Income Risk, Self-Insurance and Public Policies: Consequences for Aggregate Fluctuations and Wealth Inequality

Inaugural-Dissertation

zur Erlangung des Grades eines Doktors der Wirtschafts- und Gesellschaftswissenschaften

durch die

Rechts- und Staatswissenschaftliche Fakultät der Rheinischen Friedrich-Wilhelms-Universität Bonn

vorgelegt von

Liên Annette Phạm-Dào

aus Berlin

Bonn 2016

Dekan: Prof. Dr. Dr. Rainer Hüttemann Erstreferent: Prof. Dr. Christian Bayer Zweitreferent: Prof. Dr. Thomas Hintermaier

Tag der mündlichen Prüfung 29.7.2016

Diese Dissertation ist auf dem Hochschulschriftenserver der ULB Bonn http://hss.ulb.unibonn.de/diss-online elektronisch publiziert.

Acknowledgements

First of all, I would like to thank Christian Bayer, whom I was very lucky to have as principle supervisor. He was a constant support throughout the years, furthering me in my development and career wherever he could. Even when distant, he was available at any time for discussing ideas or answering questions. But foremost, I benefited from his broad field of interests and knowledge, and his encouragement to pursue my ideas. I am also very grateful to my second supervisor, Thomas Hintermaier, who always had an open door and ear. He was very supportive and especially helpful in giving me broad literature reviews as well as insights into small details. I am also deeply indebted to my coauthor Ralph Lütticke, from whom I learned a lot during our collaboration. He has always been a reliable and supporting friend and great company during my Master and PhD.

I want to thank Tim Kehoe for inviting me to join the University of Minnesota for a research stay, integrating me into all of the university's activities and his immense support throughout my visit. Also special thanks to Fatih Guvenen for encouraging and advising me on my last chapter. This stay has been a very valuable experience to me, I would not want to miss. I also had the chance to meet many great people and friends, whom I want to thank a lot for welcoming me with such warmth and hospitality.

I also benefited immensely from a fruitful and excellent research environment at the *Bonn Graduate School of Economics* and the *Institute of Macroeconomics and Econometrics* and want to thank the BGSE and macro department for this. In particular, I thank Keith Küster and Fabian Kindermann for their guidance and valuable suggestions on my research. Financial support from the BGSE, German Research Foundation (GRK 1707) and the European Research Council (ERC Grant agreement No. 282740) is highly acknowledged. Special thanks also to my cohort at the BGSE, of which many have become close friends and have made my time at the BGSE very special and fun. I am also extremely grateful for the company of several female BGSE friends from cohorts above and below, with whom I enjoyed numerous lunches and social gatherings.

Last, I want to thank my family and close friends for all their love and support throughout my life, and their continuous encouragement and belief in me. Without them, I would surely not be where I am today. I can't thank my husband Thanh-Long Dao enough for always backing me up and all of his considerateness. I also want to thank my friend Geovanna Erazo for making countless hours of joint studying more enjoyable and my parents, Annette and Thai Pham, for being such inspiring role models.

Contents

Introduction

1	Pı	recaution	nary Savings, Illiquid Assets, and the Aggregate Conse-	
	qυ	iences of	f Shocks to Household Income Risk	5
	1	Introduc	ction	5
	2	Related	Literature	7
	3	Model.		9
		3.1	Households	10
		3.2	Intermediate Goods Producers	13
		3.3	Entrepreneurs	13
		3.4	Goods, Money, Capital, and Labor Market Clearing	15
		3.5	Recursive Equilibrium	16
	4	Numerio	cal Implementation	16
		4.1	Krusell-Smith Equilibrium	16
		4.2	Solving the Household Planning Problem	17
	5	Calibrat		18
		5.1	Income Process	18
		5.2	Preferences and Technology	19
	6	Quantit	ative Results	21
		6.1	Household Portfolios and the Individual Response to Uncertainty	21
		6.2	Aggregate Consequences of Uncertainty Shocks	24
		6.3	Redistributive and Welfare Effects	30
	7	Conclus	ion	33
	A	opendices		35
	А	Dynami	c Planning Problem with Two Assets	35
		A.1	Properties of Primitives	36
		A.2	Properties of the Value and Policy Functions	36
		A.3	Euler Equations	39
		A.4	Algorithm	40
	В	Estimat	ion of the Stochastic Volatility Process for Household Income	41
		B.1	Income Process	41
		B.2	Data	42
		B.3	Estimation	43
		B.4	Results	43

	С	Asset D	istribution \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 46	6
	D	Quality	of the Numerical Solution	7
	Е	Welfare		1
	F	Robustn	ness Checks	3
	G	Equilibr	ium Forecasting Rules	5
2	Fi	scal Stir	nulus Payments and Precautionary Investment 59	9
	1		ztion	9
	2	Model.		1
		2.1	Households	1
		2.2	Central Bank and Government	2
		2.3	Goods, Bonds, Capital, and Labor Market Clearing	3
		2.4	Recursive Equilibrium	3
	3	Numeric	eal Implementation $\ldots \ldots 64$	4
	4	Calibrat	$ion \ldots 6$	5
		4.1	Households	5
		4.2	Central Bank and Government	
	5	Results		
		5.1	Individual Consumption Response	
		5.2	Aggregate Effects of Transfers Payments 68	
	6		ion	
	A			
	А	Equilibr	ium Forecasting Rules	3
3	Ρı	ublic Ins	surance and Wealth Inequality:	
	Α	Euro A	rea Analysis 73	5
	1		etion	
	2		Literature $\ldots \ldots .$	
	3		butive Policies and Wealth Inequality in the Euro Area	9
		3.1	Cross-country Evidence on Wealth Inequality and the Degree of	
				9
		3.2	Cross-country Evidence on Wealth Inequality and the Generosity	
				-1
	4	N (L L L L	of Public Transfers	
	4	Model.		3
	4	4.1	Household Income 84	$\frac{3}{4}$
	-	$\begin{array}{c} 4.1 \\ 4.2 \end{array}$	Household Income 84 Household Optimization Problem 84	$\frac{3}{4}$
	4 5	4.1 4.2 Calibrat	Household Income 8 Household Optimization Problem 8 Sion 8	$\frac{3}{4}$ 9
	-	4.1 4.2 Calibrat 5.1	Household Income 8 Household Optimization Problem 8 ion 8 Household Parameters 8	$ \begin{array}{c} 3 \\ 4 \\ 7 \\ 9 \\ 9 \\ 9 \end{array} $
	-	4.1 4.2 Calibrat 5.1 5.2	Household Income 8 Household Optimization Problem 8 ion 8 Household Parameters 8 Labor Market Process Before Retirement 9	$ \begin{array}{c} 3 \\ 4 \\ 7 \\ 9 \\ 9 \\ 1 \end{array} $
	5	4.1 4.2 Calibrat 5.1 5.2 5.3	Household Income 84 Household Optimization Problem 84 ion 85 Household Parameters 85 Labor Market Process Before Retirement 97 Policy Parameters 97	$ \begin{array}{c} 3 \\ 4 \\ 7 \\ 9 \\ 9 \\ 1 \\ 3 \end{array} $
	-	4.1 4.2 Calibrat 5.1 5.2 5.3 Results	Household Income 83 Household Optimization Problem 84 Household Optimization Problem 87 ion 83 Household Parameters 83 Household Parameters 84 Labor Market Process Before Retirement 94 Policy Parameters 94 . 94 . 94	$ \begin{array}{c} 3 \\ 4 \\ 7 \\ 9 \\ 9 \\ 1 \\ 3 \end{array} $
	5	4.1 4.2 Calibrat 5.1 5.2 5.3	Household Income 83 Household Optimization Problem 84 Household Optimization Problem 87 ion 88 Household Parameters 88 Labor Market Process Before Retirement 91 Policy Parameters 92	$ \begin{array}{c} 3 \\ 4 \\ 7 \\ 9 \\ 9 \\ 1 \\ 3 \\ 6 \end{array} $
	5	4.1 4.2 Calibrat 5.1 5.2 5.3 Results 6.1	Household Income83Household Optimization Problem84Household Optimization Problem87ion88Household Parameters88Household Parameters89Labor Market Process Before Retirement91Policy Parameters92Quantitative Importance of Labor Income Dynamics and WelfarePolicies94	$ \begin{array}{c} 3 \\ 4 \\ 7 \\ 9 \\ 9 \\ 1 \\ 3 \\ 6 \\ 6 \end{array} $
	5	4.1 4.2 Calibrat 5.1 5.2 5.3 Results 6.1 6.2	Household Income 83 Household Optimization Problem 84 Household Optimization Problem 87 ion 88 Household Parameters 88 Labor Market Process Before Retirement 91 Policy Parameters 92	$ \begin{array}{c} 3 \\ 4 \\ 7 \\ 9 \\ 9 \\ 1 \\ 3 \\ 6 \\ 0 \\ \end{array} $

А	Robustness of Empirical Facts	•						•		•		•						•		•	105
В	Numerical Methods	•	•				•	•		•	•	•		•				•	•	•	106
С	Calibration	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	107
Refere	ences																				111

List of Tables

1.1	Estimated parameters of the income process 19)
1.2	Calibrated parameters 20)
1.3	Alternative monetary policy rules	L
1.4	Welfare after 5 years	2
1.5	Parameter estimates 45	5
1.6	Asset distribution	3
1.7	Forecasting errors	3
1.8	Welfare after 75 years	2
1.9	Laws of motion for the price of capital	5
1.10	Laws of motion for inflation	;
1.11	Laws of motion for Fed policy rule	7
2.1	Calibrated Parameters	3
2.2	Laws of motion	3
3.1	Calibrated Parameters)
3.2	Parameters of labor market risk	3
3.3	Policy Parameters	5
3.4	Gini coefficients of private net wealth in the data and model 99)
3.5	Conditional decomposition of cross-country differences in wealth inequality 101	
3.6	Unconditional decomposition of cross-country differences in wealth in-	
~ -	equality	
3.7	Robustness 103	
3.8	Aggregate wealth and its distribution at the end of the life cycle 107	7

List of Figures

1.1	Portfolio share of liquid assets by percentiles of wealth, 2010 vs. 2004 .	7
1.2	Share of liquid assets in total net worth against percentiles of wealth .	22
1.3	Liquidity: Partial equilibrium response	23
1.4	Liquidity: General equilibrium response	24
1.5	Uncertainty shock under constant money growth	25
1.6	Uncertainty shock under inflation stabilization	27
1.7	Uncertainty shock under Fed's post-80's reaction function	28
1.8	Uncertainty shock with liquid capital	29
1.9	Welfare after 5 years	31
1.10	Idiosyncratic cross-sectional variance by age	44
1.11	Idiosyncratic income uncertainty 1939-2009	45
1.12	Share of liquid assets in total net worth	47
	Uncertainty shock with higher resolution in K and M	49
1.14	Out-of-sample performance of the forecasting rules	50
1.15	Uncertainty shock – Robustness	54
2.1	Individual consumption response to a transfer shock	68
2.2	Response to transfer shock under spending rule	70
2.3	Response to transfer shock under tax rule	71
2.4	Response to transfer shock for different paths of public debt	72
3.1	Correlation of Gini coefficients of household net wealth and percentage	
	change in income Gini coefficients due to transfers	80
3.2	Wealth inequality and minimum income benefits	82
3.3	Wealth inequality and unemployment insurance	83
3.4	Wealth inequality and public pension wealth	84
3.5	Share of wealth held by 50th percentile (bottom 95%)	105
3.6	Gini coefficient of equivalized household net wealth (bottom $95\%)$	105
3.7	Cross-sectional age-earnings profile, adjusted for euro area real wage	
	growth and PPP	108
3.8	Unemployment net replacement rate	108
3.9	Public and occupational pension replacement rate	109

Introduction

The empirical rejection of the assumption of complete financial markets and perfect risk-sharing (Deaton and Paxson, 1994), which came along with the availability of new micro data sources, paved the way for a new literature pioneered by Bewley (1983), Huggett (1993), and Aiyagari (1994). The so called "incomplete markets" literature studies the importance of idiosyncratic risks and household heterogeneity for optimal consumption and savings decisions and the aggregate economy. In particular, advances in computational speed at that time allowed economist to numerically solve multi-period optimization problems with uncertainty and heterogeneous households. The relaxing of assumptions of perfect foresight, certainty equivalence or complete markets, which in particular came in handy for deriving analytical solutions (Carroll, 2001) and to solve the problem associated with aggregation (Constantinides, 1982), also shed new light on theoretical outcomes.

In the absence of complete financial markets, households are not able to perfectly insure against risks by trading a complete set of state-contingent claims. This gives rise to precautionary savings, i.e. the additional assets households accumulate to self-insure against idiosyncratic risks. While the precautionary saving motive has been recognized by economists long ago (Keynes, 1936) and was first mathematically formalized by Leland (1968), it has now become an integral part of the theory on consumption and savings (Attanasio, 1999) and hence a central object of study in macroeconomic research. Since savings decisions are inherently linked to income and consumption. It induces a higher marginal propensity to consume out of additional income for liquidity constrained households, a finding that is in line with empirical evidence (Johnson et al., 2006) and has challenged the Permanent Income Hypothesis put forward by Friedman (1957).

The presence of liquidity constraints and the more rigorous treatment of idiosyncratic risks in incomplete markets models established new channels through which the government can enhance household insurance, and therefore introduced new motives for government intervention. Households' consumption and savings decisions critically depend, first, on the nature and magnitude of risk faced by households, and second, on the net supply of assets available for self-insurance, both of which the government can influence. For example, the government or central bank can increase the net supply of government debt or central bank money, and therefore reduce the price of self-insurance. This also implies that under incomplete markets the proposition of Ricardian equivalence (Barro, 1974) fails, and hence financing decisions by fiscal authorities matter. More directly, the government can also reduce risks households face by e.g. increasing overall risk-sharing through the provision of social insurance mechanisms. However, as noticed by Krueger and Perri (2011), taking market incompleteness as purely exogenous may also overstate the role of the government.

This thesis investigates quantitatively in a model framework the distributional and aggregate outcomes that result from the interaction of income risk, self-insurance motives and public policies. My thesis consists of three chapters and contributes to several lines of literature. One line of literature emphasizes the importance of the government in affecting the net supply of assets, as it improves household liquidity by providing additional means of saving (Woodford (1990)). In a seminal work, Aiyagari and Mc-Grattan (1998) highlight the welfare enhancing role of government debt and determine in a steady state analysis the optimal amount of public debt by weighting the gains from consumption smoothing against the costs from partly displaced capital under higher interest rates. Chapter 1 of this thesis builds a New Keynesian business cycle model with sticky prices, incomplete markets and liquid and illiquid assets. It contributes to this stream of literature by showing that in times of high income uncertainty, there is a role for central bank to intervene in asset markets. By accommodating a heightened demand for liquidity, the central bank can stabilize output and reduce the overall welfare costs from uncertainty shocks.

A second stream of literature has questioned the Permanent Income Hypothesis, i.e. that only permanent and not transitory changes in income affect household consumption. In a seminal paper, Heathcote (2005) investigates in an incomplete markets setting with heterogeneous households the effect of tax cuts on aggregate consumption, and finds that they are highly expansionary due to a reduction in distortions. However, incomplete markets contribute only little given the fact that borrowing constrained households constitute only a small fraction of consumers with low levels of consumption. A redistribution exercise from high-income to low-income and constrained households via targeted transfers by Oh and Reis (2012) confirms this result. By contrast, Kaplan and Violante (2014) demonstrate that the partial equilibrium consumption response to fiscal stimulus payments can be large in the presence of also wealthy liquidity constrained households. The model framework in Chapter 2, which builds on the one in Chapter 1, endogenously generates a large fraction of wealthy liquidity constrained households in line with empirical evidence and thus creates a large initial consumption response to fiscal transfers. More importantly, we also consider the financing in a general equilibrium set up and find that a transitory increase in government debt to finance transfers leads to a crowding in of investment, as forward-looking households want to hold on to their improved consumption-smoothing capacity.

Last, my thesis considers a third common channel in the literature through which the government can affect households' precautionary savings. İmrohoroglu et al. (1995), Heathcote (2005) and Engen and Gruber (2001) show that the government can reduce the risks faced by households and enhance their risk sharing through social insurance. Revisiting the argument by Hubbard et al. (1995) that public insurance crowds out private savings especially of the poor, I jointly model in Chapter 3 various institutions of the social security system and quantify their importance as well as those of labor market dynamics for euro area differences in private net wealth inequality.

CHAPTER 1 "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk" is joint work with Christian Bayer, Volker Tjaden and Ralph Lütticke and quantifies the business cycle consequences of fluctuations in idiosyncratic income uncertainty in a New Keynesian model with sticky prices, incomplete markets, and portfolio choice between liquid and illiquid assets. We empirically document that households face large income uncertainty that varies substantially over the business cycle. In times of heighted income uncertainty, households increase their demand for precautionary savings and rebalance their portfolio toward the liquid "paper" asset, as it provides better consumption-smoothing services. This reduces their aggregate demand for consumption goods and illiquid physical capital, leading to considerable output losses in a demand-driven economy. It is shown that in this environment, there is a role for the central bank, following a money supply rule, to alleviate output losses by accommodating this higher demand for liquid assets through the provision of central bank money. Furthermore, the welfare costs of uncertainty shocks crucially depend on the households' income and asset positions and whether the central bank engages in stabilization policies or not.

CHAPTER 2 "Fiscal Stimulus Payments and Precautionary Investment" emerged from a joint project with Christian Bayer and Ralph Lütticke and assesses, in a closely related framework as in Chapter 2, the aggregate effects of deficit-financed government transfers to households. These types of transfers have become an important fiscal policy measure by the U.S. government to stimulate the economy in the last two recessions. We match the distribution of consumption responses to fiscal transfers as provided by empirical studies. We then show that this partial equilibrium consumption response is amplified in general equilibrium, as in the presence of nominal rigidities, a higher consumption demand is met by higher production, further boosting consumption through the disposable income channel. More importantly, debt-financed transfers increase individual liquidity and enhance market liquidity through a higher net supply of government bonds. This leads to a crowding in private investment if the expansion in government debt is transitory, as households want to hold on to their improved consumption-smoothing capacity. This precautionary investment channel dominates the negative wealth effect of a future increase in distortionary labor taxes, making aggregate effects expansionary independent of the mode of financing.

In CHAPTER 3 "Public Insurance and Wealth Inequality: A Euro Area Analysis", I analyze the quantitative importance of cross-country differences in labor market risks and social security institutions for euro area differences in private net wealth inequality. I document the empirical puzzle that euro area countries with more redistributive and generous public insurance policies, robustly show higher inequality in private net wealth. Revisiting the argument by Hubbard et al. (1995) that public insurance crowds out private savings especially of the poor, I construct a life-cycle model with heterogeneous households and incomplete markets that features exogenous labor market risks, unemployment benefits, means-tested minimum income support and public and occupational pensions. Calibrating the model to the euro area differences in the net earnings process, unemployment dynamics and social security system, it can account for 70.1% of the cross-country differences in the net wealth Gini coefficients for the bottom 95% of the wealth distribution. Welfare policies contribute 57.5% to the wealth inequality differences across the euro area, while net earnings and unemployment dynamics account for 12.6%.

CHAPTER 1

Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk

Households face large income uncertainty that varies substantially over the business cycle. We examine the macroeconomic consequences of these variations in a model with incomplete markets, liquid and illiquid assets, and a nominal rigidity. Heightened uncertainty depresses aggregate demand as households respond by hoarding liquid "paper" assets for precautionary motives, thereby reducing both illiquid physical investment and consumption demand. This translates into output losses, which a central bank can prevent by providing liquidity. We show that the welfare consequences of uncertainty shocks crucially depend on a household's asset position. Households with little human capital but high illiquid wealth lose the most from an uncertainty shock and gain the most from stabilization policy.

1. Introduction

The Great Recession has brought about a reconsideration of the role of uncertainty in business cycles. Increased uncertainty has been documented and studied in various markets, but uncertainty with respect to household income stands out in its size and importance. Shocks to household income are persistent and their variance changes substantially over the business cycle. The seminal work by Storesletten et al. (2001) estimates that during an average NBER recession, income uncertainty faced by U.S. households, interpreted as income risk – i.e. the variance of persistent income shocks, is more than twice as large as in expansions.

These sizable swings in household income uncertainty lead to variations in the propensity to consume if asset markets are incomplete so that households use precautionary savings to smooth consumption. This paper quantifies the aggregate consequences of this precautionary savings channel of uncertainty shocks by means of a dynamic stochastic general equilibrium model. In this model, households have access to two types of assets to smooth consumption. They can either hold liquid money or invest in illiquid but dividend paying physical capital. This asset structure allows us to disentangle savings and physical investment and obtain aggregate demand fluctuations.¹ To obtain aggregate output effects from these fluctuations, we augment this incomplete markets framework in the tradition of Bewley (1979) by sticky prices à la Calvo (1983).

We model the illiquidity of physical capital by infrequent participation of households in the capital market, such that they can trade capital only from time to time. This can be considered as an approximation to a more complex trading friction as in Kaplan and Violante (2014), who follow the tradition of Baumol (1952) and Tobin (1956) in modeling the portfolio choice between liquid and illiquid assets.

In this economy, when idiosyncratic income uncertainty increases, individually optimal asset holdings rise and consumption demand declines. Importantly, households also rebalance their portfolios toward the liquid asset because it provides better consumption smoothing. These effects are reminiscent of the observed patterns of the share of liquid assets in the portfolios of U.S. households during the Great Recession (see Figure 1.1). According to the 2010 *Survey of Consumer Finances*, the share of liquid assets in the portfolios increased relative to 2004 across all wealth percentiles, with the strongest relative increase for the lower middle-class. In our model, this portfolio rebalancing towards liquid paper reinforces, through a decline in physical investment, the decline in consumption demand caused by higher uncertainty. Consequently, aggregate demand declines even more strongly than consumption and investment and consumption co-move.

Quantitatively, we find the following: a two standard deviation increase in household income uncertainty decreases aggregate activity by roughly 0.5% on impact and 0.4% over the first year under the assumption of a monetary policy that follows a constant nominal money growth rule (Friedman's "k% rule"). This is about half the effect size that Fernández-Villaverde et al. (2015) report for a fiscal policy uncertainty at the zero lower bound. Imposing a Taylor-type rule for monetary policy as estimated in Chowdhury and Schabert (2008), we still find a 0.3% decrease in output upon the uncertainty shock. This is more than twice as large as the effect of fiscal policy uncertainty in "normal" non zero-lower-bound times reported in Fernández-Villaverde et al. (2015). Importantly, in all cases the economy recovers only sluggishly over a five-year horizon in our model.

Since the relative price of capital falls but the value of money increases upon an uncertainty shock, such a shock has not only aggregate but also rich distributional consequences. Our welfare calculations imply that households rich in physical or human capital lose the most, because factor returns fall in times of high uncertainty. In contrast, welfare losses decline in money holdings as their value appreciates. To understand the welfare consequences of systematic policy responses to uncertainty shocks, we compare a regime where monetary policy follows Friedman's k%-rule to one where monetary policy provides additional money to stabilize inflation. Since an uncertainty shock effectively works like a demand shock in our model, monetary policy is able to reduce the negative effects on output and alleviate welfare consequences. On aver-

¹In a standard Aiyagari (1994) economy, where all savings are in physical capital, an increase in savings does not lead to a fall in total demand (investment plus consumption) because savings increase investments one-for-one.



Figure 1.1: Portfolio share of liquid assets by percentiles of wealth, 2010 vs. 2004

Notes: Portfolio share: Net liquid assets/Net total assets. Net liquid assets: cash, money market, checking, savings and call accounts, as well as government bonds and T-Bills net of credit card debt. Cash holdings are estimated by making use of the *Survey of Consumer Payment Choice* for 2008, as in Kaplan and Violante (2014). Households with negative net liquid or net illiquid wealth, as well as the top 5% by net worth, are excluded from the sample. The bar chart displays the average change in each wealth decile, and the dotted line an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15.

age, households would be willing to forgo 0.41% of their consumption over the first 20 quarters to eliminate the uncertainty shock, but this number is reduced to 0.25% with stabilization. In the latter regime, households rich in human capital pay the cost of the stabilization policy, because they save (partly in money) and thereby finance the monetary expansion. Moreover, without stabilization, these households profit from low prices of the illiquid asset in which they accumulate their long-term savings.

The remainder of the paper is organized as follows. Section 2 starts off with a review of the related literature. Section 4 develops our model, and Section 4 discusses the solution method. Section 5 introduces our estimation strategy for the income process and explains the calibration of the model. Section 6 presents the numerical results. Section 7 concludes. An Appendix follows that provides details on the properties of the value and policy functions, the numerics, the estimation of the uncertainty process from income data, and further robustness checks.

2. Related Literature

Our paper contributes to the recent literature that explores empirically and theoretically the aggregate effects of time-varying uncertainty. The seminal paper by Bloom (2009) discusses the effects of time-varying (idiosyncratic) productivity uncertainty on firms' factor demand, exploring the idea and effects of time-varying real option values of investment. This paper has triggered a stream of research that explores under which conditions such variations have aggregate effects.²

A more recent branch of this literature investigates the aggregate impact of uncertainty shocks beyond their transmission through investment and has also broadened the sources of uncertainty studied. The first papers in this vein highlight non-linearities in the New Keynesian model, in particular the role of precautionary price setting.³ Fernández-Villaverde et al. (2015), for example, look at a medium-scale DSGE model à la Smets and Wouters (2007). They find that at the zero lower bound output drops by more than 1% after a two standard deviation shock to the volatility of taxes if a countervailing fiscal policy response is ruled out. Off the ZLB the drop reduces to 0.1%.⁴ In a similar framework, Basu and Bundick (2012) highlight the labor market response to uncertainty about aggregate TFP and time preferences. They argue that, if uncertainty increases, the representative household will want to save more and consume less. Then, with King et al. (1988) preferences, the representative household will also supply more labor, which in a New Keynesian model depresses output through a "paradox of toil." When labor supply increases, wages and hence marginal costs for firms fall. This increases markups when prices are sticky, which finally depresses demand for consumption and investment, and a recession follows. Overall, they find similar aggregate effects as Fernández-Villaverde et al. (2015), in particular at the zero-lower bound.

While our paper also focuses on precautionary savings, it differs substantially in the transmission channel. We are agnostic about the importance of the "paradox of toil," because it crucially relies on a wealth effect in labor supply. We therefore assume Greenwood et al. (1988) preferences to eliminate any direct impact of uncertainty on labor supply to isolate the demand channel of precautionary savings instead.⁵ Moreover, since we focus on *idiosyncratic* income uncertainty, we can identify the uncertainty process outside the model from the Panel Study of Income Dynamics (PSID).

This focus on idiosyncratic uncertainty and the response of precautionary savings links our paper to Ravn and Sterk (2013) and Den Haan et al. (2014). Both highlight the importance of idiosyncratic unemployment risk. In their setups, households face unemployment risk in an incomplete markets model with labor market search and nominal frictions. Both papers differ in their asset market setup and the shocks considered. Ravn and Sterk (2013) look at a setup with government bonds as a means of savings. They then study a joint shock to job separations and the share of long-term

³With sticky prices, firms will target a higher markup the more uncertain future demand is.

²To name a few: Arellano et al. (2012), Bachmann and Bayer (2013), Christiano et al. (2010), Chugh (2012), Gilchrist et al. (2014), Narita (2011), Panousi and Papanikolaou (2012), Schaal (2012), and Vavra (2014) have studied the business cycle implications of a time-varying dispersion of firm-specific variables, often interpreted as and used to calibrate shocks to firm risk, propagated through various frictions: wait-and-see effects from capital adjustment frictions, financial frictions, search frictions in the labor market, nominal rigidities, and agency problems.

 $^{^{4}}$ Born and Pfeifer (2014) report an output drop of 0.025% for a similar model and a similar policy risk shock under a slightly different calibration. Regarding TFP risk they hardly find any aggregate effect.

⁵Similarly, in a search model, higher uncertainty about match quality might translate into longer search and more endogenous separation. Thus it is not clear a priori whether labor supply would increase or decrease on impact.

unemployed. This increases income risk and hence depresses aggregate demand because of higher precautionary savings. They find that such first moment shocks to the labor market can be significantly propagated and amplified through this mechanism.

Den Haan et al. (2014) consider a model with money and equity instead, where equity is not physical capital as in our model, but is equated with vacancy-ownership. In addition, they assume wage rigidity. As in our model, poorer households, in their model the unemployed, are the marginal holders of money, the low-return asset, as they effectively discount the future more. When unemployment goes up, demand for money increases. This in turn leads to deflation, pushing up real wages because nominal wages are assumed to be sticky. This has a second-round effect on money demand. Because the labor intensity of production cannot be adjusted, higher real wages depress the equity yield on existing and newly formed vacancies, which then induces portfolio adjustments by households towards money amplifying the deflations and the related output drop.

Our transmission mechanism shares to some extent this feature, but additionally highlights the importance of liquidity. Households increase their precautionary savings in conjunction with a portfolio adjustment toward the liquid asset, because its services in consumption smoothing become more valuable to households. We find that the liquidity effect is more important than the relative return effect in our model where the labor intensity of production can be adjusted.

