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Determination of Sub-daily
Earth Rotation Parameters
from VLBI Observations

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Determination of Sub-daily Earth Rotation Parameters from VLBI Observations

Summary

The work presented deals with the determination of sub-daily Earth Rotation Parameters (ERPs) from Very Long Baseline Interferometry (VLBI) observations. Monitoring and interpreting the Earth's rotation variations in general is an important task for Earth sciences, as they provide boundary which support other geophysical investigations. This holds especially for sub-daily variations of the Earth's rotation which are primarily excited by variations of the oceans. Furthermore, atmospheric impacts as well as effects of the shape of the Earth are present. VLBI observations are in particular feasible. With VLBI all five parameters of the Earth's orientation can be determined without hypothesis. Thus, VLBI derived results do not suffer from resonance effects from the modeling of artificial Earth satellites, which is the fact for ERPs derived, e.g., from observations of Global Navigation Satellite Systems (GNSS).

Although the analysis of sub-daily variations of the Earth's rotation is by no means a new scientific area of research, there is a clear need to expand the methods used. This is also obvious as further studies by other authors were performed in parallel to the results presented in this thesis. Thus, the determination and analysis of sub-daily ERPs can be considered as a vital field of research. The determination of sub-daily ERPs from VLBI observations can be divided into two areas. On the one hand, time series with a high temporal resolution, e.g., one hour, can be generated. On the other hand, an empirical model for the tidal variations of the ERPs with periods of one day and below can be determined from the VLBI observations. Both areas of research are considered within this thesis.

Concerning the time series approach, special continuous VLBI campaigns are examined as they offer ideal conditions for this approach. For the analysis of these campaigns, an optimized solution scheme is implemented. This adapts the continuous character in an optimal way and avoids negative influences of the VLBI-analysis on the subsequent examination of the sub-daily ERPs. In this way, irregular variations are confirmed and further ones are detected. However, it is pointed out that these variations can be confirmed only with continuous campaigns over longer time spans. Furthermore, a temporal resolution below one hour of the sub-daily ERPs would be desirable to detect additional short periodic variations. This is not possible with the current status of VLBI observations, but, improvement is promised by future technical concepts of VLBI. The analysis methods that are applied within this thesis, will be directly applicable to future VLBI observations.

With regard to the determination of an empirical model for tidal ERP variations, a new methodology based on the transformation of normal equation systems is developed. It is shown that this approach can be successfully applied to VLBI observations. Moreover, this approach provides a straight forward method for the combination of different space-geodetic techniques, being the most rigorous one known today, when different software packages are used to pre-process the individual techniques. Within the thesis at hand, the approach of the transformation of normal equation systems is used to estimate an empirical model for tidal ERP variations from observations of the Global Positioning Systems (GPS) as well. On this basis, the work of this thesis cumulates in a rigorous combination of GPS and VLBI observations. The combined time series with an hourly resolution as well as the determined empirical model for tidal ERP variations exhibit that the strengths of both techniques are sustained.

Bestimmung von subtäglichen Erdrotationsparametern aus VLBI Beobachtungen

Zusammenfassung

Die vorliegende Arbeit befasst sich mit der Bestimmung subtäglicher Erdrotationsparameter (ERP) aus Beobachtungen der Radio-Interferometrie auf langen Basislinien (engl.: Very Long Baseline Interferometry, VLBI). Die Beobachtung und Interpretation der Erdrotation im Allgemeinen stellt einen wichtigen Schwerpunkt der Erdwisschenschaften dar, da sie Randbedingungen für andere geowissenschaftliche Untersuchungen setzt. Dies gilt im Speziellen auch für subtägliche Variationen der Erdrotation. Diese sind vor allem bedingt durch Variationen des Ozeans, darüberhinaus lassen sich Effekte der Atmosphäre, wie auch der Gestalt der Erde feststellen. Die Beobachtungstechnik VLBI eignet sich im Besonderen für Untersuchungen der Erdrotation, da einzig mit dieser Technik alle fünf Parameter der Erdorientierung hypothesenfrei bestimmt werden können. Somit sind im Gegensatz zu Ergebnissen, die z.B. aus den Beobachtungen globaler Satellitennavigationssysteme (engl. Global Navigation Satellite System, GNSS) gewonnen werden, keine Resonanzeffekte aus der Modellierung künstlicher Erdsatelliten zu erwarten.

Obwohl die Untersuchung subtäglicher Erdrotationsvariationen keineswegs ein neuartiges wissenschaftliches Forschungsgebiet darstellt, besteht doch ein eindeutiger Bedarf bezüglich der Erweiterung der auf diesem Gebiet verwendeten Methoden. Dies wird auch daran deutlich, dass parallel zu den in dieser Arbeit dargestellten Ergebnissen weitere Untersuchungen anderer Autoren erfolgten und es sich somit um ein vitales Forschungsgebiet handelt. Die Bestimmung subtäglicher ERP aus VLBI-Beobachtungen lässt sich in zwei Teilgebiete aufgliedern. Zum Einen können Zeitreihen mit hoher zeitlicher Auflösung von beispielsweise einer Stunde generiert werden. Zum Anderen kann ein empirisches Modell für die gezeitenbedingten Variationen der ERP mit Perioden von einem Tag und weniger aus den VLBI-Beobachtungen geschätzt werden. Beide Forschungszweige sind Bestandteil dieser Arbeit.

Bezüglich des Zeitreihenansatzes werden spezielle kontinuierliche VLBI-Kampagnen untersucht, da sie die idealen Voraussetzungen für diesen Ansatz bieten. Zur Analyse dieser Kampagnen wird ein Lösungsschema entwickelt, welches optimal auf den kontinuierlichen Charakter angepasst ist. Dadurch werden negative Einflüsse der VLBI-Auswertung auf eine anschließende Untersuchung der subtäglichen ERP-Zeitreihen minimiert. Es wird gezeigt, dass sich bereits entdeckte irreguläre Variationen bestätigen und weitere detektieren lassen. Allerdings wird ebenso deutlich, dass diese Variationen nur mit kontinuierlichen Kampagnen über längere Zeiträume bestätigt werden können. Darüber hinaus wäre eine zeitliche Auflösung der ERP-Zeitreihen von deutlich unter einer Stunde wünschenswert, um weitere kurzperiodische Signale zu detektieren. Dies ist mit dem aktuellen Stand der geodätischen VLBI allerdings nicht möglich, zukünftige Entwicklungen versprechen jedoch deutliche Verbesserungen in dieser Hinsicht. Die in der vorliegenden Arbeit angewendeten Analysemethoden werden direkt auf die Analyse zukünftiger VLBI-Beobachtungen anwendbar sein.

Im Hinblick auf die Bestimmung eines empirischen Modells für ERP-Variationen wird im Rahmen dieser Arbeit eine neue Methodik entwickelt, die auf der Transformation von Normalgleichungssystemen basiert. Es wird nachgewiesen, dass sich dieser Ansatz erfolgreich auf VLBI-Beobachtungen anwenden lässt. Außerdem gewährleistet der Ansatz eine einfach zu realisierende Kombination verschiedener weltraumgeodätischer Verfahren, die zudem den strengsten zur Zeit realisierbaren Ansatz darstellt, so lange verschiedene Softwarepakete zur Vorprozessierung der einzelnen Techniken verwendet werden. Im Rahmen der vorliegenden Arbeit wird die Transformation von Normalgleichungssystemen ebenfalls zur Bestimmung eines empirischen Modells für ERP-Variationen aus Beobachtungen des Global Positioning Systems (GPS) angewendet. Auf dieser Basis erfolgt abschließend eine rigorose Kombination von GPS- und VLBI-Beobachtungen. Sowohl die kombinierten ERP-Zeitreihen mit stündlicher Auflösung als auch das gewonnene kombinierte empirische Modell für ERP-Variationen zeigen, dass die Kombination die Stärken beider Techniken nutzt.

Contents

Preface

This thesis includes the following papers, ordered chronologically and referred to as Paper A – G in the text:

Paper A:

Artz, T., S. Böckmann, A. Nothnagel, V. Tesmer (2007) *ERP time series with daily and sub-daily resolution determined from CONT05.* In: Boehm, J., A. Pany, H. Schuh (eds) Proceedings of the 18th European VLBI for Geodesy and Astrometry Working Meeting, 12 – 13 April 2007, Vienna, Geowissenschaftliche Mitteilungen, Heft Nr. 79, Schriftenreihe der Studienrichtung Vermessung und Geoinformation, Technische Universität Wien, ISSN 1811-8380, pp 69 – 74, (available electronically at `http://mars.hg.tuwien.ac.at/~evga/proceedings/S31_Artz.pdf`).

Paper B:

Artz, T., S. Böckmann, A. Nothnagel (2009) *CONT08 – First Results and High Frequency Earth Rotation.* In: Bourda, G., P. Charlot, A. Collioud (eds) Proceedings of the 19th European VLBI for Geodesy and Astrometry Working Meeting, 24 – 25 March 2009, Bordeaux, Université Bordeaux 1 - CNRS - Laboratoire d'Astrophysique de Bordeaux, pp 111 – 115, (available electronically at `http://www.u-bordeaux1.fr/vlbi2009/proceedgs/26_Artz.pdf`).

Paper C:

Artz, T., S. Böckmann, S., A. Nothnagel, P. Steigenberger (2010a) *Subdiurnal variations in the Earth's rotation from continuous Very Long Baseline Interferometry campaigns.* J Geophys Res, 115, B05404, DOI 10.1029/2009JB006834.

Paper D:

Artz, T. S. Böckmann, L. Jensen, A. Nothnagel, P. Steigenberger (2010b) *Reliability and Stability of VLBI-derived Sub-daily EOP Models.* In: Behrend, D., K.D. Baver (eds) IVS 2010 General Meeting Proceedings "VLBI2010: From Vision to Reality", 7 – 13 February 2010, Hobart, NASA/CP-2010-215864, pp 355 – 359, (available electronically at `http://ivscc.gsfc.nasa.gov/publications/gm2010/artz.pdf`)

Paper E:

Artz, T., S. Tesmer, S., A. Nothnagel (2010a) *Assessment of Periodic Sub-diurnal Earth Rotation Variations at Tidal Frequencies Through Transformation of VLBI Normal Equation Systems.* J Geod, 85(9):565 – 584, DOI 10.1007/s00190-011-0457-z.

Paper F:

Artz, T., L. Bernhard, A. Nothnagel, P. Steigenberger, S. Tesmer (2011b) *Methodology for the Combination of Sub-daily Earth Rotation from GPS and VLBI Observations* J Geod, DOI 10.1007/s00190-011-0512-9, online first.

Paper G:

Artz, T., A. Nothnagel, P. Steigenberger, S. Tesmer (2011) *Evaluation of Combined Sub-daily UT1 Estimates from GPS and VLBI Observations.* In: Alef W., Bernhart S., Nothnagel A. (eds.) Proceedings of the 20th Meeting of the European VLBI Group for Geodesy and Astrometry, Schriftenreihe des Instituts für Geodäsie und Geoinformation der Rheinischen-Friedrich-Wilhelms Universität Bonn, No. 22, ISSN 1864-1113, pp 97 – 101.

1. Introduction

As the Earth is a continually changing planet, geodetic measurements are necessary to ensure increasing knowledge about the global change of the entire system Earth. The users of these geodetic products can be found in all areas where the knowledge of any object's position is necessary. These are Earth sciences as well as societal applications, e.g., navigation and transport applications as well as early warning systems for natural hazards or weather forecasts.

Such fundamental geodetic products are the terrestrial and celestial reference frame (TRF and CRF) as well as the Earth Orientation Parameters (EOPs). The EOPs represent the rotational motion of the Earth by describing the rotational rate in terms of Universal Time (UT1) and by expressing the position of the instantaneous rotation axis with respect to (w.r.t.) the CRF (precession and nutation) and the TRF (polar motion, PM). Thus, the EOPs represent the link between TRF and CRF. The sub-group consisting of UT1 and PM is usually called Earth rotation parameters (ERPs). The rotational motion of the Earth exhibits variations on various time scales down to sub-daily phenomena which are forced by large-scale geophysical processes. These are external torques, mainly due to the gravitational attraction of the Sun and the Moon, as well internal dynamical processes. The latter can be separated into internal mass redistributions (e.g., plate tectonics or postglacial isostatic adjustment) and angular momentum exchange between the solid Earth and geophysical fluids (e.g., oceans and atmosphere). Within this thesis the focus is placed on the determination of ERPs with periods 24 hours and below, called sub-daily ERPs.

Measurements of the EOPs can be performed by space geodetic techniques like Very Long Baseline Interferometry (VLBI), Global Navigation Satellite Systems (GNSS) like the Global Positioning System (GPS) or Satellite Laser Ranging (SLR) with VLBI being the only technique that can measure all EOPs without hypothesis. Concerning sub-daily ERP variations, their major cause are tidal variations with the space geodetic techniques being sensitive to the integral effect at any tidal line. In contrast to this, present models for sub-daily variations of the ERPs only consist of gravitationally forced oceanic impacts. For example, the model for sub-daily variations that is proposed by the International Earth Rotation and Reference Systems Service (IERS) is based on an ocean tide model with additional corrections for the tri-axial shape of the Earth (libration, CHAO et al. 1991). Within this model, which is given in the IERS Conventions 2010 (Tab. 5.1a, 5.1b, 8.2a, 8.2b, 8.3a and 8.3b of PETIT and LUZUM 2010), non-tidal oceanic as well as tidal and non-tidal atmospheric effects are neglected. However, sub-daily ERP predictions based on Atmospheric Angular Momentum (AAM) and Oceanic Angular Momentum (OAM) time series exist for such effects. But, the temporal resolution of these data is six hours at the most. Hence, semi-diurnal signals can hardly be explained and pro- and retrograde semi-diurnal polar motion cannot be separated (BRZEZIŃSKI et al. 2002). As a consequence, these predictions are currently not sufficient to describe remaining effects.

In present VLBI analyses, the sub-daily ERP models are usually used to reduce the VLBI observables. This is, e.g., necessary to ensure sub-cm accuracy of the station positions (SOVERS et al. 1998). However, to fulfill future requirements, the IERS model, currently the standard, is not sufficient. For instance, the International Association of Geodesy (IAG) established the Global Geodetic Observing System (GGOS) to guarantee improved Earth sciences with a temporal resolution of 1 hour and an accuracy of 1 mm for the EOPs (GROSS et al. 2009). In this environment, the Working Group 3 of the International VLBI Service for Geodesy and Astrometry (IVS, SCHLÜTER and BEHREND 2007) developed a concept for the future VLBI system (NIELL et al. 2005). So, for the analysis of future observations, better and more consistent a priori information on the sub-daily ERP variations will be necessary.

One option to derive such information is to determine it from observations of space geodetic techniques in an inverse process as, e.g., done by GIPSON (1996). In this way, empirical models for sub-daily ERPs will amend or may even replace the theoretical models mentioned above to meet the goals of GGOS. In addition, these empirical models can be used to validate predictions that are based on ocean tidal models or AAM and OAM time series. Finally, time series of highly resolved ERPs (with a temporal resolution of, e.g., one hour) can be used to detect non-periodic phenomena which, e.g., occur due to earthquakes. However, non-periodic

effects can only be detected if the periodic part is modeled in a highly consistent manner to be able to eliminate the harmonic ERP variations that are measured by the space geodetic techniques.

Within this thesis, ERP time series with a temporal resolution of one hour as well as empirical models for tidal variations of the ERPs are determined. In this way, the current capabilities of VLBI to determine sub-daily ERPs are analyzed and a possible role of VLBI for the determination of sub-daily ERPs in the future is identified. For the estimation of time series, only the Continuous VLBI Campaigns (CONTs) are analyzed, as almost all other VLBI observing sessions are discontinuous with an average of three 24 hour blocks a week. The three most recent campaigns of the years 2002, 2005 and 2008 took place over a fortnightly timespan each, thus permitting to study VLBI-derived sub-daily ERPs. This investigation reveals significant variations of the ERPs beside the diurnal and semi-diurnal bands which are, however, not entirely consistent for the three campaigns. In contrast to investigations in the past, a consistent analysis set-up has been chosen to avoid inconsistencies. Furthermore, a new analysis strategy has been developed to cover the specific properties of these observing campaigns. Also, a new approach has been implemented to derive an empirical model for sub-daily ERPs from VLBI observations. This approach permits the combination of VLBI and GPS observations to derive sub-daily ERPs in an elegant and practical way. The application of this method cross-wise compensates geometric instabilities of the individual techniques and, thus, clearly emphasizes the importance of this type of combination.

The general structure of this thesis is as follows:

- Chapter 2, "Scientific context", describes the background in order to clarify the motivation of the thesis and gives a general overview of different approaches to assess sub-daily ERPs.

- Chapter 3, "Theory of Sub-daily Earth Rotation", gives a short overview of the theory of Earth orientation in general and the modeling of sub-daily ERP variations in particular.

- Chapter 4, "Measuring EOPs with VLBI", provides an overview of the capabilities of VLBI to determine the EOPs.

- Chapter 5, "Short description of the included papers", briefly introduces the seven papers included in this thesis.

- Chapter 6 gives a summary of the most important results and procedures of this thesis.

- Chapter 7 provides an outlook on possible further research.

- Chapter 8 provides a list of publications on related work is given to which I have contributed. These publications are not included in this thesis, but are meant to document the relevance of this work for the scientific community.

2. Scientific context

Monitoring and interpreting the Earth's rotation variations is an important task for Earth sciences. As the EOPs depend on luni-solar torques as well as on the internal structure and rheology of the Earth, they provide boundary conditions to other geophysical investigations. On daily and sub-daily time scales, the major impact on the ERPs is caused by tidal variations of the oceans. These are gravitationally forced mass redistributions as well as currents within the oceans that are forced by the tidal attraction of the Sun and the Moon. Due to the interaction of the oceans with the solid Earth, variations of the Earth's rotation are excited. Furthermore, non-tidal oceanic as well as tidal and non-tidal atmospheric excitations are expected. A detailed description of the Earth's rotation as well as the modeling of daily and sub-daily ERPs is given in Ch. 3.

From the early days of geodetic VLBI onwards, sub-daily ERPs were derived from VLBI observations. Standard 24 h VLBI experiments were split up into 2 h bins to determine sub-daily variations of the Earth's rotation rate (e.g., CARTER et al. 1985, CAMPBELL and SCHUH 1986). However, no geophysical interpretation was performed in these times. BROSCHE et al. (1991) estimate a time series of highly resolved ΔUT1 (UT1-TAI, Universal Time 1 - Atomic Time) in the same way. Subsequently, they used these time series to determine tidal effects in ΔUT1.

In the 1990s, several investigations were performed to derive tidal ERP variations in so-called empirical tidal ERP models from VLBI observations (e.g., HERRING and DONG 1991, SOVERS et al. 1993, HERRING 1993, HERRING and DONG 1994, GIPSON 1996), where GIPSON (1996) estimated 41 tidal constituents for ΔUT1 and 56 tidal constituents for PM directly from the VLBI observations (this approach is designated as *observation level* hereafter). The investigations revealed a good agreement to theoretically derived ERP models for most of the tidal terms. Some of the existing differences could be attributed to geophysical deficiencies of the theoretical models, e.g., to libration (CHAO et al. 1991). For others, however, no explanation was found. GIPSON (1996) showed that a time series of highly resolved ERPs is better explained by his empirical model in comparison to a theoretical one which is based on ocean tidal models and, thus, is more consistent to the space geodetic observations. For about 10 years, no significant improvement has been made in estimating such tidal ERP models although the increased measurement accuracy of VLBI permits a better detection of tidal ERP variations. Recently, ENGLICH et al. (2008) estimated an empirical model based on hourly resolved ERP time series (designated as *solution level* hereafter) while GIPSON and RAY (2009) presented an updated version of the 1996 model. In addition, PETROV (2007) estimated a complete Earth rotation model that also includes sub-daily variations in the form of Fourier coefficients.

Comparable empirical models were also derived from GPS and SLR observations. The SLR model of WATKINS and EANES (1994) was estimated directly from the SLR observations, whereas the GPS models (e.g., HEFTY et al. 2000, ROTHACHER et al. 2001, STEIGENBERGER et al. 2006, STEIGENBERGER 2009) were estimated from time series of highly resolved ERPs. Furthermore, STEIGENBERGER 2009 determined a combined empirical model from GPS and VLBI observations on the solution level.

Beside the determination of tidally forced ERP models, time series of highly resolved ERPs can be analyzed. Such investigations revealed the incompleteness of theoretical tidal ERP models (e.g., SCHUH and SCHMITZ-HÜBSCH 2000) where significant power was detected in the diurnal and semi-diurnal band. Furthermore, irregular quasi-periodic variations were detected for which no geophysical interpretations are available. This time series approach (see also Ch. 4.3.1) was also applied for the analysis of the CONT sessions by several authors. The CONT campaigns are scheduled in irregular intervals starting in 1994 where the observing network remains almost identical over the period of the individual campaigns. The last campaign took place in 2008 (CONT08). The aim of these campaigns is to generate continuous VLBI observations over a certain time span and to acquire the best possible VLBI data. Furthermore, the CONT sessions are designed to provide ERPs with a high accuracy and, at least in the recent decade, to permit the determination of ERPs with a high temporal resolution. As they take place at different seasons, they also permit to detect time-dependent phenomena as, e.g., atmospheric excitations. Thus, these campaigns are most suitable to estimate highly resolved ERPs from VLBI observations to analyze tidal and non-tidal variations. A multitude of

investigations has been made in analyzing sub-daily ERPs estimated from the CONT campaigns with the aim to validate ocean tidal models or to detect irregular periodic variations. The Continuous VLBI Campaign 1994 (CONT94) was, e.g, analyzed by CHAO et al. (1996) and CONT96 by SCHUH (1999) or KOLACZEK et al. (2000). Especially the very successful CONT02 revealed irregular ERP variations where a significant retrograde 8 h variation was detected (e.g., NASTULA et al. 2004, HAAS and WÜNSCH 2006, NASTULA et al. 2007). However, this variation could not be confirmed by the analysis of CONT05 (e.g., HAAS 2006) or CONT08 (e.g., NILSSON et al. 2010). Using observations over the CONT02 time span, THALLER et al. (2007) performed a rigorous combination of GPS and VLBI to derive combined EOPs by adding normal equation (NEQ) systems which contained hourly resolved ERPs.

Despite the large number of publications mentioned above, no clear and conclusive overall picture of the ERP variations as seen by VLBI has been drawn. Thus, the determination of sub-daily ERPs is still a vital research field. Parallel to the work presented in this thesis, the investigations of ENGLICH et al. (2008), GIPSON and RAY (2009), STEIGENBERGER (2009) and NILSSON et al. (2010) took place.

