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to my parents

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Introduction

Investment decisions lie at the heart of many economic activities. Each day, individual investors, banks, and fund management firms try to optimize their portfolio investments at international financial markets around the globe. Corporations buy parts of other corporations or sell divisions of their own activities in the worldwide mergers and acquisitions market in order to adjust the strategic direction of their business model. On the other hand, firms face constantly the challenge to determine their optimal investments into new factories, marketing strategies, or fundamental research activities.

Many of these investment decisions are rather complex. Usually, they involve complicated considerations on expected gains out of the transaction as well as a thorough analysis of the risk related to the investment. Hence, a sound investment evaluation is the prerequisite for any sound investment decision. Financial economics has developed numerous theoretical models as well as an abundance of empirical studies that aim to assess investment opportunities and thereby try to explain investment decisions.

This dissertation sheds light on several important aspects of investment evaluation and investment decisions. Throughout this work, I use both empirical analysis as well as theoretical models to introduce some new features into well-known topics in empirical asset pricing and the theory of optimal investment decisions. In the first three chapters I concentrate on issues surrounding investments in financial markets. First, I compare various novel approaches to estimate the equity risk premium, one of the key variables in investment theory. In the next chapter, I analyze whether it is possible to generate excess returns in equity markets by investing into stock on the basis of publicly available information in form of analysts' forecasts. In chapter three, I test whether these analysts' forecasts can also be used to predict stock returns. In the fourth chapter then, I shift the analysis towards corporate investment decisions in the real economy. More precisely, I examine how a decision maker's attitude towards uncertainty affects the investment behavior of firms. In the next paragraphs, I present the main topics addressed in this thesis in more detail. In the first chapter¹, I focus on one of the most important concepts in asset pricing theory: the equity risk premium. It is the reward that investors require to compensate them for the risk associated with holding equities compared to government securities. The equity premium plays a key role in many cost-of-capital calculations, such as those based on the capital asset pricing model (CAPM) or the Fama-French three-factor model (Fama and French, 1993).

Since the equity premium is essentially unobservable, it is also one of the most disputed concepts in finance. Not only is the magnitude of the risk premium discussed controversially among economists, but the appropriate methodology to obtain meaningful estimates thereof lies at the core of the debate. Traditionally, most academics used historical average excess returns of stocks over bonds as an appropriate proxy for the future equity risk premium.

More recently, economists put forward the so-called implied equity risk premium to estimate a forward-looking risk premium. The basic idea of the implied risk premium is to estimate the expected average cost of capital in the market with the help of present value formulas. The expected risk premium is then obtained by subtracting the prevailing yield on treasury securities.

In chapter 1, I evaluate empirically various methods that are commonly used to determine the implied equity risk premium. To this end, I compare risk premia obtained from different present value formulas, the dividend discount model and the residual income model. Then I assess them by analyzing their underlying firm-specific cost-of-capital estimates. I show that specific versions of dividend discount and residual income models lead to similar risk premium estimates. However, cross-sectional regression tests of individual firm risk premia suggest that there are qualitative differences between both approaches. Expected firm risk premia obtained from the dividend discount model are more in line with standard asset pricing models and perform better in predicting future stock returns than estimates from the residual income approach.

In the second chapter² of this thesis, I concentrate on the informational efficiency of stock markets. The semi-strong version of the efficient market hypothesis states that publicly available information about any company should already be included in current market prices. Consequently, it should not be possible to implement any profitable investment strategy that relies on publicly known company information or widely disseminated expectations on future dividends, earnings, or growth prospects.

¹This chapter is based on Schröder (2007a).

²This chapter is based on the paper by Esterer and Schröder (2007a).

INTRODUCTION

Yet, research departments of brokerage firms and investment banks put a lot of effort into equity analysis, hoping to achieve superior returns for their clients. What is more, fund management firms use information on stocks provided by such sell-side analysts to optimize their asset allocation decisions. This apparent conflict between academic theory and Wall Street practice is the compelling motivation for the study presented in chapter 2.

In this part of my thesis, I analyze whether investors can generate excess returns by implementing investment strategies that are based on analysts' forecasts. To systematically connect analysts' forecasts to current share prices, I employ various valuation models commonly used to appraise companies. More precisely, I use the so-called implied cost of capital (ICOC) as key variable to capture expectations of equity analysts. Calculated as the internal rate of return that equates share price with discounted forecasted cash-flows, the implied cost of capital allows condensing a variety of analysts' expectations about the future of any company into one single figure.

I show that a simple trading strategy based on the ICOC yields risk-adjusted excess returns with respect to several common asset pricing models, such as the CAPM, or the Fama-French three-factor model. In the examined U.S. data sample, an investment that goes long in high-ICOC stocks and short in low-ICOC stocks yields an annual return of up to 10%. When correcting the investment for the portfolio's exposure to the Fama-French risk factors, it still provides a risk-adjusted return of 6.2% p.a. These results are even more remarkable since the ICOC effect persists even after including transaction costs. Estimations across other major international capital markets confirm these findings. Hence, this study clearly rejects the joint hypothesis of efficient markets and the validity of both asset pricing models.

Chapter 3 is closely related to the previous analysis³. After having shown that analysts' forecasts can be used to generate excess returns in stock markets, I address the question whether this information can also be used to forecast individual stock returns. If analysts' forecasts contain valuable information about the companies' future prospects, the ICOC should be a good predictor of stock returns: the ICOC is just the share's rate of return that is required to reconcile its market price with future cash flows as predicted by analysts.

The objective of this chapter is hence to examine the ICOC's ability to predict stock returns. To have a benchmark, I compare the ICOC's forecasting ability with a variety of other variables that are traditionally used to predict stock returns, such as

³This analysis follows Esterer and Schröder (2007b).

the dividend-price ratio, the book-price ratio, or various implementations of the socalled value-price ratio. Since the variables' predictive power for stock returns might originate only from their underlying relation to firm risk proxies, I test the forecasting variables against both the CAPM and the three-factor model by Fama and French, which have been widely accepted as asset pricing models.

Panel regressions across the world's largest stock markets show that the ICOC is better in predicting stock returns than traditional valuation multiples - even when controlling stock returns for their risk exposure using standard asset pricing models. In the U.S. data sample, implied cost of capital estimates explain a sizable part of the cross-sectional variation in stock returns. Joint regression tests of stock returns on the ICOC estimate and standard CAPM or Fama-French risk factors highlight that ICOC coefficients remain highly significant even after controlling for the stocks' risk. Thereby, I confirm the ICOC's additional informational content obtained from equity analysts' forecasts.

In chapter 4, I finally turn to the analysis of firms' investment decisions in the real sector⁴. When companies decide about investments, they usually face uncertainty about future cash flows from their projects. This fact has been widely acknowledged by the literature on optimal investment, and every sound investment theory requires an accurate assessment of the uncertainty involved. In analyzing the uncertainty faced by a firm, researchers have essentially identified two main sources of uncertainty: risk and ambiguity. While *risk* usually refers to the return volatility of an investment project using a specific probability distribution, the notion of *ambiguity* stands for the existence of a multitude of such probability distributions to describe future profits. Whereas the impact of risk on investment decisions has been analyzed thoroughly in the past, the role of ambiguity has only recently drawn attention to the research community. How is investment actually affected by a perceived change in ambiguity?

By assuming ambiguity averse decision makers, most investment models postulate that a rise in ambiguity alienates cautions investors, which are hence less eager to invest. While this assumption facilitates the analysis, there is quite some evidence that the assumption of entirely ambiguity averse decision makers is too extreme. The survey of successful entrepreneurs by Bhidé (2000) for example shows that individuals that start businesses are highly self-confident and exhibit a very low degree of ambiguity aversion. Such behavior has also been revealed by experimental studies of Heath and Tversky (1991), underpinning the positive relationship between self-confidence

⁴The ideas of this chapter have been developed in Schröder (2007b).

and ambiguity loving behavior.

In this chapter, I propose a simple investment model that captures different personal attitudes to ambiguity, and thus allows to investigate its influence on the investment decision. I resort to the irreversible investment theory following the work of Dixit and Pindyck (1994) as framework for the model. Also known under the term real option theory, this approach offers an elegant way to determine the optimal investment strategy given an uncertain environment by applying option-pricing techniques to the investment problem. In contrast to the standard irreversible investment literature, I do not assume that the entrepreneur has perfect confidence in the perceived probability measure describing future uncertainty. Instead we assume that she considers other probability measures to be possible as well, i.e. we extend the model to include ambiguity. Finally, to examine the effect of different attitudes of the entrepreneur towards uncertainty, I assume that the preferences of the entrepreneur can be described as a convex combination of the two extreme attitudes towards ambiguity, i.e. the decision maker considers the best and the worst case only.

I find that even a very small fraction of optimism from the side of the entrepreneur can change the investment decision significantly. Most important, I show that in many cases the threshold for investing, i.e. the required expected value of a project, decreases in presence of ambiguity. As a consequence, investment is carried out at earlier stages compared to situations without uncertainty.

The next four chapters each present one idea as a self-contained unit.

Chapter 1

The Implied Equity Risk Premium: An Evaluation of Empirical Methods

A new approach of estimating a forward-looking equity risk premium (ERP) is to calculate an implied risk premium using present value (PV) formulas. This chapter compares implied risk premia obtained from different PV models and evaluates them by analyzing their underlying firm-specific cost-of-capital estimates. It is shown that specific versions of dividend discount models (DDM) and residual income models (RIM) lead to similar ERP estimates. However, cross-sectional regression tests of individual firm risk premia suggest that there are qualitative differences between both approaches. Expected firm risk premia obtained from the DDM is more in line with standard asset pricing models and performs better in predicting future stock returns than estimates from the RIM.

1.1 Introduction

The equity risk premium (hereafter ERP) is one of the most important concepts in financial economics. It is the reward that investors require to compensate them for the risk associated with holding equities compared to government securities. The equity premium¹ plays a key role in many cost-of-capital calculations, such as those based on the capital asset pricing model (CAPM) or the Fama-French three-factor model (Fama and French, 1993). Moreover, the magnitude of the ERP is critical for all investors since it substantiates decisions about asset allocation between equities and bonds.

Since the equity premium is essentially unobservable, it is also one of the most disputed concepts in finance. Not only is the magnitude of the ERP discussed contro-

¹In this study, the terms equity risk premium (ERP), risk premium, equity premium and market risk premium refer to the same concept and are used interchangeably.

versially among economists, but the appropriate methodology to calculate meaningful estimates also lies at the core of the debate. Despite certain exceptions, e.g. Blanchard (1993), most academics used historical excess returns of stocks over bonds as provided by e.g. Ibbotson Associates (2005) as an appropriate proxy for the future ERP. More recently, several economists developed a new approach to estimate the market risk premium by calculating the so-called implied ERP with the help of present value (PV) formulas. The basic idea of this concept is to estimate the expected average future cost of capital in the market, and then to subtract the prevailing yield on treasury securities.

Unfortunately, there are many different ways to estimate the implied risk premium. Whereas economists at first relied on the dividend discount model (DDM) to calculate the ERP, more recent studies opted for the residual income model (RIM), being increasingly considered to be the preferred approach. Surprisingly, a comprehensive comparison of the various approaches is still missing. The objective of this chapter is thus to examine both methods employed in the implied ERP estimation in order to contribute to the search for the most reliable approach. This evaluation is done by applying the models to the same data set concurrently. Consequently, this study is the first to allow a direct comparison of the ERP obtained from DDM and RIM.

In a first step, this study compares the magnitude of implied ERP estimates for various models across European markets. Although it is well known that infinite DDM and RIM are mathematically equivalent to each other and should therefore lead to identical ERP estimates, the empirical implementation causes the models to diverge. Hence, one focus of this study lies in examining whether and how this theoretical equivalence can be sustained in practice. To detect qualitative differences between both approaches, we then present cross-sectional regression tests to determine key factors and variables that influence the cost of capital at the firm level. Finally, we compare the different models' ability to predict individual stock returns.

This work is related to several streams of research in the literature. First, this study extends earlier works on the implied ERP: Cornell (1999) and Claus and Thomas (2001) are two of the pioneering studies in this field. More recent studies on the implied cost of capital of individual firms include Easton et al. (2002), Lee et al. (2003), Daske et al. (2004), and Pástor et al. (2006). Second, it is related to the line of research investigating the ability of DDM, RIM and DCF (discounted cash flow) formulas to explain cross-sectional returns in the context of equity valuation (Penman and Sougiannis, 1998; Courteau et al., 2001; Francis et al., 2000). Finally, this study takes up the analysis of the determinants of the implied cost of capital, as documented in Gebhardt et al. (2001), Lee et al. (2003) and Guay et al. (2005).

This chapter presents evidence that specific versions of DDMs and RIMs lead to similar implied ERP estimates. In addition, it is shown that the underlying companyspecific cost-of-capital estimates obtained from the dividend discount model can be better explained by standard asset pricing models (such as the CAPM or the Fama-French model) compared to the much more popular RIM approach². Finally, it is shown that the DDM performs better in predicting future stock returns than the RIM.

The chapter proceeds as follows. The next section presents the methodology of the implied cost of capital in more detail. Section 1.3 describes the European data sample used in this study. The ERP estimates for several European markets are presented in section 1.4. Further examinations of the models using cross-sectional regressions on firm-level cost-of-capital estimates follow in section 1.5. Section 1.6 offers a short conclusion.

1.2 The Calculation of the Cost of Capital

1.2.1 The Implied Cost of Capital

In this study, the cost of capital of individual firms is calculated using the methodology of the so-called implied cost of capital. The basic idea of this concept is to estimate the future cost of capital with the help of PV models. More precisely, the cost of equity is computed as the internal rate of return that equates discounted payoffs per share to current price. In the literature, many different versions of the present value model are employed to calculate the implied cost of capital. The two most common formulas are the DDM, as used by e.g. Cornell (1999), and the RIM, employed by Claus and Thomas (2001) or Lee et al. (2003). The general DDM can be written as follows:

$$P_0 = \sum_{t=1}^{\infty} \frac{E_0[D_t]}{(1+k)^t}$$
(1.1)

where

 $P_0 = \text{current share price, at the end of year 0,}$ $E_0[D_t] = \text{expected dividends per share at the end of year t,}$ k = cost of capital or, equivalently, shareholders' expected rate of return.

²Most empirical studies on the implied cost of capital cited above rely on the RIM.

When combined with the so-called *clean surplus* relation, the DDM can be transformed into the RIM (Feltham and Ohlson, 1995). This relation requires that all gains and losses affecting book value are also included in earnings³:

$$D_t = E_t - (B_t - B_{t-1}) \tag{1.2}$$

The RIM can be expressed as follows:

$$P_0 = B_0 + \sum_{t=1}^{\infty} \frac{E_0[R_t]}{(1+k)^t}$$
(1.3)

with

$$E_0[R_t] = E_0[E_t] - k(B_{t-1}) = (E_0[roe_t] - k)B_{t-1}$$
(1.4)

where

B_t	=	book value of equity per share at the end of year t
		$(B_0 \text{ being the current book value}),$
$E_0[R_t]$	=	expected residual income per share in year t ,
$E_0[E_t]$	=	expected earnings per share in year t ,
$E_0[roe_t]$	=	expected return on equity in year t .

Equation (1.4) demonstrates the basic idea of residual income: only if a company generates higher returns on equity than its cost of capital, it can create positive residual incomes. Otherwise the company should be valued at its book value, or even below. Since the clean surplus relation can also be written as

$$B_t = B_{t-1} + E_t - D_t = B_{t-1} + (1 - p_t)E_t$$
(1.5)

where p_t is the payout ratio of year t, future book values of equity can consequently be calculated from future earnings and retention ratios using equation (1.5).

1.2.2 Employed Models

Since exact predictions of future dividends or residual incomes cannot be made to infinity, several versions of the DDM and RIM are usually used which implement different assumptions about expected cash-flows.

Dividend Discount Models

A simple and very common version of the DDM is the Gordon (1962) growth model, assuming a constant dividend growth rate in the future. However, the limitations of

³This condition is not always met, of course. Stock options and capital increases, e.g. can affect the book value of equity while leaving earnings unchanged. Still, the relation is approximately fulfilled in most cases.

this formula are widely known, e.g. Damodaran (1994, p. 100). For most companies, the assumption of a constant dividend growth overestimates future payments, especially when employing the long-term earnings growth rate obtained from analysts as a proxy for the dividend growth rate (see below). Still, e.g. Harris and Marston (2001) rely on this model to calculate the ERP, which is hence likely to be biased upwards. Multistage DDM overcome this limitation. The two most prominent examples are a two-stage DDM, as proposed by Damodaran (1999), and a three-stage version, as used by Cornell (1999). The two-stage DDM is given by:

$$P_0 = \underbrace{\sum_{t=1}^{5} \frac{E_0[D_t]}{(1+k)^t}}_{(1-k)^t} + \underbrace{\frac{E_0[D_5](1+g_l)}{(k-g_l)(1+k)^5}}_{(k-g_l)(1-k)^5}$$
(1.6)

Growth period Stable growth

The three-stage DDM looks as follows:

$$P_0 = \underbrace{\sum_{t=1}^{5} \frac{E_0[D_t]}{(1+k)^t}}_{t=1} + \underbrace{\sum_{t=6}^{20} \frac{E_0[D_t]}{(1+k)^t}}_{t=1} + \underbrace{\frac{E_0[D_{20}](1+g_l)}{(k-g_l)(1+k)^{20}}}_{t=1}$$
(1.7)

Growth period Transition period Stable growth

Both DDM versions assume an initial 5-year phase of rather high dividend growth. In the three-stage formula, this period is followed by a transition phase in which the growth rates decline linearly to a lower, stable growth rate g_l , which is then maintained ad infinitum. In the two-stage DDM of equation (1.6), this stable growth phase follows directly after the growth phase. Thus, these equations combine the plausible conjecture of a possibly strong growth in the first years with realistic growth rates in the long run.

In the initial phase, the dividend growth is usually assumed to equal the longterm consensus earnings growth rate g, obtained from equity analysts⁴. In the stable phase following year 5 and 20 respectively, the dividend growth rate usually equals the estimated long-term GDP growth of the economy (Cornell, 1999). Note that there are two different growth rates in this study. The rate g refers to the consensus forecast of the long-term earnings growth rate by analysts, and g_l refers to the long-term nominal GDP growth rate of the economy⁵.

⁴The findings of Elton et al. (1981) suggest that analysts' forecasts are a good surrogate for investor expectations.

⁵Usually, we have $g > g_l$, in line with the basic idea of the model. For some companies however, the long-term consensus earnings growth rate g lies below the expected GDP growth rate of the

Residual Income Models

Similar to the DDM, several versions of the unrestricted model of equation (1.3) can be used. A two-stage version has been proposed by Claus and Thomas (2001):

$$P_{0} = B_{0} + \underbrace{\sum_{t=1}^{5} \frac{E_{0}[E_{t}] - k(B_{t-1})}{(1+k)^{t}}}_{\text{Growth period}} + \underbrace{\frac{E_{0}[R_{5}](1+g_{l})}{(k-g_{l})(1+k)^{5}}}_{\text{Stable growth}}$$
(1.8)

Analogous to the DDM, it is also possible to formulate a three-stage RIM:

$$P_{0} = B_{0} + \underbrace{\sum_{t=1}^{5} \frac{E_{0}[E_{t}] - k(B_{t-1})}{(1+k)^{t}}}_{\text{Growth period}} + \underbrace{\sum_{t=6}^{20} \frac{E_{0}[E_{t}] - k(B_{t-1})}{(1+k)^{t}}}_{\text{Transition period}} + \underbrace{\frac{E_{0}[R_{20}](1+g_{l})}{(k-g_{l})(1+k)^{20}}}_{\text{Stable growth}}$$
(1.9)

The two-stage model assumes an initial phase of high earnings growth rates, followed by a stable growth of residual incomes after year five. Following the practice of the DDM, earnings are expected to increase with g in the growth phase. The long-term growth rate is again presumed to equal the nominal growth of the overall economy g_l . In the three-stage version, similar to the DDM, a transition phase where the earnings growth declines to g_l , is included. All main conclusions of this work are based on these four PV formulas. Although one could think of relying on a more comprehensive set of models, we believe that the presented formulas set a reasonable frame for the objective of this chapter: the evaluation of various techniques to estimate the implied ERP.

1.2.3 Assessment of the Models

In order to assess the empirical results of this study it is essential to have a closer look at the models and their underlying assumptions.

First, note that all formulas assume constant discount rate in the future. In the view of time-varying expected returns, this might not be an appropriate assumption. Obviously, it would be possible to extend the model to incorporate time-varying discount rates by splitting up the discount factor in its two parts, i.e. the risk premium and the risk-free rate. Then one could use the information as provided by the yield curve to construct time-varying discount rates. Claus and Thomas (2001) follow this

economy g_l .

approach. However, they do not find any sizeable difference compared to the constant discount rate assumption: The shape of the yield curve in the sample period is rather flat, so that the forward rates settle rather quickly at the long-term rate. Since the yield curve was equally rather flat at the time of this study, we discarded this approach. Next, when comparing both DDM formulas, observe that due to the transition phase, the three-stage version implies higher expected cash-flows than the two-stage model by definition (in the usual case where $g > g_l$). The rather smooth transition towards the long run growth rate is probably a more realistic assumption than the sudden change in the two-stage model. In the case of the RIM, the implications for expected returns when introducing a transition period are less clear, since they depend on the relation of earnings and residual income in year 20. In some cases, the decrease of earnings in the transition phase causes very low residual incomes in year 20, which consequently lead to lower terminal values than in the two-stage version. When comparing the implicit growth assumptions of all four models it is interesting to note that the two-stage RIM and the three stage DDM implement rather similar assumptions about the expected future return on equity⁶. Consequently, the implied cost of capital derived from equations (1.7) and (1.8) should be very similar.

Moreover, two drawbacks of employing the RIM to estimate the cost of capital should be mentioned. First, applying the growth rates g and g_l to different variables (earnings and residual incomes) causes discontinuities in implied earnings growth rates in both RIMs. Such jumps, especially in the three-stage RIM, are not very plausible. Second, RIM formulas produce confusing results if the book value of equity exceeds its market capitalization. In such a case, the residual income is negative by definition. By applying g_l to negative R_t , not only is all future residual income expected to remain negative, but these abnormal losses will even increase over time. Thus, to obtain meaningful results, the RIM requires not only positive book values and earnings, but as well a book-to-market ratio smaller than one.

To conclude this section, we see that both approaches to value the cost of equity have their pros and cons. Hence, we leave the final evaluation to the empirical part of this study.

⁶Both models are functionally very different and not mathematically equivalent to each other, as compared to the unrestricted equations in (1.1) and (1.3).

1.3 Data Description

1.3.1 Data for the Cost of Capital Calculation

In this study, we focus on companies that are members of major European stock markets indices: for the Eurozone, the Euro Stoxx and the Euro Stoxx-50 are used as surrogates for the market. In the U.K., the FTSE-100 is used as a market proxy⁷. All data is as of 18. March 2003.

Most of the data is taken from the Bloomberg database, such as current share prices, the companies' market capitalizations, last cash dividends, expected earnings and the book values of equity capital.

The data obtained from any database is usually not ready to be employed in empirical studies: dividend payout dates differ across companies, or some information on book values of equity is outdated by several months. Hence, adjustments are carried out in order to improve the consistency of the data (see similar issues in Lee and Swaminathan (1999) or Gebhardt et al. (2001)).

All presented DDM require the annual dividend D_0 , which has just been paid out to the shareholders. Based on D_0 , it is then possible to calculate the series of future payments, beginning with D_1 . In this study, D_0 is calculated as follows: Bloomberg reports the payout date of the last dividend and offers a function that provides the sum of all dividends paid out in the last 12 months. This aggregate is used as a proxy when a company pays semi-annual or quarterly dividends. To overcome the problem resulting from different payout dates, the obtained PV of each projected dividend stream is compounded up to the date of this study, depending on the months that have passed since the last payment. Expressed in mathematical terms: $D_0 = D_r * (1 + k)(m/12)$, where D_r is the last reported annual dividend paid out m months before the survey date. In the case of quarterly and semi-annual dividends, a fictional pay date between the actual pay dates is used⁸.

 $^{^{7}}$ Because of missing data, the data sample is reduced quite significantly. The resulting sample selection bias could be considerable. For example, only 226 companies out of 306 Euro Stoxx member firms are included in the study. However, these companies still represent about 85% of the Euro Stoxx's market capitalization.

⁸There is some controversy in the literature about how to construct the right D_0 or D_1 , see for example Harris and Marston (1992). Moreover, the treatment of dividend taxation can have a large impact on cost-of-capital estimates. Interestingly, important empirical studies such as Dimson et al. (2002) or Cornell (1999) do not analyze the distortions caused by fiscal redistribution. Siegel (2002, p. 58) is a notable exception, stating that "the difference between before- and after tax total returns is striking". Over 200 years, the return of equity investment after taxes attains only 1/20 of the return when abstracting from taxes. This study follows the standard approach of valuation in corporate finance, which uses cash dividends (Copeland et al., 2000). The cash dividend is the

Similarly, the construction of a meaningful B_0 imposes difficulties in RIM calculations. Similar to Gebhardt et al. (2001) for instance, this study captures the problem of outdated figures by creating first a synthetic book value that updates reported book values by one year using equation (1.5). Unreported earnings since the last financial report are obtained from analysts' forecasts. The payout ratio related to past year's earnings (p_0) - generally unknown at the time of the data capture - is assumed to converge towards 50% over time. This ratio has been the average payout over the last decades in the U.S. (Claus and Thomas, 2001, p. 1638). More formally: $p_0 = (p_{-1} + 0.5)/2$, where p_{-1} is the payout rate one year before. Payout ratios above 1 are set to 1 in the subsequent year, negative ratios to 0, in line with Gebhardt et al. (2001). Future book values are also constructed using equation (1.5). Future payout ratios are assumed to decline geometrically towards 50% over the years, using the same equation as above. Regarding expected earnings, only E_1 (i.e. the earnings of the first year) are directly estimated by analysts in this study. Earnings E_2 to E_5 are approximated by projecting the growth rate q on the earnings of the year before: $E_t = E_{t-1}(1+g).^9$

The consensus forecast of long-term earnings growth g is provided by First Call. It is the arithmetic average of the expected annual increase in operating earnings of the contributing sell-side analysts. Expected nominal long-term GDP growth rates g_l are regularly published by economic consultant firms. Consensus Economics Inc. (2002) provides predictions of the estimated real GDP growth and inflation rate for all major European countries over a ten-year horizon. To obtain a forecast for the European Monetary Union (EMU), for which no estimates are directly available by Consensus Economics, a GDP-weighted average of the EMU member countries is calculated.

The equity risk premium is estimated with respect to government bonds with a term of 30 years, since these securities match the usual long-term horizon of equity investments much better than short-term bills (Dimson et al., 2002, p. 169). The ERP for the EMU is calculated using German government bonds. The yield to maturity of these securities is also provided by Bloomberg.

If quoted in deviant currencies, all company-specific data is converted into the two basic currencies of the analysis, the British Pound (GBP) in the U.K. and the Euro in the EMU. The conversion is accomplished by using the exchange rates as of 18. March 2003. Table 1.1 summarizes the aggregated data for the cost-of-capital calculation.

payment of the company to its shareholders after all corporate taxes, but before any personal taxes or tax credits. For a detailed study on taxation and implied cost of capital, see Dhaliwal et al. (2005)

⁹Although analysts usually forecast earnings beyond year 1, we had not any access to this data. Claus and Thomas (2001) use the same approach to generate missing data in their study.

			Markets and Indices	α
		U.K. FTSE-100	Euro <i>E</i> uro <i>E</i> uro <i>E</i> uro Stoxx-50	Area Euro Stoxx
Yield on 30-year	Gvt. Securities	4.56%	4.82%	4.82%
Long-Term Nom	inal GDP Growth	4.6%	4.4%	4.4%
DDM	Number of Firms	85	48	228
Calculation	Market Cap. (bn)	869.2	$1,\!216.5$	1,996.8
	Dividends D_0 (bn)	33.5	47.8	70.7
	Growth Forecast g	9.2%	9.8%	11.0%
RIM	Number of Firms	80	45	223
Calculation	Market Cap. (bn)	759.7	$1,\!129.1$	1,907.6
	Book Values B_0 (bn)	374.2	739.6	1267.0
	Earnings E_1 (bn)	61.5	101.3	172.6
	Payout Ratio p_{-1}	65.3%	48.82%	44.43%
	Growth Forecast g	9.0%	9.8%	11.0%

Table 1.1: Summary Statistics

Source: Bloomberg, Consensus Economics Inc.

Annotations to table 1.1

In the first row of table 1.1, the yields to maturity on 30-year government securities are depicted. The next row gives the expected long-term nominal GDP growth rates as provided by Consensus Economic Inc. This expected long-term nominal GDP growth is the sum of the expected long-term inflation rate and the projected real GDP growth.

The aggregated raw data of both DDM and RIM calculations is shown in the middle and lower section of the table. For both models, first the number of companies included in the calculation and their combined market capitalization is reported. The third row of the DDM section presents the aggregated reported (unadjusted) cash dividends in the 12 months prior to 18/03/2003. The last row contains the value-weighted average of the consensus growth forecast of earnings. The third row of the RIM section displays the aggregated half-year adjusted book values of equity of the respective indices. The sum of forecasted earnings for year 1 (E_1) are presented in the next row, followed by prevailing payout ratios. Payout ratios are only calculated for companies with positive earnings, since for loss firms the ratio is meaningless. Finally, the value-weighted average of the consensus growth forecast is presented.

Out of the 228 Euro Stoxx companies included in the DDM calculation, 61 are of French origin, 44 are German, 29 Dutch, 27 Italian, 24 Spanish, and the remaining 43 are from other member states of the EMU. In terms of size, 57 companies had a market capitalization over 10 billion Euro, 153 had a market capitalization between 1 and 10 billion Euro, and 18 were valued less than 1 billion Euro. The composition of the firm sample for the RIM calculation does not differ much.

All amounts are in billions, except for payout ratios, growth rates, and number of firms. In the EMU, the base currency is Euro, whereas in the U.K., all figures are expressed in GBP.

For each company, the cost of capital k is calculated by applying the equations (1.6) to (1.9) to the data. Firms with an incomplete data set, i.e. one or more missing input variables, have been ignored¹⁰. The solution of the equations is straightforward. Since they are monotone in k, they can be solved easily by iteration.

1.3.2 Data for Regression Tests

The additional data used in the cross-sectional regression tests of the implied cost of capital is presented in the next subsections. Following Lee et al. (2003), these include a measure of the historical systematic risk (market beta), the volatility of historical stock returns to account for total risk, and specific fundamental firm characteristics that have been identified as risk factors by empirical studies. Since the regressions are only carried out for the companies of the Euro Stoxx, the data has been collected for the relevant firms only.

Betas

Despite the international context, this study refrains from employing an international capital asset pricing model with separate world and local betas, as proposed by Bodnar

 $^{^{10}\}mathrm{This}$ applies also to companies which did not pay any dividends in the 12 month prior to the date of this study.

et al. (2003). Instead, a single beta factor CAPM has been chosen. The increasingly integrated capital market of the EMU suggest this step. This approach is in line with Stulz (1999), who argues that in sufficient integrated markets, there would be a tendency toward a "global CAPM". In such a setting, the covariance with the return of a European market portfolio should be the only priced risk factor. This gives following relation of systematic risk:

$$r_{it} - r_{ft} = \alpha_i + \beta_i (r_{mt} - r_{ft}) + \varepsilon_{it} \qquad t = 1, 2, \dots, T \qquad \forall i \tag{1.10}$$

where

r_{it}	=	monthly stock return of company i at time t ,
r_{ft}	=	monthly return on the risk-free asset at time t ,
α_i	=	intercept of company i ,
β_i	=	beta of company i ,
r_{mt}	=	monthly return on the market portfolio at time t ,
ε_{it}	=	error disturbance of company i at time t .

The Euro Stoxx index has been chosen as surrogate for the market portfolio. Again, the return on 30-year German government bonds is used as a proxy for a European risk-free asset¹¹. The factor model of equation (1.10) has been estimated for each company over the 60 months prior to the date of this study. The data for these regressions is taken from Datastream.

Volatility

As an additional measure of total risk, this study includes the standard deviation of monthly stock returns over the last 60 months.

Firm Characteristics

The use of specific firm characteristics as explanatory variables for the expected cost of capital has been motivated by many different empirical studies. The firm size effect has been first detected by Banz (1981). Basu (1983) presented evidence on the

¹¹In the literature, many different government securities are used to calculate excess returns. Some studies rely on short-term bills (Lee et al., 2003), others use gross returns (Fama and French, 1992). To be consistent with the implied risk premia, that are calculated with respect to long-term bonds, we opted for excess returns over long-term bonds in the beta regressions. However, the results are generally not much affected by the chosen risk-free rate, see also Grinblatt and Titman (2001). The beta estimates have a mean of 0.867, and a median of 0.829. The min of the betas is 0.250, their max is 1.685. The beta regressions have an average R^2 of 35,6% and were carried out for 224 companies.

relation of book to market value of equity and stock returns. To reduce the impact of outliers, both market capitalization and book-to-market ratio have been transformed into natural logarithms, similar to the work of Lee et al. (2003). In addition, two other characteristic variables have been included: The dividend yield and the price-earnings ratio (P/E ratio). The dividend yield, i.e. last cash dividend divided by share price, and the price-earnings ratio (calculated on the basis of next year's expected earnings) are often used as indicators for simple fundamental share price analysis. Again, the logarithm of the P/E ratio has been used in the regression analysis instead of the actual ratio in order to avoid the impact of outliers.

1.3.3 Data for Return Forecast Regressions

Historical share prices for calculating the stock returns in the 12 months following the estimation of expected returns are also taken from Datastream.

1.4 The Implied Equity Risk Premium

The equity risk premium is calculated directly from the cost-of-capital estimates of individual firms. First, the yield on government bonds is deducted to obtain the required excess return of each firm. These projected excess returns are then weighted with the companies' current market capitalization to obtain the market risk premium.

Table 1.2 summarizes the estimated implied equity premia for different European markets. The results from the two-stage DDM described by equation (1.6), and the three-stage DDM of equation (1.7) are displayed in panel A of the table. Standard errors of the weighted mean estimators are given in parenthesis¹². The results for the two-stage DDM lie at around 5%. Not surprisingly, the inclusion of a transition phase in equation (1.7) increases the estimates slightly to 6.3%.

In panel B of table 1.2, the results of the RIM analysis are presented. The estimated premia derived from the two-stage RIM (equation 1.8) following Claus and Thomas (2001) lie between 6.5% in the U.K. and 7.2% for the broad Euro Stoxx index. When calculating the ERP using the three-stage RIM of equation (1.9), the results for the Eurozone are roughly 50 basis points higher. In the U.K. however, the estimates decrease when a transition phase is included in the model. Low earnings at the end of

¹²The standard errors are calculated as the square root of the weighted variance of the expected excess returns of each company. The formula for the weighted variance is: $s^2 = \frac{n}{n-1} \sum_{i=1}^{n} w_i (e_i - erp)^2$ where e_i is the estimated excess return of company i, erp is the ERP of the index (the weighted average), n is the number of firms included in the study, s^2 is the weighted variance of the ERP and w_i is the weight of company i of the total market capitalization.

this phase cause very low residual incomes in year 20 (R_{20}) , which consequently lead to low terminal values.