Finally, our work relates to Gornemann et al. (2012). We discuss the distributional consequences of uncertainty shocks and of systematic monetary policy response. We find that both differently affect households that differ in their portfolios due to differential asset price movements. This portfolio composition aspect is new in comparison to Gornemann et al., because we introduce decisions regarding nominal versus real asset holdings to the household's problem.

3. Model

We model an economy inhabited by two types of agents: (worker-)households and entrepreneurs. Households supply capital and labor and are subject to idiosyncratic shocks to their labor productivity. These shocks are persistent and have a time-varying variance. Households self-insure in a liquid nominal asset (money) and a less liquid physical asset (capital). Liquidity of money is understood in the spirit of Kaplan and Violante's (2014) model of wealthy hand-to-mouth consumers, where households hold capital, but trading capital is subject to a friction. We model this trading friction as limited participation in the asset market. Every period, a fraction of households is randomly selected to trade physical capital. All other households may only adjust their money holdings.⁶ While money is subject to an inflation tax and pays no dividend, capital can be rented out to the intermediate-good-producing sector on a perfectly competitive rental market. This sector combines labor and capital services into intermediate goods and sells them to the entrepreneurs.

⁶We choose to exclude trading as a choice, and hence we use a simplified framework relative to Kaplan and Violante (2014) for numerical tractability. Random participation keeps the households' value function concave, thus making first-order conditions sufficient, and therefore allows us to use a variant of the endogenous grid method as an algorithm for our numerical calculations. See Appendix A for details.

Entrepreneurs capture all pure rents in the economy. For simplicity, we assume that entrepreneurs are risk neutral. They obtain rents from adjusting the aggregate capital stock due to convex capital adjustment costs and, more importantly, from differentiating the intermediate good. Facing monopolistic competition, they set prices above marginal costs for these differentiated goods. Price setting, however, is subject to a pricing friction à la Calvo (1983) so that entrepreneurs may only adjust their prices with some positive probability each period. The differentiated goods are finally bundled again to the composite final good used for consumption and investment.

The model is closed by a monetary authority that provides money in positive net supply and adjusts money growth according to the prescriptions of a Taylor type rule, which reacts to inflation deviations from target. All seigniorage is wasted.

3.1 Households

There is a continuum of ex-ante identical households of measure one indexed by i. Households are infinitely lived, have time-separable preferences with time-discount factor β , and derive felicity from consumption c_{it} and leisure. They obtain income from supplying labor and from renting out capital. A household's labor income $w_t h_{it} n_{it}$ is composed of the wage rate, w_t , hours worked, n_{it} , and idiosyncratic labor productivity, h_{it} , which evolves according to the following AR(1)-process:

$$\log h_{it} = \rho_h \log h_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_{ht}).$$
(1.1)

Households have Greenwood-Hercowitz-Huffman (GHH) preferences and maximize the discounted sum of felicity:

$$V = E_0 \max_{\{c_{it}, n_{it}\}} \sum_{t=0}^{\infty} \beta^t u \left(c_{it} - h_{it} G(n_{it}) \right).$$
(1.2)

The felicity function takes constant relative risk aversion (CRRA) form with risk aversion ξ :

$$u(x_{it}) = \frac{1}{1-\xi} x_{it}^{1-\xi}, \quad \xi > 0,$$

where $x_{it} = c_{it} - h_{it}G(n_{it})$ is household *i*'s composite demand for the bundled physical consumption good c_{it} and leisure. The former is obtained from bundling varieties *j* of differentiated consumption goods according to a Dixit-Stiglitz aggregator:

$$c_{it} = \left(\int c_{ijt}^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}}$$

Each of these differentiated goods is offered at price p_{jt} so that the demand for each of the varieties is given by

$$c_{ijt} = \left(\frac{p_{jt}}{P_t}\right)^{-\eta} c_{it},$$

where $P_t = \left(\int p_{jt}^{1-\eta} dj\right)^{\frac{1}{1-\eta}}$ is the average price level.

The disutility of work, $h_{it}G(n_{it})$, determines a household's labor supply given the

aggregate wage rate through the first-order condition:

$$h_{it}G'(n_{it}) = w_t h_{it}.$$
(1.3)

We weight the disutility of work by h_{it} to eliminate any Hartman-Abel effects of uncertainty on labor supply. Under the above assumption, a household's labor decision does not respond to idiosyncratic productivity h_{it} , but only to the aggregate wage w_t . Thus we can drop the household-specific index i, and set $n_{it} = N_t$. Scaling the disutility of working by h_{it} effectively sets the micro elasticity of labor supply to zero. Therefore, it simplifies the calibration as we can calibrate the model to the income risk that households face without the need to back out the actual productivity shocks. What is more, without this assumption, higher *realized* uncertainty leads to higher productivity inequality and hence increases aggregate labor supply.⁷

We assume a constant Frisch elasticity of aggregate labor supply with γ being the inverse elasticity:

$$G(N_t) = \frac{1}{1+\gamma} N_t^{1+\gamma}, \quad \gamma > 0,$$

and use this to simplify the expression for the composite consumption good x_{it} . Exploiting the first-order condition on labor supply, the disutility of working can be expressed in terms of the wage rate:

$$h_{it}G(N_t) = h_{it}\frac{N_t^{1+\gamma}}{1+\gamma} = \frac{h_{it}G'(N_t)N_t}{1+\gamma} = \frac{w_th_{it}N_t}{1+\gamma}.$$

In this way the demand for x_{it} can be rewritten as:

$$x_{it} = c_{it} - h_{it}G(N_t) = c_{it} - \frac{w_t h_{it} N_t}{1 + \gamma}.$$

Total labor input supplied is given by:

$$\tilde{N}_t = N_t \int h_{it} di.$$

Following the literature on idiosyncratic income risk, we assume that asset markets are incomplete. Households can only trade in nominal money, \tilde{m}_{it} , that does not bear any interest and in capital, k_{it} , to smooth their consumption. Holdings of both assets have to be non-negative. Moreover, trading capital is subject to a friction.

This trading friction allows only a randomly selected fraction of households, ν , to participate in the asset market for capital every period. Only these households can freely rebalance their portfolios. All other households obtain dividends, but may only adjust their money holdings. For those households participating in the capital market,

⁷Without this assumption, n_{it} increases in h_{it} , and hence the aggregate effective labor supply, $\int h_{it}n_{it}di$, increases when the dispersion of h_{it} increases. While it would not change the household's problem in its asset choices and the choice of x_{it} , it would complicate aggregation.

the budget constraint reads:

$$c_{it} + m_{it+1} + q_t k_{it+1} = \frac{m_{it}}{\pi_t} + (q_t + r_t)k_{it} + w_t h_{it} N_t, \quad m_{it+1}, k_{it+1} \ge 0$$

where m_{it} is real money holdings, k_{it} is capital holdings, q_t is the price of capital, r_t is the rental rate or "dividend," and $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate. We denote real money holdings of household *i* at the end of period *t* by $m_{it+1} := \frac{\tilde{m}_{it+1}}{P_t}$. Substituting the expression $c_{it} = x_{it} + \frac{w_t h_{it} N_t}{P_t}$ for consumption, we obtain:

$$m = \frac{\gamma}{1+\gamma}$$
 for consumption, we obtain

$$x_{it} + m_{it+1} + q_t k_{it+1} = \frac{m_{it}}{\pi_t} + (q_t + r_t)k_{it} + \frac{\gamma}{1+\gamma}w_t h_{it}N_t, \quad m_{it+1}, k_{it+1} \ge 0.$$
(1.4)

For those households that cannot trade in the market for capital the budget constraint simplifies to:

$$x_{it} + m_{it+1} = \frac{m_{it}}{\pi_t} + r_t k_{it} + \frac{\gamma}{1+\gamma} w_t h_{it} N_t, \quad m_{it} \ge 0.$$
(1.5)

Note that we assume that depreciation of capital is replaced through maintenance such that the dividend, r_t , is the net return on capital.

Since a household's saving decision will be some non-linear function of that household's wealth and productivity, the price level, P_t , and accordingly aggregate real money, $M_{t+1} = \frac{\tilde{M}_{t+1}}{P_t}$, will be functions of the joint distribution Θ_t of (m_t, k_t, h_t) . This makes Θ_t a state variable of the household's planning problem. This distribution evolves as a result of the economy's reaction to shocks to uncertainty that we model as time variations in the variance of idiosyncratic income shocks, σ_{ht}^2 . This variance follows a stochastic volatility process, which allows us to separate shocks to the variance from shocks to the level of household income.

$$\sigma_{ht}^2 = \bar{\sigma}^2 \exp(s_t), \quad s_t = \rho_s s_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N\left(-\frac{\sigma_s^2}{2(1-\rho_s^2)}, \sigma_s\right), \tag{1.6}$$

where $\bar{\sigma}^2$ is the steady state labor risk that households face, and s shifts this risk. Shocks ε_t to income risk are the only aggregate shocks in our model.

With this setup, the dynamic planning problem of a household is then characterized by two Bellman equations: V_a in the case where the household can adjust its capital holdings and V_n otherwise:

$$V_{a}(m,k,h;\Theta,s) = \max_{k',m'_{a}} u[x(m,m'_{a},k,k',h)] + \beta \left[\nu EV^{a}(m'_{a},k',h',\Theta',s') + (1-\nu)EV^{n}(m'_{a},k',h',\Theta',s')\right], V_{n}(m,k,h;\Theta,s) = \max_{m'_{n}} u[x(m,m'_{n},k,h)] + \beta \left[\nu EV^{a}(m'_{n},k,h',\Theta',s') + (1-\nu)EV^{n}(m'_{n},k,h',\Theta',s')\right].$$
(1.7)

In line with this notation, we define the optimal consumption policies for the adjustment and non-adjustment cases as x_a^* and x_n^* , the money holding policies as m_a^* and m_n^* , and the capital investment policy as k^* . Details on the properties of the value functions (smooth and concave) and policy functions (differentiable and increasing in total resources), the first-order conditions, and the algorithm we employ to calculate the policy functions can be found in Appendix A.

3.2 Intermediate Goods Producers

Intermediate goods are produced with a constant returns to scale production function:

$$Y_t = \tilde{N}_t^{\alpha} K_t^{(1-\alpha)}.$$

Let MC_t be the relative price at which the intermediate good is sold to entrepreneurs. The intermediate-good producer maximizes profits,

$$MC_t Y_t = MC_t \tilde{N}_t^{\alpha} K_t^{(1-\alpha)} - w_t \tilde{N}_t - (r_t + \delta) K_t,$$

but it operates in perfectly competitive markets, such that the real wage and the user costs of capital are given by the marginal products of labor and capital:

$$w_t = \alpha M C_t \left(K_t / \tilde{N}_t \right)^{1-\alpha}, \qquad (1.8)$$

$$r_t + \delta = (1 - \alpha) M C_t \left(\tilde{N}_t / K_t \right)^{\alpha}.$$
(1.9)

3.3 Entrepreneurs

Entrepreneurs differentiate the intermediate good and set prices. They are risk neutral and have the same discount factor as households. We assume that only the central bank can issue money so that entrepreneurs participate in neither the money nor the capital market. This assumption gives us tractability in the sense that it separates the entrepreneurs' price setting problem from the households' saving problem. It enables us to determine the price setting of entrepreneurs without having to take into account households' intertemporal decision making. Under these assumptions, the consumption of entrepreneur j equals her current profits, Π_{jt} . By setting the prices of final goods, entrepreneurs maximize expected discounted future profits:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Pi_{jt}.$$
 (1.10)

Entrepreneurs buy the intermediate good at a price equalling the nominal marginal costs, MC_tP_t , where MC_t is the real marginal costs at which the intermediate good is traded due to perfect competition, and then differentiate them without the need of additional input factors. The goods that entrepreneurs produce come in varieties uniformly distributed on the unit interval and each indexed by $j \in [0, 1]$. Entrepreneurs are monopolistic competitors, and hence charge a markup over their marginal costs. They are, however, subject to a Calvo (1983) price setting friction, and can only update their prices with probability θ . They maximize the expected value of future discounted profits by setting today's price, p_{jt} , taking into account the price setting friction:

$$\max_{\{p_{jt}\}} \sum_{s=0}^{\infty} (\theta\beta)^s E \Pi_{jt,t+s} = \sum_{s=0}^{\infty} (\theta\beta)^s E Y_{jt,t+s} (p_{jt} - MC_{t+s}P_{t+s})$$
(1.11)

$$s.t.: Y_{jt,t+s} = \left(\frac{p_{jt}}{P_{t+s}}\right)^{-\eta} Y_{t+s}$$

where $\Pi_{jt,t+s}$ is the profits and $Y_{jt,t+s}$ is the production level in t+s of a firm j that set prices in t.

We obtain the following first-order condition with respect to p_{jt} :

$$\sum_{s=0}^{\infty} (\theta\beta)^s EY_{jt,t+s} \left(\frac{p_{jt}^*}{P_{t-1}} - \frac{\eta}{\underbrace{\eta-1}_{\mu}} MC_{t+s} \frac{P_{t+s}}{P_{t-1}} \right) = 0,$$
(1.12)

where μ is the static optimal markup.

Recall that entrepreneurs are risk neutral and that they do not interact with households in any intertemporal trades. Moreover, *aggregate* shocks to the economy are small and homoscedastic, since the only aggregate shock we consider is the shock to the variance of housdehold income shocks. Therefore, we can solve the entrepreneurs' planning problem locally by log-linearizing around the zero inflation steady state, without having to know the solution of the households' problem. This yields, after some tedious algebra (see, e.g., Galí, 2008), the New Keynesian Phillips curve:

$$\log \pi_t = \beta E_t (\log \pi_{t+1}) + \kappa (\log M C_t + \mu), \qquad (1.13)$$

where

$$\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}.$$

We assume that besides differentiating goods and obtaining a rent from the markup they charge, entrepreneurs also obtain and consume rents from adjusting the aggregate capital stock. Since the dividend yield is below their time-preference rate, in equilibrium entrepreneurs never hold capital. The cost of adjusting the stock of capital is $\frac{\phi}{2} \left(\frac{\Delta K_{t+1}}{K_t}\right)^2 K_t + \Delta K_{t+1}$. Hence, entrepreneurs will adjust the stock of capital until the following first-order condition holds:⁸

$$q_t = 1 + \phi \frac{\Delta K_{t+1}}{K_t}.$$
 (1.14)

⁸Note that we assume capital adjustment costs only on new capital (or on the active destruction of old capital) but not on the replacement of depreciation. Depreciated capital is assumed to be replaced at the cost of one-to-one in consumption goods, and replacement is forced before the capital stock is adjusted at a cost. This differential treatment of depreciation and net investment simplifies the equilibrium conditions substantially, because the user cost of capital and hence the dividend paid to households do not depend on the next period's stock of capital, and the decisions of non-adjusters are not influenced by the price of capital q_t . Quantitatively, the fluctuations in dividends that maintenance at price q_t would bring about are negligible. Upon a 2 standard deviation shock to uncertainty, q_t falls to 0.96 – hence reducing depreciation cost by 4 basis points quarterly under the alternative specification where maintenance comes at cost q_t .

3.4 Goods, Money, Capital, and Labor Market Clearing

The labor market clears at the competitive wage given in (1.8); so does the market for capital services if (1.9) holds. We assume that the money supply is given by a monetary policy rule that adjusts the growth rate of money in order to stabilize inflation:

$$\frac{M_{t+1}}{M_t} = \left(\theta_1 / \pi_t\right)^{1+\theta_2} \left(\frac{M_t}{M_{t-1}}\right)^{\theta_3}$$
(1.15)

Here M_{t+1} is the real balances at the end of period t (with the timing aligned to our notation for the households' budget constraint). The coefficient $\theta_1 \geq 1$ determines steady-state inflation, and $\theta_2 \geq 0$ the extent to which the central bank attempts to stabilize inflation around its steady-state value: the larger θ_2 the stronger is the reaction of the central bank to deviations from the inflation target. When $\theta_2 \to \infty$ inflation is perfectly stabilized at its steady-state value. $\theta_3 \geq 0$ captures persistence in money growth. We assume that the central bank wastes any seigniorage buying final goods and choose the above functional form for its simplicity.⁹

The money market clears whenever the following equation holds:

$$(\theta_1/\pi_t)^{1+\theta_2} \left(\frac{M_t}{M_{t-1}}\right)^{\theta_3} M_t = \int \left[\nu m_a^*(m,k,h;q_t,\pi_t) + (1-\nu)m_n^*(m,k,h;q_t,\pi_t)\right] \\ \Theta_t(m,k,h) dm dk dh,$$
(1.16)

with the end-of-period real money holdings of the preceding period given by

$$M_t := \int m\Theta_t(m,k,h) dm dk dh.$$

Last, the market for capital has to clear:

$$q_{t} = 1 + \phi \frac{K_{t+1} - K_{t}}{K_{t}} = 1 + \nu \phi \frac{K_{t+1}^{*} - K_{t}}{K_{t}}, \qquad (1.17)$$

$$K_{t+1}^{*} := \int k^{*}(m, k, h; q_{t}, \pi_{t}) \Theta_{t}(m, k, h) dm dk dh,$$

$$K_{t+1} = K_{t} + \nu (K_{t+1}^{*} - K_{t}),$$

where the first equation stems from competition in the production of capital goods, the second equation defines the aggregate supply of funds from households trading capital, and the third equation defines the law of motion of aggregate capital. The goods market

⁹For the baseline calibration this is an innocuous assumption. With constant nominal money growth, the changes in seigniorage are negligible in absolute terms. Steady-state seigniorage is .64% of annual output, since money growth is 2% and the money-to-output ratio is 32%. When inflation drops, say, from 2% to 0, the real value of seigniorage increases, but only from .64% to .66% of output. As $\theta_2 \to \infty$, seigniorage occasionally turns slightly negative. It is numerically very expensive to put a constraint on M_t , and hence we abstain from doing so to keep the dynamic problem tractable. This unboundedness of seigniorage only affects the effectiveness of the stabilization policy. The central bank can commit to decrease seigniorage more in the future without the requirement of (weakly) positive seigniorage. One possible assumption to rationalize this is to assume that seigniorage is not wasted on government consumption but is used to store goods in an inefficient way.

then clears due to Walras' law, whenever both money and capital markets clear.

3.5 Recursive Equilibrium

A recursive equilibrium in our model is a set of policy functions $\{x_a^*, x_n^*, m_a^*, m_n^*, k^*\}$, value functions $\{V_a, V_n\}$, pricing functions $\{r, w, \pi, q\}$, aggregate capital and labor supply functions $\{N, K\}$, distributions Θ_t over individual asset holdings and productivity, and a perceived law of motion Γ , such that

- 1. Given $\{V_a, V_n\}$, Γ , prices, and distributions, the policy functions $\{x_a^*, x_n^*, m_a^*, m_n^*, k^*\}$ solve the households' planning problem, and given the policy functions, prices and distributions, the value functions $\{V_a, V_n\}$ are a solution to the Bellman equations (1.7).
- 2. The labor, the final-goods, the money, the capital, and the intermediate-good markets clear, i.e., (1.8), (1.13), (1.16), and (1.17) hold.
- 3. The actual law of motion and the perceived law of motion Γ coincide, i.e., $\Theta' = \Gamma(\Theta, s')$.

4. Numerical Implementation

The dynamic program (1.7) and hence the recursive equilibrium is not computable, because it involves the infinite dimensional object Θ_t .

4.1 Krusell-Smith Equilibrium

To turn this problem into a computable one, we assume that households predict future prices only on the basis of a restricted set of moments, as in Krusell and Smith (1997, 1998a). Specifically, we make the assumption that households condition their expectations only on last period's aggregate real money holdings, M_t , last period's aggregate real money growth, $\Delta(\log M_t)$, the aggregate stock of capital, K_t , and the uncertainty state, s_t . The reasoning behind this choice goes as follows: (1.16) determines inflation, which in turn depends on the beginning of period money stock and last period's money growth. Once inflation is fixed, the Phillips curve (1.13) determines markups and hence wages and dividends. These will pin down asset prices by making the marginal investor indifferent between money and physical capital. If asset-demand functions, $m_{a,n}^*$ and k^* , are sufficiently close to linear in human capital, h, and in non-human wealth, m, k, at the mass of Θ_t , we can expect approximate aggregation to hold. For our exercise, the four aggregate states $-s_t$, M_t , $\Delta(\log M_t)$, and K_t – are sufficient to describe the evolution of the aggregate economy.¹⁰

While the law of motion for s_t is pinned down by (1.6), households use the following log-linear forecasting rules for current inflation and the price of capital, where the

¹⁰Without persistence in money growth, Equation (1.16) does not depend on $\Delta(\log M_t)$ anymore making it a redundant state. In this case, we set $\beta_{\pi,q}^4 = 0$.

coefficients depend on the uncertainty state:

$$\log \pi_t = \beta_{\pi}^1(s_t) + \beta_{\pi}^2(s_t) \log M_t + \beta_{\pi}^3(s_t) \log K_t + \beta_{\pi}^4(s_t) \Delta(\log M_t), \quad (1.18)$$

$$\log q_t = \beta_q^1(s_t) + \beta_q^2(s_t) \log M_t + \beta_q^3(s_t) \log K_t + \beta_q^4(s_t) \Delta(\log M_t).$$
(1.19)

The law of motion for real money holdings, M_t , then follows from the monetary policy rule and is given by:

$$\log M_{t+1} = \log M_t + (1+\theta_2)(\log \theta_1 - \log \pi_t) + \theta_3 \Delta(\log M_t).$$

The law of motion for K_t results from (1.17).

Fluctuations in q and π happen for two reasons: As uncertainty goes up, the selfinsurance service that households receive from the illiquid capital good decreases. In addition, the rental rate of capital falls as firms' markups increase. When making their investment decisions, households need to predict the next period's capital price q' to determine the expected return on their investment. Since all other prices are known functions of the markup, only π' and q' need to be predicted.

Technically, finding the equilibrium is similar to Krusell and Smith (1997), as we need to find market clearing prices within each period. Concretely, this means the posited rules, (1.18) and (1.19), are used to solve for households' policy functions. Having solved for the policy functions conditional on the forecasting rules, we then simulate n independent sequences of economies for $t = 1, \ldots, T$ periods, keeping track of the actual distribution Θ_t . In each simulation the sequence of distributions starts from the stationary distribution implied by our model without aggregate risk. We then calculate in each period t the optimal policies for market clearing inflation rates and capital prices assuming that households resort to the policy functions derived under rule (1.18) and (1.19) from period t+1 onward. Having determined the market clearing prices, we obtain the next period's distribution Θ_{t+1} . In doing so, we obtain n sequences of equilibria. The first 250 observations of each simulation are discarded to minimize the impact of the initial distribution. We next re-estimate the parameters of (1.18)and (1.19) from the simulated data and update the parameters accordingly. By using n = 20 and T = 750, it is possible to make use of parallel computing resources and obtain 10.000 equilibrium observations. Subsequently, we recalculate policy functions and iterate until convergence in the forecasting rules.

The posited rules (1.18) and (1.19) approximate the aggregate behavior of the economy fairly well. The minimal within sample R^2 is above 99%. Also the out-of-sample performance (see Den Haan, 2010)) of the forecasting rules is good. See Appendix D.

4.2 Solving the Household Planning Problem

In solving for the households' policy functions we apply an endogenous gridpoint method as originally developed in Carroll (2006) and extended by Hintermaier and Koeniger (2010), iterating over the first-order conditions. We approximate the idiosyncratic productivity process by a discrete Markov chain with 17 states and time-varying transition probabilities, using the method proposed by Tauchen (1986). The stochastic volatility process is approximated in the same vein using 7 states.¹¹ Details on the algorithm can be found in Appendix A.4.

5. Calibration

We calibrate the model to the U.S. economy. The behavior of the model in steady state without fluctuations in uncertainty does not correspond to the time-averages of the simulated variables in the model with uncertainty shocks. Hence we cannot use the steady state to calibrate the model, but instead iterate over the full model to match the calibration targets.¹² The aggregate data used for calibration spans 1980 to 2012. One period in the model refers to a quarter of a year. The choice of parameters as summarized in Tables 1 and 2 is explained next. We present the parameters as if they were individually changed in order to match a specific data moment, but all calibrated parameters are determined jointly of course.

5.1 Income Process

We estimate the income process and hence uncertainty faced by households from income data in the Cross-National Equivalent File (CNEF) of the Panel Study of Income Dynamics (PSID), excluding the low-income sample. We construct household income as pre-tax labor income plus private and public transfers minus all taxes, and control for observable household characteristics in a first stage regression. We use the residual income to estimate the parameters governing the idiosyncratic income process ρ_s , ρ_h , $\bar{\sigma}$, and σ_s .

In a first stage regression for log-income, we control nonparametrically for the effects of age, household size, and educational attainment and parametrically with up to squared-order terms in age for the age-education interaction. We then generate variances and first and second order auto-covariances of residual income by age groups for the years 1970-2009. Based on these age-year variances and covariances, the parameters of interests are estimated by generalized method of moments (GMM). We find that the implied quarterly autocorrelation of the persistent component of income, ρ_h , is 0.976 and the average standard deviation of quarterly persistent income shocks is $\bar{\sigma} = 0.078$. The implied quarterly persistence of income risk, ρ_s , is 0.903 and thus in line with business cycle frequencies. The annual coefficient of variation for income risk, $\frac{\sigma_s}{\bar{\sigma}}$, is 0.62, which is consistent with the estimates in Storesletten et al. (2004).¹³ Table 1.1 summarizes the parameter estimates, where the values are adapted to the quarterly frequency of our model. Details on data selection and the estimation procedure can be found in Appendix B.

¹¹We solve the household policies for 30 points on the grid for money and 50 points on the grid for capital using equi-distant grids on log scale plus outliers. For aggregate money and capital holdings we use a relatively coarse grid of 3 points each. We experimented with changing the number of gridpoints without a noticeable impact on results. See Appendix D.

¹²As this is very expensive computational-wise, we match the target-ratios within +/-1%.

¹³Storesletten et al. estimate the variance of persistent shocks to annual income to be 126% higher in times of below average GDP growth than in times of above average GDP growth. This implies that the unconditional *annual* coefficient of variation of s is roughly 0.5.

Parameter	Value	Description
${ ho_h\overar\sigma}$		Persistence of income Average STD of innovations to income
$ ho_s \ \sigma_s$		Persistence of the income-innovation variance, σ_h^2 Conditional STD (log scale) of σ_h^2

Table 1.1: Estimated parameters of the income process

Notes: All values are adapted to the quarterly frequency of the model. For details on the estimation see Appendix B.

5.2 Preferences and Technology

While we can estimate the income process directly from the data, all other parameters are calibrated within the model. Table 3.1 summarizes our calibration. In detail, we choose the parameter values as follows.

Households

2

For the felicity function, $u = \frac{1}{1-\xi}x^{1-\xi}$, we set the coefficient of relative risk aversion $\xi = 4$, as in Kaplan and Violante (2014). The time-discount factor, β , and the asset market participation frequency, ν , are jointly calibrated to match the ratios of liquid and illiquid assets to output. We equate illiquid assets to all capital goods at current replacement values. This implies for the total value of illiquid assets relative to nominal GDP a capital-to-output ratio of 286%. In our baseline calibration, this implies an annual real return for illiquid assets of 3.2%. We equate liquid assets to claims of the private sector against the government and not to inside money, because the net value of inside claims does not change with inflation. Specifically, we look at average U.S. federal debt for the years 1980 to 2012 held by domestic private agents plus the monetary base. This yields an annual money-to-output ratio of 32%. For details on the steady-state asset distribution, see Appendix C. The calibrated participation frequency $\nu = 4.25\%$ is close to Kaplan and Violante's estimate for working households in their state-dependent participation framework. We take a conservative value for the inverse Frisch elasticity of labor supply, $\gamma = 2$, corresponding to the estimates by microeconometric studies. We provide a robustness check with an estimate of the inverse Frisch elasticity of labor supply, $\gamma = 1$, which follows the New Keynesian literature (Chetty et al. (2011)).

Parameter	Value	Description	Target
Household	ls		
β	0.987	Discount factor	K/Y = 286% (annual)
ν	4.25%	Participation frequency	M/Y = 32% (annual)
ξ	4	Coefficient of rel. risk av.	Kaplan and Violante (2014)
$\overset{\circ}{\gamma}$	2	Inverse of Frisch elasticity	Standard value
Intermedia	ate Goo	ods	
α	0.73	Share of labor	Income share of labor of $2/3$
δ	1.35%	Depreciation rate	NIPA: Fixed assets
Final Goo	\mathbf{ds}		
κ	0.09	Price stickiness	Mean price duration of 4 quarters
μ	0.10	Markup	10% markup (standard value)
Capital G	\mathbf{oods}		
ϕ	220	Capital adjustment costs	Relative investment volatility of 3
Monetary	Policy	(Friedman's $k\%$ rule)	
θ_1	1.005	Money growth	2% p.a.
θ_2	0	Reaction to inflation deviations	-
θ_3	0	Persistence in money growth	

Intermediate, Final, and Capital Goods Producers

We parameterize the production function of the intermediate good producer according to the U.S. National Income and Product Accounts (NIPA). In the U.S. economy the income share of labor is about 2/3. Accounting for profits we hence set $\alpha = 0.73$.