This thesis aims to produce a more unified view of the VLBI-derived sub-daily ERPs. CONT02, CONT05 and CONT08 are analyzed with an identical set-up to eliminate the impact of differing analysis options. Furthermore, prior investigations are sub-optimal in terms of the analysis set-up that does not account for the continuous character of the CONT sessions. An enhanced analysis strategy is developed within this thesis to overcome the problem. Concerning the empirical tidal ERP model, solutions were performed on the solution and on the observation level. This has been supplemented by an approach on the level of NEQ systems and all three methods have been compared intensely. Based on this NEQ approach, a combination of VLBI and GPS observations has been performed to derive sub-daily ERPs. The results clearly point out the importance of VLBI to derive consistent sub-daily ERPs without any deformations by external information. However, with the currently available VLBI observing technique improvements in sub-daily ERP models are only possible through combinations with GPS observations. The reasons for this are: the discontinuous character of VLBI observations and, more importantly, the small number of observing telescopes leading to weak and varying observing networks while GPS draws information from continuous observations and a large observing network.

In summary, this thesis documents the current status of sub-daily ERPs as derived from VLBI observations and from a sophisticated combination of VLBI and GPS observations. The concept of this thesis is represented by a hierarchical procedure starting with special fortnightly CONT sessions and culminating in a combination of observational data spanning 13 years. The methodology is based on the modification of NEQ systems to

- improve the analysis of CONT sessions

- derive an empirical model for tidal ERP variations from VLBI observations with a new method

- initially determine combined sub-daily ERPs from VLBI and GPS observations

3. Theory of Sub-daily Earth Rotation

In this chapter, the theory of the Earth's rotation is briefly described. The rotational motion can be understood as the change of the orientation of an Earth fixed reference system w.r.t. a quasi-inertial space-fixed reference system. Such reference systems are theoretical frameworks which are realized by reference frames. In this way, the Geocentric Celestial Reference System (GCRS) is accomplished by the positions of radio sources in a CRF, and the International Terrestrial Reference System (ITRS) is realized by station positions of observing sites in a TRF. The transformation between these reference frames can be realized by a rotation matrix which contains the EOPs. The characteristic of the EOPs can generally be described by the rotational motion of a gyro. This basic theoretical framework to describe the Earth's rotation is introduced in Ch. 3.1. Additional information is given, e.g., in MORITZ and MUELLER (1988).

Beside external luni-solar torques, several processes within the Earth system are present, that lead to the variability of the Earth's rotation as well. These internal processes and their interaction with sub-daily ERPs are described in Ch. 3.2. Based on the knowledge of these processes, sub-daily ERP variations were described by different authors in the last 30 years. The relevant publications are given here to provide an overview of prior research activities.

3.1 Earth Orientation Parameters

Time-dependent external torques $\boldsymbol{\tau}(t)$ act on the entire system Earth forcing the Earth's rotation to be changed. Additionally, variabilities in the Earth's mass distribution change its inertia tensor and, thus, also the rotational motion. In a rotating reference frame which is fixed to the solid Earth, the relation between external torques and changes of the angular momentum $\mathbf{L}(t)$ of the Earth under the principle of conservation of angular momentum is given by the Euler equation (e.g., MUNK and MACDONALD 1960, MORITZ and MUELLER 1988, EUBANKS 1993)

$$\frac{\partial \mathbf{L}(t)}{\partial t} + \boldsymbol{\omega}(t) \times \mathbf{L}(t) = \boldsymbol{\tau}(t) \tag{3.1}$$

where the Earth's rotation vector is given by $\boldsymbol{\omega}(t)$. The angular momentum vector $\mathbf{L}(t)$ can be separated into two parts: (1) the motion term $\mathbf{h}(t)$ which is present due to relative motions w.r.t. the Earth fixed reference frame, e.g., local winds or currents, and (2) the matter term that represents mass redistributions as changes of the Earth's tensor of inertia $\mathbf{I}(t)$

$$\mathbf{L}(t) = \mathbf{h}(t) + \mathbf{I}(t) \cdot \boldsymbol{\omega}(t). \tag{3.2}$$

These two constituents of the angular momentum are also known as wind and pressure excitation of the Earth's rotation (e.g., BARNES et al. 1983). By combining Eq. (3.1) and (3.2), the Euler-Liouville equations can be derived (e.g., GROSS 2007)

$$\frac{\partial}{\partial t} \left[\mathbf{h}(t) + \mathbf{I}(t) \cdot \boldsymbol{\omega}(t) \right] + \boldsymbol{\omega}(t) \times \left[\mathbf{h}(t) + \mathbf{I}(t) \cdot \boldsymbol{\omega}(t) \right] = \boldsymbol{\tau}(t). \tag{3.3}$$

The Euler-Liouville equations describe the relation between small Earth rotation variations and small changes in the relative angular momentum of the Earth's tensor of inertia. This basic framework is usually used for the analysis of the Earth's rotation.

Geometrically, the change of the angular momentum vector appears as a directional change in space (GROSS 1992). The resulting time-dependent orientation w.r.t. a quasi-inertial CRF generated by external torques is called precession and nutation. This movement of the instantaneous rotation axis w.r.t. the CRF can be described theoretically to a certain accuracy level (e.g., MATHEWS et al. 2002, CAPITAINE et al. 2003).

frequency in ITRS

$$-3.5 \qquad -2.5 \qquad -1.5 \qquad -0.5 \qquad +0.5 \qquad +1.5 \qquad +2.5 \text{ (cpsd)}$$

polar motion   *polar motion*

frequency in GCRS

$$-2.5 \qquad -1.5 \qquad -0.5 \qquad +0.5 \qquad +1.5 \qquad +2.5 \qquad +3.5 \text{ (cpsd)}$$
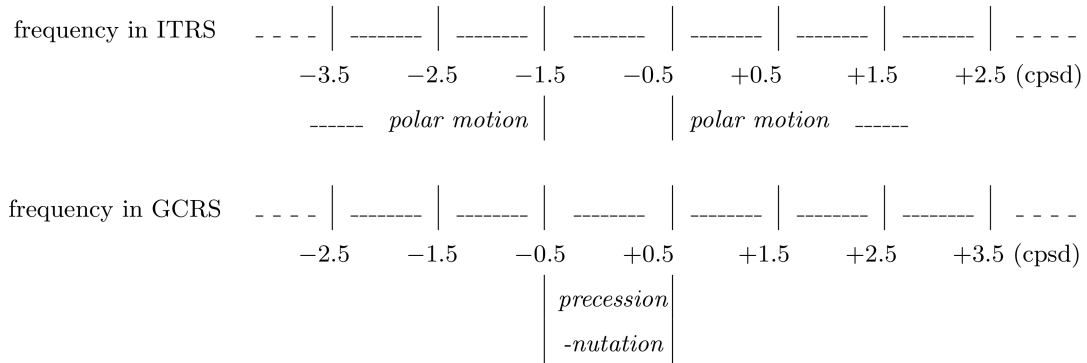
precession

-nutation

Figure 3.1: Conventional frequency separation between the precession-nutation of the CIP and its PM, either viewed in the TRF (top), or the CRF (bottom), with a 1 cpsd shift due to the rotation of the TRF w.r.t. the CRF. (PETIT and LUZUM 2010)

However, small adjustments at the 1 mas level are estimated regularly from VLBI observations (e.g., BÖCK-MANN et al. 2010). The precession-nutation model of the current version of the IERS Conventions (PETIT and LUZUM 2010) makes use of the Celestial Intermediate Pole (CIP, CAPITAINE 2002) which is closely associated with the Earth's figure axis. In fact, it is chosen to be the figure axis for the Tisserand mean outer surface of the Earth (TISSERAND 1891, WAHR 1981). With the definition of the CIP, precession and nutation are those motions with frequencies between -0.5 and +0.5 cycles per sidereal day (cpsd) in the CRF (see Fig. 3.1).

If the motion of the rotation axis w.r.t. the TRF should be studied, external torques are set to zero as these are the driving force for precession and nutation, i.e., the motion w.r.t. the CRF. Nevertheless, external torques lead to changes in the relative angular momentum and the Earth's inertia tensor and, thus, to variations of the rotation axis w.r.t. the TRF. However, these tidal variations can be considered as variations inside or between the sub-systems of the Earth as described below. The Earth's rotation vector can be expressed in a Tisserand mean-mantle frame (TISSERAND 1891) by a uniform rotation around the **z**-axis with the mean angular velocity Ω and perturbations from this uniform rotation $\boldsymbol{\Delta\omega}$

$$\boldsymbol{\omega}(t) = \boldsymbol{\omega}_0 + \boldsymbol{\Delta\omega}(t) = \boldsymbol{\omega}_0 + \Omega \cdot \mathbf{m} = \begin{pmatrix} 0 \\ 0 \\ \Omega \end{pmatrix} + \Omega \begin{pmatrix} m_x(t) \\ m_y(t) \\ m_z(t) \end{pmatrix}. \tag{3.4}$$

As the deviations from the uniform rotation are small, i.e., only a few parts of 10^8 in the rotation rate $m_z(t)$ and one part of 10^6 in the orientation of the rotation axis ($m_x(t)$ and $m_y(t)$) relative to the Earth's figure axis (GROSS et al. 2003), the Euler-Liouville equations can be linearized without loss of generality. Furthermore, the Earth can be considered as axis-symmetric as the relative difference of the equatorial moments of intertia is small: $(B - A)/A = 2.2 \cdot 10^{-5}$ (e.g., GROTEN 2004). Therefore, the first order approximation of the conservation of angular momentum equations, the linearized Euler-Liouville equations, that relate changes in the angular momentum to changes in the Earth's rotation, can be written as (e.g., GROSS 2007)

$$\frac{1}{\sigma} \frac{\partial m_x(t)}{\partial t} + m_y(t) = \chi_y(t) - \frac{1}{\Omega} \frac{\partial \chi_x(t)}{\partial t} \tag{3.5a}$$

$$\frac{1}{\sigma} \frac{\partial m_y(t)}{\partial t} + m_x(t) = -\chi_x(t) - \frac{1}{\Omega} \frac{\partial \chi_y(t)}{\partial t} \tag{3.5b}$$

$$m_z(t) = -\chi_z(t) \tag{3.5c}$$

where σ is the frequency of the relative motion of the rotation axis w.r.t. the figure axis and $\boldsymbol{\chi}(t)$ are the excitation functions. See, e.g., GROSS (2007) for a thorough explanation of the theory of the Earth's rotation based on the linearized Euler-Liouville equations. It should be noted that with this theoretical derivation of

the Earth's rotation, the variations of the Earth's rotation rate m_z can be completely separated from the variations of the rotation pole m_x and m_y.

In case of an axis-symmetric solid Earth, the motion of the rotation pole is a prograde undamped circular motion (GROSS 2007), i.e., a wobbling motion of the rotation axis w.r.t. the **z**-axis of the co-rotating reference frame called polar motion. The reason for this wobble is that the rotation axis does not agree with the Earth's figure axis (SCHÖDLBAUER 2000). EULER (1765) predicted a free wobble of the solid Earth with a period of 305 days. Due to various effects, the effective period of the free wobble is about 433 days which was first detected in astronomical observations by CHANDLER (1891). In addition to this free motion, processes within the entire system Earth are present. In contrast to the external torques, these internal processes do not change the direction of the angular momentum vector in space. However, these internal mass redistributions and angular momentum exchanges between the solid Earth and geophysical fluids lead to variations of the Earth's rotation rate and time-dependent PM (see Ch. 3.2). The definition of the CIP involves that PM is the motion of the CIP in the TRF with frequencies below -1.5 cpsd and above -0.5 cpsd, i.e., all periods beside the ones for which precession-nutation is defined (see Fig. 3.1).

As mentioned above, the EOPs build the link between the CRF and the TRF

$$\mathbf{x}_c = \mathbf{R}(t) \cdot \mathbf{x}_t \tag{3.6}$$

This transformation can be realized by a time-dependent rotation matrix $\mathbf{R}(t)$ that might be composed by three independent Euler angles. However, the EOPs are usually used due to historical reasons (see NOTH-NAGEL (1991) for the relation of these two approaches). The IERS Conventions 2010 give two equivalent ways to perform this transformation, which differ by the origin that is adopted on the CIP equator. On the one hand, the equinox is used and, on the other hand, the Celestial Intermediate Origin (CIO, CAPITAINE et al. 2000) is used, which represents a non-rotating origin (GUINOT 1979). In both cases, the general form of the transformation is (PETIT and LUZUM 2010)

$$\mathbf{x}_c = \mathbf{Q}(t) \cdot \mathbf{R}(t) \cdot \mathbf{W}(t) \cdot \mathbf{x}_t \tag{3.7}$$

where $\mathbf{Q}(t)$ and $\mathbf{W}(t)$ are matrices that describe the motion of the CIP in the CRF and the TRF and the matrix $\mathbf{R}(t)$ describes the rotation of the Earth around the **z**-axis. The matrix $\mathbf{W}(t)$ is independent of the applied procedure (MCCARTHY and CAPITAINE 2002)

$$\mathbf{W}(t) = \mathbf{R}_3(-s'(t)) \cdot \mathbf{R}_2(x_p(t)) \cdot \mathbf{R}_1(y_p(t)) \tag{3.8}$$

where $\mathbf{R}_i$ denotes a rotation matrix about the axis i. The coordinates of the CIP in the TRF are denoted by x_p and y_p and $s'(t)$ is a small correction, called "TIO locator" which depends on PM (PETIT and LUZUM 2010). TIO is the Terrestrial Intermediate Origin.

With the CIO based transformation, the IAU Resolutions 2000/2006 are implemented in accordance to the kinematic definition of the CIO as a non-rotating origin. The rotation matrix $\mathbf{R}(t)$ consists of the Earth rotation angle θ between the CIO and the TIO

$$\mathbf{R}(t) = \mathbf{R}_3(-\theta(t)) \tag{3.9}$$

and the precession-nutation matrix $\mathbf{Q}(t)$ for the CIO based transformation

$$\mathbf{Q}(t) = \mathbf{Q}(X(t), Y(t)) \cdot \mathbf{R}_3(s(t)) \tag{3.10}$$

depends on the coordinates of the CIP in the CRF $X(t)$ and $Y(t)$ and the "CIO locator" $s(t)$ (PETIT and LUZUM 2010).

When using the classical equinox based transformation, Greenwich Apparent Sidereal Time (GAST) is used in the rotation matrix

$$\mathbf{R}(t) = \mathbf{R}_3(GAST(t)) \tag{3.11}$$

The precession angles are those of LIESKE et al. (1977), and the traditional nutation parameters in longitude $\Delta\psi$ and obliquity $\Delta\epsilon$ at the time t are used (SCHUH et al. 2003)

$$\mathbf{Q}(t) = \mathbf{P}(t) \cdot \mathbf{R}_1(-\epsilon_A(t)) \cdot \mathbf{R}_3(\Delta\psi(t)) \cdot \mathbf{R}_1(\epsilon_A(t) + \Delta\epsilon(t)) \tag{3.12}$$

with the precession matrix $\mathbf{P}(t)$ (e.g., SOVERS et al. 1998) and the obliquity of the ecliptic ϵ_A. With this transformation, the rotation matrix depends on precession-nutation which is expressed by Greenwich Mean Sidereal Time (GMST) and the equation of equinoxes (e.g., WOOLARD 1953, AOKI and KINOSHITA 1983, MÜLLER 1999)

$$GAST(t) = GMST(t) + \Delta\psi(t) \cdot \cos(\epsilon_A(t)) + 0.00264'' \cdot \sin\Omega + 0.000063'' \sin 2\Omega \tag{3.13}$$

where Ω denotes the ascending node of the mean elliptical lunar path. Furthermore, the precession terms appear in the formula that links GMST and UT1 (AOKI et al. 1982, CAPITAINE and GONTIER 1993)

$$GMST(t) = GMST_{0hUT1} + c_{sid}UT1 \tag{3.14}$$

$$GMST_{0hUT1} = 6\text{h } 41\text{min } 50.54841\text{s} + 8640184.812866\text{s } T'_u + 0.093104\text{s } T'^2_u - 6.2 \cdot 10^{-6}\text{s } T'^3_u \tag{3.15}$$

with T'_u being the number of days elapsed since 2000 January 1, 12 h UT1 and c_{sid} being the conversion factor between sidereal and solar time interval (e.g., MORITZ and MUELLER 1988)

$$c_{sid} = 1.002737909350795 + 5.9006 \cdot 10^{-11}T'_u - 5.9 \cdot 10^{-15}T'^2_u \tag{3.16}$$

3.2 Excitation of the Earth Rotation Parameters

As described above, small changes in the Earth's rotation due to small changes of the relative angular momentum or the Earth's inertia tensor are studied based on the linearized Euler-Liouville equations. The rotation axis of the Earth performs a free motion w.r.t. the TRF due to the rotational behavior of the Earth. However, changes of the rotation axis as well as of the rotational rate are present on almost all time scales from decades to hours. The variations are excited by processes within the entire Earth system, e.g., mass redistributions within the geophysical fluids carry angular momenta which must be redistributed to conserve the total angular momentum and, thus, the ERPs are changed (SOVERS et al. 1998). The diversity of periods reflects the wide field of processes that force the Earth's rotation to change. These processes are mass redistributions, tidal variations and angular momentum exchanges within or between individual sub-systems of the Earth, summarized as internal processes. As sub-systems of the Earth, one can consider:
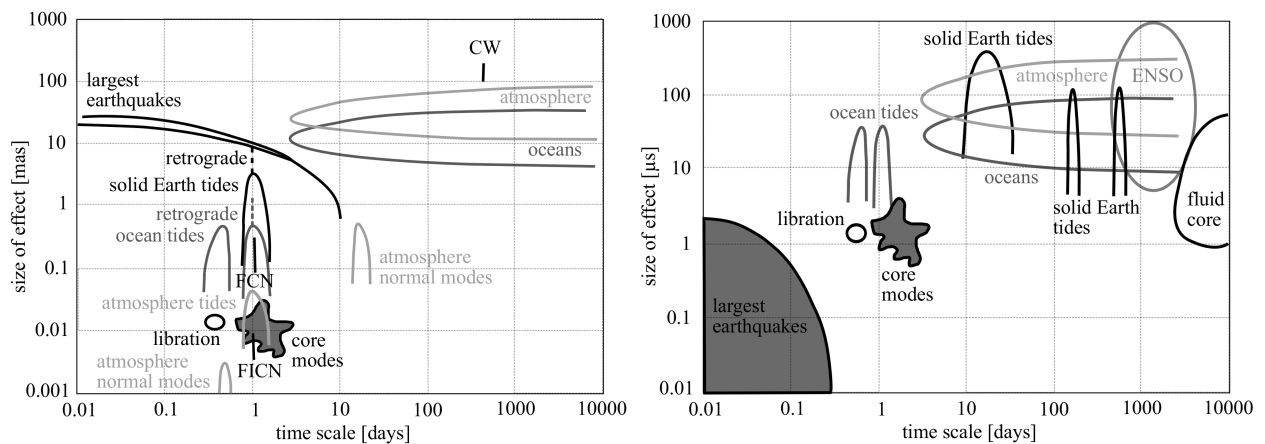

Figure 3.2: Excitation of PM (left) and UT1 (right) (`ftp://gemini.gsfc.nasa.gov/pub/core/`, SCHUH et al. 2003, modified).

the solid Earth and its fluid core, the atmosphere, the oceans, the hydrosphere and the cryosphere as well as the biosphere and the antrosphere (SCHUH et al. 2003). Several impacts on UT1 and PM are shown in Fig. 3.2 together with the relevant time scales and their magnitudes. A detailed description of the impact of each individual sub-system can be found in, e.g., GROSS (2007) or SCHUH et al. (2003).

For daily and sub-daily time-scales, tidal variations have the biggest impact. Several models were developed to describe sub-daily ERP changes, these are discussed in more detail below.

3.2.1 UT1 and Length-of-Day Variations

Variations of $\Delta UT1$ consist primarily of a linear trend. This can be seen in Fig. 3.3 where a VLBI-derived $\Delta UT1$ time series from 1994 to 2007 is displayed. Likewise, variations of length-of-day (LOD), which is the negative time derivative of $\Delta UT1$

$$LOD = -\frac{\partial \Delta UT1}{\partial t} \tag{3.17}$$

also show a linear trend of +1.8 ms/cy (MORRISON and STEPHENSON 2001). Besides this, LOD consists of decadal variations (e.g., GROSS 2001), seasonal variations (e.g., GROSS et al. 2004) and tidal variations (e.g., YODER et al. 1981).

In case of an axis-symmetric Earth, tidal LOD and $\Delta UT1$ excitations can only be generated by long-period tides, i.e., second-degree zonal components of the spherical harmonics. Nevertheless, the sources of sub-daily LOD and $\Delta UT1$ variations are primarily of tidal nature. Tesseral and sectorial spherical harmonics are present due to asymmetries which lead to diurnal and semi-diurnal tidal forcings and, thus, to LOD and $\Delta UT1$ variations at these periods as well (CHAO et al. 1996). These asymmetries of the geophysical fluids are, e.g., irregular coastlines, tidal variations in bays and the non-equilibrium behavior of the oceans (e.g. BROSCHE and SCHUH 1998). The tidally forced variations can be expressed by poly-harmonic functions (e.g., GIPSON 1996, ROTHACHER et al. 2001, PETIT and LUZUM 2010)

$$d\Delta UT1 = \sum_{i=1}^{n} u_i^c \cos \varphi_i + u_i^s \sin \varphi_i \tag{3.18a}$$

$$dLOD = \sum_{i=1}^{n} l_i^c \cos \varphi_i + l_i^s \sin \varphi_i = \sum_{i=1}^{n} \omega_i \left(u_i^c \cos \varphi_i + u_i^s \sin \varphi_i \right) \tag{3.18b}$$

where n is the number of considered tides, $\omega_i := \frac{\partial \varphi_j}{\partial t}$ the circular frequency of a tide and each tide φ_i is the sum of integers (a_i to f_i) multiplied with the so-called fundamental arguments (or Delauney variables) and GMST (usually denoted by θ)

$$\varphi_i = a_i \cdot \ell + b_i \cdot \ell' + c_i \cdot F + d_i \cdot \Omega + f_i \cdot D + (\theta + \pi) \tag{3.19}$$

The fundamental arguments ℓ, ℓ', F, D, Ω represent the mean anomaly of the Moon and the Sun, the argument of latitude of the Moon, the elongation of the Moon from the Sun and the longitude of the ascending lunar node, respectively (e.g., PETIT and LUZUM 2010). The resulting amplitudes for the sine- and cosine-component of the tidally forced $\Delta UT1$ and LOD variations are given by u_i^c and u_i^s as well as l_i^c and l_i^s in Eq. (3.18).

