Essays in Microeconomic Theory

Inaugural-Dissertation

zur Erlangung des Grades eines Doktors
der Wirtschafts- und Gesellschaftswissenschaften

durch die

Rechts- und Staatswissenschaftliche Fakultät
der Rheinischen Friedrich-Wilhelms-Universität
Bonn

vorgelegt von

Saskia Klein

aus Neunkirchen

Bonn 2019

Dekan: Prof. Dr. Jürgen von Hagen
Erstreferent: Prof. Dr. Dezsö Szalay
Zweitreferent: Prof. Dr. Stephan Lauermann

Tag der mündlichen Prüfung: 21.06.2019

Acknowledgements

≫ *Anyone who ever gave you confidence, you owe them a lot.* ≪

- Breakfast at Tiffany's

Ein besonderer Dank gilt meinen Supervisors Dezsö Szalay und Stephan Lauermann für ihre kontinuierliche Unterstützung meines Promotionsvorhabens. Dezsö Szalay hat mich bereits während meines Masters zu einer Promotion an der BGSE ermutigt und konnte mir sämtliche Zweifel hinsichtlich der Promotion nehmen. Für die stete Förderung und Betreuung meines Vorhabens bin ich ihm mehr als dankbar. Das Engagement welches Stephan Lauermann der Betreuung meiner Dissertation entgegengebracht hat, ist außerordentlich. Für die Zeit welche er in die Besprechung meiner Ideen investiert hat und für seine vielen wegweisenden Vorschläge möchte ich ihm besonders danken.

Ich hatte das große Glück Nina Bobkova kennenzulernen und gemeinsam mit ihr an meinem ersten Forschungsprojekt zu arbeiten. Durch die Zusammenarbeit mit Nina habe ich nicht nur unglaublich viel gelernt, sondern auch die Freude an meinem Promotionsvorhaben zurückgewonnen.

Meine tiefste Dankbarkeit gilt meiner Familie, die mich in allem was ich mache unterstützt und immer hinter mir steht. Meine Schwiegereltern Sigrid und Berthold Klein haben mir stets Mut gemacht und mir mit Ihrem Glaube an mich wichtigen Halt und Zuversicht gegeben.

Den Dank den ich meinen Eltern Gabi und Volker Fuchs entgegenbringe lässt sich nicht in Worte fassen. Die bedingungslose Unterstützung die ich von Euch seit jeher erhalte, hat diese Promotion überhaupt erst möglich gemacht. Ihr habt immer an mich geglaubt und nie einen Zweifel an dieser Promotion aufkommen lassen. Wenn es einen Weg gab mich in irgendeiner Form zu unterstützen, Ihr habt Ihn gefunden. Dafür bin ich Euch auf ewig mehr als dankbar.

Ich danke meinem Ehemann Jan Klein aus tiefstem Herzen für seine grenzenlose Unterstützung und seinen Glauben an mich. Du gabst mir immer die Kraft, die mir fehlte und das Vertrauen, dass ich meine Ziele erreichen werde. Ich werde nie die unzähligen Gespräche, die wir über diese Dissertation geführt haben oder meine Phasen der Frustration die Du ertragen musstest, vergessen. Du hast mich immer wieder aufgefangen und mir neue Zuversicht gegeben, egal wie verzweifelt oder frustriert ich war. Mit Dir an meiner Seite war ich erst in der Lage diese Promotion durchzustehen. Ich werde Dir für Deine Hilfe, Deinen Rückhalt und Dein Verständnis für immer dankbar sein. I owe you a lot!

Contents

List of Figures

Introduction

This dissertation is built up of four essays in Microeconomic Theory. In these essays, I analyze strategic disclosure of information to a committee and strategic communication of information within an organization. All four Chapters deal with the overall question how transmitted information influences the decisions of individuals in the presence of an underlying conflict of interest. The Chapters differ in the credibility of the transmitted information. While Chapters 1 and 2 allow the person who discloses information (*the sender*) to commit to a policy that specifies in which event which information is disclosed, in Chapters 3 and 4 the sender is missing this power to commit to a predefined communication scheme. The absence of this commitment power is crucial when it comes to how the receiving individuals update their beliefs and act afterwards. In the first Chapter of this thesis, which is joint work with Nina Bobkova, we characterize how an interested party can persuade an informed committee to vote for her preferred outcome by strategically disclosing decision relevant information. Chapter 2 is an extension of Chapter 1 and deals with the question how results change when the interested party is clueless in the sense that she no longer has access to superior information and is thus reliant on the information the committee possesses. Chapter 3 deals with the question whether an uninformed principal should delegate a decision to her better informed agent or whether the principal should consult the agent and keep the decision rights. In Chapter 4, I analyze whether a centralized organization form should be chosen over a decentralized form when important information is distributed across divisions which have different interests than the overall organization. In the following, I will provide a short summary of each chapter.

In Chapter 1, we analyze how an interested party (*information designer*) should optimally disclose decision relevant information to a group of imperfectly informed voters to obtain her preferred outcome. Given that voters know about the designer's manipulation efforts, why do voters act according to the designer's recommendations or even listen to the disclosed information? The reason is rooted in the information designer's informational advantage. In contrast to the voters, she has access to perfect information about the decision relevant subject. Since the information designer can commit to condition her recommendations on that information, voters expect to profit from following the recommended action. This way voters can be swayed in their voting behavior. We study to which extent this is possible.

Formally, we consider a biased sender that tries to persuade a committee to vote for a proposal by providing public information about its quality. Each voter has some private information about the proposal's quality. We characterize the sender-optimal disclosure policy under unanimity rule when the sender can versus cannot ask voters for a report about their private information. We find that the sender can only profit from asking agents about their private signals when the private information is sufficiently accurate. For all smaller accuracy levels, a sender who cannot elicit the private information is equally well off.

In Chapter 2, I extend this analysis to an information designer who has no informational advantage over voters. That is, all information about the quality of the proposal stems from the voters. Given that voters know all that, why is there still room for persuasion? The fact that voters are nonetheless sometimes willing to follow the recommendations of somebody uninformed is rooted in the aggregation of information: If the information designer is able to aggregate the private but imperfect knowledge of all voters, she can make recommendations based on more meaningful information. This equips the information designer with some persuasion power and leads to the question to which extent she can persuade the decision makers.

I compare a setting in which the sender is informed about the voters' signals with a setting in which the sender has to first elicit these private signals from voters. It turns out that the uninformed sender who has to ask voters for their private information is sometimes no worse off than a sender who can observe the signals of voters. Besides this, I show that the information designer cannot profit from disclosing recommendations privately to each voter. She is just equally well off when she discloses the recommended action in public.

Chapter 3 deals with the optimal allocation of decision rights when relevant information is asymmetrically distributed within the organization. Without a conflict of interest, the answer would be simple: The decision should be delegated to the person with the best knowledge. However, in the presence of a conflict of interest and in the absence of commitment power the question is no longer trivial: Is it more advantageous for a principal to delegate a decision to a better informed employee or to consult the employee and make the decision by herself? The vital point here is that there exists a trade-off between a loss of control and a loss of information. The loss of control is due to the transfer of authority to someone who has different objectives and thus will act in his full self-interest. The loss of information comes from the principal's and the agent's missing commitment power. If the principal consults the employee without being able to contract upon a predefined decision and communication protocol, the employee will adjust or withhold his information.

To address this challenging question, I analyze the decision making in an organization when there is an uninformed principal who owns the decision rights and an informed agent with a state-dependent bias. In particular, I investigate whether it is better for the principal (i) to delegate the decision to his agent, who is perfectly informed about the decision relevant state of the world, or (ii) to keep all decision rights and only consult the agent. I find that when the agent reacts significantly more sensitive to changes in the state of the world than the principal, the principal is best off when she communicates with the agent and keeps the decision rights.

In Chapter 4, I analyze which organizational form should be chosen when a company is forced to coordinate decisions for both of its divisions. That is, when a company can only make one decision that has to be implemented in both divisions. Each division manager holds a piece of decision relevant information and wants to adapt the decision more to his private information. In contrast, the principal is uninformed and wants to match the decision to the average of both pieces of information. I compare the principal's expected payoff under centralization and decentralization. Under centralization, the principal keeps the decision rights and the division managers communicate their private information to the principal. Under decentralization, division managers communicate their private information to each other and the principal delegates the decision to one of the division managers. I find that centralization performs better than decentralization in terms of maximizing the principal's expected payoff independent of how biased division managers are.

Chapter 1

Persuading an Informed Committee

1. Introduction

In voting or collective decision making, the persuasion of decision makers through a biased party plays a crucial role. To which extent a biased party can persuade decision makers might depend on how much decision-relevant knowledge they already possess. Consider for example a CEO who tries to convince a board of directors to vote for a new proposal. While the CEO wishes to always implement the proposal to improve short-term firm performance, directors only want to approve the proposal if it increases long-term performance. If directors already have some private knowledge about the long-term effects of the proposal, what is the most promising way to convince them to vote for the proposal? This is the question of this paper.

In our model an information designer requires a unanimous approval of a group of voters to implement a proposal. Depending on the proposal's unknown binary quality, voters either like or dislike the proposal. If the quality was known, all voters would agree on the optimal decision. In contrast, the information designer is biased in that she always wants the proposal to be implemented, irrespective of its quality. Each voter receives a private signal about the proposal having a high or low quality and is, according to his private signal, either optimistic or pessimistic. The information designer chooses a disclosure policy: she sends a public recommendation that is correlated with the quality of the proposal. The term *public* means that the information designer cannot make different recommendations to different voters. After having received the recommendation of the information designer, each voter decides on whether to vote for or against the proposal based on his updated belief. Although voters are aware of the information designer's interest in the proposal, they might nevertheless want to follow her recommendation. This is because the recommendation is based on the true quality of the proposal.

The main contribution of this paper is to unveil the extent to which an information designer can persuade informed voters by choosing the optimal disclosure policy. We characterize when the private information of voters restricts the information designer in her scope for persuasion.

In our benchmark case we consider an *omniscient* information designer who can observe the private signal realizations of all voters. We show that the omniscient information designer recommends to vote for the proposal with probability one when the proposal is of high quality. In the state where the proposal is of low quality, she uses a threshold policy: she recommends the proposal with probability one for any number of optimists above a certain cutoff, and recommends the status quo with certainty below the cutoff. The cutoff is such that a pessimistic voter is indifferent between the proposal and the status quo after the recommendation to vote for the proposal.

Next, we consider an *eliciting* information designer who cannot observe the private information of voters but can ask them for reports about their signal realizations. We show that the eliciting information designer cannot implement the optimal disclosure policy from the omniscient benchmark case and is always worse off compared to the omniscient information designer. This is caused by the optimists having a profitable deviation through misreporting to be pessimists. As a consequence, the eliciting information designer has to give sufficient incentives for truthful reporting by providing voters with more information. This limits the scope of the information designer for persuasion. If the probability of receiving the correct signal is below a lower threshold, the eliciting information designer always recommends to vote for the proposal in the state where voters prefer the proposal. In the state in which voters want to implement the status quo, the probability with which she recommends the proposal is stochastic and decreasing in the accuracy of the private information of voters. This optimal policy of the information designer is equivalent to maximizing the probability of a pessimist to vote for the proposal. In contrast, if the probability of receiving the correct signal is above an upper threshold, the information designer's optimal policy is to maximize the probability of an optimist to vote for the proposal.

Finally, we consider a non-eliciting information designer who can neither observe the signal realizations of voters nor ask voters for reports about their private information. If the probability of receiving the correct signal is below the same lower threshold as in the eliciting case, the optimal disclosure policy of an eliciting and of a non-eliciting information designer are equivalent. Thus, an information designer cannot profit from the ability to ask voters for their private information if the accuracy of voter's private information is not sufficiently high.

We find that voters are better off in the presence of a biased information designer compared to the situation in which they have to decide under unanimity rule only based on their private exogenous information as in Feddersen and Pesendorfer (1998).

2. Related Literature

Our paper belongs to the rapidly growing literature on information design (see Rayo and Segal, 2010; Kamenica and Gentzkow, 2011). While in Kamenica and Gentzkow (2011) there is only one agent that is uninformed, we consider persuasion of a committee of agents that is informed.

Amongst the vast emerging literature on information design, the two strands bearing most resemblance to our paper are first, private information on the receiver's side, and second, persuasion of multiple receivers. Multiple papers extend information design to a setting with *many receivers*. In these papers (Taneva, 2019; Bardhi and Guo, 2018; Alonso and Câmara, 2016; Wang, 2015; Chan et al., 2018; Heese and Lauermann, 2018) agents are aware of the payoff types of each other. There is no uncertainty about the committee constellation, and voters possess no private information about the payoff-relevant state of the world. A crucial difference between these papers and our approach is that in our model, the payoff type of each committee member bears information about the state of the world and is private information. If the committee constellation was known, all voters in our model would agree on the same election outcome.

Taneva (2019) extends the approach of a Bayes correlated equilibrium from Bergemann and Morris (2016a) to a class of Bayesian Persuasion problems with multiple receivers. She fully characterizes a binary-binary[1] model with two receivers and shows that the optimal information structure involves public signals or correlated private signals (not conditionally independent signals). Alonso and Câmara (2016) analyze how a biased sender can influence an uninformed heterogeneous committee of voters with a public signal, as in our model. They outline the scope for persuasion under different voting rules, and show when agents are worse off.[2] Chan et al. (2018) consider persuading a heterogeneous committee under the restriction to minimal winning coalitions. Wang (2015) compares private persuasion (under the restriction of conditionally independent signals) to public persuasion in collective decision making. She shows, that public persuasion performs weakly better and reveals less information than private persuasion. The closest related to our paper is Bardhi and Guo (2018). They analyze persuasion of a heterogeneous committee, and study a unanimous voting rule. They consider two persuasion regimes: general persuasion (conditional on everybody's payoff type), and individual persuasion (conditional only on own payoff type). Persuasion is private in their model: each agent does not see the messages sent to voters, neither under general nor under individual persuasion. They show that under unanimity, a restriction to a public or private persuasion regime is without loss under some assumptions. Heese and Lauermann (2018) consider persuasion of a heterogeneous committee of voters. They show that the information designer can almost surely guarantee the implementation of her preferred outcome in the limit, as the size of the committee grows sufficiently large.

Amongst the papers considering *private information* on the side of the receiver are Kolotilin et al. (2017), Kolotilin (2016), Bergemann and Morris (2016b) and Bobkova (2018). Kolotilin et al. (2017) study persuasion of one privately informed receiver, who is privately informed about his payoff type. They show that eliciting persuasion is equivalent to non-eliciting persuasion under some conditions[3]. Bergemann et al. (2018) consider a similar environment as Kolotilin et al. (2017) but add monetary transfers, which we do not allow in our framework. Kolotilin (2016) considers an information designer who tries to persuade an informed receiver. Persuasion is non-eliciting: the information designer cannot ask the receiver for his type prior to her information disclosure. Bobkova (2018) considers a stream of short-lived and privately informed buyers, that an information designer (seller) seeks to persuade into buying her product. The seller is restricted in her ability to construct experiments, and has to rely on the private information of previous receivers, that she has to elicit truthfully.

[1] Binary states and binary actions.

[2] See also Schnakenberg (2015, 2017). For example, Schnakenberg (2015) studies a cheap talk model, i.e., the expert advises the voting body by sending public cheap talk messages.

[3] Kolotilin et al. (2017) refer to eliciting and non-eliciting persuasion as public versus private persuasion. See Bergemann and Morris (2017) for a unified terminology, that we follow in this paper.

To the best of our knowledge, our framework is the first to introduce private information into persuasion of a group. We analyze the problem of an information designer when she first has to squeeze the private information out of multiple agents before she can condition her disclosure policy on it.

The idea of *omniscient* persuasion and *private* persuasion of one privately informed receiver can be found in Bergemann and Morris (2016b). We extend their discussion by providing a comparison of the cases in which the information designer is omniscient and in which the information designer has to first elicit the private signals from multiple agents. A unified perspective of the existing literature on Bayesian Persuasion and information design can be found in Bergemann and Morris (2017).

Finally, our paper relates to the literature on information aggregation in strategic voting, following Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998). Feddersen and Pesendorfer (1998) show that when voters vote strategically, voting truthfully according to one's own private information is not an equilibrium. Voters condition their strategy on pivotality events, and a unanimous voting rule is a 'uniquely bad' voting rule: it implements the inferior inefficient outcome with a higher probability than any other majority rule. In the model of Feddersen and Pesendorfer (1998), voters have to make their decision based on their private exogenous information only. In our model, we allow the designer to provide further correlated information to all agents, conditional on the true state of the world.

The paper is organized as follows. First, we introduce our model in section 3. In section 4, we first analyze the benchmark case of an omniscient information designer and characterize her optimal disclosure policy. In the subsequent section 5, we analyze information design with elicitation and show that an eliciting information designer cannot implement the optimal disclosure policy from the omniscient benchmark case. We characterize the optimal disclosure of an eliciting information designer and establish two equivalence results. In section 6 we deal with the analysis of a non-eliciting information designer and prove the equivalence of optimal disclosure policies of an eliciting and a non-eliciting information designer if the accuracy of voters' signals is below some threshold. The last section 7 deals with further remarks on a comparison to Feddersen and Pesendorfer (1998) and on the case where the omniscient sender faces only one voter whose private signal can have four (instead of two) different realizations.

3. Model

There are three voters which have to decide whether to vote for a proposal or for a status quo. An information designer tries to influence voters to vote for the proposal. In the following we use information designer and sender synonymously. The proposal requires unanimity to pass. When a voter i votes for the proposal we write $a_i = 1$, and $a_i = 0$ for the status quo. When the outcome of the ballot is the proposal we write $a = 1$, and $a = 0$ when the status quo is chosen. For example, under unanimity rule the outcome is $a = 1$ if $a_i = 1$ for all i.

Whether a voter likes or dislikes the proposal depends on an uncertain state of the world $\theta \in \{B, G\}$, where $\Pr(\theta = G) = \frac{1}{2}$. Voters have the following utility function:

$$u_i(a,\theta) = \begin{cases} \mathbb{1}_{\{\theta=G\}} - \frac{1}{2} & \text{if } a = 1 \\ 0 & \text{if } a = 0 \end{cases}$$

Hence, all voters agree on the optimal decision if the state was known: when $\theta = G$ all voters want to implement the proposal, while when $\theta = B$, all agree on the status quo. In contrast, the sender always prefers the proposal over the status quo independent of the state of the world. The sender's utility function is given by $u_S(a) = a$.

Each voter receives a private signal $z_i \in \{b, g\}$ that is correlated with the true state of the world in the following way: $\Pr(z_i = g|\theta = G) = \Pr(z_i = b|\theta = B) = p \in (\frac{1}{2}, 1)$. A voter i with a signal $z_i = g$ (referred to as a *good signal*) is more optimistic about the state of the world being $\theta = G$ than a voter with signal $z_i = b$ (referred to as a *bad signal*). Likewise, a voter with bad signal $z_i = b$ considers $\theta = B$ more likely.[4]

Denote the set of signal realizations by $Z = \{g, b\}^3$ with a typical element $z = (z_1, z_2, z_3) \in Z$. Let $k(z)$ be the number of g-signals in a typical signal realization z. By $z_{-i} \in Z_{-i}$ we refer to the signals of all voters except voter i, where Z_{-i} is the set of all signal realizations except voter i's signal. In the following we use the shortcut k to refer to $k(z)$ and k_{-i} to refer to $k(z_{-i})$.

4. Omniscient Information Design

We first analyze the benchmark case in which the sender is omniscient, i.e., observing each voter's private signal. The sender's problem is then to choose a disclosure policy $d : \Theta \times Z \to \triangle(R)$ with public recommendations $r \in R$ to maximize the probability of the event that all voters vote for the proposal. We restrict the analysis to anonymous disclosure policies, which take only the number of good and bad signals into account and not which voter has which signal.

Assumption 1. *The sender's disclosure policy is anonymous, i.e., the probability of sending any recommendation is the same for all z, z' with $k(z) = k(z')$.*

This assumption allows us to restrict attention to disclosure policies which only condition on the state of the world and on the number of good signal in the population.

The next assumption specifies how a voter behaves under indifference.[5]

Assumption 2. *If a voter is indifferent between his actions, he follows the recommendation of the sender.*

The following proposition says that we can restrict the sender to only two recommendations $r \in \{\hat{0}, \hat{1}\}$ without loss of generality. These two recommendations are direct voting recommendations, $\hat{1}$ in favor of the proposal, and $\hat{0}$ in favor of the status quo.

[4]For notational convenience we will sometimes use g_i and b_i respectively as a short cut for voter i having received signal $z_i = g$ and $z_i = b$ respectively.

[5]Note the difference to the sender-preferred equilibrium in Kamenica and Gentzkow (2011): they assumed that if indifferent, their agent votes for the sender-preferred outcome, in our case the proposal. We are interested in partial implementation, and such a one-sided tie-breaking rule pro proposal would be with loss of generality in our setting. We will see that in the optimum the sender sometimes wants voters to vote for the status quo if indifferent to achieve the highest outcome. This is driven by pivotality considerations and does not arise in Kamenica and Gentzkow (2011).

Proposition 1. *Under unanimity, it is without loss of generality to restrict the recommendation set of the omniscient sender to* $R = \{\hat{0}, \hat{1}\}$.

All omitted proofs are in the appendix. The disclosure policy of the sender is:

$$d : \Theta \times \{0, 1, 2, 3\} \to \triangle\{\hat{0}, \hat{1}\}. \tag{1.1}$$

That is, we are looking for a vector with 8 components $\{d[\hat{1}|\theta, k]\}_{k\in\{0,1,2,3\},\theta\in\{B,G\}} \in [0, 1]^8$. After the sender sends her recommendation, voters have to update their belief about θ and only follow the sender's recommendation if this yields a higher expected utility than disobeying.

Since the decision has to be made under unanimity, after recommendation $\hat{1}$ a voter is pivotal with probability one and after recommendation $\hat{0}$ he is never pivotal. This is because in equilibrium after $r = \hat{1}$ all voters follow the recommendation $\hat{1}$ which is why from the perspective of an individual voter his vote determines the outcome. Similarly, a voter will never be pivotal after $r = \hat{0}$ because all other voters already voted against the proposal which in turn makes one single vote irrelevant under unanimity. As a consequence, a voter will always follow the recommendation $\hat{0}$ and follow the recommendation $\hat{1}$ if his obedience constraint holds:

$$\begin{aligned} & \Pr(\theta = G|\hat{1}, z_i) \geq \frac{1}{2} && (OB^{\hat{1}}_{z_i}) \\ \Leftrightarrow \quad & \Pr(\theta = G|z_i) \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, k_{-i} + k(z_i)] \Pr(k_{-i}|\theta = G) \\ \geq \quad & \Pr(\theta = B|z_i) \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = B, k_{-i} + k(z_i)] \Pr(k_{-i}|\theta = B) \end{aligned}$$

The omniscient sender's maximization problem is then given by:

$$\max_d \Pr(a = 1) = \max_d \Pr(\hat{1}) = \sum_{\theta\in\{B,G\}} \sum_{k=0}^{3} d[\hat{1}|\theta, k] \Pr(k|\theta) \Pr(\theta)$$

$$\begin{aligned} s.t. \quad & 0 \leq d[r|\theta, k] \leq 1, \quad \forall r \in \{\hat{0}, \hat{1}\},\ \theta \in \{B, G\},\ k \in \{0, 1, 2, 3\} && (1.2) \\ & d[\hat{0}|\theta, k] + d[\hat{1}|\theta, k] = 1, \quad \forall \theta \in \{B, G\},\ k \in \{0, 1, 2, 3\} && (1.3) \\ & \Pr(\theta = G|\hat{1}, z_i) - \frac{1}{2} \geq 0, \qquad \forall z_i \in \{b, g\} && (OB^{\hat{1}}_{z_i}) \end{aligned}$$

The following lemma states that for the sender it is always optimal to send the recommendation to vote for the proposal when $\theta = G$.

Lemma 1. *In any optimal disclosure policy,* $d[\hat{1}|\theta = G, k] = 1$ *for all* k.

To give some intuition for Lemma 1, notice that for $\theta = G$ voters agree on the proposal being the more appropriate choice, independent of their private signal. Hence, the sender does not have to convince voters so that she can simply send $\hat{1}$ for $\theta = G$. Assume that the conjecture is false. Then, the sender could increase the probability of implementing the proposal and at the same time relax the voters' obedience constraints by simply increasing $d[\hat{1}|\theta = G, k]$ for those k for which $d[\hat{1}|\theta = G, k] \neq 1$.

The probability of sending a recommendation $\hat{1}$ for the sender is:

$$\Pr(\hat{1}) = 0.5 \Pr(\hat{1}|\theta = B) + 0.5 \underbrace{\Pr(\hat{1}|\theta = G)}_{=1} \tag{1.4}$$

$$= 0.5 \sum_{k=0}^{3} d[\hat{1}|k, \theta = B] \Pr(k|\theta = B) + 0.5 \tag{1.5}$$

Now, consider the obedience constraint for the g-type. It is easy to see that it is always satisfied if Lemma 1 holds.

Lemma 2. *The obedience constraint of the g-type is satisfied in any disclosure policy in which $d[\hat{1}|\theta = G, k] = 1$ for all k.*

Proof. The obedience constraint of the g-type is:

$$0.5p \geq 0.5(1-p) \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = B, k_{-i}+1] \Pr(k_{-i}|\theta = B) \tag{1.6}$$

Note that due to feasibility of the disclosure policy, $d[\hat{1}|\theta, k] \leq 1$ for all θ and all k. Thus,

$$\sum_{k_{-i}=0}^{2} \underbrace{d[\hat{1}|\theta = B, k_{-i}+1]}_{\leq 1} \Pr(k_{-i}|\theta = B) \tag{1.7}$$

$$\leq \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|\theta = B) = 1$$

Using this in the obedience constraint, we see that it is always satisfied as $0.5p \geq 0.5(1-p) \geq RHS$ always holds. □

The next lemma states the optimal disclosure policy if all voters have a g-signal:

Lemma 3. *In any optimal disclosure policy d, it holds that $d[\hat{1}|\theta = B, k = 3] = 1$.*

Proof. The probability $d[\hat{1}|\theta = B, k = 3]$ does not show up in the obedience constraint of the b-type, as it only applies when all voters have a g-signal. Therefore, it has no effect on the obedience of the b-type. By Lemma 2 the obedience constraint of the g-type holds in any disclosure policy that sends recommendation $\hat{1}$ whenever the state is $\theta = G$. Therefore, setting $d[\hat{1}|\theta = B, k = 3]$ increases the probability of the proposal being implemented without harming any obedience constraints. □

Using the above findings, the maximization problem of the omniscient designer becomes:

$$\max_{d} \quad \frac{1}{2}\sum_{k=0}^{3} d[\hat{1}|\theta = B, k] \Pr(k|\theta = B) + \frac{1}{2}\Pr(k = 3|\theta = B) + \frac{1}{2} \tag{1.8}$$

$$d[\hat{0}|\theta, k] + d[\hat{1}|\theta, k] = 1, \quad \forall \theta \in \{B, G\}, \ k \in \{0, 1, 2, 3\} \tag{1.9}$$

$$s.t. \quad (1-p) \geq p \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = B, k_{-i})] \Pr(k_{-i}|\theta = B) \tag{$OB_b^{\hat{1}}$}$$

By using that $p \Pr(k_{-i}|\theta = B) = \Pr(k|\theta = B)\frac{3-k}{3}$ we can rewrite $OB_b^{\hat{1}}$:

$$(1-p) \geq p \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = B, k_{-i}] \Pr(k_{-i}|\theta = B) \binom{2}{k_{-i}} \tag{1.10}$$

$$\Leftrightarrow \quad (1-p) \geq \sum_{k=0}^{3} d[\hat{1}|\theta = B, k] \Pr(k|\theta = B)\frac{3-k}{3}. \tag{1.11}$$

Increasing $d[\hat{1}|\theta = B, k]$ for any $k \in \{0, 1, 2\}$ affects the obedience constraint and the objective function of the designer differently. While an increase in $d[\hat{1}|\theta = B, k]$ for any particular $k \in \{0, 1, 2\}$ is weighted by the sender with $\Pr(k|\theta = B)$, the b-type weighs this increase with $\Pr(k|\theta = B)\frac{3-k}{3}$. In the terminology of the fractional knapsack[6], this means that different k have different value-weight ratios. As a consequence, it will matter for which k the sender will increase the probability of sending recommendation $\hat{1}$ until the constraint binds. The next proposition states that the optimal disclosure policy of the omniscient sender is a monotone threshold policy.

Proposition 2. *The unique optimal disclosure policy of the omniscient sender is a monotone cutoff policy with*

$$d[\hat{1}|\theta = G, k] = 1 \ \forall k, \quad d[\hat{1}|\theta = B, k] \begin{cases} = 1 & \text{if} \quad k > \tilde{k}, \\ \in [0, 1] & \text{if} \quad k = \tilde{k}, \\ = 0 & \text{if} \quad k < \tilde{k}, \end{cases} \tag{1.12}$$

where $\tilde{k}$ is such that $OB_b^{\hat{1}}$ binds.

Figure 1 shows the optimal policy of the omniscient sender for $p = 0.7$. If $\theta = G$, she sends with the certainty the recommendation $\hat{1}$ irrespective of the number of g-signals. If $\theta = B$, the sender uses a monotone cutoff policy, where she sends $\hat{1}$ with certainty whenever there are at least two voters with a g-signal, mixes whenever there is one voter with a g-signal, and never sends $\hat{1}$ when all have a b-signal.

5. Eliciting Information Design

In the previous section the sender could construct any experiment on the true state of the world and was able to see the private signal realizations of each voter. In this

[6]See appendix for the terminology.

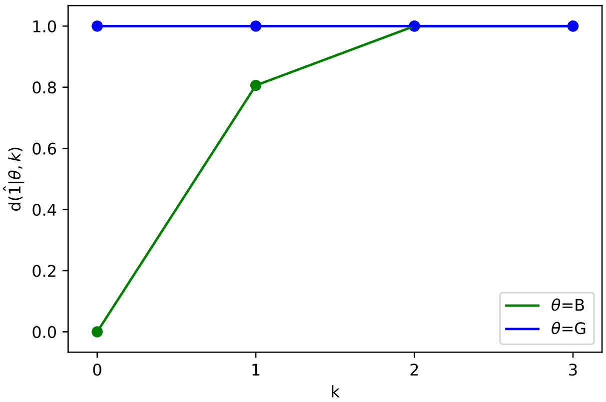


Figure 1.1: optimal disclosure policy for $p = 0.7$.

section we assume that the sender cannot see the private information of the voters, but can elicit it in an incentive compatible way. When the sender is eliciting, honesty constraints arise, and we have to check for double deviations: if an agent misreports, does he have a profitable deviation? After misreporting, the agent should not be strictly better off from any possible action after the misreport.

Each voter i sends a message to the sender, a report about his private signal realization: $\hat{z}_i \in \{\hat{g}, \hat{b}\}$. The complete profile of reported signals is then given by $\hat{z} \in \hat{Z}$. We employ the same notation as above, with the restriction, that now the sender conditions not on the number of g-signals in the true signal realizations z, but on the number of $\hat{g}$-reports in the reported signal realization $\hat{z}$. Hence, $k(\hat{z})$ denotes now the number of $\hat{g}$-reports in the reported signal realization $\hat{z}$.

As for the omniscient sender, only two signals suffice for the sender to achieve her highest implementable payoff.

Proposition 3. *Under unanimity, it is without loss of generality to restrict the recommendation set of the eliciting sender to $R = \{\hat{0}, \hat{1}\}$.*

The sender commits to a disclosure policy $d : k \in \{0, 1, 2, 3\} \to \Delta\{\hat{0}, \hat{1}\}$. Let $U(z_i, \hat{z}_i, a_i(\hat{0}, z_i), a_i(\hat{1}, z_i))$ be the expected utility of a voter with signal z_i, who reports being type $\hat{z}_i$, and votes with probability $a_i(\hat{0}, z_i)$ for the proposal after recommendation $\hat{0}$, and with probability $a_i(\hat{1}, z_i)$ for the proposal after recommendation $\hat{1}$.

We refer to a disclosure policy d of an eliciting sender as *implementable* if and only if it satisfies the obedience and the honesty constraints.

The next observation establishes, that the omniscient sender is strictly better off than the eliciting sender.

Observation 1. *The optimal disclosure policy of the omniscient sender is not implementable when the sender is eliciting.*

It is straightforward that with the optimal disclosure policy in Proposition 2 the g-type will have a profitable deviation from misreporting $\hat{b}$ and following the recommendation. Let $U(z_i, \hat{z}_i', a_i(\hat{0}, z_i), a_i(\hat{1}, z_i))$ be the expected utility of a voter with signal

z_i, who reports being type $\hat{z}'_i$, and votes with probability $a_i(\hat{0}, z_i))$ for the proposal after recommendation $\hat{0}$, and with probability $a_i(\hat{1}, z_i))$ for the proposal after recommendation $\hat{1}$. If the g-type is truthful and obedient, his expected utility is

$$\begin{aligned} U(g_i, \hat{g}_i, 0, 1) =& \Pr(\theta = G|g_i)\frac{1}{2} - \Pr(\theta = B|g_i)\frac{1}{2}\Pr(k_{-i} = 2|\theta = B) \\ &- \Pr(\theta = B|g_i)\frac{1}{2}\sum_{k_{-i}=0}^{1} d[\hat{1}|\theta = B, k_{-i} + 1]\Pr(k_{-i}|\theta = B). \end{aligned} \tag{1.13}$$

Consider the following deviation: misreport $\hat{b}$ and follow the recommendation. Then, a g-type prevents the event in which all voters have reported a g-signal, the state is $\theta = B$ and the sender sends $\hat{1}$ with probability 1. In all other states, the misreporting does not matter for the disclosure policy of the sender when $\theta = G$ because when $\theta = G$ the sender sends $\hat{1}$ with probability one for all $k \in \{0, 1, 2, 3\}$. When $\theta = B$, the g-type voter profits from misreporting since $d[\hat{1}|\theta = B, k]$ is decreasing k. Hence, the misreporting g-type will receive recommendation $\hat{1}$ with a (weakly) smaller probability than when being honest in the unfavorable state $\theta = B$. This follows because the disclosure policy is a cutoff policy: misreporting a g-signal gets a more 'favorable' cutoff than when reporting truthfully. The voter's expected payoff when being dishonest is given by

$$\begin{aligned} U(g_i, \hat{b}_i, 0, 1) =& \Pr(\theta = G|g)\frac{1}{2} \\ &- \Pr(\theta = B|g_i)\frac{1}{2}\sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = B, k_{-i}]\Pr(k_{-i}|\theta = B). \end{aligned} \tag{1.14}$$

Recommendation $\hat{1}$ is sent less frequently if $\theta = B$, hence $U(g_i, \hat{b}_i, 0, 1) > U(g_i, \hat{g}_i, 0, 1)$. The omniscient sender is strictly better off than the eliciting sender under information design. This is in line with the literature. Bergemann and Morris (2016b) show that the implementable set of equilibria is larger for an omniscient than an eliciting sender in a bank run game with one sender and one receiver. Similarly, Bobkova (2018) shows that an omniscient sender has a strictly higher probability of selling a good to a buyer when the sender is omniscient than when she is eliciting. Since an optimal disclosure policy of the omniscient sender is not implementable for the eliciting sender, we need to solve her maximization problem by taking into account the honesty constraints. The obedience constraint of the g-type after being truthful is

$$\begin{aligned} U(g_i, \hat{g}_i, 0, 1) = &\Pr(\theta = G|g_i)\sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, 1 + k_{-i}]\Pr(k_{-i}|\theta = G) \\ &- \Pr(\theta = B|g_i)\sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = B, 1 + k_{-i}]\Pr(k_{-i}|\theta = B) \geq 0 = U(g_i, \hat{g}_i, 0, 0) \end{aligned} \tag{$OB_g^{\hat{1}}$}$$

and the obedience constraint of the b-type after being truthful is

$$U(b_i, \hat{b}_i, 0, 1) = \Pr(\theta = G|b_i) \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, k_{-i}] \Pr(k_{-i}|\theta = G) \tag{$OB_b^{\hat{1}}$}$$
$$- \Pr(\theta = B|b_i) \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = B, k_{-i}] \Pr(k_{-i}|\theta = B) \geq 0 = U(b_i, \hat{b}_i, 0, 0).$$

The honesty constraint of a g-type who is obedient is then given by

$$\begin{aligned} U(g_i, \hat{g}_i, 0, 1) = & \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, 1 + k_{-i}] \Pr(k_{-i}|\theta = G)p \\ & - d[\hat{1}|\theta = B, 1 + k_{-i}] \Pr(k_{-i}|\theta = B)(1 - p) \\ \geq & \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, k_{-i}] \Pr(k_{-i}|\theta = G)p \\ & - d[\hat{1}|\theta = B, k_{-i}] \Pr(k_{-i}|\theta = B)(1 - p) = U(g_i, \hat{b}_i, 0, 1) \end{aligned} \tag{H_g}$$

and the honesty constraint of a b-type who is obedient is then given by

$$\begin{aligned} U(b_i, \hat{b}_i, 0, 1) = & \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, k_{-i}] \Pr(k_{-i}|\theta = G)(1 - p) \\ & - d[\hat{1}|\theta = B, k_{-i}] \Pr(k_{-i}|\theta = B)p \\ \geq & \sum_{k_{-i}=0}^{2} d[\hat{1}|\theta = G, k_{-i} + 1] \Pr(k_{-i}|\theta = G)(1 - p) \\ & - d[\hat{1}|\theta = B, k_{-i} + 1] \Pr(k_{-i}|\theta = B)p = U(b_i, \hat{g}_i, 0, 1). \end{aligned} \tag{H_b}$$

Note that after recommendation $r = \hat{0}$, a voter is never pivotal and hence it does not matter whether he follows or disobeys the recommendation after misreporting. That is, $U(z_i, \hat{z}_i, 1, 1) = U(z_i, \hat{z}_i, 0, 1)$. If a voter is not obedient after recommendation $\hat{1}$, i.e., $a_i(\hat{1}, z_i) = 0$, then his expected utility is simply $U(z_i, \hat{z}_i, 0, 0) = U(z_i, \hat{z}_i, 1, 0) = 0$. This takes care of all double-deviations, since the obedience constraints guarantee a non-negative payoff. The maximization problem of the eliciting sender is

$$\max_{d} \sum_{\theta \in \{B,G\}} \sum_{k=0}^{3} d[\hat{1}|\theta, k] \Pr(k|\theta) \Pr(\theta) \tag{1.15}$$
$$\text{s.t.} \quad 0 \leq d[r|\theta, k] \leq 1, \quad \forall r \in \{\hat{0}, \hat{1}\}, \theta \in \{B, G\}, k \in \{0, 1, 2, 3\} \tag{1.16}$$
$$d[\hat{1}|\theta, k] + d[\hat{0}|\theta, k] = 1 \quad \forall \theta \in \{B, G\}, k \in \{0, 1, 2, 3\} \tag{1.17}$$
$$U(g_i, \hat{g}_i, 0, 1) \geq U(g_i, \hat{g}_i, 0, 0) = 0 \tag{$OB_g^{\hat{1}}$}$$
$$U(b_i, \hat{b}_i, 0, 1) \geq U(b_i, \hat{b}_i, 0, 0) = 0 \tag{$OB_b^{\hat{1}}$}$$
$$U(g_i, \hat{g}_i, 0, 1) \geq U(g_i, \hat{b}_i, 0, 1) \tag{H_g}$$
$$U(b_i, \hat{b}_i, 0, 1) \geq U(b_i, \hat{g}_i, 0, 1). \tag{H_b}$$

Lemma 4. *If $OB_b^{\hat{1}}$ and H_g hold, then $OB_g^{\hat{1}}$ is satisfied.*

Next, we reformulate the Primal of the eliciting sender.

$$\max_{\{d[\hat{1}|\theta,k]\geq 0\}} \sum_{\substack{\theta\in\{B,G\}\\ k\in\{0,1,2,3\}}} \quad \sum_{\theta\in\{B,G\}} \sum_{k=0}^{3} d[\hat{1}|\theta,k]\Pr(k|\theta)\Pr(\theta) \tag{1.18}$$

$$\text{s.t.} \quad \sum_{k=0}^{2} \frac{3-k}{3}(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B) - d[\hat{1}|\theta=G,k])\Pr(k|\theta=G) \leq 0, \tag{$OB_b^{\hat{1}}$}$$

$$\begin{aligned}\sum_{k=1}^{3} \frac{k}{3}((d[\hat{1}|\theta=G,k-1] - d[\hat{1}|\theta=G,k])\Pr(k|\theta=G)\\ -(d[\hat{1}|\theta=B,k-1] - d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)) \leq 0,\end{aligned} \tag{H_g}$$

$$\begin{aligned}\sum_{k=0}^{2} \frac{3-k}{3}((d[\hat{1}|\theta=G,k+1] - d[\hat{1}|\theta=G,k])\Pr(k|\theta=G)\\ -(d[\hat{1}|\theta=B,k+1] - d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)) \leq 0,\end{aligned} \tag{H_b}$$

$$d[\hat{1}|\theta,k] - 1 \leq 0 \quad \forall\theta\in\{B,G\}, k\in\{0,1,2,3\}. \tag{1.19}$$

Observation 2. *The information designer's maximization problem is equivalent to maximizing* $\frac{1}{2}\left(\Pr(\hat{1}|b) + \Pr(\hat{1}|g)\right)$.

After rewriting $\frac{1}{2}\left(\Pr(\hat{1}|b) + \Pr(\hat{1}|g)\right)$ into

$$\sum_{k=0}^{3}\left(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B)\frac{1}{2} + d[\hat{1}|\theta=G,k]\Pr(k|\theta=G)\frac{1}{2}\right)\underbrace{\left(\frac{k}{3}+\frac{3-k}{3}\right)}_{=1} \tag{1.20}$$

one can directly see that maximizing the ex-ante type is equivalent to the objective function of the eliciting designer.

Proposition 4. *The optimal disclosure policy of the eliciting information designer for $p \leq \underline{p}$ and $\forall k$ is*

$$d[\hat{1}|\theta=G,k] = 1, \quad d[\hat{1}|\theta=B,k] = \frac{(1-p)}{p},$$

where $\underline{p} = \frac{1}{\sqrt{2}}$.

The optimal disclosure policy of the eliciting sender for this interval of accuracy levels does not condition on the information reported by voters. The eliciting sender sends $\hat{1}$ in each state of the world with a constant probability, i.e., independent of how many g-signals there were reported. Figure 1.2 shows the optimal disclosure policy of the eliciting sender for $p = 0.6$.

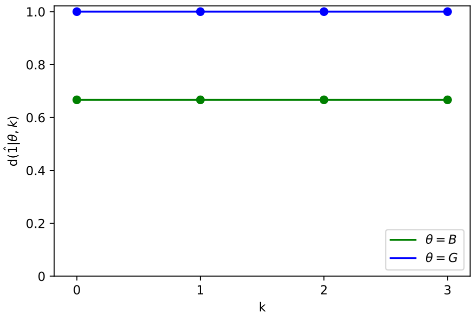


Figure 1.2: optimal policy for $p = 0.6$.

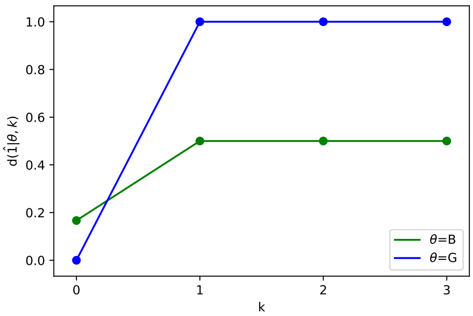


Figure 1.3: optimal policy for $p = 0.75$.

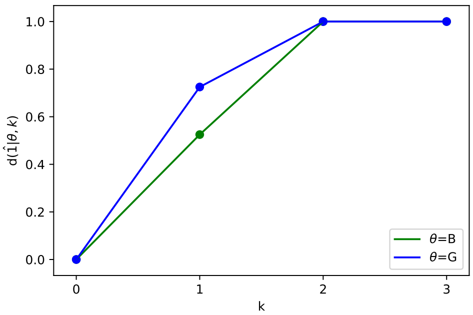


Figure 1.4: optimal policy $p = 0.9$.

Proposition 5. *The optimal disclosure policy of the eliciting information designer for $\underline{p} \leq p \leq \overline{p}$ is*

$$d[\hat{1}|\theta = B, k] = \begin{cases} \frac{(1-p)}{p}(2p-1) & \textit{if} \quad k = 0 \\ 2(1-p) & \textit{if} \quad k \neq 0 \end{cases} \quad , \quad d[\hat{1}|\theta = G, k] = \begin{cases} 0 & \textit{if} \quad k = 0 \\ 1 & \textit{if} \quad k \neq 0, \end{cases}$$

where $\underline{p} = \frac{1}{\sqrt{2}}$ *and* $\overline{p} = \frac{1+\sqrt{13}}{6}$.

In contrast to the previous proposition, for this intermediate interval of accuracy levels, the sender starts to use the information reported by voters. That is, when the private information of voters is more accurate, the sender conditions her disclosure policy on the number of reported g-signals. Moreover, the eliciting sender's optimal disclosure policy is monotone, i.e., she increases the probability with which she sends the recommendation $\hat{1}$ when the number of reported g-signals increases. Figure 1.3 shows the disclosure policy for $p = 0.75$. If $\theta = G$ and at least one $\hat{g}$-report, the sender increases the probability of sending $\hat{1}$ from 0 to 1. Similarly, she sends $\hat{1}$ in $\theta = B$ more often when there is at least one $\hat{g}$-report.

Proposition 6. *The optimal disclosure policy of the eliciting information designer for*

$\overline{p} \leq p < 1$ *is*

$$d[\hat{1}|\theta = B, k] = \begin{cases} 0 \quad if \qquad k = 0, \\ \frac{(p-\frac{1}{2})(3-p)}{2(2p-1)} \quad if\ k = 1, \\ 1 \quad if \quad k \in \{2, 3\}, \end{cases} \qquad d[\hat{1}|\theta = G, k] = \begin{cases} 0 \quad if \qquad k = 0, \\ \frac{(p-\frac{1}{2})(p+2)}{2(2p-1)} \quad if\ k = 1, \\ 1 \quad if \quad k \in \{2, 3\}, \end{cases}$$

for $\overline{p} = \frac{1+\sqrt{13}}{6}$.

As for the previous interval of accuracy levels, the sender makes use of the information reported by voters: she changes the probability with which she sends $\hat{1}$ depending on how many g-signals were reported. Moreover, the sender uses a monotone disclosure policy. Figure 3.4 shows the optimal disclosure policy of the eliciting sender for $p = 0.9$. Note the bang-bang structure of the optimal disclosure policy: In both states of the world, the eliciting sender recommends $\hat{1}$ with certainty whenever there are at least two $\hat{g}$-reports, she mixes between $\hat{0}$ and $\hat{1}$ when there is exactly one $\hat{g}$-report, and never recommends $\hat{1}$ when there is no $\hat{g}$-report.

Proposition 7. *For* $p \leq \underline{p}$*, the information designer's optimal disclosure policy is equivalent to maximizing* $\Pr(\hat{1}|b)$*.*

Proposition 8. *For* $\overline{p} \leq p < 1$*, the information designer's optimal disclosure policy is equivalent to maximizing* $\Pr(\hat{1}|g)$*.*

While Proposition 7 says that the information designer maximizes the probability of a b-type to vote for the proposal if $p \leq \underline{p}$, Proposition 8 says that her optimal disclosure policy is equivalent to maximizing the probability of a g-type to vote for the proposal if $p \geq \overline{p}$. Note that the expected utility of the sender is strictly decreasing in the accuracy of the voters' private signals, that is, the more convinced the voters, the less scope for persuasion. This is depicted in Figure 1.5.

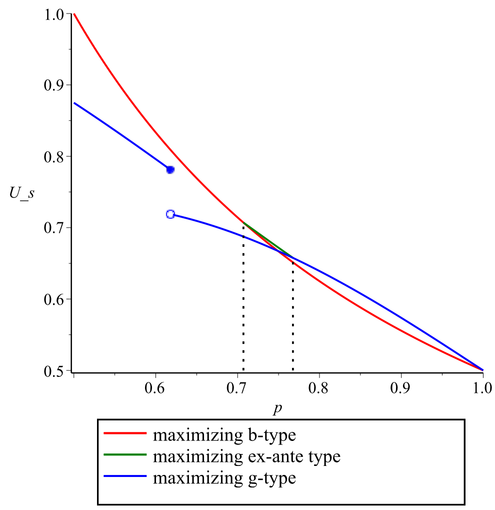


Figure 1.5: expected payoff of the eliciting sender.

6. Non-Eliciting Information Design

In this section the sender cannot ask voters for their private information. In this case the sender is not able to condition her disclosure policy on the private signals or reports of the voters. The following lemma restricts the sender's recommendation set.

Lemma 5. *Under unanimity, it is without loss of generality for optimality to restrict the recommendation set of the non-eliciting sender to* $R = \{\hat{0}, \widehat{01}, \hat{1}\}$.

As before, obedient voters vote for the proposal after $\hat{1}$ and for the status quo after $\hat{0}$. After $\widehat{01}$, only g-types vote for the proposal, and the b-types reject the proposal. To give some intuition for Lemma 3, observe that there exist only three possible cases that can occur after any recommendation: either both types weakly favor the proposal, only the g-type favors for the proposal, or both types strictly dislike the proposal. A g-type never votes against the proposal while a voter with a bad signal strictly prefers the proposal. This is because a voter with a g-signal is more optimistic about $\theta = G$ than a b-type voter. As a consequence, the above three recommendations are sufficient to capture all the possible cases that can occur.

The disclosure policy of a non-eliciting sender is:

$$d : \Theta \to \Delta\{\hat{0}, \widehat{01}, \hat{1}\}. \tag{1.21}$$

The obedience constraints for each type of voter after $\hat{1}$ are given by:

$$U_i(g_i, a_i(\hat{1}, g_i) = 1) = \sum_{\theta \in \{B,G\}} d[\hat{1}|\theta] \Pr(\theta|g_i)(\mathbb{1}_{\theta=G} - \frac{1}{2}) \geq 0 = U_i(g_i, a_i(\hat{1}, g_i) = 0) \tag{$OB_g^{\hat{1}}$}$$

$$U_i(b_i, a_i(\hat{1}, b_i) = 1) = \sum_{\theta \in \{B,G\}} d[\hat{1}|\theta] \Pr(\theta|b_i)(\mathbb{1}_{\theta=G} - \frac{1}{2}) \geq 0 = U_i(b_i, a_i(\hat{1}, b_i) = 0) \tag{$OB_b^{\hat{1}}$}$$

The obedience constraints for each type of voter after $\widehat{01}$ are given by:

$$\begin{aligned} U_i(g_i, a_i(\widehat{01}, g_i) = 1) = &\sum_{\theta \in \{B,G\}} d[\widehat{01}|\theta] \Pr(k = 3|\theta) \Pr(\theta)(\mathbb{1}_{\theta=G} - \frac{1}{2}) \\ &\geq 0 = U_i(g_i, a_i(\hat{1}, g_i) = 0), \end{aligned} \tag{$OB_g^{\widehat{01}}$}$$

$$\begin{aligned} U_i(b_i, a_i(\widehat{01}, b_i) = 1) = &\sum_{\theta \in \{B,G\}} d[\widehat{01}|\theta] \Pr(k = 2|\theta) \Pr(\theta)(\mathbb{1}_{\theta=G} - \frac{1}{2}) \\ &\leq 0 = U_i(b_i, a_i(\hat{1}, b_i) = 0). \end{aligned} \tag{$OB_b^{\widehat{01}}$}$$

The sender's maximization problem becomes:

$$\max_{d} \quad \sum_{\theta\in\{B,G\}} (d[\hat{1}|\theta] + d[\widehat{01}|\theta]\Pr(k=3|\theta))\Pr(\theta) \tag{1.22}$$

$$\text{s.t.} \quad 0 \leq d[r|\theta] \leq 1, \quad \forall r \in \{\hat{0}, \widehat{01}, \hat{1}\}, \theta \in \{B, G\} \tag{1.23}$$

$$d[\hat{1}|\theta] + d[\widehat{01}|\theta] + d[\hat{0}|\theta] = 1 \quad \forall \theta \in \{B, G\} \tag{1.24}$$

$$U_i(g_i, a_i(\hat{1}, g_i) = 1) \geq U_i(g_i, a_i(\hat{1}, g_i) = 0) = 0 \tag{$OB_g^{\hat{1}}$}$$

$$U_i(b_i, a_i(\hat{1}, b_i) = 1) \geq U_i(b_i, a_i(\hat{1}, b_i) = 0) = 0 \tag{$OB_b^{\hat{1}}$}$$

$$U_i(g_i, a_i(\widehat{01}, g_i) = 1) \geq U_i(g_i, a_i(\widehat{01}, g_i) = 0) \tag{$OB_g^{\widehat{01}}$}$$

$$U_i(b_i, a_i(\widehat{01}, b_i) = 1) \leq U_i(b_i, a_i(\widehat{01}, b_i) = 0) \tag{$OB_b^{\widehat{01}}$}$$

The next result shows the solution to the above problem of a non-eliciting information designer.

