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Contents

Introduction

Contracts are written to commit economic agents to a course of action that does not necessarily coincide with their preferred choice in all possible future contingencies. Hence, the decision whether to join a contract involves a careful evaluation of the overall expected, personal benefit. A recent literature analyzes profit-maximizing contracts for scenarios in which agents can acquire relevant information during that deliberation. The first two chapters of this dissertation contribute to this literature: I demonstrate the benefits of stochastic contracts and sequential screening mechanisms.

To this end, I use a principal-agent model that deals with bilateral trading. Both parties do initially not know the agent's preferences over possible trade agreements. While deliberating whether to accept a contract, the agent can spend resources to investigate the state. The agent's preferences might of course be relevant for the principal as well, given that she must decide about the terms of trade. But contractual clauses on information acquisition or its truthful transmission are not court-enforceable; these acts are unobservable and unverifiable, respectively. Moreover, the principal herself does not have the possibility to investigate the state.

From a design perspective, such contractual relationships involve the following problem. Initially, the agent only has an ex-ante outside option—the possibility to decide about his participation without any information about his preferences. Through precontractual investigation, however, he can acquire an ex-interim outside option—the possibility to decide conditional on his findings. Indeed, contracts that allocate all expected surplus to the principal are generally unprofitable for the agent in some states, as the terms of trade cannot directly condition on the agent's preferences. The principal has basically two alternatives. One possibility is to offer a separating contract (i.e., a contract which encourages the agent to investigate the state and reveal his findings).

But then she cannot fully appropriate the expected surplus because the contract must be ex-interim individually rational to ensure the agent's participation. Alternatively, the principal could propose a pooling contract (i.e., a contract which does not encourage investigation). In that case, she may appropriate the expected surplus, but only if investigation costs are sufficiently high relative to the traded quantity. Otherwise, the agent would find it too risky to accept the contract without having checked his payoff in advance. Thus, the principal must generally make a trade-off between efficiency and surplus extraction.

Chapters 1 and 2 describe optimal solutions to this trade-off. In Chapter 1, I consider the case that information gathering can be postponed until after signing. It turns out that the principal does actually not search for a profit-maximizing contract among the pooling and separating ones. Specifically, contracts take the form of a lottery, the lots being a pooling and separating *schedule*. After signing, one of the two schedules is implemented according to some probability distribution. The key insight is that such stochastic contracts can reconcile full surplus extraction with incentives for information gathering, for unless the separating schedule is implemented with certainty, precontractual investigation still comes at a price.

Chapter 2 qualifies a finding by Crémer and Khalil (1992), according to which precontractual investigation should never be encouraged if the agent anyway learns his preferences after signing, before the contract is carried out. I show that this result does typically not extend beyond the case in which precontractual investigation removes *all* uncertainty about the unknown state. In a nutshell, the principal offers a separating contracts if and only if she benefits from sequential screening. Sequential screening mechanisms potentially facilitate her trade-off because they entail choice under uncertainty for the agent unless he can acquire perfect information.

Chapter 3, on the other hand, offers an explanation as to why incentive pay sometimes depends on subjective performance evaluations even though comprehensive objective appraisal systems seem feasible. Subjective evaluations involve the obvious problem that compensation is left at the discretion of supervisors, who might be biased for various reasons. I argue that subjectivity can be a desirable property of a performance measure. Specifically, employers might resort to subjective evaluations in order to withhold information from their workers. I furthermore argue that this motive is particularly strong under circumstances where the credibility issue associated with subjective measures can be solved at low costs. My explanation suggests that subjective performance evaluations are used at early stages of long-term employment relationships in which details that affect the worker's productivity on the job are highly uncertain and can be inferred from performance.

Chapter 1

Endogenous Information and Stochastic Contracts

Abstract. A growing literature analyzes profit-maximizing contracts for situations in which agents can acquire private information *before* they decide whether to join the contract. It is conjectured that the results also apply to the more natural scenario where information can be acquired *either before or after* signing. This chapter shows that, in fact, the latter scenario is more favorable for the principal. Using stochastic contracts, she can induce information acquisition with some probability and yet appropriate the generated surplus.

1.1 Introduction

A growing literature (Crémer et al., 1998a, Shi, 2012, and Szalay, 2009) analyzes profit-maximizing contracts for situations with the following characteristics: (1) At the moment when the principal offers the contract, agents do not possess private information, but (2) each agent can covertly investigate his payoff from the contract *before* he decides whether to participate.[1] It is conjectured that the results also apply to the more natural scenario where information can be acquired *either before or after* the contract is

[1]A key objective of this literature is to examine comparative statics with respect to investigation costs. The standard case of adverse selection, where agents are already privately informed at the outset, obtains for zero investigation costs.

signed.[2] This chapter shows that, in fact, the difference is relevant. The latter scenario features additional implementable outcomes, which might be more profitable for the principal.

Consider the following procurement relationship. A principal demands customized parts which the agent can produce. The costs at which the agent operates are initially unknown to both parties. However, the agent can spend resources to learn the exact state before he processes the order. His findings might clearly be relevant for the principal, too, as she could tailor her demand more efficiently if she knew the agent's costs. But contractual clauses on information acquisition or its truthful transmission are not court-enforceable; these acts are unobservable and unverifiable, respectively. Moreover, the principal herself does not have the possibility to investigate the state.

Crémer et al. (1998a) study the optimal contract for this relationship under the assumption that the agent can only acquire information between contract offer and signing.[3] In that case, the agent has the (costly) option to check his payoff from the contract before he decides whether to join it. More importantly, if the contract provides incentives which induce information acquisition, the agent knows his payoff at that date. The principal has two alternatives: One possibility is to offer a separating contract (i.e., a contract which encourages the agent to investigate his costs and reveal his findings). But then she cannot extract the entire surplus because the agent has private information at signing.[4] Alternatively, the principal could propose a pooling contract (i.e., a contract which does not encourage information acquisition). In that case, she may appropriate the generated surplus, but only if investigation costs are sufficiently high relative to output. Otherwise, the agent would find it too risky to accept the contract without having checked his payoff. Thus, the principal must generally make a trade-off between

[2] See footnote 1 in Crémer et al. (1998a).

[3] Szalay (2009) considers the same scenario but assumes that the precision of information depends on the investigation effort. Shi (2012), in contrast, is concerned with optimal auction design and, thus, allows for multiple agents. In his model, the precision of information is also endogenous, but he uses a different ordering than Szalay.

[4] More precisely, the principal cannot extract the entire surplus if the agent is to produce some minimum output regardless of his costs.

efficiency and surplus extraction.

I depart from Crémer et al. (1998a) in that the agent can investigate his costs in my model either between contract offer and signing, *or afterwards.* It turns out that the principal must make the same trade-off but has more choices. First of all, the relevant class of contracts differs. The principal does not search for an optimal contract among the pooling and separating contracts. She can restrict herself to contracts which will never induce the agent to acquire information before signing. Specifically, her contract takes the form of a lottery—the lots being a pooling and a separating *schedule.* After signing, one of these two schedules is implemented according to some probability distribution.

Degenerate lotteries are essentially identical to the pooling and separating contracts. Non-degenerate lotteries, on the other hand, can implement otherwise infeasible schedules and thus extend the principal's choice set. In a nutshell, this is because such stochastic contracts entail uncertainty for the agent as to whether investigation costs will be due after signing. Put differently, since the separating schedule is not implemented with certainty, precontractual investigation still comes at a price. Therefore, stochastic contracts only need to satisfy the weak participation condition that applied to the pooling contracts. The principal may consequently appropriate the generated surplus of both schedules if the price of precontractual investigation is sufficiently high relative to output. I show that optimal contracts are stochastic under particular circumstances. Hence, the principal is better off when information can be gathered either before or after signing, rather than only at the first date.

My analysis provides some insight into the fact that procurement contracts frequently entitle the project owner to submit *change orders* as to contract amount, deadlines, designs, etc. (see Bajari and Tadelis, 2001). U.S. Government contracts, for example, must generally include "a changes clause that permits the contracting officer to make unilateral changes, in designated areas, within the general scope of the contract" (FAR 43.201a). The use of change orders is often explained with bad project management.[5] However, it introduces uncertainty for the contractor about the services

[5]E.g., a panel on the Los Angeles Community College District's rebuilding program "found that

he will ultimately provide, which makes it difficult to examine in advance whether the stipulated compensation is adequate. In this respect, contracts with changes clauses resemble the stochastic contracts in my analysis.

Formally, the benefits of stochastic contracts are due to a relaxed participation condition. Unlike with separating contracts, the agent may end up with a loss and yet acquires information with some probability. The participation constraint is in fact endogenously determined in the present situation. On the one hand, the contract must be acceptable for the agent from an ex-ante perspective (i.e., in expectation over all levels of production costs). On the other hand, it must not be too unprofitable from an ex-interim perspective (i.e., for each particular cost level) to rule out precontractual investigation. Now, since investigation costs are due anyway for the agent if the separating schedule will be implemented, a different randomization can result in a different participation constraint. This effect is clearly not related to the role of stochastic contracts as screening device in the standard case with exogenous information (see Maskin and Riley, 1984 and Strausz, 2006). In equilibrium, any contractual randomization resolves before the agent learns his type. Moreover, stochastic terms of trade cannot improve screening in my model because all agent types have a linear utility function. In different contexts (e.g., collective decision making), it has been noted as well that endogenous information may provide a rationale for the use of stochastic mechanisms (see Gerardi and Yariv, 2008, Gershkov and Szentes, 2009, and the survey by Bergemann and Välimäki, 2006).

Various papers analyze profit-maximizing contracts for related scenarios with endogenous information.[6] A key insight is that the timing of information gathering and contracting plays an important role (see Bergemann and Välimäki, 2006). In particu-

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more than a quarter of the work had been changed after contracts were awarded and designs were completed. Because of that, the [panel's] report said, the way the district handles and manages change orders was 'fraught with waste and inefficiency and should be abandoned'" (Los Angeles Times, January 12, 2012).

[6]The seminal paper of Bergemann and Välimäki (2002), on the other hand, is concerned with endogenous information and the implementation of efficient outcomes. In their model, contract design is not restricted by participation conditions.

lar, Crémer et al. (1998b), Crémer and Khalil (1994) as well as Kessler (1998) consider principal-agent relationships under the assumption that the agent can only acquire information *before the contract is offered*.[7] It is thus impossible for the principal to affect the agent's decision with her contract proposal. In the seminal paper of Lewis and Sappington (1997), the agent can only investigate the state *between contract offer and signing*. However, my result does not apply since the principal wants to induce information acquisition with certainty in their model. More closely related is the seminal analysis of Crémer and Khalil (1992), where the agent can acquire information between contract offer and signing but obtains it freely afterwards. Like in my model, information gathering before signing is just a rent-seeking activity and is deterred by the principal.[8] Comparative statics show that the principal benefits from higher investigation costs. Applied to reality, this insight suggests that she might have an interest to conceal some details about the agent's tasks before signing if uncertainty increases investigation costs. In fact, stochastic contracts serve exactly this purpose in my model. Finally, Crémer et al. (2009) as well as Krähmer and Strausz (2011) analyze the case that information can only be collected *after the contract is signed*. Here, the principal does not need to make a trade-off between efficiency and surplus extraction. This scenario is thus most favorable for her.

This chapter is organized as follows. The next section presents the model. Section 1.3 derives the main result, according to which stochastic contracts can outperform the deterministic ones. Section 1.4 concludes. Most of the proofs and some auxiliary results are relegated to Appendix A1. Appendix A2 shows the benefits of stochastic contracts in a more general model.

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[7]Crémer and Khalil (1994) and Crémer et al. (1998b) assume furthermore that the agent learns his costs freely after signing if he decides against precontractual investigation.

[8]Compte and Jehiel (2008) qualify this finding of Crémer and Khalil (1992) and argue that if several agents compete for the contract, the principal possibly induces precontractual investigation to find an agent with appropriate skills.

1.2 The model

I use a variant of the procurement model by Crémer et al. (1998a). Specifically, the agent can produce an arbitrary quantity of a good which the principal wants to consume. Given output q, marginal production costs β and transfer t, the agent's payoff is $t-\beta q$. The principal's payoff is $V(q)-t$, where V is strictly concave, continuously differentiable and satisfies $V'(0)=\infty$ and $V'(\infty)=0$.

Marginal production costs are a random variable, whose realization is low ($\beta=\underline{\beta}$) with probability $p\in(0,1)$, or high ($\beta=\overline{\beta}$), where $0<\underline{\beta}<\overline{\beta}$. (Appendix A2 demonstrates that the restriction to two levels of productions costs is not significant.) I will sometimes refer to the agent in state $\underline{\beta}$ as the *efficient type* and in state $\overline{\beta}$ as the *inefficient type*. Initially, both parties do not know which state obtains. However, the agent can acquire information, which effectively allows him to learn the state at investigation costs of $\gamma>0$. Information acquisition is unobservable, and the agent's findings cannot be verified by the principal or any third party. If the agent does not investigate, he will not learn his production costs during the interaction.

The principal has full bargaining power. At some initial date, she offers a contract to specify the quantity $q\geq 0$ to be delivered in exchange for a transfer $t\in\mathbb{R}$, both possibly contingent on arbitrary forms of communication. The principal can credibly commit to obey her contractual obligations and to decline any renegotiation. If the agent accepts the contract (i.e., if he signs it), its terms become binding for him; otherwise, the relationship ends without trade.

An important property of the model is that the agent can acquire information either after the contract is signed, or already before. This is the main difference to Crémer et al. (1998a), where information acquisition is only possible between contract offer and signing.[9] Using their terminology, I will often refer to information acquisition before signing as *precontractual investigation*.

In detail, the model has the following timing:

1. Principal offers contract, and nature selects production costs

[9]Beyond this, the two models only differ in that the agent's type set is binary in the present model whereas Crémer et al. (1998a) assume a continuum.

2. Agent can acquire information
3. Agent accepts or rejects contract
4. If contract accepted: Agent can acquire information

   If contract rejected: Relationship ends without trade

Efficiency.—From an efficiency perspective, the agent should bear the costs of information acquisition if and only if it holds that

$$E[\max_q V(q) - \beta q] - \gamma \geq \max_q V(q) - E[\beta]q.$$

The inequality means that given efficient production (i.e., $\underline{q}^* = V'^{-1}(\underline{\beta})$ for low costs, $\overline{q}^* = V'^{-1}(\overline{\beta})$ for high costs, and $q^* = V'^{-1}(E[\beta])$ if costs remain unknown), surplus will be larger if the agent investigates his type.[10] Since information costs the same before and after signing, the timing of information acquisition is irrelevant with respect to surplus.

1.3 The analysis

I now study the contracting problem from the perspective of the principal, who wants to maximize her payoff. Note that the agent is risk-neutral and not wealth-constrained. If information could only be acquired after signing, these assumptions would imply that the principal offers an efficient contract and appropriates the entire surplus.[11] On the other hand, if only precontractual investigation was possible, the principal would generally have to make a trade-off between efficiency and surplus extraction (see Crémer et al., 1998a). The analysis will show that this trade-off is also present in a situation where the agent can investigate his costs at either date. However, the principal has more choice over outcomes with stochastic contracts, which might be more profitable.

[10] Throughout the chapter, *efficient production* means the output level which maximizes surplus given what is known about production costs. An *efficient contract* implements efficient production and the surplus-maximizing information acquisition decision.

[11] E.g., the sell-the-project contract, which lets the agent choose output and specifies the transfer $t(q) = -S + V(q)$, where S is equal to the maximum possible surplus, fulfills that purpose.

1.3.1 Contracts

This section describes a class of tractable contracts to which the analysis may be restricted. Before applying the revelation principle, it is convenient to rule out contracts that induce precontractual investigation. According to Lemma 1.1, such contracts neither generate more surplus than other ones, nor allow to allocate the surplus differently.[12]

To prepare for this result, note that the contract offered by the principal generally specifies some message game, in which the history of sent messages determines the terms of trade. In fact, a larger game is played, which consists of the message game and the agent's decisions on information acquisition and signing. The two parties behave in this game according to one of the sequential equilibria that maximize the principal's payoff.

Lemma 1.1. *Suppose a contract implements an equilibrium in which, with some probability, the agent investigates his costs before signing. Then, there is a contract that implements an equilibrium with identical payoffs in which the agent stays uninformed before signing.*

Proof. The alternative contract differs from the original one in two aspects. First, for any history of the message game after which the agent might reject the original contract in the considered equilibrium, it specifies that he gets the extra option of trading $(t, q) = (0, 0)$ if he accepts. The alternative contract thus implements an equilibrium with identical payoffs in which the agent never rejects (and in which he chooses $(t, q) = (0, 0)$ after the histories where he rejected the original proposal). Second, it specifies that the message game begins at date 4, so that the agent does not have to take any decision before signing. The two differences imply that the alternative contract implements an equilibrium with identical payoffs in which the agent never gathers information before signing. □

[12]The statement is reminiscent of Lemma 1 by Crémer and Khalil (1992), who consider the situation where the agent learns his type at no cost after signing. In that setting, contracts which induce costly precontractual investigation generate strictly less surplus than other ones and are strictly suboptimal.

According to the revelation principle for multistage games (see Myerson, 1986), the search for optimal contracts can be confined to direct, incentive-compatible and individually rational contracts. Direct contracts are of the form

$$C = \{\alpha, (t, q), ((\underline{t}, \underline{q}), (\bar{t}, \bar{q}))\}.$$

Communication begins after signing. First, the principal recommends with probability α to acquire information and otherwise to stay uninformed. In the latter case, no further communication takes place, and the agent has to deliver quantity q in exchange for transfer t. In case the principal recommends information acquisition, the agent must report his finding subsequently. If he reports low costs, he has to produce quantity $\underline{q}$ and receives transfer $\underline{t}$. If he reports high costs, on the other hand, $\bar{t}$ and $\bar{q}$ must be exchanged. Such a contract is incentive-compatible if the agent finds it best to follow the principal's recommendation and to report truthfully on the equilibrium path. I also require that it does not provide an incentive to investigate production costs before signing. Finally, a contract is individually rational if the agent prefers to accept it

In the following, I will refer to (t, q) as the *pooling schedule* and to $((\underline{t}, \underline{q}), (\bar{t}, \bar{q}))$ as the *separating schedule*. Moreover, I call a contract *stochastic* if it stipulates $\alpha \in (0, 1)$ and *deterministic* if $\alpha \in \{0, 1\}$.

Remark 1.1. In general, direct contracts must also specify a recommendation as to precontractual investigation. But Lemma 1.1 implies that the agent may always be asked to stay uninformed before signing.

Remark 1.2. Tacitly, the focus has been restricted to contracts in which the delivered quantity is a deterministic function of recommendation and report. This does not imply a loss of generality. The agent is risk-neutral. Therefore, his behavior would not change if $q, \underline{q}$, and $\bar{q}$ were replaced by random variables as long as expected values remain constant. But since V is concave, the principal prefers the quantity to be deterministic.

1.3.2 Principal's problem

I now explain the trade-off that the principal must make to find an optimal contract. In the formal description of her problem, $\overline{U}(\alpha) = (1 - \alpha)(t - \overline{\beta}q) + \alpha(\bar{t} - \overline{\beta}\bar{q})$ and $\underline{U}(\alpha)$, which is defined analogously, are used to denote the agent's payoff.

