Three Essays in Macroeconomics

Implications of the Great Recession for Fiscal and Monetary Policy

Inauguraldissertation

zur Erlangung des Grades eines Doktors

der Wirtschafts- und Gesellschaftswissenschaften

durch die

Rechts- und Staatswissenschaftliche Fakultät
der Rheinischen Friedrich-Wilhelms-Universität

Bonn

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Bonn, 2020
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Tag der mündlichen Prüfung: 31.01.2020
Acknowledgements

Writing this thesis has been a very interesting and rewarding experience, which would not have been possible without the support of several people.

To begin with, I would like to express my sincerest gratitude to my primary supervisor Gernot Müller for his generous and continuous support throughout this time. His feedback, advice and guidance proved to be invaluable for my research and shaped the way I think about modern macroeconomics. It was an absolute pleasure to have him as a supervisor. I also benefited tremendously from Jürgen von Hagen, my second supervisor. His complementary views and comments on my research were extremely insightful. Additionally, I owe a debt of gratitude to Keith Küster, for his genuine comments, support and honesty.

I was delighted to have collaborated with a number of inspiring co-authors. Specifically, I would like to extent my thanks to Alexander Kriwoluzky, for the modeling and estimation skills that I was able to acquire while working with him. Also, I would like to thank Rafael Gerke and Sebastian Giesen for our detailed, profound and enlightening discussions and for giving me a warm welcome.

My heartfelt gratitude also goes to the Bonn Graduate School of Economics, the Institute for International Economic Policy and the Deutsche Bundesbank for providing the requisite environment, freedom and resources to conduct my research. During this journey of thesis writing, I have been accompanied by many fellow students. I would like to thank all of them, especially Martin Wolf, Matthias Meier and Ariel Mecikovsky for lively and thought-provoking discussions about macroeconomics.

Without doubt, I cannot thank my parents Malinka and Andre Scheer and my sister Romy Klapsch enough, who have constantly and unconditionally given me moral and emotional support throughout my life. On that note, I also owe a genuine expression of gratitude to other family members and friends who have supported me along the way.

Last but not the least, I dedicate this thesis to my wife, Julia Scheer, and my kids, Marvin and Benedikt Scheer. Their unlimited love, patience and understanding helped me tremendously to persist despite prolonged phases of frustration and despair. I could not have achieved it without you – thank you!
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Introduction

The Great Recession prompted several governments and central banks in advanced economies to take exceptional measures, whose consequences are still visible today. In particular, two developments seem to be noteworthy. First, many governments’ debt-to-GDP ratios are high with no sign of returning to pre-crisis levels, raising concerns about their sustainability. Second, interest rates are low and in some countries at the effective lower bound, reducing monetary policies’ room for manoeuvre.

This thesis investigates several implications of these two consequences of the crisis for fiscal and monetary policy. The first two chapters focus on means to reduce the debt-to-GDP ratio, either by generating primary surpluses through budgetary adjustments (chapter 1) or by directly lowering the interest rates on government debt due to financial repression (chapter 2). The last chapter evaluates the impact of interest rate forward guidance, an important tool for central banks to enhance the effectiveness of monetary policy at the effective lower bound.

In the first chapter of the thesis, entitled ‘Fiscal Consolidation with long-term Public Debt’, I compare the macroeconomic implications of expenditure-based vs. tax-based consolidation and find that tax-hikes can be less disruptive. This depends on the dynamics of the governments’ real debt-servicing costs during consolidation periods, since a lower real rate reduces the need to adjust primary surpluses and thus dampens the distortionary impact that both fiscal retrenchments entail (and vice versa). Within a New Keynesian framework which features an extended maturity structure of public debt to better account for interest rate dynamics, I show that the average maturity of government debt determines whether tax-hikes and spending-cuts increase or decrease the real rate. Calibrated to match US debt characteristics in 2018, tax-based consolidation is found to be welfare enhancing, since higher taxes lower long-term real rates. If debt would be only short-term, spending should be reduced instead.
Chapter 2 is entitled ‘Financial Repression in General Equilibrium’ and joint work with Alexander Kriwoluzky and Gernot Müller. We assess the impact of financial repression – a set of regulatory policies to keep the interest rates on government debt artificially low – in the US during the post-WW2 period. We provide a measure of the “laissez-faire interest rate”, the interest rate that would have prevailed in the absence of financial repression, on the basis of a medium-scale dynamic general equilibrium model, in which the banking sector is a captive audience for public liabilities. Our estimates indicate that the laissez-faire rate on government bonds is considerably higher than the actual interest rate at the time. All else equal, this implies that about half of the observed decline of the debt ratio was due to repression. However, once we take into account that in the absence of repression the private and public sector would have evolved differently, we find that repression actually slowed down the reduction of the debt ratio.

The last chapter of the thesis, entitled ‘The Power of Forward Guidance in a Quantitative TANK Model’ and joint work with Rafael Gerke and Sebastian Giesen, quantifies the macroeconomic effects of interest rate forward guidance within an estimated medium-scale Two-Agent New Keynesian (TANK) model of the EA economy. The mere introduction of hand-to-mouth households that cannot smooth consumption has two opposing effects on the power of forward guidance compared to a representative agent model. While the direct effect reduces its impact, as there is less intertemporal smoothing, the indirect effect raises its impact since hand-to-mouth households have a higher marginal propensity to consume. Although the overall effect is a priori ambiguous, our estimates indicate that the power of forward guidance is dampened, as there is sufficient countercyclical redistribution which weakens the indirect effect. An interaction of forward guidance with central bank asset purchases gives rise to non-linear effects that depend on the horizon of forward guidance.
Chapter 1

Fiscal Consolidation with long-term Public Debt

1.1 Introduction

Following the onset of the Great Recession, several advanced economies – apart from Germany – experienced a sharp increase in their public debt levels with no sign of returning to pre-crisis levels (see Figure 1.1). With poor growth prospects and low inflation expected in the coming years, there is a need for considerable budget cuts if these countries want to reduce their public debt ratios to more typical and sustainable levels.

In this paper, I compare the macroeconomic implications of tax-based (TB) and expenditure-based (EB) consolidations and find that TB adjustments can be less disruptive than EB ones. I arrive at this result using a framework that allows to focus on the role of (real) interest rate dynamics in the evolution of government debt. The vast (empirical) literature on the compositional effects of fiscal adjustments typically finds that spending-cuts outperform tax-hikes, especially when success is measured based on their respective impacts on GDP growth (e.g. Alesina, Favero and Giavazzi, 2019; Alesina and Ardagna, 2010; Alesina, Favero and Giavazzi, 2015; Von Hagen, Hallett and Strauch, 2002). Focusing on output is natural: if consolidation induces sharp recessions, the debt-to-GDP ratio increases.

However, to predominantly measure success by evaluating growth implications
ignores the importance of interest rates for government bonds, which also directly affect the evolution of public debt ratios. To take these two channels into account, I put forward a stylized New Keynesian model to conceptually show the way a budgetary adjustment interacts with the (real) interest rate of public liabilities. The main mechanism also holds true in a medium-scale DSGE model. I focus on a permanent reduction of the debt-to-GDP ratio and discuss the normative question of how different fiscal consolidation strategies affect welfare. To allow a meaningful trade-off, the provision of public goods enhances welfare, but it is only an imperfect substitute for private consumption. Labor taxes are proportional to the agents’ labor income and distort labor supply decisions.

I begin the analysis by assessing the long-term macroeconomic implications of permanently lower debt levels with the model calibrated to 2018 US data. The debt level has a non-trivial effect on the economy as the resulting interest expenses for the government have to be financed by distortionary taxes (Ricardian Equivalence does not apply). Hence, reducing the debt ratio results in a lower average debt servicing cost and frees up resources that can be allocated to either higher spending, lower tax rates or a combination of both. I determine the
optimal composition of tax rates and government expenditure that maximizes the households’ welfare in steady state for a given debt level, following Adam (2011). This particular allocation method is described in section 1.3. A steady-state comparison of welfare when debt ratios are 100% (roughly the current US value), 90% and 80% indicates (unsurprisingly) that households would be better off with less debt.

However, a long-term comparison disguises the fact that reaching lower public debt ratios requires cuts in expenditures or increases in tax rates during the transition (if other means like greater growth are not available). Hence, one must take the transitional adjustment costs into account for a comprehensive analysis. Fiscal consolidation is obtained using simple feedback rules that increase (decrease) the labor tax rate (government spending) if the actual debt-to-GDP ratio is above the target. The exogenous target rate is gradually reduced from 100% (roughly the current US value) to 90% within 10 years, i.e., a reduction of 10pp (Coenen, Mohr and Straub, 2008; Cogan, Taylor, Wieland and Wolters, 2013). Monetary policy follows a simple Taylor rule.

To capture the response of interest rate dynamics, the model features an extended maturity structure of public debt, following the approach by Krause and Moyen (2016). This is well suited to clarifying how various interest rates are related. Specifically, it distinguishes between the short-term policy rate that is set by the central bank, the interest rate on newly issued (long-term) government debt and the average interest rate the government has to pay on its outstanding (long-term) liabilities.¹ These distinctions are important since not every change in the policy rate or the current interest rate of newly issued bonds directly affects the average rate on outstanding bonds, unless the entire public debt has maturity of one period. However, it is the average interest rate – alongside output growth and surplus – that shapes public debt dynamics. Hence, a maturity structure for debt determines the extent to which the debt servicing costs will change in response to fiscal adjustments. Empirically, governments have issued long maturity debts quite regularly in the past. As Table 1.1 illustrates, the average maturities range between 6 and 9 years for the G7 countries as of 2018, except for the UK, where it is as high as 16 years.

The main result of this paper is that tax-based consolidations can be less disruptive than spending-based ones. I arrive at this conclusion by comparing the

¹This is not the case with (Woodford, 2001) where all the debt is reissued every period.
Table 1.1: Average maturity of debt in years as of 2018

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>UK</th>
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<td></td>
<td>8.3</td>
<td>8</td>
<td>6.3</td>
<td>7.3</td>
<td>8.9</td>
<td>15.8</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Source: ECB, Government Statistics, Average residual maturity of debt; Debt management report of the respective non-euro countries.

The mechanism behind the result is straightforward: in the model, tax-based consolidations are associated with lower average interest rates on outstanding public liabilities, and this accommodates the fiscal retrenchment. Such a benign effect lowers the need to adjust tax revenues and mitigates the adverse effects of a rising labor tax wedge. Similarly, cutting government expenditures raises the average interest rates on public debt. To counteract the impact of rising rates for the debt-to-GDP ratio, spending must be cut more forcefully, thereby aggravating the recession. The interest rate response depends on two aspects.

First, TB plans increase inflation while EB ones have a dampening effect on the aggregate price level. Through the lens of the model, raising labor tax rates lowers disposable income, inducing households to demand higher pre-tax wages to compensate part of this loss in income. Since wages are marginal costs for the firms, they charge higher prices (Eggertsson, 2011). Conversely, with a reduction in public purchases, prices become lower as firms try to attract private demand. These effects of tax- and spending-based consolidations on the inflation rate are well in line with empirical findings (see e.g. Alesina et al., 2015; Mertens and Ravn, 2013).

Second, if the average maturity of public debt is long enough, the nominal average interest rate reacts less than one-to-one with the inflation rate. This is the case even though monetary policy follows the Taylor-principle, i.e., the nominal and real (short-term) interest rates react more than one-to-one with inflation.

2 I use the terms tax-based/expenditure-based consolidation, TB/EB-plans and tax-hikes/spending-cuts interchangeably.

3 The reduction in disposable income also dampens demand which urges firms to actually cut their prices. However, the cost channel seems to dominate in general. Only for incredibly high tax hikes (over double the size) prices will fall.
However, with an extended maturity structure of public debt, only part of the outstanding stock of liabilities is reissued at new (higher/lower) interest rates. For the other part, nominal rates are predetermined. Hence, the real average interest rate – the rate the government has to pay on its outstanding debt – declines when inflation is high (i.e., with TB plans), while it increases when inflation is low (EB plans).

The above results get reversed when only short-term debt is considered. In this case, the dynamics of real average interest rates are more affected by the dynamics of the short-term policy rate and less by inflation. In such a scenario, tax-hikes actually raise the governments’ real debt servicing costs and thus cause a stronger and more persistent recession. Accordingly, the FSR is much higher than compared to the spending-based adjustment. The sacrifice ratio and welfare measure in my analysis show that for public debts with average maturities over two – four years (the current case for the US), tax-based budgetary adjustments should be favored over spending-based ones.

In a sensitivity analysis, I first quantify the impact of consolidation if the economy is stuck at the effective lower bound (ELB). I simulate a negative demand shock that brings the economy to the ELB for four quarters. I then look at the contrast between tax-based and spending-based consolidations. As mentioned above, although TB plans increase inflation and inflation expectations, monetary policy cannot counteract these pressures as it is stuck at the ELB. Nevertheless, higher expected inflation causes the real interest rate to fall, which mitigates the depth of the recession (at least in the first three quarters – a case of expansionary austerity). As a result, the economy leaves the ELB already after the third quarter. Similarly, spending-cuts aggravate the recession since monetary policy cannot accommodate the shortfall in demand and inflation, thereby raising real rates. Second, I show that the main mechanism in the paper would also hold in a medium-scale DSGE model (following Carlstrom, Fuerst and Paustian, 2017).

Several recent contributions explore various aspects of fiscal consolidation. As mentioned above, one important distinction is the composition of budgetary adjustments. In a similar vein, one could compare the magnitude of fiscal multipli-

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4 Additionally, as will be explained in detail below, the interest rate of newly issued (long-term) bonds is a weighted average of the current and expected short-term rates, which further cushions the change in average interest rates.

5 However, this view is not unchallenged. Part of the literature identifies the size of consolidation or whether consolidation has been implemented successfully as more important than the composition. See, for instance, Ardagna (2004), Holden and Midthjell (2013) or Wiese,
ers; for instance, see the recent survey by Ramey (2019). The multiplier is indeed one important ingredient to decide which fiscal composition to exploit. However, this isolated assessment overlooks the interaction with interest rate dynamics. When the average maturity of debt is high, taxes must be raised less in order to reduce the debt ratio.

An argument in favor of expenditure-based consolidations over tax-based ones is that monetary policy helps mitigate potential adverse effects in the former while exacerbating it in the latter (see for instance Guajardo, Leigh and Pescatori, 2014; Erceg and Lindé, 2013; Romei, 2017; Bi, Leeper and Leith, 2013). For example, Erceg and Lindé (2013) use a medium-scale two-country DSGE model to compare the effects of TB and EB plans with different degrees of monetary policy accommodation. They find that with an independent central bank, government spending-cuts are the less costly method of reducing public debt than tax-hikes. In a currency union, however, the central bank provides too little accommodation as it focuses on union-wide aggregates only. Therefore, expenditure-based consolidation depresses output a lot more in the short run.\(^6\)

Bi et al. (2013) study how uncertainty about the timing (at which debt levels consolidation starts) and composition (when a raise in taxes is expected but a reduction in spending is realized) affects macroeconomic performances. They set up a New Keynesian model with short-term debt and emphasize that the behavior of short real interest rate determines whether stabilizing government debt will be successful or not. In a similar vein, Romei (2017) argues that spending-based consolidation is preferable in a New Keynesian Model with heterogeneous agents where households can vote for each policy option. She states that it is especially preferable to have lower financing costs when consolidation takes place; in her paper, this is the case when spending-based adjustments are implemented.\(^7\)

All three papers emphasize that their respective result is driven by the response of real interest rates on government debt during the period of consolidation. However, they all rely on short-term debt. My paper shows how the introduction of long-term debt overturns the dynamics of the real average interest rate compared to short-term debt. It would be interesting to see how the above results might

\(^6\)The ELB scenario from above is in spirit similar to a currency union, as in both cases the monetary authority reacts relatively less strongly to the inflation rate.

\(^7\)In a related paper, Hommes, Lustenhouwer and Mavromatis (2018) show that EB plans can lead to larger recessions if heterogeneity of expectations is taken into account.
change if an extended maturity of public debt is included.

Another stream of the literature focuses on fiscal consolidations within different macroeconomic environments such as accompanying structural reforms (Vogel, 2012), political economy aspects (Pappa, Sajedi and Vella, 2015) or private debt deleveraging (Andrés, Arce and Thomas, 2018).\(^8\) The latter focus on how long-term private debt insulates the income of households from the recessive impact of fiscal retrenchment. Similarly, my model shows how the introduction of long-term bonds shields the government’s interest expenses from changes in the short-run interest rate.

In terms of an empirical (successful) debt reduction, Hall and Sargent (2011) document an important role for primary surpluses in the US after WWII even though most of the debt was reduced by steady positive GDP growth rates. They also find a non-negligible role of negative real interest rates due to realized inflation (see also Sims, 2013). Leeper and Zhou (2013) formalize the idea that revaluations of nominal debt through inflation can be a part of optimal monetary and fiscal policies when the average maturity of government debt is high. As growth is not a direct policy option (at least in the short term), and inflation might entail tremendous costs (Barro and Gordon, 1983), I focus on changes in primary surpluses.

The rest of the paper is structured as follows: The next section introduces the model in detail. Section 1.3 shows the long-run implications of different debt ratios while section 1.4 describes the transition through means of fiscal adjustments. This is followed by a section on two extensions (consolidation at the ELB and a medium-scale DSGE model) while section 1.6 concludes this paper.

### 1.2 Model set up

The model is a stylized closed economy New Keynesian model with fiscal rules and the extension of long-term bonds as in Krause and Moyen (2016). The economy consists of three agents: households, which maximize their life time utility; firms, that maximize profits and a government authority that sets fiscal policy in order to keep the actual debt level close to a target rate. The household

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\(^8\)Another interaction focuses on how the anticipation of a future debt reduction plan affects the current stimulus during a severe crisis; for instance, see Corsetti, Kuester, Meier and Müller (2010) for a case of government spending.
derives utility from consumption of private and public goods and from leisure. The asset market trades one period and long-term bonds. All households supply their labor services in a competitive labor market. On the production side there are two types of firms. The monopolistic competitive firms hire labor to produce intermediate goods and sell the goods to the final-good firm. They face nominal rigidities à la Calvo (1983) when setting their optimal price. The final-good firm uses the intermediate goods in a constant-elasticity-of-substitution (CES) production function to produce an aggregate good à la Dixit and Stiglitz (1977) which is sold to the households in a perfectly competitive market. The monetary authority follows a Taylor rule that reacts on inflation and output.

1.2.1 Long-term bonds

To better account for changes in the real interest rate of government bonds, I model an extended maturity structure for long-term bonds, following Krause and Moyen (2016). Each unit of such debt is denoted by $B_{t}^{L,n}$, where the $L$ stands for long-term and the $n$ for newly-issued debt at time $t$. Every period, the holder of $B_{t}^{L,n}$ receives an interest payment of $i_{t}^{L,n}$ (fixed rate). If the bond matures, which happens stochastically every period with probability $\gamma$, the bond pays back the principal of 1. As there is implicitly a unit-mass of $B_{t}^{L,n}$, by the law of large numbers, a fraction $\gamma$ actually matures each period. Then, the average maturity is given by $\frac{1}{\gamma}$. However, it is important to note that $\gamma$ not only determines the average maturity but also the amount of bonds maturing every period. In case of the US in 2018, this seems to be a valid approximation, see figure 1.2 below.

Given that a certain fraction of bonds matures every period, the stock of long-term debt $B_{t}^{L}$ (without $n$) evolves as

$$B_{t}^{L} = (1 - \gamma)B_{t-1}^{L} + B_{t}^{L,n}$$ (1.1)

One central object of the paper is the nominal debt-servicing cost (or interest expenses) of the government $i_{t}^{L}B_{t}^{L}$. It is obtained recursively by the sum of the average interest rate expenses of the past (weighted by the amount that did not mature) and on the newly issued bonds:

$$i_{t}^{L}B_{t}^{L} = (1 - \gamma)i_{t-1}^{L}B_{t-1}^{L} + i_{t}^{L,n}B_{t}^{L,n}$$ (1.2)
It follows that the average debt-servicing cost of the government on its outstanding debt is a weighted sum of previous debt-servicing costs, which can be seen by iterating equation (1.2):

\[ i_t^L B_t^L = i_t^{L,n} B_t^{L,n} + (1 - \gamma)i_{t-1}^{L,n} B_{t-1}^{L,n} + (1 - \gamma)^2 i_{t-2}^{L,n} B_{t-2}^{L,n} + \ldots \]  

(1.3)

Similarly, the nominal average interest rate \( i_t^L \) is a weighted sum of long-term interest rates of newly issued bonds \( i_t^{L,n} \), weighted by the relative fraction of those bonds that did not yet mature:

\[ i_t^L = i_t^{L,n} \frac{B_t^{L,n}}{B_t^L} + i_{t-1}^{L,n}(1 - \gamma) \frac{B_{t-1}^{L,n}}{B_t^L} + i_{t-2}^{L,n}(1 - \gamma)^2 \frac{B_{t-2}^{L,n}}{B_t^L} + \ldots \]  

(1.4)

This recursive formulation makes transparent that average interest rate dynamics of government debt are largely pre-determined in this setup and that the interest rate on newly issued government debt (denoted by \( i_t^{L,n} \)) is only of minor importance. This is an important feature that is shared by the data. For instance, during the height of the Greek financial crisis in 2012, 10y interest rates spiked to around 30%, i.e. \( i_{2012}^{L,n} = 30\% \). However, the average interest rate was only around 4.3%, i.e. \( i_{2012}^L = 4.3\% \). See, for instance, Deutsche Bundesbank (2017).
1.2.2 Households

Households can purchase private consumption $P_tC_t$ or invest in short- and long-term bonds, $B_t$ and $B^{L,n}_t$, respectively.\(^9\) They earn after-tax wage income $(1 - \tau_t)P_tW_tN_t$, the returns from the short-term bonds $(1 + i_{t-1})B_{t-1}$, returns from long-term bonds $(\gamma + i^L_{t-1})B^{L}_{t-1}$ and dividends from firm ownerships $P_tD_t$. Denote with $\lambda_t$ the Lagrange multiplier attached to the budget constraint and with $\mu_t$ the multiplier on the dynamics of government debt-servicing costs (equation (1.2) and (1.1)).\(^{10}\) The representative household maximizes its life time utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{\chi}{1+\phi} \right\}$$
subject to the budget constraint

$$P_tC_t + B_t + B^{L,n}_t \leq (1 - \tau_t)P_tW_tN_t + (1 + i_{t-1})B_{t-1} + (\gamma + i^L_{t-1})B^{L}_{t-1} + P_tD_t$$

and government interest rate dynamics

$$i^L_t B^{L}_t = (1 - \gamma) i^{L}_{t-1} B^{L}_{t-1} + i^{L,n}_t \left( B^{L}_t - (1 - \gamma) B^{L}_{t-1} \right) \quad (1.2)$$

Note that the interest rate on newly issued long-term debt $i^{L,n}_t$ is taken as given, similar to the short-term policy rate. However, the average rate $i^L_t$ depends on the composition of newly issued and outstanding bonds and is chosen indirectly by the household. Therefore, the household must take this into account when maximizing his welfare.

The first order conditions for the short-term bond holdings yield the familiar Euler equation:

$$C_t^{1-\sigma} = \beta E_t \left\{ \frac{1 + i_t}{1 + \pi_{t+1}} C_{t+1}^{1-\sigma} \right\} \quad (1.5)$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the net inflation rate. The optimality condition for long-term bonds is

---

\(^9\)As shown below, short-term debt will be set to zero net supply (see below), but it is needed to derive the Euler equation.

\(^{10}\)To arrive at the expression one has to scale $\mu_t$ by $\frac{\lambda_t}{\pi_t}$. 