Proposition 9. *The optimal disclosure policy of the non-eliciting sender for $p \leq \tilde{p}$ is*

$$d[\hat{1}|\theta = G] = 1, \qquad d[\hat{1}|\theta = B] = \frac{1-p}{p},$$

$$d[\hat{0}|\theta = G] = 0, \qquad d[\hat{0}|\theta = B] = \frac{2p-1}{p}.$$

The optimal disclosure policy of the non-eliciting sender for $p \geq \tilde{p}$ is

$$d[\hat{1}|\theta = G] = 1 - \frac{(2p-1)(1-p)^3}{(p^4 - (1-p)^4)}, \qquad d[\hat{1}|\theta = B] = 1 - \frac{(2p-1)p^3}{p^4 - (1-p)^4}$$

$$d[\widehat{01}|\theta = G] = \frac{(2p-1)(1-p)^3}{p^4 - (1-p)^4}, \qquad d[\widehat{01}|\theta = B] = \frac{(2p-1)p^3}{p^4 - (1-p)^4},$$

where $\tilde{p} = \sqrt[4]{\frac{1}{2}}$.

Note that for any p, the optimal disclosure policy of the designer never contains more than two messages. When comparing the optimal disclosure policy of an eliciting and a non-eliciting information designer, it becomes apparent that they are equivalent for $p \leq \underline{p} = \frac{1}{\sqrt{2}}$. The eliciting information designer has no advantage from asking voters about their private information when the accuracy of signals is sufficiently small.

Corollary 1. *Let $\underline{p} = \frac{1}{\sqrt{2}}$. For $p \leq \underline{p}$, the eliciting sender's optimal disclosure policy is equivalent to the non-eliciting sender's optimal disclosure policy.*

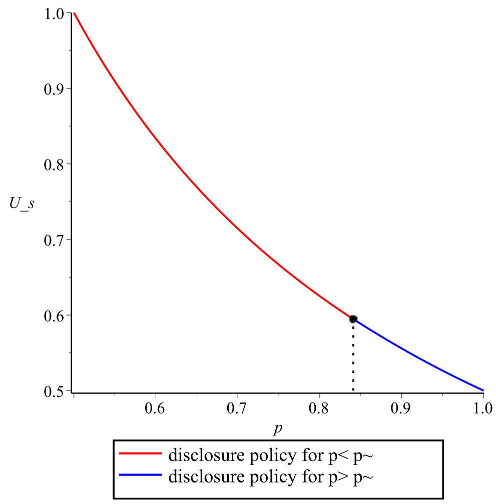


Figure 1.6: non-eliciting sender's expected payoff.

7. Further Remarks

7.1 Eliciting Sender: Comparison to Feddersen and Pesendorfer (1998)

Next, consider the error probabilities of a wrong decision when $\theta = B$ (i.e., $\Pr(a = 1|\theta = B)$) and implementing the status quo when $\theta = G$ (i.e., $\Pr(a = 0|\theta = B)$) under the optimal disclosure policy d of the sender.

We compare the probabilities of making each type of error in our setting to the corresponding probabilities of making each type of error in Feddersen and Pesendorfer (1998), where voters have to act based on their private signals without any coordination device or sender, and the decision rule is unanimity. Applied to our setting, the following is a Nash equilibrium in their model: a voter with a g-signal always votes in favor of the reform, and a b-signal voter in favor of the reform with probability $\frac{\sqrt{p}\left(p+\sqrt{p(1-p)}-1\right)}{p^{\frac{3}{2}}-(1-p)^{\frac{3}{2}}}$.

First, the probability of choosing the proposal when $\theta = B$ in our setting is bigger for all $p \in (\frac{1}{2}, 1)$ (Figure 1.7). Second, the probability of choosing the status quo when $\theta = G$ in our setting is smaller for all $p \in (\frac{1}{2}, 1)$ (Figure 1.8).

Taken together, while in Feddersen and Pesendorfer (1998) the proposal is more often implemented when $\theta = B$, in our setting the proposal is more often implemented when $\theta = G$. A manipulative information designer strictly decreases the probability of rejecting the proposal when the proposal is efficient while increasing the probability of rejecting the status quo when the status quo is efficient.

Overall, the ex-ante type (a voter whose private signal has not yet realized) receives a strictly higher expected payoff in our model than in Feddersen and Pesendorfer (1998).

Proposition 10. *Under unanimity, voters have a strictly higher expected utility with an eliciting information designer than in a symmetric equilibrium as in Feddersen and Pesendorfer (1998).*

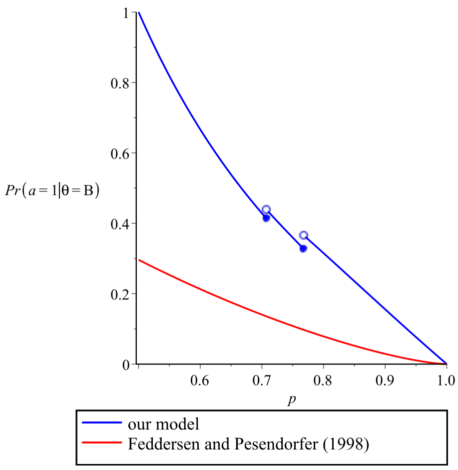


Figure 1.7: error probabilities if $\theta = B$.

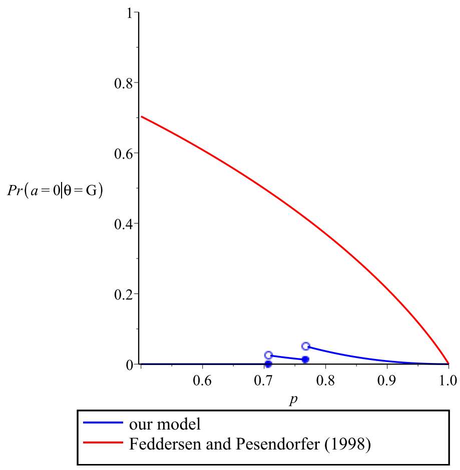


Figure 1.8: error probabilities if $\theta = G$.

Hence, even with a manipulative sender, voters are better off in expectations compared to the situation in which they have to decide on their own under unanimity.

7.2 Omniscient Sender: One Agent and Four Signal Realizations

In this section we are comparing our previous results from Section 3.1 to the case where the omniscient sender faces only one voter which receives a signal $s \in \{0, 1, 2, 3\}$. One possible interpretation for this setting is the following: Imagine voters could communicate with each other and exchange their private signals before the information designer sends her public recommendation. In this case every voter has exactly the same information, i.e., knows how many g-signals there are. As a consequence, the sender's problem is equivalent to persuading one representative voter that can possibly have four different signals realizations which correspond to the number of g-signals. Formally, the sender's problem becomes

$$\max_{d} \sum_{\theta \in \{B,G\}} \sum_{k=0}^{3} d[\hat{1}|\theta, k] \Pr(k|\theta) \Pr(\theta) \tag{1.25}$$

$$\text{s.t.} \quad \Pr(\theta = G|\hat{1}, s) - \frac{1}{2} \geq 0 \quad \forall s \in \{0, 1, 2, 3\}. \tag{$OB_s^{\hat{1}}$}$$

The obedience constraint of the representative voter can be rewritten into

$$\Pr(\theta = G|\hat{1}, s) - \frac{1}{2} \geq 0 \tag{1.26}$$

$$\Leftrightarrow \quad \frac{Pr(\hat{1}|\theta = G, s) \Pr(\theta = G, s)}{\Pr(\hat{1}, s)} \geq \frac{1}{2} \tag{1.27}$$

$$\Leftrightarrow \quad \frac{d[\hat{1}|\theta = G, k] \Pr(k|\theta = G) \Pr(\theta = G)}{\Pr(\hat{1}, k)} \geq \frac{1}{2} \tag{1.28}$$

$$\Leftrightarrow \quad d[\hat{1}|\theta = G, k] \Pr(k|\theta = G) \geq d[\hat{1}|\theta = B, k] \Pr(k|\theta = B) \tag{1.29}$$

In contrast to before, the omniscient sender who faces only one agent which has one out of four signal realizations, has to take four obedience constraints into account, one for each $s \in \{0, 1, 2, 3\}$. Due to the perfect alignment of interests of the sender and the representative voter in $\theta = G$, the omniscient sender will optimally send $\hat{1}$ in $\theta = G$ with certainty for each $s \in \{0, 1, 2, 3\}$. Simple calculations show that the voter's obedience constraint is slack if the omniscient sender also sends $\hat{1}$ with probability one for $s \in \{2, 3\}$ in $\theta = B$. For $s \in \{0, 1\}$ and $\theta = B$ she will choose $d[\hat{1}|\theta = B, k]$ such that the obedience constraint just binds. That is

$$3p^2(1-p) = d[\hat{1}|\theta = B, k = 1]3p(1-p)^2 \tag{1.30}$$

$$\Leftrightarrow \quad d[\hat{1}|\theta = B, k = 1] = \frac{(1-p)}{p} \tag{1.31}$$

and

$$p^3 = d[\hat{1}|\theta = B, k = 0](1-p)^3 \tag{1.32}$$

$$\Leftrightarrow \quad d[\hat{1}|\theta = B, k = 0] = \frac{(1-p)^3}{p^3}. \tag{1.33}$$

Hence, when the omniscient sender faces only one representative voter who knows the number of g-signals, she optimally uses the following disclosure policy

$$d[\hat{1}|\theta = G, k] = 1 \ \forall k, \quad d[\hat{1}|\theta = B, k] = \begin{cases} 1 & \text{if} \quad k \in \{2, 3\} \\ \frac{(1-p)}{p} & \text{if} \quad k = 1 \\ \frac{(1-p)^3}{p^3} & \text{if} \quad k = 0. \end{cases} \tag{1.34}$$

The omniscient sender's expected payoff in this case is given by $\frac{1}{2} + \frac{1}{2}(2-2p)$. This is always lower than the omniscient sender's expected payoff facing three informed voters who cannot communicate. Since the omniscient sender's optimal disclosure policy in $\theta = G$ is exactly the same in both settings and the total probability with which she sends $\hat{1}$ in $\theta = B$ is smaller when voters know the signals of each other, voters are better off if they know each other's signals.

8. Conclusion

We study a biased sender who tries to persuade three voters to vote for a proposal by sending public recommendations. Voters receive some private signals about the quality of the proposal and only want to approve the proposal if it is of high quality. We characterize the optimal disclosure policy under unanimity rule of a 1. omniscient, 2. eliciting, and 3. non-eliciting sender. We find that the eliciting sender can only profit from her ability to ask voters for their private signals when the accuracy of their private information is sufficiently high. Whenever the accuracy level is below some lower threshold the eliciting sender is just equally well of as the non-eliciting sender who cannot ask voters for reports about their private signals.

We show that depending on the accuracy level of the private signals of voters the optimal disclosure policy of the eliciting sender solves two other related maximization problems: For accuracy levels below some lower threshold, the eliciting sender maximizes the probability that a pessimistic voter votes for the proposal. For accuracy levels above some upper threshold, the eliciting sender maximizes the probability of an optimistic voter approving the proposal. Voters are better off in the presence of a biased information designer than in a setting where they have to vote under unanimity based on their private exogenous information only as in Feddersen and Pesendorfer (1998).

In this work we consider a sender who knows the true quality of the proposal. Extending the analysis to a restricted sender who is uninformed about the true quality and can only send public recommendations on the basis of the reports made by voters is a potential avenue for future research.

A. Appendix

A.1 Proof of Proposition 1

Proof. We show that given any outcome of disclosure policy d', the sender can implement an outcome equivalent policy d consisting of only two recommendations $R = \{\hat{0}, \hat{1}\}$. Message $\hat{0}$ is the recommendation to vote with probability 1 for the status quo irrespective of the private signal, and $\hat{1}$ is the recommendation to vote with probability 1 for proposal irrespective of the private signal.

Consider any arbitrary disclosure policy of the sender $d' : \Theta \times \{0, \dots, 3\} \to \Delta(R')$, with R' being any arbitrary message set. Denote $r' \in R'$ an element of the message space. Let $a(r', z_i)$ be the probability of a voter i with signal z_i voting for the proposal after seeing recommendation r' under disclosure policy d'.[7]

Now consider a filtering d of the original information disclosure policy d' of the following form.

$$d : R' \times K \to \Delta[\hat{0}, \hat{1}].$$

The new disclosure policy takes the realized message r' in the original disclosure policy and the number of g-signals k of the voters, and maps them into a binary voting recommendation. With slight abuse of notation, denote by $d(\hat{1}|r', k)$ the probability of sending recommendation $\hat{1}$ in favor of the proposal.

Consider the following construction for the new disclosure policy d:

$$d(\hat{1}|r', k) = a(r', g)^k a(r', b)^{3-k}. \tag{1.35}$$

It is immediate that this policy yields the same expected utility to the sender (if implementable), as the probability with which she sends signal $\hat{1}$ corresponds to the probability with which her preferred outcome would have been elected under the original disclosure policy d'.

Furthermore, this disclosure policy is designed to be proportional to the original probability of being pivotal for both voter types. In particular, for $k_{-i} \in \{0, 1, 2\}$, we have $\Pr(piv|r, k_{-i}, b) = \Pr(piv|r, k_{-i}, g) = a(r', g)^{k_{-i}} a(r', b)^{2-k_{-i}}$ and hence,

$$d(\hat{1}|r', k = k_{-i}) = a(r', g)^{k_{-i}} a(r', b)^{3-k_{-i}} = a(r', b) \Pr(piv|r', k_{-i})$$

and

$$d(\hat{1}|r', k = k_{-i} + 1) = a(r', g)^{k_{-i}+1} a(r', b)^{3-(k_{-i}+1)} = a(r', g) \Pr(piv|r', k_{-i}).$$

It is left to show that the obedience constraints are also satisfied under the new disclosure policy d'. After public recommendation $\hat{0}$ no voter is ever pivotal. It suffices to show, that both private information types have an incentive to follow the recommendation $\hat{1}$ by voting for the proposal. The obedience constraint of a voter with private signal z_i is:

[7] We assume that voters with the same signal react symmetrically to the same recommendation, $a_i(r', z_i) = a_j(r', z_j)$ if $z_i = z_j$. Hence, we drop index i.

$$\Pr(\theta = G|\hat{1}, z_i, piv) \geq \frac{1}{2} \quad \forall z_i \in \{g_i, b_i\} \tag{1.36}$$

This can be rewritten into

$$\begin{aligned}
&\Pr(\theta = G|z_i)\Pr(\hat{1}|\theta = G, z_i)\underbrace{\Pr(piv|\hat{1}, \theta = G, z_i)}_{=1} \\
\geq &\Pr(\theta = B|z_i)\Pr(\hat{1}|\theta = B, z_i)\underbrace{\Pr(piv|\hat{1}, \theta = B, z_i)}_{=1}.
\end{aligned}$$

Thus, the two obedience constraints can be expressed as

$$p\Pr(\hat{1}|\theta = G, g_i) \geq (1-p)\Pr(\hat{1}|\theta = B, g_i), \tag{$OB_g^{\hat{1}}$}$$

$$(1-p)\Pr(\hat{1}|\theta = G, b_i) \geq p\Pr(\hat{1}|\theta = B, b_i). \tag{$OB_b^{\hat{1}}$}$$

We can invoke the construction of the new disclosure policy d from d' to rewrite:

$$\begin{aligned}
\Pr(\hat{1}|\theta, g_i) &= \sum_{r' \in R'} \Pr(\hat{1}, r'|\theta, g_i) && (1.37)\\
&= \sum_{r' \in R'} \sum_{k_{-i}=0}^{2} \Pr(\hat{1}, r', k_{-i}|\theta, g_i) && (1.38)\\
&= \sum_{r' \in R'} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|\theta, g_i)\Pr(r'|k_{-i}, \theta, g_i)\Pr(\hat{1}|r', k_{-i}, \theta, g_i) && (1.39)\\
&= \sum_{r' \in R'} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|\theta, g_i)\Pr(r'|k_{-i}, \theta, g_i)d(\hat{1}|r', k = k_{-i}+1) && (1.40)
\end{aligned}$$

Analogously, for $z_i = b$ we have

$$\Pr(\hat{1}|\theta, b_i) = \sum_{r' \in R'} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|\theta, b_i)\Pr(r'|k_{-i}, \theta, b_i)d(\hat{1}|r', k = k_{-i}) \tag{1.41}$$

We can use this to rewrite the obedience constraints under the new disclosure policy:

$$\begin{aligned}
&\sum_{r' \in R'} \sum_{k_{-i}=0}^{2} p\Pr(k_{-i}|\theta = G, g_i)\Pr(r'|k_{-i}, \theta = G, g_i)d(\hat{1}|r', k = k_{-i}+1) \\
\geq &\sum_{r' \in R'} \sum_{k_{-i}=0}^{2} (1-p)\Pr(k_{-i}|\theta = B, g_i)\Pr(r'|k_{-i}, \theta = B, g_i)d(\hat{1}|r', k = k_{-i}+1)
\end{aligned}$$

$$\Leftrightarrow \sum_{r' \in R'} \sum_{k_{-i}=0}^{2} d(\hat{1}|r', k = k_{-i}+1)[p \Pr(k_{-i}|\theta = G, g_i) \Pr(r'|k_{-i}, \theta = G, g_i)$$
$$-(1-p) \Pr(k_{-i}|\theta = B, g_i) \Pr(r'|k_{-i}, \theta = B, g_i)] \geq 0. \qquad (OB_g^{\hat{1}})$$

Using the construction of the new disclosure policy, we can rewrite

$$\sum_{r' \in R'} a(r', g_i) \sum_{k_{-i}=0}^{2} \Pr(piv|r, k_{-i}, b_i)[p \Pr(k_{-i}|\theta = G, g_i) \Pr(r'|k_{-i}, \theta = G, g_i)$$
$$-(1-p) \Pr(k_{-i}|\theta = B, g_i) \Pr(r'|k_{-i}, \theta = B, g_i)] \geq 0. \qquad (1.42)$$

Analogously, for $z_i = b$ we have

$$\sum_{r' \in R'} a(r', b_i) \sum_{k_{-i}=0}^{2} \Pr(piv|r', k_{-i}, b_i)[(1-p) \Pr(k_{-i}|\theta = G, b_i) \Pr(r'|k_{-i}, \theta = G, b_i)$$
$$-p \Pr(k_{-i}|\theta = B, b_i) \Pr(r'|k_{-i}, \theta = B, b_i)] \geq 0. \qquad (OB_b^{\hat{1}})$$

Note that the original disclosure policy d' is implementable, i.e., the obedience constraint holds for each type $z_i \in \{g_i, b_i\}$ and each message $r' \in R'$, when $a_i(r', z_i) > 0$. That is,

$$\sum_{k_{-i}=0}^{2} \Pr(piv|r', k_{-i}, g_i)[p \Pr(k_{-i}|\theta = G, g_i) \Pr(r'|k_{-i}, \theta = G, g_i)$$
$$-(1-p) \Pr(k_{-i}|\theta = B, g_i) \Pr(r'|k_{-i}, \theta = B, g_i)] \geq 0. \qquad (OB_g^{r'})$$

$$\sum_{k_{-i}=0}^{2} \Pr(piv|r', k_{-i}, b_i)[(1-p) \Pr(k_{-i}|\theta = G, b_i) \Pr(r'|k_{-i}, \theta = G, b_i)$$
$$-p \Pr(k_{-i}|\theta = B, b_i) \Pr(r'|k_{-i}, \theta = B, b_i)] \geq 0. \qquad (OB_b^{r'})$$

The inner sums in the obedience constraints under the new disclosure policy d, $OB_b^{\hat{1}}$ and $OB_g^{\hat{1}}$, correspond to the original obedience constraints, $OB_g^{r'}$ and $OB_b^{r'}$, under the former disclosure policy d'. This establishes that the filtering d satisfies both obedience constraints and yields the same payoff to the designer.

□

A.2 Proof of Lemma 1

Proof. We prove this by contradiction. Assume that the disclosure policy d is optimal and that there exists k' such that $d[\hat{1}|\theta = G, k'] \neq 1$. Then, construct a new disclosure policy d' that is equal to the old disclosure policy d for all $k \neq k'$ and θ. For $k = k'$ and $\theta = G$, it sends recommendation $\hat{1}$ with probability 1. That is $d'[\hat{1}|\theta = B, k] = d[\hat{1}|\theta = B, k]\ \forall k'$, $d'[\hat{1}|\theta = G, k] = d[\hat{1}|\theta = G, k]\ \forall k' \neq k$ and $d'[\hat{1}|k'] = 1 \neq d[\hat{1}|k']$. Next we check whether the new disclosure policy d' still fulfills the voters' obedience constraints. Note that if the recommendation $\hat{0}$ was sent, no voter is ever pivotal, which is why this does not influence the obedience constraints. Sending the recommendation $\hat{1}$ for any k when $\theta = G$ increases the left hand side of the voters' obedience constraints and thus makes them easier to satisfy. The new disclosure policy relaxed the voters' obedience constraints and sends $\hat{1}$ with a strictly higher probability.

□

A.3 Proof of Proposition 2

Proof. The only part of above proposition that we have not proven is $d[\hat{1}|\theta = B, k \neq 3]$. In the following we will use the greedy algorithm (Dantzig, 1957) to solve this problem. We have a fractional knapsack problem of the following form:

Find $0 \leq x_k = d[\hat{1}|\theta = B, k] \leq 1$ for $k \in \{0, 1, 2\}$ s.t.

1) $\sum\limits_{k=0}^{2} x_k w_k \leq \frac{(1-p)}{p}$ holds and

2) $\sum\limits_{k=0}^{2} x_k v_k$ is maximized,

where $w_k = \frac{3-k}{3}\Pr(k|\theta = B)$ and $v_k = \Pr(k|\theta = B) \cdot 1$. We refer to w_k as the weight and to v_k as the value of k.

Next we calculate the value-per-weight ratio $\rho_k = \frac{v_k}{w_k}$ for $k \in \{0, 1, 2\}$:

$$\rho_k = \frac{\Pr(k|\theta = B)}{\frac{3-k}{3}\Pr(k|\theta = B)} = \frac{3}{3-k} \tag{1.43}$$

Variable ρ_k is increasing in k. Hence, if we sort the k's by decreasing ρ_k, we get the following order $2, 1, 0$. How much probability mass we can place on each k until the obedience constraint of the b-signal voter is binding, will depend on the accuracy level of the voters' private signals p.

□

A.4 Proof of Proposition 3

Proof. Consider the same filtering of the original disclosure policy d' into d as in the proof of Proposition 1:

$$d(\hat{1}|r', k) = a(r', g)^k a(r', b)^{3-k}.$$

It remains to be shown that this disclosure policy d satisfies the honesty constraints for each type. The proof of optimality for the designer, and the validity of the obedience constraints were already established in Proposition 1. We prove that the above filtering satisfies the honesty constraints of the g-type. The argument for the b-type is accordingly, and therefore omitted.

Expected utility in equilibrium with d and d'. First, we show that the old disclosure policy d' and new disclosure policy d yield exactly the same expected utility to the g-type in equilibrium, when reporting truthfully. Let R be the message set of the designer with d'.[8]

$$EU(g_i, \hat{g}_i; d') = \sum_{r \in R} \Pr(r|g_i, \hat{g}_i)(\Pr(\theta = G|r, g_i, \hat{g}_i; d') - \frac{1}{2}) \Pr(a = 1) \tag{1.44}$$

$$= \sum_{r \in R} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{g}_i, k_{-i}; d') a_i(r, g)^{k_{-i}+1} a_i(r, b)^{2-k_{-i}} q(r, g_i, \hat{g}_i, k_{-i}), \tag{1.45}$$

where $q(r, g_i, \hat{g}_i, k_{-i}) = (\Pr(\theta = G|r, g_i, \hat{g}_i, k_{-i}) - \frac{1}{2})$ is the expected net utility from implementing the proposal if a g-type voter reported truthfully, k_{-i} others also have a g-signal and the designer sent recommendation r. The factor $a_i(r, g)^{k_{-i}+1} a_i(r, b)^{2-k_{-i}}$ accounts for the probability of being pivotal $(a_i(r, g)^{k_{-i}} a_i(r, b)^{2-k_{-i}})$ times the probability of the g-type voter i voting for the reform $a_i(r, g)$ in equilibrium.

Next, consider the expected utility under the new disclosure policy d. Whenever the information designer sends recommendation $\hat{1}$ (which happens with probability $a_i(r, g)^{k_{-i}+1} a_i(r, b)^{2-k_{-i}}$ if recommendation r would have been sent in d') the reform is implemented.

$$EU(g_i, \hat{g}_i; d) = \sum_{r \in R} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{g}_i, k_{-i}; d') d(\hat{1}|r, k = k_{-i} + 1) q(r, g_i, \hat{g}_i, k_{-i})$$

$$= \sum_{r \in R} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{g}_i, k_{-i}; d') a_i(r, g)^{k_{-i}+1} a_i(r, b)^{2-k_{-i}} q(r, g_i, \hat{g}_i, k_{-i}). \tag{1.46}$$

This coincides with the utility under the original disclosure policy in Equation 2.41, $EU(g_i, \hat{g}_i; d) = EU(g_i, \hat{g}_i; d')$.

[8] For convenience of notation, we assume that R is finite.

Expected utility from misreporting in d'. Next, consider the utility of a g-voter who reports $\hat{b}$ in the original disclosure policy d'. To account for double deviations, we denote by $\tilde{a}_i(r, g_i, \hat{b}_i)$ his action after observing r when reporting $\hat{b}$.

$$EU(g_i, \hat{b}_i; d') = \sum_{r \in R} \max_{\tilde{a}_i(r,g_i,\hat{b}_i) \in [0,1]} \tilde{a}_i(r, g_i, \hat{b}_i) \cdot \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{b}_i, k_{-i}; d') a_i(r, g)^{k_{-i}} a_i(r, b)^{2-k_{-i}} q(r, g_i, \hat{b}_i, k_{-i}). \tag{1.47}$$

For simplicity of notation, denote by $EU(r|g_i, \hat{b}_i)$ the optimal utility after misreporting and observing recommendation r. Note that $EU(r|g_i, \hat{b}_i) \geq 0$ for all r, as a voter can always derive zero utility by voting against the reform. Hence,

$$EU(g_i, \hat{b}_i; d') = \sum_{r \in R} EU(r|g_i, \hat{b}_i). \tag{1.48}$$

Expected utility from misreporting in d. Finally, consider the expected utility after misreporting in the new disclosure policy d. The voter optimizes over his action $\tilde{a}(\hat{1}, g_i, \hat{b}_i)$ after recommendation $\hat{1}$ after misreporting. For simplicity, we assume that the voter votes for the reform with probability $\tilde{a}(\hat{1}, g_i, \hat{b}_i)$ after both recommendations $\hat{1}$ and $\hat{0}$, as his utility after recommendation $\hat{0}$ yields utility 0 irrespective of his action. The difference to d' is that he might not know which r lead to the recommendation $\hat{1}$.

$$EU(g_i, \hat{b}_i; d) = \max_{\tilde{a}(\hat{1},g_i,\hat{b}_i) \in [0,1]} \tilde{a}(\hat{1}, g_i, \hat{b}_i) \cdot \sum_{r \in R} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{b}_i, k_{-i}; d') \underbrace{a_i(r, g)^{k_{-i}} a_i(r, b)^{3-k_{-i}}}_{=\Pr(\hat{1}|r, k=k_{-i})} q(r, g_i, \hat{b}_i, k_{-i}). \tag{1.49}$$

If the voter knew which r of the original disclosure policy d' led to recommendation $\hat{1}$, he would be better off: he could adapt his voting decision $\tilde{a}(\hat{1}, r, g_i, \hat{g}_i)$ to each r (instead of choosing the same $\tilde{a}(\hat{1}, g_i, \hat{g}_i)$ for all r that led to $\hat{1}$). Thus,

$$EU(g_i, \hat{b}_i; d) \leq \sum_{r \in R} \max_{\tilde{a}(\hat{1},r,g_i,\hat{b}_i) \in [0,1]} \tilde{a}(\hat{1}, r, g_i, \hat{b}_i) \cdot \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{b}_i, k_{-i}; d') \underbrace{a_i(r, g)^{k_{-i}} a_i(r, b)^{3-k_{-i}}}_{=d(\hat{1}|r, k=k_{-i})} q(r, g_i, \hat{b}_i, k_{-i}) \tag{1.50}$$

$$= \sum_{r \in R} a_i(r, b) \max_{\tilde{a}(\hat{1},r,g_i,\hat{b}_i) \in [0,1]} \tilde{a}(\hat{1}, r, g_i, \hat{b}_i) \cdot \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{b}_i, k_{-i}; d') a_i(r, g)^{k_{-i}} a_i(r, b)^{2-k_{-i}} q(r, g_i, \hat{b}_i, k_{-i}), \tag{1.51}$$

where the last inequality follows by putting the non-negative factor $a_i(r,b)$ outside the maximum. But then note that the maximization problem point-wise after each r is exactly the same as in Equation 2.47 for the original disclosure policy, $EU(r|g_i,\hat{b}_i)$, which is non-negative. Hence, the optimal deviation utility is bounded above by

$$EU(g_i,\hat{b}_i;d) \leq \sum_{r\in R} \underbrace{a_i(r,b)}_{\in[0,1]} \underbrace{EU(r|g_i,\hat{b}_i)}_{\geq 0} \tag{1.52}$$

$$\leq \sum_{r\in R} EU(r|g_i,\hat{b}_i) \tag{1.53}$$

$$= EU(g_i,\hat{b}_i;d'). \tag{1.54}$$

The payoff after a misreport and an optimal best response is weakly lower than in the original disclosure policy. Note that the original disclosure policy d' by assumption satisfied all constraints, including the honesty constraint of the g-type. Hence, we established that the honesty constraint of the g-type holds by proving

$$EU(g_i,\hat{g}_i;d) = EU(g_i,\hat{g}_i;d') \geq EU(g_i,\hat{b}_i;d') \geq EU(g_i,\hat{b}_i;d).$$

□

A.5 Proof of Lemma 4

Proof. First, we rewrite the honesty constraint of g-type to sum over k instead of k_{-i}.

$$\sum_{k=1}^{3} \frac{k}{3}((d[\hat{1}|\theta=G,k-1] - d[\hat{1}|\theta=G,k])\Pr(k|\theta=G) \tag{H_g}$$

$$-(d[\hat{1}|\theta=B,k-1] - d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)) \leq 0$$

$$\Leftrightarrow \quad \underbrace{\sum_{k=1}^{3} \frac{k}{3}(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B) - d[\hat{1}|\theta=G,k])\Pr(k|\theta=G)}_{OB_g^{\hat{1}}}$$

$$\leq \quad \underbrace{\sum_{k=1}^{3} \frac{k}{3}(d[\hat{1}|\theta=B,k-1]\Pr(k|\theta=B) - d[\hat{1}|\theta=G,k-1]\Pr(k|\theta=G)}_{*}.$$

Observe that we have rewritten H_g such that the LHS of H_g is just $OB_g^{\hat{1}}$. We rewrite $(*)$, i.e., the RHS of H_g, into

$$\sum_{k=0}^{2} \frac{k+1}{3}(d[\hat{1}|\theta=B,k]\Pr(k+1|\theta=B) - d[\hat{1}|\theta=G,k]\Pr(k+1|\theta=G). \tag{1.55}$$

Next, we subtract $\sum\limits_{k=0}^{2} \frac{3-k}{3}(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B) - d[\hat{1}|\theta=G,k])\Pr(k|\theta=G) \leq$

0, which is just $OB(b) \leq 0$ rewritten, and get

$$\sum_{k=0}^{2}(d[\hat{1}|\theta = B, k](\frac{3-k}{3}\Pr(k+1|\theta = B) - \frac{k}{3}\Pr(k|\theta = B)) \qquad (1.56)$$
$$- d[\hat{1}|\theta = G, k](\frac{3-k}{3}\Pr(k+1|\theta = G) - \frac{k}{3}\Pr(k|\theta = G)).$$

Rewriting yields

$$\sum_{k=0}^{2}(d[\hat{1}|\theta = B, k]p^{2-k}(1-p)^k(1-2p) - d[\hat{1}|\theta = G, k]p^k(1-p)^{2-k}(2p-1) \leq 0.$$

Thus, we have that for all $k \leq 2$ the expression above is negative. Note that $(*) - OB_b^{\hat{1}}$ is the sum of these negative terms and is thus also negative. Moreover, $OB_g^{\hat{1}} \leq 0$ implies that $(*) \leq 0$. Since $OB_g^{\hat{1}} \leq (*)$ by equation (1.55), we have that $OB_g^{\hat{1}} \leq 0$, which proves the lemma.

□

A.6 Proof of Proposition 4

Proof. First, we take the Dual of the Primal and get

$$\min_{\substack{\lambda_{OB_b^{\hat{1}}} \geq 0,\ \lambda_{H_g} \geq 0,\ \lambda_{H_b} \geq 0 \\ \{\mu_{\theta,\,k} \geq 0\}_{\theta \in \{B,G\},\, k \in \{0,1,2,3\}}}} \quad \sum_{k=0}^{3} \mu_{\theta=B,\,k} + \sum_{k=0}^{3} \mu_{\theta=G,\,k} \qquad (1.57)$$

$$\text{s.t.} \quad p^{3-k}(1-p)^k \frac{1}{2}\left(-\binom{3}{k} + (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} + \lambda_{H_g}\binom{2}{k-1}\right) \qquad (1.58)$$
$$- \lambda_{H_g} p^{2-k}(1-p)^{k+1}\binom{2}{k} - \lambda_{H_b} p^{4-k}(1-p)^{k-1}\binom{2}{k-1}$$
$$+ \mu_{\theta=B,\,k} \ \geq\ 0 \quad \forall k \in \{0,1,2,3\}$$

$$p^k(1-p)^{3-k}\frac{1}{2}\left(-\binom{3}{k} - (\lambda_{OB_b^{\hat{1}}} - \lambda_{H_b})\binom{2}{k} - \lambda_{H_g}\binom{2}{k-1}\right) \qquad (1.59)$$
$$+ \lambda_{H_g} p^{k+1}(1-p)^{2-k}\binom{2}{k} + \lambda_{H_b} p^{k-1}(1-p)^{4-k}\binom{2}{k-1}$$
$$+ \mu_{\theta=G,\,k} \ \geq\ 0 \quad \forall k \in \{0,1,2,3\}$$

Let $\{d[\hat{1}|\theta, k] \geq 0\}_{\theta \in \{B,G\}, k \in \{0,1,2,3\}}$ be a feasible disclosure policy for the primal, and $\{\vec{\lambda}, \vec{\mu}\}$ feasible vector for the dual. Necessary and sufficient conditions for them to be optimal are

$$\lambda_{OB_b^{\hat{1}}} \cdot \left(\sum_{k=0}^{2} \binom{2}{k} (d[\hat{1}|\theta = B, k] \Pr(k|\theta = B)\frac{1}{2} - d[\hat{1}|\theta = G, k] \Pr(k|\theta = G)\frac{1}{2}) \right) = 0 \tag{1.60}$$

$$\begin{aligned} \lambda_{H_g} \cdot (\sum_{k=1}^{3} \binom{2}{k-1} ((d[\hat{1}|\theta = G, k-1] - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G)\frac{1}{2} \\ - (d[\hat{1}|\theta = B, k-1] - d[\hat{1}|\theta = B, k]) \Pr(k|\theta = B)\frac{1}{2})) = 0 \end{aligned} \tag{1.61}$$

$$\begin{aligned} \lambda_{H_b} \cdot (\sum_{k=0}^{2} \binom{2}{k} ((d[\hat{1}|\theta = G, k+1] - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G)\frac{1}{2} \\ - (d[\hat{1}|\theta = B, k+1] - d[\hat{1}|\theta = B, k]) \Pr(k|\theta = B)\frac{1}{2})) = 0 \end{aligned} \tag{1.62}$$

$$\mu_{\theta=B,\,k} \cdot \Big(d[\hat{1}|\theta = B, k] - 1 \Big) = 0 \quad \forall k \in \{0, 1, 2, 3\} \tag{1.63}$$

$$\mu_{\theta=G,\,k} \cdot \Big(d[\hat{1}|\theta = G, k] - 1 \Big) = 0 \quad \forall k \in \{0, 1, 2, 3\} \tag{1.64}$$

$$\begin{aligned} & d[\hat{1}|\theta = B, k] \cdot (p^{3-k}(1-p)^k \left(-\binom{3}{k} + (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} + \lambda_{H_g}\binom{2}{k-1} \right) \\ & - \lambda_{H_g} p^{2-k}(1-p)^{k+1}\binom{2}{k} - \lambda_{H_b} p^{4-k}(1-p)^{k-1}\binom{2}{k-1} \\ & + \mu_{\theta=B,\,k}) = 0 \quad \forall k \in \{0, 1, 2, 3\} \end{aligned} \tag{1.65}$$

$$\begin{aligned} & d[\hat{1}|\theta = G, k] \cdot (p^{k}(1-p)^{3-k} \left(-\binom{3}{k} - (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} - \lambda_{H_g}\binom{2}{k-1} \right) \\ & + \lambda_{H_g} p^{k+1}(1-p)^{2-k}\binom{2}{k} + \lambda_{H_b} p^{k-1}(1-p)^{4-k}\binom{2}{k-1} \\ & + \mu_{\theta=G,\,k}) = 0 \quad \forall k \in \{0, 1, 2, 3\}. \end{aligned} \tag{1.66}$$

The dual variables

$$\{\vec{\lambda}, \vec{\mu}\} = \left(\begin{array}{c} \lambda_{OB_b^{\hat{1}}} = \frac{1}{p} \\ \lambda_{H_g} = 1 \\ \lambda_{H_b} = 0 \\ \mu_{\theta=B,k} = 0 \quad \forall k \in \{0, \ldots, 3\} \\ \mu_{\theta=G,\,k} = p^{k-1}(1-p)^{2-k} \cdot \Big(p(1-p)\left(\tbinom{3}{k} + \tbinom{2}{k-1}\right) + (1-p-p^2)\tbinom{2}{k} \Big) \\ \forall k \in \{0, 1, 2, 3\} \end{array} \right) \tag{1.67}$$

and the disclosure policy $d[\hat{1}|\theta = G] = 1$, $d[\hat{1}|\theta = B] = \frac{(1-p)}{p}$ fulfill the above complementary slackness conditions for all $p \leq \frac{1}{\sqrt{2}}$.

After inserting $d[\hat{1}|\theta = G] = 1$ and $d[\hat{1}|\theta = B] = \frac{1-p}{p}$ in 1.60-1.62 and 1.63, one can easily see that the terms in brackets are zero. Thus, we get that $\lambda_{OB_b^{\hat{1}}} \geq 0$, $\lambda_{H_g} \geq 0$, $\lambda_{H_b} \geq 0$ and $\mu_{\theta=G,k} \geq 0 \quad \forall\ k \in \{0, \ldots, 3\}$. Since $d[\hat{1}|\theta, k] > 0 \quad \forall\, \theta \in \{B, G\}, k \in \{0, 1, 2, 3\}$, the terms in brackets must be zero. By using that

$$\left(p^3\left(-1 + \lambda_{OB_b^{\hat{1}}} + \lambda_{H_b}\right) - \lambda_{H_g} p^2(1-p) + \mu_{\theta=B,\,k=0}\right) = 0 \quad \text{and} \tag{1.68}$$

$$\left((1-p)^3\left(-1 + \lambda_{H_g}\right) - \lambda_{H_b} p(1-p)^2 + \mu_{\theta=B,\,k=3}\right) = 0 \tag{1.69}$$

we can solve for $\lambda_{OB_b^{\hat{1}}} = \frac{1}{2p}$ and $\lambda_{H_b} = \frac{(1-p)(\lambda_{H_g}-1)}{p}$. For $\lambda_{H_b} \geq 0$ we need that $\lambda_{H_g} \geq \frac{1}{2}$. Choosing $\lambda_{H_g} = \frac{1}{2}$ implies that $\lambda_{H_b} = 0$. Inserting these values for $\lambda_{OB_b^{\hat{1}}}$, λ_{H_g} and λ_{H_b} and solving for $\mu_{\theta=G,\,k=0}$ yields $\mu_{\theta=G,\,k=0} = (1-p)^2\left(\frac{1-2p^2}{p}\right)$, which is ≥ 0 if and only if $0.5 < p \leq \frac{1}{\sqrt{2}}$. For $k \in \{1, 2\}$ we get that

$$\mu_{\theta=G,\,k} = p^{k-1}(1-p)^{2-k} \cdot \left(p(1-p)(\binom{3}{k} + \binom{2}{k-1}) + (1-p-p^2)\binom{2}{k}\right) \geq 0 \tag{1.70}$$

for all $p \leq \frac{1}{\sqrt{2}}$.

□

A.7 Proof of Proposition 5

Proof. The disclosure policy in Proposition 5 and the dual variables

$$\{\vec{\lambda}, \vec{\mu}\} = \begin{pmatrix} \lambda_{OB_b^{\hat{1}}} = \frac{1}{p} \\ \lambda_{H_g} = 1 \\ \lambda_{H_b} = 0 \\ \mu_{\theta=B,k} = 0 \quad \forall k \in \{0, \ldots, 3\} \\ \mu_{\theta=G,k=0} = 0 \\ \mu_{\theta=G,k=1} = 2(1-p)(1+p-3p^2) \\ \mu_{\theta=G,\,k=2} = p(1-p)(5p+1) - p^3 \\ \mu_{\theta=G,\,k=3} = 2p^3 \end{pmatrix} \tag{1.71}$$

fulfill the above complementary slackness conditions for all $p \leq \overline{p}$.

□

A.8 Proof of Proposition 6

Proof. The disclosure policy in Proposition 6 and the dual variables

$$\{\vec{\lambda},\vec{\mu}\} = \begin{pmatrix} \lambda_{OB_b^{\hat{1}}} = \frac{3(3p^2-3p+1)}{2(2p-1)} \\ \lambda_{H_g} = \frac{3p(1-p)}{2p-1} \\ \lambda_{H_b} = 0 \\ \mu_{\theta=B,k} = 0 \quad \forall k \in \{0,1\} \\ \mu_{\theta=B,k=2} = \frac{3p(1-p)^2(3p(1+p)-1)}{2(2p-1)} \\ \mu_{\theta=B,k=3} = (1-p)^3\left(\frac{p(3p-1)-1}{2p-1}\right) \\ \mu_{\theta=G,k} = 0 \quad \forall k \in \{0,1\} \\ \mu_{\theta=G,k=2} = \frac{3p^2(1-p)(p(5-3p)-1)}{2(2p-1)} \\ \mu_{\theta=G,k=3} = p^3\left(\frac{p(5-3p)-1}{2p-1}\right) \end{pmatrix} \tag{1.72}$$

fulfill the above complementary slackness conditions for all $\overline{p} \leq p < 1$.

□

A.9 Proof of Proposition 7

Proof. We prove this proposition by the standard Primal-Dual-technique. The primal of the related problem of maximizing $\Pr(\hat{1}|b)$ is given by:

$$\max_{\substack{\{d[\hat{1}|\theta,k]\geq 0\} \\ \theta\in\{B,G\} \\ k\in\{0,1,2,3\}}} \sum_{k=0}^{3}\left(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B)\frac{1}{2} + d[\hat{1}|\theta=G,k]\Pr(k|\theta=G)\frac{1}{2}\right)\frac{3-k}{3} \tag{1.73}$$

$$\text{s.t.} \quad \sum_{k=0}^{2}\frac{3-k}{3}\left(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B) - d[\hat{1}|\theta=G,k]\Pr(k|\theta=G)\right) \leq 0 \tag{$OB_b^{\hat{1}}$}$$

$$\begin{aligned}\sum_{k=1}^{3}\frac{k}{3}\big(&(d[\hat{1}|\theta=G,k-1]-d[\hat{1}|\theta=G,k])\Pr(k|\theta=G) \\ &-(d[\hat{1}|\theta=B,k-1]-d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)\big) \leq 0\end{aligned} \tag{H_g}$$

$$\begin{aligned}\sum_{k=0}^{2}\frac{3-k}{3}\big(&(d[\hat{1}|\theta=G,k+1]-d[\hat{1}|\theta=G,k])\Pr(k|\theta=G) \\ &-(d[\hat{1}|\theta=B,k+1]-d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)\big) \leq 0\end{aligned} \tag{H_b}$$

$$d[\hat{1}|\theta,k]-1 \leq 0 \quad \forall\theta\in\{B,G\}, k\in\{0,1,2,3\} \tag{1.74}$$

Then, we take the Dual of the Primal and get:

$$\min_{\substack{\lambda_{OB_b^{\hat{1}}}\geq 0,\ \lambda_{H_g}\geq 0,\ \lambda_{H_b}\geq 0 \\ \{\mu_{\theta,\,k}\geq 0\}_{\theta\in\{B,G\},\,k\in\{0,1,2,3\}}}} \quad \sum_{k=0}^{3}\mu_{\theta=B,\,k}+\sum_{k=0}^{3}\mu_{\theta=G,\,k} \tag{1.75}$$

$$\begin{aligned}
\text{s.t.}\quad & p^{3-k}(1-p)^k\frac{1}{2}\left(-\frac{3-k}{3}\binom{3}{k}+(\lambda_{OB_b^{\hat{1}}}+\lambda_{H_b})\binom{2}{k}+\lambda_{H_g}\binom{2}{k-1}\right) & (1.76)\\
& -\lambda_{H_g}p^{2-k}(1-p)^{k+1}\binom{2}{k}-\lambda_{H_b}p^{4-k}(1-p)^{k-1}\binom{2}{k-1} \\
& +\mu_{\theta=B,\,k}\ \geq\ 0\quad \forall k\in\{0,1,2,3\}
\end{aligned}$$

$$\begin{aligned}
& p^{k}(1-p)^{3-k}\frac{1}{2}\left(-\frac{3-k}{3}\binom{3}{k}-(\lambda_{OB_b^{\hat{1}}}-\lambda_{H_b})\binom{2}{k}-\lambda_{H_g}\binom{2}{k-1}\right) & (1.77)\\
& +\lambda_{H_g}p^{k+1}(1-p)^{2-k}\binom{2}{k}+\lambda_{H_b}p^{k-1}(1-p)^{4-k}\binom{2}{k-1} \\
& +\mu_{\theta=G,\,k}\ \geq\ 0\quad \forall k\in\{0,1,2,3\}.
\end{aligned}$$

Let $\{d[\hat{1}|\theta,k]\geq 0\}_{\theta\in\{B,G\},k\in\{0,1,2,3\}}$ be a feasible disclosure policy for the primal, and $\{\vec{\lambda},\vec{\mu}\}$ feasible vector for the dual. Necessary and sufficient conditions for them to be optimal are

$$\lambda_{OB_b^{\hat{1}}}\cdot\left(\sum_{k=0}^{2}\frac{3-k}{3}(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B)\frac{1}{2}-d[\hat{1}|\theta=G,k]\Pr(k|\theta=G)\frac{1}{2})\right)=0 \tag{1.78}$$

$$\begin{aligned}
\lambda_{H_g}\cdot(&\sum_{k=1}^{3}\frac{k}{3}((d[\hat{1}|\theta=G,k-1]-d[\hat{1}|\theta=G,k])\Pr(k|\theta=G)\frac{1}{2} & (1.79)\\
&-(d[\hat{1}|\theta=B,k-1]-d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)\frac{1}{2}))=0
\end{aligned}$$

$$\begin{aligned}
\lambda_{H_b}\cdot(&\sum_{k=0}^{2}\frac{3-k}{3}((d[\hat{1}|\theta=G,k+1]-d[\hat{1}|\theta=G,k])\Pr(k|\theta=G)\frac{1}{2} & (1.80)\\
&-(d[\hat{1}|\theta=B,k+1]-d[\hat{1}|\theta=B,k])\Pr(k|\theta=B)\frac{1}{2}))=0
\end{aligned}$$

$$\mu_{\theta=B,\,k}\cdot\Big(d[\hat{1}|\theta=B,k]-1\Big)=0\quad\forall k\in\{0,1,2,3\} \tag{1.81}$$

$$\mu_{\theta=G,\,k}\cdot\Big(d[\hat{1}|\theta=G,k]-1\Big)=0\quad\forall k\in\{0,1,2,3\} \tag{1.82}$$

$$d[\hat{1}|\theta=B,k]\cdot(p^{3-k}(1-p)^k\frac{1}{2}\left(-\frac{3-k}{3}\binom{3}{k}+(\lambda_{OB_b^{\hat{1}}}+\lambda_{H_b})\binom{2}{k}+\lambda_{H_g}\binom{2}{k-1}\right) \tag{1.83}$$

$$- \lambda_{H_g} p^{2-k}(1-p)^{k+1}\binom{2}{k} - \lambda_{H_b} p^{4-k}(1-p)^{k-1}\binom{2}{k-1}$$

$$+ \mu_{\theta=B,\,k}\big) = 0 \quad \forall k \in \{0,1,2,3\}$$

$$d[\hat{1}|\theta=G,k]\cdot\Big(p^k(1-p)^{3-k}\frac{1}{2}\left(-\frac{3-k}{3}\binom{3}{k} - (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} - \lambda_{H_g}\binom{2}{k-1}\right)\Big) \tag{1.84}$$

$$+ \lambda_{H_g} p^{k+1}(1-p)^{2-k}\binom{2}{k} + \lambda_{H_b} p^{k-1}(1-p)^{4-k}\binom{2}{k-1}$$

$$+ \mu_{\theta=G,\,k}\big) = 0 \quad \forall k \in \{0,1,2,3\}$$

The disclosure policy

$$d[\hat{1}|\theta=G] = 1, \qquad d[\hat{1}|\theta=B] = \frac{(1-p)}{p} \tag{1.85}$$

and the dual variables

$$\{\vec{\lambda},\vec{\mu}\} = \left(\begin{array}{c} \lambda_{OB_b^{\hat{1}}} = 1 \\ \lambda_{H_g} = 0 \\ \lambda_{H_b} = 0 \\ \mu_{\theta=B,k} = 0 \quad \forall k \in \{0,\dots,3\} \\ \mu_{\theta=G,k=0} = 2(1-p)^3 \\ \mu_{\theta=G,\,k} = 4p^k(1-p)^{3-k} \quad \forall k \in \{1,2\} \\ \mu_{\theta=G,\,k=3} = 0 \end{array}\right) \tag{1.86}$$

fulfill the above complementary slackness conditions for all $p \in (\frac{1}{2}, 1]$.

