## Essays on Labor Market Risk and Asset-Based Income Insurance

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## Introduction

Earnings uncertainty stemming from the labor market is one of the major risks individuals face over their life-cycle. One can think of this risk belonging to one of three major categories: *unemployment risk*, *employment risk* and *wage risk*. The present work contributes to our understanding of these types of risk. Chapter 1 links cross-country variations in *unemployment risk* to variations in regulations. Chapter 2 quantifies the amount of *employment risk* and *wage risk* present in the US. Chapter 3 analyzes the welfare consequences of asset means-testing income insurance programs.

Unemployment risk arises because household members face the possibility of involuntary spells of non-employment. Labor demand of individual employers is quite volatile in industrialized countries, forcing a significant fraction of workers to leave their current employers at any period in time (see Pries and Rogerson (2005)). This phenomenon is not limited to periods of low aggregate production, but can be observed at each stage of the business cycle (see Davis and Haltiwanger (1992)). Likewise, personal circumstances, such as family related moves, force workers to give up their current employment and look for a new employer at a different location. In a Walrasian world, the Walrasian auctioneer would adjust wages to align labor demand and supply and would match all employment seeking workers to employees seeking firms instantaneously. The large observed worker flows would simply reflect reallocation of labor to new matches that yield higher match surpluses. We observe; however, workers seeking employment at the ongoing market wage, but fail to find it, indicating the presence of frictions in the labor market. Put differently, workers face the risk of involuntary spells without wage income. Unemployment duration is a main determinant of the amount of *unemployment risk* and it varies substantial

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across OECD countries. Chapter 1 addresses this issue by analyzing the amount of cross country variation that differences in product market regulation can explain within a framework of a frictional labor market model.

Besides the risk of prolonged spells of unemployment, workers face uncertainty about the quality of job offers they receive. All workers would work for the highest paying employer, and employers with lower wage offers would vanish from the market in a Walrasian labor market. It becomes optimal; however, to accept job offers below the highest possible offer whenever the worker is faced by the trade-off of accepting a current wage offer or staying in unemployment. The result is a distribution of realized employer-employee matches with different employers paying different wages to identical workers. Consequently, it becomes partially luck which type of employer the worker encounters during his search process. To highlight that the source of this labor market risk is employer specific, I refer to it as *employment risk*.

Finally, workers' productivity and hence their earnings potential is developing in a non-predictable manner. Health shocks that limit the ability of the worker to perform specific tasks are a prominent example. Also, changes in demand for worker specific skills have a stochastic component. Technological innovations make certain skills obsolete whereas increasing the demand for other skills. A prominent example is the increase in the college wage premium since the 1960s (see Katz and Murphy (1992) and Autor et al. (1998)). Similarly, changing product demand, such as cyclical variations in the construction sector, affects the labor demand for specific worker groups. Moreover, family related circumstances may temporarily affect a worker's productivity. This type of risk would be present even in a non-frictional labor market and I refer to it as *wage risk*. Chapter 2 starts from the observation that the variation in wages of observationally equivalent workers is large in the US. Volker Tjaden and I propose a model that allows us to decompose the variation into *employment risk*, *wage risk* and measurement error and to quantify the respective contributions.

The above discussion makes apparent that labor market risk is closely linked to the presence of frictions in the labor market. Therefore, any micro foundation of labor market risk has to take a stance about the source of this friction. Following the seminal work of Diamond (1982), Mortensen (1982) and Pissarides (1985), I think of the labor market being characterized by search frictions in the first two chapters. Workers do not know the location of each potential wage offer, but engage in a search process sequentially sampling job offers by firms. Provided with a job offer, they have to decide whether to accept it or continue searching. Likewise, firms have to search for a suitable worker to fill a vacant job. Posting vacancies involves costs for the firm, such as advertisement, screening and interviewing costs. Importantly, meeting a potential partner is a probabilistic event for both sides, leading to a simultaneous presence of unemployed workers and open vacancies in the labor market.

This framework provides my analysis with a great deal of structure. Most importantly, it allows me to address the issue of endogenous employment choices. Accepting a job offer, quitting into unemployment and moving to a different job are all endogenous choices. Therefore, data observables such as transition rates and the wage distribution are moments resulting from non-random events. The structure provided by the framework allows me to link the deep parameters of labor market risk to these non-random outcomes.

Chapter 3 is different from the first two in regard that it does not explicitly model the search friction and assumes labor market risk to be exogenous. Instead, it addresses the issue of insurance against the risk. A large literature finds that changes in income pass through to changes in consumption (e.g., Attanasio and Davis (1996)). Consequently, the risk affects individual and social welfare and the question arises how the government should provide insurance against it. In this chapter, I follow the incomplete markets framework proposed by Bewley (1983), Huggett (1993) and Imrohoroglu et al. (1995). Households are risk averse, but markets fail to provide a full set of Arrow-Debreu securities, an assumption that is particularly suited for insurance against labor market risk. The households can insure against the labor market risk by accumulating precautionary savings and the government can provide income transfers to households. The chapter addresses the issue whether the government should use the amount of households' savings to determine eligibility to the governmental insurance programs.

Every chapter is devised as an independent, self-contained unit. The following paragraphs preview the approach and contributions of each in some more detail and link them back to the title of the dissertation.

**Chapter 1**. This chapter investigates the amount of cross country variation in unemployment duration, the unemployment rate and average hours worked that can be accounted for by differences in product market regulation. The analysis is

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motivated by two major observations. First, the unemployment rate, the average duration of unemployment and average hours worked per worker are vastly different across major OECD countries over long time horizons<sup>1</sup>. Second, the data suggests that unemployment rates are positively correlated to product market regulation and that the variation in product market regulation is large. I think of product market regulation as entry costs into the goods market which are imposed by the government. Using a study by Djankov et al. (2002), I document entry costs to vary from 0.44 to 8.55 of monthly per capita GDP in a sample of OECD countries.

I propose a structural model, which allows me to link product market regulation to labor market outcomes. Unemployment arises from search frictions in the labor market, and labor demand is driven by firms entering into a monopolistic goods market and posting costly vacancies. When a vacant job and an unemployed worker meet, they can decide to form a match and determine the wage by generalized Nash-Bargaining. Firms enter whenever they see profits opportunities taking into account the entry costs from the governmental regulation. Therefore, the product market regulation determines the level of competition in the product market. I show that high entry costs depress labor demand leading to longer unemployment duration, a higher unemployment rate, lower wages and less average hours worked. Moreover, I show that the product market regulation interacts in a meaningful way with labor market institutions. High employment taxes and unemployment benefits amplify the response of unemployment duration to changes in product market regulation.

To assess the quantitative relevance of the differences in product market regulation, I calibrate the model to German labor market data, a country with relative high product and labor market regulations. Using the model as a laboratory allows me to quantify the changes in labor market outcomes that result from an introduction of the low level of product market regulation present in the US. I find that wages are predicted to increase by 1.6%, average hours worked by 1.5% and the unemployment rate decreased by up to 0.43 percentage points. The response of unemployment would be 50% lower when the same reduction in product market regulation would take place in a country with the weak US labor market institutions. Consequently, the degree of labor market institutions present in the country is a major determinant

<sup>&</sup>lt;sup>1</sup>Apart from inflicting major costs on a country's unemployment insurance system, the duration of unemployment is a main determinant of the amount of *unemployment risk* in a country.

for the size of the reduction in unemployment duration and *unemployment risk* that governments can achieve by decreasing product market regulation.

**Chapter 2**<sup>2</sup>. It is a well-known fact that measured wage dispersion among observationally identical workers is large and growing over a cohort's life-cycle. This residual wage dispersion may either result from identical workers that are paid differently because their employer differs (*employment risk*) because of stochastic changes in unobserved worker heterogeneity (*wage risk*), or measurement error. Quantifying the contribution of each driving force from wage data; however, suffers from endogenous employment choices. Workers are more likely to accept high wage offers when coming from unemployment and seek better paying jobs throughout their working career. Similarly, poor shocks to individual earnings potential make unemployment relatively more attractive leading to endogenous match separation. Taken together, observable wage changes, transition rates and the wage distribution are the result of an interplay of structural shocks and endogenous worker choices.

In this chapter, we build a structural model that allows us to infer from the data observables the size of *employment risk* and *wage risk* present in US data. Hornstein et al. (2011) show that our canonical search models can rationalize only a small portion of the residual wage dispersion in the data as resulting from search frictions because unemployment is not particularly painful<sup>3</sup>. Hence, using any of our canonical search models to infer the deep parameters of the labor market leads directly to the conclusion that *employment risk* must be small and measurement error must be large, to match the large variance in residual wages, and highly persistent, to match the increase over the life-cycle.

We circumvent the problem of implausible high and persistent measurement error by enriching the search problem of the worker. Workers' productivity is stochastic and unemployed workers face skill depreciation and finite unemployment benefits. Employed workers experience learning by doing on the job and can continue to search for better job prospects while working. The model is able to rationalize the large amount of residual wage dispersion and the increase over the life-cycle given a reasonable measurement error process. We find that 17.5% of wage inequality is explained by the presence of the search friction. Moreover, we argue that a realistic

<sup>&</sup>lt;sup>2</sup>This chapter is joint work with Volker Tjaden.

<sup>&</sup>lt;sup>3</sup>We use the term canonical search models here to refer to the class of search models brought forward by McCall (1970), Mortensen (1970), Lucas and Prescott (1974) and Pissarides (1985).

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quantitative appraisal of search efficiencies needs to account for the one third of job to job transitions resulting in wage losses. Ignoring this fact leads to an overestimation of the contribution of the search friction by 100%, providing an explanation for earlier findings in the literature.

**Chapter 3**. Whereas the first two chapters endogenize labor market risk, this chapter takes it as exogenous and addresses the welfare implications of different governmental insurance mechanisms. Financial markets are incomplete; therefore, labor market risk affects households' consumption and welfare (e.g., Attanasio and Davis (1996)). Households can use precautionary savings against the risk and the government can provide additional income transfers. The chapter addresses the question whether the government should use the households' wealth positions to determine eligibility to the governmental transfers.

The welfare consequences of such a policy are a priori unclear. On the one hand, for any amount of governmental expenditures, asset means-testing allows to allocate relatively high transfers to those households which are in most need for the assistance. On the other hand, the benefit scheme distorts saving decisions over the life-cycle and provides incentives to hold relatively little wealth. To study the incentives created by asset means-testing and to evaluate its welfare consequences, I introduce the insurance scheme into an otherwise standard incomplete markets model and calibrate the model to the current US system of income support programs.

I show that the marginal propensity to consume out of wealth is higher than in the standard incomplete markets framework and converges to it along the wealth state. Poor households consume relatively much and this is especially true for households with low income potential. I find that an unborn is willing to pay 0.31% of lifetime consumption to be under a regime without any asset means-testing given the same amount of total governmental expenditures. The main reason is that households with low permanent income potential have strong incentives to reduce savings and participate in the program. Average consumption of these households is higher than the unconstrained social planner solution at early stages of their life-cycle resulting in insufficient savings for retirement. Average consumption drops discontinuously by 3% around retirement and keeps decreasing at a too high rate throughout the retirement period. This failure of poor households to smooth consumption over their life-cycle leads to welfare losses that outweigh the gains from asset means-testing.

| Chapter

# Product Market Regulation, Labor Market Institutions and Labor Market Performance

## 1.1 Introduction

This chapter analyzes the role product market regulation plays in shaping differences in unemployment and average hours worked per worker between OECD countries<sup>1</sup>. Figure I shows the developments in unemployment rates and average hours worked per worker in 5 major OECD countries since the mid-80s. The figure highlights that countries like Germany and Sweden experienced persistent high unemployment rates and low amounts of average hours worked. At the same time, The UK and US faced much lower unemployment rates and workers worked more hours on average.

A growing literature investigates the role product market regulation (PMR) plays in shaping these patterns. The results; however, are inconclusive so far. Empirical work by Boeri et al. (2000), Bertrand and Kramarz (2002), Lopez-Garcia (2003) and Griffith et al. (2007) finds a positive partial correlation between PMR and unemployment rates. But, structural models by Ebell and Haefke (2009) and Fang and Rogerson (2011) indicate a negligible role<sup>2</sup>.

The main contribution of this chapter is the following: I demonstrate that the quantitative response of the unemployment rate to a change in PMR depends on

 $<sup>^{1}</sup>$ Layard et al. (1991) show that cross sectional differences in unemployment rates can mostly be explained by differences in unemployment duration.

<sup>&</sup>lt;sup>2</sup>Earlier contributions on the effect of PMR on unemployment include Blanchard and Giavazzi (2003) and Spector (2004).



Figure I: Unemployment and Hours Worked in the OECD

the strength of labor market institutions (LMIs). I use a labor market model with search and matching frictions that explains differences in the unemployment rate by differences in mean unemployment duration. I think of PMR as costs that entrepreneurs have to pay when entering into a monopolistic goods market. Decreasing these costs leads to vacancy creation, which triggers additional labor demand. The economy reaches a new equilibrium either by an increase in wages or by increases in the search costs for new employees, which I model as a decreasing function of unemployment duration. The response of unemployment is small whenever the equilibrium adjustment occurs mostly by an adjustment in wages. The relative

Notes: The figure displays unemployment rates and average yearly hours worked per worker for Germany, Italy, US, Sweden and the United Kingdom. Source: OECD Labor Force Statistics

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response of wages and search costs depends on the marginal value of an additional worker to the firm. When this value is small, the extra value created by deregulation is large relative to the existing marginal value, and relatively many firms enter into the product market and create vacancies<sup>3</sup>. I demonstrate that *LMIs* and *PMR* interact by two major channels that alter the relative response of wages and search costs. First, strong labor market institutions imply that a relatively small share of the profits generated from the product market goes to the firms, making the marginal value of a worker small. Second, strong labor market institutions decrease firms' profits, which leads to higher mark-ups in equilibrium for a given level of product market regulation. The higher mark-ups decrease the marginal revenue product of labor and hence decrease the equilibrium marginal value of an extra employer to the firm even further.

The quantitative response of unemployment also depends on the design of unemployment benefits. The response of unemployment is relatively weak when benefits are modeled as replacement rates. The reason is that deregulations in the product market trigger increases in wages, which increases unemployment benefits and counteracts the decrease in unemployment. The response of unemployment is much stronger when benefits stay constant. I additionally incorporate an intensive labor supply margin into the structural model; thus, allowing me to study the effects of PMR on the intensive labor supply margin within a frictional labor market context. PMR reduces the time a worker allocates to market work because it decreases wages<sup>4</sup>.

My work is most closely related to Ebell and Haefke (2009), who use a model with search and matching frictions in the labor market and bargaining between firms and workers over wages. The authors calibrate the model to US data following closely the calibration strategy of Shimer (2005). They show that PMR has a negligible effect on unemployment under these specifications. I calibrate my model to German data, which exhibits much stronger LMIs. Depending on the specification of unemployment benefits, a decrease in PMR from German to US level decreases the structural unemployment rate in a range from 0.1 to 0.432 percentage points in my model economy. The effect is by a factor of 2.7 larger than the one Ebell and Haefke (2009)

<sup>&</sup>lt;sup>3</sup>See Hagedorn and Manovskii (2008) and Hornstein et al. (2005) for a similar argument with respect to the relative response of unemployment and wages to productivity shocks.

<sup>&</sup>lt;sup>4</sup>The effect depends on the relative strength of the income and substitution effect. In my utility specification, the income effect dominates.

#### Chapter 1

find for a similar large deregulation in the US.

The chapter is also related to the literature that tries to understand responses at the intensive margin to changes in institutions. Prescott (2004) and Ohanian et al. (2008) examine the response of hours worked on employment taxes in a neoclassical growth model. They argue that differences between countries in employment taxes can explain most of the difference in observed hours worked. Shi and Wen (1999) and Krusell et al. (2010) incorporate the intensive margin into a search and matching model and generally find large employment responses to tax changes, too. I find differences in *PMR* explaining some of the cross country variation. Decreasing *PMR* from the German to the US level can account for up to 1.66% of differences in averaged hours worked. Consistent with the previous evidence, most differences are explained by differences in *LMIs*, especially in the tax code.

The chapter is structured as follows: The next section highlights some stylized facts about *PMR* and *LMIs* in five major OECD countries. Thereafter, I develop my model, characterize the equilibrium and explain the mechanisms at work. I calibrate the model to German data and perform the policy experiments. The last section concludes.

## **1.2 Institutions in OECD Countries**

This section presents several stylized facts about institutional differences for a sample of OECD countries. I consider the year 1997, for which detailed data on both LMIs and PMR exists<sup>5</sup>. To explain the large differences in labor market outcomes outlined above, one would expect institutions to have an economically significant size and to substantially vary between countries. This section provides estimates that verify these claims.

Looking at LMIs, my analysis focuses on cross country differences in employment taxes and replacement rates. Table 1.1 shows the substantial variation of these two measures across countries. Average gross replacement rates range from 13.9% in the US to 26.9% in Sweden. Similarly, employment taxes range from a low of 23.2% in the US to 44.4% in Sweden. There is a strong positive correlation between replacement

<sup>&</sup>lt;sup>5</sup>Institutions are quite stable over longer time horizons. For example, replacement rates were the highest in Germany and Sweden throughout the sample period (see Nickell (2006)).

Country	Replacement rate %	Employment taxes $\%$
Germany Italy Sweden UK	25.7 18 26.9 18.3	$ \begin{array}{r} 42.6 \\ 42.2 \\ 44.6 \\ 25.9 \\ \end{array} $
US	13.9	23.2

Table 1.1: Labor Market Institutions in the OECD

Notes: Replacement rates correspond to the OECD gross replacement rates. Employment taxes include direct taxes and social security contributions. Source: Nickell (2006) and McDaniel (2007)

rates and employment taxes, with Italy being the exception.

Table 1.2 summarizes the costs of entering into the product market. Djankov et al. (2002) report for my sample of countries the amount of business days required to start a medium size firm<sup>6</sup> and the costs for required fees, measured in percentage of yearly per capita GDP. The table shows that entering the product market is possible on short notice and the required costs for fees are low in Sweden, UK and the US. At the same time, Germany and Italy substantially delay business operation and it is costly to meet all legal obligations.

## 1.3 The Model

This section incorporates the above mentioned regulations into a model with a labor marker that is characterized by search and matching frictions and a monopolistic goods market. I begin by describing the household and firm problem and derive optimal policies. Using these policies, I characterize the equilibrium in the labor market for an exogenous competition level in the goods market. Afterwards, I endogenize the competition level in the goods market and relate it to the level of product market regulation.

 $<sup>^6\</sup>mathrm{Besides}$  some requirements on capital, they define a medium size firm to have between 5 - 50 employees.

Country	Lost business days	Procedural costs
Germany	90	0.085
Italy	121	0.2474
Sweden	17	0.0254
UK	11	0.0056
US	7	0.01

Table 1.2: Entry Costs in the OECD

Notes: The second column shows the amount of days it takes to set up a medium size firm. The third column reports the amount of spending required to fulfill all legal obligations that are invoked with setting up a medium size firm. The latter is measured in yearly per capita GDP. Source: Djankov et al. (2002)

### 1.3.1 Households and the Labor Market

There is a continuum of infinitely lived individuals uniformly distributed on the unit interval, who either work in period t,  $n_t$ , or are unemployed,  $u_t$ , and earn the amount of unemployment benefits b. Therefore, I have

$$1 = u_t + n_t.$$

I make the simplifying assumption that workers can unilaterally choose the fraction of time they wants to work,  $h_t$ , from the unit interval. Following Merz (1995), I assume that each individual belongs to a larger household where all labor and unemployment income is pooled and equally split. The consumption and hours worked decision is made at the household level and individuals face no idiosyncratic risk from the labor market. Search is random and the labor market is governed by a matching function  $m_t = \xi u_t^{\iota} v_t^{1-\iota}$  where  $v_t$  are the total amount of vacancies posted by firms. Defining labor market tightness as  $\theta_t = \frac{v_t}{u_t}$ , I can write the vacancy filling rate and job finding rate respectively as:

$$\frac{m_t(v_t, u_t)}{v_t} = \xi \theta_t^{-\iota} \equiv q(\theta_t)$$
$$\frac{m_t(v_t, u_t)}{u_t} = \xi \theta_t^{1-\iota} = \theta_t q(\theta_t) \equiv p(\theta_t).$$

Let  $w_t$  be the gross wage a household earns for each employed member that devotes a full a unit of time to working. Each worker faces an exogenous match separation rate, occurring at Poisson rate  $\omega$ . Moreover, firms exit the market at exogenous Poisson rate  $\delta$ . The worker joins the pool of unemployment in both cases; therefore, he only cares about the total job separation rate:  $\chi = 1 - (1 - \omega)(1 - \delta)$ . The household maximizes his life time utility by choosing the amount of consumption and hours worked by each employed member, taking the unemployment rate, the gross wage, the job finding rate and the laws of motion of the state variables as given. The maximization problem of the representative household reads:

$$\max_{c_{t},h_{t}} \left\{ \sum_{t=0}^{\infty} \beta^{t} \left[ c_{t} + \kappa [ln(1-h_{t}) + h_{t}] \right] \right\}$$
(1.1)  

$$s.t.$$

$$c_{t} = u_{t}b + w_{t}h_{t}n_{t}(1-\tau)$$

$$u_{t+1} = \chi n_{t} + (1-p(\theta_{t}))u_{t}$$

$$c_{t} \ge 0$$

$$h_{t} \in [0,1]$$

$$\{u_{t}, w_{t}, n_{t}, \theta_{t}\} given.$$

 $\beta$  is a discount factor,  $\kappa$  is a scaling parameter for the disutility of work and  $\tau$  is the employment tax. I assume that the government uses tax revenue to purchase public goods that do not enter the utility of households. The functional form ensures an interior solution for the hours choice because

$$\lim_{h \to 0} g'(h) = 0, \lim_{h \to 1} g'(h) = -\infty.$$

The first order conditions of household optimization yield an implicit solution for hours worked that is increasing in the wage<sup>7</sup>:

$$\frac{1}{1-h_t} - 1 = \frac{w_t n_t (1-\tau)}{\kappa}.$$
(1.2)

I assume that aggregate consumption composites of m goods, each having substi-

 $<sup>^{7}</sup>$ I want to stress gain that this results from utility being linear in consumption.

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tution elasticity  $\sigma$ :

$$c_t = \left[\sum_{i=1}^m c_{i,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

Each good i is produced by a separate firm leading to the individual product demand curve:

$$Y_{i,t}^D = \left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} c_t,$$

where  $P_{i,t}$  is the price firm i sets and  $P_t$  is the aggregate price level.

The sequence problem (1.1) satisfies Bellman's principle of optimality; consequently, I can write it recursively. I define separately the value of an employed household member,  $V^E$ , and an unemployed member,  $V^U$ :

$$\begin{split} V^{E} &= wh(1-\tau) + \kappa [ln(1-h) + h] + \beta [(1-\chi)V^{E'} + \chi V^{U'}] \\ V^{U} &= b + \beta [(1-p(\theta))V^{U'} + p(\theta)V^{E'}], \end{split}$$

where *primes* denote next period's values.

### 1.3.2 The Firm's Problem

Each firm *i* decides how many vacancies  $v_t$  to post in period *t* with associated costs  $\varphi$  and how much labor to hire in t + 1 taking as given the labor supply decision of the workers, the vacancy filling rate and its individual demand function:

$$V^{J}(n_{i}) = \max_{n',v} \left\{ \frac{P_{i}(y_{i})}{P} y_{i} - wn_{i}h - \varphi v_{i} + \beta(1-\delta)V^{J}(n_{i}') \right\}$$
(1.3)

$$s.t.$$

$$y_i = An_ih$$

$$\left[\frac{y_i}{c}\right]^{-\frac{1}{\sigma}} = \frac{P_i}{P}$$

$$n'_i = (1-\omega)n_i + q(\theta)v_i.$$

The first order condition with respect to vacancies yields

$$\frac{\partial V^J(n_i')}{\partial n_i'} = \frac{\varphi}{q(\theta)} \frac{1}{\beta(1-\delta)},\tag{1.4}$$

which is constant. The marginal contribution of an additional worker in the next period must equal the costs of searching for her, weighted by the discount rate and the probability of firm survival. The envelope condition of employment reads:

$$\frac{\partial V^{J}(n_{i})}{\partial n_{i}} = \underbrace{\frac{\sigma - 1}{\sigma} \underbrace{\frac{P_{i}}{P} Ah}_{MRPL_{i}} - w_{i}h - \frac{\partial w_{i}}{\partial n_{i}} hn_{i} + \frac{\varphi}{q(\theta)}(1 - \omega).$$
(1.5)

The value of an extra employee in the optimum is its marginal revenue product of labor minus the wage costs plus the search costs that would have occurred next period, if the worker had not been hired. The total wage cost from hiring an additional worker has two components. First, the firm needs to pay the additional worker the wage  $w_ih$ . Second, adding an additional worker decreases the average wage because MRPL decreases when employment increases. MRPL consists of the inverse of the mark-up a monopolistically firm sets. A higher  $\sigma$  increases MRPL because adding an additional worker has less detrimental effects on individual product demand.  $\sigma$  is a preference parameter and as  $\sigma \to \infty$  the mark-up goes to zero. I follow Blanchard and Giavazzi (2003) and Ebell and Haefke (2009) and interpret it from now on as the level of competition in the goods market. The interpretation is a natural one because higher competition implies a higher product variety, which increases the substitutability of each product.

Shifting the expression one period forward and setting it equal to (1.4) gives the optimal intertemporal firm policy:

$$\frac{\varphi}{q(\theta)} = (1-\delta)\beta[MRPL'_i - w'_i h' - \frac{\partial w'}{\partial n'} h' n'_i + \frac{\varphi}{q(\theta')}(1-\omega)].$$
(1.6)

 $\frac{\varphi}{q(\theta)}$  are the marginal costs of an extra hire, which are increasing in labor market tightness. Optimality requires that these costs must equal the value of an additional worker to the firm in the next period, weighted by the discount factor and firm survival.

Figure II plots the change in firms' labor demand as a reaction to a change

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in competition keeping labor supply and prices fixed. When competition increases, meaning  $\sigma$  increases, MRPL increases because a firm's product demand is less affected by changes in its output. This in turn leads firms to increase vacancy posting. (1.6) shows that factors decreasing the wage have the same effect on vacancy posting.





Notes: The figure displays the equilibrium changes in marginal revenue product of labor, MRPL, and the firm reaction in vacancy posting, v, to exogenous variations in the competition level,  $\sigma$ .

### 1.3.3 Wage Determination

Once a worker is matched to a firm, the two engage in generalized Nash-Bargaining over the wage:

$$\max_{w} \left\{ \Omega = \epsilon \Omega^{w} + (1 - \epsilon) \Omega_{i}^{f} \right\},$$
(1.7)

where  $\Omega^w$  and  $\Omega_i^f$  is the log of match surplus of the individual worker and the individual firm, respectively, and  $\epsilon$  is the bargaining power of workers. The match surplus for the worker is:

$$V^{W} = V^{E} - V^{U} = wh(1-\tau) - b + \kappa [ln(1-h) + h] + \beta (1-\chi - p(\theta))V^{W'}.$$
 (1.8)

The match surplus for the firm is to produce with one more worker, which is given by (1.5). Following Stole and Zwiebel (1996), the solution can be characterized by:

1.3 The Model

Lemma 1. Firm's labor demand is given by:

$$\frac{\varphi}{q(\theta)} = (1-\delta)\beta \Big[ MRPL'_i \frac{\sigma}{\sigma-\epsilon} - w'_i h' + \frac{\varphi}{q(\theta')} (1-\omega) \Big].$$
(1.9)

The wage curve is given by:

$$w_i = \frac{\epsilon}{h} \left[ \frac{\sigma}{\sigma - \epsilon} MRPL_i + \frac{\varphi}{1 - \delta} \theta \right] - \frac{1 - \epsilon}{(1 - \tau)h} \left[ \kappa [ln(1 - h) + h] - b \right].$$
(1.10)

Worker's match surplus takes the form:

$$V^{W} = wh - b + \kappa [ln(1-h) + h] + \beta [1 - \chi - p(\theta)] \frac{\epsilon}{1 - \epsilon} (1 - \tau) \frac{\varphi}{q(\theta)} \frac{1}{\beta(1 - \delta)}.$$
 (1.11)

*Proof:* See Appendix 1.A.