The dominating part of the sub-daily variations is due to the ocean tides where 90% of the measured tidal $\Delta UT1$ variations are explained by a tidal ERP model based on a theoretical ocean tidal model (CHAO et al. 1996). Such sub-daily tidal variations based on ocean tide models were first predicted by YODER et al. (1981). Afterwards, similar predictions were made by several authors. Following GROSS (2007), these investigations can be divided into two groups, those before and those after the inclusion of sea surface height altimeter observations from TOPEX/POSEIDON (FU et al. 1994) which changed the ocean tidal models significantly. Prior to TOPEX/POSEIDON, e.g., BROSCHE (1982), BAADER et al. (1983), BROSCHE et al.
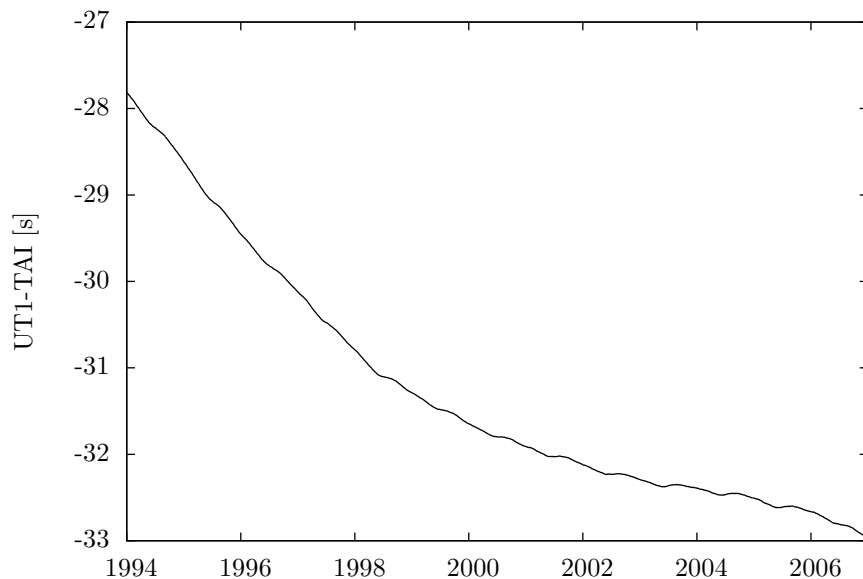
Figure 3.3: 13 year long ΔUT1 time series determined from a VLBI solution.

(1989), SEILER (1990), SEILER (1991), WÜNSCH and BUSSHOFF (1992), DICKMAN (1993), GROSS (1993) and SEILER and WÜNSCH (1995) used ocean tidal models to determine a model for tidal LOD or ΔUT1 variations. Based on assimilation models, predictions were made by, e.g., RAY et al. (1994), EGBERT et al. (1994), CHAO et al. (1995), CHAO et al. (1996) and CHAO and RAY (1997). These assimilation ocean tidal models use both, an ocean tidal model and sea-surface heights measured by TOPEX/POSEIDON. In the IERS Conventions 2010 the coefficients for diurnal and semi-diurnal variations in LOD and ΔUT1 of an updated version of the RAY et al. (1994) model are listed in Tab. 8.3 of PETIT and LUZUM (2010).

Small variations in LOD and ΔUT1, that cannot be explained by tidal ERP models based on the theoretical ocean tidal models were detected by, e.g., SCHUH and SCHMITZ-HÜBSCH (2000), HAAS and WÜNSCH (2006) and ARTZ et al. (2010). These differences might be forced by diurnal and semi-diurnal atmospheric effects as this influence is up to two orders of magnitude below the oceanic impact (BRZEZIŃSKI et al. 2002). These atmospheric effects are produced by gravitational forces and by non-tidal thermal forces. However, in the atmosphere, the thermal effects are expected to be much larger than the gravitational tides (e.g, CHAPMAN and LINDZEN 1970). These thermal effects are excited by the heating of the Sun with a basic frequency of one cycle per solar day, for which additional harmonics with integer cycles per solar day are present. However, only the waves with diurnal, semi-diurnal and ter-diurnal periods are considered as being significant (VOLLAND 1997). Furthermore, non-tidal oceanic effects might be present. This non-tidal oceanic variability is not directly related to the gravitational forcing, but generated by corresponding atmospheric tides, thus, they are called radiational ocean tides (BRZEZIŃSKI et al. 2004). The impact of the atmosphere on the Earth's rotation on time scales of one day and below was performed by, e.g., ZHAROV and GAMBIS (1996), BRZEZIŃSKI et al. (2002), DE VIRON et al. (2005) and BRZEZIŃSKI (2008). Concerning these predictions, several problems arise as they are usually based on AAM series which have a temporal resolution of six hours. Certain pairs of tidal waves are mixed together and the sampling of six hours is not sufficient to resolve them (BRZEZIŃSKI et al. 2002). Thus, it is difficult to investigate the semi-diurnal band. Nevertheless, even ter-diurnal tidal impacts were investigated by DE VIRON et al. (2005) and HAAS and WÜNSCH (2006) on a model basis.

Finally, the tri-axial shape of the Earth has to be considered, as the direct effect of external torques on the non-axis-symmetric part of the Earth leads to variations in the Earth's rotation called libration. This effect was investigated by, e.g., CHAO et al. (1991), WÜNSCH (1991), CHAO et al. (1996) as well as by BRZEZIŃSKI and CAPITAINE (2003) and BRZEZIŃSKI and CAPITAINE (2010). The most recent IERS Conventions 2010 provide such a model in Tab. 5.1b of PETIT and LUZUM (2010).

3.2.2 Polar Motion

Comparable to the excitations of the Earth's rotation rate, PM changes also exist due to various phenomena. Besides the above mentioned free wobble, several other internal forces change the position of the CIP w.r.t. the **z**-axis of the TRF. Again, mass redistributions, tidal variations and angular momentum exchanges between the solid Earth and geophysical fluids are the driving forces. Figure 3.4 shows the official IERS 05C04 PM time series (BIZOUARD and GAMBIS 2009). The most obvious characteristic is a beat with a period of about 6.3 years. This is a superimposition of the free wobble and an almost yearly signal. The period of the free wobble has been determined by, e.g., WILSON and HAUBRICH (1976) to be 433 days and its varying amplitude is around $100-200$ mas ($3-6$ m at the Earth's surface; e.g., SCHUH et al. 2000). The difference to the period of 305 days (EULER 1765) for a solid Earth depends on the internal structure and rheology of the Earth, e.g., the elasticity of the Earth, atmospheric, oceanic, and hydrological processes as well as the presence of a fluid core can be considered as causes. However, the relative contribution of each component is still unclear. In contrast, the yearly signal is a forced motion with a nearly constant amplitude of 100 mas (RUMMEL et al. 2009). The exciting processes are various geophysical and gravitational sources with an annual characteristic, where the largest impact is a high atmospheric pressure system over Siberia every winter (e.g., GROSS et al. 2003). In addition, smaller variations can be detected at decadal time scales with amplitudes around 30 mas (Markowitz wobble, GROSS 2007) and smaller variations on all measurable time scales are present. The variations with periods of one day and below, which are investigated within this thesis, belong to the latter group. They are treated in more detail below. Finally, a linear trend is present with a rate of 3.3 mas/year with a direction towards 76° longitude (e.g., SCHUH et al. 2001).

Concerning diurnal and sub-diurnal time scales, tidal variations have the biggest impact. For an axis-symmetric Earth, no PM variations are expected due to the second-degree zonal tides but asymmetries are present in the geophysical fluids as already described in Ch. 3.2.1. Due to these asymmetries, long term PM are forced by the exchange of non-axial oceanic tidal angular momentum with the solid Earth. Furthermore, diurnal and semi-diurnal ERP variations are caused by tesseral and sectorial components of the spherical harmonic expansion of the tide generating potential. The tidally induced PM variations can be modeled by a poly-harmonic representation as well (e.g., GIPSON 1996)

$$dx_p(t) = \sum_{j=1}^{n} -p_j^c \cos \psi_j + p_j^s \sin \psi_j \tag{3.20a}$$

$$dy_p(t) = \sum_{j=1}^{n} p_j^c \sin \psi_j + p_j^s \cos \psi_j \tag{3.20b}$$

with the sine- and cosine-amplitudes of the tidal PM variations p_j^s and p_j^c. All other symbols correspond to Eq. (3.18). To account for retrograde PM, the individual tides ψ_j are derived by inserting the integer factors of the fundamental arguments with opposite sign in Eq. (3.19). However, this is only done for the semi-diurnal tides, as the retrograde diurnal PM components are defined as nutation (see Fig. 3.1). CHAO et al. (1996) showed that 60% of the sub-daily PM variations can be explained by the impact of diurnal and semi-diurnal ocean tides. Remaining differences can be attributed to non-tidal atmospheric and oceanic effects. It should be mentioned, that the solid Earth tides are primarily absorbed by introducing them as station position corrections to the observables as described in Ch. 4.1.

As with ΔUT1, YODER et al. (1981) first discussed the effect of diurnal and semi-diurnal ocean tides on PM. Subsequently, models for sub-daily PM variations were derived from theoretical ocean tide models by, e.g., SEILER (1990), SEILER (1991), DICKMAN (1993), GROSS (1993), BROSCHE and WUENSCH (1994) and SEILER and WÜNSCH (1995). With the advent of TOPEX/POSEIDON and the integration of its sea surface height observations into ocean tide models, enhanced tidal PM models were calculated by CHAO et al. (1996) and CHAO and RAY (1997). In Tab 8.2 of the current IERS Conventions 2010 a model based on an ocean tide model is given where altimeter observations were assimilated.

Comparable to ΔUT1, discrepancies of these theoretical ERP models to measurements of sub-daily PM exist (e.g., SCHUH and SCHMITZ-HÜBSCH 2000, HAAS and WÜNSCH 2006, NASTULA et al. 2007 and ARTZ et al.
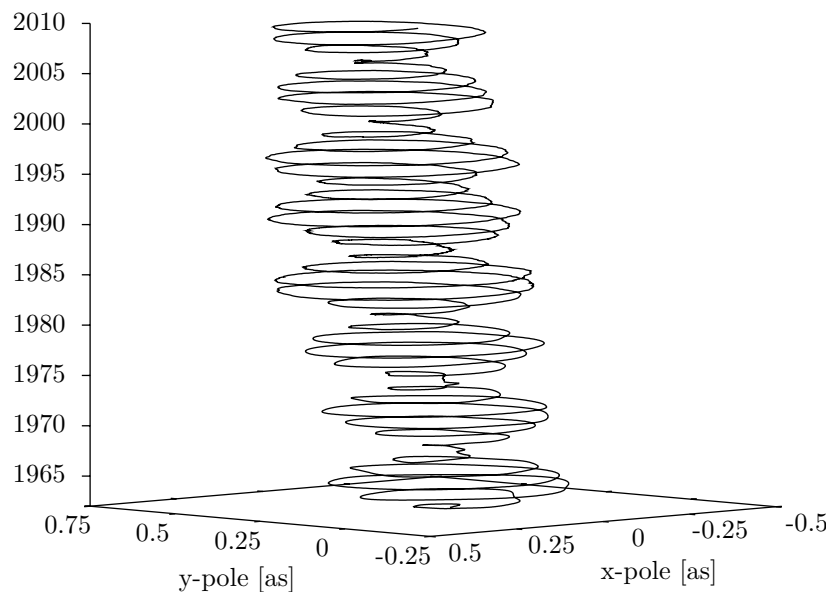
Figure 3.4: Pole spiral from the IERS 05C04 time series ranging from 1962 to 2010.

2010). The reason for these differences can again be attributed to missing non-tidal oceanic or atmospheric effects in the ERP models that are based on theoretical ocean tide models. Atmospheric excitations have been investigated by Zharov and Gambis (1996), Brzeziński et al. (2002) and de Viron et al. (2005). In addition, Brzeziński et al. (2004) and Brzeziński (2008) determined the impact of non-tidal oceanic angular momentum due to radiational atmospheric tides as well as the impact of the atmospheric tides themselves. de Viron et al. (2002) predicted diurnal PM variations based on non-tidal AAM and OAM and Haas and Wünsch (2006) reported diurnal PM excitation based on a non-tidal angular momentum approach. Concerning the semi-diurnal predictions based on AAM time series, these are uncertain as the pro- and retrograde PM components cannot be separated due to the six hour long temporal resolution of the AAM time series (Brzeziński et al. 2004). AAM series with a better resolution are currently only available for limited time spans, e.g., Salstein et al. (2008) derived hourly wind-based excitations for polar motion of the CONT02 and CONT05 periods.

Similar to ΔUT1, the effects of the tri-axial shape of the Earth are present in PM although not that pronounced. This effect was investigated by Chao et al. (1991) and Chao et al. (1996). To account for libration in the diurnal prograde PM band, the IERS Conventions 2010 provides the amplitudes for 10 tidal constituents in Tab. 5.1a of Petit and Luzum (2010).

4. Measuring Earth Rotation Parameters with VLBI

VLBI is a space geodetic technique based on radio interferometry. The interferometer is built up of two locally separated antennas with a maximum distance of about 10,000 km. These antennas observe a radio signal of currently 2.3 and 8.4 GHz that is emitted by extragalactic radio sources such as quasars or radio galaxies. A detailed description of the VLBI space segment is given by, e.g, Heinkelmann (2008). The observed radio signal is recorded digitally at each antenna together with a highly precise time information provided by a hydrogen maser. The signals are then sent to a correlation center to be cross-correlated. In this way, the primary observables for the VLBI analysis, the delay and the delay rate, are generated (e.g., Whitney 2000).

Within the IVS, a global network of about 40 VLBI stations currently exists. In a standard VLBI observing session of 24 hours duration, three to 15 of them observe in general 15 to 60 radio sources while in specific observing sessions, up to 260 radio sources are observed. The VLBI sessions, of which on average three are performed per week, are used to estimate various parameters such as the EOPs. In addition, special single baseline VLBI sessions, so-called Intensives, of one hour duration are observed almost daily to provide continuous ΔUT1 estimates (e.g., Robertson et al. 1985, Luzum and Nothnagel 2010).

In this section, the basic principle of VLBI as well as the process of parameter estimation are briefly described. Furthermore, the specifics for the determination of ERPs are presented with an emphasis on the determination of sub-daily ERP representations.

4.1 The Basic Principle of VLBI

The general configuration of VLBI consists of two antennas, building a baseline $\mathbf{b}$, which simultaneously observe the incoming electromagnetic wave front which is emitted by the same radio source and which travels along the unit vector $\mathbf{k}$. This basic principle is depicted in Fig. 4.1. Although the radio signal is initially a sphere, a planar wave front can be assumed without any loss of generality, as the radio sources are very distant (2 to 12 billion light-years). Today, the primary VLBI observable is the time delay τ, i.e., the time difference between the reception of the radio signal at antenna one and antenna two. Prior to the recording of the signal at the antennas, it traveled through the interstellar space, the Solar System as well as the Earth's atmosphere. On this path, it is affected by various electromagnetic and gravitational impact factors. Furthermore, the geometry is changing between the reception of the signal at antenna one and two, e.g., due to the Earth's rotation. All of these factors have to be considered during a VLBI analysis. Within this thesis, only the major points are mentioned, a more detailed description of general principles of VLBI can be found in many publications, e.g., Thomas (1972), Campbell (1987), Sovers et al. (1998) or Takahashi et al. (2000).

During a standard VLBI session, several radio telescopes observe several sources over a longer time span, nevertheless, the basic principle as shown in Fig. 4.1 is sufficient to describe the principle of VLBI. This holds especially, as the correlator generates the observables independently for each baseline (Sovers et al. 1998). The geometric time delay τ_g is the difference in arrival time of the radio signal between two observing telescopes, where an ideal instrumentation, synchronization and a vacuum between the source and the telescopes is assumed. As a planar wave front is assumed, the geometric delay can be computed in a right-angled triangle (e.g., Takahashi et al. 2000)

$$\tau_g = t_2 - t_1 = -\frac{1}{c}\mathbf{b} \cdot \mathbf{k} \tag{4.1}$$

with the VLBI vector baseline $\mathbf{b} = \mathbf{r}_2 - \mathbf{r}_1$ computed from the position vectors of two VLBI telescopes $\mathbf{r}_i$ and the unit vector in the direction of the radio source $\mathbf{k}$. t_1 and t_2 are the arrival times at the two telescopes
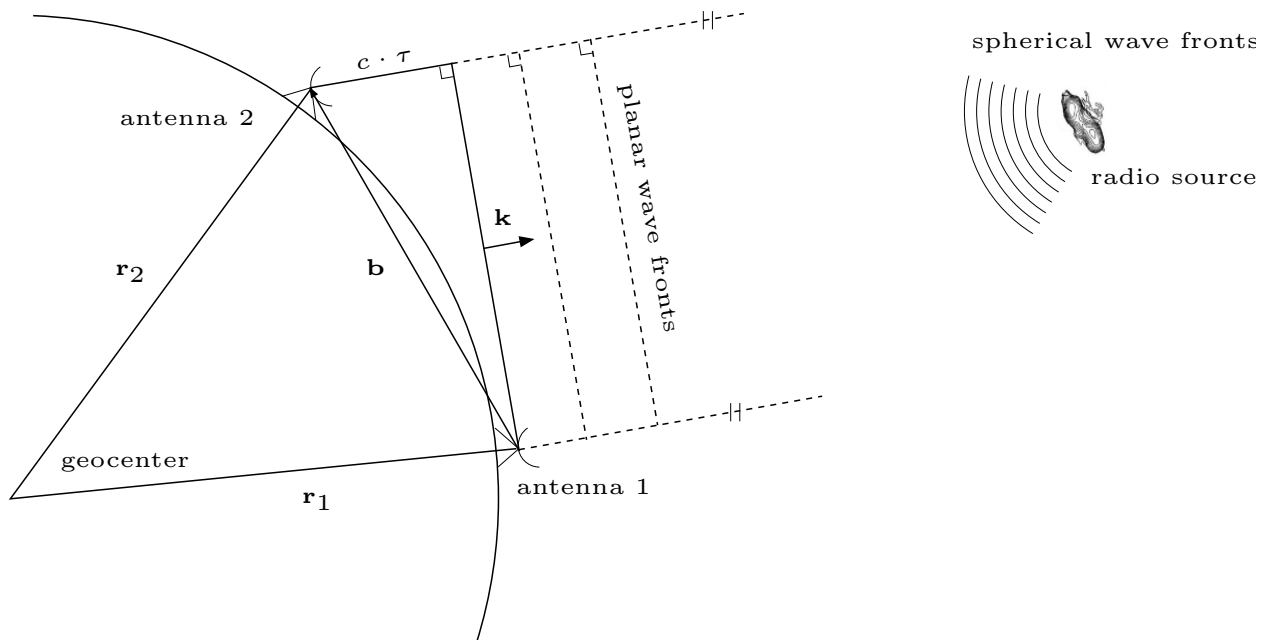
Figure 4.1: The basic principle of VLBI.

as measured on the Earth's surface, or strictly speaking the geoid. Finally, c denotes the velocity of light. However, this equation only holds if the baseline vector and the unit vector in source direction are given in the same reference system. Usually, the station positions are given in a TRF and the sources in a CRF. Thus, a transformation between these two frames is necessary. As demonstrated in Ch. 3.1, this can be realized via Eq. (3.7) by using the EOPs.

To achieve the entire accuracy of VLBI, the geometric time delay has to be expressed in a quasi-inertial barycentric system. Thus, several transformations are necessary. A detailed description of these steps can be found, e.g., in SCHUH (1987), SOVERS et al. (1998) or TAKAHASHI et al. (2000). The principle procedure is to account for the time-dependence of the baseline length $\Delta \mathbf{b}$ at the epoch t_1, where the signal has been recorded at station one, e.g., due to Earth tides or tectonic motions. Subsequently, the baseline is transformed from the geocentric TRF to the geocentric CRF

$$\mathbf{B}_g(t_1) = \mathbf{Q}(t_1) \cdot \mathbf{R}(t_1) \cdot \mathbf{W}(t_1) \cdot (\mathbf{b} + \Delta\mathbf{b}(t_1)) \tag{4.2}$$

After applying a Lorentz transformation of $B_g(t_1)$, the baseline $\mathbf{B}_{SSB}(T_1)$ is given in a frame at rest relative to the center of mass of the Solar system (Solar System Barycentric (SSB) frame) which is rotationally aligned to the geocentric CRF. This transformation is applied to account for relativistic effects that arise due to the motion of the observing sites w.r.t. the barycenter. The epoch T_1 is now representing the time where the radio signal is received at antenna one, although measured in the barycentric system. In this SSB frame, the geometric delay is calculated (e.g., SOVERS et al. 1998)

$$T_g = -\frac{1}{c}\frac{\mathbf{k} \cdot \mathbf{B}_{SSB}(T_1)}{1 - \mathbf{k} \cdot v_2} \tag{4.3}$$

where v_2 is the velocity of antenna two in the SSB frame and the source unit vector in the CRF is given by

$$\mathbf{k} = \begin{pmatrix} -\cos\alpha \cdot \cos\delta \\ \sin\alpha \cdot \cos\delta \\ \sin\delta \end{pmatrix} \tag{4.4}$$

with the source coordinates in right ascension α and declination β. This delay is corrected for the relativistic changes caused by the gravitational fields of the Sun, the Planets and the Earth. Finally, another Lorentz

transformation is performed to access the geometrical delay in a geocentric reference frame τ_g. Thus, this delay consist of the pure geometry as well as of special and general relativistic effects. Furthermore, aberration effects are comprised, to account for the rotation of the Earth and the movement of the Earth around the Sun.

In addition to the effects mentioned above, several other effects occurring on the way through the Solar System, and the Earth's atmosphere as well as geophysical phenomena have to be accounted for in order to exploit the high accuracy of the VLBI measurements. Detailed descriptions of the various effects are given in, e.g., SCHUH (1987) or TESMER (2004). According to, e.g., HAAS (1996) and CANNON (1999) the basic geometrical delay has to be extended to:

$$\tau_{obs} = \tau_g + \tau_{clock} + \tau_{instr} + \tau_{tropo} + \tau_{iono} + \ldots \tag{4.5}$$

with:

τ_{clock}	mis-synchronization of the reference clocks at each observatory,
τ_{instr}	propagation delays through on-site cable runs and other instruments,
τ_{tropo}	propagation delays through the non-ionized portions of the Earth's atmosphere,
τ_{iono}	propagation delays through the ionized portions of the Earth's atmosphere.

4.2 Parameter Estimation

When many sources are estimated for short time intervals and in many directions over a longer time span (e.g., 24 hours), the complete amount of data can be used to estimate various parameters. Based on the observation equation that was derived in Ch. 4.1, a classical least-squares adjustment (e.g., KOCH 1999)

$$\mathbf{A}^T \mathbf{\Sigma}^{-1} \mathbf{A} \mathbf{\Delta x} = \mathbf{A}^T \mathbf{\Sigma}^{-1} (\mathbf{y} - \mathbf{y_0}) \tag{4.6}$$

can be performed to derive the estimates of a specific set of parameters $\mathbf{\Delta x}$. Here, $\mathbf{y}$ denotes the vector of observed group delays and $\mathbf{y}_0$ is the vector of the theoretical delays that is derived by inserting the a priori values and model values into Eq. (4.5). $\mathbf{A}$ denotes the Jacobian matrix containing the partial derivatives of the observation equations w.r.t. the unknown parameters. $\mathbf{\Sigma}$ is the variance-covariance matrix (VCM) of the observations. The linearized least-squares algorithm is a sufficient approach as the non-linearities in the mathematical foundation of a VLBI session are small and the a priori models are good in general. The least-squares adjustment is used within this thesis, however, other estimation techniques are possible, e.g., as Kalman Filtering (e.g., HERRING et al. 1990), least squares collocation method (e.g., TITOV 2000) or the application of a square root information filter (e.g., BOLOTIN et al. 2007).

