectors. The main benefit being that particle navigation is then quite precise in all volumes. The TGEANT default is 1 mm. With the original implementation, the production cuts were enhanced for the calorimeters to around 100 mm. With the new implementation, one can safely use 1 mm across all calorimeters again without losing any computational speed.

5.3.6 Speedup of the Simulation - Tracking Detectors

In Section 5.3.5, we have seen that parameterizations are extremely useful when it comes to repetitive structures in the detectors. Especially, when each structure needs to be read out individually. It makes sense to benchmark the rest of the setup and see, what other detectors might be contributing to a considerable event time in the TGEANT simulation of the 2008 and 2009 diffractive setup and think about exploring parameterizations there as well. We found that only the MW1, MW2 and DR detectors

significantly slow down the simulations, which is remarkable, since we were not even simulating muons for the benchmark test. The MW1, MW2 and DR detectors are all built similarly. They consist of single tube like detectors that are read out individually and placed hundreds of times per detector plane. The visualization of these detectors can be seen in Fig. 5.10.

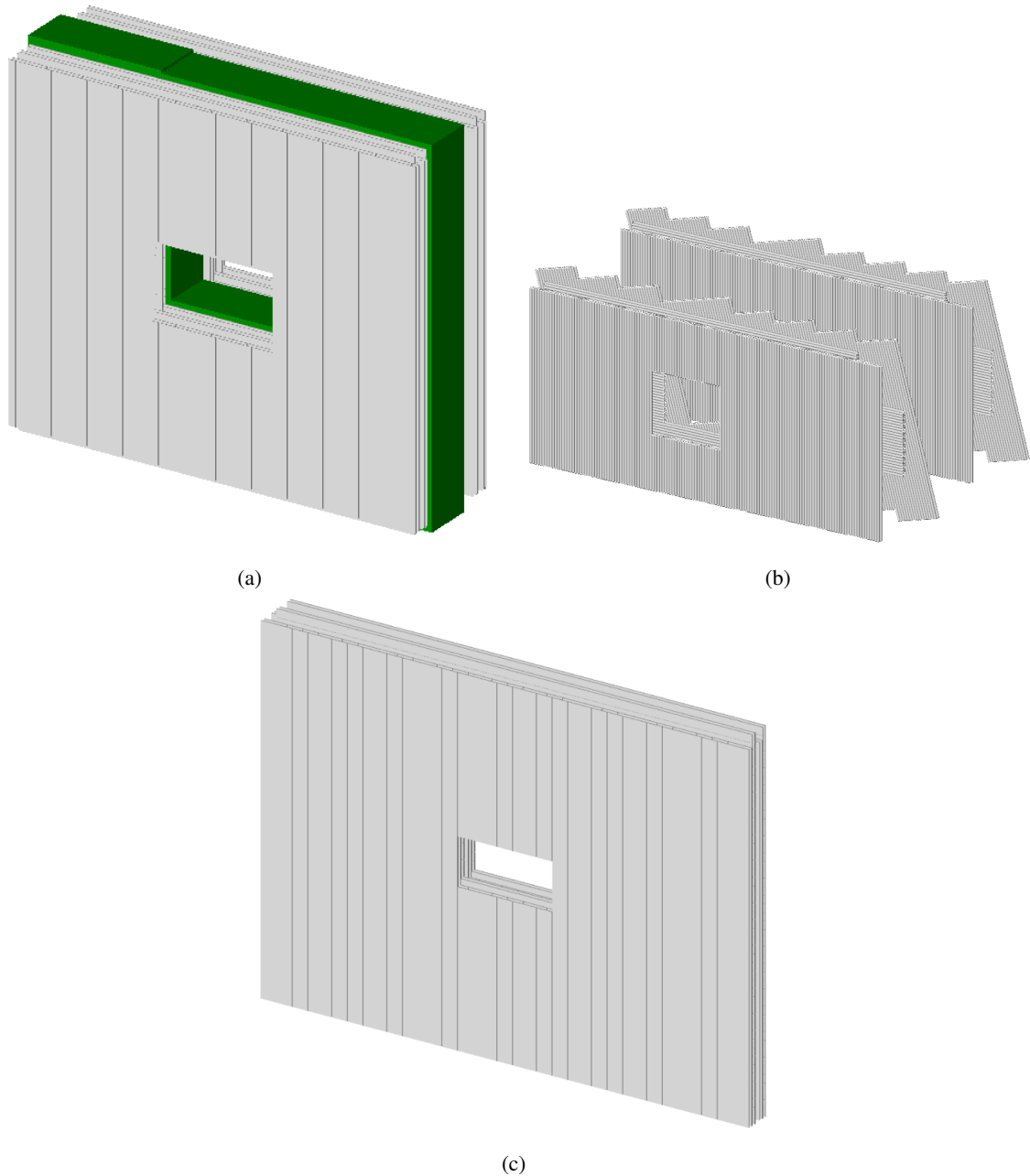


Figure 5.10: Visualization of the MW1 (a), MW2 (b) and DR (c) detector in TGEANT.

Here, one sees clearly that the detectors contain a hole for the beam particles or particles detected more downstream in case of MW1 and DR. This leads to the fact that there are two different types of

tube like detectors placed for one plane. One shorter and one larger tube like detector. Therefore, we decided to create three vessels, one for each sector (long type left, short type middle and long type right). This results again in a large reduction of logical volumes and read out objects and reduces the event times by a factor 2 again.

5.3.7 The New Way of Exporting the Detector Geometry

Sections 5.3.5 and 5.3.6 introduce the usage of parameterized volumes in TGEANT. COMPASS operates the MC chain in a way, where the output of the simulation is fed through the reconstruction the same way as one does for measured events at COMPASS. In order to do so, one needs to provide a geometry description not only for the TGEANT simulation but also for the CORAL reconstruction. Moreover, the reconstruction of COMPASS events, i.e. real events measured by the spectrometer, needs a geometry description as well. At COMPASS, we extract the geometry from TGEANT directly and convert it to a ROOT geometry. In CORAL, all matters related to the material are done with ROOT. Consequently, we need a way to translate the TGEANT geometry to ROOT. In the original implementation, this was done via the G4GDMLParser. This GEANT4 class dumps all geometry information in a GDML file. ROOT can read in GDML files and convert them to ROOT geometries directly. Unfortunately, GDML files do not support every type of parameterized volumes and the ROOT GDML to ROOT converter can read no parameterized volumes [60]. This is a pity, since in both GEANT4 and ROOT, parameterized volumes are supported.

Fortunately, there is a third party program (mostly used for A Large Ion Collider Experiment (ALICE)) that reads GEANT4 volumes from Random-Access Memory (RAM), i.e. can be used in TGEANT and converts them internally to ROOT volumes, which are then saved directly to a ROOT file. VGM unfortunately did not support every type of parameterized volume either but that code was much easier to change, than the GEANT4 source code. From release 5.2 onward [61], the code, developed to support parameterized volumes in VGM, was accepted and VGM converts GEANT4 parameterized volumes now properly to ROOT parameterized volumes. In order to use VGM, we now build it by default with TGEANT and have removed the GDML intermediate step [62].

Also in the ROOT geometry, one can feel the benefit of the parameterized volumes. The size of the TGEANT geometry for the 2008 and 2009 setup is reduced from 20 MB to 5 MB. This is especially important, since the geometry is saved in each mDST file and we save a lot of disc space with this change.

5.3.8 New Shower Profiles for ECAL2

When we simulate photons or electrons/positrons that hit the ECALs, they will shower and produce secondary particles. This will go on until the secondary particles are below a certain threshold, given via the production cuts discussed in Section 5.3.5, resulting in an energy deposition in some of the calorimeter cells. The calorimeters have only one cell in z -direction. Therefore, all z -position resolution is lost and we treat the shower profiles in two dimensions only.

We use the method described by Lednev [63] in order to characterize the shower profiles. For this, we describe the cumulative energy of a shower by

$$F(x, y) = \left(\int_{-\infty}^x \int_{-\infty}^y E(\tilde{x}, \tilde{y}) d\tilde{y} d\tilde{x} \right) / F(\infty, \infty), \quad (5.1a)$$

$$F(x) = \left(\int_{-\infty}^x \int_{-\infty}^{\infty} E(\tilde{x}, \tilde{y}) d\tilde{y} d\tilde{x} \right) / F(\infty), \quad (5.1b)$$

$$F(y) = \left(\int_{-\infty}^{\infty} \int_{-\infty}^y E(\tilde{x}, \tilde{y}) d\tilde{y} d\tilde{x} \right) / F(\infty), \quad (5.1c)$$

where $F(x, y)$ is the cumulative energy of a shower at position (x, y) and $E(\tilde{x}, \tilde{y})$ is the energy density of the shower at position $\tilde{x}, \tilde{y}$. In addition, we normalize the cumulative energy such that $F(\infty, \infty) = 1$. Infinity refers here to the full extent of the shower, i.e. every cell that measured something is included. This has the benefit that shower profiles for different shower energies can easily be compared. The ECAL2 has discrete cells, where we measure the deposited energy. Therefore, we only know the deposited energy per cell and the integrals in Eq. (5.1) change to

$$F(x, y) = \left(\sum_{i=1}^{N_x} \sum_{j=1}^{N_y} E_{ij} \right) / F(x_{\max}, y_{\max}), \quad (5.2a)$$

$$F(x) = \left(\sum_{i=1}^{N_x} \sum_{j=1}^{N_{y_{\max}}} E_{ij} \right) / F(x_{\max}), \quad (5.2b)$$

$$F(y) = \left(\sum_{i=1}^{N_{x_{\max}}} \sum_{j=1}^{N_y} E_{ij} \right) / F(y_{\max}). \quad (5.2c)$$

Here, N_x and N_y represent the number of cells, denoted with i and j , that satisfy $x_i \leq x$ and $y_j \leq y$. We denote the upper border of a cell in x -direction as x_i and in y -direction as y_j . Note that $F(x, y)$ is only well defined for a point (x, y) that is at the upper border of a cell, since we do not know the energy distribution within the cell. The deposited energy³ in cell ij is denoted as E_{ij} and $x_{\max}, y_{\max}$ indicate the upper most cell contributing to the profile. We only include cells for any shower in a 12×12 cell area around the shower. Therefore, we describe a one dimensional Lednev profile with 12 points and a two dimensional shower with 144 points. In Fig. 5.11, we see representations of typical showers in ECAL2 and their Lednev profiles in one dimension.

³ E_{ij} is the energy measured in the cell ij , which is scaled by the MC truth total shower energy over the measured total shower energy. These two quantities are not equal, since some energy of the shower may be lost in the dead material of e.g. the shashlik cells. This is accounted for in the reconstruction by the reserv parameter (see Section 5.3.9). Since we do our fits here on the TGEANT output, we need to scale the energy ourselves. Note that this does not effect the final cumulative energy distribution, since this is normalized to one, but rather effects the error calculation only, given by Eq. (5.3).

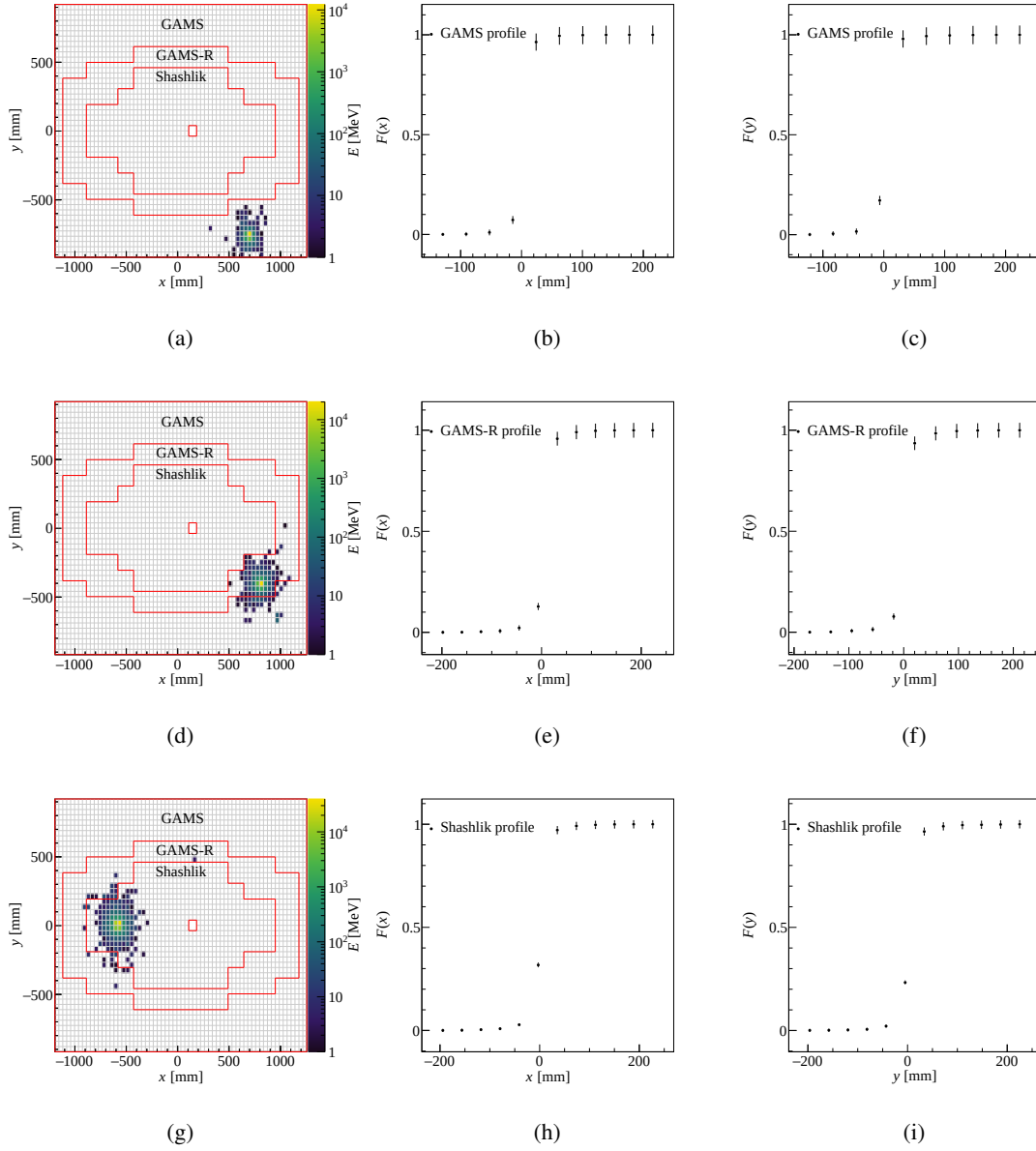


Figure 5.11: A single simulated photon shower that is contained in one of the calorimeter cell types of ECAL2. (a) for GAMS, (d) for GAMS-R and (g) for Shashlik. The borders of the different cell types are given by the red lines. Gray lines indicate the cell borders. The 2 by 2 area in the middle is the hole in ECAL2. (b), (e) and (h) show the one dimensional Lednev profile in x direction and (c), (f) and (i) the one dimensional Lednev profile in y direction of the respective showers presented in the left column.

The uncertainty on the energy measurement in a single cell is given by

$$\sigma_{E_{ij}} = \sqrt{a^2 E_{ij} + b^2 + c^2 E_{ij}^2}, \quad (5.3)$$

with $a = 0.15 \sqrt{\text{GeV}}$, $b = 0.05 \text{ GeV}$ and $c = 0.015$. This error calculation is standard for calorimeters [39] and the parameters are default at COMPASS. Parameter a reflects the uncertainty due to stochastic fluctuations of the shower development. b reflects the uncertainty due to electronic noise and c reflects the uncertainty due to irregularities of the calorimeter.

The cumulative energy of a shower (see Eq. (5.2)) is described by the following function [63]

$$F(x, y) = \frac{1}{4} + \frac{1}{2\pi} \sum_{i=1}^N a_i \left(\arctan \frac{x}{b_i} + \arctan \frac{y}{b_i} + \arctan \frac{x \cdot y}{b_i \sqrt{b_i^2 + x^2 + y^2}} \right), \quad (5.4a)$$

$$F(x) = 2F(x, 0), \quad (5.4b)$$

$$F(y) = 2F(0, y), \quad (5.4c)$$

$$\sum_{i=1}^N a_i = 1, \quad (5.4d)$$

where a_i and b_i are free parameters and the number of terms N . In Ref. [63], it is suggested to take $N = 3$, but we found that it is worth while to try to describe the showers with less parameters if possible. We will try $N = 2$ and $N = 3$ and take whatever fits the data best. In CORAL, each calorimeter cell type in ECAL2 has its own parameter set, i.e. a_i and b_i . In order to determine these parameters for our MC setup, we follow closely what was done in order to extract parameters for the real COMPASS events [64]. For each calorimeter cell type, we shoot 1000 photons with varying energies onto positions in ECAL2, where the resulting showers are contained in either the shashlik, GAMS or GAMS-R cells. In Fig. 5.12 we show the energy distributions of the photons of all selected COMPASS events in the $\eta^{(\prime)}\pi^-$ final state. We draw the energy of the 1000 photons according to the distributions of the real photons measured in the $\eta^{(\prime)}\pi^-$ final state, displayed in Fig. 5.12.

Then, we extract the center of gravity of each shower from MC truth⁴. Once, the shower is shifted by its center of gravity, all of the 1000 showers can be displayed on top of each other. Now, we have an average representation of hit angles on the calorimeter and energies of the showers. In order to extract the parameters a_i and b_i for GAMS, GAMS-R and Shashlik, we construct the quantity χ^2 according to

$$\chi_{\text{reduced}}^2 = \sum_{i=1}^{N_{\text{Points}}} \left(\frac{F_{\text{MC}}(x_i, y_i) - F_{\text{Model}}(x_i, y_i)}{\sigma_{F_{\text{MC}}(x_i, y_i)}} \right)^2, \quad (5.5)$$

where N_{points} includes all shower points for all of the 1000 showers simulated. The normalized cumulative energy of one shower point i is given by $F_{\text{MC}}(x_i, y_i)$. $F_{\text{Model}}(x_i, y_i)$ is computed by evaluating Eq. (5.4). Via Gaussian error propagation, we get $\sigma_{F_{\text{MC}}(x_i, y_i)}$ per shower point, using Eq. (5.3). We minimize Eq. (5.5) multiple times, while choosing random start parameters for a_i and b_i , until we reach 1000 successful minimizations⁵. The global minimum is defined as the fit with the lowest χ_{reduced}^2 . The final distributions are shown in Figs. 5.13 to 5.15 and the fit parameters are given in Table 5.1.

⁴ We have modified TGEANT such that one can configure it with the option `-DSaveAllCaloHits`. Then, all shower points will be saved after the TGEANT simulation. From this, it is easy to calculate the center of gravity of the shower by simply extracting the point with the most energy.

⁵ A successful minimization is given when the minimization with TMinuit [65] returns status 0.

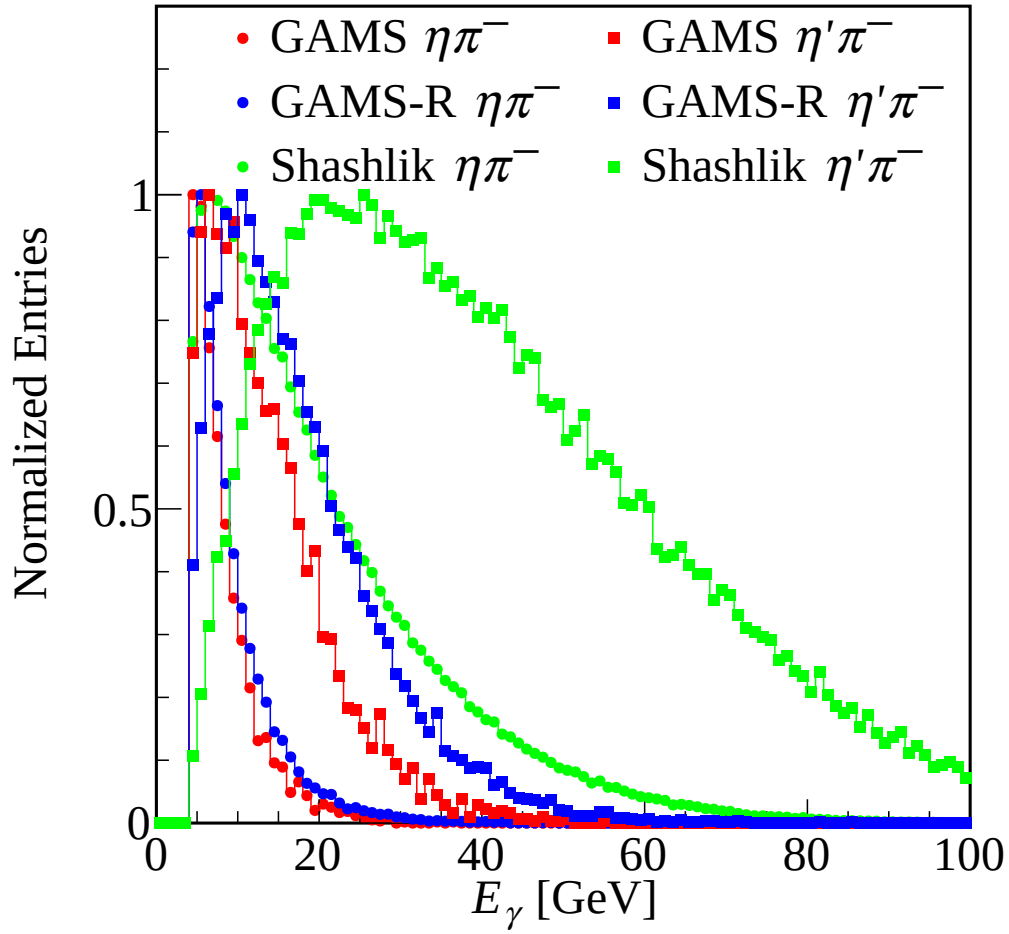
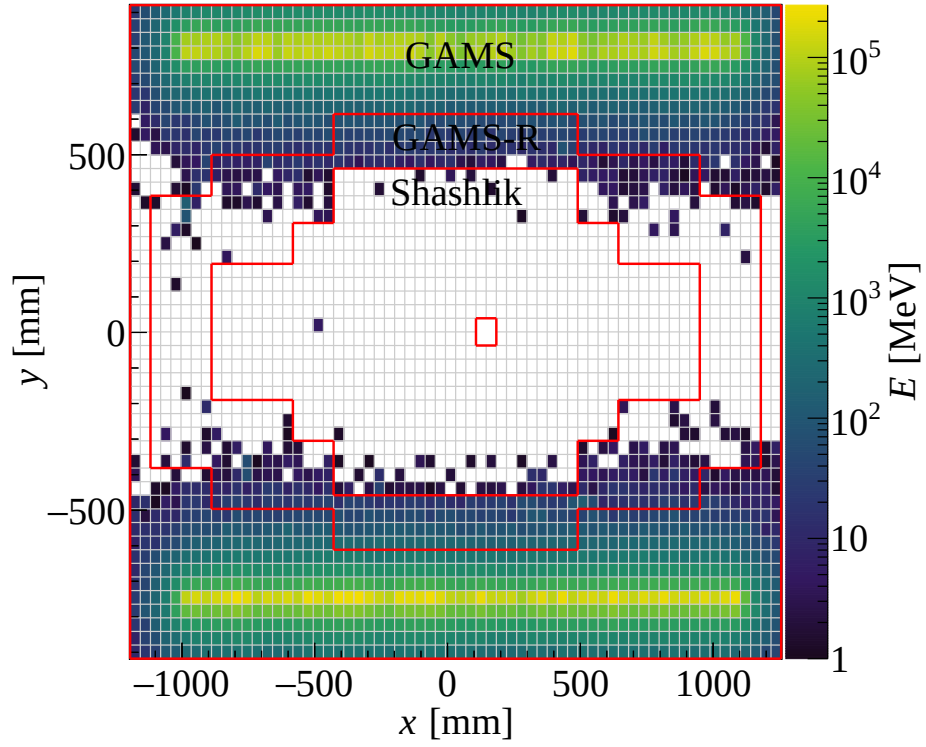
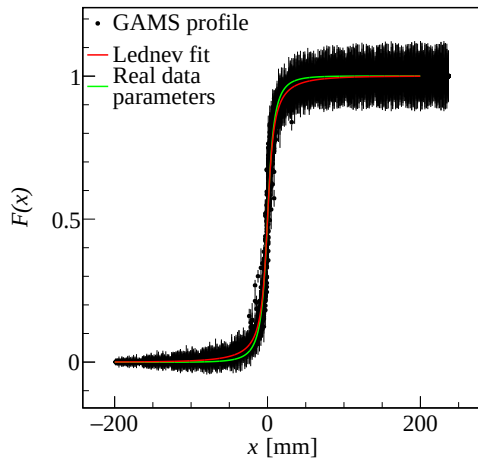


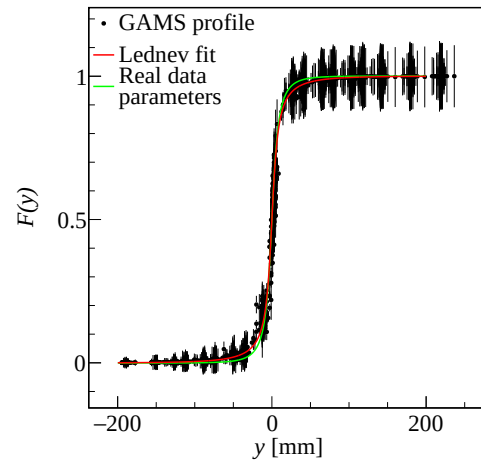
Figure 5.12: Energy distribution of photons in the different calorimeter cell types of ECAL2. Shown are all photons after the event selection in the $\eta\pi^-$ and $\eta'\pi^-$ final state for COMPASS events (see Section 5.4.4).



(a)

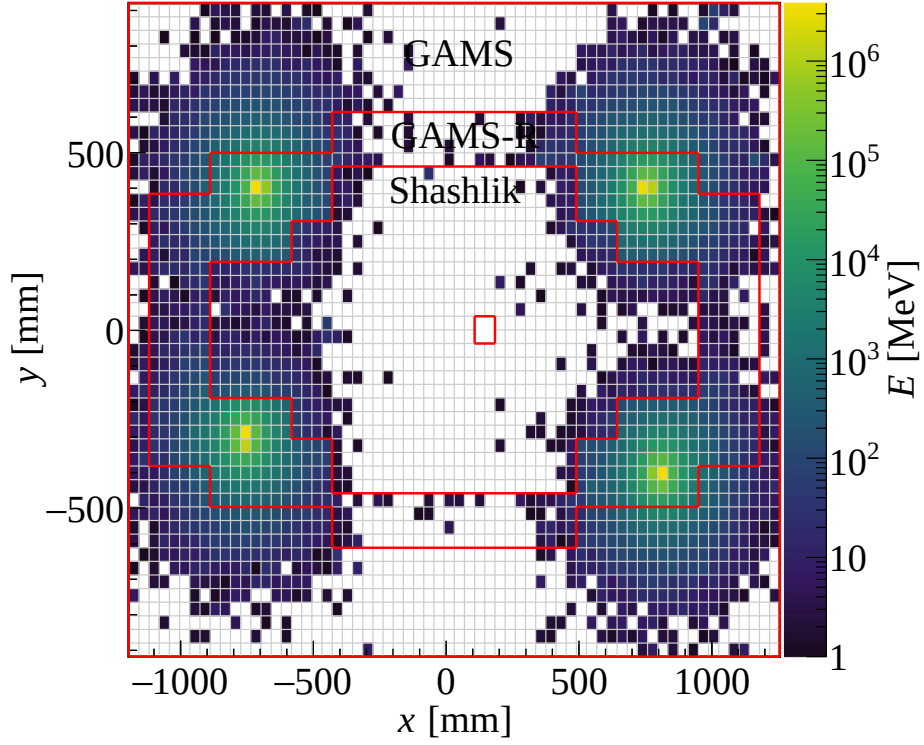


(b)

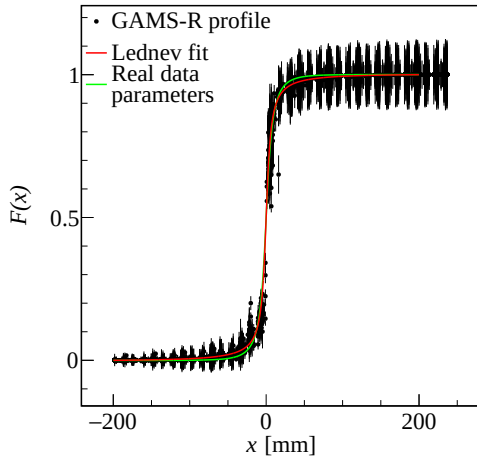


(c)

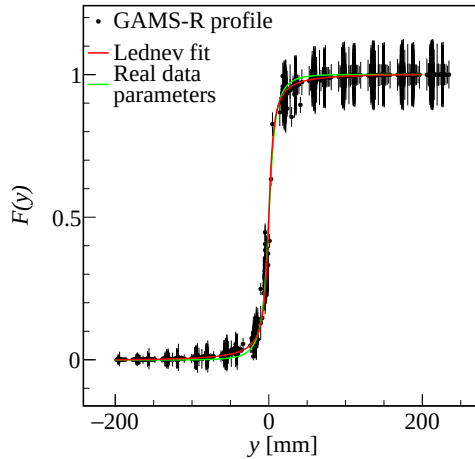
Figure 5.13: 1000 simulated photons that hit the GAMS part of ECAL2 (a), as well as the resulting Lednev profiles in x direction (b) and y direction (c). The Lednev curve, extracted by the fit is shown in red and the Lednev curve with parameters used in the reconstruction of real COMPASS events in green.



(a)

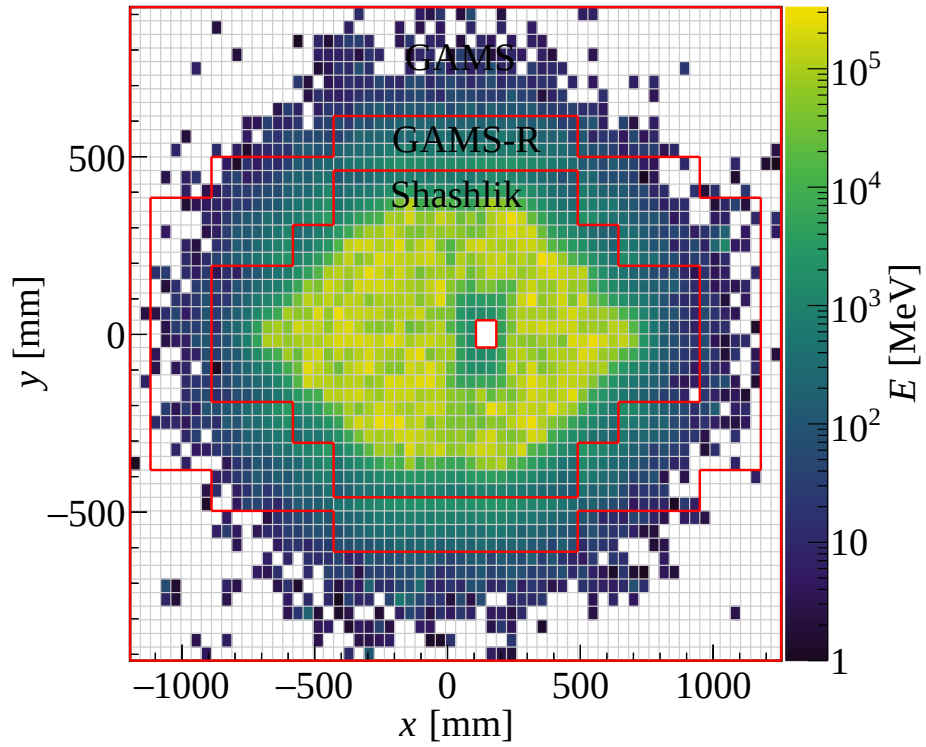


(b)

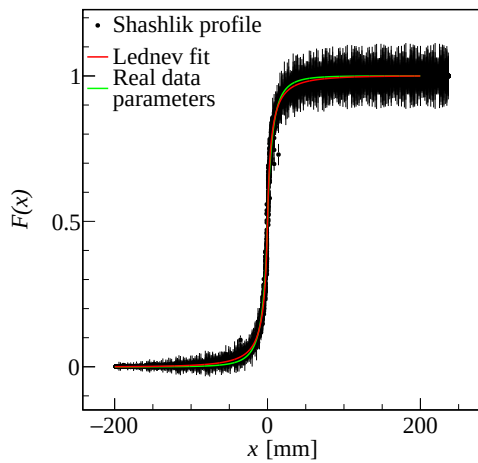


(c)

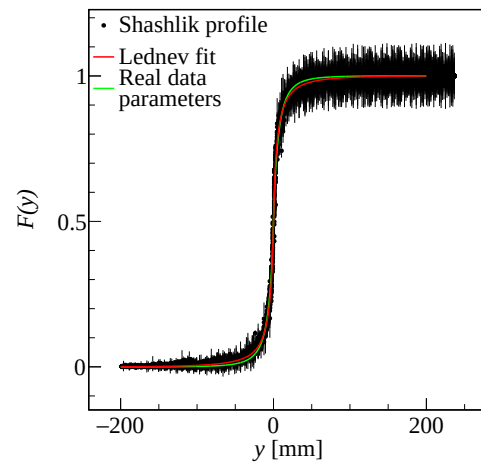
Figure 5.14: 1000 simulated photons that hit the GAMS-R part of ECAL2 (a), as well as the resulting Lednev profiles in x direction (b) and y direction (c). The Lednev curve, extracted by the fit is shown in red and the Lednev curve with parameters used in the reconstruction of real COMPASS events in green.



(a)



(b)



(c)

Figure 5.15: 1000 simulated photons that hit the Shashlik part of ECAL2 (a), as well as the resulting Lednev profiles in x direction (b) and y direction (c). The Lednev curve, extracted by the fit is shown in red and the Lednev curve with parameters used in the reconstruction of real COMPASS events in green.

Table 5.1: Fit results for $N = 2$ and $N = 3$, as well as the parameters for real data reconstruction. The χ^2_{reduced} in the real data case is calculated, using the given parameters and the MC set used for the $N = 2$ and $N = 3$ fit. We show the statistical uncertainties. Shown is the best out of 1000 successful fits.

Type	a_1	a_2	a_3	b_1	b_2	b_3	χ^2_{reduced}
GAMS (RD)	1.191	-0.191	0	6.664	43.777	0	0.890
GAMS ($N = 2$)	-0.0585(7)	1.0585(7)	0	122.3(18)	6.260(14)	0	0.565
GAMS ($N = 3$)	-0.0489(7)	1.263(31)	-0.214(31)	144.5(24)	5.26(7)	1.98(15)	0.559
GAMS-R (RD)	1.191	-0.191	0	6.664	43.777	0	0.650
GAMS-R ($N = 2$)	-0.02418(25)	1.02418(25)	0	320(8)	4.727(9)	0	0.317
GAMS-R ($N = 3$)	0.99944(19)	0.9484(21)	-0.9479(20)	63.2(11)	-71.5(12)	-3.957(18)	0.292
Shashlik (RD)	0.885	-0.140	0.255	8.104	55.860	1.520	0.969
Shashlik ($N = 2$)	1.02606(14)	-0.02606(14)	0	4.792(5)	271.6(27)	0	0.491
Shashlik ($N = 3$)	0.520(10)	-0.0443(5)	0.524(10)	2.03(4)	155.7(17)	9.45(14)	0.378

The χ^2_{reduced} values are always below 1, which hints to a slight overestimation of the uncertainties. In addition, we show only the statistical uncertainties, given by the Minuit2 fit⁶. We performed the fit for all calorimeter cell types with $N = 2$ and $N = 3$. For GAMS and GAMS-R, the χ^2_{reduced} is comparable in both cases and we choose the solution with $N = 2$. For shashlik, we use the $N = 3$ solution. In Table 5.1 we also show the parameters used for real COMPASS data. We draw them next to our best fit in Figs. 5.13 to 5.15. Although the parameters look vastly different, both curves nicely describe the Lednev profile. This signals a good agreement of our calorimeter simulation with the real detector measurements. In a real COMPASS event, we may have multiple clusters in the calorimeters and there are even cases where the photons or electrons/positrons are so close to each other, that two clusters merge to one. During the reconstruction a fit of multiple Lednev profiles is conducted to describe the energy distribution, i.e. a fit with only one profile, a fit with two, three, etc. The fit with the best χ^2 is accepted for each event and yields N reconstructed clusteres, where N is the number of Lednev profiles used.

This description of the shower profiles (see Eq. (5.4)) is only used for the reconstruction of showers in ECAL2. For ECAL1, only lower energetic particles hit the calorimeter cells, leading to a spread across less cells. Therefore, the center-of-gravity and energy of a given shower is extracted by a simple cluster finding algorithm instead. More details on the reconstruction method can be found in Ref. [66].

5.3.9 Energy Calibration of the Calorimeters

There are a many ways in which a photon or electron / positron fails to deposite all of its energy in one of the ECALs. Energy deposited in the absorber parts of the Shashlik cells is not collected. Similarly, energy deposited in the borders between cells is not collected and finally, if parts of the shower escape the calorimeter, this energy is also lost. In the reconstruction this is accounted for with the so called reserv parameter R .

$$E_{\text{final}} = \frac{E_{\text{measured}}}{R}. \quad (5.6)$$

This correction is applied for each reconstructed shower in each calorimeter. Therefore, E_{measured} is the determined energy by the shower fit in the reconstruction and E_{final} is the true energy of the

⁶ The parameters are used without any consideration of their uncertainty in the CORAL reconstruction. Therefore, we do not make detailed studies on the uncertainties here.

shower. The reserv parameter varies for every calorimeter cell type and needs to be calculated for the TGEANT implementation of the calorimeters. We use the MC sample, created for the $\eta\pi^-$ final state for this task and look at the reconstructed π^0 mass. See Section 5.4.4 for more details on the produced $\eta\pi^-$ MC sample. To the reconstructed π^0 mass, we fit a polynomial of first order as the background term, together with a Gaussian for the signal.

$$p_{\gamma\gamma}^2 = (p_{\gamma_1} + p_{\gamma_2})^2 \quad (5.7a)$$

$$\Leftrightarrow m_{\gamma\gamma}^2 = m_{\gamma_1}^2 + m_{\gamma_2}^2 + 2(E_{\gamma_1}E_{\gamma_2} - \vec{p}_{\gamma_1}\vec{p}_{\gamma_2}) \quad (5.7b)$$

$$= 2E_{\gamma_1}E_{\gamma_2}(1 - \cos(\theta_{\gamma_1\gamma_2})) \quad (5.7c)$$

$$= 2 \frac{E_{\gamma_1}^{\text{measured}}}{R_{\gamma_1}} \frac{E_{\gamma_2}^{\text{measured}}}{R_{\gamma_2}} (1 - \cos(\theta_{\gamma_1\gamma_2})) \quad (5.7d)$$

$$= \frac{1}{R_{\gamma_1}} \frac{1}{R_{\gamma_2}} \cdot \underbrace{2E_{\gamma_1}^{\text{measured}}E_{\gamma_2}^{\text{measured}}(1 - \cos(\theta_{\gamma_1\gamma_2}))}_A, \quad (5.7e)$$

where A is independent of the reserv parameters. In order to perform the reconstruction, some reserv parameter was assumed in the first place. Therefore, to extract the new reserv parameter, we write

$$m_{\gamma\gamma,\text{final}}^2 = m_{\gamma\gamma}^2 \frac{R_{\gamma_1}}{R_{\gamma_1}^{\text{new}}} \frac{R_{\gamma_2}}{R_{\gamma_2}^{\text{new}}} \quad (5.8a)$$

$$\Leftrightarrow R_{\gamma_1}^{\text{new}} R_{\gamma_2}^{\text{new}} = m_{\gamma\gamma}^2 \frac{R_{\gamma_1} R_{\gamma_2}}{m_{\gamma\gamma,\text{final}}^2}, \quad (5.8b)$$

where $m_{\gamma\gamma,\text{final}}$ is the desired end mass, i.e. literature value of the π^0 , and $m_{\gamma\gamma}$ is the mass extracted by the fit, i.e. the mean of the fitted Gaussian. The old reserv values are denoted as R_{γ_1} and R_{γ_2} , where γ_1 and γ_2 are the two reconstructed photons that emerged from the π^0 decay. We have enough statistics for each calorimeter cell type and can extract the reserv parameter separately for each calorimeter cell type. Per calorimeter cell type, Eq. (5.8) becomes

$$R^{\text{new}} = m_{\gamma\gamma} \frac{R}{m_{\gamma\gamma,\text{final}}}. \quad (5.9)$$

The newly extracted reserv parameters are tabulated in Table 5.2 and the fit, where the new paramters are extracted is shown in Fig. 5.16.

By design, the reserv parameter needs to be less than 1 to account for the energy loss. In most of the calorimeter cell types however it is close to 1, since there is no dead material in the cells. The only way to lose energy is border or escape effects. For the Shashlik cells, this is of course different, due to the absorber material which is built in and forces the reserv parameter to be smaller than 1. In Fig. 5.17, we can see the two-photon invariant mass in the different calorimeter cell types for the new reserv parameters, given in Table 5.2.

Table 5.2: Determined reserv parameters for all calorimeter cell types.

Calorimeter	Cell type	new reserv
ECAL1	OLGA	0.949497
ECAL1	Mainz	0.942732
ECAL1	GAMS	0.852755
ECAL2	GAMS	0.943747
ECAL2	GAMS-R	0.928736
ECAL2	Shashlik	0.175291

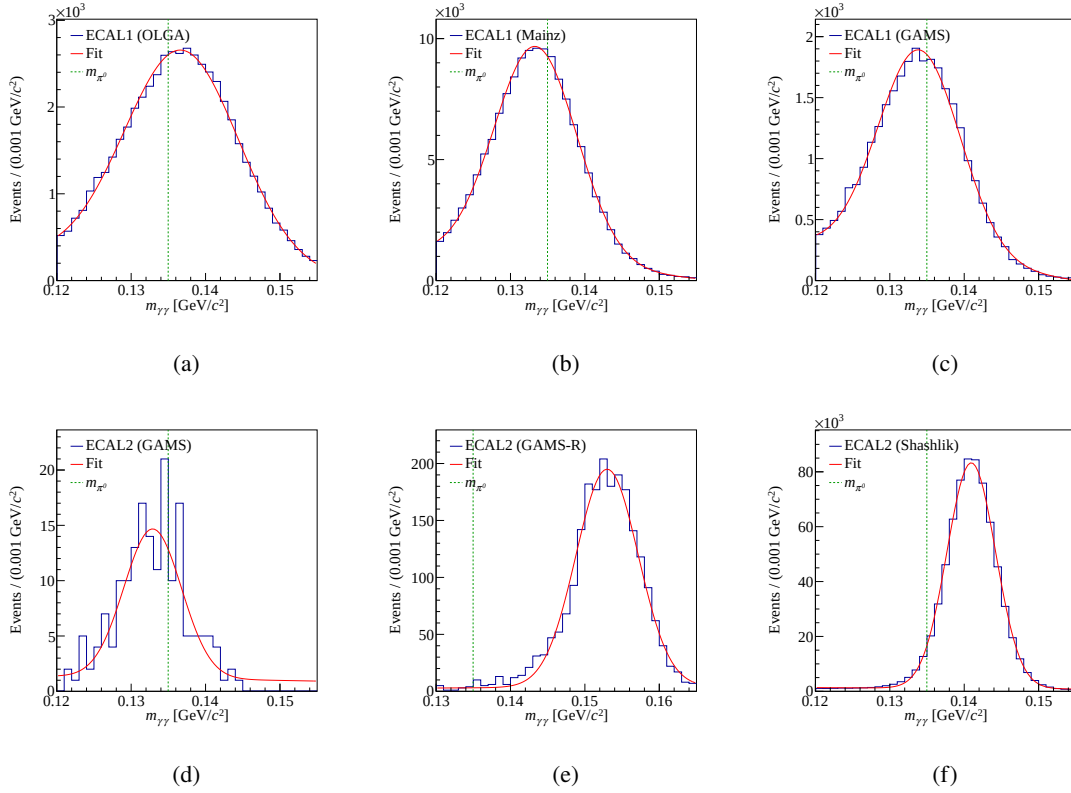


Figure 5.16: Reconstructed π^0 mass from photons hitting the OLGA (a), Mainz (b) or GAMS (c) part of ECAL1 or the GAMS (d), GAMS-R (e) or Shashlik (f) part of ECAL2. The original reserv parameters are used in the reconstruction. In red, we show the Gaussian + polynomial of first order fit. The histogram represents the simulated photons.

