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Introduction

This thesis consists of three self-contained chapters in applied microeconomic theory. A common feature of all three projects is communication about private information in different strategic environments. In Chapter 1, I analyze the problem of allocating a resource to a privately informed agent in a stochastically changing world from a contract design perspective. Chapter 2, which was written together with Paul Voß, deals with the optimal allocation of control rights on the market for corporate control. Finally, Chapter 3 analyzes how a decision-maker can optimally assign costly tasks to agents based on non-verifiable, simultaneous communication.

Commitment and the ways in which players can communicate are very different across the three chapters. In Chapter 1, the principal has commitment power in terms of her ability to specify transfers and allocations. The information provided by the agent is costly. On the other hand, in Chapter 3, no player has the power to commit to a predefined allocation or communication rule and information provision is costless and non-binding. Finally, Chapter 2 combines costly signaling via price offers by one player with responding cheap talk communication by another player in one sequential model without commitment.

To be more precise, in Chapter 1, I study dynamic contracting between a principal and an agent. The principal owns a resource of random quality and of which she has no use, but the agent can generate surplus by using it. The agent is privately informed about his ability to generate future returns and about realized returns. I investigate the question when, that is in which periods, the agent is optimally allowed to use the resource. The evolution of returns features random changes of state unobservable to both players: once the resource is in the low state, expected returns are persistently lower – a stopping problem arises endogenously. I show that the optimal contract with symmetric information is myopic (or "one-stage-look-ahead"), whereas with private information, there can exist a positive option value and thus a value from learning about the state of the resource and future revenue by allocating to the agent. My results offer a novel interpretation of contract durations: they can serve as screening device.

Chapter 2 deals with the effect of strategic information transmission on takeovers. An external bidder and incumbent management both possess private information about the firm value under their respective management. The external bidder posts a tender offer to which management responds with a cheap talk message to shareholders. We show that strategic communication can improve the allocation of control rights. In particular, the first-best allocation is attainable because management's recommendation incentivizes the bidder to fully reveal his private information via the tender offer. As management's and shareholders' interests are misaligned, shareholders prefer access to more information than management is willing to reveal. If met, this demand for information leads to inefficiently few takeovers. Excluding shareholders from obtaining additional information can thus be welfare-increasing – similar to excluding shareholders from post-takeover profits in Grossman and Hart (1980). Our paper provides a new rationale for equity compensation of managers and shows that golden parachutes can, contrary to conventional wisdom, enhance efficiency. We derive several implications for the regulation of fairness opinions and disclosure requirements during takeovers.

Finally, in Chapter 3, I study the allocation of costly tasks to two agents based on simultaneous cheap talk. Such decisions must be taken frequently and in virtually every organization: at school, one or more members of the teaching staff usually create the school timetable, a supervisor must decide who does the unpopular night shift, and in the academic environment, countless committees need representatives. Common to all these examples is that performing the particular task is costly in that it distracts from performing a more pleasant task, but the outcome may affect others. Therefore, there is an incentive to work if one expects others not to be very productive. Moreover, these tasks can be done alone or in a team. I focus on the possibility for the decision-maker to instruct the agents to work in a team and the impact on communication.

In the model, agents possess private information about the return of working on a specific task. Since working can lead to a positive externality for the other agent, there is an incentive to free-ride. The decision-maker can instruct the agents to work alone or in a team. When working together, two opposing effects can occur: teamwork can decrease or increase the own costs of working, depending on whether synergies are larger or smaller than the coordination effort. Hence, agents can be complements or substitutes. In the complements case, information transmission is limited by the teamwork technology. Specifically, not more than three distinct messages can be transmitted. In the substitutes case, agents compete with their messages not to work alone and this facilitates information transmission.

Chapter 1

When the Hype is Over: Sequential Screening with an Unobservable State

1. Introduction

Consider a principal who owns a resource of which she has no use and an agent who can rent the resource to repeatedly generate returns.[1] The quality (or state) of the resource changes randomly. Crucially, after the first time the resource is in the low state (I call this event *deterioration*) expected returns are persistently lower than before. But neither the principal nor the agent knows the state of the resource at any time. This setting models business environments that are prone to downward, unforeseeable and persistent shocks. Indeed, overhyped clothing brands and trend foods often enter the market with vast sales figures but then, once the hype is gone, revenues drop down and stay on a low level. A relevant example is franchising where a franchisor gives a franchisee the right to sell a certain product or brand (the resource) and the agent pays a franchise fee or royalty.[2] The majority of franchised chains is relatively small and not well-established[3] and such environments often entail hypes, followed by persistently low returns.

This paper investigates, from a contract-theoretic point of view, when the agent is optimally allowed to use such a resource if he is privately informed about his ability to generate returns and about the returns. The agent can be one of two types: a good (productive) or a bad (unproductive) one. Types capture the probability that the agent generates high or low average returns and model, for example, retailing and marketing skills. In a future period, the resource is more likely to be in the high state if the agent is of the good type. Independent of the agent's type, however, the probability of returning to the high state is strictly lower after

[1]Throughout the paper, I refer to the principal with the female and to the agent with the male pronoun.

[2]Franchised chains exist in a vast number of industries, such as fast food, convenience stores, car rental, hotels, automotive repair and even child education. See *https://www.franchisedirect.com/top100globalfranchises/rankings?page=1*, date 9/7/2018.

[3]Since 1986, the median size of franchised companies in the US has been below 50 units (Blair and Lafontaine 2005).

deterioration, that is when the hype is over.

The main contribution of this paper is showing that screening through the endogenous duration of a contract can be optimal – a feature not yet identified in the literature and to my knowledge, this paper is the first one with an unobservable state of a technology in a dynamic model with private information. In the main result (Theorem 1), I derive the explicit solution for the optimal contract with two payoff-relevant periods[4] and show how to design the revenue-maximizing contract in such an environment. Further, I characterize the environment such that the design problem of the principal is tractable (see Remark 1.2). Interestingly, I do not model a stopping problem, but it arises endogenously through the trade-off between not allocating to the agent in order to prevent a loss and granting the use of the resource, thereby enjoying potential future returns. Because of this, allocations can be interpreted as contract duration. Optimally, the principal offers a menu of contracts differing in durations to screen dynamically across types. The optimal contract features no distortion at the top, whereas the bad type is distorted downwards in that he is fired inefficiently early compared to the first-best contract. Even if the bad type was able to generate positive expected returns in period 1, it may be that he does not receive a contract offer at all. The driver of this effect is that the good type is allowed to use the resource in a given period if the respective expected return he generates exceeds the principal's outside option of zero. On the other hand, the return thresholds (or *distortion functions*) the bad type has to exceed are strictly positive under a regularity condition (strict monotone likelihood ratio property – henceforth *MLRP*). Because of optimal distortions, the bad type must generate higher expected returns than the good type to receive a contract (extension) which leads to an earlier termination of the contract on average.

Intuitively, one may think that – since the average quality of the resource can only become worse – the principal optimally fires the agent immediately if returns hit a lower bound because this would be a sign of deterioration. On the other hand, it is only possible to learn about the state of the resource and future revenues by allocating it to the agent, which gives an incentive to maintain the relationship. I show that the first-best contract with symmetric information is myopic (or "one-stage-look-ahead") and has no value of learning: the agent is fired irrevocably once the next period expected return conditional on previous information is smaller than her outside option. In contrast, with private information, there is a positive option value: as seen from period zero, the period-1 expected revenue generated by the bad type may be negative, but the expected sum over both periods is positive. Hence, the optimal contract dictates to hire the bad type. Then, after the first realized return, the principal faces a new decision whether to allocate or not in the next period. Learning about the return to make

[4] Analyzing only two payoff-relevant periods is surely restrictive. But if periods are long, much of the applicability is restored. For instance, the duration in franchising contracts usually varies in 5-year increments. 91 percent of franchise contracts feature a renewal period conditional on the franchisee's performance (Blair and Lafontaine 2005).

inferences about the state of the resource and thus about future revenue becomes valuable. This phenomenon is driven by the information rents the good type must be granted to make him report his type truthfully at the contracting stage. These rents are siphoned off from the bad type and determine the return thresholds he has to exceed to receive a contract (extension). As these thresholds are strictly positive under MLRP and since they differ across time, this drives a wedge between her allocation rules with and without private information.

Sequential screening via option contracts is an extensively studied feature in the literature (see, for example, Courty and Li (2000)). The main result of my paper suggests that sequential screening via varying contract durations is a further way to increase revenues. In the franchising example, one can interpret the resource as the popularity of a product or brand; and the agent's type can be seen as his privately known retailing skills or investment. From empirical work, one knows that the duration of franchising contracts is positively correlated with the franchisee's investment into human and physical capital (Brickley et al. 2006). This finding can be explained theoretically by my paper as I show that the good type receives a longer contract on average. In addition, franchised outlets are most vulnerable to closure if they perform below average. Blair and Lafontaine (2005) conclude that franchisors use termination of contracts as incentive scheme to sustain performance standards. My paper offers an alternative explanation: contract duration as screening device. In my model, it is possible that a bad type who is expected to perform well is discharged to enable screening (see Example 1.1).

The assumption that expected returns are persistently lower after the first downward state switch is restrictive. This can, however, be used to model environments that tend to hypes. Examples are selling trend foods, frozen yogurt or bubble tea. Once the hype is gone sales go down persistently and deterioration is a plausible assumption.[5]

Finally, one may ask why even the agent does not know anything about the state of the resource. The concept of product life cycles from the marketing literature helps to rationalize this assumption. It distinguishes between five stages of a product life: development, introduction, growth, maturity and decline and even well-informed product managers do not observe when a stage begins and when it ends (Kotler et al. 1991). A product could be in its maturity phase in which sales numbers and growth slow down and increasing pressure by competitors is palpable; or it could already be in its declining phase in which a competitor's product is more popular or consumers' tastes have changed fundamentally. This is where deterioration sets in. Decreasing sales figures are surely an indicative signal, but whether a hype is gone or

[5] Business start-up advisors often have a good feeling for whether a product is (over-)hyped. See *https://www.manager-magazin.de/unternehmen/handel/einzelhandel-wie-konsum-hypes-zur-ladenfalle-werden-a-908336.html*, date 12/30/2019.

not remains unknown.

The rest of this paper is organized as follows. In the remainder of this section I highlight related work. Section 2 presents the model and Section 3 provides technical results. In Section 4, I solve for the optimal contract. In Section 5, I comment on connections between my model and optimal stopping problems. Finally, in Section 6, I conclude. All proofs are delegated to Appendix A.

Literature

There is a plethora of papers about dynamic screening. All of them differ from the present paper in that they do not feature a state of the world that is unknown to all players. In general terms, these papers analyze dynamic principal-agent setups in which the agent enjoys his private valuation for a quantity once the principal allocates it to him.[6] In my model, the quality of a principal-owned resource affects the agent's valuation derived from this resource and allocations allow the agent to use the resource in the next period, contingent only on *previous* information and performance.

In their seminal paper, Courty and Li (2000) establish a dynamic principal-agent model that has since then been used and extended[7] to study dynamic contracting. A buyer faces a monopolist, learns the distribution of his valuation at the contracting stage and only after contracting does he learn his true valuation. Different from the present paper, they do not consider dynamic allocations.

Pavan et al. (2014) and Eső and Szentes (2017) analyze rich models with agents' private information arriving over an infinite time horizon and with dynamic allocations. My paper is structurally different in that allocations allow the agent to use a resource with unknown state. Thus, beyond the agent's private type there is additional uncertainty. This requires a strong form of stochastic order of the agent's private information to solve for the optimal contract. To be precise, the dynamic mechanism design literature usually requires first-order stochastic dominance (FOSD) on the underlying evolution of private information to solve for the optimal mechanism, whereas here the stronger MLRP must be assumed.

Garrett and Pavan (2012) analyze the retention policy of a firm. The incumbent manager possesses a private productivity which changes over time, exerts effort to generate cash flows and he can be replaced by a new one. Garrett and Pavan (2015) investigate a two-period model in which a manager privately observes his ability to generate cash flows. They focus on optimal compensation when the manager can exert effort and how this affects optimal distortions. If the manager is risk-averse, average distortions (in terms of effort required and

[6] See Pavan (2016) for a detailed review of the literature.

[7] See, for example, Eső and Szentes (2007a) in a multi-agent setting. Eső and Szentes (2007b) add costly and observable information acquisition for the principal; and Krähmer and Strausz (2011) allow for unobservable information acquisition by the agent.

compensation) can increase or decrease over time depending on the level of risk aversion and the persistence of his productivity. The distortion functions in the present paper (in terms of returns required for a future allocation) are positive, and in general not increasing over time.

Boleslavsky and Said (2012) analyze a repeated sales model where the buyer has, at the outset, private information that affects his private information after contracting. The latter is determined by either good or bad shocks. My model differs because the driving force of private information is not observable, whereas in their paper, a structural change of private information is observed by the buyer.

Akan et al. (2015) analyze a revenue management setup with consumers learning their valuations for future consumption at different times. The firm screens consumers with respect to the size of the refund (as in Courty and Li (2000)) and additionally on when consumers can claim the refund. Firm profits can be increased by setting an expiration date for the refund offers made to consumer types who learn early. This discourages consumers who learn later from imitating because consumers who learn late cannot know their valuation when they must choose whether to accept the refund.

Most of the papers concerned with dynamic screening (including the present one) assume that agents report their private information over time so that allocations are implemented with multiple rounds of communication. Contrarily, Kruse and Strack (2015) consider stopping problems in which revenue-maximizing allocations can be implemented with only one round of initial communication for the special case that private information evolves as random walk. In my model, a stopping problem arises endogenously via the potential deterioration of the resource.

2. Model

Environment and Resource

The model has three periods $t \in \{0, 1, 2\}$. A principal can lend a resource to an agent in each of the periods $t = 1, 2$. Only the agent can generate returns by using the resource. In every period, the resource can be in a high or a low state, $s_t \in \{H, L\}$, where $\lambda_0 := \mathbb{P}(s_0 = H) \in (0, 1]$ denotes the probability that the resource is in the high state at $t = 0$. The state of the resource is unknown to everyone and changes over time as described below.

If the agent uses the resource in period t, a random return X_t with realization x_t is generated. The X_t's are distributed according to a state-dependent cumulative distribution function (cdf), denoted by F_s. Both cdfs are defined on a set $\chi := (\underline{x}, \overline{x}) \subseteq \mathbb{R}$ with $-\infty \leq \underline{x} < 0 \leq \overline{x} \leq \infty$ and they admit a strictly positive and differentiable density f_s. I assume $-\infty < \mathbb{E}_L[X_t] < 0 \leq \mathbb{E}_H[X_t] < \infty$, where $\mathbb{E}_s$ denotes the expectation operator with respect to f_s. It will become clear that zero equals the (normalized) payoff from the principal's outside option. I assume a regularity condition for the densities, namely the strict monotone likelihood ratio property, which reflects that higher returns are associated with a higher probability of the resource being in the high state.

Assumption 1.1 (MLRP)
$\frac{f_H(x_t)}{f_L(x_t)}$ *is strictly increasing for all* $x_t \in \chi$, $t = 1, 2$.

Besides MLRP, I assume that the following condition on the derivative of the likelihood ratio is fulfilled.[8]

Assumption 1.2
There exists a $K > 0$ *such that* $\frac{d}{dx_t} \frac{f_H(x_t)}{f_L(x_t)} \leq K$ *for all* $x_t \in \chi$, $t = 1, 2$.

Information
The agent has initial and, over time, will receive additional private information about the viability of the resource. Specifically, he is privately informed in two dimensions: first, at period zero, he observes his type $\theta \in \{G, B\}$. There is a commonly known prior $\mathbb{P}(\theta = G) = \alpha \in (0, 1)$. Two parameters are associated with each type: $(\lambda_\theta, \gamma_\theta)$ with $1 > \lambda_G > \lambda_B > 0$ and $1 > \gamma_G > \gamma_B > 0$. I drop the type subscript unless there is cause for confusion. λ and γ determine the transition probabilities between the states of the resource (see Figure 1.1). The connection between both is explained in the following definition.

Definition 1.1
If $s_0 = H$*, the resource can deteriorate: with probability* $(1 - \lambda) \in (0, 1)$*, the state changes from H to L. From then on, the probability to reach state H again is* $\gamma < \lambda$.

If the resource is in the high state at the beginning, deterioration defines the first time the state switches downwards (otherwise, the resource already deteriorated). After this event, the state can return upwards, but with strictly lower probability $\gamma < \lambda$ and these probabilities are commonly known. The fact that the resource can deteriorate is also common knowledge, but since the state is unknown, it is unobservable if it occurs.

Second, the agent privately observes the realized returns x_t at time $t = 1, 2$.[9] X_1 evolves according to $F(x_1|\theta) := \lambda_\theta F_H + (1 - \lambda_\theta) F_L$ if $s_0 = H$. Otherwise, $F(x_1|\theta) := \gamma_\theta F_H + (1 - \gamma_\theta) F_L$ and the probability of $s_1 = L$ is larger here because $1 - \gamma_\theta > 1 - \lambda_\theta$.[10] $F(x_2|\theta, x_1)$ evolves

[8] This assumption will be needed to solve for the optimal period-1 transfer (see Lemma 1.6).

[9] The assumption that cash flows are private information and non-contractible is widely used in the literature; see for example Lewis and Sappington (1997) and Bester and Krähmer (2008). For instance, a franchisor will have difficulties to measure the exact returns each of her franchisees generate.

[10] Hence, it depends on the resource whether the transition probabilities are λ_θ or γ_θ, but the type affects these probabilities.

accordingly. I write $\mathbb{E}[X_2|\theta, x_1]$ as a shortcut for $\mathbb{E}[X_2|\theta, X_1 = x_1]$. Figure 1.1 illustrates the setting with $s_0 = H$ and deterioration at $t = 1$ (in red).[11]

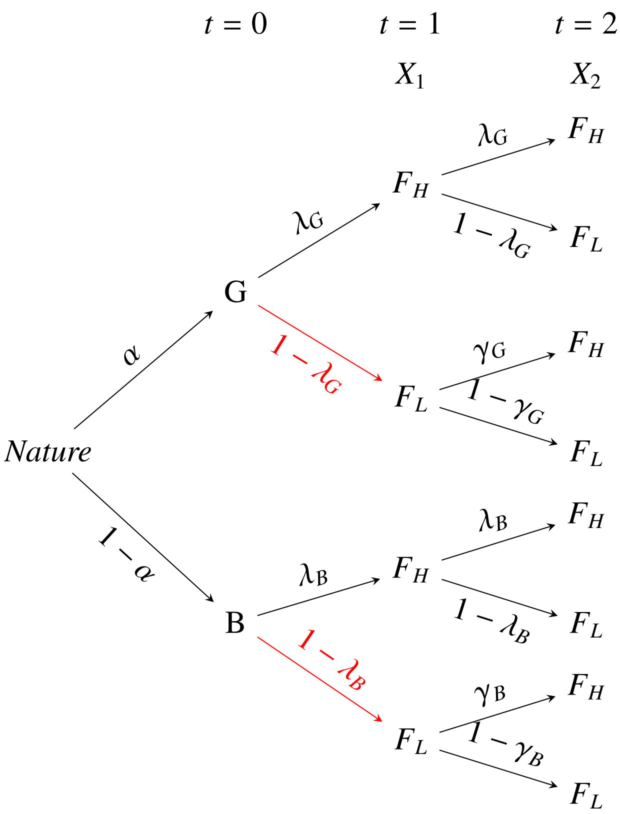

Figure 1.1: Evolution of States and Returns with $s_0 = H$ and Deterioration at $t = 1$.