According to the previous section, the contract should satisfy the following conditions. First, the agent must not gather information before signing:

$$p\underline{U}(\alpha) + (1-p)\overline{U}(\alpha) - \alpha\gamma \geq p\max\{0, \underline{U}(\alpha)\} + (1-p)\max\{0, \overline{U}(\alpha)\} - \gamma. \tag{1.1}$$

Second, the contract has to be acceptable for the uninformed agent:

$$p\underline{U}(\alpha) + (1-p)\overline{U}(\alpha) - \alpha\gamma \geq 0. \tag{1.2}$$

Third, the agent must follow the recommendation to stay uninformed. This condition is automatically satisfied because information is worthless after signing when the principal implements the pooling schedule. Fourth, the agent has to acquire information if this is recommended by the principal. More precisely, it must be unprofitable to stay uninformed and report high production costs:

$$p\underline{U}(1) + (1-p)\overline{U}(1) - \gamma \geq \overline{U}(1) + p(\overline{\beta} - \underline{\beta})\overline{q} \tag{1.3}$$

or low costs:

$$p\underline{U}(1) + (1-p)\overline{U}(1) - \gamma \geq \underline{U}(1) - (1-p)(\overline{\beta} - \underline{\beta})\underline{q}. \tag{1.4}$$

Finally, after having observed his type, the agent is to transmit his findings honestly. In case of high costs, this requires:

$$\overline{U}(1) \geq \underline{U}(1) - (\overline{\beta} - \underline{\beta})\underline{q}, \tag{1.5}$$

while for low costs it must be:

$$\underline{U}(1) \geq \overline{U}(1) + (\overline{\beta} - \underline{\beta})\overline{q}. \tag{1.6}$$

Consider now the principal's objective. Her payoff from a contract C that satisfies the conditions listed above is

$$\pi = (1-\alpha)\{V(q) - t\} + \alpha\{p[V(\underline{q}) - \underline{t}] + (1-p)[V(\overline{q}) - \overline{t}]\}.$$

Thus, the contracting problem reads

$$\mathcal{P}: \quad \max_{C} \pi \quad s.t. \quad (1.1)\text{–}(1.6).$$

The following lemma suggests a more convenient representation of the contracting problem.

Lemma 1.2. *For each contract that satisfies (1.1)–(1.6), there is a contract with identical payoffs that satisfies*

$$(1-p)\overline{U}(\alpha) + (1-\alpha)\gamma \geq 0, \tag{NIA}$$

$$\overline{U}(\alpha) + p(\overline{\beta} - \underline{\beta})[(1-\alpha)q + \alpha\overline{q}] \geq 0, \tag{IR}$$

$$(1-p)p(\overline{\beta} - \underline{\beta})(\underline{q} - \overline{q}) - \gamma \geq 0, \tag{IA}$$

and

$$\underline{U}(1) - \overline{U}(1) = (\overline{\beta} - \underline{\beta})\overline{q} + \frac{\gamma}{p}. \tag{1.7}$$

Moreover, (NIA), (IR), (IA), and (1.7) imply (1.1)–(1.6).

Proof. Follows from Claims A1.1, A1.2, and A1.3 in Appendix A1. □

Condition (1.7) can be inserted directly into the principal's objective, which I now formulate as the difference between the generated surplus and the agent's payoff:

$$\begin{aligned} \Pi =& (1-\alpha)\{V(q) - E[\beta]q\} + \alpha\{-\gamma + p[V(\underline{q}) - \underline{\beta q}] + (1-p)[V(\overline{q}) - \overline{\beta}\overline{q}]\} \\ &- \overline{U}(\alpha) - p(\overline{\beta} - \underline{\beta})[(1-\alpha)q + \alpha\overline{q}]. \end{aligned}$$

This allows me to restate the contracting problem as follows:

$$\mathcal{P}: \max_{\overline{U}(\alpha),(q,\underline{q},\overline{q})\geq 0, \alpha\in[0,1]} \Pi \quad s.t. \quad (NIA), (IR), (IA).$$

According to (IA), the *information acquisition* condition, reporting low costs obliges the agent to produce extra output. By (1.7), on the other hand, the efficient type receives extra payoff from the contract. These two conditions replace (1.3)–(1.6) to make sure that the agent is obedient and honest if the principal recommends information acquisition.[13] Note that the requirements as to extra output and payoff—(IA) and (1.7)—are stricter than in the familiar case with exogenous information, where the

[13]Two remarks are in order. First, *both* (IA) and (1.7) are information acquisition conditions. I only do not use an explicit label for (1.7) because it immediately enters the objective function. Second, the moral hazard constraints (1.3) and (1.4) directly imply the adverse selection conditions (1.5) and (1.6), for they express that the agent is even willing to bear investigation costs to report correctly. With more than two states of nature, eliciting his private information requires additional incentives (see Appendix A2).

agent is already informed from the outset (see Baron and Myerson, 1982). This is a common feature of incentive contracts for situations with endogenous information.[14]

(IR) is the *individual rationality* constraint, which replaces (1.2). It accounts for the fact that an agent with low production costs must necessarily earn more from any contract.[15] In the following, I call

$$\underline{U}(\alpha) - \overline{U}(\alpha) = (\overline{\beta} - \underline{\beta})[(1-\alpha)q + \alpha\overline{q}]$$

the efficient type's *rent*. As (IR) is an *ex-ante* participation condition (the agent is uninformed at signing), it does not rule out that the agent makes a loss if production costs turn out to be high. However, any loss must be compensated by a sufficient rent for the efficient type, so that the contract yields a positive payoff in expectation.

Finally, (NIA), the *no information acquisition* condition, replaces (1.1) to guarantee that the agent does not investigate his costs before signing. Information is valuable to the agent at signing if the contract is not profitable for him in the state with high production costs. For if he knew his type, he could avoid a loss by refusing to participate. On the other hand, information gathering is costly as well. But the agent anticipates that investigation costs might be due anyway, namely if the principal will recommend information acquisition after signing. In effect, the price of precontractual investigation is therefore only $(1-\alpha)\gamma$. To satisfy (NIA), it must be larger than the value of information at signing.

The contracting problem involves the following trade-off. To maximize the total surplus, the contract should require efficient production and information acquisition either with probability $\alpha = 0$ or $\alpha = 1$, depending on the level of investigation costs. To minimize the agent's payoff, on the other hand, the principal can exploit the agent's initial ignorance regarding production costs. Specifically, she should extract the efficient type's rent with a signing-fee.[16] Such a scheme satisfies the individual rationality

[14] See Lewis and Sappington (1997), who coined the term 'super high-powered incentive scheme', and the general analysis of Szalay (2009).

[15] The extra payoff from the pooling schedule is $\underline{U}(0) - \overline{U}(0) = (\overline{\beta} - \underline{\beta})q$. The extra payoff from the separating schedule is specified by the incentive constraint (1.7).

[16] Put differently, the principal should choose $\overline{U}(\alpha) = -p(\overline{\beta} - \underline{\beta})[(1-\alpha)q + \alpha\overline{q}]$. This loss can be interpreted as a signing-fee incorporated in the transfers $t, \overline{t}$ and $\underline{t}$.

condition (IR), but it necessarily results in a loss for the inefficient type, who will not obtain a rent. It is therefore not compatible with condition (NIA) if the principal recommends information acquisition with probability $\alpha = 1$: precontractual investigation would be valuable but effectively for free from the agent's perspective. In fact, for $\alpha = 1$ (NIA) turns into an *ex-interim* participation condition, which precludes signing-fees. Extracting the rent might also be incompatible with (NIA) given $\alpha = 0$—namely, if investigation costs are too low relative to the efficient output level.[17] Thus, the principal must generally make a trade-off between efficiency and surplus extraction.

1.3.3 Benchmark

As a benchmark, it is instructive to review the contracting problem under the common assumption in the existing literature, according to which information can only be gathered before signing. Given this assumption, my model is a special case of the one studied by Crémer et al. (1998a). Their analysis shows that the principal may restrict herself to the deterministic contracts and solve

$$\mathcal{P}_{det}: \quad \max_{\overline{U}(\alpha),(q,\underline{q},\overline{q})\geq 0,\alpha\in\{0,1\}} \Pi \quad s.t. \quad (NIA),(IR),(IA).$$

Intuitively, when the agent can only investigate the state before signing, he must also know the principal's recommendation at that date. Consequently, he makes his decisions on precontractual investigation and signing after he learns the content of the recommendation. Any schedule which is compatible with some stochastic contract can therefore be implemented with a deterministic contract, too. The principal should thus implement the best schedule with probability one. I want to find out whether she may ignore stochastic contracts in the original model as well.

1.3.4 Use of stochastic contracts

This section contains the main result, according to which optimal contracts might be stochastic. In the remaining analysis, I denote by $W(\alpha, \gamma)$ the principal's payoff from

[17] Precisely, if it holds that $\gamma < (1-p)p(\overline{\beta} - \underline{\beta})q^*$.

one of the best contracts with a particular recommendation probability $\alpha \in [0,1]$, depending on the level of investigation costs.[18]

Because of intractability issues—with α as choice variable, $\mathcal{P}$ is not convex—I will not derive the optimal contract but take the following approach to show that stochastic contracts can improve over the deterministic ones. The key step is to identify circumstances where the principal can achieve more payoff with a stochastic contract than with the corresponding lottery over deterministic contracts. After that step, I identify circumstances where such a lottery is as profitable as each involved contract. This procedure and continuity of the function W will establish the main result.

Lemma 1.3. *There exists a cutoff γ_1 such that it holds for any $\alpha \in (0,1)$:*

$$(1-\alpha)W(0,\gamma) + \alpha W(1,\gamma) < W(\alpha,\gamma) \quad \textit{if and only if} \quad \gamma > \gamma_1.$$

Proof. See Claim A1.4 in Appendix A1. □

Lemma 1.3 derives from the insight that, in principle, stochastic contracts allow to appropriate the *full* surplus—not just the one from the pooling schedule. The basic idea can be illustrated as follows. Let C_0 and C_1 be the best deterministic contracts that induce information acquisition with probability zero and one, respectively. Furthermore, let C_α be the stochastic contract which implements with probability $\alpha \in (0,1)$ the separating schedule from C_1 and otherwise the pooling schedule from C_0. Clearly, the agent would comply with C_α, and the principal would earn as much as with the corresponding lottery over the two deterministic contracts. The key observation which helps to understand Lemma 1.3 is that C_α necessarily satisfies condition (IR) with strict inequality. This is because C_1 does not include a signing-fee. Now, since (IR) is not binding for C_1, the principal can find a better stochastic contract than C_α with identical α if and only if (IR) is binding for C_0. This translates into the condition regarding investigation costs stated in the lemma.

[18] Such contracts exists, and the corresponding values $\overline{U}(\alpha), (q, \underline{q}, \overline{q})$ are unique: Given $\overline{U}(\alpha)$, the associated optimization program is convex and has a unique solution $(q, \underline{q}, \overline{q})$ due to the assumptions on V. The resulting value function is decreasing in $\overline{U}(\alpha)$, whose feasible set is closed and bounded from below.

Indeed, the benefit of stochastic contracts is due to a relaxed participation condition. If there is a chance the agent will not be asked to investigate production costs after signing, precontractual investigation comes at a price. As a consequence, the contract does not have to be profitable for the agent from an ex-interim perspective. Stochastic contracts can therefore implement outcomes that are not attainable with deterministic contracts (see Claim A1.5 in Appendix A1 for details). In particular, the principal may propose pooling and separating schedules which completely extract the generated surplus. For low investigation costs, however, such schedules involve too inefficient output levels, so that she does not use them.

The next step is to identify circumstances where a lottery over the two best deterministic contracts is as profitable as each involved contract.

Lemma 1.4. *The function W has the following properties:*

1. *There exists a unique intersection γ_2 of $W(0,\cdot)$ and $W(1,\cdot)$.*
2. *W is continuous.*

Proof. See Claims A1.6 and A1.7 in Appendix A1. □

I can now state the main result, which is an immediate consequence of Lemmas 1.3 and 1.4.

Proposition 1.1. *Suppose $\gamma_2 > \gamma_1$. Then, there exists an interval of investigation costs containing γ_2 in which optimal contracts are stochastic.*

Proof. For any $\alpha \in (0,1)$, it holds that $W(0,\gamma_2) = W(1,\gamma_2) < W(\alpha,\gamma_2)$ by Lemmas 1.3 and 1.4. Since W is continuous according to Lemma 1.4, the inequality holds for a whole interval of investigation costs containing γ_2. Finally, continuity implies that an optimal contract exists by Weierstrass' theorem. □

Proposition 1.1 essentially summarizes the preceding findings. If investigation costs are close to γ_2, the two best deterministic contracts roughly yield the same payoff to the principal. If, in addition, $\gamma > \gamma_1$ holds, stochastic contracts admit a more favorable trade-off between efficiency and surplus extraction than any of these ones. Such circumstances are for instance possible if the probability of low production costs is large (see Claim A1.8 in Appendix A1). Intuitively, the principal must then pay out rent with large probability and is therefore likely to benefit from a relaxed participation constraint.

Example.—Suppose the primitives of the model are as follows:

$$V(q) = \sqrt{q}, \quad \underline{\beta} = 1, \quad \overline{\beta} = 5, \quad p = \tfrac{1}{2}, \quad \gamma = 0.0556.$$

For this setting, the following statements can be derived. The best contract with $\alpha = 0$ is as profitable for the principal as the best contract with $\alpha = 1$. Denote these two contracts by C_0 and C_1. C_0 requires efficient production ($q = q^*$) and extracts the rent completely because (NIA) is slack. C_1, on the other hand, resembles the familiar Baron-Myerson contract.[19] Since (NIA) represents an ex-interim participation condition, the principal cannot extract the efficient type's rent. As a consequence, C_1 involves the classical trade-off between rent extraction and surplus maximization, which manifests in an inefficiently low output level $\overline{q} < \overline{q}^*$. For this contract, (IR) is slack. Consider now contract $C_{0.7}$, which implements with probability 0.7 the separating schedule from C_1 and otherwise the pooling schedule from C_0. Clearly, this proposal is as profitable for the principal as any of the two deterministic contracts. Moreover, it satisfies both (NIA) and (IR) with strict inequality. The principal can thus obtain more payoff by slightly increasing the signing-fee of $C_{0.7}$. In fact, the agent would even comply with contract $C^*_{0.7}$, which stipulates $\alpha = 0.7$, a signing-fee that extracts the rent completely, and efficient production in *all* contingencies. It yields 9.3% more payoff for the principal relative to any of the two best deterministic contracts.[20]

1.4 Conclusion

This chapter offers a new perspective on agency relationships with endogenous information. If the agent can investigate his payoff from the contract either before or after

[19] (IA) does not bind. Consequently, the agent produces the same amount as in the case with exogenous information. This reflects a finding of Crémer et al. (1998a), who show that if investigation costs are sufficiently low, the principal need not adjust her demand to induce information acquisition.

[20] $C^*_{0.7}$ may be suboptimal, too. As in the original model, the optimal contract is not obtained.

signing, it is possible to reconcile full surplus extraction with incentives for information acquisition. Appropriate contracts induce information acquisition randomly after signing, so as to create uncertainty for the agent whether investigation costs will be due.

Possibly, the principal need not resort to stochastic contracts in order to create this uncertainty if she faces more than one agent. In that case, she can induce information acquisition sequentially after signing until she meets an agent with appropriate characteristics. (Crémer et al. (2009) argue that such a procedure is optimal in an auction setting where bidders can investigate their valuations for the object after signing.) Clearly, the order according to which the agents are approached would affect the likelihood that investigation costs will be due for a particular agent.

Chapter 2

Precontractual Investigation and Sequential Screening

Abstract. The possession of private, payoff-relevant information often confers bargaining power. Should contract design induce its acquisition in the first place? Crémer and Khalil (1992) conclude: Agents should not be encouraged to investigate parameters which they will learn anyway before the contract is implemented. This chapter shows that *imperfect* investigation allows for more effective, sequential screening and can actually benefit the principal.

2.1 Introduction

Crémer and Khalil (1992), henceforth CK, suggest an alternative notion of adverse selection, according to which private information is not a priori given but acquired in response to contract design.[1] This approach begs the question of whether contract design should spur information acquisition in the first place. CK address the issue from the viewpoint of a profit-maximizing principal and conclude: Before entering into a contract, an agent should never be encouraged to investigate some unknown, relevant state of the world which he will learn anyway until the transaction takes place. The present chapter shows that this result does typically not extend beyond the case in

[1] I use the term *adverse selection* in the sense of Hart and Holmstrom (1987), referring to situations in which agents hold private, precontractual information.

which investigation removes *all* uncertainty. Generally, investigation allows for more effective, sequential screening of the agent's private information.

The basic model by CK deals with bilateral trading.[2] Initially, both parties do not know the agent's preferences over possible trade agreements. While deliberating whether to accept a contract, the agent can spend resources to investigate the state. In any case, he will fully discover its realization at some date when the contract has been signed but not yet carried out. The agent's preferences might of course be relevant for the principal, too; after all, she decides about the terms of trade. But she does not have the possibility to verify transmitted information. Also, she cannot observe whether the agent conducts an investigation.

CK mention several applications in which this information structure naturally prevails. One example is procurement of customized goods. Here, the contractor usually does not know his operating costs before inspecting the designs but can make a forecast based on experience from similar projects. Another possible interpretation of the model is that the principal actively withholds some source of private information before signing and the agent can explore some other, less informative source which the principal does not control. Prospective bidders for U.S. offshore oil and gas leases, for instance, are not permitted to conduct on-site test drills in advance but can explore neighboring tracts acquired in previous auctions (Hendricks and Porter, 1996).

From an efficiency perspective, investigation is clearly wasteful in this scenario. The two parties should simply wait until the agent learns his preferences anyway and agree then on the terms of trade. But since information will be distributed asymmetrically, contracts which allocates all expected surplus to the principal must generally be unprofitable for the agent in some states. Now, in effect, the possibility to probe the state endows the agent with the costly option to explore his payoff in advance and avoid a likely loss by refusing to participate. If information gathering entails low costs, the principal must consequently make a trade-off between efficiency and surplus extraction.

[2] They also study an extension where the principal faces several agents, for which they do not formally reestablish the result. Compte and Jehiel (2008) argue that if several agents compete for a single, bilateral contract, the principal possibly spurs investigation to find a suitable candidate.

CK study the contracting problem under the assumption that investigation allows to perfectly observe the prevailing state. Their analysis furthermore concentrates on *static* screening mechanisms, in which the agent is to transmit his private information all at once at some date when he knows his preferences with or without investigation. It turns out that, under these conditions, the principal will never use a contract which spurs the agent to observe the state in advance. Intuitively, her bargaining power is larger if the agent does not dispose of private information at the contracting stage, and the agent's incentives to reveal his preferences are unaffected by this matter.

The present chapter demonstrates that information acquisition can in general *mitigate* the information revelation problem and, therefore, benefit the principal. To this end, I extend the analysis of CK to imperfect investigations and arbitrary screening mechanisms. In fact, static mechanisms are suboptimal if the agent conducts an imperfect investigation: the contract should oblige him to transmit his findings *before* he fully learns the state. Such sequential protocols facilitate screening because the agent must decide about deviations from truthful reporting when the exact gains thereof are still uncertain. As a consequence, spurring imperfect, cheap investigations typically admits a more favorable trade-off between efficiency and surplus extraction.