5.4 Simulating Events for the 2008 and 2009 Setup

In the previous sections, we have presented all the updates to the TGEANT geometry and calibrations that come therewith that were needed to simulate diffractive final states with TGEANT, were the 2008 / 2009 setup is used. Two key features need to be explained in addition. In TGEANT, we simulate single events under the assumption of a certain beam particle hitting the target. In order to be realistic,

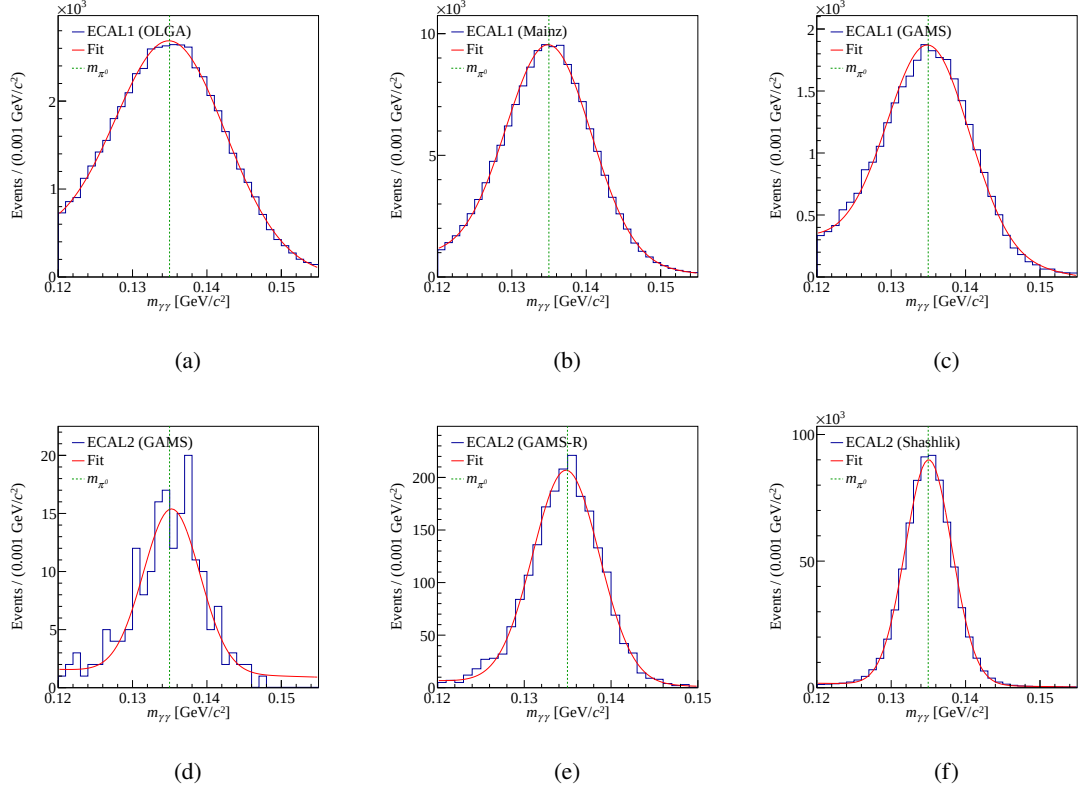


Figure 5.17: Reconstructed π^0 mass from photons hitting the OLGA (a), Mainz (b) or GAMS (c) part of ECAL1 or the GAMS (d), GAMS-R (e) or Shashlik (f) part of ECAL2. The updated reserv parameters from Table 5.2 are used in the reconstruction. In red, we show the Gaussian + polynomial of first order fit. The histogram represents the simulated photons.

we need to create a database of four momenta that mirror the real beam condition from 2008 / 2009. This will be explained in Section 5.4.1. In addition, the final state particles need to be created. For that, we have implemented our own generator in TGEANT Section 5.4.2.

5.4.1 Creating the Beam File

The MC sample for the reactions under investigations shall reflect what was measured at COMPASS, while covering the full phase space in the final state in order to describe the full acceptance of the spectrometer. Naturally, we need to start with the same initial conditions in MC as for the 2008 / 2009 COMPASS data. Each selected event starts with a beam π^- hitting the LH_2 target. In order to reflect reality as well as possible, we reconstruct real beam π^- with their four momenta while hitting the target protons and use them as starting points for the MC simulation. In order to have the largest variety, we use the selected $3\pi^-$ final state for this. The channel $p + \pi^- \rightarrow \pi^- \pi^- \pi^+ p$ is the flag ship channel at COMPASS and is analyzed in great detail in Refs. [6, 7]. We carefully follow the same selection procedure but on the Slot5 production of the 2008 and 2009 data. With this, we get a database of over 10 million beam π^- . More details on the reconstruction of the beam four momentum

can be found in Section 4.2.5.

In COMPASS, the same beam file format is used across all MC simulations. Per event, the kinetic energy E_{Kin} , x -, y -Position and the slopes (θ_{xz}) and (θ_{yz}) are stored. The beam file itself is in a binary format in order to save space and increase read in speed. The correct order of bytes per event is given in Table 5.3.

Table 5.3: Order of bytes per event in the beam file for TGEANT.

Type	Description	Value
int	Event start flag	24
int	Signals a normal beam particle	1
float	x -position	Depends on the event
float	y -position	Depends on the event
float	θ_{xz}	Depends on the event
float	θ_{yz}	Depends on the event
float	E_{Kin}	Depends on the event
int	Event end flag	24

From these variables, the full kinematics of the beam particle can be reconstructed at a given z -position. From relativistic kinematics we know

$$m^2 = E^2 - |\vec{p}|^2 \quad (5.10a)$$

$$\Leftrightarrow |\vec{p}|^2 = (m + E_{\text{Kin}})^2 - m^2 \quad (5.10b)$$

$$\Leftrightarrow |\vec{p}|^2 = E_{\text{Kin}}^2 + 2 \cdot m \cdot E_{\text{Kin}} \quad (5.10c)$$

$$\Leftrightarrow |\vec{p}| = \sqrt{E_{\text{Kin}}^2 + 2 \cdot m \cdot E_{\text{Kin}}} . \quad (5.10d)$$

From basic geometry we get

$$\tan(\theta_{xz}) = \frac{p_x}{p_z}, \quad \tan(\theta_{yz}) = \frac{p_y}{p_z} . \quad (5.11)$$

Therefore we can calculate the absolute value of the beam momentum

$$|\vec{p}| = \sqrt{p_x^2 + p_y^2 + p_z^2} \quad (5.12a)$$

$$\Leftrightarrow |\vec{p}| = \sqrt{p_z^2 \cdot \left(\frac{p_x^2}{p_z^2} + \frac{p_y^2}{p_z^2 e^2} + 1 \right)} \quad (5.12b)$$

$$\Leftrightarrow |\vec{p}| = p_z \cdot \sqrt{\left(\frac{p_x}{p_z} \right)^2 + \left(\frac{p_y}{p_z} \right)^2 + 1} \quad (5.12c)$$

$$\Leftrightarrow p_z = \frac{|\vec{p}|}{\sqrt{\left(\frac{p_x}{p_z} \right)^2 + \left(\frac{p_y}{p_z} \right)^2 + 1}} \quad (5.12d)$$

$$\Leftrightarrow p_z = \frac{|\vec{p}|}{\sqrt{\tan(\theta_{xz})^2 + \tan(\theta_{yz})^2 + 1}}. \quad (5.12e)$$

The beam momentum, together with the space point of the beam particle, is the starting point. Unfortunately, the z -position is not stored in the beam file and we need to propagate all information to a fixed starting position in z before feeding it to the beam file and TGEANT. We choose -100 cm as a starting point. The kinematics of all beam particles that are stored in the beam file are shown in Fig. 5.18.

Inside TGEANT, we calculate the number of beam particles in the beam file⁷ and draw a random starting position in the beam file. From that position onward, we read in exactly the amount of beam particles needed for the simulation (the number is given in the option file of TGEANT). This way, we ensure different beam particles for different simulations. Our beam file uses real selected events and therefore covers already the initial state acceptance, i.e. CEDAR and trigger detectors upstream of the target. For our simulation, we therefore disable any destructive interaction of the beam particle with material upstream of the target. TGEANT then propagates the beam particle to the target region and draws a random depth in the target to create the PV. Once the PV is created, we invoke the final state generator (see Section 5.4.2).

5.4.2 The Phase-Space Generator

Once, the PV is determined in TGEANT, the final state generator is invoked. In this work and together with [10], we have developed a new generator that surpasses the generator developed in Ref. [51]. In this section, we will explain its main working principle and features. The idea is to keep the generator somewhat general, while still being focused on generating reactions of the type $\pi^- p \rightarrow \text{something} + p$, i.e. where we have a beam π^- , scattering of a fixed proton target and the proton target stays in tact. We have decided to use a JavaScript Object Notation (JSON) option file to guide the generator. An example option file may look like Fig. 5.19.

⁷ This is easily possible, since every event has a fixed number of bytes. From the total size of the file and that number, the amount of beam particles in the file is directly given.

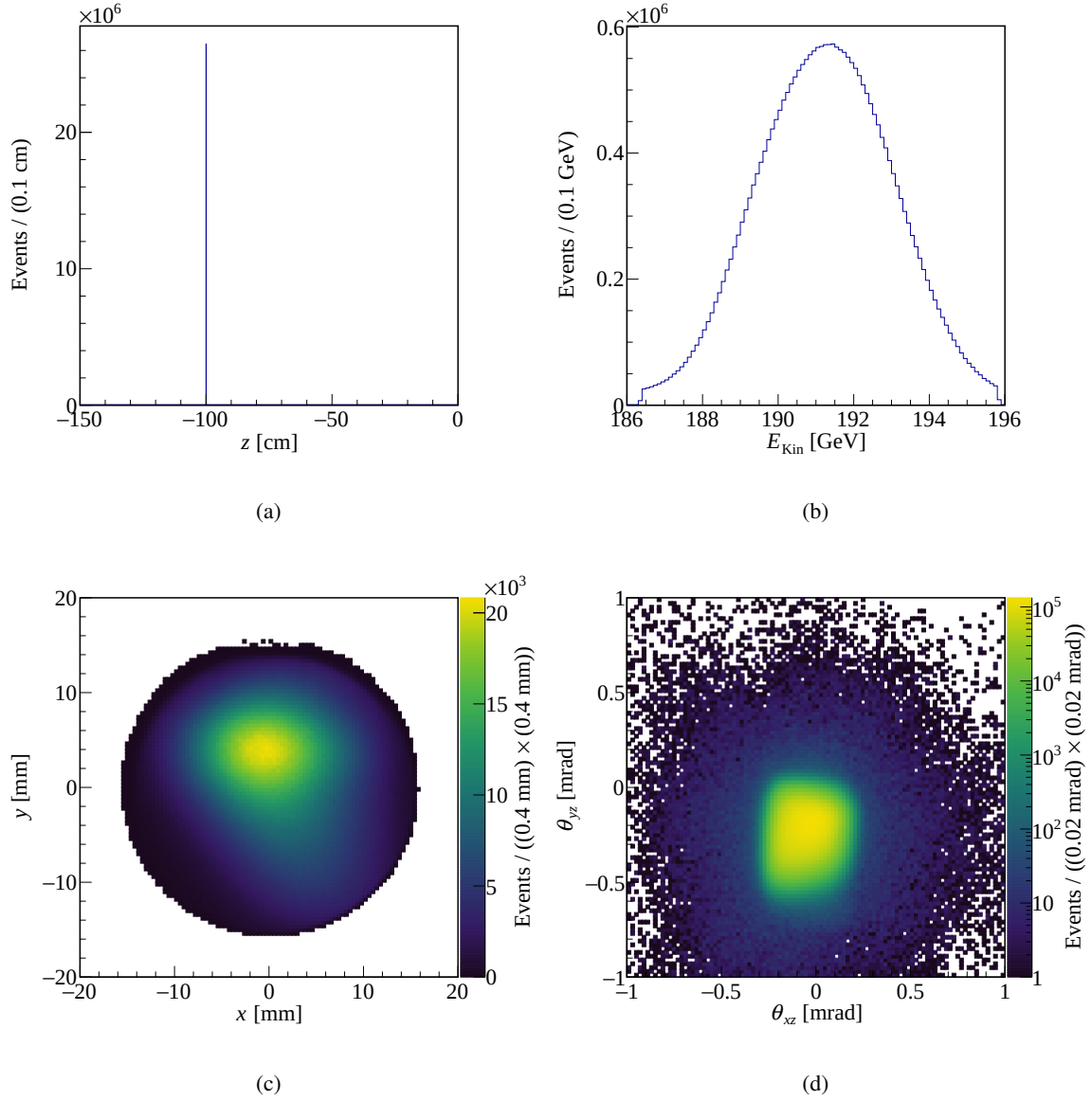


Figure 5.18: z position (a), kinetic energy (b), x vs. y position (c) and θ_{xz} vs θ_{yz} (d) for all beam file particles.

```

{
  "Finalstate": {
    "proton": {},
    "piMinus 1": {},
    "piMinus 2": {},
    "piPlus": {},
    "pi0": {
      "gamma 1": {},
      "gamma 2": {}
    }
  },
  "tprime": [0.1, 1.0],
  "mx": [0.7, 3.0],
  "mtprime hist": {
    "file": "/home/pekeler/EtaPi.root",
    "hist name": "tpmx"
  },
  "Window": {
    "UseWindow": true,
    "Sort3Pi2GBonn": true,
    "Low": 0.52,
    "High": 0.58,
    "Particles": ["piMinus", "piPlus", "gamma", "gamma"]
  },
  "Test generator with N events": 1000000
}

```

(a)

```

{
  "Finalstate": {
    "proton": {},
    "piMinus": {},
    "eta": {
      "piMinus": {},
      "piPlus": {},
      "pi0": {
        "gamma 1": {},
        "gamma 2": {}
      }
    }
  },
  "tprime": [0.1, 1.0],
  "mx": [1.8, 6],
  "costheta": 0.75,
  "Test generator with N events": 0
}

```

(b)

Figure 5.19: Example JSON files for steering of the phase-space generator. The different fields are explained in Section 5.4.2. With (a) a test set of $\pi^-\pi^-\pi^+\pi^0$ data, where the $\pi^-\pi^+\pi^0$ mass is around the η mass and with (b) a $\eta\pi^-$ simulation is initiated.

In the following, we will guide the way through the phase-space generator, while looking at the different fields in the steering file.

Final state

Starting from the kinematics of the beam particle and the target proton, a specific final state is created. Each given particle may decay further. The chain you see in Fig. 5.19(b) expands to

$$\pi^- p \rightarrow p \pi^- \eta; \quad \eta \rightarrow \pi^- \pi^+ \pi^0; \quad \pi^0 \rightarrow \gamma \gamma. \quad (5.13)$$

Due to the nature of JSON files, keys need to be unique on the same level, i.e. the π^0 decay to two photons needs to be specified such that each photon gets a unique number appended. Each of the decays shown in Eq. (5.13) is now simulated according to phase space, i.e. we use `TGenPhaseSpace` from ROOT in order to decay the initial state ($\pi^- + p$) to the final state ($p \pi^- \eta$). To be more precise, `TGenPhaseSpace` produces the final state particles with a given weight between 0 and 1. In order to have a true phase-space-like distribution, each generation has to be checked against a random number between 0 and 1. Only if the weight is smaller than the random number, can we accept the final state kinematics. One can of course achieve the same result as comparing with the random number between 0 and 1 when weighting the final state event by that weight. This however is not practical in a MC simulation where we only have the choice to simulate the event or not. This procedure is repeated for every chain implemented. Now, we have a full set of four vectors for the final state and may feed them to the simulation. To be more precise, in TGEANT we remove the navigation of the incident beam particle at the PV and start navigation of the final state particles. This whole procedure is only useful, if the decays specified in the JSON file happen at so small length scales that they happen instantly for the scope of the simulation. Usually, navigation is on a sub mm level.

tprime⁸ (optional argument)

If this field is given and **mxtprime hist** is not, t' is calculated for each generated final state. If the generated final state is not in the given t' range, the final state generation starts over.

mx (optional argument)

If this field is given and **mxtprime hist** is not, m_X (the mass of the final state without the proton) is calculated for each generated final state. If the generated final state is not in the given m_X range, the event is disregarded and the final state generation starts over.

mtpprime hist (optional argument)

If this field is given, a TH2 (two dimensional ROOT [65] diagram) is extracted from a file given in the field **file** with the name specified in the **hist name** field. From this diagram, a value for t' and m_X is drawn randomly. With this, the final state is created. This reduces the available phase space for the specified final state by a lot and makes the final state simulation converge much faster. It comes at the expense of destroying the overall phase-space distribution in m_X and t' . This may or may not be fine for different analysis and needs to be kept in mind.

Window (optional argument)

With this field, we can reduce the simulated phase space of the final state. The **UseWindow** field

⁸ See glossary for the definition of t'

switches this feature on / off. The **Low** and **High** fields specify a lower and upper border in GeV for the combined mass of all particles given in the **Particles** field. If particles are specified which are not part of the final state, an error is thrown. If there is more than one combination in the final state to achieve a combined particle given in the **Particles** field, all combinations are checked. If this field is given, the final state simulation is redone if the combined mass of the specified particles is not inside the **Low High** window. Lastly, due to the way we select our MC truth information in PHAST, we need a certain order of particles to be fed to the GEANT4 simulation, i.e. p, π^- , particles specified in the **Window** field. To force this ordering, the option **Sort3Pi2GBonn** needs to be true.

Test generator with N events

Here, any number larger than 0, produces **N** final states from the same initial state and only feeds the first one to GEANT4. The others are stored as additional MC truth information. This disables the GEANT4 simulation of the other final states but enables the analysis of the generator in the Toolbox of TGEANT.

5.4.3 Simulated Final States in This Thesis

After all updates to TGEANT, described in this chapter, a multitude of MC simulations were performed on the local cluster in Bonn [43]. We have generated approximately 120×10^6 events in the $\pi^- \pi^- \pi^+ \eta$ final state for Ref. [27] and 40×10^6 events for the $3\pi\pi^0$ final state, as well as 20×10^6 events for the $3\pi\eta$ final state for Ref. [12]. The latter two, use the **Window** option, discussed in the previous section, in order to generate reliable 4 body final state events for the $\eta\pi^-$ and $\eta'\pi^-$ final states. See Eq. (3.1) for details on the decay chain. In addition, we produced 30×10^6 events in the $\eta\pi^-$ and $\eta'\pi^-$ final state, respectively, in the kinematic range $1.8 \text{ GeV} < m_{\eta^{(\prime)}\pi^-} < 6 \text{ GeV}$, $0.1 \text{ GeV}^2/c^2 < t' < 1.0 \text{ GeV}^2/c^2$ and $|\cos \theta_{\text{GJ}}| > 0.5$. For these, we have produced true phase-space-distributed signal MC, i.e. a phase-space-distributed sample of $\eta^{(\prime)}\pi^- p$ from the $\pi^- p$ initial state, where the $\eta^{(\prime)}$ decays to $\pi^- \pi^+ \pi^0(\eta)$ according to phase space, as well as the $\pi^0(\eta)$ to two photons. We will give a short rundown of the event selection of these events and discuss key aspects of the MC generation, as well as the acceptance coverage of the $\eta^{(\prime)}\pi^-$ sample, since we will use it in Chapter 6.

5.4.4 Results from the Simulation of the $\eta\pi^-$ and $\eta'\pi^-$ Final States

The event selection of the MC data follows closely the steps in Chapter 4. There are however certain differences. First, we do not include a bad run list for MC data for obvious reasons. Second, any part of the selection that is performed upstream of the target, i.e. initial state identification with the CEDAR detectors and upstream triggers is excluded from the selection, since all of this is already considered when creating the beam file (see Section 5.4.1). In TGEANT, we do not track optical photons and thus, the RICH detector only acts as dead material in the simulation. Final state particle identification by the RICH detector, is performed using a lookup table, produced in Ref. [8], using real COMPASS events. For a given RICH likelihood-ratio threshold and particle four momentum, it can be determined if the RICH detector would correctly identify the particle as a Pion. All other parts of the event selection are performed the same way as described in Chapter 4. We use, for example, the same two-photon invariant mass cuts, to select the $\pi^0(\eta)$ as we use for real COMPASS events. In the

following, we show exemplary the kinematic distributions at different cut stages in Antok-Style and for the $\eta'\pi^-$ final state.

In Fig. 5.20(a), we see the z position of the PV. Note that the rise to larger z is due to the geometry of the target, as described in Section 5.3.2. The radial distribution is shown in Fig. 5.20(b).

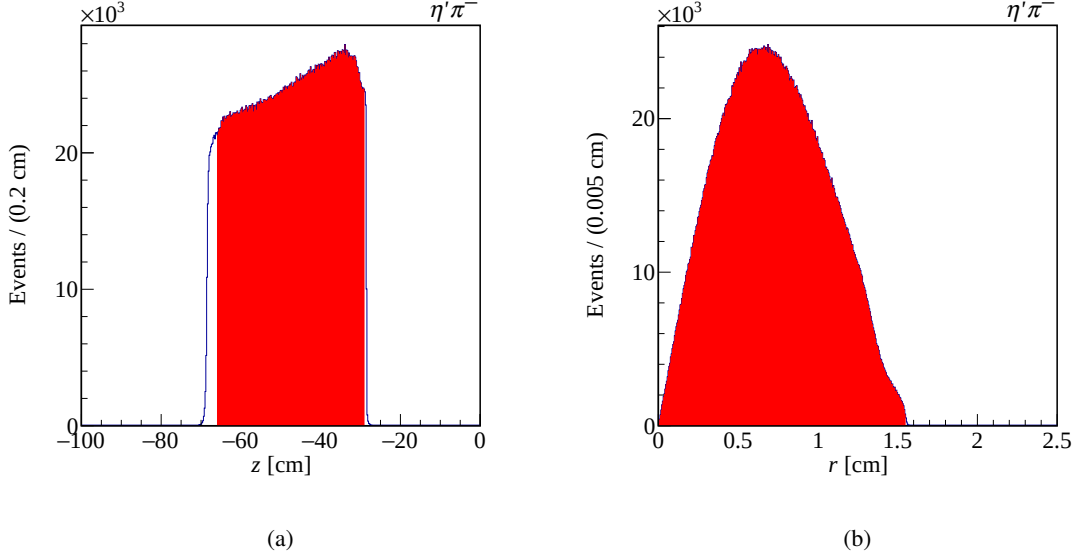


Figure 5.20: z position (a) and radial distance (b) of the PV for the MC sample of the $\eta'\pi^-$ final state in Antok-Style. The remaining events after the cut are shown in red.

In Fig. 5.21(a), we nicely see the impact of the RICH pipe and a smooth energy distribution. One can also observe the inner structure of the cells, which is a direct consequence of the improvements on ECAL2, described in Section 5.3.5. This nicely compares to the real COMPASS measurement in Fig. 4.4(a).

Fig. 5.22 shows the two-photon invariant mass distributions in the two final states. Since we created signal MC, i.e. only events where exactly one η is present (in the $\eta'\pi^-$ final state) per event, we see a clean peak around the η mass with a slight tail to lower masses.

The coplanarity (Figs. 5.23(a) and 5.23(b)) and exclusivity (Fig. 5.21(b)) distributions look clean and comparable to their real data counter parts. See Figs. 4.7(a) and 4.7(b) and Fig. 4.8(a) for comparison. Details on these quantities are explained in Section 4.2.5.

Lastly, we have a very clean $\pi^+\pi^-\gamma\gamma$ invariant mass spectrum that peaks around the $\eta^{(\prime)}$ mass in Fig. 5.24.

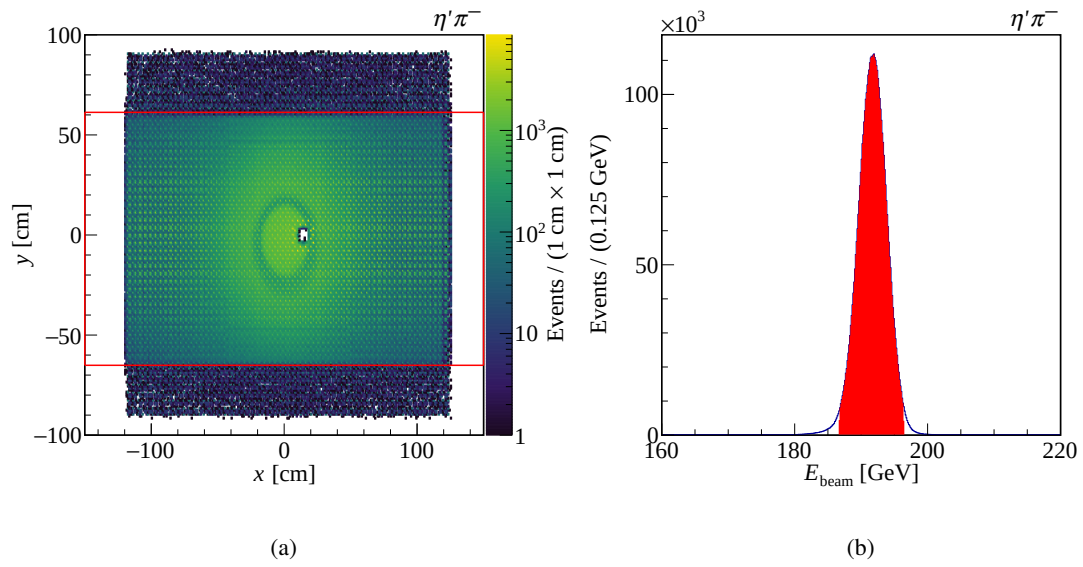


Figure 5.21: ECAL2 cluster position (a) and reconstructed beam energy (b) for the MC sample of the $\eta' \pi^-$ final state in Antok-Style. In (a), the cut ranges are drawn with red lines. In (b), the remaining events after the cut are shown in red.

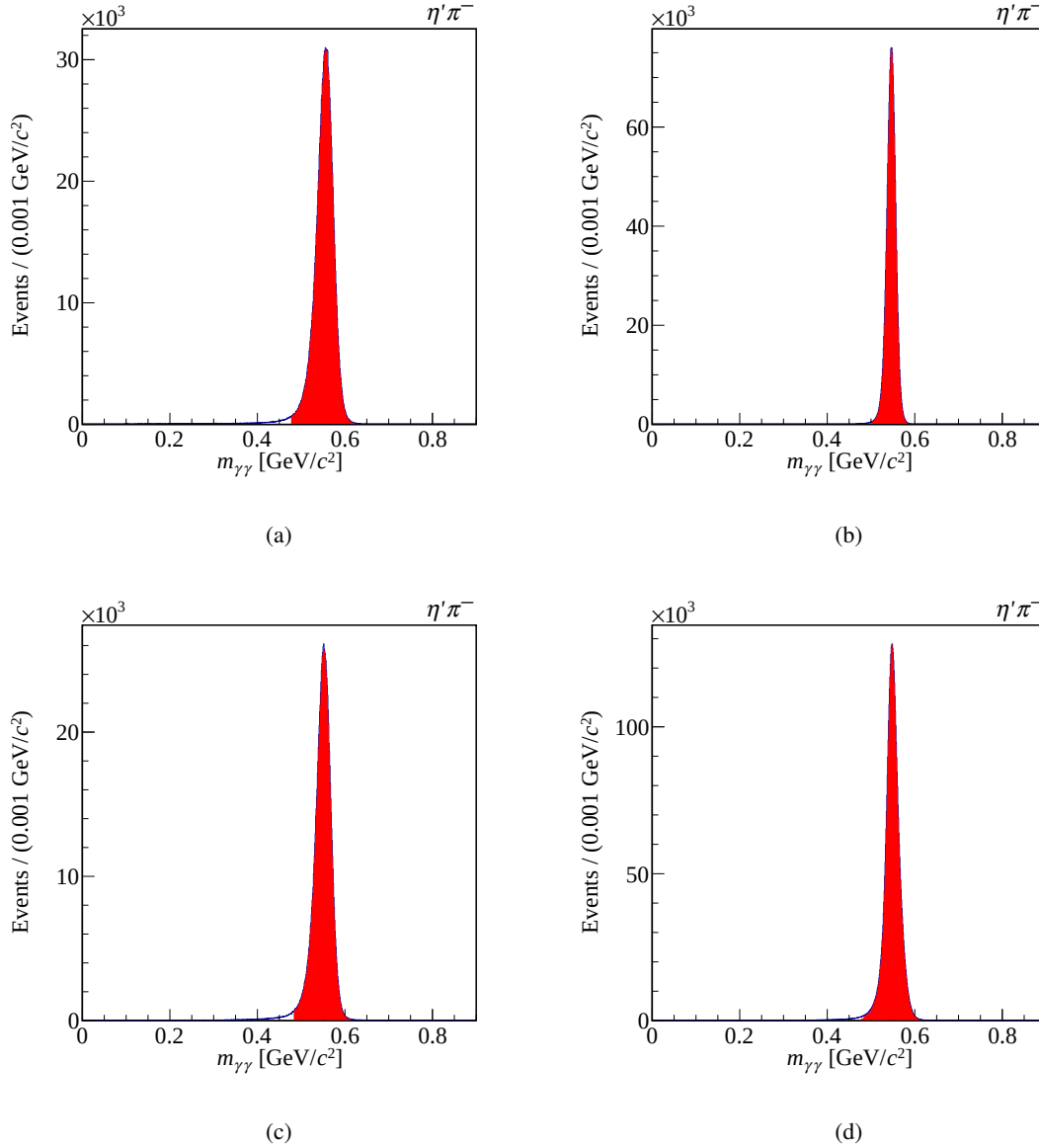


Figure 5.22: Two-photon invariant mass in the MC sample for the $\eta' \pi^-$ final state. In (a), both photons are reconstructed in ECAL1. In (b), both photons are reconstructed in ECAL2. In (c), one photon is reconstructed in ECAL1 and one in ECAL2. In (d), all combinations are shown together. The histograms are created in Antok-Style, where no kinematic fit of the two-photon system and no cut on the $\eta' \pi^-$ mass was applied. Exclusivity and coplanarity cuts are applied on the final state with raw, not yet kinematically fitted, photons. The remaining events after the cut are shown in red.

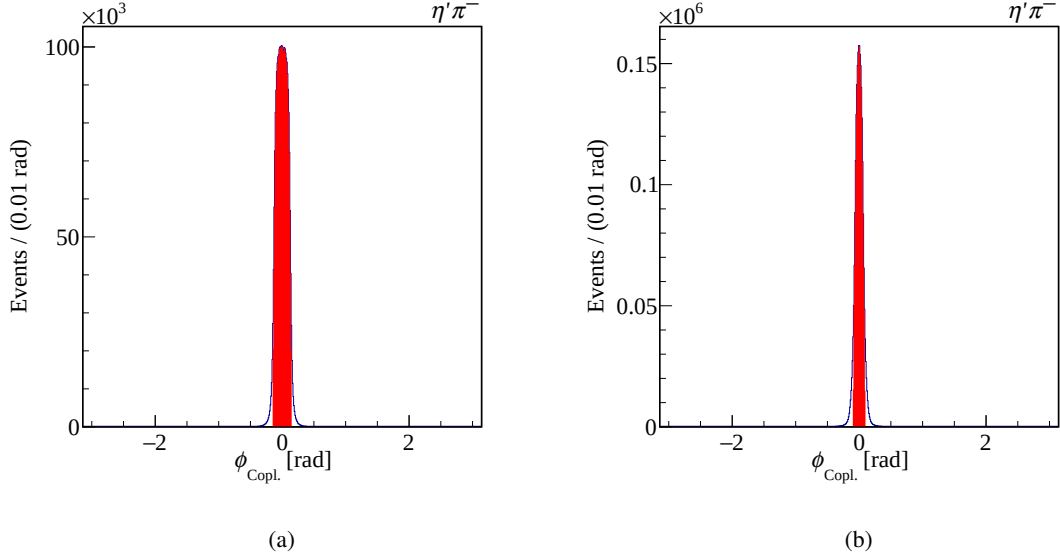


Figure 5.23: Coplanarity angle for an even slab combination in the RPD (a) and an odd one (b) for the MC sample of the $\eta'\pi^-$ final state in Antok-Style. The remaining events after the cut are shown in red.

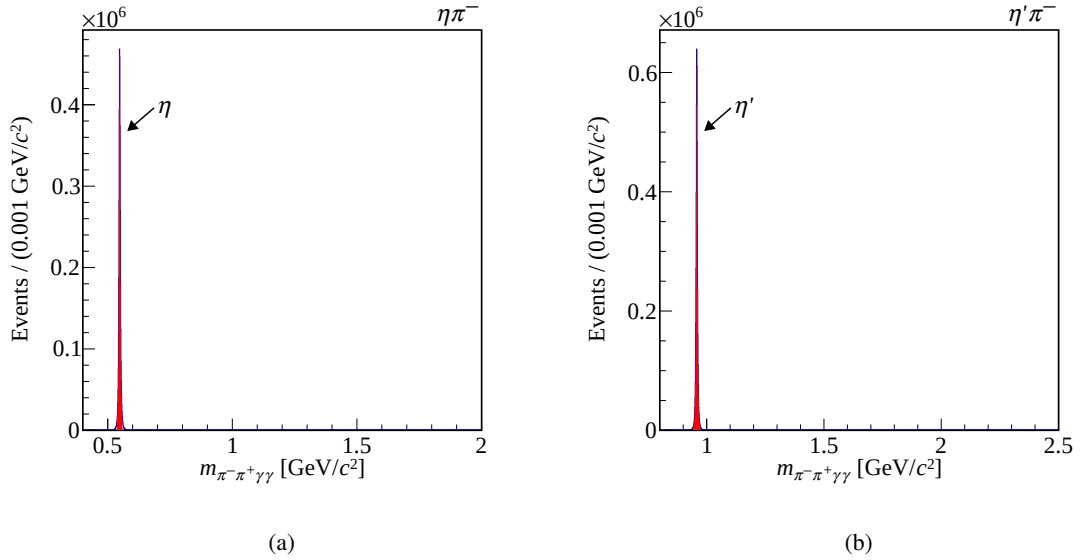


Figure 5.24: Mass of the $\pi^-\pi^+\gamma\gamma$ system for the $\eta\pi^-$ (a) and $\eta'\pi^-$ (b) final state in the MC sample, in Antok-Style. The remaining events after the cut are shown in red.

The final number of events after each cut is tabulated in Table 5.4. The largest difference between the two MC samples is the different two-photon invariant mass acceptance. It is much more likely that the COMPASS spectrometer does not detect π^0 , then η . This is due to the fact that photons that emerge from a π^0 decay are more likely to fly close to the hole in ECAL2 and are therefore missed in the reconstruction.

Table 5.4: Number of events in the $\eta^{(\prime)}\pi^-$ MC sample left after each cut in Waterfall-Style.

Cut name	$\eta\pi^-$	$\eta'\pi^-$
After preselection	30 000 384	29 999 542
Exactly one PV	29 870 016	29 890 740
Trigger	21 934 794	21 905 303
PV radius	21 926 521	21 897 065
PV z position	20 368 378	20 345 253
Three charged particles	15 109 792	14 921 095
Charge conservation	14 940 313	14 753 409
Beam time	14 940 313	14 753 409
Two photons	5 586 795	7 506 519
Recoil proton	5 433 046	7 299 329
Anti exclusivity	5 366 432	7 293 493
RICH	5 352 003	7 272 216
Two-photon invariant mass	3 965 322	5 393 258
Kinematic fit	3 964 981	5 393 239
Coplanarity	3 577 364	4 852 262
Exclusivity	3 450 388	4 685 340
$\pi^+\pi^-\gamma\gamma$ invariant mass	3 325 947	4 607 481

After the event selection, we can look at the acceptance in interesting kinematic variables and compare the amount of accepted MC events to the selected real COMPASS events. A rule of thumb is that we need roughly 10 to 20 times more accepted MC events compared to selected COMPASS events, in order to reduce the statistical uncertainty of the MC sample compared to the real data sample. In Figs. 5.25 and 5.26 we see clearly that this goal is matched for both MC samples. In addition, we see that the acceptance is rather flat in $m_{\eta^{(\prime)}\pi^-}$ and t' , while it is falling off for larger $|\cos\theta_{GJ}|$. Further details on the acceptance can be found in Appendix B.

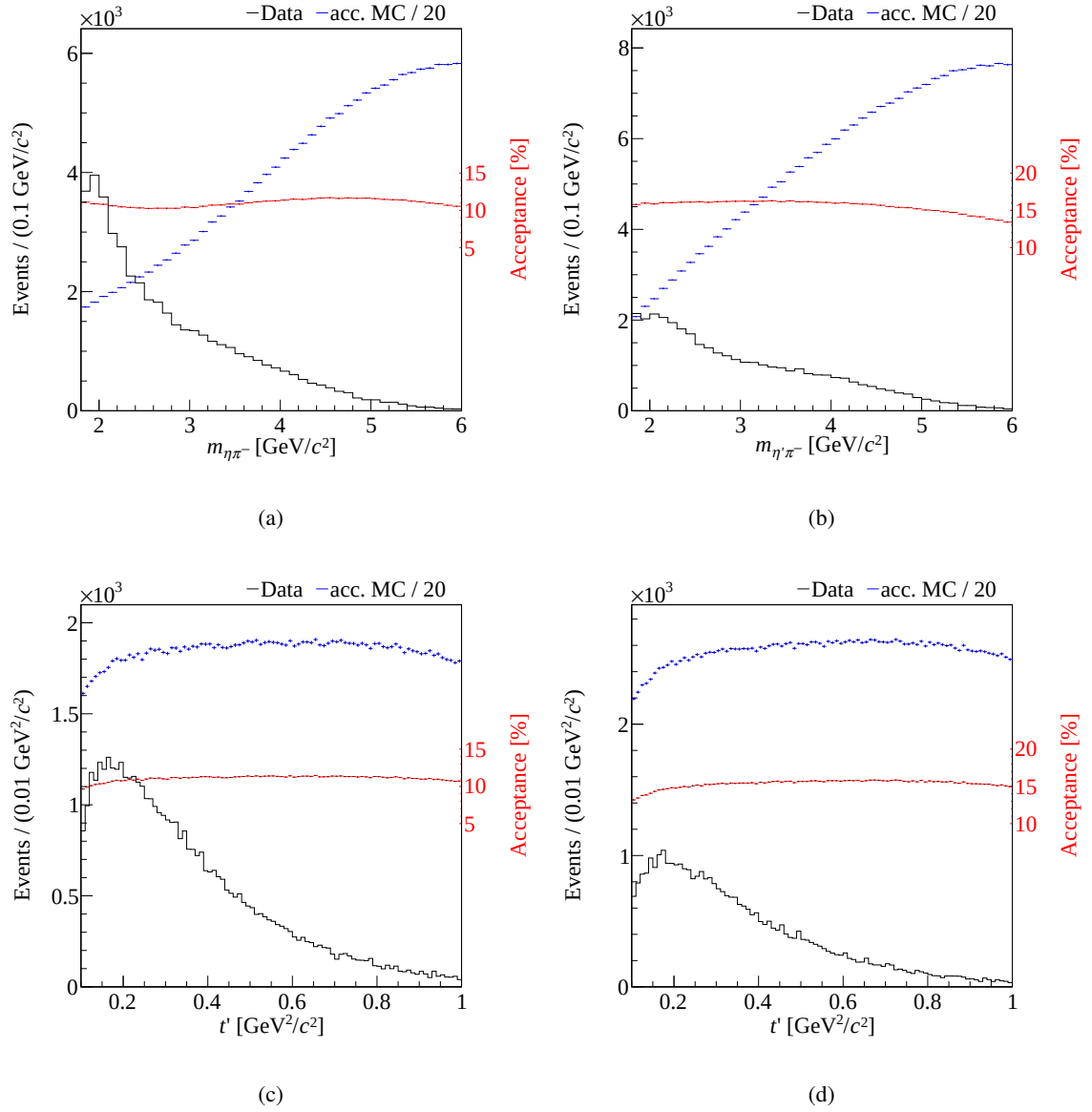


Figure 5.25: Acceptance for $m_{\eta\pi^-}$ (a) and t' (c) in the $\eta\pi^-$ and $m_{\eta'\pi^-}$ (b) and t' (d) in the $\eta'\pi^-$ final state. The data is depicted in black and the accepted MC, scaled by 1/20 in blue. The red points show the actual acceptance value.

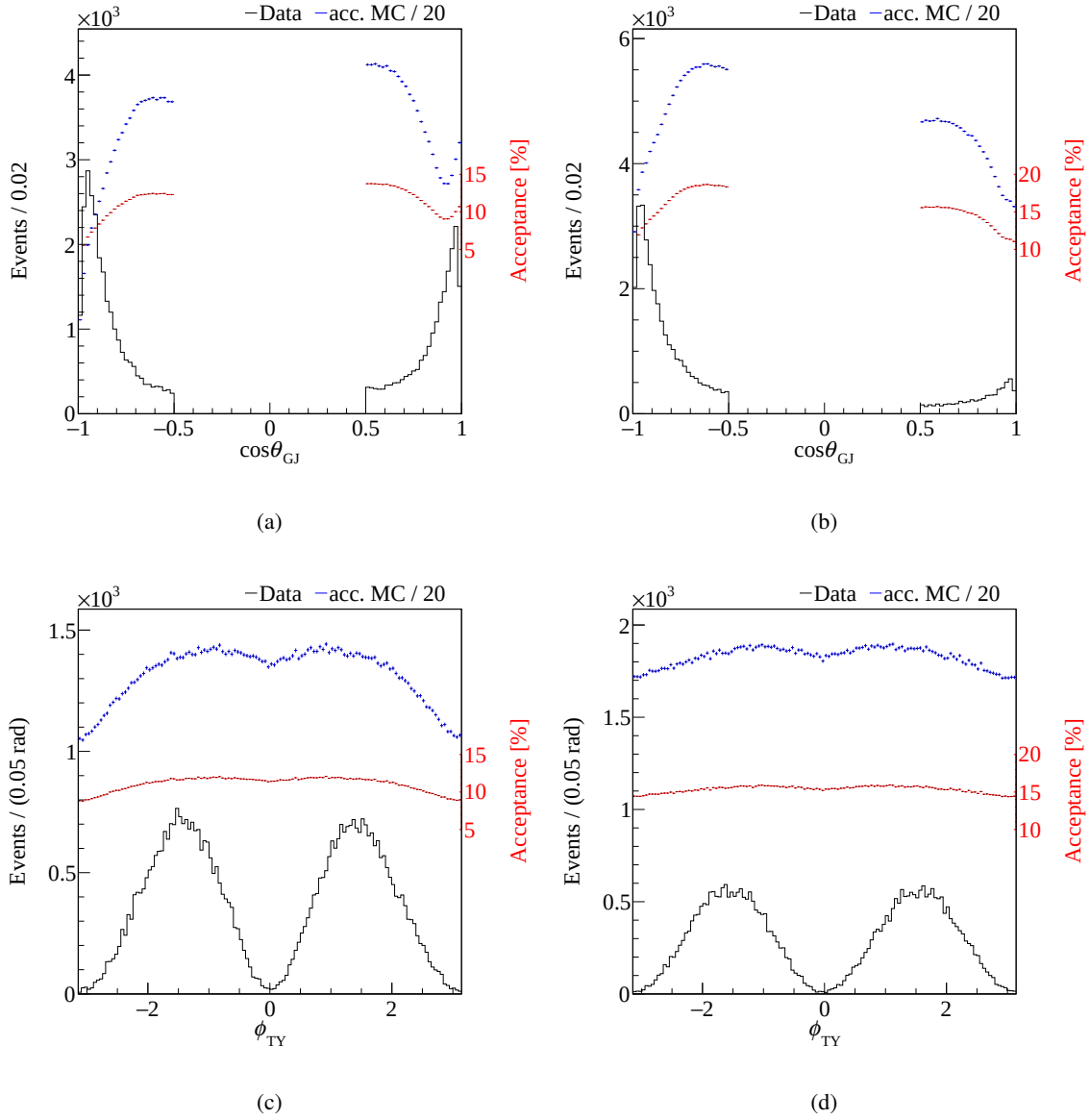


Figure 5.26: Acceptance for $\cos\theta_{GJ}$ (a) and ϕ_{GJ} (c) in the $\eta\pi^-$ and $\cos\theta_{GJ}$ (b) and ϕ_{GJ} (d) in the $\eta'\pi^-$ final state. The data is depicted in black and the accepted MC, scaled by 1/20 in blue. The red points show the actual acceptance value.

5.4.5 Comparing the Resolution from Data and Simulation

In Chapter 4, we have described the selection of data taken with the COMPASS experiment, for the reaction $\pi^- + p \rightarrow \eta^{(\prime)} + \pi^- + p$. Additionally, in Section 5.4.4, the generated phase space MC data, simulated with TGEANT, were selected in the same way. In the $\eta^{(\prime)}\pi^-$ final state, we use the calorimeters for photon reconstruction and the tracking detectors for charged particle reconstruction. In this section, we will compare the resolution of the calorimeters and the tracking system for both final states.

The resolution of the calorimeters will be investigated with events, where both photons are found in the same calorimeter. We will draw the two-photon invariant mass spectrum before the kinematic fit but with exclusivity and coplanarity cut already applied for the COMPASS and MC data. Then we fit two Gaussians and a polynomial of third order to the distribution (see Eq. (5.14)).

$$f(m) = A \left[w e^{-\frac{1}{2} \left(\frac{m-\mu}{\sigma_1} \right)^2} + (1-w) e^{-\frac{1}{2} \left(\frac{m-\mu}{\sigma_2} \right)^2} \right] + \sum_{i=0}^3 a_i m^i, \quad (5.14)$$

where A , w , μ , σ_1 , σ_2 and a_i are free parameters. The resolution is then given by

$$\sigma = \sqrt{w \cdot \sigma_1^2 + (1-w) \sigma_2^2}, \quad (5.15)$$

and the uncertainty will be calculated using gaussian error propagation. Similarly, for ECAL1, we will use events where both photons end up in ECAL1. Additionally, we will also perform this exercise for events where one photon ends up in ECAL1 and one in ECAL2.

The tracking resolution will be judged on the $m_{\pi^- \pi^+ \gamma \gamma}$ spectrum, where the two photons are already kinematically fitted and the calorimeter resolution is therefore eliminated.

Additionally, we will compare the coplanarity angle and the exclusivity distribution (see Section 4.2.5). From the coplanarity angle, we get a good idea how well the RPD is implemented in MC.

All plots are created in Antok-Style and can be found in Fig. 5.27 for the $\eta\pi^-$ final state and in Fig. 5.28 for the $\eta'\pi^-$ final state. The extracted resolutions are tabulated in Table 5.5.

Table 5.5: Extracted resolutions for the COMPASS data and the MC sample in the $\eta\pi^-$ and $\eta'\pi^-$ final state. The superscript in the two-photon invariant mass ($m_{\gamma\gamma}$) indicates in which calorimeters the photons are reconstructed.