The system of equations describes the dynamic equilibrium behavior for any level of product market competition. (1.9) determines the amount of firms' vacancy posting, given the level of product market competition, the household's hours choice and the level of unemployment. Household's hours choice follows from (1.2), and the unemployment rate follows from the law of motion for unemployment. Given the resulting labor market tightness, (1.10) characterize equilibrium wages.

### 1.3.4 Steady State Short-Run General Equilibrium

This section characterizes equilibrium wages and labor market tightness in the steady state for a given competition level and given parameters. Equilibrium implies that  $\frac{P_i}{P} = 1$  because firms' production technologies are symmetric. Therefore, (1.9) simplifies in steady state to

$$\frac{\varphi}{q(\theta)} = (1-\delta)\beta \Big[Ah\frac{\sigma-1}{\sigma}\frac{\sigma}{\sigma-\epsilon} - wh + \frac{\varphi}{q(\theta)}(1-\omega)\Big].$$

Together with (1.10), I arrive at two equations determining  $\theta$  and w:

$$\frac{\sigma - 1}{\sigma} \frac{\sigma}{\sigma - \epsilon} Ah = \frac{1}{1 - \epsilon} \frac{\varphi}{q(\theta)} \frac{1}{1 - \delta} \left[ \frac{1}{\beta} - 1 + \chi + \epsilon p(\theta) \right] - \frac{1}{1 - \tau} \left[ \kappa [ln(1 - h) + h] - b \right] \quad (1.12)$$

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$$w = \frac{\epsilon}{h} \frac{\varphi}{q(\theta)} \frac{1}{1-\delta} \frac{1}{1-\epsilon} \left[ \frac{1}{\beta} - 1 + \chi + p(\theta) \right] - \frac{1}{(1-\tau)h} \left[ \kappa [ln(1-h) + h] - b \right].$$
(1.13)

Aggregate output is given by Y = Anh, and the steady state unemployment rate balances in and outflows leading to the familiar Beveridge curve:

$$u = \frac{\chi}{\chi + p(\theta)}.$$
(1.14)

Figure III visualize the role the competition level plays in the short-run equilibrium. An increase in  $\sigma$  increases labor market tightness according to (1.12). The quantitative increase from 0.04 to 0.19 is quite sizable. According to (1.14), unemployment falls from 16% to 7.5%. The increase in  $\theta$  increases the position of the workers in the Nash-Bargaining and increases wages according to (1.13). The first order conditions of workers, (1.2), shows that this increase translates into a higher labor supply of workers.

Figure III: Short-Run Equilibrium



Notes: The figure displays the labor market equilibrium for exogenous variations in competition,  $\sigma$ . The left panel displays on the left axis the equilibrium levels of labor market tightness and on the right axis the resulting unemployment rate. The right panel shows on the left axis the equilibrium schedule for wages and on the right axis the hours worked per employed.

#### 1.3.5 Entry into the Goods Market

This section endogenizes the equilibrium level of competition and links it to PMR. Firms enter the market until profits are driven down to zero, which determines the equilibrium competition levels. Firms find it optimal to hire the steady optimal level of labor already in the first period since vacancy posting costs are linear. Therefore, the steady state value of the firm must equal the entry costs plus the costs of hiring the steady state work force. I construct the entry costs following Ebell and Haefke (2009). Djankov et al. (2002) measure the resource costs that entrepreneurs incur when starting a firm in the year 1997. These compose of costs for fees and the amount of business days lost until the firm can start operating. I evaluate the latter at the loss of per capita income per working day. Let total entry costs be denoted by  $\Gamma$ :

$$\Gamma = (d + \gamma)Y,$$

where d is the regulatory delay in months,  $\gamma$  are the fees as percentage of per capita GDP and Y is aggregate income. The free entry condition reads:

$$\Gamma(\sigma) + \frac{\varphi}{q(\theta(\sigma))} n(\sigma) = V^J(\sigma).$$
(1.15)

The equation uniquely determines  $\sigma$ . The transition to the long-run equilibrium works through the adjustment of  $\theta$  and w. To see this point, assume an increase in the regulation costs. Figure IVA displays this scenario. The right hand side of (1.15) becomes larger as  $\sigma$  decreases. The old and new long-run equilibriums are given by



Notes: Panel A plots the long-run equilibrium. The dotted line is the value of the firm for a given level of  $\sigma$ , the straight line shows the entry costs under low regulation and the dashed line the corresponding costs under high regulation. Panel B displays the relative changes of wages and labor market tightness for variations in the competition level. One calibration is done for a country with German LMIs and one for a country with US LMIs.

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the interception of the dashed and straight line with the line depicting the value of the firm. Higher entry costs lead to a higher value of the firm and a lower competition level. The reason is that an increase in entry costs requires that either the costs of hiring the steady state work force decrease, or the value of the firm increases. Both margins adjust in equilibrium. Recall from (1.12) that a decrease in  $\sigma$  lowers  $\theta$ . The lower  $\theta$  increases the vacancy contact rate, reducing the costs of hiring the steady state work force. As a result, the second term on the left hand side of (1.15) becomes smaller. Figure IVA shows; however, that this fact is dominated by the increase in the costs. Therefore, the value of the firm needs to increase additionally. The increase is obtained by adjustment at two margins. First, the lower  $\theta$  makes the replacement hiring in the future cheaper. Second, wages decrease because the relative position of the firm in the Nash-Bargaining improves, which increases future profit streams.

The discussion makes apparent that the quantitative response of unemployment to a deregulation in the product market depends on the relative response of wages and labor market tightness. The response of unemployment to a deregulation is weak, when mostly wages increase to achieve the necessary decrease in  $V^{J8}$ . This point leads me to the evaluation of sizable interaction effects between *PMR* and *LMIs*. Define  $\hat{x}$  to denote dln(x) and define the flow match surplus as

$$MS = \frac{\sigma - 1}{\sigma - \epsilon} Ah + \frac{1}{1 - \tau} \Big[ \kappa [ln(1 - h) + h] - b \Big].$$

I can approximate the effect of changes in h and  $\sigma$  on  $\theta$  using (1.12):

$$\hat{\theta} = \frac{\frac{1}{\beta} - 1 + \chi + \epsilon p(\theta)}{\epsilon p(\theta) + \iota[\frac{1}{\beta} - 1 + \chi]} \Big[ \frac{\frac{\sigma(1-\epsilon)}{(\sigma-\epsilon)^2} Ah}{MS} \hat{\sigma} + \frac{\frac{\sigma-1}{\sigma-\epsilon} Ah + \frac{\kappa}{1-\tau} [h - \frac{h}{1-h}]}{MS} \hat{h} \Big].$$
(1.16)

The equation emphasizes the two channels through which labor market institutions and product market regulation interact. First, strong labor market institutions decrease the flow surplus, leading to a larger quantitative response in  $\theta$  to a change in  $\sigma$ . A low flow match surplus implies that the changes in profits generated by changes in product market regulation are large relative to the marginal match surplus. Hence, firms adjust vacancy creation strongly to fulfill the free entry condition (1.15). Second, (1.13) implies that equilibrium wages are higher in a country with strong

<sup>&</sup>lt;sup>8</sup>See Hagedorn and Manovskii (2008) and Hornstein et al. (2005) for a similar argument for the canonical search and matching model in the case of shocks to productivity.

labor market institutions. The higher wages, in turn, decrease firms' profits. As a result, the free entry condition implies higher mark-ups in countries with strong *LMIs*. To understand the implication for the response of unemployment to product market regulation, note that for the cross partial derivative holds

$$\frac{\partial \hat{\theta}}{\partial \hat{\sigma} \partial \sigma} < 0. \tag{1.17}$$

Changes in the competition level translate stronger into changes in labor market tightness when the original competition level was low. The intuition is that the higher equilibrium mark-ups in a country with strong LMIs imply a relative low marginal revenue product of labor because individual product demand is relatively steep. Therefore, the equilibrium marginal value of an extra employer and match surplus are lower. Figure IVB visualizes these effects. It plots the percentage change in labor market tightness relative to wages  $\left(\frac{\delta\%\theta}{\delta\%w}\right)$  that results from exogenously varying  $\sigma$  for two scenarios. The straight line represents a country with German LMIs and the dashed line a country with the US LMIs. For a given change in competition, the percentage change in labor market tightness relative to wages relative to wages relative to wages is much higher in the former case, making the effects of PMR on unemployment duration relatively more severe in countries with strong LMIs.

To understand how changes in labor market tightness translate into changes in the unemployment rate, note that an approximation to (1.14) gives a one-to-one mapping between these variables that depends on the matching elasticity:

$$\hat{u} = -(1-\iota)(1-u)\hat{\theta}.$$
(1.18)

### 1.4 Calibration

I calibrate the model to match German labor market statistics. I assign values to the policy parameters from the year 1997 because the most detailed product market regulation data is available for that year. The recession during the mid-90s makes the year 1997 arguably not a steady state. Therefore, I obtain data from averages over longer time horizons for all other calibration targets. The model period is one month. In calibrating the model, I mostly follow the strategy proposed by

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Shimer (2005), opposed to the small surplus calibration proposed by Hagedorn and Manovskii (2008). Costain and Reiter (2008) show that the latter implies an elasticity of unemployment to changes in unemployment benefits that is six to 60 times larger than available estimates. (1.16) highlights that a small surplus, MS, has a similar effect on propagation from changes in  $\sigma$ . Moreover, my focus is on differences between countries and the qualitative effects are the same across the two calibration strategies. Table 1.3 summarizes all calibration parameters.

Note that the equilibrium system of equations is homogeneous of degree one in aggregate productivity. I therefore normalize A = 1. Consistent with Siegel (2002), my choice of  $\beta$  implies a yearly real interest rate of 4%. The scaling parameter for the disutility of work is set such that an employed worker devotes 17% of his available time to market work, which is the implied value from the OECD German hours worked series.

I follow Mortensen and Nágipal (2007) and identify the elasticity of the matching function using the Beveridge curve and the cyclical component of unemployment and vacancies. I obtain quarterly vacancy and unemployment data from the *Forschungs*datenzentrum der Bundesagentur für Arbeit (German Bureau of Labor (*IAB*)) for the period of 1978 – 2004<sup>9</sup>. I detrend the data using a HP-filter with a smoothing parameter  $\lambda = 10^5$ . An OLS regression of log vacancies on log unemployment yields a regression coefficient of -1.5. The Beveridge curve implies that

$$ln(\xi) + \iota ln(u_t) + (1 - \iota) ln(v_t) = ln(\chi) + ln(1 - u_t).$$

Rewriting the relation and taking the derivative with respect to log vacancies yields

$$\frac{\partial ln(v_t)}{\partial ln(u_t)} = -\frac{1}{1-\iota} \left[ \frac{u_t}{1-u_t} + \iota \right]. \tag{1.19}$$

The average HP-filtered unemployment rate over the time horizon is u = 0.078. With  $\frac{\partial ln(v)}{\partial ln(u)} = -1.5$  I obtain  $\iota = 0.57$ . The approach requires that the economy is in steady state each quarter, which might be a poor approximation for the German labor market. The estimate is close; however, to available evidence from studies that use empirical specifications of the matching function with German data. Burda and

<sup>&</sup>lt;sup>9</sup>The vacancy data uses reported job openings to the federal agency. There is no requirement for firms to report any open vacancy; thus, the measured level of vacancies is likely to be too low.

Variable	Target
$\beta = 0.9967$	4% annual real interest rate
$\kappa = 0.6635$	h = 0.169
A = 1	Normalization
$\iota = 0.57$	Equation $(1.19) = -1.5$
$\epsilon = 0.57$	Hosios condition
$\xi = 0.0468$	u = 0.078
$\varphi = 0.084$	$\theta = 0.14$
$\delta = 0.007\%$	Plant exit probability
$\omega=0.163\%$	EU = 0.17%
b = 0.0418	$\frac{b}{wh} = 25.7\%$
$\tau = 0.426$	McDaniel (2007)
d = 4.154	Djankov et al. $(2002)$
$\gamma = 1.021$	Djankov et al. $(2002)$

Table 1.3: Calibration

Notes: The left column states the calibrated variable with its value and the second states the targeted moment or the source. EU stands for the employment to unemployment transition probability.

Wyplosz (1994) obtain  $\iota = 0.7$  for the period of 1968 – 1991. Gross (1997) finds  $\iota = 0.55$  for 1972 – 1983 and 0.37 for 1984 – 1994. Finally, Fahr and Sunde (2004) suggest  $\iota = 0.48$  for the period of 1980 – 1994. To understand the implications for the choice of  $\iota$ , recall from (1.18) that a high  $\iota$  implies a relatively weak response of unemployment to changes in the institutional structure. I follow Shimer (2005) and impose the Hosios condition  $\epsilon = \iota^{10}$ .

I use the scaling efficiency of the matching function to calibrate the unemployment rate to 0.078. I set  $\varphi = 0.084$  implying a labor market tightness of 0.14, which is the average in Germany for 1978 – 2004. With regard to transition parameters, I measure from *IAB* plant level data the fraction of employment that is lost due to plant exit during the period of 1985 – 2004, which leads to  $\delta = 0.00007$ . I decide to calibrate the job separation rate from the perspective of the worker. Jung and Kuhn

<sup>&</sup>lt;sup>10</sup>Increasing workers' bargaining power would reduce marginal match surplus further, leading to stronger responses in unemployment. Again, cross country differences are unaffected by my choice and imposing the Hosios condition is attractive because it can be rationalized from a directed search perspective where firms post vacancies in sub-markets (see Shi (2006)).

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(2011) report a mean employment to unemployment (EU) transition probability of 0.17% between 1980 – 2004, implying  $\omega = 0.00163$ .

My choices for the policy parameters follow Section 1.2. I set b such that the replacement rate is 25% of nominal wages, and employment taxes are another 42.6%. Djankov et al. (2002) report that 90 business days are lost in Germany when setting-up a medium size firm. I translate this into a monthly cost by assuming 260 working days per year, leading to d = 4.154. Finally, I set  $\gamma = 1.021$  implying that regulatory fees cost 8.5% of yearly per capita GDP.

## 1.5 Out of Sample Predictions

	Germany	Italy	Sweden	UK	US
${u\% \over h}$	$7.8 \\ 0.169$	$\begin{array}{c} 5.74 \\ 0.17 \end{array}$	$9.0 \\ 0.165$	$\begin{array}{c} 4.6\\ 0.216\end{array}$	$4.2 \\ 0.221$

Table 1.4: Predictions from the Calibration

Notes: The table shows predictions of unemployment and the fraction of time an employed worker devotes to labor market activities for different countries based on a calibration that targets labor market moments in Germany.

I judge the ability of the structural model to pick up the main institutional mechanisms using the heterogeneity in institutions and labor market outcomes in the cross section of countries. I vary the institutional parameters according to Section 1.2 and assume that all preference and technology parameters are constant across countries. Table 1.4 displays the predicted unemployment rate and the hours decision for the countries under consideration. The model predicts Sweden to have a higher unemployment rate than Germany and the UK and US to have considerably lower rates, which is in line with the data. The main failure of the model is the too low prediction for the unemployment rate in Italy<sup>11</sup>. With regard to the intensive margin, the model predicts correctly that Swedish and German workers devote almost the same amount of time to market work, whereas hours worked per worker is higher

<sup>&</sup>lt;sup>11</sup>The unemployment rate in Italy varies considerably within the country, especially it is considerably higher in southern Italy.
in the UK and US. Again, the model predicts Italian workers to work relatively too little in the market. Nevertheless, I take these results as encouraging that the model is able to predict key labor market moments from the data.

# 1.6 Results

This section presents the main results of this chapter. To help focusing, I demonstrate the effects of changes in regulations using the example of German and US LMIs and PMR. I begin by demonstrating the effect of a product market deregulation on mark-ups and wages. Afterwards, I address the resulting changes in employment outcomes.

# 1.6.1 PMR, Mark-Ups and Wages

Comparing rows one and three in Table 1.5 shows that high labor market regulation increases wages and mark-ups. The increase in wages directly follows from (1.13) because the bargaining position of workers increases. The increase in wages decreases firms' profits. Mark-ups have to increase, for firms to fulfill the free entry condition (1.15). Recall from (1.17) that this difference affects the way that changes in the competition level translate into changes in labor market outcomes.

The top panel of Table 1.5 presents the predicted change in mark-ups and wages after a deregulation of the product market from the German to the US level in

		$\underline{\frac{\sigma-\epsilon}{\sigma-1}}$	w
German <i>LMIs</i>	German <i>PMR</i> US <i>PMR</i>	$1.0215 \\ 1.0051$	$0.9617 \\ 0.9775$
US <i>LMIs</i>	German <i>PMR</i> US <i>PMR</i>	$1.0191 \\ 1.0028$	$0.9498 \\ 0.9656$

Table 1.5: Changes in the Mark-Ups and Wages

Notes: The table displays the changes in the mark-ups and gross wages as a reaction to a decrease in PMR from the German to the US level. The reform is considered for a country with German and US LMIs separately.

a country with the high German *LMIs*. Mark-ups fall by 1.64 percentage points, triggering an increase in wages of about 1.63%. Rows three and four show the effects, when the same reform is implemented in a country with the low US *LMIs*. The quantitative effects turn out to be very similar to the former case. Mark-ups fall by 1.63 percentage points and wages rise by 1.66%. The slightly stronger reaction of wages in a country with low *LMIs* is in line with empirical evidence provided by Griffith et al. (2007). The decrease in mark-ups is quite sizable under both scenarios. Mark-ups decrease close to zero under US *PMR* indicating that the US level creates almost no costs in form of monopoly power<sup>12</sup>.

# 1.6.2 *PMR* and Employment Decisions

I now demonstrate how deregulation in the product market translates into quantitative changes at the extensive and intensive employment margin. I discuss two different policy regimes with respect to the treatment of unemployment benefits. First, I look at an endogenous b that reacts to the deregulation of the product market keeping the replacement rate constant. Thereafter, I discuss the case with a fixed b.

The top panel of Table 1.6 shows the effect of changes on PMR on employment choices in my baseline model with a constant replacement rate. The effect of a decrease in PMR on unemployment is almost twice as large in a country with German LMIs compared to a country with US LMIs. Thus, the propagation mechanisms analyzed in Section 1.3.5 lead to large quantitative differences between countries. The quantitative response of unemployment is; however, small overall in both cases with my calibration. The increase at the intensive margin is somewhat larger. h increases by 1.48% in the case of German LMIs and by 1.33% in the case of US LMIs.

Table 1.5 highlights that wages increase after a deregulation in the product market; therefore, b increases to keep the replacement rate constant, which counteracts the decrease in u. Expected benefits from unemployment are; however, seldom perfectly indexed to the wage in OECD countries because social assistance is usually not indexed to wages. The bottom panel of Table 1.6 demonstrates that the extensive margin reacts much stronger to changes in *PMR* when b is kept constant. In this case, the unemployment rate is predicted to decrease by 0.432 percentage points with

<sup>&</sup>lt;sup>12</sup>Part of the explanation is that firms incorporate the effect that hiring additional workers has on the average wage bill. Mark-ups would be  $\frac{\sigma}{\sigma-1} > \frac{\sigma-\epsilon}{\sigma-1}$  without this effect.

		u%	h
<i>b</i> as a replacement rate German <i>LMIs</i>	German <i>PMR</i> US <i>PMR</i>	7.8 7.7	$0.169 \\ 0.1715$
US <i>LMIs</i>	German <i>PMR</i> US <i>PMR</i>	$4.258 \\ 4.202$	$0.2182 \\ 0.2211$
b fixed German LMIs	German <i>PMR</i> US <i>PMR</i>	7.8 7.368	$0.169 \\ 0.1718$
US LMIs	German <i>PMR</i> US <i>PMR</i>	$4.258 \\ 4.177$	$0.2182 \\ 0.2211$

#### Table 1.6: Changes in Employment Decisions

Notes: The table displays the changes at the employment margin as a reaction to a decrease in PMR from the German to the US level. The reform is considered for a country with German and US LMIs separately. In the top panel, b is adjusted after the reform to keep a constant replacement rate. In the bottom panel, b is kept constant.

German LMIs and 0.081 percentage points with US LMIs. Thus, in case the size of unemployment benefits does not depend on changes in wages, PMR can have a significant effect on unemployment rates in countries with strong LMIs.

# 1.7 Conclusion

This chapter investigates the role PMR plays in explaining differences in average unemployment duration, unemployment rates and hours worked between OECD countries. I put special emphasis on the interaction effects between PMR and LMIs. PRM increases the costs of entry into a monopolistic product market; thus, increasing the mark-ups of firms. To keep firms willing to enter the product market, wages have to decrease and unemployment has to increase. I show that the relative response of wages and unemployment to a deregulation in the product market depends on the strength of labor market institutions. Strong LMIs decrease the value of an additional

worker to the firm by two distinct channels. First, there is a direct effect because they raise the relative value of unemployment. Second, there is an indirect effect. I show that strong *LMIs* lead to lower equilibrium competition in the product market. The low level of competition decreases the marginal revenue product of labor to firms and hence decreases the value of an additional worker. When this value is small, the extra value created by deregulation in the product market is large relative to the existing marginal value of an additional worker and relatively many firms enter creating relatively high additional labor demand.

I quantify the effect of PMR on unemployment and average hours worked by calibrating my model to German data. Decreasing the level of PMR present in Germany in 1997 to the corresponding US level had the potential to decrease the German unemployment rate by 0.432 percentage points, if the level of unemployment benefits would be kept constant. This effect is 2.7 times larger than previous studies suggest. The key driving force is a low marginal value of an additional worker in Germany, which increases the response of unemployment by 100% relative to a country with US *LMIs*. Moreover, workers can expect wages to rise by 1.63%, which triggers an increase at the intensive labor supply margin of about 1.66%.

Appendix

# Appendix to Chapter 1

# 1.A Proof of Lemma 1

Differentiating (1.7) yields

$$w_{i} = \frac{\epsilon}{h} \left[ MRPL_{i} - \frac{\partial w_{i}}{\partial n_{i}} hn_{i} + \frac{\varphi}{q(\theta)} (1 - \omega) \right] - \frac{1 - \epsilon}{(1 - \tau)h} \left[ -b + \kappa [ln(1 - h) + h] + \beta (1 - \chi - p(\theta))V^{W'} \right]. \quad (1.20)$$

Ignoring constant parts, for the moment, I have:

$$w = \frac{\epsilon}{h} \left[ MRPL_i - \frac{\partial w_i}{\partial n_i} hn_i \right].$$
(1.21)

Assume the differential equation has a solution to the homogeneous part, and let Z be the corresponding integration constant. Then the solution takes the form

$$w_i = Z n_i^{-\frac{1}{\epsilon}} \tag{1.22}$$

$$\frac{\partial w_i}{\partial n_i} = -\frac{1}{\epsilon} Z n^{-\frac{1+\epsilon}{\epsilon}} + \frac{\partial Z}{\partial n_i} n_i^{-\frac{1}{\epsilon}}.$$
(1.23)

Plugging (1.22) and (1.23) into the differential equation (1.21) and substituting for  $MRPL_i$  yields:

$$\frac{\partial Z}{\partial n_i} = \frac{1}{h} \frac{\sigma - 1}{\sigma} Ah \left[\frac{Ah}{c}\right]^{-\frac{1}{\sigma}} n_i^{\frac{\sigma - \epsilon \sigma - \epsilon}{\sigma \epsilon}}.$$

Integrating with respect to  $n_i$  with integration constant X yields:

$$Z = \frac{1}{h} \frac{\sigma - 1}{\sigma} Ah \left[ \frac{Ah}{c} \right]^{-\frac{1}{\sigma}} \frac{\sigma \epsilon}{\sigma - \epsilon} n_i^{\frac{\sigma - \epsilon}{\sigma \epsilon}} + X.$$

Plugging into (1.22) gives

$$w_i = \frac{\epsilon}{h} \frac{\sigma}{\sigma - \epsilon} MRPL_i + Xn_i^{-\frac{1}{\epsilon}}.$$

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To uniquely pin down the solution to the equation, I assume that the wage bill goes to zero as employment goes to zero:  $\lim_{n_i \to 0} hn_i w_i = 0$ . This results in

$$w_i = \frac{\epsilon}{h} \frac{\sigma}{\sigma - \epsilon} MRPL_i.$$

From (1.20) follows:

$$w_{i} = \frac{\epsilon}{h} \left[ \frac{\sigma}{\sigma - \epsilon} MRPL_{i} + \frac{\varphi}{q(\theta)} (1 - \omega) \right] - \frac{1 - \epsilon}{(1 - \tau)h} \left[ -b + \kappa [ln(1 - h) + h] + \beta (1 - \chi - p(\theta))V^{W'} \right] \quad (1.24)$$
$$\frac{\partial w_{i}}{\partial n_{i}} n_{i} = -\frac{\epsilon}{h} \frac{1}{\sigma - \epsilon} MRPL_{i}.$$

Plugging into (1.6) yields the labor demand curve, (1.9). Rearranging (1.24) and substituting for  $\beta(1 - \chi - p(\theta))V^{W'}$  in (1.8) yields:

$$V^W = \frac{\epsilon}{1-\epsilon} (1-\tau) \Big[ \frac{\sigma}{\sigma-\epsilon} MRPL_i - w_i h + \frac{\varphi}{q(\theta)} (1-\omega) \Big].$$

Using (1.9) and shifting one period forward gives:

$$V^{W'} = \frac{\epsilon}{1-\epsilon} (1-\tau) \frac{\varphi}{q(\theta)} (1-\omega) \frac{1}{\beta(1-\delta)}.$$

Plugging into (1.8) yields (1.11). Finally, substituting the solution for  $V^{W'}$  into (1.24) gives (1.10).

# Exploring the Causes of Frictional Wage Dispersion

# 2.1 Introduction

Residual wage dispersion is large in the US, the typical Mincer wage regression explains only around a third of observed wage variation, and growing over a cohort of workers' life-cycle. To understand this inequality, knowledge of the wage offer distribution and the distribution of idiosyncratic wage uncertainty are of first order importance. Yet, their empirical inference has proven to be non-trivial. They are inherently unobservable, and observed wage dynamics are the result of people selfselecting into and out of employment and accepting and refusing wage offers while on the job. Reduced form estimations rely on instrumental variable approaches, but credible instruments have proven hard to come by. The alternative is to make structural assumptions to infer these objects from within a model environment.

Search theory is a natural starting point for this analysis. The search friction introduces a trade-off between accepting a job offer now or waiting in unemployment to sample a better one. This induces what we refer to as *frictional wage dispersion*: workers of identical observable characteristics earn different wages in equilibrium.