My analysis suggests that adverse selection with endogenous information in the spirit of CK calls for a *dynamic* mechanism design perspective. In this respect, the chapter contributes to the growing literature which explores scope and design of sequential screening mechanisms in scenarios where agents gradually receive private information over time.[3] Most closely related within this literature is the seminal article on advanced ticket sales by Courty and Li (2000). In their model, the consumer freely obtains private information about his valuation for a ticket both before and after the contract has been signed. Conceptually, the present framework adds a moral hazard issue to that setting, as precontractual investigation entails costs and cannot be observed. A polar scenario, with *post*contractual investigation, has been analyzed by Krähmer and Strausz (2011). There, the agent's incentives to acquire information clearly differ since he cannot

[3]See, e.g., Baron and Besanko (1984), Battaglini (2005), Boleslavsky and Said (2013), Courty and Li (2000), Dai et al. (2006), Esö and Szentes (2007), Krähmer and Strausz (2012, 2011), Riordan and Sappington (1987), and the general analysis by Pavan et al. (2012).

quit the contract afterwards. Relevant is finally recent work by Krähmer and Strausz (2012). They show under general conditions that dynamic screening mechanisms cannot improve over static ones if agents are protected by limited liability. According to my analysis, a principal will then deter investigation.

Various papers study the design of profit-maximizing contracts in related scenarios with precontractual investigation.[4] The use of sequential screening mechanisms has not been explored yet. Specifically, Crémer et al. (1998a), Kessler (1998), Lewis and Sappington (1997), Shi (2012), and Szalay (2009) consider situations in which the agent does not receive information once the contract has been signed. Crémer et al. (1998b), Crémer and Khalil (1994), and Terstiege (2012), on the other hand, assume like CK that precontractual investigation yields perfect information about the unknown state. One of the literature's key objectives is to examine comparative statics with respect to investigation costs and timing. This chapter, in contrast, can be regarded as a comparative static analysis with respect to the *quality* of precontractual investigation: I show that a principal might only spur an imperfect investigation.

The chapter is organized as follows. The next section presents the model. Section 2.3 briefly studies the first-best and reviews the findings by CK. Section 2.4 derives the main result, according to which investigation can be desirable. Section 2.5 concludes. All formal proofs are relegated to Appendix A3.

2.2 The model

I use a general version of the procurement model by CK. Specifically, the principal seeks to purchase some good which the agent can produce. If the two parties agree to trade output $q \geq 0$ in exchange for transfer $t \in \mathbb{R}$, the principal's payoff is $V(q) - t$. The agent's payoff is $t - \beta q$, where β represents the marginal disutility of production. If no agreement is reached, both parties get zero payoff.

The principal can offer a contract to fix the terms of trade (t, q). At the time of

[4] The seminal article by Bergemann and Välimäki (2002), in contrast, takes the perspective of a *surplus*-maximizing principal. See Bergemann and Välimäki (2006) for a comprehensive survey of the literature on mechanism design with endogenous information.

contracting, however, no party knows the marginal disutility of production. Formally, β denotes the realization of a random variable, which is drawn from the set $\{\beta_1, \ldots, \beta_n\}$. Suppose $n > 1$ and let γ_i be the probability of realization β_i. The agent learns the value of β once the contract has been signed. Before signing, he can already acquire information about the state, but at a cost of $e \geq 0$. This act is not observable for the principal.

The model differs from the one by CK in that it allows for imperfect investigation. More precisely, I assume that information acquisition does not necessarily reveal the disutility parameter β to the agent but only the probability distribution $\gamma = (\gamma_i)_{i=1}^n$—which both parties do not know either. Formally, also γ denotes the realization of a random variable, drawn from the set $\{\gamma_1, \ldots \gamma_m\}$. Suppose $m > 1$ and let $\pi_j > 0$ be the probability of realization γ_j. The probability distribution $(\pi_j)_{j=1}^m$ is commonly known at the outset.[5] To distinguish the two possible pieces of private information, I call γ the agent's *ex-ante type* and β his *ex-post type*.

In detail, the timing of the model is as follows:

1. The principal offers a contract.
2. The agent can observe his ex-ante type γ at cost e.
3. The agent decides whether to accept (i.e., sign) the contract. If he accepts, stage 4 is reached. Otherwise, the relationship ends without trade.
4. The agent learns his ex-post type β.

Next, I state additional, technical assumptions. The function V, which determines the principal's valuation for output, is strictly concave, continuously differentiable, and satisfies $V'(0) = \infty$ and $V'(\infty) = 0$. The set of ex-post types, $\{\beta_1, \ldots, \beta_n\}$, is ordered $0 < \beta_1 < \ldots < \beta_n < \infty$. The set of ex-ante types, $\{\gamma_1, \ldots, \gamma_m\}$, is ordered in terms of first-order stochastic dominance. Specifically, for any two probability distributions $\gamma_{j'}, \gamma_{j''}$ with $j' < j''$ it holds that $\sum_{i=1}^k \gamma_{ij'} > \sum_{i=1}^k \gamma_{ij''}$ for all $k \in \{1, \ldots, n-1\}$.

Given these assumptions, investigation is *perfect* (i.e., reveals β) if and only if $n = m$ and $\gamma_{ij} = 1$ for all $i, j \in \{1, \ldots, n\}$ with $i = j$. The analysis will mainly focus on the

[5]Hence, both parties start with the prior belief that β equals β_i with probability $\sum_{j=1}^m \pi_j \gamma_{ij}$. If the agent conducts an investigation, his findings allow him to update to the posterior belief γ_i.

case that investigation is imperfect in the following, generic sense:

Condition 2.1. *Each probability distribution* $\gamma_j \in \{\gamma_1, \dots, \gamma_m\}$ *has full support on the set of ex-post types:* $\gamma_j \in (0,1)^n$.

2.3 Benchmarks

In the next section, I study the contracting problem from the perspective of the principal, who wants to maximize her profit. My aim is to examine whether contract design should provide incentives that induce the agent to acquire information. As benchmarks, it is instructive to first study the first-best and review the insights by CK into the case with perfect investigation.

First-best.—From an efficiency perspective, investigation is clearly wasteful. After all, precise information about the marginal disutility of production becomes available at no cost after signing. The two parties should consequently wait until the agent learns his ex-post type anyway and then exchange $q^\star = V'^{-1}(\beta)$.

Perfect investigation.—As shown by CK, a perfect investigation can never be in the principal's interest. Their analysis focuses on contracts of the form $C = (t_r, q_r)_{r \in R}$. After signing, at some date where he knows the value of β whether or not he acquired information, the agent must select and submit a report r from some message set R. The report obliges the parties to the terms of trade (t_r, q_r). Intuitively, with such contracts the agent might just investigate in order to acquire a more favorable bargaining position: being able to join the contract if and only if it is profitable given the true ex-ante type. The principal does not encourage information gathering because she herself could sell this option to the agent. Given any contract where an investigation pays off, she can for instance add some report s to the message set, specify $(t_s, q_s) = (0, 0)$, and charge a fee of e for the upgrade. As ex-ante and ex-post type are equivalent in this benchmark case, the modified contract replicates the outcome of the original one but deters information gathering, and the fee transfers the former investigation expenses to the principal.

2.4 The analysis

2.4.1 Contracts

I now return to the original model and argue that the principal might well ask the agent to acquire information even though investigation is inefficient and improves the agent's bargaining position. An essential step towards this insight is to apply the revelation principle for multistage games (Myerson, 1986), which allows to confine the search for optimal contracts to the direct, incentive-compatible ones.

In the present situation, there are two sorts of *direct* contracts. I call them the pooling and the separating contracts.[6] *Pooling contracts* have the form $C^P = (t_i, q_i)_{i \in I}$. Before signing, the principal recommends to not gather information. After signing, she requests a report $i \in I = \{1, \ldots, n\}$ about the ex-post type. The two parties must then adopt the terms of trade (t_i, q_i). *Separating contracts*, on the other hand, are denoted by $C^S = ((t_{ij}, q_{ij})_{i \in I_j})_{j \in J}$. Before signing, the principal recommends the agent to investigate his ex-ante type, which he must announce by sending a report $j \in J = \{1, \ldots, m\}$. Importantly, the revelation principle demands that this report is due before the agent learns the ex-post type. After signing, the principal requests a second report $i \in I_j = \{i \in I : \gamma_{ij} > 0\}$, this time about the ex-post type. Given some sequence of announcements ij, the parties finally trade (t_{ij}, q_{ij}). A direct contract is *incentive-compatible* if the agent prefers to follow the principal's recommendation and to send truthful reports on the equilibrium path.

Separating contracts fundamentally differ from the contracts described by CK in that they use a *sequential* screening mechanism, rather than a static one. In particular, the agent must select a report about the ex-ante type before learning the ex-post type. One implication of this arrangement is that even contracts which the agent will accept regardless of his findings can induce investigation, as information concerning the ex-ante type might be relevant for choosing a report. In fact, a standard revealed-preferences argument shows that the analysis of separating contracts may be restricted to such

[6] In fact, the pooling and separating contracts are the *deterministic* direct contracts. Standard arguments show that this restriction does not entail a loss of generality. See Terstiege (2012) for remarks on the use of stochastic contracts in agency relationships with endogenous information.

individually rational proposals.[7]

Clearly, a perfect investigation cannot be desirable even with this general contract space. If ex-ante and ex-post type are equivalent, the incentive-compatible, individually rational separating contracts implement outcomes $(\widetilde{t}_i, \widetilde{q}_i)_{i \in I}$ that, by revealed preferences, are also implementable using pooling contracts. The principal can actually improve on every separating contract with the corresponding pooling contract $(\widetilde{t}_i - e, \widetilde{q}_i)_{i \in I}$, for the agent saves on investigation costs.

Henceforth, I assume that investigation is imperfect in the sense of Condition 2.1. This simplifies the configuration of separating contracts, as the message sets I_j from which the agent picks the report about the ex-post type do then not depend on his report regarding the ex-ante type. More precisely, Condition 2.1 implies $I_j = I$ for all $j \in J$. As a consequence, it will be possible to pin down the agent's reporting strategy with separating contracts off the equilibrium path.[8]

2.4.2 Principal's problem

According to the previous section, the principal's problem is to find an optimal contract among the pooling and separating ones. This section provides a formal description of the problem, using the notation $U_i = t_i - \beta_i q_i$ and $U_{ij} = t_{ij} - \beta_i q_{ij}$ for the agent's payoff.

To deter investigation, the principal should offer a pooling contract that satisfies the following constraints. First, the agent must reveal the ex-post type after signing:

$$U_i \geq U_k + (\beta_k - \beta_i) q_k \quad \forall i, k \in I. \tag{P1}$$

Second, he should be willing to accept the contract:

$$\sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} U_i \right] \geq 0. \tag{P2}$$

[7]If an agent with ex-ante type γ_j rejects a given incentive-compatible contract $\widetilde{C}^S$, he would accept a contract $\widehat{C}^S$ that only differs in that $(\widehat{t}_{ij}, \widehat{q}_{ij})_{\in I_j} = (0,0)$. $\widehat{C}^S$ is incentive-compatible, too, and hence replicates the outcome of $\widetilde{C}^S$.

[8]Condition 2.1 is very common in the sequential screening literature (see, e.g., Courty and Li, 2000, Battaglini, 2005, Esö and Szentes, 2007, and Krähmer and Strausz, 2011). See Krähmer and Strausz (2008) for a model with shifting supports.

Finally, the agent must not conduct an investigation:

$$\sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} U_i \right] \geq \sum_{j=1}^{m} \pi_j \max \left\{ \sum_{i=1}^{n} \gamma_{ij} U_i,\, 0 \right\} - e. \tag{P3}$$

Consider now the principal's objective. By definition, her payoff equals the difference between the generated surplus and the agent's payoff. Formally, the best pooling contracts are therefore the solutions to

$$\mathcal{P}^P : \quad \max_{C^P} \sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} [V(q_i) - \beta_i q_i - U_i] \right] \quad s.t. \quad (P1)\text{–}(P3).$$

Next, I turn to the relevant contracts to induce investigation, the separating ones. They should satisfy the following constraints. First, the agent needs an incentive to reveal his ex-post type after signing. Given Condition 2.1, this requires

$$U_{ij} \geq U_{kj} + (\beta_k - \beta_i) q_{kj} \quad \forall i, k \in I. \tag{S1}$$

Second, he must honestly report the ex-ante type. Note that even though truth-telling is not required off the equilibrium path, (S1)—by Condition 2.1—ensures that the agent will reveal his ex-post type if he was dishonest before. Hence, the constraint reads:

$$\sum_{i=1}^{n} \gamma_{ij} U_{ij} \geq \sum_{i=1}^{n} \gamma_{ij} U_{il} \quad \forall j, l \in J. \tag{S2}$$

Third, the contract has to be acceptable for the agent:

$$\sum_{i=1}^{n} \gamma_{ij} U_{ij} \geq 0 \quad \forall j \in J. \tag{S3}$$

A separating contract must finally induce information gathering:

$$\sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} U_{ij} \right] - e \geq \max \left\{ \sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} U_{il} \right], 0 \right\} \quad \forall l \in J. \tag{S4}$$

Given these constraints, the best separating contracts are the solutions to

$$\mathcal{P}^S : \quad \max_{C^S} \sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} [V(q_{ij}) - \beta_i q_{ij} - e - U_{ij}] \right] \quad s.t. \quad (S1)\text{–}(S4).$$

2.4.3 Comparison

Before continuing with the analysis, it is helpful to compare the pooling and separating contracts in light of the insights from the study of the benchmarks.

Bargaining position.—(P2) and (P3) describe the agent's bargaining position with pooling contracts. In principle, the agent will accept any proposal that guarantees non-negative payoff in expectation over all ex-ante types. On the other hand, he has the costly option to check his ex-ante type in advance and avoid a loss by refusing to participate. This option serves as a threat to the principal. Relative to investigation costs, her proposal must not be too unprofitable for particular ex-ante types. The agent's bargaining position with separating contracts is in general stronger, as he is supposed to know the type at signing. Specifically, to ensure the agent's participation the principal must guarantee non-negative payoff for *each particular* ex-ante type (see (S3)). Only with zero investigation costs, the bargaining position does not differ between pooling and separating contracts.

Incentives to investigate.—If being offered a pooling contract, the agent might only gather information with respect to the decision whether to accept the proposal (see (P3)). This is because pooling contracts use the same, static screening mechanism as the contracts described by CK. An individually rational separating contract, on the other hand, will induce investigation if and only if information about the ex-ante type is sufficiently valuable for the agent with respect to the selection of the first report (see (S4)). Thus, the principal must generally differentiate the terms of trade among the possible ex-ante types to spur investigation. Only with zero investigation costs, differentiation is not required.

Efficiency potential.—Pooling contracts can generate the first-best surplus (e.g., let $(t_i, q_i)_{i\in I} = (V(q_i^\star), q_i^\star)_{i\in I}$). Incentive-compatible, individually rational separating contracts are generally inferior in this respect: First, the agent bears investigation costs. Second, the principal must differentiate the terms of trade among the possible ex-ante types to spur investigation. Only with zero investigation costs, separating contracts can generate the first-best surplus as well.

2.4.4 Problem with pooling contracts

I now explain why the principal might want to spur investigation. This step requires a more transparent representation of the best pooling contracts, using the notation $\Gamma_{kj} = \sum_{i=1}^{k} \gamma_{ij}$ for the cumulative probability distribution corresponding to type γ_j.

Lemma 2.1. *For each pooling contract that satisfies (P1)–(P3), there is a pooling contract with identical expected payoffs that satisfies*

$$U_i - U_{i+1} = (\beta_{i+1} - \beta_i)\, q_{i+1} \quad \forall i \in \{1, \ldots, n-1\}, \tag{2.1}$$

$$q_i - q_{i+1} \geq 0 \quad \forall i \in \{1, \ldots, n-1\}, \tag{2.2}$$

$$\sum_{j=1}^{m} p_j \left[U_n + \sum_{i=1}^{n-1} \Gamma_{ij}(\beta_{i+1} - \beta_i) q_{i+1} \right] \geq 0, \tag{2.3}$$

$$\sum_{j=l}^{m} p_j \left[U_n + \sum_{i=1}^{n-1} \Gamma_{ij}(\beta_{i+1} - \beta_i) q_{i+1} \right] + e \geq 0 \quad \forall l \in \{2, \ldots, m\}. \tag{2.4}$$

Moreover, (2.1)–(2.4) imply (P1)–(P3).

Proof. Follows from Claims A3.1–A3.3 in Appendix A3. □

Condition (2.1) can be inserted directly into the objective function of program $\mathcal{P}^P$:

$$\Pi^P = \sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} [V(q_i) - \beta_i q_i] \right] - \left[\sum_{j=1}^{m} \pi_j \left[U_n + \sum_{i=1}^{n-1} \Gamma_{ij}(\beta_{i+1} - \beta_i) q_{i+1} \right] \right].$$

This allows me to represent the best pooling contracts as the solutions to

$$\widetilde{\mathcal{P}}^P : \quad \max_{U_n,\, (q_i)_{i=1}^{n}} \Pi^P \quad s.t. \quad (2.2)\text{–}(2.4).$$

By (2.1), the agent receives rent for not exaggerating the disutility of production. According to (2.2), on the other hand, reporting less disutility obliges him to produce more output. These two, standard constraints make sure that the agent is honest when the principal asks for the ex-post type. (2.3) guarantees the agent non-negative payoff in expectation over all ex-ante types, so that he is willing to participate. The final constraint rules out investigation. As pointed out in section 2.4.3, information about the ex-ante type is valuable to the agent with pooling contracts if and only if the proposal is unprofitable for some types. (2.4) requires the value of information to be less than the level of investigation costs, taking the first-order stochastic dominance ranking into account. Namely, the ranking implies that the agent can expect a larger payoff the lower the ex-ante type, for he will earn more rent the lower the ex-post type, in turn.

Pooling contracts involve the following problem. To maximize surplus, the proposal should stipulate the efficient output schedule $(q_i^\star)_{i=1}^n$. To minimize the agent's payoff, on the other hand, the principal can take advantage of the agent's initial ignorance regarding the disutility of production. Specifically, she should appropriate the expected rent with a signing-fee.[9] Such a scheme satisfies the participation constraint, (2.3), but it necessarily results in an expected loss for high ex-ante types, which are likely to earn little rent from any contract. The agent would therefore want to know his type. If investigation costs are low, condition (2.4) thus forces the principal to make a trade-off between efficiency and surplus extraction, and offering a pooling contract might actually be suboptimal.

2.4.5 Use of separating contracts

This section establishes the main result, according to which the principal offers a separating contract and thus induces information gathering if investigation entails low costs and is imperfect in the sense of Condition 2.1. In the remaining analysis, $W^P(e)$ and $W^S(e)$ denote the principal's payoff from the best pooling and separating contracts, respectively, depending on the level of investigation costs.[10]

According to the comparison in section 2.4.3, the drawbacks of separating contracts are negligible with low investigation costs. With *zero* costs, in particular, the agent virtually knows the prevailing ex-ante type at the outset, so that his bargaining position as well as the efficiency potential do not differ relative to pooling contracts. Indeed, separating contracts give the principal more choice over outcomes in that case, for they admit—but not require—to differentiate the terms of trade among the possible ex-ante

[9] That is, she should choose $U_n = -[\sum_{j=1}^m \pi_j [\sum_{i=1}^{n-1} \Gamma_{ij}(\beta_{i+1} - \beta_i) q_{i+1}]]$, which can be interpreted as a signing-fee incorporated into transfers.