□

A.10 Proof of Proposition 8

Proof. The primal of the related problem of maximizing $\Pr(\hat{1}|g)$ is given by:

$$\max_{\substack{\{d[\hat{1}|\theta,k]\geq 0\} \\ \theta\in\{B,G\} \\ k\in\{0,1,2,3\}}} \sum_{k=0}^{3}\left(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B)\frac{1}{2} + d[\hat{1}|\theta=G,k]\Pr(k|\theta=G)\frac{1}{2}\right)\frac{k}{3} \tag{1.87}$$

$$\text{s.t.} \quad \sum_{k=0}^{2}\frac{3-k}{3}(d[\hat{1}|\theta=B,k]\Pr(k|\theta=B)\frac{1}{2} - d[\hat{1}|\theta=G,k])\Pr(k|\theta=G)\frac{1}{2} \leq 0 \quad (OB_b^{\hat{1}})$$

$$\sum_{k=1}^{3} \frac{k}{3}((d[\hat{1}|\theta = G, k-1] - d[\hat{1}|\theta = G, k] \Pr(k|\theta = G)\frac{1}{2}) \tag{H_g}$$
$$- (d[\hat{1}|\theta = B, k-1] - d[\hat{1}|\theta = B, k]) \Pr(k|\theta = B)\frac{1}{2})) \leq 0$$

$$\sum_{k=0}^{2} \frac{3-k}{3}((d[\hat{1}|\theta = G, k+1] - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G)\frac{1}{2} \tag{H_b}$$
$$- (d[\hat{1}|\theta = B, k+1] - d[\hat{1}|\theta = B, k]) \Pr(k|\theta = B)\frac{1}{2}) \leq 0$$

$$d[\hat{1}|\theta, k] - 1 \leq 0 \quad \forall \theta \in \{B, G\}, k \in \{0, 1, 2, 3\} \tag{1.88}$$

Then, we take the Dual of the Primal and get

$$\min_{\substack{\lambda_{OB_b^{\hat{1}}} \geq 0,\ \lambda_{H_g} \geq 0,\ \lambda_{H_b} \geq 0 \\ \{\mu_{\theta,\,k} \geq 0\}_{\theta \in \{B,G\},\ k \in \{0,1,2,3\}}}} \quad \sum_{k=0}^{3} \mu_{\theta=B,\,k} + \sum_{k=0}^{3} \mu_{\theta=G,\,k} \tag{1.89}$$

$$\text{s.t.} \quad p^{3-k}(1-p)^k \frac{1}{2}\left(-\frac{k}{3}\binom{3}{k} + (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} + \lambda_{H_g}\binom{2}{k-1}\right) \tag{1.90}$$
$$- \lambda_{H_g} p^{2-k}(1-p)^{k+1}\binom{2}{k} - \lambda_{H_b} p^{4-k}(1-p)^{k-1}\binom{2}{k-1}$$
$$+ \mu_{\theta=B,\,k} \ \geq\ 0 \quad \forall k \in \{0, 1, 2, 3\}$$

$$p^k(1-p)^{3-k} \frac{1}{2}\left(-\frac{k}{3}\binom{3}{k} - (\lambda_{OB_b^{\hat{1}}} - \lambda_{H_b})\binom{2}{k} - \lambda_{H_g}\binom{2}{k-1}\right) \tag{1.91}$$
$$+ \lambda_{H_g} p^{k+1}(1-p)^{2-k}\binom{2}{k} + \lambda_{H_b} p^{k-1}(1-p)^{4-k}\binom{2}{k-1}$$
$$+ \mu_{\theta=G,\,k} \ \geq\ 0 \quad \forall k \in \{0, 1, 2, 3\}.$$

Let $\{d[\hat{1}|\theta, k] \geq 0\}_{\theta \in \{B,G\}, k \in \{0,1,2,3\}}$ be a feasible disclosure policy for the primal, and $\{\vec{\lambda}, \vec{\mu}\}$ feasible vector for the dual. Necessary and sufficient conditions for them to be optimal are

$$\lambda_{OB_b^{\hat{1}}} \cdot \left(\sum_{k=0}^{2} \frac{3-k}{3}(d[\hat{1}|\theta = B, k] \Pr(k|\theta = B)\frac{1}{2} - d[\hat{1}|\theta = G, k] \Pr(k|\theta = G)\frac{1}{2})\right) = 0$$

$$\lambda_{H_g} \cdot (\sum_{k=1}^{3} \frac{k}{3}((d[\hat{1}|\theta = G, k-1] - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G)\frac{1}{2}$$
$$- (d[\hat{1}|\theta = B, k-1] - d[\hat{1}|\theta = B, k]) \Pr(k|\theta = B)\frac{1}{2})) = 0$$

$$\lambda_{H_b} \cdot (\sum_{k=0}^{2} \frac{3-k}{3}((d[\hat{1}|\theta = G, k+1] - d[\hat{1}|\theta = G, k]) \Pr(k|\theta = G)\frac{1}{2} \tag{1.92}$$
$$- (d[\hat{1}|\theta = B, k+1] - d[\hat{1}|\theta = B, k]) \Pr(k|\theta = B)\frac{1}{2})) = 0$$

$$\mu_{\theta=B,\,k} \cdot \left(d[\hat{1}|\theta = B, k] - 1\right) = 0 \quad \forall k \in \{0, 1, 2, 3\}$$

$$\mu_{\theta=G,\,k} \cdot \left(d[\hat{1}|\theta = G, k] - 1\right) = 0 \quad \forall k \in \{0, 1, 2, 3\}$$

$$d[\hat{1}|\theta = B, k] \cdot (p^{3-k}(1-p)^k \frac{1}{2}\left(-\frac{k}{3}\binom{3}{k} + (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} + \lambda_{H_g}\binom{2}{k-1}\right)$$
$$- \lambda_{H_g} p^{2-k}(1-p)^{k+1}\binom{2}{k} - \lambda_{H_b} p^{4-k}(1-p)^{k-1}\binom{2}{k-1}$$
$$+ \mu_{\theta=B,\,k}) = 0 \quad \forall k \in \{0, 1, 2, 3\}$$

$$d[\hat{1}|\theta = G, k] \cdot (p^{k}(1-p)^{3-k} \frac{1}{2}\left(-\frac{k}{3}\binom{3}{k} - (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b})\binom{2}{k} - \lambda_{H_g}\binom{2}{k-1}\right)$$
$$+ \lambda_{H_g} p^{k+1}(1-p)^{2-k}\binom{2}{k} + \lambda_{H_b} p^{k-1}(1-p)^{4-k}\binom{2}{k-1}$$
$$+ \mu_{\theta=G,\,k}) = 0 \quad \forall k \in \{0, 1, 2, 3\}.$$

The disclosure policy

$$d[\hat{1}|\theta = B, k] = \begin{cases} 0 \quad \text{if} \quad k = 0 \\ \frac{(p-\frac{1}{2})(3-p)}{2(2p-1)} \quad \text{if} \quad k = 1 \\ 1 \quad \text{if} \quad k \in \{2, 3\} \end{cases}, \quad d[\hat{1}|\theta = G, k] = \begin{cases} 0 \quad \text{if} \quad k = 0 \\ \frac{(p-\frac{1}{2})(3p^2+5p-2)}{2(6p^2-5p+1)} \text{ if} \quad k = 1 \\ 1 \quad \text{if} \quad k \in \{2, 3\}. \end{cases} \tag{1.93}$$

and the dual variables

$$\{\vec{\lambda}, \vec{\mu}\} = \begin{pmatrix} \lambda_{OB_b^{\hat{1}}} = \frac{1+3p(p-1)}{2(2p-1)} \\ \lambda_{H_g} = \frac{p(1-p)}{2p-1} \\ \lambda_{H_b} = 0 \\ \mu_{\theta=B,k} = 0 \quad \forall k \in \{0,1\} \\ \mu_{\theta=B,k=2} = \frac{3p(1-p)^2 3(p^2+p-1)}{2(2p-1)} \\ \mu_{\theta=B,k=3} = (1-p)^3\left(\frac{p^2+p-1}{2p-1}\right) \\ \mu_{\theta=G,k} = 0 \quad \forall k \in \{0,1\} \\ \mu_{\theta=G,k=2} = \frac{3p^2(1-p)(3p-p^2-1)}{2(2p-1)} \\ \mu_{\theta=G,k=3} = p^3\left(\frac{3p-p^2-1}{2p-1}\right) \end{pmatrix} \tag{1.94}$$

fulfill the above complementary slackness conditions for all $\overline{p} \leq p < 1$.

$\square$

A.11 Proof of Lemma 5

Proof. Let the designer follow a disclosure policy d', that sends messages $r' \in R'$, and voters responding optimally to this disclosure policy. Denote by $a_i(r', z_i)$ the probability, that voter i with private signal $z_i \in \{g, b\}$ votes $\hat{1}$ after receiving message r'.

The first step is to show, that the g-type is always weakly more optimistic than the b-type for any signal that is sent with strictly positive probability in some state θ. The following formulation makes use of the fact that $\Pr(r'|\theta, z_i) = \Pr(r'|\theta)$ and $\Pr(piv|r', \theta, z_i) = \Pr(piv|r', \theta)$, as the designer cannot use or elicit the private information of the agents.

Lemma 6. *Under any non-eliciting disclosure policy, in any equilibrium, the g-type is more optimistic than the b-type:* $\Pr(\theta = G|r', g, piv) \geq \Pr(\theta = G|r', b, piv)$.

Proof.

$$
\begin{aligned}
&\Pr(\theta = G|r', g, piv) \\
&= \frac{\Pr(\theta = G|g)\Pr(r'|\theta = G)\Pr(piv|r', \theta = G)}{\Pr(\theta = G|g)\Pr(r'|\theta = G)\Pr(piv|r', \theta = G) + \Pr(\theta = B|g)\Pr(r'|\theta = B)\Pr(piv|r', \theta = B)} \\
&= \frac{\frac{1}{2}p\Pr(r'|\theta = G)\Pr(piv|r', \theta = G)}{p\Pr(r'|\theta = G)\Pr(piv|r', \theta = G) + \frac{1}{2}(1-p)\Pr(r'|\theta = B)\Pr(piv|r', \theta = B)} \\
&\geq \frac{\frac{1}{2}(1-p)\Pr(r'|\theta = G)\Pr(piv|r', \theta = G)}{(1-p)\Pr(r'|\theta = G)\Pr(piv|r', \theta = G) + \frac{1}{2}p\Pr(r'|\theta = B)\Pr(piv|r', \theta = B)} \\
&= \Pr(\theta = G|r', b, piv)
\end{aligned}
$$

$\square$

Using Lemma 6, the following is a complete partition of the designer's messages:

1. $R'(\hat{0}) := \{r' \in R' : a_i(r', g) = a_i(r', b) = 0\}$

2. $R'(\hat{1}) := \{r' \in R' : a_i(r', g) > 0 \quad \wedge \quad a_i(r', b) > 0\}$

3. $R'(\hat{01}) := \{r' \notin R'(\hat{0}) \cup R'(\hat{1})\}$.

Consider the following alternative policy d, that takes the old disclosure policy d' and maps it into a message space $R = \{\hat{0}, \hat{01}, \hat{1}\}$, for all states $\theta \in \{B, G\}$:

$$d(\hat{0}|\theta, r') = \begin{cases} 1 \quad \text{if} \quad r' \in R'(\hat{0}) \\ 1 - a_i(r', g)^{n-1} \quad \text{if} \quad r' \in R'(\hat{01}) \\ 1 - \Pr(piv|r', \theta) \quad \text{if} \quad r' \in R'(\hat{1}) \\ 0 \quad \text{otherwise.} \end{cases} \tag{1.95}$$

$$d(\hat{01}|\theta, r') = \begin{cases} a_i(r', g)^{n-1} \quad \text{if} \quad r' \in R'(\hat{01}) \\ 0 \quad \text{otherwise.} \end{cases} \tag{1.96}$$

$$d(\hat{1}|\theta, r') = \begin{cases} \Pr(piv|\theta, r') \quad \text{if} \quad r' \in R'(\hat{1}) \\ 0 \quad \text{otherwise.} \end{cases} \tag{1.97}$$

Consider the following equilibrium under the new disclosure policy d:

$$a_i(r, b) = \begin{cases} 0 \quad \text{if} \quad r = \hat{0} \\ 0 \quad \text{if} \quad r = \hat{01} \\ 1 \quad \text{if} \quad r = \hat{1} \end{cases} \qquad \text{and} \qquad a_i(r, g) = \begin{cases} 0 \quad \text{if} \quad r = \hat{0} \\ 1 \quad \text{if} \quad r = \hat{01} \\ 1 \quad \text{if} \quad r = \hat{1} \end{cases} \tag{1.98}$$

First, we establish that the above policy together with the voting behavior in Equation 1.98 is an equilibrium. Then, we show that the designer weakly prefers the disclosure policy d with the restricted message set to the original disclosure policy d'.

Under the new disclosure policy d, after recommendation $\hat{0}$, no agent is ever pivotal; he therefore has no profitable deviation from voting for the status quo.

After realization $\hat{1}$, each agent is pivotal with probability 1. The next lemma is useful in establishing the obedience constraints of voters after realization $\hat{1}$.

Lemma 7. *For $r' \in R'(\hat{1})$, we have $\Pr(\theta = G|r', \hat{1}, b) = \Pr^{d'}(\theta = G|r', piv, b)$.*

For $r' \in R'(\hat{01})$ and $z_i \in \{b, g\}$, we have $\Pr(\theta = G|r', piv, \hat{01}, z_i) = \Pr^{d'}(\theta = G|r', piv, z_i)$.

Proof. Simple calculation show for $r' \in R'(\hat{1})$:

$$\Pr(\theta = G|r', \hat{1}, b) = \frac{\Pr(\theta = G|r', b) \Pr(\hat{1}|r', \theta = G)}{\Pr(\theta = G|r', b) \Pr(\hat{1}|r', \theta = G) + \Pr(\theta = B|r', b) \Pr(\hat{1}|r', \theta = B)} \tag{1.99}$$

and

$$\Pr^{d'}(\theta = G|r', piv, b) = \frac{\Pr^{d'}(\theta = G|r', b) \Pr^{d'}(piv|r', \theta = G)}{\Pr^{d'}(\theta = G|r', b) \Pr^{d'}(piv|r', \theta = G) + \Pr^{d'}(\theta = B|r', b) \Pr^{d'}(piv|r', \theta = B)} \tag{1.100}$$

The lemma follows by construction of the new disclosure policy d, where $\Pr(1|r', \hat{\theta} = G) = \Pr(piv|r', \theta = G)$ and $\Pr(1|r', \hat{\theta} = B) = \Pr(piv|r', \theta = B)$. Analogously, for $r' \in R'(\hat{01})$:

$$\Pr^{d'}(\theta = G|r', piv, b) = \frac{\Pr^{d'}(\theta = G|r', b) \Pr^{d'}(piv|r', \theta = G)}{\Pr^{d'}(\theta = G|r', b) \Pr^{d'}(piv|r', \theta = G) + \Pr^{d'}(\theta = B|r', b) \Pr^{d'}(piv|r', \theta = B)}$$

and

$$\Pr(\theta=G|r',\hat{1},b)= \tag{1.101}$$
$$\frac{\Pr(\theta=G|r',b)\Pr(\hat{01}|r',\theta=G)\Pr(piv|\theta=G,\hat{01})}{\Pr(\theta=G|r',b)\Pr(\hat{1}|r',\theta=G)\Pr(piv|\theta=G,\hat{01})+\Pr(\theta=B|r',b)\Pr(\hat{1}|r',\theta=B)\Pr(piv|\theta=B,\hat{01})}$$

Under the old disclosure policy, we have $\Pr^{d'}(piv|r',\theta=G)=p^{n-1}a_i(r',g)^{n-1}$. With the new disclosure policy, we have $\underbrace{\Pr(\hat{01}|\theta=G,r')}_{=a_i(r',g)^{n-1}}\underbrace{\Pr(piv|\theta=G,\hat{01})}_{=p^{n-1}}$, which is exactly equal to $\Pr^{d'}(piv|r',\theta=G)$. By the same argument, we have $\Pr^{d'}(piv|r',\theta=B)=\Pr(\hat{01}|\theta=B,r')$, which proves the lemma.

$\square$

Lemma 8. *For $r'\in R'(\hat{1})$, we have: $\sum_{r'\in R'(\hat{1})}\Pr(r'|\hat{1},b)=1$. For $r'\in R'(\hat{01})$, and $z_i\in\{g,b\}$, we have $\sum_{r'\in R'(\hat{1})}\Pr(r'|\hat{1},piv,z_i)=1$.*

Proof. First, consider $r'\in R'(\hat{1})$.

$$\begin{aligned}\sum_{r'\in R'(\hat{1})}\Pr(r'|\hat{1},b)&=\sum_{r'\in R'(\hat{1})}\frac{\Pr(r',\hat{1}|b)}{\Pr(\hat{1}|b)}\\&=\frac{\sum_{r'\in R'(\hat{1})}\Pr(r',\hat{1}|b)}{\sum_{r'\in R'(\hat{0})\vee R'(\hat{01})\vee R'(\hat{1})}\Pr(\hat{1},r'|b)}\\&=\frac{\sum_{r'\in R'(\hat{1})}\Pr(r',\hat{1}|b)}{\sum_{r'\in R'(\hat{0})}\underbrace{\Pr(\hat{1},r'|b)}_{=0}+\sum_{r'\in R'(\hat{01})}\underbrace{\Pr(\hat{1},r'|b)}_{=0}+\sum_{r'\in R'(\hat{1})}\Pr(\hat{1},r'|b)}\\&=1\end{aligned}$$

The last step follows, because $\hat{1}$ is only send with strictly positive probability if $r'\in R'(\hat{1})$.

Next, consider $r'\in R'(\hat{01})$.

$$\sum_{r'\in R'(\hat{01})}\Pr(r'|\hat{01},piv,z_i)=\sum_{r'\in R'(\hat{01})}\frac{\Pr(r',piv|\hat{01},z_i)}{\Pr(piv|\hat{01},z_i)}=$$
$$=\frac{\sum_{r'\in R'(\hat{01})}\Pr(r',piv|\hat{01},z_i)}{\sum_{r'\in R'(\hat{0})}\underbrace{\Pr(piv,r'|\hat{01},z_i)}_{=0}+\sum_{r'\in R'(\hat{01})}\Pr(piv,r'|\hat{01},z_i)+\sum_{r'\in R'(\hat{1})}\underbrace{\Pr(piv,r'|\hat{01},z_i)}_{=0}}$$
$$=1.$$

The last step follows because $\hat{01}$ is only send with strictly positive probability if $r'\in R'(\hat{01})$.

$\square$

The belief of each voter after $\hat{1}$ about the state being good, $\Pr(\theta = G|\hat{1}, z_i)$, is a convex combination of the beliefs under the old disclosure policy $\{\Pr(\theta = G|r', piv, z_i)\}_{r' \in R(\hat{1})}$, as the following calculation shows.

$$\begin{aligned}
\Pr(\theta = G|\hat{1}, piv, b) &= \Pr(\theta = G|\hat{1}, b) \\
&= \sum_{r' \in R'(\hat{1})} \Pr(r'|\hat{1}, b) \Pr(\theta = G|r', \hat{1}, b) \\
&\overset{\text{Lemma 7}}{=} \sum_{r' \in R'(\hat{1})} \Pr(r'|\hat{1}, b) \underbrace{\overset{d'}{\Pr}(\theta = G|r', piv, b)}_{\geq \frac{1}{2}} \geq \frac{1}{2}.
\end{aligned}$$

We have $\Pr^{d'}(\theta = G|r', piv, b) \geq \frac{1}{2}$, because $a_i(r', b) > 0$ in the original equilibrium for d': the b-type (weakly) prefers the proposal to the status quo.

Because the g-type is more optimistic under any disclosure policy (Lemma 6), we also have $\Pr(\theta = G|\hat{1}, g) \geq \Pr(\theta = G|\hat{1}, b) \geq \frac{1}{2}$. The g-type prefers the proposal to the status quo after observing $\hat{1}$. Both voters have hence no profitable deviation from voting for the proposal.

Finally, consider a signal $\hat{01}$. Using Lemma 6, as the g-type is always more optimistic than the b-type, we have $a_i(r', b) = 0$ and $a_i(r', g) > 0$ for any recommendation $r' \in R'(\hat{01})$. As both voter types are pivotal with non-zero probability (by assumption, r' is sent with strictly positive probability), we have

$$\Pr(\theta = G|\hat{01}, piv, z_i) = \sum_{r' \in R'(\hat{01})} \Pr(r'|\hat{01}, piv, z_i) \Pr(\theta = G|r', \hat{01}, piv, z_i) \tag{1.102}$$

$$= \sum_{r' \in R'(\hat{01})} \Pr(r'|\hat{01}, piv, z_i) \underbrace{\overset{d'}{\Pr}(\theta = G|r', piv, z_i)}_{\geq(\leq)\frac{1}{2} \text{ if } z_i = g(=b)} \tag{1.103}$$

$$\begin{cases} \geq \frac{1}{2} & \text{if} \quad z_i = g \\ \leq \frac{1}{2} & \text{if} \quad z_i = b \end{cases} \tag{1.104}$$

Lemma 8 establishes, that the above is a convex combination; lemma 7 binds each summand below $\frac{1}{2}$ for $z_i = b$, and above $\frac{1}{2}$ for $z_i = g$. Therefore, no voter has a profitable deviation: after $\hat{01}$, the g-type prefers the proposal, and the b-type the status quo.

The last remaining step is to show, that under the alternative constructed policy d', the designer is no worse off than under the disclosure policy d with an arbitrary message space. We prove this by showing that under the new disclosure policy d, the implementation probability of the proposal weakly increases for each r'.

Take $r' \in R'(\hat{0})$. Under both the old and the new disclosure policy, the proposal is implemented with zero probability.

Take $r' \in R'(\hat{01})$. Under the old disclosure policy, the proposal was implemented with probability $p^n a_i(r', g)^n$ if $\theta = G$, and $(1-p)^n a_i(r', g)^n$ if $\theta = B$. Under the new disclosure policy, the proposal is being implemented with probability $p^n a_i(r', g)^{n-1}$ if $\theta = G$, and $(1-p)^n a_i(r', g)^{n-1}$ if $\theta = B$. The probabilities are higher under the new disclosure policy, because $a_i(r',g)^{n-1} \geq a_i(r',g)^n$.

Finally, take $r' \in R'(\hat{1})$. Under the old disclosure policy, the proposal was implemented with probability $\Pr(piv|\theta = G, r')[pa_i(r',g) + (1-p)a_i(r',b)]$ if $\theta = G$, and with probability $\Pr(piv|\theta = B, r')[pa_i(r',b) + (1-p)a_i(r',g)]$ if $\theta = B$. Under the new disclosure policy, the proposal is implemented with probability $\Pr(piv|\theta, r')$, which is weakly higher.

□

A.12 Proof of Proposition 9

Proof. The primal of the sender's problem is

$$\max_{\{d[r|\theta]\geq 0\}_{r\in\{\widehat{01},\hat{1}\},\theta\in\{B,G\}}} \quad \sum_{\theta\in\{B,G\}} (d[\hat{1}|\theta] + d[\widehat{01}|\theta]\Pr(k=3|\theta))\Pr(\theta) \tag{1.105}$$

$$\begin{aligned}
\text{s.t.} \quad & d[\hat{1}|\theta] + d[\widehat{01}|\theta] - 1 \leq 0 \quad \forall\theta \in \{B,G\} && (1.106)\\
& d[\hat{1}|\theta = B](1-p) - d[\hat{1}|\theta = G]p \leq 0 && (OB_g^{\hat{1}})\\
& d[\hat{1}|\theta = B]p - d[\hat{1}|\theta = G](1-p) \leq 0 && (OB_b^{\hat{1}})\\
& d[\widehat{01}|\theta = B](1-p)^3 - d[\widehat{01}|\theta = G]p^3 \leq 0 && (OB_g^{\widehat{01}})\\
& d[\widehat{01}|\theta = B]p(1-p)^2 - d[\widehat{01}|\theta = G]p^2(1-p) \leq 0 && (OB_g^{\widehat{01}})
\end{aligned}$$

Next, we take the dual of the primal and get:

$$\min_{\substack{\lambda_{OBg(\hat{1})}\geq 0,\ \lambda_{OBb(\hat{1})}\geq 0\\ \lambda_{OBg(\widehat{01})}\geq 0,\ \lambda_{OBb(\widehat{01})}\geq 0\\ \mu_{\theta=B}\geq 0,\ \mu_{\theta=G}\geq 0}} \quad \mu_{\theta=B} + \mu_{\theta=G} \tag{1.107}$$

$$\begin{aligned}
\text{s.t.} \quad & -\frac{1}{2}(1 + \lambda_{OBg(\hat{1})}p + \lambda_{OBb(\hat{1})}(1-p)) + \mu_{\theta=G} \geq 0 && (1.108)\\
& -\frac{1}{2}(1 + \lambda_{OBg(\hat{1})}(1-p) - \lambda_{OBb(\hat{1})}p) + \mu_{\theta=B} \geq 0 && (1.109)\\
& -\frac{1}{2}p^2(p + \lambda_{OBg(\widehat{01})}p - \lambda_{OBb(\widehat{01})}(1-p)) + \mu_{\theta=G} \geq 0 && (1.110)\\
& -\frac{1}{2}(1-p)^2((1-p) - \lambda_{OBg(\widehat{01})}(1-p) + \lambda_{OBb(\widehat{01})}p) + \mu_{\theta=B} \geq 0 && (1.111)
\end{aligned}$$

Let $\{d[r|\theta] \geq 0\}_{r\in\{\widehat{01},\hat{1}\}}$ be a feasible disclosure policy for the primal, and $\{\vec{\lambda}, \vec{\mu}\}$ feasible vector for the dual. Necessary and sufficient conditions for the optimal are:

$$
\begin{aligned}
&\lambda_{OBg(\hat{1})} \cdot \Big(d[\hat{1}|\theta = B](1-p) - d[\hat{1}|\theta = G]p\Big) = 0,\\
&\lambda_{OBb(\hat{1})} \cdot \Big(d[\hat{1}|\theta = B]p - d[\hat{1}|\theta = G](1-p)\Big) = 0,\\
&\lambda_{OBg(\widehat{01})} \cdot \Big(d[\widehat{01}|\theta = B](1-p)^3 - d[\widehat{01}|\theta = G]p^3\Big) = 0,\\
&\lambda_{OBb(\widehat{01})} \cdot \Big(d[\widehat{01}|\theta = B]p(1-p)^2 - d[\widehat{01}|\theta = G]p^2(1-p)\Big) = 0,\\
&\mu_{\theta=B} \cdot \Big(d[\hat{1}|\theta = B] + d[\widehat{01}|\theta = B] - 1\Big) = 0,\\
&\mu_{\theta=G} \cdot \Big(d[\hat{1}|\theta = G] + d[\widehat{01}|\theta = G] - 1\Big) = 0,\\
&d[\hat{1}|\theta = G] \cdot \Big(-\frac{1}{2}(1 + \lambda_{OBg(\hat{1})}p + \lambda_{OBb(\hat{1})}(1-p)) + \mu_{\theta=G}\Big) = 0,\\
&d[\hat{1}|\theta = B] \cdot \Big(-\frac{1}{2}(1 + \lambda_{OBg(\hat{1})}(1-p) - \lambda_{OBb(\hat{1})}p) + \mu_{\theta=B}\Big) = 0,\\
&d[\widehat{01}|\theta = G] \cdot \Big(-\frac{1}{2}p^2(p + \lambda_{OBg(\widehat{01})}p - \lambda_{OBb(\widehat{01})}(1-p)) + \mu_{\theta=G}\Big) = 0,\\
&d[\widehat{01}|\theta = B] \cdot \Big(-\frac{1}{2}(1-p)^2((1-p) - \lambda_{OBg(\widehat{01})}(1-p) + \lambda_{OBb(\widehat{01})}p) + \mu_{\theta=B}\Big) = 0.
\end{aligned}
$$

It can be easily checked by substitution that the disclosure policy

$$
\{d[r|\theta] \geq 0\}_{r\in\{\widehat{01},\hat{1}\},\theta\in\{B,G\}} = \begin{cases} d[\hat{1}|\theta = G] = 1 \\ d[\hat{1}|\theta = B] = \frac{(1-p)}{p} \\ d[\widehat{01}|\theta = G] = 0 \\ d[\widehat{01}|\theta = B] = 0 \end{cases} \tag{1.112}
$$

and the dual variables

$$
\{\vec{\lambda}, \vec{\mu}\} = \begin{pmatrix} \lambda_{OBg(\hat{1})} = 0 \\ \lambda_{OBb(\hat{1})} = \frac{1}{p} \\ \lambda_{OBg(\widehat{01})} = 1 + \lambda_{OBb(\widehat{01})}\frac{p}{1-p} \\ \lambda_{OBb(\widehat{01})} = \frac{(2p^4 - \frac{1}{2})(1-p)}{p^3(1-2p)} \\ \mu_{\theta=B} = 0 \\ \mu_{\theta=G} = \frac{1}{2p} \end{pmatrix} \tag{1.113}
$$

fulfill the above complementary slackness conditions for all $p \leq \frac{1}{\sqrt[4]{2}} = \tilde{p}$. Analogously, for $p > \tilde{p}$, the disclosure policy from Proposition 10 and the duals

$$\{\vec{\lambda},\vec{\mu}\} = \begin{pmatrix} \lambda_{OBg(\hat{1})} = 0 \\ \lambda_{OBb(\hat{1})} = \frac{1}{p} - \frac{(1-p)^3\left(2p^4-1\right)}{p(p^4-(1-p)^4)} \\ \lambda_{OBg(\widehat{01})} = 1 - \frac{2p^4-1}{p^4-(1-p)^4} \\ \lambda_{OBb(\widehat{01})} = 0 \\ \mu_{\theta=B} = \frac{\frac{1}{2}(1-p)^3\left(2p^4-1\right)}{p^4-(1-p)^4} \\ \mu_{\theta=G} = \frac{1}{2}\left(\frac{1}{p} - \frac{(1-p)^4(2p^4-1)}{p(p^4-(1-p)^4)}\right) \end{pmatrix} \tag{1.114}$$

fulfill the above complementary slackness conditions.

□

A.13 Proof of Proposition 10

Proof. The error probabilities l_G (probabilities of convicting the innocent) and l_B (acquit the guilty) of a wrong decision are found in Feddersen and Pesendorfer (1998). The expected utility of an uninformed voter in Feddersen and Pesendorfer (1998) is

$$\frac{1}{2}\left(1 - l_G - l_B\right) = \frac{1}{4}\frac{(2p-1)^3}{\left(p^{3/2} + p\sqrt{1-p} - \sqrt{1-p}\right)^2}. \tag{1.115}$$

With a manipulative information designer the expected utility of a voter is

$$\frac{1}{2}\Pr(\hat{1}|\theta = G)\frac{1}{2} + \frac{1}{2}\Pr(\hat{1}|\theta = B)\left(-\frac{1}{2}\right). \tag{1.116}$$

Next, we show that the optimal disclosure policy of the designer in each of the three intervals for p yields a strictly higher utility to the voter.

Case 1: $\frac{1}{2} < p \leq \frac{1}{\sqrt{2}}$. Using the optimal disclosure policy in Proposition 4, the utility of the ex ante type is $\frac{1}{4}1 - \frac{1}{4}\frac{1-p}{p} = \frac{1}{4}\frac{2p-1}{p}$. Comparing this with Equation 1.115 shows that the utility in case 1 is strictly higher for all $p \in (\frac{1}{2}, 1)$.

Case 2: $\frac{1}{\sqrt{2}} < p \leq \frac{1+\sqrt{13}}{6}$. With the optimal disclosure policy in Proposition 5, $\Pr(\hat{1}|\theta = G) = 1 - (1-p)^3$ and $\Pr(\hat{1}|\theta = B) = (p-1)(p^2-2)$. A voter's expected utility is $\frac{1}{4}(2p-1)(2-p)$, which can be again easily checked to be larger than Equation 1.115 for all $p \in (\frac{1}{2}, 1)$.

Case 3: $\frac{1+\sqrt{13}}{6} \leq p < 1$. In this case, the probabilities of choosing the proposal are $\Pr(\hat{1}|\theta = G) = \frac{1}{4}p\left(3p^3 - 8p^2 + 3p + 6\right)$ and $\Pr(\hat{1}|\theta = B) = \frac{1}{4}(p-1)\left(3p^3 - p^2 - 4p - 4\right)$. This yields an expected utility of $\frac{1}{8}\left[(2p-1)(p+1)(2-p)\right]$. This is higher than the expected utility in Feddersen and Pesendorfer (1998) in Equation 1.115. □

Chapter 2

Clueless Persuasion

1. Introduction

Attempting to influence the beliefs of people to affect their decisions in one's own favor is common behavior. Interested parties try to *persuade* others by providing information and making recommendations. Often, these interested parties are clueless in the sense that they have no superior or even no knowledge about decision relevant issues at all, while each decision maker privately happens to be better informed. The fact that decision makers are nonetheless sometimes willing to follow the recommendations of somebody uninformed is rooted in the aggregation of information: If the interested party is able to aggregate the private but imperfect knowledge of all decision makers, she can make recommendations based on more meaningful information. This equips the interested party with some persuasion power and leads to the question to which extent she can persuade the decision makers.

Consider for example a CEO who tries to convince a board of directors to vote for a new proposal. The CEO wishes to always implement the proposal to improve short-term performance. Directors, however, only want to approve the proposal if it increases long-term performance. While the CEO has no information about the proposal's long-term consequences, directors have superior, private knowledge about the long-term effects of the proposal. We are interested in how the CEO can elicit this private information from each director and then use the aggregated information in order to convince directors to vote for the proposal.

In our model an information designer attempts to convince a committee of voters to approve a proposal with unanimity. The proposal's quality is unknown and can be either high or low. The information designer and all voters share the same prior about the proposal having a high quality. If voters knew the quality, they would agree on the optimal decision: If the quality was high, the committee would want to approve the proposal, while if it was low, the committee would want to reject it. In contrast, the information designer aims for the approval of the proposal irrespective of its quality. Each voter receives a binary signal about the quality that determines whether he is optimistic or pessimistic about the proposal. This signal is a voter's private information. The information designer does not possess any information about the quality of the proposal but she can ask voters for reports about their private information. Based on these reports, the information designer sends a public recommendation to all voters.

Upon observing the information designer's recommendation, voters update their beliefs about the quality and either vote for or against the proposal. Although voters are aware of the information designer's interest in the proposal, they might nevertheless want to follow her recommendation. This is because the recommendation is based on the aggregated information of all voters' private signals, which is a better indicator of the quality than each voter's private signal alone.

The main contribution of this paper is to unveil the extent to which an uninformed information designer can persuade imperfectly informed voters by first asking for reports about their private information and sending public recommendations based on these reports afterwards. We characterize in how far the private information of voters restricts the information designer in her scope for persuasion. In contrast to Bobkova and Klein (2019), the information designer in this paper has initially no informational advantage over voters.

In this paper, we consider restricted information design. In contrast to unrestricted information design, a *restricted* information designer is not able to condition his disclosure policy on the true state of the world. In our benchmark case we consider an *omniscient* information designer who can observe the private signal realizations of all voters. We show that the omniscient information designer uses a threshold policy: she recommends the proposal with probability one for any number of optimists above a certain cutoff, mixes at the cutoff, and recommends the status quo with certainty below the cutoff. The cutoff is such that a pessimistic voter is indifferent between the proposal and the status quo after the recommendation to vote for the proposal.

Next, we consider an *eliciting* information designer who cannot observe the private information of voters but can ask them for reports about their signal realizations. We show that the eliciting information designer is sometimes able to achieve the persuasion benchmark of the omniscient information designer: for a large range of precision levels and ex-ante probabilities, the eliciting sender can implement the optimal disclosure policy from the omniscient benchmark case and is no worse off. That is, despite the omniscient sender having the superior knowledge about the signal realizations of all voters, she has no larger scope for persuasion than the eliciting sender. Within this range of precision levels and ex-ante probabilities, the private information of voters does not restrict the eliciting information designer in her scope for persuasion. Beyond this range, the eliciting sender is no longer able to implement the omniscient sender's optimal disclosure policy.

We characterize the eliciting sender's optimal disclosure policy for the area of non-implementability of the omniscient benchmark case. One of the main observations for this area is that the eliciting sender recommends the proposal less often when there are more optimistic voters but more often when there are more pessimistic voters compared to the omniscient sender. The reason for this is that under the threshold policy of the omniscient sender, optimistic voters would always have a profitable deviation by misreporting their private signal. To counteract this effect, the eliciting sender starts to recommend the proposal more often in states with less optimistic reports of voters to increase the expected loss from misreporting. This shift in probability mass is such that optimistic voters are just indifferent between reporting truthfully and misreporting. For very high precision levels and ex-ante probabilities this even forces the eliciting sender to use non-monotone disclosure policies. That is, the sender uses policies in which she recommends the proposal with a higher probability in states where less good than bad signals have been reported.

We find that allowing the sender to make her recommendations privately to each voter and to discriminate between voters does not extend her scope for persuasion. Instead, she uses the same disclosure policy as the sender who makes her recommendations publicly and treats each voter symmetrically.

2. Related Literature

Our paper can be classified into the literature on information design and on incomplete information correlated equilibrium.

In their seminal paper Kamenica and Gentzkow (2011) solve the persuasion problem of an information designer who faces one agent that has no private information. While Kamenica and Gentzkow (2011) analyze *unrestricted* information design where the information designer is unrestricted in her ability to construct experiments conditional on the true state of the world, we focus on *restricted* information design where the information designer cannot condition her messages on the true state of the world. In our model all information stems from the agents, and the information designer has no informational advantage over these agents.

Unrestricted information design makes the additional assumption that the information designer can commit herself to any experiment with an arbitrary precision level about the true state of the world. Many authors have contributed to the unrestricted information design literature with multiple receivers; examples include Wang (2015), Chan et al. (2018), Alonso and Câmara (2016), and Bardhi and Guo (2018). Alonso and Câmara (2016) analyze how a biased politician can persuade a committee of heterogeneous voters to vote for a reform and how the voting rule in use alters the results. Chan et al. (2018) also study the persuasion of multiple voters with heterogeneous voting costs. They show that the designer is better off under private persuasion than under public persuasion as she is able to exploit the heterogeneity in voting cost when she privately communicates with the voters.

The paper that shares the most similarities to ours is Bardhi and Guo (2018). They study persuasion of a committee consisting of uninformed voters with heterogeneous but correlated payoff types under unanimity rule. They consider two different persuasion modes: under general persuasion the designer's message depends on every voter's payoff type while under individual persuasion the message that is sent to a voter depends only on that voter's payoff type. While they focus on an information designer who sends messages to each agent in private, we set priority to public messages. Bardhi and Guo (2018) find that restricting the information designer to public messages is without loss under some conditions. This finding is in line with our result on public versus private persuasion. A comparison of private and public persuasion in a collective decision making environment is conducted by Wang (2015). While in Wang (2015) signals are drawn independently under private persuasion, signals are correlated in our analysis of private persuasion. Wang (2015) finds that the information designer is weakly better off when she uses public instead of private signals.

Besides the designer's access to an arbitrary precise experiment, a further crucial difference between the above research and the present paper is that all of the above mentioned papers do not allow for private information on the receiver's side. The payoff types of all agents are common knowledge and do not carry any further information. In our model each agent possesses some private signal that is correlated with the payoff relevant state of the world. Our information designer does not know which committee constellation she is facing which is why she has to first elicit this information from the agents in an incentive compatible way.

Among the papers that add private information on the receiver side, the following are most related to ours. In a recent paper, Kolotilin et al. (2017) study the persuasion of one receiver that has private information about his own payoff type. They characterize the designer's payoff maximizing disclosure rule for the case where she elicits the agent's private information and for the case where she does not. They find that in a binary setting with two states of the world and two actions of the agent the information designer cannot profit from eliciting the agent's private information before persuading him. Since in our model all information stems from the agents, the information designer is forced to elicit this private information first. Although their work relates to ours with respect to the private information on the receiver's side, in contrast to them, we study the persuasion of multiple privately informed receivers.

Bergemann et al. (2018) characterize the revenue maximizing solution to an information design problem with elicitation and monetary transfers. In contrast to them, we do not allow for monetary transfers. Heese and Lauermann (2018) analyze the persuasion of large electorates in which voters are heterogeneous and privately informed about their payoff type. They show that when the size of the electorate grows infinitely large, the information designer is able to achieve the implementation of her preferred outcome with probability one. While they are interested in limit results and do not study the elicitation of voters' private information, we focus on small committees where the elicitation of the voters' private signals is essential.

Bergemann and Morris (2016b) motivate the concept of *omniscient* and *private* unrestricted persuasion of one agent that possesses some private information.[1] We extend their analysis to a setting in which the information designer is restricted and faces multiple privately informed agents. For an extensive overview of the existing literature on information design and its link to other literature streams see Bergemann and Morris (2017).

Our paper relates to the literature on strategic voting (see e.g. Austen-Smith and Banks, 1996; Coughlan, 2000; Feddersen and Pesendorfer, 1997, 1998; Gerardi and Yariv, 2007). Feddersen and Pesendorfer (1998) show that there does not exist an equilibrium under unanimity rule in which strategic voters vote sincerely (i.e., vote truthfully according to their own private signals). The reason for that is that strategic voters only take the events into account in which they are pivotal. They show that under unanimity rule the error probability of implementing the inefficient decision is higher than under any other majority rule. We extend the work of Feddersen and Pesendorfer (1998) by adding a manipulative information designer to their model and analyze how

[1]Their terminology private corresponds to information design with elicitation.

the designer can influence the agents' voting behavior by giving recommendations based on the aggregated information of all agents.

3. Model

There is one information designer and three voters. Each voter has to make a decision between voting for a proposal or voting for a status quo. The biased information designer tries to persuade voters to vote for the proposal. If voter $i \in \{1, 2, 3\}$ votes for the proposal, we write $a_i = 1$, and $a_i = 0$ if he votes for the status quo. The proposal requires unanimity and we write $a = 1$ if $a_i = 1\ \forall i$ and $a = 0$ if $\exists i$ s.t. $a_i = 0$.

The utility a voter receives from the proposal depends on an uncertain state of the world $\theta \in \{B, G\}$, where $\Pr(\theta = G) = q \in (0, 1)$. That is,

$$u_i(a, \theta) = \begin{cases} \mathbb{1}_{\{\theta=G\}} - \frac{1}{2} & \text{if } a = 1 \\ 0 & \text{if } a = 0. \end{cases}$$

Voters share a common interest: when $\theta = G$ all voters want to implement the proposal, while when $\theta = B$, all agree on the status quo. In contrast, the sender is biased towards the proposal and always wants the proposal to be implemented independent of the state of the world. $u_S(a) = a$ depicts the sender's utility function.

Each voter receives a private signal $z_i \in \{b, g\}$ that is correlated with the true state of the world, i.e., $\Pr(z_i = g|\theta = G) = \Pr(z_i = b|\theta = B) = p \in (\frac{1}{2}, 1)$. While a voter i with a good signal $z_i = g$ believes that $\theta = G$ is more likely than $\theta = B$ and thus is more optimistic about the proposal, a voter with a bad signal $z_i = b$ considers it more likely that $\theta = B$ and is thus rather pessimistic about the proposal. For notational convenience we will sometimes use g_i and b_i respectively as a short cut for voter i having received signal $z_i = g$ and $z_i = b$ respectively. We assume that $p + q \geq 1$ and $p \geq q$. This assumption guarantees that if a voter had to make a decision on his own, he would want to follow his own private signal.

Let $Z = \{g, b\}^3$ be the set of signal realizations with typical element $z = (z_1, z_2, z_3) \in Z$ and let $k(z)$ be the number of g-signals in a typical signal realization z. By $z_{-i} \in Z_{-i}$ we denote the signals of all voters except voter i, where Z_{-i} is the set of all signal realizations except voter i's signal. We use k as a shortcut to refer to $k(z)$ and k_{-i} to refer to $k(z_{-i})$. Denote by Z_+ the set of approval states, that is, states in which voters would want to implement the proposal if z was known. Since $\Pr(\theta = G|k) > \frac{1}{2}$ for all $k \in \{2, 3\}$, all states z with $k(z) \in \{2, 3\}$ belong to Z_+. Likewise, denote by Z_- the set of rejection states, that is, states in which voters wouldn't want to implement the proposal. Since $\Pr(\theta = G|k) < \frac{1}{2}$ for all $k \in \{0, 1\}$, all states z with $k(z) \in \{0, 1\}$ belong to Z_-, where $Z_+ = \{z|\ k \geq 2\}$ and $Z_- = \{z|\ k < 2\}$.

4. Restricted Information Design

4.1 Restricted Omniscient Information Designer

We begin our analysis with the benchmark case of an *omniscient* sender. The sender is omniscient in that she is able to observe the true signal realizations of all voters and can condition her recommendations on z. The problem of the omniscient sender is to design a disclosure policy $d : Z \to \Delta(R)$ which maximizes the probability of all voters voting for the proposal, where R is the set of all possible public recommendations. We restrict the sender to anonymous disclosure policies, i.e., disclosure policies that do not discriminate between voters and only condition on $k(z)$ but not on z.

Assumption 3. *The sender's disclosure policy is anonymous, i.e., the probability of sending any recommendation is the same $\forall z, z'$ with $k(z) = k(z')$.*

We impose the following assumption on the voting behavior of indifferent voters.

Assumption 4. *If a voter is indifferent between his actions, he follows the recommendation of the sender.*

Given these two assumptions, we find that it is without loss of generality to restrict the omniscient sender to use only two recommendations $\hat{0}$ and $\hat{1}$. $\hat{0}$ corresponds to the public recommendation that every voter, independent of his private signal, should vote for the reform, while after $\hat{1}$ every voter should vote for the proposal.

Proposition 11. *Under unanimity, it is without loss of generality to restrict the recommendation set of the restricted and omniscient sender to $R = \{\hat{0}, \hat{1}\}$.*

All omitted proofs are in the appendix. In the information design literature, it is a well-known result that if the agent has only two available actions, it is without loss of generality to restrict the recommendation set to two recommendations that correspond to the agent's actions. Note that in our setting we are persuading with one recommendation a group of voters with heterogeneous private information. As the voters hold a private piece of information (signal g or b), one might conjecture that the sender requires strictly more than two signals. After all, there are three possibilities after any recommendation: (i) both types vote for the proposal; (ii) both types vote for the status quo; (iii) only one private information type votes for the proposal, and the other against it. However, it turns out that a third message can never strictly improve the payoff of an omniscient sender under unanimous voting.

Taking everything into account we can define the disclosure policy of the sender in the following way

$$d : \{0, 1, 2, 3\} \to \triangle\{\hat{0}, \hat{1}\}. \tag{2.1}$$

That is, we aim at characterizing a vector with 4 components $\{d[\hat{1}|k]\}_{k\in\{0,1,2,3\}} \in [0, 1]^4$. Upon receiving the sender's recommendation, voters update their beliefs about θ according to Bayes Rule. Voters follow the recommendation of the sender if they receive a weakly higher expected utility from obeying than from disobeying.

Note that under unanimity rule a voter is always pivotal after recommendation $r = \hat{1}$ and never pivotal for $r = \hat{0}$. This is because after $r = \hat{1}$, given that obedience constraints hold, all voters vote for the proposal which is why a single vote determines the outcome. In contrast, after $r = \hat{0}$, everyone vote for the status quo which is why the status quo is implemented irrespective of what one single voter is doing. As a result, a voter always obeys the recommendation $\hat{0}$ and obeys the recommendation $\hat{1}$ if the following obedience constraint holds

$$\begin{aligned} & \Pr(\theta = G|\hat{1}, z_i) - \frac{1}{2} \geq 0 && (OB^{\hat{1}}_{z_i}) \\ \Leftrightarrow \quad & \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|z_i, \hat{1}) \Pr(\theta = G|z_i, k_{-i}) - \frac{1}{2} \geq 0 \quad \forall z_i \in \{g, b\}. \end{aligned}$$

For a voter with $z_i = g$ further rewriting yields[2]

$$\begin{aligned} & \sum_{k_{-i}=0}^{2} d[\hat{1}|k_{-i}, g_i] \cdot \Pr(k_{-i}, g_i) \cdot \left[\Pr(\theta = G|k_{-i}, g_i) - \frac{1}{2}\right] \geq 0 \\ \Leftrightarrow \quad & \sum_{k_{-i}=0}^{2} d[\hat{1}|k_{-i}+1] \cdot \Pr(k_{-i}+1)\frac{k_{-i}+1}{3} \cdot \left[\Pr(\theta = G|k_{-i}+1) - \frac{1}{2}\right] \geq 0 \quad \forall z_i = g \\ & && (OB^{\hat{1}}_{g}) \end{aligned}$$

and for a voter with $z_i = b$

$$\begin{aligned} & \sum_{k_{-i}=0}^{2} d[\hat{1}|k_{-i}, b_i] \cdot \Pr(k_{-i}, b_i) \cdot \left[\Pr(\theta = G|k_{-i}, b_i) - \frac{1}{2}\right] \geq 0 \\ \Leftrightarrow \quad & \sum_{k_{-i}=0}^{2} d[\hat{1}|k_{-i}] \cdot \Pr(k_{-i})\frac{3-k_{-i}}{3} \cdot \left[\Pr(\theta = G|k_{-i}) - \frac{1}{2}\right] \geq 0 \quad \forall z_i = b. && (OB^{\hat{1}}_{b}) \end{aligned}$$

The omniscient sender's problem is then given by

$$\begin{aligned} & \max_{d} \sum_{k=0}^{3} d[\hat{1}|k] \cdot \Pr(k) && (2.2) \\ \text{s.t.} \quad & 0 \leq d[r|k] \leq 1 \quad \forall r \in \{\hat{0}, \hat{1}\} \text{ and } k \in \{0,1,2,3\} && (2.3) \\ & d[\hat{1}|k] + d[\hat{0}|k] = 1 \quad \forall k \in \{0,1,2,3\} && (2.4) \\ & \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|z_i, \hat{1}) \Pr(\theta = G|z_i, k_{-i}) - \frac{1}{2} \geq 0 \quad \forall i,\ z_i \in \{g, b\}. && (OB^{\hat{1}}_{z_i}) \end{aligned}$$

The next Lemma says that it is optimal for the sender to recommend the proposal in all approval states, that is, for all $z \in Z_+$, with probability one.

Lemma 9. *It is optimal for the sender to choose $d[\hat{1}|k] = 1$ for all $k \geq 2$.*

Lemma 9 appears to be very intuitive when one notices that for all $z \in Z_+ = \{z \in Z|\ k \geq 2\}$ voters agree on the proposal being the better choice. Hence, the sender optimally recommends to vote for the proposal in all approval states and voters optimally want to follow this recommendation. Assume that $d[\hat{1}|k] \neq 1$ for some k' with

---

[2] $\Pr(k = k_{-i}+1) = \binom{3}{k_{-i}+1}(q(p^{k_{-i}+1}(1-p)^{3-(k_{-i}+1)}) + (1-q)(p^{3-(k_{-i}+1)}(1-p)^{k_{-i}+1})) = \frac{k_{-i}+1}{3}\Pr(k_{-i}, g_i) = \frac{k_{-i}+1}{3}\binom{2}{k_{-i}}(q(p^{k_{-i}+1}(1-p)^{3-(k_{-i}+1)}) + (1-q)(p^{3-(k_{-i}+1)}(1-p)^{k_{-i}+1}))$

$k' \geq 2$. Then, by increasing $d[\hat{1}|k']$, the voters' obedience constraints become more relaxed due to a higher probability of the proposal being implemented in approval states and the sender's expected utility rises due to the higher probability of the proposal being elected. Note that if the omniscient sender discloses $\hat{1}$ with certainty for all $z \in Z_+$, the obedience constraint of a g-type will always be fulfilled, independent of what the sender is recommending for $k \in \{0, 1\}$.

The next Lemma says that if the obedience constraint of a voter with $z_i = b$ is satisfied, then the obedience constraint of a voter with $z_i = g$ holds as well.

Lemma 10. *If $OB_b^{\hat{1}}$ holds, then $OB_g^{\hat{1}}$ holds as well.*

As a consequence of Lemma 9 and 10 the sender's problem reduces to

$$\max_d \sum_{k=0}^{1} d[\hat{1}|k] \cdot \Pr(k) + \Pr(k=2) + \Pr(k=3) \tag{2.5}$$

$$\text{s.t.} \quad 0 \leq d[r|k] \leq 1 \quad \forall r \in \{\hat{0}, \hat{1}\} \text{ and } k \in \{0,1,2,3\} \tag{2.6}$$

$$d[\hat{1}|k] + d[\hat{0}|k] = 1 \quad \forall k \in \{0,1,2,3\} \tag{2.7}$$

$$\sum_{k_{-i}=0}^{2} \Pr(k_{-i}|b_i, \hat{1}) \Pr(\theta = G|b_i, k_{-i}) - \frac{1}{2} \geq 0 \quad \forall i \tag{$OB_b^{\hat{1}}$}$$

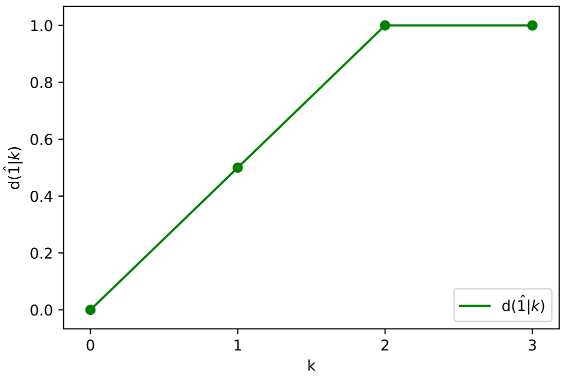
Note that an increase in $d[\hat{1}|k]$ for any $k \in \{0, 1\}$ enters the obedience constraint of a b-type and the objective function of the sender differently. The sender weighs an increase of $d[\hat{1}|k]$ for any $k \in \{0, 1\}$ with $\Pr(k)$, whereas the b-type attaches the weight of $\Pr(k)\frac{3-k}{3}$ to this increase. Due to this different effect on the sender's objective function and on the b-type's obedience constraint, it is crucial how the sender chooses the probability of sending recommendation $\hat{1}$ for $k \in \{0, 1\}$ until $OB_b^{\hat{1}}$ binds. It turns out that the optimal disclosure policy of the omniscient sender is unique and a monotone cutoff policy.

Proposition 12. *The unique optimal disclosure policy of an omniscient sender is a monotone cutoff policy with*

$$d[\hat{1}|k] = \begin{cases} 1 & \text{if } k \in \{2,3\} \\ \in [0,1] & \text{if } k = \tilde{k} \\ 0 & \text{if } k < \tilde{k}, \end{cases}$$

where $\tilde{k} \in \{0, 1\}$ and $d[\hat{1}|\tilde{k}]$ are designed such that $OB_b^{\hat{1}}$ binds.