To calibrate the parameters of the entrepreneurs' problem, we use standard values for markup and price stickiness that are widely employed in the New Keynesian literature. The Phillips curve parameter κ implies an average price duration of 4 quarters, assuming flexible capital at the firm level. The steady-state marginal costs, $exp(-\mu) = 0.91$, imply a markup of 10%. The entrepreneurs' and households' discount factor are equal.

We calibrate the adjustment cost of capital, $\phi = 220$, to match an investment to output volatility of 3.

Table 1.3: Alternative monetary policy rules

Parameter	Value	Description	Target
Inflation S	Stabiliz	ation	
$ heta_1$	1.005	Money growth	2% p.a.
$ heta_2$	1000	Reaction to inflation deviations	No deviations from target
$ heta_3$	0	Persistence in money growth	
Fed Policy	v Rule	(Post-1980)	
$ heta_1$	1.005	Money growth	2% p.a.
$ heta_2$	0.35	Reaction to inflation deviations	Chowdhury and Schabert (2008)
$ heta_3$	0.9	Persistence in money growth	Chowdhury and Schabert (2008)

Notes: For the Fed policy rule as well as all robustness checks, we recalibrate the discount factor and the participation frequency of households to match the targeted capital and money to output ratios and the capital adjustment costs to match a relative investment volatility of 3.

Central Bank

We set the average growth rate of money, θ_1 , such that our model produces an average annual inflation rate of 2%, in line with the usual inflation targets of central banks and roughly equal to average inflation in the U.S. between 1980 and 2012. To simplify the dynamics of the model and for expositional purpose, we assume in our baseline setup that the central bank follows Friedman's k% rule and hence set θ_2 and θ_3 to 0. Alternatively, we consider two additional policy rules, see Table 1.3. First, we set $\theta_2 = 1000$ and $\theta_3 = 0$, to examine uncertainty shocks without movements in the price level. Second, we calibrate towards the post-1980s money supply rule of the Federal Reserve as estimated in Chowdhury and Schabert (2008) to quantify the contribution of uncertainty shocks to the U.S. business cycle over this period. This implies $\theta_2 = 0.35$ and $\theta_3 = 0.9$.¹⁴

6. Quantitative Results

6.1 Household Portfolios and the Individual Response to Uncertainty

In our model, households hold money because it provides better short-term consumption smoothing than capital, as the latter can only be traded infrequently. This value of

¹⁴Originally, Chowdhury and Schabert report Taylor rules for money including a reaction to the output gap. We obtain $\theta_2 = 0.35$ by using the Phillips curve from our model to eliminate the output gap.

Figure 1.2: Share of liquid assets in total net worth against percentiles of wealth



Notes: For graphical illustration we make use of an Epanechnikov Kernelweighted local linear smoother with bandwidth 0.15. For the definition of net liquid assets see Figure 1.1.

liquidity decreases in the amount of money a household holds, because a household rich in liquid assets will likely be able to tap into its illiquid wealth before running down all liquid wealth. For this reason, richer households, who typically hold both more money and more capital, hold less liquid portfolios. The poorest households, on the contrary, hold almost all their wealth in the liquid asset. This holds true in the actual data as well as in our model. While our model matches relatively well the shape of the actual liquidity share of household portfolios at all wealth percentiles, it slightly underestimates the share of liquid assets for the lowest deciles; see Figure 1.2, which compares our model to the *Survey of Consumer Finances* 2004.

So what happens to total savings and its composition when uncertainty increases? In response to the increase in income uncertainty, households aim for higher precautionary savings to be in a better position to smooth their consumption. Since the liquid asset is better suited to this purpose, households first increase their demand for this asset – in fact, they even reduce holdings of the illiquid asset to increase the liquidity of their portfolio. Figure 1.3 shows how households' portfolio composition and consumption policy react to an increase in uncertainty without imposing any market clearing. The top panels displays the relative change in the consumption and portfolio liquidity compared to the average uncertainty state. For this exercise, we evaluate households' consumption policies and the portfolio choice of adjusters and non-adjusters after a 2 standard deviation shock to uncertainty, increasing the variance of idiosyncratic income shocks by 55%. We here perform a partial equilibrium analysis and compute the policies
Figure 1.3: Partial equilibrium response – Change in individual policy upon an uncertainty shock keeping prices and expectations constant at steady-state values



Notes: Top Panels: Reaction of individual consumption demand and portfolio liquidity of adjusters and non-adjusters at constant prices and price expectations relative to the respective counterpart at average uncertainty. The policies are averaged using frequency weights from the steady-state wealth distribution and reported conditional on a household falling into the x-th wealth percentile. High uncertainty corresponds to a two standard deviation shock, which is equal to a 55% increase in uncertainty.

Bottom Panels: Fraction of total demand change for money and capital accounted for by all households in a given percentile of the wealth dristribution.

As with the data, we use an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15.

under the expectation that all prices are at their steady-state values isolating hence the direct effect of income uncertainty. Across all wealth levels, households wish to increase their savings (i.e., decrease their consumption) as well as the liquidity of their portfolios when uncertainty goes up. Adjusters can do so by tipping into their capital account and thus their consumption falls less. This flight to liquidity leads to falling demand for capital even though total savings increase.

The bottom panels of Figure 1.3 display the contribution of each wealth percentile to the total change in demand for money and capital. Values above (below) one imply that a certain percentile of the wealth distribution is contributing more (less) than proportionally. We find that almost all wealth groups are equally important for the change in total asset demand. In other words, poorer households, while making up a

Figure 1.4: General equilibrium response – Change in the liquidity of household portfolios



Notes: Change in the distribution of liquidity at all percentiles of the wealth distribution at equilibrium prices and price expectations for $s = \{1, 8\}$ quarters after a two standard deviation shock to income uncertainty. The liquidity of the portfolios is averaged using frequency weights from the steady-state wealth distribution and reported conditional on a household falling into the x-th wealth percentile. The left-hand panel shows the change including changes in prices; the right-hand panel shows the pure quantity responses. As with the data, we use an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15.

smaller fraction of total asset demand, observe larger changes in their asset positions and hence are as important as richer households for the aggregate demand changes.

The change in the liquidity of household portfolios in general equilibrium is displayed in Figure 1.4; the left-hand panel shows the change in value terms; the right-hand panel shows the change in quantities, i.e., at constant prices. Portfolio liquidity initially increases at all wealth levels – in particular in value terms because the price of illiquid assets drops sharply as we will see in the next section. The increase in the share of liquid assets is least pronounced for the poorest, because of the negative income effect. After two years, the increase in liquidity is concentrated at households somewhat below median wealth. By then, rich households have partially reversed their portfolio shares as they also increase their savings in physical capital, exploiting lower capital prices. Interestingly, this picture is exactly what we found in Figure 1.1, where the increase in the liquidity of the portfolios is strongest for the lower middle class. Only the magnitude of changes in the liquidity of household portfolios during the Great Recession is much more dramatic.

6.2 Aggregate Consequences of Uncertainty Shocks

Main Findings

This simultaneous decrease in the demand for consumption and capital upon an increase in uncertainty leads to a decline in output. Figure 1.5 displays the impulse responses of output and its components, real balances and the capital stock as well as asset prices and returns for our baseline calibration. The assumed monetary policy follows a strict



Figure 1.5: Uncertainty shock under constant money growth



Impulse responses to a 2 standard deviation increase in the variance of idiosyncratic productivity. We generate these impulses by averaging over 100.000 independent simulations of the law of motions, Equations 1.18 and 1.19, that simultaneously receive the shock in T = 500. All rates (inflation, dividends, etc.) are *not* annualized.

money growth rule, i.e., it is not responsive to inflation. After a two standard deviation increase in the variance of idiosyncratic productivity shocks, output drops on impact by 0.5% and only returns to the normal growth path after roughly 20 quarters. Over the first year the output drop is 0.37% on average.

The output drop in our model results from households increasing their precautionary savings in conjunction with a portfolio adjustment toward the liquid asset. In times of high uncertainty, households dislike illiquid assets because of their limited use for short-run consumption smoothing. Conversely, the price of capital decreases on impact by 4%. Since the demand for the liquid asset is a demand for paper and not for (investment) goods, demand for both consumption and investment goods falls.

This decrease in demand puts pressure on prices. Inflation falls by 65 basis points on impact, increasing the average markup in the economy. Thus, the marginal return on capital, r_t , and consequently investment demand decline, while the return on money goes up. Thereby, the flight to liquidity increases the relative return of money, which further amplifies the portfolio adjustment. In line with the excess stock volatility puzzle, uncertainty shocks move capital prices and expected returns much more (and in the opposite direction) than they move dividends (65 vs. -4 basis points, quarterly).

Stabilization Policy

How much of this is driven by the increased value of liquidity, and how much by the differential impact of disinflation on the return of money and on dividends? We can isolate the flight to liquidity from the effect of the change in relative returns by looking at a monetary policy that is stabilizing the economy – setting $\theta_2 = 1000, \theta_3 = 0$. Under this policy, inflation is fixed and output barely moves. Also dividends are virtually constant. Thus, the relative-return effect vanishes in the case of strict inflation targeting. The corresponding impulse responses are displayed in Figure 1.6. As a consequence of the stabilization, the price of capital falls less, but it still falls by more than 2%. The expected return on capital increases by about 50 basis points. The total income of households almost stays constant in the first 5 years and hence money demand peaks at an even higher level than without stabilization.

In other words, the portfolio adjustment is to a large extent driven by a flight to liquidity. After roughly 2.5 years, real balances have increased to a point where households are well insured and want to increase their holdings of the illiquid asset again.

Quantitative Importance: Fed Policy Rule

Figure 1.7 displays the aggregate consequences of shocks to household income risk using the Fed's post-1980's money supply reaction function as estimated by Chowdhury and Schabert (2008). The results are roughly half way between perfect stabilization and constant money growth.

How Important Is the (II)liquidity of Capital?

Our calibration suggests that households can adjust their capital holdings on average every 23.5 quarters. This restricted access to savings in capital limits its use for short-



Figure 1.6: Uncertainty shock under inflation stabilization



Impulse responses to a 2 standard deviation increase in the variance of idiosyncratic productivity. We generate these impulses by averaging over 100.000 independent simulations of the law of motions, Equations 1.18 and 1.19, that simultaneously receive the shock in T = 500. All rates (inflation, dividends, etc.) are *not* annualized.

Figure 1.7: Uncertainty shock under Fed's post-80's reaction function as estimated in Chowdhury and Schabert (2008)





Impulse responses to a 2 standard deviation increase in the variance of idiosyncratic productivity. We generate these impulses by averaging over 100.000 independent simulations of the law of motions, Equations 1.18 and 1.19, that simultaneously receive the shock in T = 500. All rates (inflation, dividends, etc.) are *not* annualized.



Figure 1.8: Uncertainty shock with liquid capital ($\nu = 35\%$)

Notes: Expected net real return: $\frac{Eq_{t+1}+r_t}{q_t}$.

Impulse responses to a 2 standard deviation increase in the variance of idiosyncratic productivity. We generate these impulses by averaging over 100.000 independent simulations of the law of motions, Equations 1.18 and 1.19, that simultaneously receive the shock in T = 500. All rates (inflation, dividends, etc.) are *not* annualized.

run consumption smoothing considerably. If capital were easier to access, it would become more and more of a substitute for money in terms of its use for consumption smoothing. Hence, aggregate money holdings decline as ν increases. Figure 1.8 plots the impulse responses for an average adjustment frequency of less than a year ($\nu = 35\%$). In this case money holdings are only 8.5% of annual output on average, which corresponds to the U.S. monetary base.¹⁵

Figure 1.8 shows that the output drop is very similar with a higher portfolio adjustment frequency, although the share of money in the economy is significantly smaller and capital is very liquid in comparison to the baseline calibration. Money demand reacts more elastically to uncertainty as more households are able to adjust their portfolio. Consequently, the flight to liquidity is stronger and happens faster than with

 $^{^{15}\}mathrm{We}$ use the St. Louis Fed adjusted annual monetary base from 1980 to 2012.

more illiquid capital – in the build-up and in the reverse.

In summary, the macroeconomic effects of uncertainty shocks are robust to changes in ν . While in the limit with perfectly liquid capital money is driven out of the economy, the economy seems to not converge toward the "Aiyagari" economy without money and perfectly liquid capital. In the "Aiyagari" case, investment replaces consumption demand one-for-one when uncertainty hits. As long as households hold even tiny amounts of money for liquidity-consumption smoothing reasons, the value of money increases with income uncertainty and money demand is higher in uncertain times, which creates deflationary pressures.

In other words, and more generally speaking, uncertainty shocks will affect aggregate demand negatively only if they trigger precautionary savings in paper and not in real assets. In our model, it is the increased value of liquidity that is responsible for the portfolio adjustment toward money.

6.3 Redistributive and Welfare Effects

So far we have described the aggregate dimension of an uncertainty shock and its repercussions. Since such shocks affect the price level, asset prices, dividends, and wages differently, our model predicts that not all agents (equally) lose from the decline in consumption upon an uncertainty shock. For example, if capital prices fall, those agents that are rich in human capital but hold little physical capital could actually gain from the uncertainty shock. These agents are net savers. They increase their holdings of physical capital and can do so now more cheaply.

To quantify and understand the relative welfare consequences of the uncertainty shock and of systematic policy response, one would normally just look at the change in a household's value function. However, since solving directly for the value function is prohibitively time consuming in our model, we instead simulate and compare two sets of economies: one where the uncertainty state simply evolves according to its Markov chain properties and another set where, at time T, we exogenously increase income uncertainty, σ_{ht}^2 , by setting the shock to uncertainty to $\epsilon_T = 2\sigma_s$, a 2 standard deviation increase. We then let the economies evolve stochastically. We trace agents over the next S periods for both sets of economies, and track their period-felicity u_{iT+t} to calculate for each agent with individual state (h, m, k) in period T the discounted expected felicity stream over the next S periods as:

$$v_S(h,m,k) = E\left[\sum_{t=0}^{S} \beta^t u_{T+t} \middle| (h_T,m_T,k_T) = (h,m,k)\right],$$

where u_{T+t} is the felicity stream in period T + t under the household's optimal saving policy. For large S, v_S approximates the actual household's value function.

We then determine an equivalent consumption tax that households would be willing to face over the next S quarters in order to eliminate the uncertainty shock at time T as:

$$CE = \left(\frac{v_S^{\text{shock}}}{v_S^{\text{no shock}}}\right)^{1/(1-\xi)} - 1.$$
(1.20)



Figure 1.9: Welfare after 5 years

Notes: Welfare costs in terms of consumption equivalents (CE) as defined in (1.20). The graphs refer to the conditional expectations of CE with respect to the two displayed dimensions, respectively. The missing dimension has been integrated out. Capital and money are reported in terms of quarterly income.

	Policy regime: Constant money growth										
	Quintiles of money holdings					Qu	Quintiles of capital holdings				
	$\frac{1}{1.}$	2.	3.	4.	5.	1.	2.	3.	4.	5.	
Conditional	-0.92	-0.73	-0.55	-0.38	-0.18	-0.53	-0.47	-0.51	-0.55	-0.61	
Median	-0.96	-0.78	-0.57	-0.35	-0.02	-0.63	-0.59	-0.61	-0.64	-0.69	
	Qu	intiles	of Hum	an Cap	ital						
	0.00	0.00	0 50	0.45	0.49						
Conditional Median	-0.62	-0.62	-0.58	-0.45	-0.43						
	-0.59	-0.61	-0.60	-0.52	-0.58						
	Policy regime: Inflation stabilization										
			of mone	·			intiles of	-			
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.	
Conditional	-0.21	-0.39	-0.39	-0.35	-0.31	-0.43	-0.36	-0.32	-0.29	-0.25	
Median	-0.21 -0.41	-0.39 -0.42	-0.39 -0.37	-0.35 -0.29	-0.31 -0.14	-0.45 -0.57		-0.32 -0.37		-0.25	
Median			of hum			-0.57	-0.45	-0.57	-0.31	-0.20	
	Q(or num	an capi							
Conditional	-0.08	-0.22	-0.35	-0.40	-0.53						
Median	-0.09	-0.22	-0.39	-0.45	-0.62						
			Ро	licy reg	jime: Fed	l reactio	n funct	ion			
			of mone	0			intiles of	-			
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.	
	0.50	0 55	0.47	0.20	0.96	0.49	0.49	0.41	0.49	0.49	
Conditional Madian							-0.42 -0.52				
Median			-0.47 of hum		-0.09 tal	-0.00	-0.92	-0.30	-0.47	-0.40	
	ୁ ଏ	unnenes	or num	an capi	uai						
Conditional	-0.34	-0.42	-0.46	-0.42	-0.48						
Median		-0.42									
	0.04	0.14	0.10	0.10	0.00						

Notes: Welfare costs in terms of consumption equivalents (CE) as defined in (1.20). Conditional refers to integrating out the missing dimensions, whereas Median refers to median asset holdings of the respective other assets. We track households over 20 quarters and average over 100 independent model simulations.

Figure 1.9 displays the relative differences in v_S for S = 20 quarters in terms of consumption equivalents, CE, between the two sets of simulations of the economy. This time horizon captures the welfare consequences of the recession following the uncertainty shock. See Appendix E for an assessment of welfare after more than 75 years, when the initial position, (h_T, m_T, k_T) , has washed out in the sense that the conditional and the unconditional distributions are almost identical. Of course, in the long run there are no differences between the two sets of economies.

On average, households would be willing to forgo roughly 0.41% of their consumption over 5 years to eliminate the uncertainty shock. This average loss masks heterogeneous effects across households with different asset positions and human capital. While monetary policy can reduce the cost to roughly 0.25% on average, it also shifts the burden of the shock between households. Figure 1.9 displays the expected welfare costs of households conditioning on two of the three dimensions of the (h, m, k)-space – integrating out the missing dimension.

Without stabilization, money rich and physical asset poor households lose the least. These are households that typically acquire physical capital in exchange for their money holdings, and they can do so at favorable capital prices after the uncertainty shock. For a similar reason, the steepness of the gradient in human capital is relatively modest. After the shock, human capital rich households suffer from lower wages, but as savers they are partly compensated, because they can acquire physical capital at lower prices. Table 1.4 summarizes the figures numerically. In this table, we condition on just one dimension of the households' portfolio, and display the average relative welfare gains. We do so in two ways: First, we calculate welfare conditional on one asset taking the conditional distribution of the other two assets into account. Second, we also report welfare effects at median asset holdings of the respective other assets. The latter isolates the direct effect in the dimension of interest.

Table 1.4 and Figure 1.9 shows that the intervention of the central bank helps households with high amounts of physical assets. In particular wealthy agents with low human capital profit the most from stabilization. Conversely, the capital poor but human-capital rich households profit the least from stabilization, because it is them who finance the increased money supply and they comparatively suffer from stable prices for the physical asset.

7. Conclusion

This paper examines how variations in uncertainty about household income affect the macroeconomy through precautionary savings. For this purpose we develop a novel and tractable framework that combines nominal rigidities and incomplete markets in which households choose portfolios of liquid paper and illiquid physical assets – merging incomplete markets with wealthy hand-to-mouth consumers and New Keynesian modeling. In this model, higher uncertainty about income triggers a flight to liquidity because it is superior for short-run consumption smoothing. This reduces not only consumption but also investment and hence depresses economic activity.

Calibrating the model to match the evolution of uncertainty about household income in the U.S., we find that a spike in income uncertainty can lead to substantive output, consumption, and investment losses. This may help us to understand the slow recovery of the U.S. economy during the Great Recession, for which we document a shift toward liquid assets across all percentiles of the U.S. wealth distribution. We find that a two standard deviation increase in household income uncertainty generates output losses that are sizable.

The welfare effects of such uncertainty shocks crucially depend on a household's asset position and the stance of monetary policy. Monetary policy that drastically increases the money supply in times of increased uncertainty limits the negative welfare effects of uncertainty shocks but redistributes from the asset poor to the asset rich.

Appendices

A. Dynamic Planning Problem with Two Assets

The dynamic planning problem of a household in the model is characterized by two Bellman equations, V_a in the case where the household can adjust its capital holdings and V_n otherwise

$$V_{a}(m, k, h; \Theta, s) = \max_{k', m'_{a} \in \Gamma_{a}} u[x(m, m'_{a}, k, k', h)] + \beta \left[\nu EV^{a}(m'_{a}, k', h', \Theta', s') + (1 - \nu)EV^{n}(m'_{a}, k', h', \Theta', s')\right] V_{n}(m, k, h; \Theta, s) = \max_{m'_{n} \in \Gamma_{n}} u[x(m, m'_{n}, k, k, h)] + \beta \left[\nu EV^{a}(m'_{n}, k, h', \Theta', s') + (1 - \nu)EV^{n}(m'_{n}, k, h', \Theta', s')\right]$$
(1.21)

where the budget sets are given by

$$\Gamma_{a}(m,k,h;\Theta,s) = \{m',k' \ge 0 | q(\Theta,s)(k'-k) + m' \le \frac{\gamma}{1+\gamma} w(\Theta,s)hN + r(\Theta,s)k + \frac{m}{\pi(\Theta,s)} \}$$
(1.22)

$$\Gamma_n(m,k,h;\Theta,s) = \{m' \ge 0 | m' \le \frac{\gamma}{1+\gamma} w(\Theta,s)hN + r(\Theta,s)k + \frac{m}{\pi(\Theta,s)}\}$$
(1.23)
$$x(m,m',k,k',h) = \frac{\gamma}{1+\gamma} w(\Theta,s)hN + r(\Theta,s)k + \frac{m}{\pi(\Theta,s)} - q(\Theta,s)(k'-k) - m'$$

To save on notation, let
$$\Omega$$
 be the set of possible idiosyncratic state variables controlled
by the household, let Z be the set of potential aggregate states, let $\Gamma_i : \Omega \to \Omega$ be the
correspondence describing the feasibility constraints, and let $A_i(z) = \{(\omega, y) \in \Omega \times \Omega :$
 $y \in \Gamma_i(\omega, z)\}$ be the graph of Γ_i . Hence the states and controls of the household problem
can be defined as

$$\Omega = \{ \omega = (m,k) \in R^2_+ : m,k \le \infty \}$$

$$(1.25)$$

(1.24)

$$z = \{h, \Theta, s\} \tag{1.26}$$

and the return function $F: A \to R$ reads:

$$F(\Gamma_i(\omega, z), \omega; z) = \frac{x_i^{1-\gamma}}{1-\gamma}$$
(1.27)

Define the value before the adjustment/non-adjustment shock realizes as

$$v(\omega, z) := \nu V_a(\omega, z) + (1 - \nu) V_n(\omega, z).$$

Now we can rewrite the optimization problem of the household in terms of the

definitions above in a compact form:

$$V_a(\omega, z) = \max_{y \in \Gamma_a(\omega, z)} [F(\omega, y; z) + \beta_w Ev(y, z')]$$
(1.28)

$$V_n(\omega, z) = \max_{y \in \Gamma_n(\omega, z)} [F(\omega, y; z) + \beta_w Ev(y, z')].$$
(1.29)

Finally we define the mapping $T : C(\Omega) \to C(\Omega)$, where $C(\Omega)$ is the space of bounded, continuous and weakly concave functions.

$$(Tv)(\omega, z) = \nu V_a(\omega, z) + (1 - \nu) V_n(\omega, z)$$

$$V_a(\omega, z) = \max_{y \in \Gamma_a(\omega, z)} [F(\omega, y; z) + \beta_w Ev(y, z')]$$

$$V_n(\omega, z) = \max_{y \in \Gamma_n(\omega, z)} [F(\omega, y; z) + \beta_w Ev(y, z')].$$
(1.30)

A.1 Properties of Primitives

The following properties of the primitives of the problem obviously hold:

- **P** 1. Properties of sets Ω , $\Gamma_a(\omega, z)$, $\Gamma_n(\omega, z)$
 - 1. Ω is a convex subset of \mathbb{R}^3 .
 - 2. $\Gamma_i(\cdot, z) : \Omega \to \Omega$ is non-empty, compact-valued, continuous, monotone and convex for all z.
- **P 2.** Properties of return function F

F is bounded, continuous, strongly concave, C^2 differentiable on the interior of A, and strictly increasing in each of its first two arguments.

A.2 Properties of the Value and Policy Functions

Lemma 1. The mapping T defined by the Bellman equation for v fulfills Blackwell's sufficient conditions for a contraction on the set of bounded, continuous and weakly concave functions $C(\Omega)$.

- a) It satisfies discounting.
- b) It is monotonic.
- c) It preserves boundedness (assuming an arbitrary maximum consumption level).
- d) It preserves strict concavity.

Hence, the solution to the Bellman equation is strictly concave. The policy is a single-valued function in (m, k), and so is optimal consumption.

Proof. The proof proceeds item by item and closely follows Nancy L. Stokey (1989) taking into account that the household problem in the extended model consists of two Bellman equations.

a) Discounting

Let $a \in R_+$ and the rest be defined as above. Then it holds that:

$$\begin{aligned} (T(v+a))(\omega,z) = &\nu \max_{y \in \Gamma_a(\omega,z)} [F(\omega,y,z) + \beta_w Ev(y,z') + a] \\ &+ (1-\nu) \max_{y \in \Gamma_n(\omega,z)} [F(\omega,y,z) + \beta_w Ev(y,z') + a] \\ = &(Tv)(\omega,z) + \beta_w a \end{aligned}$$

Accordingly, T fulfills discounting.

b) Monotonicity

Let $g: \Omega \times Z \to R^2$, $f: \Omega \times Z \to R^2$ and $g(\omega, z) \ge f(\omega, z) \ \forall \omega, z \in \Omega \times Z$, then it follows that:

$$\begin{split} (Tg)(\omega,z) =& \nu \max_{y \in \Gamma_a(\omega,z)} [F(\omega,y,z) + \beta_w Eg(y,z')] \\ &+ (1-\nu) \max_{y \in \Gamma_n(\omega,z)} [F(\omega,y,z) + \beta_w Eg(y,z')] \\ \geq & \nu \max_{y \in \Gamma_a(\omega,z)} [F(\omega,y,z) + \beta_w Ef(y,z')] \\ &+ (1-\nu) \max_{y \in \Gamma_n(\omega,z)} [F(\omega,y,z) + \beta_w Ef(y,z')] \\ =& Tf(\omega,z) \end{split}$$

The objective function for which Tg is the maximized value is uniformly higher than the function for which Tf is the maximized value. Therefore, T preserves monotonicity.

c) Boundedness

From properties **P1** it follows that the mapping T defines a maximization problem over the continuous and bounded function $[F(\omega, y) + \beta_w Ev(y, z'))]$ over the compact sets $\Gamma_i(\omega, z)$ for $i = \{a, n\}$. Hence the maximum is attained. Since Fand v are bounded, Tv is also bounded.

d) Strict Concavity

Let $f \in C''(\Omega)$, where C'' is the set of bounded, continuous, strictly concave functions on Ω . Since the convex combination of two strictly concave functions is strictly concave, it is sufficient to show that $T_i[C''(\Omega)] \subseteq C''(\Omega)$, where T_i is defined by

$$T_i v = \max_{y \in \Gamma_i(\omega, z)} [F(\omega, y, z) + \beta_w Ev(y, z')], i \in \{a, n\}$$

Let $\omega_0 \neq \omega_1, \theta \in (0, 1), \omega_\theta = \theta \omega_0 + (1 - \theta) \omega_1$. Let $y_j \in \Gamma_i(\omega_j, z)$ be the maximizer of $(T_i f)(\omega_j)$ for $j = \{0, 1\}$ and $i = \{a, n\}$,

$$y_{\theta} = \theta y_{0} + (1 - \theta) y_{1}.$$

$$(T_{i}f)(\omega_{\theta}, z) \ge [F(\omega_{\theta}, y_{\theta}, z) + \beta_{w} Ef(y_{\theta}, z')]$$

$$> \theta[F(\omega_{0}, y_{0}, z) + \beta_{w} Ef(y_{0}, z'))] + (1 - \theta)[F(\omega_{1}, y_{1}, z) + \beta_{w} Ef(y_{1}, z')]$$

$$= \theta(Tf)(\omega_{0}, z) + (1 - \theta)(Tf)(\omega_{1}, z)$$

The first inequality follows from y_{θ} being feasible because of convex budget sets. The second inequality follows from the strict concavity of f. Since ω_0 and ω_1 are arbitrary, it follows that $T_i f$ is strictly concave, and since f is arbitrary that $T[C''(\Omega)] \subseteq C''(\Omega)$.

Lemma 2. The value function is C^2 and the policy function C^1 differentiable.

Proof. The properties of the choice set **P1**, of the return function **P2**, and the properties of the value function proven in (1) fulfill the assumptions of Santos's (1991) theorem on the differentiability of the policy function. According to the theorem, the value function is C^2 and the policy function C^1 differentiable.