12
$$1 = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{1}{1 + \pi_{t+1}} \left[ 1 + i_L^n t - \mu_{t+1} (1 - \gamma) \left( i_{t+1}^L - i_t^L \right) \right] \right\}$$

while $\mu_t$ evolves according to

$$\mu_t = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{1}{1 + \pi_{t+1}} (1 + (1 - \gamma) \mu_{t+1}) \right\}. \quad (1.7)$$

The first equation is the standard Euler equation, but the last two deserve a bit more attention. First, note that (1.7) in steady state results in $\mu = \frac{1}{1+\gamma}$. This reduces to the pricing function for a one-period bond if $\gamma = 1$ and also a consol if $\gamma = 0$. Since $\mu_t$ is the Lagrange multiplier on (1.2) one can interpret it as the price of the long-term bond. As can be seen from equation (1.7) the price is higher than for short-term debt. In case of $\gamma = 1$ equation (1.6) implies $i_L^n t = i_t$ and the second Euler equation collapses to the first one. The two Euler equations (1.5) and (1.6) constitute the no arbitrage condition for investing at different horizons. The right hand sight of (1.6) is the expected payoff of a long-term debt valued by the stochastic discount factor. It consists of two parts, the first, $1 + i_L^n t$ is the return if the bond would mature next period. The second, $-\mu_{t+1} (1 - \gamma) (i_{t+1}^L - i_t^L)$, can be interpreted as the capital loss (gain) that arises from a rise (fall) in the newly issued long-term rate. The no arbitrage condition implies that once the household expects a rise of newly-issued long-term interest rates, i.e. $i_{t+1}^L > i_t^L$, he asks for a premium to compensate the investment as it ties resources for several periods. It is optimal to take into account the direct return plus the opportunity costs of having resources fixed in a long-term contract. The remaining FOC yields the labor supply

$$W_t (1 - \tau_t) = \chi_n N_t^d C_t^\sigma. \quad (1.8)$$

### 1.2.3 Firms

The final good firm uses intermediate goods from the monopolistic competitive firm and produces a final good with a CES production function. Its demand for each intermediate good $j$ is given by

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} y_t^d$$
where \( y^d_t \) is the household demand for a final good. Each intermediate good firm produces its good \( y_t(j) \) according to \( y_t(j) = A_t N_t(j) \) where \( N_t(j) \) is the amount of labor and \( A_t \) aggregate technology. As the production function exhibits constant returns to scale, marginal costs are independent of the level of production and equal to

\[
mc_t = \frac{W_t}{A_t} \quad \text{(1.9)}
\]

Each firm sets a profit maximizing price subject to Calvo (1983) nominal frictions. The FOC of the firm can be cast into the following recursive forms:

\[
g^1_t = \lambda_t mc_t y^d_t + \beta \theta E_t \{ g^1_{t+1} \} \quad \text{(1.10)}
\]

\[
g^2_t = \lambda_t y^d_t + \beta \theta E_t \{ g^2_{t+1} \} . \quad \text{(1.11)}
\]

The optimal price is equal to

\[
\frac{P^*_t}{P_t} = \frac{\epsilon g^1_t}{\epsilon - 1 g^2_t} \quad \text{(1.12)}
\]

and the price index evolves according to

\[
1 = \theta(1 + \pi_t)^{1-\epsilon} + (1 - \theta) \left( \frac{P^*_t}{P_t} \right)^{1-\epsilon} . \quad \text{(1.13)}
\]

### 1.2.4 Government

Fiscal policy is captured by simple feedback rules that increase (decrease) the tax rate (government spending) if the actual debt-to-GDP ratio is above some target ratio \( \bar{d}_t \). The latter will be reduced exogenously to a lower value \( \bar{d}^{new} < \bar{d}^{old} \). Furthermore, it seems plausible that policymakers plan to reduce the target ratio gradually to avoid potentially large adverse consequences on output. To capture this gradualism I follow Coenen et al. (2008) and use the following law of motion:

\[
\bar{d}_t = (1 - \rho_d) \bar{d}^{new} + \rho_d \bar{d}_{t-1} \quad \text{(1.14)}
\]

where \( \rho_d \) is chosen such that the debt target converges to its new level of \( \bar{d}^{new} = 90\% \) after approximately 10 years.\(^{11}\) The government budget constraint is given

\(^{11}\)A linear specification or an AR(2) as in (Erceg and Lindé, 2013) did not change the results.
by

\[ B_t + B_t^{L,n} + P_t \tau_t W_t N_t = P_t G_t + (1 + i_{t-1}) B_{t-1} + (\gamma + i_{t-1}^{L}) B_{t-1}^{L}. \]  

(1.15)

Taking equation (1.1) into account, this simplifies to

\[ B_t^L + P_t \tau_t W_t N_t = P_t G_t + (1 + i_{t-1}^L) B_{t-1}^L. \]  

(1.16)

The government finances its public expenditures $G_t$ and interest payments with tax revenues or the issuance of new debt. Fiscal rules will react on the difference between the debt ratio and its respective target $\bar{d}_t$. The debt-to-GDP ratio in the model is given by

\[ d_t = \frac{B_t^L + B_t}{Y_t}. \]  

(1.17)

**Tax-based consolidation**

If consolidation is achieved by means of higher labor tax rates, the fiscal feedback rule is given by

\[ \tau_t - \tau^{new} = \phi_{\tau} (d_t - \bar{d}_t). \]  

(1.18a)

Note that the fiscal rule is in deviation from the new steady state tax rate $\tau^{new}$ that is consistent with the new, lower debt level $\bar{d}^{new}$. The parameter $\phi_{\tau}$ captures the pace of adjustment. The larger its value, the stronger taxes react on deviations from the target. As will be explained in detail in the next section, the new steady state implies a different optimal amount of public expenditures. To sharpen the intuition, I will only focus on polar cases, i.e. only tax-based adjustments or spending-based ones. Some studies that evaluate a mixed-strategy also assume that the composition of the budgetary adjustment affects the new long-term steady state. For example, Erceg and Lindé (2013) show an “intertemporal trade-off between tax-based and expenditure-based consolidation: the former induces a smaller near-term output contraction, but implies a considerably deeper output decline at longer horizons.” Therefore a mixed-strategy seems reasonable to evaluate. However, as my long-term steady state does not depend on the composition (see the next section below), there is no such reasoning a priori.

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12 This enhances transparency with respect to the instruments used, as the initial and the end steady state are similar.

13 To sharpen the intuition, I will only focus on polar cases, i.e. only tax-based adjustments or spending-based ones. Some studies that evaluate a mixed-strategy also assume that the composition of the budgetary adjustment affects the new long-term steady state. For example, Erceg and Lindé (2013) show an “intertemporal trade-off between tax-based and expenditure-based consolidation: the former induces a smaller near-term output contraction, but implies a considerably deeper output decline at longer horizons.” Therefore a mixed-strategy seems reasonable to evaluate. However, as my long-term steady state does not depend on the composition (see the next section below), there is no such reasoning a priori.
as for the evolution of the debt ratio target, namely:

\[ G_t = (1 - \rho_g)G_{new} + \rho_g G_{t-1}, \quad (1.19a) \]

where \( \rho_g \) is chosen such that \( G_t \) converges after 40 quarters to \( G_{new} \).\(^{14}\)

**Expenditure-based consolidation**

If the budgetary adjustment is achieved through a reduction in government expenditures, the spending path evolves according to

\[ G_t - G_{new} = \phi_g (d_t - \bar{d}_t). \quad (1.18b) \]

Tax rates will evolve towards their new steady state value by

\[ \tau_t = (1 - \rho_\tau)\tau_{new} + \rho_\tau \tau_{t-1}. \quad (1.19b) \]

**Monetary policy**

Monetary policy is set according to a Taylor-rule that reacts on the inflation rate and output:

\[ \frac{1 + i_t}{1 + i} = \left( \frac{1 + \pi_t}{1 + \pi} \right)^{\phi_x} \left( \frac{Y_t}{\bar{Y}} \right)^{\phi_y} \quad (1.20) \]

**1.2.5 Aggregation and exogenous rules**

Finally, the goods market must clear such that

\[ Y_t \Delta_t = A_t N_t \quad (1.21) \]

with

\[ \Delta_t = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} di \]

\(^{14}\)I provide robustness results for different transitional specifications of exogenous transition (linear, front loading or back-loading adjustments) and whether all the free resources of lower debt ratios are used to lower taxes / increase spending. Overall the results are robust to all such changes.
and by the Calvo-property

\[ \Delta_t = \theta \Delta_{t-1} (1 + \pi_t) \epsilon + (1 - \theta) \left( \frac{P^*}{P_t} \right)^{-\epsilon}. \] (1.22)

The aggregate resource constraint is

\[ Y_t = C_t + G_t. \] (1.23)

Equations (1.1) to (1.23) describe the non-linear model economy. To analyze the transition towards the new steady state I use perfect foresight (for details, see the appendix).

1.2.6 Calibration

I calibrate the model to match the debt characteristics of the US economy in 2018. For simplicity, I set the amount of short-term debt \( B_t \equiv 0 \) for all \( t \) and thus abstract from any portfolio decision taken by the government.\textsuperscript{15} The model starts with an initial debt-to-GDP level of 100% and converges to a new debt target of \( \bar{d}_{\text{new}} = 90\% \). \( \gamma \) is equal to 0.055 to match the average maturity of US debt in 2018 of 68 months.

The time preference rate \( \beta \) is chosen to match an average annual real return of 4%. The inter-temporal elasticity of substitution of private and public goods \( \sigma, \sigma_g \) as well as the inverse Frisch elasticity \( \frac{1}{\phi} \) are all set to 1. The economy is subject to a steady state mark up of 20% and an average adjustment of nominal prices that will take one year, so \( \epsilon = 6 \) and \( \theta = 0.75 \). The policy parameters for the Taylor rule are standard values that satisfy the Taylor principle with \( \phi_\pi = 1.5 \) and \( \phi_y = 0.125 \). The adjustment parameters on the fiscal feedback rules were chosen such that the actual debt level will be reduced by 10%—points within 40 quarters.

As emphasized above, \( \gamma \) also captures the average amount of debt that matures within one quarter. Figure 1.2 depicts how well that calibration fits the US data. I set government spending equal to 20% of GDP, roughly the average of post

\textsuperscript{15}Krause and Moyen (2016) set the real level of debt \( \frac{B_t}{P_t} = b_t = b \) to a constant. However, as there is no steady state inflation in my specification both specifications yield similar results. A complementary approach would be to choose a constant proportion of short- relative to long-term bonds.
WWII levels. The weighting parameters on labor and government spending are chosen such that with a debt ratio of 100% it would be optimal to spend 20% of GDP on public goods and to work $N = \frac{1}{3}$ hours (see below).\footnote{16For a robustness of some of the parameters see the conclusion. The qualitative results are basically unchanged.}

### 1.3 Long run implications

Before analyzing transitional paths one has to decide how to allocate the free resources as a result of the lower debt-to-GDP ratio and therefore lower interest expenses. In general, one can either increase public consumption, reduce the tax rate or a combination of both. I will determine the optimal composition of tax rates, government expenditure and private consumption that maximizes the households welfare for a given (lower) debt level, following Adam (2011).\footnote{17The approach usually taken in the literature is that all free resources are used for the instrument that was used in the consolidation process (see, for instance, Coenen et al., 2008; Forni, Gerali and Pisani, 2010; Glomm, Jung and Tran, 2018). That is if government spending (the tax rate) was reduced (increased) during the transition, then all the proceeds would be used to increase government spending (reduce tax rates) in the long-run. This will have a feedback effect on the household behavior, which blurs whether the overall results are driven by different fiscal measures during the transition or different steady states.} I thus assume that, for reasons outside of the model, the government decides not only to reduce debt levels but also to converge to a new steady state in which this debt level implies an optimal allocation of the other aggregate variables. The remaining task is then to assess which instrument to use in order to transit from the same steady state A to the same steady state B.\footnote{18A potential drawback is that a path for the other instrument that is not used within the consolidation phase has to be specified. However, the results do not depend on specific functional forms of the other instrument. I also checked the approach taken by the literature and again found no big difference. Hence, it seems that in the model short-term adjustments are driving the results.}

Table 1.2 shows the results of different steady state debt-to-GDP ratios (100%, 90% and 80%) for private and public consumption, hours worked and the tax rate (for details on the setup, see the appendix). The percentage change is relative to the initial debt level of 100% except for tax rates where the percentage point change is used.

The additional funds from lower debt repayments are used to reduce distortionary tax rates and to increase public good provision. As a result of both, the households will decide to work more: First, lower tax rates increase the incentive
Table 1.2: Steady state comparison for different debt levels

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>H</th>
<th>G</th>
<th>τ</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>100% debt-to-GDP (starting point)</td>
<td>0.27</td>
<td>0.33</td>
<td>0.07</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>90% debt-to-GDP</td>
<td>0.27</td>
<td>0.33</td>
<td>0.67</td>
<td>0.28</td>
<td>0.27% 0.33% 0.57% -0.43pp 0.18%</td>
</tr>
<tr>
<td>80% debt-to-GDP</td>
<td>0.27</td>
<td>0.34</td>
<td>0.07</td>
<td>0.28</td>
<td>0.54% 0.65% 1.10% -0.86pp 0.36%</td>
</tr>
</tbody>
</table>

Notes: C denotes private consumption, H hours worked, G the provision of public goods, τ the tax rate and CV, the welfare equivalent consumption variation given by equation (1.24).

to work by reducing the intratemporal labor-leisure distortions. Second, the increase in permanent government consumption constitutes c.p. a negative wealth effect that induces the agent to work more (Coenen et al., 2008). Due to lower distortionary tax rates, private consumption is crowded in my setting. The table also reports the welfare equivalent consumption variation (CV) that is required every period to make the household in the initial steady state as well off as in the new one. More precisely, denote by $V(\cdots)$ time utility, that is

$$V \left((1 + \zeta)C^{old}, N^{old}, G^{old}\right) = \sum_{t=0}^{\infty} \beta^t U \left((1 + \zeta)C^{old}, N^{old}, G^{old}\right)$$

It follows that

$$U \left((1 + \zeta)C^{old}, N^{old}, G^{old}\right) \equiv U(C^{new}, N^{new}, G^{new}) \quad (1.24)$$

$\zeta > 0$ implies that the household asks for a compensation to be indifferent between both states, that is, it prefers the new state. With a 10pp reduction in the debt ratio, households demand 0.18% of permanent consumption if debt is high to be indifferent - hence, lower debt ratios are welfare enhancing. With a 20 percentage points reduction the CV doubles to 0.36% and one can show that the linear relation persists, at least for reasonable ranges.\(^{19}\) The qualitative result is robust to different CRRA-parameters in the utility function for private or public consumption and for different mark-ups and Frisch elasticities.

\(^{19}\)In this economy, the first best would be -943% debt-to-GDP, which would raise consumption by 28.7%, hours worked by 31.1%, government spending by 40.6% and reduce the tax rate by 48.9pp (i.e. a subsidy).
1.4 Transition dynamics

While the previous section has focused on potential welfare gains from lower debt levels in the long-run, this section sheds some light on potential costs during the consolidation period and whether an equilibrium with lower debt levels is preferable relative to the status quo if the transitional adjustments are taken into account. A permanent reduction of the debt target $\bar{b}_t$ from 100% to 90% is simulated according to equation (1.14). This will induce fiscal policy to adjust accordingly which affects the economy at large. I will first present each consolidation separately, compare them, show that the results hold in medium-scale DSGE model as well, and then contrast it with a maturity-structure of only one period debt.

1.4.1 Fiscal consolidation

Figure 1.3 shows the aggregate effects for a tax-based consolidation. The fall in the debt target (upper left panel) induces the labor tax rate to rise until period 6 and then to gradually convert back to its new (lower) value $\tau^{new}$ (upper
middle panel). Higher distortionary taxes reduce the incentive to supply labor which cause a recession (lower left panel). Additionally government spending will be higher in the new long run equilibrium (lower middle panel).20 The output drop is quite pronounced and lasts for 5 to 6 years until labor supply recovers and converges to its higher long-term equilibrium. As taxes reduce the after-tax wage income, households ask for higher pre-tax wages to compensate part of the income loss. Since wages are marginal costs for firms they react to that by charging a higher prices which leads to inflation (upper right panel). The monetary authority follows the Taylor principle and raises its policy rate more than one-to-one, driving up the real short-term interest rates.

As can be seen from equation (1.4), the average interest rate $i_t$ is a weighted sum of its previous value $i_{t-1}$ and the new interest rate $i_{t,n}$. Additionally, $i_{t,n}$ depends on the expected future path of the policy rate $i_t$ if one iterates (1.6) forward. Since the model is solved under perfect foresight, agents take into account the complete path of future policy rates, not only the initial spike. Therefore, the interest rate on newly issued long-term debt $i_{t,n}^{ln}$ increases by a mere 0.4% but much less pronounced compared to the policy rate hikes of 2.5% (lower right panel). Since nominal average interest rate react less than one-to-one with inflation, the real average rate decline.21

Now contrast this scenario with an expenditure-based consolidation, depicted in figure 1.4 for the same variables. Public good provision will be reduced up to 18% and recovers gradually until it reaches its higher long run level $G^{new}$ (lower left panel). The reduction in spending forces the economy into a recession with lower inflation (upper middle and right panel).22 Monetary policy responds by cutting the short-term interest rate, but the reduction in the nominal average interest rate $i_t^L$ is muted (lower right panel) so that real ones rise.23
1.4.2 Comparing both fiscal consolidations

In the above simulations it is assumed that both consolidation strategies lead to the same long run equilibrium, but they entail different adjustment costs during the transition. While one can in principle assess the aggregate dynamics from figures 1.3 and 1.4, it is instructive to aggregate these effects into one number that is easily comparable. I use two metrics to compare the relative desirability/associated costs.

\[ \text{The increase in government spending constitutes a negative wealth effect and lowers private consumption while increasing the supply of labor. This effect lowers generally the negative effect of tax-hikes on output.} \]

\[ \text{After a while inflation is below the average long-term rate thus increasing real rates of the outstanding debt stock. However, as the total stock is already reduced by that time, this effect is rather smaller.} \]

\[ \text{The simultaneous reduction of the tax rate is helpful to partly cushion the drop in GDP. The importance of future composition of variables on current dynamics is shown, for instance, by (Cogan et al., 2013).} \]

\[ \text{The initial jump of inflation, output and the policy rate can be explained by an anticipation effect: Since households foresee that government spending will be lower during in the future, there is a positive wealth effect which raises private demand. However, in the first periods, public spending is not yet reduced as much, so the overall effect is expansionary. If spending would be cut more forcefully (increase } \phi_g \text{), there is always a recession and deflation.} \]
Figure 1.5: Fiscal Sacrifice Ratio

Notes: Fiscal Sacrifice Ratio is calculated as in (1.25).

The first is the “Fiscal Sacrifice Ratio” (FSR), a measure, that relates the output loss to the percentage point reduction of debt. For a smoother comparison I use the average output drop rather the exact drop within that period. More precisely, the ratio is defined as

$$\xi_T = \frac{1}{T} \sum_{t=1}^{T} \frac{Y_t - Y_{\text{old}}}{d_t - d_{\text{old}}}$$

(1.25)

Figure 1.5 presents the ratio at a two, three and four year horizon. Within two years, both fiscal consolidations reduce the debt-to-GDP ratio by about 3.5% points while output falls on average about 1%, consistent with a FSR of around 1/3. Increasing the time horizon reduces the sacrifice ratio as output growth increases.\(^{24}\) Over the whole time span tax-based consolidation is associated with a slightly lower FSR than a spending-based one.

A second approach is to evaluate the welfare equivalent consumption variation (CV) associated with each reduction scenario, that is the permanent amount of consumption that makes the household indifferent between remaining at the status quo (steady state with high debt) and moving to a lower debt world. It is defined as

$$V \left( (1 + \zeta)C_{\text{old}}, N_{\text{old}}, G_{\text{old}} \right) = \sum_{t=0}^{\infty} \beta^t U \left( C_t, N_t, G_t \right).$$

(1.26)

\(\zeta > 0\) implies that consolidation is actually welfare enhancing. On the other side, if \(\zeta < 0\) the status quo would be preferable. The corresponding CVs

\(^{24}\)In the first year, both sacrifice ratios are negative.
Figure 1.6: Tax-based consolidation with short-term debt

Notes: All variables are in percentage deviation from steady state, except tax rates, inflation and interest rates which are in percentage point deviations. The lower right panel displays the variable $i_t$ which coincides with the policy rate in the case of short-term debt.

are $-0.06$ for spending-based and $0.04$ for the tax-based budgetary adjustment. Thus, similar to the assessment based on the FSR, tax-based adjustments are preferable relative to spending-based ones since its CV is higher. On top, the results also imply that households actually want to consolidate when it is done by raising taxes since their life time utility is higher in that case. With spending-cuts, the transitional costs are too high such that it would be welfare detrimental. The next subsection illustrates, how much these results depend on the average maturity of public debt and the dynamics of the real average interest rates.

1.4.3 Short-term public debt

As emphasized in the introduction, it is important to properly capture the dynamics of the real average interest rate, since it has a first order effect on the debt-to-GDP ratio. To gauge the extent to which different maturities of public debt affect the interest rate, I conduct two experiments: First, I evaluate the aggregate responses for both fiscal adjustments when only short-term debt is available and second, contrast the FSR and the CV for intermediate values of maturity.
With short-term debt the dynamics of the real average interest rates correspond to the real short-term rates, with accompanying effects on public debt. Figures 1.6 and 1.7 depict the comparison of the same macroeconomic variables as above when public debt is short-term (dashed lines) and long-term (figures above, solid lines). Qualitatively, the aggregate responses are similar for both cases. The increase in the tax rate raises inflation and the nominal average interest rates and lowers GDP. Spending-cuts lower the inflation rate, the nominal interest rate and also depress output. However, for the tax-based scenario the quantitative results differ markedly.

The labor tax rate roughly doubles to 5% points (upper middle panel) which leads to an output drop of more than 3% that is also more persistent (lower left panel). In the first periods inflation actually falls (upper right panel) since the reduction in demand outweighs the increase in marginal costs. It is only after that initial period that the inflation rate is positive. Nevertheless, in the first period real average interest rates are reduced due to the Taylor-principle (lower right panel, similar as above), while higher inflation raises real rates afterwards. This also implies that taxes rise relatively more compared to the long-term debt benchmark. As a result, the recession is deeper. In the spending-based adjust-
ments, public goods still have to be cut by roughly 18% but recover much faster (lower middle panel). This lowers the real average interest rate which is beneficial for the government budget (in contrast to the TB-plan), which allows spending to increase relatively faster. Output follows that pattern quite closely (lower left panel). Compared to the tax-based consolidation, the aggregate variables do not move that much.

A comparison of both measures reveals that spending-based consolidation is now preferable. The sacrifice ratio is between 3 to 5 times smaller and the CV larger (-0.01 vs -0.13 with TB-plans). Nevertheless, as the negative value indicates, from a welfare point of view maintaining the status quo is desirable. Hence, with short-term debt the model replicates findings that EB-plans are more preferable than TB-ones.

To contrast intermediate cases and to quantify until which extended maturity TB-plans become superior, I simulate the model for a range of average maturities from one-period debt to 14 years and compare the FSR and CV at each maturity in time.\(^\text{25}\) The results are depicted in figure 1.8.

Focus on the FSR in the left panel first. It illustrates how distortionary tax-based budgetary adjustments can be when debt is only short-term. If the horizon is extended, the distortion falls relatively quickly. Overall, the maturity does not affect the FSR much in case of expenditure-based consolidation, in line with the aggregate results in the one-period debt model. The right panel shows the CV,

\(^{25}\)I focus on the 2-year horizon for the FSR.
which marks a more continuous picture: It monotonically increases (decreases) with the average maturity of debt when consolidation is tax-(spending-)based. The threshold after which tax-based consolidation is preferable is about 2 years for the CV and 4 years for the FSR. Additionally, for maturities above 3 years households are actually better off with a tax-based debt consolidation (positive CV).

1.5 Robustness

The main message in this paper is that the maturity structure of government debt can shape the aggregate dynamics of fiscal consolidation due to its induced impact on interest rates. In this section I briefly comment on some robustness checks.

The results do not change qualitatively a lot if I assume different timing assumptions on how debt is reduced and the feedback rules are set-up. Also, even though the size of consolidation reduces the overall magnitude, it does not change the relative result that TB-plans become more preferable when public debt is long-term.

I have also simulated the model with different parameter values. Broadly speaking the main result did not change although quantitatively some changes strengthen and other weaken the interest rate channel relative to the benchmark calibration. As an example, a lower Frisch elasticity renders tax-based consolidation less inflationary (muted wage response), lowering the response of the real average interest rate. This is also the case for spending-cuts, but less so. Hence, the distance between the two CV measures narrows.

1.5.1 Consolidation at the ELB

One conjecture (also raised in the literature mentioned above) is that tax-based adjustments should become more favorable when monetary policy is constrained by an effective lower bound (ELB). To quantify whether this is the case, I first simulate a negative demand shock that brings the economy to the ELB for 4 quarters. Then, I contrast tax-based vs. spending-based consolidation. Figure

\[\text{Figure 26} \]

To conduct this exercise I log-linearize the model around the steady state and follow the approach by Kulish and Pagan (2017), see the appendix for further details of the implementation.
Figure 1.9: Fiscal consolidation at the effective lower bound.

1.9 illustrates the response relative to the baseline ELB experiment, i.e. values above (below) 0 indicate that the relative response is higher (lower) than in the baseline with no extra consolidation.

Focus on the EB-plan first (red line). In order to reduce the debt ratio (above what is already implied by the recession), spending has to be cut even further (upper middle panel), which lowers GDP (bottom left panel) and inflation (upper right panel). The real average interest rate increases (bottom middle panel) quite strongly. The reason is that the short-term policy rate is stuck at the ELB which cannot accommodate the demand shortage. Therefore, nominal average interest rates react less than one-to-one for two reasons. First, as above, only part of the debt is reissued and the long-term interest rate on newly issued debt depends on todays and expected future short rates. Second, since the short-rate in the next 4 quarters is bound by the ELB, the interest rate on newly issued debt is relatively higher. This prolongs the recession.

In case of a tax-based consolidation (blue line), higher labor taxes increase inflation (falls relatively less pronounced than in baseline) and the real average interest rate falls as well. As a result, a small boom occurs in the first three quarters (bottom right panel) - a case of expansionary austerity. The reason is that the additional inflation expectations reduce short-term and long-term real rates which raise demand. This initial expansionary impact also raises short-
Figure 1.10: Comparison of tax-based (TB) and expenditure-based (EB) consolidation in the medium-scale DSGE model

**Notes:** The medium-scale DSGE model is a version of Carlstrom et al. (2017). See appendix for further details.

term rates such that the ELB is already left before the baseline of 4 quarters. However, once the tax rates keep on rising, the expansionary impact vanishes.

### 1.5.2 Medium-scale DSGE

So far, I have been using a stylized model for the simulations. To ensure that the qualitative results are not driven by the specific kind of model, I use a medium-scale DSGE model to run similar simulations. In particular, I use the model by Carlstrom et al. (2017) – it is an estimated model for the US economy that already includes long-term debt – and augment it with a fiscal sector (see the appendix for details). The results indicate that a permanent reduction of the debt-to-GDP target (upper left panel) has qualitatively similar effects to the stylized model. With tax-based consolidation, inflation and the short-term policy rate increase (bottom left and middle panel) while the real long rate decreases (bottom right panel). The opposite is true for spending-cuts. Hence, the general mechanism emphasized above holds also in more complex model environments.
1.5.3 Further sensitivity

It is straightforward to extend the model to a situation in which after consolidation begins, the government is only able to issue short-term debt. The motivation behind such a scenario is that after a crisis (like the Great Recession) investors might be weary to hold new long-term debt, especially in countries that have to consolidate. As a simplification, I assume that with the beginning of a budgetary adjustment, the government can only issue short-term debt. In this case, the interest rate of newly issued bonds and the short-term policy rate coincide, i.e. $i_t^{L,n} = i_t$. Nevertheless, the results are pretty much unchanged (illustrated in figure 1.11 in the appendix).