The optimal disclosure policy of the omniscient sender is monotone in the number of good signals, that is, the higher k, the higher the probability with which she sends the recommendation to vote for the proposal. For states in which the preferences of the sender and the voters are aligned, i.e., for $k \in Z_+$, the omniscient sender recommends the proposal with certainty. For signal realizations with exactly $\tilde{k}$ good signals, the sender mixes between recommending the proposal and the status quo and recommends the status quo with certainty for all signal realizations with strictly less than $\tilde{k}$ good



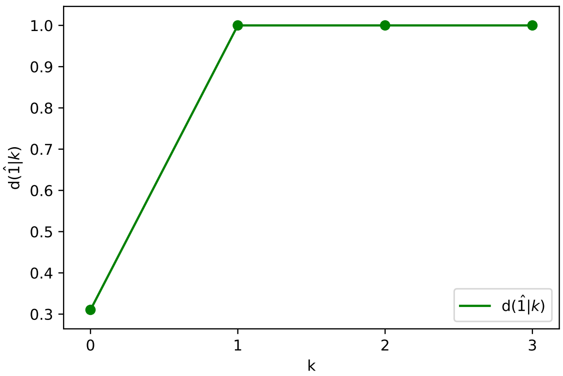


Figure 2.1: policy for $q = 0.5$ and $p = 0.7$. Figure 2.2: policy for $q = 0.8$ and $p = 0.9$.

signals. The cutoff $\tilde{k}$ depends on the specific (p, q) values and is decreasing in p and q. Figure 2.1 illustrates a case for which $\tilde{k} = 1$ and Figure 2.2 for which $\tilde{k} = 0$. When we think of the voters' obedience constraints in terms of budget constraints, it is more expensive to persuade the more pessimistic b-types. Thus, the information designer provides just sufficiently precise information to make these pessimistic b-types indifferent between the proposal and the status quo. In contrast, the g-type always strictly prefers the reform after having received the recommendation to vote for the proposal. This implies that while the b-type's expected utility is zero, the g-type receives a strictly positive expected utility from following the recommendation $\hat{1}$.

4.2 Restricted Eliciting Sender

In the previous section, the sender was omniscient: she knew all signal realizations of the voters without having to elicit them. In this section, a sender does not know the private signal realizations, but can ask voters for a report regarding their information. The restricted eliciting sender can neither construct any experiment she wants about the true state of the world nor observe the agents' signal realizations. All information she acquires has to be elicited in an incentive compatible way. If the designer has to elicit the voters' private signals, we need to impose honesty constraints and take care of double deviations: the agent must not be able to profit from any possible action, not only the obedient one, after a misreport. Each voter i makes a report $\hat{z}_i \in \{\hat{g}, \hat{b}\}$ about his private signal realization to the sender. Denote by $\hat{z} \in \hat{Z} = \{\hat{g}, \hat{b}\}^3$ the profile of all reported signals. Note that the sender only can condition her recommendations on the number of $\hat{g}$-reports in the reported signal realization $\hat{z}$ but not on the number of g-signals in the true signal realizations z. This is why we have to adjust our notation and denote now by $k(\hat{z})$ how many $\hat{g}$-reports have been reported by the voters. Restricting the eliciting sender to use only two recommendations with the same meaning as before is without loss of generality as in the omniscient sender's case.

Proposition 13. *Under unanimity, it is without loss of generality to restrict the recommendation set of the eliciting sender to $R = \{\hat{0}, \hat{1}\}$.*

The eliciting sender designs her disclosure policy $d : \{0,1,2,3\} \to \Delta\{\hat{0},\hat{1}\}$ to maximize the probability of the proposal being implemented. Let $U(z_i, \hat{z}_i, a_i(\hat{0}, z_i), a_i(\hat{1}, z_i))$ denote the expected utility of a voter with signal z_i, who reports to have signal realization $\hat{z}_i$ and who votes with probability $a_i(\hat{0}, z_i)$ for the proposal after recommendation $\hat{0}$ and with probability $a_i(\hat{1}, z_i)$ for the proposal after recommendation $\hat{1}$. We say that the eliciting sender's disclosure policy d is *implementable* if it satisfies the obedience and the honesty constraints. The obedience constraint for $r = \hat{1}$ after reporting truthfully his private signal is given by $OB_g^{\hat{1}}$ for a g-type and by $OB_b^{\hat{1}}$ for a b-type. Note that these constraints correspond to $U(g_i, \hat{g}_i, 0, 1) \geq U(g_i, \hat{g}_i, 0, 0)$ for a g-type and $U(b_i, \hat{b}_i, 0, 1) \geq U(b_i, \hat{b}_i, 0, 0)$ for a b-type when using our new terminology. The honesty constraint of a g-type who is obedient is given by

$$\begin{aligned} U(g_i, \hat{g}_i, 0, 1) = & \sum_{k_{-i}=0}^{2} d[\hat{1}|k_{-i}+1] \cdot \Pr(k = k_{-i}+1) \frac{k_{-i}+1}{3} \cdot \left[\Pr(\theta = G|k_{-i}+1) - \frac{1}{2}\right] \\ \geq & \sum_{k_{-i}=0}^{2} d[\hat{1}|k_{-i}] \cdot \Pr(k = k_{-i}+1) \frac{k_{-i}+1}{3} \cdot \left[\Pr(\theta = G|k_{-i}+1) - \frac{1}{2}\right] \\ = & U(g_i, \hat{b}_i, 0, 1). \end{aligned} \tag{H_g}$$

Likewise, the honesty constraint of a b-type who is obedient is given by

$$\begin{aligned} U(b_i, \hat{b}_i, 0, 1) = & \sum_{k_{-i}=0}^{2} d[\hat{1}|k_{-i}] \cdot \Pr(k = k_{-i}) \frac{3-k_{-i}}{3} \cdot \left[\Pr(\theta = G|k_{-i}) - \frac{1}{2}\right] \\ \geq & \sum_{k_{-i}=0}^{2} d[\hat{1}|k_{-i}+1] \cdot \Pr(k = k_{-i}) \frac{3-k_{-i}}{3} \cdot \left[\Pr(\theta = G|k_{-i}) - \frac{1}{2}\right] \\ = & U(b_i, \hat{g}_i, 0, 1). \end{aligned} \tag{H_b}$$

So far we only considered single deviations, that is, misreporting the private signal and being obedient afterwards. Note that after recommendation $r = \hat{0}$, a voter is never pivotal which is why it does not matter whether he follows or disobeys the recommendation after misreporting. That is, $U(z_i, \hat{z}_i, 1, a_i(\hat{1}, z_i)) = U(z_i, \hat{z}_i, 0, a_i(\hat{1}, z_i))$. If a voter is not obedient after recommendation $\hat{1}$, i.e., $a_i(\hat{1}, z_i) = 0$, then his expected utility is simply $U(z_i, \hat{z}_i, a_i(\hat{0}, z_i), 0) = 0$. Since by imposing the obedience constraint for each type of voter, we make sure that the expected payoff of an obedient voter is nonnegative. Hence, the obedience constraints together with the above honesty constraints, capture all possible profitable deviations.

The eliciting sender's maximization problem is then given by

$$\max_{d} \sum_{k=0}^{3} d[\hat{1}|k] \cdot \Pr(k) \tag{2.8}$$

$$\text{s.t.} \quad 0 \leq d[r|k] \leq 1 \quad \forall r \in \{\hat{0}, \hat{1}\} \text{ and } k \in \{0, 1, 2, 3\} \tag{2.9}$$

$$d[\hat{1}|k] + d[\hat{0}|k] = 1 \quad \forall k \in \{0, 1, 2, 3\} \tag{2.10}$$

$$U(b_i, \hat{b}_i, 0, 1) \geq U(b_i, \hat{b}_i, 0, 0) = 0 \quad \forall\, i \tag{$OB_b^{\hat{1}}$}$$

$$U(g_i, \hat{g}_i, 0, 1) \geq U(g_i, \hat{b}_i, 0, 1) \quad \forall\, i \tag{H_g}$$

$$U(b_i, \hat{b}_i, 0, 1) \geq U(b_i, \hat{g}_i, 0, 1) \quad \forall\, i \tag{H_b}$$

Note that Lemma 10 also holds when the sender is eliciting. We find that the eliciting sender is able to implement the same disclosure policy as the omniscient sender if a certain condition on p and q is met. In results of Bergemann and Morris (2016b) and our companion paper Bobkova and Klein (2019), an omniscient sender does strictly better than an eliciting sender.

Proposition 14. *The eliciting sender is able to achieve the persuasion benchmark of the omniscient sender if and only if $q \leq \frac{2+p}{5}$. For all $q \leq \frac{2+p}{5}$ the restricted sender's optimal disclosure policy is given by*

$$d[\hat{1}|k] = \begin{cases} 1 & \textit{if } k \in \{2, 3\} \\ \frac{p+q-1}{2(p-q)} & \textit{if } k = 1 \\ 0 & \textit{if } k = 0. \end{cases}$$

Note that for all $q \leq \frac{2+p}{5}$ the cutoff $\tilde{k}$ from Proposition 12 is one, that is, there are no (p, q) values which fulfill $q \leq \frac{2+p}{5}$ and $\tilde{k} = 0$. The reason for this is the following: If $\tilde{k} = 0$ in the omniscient sender's optimal policy, then $d[\hat{1}|k = 0] \in (0, 1)$, $d[\hat{1}|k = 1] = 1$ and $d[\hat{1}|k = 2] = 1$. Under this policy, a g-type always has a profitable deviation. Since $d[\hat{1}|k = 0] \in (0, 1)$, a g-type learns after misreporting ($\hat{z}_i = \hat{b}_i$) with a positive probability when there is no other voter with a g-signal and still learns with certainty when there is at least one other voter with a g-signal due to $d[\hat{1}|k = 1] = 1$. Put differently, by misreporting he decreases the probability of receiving the recommendation $\hat{1}$ when he would prefer the status quo ($k = 1$) by $(1 - d[\hat{1}|k = 0])$ and leaves the probability of receiving $\hat{1}$ when he prefers the proposal ($k = 2$) unchanged. Hence, by misreporting he can only gain but never lose in expectation.

In contrast, when $\tilde{k} = 1$ in the omniscient sender's optimal policy, then $d[\hat{1}|k = 0] = 0$, $d[\hat{1}|k = 1] \in (0, 1)$ and $d[\hat{1}|k = 2] = 1$. While a g-type learns from misreporting when no other voter has received a g-signal, he no longer learns with certainty when there is a further voter with a g-signal. In other words, by misreporting he decreases the probability of receiving $\hat{1}$ when he would prefer the status quo ($k = 1$) by $d[\hat{1}|k = 1]$ but at the same time he decreases the probability of receiving $\hat{1}$ when he prefers the proposal ($k = 2$) by $(1 - d[\hat{1}|k = 1])$. As long as this loss outweighs the gain from misreporting, which is the case for all $q \leq \frac{2+p}{5}$, the omniscient sender's optimal disclosure policy is implementable.

The maximum probability with which the sender can recommend the proposal when $k = 1$ is $\frac{3}{4}$ and is reached when $q = \frac{2+p}{5}$.[3] Only in this case the g-type is indifferent between reporting truthfully and misreporting. For all $q < \frac{2+p}{5}$, he strictly prefers to report his private signal truthfully, which means that the honesty constraint of the g-type is slack. When this honesty constraint becomes binding, the g-type starts to gain in expectation from misreporting and it is no longer possible to implement the omniscient sender's optimal disclosure policy.

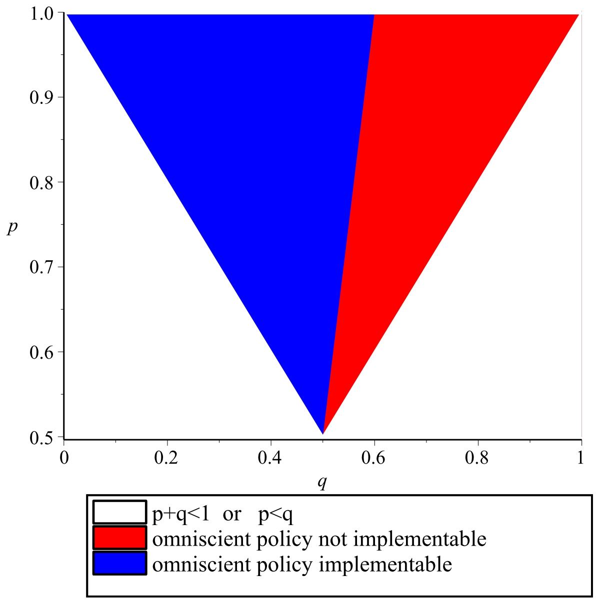


Figure 2.3: implementability

As long as $q \leq \frac{1}{2}$, the eliciting sender is able to implement the optimal omniscient disclosure policy independent of the precision level p of voters' private signals. If $q \in (\frac{1}{2}, \frac{3}{5})$, the implementability of the optimal omniscient disclosure policy depends on p. For all $q > \frac{3}{5}$ and independent of p, the eliciting sender is no longer able to achieve the omniscient benchmark case. This implies that for precision levels and ex-ante probabilities fulfilling $q \leq \frac{2+p}{5}$, neither the omniscient sender has a larger scope for persuasion compared to the eliciting sender, nor voters can profit from their private information with respect to enforcing more informative signals from the eliciting sender.

So why is it that under restricted information design and three voters, the sender is equally well off being omniscient and being eliciting? The difference stems from the preference alignment in the case when $k \in \{2, 3\}$. When $\theta = G$, both the sender and the voters, agree on the best action: implementing the proposal. When $\theta = B$, the sender and the voters disagree. However, when the sender is restricted, she has no access to any precise experiment about θ beyond k. Therefore, whenever $k \in \{2, 3\}$, this is the strongest possible signal that $\theta = G$, and both the sender and the voters agree on implementing the proposal. Having access to an experiment about θ generates a misalignment of preferences, that does not arise under restricted persuasion.

[3] $q = \frac{2+p}{5} \Rightarrow \frac{p+\frac{2+p}{5}-1}{2(p-\frac{2+p}{5})} = \frac{3}{4}$.

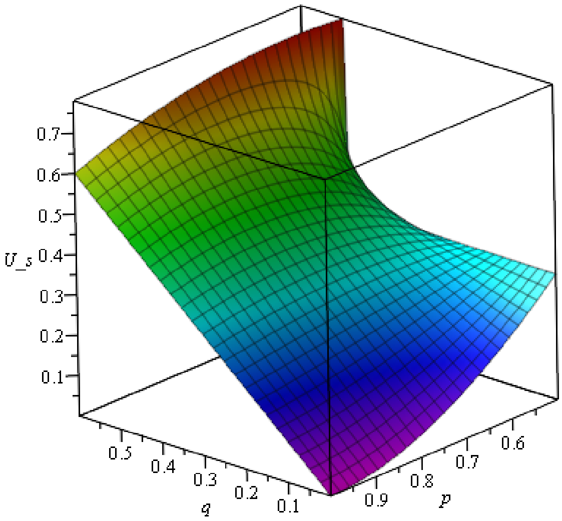


Figure 2.4: $E[U_S]$ for implementable area.

Figure 2.3 illustrates the area of implementability and Figure 2.4 the sender's expected payoff in that area. The white areas in Figure 2.3 consist of all those precision levels and ex-ante probabilities we excluded in the beginning by our two assumptions on q and p. The blue shaded area shows the set of (p, q) values for which the omniscient sender's optimal policy is implementable for the eliciting sender. The red shaded area shows the set of (p, q) values in which the eliciting sender is no longer able to do so. In the following we will further characterize how the optimal disclosure policy of the eliciting sender looks like for (p, q) values within this red area.

Proposition 15. *The optimal disclosure policy of an eliciting sender for* $\frac{2+p}{5} < q < 1$ *and* $p \leq \overline{p}(q)$ *is given by*

$$d[\hat{1}|k] = \begin{cases} 1 & \text{if } k \in \{2,3\} \\ \in (0,1) & \text{if } k \in \{0,1\}, \end{cases}$$

where $d[\hat{1}|k=1] > d[\hat{1}|k=0]$. $\overline{p}(q) > 0.9$ *and is defined in the appendix. Under this disclosure policy* $OB_b^{\hat{1}}$ *and* H_g *bind.*

As in the omniscient benchmark case, the eliciting sender's optimal disclosure policy for $\frac{2+p}{5} < q < 1$ and $p \leq \overline{p}(q)$ is monotone in k and the proposal is recommended with certainty whenever $k \in Z_+$. In contrast to the omniscient benchmark case, the honesty constraint of the g-type is binding, that is , he is indifferent between reporting truthfully and misreporting his private signal. The b-type is still indifferent between voting for the proposal and the status quo after having received the recommendation $\hat{1}$ and strictly prefers to report his private signal truthfully.

Figures 2.5 and 2.6 compare the optimal policies of the omniscient and the eliciting sender. While for $q = 0.7$ and $p = 0.9$ the omniscient sender can persuade voters with probability one to vote for the proposal when $k \geq 1$, the eliciting sender decreases the probability with which she recommends $\hat{1}$ when $k = 1$. More specifically, the eliciting sender shifts probability mass from $d[\hat{1}|k = 1]$ to $d[\hat{1}|k = 0]$. This is necessary since under the optimal omniscient disclosure policy the g-type can always strictly gain by misreporting to have a b-signal: he increases the probability with which he learns that there are no other g-types around and still learns with certainty whenever $k \in \{2, 3\}$. For the above (p, q) values, this expected gain of a g-type from misreporting grows too large and the eliciting sender can no longer implement the optimal omniscient policy.

Thus, the eliciting sender shifts probability mass from $d[\hat{1}|k = 1]$ to $d[\hat{1}|k = 0]$ in order to increase the g-type's expected loss from misreporting: If $d[\hat{1}|k = 1] < 1$, the g-type no longer receives with certainty the recommendation to vote for the reform when $k_{-i} = 1$, which increases his expected loss from misreporting.

Figure 2.7 shows the threshold $\overline{p}(q)$ which is always greater than 0.9 and strictly increasing in q. The next proposition characterizes the sender's optimal disclosure

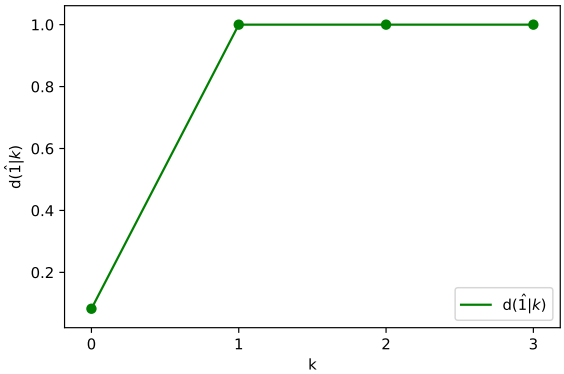


Figure 2.5: $d^{omniscient}$ for $q = 0.7$, $p = 0.9$.

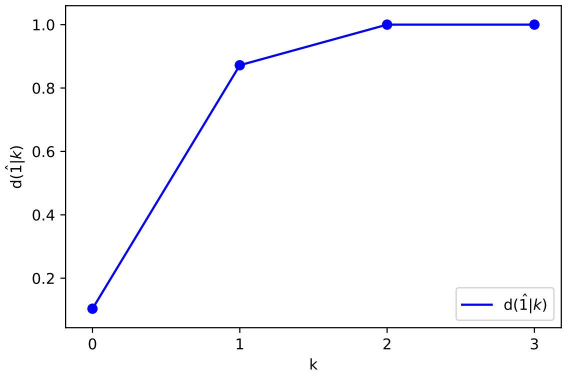


Figure 2.6: $d^{eliciting}$ for $q = 0.7$, $p = 0.9$.

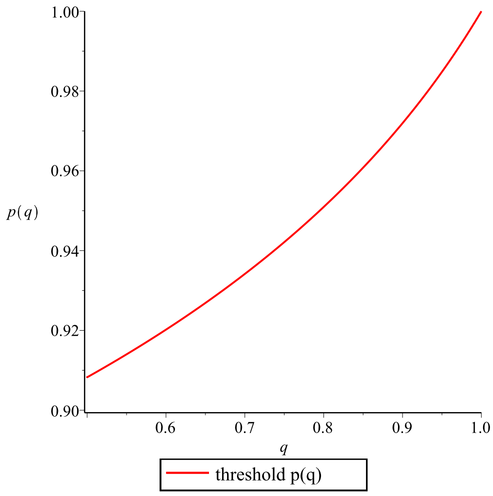


Figure 2.7: threshold $\overline{p}(q)$.

policy when voters' private information is sufficiently precise, that is, when a voter's private signal is almost fully revealing about the true state of the world.

Proposition 16. *The optimal disclosure policy of an eliciting sender for* $\frac{2+p}{5} < q \leq \overline{q}(p)$ *and* $p > \overline{p}(q)$ *is given by*

$$d_1[\hat{1}|k] = \begin{cases} 1 & \text{if } k = 3 \\ \in (0,1) & \text{if } k \in \{1,2\} \\ 0 & \text{if } k = 0, \end{cases}$$

and for $\overline{q}(p) < q < 1$ *and* $p > \overline{p}(q)$ *by*

$$d_2[\hat{1}|k] = \begin{cases} 1 & \text{if } k \in \{1,3\} \\ \in (0,1) & \text{if } k \in \{0,2\}, \end{cases}$$

where $d_2[\hat{1}|k = 2] > d_2[\hat{1}|k = 0]$. $\overline{q}(p)$ *is defined in the appendix. Under both disclosure policies* $OB_b^{\hat{1}}$ *and* H_g *bind.*

In Proposition 16, the following holds

1. if $q \leq \overline{q}(p)$, then $d_1[\hat{1}|k = 1] \leq 1$ and $d_2[\hat{1}|k = 0] < 0$ or
2. if $q > \overline{q}(p)$, then $d_1[\hat{1}|k = 1] > 1$ and $d_2[\hat{1}|k = 0] \geq 0$.

This means that if one of the two above disclosure policies is feasible in the range of $\frac{2+p}{5} < q < 1$ and $p > \overline{p}(q)$, the other one cannot be feasible.

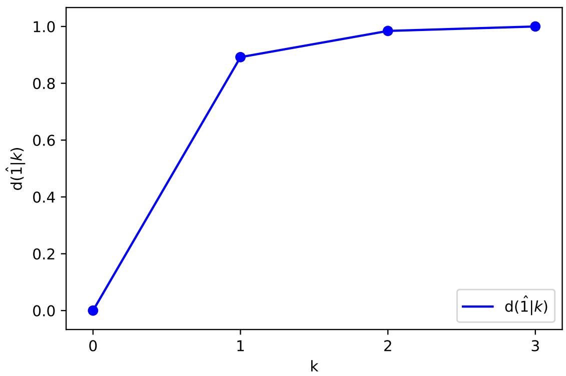


Figure 2.8: $d_1[\hat{1}|k]$ for $q = 0.63$, $p = 0.95$.

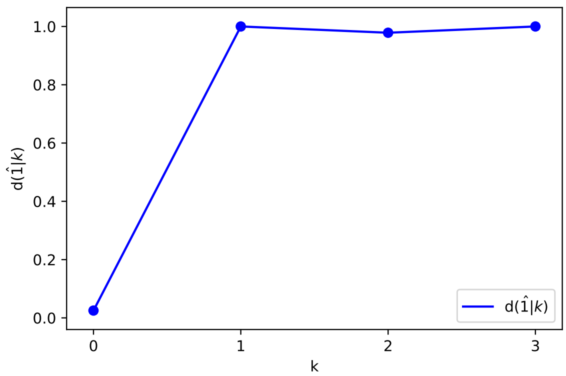


Figure 2.9: $d_2[\hat{1}|k]$ for $q = 0.7$, $p = 0.95$.

In contrast to the previous analysis, the eliciting sender's optimal disclosure policy for $\frac{2+p}{5} < q \leq 1$ and $p > \overline{p}(q)$ isn't always monotone in k (see Figure 2.9). The non-monotonicity occurs at $k = 2$: The sender mixes between recommending the proposal and the status quo when $k = 2$ and recommends with certainty the proposal when $k = 1$. At a first glance this is surprising since preferences of the sender and the voters are aligned when $k = 2$ but not when $k = 1$.

Our explanation for this observation is the following: The sender shifts probability mass from $d[\hat{1}|k=2]$ to $d[\hat{1}|k=1]$ to make misreporting unprofitable for the g-type. Since $\Pr(\theta = G|k=3) > \Pr(\theta = G|k=2)$, shifting probability mass from $d[\hat{1}|k=2]$ to $d[\hat{1}|k=1]$ increasingly hurts a misreporting g-type. He no longer learns with certainty when $k_i = 2$ ($k = 3$) but instead learns with a strictly higher probability when $k_i = 1$ ($k = 2$) from which he benefits strictly less. This way the eliciting sender reduces the g-type's expected gain from misreporting until he's indifferent between misreporting and telling the truth. When $\overline{q}(p) < q < 1$ and $p > \overline{p}(q)$ this even forces the sender to recommend the proposal with certainty for $k = 1$ and with a strictly smaller probability for $k = 2$.

Welfare Analysis

In all of the above cases, the sender's expected payoff is increasing in q and decreasing in p. The more likely $\theta = G$ is ex-ante, the easier it is for the sender to convince voters to vote for the proposal. When the precision of voters' private information increases, voters become more convinced of their private signal indicating the true state of the world. This means that while g-types become even more optimistic towards $\theta = G$, b-types become more pessimistic and thus increasingly difficult to convince. Since the proposal needs unanimity to pass, the sender is worse off with a higher accuracy of voters' private information.

We find that the b-type is always made indifferent between voting for the proposal and the status quo after he has received the recommendation $\hat{1}$, i.e., $\Pr(\theta = G|b_i, \hat{1}) = \frac{1}{2}$. That is, no matter whether the sender is omniscient or eliciting and independent how accurate the voters' private information is, a b-type never receives any information rent. In contrast, the expected payoff of the g-type is always strictly positive and increasing in p and q.

Consider the probabilities of making a wrong decision in the sense of implementing the proposal when $\theta = B$ ($\Pr(a = 1|\theta = B)$) and implementing the status quo when $\theta = G$ ($\Pr(a = 0|\theta = G)$), given the sender's optimal disclosure policy d. If we compare the probabilities of making each type of error in our setting for $q = \frac{1}{2}$ to the corresponding probabilities of making each type of error in Feddersen and Pesendorfer (1998), we find the following: First, the probability of choosing the proposal when $\theta = B$ is greater for all $p \in (\frac{1}{2}, 1)$ in our setting. Second, the probability of choosing the status quo when $\theta = G$ is smaller for all $p \in (\frac{1}{2}, 1)$ in our setting.

Figures 2.10 and 2.11 illustrate the comparison of error probabilities in both settings and the error probabilities of a benevolent sender who sends $\hat{1}$ for all $k \in \{2, 3\}$ and $\hat{0}$ for all $k \in \{0, 1\}$. While in Feddersen and Pesendorfer (1998) the proposal is more often implemented when $\theta = B$, in our setting the proposal is more often implemented when $\theta = G$ than in Feddersen and Pesendorfer (1998). In other words, a manipulative information designer strictly decreases the probability of rejecting the proposal when the proposal is indeed efficient but at the same time strictly increases the probability of rejecting the status quo when the status quo is efficient.[4] This is not surprising since the manipulative sender always wants to push voters towards voting for the proposal,

[4]By efficient we mean efficient from a voter's point of view.

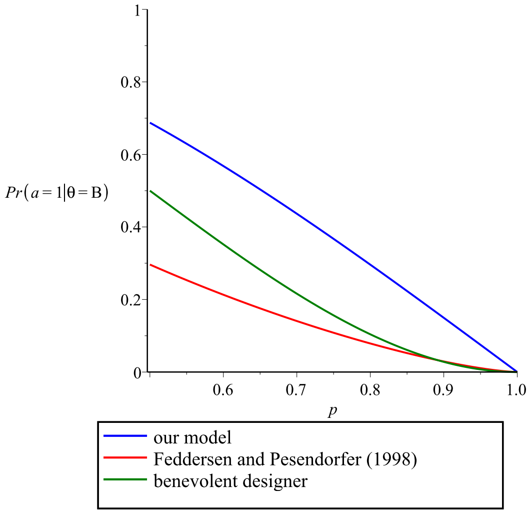


Figure 2.10: $\Pr(a = 1|\theta = B)$ for $q = 0.5$.

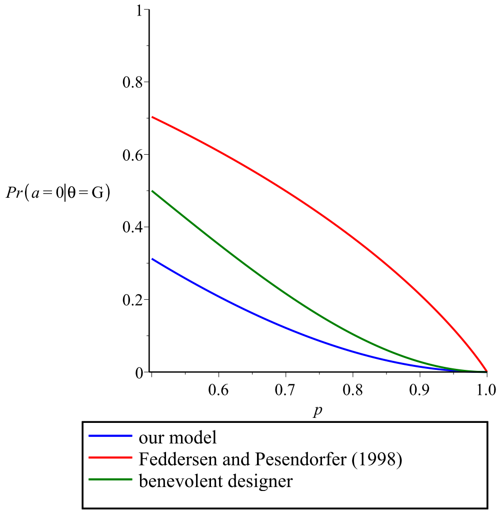


Figure 2.11: $\Pr(a = 0|\theta = G)$ for $q = 0.5$.

irrespective of the state of the world. Hence, when $\theta = B$ we make a strictly greater error than without the manipulative sender, while when $\theta = G$, preferences are aligned and we strictly decrease the probability of implementing the status quo.

When we compare the expected utility of each type of voter in Feddersen and Pesendorfer (1998) and in our model, we find that the g-type is strictly better off while the b-type is slightly worse off. The ex-ante type, that is, a voter who's private signal has not yet realized, receives a strictly higher expected payoff in our model than in Feddersen and Pesendorfer (1998). This means that even with a manipulative sender, voters are ex-ante better off compared to the situation in which they have to decide just on their own under unanimity.

5. Private versus Public Persuasion

5.1 Omniscient Sender

Can the designer do better by using private instead of public recommendations. In this section we analyze whether an omniscient sender has a larger scope for persuasion if she recommends to every voter privately whether he should vote for the proposal or for the status quo given the signal realization. We consider the case in which the sender's private recommendations to voters are correlated. That is, a private disclosure policy d is defined as a function $d : Z \to \triangle\{\hat{0}, \hat{1}\}^3$ that maps signal realizations $z \in Z$ into probability distributions over recommendation profiles $r \in \{\hat{0}, \hat{1}\}^3$. Note that the only private recommendation profile that leads to the approval of the proposal, given that all voters are obedient, is the recommendation profile where the sender recommends every voter to vote for the proposal. All other recommendation profiles in which at least one voter is recommended to vote for the status quo will lead to the rejection of the proposal. Thus, the only two events in which a voter happens to be pivotal is the recommendation profile in which the sender recommends everybody to vote for the proposal, that is, $\hat{1}_i \forall i$

and the recommendation profile in which she recommends everybody the proposal but voter i, that is, $\hat{1}_j \; \forall j \neq i$. We denote the former recommendation profile by r^a and the latter by r^{r_i}. Rewriting $\Pr(\theta = G|\hat{1}_i, piv, z_i)$ yields

$$\Pr(\theta = G|\hat{1}_i, piv, z_i) = \frac{\sum_{z_{-i}} \Pr(r^a|z_i, z_{-i}) \Pr(\theta = G|z_i, z_{-i}) \Pr(z_i, z_{-i})}{\Pr(r^a, z_i)}. \tag{2.11}$$

The only two obedience constraints which arise for each voter under private persuasion are

$$\sum_{z_{-i} \in Z_{-i}} d[r^a|z] \Pr(z_i, z_{-i})(\Pr(\theta = G|z_i, z_{-i}) - \frac{1}{2}) \geq 0 \quad \forall i, \; z_i \in \{g, b\} \tag{$OB_{z_i}^{\hat{1}_i}$}$$

$$\sum_{z_{-i} \in Z_{-i}} d[r^{r_i}|z] \Pr(z_i, z_{-i})(\Pr(\theta = G|z_i, z_{-i}) - \frac{1}{2}) < 0 \quad \forall i, \; z_i \in \{g, b\}. \tag{$OB_{z_i}^{\hat{0}_i}$}$$

The omniscient sender's maximization problem in the private persuasion case is then given by

$$\max_d \sum_{z \in Z} d[r^a|z] \cdot \Pr(z) \tag{2.12}$$

$$\text{s.t.} \quad 0 \leq d[r|z] \leq 1 \quad \forall r \in \{\hat{0}, \hat{1}\}^3 \text{ and } z \in Z \tag{2.13}$$

$$\sum_{r \in \{\hat{0}, \hat{1}\}^3} d[r|z] = 1 \quad \forall z \in Z \tag{2.14}$$

$$\sum_{z_{-i} \in Z_{-i}} d[r^a|z] \Pr(z_{-i}, z_i)(\Pr(\theta = G|z_i, z_{-i}) - \frac{1}{2}) \geq 0 \quad \forall i, \; z_i \in \{g, b\} \tag{$OB_{z_i}^{\hat{1}_i}$}$$

$$\sum_{z_{-i} \in Z_{-i}} d[r^{r_i}|z] \Pr(z_{-i}, z_i)(\Pr(\theta = G|z_i, z_{-i}) - \frac{1}{2}) < 0 \quad \forall i, \; z_i \in \{g, b\}. \tag{$OB_{z_i}^{\hat{0}_i}$}$$

By the same reasoning as in the public persuasion case it is optimal for the sender to send r^a with certainty for all $z \in Z_+$, that is, Lemma 9 goes through and we have $d[r^a|z] = 1 \; \forall z \in Z_+$. As before, if the omniscient sender recommends the proposal to everybody for all $z \in Z_+$, the obedience constraint of a g-type will always be fulfilled, independent of what the sender is recommending for $z \in Z_-$. Likewise, Lemma 10 still applies.[5] A further observation is that $OB_{z_i}^{\hat{0}_i}$ will always be fulfilled given that $d[r^a|z] = 1 \; \forall z \in Z_+$. Since the sender will only send r^{r_i} for a signal realization $z \in Z_-$, voter i realizes that when he receives the private recommendation $\hat{0}_i$ it must be the case that $z \in Z_-$. Thus, voter i wants to follow the recommendation of the sender and votes for the status quo. The problem reduces to

[5]The proof of Lemma 10 is exactly the same as before and thus left out here.

$$\max_{d} \sum_{z \in Z_-} d[r^a|z] \cdot \Pr(z) \tag{2.15}$$

$$\text{s.t.} \quad 0 \leq d[r|z] \leq 1 \quad \forall r \in \{\hat{0}, \hat{1}\}^3 \text{ and } z \in Z_- \tag{2.16}$$

$$\sum_{r \in \{\hat{0}, \hat{1}\}^3} d[r|z] = 1 \quad \forall z \in Z_- \tag{2.17}$$

$$\sum_{b_{-i} \in Z_{-i}} d[r^a|b_i, z_{-i}] \Pr(z_{-i}, b_i)(\Pr(\theta = G|b_i, z_{-i}) - \frac{1}{2}) \geq 0 \quad \forall i \tag{$OB_{b_i}^{\hat{1}_i}$}$$

Note that the private recommendation profile r^a corresponds to the public recommendation $r = \hat{1}$ and the private recommendation profile r^{r_i} can be substituted by the public recommendation $r = \hat{0}$ $\forall i$. This is due to the fact that the private recommendation profile r^{r_i} with $\hat{1}_j$ $\forall j \neq i$ and the private recommendation profile r^r with $\hat{0}_i$ $\forall i$ result in the same outcome, that is, in the rejection of the proposal. Whether the sender uses r^{r_i} or $r^r = (\hat{0}_1, \hat{0}_2, \hat{0}_3)$ does not make any difference, neither for the obedience constraint of any voter nor for the sender's expected payoff. Given that the sender only uses these two perfectly correlated private recommendation profiles r^a and r^r, voter i who observes $\hat{1}_i$ knows that all other voters $j \neq i$ also observed $\hat{1}_j$, and likewise, if he observes $\hat{0}_i$, he knows that all others also observed $\hat{0}_j$. As a consequence, the private maximization problem of the sender becomes just equivalent to the public one which is why the omniscient sender cannot profit from using private persuasion instead of public persuasion.

Proposition 17. *Under unanimity the omniscient designer can achieve the same expected payoff with public as with private persuasion. The optimal private disclosure policy of the omniscient sender is given by*

$$d[r^a|k(z)] = \begin{cases} 1 & \textit{if } k(z) > \tilde{k} \\ \in (0,1) & \textit{if } k(z) = \tilde{k} \\ 0 & \textit{if } k(z) < \tilde{k}, \end{cases}$$

where $\tilde{k}$ is such that the $OB_{b_i}^{1_i}$ binds for all i. This is the same disclosure policy as in the public case.

The omniscient sender cannot profit by discriminating between different voters. In her optimal disclosure policy she only takes the number of g signals into account but not who is having them. Why this is optimal can easily be seen. Suppose $q = \frac{1}{2}$ and the sender would want to make the obedience constraint of voter 1 with signal realization $z_1 = b$ binding. She could choose for example

$$d[r^a|z] = \begin{cases} 1 & \text{if } z \in Z_+ \cup (b_1 b_2 g_3) \\ 0 & \text{if } z \in Z_- \backslash (b_1 b_2 g_3). \end{cases}$$

Under this disclosure policy, the obedience constraint of the b-type voter 1 and the b-type voter 2 after recommendation r^a are binding. However, the obedience constraint of the b-type voter 3 is slack. The omniscient sender could extract more surplus by adjusting her disclosure policy such that the obedience constraint of every b-type binds. The following anonymous disclosure policy does the job

$$d[r^a|z] = \begin{cases} 1 & \text{if } z \in Z_+ \\ \frac{1}{2} & \text{if } z \in Z_- \backslash (b_1 b_2 b_3) \\ 0 & \text{if } z = (b_1 b_2 b_3). \end{cases}$$

This reasoning applies to all other $q \leq p$ in the same way. By treating each voter equally, the omniscient sender can make the obedience constraint of all b-types binding and extract all surplus from pessimistic voters this way. In contrast, by discriminating between voters, she can extract all of the surplus from some b-types, but leaves some surplus on the table for at least one b-type voter.

The equivalence result is driven by the pivotality considerations of voters. If the designer sends her recommendation to each voter privately conditional on the signal realization profile, each voter takes only the events into account in which he is pivotal. In the private persuasion case, the only two events in which a voter happens to be pivotal are the recommendation profiles in which the sender recommends everybody to vote for the proposal, that is, $\hat{1}_i \; \forall i$, and the recommendation profile in which she recommends everybody the proposal but voter i, that is, $\hat{1}_j \; \forall j \neq i$. As pointed out above, the sender's problem then becomes equivalent to the public persuasion case (see also Bardhi and Guo, 2018).[6]

5.2 Eliciting Sender

When the sender is eliciting we have to check whether the above optimal omniscient private policy is still implementable for $q \leq \frac{2+p}{5}$ as in the public persuasion case. It turns out that our line of argumentation from the public persuasion case also goes through in the private persuasion case. Thus, we can apply the same proof as before. Since the sender's expected payoff under the optimal omniscient policy is the highest achievable payoff for the eliciting sender, we don't have to check whether she can do better by using a different private disclosure policy for $q \leq \frac{2+p}{5}$.

The remaining question is whether the eliciting sender can profit from using a private disclosure policy instead of a public one for $q > \frac{2+p}{5}$. As before, when the sender is eliciting, honesty constraints arise for each agent in addition to the obedience constraints. In contrast to the public persuasion case, double deviations can no longer be ruled out easily. Since under private persuasion there are two recommendation profiles after which a voter is pivotal, he has more alternatives for double deviations. Yet, one possible double deviation can be ruled out immediately. If a voter, after having misreported his private signal, always votes for the status quo independent of the recommendation he has received, he receives a utility of zero. By the obedience constraints we ensure that each voter receives at least a utility of zero if he is truthful and obedient.

[6] Note that the same line of argumentation goes through when we consider independent private disclosure policies, that is, mappings from the set of signal realizations Z to the set of probability distributions over the recommendations for each voter: $d : Z \to \triangle\{\hat{0}, \hat{1}\} \; \forall i$.

Hence, misreporting and always voting for the status quo cannot be a profitable double deviation. The following three honesty constraints capture all remaining cases.

Let $c = \Pr(z_i, z_{-i})(\Pr(\theta = G|z_i, z_{-i}) - \frac{1}{2})$. First, being truthful and obedient must yield a voter a higher expected utility than misreporting and being obedient in the events in which he's pivotal

$$\sum_{z_{-i}\in Z_{-i}} d[r^a|z_i, z_{-i}] \cdot c \geq \sum_{z_{-i}\in Z_{-i}} d[r^a|\hat{z}_i, z_{-i}] \cdot c \qquad \forall i,\ z_i \in \{g, b\},\ \hat{z}_i \in \{\hat{g}, \hat{b}\} \tag{H_{z_i}}$$

Second, being truthful and obedient must yield a voter a higher expected utility than misreporting and always voting for the proposal in the events in which he's pivotal

$$\begin{aligned} &\sum_{z_{-i}\in Z_{-i}} d[r^a|z_i, z_{-i}] \cdot c \geq \\ &\sum_{z_{-i}\in Z_{-i}} (d[r^a|\hat{z}_i, z_{-i}] + d[r^{r_i}|\hat{z}_i, z_{-i}]) \cdot c \quad \forall i,\ z_i \in \{g, b\},\ \hat{z}_i \in \{\hat{g}, \hat{b}\} \end{aligned} \tag{$H_{z_i}^{\hat{0}\hat{1}}$}$$

Third, being truthful and obedient must yield a voter a higher expected utility than misreporting and voting for the proposal after $\hat{0}_i$ and for the status quo after $\hat{1}_i$ in the events in which he's pivotal

$$\sum_{z_{-i}\in Z_{-i}} d[r^a|z_i, z_{-i}] \cdot c \geq \sum_{z_{-i}\in Z_{-i}} d[r^{r_i}|\hat{z}_i, z_{-i}] \cdot c \quad \forall i,\ z_i \in \{g, b\},\ \hat{z}_i \in \{\hat{g}, \hat{b}\} \tag{$H_{z_i}^{\hat{0}}$}$$

The eliciting sender's maximization problem under private persuasion is

$$\max_d \sum_{z\in Z} d[r^a|z] \cdot \Pr(z) \quad \text{s.t.} \tag{2.18}$$

$$0 \leq d[r|z] \leq 1 \quad \forall r \in \{\hat{0}, \hat{1}\}^3 \text{ and } z \in Z \tag{2.19}$$

$$\sum_{r\in\{\hat{0},\hat{1}\}^3} d[r|z] = 1 \quad \forall z \in Z \tag{2.20}$$

$$\sum_{z_{-i}\in Z_{-i}} d[r^a|z] \Pr(b_i, z_{-i})(\Pr(\theta = G|b_i, z_{-i}) - \frac{1}{2}) \geq 0 \quad \forall i \tag{$OB_{b_i}^{\hat{1}}$}$$

$$\sum_{z_{-i}\in Z_{-i}} d[r^{r_i}|z] \cdot c \leq 0 \quad \forall i,\ z_i \in \{g, b\} \tag{$OB_{z_i}^{\hat{0}}$}$$

$$\sum_{z_{-i}\in Z_{-i}} (d[r^a|z_i, z_{-i}] - d[r^a|\hat{z}_i, z_{-i}]) \cdot c \geq 0 \quad \forall z_i \in \{g, b\},\ \hat{z}_i \in \{\hat{g}, \hat{b}\} \tag{H_{z_i}}$$

$$\sum_{z_{-i}\in Z_{-i}} (d[r^a|z_i, z_{-i}] - d[r^a|\hat{z}_i, z_{-i}] - d[r^{r_i}|\hat{z}_i, z_{-i}]) \cdot c \geq 0\ \forall z_i \in \{g, b\}, \hat{z}_i \in \{\hat{g}, \hat{b}\} \tag{$H_{z_i}^{\hat{0}\hat{1}}$}$$

$$\sum_{z_{-i}\in Z_{-i}} (d[r^a|z_i, z_{-i}] - d[r^{r_i}|\hat{z}_i, z_{-i}]) \cdot c \geq 0 \quad \forall z_i \in \{g, b\},\ \hat{z}_i \in \{\hat{g}, \hat{b}\} \tag{$H_{z_i}^{\hat{0}}$}$$

Proposition 18. *Under unanimity, the eliciting sender cannot profit from disclosing her recommendation to each voter privately. The optimal private disclosure policy of an eliciting sender is given by*

$$d[r^a|z] = \begin{cases} d[\hat{1}|k=3] & \text{if } z = (ggg) \\ \frac{1}{3} \cdot d[\hat{1}|k=2] & \text{if } z \in \{(ggb),(gbg),(bgg)\} \\ \frac{1}{3} \cdot d[\hat{1}|k=1] & \text{if } z \in \{(bgb),(bbg),(gbb)\} \\ d[\hat{1}|k=0] & \text{if } z = (bbb), \end{cases}, \qquad d[r^r|z] = 1 - d[r^a|z],$$

where $d[\hat{1}|k]$ corresponds to the optimal public disclosure policy of the eliciting sender from section 4.2.

Under the optimal disclosure policy the eliciting sender only uses the two recommendation profiles r^a and r^r, which correspond to our public recommendations $\hat{1}$ and $\hat{0}$ as explained above. Although we allow the eliciting sender to target voters with distinct recommendations, she does not make use of it. Under her optimal disclosure policy she treats each voter symmetrically by only taking the number of g signals into account but not who is having them. As a consequence, the eliciting sender who discloses her recommendation profile in public is just equally well off as an eliciting sender who discloses her recommendations to each voter privately. The intuition behind this result is the same as in the omniscient sender case. By treating each voter symmetrically, the eliciting sender can extract all surplus from the b-types. The possibility to target individual voters under private persuasion does not help the eliciting sender in extending her scope for persuasion. As a consequence, it is without loss of generality for optimality to restrict the sender in the private persuasion case to anonymous disclosure policies.

6. Conclusion

We study a restricted information designer who tries to persuade three voters to vote for a proposal. The information designer is restricted in that she cannot condition her public recommendations on the true quality of the proposal. Voters are privately but imperfectly informed about the quality of the proposal. In contrast to the designer, voters only want the proposal to be implemented if it is of high quality.

We characterize the optimal public disclosure policy under unanimity rule of an omniscient and an eliciting designer. We find that the eliciting designer is able to achieve the persuasion benchmark of the omniscient designer when the prior of the quality being high is sufficiently low. More precisely, whenever the prior is below some threshold which depends on the accuracy of the voters' private information, the eliciting designer is just equally well of as the omniscient designer who can observe the voters' signals. For prior beliefs above this threshold, the eliciting designer is worse off and her optimal disclosure policy changes depending on the accuracy of the voters' private information. While the omniscient designer's optimal disclosure policy is always monotone in the number of optimistic voters, the eliciting designer's optimal disclosure policy is sometimes non-monotone. That is, she recommends the proposal with a higher probability when there are less optimistic voters. This is necessary to provide optimistic voters with sufficient incentives to report their private signal truthfully.

We show that allowing the designer to make her recommendations privately to each voter does not extend the designer's scope for persuasion. The designer does not discriminate between different voters - even though she could. Instead, she uses the same disclosure policy as the designer who makes her recommendations in public and treats voters symmetrically. In this work we consider an information designer who does not know the true quality of the proposal. In Bobkova and Klein (2019) we analyze an unrestricted information designer who is informed about the true quality and can send public recommendations on the basis of the reports made by voters and on the true quality of the proposal.

A. Appendix

A.1 Proof of Proposition 11

Proof. We show that given any outcome of disclosure policy d', the sender can implement an outcome equivalent policy d consisting of only two recommendations $R = \{\hat{0}, \hat{1}\}$. Message $\hat{0}$ is the recommendation to vote with probability 1 for the status quo irrespective of the private signal, and $\hat{1}$ is the recommendation to vote with probability 1 for proposal irrespective of the private signal.

Consider any arbitrary disclosure policy of the sender $d' : \{0, 1, 2, 3\} \to \Delta(R')$, with R' being any arbitrary message set. Denote $r' \in R'$ an element of the message space. Let $a(r', z_i)$ be the probability of a voter i with signal z_i voting for the proposal after seeing recommendation r' under disclosure policy d'.[7]

Now consider a filtering d of the original information disclosure policy d' of the following form.

$$d : R' \times K \to \Delta[\hat{0}, \hat{1}].$$

The new disclosure policy takes the realized message r' in the original disclosure policy and the number of g-signals k of the voters, and maps them into a binary voting recommendation. With slight abuse of notation, denote by $d(\hat{1}|r', k)$ the probability of sending recommendation $\hat{1}$ in favor of the proposal.

Consider the following construction for the new disclosure policy d

$$d(\hat{1}|r', k) = a(r', g)^k a(r', b)^{3-k}. \tag{2.21}$$

It is immediate that this policy yields the same expected utility to the sender (if implementable), as the probability with which she sends signal $\hat{1}$ corresponds to the probability with which her preferred outcome would have been elected under the original disclosure policy d'.