The risk premium estimates present some evidence that the three-stage DDM (equation 1.7) and the two-stage RIM (equation 1.8) lead to similar results, as hypothesized in section II 3. Especially in the U.K., both estimates deviate by a small amount only. In the Euro area, the difference is somewhat larger, with the two-stage RIM yielding an estimate that is around 70 basis points higher compared to the three-stage DDM. Still, the estimates of both PV formulas lead to estimates in the fairly small range from 6.3% and 7.2%.

This rather close association between the two different approaches can also be found at the individual firm-level data. Table 1.3 presents the correlations of the estimated company-specific cost-of-capital estimates obtained from the different valuation models. As expected, the correlations between structurally similar models, i.e. within one of the two classes of models, are well above 90%. But the correlation between threestage DDM and two-stage RIM estimates is rather high as well, attaining 0.73 in the U.K. and 0.60 in the Euro area.

Note that the standard errors of the estimates are rather large, resulting in large confidence intervals for the point estimates. This is a common problem of implied ERP studies, since the variation of the individual implied cost of capital for the individual companies is usually large¹³. Moreover, it should be mentioned that the estimated risk premia lie above the long-year averages of the implied ERP of similar studies which are at around 3% (e.g. Claus and Thomas (2001) or Gebhardt et al. (2001)). This fact can be explained by the timing of this study. According to Siegel (2002, p. 124), rising terrorism and the economic downturn at the beginning of this century have increased the overall uncertainty of the business environment. He concludes that this rising level of uncertainty has led to a surge in the equity premium.

1.5 Analysis of Company-specific Implied Cost of Capital Estimates

After the quantitative comparison of different models to estimate an implied market risk premium, this part aims to detect qualitative differences between the models by investigating the underlying company-specific implied cost-of-capital estimates. The

¹³Since the deletion of outliers would reduce the sample size significantly in terms of the represented market capitalization, a large variation seemed to be the lesser evil. Most other studies do not report standard errors or t-statistics of risk premium estimates.
RIM
and
DDM
from
Obtained
Premium
Risk
Equity
Implied
Table 1.2 :

			Countries and Indices	
	Method	United Kingdom FTSE-100	Euro Euro Stoxx-50	Area Euro Stoxx
	Observations	84	48	228
A	2-stage DDM from equation (1.6)	$5.21\%\(2.45\%)$	5.08% (3.62%)	4.83% (3.69%)
	3-stage DDM from equation (1.7)	6.31% $(3.22%)$	6.35% (4.20%)	6.43% (5.46%)
	Observations	80	45	223
В	2-stage RIM from equation (1.8)	$6.46\%\ (2.64\%)$	6.78% (3.10%)	7.19% (4.05%)
	3-stage RIM from equation (1.9)	6.41% (3.09%)	7.05% (3.47%)	7.75% (5.23%)

Note: Standard errors are reported in parenthesis below the estimate. Economics Inc.

Table 1.3: Correlations of Cost-of-Capital Estimates

This table reports the correlations of the estimated company-specific cost-of-capital estimates across the different valuation models. Panel A reports the correlation estimates for the United Kingdom, panel B contains the correlation coefficients for the Euro Area.

	i anci ii. On	iiiea iiiigaoiii (i	151 100)	
	2-stage DDM	3-stage DDM	2-stage RIM	3-stage RIM
2-stage DDM	1.000	0.923	0.609	0.598
3-stage DDM		1.000	0.726	0.803
2-stage RIM			1.000	0.933
3-stage RIM				1.000
Observations				80

Panel A: United Kingdom (FTSE-100)

	Panel B:	Euro Area (Euro	o Stoxx)	
	2-stage DDM	3-stage DDM	2-stage RIM	3-stage RIM
2-stage DDM	1.000	0.923	0.455	0.469
3-stage DDM		1.000	0.605	0.709
2-stage RIM			1.000	0.921
3-stage RIM				1.000
Observations				223

Panel B: Euro Area (Euro Stoxx)

relatively small data set of the Euro Stoxx-50 and FTSE-100 are the reason why we focus in the remainder of the study on the rather broad Euro Stoxx index.

1.5.1 Cross-Sectional Regression Tests

This section analyzes empirically the ability of betas and firm characteristics to explain the cross-sectional variation of the European implied risk premium on the firm level. Since the implied return is essentially an expected return estimate, its magnitude should be related to common risk measures and firm characteristics, such as the market beta of the CAPM, or B/M ratio and firm size that have been identified as risk factors by Fama and French (1992, 1993).

Whereas other studies only examine the implied risk premia for firms obtained from the residual income approach, this work also analyzes the implied risk premia calculated with the help of the DDM formula. Hence, this study is the first to draw comparisons between the determinants of the implied risk premium of both models.

The Regression Setup

The relation between implied risk premia (i.e. the difference between cost of capital and the risk-free rate), betas and firm characteristics is examined using a crosssectional regression across all companies:

$$k_i - r_f = \gamma_0 + \gamma_1 \beta_i + \sum_{j=1}^J \delta_j C_{ij} + u_i \qquad i = 1, 2, ..., N$$
(1.11)

where $k_i - r_f$ is the implied risk premium estimate of firm i, β_i is its market beta estimate, C_{ij} are the characteristics j for firm i, and γ_1 and δ_j are the respective slope coefficients.

The betas that enter the cross-sectional regression (1.11) are however not the true betas, but only noisy estimates thereof, obtained from the times series regressions as displayed in (1.10). This causes an errors-in-variables (EIV) problem, leading to biased coefficients and standard errors. To correct for this bias, we employ the standard EIV regression approach as presented e.g. by Fuller (1987) or Greene (2002) by applying the so-called reliability ratio for the beta estimates. For a detailed exposition of the procedure, please see the appendix A.

The implied expected returns are regressed on the most recent available data of firm characteristics. Such a specification raises the question about spurious correlation between the dependent variable and the firm characteristics such as the book value of equity, since the latter are used to calculate the implied cost of capital. To deal with this potential problem, we employ only those firm characteristics that are not contributing to the dependent variable as regressors. The advantage of this procedure is that it allows to detect the (almost) instantaneous relation between firm risk premia and firm characteristics without any time lag¹⁴. This compares to related studies (Gebhardt et al., 2001; Lee et al., 2003), that handle this issue by introducing a one-year gap between the date of implied risk premium and firm characteristics in the regression equations¹⁵. Consequently, they examine the relationship between the expected cost of capital and prior year's fundamentals only. The sometimes sudden changes of expectations in the financial markets due to new information of the fundamental situation of the company cannot be captured in such a setting.

The cross-sectional regressions are estimated using three different specifications of the model displayed in equation (1.11). In the simplest model (S1), the risk premia are regressed on the betas only. The next specification (S2) adds the historical standard deviation of monthly returns and specific firm characteristics that are not used to calculate the risk premia to the regressors. More precisely, the DDM estimates are additionally regressed on the Fama-French risk factors size (lnMC, the log of the market capitalization), and the B/M ratio (lnBM) as well as the P/E ratio (lnPE). In turn, the RIM estimates are regressed on firm size and dividend yield (Yld)¹⁶. Since total risk should not be a priced risk factor according to any theory, finally specification (S3) omits this variable from the regressors.

Table 1.4 shows the correlations of the different risk measures and firm characteristics employed in the regression tests to explain the company-specific implied risk premia. Besides the correlation between beta and volatility whose strong relation is no surprise, we see that the P/E ratio is negatively related to both B/M ratio and dividend yield. The usual strong negative relation between B/M ratio and firm size is less pronounced in our data sample.

The empirical study of Fama and French (1992), based on average realized returns

¹⁴Still, this regression specification is not unproblematic, since the prevailing share price enters both into the formulas for the implied return and the various multiples, such as the B/M ratio or the P/E ratio. Strictly speaking, this procedure might cause spurious regression results. Nevertheless, we stick to this methodology, particularly to maintain the comparability with the works of Gebhardt et al. (2001) or Lee et al. (2003).

¹⁵Another reason put forward in other studies for introducing a one-year gap are possible publication lags, that is to ensure that the regressions are based on publicly available information only. Since this study relies on the most recent published data, this issue does not pose a problem.

¹⁶Using different explanatory variables to explain RIM and DDM based firm risk premium estimates makes it obviously more difficult to compare the regression results. Still, for the reasons mentioned above, we opted for this procedure.

	Beta (β)	Volatility	Size	P/E	B/M	D. yield
Beta (β) Return volatility Size $(lnMC)$ P/E ratio $(lnPE)$ B/M ratio $(lnBM)$ Dividend yield	1.000	0.800 1.000	0.032 -0.133 1.000	$\begin{array}{c} 0.145 \\ 0.059 \\ 0.216 \\ 1.000 \end{array}$	$\begin{array}{c} 0.033\\ 0.063\\ -0.199\\ -0.566\\ 1.000\end{array}$	$\begin{array}{c} 0.039\\ 0.177\\ -0.170\\ -0.445\\ 0.441\\ 1.000 \end{array}$
Observations						218

 Table 1.4:
 Correlations of Firm Characteristics

This table reports the correlations of firm characteristics and risk variables used as explanatory variables for the implied return. Data sample: EuroStoxx index.

presents evidence of a positive relation between cost of capital and B/M ratio, and a negative relation with firm size. The study of Gebhardt et al. (2001), analyzing the relation between implied cost of capital and firm characteristics confirms a positive relationship with B/M ratio, but a rather weak relation to firm size. Regarding the other firm characteristics, Dhaliwal et al. (2005) detect a positive relation between the implied cost of equity and the dividend yield, and Easton (2004) findings suggest a negative relation between the implied cost of equity and the P/E ratio. The study of Gebhardt et al. (2001) also detects a positive correlation between volatility and expected stock returns.

Individual Firm Regressions with Firm Betas

Table 1.5 presents the EIV estimation results when regressing individual firm risk premia on individual firm betas and individual firm characteristics. In the pure beta specification (S1) following the CAPM, only the beta coefficients in the DDM3 regression are significantly related to firm risk¹⁷. The R^2 of this regression is however very low. After controlling in addition for return volatility and other firm characteristics (S2), neither beta nor volatility is significant. This contrasts to the firm fundamentals, which exhibit significant effects on the risk premia. The P/E ratio is significantly negatively related to the implied risk premia, the B/M ratio has a positive relationship (DDM2), and the dividend yield is positively related to firm risk (RIM2). In the

 $^{^{17}\}mathrm{To}$ simplify notation, two-stage DDM is abbreviated by DDM2, three-stage DDM by DDM3, etc.

regression of the DDM2 risk premia, R^2 attains 28%. When omitting return volatility (S3), the beta coefficients of the DDM regressions are (again) significant. Firm size is not significant in any specification¹⁸.

When looking at the DDM-results, these findings provide a mixed picture in view of standard asset pricing theory: On the on hand, the mainly positive beta coefficient is in line with the CAPM. Moreover, the Fama-French risk factor B/M ratio is positively priced as well. On the other hand, firm size is not significant related to size (in contrast to the three-factor model), but P/E ratio is instead a priced risk factor. However, other studies as cited above detect similar relationships. The RIM analysis is disappointing from the point of view of betas and firm size. Moreover, the F-stat rejects the hypothesis of all variables being jointly significant at the 5% level in many RIM specifications. The strong explanatory power of the dividend yield in RIM2 confirms the findings of Dhaliwal et al. (2005). The rather poor performance of the standard regression tests for expected returns raises the question what variables determine the implied risk premium calculated from the RIM approach. Although a final answer cannot be given here, these findings suggest at least that the cost of capital obtained from the DDM method proves to be more in accordance with the CAPM or the Fama-French model.

Individual Firm Regressions With Country Betas

To further reduce the impact of noisy beta estimates, the regressions are also carried out using country betas. These country betas are calculated as the arithmetic average of the companies' betas belonging to the same out of the eleven countries in this study¹⁹. The results are reported in table 1.6.

Now, all DDM regressions indicate a positive relation between beta and firm risk premia. Moreover, the coefficient is in many cases even highly significant. Return volatility also contributes to explain the risk premia (S2). As far as other firm characteristics are concerned, P/E ratio is significantly related to firm risk. This contrasts to the RIM regressions, where beta is now negatively related to the expected implied return (RIM2). Size and B/M ratio are not related to expected returns in almost any

¹⁸Note the regression results do not change substantially when using the levels of the explanatory variables (i.e. firm size, B/M ratio, and P/E ratio) instead of using their logs. Only the B/M ratio gets (more) significant; the overall explained variance (R^2) however decreases. This applies to both the regression using firm betas as well as the subsequent regressions with country betas.

¹⁹Lee et al. (2003) carry out similar regressions using industry-country portfolios. The usual Fama and MacBeth (1973) approach of performing portfolio regressions when sorting all companies into 20 portfolios according to their beta coefficients did not yield any meaningful results.

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		\mathbf{x}	1			S2				S3		
	DDM2	DDM3	RIM2	RIM3	DDM2	DDM3	RIM2	RIM3	DDM2	DDM3	RIM2	RIM3
Intercept	0.03^{**} (2.51)	0.03^{*} (1.90)	0.08^{***} (5.53)	0.06^{***} (3.88)	0.06 (0.56)	$0.20 \\ (1.44)$	0.08 (0.87)	0.20^{*} (1.66)	0.17^{***} (2.81)	0.19^{**} (2.30)	0.15^{**} (2.27)	0.16^{**} (2.08)
Beta (β)	0.02 (1.52)	0.04^{**} (2.21)	0.00 (0.22)	0.03 (1.42)	-0.08 (-0.92)	0.07 (0.52)	-0.08 (-0.82)	0.06 (0.48)	$\begin{array}{c} 0.03^{**} \\ (2.35) \end{array}$	0.06^{***} (2.99)	0.00 (0.12)	$\begin{array}{c} 0.03 \\ (1.40) \end{array}$
Return volatility					0.99 (1.40)	-0.07 (-0.07)	$0.72 \\ (0.91)$	-0.29 (-0.30)				
Size					0.00 (0.12)	-0.00 (-0.50)	-0.00 (-0.21)	-0.01 (-1.16)	(00.0-)	-0.00 (-0.69)	-0.00 (-1.37)	-0.01 (-1.45)
P/E ratio					-0.03^{***} (-2.88)	-0.05^{***} (-3.30)			-0.04^{***} (-4.46)	-0.05^{***} (-4.13)		
B/M ratio					0.02^{***} (2.74)	0.01 (1.20)			0.01^{**} (2.44)	0.01 (1.38)		
Dividend yield							0.31^{**} (2.27)	0.28^{*} (1.68)			0.39^{***} (3.67)	0.25^{*} (1.92)
R^{2}	0.01	0.03	0.00	0.01	0.28	0.19	0.09	0.05	0.24	0.19	0.08	0.04
F-stat	2.30	4.89	0.05	2.02	15.40	9.76	4.77	2.30	16.48	11.66	5.88	3.02
u	222	222	218	218	218	218	218	218	218	218	218	218

Note: t-statistics are reported in parenthesis below the estimate. Reliability for $\beta\colon$ 0.728

* * *

- significant at the 1% level significant at the 5% level significant at the 10% level || || ||
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regression.

Regarding the DDMs, this regression approach seems to fit the data better than the previous specification (R^2 increases slightly to 30% in S2, F-stat are higher). However, return volatility is, in contrast to the CAPM, a priced risk variable. In this framework, it is interesting to note that both Fama-French factors, size and B/M ratio, are only weakly related to the implied returns. This rather unusual result might be due to the timing of the study, since share prices - close to their record lows - deviated from their usual pricing pattern. In the RIM specifications, the detected negative relationship between beta and firm risk is very clearly opposed to any theory. Together with relatively low R^2 , this finding indicates the poor explanatory power of common risk factors for the cost of capital obtained from the RIM methodology.

1.5.2 Return Prediction

In this section, we finally test the ability of the implied cost of capital to predict actual stock returns. In the regression setup, subsequent returns over 1 to 4 quarters (q) are regressed on the expected returns k_i as calculated in previous sections²⁰. The regression equation looks as follows:

$$\frac{4}{q}r_{i,q} = a_0 + a_1k_i + \varepsilon_i \tag{1.12}$$

where $r_{i,q}$ is the return of company *i* over the quarters 1 to *q*, k_i is the estimated cost of capital of firm *i* using the different DCF formulas. Note that if the estimates were perfect forecasts of stock returns and assuming constant risk premia and risk-free rates, the intercept a_0 should be zero, and the coefficient a_1 should equal 1. Again, this analysis is based on all Euro Stoxx companies with a complete data set²¹.

Table 1.7 presents the forecasting regression results. There are two main conclusions one can draw from the estimation outcome. First, the regressions present evidence that the implied cost of capital has indeed a predictive power for future stock returns. The R^2 which attain up to 21% indicate that a considerable part of the total variation of actual stock returns can be explained by the implied cost of capital, although the interrelation weakens over time. The slope coefficients in almost all regression specifications are significantly positive²². Second, the dividend discount

²⁰Lee and Swaminathan (1999) carry out similar regressions. However, they take the cost of capital as given and examine the ability of value to price ratios to explain stock returns.

 $^{^{21}}$ Compared to previous regressions, the sample size is reduced by several companies since not all firms existed 12 months after the data used for the cost of capital estimation.

 $^{^{22}}$ In many regressions, the slope coefficients are significantly higher than one (as suggested), reaching up to 7.88 in the regression of the Q1 return on the DDM2-cost of capital. In addition, the

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Intercept -0 .	M2 DDM:	S1 3 RIM2	RIM3	DDM2	S2 DDM3	RIM2	RIM3	DDM2	S3 DDM3	RIM2	RIM3
	03 -0.04 78) (-1.02	$\begin{array}{c} 0.12^{***} \\ (3.78) \end{array}$	0.09^{**} (2.33)	0.09 (1.49)	0.09 (1.06)	0.17^{**} (2.47)	$\begin{array}{c} 0.17^{*} \\ (1.96) \end{array}$	0.13^{**} (2.04)	$0.13 \\ (1.54)$	0.19^{***} (2.75)	0.19^{**} (2.26)
Beta (β) 0.0(2.4)	$)^{**}$ 0.13** (2.70) (2.70)	* -0.05 (-1.42)	0.01 (-0.19)	0.11^{***} (2.83)	0.16^{***} (3.05)	-0.08^{**} (-1.98)	-0.03 (-0.60)	0.13^{***} (3.51)	0.18^{***} (3.67)	-0.07^{*} (-1.76)	-0.01 (-0.27)
Return volatility				0.26^{**} (2.67)	0.31^{**} (2.31)	0.14 (1.32)	0.20 (1.50)				
Size				-0.00 (-0.88)	-0.00 (-0.73)	-0.00 (-0.84)	-0.00 (-1.06)	-0.00 (-1.35)	-0.00 (-1.14)	-0.00 (-1.05)	-0.00 (-1.30)
P/E ratio				-0.04^{***} (-5.24)	-0.05^{***} (-4.73)			-0.04^{***} (-5.02)	-0.05^{***} (-4.55)		
B/M ratio				0.01 (1.64)	(0.00) (0.65)			0.01^{*} (1.70)	0.00 (0.71)		
Dividend yield						0.40^{***} (3.80)	$\begin{array}{c} 0.24^{*} \\ (1.83) \end{array}$			0.42^{***} (3.97)	0.26^{**} (2.02)
R^2 0.0^{ι}	l 0.01	0.01	0.00	0.30	0.23	0.10	0.04	0.27	0.21	0.09	0.03
F-stat 5.90) 7.27	2.01	0.04	17.32	12.12	5.63	2.32	18.83	13.08	7.03	2.36
n 222	222	218	218	218	218	218	218	218	218	218	218

Note: t-statistics are reported in parenthesis below the estimate. Reliability for $\beta \colon$ 0.728

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- significant at the 1% level significant at the 5% level significant at the 10% level || || ||
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models seem to perform better in predicting future stock returns than the residual income models. Expected returns from both DDMs can explain more than twice of the variation in actual returns compared to the estimates from the RIM. Moreover, the cost of capital estimated from the popular RIM2 equation has no explanatory power for stock returns over more than two quarters, with the coefficient not being significantly different from zero. In these regressions, R^2 declines down to 1%. However, one must notice that the DDM2 is likely to underestimate the overall stock returns, with the slope coefficient being almost twice as high as the other models.

The better performance of dividend discount models to predict future stock returns can be explained by its informational advantage. Dividend policy seems to be a signalling process that conveys information on future profits (e.g. Nissim and Ziv (2001)), that appears to be crucial for accurately estimating the implied cost of capital. Very clearly, the RIM cannot capture this additional information included in dividend payments.

1.6 Conclusion

Because of the lack of alternative methods, Freeman and Davidson (1999) concluded only a few years ago that "the [traditional] excess return approach will continue to be the favored method for estimating the equity premium". With the development of forward-looking models to estimate the implied risk premium, the situation has changed discernibly in the past few years. Today there is a variety of possibilities to estimate a meaningful prospective ERP.

In contrast most other empirical works who rarely investigate the plausibility of their models to estimate the implied ERP, this study carries out an analysis of several common formulas currently used and applied them to a pan-European sample. We show that the market risk premium obtained from a two-stage RIM and a three-stage DDM are rather similar and deviate by a small amount only. The subsequent crosssectional analysis on the underlying firm-specific risk premia however detected some qualitative differences between both approaches. Surprisingly, the individual firm risk premia obtained from the RIM cannot be explained by common asset pricing models. In contrast, firm characteristics and betas explain up to 30% of the variation of the DDM risk premia. In line with the CAPM, beta is positively related to firm risk

intercept is in most regressions significantly different from zero, except for the regressions over one single quarter. These high estimates can be explained by the extraordinary recovery of share prices following the record-lows in mid-March 2003. This is of course an indication that the risk premium is *not* constant.

DCF Formula	2-stage DDM	3-stage DDM	2-stage RIM	3-stage RIM
1Q				
Intercept	-0.01 (-0.06)	$\begin{array}{c} 0.15 \\ (0.77) \end{array}$	$0.18 \\ (0.65)$	0.23 (1.21)
Expected Return k_i	7.88^{***} (3.23)	5.31^{***} (2.93)	4.73^{**} (2.09)	4.05^{***} (2.64)
R^2	0.21	0.17	0.08	0.08
2Q				
Intercept	$\begin{array}{c} 0.10 \\ (0.73) \end{array}$	0.23^{*} (1.86)	$\begin{array}{c} 0.29 \\ (1.59) \end{array}$	0.31^{**} (2.44)
Expected Return k_i	5.18^{***} (3.43)	3.32^{***} (2.90)	2.61^{*} (1.81)	2.30^{**} (2.38)
R^2	0.16	0.12	0.05	0.05
3Q				
Intercept	$\begin{array}{c} 0.14 \\ (1.52) \end{array}$	0.20^{***} (2.78)	0.34^{***} (2.96)	0.29^{***} (3.83)
Expected Return k_i	3.17^{***} (3.23)	2.16^{***} (3.30)	0.83 (0.94)	1.21^{**} (2.28)
R^2	0.10	0.08	0.01	0.02
4Q				
Intercept	$\begin{array}{c} 0.13 \\ (1.54) \end{array}$	0.18^{***} (2.64)	0.24^{**} (2.58)	0.23^{***} (3.45)
Expected Return k_i	2.66^{***} (3.16)	1.77^{***} (2.89)	1.10 (1.46)	1.13^{**} (2.23)
R^2	0.10	0.08	0.02	0.03
Observations (n)	216	216	211	211

Table 1.7: Forecasting Regressions

Note: White (1980) heteroscedasticity-consistent t-statistics are reported in parenthesis below the estimate.

*** = significant at the 1% level

** = significant at the 5% level

* = significant at the 10% level

premia in most regressions. In terms of firm characteristics, P/E ratio and, to a lesser extent, the B/M ratio, contribute to the explanation of implied firm risk premia. The Fama-French factor size is not relevant for expected firm risk. Taken together, the presented evidence casts doubt on the CAPM's ability to explain cross-sectional differences in expected stock returns. Whether such a conformity with asset pricing models is crucial for predicting actual stock returns is an empirical question. Such forecasting regressions are carried out in the last section of this chapter. It is shown that DDMs perform better in predicting future stock returns than RIMs. This result can be explained by the signalling nature of dividend payments for future earnings - an important information which the residual income model cannot make use of. Although this study reflects only the market conditions and expectations as of March 2003, the findings suggest that multistage DDMs are preferable models to estimate the implied cost of capital.

The recently developed concept of the implied equity risk premium offers a powerful tool to investors for estimating the future cost of capital. Since it is completely forward-looking, it avoids the problems related to employing historical data for future use. The practical implications of this study are straightforward: First, this work demonstrates that the selection of appropriate PV models is crucial to ensure the reliability of this instrument, given the partly large differences across the analyzed approaches. Since all models have their advantages, a sound analysis of the implied risk premium should at minimum include DDM-based approaches as well. Second, the results of other empirical studies on the implied cost of capital relying on the RIM only should be interpreted with caution. The so-obtained findings may only hold for RIM based cost-of-capital estimates, but not for the implied cost-of-capital concept in general.

Chapter 2

Implied Cost of Capital Based Investment Strategies – Evidence From International Stock Markets

This chapter demonstrates that investors can generate excess returns by implementing trading strategies that are based on publicly available analysts' forecasts. To capture expectations of equity analysts, we employ the so-called implied cost of capital (ICOC). Calculated as the internal rate of return that equates share price with discounted forecasted cash-flows, the ICOC allows condensing a variety of analysts' expectations about the future of any company into one single figure. Our analysis across the world's largest stock markets shows that a simple portfolio strategy yields significant excess returns with respect to several common asset pricing models.

2.1 Introduction

Security analysis plays an important role in the investment and fund management industry. Research departments of brokerage firms and investment banks put a lot of effort into equity analysis, hoping to achieve superior returns for their clients. Fund management firms, in turn, use - to a certain extent even explicitly - information on stocks provided by such sell-side analysts to optimize their asset allocation decisions.

Yet, from a theoretical perspective, such investment strategies should not pay off. The semi-strong version of the efficient market hypothesis states that publicly available information about expected future dividends, earnings, or growth prospects of any company should already be included in current market prices. Consequently, it should not be possible to implement any profitable investment strategy that relies on equity analysts' recommendations about the companies' expected outlook. This apparent conflict between financial theory and common business practice is the basic motivation for our study.

The objective of this chapter is to analyze empirically whether investors can generate excess returns by implementing trading strategies that are based on analysts' forecasts. In this chapter, we employ various valuation models to systematically connect analysts' forecasts to share prices. More precisely, we use the so-called implied cost of capital (ICOC) - also denoted implied return hereafter - as key variable to capture expectations of equity analysts. By relying on the ICOC, our work distinguishes itself from many related empirical studies to be briefly reviewed below. Calculated as the internal rate of return that equates share price with discounted forecasted cashflows, the ICOC allows condensing a variety of analysts' expectations about the future of any company into one single figure.

If markets are not fully efficient and thus only react sluggishly to announcements of equity analysts, then there may exist profitable investment strategies based on the companies' ICOC estimate. The underlying assumption of the trading strategy we examine is that discounted future cash flows obtained from equity analysts' provide information about a share's true value. Since the ICOC equates current market prices to the share's so-obtained fundamental value, a high implied discount rate is an indication that - holding risk characteristics fixed - a share is likely to trade below its fair value. Accordingly, if share prices converge to their true value over time, a portfolio of high-ICOC companies should outperform an investment in low-ICOC stocks.

Researchers have identified many different patterns in average stock returns that cannot be explained by the fundamental capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), or the empirically motivated three-factor model by Fama and French (1992, 1993, 1996). Prominent examples of such pricing anomalies are short-term price momentum (Jegadeesh and Titman, 1993), or the visible slow adjustment of stock prices to earnings announcements (Chan et al., 1996). First evidence that there exist profitable trading strategies based on analysts' forecasts has been put forward by the works of Womack (1996), or Barber et al. (2001), among others. Such studies concentrate either on rather simple buy- or sell recommendations (Barber et al., 2001), explicit next-year earnings forecast (Xu, 2002), or changes in these forecasts (Womack, 1996).

By using the firm's ICOC estimate to capture analysts' forecasts, our approach offers some advantages over existing studies. Since the ICOC approach is derived from fundamental valuation analysis, it enables us to include short-term earnings forecasts, mid-term growth projections, as well as long-term expectations in one single figure. Therefore, it avoids the rather short-sighted perspective of comparable measures of

2.1. INTRODUCTION

analysts' expectations. In fact, the ICOC as indicator for market mispricings is applicable over longer time horizons and thus more attractive for real-world fund managers who have to care about transaction costs¹. Finally, the implied return is essentially an implied discount factor which has a direct and intuitive interpretation as proxy for the shareholder's expected rate of return.

Under the assumption that market prices equal fundamental value as predicted by analysts' forecasts and that these projections are a good surrogate for the average investor's expectation, the ICOC's interpretation as proxy for expected stock returns is indeed valid. The primary advantage of this method to estimate expected stock returns is the ICOC's outright forward-looking perspective, since it is only relying on available estimates regarding the companies' future prospects and the current share price. Thereby this approach avoids all the problems related to using historical stock returns as proxy for expected returns.

Presumably because of this advantage, the ICOC got increasingly popular for estimating expected stock returns in the finance literature. On the one hand for example, Cornell (1999), Claus and Thomas (2001), and Gebhardt et al. (2001) use the ICOC to estimate an equity risk premium by aggregating the implied returns over entire stock markets. On the other hand, Pástor et al. (2006) rely on the implied return as a proxy for expected returns to examine the conditional mean-variance relation of individual stock returns. A primary study on the implied cost of capital methodology itself and its economic foundations is provided by Lee et al. (2003) who analyze the determinants of the ICOC at the firm level. A first application of the ICOC concept to active portfolio management is by Stotz (2005). This literature contrasts sharply to studies that challenge the ICOC's ability to be a proxy for expected stock returns. Although Gode and Mohanram (2003) document a positive relation between ICOC and stock returns, the findings of Botosan and Plumee (2005), Guay et al. (2005), or Easton and Monahan (2005) suggest indeed that the ICOC is a very poor estimate for expected stock returns.

By showing that the ICOC has the characteristic of a market anomaly with respect to both the CAPM and the Fama-French model, we also contribute to the literature that examines the relation between ICOC and expected returns. If one is to accept the Fama-French model, our findings suggest that markets are not efficient, implying that the ICOC cannot be an unbiased expected return proxy. Or put differently, the implied return can only be regarded as an expected return estimate, if one rejects the

¹The short-term horizon of the investment strategy proposed by Barber et al. (2001) requires high turnover rates. Therefore, when including transaction costs, this strategy is no longer profitable.

validity of the Fama-French model.

In addition, we extend the above studies in several directions. First, we use a much broader international set of data by investigating the ICOC concept across the stock markets of all G7 countries. Second, by employing residual income and dividend discount models to derive the ICOC we achieve more robust results and extend first comparisons of these commonly used ICOC concepts (Botosan and Plumee, 2005; Easton and Monahan, 2005). In addition to existing valuation models, we also present a modified version of the residual income model as proposed by Gebhardt et al. (2001) that is more coherent in the long run than existing formulas.

This study presents evidence that the implied cost of capital is positively related to subsequent stock returns. More importantly, we show that a simple trading strategy based on the ICOC yields risk-adjusted excess returns with respect to several common asset pricing models, such as the CAPM, or the Fama-French three-factor model. In our U.S. data sample, an investment that goes long in high-ICOC stocks and short in low-ICOC stocks yields an annual return of up to 10%. When correcting the investment for the portfolio's exposure to the Fama-French risk factors, it still provides a risk-adjusted return of 6.2% p.a. These results are even more remarkable since the ICOC effect persists even after including transaction costs - a constraint that most market anomalies do not meet. Estimations across other major international capital markets confirm these findings. Hence, our study clearly rejects the joint hypothesis of efficient markets and the validity of both asset pricing models.

This chapter proceeds as follows. In the next section, we present the estimation approach of the implied cost of capital and the present value formulas to derive it, i.e. the dividend discount model and the residual income model. Section 2.3 contains a brief description of our U.S. data sample. In section 2.4 we analyze the U.S. equity market in detail. By employing a simple portfolio investment strategy that uses the implied cost of capital estimate as stock selection variable, we present evidence that the ICOC can be regarded as a market anomaly. Finally, in section 2.5, we extend our study internationally and provide results obtained from the six other capital markets of the G7 countries. Section 2.6 concludes.

2.2 The Implied Cost of Capital

In this study, we use the firms' implied cost of capital estimates (ICOC) to implement trading strategies based on equity analysts' forecasts. The basic idea of the ICOC approach is to estimate an expected future cost of capital with the help of present value models. More precisely, the cost of equity is computed as the internal rate of return that equates expected discounted payoffs per share to current price, where expected cash flows are taken from equity analysts. Thus, the ICOC allows a systematical connection between analysts' forecasts and share prices via valuation models.

The ICOC approach shares its ability to include a variety of analysts' forecasts (such a short-term earnings forecasts, mid-term growth projections, and long-term expectations) in one single figure with forward-looking valuation concepts, such as the closely related value-to-price ratio². The primary advantage of the ICOC over this approach is that it does not require any estimate or assumption on the discount factor, thereby avoiding a potential source of errors.

In the literature, many different present value models are employed to calculate the implied cost of capital. In this work, we resort to the two most prominent models on the implied cost of capital. On the one hand, we use the dividend discount model (DDM) in the version of Cornell (1999). On the other hand, we rely on the residual income models (RIM) to evaluate the firms in our sample. We employ the model of Claus and Thomas (2001), as well as a slightly modified version of the Gebhardt et al. (2001) approach which uses a more consistent calculation of the terminal value. In the following sections, we describe the models and their implementation in more detail³. Given some assumptions such as clean surplus accounting, most of the approaches are equivalent in theory (Feltham and Ohlson, 1995). The so-called "clean surplus" assumption for example requires that all gains and losses affecting book value are also included in earnings. In practice, this condition is not always met. Stock options or capital increases e.g. can affect the book value of equity while leaving earnings unchanged. In addition, limited data availability, such as long-term earnings projections puts further restrictions on the theoretical equivalence. As a result, the structural assumptions in building a valuation model will significantly prejudice the results as we will see later.

²The value-to-price ratio (Lee et al., 1999) is the quotient of fundamental company value to market value. Similar to our approach, they derive the fundamental value of firms with the help of present value formulas, and they also use analysts' forecasts as a proxy for expected returns.

³Note that we do not present a detailed discussion about theoretical and empirical differences between residual income valuation, and the dividend discount model approach. For a thorough comparison, see for example Penman and Sougiannis (1998), Penman (2001), and Lundholm and O'Keefe (2001*b*,*a*). A short analysis of both models in the context of the ICOC approach can be found in Schröder (2007*a*).

2.2.1 The Dividend Discount Model

The general DDM states that the price of a share should equal the discounted value of future dividend payments, and can be written as follows:

$$P_0 = \sum_{t=1}^{\infty} \frac{E_0[D_t]}{(1+k)^t}$$
(2.1)

where

 $P_0 = \text{current share price, at the end of year 0,}$ $E_0[D_t] = \text{expected dividends per share at the end of year t,}$ k = cost of capital or, equivalently, shareholders' expected rate of return.

