Modeling the Beam Deflection of a Gantry Crane under Load

Christoph Holst [1], Martin Burghof [2], Heiner Kuhlmann [3]

ABSTRACT

Gantry cranes load and unload containers from container ships. To ensure safety and operation, the deflection of their main beams under load should be analyzed. This deflection depends on the weight of the loaded container as well as on the position of the trolley that moves the container. A bivariate polynomial model is build up that estimates the deflection based on these two inputs. This two-dimensional load-dependent model is developed in four steps. It determines the shape of the deflection (dependent on the trolley position) as well as its magnitude (dependent on the container weight).

A specific gantry crane was monitored with several sensors: five inclinometer sensors observing the tilt along the crane's main beam; two tacheometers observing the trolley position and the absolute deflection. Collecting the data of 18 loading operations, the parameters of the deflection model are estimated. Upon this, the deflection under load is processed dependent on the container weight and trolley position. The verification of the whole model shows deviations within $\pm$ 4mm in comparison to an independent loading operation. This quality is acceptable regarding the used measurements.

Keywords: gantry crane, beam deflection, bivariate polynomial, deformation analysis

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[1]Ph.D. Student, Institute of Geodesy and Geoinformation, University of Bonn, 53115 Bonn, Germany. E-mail: c.holst@igg.uni-bonn.de

[2]M.Sc. Student, Institute of Geodesy and Geoinformation, University of Bonn, 53115 Bonn, Germany, E-mail: macki@uni-bonn.de

[3]Professor, Institute of Geodesy and Geoinformation, University of Bonn, 53115 Bonn, Germany. E-mail: heiner.kuhlmann@uni-bonn.de

INTRODUCTION AND BACKGROUND

Gantry cranes are subject to an aging process due to their permanent utilization by loading of goods. To ensure safety and operation, the deflection of their main beams during the loading should be monitored periodically. In the present study, a mathematical model is developed that estimates the beam deflection at every longitudinal position of the beam dependent on two inputs:

1. the longitudinal position of the trolley that carries the load,
2. the weight of the load.

Here, the shape of the deflections in longitudinal direction of the gantry crane as well as the magnitude are of interest. These target variables should be determined with an accuracy of a few millimeters.

The gantry crane of investigation has a length of 134m (see Fig. 1). Its main beam is supported by two (support) beams. This construction type is known as a simply supported beam. The overhang is asymmetric and equals 20m on the one side and 40m on the opposite. The main beam is not a closed body but consists of three single beams that are connected by several cross sections. The cross beams on top of the main beam are also mounted asymmectrically. The asymmetric construction type is due to the function of the gantry crane: It loads and unloads containers with its trolley from container ships. The goods are loaded from the ships at the longer overhang, moved to the middle of the bridge and placed on a truck. Furthermore, the support beams are mounted on rails so that the whole gantry crane is moveable horizontally perpendicular to its longitudinal axis.

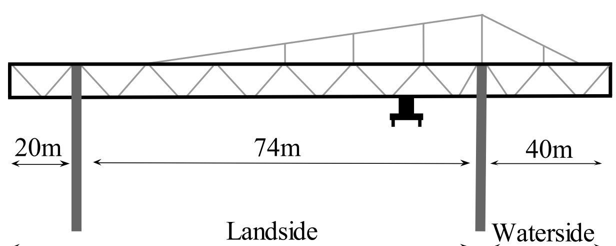

FIG. 1. Sketch of the gantry crane from side view; its main beam of investigation (black), the two vertical support beams supporting the bridge (dark gray), the different cross beams stabilizing the structure (light gray) and the trolley (black)

Based on this background, the deflection of the main beam is not expected to be symmetric even if the loaded trolley is positioned in the middle of the two support beams. The deflection of the whole beam is rather expected to be asymmetric and thus very sensitive to the position of the trolley. This has to be considered when building up a model to estimate the deflection.

Beam deflection analysis regarding gantry cranes or other bridges is widespread in literature. The corresponding studies can be grouped into the ones that are focussed on sensors and processing techniques to measure and analyze the deflection with high accuracy, precision and spatial resolution. The second group of studies builds up new models or implements existing models to mathematically and physically describe the deflection of the bridge.

Concerning the first group, promising methods are, e.g. digital image correlation (Yoneyama et al. 2007) or the sampling moiré method (Ri et al. 2012). Furthermore, ”Grating Eddy Current Sensors” (Lü et al. 2012) or range cameras (Lichti et al. 2012) are proposed. They are claimed to gain better results than total station measurements, dial gauges or laser displacement meters. Disregarding these studies, using laser scanners for measuring beam deflection is also common as an alternative to digital photogrammetry or other techniques (Gordon and Lichti 2007). The resulting benefit in accuracy and spatial resolution is shown in several studies (Rönnholm et al. 2009; Lee and Park 2011). And yet other studies install multi-sensor systems to monitor the whole bridge dependent on several inputs as wind, traffic or temperature (Wegner et al. 2006; Bogusz et al. 2012). Here, e.g. the GPS, accelerometers and inclinometers are used.

In the second group, finite element models are often considered to model the bridge (Gerdemeli et al. 2010; Pinca et al. 2009). Upon this, the strength of the bridge's structure can be analyzed (Pinca et al. 2010). This strength degrades with time and use as it can be seen in a long-run behavior analysis of a reinforced concrete beam (Seidel 2009). Moreover, the safety of gantry cranes is analyzed with a sensitivity analysis (Castillo et al. 2003) to optimize its design (Castillo et al. 2008). The assembled models are mostly based on physical parameters that describe e.g. the stress of the bridge. Thus, the physical behavior including e.g. material parameters, the weight of the bridge and its detailed structure has to be known.