A question remains how much of the results are driven by the underlying simple rules relative to some optimal policies. Even though in many applications, simple rules can approximate Ramsey policy (Schmitt-Grohe and Uribe, 2007), it is not clear in this setup. An interesting extension would be an open economy setup. This should have two countervailing effects: On the one hand, the attenuated response of monetary policy in such a union should strengthen the case for tax-based consolidation (as explained above with the ELB scenario). On the other hand, higher prices reduce the real exchange rate and lower competitiveness which dampens output. A proper analysis of how these extensions might affect the results is left for future research.

To keep the model as simple as possible the average maturity in the data is approximated by having just one bond with the exact average maturity. This seems to be a reasonably well characterization for the US. However, the shape of the CV (concave and convex) in figure 1.8 implies that an even richer maturity structure might affect especially welfare results.\(^{27}\)

1.6 Conclusion

In this paper I assess the macroeconomic implications of expenditure-based (EB) versus tax-based (TB) consolidation through the lens of a New Keynesian model.\(^{27}\) Suppose, for example, the average maturity is 10Q with 50% short-term and 50% 19Q debt. This implies a CV of -0.04 for spending-cuts and roughly 0 for tax-hikes (see figure 1.8). However, the respective number for 50% one-period debt+50% 19Q debt would imply for tax-hikes a lower and for spending-cuts a larger number, thus closing the distance between both consolidations.
with long-term debt. As it turns out, keeping track of the real average interest rate dynamics is important to determine the relative desirability of the two fiscal adjustments.

The main result of the paper is that an extended maturity of public debt can render tax-hikes superior to spending-cuts. If government bonds are short-term, the opposite holds. The reason is that the real average interest rate – the interest rate that the government pays on its outstanding liabilities and which affects the debt-to-GDP ratio next to growth and surplus – moves in opposite direction with the inflation rate, if the maturity is long enough. When consolidation is tax-based, the resulting higher inflation reduces the real interest rate which is beneficial for the government budget. On the other hand, lower inflation following EB-plans raise real rates, making consolidation harder. The results hold in a medium-scale DSGE model as well.

The present analyses is stylized. It should not be interpreted as a policy advice to always reduce the debt ratio through TB-consolidation. For instance, empirically, in many consolidations the government aims to cut rather wasteful expenditures or transfers, which I did not consider. The paper rather clarifies the important role of the interaction between real average interest rates dynamics and the composition of fiscal consolidation.
Appendices

1.A  Overview model equations stylized model

1.A.1  Households

\[ \lambda_t = C_t^{-\sigma} \]  \hspace{1cm} (27)

\[ W_t(1 - \tau_t) = \chi_n N_t^\phi C_t^\sigma \]  \hspace{1cm} (28)

\[ 1 = \beta E_t \frac{\lambda_{t+1} 1 + i_t}{\lambda_t 1 + \pi_t} \]  \hspace{1cm} (29)

\[ 1 = \beta E_t \frac{\lambda_{t+1} 1 + i_t}{\lambda_t 1 + \pi_t} \left( 1 + i_t^{L,n} - \mu_{t+1}(1 - \gamma)(i_{t+1}^{L,n} - i_t^{L,n}) \right) \]  \hspace{1cm} (30)

\[ b_t^L = \frac{(1 - \gamma)}{1 + \pi_t} b_{t-1}^L + b_t^{L,n} \]  \hspace{1cm} (31)

\[ i_t^L b_t^L = \left( \frac{1 - \gamma}{1 + \pi_t} \right) i_{t-1}^L b_{t-1}^L + i_t^{L,n} b_t^{L,n} \]  \hspace{1cm} (32)

1.A.2  Firms

\[ W_t = MC_t(1 - \alpha)A_t N_t^{-\alpha} \]  \hspace{1cm} (33)

\[ g^1_t = \lambda_t MC_t Y_t + \beta \theta E_t \left( \frac{1 + \pi_{t+1}}{1 + \pi_t} \right) g^1_{t+1} \]  \hspace{1cm} (34)

\[ g^2_t = \lambda_t Y_t + \beta \theta E_t \left( \frac{1 + \pi_{t+1}}{1 + \pi_t} \right) g^2_{t+1} \]  \hspace{1cm} (35)

\[ p^* = \frac{\epsilon}{\epsilon - 1} \frac{g^1_t}{g^2_t} \]  \hspace{1cm} (36)

\[ 1 = \theta \left( \frac{1 + \pi_t}{1 + \pi} \right)^{r-1} + (1 - \theta)(p^*)^{1-\epsilon} \]  \hspace{1cm} (37)

\[ \Delta = \theta \Delta_{t-1} \left( \frac{1 + \pi_t}{1 + \pi} \right)^{r-1} + (1 - \theta)(p^*)^{-\frac{1}{1-\alpha}} \]  \hspace{1cm} (38)

1.A.3  Government

\[ b_t^{L,n} + \tau_t W_t N_t = G_t + \frac{\gamma + i_{t-1}^L}{1 + \pi_t} b_{t-1}^L \]  \hspace{1cm} (39)
Fiscal rules, either:

\[ \tau_t - \tau_{new} = \phi \left( \frac{B_{tL}}{4Y_{new}} - d_{new} \right) \]  \hspace{1cm} (40)

\[ G_t = (1 - \rho_g)G_{new} + \rho_g G_{t-1} \]  \hspace{1cm} (41)

or

\[ G_t - G_{new} = \phi \left( \frac{B_{tL}}{4Y_{new}} - d_{new} \right) \]  \hspace{1cm} (42)

\[ \tau_t = (1 - \rho_{\tau})\tau_{new} + \rho_{\tau} \tau_{t-1} \]  \hspace{1cm} (43)

1.A.4 Monetary Policy

\[ 1 + i_t = (1 + i) \left( \frac{1 + \pi_t}{1 + \pi} \right)^\phi \left( \frac{Y_t}{Y_{new}} \right)^\phi \]  \hspace{1cm} (44)

1.A.5 Exogenous rules

\[ A_t = (A_{t-1})^{\rho_a} e^{\epsilon_t} \]  \hspace{1cm} (45)

\[ \bar{d}_t = (1 - \rho_d)\bar{d}_{new} + \rho_d \bar{d}_{t-1} \]  \hspace{1cm} (46)

1.A.6 Aggregation

\[ Y_t \Delta^{1-\alpha} = A_t N_t^{1-\alpha} \]  \hspace{1cm} (47)

\[ Y_t = C_t + G_t \]  \hspace{1cm} (48)

1.B Model solution

In a nutshell, the algorithm finds numerical values of the variables that solve the non-linear equations. The important assumption one has to impose is that the model returns to equilibrium in finite time instead of asymptotically. Taking the labor supply (1.8) as an example one rewrites

\[ W_t(1 - \tau_t) = \chi_n N_t^{\phi} C_t^{\phi} \Leftrightarrow W_t(1 - \tau_t) - \chi_n N_t^{\phi} C_t^{\phi} = 0 \]
Proceeding for equations (1.1) to (1.23) one can cast the model in time $t$
into

$$f(X_{t+1}, X_t, X_{t-1}) = f(z_t) = 0$$

with $X_t = \{W_t, \tau_t, N_t, Y_t\ldots\}$ denoting all model variables at time $t$ and $z_t = [X_{t+1}, X_t, X_{t-1}]$ collecting forward and backward-looking terms. One has to choose, first, a starting point $X_0 = X^{old}$, e.g. the initial steady state with high debt, second, an ending point $X_{T+1} = X^{new}$, the new steady state with lower debt and finally the number of periods to simulate, e.g. 2000 periods. The algorithm than stacks for $t = 1, 2, \ldots, 2000$ all the equations into one big system $F(Z) = 0, Z = [z_1 \ z_2 \ \ldots \ z_T]$, i.e. a system of $23*2000$ equations, and solves for the root.\(^{28}\)

### 1.C Long run optimization

To get the optimal allocation of variables for a given debt amount I set up a Lagrangian that maximizes the households welfare function given the constraints 1.5, 1.6, 1.8, 1.15, 1.21 and 1.23. As this is a long-run perspective only the last four equations bind. One can show that it boils down to the following Lagrangian:

$$L (N, \tau, G; \gamma^1, \gamma^2) = u(N - G) - v(N) + g(G) + \gamma^1 \left[ MC(1 - \tau) - \chi_n N^\phi(N - G)^{\sigma_c} \right] + \gamma^2 \left[ MC\tau N - G - i4Nd_{new} \right]$$

Maximization leads to 4 equations and 4 unknowns ($N, \tau, G, \gamma^2$) that can only be solved numerically (using the matlab routine \texttt{fsolve.m}). Specifically, the first order conditions are given by:

$$(N - G)^{-\sigma_c} - \chi_n N^\phi - \lambda \chi_n \left( \phi N^\phi(N - G)^{\sigma_c} + \sigma_c N^{\phi+1}(N - G)^{\sigma_c-1} \right) + \lambda \left( \tau mc - \frac{1+i}{1+i}d_{new} \right) = 0 \quad (49)$$

\(^{28}\)This algorithm is implemented in Dynare \url{http://www.dynare.org/}.  

34
\[-(N - G)^{-\sigma_c} + \chi_g G^{-\sigma_g} + \lambda (\sigma_c \chi_n N^{1+\phi} (N - G)^{\sigma_c-1} - 1) = 0\]  

(50)

\[mc(1 - \tau) - \chi_n N^\phi (N - G)^{\sigma_c} = 0\]  

(51)

\[\tau mcN - G - \frac{1 + i}{1 + \pi} 4Nd^{new} = 0\]  

(52)

1.D Robustness appendix

1.D.1 Consolidate with short-term debt

This section briefly comments on the scenario, where the government can only issue short-term debt after it begins to raise taxes or decrease spending. The average maturity from past liabilities is, however, set to the US-average. Figure 1.11 depicts the difference between this scenario and the baseline with long-term debt (i.e. figures 1.3 and 1.4). As can be seen in the bottom right panel, the nominal average interest rate indeed reacts stronger, but this does not change the
aggregate responses much. The reason is that the average interest rate reacts only sluggishly since for most liabilities, the interest rates are pre-determined while only a small part has to bear higher/lower interest rates.\textsuperscript{29}

1.D.2 ELB scenario

To conduct a ELB scenario I make the following two adjustments. First, I augment the Euler-equation with a preference/demand shock $d_t$:

$$\lambda_t = e^{D_t} c_t^{-\sigma_c}$$

with

$$D_t = \rho D_{t-1} + \varepsilon^d_t.$$ \hspace{1cm} (54)

Second, I follow Erceg and Lindé (2013) and set up the debt target as follows:

$$\bar{d}_t - \bar{d}_{t-1} = \rho_1(\bar{d}_{t-1} - \bar{d}_{t-2}) - \rho_2 \bar{d}_{t-1} + \varepsilon^b_t,$$ \hspace{1cm} (55)

where the parameters are chosen as in the paper. I then simulate a relative persistent debt reduction of up to 10% below the steady state value of 100%. Third, I have to log-linearize the model in order to follow the approach by Kulish and Pagan (2017) to simulate the model at the ELB. The size of the demand shock is then determined such that the ELB is binding for 4 quarters. This baseline scenario is depicted in figure 1.12 and shows a deep recession with negative inflation rates.

Figure 1.12: Baseline scenario when ELB is binding for 4 quarters.

The algorithm: There are two regimes, a so-called reference model, which is labeled (M1) and that holds in steady state and to which the model dynamics

\textsuperscript{29}Obviously, the shorter the average maturity would be, the closer get the short-term and long-term rate.
always converge to. For convenience, the reference model holds when the constraint is slack. Any such structural linear (or linearized) model can be written in the following (general) form:

\[ Ax_t = C + B x_{t-1} + DE_t x_{t+1} + F \varepsilon_t \]  

(M1)

which has a reduced form solution

\[ x_t = J + Q x_{t-1} + G \varepsilon_t. \]  

(M1_sol)

The ELB regime is given by the alternative model (M2):

\[ A^* x_t = C^* + B^* x_{t-1} + D^* E_t x_{t+1} + F^* \varepsilon_t \]  

(M2)

where the matrices with a star (*) account for the structural change, for instance a binding ELB or forward guidance (i.e. an interest rate peg). As will be shown below, the solution when (M2) applies, i.e. when a constraint is binding, will look as follows (see Kulish and Pagan (2017), pg. 261):

\[ x_t = J_t + Q_t x_{t-1} + G_t \varepsilon_t, \]  

(M2_sol)

where the time-varying coefficients of each matrix will be determined recursively as follows (ELB is supposed to hold from 1 to \( T \)):

**Step 1:** Simulate the model with (118): \( x_t = J + Q x_{t-1} + G \varepsilon_t, \forall t \). If the interest rate is indeed below \( i_{ELB} \) in the first period, set \( T = 2 \).

**Step 2:** Replace \( i_t = i_{ELB}, \forall t^* \in (1, \ldots, T - 1) \) and simulate the model again, now with the following solution:

\[ x_t = \begin{cases} 
J + Q x_{t-1} + G \varepsilon_t & \forall t > T - 1 \\
J_t + Q_t x_{t-1} + G_t \varepsilon_t & \forall t \leq T - 1
\end{cases} \]  

(56)

with

\[ \Xi_t = (A^* - D^* Q)^{-1}, J_t = \Xi_t (C^* + D^* J), Q_t = \Xi_t B^*, G_t = \Xi_t F^* \text{ for } t = T - 1 \]
\[ \Xi_t = (A^* - D^*Q_{t+1})^{-1}, J_t = \Xi_t (C^* + D^*J_{t+1}), Q_t = \Xi_t B^*, G_t = \Xi_t F^* \text{ for } t < T-1 \]

**Step 3:** If interest rate at time \( T \) is below \( i^{ELB} \), increase \( T \) by 1 and repeat step 2 and 3 until the interest rate is always greater or equal to \( i^{ELB} \).

### 1.E Medium-scale DSGE model

In order to compare the results to a medium-scale DSGE model I use the model by Carlstrom et al. (2017). This model is estimated on US data and features a banking sector. I introduce a fiscal sector in the spirit of above simple fiscal feedback rules. The following three equations are different to the papers’ model overview in their appendix (name of variables are the same):

\[
Y_t = C_t + G_t + I_t
\]

\[
B_t + \tau_tw_tH_t = G_t + R_l^tB_{t-1}
\]

\[
(1 - \tau_t)(w^*)^{1+\varepsilon_w\eta} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{G_{1t}}{G_{2t}}
\]
Chapter 2

Financial Repression in General Equilibrium

2.1 Introduction

Financial repression allows governments to borrow at artificially low interest rates. This may be the result of explicit or implicit ceilings on nominal interest rates or other measures. It requires that investors are somehow held captive through capital controls or regulatory policies (McKinnon, 1973; Shaw, 1973). While financial repression has traditionally been considered a phenomenon specific to developing economies, Reinhart and Sbrancia (2015) argue—based on institutional details and the observation that real interest rates have been low and often even negative—that financial repression has also been pervasive in many advanced economies during the period after WW2. Figure 2.1 provides suggestive evidence as it displays times series of US real interest rates after WW2. The data show that the yield on longer-term US government debt (solid line) has been relatively low until the late 1970s. In particular, long-term yields were not systematically higher than the short-term interest rate (dashed line) in the first half of the sample.¹

In this paper, we seek to quantify the extent of financial repression in the US

¹A similar picture emerges for nominal interest rates. The average interest rate on 10-year government bonds during 1960–1974, for instance, is very similar to average federal funds rate: 5.4 vs 5.2 percent. Instead, during 1980–2005 average long-term nominal rates were considerably higher, also vis-à-vis short term rates: 7.7 vs 6.5 percent (source: St. Louis Fed/FRED).
during the post-WW2 period—both in terms of its effects on interest rates and in term of its contribution to the sizeable “liquidation of government debt” during that period. Moreover, we ask how financial repression affected the macro-economic performance of the US economy during that period. This question is pertinent given the rather spectacular build-up of public debt in many advanced economies during recent years. In many instances, the ratio of debt-to-GDP has by now reached or even surpassed the debt levels observed at the end of WW2. It is conceivable that, as with previous episodes, financial repression may feature prominently in the mix of debt-reduction policies (Reinhart, 2012). Given poor growth prospects and low inflation, financial repression has the benefit—at least from a political-economy point of view—that it works stealthier than austerity policies. It may also be less disruptive than outright default.2

Quantifying the extent of repression is challenging, because the interest rate which would prevail in the absence of repression – say the “laissez-faire” interest rate – is not directly observable.3 Earlier studies focused on developing countries. In this case one may proxy the laissez-faire interest rate with the interest rate a government pays on world capital markets, as suggested by Giovannini and de Melo (1993). They document that the “repression tax” contributed handsomely to government revenues.4 An earlier survey by Fry (1997) concludes that financial repression contributed to government revenue in the order of 2 percent of GDP in a sample of developing economies. In their study on 12 advanced economies Reinhart and Sbrancia (2015) find that the savings of annual interest-rate expenses amounted to up to 5 percent. This result assumes a constant, repression-free interest rate in the range between 1 and 3 percent.

In our analysis we rely on a dynamic general equilibrium model in order a) to estimate of the laissez-faire interest rate and b) to study the general equilibrium effects of repression through counterfactual experiments. Our model is a conventional New Keynesian business cycle model which features leverage-constrained banks (as in Gertler and Karadi, 2011; Gertler and Karadi, 2013). The essen-

2Reinhart, Reinhart and Rogoff (2015) survey a menu of options for debt reduction which includes financial repression.

3The laissez-faire interest rate as defined in this paper will generally differ from the natural rate of interest which would prevail if prices and wages are flexible. In a flex-price world, for instance, there may still be repression which pushes the actual (and hence the natural) rate below the laissez-faire rate.

4Giovannini and de Melo (1993) investigate 24 countries during the period 1972–1987. They find several instances in which the annual amount of “revenue” that is due to financial repression amounts to 5 percent of GDP.
Figure 2.1: Short and long real interest rates on US government debt

Notes: Long real interest rates (ex post) is given by solid line, it is the return on marketable debt of government portfolio computed by Hall and Sargent (2011); dashed line is three month T-bill rate minus actual inflation (source: St. Louis Fed).

A potential feature of our model is an additional constraint under which banks operate. Specifically, we follow Chari, Dovis and Kehoe (2019) and assume that banks face a “regulatory constraint” which requires them to hold a certain fraction of their assets as government debt. As governments vary this fraction they effectively alter the yield on long term government debt. Our setup thus makes explicit that the banking sector is a captive audience for government debt. In practice, the regulatory constraint reflects a variety of measures on which the government may rely, if only unintendedly, when it auctions off its debt at elevated prices.

In the model, the government issues long-term debt only, which is held either by households or banks. Our focus on long-term government debt is motivated by the evidence shown in Figure 2.1, but also by narrative accounts of financial repression. First, in the late 1940s, the Fed, according to chairman Eccles, allowed short rates to fluctuate, but maintained a ceiling of 2.5% for the long-term rate (Chandler, 1949). This ceiling on the return of long-term debt kept to be a concern during the negotiations of the Fed Accord in 1951 which made the Federal Reserve less dependent on the Treasury. At the time the Treasury exchanged a large amount of long-term non-marketable debt for marketable debt in order to further keep long-term interest rates low (Hetzel and Leach, 2001). Similarly, during the early 1960s, the US government conducted “operation twist” in order to raise short-term rates (to attract foreign capital inflows) while keeping...
long-term rates low.

In our model, households can adjust short-term bank deposits freely. However, as in Gertler and Karadi (2013) we assume that households face transaction costs when they adjust their holdings of government debt. In equilibrium the yield on long-term debt differs from short-term interest rates on deposit because of two distinct factors. First, because banks are leverage constrained they are unable to arbitrage away yield differences between short- and long-term rates. A tighter leverage constraint because of, say, reduced net worth, raises, all else equal, the difference between short-term and long-term rates. This difference can be interpreted as a term premium due to market segmentation (Fuerst, 2015). Second, financial repression, all else equal, reduces long-term yields and hence tends to offset the term premium.

Against this background, we observe that the actual evolution of short and long-term interest rates is consistent with the predictions of our model—under the maintained hypothesis that financial repression was more pervasive in the post-WW2 period compared to the post-1970s. The model rationalize the observation that short and long-term rates differed hardly during the repression period because it predicts that repression offset the term premium.\(^5\)

Because in our analysis financial repression operates along the yield curve, it is consistent with the notion that government debt carries a “convenience yield” (Krishnamurthy and Vissing-Jorgensen, 2012). Importantly, the convenience yield reflects investors’ preference for liquid and safe assets rather than regulatory measures. However, Krishnamurthy and Vissing-Jorgensen (2012) find that the yield spreads between non-government and government debt are equally responsive to the supply of government debt in case of short and long-term debt. Also, recent estimates of the convenience yield by Del Negro, Giannone, Giannoni and Tambalotti (2017) focus on a trend that is “common across maturities”.

Through financial repression the government effectively taxes the financial sector. This is consequential for the economy at large, because banks are special in their ability to monitor firms. As in Gertler and Karadi’s original formulation we assume that all savings of households are channeled through banks in order to fund investment projects. As repression distorts banks’ portfolio choice and reduces their net worth, investment is crowded out and output and inflation

\(^5\)Estimates of the term premium (which do not account for repression) also tend to show a strong increase of the term premium after the 1970s (Adrian, Crump and Moench, 2015).
We also contrast financial repression with conventional monetary policy measures such as a cut in the short-term policy rate. Repression and conventional monetary policy may have a similar impact on public finances. Yet, monetary policy differs from financial repression or regulation in general in that it impacts short and long term real interest rates alike: “it gets in all the cracks” (Stein, 2013). More importantly still, we also show that repression and conventional monetary policy transmit through the economy in profoundly different ways.

We estimate the model on quarterly US time series data for the period 1948–1974. Our estimation is based on eight macroeconomic variables and, in addition, two financial variables, namely equity returns and banks’ net worth. We find that the model performs well. In particular the models’ prediction for the share of government debt in banks’ portfolio aligns very well with actual developments, even though those have not been considered in the estimation.

Turning to the issue at hand, we also use the model to compute the laissez-faire interest rate. It is considerably higher than actual rates, except for a few instances. However, the interest rate reduction varies considerably over time. Next we quantify the contribution of financial repression to the reduction of public debt during our sample period. We do this in two ways. First, we take an accounting perspective and compute the counterfactual evolution of debt assuming the government had paid the laissez-faire rather than the actual interest rate, keeping all else equal. We find that in the case the debt ratio would have declined by 35 rather than by 60 percentage points.

In a second experiment, we account for general equilibrium effects. Once we do that, we find that without repression the debt-to-output ratio would have declined much faster than in case of repression. Intuitively, this is because in the absence of repression the economy would have been on a more expansionary trajectory. With financial intermediation less impaired, we observe an investment boom in our counterfactual scenario. Also consumption and output are increased relative to the actual developments. As a consequence, inflation is also higher in the counterfactual scenario. These observations can explain why the debt ratio declines faster. In this sense, repression was not contributing to the liquidation of government debt at all. In our view, this finding is particularly noteworthy given the conventional view that repression is part of a toolkit to bring about a reduction of government. We find the conventional view confirmed merely from an accounting point of view.
A number of recent contributions are exploring different aspects of financial repression. Our model builds on Chari et al. (2019), notably as we rely on their regulatory constraint. Just like them our modelling of the banking sector is based on Gertler and Karadi (2011). However, they abstract from nominal rigidities and the conduct of monetary policy. Instead, they focus on the optimality of financial repression in a world where governments lack commitment to paying back its debt and may thus default on its liabilities. Importantly, they show that under commitment a repression tax is inferior to directly taxing banks’ assets because financial repression distorts not only banks’ asset holdings but also their portfolio decision. Our analysis, instead, is purely positive as it seeks to quantify the contribution of the repression tax to debt reduction and to explore counterfactual outcomes.

Roubini and Sala-i-Martin (1995) put forward a model where financial repression raises money demand, say because of regulation that limits use of checks, ATMs etc. This in turn raises the base on which the inflation tax operates. As result, their model predicts that inflation and financial repression go hand in hand, quite contrary to what our analysis suggests (see also Brock, 1989). There is also recent empirical work which suggests that repression has been under way during the recent euro area crisis (Becker and Ivashina, 2016; Ongena, Popov and Van Horen, 2016). More generally, financial regulation has been found to impact financial markets. Du, Tepper and Verdelhan (2018), for instance, rationalize large and persistent deviations from covered interest rates in light of the new regulatory environment put in place after the crisis. It seems to impair the ability of financial intermediaries to carry out arbitrage away spreads between the return of riskless securities. This mechanism operates at the heart of our model.

The remainder of the paper is structured as follows. Section 2 outlines the model and explains how financial repression works in our model. Section 3 describes our data, the estimation as well as the choice of our priors. It also presents results. We answer the main questions in Section 4 as we quantify financial repression and compute counterfactuals. A final section offers a short conclusion.
2.2 The Model

Our analysis is based on a medium-scale New Keynesian model in which the financial sector takes center stage. Here our analysis builds on earlier work of Gertler and Karadi (2013) and Chari et al. (2019). As we estimate the model in Section 2.4, we require it to be sufficiently rich to capture the dynamics of actual time-series data. In this regard we build on earlier work by Bianchi and Ilut (2017), notably as far as the fiscal sector is concerned, and on Justiniano, Primiceri and Tambalotti (2013). The economy is populated by four types of agents: households, banks, firms and a government. We discuss their decision problems in some detail below.

2.2.1 Households

There is a continuum of identical households which consume, save and supply labor to an employment agency. As in Gertler and Karadi (2011) or Gertler and Karadi (2013), a fraction $f$ of household members are bankers and a fraction $1-f$ are workers. Workers are employed by an intermediate good firm and earn wage income. Bankers manage a financial intermediary, which collects deposits from all households and funds non-financial firms and holds government bonds. There is perfect consumption smoothing within the household. Over time, each member may change its occupation, yet the fraction of household members in each occupation remains constant. In particular, with probability $1-\sigma$ a banker quits and becomes a worker next period, while with probability $f(1-\sigma)$ a worker becomes a banker. Once the banker exits its business, retained earnings are transferred to the household and the bank shuts down. Any new banker obtains a startup fund, $o_t$, from the household. This setup ensures that financial intermediaries are unable to finance all investment projects with retained earnings and thus remain dependent on deposits.