The challenge for structural approaches is to correctly identify the channels that determine the range of acceptable job offers to the worker. Yet, as Hornstein et al. (2011) (henceforth HKV) point out, a large part of the search and matching models commonly used in empirical applications cannot, by construction, account for the amount of residual inequality found in U.S. data given realistic parameterizations. One could conclude from this finding that most residual inequality is the result of measurement error in wages or the explanatory variables and that measurement error

is a highly persistent process. Indeed, estimated versions of many of the models presented in HKV either have to attribute much of wage inequality to measurement error or produce unreasonably low estimates of discount factors and/or the replacement rate. We take the view; however, that this conclusion should be drawn from a model that theoretically could account for the observed residual inequality. Otherwise, structural estimation might just be missing important dimensions to the worker problem.

Our main contribution is twofold. First, we build a structural model that is consistent with the large amount of residual wage dispersion present in our data set and its increase over a cohort's life-cycle. Our model includes a number of important channels that enlarge the set of acceptable job offers: Worker productivity is stochastic and unemployed workers face skill depreciation and finite unemployment benefits. Employed workers experience learning by doing on the job and can continue to search for better job prospects while working. We demonstrate that the skill development process and the ability to search on the job are key ingredients in generating realistic amounts of frictional wage dispersion. We also highlight that calibrated on the job search efficiencies need to account for the fact that one third of all observed job switches result in nominal wage losses. This statistic allows us to discriminate between our model and a pure job-ladder model in explaining the data. Second, we demonstrate that we can use our model to quantify the contribution of firm effects, changes in individual productivity and measurement error to wage dispersion across agents and over the life-cycle, using second moments from wage data. Across different age groups, our baseline specification finds a mean contribution of 17.5 percent of frictional wage dispersion to overall inequality.

Regarding the last result, we also shed light on why the literature so far has produced such divergent estimates. Hagedorn and Manovskii (2010) conclude that search frictions are responsible for only 6 percent of overall wage inequality. In comparison, the seminal paper by Postel-Vinay and Robin (2002) finds numbers as high as 100 percent. Our results suggest that the way the latter paper models job to job transitions is in part responsible for that finding. Like most other on the job search models, it attributes all job to job transitions to movements into more productive matches. Given the substantial size of job to job flows in the data, this implies search efficiencies have to be high. In the light of the frequent occurrence of wage losses upon movement; however, this assumption seems to exaggerate search efficiency and causes these models to overstate the importance of search frictions in generating wage dispersion. Indeed, when solving a pure on the job search calibration of our model, capable of explaining large frictional wage dispersion but neglecting wage losses, the contribution of the search friction to overall wage inequality more than doubles to over 40 percent.

Related to this chapter are a few recent contributions that augment the standard search model to replicate the empirically observed amount of residual wage dispersion. Burdett et al. (2011) and Carrillo-Tudela (2010) introduce a restricted form of experience into an on the job search model. Papp (2012) shows that wage posting combined with the assumption of Bertrand competition in the labor market can lead to almost arbitrarily large frictional wage dispersion. Also related is a line of literature that empirically asses the importance of different sources of wage inequality. Bontemps et al. (1999, 2000) set up on the job search models and structurally estimate them on French panel data. This line of literature also includes the aforementioned contribution by Postel-Vinay and Robin (2002). Hagedorn and Manovskii (2010) choose a reduced-form approach in quantifying the contributions of search frictions to wage inequality. Finally, Low et al. (2010) use an instrumental variable approach to estimate the firm offer distribution and idiosyncratic wage risk.

The remainder of the chapter is structured as follows. The following section provides some empirical motivation and takes a closer look at the efficiency of on the job search. Afterwards, we present our model and discuss parameterization. We present and analyzes our results and discuss their relation to the existing literature. The last section concludes. Additional information on the empirical part and the numerical algorithm is relegated to an appendix. All programs used for data analysis and model solution are available on the author's web pages.

# 2.2 Empirical Motivation

In this section, we introduce our data set, the Survey of Income and Program Participation (SIPP), and discuss sample selection. To motivate our further analysis, we then present two sets of facts from the data. First, we estimate residual wage dispersion and show its magnitude to be substantial. Second, we discuss wage changes

after employment to employment transitions. We argue that previous studies of on the job search have inadequately addressed the large fraction of employment to employment transitions which result in wage losses.

# 2.2.1 Data Source and Sample Creation

Our model, to be presented in the next section, places great emphasis on job to job transitions and accompanying wage changes as well as wage dynamics on the job. Our empirical analysis needs to accurately identify these phenomena. Therefore, we require longitudinal monthly wage information which identifies employer and occupation changes. The data set most adequate for these requirements is the 1993 sample of SIPP. It is a representative sample of the non-institutionalized civilian US population maintained by the US Census Bureau. The level of detail it provides in individual records allows us to accurately identify an individual's main job and hourly wages on that job. In addition, the 1993 cohort combines survey data with administrative records to accurately identify employer changes<sup>1 2</sup>.

In constructing the panel, the Census Bureau randomly assigns people to rotation groups, which are then interviewed subsequently on a four-month basis. During the interviews, the respondents give information on their labor market status for each week in the past four months separately, which is then used to assign one of eight possible activity statuses. While this form of reporting allows for a very precise labor market classification, it also constitutes one of the sample's few drawbacks. It makes it hard to compare unemployment measures based on this classification to unemployment in the Current Population Survey (CPS). It has also been shown to downward bias estimates of transition flows between employment and unemployment<sup>3</sup>. When calibrating these flow rates below, we therefore use estimates from corresponding CPS cohorts. Both panels are representative samples from the same population so this should be unproblematic.

The survey covers the years 1993-1995 (which also includes some observations from  $(1992)^4$  and therefore provides us with up to 36 months of observations per individual.

<sup>&</sup>lt;sup>1</sup>See Stinson (2003)

 $<sup>^{2}</sup>$ The survey reports at most two jobs for each four-month recording period. In case an individual has more than two jobs, the two jobs with the most hours worked are reported.

 $<sup>^3 \</sup>mathrm{See}$  Mazumder (2007) for a discussion.

<sup>&</sup>lt;sup>4</sup>Our data set grounds on CEPR SIPP extracts available for download at

We use observations from individuals aged 23-65, for whom we require complete information on the individual's employment status, age and employer id. On top of that, we only consider an individual's primary job<sup>5</sup>. We drop workers reporting to be school enrolled, the self-employed, family-workers, members of the armed forces and workers at non-profit companies. Finally, we truncate the wage distribution at the top and bottom 1% to take care of outliers and top-coding<sup>6</sup>. These restrictions leave us with 754,345 person/month observations<sup>7</sup>.

The SIPP asks respondents whether they are paid by the hour and their corresponding hourly pay rate in each month. We use this hourly pay rate whenever a worker is paid by the hour. The SIPP also reports total monthly earnings at each job, whether the job lasted the entire month and the amount of hours worked per week. We assume that workers do not alter their earning responses based on the length of a month and use smooth 4.3 weeks months when computing monthly earnings of those workers that are not paid by the hour<sup>8</sup>. SIPP records starting date and ending date of each job that is not lasting the entire month, and we use this information to calculate hourly wages for these jobs.

We identify job to job transitions as those employment changes where the worker works in two consecutive months, the technology with which the worker operates changes and the worker does not report a spell of unemployment in between<sup>9</sup>. We

http://www.ceprdata.org/sipp\_data.php.

We modify these abstracts to include further information contained in the original SIPP files but not in the abstracts.

 $<sup>{}^{5}</sup>$ As primary job we consider the position where the largest share of hours worked is spent.

 $<sup>^6\</sup>mathrm{Earnings}$  are topcoded at \$33333 for a four months period.

<sup>&</sup>lt;sup>7</sup>The programs used to prepare the data for analysis are also available from the authors' web pages. They contain further details on how we deal with missing attributes and other choices inherent in data preparation.

<sup>&</sup>lt;sup>8</sup>One issue about using the hourly pay rate is worth mentioning. In general, we are interested in total worker compensation. When workers are asked about their hourly pay rate, the interviewer asks: "What was your regular hourly pay rate at the end of month X". Therefore, respondents are unlikely to include any bonuses or performance payments. Contrary, when asking about total monthly earnings interviewers state: "Be sure to include any tips, bonuses, overtime pay, or commissions". We still decide to use the hourly pay rate, whenever available because it is likely to reduce measurement error.

<sup>&</sup>lt;sup>9</sup>Theoretically, we could use the weekly employment status and count job to job transitions only when a worker is employed in two consecutive weeks. However, we want to allow workers to spend some days between jobs, which they may need to commute, or do other pre-work required activities. Consequently, we only discard observations where the worker reports to actively seek a job during non-employment.

assume that an operating technology is job specific and record a job transition whenever the employer identification number changes, or the two digit occupational identifier changes. Appendix 2.B provides a discussion for alternative measures of job to job transitions and compares our estimate to those obtained from CPS data.

# 2.2.2 Frictional Wage Dispersion in the SIPP

For subsequent comparison with the amount of *frictional* dispersion in our model, we start by estimating the amount of *residual* wage dispersion in our data. To control for observed and unobserved worker heterogeneity as well as time effects, we employ the estimation method outlined in Hornstein et al. (2007). For each period in the sample, unique combination of year/month, we run an OLS regression of individual hourly log wages on 9 regional dummies, 14 occupational dummies<sup>10</sup>, 4 education dummies (less than high school, high school, some college, college), a dummy for marital status, one for non-white, one for disabled workers, age, age squared and the number of children. This yields a set of residuals { $\epsilon_{it}$ }. The mean  $R^2$  of these regressions is 0.36. We then compute unobserved individual effects as  $\bar{\epsilon}_i = \sum_{t=1}^{T_i} \epsilon_{it}/T_i$ . The residual wage corresponding to individual i in period t is then  $\tilde{w}_{it} = exp(\epsilon_{it} - \bar{\epsilon}_i)$ . Note that this measure is quite conservative. The distribution of residual wages has large mass around one because many workers never experience job to job transitions over the relatively short observation period and thus the individual fixed effect captures the full firm effect in wages<sup>11</sup>.

Table 2.1 summarizes a number of moments of the resulting distribution of residual wage inequality: the ratio of the mean wage to the minimum wage (Mm-ratio), the Gini coefficient and the variance of log wages. The Mm-ratio is the summary statistic advocated by HKV and for comparability to their result and other studies, it is our leading measure. Since the lowest wages in the data are likely the result of measurement error, we report a number of low percentiles as candidates for the minimum wage. The size or residual inequality is substantial and comparable to the

<sup>&</sup>lt;sup>10</sup>Occupation is likely to be correlated with individual ability as well firm effects. We want to control for the former but leave the latter untouched. Our compromise is to only include very broad occupational categories, as they should be less firm specific. Dropping occupations from our estimations altogether lowers the  $R^2$  by 0.1 without materially affecting the Mean-Min ratio, our main summary statistic for residual inequality.

<sup>&</sup>lt;sup>11</sup>We thank Tamás Papp and an anonymous referee for making us aware of this issue.

Mean	-Min l	Ratio	Gini	$Var(\log(\tilde{w}_{it}))$
Pctl.	$ \begin{array}{c} 1^{st} \\ 5^{th} \\ 10^{th} \end{array} $	2.18 1.48 1.31	0.091	0.031

Table 2.1: Residual Wage Inequality in the 1993 SIPP

Notes: The table reports summary measures of residual wage inequality in the 1993 SIPP: the mean to minimum ratio, the Gini coefficient and the variance of log wages after controlling for region, broad occupation, education, marital status, non-whites, disabilities, a quadratic in age, the number of children and individual fixed effects. Since the lowest wage observations in the data are likely the result of measurement error, we report several low percentiles as candidates for the actual minimum wage in the data.

one found by other studies (see Hornstein et al. (2007))<sup>12</sup>.

# 2.2.3 Wages and On the Job Search

### Wage Losses from Employment to Employment Transitions

One of the most important potential channels for enlarging the set of acceptable job offers to the worker is the ability to continue searching on the job. In this case, the trade-off is between earning more than unemployment benefits now and reduced search efficiency on the job. If search on the job is fairly efficient, this will substantially enlarge the range of acceptable offers<sup>13</sup>.

Studies of on the job search typically calibrate on the job search efficiency to observed employment to employment flow rates. Meanwhile, Fallick and Fleischman (2004) find in the CPS that a worker who reports to be actively searching on the job is more likely to be unemployed next month. Likewise, Fujita (2011) uses a question in the UK labor force survey that asks for the reason of on the job search. He finds that of those reporting searching, 12% do so because they fear to lose

<sup>&</sup>lt;sup>12</sup>We also perform a more standard Mincer wage regression that ignores individual fixed effects. The resulting Mm-ratio at the first and fifth percentile are 2.78 and 2.03, respectively. Hornstein et al. (2007) obtain very similar results for PSID data. Without controlling for individual effects, the Mm-ratio at the first and fifth percentile are 2.73 and 2.08, respectively. Once they control for individual effects, those numbers drop to 1.9 and 1.46.

<sup>&</sup>lt;sup>13</sup>Of course, if search efficiency on the job is as high as in unemployment, the worker will accepts any wage offer at least as high as UI benefits.

their current job and 27% because they are unsatisfied with the current job because of non-monetary reasons. This evidence seems clearly at odds with predictions of job-ladder models. Nágipal (2005) shows within a basic job-ladder model that search on the job would have to be more efficient than during unemployment in order to replicate some characteristics of flow rates. In our view, these pieces of evidence imply other mechanisms behind the magnitude of employment to employment movements than job-ladders only. They also hint at on the job search being less efficient than previously assumed, and an accurate calibration of search efficiencies should take these concerns into account.

The SIPP asks workers that terminate a job for the reason of doing so. While this appears to be the information that perfectly suits our analysis, we want to emphasis several problems with the variable. First and foremost, less than 30% of all workers making a job to job transition provide information on this variable<sup>14</sup> <sup>15</sup>. Second, the question targets solely individuals that change employer id, but not those that switch occupations within the same firm. Third, the answers are not mutually exclusive and the interviewer provides no additional guidelines to choose among these mutually non-exclusive answers<sup>16</sup>. Nevertheless, we find the information instructive to get a sense of the magnitude of job to job transitions that do not necessarily result from the desire to move up the job-ladder. Only 55% of those responding to the question and making a job to job transition state that they quit to take another job. 19% of jobs ended because the previous job did not provide the possibility to continue<sup>17</sup>. Another 20% answer either that they had "unsatisfactory work arrangements", or "quit for some other reason". The remaining worker made a transition because of personal or family related reasons. Taken together, the result supports the view that a large share of identified job to job transitions does not result in a move to a better paying job.

In Table 2.2 we supply additional evidence to support that claim using a moment that does not suffer from the drawbacks mentioned above. A pervasive phenomenon

 $^{15}\mathrm{See}$  Nágipal (2008) who also uses this information.

<sup>&</sup>lt;sup>14</sup>The question is not applicable for a negligible share of transitions because only the main job changed, but the worker stays with his old employer.

<sup>&</sup>lt;sup>16</sup>For example, a possible answer is that the individual "quit to take another job" and an alternative answer is that the individual had an "unsatisfactory work arrangements".

<sup>&</sup>lt;sup>17</sup>This includes "on layoff", "job was temporary and ended", "discharged/fired", "employer bankrupt", "employer sold business" and "slack work or business conditions"

Sample Stratification		Share loss	Mean loss
Whole sample		0.339	-0.22
Job characteristics			
	Union	0.35	-0.249
	+ Health insurance	0.355	-0.249
	+ Education	0.355	-0.249
Year	1993	0.3301	-0.2068
	1994	0.3299	-0.23
	1995	0.3638	-0.2256
Sex	Male	0.3367	-0.2261
	Female	0.3421	-0.2122
Age	23-30	0.3483	-0.2274
	31-50	0.3368	-0.2198
	51-65	0.3230	-0.1981

Table 2.2: Wage Cuts after Job to Job Transitions

Notes: The table shows the share of workers incurring a nominal cut in hourly wages after a job to job movement for the whole population and different subsamples in the 1993 SIPP. Mean loss reports the mean wage loss in log points conditional on suffering a wage cut upon movement.

in the data is that job to job transitions result in nominal wage losses. In the whole population roughly one third of all transitions result in workers earning lower hourly wages in the month after transition compared to the last month at the previous job. The months surrounding a job to job transition might be particularly prone to reporting error. Therefore, we also constructed three-month-averages of wages after and before a movement. This robustness check does not affect any of our estimates. In terms of real wage changes, the share of wage losses increases to roughly one half<sup>18</sup>.

The top panel of Table 2.2 shows that the result is robust across different stratifications of the data concerning non-monetary job characteristics. First, we exclude observations where a worker moves from a non-unionized to a unionized job. The

<sup>&</sup>lt;sup>18</sup>In principle, the worker should only care about real wages. But in the presence of some wage rigidity, the worker expects a wage loss on his current job as well and compares nominal wages.

resulting share of workers incurring a wage loss goes up by about one percentage point. Subsequently deleting transitions where a worker moves from a job that did not provide health insurance to one that does provide it, increases this share by another half percentage point. Finally, deleting observations where the former employer did not pay for educational training, but the current employer does, has no effect on the share. A likely explanation for the robustness of the result is that even though workers value these non-monetary benefits of a job, jobs that have high non-monetary benefits are usually associated with higher wage income (see for example Dey and Flinn (2008)).

The result is also robust across different stratifications of the data concerning worker characteristics. Table 2.2 splits the sample by year, sex and age, and we always find a share around 33% of workers incurring wage losses at a job to job movement. Appendix 2.B shows that the result stay robust to stratifying the data by tenure at the former job, earnings at the former job and industries and looking at real instead of nominal wage changes.

In the interpretation of this chapter, an important part of these transitions are either the result of jobs accepted within notice period of dismissal or movements for non-economic reasons (moving in with one's spouse, moving close to ones parents, etc.). To proxy for these causes, our model includes what Jolivet et al. (2006) label *forced job movements*: randomly drawn on the job offers, which the worker can accept or move into unemployment.

If idiosyncratic worker productivity uncertainty is large relative to firm dispersion, parts of these wage cuts will be the result of negative shocks to general human capital, and some of them will be simply measurement error. Consequently, our baseline model includes innovations to worker productivity and all our simulations explicitly include measurement error to account for these causes. In Appendix 2.B we give consideration to an alternative explanation brought forward by Postel-Vinay and Robin (2002) and extended by Cahuc et al. (2006). They lay out a framework, in which workers will accept wage cuts upon job to job transitions, if the option value of working at the other firm is sufficiently high. Indeed, Papp (2012) shows that this framework can rationalize a large amount of wage cuts and large frictional wage dispersion. The key operating mechanism in this class of models is that workers who experienced wage losses have on average steeper observed wage growth afterwards, i.e. wages are backloaded. As we show, there is no indication of that mechanism occurring in our data. Hagedorn and Manovskii (2010) provide further evidence against the mechanism. They show that wage growth of job stayers is uncorrelated to local labor market tightness in the US, whereas the model by Postel-Vinay and Robin (2002) predicts it to be an increasing function of the probability to receive a job offer<sup>19</sup>.

# 2.3 The Model

This section explains the worker's decision problem when employed and unemployed. We introduce a bargaining game between workers and potential employers that maps productivities into wage outcomes. We show that the resulting log wage schedule can be almost perfectly approximated by a linear function in log worker and log firm productivity. We want to stress that the additional notation coming from the general equilibrium set-up, serves only the microfoundation of the log linear wage schedule. Our quantitative results would have stayed the same, if we had taken a partial equilibrium view and had postulated a log linear wage schedule.

# 2.3.1 The Labor Market

A firm is a match producing with the worker's idiosyncratic log productivity  $A_t$ and firm specific log productivity  $\Gamma_t^{20}$ . Firms' log productivity is drawn from the distribution  $F \sim N(0, \sigma_F^2)$ . Once a worker and a firm decide to form a match, they produce output  $y_t$  according to:

$$y_t = exp(A_t + \Gamma_t).$$

We assume that search is random and the labor market is governed by a matching function  $m_t = \xi u_t^{\iota} v_t^{1-\iota}$  where  $v_t$  are vacancies and  $u_t$  is unemployment. The matching function implies an unemployed worker contact rate  $q(\theta_t)$  and a job offer probability

 $<sup>^{19}\</sup>mathrm{The}$  same holds true for models that stress the importance of learning about match quality over time.

 $<sup>^{20}\</sup>Gamma$  is the only source of match effects in our model, which we interpret as firm productivity. However, one can broaden this interpretation to include match specific effects and, as Winfried Koeniger pointed out to us, differences arising from bargaining over quasi rents from capital.

 $p(\theta_t)$  that depend only on labor market tightness  $\theta_t = \frac{v_t}{u_t}$ . Additional dimensions of worker heterogeneity are their life-cycle state  $\phi$  and unemployment benefit eligibility, indicated by  $\varpi$ .

# 2.3.2 The Households' Problem

Household period income is given by:

$$I_t(A_t, \Gamma_t, \phi_t) = \begin{cases} w_t(A_t, \Gamma_t, \phi_t) & \text{if } employed \\ \min\{b_{max}, b_t(A_t, \phi_t)\} + Z(A_t, \phi_t) & \text{if } \varpi = u_1 \\ Z_t(A_t, \phi_t) & \text{if } \varpi = u_2 \end{cases}$$

When the agent is in state  $u_1$ , he receives unemployment insurance benefits (UI), but with probability  $\lambda_l$  he loses the benefit entitlement and moves to state  $u_2$ . After match destruction, an agent is always entitled to benefits<sup>21</sup>.  $b_{max}$  are statutory maximum UI payments. Both unemployment benefits and the value of leisure depend on the worker's idiosyncratic states:

$$b(A_t, \phi_t) = rr_b \cdot \mathbb{E} \Big[ w_t(A_t, \Gamma_t, \phi_t) | A_t, \phi_t \Big]$$
$$Z(A_t, \phi_t) = rr_Z \cdot \mathbb{E} \Big[ w_t(A_t, \Gamma_t, \phi_t) | A_t, \phi_t \Big].$$

Expectations are taken over the range of acceptable job offers, which themselves depend on  $A_t$  and  $\phi_t$ . In the case of unemployment insurance, this captures the fact that benefits depend on prior contributions. More productive workers earn higher wages, and older workers likely contributed for a longer time. In the case of the value of leisure, we choose this as the closest analogy to the homogeneous agent world<sup>22</sup>. One can interpret this formulation as a reduced form for modeling wealth heterogeneity. More productive workers tend to have higher asset levels and unemployed workers deplete their assets over time.

In modeling productivity development, we are guided by the finding of Dustmann

<sup>&</sup>lt;sup>21</sup>Low et al. (2010) and Ljungqvist and Sargent (2008) assume that entitlement is conditional on the separation being involuntary on part of the worker. We choose a different path in assuming that the cause of separation is unobservable to the UI agency.

<sup>&</sup>lt;sup>22</sup>Giving everyone the same value of leisure would increase the amount of implied frictional wage dispersion.

and Meghir (2005), who show that the first two years of labor market experience raise wages substantially, 6-10% per year, whereas the return to experience is substantially lower afterwards, 0-1.2%<sup>23</sup>. We therefore introduce the life-cycle dimension  $\phi$  where agents transit through two life-cycle states with stochastic transition probabilities  $p = (p_1, p_2)$ . When the second shock hits, the agent dies and is replaced by an unemployed labor market entrant in state  $u_2$ , whose idiosyncratic log productivity is drawn from the distribution  $N \sim N(\mu_N, \sigma_N^2)$ .

The evolution of worker productivity depends on the agent's employment status, and in case the agent is employed it also depends on his life-cycle state:

$$A_{t+1} = \begin{cases} max(A_t + \nu(\phi) + \epsilon_t, pmin) & \text{if } employed \\ max(A_t - \delta + \epsilon_t, pmin) & \text{if } unemployed \end{cases}$$

 $\delta$  represents skill depreciation while being unemployed, *pmin* is a subsistence level of productivity and  $\nu(\phi)$  is a drift term that depends on the life-cycle state.  $\epsilon$  is a productivity shock with  $\epsilon \sim N(0, \sigma_{\epsilon}^2)$ . We think of wage shocks as anything altering productivity, such as demand shocks for specific skills or health shocks. The fact that net productivity growth can be negative entails that our model also features wage cuts on the job.

We assume that firms cannot commit to a wage path, and wages are renegotiated by generalized Nash-Bargaining each period. Employed workers continue to search for better job prospects, and once an outside offer arrives all parties have full information about their respective productivities. We assume that a worker cannot go back to his former employer after initiating wage bargaining with a new employer<sup>24</sup>. Consequently, his outside option is unemployment with benefit entitlement when bargaining with the new firm. Following our discussion in Section 2.2.3, we model some job to job transitions as forced movements. An employed worker receives a job offer with probability  $\lambda$  and can, in general, decide to stay with his old match or form a new

<sup>&</sup>lt;sup>23</sup>Dustmann and Meghir (2005) use German data but have the advantage of identifying effects by using displaced workers. For U.S. data, Altonji and Williams (1998) come to similar results.

<sup>&</sup>lt;sup>24</sup>In the context of on the job search, the bargaining set may be non-convex and the Nash-Bargaining solution therefore be undefined as discussed in Shimer (2006). As shown by Moscarini (2005) pp. 496, our set-up can be reconciled with Nash-Bargaining by assuming an English auction between incumbent and poaching firm in which firms do not play weekly dominant strategies. The resulting wage changes in this chapter are equivalent to that assumption, e.g., the equilibrium outcome is to bid zero in the English auction.

match. However, when receiving an outside offer, with probability  $\lambda_d$  the offer is a forced movement and the outside option becomes unemployment.