The mentioned studies of the first group are combined in the fact, that the beam deflection is analyzed for a specific load (and its corresponding weight) that acts at one specific position on the bridge. When moving the load on the bridge or when changing its weight, a new deflection curve is measured or analyzed. Mostly, no mathematical model is build up to connect the weight of the load and its position on the bridge to the deflection curve. This could be done with the finite element models of the second group. However, these models commonly suffer from the integration of physical material parameters and the resultant uncertainty due to their estimation. Thus, both groups do not allow an analysis of the deflection dependent on the loading and its position simply based on geometrical measurements without knowledege of the crane's physical structure.

A model combining the desired requirements is multi-dimensional. In general, multi-dimensional, i.e. spatial, deformation analysis in engineering geodesy is based on approximating some model to spatial observations. Afterwards, the deviations regarding another epoch or a reference can be analyzed. Usually, these models are geometric primitives as, e.g. paraboloids (Holst et al. 2012), if the geometry is known. Otherwise, if the geometry is not known, e.g. bivariate polynomials (Holst et al. 2013) or freeform surfaces as splines and NURBS (Vezočnik et al. 2009) are often used.

The identification of an appropriate model suited for the here presented application of deflection analysis is the main aspect of the present study. The challenge is to adapt this model to the input of only limited measurements and sensors as uniaxial inclinometers and tacheometers without any prior knowledge about the crane's structure. This results in a model being non-parametric following geodetic terminology of deformation analysis (Welsch and Heunecke 2001).

In the following sections, the sensors and the measurement concept are proposed. Afterwards, the model for analysis of the beam deflection is presented in four steps. Finally, the results are analyzed and discussed.

MEASUREMENT CONCEPT AND PRE-PROCESSING

The measurement concept has to consider the target variables of the deflection analysis: These are (1) the shape of the deflections in longitudinal direction of the gantry crane as well as (2) the magnitude of the deflection with an accuracy of a few millimeters. Thus, the whole main beam has to be observed to achieve the desired target. Furthermore, the used sensors have to be accurate enough to enable an analysis of the deflection with the desired accuracy of a few millimeters.

Additionally, the following main aspects have to be considered for the observation of the beam deflection:

1. The main beam is not constructed as a closed body but consists of three single beams that are connected by several cross sections.
2. Cross beams are mounted above the main beam (height of a few meters).
3. The gantry crane moves horizontally on a rail.
4. The usual operation chart of the gantry crane cannot be interrupted for the measurements.
5. The expected deflections are in the range of a few centimeters.

Upon these circumstances, several instruments cannot be considered: Laser scanners or photogrammetric sensors without object point definition suffer from aspects 1 and 3. Even if the object is signalized by defined targets, the changing distance from a fixed station of the sensors to the object could be a problem. Furthermore, dependent on the scanning frequency, aspect 4 can also be critical for laser scanner measurements. This is, because the positions of the trolley and the crane need to be assumed as being constant during the measurements. GPS with antennas fixed at several positions on the main beam would be unaffected from these aspects. Nevertheless, because of aspect 2 and the resulting significant multipath effects that cannot be separated from the real deflection, GPS also has to be dismissed.

Rather five uniaxial inclinometers (”Zerotronic 10” by Wyler), working capacitively with a pendulum, and two tacheometers (”TCA 2003” and ”TPS 1101” by Leica Geosystems) were used. The uniaxial inclinometers were positioned along the beam of the gantry crane so that they were able to measure the tilt in longitudinal direction (Fig. 2). The two outer ones were fixed above the support beams, the others equidistant inbetween so that the third one was located in the middle of the bridge. The sampling frequency was 5Hz.

The first tacheometer was stationed in front of the gantry crane on a stable position to observe the deflection of the beam in its middle position where a prism was fixed to the main beam (near the third inclinometer). The second tacheometer recorded the position of the trolley in x-direction from a station on the gantry crane itself. Thus, the relative tilt and the absolute deflection can be related to the position of the trolley along the x-axis. The two tachemoeters observed with a non-constant sampling frequency of 2–3Hz. This sampling frequency is acceptable regarding the eigenfrequency of the gantry crane of approximately 0.37Hz (see pre-processing step 2).

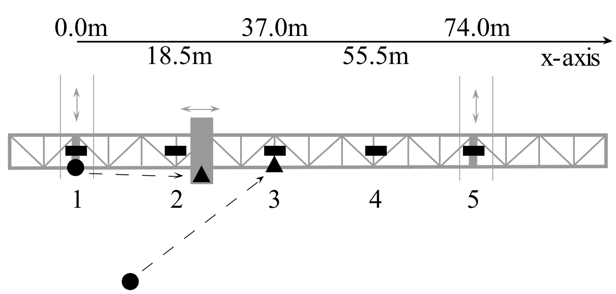

FIG. 2. Sketch of the gantry crane and the used sensors from top view; inclination sensors (black rectangles, labeled 1–5), tacheometers (black circles) and prisms (black triangles)

This measurement concept leads to the fact that the analysis was focussed on the deflection in-between the two support beams. Based on this measurement concept, 18 loading operations of the gantry crane during its normal workflow could be observed. The loaded containter weighted between 6000kg and 25000kg. Some loading operations were observed without any loaded container so that only the weight of the trolley (15000kg) deflected the bridge.