Contracts and Payoffs

The principal commits to a contract at $t = 0$ and I can invoke the dynamic revelation principle (Myerson 1986). I thus restrict the analysis to direct contracts where the agent sends reports about θ and x_t. Allocations specify the periods in which he uses the resource: q_t denotes the *probability to allocate in period t*. Let $\mathbf{q} = (q_1, q_2)$. The agent pays transfers $\mathbf{p} = (p_1, p_2)$ where p_t is *paid in period t*. If he is not allowed to use the resource in period t, define his report as $\hat{x}_t := \varnothing$. This is a placeholder and reporting $\varnothing$ provides no information. A contract is then a collection of allocations and transfers $C := \langle \mathbf{q}, \mathbf{p} \rangle$.[12] Both parties are risk-neutral and there is no discounting.[13] The agent gets to keep the returns. The principal's revenue is therefore $p_1 + p_2$, and the agent's payoff is $u = q_1 x_1 - p_1 + q_2 x_2 - p_2$. The agent has an outside option of zero at the contracting stage, but he has none after having accepted a contract. The

[11]For the case $s_0 = L$, replace all λ's with γ's as deterioration has already occurred.

[12]If for example $q_1 = q_2 = 1$, the agent is granted a long-term contract. Similarly, if $q_1 = 1$ and $q_2 = 0$, he gets a short-term contract. But in general, allocations are stochastic and need not be monotone in time: $q_t \in [0, 1]$.

[13]This does not change the essence of my results.

principal designs C to maximize expected period-0 revenue.

Timing

- At $t = 0$, $s_0 \in \{H, L\}$ realizes. Then, the agent privately observes his type $\theta \in \{G, B\}$. The principal offers a contract $C = \langle \mathbf{q}, \mathbf{p} \rangle$ and fully commits to it. If the agent accepts, he reports $\hat{\theta}$ and she allocates the resource with probability $q_1(\hat{\theta})$ for use in period 1. Otherwise, both players receive a payoff of zero.
- At $t = 1$, $s_1 \in \{H, L\}$ realizes, the agent observes x_1, then reports $\hat{x}_1$ and pays $p_1(\hat{\theta}, \hat{x}_1)$. She allocates the resource with probability $q_2(\hat{\theta}, \hat{x}_1)$ for use in period 2.
- At $t = 2$, $s_2 \in \{H, L\}$ realizes. The agent privately observes return x_2, reports $\hat{x}_2$ and pays $p_2(\hat{\theta}, \hat{x}_1, \hat{x}_2)$.

3. Technical Results

Before focusing on the contract design problem, I state implications of MLRP and the deterioration of the resource (according to Definition 1.1) that will prove useful in the later analysis. First, it is well-known that MLRP implies FOSD, i.e. $F_L(x_1) > F_H(x_1)$ for all x_1. One can readily show that MLRP also implies FOSD of the period-1 conditional cdfs:

Remark 1.1
MLRP implies $F(x_1|B) > F(x_1|G)$ for all $x_1 \in \chi$.

To see this, observe that $F(x_1|B) - F(x_1|G) > 0$ is equivalent to

$$\lambda_0(\lambda_G - \lambda_B)(F_L(x_1) - F_H(x_1)) + (1 - \lambda_0)(\gamma_G - \gamma_B)(F_L(x_1) - F_H(x_1)) > 0.$$

As the good type generates high returns with a larger probability than B does, it is an immediate implication of Remark 1.1 that expected period-1 returns are strictly larger for the good type:

Lemma 1.1
It holds that $\mathbb{E}[X_1|G] > \mathbb{E}[X_1|B]$.

In particular, Lemma 1.1 implies that whenever $\mathbb{E}[X_1|G]$ is negative, so is $\mathbb{E}[X_1|B]$.

Remark 1.2
Fix a type θ. Then, MLRP is equivalent to $\mathbb{E}[X_2|\theta, x_1]$ being strictly increasing in x_1.

This monotonicity property states that a high period-1 return is a positive signal for the expected period-2 return.[14] Remark 1.2 is not obvious and its proof can be found in Ap-

[14] In particular, *strict* MLRP is needed in the proof of Lemma 1.5 to characterize incentive compatibility.

pendix A. Before I solve for the optimal contract, I derive two useful lemmas. First, a similar statement as in Lemma 1.1 about $\mathbb{E}[X_2|\theta, x_1]$ holds.

Lemma 1.2
For all $x_1 \in \chi$, it holds that $\mathbb{E}[X_2|G, x_1] > \mathbb{E}[X_2|B, x_1]$.

Lemma 1.1 and 1.2 together give an indication why screening across types will be optimal for the principal: the good type is indeed the more efficient one in his ability to generate returns.[15] Finally, a further property arising from the (potential) deterioration of the resource will turn out to be useful.

Lemma 1.3
For all $x_1 \in \chi$ and $\theta \in \{G, B\}$, it holds that $\mathbb{E}[X_2|\theta, x_1] < \mathbb{E}[X_1|\theta]$.

Lemma 1.3 says that expected returns are strictly decreasing over time. The intuition is that with some positive probability, deterioration occurs and the state changes downwards. Hence, one can never become more optimistic about future returns. Furthermore, it follows that $\mathbb{E}[X_1|\theta] < 0$ implies $\mathbb{E}[X_2|\theta, x_1] < 0$ and this monotonicity will provide a tractable way to solve for the optimal contract (see Proposition 1.1 and Theorem 1.1).

4. The Optimal Contract

It will become clear that, endogenously, the principal faces a stopping problem. The driver of this effect is the declining return expectations and the trade-off lies between stopping too early and foregoing positive (expected) revenue and stopping too late and incurring an (actual) loss. In particular, she can learn about the state of the resource via the allocation q_1 and the realization x_1. The following two definitions are concepts well-known from the literature on optimal stopping.

Definition 1.2
The myopic (or one-stage-look-ahead) rule terminates the contract as soon as the expected revenue of the next period conditional on available information is negative.

Definition 1.3
The expected revenue gain from using the optimal allocation instead of the myopic rule is called the option value of the contract.

[15]The monotonic behavior of conditional expected returns across types yields a condition that is usual in screening models (strictly increasing differences): the difference in payoff generated via different contracting periods (or allocations) is strictly larger for G. To be precise, denote by $U(\theta, j)$ the expected sum of returns over j periods for type θ. It follows that $U(G, 2) - U(G, 2-j) > U(B, 2) - U(B, 2-j)$, $j = 1, 2$.

4.1 Benchmark: Symmetric Information

For now assume that the principal observes his type and the returns x_t. The principal can then extract total surplus and she maximizes $\mathbb{E}[q_1(\theta)X_1 + q_2(\theta, X_1)\mathbb{E}[X_2|\theta, x_1]\big|\theta]$ taking the agent's outside option into account.[16] From Lemma 1.3, it directly follows that the solution in this symmetric information case – denoted (q_1^{SI}, q_2^{SI}) – consists of three (and not four) cases:

Proposition 1.1
The optimal contract duration with symmetric information is as follows:

1. *if* $\mathbb{E}[X_2|\theta, x_1] \geq 0$, *then* $q_1^{SI} = q_2^{SI} = 1$,
2. *if* $\mathbb{E}[X_1|\theta] \geq 0$ *and* $\mathbb{E}[X_2|\theta, x_1] < 0$, *then* $q_1^{SI} = 1$ *and* $q_2^{SI} = 0$,
3. *if* $\mathbb{E}[X_1|\theta] < 0$, *then* $q_1^{SI} = q_2^{SI} = 0$.

These are the first-best allocations. Observe that a stopping problem is not modeled, but it arises endogenously which justifies the use of the term *contract duration*. In particular, payoffs are of the general quasi-linear form, so $q_1 = 0$ and $q_2 = 1$ could be feasible. Due to the potential deterioration, however, this is not optimal.[17] Allocations are deterministic, i.e. $q_1^{SI}, q_2^{SI} \in \{0, 1\}$ and the optimal contract follows the myopic rule which can be seen from part three of the proposition. Here, $\mathbb{E}[X_2|\theta, x_1]$ is also negative by Lemma 1.3. Further, in the first case $\mathbb{E}[X_1|\theta]$ is positive. Since she observes the returns, the principal terminates the contract once her updated belief about future revenues is pessimistic enough and smaller than her outside option. Finally, as the myopic rule is optimal, this contract has an option value of zero. I will further discuss this in Section 5.

4.2 The Design Problem

For simplicity, assume from now on that $\chi = [\underline{x}, \overline{x})$ with $\underline{x} > -\infty$. By invoking the revelation principle for dynamic games, I focus on direct contracts where reports $\hat{\theta}, \hat{x}_1, \hat{x}_2$ are sent that induce truth-telling on the equilibrium path, that is after truth-telling in previous periods.[18] The principal maximizes the sum of expected transfers subject to the agent reporting truthfully in all three periods and subject to period-0 participation constraints. Observe that the agent's utility in the first and second period can be written as

$$u_1(\theta, x_1) := x_1 + \mathbb{E}[q_2(\theta, x_1)X_2 - p_2(\theta, x_1, X_2)|\theta, x_1] - p_1(\theta, x_1),$$

[16] It can readily be seen that she is also able to extract the full expected surplus when the agent possesses private information only about his type and returns are observable.

[17] This property carries over to the private information case (see Theorem 1.1).

[18] Truth-telling off the equilibrium path, that is after a lie, does not follow directly from the dynamic revelation principle, but it obtains here (see Section 11 in Borgers et al. (2015) for the formal argument).

and

$$u_2(\theta, x_1, x_2) := x_2 - p_2(\theta, x_1, x_2).$$

Further, let $U(B|G)$ denote the good type's expected utility at period 0 when he lies downwards to be the bad type. $U(G|B)$ captures the converse direction. Write $U(\theta|\theta) := U(\theta)$. Finally, recall that α denotes the probability of the agent being the good type. Then, she chooses C to maximize

$$\begin{aligned} R := \; & \alpha(\mathbb{E}[p_1(G, X_1) + \mathbb{E}[p_2(G, x_1, X_2)|G, x_1]\big|G]) \\ & + (1-\alpha)(\mathbb{E}[p_1(B, X_1) + \mathbb{E}[p_2(B, x_1, X_2)|B, x_1]\big|B]), \end{aligned} \tag{1.1}$$

s.t.

$[IC_2]$ $\quad x_2 - p_2(\theta, x_1, x_2) \geq x_2 - p_2(\theta, x_1, \hat{x}_2) \; \forall\theta, \forall x_1 \in \chi \cup \{\varnothing\}, \forall x_2, \hat{x}_2 \in \chi,$

$[IC_1]$ $\quad x_1 + \mathbb{E}[q_2(\theta, x_1)X_2 - p_2(\theta, x_1, X_2)|\theta, x_1] - p_1(\theta, x_1)$

$\geq x_1 + \mathbb{E}[q_2(\theta, \hat{x}_1)X_2 - p_2(\theta, \hat{x}_1, \hat{X}_2)|\theta, x_1] - p_1(\theta, \hat{x}_1) \quad \forall\theta, \forall x_1, \hat{x}_1, X_2, \hat{X}_2 \in \chi \cup \{\varnothing\},$

$[IC_{0G}]$ $\quad U(G) \geq U(B|G),$

$[IC_{0B}]$ $\quad U(B) \geq U(G|B),$

$[IR_{0G}]$ $\quad U(G) \geq 0,$

$[IR_{0B}]$ $\quad U(B) \geq 0.$

$[IC_{0G}]$ and $[IC_{0B}]$ are period-0 incentive compatibility constraints. $[IC_2]$ and $[IC_1]$ are their period-t counterparts where it is taken into account that he has reported truthfully before. IC ensures that his expected utility cannot be raised via a deviation to any contingent reporting strategy.[19] Finally, $[IR_{0G}]$ and $[IR_{0B}]$ are his individual rationality constraints at the time of contracting.

4.3 Solution

I start solving problem (1.1) by using the period-t IC constraints to characterize transfers and to reduce the number of constraints.

Lemma 1.4
A direct contract fulfills $[IC_2]$ *if and only if* $p_2(\theta, x_1, x_2) = p_2(\theta, x_1, \underline{x}) \; \forall x_1 \in \chi \cup \{\varnothing\}$, $x_2 \in \chi, \; \theta \in \{G, B\}$.

[19] If the agent is granted to use the resource in period t, then he cannot report $\hat{x}_t = \varnothing$ by definition, but he can do so in a later period or he could have done so before.

The idea behind Lemma 1.4 is straightforward: after $t = 2$, the game ends for sure. Thus, $p_2(\theta, x_1, x_2)$ must be the maximal constant that fulfills $[IC_2]$. I continue with period 1:

Lemma 1.5
Suppose $[IC_2]$ is fulfilled. A direct contract fulfills $[IC_1]$ if and only if

1. *For every θ, $q_2(\theta, x_1)$ is increasing in x_1.*

2. *For every θ, u_1 is differentiable almost everywhere, and whenever the derivative exists:*

$$\frac{\partial u_1(\theta, x_1)}{\partial x_1} = 1 + q_2(\theta, x_1)\frac{\partial \mathbb{E}[X_2|\theta, x_1]}{\partial x_1}.$$

Lemma 1.6
Suppose $[IC_2]$ is fulfilled. If a direct contract fulfills $[IC_1]$, then for every θ, x_1:

$$p_1(\theta, x_1) = -u_1(\theta, \underline{x}) + \underline{x} + \mathbb{E}[q_2(\theta, x_1)X_2 - p_2(\theta, x_1, \underline{x})|\theta, x_1] - \int_{\underline{x}}^{x_1} q_2(\theta, z)\frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z}dz.$$

Recall that the timing of events prescribes that an allocation allows the agent to use the resource in the next period, contingent on a previous report. Therefore, the proofs of these two lemmas are similar to Proposition 11.3 in Borgers et al. (2015), but with the major difference that here, one conditions on the (true) return x_1 which makes the analysis more intricate. In particular, MLRP is needed in the proof of Lemma 1.5. This is a relatively strong form of stochastic order and the majority of papers in dynamic mechanism design (only) assumes FOSD. The stronger assumption is needed because uncertainty about the evolution of the agent's private information comes from two sources: the agent's type and the state of the resource.[20]

In the following, I make use of the first-order approach (FOA) to solve for the optimal contract. In a static environment, IC is (under regularity conditions) equivalent to a monotone allocation and an envelope condition (see for example Myerson (1981)). In the dynamic case, however, the allocation rule need not be monotone in the agent's (ex ante) information from the contracting stage. The reason is that the agent's utility is an expectation over future periods and therefore, one can only rule out that the bad type receives a longer contract than the good type on average. To tackle this issue, one usually solves a relaxed problem and optimal contracts are found by maximizing dynamic virtual surplus over all allocations disregarding some constraints.[21] Then, primitive conditions are identified that guarantee that

[20] In particular, I show that $\mathbb{E}[X_2|\theta, x_1]$ is not increasing in x_1 under FOSD (see (A.7) in the appendix).

[21] Papers that use the FOA in dynamic environments to solve for the optimal mechanism are, among others, Courty and Li (2000), Battaglini (2005), Garrett and Pavan (2012) and Pavan et al. (2014). Notice that the FOA generically fails to hold true and it is thus important to check for its validity (Battaglini and Lamba 2019). See Garrett and Pavan (2015) and Garrett et al. (2018) for an alternative variational approach.

the solution to the relaxed also solves the original problem. Such conditions require that the stochastic process of private information is sufficiently monotone. In the present setup, one can ignore B's period-0 IC constraint and G's individual rationality constraint[22] and the primitive condition for the viability of the FOA is MLRP (see Remark 1.2). Indeed, in the optimal contract q_2 is increasing in x_1 by Lemma 1.5. But it is generally not increasing in θ. The following application of the FOA circumvents this issue.

Proposition 1.2

Suppose $[IC_1]$ *and* $[IC_2]$ *are fulfilled. Then, in the optimal contract,*

1. $[IC_{0G}]$ *and* $[IR_{0B}]$ *bind.*
2. *If a contract satisfies*

$$q_1(G) \geq q_1(B) \text{ and}$$
$$q_2(G, x_1) \geq q_2(B, x_1) \; \forall x_1 \in \chi \cup \{\varnothing\},$$

then $[IC_{0B}]$ *and* $[IR_{0G}]$ *are satisfied.*

As $[IR_{0B}]$ binds, the bad type receives zero rents in the optimal contract. Moreover, it follows that ignoring $[IC_{0B}]$ and $[IR_{0G}]$ is valid if the resulting allocations are increasing in his type and then a solution to the original problem stated in (1.1) is found. Before formulating the main result, observe that Lemma 1.6 allows me to rewrite the agent's expected utility as

$$U(\theta) = \mathbb{E}[q_1(\theta)u_1(\theta, \underline{x}) + q_1(\theta)(X_1 - \underline{x}) + \int_{\underline{x}}^{x_1} q_2(\theta, z)\frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z}dz \big| \theta].$$

Substituting yields that her expected revenue can be written as

$$\begin{aligned} R =& \alpha\mathbb{E}[q_1(G)X_1 + q_2(G, X_1)\mathbb{E}[X_2|G, x_1]\big|G] - \alpha U(G) \\ &+(1-\alpha)\mathbb{E}[q_1(B)X_1 + q_2(B, X_1)\mathbb{E}[X_2|B, x_1]\big|B] - (1-\alpha)U(B). \end{aligned} \tag{1.2}$$

Hence, revenue equals the sum of expected returns (or surplus) net of the expected rent to the agent. Using equation (1.2) and substituting $U(G)$ from the binding IC constraint (the derivation and the formula itself can be found in equation (A.9) in the appendix), as well as $U(B) = 0$, one receives the following relaxed maximization problem.

[22] This is similar to static contract theory, see for example, Laffont and Martimort (2009).