We can thus define the value of employment,  $V_{\phi}^{E}$ , and the value of unemployment,  $V_{\phi,\varpi}^{U}$ , for each life-cycle state. The value of employment depends on worker and firm productivity, and the value of unemployment depends on worker productivity and benefit entitlement. We state the Bellman equations describing the problems of agents in the first life-cycle state as an example. The value of employment reads:

$$V_{1}^{E}(A_{t},\Gamma_{t}) = w_{t}(A_{t},\Gamma_{t},1) + \beta \mathbb{E}_{t} \Big\{ (1-\omega) \\ \Big[ (1-p_{1})[(1-\lambda)H(1) + \lambda[(1-\lambda_{d})\Omega_{E}(1) + \lambda_{d}\Lambda(1)]] \\ + p_{1}[(1-\lambda)H(2) + \lambda[(1-\lambda_{d})\Omega_{E}(2) + \lambda_{d}\Lambda(2)]] \Big] \\ + \omega \Big[ (1-p_{1})V_{1,u_{1}}^{U}(A_{t+1}) + p_{1}V_{2,u_{1}}^{U}(A_{t+1}) \Big] \Big\}.$$

 $\mathbb{E}_t$  is the expectation operator given all information in period t and  $\omega$  is an exogenous match destruction shock. For clarity of presentation, we define the upper envelopes for receiving a regular job offer on the job,  $\Omega_E(x)$ , receiving a forced job offer,  $\Lambda(x)$ , and the decision to quit into unemployment voluntarily, H(x), as result of a negative productivity shock.  $\Gamma'_{t+1}$  is firm productivity at an outside job offer. All are conditional on the life-cycle state:

$$\Omega_E(x) = \max \int \{ V_x^E(A_{t+1}, \Gamma_{t+1}), V_{x,u_1}^U(A_{t+1}), V_x^E(A_{t+1}, \Gamma_{t+1}') \} dF$$
  

$$H(x) = \max \{ V_x^E(A_{t+1}, \Gamma_{t+1}), V_{x,u_1}^U(A_{t+1}) \}$$
  

$$\Lambda(x) = \max \int \{ V_x^E(A_{t+1}, \Gamma_{t+1}'), V_{x,u_1}^U(A_{t+1}) \} dF.$$

There are two value functions for the unemployed, with and without benefit entitlement. Once benefits expire, the agent's flow value is reduced to the utility of leisure:

$$V_{1,u_2}^U(A_t) = Z + \beta \mathbb{E}_t \Big\{ (1 - p_1) [p(\theta) \Omega_U(1, u_2) + (1 - p(\theta)) V_{1,u_2}^U(A_{t+1})] \\ + p_1 [p(\theta) \Omega_U(2, u_2) + (1 - p(\theta)) V_{2,u_2}^U(A_{t+1})] \Big\}.$$

Conditional on receiving benefits, the value of unemployment solves:

$$\begin{split} V_{1,u_1}^U(A_t) &= b + Z + \beta \mathbb{E}_t \Big\{ (1 - \lambda_l) \\ & \Big[ (1 - p_1) [p(\theta) \Omega_U(1, u_1) + (1 - p(\theta)) V_{1,u_1}^U(A_{t+1})] \\ & + p_1 [p(\theta) \Omega_U(2, u_1) + (1 - p(\theta)) V_{2,u_1}^U(A_{t+1})] \Big] \\ & + \lambda_l \Big[ (1 - p_1) [p(\theta) \Omega_U(1, u_2) + (1 - p(\theta)) V_{1,u_2}^U(A_{t+1})] \\ & + p_1 [p(\theta) \Omega_U(2, u_2) + (1 - p(\theta)) V_{2,u_2}^U(A_{t+1})] \Big] \Big\}, \end{split}$$

where we define the conditional upper envelope for receiving a job offer:

$$\Omega_U(x,\varpi) = \max \int \{V_x^E(A_{t+1},\Gamma_{t+1}), V_{x,\varpi}^U(A_{t+1})\} dF.$$

# 2.3.3 The Firms' Problem

An entering firm's problem is described by its value to post a vacancy,  $V^{I}$ . An open vacancy entails flow costs of  $\varphi$  each period. We assume vacancies are homogeneous ex ante and the realization of the idiosyncratic productivity reveals only upon meeting a worker. When a worker is contacted,  $\Gamma$  is drawn from  $F^{25}$ . There are three ways to fill a vacancy. First, an unemployed agent might be contacted, occurring with probability  $q(\theta)$ . Second, the firm might poach a worker that is employed and make him a job offer, which happens at rate  $\frac{\lambda(1-\lambda_d)}{v}$ . Or third, a worker might be offered the vacancy by a forced job movement occurring at rate  $\frac{\lambda\lambda_d}{v}$ . Note that in any case, the ex-ante acceptance probability depends on the productivity of the vacancy. Given that firm and worker productivities are complements, more productive vacancies attract also less productive workers and are less likely to lose parts of their workforce to other firms. We relegate the further description of  $V^{I}$  to Appendix 2.A.1, as it does not provide much further insights.

The value of a filled vacancy,  $V_x^J$ , depends on the life-cycle state of the matched

<sup>&</sup>lt;sup>25</sup>This can be rationalized by assuming that there is a match specific component in productivity. The assumption assures that wages are a monotone function in firm productivity. See Eeckhout and Kircher (2011) for a simple model where match productivity is fully firm specific and wages are not an increasing function in firm productivity for all worker types.

employee and a firm employing someone in life-cycle state one has value:

$$V_1^J(A_t, \Gamma_t) = y_t - w(A_t, \Gamma_t, 1) + \beta(1 - \omega) \mathbb{E}_t \Big\{ (1 - \lambda)[(1 - p_1)\Phi(1) + p_1\Phi(2)] \\ + \lambda(1 - \lambda_d)\eta(\Gamma_{t+1})[(1 - p_1)\Phi(1) + p_1\Phi(2)] \Big\}.$$

where  $\eta(\Gamma)$  is the probability that the worker stays with the firm when contacted from an outside firm, which is increasing in  $\Gamma$ . Moreover, we define the upper envelope of match continuation conditional on the life-cycle state and productivities

$$\Phi(x) = \max\{0, V_x^J(A_{t+1}, \Gamma_{t+1})\}.$$

The equilibrium definition is standard and can be found in Appendix 2.A where we also provide a summary of within-period timing.

# 2.3.4 Approximating the Wage Schedule

To facilitate our subsequent analysis and to make our approach more comparable to standard microeconometric specifications, we approximate the equilibrium log wage schedule by a linear function. From the Nash-Bargaining solution it is obvious



Notes: The figures display the equilibrium log wage schedule for workers in the second life-cycle state. Figure I fixes firm productivity at its median level. 95 percent of all workers employed at such matches have productivity levels below the dashed line. Figure II fixes worker productivity at its median value and varies firm productivity.

that log wages are not a linear function in worker and firm productivity. Figures I and II plot ln(w) over worker and firm productivity for agents in life-cycle state 2, holding the productivity of the other fixed at its mean value. The plots indicate that these functions can still be reasonably well approximated by a linear function. We asses this more formally by fitting a linear OLS regression to an economy generated by the true non-linear dynamics of our model. To be more specific, we simulate 50000 workers for 3 years from the stationary distribution using our non-linear model. We then project the resulting data into a linear space employing the regression

$$ln(w_{i,t}) = \beta_0 + \beta_1 A_{i,t} + \beta_2 \phi_{i,t} + \beta_3 \Gamma_{i,t} + a_{i,t}.$$
(2.1)

Note, assuming the law of large number holds, the error term  $a_{i,t}$  measures the approximation error that results from the linear projection. We obtain an  $R^2$  above 0.9996, suggesting that the fit of the linear regression model is quite good. We continue to work from now on with the linear approximation (2.1) to our true non-linear model.

# 2.4 Parameterization

We pursue a dual strategy in parameterizing our model. We take a number of parameter values from other studies, which makes our results easily comparable. Also, for many of those parameters (e.g., bargaining share) our results are robust to variations. We come back to this point below. The particular focus of this chapter requires us to take great care in calibrating worker and firm productivity dispersion and flow rates in and out of employment and between firms. Wherever possible, we therefore estimate our calibration targets for the related parameters using our own data set in order to insure consistency. Although the SIPP provides very detailed and extensive coverage, we cannot estimate all of the productivity parameters on the basis of our data set. Therefore, we take additional information from other micro studies carefully discussing each of our choices. This section proceeds as follows: We first discuss our calibration regarding non-distributional parameters (preferences, institutions, flow rates) in Section 2.4.1. In Section 2.4.2, we discuss our calibration strategy regarding firm and idiosyncratic productivity dispersion. Table 2.3 summarizes our calibration.

# 2.4.1 Non-Distributional Parameters

The model period is one month. When comparing monthly wages in the model to hourly wages in the data, we assume an average of 160 work hours per month. The length of a period is of importance because it puts an upper bound on the job offer probability,  $p(\theta)$ , and the minimum duration of an unemployment spell. A maximum of one offer per month is well supported by the data<sup>26</sup>, but the second constraint is likely to be binding<sup>27</sup>.

We calculate the employment to unemployment and unemployment to employment flow rates of the US non-institutionalized population from CPS data for the years 1994-1995 following Fallick and Fleischman (2004) for reasons discussed in Section 2.2. The exogenous job destruction rate,  $\omega$ , is set such that the total job destruction rate, the sum of endogenous and exogenous movements from employment to unemployment, is 1.43 percent per month. We attach to  $\xi$  a value that implies a monthly job finding rate of 0.271.

We use SIPP data to calibrate the parameters guiding on the job search. Information on job to job movements and wage changes identify  $\lambda$  and  $\lambda_d$ . We adjust  $\lambda$  to imply that 2.45 percent of workers switch employers every period. As discussed previously, in order to correctly model the efficiency of on the job search, it is important to know how many of these movements result in wage improvements. Our identifying assumption for separating voluntary and involuntary movements is that voluntary movements always result in expected wage increases. In our data set, 34 percent of all job to job movements result in a nominal wage loss. Conditional on processes for measurement error and idiosyncratic productivity innovations, which are described below, we can use the rate of forced movements,  $\lambda_d$  to match this statistic, which implies  $\lambda_d = 0.152$ .

There is a large debate on the appropriate values for  $\alpha$ ,  $\iota$  and  $\theta$  because of their importance for business-cycle fluctuations. Fortunately, in our steady state analysis, these parameters do not affect our results because they only affect the job finding

<sup>&</sup>lt;sup>26</sup>Holzer (1988) reports based on NLSY data that 34 percent of the unemployed received at least one job offer and 12 percent received more than one offer per month.

<sup>&</sup>lt;sup>27</sup>See Clark and Summers (1979). Our model cannot by construction match the high observed outflow rates within the first month. However, time disaggregation below one month is rather costly because our numerical algorithm uses value function iteration, which converges at a rate of  $1 - \beta$ .

rate. Changing the parameters only leads to a recalibration of  $\xi$ . Henceforth, we normalize  $\alpha = \iota = 0.5$  and use  $\varphi$  to match a labor market tightness of 0.6.

Consistent with findings from Siegel (2002) for average bond and stock returns, we set  $\beta$  to imply a yearly interest rate of 4 percent. Next, we consider the flow value of unemployment. We set the replacement rate  $rr_b$  to 25 percent. As argued in Hall and Milgrom (2008), this provides a parsimonious description of the system. The maximum UI benefit payment is set to 1168 \$, which is the average across US states. The parameter determining the value of leisure,  $rr_z$ , is set to 15 percent which yields a total replacement rate of 40 percent when entering into unemployment as in Shimer (2005). Last, we fix the probability for an unemployed worker to lose his benefit entitlement such that average entitlement is six months, which is the standard length in the US system outside of economic crisis.

In the presence of tenure and selection effects, it would be very hard, and potentially produce unreliable results, to estimate mean experience gains from our data set. We therefore use life-cycle transition rates and drift terms in productivity during employment to match statistics found by Dustmann and Meghir (2005). Productivity is assumed to grow at an annual rate of 8 percent when employed during the first life-cycle state and at a rate of 1 percent during the second. The transition probability between life-cycle states, (p), is set such that agents spend on average 24 months in the first state and 480 in the second. In line with Ljungqvist and Sargent (2008), who assume that skill depreciation is twice the rate of skill accumulation, e.g., 2%. The subsistence level of log productivity, *pmin*, is set to -2, which is never binding.

# 2.4.2 Distributional Parameters

We now describe the way we calibrate the variance of log firm productivity,  $\sigma_F^2$ , idiosyncratic productivity shocks,  $\sigma_{\epsilon}^2$ , and initial worker productivities,  $\sigma_N^2$ . None of the statistics is directly observable in the data because of measurement error. Additionally, agents endogenously select themselves into and out of employment and into employment with firms of specific productivity levels in response to idiosyncratic productivity developments. Instead, we identify the moments from within our model.

Variable	Target
$\beta = 0.9967$	4 percent annual interest rate
$\varphi = 1427$	heta=0.6
$\alpha = \iota = 0.5$	Normalization
$rr_b = 0.25$	$\frac{b_{mean}}{w_{mean}} = 0.25$
$rr_{Z} = 0.15$	$\frac{Z_{mean}}{w_{mean}} = 0.15$
$b_{max}$	1168\$
$\lambda_l = 0.16$	6 month benefit duration
$\omega = 0.01$	EU flow rate of $0.0143$
$\xi = 0.48$	UE flow rate of 0.271
$\lambda = 0.0845$	JTJ flow rate of $0.0245$
$\lambda_d = 0.152$	34 percent of JTJ movements lead to wage cuts
$\nu(1) = 0.0067$	8 percent productivity growth
$\nu(2) = 0.00083$	1 percent productivity growth
$p_1 = 0.04$	2 years in 1st life-cycle
$p_2 = 0.002$	40 years in 2nd life-cycle
$\delta = 0.00167$	2 percent skill depreciation
pmin = -2	Normalization
$\sigma_F = 0.342$	Equation $(2.3) = 0.055$
$\sigma_{\epsilon} = 0.018$	Life-cycle wage profile
$\sigma_N = 0.28$	Life-cycle wage profile
$\sigma_{\iota} = 0.0236$	Estimation
$\mu_N = 6.88$	Mean monthly wage 2070\$

Table 2.3: Calibration

Notes: The left column states the calibrated variable with its value and the second states the relevant moment. EU stands for employment to unemployment, UE for unemployment to employment, and JTJ for job to job.

## Measuring Firm Heterogeneity

For identification of the firm productivity distribution, we require only a small set of assumptions. Other than specifying a general additive specification for log wages and assuming firm productivities to be log normally distributed, our identification only relies on the assumption that measurement error for job switchers is not more severe than for job stayers<sup>28</sup>.

<sup>&</sup>lt;sup>28</sup>As discussed in the appendix, we are excluding from our sample those individuals who are holding multiple jobs after a transition to rule out this source of additional reporting error.

In our SIPP data, we assume that wages are generated by

$$ln(w_{i,t}) = \alpha_0 + \alpha_1 d_t + \alpha_2 Z_i + \beta_2 \Gamma_i + e_{i,t}, \qquad (2.2)$$

where  $d_t$  captures aggregate states, such as TFP, and  $Z_i$  is a vector of idiosyncratic components. We split the unobservable,  $e_{i,t}$ , into two parts:

$$e_{i,t} = r_{i,t} + \beta_1 A_{i,t}.$$

As in the model,  $A_{i,t}$  is assumed to follow a random walk with drift and innovations  $\epsilon_{i,t}$ , and  $r_{i,t}$  captures measurement error. For our present purpose, we have to make no further assumptions regarding the distributional properties of measurement error.

First-differencing eliminates the idiosyncratic wage components<sup>29</sup>. As mentioned above, we only observe a self-selected subset of the realizations of  $\Gamma$  and  $\epsilon$  as agents can quit into unemployment after negative productivity shocks and refuse wage offers. The subsets of observed realizations,  $\Gamma^{obs}$  and  $\epsilon^{obs}$ , are themselves random variables, which follow distributions of unknown functional forms.

We can now define observed wage growth when a job to job transition takes place:

$$\Delta ln(w_{i,t}^b) = \nu + \kappa_t + \beta_2 [\Gamma_i^{obs} - \Gamma_{i-1}^{obs}] + \beta_1 \epsilon_{i,t}^{obs} + \Delta r_{i,t},$$

and when no such transition takes place:

$$\Delta ln(w_{i,t}^w) = \nu + \kappa_t + \beta_1 \epsilon_{i,t}^{obs} + \Delta r_{i,t},$$

where  $\kappa_t = \alpha_1 (d_t - d_{t-1})$ . After regressing out constant and time dummies, we obtain the residual excess variance of job movers relative to job stayers<sup>30</sup>:

$$Var\left[\Delta ln(\hat{w}_{i,t}^{b})\right] - Var\left[\Delta ln(\hat{w}_{i,t}^{w})\right]$$
$$= \beta_{2}^{2} Var\left[\Gamma_{i}^{obs} - \Gamma_{i,-1}^{obs}\right] + 2\beta_{1}\beta_{2} Cov\left[\epsilon_{i,t}^{obs}(\Gamma_{i}^{obs} - \Gamma_{i,-1}^{obs})\right], \quad (2.3)$$

<sup>&</sup>lt;sup>29</sup>In our estimations, we also checked for idiosyncratic differences in wage growth by including gender, race, industry and regional dummies. These variables were neither individually nor jointly significant.

 $<sup>^{30}</sup>$ We delete the top and bottom 0.75% of the wage growth observations to get rid of reporting error.

where we invoke the assumption that measurement error is uncorrelated with the event of job switching.

In Section 2.3.4, we demonstrate that the wage schedule in our model can be accurately approximated by a log-linear approximation analogous to equation (2.2) given by equation (2.1). Equation (2.3) therefore also approximately holds in our model and we can use it as a calibration target for  $\sigma_F^2$ . All endogenous sorting that causes the observed productivity distribution in the data to differ from the true one is also present in our model.

#### Calibrating Idiosyncratic Productivity Uncertainty and Measurement Error

In principle, we could derive a moment condition similar to the one above in order to identify idiosyncratic productivity uncertainty (see Meghir and Pistaferri (2004) for more details). Whereas the identification of firm productivity only required two consecutive wage observations, the maximum spell length of 36 months in the SIPP now becomes more of on an issue. Therefore, we opt for a different calibration strategy. We first regress out idiosyncratic wage components absent from our model (gender, race, marriage and disability)<sup>31</sup>. We then choose  $\sigma_N^2$  to match initial wage inequality and  $\sigma_{\epsilon}^2$  to match the increase in wage inequality over the life-cycle.

Lastly, an important part of wage fluctuations may actually be the result of measurement error. We therefore explicitly model it in our baseline calibration. At this point, we need to make further assumptions regarding its statistical properties. Following Meghir and Pistaferri (2004), we postulate a MA(q) process (i.e.,  $r_{i,t} = \Theta(q)\iota_{i,t} = \iota_{i,t} - \sum_{j=1}^{q} \theta_j \iota_{i,t-j}$ ). Given that studies on annual wage growth typically assume iid measurement error, we fix q at 12. Assuming  $\mathbb{E}(\epsilon_{i,t}^{obs} \epsilon_{i,t-j}^{obs}) = 0 \quad \forall j \neq 0$ , the parameters  $\Theta(12)$  and  $\sigma_{\iota}$  can be obtained using Maximum Likelihood estimation and Kalman filtering<sup>32</sup>. Appendix 2.B.3 supplies further detail on the procedure.

<sup>&</sup>lt;sup>31</sup>Wages in our model are a function of productivities. We purify our data of these effects, which are well-known drivers of wages because we think they are inadequately represented by our model set-up. Gender and race biases are likely the result more of discrimination than a representation of productivity. Marriage stands in for a joint labor supply decision absent from our model as we do not model joint intra-household decisions. Disability likely does represent productivity, but not in a way adequately captured by our model.

 $<sup>^{32}\</sup>mathrm{We}$  thank Johannes Pfeifer for providing us with the Kalman filtering routine.

# 2.5 Results

We now present the main results of this chapter. Section 2.5.1 demonstrates that the frictional wage dispersion present in our model is of the size estimated in the data. We then proceed to investigate the importance of the different channels in expanding the range of job offers acceptable to the workers. We demonstrate that our process for general human capital and the possibility to search on the job are both crucial ingredients in allowing the model to match the data. Shutting down any of the three channels: skill acquisition in employment, skill depreciation in unemployment or on the job search significantly shrinks the set of acceptable job offers and consequently frictional wage dispersion. Limited UI duration is only of second order importance. Given our calibration target, a job-ladder model with initial worker heterogeneity turns out to be a rival specification capable of producing empirically observed residual dispersion. It largely overstates; however, the gains of on the job search by neglecting the one third of job to job movements resulting in wage losses in the data.

In Section 2.5.2, we demonstrate that our model also produces a good representation of the empirical wage distribution. After discussing the structurally inferred parameters of the wage offer distribution and of idiosyncratic wage uncertainty, we determine the relative contributions of firm dispersion, productivity development and the distribution of workers over firms to overall wage dispersion. Our results attribute about 17.5 percent of wage inequality to the presence of the search friction. The on the job search model, neglecting forced movements, yields a much larger contribution of over 40 percent.

# 2.5.1 Frictional Wage Dispersion and its Causes

# Frictional Wage Dispersion in the Baseline Specification

In our model, workers of identical idiosyncratic characteristics may be earning different wages because they are employed with firms of different productivities, or because of measurement error. When measuring frictional wages, the econometrician would observe

$$ln(\tilde{w}_{i,t}) = \beta_3 \Gamma_{i,t} + r_{i,t},$$

where we again apply our approximated wage schedule from equation (2.1). All our statistics regarding frictional wages in the model are based on this expression.

Table 2.4 compares the *frictional* wage dispersion in our model to the amount of *residual* wage dispersion present in the data. Our model successfully reproduces the amount of residual inequality. In the baseline specification, the mean residual wage paid is 2.57 times the smallest observation. This is comparable, though slightly larger, to the Mm-ratio of 2.18 when taking the first percentile in the data to be the minimum wage. When looking at higher percentiles, model and data line up closely as well. Also the other statistics look favorable: the Gini coefficient matches up almost exactly and our model explains 84 percent of the variance of residual log wages in the data. When comparing different model specifications in our subsequent analysis, we only report changes in the Mm-ratio. This increases clarity of presentation and facilitates comparison with other studies, foremost HKV themselves, that report this summary statistic. When using percentiles as minimum wage, is also has the advantage of being robust to classical measurement error.

]	Mean-M	Iin Ratic	)	Gi	ni	$Var(\log$	$g(\tilde{w}_{it}))$
		Model	Data	Model	Data	Model	Data
Pctl.	$Min \\ 1^{st} \\ 5^{th} \\ 10^{th}$	$2.57 \\ 1.45 \\ 1.38 \\ 1.31$	$2.18 \\ 1.48 \\ 1.31$	0.0894	0.091	0.026	0.031

Table 2.4: Frictional Wage Dispersion

Notes: The table compares frictional wage dispersion generated by the baseline specification to residual wage dispersion in the 1993 SIPP. We report the Mm-ratio using the 1st, 5th, and 10th percentile as possible minimum wages. For comparability, we report the corresponding statistic in the data as well.

## Sources of Frictional Inequality

We now analyze how the details of our model specification interact with the range of job offers acceptable to the worker given the distribution of firm productivities in our baseline. Therefore, we resolve a number of restricted versions, each time excluding one of the main channels and recalibrating to the flow rates and the residual wage profile. In each calibration, the unemployed sample at most one job offer per month, which imposes an upper bound on the flow value of unemployment for one of the experiments. Table 2.5 reports the resulting frictional wage dispersion and replacement rates.

It turns out that the main driver behind our results is the interaction of our process for general human capital with the possibility to search on the job. Specifications **A** to **C** turn off each of those channels one after another. When setting expected experience gains during employment to zero, the Mm-ratio falls to only 1.43. As potential experience gains are equal in all firms, being employed at all becomes much more important than in which firm specifically. The same argument applies to the effects of skill depreciation, specification **B**, even though its effects are less pronounced. When setting  $\delta$  to zero, the Mm-ratio drops to 1.98, which is still sizable, but a substantial decrease from 2.57. When denying workers the possibility to search on the job, the Mm-ratio plummets to 1.19 and no positive replacement rates are able to match observed flow rates. Finally, as demonstrated in specification **D**, the limited payout duration of unemployment benefits is only of second order importance to the empirical success of our model.

	Specification	Mm-Ratio	$rr_b + rr_z$
	Baseline	2.57	0.4
٨	No learning on the job		
A	$(\nu(\phi) = 0)$	1.43	0.4
р	No skill depreciation		
Б	$(\delta = 0)$	1.98	0.4
C	No search on the job		
U	$\lambda = 0$	1.19	0
п	Infinite UI		
D	$(\lambda_l = 0)$	2.35	0.4

Table 2.5: Contributions to Frictional Wage Dispersion

Notes: The table displays the mean-min ratio and the replacement rate for four different model specifications that differ from our baseline model by some parameter restriction.

We have stressed previously the importance of accounting for job to job transitions resulting in wage losses when inferring search efficiencies from job to job flow rates. In the absence of forced movements, we could generate any value for the Mm-ratio as workers would accept even negative wages. There are two reasons for this: First, when a job offer is a forced one, moving is almost always preferred to quitting into unemployment. Second, forced job movements decrease the rate at which agents climb up the productivity ladder of firms, making future job offers more likely to be better than today's offer. In consequence, search on the job is more efficient in a model not featuring forced job movements. The value of employment increases relative to the value of unemployment, which increases frictional wage dispersion.

## A competing explanation

Our previous analysis identifies on the job search as an important channel in understanding frictional wage dispersion. Indeed, one might ask how far a more "standard" on the job search specification would go on its own in explaining the data when calibrating it to observed flow rates only and ignoring forced movements. We discuss such an experiment in this section. The combination of a job-ladder model with heterogeneous initial worker productivities and general human capital uncertainty, but no trend growth on and off the job, calibrated to our targets by itself yields an Mm-ratio of 2.83 with solidly positive replacement rates<sup>33</sup>. Given our previous estimates, this appears to be an empirically successful alternative explanation.

As discussed in Section 2.2.3; however, these model types largely overstate the efficiency of on the job search by ignoring the frequent occurrence of job to job transitions resulting in wage losses. We therefore take the ability to realistically account for wage dynamics upon job to job movement as a means of discriminating between these rival model specifications. Table 2.6 highlights the differences.

In the data, job to job movements result in wage gains of two percent on average. Conditional on suffering a wage loss upon movement, workers lose 22 percent of

<sup>&</sup>lt;sup>33</sup>Hornstein et al. (2011) also consider a job-ladder model and obtain Mm-ratios between 1.16 and 1.27 for a replacement rate of 0.4. However, the presence of individual productivity heterogeneity in our model implies heterogeneous reservation wages. In consequence, the homogeneous job offer arrival rate and the unemployment to employment flow rate are no longer identical. Moreover, the stochastic productivity process implies a higher option value of employment because workers can quit at any time into unemployment. These differences account in our case for the much larger frictional wage dispersion.

Specification	Avg. gain	Avg. loss
Data	0.0203	-0.22
Baseline	0.069	-0.21
Job-Ladder model $\nu(1) = \nu(2) = 0 = \delta \ \lambda_l = 0$	0.26	-0.07

Table 2.6: Wage Changes from Job to Job Movements

Notes: The table compares the model baseline specification with a pure on the job search specification on their implications for job to job transitions. Statistics are the resulting average wage gain upon job movement and the average wage loss conditional on observing a loss. *Data* refers to computation from the 1993 SIPP for nominal wages.

their previous wages. Our baseline specification fares quite well in reproducing these statistics. Wage gains are too high, but the order of magnitude is comparable. The model does well in reproducing the large conditional wage losses. In the job-ladder model, average wage gains of 26 percent are much too large compared to the data. Since workers in this model only transit to more productive jobs, the wage losses are only observed as result of a negative productivity shock or of measurement error. A conditional seven percent average wage loss clearly fails in this respect. We come back to this specification in Section 2.5.2 when discussing structural inference.

# 2.5.2 Wage Dispersion

### Overall Wage Dispersion in the Model and in the Data

Confident of having established the main channels shaping frictional wage inequality, we now use our calibrated baseline for structural inference regarding the sources of wage inequality. So far, the literature has suggested a wide range of estimates regarding the relative importance of differing initial abilities,  $\sigma_N$ , idiosyncratic productivity uncertainty,  $\sigma_{\epsilon}$ , the search friction,  $\sigma_F$ , and a sorting term that we introduce below.

In order to assure that our model can be used to make such statements, we first have to evaluate whether it reproduces a wage distribution comparable to what we see in the data. As discussed previously, there are a few well-known wage determinants

in the data that our model is not designed to include. In what follows; therefore, we first regress log wages in our data on a constant and dummies for disability, gender, marriage status and race. These factors account for 10 percent of log wage variation. We compare our model's wage distribution to the resulting distribution<sup>34</sup>. Figure III plots the kernel estimator of the aggregate density function of wages against its model counterpart after transforming the data back to levels<sup>35</sup>. It features the characteristic right skew of the observed wage distribution in the data. Figure IV displays the theoretical and empirical Lorenz curves of wages. Our model economy exhibits slightly more wage inequality, but the difference is negligible. Overall, the results reassure us that our model economy picks up the key moments of wage inequality present in the data.



Notes: Figure III plots the PDF of workers over wages in the model against the data. In both cases, log wages have been demeaned before transforming them back to levels and a kernel smoother has been applied. Figure IV compares Lorenz curves. The straight line designates 1993 SIPP data and the dashed line refers to the model.

## Sources of Wage Inequality

We start by discussing the wage offer distribution and the distribution of idiosyncratic wage risk. The first line of Table 2.7 displayes the results. Our estimate for  $\sqrt{\beta_1}\sigma_{\epsilon}$  implies an annual standard deviation for the permanent component of wages

<sup>&</sup>lt;sup>34</sup>This should of course not be confused with the residual distribution we used as a measure for frictional wage dispersion.