Since exact predictions of future dividends cannot be made to infinity, one has to make assumptions about expected cash-flows when implementing the model in practice. The DDM following Cornell (1999) assumes an initial 5-year phase of rather high dividend growth, which is followed by a transition phase in which the growth rates decline linearly to a lower, stable growth rate g_l , which is then maintained ad infinitum. Thus, this model combines the plausible conjecture of a possibly strong growth in the first years with realistic growth rates in the long run.

$$P_0 = \underbrace{\sum_{t=1}^{5} \frac{E_0[D_t]}{(1+k)^t}}_{t=1} + \underbrace{\sum_{t=6}^{20} \frac{E_0[D_t]}{(1+k)^t}}_{t=1} + \underbrace{\frac{E_0[D_{20}](1+g_l)}{(k-g_l)(1+k)^{20}}}_{t=1}$$
(2.2)

Growth period Transition period Stable growth

In the initial phase, the dividend growth is assume to equal the long-term consensus earnings growth rate, obtained from equity analysts⁴. In the stable phase following year 20, the dividend growth rate equals the estimated long-term GDP growth of the economy (Cornell, 1999)⁵. In this study, we use a simple moving average forecast model and calculate the expected GDP growth rate as the average geometric nominal GDP growth rate over the past 5 years. Due to its three-stage structure, we refer to this model as DDM3 hereafter⁶.

⁴The findings of Elton et al. (1981) suggest that analysts' forecasts are a good surrogate for investor expectations. The consensus growth rate is provided by IBES and is calculated as the median of the expected earnings growth rates of the contributing sell-side equity analysts.

⁵Usually, we have $g > g_l$, in line with the basic idea of the model. For some companies however, the long-term consensus earnings growth rate g lies below the expected GDP growth rate of the economy g_l .

⁶In an earlier version of this study, we evaluated also a simpler, two stage dividend discount model as proposed by Damodaran (1999). However, the results were similar to the results of the three stage model, and we will not report the results here.

2.2. THE IMPLIED COST OF CAPITAL

Note that we assume in equation (2.2), as well as in all subsequent present value formulas, an identical cost of capital k over all time periods. In the view of timevarying equity risk, this might not be an appropriate assumption. However, since the inclusion of a time-varying component usually leads to quite similar results Claus and Thomas (2001), we maintain the simple constant discount rate specification.

It is well known that due to conflicts of interest, equity analysts tend to overstate long-term growth projections (see e.g. Chan et al. (2003)), which hence biases the ICOC estimates upwards. Since this study focuses on the analysis of implied costs of capital of individual firms and differences thereof, this bias would only cause problems if there is some systematic relation between the degree of biases and some firm characteristic. We are not aware of any study documenting such a relation.

Note that the DDM requires the annual dividend D_0 which just has been paid out to the shareholders. Based on D_0 it is then possible to calculate the series of future dividend payments, beginning with D_1 . In this study, we use the sum of all dividend payments over the last 12 months, also known as 12-month trailing dividends, as D_0 . We exclude all observations from the sample, where the 12-month trailing dividend equals $0.^7$

2.2.2 The Residual Income Model

Another popular valuation formula is the RIM, stating that the value of the company equals the invested capital, plus the expected residual income from its future activities:

$$P_0 = B_0 + \sum_{t=1}^{\infty} \frac{E_0[R_t]}{(1+k)^t}$$
(2.3)

with

$$E_0[R_t] = E_0[E_t] - k(B_{t-1}) = (E_0[roe_t] - k)B_{t-1}$$
(2.4)

$$B_t = B_{t-1} + (1 - p_t)E_t \qquad \text{(clean surplus assumption)} \qquad (2.5)$$

where

⁷Of course, it would have been possible to include non-dividend paying companies into the sample by making additional assumptions on future dividends with the help of expected earnings and average payout ratios. We discarded this idea since it would have resulted in mixing different valuation indicators (dividends for the DDM vs. earnings and book values for the RIM). Moreover, the information presumably contained in dividend payments cannot be recovered by making assumptions on payout ratios.

$(B_0 \text{ being the current book value}),$ $E_0[R_t] = \text{expected residual income per share in year } t,$ $E_0[E_t] = \text{expected earnings per share in year } t,$ $E_0[roe_t] = \text{expected return on equity in year } t,$ $p_t = \text{payout ratio in year } t.$	B_t	=	book value of equity per share at the end of year t
$E_0[R_t] = \text{expected residual income per share in year } t,$ $E_0[E_t] = \text{expected earnings per share in year } t,$ $E_0[roe_t] = \text{expected return on equity in year } t,$ $p_t = \text{payout ratio in year } t.$			$(B_0 \text{ being the current book value}),$
$E_0[E_t] =$ expected earnings per share in year t , $E_0[roe_t] =$ expected return on equity in year t , $p_t =$ payout ratio in year t .	$E_0[R_t]$	=	expected residual income per share in year t ,
$E_0[roe_t] = \text{expected return on equity in year } t,$ $p_t = \text{payout ratio in year } t.$	$E_0[E_t]$	=	expected earnings per share in year t ,
$p_t = payout ratio in year t.$	$E_0[roe_t]$	=	expected return on equity in year t ,
	p_t	=	payout ratio in year t .

Similar to the DDM, assumptions about the future growth in residual incomes or earnings have to be made when implementing the model in practice. One rather simple approach is proposed by Claus and Thomas (2001), who consider a two-stage RIM (denoted RIM2 in the following), assuming an initial phase of high earnings growth rates, followed by a stable growth of residual incomes after year five:

$$P_0 = B_0 + \underbrace{\sum_{t=1}^{5} \frac{E_0[E_t] - k(B_{t-1})}{(1+k)^t}}_{(1+k)^t} + \underbrace{\frac{E_0[R_5](1+g_l)}{(k-g_l)(1+k)^5}}_{(k-g_l)(1+k)^5}$$
(2.6)

Growth period

iod Stable growth

Expected earnings for the first three years are taken from analysts' forecasts, also provided by IBES. Earnings after year 3 are estimated by applying the IBES consensus long-term earnings growth rate to the expected earnings of year 3.⁸ The growth rate in the second phase is presumed to equal the expected inflation rate calculated as the prevailing interest rate on 10-year treasury bonds less the assumed real-rate of three percent⁹.

Future expected book values of equity are calculated using equation (2.5). To that end, we have to make assumptions regarding future payout ratios. In a slight variation to the methodology of Claus and Thomas (2001), we let the current payout ratio geometrically converge towards 50% over the growth period instead of using this ratio from the first prospective year on to project future book values. This approach seemed more realistic to us. Current payout rations are calculated by dividing the 12-month trailing dividends by the 12-month trailing earnings per share. This method ensures that the payments refer to the same time period as the earnings. Payout ratios

⁸In the case where the expected earnings estimate of year 3 is missing, we also generate earnings in year 3 by applying the long-term consensus growth rate to expected earnings of year 2. If the projected earnings in year three is negative, we drop the observation from the sample.

⁹The assumption of no real growth in residual income after year 5 does not imply the absence of real earnings growth after year 5. It rather incorporates the assumption of decreasing earnings growth rates in the stable growth phase, for the usual reasons (competition, antitrust actions, etc.), similar to the transition phase of the DDM3. For more discussion, see Claus and Thomas (2001, p. 1640 and p. 1663ff).

above 1 are set to 1 in the first year, negative payout ratios are set to 0. If the payout ratio is not available due to missing dividend payments over the last 12 months, we equally set the payout ratio of year t = 1 to 0.

Gebhardt et al. (2001) rely also on the RIM to calculate the implied cost of capital. However, in contrast to Claus and Thomas (2001), they focus their assumptions not on future residual incomes, but more directly on the future return on equity (*roe* see equation (2.4)), which they assume to converge to the industry median. More formally:

$$P_{0} = B_{0} + \underbrace{\sum_{t=1}^{3} \frac{E_{0}[roe_{t}] - k}{(1+k)^{t}} B_{t-1}}_{(1+k)^{t}} + \underbrace{\sum_{t=4}^{T} \frac{E_{0}[roe_{t}] - k}{(1+k)^{t}} B_{t-1}}_{(1+k)^{t}} + \underbrace{\frac{E_{0}[iroe_{T}] - k}{k(1+k)^{T-1}} B_{T-1}}_{(1+k)^{T-1}}$$
(2.7)

Explicit forecasts Transition period Terminal value

where

T = forecast horizon of the transition period (usually 9 years) $E_0[iroe_T]$ = expected industry return on equity from period T onwards.

Similar to Gebhardt et al. (2001) we use explicit forecasts to calculate the expected return on equity for the next three years. Then we fade the roe over T-3 years to the industry *roe*. This industry *roe* is calculated as the median of all (positive) realized roe across all companies of the respective sector, over the preceding 60 months. This procedure aims to average out business cycle effects of the industry profitability¹⁰. We use the industry sector codes of the GICS classification for sorting the companies¹¹. While the use of industry averages for the long-term return on equity has some appeal due to the findings of empirical analysis (Nissim and Penman, 2001; Soliman, 2004), the assumptions in the literature regarding the payout ratios to construct future book values of equity seem somewhat arbitrary to us¹². As a consequence, we consider a slightly modified version of the three-stage RIM that avoids relying on assumptions

¹⁰Long-term changes of industry profitability, such as declining returns of current growth industries, are not captured by this approach.

¹¹In our standard implementation we fix T = 9. Instead of using the GICS classification, Gebhardt et al. (2001) rely on the 48 Fama and French (1997) industry classifications. We also tested other, more precise classifications such as the GICS Industry Group segmentation, or the Industry segmentation, but the results were very similar.

 $^{^{12}\}mathrm{As}$ mentioned earlier, Claus and Thomas (2001) assume a payout ratio of 50% across all firms, whereas Gebhardt et al. (2001) hold the prevailing payout ratio of each company constant to estimate future book values.

on future payout ratios. Following the literature on sustainable growth rates, we use the following identity between payout ratio p, return on equity *roe* and the long-term growth rate of the company g_l :

$$\frac{g_l}{roe} = 1 - p$$

$$p = 1 - \frac{g_l}{roe}$$
(2.8)

In order to estimate the future development of a company, one has to make assumptions for two out of the three parameters¹³. Instead of assuming the long-run payout ratio p, we opt to fix the long-term growth rate of the company g_l . By setting g_l equal to the expected GDP growth rate of the economy as in the earlier model, we ensure that no company will persistently grow faster than the whole economy and eventually exceed it.

Hence, we calculate the RIM following Gebhardt et al. (2001) as presented in equation (2.7), but using different projected payout ratios for each industry sector. For each sector, we calculate the long-term industry payout ratio using the relation (2.8), given the expected GDP growth of the economy and the industry *roe*. In the transition period, we then fade both payout ratio and return on equity towards their long-term levels. Since this model is basically a three-stage RIM, we denote it by the abbreviation $RIM3.^{14}$

2.3 Data and Empirical Implementation

In our detailed analysis in section 2.4, we focus on companies in the United States covered by MSCI over a time period from January 1990 to February 2006. The monthly data for prices, total returns, book values and dividends per share, market capitalization, and returns on equity are taken from MSCI. All market capitalization data are free float adjusted. The earnings estimates as well as the long-term growth rate are taken from IBES median estimates. Time series data of national accounts to calculate the expected nominal GDP growth rate is obtained from Eurostat. Stock indices to derive market betas and deflate firm size data are taken from Datastream.

¹³Of course, this procedure incorporates the implicit assumption of a constant capital structure in the future.

¹⁴In practice, our RIM3 specification does not diverge substantially from the model as proposed by Gebhardt et al. (2001). The correlation of the ICOC estimates obtained from both models is very high for the United States (greater than 0.99).

	Available observations	Mean	Standard Deviation	Min	Max
DDM3	68,759	8.87%	2.04%	4.68%	43.42%
RIM2	87,390	9.19%	2.31%	1.07%	51.10%
RIM3	$69,\!175$	8.18%	2.72%	1.00%	38.76%
All	50,402				

Table 2.1: Descriptive Statistics - United States

This table reports the number of available observations as well as the mean implied return, standard deviation, together with the minimal and maximal observation derived for the different valuation models. For the calculation of the mean, standard deviation, min, and max, only observations with values for all four models were used (50,402 observations).

Our data set contains 1,507 companies and over 122,000 monthly observations. Since the number of companies included in the study changes over time, we have an unbalanced panel data set¹⁵. Since we require five years of company data to calculate the industry *roe*, we can only estimate the ICOC from January 1995 on, leaving us with roughly 11 years of data. To calculate the implied cost of capital for the firms using the equations above, we employ the last available information as required by the formulas at the end of each calendar month. Thereby, we make sure that our ICOC estimates are based only on publicly available information. Firms with an incomplete data set, i.e. one or more missing input variables where we could not resort to approximations as explained above, have been ignored¹⁶. The solution of the equations is straightforward, since they are monotone in k, and can be solved iteratively.

In the U.S., as table 2.1 shows, the average implied return is fairly well clustered between 8% and 9% with the RIM3 having the lowest average implied return at 8.18% and the RIM2 having the highest at 9.19%. However, the standard deviation is the highest for the RIM3 model with 2.72%. The standard deviation is somewhat lower for the RIM2 model at 2.31% and the DDM3 has the lowest standard deviation at 2.04%.

Since the number of observations for the DDM3 and RIM2 ICOC is constrained by

¹⁵The available panel data set starts in 1995 with 334 monthly observations, reaches its maximum in June 2005 with 507 companies, and finally contains in February 2006 501 companies.

¹⁶Note that we do not carry out any time adjustment procedures similar to other studies on the implied cost of capital. Since we use a monthly data set, such adjustments would require the exact dividend payout dates and book value adjustments for all companies since 1995. Such data is not easy to get hold of, nor is it reliable.

	DDM3	RIM2	RIM3	MCAP	B/M
DDM3	1.000				
RIM2	0.2930	1.000			
RIM3	0.1903	0.6313	1.000		
MCAP	-0.0284	-0.1342	-0.2254	1.000	
B/M	0.2356	0.3144	0.8062	-0.2272	1.000

Table 2.2: Correlations of Implied Cost of Capital Estimates and the Fama-French Factors - United States

This table reports the correlations of ICOC estimates and the Fama-French factors B/M ratio and size (MCAP) across the different valuation models using data over all years. Only observations with data for all valuation models were used (50,402 observations).

the availability of the IBES long-term growth rate and the payout ratio, we will use for the following analysis only the data set of 50,402 observations for which all three ICOC estimate were available. However, using the more complete data set for each ICOC estimate leads to the same results in most cases.

As was to be expected, the correlations between the similar models RIM2 and RIM3 are rather high at around 63% as can be seen in table 2.2. On the other hand, the correlations between the DDM and the RIM ICOCs are very low - generally below 30%. There is little correlation with either the B/M ratio or the market capitalization with the exception of the RIM3 specification which exhibits a very high correlation of 80% with the book yield. Given the predominant role of book value of equity in both RIM formulas, this high correlation is in line with our expectations.

2.4 Tests on Implied Cost of Capital-based Investment Portfolios

If share prices do not fully reflect at all times the fundamental value of the companies as derived from analysts', then investors should be able to trade profitably on the basis of the companies' ICOC estimates. Since the ICOC equates current market prices to the share's so-obtained fundamental value, a high implied discount rate suggests that - keeping its risk exposure unchanged - a share is likely to trade below its fair value. Hence, if share prices are to converge to their fundamental value over time, a portfolio of high-ICOC companies should outperform an investment in low-ICOC stocks.

This section investigates such strategies, and analyses their ability to generate risk-adjusted excess returns with respect to common asset pricing models. Thus, this is a test of the joint hypothesis of efficient markets and the asset pricing models employed. In principle, this chapter follows the methodology of other well-known studies on profitable trading strategies such as the work on price momentum strategies by Jegadeesh and Titman (1993).

2.4.1 Portfolio Formation

At the end of each month, we rank all stocks in our sample based on their implied-costof-capital estimates. Then we group them into 8 equally weighted portfolios based on these rankings. Finally, we examine subsequent total returns, i.e. capital gains and dividend payments, over periods from 1 to 24 months. When a company drops out of the sample during the holding period, we replace its return by the market return over the period. This procedure aims to avoid a forecast bias.

Such trading strategies may be examined using either overlapping observations or non-overlapping observations. Similar to comparable studies (e.g. Fama and French (1988) in the context of long-horizon regressions), we use overlapping holding periods to increase the power of the analysis. When we consider for instance a twelve-month holding period, we use all possible portfolios that could be formed, such as those from January 1995 to January 1996, from February 1995 to February 1996, and so on.

2.4.2 Evaluation of Investment Strategies

First, we compare the profitability of investment strategies based on the various ICOC approaches as presented in section 2.2. In table 2.3 we present the mean ICOC estimates of each portfolio, the portfolio returns, and their risk characteristics. Portfolio 1 compromises the stocks with the lowest implied cost of capital estimate, and portfolio 8 consists of the high ICOC stocks. Panel A reports returns and firm characteristics of the DDM3 ICOC portfolios, panel B information on the RIM2 ICOC portfolios and panel information on the RIM3 ICOC investments.

The return of each portfolio is calculated as the equally weighted buy-and-hold return with a holding period of twelve months, using overlapping intervals. In this section, we neglect possible transaction costs. As for the portfolio risk variables, we present the mean B/M ratio, the median firm size, average market beta, and average price momentum for each of the 8 portfolios. Price momentum is calculated as change in share prices over the past six months¹⁷.

 $^{^{17}}$ The inclusion of price momentum as risk variable has been proposed by Carhart (1997). His work attempts to explain mutual fund performance by employing a four-factor model by adding price momentum to the standard three-factor Fama-French model, since the latter cannot explain

Table 2.3: Returns and Firm Characteristics of Buy-and-Hold Portfolios - United States

This table presents, along with the mean ICOC estimates and portfolio returns, the characteristics of the different portfolios. The portfolios are constructed using various ICOC approaches as sorting variable where portfolio 1 (P1) compromises the stocks with the lowest implied cost of capital estimate, and portfolio 8 (P8) consists of the high ICOC stocks. Panel A reports returns and firm characteristics of the DDM3 ICOC portfolios, panel B information on the RIM2 ICOC portfolios and panel information on the RIM3 ICOC investment portfolios.

The return of each portfolio is calculated as the equally weighted buy-and-hold return with a holding period of twelve months, using overlapping intervals. B/M is the mean book yield of the companies in a portfolio, SIZE is the median firm market capitalization (calculated as the log of the market capitalization divided by the level of the stock market index), BETA is the average five year regressed sensitivity on the market portfolio, and MOMENTUM is the average historical six-month price return. All firm characteristics are measured as of the portfolio formation date.

The two rows at the bottom of each panel show the overall averages (or medians, respectively) over the whole sample size and the average difference in returns and firm characteristics between the two extreme portfolios (P8-P1). More precisely, the return column indicates the average return that could have been generated by an investment strategy consisting in a short position of the low-ICOC portfolio P1 and a long position in the high-ICOC portfolio P8 (along with their t-statistic, which is calculated using Newey and West (1987) HAC standard errors using a lag length of 11 months). *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level. The sample period is from January 1995 to February 2006. Country: United States. Observations: 40,158.

Portfolio	ICOC	Return	Beta	B/M ratio	Size	Momentum			
Panel A: DDM3 ICOC									
P1	6.33%	14.34%	1.002	0.361	8.925	7.49%			
P2	7.43%	14.18%	0.962	0.369	8.893	7.61%			
P3	8.11%	13.32%	0.926	0.375	8.999	5.19%			
P4	8.66%	15.01%	0.848	0.369	9.179	4.57%			
P5	9.20%	14.24%	0.824	0.384	9.164	3.06%			
P6	9.76%	13.33%	0.786	0.410	9.093	1.67%			
P7	10.47%	13.42%	0.709	0.459	8.987	0.82%			
P8	12.30%	14.94%	0.752	0.568	8.679	-3.90%			
Average	9.02%	14.10%	0.852	0.412	8.982	3.34%			
P8-P1	$5.98\%^{***}$	0.60%	-0.249***	0.206^{***}	-0.245	-11.39%***			
t-stat	(29.52)	(0.16)	(-6.94)	(6.14)	(-1.38)	(-5.66)			
		Pa	anel B: RIM	2 ICOC					
P1	6.36%	10.26%	0.846	0.344	9.450	7.66%			
P2	7.65%	12.54%	0.794	0.307	9.407	7.27%			
P3	8.27%	12.22%	0.809	0.340	9.178	6.23%			
P4	8.82%	13.82%	0.791	0.376	9.070	4.89%			
P5	9.38%	15.05%	0.813	0.416	8.903	3.26%			
P6	10.05%	15.13%	0.863	0.445	8.839	1.86%			
P7	10.93%	15.32%	0.928	0.484	8.721	-0.03%			
P8	13.10%	18.60%	0.972	0.588	8.478	-4.68%			
Average	9.30%	14.10%	0.852	0.412	8.982	3.34%			
P8-P1	$6.75\%^{***}$	$8.33\%^{**}$	0.126^{**}	0.244^{***}	-0.972^{***}	-12.34%***			
t-stat	(20.39)	(2.09)	(2.06)	(4.80)	(-3.35)	(-4.08)			

Portfolio	ICOC	Return	Beta	B/M ratio	Size	Momentum				
Panel C: RIM3 ICOC										
P1	4.60%	9.80%	0.845	0.166	9.858	6.81%				
P2	5.93%	12.83%	0.857	0.222	9.473	6.52%				
P3	6.83%	12.98%	0.822	0.280	9.191	5.67%				
P4	7.63%	13.53%	0.796	0.342	8.997	5.21%				
P5	8.40%	14.53%	0.813	0.408	8.916	3.66%				
P6	9.23%	14.39%	0.851	0.485	8.773	2.07%				
$\mathbf{P7}$	10.39%	15.08%	0.901	0.571	8.657	0.10%				
P8	13.11%	19.80%	0.931	0.831	8.398	-3.52%				
Average	8.25%	14.10%	0.852	0.412	8.982	3.34%				
P8-P1	$8.51\%^{***}$	$10.00\%^{**}$	0.086^{*}	0.666^{***}	-1.460^{***}	$-10.32\%^{***}$				
t-stat	(19.13)	(2.16)	(1.76)	(12.57)	(-6.85)	(-3.42)				

The two rows at the bottom of each panel show the overall averages (or medians, respectively) over the whole sample size and the average difference in returns and firm characteristics between the two extreme portfolios (P8-P1).¹⁸ More precisely, the return column indicates the average return that could have been generated by an investment strategy consisting in a short position of the low-ICOC portfolio P1 and a long position in the high-ICOC portfolio P8 (along with their t-statistics, which are corrected for autocorrelation resulting from the use of overlapping periods). In order to allow for a comparison of the ICOC approaches, we include only observations where we have ICOC estimates for all selected approaches.

In direct comparison, we see that there is a considerable discrepancy between the models' ability to predict subsequent stock returns: Whereas the difference in returns between the two extreme ICOC portfolios obtained from the DDM3 attains only a mere 0.6% p.a., both RIM approaches are able to generate a yield of 8% to 10% when employing a long-short investment strategy. Moreover, the differences P8-P1 are significant at the 5% level for both residual income ICOCs¹⁹. When comparing to the average equally-weighted stock return over the sample period (14.1%), we can

short-term price momentum (Fama and French, 1996).

¹⁸The average market beta lies with 0.852 below the theoretical value of 1. This can be partly explained by the fact that due to missing long-term growth rates, our sample contains on average more large companies than the overall market, which usually tend to have lower beta values. From a practical perspective, it is not the average firm beta that is important, but the market beta of each portfolio, since individual correlations with the market might cancel out when pooling them into portfolios. However, estimates for portfolio betas did not differ significantly from the average firm beta of each portfolio.

 $^{^{19}}$ When using five portfolios instead of eight (similar to the analysis of many other capital markets investigated in the international section), both the spreads between the extreme portfolios and their significance reduce slightly. For example, the P5-P1 difference attains 7.3% for the RIM3 approach, and 6.2% for the RIM2 ICOC.





The figure displays the difference between returns of the high ICOC portfolio (P8) and the low ICOC portfolio (P1) over time. The returns are buy-and-hold total returns with a holding period of twelve months. The sample period is from January 1995 to February 2006.

see that both the long side (around 20%) and the short side (around 10%) of the investment contribute to this return. Hence, both the long-only investment P8 and the long-short investment P8-P1 yield returns that exceed their benchmarks, which are given by the average market return (14.1%) for the former and the average short-term risk-free rate for the zero-investment portfolio (4.2%) over this time period).

The profitability of ICOC-based investment strategies over time is displayed in figures 2.1-2.3. These graphs show the evolution of the difference between the returns of the extreme portfolios (P8-P1), where the buy-and-hold returns are calculated over twelve months. All figures show that this spread varies quite significantly, and is in many periods even below 0, i.e. the P1 performs better than the P8 portfolio. Compared to the RIM ICOC, the DDM-based approach is slightly less volatile at the cost of lower average returns. It is interesting to note that the profitability of ICOC strategies is particularly high during extreme stock market movements (record-highs in 2000, record-lows in 2003). The explanation for this pattern is straightforward: during periods that are marked by high volatility and possibly exaggerated stock price movements, the divergence between fundamental firm value (derived from analysts' forecasts) and market valuation is more pronounced. Since the ICOC captures just this discrepancy, the investment strategies are getting more profitable. Another conclusion out of the figures is that the long-short strategy has increased its profitability over time. The average P8-P1 spreads in the sub sample from 2000 until 2006 are much higher compared to the overall sample: The RIM3 P8-P1 difference attains 20.3% p.a., the RIM2 difference reaches 15.1%, the spread for the DDM3 ap-



Figure 2.2: Returns of Implied Cost of Capital Based Investment Strategies over Time - RIM2 - United States

The figure displays the difference between returns of the high ICOC portfolio (P8) and the low ICOC portfolio (P1) over time. The returns are buy-and-hold total returns with a holding period of twelve months. The sample period is from January 1995 to February 2006.

Figure 2.3: Returns of Implied Cost of Capital Based Investment Strategies over Time - RIM3 - United States



The figure displays the difference between returns of the high ICOC portfolio (P8) and the low ICOC portfolio (P1) over time. The returns are buy-and-hold total returns with a holding period of twelve months. The sample period is from January 1995 to February 2006.

	Months	1	3	6	12	18	24
DDM3	P1	1.3%	4.0%	7.6%	14.3%	21.4%	28.7%
	P8	1.5%	4.1%	7.8%	14.9%	23.1%	30.9%
	P8-P1	0.2%	0.2%	0.2%	0.6%	1.7%	2.2%
	t-stat	(0.64)	(0.18)	(0.08)	(0.16)	(0.36)	(0.44)
RIM2	P1	0.8%	2.7%	5.3%	10.3%	14.4%	18.1%
	P8	1.8%	5.0%	9.4%	18.6%	27.9%	36.4%
	P8-P1	$1.0\%^{**}$	$2.4\%^{**}$	$4.0\%^{*}$	$8.3\%^{**}$	$13.5\%^{**}$	$18.3\%^{***}$
	t-stat	(2.53)	(2.31)	(1.90)	(2.09)	(2.56)	(3.35)
		~					
RIM3	P1	0.8%	2.6%	5.2%	9.8%	13.7%	17.3%
	P8	1.8%	5.0%	9.9%	19.8%	30.2%	40.4%
	D9 D1	1 007 **	0 90% **	1 707 **	10 007 **	16 107 **	99 107 ***
	P8-P1	1.0%	2.370	4.770	10.0%	10.4%	25.170^{-11}
	t-stat	(2.38)	(2.02)	(1.99)	(2.16)	(2.52)	(2.66)
01		44.000	44.029	40 794	10 150	97 014	95 47
Observations		44,989	44,032	42,734	40,158	37,814	35,47
EWA		1.3%	3.7%	7.2%	14.1%	21.1%	28.4%

Table 2.4: Returns from Buy-and-Hold Portfolios with different Holding Periods - United States

This table reports the average, equally weighted buy-and-hold returns of the high (P8) and low (P1) ICOC portfolios, together with their average difference (P8-P1) and the t-stat thereof over holding periods of 1, 3, 6, 12, 18, and 24 months. The t-statistic is calculated using Newey and West (1987) HAC standard errors, using a lag-length corresponding the holding period minus 1. The last two lines give the number of observations (decreasing with growing investment horizon) and the equally weighted return of the whole sample. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level. The sample period is from January 1995 to February 2006. Country: United States.

proach lies at 7.3%. Hence, most of the positive results over the whole period originate from the good performance in the post 2000 period. In this sense, the results are not robust to the examined time period. Unfortunately, there is not sufficiently data of analysts' forecasts to extend the study to earlier time periods. Still, the averages are not significantly negative in the first sub period.

We now vary the buy-and-hold periods of the ICOC investment portfolios to see how their profitability changes with the investment horizon. Table 2.4 shows the average returns of the high (P8) and low (P1) ICOC portfolios, together with their average difference (P8-P1) and the t-stat thereof with holding periods of 1, 3, 6, 12, 18, and 24 months. The last two lines give the number of observations (decreasing with growing investment horizon) and the equally weighted return of the whole sample. The results essentially confirm our earlier results. The ICOC calculated by the RIM yields positive portfolio returns regardless of the buy-and-hold period used. In contrast, the DDM3-based investment strategy attains a mere 2.2% yield over a period of 24 months. The average monthly return increases with the investment horizon: over two years the p.a. return amounts to 11.5% which compares to 10% over 12 months (RIM3). Similarly, the statistical significance of a P8-P1 (RIM2) investment strategy does increase over time. This is actually good news for real-life investors who have to care about transaction costs²⁰.

2.4.3 Portfolio Risk

Firm Characteristics

It can be easily seen in table 2.3 that all ICOC approaches are related to standard firm-risk variables. There is a highly significant inverse relation between the ICOC and both past price momentum and firm size (although not significant for the DDM approach), and a positive relation to B/M ratio. Large firms, as well as firms that have seen a good share price performance over the last six months have on average smaller ICOC estimates. Growth firms, i.e. firms with a low B/M ratio, provide also lower implied returns. Especially the RIM3 seems most sensitive to the risk factors. The difference in the B/M ratio between the portfolio with the highest ICOC as calculated by RIM3 and the lowest ICOC is three times as large as for the other models. The difference in normalized market capitalization is 1.5 and seven times larger than the RIM2 and DDM3 respectively. The implied return portfolios obtained from the DDM exhibit a negative relation to market beta. In contrast, as far as the RIM approaches are concerned, we cannot detect any monotonic relationship with the beta estimates.

The relation of implied returns with B/M ratio and firm size is in line with the findings of Fama and French (1992, 1993) that have detected both variables as priced risk factors. Consequently, these results underline the standard rule that higher stock returns only come at the cost of increased portfolio risk. The association with past price momentum can be explained by the nature of the present value formula: since current share price enters the equation, companies that have experienced a rise in share prices, have ceteris paribus a lower internal rate of return. The negative relation of the

 $^{^{20}}$ A part of the good long-term performance of ICOC based investments might also originate from the use of overlapping observations to calculate the averages. Since this procedure counts the observations in the middle of the sample period (which proved to be very profitable, see figures 2.1-2.3) more often than the first and last observations (which did not particulary well), taking the simple average of all overlapping observations biases the long-term profitability upwards with increasing investment horizon.

DDM ICOC to market beta is somewhat unexpected. If asset pricing theory is right in postulating a positive relation between expected stock returns and market beta, this finding adds some doubts on the DDM ICOC's validity as proxy for expected returns.

Risk adjusted portfolio returns

In this section, we check whether the ICOC investment strategies still generate excess returns after controlling for portfolio risk. As seen in the previous section, the P8 or the P8-P1 investments are exposed both to B/M and size risk effects. This observation casts doubts over the ICOC's intrinsic ability to explain stock returns, since its relation to subsequent returns might have its origin in the ICOC's relation to underlying risk factors. To control for both risk factors, we first sort the all stocks based on B/M ratio (size, respectively), and within each B/M-quintile (size-quintile) in five portfolios based on the ICOC estimates. The P1 portfolio contains then the stocks with the lowest ICOC estimates in each quintile, the companies with the highest ICOC in each quintile end up in the P5 portfolio. All P1 to P5 portfolios will therefore contain the same number of stocks from each B/M-quintile (size-quintile), and are thus in a sense B/M (size)-neutral. Table 2.5 presents the overlapping 12-months returns of the 25 portfolios for both B/M neutral (left side of the table), and size neutral pre-sorting (right side). In panel A, the ICOC portfolios are formed using the DDM3 specification, panel B uses the RIM2 approach, and panel C the RIM3 as ICOC variable. The last row in each panel indicates the return of a P5-P1 investment within each quintile together with its associated t-statistic. If the ICOC effect is merely a reflection of both risk variables firm size and B/M ratio, the P5-P1 returns should be zero across all quintiles.