[10] Claim A3.4 in Appendix A3 confirms the existence of such contracts.

types. Hence, if investigation entails zero costs, offering a separating contract is at least not suboptimal. Lemma 2.2 makes these arguments precise.

Lemma 2.2. *Suppose Condition 2.1 is satisfied. Then, it holds that:*

1. W^P *is non-decreasing and* W^S *non-increasing.*
2. W^P *and* W^S *are continuous.*
3. *There exists a unique intersection* $\bar{e}$ *of* W^P *and* W^S .

Proof. See Claims A3.5, A3.7, and A3.8 in Appendix A3. □

By Lemma 2.2, the main result will be established if and only if separating contracts can outperform the pooling ones in the case with zero investigation costs. The following lemma therefore constitutes the final step of the analysis.

Lemma 2.3. *Suppose Condition 2.1 is satisfied. Then, it holds that* $W^S(0) > W^P(0)$.

Remark 2.1. Given $e = 0$, the contracting problem is equivalent to the one considered in the sequential screening literature (e.g., Courty and Li, 2000), where the agent is assumed to know the ex-ante type at the outset. That literature typically imposes stringent regularity conditions on the probability distributions γ_j which allow to fully characterize optimal contracts.[11] I do not follow this approach, as such conditions would not be generic with imperfect investigation. The proof just verifies the key property: optimal contracts have a sequential screening mechanism, rather than a static one.

Proof. See Claim A3.9 in Appendix A3. □

Lemma 2.3 derives from the observation that, in effect, the ex-ante type determines the agent's marginal rate of substitution between rent and signing-fee with pooling contracts.[12] The idea can be explained as follows. Suppose $\gamma \in \{\gamma_1, \gamma_2\}$ and $\beta \in \{\beta_1, \beta_2\}$. Moreover, assume zero investigation costs, so that the agent virtually knows

[11]See Battaglini and Lamba (2012) for a comprehensive account. The analytical problem with sequential screening which necessitates the regularity conditions is that truth-telling constraints of the form of (S2) commonly lack a useful characterization.

[12]Precisely, to each ex-ante type γ_j corresponds a whole vector $\Gamma_j = (\Gamma_{1j}, \dots, \Gamma_{n-1j})$ of marginal rates of substitution. Γ_{ij} is the marginal rate of substitution between fee and rent $(\beta_i - \beta_{i+1})q_{i+1}$. By the first-order stochastic dominance ranking, it holds that $\Gamma_j > \Gamma_{j+1}$ for all $j \in \{1, \dots, m-1\}$.

the ex-ante type. Under these conditions, the trade-off with pooling contracts manifests itself in an inefficiently low output level $q_2 < q_2^*$. The crucial insight is that the terms of trade should never be distorted for *both* ex-ante types in this fashion: If γ_1 prevails the agent has a larger valuation for additional rent, because he is more optimistic to receive it. Hence, if this type was to produce efficiently and pay a signing-fee that fully extracts the additional surplus, both types would comply. Such a scheme amounts to a separating contract that outperforms all pooling ones. The proof extends this reasoning to the case with an arbitrary number of types, showing that for every proposal among the best pooling contracts there exists a separating contract which exhibits 'no distortion at the top' and is more profitable.

The main result is an immediate consequence of Lemmas 2.2 and 2.3.

Proposition 2.1. *Suppose Condition 2.1 is satisfied. Then, there exists a cutoff* $\bar{e} > 0$ *such that optimal contracts are pooling contracts if and only if if* $e > \bar{e}$.

The benefit of separating contracts is basically due to a relaxed truth-telling constraint. With both classes of contracts, honesty must yield the agent at least the same payoff as the best deviation strategy. This requirement is less demanding for separating contracts because they force the agent to decide about deviations from truthful reporting *before* he learns the exact disutility parameter. Ultimately, he might not be able to obtain those terms of trade in the proposal which are most profitable to him given his true preferences. This advantage vanishes if investigation removes all uncertainty.

2.5 Conclusion

This chapter revisits the adverse selection model proposed by Crémer and Khalil (1992) and shows that the principal provides incentives for information gathering if and only if she benefits from sequential screening. Accordingly, separating contracts feature different (namely, dynamic) screening mechanisms than the pooling ones. The analysis demonstrates that separating contracts are typically advantageous if investigation costs are low unless the agent has the possibility to acquire perfect information about his preferences. This is because, with imperfect investigation, the principal can exploit the fact that sequential screening involves choice under uncertainty for the agent.

Chapter 3

Objective versus Subjective Performance Evaluations

Abstract. Why does incentive pay often depend on subjective rather than objective performance evaluations? After all, subjective evaluations entail a credibility issue. While the most plausible explanation for this practice is lack of adequate objective measures, I argue that subjective evaluations might also be used to withhold information from the worker. I furthermore argue that withholding information is particularly important under circumstances where the credibility issue is small. The statements are derived from a two-stage principal-agent model in which the stochastic relationship between effort and performance is unknown.

3.1 Introduction

Many employers try to boost employee morale by relating compensation—pay, career, power, etc.—to performance. This chapter examines how a worker's performance should be measured. One possibility is to establish in advance a comprehensible, visible evaluation procedure that disregards expertise of biased persons (*objective evaluation*). Alternatively, performance can be rated by the personal impression of the worker's supervisor, which is unverifiable (*subjective evaluation*). As an example, consider the evaluation of a project manager's work. His principal could either require initially specified, non-manipulable tests that determine the project's success at each stage until completion, or she may decide herself whether expectations are met.

If an employee's compensation depends on subjective performance evaluations, it is at the discretion of his supervisor. Honest evaluations come at a cost. An owner-manager, for instance, who would generally be tempted to understate performance and save on labor costs, might be forced to acquire credibility through obligatory bonus pools (paid out to some third party in case of negative evaluation, e.g., to charity) or up-or-out career systems (employee lost in case of negative evaluation).[1,2] If compensation depends on an objective evaluation, on the other hand, it is court-enforceable, and the employer does not incur such costs. Yet, subjective evaluations are common practice.[3]

The most plausible explanation of this fact holds that employers use subjective performance measures to obtain more accurate evaluations of their workers. More precisely, they complement objective measures which do not capture all tasks that are to be carried out or which are subject to influences beyond the agent's control. For if the evaluation is incomplete, the agent might 'game' the incentive contract and neglect tasks that are not included in the evaluation.[4] In fact, empirical studies document that

[1]The same problem occurs with an employed manager if his interests are aligned with the owner's objectives. E.g., "the NSPS [a former pay system at the U.S. Department of Defense that involved subjective evaluations] has been roundly criticized by staff members and union leaders who say subjective performance evaluations could be used to limit pay. The review panel heard many complaints that supervisors were pressured not to give too many employees a rating of 4, out of 5, apparently because money was not available" (Washington Post, July 17, 2009).

[2]Supervisors can also be be tempted to *over*state performance so as not to disgruntle employees or when they are susceptible to bribery or currying favor by employees. See Prendergast (1999) for a discussion. Finally, discretion over compensation allows supervisors to discriminate workers based on sex, nationality, etc.

[3]Due to the availability of data, most studies on performance reviews consider CEO compensation. E.g., Bushman et al. (1996) found that in 190 of 248 firms in the USA bonuses were (at least partly) at the discretion of the board of directors. Murphy and Oyer (2003) examined 280 firms in the USA; 43% displayed discretion in determining the size of bonus pools and 67% in allocating a bonus pool across participants. Gibbs et al. (2004), who surveyed car dealerships in the USA, report that subjective evaluations are also prevalent below CEO level.

[4]See Baker et al. (1994), Prendergast (1999), and the general analysis by Bernheim and Whinston (1998). This argument draws on the multi-tasking problem described by Baker (1992) and Holmstrom

subjective evaluations are more likely in jobs which comprise many tasks (see Brown, 1990 and MacLeod and Parent, 1999). If the evaluation is noisy, on the other hand, a risk-averse agent might not respond to incentive pay.[5]

However, sometimes employers seem to conduct subjective evaluations not out of necessity, as this theory suggests, but instead to *eschew* to establish comprehensive objective performance measures. The following list provides some examples:

- In 2003, the National Research Council (NRC), a think tank, examined the quality of project management within the U.S. Department of Energy. It stated a "lack of objective measures that makes it difficult to assess progress in improving project management", and "to build confidence within [...] Congress [...] and the public in the department's ability to manage the money it spends on its projects. Evidence continues to be anecdotal rather than objective, quantitative, and verifiable" (National Research Council, 2003, pp. 31–32). A subsequent report by the NRC suggested a number of appropriate measures (see National Research Council, 2005).
- A study on 17 U.S. investment banks revealed that employees in sales and trading divisions receive bonuses that largely depend on subjective performance appraisals despite "the ease with which the profitability of an individual trader can be measured each day" (Eccles and Crane, 1988, p. 170). All 17 banks had implemented bonus pools.
- The standards for associates to become partner at professional service firms are usually highly intransparent although many explicit performance measures are conceivable (see Gilson and Mnookin, 1989). (According to Morris and Pinnington (1998), who surveyed law firms in the UK, the most important promotion criteria are: getting new business, fee-earning ability, technical skill, and getting on with clients.) At such organizations, up-or-out career systems are commonplace.

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and Milgrom (1991) . E.g., as Bushman et al. (1996) note, a CEO has to plan the long-term strategy of the company. If it is concealed from the financial market, his performance should not only be measured with the stock price.

[5]See Rajan and Reichelstein (2009). This argument draws on the informativeness principle, established by Harris and Raviv (1979) and Holmstrom (1979) .

I suggest a complementary explanation, according to which subjectivity itself can be a desirable property of a performance measure. It builds on the assumption that evaluations *generate* information and that only subjective evaluations generate *exclusive* information, which can be concealed for some time.[6] My analysis applies to jobs with many tasks that are carried out sequentially (e.g., phases of project management) and, more generally, to long-term employment relationships in which performance can be assessed over time (e.g., after trading days or fiscal years until promotion decision). In a nutshell, subjective evaluations at early stages of such jobs are advantageous for the principal if details like productivity or ability are highly uncertain and can be inferred from performance.[7] This is because the worker could manipulate the incentive scheme if he learned about these details too soon and because the cost entailed by the use of subjective evaluations is actually low.

I derive my statements from a two-stage principal-agent model with hidden actions. Specifically, in each stage the wealth-constrained agent can exert effort to increase the likelihood of good performance in that stage, but effort is not observable. The principal can use incentive pay for profit maximization. Three further assumptions are important. First, the likelihood of good performance does not only depend on effort but also on an unknown, persistent parameter. Second, the principal wants the agent to work hard in each stage. Third, only the principal can evaluate performance. The third assumption is of course an extreme simplification. In many employment relationships, the worker has some idea about the produced output. Nevertheless, a comprehensive performance measure might require certain data that are not easily accessible to him (e.g., test results, sales numbers, or comparisons with other workers). I show that the assumption can be rephrased accordingly without affecting the qualitative results.

With this model, I compare the principal's profit in two scenarios that differ as to how she evaluates performance. In the *objective scenario*, the principal evaluates

[6]Put differently, concealed evaluations are often inherently susceptible to manipulation and can become unverifiable over time (i.e., equivalent to subjective evaluations).

[7]Early studies on evaluation practices by Govindarajan (1984) and Keeley (1977) indeed document significant positive correlation between the use of subjective evaluations and uncertainty about success on the job.

objectively in both stages. This means that it becomes public information at the end of each stage whether the agent was successful. In the *subjective scenario*, the principal evaluates subjectively in stage 1 and objectively in stage 2. Here, only she herself learns the outcome of stage 1, and it cannot be verified.[8]

The analysis yields that, in each scenario, the principal incurs incentive costs. In the objective scenario, contracts must take into account that the evaluation in stage 1 provides information about the unknown parameter and that the agent is in a better position to draw inference since he privately knows his effort choice. In particular, bad performance in stage 1 indicates an unfavorable state and thus calls for amplified incentive pay in stage 2. But this scheme must not tempt the agent to produce a failure on purpose. Therefore, the principal provides the same high-powered incentive also when stage 1 was successful. As a consequence, the agent can secure rent. In the subjective scenario, on the other hand, the principal optimally reveals her subjective evaluation of stage 1 only after stage 2, and only after particular histories. Incentive pay remains invariant over time, and the agent does not earn rent. However, the principal's discretion in determining the agent's performance bonus for stage 1 implies a credibility issue: ex post, the principal will always submit an evaluation which minimizes labor costs. She must therefore commit to pay the bonus regardless of the content of her evaluation—to the agent if he was successful and otherwise to a budget breaker. Hence, the use of subjective evaluations entails a cost as well, namely the payoff to the budget breaker.

I show that greater uncertainty about the stochastic relationship between effort and performance renders the subjective scenario advantageous from the principal's perspective. Intuitively, this result can be explained as follows. The model captures greater uncertainty in form of more extreme posteriors about the unknown parameter. In particular, failure in stage 1 results in a more pessimistic posterior, so that the rent in the objective scenario increases. In the subjective scenario, in contrast, the principal benefits from more extreme posteriors. This is because she optimally rewards the agent

[8]This notion of objective and subjective evaluations is consistent with the recent literature (e.g., Murphy and Oyer, 2003). Earlier papers considered subjective evaluations as being unverifiable but not necessarily private; see MacLeod (2003) and Fuchs (2007) for a discussion.

according to a wage scheme that, in fact, only requires the subjective evaluation if the agent performs well in stage 2 (otherwise, there will be no bonus for stage 1). Now, this event becomes less likely with a negative but more likely with a positive evaluation if stage 1 gets more informative. Accordingly, the principal is less likely to submit a bad rating and pay the bonus to the budget breaker. Moreover, the bonus itself can be reduced, for the agent will be more confident to earn it. Both lowers the budget breaker's payoff, the cost to overcome the principal's credibility issue.

Related literature

Most closely related is recent work on interim performance feedback during long-term relationships by Chen and Chiu (2012). They consider a similar two-stage principal-agent model in which the outcome of stage 1 can also be evaluated either objectively or subjectively. The paper examines whether an interim evaluation of either kind is desirable *at all*.[9] As the model does not assume ex-ante uncertainty about the stochastic relationship between effort and performance, the insights differ from my analysis.

According to this chapter, employers might establish subjective performance measures to withhold information from their workers and objective measures if verifiability is the major concern. Several papers indicate similar pros and cons of subjective and objective evaluations but do not explore the trade-off. Closely related are recent papers by Bashkar (2012), DeMarzo and Sannikov (2011), and Kwon (2012), who study dynamic moral hazard problems in which performance is publicly observed. As in my model, both the principal and the agent do not know the stochastic relationship between effort and output and learn about it from past performance. The papers show that incentive pay involves extra costs for the principal since just the agent knows past effort.[10] It is not examined whether the principal would gain from subjective evaluations.

Related is also the analysis by Lizzeri et al. (2002), who consider a dynamic moral

[9]In contrast to my model, pay can be made performance-contingent without interim evaluation as the agent's effort levels in the two stages are complements with respect to the outcome of stage 2.

[10]Crémer (1995), Hirao (1993), and Manso (2011) consider similar settings in which the principal avoids the extra costs by either terminating the employment relationship or implementing a different technology after failure.

hazard problem in which the principal privately observes the agent's performance.[11] They find that the principal should better not reveal the agent's interim performance before the job is finished since she would otherwise have to pay more rent for the same effort provision. (Different from my model, an interim evaluation does not have any relevant informational content but only determines the agent's continuation payoff. If it is concealed, more effective carrot-and-stick schemes are possible.) However, the principal can verify her evaluation (i.e., it is not subjective).

Incentive contracts with subjective performance measures are studied in the seminal papers by Fuchs (2007), Levin (2003), and MacLeod (2003). They consider situations in which the principal does not benefit from private instead of public information about performance but has no means to evaluate objectively. It is analyzed in detail how lack of verifiability exacerbates the contracting problem. Among these papers, Fuchs (2007) is most closely related to the present study. He also investigates a finitely repeated moral hazard problem and argues that subjective evaluations should only be revealed after the last stage. (As with Lizzeri et al. (2002), evaluations do not have relevant informational content in his model. If the agent is kept uninformed about his continuation payoff, incentive pay involves more compressed wages, so that the truth-telling constraints for the principal are less severe.)

This chapter is organized as follows. The next section presents the model. Section 3.3 first derives the best contract for the objective and the subjective scenario, respectively. Afterwards, the two scenarios are compared with respect to the principal's profit, and it is shown that more uncertainty renders the subjective scenario advantageous. Section 3.4 concludes. Lengthy proofs are relegated to Appendix A4. Appendix A5 demonstrates the benefits of subjective evaluations in a more general model.

3.2 The model

A principal in need for a project manager is matched to an agent. The quality of the match is determined by a random variable, whose distribution function is denoted by

[11] For more recent references in this strand of literature see footnote 19.

F and whose realization θ lies in the interval $[0, 1]$. Both parties do not know which state obtains.

Project management consists of two stages, and the final return to the principal depends on the agent's performance in each stage. In stage 1, the agent can exert effort $e_1 \in \{0, 1\}$ at costs $e_1 c$, where $c > 0$. The agent's effort provision is unobservable. However, the principal can collect relevant data and conduct a performance evaluation. I distinguish two scenarios, which differ as to the evaluation mode. In the *objective scenario*, an objective evaluation takes place. This means that performance is verifiable and commonly observed at the end of stage 1. In the *subjective scenario*, performance is not verifiable and privately observed by the principal at the end of the stage.[12] The relationship between effort and performance does not depend on the evaluation mode. Formally, performance is good ($x_1 = 1$) with probability $e_1\theta$ and bad ($x_1 = 0$) with probability $1 - e_1\theta$.

In stage 2, the agent exerts effort $e_2 \in \{0, 1\}$ and incurs costs $e_2 c$. Again, the principal cannot monitor whether the agent shirks. In both scenarios, the agent's performance in stage 2 is objectively evaluated.[13] It is again either good ($x_2 = 1$) or bad ($x_2 = 0$); the good outcome obtains with probability $e_2\theta$. After stage 2, the project yields the principal an unobservable, unverifiable return of $\rho(x_1, x_2)$. I normalize $\rho(0, 0) = 0$ and assume $\rho(1, x_2) - \rho(0, x_2) = \rho(x_1, 1) - \rho(x_1, 0) = R > 0$.

Both parties are risk neutral, do not discount future payoffs, and have an outside option of zero. At the outset, the principal offers a contract to specify the agent's wage. It may be contingent on any verifiable data, possibly involving messages. The agent does not dispose of own resources, so that only non-negative wages are feasible. If the contract requires a budget breaker, a third party is available. All payments are made after stage 2.

In detail, the model has the following timing:

0. Match quality θ realizes, and the principal offers a contract.

[12] Appendix A5 shows that the qualitative results also hold if the agent observes a noisy performance signal.

[13] I implicitly regard the evaluation mode as one of the principal's choice variables. The analysis will make clear that it is advantageous to evaluate objectively in stage 2.