	Reference	Final state	$\sigma_{\text{COMPASS}} [\text{MeV } c^{-2}]$	$\sigma_{\text{MC}} [\text{MeV } c^{-2}]$
$m_{\gamma\gamma}^{\text{ECAL1}}$	Fig. 5.27(a)	$\eta\pi^-$	8.26(10)	6.77(7)
$m_{\gamma\gamma}^{\text{ECAL1+ECAL2}}$	Fig. 5.27(b)	$\eta\pi^-$	7.52(5)	5.627(20)
$m_{\gamma\gamma}^{\text{ECAL2}}$	Fig. 5.27(c)	$\eta\pi^-$	3.63(5)	3.9(10)
$m_{\pi^- \pi^+ \gamma \gamma}$	Fig. 5.27(g)	$\eta\pi^-$	2.89(4)	3.360(20)
$m_{\gamma\gamma}^{\text{ECAL1}}$	Fig. 5.28(a)	$\eta'\pi^-$	24.6(9)	17.42(25)
$m_{\gamma\gamma}^{\text{ECAL1+ECAL2}}$	Fig. 5.28(b)	$\eta'\pi^-$	21.38(11)	16.13(21)
$m_{\gamma\gamma}^{\text{ECAL2}}$	Fig. 5.28(c)	$\eta'\pi^-$	11.96(14)	11.24(6)
$m_{\pi^- \pi^+ \gamma \gamma}$	Fig. 5.28(g)	$\eta'\pi^-$	3.70(9)	2.735(8)

We clearly see, that all resolutions are comparable, while the MC resolution is slightly smaller. It is important to note, that the MC sample is created from $\eta^{(\prime)}\pi^-$ events, which does not allow any

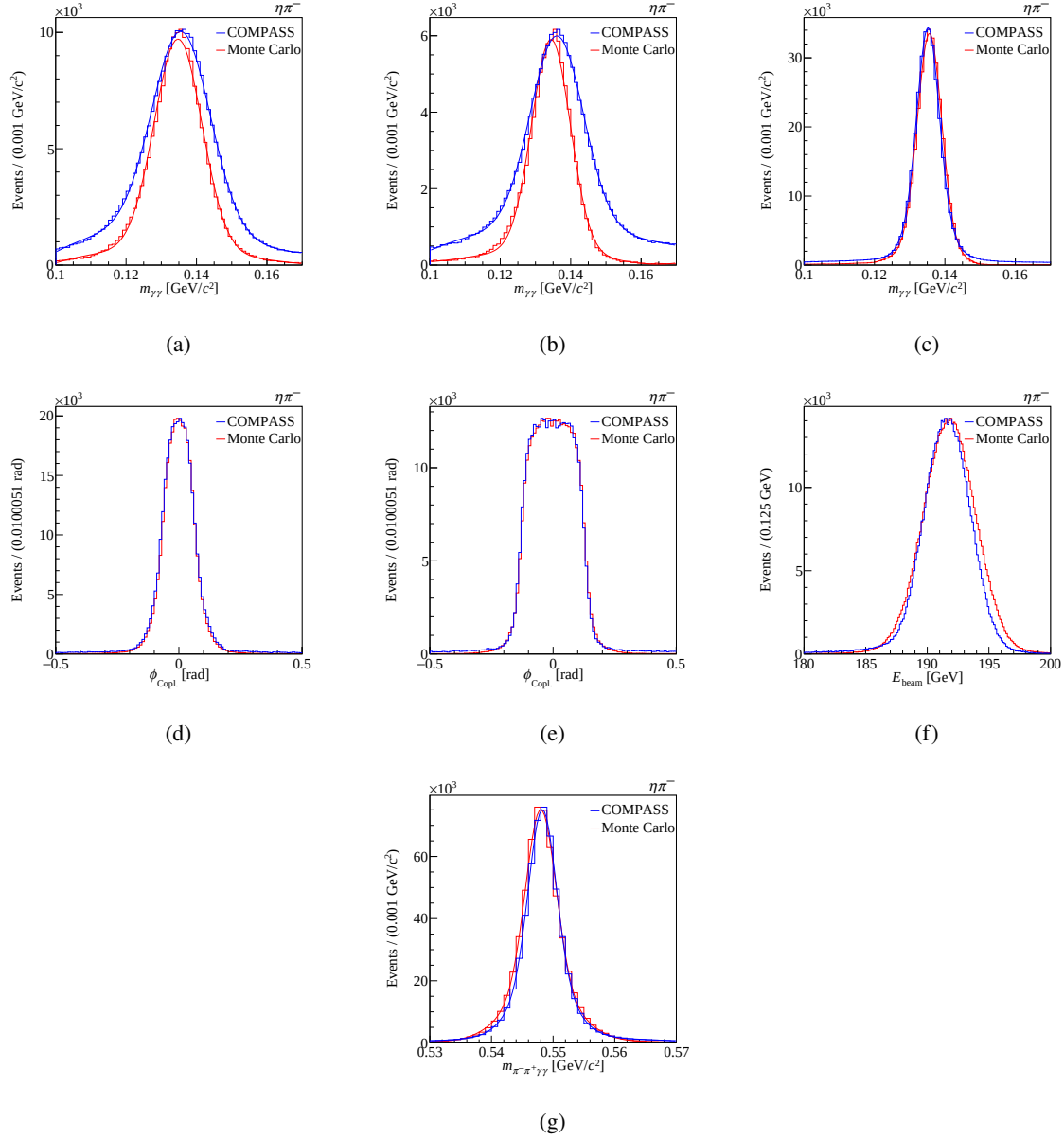


Figure 5.27: Comparison of the COMPASS resolution in blue and the MC resolution in red ($\eta\pi^-$ final state) for the two-photon invariant mass, where both photons are reconstructed in ECAL1 (a), one is reconstructed in ECAL1 and one in ECAL2 (b) and both are reconstructed in ECAL2 (c). Additionally the resolution of the coplanarity angle for an odd slab combination (d) and an even slab combination (e) as well as the reconstructed beam energy (f) and the $\pi^+\pi^-\gamma\gamma$ invariant mass. All diagrams are in Antok-Style. For the two-photon invariant mass spectra no kinematic fit of the two-photon system and no cut on the $\eta'\pi^-$ mass are applied, while exclusivity and coplanarity cuts are applied on the final state with raw, not yet kinematically-fitted, photons.

background photons. Therefore, that distribution does not exhibit a large background. This is of course not true for the COMPASS events, hence the larger background. For comparison, the COMPASS

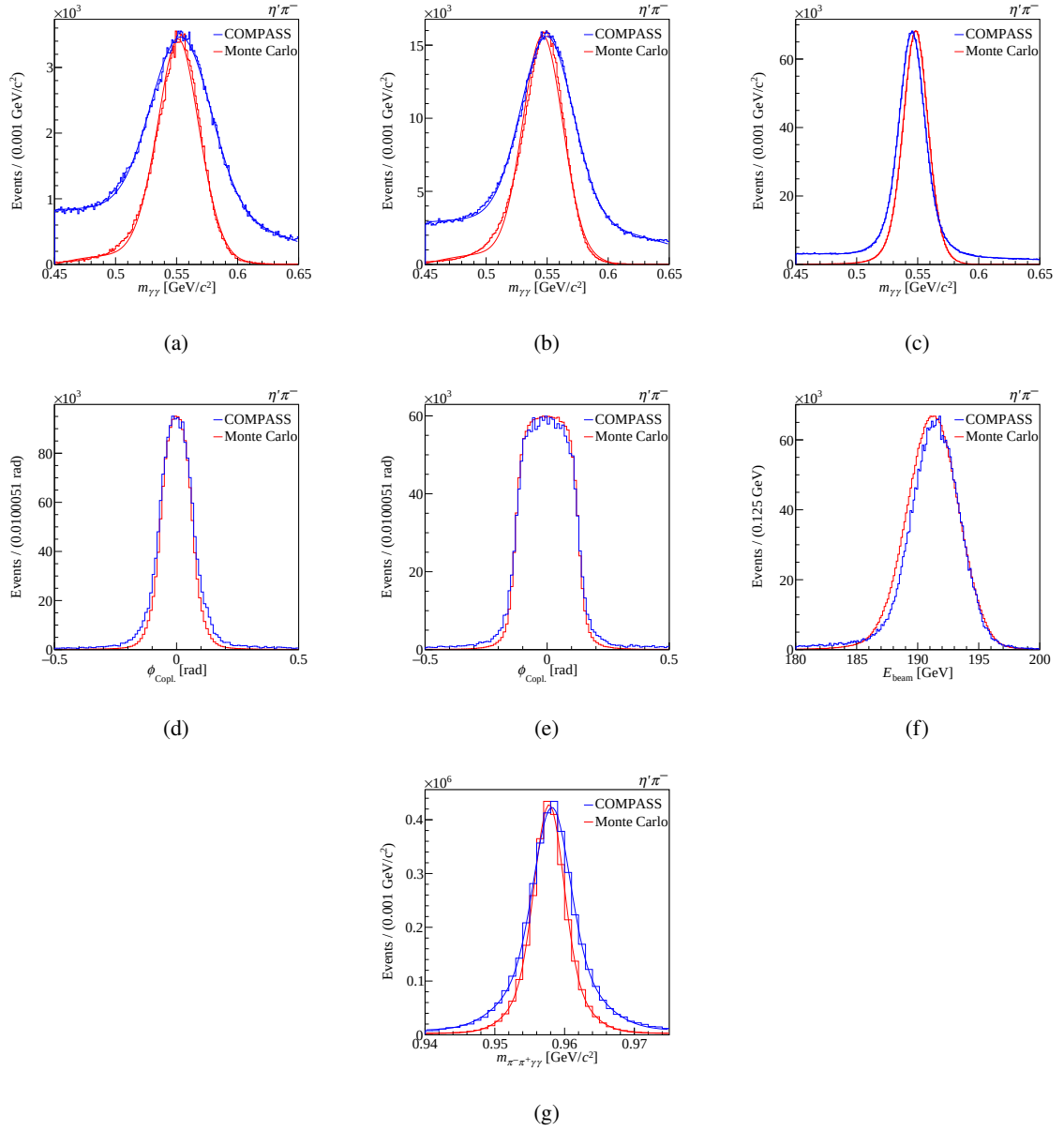


Figure 5.28: Comparison of the COMPASS resolution in blue and the MC resolution in red ($\eta'\pi^-$ final state) for the two-photon invariant mass, where both photons are reconstructed in ECAL1 (a), one is reconstructed in ECAL1 and one in ECAL2 (b) and both are reconstructed in ECAL2 (c). Additionally the resolution of the coplanarity angle for an odd slab combination (d) and an even slab combination (e) as well as the reconstructed beam energy (f) and the $\pi^+\pi^-\gamma\gamma$ invariant mass. All diagrams are in Antok-Style. For the two-photon invariant mass spectra no kinematic fit of the two-photon system and no cut on the $\eta'\pi^-$ mass are applied, while exclusivity and coplanarity cuts are applied on the final state with raw, not yet kinematically-fitted, photons.

event distribution of the two-photon invariant mass for the $\eta\pi^-$ final state are given in Fig. A.11 and for the $\eta'\pi^-$ final state in Fig. 4.5.

Fitting the Double-Regge Amplitude

In Chapter 2, the dominant production mechanisms for the reaction $\pi^- + p \rightarrow \eta^{(\prime)} + \pi^- + p$ are discussed. The low energy regime is dominated by single-Regge exchange (resonance production) and the high energy regime is dominated by double-Regge exchange. We support this claim by looking at the kinematics of the full COMPASS event sample of $\eta^{(\prime)}\pi^-$ data in the next section. The selection of the event sample is described in Chapter 4. As for most of our analyses at COMPASS [5, 7–10, 27, 67], we impose the cut¹ $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$. This cut is justified by the acceptance of the RPD, which is only able to measure proton momenta of at least $0.27 \text{ GeV } c^{-1}$ [68].

6.1 Kinematic Distributions

In Fig. 6.1, one can see the $m_{\eta^{(\prime)}\pi^-}$ vs $\cos \theta_{\text{GJ}}$ distribution for the $\eta\pi^-$ (Fig. 6.1(a)) and $\eta'\pi^-$ (Fig. 6.1(b)) final state. The $\eta\pi^-$ data are dominated by the $a_2(1320)$ in the resonance region. In order to see structure outside of the $a_2(1320)$, the $m_{\eta\pi^-}$ vs. $\cos \theta_{\text{GJ}}$ distribution is shown in log scale. The data populate the full angular range up to $2.5 \text{ GeV } c^{-2}$. From $2.5 \text{ GeV } c^{-2}$ to $6 \text{ GeV } c^{-2}$, the data favor values close to $|\cos \theta_{\text{GJ}}| \approx 1$. Additionally, one can observe an asymmetry between forward and backward intensities, which is explicitly shown in Fig. 6.3(a). This forward-backward asymmetry directly indicates the presence of odd and even partial waves (see Section 2.4.1). The forward-backward asymmetry is even more striking in the $\eta'\pi^-$ final state (see Figs. 6.1(b) and 6.3(b)). Already from the two dimensional representation, the interplay between odd and even partial waves in the resonance region is visible. The resonance region of the $\eta^{(\prime)}\pi^-$ final state was analyzed in detail in [5, 24], where only half of the COMPASS data set was used². Throughout this chapter, we will focus on the high energy regime and fit the double-Regge model, derived in Chapter 2 to the COMPASS data. In order to do so, we need to separate the resonance region and the double-Regge region. The double-Regge asymptotic limit is given in Eq. (2.27). In Figs. D.1 and D.3, we display s, s_2, t_1, t_2 and κ for the full event sample in the $\eta\pi^-$ and $\eta'\pi^-$ final state, respectively. Note, that forward and backward events define separate t_1 and s_2 variables (see Eq. (2.24)). Consequently, these variables are displayed only in the range of interest and filled with exclusively forward / backward events. Apart from small fluctuations, we observe smooth distributions, where t_1, t_2 and κ are small, while s and s_2 are large. In

¹ See glossary for the definition of t'

² The full COMPASS data set, presented in Chapter 4, will be analyzed in the resonance region by [12].

order to satisfy the double-Regge asymptotic limit (see Eq. (2.27)), we cut the data in $\cos \theta_{\text{GJ}}$ and $m_{\eta^{(\prime)}\pi^-}$. In these distributions (see Fig. 6.1) the change between the resonance and double-Regge region is best observable. We choose $|\cos \theta_{\text{GJ}}| < 0.75$ and $2.9 \text{ GeV } c^{-2} < m_{\eta\pi^-} < 6 \text{ GeV } c^{-2}$ for the $\eta\pi^-$ final state and $2.6 \text{ GeV } c^{-2} < m_{\eta'\pi^-} < 6 \text{ GeV } c^{-2}$ for the $\eta'\pi^-$ final state. The mass cut will be validated in Section 6.6. Upwards of $6 \text{ GeV } c^{-2}$, basically no events are left to analyze. Both of these cuts are indicated by red lines in Fig. 6.1 and we arrive at 19238 $\eta'\pi^-$ and 16142 $\eta\pi^-$ events.

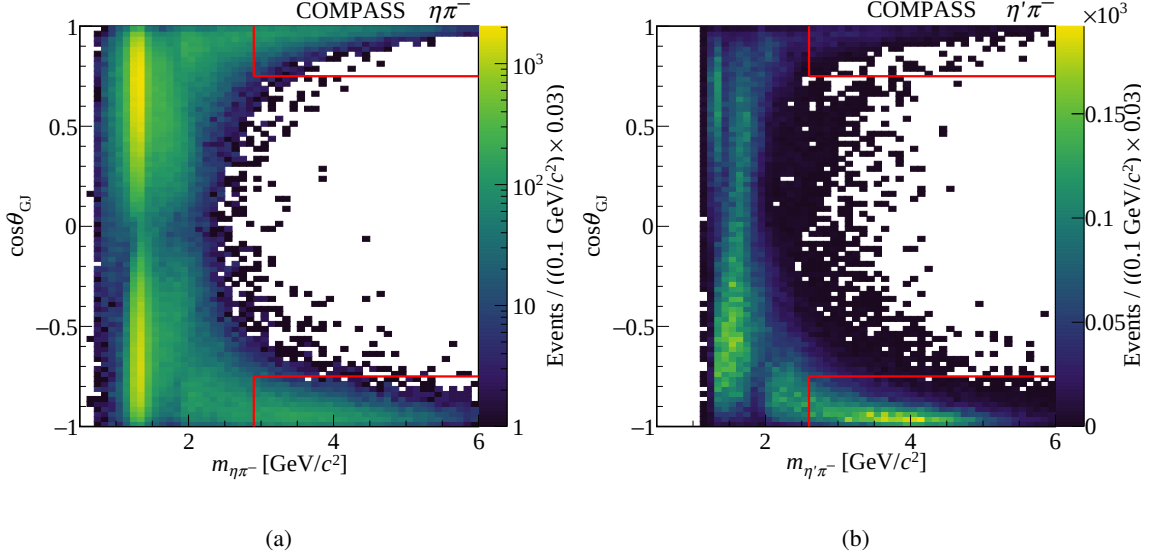


Figure 6.1: Two dimensional distribution of $m_{\eta\pi^-}$ (a) and $m_{\eta'\pi^-}$ (b) vs. $\cos \theta_{\text{GJ}}$. The red lines indicate the cut which is performed to isolate double-Regge events.

In Fig. 6.2, we present s, s_2, t_1, t_2 and κ after the cut in $\cos \theta_{\text{GJ}}$ and $m_{\eta'\pi^-}$ for the $\eta'\pi^-$ final state. We still see, apart from small fluctuations, smooth distributions without any striking features that would require to cut further into the data. The same distributions for the $\eta\pi^-$ final state are shown in Fig. D.2.

The forward-backward asymmetry is remarkably constant for the $\eta'\pi^-$ final state (Fig. 6.4(b)), while it is constant only after $4 \text{ GeV } c^{-2}$ for the $\eta\pi^-$ final state (Fig. 6.4(a)). The forward region contains much more events (relative to the backward region) in the $\eta\pi^-$ final state, compared to the $\eta'\pi^-$ final state. Lastly, one should be careful in interpreting these distributions at face value, since no acceptance correction is applied at this stage. Nevertheless, we expect little change due to that. Details about the acceptance can be found in Figs. 5.25 and 5.26 and Appendix B.

The same holds for the right-left asymmetry (Figs. 6.5(c) and 6.5(f)) and their forward (Figs. 6.5(a) and 6.5(d)) and backward (Figs. 6.5(b) and 6.5(e)) components. The corresponding plots after the cut in the double-Regge limit can be found in Fig. D.6. In all cases, we see evidence for a mixture of $m = 1$ and other m partial-waves (see Section 2.4.2 for a derivation). This is of course not very relevant for the interpretation in the double-Regge region but we will see in Fig. 6.11(c) that the double-Regge fit does not describe the ϕ_{GJ} dependence and with that the right-left asymmetry to the same accuracy as the other kinematic variables in the $\eta\pi^-$ final state.

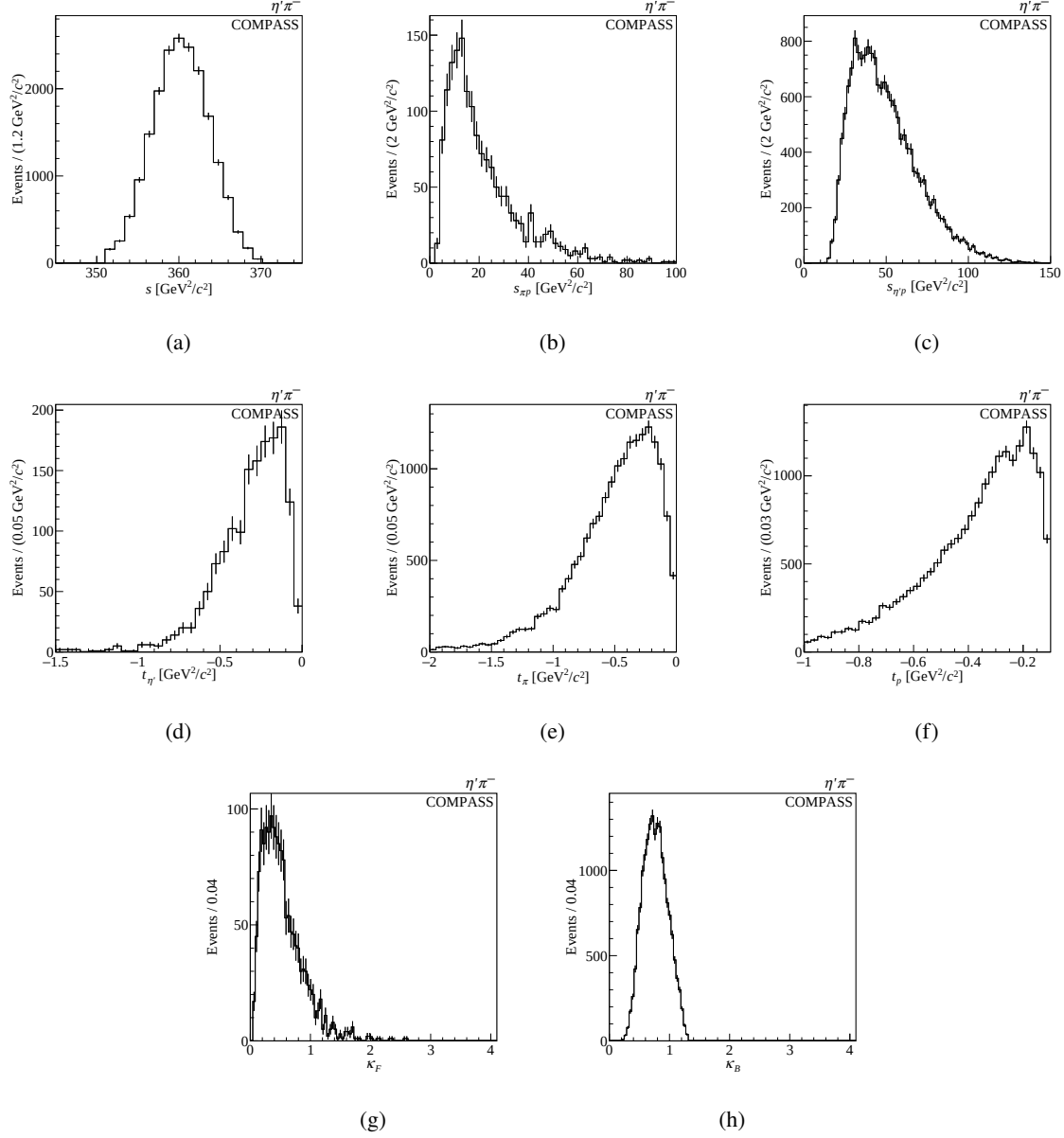


Figure 6.2: Kinematic distributions for the $\eta'\pi^-$ final state. (b), (d) and (g) represent variables, associated with $\eta^{(\prime)}$ forward production and are consequently only filled with forward events. (c), (e) and (h) represent variables, associated with $\eta^{(\prime)}$ backward production and are consequently only filled with backward events. κ_F refers to forward κ (see Eq. (2.28) with $s_2 = s_{\pi p}$). κ_B refers to backward κ (see Eq. (2.28) with $s_2 = s_{\eta' p}$). The following cuts are imposed: $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$, $2.6 \text{ GeV} c^{-2} < m_{\eta'\pi^-} < 6 \text{ GeV} c^{-2}$ and $|\cos \theta_{GJ}| > 0.75$.

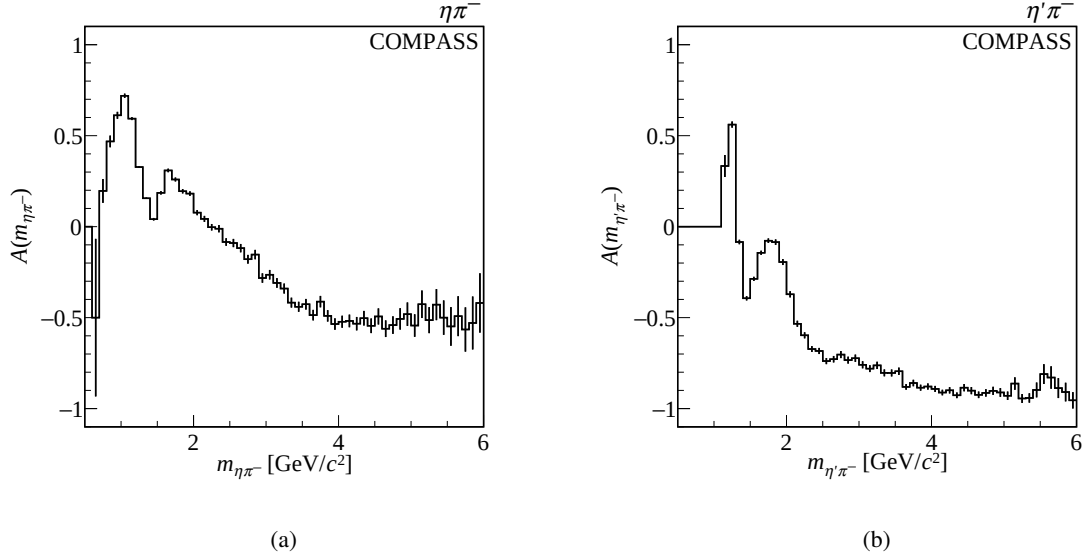


Figure 6.3: Forward-backward asymmetry for $\eta\pi^-$ (a) and $\eta'\pi^-$ (b). The following cuts are imposed: $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$.

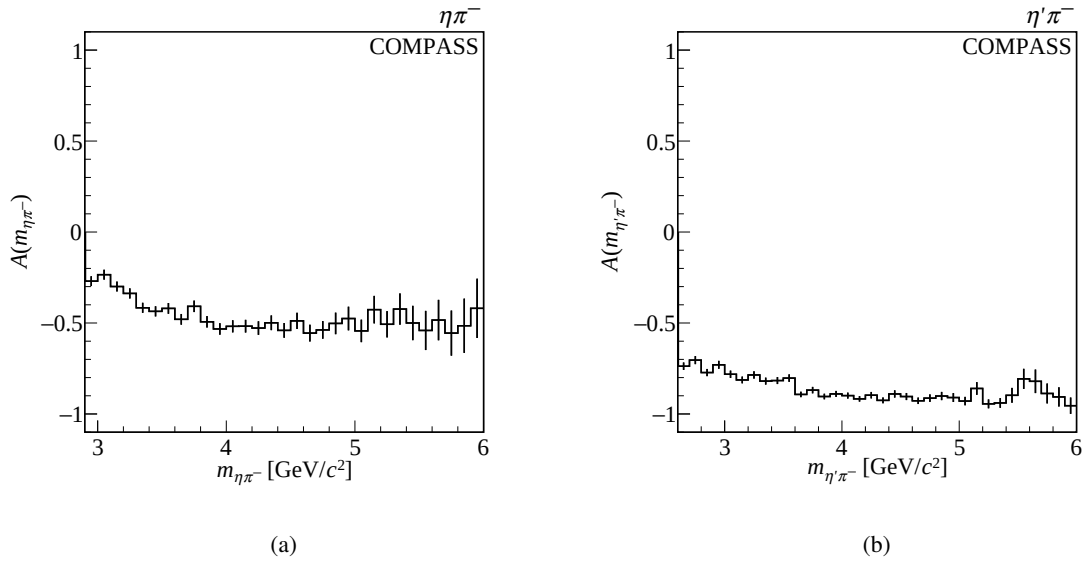


Figure 6.4: Forward-backward asymmetry for $\eta\pi^-$ (a) and $\eta'\pi^-$ (b). The following cuts are imposed: $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$, $2.9 \text{ GeV} c^{-2} < m_{\eta\pi^-} < 6 \text{ GeV} c^{-2}$, $2.6 \text{ GeV} c^{-2} < m_{\eta'\pi^-} < 6 \text{ GeV} c^{-2}$ and $|\cos \theta_{\text{GJ}}| > 0.75$.

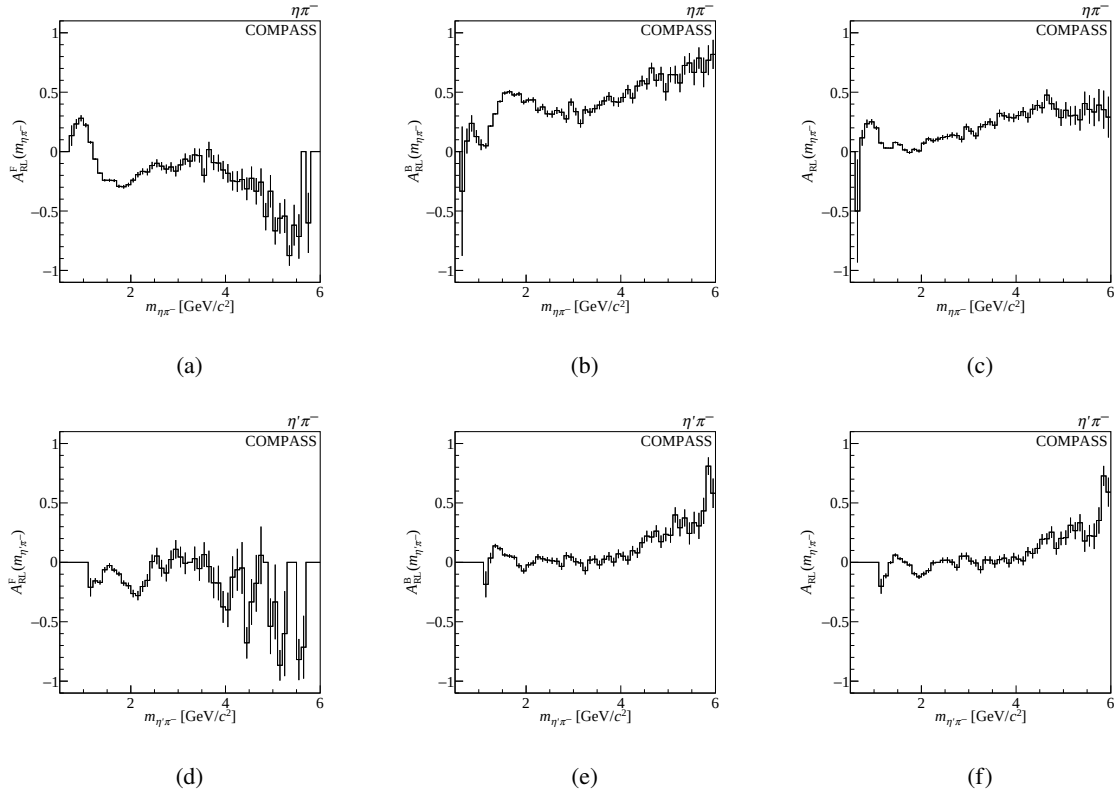


Figure 6.5: Right-left asymmetry for forward (a), backward (b) and all events (c) in the $\eta\pi^-$ final state and the right-left asymmetry for forward (d), backward (e) and all events (f) in the $\eta'\pi^-$ final state. The following cuts are imposed: $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$.

6.2 Double-Regge Intensity

The event distribution of $\pi^- + p \rightarrow \eta^{(\prime)} + \pi^- + p$ events in the double-Regge region is described by the intensity function

$$I(\vec{c}, \vec{b}, \tau) = \left| A(\vec{c}, \vec{b}, \tau) \right|^2. \quad (6.1)$$

We have shown in Section 2.3, that for forward $\eta^{(\prime)}$ the a_2 is the dominant upper exchange and an f_2 or $\mathbb{P}$ can be exchanged at the bottom. For backwards $\eta^{(\prime)}$, the top and bottom exchange is given by either an f_2 or $\mathbb{P}$. Therefore, we model the amplitude as

$$\begin{aligned} A(\vec{c}, \vec{b}, \tau) = & c_{a_2 \mathbb{P}} A_{a_2 \mathbb{P}}(\vec{b}, \tau) + c_{a_2 f_2} A_{a_2 f_2}(\vec{b}, \tau) \\ & + c_{f_2 \mathbb{P}} A_{f_2 \mathbb{P}}(\vec{b}, \tau) + c_{f_2 f_2} A_{f_2 f_2}(\vec{b}, \tau) + c_{\mathbb{P} \mathbb{P}} A_{\mathbb{P} \mathbb{P}}(\vec{b}, \tau) + c_{\mathbb{P} f_2} A_{\mathbb{P} f_2}(\vec{b}, \tau), \end{aligned} \quad (6.2)$$

where we grouped the kinematic variables s, s_1, s_2, t_1 and t_2 in the variable τ . The slope parameters, described in Eq. (2.50), are grouped in $\vec{b}$. Each unique exchange diagram, described by its amplitude A_{kl} , where k and l are the exchange Reggeon or the Pomeron, is multiplied by a strength coefficient c_{kl} . This coefficient describes the couplings at the top, middle and bottom vertex of each diagram and therefore real. The complexity of the model is fully described by the double-Regge amplitude A_{kl} . For convenience, we abbreviate $kl \rightarrow i$ and group all c_{kl} in $\vec{c}$.

The single diagram amplitudes are given by

$$A_{a_2 \mathbb{P}}(\vec{b}, \tau) = e^{b_{a_2; \text{forward}; T} t_{\eta^{(\prime)}}} e^{b_{\mathbb{P}; \text{forward}; B} t_P} \cdot A\left(\alpha_{a_2}(t_{\eta^{(\prime)}}), \alpha_{\mathbb{P}}(t_P); s_{\pi P}\right), \quad (6.3a)$$

$$A_{a_2 f_2}(\vec{b}, \tau) = e^{b_{a_2; \text{forward}; T} t_{\eta^{(\prime)}}} e^{b_{f_2; \text{forward}; B} t_P} \cdot A\left(\alpha_{a_2}(t_{\eta^{(\prime)}}), \alpha_{f_2}(t_P); s_{\pi P}\right), \quad (6.3b)$$

$$A_{f_2 \mathbb{P}}(\vec{b}, \tau) = e^{b_{f_2; \text{backward}; T} t_{\pi}} e^{b_{\mathbb{P}; \text{backward}; B} t_P} \cdot A\left(\alpha_{f_2}(t_{\pi}), \alpha_{\mathbb{P}}(t_P); s_{\eta^{(\prime)} P}\right), \quad (6.3c)$$

$$A_{f_2 f_2}(\vec{b}, \tau) = e^{b_{f_2; \text{backward}; T} t_{\pi}} e^{b_{f_2; \text{backward}; B} t_P} \cdot A\left(\alpha_{f_2}(t_{\pi}), \alpha_{f_2}(t_P); s_{\eta^{(\prime)} P}\right), \quad (6.3d)$$

$$A_{\mathbb{P} \mathbb{P}}(\vec{b}, \tau) = e^{b_{\mathbb{P}; \text{backward}; T} t_{\pi}} e^{b_{\mathbb{P}; \text{backward}; B} t_P} \cdot A\left(\alpha_{\mathbb{P}}(t_{\pi}), \alpha_{\mathbb{P}}(t_P); s_{\eta^{(\prime)} P}\right), \quad (6.3e)$$

$$A_{\mathbb{P} f_2}(\vec{b}, \tau) = e^{b_{\mathbb{P}; \text{backward}; T} t_{\pi}} e^{b_{f_2; \text{backward}; B} t_P} \cdot A\left(\alpha_{\mathbb{P}}(t_{\pi}), \alpha_{f_2}(t_P); s_{\eta^{(\prime)} P}\right), \quad (6.3f)$$

where we have introduced one formfactor per unique particle-particle-Reggeon(Pomeron) vertex. We use the notation $b_{e;f;g}$, where e is the Reggeon at the given vertex, f is either F ($\eta^{(\prime)}$ forward) or B ($\eta^{(\prime)}$ backward) and g is either T (top vertex) or B (bottom vertex). The dependencies of the single amplitudes are changed to the exchanged trajectories at the top (with their respective t_1)³ and at the bottom as well as the given s_2 of the amplitude⁴. With these dependencies, an amplitude is uniquely defined. Kinematic dependencies that are shared among all amplitudes are dropped in the dependency.

³ For $\eta^{(\prime)}$ forward, we use $t_{\eta^{(\prime)}}$ and for $\eta^{(\prime)}$ backward we use t_{π} .

⁴ For $\eta^{(\prime)}$ forward, we use $s_{\pi P}$ and for $\eta^{(\prime)}$ backward we use $s_{\eta^{(\prime)} P}$.

6.3 The Extended Negative Log Likelihood

A large sample of independent events N_m , where the probability of finding each event is small, but the number of expected events N_e is constant, follows the Poisson distribution

$$P(N_m) = e^{-N_e} \frac{N_e^{N_m}}{N_m!}, \quad (6.4)$$

The number of expected events is determined by looking at the detector acceptance. In general, we can write

$$N_e = \int I(\vec{c}, \vec{b}, \tau) \xi(\tau) d\tau, \quad (6.5)$$

where the acceptance function $\xi(\tau)$ tells us whether a specific kinematics τ can be measured within our spectrometer or not, i.e. it evaluates to 0 or 1 for fixed τ . This function is generally unknown and impossible to derive for our complex spectrometer setup. In order to approximate N_e , we generate a large set of MC data that covers the kinematics of the full phase space (see Section 5.4.4). Equation (6.5) becomes

$$N_e = \sum_{k=1}^{N_{acc}} I(\vec{c}, \vec{b}, \tau_k), \quad (6.6)$$

where the integral over the full phase space data is condensed to a sum over all accepted MC events (N_{acc}). Only for those, $\xi(\tau)$ evaluates to 1. The likelihood of measuring N_m events is the probability of measuring N_m events (Eq. (6.4)) times the probability of finding each individual event. This probability is given by the intensity (see Eq. (6.1)), divided by the number of expected events. Therefore, the likelihood reads

$$\mathcal{L}(\vec{c}, \vec{b}, N_{acc}, N_m) = e^{-N_e} \frac{N_e^{N_m}}{N_m!} \prod_{k=1}^{N_m} \frac{I(\vec{c}, \vec{b}, \tau_k)}{N_e} = \frac{e^{-N_e}}{N_m!} \prod_{k=1}^{N_m} I(\vec{c}, \vec{b}, \tau_k). \quad (6.7)$$

For a fixed event sample (N_m) and a set of MC data to describe the acceptance (N_e), the likelihood is determined by the strength coefficients of the given amplitudes ($\vec{c}$) and the slope parameters ($\vec{b}$). The task at hand is to find optimal values for those, i.e. maximize the likelihood. In order to make this task easier, we apply the logarithm to Eq. (6.7) and choose to construct the negative log likelihood, such that we can perform a minimization rather than a maximization. This is possible since the logarithm is monotonously increasing and will therefore not change the minimum.

$$-\ln \mathcal{L}(\vec{c}, \vec{b}, N_{\text{acc}}, N_{\text{m}}) = -\ln \left(\frac{e^{-N_{\text{e}}}}{N_{\text{m}}!} \prod_{k=1}^{N_{\text{m}}} I(\vec{c}, \vec{b}, \tau_k) \right) = N_{\text{e}} - \sum_{k=1}^{N_{\text{m}}} \ln I(\vec{c}, \vec{b}, \tau_k) + \ln N_{\text{m}}! \quad (6.8a)$$

$$\propto N_{\text{e}} - \sum_{k=1}^{N_{\text{m}}} \ln I(\vec{c}, \vec{b}, \tau_k) \quad (6.8b)$$

$$= \sum_{k=1}^{N_{\text{MC}}^{\text{acc}}} I(\vec{c}, \vec{b}, \tau_k) - \sum_{k=1}^{N_{\text{m}}} \ln I(\vec{c}, \vec{b}, \tau_k), \quad (6.8c)$$

where we dropped $\ln N_{\text{m}}!$ since it only introduces an absolute shift of the likelihood and does not impact the values of $\vec{c}$ and $\vec{b}$.

6.4 Fit procedure

The task at hand is now to minimize Eq. (6.8c). Due to

$$\left| \sum_i c_i A_i \right|^2 = \left| \sum_i -c_i A_i \right|^2, \quad (6.9)$$

the same minimum is reached when all strength coefficients c_i flip their sign. We fix this global sign flip by requiring $c_{a_2 \mathbb{P}}$ to be positive⁵. We use Minuit2 for the minimization of the negative log likelihood. We will perform a global fit, including all selected events, leading to a set of strength coefficients ($\vec{c}$) and slope parameters ($\vec{b}$). To be more explicit, we perform 1000 successful fits with random start parameters. A successful fit is given when Minuit2 returns with status 0. For these fits, where we fit the strength coefficients together with the slope parameters, many local minima are present and we will see high correlation between the different parameters (see Fig. 6.6). In order to converge regularly, we need to fit with Minuit2 strategy 0.⁶ In order to get a precise result with reliable error estimations, the fit is performed one final time, where the start parameters are chosen as the final parameters of the best out of the 1000 performed fits (determined by lowest negative log likelihood). Here, we use Minuit2 strategy 2, which ensures the highest fit quality and best error estimation. The result of this final fit is shown in Section 6.5.

The default settings of Minuit2 use numeric methods to determine the first and second derivatives of the negative log likelihood function. Due to the high correlation of the parameters and the large number of local minima, the numeric evaluation can be inaccurate at times. In Appendix C we have derived the first and second derivatives of the negative log likelihood. We give these first and second derivatives to Minuit2, together with some caching formalism for values that are used multiple times during the minimization, in order to speed up the process and make it more accurate⁷. The speed of the fit largely depends on the number of MC data that enters it, i.e. more MC events slow down the

⁵ This is enforced after the fit by flipping signs of all c_i , if $c_{a_2 \mathbb{P}}$ is negative.

⁶ Strategy 0 implies, that Minuit2 should not check numerical precision very carefully and should not make accurate error estimates. It serves as a quick minimization procedure, where a minimum is found to a low precision.

⁷ Using the analytic functions makes convergence into a minimum roughly 10 times more likely and reduces the computing time by an approximate factor of 2 per fit attempt.

computation. Nevertheless, wherever parts of the amplitude can be cached or pre calculated, it is done to ensure a fast minimization.

The result of one fit consists of strength parameters ($\vec{c}$) and slope parameters ($\vec{b}$) and their corresponding covariance matrix⁸.

To validate the fit, we can compare the distributions of any kinematic variable by histogramming both the COMPASS data and the accepted MC events used in the fit. Each accepted MC event is weighted by its intensity (Eq. (6.1)), evaluated with the fitted parameters ($\vec{c}$) and ($\vec{b}$). Ideally, the resulting data and MC histograms coincide across the full kinematic range. For one bin j of one such histogram, the accepted MC value is then given by

$$N^j(\vec{c}, \vec{b}) = \sum_{k=1}^{N_{\text{acc}}^j} I(\vec{c}, \vec{b}; \tau_k) = \sum_{k=1}^{N_{\text{acc}}^j} \left| \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right|^2. \quad (6.10)$$

The uncertainty of $N^j(\vec{c}, \vec{b})$ is given by

$$\Delta N^j(\vec{c}, \vec{b}) = \sqrt{\underbrace{\sum_{k=1}^{N_{\text{acc}}^j} I(\vec{c}, \vec{b}; \tau_k)^2}_{w^j(\vec{c}, \vec{b})} + J^j(\vec{c}, \vec{b}) \cdot M_{\text{cov}}(\vec{c}, \vec{b}) \cdot (J^j(\vec{c}, \vec{b}))^T}, \quad (6.11)$$

where $w^j(\vec{c}, \vec{b})$ represents the uncertainty on the MC sample, $M_{\text{cov}}(\vec{c}, \vec{b})$ is the covariance matrix of the extended negative log likelihood fit and $J^j(\vec{c}, \vec{b})$ is the Jacobian vector, i.e. the vector of partial derivatives with respect to all fit parameters, for Eq. (6.10).

$$J^j(\vec{c}, \vec{b}) = \sum_{k=1}^{N_{\text{acc}}^j} J_k(\vec{c}, \vec{b}) = \sum_{k=1}^{N_{\text{acc}}^j} \begin{pmatrix} \frac{\partial}{\partial c_1} \\ \vdots \\ \frac{\partial}{\partial c_6} \\ \frac{\partial}{\partial b_1} \\ \vdots \\ \frac{\partial}{\partial b_7} \end{pmatrix} I(\vec{c}, \vec{b}; \tau_k), \quad (6.12)$$

where we label the seven slope parameters $b_{e,f,g}$ from 1 to 7 for simplicity. In order to compute $J^j(\vec{c}, \vec{b})$, we need to compute the partial derivative with respect to all strength and slope parameters individually. See Appendix C for a detailed derivation.

In order to investigate the strength of a single diagram i , we define

$$N_i^j(\vec{c}, \vec{b}) = \sum_{k=1}^{N_{\text{acc}}^j} I_i(\vec{c}, \vec{b}; \tau_k) = \sum_{k=1}^{N_{\text{acc}}^j} \left| c_i A_{kl}(\vec{b}; \tau_k) \right|^2, \quad (6.13)$$

where the single diagram intensity only takes that diagram and no interference of other diagrams into account. Consequently, the intensity of all individual diagrams does not add up to the total intensity, if

⁸ The covariance matrix is the inverse of the Hesse matrix (matrix of second derivatives). We will use the analytic derivatives (Appendix C) to calculate this.

the interference between the different diagrams is large. Similarly, we define the bottom $\mathbb{P}$ intensity such, that only diagrams with a $\mathbb{P}$ as the bottom exchange contribute. We write

$$N_{\mathbb{B};\mathbb{P}}^j(\vec{c}, \vec{b}) = \sum_{k=1}^{N_{\text{acc}}^j} I_{\mathbb{B};\mathbb{P}}(\vec{c}, \vec{b}; \tau_k) = \sum_{k=1}^{N_{\text{acc}}^j} \left| c_{a_2\mathbb{P}} A_{a_2\mathbb{P}}(\vec{b}; \tau_k) + c_{f_2\mathbb{P}} A_{f_2\mathbb{P}}(\vec{b}; \tau_k) + c_{\mathbb{P}\mathbb{P}} A_{\mathbb{P}\mathbb{P}}(\vec{b}; \tau_k) \right|^2. \quad (6.14)$$

Consequently, the bottom f_2 intensity is defined as

$$N_{\mathbb{B};f_2}^j(\vec{c}, \vec{b}) = \sum_{k=1}^{N_{\text{acc}}^j} I_{\mathbb{B};f_2}(\vec{c}, \vec{b}; \tau_k) = \sum_{k=1}^{N_{\text{acc}}^j} \left| c_{a_2f_2} A_{a_2f_2}(\vec{b}; \tau_k) + c_{f_2f_2} A_{f_2f_2}(\vec{b}; \tau_k) + c_{\mathbb{P}f_2} A_{\mathbb{P}f_2}(\vec{b}; \tau_k) \right|^2. \quad (6.15)$$

We mentioned the dominance of Pomeron exchange in the COMPASS energy regime in Section 2.2.2. Eq. (6.14) and Eq. (6.15) will be used to determine the relative strength of the amplitudes with Pomeron exchange at the bottom vertex, as determined by the fit. The fit procedure was cross-checked by Julien Beckers (Technical University of Munich) and is documented in Appendix E.