The representative household maximizes lifetime utility subject to a budget constraint. Letting $c_t$ denote household consumption and $h_t$ hours worked, the objective is given by

$$\max_{c_t, D_t, B_t, h_t} E_t \sum_{t=0}^{\infty} \beta^t e^{\eta s_t} \left( \log(c_t - h_t c_{t-1}^a) - \chi_h \frac{h_t^{1+\varphi}}{1+\varphi} \right)$$
subject to
\[ s.t. \quad c_t + \frac{D^h_t}{P_t} + \frac{P^b_t B^b_t}{P_t} + \frac{1}{2} \kappa_b \left( \frac{P^b_t B^b_t}{P_t} - \bar{b}_h \right)^2 \leq (1 - \tau_t) \left( w_t h_t + d^firms_t \right) + \tau_{tr} t + \frac{R^d_{t-1} D^h_{t-1} P^h_{t-1}}{P_t} - \frac{\left( 1 + \rho P^b_t \right) P^h_{t-1} B^b_{t-1}}{P_t} - o_t \]

In the expression above \( E_0 \) is the expectation operator. Technological progress (defined below) is non-stationary, hence logarithmic utility ensure the existence of a balanced growth path. Additionally, there are (external) consumption habits and \( c^a_t \) denotes the average consumption in the economy. \( \beta \in (0, 1) \) is the discount factor, \( \chi_h \) is a positive constant and \( \eta_{d,t} \) a preference shock which follows an AR(1) process.

In the budget constraint \( d^h_t \) denotes real bank deposits and \( b^h_t \) holdings of real government debt, which is costly to the extent that it differs from a target level \( \bar{b}_h \) Gertler and Karadi (2013).\(^6\) \( \kappa_b \) is a positive constant. \( \tau_t \) is the tax rate, \( w_t \) the real wage, \( d^firms_t \) are dividends which accrue to households who own the different firms (see below). \( \tau^t_{tr} \) are transfers, \( R^d_t \) is the ex post real return on deposits, given by \( R^s_{t-1} \Pi_t^{-1} \), where \( R^s_{t-1} \) is the nominal interest rate on deposits contracted in period \( t - 1 \) and \( \Pi_t \) is inflation in period \( t \). \( R^b_t \) is the (gross) real return on government bonds and will be defined in detail below. \( o_t \) are transfers to family members that start a new bank. Optimality for holding government bonds requires the following condition to hold
\[ E_t \Lambda_{t,t+1} \left( R^b_{t+1} - R^d_{t+1} \right) = \kappa_b \left( b^h_t - \bar{b}_h \right), \quad (2.1) \]
where \( \Lambda_{t,t+k} = \beta^k \frac{\lambda_{t+k}}{\lambda_t} \) denotes the household’s stochastic discount factor and \( \lambda_t \) the Lagrange-Multiplier on the budget constraint.

### 2.2.2 Banks

A representative bank relies on deposits and retained earnings to fund either the capital stock of non-financial firms or purchases of government debt. Letting \( s_t \) denote the funding of non-financial firms by banks, \( b^b_t \) the stock of government debt.

\(^6\)This is meant to capture the limited participation of households in the market for government debt.
debt held by the bank and \( n_t \) the banks equity position (or net worth), we can write the balance sheet of the bank as follows:

\[
s_t + b_t^b = d_t^b + n_t.
\]

Letting \( R_t^b \) (to be defined below) denote the real return of investing in non-financial firms, net worth evolves as follows:

\[
n_{t+1} = R_{t+1}^b s_t + R_{t+1}^b b_t^b - R_{t+1}^d d_t^b,
\]

\[
= (R_{t+1}^b - R_{t+1}^d) s_t + (R_{t+1}^b - R_{t+1}^d) b_t^b + R_{t+1}^d n_t,
\]

(2.2)

where we use the bank’s balance sheet to obtain the second equation.

The expected present discounted value of a bank’s net worth at the time of exit from the banking business is given by

\[
V_t = \sum_{k=1}^{\infty} (1 - \sigma) \sigma^{k-1} E_t \Lambda_{t,t+k} n_{t+k}.
\]

(2.3)

One important friction in the banking sector is an agency problem between intermediaries and depositors, because, as in Gertler and Karadi (2011) or Gertler and Karadi (2013), bankers may divert a fraction of assets. Specifically, we assume that this fraction is \( \theta \in (0, 1) \) for private-sector funding and \( \Delta \theta \) for government debt, where \( \Delta \in (0, 1) \). The former is easier to divert, because its value is harder to observe by depositors. As a result, we require the following incentive constraint to be satisfied for depositors being willing to lend to the bank:

\[
V_t \geq \theta s_t + \Delta \theta b_t^b.
\]

(2.4)

Central to our analysis is the ability of the government to lower the yield on government debt. To allow for this possibility we assume a regulatory constraint, as suggested by Chari et al. (2019). Specifically, the following has to hold:

\[
b_t^b \geq \Gamma_t (b_t^b + s_t).
\]

Here \( \Gamma_t \) is the minimum share of government debt which banks need to hold relative to the total amount of assets. We think of this regulatory constraint as capturing a variety of measures such as those discussed by Reinhart and
Sbrancia (2015) in some detail. Such measures may not literally force financial intermediaries to hold a certain fraction of government debt in their portfolio. Still they effectively raise the demand and thus price of government debt. As we show below, this is precisely the implication of constraint (2.5). We rearrange the regulatory constraint slightly

$$b^b_t \geq \gamma_t s_t,$$  \hspace{1cm} (2.5)

with $$\gamma_t = \frac{\Gamma_t}{1-L^b_t}.$$ Maximizing (2.3) subject to (2.2), (2.4) and (2.5) yields the first order conditions:

$$E_t \tilde{\Lambda}_{t,t+1} (R^b_{t+1} - R^d_{t+1}) = \frac{\zeta_t}{1+\zeta_t} \theta + \frac{\mu_t}{1+\zeta_t} \gamma_t = \tilde{\zeta}_t + \tilde{\mu}_t \gamma_t,$$  \hspace{1cm} (2.6)

$$E_t \tilde{\Lambda}_{t,t+1} (R^b_{t+1} - R^d_{t+1}) = \Delta \tilde{\zeta}_t - \tilde{\mu}_t,$$  \hspace{1cm} (2.7)

Here $$\zeta_t$$ and $$\mu_t$$ are the multipliers on the incentive and on the regulatory constraint, respectively. $$\tilde{\Lambda}_{t,t+1}$$, in turn, is an augmented stochastic discount factor defined below. Equation (2.6) relates the (expected) excess return of investing in intermediate-good firms (relative to the deposit rate) to the tightness of the incentive constraint (2.4) and regulatory constraint (2.5). Intuitively, to the extent that bankers are leverage constrained expected excess yields persist in equilibrium. Additionally, due to the distortion of the banks’ portfolio choice through government regulation, a binding regulatory constraint (i.e. $$\mu_t > 0$$) reflects an artificially reduced demand for real capital, that results in a further elevated excess yield. This wedge rises, if the fraction of real capital that banks hold is low (i.e. a high value of $$\gamma_t$$)

Equation (2.7), in turn, relates the (expected) excess return of investing in government debt (relative to the deposit rate). Government debt is long-term, as we explain in detail below. Deposits, on the other hand, mature in the next period. Therefore, (2.7) relates the difference between long and short-term interest rates, to the tightness of the incentive constraint. Our model may thus rationalize a term premium due to market segmentation (see Fuerst, 2015). In our setup, there is market segmentation because households find it costly to adjust their debt holdings and banks are leverage constrained. As a result, there are limits to arbitrage and differences in expected yields persist in equilibrium. Yet, in addi-
tion to the excess-return component (or “term premium”) reflected by $\Delta \tilde{\varepsilon}_t$, there is a “regulatory discount” which appears in equation (2.7) via $\mu_t$. Recall that this is the multiplier on the regulatory constraint. The tighter the constraint (2.5), the lower the expected excess return on government debt. Intuitively, to the extent that regulatory constraint binds, the price of government debt is pushed up and (expected) yields are depressed because banks are incentivized to hold on to them. Note that the expected excess return on government debt given in (2.7) does not feature a liquidity premium, in line with the evidence.\footnote{Longstaff (2004) finds that liquidity premia on short-term and long-term treasuries are of similar magnitude. In our model as well as in our empirical analysis below we do not distinguish between the return on short-term deposits and the return on short-term government debt.}

It is instructive to consider a version of the complementary slackness condition associated with the regulatory constraint (2.5)

$$E_t \left\{ \hat{\Lambda}_{t,t+1} \left( R_{b,t+1}^b - \hat{R}_{t+1} \right) \right\} (b_t^b - \gamma_t s_t) = 0.$$  

(2.8)

Here $\hat{R}_{t+1}$ is the laissez-faire interest rate which would obtain if the regulatory constraint were slack ($\mu_t = 0$). This expression shows that whenever there is financial repression, that is, whenever $R_{b,t+1}^b < \hat{R}_{t+1}$, the regulatory constraint must bind. In our analysis below we assume that the regulatory constraint binds throughout. However, the extent of repression will vary over time. Either because of variations in $\gamma_t$ or because, for a given $\gamma_t$, the tightness of the constraint, captured by $\mu_t$, will generally differ across periods and states of the economy.

Following Gertler and Karadi (2013) we also assume that the incentive constraint binds always. It is then possible to define the leverage ratio $\phi_t$ as follows:

$$s_t + b_t^b = \phi_t n_t,$$  

(2.9)

where

$$\phi_t = \frac{E_t \hat{\Lambda}_{t,t+1} R_{d,t+1}^d}{\theta + \theta \Delta \gamma_t - E_t \hat{\Lambda}_{t,t+1} \left[ (R_{k,t+1}^k - R_{d,t+1}^d) + \gamma_t (R_{b,t+1}^b - R_{d,t+1}^d) \right]}.$$  

(2.10)

The leverage ratio falls in $\theta$, the fraction of assets a banker can divert. Depositors anticipate that the incentive for the banker to divert assets increase and thus ask for more “skin in the game”. The leverage ratio rises with the excess return on capital $E_t \hat{\Lambda}_{t,t+1} (R_{k,t+1}^k - R_{d,t+1}^d)$ or bonds $E_t \hat{\Lambda}_{t,t+1} (R_{b,t+1}^b - R_{d,t+1}^d)$, since that increases the value of staying a banker. Similarly, the leverage ratio rises...
with the discounted deposit rate \( E_t \hat{\Lambda}_{t,t+1} R^d_{t+1} \) as for given excess returns, the net
worth of the bank and thus the value of staying a banker increases. We can now
define the augmented discount factor as in Gertler and Karadi (2013):

\[
\hat{\Lambda}_{t,t+1} \equiv \Lambda_{t,t+1} \left( 1 - \sigma + \sigma \frac{\partial V_{t+1}}{\partial n_{t+1}} \right),
\]

(2.11)

with

\[
\frac{\partial V_t}{\partial n_t} = E_t \hat{\Lambda}_{t,t+1} \left[ (R^k_{t+1} - R^d_{t+1}) \phi_t + (R^b_{t+1} - R^d_{t+1}) \gamma_t \phi_t + R^d_{t+1} \right].
\]

(2.12)

The augmented discount factor used to price the excess return is thus a probability-
weighted average of the stochastic discount factor from the households and the
marginal increase in net-worth of the bank. Since both constraints are always
binding we arrive at

\[ V_t = \theta_s t + \Delta \theta b^b_t \]

(2.13)

and

\[ b^b_t = \gamma_t (b^b_t + s_t) \].

(2.14)

The aggregate stock of net worth \( n_t \) depends on the returns of bankers that stay
a banker (probability \( \sigma \)) and the start-up funds for new bankers:

\[ n_t = \sigma \left[ (R^k_t - R^d_t) s_{t-1} + (R^b_t - R^d_t) b^b_{t-1} + R^d_t n_{t-1} \right] + o_t. \]

(2.15)

### 2.2.3 Firms

We distinguish between four types of firms. There are intermediate good firms
which operate under perfect competition. They hire workers from the employ-
ment agencies and use the capital stock which is funded by banks. Next there a
monopolistically competitive retailers which are constrained in their price-setting
decision. Last, there are capital producers and the employment agencies.

**Intermediate good firms**

The representative intermediate good firm operates under perfect competition.
Its production function is given by

\[ y_t = (u_t k_{t-1})^\alpha (Z_t h_t(j))^{1-\alpha}, \quad \alpha \in (0,1). \]
Here, production depends on the predetermined capital stock $k_{t-1}$ and its utilization $u_t$. $Z_t$ represents exogenous labor-augmenting technological progress. We allow it be non-stationary and assume that its growth rate, $\eta_{z,t} \equiv \Delta \log Z_t$, follows an AR(1) process Justiniano et al. (2013).

As price taker, the firm’s demand for labor and capital utilization satisfies the optimality conditions

$$w_t = p_t^m (1 - \alpha) \frac{y_t}{h_t}$$

and

$$\alpha p_t^m \frac{y_t}{u_t} = \Psi'(u_t) k_{t-1}.$$  

Here $p_t^m$ denotes the real price of intermediate goods and $\Psi(u_t) = (1+\kappa)^{-1} (u_t^{1+\kappa} - 1)$ is the cost of capital utilization. We assume that in steady state $u = 1$, $\Psi(1) = 0$ and define $\kappa \equiv \frac{\Psi''(1)}{\Psi'(1)}$. After production takes place the intermediate goods producer buys new capital goods of $x_t$ at price $q_t$. Letting $\delta$ the rate of depreciation, the the law of motion of the capital stock is given by

$$k_t = (1 - \delta) k_{t-1} + x_t.$$  

The capital stock is fully funded through banks. We follow Jermann (1998) and Basu and Bundick (2017) in assuming that a constant fraction $\nu$ of the capital stock is financed through bank loans, that is $l_t = \nu q_t k_t$ and the rest by equity shares, $e_t = (1 - \nu) q_t k_t$ (also held by banks).\footnote{We normalize the amount of shares to 1 such that $e_t$ is the price of total shares.} It thus holds

$$s_t = e_t + l_t = q_t k_t.$$  

As a result, intermediate good firms are leveraged and banks’ returns from holding equity in intermediate good firms may be as volatile as in the data. In contrast to banks, firms’ leverage has no real implications due to perfect monitoring. Therefore, the firm obtains loans at the prevailing real deposit rate $R_t^d$. The firm does not keep any retained earnings. Dividend payments thus amount to $d_t^f = p_t^m y_t - w_t h_t - \Psi(u_t) k_{t-1} - q_t x_t - (R_t^d l_{t-1} - l_t)$. The return on equity reflects price changes as well as dividends and is given by:

$$R_t^e = \frac{e_t + d_t^f}{e_{t-1}} = \frac{\Psi(u_t) u_t - R_t^d \nu q_{t-1} - \Psi(u_t) + (1 - \delta) q_t}{(1 - \nu) q_{t-1}}.$$
where the first order condition (2.17) has been substituted in. From the perspective of the bank, however, the total return on funding the capital stock is key, that is, we have to add the gross return on its loan-payments less the new loan given to the firm:

\[
R^k_t = \frac{e_t + d_t + (R^d_t l_{t-1} - l_t)}{e_{t-1}} = \frac{\Psi'(u_t)u_t - \Psi(u_t) + (1 - \delta) q_t}{q_{t-1}}. \tag{2.21}
\]

**Retailers**

There is a continuum of monopolistically competitive retailers \( j \in [0, 1] \) which repackage and diversify intermediate goods. Retailers transform one unit of intermediated goods into one unit of the retail good such that marginal costs are given by \( p^m_t \).

Final goods consist of products of all retailers:

\[
y_t = \left[ \int_0^1 y_t(j) \frac{1}{1+\omega_{p,t}} \, dj \right]^{1+\omega_{p,t}}.
\]

Here \( \omega_{p,t} \) varies exogenously. Cost minimization implies that the demand for goods of a generic retailer \( j \) is given by

\[
y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\frac{1+\omega_{p,t}}{-\omega_{p,t}}} y_t, \tag{2.22}
\]

where \( P_t(j) \) is the price charged by retailer \( j \) and \( P_t \) is the price index of the final good given by

\[
P_t = \left[ \int_0^1 P_t(j)^{-\frac{1}{-\omega_{p,t}}} \, dj \right]^{-\omega_{p,t}}. \tag{2.23}
\]

\( \omega_{p,t} \) denotes the desired markup of prices over the marginal costs. We assume that \( \log (1 + \omega_{w,t}) \) follows an AR(1) process. We follow Rotemberg (1982) and assume that the adjustment of priced entails some quadratic costs for the retail firm:

\[
a_{c_t}(j) = \frac{1}{2} \varphi \left[ \frac{P_t(j)}{P_{t-1}(j)} - \Pi_{t-1}^{\xi} \Pi^{1-\xi} \right]^2 y_t(j)p_t(j),
\]

with \( \varphi \) determining the cost of price adjustments, \( \Pi \) is the steady state inflation rate, \( \xi \in (0, 1) \) captures price indexation and \( p_t(j) = \frac{P_t(j)}{P_t} \) is the price in real terms.
Retailers set prices $P_t(j)$ in order to maximize discounted life-time profits:

$$\max_{P_t(j)} E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k}^1 (1 - \tau_{t+k}) \left\{ (p_{t+k}(j) - p_{t+k}^m) y_{t+k}(j) - ac_{t+k}(j) \right\},$$

subject to the demand function (2.22).

Optimality requires the following condition to be satisfied

$$- \frac{1}{\omega_{p,t}} + p_t^m \frac{1 + \omega_{p,t}^t}{\omega_{p,t}} - \varphi \left( \Pi_t - \Pi_{t-1}^\xi \Pi_t^{1-\xi} \right) \Pi_t + \frac{1 + \omega_{p,t}^t}{\omega_{p,t}} \varphi \left( \Pi_t - \Pi_{t-1}^\xi \Pi_t^{1-\xi} \right)^2 + \frac{\Lambda_{t,t+1}^\ell}{\Lambda_{t,t}^\ell} \varphi \left( \Pi_{t+1} - \Pi_{t+1}^\xi \Pi_{t+1}^{1-\xi} \right) \Pi_{t+1} \frac{y_{t+1}}{y_t} = 0. \quad (2.24)$$

**Capital producers**

Capital producers use final goods to produce capital goods subject to an adjustment cost. They sell capital goods to intermediate good firms and distribute profits to the household sector. The objective of capital producers is given by

$$\max_{x_t} \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left\{ q_{t+k} x_{t+k} - e^{-\eta_{x,t+k}} \left[ 1 + \Theta \left( \frac{x_{t+k}}{x_{t+k-1}} \right) \right] x_{t+k} \right\}$$

Where $\eta_{x,t}$ is an investment specific shock which we specify as an AR(1), $\Theta \left( \frac{x_t}{x_{t-1}} \right) = \frac{\kappa}{2} \left( \frac{x_t}{x_{t-1}} - e^{\gamma} \right)^2$ is the adjustment cost function with $\kappa > 0$ and $\gamma$ is the steady state growth rate of neutral technology. The associated first order condition is given by

$$q_t = e^{-\eta_{x,t}} \left( 1 + \Theta \left[ \frac{x_t}{x_{t-1}} \right] \right) + \Theta' \left[ \frac{x_t}{x_{t-1}} \right] \frac{x_t}{x_{t-1}} - \Lambda_{t,t+1} e^{-\eta_{x,t+1}} \left( \Theta' \left[ \frac{x_{t+1}}{x_t} \right] \left[ \frac{x_{t+1}}{x_t} \right]^2 \right).$$

If there are no adjustment costs, i.e. $\Theta[.] = \Theta'[.] = 0$, then $q_t = \frac{1}{e^{\eta_{x,t}}}$, that is, marginal Tobin’s Q is equal to the replacement cost of capital (the relative price of capital).

**Employment agencies**

We follow Justiniano et al. (2013) and Erceg, Henderson and Levin (2000) and assume that each household is a monopolistic supplier of a differentiated labor
service which it sells to an employment agency. A unit mass of these agencies aggregates the specialized types into a homogenous labor input and sells to intermediate good firms:

\[ h_t = \left[ \int_0^1 h_t(j) \frac{1}{1+\omega_{w,t}} \right]^{1+\omega_{w,t}}. \]  

(2.25)

Here \( \omega_{w,t} \) denotes the desired markup of wages over the households’ marginal rate of substitution between labor and leisure. We assume that \( \log(1 + \omega_{w,t}) \) follows an AR(1) process. Employment agencies maximize profits such that labor demand is given by

\[ h_t(j) = \left( \frac{w_t(j)}{w_t} \right)^{-1+\frac{1}{\omega_{w,t}}} h_t. \]  

(2.26)

Here \( w_t(j) \) denotes the real wage paid to households \( j \) and \( w_t \) is the aggregate wage index given by

\[ w_t = \left[ \int_0^1 w_t(j) \frac{1}{\omega_{w,t}} \right]^{-\omega_{w,t}}. \]  

(2.27)

We further assume that each period only a constant fraction \( 1 - \theta^w \) of households/labor types can optimally adjust their nominal wages, the rest follows the simple index rule

\[ w_t(j) = w_{t-1}(j) (\Pi_{t-1}e^\eta)^{\theta^w} (\Pi_{t-1}e^\gamma)^{1-\theta^w}. \]  

(2.28)

### 2.2.4 Government

In each period the government finances purchases, transfers and interest rate payments by raising taxes and issuing nominal debt which is default free. The maturity of government may exceed one period. Specifically, as in Woodford (2001), we assume that one unit of government debt \( B_t \) issued in period \( t \) offers the following payment stream: \( \{1, \rho, \rho^2, \rho^3, \ldots \} \). Here, the decay factor \( \rho \) captures the average maturity of the bond. Letting \( p_t^l \) denote the real price for this bond, the market value of debt in real terms is given by \( b_t = p_t^l B_t \). In the absence of arbitrage across different maturities and given \( p_t^l \), the real price of a bond in period \( t + k \) must be given by \( \rho^k p_t^l \). The ex post yield of holding
long-term government debt in real terms is given by

$$R_t^b = \frac{1}{p_t} + \rho p_t^l.$$  
(2.29)

We write the budget constraint using variables measured relative to GDP. Specifically, \(R_t^b\) denotes the ex-post real interest rate of government debt, \(d_t \equiv b_t / y_t\) the debt-to-GDP ratio, \(y_t\) real output, \(e_t\) the expenditure ratio (sum of purchases and transfers) and \(\tau_t^{total}\) total tax revenues:

$$R_t^b d_{t-1} y_{t-1} + e_t = d_t + \tau_t^{total}.$$  
(2.30)

We follow the setup by Bianchi and Ilut (2017) for expenditures and purchases. Specifically, we decompose total expenditures into a short-term component \(e^s_t\) and a long-term component \(e^l_t\). The long-term component follows a highly persistent AR(1) process which is meant to capture the large and long-lasting transfer programs (Great Society), while the short-term component will react on current output to capture transfer adjustments over the business cycle. We use a hat to denote the percentage deviation of a variable from its steady state, and a tilde to denote a percentage point deviation. The process for short-term expenditures is given by:

$$\tilde{e}^s_t = \rho^{es} \tilde{e}^{s}_{t-1} + (1 - \rho^{es}) \phi_y \tilde{y}_t + \varepsilon^{es}_t.$$  
(2.31)

Government purchases \(g_t\) are given by \(g_t = \left(1 - \frac{1}{\rho_g}\right) y_t\), where \(\rho_g\) is an AR(1) government spending shock. The purchases to expenditure ratio \(g_t\) evolves according to

$$\tilde{g}_t = \rho_g \tilde{g}_{t-1} + (1 - \rho_g) \phi_y \tilde{y}_t + \varepsilon^g_t,$$  
(2.32)

which is smooth but additionally allows for a contemporaneous feedback effect of output to transfers (\(\phi_y^g\)).

The total amount of tax revenues is given by \(\tau_t^{total} = \tau_t \left(w_t h_t + d^{firms}_t\right)\). We assume that the tax rate adjusts according to the following rule:

$$\tilde{\tau}_t = \rho_{\tau} \tilde{\tau}_{t-1} + (1 - \rho_{\tau}) \left(\phi_{\tau} \tilde{d}_t + \phi^\gamma_y \tilde{y}_t\right) + \eta_{\tau,t},$$  
(2.33)

with iid stochastic disturbance \(\eta_{\tau,t}\).

Finally, monetary policy sets the nominal short-term interest rate by following
an interest rate rule

\[
\frac{R_s^t}{R_s^{t-1}} = \left( \frac{R_s^{t-1}}{R_s^t} \right)^{\rho_r} \left( \frac{\Pi_t}{\Pi^{t-1}} \right)^{\phi_r} e^{\eta_{r,t}} \tag{2.34}
\]

where \( \rho_r \) a smoothing parameter, \( \phi_r \) and \( \phi_y \) capture the reaction coefficients to inflation and output respectively and \( \eta_{r,t} \) is an iid monetary policy shock.

### 2.2.5 Market clearing

At the aggregate level, the following resource constraint needs to be satisfied

\[
y_t = c_t + g_t + e^{-\eta_{r,t}} \left[ 1 + \Theta \left[ \frac{x_t}{x_{t-1}} \right] \right] x_t + \frac{1}{2} \varphi \left[ \Pi_t - \Pi_{t-1} \Pi^{1-\xi} \right]^2 y_t + \Psi(u_t)k_{t-1}. \tag{2.35}
\]

Since households and banking sector are investing in government debt the total stock, and thus the real market value, is the sum of both

\[
b_t = b_t^y + b_t^h. \tag{2.36}
\]

### 2.3 Inspecting the mechanism

In what follows we develop some intuition for how financial repression impacts public finances in particular and the economy in general. In a first step, we take a partial equilibrium perspective and zoom in on the market for government debt. Our discussion assumes that debt is held exclusively by banks: \( B_t^b = B_t \). Further, we abstract from inflation and assume a constant fiscal surplus \( s_t = s \).