After the collection of the measurements, they are pre-processed in five steps:

1. First, gaps in the time series are filled and the tacheometer observations are interpolated to a sampling frequency of 5Hz by spline interpolation.
2. Second, the eigenfrequency of the gantry crane is detected by a frequency analysis of the measured time series. It is approximately 0.37Hz. The oscillations were initiated by the moving of the trolley, the charging and discharging of the containers. Figs. 3 and 4 show the time series of one specific loading operation of one inclination sensor as well as of the heights in the middle of the bridge measured by the tacheometer.
3. The eigenfrequency has to be filtered out of the measurements to gain the deflection or tilt, respectively, unaffected by oscillations. This is done in the third pre-processing step by a moving-average (see also Figs. 3 and 4). The moving average's length is calculated by the quotient of the sampling frequency and the eigenfrequency. At a sampling frequency of 5Hz and an eigenfrequency of 0.37Hz, the length equals approximately 13. Because this length is only an approximation, the filtering should eventually be processed more than one time per time series.

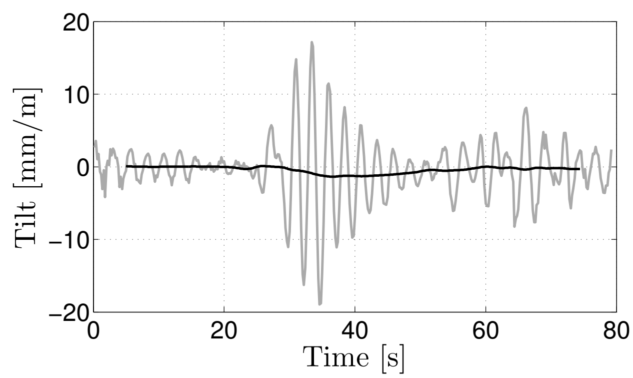

FIG. 3. Measured time series of one inclination sensor during one specific loading operation before (gray) and after filtering (black)

4. The fourth step defines one trolley position that was covered by all the 18 loading operations without any load as reference. All time series are referred to this reference regarding their temporal starting points. Furthermore, the relative observations are reset to zero at this point: From this follows that the observations being integrated in the model after pre-processing are differences to this reference.
5. In the fifth step, the measurements are finally synchronized. Afterwards, they are integrated into the deflection model of the following section.

ANALYSIS OF BEAM DEFLECTION UNDER LOAD

The discussed measurements are used to build up a mathematical model for deformation anal-

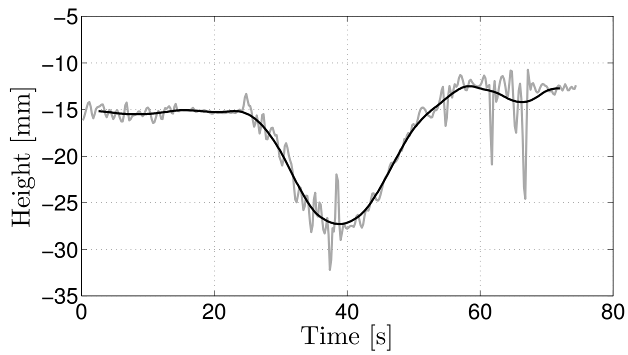

FIG. 4. Time series of the height of the main beam at its middle position, calculated by the tacheometer measurements, during one specific loading operation before (gray) and after filtering (black)

ysis. This model should be able to predict the deflection of the beam of the gantry crane at any position dependent on two inputs: the trolley position and the container weight. The target variables are the shape of the deflection as well as its magnitude that should be estimated with an accuracy of a few millimeters.

Regarding the characterization and classification of deformation models (Welsch and Heunecke 2001), the proposed one is a static model: The time component is not included and the deflections are a function of acting forces that are used as input in the model. The integration of a possible creeping into the model is not necessary based on investigated observations measured during a time of no trolley movement. This applies accordingly to a possible transition. The model is developed in four steps:

- Step 1: A one-dimensional polynomial is used to describe the beam deflection of each separate loading operation spatially dependent on the container weight and the trolley position.
- Step 2: This spatial model of the first step can be expanded to a two-dimensional model. Here, bivariate polynomials parameterize the deflections in two dimensions. The additional dimension equals the position of the trolley on the beam. This second step can be performed analogue to the first one for each separate loading operation.
- Step 3: To combine all measured loading operations into one general model in the third step, all measurements are integrated into the parameterization of the bivariate polynomial. The different loading operations are considered by a scale factor that is estimated for each loading operation in addition to the usual polynomial parameters.
- Step 4: The fourth step replaces the loading operation-dependency by a load-dependency because not the specific loading operation is of interest for the deformation but the weight of the load.

Finally, the deflection of the gantry crane can be predicted based upon several parameters dependent on the trolley position and its load. The steps 1–4 are subsequently explained in detail.

Step 1: One-dimensional Modeling

For each loading operation T, $N = 6$ observations are performed in every time step. These are five tilts by the inclination sensors ($l_{t,1}, l_{t,2}, l_{t,3}, l_{t,4}, l_{t,5}$) positioned along the x-axis (see Fig. 2) and the measured height by the first tacheometer (l_h). As described in the previous section, these observations are pre-processed: In fact, they equal differences to a defined reference. In all further investigations, these pre-processed measurements are used.