1. Stage 1: The agent exerts effort e_1. Then, performance x_1 realizes. In the objective scenario, x_1 is commonly observed. In the subjective scenario, x_1 is privately observed by the principal.
2. Stage 2: The agent exerts effort e_2. Then, performance x_2 realizes. In both scenarios, x_2 is commonly observed. Finally, payments are made.

The agent's performance in stage 1 conveys information about the unknown quality of the match, which can be used to update the initial expectation $\mu = E[\theta]$. However, one has to take the agent's effort decision in stage 1 into account to interpret this information. If the agent exerted effort ($e_1 = 1$), the posterior expectation of match quality is either μ_1 (in case $x_1 = 1$) or μ_0 (in case $x_1 = 0$). Henceforth, I assume $1 > \mu_1 > \mu > \mu_0 > 0$.[14] If the agent was lazy ($e_1 = 0$), his performance is bad regardless of the quality of the match, so that no information is conveyed and the posterior expectation remains μ. An important property of the model is that the two parties might have to base their posteriors on imperfect information: In both scenarios, the principal does not observe e_1. The agent, on the other hand, does not observe x_1 in the subjective scenario.

In the next section, I compare the two scenarios with respect to the principal's profit. As usual, the analysis involves a two-step procedure to derive optimal contracts. The first step is to assign to each effort plan a contract that implements it as cheaply as possible. The second step is to identify among these contracts a profit-maximizing one. Let (e_1, e_2) denote a deterministic effort plan that specifies the same effort level for stage 2 after all possible histories up to that stage. To concentrate the analysis on the relevant circumstances, I make the following assumption.

Assumption 3.1. *In both scenarios, the principal wants to implement effort plan* $(1, 1)$.

Claim A4.1 in Appendix A4 shows that the assumption holds if R, the principal's return from success in some stage, is sufficiently large.[15] Note that Assumption 3.1

[14] Precisely, $\mu = \int_0^1 \theta dF$, $\mu_1 = (1/\mu) \int_0^1 \theta^2 dF$, and $\mu_0 = [1/(1-\mu)] \int_0^1 \theta(1-\theta) dF$. The prescribed ordering holds for instance if F is discrete and assigns positive probability to at least three distinct realizations, or if F has a density that is supported on some non-degenerate subinterval of $[0, 1]$.

[15] I only take deterministic effort plans $(e_1, e_2(x_1))$ into account.

in particular implies that the principal does not want to make e_2 contingent on the agent's past performance. However, the evaluation in stage 1 will be required to create incentives. I want to find out whether subjective or objective evaluations are more appropriate.

3.3 The analysis

In each stage, performance is bad for sure if the agent shirks.[16] If the model did not involve uncertainty about the quality of the match, this condition would imply that in the objective scenario the principal could appropriate the entire surplus.[17] In case the principal resorts to a subjective evaluation, on the other hand, incentive contracts typically distribute some surplus to a budget breaker to confer credibility (see, e.g., MacLeod, 2003). Hence, the principal would clearly prefer the objective scenario if the quality of the match was certain. The analysis will show that this ranking might be reversed under uncertainty.

3.3.1 Objective scenario

In the objective scenario, the agent's performance is objectively evaluated in both stages. Optimal contracts neither involve communication nor a budget breaker but consist of a performance-contingent wage scheme.

Let $w = (w_{x_1 x_2})_{x_1, x_2 \in \{0,1\}}$ be a wage scheme. To implement effort plan $(1, 1)$, it must satisfy four incentive-compatibility constraints. First, the agent has to exert effort in stage 2 if he was successful in stage 1:

$$\mu_1 w_{11} + (1 - \mu_1) w_{10} - c \geq w_{10}. \tag{3.1}$$

Second, he must also work hard if his performance was bad in stage 1, holding the

[16] This assumption is to rule out the standard 'limited liability rent', which would be due in both scenarios. If anything, the assumption should favor the objective scenario, where the agent's incentive constraints are more pressing since the principal cannot withhold the evaluation of stage 1.

[17] If match quality was known to be p, for instance, the principal could achieve this by paying c/p to the agent if he is successful in stage 1 and 2, respectively.

pessimistic posterior μ_0:

$$\mu_0 w_{01} + (1-\mu_0)w_{00} - c \geq w_{00}. \tag{3.2}$$

Third, the agent must exert effort in stage 1 given that he will do so in stage 2:

$$\mu[\mu_1 w_{11}+(1-\mu_1)w_{10}-c]+(1-\mu)[\mu_0 w_{01}+(1-\mu_0)w_{00}-c]-c \geq \mu w_{01}+(1-\mu)w_{00}-c. \tag{3.3}$$

Fourth, shirking in both stages has to be unprofitable:

$$\mu[\mu_1 w_{11} + (1-\mu_1)w_{10} - c] + (1-\mu)[\mu_0 w_{01} + (1-\mu_0)w_{00} - c] - c \geq w_{00}. \tag{3.4}$$

Finally, the contract should be acceptable for the agent:

$$\mu[\mu_1 w_{11} + (1-\mu_1)w_{10} - c] + (1-\mu)[\mu_0 w_{01} + (1-\mu_0)w_{00} - c] - c \geq 0. \tag{3.5}$$

Since $\mu > \mu_0$, the last constraint is automatically satisfied with *strict* inequality by (3.3), (3.2), and the limited liability condition. Put differently, the agent can secure rent if the principal wants to implement effort plan $(1,1)$ in the objective scenario. This is because if he complies, his performance in stage 1 conveys information about the unknown quality of the match. More specifically, bad performance gives rise to the pessimistic posterior μ_0. Since the agent must continue to work hard, the principal has to offset the demotivating experience by a high reward for good performance in stage 2. But if the agent actually failed in stage 1 because he shirked, no information is conveyed via his performance, so that he is not pessimistic to be successful through hard work in stage 2. However, the principal deems the outcome of stage 1 to be informative since, in equilibrium, there is no shirking; she provides the same high-powered incentive whenever performance in stage 1 is bad. Taking limited liability into account, the agent must therefore get a strictly positive payoff if he deviates from effort plan $(1,1)$ and works according to $(0,1)$. Hence, it requires a rent to make $(1,1)$ incentive-compatible.

The cheapest wage schemes that implement effort plan $(1,1)$ thus solve

$$\min_{w \geq 0} \mu[\mu_1 w_{11} + (1-\mu_1)w_{10}] + (1-\mu)[\mu_0 w_{01} + (1-\mu_0)w_{00}] \quad s.t. \quad (3.1)\text{–}(3.4).$$

Lemma 3.1. *In the objective scenario, wage scheme w^* with*

$$(w^*_{00}, w^*_{10}, w^*_{01}, w^*_{11}) = \left(0, \frac{c}{\mu}, \frac{c}{\mu_0}, \frac{c}{\mu} + \frac{c}{\mu_0}\right).$$

implements effort plan $(1,1)$ as cheaply as possible.

Proof. Note first that (3.3) and (3.2) together ensure (3.4). Next, I reformulate (3.1)–(3.3):

$$\mu_1(w_{11} - w_{10}) \geq c, \tag{3.1'}$$

$$\mu_0(w_{01} - w_{00}) \geq c, \tag{3.2'}$$

$$\mu[\mu_1(w_{11} - w_{10}) + w_{10} - \mu_1(w_{01} - w_{00}) - w_{00}] \geq c. \tag{3.3'}$$

It is now routine to verify that (3.2') and (3.3') bind and that the proposed wage scheme indeed solves the program. □

The wage scheme in Lemma 3.1 rewards performance in stage 2 independently of performance in stage 1.[18] In particular, wages do not condition on the principal's posterior expectation of match quality, so that the agent has no incentive to manipulate it. Relative to this posterior, however, the wage scheme is too high-powered when stage 1 was successful (constraint (3.1) holds with strict inequality). As a consequence, the principal does not get the entire surplus but shares it with the agent. The following notation for the generated surplus and the agent's payoff will be helpful in stating this result:

$$S = \mu R - c \tag{3.6}$$

$$A = \left(\frac{\mu}{\mu_0} - 1\right) c. \tag{3.7}$$

Proposition 3.1. *In the objective scenario, the principal's profit is $2S - A$, the agent receives A, and the budget breaker is not involved.*

3.3.2 Subjective scenario

Consider now the subjective scenario, where the agent's performance in stage 1 is not verifiable and privately observed by the principal. Clearly, to provide an incentive to work hard repeatedly the agent needs to be rewarded depending on his performance in each stage. The principal must therefore make the wage scheme contingent on her subjective evaluation, which leads to two difficulties.

[18] The optimal wage scheme is not unique. For instance, $w = (0, 0, c/\mu_0, c/(\mu\mu_1) + c/\mu_0)$ also satisfies all constraints and is equally costly as w^*.

First, due to its subjective nature, the performance evaluation in stage 1 may lack credibility: ex post, the principal could prefer to submit a dishonest evaluation to save on wages. But the agent will shirk if he cannot be sure to receive a reward for good performance. To overcome this difficulty, the principal must involve the budget breaker. Transfers to this third party will solve her credibility problem.

The second difficulty relates to the fact that this principal-agent relationship comprises two stages, rather than just one. More precisely, it is not clear whether the principal should reveal her evaluation already before stage 2 begins, or afterwards, and in which form this should be done.[19] I take the following approach to address these issues. First, I derive a payment scheme involving wages to the agent and transfers to the budget breaker that implements effort plan $(1, 1)$ as cheaply as possible with *mediated* talk. The presence of a mediator allows for additional communication protocols and can only benefit the principal. In a second step, I show that face-to-face communication after stage 2 works as well as mediated talk. The advantage of this approach is that the benchmark case with mediator can be analyzed using the revelation principle for multistage games (Myerson, 1986).

So suppose for the moment a mediator (i.e., an impartial person with whom each party can communicate confidentially) is available to coordinate communication. According to the revelation principle, the cheapest contracts that implement effort plan $(1, 1)$ can be found in the class of contracts with the following communication protocol: Before stage 2, the principal reports her evaluation in form of a verifiable message $m \in \{0, 1\}$ to the mediator, who publicly reveals it after stage 2. No further communication takes place; in particular, the agent does not receive interim feedback about

[19]Exactly these questions are raised by the literature on interim feedback during long-term relationships. For example, Lizzeri et al. (2002), who consider a situation without uncertainty, find that the principal should not communicate the agent's interim performance before the project is completed. Suvorov and van de Ven (2009), on the other hand, show that if the principal cannot commit to a wage scheme, coarse feedback in connection with a bonus payment is beneficial. Similarly, in dynamic tournaments it might be advantageous to announce an ordinal midterm ranking (Gershkov and Perry, 2009), provide full feedback (Ederer, 2010) or partially disclose the participants' interim performance (Goltsman and Mukherjee, 2011).

his performance in stage 1.[20] The revelation principle furthermore asserts that contracts may without loss of generality specify payments which induce the principal to submit an honest report. In the following, I restrict the analysis to contracts with these properties.

Let $w = (w_{mx_2})_{m,x_2 \in \{0,1\}}$ be a wage scheme and denote by $b = (b_{mx_2})_{m,x_2 \in \{0,1\}}$ a scheme of non-negative transfers from the principal to the budget breaker. Here, $m \in \{0,1\}$ stands for the principal's reported evaluation, which will be honest in equilibrium. I refer to a combination (w, b) as a *payment scheme*.

The wage scheme must make effort plan $(1,1)$ incentive-compatible for the agent given that the principal truthfully reports to the mediator. In contrast to the objective scenario, the agent is now uninformed about past performance when choosing an effort level in stage 2. He can only use the prior expectation to estimate the quality of the match. Therefore, the incentive constraint for stage 2 pools the conditions (3.1) and (3.2):

$$\mu[\mu_1 w_{11} + (1-\mu_1)w_{10} - c] + (1-\mu)[\mu_0 w_{01} + (1-\mu_0)w_{00} - c] \geq \mu w_{10} + (1-\mu)w_{00}. \quad (3.8)$$

Furthermore, the incentive constraints (3.3) and (3.4) must be met to induce hard work in each stage. The agent's participation is then guaranteed by the limited liability assumption.

Of course, the wage scheme will only be credible for the agent if the principal is indeed willing to truthfully report her subjective evaluation to the mediator. Since the report is not transmitted to the agent before the project is completed, it cannot affect effort but only payments. Whenever wages alone would tempt the principal to cheat, transfers to the budget breaker need to be specified such that the correct report leads to the lowest total payment for the principal. Formally, in case the principal observes

[20] In general, the revelation principle prescribes that the agent receives recommendations from the mediator as to which effort to provide and that the agent sends reports to the mediator as to which effort he *did* provide. Here, recommendations would be superfluous since the agent is always supposed to work hard. Reports would also be superfluous: The principal would always expect that the agent reports hard work. The agent would have to be indifferent between all reports. Thus, neither the principal's incentives to be honest nor the agent's incentives to work hard would change with reports.

good performance this requires

$$\mu_1(w_{11}+b_{11})+(1-\mu_1)(w_{10}+b_{10}) \le \mu_1(w_{01}+b_{01})+(1-\mu_1)(w_{00}+b_{00}), \qquad (3.9)$$

whereas for bad performance it has to be

$$\mu_0(w_{01}+b_{01})+(1-\mu_0)(w_{00}+b_{00}) \le \mu_0(w_{11}+b_{11})+(1-\mu_0)(w_{10}+b_{10}). \qquad (3.10)$$

I have now identified all conditions which a payment scheme must satisfy to implement effort plan $(1,1)$ with mediated communication. Conditions (3.3), (3.4), and (3.8) rule out deviations by the agent to the effort plans (0,1), (0,0), and (1,0), respectively. Conditions (3.9) and (3.10), on the other hand, ensure that the principal honestly reports her subjective evaluation of stage 1 to the mediator. Hence, the cheapest payment schemes solve

$$\begin{aligned} \min_{(w,b)\ge 0} \quad & \mu[\mu_1(w_{11}+b_{11})+(1-\mu_1)(w_{10}+b_{10})] \\ & +(1-\mu)[\mu_0(w_{01}+b_{01})+(1-\mu_0)(w_{00}+b_{00})] \\ s.t. \quad & (3.3),\ (3.4),\ (3.8),\ (3.9),\ \textit{and}\ (3.10). \end{aligned}$$

Lemma 3.2. *In the subjective scenario with mediator, payment scheme* $(w^\star, b^\star)$ *with*

$$(w^\star_{00}, w^\star_{10}, w^\star_{01}, w^\star_{11}) = \left(0, 0, \frac{c}{\mu}, \frac{c}{\mu\mu_1}+\frac{c}{\mu}\right) \quad \textit{and} \quad (b^\star_{00}, b^\star_{10}, b^\star_{01}, b^\star_{11}) = \left(0, 0, \frac{c}{\mu\mu_1}, 0\right)$$

implements effort plan $(1,1)$ *as cheaply as possible.*

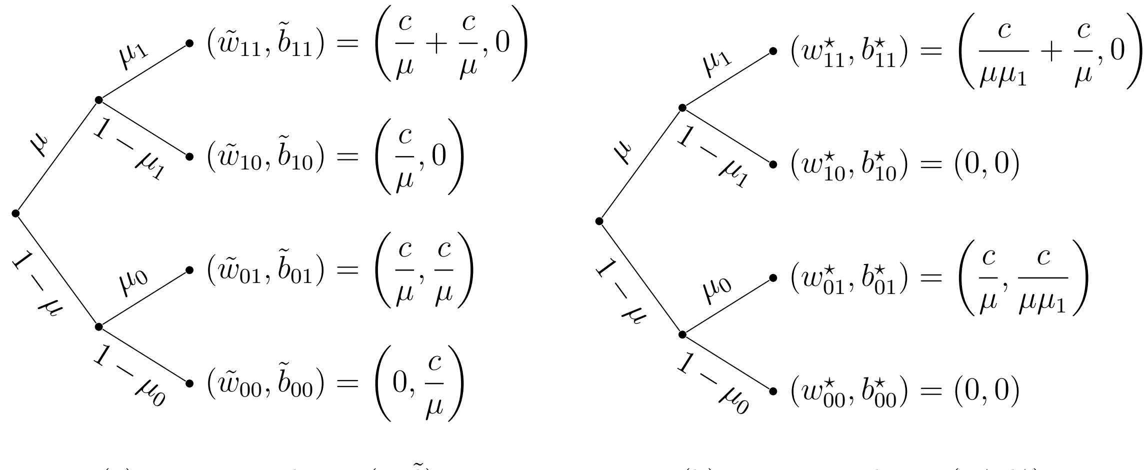
Proof. See Claim A4.2 in Appendix A4. □

The payment scheme in Lemma 3.2, $(w^\star, b^\star)$, in fact *pointwisely* satisfies the principal's truth-telling constraints for each performance outcome of stage 2:

$$w^\star_{11}+b^\star_{11} = w^\star_{01}+b^\star_{01} \quad \textit{and} \quad w^\star_{10}+b^\star_{10} = w^\star_{00}+b^\star_{00}. \qquad (3.11)$$

Thus, if the principal had to report her subjective evaluation *after* stage 2, rather than before, she would be honest as well. I now exploit this property to construct a contract that implements effort plan $(1,1)$ in the original setting without mediator at same costs as the best contracts with mediated talk.[21] Since the presence of a mediator did

[21]The optimal payment scheme with mediator is not unique. For instance, scheme (w,b) with $w=(0,c/\mu,c/\mu,2c/\mu)$ and $b=b^\star$ also satisfies all constraints and is equally costly as $(w^\star,b^\star)$.



(a) Payment scheme $(\tilde{w}, \tilde{b})$

(b) Payment scheme $(w^\star, b^\star)$

Figure 3.1: Non-contingent versus contingent reward for stage 1

not preclude any mode of face-to-face communication, it is impossible to find a better contract. Let the payment scheme be $(w^\star, b^\star)$. Communication proceeds as follows. After stage 2, the principal reports her evaluation of stage 1 in form of a verifiable message $m \in \{0, 1\}$ to the agent. No further communication takes place. From the agent's perspective, nothing has changed as compared with mediation, provided that the principal is still willing to report truthfully. And this is indeed the case, for, by (3.11), any report results in the same payment.

Without transfers to the budget breaker, the principal would always report bad performance to save on wages. The (non-mediated) contract described above confers credibility by committing the principal to pay the bonus for stage 1 regardless of the content of her subjective evaluation—to the agent if he was successful and otherwise to the budget breaker. The payoff to the budget breaker is the cost entailed by the principal's credibility issue. I state this result using the notation

$$B = \left(\frac{1}{\mu_1} - 1\right) c.$$

Proposition 3.2. *In the subjective scenario, the principal's profit is* $2S - B$*, the agent's payoff is zero, and the budget breaker receives* B*.*

I finally highlight an important property of the derived contract: it only rewards the agent for success in stage 1 if he performs well in stage 2 again. Figure 3.1 illustrates the benefit of this arrangement. The panel on the left-hand side depicts the payments under the scheme $(\tilde{w}, \tilde{b})$, which rewards good performance in stage 1 independently of stage 2 with the bonus c/μ. Here, the budget breaker's payoff is $\tilde{B} = (1/\mu - 1)c$. Scheme $(w^\star, b^\star)$, depicted on the right-hand side, only differs from $(\tilde{w}, \tilde{b})$ in that it just rewards the agent for stage 1 if stage 2 is successful as well. In expectation, $(w^\star, b^\star)$ provides the agent with the same bonus as $(\tilde{w}, \tilde{b})$, so that—given risk-neutrality—he has the same incentive to work hard. But the budget breaker's payoff is just $B < \tilde{B}$.