Furthermore, this disclosure policy is designed to be proportional to the original probability of being pivotal for both voter types. In particular, for $k_{-i} \in \{0, 1, 2\}$, we have $\Pr(piv|r, k_{-i}, b) = \Pr(piv|r, k_{-i}, g) = a(r', g)^{k_{-i}} a(r', b)^{2-k_{-i}}$ and hence,

$$d(\hat{1}|r', k = k_{-i}) = a(r', g)^{k_{-i}} a(r', b)^{3-k_{-i}} = a(r', b) \Pr(piv|r', k_{-i})$$

and

$$d(\hat{1}|r', k = k_{-i} + 1) = a(r', g)^{k_{-i}+1} a(r', b)^{3-(k_{-i}+1)} = a(r', g) \Pr(piv|r', k_{-i}).$$

It is left to show that the obedience constraints are also satisfied under the new disclosure policy d'. After public recommendation $\hat{0}$ no voter is ever pivotal. It suffices to show, that both private information types have an incentive to follow the recommendation $\hat{1}$ by voting for the proposal. The obedience constraint of a voter with private

[7]We assume that voters with the same signal react symmetrically to the same recommendation, $a_i(r', z_i) = a_j(r', z_j)$ if $z_i = z_j$. Hence, we drop index i.

signal z_i is:

$$\Pr(\theta = G|\hat{1}, z_i, piv) \geq \frac{1}{2} \quad \forall z_i \in \{g_i, b_i\} \tag{2.22}$$

This can be rewritten into

$$\underbrace{\Pr(piv|\hat{1}, \theta = G, z_i)}_{=1} \Pr(\hat{1}, \theta = G, z_i) \geq \frac{1}{2} \underbrace{\Pr(piv|\hat{1}, z_i)}_{=1} \Pr(\hat{1}, z_i) \tag{2.23}$$

$$\Leftrightarrow \quad \sum_{k_{-i}=0}^{2} \Pr(\hat{1}|\theta = G, z_i, k_{-i}) \Pr(\theta = G, z_i, k_{-i}) \geq \frac{1}{2} \Pr(\hat{1}|z_i, k_{-i}) \Pr(z_i, k_{-i}) \tag{2.24}$$

$$\Leftrightarrow \quad \sum_{k_{-i}=0}^{2} \Pr(\hat{1}|z_i, k_{-i}) \Pr(z_i, k_{-i})(\Pr(\theta = G|z_i, k_{-i}) - \frac{1}{2}) \geq 0 \tag{2.25}$$

Thus, the two obedience constraints are

$$\sum_{k_{-i}=0}^{2} \Pr(\hat{1}|k_{-i}+1) \Pr(k = k_{-i}+1)(\Pr(\theta = G|k_{-i}+1) - \frac{1}{2})\frac{k_{-i}+1}{3} \geq 0 \tag{$OB_g^{\hat{1}}$}$$

$$\sum_{k_{-i}=0}^{2} \Pr(\hat{1}|k_{-i}) \Pr(k = k_{-i})(\Pr(\theta = G|k_{-i}) - \frac{1}{2})\frac{3-k_{-i}}{3} \geq 0 \tag{$OB_b^{\hat{1}}$}$$

We can invoke the construction of the new disclosure policy d from d' to rewrite:

$$\begin{aligned}
\Pr(\hat{1}|k_{-i}, g_i) &= \sum_{r' \in R'} \Pr(\hat{1}, r'|k_{-i}, g_i) &(2.26)\\
&= \sum_{r' \in R'} \Pr(\hat{1}, r'|k_{-i}, g_i) &(2.27)\\
&= \sum_{r' \in R'} \Pr(r'|k_{-i}, g_i) \Pr(\hat{1}|r', k_{-i}, g_i) &(2.28)\\
&= \sum_{r' \in R'} \Pr(r'|k_{-i}, g_i) d(\hat{1}|r', k = k_{-i}+1) &(2.29)
\end{aligned}$$

Analogously, for $z_i = b$ we have

$$\Pr(\hat{1}|b_i) = \sum_{r' \in R'} \Pr(r'|k_{-i}, b_i) d(\hat{1}|r', k = k_{-i}) \tag{2.30}$$

We can use this to rewrite the obedience constraints under the new disclosure policy:

$$\begin{aligned}
&\sum_{r' \in R'} \sum_{k_{-i}=0}^{2} \Pr(r'|k = k_{-i}+1) d(\hat{1}|r', k = k_{-i}+1) \Pr(k = k_{-i}+1) \cdot &(OB_g^{\hat{1}})\\
&\qquad (\Pr(\theta = G|k_{-i}+1) - \frac{1}{2})\frac{k_{-i}+1}{3} \geq 0
\end{aligned}$$

$$\sum_{r' \in R'} \sum_{k_{-i}=0}^{2} \Pr(r'|k = k_{-i}) d(\hat{1}|r', k = k_{-i}) \Pr(k = k_{-i})(\Pr(\theta = G|k_{-i}) - \frac{1}{2})\frac{3 - k_{-i}}{3} \geq 0 \tag{$OB_b^{\hat{1}}$}$$

Using the construction of the new disclosure policy, we can rewrite

$$\sum_{r' \in R'} a(r', g_i) \sum_{k_{-i}=0}^{2} \Pr(piv|r, k_{-i}) \Pr(r'|k = k_{-i} + 1) \Pr(k = k_{-i} + 1) \cdot \tag{$OB_g^{\hat{1}}$}$$
$$(\Pr(\theta = G|k = k_{-i} + 1) - \frac{1}{2})\frac{k_{-i} + 1}{3} \geq 0.$$

Analogously, for $z_i = b$ we have

$$\sum_{r' \in R'} a(r', b_i) \sum_{k_{-i}=0}^{2} \Pr(piv|r, k_{-i}) \Pr(r'|k = k_{-i}) \Pr(k_{-i}) \cdot \tag{$OB_b^{\hat{1}}$}$$
$$(\Pr(\theta = G|k = k_{-i}) - \frac{1}{2})\frac{3 - k_{-i}}{3} \geq 0.$$

Note that the original disclosure policy d' is implementable, i.e. the obedience constraint holds for each type $z_i \in \{g_i, b_i\}$ and each message $r' \in R'$, when $a_i(r', z_i) > 0$. That is,

$$\sum_{k_{-i}=0}^{2} \Pr(piv|r, k_{-i}) \Pr(r'|k = k_{-i} + 1) \Pr(k_{-i} + 1) \cdot \tag{$OB_g^{r'}$}$$
$$(\Pr(\theta = G|k = k_{-i} + 1) - \frac{1}{2})\frac{k_{-i} + 1}{3} \geq 0.$$

$$\sum_{k_{-i}=0}^{2} \Pr(piv|r, k_{-i}) \Pr(r'|k = k_{-i}) \Pr(k = k_{-i}) \cdot \tag{$OB_b^{r'}$}$$
$$(\Pr(\theta = G|k = k_{-i}) - \frac{1}{2})\frac{3 - k_{-i}}{3} \geq 0.$$

The inner sums in the obedience constraints under the new disclosure policy d, $OB_b^{\hat{1}}$ and $OB_g^{\hat{1}}$, correspond to the original obedience constraints, $OB_g^{r'}$ and $OB_b^{r'}$, under the former disclosure policy d'. This establishes that the filtering d satisfies both obedience constraints and yields the same payoff to the designer. $\square$

A.2 Proof of Lemma 9

Proof. We prove this lemma by contradiction. We start by rewriting the voters' obedience constraints for the recommendation $r = \hat{1}$. We rewrite $\Pr(k_{-i}|k(z_i), \hat{1})$ into

$$\Pr(k_{-i}|k(z_i), \hat{1}) = \frac{d[\hat{1}|k(z_i), k_{-i}] \cdot \Pr(k_{-i}|k(z_i)) \cdot \Pr(k(z_i))}{\Pr(\hat{1}, k(z_i))} \tag{2.31}$$

$$= \frac{d[\hat{1}|k(z_i), k_{-i}] \cdot \Pr(k(z_i), k_{-i})}{\sum_{k_{-i}=0}^{2} d[\hat{1}|k(z_i), k_{-i}] \cdot \Pr(k(z_i), k_{-i})} \tag{2.32}$$

Then, we can rewrite a voter's obedience constraint for the recommendation $r = \hat{1}$ in the following way

$$\sum_{k_{-i}=0}^{2} \Pr(k_{-i}|k(z_i), \hat{1}) \Pr[\theta = G|k(z_i), k_{-i}] \geq \frac{1}{2} \tag{2.33}$$

$$\Leftrightarrow \sum_{k_{-i}=0}^{2} d[\hat{1}|k(z_i), k_{-i}] \cdot \Pr(k(z_i), k_{-i}) \cdot \left[\Pr(\theta = G|k(z_i), k_{-i}) - \frac{1}{2}\right] \geq 0. \tag{2.34}$$

Next assume that the disclosure policy d is optimal and that there exists k' with $k' \geq 2$ such that $d[\hat{1}|k'] \neq 1$. Then, construct a new disclosure policy d' that is equal to the old disclosure policy d for all $k \neq k'$ but sends recommendation $\hat{1}$ for k' with probability 1. That is $d'[\hat{1}|k] = d[\hat{1}|k]\ \forall k \neq k'$ and $d'[\hat{1}|k'] = 1 \neq d[\hat{1}|k']$. Next we check whether the new disclosure policy d' still fulfills the voters' obedience constraints. Note that if the recommendation $\hat{0}$ was sent, no voter is ever pivotal, which is why this does not influence the obedience constraints. If recommendation $\hat{1}$ was sent, the voters' obedience constraints are now easier to satisfy since $\left[\Pr(\theta = G|k(z_i), k_{-i}) - \frac{1}{2}\right] \geq 0$ holds for all $k \geq 2$. In other words, we just increased the probability with which a voter receives a positive utility from following the sender's recommendation and as a consequence relaxed the voter's obedience constraint. This is a contradiction. The old disclosure policy d couldn't have been optimal, because we just found a strictly better one which implements the proposal with a strictly higher probability than the old one. This proves that the sender will optimally choose $d[\hat{1}|k] = 1$ for all $k \geq 2$ to maximize the probability of implementing the proposal. □

A.3 Proof of Lemma 10

Proof. Formally we want to show that

$$\sum_{k_{-i}=0}^{2} d[\hat{1}|k_{-i}+1] \cdot \Pr(g_i, k_{-i}) \frac{k_{-i}+1}{3} \cdot \left[\Pr(\theta = G|g_i, k_{-i}) - \frac{1}{2}\right] \tag{2.35}$$

$$\geq \sum_{k_{-i}=0}^{2} d[\hat{1}|k_{-i}] \cdot \Pr(b_i, k_{-i}) \frac{3-k_{-i}}{3} \cdot \left[\Pr(\theta = G|b_i, k_{-i}) - \frac{1}{2}\right]$$

First, note that $\Pr(\theta = G|z_i, k_{-i})$ is increasing in k_{-i}. $\left[\Pr(\theta = G|z_i, k_{-i}) - \frac{1}{2}\right]$ is negative for all $k_{-i} < 1$ and positive for all $k_{-i} > 1$ for all $z_i \in \{b, g\}$. For $k_{-i} = 1$, $\left[\Pr(\theta = G|z_i, k_{-i}) - \frac{1}{2}\right]$ is positive for $z_i = g$ and negative for $z_i = b$. This means that the summands in the obedience constraint of a good-type voter ($OB_g^{\hat{1}}$) are positive for all $k_{-i} \geq 1$ and that the summands in the obedience constraint of a bad-type voter ($OB_b^{\hat{1}}$) are positive for all $k_{-i} \geq 2$. Note that the obedience constraints can equivalently be written as

$$\sum_{k=1}^{3} d[\hat{1}|k] \cdot \Pr(k)\frac{k}{3} \cdot \left[\Pr(\theta = G|k) - \frac{1}{2}\right] \tag{$OB_g^{\hat{1}}$}$$

$$\sum_{k=0}^{2} d[\hat{1}|k] \cdot \Pr(k)\frac{3-k}{3} \cdot \left[\Pr(\theta = G|k) - \frac{1}{2}\right] \tag{$OB_b^{\hat{1}}$}$$

Next, we subtract $OB_b^{\hat{1}}$ from $OB_g^{\hat{1}}$ and split the resulting difference into the following parts:

$$\sum_{k=1}^{2} d[\hat{1}|k] \cdot \Pr(k) \cdot \underbrace{\underbrace{\left[\Pr(\theta = G|k) - \frac{1}{2}\right]}_{<0 \text{ if } k=1; >0 \text{ if } k=2} \cdot \underbrace{\left[\frac{k}{3} - \frac{3-k}{3}\right]}_{\leq 0 \text{ if } k=1; \geq 0 \text{ if } k=2}}_{>0} \geq 0, \tag{2.36}$$

$$+ \quad d[\hat{1}|k=3] \cdot \Pr(k=3) \cdot \underbrace{\left[\Pr(\theta = G|k=3) - \frac{1}{2}\right]}_{>0} \geq 0, \tag{2.37}$$

$$- \quad d[\hat{1}|k=0] \cdot \Pr(k=0) \cdot \underbrace{\left[\Pr(\theta = G|k=0) - \frac{1}{2}\right]}_{<0} \quad \geq 0 \tag{2.38}$$

Adding 2.36, 2.37 and 2.38 up yields that the $OB_g^{\hat{1}} - OB_b^{\hat{1}}$ is positive. This proves the lemma. □

A.4 Proof of Proposition 12

Proof. Sender's problem is a Fractional Knapsack Problem. In the following we will use a greedy algorithm to solve this problem. The obedience constraint for a voter with a bad private signal and the recommendation $r = \hat{1}$ can be written as:

$$\sum_{k=0}^{2} d[\hat{1}|k] \cdot \Pr(k)\frac{3-k}{3} \cdot \left[\frac{1}{2} - \Pr(\theta = G|k)\right] \leq 0 \tag{2.39}$$

We have already proven that it's optimal for the sender to choose $d[\hat{1}|k] = 1$ for $k \in \{2, 3\}$. By using this result, the above obedience can be written as:

$$\sum_{k=0}^{1} d[\hat{1}|k] \cdot \Pr(k)\frac{3-k}{3} \cdot \left[\frac{1}{2} - \Pr(\theta = G|k)\right] \tag{2.40}$$
$$\leq \Pr(k=2)\left[\frac{1}{2} - \Pr(\theta = G|k=2)\right] = \frac{1}{2}p(1-p)(p+q-1)$$

Note that the sender wants to put as much weight on the recommendation $\hat{1}$ as the voters' obedience constraints allow. We showed that the sender will optimally set $d[\hat{1}|k] = 1$ for $k \in \{2, 3\}$. When the sender chooses $d[\hat{1}|k]$ for $k \in \{0, 1\}$, she can increase $d[\hat{1}|k]$ for $k \in \{0, 1\}$ as long as the voters' obedience constraints are fulfilled. We solve the problem for the obedience constraint of a b-type voter, since by Lemma 10 the obedience constraint of g-type voter holds then as well.We have a problem of the following form

Find $0 \leq x_k = d[\hat{1}|k] \leq 1$ for $k \in \{0, 1\}$ s.t.

1) $\sum\limits_{k=0}^{1} x_k w_k \leq \frac{1}{2}p(1-p)(p+q-1)$ holds and

2) $\sum\limits_{k=0}^{1} x_k v_k$ is maximized,

where $w_k = \Pr(k)\frac{3-k}{3} \cdot \left[\frac{1}{2} - \Pr(\theta = G|k)\right]$ and $v_k = \Pr(k) \cdot 1$. We refer to w_k as the weight and to v_k as the value of z_{-i}. Next we calculate the value-per-weight ratio $\rho_k = \frac{v_k}{w_k}$ for $k \in \{0, 1\}$:

$$\rho_1 = \frac{v_1}{w_1} = \frac{\Pr(k=1)}{\Pr(k=1)\frac{2}{3} \cdot \left[\frac{1}{2} - \Pr(\theta = G|k=1)\right]} = \frac{3(p+q-2pq)}{p-q}$$

$$\rho_0 = \frac{v_0}{w_0} = \frac{\Pr(k=0)}{\Pr(k=0) \cdot \left[\frac{1}{2} - \Pr(\theta = G|k=0)\right]} = \frac{2((1-q)p^3 + q(1-p)^3)}{(1-q)p^3 - q(1-p)^3}$$

We sort the k's by decreasing ρ_k. Since $\rho_1 > \rho_0$ for all $\frac{1}{2} < p < 1$ and $\rho_1 = \rho_0$ for $p = 1$, we get the ordering $1, 0$. Next we check whether the maximal allowed weight, i.e., $\frac{1}{2}p(1-p)(p+q-1)$ from the right hand side of the obedience constraint is bigger or smaller than the weight of $k = 1$. If it is smaller, we set $d[\hat{1}|k = 1] = 1$ and perform the same step with $k = 0$. If it is bigger, we set $d[\hat{1}|k = 1] = \frac{p(1-p)(p+q-1)}{2p(1-p)(p+q)} = \frac{p+q-1}{2(p-q)}$ and $d[\hat{1}|k = 0] = 0$. Note that this procedure implies that the obedience constraint of bad-type voter and the recommendation $r = \hat{1}$ is binding for this disclosure policy, i.e., a voter with a bad private signal and the recommendation $\hat{1}$ is indifferent between voting for the reform and voting for the status quo. $\square$

A.5 Proof of Proposition 13

Proof. Consider the same filtering of the original disclosure policy d' into d as in the proof of Proposition 11:

$$d(\hat{1}|r', k) = a(r', g)^k a(r', b)^{3-k}.$$

It remains to be shown that this disclosure policy d satisfies the honesty constraints for each type. The proof of optimality for the designer, and the validity of the obedience constraints were already established in Proposition 11. We prove that the above filtering satisfies the honesty constraints of the g-type. The argument for the b-type is accordingly, and therefore omitted.

Expected utility in equilibrium with d and d'. First, we show that the old disclosure policy d' and new disclosure policy d yields exactly the same expected utility to the g-type in equilibrium, when reporting truthfully. Let R be the message set of the designer with d'.[8]

$$\begin{aligned}
EU(g_i, \hat{g}_i; d') &= \sum_{r \in R} \Pr(r|\hat{g}_i, g_i)(\Pr(\theta = G|r, \hat{g}_i, g_i) - \frac{1}{2}) \Pr(a = 1) && (2.41)\\
&= \sum_{r \in R} (\Pr(\theta = G, r|\hat{g}_i, g_i) - \frac{1}{2} \Pr(r|\hat{g}_i, g_i)) \Pr(a = 1) && (2.42)\\
&= \sum_{r \in R} \sum_{k_{-i}=0}^{2} (\Pr(\theta = G, r, k_{-i}|\hat{g}_i, g_i) - \frac{1}{2} \Pr(r, k_{-i}|\hat{g}_i, g_i)) \cdot && (2.43)\\
&\qquad a(r, g)^{k_{-i}+1} a(r, b)^{2-k_{-i}} \\
&= \sum_{r \in R} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|\theta = G, k_{-i}, \hat{g}_i, g_i) \cdot && (2.44)\\
&\qquad (\Pr(\theta = G|k_{-i}, g_i) - \frac{1}{2}) a(r, g)^{k_{-i}+1} a(r, b)^{2-k_{-i}} \\
&= \sum_{r \in R} \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{g}_i, k_{-i}; d') a_i(r, g)^{k_{-i}+1} a_i(r, b)^{2-k_{-i}} q(g_i, k_{-i}), && (2.45)
\end{aligned}$$

where $q(g_i, k_{-i}) = (\Pr(\theta = G|k_{-i}, g_i) - \frac{1}{2})$ is the expected net utility from implementing the proposal if a g-type voter reported truthfully, k_{-i} others also have a g-signal and the designer sent recommendation r. The factor $a_i(r, g)^{k_{-i}+1} a_i(r, b)^{2-k_{-i}}$ accounts for the probability of being pivotal $(a_i(r, g)^{k_{-i}} a_i(r, b)^{2-k_{-i}})$ times the probability of the g-type voter i voting for the reform $a_i(r, g)$ in equilibrium.

Next, consider the expected utility under the new disclosure policy d. Whenever the designer sends recommendation $\hat{1}$ (which happens with probability $a_i(r, g)^{k_{-i}+1} a_i(r, b)^{2-k_{-i}}$

[8]For convenience of notation, we assume that R is finite.

if recommendation r would have been sent in d') the reform is implemented.

$$\begin{aligned}EU(g_i,\hat{g}_i;d) &= \sum_{r\in R}\sum_{k_{-i}=0}^{2}\Pr(k_{-i}|g_i)\Pr(r|g_i,\hat{g}_i,k_{-i};d')d(\hat{1}|r,k=k_{-i}+1)q(g_i,k_{-i})\\ &= \sum_{r\in R}\sum_{k_{-i}=0}^{2}\Pr(k_{-i}|g_i)\Pr(r|g_i,\hat{g}_i,k_{-i};d')a_i(r,g)^{k_{-i}+1}a_i(r,b)^{2-k_{-i}}q(g_i,k_{-i}).\end{aligned} \tag{2.46}$$

This coincides with the utility under the original disclosure policy in Equation 2.41, $EU(g_i,\hat{g}_i;d)=EU(g_i,\hat{g}_i;d')$.

Expected utility from misreporting in d'. Next, consider the utility of a g-voter who reports $\hat{b}$ in the original disclosure policy d'. To account for double deviations, we denote by $\tilde{a}_i(r,g_i,\hat{b}_i)$ his action after observing r when reporting $\hat{b}$.

$$\begin{aligned}EU(g_i,\hat{b}_i;d') = \sum_{r\in R}\max_{\tilde{a}_i(r,g_i,\hat{b}_i)\in[0,1]}\tilde{a}_i(r,g_i,\hat{b}_i)\cdot & \qquad (2.47)\\ \sum_{k_{-i}=0}^{2}\Pr(k_{-i}|g_i)\Pr(r|g_i,\hat{b}_i,k_{-i};d')a_i(r,g)^{k_{-i}}a_i(r,b)^{2-k_{-i}}q(g_i,k_{-i}).&\end{aligned}$$

For simplicity of notation, denote by $EU(r|g_i,\hat{b}_i)$ the optimal utility after misreporting and observing recommendation r. Note that $EU(r|g_i,\hat{b}_i)\geq 0$ for all r, as a voter can always derive zero utility by voting against the reform. Hence,

$$EU(g_i,\hat{b}_i;d')=\sum_{r\in R}EU(r|g_i,\hat{b}_i). \tag{2.48}$$

Expected utility from misreporting in d. Finally, consider the expected utility after misreporting in the new disclosure policy d. The voter optimizes over his action $\tilde{a}(\hat{1},g_i,\hat{b}_i)$ after recommendation $\hat{1}$ after misreporting. For simplicity, we assume that the voter votes for the reform with probability $\tilde{a}(\hat{1},g_i,\hat{b}_i)$ after both recommendations $\hat{1}$ and $\hat{0}$, as his utility after recommendation $\hat{0}$ yields utility 0 irrespective of his action. The difference to d' is that he might not know which r lead to the recommendation $\hat{1}$.

$$\begin{aligned}EU(g_i,\hat{b}_i;d) = \max_{\tilde{a}(\hat{1},g_i,\hat{b}_i)\in[0,1]}&\tilde{a}(\hat{1},g_i,\hat{b}_i)\cdot\\ &\sum_{r\in R}\sum_{k_{-i}=0}^{2}\Pr(k_{-i}|g_i)\Pr(r|g_i,\hat{b}_i,k_{-i};d')\underbrace{a_i(r,g)^{k_{-i}}a_i(r,b)^{3-k_{-i}}}_{=\Pr(\hat{1}|r,k=k_{-i})}q(g_i,k_{-i}).\end{aligned} \tag{2.49}$$

If the voter knew which r of the original disclosure policy d' led to recommendation $\hat{1}$, he would be better off: he could adapt his voting decision $\tilde{a}(\hat{1},r,g_i,\hat{g}_i)$ to each r

(instead of choosing the same $\tilde{a}(\hat{1}, g_i, \hat{g}_i)$ for all r that led to $\hat{1}$). Thus,

$$EU(g_i, \hat{b}_i; d) \leq \sum_{r\in R} \max_{\tilde{a}(\hat{1},r,g_i,\hat{b}_i)\in[0,1]} \tilde{a}(\hat{1}, r, g_i, \hat{b}_i) \cdot \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{b}_i, k_{-i}; d') \underbrace{a_i(r, g)^{k_{-i}} a_i(r, b)^{3-k_{-i}}}_{=d(\hat{1}|r,k=k_{-i})} q(g_i, k_{-i}) \tag{2.50}$$

$$= \sum_{r\in R} a_i(r, b) \max_{\tilde{a}(\hat{1},r,g_i,\hat{b}_i)\in[0,1]} \tilde{a}(\hat{1}, r, g_i, \hat{b}_i) \cdot \sum_{k_{-i}=0}^{2} \Pr(k_{-i}|g_i) \Pr(r|g_i, \hat{b}_i, k_{-i}; d') a_i(r, g)^{k_{-i}} a_i(r, b)^{2-k_{-i}} q(g_i, k_{-i}), \tag{2.51}$$

where the last inequality follows by putting the non-negative factor $a_i(r, b)$ outside the maximum. But then note that the maximization problem point-wise after each r is exactly the same as in Equation 2.47 for the original disclosure policy, $EU(r|g_i, \hat{b}_i)$, which is non-negative. Hence, the optimal deviation utility is bounded above by

$$EU(g_i, \hat{b}_i; d) \leq \sum_{r\in R} \underbrace{a_i(r, b)}_{\in[0,1]} \underbrace{EU(r|g_i, \hat{b}_i)}_{\geq 0} \tag{2.52}$$

$$\leq \sum_{r\in R} EU(r|g_i, \hat{b}_i) \tag{2.53}$$

$$= EU(g_i, \hat{b}_i; d'). \tag{2.54}$$

The payoff after a misreport and an optimal best response is weakly lower than in the original disclosure policy. Note that the original disclosure policy d' by assumption satisfied all constraints, including the honesty constraint of the g-type. Hence, we established that the honesty constraint of the g-type holds by proving

$$EU(g_i, \hat{g}_i; d) = EU(g_i, \hat{g}_i; d') \geq EU(g_i, \hat{b}_i; d') \geq EU(g_i, \hat{b}_i; d).$$

□

A.6 Proof of Proposition 14

Proof. For $\forall q \leq \frac{2+p}{5}$ the omniscient sender's optimal disclosure policy looks as follows:

$$d[\hat{1}|k] = \begin{cases} 1 & \text{if } k \in \{2, 3\} \\ \frac{p+q-1}{2(p-q)} & \text{if } k = 1 \\ 0 & \text{if } k = 0 \end{cases}$$

Consider first the b-type voter. If misreporting to be a g-type is not a profitable deviation, we must have that

$$\sum_{k=0}^{2}(d[\hat{1}|k]-d[\hat{1}|k+1])\cdot \Pr(k)\frac{3-k}{3}\cdot\left[\Pr(\theta=G|k)-\frac{1}{2}\right]\geq 0. \tag{H_b}$$

Given the above omniscient disclosure policy, this becomes

$$\underbrace{-\frac{p+q-1}{2(p-q)}}_{\leq 0}\cdot \Pr(k=0)\cdot \underbrace{\left[\Pr(\theta=G|k=0)-\frac{1}{2}\right]}_{\leq 0} \tag{2.55}$$

$$+\quad \underbrace{(\frac{p+q-1}{2(p-q)}-1)}_{\leq 0}\cdot \Pr(k=1)\frac{2}{3}\cdot \underbrace{\left[\Pr(\theta=G|k=1)-\frac{1}{2}\right]}_{\leq 0}\geq 0. \tag{2.56}$$

Hence, the b-type's honesty constraint is trivially fulfilled. Consider next the g-type voter. For misreporting not being a profitable deviation for the b-type, we must have that

$$\sum_{k=1}^{3}(d[\hat{1}|k]-d[\hat{1}|k-1])\cdot \Pr(k)\frac{k}{3}\cdot\left[\Pr(\theta=G|k)-\frac{1}{2}\right]\geq 0. \tag{H_g}$$

Given the above omniscient disclosure policy, this becomes

$$(1-\frac{p+q-1}{2(p-q)})\Pr(k=2)\frac{2}{3}\cdot\left[\Pr(\theta=G|k=2)-\frac{1}{2}\right] \tag{2.57}$$

$$+\quad \frac{p+q-1}{2(p-q)}\Pr(k=1)\frac{1}{3}\cdot\left[\Pr(\theta=G|k=1)-\frac{1}{2}\right]\geq 0. \tag{2.58}$$

Further rewriting yields

$$\underbrace{p(1-p)}_{\geq 0}\underbrace{(p+q-1)}_{\geq 0}\left(\frac{p+2-5q}{4(p-q)}\right)\geq 0. \tag{2.59}$$

Hence, the g-type voter has no profitable deviation if

$$\frac{p+2-5q}{4(p-q)}\geq 0 \tag{2.60}$$

$$\Leftrightarrow\quad q\leq\frac{2+p}{5}. \tag{2.61}$$

Since double deviations are neither profitable for the g-type, this proves that the g-type cannot profit from misreporting and being obedient afterwards. □

A.7 Proof of Proposition 15

Proof. The precise optimal disclosure policy of an eliciting sender for $\frac{2+p}{5} < q < 1$ and $p \leq \overline{p}(q)$ is given by

$$d[\hat{1}|k] = \begin{cases} 1 & \text{if } k \in \{2,3\} \\ \frac{(p+q-1)(p^3+p^2(1-5q+(5p-2)q))}{p^4+q(-4p^3-p^2(5q-11)+p(5q-9)-q+2)} & \text{if } k = 1 \\ \frac{(1-p)p(p+q-1)(p^4-2p^3(4q-1)+q(3p^2(5q-1)-5p(3q-1)+5q-2))}{(p^3+(-3p^2+3p-1)q)(q(4p^3+p^2(5q-11)-p(5q-9)+q-2)-p^4)} & \text{if } k = 0, \end{cases}$$

We prove this Proposition by the standard primal-dual-technique. We first formulate the primal and then the dual of the eliciting sender's problem.

$$\max_d \sum_{k=0}^{3} d[\hat{1}|k] \cdot \Pr(k) \tag{2.62}$$

$$\text{s.t.} \quad 0 \leq d[r|k] \leq 1 \quad \forall r \in \{\hat{0}, \hat{1}\} \text{ and } k \in \{0,1,2,3\} \tag{2.63}$$

$$d[\hat{1}|k] + d[\hat{0}|k] = 1 \quad \forall k \in \{0,1,2,3\} \tag{2.64}$$

$$\sum_{k=0}^{2} d[\hat{1}|k] \cdot \Pr(k) \frac{3-k}{3} \cdot \left[\frac{1}{2} - \Pr(\theta = G|k)\right] \leq 0 \quad \forall\, i \tag{$OB_b^{\hat{1}}$}$$

$$\sum_{k=1}^{3} (d[\hat{1}|k-1] - d[\hat{1}|k]) \cdot \Pr(k) \frac{k}{3} \cdot \left[\Pr(\theta = G|k) - \frac{1}{2}\right] \leq 0 \quad \forall\, i \tag{H_g}$$

$$\sum_{k=0}^{2} (d[\hat{1}|k+1] - d[\hat{1}|k]) \cdot \Pr(k) \frac{3-k}{3} \cdot \left[\Pr(\theta = G|k) - \frac{1}{2}\right] \leq 0 \quad \forall\, i \tag{H_b}$$

$$\min_{\lambda_{OB_b^{\hat{1}}} \geq 0,\ \lambda_{H_g} \geq 0,\ \lambda_{H_b} \geq 0,\ \{\lambda_k \geq 0\}_{k \in \{0,1,2,3\}}} \sum_{k=0}^{3} \lambda_k \tag{2.65}$$

$$\begin{aligned} \text{s.t.} \quad & -\Pr(k=0) - (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b}) \frac{1}{2} (q(1-p)^3 - (1-q)p^3 \qquad (2.66) \\ & + \lambda_{H_g} \frac{1}{2} p(1-p)(q-p) + \lambda_{k=0} \geq 0 \\ & -\Pr(k=1) - (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b} + \frac{1}{2}\lambda_{H_g}) p(1-p)(q-p) \qquad (2.67) \\ & + \lambda_{H_g} p(1-p)(p+q-1) + \lambda_{H_b} \frac{1}{2} (q(1-p)^3 - (1-q)p^3) + \lambda_{k=1} \geq 0 \\ & -\Pr(k=2) - (\lambda_{OB_b^{\hat{1}}} + \lambda_{H_b} + 2\lambda_{H_g}) \frac{1}{2} p(1-p)(p+q-1) \qquad (2.68) \\ & + \lambda_{H_g} \frac{1}{2} (qp^3 - (1-q)(1-p)^3) + \lambda_{H_b} p(1-p)(q-p) + \lambda_{k=2} \geq 0 \\ & -\Pr(k=3) - \lambda_{H_g} \frac{1}{2} (qp^3 - (1-q)(1-p)^3) \qquad (2.69) \\ & + \lambda_{H_b} \frac{1}{2} p(1-p)(p+q-1) + \lambda_{k=3} \geq 0. \end{aligned}$$

The complementary slackness conditions are given by

$$d[\hat{1}|k=0]\cdot(-\Pr(k=0)-(\lambda_{OB_b^{\hat{1}}}+\lambda_{H_b})\frac{1}{2}(q(1-p)^3-(1-q)p^3) \tag{2.70}$$
$$+\lambda_{H_g}\frac{1}{2}p(1-p)(q-p)+\lambda_{k=0})=0$$

$$d[\hat{1}|k=1]\cdot(-\Pr(k=1)-(\lambda_{OB_b^{\hat{1}}}+\lambda_{H_b}+\frac{1}{2}\lambda_{H_g})p(1-p)(q-p) \tag{2.71}$$
$$+\lambda_{H_g}p(1-p)(p+q-1)+\lambda_{H_b}\frac{1}{2}(q(1-p)^3-(1-q)p^3)+\lambda_{k=1})=0$$

$$d[\hat{1}|k=2]\cdot(-\Pr(k=2)-(\lambda_{OB_b^{\hat{1}}}+\lambda_{H_b}+2\lambda_{H_g})\frac{1}{2}p(1-p)(p+q-1) \tag{2.72}$$
$$+\lambda_{H_g}\frac{1}{2}(qp^3-(1-q)(1-p)^3)+\lambda_{H_b}p(1-p)(q-p)+\lambda_{k=2})=0$$

$$d[\hat{1}|k=3]\cdot(-\Pr(k=3)-\lambda_{H_g}\frac{1}{2}(qp^3-(1-q)(1-p)^3) \tag{2.73}$$
$$+\lambda_{H_b}\frac{1}{2}p(1-p)(p+q-1)+\lambda_{k=3})=0$$

$$\lambda_{OB_b^{\hat{1}}}\cdot(\sum_{k=0}^{2}d[\hat{1}|k]\cdot\Pr(k)\frac{3-k}{3}\cdot\left[\frac{1}{2}-\Pr(\theta=G|k)\right])=0 \tag{2.74}$$

$$\lambda_{H_b}\cdot(\sum_{k=0}^{2}(d[\hat{1}|k+1]-d[\hat{1}|k])\cdot\Pr(k)\frac{3-k}{3}\cdot\left[\Pr(\theta=G|k)-\frac{1}{2}\right])=0 \tag{2.75}$$

$$\lambda_{H_g}\cdot(\sum_{k=1}^{3}(d[\hat{1}|k-1]-d[\hat{1}|k])\cdot\Pr(k)\frac{k}{3}\cdot\left[\Pr(\theta=G|k)-\frac{1}{2}\right])=0 \tag{2.76}$$

$$\lambda_k\cdot(d[\hat{1}|k]-1)=0\quad\forall k\in\{0,1,2,3\}. \tag{2.77}$$

The disclosure policy from Proposition 15 and the following dual variables fulfill the above complementary slackness conditions for $\frac{1}{2}<p<\overline{p}(q)$:

$$\{\vec{\lambda}\}=\left(\begin{array}{c}
\lambda_{OB_b^{\hat{1}}}=\frac{\lambda_{H_g}p(1-p)(q-p)-2(q(1-p)^3+(1-q)p^3)}{q(1-p)^3-(1-q)p^3}\\
\lambda_{H_b}=0\\
\lambda_{H_g}=\frac{6(q+p-2qp)(q(1-p)^3-(1-q)p^3)-4(q-p)(q(1-p)^3+(1-q)p^3)}{(q(1-p)^3-(1-q)p^3)(3p+q-2)-2p(1-p)(q-p)^2}\\
\lambda_{k=0}=0\\
\lambda_{k=1}=0\\
\lambda_{k=2}=3p(1-p)(1-2qp-p-q)+(\lambda_{OB_b^{\hat{1}}}+2\lambda_{H_g})\frac{1}{2}p(1-p)(p+q-1)\\
-\lambda_{H_g}\frac{1}{2}(qp^3-(1-q)(1-p)^3)\\
\lambda_{k=3}=qp^3+(1-q)(1-p)^3+\lambda_{H_g}\frac{1}{2}(qp^3-(1-q)(1-p)^3)
\end{array}\right) \tag{2.78}$$

and $\overline{p}(q)$ solves $\lambda_{k=2} = 0$. $\square$

A.8 Proof of Proposition 16

Proof. The precise optimal disclosure policy of an eliciting sender for $\frac{2+p}{5} < q \leq \overline{q}(p)$ and $p > \overline{p}(q)$ is given by

$$d[\hat{1}|k] = \begin{cases} 1 & \text{if } k = 3 \\ \frac{(p-q)(qp^3-(1-q)(1-p)^3)}{p(1-p)(1-p-q)(1-2.5q-0.5p)+(p-q)(qp^3-(1-q)(1-p)^3)} & \text{if } k = 2 \\ \frac{(1-p-q)(p-q)(qp^3-(1-q)(1-p)^3)}{2(q-p)(p(1-p)(1-p-q)(1-2.5q-0.5p)+(p-q)(qp^3-(1-q)(1-p)^3))} & \text{if } k = 1 \\ 0 & \text{if } k = 0, \end{cases}$$

and for $\overline{q}(p) < q < 1$ and $p > \overline{p}(q)$ by

$$d[\hat{1}|k] = \begin{cases} 1 & \text{if } k \in \{1, 3\} \\ \frac{2(-p^2(1-p)^2(q-p)^2+(q(1-p)^3-(1-q)p^3)(p(1-p)(1.5p+0.5q-1)-0.5(qp^3-(1-q)(1-p)^3))}{2(q(1-p)^3-(1-q)p^3)(p(1-p)(p+q-1)-0.5(qp^3-(1-q)(1-p)^3))+p^2(1-p)^2(p+q-1)(q-p)} & \text{if } k = 2 \\ \frac{d[\hat{1}|k=2]\cdot p(1-p)(1-p-q)-2p(1-p)(q-p)}{q(1-p)^3-(1-q)p^3} & \text{if } k = 0, \end{cases}$$

The disclosure policy from Proposition 16 and the following dual variables fulfill the above complementary slackness conditions for $\frac{2+p}{5} < q \leq \overline{q}(p)$ and $p > \overline{p}(q)$:

$$\{\vec{\lambda}\} = \begin{pmatrix} \lambda_{OB_b^{\hat{1}}} = \frac{\lambda_{H_g}(\frac{3}{2}p+\frac{1}{2}q-1)-3(q+p-2qp)}{q-p} \\ \lambda_{H_b} = 0 \\ \lambda_{H_g} = \frac{6p(1-p)(q-p)(1-p-q+2qp)-3p(1-p)(p+q-1)(q+p-2qp)}{(qp^3-(1-q)(1-p)^3)(q-p)-p(1-p)(p+q-1)(\frac{5}{2}q-\frac{1}{2}p-1)} \\ \lambda_{k=0} = 0 \\ \lambda_{k=1} = 0 \\ \lambda_{k=2} = 0 \\ \lambda_{k=3} = qp^3 + (1-q)(1-p)^3 + \lambda_{H_g}\frac{1}{2}(qp^3 - (1-q)(1-p)^3) \end{pmatrix} \tag{2.79}$$

where $\overline{p}(q)$ solves $-(q(1-p)^3 + (1-q)p^3) - \lambda_{OB_b^{\hat{1}}}\frac{1}{2}(q(1-p)^3 - (1-q)p^3) + \lambda_{H_g}\frac{1}{2}p(1-p)(q-p) = 0$ and $\overline{q}(p)$ solves $d[\hat{1}|k = 1] = 1$. This $\overline{p}(q)$ is the same as in Proposition 15. The disclosure policy from Proposition 16 and the following dual variables fulfill the above complementary slackness conditions for $\overline{q}(p) < q < 1$ and $p > \overline{p}(q)$:

$$\{\vec{\lambda}\} = \begin{pmatrix} \lambda_{OB_b^{\hat{1}}} = \frac{\lambda_{H_g} p(1-p)(q-p) - 2(q(1-p)^3 + (1-q)p^3)}{q(1-p)^3 - (1-q)p^3} \\ \lambda_{H_b} = 0 \\ \lambda_{H_g} = \frac{6p(1-p)(1+2qp-p-q)(q(1-p)^3-(1-q)p^3) - 2p(1-p)(p+q-1)(q(1-p)^3+(1-q)p^3)}{(q(1-p)^3-(1-q)p^3)(qp^3-(1-q)(1-p)^3 2p(1-p)(p+q-1)) - p^2(1-p)^2(q-p)(p+q-1)} \\ \lambda_{k=0} = 0 \\ \lambda_{k=1} = 3p(1-p)(q+p-2qp) + (\lambda_{OB_b^{\hat{1}}} + \frac{1}{2}\lambda_{H_g})p(1-p)(q-p) \\ -\lambda_{H_g} p(1-p)(p+q-1) \\ \lambda_{k=2} = 0 \\ \lambda_{k=3} = qp^3 + (1-q)(1-p)^3 + \lambda_{H_g}\frac{1}{2}(qp^3 - (1-q)(1-p)^3) \end{pmatrix} \tag{2.80}$$

where $\overline{p}(q)$ solves $\lambda_{k=1} = 0$ and $\overline{q}(p)$ solves $d[\hat{1}|k=0] = 0$. $\overline{p}(q)$ and $\overline{q}(p)$ are identical to the terms we calculated in the first part of the proof of Proposition 16. □

A.9 Proof of Proposition 18

Proof. We prove this Proposition by the standard primal-dual-technique. We first formulate the primal and then the dual of the eliciting sender's problem. Let $c = \Pr(z_i, z_{-i})(\Pr(\theta = G|z_i, z_{-i}) - \frac{1}{2})$.

$$\max_{\substack{\{d[r|z]\geq 0\} \\ z\in Z \\ r\in R}} \sum_{z\in Z} d[r^a|z] \cdot \Pr(z) \tag{2.81}$$

$$\text{s.t.} \quad d[r^a|z] + d[r^{r_1}|z] + d[r^{r_2}|z] + d[r^{r_3}|z] - 1 \leq 0 \quad \forall z \in Z \tag{2.82}$$

$$\sum_{z_{-i}\in Z_{-i}} \Pr(b_i, z_{-i}) d[r^a|z](\frac{1}{2} - \Pr(\theta = G|b_i, z_{-i})) \leq 0 \quad \forall i \tag{$OB_{b_i}^{\hat{1}_i}$}$$

$$\sum_{z_{-i}\in Z_{-i}} d[r^{r_i}|z] \cdot c \leq 0 \quad \forall i,\ z_i \in \{g, b\} \tag{$OB_{z_i}^{\hat{0}_i}$}$$

$$\sum_{z_{-i}\in Z_{-i}} (d[r^a|z_i, z_{-i}] - d[r^a|\hat{z}_i, z_{-i}]) \cdot (-c) \leq 0 \tag{H_{z_i}}$$

$$\sum_{z_{-i}\in Z_{-i}} (d[r^a|z_i, z_{-i}] - d[r^a|\hat{z}_i, z_{-i}] - d[r^{r_i}|\hat{z}_i, z_{-i}]) \cdot (-c) \leq 0 \tag{$H_{z_i}^{\hat{0}\hat{1}}$}$$

$$\sum_{z_{-i}\in Z_{-i}} (d[r^a|z_i, z_{-i}] - d[r^{r_i}|\hat{z}_i, z_{-i}]) \cdot (-c) \leq 0 \quad \forall z_i, \hat{z}_i \in \{g, b\} \tag{$H_{z_i}^{\hat{0}}$}$$

$$\min_{\lambda_{OB^{\hat{1}}_{b_i}}\geq 0,\ \lambda_{H_{z_i}}\geq 0,\ \lambda_{H^{\hat{0}\hat{1}}_{z_i}}\geq 0,\ \lambda_{H^{\hat{0}}_{z_i}}\geq 0,\ \{\lambda_z\geq 0\}_{z\in Z}} \quad \sum_{z\in Z}\lambda_z \tag{2.83}$$

$$\begin{aligned}
\text{s.t.}\quad & -\Pr(ggg)+\lambda_{ggg}+\sum_i(\lambda_{H_{g_i}}+\lambda_{H^{\hat{0}\hat{1}}_{g_i}}+\lambda_{H^{\hat{0}}_{g_i}})\Pr(ggg)(\frac{1}{2}-\Pr(\theta=G|ggg))\\
& -\sum_i(\lambda_{H_{b_i}}+\lambda_{H^{\hat{0}\hat{1}}_{b_i}})\Pr(b_igg)(\frac{1}{2}-\Pr(\theta=G|b_igg))\geq 0 \qquad (k=3)\\[1em]
& -\Pr(b_igg)+\lambda_{b_igg}+(\lambda_{OB^{\hat{1}}_{b_i}}+\sum_{j\neq i}(\lambda_{H_{g_j}}+\lambda_{H^{\hat{0}\hat{1}}_{g_j}}+\lambda_{H^{\hat{0}}_{g_j}}))\Pr(b_igg)(\frac{1}{2}-\Pr(\theta=G|b_igg))\\
& -(\sum_{j\neq i}(\lambda_{H_{b_j}}+\lambda_{H^{\hat{0}\hat{1}}_{b_j}}-\lambda_{H^{\hat{0}}_{b_j}})-\lambda_{H^{\hat{0}}_{b_i}}-\lambda_{H^{\hat{0}}_{g_i}})\Pr(b_igg)(\frac{1}{2}-\Pr(\theta=G|b_igg))\\
& -(\lambda_{H_{g_i}}+\lambda_{H^{\hat{0}\hat{1}}_{g_i}})\Pr(ggg)(\frac{1}{2}-\Pr(\theta=G|ggg))\geq 0 \qquad (z_i=b,k=2)\\[1em]
& -\Pr(g_ibb)+\lambda_{g_ibb}-(\sum_{j\neq i}(\lambda_{H_{g_j}}+\lambda_{H^{\hat{0}\hat{1}}_{g_j}}))\Pr(g_ibb)(\frac{1}{2}-\Pr(\theta=G|g_ibb))\\
& +(\lambda_{H_{g_i}}+\lambda_{H^{\hat{0},\hat{1}}_{g_i}}+\lambda_{H^{\hat{0}}_{g_i}})\Pr(g_ibb)(\frac{1}{2}-\Pr(\theta=G|g_ibb))\\
& +(\sum_{j\neq i}(\lambda_{OB^{\hat{1}}_{b_j}}+\lambda_{H_{b_j}}+\lambda_{H^{\hat{0}\hat{1}}_{b_j}}-\lambda_{H^{\hat{0}}_{b_j}}))\Pr(g_ibb)(\frac{1}{2}-\Pr(\theta=G|g_ibb))\\
& -(\lambda_{H_{b_i}}+\lambda_{H^{\hat{0}\hat{1}}_{b_i}})\Pr(bbb)(\frac{1}{2}-\Pr(\theta=G|bbb))\geq 0 \qquad (z_i=g,k=1)\\[1em]
& -\Pr(bbb)+\lambda_{bbb}+\sum_i(\lambda_{OB^{\hat{1}}_{b_i}}+\lambda_{H_{b_i}}+\lambda_{H^{\hat{0}\hat{1}}_{b_i}})\Pr(bbb)(\frac{1}{2}-\Pr(\theta=G|bbb))\\
& -\sum_i(\lambda_{H_{g_i}}+\lambda_{H^{\hat{0}\hat{1}}_{g_i}})\Pr(g_ibb)(\frac{1}{2}-\Pr(\theta=G|g_ibb))\geq 0 \qquad (k=0)\\[1em]
& \lambda_{ggg}+\lambda_{OB^{\hat{0}}_{g_i}}\Pr(ggg)(\Pr(\theta=G|ggg)-\frac{1}{2})\\
& -(\lambda_{H^{\hat{0}\hat{1}}_{b_i}}+\lambda_{H^{\hat{0}}_{b_i}})\Pr(b_igg)(\frac{1}{2}-\Pr(\theta=G|b_igg))\geq 0 \qquad (r^{r_i},z_i=g,k=3)\\[1em]
& \lambda_{g_igb}+\lambda_{OB^{\hat{0}}_{g_i}}\Pr(g_igb)(\Pr(\theta=G|g_igb)-\frac{1}{2})\\
& -(\lambda_{H^{\hat{0}\hat{1}}_{b_i}}+\lambda_{H^{\hat{0}}_{b_i}})\Pr(b_igb)(\frac{1}{2}-\Pr(\theta=G|b_igb))\geq 0 \qquad (r^{r_i},z_i=g,k=2)\\[1em]
& \lambda_{g_ibb}+\lambda_{OB^{\hat{0}}_{g_i}}\Pr(g_ibb)(\Pr(\theta=G|g_ibb)-\frac{1}{2})\\
& -(\lambda_{H^{\hat{0}\hat{1}}_{b_i}}+\lambda_{H^{\hat{0}}_{b_i}})\Pr(bbb)(\frac{1}{2}-\Pr(\theta=G|bbb))\geq 0 \qquad (r^{r_i},z_i=g,k=1)
\end{aligned}$$

$$\lambda_{bbb} + \lambda_{OB^{\hat{0}}_{b_i}} \Pr(bbb)(\Pr(\theta = G|bbb) - \frac{1}{2})$$
$$- (\lambda_{H^{\hat{0}\hat{1}}_{g_i}} + \lambda_{H^{\hat{0}}_{g_i}}) \Pr(g_i bb)(\frac{1}{2} - \Pr(\theta = G|g_i bb)) \geq 0 \qquad (r^{r_i}, z_i = b, k = 0)$$

$$\lambda_{b_i bg} + \lambda_{OB^{\hat{0}}_{b_i}} \Pr(b_i bg)(\Pr(\theta = G|b_i bg) - \frac{1}{2})$$
$$- (\lambda_{H^{\hat{0}\hat{1}}_{g_i}} + \lambda_{H^{\hat{0}}_{g_i}}) \Pr(g_i bg)(\frac{1}{2} - \Pr(\theta = G|g_i bg)) \geq 0 \qquad (r^{r_i}, z_i = b, k = 1)$$

$$\lambda_{b_i gg} + \lambda_{OB^{\hat{0}}_{b_i}} \Pr(b_i gg)(\Pr(\theta = G|b_i gg) - \frac{1}{2})$$
$$- (\lambda_{H^{\hat{0}\hat{1}}_{g_i}} + \lambda_{H^{\hat{0}}_{g_i}}) \Pr(ggg)(\frac{1}{2} - \Pr(\theta = G|ggg)) \geq 0. \qquad (r^{r_i}, z_i = b, k = 2)$$

Let $c_{z_i, z_{-i}} = \Pr(z_i, z_{-i})(\Pr(\theta = G|z_i, z_{-i}) - \frac{1}{2})$. The complementary slackness conditions are the following:

$$d[r^a|ggg] \cdot (-\Pr(ggg) + \lambda_{ggg} + \sum_i (\lambda_{H_{g_i}} + \lambda_{H^{\hat{0}\hat{1}}_{g_i}} + \lambda_{H^{\hat{0}}_{g_i}}) \cdot (-c_{ggg})$$
$$- \sum_i (\lambda_{H_{b_i}} + \lambda_{H^{\hat{0}\hat{1}}_{b_i}}) \cdot (-c_{b_i gg})) = 0 \qquad (k = 3)$$
$$d[r^a|b_i gg] \cdot (-\Pr(b_i gg) + \lambda_{b_i gg} + (\lambda_{OB^{\hat{1}}_{b_i}} + \sum_{j \neq i} (\lambda_{H_{g_j}} + \lambda_{H^{\hat{0}\hat{1}}_{g_j}} + \lambda_{H^{\hat{0}}_{g_j}})) \cdot (-c_{b_i gg})$$
$$- (\sum_{j \neq i} (\lambda_{H_{b_j}} + \lambda_{H^{\hat{0}\hat{1}}_{b_j}} - \lambda_{H^{\hat{0}}_{b_j}}) - \lambda_{H^{\hat{0}}_{b_i}} - \lambda_{H^{\hat{0}}_{g_i}}) \cdot (-c_{b_i gg})$$
$$- (\lambda_{H_{g_i}} + \lambda_{H^{\hat{0}\hat{1}}_{g_i}}) \cdot (-c_{ggg})) = 0 \qquad (z_i = b, k = 2)$$
$$d[r^a|g_i bb] \cdot (-\Pr(g_i bb) + \lambda_{g_i bb} - (\sum_{j \neq i} (\lambda_{H_{g_j}} + \lambda_{H^{\hat{0}\hat{1}}_{g_j}})) \cdot (-c_{g_i bb})$$
$$+ (\lambda_{H_{g_i}} + \lambda_{H^{\hat{0},\hat{1}}_{g_i}} + \lambda_{H^{\hat{0}}_{g_i}}) \cdot (-c_{g_i bb})$$
$$+ (\sum_{j \neq i} (\lambda_{OB^{\hat{1}}_{b_j}} + \lambda_{H_{b_j}} + \lambda_{H^{\hat{0}\hat{1}}_{b_j}} - \lambda_{H^{\hat{0}}_{b_j}})) \cdot (-c_{g_i bb})$$
$$- (\lambda_{H_{b_i}} + \lambda_{H^{\hat{0}\hat{1}}_{b_i}}) \cdot (-c_{bbb})) = 0 \qquad (z_i = g, k = 1)$$
$$d[r^a|bbb] \cdot (-\Pr(bbb) + \lambda_{bbb} + \sum_i (\lambda_{OB^{\hat{1}}_{b_i}} + \lambda_{H_{b_i}} + \lambda_{H^{\hat{0}\hat{1}}_{b_i}}) \cdot (-c_{bbb})$$
$$- \sum_i (\lambda_{H_{g_i}} + \lambda_{H^{\hat{0}\hat{1}}_{g_i}}) \cdot (-c_{g_i bb})) = 0 \qquad (k = 0)$$
$$d[r^{r_i}|ggg] \cdot (\lambda_{ggg} + \lambda_{OB^{\hat{0}}_{g_i}} \cdot c_{ggg} - (\lambda_{H^{\hat{0}\hat{1}}_{b_i}} + \lambda_{H^{\hat{0}}_{b_i}}) \cdot (-c_{b_i gg})) = 0 \qquad (r^{r_i}, z_i = g, k = 3)$$
$$d[r^{r_i}|g_i gb] \cdot (\lambda_{g_i gb} + \lambda_{OB^{\hat{0}}_{g_i}} \cdot c_{g_i gb} - (\lambda_{H^{\hat{0}\hat{1}}_{b_i}} + \lambda_{H^{\hat{0}}_{b_i}}) \cdot (-c_{b_i gb}) = 0 \qquad (r^{r_i}, z_i = g, k = 2)$$
$$d[r^{r_i}|g_i bb] \cdot (\lambda_{g_i bb} + \lambda_{OB^{\hat{0}}_{g_i}} \cdot c_{g_i bb} - (\lambda_{H^{\hat{0}\hat{1}}_{b_i}} + \lambda_{H^{\hat{0}}_{b_i}}) \cdot (-c_{bbb})) = 0 \qquad (r^{r_i}, z_i = g, k = 1)$$
$$d[r^{r_i}|bbb] \cdot (\lambda_{bbb} + \lambda_{OB^{\hat{0}}_{b_i}} \cdot c_{bbb} - (\lambda_{H^{\hat{0}\hat{1}}_{g_i}} + \lambda_{H^{\hat{0}}_{g_i}}) \cdot (-c_{g_i bb})) = 0 \qquad (r^{r_i}, z_i = b, k = 0)$$
$$d[r^{r_i}|b_i bg] \cdot (\lambda_{b_i bg} + \lambda_{OB^{\hat{0}}_{b_i}} \cdot c_{b_i bg} - (\lambda_{H^{\hat{0}\hat{1}}_{g_i}} + \lambda_{H^{\hat{0}}_{g_i}}) \cdot (-c_{g_i bg})) = 0 \qquad (r^{r_i}, z_i = b, k = 1)$$

$$d[r^{r_i}|b_igg]\cdot(\lambda_{b_igg}+\lambda_{OB^{\hat{0}}_{b_i}}\cdot c_{b_igg}-(\lambda_{H^{\hat{0}\hat{1}}_{g_i}}+\lambda_{H^{\hat{0}}_{g_i}})\cdot(-c_{ggg}))=0 \qquad (r^{r_i},z_i=b,k=2)$$

$$\lambda_z\cdot(d[r^a|z]+d[r^{r_1}|z]+d[r^{r_2}|z]+d[r^{r_3}|z]-1)=0 \quad \forall z\in Z \tag{F}$$

$$\lambda_{OB^{\hat{1}}_{b_i}}\cdot(\sum_{z_{-i}\in Z_{-i}}\Pr(b_i,z_{-i})d[r^a|z](\frac{1}{2}-\Pr(\theta=G|b_i,z_{-i}))=0 \quad \forall i \tag{$OB^{\hat{1}}_{b_i}$}$$

$$\lambda_{OB^{\hat{0}}_{z_i}}\cdot(\sum_{z_{-i}\in Z_{-i}}d[r^{r_i}|z]\cdot c)=0 \quad \forall i,\ z_i\in\{g,b\} \tag{$OB^{\hat{0}}_{z_i}$}$$

$$\lambda_{H_{z_i}}\cdot(\sum_{z_{-i}\in Z_{-i}}(d[r^a|z_i,z_{-i}]-d[r^a|\hat{z}_i,z_{-i}])\cdot(-c))=0 \tag{H_{z_i}}$$

$$\lambda_{H^{\hat{0}\hat{1}}_{z_i}}\cdot(\sum_{z_{-i}\in Z_{-i}}(d[r^a|z_i,z_{-i}]-d[r^a|\hat{z}_i,z_{-i}]-d[r^{r_i}|\hat{z}_i,z_{-i}])\cdot(-c))=0 \tag{$H^{\hat{0}\hat{1}}_{z_i}$}$$

$$\lambda_{H^{\hat{0}}_{z_i}}\cdot(\sum_{z_{-i}\in Z_{-i}}(d[r^a|z_i,z_{-i}]-d[r^{r_i}|\hat{z}_i,z_{-i}])\cdot(-c))=0 \quad \forall z_i,\hat{z}_i\in\{g,b\} \tag{$H^{\hat{0}}_{z_i}$}$$

The modified disclosure policy of the eliciting sender from the public persuasion case and the following modified dual variables from the public persuasion case fulfill the above complementary slackness conditions:

$$\{\vec{\lambda}\}=\begin{pmatrix} \lambda_{OB^{\hat{1}}_{b_i}}=\lambda_{OB^{\hat{1}}_{b}} \ \forall i\in\{1,2,3\} \\ \lambda_{H_{b_i}}=\lambda_{H_b} \ \forall i\in\{1,2,3\} \\ \lambda_{H_{g_i}}=\lambda_{H_g} \ \forall i\in\{1,2,3\} \\ \lambda_{bbb}=\lambda_{k=0} \\ \lambda_{g_ibb}=\frac{1}{3}\lambda_{k=1} \ \forall i\in\{1,2,3\} \\ \lambda_{b_igg}=\frac{1}{3}\lambda_{k=2} \ \forall i\in\{1,2,3\} \\ \lambda_{ggg}=\lambda_{k=3} \\ \lambda_{H^{\hat{0}\hat{1}}_{z_i}}=\lambda_{H^{\hat{0}}_{z_i}}=\lambda_{OB^{\hat{0}}_{z_i}}=0 \quad \forall z_i\in\{b,g\},i\in\{1,2,3\} \end{pmatrix} \tag{2.84}$$

and $\overline{p}(q)$ solves $\lambda_{k=2}=0$. □

Chapter 3

Communication versus Delegation for State-Dependent Biases

1. Introduction

One of the most crucial challenges an organization faces is how decisions should be made when the decision relevant information is asymmetrically distributed across the different organization levels. To address this challenge, we analyze the decision making in an organization when there is an uninformed principal who owns the decision rights and an informed agent. In particular, we investigate whether it is better for the principal to delegate the decision to his agent, who is perfectly informed about the decision relevant state of the world, or to keep all decision rights and only consult the agent.