In principle, all components that are present in the observation equations can be estimated. However, the parameters that are typically determined in a VLBI analysis are: Station positions and their velocities, source positions as well as the EOPs. Furthermore, clock parameters are estimated to account for the deterministic and stochastic clock behavior (e.g., MA et al. 1990), and troposphere parameters are set up to account for the variability of the wet part of the troposphere (e.g., BOEHM et al. 2006, CHEN and HERRING 1997). An almost complete description of the VLBI target parameters can be found in SOVERS et al. (1998). The partial derivatives for the most common parameters are given in, e.g., SCHUH (1987), NOTHNAGEL (1991) and HAAS (1996). The observation equation given in Eq. (4.5) is simplified by ignoring the detailed Lorentz-Transformation but accounting for aberration

$$F(v_i, v_i^b) = \frac{(v_i + v_i^b) \cdot k_i}{c} + \frac{(v_i \cdot k_i)^2 + 2(v_i \cdot k_i)(v_i^b \cdot k_i)}{c^2} + \frac{(b_i \cdot v_i)(v_i^b \cdot k_i)}{c^3} + \frac{(b_i \cdot v_i)(v_i \cdot k_i)}{2c^3} \tag{4.7}$$

for a standard set-up in a single 24 hour VLBI session reads as follows

$$
\begin{aligned}
\tau_{obs}(t) = & -\frac{1}{c}(\mathbf{b} + \Delta\mathbf{b}(t)) \cdot \mathbf{W}(t) \cdot \mathbf{R}(t) \cdot \mathbf{Q}(t) \cdot \mathbf{k} \cdot \left(1 - F(v_i, v_i^b)\right) + \boldsymbol{\tau}_{other} + \boldsymbol{\epsilon} \\
& - \left(\mathcal{T}_0^a + \mathcal{T}_1^a \cdot (t - t_0) + \mathcal{T}_2^a \cdot (t - t_0)^2\right) \\
& + \left(\mathcal{T}_0^b + \mathcal{T}_1^b \cdot (t - t_0) + \mathcal{T}_2^b \cdot (t - t_0)^2\right) \\
& - \left(T^a(t_0) + \frac{T^a(t_1) - T^a(t_0)}{t_1 - t_0}(t - t_0) + ... + \frac{T^a(t_i) - T^a(t_{i-1})}{t_i - t_{i-1}}(t - t_i)\right) \\
& + \left(T^b(t_0) + \frac{T^b(t_1) - T^b(t_0)}{t_1 - t_0}(t - t_0) + ... + \frac{T^b(t_i) - T^b(t_{i-1})}{t_i - t_{i-1}}(t - t_i)\right) \\
& + \mathcal{MF}_w^a \cdot \left[\mathcal{ZD}_w^a(t_0) + \frac{\mathcal{ZD}_w^a(t_1) - \mathcal{ZD}_w^a(t_0)}{t_1 - t_0}(t - t_0) + ... + \frac{\mathcal{ZD}_w^a(t_i) - \mathcal{ZD}_w^a(t_{i-1})}{t_i - t_{i-1}}(t - t_i)\right] \\
& + \mathcal{MF}_w^b \cdot \left[\mathcal{ZD}_w^b(t_0) + \frac{\mathcal{ZD}_w^b(t_1) - \mathcal{ZD}_w^b(t_0)}{t_1 - t_0}(t - t_0) + ... + \frac{\mathcal{ZD}_w^b(t_i) - \mathcal{ZD}_w^b(t_{i-1})}{t_i - t_{i-1}}(t - t_i)\right] \\
& + \mathcal{MF}_w^a(\varepsilon^a) \cdot \cot\varepsilon^a \cdot \left[\mathcal{G}_{North}^a \cos\alpha^a + \mathcal{G}_{East}^a \sin\alpha^a\right] \\
& + \mathcal{MF}_w^b(\varepsilon^b) \cdot \cot\varepsilon^b \cdot \left[\mathcal{G}_{North}^b \cos\alpha^b + \mathcal{G}_{East}^b \sin\alpha^b\right]
\end{aligned}
\tag{4.8}
$$

with:

v_i	x-, y- or z-velocity component of the geocenter
v_i^b	x-, y- or z-velocity of station b w.r.t. geocenter
$\mathbf{k}$ and k_i	source unit vector (see also Eq. (4.4)) and one of its 3 components
$\mathbf{b}$ and b_i	baseline vector $\mathbf{b} = (x_a - x_b, y_a - y_b, z_a - z_b)^T$ and one of its 3 components
$\Delta\mathbf{b}(t)$	baseline vector corrections at epoch t for effects like, e.g., solid Earth tides
c	velocity of light
$\mathbf{W}(t)$	PM matrix as defined in Eq (3.8) in Ch. 3
$\mathbf{R}(t)$	rotation matrix as defined in Eq (3.9) or Eq (3.11) in Ch. 3
$\mathbf{Q}(t)$	precession and nutation matrix as defined in Eq (3.10) or Eq (3.12) in Ch. 3
$\mathcal{T}_j^*$	parameter of the clock polynomial at station a or b
$T^*(t_i)$	additional clock parameters at station a or b for a specific epoch t_i parameterized as continuous piecewise linear functions (CPWLF), i.e., linear splines (e.g., DE BOOR 1978, DAHMEN and REUSKEN 2006)
$\mathcal{MF}_w^*$	mapping function of the wet part of the atmosphere at station a or b
$\mathcal{ZD}_w^*(t_i)$	CPWLF zenith wet delay at station a or b for a specific epoch t_i
$\mathcal{G}_{North/East}^*$	atmospheric gradients in North or East direction at station a or b
$\boldsymbol{\tau}_{other}$	other effects as given in Eq. (4.5), e.g., ionospheric corrections τ_{iono}
$\boldsymbol{\epsilon}$	measurement noise

The stochastic model of the least-squares adjustment is described by the VCM of the observations $\boldsymbol{\Sigma}$. It is usually a major diagonal matrix with differing variances for each observation. These variances are determined during the correlation process (e.g., CLARK et al. 1985), which is applied to determine the group delay from the recorded radio signal. Following SCHUH and CAMPBELL (1994), the standard deviations of the group delay can be approximated by the signal to noise ratio (SNR) and the effective bandwidth B_{eff} (e.g., ROGERS 1970)

$$
\sigma_\tau = \frac{1}{2\pi \cdot SNR \cdot B_{eff}}
\tag{4.9}
$$

A detailed description of the SNR can be found in HASE (1999). In brief, the SNR depends on the quality of the recording, the strength of the radio source, the size of the antennas, the noise temperature of the recording systems as well as on B_{eff} and the integration time of a particular observation. In the VLBI analysis software Calc/Solve (MA et al. 1990; BAVER 2010) the observations are interactively re-weighted in a baseline dependent manner to achieve a χ^2 of approximately one for each baseline[1]. Several investigations have been

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[1] a detailed description of the algorithm is given in http://lacerta.gsfc.nasa.gov/mk5/help/upwei_02_hlp.ps.gz

performed by different authors to improve the stochastic model. Initially, Qian (1985) and Schuh and Wilkin (1989) obtained correlation coefficients from several VLBI sessions. Later, Schuh and Schmitz-Hübsch (2000) applied a modified VCM of the observations according to the approaches in the 1980s. Another approach to improve the stochastic model empirically was given by Tesmer (2004) and Tesmer and Kutterer (2004), where the stochastic model of the VLBI analysis was refined by estimating variance and covariance components for different groups which represent specific stochastic properties. Especially station and elevation dependent constituents were considered. Finally, Gipson (2006), Gipson (2007) and Gipson (2008) proposed to account for unmodeled variances and covariances by adding clock-like and elevation dependent noise. Overall, the introduction of a modified VCM leads to slightly improved baseline repeatabilities, more realistic formal errors and to an a posteriori variance factor of $\chi^2 \simeq 1$. Within this thesis the standard Calc/Solve procedure of iteratively re-weighting the observations was used in general. Only for the analysis presented in Paper C (Artz et al. 2010), the approach of Gipson (2007) was applied and slightly modified by a relative weighting procedure.

The least-squares adjustment within a VLBI analysis can be conceptually divided into two groups. On the one hand, each session can be analyzed independently, and on the other hand, a so-called global solution of all sessions can be performed. In a global solution, some parameters are estimated only once for the entire time span and others are treated session-wise (e.g., Ma et al. 1990). Several parameters, like the station velocities, cannot be estimated from observations spanning only 24 h since the sensitivity is not high enough. Others, as the clock parameters are only useful when estimated for each session independently. A global solution is performed for reference frame solutions, where generally only a linear representation for each station position component and one constant offset for the source coordinates is estimated. Likewise, for the estimation of a model for sub-daily ERP variations (see Ch. 4.3.2), a sufficient long time span is necessary to resolve the individual tidal constituents.

Considering that VLBI measurements have only relative character, i.e., only the baseline vector, the orientation of the baseline w.r.t. the CRF or the relative position of the sources are determined, a rank deficiency exists. Estimating station positions and the EOPs simultaneously leads to a rank deficiency of six, as the delay does not comprise any information on a translational or rotational change. This may be cured by applying No-Net-Translation and No-Net-Rotation (NNR/NNT, e.g., Angermann et al. 2004) conditions w.r.t. the a priori TRF, i.e., minimizing this part of the trace of the VCM which corresponds to the datum defining station positions. Geometrically speaking, the Helmert parameters between the a priori TRF and the estimated TRF are forced to be close to zero. Estimating source positions and station velocities as well increases the rank deficiency to 15. Six of them are present due to the estimation of the station velocities. As this can be seen as the temporal evolution of the TRF, additional NNR/NNT rate conditions on the velocities can be introduced to eliminate this singularity. The remaining rank deficiency of three appears in turn to the estimation of the CRF, which is only of rotational nature. Thus, additional NNR conditions are applied on the source positions. Such a solution has, e.g., been performed to estimate the second realization of the International Celestial Reference Frame (ICRF2, Fey et al. 2009).

4.3 Determination of Earth Rotation Parameters

VLBI is a fundamental technique to observe the EOPs. It is the only technique that links the TRF, represented by the VLBI antennas, with the quasi-inertial CRF, represented by the radio sources. Thus, it is the only technique which is able to determine all EOPs without hypothesis. ERPs are determined from VLBI to be used in various fields of research (e.g., Ma and MacMillan 2000) and the IVS produces an official combined ERP series with daily offsets and rates (Böckmann et al. 2010).

Typically, ERPs as determined by VLBI observations are estimated as one offset and one time derivative (rate) averaged over one observing session. The precision of these estimates has reached the 0.1 mas level in PM (Schuh et al. 2003). This precision of the ERPs is at first glance depending on the geometry of the observing network. For the determination of ΔUT1, long East-West baselines are important, whereas long North-South baselines allow precise PM estimates (e.g., Nothnagel et al. 1992). This sensitivity can be

assessed by the partial derivatives as these give the relation between a change in, e.g., $\Delta UT1$ and a resulting change of the delay

$$\Delta\tau_{\Delta UT1} = \frac{\partial\tau}{\partial\Delta UT1}\Delta\Delta UT1 \tag{4.10}$$

With the partial derivatives (e.g., NOTHNAGEL 1991)

$$\frac{\partial\tau}{\partial\Delta UT1} = -\frac{1}{c}\Omega\cos\delta(b_x\sin h - b_y\cos h) \tag{4.11a}$$

$$\frac{\partial\tau}{\partial x_p} = -\frac{1}{c}(b_x\sin\delta - b_z\cos\delta\cos h) \tag{4.11b}$$

$$\frac{\partial\tau}{\partial y_p} = -\frac{1}{c}(b_y\sin\delta - b_z\cos\delta\sin h) \tag{4.11c}$$

depending on the baseline components b_x, b_y, b_z, the declination δ and the Greenwich hour angle h of the observed source. Here, Ω denotes the conversion factor from UT1 to sidereal time. Thus, long extensions of the baseline in x- and y-direction are also important for a precise determination of x-pole and y-pole, respectively. This seems to give rise to noticeable systematic variations more prominent in the y-pole component, which have been identified to belong to changes in the IVS network constellations (ARTZ et al. 2008).

In addition to the geometry of the observing network, the general quality of the VLBI observations is also relevant for the precision of the ERP estimates. Especially until 1993, clear improvements of the VLBI observing technology can be seen. The major cause for these more accurate observations can be found in the improved microwave receiving systems with cryogenic amplifiers being employed more and more. Other technological changes like the increased recording rate are not that relevant for ERPs which are estimated as an average over one 24 hour session. So, from 1993 until 2010, a timespan which covers about two thirds of the whole number of observing sessions used, the initial group delay observables can be considered as being of a rather homogeneous quality with only gradual improvements over time.

Another crucial point when estimating VLBI-derived ERPs is the datum definition. Estimating station positions and ERPs simultaneously on a session-wise basis leads to irregular ERPs caused by significant correlations between the ERPs and all other parameters (see Ch. 6.1). Thus, station and source positions are usually fixed to the a priori values when ERPs are determined. However, performing a TRF-CRF-EOP solution where the stations and sources are kept as global parameters produces the most consistent set of results (e.g., TESMER et al. 2004).

4.3.1   Estimating Time Series of Highly Resolved Earth Rotation Parameters

There are various options to estimate ERPs with a sub-daily resolution. The most common way is to parameterize the ERPs as continuous piecewise linear functions (CPWLF), i.e., linear splines (e.g., DE BOOR 1978, DAHMEN and REUSKEN 2006). Thus, estimates are derived at interval borders and a session with a duration of 24 hours leads to 25 parameters if an hourly resolution is strived for. The partial derivatives can be calculated by a second derivation step based on the partial derivatives of the delay w.r.t. a constant daily offset given in Eq. (4.11). Thus, e.g., a delay at time t contributes to two subsequent CPWLF x-pole parameters x_p^i and x_p^{i+1} by

$$\frac{\partial\tau}{\partial x_p^i} = \frac{\partial\tau}{\partial x_p}\cdot\frac{\partial x_p}{\partial x_p^i} \tag{4.12a}$$

$$\frac{\partial\tau}{\partial x_p^{i+1}} = \frac{\partial\tau}{\partial x_p}\cdot\frac{\partial x_p}{\partial x_p^{i+1}} \tag{4.12b}$$

where the derivatives of the daily w.r.t. the highly resolved parameters are given by

$$\frac{\partial x_p}{\partial x_p^i} = \begin{cases} 1 - \frac{t-t_i}{t_{i+1}-t_i}, & \text{if } t \in [t_i, t_{i+1}] \\ 0, & \text{if } t \notin [t_i, t_{i+1}] \end{cases} \tag{4.13a}$$

$$\frac{\partial x_p}{\partial x_p^{i+1}} = \begin{cases} \frac{t-t_i}{t_{i+1}-t_i}, & \text{if } t \in [t_i, t_{i+1}] \\ 0, & \text{if } t \notin [t_i, t_{i+1}] \end{cases} \tag{4.13b}$$

Highly resolved sub-daily ERPs can be determined from VLBI observations with this approach where the geometric stability of the VLBI solutions permits a minimum interval length of one hour. Compared to a single set of ERPs per 24 h session, the formal errors of course deteriorate and are even worse at the session boundaries (see Ch. 6.1). The reason is that the first and the last interval are estimated from at most half of the observations compared to the other intervals as VLBI sessions often start and end at half UTC hours.

When estimating hourly ERPs from VLBI observations, additional stabilizing constraints are often necessary especially when using older data where too few observations and small observing networks lead to a badly determined equation system. Thus, pseudo-observations have to be added to stabilize the equation system. A common approach for this stabilization is to force two subsequent parameters to be equal attributed with a small weight. Also, for hourly resolved ERPs, the recording rate is not that crucial. As described in Paper C, the amount of observations was significantly increased from CONT02 over CONT05 to CONT08 mainly due to the increased recording rate up to 1024 Mbit/sec. Nevertheless, the repeatability as a measure of the precision of the ERPs has not improved accordingly. However, it becomes apparent from Fig. 1 and Fig 4 of Paper C, that the network size is the dominating factor for the precision of the ERPs. Especially the volume of the observing networks has been significantly improved from the early 1980s until today.

Finally, one has to consider dependencies between nutation and PM, when estimating sub-daily ERPs and nutation corrections simultaneously. As presented in Ch. 3.1, a one-to-one correlation between the nutation angles and a retrograde PM term is present. THALLER et al. (2007) have studied these correlations in depth. If daily EOPs are estimated in a session that lasts for 24 hours, the de-correlation is warranted by the fact that the Earth performs an entire revolution during the observation time. For the estimation of sub-daily ERPs, TESMER et al. (2001) proposed to fix nutation and, thus, keeping the a priori nutation model. As a consequence, deficiencies of the nutation model would be present as retrograde diurnal PM variations. Another possibility to eliminate these correlations is to constrain all retrograde diurnal signals in PM to zero to estimate the entire set of EOPs including sub-daily ERPs (e.g., BROCKMANN 1997, NILSSON et al. 2010). These constraints are built following Eq. (3.20). The amplitudes of a retrograde signal in PM p^c_{-1} and p^s_{-1} with $\psi_{-1}(t) = -2\pi/T \cdot t$ are constrained to zero, where T denotes one sidereal day and t is the epoch of the PM parameter.

It should be noted that the generation of time series of highly-resolved ERPs with VLBI is somewhat ambivalent. On the one hand several sessions, especially the older ones, do not permit the estimation of hourly ERPs, thus, stabilizing constraints are necessary. However, these constraints might deform the final results. Furthermore, on average only three VLBI observing sessions with a duration of 24 hours are performed per week. Thus, the resulting time series has substantial gaps and the detection of harmonic variations has to account for these gaps, e.g, by calculating a Lomb-Scargle periodogram (e.g, LOMB 1976, SCARGLE 1982, PRESS et al. 2007). On the other hand, non-periodic changes in the Earth's rotation can only be investigated using the time series approach. In this way, SCHUH and TITOV (1999) and SCHUH and SCHMITZ-HÜBSCH (2000) detected irregular quasi-periodic variations beside the well known tidal bands. Furthermore, this approach to derive sub-daily ERPs is valuable and useful especially for the CONT sessions. The generation of highly-resolved ERP time series is also relevant for the estimation of an empirical model of tidal ERP variations as presented below as such a model might be estimated from these time series.

4.3.2 Estimating an Empirical Model for Tidal Variations of the Earth Rotation Parameters

Several possible ways exist to estimate an empirical model for tidal ERP variations from VLBI observations. The functional dependence is given by Eq. (3.18) and Eq. (3.20). This relationship can be used to replace the daily ERPs in the transformation between the CRF and the TRF. Thus, the tidal ERP coefficients can

be estimated directly from the VLBI observations. Again, this can be expressed as a second derivation step. So, the partial derivatives of the delay w.r.t the coefficients of the j$^{\text{th}}$ tidal component reads

$$\frac{\partial\tau}{\partial p_j^c} = \frac{\partial\tau}{\partial x_p}\cdot\frac{\partial x_p}{\partial p_j^c} + \frac{\partial\tau}{\partial y_p}\cdot\frac{\partial y_p}{\partial p_j^c} = \frac{\partial\tau}{\partial x_p}(-\cos\psi_j) + \frac{\partial\tau}{\partial y_p}\sin\psi_j$$

$$= \frac{1}{c}(b_x\sin\delta - b_z\cos\delta\cos h)\cos\psi_j - \frac{1}{c}(b_y\sin\delta - b_z\cos\delta\sin h)\sin\psi_j \tag{4.14a}$$

$$\frac{\partial\tau}{\partial p_j^s} = \frac{\partial\tau}{\partial x_p}\cdot\frac{\partial x_p}{\partial p_j^s} + \frac{\partial\tau}{\partial y_p}\cdot\frac{\partial y_p}{\partial p_j^s} = \frac{\partial\tau}{\partial x_p}\sin\psi_j + \frac{\partial\tau}{\partial y_p}\cos\psi_j$$

$$= -\frac{1}{c}(b_x\sin\delta - b_z\cos\delta\cos h)\sin\psi_j - \frac{1}{c}(b_y\sin\delta - b_z\cos\delta\sin h)\cos\psi_j \tag{4.14b}$$

$$\frac{\partial\tau}{\partial u_j^c} = \frac{\partial\tau}{\partial\Delta UT1}\frac{\partial\Delta UT1}{\partial u_j^c} = \frac{\partial\tau}{\partial\Delta UT1}\cos\psi_j$$

$$= -\frac{1}{c}\Omega\cos\delta(b_x\sin h - b_y\cos h)\cos\psi_j \tag{4.14c}$$

$$\frac{\partial\tau}{\partial u_j^s} = \frac{\partial\tau}{\partial\Delta UT1}\frac{\partial\Delta UT1}{\partial u_j^s} = \frac{\partial\tau}{\partial\Delta UT1}\sin\psi_j$$

$$= -\frac{1}{c}\Omega\cos\delta(b_x\sin h - b_y\cos h)\sin\psi_j \tag{4.14d}$$

The other coefficients can be derived in the same way. This approach on the observation level is the most strict one as the information of each observation is maintained.

Another option to derive the tidal ERP model is to use time series of highly resolved ERPs as pseudo observations for a second adjustment. The partial derivatives are those of the ERPs w.r.t. the model coefficients. Using GPS observations, this is the general approach. The main disadvantage of this approach at the solution level is that correlations inbetween the ERPs and between the ERPs and other parameters are neglected. The only stochastic information is given by the formal errors of the highly resolved ERPs. Furthermore, information is lost as a consequence of the intermediate generation of the highly-resolved time series.

Finally, a third option was developed and applied within this thesis. This approach is based on the transformation of NEQ systems that originally contain highly resolved ERPs. These NEQ systems are then transformed to a representation that contains the coefficients of the tidal ERP model (see Ch. 6.2). A similar approach has also been applied by NILSSON et al. (2010) to estimate the Fourier spectrum of the EOPs for CONT08.

State of the art empirical tidal ERP models comprise not only the major tidal terms. Smaller terms and sidebands of the major tidal terms are included as well. GIPSON (1996) estimated 41 tidal constituents for ΔUT1 and 56 tidal constituents for PM. Estimating these tidal terms is only possible with observations covering more than 18.6 years. If this is not fulfilled, additional information is necessary to de-correlate the coefficients. GIPSON (1996) proposed the use of so-called sideband constraints based on the tidal admittance principle. These are imposed for terms differing from each other by less than one cycle in the observational time span:

$$\frac{a_{j'}}{a_j} = \frac{V_{j'}}{V_j} \tag{4.15}$$

where a_j represents any model coefficient for the major tide j and $a_{j'}$ stands for the corresponding coefficients of its sideband. V and $V_{j'}$ are the corresponding amplitudes in the tide generating potential.

5. Short description of the included papers

In this section, the papers of this thesis are very briefly introduced following the chronological sequence of progress. All of them focus on Earth rotation variations with periods around one day and below. On the one hand, they focus on the generation of ERP time series with a temporal resolution of one hour (see Sec. 5.1 – 5.3 as well as Sec. 5.6 and 5.7). On the other hand, empirical tidal ERP models are estimated (see Sec. 5.4 – 5.7). Three of these papers are published or at least submitted as reviewed papers (one in Journal of Geophysical Research – Solid Earth and two in Journal of Geodesy), the other four papers are published as summaries of research work, presented during international VLBI working meetings (number of pages strictly limited to five or six). Paper A to Paper G present a continuous research progress starting from the time series approach, which was applied to the CONT sessions, over the development and application of the transformation of NEQ systems to determine an empirical model for tidal ERP variations from all available VLBI observations culminating in the combination of VLBI and GPS observations to derive sub-daily ERPs.