Additionally to these observations, one further measurement is regarded as deterministic input: The position of the trolley (y) along the x-axis measured by the second tacheometer. This is not included as observation because it is not integrated in the adjustment of the one-dimensional modeling: one separate polynomial is estimated for each trolley position y of each loading operation T. Thus, the modeling of each deflection curve $z(x)$ depends on the loading operation T and the trolley position y along the x-axis: $z_{y,T}(x)$.

For the parameterization of the deflection curves, a polynomial of order a_x – as will be explained later in this subsection – is chosen:

$$z_{y,T}(x) = \sum_{i=1}^{a_x} p_i \cdot x^i. \tag{1}$$

Here, $\mathbf{p} = [p_1, ..., p_{a_x}]^T$ are the parameters that are to be estimated. Because the offset of the polynomial at $x = 0\text{m}$, which is above the left support beam, is assumed to be zero $z_{y,T}(x = 0\text{m}) = 0\text{m}$, the parameter p_0 can be neglected. This reduces the number of parameters to a_x. For the estimation of these parameters, the observations

$$\mathbf{l} = [l_{t,1}, l_{t,2}, l_{t,3}, l_{t,4}, l_{t,5}, l_h, l_o]^T \tag{2}$$

are used. Here, the pseudo-observation l_o forces the end of the polynomial, which is above the right support beam, to zero: $z_{y,T}(x = 74\text{m}) = 0\text{m}$. This fixation of the polynomial at both positions above the two support beams leads to a more reliable estimation due to the limited number of observations. Because this pseudo-observation is stochastic as the other real observations, the polynomial is only fixed to zero at $x = 74\text{m}$ within its standard deviation σ_o. This would be different if the pseudo-observation was introduced as condition equation instead.

The corresponding covariance matrix $\mathbf{\Sigma}_{ll}$ is given by

$$\mathbf{\Sigma}_{ll} = \text{diag}\{[\sigma_t^2, \sigma_t^2, \sigma_t^2, \sigma_t^2, \sigma_t^2, \sigma_h^2, \sigma_o^2]\}. \tag{3}$$

The standard deviations are $\sigma_t = 0.1\text{mm/m}$ for the inclinometer measurements, $\sigma_h = 2.5\text{mm}$ for the measured height by the tacheometer and $\sigma_o = 0.1\text{mm}$ for the pseudo-observation. The standard deviation of the inclinometers σ_t is presumed very pessimistically. This is not based upon the precision of the inclinometer sensor itself but rather integrates the misalignment error and the uncertainty due to the oscillations of the beam. After all, all of these standard deviations are empirical assessments based on manufacturer's specifications and geometrical considerations (distances measured by tacheometer, misalignment of inclinometers).

Integrating the observations $\mathbf{l}$ and their covariance matrix $\mathbf{\Sigma}_{ll}$ into a least-squares fit in a linear Gauss-Markov model (Koch 1988)

$$\mathbf{l} + \hat{\mathbf{v}} = \mathbf{A}\hat{\mathbf{p}}, \tag{4}$$

the parameters are estimated by

$$\hat{\mathbf{p}} = \left(\mathbf{A}^T \boldsymbol{\Sigma}_{ll}^{-1} \mathbf{A}\right)^{-1} \mathbf{A}^T \boldsymbol{\Sigma}_{ll}^{-1} \mathbf{l}. \tag{5}$$

The estimated residuals are defined by $\hat{\mathbf{v}}$ and the design matrix by $\mathbf{A}$. The design matrix connects the observations $\mathbf{l}$ with the functional model of the deflection curve. For the measured height l_h and the pseudo-observation l_o, this functional model equals Eq. (1). The functional model for the measured tilts $l_{t,1} - l_{t,5}$ equals the derivative of the deflection curve of Eq. (1): $\partial z_{y,T} / \partial p_i$.

For the one-dimensional beam modeling, the order a_x of the polynomial has to be determined. This is done both by geometrical and statistical investigations and leads to an optimal order of $a_x = 5$ independent from the position of the trolley and the specific loading operation. Fig. 5 shows the coefficient of determination – also known as goodness of fit – of three different polynomial orders (4,5,6) corresponding to one specific loading operation. A coefficient near 1 attests a good fit whereas a value near 0 suggests a revision of the fit (Borradaile 2003). As can be seen, order 5 improves the fit significantly compared to order 4, but order 6 does not lead to a significant enhancement of the fit anymore.

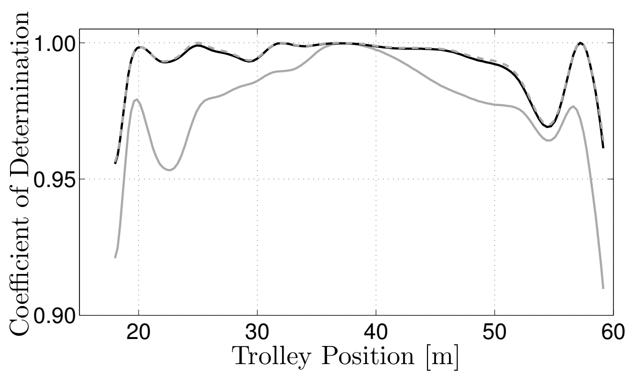

FIG. 5. Coefficient of determination of polynomial approximation of one specific loading operation; polynomial of order 4 (gray), 5 (black) and 6 (gray dashed)