The systematic uncertainty is determined by varying the Regge-trajectory parameters. The Pomeron parameters are well established but the exact values for the a_2 and f_2 parameters vary in the literature [14, 69]. We vary the trajectory parameters α_0 to 0.45 and 0.55 and α_1 to 0.85 and 0.95 and perform the fit for each variation, i.e. eight additional times⁹. For each fit we compute the MC accepted intensity per kinematic variable. The systematic uncertainty will be shown as an error band on the distributions, where the minimum and maximum of that band is extracted as the minimum or maximum of any of the systematic studies for that particular point. We present the c_i and b_i values for all systematic studies in Tables 6.1 and 6.2. The statistical uncertainty is omitted for the systematic studies, since it does not enter the estimate of the systematic uncertainty.

We will represent the fit result with the following weighted intensity color code. The data will be shown in a black histogram and the full intensity as red circles. Additionally, we show the intensity of the single diagrams, where top a_2 diagrams are shown in green, top f_2 in orange and top $\mathbb{P}$ in blue. Bottom $\mathbb{P}$ diagrams will be represented by triangles and bottom f_2 by squares. The bottom $\mathbb{P}$ intensity, i.e. the coherent sum over all bottom $\mathbb{P}$ diagrams will be represented by blue squares and the bottom f_2 intensity by orange triangles. The systematic uncertainty is represented as an error band and the statistical error as an error bar, which might be too small to be visible at times.

6.5 Results of the Fit

In Fig. 6.1 we see, that the forward and backward region is kinematically disconnected. This is also clear when looking at the different diagrams that are involved. We have dedicated diagrams for forward and backward events. The parameter set of the best fit is documented in Table 6.3. The quoted uncertainty is of statistical nature and given by Minuit2.

Due to the high correlation of the c_i and b_i , it is not meaningful to quote an error without the

⁹ Each fit is performed in the same way as the main fit, i.e. 1000 successful fit attempts to find the global minimum. The global minimum is shown in Tables 6.1 and 6.2.

Table 6.1: Systematic studies for the $\eta\pi^-$ final state.

	$\alpha_0(a_2)$ = 0.45	$\alpha_0(a_2)$ = 0.55	$\alpha_1(a_2)$ = 0.85	$\alpha_1(a_2)$ = 0.95	$\alpha_0(f_2)$ = 0.45	$\alpha_0(f_2)$ = 0.55	$\alpha_1(f_2)$ = 0.85	$\alpha_1(f_2)$ = 0.95
$c_{a_2\mathbb{P}}$	0.0154	0.0133	0.0142	0.0134	0.0140	0.0128	0.0137	0.0138
$c_{a_2f_2}$	0.078	0.034	0.037	0.046	0.042	0.0363	0.042	0.040
$c_{f_2\mathbb{P}}$	-0.0141	-0.0141	-0.0142	-0.0138	-0.0140	-0.0132	-0.0136	-0.0146
$c_{f_2f_2}$	-0.203	-0.205	-0.200	-0.21	-0.25	-0.082	-0.206	-0.21
$c_{\mathbb{P}\mathbb{P}}$	0.00131	0.00132	0.00130	0.00135	0.00130	0.00126	0.00133	0.00132
$c_{\mathbb{P}f_2}$	-0.0091	-0.0090	-0.0093	-0.0085	-0.0082	-0.0098	-0.0090	-0.0085
$b_{a_2;\mathbb{F};\mathbb{T}}$	-0.075	-0.153	-0.006	-0.241	-0.134	-0.152	-0.141	-0.133
$b_{\mathbb{P};\mathbb{F};\mathbb{B}}$	2.01	2.25	2.26	2.16	2.22	2.15	2.22	2.20
$b_{f_2;\mathbb{F};\mathbb{B}}$	1.20	0.99	1.05	1.04	0.98	1.26	1.21	0.79
$b_{f_2;\mathbb{B};\mathbb{T}}$	1.28	1.28	1.25	1.35	1.32	0.95	1.36	1.20
$b_{f_2;\mathbb{B};\mathbb{B}}$	1.20	1.20	1.18	1.23	1.24	0.87	1.33	1.03
$b_{\mathbb{P};\mathbb{B};\mathbb{T}}$	0.62	0.63	0.62	0.64	0.63	0.56	0.63	0.63
$b_{\mathbb{P};\mathbb{B};\mathbb{B}}$	1.57	1.57	1.57	1.57	1.54	1.67	1.56	1.58

Table 6.2: Systematic studies for the $\eta'\pi^-$ final state.

	$\alpha_0(a_2)$ = 0.45	$\alpha_0(a_2)$ = 0.55	$\alpha_1(a_2)$ = 0.85	$\alpha_1(a_2)$ = 0.95	$\alpha_0(f_2)$ = 0.45	$\alpha_0(f_2)$ = 0.55	$\alpha_1(f_2)$ = 0.85	$\alpha_1(f_2)$ = 0.95
$c_{a_2\mathbb{P}}$	0.0081	0.0074	0.0079	0.0073	0.0078	0.0071	0.0076	0.0077
$c_{a_2f_2}$	0.051	0.022	0.024	0.029	0.027	0.024	0.027	0.026
$c_{f_2\mathbb{P}}$	-0.0036	-0.0036	-0.0036	-0.0035	-0.0034	-0.0039	-0.0034	-0.0036
$c_{f_2f_2}$	-0.138	-0.138	-0.137	-0.138	-0.161	-0.074	-0.138	-0.139
$c_{\mathbb{P}\mathbb{P}}$	0.00067	0.00067	0.00067	0.00067	0.00066	0.00070	0.00063	0.00072
$c_{\mathbb{P}f_2}$	-0.0066	-0.0066	-0.0066	-0.0066	-0.0068	-0.0057	-0.0065	-0.0066
$b_{a_2;\mathbb{F};\mathbb{T}}$	0.32	0.25	0.42	0.12	0.27	0.25	0.26	0.27
$b_{\mathbb{P};\mathbb{F};\mathbb{B}}$	1.80	2.14	2.13	2.01	2.10	2.00	2.08	2.08
$b_{f_2;\mathbb{F};\mathbb{B}}$	1.60	1.7	1.7	1.6	1.6	1.9	1.8	1.4
$b_{f_2;\mathbb{B};\mathbb{T}}$	0.67	0.67	0.67	0.67	0.68	0.61	0.70	0.63
$b_{f_2;\mathbb{B};\mathbb{B}}$	0.10	0.11	0.10	0.11	0.13	-0.00	0.23	-0.07
$b_{\mathbb{P};\mathbb{B};\mathbb{T}}$	0.29	0.29	0.29	0.29	0.29	0.28	0.27	0.32
$b_{\mathbb{P};\mathbb{B};\mathbb{B}}$	1.42	1.42	1.42	1.42	1.39	1.52	1.39	1.45

covariance matrix. Since there is no reasonable way of constructing a covariance matrix for systematic studies, we do not quote the systematic uncertainty here. The statistical covariance matrix is directly given by Minuit2 (via the inverse of the hessian matrix). We use it to calculate the appropriate errors (see Eq. (6.11)) for the result plots. A nice way of displaying the covariance matrix is the correlation

Table 6.3: Determined c_i and b_i parameters for the best fit for the $\eta\pi^-$ and $\eta'\pi^-$ final state with statistical uncertainties.

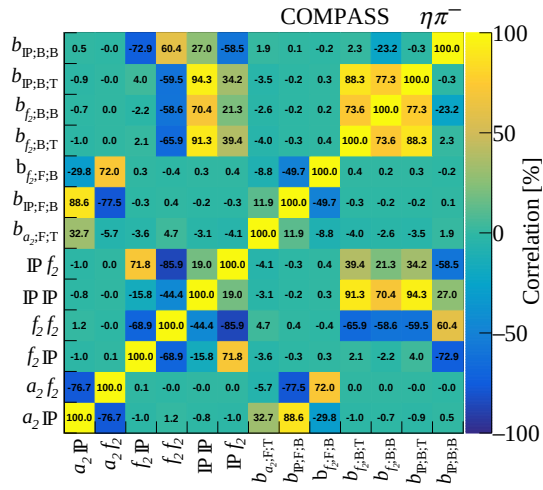
	$\eta\pi^-$	$\eta'\pi^-$
$c_{a_2\mathbb{P}}$	0.0137(7)	0.0076(6)
$c_{a_2f_2}$	0.041(4)	0.027(5)
$c_{f_2\mathbb{P}}$	-0.0141(16)	-0.0036(8)
$c_{f_2f_2}$	-0.204(33)	-0.138(9)
$c_{\mathbb{P}\mathbb{P}}$	0.00132(12)	0.00067(5)
$c_{\mathbb{P}f_2}$	-0.0090(23)	-0.0066(7)
$b_{a_2;\text{F};\text{T}}$	-0.138(28)	0.27(6)
$b_{\mathbb{P};\text{F};\text{B}}$	2.21(8)	2.08(13)
$b_{f_2;\text{F};\text{B}}$	1.04(26)	1.7(5)
$b_{f_2;\text{B};\text{T}}$	1.28(19)	0.67(10)
$b_{f_2;\text{B};\text{B}}$	1.20(19)	0.11(8)
$b_{\mathbb{P};\text{B};\text{T}}$	0.62(6)	0.29(4)
$b_{\mathbb{P};\text{B};\text{B}}$	1.57(6)	1.42(6)

matrix. An element of the correlation matrix is given by

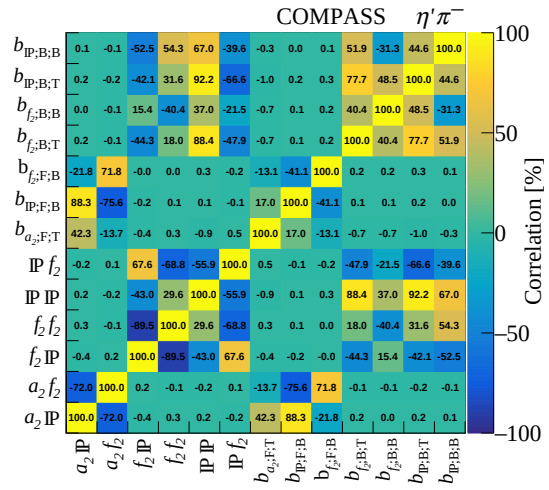
$$C_{ij} = \frac{\text{cov}_{ij}}{\sqrt{\text{cov}_{ii}\text{cov}_{jj}}}, \quad (6.16)$$

where i and j are the row and column indices of the matrix, i.e. they represent different fit parameters. The covariance between two fit parameters is denoted as (cov_{ij}) . We chose to display the correlation in % and therefore multiply C_{ij} with 100. The correlation matrix is given in Fig. 6.6.

We see a high correlation in the forward and backward sectors and less than 1% correlation in between the sectors. This strengthens the claim that both sectors are almost completely disconnected.



(a)



(b)

Figure 6.6: Correlation matrix for the $\eta\pi^-$ (a) and for the $\eta'\pi^-$ (b) final state.

6.5.1 The Forward Sector

The forward sector only contains two leading diagrams, i.e. $a_2 \mathbb{P}$ and $a_2 f_2$. Figure 6.7 shows the key kinematic variables. We deliberately only show the kinematic range in $\cos \theta_{\text{GJ}}$ (Figs. 6.7(a) and 6.7(b)) and t_1 (Figs. 6.7(e) and 6.7(f)), of the forward events. As expected, only forward amplitudes contribute and we see a nice agreement of the full fit (red spheres) and the COMPASS data (black histogram). Varying the Regge-trajectory parameters for the systematic uncertainty shifts the significance of the different contributing diagrams but has little to no effect on the overall fit. We note a strong dominance of the $a_2 \mathbb{P}$ diagram over the $a_2 f_2$ one. If one observes carefully the forward intensity in the $\eta\pi^-$ final state (Fig. 6.7(c)) compared to that of the $\eta'\pi^-$ final state (Fig. 6.7(d)) one can make out a worse description in the $\eta\pi^-$ final state. In the $\eta'\pi^-$ final state, the intensity follows the data very nicely but in the $\eta\pi^-$ final state, the intensity is underestimated at low masses and overestimated at high masses. In this diagram, the effect is minimal but we will see in Fig. 6.10(a) that the forward-backward asymmetry description will suffer from this. Since the $\eta'\pi^-$ final state is described well, this might indicate that the $\eta\pi^-$ sector is contaminated by more non double-Regge events or that we need sub leading diagrams to describe the forward sector to a higher accuracy. An inclusion of a daughter trajectory of the a_2 for the $\eta\pi^-$ final state might be interesting in the future. Due to the small incompatibility in the forward intensity, we would expect only a small intensity of the daughter trajectory diagrams.

6.5.2 The Backward Sector

The backward sector contains four diagrams, i.e. $f_2 \mathbb{P}$, $f_2 f_2$, $\mathbb{P}\mathbb{P}$ and $\mathbb{P} f_2$. Figure 6.8 shows the key kinematic variables. We deliberately only show the kinematic range in $\cos \theta_{\text{GJ}}$ (Figs. 6.8(a) and 6.8(b)) and t_1 (Figs. 6.8(e) and 6.8(f)), of the backward events. As expected, only backward amplitudes contribute and we see a nice agreement of the full fit (red spheres) and the COMPASS data (black histogram). Varying the Regge-trajectory parameters for the systematic uncertainty shifts the significance of the different contributing diagrams but has little to no effect on the overall fit. The backward sector is described to a higher accuracy by the model, compared to the forward sector. This can have multiple explanations. We have double the amount of amplitudes in the backward sector and more noticeably two different exchanges at the top vertex compared to only the a_2 in the forward sector.

6.5.3 Combined Kinematics

In Figs. 6.9(a) and 6.9(b) we see the overall $m_{\eta^{(\prime)}\pi^-}$ distribution. It is nicely described in both final states, which might come as a surprise since we saw small inaccuracies in the forward intensity for the $\eta\pi^-$ final state. However, both final states have considerably more events in the backward sector. Therefore, this plot is mostly dominated by the description of the backward sector. More interesting is the description of $t_2 = t_p$ and the implications that come with it. Apart from $t_2 \approx -0.1$, the distribution is nicely described by the fit (see Figs. 6.9(c) and 6.9(d)). In Section 2.2.2 we find, that single-Regge exchange is dominated by Pomeron exchange. Therefore, we also expect Pomeron exchange to be dominant in the double-Regge regime. One can probe this assumption by displaying the bottom Pomeron intensity (Eq. (6.14)), relative to the bottom f_2 intensity (Eq. (6.15)), while looking at the t_2 variable. We draw these contributions in Figs. 6.9(e) and 6.9(f) and observe a larger bottom $\mathbb{P}$ contribution in the $\eta\pi^-$ final state. Additionally, a considerable bottom f_2 intensity seem to be needed to describe the t_2 kinematics. In Table 6.4, we show the relative strength of all amplitudes and the

bottom $\mathbb{P} / f_2$ intensities. It is clear that much intensity is also hidden in the interference of bottom $\mathbb{P}$ and bottom f_2 amplitudes. The fit clearly needs an interplay of the two to describe the data.

Table 6.4: Relative strength of the single amplitudes in the projection to t_p . The incoherent sum is calculated, using all single amplitudes. The bottom $\mathbb{P} / f_2$ take interference between bottom $\mathbb{P}$ or bottom f_2 , respectively, into account. All values are in % relative to the full model intensity.

	$c_{a_2 \mathbb{P}}$	$c_{a_2 f_2}$	$c_{f_2 \mathbb{P}}$	$c_{f_2 f_2}$	$c_{\mathbb{P} \mathbb{P}}$	$c_{\mathbb{P} f_2}$	Incoherent sum	bottom $\mathbb{P}$	bottom f_2
$\eta\pi^-$	17.3	2.6	7.3	5.8	36.3	3.2	72.5	58.1	14.9
$\eta'\pi^-$	5.0	0.6	2.8	17.1	30.7	7.2	63.4	36.6	32.5

We see the forward-backward asymmetry in Figs. 6.10(a) and 6.10(b), where we find a remarkably good agreement within the $\eta'\pi^-$ final state and a slightly worse description in the $\eta\pi^-$ final state. This is due to the worse description of the forward intensity (Fig. 6.7(a)) for the $\eta\pi^-$ final state compared to the $\eta'\pi^-$ final state.

Lastly, we discuss the ϕ_{GJ} description of the fit. We are aware that the model is not meant to capture irregularities in the ϕ_{GJ} distribution. The only way to account for ϕ_{GJ} is in the K factor (Eq. (2.42)). Dependencies of $\cos \theta_{\text{GJ}}$ for example are covered due to the description of t_1 , which directly depends on $\cos \theta_{\text{GJ}}$. There is no such equivalent for the ϕ_{GJ} in the model. In Figs. 6.11(a) and 6.11(b) we show the ϕ_{GJ} distributions. In both final states, the ϕ_{GJ} dependence of the $a_2 \mathbb{P}$ is largely different than that of the other amplitudes. The ϕ_{GJ} description is much worse in the $\eta\pi^-$, compared to the $\eta'\pi^-$ final state. The number of forward events is much larger in the $\eta\pi^-$ final state, compared to the $\eta'\pi^-$ final state and the description of the forward intensity is not as convincing for the $\eta\pi^-$ final state, compared to the $\eta'\pi^-$ final state (see Section 6.5.1). Therefore, it makes sense that the ϕ_{GJ} distribution is described to a lesser degree in the $\eta\pi^-$ final state. This is consequently also reflected in the right-left asymmetry (Figs. 6.11(c) and 6.11(d)), which is underestimated for the $\eta\pi^-$ final state. For completeness, we have drawn the forward and backward components of the right-left asymmetry in Fig. 6.12.

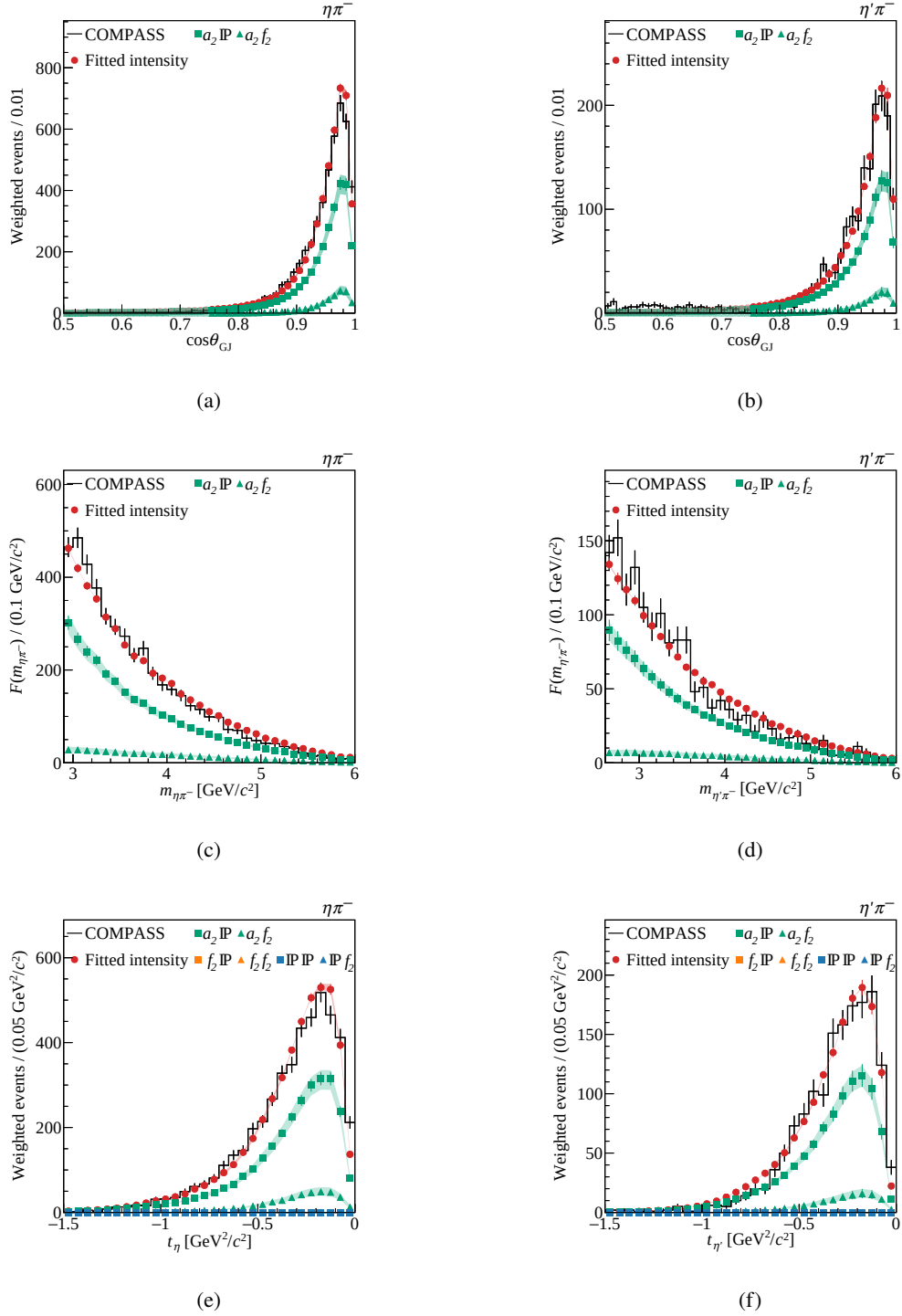


Figure 6.7: $\cos\theta_{GJ}$ distribution in the $\eta\pi^-$ (a) and $\eta'\pi^-$ (b) final state, the forward intensity in the $\eta\pi^-$ (c) and $\eta'\pi^-$ (d) final state and the $t_1 = t_\eta$ (e) and $t_1 = t_{\eta'}$ (f) distribution, zoomed for only forward events. We use the weighted intensity color code in the shown diagrams.

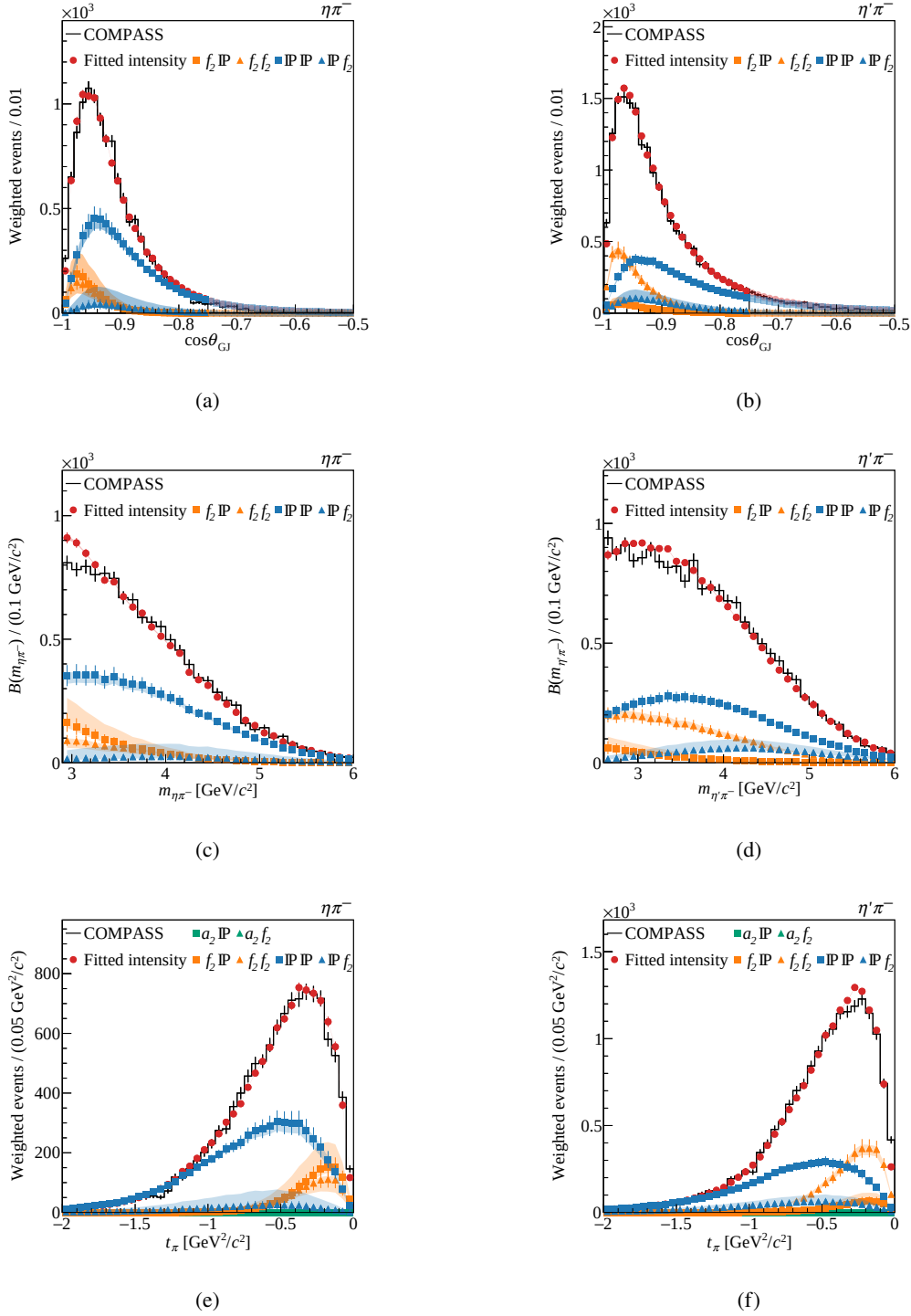


Figure 6.8: $\cos\theta_{GJ}$ distribution in the $\eta\pi^-$ (a) and $\eta'\pi^-$ (b) final state, the backward intensity in the $\eta\pi^-$ (c) and $\eta'\pi^-$ (d) final state and the $t_1 = t_\eta$ (e) and $t_1 = t_{\eta'}$ (f) distribution, zoomed for only backward events. We use the weighted intensity color code in the shown diagrams.

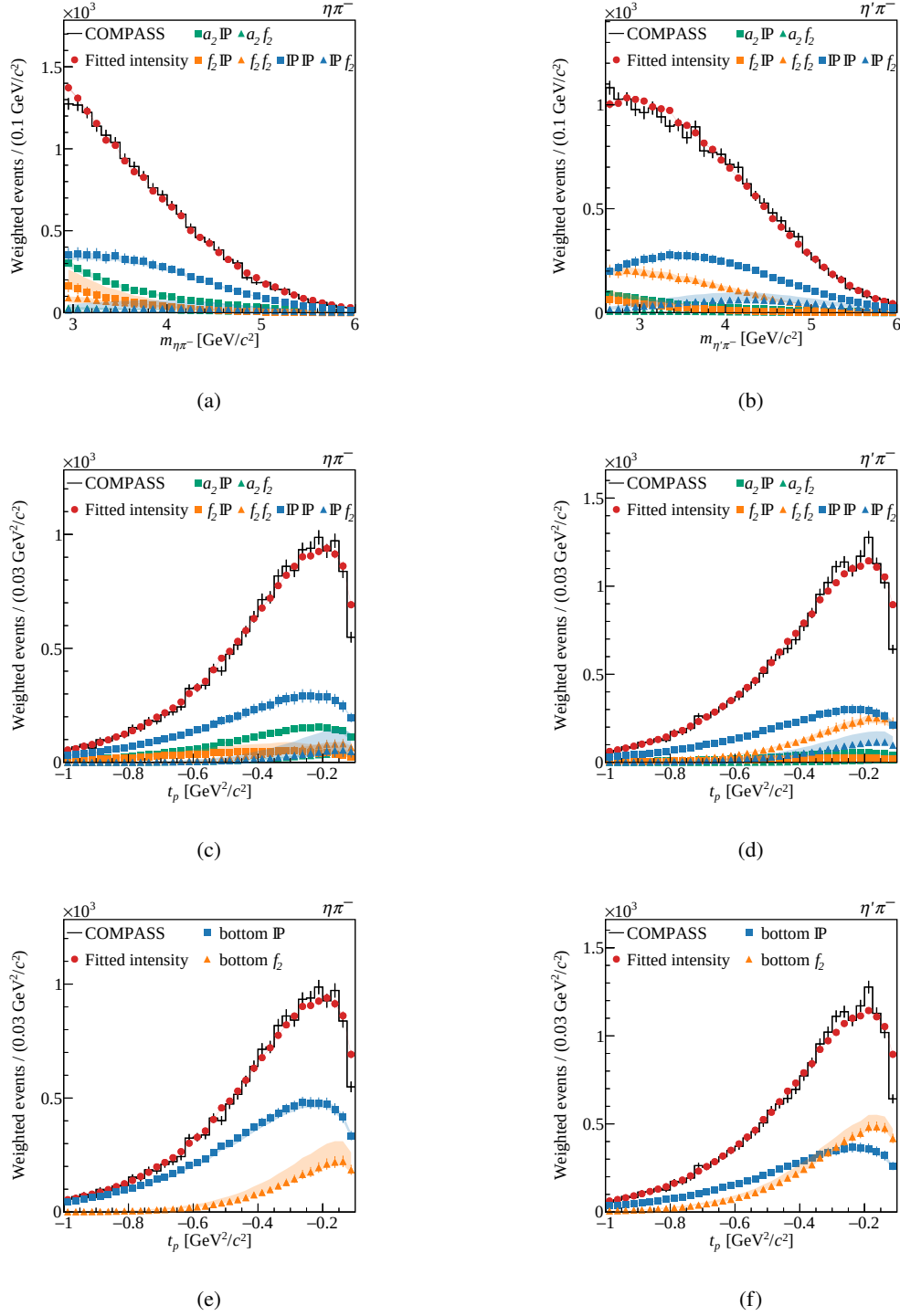


Figure 6.9: $m_{\eta\pi^-}$ (a), $m_{\eta'\pi^-}$ (b), $t_2 = t_p$ distribution in the $\eta\pi^-$ (c) and $\eta'\pi^-$ (d) final state and the $t_2 = t_p$ distribution for the $\eta\pi^-$ (e) and $\eta'\pi^-$ (f) final state, where the amplitude, used to produce the curves, is constructed by only diagrams with a $\mathbb{P}$ or f_2 at the bottom vertex. We use the weighted intensity color code in the shown diagrams.

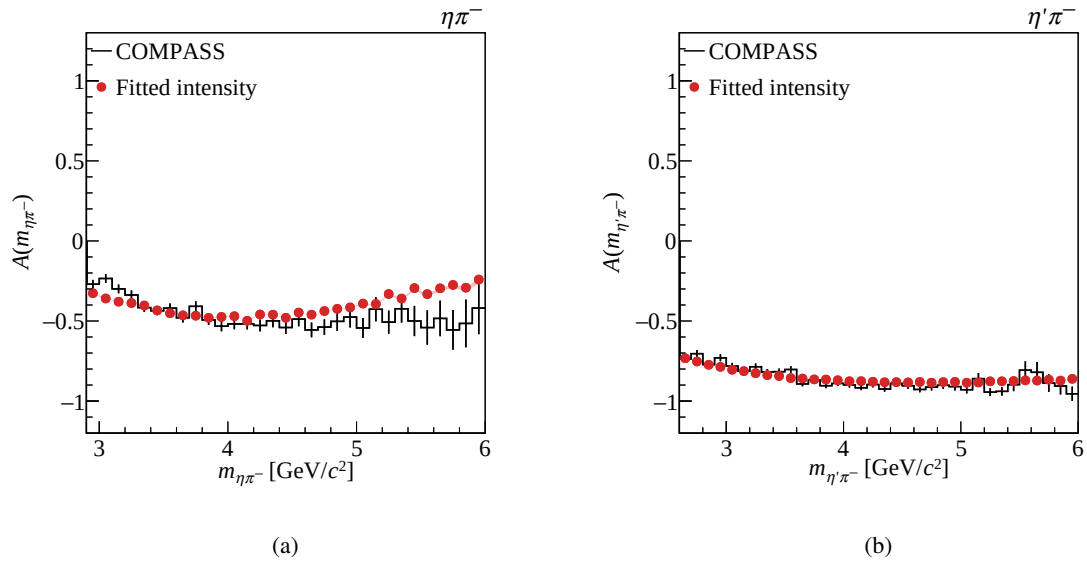


Figure 6.10: Forward-backward intensity in the $\eta\pi^-$ (a) and $\eta'\pi^-$ (b) final state. We use the weighted intensity color code in the shown diagrams.

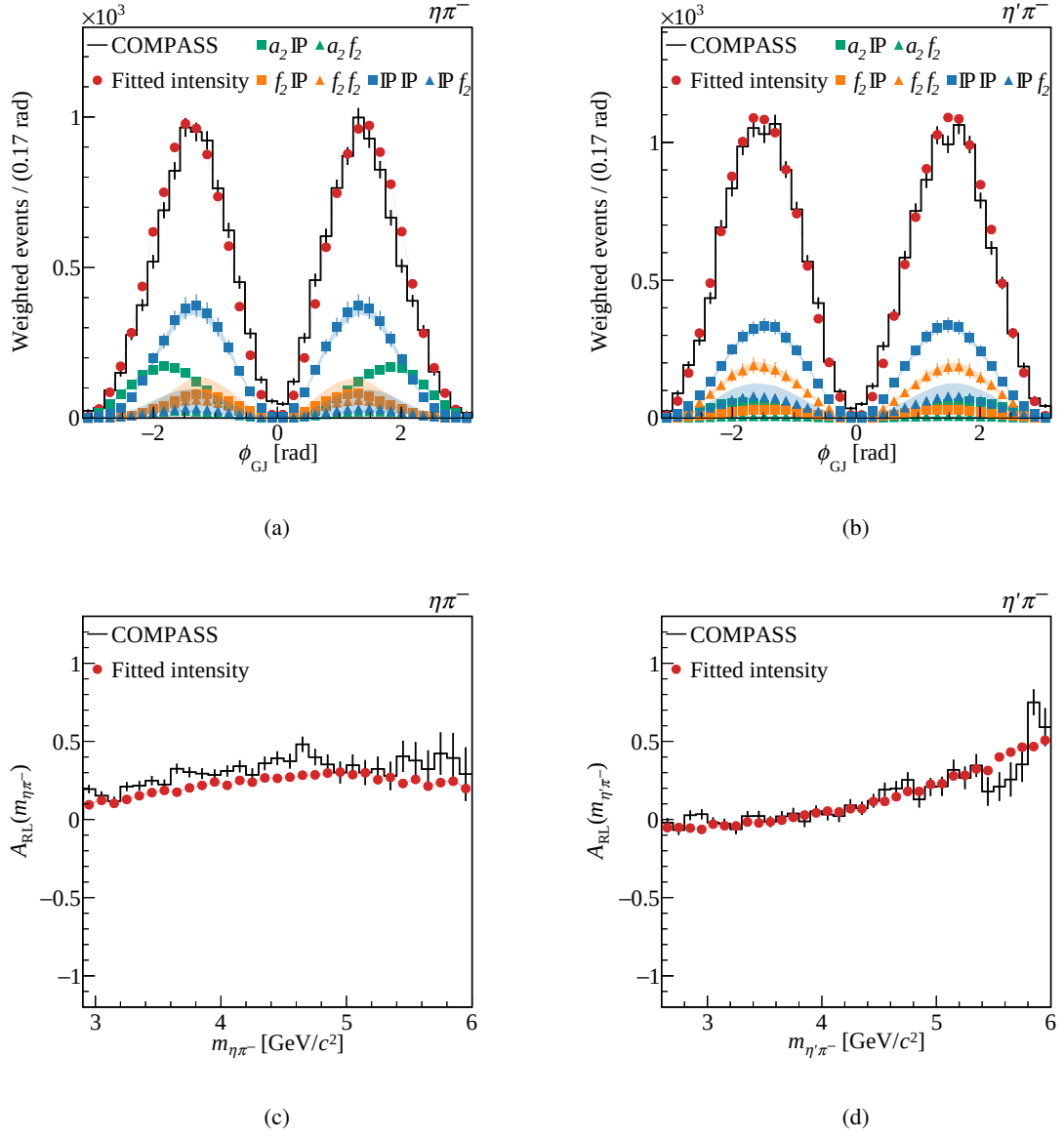


Figure 6.11: ϕ_{GJ} distribution in the $\eta\pi^-$ (a) and $\eta'\pi^-$ (b) final state and the right-left asymmetry in the $\eta\pi^-$ (c) and $\eta'\pi^-$ (d) final state. We use the weighted intensity color code in the shown diagrams.

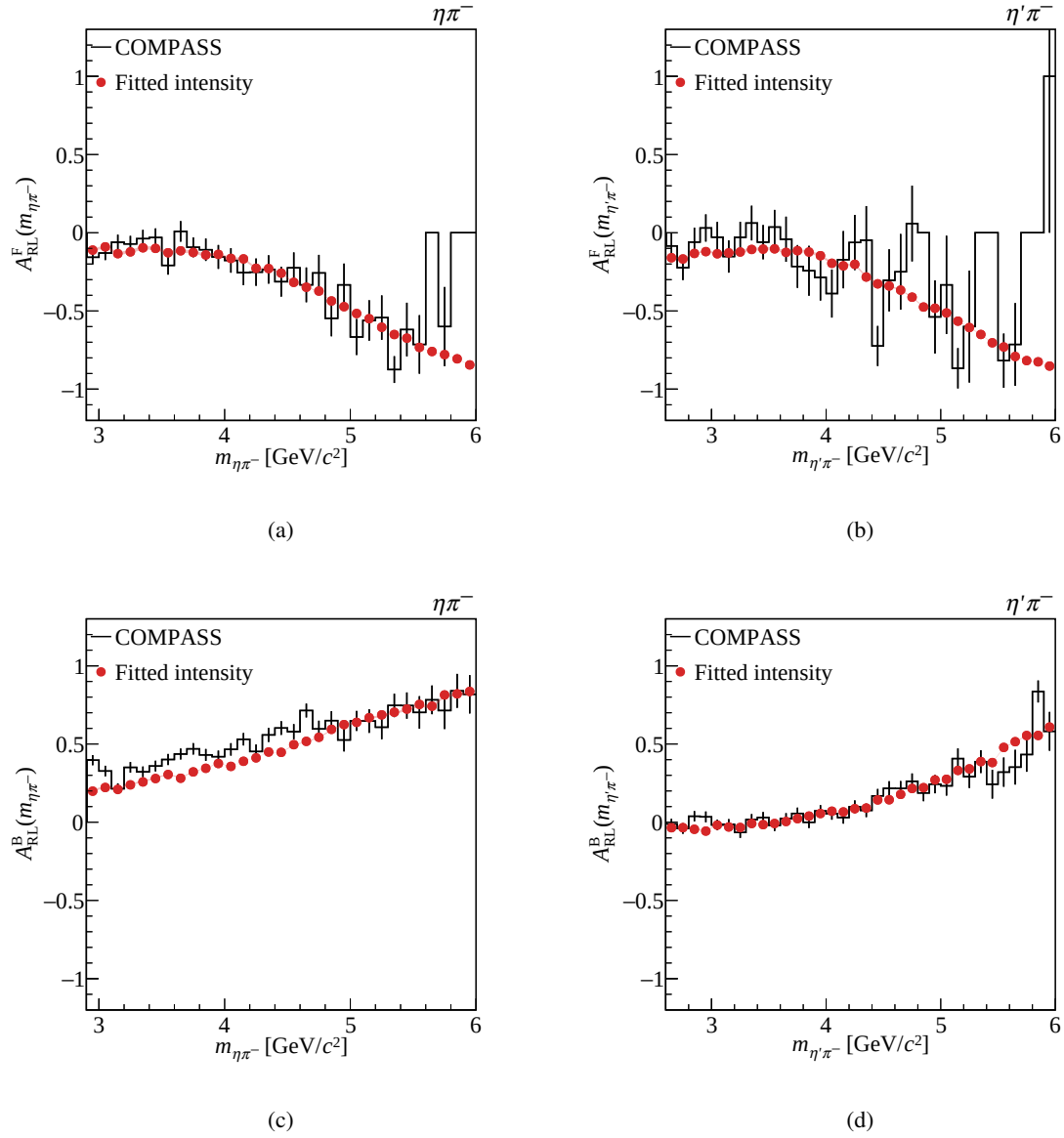


Figure 6.12: Right-left asymmetry for forward events in the $\eta\pi^-$ (a) and $\eta'\pi^-$ (b) final state, as well as the right-left asymmetry for the backward events in the $\eta\pi^-$ (c) and $\eta'\pi^-$ (d) final state. We use the weighted intensity color code in the shown diagrams.

6.6 Validation of the $m_{\eta^{(\prime)}\pi^-}$ Cut

The double-Regge amplitude (Eq. (2.49)) fully describes the double-Regge kinematics in $m_{\eta^{(\prime)}\pi^-} = \sqrt{s_1}$. Therefore, a fit in bins of s_1 , should yield consistent results across all bins and with the global fit. We test this by imposing a binning of $0.1 \text{ GeV } c^{-2}$ below $m_{\eta'\pi^-} = 5 \text{ GeV } c^{-2}$ and $0.25 \text{ GeV } c^{-2}$ above. We increase the bin width at higher mass, since the amount of data is very low there. For every bin, the slope parameters ($\vec{b}$) are fixed to that of the unbinned fit. We perform 20 000 successful fits per bin¹⁰ and show the strength coefficients c_i for every diagram, bin and fit in Fig. 6.13.

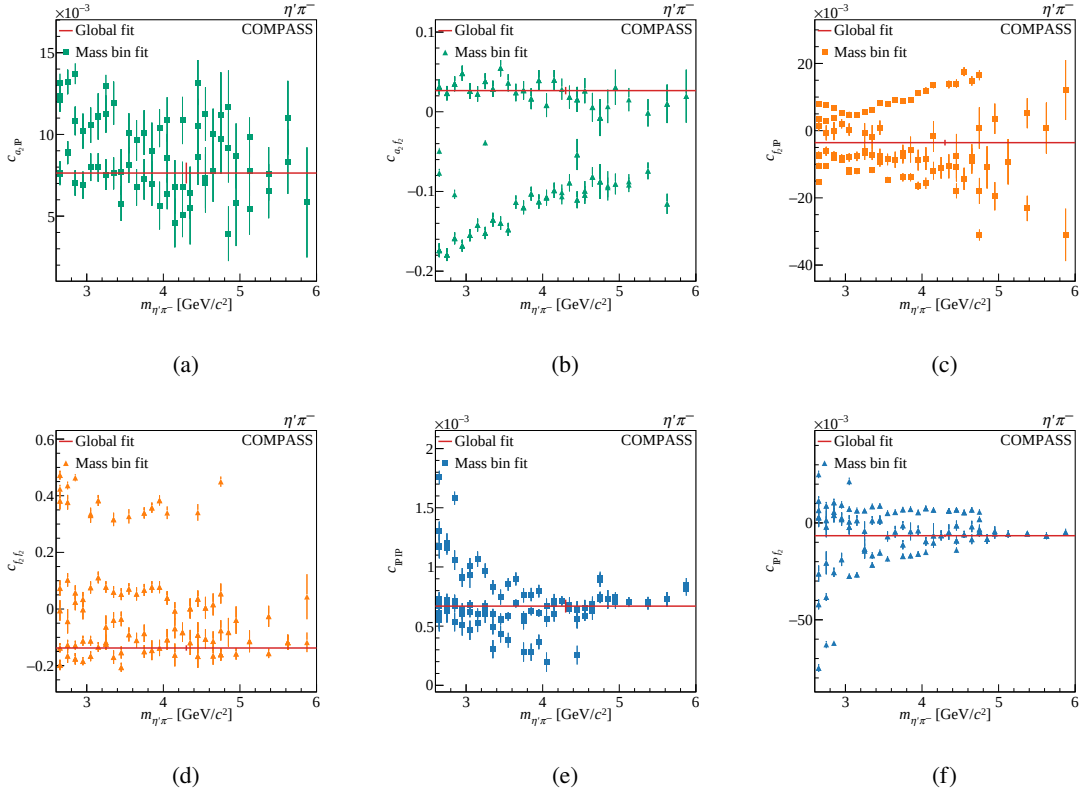


Figure 6.13: Fitted strength coefficients $c_{a_2\mathbb{P}}$ (a), $c_{a_2f_2}$ (b), $c_{f_2\mathbb{P}}$ (c), $c_{f_2f_2}$ (d), $c_{\mathbb{P}\mathbb{P}}$ (e) and $c_{\mathbb{P}f_2}$ (f) for the mass bin fit in the $\eta'\pi^-$ final state for all successful fits. The global minimum from the unbinned fit is drawn as a red line on top of the diagrams.

From these plots we see, that a lot of local minima are found by the different fits. In order to select the fit, closest to the result of the global fit (shown with a red line in the plots), we define, in agreement with Ref. [70], the Residual as

¹⁰ In Eq. (C.9), we derive a different representation of the the intensity (Eq. (6.1)), which allows us to perform the sum over all events before the minimization. Since we fix the slope parameters, the amplitude per event is constant with respect to the minimization. Therefore, it makes sense to use this trick here. Consequently, the number of MC events becomes irrelevant to the minimization, which reduces the computing time by a large factor, when dealing with the sum over all MC events and allows us to perform a high number of fits per bin.

$$\text{Residual}(b) = T^{-1}(\vec{c}^b, \vec{c}^g) M_{\text{cov}}^{-1}(b) T(\vec{c}^b, \vec{c}^g), \quad (6.17a)$$

$$T(\vec{c}^b, \vec{c}^g) = \begin{pmatrix} c_1^b - c_1^g \\ c_2^b - c_2^g \\ c_3^b - c_3^g \\ c_4^b - c_4^g \\ c_5^b - c_5^g \\ c_6^b - c_6^g \\ c_7^b - c_7^g \end{pmatrix}, \quad (6.17b)$$

where b indicates the bin and g the global unbinned fit result. Per bin, the fit with the lowest Residual is selected. The final strength coefficient distribution is shown in Fig. 6.14 for the $\eta'\pi^-$ and in Fig. 6.15 for the $\eta\pi^-$ final state. In both final states, we see no strong mass-dependence for most of the amplitudes. One could make the case, that the $a_2 f_2$ amplitude and the $\mathbb{P} f_2$ amplitude exhibit a slight falling behavior towards higher masses. However, these amplitudes catch very little intensity in the global fit.