The principal essentially benefits from making the bonus for stage 1 contingent on good performance in stage 2 because that event is more likely with a *positive* subjective evaluation, which indicates *high* match quality. Hence, the probability that the bonus will be paid out to the budget breaker, rather than the agent, reduces.

3.3.3 Comparison of the scenarios

The previous findings can be used to compare the two scenarios with respect to the principal's profit. By Propositions 3.1 and 3.2, implementing hard work in each stage requires incentive costs, namely A in the objective and B in the subjective scenario. Proposition 3.3 follows immediately.

Proposition 3.3.

1. *Suppose $A = B$. The scenarios are equivalent with respect to the principal's profit.*
2. *Suppose $A < B$. The principal's profit is strictly larger in the objective scenario.*
3. *Suppose $A > B$. The principal's profit is strictly larger in the subjective scenario.*

Each scenario has its own problem if the agent is to work hard repeatedly. In case performance in stage 1 is objectively evaluated (objective scenario), the outcome of stage 1 cannot be concealed from the agent. It conveys information about the unknown quality of the match, which just the agent can interpret correctly with certainty. Therefore, the agent receives private information before the project is completed, and it secures him a rent. With a subjective evaluation (subjective scenario), on the other hand, this problem does not arise. It is the principal who obtains private information, and she may communicate with the agent only after his job is done. But in contrast to the objective scenario, the principal must involve the budget breaker to make the evaluation credible. Each problem can be more substantial, and each ranking of the scenarios can arise.

Uncertainty about match quality causes the problem in the objective scenario. More uncertainty, in form of more extreme posteriors after stage 1, increases the rent that the agent can secure by pretending to have a pessimistic belief. In contrast, more uncertainty *diminishes* the problem that arises in the subjective scenario. Recall that, in this scenario, the principal optimally rewards the agent for stage 1 only if stage 2 is successful. Now, this event becomes less likely after a negative and more likely after a positive evaluation if stage 1 gets more informative about match quality. Hence, the probability with which the principal pays the bonus to the budget breaker decreases. Moreover, the principal can decrease the bonus itself, for the agent will be more confident to earn it. Both lowers the budget breaker's payoff, the cost entailed by the credibility issue. In summary, more uncertainty is detrimental in the objective but beneficial in the subjective scenario. I state this result using the concept of mean preserving spread (MPS) as a criterion for differences in uncertainty.[22]

Proposition 3.4. *Suppose F is replaced by an MPS.*

1. *In the objective scenario, the principal's profit decreases.*
2. *In the subjective scenario, the principal's profit increases.*

Proof. By the definition of MPS, the prior μ does not change. The posteriors μ_1 and μ_0, on the other hand, are expectations of a convex and a concave function, respectively (see footnote 14). It follows that μ_1 is greater and μ_0 smaller than given F. Thus, the MPS leaves S unchanged, raises A, and lowers B. In light of Propositions 3.1 and 3.2, this finding concludes the proof. □

3.4 Conclusion

This chapter suggests an explanation as to why incentive pay sometimes depends on subjective performance evaluations even though comprehensive objective appraisal sys-

[22]Given two distribution functions F_1 and F_2 of match quality, F_2 is an MPS of F_1 if and only if $\int_0^1 h(\theta)dF_2 \leq \int_0^1 h(\theta)dF_1$ for any concave function h over $[0, 1]$; see, e.g., Proposition 6.D.2 in Mas-Colell et al. (1995).

tems seem feasible. My analysis builds on two central assumptions. First, an evaluation is considered as a means to generate information about the worker's performance that would remain unknown otherwise. Second, the acquired information is either private and unverifiable or public and verifiable, depending on whether the employer rates subjectively or objectively. Verifiability is clearly an important concern since discretionary compensation entails a credibility issue for the employer. I show that, nevertheless, subjective evaluations can be advantageous because they allow to withhold information from the worker. Indeed, withholding information seems particularly important under circumstances where the credibility issue is small. According to my analysis, subjective evaluations are used at early stages of employment relationships in which characteristics that determine the worker's success on the job are highly uncertain.

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Appendices

A1 Chapter 1: Formal results

Claim A1.1. *(1.3)–(1.6) hold if and only if (IA) and*

$$\underline{U}(1) - \overline{U}(1) \in \left[(\overline{\beta} - \underline{\beta})\overline{q} + \frac{\gamma}{p}, (\overline{\beta} - \underline{\beta})\underline{q} - \frac{\gamma}{1-p}\right] \tag{A1.1}$$

are satisfied.

Proof. Conditions (1.3) and (1.4), which ensure that the agent acquires information if this is recommended, are equivalent to

$$\begin{aligned}\overline{U}(1) &\leq \underline{U}(1) - (\overline{\beta} - \underline{\beta})\overline{q} - \frac{\gamma}{p} \\ \underline{U}(1) &\leq \overline{U}(1) + (\overline{\beta} - \underline{\beta})\underline{q} - \frac{\gamma}{1-p}.\end{aligned}$$

These two inequalities guarantee the adverse selection conditions (1.5) and (1.6) and are equivalent to (IA) together with (A1.1) . □

Claim A1.2. *If a contract satisfies conditions (1.2), (IA), and (A1.1), then* $\underline{U}(\alpha) \geq 0$.

Proof. If the agent is to stay uninformed after signing and production costs are low, he gets the extra payoff

$$\underline{U}(0) - \overline{U}(0) = (\overline{\beta} - \underline{\beta})q \geq 0.$$

On the other hand, conditions (IA) and (A1.1) require

$$\underline{U}(1) - \overline{U}(1) > 0,$$

so that $\underline{U}(\alpha) \geq \overline{U}(\alpha)$. If the contract satisfies the individual rationality condition (1.2), this inequality implies $\underline{U}(\alpha) \geq 0$. □

Claim A1.3. *Suppose a contract satisfies (1.1), (1.2), (IA), and (A1.1). Then, there is a contract which satisfies (1.1), (1.2), (IA), and*

$$\underline{U}(1) - \overline{U}(1) = (\overline{\beta} - \underline{\beta})\overline{q} + \frac{\gamma}{p} \tag{1.7}$$

and results in identical payoffs.

Proof. In particular, suppose the original contract, denoted by C, implies

$$\underline{U}(1) - \overline{U}(1) = (\overline{\beta} - \underline{\beta})\overline{q} + \frac{\gamma}{p} + r,$$

where r is such that (A1.1) holds. The alternative contract, denoted by C', differs from C in two aspects. First, it specifies $\underline{t}' = \underline{t} - r$. Because of this difference, equation (1.7) holds. Second, it specifies $t' = t + \frac{p\alpha r}{1-\alpha}$. Together, these two differences imply,

$$p\underline{U}'(\alpha) + (1-p)\overline{U}'(\alpha) = p\underline{U}(\alpha) + (1-p)\overline{U}(\alpha),$$

so that the alternative contract C' results in identical payoffs for both parties given that it satisfies (1.1), (1.2), and (IA). Conditions (1.2) and (IA) hold because of the preceding equation and since the two contracts specify the same output levels, respectively. Condition (1.1) holds because $\underline{U}'(\alpha) \geq 0$ by Claim A1.2 and $\overline{U}'(\alpha) \geq \overline{U}(\alpha)$ by the specification of C'. □

Claim A1.4. *There exists a cutoff γ_1 such that it holds for any $\alpha \in (0,1)$:*

$$(1-\alpha)W(0,\gamma) + \alpha W(1,\gamma) < W(\alpha,\gamma) \quad \text{if and only if} \quad \gamma > \gamma_1.$$

Proof. Take any $\alpha \in (0,1)$. Without loss of generality, I may relabel $\overline{U}(\alpha) = (1-\alpha)\overline{U}(0)$. Then, by definition,

$$\begin{aligned} W(\alpha,\gamma) = \max_{\overline{U}(0),(q,\underline{q},\overline{q})\geq 0} & (1-\alpha)[V(q) - \overline{\beta}q - \overline{U}(0)] \\ & + \alpha\{-\gamma + p[V(\underline{q}) - \underline{\beta q}]\} \\ & + \alpha(1-p)[V(\overline{q}) - (\overline{\beta} + \tfrac{p}{1-p}(\overline{\beta} - \underline{\beta}))\overline{q}] \\ & + \lambda_1[(1-p)\overline{U}(0) + \gamma] \\ & + \lambda_2[\overline{U}_0 + p(\overline{\beta} - \underline{\beta})(q + \tfrac{\alpha}{1-\alpha}\overline{q})] \\ & + \lambda_3[(1-p)p(\overline{\beta} - \underline{\beta})(\underline{q} - \overline{q}) - \gamma], \end{aligned}$$

where the λs are non-negative Lagrange multipliers. On the other hand, the left hand side of the inequality in the claim can be stated as

$$\begin{aligned}(1-\alpha)W(0,\gamma)+\alpha W(1,\gamma) = \max_{\overline{U}(0),(q,\underline{q},\overline{q})\geq 0} & (1-\alpha)[V(q)-\overline{\beta}q-\overline{U}(0)] \\ & + \alpha\{-\gamma + p[V(\underline{q})-\underline{\beta q}]\} \\ & + \alpha(1-p)[V(\overline{q})-(\overline{\beta}+\tfrac{p}{1-p}(\overline{\beta}-\underline{\beta}))\overline{q}] \\ & + \mu_1[(1-p)\overline{U}(0)+\gamma] \\ & + \mu_2[\overline{U}(0)+p(\overline{\beta}-\underline{\beta})q] \\ & + \mu_3[(1-p)p(\overline{\beta}-\underline{\beta})(\underline{q}-\overline{q})-\gamma],\end{aligned}$$

where the μs are non-negative Lagrange multipliers. The two programs only differ as to the second constraint, and the inequality in the claim holds if and only if $\mu_2 > 0$. One can verify that this is equivalent to $\gamma > (1-p)p(\overline{\beta}-\underline{\beta})\overline{q}^* \equiv \gamma_1$. □

Claim A1.5. *Define*

$$\begin{aligned} S &= \{((\underline{t},\underline{q}),(\overline{t},\overline{q})) : \overline{U}(1)\geq 0, \quad (IA), \quad (1.7)\}, \\ P &= \{(t,q) : (1-p)\overline{U}(0)+\gamma \geq 0, \quad \overline{U}(0)+p(\overline{\beta}-\underline{\beta})q\geq 0\},\end{aligned}$$

and

$$\begin{aligned}\widetilde{P}(\alpha,((\underline{t},\underline{q}),(\overline{t},\overline{q}))) = \{(t,q) : & (1-p)[\overline{U}(0)+\tfrac{\alpha}{1-\alpha}\overline{U}(1)]+\gamma\geq 0, \\ & \overline{U}(0)+\tfrac{\alpha}{1-\alpha}\overline{U}(1)+p(\overline{\beta}-\underline{\beta})[q+\tfrac{\alpha}{1-\alpha}\overline{q}]\geq 0\}.\end{aligned}$$

1. *For each contract that satisfies (NIA), (IR), (IA), and (1.7), there is a contract that satisfies $\overline{U}(1)\geq 0$ in addition and results in identical payoffs.*
2. *Given $\overline{U}(1)\geq 0$, a contract satisfies (NIA), (IR), (IA), and (1.7) if and only if*
   - *the separating schedule is in S if $\alpha > 0$*
   - *the pooling schedule is in P if $\alpha = 0$ and in $\widetilde{P}(\alpha,((\underline{t},\underline{q}),(\overline{t},\overline{q})))$ if $\alpha\in(0,1)$.*

Proof. Given $\alpha = 1$, condition (NIA) requires $\overline{U}(\alpha)\geq 0$. For $\alpha < 1$, on the other hand, one may without loss of generality relabel $\overline{U}(\alpha) = (1-\alpha)\overline{U}(0)$, so that $\overline{U}(1) = 0$. The implications for the implementable schedules now follow from (NIA), (IR), (IA), and (1.7). □

Claim A1.6. *W is continuous.*

Proof. Note first that, formally, $W(\alpha,\gamma)$ is the maximum value of program

$$\mathcal{P}(\alpha,\gamma): \quad \max_{\overline{U}(\alpha),(q,\underline{q},\overline{q})\geq 0} \Pi \quad s.t. \quad (NIA),(IR),(IA),$$

where $\alpha \in [0,1]$. Due to the assumptions on V, there exists a unique solution to this program (see footnote 18).

The claim holds by the maximum theorem (e.g., de la Fuente, 2000, p. 301) if the correspondence that assigns to each (α,γ) the set of feasible choices in $\mathcal{P}(\alpha,\gamma)$ is compact-valued and continuous. I denote this correspondence by F. It is not compact because there are no upper bounds on $\overline{U}(\alpha), q$, and $\underline{q}$.[23] By adding non-binding constraints, I will replace it by another correspondence f such that maximization with respect to f yields the same maximum value $W(\alpha,\gamma)$ and such that f is compact-valued and continuous. Consequently, $W(\alpha,\gamma)$ has to be continuous by the maximum theorem. I will use the following lemma (which is a special case of Thm. 2.2, p. 303, in de la Fuente, 2000):

(I) *For $i=1,\ldots,I$, let $g^i(\overline{U}(\alpha),q,\overline{q},\underline{q},\alpha,\gamma):\mathbb{R}^4\times[0,1]\times\mathbb{R}_{>0}\to\mathbb{R}$ be continuous functions that are affine given (α,γ), and define the correspondence $f:[0,1]\times\mathbb{R}_{>0}\rightrightarrows\mathbb{R}^4$ by*

$$f(\alpha,\gamma)=\{(\overline{U}(\alpha),q,\overline{q},\underline{q}):g^i(\overline{U}(\alpha),q,\overline{q},\underline{q},\alpha,\gamma)\geq 0 \quad \text{for all} \quad i=1,\ldots,I\}.$$

Let $f(\widehat{\alpha},\widehat{\gamma})$ be compact, and assume that there is some point $(\overline{U}'(\widehat{\alpha}),q',\overline{q}',\underline{q}')\in f(\widehat{\alpha},\widehat{\gamma})$ such that $g^i(\overline{U}'(\widehat{\alpha}),q',\overline{q}',\underline{q}',\widehat{\alpha},\widehat{\gamma})>0$ for all i. Then, f is continuous at $(\widehat{\alpha},\widehat{\gamma})$.

It can be checked that for $k>0$ and $\phi=(1-p)p(\overline{\beta}-\underline{\beta})$, the following conditions would be non-binding as additional constraints in $\mathcal{P}(\alpha,\gamma)$:

$$\overline{U}(\alpha)\leq k, \quad q\leq q^*+k, \quad \text{and} \quad \underline{q}\leq\max\{\underline{q}^*,\overline{q}^*+\tfrac{\gamma}{\phi}\}+k.$$

Add them to $\mathcal{P}(\alpha,\gamma)$. This yields a bounded feasible set, which I denote by $f(\alpha,\gamma)$. It can be described by level sets of continuous functions $g^i(\overline{U}(\alpha),q,\overline{q},\underline{q},\alpha,\gamma)$, where

[23] In contrast, (IA) provides an upper bound for $\overline{q}$.

the functions are affine given (α, γ). Being an intersection of closed sets, $f(\alpha, \gamma)$ is closed and therefore compact. At the point $(\overline{U}'(\alpha), \underline{q}', \overline{q}', \underline{q}') = (0, q^*, \overline{q}^*, \max\{\underline{q}^*, \overline{q}^* + \frac{\gamma}{\phi}\})$, all constraints are satisfied with strict inequality. Hence, the correspondence f is continuous by **(I)**. If the feasible set $F(\alpha, \gamma)$ in problem $\mathcal{P}(\alpha, \gamma)$ is replaced by $f(\alpha, \gamma)$, one obtains the same maximum value $W(\alpha, \gamma)$ because the solution with respect to $F(\alpha, \gamma)$ is contained in $f(\alpha, \gamma)$ and $f(\alpha, \gamma)$ itself is contained in $F(\alpha, \gamma)$. □

Claim A1.7. *There exists a unique intersection γ_2 of $W(0, \cdot)$ and $W(1, \cdot)$.*

Proof. By definition,

$$\begin{aligned} W(0, \gamma) = \max_{\overline{U}(0), q \geq 0} \; & V(q) - E[\beta]q - \overline{U}(0) - p(\overline{\beta} - \underline{\beta})q \\ & + \lambda_1[(1-p)\overline{U}(0) + \gamma] \\ & + \lambda_2[\overline{U}(0) + p(\overline{\beta} - \underline{\beta})q], \end{aligned}$$

where the λs are non-negative Lagrange multipliers. Also by definition,

$$\begin{aligned} W(1, \gamma) = \max_{\overline{U}(1), (\underline{q}, \overline{q}) \geq 0} \; & -\gamma + p[V(\underline{q}) - \underline{\beta q}] + (1-p)[V(\overline{q}) - \overline{\beta q}] \\ & - \overline{U}(1) - p(\overline{\beta} - \underline{\beta})\overline{q} \\ & + \mu_1 \overline{U}(1) \\ & + \mu_2[(1-p)p(\overline{\beta} - \underline{\beta})(\underline{q} - \overline{q}) - \gamma], \end{aligned}$$

where the μs are non-negative Lagrange multipliers. $W(0, \cdot)$ and $W(1, \cdot)$ are concave functions of γ (e.g., de la Fuente, 2000, p. 313, Thm. 2.12) and therefore differentiable almost everywhere. At points where they are differentiable, it holds that

$$\begin{aligned} \frac{dW(0, \gamma)}{d\gamma} &= \lambda_1 \geq 0 \\ \frac{dW(1, \gamma)}{d\gamma} &= -1 - \mu_2 < 0, \end{aligned}$$

so that (by continuity of W, see Claim A1.6) there can be at most one intersection. Furthermore, one can check that (i) $W(1, 0) > W(0, 0)$, and that (ii) $W(1, \widehat{\gamma}) < W(0, \widehat{\gamma})$, where $\widehat{\gamma} = W(1, 0) - W(0, 0) + k$ with $k > 0$.[24] Therefore, any intersection must lie in the interval $(0, \widehat{\gamma})$. Since the functions $W(0, \cdot)$ and $W(1, \cdot)$ are continuous by Claim A1.6, the intermediate value theorem implies that an intersection exists. □

[24] $W(\cdot, 0)$ is to denote $\lim_{\gamma \to 0} W(\cdot, \gamma)$.

Claim A1.8. *There exists a cutoff* $p_0 < 1$ *such that* $\gamma_2 > \gamma_1$ *if* $p > p_0$.

Proof. Recall from the proof of Claim A1.4 that $\gamma_1 \equiv (1-p)p(\overline{\beta}-\underline{\beta})\overline{q}^*$; γ_2, on the other hand, denotes the intersection of $W(0,\cdot)$ and $W(1,\cdot)$.

According to the proof of Claim A1.7, $W(0,\cdot)$ is increasing and $W(1,\cdot)$ decreasing. Moreover, both functions are continuous by Claim A1.6. Therefore, $\gamma_2 > \gamma_1$ holds if and only if $W(1,\gamma_1) > W(0,\gamma_1)$.