We then consider a simplified version of the government budget constraint:

\[
P_t^b B_t = \frac{1 + \rho P_t^b}{P_{t-1}^b} P_{t-1}^b B_{t-1} - s.
\]

This expression implicitly defines the supply curve of government debt: it relates the current price of debt \( P_t^b \) to the quantity of bonds \( B_t \), given outstanding liabilities and bond prices in the previous period. The supply curve is downward sloping because a higher bond price reduces the amount of debt which needs to be placed with banks in order to redeem a given amount of outstanding debt net
Figure 2.2: Stylized representation of market for government debt

Notes: Supply (demand) of debt represented by blue (black) line. Regulatory constraint represented by RC curve. \( \tilde{P}^b \) is the laissez-faire price of debt. \( P^b \) is the actual price. Repression shifts demand for government debt upward.

of the surplus. We depict the supply curve as the blue solid in Figure 2.2. It is labeled “S”.

The same figure also features a demand curve for government debt, labeled “D”, that determines the demand for government debt in the absence of repression or, equivalently, in case the regulatory constraint is not binding. Without loss of generality we assume it to be horizontal. As \( R_t^b = \frac{1+\rho P_t^b}{P_{t-1}} \), it is implicitly determined by the bankers’ optimality condition (2.7) that ties the return on government debt to the deposit rate. The deposit rate, in turn, is proportional to the time-discount factor thanks to optimality condition (2.1). The intersection of “D” and “S” in Figure 2.2 determines the “laissez-faire” price \( \tilde{P}^b \) of debt that prevails in the absence of repression.

Because \( b_t = P_t^b B_t \), the regulatory constraint (2.5) implies

\[
P_t^b B_t \geq \gamma t s_t.
\]

For a given market value of firms, \( s_t \), the regulatory constraint defines a downward sloping relationship between the price of debt and the amount of debt that needs to be held by banks whenever the regulatory constraint binds. It is shown as a hyperbola in Figure 2.2, depicted in red and labeled \( RC \). Intuitively, the constraint is satisfied with equality if either the volume held by banks is high and the price is low, or vice versa.
What determines the equilibrium in the market for government debt? Since we assume that the regulatory constraint binds, the equilibrium price $P^b$ is given by the intersection of $RC$ and $S$. It exceeds the laissez-faire price. This price is consistent with the demand curve, because in case of repression the demand curve for government debt shifts upward (from $D$ to $D'$). Formally, this is brought about by a positive realization of the Lagrange multiplier $\mu$ in the bankers’ optimality condition (2.7). Because holding an additional unit of government debt provides additional value to the bank if the regulatory constraint binds, bankers are ready to purchase government debt at a price which exceeds the laissez-faire price. Equivalently, for a given price of government debt in the next period, repression lowers the yield on government debt. At the same time, due to repression the government needs to issue less debt in order to meet a given financing requirement.

How does the economy adjust to financial repression? In order to illustrate essential aspects of the transmission mechanism we simulate a simplified version of the model outlined in Section 2.2.\textsuperscript{9} Specifically, we assume that the economy is initially in steady state as the regulatory constraint tightens temporarily. There is in other words a shock to $\gamma_t$. We contrast the effects of this repression shock with those of a conventional monetary policy shock, that is, an exogenous reduction of the short-term policy rate.

Figure 2.3 shows the impulse response functions. The blue solid line is the response to the repression shock. The red dashed line is the response to the (expansionary) monetary policy shock. Here vertical axes indicate deviations from steady state and horizontal axes indicate time in quarters. Focus first on the repression shock. The upper panel of figure 2.3 shows the implications for public finances and inflation. Increased financial repression—via the regulatory constraint—requires banks to hold a higher fraction of their portfolio in government debt. All else equal they increase their demand for debt which in turn raises its price (not shown) and lowers the expected return (upper right graph). The reduction of the interest rate, all else equal, reduces the debt-to-GDP ratio (lower middle graph). However, initially public debt increases because repres-

\textsuperscript{9}Specifically, we assume that debt is held exclusively by banks, there is no habit persistence, wages are set in a perfectly competitive way, there are no government expenditures (purchases or transfers), monetary and fiscal rules have no smoothing terms, monetary policy adjusts interest rates only in response to inflation and there are taxes are lump-sum. We also assume that monetary policy is active and fiscal policy is passive, following the notation by Leeper (1991). Results are qualitatively similar for the estimated model as we show below.
Notes: Responses to repression shock (blue solid line) vs cut of policy rate (red dotted line). Vertical axis measures deviations from steady state, horizontal axis measures time in quarters.
sion raises the price of outstanding debt, and thus the holding period return HPR (lower left graph). Furthermore the reduction in inflation increases the real market value of debt. Therefore, the net effect of a repression shock on the debt ratio is ambiguous and likely to change over time.

The lower panel of Figure 2.3 shows how the repression shock transmits into the economy. As the regularity constraint tightens, banks respond by rebalancing their portfolios: they reduce their funding of firms as they are forced to hold more government debt. As a result, the price of investment (Tobins Q) declines (upper left graph) as does investment (lower left graph). As stressed by Chari et al. (2019), repression distorts the optimal allocation of capital. Repression crowds out investment via this portfolio effect. In addition, there is a net worth effect: since the value of investment and the return on government decline, banks net worth declines (upper middle graph).\footnote{This is despite the increase of the HPR on government debt in the first period, as it is more than offset by the loss in market value of the investment into real firms.} This kicks off a second round effect. First, since banks’ equity is directly linked to real lending (due to market segmentation), investment drops even further which drives the economy into a prolonged recession (lower right graph). Second, since net equity is reduced, households withdraw their deposits from the banks, as the value of staying a banker falls and households only have limited enforcement capabilities. This additionally reduces lending and thus enhances the drop in investment and output. Even though the increase in repression dies out after 20 quarters, output is still below its steady-state value (lower right graph). This, in turn, may put upward pressure on the debt-to-GDP ratio, as we discuss below.

At times, some commentators also refer to low policy rates as “financial repression”. In our model, financial repression shares some features with conventional monetary policy, but is fundamental different in other dimensions. To illustrate this, the red dashed line in Figure 2.3 shows the responses to a cut in the interest rate. Focus on the upper panel first: we consider a cut in the policy rate which induces a decline of ex ante interest rates comparable to the one observed in response to the repression shock (upper right graph). The ex ante interest rate declines because prices are sticky and hence expected inflation (lower right graph) does not fully offset the change in the policy rate. The long-term rate declines with the short-term rate thanks to no-arbitrage conditions. Also, the holding period return on government debt evolves similar to what happens in response to repression (lower left graph). The public-debt-to-output ratio (lower
middle panel) declines more strongly in response to the monetary policy shock, because the cut in the policy rate is expansionary. Yet, in sum, as far as public finances are concerned, the effects of the monetary shock are comparable to the repression shock.

There are stark differences when it comes to the transmission into the economy, shown in the lower panel of Figure 2.3. The reduction of interest rates stimulates household consumption (lower middle graph). In the process, households reduce their savings with banks (upper right graph). To meet the regulatory constraint banks have to reduce their funding of firms, as the increase in net worth (upper middle graph) is more than compensated by the fall in deposits. As a result investment declines somewhat (lower left graph).\footnote{Since the liability side shrinks, so does the asset side. Financial repression is still present in steady state, however, therefore the relative share of public debt and funding of firms has to be kept similar. The reduction in liabilities is thus met by a reduction of investment and government debt (not shown) of a similar size.} However, the economy expands due to increased consumption. In contrast to the repression shock, the effect of the monetary policy shock on output is less persistent than in case of the repression shock because bankers’ net worth is much less affected.

\section*{2.4 Estimation}

We estimate the model using Bayesian estimation techniques. In this section, we first describe the dataset used to estimate the parameters of model. Afterwards, we outline the choice of the prior distribution of the parameters and report the corresponding posterior distributions.

\subsection*{2.4.1 Data}

We estimate the model using ten time series of US quarterly data from 1948Q2–1974Q4. Four of these series are macro time series, four are fiscal time series and two time series capture the financial sector.

We obtain real per capita GDP growth from NIPA (nominal GDP: Table 1.1.5, line 1 and GDP deflator: Table 1.1.4, line 1). We follow (Leeper, Plante and Traum, 2010) in constructing the population series. Furthermore, we use the real per capita growth rate of private investment, which is the sum of personal...
consumption expenditures on durable goods (Table 1.1.5, line 4) and gross private domestic investment (Table 1.1.5, line 7). Additionally, we include the inflation rate, measured as the quarterly log difference of the GDP deflator. Since the Federal Reserve started targeting a specific rate only from June 1954, we use as measures for the nominal interest rate the secondary market rate of the 3m Treasury Bill until 1954Q2 and thereafter by the effective Federal Funds rate.\textsuperscript{12}

As fiscal time series we include the market value of debt relative to GDP\textsuperscript{13} government purchases, government expenditures and government revenues. We obtain the market value of debt from Cox and Hirschhorn (1983), data for government purchases, expenditures and revenues from NIPA. We transform government purchases to be consistent with \( q_t \) in the model and define government expenditures as the sum of purchases and transfers, all relative to GDP.\textsuperscript{14} We compute tax revenues as the difference between current receipts (NIPA Table 3.2, line 37) and current transfer receipts (NIPA Table 3.2, line 16).

As financial frictions are at the heart of our analysis, we also use two financial time series in the estimation: bank equity and equity returns.\textsuperscript{15}

\textsuperscript{12}Both rates are not completely identical but follow a very close pattern as demonstrated by a correlation coefficient of 0.98 for 1954Q3 to 1974Q4.

\textsuperscript{13}We use the market value of privately held government debt. Since we assume a consolidated budget we abstract from debt held by the Federal Reserve or U.S. government accounts.

\textsuperscript{14}Data for purchases are from NIPA tables Table 3.2, line 21 (consumption) Table 3.2, line 41 (investment) and Table 3.2, line 43 (net purchases of non-produced assets) minus Table 3.2, line 44 (consumption of fixed capital). Transfers are given by the sum of net current transfer payments (Table 3.2, line 22 and line 16), subsidies (Table 3.2, line 32), and net capital transfers (Table 3.2, line 42 and line 38).

\textsuperscript{15}We retrieve bank equity from the Board of Governors of the Federal Reserve System. Specifically, we use total capital accounts for all commercial banks from H.8. - Assets and Liabilities of Commercial Banks in the U.S., transform it into real per capita values as explained below and take the growth rates as observable. We always use the last available entry for each quarter. To compute equity returns we use the mean quarterly price and dividend data on the US stock market provided by Shiller (2005), deflate it and calculate the return on equity including the dividend payments.
where $dl$ is 100 times the log difference of each variable while the rest is the observed ratio. Remember that a hat (\(\hat{\cdot}\)) denotes the log-deviation and tilde (\(\tilde{\cdot}\)) the linear deviation from steady state. Note that in the model most variables inherit the non-stationarity of the technological progress $Z_t$. We therefore express variables in deviations from the non-stationary trend ($a$). Then, we (log-)linearize the model around its non-stochastic steady state.

The estimation sample starts in 1948Q2 because the population series only goes back until 1948. It ends in 1974Q4 for two reasons: First, in 1974 the federal debt to GDP ratio is the lowest after WWII and thus this period is characterized by a large reduction in the debt to GDP ratio from 75.5% to 16.9%. Second, the period afterwards, especially after the appointment of Volcker 1979 marks a shift in the conduct of monetary policy, see for example Clarida, Gali and Gertler (2000) or Bianchi and Ilut (2017).

### 2.4.2 Choice of prior distribution

Most of the parameters have been estimated before and we therefore follow the choices of the corresponding literature, e.g. Justiniano et al. (2013) and Bianchi and Ilut (2017). As for the banking sector we follow the suggestions by Gertler and Karadi (2013). The left panel of Table 2.1 summarizes the the prior distribution of the model parameters.

The first block contains parameters which characterize the behavior of policy, starting with the amount of regulation in steady state, $\gamma$. For the prior mean (and the external validation of the time series below) we use the available date
Table 2.1: Prior and posterior distribution of estimated parameters

<table>
<thead>
<tr>
<th>Dist</th>
<th>Mean</th>
<th>SE</th>
<th>Mode</th>
<th>Mean</th>
<th>5 percent</th>
<th>95 percent</th>
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<td>$\Gamma$</td>
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<td>$\tau_d$</td>
<td>Tax on debt</td>
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<td>0.05</td>
<td>0.0569</td>
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<td>$\tau_y$</td>
<td>Tax on output</td>
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<td>0.5717</td>
<td>0.5235</td>
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<tr>
<td>$\phi_y$</td>
<td>MP on output</td>
<td>G</td>
<td>0.25</td>
<td>0.10</td>
<td>0.1665</td>
<td>0.1585</td>
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<td>$\nu_y$</td>
<td>Exp share on output</td>
<td>N</td>
<td>0.10</td>
<td>0.20</td>
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<td><strong>Posterior</strong></td>
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<td>$\xi^\pi$</td>
<td>SR-exp on output</td>
<td>N</td>
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<td>-0.5984</td>
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<td>$\kappa^\pi$</td>
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<td>$\varphi$</td>
<td>Surv. rate banker</td>
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<td>$\nu$</td>
<td>Loan share</td>
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<td>$\xi^\pi$</td>
<td>Inflation</td>
<td>N</td>
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<td>0.7533</td>
</tr>
<tr>
<td>$\xi^w$</td>
<td>Growth</td>
<td>N</td>
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<td>0.05</td>
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<td>0.5278</td>
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<td>$\varphi$</td>
<td>Debt to GDP</td>
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<td>1.6526</td>
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<td>Purchases</td>
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<td>0.01</td>
<td>1.1113</td>
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<td>$\tau$</td>
<td>Tax revenue</td>
<td>N</td>
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<td>0.01</td>
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<td><strong>AR(1) shocks</strong></td>
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<td>Regulation</td>
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<td>0.20</td>
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<td><strong>Std shocks</strong></td>
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<td>$\sigma_{\tau}$</td>
<td>Regulation</td>
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<td>1.00</td>
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</table>

Notes: N stands for the Normal, B the Beta, G the Gamma and IG the inverted Gamma distribution.
from H.8 above and calculate the share of government debt relativ to investment for the commercial banks. The sample mean is a little above 0.3 and a standard deviation of 0.05, hence the values for the prior beta distribution. We allow in our analysis for different regimes of monetary and fiscal interaction. In particular, we will concentrate on uniquely determined bounded rational expectation equilibria. These regimes exhibit either an active monetary authority coupled with a passive fiscal authority (regime $M$) or a passive monetary authority coupled with an active fiscal authority (regime $F$). Regarding the parameters in the policy function we set a prior distribution such that both regimes can potentially prevail. The prior distribution of the monetary reaction coefficient on inflation, $\phi_{\pi}$, is a normal distribution centered around 1 with a standard deviation of 0.5 and on output, $\phi_y$, a gamma distribution with mean 0.25 and standard deviation of 0.1. The prior distribution of the coefficient on debt in the tax rule, $\tau_d$, is a normal distribution with mean 0 and a standard deviation of 0.05 and on output, $\tau_y$, is a normal distribution with mean 0.2 and a standard deviation of 0.2. The response coefficient of government spending and the coefficient determining the response of short-run expenditures to the output gap are assumed to have a normal distribution with mean 0.1 and 0.2 respectively.

The next block deals with parameters for the banking sector. The amount of repression discount $\mu$ in steady state has a gamma distribution with mean 0.5 and standard deviation of 0.2. The mean was chosen in line with Reinhart and Sbrancia (2015) who find that repression is around 2% annually for the United States. We use a normal distribution for the the term premium, $\tilde{\zeta}$, and the steady state leverage ratio $\phi$ with mean 1 and 6 and standard deviations of 0.25 and 1 respectively. We choose a beta distribution with mean 0.5 and standard deviation of 0.25 for the probability of staying a banker, $\sigma$ and the amount of bonds that is financed by loans, $\nu$. We include a measurement error in the series of networth with an inverse gamma centered tightly at 0.5. We follow Gertler and Karadi (2013) and calibrate $\kappa_b$ to 1, since the data was not informative about its value.

The indexation parameter for wage as well as price indexation follow a beta distribution with mean 0.5 and a standard deviation of 0.15. For the slope coefficient in the Phillips curve we specify a Gamma distribution with a mean

\[^{16}\text{Specifically, we use the item “U.S. Govt. obligations” and “Loans and investments” on a quarterly basis as above.}\]

\[^{17}\text{Our results are robust to alternative values.}\]
of 0.3 and standard distribution 0.15. For the parameter controlling the wage stickiness we formulate a beta distribution with mean 0.66 and standard deviation 0.1. For the inverse of the Frisch elasticity, for the parameters governing the investment adjustment costs and capacity utilization costs we select a gamma distribution with a mean of 2, 4 and 5 respectively. The standard deviations of these distributions imply a wide prior distribution.

We specify values for steady-state inflation, GDP growth, the steady-state values of the debt-to-GDP ratio, the government purchases-to-GDP ratio and the tax-to-GDP ratio according to a normal distribution centered around the sample means.

We choose an beta distribution with mean 0.6 and a standard deviation of 0.2 for the autoregressive parameters, which are not related to government expenditures. In order to ensure the identification of the short- and long-run components of government expenditures, we follow Bianchi and Ilut (2017) and specify a beta distribution with mean of 0.2 and a standard deviation of 0.05 for the autoregressive parameter of the short-run expenditure shock and the growth-rate of total factor productivity. The autoregressive coefficient of the long-run component is calibrated to 0.99. As prior distributions for the standard deviations of the structural shocks we employ inverted-gamma distributions and use the same mean and standard deviations as Bianchi and Ilut (2017) when using the same shocks. The prior of the investment specific shock has a mean of 10 with a standard deviation of 2 since previous literature usually finds large posterior means. The prior of the wage mark-up shock is centered at 2 with standard deviation 2. Furthermore, we calibrate the discount factor $\beta$ to 0.995, the share of capital $\alpha$ to 0.3, the amount of habit $h$ to 0.9 and the average maturity to its sample mean of 5 years.

Before we estimate the model, we verify that all parameters are identified locally, using the method by Iskrev (2010).\textsuperscript{18}

\subsection*{2.4.3 Results}

We approximate the posterior distribution of the estimated parameters using a random walk Metropolis-Hastings algorithm. We run two chains with 2,000,000 draws each. In order to assess convergence of the chains, we compute several

\textsuperscript{18}The results and statistics are available upon request.
measures following Brooks and Gelman (1998). We find that the interval of the posterior distribution which is covered by the chains as well as the second moment of the posterior distribution are stable after approximately 1,000,000 draws. We report results based on every second draw of the last 250,000 draws of each chain.

The right panel of Table 2.1 reports the compares the posterior mode, mean and the 90-percent credible intervals. Most of our estimates of structural parameters are in line with the literature for similar kinds of medium-scaled DSGE models, e.g. Bianchi and Ilut (2017) or Justiniano et al. (2013). In the Appendix we plot the prior and the posterior distribution of each parameter.

Our estimates indicate that the sample period is described by regime F. This finding is in line with Bianchi and Ilut (2017) and Davig and Leeper (2006). More precisely, we estimate the reaction coefficient of monetary policy on inflation smaller than 1 which implies a passive monetary policy regime. The estimated coefficient of the tax rate on the debt to GDP ratio might appear at first high with its mean of 0.05 for a fiscal authority which does not adjust taxes in order to stabilize outstanding debt. However, this is the coefficient on the tax rate and not on total taxes. Moreover, what matters for determinacy is the response of the government surplus. We estimate that government expenditures increase strongly whenever output falls (the reaction coefficient is roughly $-0.6$). Thus in total, the surplus does not sufficiently adjust to government debt in order to stabilize outstanding debt.

### 2.4.4 External validation

It is instructive to compare the predictions of the model with data which have not been used in the estimation. For this purpose, we display, in the left graph of figure 2.4, the share of public debt in the portfolio of the banking sector. The solid line represents the actual data, as provided by Board of Governors (see Section 2.4.1 above). During our sample period the share of debt declines from some 55 percent to less than 10 percent. The dashed line shows the model prediction for the share of public debt in the portfolio of the banking sector ($\Gamma_t$). While the model underpredicts the share of public debt at the beginning of the sample somewhat, the model predictions aligns fairly well with actual developments.

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19We provide the univariate convergence in the Appendix.
Figure 2.4: Data vs. model prediction

Notes: Data (solid line) vs. model prediction (dashed line). Left panel shows share of public debt in banks’ portfolio estimated (data source: Board of Governors), right panel shows return on government debt (data source: Hall and Sargent (2011)). Note that the time series was not used in the estimation and hence the model performance in this regard provides some external validation.

The right panel of Figure 2.4 shows the ex-post real return on government debt predicted by the model and contrasts it with a time series compiled by Hall and Sargent (2011). They use market prices for all marketable public bonds and calculate the (ex-post real) holding period return. Their measure thus captures changed in bond valuations which are not reflected in interest rate expenses computed on actual coupon payments. We find once more that the prediction of the model performs quite well.

Finally, we also compare the ownership structure of government debt in the model (households and financial institutions) to an empirical counterpart. In principle US government debt is also held by foreign investors. However, the share of debt held by foreigners is very small in our sample period (approximately 6 percent). Regarding the debt holdings of households and the financial sector we find again that the model performs well. It predicts that about 20 percent of government debt were held by households (and 80 percent by financial institutions). This share is relatively stable during our sample period. In the data the share is 22 and 74 percent respectively.

The empirical estimates are taken from table OFS-2 in the Treasury Bulletin, specifically from the volumes of December 1964, December 1979 and November 1982, published by the St. Louis Fed. The average share of household holdings is calculated as the share of total individuals relativ to total Federal securities outstanding. The average share of household (or foreign) holdings is calculated as the share of total individuals (Foreign and international) relative to total Federal securities outstanding.
2.5 Financial repression in the United States 1948–1974

We are finally in a position to address the questions that motivate our analysis: how large was financial repression during our sample period and what was its effect on macroeconomic performance? By construction the estimated model accounts for the strong decline of public debt during our sample period. However, as we argued above, the model also predicts the behavior of other important variables quite well, even though they have not been included in the estimation. We are thus confident that the estimated model allows us to answer these questions accurately.

In a first step towards quantifying the contribution of financial repression, we compute the laissez-faire interest rate, that is, the interest which would have prevailed in the absence of repression. We obtain it, as we turn to equation (2.7) and set $\tilde{\mu}_t$ to zero. The left graph of Figure 2.5 contrasts the laissez-faire interest to the actual interest rate, both measured from an ex post point of view in real terms. The laissez-faire rate exceeds the actual interest rate by several percentage points, notably in the early sample period. The gap between the two rates declines over time, but it is not trivial in most periods. This suggests that financial repression was sizeable, in line with the findings of Reinhart and Sbrancia (2015).

Put differently, our estimates suggests that the US government was able to borrow at artificially low interest rates. How strongly did this contribute to the reduction of public debt? There are different ways to approach this question. The first approach relies on simply accounting. Namely, we can compute the evolution of debt under the assumption that, all else equal, the government would have borrowed at the laissez-faire interest rate. The right graph of Figure

---

\[21\] Our estimates indicate that $\tilde{\mu}_t$ is negative most of the time, i.e. banks wanted to hold more public debt than allowed by the constraint. Hence, through the lens of the model, the observed low HPR was not caused by low ex-ante returns but the result of surprise reductions of repression over time (negative shocks lower $\tilde{\mu}_t$). This result is partly driven by the underlying maturity structure of Woodford (2001), in which the HPR is affected strongly by capital gains and losses, since all bonds are traded each period. We chose Woodford (2001), as it seems to be the most common approach to modelling long-term debt. However, in future work one should model the maturity structure of government bonds following Krause and Moyen (2016). In such a framework, long-term interest rates are a weighted average of past and new rates, which puts less weight on capital gains and losses. Empirically, this seems to be more relevant, as not all debt is rolled over every period.
Figure 2.5: Repression in the US 1948–1974

Notes: Left panel shows actual (solid line) and laissez-faire (dashed line) interest rate (real, ex post); right panel shows actual evolution of debt-to-GDP ratio (solid line) and counterfactual evolution assuming the laissez-faire interest rate (dashed line).

2.5 shows the result as it contrasts the evolution of debt under this assumption (dashed line) to the actual development (solid line). We find that while the actual decline of debt amounted to some 60 percentage points during our sample period, the decline would have been only about 35 percentage point if the government would have paid the laissez-faire rate. Hence, all else equal, the debt-to-GDP ratio would have been about 25 percentage points higher at the end of 1974.\footnote{Hall and Sargent (2011) consider the period 1945-74 during which public debt fell by 80 percentage points. Growth in real GDP and primary surpluses each contributed roughly 40 percent to the reduction of the debt-to-GDP ratio. 20 percent of the decline, however, were due to negative real returns. Note that our analysis differs in that we contrast the effect of repression by comparing actual interest rates to the laissez-faire interest rates (which is generally larger than zero).}

However, it is unlikely that other things would have been equal because repression impacts not only public finances, but also the economy in general. We now develop a counterfactual scenario which accounts for this possibility. Specifically, we simulate a counterfactual scenario based on our model economy but we assume that there is no regulatory constraint. Otherwise we leave the model unchanged as we compute the equilibrium outcome. In particular, we assume that the model economy is exposed to the same shocks and governed by the same parameter and policy rules as the estimated model.

Figure 2.6 shows the results. It displays the behavior of four selected time series under the counterfactual (dashed line), contrasting it to the actual outcome (solid line). Two observations stand out. First, the debt-to-output ratio would have declined \textit{faster} in the absence of repression. This result is perhaps surprising,
Figure 2.6: Actual data vs. counterfactual in the absence of repression

Notes: Actual time series (black solid line) vs. counterfactual outcome in the absence of repression (red dashed line).

but can be rationalized in light of the second observation: we find that, by and large, the economy would have been on a more expansionary path in the absence of repression.