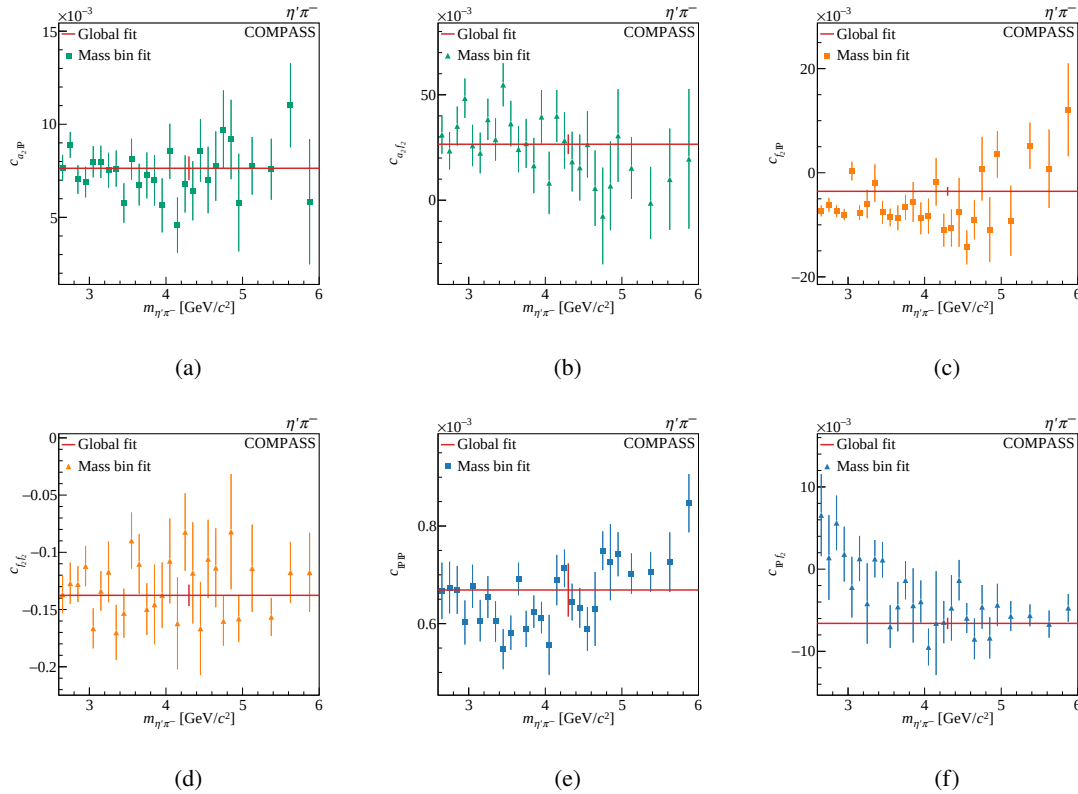


Figure 6.14: Fitted strength coefficients $c_{a_2 \mathbb{P}}$ (a), $c_{a_2 f_2}$ (b), $c_{f_2 \mathbb{P}}$ (c), $c_{f_2 f_2}$ (d), $c_{\mathbb{P} \mathbb{P}}$ (e) and $c_{\mathbb{P} f_2}$ (f) for the mass bin fit in the $\eta'\pi^-$ final state for the best successful fits. The global minimum from the unbinned fit is drawn as a red line on top of the diagrams.

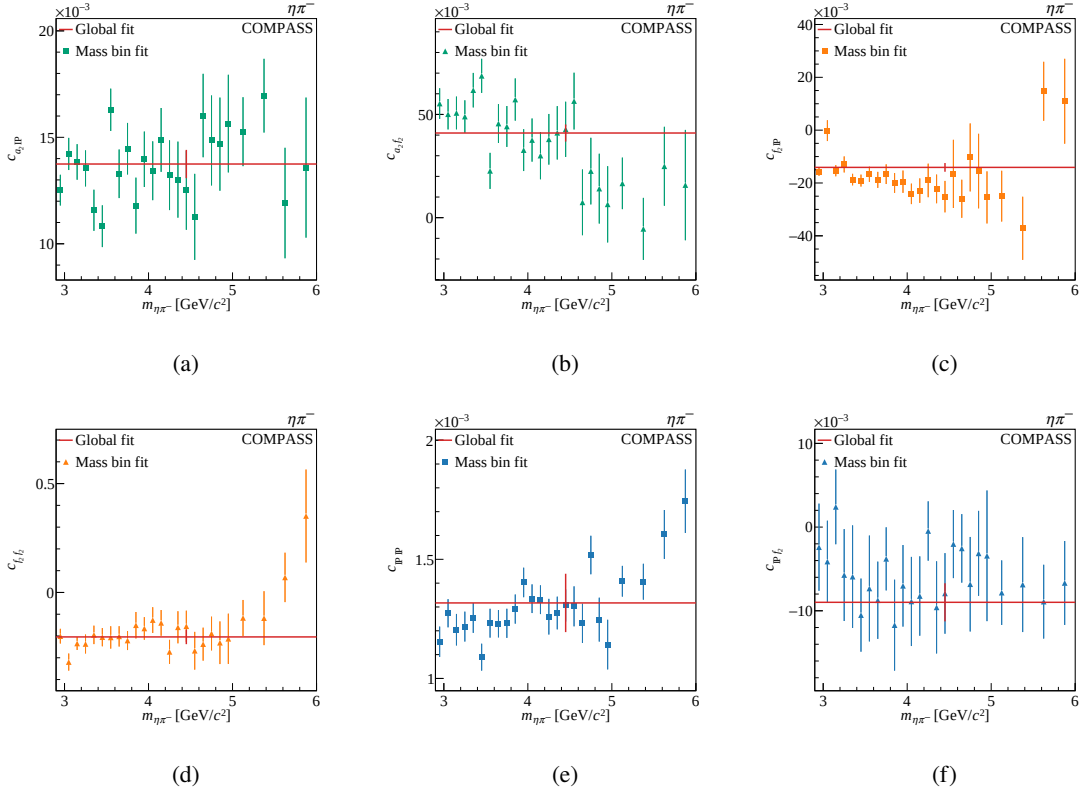


Figure 6.15: Fitted strength coefficients $c_{a_2\mathbb{P}}$ (a), $c_{a_2f_2}$ (b), $c_{f_2\mathbb{P}}$ (c), $c_{f_2f_2}$ (d), $c_{\mathbb{P}\mathbb{P}}$ (e) and $c_{\mathbb{P}f_2}$ (f) for the mass bin fit in the $\eta\pi^-$ final state for the best successful fits. The global minimum from the unbinned fit is drawn as a red line on top of the diagrams.

In order to judge the goodness of the fit, we extend Eq. (6.17) to include all bins and define

$$\text{Residual}_{\text{weighted}} = \frac{\sum_b \text{Residual}(b)}{6 \cdot N_b}. \quad (6.18)$$

We can validate the $m_{\eta^{(\prime)}\pi^-}$ cut further by repeating the global fit for a different mass range, extracting slope parameters and strength coefficients and repeating the binned fit for the new mass range. We perform 19 such studies, where $m_{\eta^{(\prime)}\pi^-}$ ranges from 1.8 to $6 \text{ GeV } c^{-2}$, 1.9 to $6 \text{ GeV } c^{-2}$, 2.0 to $6 \text{ GeV } c^{-2}$, ..., 3.5 to $6 \text{ GeV } c^{-2}$, 4.0 to $6 \text{ GeV } c^{-2}$. For each study we construct a $\text{Residual}_{\text{weighted}}$ and show it in Fig. 6.16(a) for the $\eta\pi^-$ and Fig. 6.16(b) for the $\eta'\pi^-$ final state.

In both final state, we see a clear falling trend of the $\text{Residual}_{\text{weighted}}$. In the $\eta'\pi^-$ final state, it starts to saturate around $2.6 \text{ GeV } c^{-2}$. In the $\eta\pi^-$ final state at around $2.9 \text{ GeV } c^{-2}$. We chose these values for our main fit. It is important to note, that the $\text{Residual}_{\text{weighted}}$ gets even smaller for $4.0 \text{ GeV } c^{-2}$. However, in that mass region the amount of data also gets smaller and we decide to describe a larger kinematics and start the fit a little earlier throughout this work. The rise in the $\text{Residual}_{\text{weighted}}$ around $2.7 \text{ GeV } c^{-2}$ is due to the fact, that the fit finds the best minimum to be one where certain amplitudes are deselected, i.e. their c_i is 0 in the global fit. The mass binned fit does not agree with this, which makes the $\text{Residual}_{\text{weighted}}$ rise.

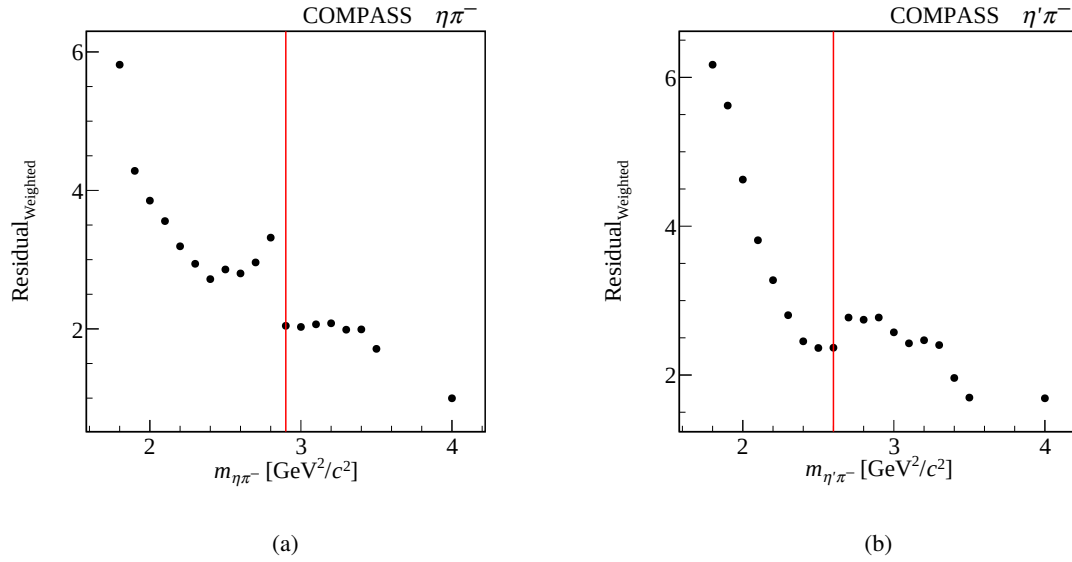


Figure 6.16: The $\text{Residual}_{\text{weighted}}$ distribution for the $\eta\pi^-$ (a) and $\eta'\pi^-$ (b) final state for different $m_{\eta^{(\prime)}\pi^-}$ ranges. The lower limit of the main fit is indicated by the red line.

Summary and Outlook

In this work, we investigated the production of the $\eta^{(\prime)}\pi^-$ system at COMPASS, with particular focus on the high-energy regime where double-Regge exchange dominates. We began in Chapter 2 by introducing the quark model, scattering theory, and deriving the general double-Regge amplitude from Regge theory. The forward-backward asymmetry was defined. Its deviation from zero directly indicates the presence of both odd and even partial waves. Odd partial waves in the $\eta^{(\prime)}\pi^-$ system correspond to spin-exotic quantum numbers.

In Chapter 3, the COMPASS experiment was introduced, which collected two years of dedicated high-energy π^- scattering data on a fixed proton target. In Chapter 4, the selection of the exclusive $\eta^{(\prime)}\pi^- + p$ final state is described, yielding approximately 270 000 events for $\eta\pi^-$ and 90 000 events for $\eta'\pi^-$. We reconstruct the $\eta^{(\prime)}\pi^-$ system in the decay $\eta^{(\prime)}\pi^- \rightarrow \pi^-\pi^+\pi^0(\eta)$, $\pi^0(\eta) \rightarrow \gamma\gamma$.

Accurate acceptance modeling is crucial for this analysis, especially in the calorimeters, where the two photons are reconstructed. In Chapter 5, we describe the COMPASS MC simulation program TGEANT. We described updates to the implementation of the trigger logic and implemented missing detectors. The calorimeter simulation previously relied on a slow and approximate implementation. We redesigned it using parameterized module structures, allowing memory reuse and thereby achieving an efficient and fast simulation. Additionally, this allowed us to describe each calorimeter module in much more detail, including internal structures, which improves the accuracy of the shower simulation. Finally, we created a large phase-space distributed $\eta^{(\prime)}\pi^-$ signal MC sample and propagated it through the MC chain. With this, we are able to describe the acceptance for the $\eta^{(\prime)}\pi^-$ final state with high accuracy.

In Chapter 6, we applied the double-Regge amplitude to fit the data. The resonance and double-Regge regions were separated using cuts in $m_{\eta^{(\prime)}\pi^-}$ and $\cos\theta_{\text{GJ}}$: $2.9 \text{ GeV } c^{-2} < m_{\eta\pi^-} < 6 \text{ GeV } c^{-2}$, $2.6 \text{ GeV } c^{-2} < m_{\eta'\pi^-} < 6 \text{ GeV } c^{-2}$, and $|\cos\theta_{\text{GJ}}| > 0.75$. Six double-Regge amplitudes were used: two for forward η ($a_2\mathbb{P}$ and a_2f_2) and four for backward η ($f_2\mathbb{P}$, f_2f_2 , $\mathbb{P}\mathbb{P}$, and $\mathbb{P}f_2$). The fit was achieved with only 13 free parameters: six strength coefficients (one per amplitude) that represent the different particle-Reggeon-particle and Reggeon-particle-Reggeon couplings per amplitude, and seven slope parameters describing the t -dependence at each unique particle-Reggeon-particle vertex. We found that accurate t -dependence modeling is critical for extracting the relative strength of amplitudes and is a novel aspect of this study, compared to Ref. [11].

The forward region was found to be dominated by $a_2\mathbb{P}$ exchange, with a small a_2f_2 component that was mainly needed to account for the ϕ_{GJ} dependence. The forward intensity in $\eta'\pi^-$ is slightly better

described than in $\eta\pi^-$, suggesting a possible small contribution from unnatural exchanges or daughter trajectories in the $\eta\pi^-$ final state. The backward sector is accurately modeled in both final states, with $\mathbb{P}\mathbb{P}$ carrying the largest intensity, while interference among amplitudes is essential to describe the data. Especially interesting is the importance of bottom f_2 diagrams, which are found to be of similar importance as bottom $\mathbb{P}$ diagrams. These results differ from Ref. [11], which found f_2f_2 dominance, highlighting the importance of modeling the correct t -dependence at the vertices.

Future work could include extracting the coupling strengths at each vertex using the fitted coefficients c_i . Double-Regge exchange also serves as a background in the resonance region, and theoretical approaches such as finite energy sum rules could connect resonance and double-Regge regions once applicable to $2 \rightarrow 3$ scattering. The results of this thesis provide a foundation for such studies and demonstrate the first event-based double-Regge fit of the $\eta^{(\prime)}\pi^-$ final state.

Further Details on the Event Selection for $\eta^{(\prime)}\pi^-$ Data

A.1 Determination of the Cut Ranges

If there is no clear reason to define the cut window otherwise, it is chosen such that 99 % of the signal is captured within the cut. First, we fit an appropriate signal and background curve to the data. This is a double Gaussian for the signal, which is mostly dominated by detector resolution, and a polynomial of third order for the background description. The only exception is the exclusivity peak, where the fit function is defined in Eq. (4.18). Then, we determine the left and right width at half maximum (LWHM and RWHM, respectively) and figure out a factor k with whom we multiply both widths in order to arrive at the cut window.

$$\text{Lower cut boarder: } \text{mean} - k \cdot \text{LWHM}, \quad (\text{A.1a})$$

$$\text{Upper cut boarder: } \text{mean} + k \cdot \text{RWHM}. \quad (\text{A.1b})$$

The fits to determine the cut ranges are performed on the Antok-Style histograms while using the Slot4 cut ranges [44] for all other cuts. They are shown in Figs. A.1 and A.2 for the $\eta\pi^-$ and $\eta'\pi^-$ final state respectively.

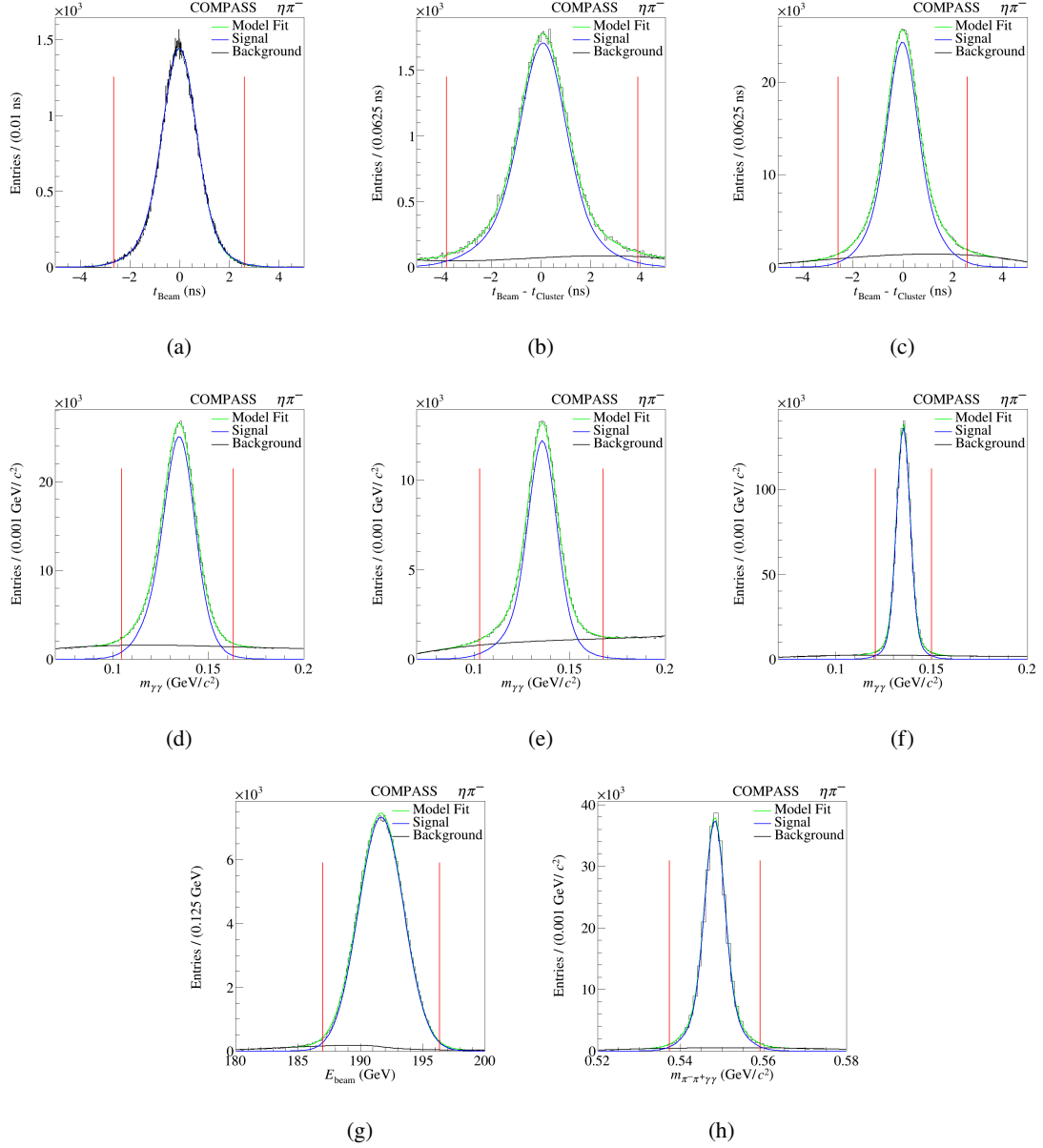


Figure A.1: Slot5 cut ranges for the beam time (a), the ECAL1 cluster time (b), ECAL2 cluster time (c), the two photon invariant mass where both photons are reconstructed in ECAL1 (d), ECAL2 (f) and one in ECAL1 and one in ECAL2 (e). Additionally, the Slot5 cut ranges for the reconstructed beam energy (g) and the η invariant mass spectrum. All plots are shown for the $\eta\pi^-$ final state.

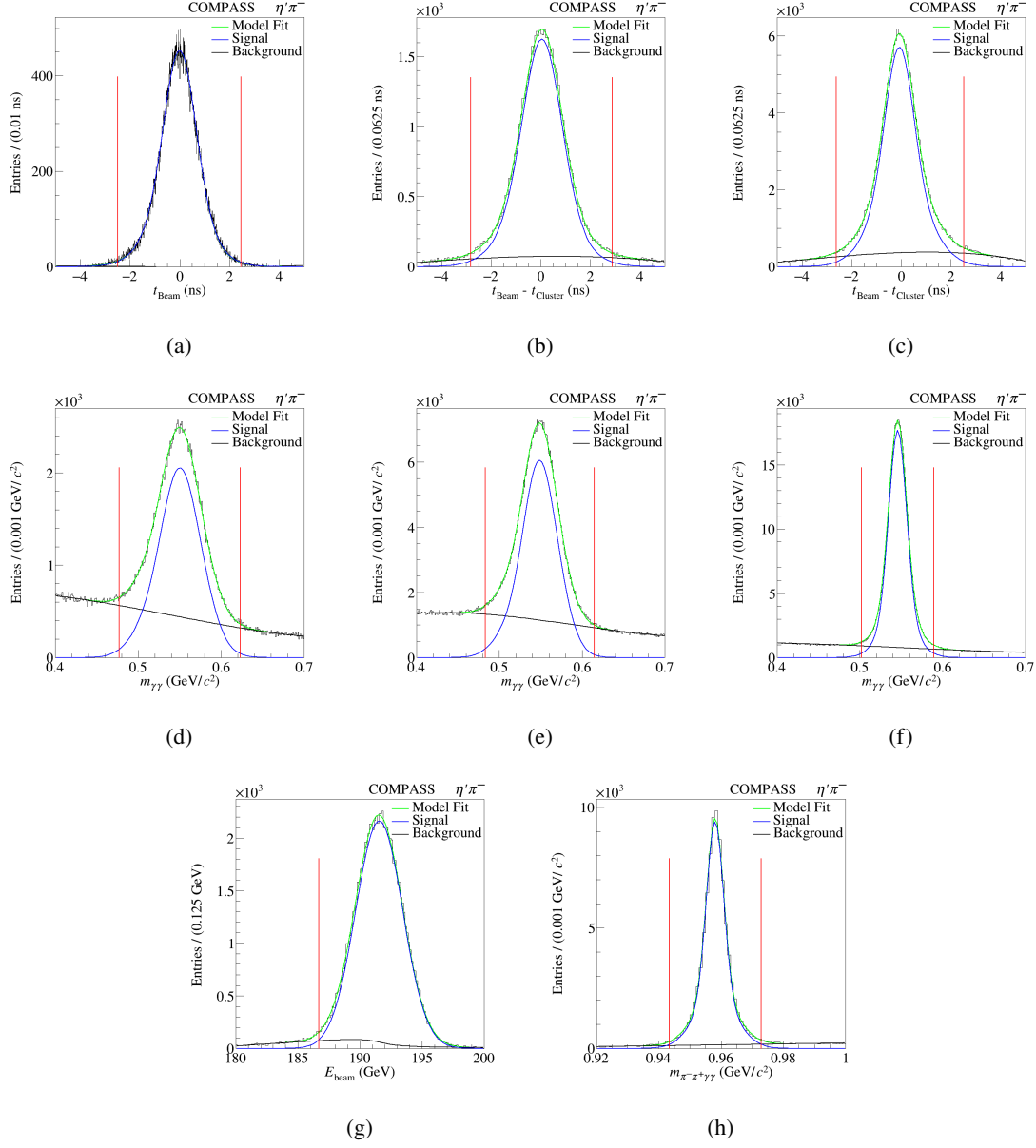


Figure A.2: Slot5 cut ranges for the beam time (a), the ECAL1 cluster time (b), ECAL2 cluster time (c), the two photon invariant mass where both photons are reconstructed in ECAL1 (d), ECAL2 (f) and one in ECAL1 and one in ECAL2 (e). Additionally, the Slot5 cut ranges for the reconstructed beam energy (g) and the $\eta'\pi^-$ invariant mass spectrum. All plots are shown for the $\eta'\pi^-$ final state.

A.2 Full Event Selection Plots in Antok-Style and Waterfall-Style

In Chapter 4, we document the event selection in great detail, with a focus on the $\eta'\pi^-$ final state within the plots, while focusing on the Antok-Style. Here, we display a complete set of plots per cut that reflects the Waterfall-Style and Antok-Style for the $\eta\pi^-$ and $\eta'\pi^-$ final state.

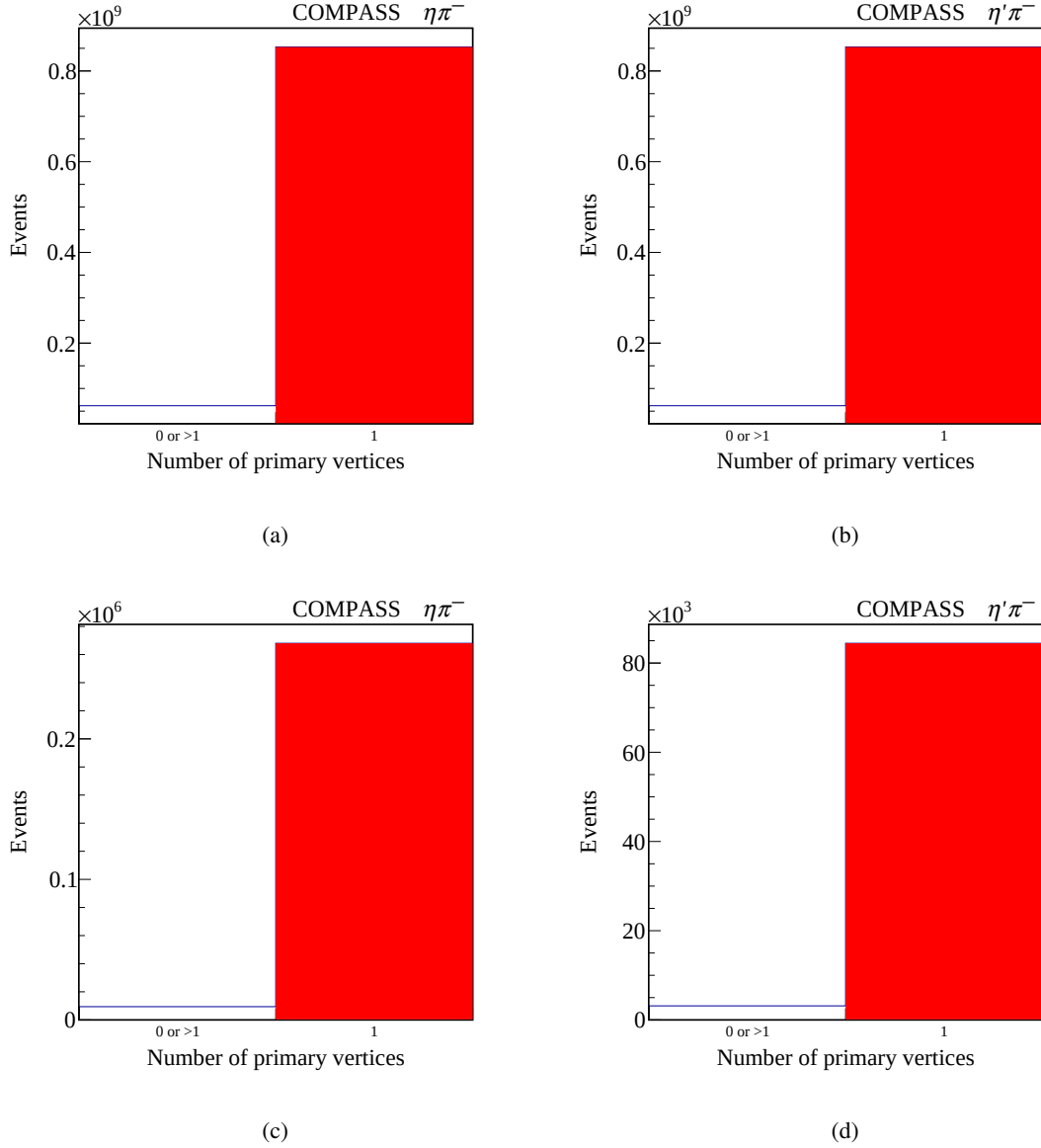


Figure A.3: Slot5 cut on one and exactly one PV. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style. The remaining events after the cut are shown in red.

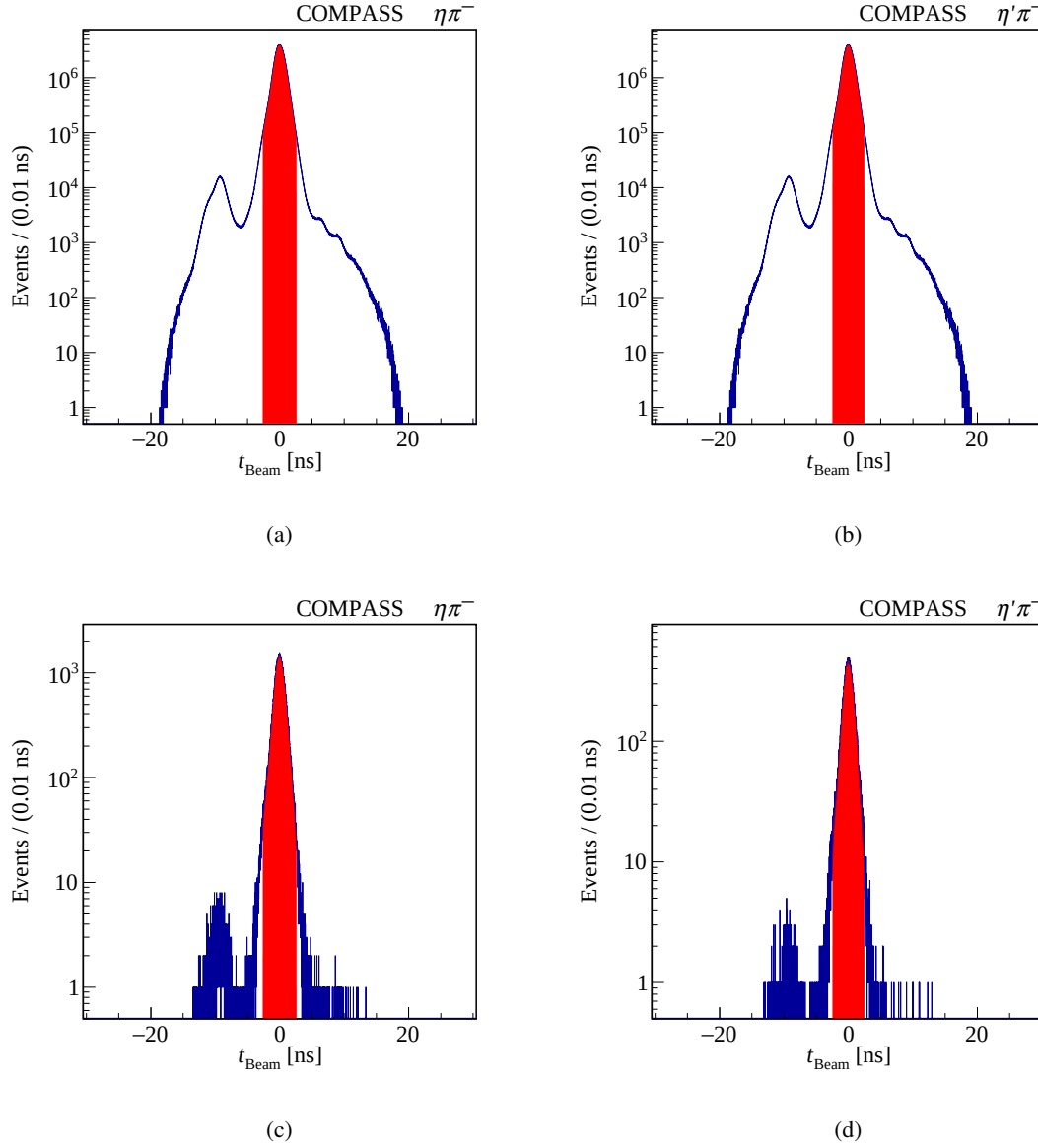


Figure A.4: Slot5 cut on the relative time of the beam particle to the trigger. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style. The remaining events after the cut are shown in red.

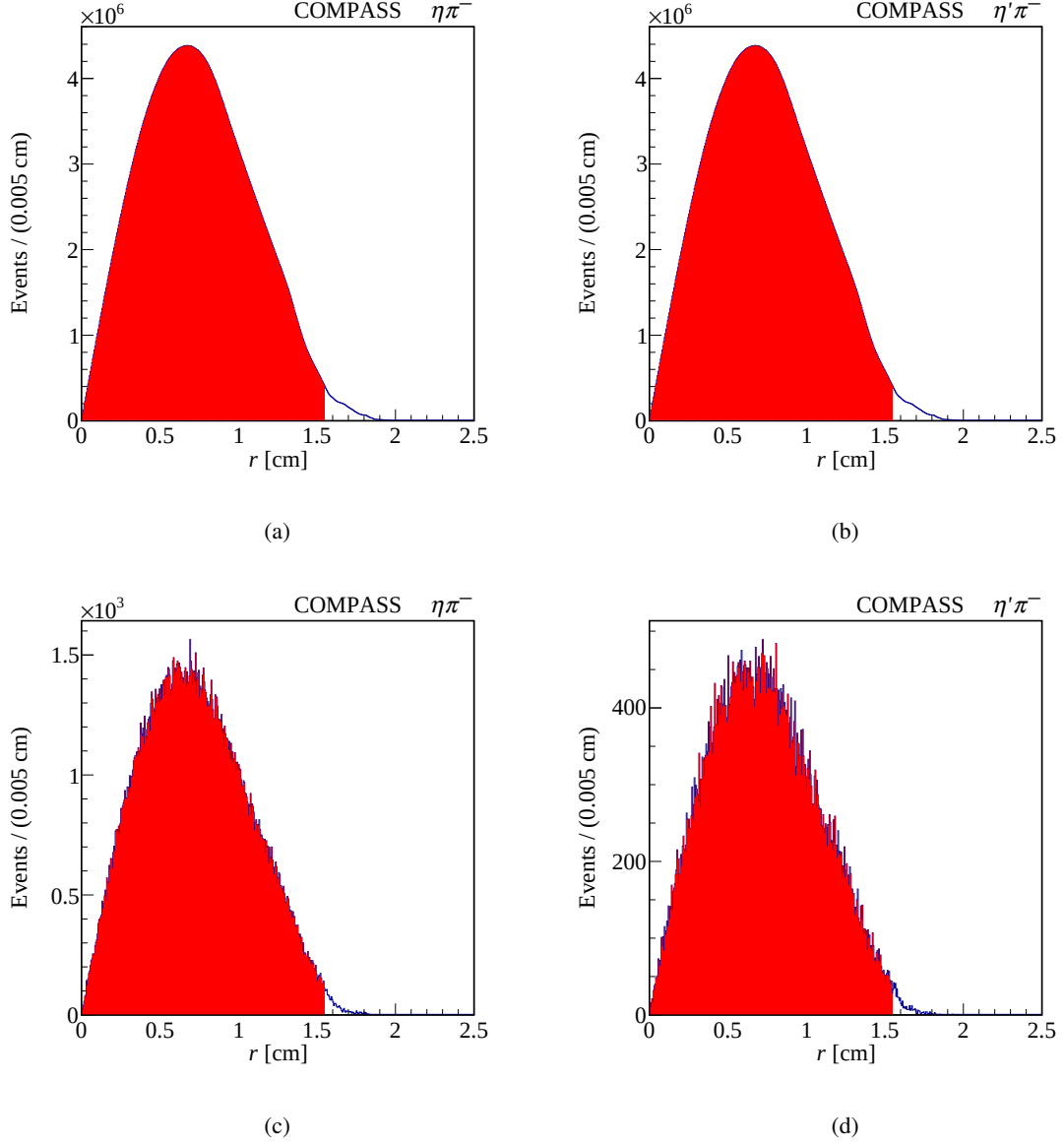


Figure A.5: Slot5 cut on the radial distance of the PV to the z axis. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style. The remaining events after the cut are shown in red.

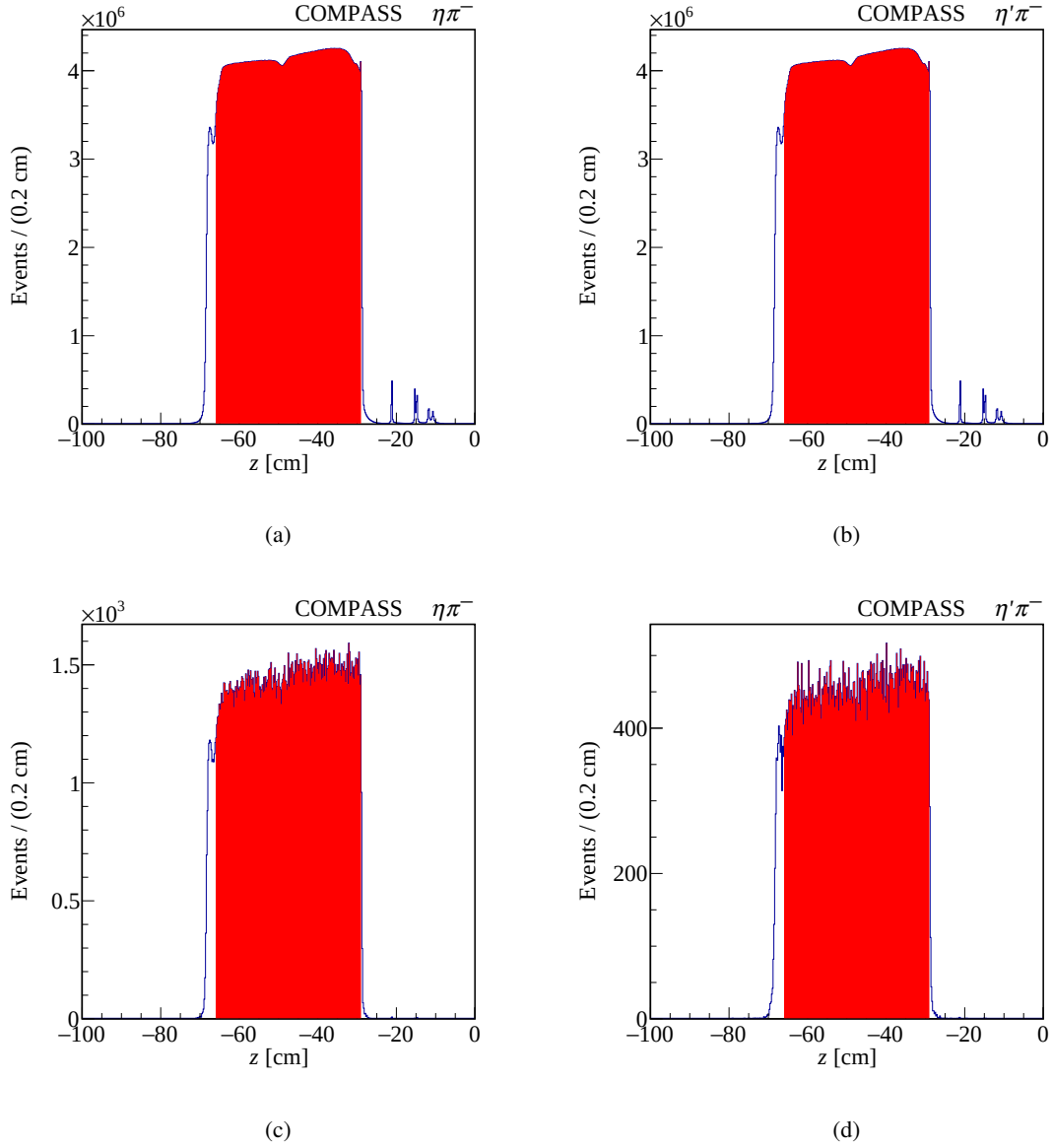


Figure A.6: Slot5 cut on the z position of the PV. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style. The remaining events after the cut are shown in red.

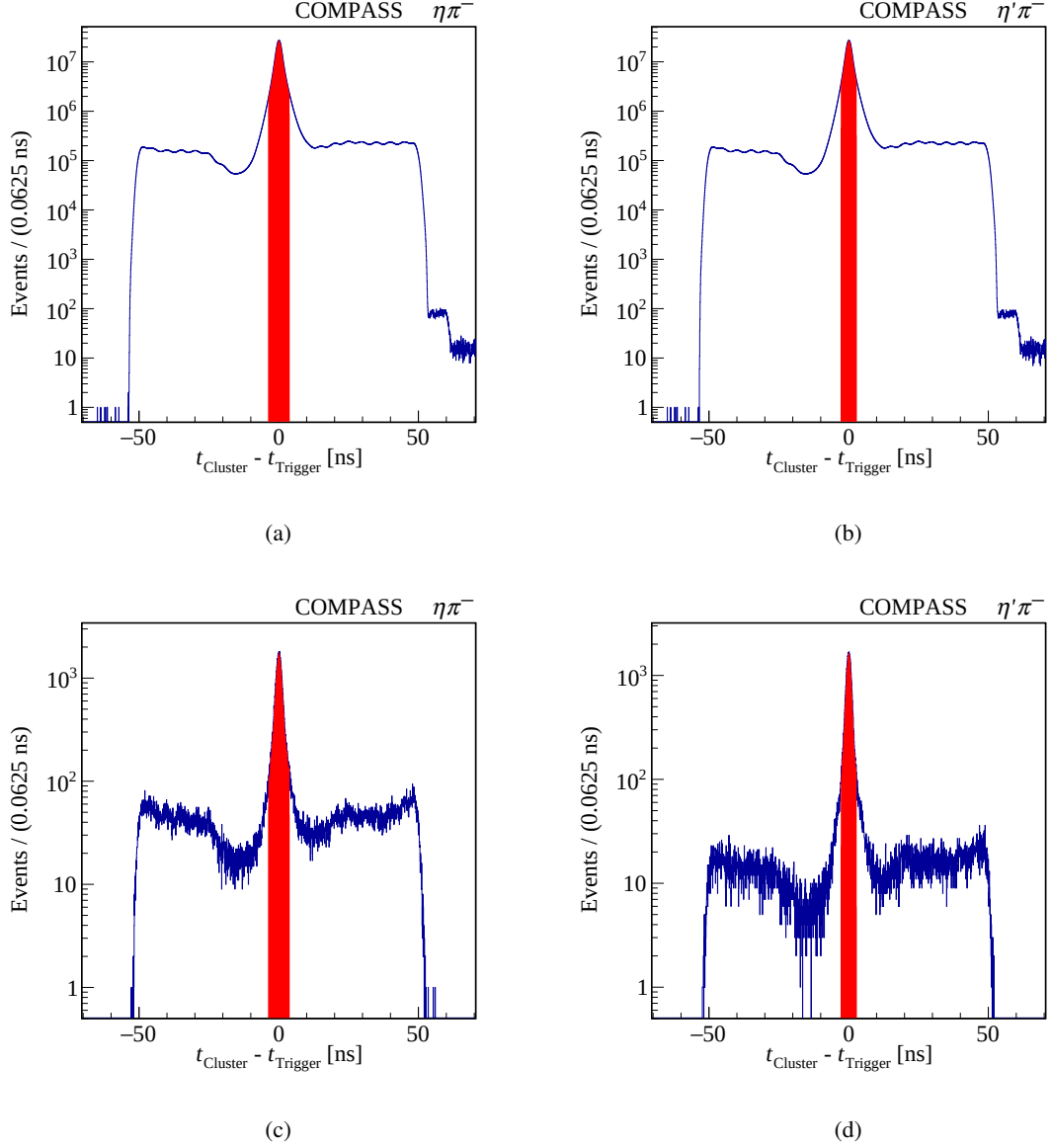


Figure A.7: Slot5 cut on the difference in cluster time to trigger time for ECAL1. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style. The remaining events after the cut are shown in red.

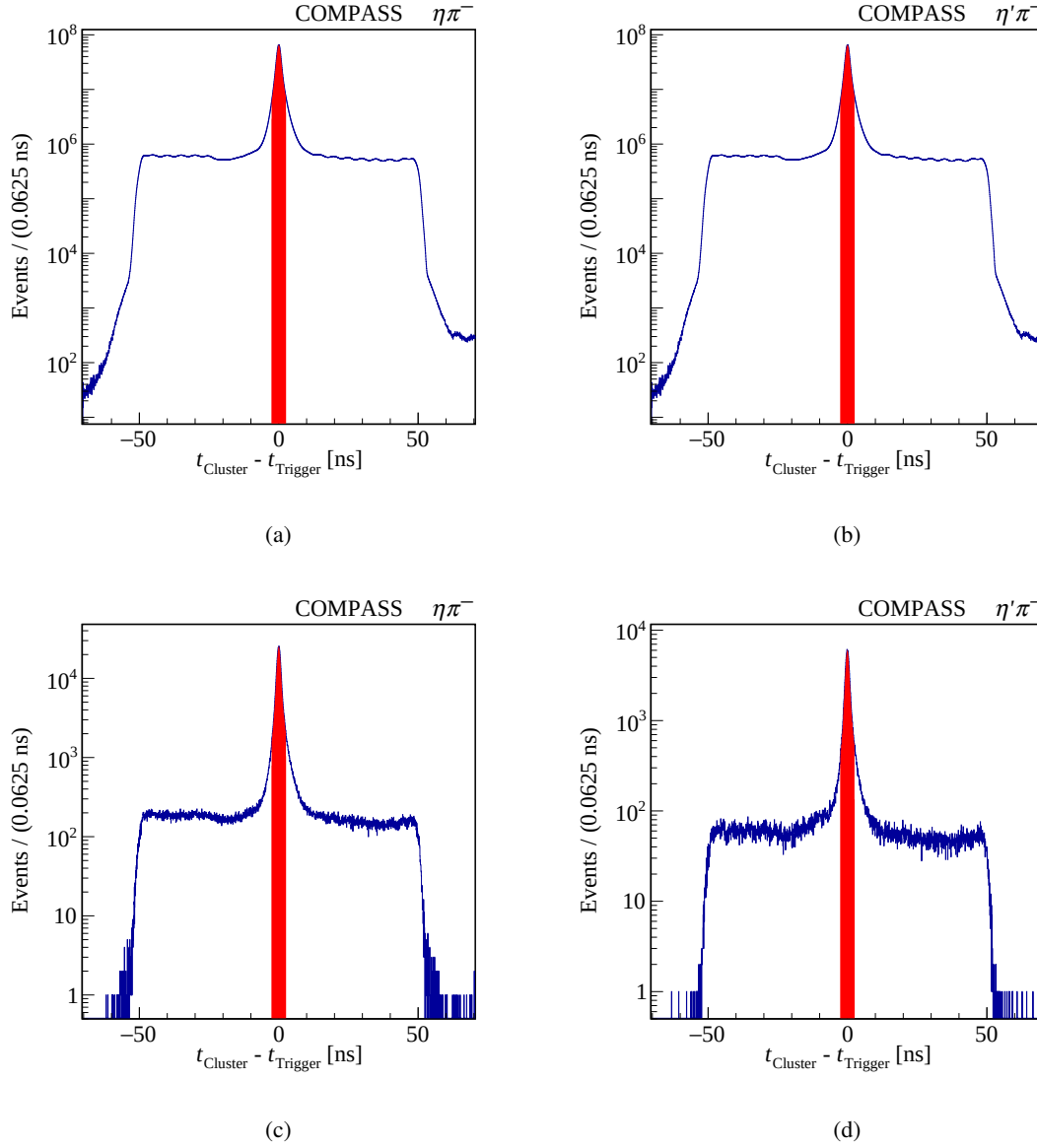


Figure A.8: Slot5 cut on the difference in cluster time to trigger time for ECAL2. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style. The remaining events after the cut are shown in red.