As can be verified,

$$\begin{aligned} W(0,\gamma_1) &= \max_{q\geq 0} V(q) - \overline{\beta}q + p(\overline{\beta}-\underline{\beta})\overline{q}^* \\ &= V(\overline{q}^*) - \underline{\beta}\overline{q}^* - (1-p)(\overline{\beta}-\underline{\beta})\overline{q}^*. \end{aligned}$$

Consider now $W(1,\gamma_1)$. At this level of investigation costs, condition (IA) reads $\underline{q} - \overline{q} - \overline{q}^* \geq 0$. Therefore,

$$\begin{aligned} W(1,\gamma_1) = \max_{(\underline{q},\overline{q})\geq 0} & -\gamma_1 + p[V(\underline{q}) - \underline{\beta q}] + (1-p)[V(\overline{q}) - \overline{\beta q}] - p(\overline{\beta}-\underline{\beta})\overline{q} \\ s.t. \quad & \underline{q} - \overline{q} - \overline{q}^* \geq 0 \\ \geq & -(1-p)p(\overline{\beta}-\underline{\beta})\overline{q}^* + p[V(\underline{q}^*) - \underline{\beta q}^*], \end{aligned}$$

which concludes the proof (e.g., define $p_0 \equiv [V(\overline{q}^*) - \underline{\beta}\overline{q}^*]/[V(\underline{q}^*) - \underline{\beta q}^*]$). □

A2 Chapter 1: Arbitrary number of types

This appendix shows that the benefits of stochastic contracts are not a peculiarity of the original model with its two states of nature. To this end, consider now the more general case in which the agent's production costs are drawn from the set $\{\beta_1,\ldots,\beta_n\}$. Suppose $0 < \beta_1 < \ldots < \beta_n$, and let $p_i > 0$ be the probability of state i.

The arguments in section 1.3.1 as to the relevant class of contracts do not depend on the number of states. Thus, only the structure of the separating schedule has to be modified, and the principal can confine herself to contracts of the form

$$c = \{\alpha, (t,q), (t_i,q_i)_{i=1}^n\}.$$

As before, the proposed contract must deter precontractual investigation, be acceptable, and induce the agent to be obedient and honest if he is asked to investigate his costs. Using the notation $U_i(\alpha) = (1-\alpha)(t - \beta_i q) + \alpha(t_i - \beta_i q_i)$, these conditions read:

$$\sum_{i=1}^{n} p_i U_i(\alpha) - \alpha\gamma \geq \sum_{i=1}^{n} p_i \max\{U_i(\alpha), 0\} - \gamma \tag{A2.1}$$

$$\sum_{i=1}^{n} p_i U_i(\alpha) - \alpha\gamma \geq 0 \tag{A2.2}$$

$$\sum_{i=1}^{n} p_i U_i(1) - \gamma \geq U_j(1) - (E[\beta] - \beta_j)q_j \quad \forall j \in \{1, \ldots, n\} \tag{A2.3}$$

$$U_i(1) \geq U_j(1) - (\beta_i - \beta_j)q_j \quad \forall i, j \in \{1, \ldots, n\}. \tag{A2.4}$$

The principal's payoff from such a contract is

$$\nu = (1-\alpha)[V(q) - t] + \alpha \left[\sum_{i=1}^{n} V(q_i) - t_i \right],$$

so that the contracting problem can be stated as

$$\max_{c} \nu \quad s.t. \quad (A2.1)\text{–}(A2.4).$$

The analysis of the case with two levels of production costs—the original model—is special in that the moral hazard constraints (A2.3) imply the adverse selection constraints (A2.4). But the benefits of stochastic contracts are robust: according to Claim A2.1, stochastic contracts generally allow to implement schedules that are incompatible with deterministic contracts (cf. Claim A1.5 in Appendix A1). An analogous procedure as in the main text can be applied to show that the additional choices may indeed be relevant for the principal.

Claim A2.1. *Define*

$$E = \left\{ (t_i, q_i)_{i=1}^{n} : U_n(1) \geq 0, \quad (A2.3), \quad (A2.4) \right\},$$

$$F = \left\{ (t, q) : \sum_{i=j+1}^{n} p_i U_i(0) + \gamma \geq 0 \ \forall j \in \{1, \ldots, n-1\}, \quad \sum_{i=1}^{n} p_i U_i(0) \geq 0 \right\},$$

and

$$\begin{aligned}\widetilde{F}(\alpha,(t_i,q_i)_{i=1}^n) =\big\{(t,q) :\\ &\sum_{i=j+1}^n p_i\left[U_i(0)+\frac{\alpha}{1-\alpha}U_i(1)\right]+\gamma\geq 0\,\forall j\in\{1,\dots,n-1\},\\ &\sum_{i=1}^n p_iU_i(0)+\frac{\alpha}{1-\alpha}\left[\sum_{i=1}^n p_iU_i(1)-\gamma\right]\geq 0\big\}.\end{aligned}$$

1. *For each contract that satisfies (A2.1)–(A2.4), there is a contract that satisfies* $U_n(1)\geq 0$ *in addition and results in identical payoffs.*
2. *Given* $U_n(1)\geq 0$*, a contract satisfies (A2.1)–(A2.4) if and only if*
   - *the separating schedule is in* E *if* $\alpha>0$
   - *the pooling schedule is in* F *if* $\alpha=0$ *and in* $\widetilde{F}(\alpha,(t_i,q_i)_{i=1}^n)$ *if* $\alpha\in(0,1)$.

Proof. I first derive some auxiliary results.

(II) By standard arguments, the adverse selection constraints (A2.4) are equivalent to

$$U_i(1)-U_{i+1}(1)\in[(\beta_{i+1}-\beta_i)q_{i+1},(\beta_{i+1}-\beta_i)q_i]\quad\forall i\in\{1,\dots,n-1\}$$

together with $q_1\geq\cdots\geq q_n$. This implies $U_1(1)\geq\ldots\geq U_n(1)$. Moreover, by definition it holds that

$$U_i(0)-U_{i+1}(0)=(\beta_{i+1}-\beta_i)q\quad\forall i\in\{1,\dots,n-1\},$$

so that any contract which satisfies (A2.4) involves $U_1(\alpha)\geq\ldots\geq U_n(\alpha)$. Condition (A2.1) may therefore be reformulated as

$$\sum_{i=1}^n p_iU_i(\alpha)-\alpha\gamma\geq\sum_{i=1}^j p_iU_i(\alpha)-\gamma\quad\forall j\in\{1,\dots,n-1\}.$$

The two statements of the claim can now be proved.

1. Suppose first $\alpha=1$. According to **(II)**, (A2.1) requires $U_n(1)\geq 0$ if (A2.4) is to hold. Next, consider the case $\alpha<1$ and suppose the original contract, denoted by c, implies $U_n(1)=-r<0$. The alternative contract, denoted by c', differs in two aspects. First, it specifies $t_i'=t_i+r$ for all i, so that $U_n'(1)=0$. Second, it specifies $t'=t-\frac{\alpha}{1-\alpha}r$. These modifications imply

$$U_i'(\alpha)=U_i(\alpha)\quad\forall i\in\{1,\dots,n\}.\tag{A2.5}$$

Thus, the alternative contract results in identical payoffs given that it satisfies (A2.1)–(A2.4). Conditions (A2.1) and (A2.2) are met because of (A2.5). Condition (A2.3) holds since both sides of these inequalities are raised by r. Finally, (**II**) implies that condition (A2.4) is satisfied because the two contracts specify the same output levels and

$$U_i'(1) - U_{i+1}'(1) = U_i(1) - U_{i+1}(1) \quad \forall i \in \{1, \ldots, n-1\}.$$

2. Follows from (A2.1)–(A2.4) and **(II)**.

□

A3 Chapter 2: Formal results

Claim A3.1. *(P1) is satisfied if and only if it holds that*

$$U_i - U_{i+1} \in [(\beta_{i+1} - \beta_i)\, q_{i+1},\, (\beta_{i+1} - \beta_i)\, q_i] \quad \forall i \in \{1, \ldots, n-1\} \tag{A3.1}$$

$$q_i - q_{i+1} \geq 0 \quad \forall i \in \{1, \ldots, n-1\} \tag{2.2}$$

Proof. The proof is standard and, therefore, omitted. □

Claim A3.2. *If C^P satisfies (P1) and (P2), condition (P3) holds as well if and only if*

$$\sum_{j=l}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} U_i \right] + e \geq 0 \quad \forall l \in \{2, \ldots, m\}. \tag{A3.2}$$

Proof. I first derive some auxiliary results.

(I) First, straightforward algebra shows that (P3) is equivalent to

$$\sum_{j=1}^{m} \pi_j \min \left\{ 0, \sum_{i=1}^{n} \gamma_{ij} U_i \right\} + e \geq 0.$$

Second, according to Claim A3.1 (P1) implies $U_i \geq U_{i+1}$ for all $i \in \{1, \ldots, n-1\}$ and hence, by the first-order stochastic dominance ranking of the ex-ante types,

$$\sum_{i=1}^{n} \gamma_{ij} U_i \geq \sum_{i=1}^{n} \gamma_{i(j+1)} U_i \quad \forall j \in \{1, \ldots, m-1\}. \tag{A3.3}$$

Third, given (A3.3) a necessary condition for (P2) is $\sum_{i=1}^{n} \gamma_{i1} U_i \geq 0$.

The claim can now be proved. Suppose C^P satisfies (P1) and (P2) but violates (P3). By **(I)**, there is an $l \in \{2, \ldots, m\}$ such that

$$\sum_{j=1}^{m} \pi_j \min\left\{0, \sum_{i=1}^{n} \gamma_{ij} U_i\right\} + e = \sum_{j=l}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} U_i\right] + e < 0;$$

hence, (A3.2) does not hold. Suppose C^P satisfies (P1) and (P2) but violates (A3.2). Let $l \in \{2, \ldots, m\}$ be the smallest j such that $\sum_{i=1}^{n} \gamma_{ij} U_i < 0$. By **(I)**,

$$0 > \sum_{j=l}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} U_i\right] + e = \sum_{j=1}^{m} \pi_j \min\left\{0, \sum_{i=1}^{n} \gamma_{ij} U_i\right\} + e;$$

that is, (P3) does not hold. □

Claim A3.3. *Suppose C^P satisfies (A3.1), (2.2), (P2), and (P3). Then there is a pooling contract which satisfies (2.2), (P2), (P3), and*

$$U_i - U_{i+1} = (\beta_{i+1} - \beta_i)\, q_{i+1} \quad \forall i \in \{1, \ldots, n-1\} \tag{2.1}$$

and results in identical expected payoffs.

Proof. Suppose there exists an $i' \in I$ for which the original contract C^P implies

$$U_{i'} - U_{i'+1} = (\beta_{i'+1} - \beta_{i'})\, q_{i'+1} + \upsilon,$$

where υ is such that (A3.1) holds. The alternative contract, denoted by $\widetilde{C}^P$, differs from C^P only with respect to transfers. Specifically,

$$\widetilde{t}_k = \begin{cases} t_k - \left(1 - \left[\sum_{j=1}^{m} \pi_j \Gamma_{i'j}\right]\right) \upsilon & \textit{if } k \in \{1, \ldots, i'\} \\ t_k + \sum_{j=1}^{m} \pi_j \Gamma_{i'j} \upsilon & \textit{if } k \in \{i'+1, \ldots, n\}. \end{cases}$$

This difference implies

$$\widetilde{U}_{i'} - \widetilde{U}_{i'+1} = (\beta_{i'+1} - \beta_{i'})\, q_{i'+1}$$

and

$$\widetilde{U}_i - \widetilde{U}_{i+1} = U_i - U_{i+1} \quad \forall i \in \{1, \ldots, n-1\} \setminus i'.$$

Furthermore,

$$\sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} \widetilde{t}_i\right] = \sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} t_i\right],$$

so that the alternative contract satisfies (P2) and—if (P3) is also met—results in identical expected payoffs as the original one. Since, for any $l \in \{2, \dots, m\}$,

$$\begin{aligned}\sum_{j=l}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} \widetilde{U}_i\right] + e &= \sum_{j=l}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} U_i - \Gamma_{i'j} \upsilon + \sum_{j=1}^{m} \pi_j \Gamma_{i'j} \upsilon\right] + e \\ &> \sum_{j=l}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} U_i\right] + e \quad \geq \quad 0,\end{aligned}$$

where the first inequality follows from the first-order stochastic dominance ranking and the second one from the hypothesis, Claim A3.2 ensures that condition (P3) is met. □

Claim A3.4. *If Condition 2.1 holds, there exist solutions to both $\mathcal{P}^P$ and $\mathcal{P}^S$.*

Proof. I will use the following theorem (e.g., Rockafellar, 1970, Thm. 27.3):

(II) *Let $h : \mathbb{R}^I \rightarrow \mathbb{R} \cup \{-\infty\}$ be a closed proper concave function, and let $D \subseteq \mathbb{R}^I$ be a non-empty closed convex set over which h is to be maximized. If h and D have no direction of recession in common, then h attains its supremum over D.*[25]

Consider first program $\mathcal{P}^P$. The objective function can be restated as follows:

$$h(C^P) = \begin{cases} \sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} [V(q_i) - t_i]\right] & \textit{if } q_i \geq 0 \, \forall i \in I \\ -\infty & \textit{else.} \end{cases}$$

Due to the assumptions on V, h is a closed proper concave function. Next, let D be the set of all combinations C^P that satisfy (P1)–(P3). These constraints can be described by closed level sets of affine functions, so that D must be closed and convex. D is furthermore non-empty—for instance, C^P with $(t_i, q_i) = (0, 0)$ for all $i \in I$ satisfies (P1)–(P3).

I now apply **(II)** and show that h and D have no directions of recession in common. As $V'(\infty) = 0$, and because $h(C^P) = -\infty$ if C^P contains some negative q_i, any direction of recession of h satisfies

$$\sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} t_i\right] \leq 0 \quad \textit{and} \quad q_i \geq 0 \, \forall i \in I.$$

[25] According to Rockafellar (1970), a concave function $h : \mathbb{R}^I \rightarrow \mathbb{R} \cup \{-\infty\}$ is *proper* if $h(x) > -\infty$ for at least one x, and it is *closed* if $\{x : h(x) \geq \alpha\}$ is closed for every $\alpha \in \mathbb{R}$. A vector $y \neq 0$ is a *direction of recession of a convex set* D if $x + \lambda y \in D$ for every $\lambda \geq 0$ and every $x \in D$. The *directions of recession of a closed proper concave function* h are the directions of recession of the sets $\{x : h(x) \geq \alpha\}$, where $\alpha \in \mathbb{R}$.

But even the set of all C^P that are compatible with (P2) cannot have such directions of recession.

Consider now program $\mathcal{P}^S$. The set of all C^S that satisfy (S1)–(S4) is non-empty—for instance, any C^S with

$$(t_{ij}, q_{ij}) = \begin{cases} (0,0)\,\forall\, i \in I & \textit{if } j \neq 1 \\ (t,q)\,\forall\, i \in I & \textit{if } j = 1, \end{cases}$$

where (t, q) fulfills

$$\pi_1 \left[t - \left[\sum_{i=1}^{n} \gamma_{i1}\beta_i q \right] \right] - e \geq 0 \quad \textit{and} \quad t - \left[\sum_{i=1}^{n} \gamma_{i2}\beta_i q \right] \leq 0,$$

satisfies (S1)–(S4). Such combinations exist due to the first-order stochastic dominance ranking of the ex-ante types. Existence of solutions to program $\mathcal{P}^S$ can now be proved analogously to program $\mathcal{P}^P$. □

Claim A3.5. *If Condition 2.1 holds, W^P is non-decreasing and W^S non-increasing.*

Proof. If e increases, the feasible sets in $\mathcal{P}^P$ and $\mathcal{P}^S$ expand and contract, respectively. The corresponding objective functions, on the other hand, do not vary with e. □

Claim A3.6. *There exists a cutoff $\eta > 0$ such that for any $e < \eta$*

$$\begin{aligned} W^P(e) = \max_{(q_i)_{i=1}^n} \quad & \sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij} [V(q_i) - \beta_i q_i] \right] \\ & - \left[-\frac{e}{\pi_m} + \sum_{j=1}^{m-1} \pi_j \left[\sum_{i=1}^{n-1} (\Gamma_{ij} - \Gamma_{im})(\beta_{i+1} - \beta_i) q_{i+1} \right] \right] \quad s.t. \quad (2.2). \end{aligned}$$

Proof. I first derive some auxiliary results, using insights by CK.

(III) *There exists a value $\underline{q} > 0$ such that for any e solutions to $\widetilde{\mathcal{P}}^P$ satisfy $q_i \geq \underline{q}$ for all $i \in I$.*

Proof. This is Claim 1 of the appendix by CK. Their proof applies to the current setting.

(IV) *There exists a cutoff $\psi > 0$ such that for any $e < \psi$ solutions to $\widetilde{\mathcal{P}}^P$ satisfy*

$$\sum_{j=l}^{m} \pi_j \left[U_n + \sum_{i=1}^{n-1} \Gamma_{ij}(\beta_{i+1} - \beta_i) q_{i+1} \right] + e > 0 \quad \forall l \in \{2, \ldots, m-1\}.$$

Proof. This is analogous to Claim 2 of the appendix by CK. By **(III)** and the first-order stochastic dominance ranking of the ex-ante types, solutions involve

$$\sum_{i=1}^{n-1} \Gamma_{ij}(\beta_{i+1}-\beta_i)q_{i+1} > \sum_{i=1}^{n-1} \Gamma_{i(j+1)}(\beta_{i+1}-\beta_i)q_{i+1} \quad \forall j \in \{1,\dots,m-1\} \tag{A3.4}$$

To prove the statement it thus suffices to show that, for low e,

$$\sum_{j=m-1}^{m} \pi_j \left[U_n + \sum_{i=1}^{n-1} \Gamma_{ij}(\beta_{i+1}-\beta_i)q_{i+1}\right] + e = 0$$

is impossible. Suppose, by contradiction, this equality holds. Then

$$\begin{aligned} & \pi_m \left[U_n + \sum_{i=1}^{n-1} \Gamma_{im}(\beta_{i+1}-\beta_i)q_{i+1}\right] \\ \leq \quad & \pi_m \left[\frac{\pi_{m-1}}{\pi_m+\pi_{m-1}} \sum_{i=1}^{n-1} (\Gamma_{im}-\Gamma_{i(m-1)})(\beta_{i+1}-\beta_i)q_{i+1}\right] \\ < \quad & \pi_m \left[\frac{\pi_{m-1}}{\pi_m+\pi_{m-1}} \sum_{i=1}^{n-1} (\Gamma_{im}-\Gamma_{i(m-1)})(\beta_{i+1}-\beta_i)\underline{q}\right] \quad < \quad 0, \end{aligned}$$

where the first inequality follows by hypothesis and the latter ones by **(III)** and the first-order stochastic dominance ranking of the ex-ante types. Hence, the term $\pi_m \left[U_n + \sum_{i=1}^{n-1} \Gamma_{im}(\beta_{i+1}-\beta_i)q_{i+1}\right]$ is bounded above away from zero. This implies that constraint (2.4) cannot hold for low e; a contradiction.