 $<sup>^{35}</sup>$ We truncate our observed wage data at the bottom and top 1% wage observations to delete outliers. We do the same adjustment to our simulated data in this section.

Specification	$\sqrt{eta_3}\sigma_F$	$\sqrt{\beta_1}\sigma_\epsilon$
Baseline	0.29	0.0180
Job-Ladder model $ u(1) = \nu(2) = 0 = \delta \ \lambda_l = 0 $	0.44	0.0156

Table 2.7: Wage Offer Distribution and Idiosyncratic Risk

Notes: The table displays the standard deviations of the wage offer distribution and of the idiosyncratic wage shock. The first line refers to the baseline specification and the second one to a calibration of on the job search only.

of 0.0624. To put our results into perspective, Low et al. (2010), also using the 1993 SIPP, estimate a standard deviation for the wage offer distribution of 0.23 and of 0.103 for annual productivity innovations. Our estimates attribute more ex-ante wage uncertainty to the firm component as opposed to idiosyncratic productivity uncertainty.

To evaluate the contributions of idiosyncratic productivity innovations, firm differences, and worker selection into matches, we simulate a panel of 15000 workers' histories for 43 years. Consider the following variance decomposition based on a slightly modified version of (2.1), which we estimate separately for each age group in our simulated data:

$$Var(ln(w_i)) = \beta_1^2 Var(A_i) + \beta_2^2 Var(\Gamma_i) + 2\beta_1\beta_2 Cov(A_i, \Gamma_i) + Var(r_i)$$

The left panel of Figure V displays the results. Measurement error does not appear to be very important. Sorting of workers over firm productivities has a mild negative effect. For young workers, firm heterogeneity explains more than forty percent of the overall log wage variance, but that number quickly drops as workers' employment histories become more diverse. Our model identifies worker heterogeneity as the dominant factor in explaining variations in wages and this effect is increasing in age<sup>36</sup>. In a population weighted average, frictional wage dispersion accounts for 19.45

<sup>&</sup>lt;sup>36</sup>Note that this finding is not in contrast to the fact that a Mincer wage equation with worker fixed effects explains only little variation in wages. Individual productivity is only partially correlated with initial productivity and all changes in productivity are time varying unobservables to the econometrician. The typical worker observables included in the Mincer wage equation can at

percent of wage inequality within our model. Given that we eliminated 10 percent of wage variation through our fixed effect regression, this implies frictional inequality to account for 17.5 percent of overall wage inequality present in our data.

## Neglecting Wage Cuts in On the Job Search Models

The literature so far has produced a wide range of estimates regarding the contribution of the search friction to overall inequality. Estimates range from 6 percent in Hagedorn and Manovskii (2010) to as high as 100 percent in Postel-Vinay and Robin (2002) for low skilled workers. Our estimate comes out in the lower part of that spectrum. Using our model, we can show why on the job search models, like the one estimated in Postel-Vinay and Robin (2002), are likely to produce higher estimates for the contribution of frictional wage dispersion<sup>37</sup>. These models have so far attributed all employment to employment transitions to upwards movements on

Figure V: Contribution of Search Frictions to Overall Wage Dispersion Baseline vs. Job-Ladder Model



Notes: The graphs display the contribution of sorting (black area), firm effects (dark gray area), and measurement error (medium gray area) to the variance of log wages conditional on age. The left panel is from our baseline specification, the right panel results from a job-ladder model with idiosyncratic productivity risk. The residual variance is resulting from dispersion in worker productivity.

best proxy for these variations.

<sup>&</sup>lt;sup>37</sup>The comparison to the results in Hagedorn and Manovskii (2010) is less straightforward. Their approach has the advantage of not having to make distributional assumptions, whereas we have to specify log-normality for the wage offer distributions. However, they need to assume that endogenous quitting is absent and that innovations to wages are non-permanent. It is trivial to show that using their estimation technique in our setting leads to a considerable reduction in the estimated importance of search frictions.
a wage ladder, at least in expectations. As argued above, this implicit assumption overstates the efficiency of on the job search.

We perform the same structural decomposition for the job-ladder version of our model. The result can be seen in the right panel of Figure V. The cross-sectional average for the contribution of frictional wage dispersion more than doubles to about 45 percent (40 percent of wage variation in the data) with values as high as 50 percent for the youngest workers and decreasing much slower over the life-cycle. The bottom line in Table 2.7 tells a similar story. The calibrated standard deviation for the wage offer distribution increases by over 50 percent while idiosyncratic wage uncertainty drops by 15 percent. The two model versions tell rather different stories about the sources of life-time wage inequality.

### 2.6 Conclusion

Structural estimation of search models has frequently been used to circumvent the problem of finding instruments in quantifying sources of wage risk and inequality. One empirical appeal in using a search structure for estimation is its theoretical ability to rationalize the large amount of wage inequality that cannot be explained by worker observables. The search friction makes looking for the best possible wage offer costly and induces workers of identical characteristics to accept a range of different job offers.

Yet, as Hornstein et al. (2011) point out in a recent contribution, it is a builtin feature in many of the commonly used search frameworks that they can only rationalize a small portion of the empirically observed *residual inequality* as *frictional inequality* given reasonable parameter values for discount factor and replacement rate. When using them in structural estimations on wage data, the researcher is therefore bound to either obtain unreasonably low estimates for discount factor and replacement rate or to fix them a priori and attribute most of wage inequality to measurement error. In our view; however, this conclusion should be drawn from a model that theoretically could account for the observed residual inequality.

In this chapter, we therefore build a rich structural model capable of rationalizing empirically observed residual inequality as frictional while also estimating and including measurement error. We trace out the different channels influencing the worker

decision and conclude that idiosyncratic productivity development and on the job search are the driving factors behind frictional inequality. Concerning the latter, we argue that a model featuring job to job transitions needs be able to simultaneously account for wage movements upon transition. In particular, it must also address the one third of job switches which result in workers taking pay cuts. This feature allows us to discriminate between our model and a more standard job-ladder model in terms of their ability to best match the data.

It also leads us to make a second more general point regarding the structural inference of sources of wage inequality. Job to job transitions in the data are large and an obvious source of wage mobility. Yet, many on the job search models make the implicit assumptions that outside offers on the job are only accepted when they are associated with expected wage improvements. In order to rationalize the size of worker flows, these models therefore end up with high estimated search efficiency. When simultaneously inferring the wage offer distribution from wage volatility for job switchers, these models are bound to exaggerate the importance of the search friction in generating overall inequality. We find search-related inequality to be responsible for 17.5 percent of overall inequality. When inferring the same number from a job-ladder model neglecting that job to job transitions frequently result in wage losses, it doubles to more than 40 percent. This finding explains some of the higher estimates in the literature regarding the importance of search frictions for wage inequality.

# **Appendix to Chapter 2**

# 2.A Further Model Details

### 2.A.1 The Value of a Vacancy

Here, we supply the calculation of the value of a vacancy, which for reasons of parsimony we excluded from the main text. To evaluate future profit prospects and acceptance probabilities, the entrepreneur needs to know the stationary distribution of the unemployed over productivity, benefit states and life cycle states. Moreover, he needs to know the distribution of workers over their productivities, life cycle states and other firms' productivities. Summarizing the productivity states in  $s = (A, \Gamma)$ , the value of posting a vacancy  $V^I$  is the expectation of firm value  $V_x^J$  over productivity and life cycle states, minus the vacancy posting costs  $\varphi$ :

$$V^{I} = -\varphi + \beta \mathbb{E}_{t} \left\{ q_{x}^{1}(s)q(\theta)V_{x}^{J}(s') + q_{x}^{2}(s)\frac{\lambda(1-\lambda_{d})}{v}V_{x}^{J}(s') + q_{x}^{3}(s)\frac{\lambda\lambda_{d}}{v}V_{x}^{J}(s') \right\},$$

where  $q_x^1, q_x^2, q_x^3$  are the probabilities that a worker accepts the respective job offer given that he is of type A and in life cycle x and the firm is of type  $\Gamma$ . These probabilities are strictly increasing in  $\Gamma$ , as a more productive firm finds it easier to attract workers. We set the continuation value of a vacancy to zero, which is true in equilibrium, because of free entry into the market and our assumption that  $\Gamma$  is redrawn after each contact.

### 2.A.2 Equilibrium Definition

A stationary equilibrium consists of

- Value functions for the employed, unemployed and the firm value.
- Free entry drives profits for newly posted vacancies to zero:  $V^{I} = 0$ .
- Wages solve

$$\max_{w_t} : \{ \alpha log(V_x^E - V_{x,u_1}^U) + (1 - \alpha) log(V_x^J) \},\$$

where  $\alpha$  is the bargaining power of workers.

- A policy function that is consistent with the value functions and that maps worker productivity, firm productivity, benefit entitlement, and the life cycle state into a decision, whether a match is formed or not.
- Stationary distributions of the employed and unemployed over worker productivities, employment states, life cycle states, benefit entitlement states and firm productivities.

### 2.A.3 Model Timing

- 1. The employed workers negotiate a wage with their firm.
- 2. Production takes place.
- 3. Some unemployed transit from  $u_1$  to  $u_2$ .
- 4. The employed and unemployed experience productivity transitions according to their laws of motion.
- 5. Life cycle transitions take place. Agents die and are replaced.
- 6. Exogenous job destruction occurs. Agents becoming unemployed cannot search for employment within this period.
- 7. On the job offers realize.
- 8. Employed agents decide whether to quit, and the unemployed with job offers decide whether to accept the job.

# 2.B More on the Empirics of On the Job Search

### 2.B.1 Measuring Job to Job Flows

In order to assess the efficiency of search on the job, it is crucial to accurately identify job to job transitions in the data. One of the biggest advantages in working with SIPP data is that workers are asked to report an employment status for each week of the reporting period separately. While a higher degree of time aggregation

JTJ1	JTJ2	JTJ3	JTJ4	CPS
1.87	1.68	2.45	1.11	2.82

Table 2.8: Different Definitions of JTJ Flow Rates

Notes: The table shows percentage probabilities for job to job transitions based on SIPP data from end of 1992 to 1995. For reference we also quote monthly averages from Fallick and Fleischman (2004) for the years 1994-1995. We differentiate between four different measures of job to job transitions: JTJ1 identifies a job to job transition when a worker is employed at a different firm between two consecutive months. JTJ2 identifies a job to job transition when the worker's 2 digit occupation code changed between two consecutive months.  $JTJ3 = JTJ1 \cup JTJ2$ .  $JTJ4 = JTJ1 \cap JTJ2$ .

may mask intermittent unemployment spells, we can identify any unemployment spell lasting longer than one workweek.

In a given month, we count as employed someone who reports holding a job for the entire month. This definition includes paid as well as unpaid absences as result of vacations, illnesses or labor disputes. It does exclude; however, those who report having been on layoff for at least a week. There is no standard definition for job to job movements in empirical work. We therefore experiment with several different definitions. Our first measure is analogous to the definition in Fallick and Fleischman (2004) and equates job to job transitions with firm changes. We use a monthly employer identifier based on company names created by Stinson (2003). We refer to this definition by JTJ1. Given that a firm is a match in our model and given that employees may transit between jobs within a given firm, we find it useful to somewhat broaden the concept beyond employer id changes. For JTJ2 we therefore follow Moscarini and Thomsson (2007) in identifying job to job movements by changes in the two digit occupational code. Moreover, we define  $JTJ3 = JTJ1 \cup JTJ2$  and  $JTJ4 = JTJ1 \cap JTJ2$ .

Table 2.8 lists job to job flow rates based on the different definitions. For comparison, we also report averages from monthly estimates for the years 1994 and 1995 taken from Fallick and Fleischman (2004), who use CPS data. Identifying job to job movements by employer changes or changes in the occupational code alone yields roughly comparable flow sizes. However, only our broadest definition of job to job employment transitions comes close to the magnitude found using CPS. In order to ensure comparability of our results with studies based on CPS data and following

the arguments made above, we calibrate our model baseline specification on the 2.45 percent based on definition JTJ3.

### 2.B.2 Wages and On the Job Search

We argue in this chapter that the magnitude of job to job flows in itself is insufficient to evaluate the efficiency of on the job search. Instead, the question is how many of these job changes actually yield higher wages for the worker. In this section, we extend the analysis of the main text. We demonstrate that the results are mostly unchanged when looking at different definitions of job to job transitions and different data stratifications. Moreover, we compute the statistics in question for real wage changes.

We obtain real wages by deflating nominal wages with the CPI. For the present purpose and all subsequent exercises, we drop any person/month observation for which we cannot determine an hourly wage. In addition, we drop observations without industry identifier and job to job transitions which result in the individual holding more than one job after transiting<sup>38</sup>.

### Wage Gains from Employment Changes

First, we consider the mean change in log wages that results from a job to job transition. Our results depend somewhat on whether we consider nominal or real wage changes. Of course, the worker should only care about real wages in making his decision. Meanwhile, an argument can be made that in the presence of some wage rigidity, the worker expects a real wage loss on his current job as well and therefore compares nominal wages. Table 2.9 shows mean nominal and real wage gains for our different definitions of job to job movements.

Wage gains after a job to job transition average only to about two percent. As shown in Table 2.9, this is because roughly thirty-four percent of these transitions actually yield nominal wage losses. The figure increases to about fifty-two percent when considering real wages. Wage losses are not just frequent, they are also sizable. Conditional upon taking a cut after a job to job transition, losses average to twentythree percent for nominal and seventeen percent for real wages. Reassuringly, these

<sup>&</sup>lt;sup>38</sup>An individual working two jobs simultaneously may have trouble correctly attributing hours worked to the different jobs. This could potentially add noise to the data.

		Nominal			Real	
	Mean	Share loss	Mean loss	Mean	Share loss	Mean loss
JTJ1	0.0224	0.3444	-0.2362	0.0199	0.5386	-0.1534
JTJ2	0.0194	0.3738	-0.2343	0.0171	0.5147	-0.1725
JTJ3	0.0203	0.3390	-0.2200	0.0179	0.5386	-0.1409
JTJ4	0.0224	0.4046	-0.2660	0.0202	0.5	-0.2174

Table 2.9: Aggregate Changes in Wages after Job to Job Transitions

Notes: The table reports wage changes resulting from a job to job transition for real and nominal wages, respectively. The statistics under consideration are: The average change in log wages, the share of workers incurring a wage loss, and the average change in log wages, given that the observed change is a loss. We differentiate between four different measures of job to job transitions: JTJ1 identifies a job to job transition when a worker is employed at a different firm between two consecutive months. JTJ2 identifies a job to job transition when the worker's 2 digit occupation code changed between two consecutive months.  $JTJ3 = JTJ1 \cup JTJ2$ .

figures are largely invariant to which definition we use. From now on, all statistics reported are therefore based on JTJ3 only.

We also stratify our sample by different observable characteristics to show that the phenomenon we just described is not driven by a specific population sub-group, but is a key characteristic of the entire labor market. The results are summarized in Table 2.10.

We first split our sample into different years. The willingness of workers to accept a wage reduction upon transition might depend on the aggregate state of the economy. In the years 1993 to 1995, the time of our sample, the US economy was gradually moving out of the post-Gulf War I recession and unemployment was steadily falling throughout the sample period. Still, as indicated in the first panel of Table 2.10, there is now discernible time trend in the data. By 1995, unemployment had reached a historic low, but workers still accepted a wage cut when making a job to job transition about one third of the time.

Women are known to have less stable work relationships than men and might therefore be responsible for an overproportional share of loss making job to job transitions. Nonetheless, in the data both sexes have an equal probability of experiencing a wage cut after moving. The same holds for stratifications by age groups. Young workers have a looser attachment to the labor market and may initially experiment with different career paths or search for jobs with higher non-monetary benefits. But none

		Nominal		Real		
Stratify by:		Share loss	Obs.	Share loss	Obs.	
Year						
	1993	0.3301	4649	0.5468	4650	
	1994	0.3299	3892	0.5267	3889	
	1995	0.3638	2959	0.5431	2959	
Sex						
	Male	0.3367	6351	0.5335	6347	
	Female	0.3421	5176	0.5449	5178	
Age						
	23-30	0.3483	3659	0.5183	3658	
	31-50	0.3368	6470	0.5419	6468	
	51-65	0.3230	1398	0.5819	1399	
Industry						
	Agriculture	0.3999	119	0.5719	119	
	Manufacturing	0.3173	4274	0.5241	4271	
	Trade	0.3550	3083	0.5434	3080	
	Services	0.3687	1287	0.5971	1287	
	Government	0.3379	2767	0.5259	2768	
Income						
	Lowest $25\%$	0.2316	3125	0.4191	2958	
	25-75%	0.3514	5567	0.5541	5716	
	Top $25\%$	0.4428	2835	0.6409	2815	
Tenure						
	Less than 6 months	0.352	4932	0.526	4932	
	6-12  months	0.304	170	0.523	170	
	1-3 years	0.296	410	0.537	410	
	3-10 years	0.296	694	0.577	694	
	10 and more years	0.334	640	0.605	640	

Table 2.10:	Share	of	Wage	Cuts	After	Job	$\operatorname{to}$	Job	Transitions	$\mathrm{in}$	Different
	Subsa	$^{\mathrm{mp}}$	les								

Notes: The table shows the share of workers incurring a wage cut after a job to job transition for a number of different sample stratifications. The column "Obs." reports the number of observed job to job transitions in the specific sub sample. Due to different outlier identifications, this number does not need to match exactly between the cases of nominal and real wages.

of these phenomena cause the youngest age group to experience markedly more job to job transitions with wage losses.

### 2.B More on the Empirics of On the Job Search

We try out three more relevant data subsets. The first concerns the industry the worker moves to. Some industries may offer substantial non-monetary benefits compared to others. Of course, this exercise is not only subject to selection issues, it is also well-known that wages show industry differentials. In consequence, we should be expecting to identify industry pairs where wages fall in expectations when moving from one industry to the other. In order to have sufficiently many observations for all subsamples, we group industries into four broad sectors using their three digit industry codes: Agriculture, Manufacturing, Trade, Private Services, and Government. There are notable differences between sectors. Still, the share of workers incurring a wage cut after a job to job transition never falls below 31.73 percent.

We also stratify our sample by earnings. We split the main sample into its lowest and highest quartile and the observations in-between. Again, we do not expect the outcome to be random, because high wage earners are more likely to incur a loss when they are forced to look for alternative employment. In a simple employment lottery, where all workers sample wages from the same random distribution, the probability of incurring a wage loss is an increasing function of the current wage. Nonetheless, low wage earners are far from insulated to wage losses when switching jobs and even in the lowest quartile, 23 percent of all job to job transitions result in nominal wage losses.

Finally, we split the sample by tenure at the previous job. High tenured workers are likely to have been in a good match previously. Therefore, observing a job to job transition conditional on high tenure may suggest that the movement must have been a forced one. Our data provides little support for this view. Workers with more than 10 years of tenure have indeed somewhat higher probabilities to incur a wage cut upon a job to job transition, but the differences are negligible. A possible explanation is that the high match surplus makes it unlikely for the firm to exit the market and hence the probability of a forced job movement decreases. Besides these considerations, there are two major measurement issues with tenure in our data set. First, the SIPP asks respondents about the starting date with a specific employer. Hence, prior to the start of the observation period, tenure is solely employer and not occupation specific. Second, of those employed at their first observation month, almost 10% report to have zero tenure implying unreasonably high turnover rates at a monthly frequency.

### **Alternative Explanations**

Postel-Vinay and Robin (2002) and Cahuc et al. (2006) propose an alternative explanation for those wage losses. They lay out a model where wages can only be renegotiated by mutual agreement, and the firm has all the bargaining power, in Postel-Vinay and Robin (2002), or part of the bargaining power, in Cahuc et al. (2006). Wage raises on the job occur as a result of counter-offers to bids by other firms. They demonstrate that in such a framework workers will accept wage cuts upon job to job transitions, if the option value of working at the other firm is sufficiently high. Workers only move to firms more productive than their current employer and very productive firms offer the potential of large future wage gains.

A testable implication of these types of models is that expected future wage growth with the new employer should be an increasing function of the wage cut accepted. The left panel of Figure VI plots cumulative wage growth with the new job against the initial wage change for our population of job to job transitions. There is no relationship between the initial wage change and consecutive wage growth. In the right panel, we restrict the sample to agents whom we observe for at least two years with their new job (This time, the initial wage cut is included in the sum). We again find no evidence, that agents that accepted an initial wage cut are compensated by



Figure VI: Initial Wages Change and Subsequent Wage Growth

Notes: The left panel plots cumulative wage growth in the months after a job to job movement against the initial wage change, excluding the latter from the calculation. The figure was generated using all observed job to job transitions. In the right panel, we only include job to job transitions where the worker was subsequently observed for at least 24 months. The cumulation of wage growth now includes the initial change upon transition.

steeper wage profiles on the new  $job^{39}$ .

### 2.B.3 Estimating the Measurement Error Process

To ensure that it is not measurement error that drives the fraction of agents accepting wage cuts upon job to job transitions and in order to quantify its contribution to observed (frictional) wage dispersion, we simulate our model with measurement error. Recall that wages in the data are given by (2.2). To simulate our model with the same measurement error process, we require estimates of  $\Theta(12)$  and  $\sigma_{\iota}$ . We obtain these by maximizing the sum of individual likelihoods of within job wage growth in the data. More specifically, we treat  $\iota_{i,t}$  as unobserved state and obtain the individual likelihood for wage growth of individual i from the following state space representation:

$$g_{it} = \begin{bmatrix} 1 \\ \theta_1 - 1 \\ \theta_2 - \theta_1 \\ \theta_3 - \theta_2 \\ \theta_4 - \theta_3 \\ \theta_5 - \theta_4 \\ \theta_6 - \theta_5 \\ \theta_7 - \theta_6 \\ \theta_8 - \theta_7 \\ \theta_9 - \theta_8 \\ \theta_{10} - \theta_9 \\ \theta_{11} - \theta_{10} \\ \theta_{12} - \theta_{11} \\ -\theta_{12} \end{bmatrix} \rho_{it} + \beta_1 \epsilon_{it}$$

<sup>&</sup>lt;sup>39</sup>A caveat in interpreting this finding should be mentioned. It is of course possible that the higher expected wage increases lie further in the future than the two years we observe. Given that Dustmann and Meghir (2005) find wage-tenure profiles to be basically flat after two years; however, we find this not very likely.

Our calibration imposes the following moment restriction:  $\beta_1^2 \sigma_{\epsilon}^2 = 0.00032$ . Table 2.11 reports our estimation results.

# 2.C Numerical Algorithm

The numerical algorithm consists of two nested loops followed by simulations. Codes are available on the authors' webpages.

- We begin the algorithm by guessing a labor market tightness  $\theta$ .
- Next, we guess the wage function and discretize the workers' log productivity by 1500 grid points. We find 15 to be a non-binding upper bound. The distribution of log firm productivities is discretized into 50 equi-likely grid points. The third dimension of the wage function is the two life cycle states.
- Given the initial guesses, we can start the inner loop, which calculates the value functions using value function iteration. Expectations regarding next period's idiosyncratic productivity are calculated using Gaussian quadrature with 10 nodes for evaluating the productivity innovations and cubic spline interpolation between productivity grid points.

Parameter	Estimate
$\sigma_\iota$	0.0236
$ heta_1$	0.066
$ heta_2$	-0.4426
$ heta_3$	0.9846
$ heta_4$	0.0779
$ heta_5$	3.5932
$ heta_6$	2.7587
$ heta_7$	1.4039
$ heta_8$	1.3519
$ heta_9$	1.2144
$ heta_{10}$	-0.1461
$ heta_{11}$	-0.0096
$ heta_{12}$	0.4869

### Table 2.11: Estimates for Measurement Error

Notes: The table shows the estimation results for the measurement error process.  $\sigma_{\iota}$  is the standard deviation of the MA(12) process and  $\theta_i$  the corresponding coefficients.

- The value functions of the workers allow us to update the wage function. The value of the firm is implied by Nash-Bargaining:  $V_x^J(s) = \frac{1-\alpha}{\alpha}(V_x^E(s) V_x^U(s))$ . For obtaining the expected value of the firm next period, we again use Gaussian quadrature and spline interpolation. We then update policy functions.
- Solving the value of the firm function for wages yields the implied wage schedule for each grid point ( $w_{computed}$ ). Wages are only determined by Nash-Bargaining in equilibrium. However, worker heterogeneity implies that in equilibrium there will be certain potential matches whose surplus is negative. In order to be able to compute meaningful values of employment at these firms we set wages equal productivity or, put differently, we set the firm value to zero. Afterwards, we update wages by  $w_{new} = \rho w_{initial} + (1 - \rho) w_{computed}$  until convergence.  $\rho$  is the updating weight and we find 0.75 to work fine at the beginning and increase it to 0.9 towards convergence.

- Upon convergence, we calculate  $\theta$  implied by the free entry condition. This requires the stationary distributions of the employed and the unemployed. We compute these by distribution function iteration, using the policy functions. For the distribution function we use a finer grid for worker productivities of 5000 grid points. Using the results, we update  $\theta$  until convergence.
- The last step are the simulations that employ the policy functions and equilibrium job offer rates. We use linear inter and extrapolation on the worker and firm productivity grid<sup>40</sup>.

<sup>&</sup>lt;sup>40</sup>We opt for linear interpolation at this step, as it considerably decreases the computational burden and does not appear to alter the results compared to spline interpolation. Also, spline extrapolation is known to be unreliable.

# Savings Behavior and Means-Tested Programs

# 3.1 Introduction

In the US, several of the major income support programs pay benefits to households conditional on their earnings and wealth being below certain thresholds. A recent public debate surrounds the welfare consequences of asset means-testing. On the one hand, asset means-testing allows for relatively high allotments for a given amount of governmental expenditures; therefore, mitigates the financial constraints of poor households<sup>1</sup>. Moreover, wealth reflects in part the households' past history of luck in the labor market. Hence, asset means-testing allows allocating the scarce resources explicitly to those households that suffered a series of poor labor market outcomes and are in need for the insurance. On the other hand, the insurance scheme provides strong incentives to the households to hold little savings leading to substantial consumption drops during periods with poor labor market outcomes. Furthermore, the benefit scheme distorts saving decisions over the life-cycle leading to a reduction in retirement savings<sup>2</sup>.

The main contribution of this chapter is to introduce the current US means-tested income support programs into the standard incomplete markets model (SIM) and evaluate the welfare consequences of the programs within the laboratory of the structural model. I argue that asset means-testing income support decreases social welfare compared to a program that pays unconditional benefits to low income

<sup>&</sup>lt;sup>1</sup>Examples for arguments along this line are Ryan (2011) and Cowen (2008).

<sup>&</sup>lt;sup>2</sup>Examples for arguments along this line are the Federal Government (2010) and The Retirement Security Project (2007).

households. Given the same amount of total governmental expenditures, an unborn is willing to pay 0.31% of lifetime consumption to be under a regime without asset means-testing. Especially households with low permanent labor market income suffer from asset means-testing.

The welfare costs result from households failing to engage in full intertemporal consumption smoothing. Means-testing induces savings of eligible households to become flat in a range of the state space where they choose the maximum asset position that still entitles them for the program. I show that this asset choice and the minimum available asset choice are the only choices that are not characterized by first order conditions. However, also choices that equate expected marginal utilities across two periods are affected by the program. More specifically, I prove that the savings function features discontinuous increases along the wealth state. Intuitively, households are forward looking and expect with positive probability that they want to participate in the program in the future; therefore, increase consumption already today to have more intertemporal consumption smoothing. As a consequence, households hold on average fewer savings than under a regime without asset means-testing. I show that a significant fraction of low income households fails to build up sufficient savings to sustain their current consumption levels during retirement. The average life-cycle consumption profile becomes concave with a discontinuous decline at retirement, a finding that is in line with the well-known retirement consumption puzzle. The failure of low income households to smooth consumption over the life-cycle is the main reason for the lower welfare under means-testing.