Note that strong concavity of the return function holds for CRRA utility, because of the arbitrary maximum we set for consumption. $\hfill \Box$

Lemma 3. The total savings $S_i^* := m_i^*(\omega, z) + q(z)k_i^*(\omega, z)$ and consumption $c_i^*, i \in \{a, n\}$ are increasing in ω if r(z) is positive. In the adjustment case total savings and consumption are increasing in total resources $R_a = [q(z) + r(z)]k + m/\pi(z)$ for any r(z).

Proof. Define $\tilde{v}(S, z) := \max_{\{m,k|m+q(z)k \leq S\}} Ev(m, k; z')$ and resources in the case of no adjustment $R_n = r(z)k + m/\pi(z)$. Since v is strictly concave and increasing, so is \tilde{v} by the line of the proof of Lemma 1.d). Now we can (re)write the planning problem as

$$V_{a}(m,k;z) = \max_{\substack{S \le \frac{\gamma}{1+\gamma}w(z)hN + R_{a}}} [u(\frac{\gamma}{1+\gamma}w(z)hN + [q(z) + r(z)]k + m/\pi(z) - S) + \beta_{W}\tilde{v}(S,z)]$$
$$V_{n}(m,k;z) = \max_{\substack{m' \le \frac{\gamma}{1+\gamma}w(z)hN + R_{a}}} [u(\frac{\gamma}{1+\gamma}w(z)hN + r(z)k + m/\pi(z) - m') + \beta_{W}Ev(m',k;z')].$$

Due to differentiability we obtain the following (sufficient) first-order conditions:

$$\frac{\partial u\left(\frac{\gamma}{1+\gamma}w(z)hN + [q(z)+r(z)]k + m/\pi(z) - S\right)}{\partial c} = \beta_W \frac{\partial \tilde{v}(S,z)}{\partial S}$$
$$\frac{\partial u\left(\frac{\gamma}{1+\gamma}w(z)hN + r(z)k + m/\pi(z) - m'\right)}{\partial c} = \beta_W \frac{\partial v(m',k;z)}{\partial m'}.$$
(1.31)

Since the left-hand sides are decreasing in $\omega = (m, k)$, and increasing in S (respectively m'), and the right-hand side is decreasing in S (respectively m'), $S_i^* = \begin{cases} qk' + m' \text{ if } i = a \\ qk + m' \text{ if } i = n \end{cases}$ must be increasing in ω .

Since the right-hand side of (1.31) is hence decreasing in ω , so must be the left-hand

A.3 Euler Equations

Denote the optimal policies for consumption, for money holdings and capital as $x_i^*, m_i^*, k^*, i \in \{a, n\}$ respectively. The first-order conditions for an inner solution in the (non-)adjustment case read:

$$k^* : \frac{\partial u(x_a^*)}{\partial x} q \qquad = \beta E \left[\nu \frac{\partial V_a(m_a^*, k^*; z')}{\partial k} + (1 - \nu) \frac{\partial V_n(m_a', k'; z')}{\partial k} \right] \tag{1.32}$$

$$m_a^* : \frac{\partial u(x_a^*)}{\partial x} = \beta E \left[\nu \frac{\partial V_a(m_a^*, k^*; z')}{\partial m} + (1 - \nu) \frac{\partial V_n(m_a^*, k^*; z')}{\partial m} \right]$$
(1.33)

$$m_n^* : \frac{\partial u(x_n^*)}{\partial x} = \beta E \left[\nu \frac{\partial V_a(m_n^*, k; z')}{\partial m} + (1 - \nu) \frac{\partial V_n(m_n^*, k; z')}{\partial m} \right]$$
(1.34)

Note the subtle difference between (1.33) and (1.34), which lies in the different capital stocks k' vs. k in the right-hand side expressions.

Differentiating the value functions with respect to k and m, we obtain:

$$\frac{\partial V_a(m,k;z)}{\partial k} = \frac{\partial u[x_a^*(m,k;z)]}{\partial x}(q(z) + r(z))$$
(1.35)

$$\frac{\partial V_a(m,k;z)}{\partial m} = \frac{\partial u[x_a^*(m,k;z)]}{\partial x} \pi(z)^{-1}$$
(1.36)

$$\frac{\partial V_n(m,k;z)}{\partial m} = \frac{\partial u[x_n^*(m,k;z)]}{\partial x} \pi(z)^{-1}$$
(1.37)

$$\frac{\partial V_n(m,k;z)}{\partial k} = r(z) \frac{\partial u[x_n^*(m,k;z)]}{\partial x}$$
(1.38)

$$+\beta E \left[\nu \frac{\partial V_a[m_n^*(m,k;z),k;z']}{\partial k} + (1-\nu) \frac{\partial V^n[m_n^*(m,k;z),k;z']}{\partial k} \right]$$
$$= r(z) \frac{\partial u[x_n^*(m,k;z)]}{\partial x} + \beta \nu E \frac{\partial u\{x_a^*[m_n^*(m,k;z),k;z],k;z'\}}{\partial x} (q(z') + r(z'))$$
$$+ \beta (1-\nu) E \frac{\partial V_n\{[m_n^*(m,k;z),k;z],k;z'\}}{\partial k}$$

such that the marginal value of capital in non-adjustment is defined recursively.

Now we can plug the second set of equations into the first set of equations and

obtain the following Euler equations (in slightly shortened notation):

$$\begin{aligned} \frac{\partial u[x_{a}^{*}(m,k;z)]}{\partial x}q(z) &=\beta E \left[\nu \frac{\partial u[x_{a}^{*}(m_{a}^{*},k^{*};z')]}{\partial x}[q(z')+r(z')] + (1-\nu) \frac{\partial V^{n}(m_{a}^{*},k';z')}{\partial k'} \right] \end{aligned} \tag{1.39} \\ \frac{\partial u[x_{a}^{*}(m,k;z)]}{\partial x} &=\beta E \pi'(z')^{-1} \left[\nu \frac{\partial u[x_{a}^{*}(m_{a}^{*},k^{*};z')]}{\partial x} + (1-\nu) \frac{\partial u[x_{n}^{*}(m_{a}^{*},k';z')]}{\partial x} \right] \end{aligned} \tag{1.40} \\ \frac{\partial u[x_{n}^{*}(m,k,;z)]}{\partial x} &=\beta E \pi'(z')^{-1} \left[\nu \frac{\partial u[x_{a}^{*}(m_{a}^{*},k^{*};z')]}{\partial x} + (1-\nu) \frac{\partial u[x_{n}^{*}(m_{n}^{*},k;z')]}{\partial x} \right] \end{aligned} \tag{1.41}$$

A.4 Algorithm

The algorithm we use to solve for optimal policies given the Krusell-Smith forecasting rules is a version of Hintermaier and Koeniger's (2010) extension of the endogenous grid method, originally developed by Carroll (2006).

It works iteratively until convergence of policies as follows: Start with some guess for the policy functions x_a^* and x_n^* on a given grid $(m, k) \in M \times K$. Define the shadow value of capital

$$\beta^{-1}\psi(m,k;z) := \nu E \left\{ \frac{\partial u \{x_a^*[m_n^*(m,k,z),k;z']\}}{\partial x} [q(z') + r(z')] \right\}$$
(1.42)
+ $(1-\nu)E \frac{\partial V_n[m_n^*(m,k,z),k;z']}{\partial k}$
= $\nu E \left\{ \frac{\partial u \{x_a^*[m_n^*(m,k,z),k;z']\}}{\partial x} [q(z') + r(z')] \right\}$
+ $(1-\nu)E \left\{ \frac{\partial u \{x_n^*(m_n^*(m,k,z),k;z')\}}{\partial x} r(z') \right\}$
+ $(1-\nu)E \left\{ \psi[m_n^*(m,k,z),k;z'] \right\}.$

Guess initially $\psi = 0$. Then

- 1. Solve for an update of x_n^* by standard endogenous grid methods using equation (1.41), and denote $m_n^*(m,k;z)$ as the optimal money holdings without capital adjustment.
- 2. Find for every k' on-grid some (off-grid) value of $\tilde{m}_a^*(k'; z)$ such that combining (1.40) and (1.39) yields:

$$0 = \nu E \left\{ \frac{\partial u[x_a^*(\tilde{m}_a^*(k',z),k';z')]}{\partial x} \left[\frac{q(z') + r(z')}{q(z)} - \pi(z')^{-1} \right] \right\}$$
(1.43)
+ $(1-\nu)E \left\{ \frac{\partial u[x_a^*(\tilde{m}_a^*(k',z),k';z')]}{\partial x} \left[\frac{r(z')}{q(z)} - \pi(z')^{-1} \right] \right\} + (1-\nu)E \left[\frac{\psi(\tilde{m}_a^*(k',z),k';z')}{q(z)} \right]$

N.B. that $E\psi$ takes the stochastic transitions in h' into account and does not replace the expectations operator in the definition of ψ . If no solution exists, set $\tilde{m}_a^* = 0$. Uniqueness (conditional on existence) of \tilde{m}_a^* follows from the strict concavity of v.

3. Solve for total initial resources, by solving the Euler equation (1.40) for $\tilde{x}^*(k', z)$, such that:

$$\tilde{x}^{*}(k',z) = \frac{\partial u^{-1}}{\partial x} \left\{ \beta E \pi(z')^{-1} \left[\nu \frac{\partial u \{x_{a}^{*}[m_{a}^{*}(k',z),k';z']\}}{\partial x} + (1-\nu) \frac{\partial u \{x_{a}^{*}[m_{a}^{*}(k',z),k';z']\}}{\partial x} \right] \right\}$$
(1.44)

where the right-hand side expressions are obtained by interpolating $x_a^*(m_a^*(k', z), k', z')$ from the on-grid guesses $x_a^*(m, k; z)$ and taking expected values with respect to z'.

This way we obtain total non-human resources $\tilde{R}_a(k', z)$ that are compatible with plans $(m^*(k'), k')$ and a consumption policy $\tilde{\tilde{x}}_a^*(\tilde{R}_a(k', z), z)$ in total resources.

- 4. Since (consumption) policies are increasing in resources, we can obtain consumption policy updates as follows: Calculate total resources for each (m, k) pair $R_a(m, k) = (q+r)k + m/\pi$ and use the consumption policy obtained before to update $x_a^*(m, k, z)$ by interpolating at $R_a(m, k)$ from the set $\{(\tilde{x}_a^*(\tilde{R}_a(k', z), z), R_a(k', z)) | k' \in K\}$.¹⁶
- 5. Update ψ : Calculate a new value of ψ using (1.38), such that:

$$\psi^{new}(m,k,z) = \beta \nu E \left\{ \frac{\partial u \{x_a^*[m_n^*(m,k,z),k;z']\}}{\partial x} [q(z') + r(z')] \right\} + \beta (1-\nu) E \left\{ \frac{\partial u \{x_n^*(m_n^*(m,k,z),k;z')]}{\partial x} r(z') \right\} + \beta (1-\nu) E \left\{ \psi^{old}[m_n^*(m,k,z),k;z'] \right\}.$$
(1.45)

making use of the **updated** consumption policies.

B. Estimation of the Stochastic Volatility Process for Household Income

B.1 Income Process

We assume that the observed log-income of a household, $y_{i,a,t}$, is composed of four components: a deterministic part $f(o_{i,a,t})$, a transitory part $\tau_{i,a,t}$, a persistent part

¹⁶If a boundary solution $\tilde{m}^*(0) > 0$ is found, we use the "n" problem to obtain consumption policies for resources below $\tilde{m}^*(0)$.

 $h_{i,a,t}$, and a permanent part μ_i such that:

$$y_{i,a,t} = f(o_{i,a,t}) + y_{i,a,t}^*, (1.46)$$

$$y_{i,a,t}^* = \tau_{i,a,t} + h_{i,a,t} + \mu_i, \qquad (1.47)$$

$$h_{i,a,t} = \rho_h h_{i,a-1,t-1} + \epsilon_{i,a,t},$$
 (1.48)

where $o_{i,a,t}$ is observable characteristics of the household's head, $y_{i,a,t}^*$ is the stochastic component of a household's income ("residual income"), t is calendar time, and a is the household's years of labor market experience. We assume that all households start with $h_{i,0,t} = 0$ when they enter the labor market.

For the shocks $\epsilon_{i,a,t}$ to the persistent part h we assume them to be Gaussian, $\epsilon_{i,a,t} \sim N(0, \sigma_t^{\epsilon,2})$, with a time-varying variance that follows an AR(1) process (in logs) plus quadratic trend.

$$\log \sigma_t^{\epsilon,2} = (1 - \rho_s)\mu_s + \xi_1 t + \xi_2 t^2 + \rho_s \log \sigma_{t-1}^{\epsilon,2} + \varepsilon_t, \qquad (1.49)$$

$$\varepsilon_t \sim N(0, \sigma_s^2).$$
 (1.50)

For the variances of the fixed effect μ_i we assume them to be cohort specific, such that $\mu_i \sim N(0, \sigma_{t-a}^{\mu,2})$, where t-a denotes the birth cohort. We assume the transitory component, $\tau_{i,a,t} \sim N(0, \sigma_{\tau}^2)$, to have a constant variance.

Income Variances

Under the above assumptions, the variance of residual income, $y_{i,a,t}^*$, is given by

$$\sigma_{a,t}^{y,2} = \sigma_{\tau}^2 + \sigma_{\mu,t-a}^2 + \sigma_{a,t}^{h,2}, \qquad (1.51)$$

$$\sigma_{a,t}^{h,2} = \rho_h^2 \sigma_{a-1,t-1}^{h,2} + \sigma_t^{\epsilon,2}; \quad \sigma_{0,t}^{h,2} = 0,$$
(1.52)

$$\log \sigma_t^{\epsilon,2} = (1 - \rho_s)\mu_s + \xi_1 t + \xi_2 t^2 + \rho_s \log \sigma_{t-1}^{\epsilon,2} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_s^2).$$
(1.53)

We use the above equations to identify the parameters governing the stochastic volatility process, Equation (1.6), $\{\rho_h, \rho_s, \bar{\sigma}, \sigma_s^2\}$ from the data.¹⁷

B.2 Data

We take the 1970-2009 Cross-National Equivalent File (CNEF) of the Panel Study of Income Dynamics (PSID) and drop the low income sample. We keep all households in the sample that have at least two but no more than 10 household members who work combined at least 1040 hours per year and a male household head no younger than 25 and not older than 55. We focus on the age of 25 to 55 to abstract from the effects of household formation and retirement. We construct household income as pre-tax labor income plus private and public transfers minus all taxes. These selection criteria yield a sample that has on average about 1815 observations for each year of the survey.

¹⁷Where $\bar{\sigma} = sqrt(exp(\mu_s + \frac{\sigma_s^2}{(1-\rho_s^2)})$ corresponds to the level-mean of Equation (1.53).

B.3 Estimation

Our estimation procedure proceeds in two steps. First we estimate the deterministic component, $f(o_{i,a,t})$ (Eq. (1.54)), by running an OLS regression of log household income on time dummies, age dummies, schooling dummies interacted with up to a quadratic age trend, and household size dummies.

$$f(o_{i,a,t}) = \theta_0 + \theta_1^T \mathbf{D}_t + \theta_2^T \mathbf{x}_{i,a,t}, \qquad (1.54)$$

where \mathbf{D}_t is a vector of year dummy variables, $t = \{1970, ..., 1997, 1999, 2001, 2003, 2005, 2007, 2009\}$, and $\mathbf{x}_{i,a,t}$ is a vector containing all remaining regressors for household i with a years of labor market experience at date t. We eliminate any observation where the residual of this regression, $y_{i,a,t}^*$, belongs to the bottom or top per percent of all residuals for an age group.

From the residuals of this regression, we then calculate the sample variance within an age-year cell, $s_{a,t}^2$, across ages, $a = \{1, \ldots, 31\}$, and times t as well as covariances $c_{a,t}^1 = cov(y_{i,a,t}^*, y_{i,a-1,t-1}^*)$ and $c_{a,t}^2 = cov(y_{i,a,t}^*, y_{i,a-2,t-2}^*)$. This yields 992 sample-variance and 1798 sample-covariance estimates, where each estimate is constructed from on average 55 observations on the log-income residual.

Given the income process as laid out above, we can derive the moment conditions corresponding to the estimates for empirical variance s^2 and first and second order auto covariances in residual household income c^1, c^2 for each age-year combination.

$$s_{a,t}^2 = \sigma_{t-a}^{\mu,2} + \sigma_{\tau}^2 + \sum_{j=1}^{a-1} \rho_h^{2j} \sigma_{a-j,t-j}^{h,2} + \psi_{a,t}^s$$
(1.55)

$$c_{a,t}^{1} = \sigma_{t-a}^{\mu,2} + \rho_{h} \sigma_{a-1,t-1}^{h,2} + \psi_{a,t}^{c1}$$
(1.56)

$$c_{a,t}^2 = \sigma_{t-a}^{\mu,2} + \rho_h^2 \sigma_{a-2,t-2}^{h,2} + \psi_{a,t}^{c2}$$
(1.57)

where $\sigma_{a,t}^{h,2}$ obeys Equations (1.52) and (1.53) and ψ are the residuals.

B.4 Results

First-Stage Regression

The first-stage regression, Equation (1.54), controls for observable household characteristics and hence filters out the deterministic cross-sectional variation in household income. The results are comparable to existing studies, implying a concave earnings function in age and education. The inclusion of age-education interactions as well as controlling for age, education, and household size nonparametrically considerably raises the $R^2 = 0.6$.

The residuals of this regression yield the idiosyncratic component of income, $y_{i,a,t}^*$, from which we obtain the idiosyncratic cross-sectional variation in household income. Figure 1.10 depicts the variance of idiosyncratic income by age averaged across 1970-2009. The variance at labor market entry is already substantial and it increases by about 50% after 30 years of labor market participation. The initial dispersion helps to identify $\sigma_{\tau}^2 + \sigma_{t-a}^{\mu,2}$, whereas the rate of increase contains information on $\sigma_{a,t}^{h,2}$.

Figure 1.10: Idiosyncratic cross-sectional variance by age



Notes: Cross-sectional variance averaged across time calculated from the residuals of the first-stage regression.

Parameter Estimates

We estimate the following parameters by the generalized method of moments (GMM) minimizing the sum of squared residuals ψ^2 given the moment conditions in Equations (1.55), (1.56), and (1.57):

$$\left(\rho_h \quad \rho_s \quad \mu_s \quad \xi_1 \quad \xi_2 \quad \sigma_t^{\mu,2} \quad \sigma_\tau^2 \quad \sigma_t^{\epsilon,2}\right), \tag{1.58}$$

for $t = \{1939, \dots, 1969, \dots, 2009\}.$

We can track the history of the variance of persistent income shocks, $\sigma_t^{\epsilon,2}$, back to the year when the oldest cohorts at the start of the survey in 1970 entered the labor market. This way we obtain a time series for income uncertainty going back to 1939, see Figure 1.11.

Table 1.5 summarizes the parameters values of interest. Persistent shocks to idiosyncratic income have an annual autocorrelation of $\rho_h = 0.9069$ and an average standard deviation of $\bar{\sigma} = sqrt(exp(\mu_s + \frac{var(\sigma_t^{\epsilon,2})}{2(1-\rho_s^2)}) = 0.1483$ – similar to the estimates by Storesletten et al. (2004). For shocks to the variance of persistent income shocks, we estimate an annual autocorrelation of $\rho_s = 0.6651$ and a coefficient of variation of $\frac{\sigma_s}{\sqrt{1-\rho_s^2}} = sqrt(exp(\frac{var(\sigma_t^{\epsilon,2})}{(1-\rho_s^2)}) - 1) = 0.607.$

 Table 1.5: Parameter estimates

$ ho_h$	$ ho_s$	$\bar{\sigma}$	$rac{\sigma_s}{\sqrt{1- ho_s^2}}$
0.9069	0.6651	0.1483	0.6070

Notes: Where $\bar{\sigma}$ in Equation 1.6 corresponds to the (level-)mean of the persistent component, $sqrt(exp(\mu_s + \frac{var(\sigma_t^{\epsilon,2})}{(1-\rho_s^2)}))$, and the riskshifting parameter s follows from the coefficient of variation implied by the variation in the persistent component, $sqrt(exp(\frac{var(\sigma_t^{\epsilon,2})}{(1-\rho_s^2)}) - 1))$. These annual estimates are then converted to quarterly frequency.

Figure 1.11: Idiosyncratic income uncertainty 1939-2009



Notes: Constructed time series for the variance of persistent idiosyncratic income shocks based on PSID data. The second panel is without the linear-quadratic trend.

C. Asset Distribution

Table 1.6 summarizes the wealth distribution implied by our model (i.e., for the baseline calibration without fluctuations in uncertainty). As with any incomplete markets model that does not resort to heterogeneity in preferences or extremely skewed processes for idiosyncratic productivity, we fail to match the skewness in wealth documented for the U.S. Whereas the fraction of wealth held by the richest quintile is about 80% in the U.S., the top quintile in our model holds only 41% of total wealth. The same discrepancy holds for the Gini coefficient, where our model falls short as well – 0.38 versus 0.8 in the data.

Table	1.6:	Asset	distribution
-------	------	-------	--------------

2nd	3rd	4th	F 1 1	
		-1011	5th	
10.93	17.98	26.16	41.31	0.38
18.54 81.46	$12.56 \\ 87.44$	$8.32 \\ 91.68$	$5.54 \\ 94.46$	
1.00	1.61	2.42	2.65	
	18.54 81.46	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

These shortcomings are, however, not of great importance for our transmission mechanism. The top quintile is well insured, because they hold a sizable amount of liquid assets. Hence, they are least affected by ups and downs in uncertainty. The lower quintiles are the ones building up precautionary savings and thus the ones that react strongest to changes in uncertainty. In this dimension our model replicates the data fairly well. The poorest quintile in the U.S. has about zero wealth on average – including indebted households. The poorest households in our model hold only few assets – 3.6% of total wealth.

Our model also has implications for the ratio of liquid to illiquid assets conditional on how rich households are in total. Households save in money because it provides better short-term consumption smoothing than capital. This value of liquidity decreases in the amount of money a household holds. Hence, our model implies that the share of liquid assets in the portfolio declines in total wealth. Figure 1.12 plots the prediction of our model and the data equivalent taken from the Survey of Consumer Finances 2004 (SCF) according to the definitions by Kaplan and Violante (2014). The poorest households in the U.S. and in our model predominantly hold liquid assets. The share of liquid assets then rapidly falls below 20% in both graphs, but rises again in the SCF for the richest households. This is because stocks, mutual funds, and non-governmental bond holdings are concentrated at the top quintiles as can be seen by comparing the broad liquidity measure, which includes all of those, to the narrow definition. If we Figure 1.12: Share of liquid assets in total net worth against percentiles of total wealth in 2004



Notes: We compare our measure of liquid net worth (see Figure 1.1) to a broader definition of liquid assets that includes mutual funds, stocks, and non-governmental bonds as in Kaplan and Violante (2014). For graphical illustration we make use of an Epanechnikov Kernel-weighted local linear smoother with bandwidth 0.15.

also exclude those assets that usually induce some transaction cost (e.g., a commission) when acquiring them from a bank or broker, the share of liquid assets is substantially reduced for the asset rich.

D. Quality of the Numerical Solution

The equilibrium forecasting rules are obtained by regressing them in each iteration of the algorithm on 10.000 observations. We generate the observations by simulating the model in parallel on 20 machines, letting each economy run for 750 periods and discarding the first 250 periods. The R^2 is generally above 99% for all calibrations; see Tables 1.9 and 1.10. In the case of perfect stabilization, π_t is virtually constant, such that the R^2 of the π -forecasting is a nonsensical statistic. Figure 1.13 shows that our results are robust to increasing the resolution for the aggregate state variables.

Following Den Haan (2010), we also test the out-of-sample performance of the forecasting rules. For this we initialize the model and the forecasting rules at steady state values, feed in the same shock sequence, but otherwise let them run independently. Figure 1.14 plots time series of the prices q and π as well as the states K and M taken from the simulation of the model and the forecasting rules. The equilibrium forecasting rules track the evolution of the underlying model without any tendency of divergence. Table 1.7 summarizes the mean and maximum difference between the series generated by the model and the forecasting rules. The mean error for all four time series is less than 0.3%. The maximum errors are small, too.

	Price of capital q_t	Capital K_t	Inflation π_t	Real money M_t
Mean Error Max Error	$\begin{array}{c} 0.28 \\ 1.20 \end{array}$	$0.07 \\ 0.20$	$\begin{array}{c} 0.01 \\ 0.07 \end{array}$	$0.22 \\ 0.69$

Table 1.7: Forecasting errors

Notes: Percentage differences in out-of-sample forecasts between forecasting rules and model for $t = \{1, ..., 1.500\}$; see Den Haan (2010).



Figure 1.13: Uncertainty shock with higher resolution in K and M

Notes: See figure 1.5. We increase the number of gridpoints to 5 for capital and 7 for money.



Figure 1.14: Out-of-sample performance of the forecasting rules

Notes: Out-of-sample comparison between law of motions and model zoomed in at $t = \{1000, ..., 1.500\}$ for visibility; see Den Haan (2010).

E. Welfare

Table 1.8 provides the long run welfare effects with and without stabilization after 75 years when the economy is back at its steady state.

			Poli	cy regii	me: Con	stant mo	oney gro	owth				
	Qu	intiles o	of mone	ev holdi	ngs	Qu	Quintiles of capital holdings					
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.		
Conditional	-0.20	-0.18	-0.15	-0.11	-0.05	-0.11	-0.10	-0.12	-0.14	-0.18		
Median	-0.23	-0.18	-0.13	-0.08	0.02	-0.14	-0.13	-0.14	-0.16	-0.18		
	Qu	intiles of	of Hum	an Cap	ital							
Conditional	-0.09	-0.13	-0.14	-0.14	-0.16							
Median	-0.05	-0.15	-0.14 -0.15	-0.14 -0.16	-0.10							
			Ро	licy reg	gime: Inf	flation st	abilizat	ion				
	Quintiles of money holdings			ngg	Quintiles of capital holdings				nge			
	$\frac{\sqrt{2}}{1}$	$\frac{1111105}{2}$	$\frac{31 \text{ mone}}{3.}$	<u>4.</u>	$\frac{11gs}{5.}$	$\frac{\sqrt{2}}{1}$	$\frac{1111105}{2}$	$\frac{3}{3}$.	<u>4.</u>	$\frac{11gs}{5.}$		
			0.	1.	0.	1.		0.	1.	0.		
Conditional	-0.03	-0.06	-0.06	-0.06	-0.07	-0.06	-0.05	-0.06	-0.06	-0.07		
Median	-0.06	-0.05	-0.04	-0.02	-0.00	-0.07	-0.05	-0.04	-0.04	-0.03		
	Qu	untiles	of hum	an capi	tal							
Conditional	0.01	-0.01	-0.05	-0.08	-0.14							
Median	0.02	-0.01	-0.05	-0.08	-0.17							
			Do	liou noo	ima. Fa	d reactio	n funct	ion				
			PO	ncy reg	,ime: re	d reactio	n iunci	1011				
	Qu	intiles of	of mone	ey holdi	ngs	Qu	intiles o	of capit	al holdi	ngs		
	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.		
Conditional	0.11	0.11	0.10	0.08	0.07	-0.08	0.08	0.00	0.10	0.11		
Median			-0.10		0.00	-0.08			-0.10			
Wiedlan			of hum			0.10	0.00	0.05	0.00	0.10		
				±								
Conditional	-0.04	-0.07	-0.09	-0.10	-0.15							
Median	-0.01	-0.06	-0.09	-0.12	-0.20							

Notes: Welfare costs in terms of consumption equivalents (CE) as defined in (1.20). Conditional refers to integrating out the missing dimensions, whereas Median refers to median asset holdings of the respective other assets. We track households over 300 quarters and average over 100 independent model simulations.

F. Robustness Checks

For the risk aversion parameter, the Frisch elasticity of labor supply, the average markup, and the frequency of price adjustment, we take standard values from the literature as there is no direct counterpart in the data. To account for this loose calibration strategy, we check the robustness of our findings with respect to the assumed parameter values. We do so by varying one of the parameters at a time while recalibrating the discount factor and the participation frequency of households to match the targeted capital and money to output ratios and the capital adjustment costs to match a relative investment volatility of $3.^{18}$

We find our results to be robust to all the considered parameter variations. The impulse response functions for output, consumption, investment, and inflation are displayed in Figure 1.15. The output drop on impact always remains around 0.5% – the result of our baseline calibration. Key for this robustness is the recalibration of other parameters. For example, if households are assumed to be less risk averse, then capital must be less liquid to match the observed holdings of liquid assets. Therefore, while a lower risk aversion makes the increase in precautionary savings less pronounced, the inferred lower liquidity of capital intensifies the liquidity effect. This leaves the output effect almost unchanged. In other words, the stability of our results stems from the model inherent trade-offs.

¹⁸For the robustness check where we set the inverse Frisch elasticity to 1, physical capital becomes so attractive to households, that the calibration forces capital to become very illiquid in order to match the observed money to output ratio. Then, however, investment moves so little (as close to nobody can trade) that there is no positive adjustment cost such that the relative investment volatility is 3. In these cases we assume no capital adjustment costs.



Notes: See Figure 1.5 or 1.6.

G. Equilibrium Forecasting Rules

Tables 1.9 to 1.11 display the equilibrium laws of motion for the Krusell-Smith equilibrium.