The DDM3 approach in the upper panel does not show any significant positive P5-P1 returns, as was to be expected from the previous analysis. In many B/M or size quintiles this difference is even negative. In contrast, the results of the two lower panels of the table indicate that RIM based ICOC trading strategies have almost the same magnitude when investment strategies are carried out on the different risk-neutral subsamples of stocks as when they are employed on the full sample. Long-short investments yield almost in all quintiles positive returns; in some quintiles these returns are even highly significant. Especially in the extreme quintiles of both the B/M and size pre-sorting, the ICOC effect is particularly high. Since this relation between B/M and size with the magnitude of the ICOC effect is not monotonic, this observation is only a weak indication that RIM ICOC effect is related to both B/M

	Bo	pok-to-M	arket Eq	uity Quir	ntiles		Size	e Quintile	es	
				Pane	el A: DDM3	B ICOC				
	Low	2	3	4	High	Small	2	3	4	Big
P1	14.2%	10.9%	14.3%	16.4%	20.3%	16.4%	15.5%	16.3%	16.7%	10.8%
P2	10.0%	15.5%	13.5%	13.4%	19.4%	14.7%	16.1%	14.1%	11.4%	12.3%
P3	13.8%	11.7%	13.4%	16.1%	15.7%	19.0%	16.0%	12.5%	13.7%	12.3%
P4	13.6%	12.0%	12.1%	13.3%	16.6%	13.8%	15.0%	11.6%	11.6%	12.5%
P5	11.6%	11.9%	14.2%	15.6%	17.2%	15.5%	15.9%	15.6%	13.0%	14.3%
P5-P1	-2.6%	1.1%	-0.1%	-0.8%	-3.1%	-1.0%	0.4%	-0.6%	-3.7%	3.5%
t-stat	-0.51	0.51	-0.02	-0.38	-1.08	-0.46	0.10	-0.23	-1.65	0.92
				Pan	el B: RIM2	ICOC				
	Low	2	3	4	High	Small	2	3	4	Big
P1	9.9%	9.1%	14.1%	15.2%	13.3%	14.7%	15.9%	13.0%	11.3%	9.9%
P2	12.6%	12.6%	12.3%	14.2%	14.3%	14.3%	14.6%	12.1%	13.1%	11.1%
P3	14.1%	14.3%	14.3%	14.6%	18.7%	16.2%	15.1%	14.7%	13.7%	12.1%
P4	13.5%	15.7%	13.9%	14.8%	20.4%	14.5%	15.9%	15.2%	14.2%	12.9%
P5	13.1%	10.4%	12.8%	16.2%	23.1%	19.9%	17.2%	15.3%	14.3%	16.1%
P5-P1	3.2%	1.8%	-1.3%	1.0%	$9.8\%^{***}$	$5.2\%^{*}$	1.3%	2.3%	3.0%	$6.1\%^{*}$
t-stat	0.70	0.37	-0.41	0.38	3.02	1.69	0.35	0.72	0.82	1.68
				Pan	el C: RIM3	ICOC				
	Low	2	3	4	High	Small	2	3	4	Big
P1	8.6%	9.5%	12.3%	13.7%	13.5%	12.6%	13.7%	12.3%	11.7%	10.9%
P2	12.2%	12.0%	13.1%	14.9%	15.3%	13.6%	16.0%	12.6%	15.3%	9.8%
P3	13.1%	13.1%	13.2%	14.2%	17.1%	14.7%	14.4%	13.7%	13.6%	13.0%
P4	15.3%	14.2%	16.0%	15.6%	20.8%	17.0%	15.2%	14.8%	12.9%	11.9%
P5	14.0%	13.5%	12.9%	16.5%	23.1%	21.8%	19.5%	16.9%	12.9%	16.4%
P5-P1	5.4%	4.0%	0.6%	2.8%	9.6%***	9.3%***	5.7%	4.6%	1.2%	$5.4\%^{*}$
t-stat	1.13	1.30	0.22	0.89	2.95	2.63	1.51	1.23	0.32	1.81

Table 2.5: Returns of Implied Cost of Capital Based Portfolios that Control for B/M ratio and Firm Size - United States

At the end of each month all stocks are ranked in ascending order based on the B/M ratio (columns on the left) or firms size (columns on the right). The "Low" quintile contains stocks with a low B/M ratio, the "High" quintile contains stocks with a high B/M ratio. Similarly, the "Small" quintile contains smaller stocks, the "Big" quintile containing stocks of large companies. Within each quintile, the firms are sorted in five portfolios based on their ICOC estimates. The P1 portfolio contains then the stocks with the lowest ICOC estimates in each quintile, the companies with the highest ICOC in each quintile end up in the P5 portfolio. All P1 to P5 portfolios will therefore contain the same number of stocks from each B/M-quintile (size-quintile), and are thus B/M (size)neutral. The table presents the overlapping 12-months returns of the so-obtained 25 portfolios for both B/M and size neutral pre-sorting. In panel A, the ICOC portfolios are formed using the DDM3 specification, panel B uses the RIM2 approach, and panel C the RIM3 as ICOC variable. The last row in each panel indicates the return of a P5-P1 investment within each quintile together with its associated t-statistic. The t-statistic is calculated using Newey-West (1987) HAC standard errors, using 11 lags. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level. The sample period is from January 1995 to February 2006. Country: United States. Observations: 40,158.

and size effect. The presence of a strong ICOC effect within the extreme portfolios suggests rather that the ICOC effect is particularly pronounced for companies that exhibit extraordinary firm characteristics, i.e. for example very small firms or very big firms. The bottom line of this double-sort portfolio analysis is that the profitability of RIM-based ICOC strategies is not confined to a particular subsample of companies, i.e. the ICOC effect is not a simple transformation of both risk proxies firm size and B/M ratio.

Another standard methodology to control the portfolio returns for their inherent risk positions is to calculate their portfolio alphas with respect to common asset pricing models, such as the CAPM or the Fama-French factor model. In table 2.6, we report the results from time-series regressions based on the CAPM and the Fama and French (1993) three-factor model. More precisely, we regress the portfolio excess returns over the risk-free rate on the market excess return and the Fama-French factors:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{m,t} - r_{f,t}) + \gamma_i SMB_t + \delta_i HML_t + \varepsilon_{i,t}$$
(2.9)

where $r_{i,t}$ is the 12-month return of portfolio i; $r_{f,t}$ is the risk-free rate and $r_{m,t}$ the market return over the same time period; SMB_t is the Fama-French small firm factor, and HML_t is the book-to-market factor. Since we use overlapping periods, we use Newey-West HAC standard errors to calculate the t-statistics²¹. In panel B and C, the CAPM alpha of the long-short P8-P1 portfolio are both significantly positive, providing a risk-adjusted portfolio return of 7.6% for the RIM2, and 11.5% for the RIM3, respectively. When adopting the three-factor model by Fama-French, and thereby also controlling for average size and B/M ratio of the portfolios, the long-short investment in both extreme portfolios remain positive, in the case of the RIM3 even highly significant, attaining a risk adjusted annual return of 6.2%. Both RIM measures of the ICOC display significant factor loadings for the B/M ratio, but not to firm size. Again, the DDM3 ICOC provides no abnormal returns with respect to both asset pricing models, although the returns are significantly related to all risk factors²².

²¹When estimating equation (2.9) for the zero investment portfolio return $r_{i,t}$ of the long-short strategy (P8-P1), we obviously do not subtract the risk-free rate. In order to preserve a better comparison to the standard stock return regression of Fama and French (1993), we refrain from estimating a four factor model including price momentum as fourth risk factor, similar to Carhart (1997). Such a regression test would likely strengthen the ICOC effect, since the ICOC is negatively related to price momentum (see table 2.3), and therefore would increase the estimated portfolio alphas when controlling for price momentum (momentum is usually positively related to stock returns).

²²It is important to note that these findings are robust to changes in the holding period of the portfolios. Especially regression tests on non-overlapping monthly portfolio returns yield similar

Portfolio	α	$t(\alpha)$	β	$t(\beta)$	γ	$t(\gamma)$	δ	$t(\delta)$		
Panel A: DDM3 ICOC										
P1 (low ICOC)	6.2%	2.76	0.79	9.31						
P8 (high ICOC)	9.2%	2.38	0.38	1.67						
P8-P1	3.0%	0.82	-0.41	-2.18						
P1 (low ICOC)	5.9%	3.07	0.77	13.78	0.43	3.79	0.06	0.83		
P8 (high ICOC)	5.0%	3.08	0.49	6.26	0.07	0.44	0.73	5.17		
P8-P1	-0.9%	-0.35	-0.32	-3.53	-0.37	-3.04	0.67	5.41		
		Panel	B: RIM	12 ICO	C					
P1 (low ICOC)	2.3%	1.35	0.77	12.39						
P8 (high ICOC)	9.9%	2.18	0.69	2.72						
P8-P1	7.6%	1.68	-0.08	-0.30						
P1 (low ICOC)	2.3%	1.22	0.75	15.59	0.24	1.50	0.01	0.04		
P8 (high ICOC)	5.2%	3.04	0.76	8.31	0.30	1.17	0.84	4.98		
P8-P1	2.9%	1.25	0.00	0.03	0.06	0.22	0.83	3.83		
		Panel	C: RIM	13 ICOO	C					
P1 (low ICOC)	1.0%	0.80	0.83	15.20						
P8 (high ICOC)	12.5%	2.68	0.59	2.18						
P8-P1	11.5%	2.25	-0.24	-0.78						
P1 (low ICOC)	1.3%	1.02	0.82	16.61	0.01	0.12	-0.05	-0.56		
P8 (high ICOC)	7.5%	4.57	0.67	8.63	0.23	4.13	0.88	4.96		
P8-P1	6.2%	2.91	-0.16	-1.39	0.22	0.88	0.93	4.14		

Table 2.6: Risk-Adjusted Excess Returns - United States

This table reports the results from time-series regressions based on the CAPM and the Fama and French (1993) three-factor model. We regress the portfolio excess return (over the U.S. risk-free rate) of the low-ICOC portfolio P1, the portfolio excess return of the high-ICOC portfolio P8, as well as the return of the long-short investment (P8-P1) on the market excess return and the Fama-French factors:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{m,t} - r_{f,t}) + \gamma_i SMB_t + \delta_i HML_t + \varepsilon_{i,t}$$

$$(2.9)$$

where $r_{i,t}$ is the 12-month return of portfolio i; $r_{f,t}$ is the U.S. risk-free rate, and $r_{m,t}$ the market return (Standards&Poor's stock index) over the same time period; SMB_t is the Fama-French small firm factor (the excess return of a portfolio of small firms over a portfolio of large stocks), and HML_t is the book-to-market factor (the excess return of a portfolio of high-value firms over a portfolio of low-value firms). SMB is constructed by ranking all stocks in ascending order on market equity value at the beginning of the twelve-month period. The stocks below the median size end up in the portfolio S, the stock above the median in B. Similarly, HML is formed by ranking all stocks in ascending order on book value of equity divided by its market value at the beginning of the twelvemonth period. The stocks below the median end up in the portfolio L, the stock above the median in H. For details on portfolio construction, see Fama and French (1993).

The analysis is carried out over whole overlapping data set. Thus, the t-statistics are calculated using Newey-West (1987) HAC standard errors, using 11 lags. The sample period is from January 1995 to February 2006. Country: United States. Observations: 40,158.

The overall conclusion from table 2.6 is that a risk adjustment for the three Fama-French factors makes at least the RIM3 ICOC effect to appear even more in conflict with the joint hypothesis of market efficiency and both Fama-French model and CAPM theory. Consequently, the implied costs of capital cannot be regarded as a mere transformation of the risk factors B/M ratio and firm size, but rather as a pricing anomaly with respect to these commonly used asset pricing models.

2.4.4 The Impact of Transaction Costs

The previous analysis makes a case for investing according to the implied cost of capital as stock selection principle. However, in real life, the required regular portfolio adjustments do not come for free, but give rise to transaction costs that reduce the risk-adjusted excess returns. Hence, in this chapter we re-examine the profitability of ICOC-based trading strategies including transaction costs.

We assume that the so-called "round-trip" transaction costs, i.e. the cost of replacing a share in a portfolio with a new one, amount to 2 percent of the share price. This is a very conservative assumption, especially since our investments consist mainly of blue-chip stocks²³. In order to assess the impact of transaction costs induced by portfolio rebalancing, we first calculate the turnover ratios of the trading strategies considered. We analyze two alternative investment strategies. One approach examines the previously examined long-short investment (P8-P1), the other one consists just in holding a long position in the high ICOC portfolio (P8). Since the DDM strategies have shown to perform very badly, we do not examine them here. Again, we consider annual portfolio reshufflings. The long-short investments require an annual turnover of 45% for the RIM3 ICOC strategy and 56% for the RIM2 ICOC portfolio. Long-only investments necessitate slightly higher turnover rates, attaining 48% for the RIM3 strategy and 61% for the RIM2 ICOC.

Round-trip transaction costs of 2% then mean, for each investment strategy, annual transaction costs equal to 2% of its annual turnover. Hence, the transaction costs reduce the risk-adjusted excess returns as shown in table 2.6 by roughly 1 to 1.2 percent, depending on the strategy examined.

The long-short RIM2 investment yields hence a risk-adjusted return of 2% p.a. with respect to the Fama-French model after considering the effect of transaction

estimation results.

 $^{^{23}}$ Related papers assume much lower transaction costs. Barber et al. (2001) e.g. use 1.31 percent cost per round-trip transaction. Moreover, they indicate that transaction costs amount to only 0.727 percent for large firms.
costs. When assuming that the standard deviation of the portfolio's risk-adjusted net return is equal to its risk-adjusted return before transaction costs, this yield is however not statistically different from zero, with a t-statistic of only 0.88.²⁴ The RIM3 ICOC long-short strategy yields in contrast 5.3% p.a. after transaction costs; a yield that is statistically significant from zero with a t-value of 2.59. Similar to the preceding analysis, long-only investments yield higher returns, although below the benchmark that is given by the market return. The RIM2 long-only investment attains 4.0% after considering transaction costs (with a t-statistic of 2.42), and the RIM3 longonly strategy yields 6.3%, also highly significant with a t-statistic of 3.93. Hence, the ICOC effect is even profitable for real-world investors that face both transaction costs and short selling constraints in stock markets.

2.5 International Implied Cost of Capital Strategies

In this section, we investigate the ICOC effect across other large equity markets. Thus, we repeat the previous portfolio analysis for other important international capital markets, i.e. the remaining G7 countries Canada, France, Germany, Italy, Japan, and the U.K.

As with the U.S. data, monthly data for prices, total returns, book values per share, dividends per share, market capitalization, and returns on equity are taken from MSCI. Earnings estimates as well as the long-term growth rate are taken from IBES median estimates. The time series data of national accounts to calculate the expected nominal GDP growth rate is obtained from Eurostat again²⁵. All data are denoted in local currency. If quoted in deviant currencies, the data is converted into the local currency, where the conversion is accomplished by using the WM Company exchange rate as of the date of the data. With over 100,000 monthly observations, our data sample is the largest for Japan, and the smallest for Italy, with just over 22,000 monthly observations.

The concept of the implied cost of capital should work equally well in all equity markets. Therefore, we would expect similar results among all G7 countries. However, there are large differences in data availability and the underlying economies across

²⁴Since turnover ratios are not constant, the standard deviation of the net excess returns should be higher than that of the excess returns before transaction costs. Thus, the statistical significance of the long-only investment returns calculated here might be overstated.

²⁵Due to the long-lasting recession in Japan, the geometric nominal GDP growth rate over the past 5 years (used as a proxy of the expected GDP growth), is not always positive. In order to ensure the existence of a positive root of the present value formulas, we replaced negative geometric nominal GDP growth rates by an expectation of 1%.

the various markets. First, the seven countries exhibit considerable discrepancies in performance over the sample period from 1995 to 2006, such as the long-lasting recession in Japan or the stock market bubble in the western countries. A second source of possible divergence is the availability of IBES data. The two-stage RIM as well as the DDM rely heavily on the long-term consensus growth rate of equity analysts. Estimating such growth rates has a long tradition in the U.S., but not in European countries, where analysts have rarely published such forecasts over the early years in our study period and have concentrated instead on explicit earnings forecasts for the next years. Hence, implementing these models in European (and the Japanese) markets leads to a considerable decrease in the available data set. Finally, deviating accounting standards for book value of equity across the countries considered add some uncertainty to the implied returns derived from the RIMs.

2.5.1 Tests on Implied Cost of Capital-based Investment Portfolios

We replicate the portfolio analysis as presented in section 2.4 for the six other capital markets. The portfolio formation is carried out as for the U.S. data sample. Table 2.7 reports the average total returns of the low-ICOC and high-ICOC portfolios, together with their average difference. The rebalancing interval is again twelve months. Note that because of smaller data sets, we reduced the number of portfolios for Canada, France, Germany, and Italy to 5 only. In addition, we report in table 2.7 the portfolio alphas for a long-short strategy corrected for risk exposures using the Fama-French model, similar to table 2.6. The last two lines give the number of observations and the equally weighted return of the whole sample²⁶.

When looking at the raw-returns, i.e. returns not corrected for their respective risk exposure, we can detect large differences across countries and ICOC approaches. In Germany, the ICOC effect is insignificant or even negative, in Canada, the U.K., and especially in Japan, the ICOC effect is highly significant, attaining a yield up to 25.3% p.a. for the RIM2 in Japan. Interestingly, in the continental European countries the ICOC obtained from the dividend discount model causes larger returns of long-short investments compared to their RIM counterparts. This difference between both ICOC approaches becomes even more apparent when adjusting the portfolio returns for risk exposure using the Fama-French factors. Risk-adjusted returns obtained from the DDM ICOC are significant in all countries, yielding 5%-16% of excess returns annually.

 $^{^{26}}$ We can give only a summary of the main findings here. We refrain from displaying portfolio characteristics, neither do we show the returns of investment strategies using different holding periods.

	Country	CA	FB	GE	IT	IP	UK
DDM3	P1 (low ICOC)	7 9%	14.9%	8.2%	15.9%	3.1%	6.6%
DDM0	P8 (high ICOC)	19.5%	23.6%	13.2%	31.1%	16.0%	12.9%
	1 0 (iligii 1000)	15.570	20.070	10.270	01.170	10.070	12.570
	Difference (P8-P1)	11.7%**	8.7%***	$5.0\%^{*}$	15.1%**	12.8%***	$6.3\%^{*}$
	t-stat	2.40	2.71	1.69	2.12	7.31	1.80
	FF-alpha (P8-P1)	$11.8\%^{***}$	$8.9\%^{***}$	$5.0\%^{***}$	$16.8\%^{***}$	$12.2\%^{**}$	$7.4\%^{*}$
	t-stat	3.39	6.96	3.12	3.29	4.84	1.93
RIM2	P1 (low ICOC)	8.6%	13.7%	10.2%	13.3%	-2.0%	7.0%
	P8 (high ICOC)	17.2%	20.0%	14.0%	22.4%	23.3%	20.0%
	Difference (P8-P1)	$8.6\%^{*}$	6.3%	3.7%	9.1%	$25.3\%^{***}$	$13.9\%^{***}$
	t-stat	1.88	1.27	0.97	1.49	5.61	3.36
	FF-alpha (P8-P1)	$5.0\%^{*}$	$6.6\%^{***}$	3.0%	4.1%	$17.5\%^{***}$	$10.0\%^{**}$
	t-stat	1.84	2.60	1.03	1.16	4.74	2.21
RIM3	P1 (low ICOC)	10.9%	15.2%	13.6%	14.5%	0.2%	6.1%
	P8 (high ICOC)	20.8%	22.0%	13.0%	26.3%	15.3%	21.2%
	, <u> </u>						
	Difference (P8-P1)	$9.9\%^{*}$	6.8%	-0.6%	11.7%	$15.1\%^{***}$	$15.1\%^{***}$
	t-stat	1.93	0.89	-0.12	1.13	3.85	2.89
	FF-alpha (P8-P1)	2.8%	$5.1\%^{*}$	3.0%	5.0%	4.0%	5.3%
	t-stat	0.62	1.79	0.69	0.76	1.27	1.21
Portfolie	OS	5	5	5	5	8	8
Observa	tions	$5,\!03$	8,58	$6,\!485$	2,926	16,713	13,768
EWA		15.7%	17.8%	12.9%	18.6%	8.5%	11.5%

Table	e 2.7:	Raw	Returns	and	Risk-ad	justed	Excess	Returns	from	Buy-an	d-Hold	Port-
folios	- Int	ernat	ional An	alysi	\mathbf{S}							

This table reports the average, equally weighted buy-and-hold returns of the high and low ICOC portfolios, together with their average difference and t-stat thereof for the other G7 stock markets: Canada, France, Germany, Italy, Japan, and the U.K. The holding period of the portfolios is twelve months. The averages are calculated using overlapping observations. Hence, the t-statistic of the difference is calculated using Newey-West (1987) HAC standard errors. In addition, we also report the portfolio alphas for a long-short investment strategy with respect to the Fama-French model to examine the ICOC effect when correcting for portfolio risk, using the same regression methodology as for the U.S. data (see table 2.6 for a detailed explanation). The last three lines give the number of portfolios employed to sort the companies in (8 for Japan and the U.K., otherwise 5), the number of the overall observations available for each market and the equally weighted return over the whole sample. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level. The sample period is from January 1995 to February 2006.

Compared to this, the ICOC of the RIM3 model - performing well in the U.S. - is only statistically different from 0 in France with a long-short return of around 5%. The RIM2 ICOC yields variable returns, exhibiting large differences across countries: a mere 3% abnormal return in Germany to more than 17% in Japan.

The results indicate that the ICOC effect is not limited to the U.S. capital market, but is also present in all important stock markets. In addition, the international analysis shows that in many countries this stock market anomaly with respect to the Fama-French model is even more pronounced than in the U.S, at least when referring to the DDM-based ICOC. The fact that the efficacy of the ICOC effect is not equally strong in all stock markets should not be too surprising: varying accounting standards and large differences in the availability of IBES data is certainly contributing to some of the discrepancies. Furthermore, it seems that in continental European countries, the dividend policy conveys more information about future profits (see e.g. Nissim and Ziv (2001)) than in the U.S. Such a difference could explain the better performance of DDM-based strategies in these countries. Moreover, the low correlation between DDM-ICOC and risk factors (in all countries) results in a rather low risk-exposure of the respective investment portfolios, whereas the RIM portfolios have rather high risk loadings.

2.6 Conclusion

The recently developed concept of the implied cost of capital has become a popular tool for estimating expected stock returns both in academia and practice. By aggregating individual stock returns over the entire market, this approach is used extensively in economic research to derive a forward-looking equity risk premium estimate. Fund managers try to exploit the so-obtained expected returns to improve the performance of their investment portfolios.

Surprisingly, a sound econometric foundation of investment strategies based on analysts' forecasts derived from the implied cost of capital methodology is still missing. In this work, we employ a simple trading strategy to test the ICOC's ability to generate excess returns. Using three different specifications of the implied return, we show that an investment that buys high-ICOC stocks and sells short low-ICOC stocks yields an annual return of up to 10% in the U.S. equity market. When correcting the investment for the portfolio's exposure to the Fama-French risk factors, it still yields a risk-adjusted return of 6.2% p.a. The international extension of this study to other large capital markets confirms the results from U.S. data sample. Although the magnitude of the ICOC effect varies across present value models employed and capital markets examined, the observed stock returns are inconsistent with the joint hypothesis of efficient markets and standard asset pricing models.

The implication of our findings is not clear-cut - similar to any asset pricing anomaly. Either the asset pricing models are wrong, or markets are not efficient, or - as Fama (1998) claims - the detected anomaly is a result of data-snooping. Since the ICOC effect is present across all large stock markets, we do not believe in the latter explanation. Moreover, related studies like Womack (1996) and Barber et al. (2001) document similar evidence. Hence, if one is then to join the majority in accepting the Fama-French asset pricing model, our findings suggest that markets are not efficient. This imperfect market efficiency is even more remarkable since the ICOC effect persists even after including transaction costs - a requirement that other market anomalies often do not meet. There are several reasons to explain this market inefficiency. Analysts might have better access to relevant company-specific news, or they are superior in processing such information. Or market prices may react either too slowly or too excessively to publicly available information, such as company announcements or changes in the general economic outlook. The fact that ICOC strategies are particularly profitable during periods of possibly exaggerated stock market movements might underpin this hypothesis. In either case, this would underline the additional information provided by analysts' forecasts and thus the importance of equity analysis. Inefficient markets also imply that the ICOC cannot be an expected return proxy, since the assumption of equalization of market prices and fundamental firm value is violated. Thus, this study contributes to more recent empirical evidence that challenges the use of ICOC as expected return estimate in the academic literature.

Our results have also important practical implications for portfolio managers. This chapter puts forward evidence for the profitability of ICOC-based investment strategies, even after correcting for risk exposure and transaction costs, which makes this study distinct from comparable trading strategies based on equity analysts' recommendations. Still, large discrepancies between the various ICOC approaches underpin the importance to carefully select the right approach.

Chapter 3

Predicting Equity Returns with the Implied Cost of Capital

We show that investors can use publicly available information of equity analysts to successfully predict stock returns. More precisely, we employ the so-called implied cost of capital (ICOC) to forecast stock returns. Calculated as the internal rate of return that equates share price with discounted analysts' forecasts, the ICOC represents a convenient transformation of a variety of analysts' expectations into a single forecasting variable. Panel regressions across the world's largest capital markets show that the ICOC is better in predicting individual stock returns than traditional valuation multiples - even after taking into account the stocks' riskiness, as implied by standard asset pricing models.

3.1 Introduction

Fundamental equity research lies at the heart of any successful fund management and stock broking business. Analysts put a lot of effort into estimating future earnings per share, dividend payments, and long-term earnings growth rates for many listed companies around the globe. In combination with a model for risk-adjusted expected returns, these forecasts are then usually used to derive for each security an estimate of its underlying fundamental value, also called target price or fair value. Based on these estimates, brokerage firms formulate recommendations about the optimal composition of their clients' portfolios. If a share trades below its estimated fair value, the stock is recommended to be added to the portfolio - if not, the share is not a suggested investment.

In this chapter we take a different approach. Instead of employing analysts' forecasts to derive estimates for the shares' fair values, we use these expectations to compute for each share its so-called implied cost of capital (ICOC). Calculated as the internal rate of return¹ that equates current share price with discounted future analysts' forecasts, the ICOC represents a convenient transformation of analysts' expectations about future dividends, earnings, and growth prospects into a single forecasting variable. If analysts' forecasts contain valuable information about the companies' future prospects, the ICOC should be a good predictor of stock returns: the ICOC is just the share's rate of return that is required to reconcile its market price with future cash flows as predicted by analysts.

To see this more clearly, consider the example of a firm that operates for one period only. At the end of the period, the firm is assumed to pay a final dividend D_1 . Suppose that analysts predict a dividend D_1 of 11 USD, and that the shares trade at the beginning of the period at 10 USD. If analysts are right in their prediction and markets are fully efficient, the share has to yield a return equal to the ICOC of 10% over this period. Of course, in case that analysts' forecasts turn out not to be fully correct, the realized return will diverge from the ICOC's prediction. But unless the analysts' forecasting error is too large, the ICOC will not differ by much from the subsequent stock return, and should therefore be a good predicting variable.

The objective of this chapter is to analyze empirically whether the ICOC as derived from analysts' forecasts is a good predictor of stock returns. To have a benchmark, we compare the ICOC's forecasting ability with a variety of variables that are traditionally used to predict stock returns, such as the dividend-price ratio, the book-price ratio, or various implementations of the so-called value-price ratio². We examine the relation between stock returns and the ICOC as well as other forecasting variables by carrying out panel regression tests across the world's largest equity markets. Since the variables' predictive power for stock returns might originate only from their underlying relation to firm risk proxies, we test the forecasting variables against both the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) and the three-factor model by Fama and French (1992, 1993, 1996), which have been widely accepted as asset pricing models. When controlling for the standard risk-return relation implied by these models, only independent forecasting variables should be significantly connected to stock returns - otherwise they prove to be only a transformation of the models' risk proxies. Thus, our stock return regressions provide also a test of the informational content of analysts' forecasts. Only if the ICOC coefficients remain significant in regression tests including firm risk variables, analysts' expectations contain

¹In this study, the terms *implied cost of capital* (ICOC) and *implied return* refer to the same concept and are used interchangeably.

 $^{^{2}}$ The value-to-price ratio (Lee et al., 1999) is the quotient of estimated fundamental company value to market value. This approach is to be briefly reviewed below.

3.1. INTRODUCTION

indeed additional value for investors beyond risk-adjusted expected returns.

A first series of papers that use analysts' forecasts to predict stock returns is by Lee and Swaminathan (1999) and Lee et al. (1999). These empirical studies derive - similar to the standard approach of equity analysts - an estimate of the Dow Jones Industrial Average's intrinsic values' by discounting analysts' forecasts with appropriate risk-adjusted expected returns. They show that the ratio of estimated value to market price of this set of companies (V/P ratio) is better in predicting average stock market returns than traditional valuation multiples such as the book-price, earnings-price, or the dividend-price ratios.

Compared to the V/P ratio, the ICOC's primary advantage is that it does not require any estimate or assumption on the discount factor for its calculation. It thereby avoids a potential source of errors, which should consequently result in a higher accuracy of forecast. Our work enlarges the scope of existing studies by extending the analysis to many different versions of both the ICOC and the V/P ratio. By testing the predictive power of forecasting variables against the risk-return relation of common asset pricing models, our study allows to disentangle the variables' intrinsic forecasting ability from standard firm risk effects - a crucial point that is often neglected in stock return regressions. In addition, our study uses a much broader international set of data by investigating both price-value and ICOC concepts for individual stocks across the capital markets of all G7 countries.

The fact that equity analysts' recommendations provide valuable information for investors has been underlined in many empirical studies. Womack (1996) or Barber et al. (2001) are among the first to show that there exist profitable trading strategies based on analysts' forecasts, which cannot be explained by standard asset pricing models, such as the CAPM or the Fama-French three factor model. Such studies concentrate either on the information contained in rather simple buy- or sell recommendations (Barber et al., 2001), explicit next-year earnings forecast (Xu, 2002), or changes in these forecasts (Womack, 1996).

This study contributes also to this stream of literature. By showing that the ICOC is a significant variable in explaining the cross-sectional variation in stock returns after controlling for standard firm risk factors, we underline the additional information contained in analysts' forecasts. Hence the ICOC effect is even present at the level of individual firm data, and not an artefact of the portfolio strategy employed in Esterer and Schröder (2007a).

More recently, the ICOC has been used as a proxy for expected stock returns. Under the assumptions that market prices equal fundamental value as predicted by analysts' forecasts and that these projections are a good surrogate for the average investor's expectation, this interpretation is indeed valid. The primary advantage of this method to estimate expected stock returns is the ICOC's outright forward-looking perspective, since it is only relying on available estimates regarding the companies' future prospects and the current share price. Thereby this approach avoids all the problems related to using historical stock returns as proxy for expected returns. Presumably because of this advantage, the ICOC got increasingly popular for estimating expected stock returns in the finance literature. On the one hand for example, Cornell (1999), Claus and Thomas (2001), and Gebhardt et al. (2001) use the ICOC to estimate an equity risk premium by aggregating the implied returns over entire stock markets. On the other hand, Pástor et al. (2006) rely on the implied return as a proxy for expected returns to examine the conditional mean-variance relation of individual stock returns. However, this literature contrasts sharply to studies that challenge the ICOC's ability to be a proxy for expected stock returns. Although Gode and Mohanram (2003) document a positive relation between ICOC and stock returns, the findings of Botosan and Plumee (2005), Guay et al. (2005), or Easton and Monahan (2005) suggest that the ICOC is a very poor estimate for expected stock returns. Since in equilibrium expected returns equal realized returns, our study can also be conceived as a test whether the ICOC is a good proxy for expected returns. However, many previous attempts to explain stock returns with ICOC estimates failed because of insufficient power of the Fama and MacBeth (1973) regressions, a point remarked by e.g. Guav et al. (2005). The use of panel regressions to analyze the ICOC's relation to stock returns enables us to provide considerably clearer evidence on the determinants of stock returns compared to existing studies³.

This study documents that the implied cost of capital is positively related to stock returns. Implied cost of capital estimates can explain up to 13% of the cross-sectional variation in stock returns in the U.S. American stock market over a time horizon of 24 months. More important, the ICOC's ability to predict stock returns is higher than traditional valuation multiples and the V/P ratio as developed by Lee et al. (1999). Joint regression tests of stock returns on the ICOC estimate and standard CAPM or Fama-French risk factors highlight that ICOC coefficients remain highly significant even after controlling for the stocks' risk. Thereby, we confirm the ICOC's additional informational content obtained from equity analysts' forecasts. These findings com-

³Our analysis provides however only a rather simple test whether the ICOC is a good proxy for expected stock returns. Easton and Monahan (2005) point out that realized returns differ from expected returns even on average and over longer time horizons, due to information surprises. A more exact way to analyze the relation of ICOC to stock returns would be to carry out the decomposition of stock returns into expected returns, return news, and cash flow news, following Voulteenaho (2002). We do not provide such a detailed analysis here.

pare to the much weaker support of traditional multiples which are often less related to stock returns after controlling for firm risk. Estimations across other international capital markets confirm these findings.

This chapter proceeds as follows. In the next section, we first present the concept of the implied cost of capital and the value-price ratio in more detail. Then we introduce the present value formulas to derive them, i.e. the dividend discount model and the residual income model. Section 3.3 contains a brief description of our U.S. data sample. In section 3.4 we analyze the U.S. equity market in detail. By performing panel regression tests, we show that the ICOC is better in predicting equity returns than many other common forecasting variables, including the value-price ratio. Finally, in section 3.5, we extend our study internationally and provide results obtained from the six other capital markets of the G7 countries. Section 3.6 concludes.

3.2 Implied Cost of Capital and Value-Price Ratio

In this study, we use publicly available information of equity analysts to predict stock returns. To capture analysts' forecasts into one single forecasting variable, we rely on the firms' implied cost of capital estimates (ICOC). The basic idea of the ICOC approach is to estimate an expected future cost of capital with the help of present value models. More precisely, the cost of equity is computed as the internal rate of return that equates expected discounted payoffs per share to current price, where expected cash flows are taken from equity analysts. Thus, the ICOC allows a systematical connection between analysts' forecasts and share prices via valuation models. The ICOC approach shares its ability to transform analysts' forecasts into one single predictive variable with the closely related value-to-price ratio (Lee et al., 1999). The value-to-price (V/P) ratio is defined as the ratio of fundamental firm value to current market capitalization. Similar to the ICOC, this approach uses present value models and analysts' forecasts to estimate the fundamental value of firms. More specifically, fundamental firm value is calculated by discounting analysts' forecasts with an appropriate required rate of return. Then this fundamental value is divided by the companies' market capitalization to obtain the value-price ratio⁴. Because of its similar structure, the V/P ratio can also be conceived as the "inverse" of the ICOC. The former uses the present value relation for estimating the fair value of the firm with the help of an assumed discount rate, whereas the latter solves the equation for the discount factor. Hence, the primary advantage of the ICOC concept over the V/P ratio

 $^{^4\}mathrm{For}$ a more detailed discussion of the V/P ratio, please refer to Lee et al. (1999).

is that it does not require any estimate or assumption on the discount factor for its calculation. It thereby avoids a potential source of errors, which should consequently result in a higher accuracy of forecast⁵.

3.2.1 The Present Value Formulas

Many different present value models can be used to estimate the implied cost of capital or the value-price ratio, respectively. In this work we resort to the two most common present value models, as employed in prominent studies on the implied cost of capital. On the one hand, we use the dividend discount model (DDM) in the version of Cornell (1999). On the other hand, we rely on the residual income models (RIM) to appraise the firms in our sample. We employ the model of Claus and Thomas (2001), as well as a slightly modified version of the Gebhardt et al. (2001) approach which uses a more consistent calculation of the terminal value⁶. Given some assumptions such as clean surplus accounting, most of the approaches are equivalent in theory (Feltham and Ohlson, 1995). The so-called "clean surplus" assumption for example requires that all gains and losses affecting book value are also included in earnings. In practice, this condition is not always met. Stock options or capital increases e.g. can affect the book value of equity while leaving earnings unchanged. In addition, limited data availability such as missing long-term earnings projections put further restrictions on the theoretical equivalence. As a result, the structural assumptions in building a valuation model significantly prejudice the results as we will see later.

The Dividend Discount Model

The general DDM states that the price of a share should equal the discounted value of future dividend payments, and can be written as follows:

$$P_0 = \sum_{t=1}^{\infty} \frac{E_0[D_t]}{(1+k)^t}$$
(3.1)

⁵The fact that the ICOC does not require any assumed discount rate for its calculation does not imply that we do not need a discount factor (or a model for a discount factor) at all when applying the ICOC. We have to rely on some discount factor model to evaluate and interpret the ICOC, as we will see later.

⁶Note that we do not present a detailed discussion about theoretical and empirical differences between residual income valuation, and the dividend discount model approach. For a thorough comparison, see for example Penman and Sougiannis (1998), Penman (2001), and Lundholm and O'Keefe (2001*b*,*a*). A short analysis of both models in the context of the ICOC approach can be found in Schröder (2007*a*).

where

$$P_0$$
 = current share price, at the end of year 0,
 $E_0[D_t]$ = expected dividends per share at the end of year t,
 k = cost of capital or, equivalently, shareholders' expected rate of return.