There is an additional, yet by an order of magnitude smaller, welfare cost implied by means-testing. The reduced saving incentives of the asset poor lead to a relatively high wealth inequality. This in turn translates into utility of consumption being more unequally distributed under means-testing. The costs of consumption inequality decrease conditional on a permanent income potential; however, between group differences become larger leading to a net increase.

The paper most closely related to my work is a contribution by Hubbard et al. (1995). They show that introducing an income floor at zero assets helps to understand why a large fraction of low income households holds little precautionary savings in the US. The income floor acts very similar to asset means-testing and inflicts households with low life-time income to choose zero savings for a range of the asset

space. While computationally attractive, approximating asset means-testing by an income floor does not lend itself to welfare analysis because it implies an unreasonable high amount of agents with zero assets in bad income states; therefore, exaggerates the welfare costs from means-testing. My work is also related to a contribution by Heer (2002), who explores the implications that the two tier German unemployment system, which is partly means-tested, has on aggregate savings and inequality. He shows that the program induces some agents to choose low levels of assets leading to an increased wealth inequality. Recent contributions by Koehne and Kuhn (2012) and Rendahl (2012) study the trade-off between additional insurance and moralhazard in search effort when unemployment insurance depends on assets. Their focus is on differentiable benefit schemes and they do not study the savings decision for retirement, which is the main driving force for welfare differences in my set-up. My analytical results are closely related to Clausen and Strub (2012). They show that a non-binding continuous choice always satisfies the first order conditions in a mixed continuous discrete choice model. Households are never indifferent between discrete choices, and the expected value function has hence only downward kinks. My value function has the same properties, and I make use of their concept of sub and super-differentiability in my proofs.

The chapter is structured in the following way. I briefly review current means-tested programs in the US and describe how I choose to introduce them into the *SIM*. The following section characterizes the solution to household behavior analytically, provides intuition for the main novel mechanisms and explains their implications for social welfare. Afterwards, I briefly introduce the data set I am using and discuss calibration of the model. I then argue that the novel mechanisms of my model find support from stylized facts in the data. Last, I conduct the welfare analysis and conclude.

# 3.2 Introducing a Means-Tested Program

In this section, I introduce a means-tested program into the SIM that is intended to mimic the US system. The next section briefly reviews the main US programs that

place a limit on households' income and savings<sup>3</sup><sup>4</sup>. I provide an overview about the size of the programs, the benefits they provide and the eligibility criterion. For a more detailed review see Moffitt (2003). I then argue that the programs are sufficiently similar to treat them within a unified framework. Finally, I introduce my model and explain how household behavior is affected by the program.

### 3.2.1 Means-Tested Programs is the US

The Supplemental Nutrition Assistance Program (SNAP) provides households with vouchers for food. The goal of the program is to make high quality nutrition food available to low income households. A wide range of stores accepts these vouchers; therefore, the in kind benefits are similar to actual cash payments to the household. 44.7 million individuals participated in the program, and total costs summed to 75.7 billion dollars in 2011. Households' savings must typically not exceed \$2000, but housing property and car value are not counted to some degree in most states. The gross monthly households' income must not exceed 130% of poverty income, and about 30% of net income is deducted from the allotment<sup>5</sup>. The program is expanded by the Special Supplemental Nutrition Program for Women, Infants and Children. Females who are pregnant or have children less than five years of age can apply. 8.96 million people received benefits in 2011 leading to total spending of 7.2 billion dollars. The eligibility requirements for this program are less stringent than for the Supplemental Nutrition Assistance Program, but it only provides food benefits for the child.

The Temporary Assistance to Needy Families program provides income support to families with children under 19 years of age<sup>6</sup>. The program provides both cash and in kind transfers. The latter serves basic needs such as child care, education and transportation. A unique feature of the program is that it is designed to promote labor force participation. Household members must either be working or prove to be

<sup>4</sup>The amount of benefits may also depend on the size of the household.

<sup>&</sup>lt;sup>3</sup>All programs that I consider are initiated by the Federal Government. However, the Federal Government only provides a general framework and caries part of the total costs. The individual states are free to design the details of the legislation. Hence, eligibility criterion differ across states, both concerning the level of allowed income and resources.

<sup>&</sup>lt;sup>5</sup>Net income is gross income after some allowances.

<sup>&</sup>lt;sup>6</sup>Some states want to promote "regular families", excluding teenage mothers who do not live with their parents.

actively searching for employment. The typical asset limit for households is \$2000, and most states allow a partial deduction for the vehicle value. Gross income may typically not exceed 100% of poverty level, and allotments typically decrease based upon the households' net income. Total spending for this program totaled to 33.3 billion dollars in 2011.

The Low Income Home Energy Assistance Program provides energy assistance to eligible households. Eligibility is usually guaranteed when a household participates in another welfare program. Total costs of this program are close to 4.48 billion dollars in 2011.

The two major means-tested programs that are not included in my analysis are *Medicaid* and *subsidized housing*. I do not have data regarding the amount of benefits that households receive from these programs. Moreover, these programs are less suitable as insurance against income shocks. *Medicaid* provides medical support for eligible households in case of bad health. Housing benefits may take the form of subsidized rents in state owned housing projects or in the private market. Eligibility is not guaranteed, and the waiting lists can be extensive.

Henceforth, my analysis considers programs that provide either cash transfers or in-kind benefits that are quick to access and serve everyday basic needs. Therefore, I make the assumption that I can approximate these transfers by direct cash transfers to households that consume a single good.

### 3.2.2 The Model

The economy is populated by a unit mass of households, each living for T periods. During the first W periods, the household works in the market and retires with certainty in period W + 1 living until period T. Once a household vanishes, he is replaced by a newborn household. The household takes as given initial total assets  $a_1$  and the laws of motion of the state variables. He chooses each period end of period assets  $k_t$ , which pay certain return from the world capital market (1+r). The household maximizes the stream of isoelastic period utility U from consumption over his lifetime:

$$\max_{k_t} \left\{ \sum_{t=1}^T \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \right\}$$
(3.1)

$$s.t.$$

$$c_t = a_t + w_t - k_t$$

$$a_{t+1} = (1+r)k_t + F(k_t, w_t)$$

$$a_{t+1} \ge 0, \quad c_t \ge 0$$

$$\{a_1, w_t\} \text{ given.}$$

The law of motion of assets depends on the eligibility for the means-tested program.  $F(k_t, w_t)$  is thought to capture the two main features outlined in Section 3.2.1, income and asset means-testing:

$$F(k_t, w_t) = \begin{cases} 0 & \text{if } k_t > \frac{\bar{a}}{1+r} \cup w_t > w_{elig}^t \\ S(w_t) & \text{if } k_t \le \frac{\bar{a}}{1+r} \cap w_t \le w_{elig}^t \end{cases}$$

 $w_{elig}^{t}$  is the maximum amount of income a household may receive to still be eligible for the program, which may depend on households' age, and  $\bar{a}$  is the asset limit<sup>7</sup>. Several points are worth mentioning about the way I model the means-tested program. First, I allow households to borrow against end of period means-tested income. Second, the *SIM* is nested within my model with F not being a function of  $k_t$ . Third, I assume that the government can perfectly observe savings  $k_t$ . I argue in Appendix 3.B that the key mechanisms that I explain in Section 3.2.3 will prevail, if households can hide a fixed amount of savings. Moreover, the results will be identical to the present set-up for a substantial range of parameterization, if the technology for hiding savings is probabilistic. Last, note that I omit any means of financing for the governmental program, which is mainly to keep notation simple. Having labor taxation to finance the program would leave my results practically unchanged because there is no employment decision.

Households' log period income is additive in a deterministic component  $(\mu_t)$  and a transitory component  $(\varphi_t)$ .  $\mu_t$  takes values from the discrete, ordered set with entries  $\mu_t^m$  and  $m = \{1, n\}$ . The path of  $\mu_t$  is given by a function  $\mathcal{F}$  that depends on the income potential drawn upon birth and time:  $\mu_t = \mathcal{F}(\mu_1, t)$ . The transitory

<sup>&</sup>lt;sup>7</sup>Some readers may want to compare my specification to the one put forward by Hubbard et al. (1995). Abstracting from medical expenses that they have in their model, they specify  $F(k_t, w_t) = max\{0, \overline{C} - [(1+r)k_t + w_t]\}$  where  $\overline{C}$  is a guaranteed consumption floor. It is straightforward to see that in this set-up all households participating in the program choose  $k_t = 0$ .

component follows an exogenous mean-zero, N - state Markov processes during working life. The vector of values is denoted by  $\varphi^v$ . The elements of the transition matrix,  $\Pi^W$ , are common among households:

$$\pi_{j,k}^W = prob[\varphi_t = \varphi^k | \varphi_{t-1} = \varphi^j].$$

The process is intended to capture the uncertainty from changes in households' labor market earnings and possible incidences of unemployment. During retirement, the household receives a constant fraction of his last gross income. The transition matrix during retirement,  $\Pi^R$ , is simply the identity matrix with elements  $\pi^R$ . Thus,

$$ln(w_{t+1}^{i}(\varphi_{t+1}, \mu_{1})) = \begin{cases} \mathcal{F}(\mu_{1}, t+1) + \Pi^{W}\varphi_{t} & \text{if } t \leq W\\ ln(\kappa(w_{W}^{i})) & \text{if } t > W \end{cases}$$

Note the way I introduce retirement income. The part replacing income,  $\kappa(w_W^i)$ , is paid unconditional on participating in the program. However, individual savings for retirement in form of assets influence ones eligibility to the means-tested program.

The problem (3.1) satisfies Bellman's principle of optimality; consequently, I can write it recursively. The value function of a household currently in period t with asset position  $a_t$ , permanent initial income potential  $\mu_1$  and current transitory income  $\varphi_t$  reads:

$$V_t(a,\varphi,\mu_1) = \max_{k\in\Gamma(a,w)} \left\{ \frac{(a+w-k)^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_t \{ V_{t+1}(\phi(k),\varphi',\mu_1) \} \right\}$$
(3.2)

 $\phi(k) = (1+r)k + F(k,w)$   $log(w) = \mu + \varphi$ (3.3)

$$\mu' = \mathcal{F}(\mu_1, t+1)$$
$$\varphi' = \begin{cases} \Pi^W \varphi & \text{if } t \le W \\ \varphi & \text{if } t > W \end{cases}$$
$$\Gamma(a, w) = \begin{cases} a+w \\ \phi(k) \ge 0 \end{cases}$$

I refer to  $\Gamma(a, w)$  as feasibility correspondence, which is compact and continuous. Note that  $V_t$  is a function of time indicated by the t subscript. To make the notation more compact for the subsequent analysis, define conditional on  $\varphi_t = \varphi^k$ :

$$\mathbb{E}_{t}\{V_{t+1}(\phi(k),\varphi',\mu_{1})\} = I_{W=1}\sum_{j=1}^{N}\pi_{k,j}^{W}V_{t+1}(\phi(k),\varphi^{j},\mu_{1}) + I_{R=1}\sum_{j=1}^{N}\pi_{k,j}^{R}V_{t+1}(\phi(k),\varphi^{j},\mu_{1})$$
$$= \mathbb{V}_{t+1}(\phi(k),\varphi',\mu_{1}),$$

where  $I_{W=1}$  and  $I_{R=1}$  are indicator functions that are one during working life and retirement, respectively.

### 3.2.3 Characterizing the Value Function

This section characterizes  $V_t(a, \varphi, \mu_1)$  and optimal choices  $k_t$ . It is crucial to understand the effects means-testing has on optimal choices in order to understand its welfare implications. I provide four main theorems that give conditions for first order conditions to be optimal, and I show when households deviate from first order conditions. Furthermore, I explain why household behavior is affected by the meanstested regime even when first order conditions hold. I delegate all proofs to Appendix 3.A.

Let me begin by introducing some notation. Let  $k_t(a, \varphi, \mu_1)$  be the optimal choice for end of period assets induced by the vector  $(a, \varphi, \mu_1)$  in period t. Likewise, let  $a_{t+1}(a, \varphi, \mu_1)$  be the optimal choice for next period assets. I cannot establish that these correspondences are single valued for a range of the state space given the problem stated in (3.2). I assume that the household chooses the larger  $k_t$  when he is indifferent between choices. Consequently,  $k_t(a, \varphi, \mu_1)$  and  $a_{t+1}(a, \varphi, \mu_1)$  are indeed unique and I refer to them as policy functions. At the end of this section, I show why non-uniqueness can arise and argue that it is of little practical relevance. Let me define  $S^{sup} = max\{S(w)\}$ . Last, I define the range with length  $\epsilon$  and center  $k^0$ as  $B_{\epsilon}(k^0)$ . For most of this section, my concerns are about the asset dimension as income is exogenous to the household. Therefore, I sometimes fix  $\varphi$  and  $\mu_1$  at some  $(\varphi^0, \mu_1^0)$ .

**Lemma 1:**  $V_t(\cdot, \varphi, \mu_1)$  is strictly increasing  $\forall a$ .

Note the importance of defining a as total assets, including  $S(w_t)$  and defining  $k_t$  as the choice excluding  $S(w_t)$ . Defining the state as assets excluding  $S(w_t)$  would lead to a downward jump in  $V_t$ . The next Lemma establishes weak monotonicity of the policy  $k_t(\cdot, \varphi, \mu_1)$ , which is a direct result from the strictly concave period utility function.

**Lemma 2:**  $k_t(\cdot, \varphi, \mu_1)$  is increasing.

I am now ready to establishes continuity of  $V_t(\cdot, \varphi, \mu_1)$ . Intuitively, optimal choices imply that a small change in the asset position does not lead to large changes in the value function, even though the law of motion of the endogenous state variable is not continuous.

**Lemma 3:**  $V_t(\cdot, \varphi, \mu_1)$  is continuous  $\forall a$ .

The following Lemma establishes that the policy function is not strictly increasing for agents with  $w_t^0 \leq w_{elig}^t$ . More specific, I show that for a range of  $(a, \varphi^0, \mu_1^0)$ the policy  $k_t(a, \varphi^0, \mu_1^0)$  is flat with choice  $k_t = \frac{\bar{a}}{1+r}$ . The intuition is simple: The

Figure I: Policy Function in T-1



Notes: The figure displays the policy function of a household with state vector  $(a, \varphi^1, \mu_1^1)$  in period T - 1. Wages are such that the household is eligible to the means-tested program.

household has to weight the extra utility he gets from consumption smoothing against the income loss he incurs from choosing  $k_t > \frac{\bar{a}}{1+r}$ .

**Lemma 4:**  $k_t(a, \varphi^0, \mu_1^0)$  is not strictly increasing  $\forall w_t^0 \leq w_{elig}^t$ . More specific,  $\exists k_t(a, \varphi^0, \mu_1^0) \in B_{\epsilon}(k_t^0(a^0, \varphi^0, \mu_1^0))$  with  $k_t^0 = \frac{\bar{a}}{1+r}$ .

Figure I Panel A highlights this point graphically for period T-1. The optimal policy is to choose  $k_{T-1} = \frac{\bar{a}}{1+r}$  in a range of the asset state. Note that this behavior inflicts a cost on social welfare. The social planner always prefers that each individual household equates the expected marginal utility of consumption.

The next Lemma highlights a point already apparent in the figure. It is optimal to choose  $k_t > \frac{\bar{a}}{1+r}$  for a large enough. The economic intuition behind the result is that the gains from consumption smoothing become larger than the income effect from the forgone income for sufficiently high asset level.

**Lemma 5:**  $\exists \tilde{a}_t(\varphi^0, \mu_1^0) \ s.th. \ \tilde{k}_t(\tilde{a}_t, \varphi^0, \mu_1^0) > \frac{\bar{a}}{1+r} \ \forall \ a > \tilde{a}_t(\varphi^0, \mu_1^0).$ 

Equipped with these Lemmas, I can state my main theorems that are about conditions for first order conditions to be either necessary or sufficient. The first theorem deals with the retirement period.

**Theorem 1:** Let t > W and  $w_t^0 > w_{elig}^t$ . Then  $\frac{\partial V_t(\cdot,\varphi^0,\mu_1^0)}{\partial k_t}$  exists and  $\frac{\partial V_t(\cdot,\varphi^0,\mu_1^0)}{\partial k_t} = 0$  is sufficient for an optimum  $\forall a_{t+1}(a,\varphi^0,\mu_1^0) > 0$  and  $\forall t \in \{W+1,T\}$ .

The result follows from the assumption that income is fixed during retirement. Therefore,  $\forall w_W > w_{elig}^t$  choices cannot be disturbed in any period in the future. I need some more notations before stating my second main theorem. Let  $(\dot{a}_s(\varphi^0, \mu_1^0), \varphi^0, \mu_1^0)$ be the state vector in period *s s.th*. under no possible realization of the world in  $t \in \{s, T\}$  the household wants to chooses  $k_t \leq \frac{\ddot{a}}{1+r}$ . Thus, from today on, the household chooses with certainty a policy that makes him never eligible to the transfer.

**Theorem 2:** Let  $\dot{a}_s(\varphi^0, \mu_1^0)$  be defined as above. Then  $\frac{\partial V_s(\cdot, \varphi^0, \mu_1^0)}{\partial k_s}$  exists and  $\frac{\partial V_s(\cdot, \varphi^0, \mu_1^0)}{\partial k_s} = 0$  is sufficient for an optimum  $\forall a_{s+1}(a, \varphi^0, \mu_1^0) > 0$  and  $a_s > \dot{a}_s(\varphi^0, \mu_1^0)$ .

Theorem 2 is a powerful result because it implies that the means-tested program has no impact on optimal choices for sufficiently rich households. The next theorem argues that for non-binding choices the same holds true when the asset state leads to choices strictly less than  $\frac{\bar{a}}{1+r}$ .

**Theorem 3:** Let  $(\ddot{a}_s(\varphi^0, \mu_1^0), \varphi^0, \mu_1^0)$  be the state vector in period s s.th. under no possible realization of the world in  $t \in \{s, T\}$  the household chooses  $k_t \geq \frac{\ddot{a}}{1+r}$ . Consider all a s.th.  $a_{s+1}(a, \varphi^0, \mu_1^0) > 0$  and  $a \leq \ddot{a}_s(\varphi^0, \mu_1^0)$ . Then  $\frac{\partial V_s(\cdot, \varphi^0, \mu_1^0)}{\partial k_s}$  exists and  $\frac{\partial V_s(\cdot, \varphi^0, \mu_1^0)}{\partial k_s} = 0$  is a sufficient condition for a maximum.

My last theorem is concerned with choices that are to the right of  $a \geq \tilde{a}_t(\varphi^0, \mu_1^0)$  but to the left of  $\dot{a}_t(\varphi^0, \mu_1^0)$ . I argue that first order conditions are still necessary for an optimum. The main issue in proving the result is that  $\mathbb{V}_t(\cdot, \varphi, \mu_1)$  is not differentiable at all points in this range. I show the result by demonstrating that these points must be downward kinks. Because these cannot be optimal choices, it follows that the function is differentiable at all optimal choices. Standard variation arguments then lead to the necessity of first order conditions.

**Theorem 4:**  $\frac{\partial V_t(\cdot,\varphi^0,\mu_1^0)}{\partial k_t} = 0$  is a necessary condition for  $k_t(a,\varphi^0,\mu_1^0)$  to solve (3.2)  $\forall a \geq \tilde{a}_t(\varphi^0,\mu_1^0)$  and  $a_{t+1}(a,\varphi^0,\mu_1^0) > 0$ .

Figure II Panel A shows the value and policy function of an eligible household in period T-1. One can see how an upward jump in the policy function translates into a downward kink in the value function. To provide a better understanding for the trade-off the household faces between the income effect and the consumption smoothing effect, let me define the following function at  $(\tilde{a}_{T-1}, \varphi^0, \mu_1^0)$ :

$$W_{T-1}(K_{T-1}, \tilde{a}_{T-1}, \varphi^0, \mu_1^0) = U(\tilde{a}_{T-1} + w_t^0 - K_{T-1}) + \beta \mathbb{V}_T((1+r)K_{T-1} + F(K_{T-1}, w_t^0), \varphi', \mu_1^0)$$
$$K_{T-1} \le \tilde{a}_{T-1} + w_t^0,$$

the return function from different admissible strategies this period and following

optimal policy next period. Obviously,

$$W_{T-1}(K_{T-1}, \tilde{a}_{T-1}, \varphi^0, \mu_1^0) \le V_{T-1}(\tilde{a}_{T-1}, \varphi^0, \mu_1^0)$$

with equality at  $K_{T-1} = k_{T-1}(\tilde{a}_{T-1}, \varphi^0, \mu_1^0)$ . The function is depicted in Figure II, Panel B. The first local maximum is the choice  $K_{T-1} = \frac{\bar{a}}{1+r}$ . Choices just above this point lead to lower returns because the negative income effect dominates the additional consumption smoothing effect. Larger choices lead to additional consumption smoothing gains, which are largest at the second local maximum, where (3.2) satisfies the first order conditions.

The fact that households satisfy first order conditions to the right of  $a \ge \tilde{a}_t(\varphi^0, \mu_1^0)$ does not imply that their choices are not affected by the means-tested regime. To see that point, note that the life-cycle dimension and stochastic income imply that households possibly attach positive probability to a state where they want to participate in the means-tested regime in the future. They adjust their savings decisions already today to fulfill the asset requirements in that case.

Let me first elaborate on the role of the life-cycle dimension. Consider a household in period T-2 that has income  $w_t^0 \leq w_{elig}^t$ . Lemma 4 and Lemma 5 establish





Notes: Panel A displays the policy and value function of a household with state vector  $(a, \varphi^1, \mu_1^1)$  in period T-1. Wages are such that the household is eligible to the means-tested program. Panel B depicts the return function in T-1, i.e., the return from different admissible strategies in T-1,  $K_{T-1}$ , and following optimal policy in T, for the same type of household.





Notes: Panel A displays the return function in T-2, i.e., the return from different admissible strategies in T-2,  $K_{T-2}$ , and following optimal policy in T-1 for an agent choosing between the left and right of a non-differentiable point. Panel B displays the resulting value and policy function.

that the policy function has a flat part and  $\exists \tilde{a}_{T-2}(\varphi^0, \mu_1^0)$  s.th.  $\tilde{k}_{T-2}(\tilde{a}_{T-2}, \varphi^0, \mu_1^0) > \frac{\bar{a}}{1+r} \forall a > \tilde{a}_{T-2}(\varphi^0, \mu_1^0)$ . Moreover, *Theorem* 4 establishes that the value function has a downward kink at  $\tilde{a}_{T-2}(\varphi^0, \mu_1^0)$ . Note that  $V_{T-1}$  has a downward kink at  $\tilde{a}_{T-1}(\varphi^0, \mu_1^0)$ , implying that the value function becomes steeper to the right of the non-differentiability. Consider the point  $\tilde{a}_{T-2}(\varphi^0, \mu_1^0)$  s.th.  $a_{T-1}(\tilde{a}_{T-2}(\varphi^0, \mu_1^0)) > \tilde{a}_{T-1}(\varphi^0, \mu_1^0)$ . To understand the decision the household has to make, let me define the following return function:

$$W_{T-2}(K_{T-2},\tilde{\tilde{a}}_{T-2}(\varphi^0,\mu_1^0),\varphi^0,\mu_1^0) = U(\tilde{\tilde{a}}_{T-2}(\varphi^0,\mu_1^0) + w_t^0 - K_{T-2}) + \beta \mathbb{V}_{T-1}((1+r)K_{T-2} + F(K_{T-2},\varphi^0,\mu_1^0),\varphi',\mu_1^0).$$

Figure III Panel A shows the two local maxima of the return function. The first implies that the household satisfies the first order conditions by choosing to the left of  $\tilde{a}_{T-1}(\varphi^0, \mu_1^0)$  and receives means-tested transfers at end of period T-1. The second local maximum satisfies the first order conditions by choosing to the right of  $\tilde{a}_{T-1}(\varphi^0, \mu_1^0)$  and the household never participates in the means-tested program<sup>8</sup>. The policy function makes a second jump at  $\tilde{\tilde{a}}_{T-2}(\varphi^0, \mu_1^0)$ , and the value function

<sup>&</sup>lt;sup>8</sup>The figure highlights that non-uniqueness in  $k_t$  can arise when the household is exactly indifferent between choosing to the left and the right of a non-differentiability.



#### Figure IV: Consumption Behavior

Notes: The graph compares two consumption functions in T-3 where in the one state the agent has wage income such that he is eligible for the means-tested program (straight line) and in the other he is not eligible (dashed line).

becomes non-differentiable at this point, which I highlight graphically in Panel B.

Uncertain income has a very similar effect, but households with  $w > w_{elig}^t$  also become affected. These households place positive probability on becoming eligible for means-testing in the future. Consequently, they adjust their savings behavior today to have in expectations the most possible intertemporal consumption smoothing. Note, this leads to a rapid increase in the number of non-differentiabilities in the value function because *any* path of the state variables that makes the household at *any* point in the future eligible to means-testing has to be considered.

Finally, Figure IV shows how the non-differentiabilities in the value function translate into optimal consumption behavior. The calibration is such that agents are not eligible for the program in the high income state, but they are eligible in the low income state. Concerning the latter, the following behavior arises: In the left most asset section, workers are borrowing constrained leading to a relatively steep consumption rise (Not visible in the figure). Afterwards, decisions are characterized by *Theorem 3* implying that first order conditions hold, and the consumption profile becomes flatter. *Lemma 4* characterizes the following section where savings are constant and all additional assets are consumed. The following three discontinuities are induced by downward kinks in the value function. Therefore, the slope of the

value function increases after each jump leading to an extra incentive to accumulate assets and decrease consumption. Recall, *Theorem* 4 establishes that first order conditions still hold in this section. Finally, *Theorem* 2 establishes sufficiency of first order conditions in the last section, where behavior is identical to behavior in the *SIM*.

# 3.3 Data Description, Sample Selection and Calibration

This section introduces the data set I am using and explains my calibration strategy for the model. Appendix 3.C provide a description of the numerical algorithm used to solve the model.

### 3.3.1 Data Description

My analysis requires longitudinal household data on components of income, assets and means-tested transfers. The dataset best meeting these requirements is the *Survey of Income and Program Participation* (SIPP). The SIPP is a representative sample of the non-institutionalized civilian US population maintained by the US Census Bureau. I use the 1996 (1996-1999), 2001 (2001-2003) and 2004 (2004-2007) samples, deflating all data with the CPI<sup>9</sup>. The SIPP provides monthly information on income, transfers from different means-tested programs and household affiliation. Moreover, it provides detailed information on different liabilities and asset holdings<sup>10</sup> on a yearly basis.

The asset and liability information allow me to split them into short term, long term and property value, such as housing and different kind of vehicles. My data counterpart to households' precautionary savings in the model is the sum of all assets and liabilities the household has. Two problems arise with this measure. First, durable goods should be counted as savings, but the data has only information on housing and vehicles, inducing an underestimation of precautionary savings. Second, savings

<sup>&</sup>lt;sup>9</sup>The 1996 panel oversamples households close to poverty. I use household weights provided by the SIPP in all samples to correct for this issue.

<sup>&</sup>lt;sup>10</sup>The 1996 sample provides four times data on assets and liabilities, the 2001 sample three times and the 2006 panel twice.

may not reflect precautionary saving motives or retirement saving decisions, but necessary business equity that a household holds resulting from incomplete markets for business financing. I drop all households holding business equity to account for this latter concern.