		Baselin	e	٤	Ċ	,	κ	ļ	u	γ
	-	$\theta_2 = 10^3$ $\theta_3 = 0$	$\theta_2 = .35$ $\theta_2 = .9$	3^*	5	0.04	0.5	5^*	15	1^*
	03 0	<u> </u>	03 .0							
β_q^1										
		1.00			0.00	0.40	0.44	0.15	0.04	K 10
s_1	1.71	1.39	1.75	0.79	2.03		-0.44	0.15	2.36	5.18
s_2		1.41	1.75	0.87	2.01		-0.36	0.67	2.34	5.70
s_3		1.50	1.77	0.89	1.99		-0.39	0.53	2.33	5.85
s_4		1.55	1.78	0.90	2.00		-0.37	0.35	2.33	5.96
s_5		1.62	1.79	0.91	2.03		-0.35	0.22	2.34	6.04
s_6		1.66	1.80	0.93	2.05		-0.39	0.33	2.35	6.29
s_7	1.82	1.75	1.82	1.00	2.14	3.06	-0.25	0.51	2.40	6.58
β_q^2	-									
s_1	0.50	0.12	0.22	0.14	0.63	0.65	0.46	0.08	0.74	1.02
s_2		0.13	0.24	0.16	0.65	0.68	0.47	0.10	0.75	1.17
s_3		0.14	0.26	0.17	0.66	0.69	0.48	0.12	0.77	1.30
s_4		0.15	0.27	0.19	0.68	0.69	0.50	0.13	0.78	1.45
s_5		0.16	0.29	0.22	0.69	0.69	0.51	0.15	0.78	1.60
s_6	0.58	0.18	0.31	0.24	0.71	0.69	0.54	0.17	0.78	1.80
s_7		0.20	0.34	0.29	0.74	0.73	0.57	0.20	0.79	2.04
β_q^3										
s_1	-0.86	-0.83	-1.03	-0.45	-1.00	-1.97	0.55	-0.04	-1.22	-2.87
s_1 s_2		-0.85	-1.03		-0.99	-1.86			-1.21	-3.18
	-0.87	-0.90	-1.04		-0.99		0.49		-1.21	-3.25
s_4		-0.94	-1.05		-1.00		0.48		-1.22	-3.29
	-0.88	-0.99	-1.06		-1.03		0.46		-1.23	-3.30
~			-1.07							
-			-1.10						-1.31	
R^2	2									
	99.55	98.54	99.70	98.52	99.74	99.58	99.40	98.77	99.57	99.00

Table 1.9: Laws of motion for the price of capital

* For readability the coefficients of the law of motion are multiplied by 10.000.

		Baselin	е	ξ	- - -		κ	μ	l	γ
	$\theta_2 = 0$	$\theta_2 = 10^3$	$\theta_2 = .35$	3	5	0.04	0.5	5	15	1
	$\theta_3 = 0$	$\theta_3 = 0$	$\theta_3 = .9$							
01										
β_{π}^{1}										
s_1	2.80	0.48	0.08	12.46	2.65	44.20	-19.49	40.74	3.76	6.07
s_2	2.37	0.49	0.50	13.19			-19.30	33.41	3.54	6.34
$\overline{s_3}$	2.16	0.49	1.20	14.36		44.14	-19.31	27.85	3.56	6.57
s_4	1.67	0.49	1.49	14.33	2.17	41.17	-19.21	21.63	3.26	6.55
s_5	1.19	0.49	1.82	14.08	1.86	37.87	-19.55	18.86	2.87	6.63
s_6	1.00	0.49	2.20	13.99	1.41	34.76	-19.92	2.06	2.54	6.56
s_7	0.59	0.49	2.18	13.54	0.77	28.95	-20.62	-39.60	2.29	6.28
- 0										
β_{π}^2										
s_1	4.19	-0.00	0.53	4.18	3.87	10.06	2.83	3.36	4.03	2.79
s_2	4.31	0.00	0.71	4.64	3.97	11.44	2.85	3.87	4.12	3.09
$\overline{s_3}$	4.39	0.00	0.87	5.04	4.06	11.97	2.89	4.30	4.23	3.37
s_4		0.00	1.02	5.42	4.14	12.07	2.96	4.69	4.28	3.65
s_5	4.57	0.00	1.19	5.85	4.26	12.19	3.06	5.13	4.36	3.98
s_6	4.76	0.00	1.38	6.39	4.42	12.34	3.21	5.64	4.49	4.39
s_7	5.17	0.00	1.67	7.27	4.80	13.19	3.49	6.44	4.88	5.03
~?										
β_{π}^3										
s_1	1.17	0.01	0.78	-5.25	1.14	-22.92	14.99	-22.73	0.71	-1.78
s_2	1.27	0.01	0.49		1.11	-22.25		-18.23		-2.01
$\overline{s_3}$	1.22	0.01	0.00	-6.58			14.62	-14.83		-2.22
s_4	1.36	0.01	-0.22	-6.59		-21.24		-11.04		-2.26
s_5	1.48	0.01	-0.47	-6.46	0.95	-19.50	14.60	-9.37	0.81	-2.34
s_6	1.40	0.01	-0.78	-6.43	1.05	-18.03	14.75	0.91	0.88	-2.34
s_7	1.48	0.01	-0.83	-6.08	1.29	-14.71	15.13	26.52	0.97	-2.13
R^2	-									
	99.87	88.77	99.66	99.89	99.84	99.08	99.81	99.89	99.73	99.87

Table 1.10: Laws of motion for inflation

Notes: For readability all values are mutiplied by 100.

Table 1.11: Laws of motion for Fed policy rule continued

	Baseline					
	_	$\theta_2 = .35$ $\theta_3 = .9$				
	β_q^4	β_{π}^4				
s_1	5.65	19.43				
$egin{array}{c} s_2 \ s_3 \ s_4 \end{array}$	25.18 33.51 36.83	20.84 20.71 20.32				
s_5 s_6	$47.80 \\ 70.02$	19.39 18.78				
s_7	116.69	19.15				

Notes: For readability all values are mutiplied by 100.
CHAPTER 2

Fiscal Stimulus Payments and Precautionary Investment

Deficit-financed government transfers to households have been an important part of the fiscal response to the last two recessions in the United States. This paper assesses the aggregate effects of this type of fiscal intervention in a New Keynesian business cycle model with incomplete markets and portfolio choice between liquid public debt and illiquid physical assets. In this environment, transfers do not only work through the disposable income channel but also by affecting household liquidity. Transfers increase individual liquidity and debt finance enhances market liquidity. This has consequences when the government retires this debt. Then households shift their savings into the physical asset to smooth consumption. This leads to a prolonged increase in capital and output. This precautionary investment channel dominates negative wealth effects of distortionary taxes, making aggregate effects expansionary independent of the mode of financing.

1. Introduction

Deficit-financed government transfers to households, so called fiscal stimulus payments, have become part and parcel of the fiscal response to recessions. In the last two recession of 2001 and 2007-2009, U.S. households received one-off payments between \$500 to \$1000 amounting to fiscal outlays of 0.4 - 0.7% of annual GDP. Household-level data from both episodes reveal that households spent on average around 25% of those payments on consumption.¹ The average size and the distribution of the consumption response can be rationalized through liquidity constraints (see Kaplan and Violante, 2014). Whether this type of fiscal intervention is successful in stabilizing output, however, depends on the joint-response of consumption and investment. A key argument against government transfers is that government deficits may crowd out private invest-

¹See e.g. Johnson et al. (2006) and Parker et al. (2013). These studies have exploited the fact that the timing of receipt of payment was based on the last two digits of individual Social Security Numbers and, thus, effectively random. This randomization allows to estimate the causal effect of receipt of payment on consumption relative to the control group of households that received the payment in a different quarter.

ment potentially offsetting any increase in consumption.

This paper accounts for both household consumption and portfolio decisions in assessing the aggregate effects of fiscal stimulus payments. Toward this end, we build a New Keynesian business cycle model with incomplete markets and portfolio choice between liquid public debt and illiquid physical assets. Public debt can be traded without frictions, whereas physical capital can only be adjusted with a fixed probability each period. As in Aiyagari and McGrattan (1998), households value public debt as an additional means of smoothing consumption.

Despite of public debt and capital being substitutes, we find that deficit-financed government transfers may actually lead to a crowding-in of investment. Key for this result is the transient nature of the increase in public debt. On impact, when fiscal transfers and the increase in public debt take place, public debt only partly replaces capital while total savings increase. When - foreseeable - the government starts to retire this debt, households would like to hang on to their improved consumption-smoothing capacity. This crowds in private investment. We find that a faster reversal in public debt leads to a stronger boom in investment.

What is more, this precautionary investment channel is little affected by how public debt is financed. This is in stark contrast to an economy without the liquidity effect of public debt, in which financing by tax hikes or spending cuts lead to opposite investment responses. When transfers are financed by distortionary taxes, the net wealth effect of transfers is negative and, thus, savings fall in an economy with complete markets. When financed by government spending cuts, transfers imply an increase in wealth and, thus, a positive investment response. When markets are incomplete, however, the liquidity effect dominates the wealth effect such that investment always increases independent of the financing.

With these results, this paper contributes to the literature on the aggregate effects of fiscal stimulus payments by highlighting the liquidity channel of public debt. Oh and Reis (2012) build a model with incomplete markets and sticky prices, but abstract from public debt and instead look into the effects of redistribution across households within a period. Other studies feature Ricardian households to simplify the role of public debt. Giambattista and Pennings (2013) compare the multiplier of government purchases and transfer in a model with Ricardian and rule-of-thumb households. In a similar vein, Mehrotra (2014) compares both multipliers in a borrower-lender economy. McKay and Reis (2016) assess automatic fiscal stabilizers in a model in which Ricardian households own the capital stock.

We also contribute to the literature that discusses the effects of public debt by raising the supply of assets. We share with Aiyagari and McGrattan (1998) the focus on precautionary motives, but go beyond their steady state analysis. We show that transitory increases in public debt do not crowd-out capital because of an investment boom during the transition back to steady state. In a model without precautionary motives, Woodford (1990) shows that higher public debt may crowd-in investment through loosening liquidity constraints. The spirit of the analysis is closest to Challe and Ragot (2011). They investigate a similar liquidity channel in the case of deficit-financed government spending shocks with a focus on the consumption response. They find as well that liquidity effects dominate wealth effects leading to a crowding-in of consumption. The remainder of the paper is organized as follows. Section 2, 3, and 4 explain the model, solution method, and calibration. Section 5 presents the quantitative results, and Section 6 concludes.

2. Model

The model economy builds on Bayer et al. (2015) and, thus, we only describe in the following the changes made to the model framework presented in the previous chapter. The economic environment differs as we consider an unexpected transfer payment of 0.5% of annual GDP, and explicitly model besides a monetary authority a government sector collecting taxes and following strict fiscal rules. Fiscal policy is described either by a tax or a spending rule reacting to public debt deviations from steady state. Furthermore, the central bank conducts monetary policy in a conventional way and sets the nominal interest rate according to a Taylor (1993)-type rule. In contrast to Chapter 1, a worker-household can now either stay a worker in the next period or transit into entrepreneurship with some exogenous probability, which is independent from its productivity level. In this approach, we follow Luetticke (2015). We next describe the modified household problem and the government sector in more detail. The problem of firms is unchanged.

2.1 Households

The household problem is modified in various ways. There are two types of households, entrepreneurs $(s_{it} = 0)$ and workers $(s_{it} = 0)$. They switch from one type to the other according to an idiosyncratic shock s_{it} .² At all times, the fraction of entrepreneurial households remains constant. Worker-households supply labor on a perfectly competitive market and their idiosyncratic labor productivity, h_{it} , follows an autoregressive process of order one. Compared to Chapter 1, we abstract from persistent shocks to productivity risk, which remains constant at $\bar{\sigma}$:

$$\log h_{it} = \rho_h \log h_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \bar{\sigma}).$$
(2.1)

Entrepreneurial households do not participate in the labor market and instead hold an equal stake in the economy's total profits Π_t , generated by monopolistic resellers. Both, entrepreneur- and worker-households, pay proportional taxes, τ_t , either on their earned profits, Π_t , or on their labor income, $w_t h_{it} N_t$. Depending on the fiscal rule in place, the government might adjusts taxes in response to changes in government debt. In particular, households now receive an unexpected lump-sum transfer, τ_0 in period t = 0 from the government.

For those households participating in the capital market, the modified budget con-

²Please note that s_{it} now denotes an idiosyncratic shock to the employment status of household i and no longer refers to the aggregate uncertainty shock.

straint reads:³

$$c_{it} + b_{it+1} + q_t k_{it+1} = \frac{R_{t-1}^B}{\pi_t} b_{it} + (q_t + r_t) k_{it} + \tau_t \left[s_{it} w_t h_{it} N_t + (1 - s_{it}) \Pi_t \right] + \tau_0,$$

$$k_{it+1} \ge 0, b_{it+1} \ge 0, \qquad (2.2)$$

where b_{it} is the real value of nominal bond holdings, k_{it} are capital holdings, q_t is the price of capital, r_t is the rental rate or "dividend", R^B_{t-1} is the gross nominal return on bonds, and $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate. We denote real bond holdings of household i at the end of period t by $b_{it+1} := \frac{\tilde{b}_{it+1}}{P_t}$.

For those households that cannot trade in the market for capital, the modified budget constraint simplifies to:

$$c_{it} + b_{it+1} = \frac{R_{t-1}^B}{\pi_t} b_{it} + r_t k_{it} + \tau_t \left[s_{it} w_t h_{it} N_t + (1 - s_{it}) \Pi_t \right] + \tau_0,$$

$$b_{it+1} \ge 0.$$
 (2.3)

With this setup, two Bellman equations characterize the dynamic planning problem of a household; V_a in case the household can adjust its capital holdings and V_n otherwise:⁴

$$V_{a}(b,k,h,s;\Theta) = \max_{k',b'_{a}} u[c(b,b'_{a},k,k',h,s)] + \beta[\nu EV^{a}(b'_{a},k',h',s',\Theta') + (1-\nu)EV^{n}(b'_{a},k',h',s',\Theta')],$$

$$V_{n}(b,k,h,s;\Theta) = \max_{b'_{n}} u[c(b,b'_{n},k,h,s)] + \beta[\nu EV^{a}(b'_{n},k,h',s',\Theta') + (1-\nu)EV^{n}(b'_{n},k,h',s',\Theta')].$$
(2.4)

2.2 Central Bank and Government

Following the modifications made by Luetticke (2015) to our model framework in Chapter 1, the central bank sets the nominal interest rate R_t^B on government bonds according to a Taylor rule that reacts to inflation more than one-to-one ($\theta_{\pi} > 1$), whenever inflation exceeds its target value, π :

$$\frac{R_t^B}{R^B} = \left(\frac{R_{t-1}^B}{R^B}\right)^{\rho_R B} \left(\frac{1+\pi_t}{1+\pi}\right)^{\theta_\pi}.$$
(2.5)

The parameter ρ_{R^B} captures the policy inertia of the nominal interest rate, empirically documented by Clarida et al. (2000).

The fiscal authority pays lump-sum transfers $\tau_0 > 0$ to households in period t = 0 that are financed by debt issuance. In all other periods, τ_0 equals zero. Let B_{t+1} denote

³The household problem can be expressed in terms of composite good x_{it} by making use of $c_{it} = x_{it} + \frac{\tau_t w_t h_{it} N_t}{1+\gamma}$.

⁴No conditioning on aggregate shocks is required. The transfer shock only occurs at t = 0 and then the economy deterministically reverts back to steady state.

time t real value of public debt. The government budget constraint reads:

$$B_{t+1} = \frac{R_{t-1}^B}{1+\pi_t} B_t + G_t + \tau_0 - T_t, \qquad (2.6)$$

where real tax revenues are given by:

$$T_t = (1 - \tau_t) \left[(N_t W_t \int s_i h_i \Theta_t(b, k, h, s)) + \Pi_t \right].$$

$$(2.7)$$

The government either adjusts taxes, τ_t , or government spending, G_t , to bring debt back to its steady state value from t = 1 onwards. We assume simple linear rules similar to the ones estimated by Leeper et al. (2010):

$$G_t = \gamma_1 - \gamma_2 (B_t - B), \qquad (2.8)$$

$$\tau_t = \gamma_3 + \gamma_4 \log(B_t/B), \tag{2.9}$$

with B equal to the steady state debt level. The parameters $\gamma_2, \gamma_4 > 0$ measure the speed at which public debt returns to its steady state value.

2.3 Goods, Bonds, Capital, and Labor Market Clearing

The labor market clears at the competitive wage given in (1.8) in Chapter 1; so does the market for capital services if (1.9) from Chapter 1 holds.

The nominal bonds market clears, whenever the following equation holds:

$$B_{t+1} = \int \left[\nu b_a^*(b, k, h, s; q_t, \pi_t) + (1 - \nu) b_n^*(b, k, h, s; q_t, \pi_t)\right] \\\Theta_t(b, k, h, s) db dk dh ds,$$
(2.10)

Last, the market for capital has to clear:

$$q_{t} = 1 + \phi \frac{K_{t+1} - K_{t}}{K_{t}} = 1 + \nu \phi \frac{K_{t+1}^{*} - K_{t}}{K_{t}}, \qquad (2.11)$$

$$K_{t+1}^{*} := \int k^{*}(b, k, h, s; q_{t}, \pi_{t}) \Theta_{t}(b, k, h, s) db dk dh ds,$$

$$K_{t+1} = K_{t} + \nu (K_{t+1}^{*} - K_{t}),$$

where the first equation stems from competition in the production of capital goods, the second equation defines the aggregate supply of funds from households trading capital, and the third equation defines the law of motion of aggregate capital. The goods market then clears due to Walras' law, whenever both, bonds and capital markets, clear.

2.4 Recursive Equilibrium

A recursive equilibrium in our model is a set of policy functions $\{x_a^*, x_n^*, b_a^*, b_n^*, k^*\}$, value functions $\{V_a, V_n\}$, pricing functions $\{r, R^B, w, \pi, q\}$, aggregate bonds, capital and labor supply functions $\{B, N, K\}$, distributions Θ_t over individual asset holdings, types, and productivity, and a perceived law of motion Γ , such that

- 1. Given $\{V_a, V_n\}$, Γ , prices, and distributions, the policy functions $\{x_a^*, x_n^*, b_a^*, b_n^*, k^*\}$ solve the households' planning problem, and given the policy functions, prices and distributions, the value functions $\{V_a, V_n\}$ are a solution to the Bellman equations (1.7).
- 2. The labor, the final-goods, the bonds, the capital, and the intermediate-good markets clear, i.e., (1.8), (1.13), (2.10), and (2.11) hold.
- 3. The actual law of motion and the perceived law of motion Γ coincide, i.e., $\Theta' = \Gamma(\Theta, s')$.

3. Numerical Implementation

We compute the transitional dynamics after an unexpected one-off fiscal stimulus shock with the help of Krusell and Smith (1998a)-rules. We consider an economy that is in steady state before period t = 0. In t = 0, all households receive an unexpected fiscal transfer, τ_0 . There are no more shocks from t = 1 onwards. From then on, households anticipate how prices evolve on the path back to the long-run equilibrium of the economy. These prices are, of course, a function of all states including the joint distribution $\Theta_t(b, k, h, s)$. Hence, we assume that households predict future prices on the basis of a restricted set of moments as in Krusell and Smith (1997, 1998a).

Specifically, we make the assumption that households condition their expectations on last period's aggregate real bond holdings, B_t , last period's nominal interest rate, R_{t-1}^B , and the aggregate stock of capital, K_t . If asset-demand functions, $b_{a,n}^*$ and k^* , are sufficiently close to linear in human capital, h, and in non-human wealth, b and k, at the mass of Θ_t , we can expect approximate aggregation to hold. For this exercise, the three aggregate states $-B_t$, R_{t-1}^B , K_t – are sufficient to describe the evolution of the aggregate economy.

Households use the following log-linear forecasting rules for current inflation and the price of capital:

$$\log \pi_t = \beta_{\pi}^1 + \beta_{\pi}^2 \hat{B}_t + \beta_{\pi}^3 \hat{K}_t + \beta_{\pi}^4 \hat{R}_{t-1}^B, \qquad (2.12)$$

$$\log q_t = \beta_q^1 + \beta_q^2 \hat{B}_t + \beta_q^3 \hat{K}_t + \beta_q^4 \hat{R}_{t-1}^B, \qquad (2.13)$$

where (.) refers to log-differences from the steady state value of each variable. The law of motion for aggregate real bonds, B_t , then follows from the government budget constraint (2.6). The Taylor-rule (2.5) determines the motion of the nominal interest rate, R_t^B . The law of motion for K_t results from (2.11).

To find the deterministic law of motion in response to a zero-probability fiscal stimulus payment shock, we need to solve for the market clearing prices each period. Concretely, this means the posited rules, (2.12) and (2.13), are used to solve for the households' policy functions. Having solved for the policy functions conditional on the forecasting rules, we then simulate the model for $t = 0, \ldots, T$ periods, keeping track of the actual distribution Θ_t . The simulation starts in steady state and the transfer shock hits in t = 0. We then calculate in each period t the optimal policies for market clearing inflation rates and asset prices assuming that households resort to the policy functions.

derived under rule (2.12) and (2.13) from period t + 1 onwards. Having determined the market clearing prices, we obtain next period's distribution Θ_{t+1} . We next re-estimate the parameters of (2.12) and (2.13) from the simulated data and update the parameters accordingly. Subsequently, we recalculate policy functions and iterate until convergence in the forecasting rules.

The posited rules (2.12) and (2.13) approximate the aggregate behavior of the economy well. The within sample R^2 is well above 99% (see Appendix A).

4. Calibration

One period in the model is a quarter. We adopt most of the calibration of Chapter 1 for the household and firm side, and in the following only elaborate on the differences in the calibration strategy relative to the previous chapter. In particular, we now explicitly target the share of poor and wealthy hand-to-mouth households as estimated in Kaplan and Violante (2014).

4.1 Households

For the felicity function, $u = \frac{1}{1-\xi}x^{1-\xi}$, we set the coefficient of relative risk aversion to $\xi = 1.5$. We jointly choose this parameter value with the standard deviation of the income process to match the percent of poor and wealthy liquidity constrained households, which jointly make up around 33% of the households in the Survey of Consumer Finances (SCF) in 2004, and of which 2/3 are defined as wealthy. Households are considered to be liquidity constrained, if their liquid assets are half of their monthly income. In accordance with Kaplan and Violante (2014), a household is defined as wealthy, if it owns positive amounts of illiquid wealth. We set the implied annual persistence of income shocks to a common estimate of 0.95. As in Luetticke (2015), we calibrate the share of entrepreneurs in the economy to be 1% in order to match the U.S. net wealth Gini coefficient of 0.82 in the SCF.

The aggregate data used for the calibration of the capital-to-output ratio and the bonds-to-output ratio in the model spans 1984Q1 to 2008Q3. This way, we now match an annual debt-to-GDP ratio of 23%, which reflects the net asset of government bonds available to households prior to the large expansion of public balance sheets in response to the Financial Crisis. In particular, we consider average U.S. federal debt held by domestic private agents. The parameter governing the adjustment costs of capital, $\phi = 10$, is chosen as in Luetticke (2015), where he targets a relative investment volatility of 3.

4.2 Central Bank and Government

As in Chapter 1, the annual inflation rate is at 2%. The steady state nominal return on government bonds is set to 4% p.a. We adopt the parameter, $\rho_{R^B} = 0.95$ describing the policy inertia inherent to the nominal interest rate from Nakamura and Steinsson (2013) and set the central bank's reaction parameter to inflation deviations from target to $\theta_{\pi} = 1.5$, a common value in the New Keynesian literature. =

Parameter	Value	Description	Target	
Household	ls			
β	0.985	Discount factor	K/Y = 290% (annual)	
ν	4.5%	Participation frequency	B/Y = 23% (annual)	
ξ	1.5	Coefficient of rel. risk av.	Share of liquidity constrained households	
γ	0.5	Inv. Frisch elasticity	Standard value	
Intermedia				
α	72%	Share of labor	Income share of labor of 66%	
δ	1.35%	Depreciation rate	NIPA: Fixed assets & durables	
Final Goo	\mathbf{ds}			
κ	0.08	Price stickiness	Avg. price duration of 4 quarter	
μ	0.06	Markup	6% markup (standard value)	
Capital G	oods			
ϕ	10	Capital adjustment costs	STD(I)/STD(Y)=3	
Fiscal Poli	icy			
γ_1	0.05	G in steady state	G/Y = 20%	
γ_2	0.2	G reaction function	Path of debt	
γ_3	0.3	Tax rate in steady state	Budget balance	
γ_4	0.2	τ reaction function	Path of debt	
Monetary	Policy			
Π	1.005	Inflation	2% p.a.	
R^B	1.01	Nominal interest rate	4% p.a.	
θ_{π}	1.5	Reaction to inflation	Standard value	
$ ho_{R^B}$	0.95	Interest rate smoothing	Nakamura and Steinsson (2013)	
Income Pr	cocess			
$ ho_h$	0.987	Persistence of productivity	Standard value	
$\bar{\sigma}$	0.08	STD of innovations	Share of poor liquidity constrained households	

The proportional tax rate, $1 - \tau = 0.3$, levied on labor income and profits is chosen such that in the stationary equilibrium total tax revenues cover total government expenses on purchases and interest payments on government debt. Government purchases amount to 20% of annual output. Fiscal policy is described either by a tax or spending rule reacting to public debt deviations from steady state, in order to rule out an explosive path of government debt. We pick the reaction parameters in the baseline calibration such that government debt is back at its steady state debt level under both rules after 40 quarters. Table 2.1 summarizes the calibration.

5. Results

In the following, we consider a policy experiment that consists of an unexpected, one-off payment of about \$500 (0.5% of annual output) to each household in the economy paid out in t = 0. We assume that the policy is deficit financed in t = 0 and that from t = 1 onwards either labor taxes or government spending react to debt deviations from steady state according to a fiscal rule that brings debt back to its steady state value.

We first show that the model replicates the distribution of marginal propensities to consume across households as documented by the empirical literature. We then assess the aggregate effects of this type of fiscal intervention under both financing schemes while keeping the path of debt constant. We do so in the full model in which public debt affects household liquidity and in a representative agent version of the model to highlight the importance of liquidity effects for aggregate outcomes.

5.1 Individual Consumption Response

Figure 2.1 plots the marginal propensity to consume (MPC) as a function of a household's asset position. Throughout most of the asset-space households consume very little out of the \$500 payment by the government. This includes the diagonal along which all households would be clustered if we were to consider net worth only. With two assets, however, a large fraction of wealthy households prefers to hold high-return illiquid over liquid assets (capital k_i over bonds b_i). This makes them potentially constrained in their consumption. In addition, these "wealthy hand-to-mouth" households have a higher MPC than their poor counterparts without any assets. The MPC increases in capital because richer households have a higher target for their consumption path. For households very rich in capital but without any liquid assets it actually reaches 90%, whereas households with no assets at all consume around 25% out of extra cash.

These patterns are reminiscent of recent empirical findings on household consumption behavior. Misra and Surico (2014), who use quantile regressions to identify heterogeneity in consumption responses, document that the households with the highest MPCs hold little liquid assets. The model is able to replicate this finding because of two features: First, markets are incomplete and households face idiosyncratic income risk making them ex-post heterogeneous. Second, portfolio choice between liquid and illiquid assets renders a large fraction of households constrained in their consumption each period. Short-run fluctuations in marginal utility are less costly then foregoing the higher return on the illiquid asset.⁵

When it comes to the total effect of transfers, incomplete markets imply that it is important to take into account the path of public debt. An increase in public debt enhances household liquidity by effectively loosening borrowing constraints (see Aiyagari and McGrattan, 1998). When markets are incomplete, this has first order effects on consumption and savings. In the following section, we discuss the role of public debt for the aggregate effects of transfers.

Figure 2.1: Individual consumption response to a transfer shock



Notes: Marginal propensities to consume in partial equilibrium with fixed prices. The bottom of the graph depicts the distribution of households over capital and bond holdings relative to annual income in steady state.

5.2 Aggregate Effects of Transfers Payments

This section assesses the aggregate effects of deficit-financed transfer payments in a model in which public debt affects household liquidity. We compare two different financing scenarios with opposing wealth effects while keeping the path of public debt the same. We first discuss the effects of transfer payments with government spending adjusting from t = 1 onwards. Under this scenario, the present-value wealth effect of the fiscal intervention is positive.

Figure 2.2 shows the response of aggregate prices and quantities to the transfer in the case of lower government spending in the future. The first row of Figure 2.2 depicts output and its components (consumption, investment, government spending) as a percent of transfer. The solid line corresponds to the economy with endogenous heterogeneity, whereas the dashed line shows the response of an economy with two types

⁵The idea that small deviations from optimal consumption imply negligible utility costs goes at least back to Cochrane et al. (1989).

of households: Ricardian households and hand-to-mouth households without access to financial markets. We determine the share of hand-to-mouth households (about 30%) in the two-agent model by matching the consumption response of the full model in the first period.