Fig. 5 reveals the fact that the approximation is best if the trolley is positioned in the middle of the bridge at 37m. The quality of the fit seems to decrease when moving the trolley away from the middle. But because the coefficient of determination is also sensitive to the magnitude of the deflection, further parameters should be considered when judging the quality of the fit. These can be the estimated residuals $\hat{\mathbf{v}}$ and the a posteriori variance of the adjustment $\hat{s}^2$. Both support the previous result (graphs not shown here): The quality of the fit decreases when moving the trolley to the endings of the beam. These findings are due to two reasons: (1) The used polynomial model gets less adequate for parameterization if the trolley moves to the endings of the crane. (2) The signal-to-noise ratio gets worse when moving the trolley to the endings because the deflection's magnitude decreases. Thus, to gain reliable results by the approximation, all further investigations are limited to trolley positions between 20m to 50m.

Nevertheless, it has to be mentioned that a statistical analysis based on these values should be done carefully: Only six observations plus one pseudo-observation are integrated in each adjustment so that the redundancy of $r = N + 1 - a_x = 2$ is not very high.

The chosen polynomial model of order $a_x = 5$ is also reasonable regarding physical beam deflection theory (Beer and Johnston 1992). The deflection of a simply supported beam being depended on the trolley position is modeled based on physical parameters as a polynomial of third order (Gordon and Lichti 2007). But this model does not incorporate asymmetric overhangs as in the present gantry crane. Following, the order should be raised by two in comparison to a simply supported beam. This considers the deflection at the overhangs as well as the asymmetry of the deflection due to the overhangs of different length.

Fig. 6 shows the resulting deflection curves of one specific loading operation dependent on three different trolley positions. As can be seen, the magnitude of the deflection as well as the position of maximal deflection depends on the position of the trolley. Furthermore – as already assumed because of the asymmetric construction of the gantry crane – the deflection curves are also asymmetric to the middle of the beam.

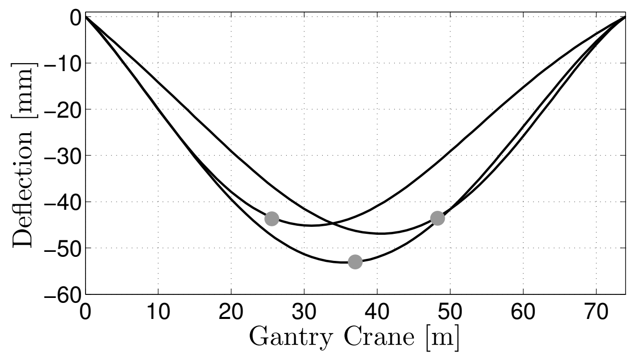

FIG. 6. Estimated polynomials (black) dependent on the trolley position (gray) of 25.5m, 37.0m and 48.5m of one specific loading operation

Step 2: Two-dimensional Modeling

So far, one deflection curve $z_{y,T}(x)$ exists for each time step (trolley position) y and each loading operation T (see Fig. 6). Now, the deflection curve $z_{y,T}(x)$ is expanded by the trolley position y along the x-axis to a two-dimensional model $z_T(x, y)$. Therefore, the polynomial is simply expanded to a bivariate polynomial (Lancaster and Salkauskas 1986)

$$z_T(x, y) = \sum_{i=0}^{a_x} \sum_{j=0}^{a_y - i} p_{i,j} \cdot x^i y^j; \qquad i + j \neq 0. \tag{6}$$

The order of this bivariate polynomial remains $a_x = 5$ in the already parameterized x-direction whereas it is increased to $a_y = 6$ in y-direction. This is due to a better coefficient of determination and will be explained later in the discussion. Eq. (6) thus consists of $(a_y + 1) \cdot (a_y + 2)/2 - 2 = 26$ parameters (Lancaster and Salkauskas 1986). The reduction by two is based on the neglection of parameter $p_{0,0}$ again as well as of $p_{6,0}$ due to the limitation of $a_x = 5$.

Now, because the position of the trolley y and the deflection along the longitudinal direction of the bridge x is integrated in one model, one combined bivariate polynomial exists for each loading operation. Nevertheless, there is still no link between the modeling and the polynomial parameters of different loading operations T.

Step 3: Two-dimensional Loading Operation-Dependent Modeling

To investigate a model that predicts the deflection of the main beam of the gantry crane, all loading operations T have to be assimilated into one single model. Otherwise, the parameters and the deflection model are only valid for one specific loading operation not being able to be generalized and predicted to other loading operations. To overcome this drawback, the polynomial parameters $p_{i,j}$ are only estimated uniquely for all loading operations. The different loading operations are incorporated by a loading operation-specific scale factor M_T. This suggests that the shape of each estimated two-dimensional deflection model $\hat{z}_T(x, y)$ stays the same, independent from the specific loading operation – only the magnitude of the deflection varies by the modeled scale factor M_T:

$$z(x, y, T) = M_T \cdot \sum_{i=0}^{a_x} \sum_{j=0}^{a_y - i} p_{i,j} \cdot x^i y^j; \qquad i + j \neq 0. \tag{7}$$

This model includes the measurements of all 18 loading operations T and all trolley positions y to estimate the deflection. It consists of $(a_y + 1) \cdot (a_y + 2)/2 - 2 + 18 = 44$ parameters including one scale factor M_T for each loading operation T. Because of the scale factors, Eq. (7) is nonlinear. Thus, the parameters are estimated iteratively by a nonlinear Gauss-Markov model with a Taylor series approximation (Mikhail and Ackermann 1976) instead of using Eq. (5).