Given our earlier discussion about the economy-wide effects of financial repression in section 2.3 above this is hardly surprising: financial repression distorts financial intermediation. It constrains banks in their ability to channel funds from households to firms. Confirming this insight, we find that in the counterfactual scenario, investment is much higher without repression. In addition, we observe that there is a consumption boom.

Higher consumption and investment implies that output is higher as well as inflation. Lastly, we also observe that fiscal surpluses are higher in the counterfactual. Overall, these developments rationalize why public debt would have declined more strongly. In the absence of repression.
2.6 Conclusion

How large was financial repression in the US in the aftermath of WW2? We find that repression lowered the interest rate at which the government borrowed by several percentage points. The actual interest rate in our sample period was considerably lower than the laissez-faire rate. We define the laissez-faire rate as a counterfactual object: the interest which would have been observed in the absence of repression. We can recover it on the basis of our estimated model and it fluctuates over time just like the actual interest rate.

Did repression make an important contribution to the decline of public debt? Here the answer is: “it depends”. In an accounting sense the contribution was rather large. If we compute the evolution of public debt on the basis of the laissez-faire rate and keep everything else equal, the decline of the debt-to-GDP ratio during our sample period would have been less pronounced: the ratio would have declined by approximately 35, rather than by 60 percentage points. However, a full-fledged counterfactual should also take into account the broader implications of financial repression for economic performance. Once we do that, we find that repression slowed down the decline of public debt relative to output. This finding can be rationalized in light of the answer to a third question.

What was the impact of repression on the economy? Repression distorts financial intermediation and thus hampers investment and growth. We illustrate this effect through model simulations. For our counterfactual we also find that this effect has been large during our sample period. Absent repression the economy would have expanded more strongly and this is why the debt-to-GDP would have declined more strongly in the absence of repression.
Appendices

2.A Debt accounting

In order to account for debt dynamics we decompose the change in the debt to GDP ratio between 1948Q2 and 1979Q3 into nominal return, inflation, growth and deficits. We use the linearized government budget constraint

\[
\hat{d}_t = \frac{R^l}{\Pi e^{\gamma}} d(\hat{R}_t^l - \hat{\pi}_t - \hat{y}_t^{growth}) + \frac{R^l}{\Pi e^{\gamma}} \hat{d}_{t-1} - \hat{s}_t
\]

(37)

with \(\hat{\cdot}\) denoting linear deviations from steady state and \(^\hat{\cdot}\) log-deviations, \(d\) the debt to GDP ratio, \(R^l\) the nominal return on debt, \(\pi\) the inflation rate, \(y^{growth}\) the per-capita growth rate, and \(s\) surplus to GDP ratio. We can re-write the budget constraint in terms of debt to GDP differences, taking advantage that \(\hat{d}_t + 1 - \hat{d}_t\) is actual public debt ratios.

\[
d_{t+k} - d_{t-1} = \sum_{i=0}^{k} \frac{R^l}{\Pi e^{\gamma}} \hat{R}_{t+i} - \sum_{i=0}^{k} \frac{R^l}{\Pi e^{\gamma}} \hat{\pi}_{t+i} - \sum_{i=0}^{k} \frac{R^l}{\Pi e^{\gamma}} \hat{y}_{t+i}^{growth} + \sum_{i=0}^{k} \hat{s}_{t+i} + rest_{t-1+i}
\]

(38)

with \(rest_{t-1+i} = \sum_{i=0}^{k} \left( \frac{R^l}{\Pi e^{\gamma}} - 1 \right) \hat{d}_{t-1+i}\), but it accounts for less than 0.5% of the reduction. Table A.1 depicts the results: The debt-ratio was reduced by roughly 53%-points, most of it was due to inflation and growth, surplus did reduce the ratio especially in the beginning of the sample but the deficits in the late 1970s increased the debt ratio again.\(^{23}\) Since we put in all components of the government budget constraint as observables, expect the return, the results in table A.1 are basically data-driven, the estimation itself does not affect the numbers much. However, what is unobservable, is the return on government debt in the

\(^{23}\)We add the respective level effects, as for example a log-linear inflation rate of -0.5% is still positive inflation if the steady state of inflation is 1%. Hall Sargent find that surplus did contribute quite significantly to the debt reduction. When inspecting both surplus series we noticed a difference which is not due to different definitions of expenditures and tax revenues but data revision.
Table A.1: Debt reduction accounting

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Decomposition debt evolution</td>
<td></td>
</tr>
<tr>
<td>Debt in 1948Q2</td>
<td>75.5%</td>
</tr>
<tr>
<td>Debt in 1979Q3</td>
<td>16.6%</td>
</tr>
<tr>
<td>Change</td>
<td>-59.0%</td>
</tr>
<tr>
<td>Nominal return</td>
<td>21.6%</td>
</tr>
<tr>
<td>Inflation</td>
<td>-30.7%</td>
</tr>
<tr>
<td>Growth</td>
<td>-22.9%</td>
</tr>
<tr>
<td>Deficit / GDP</td>
<td>-27.0%</td>
</tr>
</tbody>
</table>

Notes: Accounting is similar to Hall and Sargent (2011). Surplus not as important due to updated time series which lowered surplus rate.

absence of repression.

2.B Estimation details

Figure 2.7: Prior vs. posterior distribution

(a)
<table>
<thead>
<tr>
<th>Variable</th>
<th>Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>XIPC</td>
<td>0.4, 0.8</td>
<td></td>
</tr>
<tr>
<td>KAPPAPC</td>
<td>0</td>
<td>1000</td>
</tr>
<tr>
<td>PHILABOR</td>
<td>2, 4</td>
<td></td>
</tr>
<tr>
<td>XIX</td>
<td>2, 4</td>
<td>500, 1000</td>
</tr>
<tr>
<td>KAPPA2</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>RHO_X</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>RHO_ZETA</td>
<td>0.5, 1</td>
<td>20</td>
</tr>
<tr>
<td>RHO_Z</td>
<td>0.2, 0.4, 0.6, 0.8, 1</td>
<td></td>
</tr>
<tr>
<td>RHO_D</td>
<td>0.2, 0.4, 0.6, 0.8, 1</td>
<td></td>
</tr>
<tr>
<td>RHO_ES</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>RHO_MU</td>
<td>0.5, 1</td>
<td>100</td>
</tr>
<tr>
<td>RHO_W</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>RHO_PSI</td>
<td>0.5, 1</td>
<td>100</td>
</tr>
<tr>
<td>HUNDRED_PI</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>GAMMA_GROWTH</td>
<td>1.2, 1.4, 1.6, 1.8, 2, 2.2</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>1.1, 1.15</td>
<td>50</td>
</tr>
<tr>
<td>g</td>
<td>1.1, 1.15</td>
<td>2</td>
</tr>
<tr>
<td>mutilde_100</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Figure 2.8: Univariate convergence statistics.
\begin{align*}
\text{XIPC (Interval)} & \quad \text{XIPC (m2)} & \quad \text{XIPC (m3)} \\
\text{KAPPAPC (Interval)} & \quad \text{KAPPAPC (m2)} & \quad \text{KAPPAPC (m3)} \\
\text{PHILABOR (Interval)} & \quad \text{PHILABOR (m2)} & \quad \text{PHILABOR (m3)} \\
\end{align*}
Figure 2.9: Multivariate convergence statistics.
Chapter 3

The Power of Forward Guidance in a Quantitative TANK Model

3.1 Introduction

Interest rate forward guidance has become an important tool for central banks to enhance the effectiveness of monetary policy at the zero lower bound (Fed, 2008; Deutsche Bundesbank, 2013). In this paper, we quantify the macroeconomic effects of forward guidance within an estimated medium-scale two-agent New Keynesian (TANK) model. This framework serves as a simple approximation to a fully-fledged heterogeneous agent New Keynesian model (see for instance Bilbiie, 2019b; Debortoli and Galí, 2018). Such models can dampen the strong aggregate effect of forward guidance, that is inherent in many complete market or representative agent models. One reason is that full heterogeneity features lower intertemporal substitution of households, which reduces the responsiveness of present macroeconomic aggregates to changes in future interest rates (McKay, Nakamura and Steinsson, 2016; Bilbiie, 2019a).\textsuperscript{1}

As is already well known, strong intertemporal substitution is caused by forward-looking behavior (Del Negro, Giannoni and Patterson, 2015; Kiley, 2016).\textsuperscript{2} One

\textsuperscript{1}This introduces some form of discounting into the Euler equation (McKay, Nakamura and Steinsson, 2017). Discounting can be also achieved through deviations from rational expectations, as for instance with incomplete information (Angeletos and Lian, 2018), bounded rationality (Gabaix, 2018) or level-k thinking (García-Schmidt and Woodford, 2019).

\textsuperscript{2}Take as a baseline the following forward guidance scenario: the central bank pegs the interest rate at a low level for the next $T$ quarters, which will lead to an expansion and inflation in all $T$ quarters. Now suppose that the peg is extended by one period, i.e. until $T + 1$. This
possibility to attenuate the forward-looking behavior within the model would therefore be to introduce some heterogeneity on the households side, in which one type of household behaves “as usual” and another type does not smooth consumption intertemporally. These latter agents are typically called hand-to-mouth households, which have no access to financial markets and can thus neither borrow nor save. Therefore, they are not forward-looking.

However, having a model with two agents does not automatically imply a reduction in the power of forward guidance compared to the representative agent model. As shown analytically by Bilbiie (2008, 2019b), the overall strength of intertemporal substitution depends on the elasticity of the hand-to-mouth agents’ income to aggregate income. The reason is that this elasticity shapes the relative strength of the so-called direct and indirect effects of forward guidance (for such a distinction, see also Kaplan, Moll and Violante, 2018).

The direct effect of changes in interest rates refers to the impact in the absence of changes in household income / general equilibrium and usually works via intertemporal substitution. As an example, when nominal interest rates fall, all else equal, real rates fall as well. This induces households to save less and to increase their demand for consumption. Complementary to this direct effect is the indirect effect of monetary policy. It operates through the general equilibrium increase in labor demand and thus income which is necessary to satisfy the increase in consumption demand. Higher household income raises consumption even further and so on. As discussed in Kaplan et al. (2018) or Luetticke (2019), in representative agent models most of the transmission of monetary policy on output and inflation is due to the direct effect of intertemporal substitution. In contrast, in heterogeneous agent models the indirect effect dominates.

Taken together, the introduction of hand-to-mouth households can increase or decrease the power of forward guidance. This depends on which of the two effects dominates. As Bilbiie (2019b) shows analytically for a simple two-agent model, the introduction of hand-to-mouth households reduces the direct effect of forward guidance, as only a smaller fraction of households smooths intertemporally.

will lead to a stimulus in \( T + 1 \) which raises inflation in \( T + 1 \) and thus lowers the real rate in \( T \). Since monetary policy is constrained by the peg there is a further stimulus in period \( T \) which also raises inflation in \( T \) and lowers real rates in \( T - 1 \). This process continues until the present. As monetary policy does not counteract any stimulus until \( T + 1 \), the cumulative effect of a future expansion rises more, the longer the peg and thus the further away the marginal extension. If monetary policy would not be constrained by a peg, it would simply raise its policy rate and thus limit the aggregate response.
However, it enhances the indirect, i.e. general-equilibrium effect, since the higher marginal propensity to consume of hand-to-mouth households raises per se their own (and thus total) consumption. If, in addition, their income “over-reacts” to changes in aggregate income (which happens with no or too little redistribution), the amplification through the indirect effect dominates and the power of forward guidance increases compared to the representative agent model.\textsuperscript{3}

Our contribution is to illustrate under which conditions the direct or indirect effect dominates in an empirically realistic two-agent model. We estimate a medium-scale version on eight euro area time series and evaluate the quantitative implications of hand-to-mouth households to dampen the power of forward guidance. For plausible ranges of parameters the power of forward guidance is indeed reduced compared to our representative agent benchmark version. The amount of attenuation depends on the degree of countercyclical transfers (similar to automatic stabilizers in McKay and Reis, 2016) and the share of hand-to-mouth households. If there is no or “too little” redistribution, our model amplifies the impact of forward guidance on the economy relative to the benchmark representative agent model. Moreover, we evaluate the combined effects of forward guidance and the Eurosystem’s asset purchase program. We find that the combined impact of asset purchases and forward guidance is higher than the sum of each policy used in isolation. This difference increases with the horizon of forward guidance.

Although our two-agent model can dampen the power of forward guidance, it does not feature a mechanism to solve the so-called forward guidance puzzle: an unreasonably large response of inflation and output that rises exponentially if the horizon of interest rate guidance is extended.\textsuperscript{4} This paper rather emphasizes a simple, yet empirically realistic, extension of medium-scale New Keynesian models – heterogeneity and countercyclical transfers – that allows to substantially tame the power of forward guidance.\textsuperscript{5} For realistic values of the share of hand-to-

\textsuperscript{3}Note that Kaplan et al. (2018) point to fiscal policy as an important driving force in their heterogeneous agent model.

\textsuperscript{4}A possibility to solve the forward guidance puzzle is to include uninsurable income risk, an essential feature of heterogeneous agent models (e.g. McKay et al., 2016; Werning, 2015). Bilbiie (2019a) shows how uninsurable income risk generates discounting (or compounding) in the Euler equation and that this can solve the forward guidance puzzle if it is combined with procyclical income inequality (i.e. hand-to-mouth households income decreases/increases relative to unconstrained households income in a boom/recession). Acharya and Dogra (2019) show within a setup of special preferences that adding (procyclical) income risk can solve the forward guidance puzzle even absent heterogeneity in the marginal propensities to consume.

\textsuperscript{5}The odds ratio favors our two-agent model over its representative agent version essentially
mouth households and countercyclical transfers our model attenuates the impact of forward guidance by up to 40% compared to the representative agent version.6

The next section describes the framework used with a special emphasis on the two crucial features of rule-of-thumb households and the transfer scheme. Section 3.3 gives an overview of the data and the estimation results. Section 3.4 describes the forward guidance simulations conducted in this paper before the final section concludes.

3.2 Framework

The model builds heavily on the medium-scale New Keynesian model of Carlstrom et al. (2017) that features a rich financial sector which allows to analyze the effects of unconventional monetary policy measures. We augment their framework by rule-of-thumb consumers in the spirit of Galí, López-Salido and Vallés (2007) and a simple transfer rule (Bilbiie, 2008). The economy consists of households, firms and a banking sector, which will be explained in detail below. In a nutshell, real investment is ultimately financed by financial intermediaries, whose lending capacities are constrained by their net worth.

3.2.1 Households

The economy is populated by two types of households: A measure $1 - \lambda$ of households has complete access to financial markets and can smooth consumption through short-term deposits and the accumulation of real capital – we call them Ricardian households. The remaining fraction $\lambda$ has no access to financial markets (it can neither borrow nor save) and consumes its wage income and transfers – we call them hand-to-mouth (rule-of-thumb or constrained) households.

Each Ricardian household maximizes lifetime utility with probability one.

6The richer model allows us to show that the mere introduction of hand-to-mouth households can in principle attenuate the impact of forward guidance, i.e. without the necessity to introduce countercyclical transfers. For example, if wages are assumed to be very sticky (re-optimization only every 20 quarters), the (above mentioned) indirect effect is much weaker while the direct effect is still in place (for the implications of sticky wages and hand-to-mouth households, see Colciago, 2011).
\[ E_t \sum_{s=0}^{\infty} \beta^s d_{t+s} \left\{ \ln \left( C_{t+s}^\alpha - h C_{t+s-1}^\alpha \right) - B \frac{H_{t+s}^{1+\eta}}{1+\eta} \right\} , \tag{3.1} \]

where \( C_t^\alpha \) denotes private consumption, \( h \) degree of habit, \( H_t \) the (individual) labor input (scaled by \( B \) to normalize labor input in steady state) and \( d_t \) a shock to the linearized discount factor given by:

\[ d_t = (1 - \rho_d) \ln(d) + \rho_d d_{t-1} + \epsilon_{d,t} \tag{3.2} \]

The budget constraint is given by

\[ C_t^\alpha + P_t^k I_t^\alpha + \frac{D_t}{P_t} + (1 + \kappa Q_t) \frac{F_{t-1}}{P_t} = w_t H_t + R_t^k K_t + \frac{D_{t-1}}{P_t} R_{t-1}^d + \text{div}_t - T_t^\alpha + \frac{Q_t F_t}{P_t} \tag{3.3} \]

Households invest in real capital \( P_t^k I_t \), save deposits \( \frac{D_t}{P_t} \) and repay their outstanding debt including a coupon payment of 1, \( (1 + \kappa Q_t) \frac{F_{t-1}}{P_t} \) (see below).\(^7\) They earn labor income \( w_t H_t \) (to be specified below), a return on capital \( R_t^k K_t \) and deposits \( R_{t-1}^d \frac{D_{t-1}}{P_t} \) and dividends \( \text{div}_t \) net of taxes \( T_t^\alpha \) (which consists of a lump-sum part and a re-distributive part, see section 3.2.5 for details). \( \text{div}_t \) includes dividends from the FI (\( \text{div}_t^{FI} \)), capital goods producer (\( \text{div}_t^{CP} \)) and intermediate goods producer (\( \text{div}_t^{IP} \)).

There is a need for intermediation through the financial system since all investment purchases of the household must beforehand be financed by issuing new investment bonds (hence, there is loan in advance constraint). The price of such bonds is denoted by \( Q_t \) and offers the following payment stream of the household, following Woodford (2001): 1, \( \kappa, \kappa^2, \ldots \) etc.\(^8\) Let \( CI_t \) denote the number of new perpetuities issued in time \( t \), then the household’s stock of nominal liabilities \( F_t \) is given by

\[ F_t = \kappa F_{t-1} + CI_t \iff CI_t = F_t - \kappa F_{t-1}. \tag{3.4} \]

The loan in advance constraint is then given by:

\[ P_t^k I_t \leq \frac{Q_t CI_t}{P_t} \tag{3.5} \]

\(^7\)Note that they have also access to short-term government bonds, but those are perfect substitutes with deposits. \( D_t \) can thus be interpreted as the households net resource flow into the FIs (Carlstrom et al., 2017).

\(^8\)Due to the recursive structure, \( \kappa^h Q_t \) is the time \( t \) price of such a bond that was issued in period \( t - h \).
The law of motion for capital follows:

\[ K_t = (1 - \delta) K_{t-1} + I_t. \]  

(3.6)

The representative Ricardian household therefore maximizes utility (3.1) subject to the budget constraint (3.3, multiplier \( \vartheta_t \)), the loan in advance constraint (3.5) and the law of motion for capital (3.6). The first order conditions are given by:

\[ \Lambda_t = \frac{b_t}{C_t - hC_{t-1}^\alpha} - \frac{\beta h b_{t+1}}{C_{t+1} - hC_t^\alpha} \]  

(3.7)

\[ \Lambda_t = E_t \beta \frac{\Lambda_{t+1} P_t^d}{\Pi_{t+1}} \text{ with } \Pi_{t+1} = \frac{P_{t+1}}{P_t} \]  

(3.8)

\[ \Lambda_t M_t Q_t = E_t \beta \frac{\Lambda_{t+1} (1 + \kappa Q_{t+1} M_{t+1})}{\Pi_{t+1}} \]  

(3.9)

\[ \Lambda_t M_t P_t^k = E_t \beta \Lambda_{t+1} \left[ R_{t+1}^k + M_{t+1} P_{t+1}^k (1 - \delta) \right] \]  

(3.10)

with \( M_t = 1 + \frac{\vartheta_t}{\Lambda_t} \) or \( \Lambda_t M_t = \Lambda_t + \vartheta_t \). The first two equations comprise the typical Euler-equation for deposits, the third one for investment bonds. Note that the demand for capital (last equation) is distorted by the time-varying distortion \( M_t \) which depends on the multiplier of the loan-in-advance constraint (3.5). As discussed in great detail in Carlstrom et al. (2017), this distortion acts like a mark-up on the price of new capital and is basically the term premium that exists due to the segmented markets and the leverage constraint of the banks that limit the arbitrage across the term structure (see next subsection).

The budget constraint of hand-to-mouth agents is much simpler as they neither borrow nor save and only consume their labor income less taxes:\footnote{Such a behavior can be rationalized for instance by myopic behavior, a lack of access to capital markets or ignorance of intertemporal trading opportunities. As pointed out by Galí et al. (2007), this is a rather extreme form of non-Ricardian behavior, which nevertheless capture the observed heterogeneity in consumption responses and income as found in the data.}

\[ C_t^h = w_t H_t - T_t^h, \]  

(3.11)

where their consumption is \( C_t^h \), labor income is \( w_t H_t \) (see below) and \( T_t^h \) are
taxes that hand-to-mouth households have to pay. Overall taxes are given by a time-invariant component $T^h$ and a countercyclical transfer scheme:\footnote{The time-invariant component ensures that in steady state consumption is similar across households (i.e. $C^h = C^o$, see also Galí et al., 2007).}

$$T^h_t = \frac{\tau}{\lambda} (Y_t - Y) + T^h.$$  \hspace{1cm} (3.12)

$\tau \geq 0$ captures the degree of countercyclical transfers which rebates income whenever aggregate output is different from steady state $(Y_t - Y)$ – see section 3.2.5 for more details and how the optimizers pay for that transfer.\footnote{Bilbiie (2019a) proposes to rebate firm profits. In our model not all kinds of profits would imply a taming of the impact of forward guidance. For instance, bank profits would amplify the aggregate effects because they are strongly procyclical (reduction of interest rates constitutes a capital gain for banks, raising profits). In contrast, intermediate good profits are countercyclical (similar to mark-ups after demand-type driven shocks).} Although this transfer scheme is stylized, it captures in a parsimonious way automatic stabilizers that are found in more complex settings (see for instance Leeper et al., 2010).\footnote{The study of more complex transfer rules or distortionary taxes seems interesting, but is beyond the scope of the paper.} Additionally, it seems the most direct way to introduce redistribution within the two heterogeneous agents.

### 3.2.2 Labor agencies

Each household supplies a specialized type of labor $H^j_t$, independent of whether it is a Ricardian or a rule-of-thumb household (in the spirit of Erceg et al., 2000). Since firms do not differentiate between the two households when hiring labor for a specialized type $j$, the supply of hours and the wage rate is the same for both groups. The labor agencies bundle the specialized labor inputs into a homogeneous labor output that it sells to the intermediate good firm according to

$$H_t = \left[ \int_0^1 (H^j_t)^{1/(1+\lambda_{w,t})} dj \right]^{1+\lambda_{w,t}}$$  \hspace{1cm} (3.13)

where $\lambda_{w,t}$ is the wage mark-up, following (in linearized form)

$$\lambda_{w,t} = (1 - \rho_{\lambda_w}) \ln(\lambda_w) + \rho_{\lambda_w}(\lambda_{w,t-1}) + \epsilon_{\lambda_{w,t}}.$$  \hspace{1cm} (3.14)
The demand for the different types of labor inputs is given by

$$H^j_t = \left(\frac{W^j_t}{W_t}\right)^{1+\lambda w,t} H_t$$ \hspace{1cm} (3.15)$$

In each period, the probability of resetting the wage is \((1-\theta_w)\), while with the complementary probability \((\theta_w)\) the wage is automatically increased following the indexation rule:

$$W^j_t = \Pi_{t-1}^w W^j_{t-1}$$

The maximization problem of a given union for the specialized labor input \(j\) is given by (similar to Colciago, 2011):

$$\max_{\hat{W}_t} E_t \sum_{s=0}^{\infty} (\beta \theta_w)^s \left\{ (1-\lambda) u \left(C_t^{\ell,s}\right) + \lambda u \left(C_t^{h,s}\right) - d_{t+s} \Lambda^{a}_{t+s} B \frac{H_{t+s}^{1+\eta}}{1+\eta} \right\}$$

s.t. the budget constraints (3.3), (3.11) and labor demand (3.15) and with \(\Lambda^{a}_{t+s} = (1-\lambda) \Lambda^{o}_{t+s} + \lambda \Lambda^{h}_{t+s}\). \hspace{1cm} (3.17)

### 3.2.3 Financial intermediaries

The financial intermediaries (FI) in the model use accumulated net worth \(N_t\) and short-term deposits \(D_t\) to finance investment bonds \(F_t\) and long-term government bonds \(B_t\). Their balance sheet is given by:

$$Q_t \frac{B_t}{P_t} + Q_t \frac{F_t}{P_t} = N_t + \frac{D_t}{P_t} = L_t N_t,$$

where \(L_t\) denotes leverage. Note that investment and government bonds are perfect substitutes since they offer the same payment streams and thus are valued at the same price \(Q_t\). Define the return on those bonds as \(R^L_t\):

$$R^L_t = \frac{1 + \kappa Q_t}{Q_{t-1}}.$$  \hspace{1cm} (3.17)

\(13\) We define \(\Lambda^{h}_{t+s} = d_{t+s} \frac{1}{1+\eta_{t+s}}\), i.e. without habit. The simulation results do not change qualitatively if we also introduce habit there.
Every period a financial intermediary receives the coupon payment of 1 from its old assets in $t-1$, which it additionally sells completely. Its income is thus $(1 + \kappa Q_t) \left( \frac{B_{t-1}}{P_t} + \frac{F_{t-1}}{P_t} \right)$. It purchases new assets at price $Q_t$, such that the real value of these purchases is $Q_t \left( \frac{F_t}{P_t} + \frac{B_t}{P_t} \right)$. It further collects new deposits $D_t$ and has to pay out interest rate expenses on the deposits of the previous period $R_{t-1} \frac{D_{t-1}}{P_t}$. Any change in the net worth from steady state will be costly: $f(N_t) N_t$, with $f(N_t) = \frac{\Psi_n}{2} \left( \frac{N_t - N}{N} \right)^2$. Thus, the remaining dividend payments are given by interest income less the expenditures:

$$
div_{t}^{FI} = (1 + \kappa Q_t) \left( \frac{B_{t-1}}{P_t} + \frac{F_{t-1}}{P_t} \right) + D_t - Q_t \left( \frac{F_t}{P_t} + \frac{B_t}{P_t} \right) - R_{t-1}^{d} \frac{D_{t-1}}{P_t} - f(N_t) N_t
$$

$$
\Leftrightarrow div_{t}^{FI} + (1 + N_t) f(N_t) = \frac{P_{t-1}}{P_t} \left( (R_{t}^{L} - R_{t-1}^{d}) L_{t-1} + R_{t-1}^{d} \right) N_{t-1},
$$

(3.18)

where the definition of the return $R_{t}^{L}$ and the banks’ balance sheet (3.16) were substituted. This equation shows that profits will be partly paid out as dividends $div_{t}^{FI}$ to the (Ricardian) households while the rest is retained as net worth for subsequent activity. The FI discounts dividend flows using the (Ricardian) household’s pricing kernel augmented with additional impatience $\zeta < 1$, which allows for a positive excess return of long-term debt over deposits in steady state.15

The FI then chooses dividends $div_{t}^{FI}$ and net worth $N_t$ to maximize expected dividend payments

$$
V_t = E_t \sum_{s=0}^{\infty} (\beta \zeta)^s \Lambda_{t+s} div_{t+s}^{FI}
$$

subject to (3.18). This yields the following first-order condition:

$$
\Lambda_t \left[ 1 + f (N_t) + N_t f' (N_t) \right] = E_t \Lambda_{t+1} \beta \zeta \frac{P_t}{P_{t+1}} \left[ (R_{t+1}^{L} - R^d_t) L_t + R^d_t \right].
$$

(3.20)

The FIs are subject to a simple hold-up problem which limits their ability to attract deposits (in spirit similar to Gertler and Karadi, 2013). We follow the

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14 As will be shown below, a leverage constraint (due to a “hold-up” problem) limits the ability of the FI to attract deposits and thus eliminates the arbitrage opportunity between long and short rates. However, this limit to arbitrage could be undone by an increase in net worth (implicitly, that would be a lump-sum transfer (tax) on the (Ricardian) households). The net worth adjustment cost ensure that this does not happen.