Since exact predictions of future dividends cannot be made to infinity, one has to make assumptions about expected cash-flows when implementing the model in practice. The DDM following Cornell (1999) assumes an initial 5-year phase of rather high dividend growth, which is followed by a transition phase in which the growth rates decline linearly to a lower, stable growth rate g_l , which is then maintained ad infinitum. Thus, this model combines the plausible conjecture of a possibly strong growth in the first years with realistic growth rates in the long run.

$$P_0 = \underbrace{\sum_{t=1}^{5} \frac{E_0[D_t]}{(1+k)^t}}_{t=1} + \underbrace{\sum_{t=6}^{20} \frac{E_0[D_t]}{(1+k)^t}}_{t=1} + \underbrace{\frac{E_0[D_{20}](1+g_l)}{(k-g_l)(1+k)^{20}}}_{(k-g_l)(1+k)^{20}}$$
(3.2)

Growth period Transition period Stable growth

In the initial phase, the dividend growth is assumed to equal the long-term consensus earnings growth rate, obtained from equity analysts⁷. In the stable phase following year 20, the dividend growth rate equals the estimated long-term GDP growth of the economy (Cornell, 1999)⁸. In this study, we use a simple moving average forecast model and calculate the expected GDP growth rate as the average geometric nominal GDP growth rate over the past 5 years. Due to its three-stage structure, we refer to this model as DDM3 hereafter.

Note that we assume in equation (3.2), as well as in all subsequent present value formulas, an identical cost of capital k over all time periods. In the view of timevarying equity risk, this might not be an appropriate assumption. However, since the inclusion of a time-varying component usually leads to quite similar results (Claus and Thomas, 2001), we maintain the simple constant discount rate specification.

It is well known that due to conflicts of interest, equity analysts tend to overstate long-term growth projections (see e.g. Chan et al. (2003)), which hence biases the ICOC estimates upwards. Since this study focuses on the analysis of implied costs of

⁷The findings of Elton et al. (1981) suggest that analysts' forecasts are a good surrogate for investor expectations. The consensus growth rate is provided by IBES and is calculated as the median of the expected earnings growth rates of the contributing sell-side equity analysts.

⁸Usually, we have $g > g_l$, in line with the basic idea of the model. For some companies however, the long-term consensus earnings growth rate g lies below the expected GDP growth rate of the economy g_l .

capital of individual firms and differences thereof, this bias would only cause problems if there was some systematic relation between the degree of biases and some firm characteristic. We are not aware of any study documenting such a relation.

Note that the DDM requires the annual dividend D_0 which just has been paid out to the shareholders. Based on D_0 it is then possible to calculate the series of future dividend payments, beginning with D_1 . In this study, we use the sum of all dividend payments over the last 12 months, also known as 12-month trailing dividends, as D_0 . We exclude all observations from the sample, where the 12-month trailing dividend equals 0.9

The Residual Income Model

Another popular valuation formula is the RIM, stating that the value of the company equals the invested capital, plus the expected residual income from its future activities:

$$P_0 = B_0 + \sum_{t=1}^{\infty} \frac{E_0[R_t]}{(1+k)^t}$$
(3.3)

with

$$E_0[R_t] = E_0[E_t] - k(B_{t-1}) = (E_0[roe_t] - k)B_{t-1}$$
(3.4)

$$B_t = B_{t-1} + (1 - p_t)E_t \qquad \text{(clean surplus assumption)} \tag{3.5}$$

where

B_t	=	book value of equity per share at the end of year t
		$(B_0 \text{ being the current book value}),$
$E_0[R_t]$	=	expected residual income per share in year t ,
$E_0[E_t]$	=	expected earnings per share in year t ,
$E_0[roe_t]$	=	expected return on equity in year t ,
p_t	=	payout ratio in year t .

Similar to the DDM, assumptions about the future growth in residual incomes or earnings have to be made when implementing the model in practice. One rather

⁹Of course, it would have been possible to include non-dividend paying companies into the sample by making additional assumptions on future dividends with the help of expected earnings and average payout ratios. We discarded this idea since it would have resulted in mixing different valuation indicators (dividends for the DDM vs. earnings and book values for the RIM). Moreover, the information presumably contained in dividend payments cannot be recovered by making assumptions on payout ratios.

simple approach is proposed by Claus and Thomas (2001), who consider a two-stage RIM (denoted RIM2 in the following), assuming an initial phase of high earnings growth rates, followed by a stable growth of residual incomes after year five:

$$P_0 = B_0 + \underbrace{\sum_{t=1}^{5} \frac{E_0[E_t] - k(B_{t-1})}{(1+k)^t}}_{(1+k)^t} + \underbrace{\frac{E_0[R_5](1+g_l)}{(k-g_l)(1+k)^5}}_{(k-g_l)(1+k)^5}$$
(3.6)

Growth period Stable growth

Expected earnings for the first three years are taken from analysts' forecasts, also provided by IBES. Earnings after year 3 are estimated by applying the IBES consensus long-term earnings growth rate to the expected earnings of year 3.¹⁰ The growth rate in the second phase is presumed to equal the expected inflation rate calculated as the prevailing interest rate on 10-year treasury bonds less the assumed real-rate of three percent¹¹. Future expected book values of equity are calculated using equation (3.5). To that end, we have to make assumptions regarding future payout ratios. In a slight variation to the methodology of Claus and Thomas (2001), we let the current payout ratio geometrically converge towards 50% over the growth period instead of using this ratio from the first prospective year on to project future book values. This approach seemed more realistic to us. Current payout rations are calculated by dividing the 12month trailing dividends by the 12-month trailing earnings per share. This method ensures that the payments refer to the same time period as the earnings. Payout ratios above 1 are set to 1 in the first year, negative payout ratios are set to 0. If the payout ratio was missing due to missing dividend payments over the last 12 months, we equally set the payout ratio of year t = 1 to 0.

Gebhardt et al. (2001) rely also on the RIM to calculate the implied cost of capital. However, in contrast to Claus and Thomas (2001), they focus their assumptions not on future residual incomes, but more directly on the future return on equity (*roe* - see equation (3.4), which they assume to converge to the industry median. More formally:

¹⁰In the case where the expected earnings estimate of year 3 was missing, we also generated earnings in year 3 by applying the long-term consensus growth rate to expected earnings of year 2. If the projected earnings in year three were negative, we dropped the observation from the sample.

¹¹The assumption of no real growth in residual income after year 5 does not imply the absence of real earnings growth after year 5. It rather incorporates the assumption of decreasing earnings growth rates in the stable growth phase, for the usual reasons (competition, antitrust actions, etc.), similar to the transition phase of the DDM3. For more discussion, see Claus and Thomas (2001, p. 1640 and p. 1663ff).

$$P_{0} = B_{0} + \underbrace{\sum_{t=1}^{3} \frac{E_{0}[roe_{t}] - k}{(1+k)^{t}} B_{t-1}}_{\text{Explicit forecasts}} + \underbrace{\sum_{t=4}^{T} \frac{E_{0}[roe_{t}] - k}{(1+k)^{t}} B_{t-1}}_{\text{Transition period}} + \underbrace{\frac{E_{0}[iroe_{T}] - k}{k(1+k)^{T-1}} B_{T-1}}_{\text{Terminal value}}$$
(3.7)

where

T = forecast horizon of the transition period (9 years) $E_0[iroe_T]$ = expected industry return on equity from period T onwards.

Similar to Gebhardt et al. (2001) we use explicit forecasts to calculate the expected return on equity for the next three years. Then we fade the roe over T-3 years to the industry *roe*. This industry *roe* is calculated as the median of all (positive) realized roe across all companies of the respective sector, over the preceding 60 months. This procedure aims to average out business cycle effects of the industry profitability¹². We use the industry sector codes of the GICS classification for sorting the companies¹³. While the use of industry averages for the long-term return on equity has some appeal due to the findings of empirical analysis (Nissim and Penman, 2001; Soliman, 2004), the assumptions in the literature regarding the payout ratios to construct future book values of equity seem somewhat arbitrary to us¹⁴. As a consequence, we consider a slightly modified version of the three-stage RIM that avoids relying on assumptions on future payout ratios. Following the literature on sustainable growth rates, we use the following identity between payout ratio p, return on equity *roe* and the long-term growth rate of the company g_l :

$$\frac{g_l}{roe} = 1 - p$$

$$p = 1 - \frac{g_l}{roe}$$
(3.8)

 $^{^{12}}$ Long-term changes of industry profitability, such as declining returns of current growth industries, are not captured by this approach.

¹³In our standard implementation we fix T = 9. Instead of using the GICS classification, Gebhardt et al. (2001) rely on the 48 Fama and French (1997) industry classifications. We also tested other, more precise classifications such as the GICS Industry Group segmentation, or the Industry segmentation, but the results were very similar.

 $^{^{14}}$ As mentioned earlier, Claus and Thomas (2001) assume a payout ratio of 50% across all firms, whereas Gebhardt et al. (2001) hold the prevailing payout ratio of each company constant to estimate future book values.

In order to estimate the future development of a company, one has to make assumptions for two out of the three parameters¹⁵. Instead of assuming the long-run payout ratio p, we opt to fix the long-term growth rate of the company g_l . By setting g_l equal to the expected GDP growth rate of the economy as in the earlier model, we ensure that no company will persistently grow faster than the whole economy and eventually exceed it.

Hence, we employ the RIM following Gebhardt et al. (2001) as presented in equation (3.7), but using different projected payout ratios for each industry sector. For each sector, we calculate the long-term industry payout ratio using the relation (3.8), given the expected GDP growth of the economy and the industry *roe*. In the transition period, we then fade both payout ratio and return on equity towards their long-term levels. Since this model is basically a three-stage RIM, we denote it by the abbreviation RIM3.¹⁶

3.3 Data and Empirical Implementation

In our detailed analysis in section 3.4, we focus on companies in the United States covered by MSCI over a time period from January 1990 to February 2006. The monthly data for prices, total returns, book values and dividends per share, market capitalization, and returns on equity are taken from MSCI. All market capitalization data are free float adjusted. The earnings estimates as well as the long-term growth rate are taken from IBES median estimates. Time series data of national accounts to calculate the expected nominal GDP growth rate is obtained from Eurostat. Stock indices to derive market betas and deflate firm size data are taken from Datastream.

Our data set contains 1,507 companies and over 122,000 monthly observations. Since the number of companies included in the study changes over time, we have an unbalanced panel data set¹⁷. Since we require five years of company data to calculate the industry roe, we can only estimate the ICOC and valuation multiples from January 1995 on, leaving us with roughly 11 years of data. To calculate the implied cost of capital and the V/P ratio using the equations above, we employ the last available

 $^{^{15}{\}rm Of}$ course, this procedure incorporates the implicit assumption of a constant capital structure in the future.

¹⁶In practice, our RIM3 specification does not diverge substantially from the model as proposed by Gebhardt et al. (2001). The correlation of the ICOC estimates obtained from both models is very high for the United States (greater than 0.99).

¹⁷The available panel data set starts in 1995 with 334 monthly observations, reaches its maximum in June 2005 with 507 companies, and finally contains in February 2006 501 companies.

information as required by the formulas at the end of each calendar month. Thereby, we make sure that the ICOC estimates and valuation multiples are based only on publicly available information. Firms with an incomplete data set, i.e. one or more missing input variables where we could not resort to approximations as explained above, have been ignored¹⁸.

To estimate the ICOC, we solve the equations for the internal rate of return, given the prevailing share price and cash flow forecasts. The solution is straightforward, since they are monotone in k, and can thus be solved iteratively. To calculate the V/P ratio, we first estimate the value of a share with the help of the same present value formulas. In addition to the cash flow forecasts, we need also an assumption on the discount rate, as indicated above. We adopt a simple approach, as suggested by Lee et al. (1999), who employ a constant equity risk premium over the risk-free rate. We fix the risk premium at 5%. Similar to their approach, we use both longterm government bonds and short-term bills as risk-free rate. Finally, we divide the estimate for fundamental share value by its market price to obtain the value-price ratio. Negative value-price ratios have been dropped from the sample. Given the three valuation models above, we thus have six different estimates for the V/P ratio. The abbreviation RIM3 VPS indicates the V/P ratio derived from the RIM3 formula using short-term bills plus 5 percent as discount rate; RIM3 VPL stands for the V/P ratio derived from the RIM3 formula with a discount rate of long-term bonds plus 5 percent, and so on^{19} .

In the U.S., as table 3.1 shows, the average ICOC is clustered around 8% to 9% with the RIM3 having the lowest average expected return at 8.16% and the RIM2 having the highest at 9.20%. The standard deviation is the highest for the RIM3 model with 2.70%. The standard deviation is somewhat lower for the RIM2 model at 2.29% and the DDM3 has the lowest standard deviation at 2.04%. The mean V/P ratio lies between 0.7 and 1.3, i.e. around their theoretical value of 1. Since short-term interest rates have been lower than the long-term rate during the examined time period, the mean V/P ratio is higher when using short-term rates to discount future

¹⁸Note that we do not carry out any time adjustment procedures similar to other studies on the implied cost of capital. Since we use a monthly data set, such adjustments would require the exact dividend payout dates and book value adjustments for all companies since 1995. Such data is not easy to get hold of, nor is it reliable.

¹⁹The simple 5% assumption for the risk premium might appear somewhat arbitrary. However, Lee et al. (1999) do not find any significant differences when changing the risk premium assumption. They also use more complex models to calculate risk-adjusted discount rates for each industry group separately, using the Fama-French three-factor model. They find no substantial differences in their results.

	Mean	Standard Deviation	Min	Max
DDM3 ICOC	8.88%	2.04%	4.68%	43.42%
RIM2 ICOC	9.20%	2.29%	1.07%	51.10%
RIM3 ICOC	8.16%	2.70%	1.00%	38.76%
DDM3 VPS	1.29	1.39	0.01	55.40
DDM3 VPL	0.73	0.61	0.01	48.95
RIM2 VPS	1.08	0.52	0.01	18.32
RIM2 VPL	0.85	0.33	0.01	12.26
RIM3 VPS	0.98	0.63	0.08	22.61
RIM3 VPL	0.78	0.40	0.09	14.67

Table 3.1: Descriptive Statistics - United States

This table gives and overview about both the implied return and the value-price ratio for the U.S. data sample. The first three lines report the means, standard deviations, and the minimal and maximal observation of the implied return derived from the different valuation models. The last six lines show a summary of the value-price estimates. VPL stands for the value-price ratios using long-term government bonds as interest rate, VPS uses short-term bills. Observations: 49,539.

cash flows. The V/P ratios are the highest (and most volatile) for the DDM3 formula, and the lowest (and least volatile) when using the RIM3 approach.

Since the number of observations based on the DDM3 and RIM2 formulas is constrained by the availability of the IBES long-term growth rate and the payout ratio, we will use for the following analysis only the data set of 49,539 observations for which all forecasting variables were available. However, using the more complete data set for each variable leads to the same results in most cases.

As was to be expected, the correlation between structurally similar models is rather high, as can be seen in table 3.2. For example, the ICOC from RIM2 and RIM3 are highly correlated (0.64), the V/P ratios using different interest rates, such as RIM3 VPL and RIM3 VPS (0.92), or the V/P ratio with the ICOC estimate when obtained from the same present value formula (RIM3: 0.92). On the other hand, correlations between different models are generally rather low, such as the ICOC estimates from DDM3 and the RIM3 (at around 0.20). There is little correlation between the ICOC and V/P estimate with the Fama-French risk variables, B/M ratio and market capitalization. The only notable exception is the estimates using the RIM3, which exhibit a very high correlation up to 80% with the book yield. Given the predominant role of book value of equity in the RIM3 formula, this high correlation is in line with our expectations.

-0.18	-0.1362	-0.1334	-0.0898	-0.0231	0.0093	-0.2271	-0.1373	-0.0324	MCAP
	0.7095	0.3830	0.3575	0.2087	0.2231	0.8071	0.3254	0.2513	BM
	0.9237	0.6567	0.6430	0.1992	0.2994	0.9222	0.5005	0.1479	RIM3 VPL
	1.0000	0.5715	0.7405	0.1722	0.4404	0.7818	0.3555	0.0525	RIM3 VPS
		1.0000	0.8584	0.2620	0.2828	0.6599	0.8949	0.2144	RIM2 VPL
			1.0000	0.2329	0.5101	0.5624	0.6595	0.1048	RIM2 VPS
				1.0000	0.7482	0.1532	0.1807	0.7707	DDM3 VPL
					1.0000	0.1698	0.0867	0.5415	DDM3 VPS
						1.0000	0.6353	0.1994	RIM3 ICOC
							1.0000	0.2994	RIM2 ICOC
								1.0000	DDM3 ICOC
	VPS	VPL	VPS	VPL	VPS	ICOC	ICOC	ICOC	
	RIM3	RIM2	RIM2	DDM3	DDM3	RIM3	RIM2	DDM3	

This table reports the correlations of both the ICOC and V/P ratios with the Fama-French factors B/M ratio and size (MCAP) across the different valuation models using data over all years. Observations: 49,539.

States

Table 3.2: Correlations of Implied Cost of Capital Estimates and Value-Price ratios with the Fama-French Factors - United

3.4 Forecasting Regression Tests

Given the assumption that analysts' forecasts contain valuable information about the companies' future prospects, the implied cost of capital is just the share's rate of return that is required to reconcile its market price with its future cash flows as predicted by analysts. Hence, the ICOC should be a good forecasting variable for stock returns.

In this section, we use panel regression tests to analyze whether the ICOC is a good predictor of stock returns. In a first step we investigate the ICOC's direct ability to predict stock returns by regressing multiperiod stock returns on the current firm's ICOC estimate, similar to the approaches of Fama and French (1988), Chan et al. (1996), or Lee et al. (1999). To have an empirical benchmark, we compare the ICOC's forecasting ability with a variety of variables that are traditionally used to predict stock returns, such as the dividend-price ratio, the book-price ratio, and various implementations of the value-price (V/P) ratio, as brought forward by Lee et al. (1999).

Since the variables' predictive power for stock returns might originate only from their underlying relation to firm risk proxies, we then extent the scope of the analysis and look at the more fundamental relation between implied return and cross-sectional variation in stock returns within the widely accepted Fama-French asset pricing framework. More precisely, we expand the pure forecasting regressions to include the firm's market factor loading and the Fama-French risk variables size and B/M ratio, similar to Fama and French (1992), or Brennan et al. (1998). Consequently, forecasting variables will only be significant if they contain information beyond the standard riskreturn relation implied by these models. Thus, this approach allows us to disentangle the variables' predictive power from standard risk factors to explain the variation in stock returns. Given that anticipations are the driving force of any financial market, we hypothesize that including the expectations-driven ICOC as derived from analysts' forecasts should improve the models' ability to explain the cross-sectional variation in stock returns significantly.

In the literature, there are two common econometric approaches to carry out regression tests on stock returns. Researchers either employ the Fama and MacBeth (1973) cross-sectional regressions, or they rely on the panel regression approach. Especially in asset pricing studies the Fama and MacBeth (1973) cross-sectional regressions are the predominant way to perform stock return regressions, such as in the papers of Fama and French (1992) or Chan et al. (1996). Some recent studies however point out some possible weaknesses of this popular estimation method. Usually the FamaMacBeth regressions are carried out not on individual firm-level data, but on portfolios constructed according to some exogenously specified sorting-variable. This raises the issue which sorting variable to select. Brennan et al. (1998) argue that selecting some out of many possible explanatory variables creates a "data-snooping bias that is inherent in all portfolio based approaches", since the selection of the sorting variable as well as the sorting order can influence the results significantly. Especially the common practice to construct portfolios according to B/M ratio and size is likely to overestimate the regression results (Lewellen et al., 2007). To avoid such a bias, one could use many different firm characteristics or risk factors to construct the portfolios. But as Bauer et al. (2004) emphasize, the number of portfolios needed increases exponentially with the number of firm characteristics examined. With 5 groups for 5 different characteristics e.g., we would need 55 portfolios. Given our data, many of them would contain none or few stocks.

To avoid the problems that are inherent in the portfolio regression approach some researchers carry out the cross-sectional regressions on individual firm data, such as e.g. Chan et al. (1996), or Subrahmanyam (2005). In general however, these regressions have very low power and insignificant coefficient estimates because of a rather short time-dimension of the examined data sets, i.e. the average slope coefficients are small compared to their standard errors²⁰.

Presumably because of these shortcomings of the Fama-MacBeth methodology, many researchers have shifted their focus on performing panel regressions of individual firm data when examining determinants of stock returns (Pandey, 2001; Subrahmanyam, 2005). The main advantage of the panel regression methodology is its ability to use the whole information conveyed in the data in one regression step. Thus, panel analysis usually provides more significant coefficient estimates (Baltagi, 2005) without imposing a data-snooping bias through the construction of portfolios²¹.

Since the results of the Fama-MacBeth stock return regressions for our data set are only marginally significant, we adopt the panel approach in our study and relegate a short description of the Fama-MacBeth approach and its outcomes to the appendix B.

²⁰Many empirical studies relying on cross-sectional regressions on individual firm data achieve only very low or even insignificant t-statistics for market beta, firm size, or B/M ratio, e.g. Chan et al. (1996) or Lee et al. (1999). The latter try to overcome the problem of insignificant beta coefficients by forming country-industry portfolios to estimate a country-industry beta, thereby hoping to increase the accuracy of the beta estimate. Then they run individual firm regressions but include the industry beta as a proxy for the company beta as explaining variable.

 $^{^{21}}$ For a detailed comparison of the Fama and MacBeth (1973) and panel regression methods, see Petersen (2006).

3.4.1 The Panel Regression Methodology

Our panel regression approach is the natural extension of the Fama and MacBeth (1973) cross-sectional regressions to multiple time periods. Instead of regressing each cross-section individually, we combine all monthly cross-sections to estimate the model (pooled time-series cross-section). In analogy to Fama and French (1992), we regress the firm's individual stock return $r_{i,t}$ on its ICOC estimate $k_{i,t}$, the market beta $\beta_{i,t}$. (the firm's factor loading on the market excess return), and firm characteristics $X_{i,t}$ that have been identified as risk-factor in the literature:

$$r_{i,t} = \alpha + \delta k_{i,t} + \varphi d_t \beta_{i,t} + \gamma' X_{i,t} + u_{i,t}$$
(3.9)

where the subscript *i* denotes the company (cross-section dimension) and *t* denotes the time period of the observation (time-series dimension)²². The subsequent total stock return measured after having observed the risk-factors and firm characteristics is denoted by $r_{i,t}$. The firm characteristics include the B/M ratio, firm size (calculated as the log of the market capitalization divided by the level of the stock market index), and the historical six-month price momentum²³. The variable d_t is a dummy for the ex post observed market risk premium (i.e. the difference between the market return and the risk-free rate), having a value of 1 when the market risk premium is positive during the holing period, and -1 if it is negative. This dummy is necessary to account for the fact that during periods when the realized market return is less than the risk-free rate, the relationship between predicted return and beta is reversed. More precisely, high-beta stocks should have lower returns when the ex post risk premium is negative (Pettengill et al., 1995).

The general specification in (3.9) is known as pooled regression model. In many cases, however, the underlying assumption (when estimating the pooled data set by standard OLS) that the observation of a company at time t is independent of an observation of the same company at a different point in time s is not met. Hence, the regression equation is modified to allow for individual effects for each company. This individual effect can account for firm characteristics that are not included in the

 $^{^{22}}$ For similar regressions, see Brennan et al. (1998), or Lee et al. (1999, 2003). The use of the firm characteristics size and B/M ratio instead of the respective factor loadings is motivated by the work of Daniel and Titman (1997) who argue that it is rather the characteristics than the covariance structure that explains the variation in stock returns.

²³The inclusion of price momentum as risk variable has been proposed by Carhart (1997). His work attempts to explain mutual fund performance by employing a four-factor model by adding price momentum to the standard three-factor Fama-French model, since the latter cannot explain short-term price momentum (Fama and French, 1996).

regression equations such as the industry sector, or unobservable factors. This model is known as the one-way individual effects model:

$$r_{i,t} = \alpha_i + \delta k_{i,t} + \varphi d_t \beta_{i,t} + \gamma' X_{i,t} + u_{i,t}$$

$$(3.10)$$

One of the most common approaches to estimate such an individual effects model is to assume that the individual effect α_i of each firm is constant over time. Relying on such a one-way fixed effect (FE) model hence implies that the returns of some companies are on average higher than the return of the market, whereas some other companies underperform the market on average. Another variant of the one-way individual effects model is the random-effects (RE) model, that similarly assumes that the individual effects α_i are constant, but that these individual effects are distributed randomly across firms with a zero mean: $\alpha_i \sim \mathcal{N}(0, \sigma_{\alpha}^2)$.

To detect which model is appropriate for out data set, we carry out several common statistical tests. First we rely on an F-test to see whether the estimated fixed effects are jointly significantly different from zero. If the H_0 (no significance of the individual fixed effects) is rejected, we can conclude that a fixed-effect model is preferred to the simple pooled OLS estimation. Second, we perform a Breusch-Pagan test on the random effects model, to see whether the random effects are significantly different from zero. If the H_0 (no significance of individual random effects) is rejected, we can conclude that a random-effect model is preferred to the simple pooled OLS estimation. Third, we conduct a Hausmann specification test, to see whether the coefficients of the FE and RE estimation differ significantly from each other. If the H_0 (no significant difference between the estimated coefficients) is rejected, we can conclude that a fixedeffect model is preferred to the random effects estimation²⁴. Note that we carry out these specification tests on non-overlapping data sets since the tests (F-test, Breusch-Pagan test, and Hausmann test) implemented by statistical packages do not correct for $autocorrelation^{25}$. The test results of all samples showed that the fixed effects model is the preferred approach to estimate the panel regressions. We hence present only the results of the FE-panel regressions.

Stock return regressions may be carried out using either overlapping observations or non-overlapping observations. Since Campbell (2001) shows that the use of over-

²⁴Fixed-effects estimates are always consistent, but random-effects estimates might be more efficient.

 $^{^{25}}$ We conduct these tests for all possible non-overlapping panel data sets. In most cases however, the results do not differ across the various non-overlapping panels and lead to the same conclusion of employing a one-way fixed effects model.

lapping observations increases the power of the regression, it is standard to run the regression over the whole overlapping data set²⁶. Fama and French (1988) or Chan et al. (1996) rely on the same approach. To correct for the so induced serial correlation in the regression residuals, we calculate the t-statistics of the coefficient estimates using the FE, heteroscedasticity robust and autocorrelated-adjusted standard errors following Rogers (1993).

3.4.2 Univariate Regressions

Table 3.3 presents the regression estimates and t-statistics for the univariate one-way fixed firm effects model of equation (3.10) without controlling for firm-risk variables such as market beta or specific firm characteristics. The explained variance of each model is given in the right column of each panel. Panel A on the left shows the estimation output when regressing 12-month stock returns on the forecasting variables; panel B displays the results when using 24-month stock returns.

In the simple univariate forecasting regressions, all coefficients are highly significant. In terms of t-statistics, on the one hand, the ICOC obtained from the RIM perform best in predicting stock returns. On the other hand, the B/P ratio explains most of the variance of the cross-sectional stock returns with an R-squared of 8.71%. In the 12-month return regression specification The E/P ratio and - to a lesser extent - the D/P ratio perform almost similarly well in predicting stock returns. As far as the V/P ratio is concerned, it depends on the valuation model that is used to estimate the intrinsic value, whether this forecasting measure is good in predicting stock return. DDM-based approaches are less related to stock returns than those obtained from residual income models. In general, the statistical significance of the coefficients as well as the explained variance (R2) increases when enlarging the time horizon of subsequent stock returns from 12 to 24 months (see panel B). Interestingly, V/P ratios using a risk premium plus the long-term interest rate to discount expected cash flows perform better than when using short-term rates. This result is opposed to the findings of Lee et al. (1999) who detect a better forecasting ability when using discount factors based on the short-term rate. More importantly, we do not find that the V/P ratio clearly outperforms traditional measures to predict stock returns, such as the D/P, E/P, and B/P ratio. With regard to the explained variance, the V/P is even worse in predicting stock returns than the simple B/P ratio.

²⁶The increased power stems mainly from two sources: the average return over a longer horizon provides a better proxy of conditional expected returns than short-period returns, and the regression standard errors get smaller because of the negative correlation between future expected returns and current unexpected stock returns (for more details, please refer to Campbell (2001)).

	Panel A: 12-	-month returns	Panel B: 24	-month returns
Variable	Coefficient	R^2	Coefficient	R^2
DDM3 ICOC	4.46	2.65	9.56	4.22
	(6.65)		(7.19)	
RIM2 ICOC	4.59	6.00	9.15	8.98
	(13.98)		(13.55)	
RIM3 ICOC	5.50	8.43	11.07	12.97
	(15.17)		(16.60)	
DDM3 VPS	0.01	0.19	0.05	1.46
	(3.06)		(7.26)	
DDM3 VPL	0.14	1.68	0.33	3.31
	(6.12)		(8.26)	
RIM2 VPS	0.95	1.72	0.26	5.09
	(9.27)		(12.25)	
RIM2 VPL	0.27	4.40	0.55	7.19
	(11.35)		(10.93)	
RIM3 VPS	0.11	3.05	0.26	6.58
	(8.61)		(10.58)	
RIM3 VPL	0.28	5.80	0.55	8.80
	(10.13)		(10.31)	
EP	2.42	4.78	4.34	6.18
	(8.28)		(7.52)	
DP	10.01	6.19	20.22	9.46
	(9.08)		(9.50)	
BP	0.53	8.71	1.01	12.16
	(8.77)		(7.81)	
Observations	38,014		32,191	

Table 3.3: Univariate Panel Regressions with One-Way Fixed Effects - United States

This table presents the regression estimates and t-statistics for the one-way fixed firm effects model:

$$r_{i,t} = \alpha_i + \delta k_{i,t} + u_{i,t}$$

where the subscript *i* denotes the company (cross-section dimension) and *t* denotes the time period of the observation (time-series dimension). The subsequent total stock return after having observed the forecasting variable is denoted by $r_{i,t}$. In panel A, total stock returns are measured over 12 months, in panel B, subsequent stock returns are measured over 24 months. The forecasting variables are denoted by $k_{i,t}$. Finally, $u_{i,t}$ is the error term. The first three rows report the results when regressing stock returns on the ICOC estimates. The subsequent six lines show the regression results when using the V/P ratio as predictive variable. The abbreviation *RIM3VPS* indicates the V/P ratio derived from the RIM3 formula using short-term bills plus 5 percent as discount rate; *RIM3VPL* stands for the V/P ratio derived from the RIM3 formula with a discount rate of long-term bonds plus 5 percent, and so on. Finally, the last three rows give the explanatory power of the D/P, B/P, and E/P ratio. The explained variance of each model is given in the right column.

The regressions are carried out over whole overlapping panel data set. The t-statistics below the coefficient estimates are calculated on the basis of heteroscedasticity- and autocorrelation-consistent (HAC) standard errors following Rogers (1993). The regressions are run over the full cross-section of companies. All coefficients are significant at the 1% level. The sample period is from January 1995 to February 2006. Country: United States.

3.4. FORECASTING REGRESSION TESTS

The primary difference between our approach and the study by Lee et al. (1999) is that their work does not use individual stock return data, but the return of the Dow Jones Industrial Average (DJIA) portfolio. Since the DJIA contains only the 30 largest stocks of the U.S. equity market, a possible explanation for the drastically different results compared to our work is the inclusion of smaller stocks in our analysis. We hence repeat the regressions for several subsections of large stocks, i.e. the largest quintile and the largest decile of all stocks. The results are rather identical as compared to the full sample. Again, the ICOC and E/P, B/P, and B/P ratios perform better in predicting stock returns than the V/P ratio. Hence, firm size cannot explain the differences between the two studies.

In general, our results confirm the relationship between ICOC and subsequent stock returns. Very clearly, the implied return is superior in predicting equity returns than the V/P ratio. However, the power of the regressions is rather small, attaining a maximal R^2 of 13% for the ICOC in the RIM3 specification.

3.4.3 Multivariate Fama-French Regressions

Next, we include the Fama-French risk factors into the regression equation. This allows us to disentangle the forecasting variables' predictive power from their underlying relation to known firm-risk effects. Table 3.4 presents the regression estimates and t-statistics for the one-way fixed firm effects model of equation (3.10). The explained variance of each model is given in the right column. Again, panel A shows the estimation output when regression 12-month stock returns on the forecasting variables; panel B displays the results when using 24-month stock returns.

As benchmark, the first lines of both panels provide the standard Fama-French (1992) regression specification without any forecasting variable. All Fama-French regressors prove to be highly significant. Market beta and B/M ratio are positively related to stock returns, whereas firm size exhibits a negative relationship to stock returns. Thereby, this result confirms many prior studies using the Fama-French factors to explain stock returns. In contrast to Fama and French (1996) or Chan et al. (1996) however, price momentum is not related to stock returns. The explained variance of this standard specification reaches 18% in the 12-month stock return regression test and 28% in the 24-month regressions.

The next three lines give the results when adding the implied return to this riskadjusted regression equation. The ICOC coefficients are less significant than in the univariate specification, but remain significant. Especially the ICOC from the RIM2 proves to be rather independent from the risk-return adjustment of the Fama-French

Table 3.4:Fama-French Panel Regressions with One-Way Fixed Effects - UnitedStates

This table presents the regression estimates and t-statistics for the one-way fixed firm effects model:

$$r_{i,t} = \alpha_i + \delta k_{i,t} + \varphi d_t \beta_{i,t} + \gamma' X_{i,t} + u_{i,t}$$

$$(3.10)$$

where the subscript *i* denotes the company (cross-section dimension) and *t* denotes the time period of the observation (time-series dimension). The subsequent total stock return after having observed forecasting variables and risk-factors is denoted by $r_{i,t}$. The forecasting variables are denoted by $k_{i,t}$, and the market beta estimate by $\beta_{i,t}$. The variable d_t is a dummy for the expost observed market risk premium, having a value of 1 when the market risk premium is positive during the holing period, and -1 if it is negative. Finally, $X_{i,t}$ is a vector of firm characteristics that have been identified as risk-factors, and $u_{i,t}$ is the error term. The Fama-French factors are abbreviated as follows: B/M is the book yield, SIZE is calculated as the log of the market capitalization divided by the level of the stock market index, BETA is the five year regressed sensitivity on the market portfolio, and momentum (MOM) is the historical six-month price return.

The upper line reports the results of the standard Fama and French (1992) regression specification, including price momentum without any forecasting variable. In the next three rows, the table contains the results when regressing stock returns on both the ICOC estimates and the Fama-French factors. The subsequent six lines show the regression results when using the V/P ratio as predictive variable. The abbreviation VPS indicates the V/P ratio using short-term bills plus 5 percent as discount rate; VPL stands for the V/P ratio employing a discount rate of long-term bonds plus 5 percent. Finally, the last three rows give the joint explanatory power of the D/P, B/P, and E/P ratio with the Fama-French factors. The regressions are carried out over whole overlapping panel data set. The t-statistics below the coefficient estimates are calculated on the basis of heteroscedasticityand autocorrelation-consistent (HAC) standard errors following Rogers (1993). The regressions are run over the full cross-section of companies. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level. The sample period is from January 1995 to February 2006.