Proposition 1.3
The principal solves

$$\max_{q_1, q_2} R$$

subject to $[IC_{0G}]$, $[IR_{0B}]$, q_2 *being increasing in* x_1 *and* $q_1, q_2 \in [0, 1]$.
R can be written as

$$\begin{aligned} &\alpha\Big(q_1(G)\mathbb{E}[X_1|G] + \mathbb{E}[q_2(G, X_1)\mathbb{E}[X_2|G, x_1]\Big|G]\Big) \\ &\quad + (1-\alpha)\Big(q_1(B)\mathbb{E}[X_1 - D_1|B] + \mathbb{E}\Big[q_2(B, X_1)\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1]\Big|B\Big]\Big), \end{aligned} \tag{1.3}$$

where

$$D_1 = \frac{\alpha}{1-\alpha}(\mathbb{E}[X_1|G] - \mathbb{E}[X_1|B]) > 0, \tag{1.4}$$

$$D_2^1(x_1) = \frac{\alpha}{1-\alpha}\frac{f(x_1|G)}{f(x_1|B)}\Big(\mathbb{E}[X_2|G, x_1] - \mathbb{E}[X_2|B, x_1] + \frac{\partial \mathbb{E}[X_2|B, x_1]}{\partial x_1}\frac{F(x_1|B) - F(x_1|G)}{f(x_1|G)}\Big) > 0. \tag{1.5}$$

I call D_1 and $D_2^1(x_1)$ *distortion functions* and their derivation can be found in the proof of Lemma 1.7 in the appendix. More precisely, $D_2^1(x_1)$ is the period-2 distortion function as seen from period 1 after the realization of x_1. In particular, the expected revenue in (1.3) takes the well-known form of (dynamic) virtual surplus and the relaxed problem can now be solved point-wise. Before doing so, notice that R consists of the expected surplus per type net of the distortions for B (G's distortions are equal to zero in both periods). These distortion functions consist of the inverse hazard rate $\frac{\alpha}{1-\alpha}$ – which accounts for the trade-off between rent extraction and efficiency – times an impulse response term that accounts for a change of the current private information and its impact on the evolution of later private information. This measures the effect of distorting a period-t allocation on the agent's expected rent. Notice that both distortion functions are positive because

$$\frac{\partial \mathbb{E}[X_2|B, x_1]}{\partial x_1} = \frac{\lambda_B(1-\lambda_B)\frac{d}{dx_1}\frac{f_H(x_1)}{f_L(x_1)}}{\left(\lambda_B\frac{f_H(x_1)}{f_L(x_1)} + (1-\lambda_B)\right)^2}(\lambda_B - \gamma_B)(\mathbb{E}_H(X_2) - \mathbb{E}_L(X_2)). \tag{1.6}$$

The derivation of the last expression can be found in (A.7). Recall that $\mathbb{E}_H(X_2) > \mathbb{E}_L(X_2)$ and $\lambda_B > \gamma_B$. Hence, (1.6) is positive if and only if MLRP (or $\frac{d}{dx_1}\frac{f_H(x_1)}{f_L(x_1)} > 0$) holds. $\lambda_B - \gamma_B$ is the intensity of deterioration for B. Therefore, the greater the impact of deterioration on his productivity is, the more is he distorted (and in turn, the good type receives a higher expected rent).

The problem of the principal is intertemporal and she must base her decision given the information at hand. As distortion functions are different across time there are some cases to distinguish. But since they are positive and due to Lemmas 1.1 - 1.3, the following main result, a complete solution to the principal's problem, obtains.

Theorem 1.1
The optimal contract exhibits the following features:

- *allocations have no time gap, that is whenever she allocates to him in period 2, then she did so in period 1.*
- *The good type receives the first-best allocation and the bad type is distorted downwards with respect to time as described below.*
- *It is generally not myopic.*
- *The optimal contract durations are as follows:*

1. *If* $\mathbb{E}[X_2|G, x_1] \geq 0$*, then* $q_1(G) = q_2(G, x_1) = 1$*; and*
   (a) *if* $\mathbb{E}[X_1 - D_1|B] \geq 0$ *and*
      i. $\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1] \geq 0$*, then* $q_1(B) = q_2(B, x_1) = 1$*,*
      ii. $\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1] < 0$*, then* $q_1(B) = 1, q_2(B, x_1) = 0$*,*
   (b) *if* $\mathbb{E}[X_1 - D_1|B] < 0$*, but* $\mathbb{E}[X_1 - D_1|B] + \mathbb{E}[\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1]\big|B] \geq 0$ *and*
      i. $\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1] \geq 0$*, then* $q_1(B) = 1, q_2(B, x_1) = 1$*,*
      ii. $\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1] < 0$*, then* $q_1(B) = 1, q_2(B, x_1) = 0$*,*
   (c) *if* $\mathbb{E}[X_1 - D_1|B] < 0$ *and* $\mathbb{E}[X_1 - D_1|B] + \mathbb{E}[\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1]\big|B] < 0$*, then* $q_1(B) = 0, q_2(B, x_1) = 0$.
2. *If* $\mathbb{E}[X_1|G] \geq 0$ *and* $\mathbb{E}[X_2|G, x_1] < 0$*, then* $q_1(G) = 1, q_2(G, x_1) = 0$*; and*
   (a) *if* $\mathbb{E}[X_1 - D_1|B] \geq 0$*, then* $q_1(B) = 1, q_2(B, x_1) = 0$*,*
   (b) *if* $\mathbb{E}[X_1 - D_1|B] < 0$*, then* $q_1(B) = q_2(B, x_1) = 0$.
3. *If* $\mathbb{E}[X_1|G] < 0$*, then* $q_1(G) = q_1(B) = q_2(G, x_1) = q_2(B, x_1) = 0$.

To be more succinct, the following table reduces the three main cases to their contract durations for both types.

Case	G	B
1.(a)	2	*2/1*
1.(b)	2	*2/1*
1.(c)	2	*0*
2.(a)	*1*	*1*
2.(b)	*1*	*0*
3.	*0*	*0*

Table 1.1: Overview of Optimal Contract Durations.

The case *1.(b)* features a positive option value which was not prevailing with symmetric information. In the next section, I will investigate this in detail. Observe that cases one and three are mutually exclusive since Lemma 1.3 implies that $\mathbb{E}[X_1|G] < 0$ also yields $\mathbb{E}[X_2|G, x_1] < 0$. Similarly, in case two, $\mathbb{E}[X_2|G, x_1] < 0$ implies $\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1] < 0$ because distortions are positive. The solution is deterministic as in the symmetric information benchmark. The major difference to the benchmark is that the threshold in terms of expected returns the bad type has to exceed to start the relationship ($q_1 = 1$) or to extend it ($q_2 = 1$) is strictly larger than the outside option (and depends on x_1 in the case of D_2^1). Importantly, the solution is monotone in the agent's type: every time B receives an allocation the good type does so, as well. Therefore, this solution also solves the original problem stated in (1.1).

The general structure of the optimal contract is well-known from contract theory: it features no distortion at the top, that is the good type can use the resource efficiently. On the other hand, there are return realizations that would allow the bad type to use the resource in the first-best contract, but not here in the second-best one. Theorem 1.1 also shows why it is not optimal to sell the resource to the agent at period zero – although both players are risk-neutral: learning about x_1 is valuable for the principal and she optimally stops or prolongs the contract based on her updated belief about the next period. I will analyze this phenomenon in the next section.

5. Discussion

5.1 Private Information and Option Value – An Optimal Stopping Approach

Following Proposition 1.1, the optimal contract with symmetric information is myopic. To link this fact to a more general observation from optimal stopping in decision theory, define the events $S_0 := \{\mathbb{E}[X_1|\theta] < 0\}$ and $S_1 := \{\mathbb{E}[X_2|\theta, x_1] < 0\}$. Following Lemma 1.3, it holds that $S_0 \subset S_1$ is true for all x_1 and θ. In many optimal stopping problems[23], the myopic rule is optimal if such a set monotonicity holds and expected payoffs do not fluctuate over time given the information at hand. The fact that the myopic rule is optimal in my model without private information can be explained by this observation. Put differently, if the myopic rule dictates not to allocate in period 1, so does it in period 2, irrespective of the realization of X_1.

With private information, the above monotonicity still holds, but it becomes irrelevant as allocation rule for the bad type. The reason is that B is distorted downwards in order to award information rents to the good type for making him report his type truthfully. In this context, case *1.(b)* of Theorem 1.1 is worth analyzing in detail because with private information, the optimal contract is no longer myopic. It may be that the period-1 expected revenue (or virtual surplus) by B is negative and the myopic rule would dictate not to allocate at all. The expected revenue over both periods as seen from period zero, however, can be positive ($\mathbb{E}[X_1 - D_1|B] + \mathbb{E}[\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1]\big|B] \geq 0$) and the optimal contract prescribes to hire the agent. At $t = 1$, after the realization x_1, the principal faces a new decision based on the period-2 expected revenue $\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1]$. Driven by the distortion function $D_2^1(x_1)$, revenue as seen from the contracting stage and updated revenue in period 1 are structurally different depending on the realization x_1. This is not the case with symmetric information where updating was trivial because the threshold that determines optimal allocations is equal to zero in both periods. Consequently, an option value arises for two reasons: first, the return thresholds B must exceed to receive an allocation are different across periods. Second, D_2^1 is a function of X_1 and with knowledge of x_1, this distortion is different from the ex-ante view. Besides the possibility to screen across types, allocations therefore have a positive value of learning. In the next subsection, I will be more precise about *when* this occurs. Before, the following example shows the existence of case *1.(b)* in Theorem 1.1.

Example 1.1

Suppose that $\chi = [-1, 1)$ *and* $f_H(x) = \frac{1}{2}(x+1)$, $f_L(x) = \frac{1}{\sqrt{8}}(x+1)^{-0.5}$. *Then,* $F_H(x) = \frac{1}{4}(x+1)^2$, $F_L(x) = \frac{1}{\sqrt{2}}(x+1)^{0.5}$, $\mathbb{E}_H[X] = \frac{1}{3}$, $\mathbb{E}_L[X] = -\frac{1}{3}$, $\frac{f_H(x)}{f_L(x)} = \sqrt{2}(x+1)^{1.5}$ *which fulfills MLRP. For simplicity, set* $\lambda_0 = 1$. *Moreover, assume that* $\alpha = 0.5, \gamma_G = 0.7, \gamma_B = 0.68, \lambda_G = 0.9$ *and* $\lambda_B = 0.7$. *These parameters result in the following:*
$\mathbb{E}[X_1 - D_1|B] < 0$, *but* $\mathbb{E}[X_1 - D_1|B] + \mathbb{E}[\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1]\big|B] = 0.0053$. *Moreover,* $\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1]$ *is positive for all* $x_1 \leq -0.127$ *and negative otherwise, so for large realizations* x_1, *the optimal contract distorts intensely in period 2.*

[23] See, for example, Ferguson and Hardwick (1989) for an application to optimal proofreading.

5.2 Learning and Screening

The principal learns about the agent's type at period zero but nothing about the resource yet. She can, however, allocate in period 1 to gain information about X_1 and infer about the state of the resource to assess future revenue. By doing so, she can adjust her decision whether to allocate in period 2 and optimally distort the bad type. I now investigate when exactly the option value is strictly positive. First, by case one in Theorem 1.1, the good type must receive a two-period contract. Further observe that D_1 and D_2^1 consist of the difference in period surplus across types, $\mathbb{E}[X_1|G] - \mathbb{E}[X_1|B]$ and $\mathbb{E}[X_2|G, x_1] - \mathbb{E}[X_2|B, x_1]$. One can show that D_1 is increasing in $(\lambda_G - \lambda_B)$.[24] For $\mathbb{E}[X_1 - D_1|B] + \mathbb{E}\big[\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1]\big|B\big] \geq 0$ to hold, both distortion functions must not be too large. In particular, the productivity advantage of G, $(\lambda_G - \lambda_B)$, must be small. Second, D_2^1 is increasing in $(\lambda_B - \gamma_B)$ (see (1.6)) and small if the bad type's ability to generate positive returns after deterioration, γ_B, is large.

The bad type is optimally distorted to screen types. This becomes harder when types are similar. Here, as the good type is very productive and $(\lambda_G - \lambda_B)$ is small, the principal knows that the bad type is also able to generate positive returns. She can therefore benefit from allocating and learn how to distort optimally via x_1. In Example 1.1, she stops the contract inefficiently early via a large distortion $D_2^1(x_1)$ after observing a large x_1. In contrast, after a low realization, she distorts less severe resulting in a positive expected revenue in period 2, and she prolongs the contract to realize the option value amounting to $\mathbb{E}[X_2 - D_2^1(x_1)|B, x_1]$. In this case, B receives a two-period contract, whereas the myopic rule would have caused no allocation at all.

5.3 Unobservability of the State

Now, I briefly comment on the assumption that the state of the resource is unobservable. If the principal and the agent observe the state the problem becomes trivial: once it is in the low state, the expected belief for the next period becomes $\gamma\mathbb{E}_H[X_t] + (1 - \gamma)\mathbb{E}_L[X_t]$. Moreover,

$$\mathbb{E}[X_2|\theta, x_1, s_0 = H, s_1 = H] = \lambda\mathbb{E}_H[X_2] + (1 - \lambda)\mathbb{E}_L[X_2] = \mathbb{E}[X_1|\theta, s_0 = H],$$

and expected beliefs become independent of the realization x_1. D_1 and D_2^1 are identical and an option value can never arise. Technically, MLRP is not needed in this scenario and characterizing IC becomes straightforward. If only the principal observes the state, her revenue can be weakly increased by offering a state-dependent contract. Finally, if only the agent observes the state, the good type (weakly) gains rents because the optimal contract can extract this information via incentive constraints with respect to the state.

[24] $\mathbb{E}[X_1|G] - \mathbb{E}[X_1|B]$ can be written as $\lambda_0(\lambda_G - \lambda_B)(\mathbb{E}_H[X_1] - \mathbb{E}_L[X_1]) + (1 - \lambda_0)(\gamma_G - \gamma_B)(\mathbb{E}_H[X_1] - \mathbb{E}_L[X_1])$.

6. Concluding Remarks

Many contractual agreements specify the duration of the relationship and extension periods. In this paper, I rationalize this structure and show that screening with respect to the contract duration can be optimal in a relationship with random change of quality of a principal-owned resource. Allocating such a resource to a privately informed agent and the randomness of its quality can generate a positive option value. A stopping problem arises and the agent is fired once and for all when the option value becomes zero. A negative expected short-term revenue may be accepted by the principal to learn about the quality of the resource and future revenue via allocating to the agent.

A. Appendix: Proofs of Chapter 1

Proof of Remark 1.2. $\mathbb{E}[X_2|\theta, x_1]$ can be written as (I drop the subscript when there is no cause for confusion):

$$\begin{aligned}
&\frac{\lambda_0 \lambda f_H(x_1)}{\lambda_0 \lambda f_H(x_1) + \lambda_0(1-\lambda) f_L(x_1)} (\lambda \mathbb{E}_H(X_2) + (1-\lambda)\mathbb{E}_L(X_2)) \\
&+ \frac{\lambda_0(1-\lambda) f_L(x_1)}{\lambda_0 \lambda f_H(x_1) + \lambda_0(1-\lambda) f_L(x_1)} (\gamma \mathbb{E}_H(X_2) + (1-\gamma)\mathbb{E}_L(X_2)) \\
&+ \frac{(1-\lambda_0)\gamma f_H(x_1)}{(1-\lambda_0)\gamma f_H(x_1) + (1-\lambda_0)(1-\gamma) f_L(x_1)} (\gamma \mathbb{E}_H(X_2) + (1-\gamma)\mathbb{E}_L(X_2)) \\
&+ \frac{(1-\lambda_0)(1-\gamma) f_L(x_1)}{(1-\lambda_0)\gamma f_H(x_1) + (1-\lambda_0)(1-\gamma) f_L(x_1)} (\gamma \mathbb{E}_H(X_2) + (1-\gamma)\mathbb{E}_L(X_2)),
\end{aligned}$$

or equivalently,

$$\begin{aligned}
&\frac{\lambda \frac{f_H(x_1)}{f_L(x_1)}}{\lambda \frac{f_H(x_1)}{f_L(x_1)} + (1-\lambda)} (\lambda \mathbb{E}_H(X_2) + (1-\lambda)\mathbb{E}_L(X_2)) \\
&+ \frac{1-\lambda}{\lambda \frac{f_H(x_1)}{f_L(x_1)} + (1-\lambda)} (\gamma \mathbb{E}_H(X_2) + (1-\gamma)\mathbb{E}_L(X_2)) \\
&+ \gamma \mathbb{E}_H(X_2) + (1-\gamma)\mathbb{E}_L(X_2).
\end{aligned}$$

It follows

$$\frac{\partial \mathbb{E}[X_2|\theta, x_1]}{\partial x_1} = \frac{\lambda(1-\lambda)\frac{d}{dx_1}\frac{f_H(x_1)}{f_L(x_1)}}{\left(\lambda \frac{f_H(x_1)}{f_L(x_1)} + (1-\lambda)\right)^2} (\lambda - \gamma)(\mathbb{E}_H(X_2) - \mathbb{E}_L(X_2)). \tag{A.7}$$

Recall that $\lambda > \gamma$ and that $\mathbb{E}_H(X_2) > \mathbb{E}_L(X_2)$. Hence, (A.7) is (strictly) positive if and only if $\frac{d}{dx_1}\frac{f_H(x_1)}{f_L(x_1)} > 0$, that is if and only if MLRP holds. □

Proof of Lemma 1.3. I prove this result first because the following proof of Lemma 1.2 works similarly, but is fairly longer. Fix a θ and an x_1. It holds that

$$\begin{aligned}
\mathbb{E}[X_2|\theta, x_1] &= \lambda_0\mathbb{P}(s_1 = H|\theta, x_1)(\lambda\mathbb{E}_H[X_2] + (1-\lambda)\mathbb{E}_L[X_2]) \\
&+ \lambda_0(1 - \mathbb{P}(s_1 = H|\theta, x_1))(\gamma\mathbb{E}_H[X_2] + (1-\gamma)\mathbb{E}_L[X_2]) \\
&+ (1-\lambda_0)\mathbb{P}(s_1 = H|\theta, x_1)(\gamma\mathbb{E}_H[X_2] + (1-\gamma)\mathbb{E}_L[X_2]) \\
&+ (1-\lambda_0)(1 - \mathbb{P}(s_1 = H|\theta, x_1))(\gamma\mathbb{E}_H[X_2] + (1-\gamma)\mathbb{E}_L[X_2]) \\
&= \lambda_0\mathbb{P}(s_1 = H|\theta, x_1)(\lambda\mathbb{E}_H[X_1] + (1-\lambda)\mathbb{E}_L[X_1]) \\
&+ \lambda_0(1 - \mathbb{P}(s_1 = H|\theta, x_1))(\gamma\mathbb{E}_H[X_1] + (1-\gamma)\mathbb{E}_L[X_1]) \\
&+ (1-\lambda_0)(\gamma\mathbb{E}_H[X_1] + (1-\gamma)\mathbb{E}_L[X_1]) \\
&< (\lambda\mathbb{E}_H[X_1] + (1-\lambda)\mathbb{E}_L[X_1])(\lambda_0\mathbb{P}(s_1 = H|\lambda, x_1) + \lambda_0 - \lambda_0\mathbb{P}(s_1 = H|\lambda, x_1)) \\
&+ (\gamma\mathbb{E}_H[X_1] + (1-\gamma)\mathbb{E}_L[X_1])(1-\lambda_0) \\
&= \mathbb{E}[X_1|\theta],
\end{aligned}$$

where the second equality comes from the fact that $\mathbb{E}_s[X_2] = \mathbb{E}_s[X_1]$ and the inequality holds because $\gamma < \lambda$. □