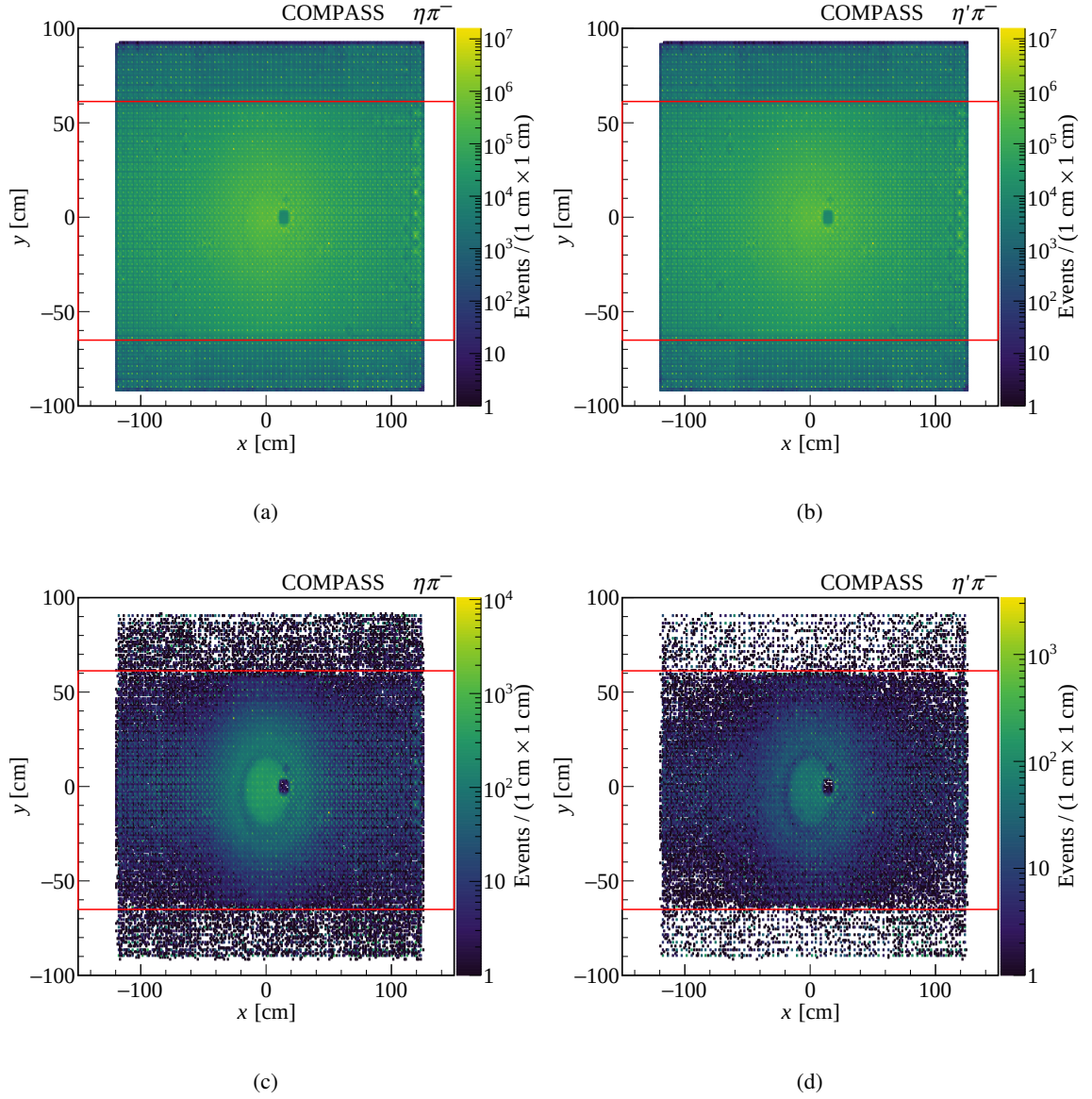


Figure A.9: Slot5 cut on the shadow of the HCAL1. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style. The cut ranges are shown with red lines.

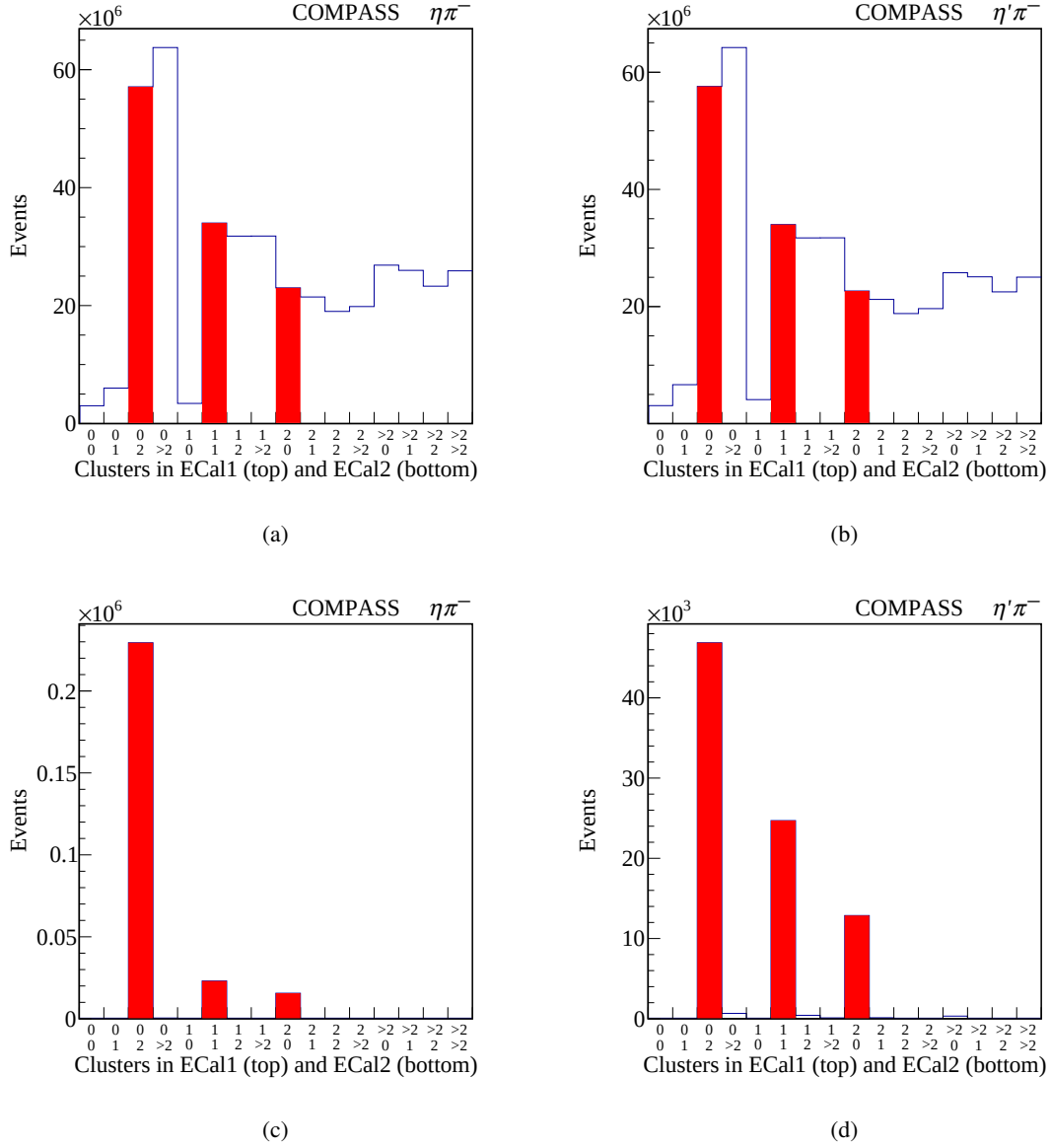


Figure A.10: Slot5 cut on the number of photons found after all photon cuts. The plot is divided into all relevant combinations of photons found in the different ECALs. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style. The remaining events after the cut are shown in red.

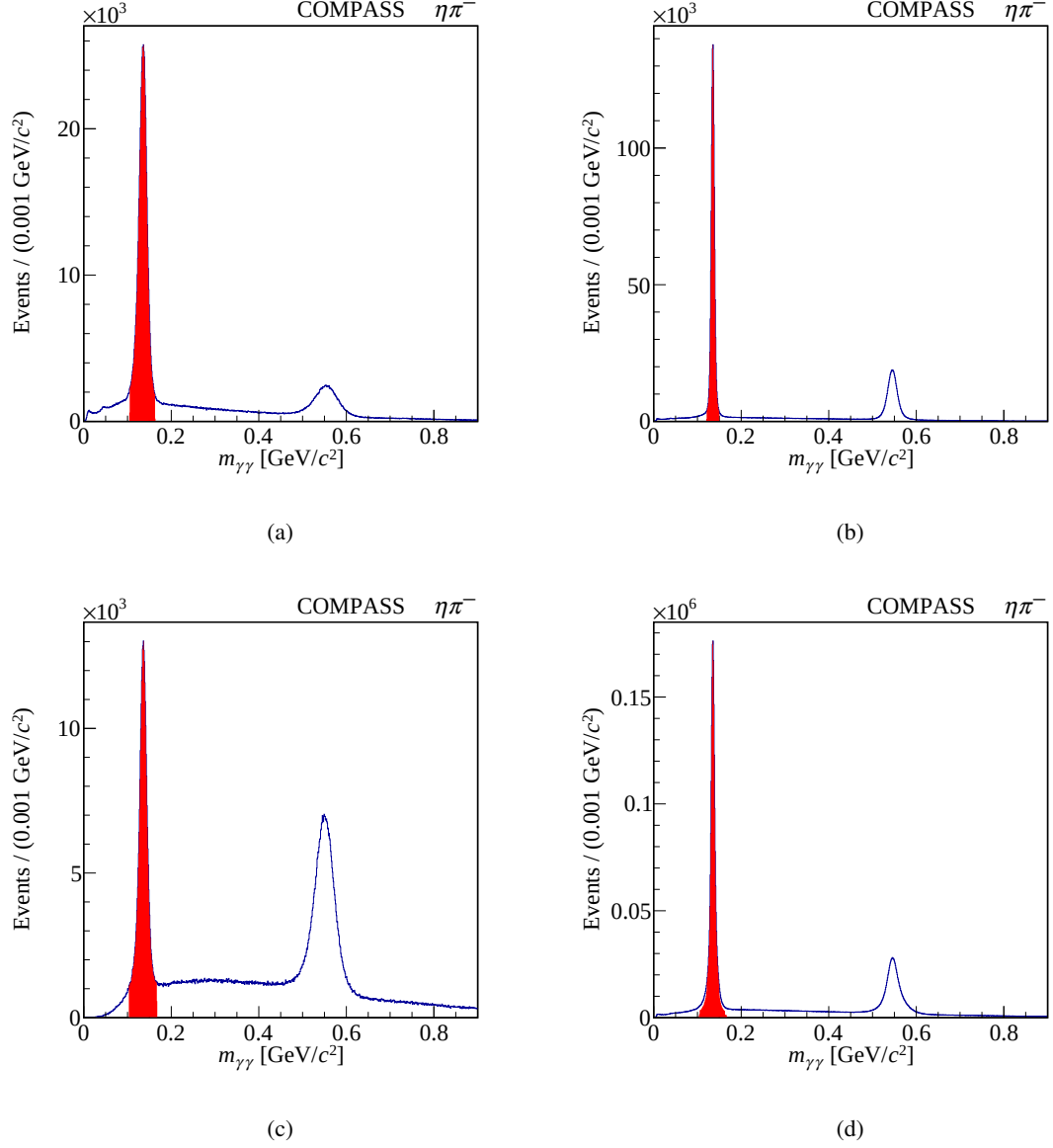


Figure A.11: Two-photon invariant mass for the $\eta\pi^-$ final state. In (a), both photons are reconstructed in ECAL1. In (b), both photons are reconstructed in ECAL2. In (c), one photon is reconstructed in ECAL1 and one in ECAL2. In (d), all combinations are shown together. The histograms are created in Antok-Style, where no kinematic fit of the two-photon system and no cut on the $\eta\pi^-$ mass was applied. Exclusivity and coplanarity cuts are applied on the final state with raw, not yet kinematically-fitted, photons. The remaining events after the cut are shown in red.

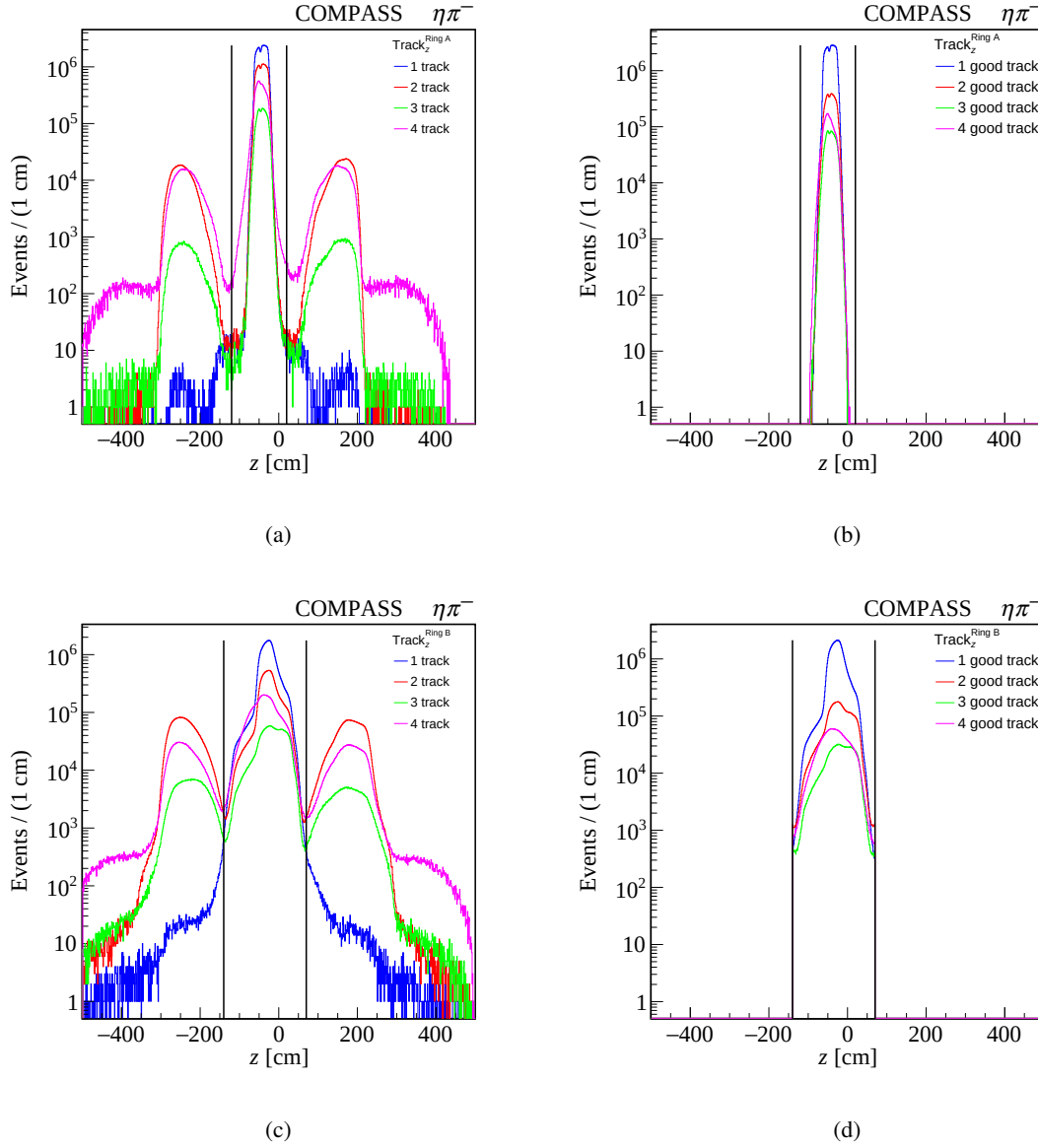


Figure A.12: z position of the recoil-proton tracks for the $\eta\pi^-$ final state at Ring A (a) and Ring B (c). In (b) and (d) the z position of good recoil-proton tracks, as defined in Eq. (4.6), is shown at Ring A and B, respectively. Cut ranges are indicated with black lines and all plots shown are in Waterfall-Style.

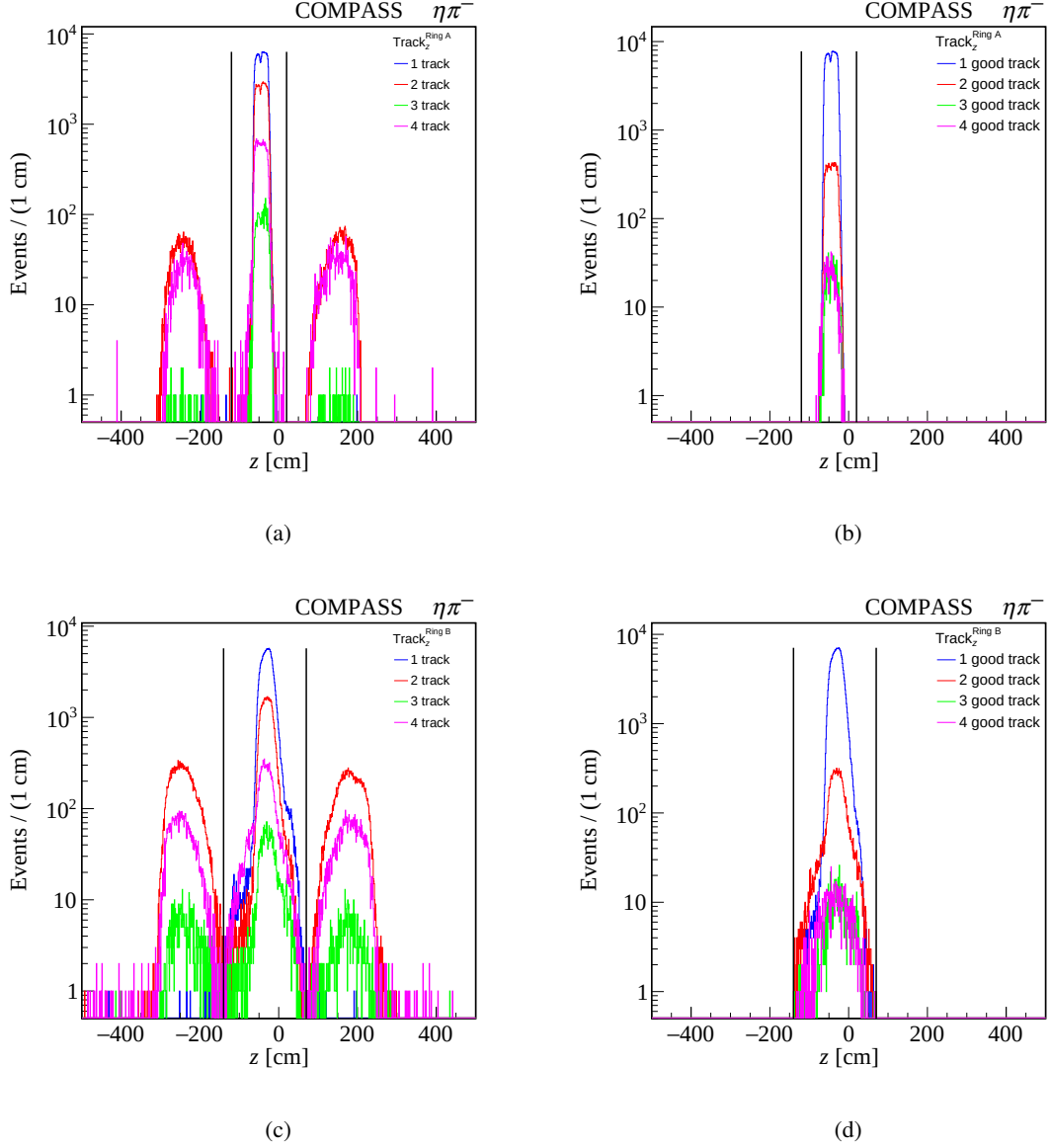


Figure A.13: z position of the recoil-proton tracks for the $\eta\pi^-$ final state at Ring A (a) and Ring B (c). In (b) and (d) the z position of good recoil-proton tracks, as defined in Eq. (4.6), is shown at Ring A and B, respectively. Cut ranges are indicated with black lines and all plots shown are in Antok-Style.

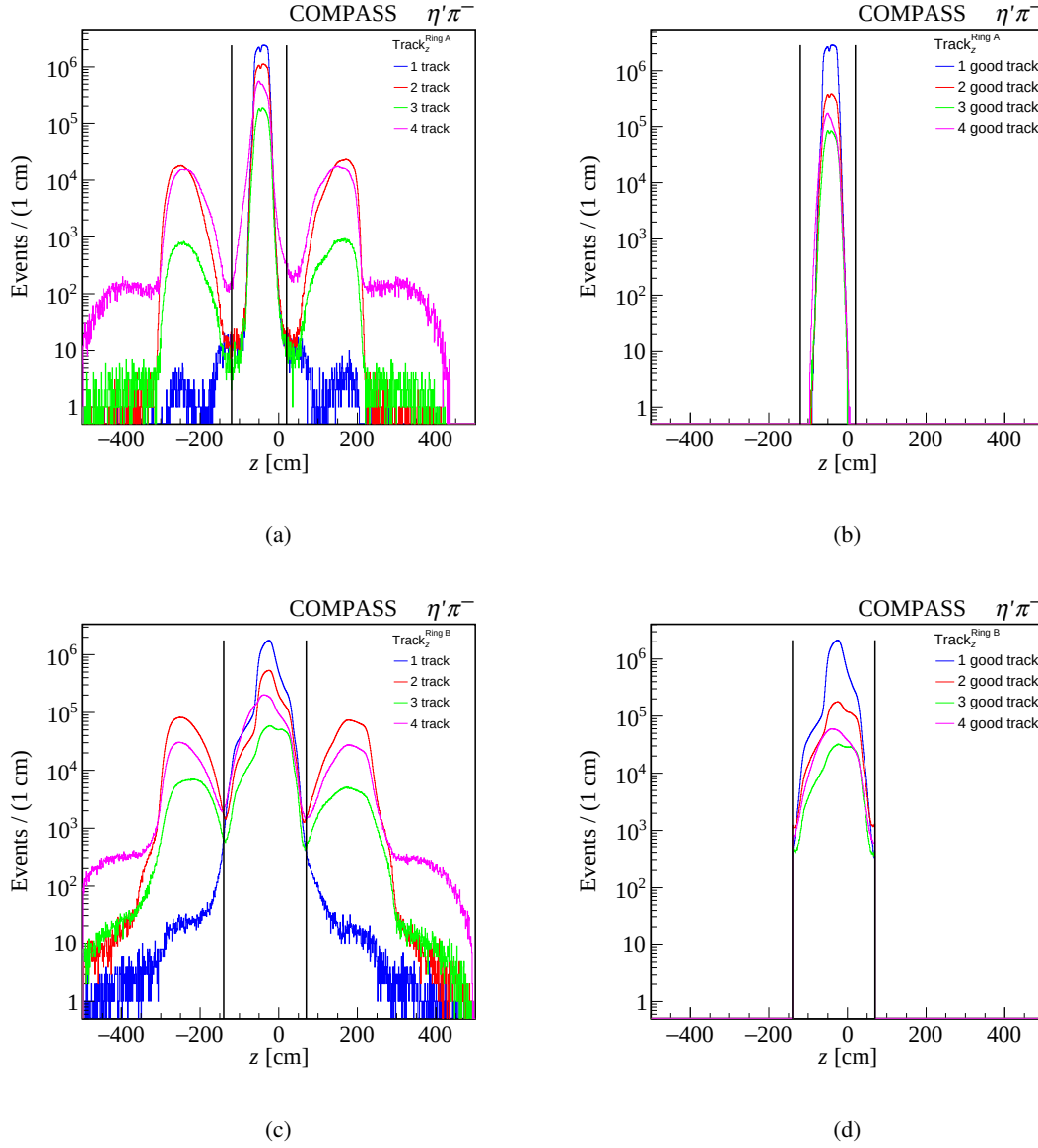


Figure A.14: z position of the recoil-proton tracks for the $\eta'\pi^-$ final state at Ring A (a) and Ring B (c). In (b) and (d) the z position of good recoil-proton tracks, as defined in Eq. (4.6), is shown at Ring A and B, respectively. Cut ranges are indicated with black lines and all plots shown are in Waterfall-Style.

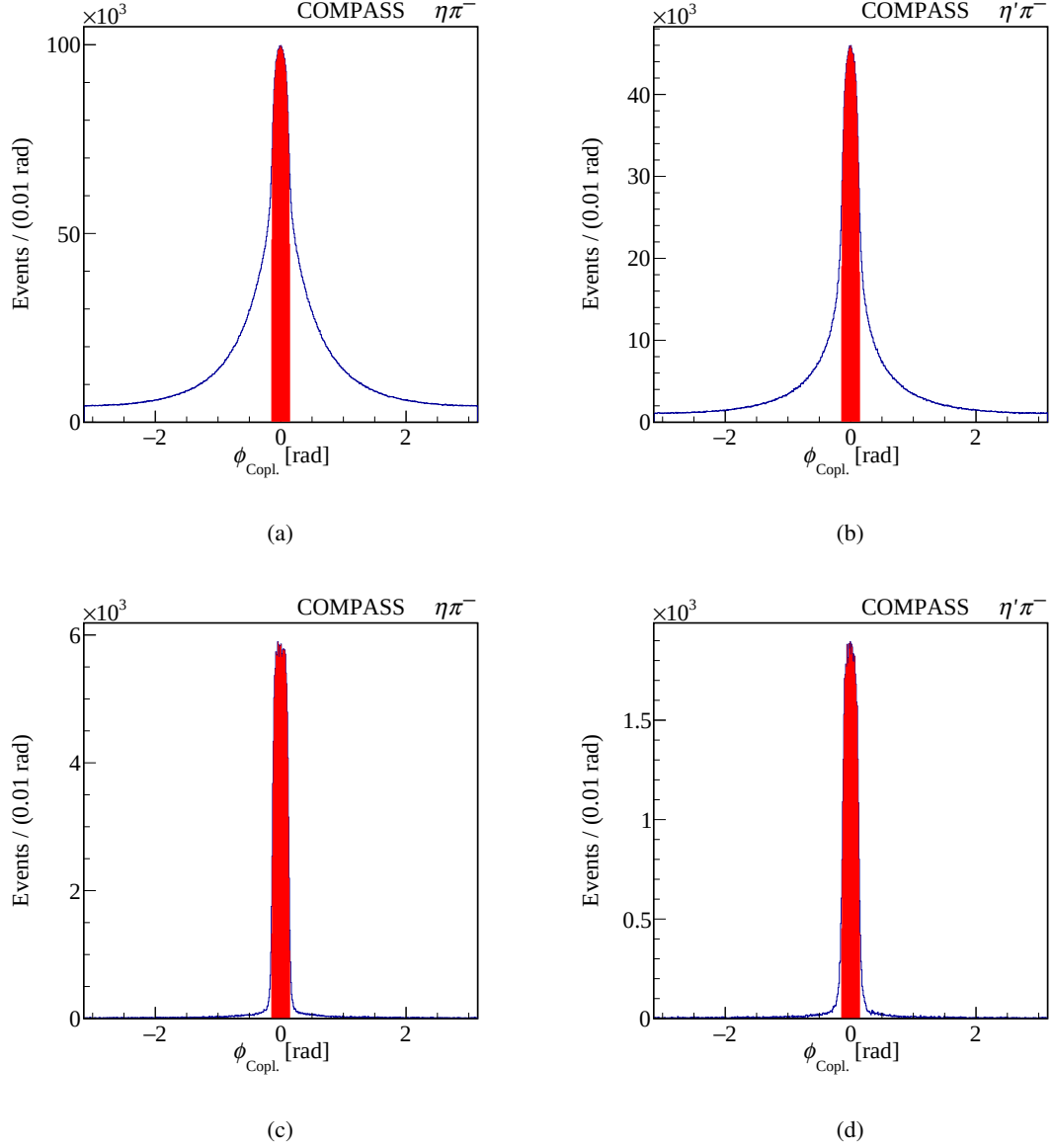


Figure A.15: Slot5 cut on the coplanarity angle for even slabs in the RPD. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style. The remaining events after the cut are shown in red.

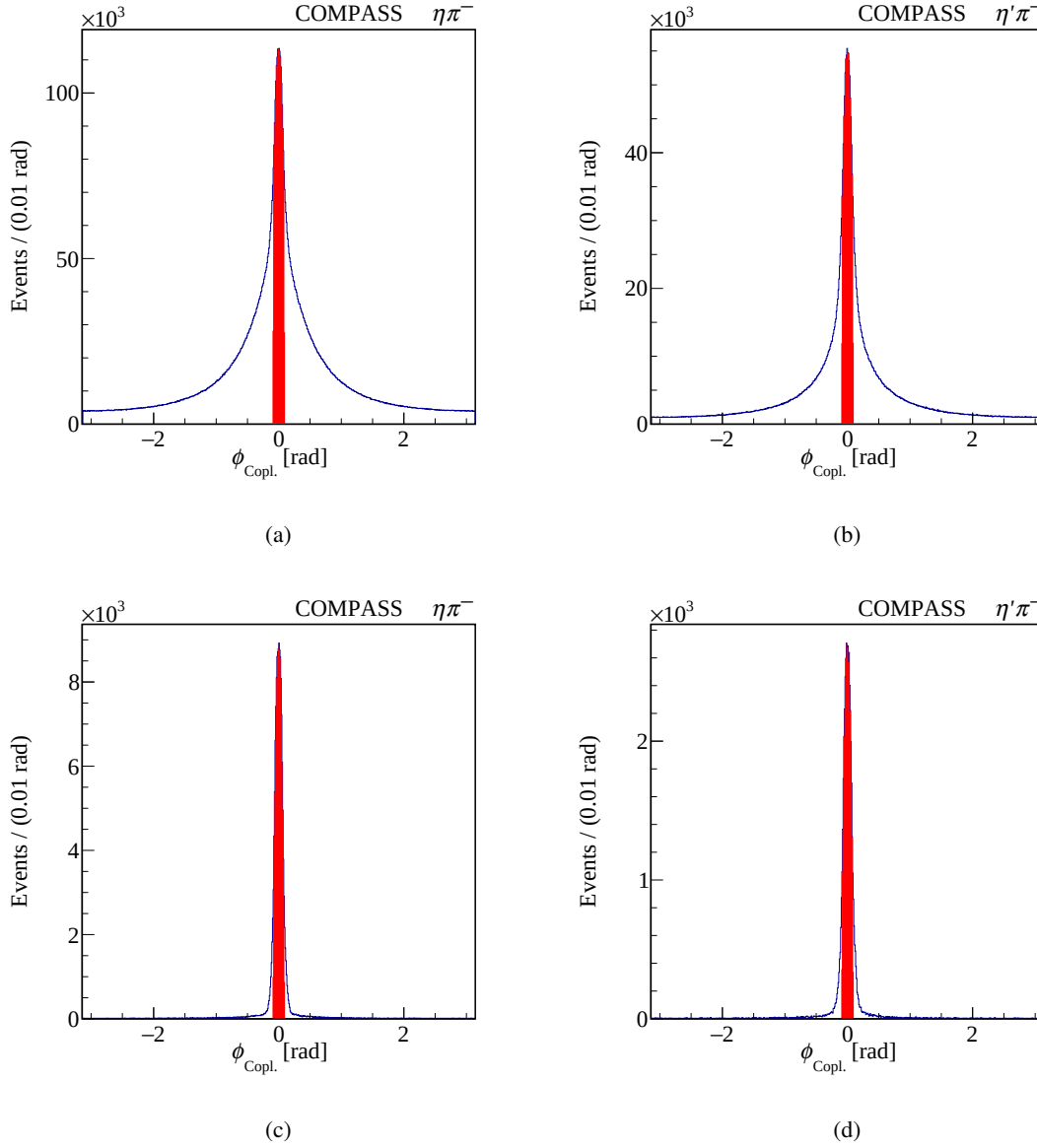


Figure A.16: Slot5 cut on the coplanarity angle for odd slabs in the RPD. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style. The remaining events after the cut are shown in red.

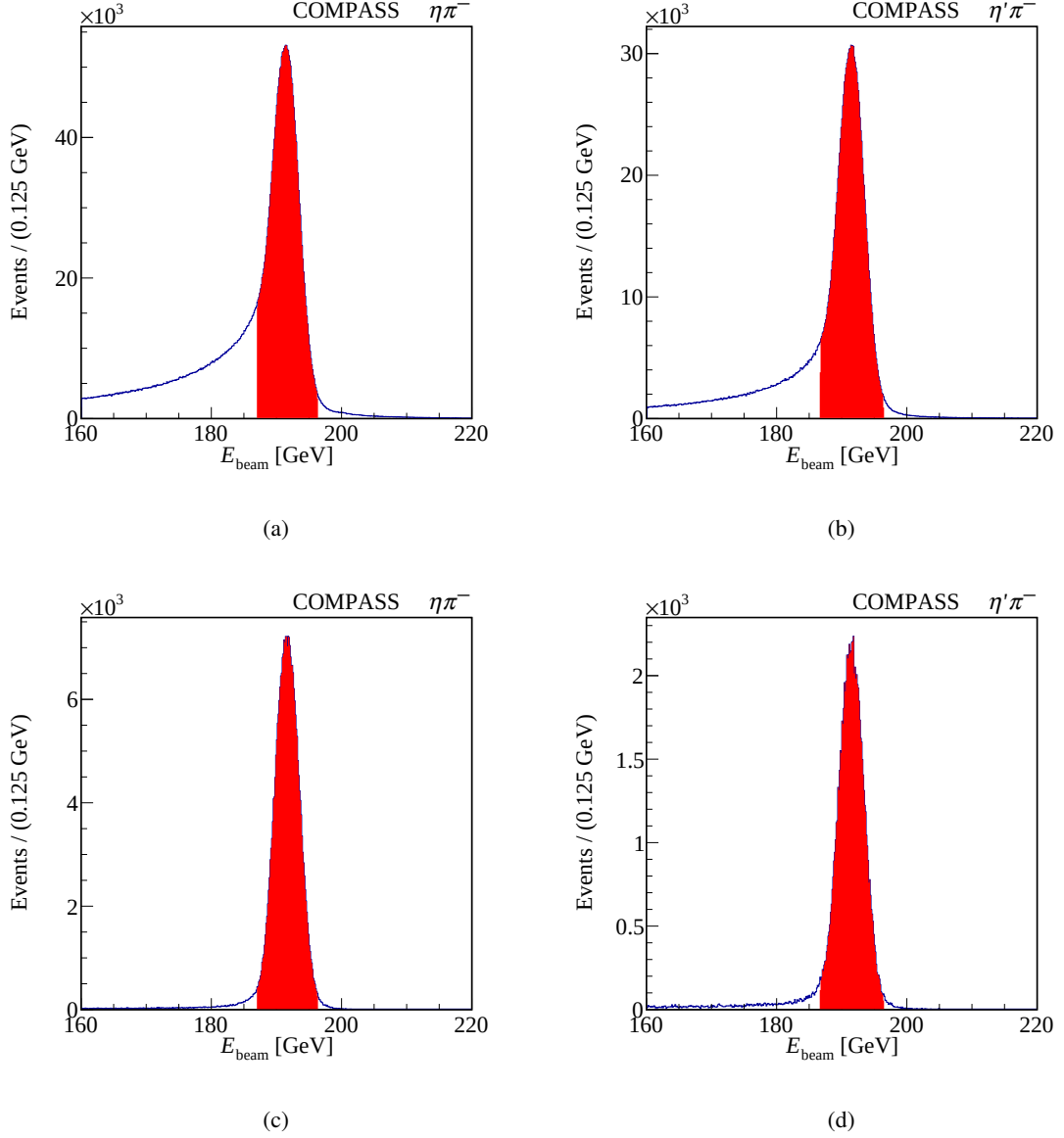


Figure A.17: Slot5 cut on the exclusivity of the final state. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style. The remaining events after the cut are shown in red.

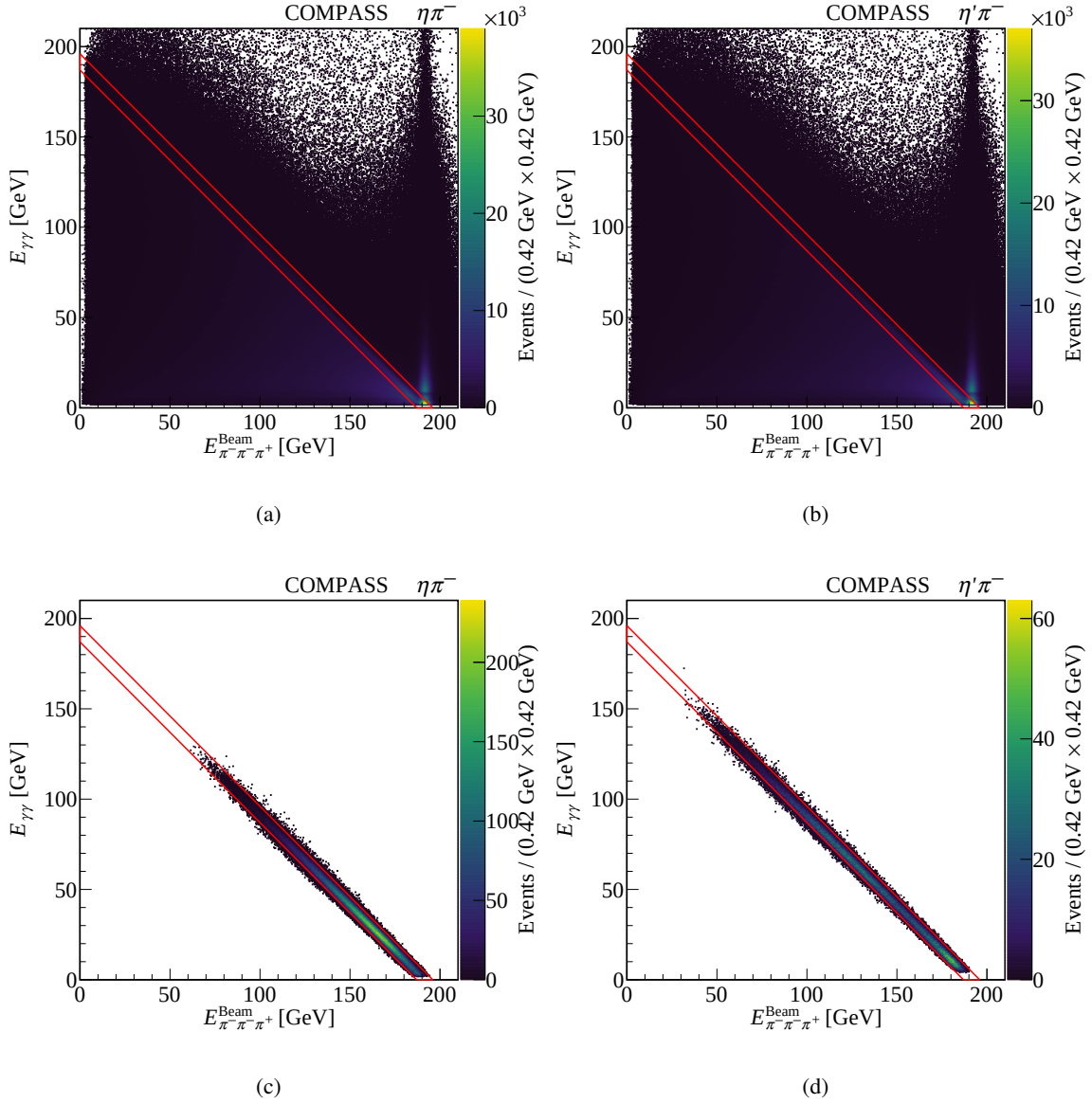


Figure A.18: Slot5 cut on the anti exclusivity. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style. The cut ranges are shown with red lines.

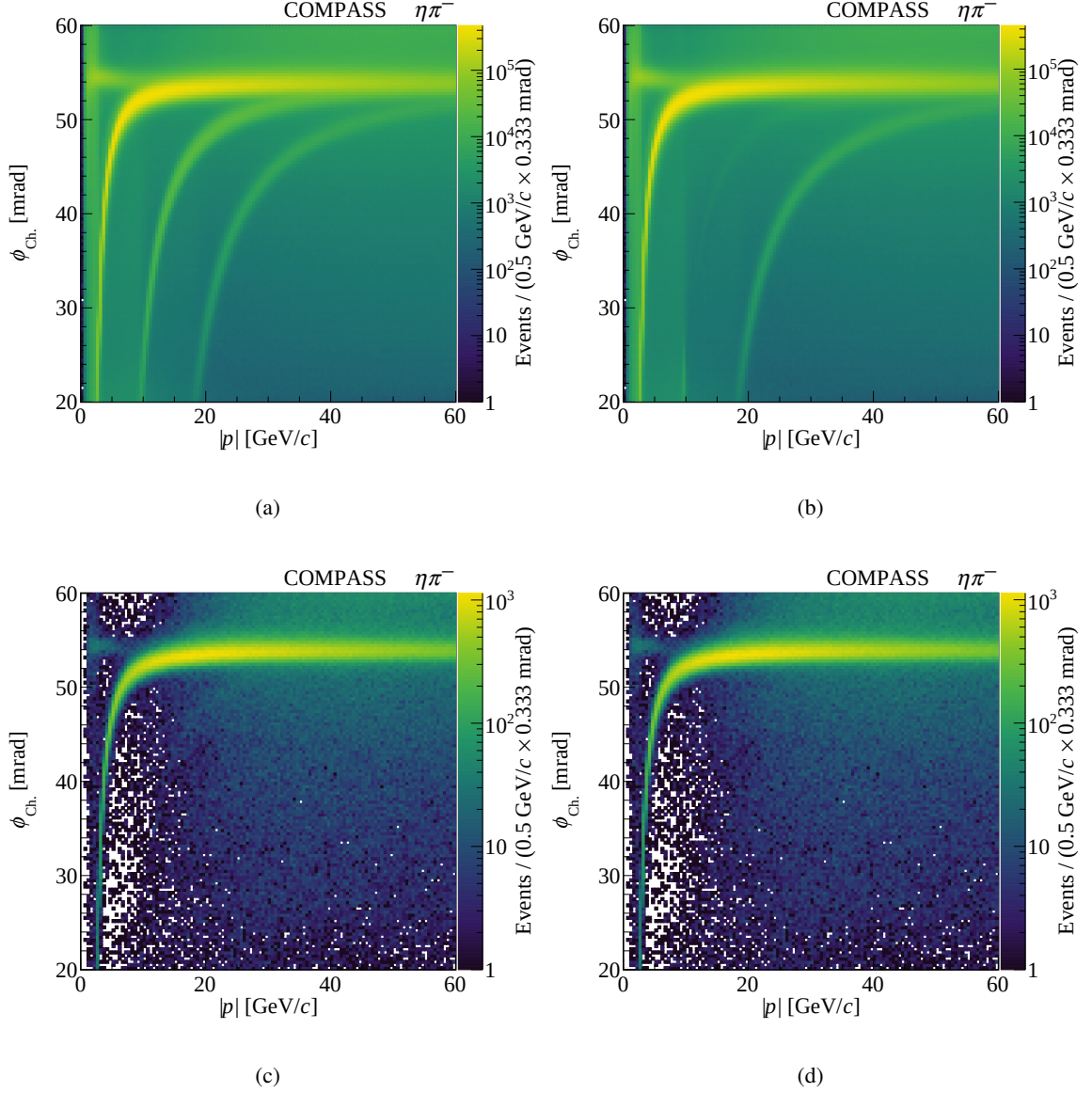


Figure A.19: Slot5 cut on final-state kaons identified by the RICH in the $\eta\pi^-$ final state. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style.