If the preferences of the principal and the agent coincided, they would always agree on the best possible decision and no problem would arise. However, the agent usually has different objectives and the principal cannot commit on anything except for the delegation of decision rights. In this case, strategic considerations take place and information about the state of the world will never be fully revealed by the agent. Thus, the question whether it is more advantageous for the principal to delegate or to communicate requires an elaborate comparison of these two organizational structures.

Dessein (2002) conducts a comparison of delegation and communication for constant biases of the agent. A constant bias implies that the difference in preferences between the principal and the agent does not change with the state of the world. The vital point in Dessein (2002) is that there exists a trade-off between a loss of control and a loss of information. The loss of control is due to the transfer of authority to the biased agent. As in this case the principal commits to never overrule any decision, the agent acts in his full self-interest without internalizing the principal's preferences. Hence, the principal always experiences a loss due to the discrepancy in objectives. On the other hand, if the principal only consults the agent, she keeps the control but faces a loss of information due to the agent's retention of information. This loss of information is not apparent when decision rights have been delegated because in this case all available information is used by the agent. Dessein (2002) finds that in the context of the uniform-quadratic example of Crawford and Sobel (1982) full delegation always beats communication whenever the latter is informative.

However, differences in preferences might not be constant but can vary. This important aspect is neglected in the analysis of Dessein (2002) and has not been investigated for the classic principal-agent model in existing literature. We seek to address this limitation by modeling the agent's bias as state-dependent and investigate whether Dessein's (2002) result still holds. That is, we analyze whether delegation still outperforms communication when the difference in preferences of the principal and the agent varies with the state of the world. This analysis requires a distinction between two different types of biases, that is, biases for which the agent reacts either more or less sensitive than the principal to changes in the state of the world. Our findings run contradictory to Dessein (2002) as we show that communication can in fact beat delegation if the agent reacts more sensitive than the principal.

Our resulting communication equilibria correspond to partition equilibria, where the agent divides the state space into subintervals and only communicates the subinterval in which his privately observed state lies. Unlike the results of Crawford and Sobel (1982), in our model (i) communication is always informative and (ii) there does not exist an upper bound on the size of the partition equilibrium whenever the agent's degree of sensitivity is higher than the principal's. When the agent reacts relatively less sensitive than the principal to changes in the state, there do no longer exist partition equilibria of every size for every bias of the agent.

We illustrate that when the agent reacts significantly more sensitive to changes in the state than the principal, increasing the equilibrium size predominantly refines the transmitted information around the state of the world for which preferences coincide. The impact on the quality of the transmitted information for states further away from the point of agreement is significantly smaller and almost negligible for large enough biases. However, the limit-equilibrium in which we let the partition size converge to infinity yields the principal and the agent a higher expected payoff than any other partition equilibrium of a smaller size. Thus, it is the best partition equilibrium the principal and the agent can agree upon.

We show that if the agent's bias exceeds a certain threshold, the principal's expected payoff under communication is higher than under delegation. Consequently, Dessein's (2002) result, that for the uniform-quadratic example delegation always beats communication whenever the latter is informative, does not hold for state-dependent biases.

2. Related Literature

This paper builds upon different streams of existing literature.

First and foremost, this paper is related to the work of Crawford and Sobel (1982) (henceforth CS). Their seminal paper 'Strategic Information Transmission' analyzes cheap talk, that is, costless, non-binding and unverifiable communication (Alonso et al., 2008). More specifically, in the context of a sender-receiver game CS analyze how information about a payoff relevant parameter gets transmitted when there is only one informed agent whose preferences are different from those of an uninformed decision maker. They show that all communication equilibria must take the form of partitions and that there exists an upper bound on the equilibrium size that gets determined by the agent's bias. The smaller the difference in preferences, the more elements a partition can have. Additionally, they find that the communicated information gets more precise as the number of partition elements increases, or equivalently, as the bias of the agent decreases. The main technical difference between their and our analysis lies in the nature of the agent's bias. While they assume the agent's bias to be constant, we consider a state-dependent bias and let the difference in preferences of the principal and the agent vary with the state of the world.

Since the seminal paper of CS, many authors have contributed to the cheap talk literature (e.g., Aumann and Hart, 2003; Deimen and Szalay, 2019b; Melumad and Shibano, 1991; Krishna and Morgan, 2004; Stein, 1989). In particular, Stein (1989) studies how the Federal Reserve releases private information about its policy objectives. Due to the Federal Reserve's incentive to manipulate the public's expectation, full revelation of its future policy cannot take place. As in our model, the conflict of interest is not constant but can change. This allows for the existence of infinitely many partition equilibria. Stein (1989) further points out that the amount of information that can be transmitted via cheap talk is nevertheless still bounded as described in CS.

An extension of the classic cheap talk model is done by Melumad and Shibano (1991). They extend the model of CS by introducing, in addition to the unconditional (or constant) bias, a conditional (or state-dependent) preference divergence which allows for preferences to coincide. As they include both, a constant and a state-dependent bias, their model is more general than ours. However, as they concentrate on analyzing finite equilibria, they do not consider the expected payoff in the limit-equilibrium.

Dimitrakas and Sarafidis (2005) modify the classic model of CS by adding uncertainty among the receiver about the sender's bias and analyze how this modification affects the quality of information transmission. They find that there exists an infinity of partition equilibria that converge very fast towards a limit-equilibrium. However, while they only seek to answer the question how uncertainty about the sender's bias influences the quality of communication, we are interested in how a state-dependent bias influences the quality of communication.

Besides these rather specific differences, the common difference between these papers and our work is manifested in the analysis of how the expected payoff under delegation compares to the expected payoff under communication.[1] As such this paper also contributes to the literature on delegation and the comparison of delegation and communication (e.g., Alonso and Matouschek, 2008; Dessein, 2002; Holmström, 1984; Ludema and Olofsgård, 2008). In particular, this paper is related to the work of Dessein (2002). Dessein (2002) also analyzes for the uniform-quadratic example of CS whether delegation or communication is the better choice for the principal if the agent has a constant bias.[2] One of Dessein's (2002) findings is that for the uniform-quadratic example, delegation always beats communication whenever the latter is informative. We also compare communication with delegation, but in contrast to Dessein (2002), we model the agent's bias as state-dependent and allow for preferences to coincide.

Alonso et al. (2008) analyze decision making in an organization where an uninformed

[1]Only Li et al. (2016) briefly discuss that for their model communication and delegation are outcome equivalent, which significantly differs from our results.

[2]Harris and Raviv (2005) use an analogous model, but in contrast to Dessein (2002), they assume that both, the principal and the agent, have some private information.

headquarter manager can either consult his two perfectly informed, but biased, division managers and make the decision for both divisions on his own (centralization) or fully delegate the decisions to them (decentralization). Since in Alonso et al.'s (2008) model the decisions of the two divisions need to be coordinated, the two division managers also need to communicate with each other under decentralization. Division managers are biased in that they care more about maximizing their own division's than the over-all organization's payoff. Analogous to our model, the incentive to misrepresent private information varies with the state of the world and a full alignment of preferences is possible in exactly one state of the world. Alonso et al. (2008) find that the communication equilbria are equivalent to partitions which can possibly have infinitely many elements. They show that vertical communication is in general more efficient than horizontal communication.[3] However, they find that whether centralization or decentralization performs better depends on the coordination parameter. In other words, if the need for coordination between the two divisions increases, decentralization can perform better than centralization. While in Alonso et al. (2008) there exists a trade-off between coordination and adaption when decisions have to be made, in our model there is no need for coordination.

In addition to the above paper, Alonso and Matouschek (2008) analyze how the optimal delegation scheme should be designed by the principal when the agent is biased.[4] Since they allow for any continuous difference of preferences, the state-dependent preference divergence as analyzed in this paper is included in their analysis. However, in contrast to us, they concentrate on the mere delegation problem. More specifically, they investigate how much discretion should be given to the agent without considering the option of engaging in cheap talk communication with the agent.

A further comparison of communication and delegation for constant biases is done by Ottaviani (2000). He introduces a varying degree of strategic sophistication for the receiver. If the receiver is fully sophisticated, cheap talk takes place and the resulting equilibria correspond to the partition equilibria from CS. If, however, the receiver is totally unsophisticated, communication takes the form of delegation because the receiver simply implements everything the sender advises him to do. Since the main goal of Ottaviani (2000) is to show how the degree of sophistication influences the sender's strategies and the resulting equilibria, our work differs significantly from his.

A recent paper that is related to ours is Deimen and Szalay (2019a). They study a decision maker that is reliant on the information acquisition of one expert. The conflict of interest between the decision maker and the expert is state-dependent and endogenously influenced by the information the expert acquires. They compare two settings: one in which the decision maker delegates the decision to the expert and one in which she keeps the decision rights and only communicates with the expert. While in Deimen and Szalay (2019a) the analysis of how the delegation of authority influences the information acquisition of the expert is crucial, in our paper the agent already has

[3]Note that vertical and horizontal communication correspond to centralization and decentralization, respectively.

[4]Their paper builds strongly on the work of Holmström (1984) who was the first to examine this delegation problem. In contrast to Holmström (1984), Alonso and Matouschek (2008) do not impose any restrictions on the feasible delegation sets. As a consequence, they are able to derive more general characterization results for the delegation problem.

all relevant information and his bias is known by the principal.

3. Model

An organization consists of an uninformed principal (she) and a perfectly informed agent (he). The principal has to decide which project the organization should undertake. There is an infinite number of possible projects available and all of them differ only in one dimension. Suppose that each project can be represented by a real number $y \in [-1, 1]$. Furthermore, the project decision needs to be adapted to the current state of the world θ, a random variable which is uniformly distributed on $[-1, 1]$. While the principal cannot observe the decision relevant state and thus only holds a prior belief about its realization, the agent is perfectly informed about θ. Except for the state of the world, everything is common knowledge. The principal's and the agent's utility functions are given by

$$U_P(y, \theta) = -(y - \theta)^2 \tag{3.1}$$

$$U_A(y, \theta, b) = -(y - b\theta)^2. \tag{3.2}$$

These utility functions have a unique maximizer for each θ, denoted by $y_P^*(\theta) = \theta$ and $y_A^*(\theta, b) = b\theta$. We refer to them as the principal's and the agent's preferred project decisions. The scalar $b \in (0, 1) \cup (1, \infty)$ is a parameter that measures how sensitive the agent reacts to changes in the state of the world. We call b the agent's state-dependent bias. The principal's sensitivity is normalized to one. We exclude $b = 1$ in our analysis since in this case preferences perfectly coincide and both the principal and the agent agree on the best possible project choice for every state.

At $\theta = 0$ a reversal of preferences takes place: If $b > 1$, the agent prefers smaller projects relative to the principal for $\theta \in [-1, 0)$ and larger projects than the principal for $\theta \in (0, 1]$. To put this in formal terms, if $b > 1$, then $y_A^*(\theta, b) < y_P^*(\theta)$ for $\theta \in [-1, 0)$ and $y_A^*(\theta, b) > y_P^*(\theta)$ for $\theta \in (0, 1]$. For $b \in (0, 1)$ the statement inverts and it holds that $y_A^*(\theta, b) > y_P^*(\theta)$ for $\theta \in [-1, 0)$ and $y_A^*(\theta, b) < y_P^*(\theta)$ for $\theta \in (0, 1]$. At $\theta = 0$, preferences coincide, that is, $y_A^*(0, b) = y_P^*(0) = 0$ (Melumad and Shibano, 1991).

This latter aspect is crucial for our analysis and distinguishes our work from that of CS as they do not allow for preferences to coincide. In the model of CS the agent's utility function is given by $U_A(y, \theta, b) = -(y - (\theta + b))^2$, where $b > 0$, and the principal's utility function is identical to ours. The preferred choices of the agent and the principal are $y_A^*(\theta, b) = \theta + b$ and $y_P^*(\theta) = \theta$, respectively, and the difference in optimal actions (i.e., $y_A^*(\theta, b) - y_P^*(\theta)$) is constantly b. Hence, the agent always prefers larger project decisions relative to the principal independent of the state of the world. Instead, in our model the difference in optimal actions is $(b - 1)\theta$. This implies that the conflict of interest is smaller, the smaller $|\theta|$ is and vanishes completely at $\theta = 0$.

We assume that contracts are incomplete and consider a setting with no transfers. These assumptions imply that the principal can neither commit to a decision rule that depends on the transmitted information nor commit to pay transfers to the agent. However, we assume that the principal and the agent can contract upon the allocation of the ex-ante decision rights. In other words, the principal might transfer all authority to the agent in the beginning. In this case the agent is free to choose his preferred project and does not need to obtain a final approval by the principal. As a result, under full delegation the agent will make the decision in his best self-interest and will not take the principal's preferences into account when choosing a project. As the principal has irrevocably lost the control over the decision, she cannot overrule the decisions made by the agent and thus will always experience a loss due to the agent's bias (except for $\theta = 0$).

Instead, if the principal keeps the authority in the beginning, she has the option of consulting the agent. In this case the agent gives the principal an advice what to do, which means that the agent sends the principal a costless, unverifiable message with information about the state of the world. Upon receiving the agent's message the principal needs to extract the embedded information in order to subsequently choose a project based on her updated beliefs about θ. Since the principal cannot commit to a decision rule conditional on the received message, the agent correctly anticipates that the principal will adjust the advice for his bias. Thus, communication takes the form of cheap talk, which is why the agent will never fully reveal his private information. Hence, relevant information gets lost and cannot be used by the principal when she makes the decision.

From this line of reasoning, it becomes clear that there exists a trade-off between a loss of control and a loss of information (Dessein, 2002). Should the principal fully delegate the decision rights in order to make use of all the available information, while losing the control over the project choice? Or should she just consult the agent and make the decision on her own, while facing a loss of information? In order to answer this question, we calculate the principal's expected payoff under both organizational structures and compare them afterwards.

4. Communication versus Delegation

4.1 Delegation

After having introduced the model, in this section we calculate the principal's expected payoff when she decides to delegate full authority to the agent ex-ante. Since we assume that the principal can only delegate all decisions rights, the agent is free to choose every available project. After the decision has been made, the principal cannot overrule the agent's choice anymore. Therefore, the agent will choose the decision that maximizes his payoff given θ, that is, $y_A^*(\theta) = b\theta$. Given the agent's strategy, the expected payoff of the principal under full delegation is

$$E[U_P^D] = -\int_{-1}^{1} (b\theta - \theta)^2 \, \frac{1}{2} \, d\theta = -\frac{1}{2}(b-1)^2 \int_{-1}^{1} \theta^2 \, d\theta \; = -\frac{1}{3}(b-1)^2. \tag{3.3}$$

While the loss the principal expects under delegation is very small for biases close to one, the principal's expected loss increases very fast as the agent's bias increases. This is not surprising as the difference in preferred project choices of the principal and the agent, i.e., $(b-1)\theta$, is smaller the closer b is to one. There exists a threshold $\bar{b}$ such that for all $b > \bar{b}$ the principal never delegates the decision rights to the agent in the beginning. This is due to the fact that for biases greater than this threshold, delegation always performs worse than making a totally uninformed decision. The principal can do better than delegating the decision rights by simply implementing her prior of θ, i.e., $y_P = E[\theta] = 0$. Her expected payoff in this case is

$$E[U_P] = -\int_{-1}^{1} (0-\theta)^2 \, \frac{1}{2} \, d\theta = -\frac{1}{3}. \tag{3.4}$$

Hence, if $-\frac{1}{3}(b-1)^2 < -\frac{1}{3}$ or, respectively, if $b > 2 = \bar{b}$, the principal will never transfer full authority to the agent as in this case making an uninformed decision always yields her a higher expected payoff. Note that this result implies that also communication performs better than delegation for all $b > \bar{b}$. Even if the agent's message is uninformative and no information at all is transmitted, the principal's beliefs stay unchanged and she would just implement her prior of θ. We derive the following proposition (see also Dessein, 2002).

Proposition 19. *If θ is uniformly distributed on $[-1, 1]$ and $b > 0$, then there exists a threshold $\bar{b}$ such that the principal will never delegate all decision rights to the agent if $b > \bar{b} = 2$.*

4.2 Communication for $b > 1$

The advantage of communication compared to delegation is that the principal keeps the authority and is not forced to implement the biased project choice of the agent. Instead, after having received the message of the agent, the principal can choose the project that maximizes her expected payoff given her updated beliefs about the state of the world. In this sender-receiver game, the agent, as the sender, first privately observes the state of the world which is relevant for both. After having observed the state he sends a possibly noisy signal to the principal, the receiver, who then makes a decision based on the received information. This decision determines the welfare of both. Similar to Alonso et al. (2008), we use the concept of a Perfect Bayesian Nash Equilibrium. This concept requires that the agent's message maximizes his expected payoff given the principal's decision rule, and that the principal's project choice maximizes her expected payoff given the agent's signaling rule and her belief function. The next definition puts these characteristics in formal terms.

Definition 1. *An equilibrium consists of a family of signaling rules $q(\cdot)$ for the agent, where for each $\theta \in [-1, 1]$, $q(m|\theta)$ is the conditional probability of sending a message m given the state θ, a belief function $g(\cdot)$ for the principal, where $g(\theta|m)$ denotes the probability of θ given message m, and a decision rule $y(\cdot)$ for the principal, where $y(m)$ is a mapping from the set of feasible signals M to the set of actions $[-1, 1]$, such that:*

(i) for each $\theta \in [-1, 1]$, if $q(m|\theta) > 0$, then m maximizes the expected utility of the agent, given the principal's decision rule $y(\cdot)$, and

(ii) for each m, $y(m)$ maximizes the expected utility of the principal given her belief function.

(iii) the belief function $g(\theta|m)$ is derived from $q(m|\theta)$ using Bayes Rule whenever possible.

As shown by CS for constant biases and by Melumad and Shibano (1991) for state-dependent biases, the resulting equilibria of the communication game are partition equilibria. This means that each equilibrium is characterized by a partition $a^N = (a_0, a_1, \ldots, a_N)$ of the state space, where $[-1, 1]$ is divided into $N \in \mathbb{N}$ intervals. To be qualified as an equilibrium partition, the dividing thresholds must be monotonically ordered and have to lie in between the limits of the state space, that is, $-1 = a_0 < a_1 < \cdots < a_N = 1$. We call these two conditions the monotonicity and the boundary condition. In such a partition equilibrium, the agent introduces noise into his signal by solely revealing in which interval the observed state lies. That is, he reports the same signal for all states that belong to the same partition element. Consequently, relevant information gets lost. After having received the agent's message, the principal updates her prior belief about θ by using the information transmitted by the agent. That is, the principal forms her expectation about θ conditional on knowing to which interval θ belongs. Given her updated belief about θ, the principal subsequently chooses the project that maximizes her expected payoff. The following proposition summarizes the characteristics of partition equilibria for our setting.[5]

Define for all $\underline{a}, \overline{a} \in [-1, 1]$, $\underline{a} < \overline{a}$,

$$\overline{y}(\underline{a}, \overline{a}) \equiv \operatorname*{argmax}_{y} \int_{\underline{a}}^{\overline{a}} U_P(y, \theta)\frac{1}{2}d\theta = \frac{\underline{a} + \overline{a}}{2}.$$

Proposition 20. *If $b > 1$, then there exists at least one equilibrium $(y(\cdot), q(\cdot), g(\cdot))$ for every $N \in \mathbb{N}$, where*

(i) $q(m|\theta)$ is uniform, supported on $[a_i, a_{i+1}]$ if $\theta \in (a_i, a_{i+1})$

(ii) $g(\theta|m)$ is uniform, supported on $[a_i, a_{i+1}]$ if $m \in (a_i, a_{i+1})$

(iii) $y(m) = \overline{y}(a_i, a_{i+1})$ for all $m \in (a_i, a_{i+1})$

(iv) $a_{i+1} - a_i = a_i - a_{i-1} + 4(b-1)a_i$ for $i = 1, ..., N-1$

(v) $a_0 = -1$ and $a_N = 1$.

Given a partition of size N, an arbitrary threshold a_i is defined by

$$a_i = \frac{x^i(1 + y^N) - y^i(1 + x^N)}{x^N - y^N} \quad \text{for } 0 \leq i \leq N, \tag{3.5}$$

[5] Proposition 20 is a modified version of Theorem 1 from CS and Proposition 1 from Alonso et al. (2008).

where $x = \sqrt{(1-2b)^2 - 1} + 2b - 1$ and $y = -\sqrt{(1-2b)^2 - 1} + 2b - 1$. These thresholds fulfill the monotonicity and the boundary conditions for every partition size N if $b > 1$.

The difference equation from part (iv) of Proposition 20 determines the length of each interval in our equilibrium. In particular, the length of the interval $[a_i, a_{i+1}]$ equals the length of the preceding interval $[a_{i-1}, a_i]$ plus $4(b-1)a_i$. In contrast, the difference equation in CS is given by $a_{i+1} - a_i = a_i - a_{i-1} + 4b$ for which successive intervals grow by a constant term $4b$. In our model, the rate at which the intervals increase depends in addition to the agent's bias also on the threshold a_i. Since $4(b-1) > 0$ for all $b > 1$, the length of successive intervals decreases if $a_i < 0$, and increases if $a_i > 0$. This implies that intervals are small around zero and increase towards the lower and upper interval limit of $[-1, 1]$. The rate at which intervals decrease and increase is higher, the larger the agent's bias and the greater $|a_i|$. We find that $-a_i = a_{N-i}$ for each $i \leq N$, that is, intervals are symmetrically distributed around zero. Two different cases can arise: Either $N = 2n$, which means that the partition has an even number of elements, or $N = 2n + 1$, meaning that the partition has an odd number of elements, where $n \in \mathbb{N}$. In the first case it holds that $a_{\frac{N}{2}} = 0$, whereas in the latter $a_{\frac{N-1}{2}}$ and $a_{\frac{N+1}{2}}$ form a symmetric interval around zero where the principal's expected value of θ is zero.[6]

There is a rational for these observations: If $b > 1$, the agent reacts more sensitive to changes in the state of world relative to the principal and thus always prefers more extreme projects than the principal. This means that when the difference in optimal actions (i.e., $y_A^*(\theta, b) - y_P^*(\theta) = \theta(b-1)$) is positive, the agent wants to lie to the right of the true state, while when the difference is negative, he wants to lie to the left of θ. This difference in optimal actions is greater, the greater $|\theta|$. As a consequence, the agent introduces more noise the farther away states are from zero if $b > 1$.

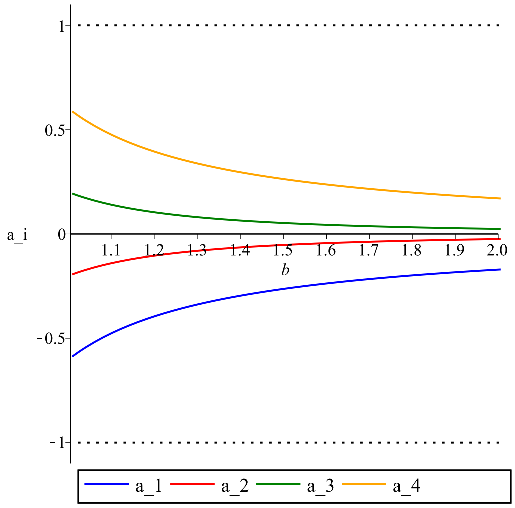


Figure 3.1: $a^5(b)$ for $N = 5$ and $b > 1$

Figure 3.1 illustrates these characteristics for a partition equilibrium of size $N = 5$. All thresholds are depicted as functions of the bias, that is, $a_i(b)$ for $i \in \{0, \ldots, 5\}$, and

[6]We share these characteristics of the agent's signaling strategy with Alonso et al. (2008).

$a_0 = -1$ and $a_5 = 1$ are represented by the two dotted lines. It can easily be seen that $-1 = a_0 < a_1 < a_2 < a_3 < a_4 < a_5 = 1$ holds for all $b > 1$. Hence, the monotonicity and boundary conditions are fulfilled and a partition equilibrium of size $N = 5$ exists for all $b > 1$.

As stated above, the characteristics of the partition equilibria for state-dependent biases differ from those for constant biases. In CS the size of a partition equilibrium is bounded and the maximum number of partition elements is determined by the difference in preferences b. They find that the greater the agent's bias, the larger are the intervals and thus, the smaller is the maximum number of intervals. Note that this further implies that in equilibrium also the amount of transmittable information is bounded. In contrast, in our model there does not exist an upper bound on the equilibrium size. This difference is due to two characteristics of our model: (i) the possibility of full aligned preferences at $\theta = 0$ and (ii) the agent's preference for relatively more extreme projects compared to the principal. For θ converging to zero, the conflict of interest vanishes and at the same time the agent's incentive to exaggerate or understate, his private information decreases (see also Alonso et al., 2008). These two effects combined allow intervals around zero to get arbitrarily small and equilibria with any, possibly infinite, number of partition elements to arise. At a first glance, this seems to contradict the findings of CS, who show that there exists a limit on how much information can be transmitted in equilibrium. Even though, there does not exist a limit on the equilibrium size in our setting, the amount of transmittable information is still limited if the preferences of the principal and the agent are not too closely aligned.

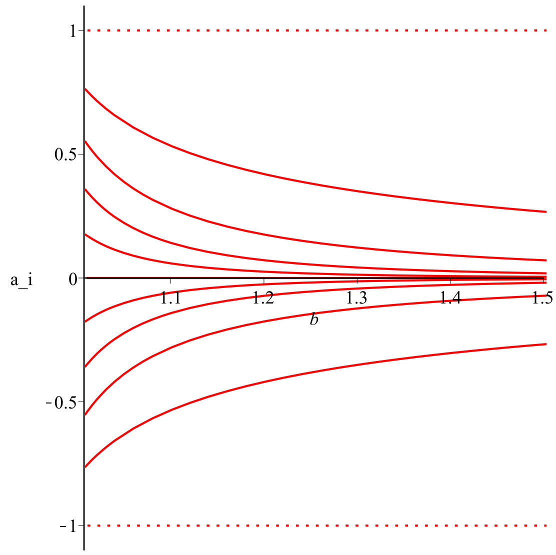


Figure 3.2: $a^{10}(b)$ for $b > 1$.

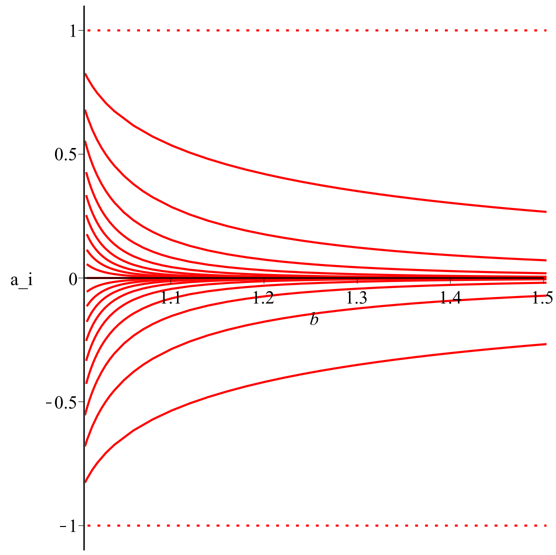


Figure 3.3: $a^{20}(b)$ for $b > 1$.

Figures 3.2 and 3.3 display the thresholds a_i as a function of the agent's bias b for partitions of size $N = 10$ and $N = 20$. When comparing figures 3.2 and 3.3, it becomes apparent that the additional thresholds partition the intervals close to zero to a greater extent than those that are farther away from zero. As the agent's bias increases, the impact of additional partition elements on the outer intervals has relatively low influence. For example, for $b \in (1.1, \infty)$, the length of outer intervals in a partition of size $N = 20$ is approximately the same as the length of outer intervals in a partition of size $N = 10$, despite having ten more elements. While these two partitions differ significantly in the small neighborhood of zero, they are almost identical in the area farther apart from zero (see Figure 3.4 and 3.5). This implies that if the conflict of

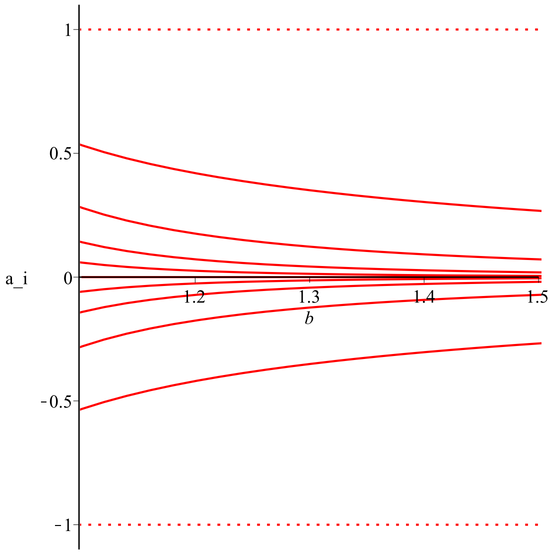


Figure 3.4: $a^{10}(b)$ for $b > 1.1$.

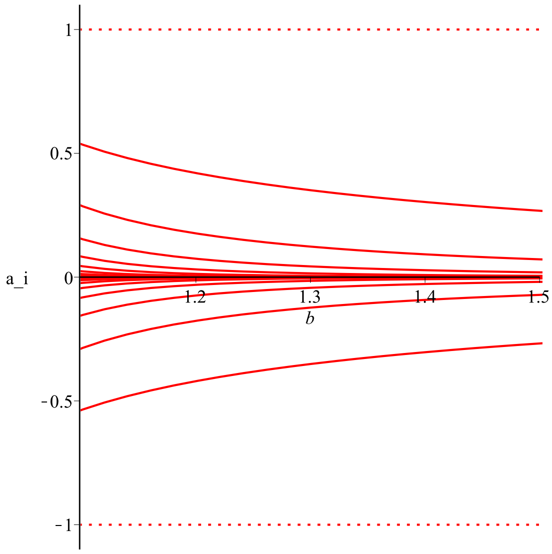


Figure 3.5: $a^{20}(b)$ for $b > 1.1$.

interest is severe enough, increasing the number of partition elements has a negligible effect on the precision of the transmitted information for states farther away from zero but refines the transmitted information for states close to zero. Observations do not change if we further increase N. For significantly large biases the length of outer intervals remains almost the same whereas the length of intervals in the small neighborhood of zero gets infinitesimal small. Hence, even for partitions with infinitely many elements there exists an upper bound on the amount of information that can be communicated given that the agent's bias is severe enough.[7]

The following proposition states that the expected payoffs of the principal and the agent are increasing the size of the partition equilibrium.[8]

Proposition 21. *In the limit equilibrium, that is, $(q(m|\theta), y(m))$ for $N \to \infty$ and $b > 1$, the expected payoffs $E[U_P^{C,\infty}]$ and $E[U_A^{C,\infty}]$ are higher than in any other equilibrium with a finite N. The principal's expected payoff in the limit equilibrium is given by*

$$E[U_P^{C,\infty}] = \frac{1}{3}\left(\frac{1-b}{4b-1}\right). \tag{3.6}$$

[7]We have this feature in common with Stein (1989). He finds that there exists a limit on the precision with which information is communicated when the Fed's target value is not too small.

[8]Proposition 21 is based on Proposition 2 from Alonso et al. (2008) and adjusted to our setting.

and the agent's expected payoff is given by

$$E[U_A^{C,\infty}] = \frac{1}{3}\left(\frac{b(4b-3)(1-b)}{4b-1}\right). \tag{3.7}$$

For the proof that the limit of strategies and beliefs constitutes a Perfect Bayesian equilibrium we refer the reader to Alonso et al. (2008) as this proof also applies to our setting.

4.3 Comparison of Expected Payoffs for $b > 1$

To answer the question whether it is better for the principal to delegate the decision or to communicate with the agent, we compare the expected payoff of the principal for both organizational structures. It is reasonable to assume that the principal and the agent coordinate on the limit-equilibrium as it gives both the highest possible payoffs. Comparing the principal's expected payoffs under communication and delegation yields

$$E[U_P^{C,\infty}] = \frac{1}{3}\left(\frac{1-b}{4b-1}\right) \geq -\frac{1}{3}(b-1)^2 = E[U_P^D] \tag{3.8}$$

$$\Leftrightarrow \qquad b \geq \frac{5}{4}. \tag{3.9}$$

Proposition 22. *While delegation beats communication* $\forall b \in (1, \frac{5}{4})$*, communication beats delegation for all* $b \in [\frac{5}{4}, \infty)$*.*

We conclude that full delegation strictly dominates communication for all $b \in (1, \frac{5}{4})$. That is, the principal receives a strictly higher expected payoff if she fully delegates all decision rights to the agent for all $b \in (1, \frac{5}{4})$. In contrast, for all $b \in (\frac{5}{4}, \infty)$, the principal is strictly better off if she communicates with the agent and uses the transmitted information to make the decision by herself. At $b = \frac{5}{4}$ the principal is just equally well off under both organizational forms. Since $b = \frac{5}{4} < 2 = \bar{b}$ (see section 3.1), delegation also beats making a totally uninformed decision for $b \in (1, \frac{5}{4})$. This observation contradicts Dessein's (2002) finding that for the uniform-quadratic-example with a constant bias full delegation always strictly dominates communication whenever the latter is informative. We conclude that the changes in results follow from the perfect alignment of preferences at $\theta = 0$ as well as from the agent reacting relatively more sensitive to changes in the state of the world than the principal.

Figure 3.6 shows the principal's expected payoff under communication and delegation. It can easily be seen that both expected payoffs are decreasing as the agent's bias increases and that they only intersect once at $b = \frac{5}{4}$. While delegation performs better than communication for biases $b < \frac{5}{4}$, communication beats delegation for all biases $b > \frac{5}{4}$. For biases to the left of the intersection point the difference in expected payoffs is small, whereas for biases to the right of the intersection point the expected payoffs diverge very fast and delegation performs significantly worse than communication.

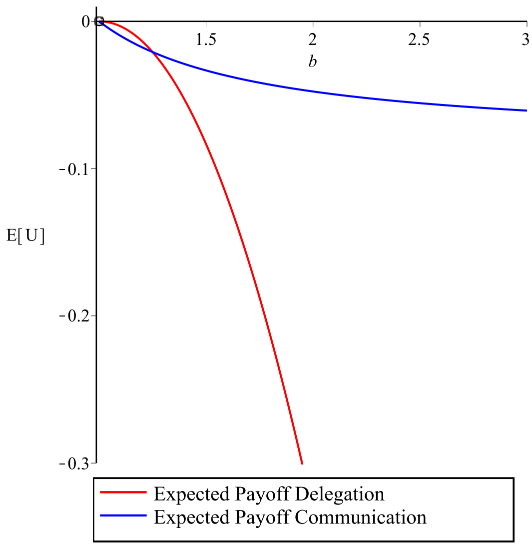


Figure 3.6: comparison of expected payoffs for $N \to \infty$ and $b > 1$

4.4 Communication for $b \in (0, 1)$

In this section, we discuss the characteristics of partition equilibria when the agent reacts less sensitive than the principal to changes in the state, that is, when $b \in (0, 1)$. As starting point, we take the difference equation $a_{k+1} - a_k = a_k - a_{k-1} + 4(b-1)a_k$ from section 4.2 which determines how the length of succeeding intervals evolves.[9] If a partition equilibrium of size N exists for $b \in (0, 1)$, then the growth rate (i.e., $4(b-1)a_k$) is negative if $a_k > 0$ and positive if $a_k < 0$. This implies that the length of intervals decreases as intervals approach the limits of $[-1, 1]$ and increases as they approach zero. This differs from our findings for $b > 1$, where the length of intervals increases in the direction of the interval limits of $[-1, 1]$.

The reason for this is analogous to the one in section 4.2. If $b \in (0, 1)$, the agent responses less sensitive to the state and therefore prefers less extreme projects than the principal. Thus, when $b \in (0, 1)$ the agent's incentive to lie inverts and - in contrast to the case where $b > 1$ - the agent introduces more noise the closer states are to zero. This explains why intervals increase towards zero for $b \in (0, 1)$. Note that the difference in optimal actions of the principal and the agent is smaller the closer θ is to zero. That is, the effect of the agent's sensitivity level and the effect of the preference alignment at zero work in opposite directions. Hence, for $b \in (0, 1)$ we expect the rate at which intervals increase to be smaller than for $b > 1$ where both of the above effects work in the same direction.

Since the derivation of a_k for $b \in (0, 1)$ requires a case distinction, we get different threshold equations for $b \in (0, \frac{1}{2})$ and $b \in (\frac{1}{2}, 1)$. For mathematical tractability we do

[9] For notational convenience, we use k instead of i as subscript.

not consider the case $b = \frac{1}{2}$. Let $q = \arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right)$. For $b \in (0, \frac{1}{2})$ we get

$$a_k = -\cos((q+\pi)k) + \frac{1+\cos((q+\pi)N)}{\sin((q+\pi)N)}\sin((q+\pi)k)$$

and for $b \in (0.5, 1)$ we obtain

$$a_k = -\cos(q \cdot k) + \frac{1+\cos(q \cdot N)}{\sin(q \cdot N)}\sin(q \cdot k).$$

In section 4.2 we find that if $b > 1$ partitions of every size N fulfill the monotonicity and boundary condition. This is no longer true if $b \in (0, 1)$. Subsequently, we discuss how the thresholds evolve in partitions of different sizes N. We check for which $b \in (0, 1)$ partitions of different size $N \geq 2$ fulfill the monotonicity and the boundary constraints, that is, when $-1 = a_0 < a_1 < \cdots < a_{N-1} < a_N = 1$ holds. In order to cope with this simultaneously we use the graphical illustration of the thresholds for different partition sizes and different biases. Figures 3.7 to 3.11 illustrate a_k as a function of b in partitions of different size $N \in \{2, 3, 4, 5, 10\}$. The dotted lines depict $a_0 = -1$ and $a_N = 1$.

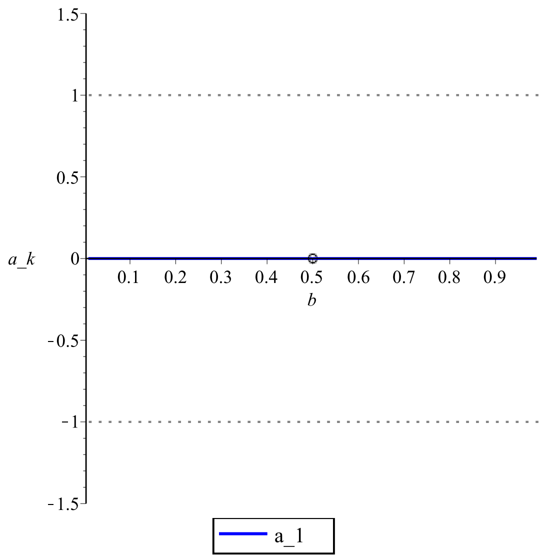


Figure 3.7: $a^2(b)$ for $b \in (0, 1)$.

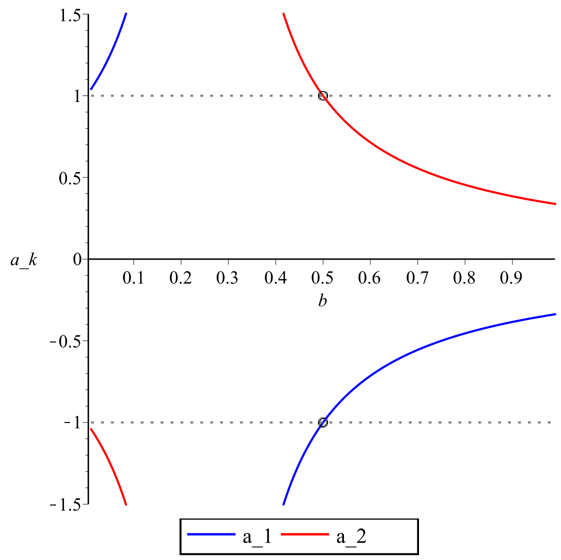


Figure 3.8: $a^3(b)$ for $b \in (0, 1)$.

Since for $N = 2$ the threshold a_1 equals zero for all $b \in (0, 1)$, the state space is partitioned into two equally sized and symmetrically distributed intervals around zero. The monotonicity and the boundary conditions are trivially fulfilled and communication is informative for all $b \in (0, \frac{1}{2}) \cup (\frac{1}{2}, 1)$. Note that, for $b > 1$ we also obtain that a_1 equals zero independent of b. Thus, for $N = 2$ the partition equilibria for $b > 1$ and $b \in (0, 1)$ are equivalent.

For all $N > 2$ we find that the thresholds depend on the agent's bias (see Figures 3.8-3.11). This has a considerable influence on the existence of a partition equilibrium.

For example, if $N = 3$, then for all $b \in (0, \frac{1}{2})$ both thresholds a_1 and a_2 do not lie in $(-1, 1)$ which is why there does not exist a partition equilibrium for this interval of biases. In contrast, for all $b \in (\frac{1}{2}, 1)$ it holds that $-1 < a_1 < a_2 < 1$ and $a_1 = -a_2$. Hence, for all $b \in (\frac{1}{2}, 1)$ there exists a partition equilibrium of size $N = 3$, in which the intervals are symmetrically distributed around zero.

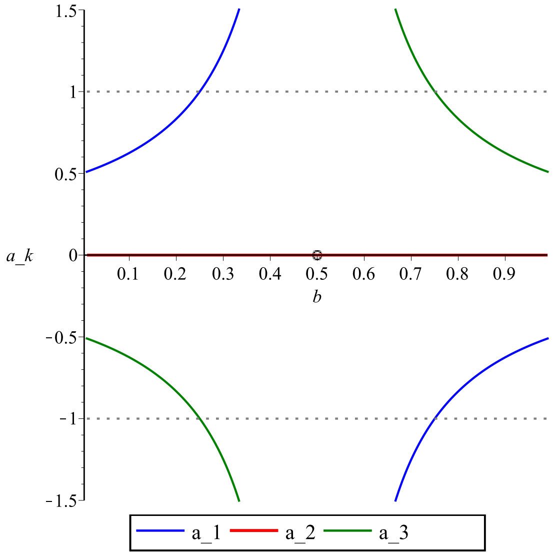


Figure 3.9: $a^4(b)$ for $b \in (0, 1)$.

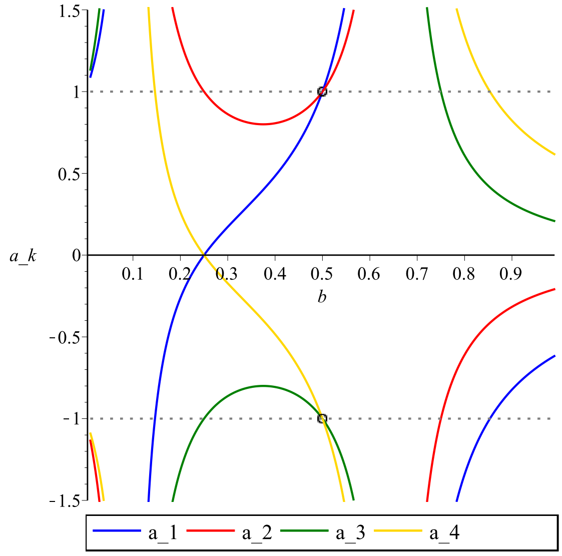


Figure 3.10: $a^5(b)$ for $b \in (0, 1)$.

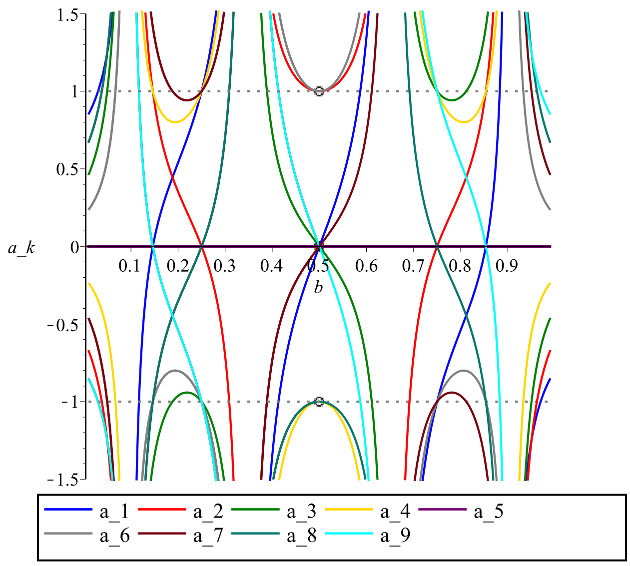


Figure 3.11: $a^{10}(b)$ for $b \in (0, 1)$.

From Figures 3.8-3.11 we can see, that the range of biases for which a partition equilibrium exists decreases in N. The greater N, the smaller is the range of biases for which thresholds fulfill the monotonicity constraint as well as the boundary conditions. For example, in the partition equilibrium of size $N = 4$, either the monotonicity constraint is violated as $a_1 > a_2 > a_3$ or the boundary condition is violated for $b \in (0, 0.75)$.

Only for $b \in (0.75, 1)$ a partition equilibrium of size $N = 4$ exists in which intervals remain symmetrically distributed around zero. The analysis of greater sized equilibria is analogous and shows that the interval of biases for which both existence conditions are fulfilled decreases in N.

We find that the boundary condition determines the lower interval limit of biases for which an equilibrium of size $N \geq 3$ exists. For each N, the lower interval limit is determined by the last time where threshold a_1 crosses -1 and starts to lie within the boundaries. Let $\bar{b}(N)$ be such that a partition equilibrium of size N exists for all $b \in [\bar{b}(N), 1)$. This lower interval limit is given by $\bar{b}(N) = \max\{b \mid a_1(b) = -1\}$.[10] As depicted by figures 3.9-3.11, $\bar{b}(N)$ is increasing in N and converges to one as N increases. This means that a greater sized partition requires preferences of the principal and the agent to be almost perfectly aligned such that an equilibrium of this size exists.

This result stands in contrast to our previous findings that an equilibrium of every size always exists for biases greater than 1. A rational for this is the following: In both cases, for $b > 1$ and for $b \in (0, 1)$, the difference in preferences is smaller, the closer θ is to zero. As a consequence, while for $b > 1$ the effects of the agent's sensitivity and of the state-dependent preference divergence trigger increasingly small intervals in the direct neighborhood of zero, for $b \in (0, 1)$ they work in opposite directions. This hinders partition equilibria of a greater size N to exist for small sensitivity levels of the agent and only allows them to exist when preferences are almost aligned.

4.5 Comparison of Expected Payoffs for $b \in (0, 1)$

Analogous to the case $b > 1$, we compare the expected payoffs under communication and delegation for $b \in (0, \frac{1}{2}) \cup (\frac{1}{2}, 1)$. In section 4.4 we find that there does not exist a partition equilibrium of every size N for every $b \in (0, 1)$. To perform a reasonable analysis, we need to adjust the range of biases when comparing the two organizational forms. As calculated in Section 4.1, the expected payoff of the principal under delegation is given by $E[U_P^D] = -\frac{1}{3}(b-1)^2$. We start our comparison by considering the smallest partition for which communication is just informative, that is, for $N = 2$. We find in section 4.4 that in this case $a_1 = 0$ for all $b \in (0, \frac{1}{2}) \cup (\frac{1}{2}, 1)$. Thus, for $N = 2$, the principal's expected payoff under communication is constant and given by

$$E[U_P^{C,2}] = \sum_{i=1}^{2} \int_{a_{i-1}}^{a_i} -(y_i - \theta)^2 \frac{1}{2} \, d\theta = -\frac{1}{12}. \tag{3.10}$$

Solving $-\frac{1}{3}(b-1)^2 < -\frac{1}{12}$ for $b \in (0, 1)$ yields $b < \frac{1}{2}$. Since $E[U_P^D]$ is strictly increasing in b for all $b \in (0, 1)$, we conclude that communication is the better choice for the principal for all $b \in (0, \frac{1}{2})$. If $b \in (\frac{1}{2}, 1)$, the principal is better off if she delegates the decision rights to the agent (see Figure 3.12). Subsequently, we illustrate graphically how the expected payoffs under communication and delegation compare for $N > 2$. Each figure depicts the comparison of both organizational forms for a different partition size $N \in \{3, 4, 5\}$ and the adjusted interval of biases $(\bar{b}(N), 1)$.

[10]Note that by the symmetry of the thresholds we could have defined $\bar{b}(N)$ also by $\bar{b}(N) = \max\{b \mid a_{N-1}(b) = 1\}$.