In the following, my own and the (co-)authors' contributions to the papers are briefly described.
Axel Nothnagel is the supervisor of this thesis and, consequently, of the whole work contributing to prepare the papers. Furthermore, he improved the text of the papers by organizational, linguistic and grammatical corrections.
Sarah Tesmer née Böckmann contributed to all papers through fundamental discussions about the scientific contents and its presentation. In addition she provided a preliminary review of the manuscripts and, thus, helped to improve the text.
Peter Steigenberger provided the GPS data sets. These were used for comparisons in Paper C and as input to the combination efforts in Paper F and Paper G. Furthermore, he performed the estimation of the empirical tidal ERP model on the solution level in Paper D.
Laura Jensen implemented the initial steps for the estimation of an empirical ERP model on the basis of the transformation of NEQ systems. This was done in a course within the masters studies at IGG and was supervised by myself. Some of the solutions presented in Paper D were performed following this work.
Lisa Bernhard performed the estimation of an empirical ERP model from GPS observations in her masters thesis, which was supervised by myself. This was one of the preliminary works for the estimation of a combined tidal ERP model. Furthermore, she prepared some of the comparisons presented in Paper F.
My own contribution to all papers consists of the development of the analysis strategy and theoretical considerations. I performed all initial VLBI analysis with the software package Calc/Solve (Ma et al. 1990; Baver 2010). Furthermore, I modified Calc/Solve to export the completely unconstrained observation equations and implemented almost all additional analysis tools in C++ and Perl. Beside the estimation of the empirical ERP model on the solution level in Paper D, I did all the computations, elaborated the presentation of the results, and wrote the text.

5.1   Main Points of Paper A

Artz, T., S. Böckmann, A. Nothnagel, V. Tesmer (2007) *ERP time series with daily and sub-daily resolution determined from CONT05.* In: Boehm, J., A. Pany, H. Schuh (eds) Proceedings of the 18th European VLBI for Geodesy and Astrometry Working Meeting, 12 – 13 April 2007, Vienna, Geowissenschaftliche Mitteilungen, Heft Nr. 79, Schriftenreihe der Studienrichtung Vermessung und Geoinformation, Technische Universität Wien, ISSN 1811-8380, pp 69 – 74, (available electronically at `http://mars.hg.tuwien.ac.at/~evga/proceedings/S31_Artz.pdf`).

This paper provides initial investigations concerning the simultaneous estimation of ERPs and station positions. Analysis of the correlation matrix of estimated parameters is performed to provide an impression of the dependencies between them. It is documented that VLBI-derived ERPs are not reliable if they are estimated

simultaneously with station positions on a session-wise basis. However, estimating the station positions as an average for a fortnightly time span is sufficient to de-correlate the parameters. In addition, a stacking approach for the hourly resolved ERPs is developed. Thereby, the individual sessions of the continuous campaign are concatenated, deficiencies at the session borders are minimized and a consistent time series over the whole CONT05 period is produced.

5.2   Main Points of Paper B

ARTZ, T., S. BÖCKMANN, A. NOTHNAGEL (2009) *CONT08 – First Results and High Frequency Earth Rotation.* In: Bourda, G., P. Charlot, A. Collioud (eds) Proceedings of the 19th European VLBI for Geodesy and Astrometry Working Meeting, 24 – 25 March 2009, Bordeaux, Université Bordeaux 1 - CNRS - Laboratoire d'Astrophysique de Bordeaux, pp 111 – 115, (available electronically at `http://www.u-bordeaux1.fr/vlbi2009/proceedgs/26_Artz.pdf`).

Here, first results from the most recent continuous VLBI campaign (CONT08) are presented. The analysis strategy is based on Paper A. However, it is shown that the stacking of the ERPs is not that important compared to prior CONT sessions, as the observing schedule of CONT08 was changed based on the experiences gained from CONT05. As a basis for further analysis, a general quality assessment of CONT08 is performed investigating station position variations and daily ERPs. In addition, high-resolution Earth rotation time series are generated in a way that ensures consistency over the whole time span. Based on this time series, the enhancement of the new scheduling is demonstrated.

5.3   Main Points of Paper C

ARTZ, T., S. BÖCKMANN, S., A. NOTHNAGEL, P. STEIGENBERGER (2010a) *Subdiurnal variations in the Earth's rotation from continuous Very Long Baseline Interferometry campaigns.* J Geophys Res, 115, B05404, DOI 10.1029/2009JB006834.

This paper presents the state-of-the-art analysis of the CONT sessions. The analysis is based on the two papers mentioned above. Sub-diurnal periodic effects in the Earth's rotation as seen by VLBI are investigated on the basis of the campaigns that took place in 2002, 2005 and 2008. In this publication, an analysis which is based on hourly estimates of PM and ΔUT1 and a subsequent spectral analysis is presented. The noticeable facts are discussed in comparison with theoretical predictions and GPS-derived results. The intermittent detection of periodicities as reported by HAAS and WÜNSCH (2006) applies in this analysis for CONT02 as well. For CONT05 and CONT08 minute signals at periods of around 6 hours are contained in the amplitude spectra. The origin of these signals is still questionable.

5.4   Main Points of Paper D

ARTZ, T. S. BÖCKMANN, L. JENSEN, A. NOTHNAGEL, P. STEIGENBERGER (2010b) *Reliability and Stability of VLBI-derived Sub-daily EOP Models.* In: Behrend, D., K.D. Baver (eds) IVS 2010 General Meeting Proceedings "VLBI2010: From Vision to Reality", 7 – 13 February 2010, Hobart, NASA/CP-2010-215864, pp 355 – 359, (available electronically at `http://ivscc.gsfc.nasa.gov/publications/gm2010/artz.pdf`)

As shown in Paper C, the IERS model for tidal ERP variations with periods around one day and below does not describe all effects that are measured by VLBI. The main reason is that VLBI measures the integral excitation at any tidal line. Thus, it is reasonable to estimate an empirical model for these variations from the VLBI observations. The aim of this paper is to evaluate the quality and reliability of such a model. Therefore, the impact of the estimation method and the analysis options as well as the temporal stability of empirical

ERP models are investigated. Concerning the estimation method, this paper provides the first documentation of an approach that is based on the transformation of NEQ systems. It is shown that the formal errors should be inflated by a factor of three to provide a realistic accuracy measure of the model coefficients. This coincides with the noise floor which is derived by terms that have no amplitude in the tide generating potential. Furthermore, the repeatability of the model coefficients is derived to confirm these results. The impact of various analysis options is analyzed. It is small but significant when changing troposphere parameterization or including harmonic station position variations. Furthermore, different estimation techniques are compared. The approach on the observation level and on the NEQ level do reveal almost no differences within the derived accuracy level. The differences of the solution level approach are only slightly bigger.

5.5 Main Points of Paper E

Artz, T., S. Böckmann, S., A. Nothnagel (2010a) *Assessment of Periodic Sub-diurnal Earth Rotation Variations at Tidal Frequencies Through Transformation of VLBI Normal Equation Systems.* J Geod, 85(9):565 – 584, DOI 10.1007/s00190-011-0457-z. 2011.

Based on Paper D, an empirical model for periodic variations of diurnal and sub-diurnal ERPs is presented that is derived based on the transformation of NEQ systems of VLBI observing sessions. NEQ systems that contain highly-resolved PM and ΔUT1 with a temporal resolution of 15 minutes are generated and then transformed to the coefficients of the tidal ERP model to be solved for. To investigate the quality of this model, comparisons with empirical models from GPS, another VLBI model and the model adopted by the IERS Conventions are performed. The absolute coefficients of these models agree almost completely within 7.5 μas in PM and 0.5 μs in ΔUT1. Several bigger differences exist, which are discussed in this paper. To be able to compare the model estimates with results of the continuous VLBI campaigns, where signals with periods of eight and six hours were detected, terms in the ter- and quarter-diurnal band are included into the tidal ERP model. However, almost no common features with the results of continuous VLBI campaigns or ERP predictions in these tidal bands can be seen.

5.6 Main Points of Paper F

Artz, T., L. Bernhard, A. Nothnagel, P. Steigenberger, S. Tesmer (2011b) *Methodology for the Combination of Sub-daily Earth Rotation from GPS and VLBI Observations* J Geod, DOI 10.1007/s00190-011-0512-9, online first.

A combination procedure of EOPs from GPS and VLBI observations is developed on the basis of homogeneous NEQ systems. The emphasis and purpose of the combination is the determination of sub-daily PM and ΔUT1. Time series with an hourly resolution and a model for tidal variations of PM and ΔUT1 are estimated. As in both cases 14-day nutation corrections are estimated simultaneously, the derived EOPs represent the complete transformation between the CRF and the TRF. Due to the combination procedure, it is warranted that the strengths of both techniques are preserved. At the same time, only a minimum of de-correlating or stabilizing constraints are necessary. Hereby, a PM time series is determined, whose precision is mainly dominated by GPS observations. However, this setup benefits from the fact that VLBI delivers nutation and ΔUT1 estimates at the same time. An even bigger enhancement can be seen for the ΔUT1 estimation where the high-frequency variations are provided by GPS, while the long term trend is defined by VLBI. The combined tidal PM and ΔUT1 model, which is estimated, is predominantly determined from the GPS observations. Overall, the combined tidal model for the first time completely comprises the geometrical benefits of VLBI and GPS observations.

5.7   Main Points of Paper G

ARTZ, T., A. NOTHNAGEL, P. STEIGENBERGER, S. TESMER (2011) *Evaluation of Combined Sub-daily UT1 Estimates from GPS and VLBI Observations.* In: Alef W., Bernhart S., Nothnagel A. (eds.) Proceedings of the 20th Meeting of the European VLBI Group for Geodesy and Astrometry, Schriftenreihe des Instituts für Geodäsie und Geoinformation der Rheinischen-Friedrich-Wilhelms Universität Bonn, No. 22, ISSN 1864-1113, pp 97 – 101

The combination procedure, which was presented in Paper E was applied to determine a time series of combined hourly resolved ΔUT1 time series from VLBI and GPS observations. Based on these time series, it is demonstrated that the combination sustains the strengths of both techniques. Furthermore, the impact of different VLBI session types is evaluated by ΔUT1 LOD comparisons. On the one hand, ΔUT1 is compared to the IERS 05C04 series and, on the other hand, LOD is compared to time series based on geophysical fluids (OAM and AAM). The noise of the combined ΔUT1 results decreases the more VLBI sessions are used, in contrary, the LOD consistency with geophysical fluids slightly decreases. Finally, an empirical model for sub-daily variations of the ERPs was determined based on transformations of the given NEQ systems. Using this model for the estimation of the time series clearly reduces the remaining sub-daily UT1 variations.

6. Summary of the most important results

6.1 Analysis of Continuous VLBI Campaigns

VLBI observations that last for 24 hours are performed by the observing networks of the IVS on average on three non-consecutive days every week. For organizational reasons, the networks vary from session to session. Starting in the year 1994, several continuous VLBI campaigns of about two weeks each have been scheduled in irregular intervals with almost identical networks over the period of the campaigns. Currently the CONT sessions can not be maintained for a longer period than two weeks or more often than in the past, since they place a heavy observing load on the participating radio telescopes. However, this will change with the concept of VLBI2010 (NIELL et al. 2005) becoming reality. Thus, a comprehensive analysis of the CONT sessions is a hint for future VLBI analyses, although VLBI2010 will become more extensive and comprehensive (e.g., WRESNIK et al. 2009).

In part, the investigations within this thesis focus on CONT02, CONT05 and CONT08. A detailed description of these three campaigns is given in Paper C (ARTZ et al. 2010). Due to the standard analysis procedure in the Mark IV analysis chain, these fortnightly time spans are divided into 15 independent datasets for a duration of 24 hours each. Therefore, parameters that are estimated with a sub-daily resolution occur twice at the same epoch, i.e., at the end of one and the beginning of the successive session, with results often differing by more than 1 mas in PM.

Based on the highly precise data set of CONT05, investigations concerning the datum are performed with this series as a test case (paper A, ARTZ et al. 2007). The correlations of daily ERPs to all other parameters for one single session are displayed in Fig. 6.1 for three different solution approaches: (1) the datum is defined by NNR/NNT conditions w.r.t. the a priori positions only for this session, i.e., station positions are estimated session-wise (dark gray); (2) NNR/NNT conditions are applied for the entire CONT05 time span, thus, station positions are estimated as an average over the whole campaign (black); (3) station positions are fixed to a priori values (light gray). Obviously, it is not sufficient to apply NNR/NNT conditions session-wise as this leads to significant correlations of the ERPs with the station positions and also with the ZWDs at some stations (e.g., HR: Hartebeesthoek, South Africa). Furthermore, higher correlations between all other parameters are present[1]. The correlations appearing in the individual NNR/NNT solution (dark gray) are reduced either by fixing sites (light gray) or by calculating the complete solution (black). Stacking the NEQ over two weeks and estimating site positions once for the mid epoch is sufficient to de-correlate the equation system. However, when solving individual sessions to estimate the ERPs, station positions should be fixed. These results are valid for the determination of sub-daily ERPs as well[2]. It should be mentioned that for standard sessions with smaller observing networks the correlations are even more distinct if only NNR/NNT conditions are applied.

After performing an extensive quality assessment initially presented in Paper B (ARTZ et al. 2009) for CONT08 and enlarged in Paper C, highly consistent time series are generated for all three campaigns. The NEQ systems of each individual 24 hour session are added to one big equation system for the whole campaign where observations of neighboring sessions contribute to parameters at session borders simultaneously. The NEQ systems of adjacent sessions are set up to be linked by stacking the respective equation elements. First of all, the NEQ systems of the classical least-squares adjustment given by Eq. (4.6) are built up for each single session

$$\mathbf{N}_i = \mathbf{A}_i^T \cdot \mathbf{\Sigma}_i^{-1} \cdot \mathbf{A}_i \qquad (6.1a)$$

$$\mathbf{n}_i = \mathbf{A}_i^T \cdot \mathbf{\Sigma}_i^{-1} \cdot (\mathbf{y}_i - \mathbf{y_{i,0}}) \qquad (6.1b)$$

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[1] see Fig. $2-4$ of paper A for more details
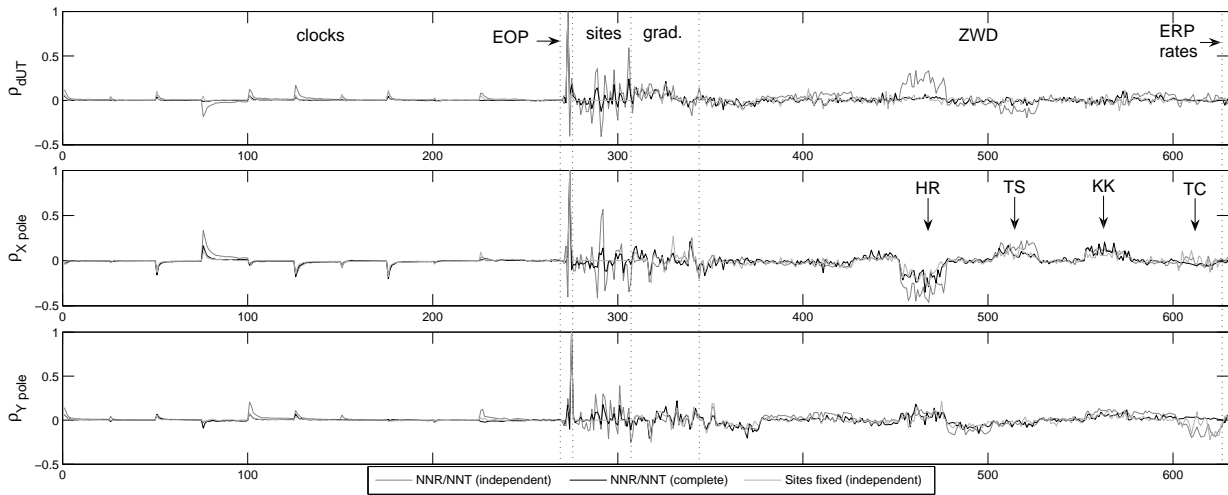[2] see also Fig. 6 of paper A

Figure 6.1: Coefficients of correlation of daily ERPs with all other parameters for three different solutions. (Artz et al.2007)

In order to establish the continuity of the campaign and to stabilize parameters at the session borders, the individual NEQs are added to one single NEQ for the complete campaign by adding elements of parameters of the same type and the same epoch. Assuming that two parameters in session one and two $\Delta x_n^{(1)}$ and $\Delta x_1^{(2)}$ are equal, the modification from completely independent to the stacked NEQ reads as follows:

$$\mathbf{N}: \begin{pmatrix} N_{11}^{(1)} & \cdots & N_{1n}^{(1)} & & & \\ \vdots & \ddots & \vdots & & \mathbf{0} & \\ N_{n1}^{(1)} & \cdots & N_{nn}^{(1)} & & & \\ & & & N_{11}^{(2)} & \cdots & N_{1n}^{(2)} \\ & \mathbf{0} & & \vdots & \ddots & \vdots \\ & & & N_{n1}^{(2)} & \cdots & N_{nn}^{(2)} \end{pmatrix} \rightarrow \begin{pmatrix} N_{11}^{(1)} & \cdots & N_{1n}^{(1)} & & \mathbf{0} & \\ \vdots & \ddots & \vdots & & & \\ N_{n1}^{(1)} & \cdots & N_{nn}^{(1)}+N_{11}^{(2)} & N_{12}^{(2)} & \cdots & N_{1n}^{(2)} \\ & & N_{21}^{(2)} & N_{22}^{(2)} & \cdots & N_{1n}^{(2)} \\ & \mathbf{0} & \vdots & \vdots & \ddots & \vdots \\ & & N_{n2}^{(2)} & N_{n2}^{(2)} & \cdots & N_{nn}^{(2)} \end{pmatrix} \quad (6.2)$$

Through this procedure, the parameters at the session borders are stabilized and estimated only once. These parameters are the ERPs, ZWDs and troposphere gradients. Furthermore, station positions are transformed to the mid epoch of the campaign and stacked as well. In contrast, clock parameters are treated as session parameters and have, thus, been pre-reduced from the individual NEQ systems. Reduction of parameters from the NEQ systems means to reduce the number of parameters without changing the solution. The reduced parameters are estimated only implicitly, as this procedure maintains the functional model of the adjustment by transferring the properties of parameters to be reduced to the remaining ones (see e.g., Vennebusch et al. 2007).

The modified solution approach of stacking the NEQs sustains the character of continuous campaigns also within the analysis process. Two advantages can be drawn from this method. First, the correlations between EOPs and other parameters are minimized without fixing stations to their a priori values as described above. Second, and even more important, periods without observations can be covered more satisfactorily. Thus, weak estimates of parameters with a sub-daily resolution that are present in the conventional session-wise analysis scheme are eliminated if the modified analysis scheme is applied. Figure 6.2 exemplarily shows the improvement for the CONT05 x-pole component through the stacking method. A time series generated in a standard session-wise way without any modifications is displayed by a gray line. It exhibits significant outliers at the session boundaries. For the time series derived with the modified solution approach (black line), the outliers at the session borders are removed. This effect can be seen for the sub-daily estimated ERPs and for hourly ZWD estimates as well. Thus, the time series of highly resolved ERPs are consistent over the
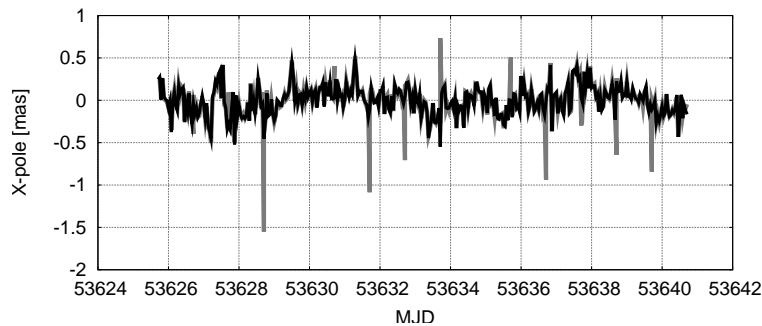
Figure 6.2: X-pole component derived from CONT05 with (black) and without (gray) stacked NEQ systems, reduced by IERS 05 C04 ERPs and a priori subdaily ERP model of McCarthy and Petit2004. (Artz et al.2010b)

whole time spans of the continuous VLBI campaigns and no artifacts will influence subsequent analyses. The ERP time series are validated by a comparison with sub-daily GPS-derived ERPs and interpolated IERS 05 C04 ERPs that has been performed in Paper B and Paper C. This external validation now yields an agreement of around 200 μas in terms of weighted root mean squared (WRMS) of the PM differences. As it can be assumed that GPS determines PM with a higher precision (e.g., Steigenberger et al. 2006), it can be concluded that the configuration of the VLBI observing network as it was employed in CONT05 and CONT08 provides a sufficient geometrical stability to derive sub-daily ERPs.