Assuming a scale factor of $M_T = 1$, the corresponding estimated bivariate polynomial is shown in Fig. 7. As already described, the trolley positions are limited to the interval of 20m to 50m because of an analysis of the goodness of fit (see Fig. 5), the estimated residuals $\hat{\mathbf{v}}$ and the a posteriori variance $\hat{s}^2$. Analogue to Fig. 6, the deflection is asymmetric to the middle of the beam because of the asymmetric construction of the gantry crane.

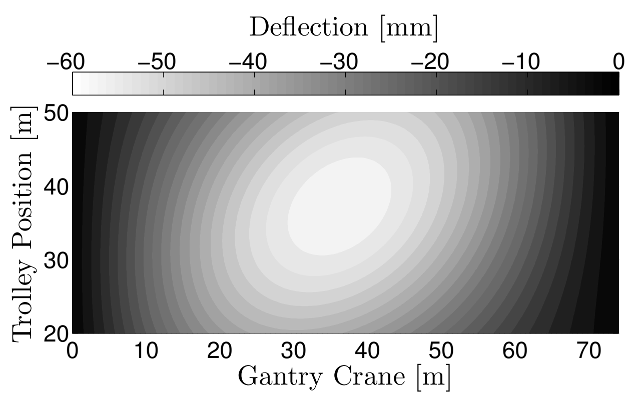

FIG. 7. Two-dimensional load-dependent polynomial model of the beam deflection assuming a scale factor of $M_T = 1$

Step 4: Two-dimensional Load-dependent Modeling

So far, all measurements are included in the bivariate polynomial model (7) to estimate the deflection $\hat{z}(x, y, T)$ along the x-axis dependent on the trolley position y and one of the 18 loading operations T. The individual container weights of the different loading operations have been neglected until now. The estimated scale factors are thus not related to the container weight.

Thus, instead of using a scale factor for each loading operation, the container weights of the loaded goods during each loading operation should be integrated. Tab. 1 shows these container weights. The weight of 0kg indicates that no container was loaded so that only the trolley was moved without any goods. As can be seen, the 18 estimated scale factors could have been grouped to seven classes of weights so that only 7 scale factors would have been estimated. While this would lead to less scale factors of higher precision, the final model quality resulting from the present step of modeling should stay unaffected from this modification.

Container Weight [kg]	0	6000	8000	15000	20000	22000	25000
Number of Loading Operations	5	2	2	2	3	2	2

TABLE 1. Number of loading operations that were traced and the corresponding container weights (total number of 18 loading operations)

The relation between the scale factors and the container weights of the loading operations is presented in Fig. 8. Here, to focus on the maximal deflection dependent on the container weight, the scale factors are multiplied by the maximal deflection using model (7) and a scale factor of $M_T = 1$. This relation can be approximated by a polynomial of order 2 and three parameters b_0, b_1, b_2 with a very high coefficient of determination. The residuals between this model and the container weights are mostly due to the uncertainty of the developed model.

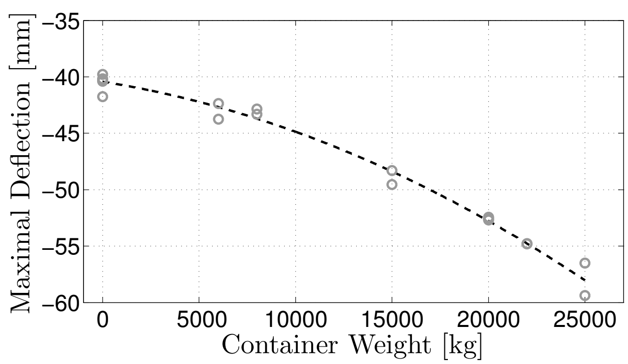

FIG. 8. Estimated scale factors multiplicated with the maximal deflection (gray) and the approximated polynomial of order 2 (black)

Following these investigations, the polynomial parameters b_0, b_1, b_2 are now integrated into the model instead of using one scale factor M_T for each loading operation:

$$z(x,y,g) = (b_0 + b_1 \cdot g + b_2 \cdot g^2) \cdot \sum_{i=0}^{a_x} \sum_{j=0}^{a_y - i} p_{i,j} \cdot x^i y^j; \qquad i + j \neq 0. \tag{8}$$

This enables a modeling dependent on the container weight g instead of the specific loading operation T where the container weights were disregarded. Furthermore, the number of parameters is reduced to $(a_y + 1) \cdot (a_y + 2)/2 - 2 + 3 = 29$.

As a conclusion, after estimating the 29 parameters $\hat{\mathbf{p}} = [\hat{p}_{0,1}, ..., \hat{p}_{5,1}]^T$ and $\hat{\mathbf{b}} = [\hat{b}_0, \hat{b}_1, \hat{b}_2]^T$, the deflection of the main beam of the gantry crane can be predicted by $\hat{z}(x,y,g)$, Eq. (8), at any longitudinal position x. Inputs are the container weight g and the trolley position y.