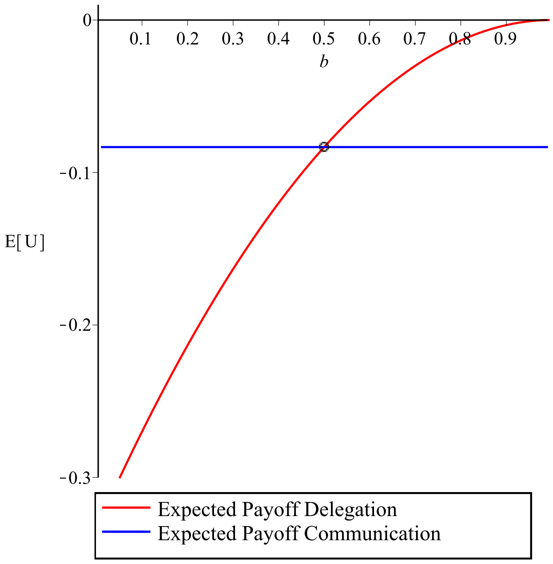


Figure 3.12: $N = 2$ and $b \in (0, 1)$.

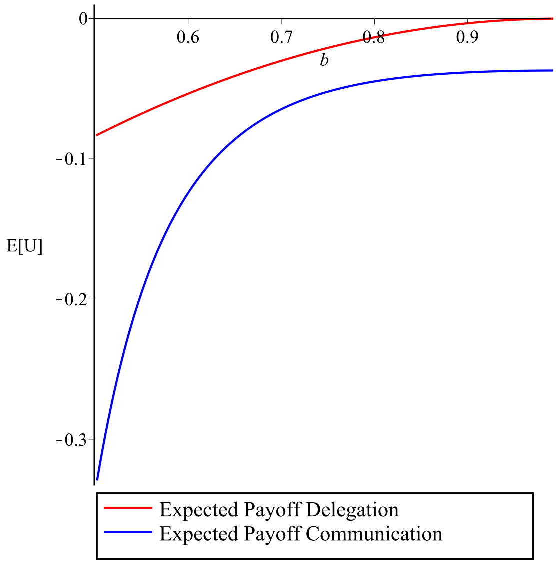


Figure 3.13: $N = 3$ and $b \in (\bar{b}(3), 1)$.

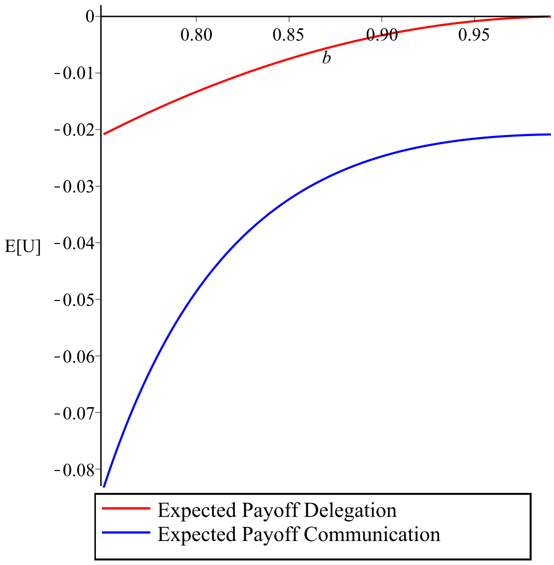


Figure 3.14: $N = 4$ and $b \in (\bar{b}(4), 1)$.

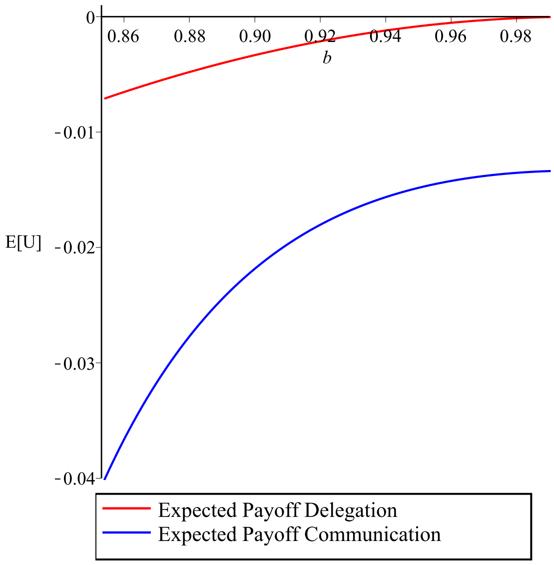


Figure 3.15: $N = 5$ and $b \in (\bar{b}(5), 1)$.

We find that for $b \in (\frac{1}{2}, 0.66)$ the principal's expected payoff under communication for $N = 3$ is even smaller compared to her expected payoff for $N = 2$. This is contrary to our findings for $b > 1$, as in this case the expected payoff is increasing in N for all $b > 1$. This difference is due to the fact that for $N = 3$ and $b \in (\frac{1}{2}, 0.66)$ the thresholds a_1 and a_2 are too far apart from each other, such that the information loss in the interval $[a_1, a_2]$ outweighs the information gain resulting from the additional interval (see Figure 3.8). As a consequence, the expected payoff in the partition of size $N = 3$ cannot increase for $b \in (\frac{1}{2}, 0.66)$. Only for $b \in (0.66, 1)$ when the precision of the transmitted information improves as the thresholds a_1 and a_2 move closer to each other, the expected payoff in the equilibrium of size $N = 3$ exceeds the expected payoff for $N = 2$. Nevertheless, this improvement does not suffice for communication to beat delegation (see Figure 3.13).

For greater sized partitions we observe the same. For example, the principal's expected payoff in a partition equilibrium of size $N = 3$ is greater than her expected payoff in a partition of size $N = 4$ for all $b \in (\bar{b}(4), 081)$. Likewise, the principal is better off in an equilibrium of size $N = 4$ than in one equilibrium of size $N = 5$ for all $b \in (\bar{b}(5), 088)$. Only when preferences become more aligned, the greater sized communication equilibria yield the principal a higher expected payoff than the smaller once.

The dominance of delegation over communication persists also for greater sized partitions. The reason for this is that for greater sized partition equilibria to exist, preferences must be increasingly aligned and a comparison of both organizational forms is only possible for $b \in (\bar{b}(N), 1)$. However, when preferences are close to alignment, delegation yields the principal expected payoffs close to her highest achievable payoff and communication cannot beat delegation as illustrated in Figures 3.14 and 3.15.

5. Conclusion

We study the comparison of delegation versus communication for state-dependent biases. More precisely, we address the question whether an uninformed principal should delegate a decision to her perfectly informed but biased agent or whether she is better off when she only consults the agent and makes the decision on her own. Our work extends the analysis of Dessein (2002) who addresses this problem for constant biases. However, contrary to Dessein (2002), we model the agent's bias as state-dependent so that the difference in preferences of the principal and the agent changes as the state of the world changes. This modification changes results significantly. While Dessein (2002) shows that for the uniform-quadratic example of CS, delegation always performs better than communication if the latter is informative, we find that communication can beat delegation for a large range of biases. We illustrate that communication always performs better than delegation when the agent's bias exceeds a certain threshold (i.e., $b > \frac{5}{4}$). If, instead, the agent reacts less sensitive to changes state of the world, results are similar to Dessein's (2002) findings and delegation is generally the better choice for the principal. The differing results are due to our consideration of a state-dependent preference divergence and the resulting possibility of full aligned preferences. In case of an agent that reacts more sensitive to changes in the state of the world than the principal, the consideration of a state-dependent bias allows for communication equilibria of an infinite size to exist. This way the precision of the transmitted information improves significantly. As a result, the principal's expected payoff under communication outperforms her expected payoff under delegation for a sufficiently sensitive reacting agent. In contrast, when the agent reacts less sensitive to changes in the world than the principal, delegation is in general the better choice for the principal.

The characteristics of the communication equilibria as illustrated in this paper particularly complement the findings of Alonso et al. (2008). However, a comparison of their results regarding the analysis of centralization versus decentralization with our results regarding the analysis of communication versus delegation for the setting with one agent is not quite applicable. This is due to the fact that they consider a sender-receiver game with two senders, and, additionally, introduce a need for coordination. Thus, their main findings are formulated for different combinations of the division manager's bias and an additional coordination parameter. Even if we set the coordination parameter to zero and only take one division manager into account, results are not comparable: the preferences of the headquarter manager and the division manager would be perfectly aligned for every state of the world. Thus, our analysis of state-dependent biases advances the current understanding of the literature on the comparison of delegation and communication in the context of the classic principal-agent model.

A. Appendix

A.1 Proof of Proposition 20

Proof. In the following we will illustrate that the described strategies constitute a partition equilibrium of the communication game and we will characterize it for the case $b > 1$.

First, we have to show that the principal's decision $\overline{y}(a_i, a_{i+1})$ is a best response to the agent's message $m \in (a_i, a_{i+1}), \forall i$. Since the agent uniformly randomizes his message over $[a_i, a_{i+1}]$ if $\theta \in (a_i, a_{i+1})$, the principal's best response upon receiving $m \in (a_i, a_{i+1})$ is

$$\underset{y}{\operatorname{argmax}} \int_{a_i}^{a_{i+1}} U_P(y, \theta)\frac{1}{2}d\theta = \frac{a_i + a_{i+1}}{2} = \overline{y}(a_i, a_{i+1}). \tag{3.11}$$

Second, in order for the agent's signaling rule to be a best response to the principal's decision $\bar{y}(a_i, a_{i+1}), \forall i$, the agent must not have an incentive to lie about the interval in which the true θ lies. This requires that the agent is indifferent between sending messages $m_i \in (a_{i-1}, a_i)$ and $m_{i+1} \in (a_i, a_{i+1})$ when θ coincides with the threshold a_i. Formally this means that

$$U_A(y_i, a_i, b) = U_A(y_{i+1}, a_i, b), \tag{3.12}$$

or

$$-\left(\frac{a_{i-1} + a_i}{2} - ba_i\right)^2 = -\left(\frac{a_i + a_{i+1}}{2} - ba_i\right)^2, \tag{3.13}$$

where $y_i = \overline{y}(a_{i-1}, a_i)$ and $y_{i+1} = \overline{y}(a_i, a_{i+1})$. Rearranging this indifference condition yields

$$a_{i+1} - a_i = a_i - a_{i-1} + 4(b-1)a_i. \tag{3.14}$$

Next, we need to solve the difference equation. Rearranging the above indifference condition yields

$$a_{i+1} + (2 - 4b)a_i + a_{i-1} = 0. \tag{3.15}$$

The characteristic equation of this second-order linear difference equation is given by

$$z^2 + (2 - 4b)z + 1 = 0. \tag{3.16}$$

The respective roots of the characteristic equation are

$$x = \sqrt{(1-2b)^2 - 1} + 2b - 1 \quad \text{and} \quad y = -\sqrt{(1-2b)^2 - 1} + 2b - 1, \tag{3.17}$$

where $xy = 1$ and $x > 1$. Consequently, the general solution of the difference equation is

$$a_i = \lambda x^i + \mu y^i. \tag{3.18}$$

In order to determine λ and μ we use the two initial conditions, $a_0 = -1$ and $a_N = 1$, and solve the corresponding equations (i) $a_0 = \lambda + \mu = -1$ and (ii) $a_N = \lambda x^N + \mu y^N = 1$ for λ and μ. This yields

$$\lambda = \frac{1+y^N}{x^N - y^N} \quad \text{and} \quad \mu = -\frac{1+y^N}{x^N - y^N} - 1. \tag{3.19}$$

Finally, we substitute for λ and μ in the general solution and obtain the solution of the difference equation. That is,

$$a_i = \frac{x^i(1+y^N) - y^i(1+x^N)}{x^N - y^N} \quad \text{for } 0 \leq i \leq N. \tag{TH}$$

After inserting $x = \sqrt{(1-2b)^2 - 1} + 2b - 1$ and $y = -\sqrt{(1-2b)^2 - 1} + 2b - 1$ it can easily be checked that $-a_i = a_{N-i}$ for each $i \leq N$ and that $a_{\frac{N}{2}} = 0$ if N is even. If N is odd, then $a_{\frac{N-1}{2}}$ and $a_{\frac{N+1}{2}}$ form a symmetric interval around zero.

□

A.2 Proof of Proposition 21

Proof. In order to prove Proposition 21 for our setting we will first calculate the expected payoffs of the principal and the agent for a finite N and then let N converge towards infinity. Given the equilibrium strategies and beliefs, the principal's expected payoff in a partition equilibrium of size N can be expressed in terms of the residual variance of θ (Crawford and Sobel, 1982). That is,

$$E[U_P^N] = E[-(y_i - \theta)^2] = -\sum_{i=1}^{N} \int_{a_{i-1}}^{a_i} (y_i - \theta)^2 \frac{1}{2}\, d\theta = -\sum_{i=1}^{N} \int_{a_{i-1}}^{a_i} \left(\frac{a_{i-1} + a_i}{2} - \theta\right)^2 \frac{1}{2}\, d\theta. \tag{3.20}$$

Analogously, the agent's expected payoff is given by

$$E[U_A^N] = E[-(y_i - b\theta)^2] = -\sum_{i=1}^{N} \int_{a_{i-1}}^{a_i} \left(\frac{a_{i-1} + a_i}{2} - b\theta\right)^2 \frac{1}{2}\, d\theta. \tag{3.21}$$

We use the following Lemma to calculate the expected payoffs. This Lemma is based on Lemma 1, A1 and A2 from Alonso et al. (2008) and adjusted to our setting.

Lemma 11.

(i) $E[y_i\theta] = E[y_i^2]$,

(ii) $E[y_i^2] = \frac{1}{4}\left[\frac{(x^{3N}-1)(x+1)^2}{(x^N-1)^3(x^2+x+1)} - \frac{x^N(x+1)^2}{x(x^N-1)^2}\right]$,

(iii) $E[y_i^2]$ *is strictly increasing in* N*, and*

(iv) $\lim\limits_{N\to\infty} E[y_i^2] = \frac{(x+1)^2}{4(x^2+x+1)} = \frac{b}{4b-1}$.

Proof. We use the same approach as in Alonso et al. (2008). (i) Given that $y_i = \frac{a_{i-1}+a_i}{2} = E[\theta|\theta \in (a_{i-1}, a_i)]$, we can use the law of iterated expectation and obtain

$$E[y_i\theta] = E[E[y_i\theta|\theta \in (a_{i-1}, a_i)]] = E[y_i E[\theta|\theta \in (a_{i-1}, a_i)]] = E[y_i^2]. \tag{3.22}$$

To prove part *(ii)* and *(iii)* we need to calculate $E[y_i^2]$ which can be expressed as follows

$$\begin{aligned} E[y_i^2] &= \sum_{i=1}^{N} \int\limits_{a_{i-1}}^{a_i} (y_i)^2 \frac{1}{2}\, d\theta = \sum_{i=1}^{N} \int\limits_{a_{i-1}}^{a_i} \left(\frac{a_{i-1}+a_i}{2}\right)^2 \frac{1}{2}\, d\theta \\ &= \frac{1}{8}\sum_{i=1}^{N}(a_i - a_{i-1})(a_i^2 + 2a_i a_{i-1} + a_{i-1}^2) \end{aligned} \tag{3.23}$$

After substituting equation 4.16 for a_i, we obtain

$$\begin{aligned} E[y_i^2] = \frac{1}{8}\sum_{i=1}^{N} & \frac{(x^i(1+y^N) - y^i(1+x^N))^3}{(x^N-y^N)^3} - \frac{(x^{i-1}(1+y^N) - y^{i-1}(1+x^N))^3}{(x^N-y^N)^3} \\ &+ \frac{(x^i(1+y^N) - y^i(1+x^N))^2(x^{i-1}(1+y^N) - y^{i-1}(1+x^N))}{(x^N-y^N)^2(x^N-y^N)} \\ &- \frac{(x^i(1+y^N) - y^i(1+x^N))(x^{i-1}(1+y^N) - y^{i-1}(1+x^N))^2}{(x^N-y^N)(x^N-y^N)^2}. \end{aligned} \tag{3.24}$$

Rewriting yields

$$\begin{aligned} E[y_i^2] = \frac{1}{8(x^N-y^N)^3} \sum_{i=1}^{N} \; & x^{3i}(1+y^N)^3 \; - \; 3x^{2i}(1+y^N)^2 y^i(1+x^N) \\ &+ 3x^i(1+y^N)y^{2i}(1+x^N)^2 \; - \; y^{3i}(1+x^N)^3 + x^{3i-1}(1+y^N)^3 \\ &- x^{2i}(1+y^N)^2 y^{i-1}(1+x^N) \; - \; 2x^{2i-1}(1+y^N)^2 y^i(1+x^N) \\ &+ 2x^i(1+y^N)y^{2i-1}(1+x^N)^2 \; + \; x^{i-1}(1+y^N)y^{2i}(1+x^N)^2 \\ &- y^{3i-1}(1+x^N)^3 - x^{3i-2}(1+y^N)^3 + 2x^{2i-1}(1+y^N)^2 y^{i-1}(1+x^N) \\ &- x^i(1+y^N)y^{2(i-1)}(1+x^N)^2 + x^{2(i-1)}(1+y^N)^2 y^i(1+x^N) \\ &- 2x^{i-1}(1+y^N)y^{2i-1}(1+x^N)^2 \; + \; y^{3i-2}(1+x^N)^3 \\ &- x^{3(i-1)}(1+y^N)^3 + 3x^{2(i-1)}(1+y^N)^2 y^{i-1}(1+x^N) \\ &- 3x^{i-1}(1+y^N)y^{2(i-1)}(1+x^N)^2 + y^{3(i-1)}(1+x^N)^3. \end{aligned} \tag{3.25}$$

Now we use that $xy = 1$ and get the following expression

$$\begin{aligned} E[y_i^2] = \frac{1}{8(x^N - y^N)^3} \; &\sum_{i=1}^{N} (1+y^N)^3 x^{3(i-1)} (x+1)^2 (x-1) \\ &+ (1+x^N)^3 y^{3(i-1)} (y+1)^2 (1-y) \\ &+ (1+y^N)(1+x^N) y^{i-1} (y^2-1)(x+1) \\ &- (1+y^N)^2 (1+x^N) x^{i-1} (x^2-1)(y+1). \end{aligned} \tag{3.26}$$

Rearranging and performing some calculations yields

$$\begin{aligned} E[y_i^2] =& \frac{(1+y^N)^3 (x+1)^2 (x-1)}{8\left(x^N - y^N\right)^3 x^3} \left(\frac{x^3(1-x^{3N})}{1-x^3} \right) \\ &+ \frac{(1+x^N)^3 (y+1)^2 (1-y)}{8\left(x^N - y^N\right)^3 y^3} \left(\frac{y^3(1-y^{3N})}{1-y^3} \right) \\ &+ \frac{(1+y^N)(1+x^N)^2 (y^2-1)(x+1)}{8\left(x^N - y^N\right)^3 y} \left(\frac{y(1-y^N)}{1-y} \right) \\ &+ \frac{-(1+y^N)^2 (1+x^N)(x^2-1)(y+1)}{8\left(x^N - y^N\right)^3 x} \left(\frac{x(1-x^N)}{1-x} \right). \end{aligned} \tag{3.27}$$

Next, we use that $y = \frac{1}{x}$, and simplify the above expressions. That is,

$$\begin{aligned} E[y_i^2] =& - \frac{(x^N+1)^3 (x+1)^2 (1-x^{3N})}{8\left(x^{2N}-1\right)^3 \left(x^2+x+1\right)} + \frac{(1+x^N)^3 (x+1)^2 (x^{3N}-1)}{8\left(x^{2N}-1\right)^3 \left(1+x+x^2\right)} \\ &- \frac{(1+x^N)^3 (x+1)^2 (x^N-1) x^N}{8x\left(x^{2N}-1\right)^3} + \frac{(1+x^N)^3 (x+1)^2 (1-x^N) x^N}{8x\left(x^{2N}-1\right)^3}. \end{aligned} \tag{3.28}$$

$$= \frac{(1+x^N)^3 (1+x)^2 (x^{3N}-1)}{4(x^{2N}-1)^3 (x^2+x+1)} + \frac{(x^N+1)^3 (x+1)^2 (1-x^N) x^N}{4x(x^{2N}-1)^3} \tag{3.29}$$

$$= \frac{1}{4} \left[\frac{(x^{3N}-1)(x+1)^2}{(x^N-1)^3 (x^2+x+1)} - \frac{x^N (x+1)^2}{x(x^N-1)^2} \right]. \tag{3.30}$$

After having calculated $E[y_i^2]$, we continue by showing that $E[y_i^2]$ is strictly increasing in N. For this purpose we define

$$f(p) = \frac{1}{4} \left[\frac{(p^3-1)(x+1)^2}{(p-1)^3 (x^2+x+1)} - \frac{p(x+1)^2}{x(p-1)^2} \right] \tag{3.31}$$

and calculate the derivative of $f(p)$

$$f'(p) = \frac{1}{4}\left[\frac{(x+1)^2}{(x^2+x+1)}\left(\frac{3p^2(p-1)^3-3(p^3-1)(p-1)^2}{(p-1)^6}\right)\right] \tag{3.32}$$
$$-\frac{1}{4}\left[\frac{(x+1)^2}{x}\left(\frac{(p-1)^2(p+1)}{(p-1)^4}\right)\right]$$
$$=\frac{1}{4}\left[\frac{(p+1)(x^4-2x^2+1)}{(p-1)^3(x^2+x+1)x}\right].$$

The derivative of $f(p)$ is strictly positive for all $p > 1$. Since $E[y_i^2] = f(x^N)$ and $x^N > 1$, it follows that $E[y_i^2]$ is strictly increasing in N. Finally, the we calculate the value of $E[y_i^2]$ in the limit-equilibrium. That is,

$$\lim_{N\to\infty} E[y_i^2] = \lim_{N\to\infty} \frac{1}{4}\left[\frac{(x^{3N}-1)(x+1)^2}{(x^N-1)^3(x^2+x+1)} - \frac{x^N(x+1)^2}{x(x^N-1)^2}\right] = \frac{(x+1)^2}{4(x^2+x+1)}. \tag{3.33}$$

After substituting for $x = \sqrt{(1-2b)^2-1}+2b-1$ and rewriting, we obtain

$$\lim_{N\to\infty} E[y_i^2] = \frac{4b(\sqrt{(1-2b)^2-1}+2b-1)}{4(4b-1)(\sqrt{(1-2b)^2-1}+2b-1)} = \frac{b}{4b-1}. \tag{3.34}$$

□

The expected payoff of the principal can then be written as

$$E[U_P^N] = -E[(y_i-\theta)^2] = -E[\theta^2] + E[y_i^2]. \tag{3.35}$$

In the limit-equilibrium, that is for $N \to \infty$, the principal consequently expects to get

$$E[U_P^\infty] = -E[\theta^2] + \lim_{N\to\infty} E[y_i^2] = -\sigma^2 + \frac{b}{4b-1} = \frac{1}{3}\left(\frac{1-b}{4b-1}\right). \tag{3.36}$$

In an analogous way we can rewrite the agent's expected payoff as

$$E[U_A^N] = E[-(y_i-b\theta)^2] = -b^2E[\theta^2] + (2b-1)E[y_i^2]$$

and calculate his expected payoff in the limit-equilibrium. That is,

$$E[U_A^\infty] = -b^2E[\theta^2] + \lim_{N\to\infty}(2b-1)E[y_i^2] = -b^2\sigma^2 + \frac{(2b-1)b}{4b-1} \tag{3.37}$$
$$= \frac{1}{3}\left(\frac{-4b^3+7b^2-3b}{4b-1}\right).$$

Since $E[y_i^2]$ is increasing in N according to Lemma 1, the principal's and the agent's expected payoffs in the limit-equilibrium are higher than in any other finite partition equilibrium. □

A.3 Solution of the Difference Equation for $b \in (0,1)$

Proof. The roots of the characteristic equation are

$$x = \sqrt{(1-2b)^2 - 1} + 2b - 1 \quad \text{and} \quad y = -\sqrt{(1-2b)^2 - 1} + 2b - 1. \tag{3.38}$$

It is obvious that for $b \in (0,1)$ these roots are complex numbers, that is,

$$x = 2b - 1 + i\sqrt{1-(1-2b)^2} \quad \text{and} \quad y = 2b - 1 - i\sqrt{1-(1-2b)^2}. \tag{3.39}$$

Expressing these complex numbers in polar coordinates gives

$$x = r(\cos(\varphi) + i\sin(\varphi)) \quad \text{and} \quad y = r(\cos(\varphi) - i\sin(\varphi)), \tag{3.40}$$

where,

$$r = \sqrt{(2b-1)^2 + \left(\sqrt{1-(1-2b)^2}\right)^2} = 1 \tag{3.41}$$

and

$$\varphi = \begin{cases} \arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right) + \pi & \text{for } b \in (0, 0.5) \\ \arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right) & \text{for } b \in (0.5, 1). \end{cases} \tag{3.42}$$

In the following we exclude $b = 0.5$ for mathematical tractability. Consequently, for $b \in (0, 0.5)$ the polar coordinates of the complex numbers are then given by

$$\cos((\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right) + \pi)) \pm i\sin((\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right) + \pi)) \tag{3.43}$$

and for $b \in (0.5, 1)$ by

$$\cos(\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right)) \pm i\sin(\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right)). \tag{3.44}$$

As general solution of the difference equation we then obtain for $b \in (0, 0.5)$

$$a_k = \lambda\cos((\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right) + \pi)k) + \mu\sin((\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right) + \pi)k) \tag{3.45}$$

and for $b \in (0.5, 1)$

$$a_k = \lambda \cos(\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right)k) + \mu \sin(\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right)k). \tag{3.46}$$

Note that 3.45 and 3.46 are real-valued. We use the initial conditions $a_0 = -1$ and $a_N = 1$ in order to determine λ and μ. This yields for $b \in (0, 0.5)$

$$\lambda = -1 \quad \text{and} \quad \mu = \frac{1 + \cos((\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right) + \pi)N)}{\sin((\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right) + \pi)N)}$$

and for $b \in (0.5, 1)$

$$\lambda = -1 \quad \text{and} \quad \mu = \frac{1 + \cos(\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right)N)}{\sin(\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right)N)}. \tag{3.47}$$

Consequently, the solution of the difference equation for $b \in (0, 0.5)$ is

$$\begin{aligned} a_k =& -\cos((\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right) + \pi)k) \\ &+ \frac{1 + \cos((\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right) + \pi)N)}{\sin((\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right) + \pi)N)} \sin((\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right) + \pi)k) \end{aligned} \tag{3.48}$$

and for $b \in (0.5, 1)$

$$\begin{aligned} a_k =& -\cos(\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right)k) \\ &+ \frac{1 + \cos(\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right)N)}{\sin(\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right)N)} \sin(\arctan\left(\frac{\sqrt{1-(1-2b)^2}}{2b-1}\right)k). \end{aligned} \tag{3.49}$$

$\square$

Chapter 4

A Note on Centralization versus Decentralization

1. Introduction

Multi-divisional organizations can be forced to perfectly coordinate their decisions. In such a case, a principal makes one decision that will be valid for all divisions, as extending the scope of decision making in such multi-divisional organizations would create more harm than good. This can, for example, be due to managerial considerations with respect to diseconomies of scope, brand image or cost saving motives. Even though this decision might be optimal on the firm-level, for each division it can be suboptimal as divisions usually make more profit when they adopt the decision to their local environment (e.g., specific customer taste). Compared to the principal, division managers are typically better informed about their division-specific environment and biased in that they care more about maximizing their division-specific profit than the overall organization's profit.

In the context of these information asymmetries and conflicts of interest, we seek to answer the question which organizational form should be chosen when a company faces a need to coordinate decisions and, importantly, is forced to perfectly coordinate its decisions. Given that only one decision can be made and that the optimal decision depends on all division-specific environments which are only known by the biased division managers: Should decision making be centralized in a headquarter or decentralized in the divisions? To answer this question we analyze a two-divisional organization with one principal and two division managers, henceforth agents. There are two environmental states of the world, one for each division. Each division manager is perfectly informed about his own division-specific environment, whereas the principal is totally uninformed. We assume that only one decision can be made and must be implemented in both divisions. The principal wants to maximize the organizations overall profit - the sum of both divisions' profits - which requires matching the final decision to the average of both pieces of information. In contrast, agents are biased in the sense that they care more about the payoff of their own division and thus want to adapt the final decision more to their private information. We analyze two different organizational forms: centralization and decentralization. Under centralization agents communicate their private information vertically to the principal who then makes the final decision.

Under decentralization agents exchange their private information horizontally with each other and the decision rights are delegated to one of the agents.

We find that centralization performs strictly better than decentralization in terms of maximizing the principal's expected payoff for all adaption biases of the agents. This result continues to hold even when we allow agents to have biases that differ in their level of intensity.

Our model builds on the framework of Alonso et al. (2008), but differs in the following two ways: First, although in Alonso et al. (2008) the organization faces a need to coordinate their decisions, the organization is nevertheless allowed to make different decisions for its divisions. Second, Alonso et al. (2008) analyze for which adaption and coordination biases which organizational form performs best. In our model only one decision is made and implemented in both divisions. Thus, we ask whether centralization or decentralization performs better when a company is forced to coordinate its decisions. As coordination is prescribed and thus only one decision is made in our model, we do not have the need for coordination bias as in Alonso et al. (2008). These differences in the payoff functions of the principal and the agents change results in the following way: While Alonso et al. (2008) find that decentralization can dominate centralization even when the need for coordination is very high, we find that centralization always performs better for all adaption biases of the agents.

The driving force behind our result is the quality of communication. The quality of communication is the highest under vertical communication while under horizontal communication the quality of communication is lower for all biases of the agents. This is also in general true for Alonso et al. (2008) - However in their setting the difference in quality vanishes as their need for coordination bias converges to infinity. In our model there always exists a conflict of interest between the principal and the division managers due to the agents' bias. Thus, the loss of authority under decentralization always leads to a loss in the principal's payoff due this conflict of interest. In Alonso et al. (2008) the optimal decisions of the headquarter manager and division managers become the same when the need for coordination bias converges to infinity. This alignment of interests causes decentralization and centralization to perform equally well for coordination biases converging to infinity in their model. For arbitrarily high but finite coordination needs and small biases of the agents Alonso et al. (2008) find that decentralization even outperforms centralization. In contrast, our result implies that when coordination is prescribed ex ante, the principal is always better off when she makes the decision herself.

2. Related Literature

This paper belongs to the literature on strategic information transmission (e.g. Crawford and Sobel, 1982; Li et al., 2016; Melumad and Shibano, 1991; Rantakari, 2013, 2019; Deimen and Szalay, 2019b) and on organizational design (e.g. Alonso et al., 2008, 2015; Athey and Roberts, 2001; Deimen and Szalay, 2019a; Dessein et al., 2010; Liu and Migrow, 2019). Crawford and Sobel (1982) analyze how information about a payoff relevant state of the world gets transmitted from an informed agent (sender) to an uninformed decision maker (receiver) when there is an underlying conflict of interest. One of their main findings is that all communication equilibria are partitions of the state space and that the size of these partitions is bounded. They show that the smaller the conflict of interest between the sender and the decision maker, the more elements a partition can have and the more precise is the transmitted information. In contrast to Crawford and Sobel (1982), we analyze a setting with two senders and two states of the world and allow for the conflict of interest to change with the state of the world.

Another related paper is Rantakari (2019). He studies a principal who receives recommendations from two agents regarding which project to implement. Each agent is perfectly informed about the quality of his own project and biased towards his own project. The principal is totally uninformed but can evaluate the agents' projects against some cost. While Rantakari (2019) is interested in how the quality of information transmission is influenced by the principal's possibility of additional investigations, we do not allow for the possibility of further information acquisition.

In a similar model as in Rantakari (2019), Li et al. (2016) study competitive cheap talk without allowing for the principal to acquire further information. While in their model the principal is faced with the decision which of the two available projects should optimally be implemented, the principal in our model has to match the decision to the private information of both agents equally. Due to this difference in the nature of decision making and in the payoff structures, the analysis of communication equilibria in Li et al. (2016) is different from ours.

A common difference between the above mentioned papers and our work lies in the comparison of two different organizational forms, that is, the comparison of centralization and decentralization.[1] Within the literature of organizational design, Alonso et al. (2008) analyze decision making in an organization where an uninformed headquarter manager can either consult his two perfectly informed, but biased, division managers and make the decision for both divisions on his own (centralization) or fully delegate the decisions to them (decentralization). In Alonso et al. (2008) two decisions are made and the organization faces a need to coordinate these two decisions. While the headquarter manager wants to maximize the over-all organization's payoff, division managers care more about maximizing their own division's payoff. Alonso et al. (2008) demonstrate that the performance of centralization and decentralization depends on the severity of the need for coordination and on how biased agents are towards maximizing their own division's profit. For example, they show that if the need for coordination between the two divisions increases, decentralization can perform better than centralization. While in Alonso et al. (2008) there exists a trade-off between coordination and adaption, in our model coordination is prescribed as only one decision is made and implemented in both divisions. In contrast to Alonso et al. (2008), we find that centralization always beats decentralization independent of the agents' bias.

Recently, Liu and Migrow (2019) analyze how a multi-divisional organization should optimally allocate its decision making rights when operating in volatile markets. As in Alonso et al. (2008) there exists a trade-off between coordinating decisions and adapting decisions to local conditions. Division managers can acquire decision relevant information against some cost and are able to verify the information acquired. They compare the performance of a centralized and a decentralized organization form. While the

[1] Only Li et al. (2016) briefly discuss that communication and delegation are outcome equivalent. This significantly differs from our results.

quality of communication is the same under both organizational forms, the division manager's acquisition of information is better under decentralization. They find that when the need for coordination is very high, decentralization can beat centralization in volatile markets. In contrast, in our setting it is ex-ante known that each division contributes equally to the organizations overall profit and each division manager is perfectly informed about his local conditions but cannot verify his transmitted information.

Deimen and Szalay (2019a) consider a decision maker who depends on decision relevant information that can only be acquired by a biased expert. The conflict of interest between the decision maker and the expert is endogenous and affected by the expert's information acquisition. They analyze whether the decision maker is better off if he delegates the decision to the expert or if he communicates with him and makes the decision on his own. In contrast, we study a setting with two experts, each of them carrying only a piece of relevant information and whose biases are common knowledge. In Deimen and Szalay (2019a) the delegation of authority has a significant influence on the information acquisition of the expert. In our paper the agents already possess the relevant information and the focus lies on how the allocation of authority influences the information that is transmitted.

3. Model

There is one organization which consists of two divisions $i \in \{1, 2\}$. Each division is managed by a division manager, which we denote by agent $i \in \{1, 2\}$. The principal of the organization has to decide which project $y \in [0, 1]$ the organization should undertake. The profit of division i is given by $-(y - \theta_i)^2$. That is, the profit division i generates depends on how well the project choice y matches its local condition θ_i, where θ_i is uniformly and independently distributed on $[0, 1]$. While the principal cannot observe the decision-relevant local conditions (i.e. states of the world), agent i is perfectly informed about θ_i. The principal wants to maximize the overall profit of the organization, which is simply given by the sum of both divisions' payoffs. We assume that the organization is forced to perfectly coordinate the decisions of its divisions by prescribing that only one project is chosen and implemented in both divisions. This assumption distinguishes our work from Alonso et al. (2008) who allow the organization to make two different decisions, one for each division.

Since the organization's profit is composed of the sum of both divisions' payoffs, the principal wants to adapt this project choice equally to both local conditions. In contrast, each agent is biased towards maximizing the profit of its own division and wants to adjust the decision more to his local conditions. The principal's and the agents' utility functions are given by

$$U_P(y, \theta_1, \theta_2) = -(y - \theta_1)^2 - (y - \theta_2)^2 \qquad (U_P)$$

$$U_1(y, \theta_1, \theta_2, \lambda) = -\lambda(y - \theta_1)^2 - (1 - \lambda)(y - \theta_2)^2 \qquad (U_1)$$

$$U_2(y, \theta_1, \theta_2, \lambda) = -(1 - \lambda)(y - \theta_1)^2 - \lambda(y - \theta_2)^2. \qquad (U_2)$$

These utility functions have a unique maximizer for each (θ_1, θ_2) combination, which are given by $y_P^*(\theta_1, \theta_2) = \frac{1}{2}(\theta_1 + \theta_2)$ for the principal and $y_1^*(\theta_1, \theta_2, \lambda) = \lambda\theta_1 + (1-\lambda)\theta_2$ and $y_2^*(\theta_1, \theta_2, \lambda) = (1-\lambda)\theta_1 + \lambda\theta_2$ for agent 1 and agent 2, respectively. We refer to them as the principal's and the agents' preferred project decisions. The scalar $\lambda \in (\frac{1}{2}, 1)$ measures how biased each agent is towards adapting the decision to his private information. That is, each agent puts more weight on his private information than on the information of the other agent. It is obvious that when $\lambda = \frac{1}{2}$, preferences of the principal and the agents are perfectly aligned. Except for the local conditions everything is common knowledge. In the following we study two different organizational forms and analyze which form performs best when the organization is forced to coordinate on one decision that is implemented in both of its divisions.

4. Centralization versus Decentralization

4.1 Centralization

Under centralization agents communicate their private information vertically to the principal; the principal then makes the final decision. Except for the allocation of decision rights, nothing can be contracted upon ex ante. Neither the principal can commit to a predefined decision scheme nor the agents can commit to a communication scheme. The timing in this cheap talk game is as follows: After the decision rights have been allocated to the principal, agent 1 and agent 2 simultaneously send their messages m^1 and m^2, respectively, to the principal. The principal then chooses a project based on those messages. The advantage of centralization is rooted in the fact that the principal keeps the authority rights. This way she can perfectly adjust the final decision to the information she has received from the agents. The downside of centralization is that agents have an incentive to misrepresent their private information. More specifically, agents tend to either exaggerate or understate the information about their local conditions. This is due to the fact that agents want the principal's posterior of θ_i to be more extreme than the actual realization of θ_i. To see this, consider the agent 1's expected utility given the principal's decisions y_P

$$\mathbb{E}[U_1(y_P, \theta_1, \theta_2, \lambda)|\theta_1] = \mathbb{E}[-\lambda(y_P - \theta_1)^2 - (1-\lambda)(y_P - \theta_2)^2|\theta_1]. \tag{4.1}$$

After having received the agents' messages, the principal will optimally choose the average of both posteriors, that is, $y_P(m^1, m^2) = \frac{\mathbb{E}[\theta_1|m^1] + \mathbb{E}[\theta_2|m^2]}{2}$. Denote by $\rho_1 = \mathbb{E}[\theta_1|m^1]$ the principal's posterior of θ_1 given the message of agent 1. If agent 1 was able to determine ρ_1 by sending message m_1, he would want the principal to hold a posterior belief of θ_1 that maximizes his expected utility. That is,

$$\rho_1^* = \arg\max \mathbb{E}[-\lambda(y_P - \theta_1)^2 - (1-\lambda)(y_P - \theta_2)^2|\theta_1]. \tag{4.2}$$

Using in advance that in equilibrium $\mathbb{E}_{m_2}[\mathbb{E}[\theta_2|m^2]] = \mathbb{E}[\theta_2] = \frac{1}{2}$ will hold, we can calculate that $\rho_1^* = \frac{1}{2} + 2\lambda\theta_1 - \lambda$. This relationship reveals that for $\theta_1 > \frac{1}{2}$, it holds that $\rho_1^* > \theta_1$, implying that the agent wants the principal's posterior of θ_1 to be greater than the true θ_1. This means that the agent has an incentive to exaggerate his private information for all $\theta_1 > \frac{1}{2}$. In contrast, when $\theta_1 < \frac{1}{2}$, it holds that $\rho_1^* < \theta_1$ and the agent has an incentive to understate his private information to decrease the principal's posterior of θ_1. Only at $\theta_1 = \frac{1}{2}$, the agent is willing to report his private information truthfully as in this case $\rho_1^* = \theta_1$ holds.[2] For agent 2, the incentives to misrepresent his private information are analogous. Hence, under centralization, agents either want to exaggerate or understate their private information, which causes a loss of decision relevant information. The characteristics of a Perfect Bayesian communication equilibrium under centralization are summarized in the subsequent definition.

Definition 2. *An equilibrium consists of a signaling rule $q_i^C(\cdot)$ for each agent $i \in \{1,2\}$, where for each $\theta_i \in [0,1]$, $q_i^C(m^i|\theta_i)$ is the conditional probability of sending a message $m^i \in M_i$ given the local condition θ_i, two belief functions $g_i(\cdot)$ for the principal, where $g_i(\theta_i|m^i)$ denotes the probability of θ_i given message m^i, and a decision rule $y_P(\cdot)$ for the principal, where $y_P(m^1, m^2)$ is a mapping from the sets of feasible signals M_1 and M_2 to the set of actions $[0,1]$, such that:*

(i) for each $\theta_i \in [0,1]$, if $q_i^C(m^i|\theta_i) > 0$, then m^i maximizes the expected utility of agent i given the principal's decision rule $y_P(\cdot)$,

(ii) for each (m^1, m^2), $y_P(m^1, m^2)$ maximizes the expected utility of the principal given her belief functions $g_1(\theta_1|m^1)$ and $g_2(\theta_2|m^2)$, and

(iii) each belief function $g_i(\theta_i|m^i)$ is derived from $q_i^C(m^i|\theta_i)$ using Bayes Rule whenever possible.

We show that there always exist a partition equilibrium of any size $N \in \mathbb{N}$ (see Appendix). In these partition equilibria the domain of local conditions (i.e., $[0,1]$) is broken down into (sub)intervals and each agent only communicates the (sub)interval his true local condition lies in. Let $a^{C,N} = (a_{i,0}, \ldots, a_{i,k}, \ldots, a_{i,N})$ denote a partition of $[0,1]$ for agent i under centralization, where $0 < k < N$. Define for all $\underline{a}, \overline{a}, \underline{b}, \overline{b} \in [0,1]$, $\underline{a} < \overline{a}$, and $\underline{b} < \overline{b}$,

$$\overline{y}(\underline{a}, \overline{a}, \underline{b}, \overline{b}) \equiv \underset{y}{\operatorname{argmax}} \int_{\underline{a}}^{\overline{a}} \int_{\underline{b}}^{\overline{b}} U_P(y, \theta_1, \theta_2) d\theta_1 d\theta_2 = \frac{1}{2}\left(\frac{\underline{a} + \overline{a}}{2} + \frac{\underline{b} + \overline{b}}{2}\right).$$

The following proposition specifies the characteristics of partition equilibria under centralization.

Proposition 23. *Let $\lambda \in (\frac{1}{2}, 1)$, then for every $N \in \mathbb{N}$, there exists at least one equilibrium $(y_P(\cdot), q_1(\cdot), q_2(\cdot), g_1(\cdot), g_2(\cdot))$, where*

(i) $q_i(m^i|\theta_i)$ is uniform, supported on $[a_{i,k}, a_{i,k+1}]$ if $\theta_i \in (a_{i,k}, a_{i,k+1})$, $i \in \{1,2\}$

(ii) $g_i(\theta_i|m^i)$ is uniform, supported on $[a_{i,k}, a_{i,k+1}]$ if $m^i \in (a_{i,k}, a_{i,k+1})$, $i \in \{1,2\}$

(iii) $y_P(m^1, m^2) = \overline{y}(a_{1,k}, a_{1,k+1}, a_{2,l}, a_{2,l+1})$ if $m^1 \in (a_{i,k}, a_{i,k+1})$, $m^2 \in (a_{i,l}, a_{i,l+1})$

[2]The above described incentives of the agents to misrepresent their private information are analogous to the ones described in Alonso et al. (2008).

(iv) $a_{i,k+1} - a_{i,k} = a_{i,k} - a_{i,k-1} + 4(2\lambda - 1)(a_{i,k} - \frac{1}{2})$ *for* $k = 1, ..., N-1$, $i \in \{1, 2\}$

(v) $a_{i,0} = 0$ *and* $a_{i,N} = 1$, $i \in \{1, 2\}$.

All omitted proofs are in the appendix. Analyzing part (*iv*) from Proposition 23, i.e.,

$$a_{i,k+1} - a_{i,k} = a_{i,k} - a_{i,k-1} + \underbrace{4(2\lambda - 1)(a_{i,k} - \frac{1}{2})}_{\geq 0 \text{ iff } a_{i,k} \geq \frac{1}{2}}, \tag{4.3}$$

reveals how intervals evolve for the optimal signaling strategy of agent 1 and agent 2 in this equilibrium. The length of successive intervals decreases if $a_k < \frac{1}{2}$, and increases if $a_k > \frac{1}{2}$. This implies that intervals are small around $\frac{1}{2}$ and increase towards the lower and upper interval limit of $[0, 1]$. The rate at which the intervals decrease or increase is higher, the larger the agent's bias λ and the larger the larger $|a_{i,k} - \frac{1}{2}|$. We find that $\frac{1}{2} - a_{i,k} = a_{i,N-k} - \frac{1}{2}$ for each $k \leq N$, which means that intervals are symmetrically distributed around $\frac{1}{2}$. Two different cases can arise: When the partition has an even number of elements (i.e., $N = 2n$), it holds that $a_{\frac{N}{2}} = \frac{1}{2}$. Instead, if the partition has an odd number of elements (i.e., $N = 2n - 1$), there is a symmetric interval around $\frac{1}{2}$ where the principal's expected value of θ_1 is $\frac{1}{2}$.

The intuition for this result is rooted in the above discussed interest of the agents to either exaggerate or understate their private information when $\theta_i \neq \frac{1}{2}$. When sending their message agents internalize that the principal will adjust her decision equally to both received messages. Remember that each agent's prior belief of the other division's local condition is $\frac{1}{2}$. As a consequence, if $\theta_1 > \frac{1}{2} = \mathbb{E}[\theta_2]$, then $y_1^* > y_P^*$ and agent 1 lies to right of θ_1 to increase the principal's posterior of θ_1. In contrast, if $\theta_1 < \frac{1}{2}$, then $y_1^* < y_P^*$ and agent 1 lies to left of θ_1 to decrease the principal's posterior of θ_1. By overemphasizing his private information, agent 1 makes the principal implement a more extreme project choice either to the left or right of the true θ_1. The incentive to overemphasize his private information is larger the larger the agent's bias and the further away θ_1 is from $\frac{1}{2}$. Only at $\theta_1 = \frac{1}{2}$, preferences of the agent and the principal are aligned (i.e.,$y_1^* = y_P^*$) and the agent has no incentive to lie. This line of reasoning explains why intervals are small around $\frac{1}{2}$ and increase towards the extremer project decision 0 and 1.[3] The intuition for agent 2's signaling strategy is analogous.

4.2 Decentralization

Under decentralization, the principal delegates the decision rights to one of the agents and the other agent communicates horizontally his information to this authorized agent. We allow the principal to use a random delegation policy where agent 1 makes the decision with some probability α and agent 2 with the residual probability $1 - \alpha$. Note, however, that such a random assignment of authority does not influence the signaling strategies of the agents. This is due to the fact, that each agent realizes that his message only matters when the other agent makes the decision. That is, when an agent

[3] See also Alonso et al. (2008) for these characteristics and the intuition for the results.

i is authorized to make the decision he can perfectly adjust the decision to his true θ_i. In contrast, when the other agent makes the decision, his message is important as it influences the posterior belief of the authorized agent about θ_i. Since every random delegation scheme will result in the same signaling strategies of the agents, it is without loss of generality to restrict attention to deterministic delegation policies. Note that due to the symmetry in our model, it is irrelevant for the principal's expected payoff which agent makes the final decision. We conduct the analysis for agent 1 making the final decision and agent 2 communicating to agent 1. The analysis for agent 2 making the final decision and agent 1 communicating to agent 2 is analogous. The characteristics of a Perfect Bayesian communication equilibrium under decentralization are summarized in the next definition.

Definition 3. *An equilibrium consists of a family of a signaling rule $q_2^D(\cdot)$ for agent 2, where for each $\theta_2 \in [0,1]$, $q_2^D(m^2|\theta_2)$ is the conditional probability of sending a message m^2 given agent 2's local condition θ_2, a belief function $g_2(\cdot)$ for agent 1, where $g_2(\theta_2|m^2)$ denotes the probability of θ_2 given message m^2, and a decision rule $y_1(\cdot)$ for agent 1, where $y_1(m^2)$ is a mapping from the set of feasible signals M_2 to the set of actions $[0,1]$, such that:*

(i) for $\theta_2 \in [0,1]$, if $q_2^D(m^2|\theta_2) > 0$, then m^2 maximizes the expected utility of the agent 2 given agent 1's decision rule $y_1(\cdot)$, and

(ii) for each m^2, $y_1(m^2)$ maximizes the expected utility of agent 1 given his belief function $g_2(\theta_2|m^2)$, and

(iii) the belief function $g_2(\theta_2|m^2)$ is derived from $q_2^D(m^2|\theta_2)$ using Bayes Rule whenever possible.

Due to the delegation of decision rights to agent 1 under decentralization, agent 1 is free to choose the decision that maximizes his expected utility. Since this optimal decision from agent 1's point of view differs from the principal's optimal decision, the principal incurs a loss. Despite this disadvantage, there is also the advantage that the principal no longer experiences a loss due to noisy communication about θ_1 under decentralization. Agent 1 has perfect knowledge about θ_1 and can use all this information when making the decision. However, note that when the principal delegates the decision to one of her agents, she still faces a loss of information. Since the optimal decision depends on both pieces of information, agent 1 is reliant on the information transmitted by agent 2. By the same reason we gave in the previous section, agent 2 will introduce noise into his communication which causes a loss of information about θ_2. Let $\rho_2 = \mathbb{E}[\theta_2|m^2]$. Then, agent 2 wants agent 1 to have a posterior of θ_2 that maximizes his expected utility, i.e.,

$$\rho_2^* = \arg\max \mathbb{E}[-\lambda(y_1 - \theta_1)^2 - (1-\lambda)(y_1 - \theta_2)^2|\theta_2], \tag{4.4}$$

where $y_1 = \lambda\theta_1 + (1-\lambda)\rho_2$. For the optimal posterior from agent 2's perspective we get that $\rho_2^* = \frac{\frac{1}{2}-\lambda+\lambda\theta_1}{1-\lambda}$. Since either $\rho_2^* > \theta_2 > \frac{1}{2}$ or $\rho_2^* < \theta_2 < \frac{1}{2}$ holds, agent 2 wants to either exaggerate or understate his private information whenever $\theta_2 \neq \frac{1}{2}$. At $\theta = \frac{1}{2}$, it holds that $\rho_2^* = \frac{1}{2}$ and agent 2 has no incentive to misrepresent his private information. Hence, the loss the principal incurs under decentralization is a mixture of the loss of control - due to the biased agent 1 making the decision - and the loss of information - due to agent 2's incentive to misrepresent decision relevant information to agent 1.

We find that also under decentralization there always exist a partition equilibrium of any size $N \in \mathbb{N}$. The next proposition summarizes this result. Let $b^{D,N} = (b_0, \ldots, b_j, \ldots, b_N)$ denote a partitioning of $[0,1]$ for agent 2 under decentralization, where $0 < j < N$, and define for all $\underline{b}, \overline{b} \in [0,1]$, $\underline{b} < \overline{b}$,

$$\overline{y}(\underline{b}, \overline{b}) \equiv \underset{y}{\text{argmax}} \int_{\underline{b}}^{\overline{b}} U_1(y, \theta_1, \theta_2, \lambda) d\theta_2 = \lambda\theta_1 + (1-\lambda)\frac{\underline{b} + \overline{b}}{2}.$$

Proposition 24. *Let $\lambda \in (\frac{1}{2}, 1)$, then for every $N \in \mathbb{N}$, there exists at least one equilibrium $(y_1(\cdot), q_2(\cdot), g_2(\cdot))$, where*

(i) $q_2(m^2|\theta_2)$ *is uniform, supported on* $[b_j, b_{j+1}]$ *if* $\theta_2 \in (b_j, b_{j+1})$

(ii) $g_2(\theta_2|m^2)$ *is uniform, supported on* $[b_j, b_{j+1}]$ *if* $m^2 \in (b_j, b_{j+1})$

(iii) $y_1(m^2) = \overline{y}(b_j, b_{j+1})$ *for all* $m^2 \in (b_j, b_{j+1})$

(iv) $b_{j+1} - b_j = b_j - b_{j-1} + \frac{4(2\lambda-1)}{1-\lambda}(b_j - \frac{1}{2})$ *for* $j = 1, ..., N-1$

(v) $b_0 = 0$ *and* $b_N = 1$.

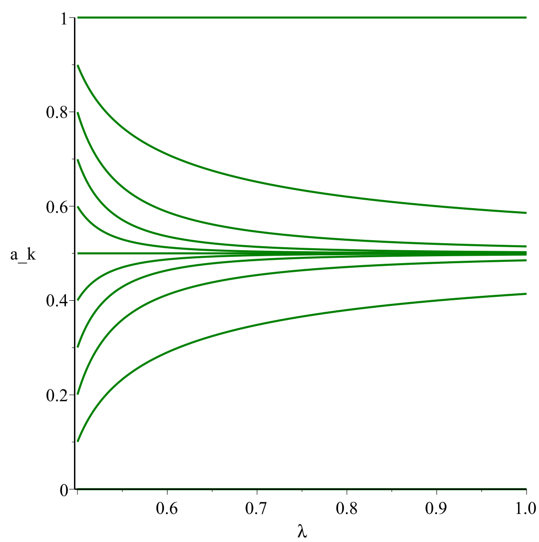


Figure 4.1: $a^{C,10}(b)$ for $N = 10$.