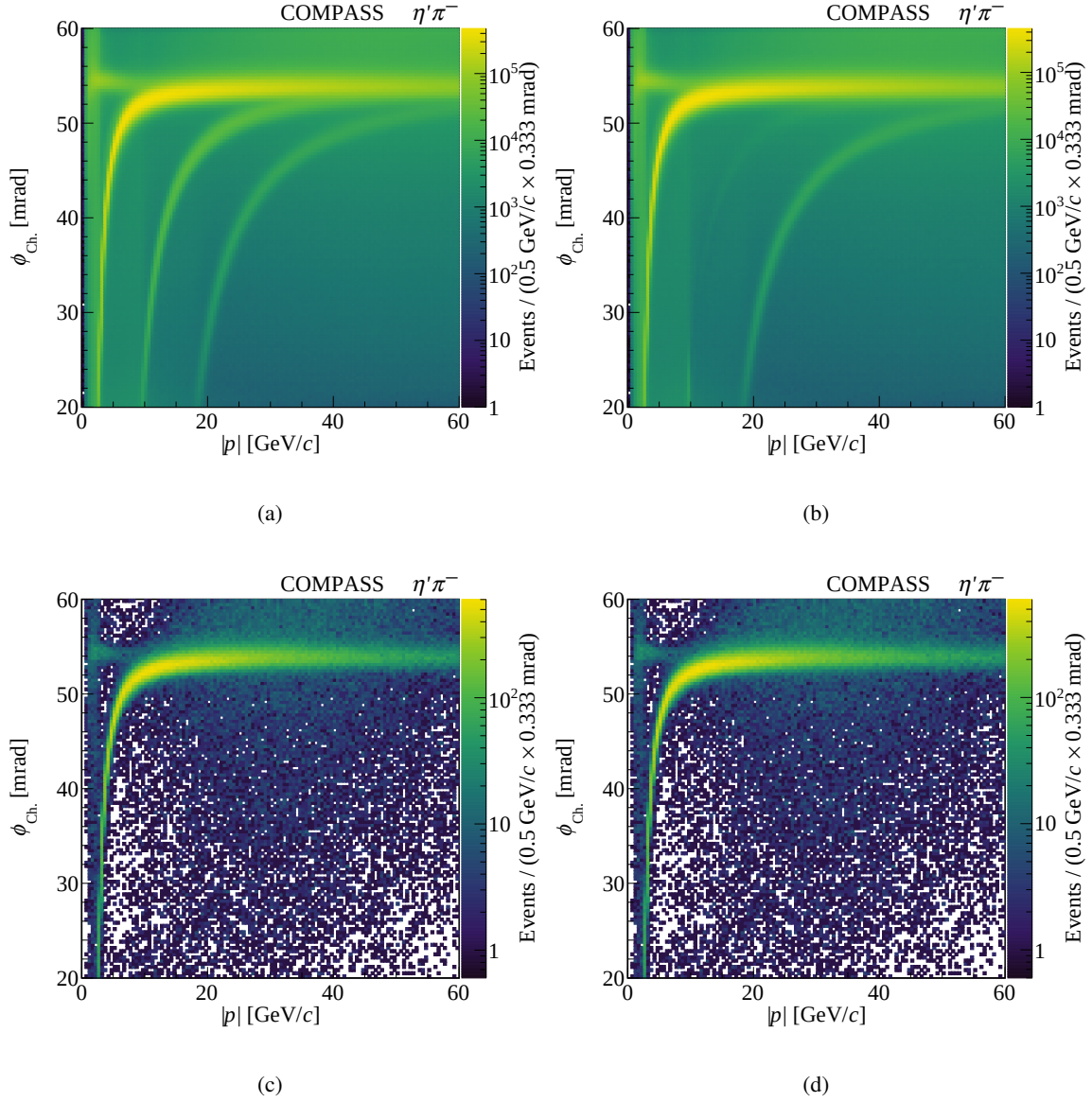


Figure A.20: Slot5 cut on final-state kaons identified by the RICH in the $\eta'\pi^-$ final state. (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) feature the selection in Waterfall-Style. (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state) feature the selection in Antok-Style.

A.3 Cross Check of the Event Selection

The cross check was performed by Philipp Haas, using an independent implementation of the event selection but using the same cuts as discussed during this thesis. One can see in Figs. A.21 to A.24 that the difference between the two selections is less than 1%.

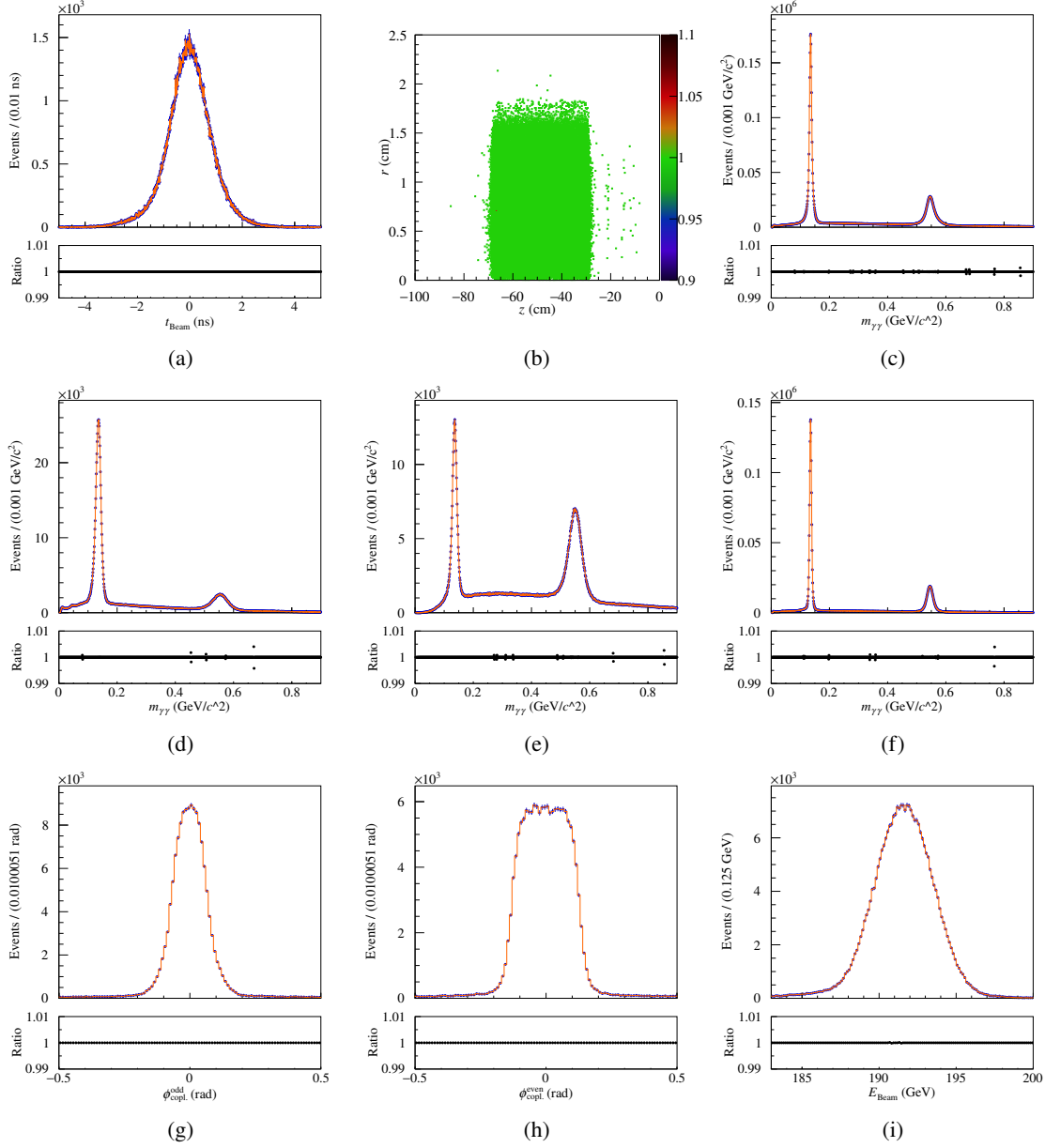


Figure A.21: Blue represents this work, orange represents the cross check by Philipp Haas. Cross checked is the beam time (a), the z vs. r distribution of the PV (b), the two-photon invariant mass for all combinations (c), events where both photons are reconstructed in ECAL1 (d), one is reconstructed in ECAL1 and one in ECAL2 (e) and both in ECAL2 (f). Additionally, the coplanarity angle for an odd slab combination (g) and an even slab combination (h), as well as the reconstructed beam energy (i) is checked. The two-photon invariant mass histograms are without the following cuts: kinematic fit and $m_{\pi^+\pi^-\gamma\gamma}$, in addition, the coplanarity and exclusivity cuts are applied with the non-kinematically-fitted π^0 . All histograms are in Antok-Style and for the $\eta\pi^-$ final state.

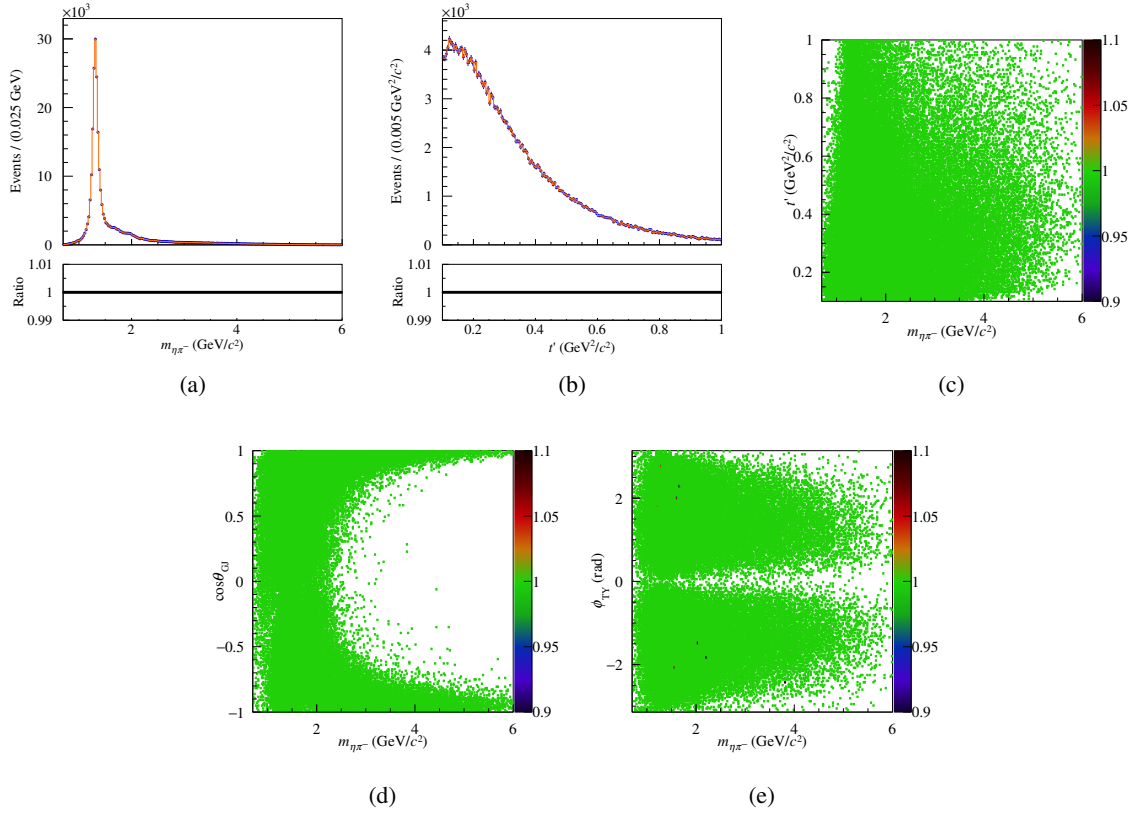


Figure A.22: Blue represents this work, orange represents the cross check by Philipp Haas. Cross checked is the $m_{\eta\pi^-}$ (a), the t' (b), the $m_{\eta\pi^-}$ vs. t' (c), the $m_{\eta\pi^-}$ vs. $\cos\theta_{\text{GJ}}$ (d) and the $m_{\eta\pi^-}$ vs. ϕ_{GJ} (e) distribution. All histograms are produced after the event selection and for the $\eta\pi^-$ final state, together with the cut on $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$.

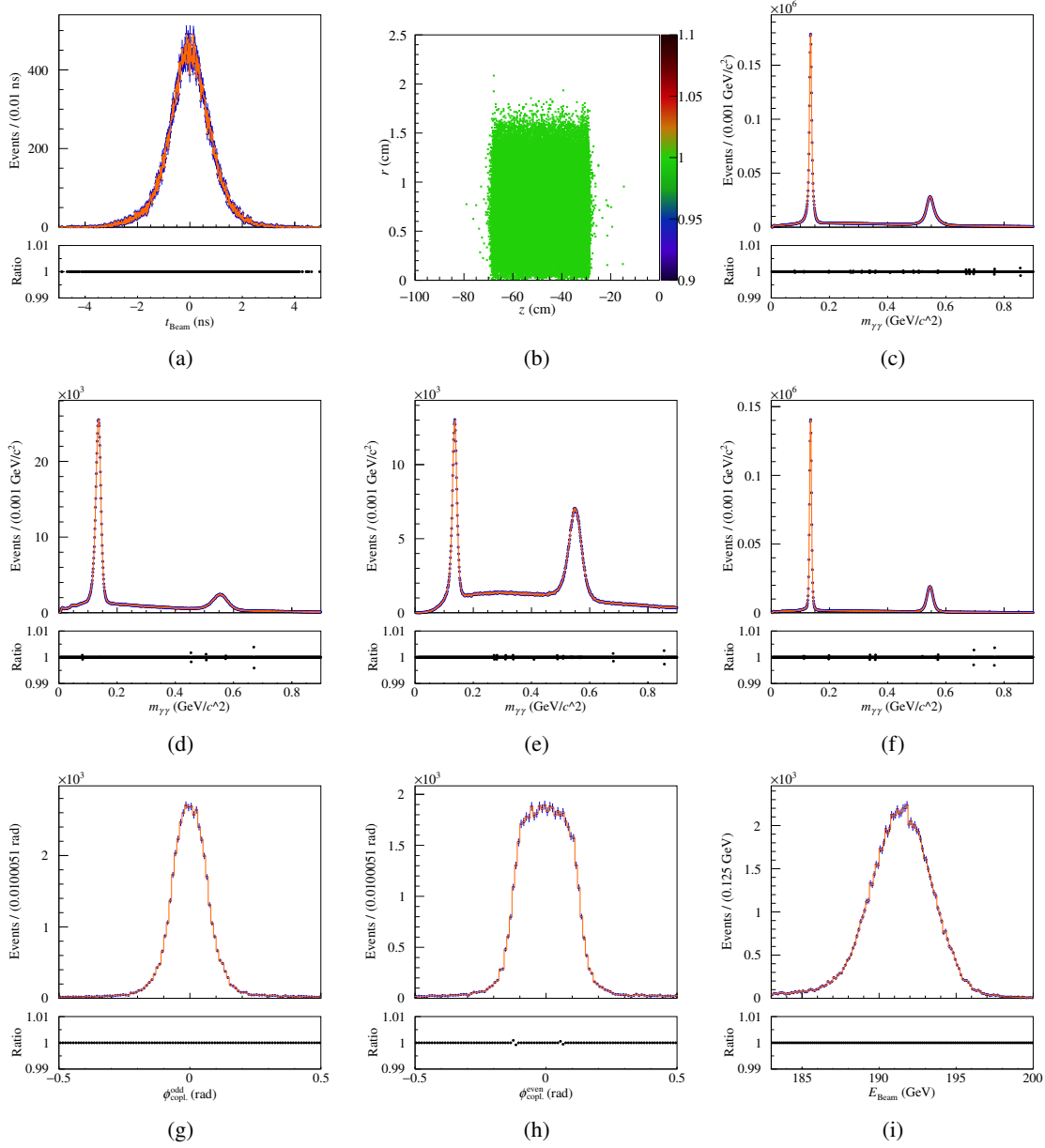


Figure A.23: Blue represents this work, orange represents the cross check by Philipp Haas. Cross checked is the beam time (a), the z vs. r distribution of the PV (b), the two-photon invariant mass for all combinations (c), events where both photons are reconstructed in ECAL1 (d), one is reconstructed in ECAL1 and one in ECAL2 (e) and both in ECAL2 (f). Additionally, the coplanarity angle for an odd slab combination (g) and an even slab combination (h), as well as the reconstructed beam energy (i) is checked. The two-photon invariant mass histograms are without the following cuts: kinematic fit and $m_{\pi^+\pi^-\gamma\gamma}$, in addition, the coplanarity and exclusivity cuts are applied with the non-kinematically-fitted π^0 . All histograms are in Antok-Style and for the $\eta'\pi^-\pi^+$ final state.

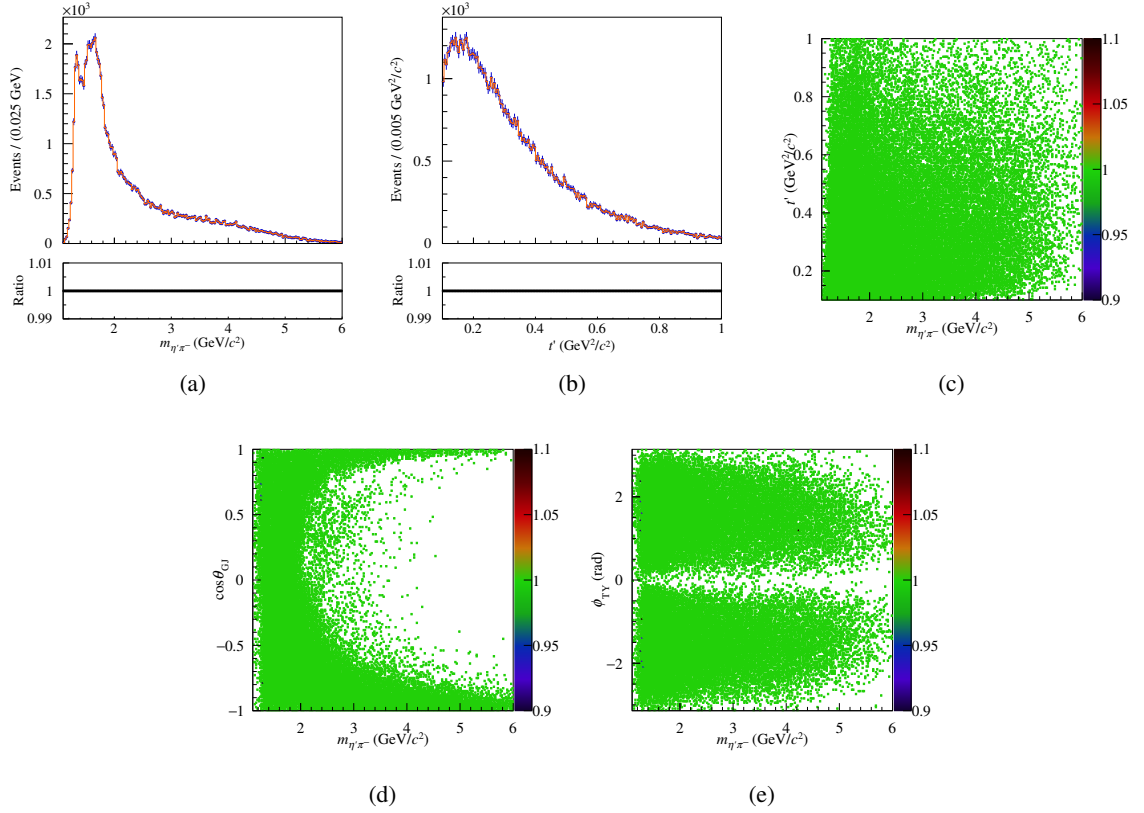


Figure A.24: Blue represents this work, orange represents the cross check by Philipp Haas. Cross checked is the $m_{\eta'\pi^-}$ (a), the t' (b), the $m_{\eta'\pi^-}$ vs. t' (c), the $m_{\eta'\pi^-}$ vs. $\cos\theta_{\text{GJ}}$ (d) and the $m_{\eta'\pi^-}$ vs. ϕ_{GJ} (e) distribution. All histograms are produced after the event selection and for the $\eta'\pi^-$ final state, together with the cut on $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$.

Further Details on the Acceptance of the $\eta^{(\prime)}\pi^-$ Monte Carlo Sample

In this chapter, we present the two dimensional acceptance of the two 30×10^6 MC events sample in the $\eta^{(\prime)}\pi^-$ final state. Note that no events were simulated with $|\cos \theta_{GJ}| < 0.5$. In addition to the acceptance, we show the distribution of the data in the same variables and the generated distributions.

In Fig. B.1, we see the mass of the $\eta^{(\prime)}\pi^-$ system vs. t' . Since the event sample is created according to phase space, the amount of data simulated for higher masses is higher than for lower masses, while the distribution is flat in t' . One can see a slight acceptance drop of in the $\eta\pi^-$ final state around 3 GeV. Since this is not visible in the $\eta'\pi^-$ final state, this is most likely due to the fact that the reconstructed photons (emerging from the π^0 decay) are slightly more likely to not be seen by the detector in the mass region.

In Fig. B.2, we see $\cos \theta_{GJ}$ vs. ϕ_{GJ} . Due to the phase space generation, the event sample is flat during generation of the events. The acceptance clearly shows that we have a low acceptance for events with $\cos \theta_{GJ} \approx -1$ and $\phi_{GJ} \approx \pm\pi$, as well as $\cos \theta_{GJ} \approx 1$ and $\phi_{GJ} \approx 0$. These areas are also less populated in the data diagrams.

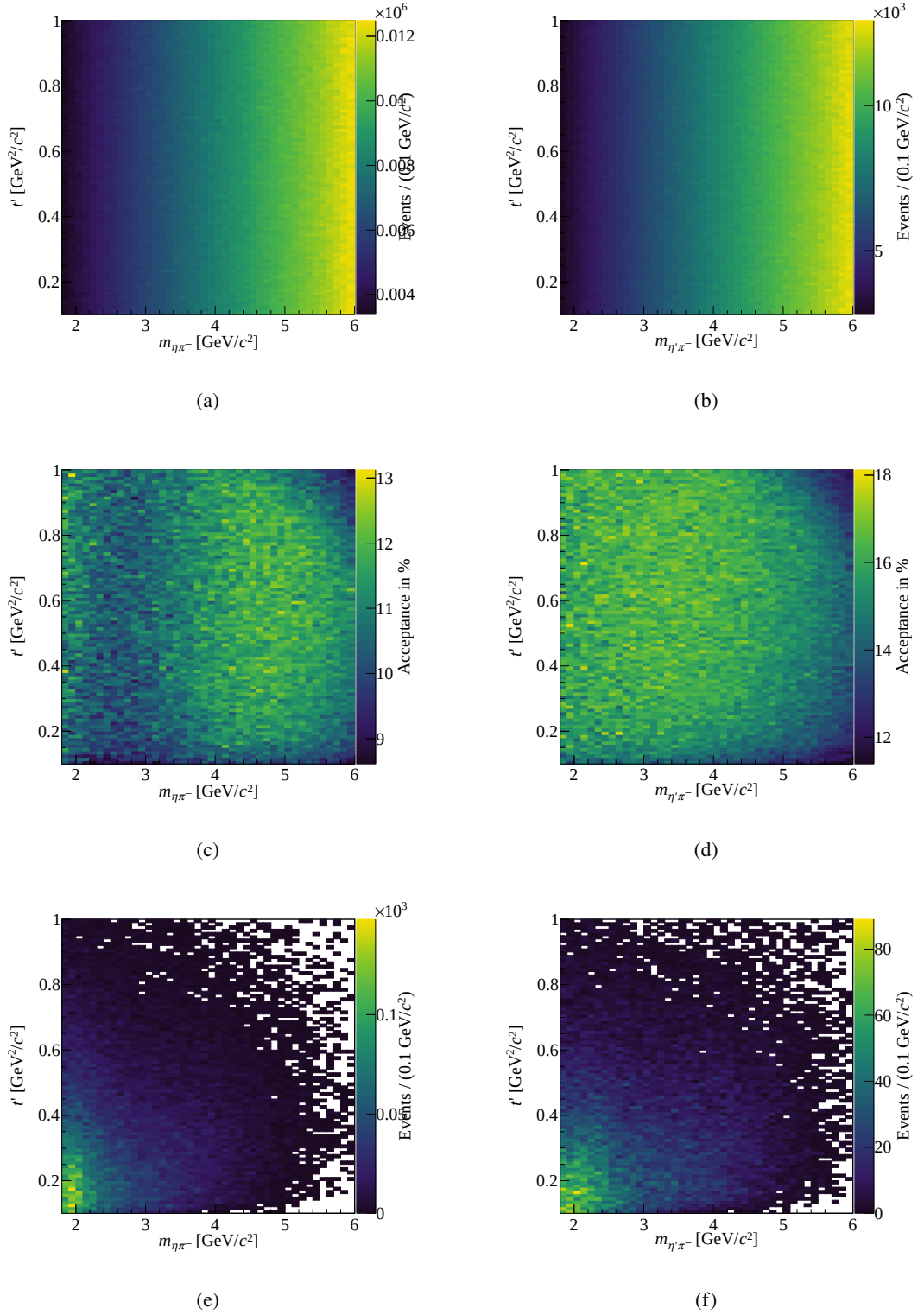


Figure B.1: Two dimensional distribution of the generated MC data (a) and (b), their acceptance (c) and (d) and their data distribution (e) and (f) in $m_{\eta^{(\prime)}\pi^-}$ vs t' .

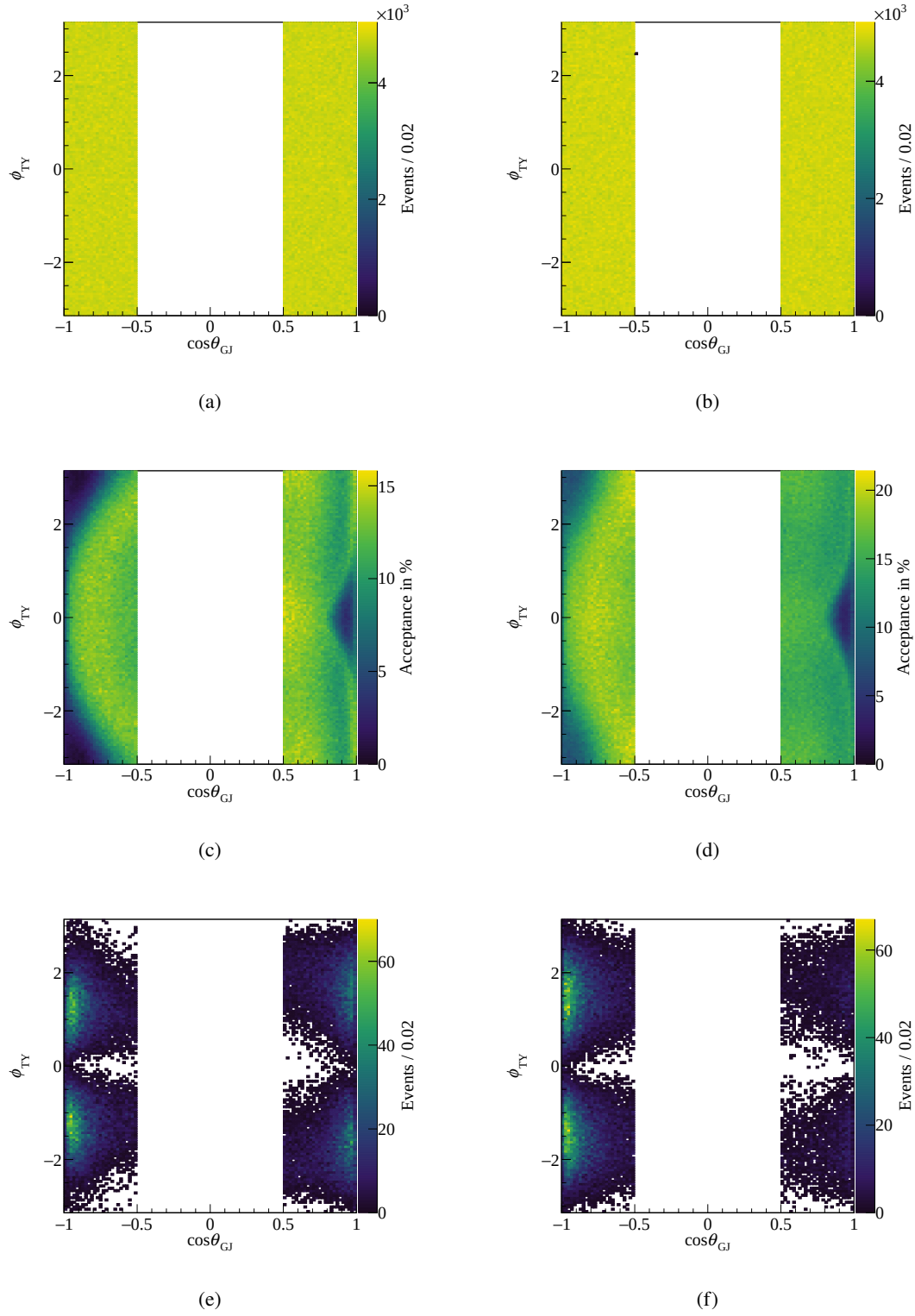


Figure B.2: Two dimensional distribution of the generated MC data (a) and (b), their acceptance (c) and (d) and their data distribution (e) and (f) in $\cos\theta_{GJ}$ vs ϕ_{GJ} .

Mathematical Properties of the Negative Log Likelihood and the Intensity

In this chapter, we will first discuss some useful properties of the negative log likelihood (Eq. (6.8c)). The first derivative with respect to one fit parameter x_l is given by

$$\frac{\partial}{\partial x_l} \left(-\ln \mathcal{L}(\vec{c}, \vec{b}, N_{\text{acc}}, N_m) \right) = \sum_{k=1}^{N_{\text{acc}}} \frac{\partial}{\partial x_l} I(\vec{c}, \vec{b}, \tau_k) - \sum_{k=1}^{N_m} \frac{\partial}{\partial x_l} \ln I(\vec{c}, \vec{b}, \tau_k) \quad (\text{C.1a})$$

$$= \sum_{k=1}^{N_{\text{acc}}} \frac{\partial}{\partial x_l} I(\vec{c}, \vec{b}, \tau_k) - \sum_{k=1}^{N_m} \frac{\frac{\partial}{\partial x_l} I(\vec{c}, \vec{b}, \tau_k)}{I(\vec{c}, \vec{b}, \tau_k)}, \quad (\text{C.1b})$$

where x_l is either one strength coefficient (c_l) or one slope parameter (b_l). The second derivative with respect to two fit parameters x_l and x_m is

$$\frac{\partial^2}{\partial x_l \partial x_m} \left(-\ln \mathcal{L}(\vec{c}, \vec{b}, N_{\text{acc}}, N_m) \right) = \frac{\partial}{\partial x_m} \left(\sum_{k=1}^{N_{\text{acc}}} \frac{\partial}{\partial x_l} I(\vec{c}, \vec{b}, \tau_k) - \sum_{k=1}^{N_m} \frac{\frac{\partial}{\partial x_l} I(\vec{c}, \vec{b}, \tau_k)}{I(\vec{c}, \vec{b}, \tau_k)} \right) \quad (\text{C.2a})$$

$$= \sum_{k=1}^{N_{\text{acc}}} \frac{\partial^2}{\partial x_l \partial x_m} I(\vec{c}, \vec{b}, \tau_k) \quad (\text{C.2b})$$

$$- \sum_{k=1}^{N_m} \left(\frac{\frac{\partial^2}{\partial x_l \partial x_m} I(\vec{c}, \vec{b}, \tau_k)}{I(\vec{c}, \vec{b}, \tau_k)} - \frac{\frac{\partial}{\partial x_l} I(\vec{c}, \vec{b}, \tau_k) \frac{\partial}{\partial x_m} I(\vec{c}, \vec{b}, \tau_k)}{I(\vec{c}, \vec{b}, \tau_k)^2} \right). \quad (\text{C.2c})$$

In order to compute the derivative of the intensity function Eq. (6.1), we first note down the general behavior of the derivative of the absolute value squared of a complex number

$$\frac{\partial}{\partial x} |f(x)|^2 = \frac{\partial}{\partial x} (f(x) \cdot f^*(x)) \quad (\text{C.3a})$$

$$= \left(\frac{\partial}{\partial x} f(x) \right) f^*(x) + \left(\frac{\partial}{\partial x} f^*(x) \right) f(x) \quad (\text{C.3b})$$

$$= \left(\frac{\partial}{\partial x} f(x) \right) f^*(x) + \left[\left(\frac{\partial}{\partial x} f(x) \right) f^*(x) \right]^* \quad (\text{C.3c})$$

$$= 2 \operatorname{Re} \left\{ \left(\frac{\partial}{\partial x} f(x) \right) f^*(x) \right\}, \quad (\text{C.3d})$$

where $f(x)$ is any complex function and the $*$ denotes the complex conjugate. Now, we write the derivatives of the intensity function. First we make the derivative after one strength coefficient c_l

$$\frac{\partial}{\partial c_l} I(\vec{c}, \vec{b}, \tau_k) = \frac{\partial}{\partial c_l} \left| \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right|^2 \quad (\text{C.4a})$$

$$= 2 \operatorname{Re} \left\{ \left(\frac{\partial}{\partial c_l} \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right) \cdot \left(\sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right)^* \right\} \quad (\text{C.4b})$$

$$= 2 \operatorname{Re} \left\{ A_l(\vec{b}; \tau_k) \cdot A(\vec{c}, \vec{b}, \tau_k)^* \right\}. \quad (\text{C.4c})$$

We use the short notation of the amplitude, where we assigned a number to each individual amplitude from 1 to 6 in any order. See Eq. (6.2) for comparison. Secondly we write the derivative for one slope parameter b_l

$$\frac{\partial}{\partial b_l} I(\vec{c}, \vec{b}, \tau_k) = \frac{\partial}{\partial b_l} \left| \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right|^2 \quad (\text{C.5a})$$

$$= 2 \operatorname{Re} \left\{ \left(\frac{\partial}{\partial b_l} \sum_{i=1}^6 c_i e^{b_{i,1}t_1} e^{b_{i,2}t_2} A_i(\alpha_1(t_1), \alpha_2(t_2), s_2) \right) \cdot \left(\sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right)^* \right\} \quad (\text{C.5b})$$

$$= 2 \operatorname{Re} \left\{ t_l \left(\sum_{i \in L} c_i A_i(\vec{b}; \tau_k) \right) \cdot A(\vec{c}, \vec{b}, \tau_k)^* \right\}. \quad (\text{C.5c})$$

We have introduced the notation $e^{b_{i,j}t_j}$ for the form factors. The index i in this case is to be understood as the slope parameter, connected to the vertex at the top ($j = 1$) or at the bottom ($j = 2$). Since the slope parameter can be shared between multiple amplitudes, we introduce the notation $i \in L$ in Eq. (C.5c). L is the set of amplitudes, where the slope parameter b_l is involved. The Mandelstam variable t_l is the one that is connected to b_l in the form factor. Slope parameters are unique at the top or bottom vertex and therefore their connection to t is also unique and we can make this statement. Lastly we write the second derivative after two strength coefficients

$$\frac{\partial^2}{\partial c_l \partial c_m} I(\vec{c}, \vec{b}, \tau_k) = \frac{\partial}{\partial c_m} 2 \operatorname{Re} \left\{ A_l(\vec{b}; \tau_k) \cdot \left(\sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right)^* \right\} \quad (\text{C.6a})$$

$$= 2 \operatorname{Re} \left\{ A_l(\vec{b}; \tau_k) A_m(\vec{b}; \tau_k)^* \right\}, \quad (\text{C.6b})$$

and two slope parameters

$$\frac{\partial^2}{\partial b_l \partial b_m} I(\vec{c}, \vec{b}, \tau_k) = \frac{\partial}{\partial b_m} 2 \operatorname{Re} \left\{ t_l \left(\sum_{i \in L} c_i A_i(\vec{b}; \tau_k) \right) \cdot \left(\sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right)^* \right\} \quad (\text{C.7a})$$

$$= 2 \operatorname{Re} \left\{ t_l \left(\frac{\partial}{\partial b_m} \sum_{i \in L} c_i A_i(\vec{b}; \tau_k) \right) \cdot A(\vec{c}, \vec{b}, \tau_k)^* \right\} \quad (\text{C.7b})$$

$$+ 2 \operatorname{Re} \left\{ t_l \left(\sum_{i \in L} c_i A_i(\vec{b}; \tau_k) \right) \cdot \left(\frac{\partial}{\partial b_m} \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right)^* \right\} \quad (\text{C.7c})$$

$$= 2 \operatorname{Re} \left\{ t_l \left(t_m \sum_{\substack{i \in L \\ i \in M}} c_i A_i(\vec{b}; \tau_k) \right) \cdot A(\vec{c}, \vec{b}, \tau_k)^* \right\} \quad (\text{C.7d})$$

$$+ 2 \operatorname{Re} \left\{ t_l \left(\sum_{i \in L} c_i A_i(\vec{b}; \tau_k) \right) \cdot \left(t_m \sum_{i \in M} c_i A_i(\vec{b}; \tau_k) \right)^* \right\}, \quad (\text{C.7e})$$

and a combination of the two

$$\frac{\partial}{\partial b_m \partial c_l} I(\vec{c}, \vec{b}, \tau_k) = \frac{\partial^2}{\partial c_l \partial b_m} I(\vec{c}, \vec{b}, \tau_k) = \frac{\partial}{\partial b_m} 2 \operatorname{Re} \left\{ A_l(\vec{b}; \tau_k) \cdot \left(\sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right)^* \right\} \quad (\text{C.8a})$$

$$= 2 \operatorname{Re} \left\{ \frac{\partial}{\partial b_m} A_l(\vec{b}; \tau_k) \cdot A(\vec{c}, \vec{b}, \tau_k)^* \right\} \quad (\text{C.8b})$$

$$+ 2 \operatorname{Re} \left\{ A_l(\vec{b}; \tau_k) \cdot \left(\frac{\partial}{\partial b_m} \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right)^* \right\} \quad (\text{C.8c})$$

$$= 2 \operatorname{Re} \left\{ \left(\delta_{ml} t_m A_l(\vec{b}; \tau_k) \right) \cdot A(\vec{c}, \vec{b}, \tau_k)^* \right\} \quad (\text{C.8d})$$

$$+ 2 \operatorname{Re} \left\{ A_l(\vec{b}; \tau_k) \cdot \left(t_m \sum_{i \in M} c_i A_i(\vec{b}; \tau_k) \right)^* \right\}. \quad (\text{C.8e})$$

We can see that most of these derivatives depend on the intensity itself or a function similar to the intensity, i.e. a sum over less amplitudes. This fact will be used during the minimization of the negative log likelihood, in order to speed up the minimization, while caching these (sub)intensities per minimization.

At times it might be useful to rewrite the sum over all accepted MC events as

$$\begin{aligned}
 \sum_{k=1}^{N_{\text{acc}}} \left| \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right|^2 &= \sum_{k=1}^{N_{\text{acc}}} \left(\left(\sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right) \left(\sum_{j=1}^6 c_j A_j(\vec{b}; \tau_k) \right)^* \right) \\
 &= \sum_{k=1}^{N_{\text{acc}}} \left(\sum_{i=1}^6 \left(c_i A_i(\vec{b}; \tau_k) \sum_{j=1}^6 c_j^* A_j(\vec{b}; \tau_k)^* \right) \right) = \sum_{k=1}^{N_{\text{acc}}} \left(\sum_{i=1}^6 \sum_{j=1}^6 c_i A_i(\vec{b}; \tau_k) c_j^* A_j(\vec{b}; \tau_k)^* \right) \\
 &= \sum_{k=1}^{N_{\text{acc}}} \left(\sum_{i=1}^6 \left(\sum_{j=1}^{i-1} c_i A_i(\vec{b}; \tau_k) c_j^* A_j(\vec{b}; \tau_k)^* + \sum_{j=i+1}^6 c_i A_i(\vec{b}; \tau_k) c_j^* A_j(\vec{b}; \tau_k)^* + c_i A_i(\vec{b}; \tau_k) c_i^* A_i(\vec{b}; \tau_k)^* \right) \right) \\
 &= \sum_{k=1}^{N_{\text{acc}}} \left(\sum_{i=1}^6 \sum_{j=1}^{i-1} c_i A_i(\vec{b}; \tau_k) c_j^* A_j(\vec{b}; \tau_k)^* + \sum_{i=1}^6 \sum_{j=i+1}^6 c_i A_i(\vec{b}; \tau_k) c_j^* A_j(\vec{b}; \tau_k)^* + \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) c_i^* A_i(\vec{b}; \tau_k)^* \right) \\
 &= \sum_{k=1}^{N_{\text{acc}}} \left(\sum_{i=1}^6 \sum_{j=1}^{i-1} \left(c_i^* A_i(\vec{b}; \tau_k)^* c_j A_j(\vec{b}; \tau_k) \right)^* + \sum_{i=1}^6 \sum_{j=i+1}^6 c_i A_i(\vec{b}; \tau_k) c_j^* A_j(\vec{b}; \tau_k)^* + \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) c_i^* A_i(\vec{b}; \tau_k)^* \right) \\
 &= \sum_{k=1}^{N_{\text{acc}}} \left(\sum_{j=1}^6 \sum_{i=j+1}^6 \left(c_i^* A_i(\vec{b}; \tau_k)^* c_j A_j(\vec{b}; \tau_k) \right)^* + \sum_{i=1}^6 \sum_{j=i+1}^6 c_i A_i(\vec{b}; \tau_k) c_j^* A_j(\vec{b}; \tau_k)^* + \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) c_i^* A_i(\vec{b}; \tau_k)^* \right) \\
 &= \sum_{k=1}^{N_{\text{acc}}} \left(\sum_{i=1}^6 \sum_{j=i+1}^6 \left(c_j^* A_j(\vec{b}; \tau_k)^* c_i A_i(\vec{b}; \tau_k) \right)^* + \sum_{i=1}^6 \sum_{j=i+1}^6 c_i A_i(\vec{b}; \tau_k) c_j^* A_j(\vec{b}; \tau_k)^* + \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) c_i^* A_i(\vec{b}; \tau_k)^* \right) \\
 &= \sum_{k=1}^{N_{\text{acc}}} \left(\sum_{i=1}^6 \sum_{j=i+1}^6 \left[\left(c_j^* A_j(\vec{b}; \tau_k)^* c_i A_i(\vec{b}; \tau_k) \right)^* + c_i A_i(\vec{b}; \tau_k) c_j^* A_j(\vec{b}; \tau_k)^* \right] + \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) c_i^* A_i(\vec{b}; \tau_k)^* \right) \\
 &= \sum_{k=1}^{N_{\text{acc}}} \left(\sum_{i=1}^6 \sum_{j=i+1}^6 2 \operatorname{Re} \left\{ c_i A_i(\vec{b}; \tau_k) c_j^* A_j(\vec{b}; \tau_k)^* \right\} + \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) c_i^* A_i(\vec{b}; \tau_k)^* \right) \\
 &= \sum_{k=1}^{N_{\text{acc}}} \left(\sum_{i=1}^6 \sum_{j=i+1}^6 2 \operatorname{Re} \left\{ c_i c_j^* A_i(\vec{b}; \tau_k) A_j(\vec{b}; \tau_k)^* \right\} + \sum_{i=1}^6 c_i c_i^* A_i(\vec{b}; \tau_k) A_i(\vec{b}; \tau_k)^* \right) \\
 &= \sum_{i=1}^6 \sum_{j=i+1}^6 2 \operatorname{Re} \left\{ c_i c_j^* \sum_{k=1}^{N_{\text{acc}}} \left(A_i(\vec{b}; \tau_k) A_j(\vec{b}; \tau_k)^* \right) \right\} + \sum_{i=1}^6 c_i c_i^* \sum_{k=1}^{N_{\text{acc}}} \left(A_i(\vec{b}; \tau_k) A_i(\vec{b}; \tau_k)^* \right) \\
 &= \sum_{i=1}^6 \sum_{j=i+1}^6 2 \operatorname{Re} \{ c_i c_j^* A_{ij} \} + \sum_{i=1}^6 c_i c_i^* A_{ii} .
 \end{aligned}
 \tag{C.9}$$

Now, we can compute the sum over all accepted events before we do the minimization. With $c_i c_j^* \equiv c_{ij}$ and $\sum_{k=1}^{N_{\text{acc}}} A_i(\vec{b}; \tau_k) A_j(\vec{b}; \tau_k)^* \equiv A_{ij}$. We can simplify even further

$$\begin{aligned}
2 \operatorname{Re}\{c_{ij} A_{ij}\} &= 2 \operatorname{Re}\{\operatorname{Re}\{c_{ij}\} \operatorname{Re}\{A_{ij}\} - \operatorname{Im}\{c_{ij}\} \operatorname{Im}\{A_{ij}\} \\
&\quad + i \left(\operatorname{Re}\{c_{ij}\} \operatorname{Im}\{A_{ij}\} + \operatorname{Re}\{A_{ij}\} \operatorname{Im}\{c_{ij}\} \right)\} \\
&= 2 \left(\operatorname{Re}\{c_{ij}\} \operatorname{Re}\{A_{ij}\} - \operatorname{Im}\{c_{ij}\} \operatorname{Im}\{A_{ij}\} \right) \\
&\Leftrightarrow c_{ii} A_{ii} = \operatorname{Re}\{c_{ii}\} \operatorname{Re}\{A_{ii}\} .
\end{aligned} \tag{C.10}$$

For c_i real, the final simplification reads

$$\sum_{k=1}^{N_{\text{acc}}} \left| \sum_{i=1}^6 c_i A_i(\vec{b}; \tau_k) \right|^2 = \sum_{i=1}^6 \sum_{j=i+1}^6 2c_i c_j \operatorname{Re}\{A_{ij}\} + \sum_{i=1}^6 c_i^2 A_{ii} , \tag{C.11}$$

where the first derivative is given by

$$\frac{\partial}{\partial c_l} \left(\sum_{i=1}^6 \sum_{j=i+1}^6 2c_i c_j \operatorname{Re}\{A_{ij}\} + \sum_{i=1}^6 c_i^2 A_{ii} \right) \tag{C.12a}$$

$$= \sum_{i=1}^{l-1} 2c_i \operatorname{Re}\{A_{il}\} + \sum_{j=l+1}^6 2c_j \operatorname{Re}\{A_{lj}\} + 2c_l A_{ll} \tag{C.12b}$$

$$= \sum_{i=1}^6 2c_i \operatorname{Re}\{A_{il}\} , \tag{C.12c}$$

and the second derivative is by

$$\frac{\partial^2}{\partial c_l \partial c_m} \left(\sum_{i=1}^6 \sum_{j=i+1}^6 2c_i c_j \operatorname{Re}\{A_{ij}\} + \sum_{i=1}^6 c_i^2 A_{ii} \right) = 2 \operatorname{Re}\{A_{ml}\} . \tag{C.13}$$

Unsurprisingly this is the same as calculated in Eq. (C.6b).