My model is about behavior at the household level. I define a household as a group of persons living at a common address<sup>11</sup>, and I define the head as the person in whose name the place is owned or rented<sup>12</sup>. I aggregate income and asset data of head and spouse to mimic the within household insurance present in the model. I assume that a household enters the labor market with age 25 and his economical live ends with 81<sup>13</sup>. To focus on the part of the population that is affected by the means-tested scheme, I drop all households that have average income in excess of four times the federal poverty limit<sup>14</sup>.

### 3.3.2 Calibration

Table 3.1 summarizes the calibration. The decision period is one year. The agent works for 43 years and lives in retirement for 15 more years. Consistent with Siegel (2002), I set the yearly world interest rate at 4%. The ability of households to smooth consumption across periods depends greatly on their asset holdings. Therefore, I use the discount factor  $\beta$  to match the median wealth to income ratio, which is 2.2 in my dataset. I set  $\gamma = 1.5$ .

The SIM is intended to capture consumption decisions given an exogenous income process. I take as data counterpart earnings in the labor market and unemployment compensation<sup>15</sup>. To estimate the income process, I restrict the sample to households with prime aged heads (25 - 50). I postulate the following log income process for

<sup>13</sup>I drop observations where the household is school enrolled, or works as a family worker.

 $<sup>^{11}\</sup>mathrm{See}$  the SIPP User Guide for detailed information about the definition of an address.

<sup>&</sup>lt;sup>12</sup>I change the head of a household when the default head lives non-married in a household together with his parents who have higher income and are younger than 67. Moreover, I define a new household every time that the composition of the household changes.

 $<sup>^{14}\</sup>mathrm{Less}$  than 1% of these households ever receive means-tested income in my sample.

<sup>&</sup>lt;sup>15</sup>I aggregate earnings from all jobs that the individual holds and add "incidental earnings" and sickness payments.

Variable	Target
r = 0.04	4% Yearly interest rate
$\beta = 0.961$	Median wealth to income ratio of 2.2
$\gamma = 1.5$	
$\kappa(w_W^i)$	Distribution of retirement replacement rates
$\mu_1$	20,40,60,80% of initial income distribution
$\mathcal{F}(\mu_1, t)$	Age-income profile for two education groups
N = 5	
$\Pi^W, \ \varphi$	$\rho = 0.94, \ \sigma^2 = 0.022$
$\lambda_1(\cdot, \varphi, \mu_1)$	Density of asset holdings of 25 years old
$w^t_{eliq}$	130% of poverty income during retirement and working life
$\bar{a} = 9.64$	2000\$ in 2011

Table 3.1: Calibration

Notes: The left column states the calibrated variable with its value and the second states the relevant moment.

individual i in the data:

$$ln(w_{i,t}) = \phi_i + z_{i,t} + \iota_{i,t}$$
(3.4)

$$z_{i,t} = \rho z_{i,t-1} + \epsilon_{i,t}, \tag{3.5}$$

where  $\epsilon_{i,t} \sim N(0, \sigma^2)$ ,  $\iota_{i,t} \sim N(0, \sigma_{\iota}^2)$ . In a first step, I obtain residuals  $(\tilde{w}_{i,t})$  from regressing individual household log income on time dummies, two race dummies (white, non-white), a gender dummy, a dummy for being disabled, age dummies, four education dummies (less than high school, high school, some college, college) and an interaction between education and age dummies, each using the head of the household. The cross sectional dimension far exceeds the time dimension in my data set and my panel is not balanced. Therefore, I opt to identify  $\rho$  and  $\sigma$  by matching cross sectional moments of the age distribution, as in Storesletten et al. (2004), instead of matching moments of the autocorrelation function. Note that cross sectional income dispersion evolves over the life-cycle according to

$$Var(ln(w_{i,t})) = \sigma_{\phi}^{2} + \sigma_{\iota}^{2} + \sigma^{2} \sum_{s=0}^{t-1} \rho^{2s}.$$
 (3.6)

### Figure V: Income Process



Notes: *Panel A* displays the cross sectional income dispersion over age from the data and the theoretical moment (3.6) for the optimal choice of autocorrelation and variance of income shocks. *Panel B* shows the estimated income growth of households over the life-cycle for four education groups. L HS: Less than high-school, HS: High school diploma, S C: Some college, C: College Degree. Source: SIPP (1996, 2001, 2004)

Hence,  $\rho$  controls the curvature of the profile and  $\sigma$  the increase over time. I match these moments by minimizing the area between the theoretical moment (3.6) and the income residuals estimated from my data:

$$\min_{\rho,\sigma} \left\{ \int_{25}^{50} \left| Var \left( ln(w_{i,t}(\rho,\sigma)) \right) - Var \left( ln(\tilde{w}_{i,t}) \right) \right| \right\}.$$
(3.7)

Figure V Panel A plots the data and the resulting profile with  $\rho = 0.931$  and  $\sigma^2 = 0.023$ .

Turning to the model, I use the Markov process with N = 5 states to match the stochastic component of income. Following Tauchen (1986), I use the entries of the vector of values and the transition matrix to match the moments of the AR(1)process<sup>16</sup>. I allow households to have four different initial permanent income states,  $\mu_1$ , and use these to match the 20<sup>th</sup>, 40<sup>th</sup>, 60<sup>th</sup> and 80<sup>th</sup> percentile of the income distribution for those younger than 27 in their first month of observation. My reduced form regression provides me with profiles for income growth over the life-cycle for each education group. I approximate the profiles by a fourth order polynomial, impose monotonicity and assume that income does not grow any longer after the age of 50.

<sup>&</sup>lt;sup>16</sup>The reason for the relatively low number of income states is the computational burden.

Figure V Panel B shows the resulting profiles. Households with a college degree have higher initial income and more income growth over the life-cycle. All other profiles are remarkably similar<sup>17</sup>. Households with a college degree compose 35% of my population. Therefore, I opt to give the two high initial income states the income growth profile of college graduates and the two lower states the profile of high school graduates.

The amount of households' savings for retirement, and hence their incentive to participate in the means-tested program, depends on their replacement rate during retirement. Retirement savings can be exempt from the means-test when they are not readily available<sup>18</sup>. As a result, all promises obtained from social security, most defined pension plans, and retirement plans managed by the employer (401k plans) are not counted into the asset limit. However, individual retirement plans (IRA) and retirement plans of the self-employed (KEOGH) usually do count. Moreover, 401k plans are transferred under some conditions into an IRA account in the case of unemployment<sup>19</sup>. Henceforth, I define retirement income as the sum of social security income and pensions from former employers, unions or the government<sup>20</sup>. I exclude from retirement income payments received from IRA and KEOGH accounts and 401k plans. I then compute the distribution of the retirement replacement rate in my population and use  $\kappa(w_W^i)$  to match this distribution. Appendix 3.D provides further information on this procedure and provides the entire distribution.

Call the stationary distribution of agents with age t,  $\lambda_t(a, \varphi, \mu_1)$ . I assume each age state is populated by the same amount of agents, which leads to a unique stationary distribution. I calibrate  $\lambda_1(\cdot, \varphi, \mu_1)$  to the unconditional density of wealth holdings of those 25 years old without any distributional assumptions.

Next, I need to calibrate the parameters for the law of motion of the endogenous state  $(S(w_t), w_{elig}^t \text{ and } \bar{a})$ . I want to match the amount of insurance means-tested programs provide on average to households. I discretize the income distribution into

<sup>&</sup>lt;sup>17</sup>High school dropouts have a slightly higher intercept compared to those that complete high school. This fact is likely resulting from dropouts having more labor market experience at age 25.

 <sup>&</sup>lt;sup>18</sup>The individual states have some freedom in determining which savings are *readily available*.
 <sup>19</sup>The 2008 Farm Bill exempts all tax preferred retirement accounts from the means-test for SNAP from October 2008 onwards. I find it unlikely that households in my data adjusted their savings

in anticipation of this bill given that retirement savings are long-term investment decisions. <sup>20</sup>I define a household as retired when he has positive retirement income, is older than 61 and does not receive income from the labor market.

200 income bins and compute for each bin the average households' means-tested income for those that are non-retired and retired separately. I approximate these schedules by a 4<sup>th</sup> order polynomial and impose monotonicity. I set  $w_{elig}^t = 133$  during working life, which is about 130% of poverty income for a four person household<sup>21</sup>. I set  $w_{elig}^t$  10% lower during retirement reflecting that the poverty income threshold is lower during retirement. Figure VI *Panel B* displays the resulting schedule for the income states under consideration<sup>22</sup>. To obtain  $S(w_t)$ , I interpolate the values from the approximated schedules. Finally, I calibrate the maximum amount of savings to \$2000 in 2011 nominal value, which represents the cut-off for most of the programs in place.





Notes: *Panel A* displays the average means-tested income a household receives given his labor market income percentile together with a monotonic, fourth order polynomial approximation to this schedule. *Panel B* displays the same for retirement. Source: SIPP (1996, 2001, 2004)

<sup>&</sup>lt;sup>21</sup>There are a few households in the data that receive means-tested income with higher incomes. These are either relatively large families, or live in a state with especially loose income testing.

<sup>&</sup>lt;sup>22</sup>Average means-tested income start to increase for relatively rich households in the data during retirement. I impose monotonicity, which is more closely in line with regulation and hence cannot account for this fact.

# 3.4 Comparing Implications of the Model with the Data

The next section shows that changes in individual saving incentives induced by the means-tested programs are key to understand the welfare implications of the program. This section argues that these mechanisms are also present in the data. Moreover, I provide whenever possible a comparison to the SIM for the moments of interest, which gives some intuition to the factors driving the difference in welfare between an asset means-tested and a non means-tested regime<sup>23</sup>.

Figure VII displays the share of households receiving means-tested income in the model, the share of households that would be eligible for the program due to sufficient low income and the share of households receiving any of the transfers outlined in Section 3.2.1. The model almost matches the overall mean and does a good job in matching the decreasing share over the working life. However, the model fails in two respects. First, it predicts that the share decreases during working life discontinuously. The low discrete state approximation to income is hence a limitation to my model. Second, the share of households receiving means-tested transfers is increasing in the model once retirement is reached, but is flat during retirement in the data. Certain length of life and the abstraction from a bequest motive are the most likely reasons for this variation.

Figure VIII shows that average consumption is hump-shaped in my model with a discontinuous decline of 1.5% once retirement is reached. This drop increases to more than 3% when considering the decline from the year before retirement to the first year into retirement. The reason for the consumption drop is that a significant portion of households holds few assets when entering retirement because they want to participate in the means-tested program prior to it. When income drops due to retirement, these agents must adjust consumption downwards. A large literature finds that consumption drops substantially upon retirement, a finding often referred to the "retirement consumption puzzle". In a recent study, Haider and Stephens (2007) find an average 10% consumption drop upon retirement using expected retirement age as an instrument. Hubbard et al. (1994), using PSID data, and Bernheim

 $<sup>^{23}\</sup>text{This}$  section uses  $\beta$  to match the same wealth to income ratio in the SIM as in my baseline model.



Figure VII: Participation in the Means-tested Program

Notes: The graph displays the faction of households that participates in means-tested programs. *Data* refers to SIPP households that participate in any of the programs outlined in Section 3.2.1. *Model* refers to the equilibrium distribution from my model. The dotted line shows the fraction of households that has income below the eligibility threshold in my model.

et al. (2001), using PSID and CEX data, show that wealth holdings are particularly low for households with low retirement income, implying large consumption drops for these households. I argue that my model features the same qualitative feature in Section 3.5.1. On the contrary, households in the SIM are very successful in smoothing consumption in the event of retirement. Note the implications for social welfare. Households fail to equate marginal utilities under means-testing implying a reduction in social welfare.

Panel B of Figure VIII shows the Lorenz curve of wealth in the data and the two models<sup>24</sup>. Recall, the calibration assures that the models match mean asset holdings. My baseline model does a good job in matching the inequality at the bottom of the wealth distribution; however, it implies too little inequality at the top of the distribution. My model implies that extremely rich households are nearly unaffected by the means-tested program making me confident that the too little inequality at the top has little impact on my welfare analysis<sup>25</sup>. Put differently, my model implies reasonable amounts of self-insurance and retirement savings, both on average and in

 $<sup>^{24}\</sup>mathrm{The}$  wealth data is top truncated, leading to too little wealth inequality in the data.

<sup>&</sup>lt;sup>25</sup>I discuss the individual policy functions and the transmission of means-testing to high wealth states more thoroughly in Appendix 3.E.
#### 3.5 Welfare Analysis



#### Figure VIII: Mean Consumption

Notes: Panel A displays the average consumption profiles in the SIM and the baseline model over the life-cycle. Panel B displays the Lorenz curve of wealth for the two models and the SIPP data.

the cross section; thus, allowing me to assess welfare consequences of means-testing. The SIM implies much less wealth inequality at the bottom of the wealth distribution and similar inequality at the top of the distribution.

# 3.5 Welfare Analysis

This section discusses the welfare implications of means-tested programs. I compare the current regime to a regime without asset means-testing, which I simply refer to as *alternative regime*. The model environment implies that full insurance is optimal from a perspective of an unborn. Henceforth, I always keep the total value or resources needed to finance the system constant. All other calibration parameters are left unchanged. Put differently, I do not address the question whether the level of current governmental spending is optimal, but ask whether an alternative insurance scheme can increase social welfare given the same amount of expenditures. Furthermore, I avoid redistribution between income states and keep total resources in retirement and working life constant. I begin by comparing welfare across two economies in steady state. Afterwards, I take transition dynamics from one steady state to the other into account.

### 3.5.1 Abolishing Means-Testing (Steady State)

In order to get a deeper understanding for the implications of means-testing on welfare, it is instructive to first compare two economies in steady state. To avoid any redistributional effects across income states, I compute for each vector  $(\varphi_t, \mu_1, t)$  the amount of transfers that agents receive in this state,  $\mathcal{B}_t(a, \varphi_t, \mu_1)$ , under means-testing. The *alternative regime* provides during working life and retirement for each state  $(\varphi, \mu_1)$  the constant benefit streams  $(b^W(\varphi, \mu_1), b^R(\varphi, \mu_1))$  that have the same net present value as the streams received from means-testing:

$$\sum_{t=1}^{W} \frac{\int \mathcal{B}_t(a,\varphi_t,\mu_1) d\lambda_t(a,\varphi_t,\mu_1)}{(1+r)^{t-1}} = \int b^W(\varphi,\mu_1) \sum_{t=1}^{W} \frac{d\hat{\lambda}_t(a,\varphi_t,\mu_1)}{(1+r)^{t-1}}$$
$$\sum_{t=W+1}^{T} \frac{\int \mathcal{B}_t(a,\varphi_t,\mu_1) d\lambda_t(a,\varphi_t,\mu_1)}{(1+r)^{t-1}} = \int b^R(\varphi,\mu_1) \sum_{t=W+1}^{T} \frac{d\hat{\lambda}_t(a,\varphi_t,\mu_1)}{(1+r)^{t-1}},$$

where  $\lambda_t(a, \varphi, \mu_1)$  and  $\hat{\lambda}_t(a, \varphi, \mu_1)$  are the distribution functions of households with age t scaled to one. There exists a trade-off from a welfare perspective because end of period benefits are lower under the *alternative regime* and hence less insurance is provided in the low income states. However, all agents in the low income states benefit from the program. Moreover, the distortions of consumption behavior are different across the two regimes.

#### Consumption over the Life-cycle

To understand the welfare implications, it is useful to understand how average consumption over the life-cycle behaves in the two regimes compared to the unconstrained social planner solution. The social planner, who can pool labor and means-tested income from different agents, wants to provide full consumption insurance. Figure IX plots the solution over the life-cycle for the lowest and highest permanent income state and compares it to the consumption profiles from the two policy regimes. Average consumption profiles are too flat under the *alternative regime* compared to the social planner solution. The reason is that households build up precautionary savings against income uncertainty. Consequently, there is potential for means-testing to increase welfare by providing more insurance in particularly bad states of the world. However, average consumption turns out to be too high under



Figure IX: Comparing Average Consumption Profiles

Notes: The graph displays average consumption profiles for the lowest and highest permanent income states. It displays the social planner solution (SP), the solution with means-testing and the solution in the regime without asset means-testing.

means-testing in the low permanent income state initially and hence too low later in life. Moreover, average consumption drops significantly in the low income state at retirement under means-testing. Households choose a somewhat steeper profile under means-testing in the highest permanent income state, but the profiles are almost identical across the two regimes.

#### **Quantifying Welfare Changes and Decomposing Effects**

I express the welfare gain as fraction of lifetime consumption that makes the average household of age s indifferent between the two regimes. To be more specific, denote by  $c_t(a, \varphi, \mu_1)$  the optimal consumption function of an agent under the regime with the means-tested program and let  $\hat{c}_t(a, \varphi, \mu_1)$  be the corresponding function under the *alternative regime*. Hence, I am interested in the  $\omega_t^U$  that solves

$$\int E_t \sum_{s=t}^T \beta^{s-t} U([1+\omega_s^U]c_s(a,\varphi,\mu_1)) d\lambda_s(a,\varphi,\mu_1)$$
$$= \int E_t \sum_{s=t}^T \beta^{s-t} U(\hat{c}_s(a,\varphi,\mu_1)) d\hat{\lambda}_s(a,\varphi,\mu_1). \quad (3.8)$$

One can show that

$$\omega_s^U = \left(\frac{\int \hat{V}_s d\hat{\lambda}_s}{\int V_s d\lambda_s}\right)^{\frac{1}{1-\gamma}} - 1.$$

 $\omega_1^U$  measures the average willingness to pay of expected lifetime consumption of an unborn to live under the *alternative regime*. Flodén (2001) shows how to decompose the utilitarian welfare gain into gains from an increase in level consumption, gains from reduced consumption uncertainty and gains from reduced consumption inequality in an infinite horizon problem. I can perform the same decomposition for each cohort with age s.

Denote by  $\hat{c}_t(a, \varphi, \mu_1)$  the expected mean consumption from period t to end of life induced by the current state vector  $(a, \varphi, \mu_1)$ . Define  $C_s$  as mean expected lifetime consumption for a household with age s. Formally,

$$C_s = \int \hat{c}_s(a,\varphi,\mu_1) d\lambda_s.$$

The percentage increase in mean expected consumption between the two regimes for each cohort with age s reads:

$$(1+\omega_s^L)C_s = \hat{C}_s. \tag{3.9}$$

Define by  $V_t(c_{t:T}(a, \varphi, \mu_1))$  the value function in period t given the state  $(a, \varphi, \mu_1)$ , expressed in the optimal consumption policy from period t onwards. Certainty equivalent consumption for each point in the state space  $(\bar{c}_s(a, \varphi, \mu_1))$  solves

$$V_s(\bar{c}_s(a,\varphi,\mu_1)) = V_s(c_{s:T}(a,\varphi,\mu_1)).$$

Average certainty equivalent consumption of cohort s is then given by:  $\bar{C}_s = \int \bar{c}_s(a,\varphi,\mu_1)d\lambda_s$ . I define now the costs of consumption uncertainty for each cohort s as the deviation from the value of mean expected consumption and certainty equivalent consumption:

$$p_s^{unc} = 1 - \left(\frac{V_s(\bar{C}_s)}{V_s(C_s)}\right)^{\frac{1}{1-\gamma}}.$$
 (3.10)

Similarly, I define the costs of consumption inequality as:

$$p_s^{inq} = 1 - \left(\frac{\int V_s(\bar{c}_s(a,\varphi,\mu_1))d\lambda_s}{V_s(\bar{C}_s)}\right)^{\frac{1}{1-\gamma}}.$$

I can express the welfare gains of decreased consumption uncertainty and decreased consumption inequality that result from moving to the non-means-tested regime as:

$$\begin{split} \omega_s^{unc} &= \frac{1-\hat{p}_s^{unc}}{1-p_s^{unc}}-1\\ \omega_s^{inq} &= \frac{1-\hat{p}_s^{inq}}{1-p_s^{inq}}-1. \end{split}$$

Similarly to Flodén (2001), I can decompose the total utilitarian welfare gain of each cohort s:

$$\boldsymbol{\omega}_s^U = (1+\boldsymbol{\omega}_s^L)(1+\boldsymbol{\omega}_s^{unc})(1+\boldsymbol{\omega}_s^{inq})-1.$$

Before turning to the results, it is worth elaborating on the relationship of (3.9) and (3.10). Consider a world where income uncertainty is exogenously reduced. Households react by decreasing precautionary savings and move closer to the social planner solution. The decrease in savings decreases life-time income, which would show up as a negative level effect in (3.9). However, welfare would increase, because the increase in (3.10) would outweigh the level effect.

Equipped with these considerations, I now turn to the welfare effects of abolishing means-testing. Table 3.2 shows the result from the policy experiment. An unborn is willing to give up 0.31% of life-time consumption in order to live under the *alternative regime*, besides facing considerably higher costs of consumption uncertainty. Agents that receive temporary low income but have high savings have little problem to keep consumption stable. The higher allotment under the means-tested regime allows the same for agents with low assets.

Wealth inequality as measured by the Gini coefficient is higher under means-testing. This has two reasons: First, there is a direct effect on income of asset rich households to be more equal. There is also an effect arising from endogenous choices that is already discussed by Heer (2002). Households select themselves into those holding little wealth and participate in the means-tested program and those that hold more

Gini coefficient Wealth income ratio		Means-tested 0.58 2.196	Unconditional 0.52 2.574
$egin{array}{l} \omega_1^U \ \omega_1^L \ \omega_1^{unc} \ \omega_1^{inq} \ \omega_1^{inq} \end{array}$	0.31% 1.08% -0.84% 0.08%		

Table 3.2: Welfare Analysis

Notes: The top panel displays the Gini coefficient of wealth and the mean wealth to income ratio for the two model specifications. The second column refers to the baseline, means-tested program and the third column to a program that pays benefits unconditionally to low income states. The bottom panel shows the welfare gain of switching from the means-tested programs to an unconditional program.  $\omega_1^{U}$ : utilitarian welfare gain,  $\omega_1^{L}$ : gain from consumption increase,  $\omega_1^{unc}$ : gains from reduced consumption uncertainty,  $\omega_1^{inq}$ : gains from reduced consumption inequality.

wealth and do not participate. Reflecting this, households hold on average less precautionary savings under the means-tested regime and the wealth to median income ratio is lower than under the *alternative regime*. The increase in wealth inequality translates into somewhat higher costs of consumption inequality.

The decrease in households' savings over the life-cycle reduces lifetime income from asset holdings. Thus, an unborn has significantly lower expected consumption under the means-tested regime. The inflicted welfare costs outweigh the gains from reduced consumption uncertainty.

My decomposition allows me to compare welfare in the two economies over the life-cycle. Figure X shows the components of social welfare for each age. Expected consumption is always higher under the *alternative regime* and the gains are increasing over the life-cycle. Similarly, total welfare gains from moving to the *alternative regime* are increasing over the life-cycle, even faster than the gains from expected consumption increases. The reason is that the costs of increased consumption uncertainty decrease until age 37 and turn into gains for agents close to retirement. The term "costs of consumption uncertainty" may be confusing in this respect. Recall from (3.10) that I define these costs as deviations of expected consumption from certainty equivalent



#### Figure X: Decomposing Welfare Gains

Notes: The figures display the welfare gain of switching from the means-tested programs to an unconditional program. Panel A shows conditional on cohort the utilitarian welfare gain,  $\omega_s^U$ , and the gain from higher average consumption,  $\omega_s^L$ . Panel B displays the gains from the change in the costs of consumption uncertainty,  $\omega_s^{unc}$ , and the change in the costs of consumption inequality,  $\omega_s^{inq}$ .

consumption. Figure IX shows that expected consumption drops discontinuously at retirement. The means-tested program induces some agents to deviate from full intertemporal consumption smoothing, which shows up as a cost of consumption uncertainty.  $\omega_s^{unc}$  is slightly positive during retirement for a similar reason. Agents under the means-tested regime choose consumption profiles that deviate from full intertemporal consumption smoothing to increase lifetime income. The gains of reduced consumption inequality are increasing throughout the life-cycle.

Given my income process, it is obvious that conditional on any  $\mu_1^0$ 

$$\omega_s^U(\mu_1^0) = (1 + \omega_s^L(\mu_1^0))(1 + \omega_s^{unc}(\mu_1^0))(1 + \omega_s^{inq}(\mu_1^0)) - 1$$

This allows me to analyze further which subpopulation loses most from meanstesting. Table 3.3 displays the total welfare change and its components for the four different permanent income states. Households with expected low permanent income are the ones suffering from means-testing. Recall from Figure IX that these households are least smoothing consumption over the life-cycle. In fact, all other groups have welfare gains from means-testing, but these are comparably small. Again, Figure IX shows that higher income states find it easier to smooth consumption over

	$\mu_1 = 88$	$\mu_1 = 127$	$\mu_1 = 170$	$\mu_1 = 227$
$\omega_1^U$	1.22%	-0.21%	$-4*10^{-3}\%$	0
$\omega_1^L$	6.65%	0.56%	$2*10^{-3}\%$	0
$\omega_1^{unc}$	-4.68%	-0.66%	$-4*10^{-3}\%$	0
$\omega_1^{inq}$	-0.44%	-0.11%	$-2*10^{-3}\%$	0

Table 3.3: Welfare of Sub-Populations

Notes: The table displays different welfare components from moving to a non meanstested regime for the four different permanent income states.  $\omega_1^U$ : utilitarian welfare gain,  $\omega_1^L$ : gain from consumption increase,  $\omega_1^{unc}$ : gains from reduced consumption uncertainty,  $\omega_1^{inq}$ : gains from reduced consumption inequality. Welfare changes for  $\mu_1 = 227$  are numerically not distinguishable from zero.

the life-cycle and the reduced costs of consumption uncertainty allow them to reduce precautionary savings. Therefore, if the government could condition on permanent income instead of overall income, it could increase social welfare by introducing it only for high permanent income states. The costs of within-group consumption inequality are lower under means-testing for all income groups. Hence, the higher costs of consumption inequality are arising from an increase in between group consumption inequality.

### 3.5.2 Abolishing Means-Testing (Transition Dynamics)

Section 3.5.1 shows that welfare is higher in a world without asset means-testing. However, this does not directly imply that society is better off from abolishing means-testing. Agents living under means-testing hold relatively little assets and the asset poor receive less transfers under the *alternative regime* forcing them to lower consumption. These temporary welfare costs may well outweigh the long-term gains from abolishing means-testing.