Panel	A: 12-month stock	k return re	egressions.	Observatio	ons: 38,014.	
ariable	Coefficient	BETA	BM	SIZE	MOM	j

Variable	Coefficient	BETA	BM	SIZE	MOM	R^2
		0.10***	0.37***	-0.20***	0.00	18.00
		(14.21)	(6.61)	(-8.84)	(0.11)	
DDM3 ICOC	1.39^{**}	0.10^{***}	0.34^{***}	-0.20***	0.01	18.21
	(1.97)	(14.83)	(5.72)	(-8.43)	(0.97)	
RIM2 ICOC	3.10^{***}	0.10^{***}	0.32^{***}	-0.18***	0.05^{***}	20.34
	(7.40)	(14.51)	(5.03)	(-7.68)	(3.13)	
RIM3 ICOC	2.94^{***}	0.10^{***}	0.18^{**}	-0.18***	0.03^{**}	18.85
	(4.49)	(15.37)	(2.56)	(-7.99)	(2.10)	
DDM3 VPS	-0.01**	0.10***	0.39^{***}	-0.20***	-0.00	18.10
	(-1.98)	(14.08)	(6.90)	(-8.66)	(-0.10)	
DDM3 VPL	-0.00	0.10^{***}	0.37^{***}	-0.20***	0.00	18.10
	(-0.16)	(14.21)	(6.24)	(-8.84)	(0.06)	
RIM2 VPS	0.03^{*}	0.10^{***}	0.33^{***}	-0.21***	0.01	18.19
	(1.94)	(14.49)	(5.84)	(-9.16)	(0.64)	
RIM2 VPL	0.14^{***}	0.10***	0.30***	-0.20***	0.04**	18.97
	(3.98)	(14.63)	(4.96)	(-8.35)	(1.97)	
RIM3 VPS	-0.01	0.10^{***}	0.38^{***}	-0.20***	0.00	18.01
	(-0.35)	(14.21)	(5.73)	(-8.69)	(0.07)	
RIM3 VPL	0.03	0.10^{***}	0.33***	-0.10***	0.01	18.03
	(0.82)	(14.46)	(4.35)	(-8.80)	(0.35)	

$\frac{\text{oefficient}}{0.87^{***}}$	BETA	BM	SIZE	MOM	R^2
0.87***	0 10***				-
0.0.	0.10	0.29^{***}	-0.20***	0.02	18.38
(2.81)	(14.60)	(5.43)	(-8.59)	(0.96)	
3.59^{***}	0.10^{***}	0.30^{***}	-0.19***	0.01	18.56
(2.87)	(13.91)	(4.45)	(-8.25)	(0.95)	
0.37^{***}	0.10^{***}	-	-0.20***	0.00	18.00
(6.61)	(14.21)	-	(-8.84)	(0.11)	
	$\begin{array}{c}(2.81)\\3.59^{***}\\(2.87)\\0.37^{***}\\(6.61)\end{array}$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$\begin{array}{ccccccc} (2.81) & (14.60) & (5.43) \\ 3.59^{***} & 0.10^{***} & 0.30^{***} \\ (2.87) & (13.91) & (4.45) \\ 0.37^{***} & 0.10^{***} & - \\ (6.61) & (14.21) & - \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

Panel B: 24-month stock return regressions. Observations: 32,191.

Variable	Coefficient	BETA	BM	SIZE	MOM	R^2
		0.21^{***}	0.58^{***}	-0.46***	0.00	28.46
		(16.17)	(5.47)	(-9.30)	(0.08)	
DDM3 ICOC	2.24*	0.21^{***}	0.54^{***}	-0.44***	0.02	28.63
	(1.76)	(16.24)	(4.78)	(-9.08)	(0.89)	
RIM2 ICOC	5.18^{***}	0.20^{***}	0.50^{***}	-0.41***	0.09^{***}	30.85
	(7.06)	(15.92)	(4.12)	(-8.29)	(3.01)	
RIM3 ICOC	5.46^{***}	0.21^{***}	0.24^{**}	-0.41***	0.06^{**}	29.56
	(5.01)	(16.96)	(2.00)	(-8.51)	(2.48)	
DDM3 VPS	0.01	0.21^{***}	0.57^{***}	-0.46***	0.00	28.47
	(0.67)	(16.14)	(5.18)	(-9.27)	(0.18)	
DDM3 VPL	0.01	0.21^{***}	0.58^{***}	-0.46***	0.00	28.46
	(0.20)	(16.17)	(5.15)	(-9.33)	(0.15)	
RIM2 VPS	0.12^{***}	0.21^{***}	0.46^{***}	-0.47***	0.03	29.28
	(3.62)	(15.95)	(4.06)	(-9.63)	(1.28)	
RIM2 VPL	0.25^{***}	0.21^{***}	0.46^{***}	-0.44***	0.06^{**}	29.62
	(4.11)	(16.09)	(3.93)	(-8.77)	(2.09)	
RIM3 VPS	0.06^{*}	0.21^{***}	0.46^{***}	-0.47***	0.01	28.62
	(1.68)	(16.25)	(3.63)	(-9.44)	(0.33)	
RIM3 VPL	0.09	0.21^{***}	0.47^{***}	-0.45***	0.01	28.53
	(1.22)	(16.50)	(3.37)	(-9.26)	(0.52)	
EP	1.05^{**}	0.21^{***}	0.48^{***}	-0.45***	0.02	28.67
	(1.98)	(16.39)	(4.58)	(-9.18)	(0.72)	
DP	6.09^{***}	0.20^{***}	0.47^{***}	-0.43***	0.02	29.04
	(2.79)	(15.69)	(3.84)	(-8.68)	(0.92)	
BP	0.58^{***}	0.21^{***}	-	-0.46***	0.00	28.46
	(5.47)	(16.17)	-	(-9.30)	(0.08)	

factors. In the 12-month regression specification, the explained variance of the joint regressions model also increases substantially from 18% to 20.3%.

The six subsequent rows present the regression results when controlling the V/P ratio for the Fama-French variables. Here, only the V/P ratio obtained from the RIM2 is significantly (positively) related to stock returns. The DDM V/P multiples have even a negative regression coefficient. Apparently, the V/P ratios have their predictive power for stock returns - as documented in table 3.3 - only due to their strong relation to standard firm-risk variables. Put differently, the V/P ratios might be considered more as a transformation of these risk proxies since they do not add any explanatory power to the Fama-French model. This can also be seen from the rather marginal increase of the R^2 from 18% to 18.10% - with the exception of the V/P ratio of the RIM2. Still, the RIM2 V/P is definitely less related to stock returns than the ICOC estimate based on the same model.

Finally, the last three rows give the joint explanatory power of the E/P, D/P, and B/P ratio with the Fama-French factors. Note that the last specification is identical to the pure Fama-French specification since the B/P ratio is nothing else than the risk variable B/M ratio. All three multiples are significantly related to stock returns. They perform better than many of the V/P ratios. Still, the ICOC estimate from the RIM2 proves to have a higher added-value in explaining the cross-sectional variation in stock returns.

When comparing the regressions results in both panels, i.e. the 12-month regressions with the 24-months regressions, we find that the statistical significance of the coefficients does not differ much. However, the R^2 increases when extending the time horizon.

The overall conclusion of this table is that implied return is much more of an independent factor to predict stock returns than all other multiples analyzed. At first sight, this result might appear peculiar, since the V/P ratio is calculated from identical analysts' data using the same valuation models than the implied cost of capital. However, the ICOC is a much more direct transformation of the data; the V/P ratio needs an additional assumption about the discount rate to derive the estimate of intrinsic value. This "detour" might cause the differences. If one is to believe in the Fama-French model, the coefficients of all forecasting variables - including the ICOC - should be close to zero. Hence, the highly significant ICOC coefficients indicate very clearly that the implied return contains some important additional information for explaining the variation in stock returns. In a more general perspective, the significant ICOC coefficients question the joint hypothesis of efficient markets and the validity of the Fama-French model.

3.4.4 Robustness tests

In this section, we test the robustness of previous results by examining either data subsamples or using different regression specifications. First, we divide the data sample in two sub periods. The first includes the years from 1995 up to 1999, the second periods contains the remaining month until 2006. The cutting point in January 2000 is very close to the record-highs of the U.S. stock market in March 2000. By analyzing the time periods before and after this turning point, we can see whether the predictive power of the variables depends on the stock market environment. However, the results do not change substantially compared to the full time data-set. In univariate regressions, all variables in both sub periods are still highly significant. Again, the ICOC obtained from the RIM3 performs best, attaining t-values of around 14.50 in both periods. DDM-based approaches fall behind their RIM counterparts. When controlling the forecasting variables for the Fama-French factors, it is interesting to note that the additional power of almost all predictive variables is much smaller in the second time period. For example, the RIM3 ICOC is only significant at the 10% level, and DDM3 ICOC, D/P ratio, and E/P ratio are no longer significantly related to stock returns after correcting for the required return compensation of the Fama-French model. In this sense, the results are not fully robust to the examined data sample. Unfortunately, there is not sufficiently data of analysts' forecasts to extend the study to earlier time periods. Second, we check whether the results are only driven by the methodology of using overlapping holding periods. Since overlapping regressions might overstate the true relationship between returns and the ICOC, we reduce the forecasting horizon to one month. Thus, the regression test can be conducted using non-overlapping stock returns. Yet, we do not find any sizeable difference. The coefficient estimates are slightly less significant, but the general picture remains unchanged: the ICOC performs best in predicting stock returns.

3.5 International Analysis

Now we extend our analysis to test the ICOC's ability to predict subsequent stock return across other large equity markets. We hence repeat the previous regression tests for all capital markets of the remaining G7 countries, i.e. Canada, France, Germany, Italy, Japan, and the U.K. As with the U.S. data, monthly data for prices, total returns, book values per share, dividends per share, market capitalization, and returns on equity are taken from MSCI. Earnings estimates as well as the long-term growth rate are taken from IBES median estimates. The time series data of national accounts to calculate the expected nominal GDP growth rate is obtained from Eurostat again²⁷. All data are denoted in local currency. If quoted in deviant currencies, data is converted into the local currency, where the conversion is accomplished by using the WM Company exchange rate as of the date of the data. With over 100,000 monthly observations, our data sample is the largest for Japan, and the smallest for Italy, with just over 22,000 monthly observations.

In theory, the predictive power of the ICOC for future stock returns should be existent in all equity markets. Consequently, we would expect similar regression results among all G7 countries. However, the countries analyzed differ highly in their underlying economies and - more important - in their data availability. First, the seven countries exhibit considerable discrepancies in stock market performance over the sample period from 1995 to 2006, such as the long-lasting bear market in Japan or the stock market bubble in the western countries. A second source of possible divergence is the availability of IBES data. The two-stage RIM as well as the DDM rely heavily on the long-term consensus growth rate of equity analysts. Estimating such growth rates has a long tradition in North America, but not in European countries, where analysts have rarely published such forecasts over the early years in our sample period and have concentrated instead on explicit earnings forecasts for the next years. Hence, implementing these models in European (and the Japanese) markets leads to a considerable decrease in the available data set. Finally, deviating accounting standards for book value of equity across the countries considered add some uncertainty to the implied returns derived from the RIMs. Similar to the analysis of the U.S. American equity market, we first test the ICOC's and the other forecasting variables' direct ability to predict stock returns in simple univariate regression tests. Then, in a second step, we control the forecasting regressions for the standard risk-return relation of the Fama-French model to disentangle the variables' predictive power and risk factors to explain the variation in stock returns. The analysis follows the same methodology as the regression tests of the U.S. data sample.

3.5.1 Univariate Forecasting Regressions

Table 3.5 shows the coefficient estimates and t-statistics for the one-way fixed effects model in equation (11) for the non-U.S. G7 countries. The regressions are carried out using twelve-month stock returns over whole overlapping panel data set. The

 $^{^{27}}$ Due to the long-lasting recession in Japan, the geometric nominal GDP growth rate over the past 5 years (used as a proxy of the expected GDP growth), is not always positive. In order to ensure the existence of a positive root of the present value formulas, we replaced negative geometric nominal GDP growth rates by an expectation of 1%.

Variable	CA	\mathbf{FR}	GE	IT	JP	UK
DDM3 ICOC	1.26^{**}	0.80^{*}	1.63^{***}	1.99^{*}	1.09^{***}	2.48^{***}
t-stat	(1.99)	(1.66)	(3.21)	(1.65)	(3.35)	(4.66)
R^2	0.80	0.44	1.91	1.98	1.15	4.26
RIM2 ICOC	3.40^{***}	2.69^{***}	3.06^{***}	5.40^{***}	6.04^{***}	5.13^{***}
t-stat	(5.21)	(3.48)	(3.78)	(4.07)	(11.93)	(10.65)
R^2	4.19	02.08	3.40	4.61	10.24	10.77
RIM3 ICOC	5.28^{***}	4.23***	3.39^{***}	8.36^{***}	17.17^{***}	4.91^{***}
t-stat	(4.38)	(4.96)	(5.64)	(3.43)	(16.86)	(10.40)
R^2	5.78	4.45	5.28	8.37	18.51	10.81
DDM3 VPS	0.01	0.01*	0.01**	0.01*	0.00	0.02*
t-stat	(1.17)	(1.86)	(1.96)	(1.72)	(1.43)	(1.82)
R^2	0.37	0.28	0.55	03.01	0.18	0.87
DDM3 VPL	0.02^{*}	0.01	0.02	0.02	0.00	0.02^{**}
t-stat	(1.86)	(1.45)	(1.56)	(1.51)	(1.56)	(2.16)
R^2	0.60	0.15	0.43	2.59	0.24	01.06
RIM2 VPS	0.12^{***}	0.14^{***}	0.14^{***}	0.28^{***}	0.31^{***}	0.28^{***}
t-stat	(3.21)	(3.55)	(3.54)	(5.57)	(11.57)	(7.08)
R^2	2.33	2.95	3.71	5.81	9.47	7.89
RIM2 VPL	0.20^{***}	0.14^{***}	0.14^{***}	0.30^{***}	0.36^{***}	0.30^{***}
t-stat	(3.72)	(2.85)	(3.25)	(2.76)	(12.00)	(7.39)
R^2	2.65	1.76	2.21	3.40	10.10	7.57
RIM3 VPS	0.22^{***}	0.27^{***}	0.21^{***}	0.46^{***}	0.68^{***}	0.27^{***}
t-stat	(3.32)	(4.63)	(6.07)	(3.57)	(12.73)	(8.56)
R^2	3.48	5.46	7.23	7.53	15.72	08.08
RIM3 VPL	0.36^{***}	0.31^{***}	0.23^{***}	0.51^{***}	0.79^{***}	0.27^{***}
t-stat	(3.83)	(4.12)	(5.95)	(2.86)	(13.44)	(7.94)
R^2	4.38	4.48	5.70	5.22	16.26	7.53
EP	2.42^{***}	1.57^{*}	2.05^{***}	3.01^{***}	2.15^{***}	3.67^{***}
t-stat	(4.22)	(1.94)	(3.91)	(2.69)	(4.48)	(8.35)
R^2	2.62	1.24	2.79	2.65	1.75	8.90
DP	11.03^{***}	4.05^{***}	6.99^{***}	4.55	37.23^{***}	8.52***
t-stat	(4.09)	(2.99)	(4.18)	(1.32)	(10.68)	(9.60)
R^2	5.00	2.71	5.13	01.02	13.57	12.93
BP	0.47^{***}	0.30^{**}	0.40^{***}	0.70^{***}	0.61^{***}	0.49^{***}
t-stat	(5.35)	(2.45)	(7.20)	(3.56)	(8.89)	(8.37)
\mathbb{R}^2	6.71	3.41	8.68	12.71	14.30	9.27
Observations	4,563	$7,\!879$	6,021	2,692	14,586	12,852

Table 3.5: Univariate Panel Regressions with One-Way Fixed Effects - International Analysis

This table presents the coefficient estimates and t-statistics of each forecasting variable for the oneway fixed firm effects model of equation (3.10) without controlling for firm-risk. Each column gives the results for one of the other six G7 countries, i.e. Canada, France, Germany, Italy, Japan, and the U.K. A more detailed description of the table is given in the explanatory notes of table 3.3.

The regressions are carried out over whole overlapping panel data set. The t-statistics below the coefficient estimates are calculated on the basis of heteroscedasticity- and autocorrelation-consistent (HAC) standard errors following Rogers (1993). The regressions are run over the full cross-section of companies. The sample period is from January 1995 to February 2006.

explained variance of each model is given in the third line of each regression output.

Many of the forecasting variables prove again to be good explanatory variables for stock returns across all countries. Especially the ICOC coefficients are always significantly positive. In contrast, the V/P ratio obtained from the DDM3 is only weakly related to stock returns - and in many countries this coefficient is not even significant. Similarly, the DDM-based ICOC performs worse than the implied return obtained from the residual income model. Interestingly, when comparing the predictive power of the V/P ratios based on long-term and short-term interest rates with each other, we do not find that one approach is generally superior across the countries - again opposed to the results of Lee et al. (1999), who detect a better forecasting ability of the V/P ratio using the short-term rate.

In many countries the ICOC performs better, but in France and Germany, the V/P ratio exhibits a higher forecasting power. As far as the other multiples are concerned, the B/P ratio performs best in many countries, even exceeding the analysts driven multiples (Canada and Germany). The explained variance (R^2) of all models is generally rather low, especially for the DDM approach. Only in the U.K. and the Japanese market, the RIM-based models seem to capture a good part of the variation of stock returns over the whole data panel, with an R-squared attaining over 18%.

3.5.2 Multivariate Forecasting Regressions

In table 3.6 we report the results of the panel regressions when we control stock returns for the influence of firm-specific risk factors. In the upper part of the panel, we show first the results of the Fama-French (1992) regression specification without any forecasting variable. As was to be expected, the risk factors of the Fama-French model exhibit their standard relation to stock returns across all countries. Market beta and B/M ratio are positively related to stock returns, whereas firm size exhibits a negative relationship to stock returns. However, not in all countries the coefficients are significant. Similar to the analysis of the U.S. data, price momentum is not generally related to stock returns. The explained variance of this standard specification attains a mere 16% in Canada and reaches up to 32% in Japan. In the lower part of the table, we present the coefficient estimates of the various forecasting variables when adding them to the Fama-French regressions above. Compared to table 5, almost all coefficients loose some of their explanatory power for stock returns. Although there are large differences between the various predictive variables and countries, we can observe some patterns in the regression output. First, similar to the case in the U.S. equity markets, DDM based multiples perform rather badly in predicting stock returns compared to

Table 3.6: Multivariate Fama-French Panel Regressions with One-Way Fixed Effects- International Analysis

This table presents the coefficient estimates and t-statistics of each forecasting variable for the oneway fixed firm effects model of equation (3.10) including the Fama-French factors to account for firm-risk. Each column gives the results for one of the other six G7 countries, i.e. Canada, France, Germany, Italy, Japan, and the U.K. The first lines contain for each country the results of the standard Fama and French (1992) regression specification, including price momentum without any forecasting variable. Then, in the next set of rows, the table contains the regression coefficients of the ICOC forecasting variable when controlling the stock returns for the Fama-French factors and price momentum (note that the regression coefficients are not shown to save space). The subsequent lines show the regression results when using the V/P ratio as predictive variable. Finally, the last rows give the regression results when using the E/P and the D/P ratio to predict stock returns. A more detailed description of the table is given in the explanatory notes of table 3.4.

The regressions are carried out over whole overlapping panel data set. The t-statistics below the coefficient estimates are calculated on the basis of heteroscedasticity- and autocorrelation-consistent (HAC) standard errors following Rogers (1993). The regressions are run over the full cross-section of companies. The sample period is from January 1995 to February 2006.

Variable	CA	\mathbf{FR}	GE	IT	JP	UK
Beta	0.11***	0.16***	0.20***	0.16***	0.15***	0.10***
t-stat	(4.20)	(11.42)	(13.44)	(8.14)	(12.69)	(14.18)
BM	0.29^{***}	0.08	0.17^{***}	0.47^{**}	0.29^{***}	0.28***
t-stat	(3.99)	(1.38)	(2.86)	(2.64)	(5.76)	(4.59)
Size	-0.15***	-0.21^{***}	-0.28***	-0.20	-0.25^{***}	-0.23***
t-stat	(-2.66)	(-6.15)	(-5.39)	(-1.60)	(-7.87)	(-8.27)
Momentum	0.07^{*}	-0.01	-0.03	0.03	0.00^{**}	-0.02
t-stat	(1.66)	(-0.21)	(-1.04)	(0.81)	(-2.51)	(-0.59)
R^2	16.17	21.01	28.73	23.95	32.28	24.21
DDM3 ICOC	0.49	0.69	0.99**	1.60	0.53^{*}	0.76^{*}
t-stat	(0.68)	(1.50)	(2.21)	(1.44)	(1.73)	(1.67)
R^2	16.28	21.31	29.41	25.18	32.54	24.54
RIM2 ICOC	2.06^{***}	2.18^{***}	0.65	2.85^{**}	3.15^{***}	2.96^{***}
t-stat	(2.88)	(4.19)	(0.94)	(2.40)	(6.59)	(6.31)
R^2	17.60	22.21	28.86	25.04	34.71	27.21
RIM3 ICOC	3.00**	2.14**	-0.69	-2.33	9.35***	3.56^{***}
t-stat	(1.94)	(2.49)	(-1.00)	(-1.18)	(4.64)	(4.06)
R^2	17.27	21.50	28.82	24.16	33.88	25.85
DDM3 VPS	0.01	0.00	0.00	0.01	0.00	0.00
t-stat	(1.11)	(1.19)	(0.77)	(1.19)	(0.45)	(0.68)
R^2	16.50	21.06	28.80	25.21	32.30	24.25
RIM2 VPS	0.09**	0.08***	0.01	0.10**	0.17***	0.16***
t-stat	(2.07)	(3.34)	(0.32)	(2.22)	(6.70)	(4.04)
R^2	17.36	21.92	28.74	24.52	34.77	26.35

(Table 3.6 continued)						
Variable	CA	\mathbf{FR}	GE	IT	$_{\rm JP}$	UK
EP	1.11**	0.77	0.11	0.33	0.73*	1.76^{***}
t-stat	(2.11)	(1.55)	(0.35)	(0.82)	(1.75)	(3.91)
R^2	16.62	21.26	28.74	23.98	33.88	25.60
DP	2.89	1.35^{*}	3.09^{***}	3.97^{**}	13.91^{***}	3.96^{***}
t-stat	(0.95)	(1.81)	(2.72)	(2.36)	(3.52)	(4.11)
R^2	16.37	21.25	29.41	24.52	33.31	25.77
Observations	4,563	$7,\!879$	6,021	$2,\!692$	$14,\!586$	12,852

(Table 3.6 continued)

the RIM approaches. Only in Germany, the DDM3 ICOC coefficient is still significant Second, in this risk-adjusted environment, the two-stage RIM outperforms the RIM3 in explaining additional variation in stock returns. Since the RIM3 is rather highly correlated with the B/M ratio, it is much less of an independent predictor than the RIM2. Finally, the ICOC is in many countries (Canada, France, Germany, and U.K.) better than the V/P ratios. When looking at the predictive power of the E/P and D/Pratio (note that the B/P ratio is omitted since it is already included in the B/M ratio of the Fama-French factors), we can see that they do not outperform those derived from analysts' forecasts in any country. The R-squared increases in many countries when adding the ICOC and V/P to the regression specification, most notably in Japan and the U.K. In essence, this international analysis confirms the outcome of the U.S. data sample: significant ICOC coefficients present evidence that the implied return is contributing some explanatory power for stock returns that goes beyond the standard risk-return relation captured by the Fama-French factors. Moreover, the additional information originating from analysts is better retrieved by the ICOC than by the V/P ratio.

The estimated coefficients of both RIM3 V/P ratio and RIM3 ICOC regressions for Germany and Italy have a somewhat peculiar feature. Although these variables are positively related to stock returns - as can be seen in the univariate regression tests - they exhibit negative coefficients in the joint regressions with the Fama-French risk characteristics. In the case of the Italian stock market, this negative relation is even significant. In fact, these regression specifications exhibit a multicollinearity problem. Since B/M and the forecasting variables obtained from the three-stage RIM are highly correlated (up to 90%, depending on the country), they contain essentially identical information. The negative coefficients are a direct consequence of this high correlation. Since the B/M ratio is sufficient to capture almost all of the explanatory power, the ICOC (or V/P ratio, respectively) captures then the remaining (negative) effects. The interpretation of this pattern is not obvious. If one is to accept the Fama-
French model a priori, this result challenges the RIM3 approach as independent factor to explain stock returns in Italy and Germany: The regressions basically indicate that the implied return (and the V/P ratio) owes its positive relation to stock returns only due to its close relation to the B/M ratio. On the other hand, at the aggregate of this international extension, the additional explanatory power of the implied cost of capital for stock returns is still evident. What is more, in the largest stock markets apart from the U.S. (France, Japan, and the United Kingdom) the RIM3 implied return even subsumes the B/M effect, i.e. the coefficients of the B/M effect are no longer related to stock return (not shown). Since - as mentioned earlier - the ICOC has a solid foundation in valuation analysis as compared to the ad-hoc multiple B/M ratio, the implied cost of capital might be a potential replacement for the B/M ratio as risk factor in these markets.

3.6 Conclusion

The recently developed concept of the implied cost of capital has become a popular tool for estimating expected stock returns both in academia and practice. By aggregating individual stock returns over the entire market, this approach is used extensively in economic research to derive a forward-looking equity risk premium estimate. Fund managers try to exploit the so-obtained expected returns to improve the performance of their investment portfolios (Stotz, 2005). This chapter shows that the implied cost of capital is as well a good predictor for stock returns. Over a time horizon of 24 months, implied cost of capital estimates can explain up to 13% of the cross-sectional variation of stock returns in the U.S. American stock market. What is more, the ICOC's ability to predict stock returns is higher than traditional valuation multiples and the V/P ratio as developed by Lee et al. (1999). Hence, this study extends previous evidence on the marketwide predictability of stock returns with the help of analysts' recommendations to individual securities. Joint regression tests of stock returns on the ICOC estimate and standard CAPM or Fama-French risk factors highlight that ICOC coefficients remain highly significant even after controlling for the stocks' risk. Estimations across other major international capital markets confirm our findings.

These results have various important implications. First, by showing that ICOC improves the ability of standard asset pricing models to explain the cross-sectional variation in stock returns significantly, we underline the additional informational content included in equity analysts' forecasts. Thus, detailed equity analysis as provided by investment banks and stock brokers is not a futile service. At the same time,

significant coefficient estimates of the ICOC are opposed to the predictions of the Fama-French asset pricing model. Hence, our results question the joint hypothesis of efficient markets and the validity of the Fama-French model. Consequently, this study also contributes to the question whether the ICOC can be used as a proxy for expected stock returns. Significant ICOC coefficients mean that the ICOC can only be regarded as estimate for the expected return, if one is to reject the Fama-French model. Thus, our work contributes to the more recent empirical evidence that challenges the use of ICOC as expected return estimate in the academic literature.

Chapter 4

Investment Under Ambiguity With the Best and Worst in Mind

Recent literature on optimal investment has stressed the difference between the impact of risk compared to the impact of ambiguity - also called Knightian uncertainty - on investors' decisions. In this chapter, we show that a decision maker's attitude towards ambiguity is crucial for his investment evaluation and decision. By introducing an individual parameter reflecting personal characteristics of the entrepreneur, our simple investment model helps to explain differences in investment behavior in situations which are objectively identical. We show that the presence of ambiguity leads in many cases to an increase in the subjective project value, and entrepreneurs are more eager to invest.

4.1 Introduction

When firms decide about investments, they usually face uncertainty about future cash flows from their projects. This fact has been widely acknowledged by the literature on optimal investment, and every sound investment theory requires an accurate assessment of the uncertainty involved. In analyzing the uncertainty faced by a firm, researchers have essentially identified two main sources of uncertainty: risk and ambiguity. While *risk* usually refers to the return volatility of an investment project using a specific probability distribution, the notion of *ambiguity*¹ stands for the existence of a multitude of such probability distributions to describe future profits. Whereas the impact of risk on investment decisions has been analyzed thoroughly in the past, the role of ambiguity has only recently drawn attention to the research community. How is investment actually affected by a perceived change in ambiguity?

¹Sometimes ambiguity is also called *Knightian uncertainty*, following the work of Knight (1921). In this study, both terms refer to the same concept and are used interchangeably.

The answer is not obvious. Standard investment models that allow for ambiguity assume that decision makers are completely averse to ambiguity. Most papers hence postulate that a rise in ambiguity alienates investors, which are consequently less eager to invest. As an example, think of the development of a crude oil field. If the development of future oil prices seems to become less predictable, the investment project is more likely to get abandoned, since it might not be possible to recover the sunk investment costs. At least, it is likely to get postponed to see whether prices stabilize at a profitable level or not.

There are, however, situations in which the assumption of complete ambiguity averse decision makers seems too extreme. In a survey of successful entrepreneurs Bhidé (2000) shows that individuals that start businesses are highly self-confident and exhibit a very low degree of ambiguity aversion. When they spot some business opportunity in a new booming industry sector, they embrace rising uncertainty and interpret it as an increase in possible profits. That was obviously the case during the new economy bubble, when many entrepreneurs - fully confident about their business idea - invested into the internet just because of the fact that it was new and rather unknown. These two examples suggest that it is essential to include some individual parameter into the decision model in order to capture the entrepreneur's attitude towards the ambiguity he faces. Most important, traditional investment models that abstract from the presence of ambiguity cannot distinguish between these two different situations, since they are in terms of risk objectively identical².

The objective of this chapter is to propose a simple investment model that captures different personal attitudes towards ambiguity and thus allows to investigate its influence on the investment decision. We resort to the irreversible investment theory following the works of McDonald and Siegel (1986) and Dixit and Pindyck (1994) as framework for the model. Also known under the term real option theory, this approach offers an elegant way to assess the optimal investment strategy given an uncertain environment by applying option-pricing techniques to the investment problem. In contrast to the standard irreversible investment literature, we do not assume that the entrepreneur has perfect confidence in the perceived probability measure describing future uncertainty. Instead we assume that she considers other probability measures to be possible as well, i.e. we extend the model to include ambiguity. By applying various individual preferences to the resulting decision problem under ambiguity, we can examine the effect of different attitudes of the entrepreneur towards

²All throughout this chapter, we assume that the riskiness, i.e. volatility, of an investment project is constant and fully known to the decision maker. It should be noted that there might be reasons beyond the entrepreneur's attitude towards ambiguity that can influence his investment decision.

ambiguity. More precisely, we intend to determine the optimal investment strategy given ambiguity when the preferences of the entrepreneur can be described as a convex combination of the two extreme attitudes towards ambiguity, i.e. looking at the best and the worst case only. Such preferences have been proposed among others by Ghiradato et al. (2004) and Olszewski (2007), and are also known under the label α -MEU preferences.

In the first part of the chapter, we reduce the general irreversible investment problem to include only an all-or-nothing decision, i.e. the entrepreneur has the possibility to decide positively about the investment, or to abandon the whole project. In section 4.6, we then introduce the possibility to defer the investment project to a later point of time. Unfortunately, α -MEU preferences generally violate the dynamic consistency requirement to solve the resulting intertemporal optimization problem. Nevertheless, we present solutions to some important special cases.

We find that even a very small fraction of optimism from the side of the entrepreneur can change the investment decision significantly. Most important, we show that in many cases the threshold for investing, i.e. the required expected value of a project, decreases in presence of ambiguity. As a consequence, investment is carried out at earlier stages compared to situations without uncertainty.

The model we present in this work is related to several streams of literature. On the one hand, it follows the standard models of irreversible investment under uncertainty succeeding the seminal article by McDonald and Siegel (1986) who paved the way for applying option-pricing techniques to the investment problem. On the other hand, this work relates to the literature of decision making under ambiguity. The fact that the distinction between risk and uncertainty is behaviorally meaningful was first shown by the Ellsberg (1961) paradox. Out of the various theories that allow for ambiguity, the Choquet Expected Utility theory by Schmeidler (1989) and the Multiple Expected Utility theory by Gilboa and Schmeidler (1989) are the most prominent. In this chapter, we rely on the formulation of the latter, using the continuous time implementation by Chen and Epstein (2002). As such, this work finally relates to the literature on optimism, overconfidence and decision making in behavioral economics. When facing ambiguous situations, Heath and Tversky (1991) argue that the subject's attitude towards ambiguity depends on their perceived competence level. They show that individuals feeling competent react more favorite to ambiguous situations and are even ambiguity seeking. Since the fact that people tend to overestimate their subjective knowledge or competence is widely acknowledged in the psychology literature (overconfidence), see e.g. DeBondt and Thaler (1995), we should expect to observe some ambiguity loving behavior when facing investment decisions.

In fact, the experimental study by Gysler et al. (2002) finds some evidence for the alleged close relationship between ambiguity attitude and perceived competence and overconfidence³.

To my knowledge, this is the first theoretical model to include ambiguity loving features into the investment problem in continuous time given ambiguity⁴. As such, it is inspired by the work of Nishimura and Ozaki (2007) who first combined these two streams of research, assuming completely ambiguity averse decision makers. In fact, our model offers a generalization of their work, including ambiguity averse investors as a special case. Other recent works in this direction are the papers by Miao and Wang (2007), Asano (2005), and Trojanowska and Kort (2006). Finally, this work also contributes to the important issue of dynamic optimization under ambiguity. The α -MEU preferences of Ghiradato et al. (2004) are very popular in assessing decisions under ambiguity, since they allow a simple framework to differentiate the magnitude of ambiguity from the decision marker's attitude towards ambiguity. However, as this study shows, α -MEU preferences are generally not dynamically consistent, i.e. they cannot be represented in a recursive way. This drawback implies that this preference model cannot be used when facing dynamic optimization problems.

This chapter proceeds as follows. In the next section, we present the simple investment problem of the entrepreneur, who faces the decision either to invest into a given project, or to abandon it. Then we describe the Chen and Epstein (2002) model of ambiguity in continuous time, that we employ in this work. The decision maker's attitude towards this ambiguity is formulated in section 4.4. The solution of the simple investment problem is then presented in section 4.5. First, we derive the value of the investment project, then we turn to the investment decision by answering the important question of whether to invest or not. Finally, in section 4.6, we lift the restriction of immediate investment requirement. Section 4.7 concludes.

4.2 The Firm's Investment Problem

Time t evolves over $[0, \infty)$ and uncertainty is described by a probability space (Ω, \mathcal{F}, P) . Let B_t be a standard Brownian motion defined on this probability space, and $\mathbb{F} = \{\mathcal{F}_t, t \geq 0\}$ its augmented filtration, i.e. the σ -algebra generated by $(B_s)_{s \leq t}$

³There is however no universal definition of the terms overconfidence and optimism in the presence of ambiguity. Epstein and Schneider (2006) for example present a setting which allows for the combination of ambiguity averse and overconfident agents.

⁴Basili and Fontini (2006) provide an empirical approach to estimate the degree of ambiguity by deriving the best and the worst case from ask and bid prices of the oil market.

and the P-null sets.

Think of a risk-neutral entrepreneur who wants to set up a start-up venture and intends to evaluate his investment project in order to decide whether to invest or not. We assume that the entrepreneur's project can be characterized by an uncertain profit flow which follows a geometric Brownian motion:

$$d\pi_t = \mu \pi_t dt + \sigma \pi_t dB_t \tag{4.1}$$

with π_0 and $\sigma > 0$. Investment costs are denoted by I. In the first part of the analysis, we assume that the investor faces an all-or-nothing decision: He can either decide to invest immediately, or abandon the project completely. Later, in section 4.6, we relax this assumption. The investment problem is hence given by choosing the optimum between the expected value when investing (denoted by V_t) less investment costs I, and not investing, which yields a profit of 0:

$$F_t = \max\{V_t - I, 0\} = \max\left\{E_t\left[\int_t^\infty e^{-\rho(s-t)}\pi_s ds\right] - I, 0\right\}$$
(4.2)

where ρ is the continuously compounded interest rate. Since the decision maker has the right, but not the obligation to invest, this expression is also called the investment option, and is denoted by F_t . For this project evaluation to make sense, we assume in addition that $\mu < \rho$ - otherwise the expected project value could get infinitely large (for $\mu \to \rho$ or $\mu > \rho$).