The claim can now be proved. By (A3.4), solutions to $\widetilde{\mathcal{P}}^P$ involve

$$\sum_{j=1}^{m} \pi_j \left[U_n + \sum_{i=1}^{n-1} \Gamma_{ij}(\beta_{i+1}-\beta_i)q_{i+1}\right] > U_n + \sum_{i=1}^{n-1} \Gamma_{im}(\beta_{i+1}-\beta_i)q_{i+1}.$$

Since U_n enters the objective function with negative sign, this inequality and **(IV)** imply that there exists an $\eta > 0$ such that constraint (2.3) holds with strict inequality and

$$U_n + \sum_{i=1}^{n-1} \Gamma_{im}(\beta_{i+1}-\beta_i)q_{i+1} = -\frac{e}{\pi_m}.$$

Inserting this expression into the objective function establishes the claim. □

Claim A3.7. *If Condition 2.1 holds, both W^P and W^S are continuous.*

Proof. W^P and W^S are concave, for the objective functions and constraints of the corresponding maximization problems are concave in both the parameter e and the choice variables (e.g., de la Fuente, 2000, p. 313, Thm. 2.12). Hence, W^P and W^S are continuous on $(0,\infty)$. Claim A3.6 implies that W^P must be continuous at $e = 0$. Suppose W^S is not continuous at $e = 0$. By concavity, $\lim_{e\downarrow 0} W^S(e)$ exists and $\lim_{e\downarrow 0} W^S(e) > W^S(0)$. This inequality contradicts Claim A3.5. □

Claim A3.8. *If Condition 2.1 holds, there exists a unique intersection of W^S and W^P.*

Proof. The following expression will be helpful:

$$W^\star = \sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n} \gamma_{ij}[V(q_i^\star) - \beta_i q_i^*] \right].$$

Since $q^\star$ maximizes surplus, $W^P(e) \leq W^*$ and $W^S(e) \leq W^* - e$ for all e.

I first argue that an intersection exists. If $e = 0$, to any $\widetilde{C}^P$ which satisfies (P1)–(P3) there corresponds a $\widetilde{C}^S$ which satisfies (S1)–(S4) such that $(\widetilde{t}_{ij}, \widetilde{q}_{ij})_{i\in I} = (\widetilde{t}_i, \widetilde{q}_i)_{i\in I}$ for all $j \in J$. Hence, $W^S(0) \geq W^P(0)$. Suppose $W^S(0) > W^P(0)$, and define $\widehat{e} = W^\star - W^P(0)$. As W^P is weakly increasing by Claim A3.5, $W^S(\widehat{e}) \leq W^P(\widehat{e})$. Since W^S and W^P are continuous by Claim A3.7, the intermediate value theorem implies that an intersection exists.

I now argue that there cannot be several intersections. Let $e^\star$ be the cutoff level of investigation costs such that constraint (2.4) binds in program $\widetilde{\mathcal{P}}^P$ if and only if $e < e^*$. (Precisely, $e^\star = -\max_{l\in\{1,\dots,m-1\}} \sum_{j=l}^{m} \pi_j \left[U_n^\star + \sum_{i=1}^{n-1} \Gamma_{ij}(\beta_{i+1} - \beta_i) q_{i+1}^\star\right]$, where $U_n^\star = -\left[\sum_{j=1}^{m} \pi_j \left[\sum_{i=1}^{n-1} \Gamma_{ij}(\beta_{i+1} - \beta_i) q_{i+1}^\star\right]\right]$.) That is,

$$W^P(e) \begin{cases} < W^\star & \textit{if } e < e^\star \\ = W^\star & \textit{else.} \end{cases}$$

Since $W^S(e) < W^\star$ for all e, any intersection of W^S and W^P must be strictly smaller than $e^\star$. So suppose $e < e^\star$, and recall from the proof of Claim A3.7 that W^P is concave and therefore differentiable almost everywhere. At points where it is differentiable, it holds that

$$\frac{dW^P(e)}{de} = \sum_{j=1}^{m-1} \kappa_j,$$

where the $\kappa's$ denote non-negative Lagrange multipliers associated to constraint (2.4). By definition of $e^\star$, at least one multiplier is strictly positive. Since the function W^P is continuous by Claim A3.7, it must hence be strictly increasing on $[0, e^\star)$. This implies that there cannot be several intersections between W^S and W^P, for W^S is weakly decreasing by Claim A3.5. $\square$

Claim A3.9. *Suppose Condition 2.1 is satisfied. Then, it holds that* $W^S(0) > W^P(0)$.

Proof. I first derive some auxiliary results.

(V) *Suppose* $e = 0$. *Let* $\overline{C}^P$ *be one of the best pooling contracts that satisfy (2.1)–(2.4). Then, it holds that* $\overline{q}_i < q_i^\star$ *for all* $i > 1$ *and* $\overline{q}_1 = q_1^\star$.

Proof. By Claim A3.6,

$$\overline{q}_n \in \arg\max_q \sum_{j=1}^{m} \pi_j \gamma_{nj} \left[V(q) - \beta_n q\right] - \left[\sum_{j=1}^{m-1} \pi_j \left(\Gamma_{(n-1)j} - \Gamma_{(n-1)m}\right) (\beta_n - \beta_{n-1}) q\right] + \lambda_{n-1}(q_{n-1} - q),$$

$$\overline{q}_i \in \arg\max_q \sum_{j=1}^{m} \pi_j \gamma_{ij} \left[V(q) - \beta_i q\right] - \left[\sum_{j=1}^{m-1} \pi_j \left(\Gamma_{(i-1)j} - \Gamma_{(i-1)m}\right) (\beta_i - \beta_{i-1}) q\right] + \lambda_i(q - q_{i+1}) + \lambda_{i-1}(q_{i-1} - q) \quad \forall\, i \in \{2, \ldots, n-1\},$$

and

$$\overline{q}_1 \in \arg\max_q \sum_{j=1}^{m} \pi_j \gamma_{1j} \left[V(q) - \beta_1 q\right] + \lambda_1(q - q_2),$$

where the λs are non-negative Lagrange multipliers associated to constraint (2.2). It follows that $\overline{q}_1 = q_1^\star$ and $\overline{q}_n < q_n^\star$. Consider an $i \in \{2, \ldots, n-1\}$ and suppose $\overline{q}_{i+1} < q_{i+1}^\star$. If $\lambda_i = 0$, it holds that $\overline{q}_i < q_i^\star$. If $\lambda_i > 0$, on the other hand, complementary slackness implies $\overline{q}_i = \overline{q}_{i+1}$, so that $\overline{q}_i < q_i^\star$ by the induction hypothesis.

(VI) *Suppose* $e = 0$. *Let* $\overline{C}^P$ *satisfy (2.1)–(2.4), and let* $\underline{C}^P$ *satisfy (2.1) and (2.2). Moreover, suppose* $\underline{q}_i \geq \overline{q}_i$ *for all* $i \in I$ *and*

$$\underline{U}_n + \sum_{i=1}^{n-1} \Gamma_{i1} \left(\beta_{i+1} - \beta_i\right) \underline{q}_{i+1} = \overline{U}_n + \sum_{i=1}^{n-1} \Gamma_{i1} \left(\beta_{i+1} - \beta_i\right) \overline{q}_{i+1}. \tag{A3.5}$$

Then, there exists a separating contract $\widehat{C}^S$ *satisfying (S1)–(S4) such that*

$$(\widehat{t}_{i1}, \widehat{q}_{i1})_{i=1}^n = (\underline{t}_i, \underline{q}_i)_{i=1}^n \quad \textit{and} \quad (\widehat{t}_{ij}, \widehat{q}_{ij})_{i=1}^n = (\overline{t}_i, \overline{q}_i)_{i=1}^n \quad \forall j \in \{2, \ldots, m\}.$$

Proof. Since both $\overline{C}^P$ and $\underline{C}^P$ satisfy (2.1) and (2.2), C^S satisfies (S1) by Claim A3.1 (resp. the analogue of the claim for (S1)). I next verify that C^S satisfies (S2). By (A3.5), ex-ante type γ_1 has no incentive to send a dishonest report. For all other ex-ante types, the gain from deviating is negative as well:

$$\begin{aligned} & \sum_{i=1}^{n} \gamma_{ij}[\underline{t}_i - \beta_i \underline{q}_i] - \left[\sum_{i=1}^{n} \gamma_{ij}[\overline{t}_i - \beta_i \overline{q}_i]\right] \\ = \quad & \underline{U}_n + \sum_{i=1}^{n-1} \Gamma_{ij}\left(\beta_{i+1} - \beta_i\right) \underline{q}_{i+1} - \overline{U}_n - \left[\sum_{i=1}^{n-1} \Gamma_{ij}\left(\beta_{i+1} - \beta_i\right) \overline{q}_{i+1}\right] \\ = \quad & \sum_{i=1}^{n} (\Gamma_{ij} - \Gamma_{i1})\left(\beta_{i+1} - \beta_i\right)\left(\underline{q}_{i+1} - \overline{q}_{i+1}\right) \quad \leq \quad 0, \end{aligned}$$

where the inequality follows from the first-order stochastic dominance ranking of the ex-ante types. Finally, given that investigation costs are zero, (S3) holds since $\overline{C}^P$ satisfies (2.4) and no ex-ante type has an incentive to deviate, and (S2) implies (S4).

The claim can now be proved. Let $\overline{C}^P$ be one of the best pooling contracts that satisfy (2.1)–(2.4). Consider the combination $\underline{C}^P$, defined as follows:

$$\begin{aligned} & (\underline{q}_i)_{i=1}^{n} = (q_i^*)_{i=1}^{n}, \\ & (\underline{t}_i)_{i=1}^{n-1} \quad s.t. \quad \underline{U}_i - \underline{U}_{i+1} = \left(\beta_{i+1} - \beta_i\right) \underline{q}_{i+1} \quad \forall\, i \in \{1, \ldots, n-1\}, \\ & \underline{t}_n \quad s.t. \quad \underline{U}_n + \sum_{i=1}^{n-1} \Gamma_{i1}\left(\beta_{i+1} - \beta_i\right) \underline{q}_{i+1} = \overline{U}_n + \sum_{i=1}^{n-1} \Gamma_{i1}\left(\beta_{i+1} - \beta_i\right) \overline{q}_{i+1}. \end{aligned}$$

By **(V)**, $\underline{q}_i \geq \overline{q}_i$ for all $i \in I$. Therefore, **(VI)** ensures the existence of a separating contract $\widehat{C}^S$ satisfying (S1)–(S4) such that

$$(\widehat{t}_{i1}, \widehat{q}_{i1})_{i=1}^{n} = (\underline{t}_i, \underline{q}_i)_{i=1}^{n} \quad and \quad (\widehat{t}_{ij}, \widehat{q}_{ij})_{i=1}^{n} = (\overline{t}_i, \overline{q}_i)_{i=1}^{n} \quad \forall j \in \{2, \ldots, m\}.$$

By **(V)**, $\widehat{C}^S$ generates more expected surplus than the best pooling contracts. By construction, it gives the same expected payoff to the agent. It thus improves over all pooling contracts. □

A4 Chapter 3: Formal results

Claim A4.1. *There exists a cutoff R' such that effort plan $(1, 1)$ is optimal among all plans $(e_1, e_2(x_1))$ if $R > R'$.*

Proof. According to Propositions 3.1 and 3.2, the principal's profit with effort plan $(1, 1)$ is $2S - A$ in the objective and $2S - B$ in the subjective scenario. The maximum *surplus* with other plans $(e_1, e_2(x_1))$ is $\max\{0, S + \mu(\mu_1 R - c)\}$, where $S + \mu(\mu_1 R - c)$ is the surplus generated with plan $(e_1, e_2(x_1 = 1), e_2(x_1 = 0)) = (1, 1, 0)$. A sufficient condition for $(1, 1)$ being optimal among all plans $(e_1, e_2(x_1))$ is therefore

$$(1 - \mu)(\mu_0 R - c) \geq \max\{A, B\},$$

which concludes the proof. □

Claim A4.2. *In the subjective scenario with mediator, payment scheme* $(w^\star, b^\star)$ *with*

$$(w^\star_{00}, w^\star_{10}, w^\star_{01}, w^\star_{11}) = \left(0, 0, \frac{c}{\mu}, \frac{c}{\mu\mu_1} + \frac{c}{\mu}\right) \quad \textit{and} \quad (b^\star_{00}, b^\star_{10}, b^\star_{01}, b^\star_{11}) = \left(0, 0, \frac{c}{\mu\mu_1}, 0\right)$$

implements effort plan $(1, 1)$ *as cheaply as possible.*

Proof. I first derive some auxiliary results.

(IX) Consider the choice variables w_{00}, b_{00}, and b_{11}. Suppose $(\hat{w}, \hat{b})$ satisfies all constraints. Then the alternative payment scheme (w', b') with

$$w' = (0, \hat{w}_{10}, \hat{w}_{01}, \hat{w}_{11} + \hat{b}_{11}) \quad \textit{and} \quad b' = \left(0, \hat{b}_{10}, \hat{b}_{01} + \frac{1 - \mu_0}{\mu_0}\hat{b}_{00}, 0\right)$$

also satisfies all constraints and yields the same value of the objective function. Hence, I may without loss of generality set $w_{00} = b_{00} = b_{11} = 0$.

(X) Next, I show that the truth-telling constraint (3.9) binds. Suppose not. Then $(\tilde{w}, \tilde{b})$ with

$$\hat{w} = \left(0, \frac{c}{\mu}, \frac{c}{\mu}, \frac{2c}{\mu}\right) \quad \textit{and} \quad \hat{b} = (0, 0, 0, 0)$$

would be an optimal solution. Since it violates (3.9) and as the program is linear, (3.9) must bind.

(XI) Consider now the choice variables w_{10}, b_{10}, and b_{01}, and take **(IX)** and **(X)** into account. The binding truth-telling condition (3.9) reads

$$\mu_1 w_{11} + (1 - \mu_1)(w_{10} + b_{10}) = \mu_1(w_{01} + b_{01}),$$

and the second truth-telling condition, (3.10), reads

$$\mu_0(w_{01} + b_{01}) \leq \mu_0 w_{11} + (1 - \mu_0)(w_{10} + b_{10}).$$

For the moment, ignore (3.10). Suppose $(\overline{w}, \overline{b})$ satisfies all other constraints. Then, the alternative payment scheme $(\underline{w}, \underline{b})$ with

$$\underline{w} = \left(0, 0, \overline{w}_{01}, \overline{w}_{11} + \frac{1-\mu_1}{\mu_1}\overline{w}_{10}\right) \quad and \quad \underline{b} = \left(0, 0, \overline{w}_{11} + \frac{1-\mu_1}{\mu_1}w_{10} - \overline{w}_{01}, 0\right)$$

also satisfies all constraints and yields at least the same value of the objective function ('at least' because $b_{10} = 0$ is optimal). Here,

$$\underline{b}_{01} = \overline{w}_{11} + \frac{1-\mu_1}{\mu_1}w_{10} - \overline{w}_{01}$$

is indeed a non-negative transfer because $\overline{w}$ satisfies (3.3), which requires

$$\mu[\mu_1 w_{11} + (1-\mu_1)w_{10} - \mu_1 w_{01}] \geq c.$$

The alternative payment scheme actually also satisfies the ignored truth-telling constraint (3.10). I may hence set $w_{10} = b_{10} = 0$ and $b_{01} = w_{11} - w_{01}$.

According to **(IX)**–**(XI)**, I may search for an optimal solution among all payment schemes (w, b) that satisfy

$$w = (0, 0, w_{01}, w_{11}) \quad and \quad b = (0, 0, w_{11} - w_{01}, 0).$$

The optimal values w_{01}, w_{11} can be derived from

$$\begin{aligned} \min_{w_{11}\geq 0, w_{01}\geq 0} \quad & \mu w_{11} \\ s.t. \quad & \mu\mu_1(w_{11} - w_{01}) \geq c && \text{(3.3')} \\ & \mu\mu_1 w_{11} + (1-\mu)\mu_0 w_{01} \geq c + c && \text{(3.4')} \\ & \mu\mu_1 w_{11} + (1-\mu)\mu_0 w_{01} \geq c. && \text{(3.8')} \end{aligned}$$

This program is solved by $w_{01} = c/\mu$, $w_{11} = c/(\mu\mu_1) + c/\mu$. □

A5 Chapter 3: Alternative subjective scenario

This appendix shows that the benefits of subjective evaluations are not a peculiarity of the original model, in which only the principal can evaluate performance. To this end, consider the *alternative subjective scenario*, which differs from the original one in that the agent privately observes a performance signal $s \in \{0, 1\}$ at the end of stage

1. Let $Pr(x_1 = s) = \alpha \in (1/2, 1)$ be the probability that the signal coincides with the principal's subjective evaluation.

The main result of this section, Claim A5.1, establishes a lower bound on the principal's profit. I use the notation

$$\alpha_0 = \min\{\mu, (1-\alpha)\mu_1 + \alpha\mu_0\}$$
$$A' = \left(\frac{\mu}{\alpha_0} - 1\right) c.$$

Claim A5.1. *In the alternative subjective scenario, the principal's profit is at least* $2S - A' - B$.

Observe that A' is lower than A, the agent's payoff in the objective scenario. In particular, it holds that $A' + B < A$ if the posteriors μ_0 and μ_1 are sufficiently low and high, respectively. More uncertainty thus also renders the alternative subjective scenario advantageous for the principal.

Claim A5.1 follows from Claim A5.2. I construct a contract that implements effort plan $(1, 1)$ in the alternative subjective scenario using the same communication protocol as the non-mediated contract derived in section 3.3.2. Accordingly, the payments do not depend on the agent's signal but only on the the principal's reported evaluation of stage 1 and on the objective evaluation of stage 2.[26]

Claim A5.2. *In the alternative subjective scenario, payment scheme* (w', b') *with*

$$(w'_{00}, w'_{10}, w'_{01}, w'_{11}) = \left(0, 0, \frac{c}{\alpha_0}, \frac{c}{\mu\mu_1} + \frac{c}{\alpha_0}\right) \quad \text{and} \quad (b'_{00}, b'_{10}, b'_{01}, b'_{11}) = \left(0, 0, \frac{c}{\mu\mu_1}, 0\right)$$

implements effort plan $(1, 1)$.

Proof. First, the agent has an incentive to exert effort in stage 2 if he worked hard in stage 1 and observes $s = 1$:

$$\alpha[\mu_1 w'_{11} + (1-\mu_1)w'_{10} - c] + (1-\alpha)[\mu_0 w'_{01} + (1-\mu_0)w'_{00} - c] \geq \alpha w'_{10} + (1-\alpha)w'_{00}.$$

[26]As the agent's signal is only imperfectly correlated with the subjective evaluation, the budget breaker's payoff should be greater than zero also if the contract takes the signal into account.

Second, the agent also has an incentive to exert effort in stage 2 if he worked hard in stage 1 and observes $s = 0$:

$$(1-\alpha)[\mu_1 w'_{11} + (1-\mu_1)w'_{10} - c] + \alpha[\mu_0 w'_{01} + (1-\mu_0)w'_{00} - c] \geq (1-\alpha)w'_{10} + \alpha w'_{00}.$$

Moreover, w' satisfies the conditions (3.3)–(3.5). The contract thus implements effort plan $(1,1)$ if the principal is willing to submit an honest evaluation. Indeed, any report results in the same payment for the principal:

$$w'_{11} + b'_{11} = w'_{01} + b'_{01} \quad \textit{and} \quad w'_{10} + b'_{10} = w'_{00} + b'_{00},$$

which concludes the proof. □