Accordingly, consumption increases by 50% of the transfer in both economies. This is about twice as much as the partial equilibrium response of consumption discussed in the previous section. With sticky prices the initial increase in consumption is met by higher production and, thus, higher income amplifying the direct effect of the transfer on consumption. This disposable income channel, which relies on the presence of households with high marginal propensities to consume, drives the output response in the first period.

After the first period, the response of unconstrained households becomes central for the path of output. Unconstrained households respond to the positive wealth effect by increasing consumption permanently. To do so, households move wealth into the future. In particular, they increase their holdings of physical assets as the government reduces the amount of outstanding debt at the same time. This investment boom is stronger with complete markets. Under incomplete markets, household portfolio choices are influenced by precautionary motives, and investment therefore responds less strongly because the portfolio composition matters for consumption smoothing (see Luetticke (2015) for a similar argument in the case of monetary policy shocks).

The role of precautionary motives in shaping the investment response becomes more evident in the case of higher future taxes (see Figure 2.3). Under this scenario, the present-value wealth effect of the fiscal intervention is negative because of higher distortionary taxes. In the economy with limited heterogeneity, Ricardian households react to the negative wealth effect by lowering their consumption and savings. This leads to a crowding out of investment given the expansion of public debt. In the economy with endogenous heterogeneity, however, households still save in physical capital because of precautionary motives. Households would like to hang on to their improved consumption-smoothing capacity that the government brought about by increasing the aggregate supply of savings devices. As the government retires its debt, this crowds in private investment. The precautionary motives are also reflected in a lower liquidity premium.

This precautionary investment channel breaks the downward spiral of lower capital and lower labor supply, which occurs under complete markets. As a result, the output response is positive despite the negative wealth effect. Without precautionary savings, by contrast, increasing labor taxes to finance transfers leads to a long lasting recession by crowding out capital.

Key for the crowding in of capital is the transient nature of the increase in public debt. As in Aiyagari and McGrattan (1998), a permanent increase in public debt would displace capital while total savings increase. Figure 2.2 and 2.3 show that a similar logic applies to deficit-financed transfers. Investment initially falls but by substantially less than the increase in bond holdings so that total savings increase significantly. In contrast to a permanent increase in public debt, however, investment immediately recovers as households respond to the reversal in public debt by shifting savings from bonds to capital. After 40 quarters public debt is back at its steady state value and the



Figure 2.2: Response to transfer shock (0.5% of annual output) under spending rule

Notes: Impulse responses to a deficit-financed one-off transfer of 0.5% of annual output with tax rule stabilizing debt. Solid line: the model with heterogeneous households. Dashed line: same calibration with a representative household and rule-of-thumb households. $*LP = (q_{t+1} + r_{t+1})/q_t - R_t^B/\pi_{t+1} * X_t = \int (c_{it} - h_{it} \frac{n_{it}^{1+\gamma}}{1+\gamma}) di$



Figure 2.3: Response to transfer shock (0.5% of annual output) under tax rule

Notes: Impulse responses to a deficit-financed one-off transfer of 0.5% of annual output with tax rule stabilizing debt. Solid line: the model with heterogeneous households. Dashed line: same calibration with a representative household and rule-of-thumb households. $*LP = (q_{t+1} + r_{t+1})/q_t - R_t^B/\pi_{t+1} * X_t = \int (c_{it} - h_{it} \frac{n_{it}^{1+\gamma}}{1+\gamma}) di$

Figure 2.4: Response to transfer shock (0.5% of annual output) for different paths of public debt



Notes: Impulse responses to a deficit-financed one-off transfer of 0.5% of annual output for different parameterizations of the tax rule stabilizing debt. Solid line: Debt back at steady state in 40 quarters. Uneven-dashed line: Debt back at steady state in 30 quarters. Even-dashed line: Debt back at steady state in 20 quarters.

boom in investment comes to an end.

Figure 2.4 compares the output, consumption, and investment response for different paths of public debt. We find that the investment boom is more pronounced for a faster reversal in debt as implied by the precautionary investment motive. Households rely on their savings to smooth consumption in the presence of idiosyncratic income shocks. Hence, households react to a faster reversal in public debt by faster accumulating physical assets. In the case of a return to the steady state debt level in 20 quarters, investment does not fall in the first period and the output response is 40% larger than in the baseline.

6. Conclusion

Deficit-financed fiscal stimulus payments have become an important policy measure to counteract recessions. In this paper, we ask whether the empirical evidence on a sizable consumption response to such transfers at the household level implies that this type of fiscal intervention is indeed expansionary? We do so by building a New Keynesian business cycle model with heterogeneous households that takes into account the financing of transfers and matches the empirical evidence on the individual consumption response. Importantly, in this environment, transfers not only affect the aggregate economy through the disposable income channel but also by enhancing household and market liquidity because of debt finance.

To highlight the importance of this liquidity channel for the aggregate economy, we

contrast our model results to a two-agent model with Ricardian and hand-to-mouth households, which replicates the consumption response to transfers but lacks the liquidity effect of public debt. In the two-agent model, Ricardian households would like to reduce their consumption and savings in response to the negative wealth effect of higher future distortionary taxes, inducing a persistent decline in investment and a prolonged recession. In contrast, in the presence of potentially binding borrowing constraints, a precautionary investment motive overturns this result as households would like to hang on to their improved consumption smoothing capacity and, thus, shift their savings into the physical asset when the government starts to retire its debt.

We find that this liquidity channel is stronger than wealth effects induced by government fincancing decisions and, thus, makes the aggregate effects of transfers expansionary independent of the mode of financing.

Appendices

A. Equilibrium Forecasting Rules

Tables 2.2 displays the equilibrium laws of motion for the Krusell-Smith equilibrium.

β^1	β^2	eta^3	β^4	R^2	
	Spend	ling Rule	e.		
	opone		~		
0.50	-1.23	2.10	-23.69	99.73	
0.01	5.05	-43.21	-38.59	99.95	
	T				
Tax Rule					
0.50	-0.70	0.27	-24.09	99.76	
0.00	3.77	-28.61	-41.56	99.95	
	0.50 0.01 0.50	p p 0.50 -1.23 0.01 5.05 Ta: 0.50 -0.70	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	p p p Spending Rule Spending Rule 0.50 -1.23 2.10 -23.69 0.01 5.05 -43.21 -38.59 Tax Rule 0.50 -0.70 0.27 -24.09	

Table 2.2: Laws of motion

All values are multiplied by 100 for readability.

The equilibrium forecasting rules are obtained by regressing them in each iteration of the algorithm on the response of the economy to the transfer shock. The R^2 is above 99%.

CHAPTER 3

Public Insurance and Wealth Inequality: A Euro Area Analysis

This paper assesses the quantitative importance of cross-country differences in labor market dynamics and social security institutions for euro area differences in private net wealth inequality. I document the empirical puzzle that euro area countries with the largest reduction in the income Gini coefficient through public transfers robustly show higher inequality in private net wealth. Revisiting the argument by Hubbard et al. (1995) that public insurance crowds out private savings especially of the poor, I construct a life-cycle model with heterogeneous households and incomplete markets that features exogenous labor market risks, unemployment benefits, mean-tested minimum income support and public and occupational pensions. Calibrating the model to the euro area differences in the net earnings process, unemployment dynamics and social security system, it can account for 70.1%of the cross-country differences in the net wealth Gini coefficients for the bottom 95% of the wealth distribution. Welfare policies contribute 57.5%to the wealth inequality differences across the euro area, while net earnings and unemployment dynamics account for 12.6%.

1. Introduction

The first wave of the Household Finance and Consumption Survey $(HFCS)^1$, mainly conducted in the period from 2009 to 2011, reveals that there are large cross-country variations in household private net wealth inequality for the ten largest economies in the euro area. The Gini coefficient of household net wealth ranges from 0.76 for Austria to 0.56 in Greece. Since the release of the HFCS in 2013, the causes of the large euro area differences in private net wealth inequality and the surprisingly low median wealth in some euro area countries with high GDP per capita have been at the forefront of public and political debates. In particular, there has been a discussion about the role played by institutional factors. This paper aims to contribute to this debate by assessing the

¹This paper uses data from the HFCS and EU-SILC. The results published and the related observations and conclusions are mine and do not correspond to results or analysis of the HFCN, Eurostat, the European Commission or any other national statistical authorities.

quantitative importance of cross-country differences in social security institutions and labor market dynamics for euro area differences in private net wealth inequality, as measured by the Gini coefficient.

The interest in social security as a potential determinant of the wealth distribution is motivated by the surprising finding that those countries with a larger reduction in the income Gini coefficient through redistributive public transfers to households robustly show higher inequality in private net wealth. On its own, this correlation falls short of providing a quantitative assessment of the importance of these policies. Therefore, going back to the theoretical contribution by Hubbard et al. (1995) that public insurance crowds out private savings, especially of the poor, I construct a life cycle model with heterogeneous households and incomplete markets that features exogenous labor market risks, unemployment insurance, means-tested minimum income support, and pension benefits. There are three key determinants for wealth accumulation in the model, namely old-age provision, leaving bequests, and precautionary savings to self-insure against gross earnings, unemployment and life-span risk. The more redistributive transfers are across households, the more pronounced the crowding-out effect is on private precautionary savings for low-income households, as there is a relatively greater reduction in their need for self-insurance.

Calibrating the model to the euro area differences in net earnings processes and unemployment dynamics and to the institutional differences in public insurance, the model can account for 70.1% of the euro area variation in private net wealth inequality. Moreover, by adopting a modified method based on Guvenen et al. (2014), I provide a decomposition that disentangles how much each factor contributes individually to this fraction. The model results suggest that welfare policies contribute 57.5% to euro area differences in net wealth Gini coefficients for the bottom 95% of the wealth distribution. It turns out that the most important institution of the social security system for determining wealth inequality differences across the euro area is means-tested minimum income support. It contributes the lion's share of 44.8%, followed by public and occupational pension schemes, which account for 10.7%. Differences in unemployment benefits, by contrast, play only a minor role, contributing 2%. Furthermore, the net earnings processs and unemployment dynamics can jointly rationalize 12.6% of the euro area differences in private net wealth inequality. This important role of public insurance for wealth inequality patterns in the euro area also sheds light on why welfare states with more redistributive transfers show higher wealth inequality. While transfers directly mitigate income differences across households, their general availability leads to a more unequal wealth distribution in the long run.

The importance of minimum income support programs for the wealth distribution relative to other policies is rooted in several distinct features. First, minimum income benefits are not dependent on past contributions and hence are more redistributive across individuals compared to unemployment benefits or pensions, which are instead more redistributive over the life cycle in the euro area. The lower bound on consumption leaves households with high expected life-time income relatively unaffected in their precautionary savings decision, while the need for self-insurance of households in the lower part of the income distribution is substantially reduced, thereby increasing wealth inequality. Second, minimum income assistance guarantees a certain lump sum transfer, while future potential unemployment benefits replace a constant fraction of previous net income and are hence still dependent on uncertain net income. Similarly, there remains some uncertainty about the exact pension level during retirement, as it will depend on the household's pre-retirement labor market performance. As households are risk averse, the effects of unemployment and pension benefits on wealth inequality turn out weaker despite strong crowding out effects on aggregate private savings. Third, the asset-based means-testing of minimum income support introduces an implicit tax on savings, such that low-wealth households face a trade-off between saving for bad income states and dissaving to become eligible for income support. And last, minimum income benefits are of unlimited duration, while unemployment benefits mitigate earnings losses only temporarily.

Regarding the calibration of the model, I assume that all countries share the same key technological and preference parameter values and allow them to differ only in the parameters describing the unemployment and net earnings process, as well as the social security system. The coefficient of determination then provides a measure to quantify what fraction of cross-country differences in wealth inequality are generated solely by these country-specific features of the labor market process and social security institutions. For the decomposition exercise, I construct a fictive euro area country as a reference unit, whose parameters correspond to the average of the individual countries' parameters. I then sequentially set the country-specific parameters to the parameters of the constructed reference euro area country until no parameter differences between countries remain, and each time determine the explanatory power of the model. This way, I can quantify how much each factor contributes individually to the overall fraction of cross-country differences explained by the model. The variances of the gross earnings processes are estimated from household income data of the European Survey on Income and Living Conditions (EU-SILC) for 2004 until 2010. The calibration of labor income tax schedules and welfare policies is mainly based on estimates from the OECD benefit and tax model, or own estimates from the EU-SILC.

While several studies have highlighted the distortionary effects of certain institutions of the social security system on aggregate savings, less research has concentrated on the consequences of public insurance for private net wealth inequality. This is because research on wealth inequality has so far mainly focused on the upper tail of the wealth distribution. In contrast, this paper sheds light on the remaining part of the wealth distribution. It is shown that, when considering the bottom 95% of the wealth distribution, large euro area differences in wealth inequality remain, and that public transfers and labor market dynamics are indeed central for determining wealth inequality patterns across the euro area.

The paper is organized as follows. Section 2 discusses the current literature, and section 3 presents empirical evidence. Section 4 introduces the model, section 5 the calibration strategy, and section 6 presents the quantitative results. Section 7 concludes.

2. Related Literature

There is a large theoretical literature on the determinants of wealth inequality, with a particular focus on the high wealth concentration in the United States. In fact, the vast majority of the work has focused on the upper tail of the wealth distribution and, in particular, on explaining high savings rates of wealthy people documented in the empirical literature (Dynan et al., 2000). To account for these, standard incomplete markets models have been extended by various features such as heterogeneity in patience (Krusell and Smith, 1998b), transmission in human capital and voluntary bequests across generations (De Nardi, 2004), entrepreneurship (Quadrini et al., 1999), high returns on capital in the presence of borrowing constraints (Cagetti and De Nardi, 2006) or high earnings risk for top earners (Castaneda et al., 2003).

In contrast, less research has concentrated on the remaining part of the wealth distribution and, in particular, on the low savings of income poor households. In a seminal paper, Hubbard et al. (1995) highlight the distortionary effects of social insurance on households savings behavior through the reduction of income risk and an implicit tax on savings in the presence of asset means-testing. In particular, they aim to explain why many low-income households accumulate only little wealth over the life cycle, much less than a standard life cycle model would suggest. Other work relates precautionary savings to other institutions of social security systems, such as health insurance (Kotlikoff, 1986), public pension systems or unemployment insurance. Engen and Gruber (2001) show theoretically and empirically that higher unemployment replacement rates crowd out aggregate savings. For the empirical analysis, they exploit differences in unemployment generosity across U.S. states.

While most of those papers shed light on the distortion of aggregate savings through specific institutions of the social security system, I jointly model three of them and analyze their implications for wealth inequality. In a similar vein, two papers explicitly relate wealth inequality to public insurance and more concretely public pension systems for specific countries. Domeij and Klein (2002) show that wealth inequality in Sweden is driven to a large extent by its very redistributive public pension scheme. Given the current discussion about wealth inequality fueled by Thomas Piketty, Kaymak and Poschke (2016) made a recent contribution, analyzing the extent to which institutional changes from 1960 to 2010 in the U.S. can explain the increased share of wealth held by the top percentiles. They find that despite the dominant role of changing wage inequality, the expansion of social security in terms of more generous pensions and Medicare can account for an important portion. Hintermaier and Koeniger (2011) also attribute changes in the U.S. net wealth distribution across time to increases in income risk. Considering similar savings motives in an incomplete markets life-cycle model as in the present paper, they also abstract from the upper tail of the wealth distribution for their analysis.

Regarding the euro area, Fessler and Schuerz (2015) provide empirical evidence based on the HFCS for the role of welfare state policies in explaining cross-country differences in household net wealth. Controlling for various household characteristics and inheritance, they find in a multilevel cross-country regression that welfare state expenditures across countries are negatively correlated with household net wealth and hence a substitute for private wealth. Moreover, they show that the substitution effect of pension and social security expenditures with regard to private wealth holdings is significant along the wealth distribution, but relatively lower at high wealth levels. While the latter result empirically confirms the hypothesis that more generous public insurance increases wealth inequality, this present paper explicitly models various features of the social security systems in the euro area and quantifies and decomposes their importance for cross-country differences in wealth inequality.

The only other paper explicitly analyzing wealth inequality differences in the euro area is empirical in nature and has stressed the importance of cross-country differences in home ownership rates (Kaas et al., 2015) and the fact that tenants accumulate less wealth on average than homeowners. I see my paper as complementary to their work, as investment in housing is one way to accumulate wealth. Furthermore, given that empirical evidence suggests a positive correlation of household income and home ownership status and that tenants are commonly low-income households, social security might shed light on the still puzzling question of why tenants accumulate so little wealth compared to homeowners.

3. Redistributive Policies and Wealth Inequality in the Euro Area

3.1 Cross-country Evidence on Wealth Inequality and the Degree of Redistribution of Public Transfers

The newly available household data on private wealth from the Household Finance and Consumption Survey conducted by the ECB (2013) allows a euro area wide comparison of household wealth, given the ex ante coordination of the survey questionnaire and methodology and its emphasis on output harmonization. As in the Survey of Consumer Finances, oversampling procedures of wealthy households are applied in order to achieve unbiased estimates of wealth and its distribution. The release of the first wave of the HFCS in 2013, covering household interviews mainly conducted in the period from 2009 to 2011, allows a reasonable comparative analysis of the distribution of household wealth across the euro area.²

First, I document along the vertical axis of Figure 3.1 that there are large crosscountry variations in household private net wealth inequality for the ten largest economies in the euro area.³ Household net wealth is defined as the household's total assets, i.e. real and financial assets,⁴ net of its total liabilities, and excludes wealth from public or occupational pension plans. The Gini coefficient of household private net wealth ranges from 0.76 for Austria to 0.56 in Greece.⁵ The cross-country evidence on wealth

²The data for Spain in the first wave refers to the year 2008. Since the Spanish survey of household finances (EFF) is the only survey which was conducted before the financial crisis, I use already published data from the second wave which corresponds to the year 2011 and hence allows for a post-crisis analysis in all countries. A comparison of the wealth Gini coefficients for the two waves in Spain indicates an increase in wealth inequality from 0.581 to 0.608 after the bursting of the housing bubble.

³In my analysis, I focus on ten euro area countries, for which I have complete data on all parameters of interest: Austria (AT), Belgium (BE), Finland (FI), France (FR), Germany (DE), Greece (GR), Italy (IT), Netherlands (NL), Portugal (PT) and Spain (ES).

⁴Real assets cover the household's main residence, other real estate, vehicles, valuables, and selfemployment business wealth, while financial assets are composed of deposits, mutual funds, bonds, publicly traded shares, voluntary pensions etc.

⁵Since the HFCS is a multiply imputed dataset, I take the average Gini coefficient over all five implicates.

inequality becomes particularly striking when it is depicted in relation to the degree of redistribution of transfers, which generally aim at reducing income inequality. Based on income data from the European Survey on Income and Living Conditions (EU-SILC) for 2004 to 2010, I first document that, within the euro zone, those countries with more generous and redistributive transfers to households robustly show higher inequality in private net wealth. The degree of redistribution of transfers is measured by the reduction in the Gini coefficient of after-tax earnings when augmenting it with public transfers.⁶ Figure 3.1 shows that there is a negative correlation of -0.5.⁷

Figure 3.1: Correlation of Gini coefficients of household net wealth and percentage change in income Gini coefficients due to transfers



Sources: HFCS (~2010), Encuesta financiera de las familias espanolas (EFF, 2011), EU-SILC (2004-2010)

The redistributive effect of transfers in some countries is so substantial, that even the cross-country correlation of the Gini coefficients of after tax earnings and the Gini coefficients of net wealth switches from positive (0.12) to negative (-0.62) when adding transfers to the income measure. So interestingly, in the euro area countries where inequality in households' earnings after tax and transfers is lowest and transfers reduce the income Gini coefficient to the largest extent, private net wealth is most unequally distributed.

Since the proposed model mechanism relates wealth inequality to public insurance and, in particular, implies that there is a larger share of low-wealth households in more

 $^{^{6}\}mathrm{Public}$ transfers are composed of unemployment benefits, old-age and survivors' benefits, family allowances, housing allowances, and minimum income benefits.

⁷In particular, this negative correlation also holds for the change in the income Gini coefficient of only working households.

generous welfare states, it is important to ascertain that the differences in the wealth Gini coefficients within the euro area are not driven by differences in the share of wealth held by the richest households. Hence, the top 5th percentile of the wealth distribution in each country, owning on average about 50% of total wealth, is discarded. Importantly, their wealth accumulation process is unlikely to be affected by social insurance. The results remain robust and the correlation even increases to -0.69.⁸ It is shown in Appendix A that this result is also robust to other measures of wealth inequality, such as the share of wealth held by the bottom 50%, excluding the top 5th percentile (see Figure 3.5). Accounting for cross-country differences in household compositions through equivalization of net wealth also preserves the result (see Figure 3.6). This robustness of cross-country differences in wealth inequality with respect to the household structure has also been documented by Fessler et al. (2014) in a more profound analysis.

At first, these empirical facts are surprising, as the reduction in income differences across households through transfers would be expected to also translate into lower wealth differences. However, this paper demonstrates, in line with the argument by Hubbard et al. (1995), that generous public insurance crowds out private savings, especially of the poor, and thereby creates a larger of fraction of low-wealth households. This increase in overall wealth inequality through an expansion of the left tail of the wealth distribution in the long run outweighs the mitigating effects of lower income differences through transfers in the short run, leading to higher wealth inequality in more generous welfare states.

3.2 Cross-country Evidence on Wealth Inequality and the Generosity of Public Transfers

In order to analyze differences in the generosity of various social security institutions in the euro zone, I consider statistics on several social security payments. Plotting those against the countries' private net wealth Gini coefficients suggests that there might be some systematic relationship between social security generosity and private net wealth inequality.

Minimum Income Support Programs

Figure 3.2 shows a positive correlation of 0.81 between the Gini coefficient of private net wealth (of the bottom 95%) and the absolute amount of minimum income benefits of means-tested income support programs expressed as a percentage of median net labor income of employed working-age households according to the EU-SILC. Since the EuMin database on minimum income protection in Europe (Bahle and Huble, 2012) provides the legally stipulated amounts of minimum income benefits for various household types, benefits are weighted by the household composition in the HFCS. This positive correlation is in line with the model predictions that in countries with more generous minimum income benefits, poor households have lower incentives to save for

⁸Discarding the top 10th or 20th percentile from the wealth distribution does not considerably change the relative order and level of the Gini coefficients for private net wealth and only further increases the negative correlation to -0.71 and -0.72, respectively.

old-age and precautionary motives and hence wealth will be distributed more unequally. Notably, Italy and Greece are the only two countries which do not provide universal minimum income support to their citizens.⁹





Sources: HFCS (~2010), EEF (2011), EuMin database/OECD "Benefits and Wages" database (2004-2008, 2010)

Unemployment Insurance

Furthermore, I document in Figure 3.3 a positive correlation of 0.45 between private net wealth inequality for the bottom 95% and the average unemployment net replacement rate over the first 60 months. Similarly to minimum income benefits, the average unemployment benefit replacement rates are weighted by the household compositions in the HFCS. It should be kept in mind that the average net replacement rate alone is not indicative of which benefit system provides better insurance to households, since

⁹ In 1998, Italy implemented local minimum income schemes in some municipalities as an experiment. However, this policy was abolished again in 2003 and replaced with an optional and poorly subsidized policy, which enabled only some wealthier regions to implement the scheme (Casas, 2005). Only in 2014, the Italian government decided to introduce a pilot project called "support for active inclusion". However, so far support is linked to previous labor market participation and focuses on families with children (Social Protection Committee, 2014). In Greece, the provision of income support is up to regional authorities and mainly targets specific groups, e.g. in old age, in mountainous regions or poor households with children. However, as can be seen in the graph, these child benefits are very low and amounted from 2004 until 2010 on average to 55 Euro per month. The main political arguments in Greece against minimum income support are budgetary constraints and high regional income diversity which hinders the government from setting one universal minimum income standard (Casas, 2005).

the insurance effect also depends on the duration of benefit eligibility and the expected length of the unemployment spell.



Figure 3.3: Wealth inequality and unemployment insurance

Sources: HFCS (~2010), EEF (2011), OECD "Benefits and Wages" database (2004/2010)

Public and Occupational Pension Entitlements

Figure 3.4 depicts cross-country differences in pension generosity and suggests a strong positive relationship of 0.73 with private net wealth inequality, a finding that is in line with the theoretical predictions of the model. The generosity of the pension system is measured by the net pension replacement rate and defined as median net old-age/survivors' benefits of retired households aged 65 to 75 relative to median net earnings of employed and unemployed households aged 50 to 60. It is calculated from the annual waves of the EU-SILC (2004-2010). Besides public pension entitlements, it also captures survivors' benefits and payments by occupational pension plans.

4. Model

Following Hubbard et al. (1995), I consider a partial equilibrium life-cycle model with incomplete markets and a small open economy.

The household sector faces idiosyncratic earnings risk, i.e. stochastic fluctuations in gross earnings, but also an exogenous risk of becoming unemployed as in Wellschmied (2015). Unemployment risk is explicitly modeled to analyze the role of unemployment insurance.



Figure 3.4: Wealth inequality and public pension wealth

Sources: HFCS (~2010), EEF (2011), EU-SILC (2004-2010)

Households save for old age, and for precautionary and bequest motives. The government may provide public insurance to households such as asset-based means-tested minimum income support, unemployment insurance or pension schemes. Households pay progressive labor income taxes and make social security contributions.

4.1 Household Income

Households' heads enter the labor market at age h_1 and work for 40 years. Until retirement at age h_{40} , households' gross earnings during employment are composed of a deterministic part, $\mu_{c,h}$, determined by age, and a stochastic component, z_h , that captures the uncertainty and persistence of earnings shocks:

$$\omega_h = \mu_{c,h} + z_h$$

The stochastic component evolves according to an AR(1) process:

$$z_h = \rho z_{h-1} + \nu_h, \ \nu_h \sim N(0, \sigma_c^2)$$

$$c \in \{AT, BE, FI, FR, DE, GR, IT, NL, PT, ES\}$$

Employed (e) households pay progressive labor income taxes, which also include social security contributions, and obtain net earnings:

$$w_{h,e}^{net} = (1 - \tau(\omega_h))\omega_h$$

where $\tau_c(\omega_h)$ is a tax rate function of gross earnings. Given that a household's employment status and earnings determine its potential to accumulate wealth, a steeper age-earnings profile, $\mu_{c,h}$, higher gross earnings risk, σ_c^2 , and unemployment risk also translate into higher wealth inequality. However, the effect will be muted under more progressive income taxation.

If the household becomes unemployed, the government provides unemployment insurance up to the retirement age of h_{40} . In general, unemployment benefits are modeled to replace a constant fraction of the previous period's net earnings. To avoid an additional state variable in the household's problem, previous period's net earnings are approximated by today's earnings, which would have realized if the household had not become unemployed. Since the earnings process is quite persistent, last period's net earnings are fairly well approximated.

The country-specific initial net unemployment replacement rate $rr_c(\omega_h)$ is a function of gross earnings. Households receive benefits in the first period of unemployment with certainty and keep them in the following period with some positive probability p_c . This modeling approach is meant to capture cross-country differences in the duration of benefit eligibility.

$$b_h = \begin{cases} rr_c(\omega_h) w_{h,e}^{net}, \text{ if eligible} \\ 0, \text{ if not eligible} \end{cases}$$

Engen and Gruber (2001) have shown theoretically and empirically that higher unemployment replacement rates decrease aggregate precautionary savings. Consequently, in the case of redistributive unemployment schemes, i.e. households with lower gross income have higher replacement rates, private savings of the low-income are crowded out relatively more, thereby increasing overall wealth inequality.

However, the degree of insurance through unemployment benefits also critically depends on the overall unemployment risk, i.e. the joint-job finding and job-separation rate. Moreover, the impact of the unemployment insurance system on wealth inequality also depends on how well benefits insure households during their expected spell of unemployment. If the expected unemployment duration is relatively long or the replacement rate low, households will be incentivized to increase self-insurance and hence wealth inequality will be lower.

The household stops working at retirement age and receives public pension payments, which are a concave function f_c of the household's pre-retirement net earnings.

$$w_h^r = f_c(w_{h_{40},e}^{net}), \text{ if } h > 40$$

Consequently, the implied retirement net replacement rate declines with pre-retirement net earnings. If the household is unemployed prior to retirement, the pension payment replaces a fraction of the net earnings that would have realized according to the stochastic process if the household had been employed in the period prior to retirement. Otherwise, pension losses due to unemployment shocks would be highly overestimated, as pensions usually replace a fraction of life-time earnings. Since earnings are calibrated to be quite persistent, pre-retirement earnings are used to proxy life-time earnings. There is no explicit retirement decision.

In my framework, there are two features of the pension system affecting wealth inequality, namely its generosity and its degree of progressivity. The more generous and redistributive the pension entitlements, the more unequally wealth should be distributed among households.

First, the model predicts a positive relationship between the generosity of pension entitlements and wealth inequality. The higher expected pension payments in the future, the less private wealth households will accumulate for old-age provision. This displacement effect does not only matter for aggregate savings, but also for wealth inequality, because households cannot access their public and occupational pension accounts during working life. The higher the public pension entitlements, the lower the overall stock of private net wealth that can be used for consumption smoothing in case of negative income shocks. Households are therefore more likely to deaccumulate assets and become borrowing-constrained in the presence of more generous pension schemes, because they can no longer pool their precautionary and old-age savings.