DISCUSSION

The presented analysis shows that the parameterization of the deflection of the main beam of the described gantry crane is possible. A bivariate polynomial model of orders $a_x = 5$ and $a_y = 6$, expanded by an additonal regression, enables the estimation of the deflection at any position of the main beam dependent on the container weight and the trolley position. The model is developed in four steps. Each step is reasonably chosen but some points should be analyzed deeper. This is done in the present section.

Measurement Concept

It is obvious that a more accurate model could be built up upon an improved measurement concept. E.g. more inclination sensors could have been fixed on the bridge to gain more continuous observations and a higher spatial resolution of the deflection. Also more prisms could have been used as absolute reference. Alternatively, other sensors could have been considered. Nevertheless, this was not feasible because of the surrounding conditions. E.g. the measurements had to take place during the gantry crane's normal workflow. Based on this, the present study was not focussed on an improvement of the measurement concept. Rather, the task was to find an appropriate model that can be processed simply upon the given observations.

One-dimensional Modeling with Polynomial

In the first step, the measurements are approximated by the polynomial model of Eq. (1). The order $a_x = 5$ was chosen because of the coefficient of determination. An analysis of the residuals of the fit has not been given so far. Figs. 9 and 10 show the estimated residuals $\hat{\mathbf{v}} = \mathbf{A}\hat{\mathbf{p}} - \mathbf{l}$ (see Eq. 4) of all estimated polynomials corresponding to the varying trolley positions of one specific loading operation. As can be seen, these residuals depend on the trolley position. Thus, the quality of the polynomial approximation depends on the trolley position.

Regarding the inclination sensors, the best approximation is given if the trolley is positioned near the middle of the beam at $x = 37$m. This was already investigated based on the coefficient of determination (see Fig. 5). Furthermore, the choice of limiting the analysis to trolley positions between 20m to 50m can be confirmed. Nevertheless, especially the residuals of the measured heights by the tacheometer are between -1mm and -2mm. This implies, as the tacheometer is the only absolute reference, that the whole gantry crane sinks and deforms under load. This assumption can be encouraged by the fact that one support beam is a pendulum beam so that the main beam is supported flexible. The inclinometer sensors are not sensitive to this flexible structure because they are reset to zero in the pre-processing before each new loading operation. The absolute tilt is thus eliminated in the pre-processing.

Disregarding these facts, the polynomial model still seems to be a sufficient approximation of the measurements. Investigations and results concerning the approximation of a traverse or splines do not lead to better results.

Choice of Polynomial Order for Two-dimensional Modeling

When expanding the one-dimensional model to a two-dimensional model, the order of the polynomial in y-direction $a_y = 6$ is larger than the one in x-direction $a_x = 5$. This is due to a better coefficient of determination with order $a_y = 6$. From this follows that – although the one-dimensional approximation is sufficient with order $a_x = 5$ – the expansion of the polynomial

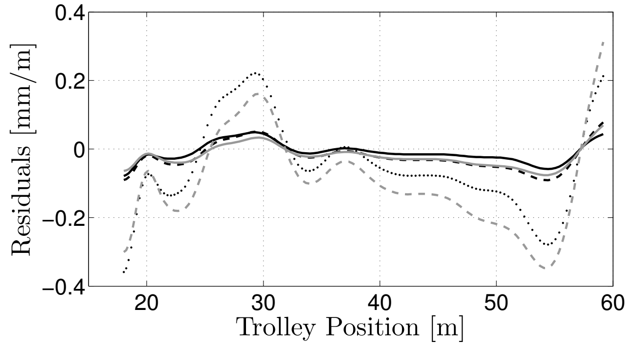

FIG. 9. Residuals of inclinometer sensors of polynomial approximations corresponding to the varying trolley positions of one specific loading operation; first (black lined), second (black dotted), third (black dashed), fourth (gray dashed) and fifth (gray lined) inclinometer

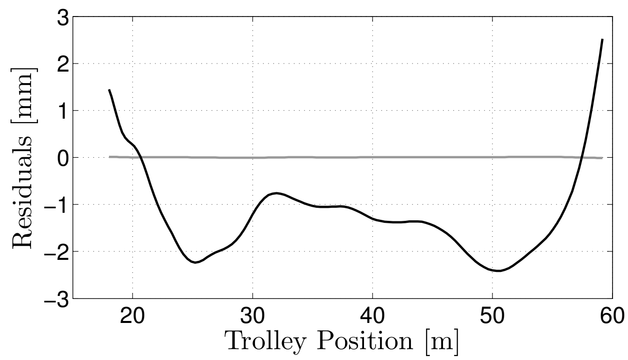

FIG. 10. Residuals of calculated heights (black), measured by tacheometer, and pseudo-observations ($z_{y,T}(x = 74$m$) = 0$, gray) of polynomial approximations corresponding to the varying trolley positions of one specific loading operation

by the trolley position y to a bivariate approximation cannot be modeled sufficiently with order 5 anymore.

This can also be confirmed by a further analysis: The one-dimensional estimated polynomials $\hat{z}_{y,T}(x)$, Eq. (1), of one specific loading operation are estimated for all trolley positions y. These can be compared to the two-dimensional loading operation-dependent model $\hat{z}(x, y, T)$, Eq. (7), of the same loading operation. Fig. 11 shows the residuals between these two models. As can be seen, the residuals are within $\pm$ 4mm. The systematics in the residuals are due to the distinction of the different loading operations only by one single scale factor.