More Details on the Kinematic Distributions

In this chapter we present some kinematic distributions before and after the double-Regge limit cuts. They complement their counterparts in Section 6.1.

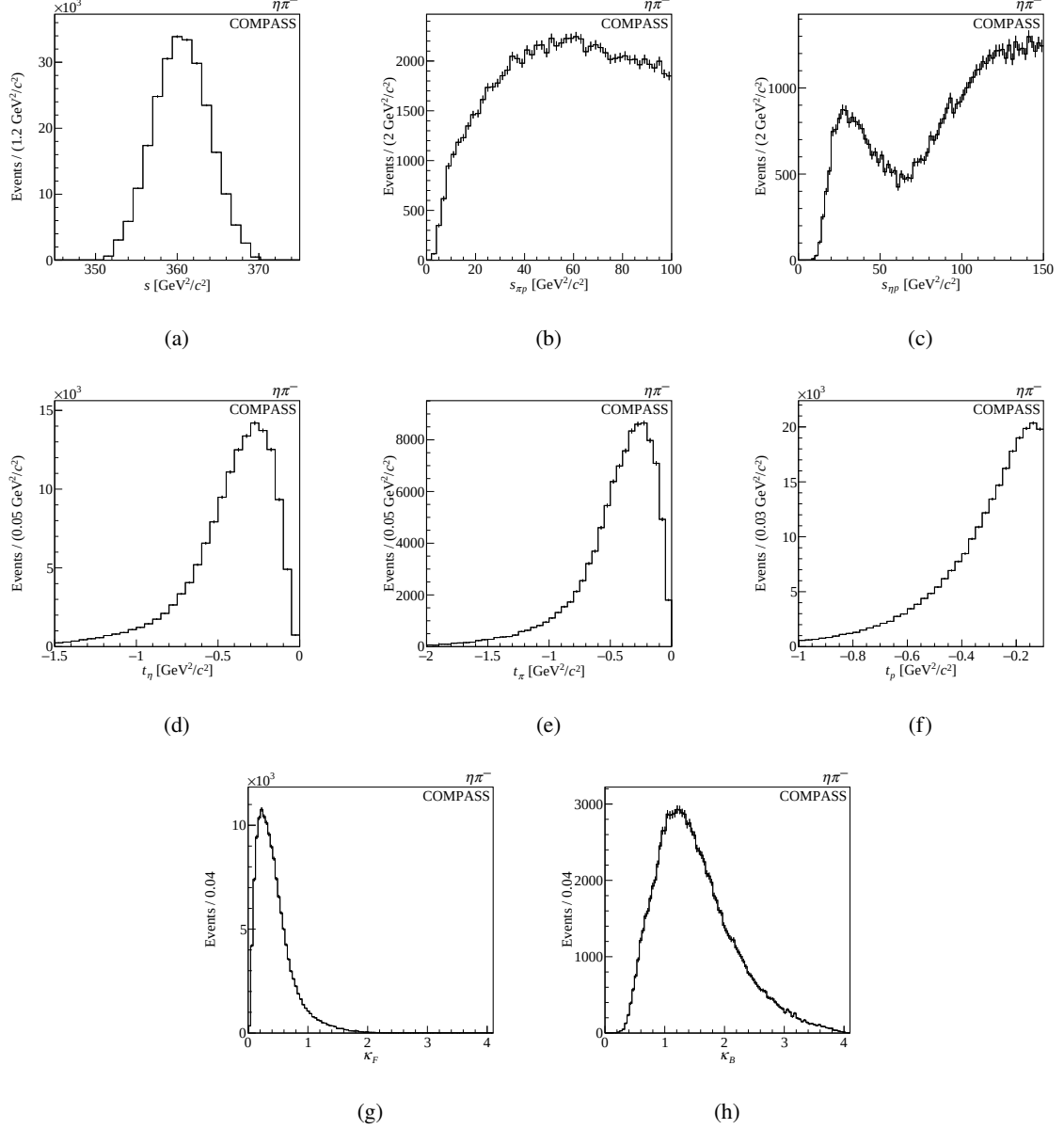


Figure D.1: Kinematic distributions for the $\eta\pi^-$ final state. (b), (d) and (g) represent variables, associated with $\eta^{(\prime)}$ forward production and are consequently only filled with forward events. (c), (e) and (h) represent variables, associated with $\eta^{(\prime)}$ backward production and are consequently only filled with backward events. κ_F refers to forward κ (see Eq. (2.28) with $s_2 = s_{\pi p}$). κ_B refers to backward κ (see Eq. (2.28) with $s_2 = s_{\eta p}$). The following cuts are imposed: $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$.

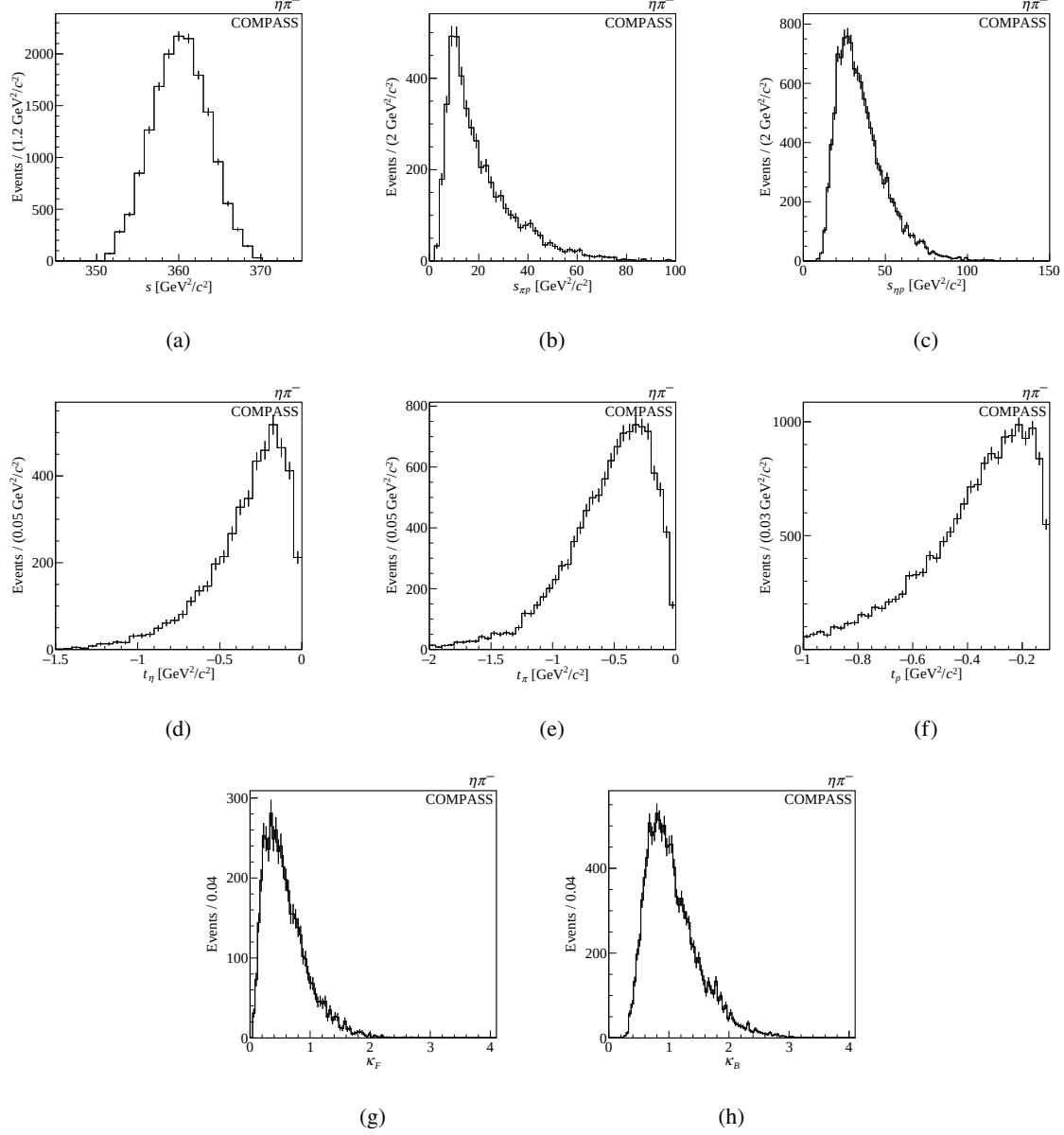


Figure D.2: Kinematic distributions for the $\eta\pi^-$ final state. (b), (d) and (g) represent variables, associated with $\eta^{(\prime)}$ forward production and are consequently only filled with forward events. (c), (e) and (h) represent variables, associated with $\eta^{(\prime)}$ backward production and are consequently only filled with backward events. κ_F refers to forward κ (see Eq. (2.28) with $s_2 = s_{\pi p}$). κ_B refers to backward κ (see Eq. (2.28) with $s_2 = s_{\eta p}$). The following cuts are imposed: $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$, $2.9 \text{ GeV} c^{-2} < m_{\eta\pi^-} < 6 \text{ GeV} c^{-2}$ and $|\cos \theta_{\text{GJ}}| > 0.75$.

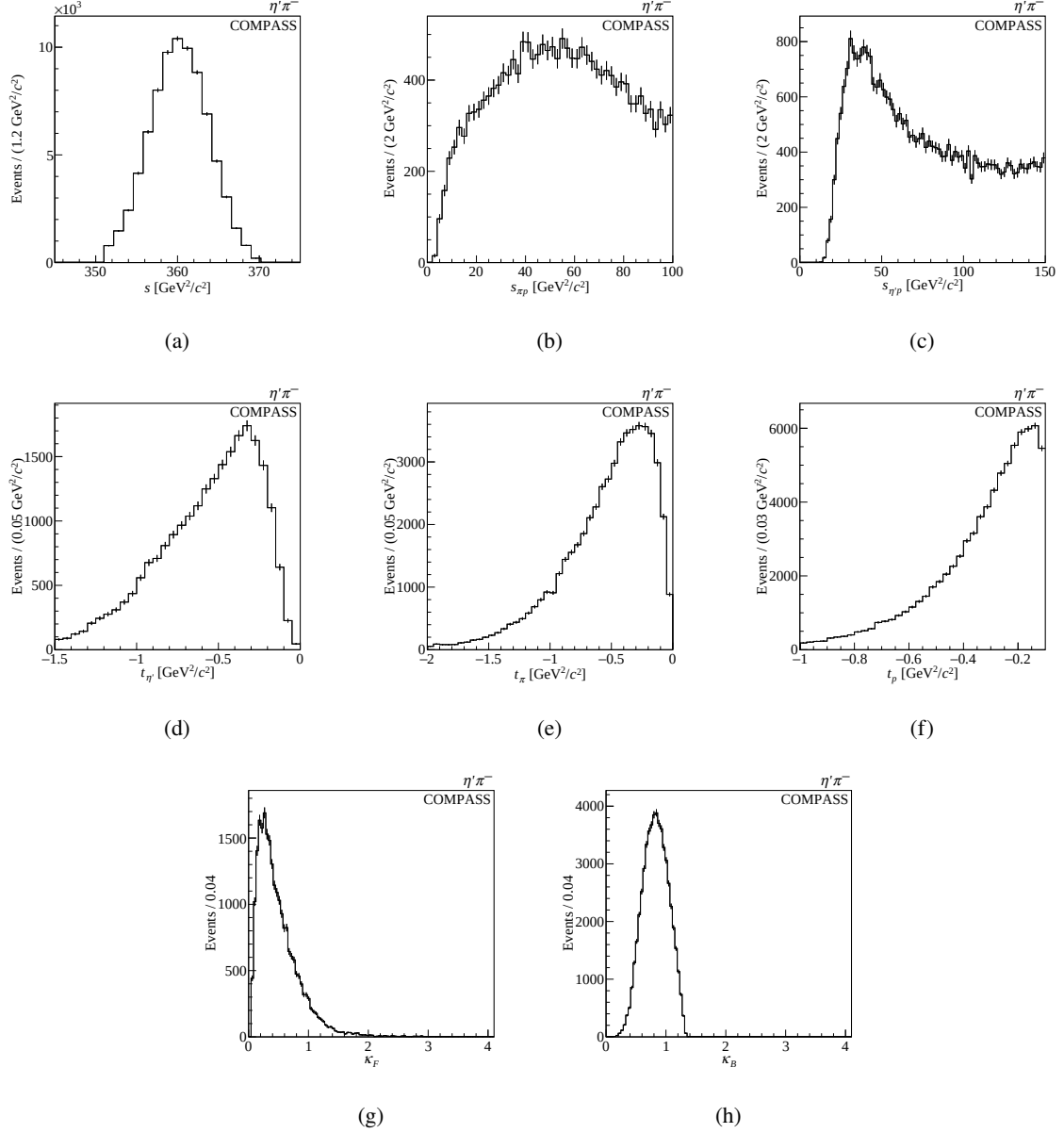
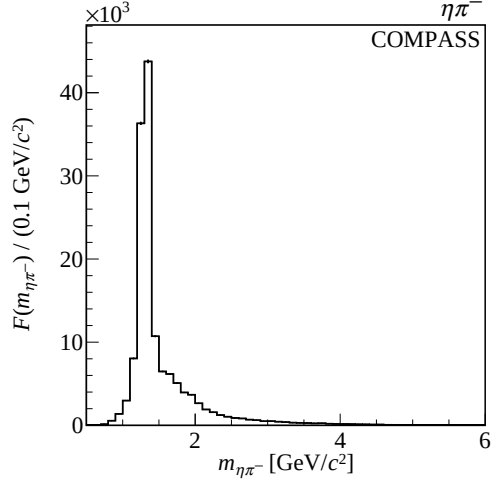
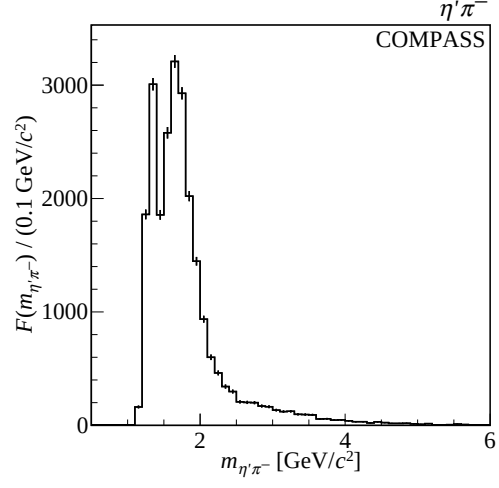


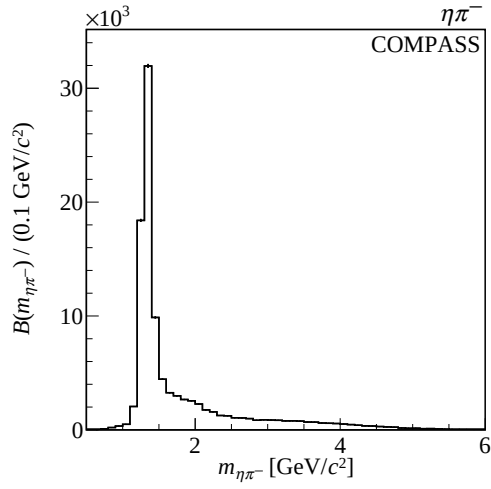
Figure D.3: Kinematic distributions for the $\eta' \pi^-$ final state. (b), (d) and (g) represent variables, associated with $\eta^{(\prime)}$ forward production and are consequently only filled with forward events. (c), (e) and (h) represent variables, associated with $\eta^{(\prime)}$ backward production and are consequently only filled with backward events. κ_F refers to forward κ (see Eq. (2.28) with $s_2 = s_{\pi p}$). κ_B refers to backward κ (see Eq. (2.28) with $s_2 = s_{\eta' p}$). The following cuts are imposed: $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$.



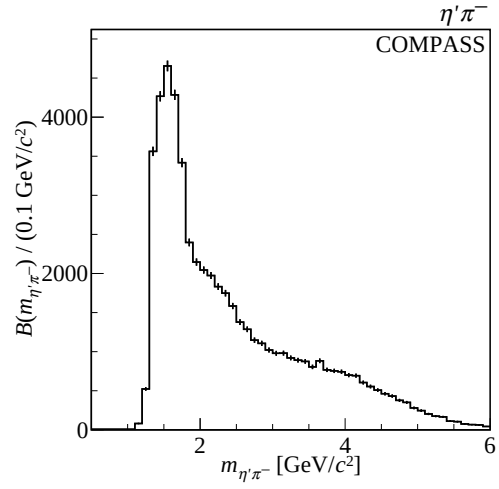
(a)



(b)



(c)



(d)

Figure D.4: Forward intensity (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) and the backward intensity (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state). The following cuts are imposed: $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$.

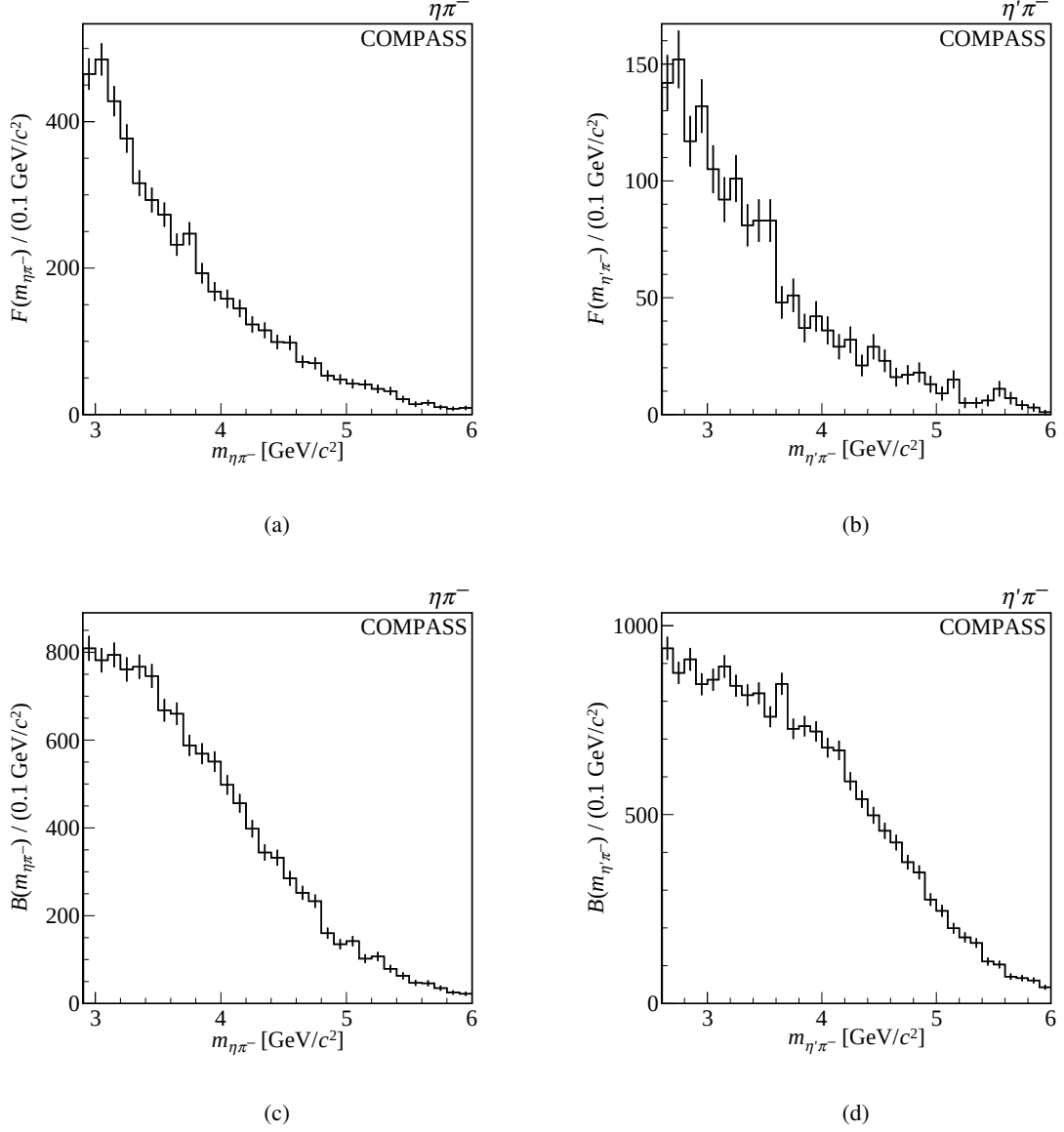


Figure D.5: Forward intensity (a) ($\eta\pi^-$ final state) and (b) ($\eta'\pi^-$ final state) and the backward intensity (c) ($\eta\pi^-$ final state) and (d) ($\eta'\pi^-$ final state). The following cuts are imposed: $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$, $2.9 \text{ GeV} c^{-2} < m_{\eta\pi^-} < 6 \text{ GeV} c^{-2}$, $2.6 \text{ GeV} c^{-2} < m_{\eta'\pi^-} < 6 \text{ GeV} c^{-2}$ and $|\cos \theta_{\text{GJ}}| > 0.75$.

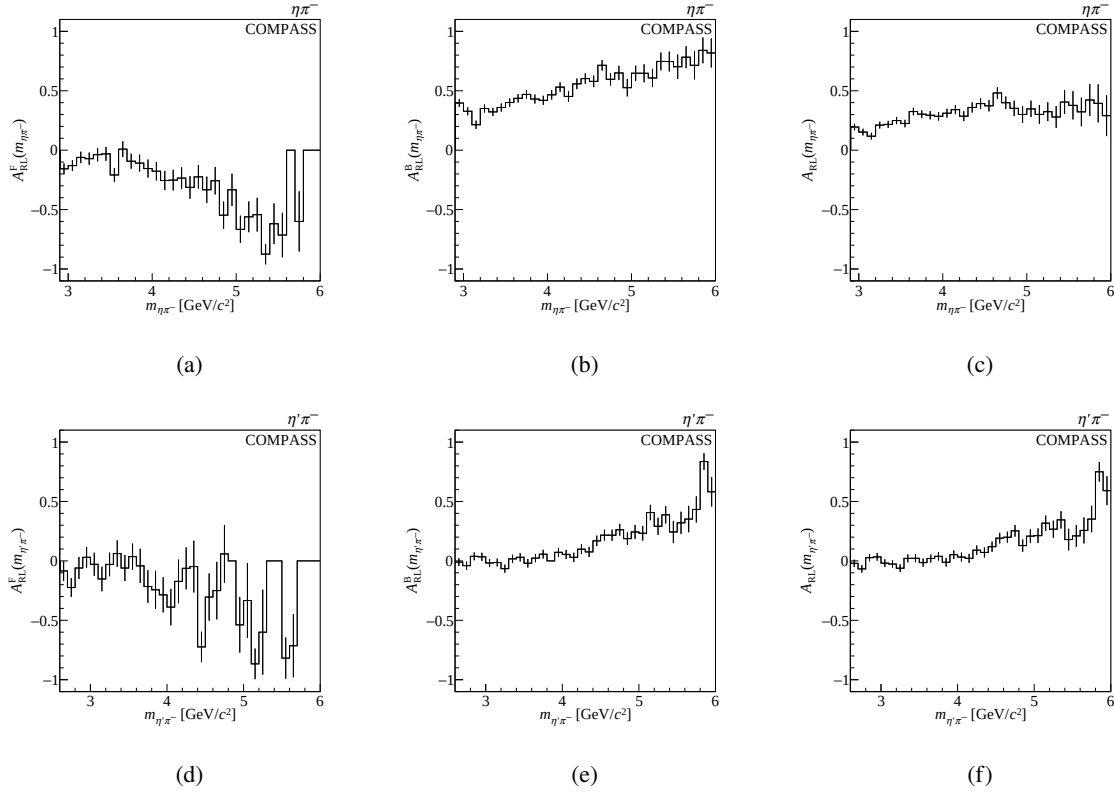


Figure D.6: Right-left asymmetry for forward (a), backward (b) and all events (c) in the $\eta\pi^-$ final state and the right-left asymmetry for forward (d), backward (e) and all events (f) in the $\eta'\pi^-$ final state. The following cuts are imposed: $0.1 \text{ GeV}^2 c^{-2} < t' < 1 \text{ GeV}^2 c^{-2}$, $2.9 \text{ GeV} c^{-2} < m_{\eta\pi^-} < 6 \text{ GeV} c^{-2}$, $2.6 \text{ GeV} c^{-2} < m_{\eta'\pi^-} < 6 \text{ GeV} c^{-2}$ and $|\cos \theta_{GJ}| > 0.75$.

Cross Check of the Fit

In order to ensure that there are no bugs in the way the double-Regge amplitudes are calculated and used, a full cross-check of the following things was performed by Julien Beckers. We have documented the cross-check in a specific internal gitlab repository [71].

The following points have been cross-checked for the global fit (2.4 to 6 GeV) and both final states ($\eta\pi^-$ and $\eta'\pi^-$). Note that this fit range was chosen because the cross check was performed for Ref. [72], where we discussed a slightly different mass start of the fit. However, the fit itself did not change and therefore this cross check is still valid. The results match up to machine precision.

- Calculation of all kinematic variables and double-Regge amplitudes for one event
- Calculation of the integral matrices
- Calculation of the likelihood with given parameters c_i and b_i

In addition, we have cross-checked the way how the MC weighting diagrams (see Eq. (6.10)) are constructed from the resulting coefficients c_i and b_i , including the error propagation described in Eq. (6.11).

In Fig. E.1, one can see that the absolute value of all weight diagrams, considering $F(m_{\eta\pi^-})$ match between this study and the cross check in the $\eta\pi^-$ final state. Note that also the errors match. See Fig. E.2. This study is drawn in blue, while the cross check is drawn in orange. The ratio at the bottom of the plot is either the value or error ratio. The diagrams are checked for the $\eta\pi^-$ final state only. The $\eta'\pi^-$ final state works conceptional exactly the same and therefore there is no need to check it also.

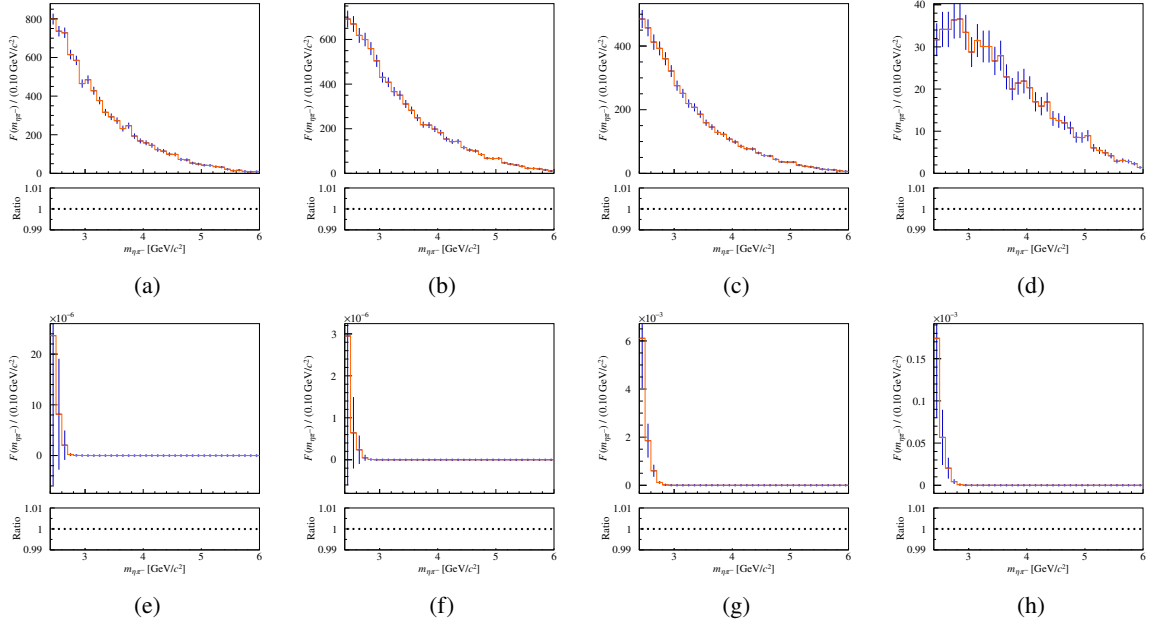


Figure E.1: $\eta\pi^-$ final state. Blue represents this work, orange represents the work by Julien Beckers. Ratio is the ratio of values. Weighting of $F(m_{\eta\pi^-})$ is compared for COMPASS data (a), the full intensity (b), the $a_2\mathbb{P}$ intensity (c), the a_2f_2 intensity (d), the $f_2\mathbb{P}$ intensity (e), the f_2f_2 intensity (f), the $\mathbb{P}\mathbb{P}$ intensity (g) and the $\mathbb{P}f_2$ intensity (h).

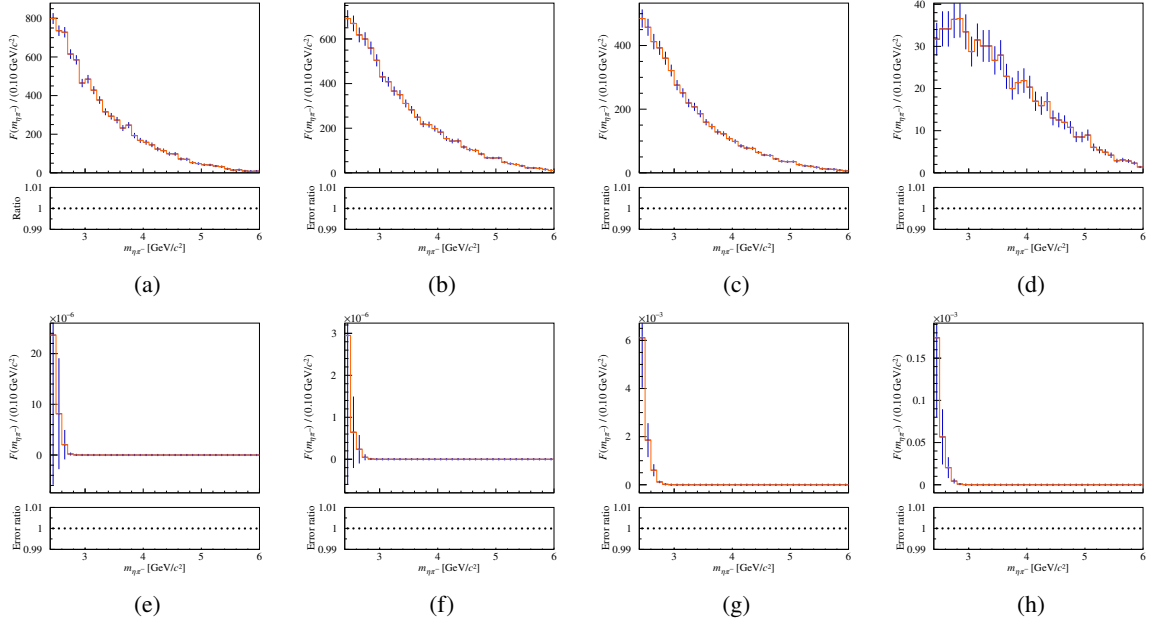


Figure E.2: $\eta\pi^-$ final state. Blue represents this work, orange represents the work by Julien Beckers. Ratio is the ratio of errors. Weighting of $F(m_{\eta\pi^-})$ is compared for COMPASS data (a), the full intensity (b), the $a_2\mathbb{P}$ intensity (c), the a_2f_2 intensity (d), the $f_2\mathbb{P}$ intensity (e), the f_2f_2 intensity (f), the $\mathbb{P}\mathbb{P}$ intensity (g) and the $\mathbb{P}f_2$ intensity (h).

Own Contributions

Within this work, not only the described physics results were achieved but numerous other important tasks were performed. I was considerably involved or am the main author of software packages used within the group (see Table F.1).

Table F.1: Software, where I contributed considerably.

Software	Contribution type	Ref.
TGEANT (COMPASS MC framework)	developer	[73]
Regge fits of COMPASS data	author	[72]
Wave Analysis Software Package (WASP)	original author / developer	[74]
BAF2 cluster [43] automatic job submission	author	[75]
Event selection for $3\pi\gamma\gamma$ -like final states in PHAST [46]	author	[40]

I was the day-to-day supervisor (sometimes shared with David Spülbeck) for the theses in Table F.2.

Table F.2: Day-to-day supervised theses. In all theses, we used COMPASS data or performed simulations for the 2008 and 2009 COMPASS setup.

Thesis type	Supervision type	Topic	Ref.
Internship	Main	Stability test of 2008 COMPASS data	[76]
Bachelor thesis	Main	Data stability in 2008 and 2009	[47]
WHF	Main	RPD: Studies on Calibration and Reconstruction	[32]
Master thesis	Main	First analysis of the $f_1(1285)\pi^-$ final state	[77]
Bachelor thesis	Main	GFlash for COMPASS calorimeters	[59]
Bachelor thesis	Shared	Event selection for $K^- + \pi^- \pi^+ \pi^0$	[78]
Master thesis	Shared	Partial-wave analysis of the $\eta^{(\prime)}\pi^-$ final state	[12]

In Table F.3, I lists the publications during this thesis.

Table F.3: List of publications during this thesis in chronological order.

Publication type	Details	Ref.
Conference talk	DPG 2019	[79]
Release note	Event selection 2022	[44]
Conference talk	DPG 2022	[80]
Workshop talk	2nd COMAP 2023	[81]
Conference talk	DPG 2023	[82]
Conference talk	HQL 2023	[83]
Conference proceedings	HQL 2023	[84]
Release note	Cross check for $K_S^0 K^-$ PWD 2024	[67]
Workshop talk	FDSA 2024	[85]
Conference talk	DPG 2024	[86]
Lecture	Monte-Carlo school 2024	[87]
Conference talk	DPG 2025	[88]
Release note	Double-Regge 2025	[72]
Conference talk	Hadron 2025	[89]
Conference proceedings	Hadron 2025	[90]
Workshop talk	IWHSS 2025	[91]

For the COMPASS and Apparatus for Meson and Baryon Experimental Research (AMBER) collaboration, where I was a part of, I was “Gem On-Call expert” from 2018 to 2025 together with my colleagues from the Ketzer workgroup in Bonn. I was on shift duty every datataking year and was supervising students from Bonn during their stay at CERN. In addition, I was part of the COMPASS spokesperson search committee in 2023 as junior representative. As a working group we were responsible for keeping the GEMs operational and calibrating them. I helped with the time calibration and reshaped the software used for that [92].

For the working group of Bernhard Ketzer, I was IT-support from 2019 to 2025 together with Mathias Wagner (untill 2023), alone for 2024 and together with Lars Döpfer since 2025. This included the development of the group website [93] and the FTD website [94], as well as maintenance of the group server (fsketzer). Additionally, I set up many new computers and helped to accomplished an automatic setup procedure, which included the integration into the institute infrastructure. I made system-wide installation of custom software (TGEANT, CORAL, PHAST, etc.) doable with simple scripts and was tasked with hosting and installing a GEM production database on one of our group computers. I provided continuous support to the subdevelopers of the FTD website and in C++ coding for many students within the group. I was an assessor in oral examinations and a tutor for the courses in Table F.4.

Table F.4: My active tutor duties during this thesis. Winter semester is abbreviated by WS and summer semester by SS.

Semester	Course
WS 2019/2020	Experiments on the structure of hadrons
SS 2020	Physics of hadrons
WS 2020/2021	Physics 1
SS 2021	Physics 2
WS 2021/2022	Physics 5
SS 2022	Physics of hadrons
WS 2022/2023	Physics of particle detectors
SS 2023	Advance gaseous detectors
WS 2023/2024	Physics 1
WS 2024/2025	Physics of particle detectors
SS 2025	Physics of hadrons

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Glossary

t' In general, the momentum transfer t can be described as follows

$$t = (p_1 - p_3)^2 = m_1^2 + m_3^2 - 2(E_1 E_3 - |\vec{p}_1| |\vec{p}_3| \cos \theta_{13}) \quad (1)$$

with the first particle being the beam particle, the second one is the target proton, the third one is the $\eta^{(\prime)} \pi^-$ system and the fourth one is the recoil proton. In the center-of-momentum frame, we can calculate the energy via

$$E_{1,\text{CM}} = \frac{s + m_2^2 - m_1^2}{2\sqrt{s}}. \quad (2)$$

With $\cos \theta_{13} = \pm 1$, the minimum and maximum values of t in the center-of-momentum frame are

$$t_{\text{min,max}}^{\text{CM}} = m_1^2 + m_3^2 \mp 2 \left(E_{1,\text{CM}} E_{3,\text{CM}} - \sqrt{E_{1,\text{CM}}^2 - m_1^2} \sqrt{E_{3,\text{CM}}^2 - m_3^2} \right). \quad (3)$$

These values exhibit an event-by-event bias which we can correct for by introducing the variable t' as

$$t' = |t| - |t_{\text{max}}| \quad (4)$$

. 83, 84, 89, 90, 95, 97–99, 146, 148–150, 160–165, 184, 185

Antok-Style During an event selection, multiple cuts are performed. In Antok-Style, for every cut, all other cuts are applied first, followed by the cut under test. viii, 34–38, 40–42, 44, 46–48, 85–88, 92–94, 123, 126–143, 145, 147, 183, 185

C++ A compiled programming language used in almost all software during this thesis. 31, 34, 51, 52, 54, 170, 194

detector.dat A plain text file, where detector information, positioning and other properties are stored. It's used in CORAL and TGEANT as an alignment file. 31, 55, 57

FORTRAN A high-level programming language especially suited to numeric computation and scientific computing. It is not used during this thesis but still used by older analysis frameworks. 51, 193

- GFlash** A parameterized model in GEANT4 for simulating electromagnetic showers in calorimeters. It can be used in TGEANT, where an effective description of the calorimeter material is used [58]. 2, 55, 62, 63, 169
- Minuit2** A C++ numerical minimization library used for function minimization and error analysis. We use it within the framework [95]. 102, 104, 105
- Python** An interpreted programming language known for its readability and widespread use in data analysis. 51
- reserv** A scaling parameter to account for energy loss in the calorimeter. There is one reserv parameter per calorimeter cell type and the shower energy is calculated with Eq. (5.6).. 63, 68, 75–78, 184, 187
- ROOT** An open-source data analysis framework developed at CERN, widely used within this thesis for histograms, data storage and analysis [65]. 31, 55, 67, 83, 190, 195
- run** COMPASS data taking increment. One run is composed of up to 200 spills. The main purpose of this is to have an organized data taking format. 21, 34
- Slot4** Fourth data production of the 2008 and 2009 diffractive data. This is an informal counting scheme within COMPASS. A higher slot number indicates a later / more up to date production. 33, 34, 123
- Slot5** Fifth data production of the 2008 and 2009 diffractive data. This is an informal counting scheme within COMPASS. A higher slot number indicates a later / more up to date production. viii, 33, 34, 78, 123–134, 136, 138–144, 146, 148, 183–185
- spill** One extraction period from the SPS to the COMPASS experiment. It is usually about 5 s. 21, 34, 190
- Toolbox** A plugin based system, deployed with TGEANT, that can read output files from TGEANT and access all generated information. Within this thesis the `Lednev` and `BonnPhaseSpaceTest` plugins were developed and used to extract the shower profiles in ECAL2 and test the phase space generator, described in Section 5.4.2.. 84
- Waterfall-Style** During an event selection, multiple cuts are performed. In Waterfall-Style, every cut is performed in a fixed order. viii, 34, 41, 47, 49, 89, 126–133, 135, 137–143, 185
- weighted intensity color code** While representing our fit result, we have a specific color code in which we show the results. The intensity (see Eq. (6.1)) with $\vec{c}$ and $\vec{b}$ from the best fit is histogrammed for all MC accepted events (see Eq. (6.10)). The color code is

Black histogram → data distribution
 Red circle (●) → **fitted intensity** (Eq. (6.2))
 Green square (■) → intensity of the $a_2 \mathbb{P}$ diagram.
 Green triangle (▲) → intensity of the $a_2 f_2$ diagram.
 Orange square (■) → intensity of the $f_2 \mathbb{P}$ diagram.
 Orange triangle (▲) → intensity of the $f_2 f_2$ diagram.
 Blue square (■) → intensity of the $\mathbb{P} \mathbb{P}$ diagram.
 Blue triangle (▲) → intensity of the $\mathbb{P} f_2$ diagram.

Blue square (■) → intensity of all **bottom** $\mathbb{P}$ diagrams, i.e. the amplitude is the coherent sum over all diagram with a $\mathbb{P}$ at the bottom vertex.

Orange triangle (▲) → intensity of all **bottom** f_2 diagrams, i.e. the amplitude is the coherent sum over all diagram with an f_2 at the bottom vertex.

Diagrams with single intensities and bottom $\mathbb{P} / f_2$ never mix on the same canvas.

The systematic error will be shown as an error band. The statistical error might be too small at times to be visible beyond the markers. If visible, one can see it as an error bar.. 104, 110–115

Acronyms

ALICE A Large Ion Collider Experiment. 67

AMBER Apparatus for Meson and Baryon Experimental Research. 170

CEDAR Cherenkov Detector with Achromatic Ring focus. 21, 27, 46, 47, 80, 84

CERN Conseil Européen pour la Recherche Nucléaire. 21, 51, 52, 190, 193, 194

COMGEANT COMpass GEometry ANd Tracking [48] is the MC framework used in COMPASS prior to TGEANT. It uses GEANT3 for the detector simulation and is written in . 51, 52, 62

COMPASS COMmon Muon Proton Apparatus for Structure and Spectroscopy. vii, viii, 1, 8, 12, 14, 15, 17, 19, 21, 22, 24, 26, 28–35, 43, 45, 51, 52, 54–56, 58, 60–68, 70–76, 78–80, 82, 84–86, 88–90, 92–95, 103, 104, 108, 121, 168–170, 183, 184, 187, 190, 193–195

CORAL COMpass Reconstruction ALgorithm [96] is the reconstruction program used at COMPASS. 31, 32, 35, 55, 67, 70, 75, 170, 189

DC Drift Chamber. 21

DR Rich Wall. 22, 65, 66

ECAL1 Electromagnetic CALorimeter 1. 22, 28, 29, 36–40, 62, 75, 77, 78, 87, 92–94, 124, 125, 130, 134, 145, 147, 183, 185

ECAL2 Electromagnetic CALorimeter 2. viii, 22, 26, 29, 30, 36–40, 62, 63, 67–75, 77, 78, 85–87, 89, 92–94, 124, 125, 131, 134, 145, 147, 183–185, 190

ECALs Electromagnetic CALorimeters 1 and 2. 33, 36, 67, 75, 133

FH Forward Hodoscope. 22, 26, 27, 59

GDML Geometry Description Markup Language [97], an XML-based format for describing detector geometry, used in GEANT4. 55, 67

GEANT3 GEometry ANd Tracking 3 [49], developed at CERN and based on is used to simulate particle detectors to a high accuracy. It is superseded by GEANT4. 51, 52, 193, 194

- GEANT4** GEometry ANd Tracking 4 [50], developed at CERN and based on is used to simulate particle detectors to a high accuracy. It is the successor of GEANT3. viii, 31, 52, 53, 55, 62, 63, 65, 67, 84, 190, 193, 195
- GEM** Gas Electron Multiplier. 21, 170
- GJ** Gottfried-Jackson. 11, 12, 16–19, 84, 89, 91, 95–98, 108–111, 114, 121, 146, 148, 149, 151, 161, 164, 165, 183–185
- GUI** Graphical User Interface. 54–58, 183
- HCAL1** Hadronic CALorimeter 1. 22, 37, 132, 185
- HCAL2** Hadronic CALorimeter 2. 22, 27
- JSON** JavaScript Object Notation [98], a lightweight data-interchange format that is commonly used in this thesis for option and steering files. 80, 82, 83, 184
- LAS** Large Angle Spectrometer. 21, 22
- MC** Monte Carlo. viii, 2, 31, 51, 52, 54–56, 58, 60–62, 64–68, 70, 72, 74–76, 78–80, 82–94, 101–104, 116, 121, 149–151, 155, 167, 169, 184, 185, 187, 190, 193, 195
- mDST** mini Data Storage Tree. 31, 32, 67
- MicroMegas** Micromesh Gaseous Structure. 21
- MuC** Munich Counter. 26–28, 59
- MW1** Muon Wall 1. 22, 65, 66
- MW2** Muon Wall 2. 22, 65, 66
- MzC** Mainz Counter. 22, 26–28, 59
- PHAST** PHysics Analysis Software Tools [46] is the analysis software for event selections used at COMPASS. 31, 32, 34, 35, 84, 170
- PV** A Primary Vertex is an interaction point with the beam particle and at least one other particle. 33–38, 41, 49, 60, 63, 80, 83, 85, 89, 126, 128, 129, 145, 147, 183–185
- QCD** Quantum Chromodynamics. 1
- RAM** Random-Access Memory. 67
- RICH** Ring Imaging CHerenkov detector. viii, 22, 47, 55, 57, 61, 84, 85, 142, 143, 183, 185
- RPD** Recoil Proton Detector. viii, 21–23, 25–28, 33, 41, 43, 44, 60, 61, 88, 92, 95, 169, 183, 184

SAS Small Angle Spectrometer. 21, 22

SPS Super Proton Synchrotron. 21, 190

TGEANT Total GEometry ANd Tracking [13, 73] is the modern MC simulation framework of COMPASS, based on GEANT4. 22, 31, 52, 54–62, 64–68, 70, 76–80, 83, 84, 92, 121, 170, 183, 184, 187, 189, 190, 193

VGM Virtual Geometry Model [53], is used in this thesis as an interface between GEANT4 detector description and geometry description. 55, 67

WLS wave-length shifting. 30, 62–64