Second, as with unemployment benefits, a redistributive pension scheme will particularly discourage savings of low-income households (Domeij and Klein, 2002).

More generally, household net earnings before minimum income benefits are defined as

$$w_h^{net} = \begin{cases} \omega_h (1 - \tau_c(\omega_h)), \text{ if e and } h \le 40 \\\\ b_h, \text{ if } u_b \text{ and } h \le 40 \\\\ \omega_{min}, \text{ if } u \text{ and } h \le 40 \\\\ w_h^r, \text{ if } h > 40 \end{cases}$$

where e stands for being employed, u_b for being unemployed and eligible for unemployment benefits and u for being unemployed and no longer eligible. ω_{min} is meant to capture a minimum income that can be privately obtained by the household when unemployed, e.g. through private transfers from family members outside the household.

If unemployment benefits expire or the working income is too low to cover basic household expenses, households may become eligible for minimum income benefits, i.e. the government guarantees a minimum consumption floor, $T\bar{R}_c$. Minimum income support programs are considered to be households' last public safety net. In contrast to unemployment benefits and pension payments, they are universal and hence do not depend on past contributions, but only on the households' current means in terms of assets and income. Therefore, the actual transfer, TR_c , made to the household negatively depends on its choice of end-of-period wealth, k_h , and its net income, w_h^{net} , and is zero, if both exceed TR_c .

$$TR_{c}(k_{h}, w_{h}^{net}) = max\{0, TR_{c} - k_{h}(1+r) - w_{h}^{net}\}$$

Asset-tested minimum income benefits were first introduced into a life-cycle model in a seminal paper by Hubbard et al. (1995). The effect of means-tested minimum income benefits on wealth inequality is twofold. First, minimum income support reduces, in particular, the downward income risk for low-income households and thereby lowers their need to self-insure to a relatively greater extent. They show that households with high expected life-time income still maintain the usual incentives to save for precautionary purposes, while this motive is highly distorted for the lower part of the income distribution for whom minimum income benefits make up a larger fraction of their life-time income, thus amplifying the wealth gap between high and low-income households. Second, asset-based means-testing introduces an implicit tax on savings and hence households face a trade-off between saving for precautionary motives and dissaving to become eligible for income support.

4.2 Household Optimization Problem

Each period, the household chooses its total consumption, c, and end-of-period assets, k, given its beginning-of-period assets, a, labor income, z, and employment status. In particular, it forms expectations about whether it will be alive and employed in the next period and if so, what the resulting labor market income will be. In case the household was employed the previous period and becomes unemployed, it will always receive unemployment benefits in the first year. The dynamic planning problem of the household subject to the budget and non-negativity constraint will be presented for each labor market status $\{e, u_b, u\}$.

The Bellman equation of the employed household at age h is:

$$V(h, a, z, e) = \max_{c,k} \left\{ u(c) + \beta \mathbf{E} \left\{ (1 - \iota_h) [(1 - \delta_c) V(h + 1, a', z', e) + \delta_c V(h + 1, a', z', u_b)] + \iota_h \phi(a') \right\} \right\}$$

$$s.t.: c + \frac{a'}{1+r} = a + w^{net} + \frac{TR_c(k, w^{net})}{1+r}$$
$$a' = (1+r)k + TR_c(k, w^{net}) \ge 0$$

where δ_c is the country-specific probability of job separation and E is the expectation operator. The household dies with probability ι_h at age h and with certainty at the age of 84. For every deceased household, a new household will be born. Beginning-ofperiod assets in the next period, a', correspond to the end-of-period asset choice of the household, the earned dividend and potential end-of-period transfers, and have to be non-negative. The fact that low net wealth households close to the borrowing constraint are likely to be income poor households with limited access to credit, motivates the assumption of zero borrowing across all countries. The utility function displays constant relative risk aversion (CRRA) with risk aversion parameter ξ :

$$u(u) = \frac{1}{1-\xi}u^{1-\xi}, \quad \xi > 0,$$

For the bequest function, I choose the specification as in De Nardi and Yang (2015):

$$\phi(a) = \phi_1 \frac{(a+\phi_2)^{1-\xi}}{1-\xi}, \quad \xi > 0,$$

The parameter ϕ_1 governs the desire to leave bequests, while the parameter ϕ_2 reflects the extent to which bequests are luxury goods. Note that households' preferences for leaving bequests are assumed to be equal across all countries, as the analysis of crosscountry differences in the bequest motives are beyond the scope of this paper.

The optimization problem of a currently unemployed household receiving benefits is:

$$V(h, a, z, u_b) = \max_{c,k} \left\{ u(c) + \beta \mathbf{E} \left\{ (1 - \iota_h) \left[\gamma_c V(h + 1, a', z', e) + (1 - \gamma_c) [p_c V(h + 1, a', z', u_b) + (1 - p_c) V(h + 1, a', z', u)] \right] + \iota_h \phi(a') \right\} \right\}$$

$$s.t.: c + \frac{a'}{1+r} = a + b + \frac{TR_c(k,b)}{1+r}$$
$$a' = (1+r)k + TR_c(k, w^{net}) \ge 0$$

The household either finds a new job with probability γ_c or stays in unemployment, in which case it keeps its benefits with probability p_c . This modeling approach is meant to capture cross-country differences in the duration of benefit eligibility. I also allow for cross-country differences in unemployment rates through country-specific parameters for the job separation rate, δ_c , and job finding rate, γ_c , as the ultimate effect of the unemployment insurance system on wealth inequality also depends on how well benefits insure households during their expected period of unemployment.

The optimization problem of a currently unemployed household no longer receiving unemployment benefits is:

$$V(h, a, z, u) = \max_{c,k} \left\{ u(c) + \beta \mathbf{E} \left\{ (1 - \iota_h) \left[\gamma_c EV(h + 1, a', z', e) + (1 - \gamma_c) V(h + 1, a', z', u) \right] + \iota_h \phi(a') \right\} \right\}$$

$$s.t.: c + \frac{a'}{1+r} = a + \omega_{min} + \frac{TR_c(k, \omega_{min})}{1+r}$$
$$a' = (1+r)k + TR_c(k, \omega_{min}) \ge 0$$

In the present paper, I allow for a simple "warm-glow" type of bequest motive to ensure that not all households deaccumulate their wealth during retirement. In order to analyze the role of bequests for euro area differences in wealth inequality, and in particular the interaction between bequests and social security provision, a more enhanced model of bequests would be needed, where young households anticipate and inherit only the wealth accumulated and left by their parents. In this kind of framework, bequests generally lead to an amplification of wealth inequality in the presence of public insurance, since social security "disinherits" the poor (Gokhale et al., 2001). However, allowing for this type of bequest here would substantially increase the computational burden of the model, because young households also would have to take into account their parents' state variables in order to form expectations about the size of their future bequest. Therefore, the analysis of the role of bequests for cross-country wealth inequality differences is left for future work.

5. Calibration

5.1 Household Parameters

The model's parameters are calibrated at an annual frequency and the baseline parameters are reported in Table 3.1. Despite the fact that each parameter is presented to be calibrated individually to match a specific data moment, it has to be kept in mind that all parameters are of course calibrated jointly.

For the felicity and bequest function, I calibrate the coefficient of relative risk aversion to a value of $\xi = 1.5$, as in a closely related paper on minimum income benefits by Wellschmied (2015). The two parameters ϕ_1 and ϕ_2 governing the bequest motive are pinned down by matching the average and median wealth of households at the end of the life cycle relative to average and median wealth of younger households in the euro area. ϕ_1 determines the overall strength of the bequest motive, and is chosen to match the average across all euro area countries' ratios of mean wealth of households older or equal to 84 years relative to households younger than 84 of 0.56 (see Table 3.8 Appendix C). ϕ_2 mainly affects the distribution of wealth at older ages, as for $\phi_2 > 0$ bequests become a luxury good and only households at a certain treshold of wealth will have the desire to leave bequests. Hence, I match in the model, as a second moment, the average across all countries' ratios of median wealth of households older or equal to 84 years relative to median wealth of their younger counterpart.

The annual real interest rate is set to 2.5%, which corresponds to the average across the countries' annual yields of 10-year national government bonds traded in the secondary market and is adjusted for inflation as measured by the annual rate of change of the Harmonised Index of Consumer Prices.¹⁰ The choice of this specific annual real

 $^{^{10}{\}rm Government}$ bond yields and inflation rates cover the years from 1997 to 2010 and are provided by the ECB Statistical Data Warehouse.

interest rate is however not crucial for the model's implied cross-country differences in wealth inequality, as e.g. a higher real interest rate of 4% would leave the explanatory power of the model unchanged. The model's time preference parameter, β , is set to the same value of 0.983 across all ten euro area countries (N=10) and chosen to equalize the average over all countries' Gini coefficients in the data and in the model:

$$\bar{\lambda} = \frac{\sum_{c=1}^{N} \lambda_c}{N} = \frac{\sum_{c=1}^{N} \hat{\lambda}_c(\beta, \boldsymbol{\theta}_c)}{N}$$

where λ_c is the Gini coefficient of private net wealth in country c according to the HFCS, after discarding the top 5th percentile from the wealth distribution in each country. The model's predicted wealth Gini coefficient, $\hat{\lambda}_c(\boldsymbol{\theta}_c)$, for country c is a function of M country-specific parameters, $\boldsymbol{\theta}_c = \left\{ \theta_c^1, \dots, \theta_c^M \right\}$, describing its labor market process and welfare policies. In the baseline analysis, I choose to discard the top 5th percentiles from the actual wealth distributions for the calculation of λ_c , as the model is, like most incomplete markets models, incapable of matching the high wealth inequality levels observed in the data without generating an overly large fraction of zero wealth households.

Parameter	Description	Target/Source		
β 0.9	3 Discount factor	Mean over countries' Gini coefficients of private net wealth of 0.573 (HFCS, ~ 2010)		
ξ 1	.5 Coefficient of RRA	Wellschmied (2015)		
r 2.5	% Annual real interest rate	Mean over countries' real annual yields of 10-year government bonds traded on secondary market (ECB SDW, 1999-2010)		
ϕ_1	28 Bequest utility	Avg. euro area ratio of median/mean		
ϕ_2 8	.8 Bequest utility shifter	wealth of heads aged ≥ 84 relative median/ mean wealth of heads aged < 84 : 0.3167/0.5559		
$a_{h_1} \in 58$	3 Initial asset level at age 22	Mean over countries' median asset holdings of households with heads aged 20-25		
ω_{min}	0 Private transfers to unemployed household	Median net private transfers received by		
ι_h	Probability of dying	Life tables for euro area countries, Eurostat (1995-2010)		
h_1	2 Age of labor market entr			
h_{40}	52 Age of retirement	Avg. pensionable age in (OECD, 1999-2010)		
h_{62}	Age of decease	Oldest age in life table		

All countries are assumed to have the same annual survival probabilities for households' heads at a given age. They are set to the average across all country-specific survival probability rates. This data is provided by the Eurostat life tables for the time span from 1995 to 2010. The survival probability of households at age 84 is set to zero. To account for the current age structure of the population in euro area, I average across the countries' age distributions in the HFCS, approximate the resulting euro area age distribution with a polynomial function of order five and apply the obtained age weights to the model before computing the Gini coefficients.

Households at the age of 22 start with an initial asset level of $a_{h_1} = €5813$, which corresponds to the mean over countries' median asset holdings of households with heads aged between 20 and 25.¹¹ The median unemployed household in the EU-SILC received zero net private transfers from 2004 to 2010. To test for robustness, I will also allow for positive private transfers in section 6.

I then assume that all countries share the same baseline parameters shown in Table 3.1 and I allow them to differ only in the parameters describing the unemployment and net earnings process, as well as the social security system. This way, I can examine the variation in cross-country wealth inequalities that are generated by the countries' specificities of their social security institutions and labor markets.

5.2 Labor Market Process Before Retirement

Earnings and unemployment dynamics are assumed to be purely exogenous. The job separation and finding rates are chosen to match the average unemployment rate and percentage share of long-term unemployed in each country. The household unemployment rates for the time span 2004 to 2010 in the ten euro area countries are calculated from the EU-SILC dataset using weights and are displayed in Table 3.2. They are derived from the employment status of the household's head who is identified as the household member with the highest personal income, in terms of gross earnings and public individual transfers such as e.g. unemployment benefits. The percentage of long-term unemployed households, i.e. households with heads being unemployed for more than a year, cannot be determined from the EU-SILC directly and is hence approximated with the average percentage share of long-term unemployed individuals reported by Eurostat from 2004 to 2010.

The earnings risk is estimated for each country from EU-SILC gross earnings data of employed households' heads between 25 and 60 years. I assume that the observed logincome of a household, $y_{i,h,t}$ is composed of a deterministic part, $f(o_{i,h,t})$, determined by observable household characteristics of the household head, $o_{i,h,t}$, and a stochastic component, $y_{i,h,t}^*$, which follows an AR(1) process with persistence ρ :

$$y_{i,h,t} = f(o_{i,h,t}) + y_{i,h,t}^*,$$
(3.1)

$$y_{i,h,t}^* = \rho y_{i,h-1,t-1}^* + \nu_{i,h,t}, \qquad (3.2)$$

$$\nu_{i,h,t} \sim N(0,\hat{\sigma}_c^2) \tag{3.3}$$

where t is the year and h is the age of household i. Earnings shocks, $\nu_{i,h,t}$, are assumed to be drawn from a log-normal distribution. This log-normality assumption, although it usually does not hold perfectly, allows me to approximate the earnings process using a Markov chain with seven income states.

 $^{^{11}\}mathrm{Note}$ that households' initial wealth does not necessarily coincide with the amount of bequests left by deceased households.

I estimate the deterministic component, $f(o_{i,h,t})$ by regressing log-earnings on an age polynomial of order 4, education and gender dummies, and the household composition. In order to control for variations in household size and composition over the life cycle, I include the number of heads (single or couple), the number of children younger than 18 and the number of other dependent adults in each household. I eliminate any observation where the residual of this regression, $y_{i,h,t}^*$, belongs to the bottom or top 0.5 % of all residuals.

Following Hintermaier and Koeniger (2016), the sample variance of the residuals, $y_{i,h,t}^*$, from this regression is used to derive the country-specific short-run variance of gross earnings shocks, $\hat{\sigma}_c^2$, assuming the same persistence of $\rho = 0.95$ of earnings shocks across all countries, a common estimate in the empirical literature. This parameter is calibrated rather than estimated, such that the cross-country differences predicted by the model do not depend on imprecise estimates of this parameter, given the short time horizon of only 3 years available for estimation in some countries.¹² While controlling for household characteristics leads to an underestimation of overall earnings heterogeneity in the model, it allows me to obtain a better proxy for pure earnings risk. This is important because the degree of earnings risk matters for the accumulation of precautionary savings and its distribution, which aside from savings for old age is of main interest in this paper.

Since the EU-SILC panel only follows a household for three consecutive years, it is not possible to observe the full life-cycle earnings of a cohort. However, I make use of the cross-sectional age-earnings patterns to approximate deterministic life-cycle profiles, $\mu_{c,h}$, in the respective countries. Therefore, I determine median gross earnings at each age and smooth the profile using a forth order polynomial. Since, due to positive real earnings growth e.g. cohorts at the age of 30 have on average a higher nominal median earnings level than cohorts of currently 50-years did have 20 years ago, this cross-sectional age-earnings profile underestimates earnings growth over the life cycle. Hence, in order to make average earnings comparable across cohorts, I need to transform median earnings at a given age relative to the base age of 50 by multiplying it with the real wage growth factor $(1+q)^{age-50}$. I adjust all earnings profiles with the euro area average real growth rate of 1% calculated from average real wages from 1990 until 2010 provided by the OECD Database.¹³ Figure 3.7 shows the countries' crosssectional age-earnings profiles, adjusted for real euro area wage growth. Furthermore, they are divided by the average Purchasing Power Parity index from 2004 to 2010 of the respective country to ensure comparability across the euro area. Note that the results are also robust to using average gross earnings instead of median earnings for the calibration of the age-earnings profile.

As the EU-SILC is a survey specifically designed to measure household income, gross earnings data from the EU-SILC is preferred over data from the HFCS. While house-

¹²The EU-SILC Survey was implemented in 2004 in Austria, Belgium, Finland and France, in 2005 in Germany and the Netherlands, in 2006 in Spain, and in 2007 in Greece, Italy and Portugal.

¹³Average real wages are obtained by dividing the national-accounts-based total wage bill by the average number of employees in the total economy, which is then multiplied by the ratio of the average usual weekly hours per full-time employee to the average usual weekly hours for all employees. They are measured in USD constant prices using 2012 as a base year and Purchasing Power Parities for private consumption in the same year.

holds interviewed in the EU-SILC are explicitly asked for all potential income sources, earnings questions in the HFCS are more broadly categorized, thereby increasing the risk of imprecise measurement.

Country	Std. of earnings shocks σ	Job sep. rate δ	Job find. rate γ	Unempl. rate	Fract. of long-term unemployed $(>1 y)$
AT	0.1809	0.0351	0.7450	6.4%	53%
BE	0.1501	0.0405	0.4590	9.8%	23%
\mathbf{FI}	0.1539	0.0840	0.7560	8.3%	30%
\mathbf{FR}	0.1599	0.0611	0.6100	7.8%	23%
DE	0.1802	0.0447	0.4810	10.5%	40%
GR	0.1885	0.0578	0.5090	5.6%	48%
IT	0.1723	0.0417	0.4220	6.7%	49%
NL	0.1533	0.0370	0.6610	2.9%	48%
\mathbf{PT}	0.1758	0.0437	0.5550	7.6%	49%
ES	0.1750	0.1021	0.5880	9.9%	27%

Table 3.2: Parameters of labor market risk

5.3 Policy Parameters

Minimum Income Support

Information on the absolute amount of minimum income benefits is taken from a comparative database on minimum income protection in Europe, which provides annual data from 2004 until 2008. The data is based on the OECD tax and benefit model which simulates minimum income benefits for various household types (single person or married couple, without children or with 2 children). However, the corresponding OECD "Benefits and Wages" database only provides information for the years 2005, 2007 and 2010. Therefore, I make use of the data provided by the EuMin database until 2008 and complement it with OECD data for 2010 and interpolate in-between for 2009. Minimum income benefits cover cash benefits, including housing benefits as well as child benefits. Since e.g. married couples with children are entitled to more generous social assistance, I take the different household compositions in the euro area countries into account when computing the average expected entitlements by weighting with the corresponding percentage of households of each type in the sample.¹⁴ Figure 3.2 shows the weighted absolute amount of minimum income benefits expressed as a percentage

¹⁴Using information on the relationship and age of household members in the HFCS, I assign all households in each country to 4 different types which are meant to approximate the aforementioned types stipulated in the OECD benefit model: Single head or head in partnership/marriage, without children or with at least one child.

of median net earnings of employed households aged 25 to 60 in the EU-SILC sample.¹⁵

Unemployment Insurance

The OECD database "Benefits and Wages" reports the initial unemployment net replacement rates for multiples of average worker (AW) gross earnings and 6 different household types (single person, one-earner married couple or two-earner married couple, without children or with two children). Since unemployment net replacement rates differ for distinct household types due to e.g. potential family, childcare or lone-parent benefits, the average net replacement rate used for the calibration of the model is obtained by weighting with the corresponding fraction of households of each approximate type in the sample. Similarly to the weighting of minimum income benefits, I use information from the HFCS on age, and employment status and relationships of household members, to assign households to 6 categories. Since unemployment benefits depend on an additional characteristic of the household, namely whether the household is a one-earner or two-earner married couple, I also use information on the employment status to classify households.

Figure 3.8 in Appendix C depicts the initial net replacement rate, averaged over the years 2004 and 2010, as a function of multiples of average gross earnings and reveals that unemployment benefits replace a larger fraction of previous earnings for low-income households.¹⁶ The country-specific probability, p_c , of keeping benefits from the second year on is chosen to match the average net replacement rates over the first 5 years of unemployment, a statistic also provided by the OECD database.

Public and Occupational Pension Scheme

The net pension replacement rate as shown in Figure 3.4 is defined as the median net old-age and survivors' benefits of retired households aged 65 to 75 relative to median net earnings of employed and unemployed households aged 50 to 60 in the EU-SILC. Since most occupational pension plans held by households are still of type defined benefit and future payments are therefore dependent on unknown future conditions, the HFCS measures households' entitlements to their occupational pension plans relatively

¹⁵Spain constitutes a special case, as it is the only country where minimum income provision is a regional competence and conditions of payment can hence vary across regions. Therefore, the minimum income support for Spain reported in the EuMin database only refers to the amount of minimum income benefits available to households resident in the community of Madrid. Moreover, minimum income benefits are only of unlimited duration in the six autonomous communities of Asturias, Castilla y Leon, Madrid, Cataluna, Extremadura and Valencia (length of 3 years), which were inhabited by 50.7% of Spain's total population in 2011. The calibration strategy for Spain is to solve the model twice, first for regions for which the model assumes no minimum income scheme for simplicity, and second, for the remaining regions under the assumption that these provide unlimited minimum income benefits of an amount equal to that of Madrid. The overall implied Gini coefficient of private net wealth in Spain is computed from the weighted average of the two resulting wealth distributions.

¹⁶For the model's calibration, I assume that households with previous gross earnings levels smaller than 67% of average gross earnings have the same net replacement rate as households with previous earnings equal to 67% of average gross earnings. The net replacement rates of households with previous gross earnings larger than 1.5 times of average gross earnings are extrapolated and stay constant.
Table 3.3	8: Policy	Parameters
-----------	-----------	------------

Parameter	Target	Data Source
Minimum inco	ome benefits	
$T\bar{R}_c$	Income floor (weighted	EuMin database (2004-2008),
-	by household composition)	OECD "Benefits and Wages" database (2010)
Unemploymer	nt insurance	
$rr_c(\omega_h)$	Initial net replacement rate (weighted)	OECD "Benefits and Wages"
\mathbf{p}_c	Avg. net replacement rate over first 5 years of unemployment	database (2004/2010)
Pensions		
$f_c(x) = a_c x^{\frac{1}{b_c}}$	Median net pension replacement rate	EU-SILC (2004-2010), adjusted for
	Pension progressivity index $1 - \frac{gini(w_{1}^{r})}{gini(w_{net}^{net})}$	real wage growth
Earnings tax	$gini(w_{40}^2)$	
$ au_c(\omega_h)$	Avg. tax rate schedule defined on multiples of average earnings	OECD tax database (2010)

poorly. Therefore, I follow the Household Finance and Consumption Network and only include private pension wealth in the calculation of total private net wealth. Instead, the net pension replacement rate includes income from both public and occupational pension plans, but excludes pension income from individual private plans. In order to allow for comparability of incomes of different cohorts, I adjust earnings and pensions for the average euro area real wage growth of 1%, as described in Section 5.2. In order to capture the extent to which the public and occupational pension systems are redistributive, the pension progressivity index is calculated. In the model, the index is defined as 1 minus the ratio of the Gini coefficient of net pension income w_{41}^r relative to the Gini coefficient of pre-retirement net income of employed and unemployed households w_{40}^{net} (see formula in Table 3.3). For this measure, I again refer to the two age groups used for the calculation of the net replacement rate. If pensions are perfectly proportional to pre-retirement income and hence not redistributive, the Gini coefficient of pensions is equal to the Gini coefficient of pre-retirement earnings and the progressivity index corresponds to 0. If there were a flat-rate pension scheme instead, the Gini coefficient of pensions is zero and the index would take a value of 1. I assume that pension payments are a concave function of the household's pre-retirement net earnings: $f_c(x) = a_c x^{\frac{1}{b_c}}$. I choose the constant a_c and b_c in order to match the pension progressivity index and median net replacement rate in the EU-SILC. The resulting net replacement rates are declining convex functions of pre-retirement net earnings and are shown in Figure 3.9 of Appendix C. For the calibration, I choose an empirical estimate over the theoretical net replacement rates provided by the OECD, because the latter neglect public pensions paid to state employees, which are quite generous in Germany and France.¹⁷

¹⁷The OECD replacement rates measure the theoretical net pension replacement rate of a representative worker who works a full career and enters the labor market today taking into account all to

Labor Income Taxes and Social Security Contributions

The OECD income tax database provides the average labor income tax rates for various levels of gross earnings. It includes central and sub-central government income taxes as well as employee social security contributions. The tax schedule which is defined over multiples of average earnings is directly applied in the model and interpolated in-between. Negative average tax rates are ruled out. Since the OECD tax database only provides the average tax rates for up to twice the average earnings, top marginal tax rates are used to infer the average tax rate for higher earnings (see Guvenen et al. (2014) for the applied method).

6. Results

This section presents the main results. In the following, I seek to answer which fraction of euro area variation in wealth inequality can be attributed to differences in labor market dynamics and the various features of the social security system. Furthermore, I will identify how much each factor contributes individually to this fraction.

6.1 Quantitative Importance of Labor Income Dynamics and Welfare Policies

First, I will determine which fraction of euro area variation in wealth inequality can be explained by all factors of interest. In order to determine the predictive power of the model for cross-country differences in wealth inequality, I make use of the coefficient of determination. It is a common measure to determine the goodness of fit of forecasting models. The model's predicted wealth Gini coefficient, $\hat{\lambda}_c(\boldsymbol{\theta}_c)$, for country c is based on a parameter vector of all the country-specific parameters, $\boldsymbol{\theta}_c$, describing its welfare policies and labor market process, and can be interpreted as a forecast of the actual wealth Gini coefficient, λ_c . Let $\hat{\epsilon}_c$ denote the forecast error of the model when predicting the wealth Gini coefficient of country c:

$$\hat{\epsilon}_c = \lambda_c - \hat{\lambda}_c(\boldsymbol{\theta_c})$$

The model's time preference parameter, β , is set equally across all ten countries (N=10) and calibrated to equalize the average across all country's Gini coefficients in the data and in the model, given the country-specific parameterization. This implies an average model prediction error, $\bar{\hat{\epsilon}}$, of zero and hence no systematic over- or underestimation of wealth inequality levels for the euro area countries considered. Assuming that all the countries share the same baseline parameter values shown in Table 3.1, I allow them to differ only in the parameters describing the unemployment and net earnings process, as well as the social security system. The coefficient of determination is used to quantify the predictive power of the model for cross-country variations in wealth inequality.

date enacted pensions reforms. So while the OECD net replacement rates refers to individuals, the EU-SILC median net replacement rate refers to a household unit. Furthermore, the OECD theoretical replacement rate might overestimate pension entitlements in countries with higher unemployment rates or lower labor market participation rates.

The coefficient of determination, R^2 , relates the total sum of squared forecast errors generated by the calibrated model to the total sum of squared forecast errors implied by a benchmark model predicting the same Gini coefficient, namely the mean, for each country. However, squaring forecast errors leads to an unequal weighting of small and large forecast errors. Therefore, in order to equally weight each country's forecast for the overall assessment of the model, a modified coefficient of determination, R, is introduced which expresses forecast errors in absolute instead of squared terms. It is defined as:

$$R = 1 - \frac{\sum_{c=1}^{N} |\hat{\epsilon}_c|}{\sum_{c=1}^{N} |\lambda_c - \bar{\lambda}|}$$

Relative to other constants, the mean is the most suitable benchmark forecast to evaluate the model's predictive power due to its property of minimizing the sum of absolute forecast errors:

$$\bar{\lambda} \in \operatorname{argmin}\left\{\Sigma_{c=1}^{N} |\lambda_{c} - x|\right\}$$

Table 3.4 quantifies the overall importance of welfare policies and labor income dynamics for cross-country variations in wealth inequality using R. Overall, the model results suggest that those factors can explain 70.1% of the differences in wealth inequality across the euro area for the bottom 95% in each country's wealth distribution. Importantly, note that the parameter vector is not chosen to maximize this statistic, but is calibrated to the observed differences in welfare policies and labor market dynamics across countries. As it turns out, the modified measure, R, is more conservative compared to R^2 , which implies for the same predictions an explanatory power of 89.2%. The higher R^2 originates from the fact that the model performs particularly well in forecasting the large differences in wealth Gini coefficients across countries.

The first two columns of Table 3.4 report the 95%-wealth Gini coefficient for all countries according to the HFCS, first in levels, in column (a), and then in column (b) and (c) expressed as a deviation and squared deviation from the mean across all euro area Gini coefficients. Column (d) and (e) depict the same statistics as column (a) and (b) for the model predictions, $\hat{\lambda}_c(\beta, \theta_c)$. Negative deviations imply that the wealth Gini coefficient in the respective country is below average, while positive deviations indicate countries in the euro area with above-average wealth inequality. Comparing the signs of the deviations in column (b) and (e) reveals that for every country the model correctly predicts the relative ranking of wealth inequality with respect to the mean. Furthermore, the table shows in column (f) the prediction errors of the model's forecasts, as well as absolute errors in column (g). The row labeled "Mean" in Table 3.4 demonstrates in column (a) and (d) that the average of Gini coefficients λ_c in the data of 0.573 equals, through the calibration of β , the mean of the model's implied Gini coefficients. For the same reason, the average forecast error of the model in column (f) is zero.