When performing the same comparison with order 5 in y-direction of the two-dimensional model, the deviations increase significantly. An analysis of the other loading operations leads to similar results. Both these findings, the better coefficient of determination and the smaller residuals, suggest using an order of $a_y = 6$ for the two-dimensional models of Eqs. (6), (7) and (8).

It should be mentioned that the here examined specific loading operation is not part of the parameter identification process when performing this comparison. Otherwise, the specific loading

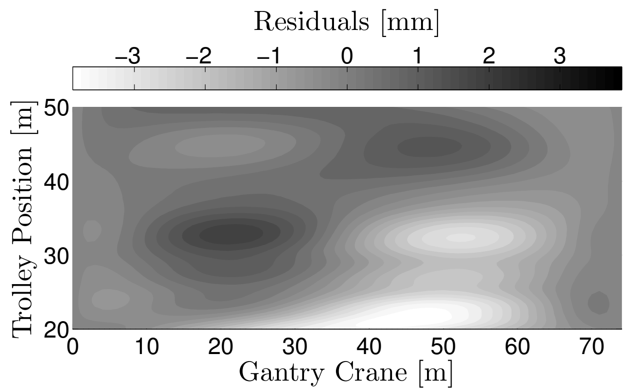

FIG. 11. Comparison between the one-dimensional model, Eq. (1), estimated at different trolley positions, and the two-dimensional loading operation-dependent model, Eq. (7), of the same loading operation

operation would be highly correlated with the model parameters and the comparison would lose its validity. Thus, only 17 loading operations are used for parameter estimation and one is used for validation during this comparison. The estimated model parameters are not affected significantly by this procedure.

Process of Parameter Estimation

Theoretically, the two-dimensional load-dependent model $\hat{z}(x, y, g)$ could be estimated simply upon the original measurements. Then, because only one adjustment model is used, no variance propagation would be necessary. Nevertheless, the two-dimensional load-dependent model is processed step by step in this study because it was developed in these individual steps. First, the original measurements are approximated by the one-dimensional model $\hat{z}_{y,T}(x)$. Upon this, the deflection curves can be build up. They are used afterwards to approximate them by the two-dimensional loading operation-dependent model $\hat{z}(x, y, T)$, before the regression parameters $\hat{b}_0, \hat{b}_1, \hat{b}_2$ are estimated in the last step for the two-dimensional load-dependent model $\hat{z}(x, y, g)$. Between these steps, an integrated variance propagation is necessary (Mikhail and Ackermann 1976). Thus, based upon the covariance matrix $\mathbf{\Sigma}_{ll}$ of the original measurements of each loading operation (Eq. 3), the variances are propagated according to the mentioned individual steps.

Some measurements used in the developed model during the different steps are assumed to be error-free (see Tab. 2). In fact, this assumption is not applicable regarding especially the accuracy of the container weights. This value of approximately 1000kg is very low because the weight was measured only by a balance on the trolley during the gantry crane's normal workflow. It was not possible to determine the weight more precisely. Following, these uncertainties due to the measurements should technically be considered in the modeling. This can be done by a Gauss-Helmert model (Mikhail and Ackermann 1976) or a total least-squares model (Markovsky and Huffel 2007) where errors are assumed to be in all variables. However, the developed model is not sensitive to the simplifications when neglecting these errors. This was proved in every step of modeling; no significant effect could be observed.

CONCLUSION

The analysis presented a bivariate polynomial model that is able to estimate and predict the

Step	Measurement	Instrument	Accuracy
1	position of inclinometer sensors on x-axis	t (unique)	mm–cm
1	position of prism on x-axis	t (unique)	mm–cm
2	trolley position on x-axis	t (continuous)	mm–cm
4	container weights	b (unique per load)	1000kg

TABLE 2. Error-free assumed measurements that are used as input in the different steps of modeling; corresponding instruments (t = tacheometer, b = balance on trolley) and approximate accuracies

deflection of the main beam of a gantry crane. Based upon the measurements of five inclinometers and one tacheometer, the deflection curve is modeled two-dimensionally. Input parameters are the container weight and the position of the trolley. The model is defined by 29 estimated parameters whereupon 26 polynomial parameters describe the shape of the deflection and 3 regression parameters its magnitude. Based on geodetic terminology for deformation analysis (Welsch and Heunecke 2001),

- the container weight and the trolley position equal the system input,
- the polynomial model equals the object behavior and
- the deflection, measured by inclination and vertical displacement, equals the system output or system reaction.

The system input as well as the system output are measured only to a certain precision. The container weight and the trolley position are measured by a balance on the trolley and a tacheometer, respectively. Especially the accuracy of the balance is very low. Nevertheless, the simplification of neglecting these uncertainties and regarding the input as deterministic does not lead to a loss in accuracy of the model.

The prediction of the deflection fulfills the desired accuracy: Deviations between the deflection curves of a specific loading operation, estimated directly based on the measurements, and the final model are in the range of a few millimeters. To ensure the independency between the model and the reference, this specific loading operation is not part of the parameter estimation during this comparison. Thus, the goal of estimating the deflection, i.e. its shape and magnitude, with an accurcay of a few millimeters upon several input factors is reached.

The proposed model quality is limited to the central section of the gantry crane between 20m to 50m. The outer sections cannot be approximated by the polynomial model with the same quality. This limitation could possibly be avoided when using a finite element model of the gantry crane. This model could have handled the special asymmetric geometry of the crane better if its structure had been known in detail. However, the developed model is sufficient for the proposed goal of the present study.

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