Proof of Lemma 1.2. I want to verify that $\mathbb{E}[X_2|G, x_1] > \mathbb{E}[X_2|B, x_1]$. By the proof of Lemma 1.3, this is equivalent to showing the following:

$$\begin{aligned}
&\lambda_0\mathbb{P}(s_1 = H|s_0 = H, \theta = G, x_1)(\lambda_G\mathbb{E}_H[X_1] + (1-\lambda_G)\mathbb{E}_L[X_1]) \\
&\quad + \lambda_0(1 - \mathbb{P}(s_1 = H|s_0 = H, \theta = G, x_1))(\gamma_G\mathbb{E}_H[X_1] + (1-\gamma_G)\mathbb{E}_L[X_1]) \\
&\quad + (1-\lambda_0)(\gamma_G\mathbb{E}_H[X_1] + (1-\gamma_G)\mathbb{E}_L[X_1]) \\
&> \lambda_0\mathbb{P}(s_1 = H|s_0 = H, \theta = B, x_1)(\lambda_B\mathbb{E}_H[X_1] + (1-\lambda_B)\mathbb{E}_L[X_1]) \\
&\quad + \lambda_0(1 - \mathbb{P}(s_1 = H|s_0 = H, \theta = B, x_1))(\gamma_B\mathbb{E}_H[X_1] + (1-\gamma_B)\mathbb{E}_L[X_1]) \\
&\quad + (1-\lambda_0)(\gamma_B\mathbb{E}_H[X_1] + (1-\gamma_B)\mathbb{E}_L[X_1]).
\end{aligned}$$

Since $(\lambda_G\mathbb{E}_H[X_1] + (1-\lambda_G)\mathbb{E}_L[X_1]) > (\lambda_B\mathbb{E}_H[X_1] + (1-\lambda_B)\mathbb{E}_L[X_1])$ and analogously for γ_G

and γ_B, it holds that

$$\begin{aligned}
&\lambda_0 \mathbb{P}(s_1 = H | s_0 = H, \theta = G, x_1)(\lambda_G \mathbb{E}_H[X_1] + (1-\lambda_G)\mathbb{E}_L[X_1]) \\
&\quad + \lambda_0(1 - \mathbb{P}(s_1 = H | s_0 = H, \theta = G, x_1))(\gamma_G \mathbb{E}_H[X_1] + (1-\gamma_G)\mathbb{E}_L[X_1]) \\
&\quad + (1-\lambda_0)(\gamma_G \mathbb{E}_H[X_1] + (1-\gamma_G)\mathbb{E}_L[X_1]) \\
>&\lambda_0 \mathbb{P}(s_1 = H | s_0 = H, \theta = G, x_1)(\lambda_B \mathbb{E}_H[X_1] + (1-\lambda_B)\mathbb{E}_L[X_1]) \\
&\quad + \lambda_0(1 - \mathbb{P}(s_1 = H | s_0 = H, \theta = G, x_1))(\gamma_B \mathbb{E}_H[X_1] + (1-\gamma_B)\mathbb{E}_L[X_1]) \\
&\quad + (1-\lambda_0)(\gamma_B \mathbb{E}_H[X_1] + (1-\gamma_B)\mathbb{E}_L[X_1]).
\end{aligned}$$

Moreover, $\lambda_B \mathbb{E}_H[X_1] + (1-\lambda_B)\mathbb{E}_L[X_1] > \gamma_B \mathbb{E}_H[X_1] + (1-\gamma_B)\mathbb{E}_L[X_1]$ and $\mathbb{P}(s_1 = H | s_0 = H, \theta = G, x_1) > \mathbb{P}(s_1 = H | s_0 = H, \theta = B, x_1)$. To verify the latter:

$$\begin{aligned}
\mathbb{P}(s_1 = H | s_0 = H, \theta = G, x_1) &= \frac{\lambda_G f_H(x_1)}{\lambda_G f_H(x_1) + (1-\lambda_G) f_L(x_1)} \\
> \mathbb{P}(s_1 = H | s_0 = H, \theta = B, x_1) &= \frac{\lambda_B f_H(x_1)}{\lambda_B f_H(x_1) + (1-\lambda_B) f_L(x_1)},
\end{aligned}$$

is equivalent to $\lambda_G > \lambda_B$ and thus true. Hence,

$$\begin{aligned}
&\mathbb{P}(s_1 = H | s_0 = H, \theta = G, x_1)(\lambda_B \mathbb{E}_H[X_1] + (1-\lambda_B)\mathbb{E}_L[X_1]) \\
+&(1 - \mathbb{P}(s_1 = H | s_0 = H, \theta = G, x_1))(\gamma_B \mathbb{E}_H[X_1] + (1-\gamma_B)\mathbb{E}_L[X_1]) \\
>&\mathbb{P}(s_1 = H | s_0 = H, \theta = B, x_1)(\lambda_B \mathbb{E}_H[X_1] + (1-\lambda_B)\mathbb{E}_L[X_1]) \\
+&(1 - \mathbb{P}(s_1 = H | s_0 = H, \theta = B, x_1))(\gamma_B \mathbb{E}_H[X_1] + (1-\gamma_B)\mathbb{E}_L[X_1]),
\end{aligned}$$

which shows the claim. □

Proof of Lemma 1.4. Necessity: By $[IC_2]$, any type will announce $\hat{x}_2$ that induces the lowest possible period-2-transfer: $p_2(\theta, x_1, \hat{x}_2) = p_2(\theta, x_1, x_2)$ for all θ and for all $x_1 \in \chi \cup \{\varnothing\}, x_2, \hat{x}_2 \in \chi$. Hence, p_2 is constant in $\hat{x}_2$ for all previous realizations. Showing that p_2 is determined by the lowest type $\underline{x}$ is a standard argument (see, for example, Subsection 2.2 in Borgers et al. (2015)).

Sufficiency: This direction is clear. □

Proof of Lemma 1.5. $[IC_2]$ is fulfilled by assumption and I can ignore the constant transfer p_2 if convenient.

Necessity: Fix a θ and suppose $x_1 > x_1'$. By $[IC_1]$:

$$q_2(\theta, x_1)\mathbb{E}[X_2|\theta, x_1] - p_1(\theta, x_1) \geq q_2(\theta, x_1')\mathbb{E}[X_2|\theta, x_1] - p_1(\theta, x_1') \text{ and}$$
$$q_2(\theta, x_1')\mathbb{E}[X_2|\theta, x_1'] - p_1(\theta, x_1') \geq q_2(\theta, x_1)\mathbb{E}[X_2|\theta, x_1'] - p_1(\theta, x_1).$$

Rearranging and summing both yields

$$q_2(\theta, x_1)(\mathbb{E}[X_2|\theta, x_1] - \mathbb{E}[X_2|\theta, x_1']) \geq q_2(\theta, x_1')(\mathbb{E}[X_2|\theta, x_1] - \mathbb{E}[X_2|\theta, x_1']),$$

which shows part 1. since $\mathbb{E}[X_2|\theta, x_1]$ is strictly increasing by Remark 1.2.
To show part 2., observe that $[IC_1]$ implies

$$u_1(\theta, x_1) = \max_{\hat{x}_1 \in \mathcal{X}} \{x_1 - p_1(\theta, \hat{x}_1) + \mathbb{E}[q_2(\theta, \hat{x}_1)X_2 - p_2(\theta, \hat{x}_1, \underline{x})|\theta, x_1]\}, \tag{A.8}$$

for any fixed θ. This function is increasing in x_1 and continuous. It follows that u_1 is differentiable almost everywhere on $(\underline{x}, \overline{x})$ and by $[IC_1]$:

$$\begin{aligned}
\lim_{h\to 0} \frac{u_1(\theta, x_1 + h) - u_1(\theta, x_1)}{h} \geq & \lim_{h\to 0} \frac{1}{h}((x_1 + h) - p_1(\theta, x_1) + \mathbb{E}[q_2(\theta, x_1)X_2 - p_2(\theta, x_1, \underline{x})|\theta, x_1 + h] \\
& - (x_1 - p_1(\theta, x_1) + \mathbb{E}[q_2(\theta, x_1)X_2 - p_2(\theta, x_1, \underline{x})|\theta, x_1])) \\
= & \lim_{h\to 0} \frac{1}{h}\Big(h + q_2(\theta, x_1)(\mathbb{E}[X_2|\theta, x_1 + h] - \mathbb{E}[X_2|\theta, x_1])\Big) \\
= & 1 + q_2(\theta, x_1)\frac{\partial \mathbb{E}[X_2|\theta, x_1]}{\partial x_1}.
\end{aligned}$$

The same argument is valid for $\lim_{h\to 0} \frac{u_1(\theta, x_1) - u_1(\theta, x_1 - h)}{h}$. This shows part 2. and thus necessity.

Sufficiency: Suppose $q_2(\theta, x_1)$ is increasing in x_1 and part 2. holds. To show $[IC_1]$, take $x_1 > x_1'$, so that

$$\begin{aligned}
u_1(\theta, x_1) = & x_1 - p_1(\theta, x_1) + \mathbb{E}[q_2(\theta, x_1)X_2 - p_2(\theta, x_1, \underline{x})|\theta, x_1] \\
\geq & x_1 - p_1(\theta, x_1') + \mathbb{E}[q_2(\theta, x_1')X_2 - p_2(\theta, x_1', \underline{x})|\theta, x_1] \\
= & x_1 - p_1(\theta, x_1') + \mathbb{E}[q_2(\theta, x_1')X_2 - p_2(\theta, x_1', \underline{x})|\theta, x_1] \\
& + x_1' + \mathbb{E}[q_2(\theta, x_1')X_2 - p_2(\theta, x_1', \underline{x})|\theta, x_1'] \\
& - x_1' - \mathbb{E}[q_2(\theta, x_1')X_2 - p_2(\theta, x_1', \underline{x})|\theta, x_1'] \\
\Leftrightarrow u_1(\theta, x_1) \geq & u_1(\theta, x_1') + x_1 + \mathbb{E}[q_2(\theta, x_1')X_2 - p_2(\theta, x_1', \underline{x})|\theta, x_1] \\
& - x_1' - \mathbb{E}[q_2(\theta, x_1')X_2 - p_2(\theta, x_1', \underline{x})|\theta, x_1'],
\end{aligned}$$

and thus

$$u_1(\theta, x_1) - u_1(\theta, x_1') \geq (x_1 - x_1') + q_2(\theta, x_1')(\mathbb{E}[X_2|\theta, x_1] - \mathbb{E}[X_2|\theta, x_1']).$$

Using that $\mathbb{E}[X_2|\theta, x_1]$ is absolutely continuous together with part 2. yields

$$\int_{x_1'}^{x_1} 1 + q_2(\theta, z)\frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z} dz \geq \int_{x_1'}^{x_1} 1 + q_2(\theta, x_1')\frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z} dz$$
$$\iff \int_{x_1'}^{x_1} q_2(\theta, z)\frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z} dz \geq q_2(\theta, x_1') \int_{x_1'}^{x_1} \frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z} dz.$$

The two integrals have the same integration limits and $\frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z}$ is positive. By part 1., q_2 is increasing in x_1. Therefore, the inequality

$$u_1(\theta, x_1) \geq x_1 - p_1(\theta, x_1') + \mathbb{E}[q_2(\theta, x_1')X_2 - p_2(\theta, x_1', \underline{x})|\theta, x_1]$$

is true for all $x_1 > x_1'$. The argument for $x_1 < x_1'$ is the same. This shows $[IC_1]$. □

Proof of Lemma 1.6. To solve for p_1, use Assumption 1.2 and (A.7) and it follows that $\mathbb{E}[X_2|\theta, x_1]$ has a bounded derivative with respect to x_1 and is thus Lipschitz-continuous; so is the function x_1 in equation (A.8). Take the maximum of both Lipschitz constants and it follows that u_1 is Lipschitz-continuous (see Theorem 4.6.3. in Sohrab (2003)). Lipschitz-continuity implies absolute continuity. Hence, by part 2. of Lemma 1.5, I can write

$$u_1(\theta, x_1) = u_1(\theta, \underline{x}) + \int_{\underline{x}}^{x_1} 1 + q_2(\theta, z)\frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z} dz.$$

Plugging u_1 into the left-hand-side yields

$$x_1 - p_1(\theta, x_1) + \mathbb{E}[q_2(\theta, x_1)X_2 - p_2(\theta, x_1, \underline{x})|\theta, x_1] = u_1(\theta, \underline{x}) + \int_{\underline{x}}^{x_1} 1 + q_2(\theta, z)\frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z} dz.$$

Rearranging yields

$$\begin{aligned} p_1(\theta, x_1) = & - u_1(\theta, \underline{x}) + \underline{x} \\ & + \mathbb{E}[q_2(\theta, x_1)X_2 - p_2(\theta, x_1, \underline{x})|\theta, x_1] \\ & - \int_{\underline{x}}^{x_1} q_2(\theta, z)\frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z} dz. \end{aligned}$$

□

Proof of Proposition 1.2. The proof is performed in three steps and I start by assuming that

$[IC_{0G}]$ and $[IR_{0B}]$ both hold and show that the other constraints are fulfilled.
Step 1: $[IC_{0G}]$ and $[IR_{0B}]$ imply $[IR_{0G}]$.
From $[IC_{0G}]$ and adding a zero, it follows that

$$\begin{aligned}
U(G) =&\mathbb{E}\Big[q_1(G)X_1 - p_1(G, X_1) + \mathbb{E}[q_2(G, x_1)X_2 - p_2(G, x_1, \underline{x})\Big|G, x_1]\Big|G\Big] \\
\geq& U(B|G) \\
=&\mathbb{E}\Big[q_1(B)X_1 - p_1(B, X_1) + \mathbb{E}[q_2(B, X_1)X_2 - p_2(B, x_1, \underline{x})\Big|G, x_1]\Big|G\Big] \\
&+ \Big(\mathbb{E}[\mathbb{E}[q_2(B, x_1)X_2 - p_2(B, x_1, \underline{x})|B, x_1]\Big|B] - \mathbb{E}[p_1(B, X_1)|B] \\
&+ \mathbb{E}[q_1(B)X_1|B]\Big) \\
&- \Big(\mathbb{E}[\mathbb{E}[q_2(B, x_1)X_2 - p_2(B, x_1, \underline{x})|B, x_1]\Big|B] - \mathbb{E}[p_1(B, X_1)|B] \\
&+ \mathbb{E}[q_1(B)X_1|B]\Big) \\
=&\mathbb{E}[q_1(B)X_1 - p_1(B, X_1) + \mathbb{E}[q_2(B, x_1)X_2 - p_2(B, x_1, \underline{x})|B, x_1]\Big|B] \\
&+ \mathbb{E}[q_1(B)X_1 - p_1(B, X_1) + \mathbb{E}[q_2(B, x_1)X_2 - p_2(B, x_1, \underline{x})|G, x_1]\Big|G] \\
&- \mathbb{E}[q_1(B)X_1 - p_1(B, X_1) + \mathbb{E}[q_2(B, x_1)X_2 - p_2(B, x_1, \underline{x})|B, x_1]\Big|B] \\
=&U(B) \\
&+ \mathbb{E}[q_1(B)X_1 - p_1(B, X_1)|G] - \mathbb{E}[q_1(B)X_1 - p_1(B, X_1)|B] \\
&+ \mathbb{E}[\mathbb{E}[q_2(B, x_1)X_2 - p_2(B, x_1, \underline{x})|G, x_1]|G] \\
&- \mathbb{E}[\mathbb{E}[q_2(B, x_1)X_2 - p_2(B, x_1, \underline{x})|B, x_1]|B].
\end{aligned}$$

Now use Lemma 1.6 and plug in p_1. The last expression becomes:

$$\begin{aligned}
&U(B) + q_1(B)\mathbb{E}[X_1|G] - q_1(B)\mathbb{E}[X_1|B] \\
&- \mathbb{E}[\underline{x} - u_1(B,\underline{x}) + \mathbb{E}[q_2(B,x_1)X_2 - p_2(B,X_1,\underline{x})|B,x_1] \\
&\qquad - \int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz\Big|G] \\
&+ \mathbb{E}[\underline{x} - u_1(B,\underline{x}) + \mathbb{E}[q_2(B,x_1)X_2 - p_2(B,X_1,\underline{x})|B,x_1] \\
&\qquad - \int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz\Big|B] \\
&+ \mathbb{E}[\mathbb{E}[q_2(B,x_1)X_2 - p_2(B,x_1,\underline{x})|G,x_1]|G] \\
&- \mathbb{E}[\mathbb{E}[q_2(B,x_1)X_2 - p_2(B,x_1,\underline{x})|B,x_1]|B]. \\
=&U(B) + q_1(B)(\mathbb{E}[X_1|G] - \mathbb{E}[X_1|B]) \\
&- \mathbb{E}[q_2(B,x_1)\mathbb{E}[X_2|B,x_1] - \int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz\Big|G] \\
&+ \mathbb{E}[q_2(B,x_1)\mathbb{E}[X_2|B,x_1] - \int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz\Big|B] \\
&+ \mathbb{E}[\mathbb{E}[q_2(B,x_1)X_2|G,x_1]|G] \\
&- \mathbb{E}[\mathbb{E}[q_2(B,x_1)X_2|B,x_1]|B] \\
=&U(B) + q_1(B)(\mathbb{E}[X_1|G] - \mathbb{E}[X_1|B]) \\
&+ \Big(\mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz\Big|G] - \mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz\Big|B]\Big) \\
&+ \mathbb{E}[\mathbb{E}[q_2(B,x_1)X_2|G,x_1]|G] \\
&- \mathbb{E}[\mathbb{E}[q_2(B,x_1)X_2|B,x_1]|G] \\
=&U(B) + q_1(B)(\mathbb{E}[X_1|G] - \mathbb{E}[X_1|B]) \\
&+ \mathbb{E}[\mathbb{E}[q_2(B,x_1)X_2|G,x_1] - \mathbb{E}[q_2(B,x_1)X_2|B,x_1]\Big|G] \\
&+ (\mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz\Big|G] - \mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz\Big|B]).
\end{aligned} \tag{A.9}$$

$U(B)$ is positive by $[IR_{0B}]$. The second summand is positive by Lemma 1.1 and the third one is so by Lemma 1.2, as well. By interchanging the order of integration, the last summand equals

$$\begin{aligned}
&\int_{\mathcal{X}} f(x_1|\theta) \int_{\underline{x}}^{x_1} q_2(\theta, z) \frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z} dz dx_1 \\
&= \int_{\mathcal{X}} \int_{z}^{\overline{x}} f(x_1|\theta) q_2(\theta, z) \frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z} dx_1 dz \\
&= \int_{\mathcal{X}} (1 - F(z|\theta)) q_2(\theta, z) \frac{\partial \mathbb{E}[X_2|\theta, z]}{\partial z} dz.
\end{aligned}$$

And therefore,

$$\begin{aligned}
&(\mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B, z) \frac{\partial \mathbb{E}[X_2|B, z]}{\partial z} dz \big| G] - \mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B, z) \frac{\partial \mathbb{E}[X_2|B, z]}{\partial z} dz \big| B]) \qquad \text{(A.10)} \\
&= \int_{\mathcal{X}} (F(x_1|B) - F(x_1|G)) q_2(B, x_1) \frac{\partial \mathbb{E}[X_2|B, x_1]}{\partial x_1} dx_1,
\end{aligned}$$

which is positive by Remark 1.2. Hence, $[IR_{0G}]$ is fulfilled.