15 It can be shown that $R^L = R^d + \frac{1-\zeta}{\zeta} R^d > R^d$ if $\zeta < 1$. 

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approach by Carlstrom et al. (2017) completely and arrive at the following expression for the leverage constraint $L_t$: \[ L_t = \frac{1}{1 + (\Phi_t - 1) E_t \frac{R_{t+1}}{R_t}}, \] (3.21)

where $\Phi_t$ measures exogenous changes in the financial friction:

\[ \Phi_t = (1 - \rho_\Phi) \Phi + \rho_\Phi \Phi_{t-1} + \varepsilon_{\Phi,t}. \] (3.22)

### 3.2.4 Goods market

Perfectly competitive final goods producers combine differentiated intermediate goods $Y_t(i)$ into a homogeneous good $Y_t$ according to the technology:

\[ Y_t = \left[ \int_0^1 Y_t(i) \frac{1}{1 + \lambda_{w,t}} di \right]^{1+\lambda_{p,t}}, \]

where $\lambda_{p,t}$ is the time-varying price mark-up that evolves according to

\[ \lambda_{p,t} = (1 - \rho_{\lambda_p}) \ln(\lambda_p) + \rho_{\lambda_p} \lambda_{p,t-1} + \varepsilon_{\lambda_{p,t}}. \] (3.23)

Profit maximization leads to the following demand function:

\[ Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\frac{1 + \lambda_{w,t}}{\lambda_{w,t}}} Y_t, \] (3.24)

with

\[ P_t = \left[ \int_0^1 P_t(i)^{-\frac{1}{\lambda_{w,t}}} di \right]^{-\lambda_{w,t}}. \] (3.25)

A continuum of monopolistic competitive firms combines capital $K_{t-1}$ and labor $H_t$ to produce intermediate goods according to a standard Cobb-Douglas technology. The production function is given by:

\[ Y_t(i) = A_t K_{t-1}(i)^{\alpha} H_t(i)^{1-\alpha} \] (3.26)

\[ ^{16}\text{Details of the derivation can be found in their paper.} \]
with
\[ A_t = (1 - \rho_a) \ln(A) + \rho_a A_{t-1} + \epsilon_{A,t}. \quad (3.27) \]

The intermediate goods producers set prices based on Calvo contracts. In each period firms adjust their prices with probability \((1 - \theta_p)\) independently from previous adjustments. Those firms that cannot adjust their prices in a given period will re-set their prices according to the following indexation rule:

\[ P_t(i) = \Pi_{t-1}^{\rho_p} P_{t-1}(i). \]

Firms that can adjust their prices face the following problem:

\[
\max \ E_t \sum_{s=0}^{\infty} \theta_p \beta^s \Lambda_{t+s} \left[ \frac{P_t \left( \prod_{k=1}^{s} \Pi_{t+k-1}^{\rho_p} \right)}{P_{t+s}} Y_{t+s}(i) - \frac{W_{t+s}}{P_{t+s}} H_{t+s}(i) - R_{t+s}^k K_{t+s}(i) \right],
\]

subject to labor demand \((3.15)\) and \(Y_t(i) = \left( \frac{P_t(i)}{P_0} \right)^{-\varepsilon_{p,t}} Y_t\). It holds that dividends are given by \(\text{div}_t^{IG} = Y_t - w_t H_t - R_t^k K_{t-1}\).

The capital goods producers take final output \(I_t\) and sell it (with a mark-up) subject to adjustment costs to the households, therefore dividends \(\text{div}_t^{CP} = P_t^k I_t^p - I_t = P_t^k \mu_t \left[ 1 - S \left( \frac{I_{t+s}}{I_{t+s-1}} \right) \right] I_t - I_t\), where the investment specific technology shock follows an AR(1) process:

\[ \mu_t = (1 - \rho_\mu) \ln(\mu) + \rho_\mu \mu_{t-1} + \epsilon_{\mu,t}. \quad (3.28) \]

The profit maximization is then described by

\[
\max_{I_t} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \left[ P_{t+s}^k \mu_{t+s} \left[ 1 - S \left( \frac{I_{t+s}}{I_{t+s-1}} \right) \right] I_{t+s} - I_{t+s} \right]. \quad (3.29)
\]
3.2.5 Government policies

The central bank follows a Taylor rule when setting its short-term policy rate $R_t$: \[ \ln (R_t) = (1 - \rho) \ln (R) + \rho \ln (R_{t-1}) + (1 - \rho) (\tau_\pi (\pi_t - \pi) + \tau_y (y_t - y_{t-1})) + R_t^c \]

with \[ R_t^c = (1 - \rho_m) \ln (R^c) + \rho_m R_{t-1}^c + \varepsilon_{R,t}. \] (3.30)

The government collects taxes $T_t$ in a lump-sum fashion and issues government bonds $Q_t B_t / P_t$ to finance its outstanding debt including coupon payments $(1 + \kappa Q_t) B_{t-1} / P_t$. \[ Q_t B_t / P_t + T_t = (1 + \kappa Q_t) B_{t-1} / P_t. \] (3.31)

Note that tax-income $T_t = \lambda T_t^h + (1 - \lambda) T_t^o$ is net of the countercyclical transfers paid to hand-to-mouth households. Implicitly, there is redistribution of countercyclical transfers $\tau (Y_t - Y)$ from optimizing to hand-to-mouth households (via the government). The respective tax rules for both agents are given by the following two equations:

\[ T_t^o = \frac{1}{1 - \lambda} \left( T_t^h + T_t^o - \tau (Y_t - Y) \right) \] (3.32) \[ T_t^h = T_t^h + \frac{\tau}{\lambda} (Y_t - Y). \] (3.33)

For simplicity, only the Ricardian households finance the government. Additionally, they are involved in the countercyclical transfer system in which the hand-to-mouth households participate as well. The degree of countercyclicality is given by $\tau$. $T_t^o$ and $T_t^h$ are chosen such that consumption of hand-to-mouth households and Ricardian households coincide in steady state. \[ 17 \] Since short-term government debt and bank deposits are perfect substitutes it holds that $R_t^d = R_t$. \[ 18 \] Since debt-stabilizing taxes are levied on Ricardian households only, there is no feedback of debt-dynamics on decisions due to Ricardian equivalence. However, this does not apply to the redistribution scheme. \[ 19 \] As the focus of this paper is on the effect of forward guidance when a fraction of households does not feature forward-looking behavior – and not so much about different consumption distributions – we view that assumption as being largely justifiable.
3.2.6 Aggregation

Taking the household and the government budget constraint, as well as all dividend payments, one arrives at the aggregate resource constraint

\[
Y_t = C_t + I_t + f(N_t)N_t, \tag{3.34}
\]

where aggregate consumption and investment are given by a weighted average of the respective variables for optimizer and rule-of-thumb households:

\[
C_t = (1 - \lambda) C_t^o + \lambda C_t^h \tag{3.35}
\]

and

\[
I_t = (1 - \lambda) I_t^o. \tag{3.36}
\]

Similarly, the aggregate capital stock is given by

\[
K_t = (1 - \lambda) K_t^o. \tag{3.37}
\]

3.3 Estimation

After linearizing the model around the steady state we estimate it using Bayesian estimation methods. We use eight quarterly euro area time series with the sample period 1999Q1 to 2014Q4.\footnote{We stopped the estimation before interest rates (especially the 3-month Euribor) turned negative in 2015Q2 in our sample.} In this section, we first describe the dataset, followed by description of the calibration and prior distributions of the respective parameters. Finally, we report the corresponding posterior distributions.

3.3.1 Data

We use a total of eight observables for the euro area: real GDP per capita, real investment, gross inflation, employment growth, real wage growth, the first difference of the short- and long-term interest rate, and real bank net worth growth. The time series on bank net worth is taken from the European Central Bank’s MFI Balance Sheet Items Statistics. All the other variables are taken from the
Area-wide Model database of the ECB.\footnote{21} Since we have only seven structural shocks in the model, we add a measurement error to the observations equation for bank net worth in order to avoid stochastic singularity.

Per capita output and investment are obtained by dividing real GDP (YER) and investment (ITR) by the number in the labor force (LFN). Growth rates are log-differences. Inflation is measured as the growth rate of the seasonally adjusted Harmonised Index of Consumer Prices (HICPSA). Employment growth is the log-difference of the total employment (LNN). For the real wage series we first divide the nominal wage rate per head (WRN) by the HICPSA and then take the log-difference. Our short-term nominal interest rate is the 3-month Euribor rate (STN) and our long-term nominal interest rate the euro area 10-year government benchmark bond yield (LTN). Real bank net worth is obtained by dividing the nominal capital and reserves of euro area monetary financial institutions (excluding eurosystem) (NWB) by HICPSA and taking the log-difference. All series are demeaned with their respective sample mean.\footnote{22}

\[
\begin{bmatrix}
\text{dlGDP}_t \\
\text{dlInvestment}_t \\
\text{dlGDPDeflator}_t \\
\text{ShortInterestRate}_t \\
\text{LongInterestRate}_t \\
\text{dlHours}_t \\
\text{dlWages}_t \\
\text{dlNetworth}_t
\end{bmatrix} = 100 \cdot \begin{bmatrix}
0 \\
0 \\
\log(\Pi) \\
\log(\Pi/\beta) \\
\log(\Pi/\beta) + 0.01/4 \\
0 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
\hat{y}_t - \hat{y}_{t-1} \\
\hat{x}_t - \hat{x}_{t-1} \\
\hat{\pi}_t \\
\hat{r}_t \\
\hat{r}^{L,10}_t \\
\hat{h}_t - \hat{h}_{t-1} \\
\hat{w}_t - \hat{w}_{t-1} \\
\hat{n}_t - \hat{n}_{t-1} + \varepsilon_{n,t}
\end{bmatrix},
\]

We match the long-term interest rate time series to the yield-to-maturity of the 10 year government bond \( r^{L,10}_t = \log R^{L,10}_t - \log R^L_{t-1} \), with \( R^{L,10}_t = \epsilon_t + \kappa \) (see Carlstrom et al., 2017).

### 3.3.2 Calibration and prior distributions

As is common in the literature, we calibrate a subset of the structural parameters to ensure identification. We follow mostly the calibration of Carlstrom et al.\footnote{21} We use the 18th update of the Area-wide Model (AWM) database from August 2018.\footnote{22} An estimation of the steady states (for instance inflation) did not change the results much.
The time preference $\beta$ is set to 0.99, yielding a steady state annual real interest rate of roughly 4%. The labor income share $\alpha$ is set to 0.33 and the depreciation rate to $\delta = 0.025$, which implies a 10% annual depreciation of the capital stock. The steady state mark-ups of prices and wages are set to 20%\(^\text{, i.e.} \lambda_w = \lambda_p = 0.2\). The leverage ratio is set to 6 which implies $\zeta = 0.9854$. We impose that in steady state the annual long-term rate $R^L$ is one percentage point above the short-term one, i.e. $R^L = R^{L,10} = R + 0.01/4$ (see the observation equation).\(^\text{24}\) In order to estimate the model with a 10-year government bond (similar to its empirical counterpart) we set $\kappa = 0.975$. It was not possible to identify the share of hand-to-mouth households $\lambda$ and the redistribution coefficient $\tau$ simultaneously in the data. We therefore calibrate the share of constrained households to 30\% since there is empirical evidence for such a share (e.g. Dolls, Fuest and Peichl, 2012; Bilbiie and Straub, 2013; Fève and Sahuc, 2017).\(^\text{25}\) The prior choices are largely taken from Carlstrom et al. (2017) and are summarized in columns 2 to 4 of Table 3.1. The first block of parameters determine the shape of the utility and cost functions. For the amount of habit $h$, we use a beta distribution with mean 0.5 and standard deviation of 0.2. The inverse Frisch elasticity $\eta$ has a relatively flat prior centered around 2. The prior mean and standard deviation for the investment adjustment costs $\Psi_I$ are taken from the posterior mode of Coenen, Karadi, Schmidt and Warne (2018). For the amount of indexation and the amount of stickiness we use a beta distribution centered around 0.6 and 0.7, respectively, with a standard deviation of 0.1 for all four parameters. The prior of the degree of monetary persistence is a beta distribution with mean 0.7 and standard deviation of 0.1. The two Taylor coefficients on inflation and output follow both a normal distribution centered around 1.5 and 0.5 respectively. For the size of redistribution we took a relatively flat prior around 0.3 (the share of hand-to-mouth households).

We specify for all autocorrelations of the shocks a beta distribution which is centered around 0.6 with a standard deviation of 0.2.\(^\text{26}\) All priors for the standard

\(^{23}\)We cross-check with values from Smets and Wouters (2003) which studied the euro area, but the results were largely unchanged.

\(^{24}\)In the data the long-term rate for the sample period was roughly 1.5pp higher than the short-term rate. However, results were basically unchanged when we estimated the model with this higher value.

\(^{25}\)As a cross check we estimated the model with the calibrated redistribution $\tau$ (at the posterior mean of Table 3.1) and found a share of hand-to-mouth households of around 0.35.

\(^{26}\)Note that for better identification of the autocorrelation of the monetary policy shock and the persistence in the Taylor rule we use a slightly tighter prior on the persistence, see above.
Table 3.1: Prior and posterior distribution of estimated parameters

<table>
<thead>
<tr>
<th></th>
<th>Prior</th>
<th></th>
<th>Posterior</th>
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<tbody>
<tr>
<td></td>
<td>Dist</td>
<td>Mean</td>
<td>SE</td>
<td>Mode</td>
</tr>
<tr>
<td>Utility &amp; technology</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$ Habit</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.7721</td>
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<tr>
<td>$\eta$ Inverse Frisch</td>
<td>G</td>
<td>2</td>
<td>0.5</td>
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<tr>
<td>$\psi_I$ Investment adj. costs</td>
<td>G</td>
<td>10</td>
<td>1.0</td>
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<tr>
<td>$\psi_N$ Net worth adj. costs</td>
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<td>3</td>
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<tr>
<td>Stickiness</td>
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<td></td>
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<tr>
<td>$\iota_p$ Price indexation</td>
<td>B</td>
<td>0.6</td>
<td>0.1</td>
<td>0.4979</td>
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<tr>
<td>$\iota_w$ Wage indexation</td>
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<td>0.6</td>
<td>0.1</td>
<td>0.3079</td>
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<tr>
<td>$\theta_p$ Price stickiness</td>
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<td>0.1</td>
<td>0.8046</td>
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<tr>
<td>$\theta_w$ Wage stickiness</td>
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<td>0.7</td>
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<td>Government policy</td>
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<tr>
<td>$\rho$ MP smoothing</td>
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<tr>
<td>$\tau_p$ MP on inflation</td>
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<tr>
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<td>$\tau$ Size redistribution</td>
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<td>AR(1) shocks</td>
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<tr>
<td>$\rho_a$ TFP</td>
<td>B</td>
<td>0.60</td>
<td>0.20</td>
<td>0.9876</td>
</tr>
<tr>
<td>$\rho_\phi$ Financial friction</td>
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<td>0.20</td>
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<td>$\rho_\mu$ Investment specific</td>
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<td>$\rho_{\lambda_w}$ Wage markup</td>
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<td>0.20</td>
<td>0.1803</td>
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<tr>
<td>$\rho_{\lambda_p}$ Price markup</td>
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<tr>
<td>$\rho_d$ Demand</td>
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<td>0.20</td>
<td>0.4710</td>
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<td>$\rho_m$ Monetary policy</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_a$ TFP</td>
<td>IG</td>
<td>0.50</td>
<td>1.00</td>
<td>0.0057</td>
</tr>
<tr>
<td>$\sigma_\phi$ Financial friction</td>
<td>IG</td>
<td>0.50</td>
<td>1.00</td>
<td>0.1844</td>
</tr>
<tr>
<td>$\sigma_\mu$ Investment specific</td>
<td>IG</td>
<td>0.50</td>
<td>1.00</td>
<td>0.0982</td>
</tr>
<tr>
<td>$\sigma_{\lambda_w}$ Wage markup</td>
<td>IG</td>
<td>0.10</td>
<td>1.00</td>
<td>0.8051</td>
</tr>
<tr>
<td>$\sigma_{\lambda_p}$ Price markup</td>
<td>IG</td>
<td>0.10</td>
<td>1.00</td>
<td>0.0466</td>
</tr>
<tr>
<td>$\sigma_d$ Demand</td>
<td>IG</td>
<td>0.10</td>
<td>1.00</td>
<td>0.6141</td>
</tr>
<tr>
<td>$\sigma_r$ Monetary policy</td>
<td>IG</td>
<td>0.10</td>
<td>1.00</td>
<td>0.0283</td>
</tr>
<tr>
<td>$\sigma_{ME}$ ME on net worth</td>
<td>IG</td>
<td>0.001</td>
<td>1.00</td>
<td>0.0117</td>
</tr>
</tbody>
</table>

Notes: N stands for the Normal, B the Beta, G the Gamma and IG the inverted Gamma distribution.
deviations of shocks follow a relatively flat inverse gamma distribution with standard deviation of 1. The prior of the wage markup, price markup, demand and monetary policy are all centered around 0.1. For TFP, financial friction and the investment specific technology we use slightly higher values of 0.5. The mean for the measurement error on net worth is taken from the variance of the underlying data sample.

### 3.3.3 Posterior distribution

With the above specified prior distributions, we draw from the posterior distributions using the Metropolis-Hastings algorithm with two chains, each with 1,000,000 draws. In order to assess the convergence of the chains, we compute several measures following (Brooks and Gelman, 1998). The interval of the posterior distribution which is covered by the chains, as well as the second moment of the posterior distribution, seem to be stable for most parameters after approximately 500,000 draws. We report results based on the last 100,000 draws of each chain.

The last columns of Table 3.1 report the posterior mode, the posterior mean, and the lower and upper bounds of the 90% posterior density interval of the estimated parameters obtained by the Metropolis-Hastings algorithm. Most of our estimates are largely in line with similar estimates for the euro area (e.g. Smets and Wouters, 2003; Coenen et al., 2018). In the Appendix we plot the prior and the posterior distribution of each parameter.

Compared to the above two studies, we find for our data a slightly higher value of habit and wage stickiness and much lower persistence of monetary policy (around 0.72 compared to above 0.9 in the other two studies). However, note that the monetary policy shock is also persistent, therefore our parameters actually imply a more persistent monetary policy response for a monetary policy shock. Additionally, our quantitative simulations below do not change qualitatively if we assume a higher monetary policy persistence. We estimate the degree of redistribution $\tau \sim 0.15^{27}$. This is relatively close to Leeper et al. (2010), who find $\tau$ in the range 0.05 to 0.25 with a mean of 0.13 in a similar transfer rule for a representative agent model.

---

27This is in principle the same number as in Gerke, Giesen and Scheer (2020). However, in that paper $\tau_{GGS20}^{G}$ is around 0.5 with the transfer rule $T_{i}^{h} = \tau_{GGS20}^{G} (Y_{i} - Y) + T^{h}$. Here, we use the transfer rule $T_{i}^{h} = \frac{\lambda}{\tau} (Y_{i} - Y) + T^{h}$. Hence our value of $\tau$ is simply a scaled version from Gerke et al. (2020), as the following holds: $\tau_{GGS20}^{G} = \frac{\lambda}{\tau}$. 

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For robustness, we estimated the model from 1999 to 2007 to check that the estimates are not distorted by the financial crisis. Overall the parameters are not that different and are within the posterior bands: there is in general less persistence in the system (smaller AR(1) coefficients) and smaller nominal rigidities (although the indexation parameter is high in either case). The redistribution parameter $\tau$ is smaller (0.143 instead of 0.157). For a second robustness check, we estimated the model from 1999 to 2014 with consumption instead of GDP as observable. Again, the estimated parameters are not that different. A notable exception is the size of redistribution, $\tau$, which is reduced to 0.117. However, even this smaller value for redistribution implies a reduction compared to the representative version, as the next section makes clear (specifically, Figure 3.3).

### 3.4 Simulations

In order to assess the quantitative implications of hand-to-mouth consumers we run several forward guidance simulations using the anticipated news approach of Laséen and Svensson (2011). We start with the impact forward guidance, implemented as an interest rate peg of 25bps annually below steady state for six quarters. This scenario is (as of June 2019) in essence similar to a cut in the deposit facility rate of 25bps with an extension of forward guidance “until the end of 2020”. According to recent estimates of EONIA forward curves, this seems to be a plausible scenario for the euro area, see Lane (2019). We compare this scenario within three models: ‘TANK + transfers’ (blue dotted line, i.e. our estimated two-agent New Keynesian (TANK) model specified in section 3.2), ‘TANK’ (red solid-dotted line, no transfers, i.e. $\tau = 0$) and ‘RANK’ (black solid line, the Representative Agent New Keynesian (RANK) model with no hand-to-mouth households, i.e. $\tau = \lambda = 0$).

Figure 3.1 illustrates how such an expansionary forward guidance (upper left panel) lowers long-term interest rates (upper middle panel) and thus stimulates investment (upper right panel) and consumption (bottom left panel). This raises real GDP (bottom middle panel) and inflation (bottom right panel). As one can see, both TANK variants encompass the RANK model: the impact of forward guidance is more pronounced in the TANK model (hence, the above mentioned indirect effect dominates as in Bilbiie, 2019b) but less pronounced in our TANK model with transfers (the direct effect dominates).
Figure 3.1: Simulated quarterly responses of aggregates in %-deviation from steady state, if the interest rate is held 25bps annually below steady state for six quarters.

Figure 3.2: Simulated quarterly responses for an interest rate peg of 25bps below steady state for six quarters in %-deviation from steady state. It depicts aggregate variables for RANK and the response of hand-to-mouth households on TANK (red solid-dotted line) and Tank with transfers (blue dotted line) over time (quarters).

The amplified (dampened) aggregate response can be explained if we examine the strength of the indirect effect, i.e. by inspecting the constrained households’ total income and their consumption demand. Figure 3.2 depicts the response of labor income, transfers and consumption only for the constrained households (for both TANK models) and contrasts them to (the aggregate response in) RANK.

Focus on TANK first (no transfers, red solid-dotted line). The left panel reveals that labor (and thus total) income of hand-to-mouth households increases com-
pared to RANK, which raises their consumption demand (right panel) and thus aggregate consumption, investment and income (red solid-dotted line in Figure 3.1). Therefore, although a smaller fraction of households smooths intertemporally (which should per se dampen the aggregate effects of forward guidance), the higher marginal propensity to consume of constrained households predominates.

Now, contrast these dynamics with the empirical TANK that includes countercyclical transfers (blue dotted line). As the expansionary policy leads to a boom, hand-to-mouth households receive less countercyclical transfers (middle panel), so they reduce (relatively) their consumption demand (right panel). This feeds back into a relatively smaller aggregate response and thus lower wage income (left panel). The (relative) fall in labor income and transfers results in a relatively small consumption response of hand-to-mouth households, which dampens the impact on aggregate consumption (bottom left panel in Figure 3.1). Hence, the direct effect outweighs the indirect one and the power of forward guidance is attenuated.

To assess the contribution of the share of hand-to-mouth households $\lambda$ and the associated degree of countercyclical redistribution $\tau$ that is necessary to reduce the power of forward guidance, Figure 3.3 contrasts the relative peak response of inflation (left panel) and output (right panel) for eight quarters of forward guidance with different combinations of $\lambda$ and $\tau$. A value above 1 (depicted in bright yellow) indicates an amplification and a value below 1 a dampening.
relative to RANK.

There are two takeaways. First, there is a non-negligible parameter region where the introduction of hand-to-mouth households amplifies the effects of forward guidance, especially when redistribution is low. Second, the combination of a high share of constrained households and significant redistribution leads to the strongest reduction of the power of forward guidance. The crosses in the figure highlight the values that were used for Figure 3.1 and 3.2. For this parameter combination, the power of forward guidance is reduced by approximately 40% compared to our RANK benchmark. However, the amount of attenuation depends on the length of forward guidance. In case the central bank promises an expansionary stance for only 6 quarters, the peak impact of inflation is reduced by approximately 22%.