For simplicity, I make the payment under the *alternative regime* age dependent in this section. More specifically, again denote functions in the *alternative regime* by "hat". I solve for the payment  $b(\varphi_t, \mu_1)$  s.th.

$$\int \mathcal{B}_t(a,\varphi_t,\mu_1)d\lambda_t(a,\varphi_t,\mu_1) = \int b_t(\varphi_t,\mu_1)d\hat{\lambda}_t(a,\varphi_t,\mu_1).$$

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Households in my economy do not care about future generations. However, I assume that a social planner incorporates the long-run effects that a policy change has on future generations. More specifically, I assume that the social planner compares the discounted sum of average willingness to pay from now to infinity. Let me expand the state space by the time that a specific cohort is born  $B \in (0, \infty)$ :  $(a, \varphi, \mu_t, B)$ where I index the period in which the policy change occurs by B = 0. The discounted sum of average willingness to pay (AWP) is the sum of the willingness to pay of the currently living households, CL, and all future households, FL:

$$AWP = \underbrace{\left(\frac{\int \hat{V}(\cdot, \cdot, \cdot, 0) d\hat{\lambda}(\cdot, \cdot, \cdot, 0)}{\int V(\cdot, \cdot, \cdot, 0) d\lambda(\cdot, \cdot, \cdot, 0)}\right)^{\frac{1}{1-\gamma}}_{CL}}_{CL} + \underbrace{\sum_{s=1}^{\infty} \beta^s \left(\frac{\int \hat{V}(\cdot, \cdot, \cdot, s) d\hat{\lambda}(\cdot, \cdot, \cdot, 0)}{\int V(\cdot, \cdot, \cdot, s) d\lambda(\cdot, \cdot, \cdot, s)}\right)^{\frac{1}{1-\gamma}}_{FL}}_{FL}.$$

Because a change in the regime has no general equilibrium price effects this expression simplifies to

$$AWP = \underbrace{\left(\frac{\int \hat{V}(\cdot, \cdot, \cdot, 0) d\hat{\lambda}(\cdot, \cdot, \cdot, 0)}{\int V(\cdot, \cdot, \cdot, 0) d\lambda(\cdot, \cdot, \cdot, 0)}\right)^{\frac{1}{1-\gamma}} - 1}_{CL} + \underbrace{\sum_{s=1}^{\infty} \beta^s \omega_1^U}_{FL}.$$
(3.11)

Table $3.4$ :	Welfare	Component	with		
Transition Dynamics					

CL	FL
0.66%	0.35%

Notes: The table displays the welfare effects for a change from the means-tested regime to a non means-tested regime. CL: welfare effect on the currently living. FL: welfare effects on all future living generations.

Table 3.4 shows the two components of (3.11). Both, the gain of the current population and the gains of future generations are positive. This implies that a change to a non-asset-based system is implementable, given appropriate transfers among the current population.

### 3.6 Conclusion

A prevailing feature in US income support programs is that they pay benefits to households conditional on their earnings and wealth being below certain thresholds. The common rationale behind asset means-testing is that the insurance scheme allows targeting relatively high benefits to those households that are in most need for them. Households with low assets are likely to have had a series of poor earnings outcomes, and targeting benefits to these households has the potential to mitigate the adverse effects of incomplete financial markets. This chapter introduces the current US income support programs into an incomplete markets model and evaluates the welfare consequences of the programs' asset means-testing. Given total current expenditures, social welfare would increase from abolishing the asset means-test. Comparing steady states, an unborn is willing to pay 0.31% of lifetime consumption to be under a regime without asset means-testing. The welfare gains remain present when taking transition dynamics of the economy into account.

The welfare costs of asset means-testing arise from households altering their savings decisions as a response to the insurance scheme. I show that the propensity to consume out of wealth decreases discontinuously along the wealth state and converges to the behavior of agents in a world without asset means-testing. Put differently, asset poor households save relatively little under asset means-testing. Consequently, households hold on average less precautionary savings and less savings for retirement. The reason is that they either want to participate in the income support program today and fulfill the eligibility requirements, or they expect to participate in it at any point in the future; therefore, increase consumption already today to have more intertemporal consumption smoothing.

This behavior leads to two major channels decreasing social welfare under asset means-testing. First and most importantly, a significant fraction of households fails to build up sufficient savings to retain their consumption levels during retirement. Average consumption declines by more than 3% around retirement and keeps decreasing throughout it. The decline in consumption is most pronounced for households with low income potential, who have the strongest incentives to participate in the program, inflicting a major cost to social welfare. In fact, the insurance scheme increases welfare of households with high income potential, who have less trouble to smooth consumption over the life-cycle, but the gains are negligible. Second, means-testing increases wealth dispersion, which increases the dispersion of utility from consumption. Because the social welfare function is concave, this decreases social welfare. However, this latter effect turns out to be small compared to the welfare costs arising from the failure to smooth consumption intertemporally.

### Appendix to Chapter 3

### **3.A Proofs**

Proof of Lemma 1: Let  $k_t^j(a_t^j, \varphi, \mu_1)$  be the optimal policy and let  $a_t^k > a_t^j$ . By the definition of  $\Gamma$ , I have  $\Gamma^j \subset \Gamma^k$ . Thus,  $k_t^j$  is an admissible policy for  $a_t^k$  with strictly larger current consume this period. Because U is increasing in current consumption and  $k_t^k(a_t^k, \varphi, \mu_1)$  maximizes  $V_t$ ,  $V(a^k, \varphi, \mu_1) > V(a^j, \varphi, \mu_1)$ .

Proof of Lemma 2: Let  $a_t^k > a_t^j$  and hence  $\Gamma^j \subset \Gamma^k$ . Let  $k_t^j(a_t^j, \varphi, \mu_1)$  and it follows that  $k_t^j$  is an admissible choice for  $(a_t^k, \varphi, \mu_1)$ . Assume the optimal policy for state  $(a_t^k, \varphi, \mu_1)$  is such that  $k_t^j > k_t^k$ . It directly follows that  $F(k_t^k) \ge F(k_t^j)$ . First, assume they are equal. Because  $V(\cdot, \varphi, \mu_1)$  is strictly increasing, I have  $\mathbb{V}_{t+1}(\phi(k_t^j), \varphi', \mu_1) > \mathbb{V}_{t+1}(\phi(k_t^k), \varphi', \mu_1)$ . Resulting from the concavity of  $U, k_t^k$ cannot be optimal given the optimality of  $k_t^j$ . Assume now  $F(k_t^k) > F(k_t^j)$ . Hence,  $F(k_t^j) = 0$  and  $F(k_t^k) = S(w_t^0)$ . This again contradicts the concavity of U because the marginal gain from consuming more today are larger for the lower asset position.

Proof of Lemma 3: I proof that  $V_{T-1}$  is continuous. Once established, the same logic carries through for all periods. Consider all  $a_{T-1}^+$  s.th.  $k_{T-1}(a_{T-1}^+, \varphi, \mu_1) > \frac{\bar{a}}{1+r}$ . Then by Lemma 2  $F(k_{T-1}(a_{T-1}^+, \varphi, \mu_1)) = 0$ , and there exists a smallest point for which this condition still holds, which I call  $\hat{a}_{T-1}^+$ . Moreover, both U and  $\mathbb{V}_T$  are continuous (The continuity of  $\mathbb{V}_T$  follows trivially from the fact that  $V_T(\cdot, \varphi, \mu_1)$  is continuous and  $\phi(k)$  is a constant.). Therefore, I maximize a continuous function over a continuous correspondence and by Berge's theorem of the maximum the resulting value function is continuous. By the same logic, I can establish continuity at all points  $a_{T-1}^-$  s.th.  $k_{T-1}(a_{T-1}^-, \varphi, \mu_1) \leq \frac{\bar{a}}{1+r}$ . Consequently, I only need to establish continuous, it features either an upward or downward jump at  $\hat{a}_{T-1}^+$ . Due to Lemma 1,  $V_{T-1}$ cannot have a downward jump. Next, assume it would have an upward jump. Then by the continuity of U, it must be that  $k_{T-1}(\hat{a}_{T-1}^+ - \epsilon, \varphi, \mu_1) > \frac{\bar{a}}{1+r}$  because such a policy would bring the implied value arbitrary close to the upward jump. But this contradicts the fact that  $\hat{a}_{T-1}^+$  is the least such point. Proof of Lemma 4: The proof goes by contradiction. Fix some  $w_t^0 \leq w_{elig}^t$ .  $k_t(a_t, \varphi^0, \mu_1^0)$  would be strictly increasing when  $\exists$  a tuple  $(\hat{a}_t - \epsilon, \varphi^0, \mu_1^0)$  s.th.  $k_t(\hat{a}_t - \epsilon, \varphi^0, \mu_1^0) = \frac{\bar{a}}{1+r}$  and  $\forall \epsilon$  the tuple  $(\hat{a}_t, \varphi^0, \mu_1^0)$  leads to  $k_t(\hat{a}_t, \varphi^0, \mu_1^0) > \frac{\bar{a}}{1+r}$ . Moreover,  $F(k_t(\hat{a}_t - \epsilon, \varphi^0, \mu_1^0)) = S(w_t^0)$  and  $F(k_t(\hat{a}_t, \varphi^0, \mu_1^0)) = 0$ . I now show that for this case  $k_t(\hat{a}_t, \varphi^0, \mu_1^0) > \frac{\bar{a}}{1+r}$  cannot be an optimal policy  $\forall \epsilon$ . The policy  $\tilde{k}_t(\hat{a}_t, \varphi^0, \mu_1^0) = \frac{\bar{a}}{1+r}$  was preferred iff  $\exists$  an  $\epsilon$  s.th.

$$\underbrace{U(\hat{a}_t + w_t^0 - k_t(\hat{a}_t, \varphi^0, \mu_1^0)) - U(\hat{a}_t + w_t^0 - \tilde{k}_t(\hat{a}_t, \varphi^0, \mu_1^0) + \epsilon)}_{<0}}_{<0} \\ < \underbrace{\beta[\mathbb{V}_{t+1}((1+r)[k_t - \epsilon] + S(w_t^0), \varphi', \mu_1^0) - \mathbb{V}_{t+1}((1+r)k_t, \varphi', \mu_1^0)]}_{>0},$$

where the inequality on the right hand side comes from the fact that V is increasing and  $S(w_t^0) > 0$ .

Proof of Lemma 5: The proof proceeds by contradiction. Assume  $\forall a_t(\varphi^0, \mu_1^0)$ ,  $k_t(a_t(\varphi^0, \mu_1^0)) \leq \frac{\bar{a}}{1+r}$ . For expositional reasons, I assume the equality holds. Moreover, assume  $w_t^0 \leq w_{elig}^t$ . The result for all other w follow trivially. Now consider the alternative policy  $k_t^0(a_t^0, \varphi^0, \mu_1^0) = \frac{\bar{a}+x}{1+r}$  for some state  $(a_t^0, \varphi^0, \mu_1^0)$  and  $x > S^{sup}$ . This alternative policy is better for some  $(a_t^0, \varphi^0, \mu_1^0)$  iff the following inequality holds:

$$\underbrace{U(a^{0} + w_{t}^{0} - \frac{\bar{a}}{1+r}) - U(a^{0} + w_{t}^{0} - \frac{\bar{a} + x}{1+r})}_{\rightarrow 0 \text{ for } a^{0} \text{ large enough}} < \underbrace{\beta[\mathbb{V}_{t+1}(\bar{a} + x, \varphi', \mu_{1}^{0}) - \mathbb{V}_{t+1}(\bar{a} + S(w_{t}^{0}), \varphi', \mu_{1}^{0})]}_{>0}$$

The convergence to 0 of the left hand side results from the concavity of U and the inequality on the right hand side results from *Lemma 1*. The second part of the Lemma results from the monotonicity of the policy function.

Proof of Theorem 1: Call a typical element from  $\Gamma(a_t, \varphi^0, \mu_1^0) \, \check{a}_t$ . By assumption

$$\phi(k_t(\breve{a}_t,\varphi^0,\mu_1^0)) = (1+r)k_t(\breve{a}_t,\varphi^0,\mu_1^0),$$

which is a continuous function. Both, the feasibility correspondence and the law of motion are concave. Moreover, the inside of (3.2) is just the sum of concave functions and hence concave itself. Thus,  $V_t$  is concave in this range. To proof the Theorem, I apply the Benveniste and Scheinkman (1979) Lemma. Let  $\check{k}_t(\check{a}_t(\varphi^0, \mu_1^0), \varphi^0, \mu_1^0)$  solve (3.2). Now define  $A_t \in B_{\epsilon}(\check{a}_t)$  where  $\epsilon$  is chosen s.th.  $\check{k}_t$  is still feasible  $\forall A_t$ . Define the function

$$W(A_t, \varphi^0, \mu_1^0) = U(A_t + w_t^0 - \breve{k}_t) + \beta \mathbb{V}_{t+1}((1+r)\breve{k}_t, \varphi', \mu_1^0).$$

Note that  $W(\cdot, \varphi^0, \mu_1^0)$  is continuous and concave because U is continuous and concave and  $\beta \mathbb{V}_{t+1}((1+r)\check{k}_t, \varphi', \mu_1^0)$  is a constant. It follows that  $W(A_t, \varphi^0, \mu_1^0) \leq V_t(A_t, \varphi^0, \mu_1^0)$ with equality at  $\check{a}_t \in A_t$ . Thus, the *Benveniste and Scheinkman Lemma* establishes differentiability of  $V_t(A_t, \varphi^0, \mu_1^0)$ . Because the function is concave and by assumption the borrowing constraint is slack, the first order conditions are sufficient for a maximum.

Proof of Theorem 2: By assumption  $F(k_t(\dot{a}_s(\varphi^0, \mu_1^0), \varphi^0, \mu_1^0)) = 0 \ \forall t \ge s$  and by Lemma 2 this holds  $\forall a_s > \dot{a}_s(\varphi^0, \mu_1^0)$ . Call a typical element from this later set  $\check{a}_t$ . Thus, for  $\check{a}_t$ :

$$\phi(k_t(\breve{a}_t)) = (1+r)k_t(\breve{a}_t),$$

which is a continuous function. Consequently, the same logic as in *Theorem 1* applies.

Proof of Theorem 3: It is sufficient to show that  $V_t$  is concave. Then the result follows by the same logic as in Theorem 1. Call the set of  $a_t$  satisfying the above conditions  $A_t$ . Because  $k_t(a, \varphi^0, \mu_1^0) < \frac{\bar{a}}{1+r}$  I have that  $\forall a \in A_t \ \phi(k_t) = (1+r)k_t + S(w_t^0)$ . Moreover, by assumption for each induced  $a_s$ ,  $\phi(k_s) = (1+r)k_s + S(w_t^0) \ \forall s > t$ . Hence,  $\phi(A_s)$  is a concave function and  $V_{t+1}$  is just the sum of concave functions, which is concave. Therefore, the function inside the max operator in (3.2) is concave and the constraints are concave, assuring concavity of  $V_t(A_t, \varphi^0, \mu_1^0)$ .

Proof of Theorem 4: Theorem 2 establishes the result  $\forall \dot{a}_s(\varphi^0, \mu_1^0)$ ; hence, I focus here on all other points. Clausen and Strub (2012) show that non-differentiable points can be classified into upward, the function is not sub-differentiable, and downward kinks, the function is not superdifferentiable. As they demonstrate, choosing  $k_t$  at a downward kink cannot be optimal because the slope of  $V_t(\cdot, \varphi^0, \mu_1^0)$ is increasing to the right. Therefore, it is sufficient for me to show that all points of discontinuity of  $V_t(\cdot, \varphi^0, \mu_1^0)$  are downward kinks or equivalently that  $V_t$  is subdifferentiable  $\forall a_t \geq \tilde{a}_t(\varphi^0, \mu_1^0)$ . Following the notation of Clausen and Strub (2012), call  $\partial_D V_t(a^0, \varphi^0, \mu_1^0)$  the sub-differentiable of  $V_t$  at  $a^0$ :

$$\partial_D V_t(a^0, \varphi^0, \mu_1^0) = \left\{ m \in \Re : \limsup_{\Delta a^0 \to 0^-} \left\{ \frac{V_t(a^0 + \Delta a^0, \varphi^0, \mu_1^0) - V_t(a^0, \varphi^0, \mu_1^0)}{\Delta a^0} \right\} \le m$$
$$\le \liminf_{\Delta a^0 \to 0^+} \left\{ \frac{V_t(a^0 + \Delta a^0, \varphi^0, \mu_1^0) - V_t(a^0, \varphi^0, \mu_1^0)}{\Delta a^0} \right\} \right\}. \quad (3.12)$$

 $V_t(a^0, \varphi^0, \mu_1^0)$  is sub-differentiable at  $a^0$  iff  $\partial_D V_t(a^0, \varphi^0, \mu_1^0)$  is non-empty. Intuitively, a function is sub-differentiable at a point when its slope approaching the point from the right is larger than the slope approaching from the left.

I first argue that the upward jump in the policy function at  $\tilde{a}_t(\varphi^0, \mu_1^0)$  leads to  $V_t$ being still sub-differentiable. For the ease of presentation, I omit the dependence of  $\tilde{a}_t$  on the exogenous state vector  $(\varphi^0, \mu_1^0)$  from here on. Lemma 5 establishes that  $k_t(\tilde{a}_t, \varphi^0, \mu_1^0) = k_t(\tilde{a}_t - \epsilon, \varphi^0, \mu_1^0)$ . Therefore, the first part of (3.12) simplifies to

$$\lim_{\Delta \tilde{a}_t \to 0^-} \left\{ \frac{U(\tilde{a}_t + \Delta \tilde{a}_t + w_t^0 - k_t) - U(\tilde{a}_t + w_t^0 - k_t)}{\Delta \tilde{a}_t} \right\}.$$
 (3.13)

The second part of (3.12) becomes

$$\lim_{\Delta \tilde{a}_{t} \to 0^{+}} \left\{ \frac{U(\tilde{a}_{t} + \Delta \tilde{a}_{t} + w_{t}^{0} - k_{t}(\tilde{a}_{t} + \Delta \tilde{a}_{t}, \varphi^{0}, \mu_{1}^{0}))}{\Delta \tilde{a}_{t}} - \frac{U(\tilde{a}_{t} + w_{t}^{0} - k_{t}(\tilde{a}_{t}, \varphi^{0}, \mu_{1}^{0}))}{\Delta \tilde{a}_{t}} + \beta \left[ \frac{\nabla_{t+1}((1+r)k_{t}(\tilde{a}_{t} + \Delta \tilde{a}_{t}, \varphi^{0}, \mu_{1}^{0}), \varphi', \mu_{1}^{0})}{\Delta \tilde{a}_{t}} \right] - \frac{\nabla_{t+1}((1+r)k_{t}(\tilde{a}_{t}, \varphi^{0}, \mu^{0}), \varphi', \mu_{1}^{0})}{\Delta \tilde{a}_{t}} \right] \right\}. \quad (3.14)$$

Because  $k_t(\tilde{a}_t + \Delta \tilde{a}_t, \varphi^0, \mu_1^0)$  is optimal, it must be that

$$U(\tilde{a}_{t} + \Delta \tilde{a}_{t} + w_{t}^{0} - k_{t}(\tilde{a}_{t} + \Delta \tilde{a}_{t}, \varphi^{0}, \mu_{1}^{0})) + \beta \mathbb{V}_{t+1}((1+r)k_{t}(\tilde{a}_{t} + \Delta \tilde{a}_{t}, \varphi^{0}, \mu_{1}^{0}), \varphi', \mu_{1}^{0}) \geq U(\tilde{a}_{t} + \Delta \tilde{a}_{t} + w_{t}^{0} - k_{t}(\tilde{a}_{t}, \varphi^{0}, \mu_{1}^{0})) + \beta \mathbb{V}_{t+1}((1+r)k_{t}(\tilde{a}_{t}, \varphi^{0}, \mu_{1}^{0}), \varphi', \mu_{1}^{0}).$$

Together with the fact that  $k_t(\cdot, \varphi^0, \mu_1^0)$  is weakly increasing and  $V_{t+1}(\cdot, \varphi', \mu_1^0)$  is strictly increasing implies (3.14)  $\geq$  (3.13) as was to be shown.

I still need to show that  $V_t$  is sub-differentiable, given that  $\mathbb{V}_{t+1}$  is sub-differentiable. Clausen and Strub (2012) show that kinks do not cancel out under addition. Hence, it is sufficient to show that the upper envelope of a sub-differentiable function is subdifferentiable<sup>26</sup>. When  $V(\cdot, \varphi^0, \mu^0)$  is the upper envelope of some sub-differentiable function, f(a, K), with  $V_t(a^0, \varphi^0, \mu^0) = f(a^0, k)$ :

$$f(a + \Delta a, k) - f(a, k) \le V(a^0 + \Delta a, \varphi^0, \mu_1^0) - V_t(a^0, \varphi^0, \mu_1^0).$$

It follows that  $\partial_D f \in \partial_D V_t(\cdot, \varphi^0, \mu_1^0)$  and consequently  $V_t(\cdot, \varphi^0, \mu_1^0)$  is sub-differentiable. The desired result follows directly: All non-differentiable points cannot be a solution to (3.1).

# 3.B Hidden Savings

This section relaxes the assumption that the government can perfectly observe savings  $k_t^{27}$ . A full characterization of the household problem is beyond the aim of this chapter. Instead, I provide intuition for some specifications of particular interest. I first show that a specification where households can hide a fixed amount of savings does not alter the main mechanisms of my model. I can construct examples where savings behavior is significantly different from my baseline model when the government observes hidden savings only with a certain probability. Nevertheless, a significant range of parameterization implies the same household behavior as in my baseline model even in that case.

 $<sup>^{26}</sup>$ My proof follows their *Lemma* 4 where I replace the derivative with the sub-differential.

<sup>&</sup>lt;sup>27</sup>The section uses notation and refers to results from Section 3.2.3 and I advise to read that section first.

Consider the following modification for the means-tested transfer:

$$F(k_t, w_t) = \begin{cases} 0 & \text{with } 1 - P(k_t) \cup \text{ if } w_t > w_{elig}^t \\ S(w_t) & \text{with } P(k_t) \cap \text{ if } w_t \le w_{elig}^t \end{cases}$$

where  $1 - P(k_t)$  is the probability that the government observes that the household has savings exceeding  $\frac{\bar{a}}{1+r}$ . It is straightforward to see that the logics of Lemma 4 and Lemma 5 still apply  $\forall P(k_t) < 1$ .  $\forall w_t^0 \leq w_{elig}^t$  the policy function is flat in a range of the asset state and makes a jump at some  $a(\varphi^0, \mu_1^0)$ .

Consider now a special case where households can hide savings  $\dot{k}_t$ . So  $P(k_t) = 1 \forall k_t \leq \dot{k}_t$  and zero thereafter. In this simple case, the proofs from Section 3.2.3 still apply. The only modification is that the flat region characterized by *Lemma 4* and the jump point characterized by *Lemma 5* are to the right in the asset state compared to my baseline model.

Now consider the general case with an arbitrary  $P(k_t)$ . It is obvious that I can find a schedule *s.th*. the solution with hidden savings coincides exactly with the solution of my main model. Crucial for this result is that  $P(k_t)$  is sufficiently small close to  $\tilde{a}_t(\varphi^0, \mu_1^0)$ . To see this point take an extreme case where  $P(k_t(\cdot, \varphi^0, \mu_1^0)) = 0.99$ 

Figure XI: Policy with Hidden Savings



Notes: Panel A displays the policy function of a household where the probability to successfully hide savings below the point  $\tilde{a}_t(\varphi^1, \mu_1^1)$  is almost one and zero for all higher savings. For comparison, it also plots the policy function from my baseline model. Panel B shows the policy function when the ability to hide savings is decreasing slowly along the asset dimension.

 $\forall \frac{\tilde{a}}{1+r} < k_t < \tilde{a}_t(\varphi^0, \mu_1^0)$  and  $P(k_t(\tilde{a}_t(\varphi^0, \mu_1^0), \varphi^0, \mu_1^0) + \epsilon) = 0$ . Figure XI Panel A plots the policy function in T-1 together with the policy function from my baseline model. Note that the region characterized by Lemma 4 becomes quite small because taking the risk of increasing savings becomes attractive quickly. Moreover, the policy function becomes flat in a second region of the asset state. Agents choose the maximum savings that have positive probability of not being detected in this region. To get an intuition for the robustness of my results to a more general specification of hidden savings, consider the following parameterization:

$$P(k_t) = \begin{cases} 1 & \text{if } k_t \le \frac{\bar{a}}{1+r} \\ max(1 - \frac{k_t - \frac{\bar{a}}{1+r}}{\alpha}, 0) & \text{if } k_t > \frac{\bar{a}}{1+r} \end{cases}$$

 $\alpha$  controls the ability of the government to accurately observe savings. My baseline model is the limit case with  $\alpha \to 0$ . Figure XI *Panel B* plots the value and policy function of an eligible household in T-1 for two different values of  $\alpha$ . With  $\alpha = 22.3$ the households attach positive probability of successfully hiding assets in the range  $9.3 \leq k_{T-1} \leq 31.5$  and the resulting policy function is identical to my baseline model. The range expands to  $9.3 \leq k_{T-1} \leq 37.1$  with  $\alpha = 27.8$  and households policy starts to deviate slightly from my baseline specification.

## 3.C Numerical Algorithm

My algorithm differs markedly between retirement and working life. Consider first the case during retirement, where I allow for off grid choices. This raises the issue of finding maxima. My theoretical results show under simply verifiable conditions that  $k_t = \frac{\bar{a}}{1+r}$  is the only non-binding choice that does not satisfy the first order conditions. Hence, the optimal choice satisfying the first order conditions can be simply compared to  $k_t = \frac{\bar{a}}{1+r}$ . Recall from Figure III *Panel B* that the return function is not strictly concave over the asset grid. However, it is strictly concave between all non-differentiable points in  $\mathbb{V}_{t+1}$ , which are known when computing  $V_t$ . Therefore, using standard maximum search algorithms, I can find the local maximum between all non-differentiable points and compare them. The choice associated with the highest value solves (3.2). *Golden section search* is a standard algorithm to perform this task. The numerical burden is still large, as there are easily more than 100 discontinuities making it necessary to solve for 101 possible candidates. Therefore, I employ the endogenous grid point method developed by Caroll (2006) to find the candidate points.

Recall, uncertain income increases the number of non-differentiabilities quickly making it computationally extremely expensive to allow for off-grid choices. As a compromise, I allow during working life only for on-grid choices given the value and policy function from retirement. Normalized Euler errors increase by a factor of around 1000 compared to the more exact computation during retirement, but they are still in an admissible range around  $10^{-3}$ .

# 3.D Computing Retirement Replacement Rates

This section provides further information on the way I compute the retirement replacement rate. To have a more homogeneous group of agents, I further restrict households' age for this analysis. I define a household as *working* when head or spouse report to be in the labor force, have income from the labor market in any month during the observation period, the head is between 55 and 61 and neither head, nor spouse receive social security income. I define a household as *retired* in the data when head and spouse report to be out of the labor force, have no income from the labor market, the head is above 65 and head or spouse receive some social security income during any month of the observation period. I calculate the distribution of income in both subpopulations and compute the replacement rate at each point in the income distribution. Figure XII displays the result for the range of incomes that I consider in my model.

Retirement income replaces between 57 and 39% of working income and is downward sloping in income. The reason is foremost that social security replaces relatively little income for the income rich. I obtain  $\kappa(w_W^i)$  for each income state by interpolating on the retirement replacement rate schedule.

# 3.E Individual Policy Functions

This section looks at the individual saving decisions that lead to the aggregate dispersion in wealth. Let  $g_t(a_t, \varphi, \mu_1)$  denote the growth given the state vector

#### Figure XII: Retirement replacement rate



Notes: The graph displays the replacement rate provided by retirement income conditional on income during working-life.

 $[a_t, \varphi, \mu_1]$ . Figure XIII displays the functions under means-testing and in the SIM.

Let me first discuss the mechanisms that are inherent to the *SIM*. Asset growth is an increasing function of wealth early in life, because agents want to build up precautionary savings. As agents age, it becomes increasing unlikely that future income changes. Moreover, the saving for retirement motive starts to dominate, inducing a downward sloping relationship for all income states. Note that the conditional asset growth becomes almost flat very quickly.

In contrast, the model with means-testing behaves very differently, especially for young and poor households. After an initial drop, the decreasing propensity to consume out of wealth leads to an increasing conditional asset growth for the low income households and the medium income households at young ages. Saving decisions for high income households are almost undistorted, reflecting that these households have little incentive to participate in the means-tested program.



Figure XIII: Comparing Asset Growth across Models

Notes: The figures display asset growth in the SIM and the model with means-testing (MT) for workers in their first year and in the last year before retirement and for different permanent income states.

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