In the last row of the table, labeled R, the modified coefficient of determination is reported. It indicates that the model can explain $70.1\% (= 1 - \frac{0.0172}{0.0575})$ of the crosscountry differences in wealth inequality for the bottom 95% of the private net wealth distributions in 2010. Column (h) reports how well the model performs for the respective countries and the explanatory power of the model ranges from 37.4% for France to 97.5% for Italy. When including the richest 5% in the calculation of the Gini coefficients, the explanatory power of the model drops to 27.8% after recalibrating the preference parameter to $\beta = 0.961$ in order to match the average across all country's wealth Gini coefficients of 0.654 in the data. This finding is consistent with the notion that the wealth accumulation process of the wealthiest 5% is unlikely to be driven by one of the savings motives considered in this model.

	bottom 95%						all		
		Data		Model		Forecast Error		Goodness of fit	
	λ_c	$\lambda_c - ar{\lambda}$	$ \lambda_c - ar{\lambda} $	$\hat{\lambda}_{c}(eta,oldsymbol{ heta}_{c})$	$\hat{\lambda}_{c}(eta, oldsymbol{ heta}_{c}) - ar{\lambda}$	$\hat{\epsilon}_c$	$ \hat{\epsilon}_c $	$1 - \frac{(g)}{(c)}$	
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
Austria	0.656	-0.083	0.0833	0.634	-0.061	-0.022	0.0219	73.8%	
Belgium	0.522	0.051	0.0508	0.535	0.037	0.013	0.0135	73.5%	
Finland	0.612	-0.040	0.0399	0.608	-0.035	-0.005	0.0048	88.1%	
France	0.591	-0.019	0.0190	0.603	-0.031	0.012	0.0119	37.4%	
Germany	0.661	-0.088	0.0884	0.624	-0.052	-0.037	0.0366	58.6%	
Greece	0.499	0.074	0.0738	0.525	0.048	0.026	0.0262	64.5%	
Italy	0.516	0.056	0.0561	0.518	0.055	0.001	0.0014	97.5%	
Netherlands	0.629	-0.057	0.0569	0.607	-0.034	-0.023	0.0227	60.2%	
Portugal	0.540	0.033	0.0326	0.547	0.025	0.007	0.0073	77.5%	
Spain	0.498	0.074	0.0741	0.524	0.048	0.026	0.0256	65.4%	
Mean	0.573	0.000	0.0575	0.573	0.000	0.000	0.0172		
R								70.1%	27.8%

Table 3.4: Gini coefficients of private net wealth in the data and model

6.2 Decomposition

This section introduces a decomposition method which quantifies the separate effects of labor market dynamics, the pension system, unemployment insurance and minimum income benefits for the fraction explained by the model of 70.1%, as reported in Table 3.4. The decomposition method presented here is most closely related to one adopted in a paper by Guvenen et al. (2014) in which they analyze the extent to which differences in wage inequality between the US, their chosen reference country, and six other central European countries can be attributed to differences in labor income tax progressivity and also provide a decomposition.

For the decomposition exercise, I construct a fictive euro area country (EA) that serves as a benchmark country. Its policy parameters are defined as the average across all country-specific individual parameters, θ_c^i :

$$\theta_{EA}^i = \frac{\Sigma_{c=1}^N \theta_c^i}{N}, \ i = 1, ..M$$

By sequentially setting the country-specific parameters to those of the reference country and recalibrating the discount factor, β , to match the mean wealth Gini coefficient $\bar{\lambda}$ in the data, the contribution of the income process and each policy to the euro area variation in wealth inequality can be determined. Finally, when setting all parameters to those of the fictive euro area benchmark country, all countries will exhibit the same mean Gini coefficient, $\lambda_{EA}(\boldsymbol{\theta}_{EA}) = \bar{\lambda}$, as no differences remain.

Table 3.5 demonstrates the decomposition method which disentangles the contribution of each factor to the explanatory power of the model for cross-country wealth inequality differences. First, in column (1), I set the income process and all welfare policies to the country-specific parameters and obtain R as reported in Table 3.4.

In the next column (2), I then assume that all countries have the same labor income and unemployment dynamics as in the EA reference country, but still differ in the social security dimensions considered. At this stage the discount factor, β , is recalibrated in order to equalize the average of actual and predicted wealth Gini coefficients across the euro area and R is again determined. This step allows me to separate the role of those three policies from that of the labor market dynamics. The difference of the coefficient of determination, R, in columns (1) and (2) in the last row provides a useful measure of the role of the income process, which accounts for 12.2% (= 70.1% - 57.9%) of euro area differences in wealth inequality. Note that the share contributed by the income process is conditional on the order in which the country-specific parameters are set to those of the EA benchmark country. This is due to an interdependence of all policies with each other and with labor market dynamics. To exemplify the interaction of labor market dynamics with the unemployment insurance or public pension scheme, one can consider two countries with different degrees of net earnings risk. The same net unemployment or pension replacement rate would lead to a larger crowding-out of savings in the country with low income risk, because future benefits are less uncertain. Next in column (3), I also set the unemployment insurance system of each country equal to the one in the fictive EA reference country, but each country retains its own public and occupational pension scheme and minimum income support program. Taking the

	(1)	(2)	(3)	(4)	Fraction explained
MI* Pensions	- -	-		set to EA	42.6% 10.6%
UI* Income	-	set to EA	set to EA set to EA	set to EA set to EA	4.7% 12.2%
Austria	0.634	0.600	0.601	0.586	
Belgium	0.535	0.566	0.568	0.574	
Finland	0.608	0.605	0.601	0.601	
France	0.603	0.596	0.594	0.581	
Germany	0.624	0.603	0.602	0.598	
Greece	0.525	0.507	0.519	0.528	
Italy	0.518	0.521	0.530	0.525	
Netherlands	0.607	0.629	0.620	0.619	
Portugal	0.547	0.563	0.559	0.570	
Spain	0.524	0.534	0.531	0.543	
R	70.1%	57.9%	53.1%	42.6%	
β	97.3	97.5	97.6	97.6	

Table 3.5: Conditional decomposition of cross-country differences in wealth inequality

Notes: $UI^* = Unemployment insurance, MI^* = Minimum-income support.$

difference between columns (2) and (3) reveals that the unemployment insurance system contributes 4.7%, conditional on the income process being equal across all countries. Continuing in this manner, I am ultimately able to separate the conditional role of each institution for euro area wealth inequality differences. Finally, setting all the parameters to the one of the euro area reference country will lead to an R of zero. This is because R assesses the model's forecasts, $\hat{\lambda}_c(\boldsymbol{\theta}_c)$, against a simple benchmark model predicting the mean, $\bar{\lambda}$, for each country and hence zero variation in cross-country differences in wealth inequality, a prediction that my model exactly makes if there are no euro area differences in country-specific parameters, $\boldsymbol{\theta}_c$.

To get an estimate of the effects of income risk and the welfare policies that is not dependent on the specific ordering, I will determine the contribution to the overall fraction explained by each factor for every possible ordering and take the average. In total, there are four different factors considered in the decomposition, which leads to 16 possible orders, each providing an estimate for the contribution of one determinant. Table 3.6: Unconditional decomposition of cross-country differences in wealth inequality

	Fraction explained
MI* Pensions UI* Income	$\begin{array}{c} 44.8\% \\ 10.7\% \\ 2.0\% \\ 12.6\% \end{array}$

Notes: $UI^* = Unemployment insurance, MI^* = Minimum-income support.$

Table 3.6 provides the results of this final unconditional decomposition method.

The decomposition results indeed change slightly compared to Table 3.5 and suggest that welfare policies contribute 57.5% to the euro area differences in the net wealth Gini coefficients for the bottom 95% of the wealth distribution. It turns out that the most important drivers of the social security system for determining wealth inequality differences across the euro area are means-tested minimum income support programs and pension schemes. While the pension system can rationalize 10.7%, minimum income support programs stand out by far, accounting for 44.8% of the differences. Institutional differences in unemployment insurance systems across the euro area, by contrast, play with 2% only a minor role. Furthermore, 12.6% of the cross-country differences in wealth inequality can be attributed to the net earnings process and unemployment dynamics.

The strong effect of minimum income support programs on the wealth distribution relative to other policies is due to several distinct features. First, means-tested minimum income benefits do not depend on any past contributions, but only on the households' current means. Hence, they are much more redistributive across individuals compared to unemployment benefits or pensions, which are in the euro area rather redistributive over the life cycle. Hubbard et al. (1995) have shown that the lower bound on consumption leaves the precautionary savings decision of households with high expected life-time income relatively unaffected, while the need for self-insurance for households in the lower part of the income distribution strongly reduces, thereby increasing wealth inequality. Second, minimum income assistance guarantees a certain lump sum transfer, while future potential unemployment benefits replace a constant fraction of previous net income and are hence still dependent on uncertain net income. While the receipt of pension payments during retirement is certain, also some uncertainty about the exact pension level during retirement remains, as pensions will depend on the household's pre-retirement labor market performance. Since households are risk averse, this uncertainty characteristic of unemployment and pension benefits leads to weaker effects on wealth inequality despite sizeable aggregate effects on wealth. Third, the asset-test of minimum income support introduces an implicit tax on savings, such that low-wealth households face a trade-off between saving for bad income states and

Table 3.7: Robustness

Specification	R
 No asset test Private transfers Share held by bottom 50% 	$70.8\% \\ 68.2\% \\ 69.7\%$

dissaving to become eligible for income support. And last, minimum income benefits are of unlimited duration, while unemployment benefits mitigate earnings losses only temporarily.

It is shown in Table 3.7 that the importance of minimum income support programs for determining wealth inequality differences does not crucially depend on the transfers' characteristic of being asset-tested. It can be shown that the explanatory power of the model remains high at 70.8% when assuming the extreme case of 100% asset exemption levels for all euro area countries. While e.g. the predicted wealth inequality for Germany slightly worsens after abolishing the asset-test and recalibrating the time preference parameter, the model prediction of the Gini wealth coefficient for Greece moves under this assumption closer to the actual value. Next, I also relax the assumption of zero income during unemployment and allow for positive private transfers of 10% of the country's median net earnings to unemployed households. Relative to the baseline results, the explanatory power of the model only slightly decreases by 2%. As a further robustness test, the share of total net wealth held by the bottom 50% of the population is considered as an alternative measure for wealth inequality in Table 3.7, again excluding the top 5th percentile of the wealth distribution in the data. The results are also robust to this measure. On average, policies and income processes can also account for 69.7% of the cross-country variation in the net wealth share held by the poorest 50%.¹⁸

These results also shed light on the documented empirical puzzle that countries with a larger reduction in the income Gini coefficient through transfers, show higher wealth inequality. Since higher after-tax earnings inequality and unemployment rates lead to higher wealth inequality, one would expect that transfers by reducing income differences, lower wealth inequality in turn. In fact, the opposite is true. While transfers temporarily mitigate income differences across households, their general availability leads to a more unequal wealth distribution in the long run. Furthermore, the analysis also questions a common practice in the incomplete markets literature of estimating the standard deviation of household income shocks directly from after-tax and transfers income data

 $^{^{18}}$ While the 90/10 and the 50/10 percentile ratios are suitable and commonly used to analyze inequality of income distributions, they are less suited when working with wealth distributions, because the 10th percentile can be negative or close to zero and hence the ratio can be negative or infinite.

for the model's calibration, as this leads to an overly low implied wealth inequality. It is therefore important to model both the earnings process and transfers explicitly if the wealth distribution is essential for the question at hand.

7. Conclusion

Consistent with the theory of Hubbard et al. (1995) that public insurance distorts private savings decisions especially of low-income and low-wealth households, I empirically document that countries with more generous and redistributive welfare policies have considerably higher wealth inequality. Using a structural life-cycle model featuring labor market risk, bequests, and various institutions of the social security system, it is shown that labor market dynamics and redistributive policies can account for 70.1% of the euro area variation in wealth inequality for the bottom 95% of the wealth distribution. Furthermore, I provide a decomposition of the individual roles of the factors considered. It is shown that welfare policies can account for 57.5% of the euro area differences in wealth inequality and that the most important institution of the social security system driving these results is means-tested minimum income support provided by the government. Since minimum income support benefits are highly redistributive across individuals, certain, asset-tested and of unlimited duration, they strongly affect wealth inequality, and euro area differences in this institution can account for 44.8%of the differences in the net wealth Gini coefficients. Public and occupational pension entitlements and labor market dynamics can rationalize 10.7% and 12.6%, respectively. In contrast, cross-country differences in unemployment insurance systems have with 2%only little explanatory power. I also demonstrate that the asset-based means-testing of minimum income provision is not central to the overall explanatory power of minimum income benefits, as the model's explanatory power remains unchanged when minimum income benefits are assumed to be 100% exempt from the asset-test.

While many studies on wealth inequality focus on determinants which influence the upper tail of the wealth distribution, this analysis sheds light on the remaining part, and in particular, the role of public insurance. When considering the bottom 95% of the wealth distribution, there are still large cross-country differences in wealth inequality to be understood and this analysis reveals that redistributive welfare policies are indeed central in determining wealth inequality patterns across the euro area.

Appendices

A. Robustness of Empirical Facts

Figure 3.5: Share of wealth held by 50th percentile (bottom 95%)



Sources: HFCS (~2010), EEF (2011), EU-SILC (2004-2010)

Figure 3.6: Gini coefficient of equivalized household net wealth (bottom 95%)



Sources: HFCS (~2010), EEF (2011), EU-SILC (2004-2010),

B. Numerical Methods

The household problem is solved backwards by starting in the last period of life, h_{62} . Optimal consumption and savings choices in previous periods are then derived, given subsequent optimal consumption choices and corresponding value functions. I solve for the optimal policies of households whose income is sufficiently low to be eligible for minimum income support, but whose current wealth is such that they never want to take up this support and hence choose end of period wealth holdings $k_h > TR_c$ – w_{h}^{net} , by applying the endogenous gridpoint method as originally developed in Carroll (2006). For lower current wealth holdings, multiple local maxima can emerge and hence, following Wellschmied (2015), I solve for the global maximum via value function iteration and allow for at least 2000 asset choices with a very fine asset grid at the low end of the asset distribution. Increasing the number of gridpoints did not have a noticeable effect on the model-implied Gini coefficients of private net wealth. Also if households are not currently eligible for minimum income support since $w_h^{net} \geq TR_c$, multiple maxima and distortions can arise for households with sufficiently low current wealth levels and hence optimal policies are in this region determined by value function iteration. These distortions arise due to the life-cycle dimension and stochastic nature of earnings. Households place a positive probability on entering a low income state that could potentially make them eligible for means-tested income support in the future. Therefore, today's value function inherits the kinks in the expected value functions of states when households are eligible for minimum income benefits.

I approximate the idiosyncratic gross earnings process using a discrete Markov chain with 7 states, using the method proposed by Tauchen (1986).

C. Calibration

Country	$\frac{A_{mean,h\geq 84}}{A_{mean,h< 84}}$	$\frac{A_{median,h\geq 84}}{A_{median,h< 84}}$
AT	0.54	0.43
BE	0.33	0.14
FI	0.66	0.37
\mathbf{FR}	0.50	0.27
DE	0.71	0.64
GR	0.67	0.29
IT	0.39	0.17
NL	0.59	0.36
PT	0.81	0.39
\mathbf{ES}	0.37	0.16
EA Avg.	0.56	0.32

Table 3.8: Aggregate wealth and its distribution at the end of the life cycle

Sources: Own calculations, HFCS (~2010, excl. top 5th percentile)

Notes: $\frac{A_{mean,h\geq 84}}{A_{mean,h< 84}}$ =ratio of mean wealth of households older or equal to 84 years relative to mean wealth of households younger than 84; $\frac{A_{median,h\geq 84}}{A_{median,h< 84}}$ =ratio of mean wealth of households older or equal to 84 years relative to mean wealth of households younger than 84. The average values for the euro area shown in the last row are used for the calibration of the bequest function.

Figure 3.7: Cross-sectional age-earnings profile, adjusted for euro area real wage growth and PPP



Sources: Own estimations, EU-SILC (2004-2010)

Figure 3.8: Unemployment net replacement rate



Sources: OECD "Benefits and Wages" database (2004/2010)

Figure 3.9: Public and occupational pension replacement rate



Sources: Own calculations, EU-SILC (2004-2010)

Bibliography

- Aiyagari, S. R. (1994). Uninsured Idiosyncratic Risk and Aggregate Saving. Quarterly Journal of Economics, 109(3):659–684.
- Aiyagari, S. R. and McGrattan, E. R. (1998). The optimum quantity of debt. Journal of Monetary Economics, 42(3):447–469.
- Arellano, C., Bai, Y., and Kehoe, P. (2012). Financial markets and fluctuations in uncertainty. Federal Reserve Bank of Minneapolis Research Department Staff Report 466.
- Attanasio, O. P. (1999). Consumption. Handbook of macroeconomics, 1:741–812.
- Bachmann, R. and Bayer, C. (2013). Wait-and-see' business cycles? Journal of Monetary Economics, 60(6):704–719.
- Bahle, T. and Huble, V. (2012). Eumin: Mindestsicherung in Europa. MZES Datenbankdokumentation.
- Barro, R. J. (1974). Are government bonds net wealth? *Journal of Political Economy*, 82(6):1095–1117.
- Basu, S. and Bundick, B. (2012). Uncertainty shocks in a model of effective demand. *NBER Working Paper No. 18420.*
- Baumol, W. J. (1952). The transactions demand for cash: An inventory theoretic approach. *Quarterly Journal of Economics*, 66(4):pp. 545–556.
- Bayer, C., Lütticke, R., Pham-Dao, L., and Tjaden, V. (2015). Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk. *CEPR Discussion Paper No. DP10849.*
- Bewley, T. (1979). The Optimum Quantity of Money. Discussion Paper No 383, Northwestern University, Center for Mathematical Studies in Economics and Management Science.
- Bewley, T. (1983). A difficulty with the optimum quantity of money. *Econometrica*, pages 1485–1504.

Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica*, 77(3):623–685.

- Born, B. and Pfeifer, J. (2014). Policy risk and the business cycle. *Journal of Monetary Economics*, 68:68–85.
- Cagetti, M. and De Nardi, M. (2006). Entrepreneurship, frictions, and wealth. Journal of political Economy, 114(5):835–870.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. Journal of Monetary Economics, 12(3):383–398.
- Carroll, C. (2006). The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters*, 91(3):312–320.
- Carroll, C. D. (2001). A theory of the consumption function, with and without liquidity constraints. *The Journal of Economic Perspectives*, 15(3):23–45.
- Casas, R. P. (2005). Minimum income standards in enlarged EU: Guaranteed minimum income schemes. *Transnational Exchange Project. Setting Minimum Social Standards across Europe, EAPN, Working Paper,* 2.
- Castaneda, A., Díaz-Giménez, J., and Ríos-Rull, J.-V. (2003). Accounting for the U.S. earnings and wealth inequality. *Journal of Political Economy*, 111(4):818–857.
- Challe, E. and Ragot, X. (2011). Fiscal policy in a tractable liquidity-constrained economy^{*}. *The Economic Journal*, 121(551):273–317.
- Chetty, R., Guren, A., Manoli, D., and Weber, A. (2011). Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins. *American Economic Review*, 101(3):471–475.
- Chowdhury, I. and Schabert, A. (2008). Federal reserve policy viewed through a money supply lens. *Journal of Monetary Economics*, 55(4):825–834.
- Christiano, L., Motto, R., and Rostagno, M. (2010). Financial factors in economic fluctuations. *ECB Working Paper 1192*.
- Chugh, S. (2012). Firm risk and leverage-based business cycles. *mimeo*, University of Maryland.
- Clarida, R., Gali, J., and Gertler, M. (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory. *The Quarterly Journal of Economics*, 115(1):147–180.
- Cochrane, J. H. et al. (1989). The sensitivity of tests of the intertemporal allocation of consumption to near-rational alternatives. *American Economic Review*, 79(3):319–37.
- Constantinides, G. M. (1982). Intertemporal asset pricing with heterogeneous consumers and without demand aggregation. *Journal of Business*, pages 253–267.

- De Nardi, M. (2004). Wealth inequality and intergenerational links. *The Review of Economic Studies*, 71(3):743–768.
- De Nardi, M. and Yang, F. (2015). Wealth inequality, family background, and estate taxation. *Journal of Monetary Economics*.
- Deaton, A. and Paxson, C. (1994). Intertemporal choice and inequality. Journal of Political Economy, 102(3):437–67.
- Den Haan, W., Riegler, M., and Rendahl, P. (2014). Unemployment (fears), precautionary savings, and aggregate demand. In 2014 Meeting Papers, number 1062. Society for Economic Dynamics.
- Den Haan, W. J. (2010). Assessing the accuracy of the aggregate law of motion in models with heterogeneous agents. *Journal of Economic Dynamics and Control*, 34(1):79–99.
- Domeij, D. and Klein, P. (2002). Public pensions: To what extent do they account for Swedish wealth inequality? *Review of Economic Dynamics*, 5(3):503–534.
- Dynan, K. E., Skinner, J., and Zeldes, S. P. (2000). Do the rich save more? *NBER* Working Paper No. 7906.
- Engen, E. M. and Gruber, J. (2001). Unemployment insurance and precautionary saving. *Journal of Monetary Economics*, 47(3):545–579.
- Fernández-Villaverde, J., Guerrón-Quintana, P., Kuester, K., and Rubio-Ramírez, J. (2015). Fiscal volatility shocks and economic activity. *American Economic Review*, 105(11):3352–3384.
- Fessler, P., Lindner, P., and Segalla, E. (2014). Net wealth across the euro area: why household structure matters and how to control for it. *ECB Working Paper No. 1663*.
- Fessler, P. and Schuerz, M. (2015). Private wealth across European countries: The role of income, inheritance and the welfare state. *ECB Working Paper No. 1847.*
- Friedman, M. (1957). A theory of the consumption function. Princton University Press.
- Galí, J. (2008). Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework. Princeton University Press.
- Giambattista, E. and Pennings, S. (2013). When is the government transfer multiplier large. *mimeo, New York University.*
- Gilchrist, S., Sim, J. W., and Zakrajšek, E. (2014). Uncertainty, financial frictions, and investment dynamics. NBER Working Paper No. 20038.
- Gokhale, J., Kotlikoff, L. J., Sefton, J., and Weale, M. (2001). Simulating the transmission of wealth inequality via bequests. *Journal of Public economics*, 79(1):93–128.

- Gornemann, N., Kuester, K., and Nakajima, M. (2012). Monetary policy with heterogeneous agents. *Federal Reserve Bank of Philadelphia*, WP No. 12-21.
- Greenwood, J., Hercowitz, Z., and Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. *American Economic Review*, 78(3):402–417.
- Guvenen, F., Kuruscu, B., and Ozkan, S. (2014). Taxation of human capital and wage inequality: A cross-country analysis. *The Review of Economic Studies*, 81:818–850.
- Heathcote, J. (2005). Fiscal policy with heterogeneous agents and incomplete markets. *The Review of Economic Studies*, 72(1):161–188.
- Hintermaier, T. and Koeniger, W. (2010). The method of endogenous gridpoints with occasionally binding constraints among endogenous variables. *Journal of Economic Dynamics and Control*, 34(10):2074–2088.
- Hintermaier, T. and Koeniger, W. (2011). On the evolution of the U.S. consumer wealth distribution. *Review of Economic Dynamics*, 14(2):317–338.
- Hintermaier, T. and Koeniger, W. (2016). Debt portfolios and homestead exemptions. American Economic Journal: Macroeconomics (forthcoming), 81.
- Hubbard, R. G., Skinner, J., and Zeldes, S. P. (1995). Precautionary saving and social insurance. *Journal of Political Economy*, 103(2):360–399.
- Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17(5-6):953–969.
- Imrohoroglu, A., Imrohoroglu, S., and Joines, D. H. (1995). A life cycle analysis of social security. *Economic theory*, 6(1):83–114.
- Johnson, D. S., Parker, J. A., and Souleles, N. S. (2006). Household expenditure and the income tax rebates of 2001. American Economic Review, 96(5):1589–1610.
- Kaas, L., Kocharkov, G., and Preugschat, E. (2015). Wealth inequality and homeownership in Europe. *CESifo Working Paper No. 5498*.
- Kaplan, G. and Violante, G. L. (2014). A model of the consumption response to fiscal stimulus payments. *Econometrica*, 82:1199–1239.
- Kaymak, B. and Poschke, M. (2016). The evolution of wealth inequality over half a century: The role of taxes, transfers and technology. *Journal of Monetary Economics*, 77:1–25.
- Keynes, J. M. (1936). General theory of employment, interest and money. Palgrave Macmillan.
- King, R. G., Plosser, C. I., and Rebelo, S. T. (1988). Production, growth and business cycles: I. The basic neoclassical model. *Journal of Monetary Economics*, 21(2):195 – 232.

- Kotlikoff, L. J. (1986). Health expenditures and precautionary savings. *NBER Working Paper No. 2008.*
- Krueger, D. and Perri, F. (2011). Public versus private risk sharing. Journal of Economic Theory, 146(3):920–956.
- Krusell, P. and Smith, A. A. (1997). Income and wealth heterogeneity, portfolio choice, and equilibrium asset returns. *Macroeconomic Dynamics*, 1(02):387–422.
- Krusell, P. and Smith, A. A. (1998a). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896.
- Krusell, P. and Smith, Jr, A. A. (1998b). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896.
- Leeper, E. M., Plante, M., and Traum, N. (2010). Dynamics of fiscal financing in the united states. *Journal of Econometrics*, 156(2):304–321.
- Leland, H. A. (1968). Saving and uncertainty: The precautionary demand for saving. *Quarterly Journal of Economics*, pages 465–473.
- Luetticke, R. (2015). Transmission of monetary policy with heterogeneity in household portfolios. *mimeo*, University of Bonn.
- McKay, A. and Reis, R. (2016). The role of automatic stabilizers in the U.S. business cycle. *Econometrica*, 84(1):141–194.
- Mehrotra, N. R. (2014). Fiscal policy stabilization: Purchases or transfers? *mimeo*, Boston University.
- Misra, K. and Surico, P. (2014). Consumption, income changes, and heterogeneity: Evidence from two fiscal stimulus programs. *American Economic Journal: Macroe*conomics, 6(4):84–106.
- Nakamura, E. and Steinsson, J. (2013). High frequency identification of monetary non-neutrality. NBER Working Paper No. 19260.
- Nancy L. Stokey, Robert E. Lucas, E. C. P. (1989). *Recursive Methods in Economic Dynamics*. Harvard University Press.
- Narita, F. (2011). Hidden actions, risk-taking and uncertainty shocks. *mimeo, University of Minnesota*.
- Oh, H. and Reis, R. (2012). Targeted transfers and the fiscal response to the great recession. *Journal of Monetary Economics*, 59:50–64.
- Panousi, V. and Papanikolaou, D. (2012). Investment, idiosyncratic risk, and ownership. Journal of Finance, 67(3):1113–1148.

- Parker, J. A., Souleles, N. S., Johnson, D. S., and McClelland, R. (2013). Consumer spending and the economic stimulus payments of 2008. *American Economic Review*, 103(6):2530–53.
- Quadrini, V. et al. (1999). The importance of entrepreneurship for wealth concentration and mobility. *Review of income and Wealth*, 45(1):1–19.
- Ravn, M. O. and Sterk, V. (2013). Job uncertainty and deep recessions. *mimeo*, *University College London*.
- Santos, M. S. (1991). Smoothness of the policy function in discrete time economic models. *Econometrica*, 59(5):1365–1382.
- Schaal, E. (2012). Uncertainty, productivity and unemployment in the great recession. mimeo, Federal Reserve Bank of Minneapolis.
- Smets, F. and Wouters, R. (2007). Shocks and frictions in U.S. business cycles: A bayesian DSGE approach. American Economic Review, 97(3):586–606.
- Social Protection Committee (2014). Minimum income schemes in the euro area member states. *Council of the European Union*.
- Storesletten, K., Telmer, C. I., and Yaron, A. (2001). The welfare cost of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk. *European Economic Review*, 45(7):1311–1339.
- Storesletten, K., Telmer, C. I., and Yaron, A. (2004). Cyclical dynamics in idiosyncratic labor market risk. *Journal of Political Economy*, 112(3):695–717.
- Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20(2):177 181.
- Taylor, J. B. (1993). Discretion versus policy rules in practice. In *Carnegie-Rochester* conference series on public policy, volume 39, pages 195–214. Elsevier.
- Tobin, J. (1956). The interest-elasticity of transactions demand for cash. *The Review* of *Economics and Statistics*, 38(3):pp. 241–247.
- Vavra, J. (2014). Inflation dynamics and time-varying volatility: New evidence and an ss interpretation. Quarterly Journal of Economics, 129(1):215–258.
- Wellschmied, F. (2015). The welfare effects of asset means-testing income support. IZA Discussion Paper No. 8838.
- Woodford, M. (1990). Public debt as private liquidity. *American Economic Review*, 80(2):382–388.