Although our TANK model with transfers can thus attenuate the power of forward guidance, it does not resolve the so-called forward guidance puzzle (Del Negro et al., 2015), see Figure 3.4. This figure depicts for all three model variants (‘RANK’, ‘TANK’, ‘TANK+transfer’) an exponentially increasing peak impact of three consecutive rate cuts on consumption (left panel), inflation (middle panel) and output (right panel), at different horizons at which these cuts occur. As one can see, even though the peak impact is reduced in the estimated TANK model with transfers (blue dotted line) compared to the pure TANK model (red solid-dotted line), the impact is still increasing the further away the cuts occur. Hence, to actually resolve the puzzle in our model, one would probably have to add uninsurable idiosyncratic income risk to the model (as in a HANK model), that triggers a yet missing self-insurance mechanism (as shown for a simple TANK analytically by Bilbiie, 2019a).

In a last scenario we illustrate the interaction of forward guidance (FG) with asset purchases (APP), as observed in recent years. As a baseline, we simulate the impact of the Eurosystem’s asset purchase program as of early 2015 (similar to Sahuc, 2016), within our estimated TANK model with transfers (blue solid line in Figure 3.5). The purchases (upper left panel) stimulate investment through portfolio-rebalancing (not shown), which raises real GDP (not shown) and inflation (bottom left and right panel). As a result, the policy rate also increases (upper right panel).

\footnote{In McKay et al. (2016) the HANK model reduces the initial impact of their forward guidance experiment by 60% compared to their RANK model.}
Figure 3.4: Simulated peak responses for different horizons of forward guidance on three consecutive rate cuts of 25bps. The horizontal axis depicts the respective horizon when the cuts occur, the vertical one the impact in % relative to steady state.

Figure 3.5: Simulated response of APP and APP with FG (interest rate set 25bps below the steady state for six and eight quarters). The horizontal axis depicts quarters, the vertical one the impact in % relative to steady state. The blue solid line depicts the response of the APP-baseline scenario, the blue dashed (blue dotted) line the interaction with six (eight) quarters FG. The green solid-cross and solid-circle lines depict the sum of the APP-baseline and an isolated FG impact of a 25bps cut for six and eight quarters, respectively.

We compare these responses with a scenario where we additionally keep the interest rate 25bps annually below steady state for six quarters (blue dashed line) and eight quarters (blue dotted line). In both cases, as expected, the simulta-
neous use of APP and FG raises GDP (not shown) and inflation (bottom left and right panel, respectively) above the baseline scenario. However, it is noteworthy that the impact is higher than the sum of each policy used in isolation (the green solid-cross line for six quarters FG and the solid-circle line for eight quarters, respectively). The reason is twofold. First, monetary policy is more accommodative as it lowers interest rates by more than 25bps (APP per se raises policy rates). Second, this additional stimulus becomes reinforced due to the interest rate peg, as the rise of the inflation rate induces a further reduction in real interest rates, which amplifies the stimulus. This second amplification channel explains why the difference between the interaction and the sum of the isolated policies increases with the horizon of forward guidance (i.e. the difference between ‘APP + 8QFG’ and ‘sum(APP, 8QFG)’ is higher than between ‘APP + 6QFG’ and ‘sum(APP, 6QFG)’).

3.5 Conclusion and discussion

We have introduced hand-to-mouth households into a medium-scale New Keynesian DSGE model with banks to study the quantitative implications of forward guidance. We also study its combination with asset purchases. Such a two-agent New Keynesian model approximates the aggregate effects of heterogeneous agents models in a parsimonious way. We show that for plausible ranges of parameters the power of forward guidance can be dampened compared to our representative agent benchmark model. However, the amount of attenuation depends on the degree of countercyclical transfers and the share of hand-to-mouth households. This is because the two parameters shape the relative strength of the direct and indirect effects of interest rate forward guidance (Bilbiie, 2019b). If there is no or “too little” redistribution, models with hand-to-mouth households amplify the impact of forward guidance on the economy relative to a representative agent benchmark.

A further taming is possible if monetary policy is history dependent. An inertial reaction of the central bank will carry its endogenous feedback of interest rates into the future after the forward guidance period (similar to Bilbiie, 2019a).

29The isolated impact of the APP on inflation is around 0.14%. The isolated impact of a 25bps reduction in the short rate for six / eight quarters is around 0.06% / 0.12%, respectively. However, the combination of APP and forward guidance raises inflation by 0.39% and 0.82%, respectively.
As many central banks indeed emphasize a medium-term goal of their inflation targets, we estimated a version of the above model with a Taylor rule that reacts to a four-quarter average of the past inflation rates (e.g. Justiniano et al., 2013). Our results indeed indicate a further taming of the power of forward guidance. However, the forward guidance puzzle remains unsolved. We leave a thorough analysis of the quantitative implications of different monetary policy rules and strategies for future work.
Appendices

3.A Model appendix

\[ \Lambda_t^o = d_t \frac{1}{C_t^o - hC_{t-1}^o} - E_t d_{t+1} \frac{h \beta}{C_{t+1}^o - hC_t^o} \quad (38) \]

\[ \Lambda_t^o = \beta \Lambda_{t+1}^o \frac{R_t}{\Pi_{t+1}} \quad (39) \]

\[ (w_t^*)^{1+\varepsilon_w} = \frac{\varepsilon_w}{\varepsilon_w - 1} G_t^I \quad (40) \]

\[ G_t^I = \lambda_{w,t} d_t B w^{\varepsilon_w(1+\eta)} H_t^{1+\eta} + \theta_w \beta E_t \left( \frac{\Pi_{t+1}}{\Pi_t^{\varepsilon_w}} \right)^{(1+\eta)\varepsilon_w} G_{t+1}^I \quad (41) \]

\[ G_t^{II} = \Lambda_t w^{\varepsilon_w} H_t + \theta_w \beta \left( \frac{\Pi_{t+1}}{\Pi_t^{\varepsilon_w}} \right)^{\varepsilon_w-1} G_{t+1}^{II} \quad (42) \]

\[ w_t^{1-\varepsilon_w} = (1 - \theta_w) (w_t^*)^{1-\varepsilon_w} + \theta_w \left( \frac{\Pi_{t-1}^{\varepsilon_w}}{\Pi_{t-1}} \right) w_t^{1-\varepsilon_w} \quad (43) \]

\[ \Lambda^o P_t^k M = \beta \Lambda_{t+1}^o \left( R_{t+1}^k + (1 - \delta) P_{t+1}^k M_{t+1} \right) \quad (44) \]

\[ \Lambda_t^o Q_t M_t = \beta \Lambda_{t+1}^o \left( \frac{(1 + \kappa Q_{t+1}) M_{t+1}}{\Pi_{t+1}} \right) \quad (45) \]

\[ V_t^{kh} = d_t \left( \log (C_t^o - hC_{t-1}^o) - d_t^w B H_t^{1+\eta} \right) + \beta V_{t+1}^{kh} \quad (46) \]

\[ R_t^k = mc_t MPK_t \quad (47) \]

\[ w_t = mc_t MPL_t \quad (48) \]
\[
\Pi'_t = \frac{\varepsilon_p}{\varepsilon_p - 1} X'_t \Pi_t
\]

\[
X'_t = Y_t \lambda_{p,t} m c_t + \theta_p \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Pi_{t-\varepsilon_p}^t \Pi_{t+1}^e X'_{t+1}
\]

\[
X''_t = Y_t + \theta_p \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Pi_{t-\varepsilon_p}^t \Pi_{t+1}^e X''_{t+1}
\]

\[
\Pi_{t-\varepsilon_p} = (1 - \theta_p) (\Pi'_t)^{1-\varepsilon_p} + \theta_p \Pi_{t-1}^{1-\varepsilon_p}
\]

\[
d''_t = \Pi''_t \left( (1 - \theta_p) (\Pi'_t)^{-\varepsilon_p} + \theta_p \Pi_{t-1}^{-\varepsilon_p \varepsilon_p} d''_{t-1} \right)
\]

\[
d''_t = \theta_w \Pi_{t-1}^{-\varepsilon_w} \left( \frac{w_t}{w_{t-1}} - \Pi_t \right)^{\varepsilon_w} d''_{t-1} + (1 - \theta_w)^{1-\varepsilon_w} \left( 1 - \theta_w \left( \Pi''_{t-1} \frac{w_{t-1}}{w_t} \right)^{1-\varepsilon_w} \right)
\]

\[
Y_t = C_t + I_t + f(N_t) N_t
\]

\[
d''_t Y_t = A_t \kappa_{t-1}^{\alpha} H''_t^{1-\alpha}
\]

\[
K''_t = (1 - \delta) K''_{t-1} + \mu_t \left( 1 - \frac{\Psi_I}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I''_t
\]

\[
P^k_t \mu_t \left\{ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right\} = 1 - \beta P^k_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \mu_{t+1} \left\{ -S^t \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\}
\]

\[
\bar{B}_t + \bar{F}_t = N_t L_t
\]
\[ L_t = \frac{1}{1 + (\Phi_t - 1) \frac{R_{t+1}^L}{R_t}} \]  

\[ P_t^k I_t^o = \bar{F}_t - \kappa \frac{\bar{F}_{t-1}}{\Pi_t} \frac{Q_t}{Q_{t-1}} \]  

\[ \Lambda_t^o \left( 1 + N_t \left( \Psi_n \frac{N_t - N}{N} \right) + \frac{\psi N}{2} \left( \frac{N_t - N}{N} \right)^2 \right) = \beta \zeta \Lambda_{t+1}^o \frac{1}{\Pi_{t+1}} \left( (R_{t+1}^L - R_t) L_t + R_t \right) \]  

\[ R_t^L = \frac{1 + \kappa Q_t}{Q_{t-1}} \]  

\[ MPL_t = (1 - \alpha) A_t \left( \frac{K_t-1}{H_t} \right)^\alpha \]  

\[ MPK_t = \alpha A_t \left( \frac{K_t-1}{H_t} \right)^{\alpha - 1} \]  

\[ R_t = (R_{t-1})^\rho \left( R \Pi_t^{\tau_y} \left( \frac{Y_t}{Y_{t-1}} \right)^{\tau_y} \right)^{1-\rho} \varepsilon_t^R \]  

\[ I_t = (1 - \lambda) I_t^o \]  

\[ K_t = (1 - \lambda) K_t^o \]  

\[ C_t = (1 - \lambda) C_t^o + \lambda C_t^h \]  

\[ \Lambda_t^h = d_t \frac{1}{C_t^h} \]  

\[ C_t^h = d_t^w w_t H_t - T_t^h - \tau (Y_t - Y) \]
\[
d_{t}^{w} = \theta_w \Pi_{t-1}^{1-\varepsilon_w} \left( \frac{w_t}{w_{t-1}} \Pi_t \right)^{\varepsilon_w} d_{t-1}^{w} + (1 - \theta_w \left( \Pi_t^{1-\varepsilon_w} \frac{w_t}{w_{t-1}} \Pi_t \right)^{1-\varepsilon_w})^{\varepsilon_w}_{w_{t-1}}^{w_t} \\
\Lambda_t = (1 - \lambda) \Lambda_t^t + \lambda \Lambda_t^h \\
R_t^e = (1 - \rho_m) \ln(R^e) + \rho_m R_{t-1}^e + \varepsilon_{R,t} \\
\mu_t = (1 - \rho_{\mu}) \ln(\mu) + \rho_{\mu} \mu_{t-1} + \varepsilon_{\mu,t} \\
A_t = (1 - \rho_a) \ln(A) + \rho_a A_{t-1} + \varepsilon_{A,t} \\
\lambda_{p,t} = (1 - \rho_{\lambda_p}) \ln(\lambda_p) + \rho_{\lambda_p} \lambda_{p,t-1} + \varepsilon_{\lambda_p,t} \\
\Phi_t = (1 - \rho_{\Phi}) \Phi + \rho_{\Phi} \Phi_{t-1} + \varepsilon_{\Phi,t} \\
\lambda_{w,t} = (1 - \rho_{\lambda_w}) \ln(\lambda_w) + \rho_{\lambda_w} (\lambda_{w,t-1}) + \varepsilon_{\lambda_w,t} \\
d_t = (1 - \rho_d) \ln(d) + \rho_d d_{t-1} + \varepsilon_{d,t} \\
\begin{align*}
3.1.1 \text{ Steady states} \\
\Pi^* = \left( \frac{1 - \theta_p \Pi^{-(1-\varepsilon_p)(1-\varepsilon_p)}}{1 - \theta_p} \right)^{1-\varepsilon_p} \Pi \\
P^k = \frac{1}{\mu}
\end{align*}
\]
\[ R = \frac{\Pi}{\beta} \] (83)

\[ d^p = \frac{(1 - \theta_p)^{1 - \tau_p} (1 - \theta_p \Pi \varepsilon_p^{\varepsilon_p - 1}(1 - \varepsilon_p))^{\varepsilon_p/\varepsilon_p - 1}}{(1 - \theta_p \Pi \varepsilon_p^{1 - \varepsilon_p})} \] (84)

\[ d^w = \frac{(1 - \theta_w)^{1 - \tau_w} (1 - \theta_w \Pi \varepsilon_w^{\varepsilon_w - 1}(1 - \varepsilon_w))^{\varepsilon_w/\varepsilon_w - 1}}{(1 - \theta_w \Pi \varepsilon_w^{1 - \varepsilon_w})} \] (85)

\[ mc = \frac{\varepsilon_p - 1}{\varepsilon_p} \left( 1 - \theta_p \beta \Pi \varepsilon_p^{\varepsilon_p - 1}(1 - \varepsilon_p) \right) \varepsilon_p/\varepsilon_p - 1 \] (86)

\[ R^L = (0.01/4) + R \] (87)

\[ Q = \frac{1}{R^L - \kappa} \] (88)

\[ \zeta = \frac{1}{\Pi((R^L - R)L + R)} \] (89)

\[ \Phi = 1 + \frac{1 - L}{(R^L / R)L} \] (90)

\[ M = \frac{\beta}{(\Pi - \beta \kappa)Q} \] (91)

\[ R^k = MP^k \frac{1 - \beta(1 - \delta)}{\beta} \] (92)

\[ \left( \frac{K}{H} \right) = \left( \frac{R^k}{mc \alpha} \right)^{\lambda - 1} \] (93)

\[ K = \left( \frac{K}{H} \right) H \] (94)

\[ K^o = \frac{K}{1 - \lambda} \] (95)
\[ w = mc(1 - \alpha) \left( \frac{K}{H} \right)^{\alpha} \]  
(96)

\[ w^* = w \left( \frac{1 - \theta_w}{1 - \theta_w \Pi(\alpha - 1)(1 - \varepsilon_w)} \right)^{\frac{1}{\varepsilon_w}} \]  
(97)

\[ Y = \frac{1}{d^\alpha} AK^\alpha H^{1-\alpha} \]  
(98)

\[ I^o = \delta K^o \frac{1}{\mu} \]  
(99)

\[ I = (1 - \lambda) I^o \]  
(100)

\[ C = Y - I \]  
(101)

\[ C^o = C \]  
(102)

\[ C^h = C \]  
(103)

\[ \Lambda^o = \frac{1 - h\beta}{(1 - h)C^o} \]  
(104)

\[ \Lambda^h = \frac{1}{C^h} \]  
(105)

\[ \Lambda = (1 - \lambda)\Lambda^o + \lambda\Lambda^h \]  
(106)

\[ X^I = \frac{Y \lambda_p mc}{1 - \beta \theta_p \Pi(\varepsilon_p)(1 - \varepsilon_p)} \]  
(107)

\[ X^{II} = \frac{Y}{1 - \theta_p \beta \Pi(\varepsilon_p - 1)(1 - \varepsilon_p)} \]  
(108)
\[
B = \frac{\varepsilon_w - 1}{\varepsilon_w} \left( \frac{1 - \beta \theta_w \Pi^{1+\varepsilon_w \eta - \varepsilon_w (1+\eta)}}{1 - \beta \theta_w \Pi^{1-\varepsilon_w}} \right) \left( w^* \right)^{(1+\varepsilon_w \eta)} \Lambda \lambda_w w^{\varepsilon_w \eta} H^n \tag{109}
\]

\[
G^I = \frac{\lambda_w^{\varepsilon_w (1+\eta)} H^{1+\eta}}{1 - \beta \theta_w \Pi^{1+\varepsilon_w \eta - \varepsilon_w (1+\eta)}} \tag{110}
\]

\[
G^{II} = \frac{\varepsilon_w}{\varepsilon_w - 1} \left( w^* \right)^{1+\varepsilon_w \eta} \tag{111}
\]

\[
\bar{F} = \frac{I^o}{1 - \frac{\alpha}{H}} \tag{112}
\]

\[
N = \frac{\bar{B} + \bar{F}}{L} \tag{113}
\]

\[
MPL = (1 - \alpha) \left( \frac{K}{H} \right)^{\alpha} \tag{114}
\]

\[
MPK = \alpha \left( \frac{K}{H} \right)^{\alpha - 1} \tag{115}
\]

\[
V^h = \frac{\log(C - hC) - B H^{1+\eta} \frac{H^{1+\eta}}{1+\eta}}{1 - \beta} \tag{116}
\]

3.B Numerical implementation of anticipated shocks

To simulate forward guidance paths we follow the approach by Laséen and Svensson (2011), which was implemented for instance by Krause and Moyen (2016). Start with a model without news shocks that (in its linear form and no constant) can be written as follows:

\[
A x_t = B x_{t-1} + DE_t x_{t+1} + F \varepsilon_t \tag{117}
\]
with the reduced form solution

\[ x_t = Q x_{t-1} + G \varepsilon_t \]  
(118)

To conduct forward guidance in such a model, the monetary policy rule (which, for simplicity, is given by \( r_t = \phi_{\pi} \pi_t \)) will be augmented by past announcements of changes in the interest rate, that all realize in \( t \) (Harrison, 2015):

\[ r_t = \phi_{\pi} \pi_t + \bar{\varepsilon}_t^r \]  
(119)

with

\[ \bar{\varepsilon}_t^r = \sum_{j=0}^{J-1} \nu_{j,t-j}^r. \]  
(120)

The general notation is that the value after the comma denotes the time of announcement \( \cdot, t-j \) and the value before the comma the time until the announcement realizes \( \cdot, i \), i.e. in \( t-j+j=t \). Thus, the disturbance \( \nu_{j,t-j}^r \) represents an announcement in \( t-j \) that affects the policy rate in \( j \) periods. Put differently, the shock is known by the agents in \( t-j \), but the change in the interest rate takes place in period \( (t-j)+j=t \).\(^{30}\) The term \( \nu_{0,t}^r \) is similar to a monetary policy shock.

Past announcements of future interest rate adjustments become part of the state space and are denoted by \( \nu^{i-1} \):

\[ \nu^i = \{ \nu_{i,t-j}^r \}_{i,j \in \{1, \ldots, J-1\}, \ i \geq j} \]  
(121)

This vector includes all announcements from the past, i.e. in \( t-1 \) or earlier (captured by \( \cdot, t-j \)) that affect the policy rate in \( t \) or later (captured by \( \cdot, i \), the time until it realizes).

The model with news shocks is then an extended version of eq (117), to include a block of policy rule shocks \( \nu_t \) and states \( \nu^i \):

\[ [A \ A_{\nu}][x_t \ \nu^i]' = [B \ B_{\nu}][x_{t-1} \ \nu^{i-1}]' + [D \ D_{\nu}]E_t[x_{t+1} \ \nu^{i+1}]' + [F \ F_{\nu}][\varepsilon_t \ \nu_t]' \]  
(122)

with

\[ \nu_t = \{ \nu_{0,t}^r, \nu_{1,t}^r, \ldots, \nu_{J-1,t}^r \} \]  
(123)

\(^{30}\)However, as the model is forward-looking, such an announcement has an impact on the economy already from \( t-j \) onwards.
In short:
\[
\tilde{A}\tilde{x}_t = \tilde{B}\tilde{x}_{t-1} + \tilde{D}\tilde{E}_t\tilde{x}_{t+1} + \tilde{F}\tilde{e}_t
\]  
(124)

with the solution given by:
\[
\tilde{x}_t = \tilde{Q}\tilde{x}_{t-1} + \begin{bmatrix} G & G \end{bmatrix} [\varepsilon_t \; \nu_t]'
\]  
(125)

We assume now that the central bank can actually choose the news shocks \(\nu_t\) directly and denote such a vector with:
\[
\bar{\nu}_T = \begin{bmatrix} \bar{\nu}_{0,T} & \bar{\nu}_{1,T} & \ldots & \bar{\nu}_{J-1,T} \end{bmatrix}'.
\]  
(126)

These are \(J\) announcements for the policy rate that are announced in the beginning of period \(T\). Importantly, households take these announcements into account, i.e. – in contrast to exogenous shocks – \(E_{T-1}\tilde{\nu}_T = \tilde{\nu}_T\). Therefore, the \(h\)-period ahead model-based forecast \(\tilde{x}_{T+h,T-1}\) is now given by:
\[
\tilde{x}_{T+h,T-1} = \tilde{Q}^h \left( \tilde{Q}\tilde{x}_{T-1} + G\nu\bar{\nu}_T \right) = \tilde{Q}^{h+1}\tilde{x}_{T-1} + \tilde{Q}^h G\nu\bar{\nu}_T, \quad h = 0, \ldots, J - 1. \quad \text{\footnote{Since the central bank announces its path for \(J\) periods, we focus on the forecast horizon until \(T + J - 1\). Harrison (2015) also focuses on situations in which \(H \neq J\), where \(H\) is the forecasting horizon.}}
\]  
(127)

Since \(r_t\) is part of \(\tilde{x}_t\), it holds for those periods
\[
r_{T+h,T-1} = \tilde{Q}^{h+1}\tilde{x}_{T-1} + \tilde{Q}^h G_{\nu,r}\bar{\nu}_T,
\]  
(128)

with \(\tilde{Q}^h\) denoting the respective row of matrix \(\tilde{Q}\) and \(G_{\nu,r}\) of \(G\). This implies that for a given vector \(\bar{\nu}_T\), equation (128) determines the (anticipated) path of the policy rate. Put differently, if the policymaker wants the interest rate to follow a pre-specified path, like
\[
r_{T+h,T-1} = \bar{r}_{T+h}, \quad h = 0, \ldots, J - 1.
\]  
(129)

he has to choose the announcements \(\bar{\nu}_T\) such that (129) is satisfied.

Stacking all \(J\) equations from (128) into a single matrix leads to:
\[
\begin{bmatrix}
\tilde{Q}_r \\
\tilde{Q}_r^2 \\
\vdots \\
\tilde{Q}_r^J
\end{bmatrix}
\cdot \bar{x}_{T-1} +
\begin{bmatrix}
G_{\nu,r} \\
\tilde{Q}_r G_{\nu,r} \\
\vdots \\
\tilde{Q}_r^{J-1} G_{\nu,r}
\end{bmatrix}
\cdot \tilde{\nu}_T =
\begin{bmatrix}
\tilde{r}_T \\
\tilde{r}_{T+1} \\
\vdots \\
\tilde{r}_{T+J-1}
\end{bmatrix}
\]

Since this is a linear model (with unique solution), one can invert matrix \( B^* \) to solve for the policy vector \( \tilde{\nu}_T \):

\[
\tilde{\nu}_T = (B^*)^{-1} (\tilde{r}^T - A^*)
\]  

(130)

3.C Additional Figures

Figure 3.6: Prior vs. posterior distribution
Notes: Prior (grey) vs. posterior (black) distribution. Green dashed line depicts the mode of the posterior distribution.
Figure 3.7: Univariate convergence statistics.
\textbf{SE\_epsilon\_D (Interval)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 0.02 & 0.04 \\
\hline
5 & & & & \\
10 & & & & \\
0 & & & & \\
0.1 & & & & \\
0.2 & & & & \\
0.3 & & & & \\
0.4 & & & & \\
\end{tabular}

\textbf{SE\_epsilon\_D (m2)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 0.5 & 1 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
0.1 & & & & & \\
0.2 & & & & & \\
0.3 & & & & & \\
0.4 & & & & & \\
\end{tabular}

\textbf{SE\_epsilon\_D (m3)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 2 & 4 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
2 & & & & & \\
4 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{SE\_growth\_dat\_eps (Int)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 2 & 4 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
2 & & & & & \\
4 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{SE\_growth\_dat\_eps (m2)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 2 & 4 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
2 & & & & & \\
4 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{SE\_growth\_dat\_eps (m3)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 2 & 4 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
2 & & & & & \\
4 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{h (Interval)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 0.2 & 0.4 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
0.1 & & & & & \\
0.2 & & & & & \\
0.3 & & & & & \\
0.4 & & & & & \\
\end{tabular}

\textbf{h (m2)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 10^{-3} & 10^{-4} \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{h (m3)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 10^{-3} & 10^{-4} \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{eta (Interval)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 0.2 & 0.4 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
0.1 & & & & & \\
0.2 & & & & & \\
0.3 & & & & & \\
0.4 & & & & & \\
\end{tabular}

\textbf{eta (m2)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 10^{-3} & 10^{-4} \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{eta (m3)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 10^{-3} & 10^{-4} \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{jota\_p (Interval)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 0.01 & 0.02 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
0.5 & & & & & \\
1 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{jota\_p (m2)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 0.01 & 0.02 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
0.5 & & & & & \\
1 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{jota\_p (m3)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 0.01 & 0.02 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
0.5 & & & & & \\
1 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{jota\_w (Interval)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 0.005 & 0.01 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
0.005 & & & & & \\
0.01 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{jota\_w (m2)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 0.005 & 0.01 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
0.005 & & & & & \\
0.01 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}

\textbf{jota\_w (m3)}
\begin{tabular}{c c c c}
& 5 & 10 & 0 & 0.005 & 0.01 \\
\hline
5 & & & & & \\
10 & & & & & \\
0 & & & & & \\
0.005 & & & & & \\
0.01 & & & & & \\
10^{-3} & & & & & \\
10^{-4} & & & & & \\
\end{tabular}
Figure 3.8: Multivariate convergence statistics.
Bibliography