Furthermore, these highly consistent and entirely continuous ERP time series are analyzed in the frequency domain to investigate harmonic or irregular quasi-harmonic variations. Haas and Wünsch (2006) and Nastula et al. (2007) reported significant ter-diurnal retrograde PM variations during CONT02. These were not detected in CONT05 (Haas 2006). However, it is not clear whether these variations are real geophysical phenomena or due to deficiencies in the analysis chain. With the optimized analysis applied within this thesis, the presence of the retrograde 8 h term was confirmed. Thus, the processing of individual sessions is not responsible for these variations. As all three campaigns are analyzed in a completely identical way, it can also be stated that this retrograde signal is not present in CONT05 and CONT08. In Fig. 6.3, the amplitude spectra for all three campaigns are shown for PM exemplarily.[3] To derive these spectra, the ERPs are detrended by continuous piece-wise polynomials to account for long term ERP variations. In the next step, amplitudes of all periodicities contained in each data set were estimated from the derived time series in least squares adjustments. This is done for x-pole, y-pole and ΔUT1 independently with the observation equations

$$x_p = \sum_{k=1}^{k_{max}} S_{k,xp} \cdot \sin(\Omega_k t) + C_{k,xp} \cdot \cos(\Omega_k t) \tag{6.3a}$$

$$y_p = \sum_{k=1}^{k_{max}} S_{k,yp} \cdot \sin(\Omega_k t) + C_{k,yp} \cdot \cos(\Omega_k t) \tag{6.3b}$$

$$\Delta UT = \sum_{k=1}^{k_{max}} S_{k,ut} \cdot \sin(\Omega_k t) + C_{k,ut} \cdot \cos(\Omega_k t) \tag{6.3c}$$

for arbitrary frequencies Ω_k. Here, 140 equidistant frequencies have been chosen between the lowest possible frequency and the Nyqvist frequency $\Omega_{max} = 1/(2\Delta t)$ The lowest frequency corresponds to a signal whose period contains the whole campaign $\Omega_{min} = 1/(15d)$. Since the EOP estimates have a temporal resolution Δt of one hour, Ω_{max} is $1/(2\,h)$. From Eq. (6.3) the sine and cosine coefficients (S_k, C_k) are estimated. Then, the amplitude of a signal corresponding to Ω_k can be calculated from these two coefficients

$$A_k = \sqrt{S_k^2 + C_k^2} \quad . \tag{6.4}$$

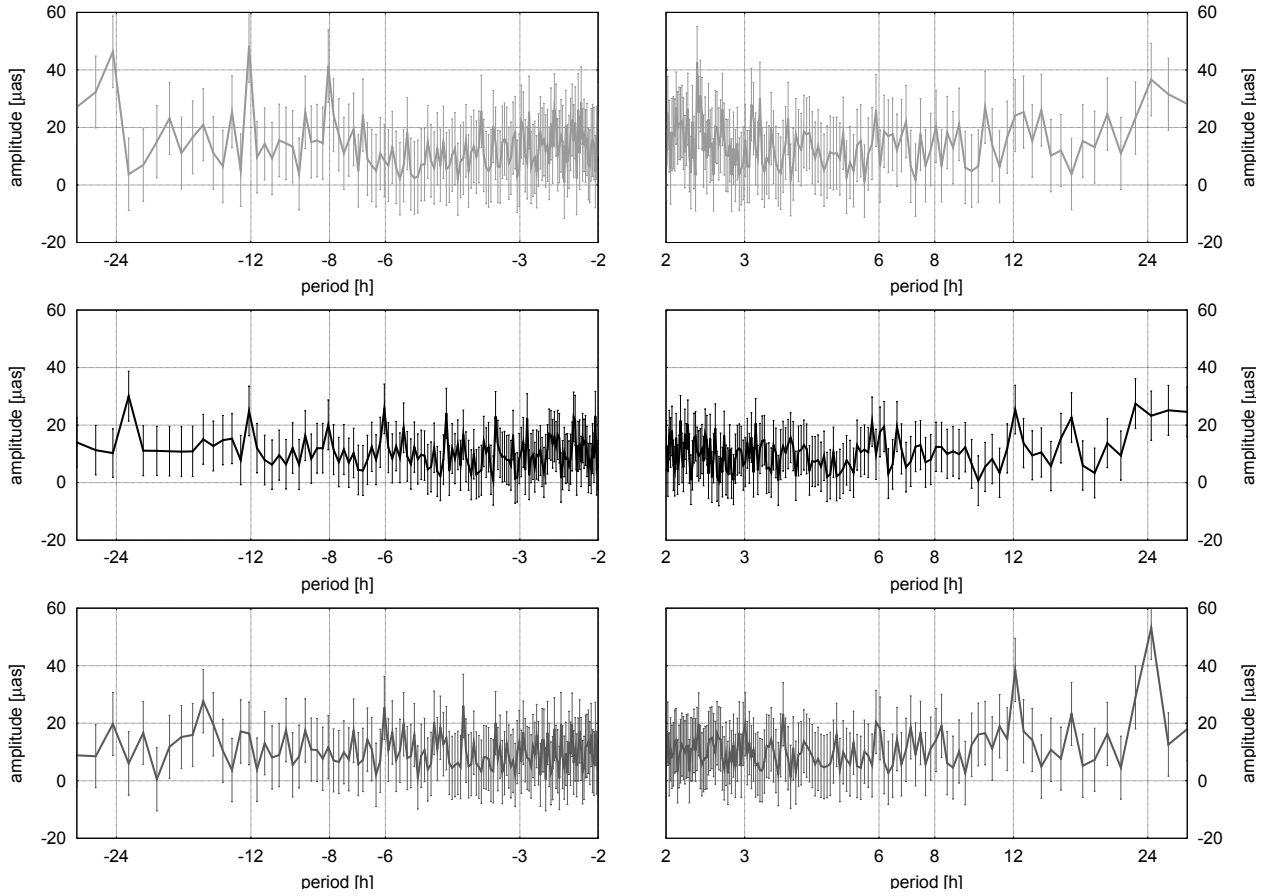[3]The ΔUT1 spectra are given in Fig. 10 of Paper C.

Figure 6.3: Least squares estimated retrograde (left) and prograde (right) amplitude spectra of residual PM after applying the IERS2003 sub-daily model ERP (Tab. 5.1 and 8.2 of McCarthy and Petit 2004) and a detrending function to reduce the long term variations (top: CONT02, middle: CONT05, bottom: CONT08).(Artz et al.2010b)

Finally, for the polar motion variations, the pro- and retrograde amplitudes are evaluated from S_k and C_k

$$A_{k,ret} = \frac{1}{2} \cdot \sqrt{(C_{k,xp} + S_{k,yp})^2 + (S_{k,xp} - C_{k,yp})^2} \tag{6.5a}$$

$$A_{k,pro} = \frac{1}{2} \cdot \sqrt{(C_{k,xp} - S_{k,yp})^2 + (S_{k,xp} + C_{k,yp})^2} \tag{6.5b}$$

The amplitudes of several oscillations can be identified as being significant and it is assured that there are no significant correlations between the derived amplitudes. The residual diurnal and semi-diurnal amplitudes could be explained to some extent by geophysical excitations caused by the tri-axial shape of the Earth or by atmospheric and non-tidal oceanic variations. A broad discussion of significant amplitudes and possible causes is given in Sec. 5.2 of Paper C. Here, only sub-daily ERP signals that are not related to gravitationally forced diurnal and semi-diurnal ocean tides are listed in Tab. 6.1 for completeness. Variations not explainable at this stage may also be attributable to errors in the general processing scheme of the geodetic observations and errors in the applied a priori ocean tidal model. Beside the diurnal and semi-diurnal terms, variations with periods of eight hours can be seen in the retrograde CONT02 PM spectrum with an amplitude of 42 μas. Furthermore, in CONT05, a retrograde 8 h term also appears, but with an amplitude of just 22 μas, while in CONT08 it is not visible at all. Peaks at 6 h are discernible in the pro- and retrograde PM spectrum of CONT08. The retrograde amplitude of 26 μas is confirming the peak in the spectrum of CONT05 (30 μas). In addition, in the prograde band, the 6 h term is present as well with amplitudes of 20 μas in CONT08 and

Table 6.1: Sub-daily ERP signals that are not related to gravitationally forced diurnal and semi-diurnal ocean tides. (ARTZ et al.2010B)

Period [h]	Amplitude PM [μas]	Amplitude UT1 [μs]	Comment	Reference
8.0000	0.46(p)/0.57(r)	0.48	S_3 atmospheric tide	DE VIRON et al. (2005)
8.28	0.43(p)/0.30(r)	0.57	M_3 from hydrodynamical model	HAAS and WÜNSCH (2006)
11.9672	-	0.2	K_2 libration	CHAO et al. (1991)
12.0000	2.9	0.5	S_2 atmospheric tide	BRZEZIŃSKI et al. (2002)
	-	0.9	S_2 libration	CHAO et al. (1991)
12.4206	-	1.9	M_2 libration	CHAO et al. (1991)
12.6584	-	0.3	N_2 libration	CHAO et al. (1991)
14.6	3	-	ξ_2^1 atmospheric normal mode	BRZEZIŃSKI et al. (2002)
23.0985	0.8	-	J_1 libration	CHAO et al. (1991)
23.8693	0.7	-	ϕ_1 atmospheric tide	BRZEZIŃSKI et al. (2002)
23.9345	1.6	-	K_1 atmospheric tide	BRZEZIŃSKI et al. (2002)
	0.8	-	K_1 atmospheric tide	BRZEZIŃSKI et al. (2004)
	15	-	K_1 libration	CHAO et al. (1991)
24.0000	7.1	0.5	S_1 atmospheric tide	BRZEZIŃSKI et al. (2002)
	7.8	-	S_1 atmospheric tide	BRZEZIŃSKI et al. (2004)
	7.8	-	non-tidal AAM	DE VIRON et al. (2002)
	17	-	non-tidal OAM	DE VIRON et al. (2002)
	24.7	0.6	non-tidal angular momentum	HAAS and WÜNSCH (2006)
24.0659	1.3	-	P_1 atmospheric tide	BRZEZIŃSKI et al. (2002)
	1.2	-	P_1 atmospheric tide	BRZEZIŃSKI et al. (2004)
	4.7	-	P_1 libration	CHAO et al. (1991)
24.8333	0.8	-	M_1 libration	CHAO et al. (1991)
25.8193	10	-	O_1 libration	CHAO et al. (1991)
26.8684	0.8	-	Q_1 libration	CHAO et al. (1991)
28.8	31	-	ϕ_1^1 atmospheric normal mode	BRZEZIŃSKI et al. (2002)

around 20 μas in CONT05 for periods of 6.1 and 6.6 h. Finally, there is a prograde signal with an amplitude of 29 μas at a period of 6.8 h in the CONT02 spectrum. For ΔUT1, no significant amplitudes are present for these periods.

The analysis of comparable GPS ERP estimates brought up the same period of 6 h[4]. This might be considered as a confirmation of the real existence of a harmonic at such a period but, due to the existence of spurious signals at $24/n$ h, with n being integer values, caution is still advisable.

6.2   An Empirical Tidal Model for Variations of the Earth Orientation Parameters from VLBI Observations

As presented by the time series approach applied to the CONT sessions, the official IERS sub-daily ERP model is not valid for all harmonic variations measured by VLBI. The main portion of the tidally driven ERP variations is caused by the ocean tides and, thus, explained by the IERS2010 models. Within this thesis, the sub-diurnal ocean tidal impact on the ERPs is denoted with IERS2010–A (Tab. 8.2a, 8.2b, 8.3a and 8.3b of PETIT and LUZUM 2010), whereas, the model called IERS2010–B (Tab. 5.1a, 5.1b, 8.2a, 8.2b, 8.3a and 8.3b

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[4]In Sec. 5.23 of paper C the results of the spectral analysis of comparable GPS time series is given.

of PETIT and LUZUM 2010) also contains the effect of external torques acting on the non-axis-symmetric Earth. However, model errors or inconsistencies might cause differences at the diurnal and semi-diurnal bands. Furthermore, the space geodetic techniques are sensitive to the integral excitation at any tidal wave leading to non-tidal oceanic or atmospheric excitations that can hardly be explained by present predictions. Thus, the availability of more consistent models is necessary to detect other effects as, e.g., episodic ERP variations. These models might be of empirical nature to ensure the highest consistency within the analysis. However, using VLBI-derived empirical models for a subsequent VLBI analysis, may always imply the risk of unintentionally absorbing effects of other geophysical nature.

Within this thesis, a model is determined that differs from the approaches presented in the past where, e.g., GIPSON (1996), GIPSON and RAY (2009) estimated a model on the observation level and ENGLICH et al. (2008) used the solution level approach. The details of estimating such empirical tidal ERP models are explained in Ch. 4.3.2. Here, the transformation of NEQ systems is developed and applied primarily. However, the other approaches have also been used to investigate the impact of the different estimation techniques.

When applying the approach on the NEQ level, a matrix consisting of highly resolved ERPs is transformed to a representation with the coefficients of the tidal ERP model. For this, the poly-harmonic representation given in Eq. (3.18) and (3.20) is expanded to

$$\Delta x_p(t) = a_x + b_x \cdot \Delta t + \sum_{j=1}^{n} -p_j^c \cos \psi_j(t) + p_j^s \sin \psi_j(t) \tag{6.6a}$$

$$\Delta y_p(t) = a_y + b_y \cdot \Delta t + \sum_{j=1}^{n} p_j^c \sin \psi_j(t) + p_j^s \cos \psi_j(t) \tag{6.6b}$$

$$\Delta UT1(t) = a_u + b_u \cdot \Delta t + \sum_{j=1}^{n} u_j^c \cos \psi_j(t) + u_j^s \sin \psi_j(t). \tag{6.6c}$$

where the coefficients a and b can be set up either to represent daily ERPs or to account only for a linear ERP correction over the entire solution. More details on the handling of these coefficients is given in Paper E (ARTZ et al. 2011A). The model coefficients p_j^c, p_j^s, u_j^c and u_j^s as well as the linear ERP corrections a and b are estimated by transforming the NEQ elements for highly-resolved ERPs to those of the model coefficients according to Eq. (6.6). Let $\mathbf{\Delta x}$ denote the vector of the original parameters (i.e., the ERPs at sub-daily epochs, e.g., every 15 minutes) and $\mathbf{\Delta y}$ the vector of the new parameters (i.e., the coefficients of Eq. (6.6)), which will replace the original ones. Assuming a linear or linearized functional dependence

$$\mathbf{\Delta x} = \mathbf{B} \cdot \mathbf{\Delta y} \tag{6.7}$$

the new NEQ system can be derived by (e.g., BROCKMANN 1997)

$$\widetilde{\mathbf{N}} = \mathbf{B}^T \cdot \mathbf{N} \cdot \mathbf{B}, \qquad \widetilde{\mathbf{n}} = \mathbf{B}^T \cdot \mathbf{n} \tag{6.8}$$

where the matrix $\mathbf{B}$ consists of the partial derivatives of the ERPs w.r.t. the new parameters. This partial derivatives are given in Eq. (4.14). For all parameters that should remain unchanged, an identity matrix is inserted.

To investigate the reliability and stability of VLBI-derived models for tidal ERP variations, several comparisons have been presented in Paper D (ARTZ et al. 2010) and Paper E. Windowed solutions of limited time spans covering 13 years of VLBI observations have been performed to analyze the stability of the model coefficients. In this way, it has been pointed out that the formal errors should be increased by a factor of three to represent a reliable accuracy measure.[5] This is sustained by the analysis of terms that have no amplitude in the tide generating potential. These have been added to the tidal ERP model according to GIPSON (1996). For these small terms, no amplitude in the tidal ERP model is expected and, therefore, their estimates can be used to assess the noise level of the models. The comparison of the small terms with

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[5]see Sec. 3 of Paper D for a detailed description of finding a stable solution and the corresponding results.
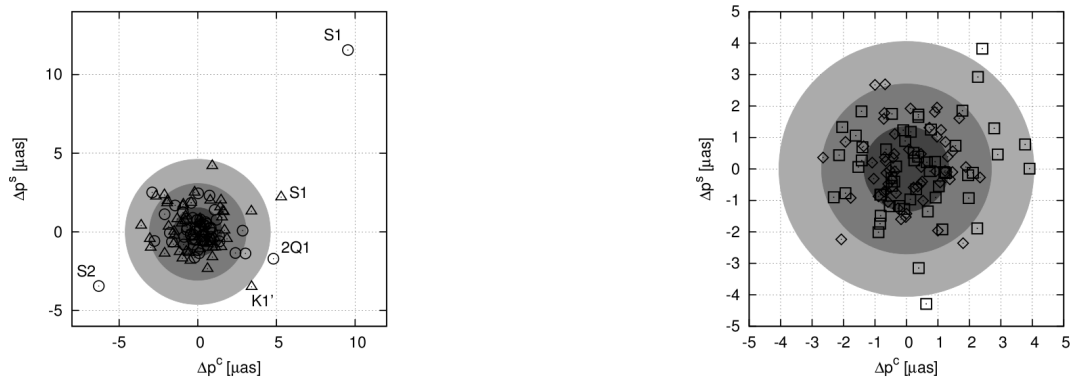
Figure 6.4: Phasor diagrams with differences of model coefficients. On the left: differences of the PM model terms of a standard solution to a solution where harmonic site positions (circles) or a different parameterization of the troposphere (triangles) is used. On the right: PM model differences of the observation level approach to the NEQ (diamonds) and solution (squares) level approaches. The circles show one (dark gray) to three (light gray) times the standard deviation of the differences of the estimated model coefficients. (ARTZ et al.2010a)

the formal errors of the model coefficients also exhibits that the formal errors are too optimistic by a factor of three[6].

This analysis presented in paper D also includes a comparison of the three different approaches to determine the model coefficients with an exactly identical set-up and selection of sessions. The approach on the observation level is the most strict one, as the information of each observation is preserved. For the two other approaches, a discretization to hourly spaced ERPs is done first, thus information is lost. The advantage of the NEQ procedure above the solution level approach is the conservation of the full variance and covariance information. The differences are shown in the right part of Fig. 6.4 in the form of a phasor plot, where the cosine-components are plotted against the sine-components. These differences are, in general, below the level of the threefold standard deviations, i.e., approximately 4 μas, and thus, in the range of the model precision.

Furthermore, the assumptions concerning the strictness of the estimation procedure are fulfilled as the model on the observation level is slightly closer to the one on the NEQ level compared to the solution level. Furthermore, the impact of several analysis options has been tested. The major impact can be seen when changing the temporal resolution of the ZWDs from 20 to 60 min and when introducing harmonic station variations (PETROV and MA 2003). Obviously, station motions are able to absorb ERP variations and vice versa. These effects are depicted on the left of Fig. 6.4.[7]

Based on these investigations, a final version of the VLBI-only tidal ERP model is estimated and presented in Paper E. No sideband constraints have been imposed as the considered time span is significantly longer than 18.6 years. To reach the highest possible consistency within the solution, station and source positions, nutation offsets as well as daily PM and ΔUT1 offsets and rates have been estimated. Furthermore, only terms that have been considered to be significantly estimated are set-up. These are terms with estimates bigger than the threefold formal errors in an initial solution. This model generally agrees to other models on a level of 7.5 μas in PM and 0.5 μs in ΔUT1. The models used for comparison are another VLBI model calculated at the Goddard Space Flight Center (GSFC) and a GPS model calculated at the Technische Universität München (TUM). The GSFC model is an updated version of GIPSON and RAY (2009) (J. GIPSON, personal communication, 2010) and the TUM model is an updated version of STEIGENBERGER (2009) (P. STEIGENBERGER, personal communication, 2010). The thresholds mentioned above are exceeded by 5% of the coefficient differences between the IGG model and the two reference models.[8] Especially large differences are present, e.g., at the diurnal S1 and the semi-diurnal S2 term in PM. These differences can be

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[6]see Sec. 4.2 of paper E

[7]In addition, the impact of sideband constraints and the EOP handling are presented in Sec. 4.1 of Paper E.

[8]Larger deviations for some individual terms are present. For a detailed discussion of these terms see Sec. 4.3.1 of Paper E.
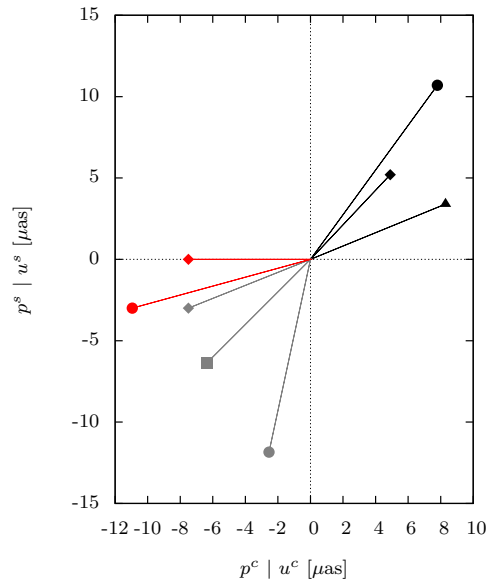
Figure 6.5: Phasor diagrams for various predictions of the PM S1-tide (black), ΔUT1 S1-tide (gray) and ΔUT1 S2-tide (red). The circles show the IGG - IERS2010-A differences, the diamonds the results of BRZEZIŃSKI et al. (2002), the triangle the result of BRZEZIŃSKI et al. (2004) and the square the result of HAAS and WÜNSCH (2006). (ARTZ et al.2011a)

attributed to the different handling of the long term PM variations. Furthermore, diurnal and semi-diurnal station motions are applied for the GSFC solution while they are not applied for the solution determined within this thesis. As evaluated in paper D, these harmonic station position variations are supposed to force changes of the S1 and S2 terms within the tidal ERP models. This suggestion is supported by the work of SCHERNECK and HAAS (1999) who investigated the impact of tidally driven station position changes and their interaction with derived ERP variations.

In comparison to the IERS models for tidal ERP variations, the combined model shows significant differences at S1 for PM. These have been expected because for S1 strong atmospherically forced variations were predicted by other authors. These predictions are rather diverse, however, the amplitudes of the IGG model and those reported by BRZEZIŃSKI et al. (2004) differ by only 4 μas and the phase shift is 30°. The differences to the IERS2010–A model are displayed in Fig. 6.5 together with predictions of BRZEZIŃSKI et al. (2002), BRZEZIŃSKI et al. (2004) and HAAS and WÜNSCH (2006). In addition, for some terms the differences to the IERS2010–A model are out of phase with the libration corrections that are included in the IERS2010–B model. Thus, for K1 the estimates are almost doubled, when the libration corrections are introduced. For the ΔUT1 comparisons, the empirical model shows an unexplained amplitude difference at O1 of approximately 1.8 μs w.r.t. the IERS2010 models. This is also present for the two empirical models used for comparison. The predicted atmospherically forced differences in ΔUT1 at S1 and S2 are in more or less acceptable agreement with the existing predictions (see Fig. 6.5).[9]

Ter- and quarter diurnal terms are included in the model to validate the CONT results with a long time span. The coefficients of these terms are depicted in Fig. 6.6 as well. None of the estimated coefficients has the power to explain any of the effects which were seen in the analysis of the CONT sessions. In addition, there is almost no agreement with model predictions[10]. However, several harmonic variations in the ter- and quarter diurnal band are estimated with significant amplitudes within this thesis. These are, e.g., K3 and S3 as well as M4.

When the empirical model for the diurnal and sub-diurnal variability of ERPs is used for the analysis of the CONT sessions, the power of the highly- resolved ERP estimates significantly decreases. Figure 6.7 shows

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[9]All other differences bigger than 7.5 μas in PM and 0.5 μas in ΔUT1 are discussed in more detail in Sec. 4.3.2 of Paper E.