In absence of ambiguity, we know the true parameters of the geometric Brownian motion as presented in (4.1) that describe the future profit flow. Thus, we can calculate the expected investment value V_t quite easily, e.g. as presented in Dixit and Pindyck (1994, p. 72),

$$V_t = \frac{\pi_t}{\rho - \mu} \tag{4.3}$$

which is just the standard expected value of an infinite profit stream. If V_t exceeds the investment costs I, the entrepreneur engages in the project - otherwise he will not undertake the venture.

4.3 Ambiguity in Continuous Time

Now, we assume that the entrepreneur is not perfectly sure about the parameters governing future profit flows, but faces ambiguity over the probability measure P. Instead of a single probability measure, we assume hence that the decision maker

considers now a set of probability measures \mathcal{P} as possible⁵. The model of ambiguity in continuous time presented here follows the work of Chen and Epstein (2002). It is based on the recursive multiple priors utility model developed by Epstein and Wang (1994), in which the decision maker's beliefs are represented by a suitable collection of sets of one-step-ahead conditional probabilities to allow for ambiguity.

We define the set \mathcal{P} to be mutually absolutely continuous with respect to P, which we call the original probability measure. This set can be defined by so-called density generators $\theta = (\theta_t)$, a class of stochastic processes that can be used to generate probability measures \mathcal{Q}^{θ} out of the original P by defining the densities of the probability distributions. Moreover, we assume that the density generators $(\theta_t) \in \Theta$ are restricted to the non-stochastic range $K = [-\kappa, \kappa]$. This definition of the set \mathcal{P} translates into a constant ambiguity interval around the original measure P, thereby facilitating analytical solutions of dynamic optimization problems. Furthermore, it ensures that \mathcal{P} is rectangular (Chen and Epstein, 2002), a necessary condition for intertemporal optimization problems to be dynamically consistent. For a formal derivation see the appendix C.1.

It follows from the specification of ambiguity, using Girsanov's theorem (see e.g. Duffie (2001, p. 337)), that a stochastic process $(B_t^{\theta})_{0 \le t < \infty}$ defined as

$$(\forall t \ge 0, \forall \theta \in \Theta) \quad B_t^{\theta} = B_t + \int_0^t \theta_s ds$$
 (4.4)

is a standard Brownian motion with respect to \mathbb{F} on $(\Omega, \mathcal{F}, \mathcal{Q}^{\theta})$. Now we use this result (note that (4.4) is equivalent to $dB_t^{\theta} = dB_t + \theta_t dt$) to generalize the stochastic Ito process of the profit flow given by equation (4.1) to the general set \mathcal{P} :

$$(\forall t \ge 0, \forall \theta \in \Theta) \quad d\pi_t = (\mu - \sigma\theta_t)\pi_t dt + \sigma\pi_t dB_t^\theta \tag{4.5}$$

Hence, we see that the all stochastic processes to describe the profit flow π_t differ only in the drift term from each other. Thus, the multiplicity of measures in \mathcal{P} translates in modeling ambiguity about the drift of the profit flow, which can vary according to the range K. Finally, we recall the solution for π_t :

⁵The use of a set of probability measures \mathcal{P} is motivated by Ellsberg (1961): He showed that the conventional approach of choosing a subjective probability distribution in the absence of an objective distribution (Subjective Utility Theory, Savage (1954)) is in conflict with the observed behavior of individuals. In this light, the use of a set of distributions imposes a less rigid framework on the decision maker.

$$(\forall t \ge 0, \forall \theta \in \Theta) \quad \pi_t = \pi_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t - \sigma\int_0^t \theta_s ds + \sigma B_t^\theta\right) \tag{4.6}$$

It is important to keep in mind that there is only one observable stochastic profit flow π_t , but many different stochastic differential equations (the set defined in (4.6)) that can describe it. Ambiguity over this set does not vanish over time, since we assume that all θ_t vary within the range K in an independent and indistinguishable way (IID ambiguity, Epstein and Schneider (2003)). Accordingly, it is not possible to learn the distribution of $\theta_t \in \Theta$, neither to reduce the set \mathcal{P} over time.

4.4 Decision making under Ambiguity

How is a decision maker affected by the presence of ambiguity? In his famous urn experiment, Ellsberg (1961) showed not only that there is an impact of *uncertainty* on decisions which is different from *risk*, but his findings suggested as well that decision makers tend to be uncertainty or ambiguity averse. Hence, theory followed experiment, and economists came up with some ideas how to include ambiguity, as well as the decision makers attitude towards ambiguity into the standard expected utility model. One of the most popular models is the Multiple Expected Utility theory by Gilboa and Schmeidler (1989) that replaces the usual unique probability distribution with a set of probability distributions. By maximizing utility over the worst possible probability distribution (maximin preferences), they offer an approach to model the decision making under complete ambiguity aversion.

In this work, we model decision making under ambiguity by applying a convex combination of two extreme preferences over the set of probability distributions. One part of the weight is attributed to the worst probability distribution, similar to the Gilboa and Schmeidler (1989) model, accounting for ambiguity aversion. However, some fraction of the weight is given to the best possible probability distribution, reflecting ambiguity loving characteristics of the decision maker. Such a convex combination is also known as Hurwitz criterion. In fact, this model is inspired by the α -MEU preferences as presented by Ghiradato et al. (2004) and the *neo-additive capacities* by Chateauneuf et al. (2006). The latter model decision making under ambiguity by evaluating a convex combination of the two extreme outcomes, the best and the worst scenario, and a usual additive probability distribution⁶. As noted earlier, the

 $^{^{6}}$ When setting the parameters attributed to the best and the worst case such that they sum up

inclusion of such ambiguity *loving* features was shown to behaviorally meaningful by e.g. Heath and Tversky (1991) or Kilka and Weber (1998). Especially in the context of start-up investments, the incorporation of ambiguity love from the side of the entrepreneur seems to be evident, at least to a part⁷. Such a convex combination of two extreme preferences over uncertain events is characterized by the parameter $\alpha \in [0, 1]$, describing the decision maker's attitude towards ambiguity. Since it is attributed to the best case, we call this parameter also the degree of optimism. Given that a decision maker's preferences can be characterized by such a parameter, the so-called α -expected value of a function can be represented by the following expression:

$$E^{\alpha}[f(x)] = \alpha \sup_{p \in \mathcal{P}} E^{p}[f(x)] + (1 - \alpha) \inf_{p \in \mathcal{P}} E^{p}[f(x)]$$

$$(4.7)$$

where \mathcal{P} is the set of possible probability measures, and $f: x \to \mathbb{R}$ is the stochastic payoff function. The intuition of this α -expected utility is rather simple: the decision maker puts some of his weight (α) to the best possible scenario, and the rest of his weight to the worst possible scenario $(1 - \alpha)$.

4.5 Optimal Investment under Ambiguity

Now we are ready to analyze the optimal investment decision under ambiguity. First, we present the evaluation of the investment project, given the observable current profit flow π_t . Then we examine the investor's investment problem, i.e. his decision to carry out the project or to abandon it. Finally, we examine the implications of changes in the entrepreneur's perceived uncertainty (the parameter κ) and level of optimism (represented by α) on his project evaluation and investment decision.

to one $(\gamma + \lambda = 1)$, our formulation coincides with their model in a finite state space environment. For a detailed presentation of the neo-additive capacities, please refer to Chateauneuf et al. (2006). A more axiomatic treatment is included in Ghiradato et al. (2004).

⁷It might appear somewhat peculiar to the reader that we focus on the two extreme attitudes towards ambiguity only. There are several reasons why we opt for this approach: first, the results are driven by the extremes of the range of uncertainty. Including the whole range between the best and worst case (similar to e.g. Klibanoff et al. (2005)) would basically lead to similar, but of course weaker results. Moreover, it is important to keep in mind that - although selecting only two possible probability distributions out of a large set of distributions - the α -MEU preferences do not imply that the decision maker considers these two distributions to be in a sense more correct than others. These two extreme distributions represent only an assessment of the best and the worst possible scenarios. The α -MEU preferences are then a way to combine theses assessments into one a single preference order. Finally, the solution of intertemporal optimization problems in a dynamically consistent way is much easier when focussing on the extremes.

4.5.1 Project Evaluation

According to equation (4.2), the payoff function f is given here by the present value of future profits:

$$f(\pi_t) = E_t \left[\int_t^\infty e^{-\rho(s-t)} \pi_s ds \right]$$
(4.8)

Ambiguity is modeled by a set of probability distributions $\mathcal{P} = \{\mathcal{Q}^{\theta} | \theta \in \Theta\}$, as defined in section 4.3. Remember that the degree of ambiguity is specified by the interval of the density generator $K = [-\kappa, \kappa]$.⁸ The α -expected value of the installed investment project V can thus be calculated as

$$V(\pi_t|\alpha) = \alpha \sup_{Q^\theta \in \mathcal{P}} E_t^{Q^\theta} \left[\int_t^\infty e^{-\rho(s-t)} \pi_s ds \right] + (1-\alpha) \inf_{Q^\theta \in \mathcal{P}} E_t^{Q^\theta} \left[\int_t^\infty e^{-\rho(s-t)} \pi_s ds \right]$$
(4.9)

which has the following solution:

Proposition 1. The α -expected value of the investment project with an infinite profit stream is given by:

$$V(\pi_t | \alpha) = \pi_t \left(\frac{\alpha}{\rho - (\mu + \kappa \sigma)} + \frac{1 - \alpha}{\rho - (\mu - \kappa \sigma)} \right)$$
(4.10)

Proof. See appendix C.2.

What can we learn from this expression? First, look at the case without ambiguity, i.e. $\kappa = 0$. Then the term reduces to $V_t = \pi_t/(\rho - \mu)$, which is identical to the standard expression in (4.3) for the expected present value of an infinite profit stream. If we let ambiguity gradually increase ($\kappa > 0$), the decision maker adds the sum of the two terms of (4.10), depending on the parameter α . In the case of complete ambiguity aversion, also called pessimism, α equals 0 and the value of the installed investment project coincides with the one under maximin preferences, as analyzed by Nishimura and Ozaki (2007): $V(\pi_t | \alpha = 0) = \frac{\pi_t}{\rho - (\mu - \kappa \sigma)}$. For notational convenience, we replace the term in the brackets of (4.10) by the parameter ϕ :

$$\phi = \frac{\alpha}{\rho - (\mu + \kappa\sigma)} + \frac{1 - \alpha}{\rho - (\mu - \kappa\sigma)}$$
(4.11)

This gives: $V(\pi_t | \alpha) = \pi_t \phi$.

⁸Note that for the evaluation problem to make sense, the admissible range of κ is restricted to $\kappa < (\rho - \mu)/\sigma$, as we will see in the following.

4.5.2 Investment Decision

After having analyzed the project value, the entrepreneur compares the expected profits of the investment $V(\pi_t|\alpha)$ with the related investment costs I, to determine the value of his option to invest:

$$F(\pi_t | \alpha) = \max \{ V(\pi_t | \alpha) - I, 0 \} = \max \{ \pi_t \phi - I, 0 \}$$
(4.12)

In case of $\phi \pi_t > I$, investment is carried out. When solving this condition for the current profit level π_t , it is possible to calculate the critical level of current profits, that must prevail in order to invest.

Corollary 1. The critical level of profits π_t , that is required in order to invest, is given by:

$$\pi^* = \frac{I}{\phi} = \frac{I}{\frac{\alpha}{\rho - (\mu + \kappa \sigma)} + \frac{1 - \alpha}{\rho - (\mu - \kappa \sigma)}}$$
(4.13)

Put differently, the decision maker decides to invest if and only if the current observable profits π_t exceed π^* . Otherwise, he will abandon the project.

4.5.3 Comparative Statics

In this section, we look at the effects of changes in the perceived level of ambiguity (κ) and the decision maker's attitude towards ambiguity (α) on the expected value of an investment project $V(\pi_t)$, the option to invest $F(\pi_t)$, and the investment threshold π^* . Since most of the interrelations are rather complex, we refer to numerical examples in order to demonstrate the different effects. In the illustrations presented below, we fix the investment parameters, unless otherwise stated, as follows: $\rho = 0.1$, $\mu = 0.05$, I = 10, $\pi_t = 1$, and $\sigma = 0.2$. The figures and results are however robust to changes in these assumptions.

An increase in Ambiguity

What happens if the decision maker perceives an increase in ambiguity? Since the expected value of the project $V(\pi_t)$ is linear in the parameter ϕ , we analyze the derivative of the parameter ϕ with respect to κ :

$$\frac{\partial \phi}{\partial \kappa} = \frac{\sigma \left(\alpha (\rho - (\mu - \kappa \sigma))^2 - (1 - \alpha) (\rho - (\mu + \kappa \sigma))^2 \right)}{(\rho - (\mu + \sigma \kappa))^2 (\rho - (\mu - \sigma \kappa))^2}$$

This derivative is nonnegative if and only if

$$\alpha \ge \frac{1}{2} - \frac{\kappa \sigma(\rho - \mu)}{(\rho - \mu)^2 + \kappa^2 \sigma^2} \tag{4.14}$$

The term on the right hand side of the inequality can reach values in the range between 0 and 0.5. In the absence of either risk (i.e. $\sigma = 0$) or ambiguity ($\kappa = 0$), α must be larger than 0.5 such that increasing ambiguity has a positive effect on the expected investment value. For all strictly positive values of both κ and σ , the required level of optimism is smaller, attaining a minimum value of $\alpha \to 0$ in the limit when increasing κ towards its maximal admissible range $(\rho - \mu)/\sigma$. Consequently, we can state the following lemma:

Lemma 1. For all positive values of optimism $\alpha > 0$, there exists a threshold level of ambiguity κ^* such that for all levels of ambiguity $\kappa > \kappa^*$, a perceived increase in ambiguity has a positive impact on the expected investment value $V(\pi_t)$.

Proof. For $\alpha \geq 0.5$, $\kappa^* = 0$ since we know from condition (4.14) that $\partial V(\pi_t)/\partial \kappa$ is nonnegative for all values of κ if $\alpha \geq 0.5$. For $\alpha \in (0, 0.5)$, κ^* is given by:

$$\kappa^* = \frac{(\rho - \mu) \left(\sqrt{\alpha - \alpha^2} - \frac{1}{2}\right)}{\sigma \left(\alpha - \frac{1}{2}\right)} \tag{4.15}$$

which is the value of κ where $\partial V(\pi_t)/\partial \kappa = 0$. When $\alpha \to 0.5$, $\kappa^* \to 0$, i.e. no ambiguity; when $\alpha \to 0$, $\kappa^* \to (\rho - \mu)/\sigma$, the maximal admissible level of ambiguity.

We can see this effect more easily in figure 4.1. This graph plots the natural logarithm of the expected value of the investment $V(\pi_t)$ as a function of ambiguity (κ) for different levels of optimism $(\alpha)^9$. For most parameter values of optimism, increasing ambiguity leads a decision maker to value the investment project higher than before, especially when ambiguity is already quite high. Only almost completely ambiguity averse decision makers relate more uncertainty to a lower project value. The reason for this rather positive response to ambiguity lies in the convex combination of the best and the worst case. A small fraction of optimism suffices to induce an overall positive picture of the investment project, even for rather pessimistic entrepreneurs.

Since the value of the option to invest $F(\pi_t)$ is just equal to the expected project value less investment costs, a perceived increase in ambiguity has a similarly positive effect on $F(\pi_t)$. Figure 4.2 displays the value of the investment project $V(\pi_t)$, and the option to invest $F(\pi_t)$ as a function of the current profit level π_t for different degrees

 $^{^{9}}$ We use the logarithm of the expected investment value because of scaling reasons in the graph.



Figure 4.1: Value of Investment Project as a Function of Ambiguity

The natural logarithm of the value of the installed investment project $V(\pi_t)$ as a function of the level on ambiguity (κ) for different values of optimism (α). In case of complete optimism ($\alpha = 1$), ambiguity increases the project value, in case of complete pessimism ($\alpha = 0$), ambiguity decreases its value. For all parameter values in-between, it depends on the level of ambiguity, whether increasing ambiguity rises or lowers the project value. The value of the installed project value is given by:

$$V(\pi_t | \alpha) = \pi_t \left(\frac{\alpha}{\rho - (\mu + \kappa \sigma)} + \frac{1 - \alpha}{\rho - (\mu - \kappa \sigma)} \right)$$
(4.10)



Figure 4.2: Value of Investment Project as a Function of Current Profit

The value of the investment project $V(\pi_t)$, and the option to invest $F(\pi_t)$ as a function of the current profit level π_t for different degrees of ambiguity. The level of optimism is fixed at $\alpha = 0.5$. The value of the option to invest is given by:

$$F(\pi_t | \alpha) = \max \{ V_t - I, 0 \} = \max \{ \pi_t \phi - I, 0 \}$$
(4.12)

of ambiguity. The level of optimism is fixed at $\alpha = 0.5$. The value of both the option to invest as well the expected project value increase when the perceived ambiguity, i.e. κ , rises. As a consequence, the critical value of current profits π^* , that must prevail so that the decision maker wants to invest, reduces with increased ambiguity. More formally, we can state the following corollary, derived directly from the Lemma:

Corollary 2. For all positive values of optimism $\alpha > 0$, there exists a level of ambiguity κ^* such that for all degrees of ambiguity $\kappa > \kappa^*$, a perceived increase in ambiguity has a negative impact on the investment threshold π^* . κ^* is given by (4.15)

This effect can also be seen in figure 4.3, which presents the investment threshold (π^*) as a function of the level of ambiguity (κ) for different values of optimism (α) . Analogously to figure 4.1, we can see that for most parameter values of optimism, increasing ambiguity has a negative effect on the investment threshold (π^*) , most predominantly in the case of complete optimism $(\alpha = 1)$. Only in the case of very pessimistic decision makers, i.e. α is close to 0, the presence of ambiguity increases investment threshold. In the absence of ambiguity, the threshold lies at 0.5. Thus, the bottom line of this analysis is that for most decision makers, an increase in ambiguity has a positive impact on the investment decision.

An increase in optimism

Next, we can look at the effect of an increase in optimism, i.e. an increase in α , on the subjective project evaluation and investment decision. How is the value of the investment project $V(\pi_t|\alpha)$ influenced by an increase in optimism? To see this, we look at the relation between $V(\pi_t)$ and α . Consider the derivative of the coefficient ϕ with respect to α :

$$\frac{\partial \phi}{\partial \alpha} = \frac{1}{\rho - (\mu + \kappa \sigma)} - \frac{1}{\rho - (\mu - \kappa \sigma)} \ge 0$$

This expression is unambiguously positive as long as κ and σ are both strictly positive. Since the value of the investment project is linear in ϕ , increasing optimism always leads to an increase in the perceived value of the project. This is quite intuitive: Future growth opportunities are considered to be more and more likely, and hence the expected present value of future profits rises. Again, this effect translates directly in the same manner on to the value of the option to invest, $F(\pi_t | \alpha)$.

How is the investment threshold effected by an increase in optimism? Figure 4.4 plots the investment threshold (π^*) as a function of the value of optimism (α) for



Figure 4.3: Investment Threshold as a Function of Ambiguity

The investment threshold (π^*) as a function of the level of ambiguity (κ) for different values of optimism (α) . In case of complete optimism $(\alpha = 1)$, ambiguity decreases the investment threshold π^* , in case of complete pessimism $(\alpha = 0)$, the presence of ambiguity increases its value. For parameter values in-between, ambiguity decreases the investment threshold as well, but not as much as in the optimistic case. In the absence of ambiguity, the threshold lies at 1. The function for the threshold level π^* is given by:

$$\pi^* = \frac{I}{\phi} = \frac{I}{\frac{\alpha}{\rho - (\mu + \kappa \sigma)} + \frac{1 - \alpha}{\rho - (\mu - \kappa \sigma)}}$$
(4.13)



Figure 4.4: Investment Threshold as a Function of Optimism

The investment threshold (π^*) as a function of the value of optimism (α) for different levels of ambiguity (κ) . For all positive degrees of ambiguity, decreasing optimism lowers the threshold level. For low values of α , i.e. an pessimist, the threshold is higher than without ambiguity; for high values of α , i.e. an optimist, the threshold is below. The function for the threshold level π^* is given by:

$$\pi^* = \frac{I}{\phi} = \frac{I}{\frac{\alpha}{\rho - (\mu + \kappa\sigma)} + \frac{1 - \alpha}{\rho - (\mu - \kappa\sigma)}}$$
(4.13)

different levels of ambiguity (κ). For all strictly positive degrees of ambiguity, decreasing optimism lowers the threshold level. For low values of α , i.e. an pessimist, the threshold is higher than without ambiguity; for high values of α , i.e. an optimist, the threshold is below. Again, this negative relation between optimism is in line with our intuition: The more optimistic the investor is, the higher he valuates his project, given the current profit flow. Thus, the investment threshold decreases when optimism rises. The higher the level of ambiguity, the more pronounced is this effect.

4.6 Flexible Investment Timing

In the previous analysis we have focussed on the optimal investment decision when the entrepreneur faces a pass-fail decision. He can either decide to invest immediately, or abandon the whole project. More precisely, we discarded the entrepreneur's possibility to put off the investment project for a while and to decide upon the venture at a later point of time. Yet, in many practical examples, investment projects have a higher degree of flexibility regarding the investment timing. Quite often the decision maker has the possibility to defer the project for a certain time period, in order to wait for - hopefully - some more favorable investment conditions.

In this section, we thus extend our model of investment under ambiguity by lifting the restriction of the immediate investment requirement. Once we allow the investment decision and the investment itself to be carried out at later points of time, the determination of the optimal investment time becomes paramount. The investment problem changes accordingly. Now, the investor wants to maximize the value of the project over the investment time $\tau \in [t, \infty)$. In absence of ambiguity, the problem is as follows:

$$F_t = \max_{\tau} E_t \left[\int_{\tau}^{\infty} e^{-\rho(s-t)} \pi_s ds - e^{-\rho(\tau-t)} I \right]$$
(4.16)

Since the investor faces a project generating an infinite stream of profits, we have a stationary optimal stopping problem in continuous time which allows us to derive a critical level of current profits π' (independent of time), that must be surpassed in order to invest. In contrast to π^* , which gives the critical level of profits when having only the possibility to invest immediately or to abandon the projet, π' reflects the possibility of delaying the investment project for a certain time. We have therefore $\pi' > \pi^*$. The difference captures the so-called "value of waiting", i.e. having the possibility to carry out the project when investment conditions are better increases the investment threshold.

The problem of (4.16) is generally known as irreversible investment problem, see e.g. Dixit and Pindyck (1994). Most important, this methodology allows to determine the optimal timing for investment opportunities that do not vanish at once but can be carried out over an extended period. Because of the infinite time horizon, the investment problem has a recursive structure and can thus be solved by dynamic programming, leading to a analytical characterization of the solution¹⁰.

¹⁰See Dixit and Pindyck (1994) for a presentation and full characterization of the general solution of the investment problem using dynamic programming.

Now, the question is how this problem can be formulated in an ambiguous environment. Compared to the maximization problem in (4.16), the decision maker now faces a set of probability distributions \mathcal{P} , as specified in section 4.3. Applying the α -expected utility, we get the following maximization problem:

$$F(\pi_t | \alpha) = \max_{\tau \in [t,\infty)} E_t^{\alpha} \left[\int_{\tau}^{\infty} e^{-\rho(s-t)} \pi_s ds - e^{-\rho(\tau-t)} I \right]$$
(4.17)

However, it is generally not possible to derive a solution to this problem using dynamic programming similar to the investment problem without ambiguity. Although we face an infinite time horizon and a constant level of ambiguity (specified by the constant ambiguity range K), and thus have at each point of time a priori an identical decision problem only depending on the state variable π_t , the optimization problem of (4.17) cannot be represented in a recursive way. The reason is that the preferences themselves have to be recursive, i.e. dynamically consistent for dynamic programming techniques to be applicable.

Unfortunately, the α -expected utility model based on the α -MEU preferences as presented by Ghiradato et al. (2004) do generally not exhibit such a recursive structure¹¹. In general, for preferences to be recursive, the law of iterated expectations, which is usually given by $E_t[x] = E_t[E_s[x]] \quad \forall s > t$ in this context, must be fulfilled. In the multiple priors model, an equivalent condition must hold, taking into account the multiplicity of probability measures at each point of time, and the expectation operator defined on these probability measures. In our case of α -MEU preferences, this condition is given by:

$$\forall s > t \qquad E_t^{\alpha}[x] = E_t^{\alpha}[E_s^{\alpha}[x]]$$

Or, equivalently, using the explicit notation as presented in (4.7):

¹¹It is well known that dynamic consistency is very difficult to reconcile with ambiguity. For a detailed discussion, see e.g. Klibanoff et al. (2006). Actually, Klibanoff et al. (2006) propose a model of ambiguity preferences that allow for a multitude of ambiguity attitudes and while preserving dynamic consistency. However, their model is rather complex and therefore not suitable for continuous time problems.

$$\alpha \sup_{p \in \mathcal{P}} E_t^p [x] + (1 - \alpha) \inf_{p \in \mathcal{P}} E_t^p [x]$$

$$= \alpha \sup_{p \in \mathcal{P}} E_t^p \left[\alpha \sup_{p \in \mathcal{P}'} E_s^p [x] + (1 - \alpha) \inf_{p \in \mathcal{P}'} E_s^p [x] \right]$$

$$+ (1 - \alpha) \inf_{p \in \mathcal{P}} E_t^p \left[\alpha \sup_{p \in \mathcal{P}'} E_s^p [x] + (1 - \alpha) \inf_{p \in \mathcal{P}'} E_s^p [x] \right]$$

$$(4.18)$$

where \mathcal{P}' denotes the set of probability measures at time s > t, derived from \mathcal{P} by the set of conditional probabilities imposed by strong rectangularity (\mathcal{P} denoting as usual the probability set at time t). In general, the recursive structure imposed by this condition is not met. To see this more clearly, note that the second term can be transformed into

$$\alpha^{2} \sup_{p \in \mathcal{P}} E_{t}^{p}[x] + 2\alpha(1-\alpha) \sup_{p \in \mathcal{P}} E_{t}^{p} \left[\inf_{p \in \mathcal{P}'} E_{s}^{p}[x] \right] + (1-\alpha)^{2} \inf_{p \in \mathcal{P}} E_{t}^{p}[x]$$
(4.19)

which is generally different from the left hand side of (4.18). For a more detailed derivation, see appendix C.3.

More intuitively, the decision maker evaluates x at each point of time as the weighted sum of the best and worst case scenario. So he evaluates for example, at time s > t, the remaining decision tree under both the best case $(\sup_{p \in \mathcal{P}'} E_s[x])$ and the worst case scenario $(\inf_{p \in \mathcal{P}'} E_s[x])$, and combines them together to $E_s^{\alpha}[x]$. However, when evaluating x at an earlier time t, the decision maker will not take into account the best case scenario at time t of the worst case scenario at time s, since the α -MEU preferences do not allow for intertemporal weighting of best and worst cases, which are given by the term in the middle of expression (4.19). Instead, α -MEU preferences reflect only the intratemporal weighting of the best and the worst case. Hence, the decision maker evaluates x at time t for the best case scenario in s and the worst case of the worst case of the worst case scenario in s, i.e. the two other terms of (4.19). Consequently, $E_t^{\alpha}[x]$ cannot use all the information as $E_t^{\alpha}[E_s^{\alpha}[x]]$ can incorporate, so that both expressions differ from each other.

4.6.1 Special Cases

Fortunately, there some special cases in which it is possible to derive a solution for the optimal investment strategy under ambiguity, as presented in (4.17). In the case of complete pessimism ($\alpha = 0$) or complete optimism ($\alpha = 1$) the condition (4.18) is fulfilled, as can be easily verified in (4.19)¹². The convex combination of the α -expected utility reduces then to a single term, consisting either of the best case scenario, or the worst case. As Chen and Epstein (2002) show, each of the two terms has a recursive structure - given the rectangularity assumption on the set \mathcal{P} . Using this fact, dynamic programming yields, similarly to the standard real option theory, an analytical solution in both extreme cases. In their paper, Nishimura and Ozaki (2007) examine the worst case scenario, i.e. the pessimistic decision maker, which corresponds our case of $\alpha = 0$. The solution of the optimistic case is just analogous. We can hence state:

Proposition 2. Given a complete optimistic decision maker ($\alpha = 1$), or a complete pessimistic decision maker ($\alpha = 0$), the critical level of current profits π_t , that must be attained in order to invest is given as:

$$\pi' = \frac{b}{(b-1)\phi}I\tag{4.20}$$

where b is given by:

$$b = \frac{1}{2} - \frac{\gamma}{\sigma^2} + \sqrt{\left(\frac{\gamma}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1$$
(4.21)

and γ is given by:

$$\gamma = \alpha(\mu + \sigma\kappa) + (1 - \alpha)(\mu - \sigma\kappa) \tag{4.22}$$

and ϕ is given by (4.11).

Proof. See appendix C.4

Once π_t exceeds π' , the option to invest is executed and investment is made. Note that equation (4.20) is rather similar to the usual expression for the investment thresh-

¹²Another trivial case in which condition (4.18) is fulfilled is given if the set \mathcal{P} is singleton, i.e. in the absence of ambiguity. In this case, the infinimum and supremum terms coincide. If deviating from the strong rectangularity assumption, one can think of other conditions in which (4.18) holds. As a simple example consider the case when \mathcal{P}' reduces to a singleton, either state dependent or not. However, in the absence of the strong rectangularity structure, it is not possible to derive analytical solutions. In addition, such models rarely make economically sense - especially in the continuous time context.

old without ambiguity, where π' is given by: $\frac{b}{(b-1)}I(\rho-\mu)$.¹³ It depends on the parameters α and κ , whether the critical level is higher or lower than in the standard case. In the case of complete ambiguity aversion ($\alpha = 0$), the critical level π' is always higher in presence of uncertainty than without it. Because future prospects are evaluated very pessimistic, current conditions must reach higher levels in order to decide positively about the investment. Of course, in case of $\kappa = 0$ the critical levels coincide. One can also show that the value of the option to invest is given by:

for
$$\alpha \in \{0;1\}$$
 $F(\pi_t | \alpha) = \frac{(b-1)^{b-1}}{I^{(b-1)}b^b} (\pi_t \phi)^b = \frac{(b-1)^{b-1}}{I^{(b-1)}b^b} (V(\pi_t))^b$ (4.23)

4.6.2 Comparative Statics

Again, we now have a look at some comparative statics in order to get a better understanding of the model's predictions. Since we restrict the attitude towards ambiguity to the two extreme cases only, we present the effect of changes in the level of ambiguity (κ) only.

An Increase in Ambiguity

How is the decision maker's evaluation of the investment option affected by an perceived increase in ambiguity? Figure 4.5 displays the natural logarithm of the value of the option to invest $F(\pi_t|\alpha)$ as a function of the level on ambiguity (κ) for the case of complete optimism ($\alpha = 1$), and complete pessimism ($\alpha = 0$). In case of complete optimism ($\alpha = 1$), ambiguity increases the option value, in case of complete pessimism ($\alpha = 0$), ambiguity decreases its value. This is fairly similar to figure 4.1 which plots the value of the project V_t . This pattern is quite intuitive, since - besides the coefficient - the value of the option to invest equals the value of the installed investment to the power of *b* (see equation (4.23)). Since *b* is larger than 1, the responsiveness (positive or negative) of the value if the investment is enforced in the option formula.

Another interesting issue to look at is the investment timing and how it is affected by a perceived change in ambiguity. Since the timing of the investment is fully determined by the instant current profits π_t exceed the threshold level π' for the first time, all that matters is to examine the relation between threshold level π' and ambiguity. Figure 4.6 displays the investment threshold π' as a function of the level of ambiguity for the case of complete optimism ($\alpha = 1$), and complete pessimism ($\alpha = 0$). In case

¹³Of course, b is then calculated by replacing γ with μ .



Figure 4.5: Option to Invest as a Function of Ambiguity

The natural logarithm of the value of the option to invest $F(\pi_t)$ as a function of the level of ambiguity (κ) for the case of complete optimism $(\alpha = 1)$, and complete pessimism $(\alpha = 0)$. In case of complete optimism $(\alpha = 1)$, ambiguity increases the option value, in case of complete pessimism $(\alpha = 0)$, ambiguity decreases its value. The value of the option to invest is given by:

$$F(\pi_t|\alpha) = \frac{(b-1)^{b-1}}{I^{(b-1)}b^b} (\pi_t \phi)^b = \frac{(b-1)^{b-1}}{I^{(b-1)}b^b} V(\pi_t)^b$$
(4.23)



Figure 4.6: Investment Threshold as a Function of Ambiguity

The investment threshold (π') as a function of the level of ambiguity (κ) for the case of complete optimism $(\alpha = 1)$, and complete pessimism $(\alpha = 0)$. In case of complete optimism $(\alpha = 1)$, ambiguity decreases the investment threshold π' , in case of complete pessimism $(\alpha = 0)$, the presence of ambiguity increases its value. The function for the threshold level of π^* is given by:

$$\pi' = \frac{b}{(b-1)\phi}I\tag{4.20}$$

of complete optimism ($\alpha = 1$), ambiguity decreases the investment threshold π' , in case of complete pessimism ($\alpha = 0$), the presence of ambiguity increases its value. Intuitively, the reason for this effect is clear: Due to uncertainty, the optimist perceives the value of the investment project to get higher (lower for the pessimist), as seen before. Hence, it becomes more costly (less costly) to wait rather than to invest and receiving the stream of profits.

4.7 Conclusion

At the latest from the literature in behavioral economics and psychology we know that individuals tend to overestimate their competence. When facing ambiguous situations, this overconfidence can lead to ambiguity loving behavior. This study shows that an entrepreneur's attitude towards ambiguity is crucial when analyzing his investment decisions given an uncertain environment. By introducing an individual parameter reflecting the personal attitudes of the decision maker, our simple irreversible investment model helps to explain differences in investment behavior in situations which are objectively identical. What is more, this analysis shows that the presence of ambiguity leads in many cases to an increase in the subjective project value, and entrepreneurs are more eager to invest - even when they are ambiguity averse up to a very small fraction of optimism.

For traditional investment models, it does not make any difference when investigating an optimal investment strategy whether you are an enthusiastic entrepreneur in the internet start-up business, or a faithful accountant in the petroleum industry. However, in reality this difference seems to matter.

Appendix

A Appendix to Chapter 1

A.1 The EIV Regression Approach

Since the betas that enter the cross-sectional regression (1.11) are not the true betas, but only noisy estimates thereof (obtained from the times series regressions as displayed in (1.10)), we face an errors-in-variables (EIV) problem in the second pass of the regression approach.

$$r_{it} - r_{ft} = \alpha_i + \beta_i (r_{mt} - r_{ft}) + \varepsilon_{it} \qquad t = 1, 2, \dots, T \qquad \forall i \tag{1.10}$$

More precisely, standard OLS estimation in (1.11) would underestimate the effect of the beta estimates, since the true betas have a lower standard errors than the estimated betas. In addition, other coefficients in the model can be biased to the extent they are correlated with this poorly measured variable.