Step 2: $[IR_{0B}]$ and $[IC_{0G}]$ bind at the optimum.

Similar to static contract theory, one can write the principal's revenue as a function of both types' expected utilities (see Lemma 1.7 below). Expected revenue can then be written as

$$\begin{aligned}
R =& \alpha \mathbb{E}[q_1(G) X_1 + q_2(G, X_1) \mathbb{E}[X_2|G, x_1] \big| G] - \alpha U(G) \\
&+ (1 - \alpha) \mathbb{E}[q_1(B) X_1 + q_2(B, X_1) \mathbb{E}[X_2|B, x_1] \big| B] - (1 - \alpha) U(B).
\end{aligned}$$

a) Now suppose $[IR_{0B}]$ does not bind, i.e. $U(B) = \epsilon > 0$. Then she can decrease $U(B)$ by ϵ and gain positive revenues of $(1 - \alpha)\epsilon$. Hence, $U(B) = 0$ is optimal.
b) Suppose $[IC_{0G}]$ does not bind and $U(G)$ is equal to the final (positive) expression in equation (A.9) plus some $\epsilon > 0$. Again, she can decrease $U(G)$ by ϵ and gain strictly positive revenues of $\alpha\epsilon$ – a contradiction.

Step 3: $[IC_{0B}]$ is fulfilled.

I make use of an analogous argument as in step 1.

$$
\begin{aligned}
& U(B) \geq U(G|B) \\
\iff & U(B) \geq \mathbb{E}[q_1(G)X_1 - p_1(G,X_1) + \mathbb{E}[q_2(G,X_1)X_2 - p_2(G,X_1,\underline{x})|B,x_1]|B] \\
& \quad = U(G) - q_1(G)(\mathbb{E}[X_1|G] - \mathbb{E}[X_1|B]) \\
& \quad - \mathbb{E}[q_2(B,x_1)(\mathbb{E}[X_2|G,x_1] - \mathbb{E}[X_2|B,x_1])\Big|G] \\
& \quad - \Big(\mathbb{E}[\int_{\underline{x}}^{x_1} q_2(G,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz|G] - \mathbb{E}[\int_{\underline{x}}^{x_1} q_2(G,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz|B]\Big).
\end{aligned}
$$

The last expression equals $U(G) - U(B|G)$. According to step 2, $[IR_{0B}]$ and $[IC_{0G}]$ both bind, so $U(G) = U(B|G)$. In particular, one has zero on both sides of the inequality which completes the proof. □

Lemma 1.7
The distortion functions D_1 and D_2^1 from (1.3) *are given by*

$$D_1 = \frac{\alpha}{1-\alpha}(\mathbb{E}[X_1|G] - \mathbb{E}[X_1|B])$$

and

$$
\begin{aligned}
D_2 &:= \mathbb{E}[D_2^1(x_1)|B] \\
&= \mathbb{E}\Big[\frac{\alpha}{1-\alpha}\frac{f(x_1|G)}{f(x_1|B)}(\mathbb{E}[X_2|G,x_1] - \mathbb{E}[X_2|B,x_1] + \frac{\partial \mathbb{E}[X_2|\theta,x_1]}{\partial x_1}\frac{F(x_1|B)-F(x_1|G)}{f(x_1|G)})\Big|B\Big].
\end{aligned}
$$

Proof of Lemma 1.7. First, observe that by equation (1.2) and the binding IC constraint for the good type, one can write the principal's revenue as

$$
\begin{aligned}
& \alpha(q_1(G)\mathbb{E}[X_1|G] + \mathbb{E}[q_2(G,X_1)\mathbb{E}[X_2|G,x_1]\Big|G]) \\
& + (1-\alpha)\Big\{q_1(B)(\mathbb{E}[X_1|B] - \frac{\alpha}{1-\alpha}(\mathbb{E}[X_1|G] - \mathbb{E}[X_1|B])) \\
& + \Big[\mathbb{E}[q_2(B,X_1)\mathbb{E}[X_2|B,x_1]\Big|B] \\
& - \frac{\alpha}{1-\alpha}\Big(\mathbb{E}[q_2(B,X_1)(\mathbb{E}[X_2|G,x_1] - \mathbb{E}[X_2|B,x_1])\Big|G] \\
& + \mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz|G] - \mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz|B]\Big)\Big]\Big\}.
\end{aligned}
$$

For tractability, I rewrite the revenue function, but the period-1 distortion function can immediately be identified as

$$D_1 = \frac{\alpha}{1-\alpha}(\mathbb{E}[X_1|G] - \mathbb{E}[X_1|B]) > 0.$$

Furthermore, one sees that

$$D_2 = \frac{\alpha}{1-\alpha}\Big(\mathbb{E}[\mathbb{E}[q_2(B,X_1)X_2|G,x_1] - \mathbb{E}[q_2(B,X_1)X_2|B,x_1]\big|G] + \mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz|G] - \mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz|B]\Big). \tag{A.11}$$

Now, I can change the order of integration to obtain

$$\begin{aligned}\mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz\big|G] &= \int_{\mathcal{X}} f(x_1|G)\int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dzdx_1 \\ &= \int_{\mathcal{X}} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}\int_z^{\overline{x}} f(x_1|G)dx_1dz \\ &= \int_{\mathcal{X}} q_2(B,z)(1-F(z|G))\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz \\ &= \int_{\mathcal{X}} q_2(B,x_1)(1-F(x_1|G))\frac{\partial \mathbb{E}[X_2|B,x_1]}{\partial x_1}dx_1,\end{aligned} \tag{A.12}$$

where the last equality is merely a relabeling. It follows that

$$\begin{aligned}&\mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz|G] - \mathbb{E}[\int_{\underline{x}}^{x_1} q_2(B,z)\frac{\partial \mathbb{E}[X_2|B,z]}{\partial z}dz|B] \\ &= \int_{\mathcal{X}} q_2(B,x_1)(1-F(x_1|G))\frac{\partial \mathbb{E}[X_2|B,x_1]}{\partial x_1}dx_1 - \int_{\mathcal{X}} q_2(B,x_1)(1-F(x_1|B))\frac{\partial \mathbb{E}[X_2|B,x_1]}{\partial x_1}dx_1 \\ &= \int_{\mathcal{X}} q_2(B,x_1)\frac{F(x_1|B)-F(x_1|G)}{f(x_1|G)}f(x_1|G)\frac{\partial \mathbb{E}[X_2|B,x_1]}{\partial x_1}dx_1 \\ &= \mathbb{E}[q_2(B,x_1)\frac{\partial \mathbb{E}[X_2|B,x_1]}{\partial x_1}\frac{F(x_1|B)-F(x_1|G)}{f(x_1|G)}\big|G].\end{aligned} \tag{A.13}$$

Using equation (A.7):

$$D_2 = \frac{\alpha}{1-\alpha}\mathbb{E}\Bigg[q_2(B,X_1)\Big(\mathbb{E}[X_2|G,x_1] - \mathbb{E}[X_2|B,x_1] + \frac{\lambda_B(1-\lambda_B)\frac{d}{dx_1}\frac{f_H(x_1)}{f_L(x_1)}}{(\lambda_B\frac{f_H(x_1)}{f_L(x_1)} + (1-\lambda_B))^2}(\lambda_B-\gamma_B)(\mathbb{E}_H(X_2)-\mathbb{E}_L(X_2))\frac{F(x_1|B)-F(x_1|G)}{f(x_1|G)}\Big)\Big|G\Bigg]. \tag{A.14}$$

This term conditions on $\theta = G$. The problem is that the (undistorted) expected surplus for the second period, $\mathbb{E}[q_2(B,X_1)\mathbb{E}[X_2|B,x_1]\big|B]$, conditions on $\theta = B$ which makes it intractable to solve the principal's problem point-wise, i.e. to write the virtual surplus for the second period under one integral with exactly one allocation function. I thus rewrite the revenue with respect to B from the second period as follows (I omit the probability $(1-\alpha)$ for simplicity):

$$\begin{aligned}
&\Big[\int_{\mathcal{X}} q_2(B,x_1)\mathbb{E}[X_2|B,x_1]f(x_1|B)dx_1 - \frac{\alpha}{1-\alpha}\int_{\mathcal{X}} q_2(B,x_1)\big(\mathbb{E}[X_2|G,x_1]-\mathbb{E}[X_2|B,x_1] \\
&+\mathbb{E}[q_2(B,x_1)\frac{\partial\mathbb{E}[X_2|B,x_1]}{\partial x_1}\frac{F(x_1|B)-F(x_1|G)}{f(x_1|G)}\big)f(x_1|G)dx_1\Big] \\
&=\mathbb{E}\Big[q_2(B,x_1)\{\mathbb{E}[X_2|B,x_1] - \frac{\alpha}{1-\alpha}\frac{f(x_1|G)}{f(x_1|B)}(\mathbb{E}[X_2|G,x_1]-\mathbb{E}[X_2|B,x_1] \\
&+\mathbb{E}[q_2(B,x_1)\frac{\partial\mathbb{E}[X_2|B,x_1]}{\partial x_1}\frac{F(x_1|B)-F(x_1|G)}{f(x_1|G)})\}\Big|B\Big] \\
&=:\mathbb{E}\Big[q_2(B,x_1)(\mathbb{E}[X_2|B,x_1]-D_2^1(X_1))\Big|B\Big] \\
&=\mathbb{E}\Big[q_2(B,x_1)\mathbb{E}[X_2-D_2^1(x_1)|B,x_1]\Big|B\Big],
\end{aligned}$$

where the last line follows from the law of total expectation. □

CHAPTER 2

Strategic Information Transmission and Efficient Corporate Control

1. Introduction

The limitations of takeovers as a means to allocate corporate resources to the most efficient management have been studied extensively. Starting with Grossman and Hart (1980), research centered primarily around collective action problems undermining the effectiveness of the market for corporate control. Numerous ways have been suggested to deal with the free-rider problem in the realm of takeovers.[1] So far, little emphasis has been placed on the information asymmetries that naturally arise during a takeover. Further, little is known about how information can be transmitted to resolve informational frictions. To be precise, the literature mainly considered uncertainty about how much the external bidder is able to improve the target firm value but not uncertainty about *whether* he is able to improve it at all. This information structure poses (among others) one puzzle yet to be solved: if it was common knowledge that the external bidder will be value-improving, proxy fights would guarantee efficient control as they circumvent the free-rider problem. This, however, makes the infrequent[2] occurrence of proxy fights rather puzzling (Bebchuk and Hart 2001).

In this paper, we study a situation with uncertainty about whether a potential takeover is profitable and ways to resolve this uncertainty. To this end, we introduce a framework which allows for two-sided private information and different forms of information transmission. In particular, not only the bidder but also the incumbent management possesses private 'inside'

[1]Grossman and Hart (1980) show how the exclusion of initial shareholders from takeover gains can circumvent the free-rider problem. Bagnoli and Lipman (1988) show that free-riding is not an issue in models with finitely many shareholders. Shleifer and Vishny (1986) show that toehold acquisitions prior to the takeover attempt can make takeovers profitable and Müller and Panunzi (2004) demonstrate how dilution of the target firm's share value can be attained via leveraged *bootstrap* acquisitions.

[2]See Mulherin and Poulsen (1998) and Bebchuk (2007) for evidence on the rare use of proxy fights for the time periods 1979 – 1994 and 1996 – 2005, respectively.

information about the future profitability if it remains in charge.[3] The shareholders making the final decision often have only crude estimates of both pieces of information at their disposal. Shareholders' uncertainty about the firm value under either management can explain the low frequency of proxy fights: if expected firm value under incumbent management is larger than under an external bidder, a proxy fight will not succeed. These informational frictions may be overcome via three channels. First, the external bidder can signal private information via his tender offer. Second, frequently observed management recommendations can provide some of the insider's private information. Third, shareholders can acquire additional information from other sources. Of course, all parties are interested in maximizing their individual payoffs, thereby impeding information transmission.

The main contribution of this paper is to show that management recommendations, even though never fully informative, can implement first-best control allocation only if shareholders cannot acquire additional information about the firm value. In particular, we show that management's strategic communication serves a dual role: on the one hand, it provides information regarding management's inside information. On the other hand, it can be used to incentivize the bidder to fully reveal his private information. Conversely, if shareholders have access to more, albeit costless information, too few takeovers occur in equilibrium. Similar to Grossman and Hart (1980) who argue in favor of (partial) exclusion of initial shareholders from post-takeover profits, we show that excluding shareholders from learning about the value of the firm can be welfare-improving.

In our basic model, an external bidder is privately informed about his ability to manage the company once he is in charge. To obtain control, he can submit a public tender offer to acquire a controlling stake in the company from the single initial shareholder. After the bidder's tender offer, the incumbent manager sends a cheap talk message to the shareholder which is based on his private information and the bidder's offer. The manager, being compensated with shares[4], compares the firm value under his with the firm value under the external bidder's management when he sends his message. In contrast, the shareholder wants to tender only if her expected payoff from selling shares (which contains the price offer) exceeds the expected firm value under incumbent management. The level of (dis)agreement in the cheap talk stage is thus given by the difference of expected bidder type (incumbent's view) and tender offer (shareholder's objective). As the tender offer is an equilibrium object, the level of conflict in the cheap talk stage arises endogenously.

[3] During the takeover of BEA Systems, Inc. by Oracle in 2007/08, BEA's management allegedly declined a takeover offer due to its private information: "BEA has said it cannot fully disclose to the public why it rejected Oracle's offer because the information is confidential [...]. Some analysts have speculated that the company may have secret products in development that it believes will be blockbusters." *https://www.reuters.com/article/us-bea-icahn/bea-giving-confidential-information-to-carl-icahn-idUSWNAS031920071105* , date 9/30/2019.

[4] We further extend our model and introduce private benefits the manager enjoys from being in charge and show how golden parachutes can be used to mitigate the problems associated with private benefits.

As a benchmark, we let the shareholder freely choose the level of information she obtains. As she faces a pure decision problem at the tendering stage, she will always choose to become fully informed.[5] We show that this is, however, not efficient and leads to misallocations of control: too few takeovers occur in equilibrium.

Without strategic management recommendations and in absence of shareholder learning, there only exist equilibria where no shareholder ever tenders, and equilibria where all bidder types above some cutoff take over the company with certainty (partial pooling). All types below the cutoff never gain control. Not surprisingly, such cutoff equilibria never attain the optimal control allocation.

In the presence of cheap talk by the incumbent, we prove existence of an equilibrium in which the manager sends a binary recommendation in favor or against a takeover which is followed by the shareholder. The anticipation of this message makes the bidder fully reveal his type via his tender offer. Thus, cheap talk enables both information provision regarding the incumbent's type and screening of the bidder's type. This is feasible because anticipating the informative management recommendation, the bidder trades off the probability of a takeover with profits earned from a takeover. Higher prices are costly to the bidder, but they will, in equilibrium, imply a higher takeover probability because they signal a higher type. We show that the first-best control allocation is attained with management recommendation. Strategic information transmission by the incumbent management thus improves the allocation of control rights compared to both, a fully informed and an uninformed shareholder.

To gain intuition for this result, notice that with strategic communication, the shareholder only obtains a binary message regarding the firm value under incumbent management. As interests of shareholder and manager are not perfectly aligned, more precise strategic information transmission is not feasible. It is also not desirable because with cheap talk the manager optimally obfuscates information and implements first-best by letting the external bidder extract all gains from trade. On the other hand, if the shareholder can freely choose the level of information she receives, she will become fully informed. In this case, a takeover occurs only if the incumbent's type is below the price offer (as opposed to the signaled bidder's type). It can be shown that first-best in this case requires all bidder types to earn zero profits on the takeover. This can, however, never be an equilibrium as imitating lower bidder types will yield strictly positive profits. Hence, there is a tension between the owners of the firm and society regarding the optimal form and level of information provision.

We extend our model to a general ownership structure with finitely many shareholders. Further, we introduce private benefits from retaining control for incumbent management. Two differences arise: multiple shareholders give rise to equilibria suffering from coordination

[5] To focus on allocative efficiency, we abstract from any costs associated with additional information acquisition.

failures; and private benefits from remaining in charge hamper communication and introduce a wedge between the incumbent's incentives and first-best. We show, however, that also with finitely many shareholders, the informative cheap talk equilibrium exists for sufficiently small private benefits. We further establish that if the private benefits are not too large, then this equilibrium dominates the setting with fully informed shareholders in terms of welfare. In that sense, the equilibrium with informative cheap talk is robust in both dimensions.

Our paper has implications on optimal managerial salary schemes during takeovers, regulation of fairness opinions[6] and disclosure requirements. First, we provide a novel rationale for equity compensation of managers which does not rely on the typical moral hazard argument. In our model, it is the management's advisory role in takeovers that requires equity compensation to achieve efficiency. Further, it is crucial that the manager maintains his share position for a holding period after he steps down from office.[7] Indeed many companies offer vested shares to their named executive officers as part of the compensation package. Holders of these shares become owner of the asset only gradually over time to provide incentives to remain with the company. Often, compensation agreements specify that the shares – after termination of employment following the change in control – do not vest immediately, but within a specified time period up to two years (Shearman & Sterling LLP 2016). Management's advisory role may also be strengthened by increasing the benefits from being replaced due to a takeover. Golden parachutes are often subject to public criticism and seen as sign of management entrenchment. A recent example includes the takeover of Mead Johnson by Reckitt Banckiser as an article in the Financial Times states:

'*Mead introduced a "golden parachute" pay scheme if [executives] are let go within two years of a takeover… [T]he prospect of being paid because you decide to leave a job may seem decidedly odd. Not, sadly, in the wider context of executive pay agreements, where Mead's example is anything but unusual.*'.[8]

Through the lens of our model, however, golden parachutes can be efficient. They serve to improve the advisory role of management which typically obtains some private benefits from remaining in charge. Rewarding incumbent management after a successful takeover

[6]A fairness opinion comprises a brief letter stating the fairness of the offered price and supplementing material such as data, methods and computations used for valuation (Bebchuk and Kahan 1989). In 1986, for example, Connecticut National Bank issued a fairness opinion for the takeover of Nutri/System, Inc. stating that the "$7.16 a share price was fair to shareholders because the company was worth between $6.50 and $8.50 a share." See *https://casetext.com/case/herskowitz-v-nutrisystem-inc*, date 3/19/2019.