[10]A detailed comparison of present model predictions with the estimated model coefficients is given in Sec. 4.3.3 of Paper E
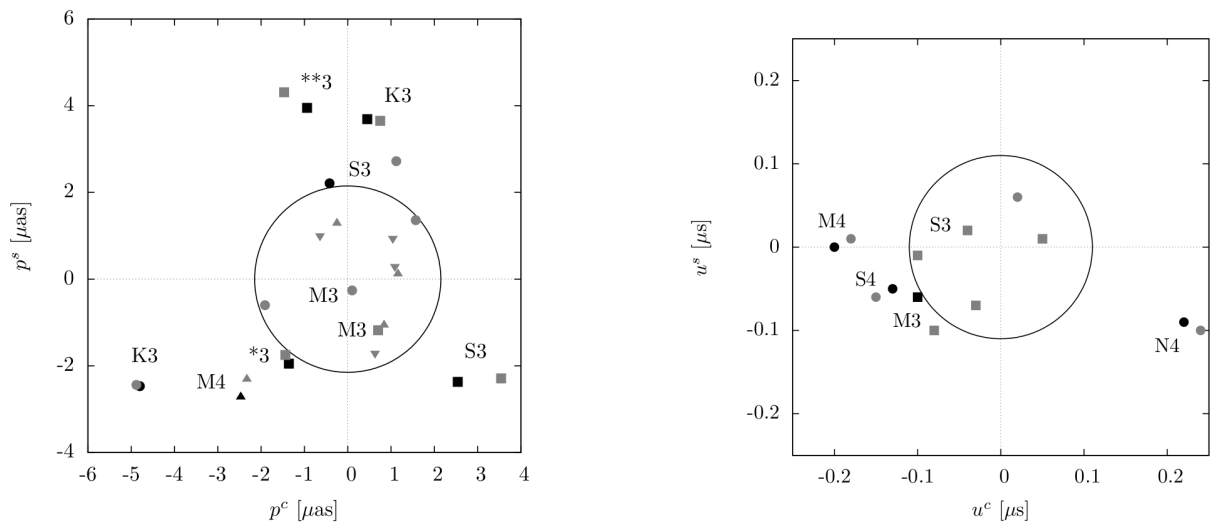
Figure 6.6: Phasor diagrams of ter- and quarter-diurnal terms for PM (left) and ΔUT1 (right) black symbols are in the IGG model and the gray ones are taken from the initial model. Ter-diurnal prograde PM and ter-diurnal ΔUT1 are depicted by squares, ter-diurnal retrograde PM by circles, quarter-diurnal prograde PM and ΔUT1 by lower and quarter-diurnal retrograde PM by upper triangles. The abbreviations *3 and **3 mark the estimates of the terms with periods of 8.0764 h and 8.3854 h. (ARTZ et al. 2011A)
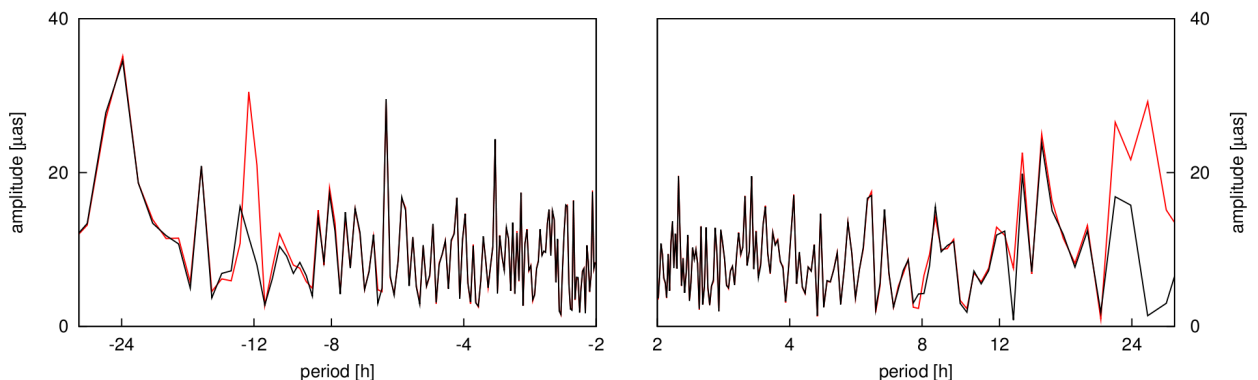
Figure 6.7: Least-squares estimated power spectrum of residual PM during CONT05 for an analysis with the IERS2010–B (red) and the empirical IGG model (black) as a priori information. (ARTZ et al. 2011A)

the spectra of residual PM during CONT05 w.r.t. the IERS2010–B model in red and the empirical IGG model in black. For the diurnal and semi-diurnal terms, the noise level is almost reached, while the six and eight hour signal is almost unchanged. These characteristics can be seen for CONT02 and CONT08 as well. Furthermore, in the ΔUT1 spectra (not shown here) similar effects can be seen with significant changes in the spectra at the diurnal and semi-diurnal periods. This indicates on the one hand that the empirical model includes almost all ERP variations in the diurnal and semi-diurnal band that are measured by VLBI. On the other hand, the ter- and quarter diurnal variations that are present for the CONT sessions cannot be explained by a model that is derived from a global VLBI solution with data of 30 years. This empirical model is the most consistent representation of the sub-daily ERP variations that can be applied to the VLBI analysis. Thus, it becomes clear that the ter- and quarter-diurnal variations are either time specific phenomena or subject to the limited CONT time spans. One possible reason might be seen in the ionosphere calibration where only the first order effect is accounted for by building a linear combination of the X-band and S-band delays (e.g., SOVERS et al. 1998). However, including the second order effect of the ionospheric correction (e.g., KEDAR et al. 2003, HAWAREY et al. 2005) did not lead to a significant changes.

6.3 Inter-technique Combination to Estimate Sub-daily Earth Orientation Parameters

VLBI is the only technique that is able to measure all components of the EOPs. However, the analyses of VLBI observations suffer for various reasons. The number of observing radio telescopes in a global VLBI session for EOP determination is about six to eight (Schlüter and Behrend 2007) and their participation is varying as well. As a consequence, the station positions have to be fixed when the EOPs are estimated as shown in Sec. 6.1. Furthermore, it is difficult to find a set of stations that might be used for the datum definition when stacking over a longer time span. In addition, the VLBI telescopes observe only one source at a specific time and about 20 are observed within an hour. Thus, the ratio of observations to unknowns is unfavorable. As a consequence, a large number of constraints have to be added to the equation system.

The situation is almost complementary when using GPS observations. See, e.g., Steigenberger (2009) for a detailed description of the GPS analysis chain and options. The GPS network of the International GNSS Service (IGS, Dow et al. 2009) is big with about 165 globally distributed receivers recording the observations each day (Steigenberger 2009). Furthermore, several GPS satellites are observed at each epoch with a sampling of, e.g., 3 minutes in the final processing (Steigenberger 2009). Thus, a huge redundancy is produced. Especially the bigger underlying observing network leads to numerous advantages of GPS compared to VLBI. For instance, PM is determined with a significantly better reliability and especially a sub-daily resolution is possible without applying stabilizing constraints. However, when using GPS measurements alone, only the time derivatives of the nutation and ΔUT1 parameters can be determined unambiguously. This is a consequence of the one-to-one correlation between the satellite orbits and nutation or ΔUT1 (e.g., Rothacher et al. 1999). Thus, trying to estimate the absolute parameters from GPS observations leads to random offsets and drifts w.r.t. VLBI-derived parameters. Furthermore, an additional (quasi-)singularity is caused by the correlation between an arbitrary retrograde diurnal PM and the orbit parameters (Hefty et al. 2000). Finally, in the diurnal and semi-diurnal period range most of the satellite techniques suffer from resonances with the satellite orbital periods, a problem that does not occur in VLBI. Thus, GPS is not able to resolve several of the diurnal periods of the ERPs, due to resonances with the orbital period (Schuh et al. 2002). This is sustained by the investigations of Steigenberger (2009) who revealed a bad repeatability of the S1 and ψ1 coefficients of the tidal ERP model in comparison to the other terms.

Due to the substantial differences between the space geodetic techniques, inter-technique combinations are performed in the framework of the IERS to determine the most reliable parameters. In this way, the ITRF is determined as a multi-technique reference frame (e.g., Altamimi et al. 2007). Furthermore, the IERS Earth Orientation Product Centre (Observatoire de Paris, France) and the IERS Rapid Service and Prediction Center (United States Naval Observatory, USA) combine the output of the services and other analysis centers in a sense of an inter-technique combination (e.g., Bizouard and Gambis 2009). Despite this standard procedure, only two investigations were done to derive sub-daily ERPs in a combination process. Thaller et al. (2007) performed a robust combination of GPS and VLBI NEQ systems for the time span of the CONT02 campaign. However, this investigation provided no long term information on sub-daily ERPs. Steigenberger (2009) described the effect of combining two empirical tidal ERP models that were derived from GPS and VLBI ERP time series.

To densify this research area, a combination of VLBI and GPS observations is performed within this thesis. For this purpose, GPS and VLBI NEQ systems taken from the project GGOS-D (Integration of Space Geodetic Techniques as the Basis for a Global Geodetic-Geophysical Observing System, Rothacher et al. 2011) are used to estimate sub-daily ERPs for the timespan 1994.0 – 2007.0. On the one hand combined long term PM and ΔUT1 time series with an hourly resolution are derived. On the other hand, a combined empirical ERP model containing tidal components with diurnal and semi-diurnal terms is estimated. One of the aims of GGOS-D was the homogeneous modeling and parameterization among the different techniques leading to highly homogeneous and consistent contributions of the different techniques in the form of NEQ systems. A detailed description of the analysis set-up within GGOS-D is given in Rothacher et al. (2011).

The detailed processing steps applied for the combination are described in Sec. 2.2 of Paper F (Artz et al. 2011b). In brief, stable station and source positions are fixed to the a priori values, and the troposphere

parameters, clocks and satellite orbits are pre-reduced from the NEQ systems. Stabilizing constraints are imposed for these pre-reduced parameters. Furthermore, unstable stations or sources were pre-reduced as well. These have not enough observations for reliable global estimates or show discontinuities, thus, their coordinates in the a priori reference frames are not good enough for fixing them. Finally, nutation corrections are transformed to a linear representation over fortnightly time spans. Then the individual daily (GPS) or session-wise (VLBI) NEQ systems are added to one fortnightly NEQ system and these two NEQ systems are added to form a combined one. This combination on a fortnightly basis is not performed by simply adding the individual NEQ systems due to various reasons. The variance level of the GPS and the VLBI solutions different, due to a discordance in the number of observations. Furthermore, the number of VLBI sessions included in a fortnightly time-span n_V is varying from two to twenty while the number of GPS solutions n_G is always fourteen. As two or more different VLBI networks might observe on single days, more than fourteen VLBI sessions can be present within two weeks. Finally, the observing geometry of the VLBI networks is very inhomogeneous, thus, the relative weighting of GPS and VLBI should also account for this time dependence. Therefore, a simple scaling has been applied for the individual 14 day NEQ systems that adjusts the different variance levels and considers the number of VLBI sessions:

$$\overline{tr} = \frac{1}{2}\left(tr(\mathbf{N}_G) + tr(\mathbf{N}_V)\right) \tag{6.9a}$$

$$\mathbf{N}_C = \frac{n_G}{n_V}\frac{\overline{tr}}{tr(\mathbf{N}_G)}\mathbf{N}_G + \frac{\overline{tr}}{tr(\mathbf{N}_V)}\mathbf{N}_V. \tag{6.9b}$$

Here, $\mathbf{N}_G$ represents a GPS NEQ matrix and $\mathbf{N}_V$ as well as $\mathbf{N}_C$ denote the VLBI and combined NEQ matrices, respectively. Thus, the individual normal matrices are scaled in a way that their trace is equal to the trace $\overline{tr}$, where only the EOPs have been considered. Finally, the scaled normal matrices have been added with relative weights corresponding to the number of VLBI and GPS solutions in the respective interval. The same scaling was done for the right hand side of the NEQ system. As this scaling was done independently for each 14 day interval, the NEQ systems were scaled with different factors. These combined NEQ systems, that are valid for two weeks, are either solved to determine the hourly resolved time series or are transformed to the representation of the tidal ERP model. In the latter case the procedure described in Ch. 6.2 by Eq. (6.6) – (6.8) is applied and the transformed NEQ systems are added to one NEQ system for the entire time span. In both cases nutation corrections linear over two-week intervals are estimated simultaneously.

6.3.1 Long Term Time Series

Estimating time series of hourly resolved ERPs independently for both techniques requires several constraints[11]. These constraints are not necessary for the combined solution as geometric instabilities of the techniques are cross-wise compensated by the combination algorithm. Thus, GPS provides precise short period variations, while VLBI provides the absolute information on ΔUT1 and the nutation corrections. In this way, combined time series are determined where only the retrograde diurnal PM components are blocked and no other constraints are necessary. Table 6.2 shows the root mean squared (RMS) differences between sub-daily solutions and corresponding daily solutions. The daily solutions are subtracted to account for the long term behavior of the ERPs. These combined daily time series have been derived from the same set of NEQ systems by the transformation described in Sec. 6.2 where the sums in Eq. (6.6) have been neglected. Assuming that a smaller scatter means a better solution and more weight, the RMS differences can be used for a quality assessment. This assumption holds as remaining harmonic differences, i.e., harmonic components measured by VLBI and GPS that are not part of the a priori model, are significantly smaller compared to the noise. The RMS of the PM differences are almost identical for the GPS-only and the combined solutions and not depending on the daily reference series. Thus, no reduction of the noise within the hourly resolved PM series has been achieved by the combination. But, beside the retrograde block the VLBI observations fully replace the constraints that are necessary for a GPS-only solution. Nevertheless, the GPS observations totally dominate the combination in view of PM determination.

Particular attention should be paid to the ΔUT1 time series. In Fig. 6.8, a one-month excerpt of the whole time series is given. Although the VLBI time series is not continuous and the GPS time series is rather

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[11]see Sec. 3 of Paper F for a detailed descriptions of the necessary constraint equations.

Table 6.2: RMS values of the differences between the estimated time series and several reference series. The corresponding time series are plotted in Fig. 1 of Paper F.

	x-pole [μas]			y-pole [μas]			ΔUT1 [μs]			
	COMBI[a]	IGS[b]	C04	COMBI[a]	IGS[b]	C04	COMBI[a]	COMBI [ac]	IVS[dc]	C04[c]
GPS	190	181	236	195	190	247	163	162	143	158
VLBI	1138	1184	1141	753	724	760	41	41	48	43
COMBI	181	183	232	190	193	240	18	11	30	18

[a]special combined solution with daily ERPs

[b]`ftp://cddis.gsfc.nasa.gov/pub/gps/products/igs00p03.erp.Z`

[c]calculated only at VLBI epochs

[d]`http://vlbi.geod.uni-bonn.de/IVS-AC/combi-sinex/QUAT/COMBI/series_eop_combi_COMBI_1.txt`
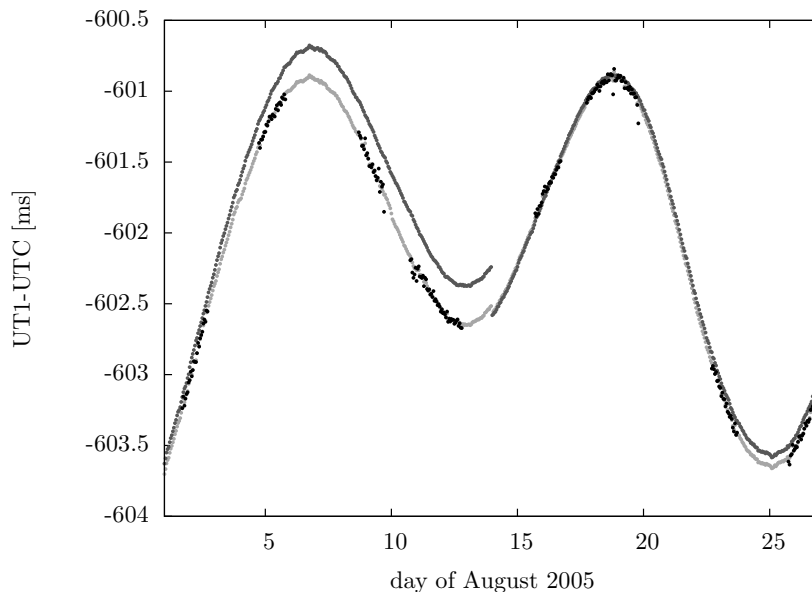
Figure 6.8: One-month excerpt of GPS (dark gray), VLBI (black) and combined (light gray) ΔUT1 time series. The jump within the GPS time series appears due to the beginning of a new 14-day interval. Such jumps are minimized between 14-day intervals of the combined time series.

irregular due to significant offsets and drifts w.r.t. the VLBI results, the combined time series conserves the short-period variations derived by GPS and shifts them to the level of the VLBI estimates. Obviously, time correlated biases which are present in the GPS LOD estimates (e.g., RAY 1996) – and, thus, also in the GPS ΔUT1 estimates – are compensated by occasionally occurring VLBI sessions. As a consequence, jumps within the GPS-only time series between subsequent 14-day intervals were significantly reduced by the combination procedure. The mean value of the jumps between the individual 14 d intervals of the GPS-only time series of 197 μs with a standard deviation of 158 μs is reduced to 68 μs with a standard deviation of 66 μs for the combined solution. Furthermore, the combined time series is robust against VLBI sessions which lead to unreliably high ΔUT1 estimates as the relative information from GPS has a notable impact on the combined result. This is again represented by the RMS differences listed in Tab. 6.2 as well. These are substantially reduced, indicating a great benefit of the combination procedure for the ΔUT1 estimates.

As a reliable quality assessment of ΔUT1 is almost not possible due to a missing independent reference series, an LOD comparison with geophysical fluid time series has been performed in Paper G (ARTZ et al.

2011c)[12]. In contrast to the ΔUT1 comparisons presented above, this represents an independent validation of the hourly resolved time series. The AAM values were taken from the NCEP Reanalysis[13] (SALSTEIN and ROSEN 1997), where the inverted barometer assumption was applied, and the OAM values were taken from the ECCO_kf066b_6hr model[14] (GROSS 2009). For generating the LOD time series, the ΔUT1 time series has been reduced by zonal tides in a first step. For this purpose, the elastic body and equilibrium ocean tide model of YODER et al. (1981), the in-phase and out-of-phase components of the WAHR and BERGEN (1986) inelastic body tide model and the dynamic ocean tide model "A" of KANTHA et al. (1998) have been used. From these time series, the LOD time series has been derived and smoothed to a temporal resolution of 6 h. The best agreement with the AAM/OAM series can be achieved by performing an unconstrained GPS-only solution. By performing a combination, this consistency is slightly degraded by at most 5% depending on the types of the VLBI sessions (see Ch. 6.3.2). However, this reduced consistency to the geophysical data is not supposed to be forced by the amount of VLBI data used rather than to be attributed to the VLBI observing networks. Nevertheless it is remarkable, that the noise of the ΔUT1 time series is improved while the unconstrained GPS-only solution is not useful to perform ΔUT1 investigations.

6.3.2 Impact of various VLBI observing session types

SENIOR et al. (2010) found that the rejection of some VLBI data led to an improved agreement of combined LOD with AAM/OAM series. This hypothesis is validated by the combination approach which was developed within this thesis. In Paper G the impact of various VLBI observing sessions types on the combined time series is investigated. For this purpose, solutions with IVS rapid turnaround sessions (R1 and R4, SCHLÜTER and BEHREND 2007) and Intensive sessions as well as all possible VLBI sessions have been calculated. The limitation of using only Intensive sessions leads to a significant unbalance of the VLBI and the GPS contribution. Thus, within the combination represented by Eq. (6.9) the GPS contribution is often overweighted. To avoid worse ΔUT1 results, the factor n_G/n_V in Eq. (6.9b) has been set to one. However, the differences to the ΔUT1 time series shown in Sec. 6.1 are not significant if all possible VLBI sessions are used. The results of this analysis are different for the ΔUT1 comparisons with IERS 05C04 and the LOD comparisons with AAM/OAM time series.

Concerning the combined ΔUT1 time series with hourly resolution, a better agreement to IERS 05C04 is achieved by the combination compared to technique-independent solutions[15]. This better agreement in terms of RMS differences implies a higher quality of the derived time series as the roughness of the hourly estimates is reduced. A significant reduction of the noise within the time series is achieved by the combination procedure. The combined series is less noisy the more VLBI sessions are used, however, using only VLBI Intensive sessions for the combination already leads to an improved UT1 series.

Concerning the LOD comparisons as described in the previous chapter, the best agreement to the AAM/OAM series can be obtained by performing an unconstrained GPS-only solution or a combination of GPS with VLBI R1 sessions[16]. The unconstrained GPS-only solution is a special approach to determine LOD where the ΔUT1 estimates are not fixed to IERS C04 every fortnightly time span. Thus, the ΔUT1 time series consists of a random walk behavior w.r.t. a VLBI-only solution, however, the derived LODs are unaffected. When only R1 sessions are used in the combination, the agreement to AAM/OAM is equal to using GPS-only. Adding other VLBI session types leads to a small degradation of at most 5% when using all VLBI data. However, this reduced consistency to the geophysical data is not supposed to be forced by the amount of VLBI data used rather than to be attributed to the VLBI observing networks. When using only VLBI Intensive sessions for the combination, the RMS residuals get worse by about 30%.

As expected, relative variations were primarily defined by the GPS observations while the VLBI observations define the absolute information. Therefore, the ΔUT1 estimation is significantly improved by adding more

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[12]Although a slightly different relative weighting of GPS and VLBI has been used in Paper G −i.e., the number of VLBI and GPS solutions in each 14 day interval has not been accounted for− the ΔUT1 results are comparable to those which have been derived in Paper H.

[13]`ftp.aer.com/pub/anon_collaborations/sba/aamf.ncep.reanalysis.1948.2009`

[14]`ftp://euler.jpl.nasa.gov/sbo/oam_global/ECCO_kf066b_6hr.chi`

[15]see Tab. 1 of Paper G for a listing of the RMS ΔUT1 differences.

[16]see Tab. 2 of Paper G for a listing of the RMS LOD differences.

VLBI sessions of any type. At the same time, the differential consistency is slightly degraded when using various VLBI session types especially with smaller observing networks. This supports the findings of SENIOR et al. (2010) who suggested that the ΔUT1 errors of the Intensive and weaker 24 h VLBI sessions are heterogeneous, possibly due to the diversity of observing geometries used.

6.3.3   Empirical Model for Tidal Variations of the Earth Orientation Parameters

The combined tidal ERP model for sub-daily variations has been estimated for 74 tidal ΔUT1 components and 97 PM components. These are the significant subset of an initial set-up consisting of 103 terms for ΔUT1 and 152 terms for PM. The components of the initial model have been chosen by their amplitude in the tide generating potential. Therefore, a threshold of 1 mm has been chosen from the tidal potential developed of BÜLLESFELD (1985).

For the combined model, the stability investigations presented in Paper D and Paper E have been repeated. The noise floor of the tidal ERP model has been investigated in three ways yielding about 1 μas for diurnal PM and 0.7 μs for diurnal dUT1. The semi-diurnal components have a slightly better accuracy. Thus, the conclusion that the formal errors are too optimistic could be confirmed.[17] For the combined model, the formal errors are too optimistic by a factor of two.

By comparing the combined model with the individual models, i.e., VLBI-only and GPS-only, the success of the combination can be demonstrated. In Fig. 6.9 the coefficient differences of the combined model to the individual models are shown for the major tidal terms. Smaller tidal constituents are left out for clarity here. Furthermore, Tab. 6.3 lists tidal terms with coefficient differences between the GPS-only and the VLBI-only coefficients above 6 μas in PM and above 0.4 μs in ΔUT1. These thresholds are exceeded by only 10% of the model components. These deviations are supposed to result from technique-specific effects as the modeling of the satellite orbits, in particular deficiencies in the radiation pressure modeling, might be present. Furthermore, ROTHACHER et al. (2001) reported systematic errors of GPS-derived empirical tidal ERP models that result in a rail of S1, S2, S3, etc. for which no explanation exists. Moreover, the homogeneous reprocessing with state-of-the-art models apparently did not reduce these artifacts significantly. Especially for the S1 component in PM, these errors of the GPS observations seem to force a partial degradation of the combined result. Finally, the influence of non-linear station motions was not accounted for directly within this analysis. Thus, diurnal network variations that are not absorbed by the daily ERPs or semi-diurnal network variations might have corrupted the tidal ERP model.