To correct for this possible bias, we employ the standard EIV regression approach as presented e.g. by Greene (2002) or Fuller (1987): we know that the betas, one of the explanatory variables in the subsequent regressions, are measured with error, since the true value of the variables cannot be observed directly. Instead of observing β , one can only observe the sum:

$$\hat{\beta} = \beta + u$$

where u is a normally distributed random variable with $cov(\beta, u) = 0$, and $\hat{\beta}$ are the estimates obtained from (1.10). One can adjust for the bias caused by the measurement error in the second pass with the help of its reliability ratio, which is defined as follows:

$$r = 1 - \frac{var(u)}{var(\hat{\beta})}$$

The (empirical) variance of the estimated variable $\hat{\beta}$ is easily calculated. The variance of the error u is more difficult to estimate, since the true values of beta (β) and the variance thereof are unknown. However, we can make use of the information we have about the variance of each individual $\hat{\beta}_i$ in the first pass, which is given by:

$$var(\hat{\beta}_i) = \sigma_i^2[(X'X)^{-1}]_{22} \qquad \forall i$$

where X is the matrix of explaining variables in (1.10), i.e. the combination of a ones vector (for the intercept α_i) and the vector of market excess returns $(r_{mt} - r_{ft})$. $[(X'X)^{-1}]_{22}$ then denotes the second row, second column element of the inverted matrix of X'X. Since we have for each observation *i* the relation $\hat{\beta}_i = \beta_i + u_i$, this gives (the true β_i are fixed):

$$var(u_i) = var(\hat{\beta}_i) = \sigma_i^2 [(X'X)^{-1}]_{22} \qquad \forall i$$

Note that the term $[(X'X)^{-1}]_{22}$ is identical for each *i*. Hence, all what we need for estimating var(u) is an estimate of σ^2 . For large *N*, a consistent estimator of σ^2 is given by the average σ_i^2 :

$$\hat{\sigma}^2 = \bar{\sigma}^2 = \frac{1}{N} \sum_N \hat{\sigma}_i^2$$

Under the assumption of independence, and existence of the fourth moment of the error terms, this average converges by WLLN in the limit to the true value of σ^2 . This gives us finally:

$$var(u) \approx \bar{\sigma}^2 [(X'X)^{-1}]_{22}$$

= $\left(\frac{1}{N} \sum_{n=1}^N \hat{\sigma}_i^2\right) [(X'X)^{-1}]_{22}$ (A.1)

In other words, we use the average variance of the individual errors $\hat{\sigma}_i^2$ together with the similar structure of our N times series regressions to estimate the variance of the disturbance u. Calculating and inserting in (A.1), we get:

$$var(u) = 0.00944 \cdot 2.614 = 0.0247$$

Since the empirical variance of $\hat{\beta}$ is 0.0906, we obtain a reliability ratio of

$$r = 1 - \frac{0.0247}{0.0906} = 0.728$$

The cross-sectional regressions are hence carried out using a reliability ratio 0.728 for the beta estimates.

The EIV regressions take explicitly into account the fact that the betas in (1.11) are not the true betas (but only estimated betas) by adjusting the standard error of the betas in the regression. For example, using EIV regressions, the slope coefficients is given by: $\gamma_1^{EIV} = \gamma_1^{OLS}/r$. Hence, when considering that we use estimated betas, we increase their relative importance in the regression model. For a theoretical derivation of EIV regression, see Fuller (1987).

B Appendix to Chapter 3

B.1 Fama-MacBeth Regressions

In addition to the panel regressions displayed in the body of the chapter 3, we present here methodology and results of the Fama-MacBeth regressions, since they are very common in the empirical finance literature.

Methodology

For each month, we first regress the firm's individual stock return on its forecasting variable (e.g. the ICOC or V/P estimate) and firm-specific risk factors, similar to Fama and French (1992):

$$r_i = \alpha + \delta k_i + \varphi d\beta_i + \gamma' X_i + u_i \tag{B.1}$$

where r_i is the subsequent total stock return measured after having measured the forecasting variable (denoted by k_i), the market beta estimate β_i , the vector of firm characteristics that have been identified as risk-factors X_i . Finally, u_i is the error term. Again, the variable d is a dummy for the ex post observed market risk premium (i.e. the difference between the market return and the risk-free rate), having a value of 1 when the market risk premium is positive during the holing period, and -1 if it is negative (see section 3.4.1 for more details). In the next step, we then test whether the average coefficient estimates δ , φ , and γ' are significantly different from zero. The t-statistics are calculated by regressing the time series of the coefficient estimates on a constant. Since we use again overlapping data, the t-statistic is calculated on the basis of heteroscedasticity- and autocorrelation-consistent (HAC) standard errors (Newey and West, 1987) with 11 lags. In line with our discussion in section 3.4, we refrain from sorting the companies into portfolios before carrying out the Fama-MacBeth regressions, using individual firm data instead.

Results

Table B.1 reports the time-series averages of the slope coefficients of monthly crosssectional regressions when using the forecasting variable, such as the ICOC or V/P estimate, as only explanatory variable. In this simple specification, only the RIM2 ICOC is weakly related to stock returns.

Table B.2 then shows the results of the cross-sectional Fama-MacBeth Regressions when including other firm risk variables. The first row gives the results of the standard

Table B.1: Univariate Cross-sectional Fama-MacBeth Regressions (1973) - United States

Variable	Coefficient	t-stat
DDM3 ICOC	-0.22	(-0.35)
RIM2 ICOC	1.06^{*}	(1.66)
RIM3 ICOC	0.74	(1.43)
DDM3 VPS	-0.00	(-0.18)
DDM3 VPL	-0.01	(-0.38)
RIM2 VPS	0.06	(1.40)
RIM2 VPL	0.07	(1.58)
RIM3 VPS	0.03	(0.89)
RIM3 VPL	0.04	(1.09)
EP	0.37	(0.86)
DP	-0.27	(-0.34)
BP	0.03	(0.65)

This table shows the results of different univariate Fama-MacBeth regressions, i.e. the time-series averages of the slope coefficients of the cross-sectional regressions along with their t-statistics. The first three rows report the results when regressing stock returns on the ICOC estimates. The subsequent six lines show the regression results when using the V/P ratio as predictive variable. Finally, the last three rows give the explanatory power of the D/P, B/P, and E/P ratio.

The t-statistic is calculated on the basis of heteroscedasticity- and autocorrelation-consistent (HAC) standard errors following Newey-West (1987). The regressions are run over the full cross-section of companies. Stock returns are measured over 12 months. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level. The sample period is from January 1995 to February 2006. Country: United States. Observations: 38,014.

Fama and French (1992) regression (only the ex post risk premium dummy and price momentum is added) without any forecasting variable. Besides market beta, none of the explanatory variables is significant. This rather poor result is however in accordance with many studies using Fama-MacBeth regressions with individual data, e.g. Chan et al. (1996), or Subrahmanyam (2005). Then, below, the regression results including the forecasting variables are displayed. After controlling for the Fama-French factors and the short-term price momentum, none of the forecasting variables is significant. Again, only market beta is significantly related to stock returns.

Variable	Coefficient	BETA	BM	SIZE	MOM
		0.06^{***}	0.03	-0.00	0.03
		(3.60)	(1.03)	(-0.27)	(0.95)
DDM3 ICOC	-0.50	0.05***	0.04	-0.00	0.03
	(-1.34)	(3.16)	(1.22)	(-0.24)	(0.80)
RIM2 ICOC	0.55	0.06^{***}	0.03	-0.00	0.05
	(1.22)	(3.69)	(0.95)	(-0.13)	(1.64)
RIM3 ICOC	0.73	0.06^{***}	-0.01	-0.00	0.04
	(1.03)	(3.40)	(-0.21)	(-0.18)	(1.33)
DDM3 VPS	-0.01	0.05***	0.04	-0.00	0.03
	(-0.83)	(3.22)	(1.24)	(-0.23)	(0.84)
DDM3 VPL	-0.02	0.06^{***}	0.04	-0.00	0.03
	(-1.39)	(3.18)	(1.26)	(-0.22)	(0.83)
RIM2 VPS	0.03	0.06^{***}	0.03	-0.00	0.05
	(1.14)	(3.70)	(0.92)	(-0.13)	(1.63)
RIM2 VPL	0.038	0.06^{***}	0.03	-0.00	0.05
	(1.20)	(3.70)	(0.91)	(-0.13)	(1.63)
RIM3 VPS	0.057	0.06^{***}	-0.01	-0.00	0.04
	(1.07)	(3.43)	(-0.20)	(-0.29)	(1.27)
RIM3 VPL	0.05	0.06^{***}	-0.01	-0.00	0.04
	(0.95)	(3.40)	(-0.17)	(-0.30)	(1.26)
EP	0.19	0.06***	0.02	-0.00	0.04
	(0.68)	(3.55)	(0.55)	(-0.32)	(1.49)
DP	-0.73	0.05***	0.05^{*}	-0.00	0.03
	(-1.39)	(2.93)	(1.68)	(-0.14)	(0.86)
BP	0.03	0.06^{***}	-	-0.00	0.03
	(1.03)	(3.60)	-	(-0.27)	(0.95)

Table B.2: Fama-French Cross-sectional Fama-MacBeth Regressions (1973) - United States

This table shows the results of different Fama-MacBeth regressions, i.e. the time-series averages of the slope coefficients of the cross-sectional regressions along with their t-statistics. The Fama-French factors are abbreviated as follows: B/M ist the book yield, SIZE is calculated as the log of the market capitalization divided by the level of the stock market index, BETA is the five year regressed sensitivity on the market portfolio, and momentum (MOM) is the historical six-month price return. The upper line reports the results of the standard Fama and French (1992) regression specification, including price momentum but without any forecasting variable. Then, in the next three rows, the table contains the results when regressing stock returns on both the ICOC estimates and the Fama-French factors. The subsequent six lines show the regression results when using the V/P ratio as predictive variable. Finally, the last three rows give the joint explanatory power of the D/P, B/P, and E/P ratio with the Fama-French factors.

The t-statistic is calculated on the basis of heteroscedasticity- and autocorrelation-consistent (HAC) standard errors following Newey-West (1987). The regressions are run over the full cross-section of companies. Stock returns are measured over 12 months. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level. The sample period is from January 1995 to February 2006. Country: United States. Observations: 38,014.

C Appendix to Chapter 4

C.1 Ambiguity in Continuous Time

In this appendix, we present the approach of Chen and Epstein (2002) to model ambiguity in continuous time. Most important, we have to specify the set \mathcal{P} of measures equivalent to P by using suitable density generators.

Time¹⁴ t evolves over [0,T] and uncertainty is described by a probability space (Ω, \mathcal{F}, P) . Let \mathcal{L} be the set of real-valued, measurable, and \mathbb{F} -adapted stochastic processes on (Ω, \mathcal{F}, P) and let \mathcal{L}^2 be a subset of \mathcal{L} which is defined by

$$\mathcal{L}^{2} = \left\{ (\theta_{t})_{0 \leq t \leq T} \in \mathcal{L} \middle| \int_{0}^{T} \theta_{t}^{2} dt < +\infty \quad P - \text{a.s.} \right\}$$

A density generator is a stochastic process $\theta = (\theta_t) \in \mathcal{L}^2$ for which the process (z_t^{θ}) is a \mathbb{F} -martingale, where

$$(\forall t) \quad z_t^{\theta} = \exp\left(-\frac{1}{2}\int_0^t \theta_s^2 ds - \int_0^t \theta_s dB_s\right)$$

A sufficient condition for (z_t^{θ}) to be a \mathbb{F} -martingale and thus for (θ_t) to be a density generator is Novikov's condition:

$$E^P\left[\exp\left(\frac{1}{2}\int_0^T \theta_s^2 ds\right)\right] < +\infty$$

With density generators we can construct other probability measures from a given probability measure:

$$(\forall A \in \mathcal{F}_T) \quad \mathcal{Q}^{\theta}(A) = \int_A z_T^{\theta}(\omega) dP(\omega)$$

where Q^{θ} is the new probability measure which is absolutely continuous with respect to P.

Thus, given a set Θ of density generators, the corresponding set of probability measures is

$$\mathcal{P} = \{ \mathcal{Q}^{\theta} | \theta \in \Theta \} \tag{C.1}$$

Hence, ambiguity is characterized by \mathcal{P} for some set Θ . Finally, we have to specify the set of density generators Θ that generate the set of probability measures \mathcal{P} . We

¹⁴Although we rely in this appendix for simplicity on a finite time horizon, the results also hold true for the infinite time horizon.

rely on the definition of κ -ignorance and *IID ambiguity* by Chen and Epstein (2002) and specify Θ as follows:

$$\Theta = \{ (\theta_t) \in \mathcal{L}^2 | \theta_t(\omega) \in [-\kappa, \kappa] \ (m \otimes P) - a.s. \}$$
(C.2)

where m denotes the Lebesgue measure restricted on $\mathcal{B}([0,T])$.

This definition ensures that any element of Θ is restricted to the non-stochastic interval $K = [-\kappa, \kappa]$. Consequently, the corresponding set of measures \mathcal{P} is clustered within a constant ambiguity interval around the original measure P, thereby facilitating analytical solutions of dynamic optimization problems. Furthermore, this restriction assures that \mathcal{P} is *rectangular* in the terminology of Chen and Epstein (2002), and *strongly rectangular* in the terminology of Nishimura and Ozaki (2007). Rectangularity is a necessary condition for intertemporal optimization problems - in the ambiguity framework of Nishimura and Ozaki (2007) - to be dynamically consistent. For are more detailed exposition, see also Asano (2005).

C.2 Proof of Proposition 1

In this appendix, we prove proposition 1 (4.10) and derive the expected value of the installed investment project. The expression for the α -expected value is given as follows:

$$V(\pi_t|\alpha) = \alpha \sup_{Q^\theta \in \mathcal{P}} E_t^{Q^\theta} \left[\int_t^\infty e^{-\rho(s-t)} \pi_s ds \right] + (1-\alpha) \inf_{Q^\theta \in \mathcal{P}} E_t^{Q^\theta} \left[\int_t^\infty e^{-\rho(s-t)} \pi_s ds \right]$$
(4.9)

For notational convenience, we set t = 0, that is the level of current profits are denoted by π_0 . As a first step to derive the solution of (4.9), we show that

$$\sup_{\theta \in \Theta} E_0^{Q^{\theta}} \left[\exp\left(\left(B_t^{\theta} - \int_0^t \theta_s ds \right) \sigma \right) \right] = E_0^{Q^{-\kappa}} \left[\exp\left(\left(B_t^{-\kappa} + \kappa t \right) \sigma \right) \right]$$
(C.3)

where $E^{Q^{-\kappa}}$ denotes the expectation with respect to the probability measure generated by the non-stochastic density generator $-\kappa \in K$. To see this, note that $\forall (\theta_t) \in \Theta$, that is $\forall (\theta_t) \in [-\kappa, \kappa]$, it is true that:

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$$E_0^{Q^{\theta}} \left[\exp\left(\left(B_t^{\theta} - \int_0^t \theta_s ds \right) \sigma \right) \right] \leq E_0^{Q^{\theta}} \left[\exp\left(\left(B_t^{\theta} - \int_0^t -\kappa ds \right) \sigma \right) \right]$$
$$= E_0^{Q^{\theta}} \left[\exp\left(\left(B_t^{\theta} + \kappa t \right) \sigma \right) \right]$$
$$= \exp\left(\left(\left(\frac{1}{2} \sigma t + \kappa t \right) \sigma \right) \right]$$
$$= E_0^{Q^{-\kappa}} \left[\exp\left(\left(B_t^{-\kappa} + \kappa t \right) \sigma \right) \right]$$

Now, we are ready to transform the supremum expression in the first term of equation (4.9) above as follows:

$$\begin{split} \sup_{Q^{\theta} \in \mathcal{P}} E_{0}^{Q^{\theta}} \left[\int_{0}^{\infty} e^{-\rho t} \pi_{t} dt \right] \\ &= \sup_{Q^{\theta} \in \mathcal{P}} \int_{0}^{\infty} E_{0}^{Q^{\theta}} \left[e^{-\rho t} \pi_{0} \exp\left(\left(\mu - \frac{1}{2}\sigma^{2}\right)t - \sigma\int_{0}^{t} \theta_{s} ds + \sigma B_{t}^{\theta}\right) \right] dt \\ &= \int_{0}^{\infty} \pi_{0} e^{\left(\mu - \rho - \frac{1}{2}\sigma^{2}\right)t} \sup_{\theta \in \Theta} E_{0}^{Q^{\theta}} \left[\exp\left(\left(B_{t}^{\theta} - \int_{0}^{t} \theta_{s} ds\right)\sigma\right) \right] dt \\ &= \pi_{0} \int_{0}^{\infty} e^{\left(\mu - \rho - \frac{1}{2}\sigma^{2}\right)t} E_{0}^{Q^{-\kappa}} \left[\exp\left(\left(B_{t}^{-\kappa} + \kappa t\right)\sigma\right) \right] dt \\ &= \pi_{0} \int_{0}^{\infty} e^{\left(\mu - \rho - \frac{1}{2}\sigma^{2}\right)t} \exp\left(\sigma\left(\kappa t + \frac{1}{2}\sigma t\right)\right) dt \\ &= \pi_{0} \int_{0}^{\infty} e^{-\left(\rho - \kappa \sigma - \mu\right)t} dt \\ &= \frac{\pi_{0}}{\rho - \left(\mu + \kappa \sigma\right)} \end{split}$$
(C.4)

where we use the relation (C.3) to establish the third equality sign.

Next, we look at the infinimum expression in the second term of (4.9), for which we can do the analogous steps as for the first term, by replacing the supremum operator with infinimum operator. This gives us:

$$\inf_{Q^{\theta} \in \mathcal{P}} E_0^{Q^{\theta}} \left[\int_0^\infty e^{-\rho t} \pi_t dt \right] = \frac{\pi_0}{\rho - (\mu - \kappa \sigma)} \tag{C.5}$$

Inserting the terms (C.4) and (C.5) into (4.9), we finally obtain following expression for the expected value of the installed investment project:
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$$V(\pi_0|\alpha) = \alpha \frac{\pi_0}{\rho - (\mu + \kappa \sigma)} + (1 - \alpha) \frac{\pi_0}{\rho - (\mu - \kappa \sigma)}$$
$$= \pi_0 \left(\frac{\alpha}{\rho - (\mu + \kappa \sigma)} + \frac{(1 - \alpha)}{\rho - (\mu - \kappa \sigma)} \right)$$
(C.6)

which is identical to the expression (4.10) in the Proposition 1 we wanted to prove (remember that we set t = 0 for notational convenience). Here at the latest, we can see that the condition $\kappa < (\rho - \mu)/\sigma$ must be fulfilled in order that the problem makes sense.

C.3 Derivation of Expression (4.19)

In this appendix, we present the derivation of expression (4.19) from the right hand side of the condition (4.18). First, we show the following lemma:

Lemma 2. For a random variable x, which is real-valued, measurable, and \mathbb{F} -adapted on (Ω, \mathcal{F}, P) , and given that \mathcal{P} is strongly rectangular, it holds $\forall s > t$:

$$\sup_{p \in \mathcal{P}} E_t^p \left[\sup_{p \in \mathcal{P}'} E_s^p \left[x \right] \right] = \sup_{p \in \mathcal{P}} E_t^p [x]$$
(C.7)

Similarly, it holds that:

$$\inf_{p \in \mathcal{P}} E_t^p \left[\inf_{p \in \mathcal{P}'} E_s^p \left[x \right] \right] = \inf_{p \in \mathcal{P}} E_t^p [x]$$
(C.8)

Proof. The proof follows Appendix B in Nishimura and Ozaki (2007). We only present the derivation of equation (C.7) here, since the proof of equation (C.8) is analogous by replacing the supremum operator with infinimum operator.

To show that " \leq " holds, let $\theta^* \in \Theta$ be a density generator such that

$$E_t^{Q^{\theta^*}}\left[E_s^{Q^{\theta^*}}[x]\right] = \sup_{\theta \in \Theta} E_t^{Q^{\theta}}\left[E_s^{Q^{\theta}}[x]\right]$$

where the set Θ is defined by (C.2). Then, we can establish the following relation:

$$\sup_{\theta \in \Theta} E_t^{Q^{\theta}} \left[E_s^{Q^{\theta}}[x] \right] = E_t^{Q^{\theta^*}} \left[E_s^{Q^{\theta^*}}[x] \right]$$
$$\leq E_t^{Q^{\theta^*}} \left[\sup_{\theta' \in \Theta} E_s^{Q^{\theta'}}[x] \right]$$
$$\leq \sup_{\theta \in \Theta} E_t^{Q^{\theta}} \left[\sup_{\theta' \in \Theta} E_s^{Q^{\theta'}}[x] \right]$$

where $\theta' \in \Theta$ denotes a density generator at time s > t. Using the definition of \mathcal{P} in (C.1), we have:

$$\sup_{p \in \mathcal{P}} E_t^p \left[E_s^p[x] \right] \le \sup_{p \in \mathcal{P}} E_t^p \left[\sup_{p \in \mathcal{P}'} E_s^p[x] \right]$$

To show that " \geq " holds, let θ^* and θ'^* be density generators $\in \Theta$ such that

$$E_t^{Q^{\theta^*}}\left[E_s^{Q^{\theta'^*}}[x]\right] = \sup_{\theta \in \Theta} E_t^{Q^{\theta}}\left[\sup_{\theta' \in \Theta} E_s^{Q^{\theta'}}[x]\right]$$

Also define θ^{**} by: $(\theta_u^{**})_{0 \le u < s} = (\theta_u^*)_{0 \le u < s}$ and $(\theta_u^{**})_{s \le u < T} = (\theta_u^{**})_{s \le u < T}$. Then strong rectangularity implies that $\theta^{**} \in \Theta$. Furthermore,

$$\begin{split} \sup_{\theta \in \Theta} E_t^{Q^{\theta}} \left[\sup_{\theta' \in \Theta} E_s^{Q^{\theta^*}}[x] \right] &= E_t^{Q^{\theta^*}} \left[E_s^{Q^{\theta'^*}}[x] \right] \\ &= E_t^{Q^{\theta^{**}}} \left[E_s^{Q^{\theta^{**}}}[x] \right] \\ &\leq \sup_{\theta \in \Theta} E_t^{Q^{\theta}} \left[E_s^{Q^{\theta}}[x] \right] \end{split}$$

where the second equality holds by Lemma B2 in Nishimura and Ozaki (2007) and the inequality holds by the fact that $\theta^{**} \in \Theta$. Using the definition of \mathcal{P} , we have:

$$\sup_{p \in \mathcal{P}} E_t^p \left[\sup_{p \in \mathcal{P}'} E_s^p \left[x \right] \right] \le \sup_{p \in \mathcal{P}} E_t^p \left[E_s^p \left[x \right] \right]$$

which completes the proof.

Next, we use the minimax theorem for continuous stochastic processes under multiple priors and strong rectangularity:

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$$\sup_{p \in \mathcal{P}} E_t^p \left[\inf_{p \in \mathcal{P}'} E_s^p[x] \right] = \inf_{p \in \mathcal{P}} E_t^p \left[\sup_{p \in \mathcal{P}'} E_s^p[x] \right]$$
(C.9)

For a proof, see e.g. Proposition 5.14 in Karatzas and Kou (1998). Finally, we can present the full derivation:

$$\begin{split} &\alpha\sup_{p\in\mathcal{P}}E_t^p\left[\alpha\sup_{p\in\mathcal{P}'}E_s^p\left[x\right]+\left(1-\alpha\right)\inf_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]+\\ &\left(1-\alpha\right)\inf_{p\in\mathcal{P}}E_t^p\left[\alpha\sup_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]+\left(1-\alpha\right)\inf_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]\\ &= &\alpha\sup_{p\in\mathcal{P}}E_t^p\left[\alpha\sup_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]+\alpha\sup_{p\in\mathcal{P}}E_t^p\left[\left(1-\alpha\right)\inf_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]+\\ &\left(1-\alpha\right)\inf_{p\in\mathcal{P}}E_t^p\left[\alpha\sup_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]+\left(1-\alpha\right)\sup_{p\in\mathcal{P}}E_t^p\left[\left(1-\alpha\right)\inf_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]\\ &= &\alpha^2\sup_{p\in\mathcal{P}}E_t^p\left[\sup_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]+\alpha(1-\alpha)\sup_{p\in\mathcal{P}}E_t^p\left[\inf_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]+\\ &\left(1-\alpha\right)\alpha\inf_{p\in\mathcal{P}}E_t^p\left[\sup_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]+\left(1-\alpha\right)^2\inf_{p\in\mathcal{P}}E_t^p\left[\inf_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]\\ &= &\alpha^2\sup_{p\in\mathcal{P}}E_t^p[x]+\alpha(1-\alpha)\left(\sup_{p\in\mathcal{P}}E_t^p\left[\inf_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]+\inf_{p\in\mathcal{P}}E_t^p\left[\sup_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]\right)+\\ &\left(1-\alpha\right)^2\inf_{p\in\mathcal{P}}E_t^p[x]\\ &= &\alpha^2\sup_{p\in\mathcal{P}}E_t^p[x]+2\alpha(1-\alpha)\sup_{p\in\mathcal{P}}E_t^p\left[\inf_{p\in\mathcal{P}'}E_s^p\left[x\right]\right]+\left(1-\alpha\right)^2\inf_{p\in\mathcal{P}}E_t^p[x]\end{split}$$

where we used the relations (C.7) and (C.8) to establish the third equality sign, and the minimax theorem (C.9) to finally obtain expression (4.19) in the last line.

C.4 Proof of Proposition 2

In this appendix, we prove Proposition 2 and derive the investment threshold π' , i.e. the critical level of current profits π_t , that must be attained in order to invest. Applying α -expected utility, the value of the option to invest is given by the solution to the following maximization problem:

$$F(\pi_t | \alpha) = \max_{\tau \in [t,\infty)} E_t^{\alpha} \left[\int_{\tau}^{\infty} e^{-\rho(s-t)} \pi_s ds - e^{-\rho(\tau-t)} I \right]$$
(4.17)

Or, in explicit notation using the definition as presented in (4.7):

$$F(\pi_t | \alpha) = \max_{\tau \in [t,\infty)} \left\{ \alpha \sup_{p \in \mathcal{P}} E_t^p \left[\int_{\tau}^{\infty} e^{-\rho(s-t)} \pi_s ds - e^{-\rho(\tau-t)} I \right] + (C.10) \right.$$
$$\left. (1-\alpha) \inf_{p \in \mathcal{P}} E_t^p \left[\int_{\tau}^{\infty} e^{-\rho(s-t)} \pi_s ds - e^{-\rho(\tau-t)} I \right] \right\}$$

This proof follows the presentation in Nishimura and Ozaki (2007). We only show the proof for the case of complete optimism ($\alpha = 1$), since the case of $\alpha = 0$ is analogous by replacing the supremum term with the infinimum term. In case of $\alpha = 1$, the maximization problem (C.10) reduces to:

$$F(\pi_t | \alpha = 1) = \max_{\tau \in [t,\infty)} \sup_{p \in \mathcal{P}} E_t^p \left[\int_{\tau}^{\infty} e^{-\rho(s-t)} \pi_s ds - e^{-\rho(\tau-t)} I \right]$$
(C.11)

In this maximization problem, there is always a decision to be made between stopping (i.e. investing at time t) and continuation (i.e. waiting for a short time interval Δt and reconsidering whether to invest or not at time $t + \Delta t$). Thus, we can transform the maximization problem in (C.11) into a Bellman equation¹⁵:

$$F_{0} = \max_{\tau \in [0,\infty)} \sup_{p \in \mathcal{P}} E_{0}^{p} \left[\int_{\tau}^{\infty} e^{-\rho t} \pi_{t} dt - e^{-\rho \tau} I \right]$$

$$= \max \left\{ \sup_{p \in \mathcal{P}} E_{0}^{p} \left[\int_{0}^{\infty} e^{-\rho t} \pi_{t} dt - I \right], \max_{\tau \in [0+\Delta t,\infty)} \sup_{p \in \mathcal{P}} E_{0}^{p} \left[\int_{\tau}^{\infty} e^{-\rho t} \pi_{t} dt - e^{-\rho \tau} I \right] \right\}$$

$$= \max \left\{ V_{0} - I, \max_{\tau \in [0+\Delta t,\infty)} \sup_{p \in \mathcal{P}} E_{0}^{p} \left[\int_{\tau}^{\infty} e^{-\rho t} \pi_{t} dt - e^{-\rho \tau} I \right] \right\}$$

$$= \max \left\{ V_{0} - I, e^{-\rho \Delta t} \max_{\tau \in [0+\Delta t,\infty)} \sup_{p \in \mathcal{P}} E_{0}^{p} \left[\int_{\tau}^{\infty} e^{-\rho (t-\Delta t)} \pi_{t} dt - e^{-\rho (\tau-\Delta t)} I \right] \right\}$$

$$= \max \left\{ V_{0} - I, e^{-\rho \Delta t} \max_{\tau \in [0+\Delta t,\infty)} \sup_{p \in \mathcal{P}} E_{0}^{p} \left[\sup_{p \in \mathcal{P}'} E_{0+\Delta t}^{p} \left[\int_{\tau}^{\infty} e^{-\rho (t-\Delta t)} \pi_{t} dt - e^{-\rho (\tau-\Delta t)} I \right] \right] \right\}$$

$$= \max \left\{ V_{0} - I, e^{-\rho \Delta t} \sup_{p \in \mathcal{P}} E_{0}^{p} \left[\max_{\tau \in [0+\Delta t,\infty)} \sup_{p \in \mathcal{P}'} E_{0+\Delta t}^{p} \left[\int_{\tau}^{\infty} e^{-\rho (t-\Delta t)} \pi_{t} dt - e^{-\rho (\tau-\Delta t)} I \right] \right] \right\}$$

$$= \max \left\{ V_{0} - I, e^{-\rho \Delta t} \sup_{p \in \mathcal{P}} E_{0}^{p} \left[\max_{\tau \in [0+\Delta t,\infty)} \sup_{p \in \mathcal{P}'} E_{0+\Delta t}^{p} \left[\int_{\tau}^{\infty} e^{-\rho (t-\Delta t)} \pi_{t} dt - e^{-\rho (\tau-\Delta t)} I \right] \right] \right\}$$

$$(C.12)$$

where \mathcal{P}' denotes the set of probability measures at time $t = 0 + \Delta t$, derived from

¹⁵For notational convenience, we set t = 0, that is the level of current profits are denoted by π_0 . Moreover, we omit the condition $\alpha = 1$, so that we can write $F(\pi_t | \alpha = 1) = F(\pi_0) = F_0$.

 \mathcal{P} by the set of conditional probabilities imposed by strong rectangularity. The first equation holds by the definition of F_0 in (C.11), the second by splitting up the decision into the two possibilities (i.e. investing or waiting), the third by the definition of V_0 in the case of $\alpha = 1$ (see equation (4.9)), the fifth by strong rectangularity (see the Lemma in Appendix C), the sixth by the fact that τ is restricted to the interval $[0 + \Delta t, \infty)$, and the seventh by the definition of $F_{0+\Delta t}$ in analogy to (C.11).

To solve the optimal stopping problem, we conjecture a solution in the form of a critical threshold value π^* for current values of π_t such that investment is optimal if $\pi_t > \pi^*$, and waiting is optimal if $\pi_t < \pi^*$. Suppose we are in a region where the value of the option to invest is such that we do not want to invest. Then we can transform (C.12) as follows:

$$F_{0} = e^{-\rho\Delta t} \sup_{p \in \mathcal{P}} E_{0}^{p} [F_{0+\Delta t}]$$

$$\approx \sup_{p \in \mathcal{P}} E_{0}^{p} [F_{0+\Delta t}] \frac{1}{1+\rho\Delta t}$$
(C.13)

This approximation holds by the linear approximation of the exponential function, and is valid for small Δt . Multiplying with $1 + \rho \Delta t$, rearranging, and letting Δt go to zero (*dt* denoting the limit):

$$0 = \frac{1}{dt} \sup_{p \in \mathcal{P}} E_0^p \left[dF \right] - \rho F \tag{C.14}$$

We calculate $\sup_{p \in \mathcal{P}} E_0^p [dF]$ with the help of Ito's Lemma, since the value of the option F is a function of the profit flow¹⁶ π . The solution is given as follows, similar to the proof in appendix C.2:

$$\sup_{p\in\mathcal{P}} E_0^p[dF] = \sup_{p\in\mathcal{P}} E_0^p\left[(\mu - \sigma\theta_t)\pi F'dt + \sigma\pi F'dB_t^\theta + \frac{1}{2}\sigma^2\pi^2 F''(dB_t^\theta)^2\right]$$
$$= (\mu + \sigma\kappa)\pi F'(\pi)dt + \frac{1}{2}\sigma^2\pi^2 F''(\pi)dt$$

Inserting into (C.14):

¹⁶Since the project yields an infinite stream of profits, the threshold level for investing (π^*) is independent of time so that we can omit the time subscript.

$$0 = (\mu + \sigma \kappa)\pi F'(\pi) + \frac{1}{2}\sigma^2 \pi^2 F''(\pi) - \rho F(\pi)$$

$$0 = \frac{1}{2}\sigma^2 \pi^2 F''(\pi) + \gamma \pi F'(\pi) - \rho F(\pi)$$

where we replaced the term in the brackets by γ .¹⁷ We guess a solution for this differential equation, with the help of the condition F(0) = 0:

$$F(\pi) = A\pi^b$$

If we substitute this solution into the differential equation, we get:

$$\begin{split} \frac{1}{2}\sigma^2\pi^2(b-1)bA\pi^{b-2} + \gamma\pi bA\pi^{b-1} - \rho A\pi^b &= 0\\ \frac{1}{2}\sigma^2b(b-1) + \gamma b - \rho &= 0\\ \frac{1}{2}\sigma^2b^2 + \left(\gamma - \frac{1}{2}\sigma^2\right)b - \rho &= 0 \end{split}$$

Hence,

$$b = \frac{1}{2} - \frac{\gamma}{\sigma^2} + \sqrt{\left(\frac{\gamma}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1$$
(4.21)

To solve the differential equation, we use the value-matching and the smoothpasting condition:

$$F(\pi') = V(\pi') - I$$

$$F'(\pi') = V'(\pi')$$

where $V(\pi_t) = \pi_t \phi$, and ϕ is given by (4.11). We obtain:

$$\pi' = \frac{b}{(b-1)\phi}I\tag{4.20}$$

The coefficient A equals:

¹⁷Note that in Proposition 2, $\gamma = \alpha(\mu + \sigma\kappa) + (1 - \alpha)(\mu - \sigma\kappa)$. Since we have $\alpha = 0$, the two terms coincide.

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$$A = \frac{(b-1)^{b-1}}{I^{(b-1)}b^b}\phi^b$$

Hence, the value of the option to invest is finally given by:

$$F(\pi_t | \alpha) = \frac{(b-1)^{b-1}}{I^{(b-1)}b^b} (\pi_t \phi)^b$$
(4.23)

which completes the proof.

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