[7]An alternative would be to pay the manager a bonus for a high post-takeover shareholder value. In the present paper, this holding period need not necessarily be required by law since ex post, it is in the manager's best interest to tender none of his shares.

[8]See *https://www.ft.com/content/c63591b0-ea08-11e6-893c-082c54a7f539*, date 12/2/2019.

may thus help to balance management's interests between remaining in charge and stepping down. Ultimately, this helps to maximize firm value. Of course, the golden parachute should be contingent on a takeover and not be triggered by a dismissal due to mismanagement or similar reasons.[9]

Second, consulting an outside advisor (such as an investment bank) who provides information beyond the manager's recommendation is common within the realm of corporate takeovers (Kisgen et al. 2009). Furthermore, management may be subject to mandatory disclosure rules (Bainbridge 1999).[10] Such fairness opinions and similar disclosure of information should not be required by law since they may destroy firm value.[11] Importantly, as the current shareholders in our model want more information at the time of their tendering decision, they may be prone to force management to procure an expert opinion or provide additional disclosure by the threat of a lawsuit. Eliminating the possibility of successful lawsuits may increase allocative efficiency. Our model also provides a rationale for uninformative fairness opinions: uninformative rubber-stamping of management's recommendation can actually be an optimal response to legally required fairness opinions provided management has discretion over how informative the report is.

The rest of this paper is organized as follows. In the remainder of this section we highlight the relationship between our results and related work. Section 2 introduces our basic model. We present a benchmark in Section 3. In section 4, we solve our main model. In Section 5, we investigate several extensions to our basic model. In Section 6, we show how our results can be used for the optimal use of golden parachutes in takeovers. Section 7 comments on a connection of our model with auction theory. Finally, Section 8 concludes. All proofs are delegated to Appendix A.

Literature on Corporate Takeovers

In the following, we highlight papers from the literature on corporate takeovers that are most related to ours. For a detailed review of the literature, see for example Burkart and Panunzi (2008).

In their seminal paper, Grossman and Hart (1980) argue that widely held companies are less prone to takeovers because shareholders can free-ride by not selling their shares and benefit from post-takeover profits. To make efficient takeovers possible, a corporate charter can incorporate exclusionary devices such as dilution of property rights to overcome the free-rider problem. Bagnoli and Lipman (1988) have shown that profitable takeovers of widely

[9]This was true in the case of Mead Johnson.

[10]In the US, if an attempt to purchase more than five percent of the shares of a target company is initiated, both the bidder as well as current management are legally compelled to disclose a statement (Bainbridge 1999).

[11]Although not explicitly required by law, there is evidence that managers acquire fairness opinions as protection against lawsuits initiated by unsatisfied shareholders (Kisgen et al. 2009).

held firms are possible without exclusion. The crucial feature are *finitely* many shareholders which enables the bidder to make some shareholders pivotal to impede free-riding. As our model contains a finite number of shareholders, we abstract from the free-rider problem and focus instead on informational frictions. Similar to the exclusion of shareholders to overcome the free-rider problem as in Grossman and Hart (1980), we show that excluding shareholders from learning additional information can be welfare-increasing.

Our paper is related to Levit (2017) where only one party (a board) has private information and advises shareholders about a potential takeover in form of cheap talk communication. The bidder in Levit (2017) does not possess private information which shuts down signaling. In our model, signaling is crucial as the interaction of costly signaling by the bidder and cheap talk by the incumbent drives our main result.

Marquez and Yılmaz (2008) analyze a framework in which shareholders privately observe conditionally independent signals about the potential value improvement of a takeover with an uninformed bidder. Takeovers may not be feasible as the bidder faces a lemons problem. Ekmekci and Kos (2016) are able to resolve this issue by introducing a large minority shareholder. Ekmekci and Kos (2014) allow for information acquisition by the bidder and the shareholders. It is shown that unilateral access to information for the bidder is of no use to him because all his information will be encoded in the price offer. Shareholders in their model prefer imprecise information because very detailed information provision may lead to a complete market breakdown. Ekmekci et al. (2016) derive the optimal mechanism for the sale of a company when the buyer privately knows both, the security benefits he will create and his private benefits of control. Current management and owner of the target firm are identical and the current value of the firm is commonly known.

In our model, the bidder signals his private information via his tender offer and we construct a fully revealing equilibrium (on the bidder's side). In this way, our model is related to Hirshleifer and Titman (1990) and Burkart and Lee (2015). In Hirshleifer and Titman (1990), there exist mixed equilibria by which the bidder is induced to completely reveal his type. In our setting, mixed strategies cannot be used to attain separation without a cheap talk recommendation by the incumbent. Further, Burkart and Lee (2015) show how an external bidder can reveal his type by committing to relinquish private benefits. The bidder in their paper is commonly known to be value-improving. We find an alternative way of inducing bidder separation: strategic management recommendations. In our setting, it is not known ex ante who is better equipped to steer the company and thus separation is a necessary condition for efficient control allocation.

In the context of mergers, Hansen (1987), Berkovitch and Narayanan (1990) and Eckbo et al. (1990) study a setting in which separation can be obtained by a mix of cash and equity offers. We are interested in the allocation of control rights, whereas they consider the case in which two companies want to exploit synergies of a merging asset.

Literature on Communication and Corporate Governance

Up to now, a plethora of papers has analyzed strategic communication in manifold economic environments. The seminal paper on cheap talk by Crawford and Sobel (1982) analyzes a situation with one informed sender and one uninformed receiver with a continuous action space. We combine costly signaling and cheap talk in a sequential model: an informed sender (the bidder) sends a costly message (his price) to which an informed receiver (the incumbent manager) reacts by sending a cheap talk recommendation. Accordingly, the manager is sender and receiver of information in one. Our paper features an endogenous conflict of interest of shareholder and management as in Antic and Persico (2018; 2019). They provide a model of information transmission in which an expert shareholder chooses how much information to communicate about the return of an investment to a controlling shareholder who then decides on the investment strategy. A main innovation is that the bias is determined endogenously in the cheap talk stage through share acquisitions in a competitive market prior to the communication stage. As a result, perfect information transmission is obtained. In our model, the conflict of interest is not determined by the communicating parties, but through the price offer of the external bidder. Hence, full information transmission is in general not feasible.

Malenko and Tsoy (2019) show that advisors in English auctions (such as managers in takeovers) who are biased towards overbidding can increase expected revenues and allocative efficiency via cheap talk messages. In their paper, cheap talk advice influences the bidders' optimal price offer whereas in our model, the bidder's price offer affects the cheap talk message. Other papers that analyze strategic information transmission in the realm of corporate governance include: Adams and Ferreira (2007) analyze the monitoring and advisory role of a board. It is shown that, to facilitate communication between board and CEO, the optimal board is not completely independent. Harris and Raviv (2008) examine the optimal board size and composition in the light of communication within the board. Kakhbod et al. (2019) study the design of an advisory committee when heterogeneous shareholders can acquire information and communicate. Malenko (2013) considers communication by directors of a company board in the presence of conformity motives. Finally, Levit (2018) shows how the threat of voice and exit can help activist shareholders to communicate more effectively.

2. Model

Environment

An external bidder E considers the acquisition of a company. The target has a continuum of shares of measure one outstanding. The bidder makes a publicly observable tender offer by posting a price $p_E \in \mathbb{R}_+$. For a successful takeover he must acquire at least a fraction $\lambda > 0$ of the outstanding shares. The offer is conditional: if a fraction less than λ of the shares is tendered, the offer becomes void.

The company is currently owned by a single (initial) shareholder (she) and the incumbent manager (I). We generalize the ownership structure to any finite number of shareholders in Section 5. Manager I owns a fraction $s \in (0, \lambda)$ of the shares making the initial shareholder hold a controlling stake in the company of $1 - s$.[12] The incumbent cannot make a counteroffer and he is not allowed to tender his shares.[13] It will become clear that I has, endogenously, no incentive to trade his shares during the takeover.

The game has three periods indexed by $t \in \{1, 2, 3\}$. At $t = 1$, the external bidder posts his tender offer p_E. At $t = 2$ and after observing the price, I sends a cheap talk message m_I. Finally at $t = 3$ and given p_E and m_I, the shareholder decides which fraction $\gamma \in [0, 1]$ of her share endowment $1 - s$ to tender. Neither the incumbent manager can commit to tell the truth nor can the shareholder commit to a tender rule ex ante. The timing of events is summarized in Figure 2.1.

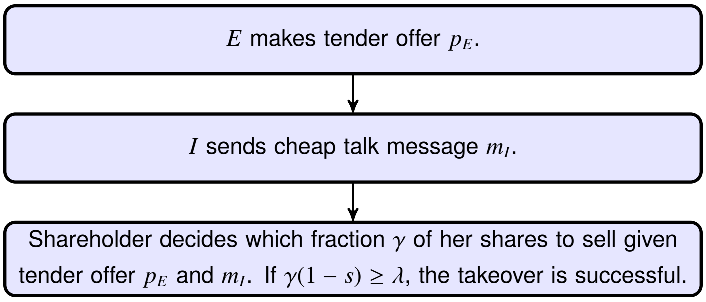

Figure 2.1: Timing of the Game.

Information

As a novelty in the literature on corporate takeovers, whether a takeover is socially efficient depends on both the bidder's and the incumbent's private information. The bidder privately observes his type ω_E which comprises information about his ability to run the company after a successful takeover. Furthermore, the manager has private *inside* information about the

[12]As noted in the Introduction, the shareholder may also own all shares if I is interested in the well-being of the company even after a successful takeover due to compensation schemes such as gradually vesting equity, stock options or bonus payments.

[13]The reasons for this selling restriction are manifold and include, for instance, insider trading restrictions and incentive features in his employment contract such as stock options and vesting equity not immediately tradable. Further, employment contracts often specify retention periods even after the managers leave the company. Our results will imply that these features are highly desirable to increase efficiency in takeovers.

company's future profits under his management denoted by ω_I.[14] The shareholder does not know either of the two types. The bidder's and the incumbent's types are independently[15] distributed on $[0, 1]$ according to continuous and commonly known cdfs F_E and F_I. Both cdfs admit densities f_E and f_I with full support. Finally, we denote $\mu_I := \mathbb{E}[\omega_I]$.

Payoffs

The firm's profits π are given by ω_E if the takeover attempt is successful and ω_I if the incumbent stays in charge. If no takeover occurs, the shareholder will earn ω_I per share irrespective of her tendering decision. Conditional on a successful takeover, tendering a fraction γ of her share endowment yields p_E per share and security benefits (as residual claim on the firm's assets) of ω_E on the remaining $(1-\gamma)(1-s)$ shares. This results in the following shareholder utility:

$$v = \begin{cases} (1-s)\big(\gamma p_E + (1-\gamma)\omega_E\big), & \text{if takeover successful} \\ (1-s)\omega_I, & \text{otherwise.} \end{cases}$$

The incumbent's utility is given by his share endowment under either control allocation. In Section 5, we generalize his payoff structure and include private benefits of control. These may accrue due to a fixed above market rate salary or reputational concerns. In the current version of the model, the incumbent's utility is given by:

$$u_I = \begin{cases} s\omega_E, & \text{if takeover successful} \\ s\omega_I, & \text{otherwise.} \end{cases}$$

Observe that even without private benefits, the interests of the incumbent and the shareholder are generally not perfectly aligned because the shareholder's payoff is a function of the tender offer p_E which is an equilibrium outcome. Conversely, the incumbent is solely interested in

[14]Even though the manager runs the company at the time of the tender offer, he still may possess superior inside information about the future profitability under his management. He may know, for example, about the state of an R&D project or secret negotiations with a large potential customer. In general, the empirical literature suggests that the strongest form of the efficient market hypothesis does not hold true and not all insider information is incorporated in the market price. Since ω_I and ω_E have different distributions, the uncertainty regarding ω_I could be smaller and the expected value of ω_E could be larger.

[15]The firm value under the different managements can be correlated. The correlated part, however, is likely to be publicly observable (for example via information contained in the annual financial statements). Hence, one can normalize the common component to zero.

the bidder's type. The bidder's utility is given by:

$$u_E = \begin{cases} \gamma(1-s)(\omega_E - p_E), & \text{if takeover successful} \\ 0, & \text{otherwise.} \end{cases}$$

E derives constant utility normalized to zero if no takeover occurs. And if the tender offer is successful, E buys a fraction of $\gamma(1-s) \geq \lambda$ shares from the shareholder at per share costs of p_E and gains ω_E on the shares acquired.[16]

Strategies

Given the observed tender offer p_E and the incumbent's message m_I, a (pure) strategy for the shareholder specifies a fraction γ of tendered shares, i.e. $\gamma : \mathbb{R}_+ \times M_I \to [0,1]$ where $M_I = [0,1]$ denotes the message space. An incumbent's strategy is a mapping from the set of price offers and his type space into the message space, i.e. $m_I : \mathbb{R}_+ \times [0,1] \to [0,1]$. Finally, a (pure) strategy for the bidder, $p_E : [0,1] \to \mathbb{R}_+$, specifies a tender offer for any type ω_E. Throughout this paper, we assume that indifference on the shareholder side is broken in favor of a takeover.[17] Our solution concept is perfect Bayesian equilibrium in pure strategies – henceforth referred to as *equilibrium.* Whenever necessary, we restrict attention to off-path beliefs satisfying the intuitive criterion by Cho and Kreps (1987). An equilibrium requires that (equilibrium objects are denoted with a star):

1. given tender offer p_E^* and message m_I^*, the shareholder chooses optimally how many shares to tender, i.e. she chooses γ^* to maximize $\mathbb{E}[v|p_E^*, m_I^*]$.

2. Given p_E^* and γ^*, I chooses $m_I^* \in argmax\, \mathbb{E}[u_I|p_E^*, \omega_I, \gamma^*]$.

3. Given m_I^* and γ^*, E chooses p_E^* to maximize his expected profits $\mathbb{E}[u_E|\omega_E, m_I^*, \gamma^*]$.

4. Whenever possible, all players update their posterior belief according to Bayes' rule.

First-best Allocation

In our setting, ex post efficiency requires that the potential manager with the higher type leads the company. The following definition establishes the notion of first-best in our setting.

Definition 2.1

We call any equilibrium (firm value-) optimal or first-best if it leads to a takeover if and only if $\omega_E \geq \omega_I$.

[16]As we abstract from the free-rider problem, there is no need to model private benefits for the external bidder to make takeovers feasible.

[17]This assumption is made to circumvent an openness problem and to ensure existence of equilibria.

3. Informed Shareholder

Before we analyze the implications of strategic information transmission by the incumbent, we turn to the case of an informed shareholder who knows ω_I. In Section 4.3, we argue that, endogenously, the shareholder prefers to be well-informed.[18] For a given price offer p_E and induced posterior type $\mathbb{E}[\omega_E|p_E]$, a shareholder who knows ω_I will want to tender whenever there is some $\gamma(1-s) \geq \lambda$ such that

$$\gamma p_E + (1-\gamma)\mathbb{E}[\omega_E|p_E] \geq \omega_I. \tag{3.1}$$

A takeover is desired by the shareholder if there is a convex combination of the posted price and posterior expected bidder type that weakly exceeds the benefits from leaving the incumbent in charge. Given the tendering decision of the shareholder and his private type ω_E, the external bidder chooses a price $p_E \in \mathbb{R}_+$ to maximize his expected utility. The following proposition establishes that, in any equilibrium, the bidder's tender offer and the shareholder's tendering decision are jointly inconsistent with the first-best allocation, i.e. ex post inefficient.

Proposition 2.1
Suppose the shareholder is perfectly informed about ω_I. Then, there is no equilibrium in which the first-best allocation implemented.

The intuition behind Proposition 2.1 is as follows. In order to obtain first-best, the shareholder's tendering inequality (3.1) must be equivalent to $\omega_E \geq \omega_I$. The proof shows that this is only the case if $p_E = \omega_E$, i.e. first-best is only attainable if E makes zero profits and fully reveals his type. We show, however, that zero profits cannot be part of an equilibrium with full separation that is ex post efficient because higher types would imitate price offers of lower types: in a fully separating equilibrium that implements the first-best allocation, every bidder type has a strictly positive takeover probability. Consequently, for all $\omega_E > 0$ there is a profitable downward deviation. First-best is therefore not attainable with full information about ω_I.

Remark 2.1
Our setting is restricted to price offers and there is no commitment regarding the allocation rule: the shareholder will tender only if she finds it optimal given p_E and ω_I. For the case where all shares must be traded for a change in control, i.e. $\lambda = 1 - s$, Proposition 2.1 follows from the classical impossibility result in bilateral trade by Myerson and Satterthwaite (1983) and (ex post) efficient trade is also not feasible in the more general mechanism design problem. For $\lambda < 1 - s$, the impossibility of first-best does not follow from Myerson and

[18]We complement the analysis with a discussion of potential information channels.

Satterthwaite (1983) because we consider interdependent values. If the shareholder does not tender her entire share endowment, i.e. $\gamma < 1$, the shareholder participates on the expected value improvement by the bidder. Hence, there is some degree of alignment of interests among shareholder and external bidder which may give rise to efficient trade. Proposition 2.1 shows, however, that ex post efficiency is still not attainable with take-it-or-leave-it price offers.

4. Strategic Management Recommendation

We now analyze the case in which the shareholder's only source of information regarding ω_I is the incumbent's cheap talk message. We show that there exists an equilibrium in which the bidder perfectly reveals his type *because of* the incumbent's cheap talk recommendation. Beyond this, we establish that informative cheap talk can implement the first-best control allocation and thus dominates a setting where the shareholder is fully informed in terms of welfare. Then, we derive the set of equilibria when cheap talk is uninformative and show that separation of the bidder's type cannot be attained in this case.

4.1 Informative Cheap Talk

Cheap talk not only (partially) informs the shareholder about ω_I, but also induces the bidder to fully reveal his type. As a short cut, we will refer to an equilibrium with full information about the bidder's type as *fully revealing* or *fully separating*. In contrast to the previous benchmark, since shareholder's and incumbent's interests are not completely aligned, cheap talk prevents the shareholder from becoming fully informed. This, however, will turn out to be beneficial for the control allocation.

Tendering Decision and Cheap Talk Message

As the shareholder plays a pure strategy in $t = 3$, there are only two outcomes with respect to the final control allocation given p_E and m_I and the associated posteriors: either a takeover occurs with certainty or never. At the cheap talk stage, the manager knows p_E and therefore, he knows (on the equilibrium path) whether a takeover will occur if he sends some message m_I. He is indifferent between both outcomes whenever $s\mathbb{E}[\omega_E|p_E] = s\omega_I$ which in turn implies that a takeover is endorsed by I whenever

$$\omega_I \leq \omega_I^*(p_E) := \mathbb{E}[\omega_E|p_E]. \tag{4.1}$$

The indifference type ω_I^* equals the posterior expected type of E and is thus a function of p_E. When it is clear from the context, we drop the price. Notice that, by the common support assumption, for any p_E and induced posterior belief about ω_E there is a unique cutoff

$\omega_I^* \in [0, 1]$ at which the incumbent is indifferent. The implication of informative cheap talk is illustrated in Figure 2.2. If the incumbent manager is not well-equipped to steer the company (low ω_I) and if he has a sufficiently high posterior expectation about the bidder's type, he prefers the shareholder to tender her shares. Conversely, if the manager knows that he is very skilled he recommends not to tender. It follows that he sends at most two non-outcome equivalent messages.