Although a few terms differ between the two individual tidal ERP models with deviations above the uncertainty level, a combination effort is legitimate, especially as the combination approach minimizes technique-specific shortcomings as shown in Ch. 6.1. Since a homogeneous processing of the individual contributions had been performed within the GGOS-D project, all sorts of presently known inconsistencies should have been debarred. In Fig. 6.9, the combined model is present implicitly represented by the origins of all sub-figures. It can be seen that the combined solution is almost always between the individual solutions and, as expected, GPS has a bigger impact on the combination than VLBI in most of the cases. Again, the reasons are the number of observations and the more stable underlying observing network. Both criteria lead to a higher sensitivity of the GPS observations to the short period variations of the Earth's rotation. For a few terms, the combined result is not between the individual VLBI and GPS results, see e.g., K1 in prograde PM or S2 in retrograde PM. This might arise from remaining correlations of the major tidal constituents and its sidebands, as K1 has two sidebands with big amplitudes in the tidal potential. For S2, the reason might be attributed to the handling of the station positions. These results for the major tidal terms are representative for all estimated terms.

Comparisons with other tidal ERP models demonstrate the success of the combination as well. The same reference models as in Ch. 6.2 are used. The RMS coefficient differences between the combined and the two empirical reference models in general show a good agreement (see Tab. 6.4). The combined model agrees

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[17]The detailed results of the stability and reliability investigations for the combined model are presented in Sec. 4.1 of Paper F.
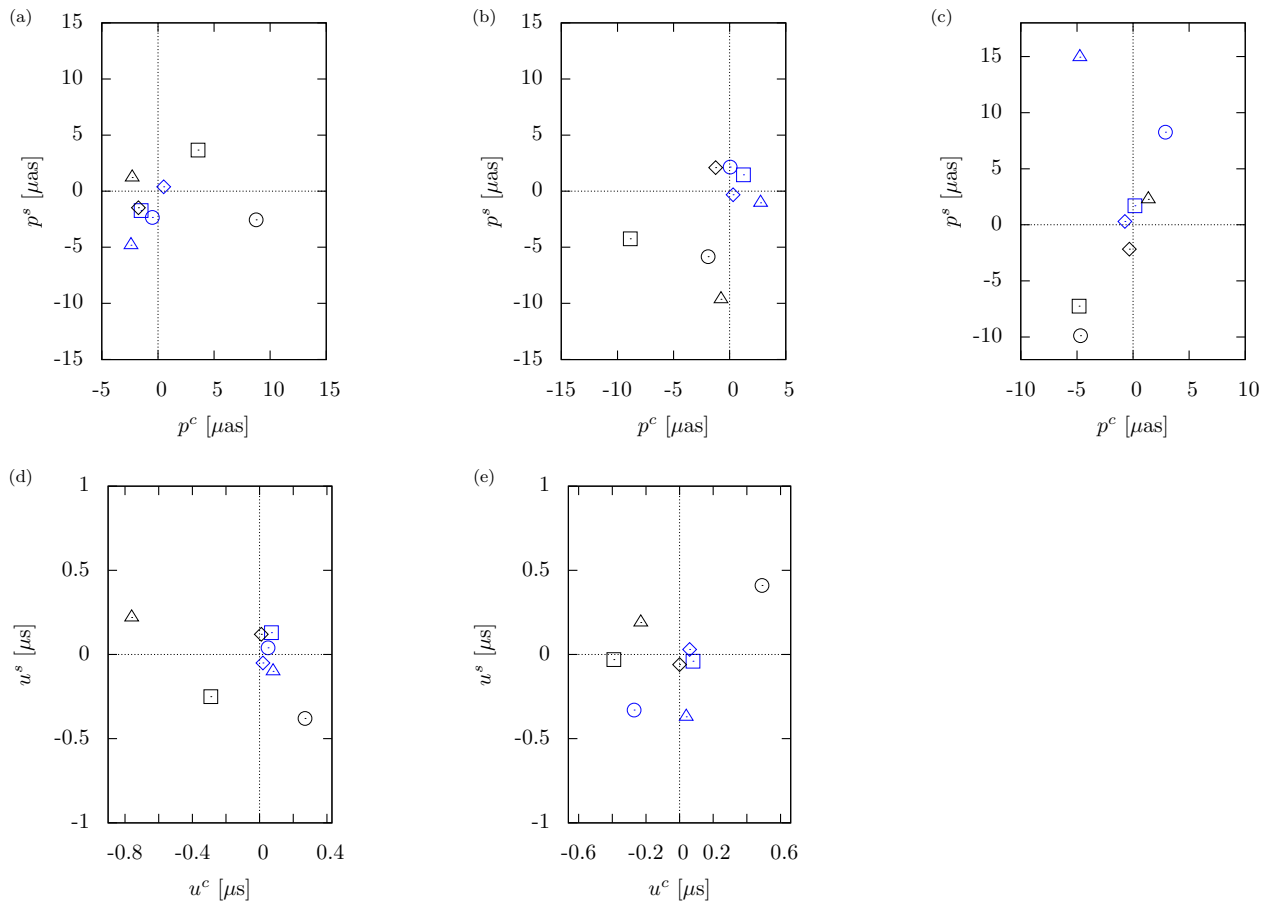
Figure 6.9: Coefficient differences for the major tidal terms of the individual GPS (blue) and the individual VLBI model w.r.t. the combined model. The sub-figures are separated for prograde diurnal (a), prograde semi-diurnal (b), retrograde semi-diurnal PM (c) as well as diurnal (d) and semi-diurnal dUT1 components (e). In the prograde band the symbols denote O1 (squares), P1 (triangles), Q1 (diamonds) and K1 (circles). In the semi-diurnal bands: M2 (squares), S2 (triangles), N2 (diamonds) and K2 (circles). (ARTZ et al.2011B)

with the VLBI-only and the GPS-only model at the level of $3-4.5$ μas for PM and $0.2-0.3$ μs for ΔUT1, respectively. The differences to the GPS model are slightly smaller in PM, while a reverse situation can be seen for ΔUT1. In comparison to the IERS2010–A model, the combined model shows the best agreement for ΔUT1. Concerning PM, RMS differences are slightly bigger than the GSFC ones, however, not significant.

Table 6.3: Tidal terms with coefficient differences between the GPS-only and the VLBI-only coefficients above 6 μas in PM and above 0.4 μs in ΔUT1. Retrograde terms are labeled with (r). (ARTZ et al.2011B)

Name	Doodson Number	Δp^c [μas]	Δp^s [μas]	Δu^c [μs]	Δu^s [μs]
$\sigma 1$	127.555	-6.86	2.50	0.42	0.40
Q1	135.655	6.40	-0.35	0.09	-0.42
	144.556	-10.85	3.57	-	-
O1	145.555	-	-	-0.22	-0.58
	145.755	-	-	0.11	0.76
	146.554	-6.94	-5.38	-	-
T01	147.555	-7.30	6.95	-	-
	147.565	-	-	0.25	-0.56
	155.455	-	-	-0.55	-0.12
M1	155.655	-6.87	-2.40	-	-
$\pi 1$	162.556	-4.58	10.70	-	-
P1	163.555	-	-	-0.69	-0.02
S1	164.556	30.84	-12.99	-0.42	0.24
K1	165.555	-	-	-0.24	-0.66
$\psi 1$	166.554	-8.12	-7.96	0.60	-0.23
TT1	173.655	-0.37	-6.15	-0.61	-0.37
(r)	244.656	6.52	-5.64		
N2	245.655	-	-	-0.44	0.00
	254.556	-	-	-0.20	0.44
M2	255.555	-8.22	-4.13	-0.68	0.17
M2 (r)	255.555	-3.92	-11.07	-	-
$\lambda 2$	263.655	7.85	1.11		
L2 (r)	265.455	-1.04	9.34	-	-
(r)	265.555	-6.24	-3.67	-	-
T2	272.556	-	-	-1.07	1.28
T2 (r)	272.556	2.91	-11.40	-	-
S2	273.555	-1.23	-13.45		
S2 (r)	273.555	2.89	-8.41	-	-
R2	274.554	-	-	1.24	0.17
K2	275.555	0.88	-17.28	-0.35	0.53
K2 (r)	275.555	-6.33	-16.43	-	-

Table 6.4: Mean RMS amplitude differences between different models. The differences are calculated only for terms that are estimated for all three empirical models. (ARTZ et al.2011B)

	ΔUT1 [μs]			PM [μas]		
	IERS	TUM	GSFC	IERS	TUM	GSFC
TUM	0.41	-	0.30	5.3	-	5.1
GSFC	0.46	0.30	-	4.7	5.1	-
COMBI	0.39	0.27	0.20	4.8	4.4	3.0

7. Summary and Outlook

Within this thesis, an extensive analysis of VLBI observations has been performed to derive ERPs with a sub-daily resolution applying various geodetic methods adapted to the advantage of the investigations. This research field has to be subdivided into two areas. First, time series of highly resolved ERPs can be determined and second, a model for ERP variations with periods at the well known tidal bands can be estimated.

In the past, several investigations were performed to determine sub-daily variations of the ERPs with VLBI observations and to analyze these results. Furthermore, sub-daily ERPs were also estimated from GPS and SLR observations. However, there are several motivations to reassess the role of VLBI for the determination of sub-daily ERPs. First of all, the publications concerning VLBI-derived empirical tidal ERP models were somewhat outdated. Following GIPSON (1996), for more than a decade no improvement was shown although more valuable observations were available. Furthermore, almost all analyses of the CONT sessions were imperfect as they did not account entirely for the continuous behavior of these campaigns. Finally, and most importantly, the sub-daily ERPs as derived from the different space geodetic techniques were only compared to each other. In this way, some common features and some discrepancies were found. However, no broad investigations concerning inter-technique combinations were performed although this is the common practice for all important products of the IERS. All of these aspects have been considered within this thesis to provide the cutting-edge in terms of the determination of VLBI-derived sub-daily ERPs. This expertise is also used to provide an outlook for further analysis options.

As VLBI observations are usually performed about three times per week for 24 hours, the standard VLBI sessions alone are normally not ideal for a detailed analysis of sub-daily ERP variations. Furthermore, standard VLBI sessions are performed employing observing networks that are often not sufficient for a reliable estimation of highly-resolved ERPs. To overcome these deficits, the CONT sessions are used within this thesis to determine continuous highly resolved ERP time series from VLBI. An analysis chain has been developed to account for the specifics of these campaigns. Based on these investigations, deficiencies of the IERS sub-daily models became obvious. The results of prior investigations have been verified and other irregularities as the 6 h variations in CONT05 and CONT08 have been detected. However, several concerns exist due to the short time spans of only 14 days. For further continuous VLBI campaigns, longer time spans should be considered to ensure a more reliable analysis of the highly resolved ERPs. Thus, the irregular variations that have been detected within this work might be confirmed or rejected. The success of a new scheduling approach applied in CONT08 might be extended to balance the observing load in such an extended CONT.

Within this thesis, the determination of a VLBI-only tidal ERP model based on the transformation of NEQ systems has been developed and applied for the first time. The comparison of this model with other state-of-the-art models revealed the suitability of this approach. With the currently available VLBI observing technique no improvements for the determination of such a VLBI-only model can be expected for the future. However, the initiation of the VLBI2010 concepts might change the situation significantly. Until then, especially the number of observing sites and the insufficient network stability leads to insuperable restrictions on the sub-daily ERPs derived from VLBI observations.

Although VLBI is restricted by the present peculiarities, it is a valuable tool for the ERP determination. Most notably, VLBI is the only space geodetic technique that is able to measure all EOPs. Concerning sub-daily ERPs, the results are weak, however, GPS-determined sub-daily ERPs suffer due to the resonance with the satellite orbits. Thus, an inter-technique combination of VLBI and GPS is performed within this thesis to get the most of both techniques. Thereby, remaining technique-specific errors are reduced. Furthermore, geometric instabilities of the techniques are cross-wise compensated and the stochastic noise is reduced which is the very purpose of combinations. Furthermore, long term time series of hourly spaced ERPs are estimated with a minimum set of stabilizing or de-correlating constraint equations. This time series stands for the success of the combination approach, as it exhibits the high relative quality of GPS and the high

absolute quality of VLBI in terms of ΔUT1 and nutation. However, especially the GPS-specific errors lead to a partial degradation of the combined model.

Future investigations should be focused on the combination of space geodetic techniques. In this process, a more sophisticated relative weighting of the VLBI and GPS observations should be implemented. In addition, SLR observations could be used as well. If SLR is used instead of GPS, the impact of the orbits might be investigated as different resonances are expected. Furthermore, the TRF and CRF should be estimated simultaneously with the sub-daily ERPs to achieve the most consistent set-up. This would demand the inclusion of the relative positions of co-located VLBI and GPS sites, i.e, the local ties. Finally, non-linear station motions have been identified to influence the sub- daily ERPs. It has been shown in this thesis, that a harmonic station position model absorbs tidal ERP variations and vice versa. As a consequence, global parameters as daily estimated ERP rates change significantly. Estimating station positions on a weekly or fortnightly basis might be a preferable approach. However, due to the changing VLBI station constellation, the impact of the local ties would even be increased. This will be overcome or at least mitigated by the VLBI observation scenarios of the future which follow the VLBI observing concept with observations for 24 h a day and on seven days a week with $12-16$ radio telescopes observing quasars in a much more rapid sequence than today. With this realized, another significant step in accuracy and resolution can be expected for research in sub-daily ERP variations.

8. List of publications relevant to the thesis work

Below is a list of publications on related work to which I have contributed. These publications are not included in this thesis, but document the relevance of this work in the scientific community.

BÖCKMANN, S., T. ARTZ, A. NOTHNAGEL, V. TESMER (2007) *Comparison and combination of consistent VLBI solution.* In: Boehm, J., A. Pany, H. Schuh (eds) Proceedings of the 18th European VLBI for Geodesy and Astrometry Working Meeting, 12 – 13 April 2007, Vienna, Geowissenschaftliche Mitteilungen, Heft Nr. 79, Schriftenreihe der Studienrichtung Vermessung und Geoinformation, Technische Universität Wien, ISSN 1811-8380, pp 82 – 87, (available electronically at `http://mars.hg.tuwien.ac.at/~evga/proceedings/S34_Boeckmann.pdf`).

ARTZ, T., S. BÖCKMANN, A. NOTHNAGEL, V. TESMER (2008) Comparison and Validation of VLBI Derived Polar Motion Estimates. In: Finkelstein, A. and D. Behrend (eds), *Proceedings of the Fifth IVS General Meeting "Measuring the Future",* , 2 – 6 March 2008, St. Petersburg, Nauka, ISBN 978-5-02-025332-2 pp 324 – 328. (available electronically at `ftp://ivscc.gsfc.nasa.gov/pub/general-meeting/2008/pdf/artz.pdf`).

NOTHNAGEL, A., A. BERTARINI, C. DULFER, T. ARTZ, J. WAGNER, G. MOLERA, J. RITAKARI (2008) Comparisons of Correlations Using Disk-Transfer and e-VLBI-Transfer. In: Finkelstein, A. and D. Behrend (eds), *Proceedings of the Fifth IVS General Meeting "Measuring the Future",* , 2 – 6 March 2008, St. Petersburg, Nauka, ISBN 978-5-02-025332-2 pp 63 – 67. (available electronically at `ftp://ivscc.gsfc.nasa.gov/pub/general-meeting/2008/pdf/nothnagel1.pdf`).

BÖCKMANN, S., T. ARTZ, A. NOTHNAGEL (2009) *IVS' contribution to ITRF2008 - Status & Results.* In: Bourda, G., P. Charlot, A. Collioud (eds) *Proceedings of the 19th European VLBI for Geodesy and Astrometry Working Meeting*, Université Bordeaux 1 - CNRS - Laboratoire d'Astrophysique de Bordeaux, pp 102-106, (available electronically at `http://www.u-bordeaux1.fr/vlbi2009/proceedgs/24_Boeckmann.pdf`).

BÖCKMANN, S., T. ARTZ, A. NOTHNAGEL, V. TESMER (2010a) *International VLBI Service for Geodesy and Astrometry: Earth orientation parameter combination methodology and quality of the combined products.* J Geophys Res, 115:4404, DOI 10.1029/2009JB006465.

BÖCKMANN, S., T. ARTZ, A. NOTHNAGEL (2010b) *VLBI terrestrial reference frame contributions to ITRF2008.* J Geod 84:201-219, DOI 10.1007/s00190-009-0357-7.

BÖCKMANN, S., T. ARTZ, A. NOTHNAGEL (2010c) *Correlations between the contributions of individual IVS analysis centers.* In: Behrend, D., K.D. Baver (eds) IVS 2010 General Meeting Proceedings "VLBI2010: From Vision to Reality", 7 – 13 February 2010, Hobart, NASA/CP-2010-215864, pp 222 – 226, (available electronically at `http://ivscc.gsfc.nasa.gov/publications/gm2010/boeckmann.pdf`)

STEIGENBERGER, P., T. ARTZ, S. BÖCKMANN, R. KELM, R. KÖNIG, B. MEISEL, H. MÜLLER, A. NOTHNAGEL, S. RUDENKO, V. TESMER, D. THALLER (2009) GGOS-D Consistent, High-Accuracy Technique-Specific Solutions. In: Flechtner, F.M., M. Mandea, T. Gruber, M. Rothacher, J. Wickert, A. Güntner, T. Schöne (eds), *System Earth via Geodetic-Geophysical Space Techniques*, Advanced Technologies in Earth Sciences, Springer Berlin Heidelberg, ISBN 978-3-642-10228-8, 545 – 554, DOI 10.1007/978-3-642-10228-8_45.

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Geodäsie und Geoinformation der Rheinischen-Friedrich-Wilhelms Universität Bonn, No. 22, ISSN 1864-
1113, pp 123 – 127.

Abbreviations

AAM	Atmospheric Angular Momentum
CIO	Celestial Intermediate Origin
CIP	Celestial Intermediate Pole
CONT	Continuous VLBI Campaign
cpsd	cycles per sidereal day
CRF	celestial reference frame
EOP	Earth Orientation Parameter
ERP	Earth rotation parameter
GAST	Greenwich Apparent Sidereal Time
GGOS-D	Integration of Space Geodetic Techniques as the Basis for a Global Geodetic-Geophysical Observing System
GCRS	Geocentric Celestial Reference System
GGOS	Global Geodetic Observing System
GMST	Greenwich Mean Sidereal Time
GNSS	Global Navigation Satellite Systems
GPS	Global Positioning System
GSFC	Goddard Space Flight Center
IAG	International Association of Geodesy
IERS	International Earth Rotation and Reference Systems Service
IGG	Institute of Geodesy and Geoinformation
IGS	International GNSS Service
ITRS	International Terrestrial Reference System
IVS	International VLBI Service for Geodesy and Astrometry
LOD	length-of-day
NEQ	Normal Equation
OAM	Oceanic Angular Momentum
PM	polar motion
RMS	root mean squared
SLR	Satellite Laser Ranging
SNR	signal to noise ratio
TAI	Atomic Time (Temps Atomique International)
TIO	Terrestrial Intermediate Origin
TRF	terrestrial reference frame
TUM	Technische Universität München
UT1	Universal Time
ΔUT1	UT1 - TAI
VLBI	Very Long Baseline Interferometry
WRMS	weighted root mean squared

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A. Appended Papers

A.1   Paper A

Originally published as:

ARTZ, T., S. BÖCKMANN, A. NOTHNAGEL, V. TESMER (2007) *ERP time series with daily and sub-daily resolution determined from CONT05.* In: Boehm, J., A. Pany, H. Schuh (eds) Proceedings of the 18th European VLBI for Geodesy and Astrometry Working Meeting, 12 – 13 April 2007, Vienna, Geowissenschaftliche Mitteilungen, Heft Nr. 79, Schriftenreihe der Studienrichtung Vermessung und Geoinformation, Technische Universität Wien, ISSN 1811-8380, pp 69 – 74, (available electronically at `http://mars.hg.tuwien.ac.at/~evga/proceedings/S31_Artz.pdf`).

A.2 Paper B

Originally published as:

Artz, T., S. Böckmann, A. Nothnagel (2009) *CONT08 – First Results and High Frequency Earth Rotation.* In: Bourda, G., P. Charlot, A. Collioud (eds) Proceedings of the 19th European VLBI for Geodesy and Astrometry Working Meeting, 24 – 25 March 2009, Bordeaux, Université Bordeaux 1 - CNRS - Laboratoire d'Astrophysique de Bordeaux, pp 111 – 115, (available electronically at `http://www.u-bordeaux1.fr/vlbi2009/proceedgs/26_Artz.pdf`).

A.3 Paper C

ARTZ, T., S. BÖCKMANN, S., A. NOTHNAGEL, P. STEIGENBERGER (2010a) *Subdiurnal variations in the Earth's rotation from continuous Very Long Baseline Interferometry campaigns.* J Geophys Res, 115, B05404, DOI 10.1029/2009JB006834.

A.4 Paper D

Originally published as:

ARTZ, T. S. BÖCKMANN, L. JENSEN, A. NOTHNAGEL, P. STEIGENBERGER (2010B) *Reliability and Stability of VLBI-derived Sub-daily EOP Models.* In: Behrend, D., K.D. Baver (eds) IVS 2010 General Meeting Proceedings "VLBI2010: From Vision to Reality", 7 – 13 February 2010, Hobart, NASA/CP-2010-215864, pp 355 – 359, (available electronically at `http://ivscc.gsfc.nasa.gov/publications/gm2010/artz.pdf`)

A.5 Paper E

Originally published as:

Artz, T., S. Tesmer, S., A. Nothnagel (2010a) *Assessment of Periodic Sub-diurnal Earth Rotation Variations at Tidal Frequencies Through Transformation of VLBI Normal Equation Systems.* J Geod, 85(9):565 – 584, DOI 10.1007/s00190-011-0457-z.

The publication is available at `www.springerlink.com`

A.6   Paper F

Originally published as:

Artz, T., L. Bernhard, A. Nothnagel, P. Steigenberger, S. Tesmer (2011b) *Methodology for the Combination of Sub-daily Earth Rotation from GPS and VLBI Observations* J Geod, DOI 10.1007/s00190-011-0512-9, online first.

The publication is available at `www.springerlink.com`

A.7   Paper G

Originally published as:

Artz, T., A. Nothnagel, P. Steigenberger, S. Tesmer (2011) *Evaluation of Combined Sub-daily UT1 Estimates from GPS and VLBI Observations.* In: Alef W., Bernhart S., Nothnagel A. (eds.) Proceedings of the 20th Meeting of the European VLBI Group for Geodesy and Astrometry, Schriftenreihe des Instituts für Geodäsie und Geoinformation der Rheinischen-Friedrich-Wilhelms Universität Bonn, No. 22, ISSN 1864-1113, pp 97 – 101.