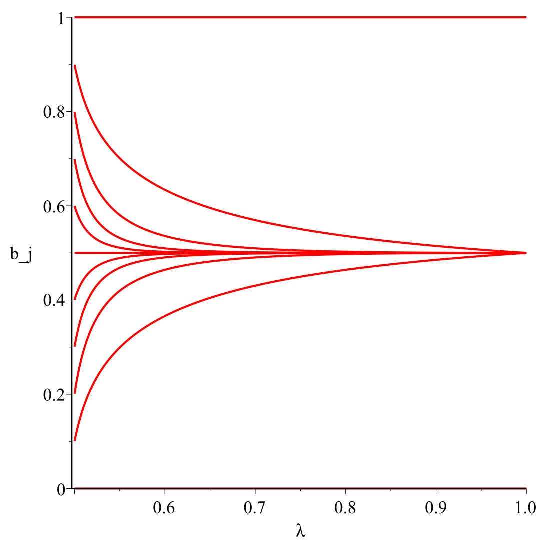


Figure 4.2: $b^{D,10}(b)$ for $N = 10$.

Part (iii) of Proposition 24 shows that the characteristics of intervals under decentralization are the same as under centralization. We refer the reader to section 4.1, where we discuss these characteristic in detail. Also the intuition for these properties of intervals is analogous to before. The difference under decentralization is that instead of the principal, agent 1 makes the decision. Since the conflict of interest between agents is more severe than between the principal and the agents, agent 2's incentive to overemphasize his private information is even stronger under decentralization than under centralization. That is, $y_2^* > y_P^* > y_1^*$ if $\theta_2 > \frac{1}{2}$ and $y_2^* < y_P^* < y_1^*$ if $\theta_2 < \frac{1}{2}$. At θ_2 the preferences of both agents are aligned and there is no need for agent 2 to lie to the right or left of $\frac{1}{2}$. Figures 4.1 and 4.2 depict how intervals evolve under centralization and decentralization in an equilibrium of size $N = 10$.

4.3 Expected Payoff under Centralization and Decentralization

In this section we analyze under which organizational form the principal is better off. For this purpose we calculate the principal's expected payoff under centralization and decentralization and analyze how it changes with an increasing partition size.

Proposition 25. *In the centralized limit equilibrium* $(q_1^C(\cdot), q_2^C(\cdot), g_1(\cdot), g_2(\cdot), y_P(\cdot))$ *and the decentralized limit equilibrium* $(q_2^D(\cdot), g_2(\cdot), y_1(\cdot))$ *for* $N \to \infty$*, the principal's and the agents' expected payoffs* $E[U_P^\infty]$ *and* $E[U_{A_i}^\infty]$ *are higher than in any other equilibrium with a finite* $N \in \mathbb{N}$*. The expected payoff of the principal in the limit equilibrium under centralization is*

$$E[U_P^{C,\infty}] = -\frac{5\lambda - 1}{6(8\lambda - 1)} \tag{4.5}$$

and under decentralization

$$E[U_P^{D,\infty}] = \frac{\lambda((9 - 8\lambda)\lambda - 6) + 2}{6(5\lambda - 1)}. \tag{4.6}$$

Proposition 25 says that the greater N, that is, the more messages the agents use in equilibrium, the greater is the principal's and the agents' expected payoff.[4] Having established that under both organizational it is best for the principal and the agents to coordinate on the limit equilibrium, we are now ready to analyze under which organizational form communication is more meaningful. That is, which organizational form allows for a higher quality of communication. As Alonso et al. (2008), we measure the quality of communication by the residual variance $E[(\theta_i - E[\theta_i|m_i])^2] = E[(\theta_i^2 - 2\theta_i\overline{m}_i + \overline{m}_i^2)] = \frac{1}{3} - E[\overline{m}_i^2]$ for $i \in \{1, 2\}$.

Proposition 26. *The quality of communication is always strictly higher under centralization than under decentralization. The residual variances under both organizational forms is given by*

$$E[(\theta_i - E[\theta_i|m_i])^2] = \begin{cases} \frac{1}{3} - \frac{10\lambda-1}{4(8\lambda-1)} & \textit{if centralization} \\ \frac{1}{3} - \frac{6\lambda-1}{4(5\lambda-1)} & \textit{if decentralization.} \end{cases}$$

[4] The proof that the limit of strategies and beliefs constitute a Perfect Bayesian Equilibrium in Alonso et al. (2008) applies to our setting. For the proof we refer the reader to Alonso et al. (2008).

We find that the quality of communication is always higher (i.e., the residual variance is always smaller) under centralization than under decentralization. Under centralization each agent transmits more precise information to the principal than agent 2 to agent 1 under decentralization. This result is driven by the misalignment of interest between the principal and the agents. The conflict of interest between the agents is more severe than the mismatch of preferences between the principal and the agents. As a consequence, agents introduce less noise when communicating information to the principal than when communicating to each other. Aside from communication under centralization always having a smaller residual variance than under decentralization, the difference in residual variances grows larger as the agents' biases increase. A higher bias has a more detrimental effect on the precision of the transmitted information under decentralization than under centralization (see also Figures 4.1 and 4.2). This is in line with Alonso et al. (2008). In Alonso et al. (2008), vertical communication to a headquarter is in general more efficient than the horizontal communication between division managers. However, the difference in the quality of communication vanishes as their need for coordination bias converges to infinity. In our setting coordination is prescribed. Thus, the difference in the quality of communication always exists. Figure 4.3 shows the residual variance of both, communication under centralization and decentralization.

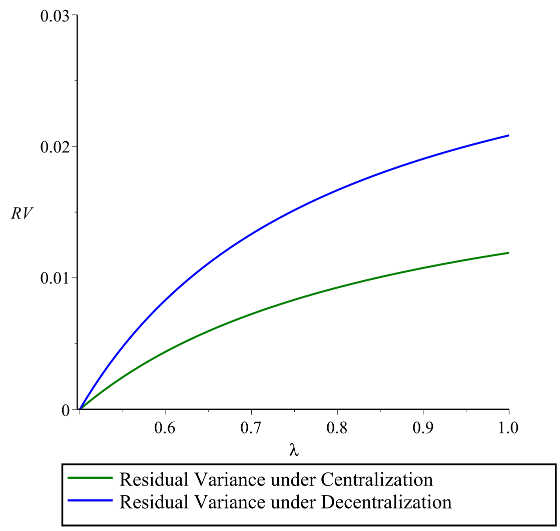


Figure 4.3: residual variance

Note that although the transmitted information is less precise for one local condition under decentralization, there is perfect knowledge about the local condition of the authorized agent. This means that under decentralization more information is lost for one state of the world but no information is lost for the other state of the world. As a consequence, under centralization the principal suffers from a higher loss of information but profits from a better quality of communication. Under decentralization she experiences a loss of authority and a lower quality of communication but benefits from agent 1's perfect knowledge about his local condition. We find that when the principal and the agents coordinate on the limit equilibrium, centralization beats decentralization for all possible biases $\lambda \in (\frac{1}{2}, 1)$.

Proposition 27. *Let $N \to \infty$. Centralization yields the principal a higher expected payoff than decentralization for all $\lambda \in (\frac{1}{2}, 1)$.*

This contrasts the results of Alonso et al. (2008), who find that decentralization can dominate centralization even when the coordination of decisions is extremely important. When the need for coordination bias in Alonso et al. (2008) converges to infinity, both organizational forms perform equally well. In contrast, we find that when decisions must be coordinated, decentralization cannot dominate centralization even if the agents' bias is small. While for small biases (λ close to $\frac{1}{2}$) the difference in the principal's expected payoff under centralization and decentralization is small, this difference grows larger as λ increases. More specifically, as the agents' bias increases, the principal's expected payoff under centralization decreases far more slowly than her expected payoff under decentralization. While under centralization the principal does not suffer significantly from larger biases, under decentralization she is even as worse off as in the case where she makes a totally uninformed decision. Figure 4.4 illustrates the principal's expected payoff under the three different organizational forms: centralization, decentralization and uninformed decision making for all λ.[5]

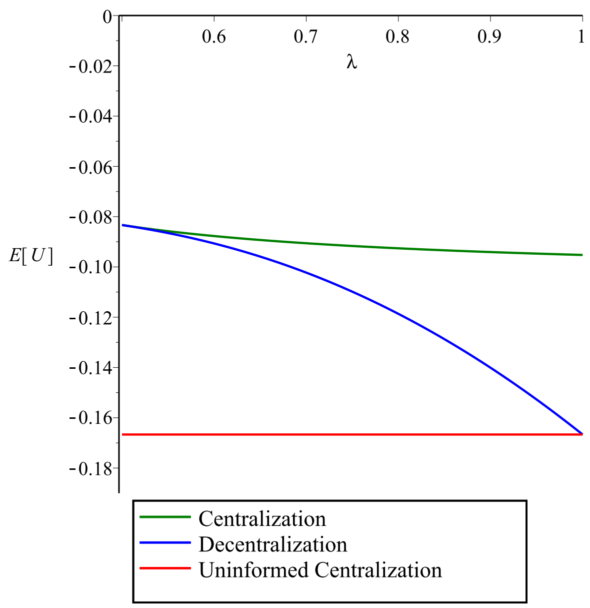


Figure 4.4: principal's expected payoff

Why can decentralization never perform better than centralization in our model? Our explanation is the following: In Alonso et al. (2008) the optimal decisions of the headquarter manager and division managers are the same when the need for coordination converges to infinity. That is, they all want to perfectly coordinate their decisions. At the same time the difference in the quality of communication vanishes as the need

[5] When the principal makes an uninformed decision she chooses $y_P = \frac{1}{2}(E[\theta_1 + \theta_2])$. This yields her a constant expected payoff of $\frac{1}{12}$.

for coordination parameter converges to infinity. As a consequence, centralization and decentralization perform equally well in the limit. In our model, in contrast, the principal and the agents' optimal decisions can never match. The principal always wants to choose the average of the posteriors whereas the agents always want to choose the weighted average of posteriors. Since agents communicate to the principal with less noise than to each other and the principal keeps the authority, centralization dominates decentralization even for very small biases of the agents. The perfect adaption to the true θ_1 under decentralization does not outweigh the loss due to more noisy communication by the agent 2 and the biased decision making of agent 1. Even when the preferences of the principal and the agents are close to alignment, the delegation of decision rights to the agents is never optimal.

Considering Different Biases

Next, we allow for agent 1 and agent 2 to have different biases $\lambda_1,\ \lambda_2 \in (\frac{1}{2}, 1)$. That is

$$U_1(y, \theta_1, \theta_2, \lambda_1) = -\lambda_1(y-\theta_1)^2 - (1-\lambda_1)(y-\theta_2)^2 \tag{U_1}$$

$$U_2(y, \theta_1, \theta_2, \lambda_2) = -(1-\lambda_2)(y-\theta_1)^2 - \lambda_2(y-\theta_2)^2. \tag{U_2}$$

The analysis for centralization and decentralization proceeds in the same way as before. However, the corresponding cutoffs for the communication equilibria under both organizational forms change as the agents are now differently biased. Our setting is no longer symmetric. Thus, it is no longer without loss of generality to focus on delegating the decision to one agent with certainty. Given the random delegation scheme $(\alpha, 1-\alpha)$, both agents communicate their private information to each other and agent 1 makes the final decision with probability α and agent 2 with probability $1-\alpha$. Note that the randomness does not affect the cutoffs of the agent's signaling strategy since they condition their message on the event of being pivotal, i.e., the event in which their message matters. Hence, for the calculation of the communication cutoffs under decentralization we can again focus on agent 2 communicating to agent 1 and agent 1 making the decision with probability one.[6] We find that when agents differ in their biases, our previous results still hold. The principal's and the agents' expected payoffs are increasing in the equilibrium size and centralization beats decentralization for all biases of the agents.

Proposition 28. *If agents have different biases $\lambda_1, \lambda_2 \in (\frac{1}{2}, 1)$, then Proposition 25 still holds. The expected payoff of the principal in the limit equilibrium under centralization is*

$$E[U_P^{C,\infty}] = -\frac{1}{48} + \sum_{i=1}^{2} \frac{1}{32(8\lambda_i - 1)}$$

[6]In order to get the thresholds for the case in which agent 1 communicates to agent 2 and agent 2 makes the final decision with probability one, we just have to switch λ_1 and λ_2 in the formulas.

and under decentralization

$$E[U_P^{D,\infty,\alpha}] = -\frac{1}{3}$$
$$+ 2\sum_{i=1}^{2}(\mathbb{1}_{i=1}\alpha + \mathbb{1}_{i=2}(1-\alpha))(\frac{1}{12}(1-2\lambda_i+2\lambda_i^2) + \lambda_i(1-\lambda_i)\frac{5\lambda_{-i}+\lambda_i-1}{4(4\lambda_{-i}+\lambda_i-1)}),$$

where $E[U_P^{D,\infty,\alpha}]$ is strictly increasing in the probability with which the less biased agent makes the decision. Centralization yields the principal a higher expected payoff than decentralization for all $\lambda_1, \lambda_2 \in (\frac{1}{2}, 1)$.

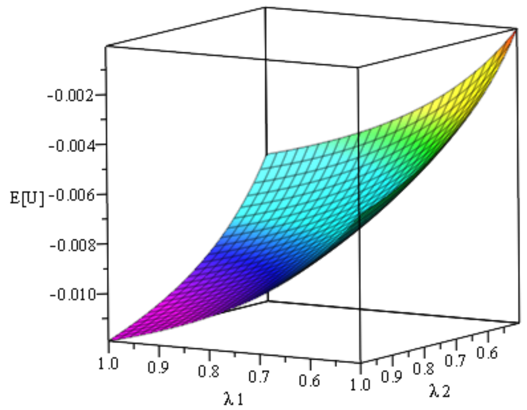


Figure 4.5: $E[U_P^{C,\infty}]$.

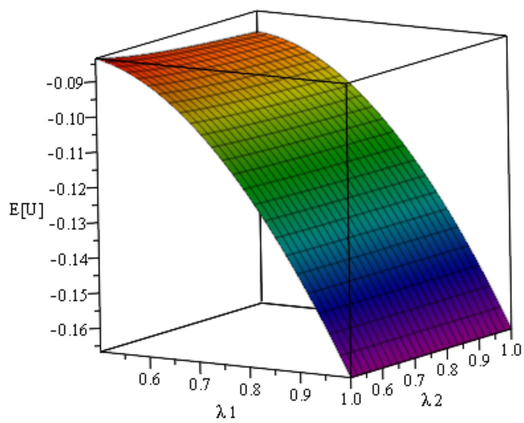


Figure 4.6: $E[U_P^{D,\infty,\frac{1}{2}}]$.

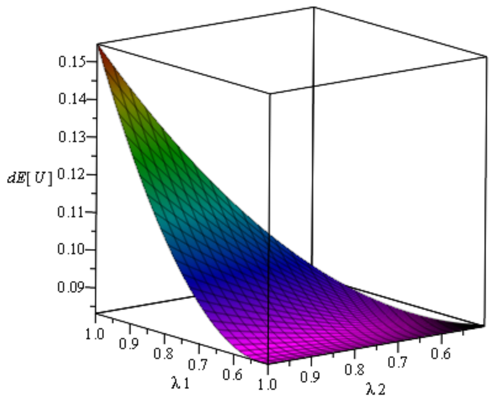


Figure 4.7: $E[U_P^{C,\infty}] - E[U_P^{D,\infty,1}]$.

Figure 4.5 illustrates the principal's expected payoff under centralization if the agents have different biases. Figure 4.6 illustrates the principal's expected payoff under decentralization for $\alpha = \frac{1}{2}$, that is, when both agents make the decision with probability

$\frac{1}{2}$. The principal's expected payoff under decentralization increases in the probability with which the less biased agent makes the decision. As a consequence, the principal optimally chooses $\alpha = 1$ when $\lambda_1 \leq \lambda_2$ and $\alpha = 0$ when $\lambda_2 < \lambda_1$. Assume without loss of generality that $\frac{1}{2} < \lambda_1 \leq \lambda_2 < 1$. Comparing the principal's expected payoff under centralization with his expected payoff under decentralization for $\alpha = 1$ shows that centralization beats decentralization for all biases of the agents. Figure 4.7 illustrates the difference in expected payoffs under centralization and decentralization and $\alpha = 1$.

5. Conclusion

In the context of information asymmetries and conflicting interests, we analyze which organizational form should be chosen by a principal - given that her company is forced to coordinate the decisions of its two divisions. The optimal decision depends on two pieces of information and each division manager is privately informed about one piece. While the principal wants to adapt the decision to both pieces of information equally, the division managers are biased towards matching the decision more to their private information. The principal can either communicate with both division managers and make the decision herself (centralization) or delegate the decision to one of the division managers and let division managers exchange their information with each other (decentralization). We find that centralization outperforms decentralization for every bias of the division managers.

Our work extends the analysis of Alonso et al. (2008) who address this problem for a company that is free to choose two different decisions, one for each division, but that also faces a need for coordination. In contrast to our result, in Alonso et al. (2008), decentralization can dominate centralization even when the need for coordination is highly important. When the need for coordination converges to infinity, both organizational forms perform equally well. In this case the quality of communication under centralization and decentralization is essentially the same and the interests of the principal and the division managers are aligned. Our model differs in two ways: First, the quality of of communication under centralization is always better than under decentralization. Second, there always exists a conflict of interest between the principal and the agents. These two differences change results such that centralization always performs strictly better than decentralization when the organization can only make one single decision.

A. Appendix

A.1 Proof of Proposition 23

In a first step we have to show that the principal's decision $\overline{y}(a_k, a_{k+1}, b_j, b_{j+1})$ is a best response to the agents' messages $m^1 \in (a_k, a_{k+1})$ and $m^2 \in (b_j, b_{j+1})$, $\forall k, j \in \{0, \dots, N\}$. Since agent 1 uniformly randomizes his message over $[a_k, a_{k+1}]$ if $\theta_1 \in (a_k, a_{k+1})$ and agent 2 over $[b_j, b_{j+1}]$ if $\theta_2 \in (b_j, b_{j+1})$, the principal's best response upon receiving $m^1 \in (a_k, a_{k+1})$ and $m^2 \in (b_j, b_{j+1})$ is

$$\underset{y}{\operatorname{argmax}} \int_{a_k}^{a_{k+1}} \int_{b_j}^{b_{j+1}} U_P(y, \theta_1, \theta_2) d\theta_1 d\theta_2 = \frac{1}{2}\left(\frac{a_k + a_{k+1}}{2} + \frac{b_j + b_{j+1}}{2}\right). \tag{4.7}$$

Second, in order for an agent's signaling rule to be a best response to the principal's decision $\overline{y}(a_k, a_{k+1}, b_j, b_{j+1}), \forall k, j \in \{0, \dots, N\}$, no agent may have an incentive to lie about the interval in which the true θ_i lies. We conduct this analysis for agent 1. By the symmetry of agents in our model the analysis for agent 2 is exactly the same. As before, agent 1 must be indifferent between sending messages $m_k \in (a_{k-1}, a_k)$ and $m_{k+1} \in (a_k, a_{k+1})$ when θ_1 coincides with the threshold a_k.[7] Define $\overline{m}_k = \frac{a_{k-1}+a_k}{2}$, $\overline{m}_{k+1} = \frac{a_k+a_{k+1}}{2}$, $\overline{m}_j = \frac{b_{j-1}+b_j}{2}$, and $\overline{m}_{j+1} = \frac{b_j+b_{j+1}}{2}$.

Formally this means that

$$U_1(y_{k,j}, a_k, \theta_2, \lambda) = U_1(y_{k+1,j}, a_k, \theta_2, \lambda), \tag{4.8}$$

$$\Leftrightarrow \quad -\lambda \sum_{j=0}^{N} \int_{b_{j-1}}^{b_j} (\underbrace{\frac{1}{2}(\overline{m}_k + \overline{m}_j)}_{y_{k,j}} - a_k)^2 d\theta_2 - (1-\lambda) \sum_{j=0}^{N} \int_{b_{j-1}}^{b_j} (\underbrace{\frac{1}{2}(\overline{m}_k + \overline{m}_j)}_{y_{k,j}} - \theta_2)^2 d\theta_2 \quad (IC_1)$$

$$= \quad -\lambda \sum_{j=0}^{N} \int_{b_{j-1}}^{b_j} (\underbrace{\frac{1}{2}(\overline{m}_{k+1} + \overline{m}_j)}_{y_{k+1,j}} - a_k)^2 d\theta_2 - (1-\lambda) \sum_{j=0}^{N} \int_{b_{j-1}}^{b_j} (\underbrace{\frac{1}{2}(\overline{m}_{k+1} + \overline{m}_j)}_{y_{k+1,j}} - \theta_2)^2 d\theta_2, \tag{4.9}$$

where $y_{k,j} = \overline{y}(a_{k-1}, a_k, b_{j-1}, b_j)$ and $y_{k+1,j} = \overline{y}(a_k, a_{k+1}, b_{j-1}, b_j)$. After some calculations we get that this indifference condition yields

$$a_{k+1} + 8(\frac{1}{4} - \lambda)a_k + a_{k-1} = 2(1 - 2\lambda) \tag{4.10}$$

$$\Leftrightarrow \quad a_{k+1} - a_k = a_k - a_{k-1} + 4(2\lambda - 1)(a_k - \frac{1}{2}) \tag{4.11}$$

Next we solve the inhomogeneous second-order difference equation 4.22 to get the thresholds for agent 1's signaling strategy. The characteristic equation of the homoge-

[7] Likewise, agent 2 must be indifferent between sending $m_j \in (b_{j-1}, b_j)$ and $m_{j+1} \in (b_j, b_{j+1})$ when θ_2 coincides with the threshold b_j.

neous difference equation is given by

$$z^2 - 8(\lambda - \frac{1}{4})z + 1 = 0. \tag{4.12}$$

The respective roots of this characteristic equation are

$$x = \sqrt{16(\lambda - \frac{1}{4})^2 - 1} + 4(\lambda - \frac{1}{4}) \quad \text{and} \quad y = -\sqrt{16(\lambda - \frac{1}{4})^2 - 1} + 4(\lambda - \frac{1}{4}), \tag{4.13}$$

where $xy = 1$ and $x > 1$. A particular solution ϕ to the inhomogeneous indifference equation is given by $\phi = \frac{1}{2}$. Consequently, the general solution of the difference equation is

$$a_k = \nu x^k + \mu y^k + \frac{1}{2}. \tag{4.14}$$

In order to determine ν and μ we use the two initial conditions, $a_0 = 0$ and $a_N = 1$, and solve the corresponding equations (i) $a_0 = \nu + \mu + \frac{1}{2} = 0$ and (ii) $a_N = \nu x^N + \mu y^N + \frac{1}{2} = 1$ for ν and μ. This yields

$$\nu = \frac{1 + y^N}{2(x^N - y^N)} \quad \text{and} \quad \mu = -\frac{1 + x^N}{2(x^N - y^N)}. \tag{4.15}$$

Finally, we substitute for ν and μ in the general solution and obtain the solution of the difference equation. That is,

$$a_k = \frac{1}{2}\left(\frac{x^k(1 + y^N) - y^k(1 + x^N)}{x^N - y^N} + 1\right) \quad \text{for } 0 \leq k \leq N. \tag{4.16}$$

As stated before, the analysis for agent 2 is analogous and we get exactly the same thresholds for the signaling strategy of agent 2. That is,

$$b_j = \frac{1}{2}\left(\frac{x^k(1 + y^N) - y^k(1 + x^N)}{x^N - y^N} + 1\right) \quad \text{for } 0 \leq j \leq N. \tag{4.17}$$

A.2 Proof of Proposition 24

First, we have to show that agent 1's decision $\overline{y}(b_j, b_{j+1})$ is a best response to agent 2's messages $m^2 \in (b_j, b_{j+1})$ $\forall j$. Since agent 2 uniformly randomizes his message over $[b_j, b_{j+1}]$ if $\theta_2 \in (b_j, b_{j+1})$, agent 1's best response upon receiving $m^2 \in (b_j, b_{j+1})$ is

$$\underset{y}{\text{argmax}} \int_{b_j}^{b_{j+1}} U_1(y, \theta_1, \theta_2, \lambda) d\theta_2 = \lambda\theta_1 + (1 - \lambda)\frac{b_j + b_{j+1}}{2}. \tag{4.18}$$

Second, in order for agent 2's signaling rule to be a best response to agent 1's decision $\overline{y}(b_j, b_{j+1})$ $\forall j$, agent 2 must not have an incentive to lie about the interval in which the true θ_2 lies. This requires that agent 2 is indifferent between sending messages

$m_j \in (b_{j-1}, b_j)$ and $m_{j+1} \in (b_j, b_{j+1})$ when θ_2 coincides with the threshold b_j.

Formally this means that

$$U_2(y_j, \theta_1, b_j, \lambda) = U_1(y_{j+1}, \theta_1, b_j, \lambda), \tag{4.19}$$

or

$$-(1-\lambda)\int_0^1 (\underbrace{\lambda\theta_1 + (1-\lambda)m_j}_{y_j} - \theta_1)^2 d\theta_1 - \lambda \int_0^1 (\underbrace{\lambda\theta_1 + (1-\lambda)m_j}_{y_j} - b_j)^2 d\theta_1 \tag{4.20}$$

$$= -(1-\lambda)\int_0^1 (\underbrace{\lambda\theta_1 + (1-\lambda)m_{j+1}}_{y_{j+1}} - \theta_1)^2 d\theta_1 - \lambda \int_0^1 (\underbrace{\lambda\theta_1 + (1-\lambda)m_{j+1}}_{y_{j+1}} - b_j)^2 d\theta_1, \tag{4.21}$$

where $y_j = \overline{y}(b_{j-1}, b_j)$ and $y_{j+1} = \overline{y}(b_j, b_{j+1})$. After some calculations we get that this indifference condition yields

$$b_{j+1} - \frac{2(3\lambda-1)}{1-\lambda} b_j + b_{j-1} = \frac{2(1-2\lambda)}{1-\lambda} \tag{4.22}$$

$$\Leftrightarrow \quad b_{j+1} - b_j = b_j - b_{j-1} + \underbrace{\frac{4(2\lambda-1)}{1-\lambda}(b_j - \frac{1}{2})}_{\geq 0 \text{ iff } b_j \geq \frac{1}{2}} \tag{4.23}$$

Next we solve the difference equation. The characteristic equation of this inhomogeneous second-order linear difference equation 4.22 is given by

$$z^2 - \frac{2(3\lambda-1)}{1-\lambda} z + 1 = 0. \tag{4.24}$$

The respective roots of the characteristic equation are

$$x = \sqrt{\left(\frac{3\lambda-1}{1-\lambda}\right)^2 - 1} + \frac{3\lambda-1}{1-\lambda} \quad \text{and} \quad y = -\sqrt{\left(\frac{3\lambda-1}{1-\lambda}\right)^2 - 1} + \frac{3\lambda-1}{1-\lambda}, \tag{4.25}$$

where $xy = 1$ and $x > 1$. An individual solution ϕ to the inhomogeneous indifference equation is given by $\phi = \frac{1}{2}$. Consequently, the general solution of the difference equation is

$$b_j = \nu x^j + \mu y^j + \frac{1}{2}. \tag{4.26}$$

In order to determine ν and μ we use the two initial conditions, $a_0 = 0$ and $a_N = 1$, and solve the corresponding equations (i) $a_0 = \nu + \mu + \frac{1}{2} = 0$ and (ii) $a_N = \nu x^N + \mu y^N + \frac{1}{2} = 1$ for ν and μ. This yields

$$\nu = \frac{1 + y^N}{2(x^N - y^N)} \quad \text{and} \quad \mu = -\frac{1 + x^N}{2(x^N - y^N)}. \tag{4.27}$$

Finally, we substitute for ν and μ in the general solution and obtain the solution of the difference equation. That is,

$$b_j = \frac{1}{2}\left(\frac{x^j(1+y^N) - y^j(1+x^N)}{x^N - y^N} + 1\right) \quad \text{for } 0 \leq j \leq N. \tag{4.28}$$

A.3 Proof of Proposition 25

Proof. To prove Proposition 25 we first calculate the expected payoff of the principal for a finite N and then let N converge towards infinity. Given the equilibrium strategies and beliefs, the principal's expected payoff in the centralized communication equilibrium of size N can be expressed in terms of the residual variance of θ_1 and θ_2 (Crawford and Sobel, 1982). That is,

$$\begin{aligned} E[U_P^{C,N}] &= -E[(y_{k,j}-\theta_1)^2 + (y_{k,j}-\theta_2)^2] & (4.29)\\ &= -(E[(\frac{1}{2}(\overline{m}_k + \overline{m}_j) - \theta_1)^2] + E[(\frac{1}{2}(\overline{m}_k + \overline{m}_j) - \theta_2)^2]) & (4.30)\\ &= -\left(\sum_{k=1}^{N}\sum_{j=1}^{N}\int_{a_{k-1}}^{a_k}\int_{b_{j-1}}^{b_j}\left(\frac{1}{2}(\overline{m}_k + \overline{m}_j) - \theta_1\right)^2 + \left(\frac{1}{2}(\overline{m}_k + \overline{m}_j) - \theta_2\right)^2 d\theta_1 d\theta_2\right). & (4.31)\end{aligned}$$

Analogously, the principal's expected payoff in the decentralized communication equilibrium of size N is given by[8]

$$\begin{aligned} E[U_P^{D,N}] &= -E[(y_j-\theta_1)^2 + (y_j-\theta_2)^2] & (4.32)\\ &= -\left(\sum_{j=1}^{N}\int_{0}^{1}\int_{b_{j-1}}^{b_j}(\lambda\theta_1 + (1-\lambda)\overline{m}_j - \theta_1)^2 + (\lambda\theta_1 + (1-\lambda)\overline{m}_j - \theta_2)^2\ d\theta_1 d\theta_2\right). & (4.33)\end{aligned}$$

To calculate the expected payoffs under centralization and decentralization we need the following Lemma which is based on Lemma 1, A1 and A2 from Alonso et al. (2008) and adjusted to our setting.

Lemma 12.

(i) $E[\overline{m}_k\theta_1] = E[\overline{m}_k^2]$ *and* $E[\overline{m}_j\theta_2] = E[\overline{m}_j^2]$,

(ii) $E[\overline{m}_k^2] = E[\overline{m}_j^2] = \frac{1}{16}\left[\frac{(x^{3N}-1)(x+1)^2}{(x^N-1)^3(x^2+x+1)} - \frac{x^N(x+1)^2}{x(x^N-1)^2} + 4\right]$,

(iii) $E[\overline{m}_k^2]$ *is strictly increasing in* N, *and*

[8]Note that we focus here without loss of generality on the decentralized communication equilibrium in which only agent 2 communicates his private information to agent 1 and agent 1 makes the decision afterwards

(iv) $\lim_{N\to\infty} E[\overline{m}_k^2] = \frac{1}{16}\left(\frac{(x+1)^2}{4(x^2+x+1)}+4\right) = \begin{cases} \frac{10\lambda-1}{4(8\lambda-1)} & \textit{if centralization} \\ \frac{6\lambda-1}{4(5\lambda-1)} & \textit{if decentralization} \end{cases}.$

Proof. Note that the approach is the same as the one used in Alonso et al. (2008). Given that $\overline{m}_k = \frac{a_{k-1}+a_k}{2} = E[\theta_1|\theta_1 \in (a_{k-1}, a_k)]$, we can use the law of iterated expectation and obtain

$$E[\overline{m}_k\theta_1] = E[E[\overline{m}_k\theta_1|\theta_1 \in (a_{k-1}, a_k)]] = E[\overline{m}_k E[\theta_1|\theta_1 \in (a_{k-1}, a_k)]] = E[\overline{m}_k^2]. \quad (4.34)$$

This proves part (i). To prove part (ii) and (iii) we need to calculate $E[\overline{m}_k^2]$ which can be expressed as follows

$$E[\overline{m}_k^2] = \sum_{i=1}^{N}\int\limits_{a_{k-1}}^{a_k}(\overline{m}_k)^2\,d\theta = \sum_{i=1}^{N}\int\limits_{a_{k-1}}^{a_k}\left(\frac{a_{k-1}+a_k}{2}\right)^2\,d\theta \quad (4.35)$$

$$= \frac{1}{4}\sum_{i=1}^{N}(a_k - a_{k-1})(a_k^2 + 2a_ka_{k-1} + a_{k-1}^2) \quad (4.36)$$

$$= \frac{1}{4}\sum_{i=1}^{N}(a_k^3 + a_k^2a_{k-1} - a_ka_{k-1}^2 - a_{k-1}^3) \quad (4.37)$$

Next we substitute equation 4.16 for a_k and get

$$E[\overline{m}_k^2] = \frac{1}{32}\sum_{i=1}^{N}\left(\frac{x^i(1+y^N)-y^i(1+x^N)}{x^N-y^N}+1\right)^3 \quad (4.38)$$

$$+\left(\frac{x^i(1+y^N)-y^i(1+x^N)}{x^N-y^N}+1\right)^2\cdot\left(\frac{x^{i-1}(1+y^N)-y^{i-1}(1+x^N)}{x^N-y^N}+1\right) \quad (4.39)$$

$$-\left(\frac{x^i(1+y^N)-y^i(1+x^N)}{x^N-y^N}+1\right)\cdot\left(\frac{x^{i-1}(1+y^N)-y^{i-1}(1+x^N)}{x^N-y^N}+1\right)^2 \quad (4.40)$$

$$-\left(\frac{x^{i-1}(1+y^N)-y^{i-1}(1+x^N)}{x^N-y^N}+1\right)^3 \quad (4.41)$$

Let $c = \frac{1}{32}\frac{1}{(x^N-y^N)^3}$, $d = \frac{1}{8}\frac{1}{(x^N-y^N)^2}$ and $e = \frac{1}{8}\frac{1}{x^N-y^N}$. After several rewriting and calculation steps we get

$$E[\overline{m}_k^2] =$$

$$c\cdot[\frac{(1+y^N)^3(x-1)(x+1)^2(x^{3N}-1)}{x^3-1} - \frac{(1+x^N)^3(y-1)(y+1)^2(y^{3N}-1)}{y^3-1} \quad (4.42)$$

$$-\frac{(1+y^N)^2(1+x^N)(x+1)^2x(x^N-1)}{x^2} + \frac{(1+x^N)^2(1+y^N)(y+1)^2y(y^N-1)}{y^2}]$$

$$+\,d\cdot[(1+y^N)^2(x^{2N}-1)+(1+x^N)^2(y^{2N}-1)] \quad (4.43)$$

$$+\,e\cdot[(1+y^N)(x^N-1)-(1+x^N)(y^N-1)] \quad (4.44)$$

By using that $y = \frac{1}{x}$ and $xy = 1$ and by performing similar calculations as in the proof of Lemma 11, we obtain

$$E[\overline{m}_k^2] = \frac{1}{16}\left[\frac{(1+x^N)^3(1+x)^2(x^{3N}-1)}{(x^{2N}-1)^3(x^2+x+1)} - \frac{(x^N+1)^3(x+1)^2(1-x^N)x^N}{x(x^{2N}-1)^3} + 4\right] \tag{4.45}$$

$$= \frac{1}{16}\left[\frac{(x^{3N}-1)(x+1)^2}{(x^N-1)^3(x^2+x+1)} - \frac{x^N(x+1)^2}{x(x^N-1)^2} + 4\right]. \tag{4.46}$$

This proves part (*ii*).

To prove part (*iii*) we use the same approach as in the proof of Lemma 11 part (*iii*). First, we define

$$f(p) = \frac{1}{16}\left[\frac{(p^3-1)(x+1)^2}{(p-1)^3(x^2+x+1)} - \frac{p(x+1)^2}{x(p-1)^2} + 4\right] \tag{4.47}$$

and then calculate the derivative of $f(p)$

$$\begin{aligned} f'(p) &= \frac{1}{16}\left[\frac{(x+1)^2}{(x^2+x+1)}\left(\frac{3p^2(p-1)^3 - 3(p^3-1)(p-1)^2}{(p-1)^6}\right)\right] \qquad (4.48)\\ &= \frac{1}{16}\left[\frac{(x+1)^2}{x}\left(\frac{(p-1)^2(p+1)}{(p-1)^4}\right)\right]\\ &= \frac{1}{16}\left[\frac{-3(x+1)^2-(p+1)}{(x^2+x+1)(p-1)^3} + \frac{(x+1)^2(p+1)}{x(p-1)^3}\right]\\ &= \frac{1}{16}\left[\frac{(p+1)(x^4-2x^2+1)}{(p-1)^3(x^2+x+1)x}\right]. \end{aligned}$$

The derivative of $f(p)$ is strictly positive for all $p > 1$. Since $E[\overline{m}_k^2] = f(x^N)$ and $x^N > 1$, it follows that $E[\overline{m}_k^2]$ is strictly increasing in N.

To show part (*iv*) we calculate the value of $E[\overline{m}_k^2]$ in the limit-equilibrium under centralization and decentralization. That is,

$$\lim_{N\to\infty} E[\overline{m}_k^2] = \lim_{N\to\infty}\frac{1}{16}\left[\frac{(x^{3N}-1)(x+1)^2}{(x^N-1)^3(x^2+x+1)} - \frac{x^N(x+1)^2}{x(x^N-1)^2} + 4\right] \tag{4.49}$$

$$= \frac{1}{16}\left(\frac{(x+1)^2}{(x^2+x+1)} + 4\right). \tag{4.50}$$

We substitute for the root of the characteristic equation of the homogeneous difference equation under centralization, i.e., $x = \sqrt{16(\lambda-\frac{1}{4})^2-1} + 4(\lambda-\frac{1}{4})$, to get the corresponding value of $\lim_{N\to\infty} E[\overline{m}_k^2]$ under centralization.

$$\lim_{N\to\infty} E[\overline{m}_k^2] = \frac{10\lambda-1}{4(8\lambda-1)}. \tag{4.51}$$

Analogously, substituting for the root of the characteristic equation of the homogeneous difference equation under decentralization, i.e., $x = \sqrt{\left(\frac{3\lambda-1}{1-\lambda}\right)^2 - 1} + \frac{3\lambda-1}{1-\lambda}$, yields the value of $\lim\limits_{N\to\infty} E[\overline{m}_k^2]$ under decentralization. That is,

$$\lim_{N\to\infty} E[\overline{m}_k^2] = \frac{6\lambda - 1}{4(5\lambda - 1)}. \tag{4.52}$$

□

Using Lemma 12, the expected payoff of the principal under centralization is given by

$$E[U_P^{C,N}] = -\frac{5}{12} + E[\overline{m}_k^2]. \tag{4.53}$$

Since $E[\overline{m}_k^2]$ is strictly increasing in N, $E[U_P^{C,N}]$ is strictly increasing in N and thus the highest in the limit equilibrium, i.e., for $N \to \infty$. Substituting for $E[\overline{m}_k^2] = \frac{10\lambda-1}{4(8\lambda-1)}$, yields the principal's expected payoff in this centralized limit-equilibrium.

$$E[U_P^{C,\infty}] = -\frac{5}{12} + \lim_{N\to\infty} E[\overline{m}_k^2] = -\frac{5\lambda - 1}{6(8\lambda - 1)}. \tag{4.54}$$

Analogously, by applying Lemma 12, the principal's expected payoff under decentralization can then be written as

$$E[U_P^{D,N}] = -\frac{1}{6}(1 + 2\lambda - 2\lambda^2) + 2\lambda(1-\lambda)E[\overline{m}_j^2]. \tag{4.55}$$

By Lemma 12, $E[U_P^{D,N}]$ is strictly increasing in N. As a consequence, the principal achieves the highest expected payoff in the decentralized limit-equilibrium. That is,

$$E[U_P^{D,\infty}] = -\frac{1}{6}(1 + 2\lambda - 2\lambda^2) + 2\lambda(1-\lambda) \lim_{N\to\infty} E[\overline{m}_j^2] = \frac{\lambda((9 - 8\lambda)\lambda - 6) + 1}{6(5\lambda - 1)}. \tag{4.56}$$

The calculation of the agents' expected payoffs under both organizational forms is analogous. Under centralization, the agents' expected payoff is the same as the principal's expected payoff. Under decentralization, the agent who is making the decision receives an expected payoff of

$$E[U_A^{D,\infty}] = E[\overline{m}_k^2](1-\lambda)^2 - \frac{1}{6}(\lambda - 1)(\lambda - 2) \tag{4.57}$$

and the agent who is only communicating to the decision making agent expects to get

$$E[U_A^{D,\infty}] = E[\overline{m}_k^2](1-\lambda)(3\lambda - 1) + \frac{1}{6}(3\lambda^2 - 5\lambda - 1). \tag{4.58}$$

□

A.4 Proof of Proposition 26

Proof. Since $\frac{1}{3} - \frac{10\lambda-1}{4(8\lambda-1)} < \frac{1}{3} - \frac{6\lambda-1}{4(5\lambda-1)}$ holds for all $\lambda \in (\frac{1}{2}, 1)$, we conclude that the quality of communication under centralization is always higher than the quality of communication under decentralization. □

A.5 Proof of Proposition 27

Proof. Comparing $E[U_P^{C,\infty}]$ with $E[U_P^{D,\infty}]$ shows that $-\frac{5\lambda-1}{6(8\lambda-1)} > \frac{\lambda((9-8\lambda)\lambda-6)+1}{6(5\lambda-1)}$ for all $\lambda \in (\frac{1}{2}, 1)$. □

A.6 Proof of Proposition 28

Proof. After conducting the same analysis as above, we get for the centralized limit equilibrium the following respective roots of the characteristic equation for agent $i \in \{1, 2\}$

$$x = \sqrt{16(\lambda_i - \frac{1}{4})^2 - 1} + 4(\lambda_i - \frac{1}{4}) \quad \text{and} \quad y = -\sqrt{16(\lambda_i - \frac{1}{4})^2 - 1} + 4(\lambda_i - \frac{1}{4}). \tag{4.59}$$

For the decentralized communication equilibrium we then get the following respective roots of the characteristic equation

$$\begin{aligned} x &= \sqrt{\left(\frac{1-\lambda_1-2\lambda_2}{1-\lambda_1}\right)^2 - 1} - \frac{1-\lambda_1-2\lambda_2}{1-\lambda_1} \quad \text{and} \\ y &= -\sqrt{\left(\frac{1-\lambda_1-2\lambda_2}{1-\lambda_1}\right)^2 - 1} - \frac{1-\lambda_1-2\lambda_2}{1-\lambda_1} \end{aligned} \tag{4.60}$$

The expected payoff of the principal under centralization can then be written as

$$E[U_P^{C,N}] = -\frac{1}{3} + \frac{1}{2}(E[\overline{m}_k^2] + E[\overline{m}_j^2]). \tag{4.61}$$

By the same reasoning as in the proof of Proposition 25, the principal and the agents' expected payoffs are increasing in N. In the limit-equilibrium, that is for $N \to \infty$, the principal then receives an expected payoff of

$$E[U_P^{C,\infty}] = -\frac{1}{3} + \frac{1}{2}\lim_{N\to\infty}(E[\overline{m}_k^2] + E[\overline{m}_j^2]) = -\frac{1}{3} + \underbrace{\frac{1-10\lambda_1}{8-64\lambda_1}}_{\lim_{N\to\infty} E[\overline{m}_k^2]} + \underbrace{\frac{1-10\lambda_2}{8-64\lambda_2}}_{\lim_{N\to\infty} E[\overline{m}_j^2]} \tag{4.62}$$

$$\Leftrightarrow -\frac{1}{48} + \frac{1}{32(8\lambda_1-1)} + \frac{1}{32(8\lambda_2-1)}.$$

The expected payoff of the principal under decentralization can then be written as

$$E[U_P^{D,N}] = -\frac{1}{3} + \frac{1}{6}(\alpha(1-2\lambda_1+2\lambda_1^2) + (1-\alpha)(1-2\lambda_2+2\lambda_2^2)) + 2\alpha\lambda_1(1-\lambda_1)E[\overline{m}_j^2] \tag{4.63}$$
$$+ 2(1-\alpha)\lambda_2(1-\lambda_2)E[\overline{m}_k^2].$$

In the limit-equilibrium, that is for $N \to \infty$, the principal consequently expects to get

$$E[U_P^{D,\infty}] = -\frac{1}{3} + \frac{1}{6}(\alpha(1-2\lambda_1+2\lambda_1^2) + (1-\alpha)(1-2\lambda_2+2\lambda_2^2)) \tag{4.64}$$
$$+ 2\alpha\lambda_1(1-\lambda_1)\lim_{N\to\infty} E[\overline{m}_j^2] + 2(1-\alpha)\lambda_2(1-\lambda_2)\lim_{N\to\infty} E[\overline{m}_k^2]$$
$$= -\frac{1}{3} + \frac{1}{6}(\alpha(1-2\lambda_1+2\lambda_1^2) + (1-\alpha)(1-2\lambda_2+2\lambda_2^2)) \tag{4.65}$$
$$+ \alpha\lambda_1(1-\lambda_1)\underbrace{\frac{5\lambda_2+\lambda_1-1}{2(4\lambda_2+\lambda_1-1)}}_{\lim_{N\to\infty} E[\overline{m}_j^2]} + (1-\alpha)\lambda_2(1-\lambda_2)\underbrace{\frac{5\lambda_1+\lambda_2-1}{2(4\lambda_1+\lambda_2-1)}}_{\lim_{N\to\infty} E[\overline{m}_k^2]}.$$

Assume without loss of generality that $\lambda_1 < \lambda_2$. Since $E[U_P^{D,\infty}]$ is increasing in α, the principal is best off if she fully delegates the decision to the less biased agent, i.e., $\alpha = 1$, and lets agent 2 communicate his private information to agent 1.

Comparing $E[U_P^{C,\infty}]$ with $E[U_P^{D,\infty}]$ for $\alpha = 1$ shows that $-\frac{1}{48} + \frac{1}{32(8\lambda_1-1)} + \frac{1}{32(8\lambda_2-1)} > -\frac{1}{3} + \frac{1}{6}(1-2\lambda_1+2\lambda_1^2) + \lambda_1(1-\lambda_1)\frac{5\lambda_2+\lambda_1-1}{2(4\lambda_2+\lambda_1-1)}$ for all $\frac{1}{2} < \lambda_1 < \lambda_2 < 1$. □

References

Alonso, R. and Câmara, O. (2016). Persuading Voters. *American Economic Review*, 106(11):3590–3605.

Alonso, R., Dessein, W., and Matouschek, N. (2008). When Does Coordination Require Centralization? *American Economic Review*, 98(1):145–179.

Alonso, R., Dessein, W., and Matouschek, N. (2015). Organizing to Adapt and Compete. *American Economic Journal: Mircoeconomics*, 7(2):158–187.

Alonso, R. and Matouschek, N. (2008). Optimal Delegation. *Review of Economic Studies*, 75(1):259–293.

Athey, S. and Roberts, J. (2001). Organizanation Design: Decision Rights and Incentive Contracts. *American Economic Review*, 91:200–205.

Aumann, R. J. and Hart, S. (2003). Long Cheap Talk. *Econometrica*, 71(6):1619–1660.

Austen-Smith, D. and Banks, J. S. (1996). Information Aggregation, Rationality and the Condorcet Jury Theorem. *American Political Science Review*, 90(1):34–45.

Bardhi, A. and Guo, Y. (2018). Modes of Persuasion Towards Unanimous Consent. *Theoretical Economics*, 13:1111–1149.

Bergemann, D., Bonatti, A., and Smolin, A. (2018). The Design and Price of Information. *American Economic Review*, 108:1–48.

Bergemann, D. and Morris, S. (2016a). Bayes Correlated Equilibrium and the Comparison of Information Structures in Games. *Theoretical Economics*, 11(2):487–522.

Bergemann, D. and Morris, S. (2016b). Information Design, Bayesian Persuasion and Bayes Correlated Equilibrium. *American Economic Review: Papers and Proceedings*, 106(5):586–591.

Bergemann, D. and Morris, S. (2017). Information Design: a Unified Perspective. Discussion paper.

Bobkova, N. (2018). Learning to Persuade. Discussion paper.

Bobkova, N. and Klein, S. (2019). Persuading an Informed Committee. Discussion paper.

Chan, J., Gupta, S., Li, F., and Wang, Y. (2018). Pivotal Persuasion. *Journal of Economic Theory*, 180:178–202. Discussion paper.

Coughlan, P. J. (2000). In Defense of Unanimous Jury Verdicts: Mistrials, Communication, and Strategic Voting. *American Political Science Review*, 94(2):375–393.

Crawford, V. P. and Sobel, J. (1982). Strategic Information Transmission. *Econometrica*, 50(6):1431–1451.

Deimen, I. and Szalay, D. (2019a). Delegated Expertise, Authority, and Communication. *American Economic Review*, forthcoming.

Deimen, I. and Szalay, D. (2019b). Information and Communication in Organizations. *AEA Papers and Proceedings*, forthcoming.

Dessein, W. (2002). Authority and Communication in Organizations. *The Review of Economic Studies*, 69(4):811–838.

Dessein, W., Garicano, L., and Gertner, R. (2010). Organizing for Synergies. *American Economic Journal: Mircoeconomics*, 2(4):77–114.

Dimitrakas, V. and Sarafidis, Y. (2005). Advice from an Expert with Unknown Motives. *INSEAD, Discussion paper.*

Feddersen, T. and Pesendorfer, W. (1997). Voting Behaviour and Information Aggregation in Elections With Private Information. *Econometrica*, 65(5):1029–1058.

Feddersen, T. and Pesendorfer, W. (1998). Convicting the Innocent: the Inferiority of Unanimous Jury Verdicts Under Strategic Voting. *The American Political Science Review*, 92:23–35.

Gerardi, D. and Yariv, L. (2007). Deliberative Voting. *Journal of Economic Theory*, 134:317–338.

Harris, M. and Raviv, A. (2005). Allocation of Decision-Making Authority. *Review of Finance*, 9(3):353–383.

Heese, C. and Lauermann, S. (2018). Persuasion of a Group of Heterogeneous Voters. Discussion paper.

Holmström, B. (1984). On the Theory of Optimal Delegation. In Boyer, M. and Kihlstrom, R., editors, *Bayesian Models in Economic Theory*, pages 115–141. North-Holland Pub. Co.

Kamenica, E. and Gentzkow, M. (2011). Bayesian Persuasion. *American Economic Review*, 101:2590–2615.

Kolotilin, A. (2016). Optimal Information Disclosure: a Linear Programming Approach. Discussion paper.

Kolotilin, A., Li, M., Mylovanov, T., and Zapechelnyuk, A. (2017). Persuasion of a Privately Informed Receiver. *Econometrica*, 85(6):1949–1964. Discussion paper.

Krishna, V. and Morgan, J. (2004). The Art of Conversation: Eliciting Information from Experts Trough Multi-Stage Communication. *Journal of Economic Theory*, 117(2):147–179.

Li, Z., Rantakari, H., and Yang, H. (2016). Competitive Cheap Talk. *Games and Economic Behavior*, 96:65–89.

Liu, S. and Migrow, D. (2019). Designing Organizations in Volatile Markets. Discussion paper.

Ludema, R. D. and Olofsgård, A. (2008). Delegation versus Communication in the Organization of Government. *Journal of Public Economics*, 92(1-2):213–235.

Melumad, N. D. and Shibano, T. (1991). Communication in Settings with No Transfers. *The RAND Journal of Economics*, 22(2):173–198.

Ottaviani, M. (2000). The Economics of Advice. *University College London, Discussion paper.*

Rantakari, H. (2013). Organizational Design and Environmental Volatility. *Journal of Law, Economics and Organizations*, 29(3):569–607.

Rantakari, H. (2019). Soliciting Advice: Active versus Passive Principals. *Journal of Law, Economics and Organizations*, Forthcoming.

Rayo, L. and Segal, I. (2010). Optimal Information Disclosure. *Journal of Political Economy*, 118(5):949–987.

Schnakenberg, K. E. (2015). Expert Advice to a Voting Body. *Journal of Economic Theory*, 160:102–113.

Schnakenberg, K. E. (2017). Informational Lobbying and Legislative Voting. *American Journal of Political Science*, 61(1):129–145.

Stein, J. C. (1989). Cheap Talk and the Fed: A Theory of Imprecise Policy Announcements. *The American Economic Review*, 79(1):32–42.

Taneva, I. (2019). Information Design. *American Economic Journal: Mircoeconomics*, forthcoming.

Wang, Y. (2015). Bayesian Persuasion with Multiple Receivers. Discussion paper.