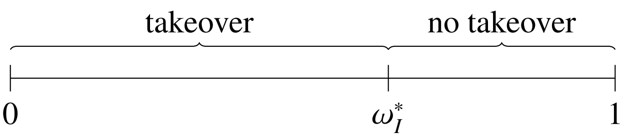

Figure 2.2: The ω_I-Type Space with Cutoff Type ω_I^*.

Bidder's Payoff
If the shareholder follows I's recommendation, the bidder's expected utility is given by:

$$F_I(\omega_I^*(p_E))\,\gamma(p_E)(1-s)(\omega_E - p_E). \tag{4.2}$$

When the bidder chooses his tender offer at $t = 1$, the incumbent's message is not known since it will depend on I's private type ω_I. The bidder's expected utility thus equals the probability that the incumbent's type is below the cutoff type – $F_I(\omega_I^*(p_E))$ – and the amount of shares tendered, $\gamma(p_E)(1-s)$, times the profit earned on each share acquired by the bidder, $(\omega_E - p_E)$. Equation (4.2) illustrates that, if the shareholder follows I's message, the final allocation is fixed by the incumbent's indifference type $\omega_I^*(p_E)$ for any p_E and the corresponding posterior type $\mathbb{E}[\omega_E|p_E]$. The following main result characterizes a fully separating equilibrium with informative cheap talk.

Theorem 2.1
There is an equilibrium in which E fully reveals his type by posting

$$p_E^* = \mathbb{E}[\omega_I|\omega_I \leq \omega_I^*(\omega_E)].$$

Furthermore,

1. *if $\omega_I \leq \omega_I^*(p_E)$, then $m_I^* \in [0, \omega_I^*(p_E)]$, and a takeover occurs with probability one;*
2. *if $\omega_I > \omega_I^*(p_E)$, then $m_I^* \in (\omega_I^*(p_E), 1]$, and a takeover occurs with probability zero.*

Finally, it holds that $\gamma^* = \frac{\lambda}{1-s}$.

Theorem 2.1 establishes that there exists an equilibrium in which the bidder fully reveals his type via his tender offer. Given p_E^*, the incumbent's posterior belief assigns probability one to the true bidder type on the equilibrium path and I's indifferent type becomes $\omega_I^* = \omega_E$. The manager sends a binary cheap talk message in favor or against the takeover. And finally, the shareholder finds it optimal to follow I's message given p_E^* and her posterior beliefs of ω_E and ω_I. If a takeover occurs she tenders as few shares as possible, i.e. $\gamma^* = \frac{\lambda}{1-s}$.

Tender Offer

After informative cheap talk, the fully revealing equilibrium exists because of the recommendation by the manager: it enables separation by introducing a way to compensate higher bidder types for posting higher prices. To see this, consider the bidder's *per share* profit $F_I(\omega_I^*(p_E))[\omega_E - p_E]$. If I's type is below ω_I^*, he recommends a takeover. And if the shareholder follows I's message, the takeover probability is given by $F(\omega_I^*)$. As $\omega_I^* = \mathbb{E}[\omega_E|p_E]$, the takeover probability strictly increases in the posterior expected bidder type induced by the tender offer p_E. Separation is feasible because increasing p_E induces a higher posterior expectation and therefore a higher takeover probability.

More precisely, for a fully separating equilibrium to exist, there has to be a strictly increasing (and thus invertible) function $p_E : [0,1] \to \mathbb{R}_+$ such that, given any ω_E, E chooses his bid $p \in \mathbb{R}_+$ optimally:

$$p = p_E(\omega_E) \in argmax \;\; F_I[\omega_I^*(p_E^{-1}(p))](\omega_E - p). \tag{4.3}$$

For any ω_E, this maximization yields the bidder-optimal price offer given that the shareholder follows I's message. For any particular bid p, the takeover probability is thus determined by $F_I[\omega_I^*(p_E^{-1}(p))]$. The unique solution to (4.3) is given by $p_E^*(\omega_E) = \mathbb{E}[\omega_I|\omega_I \leq \omega_I^*]$, where, in the fully separating equilibrium, $\omega_I^* = \omega_E$. It is then easy to verify that given incumbent and shareholder form beliefs according to $p_E^*(\omega_E)$ it is indeed optimal for type ω_E to bid $p = p_E^*(\omega_E)$ relative to any other $p \in [p_E^*(0), p_E^*(1)]$.

Moreover, no bidder type wants to deviate to an (off-path) tender offer above $p_E^*(1)$ because $p_E^*(1)$ ensures a takeover with probability one. Hence, independent of off-path beliefs, deviating to a higher price only increases the costs but leaves the benefits unaffected. Further, as $p_E^*(0) = 0$ and $p_E \in \mathbb{R}_+$, we need not consider downward deviations to off-path prices.

Cheap Talk Constraints

In the equilibrium constructed in Theorem 2.1, the shareholder follows the incumbent's recommendation. To verify this, one has to show that, given the equilibrium price p_E^* and $m_I^*(\omega_I \leq \omega_I^*)$ such that the incumbent endorses a takeover, the shareholder prefers tendering $\gamma \geq \frac{\lambda}{1-s}$ shares over leaving the incumbent in charge. That is, for some $\gamma \geq \frac{\lambda}{1-s}$, it has to hold that

$$\gamma p_E^*(\omega_E) + (1-\gamma)\mathbb{E}[\omega_E | p_E^*(\omega_E)] \geq \mathbb{E}[\omega_I | \omega_I \leq \omega_I^*(\omega_E)]. \tag{4.4}$$

Inequality (4.4) implies that, given the manager favors the takeover, the shareholder finds it indeed optimal to tender sufficiently many shares to enable a successful takeover. Conversely, suppose that the manager does not recommend a takeover at $p_E^*(\omega_E)$, (i.e. $m_I^*(\omega_I > \omega_I^*)$). Then, the shareholder finds it optimal to follow the recommendation if

$$\gamma p_E^*(\omega_E) + (1-\gamma)\mathbb{E}[\omega_E | p_E^*(\omega_E)] < \mathbb{E}[\omega_I | \omega_I > \omega_I^*(\omega_E)]. \tag{4.5}$$

It is sufficient to check inequality (4.5) for $\gamma = \frac{\lambda}{(1-s)}$ because $\mathbb{E}[\omega_E | p_E] \geq p_E$ has to hold in equilibrium as otherwise, some bidder type would make strictly negative profits. Finally, observe that the bidder's tender offer, $p_E^* = \mathbb{E}[\omega_I | \omega_I \leq \omega_E]$, is the shareholder's outside option of leaving the incumbent in charge given that he sends a message in favor of a takeover. As the shareholder receives exactly her outside option on the shares tendered, E obtains all expected gains he creates by taking control over the company. The shareholder participates on the bidder's value improvement via the shares that are not tendered $(1 - s - \lambda)$.

Efficient Control Allocation

An immediate corollary of Theorem 2.1 is that this fully revealing equilibrium induces the first-best allocation of control rights and consequently, is more efficient than a situation with a fully informed shareholder (Section 3).

Corollary 2.1

The equilibrium with informative cheap talk in Theorem 2.1 induces the first-best control allocation. In particular, it exhibits a strictly larger expected firm value than any equilibrium in which the shareholder is fully informed about ω_I.

The intuition is straightforward: as $\omega_I^* = \omega_E$, the incumbent recommends a takeover if and only if it is efficient. As the shareholder finds it in her best interest to follow the recommendation, the first-best control allocation is obtained. Observe that there will never be perfect information transmission in the separating equilibrium: the cutoff type ω_I^* equals ω_E and I merely sends a cutoff message revealing whether $\omega_I \leq \omega_E$ or not. Rather surprisingly, the equilibrium with informative cheap talk welfare-dominates our benchmark setup in which the shareholder is fully informed about ω_I. The intuition is as follows. The external bidder will post prices below his true type to make a profit on the takeover. If information is controlled by the incumbent manager via his message, he recommends a takeover whenever $\mathbb{E}[\omega_E | p_E^*] \geq \omega_I$. In equilibrium, the shareholder cannot do better than following I's recommendation. Conversely, if the shareholder is fully informed about ω_I and the bidder's price offer is fully separating[19], she tenders if and only if $\frac{\lambda}{1-s}p_E^*(\omega_E) + (1 - \frac{\lambda}{1-s})\omega_E \geq \omega_I$. Denote by $\tilde{\omega}_I := \frac{\lambda}{(1-s)}p_E^* + (1 - \frac{\lambda}{(1-s)})\omega_E$ the incumbent type at which a fully informed shareholder is exactly indifferent between a takeover and leaving the incumbent in charge. Then, $\tilde{\omega}_I < \omega_E$ holds because $p_E(\omega_E) = \omega_E$ can never be part of an equilibrium because this would imply zero profits (see Section 3). Therefore, there are types $\omega_I \in (\tilde{\omega}_I, \omega_E)$ for which a takeover does not occur with a fully informed shareholder but the first-best allocation would require it.

Put differently, the equilibrium message of I pools cases where the shareholder prefers to tender with cases where the shareholder would be better off not tendering.[20] To see this, note that $\tilde{\omega}_I < \omega_E = \omega_I^*$. Consequently, given ω_E and $p_E^*(\omega_E)$, for all $\omega_I \leq \tilde{\omega}_I$, the shareholder would tender if she knew ω_I. Conversely, for all $\omega_I > \tilde{\omega}_I$, the shareholder would leave the incumbent in charge as she does not fully internalize all gains from trade. If the shareholder can base her decision solely on m_I, she can only tell whether ω_I is larger or smaller than ω_I^*, but – as $\tilde{\omega}_I < \omega_I^*$ – she never infers if $\omega_I \in (\tilde{\omega}_I, \omega_I^*]$, where she would keep her shares with full information. The fact that she is *not perfectly informed* about the firm value is what enables the first-best allocation of control rights.

Remark 2.2
In our setting, we focus on cheap talk to alleviate the informational frictions because this seems to be the prevalent channel in practice. Alternatively, a shareholder could delegate (without commitment) the control right to the incumbent manager who then decides whether a takeover occurs or not at a given price offer. Due to the binary action, delegation and informative management recommendations are outcome-equivalent in our setting.[21] *In this sense, delegation can be an alternative instrument to achieve the first-best control allocation.*

Remark 2.3
Interestingly, the equilibrium in Theorem 2.1 is robust to the possibility of the bidder revising his offer after the incumbent's cheap talk message. To see this, suppose that after observing I's message E posts a new price $p_E(\omega_E, m_I^*)$, where m_I^* is the equilibrium message according to Theorem 2.1. This case is only relevant if $m_I^* \in (\omega_I^*, 1]$, because otherwise a takeover already occurs at the original price. But then, as $\omega_I^* = \omega_E$, the bidder knows that his type is smaller than ω_I and he cannot profit from revising his offer: this offer would need to exceed the shareholder's posterior expectation of ω_I, yielding a loss for E.

[19] If it was not fully separating, the efficient control allocation cannot be implemented (see Section 3).

[20] This misalignment is the reason why the message by the incumbent can never be fully revealing.

[21] See Dessein (2002) for an analysis of communication versus delegation with commitment and continuous action space.

4.2 Uninformative Cheap Talk

Since the recommendation of the manager is cheap talk, there always exists an equilibrium in which his message is uninformative. A message $m_I(p_E, \omega_I)$ is uninformative (or *babbling*) if for all $p_E \in \mathbb{R}_+$, $m_I(p_E, \omega_I)$ is independent of ω_I. Alternatively, one can interpret the results of this subsection as a benchmark in which the incumbent manager is not able to give a recommendation to the shareholder. Given an uninformative message of the manager, the next proposition characterizes the set of equilibria.

Proposition 2.2
In any babbling equilibrium, there exists a cutoff price $\hat{p}_E < 1$ such that:
if $\omega_E < \hat{p}_E$, a takeover never occurs;
if $\omega_E \geq \hat{p}_E$, E posts $\hat{p}_E$ and a takeover occurs with probability one.
Finally, it holds that $\gamma^* = \frac{\lambda}{1-s}$.

The result states that all equilibria with uninformative cheap talk are partially pooling in that all bidder types larger than some cutoff post the same price resulting in a takeover. For simplicity, we simply call these *pooling equilibria*. Further, in every pooling equilibrium, the shareholder tenders as few shares as possible such that a takeover still occurs. $\gamma^* = \frac{\lambda}{1-s}$ holds true in any pooling equilibrium because $\hat{p}_E < 1$ implies that $\mathbb{E}[\omega_E|\hat{p}_E] > \hat{p}_E$. Consequently, whenever $\gamma^* > \frac{\lambda}{1-s}$, then the shareholder could profitably deviate to tendering fewer shares gaining the security benefits $\mathbb{E}[\omega_E|\hat{p}_E]$ while losing $\hat{p}_E$ on the residual shares and still making the takeover successful. Moreover, Proposition 2.2 shows that without informative cheap talk, no separation can be induced with respect to the bidder's type apart from a single cutoff. The intuition behind this observation is that for any finer separation to exist, in order to post larger prices one has to incentivize higher bidder types with a higher probability of obtaining control. But such a screening device is missing here.

Hirshleifer and Titman (1990) show that in a model with a continuum of shareholders, separation of the bidder may be attainable if shareholders play mixed strategies.[22] Although we abstract from mixing, observe that even if we allowed the shareholder to play mixed strategies, Proposition 2.2 would still hold. To see this, note that the shareholder is indifferent between selling and keeping her shares if and only if

$$\gamma p_E + (1 - \gamma)\mathbb{E}[\omega_E|p_E] = \mu_I, \tag{4.6}$$

for some $\gamma \geq \frac{\lambda}{1-s}$. The first observation is that if there was separation, zero profits for bidder

[22] It is noteworthy, however, that mixing will always cause welfare losses and first-best can never be implemented as the allocation of control is probabilistic.

types $\omega_E > 0$ cannot be part of an equilibrium.[23] Hence, $\omega_E > p_E$ holds and therefore, the shareholder tenders as few shares as possible, i.e. $\gamma = \frac{\lambda}{(1-s)}$. Now denote the probability of a takeover, given that she is indifferent at p_E, by $\phi(p_E)$. By monotonicity of the bidder's payoff, higher types have a higher willingness to pay for a given takeover probability. To induce full separation, one needs that the bidder's strategy is strictly increasing in ω_E. For this to be optimal, higher types need to be compensated with a higher takeover probability. As the shareholder needs to mix at any price after which a takeover occurs with non-zero probability except for the price posted by $\omega_E = 1$, the indifference constraint (4.6) would need to hold for any type pair $0 < \omega_E < \omega'_E < 1$ posting prices $p_E < p'_E$ with $0 < \phi(p_E) < \phi(p'_E)$.[24] But since $\frac{\lambda}{(1-s)}p'_E + (1 - \frac{\lambda}{(1-s)})\omega'_E > \frac{\lambda}{(1-s)}p_E + (1 - \frac{\lambda}{(1-s)})\omega_E = \mu_I$, she cannot be indifferent at both prices which yields a contradiction. Therefore, in contrast to Hirshleifer and Titman (1990), full separation is not feasible through mixing.

Figure 2.3 shows the control allocation in a pooling equilibrium as described in Proposition 2.2.

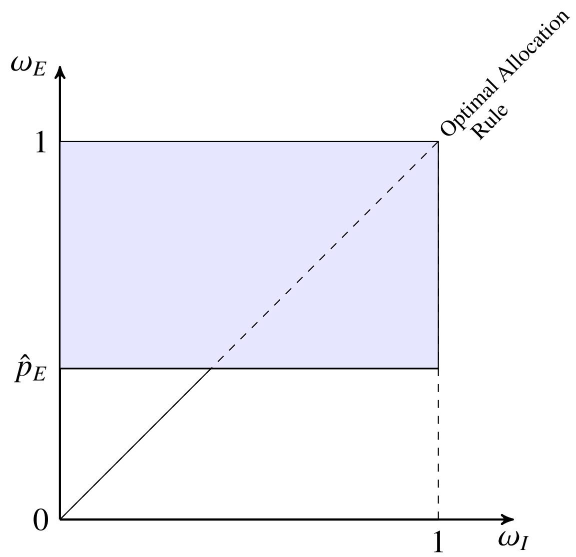

Figure 2.3: Optimal Allocation vs. Pooling Equilibria.

Independent of ω_I, a takeover occurs whenever $\omega_E \geq \hat{p}_E$, so the blue area depicts those type pairs for which a takeover is realized. All optimal allocations, however, lie above the 45 degree line. Thus, there are pairs for which inefficient takeovers occur (blue triangle below the 45 degree line) and pairs for which I remains in charge although E would be optimal

[23]The precise argument requires some work. If there is full separation, we know that there exists an $\tilde{\omega}_E < 1$ such that all $\omega_E \in [\tilde{\omega}_E, 1]$ have a strictly positive takeover probability. If this was not true, any type close enough to 1 could offer μ_I and take over the company with certainty – making strictly positive profits. Hence, for all $\omega_E > \tilde{\omega}_E$, zero profits cannot be an equilibrium outcome as these types could deviate to the price offer $p_E(\tilde{\omega}_E)$ and realize a strictly positive profit.

[24]Such a type pair always exists because $\mu_I < 1$.

(white triangle above the 45 degree line). Not surprisingly, first-best cannot be attained in a pooling equilibrium as no information is transmitted about ω_I and only very limited about ω_E.

Remark 2.4

Without informative cheap talk, the first-best allocation of control rights is not attainable.[25]

4.3 Endogenous Shareholder Learning

As noted in Section 3, a shareholder who is fully informed about the current firm value prevents the first-best allocation of control rights whereas cheap talk is able to implement first-best. A problem arises when shareholders themselves can choose the information they obtain. In practice, when a corporate bidder aims at taking over a target company, outside experts or advisors such as investment banks and consulting firms are frequently hired to conduct a fairness opinion. The aim of such assessments is to credibly inform the shareholders about the value of the company (Kisgen et al. 2009). Another interpretation of shareholders' additional learning is that regulation forces management to provide (credible) information to shareholders. Corporate law gives shareholders the opportunity to enforce a fairness opinion and/or management disclosure (Kisgen et al. 2009; Bainbridge 1999).

Irrespective of the source of information, consider now a situation where the shareholder has observed p_E and m_I. Then, if she can freely choose the level of information about ω_I, she will always choose the fully informative signal because she faces a pure decision-theoretic problem at this stage (a formal treatment can be found in Lemma 2.1 in Appendix B):

Remark 2.5

If possible, the shareholder acquires the fully informative signal about ω_I.

When the shareholder can learn ω_I perfectly, the message m_I is irrelevant and E will anticipate that the shareholder will become fully informed. From Section 3, we know that first-best is not attainable in this situation. Through the lens of our model, a setting in which shareholders can force management to conduct a fairness opinion or disclose additional information is welfare-destroying. Our results therefore suggest that management recommendations may suffice to overcome the informational frictions in the market for corporate control and additional sources of information may in fact harm efficiency.

5. Extensions

We now generalize the model in two important directions. First, most companies are not owned by a single shareholder but have multiple owners. We allow for this possibility by

[25] We only illustrate the point graphically. The formal proof is obvious.

assuming that the target firm is owned by some finite number of shareholders. It will turn out that our results remain true with any finite number of shareholders. The only difference is that there exist equilibria exhibiting coordination failures.

Second, typically, the incumbent manager of a company will enjoy private benefits B_I from remaining in charge. For instance, B_I may stem from a fixed above market wage or general benefits from being in charge (such as status, amenities etc.). Private benefits will make the manager more reluctant to recommend a takeover and drive a wedge between the optimal allocation rule and the preferences of the incumbent. We will prove, however, that an equilibrium similar to Theorem 2.1 still exists and this can again welfare-dominate a situation with informed shareholders. In Section 6, we discuss how the managerial salary scheme can be adjusted to implement first-best in presence of private benefits.

5.1 A Model with Multiple Shareholders and Private Benefits

The company is now owned by $j \in \{1, \ldots, J\}$ initial shareholders and I. A typical shareholder j owns a fraction of s_j shares and all shareholders jointly own $\sum_{j=1}^{J} s_j = 1 - s > \lambda$. The incumbent still owns the remaining $s < \lambda$ shares. The game evolves as before: first, E posts a tender offer p_E to which I responds with a cheap talk message m_I. In the final stage of the game, the shareholders decide individually and simultaneously which fraction $\gamma_j \in [0, 1]$ of their share endowment s_j to tender given p_E and m_I. Let T denote the total amount of shares tendered, i.e. $T := \sum_{j=1}^{J} s_j\gamma_j$.

The payoff of shareholder j is composed as follows. If no takeover occurs, shareholder j will earn ω_I per share irrespective of her tendering decision. Conditional on a successful takeover, tendering γ_j of the s_j shares yields p_E per share and security benefits of ω_E on the residual $1 - \gamma_j$ shares. This results in the following utility of shareholder j:

$$v_j = \begin{cases} s_j\big(\gamma_j p_E + (1-\gamma_j)\omega_E\big), & \text{if takeover successful} \\ s_j\omega_I, & \text{otherwise.} \end{cases}$$

As noted above, besides being interested in the value of his shares, the incumbent also enjoys private benefits $B_I \geq 0$ from being in charge. B_I is common knowledge. Let $b_I := \frac{B_I}{s}$ denote I's private benefit per share. We will refer to b_I as I's *bias*. The incumbent's utility is given by:

$$u_I = \begin{cases} s\omega_E, & \text{if takeover successful} \\ s\omega_I + B_I, & \text{otherwise.} \end{cases}$$

The bidder's utility is as follows

$$u_E = \begin{cases} T(\omega_E - p_E), & \text{if takeover successful} \\ 0, & \text{otherwise.} \end{cases}$$

Strategies
Given the observed tender offer p_E and the incumbent's message m_I, a (pure) strategy for shareholder j specifies a fraction γ_j of tendered shares, i.e. $\gamma_j : \mathbb{R}_+ \times M_I \to [0,1]$. We continue to assume that $M_I = [0,1]$. An incumbent's strategy is a mapping from the set of price offers and his type space into the message space, i.e. $m_I : \mathbb{R}_+ \times [0,1] \to [0,1]$. Finally, a (pure) strategy for the bidder $p_E : [0,1] \to \mathbb{R}_+$ specifies a tender offer for a type ω_E. We still assume that indifference on the shareholder side is broken in favor of a takeover. The solution concept remains perfect Bayesian equilibrium in pure strategies and if necessary, we keep restricting attention to off-path beliefs satisfying the intuitive criterion by Cho and Kreps (1987). An equilibrium requires:

1. given tender offer p_E^*, message m_I^*, and given the tendering decision of the other shareholders, γ_{-j}^*, any shareholder $j \in \{1, \ldots, J\}$ chooses optimally how many shares to tender, i.e. she chooses γ_j^* that maximizes $\mathbb{E}[v_j | p_E^*, m_I^*, \gamma_{-j}^*]$.

2. Given p_E^* and γ_j^*, I chooses $m_I^* \in \mathit{argmax}\, \mathbb{E}[u_I | p_E^*, \omega_I, \gamma_j^*]$ for all $j = 1, \ldots, J$.

3. Given m_I^* and γ_j^*, E chooses p_E^* to maximize his expected profits $\mathbb{E}[u_E | \omega_E, m_I^*, \gamma_j^*]$ for all $j = 1, \ldots, J$.

4. Whenever possible, all players update their posterior belief according to Bayes' rule.

5.2 Results

Fully Informed Shareholders
As before, if shareholders were perfectly informed about ω_I, the first-best allocation of control rights is not attainable. Observe that in this scenario, the incumbent and thus also his bias have no influence. The only difference is at the tendering stage. Since the company is owned by multiple shareholders, it may be the case that no single shareholder holds a majority stake individually ($s_j < \lambda$ for all j). Hence, now there also exist equilibria exhibiting a coordination failure as follows: if a shareholder expects all other shareholders not to tender, her decision does not have any influence on the outcome and thus she may as well not tender. In equilibrium, no shareholder ever tenders.[26] It is intuitive that the potential coordination

[26]This relies on the conditional form of the offer which becomes void if a total fraction less than λ shares is tendered.

failure will not improve welfare in our setting. The following proposition extends the result from Section 4 to the general ownership structure.

Proposition 2.3
Suppose shareholders are perfectly informed about ω_I. Then, there is no equilibrium in which the first-best allocation implemented.

The same logic as in the proof of Proposition 2.1 obtains here (and the proof is thus omitted): a necessary condition for first-best is full separation on the bidder's side, but in any ex post efficient fully separating equilibrium, the bidder must gain strictly positive expected profits. Thus, the equilibrium price must be lower than the bidder type. As shareholders compare a convex combination of price and expected security benefits with firm value under incumbent management, there will always be misallocations of control.

Uninformative Cheap Talk
There are of course always babbling equilibria. Since no information is transmitted in such equilibria, I's bias b_I does again not matter for the equilibrium outcome. b_I will, however, define a set in which babbling is the unique outcome of the cheap talk stage. Babbling equilibria will, similar to the basic model, either feature a cutoff structure or have no takeover as the certain outcome. The next proposition characterizes the set of these equilibria.

Proposition 2.4
There always exists a babbling equilibrium. In any such equilibrium,

1. *either a takeover never occurs;*
2. *or there exists a cutoff price $\hat{p}_E < 1$ such that:*
   if $\omega_E < \hat{p}_E$, a takeover occurs with probability zero;
   if $\omega_E \geq \hat{p}_E$, E posts $\hat{p}_E$ and a takeover occurs with probability one;
   further, it holds that $T^*(\hat{p}_E) = \lambda$;
3. *or $\hat{p}_E = 1$ and a takeover occurs if and only if $\omega_E = 1$. It holds that: $T^*(\hat{p}_E) \geq \lambda$.*

Proposition 2.4 shows existence of three different kinds of equilibria: first, a takeover may never occur if no shareholder individually holds a majority stake. As no shareholder is pivotal on her own, never selling any shares constitutes an equilibrium – independent of price offers and beliefs about ω_E and ω_I.

Second, there are cutoff equilibria as in Proposition 2.2. In those, shareholders jointly tender $T^* = \lambda$ shares whenever a takeover occurs. The underlying argument goes back to Bagnoli and Lipman (1988) who analyze a complete information takeover game with finitely many shareholders. The idea is that in equilibrium, whenever the price p_E is strictly below the security benefits after a successful takeover, the gain on keeping a share is larger than tendering if this decision does not affect the overall success of the takeover. Hence, in any pure strategy equilibrium with a takeover, every shareholder is pivotal with all the shares she tenders. If any shareholder would tender more shares, she would have a profitable deviation to tender strictly less while still making the takeover successful. As our setting entails asymmetric information, the true security benefits are not necessarily known by the shareholders. One can, however, easily see that whenever $p_E < \mathbb{E}[\omega_E|p_E]$, the logic by Bagnoli and Lipman (1988) applies ($p_E < \mathbb{E}[\omega_E|p_E]$ arises endogenously from the bidder's incentives to lower his price in order to maximize profits).

As the first equilibrium type, case three only exists if no shareholder individually holds a majority stake. Then, for all $p_E < 1$ no shareholder ever tenders sufficiently many shares to make another shareholder pivotal. Thus, at any $p_E < 1$ selling no shares is a best response for shareholders. $p_E = 1$ is only posted by the highest type $\omega_E = 1$ because all other types would make strictly negative profits. As post-takeover security benefits equal the price offer, i.e. $p_E = \mathbb{E}[\omega_E|p_E] = 1$, shareholders are indifferent between any γ_j that makes the takeover succeed and therefore, $T^*(1) \in [\lambda, 1-s]$.

Informative Cheap Talk

We now analyze equilibria with informative cheap talk. As the incumbent enjoys private benefits $B_I \geq 0$ from remaining in charge, I is now indifferent between a takeover and remaining in charge if $s\omega_I + B_I = s\mathbb{E}[\omega_E|p_E]$. Recalling that $b_I = \frac{B_I}{s}$, his indifferent type is then

$$\omega_I^* := \max\{\mathbb{E}[\omega_E|p_E] - b_I; 0\}.$$

The intuition is the same as before: whenever $\omega_I \leq \omega_I^*$, the incumbent favors a takeover. In contrast to the basic model without bias, informative cheap talk is harder to attain. Intuitively, if the incumbent only cares about remaining in charge, independent of ω_E and ω_I, there cannot be any meaningful communication. The following result shows that also with multiple shareholders and strictly positive bias, there exists an equilibrium with informative cheap talk in which the bidder fully reveals his type via his tender offer.

Theorem 2.2

There exists a $\overline{b}_I > 0$ such that for all $b_I \leq \overline{b}_I$, there is an equilibrium in which E fully reveals his type by posting

$$p_E = \begin{cases} \mathbb{E}[\omega_I|\omega_I \leq \omega_I^*(\omega_E)] + b_I, & \text{if } \omega_E \geq b_I \\ \omega_E, & \text{otherwise.} \end{cases}$$

Furthermore,

1. *if* $\omega_I \leq \omega_I^*(p_E)$, *then* $m_I^* \in [0, \omega_I^*(p_E)]$, *and a takeover occurs with probability one;*

2. *if* $\omega_I > \omega_I^*(p_E)$, *then* $m_I^* \in (\omega_I^*(p_E), 1]$, *and a takeover occurs with probability zero;*

and $T^* = \lambda$.

The statement of Theorem 2.2 is similar to Theorem 2.1. E fully reveals his type via the price offer. The incumbent sends, conditional on p_E, a binary cheap talk message in favor or against the takeover. And shareholders follow I's message in equilibrium and tender jointly as few shares as possible such that the takeover is realized.

The equilibrium only exists for small enough biases. Intuitively, if b_I grows very large (the private benefit B_I is large relative to the share endowment s), the incumbent always prefers retaining control. Hence, his message is never informative and there is no scope to screen the bidder's type.

If the equilibrium exists, i.e. b_I is smaller than $\overline{b}_I$, there are some noteworthy differences relative to the basic model.[27] The allocation is still determined by an incumbent's indifference type. As the incumbent is now biased against a takeover, this type has shifted downwards to $\omega_I^* = max\{\omega_E - b_I; 0\}$. As a consequence, there is an interval of bidder types $\omega_E \in [0, b_I)$ for which the incumbent never recommends a takeover. As shareholders still follow the message in equilibrium, these bidder types will never obtain control over the target company. Therefore, in equilibrium, they are indifferent between posting any price $[0, b_I)$ as all imply zero profits and it is a best response to post their true type as tender offer. The interesting case contains the bidder types strictly larger than b_I.[28] These have, on the equilibrium path, a strictly positive takeover probability. The equilibrium price changes in two dimensions. First, note that $\mathbb{E}[\omega_I|\omega_I \leq \omega_I^*(\omega_E)] = \mathbb{E}[\omega_I|\omega_I \leq \omega_E - b_I]$. Conditional on a message in favor of the takeover by the incumbent, shareholders learn that $\omega_I \leq \omega_I^* = \omega_E - b_I$, i.e. shareholders are more pessimistic about their outside option of leaving the incumbent in charge. This decreases the first component of the price relative to Theorem 2.1. On the other hand, the price now includes b_I itself with an additive component. The intuition is that a large bias will make the incumbent less likely to endorse the takeover. As shareholders follow I's message in equilibrium, this makes it more difficult for the bidder to realize the takeover and he is willing to increase his price offer relative to $\mathbb{E}[\omega_I|\omega_I \leq \omega_E - b_I]$.

Further and similar to the basic model, $T^* = \lambda$ such that all shareholders are pivotal with all the shares they tender. Hence, given the other shareholders' strategy, no shareholder wants

[27] $\overline{b}_I$ is defined in the proof of Theorem 2.2.

[28] If $\omega_E = b_I$, the takeover probability is exactly zero. Further, the equilibrium price is continuous and $p_E(b_I) = b_I$.

to tender fewer shares as this would make the takeover fail.

Welfare Comparison

As the incumbent is biased against the takeover, first-best will generally not be implementable with informative cheap talk. We can, however, show that there is an interval of biases $[0, \overline{b}_I^{FV}]$ such that if $b_I \in [0, \overline{b}_I^{FV}]$, the equilibrium with informative cheap talk from Theorem 2.2 improves the allocation of control rights compared to a situation where 1) shareholders are not informed at all (babbling equilibrium), and 2) shareholders are fully informed about the current firm vale (for example through endogenous learning).

Proposition 2.5

There exists a $\overline{b}_I^{FV} > 0$ such that for all $b_I \leq \overline{b}_I^{FV}$, there is an equilibrium with informative cheap talk by the incumbent that improves expected firm value compared to

1. *any equilibrium without (informative) communication;*
2. *any equilibrium with fully informed shareholders.*

Further, if b_I vanishes, expected firm value approaches first-best with informative cheap talk.

Proposition 2.5 establishes that even for a biased incumbent manager, cheap talk outperforms both equilibria where shareholders are fully informed or completely uninformed about ω_I. The intuition is again that in both cases the optimal allocation is bounded away from first-best, whereas welfare in the informative cheap talk equilibrium approaches first-best as b_I converges to zero: according to Theorem 2.2, a takeover occurs if and only if $\omega_I \leq max\{\omega_E - b_I; 0\}$. And as b_I converges to zero, this clearly becomes the first-best allocation rule.

The following section gives precise solutions for the case of uniformly distributed types. It turns out that welfare with informative cheap talk dominates the other two informational regimes for a relatively large interval of biases.

5.3 The Uniform Case

We now provide a numerical example of our results for the uniform case. To be precise, in this subsection we assume that ω_I and ω_E are i.i.d. random variables that are distributed according to the uniform distribution on $[0, 1]$.[29] For simplicity, we further assume that $J = 1$ and $\lambda = 1 - s$. We identify welfare with ex ante firm value and do not include B_I in this welfare measure. The equilibrium with informative cheap talk characterized in Theorem 2.2 exists for $b_I \leq \overline{b}_I = \frac{3}{2} - \sqrt{\frac{5}{4}} \approx 0.382$. On that account, a separating equilibrium can be

[29] So far, identical distributions were not needed.

supported for relatively large biases. The tender offer price is then given by $p_E = \frac{1}{2}\omega_E + \frac{1}{2}b_I$. Expected welfare is $\frac{2}{3} - b_I^2$ which converges to $\frac{2}{3}$ as b_I goes to zero – the first-best firm value.[30]

We now derive the maximal bias such that informative cheap talk increases firm value. To this end, we consider the (unique)[31] equilibrium without informative cheap talk: E offers $p_E = \frac{1}{2} = \mu_I$ if $\omega_E \geq \frac{1}{2}$ and a takeover occurs; otherwise, no takeover occurs. The (highest) ex ante firm value without communication equals $\mathbb{E}[\omega_I \mathbf{1}_{\{\omega_E < \mu_I\}}] + \mathbb{E}[\omega_E \mathbf{1}_{\{\omega_E \geq \mu_I\}}] = \frac{5}{8}$ which is smaller than $\frac{2}{3} - b_I^2$ for $b_I \leq \frac{1}{\sqrt{24}} \approx 0.204$.

If the shareholder knew the current firm value, she would tender if and only if the tender offer is larger than ω_I. The equilibrium price in this setting is identical to the price a monopolist would choose: $p_E^* = \frac{1}{2}\omega_E$. Thus, expected welfare equals $\mathbb{E}[\omega_I \mathbf{1}_{\{\omega_I > \frac{1}{2}\omega_E\}}] + \mathbb{E}[\omega_E \mathbf{1}_{\{\frac{1}{2}\omega_E \geq \omega_I\}}] = \frac{5}{8}$ which is – maybe surprisingly – the same as under uninformative cheap talk. It follows that for $b_I \leq \frac{1}{\sqrt{24}}$, informative cheap talk improves welfare compared with a situation where the shareholder becomes fully informed about ω_I.

Apart from aggregate welfare considerations, the numerical example allows us to shed light on the distribution of payoffs among I, E and the initial shareholder: if both equilibria exist, i.e. $b_I \leq \overline{b}_I$, the manager always prefers informative cheap talk compared with the babbling equilibrium. His ex ante payoff in the fully revealing equilibrium with cheap talk is $s\mathbb{E}[\omega_I \mathbf{1}_{\{\omega_I > \omega_I^*\}}] + s\mathbb{E}[\omega_E \mathbf{1}_{\{\omega_I \leq \omega_I^*\}}] + \mathbb{P}(\omega_I > \omega_I^*)B_I = \frac{2}{3}s + \frac{1}{2}B_I$ which clearly exceeds his payoff for the case without cheap talk $s\mathbb{E}[\omega_I \mathbf{1}_{\{\omega_E < \mu_I\}}] + \mathbb{P}(\omega_E < \mu_I)B_I + s\mathbb{E}[\omega_E \mathbf{1}_{\{\omega_E \geq \mu_I\}}] = \frac{5}{8}s + \frac{1}{2}B_I$. Further, as the manager can only communicate if $b_I \leq \overline{b}_I$, increasing his private benefits B_I and thereby b_I slightly, leads to a discontinuous drop in his payoff. Hence, the manager would like to limit his private benefits of control at $\overline{b}_I$.[32] The shareholder obtains an expected payoff of $(1-s)(\frac{1}{2} + \frac{b_I}{2} - \frac{5}{4}b_I^2)$ with informative cheap talk and $\frac{1}{2}(1-s)$ without cheap talk. As a consequence, whenever cheap talk is feasible, the shareholder prefers it. The intuition behind this is that she only follows the manager's recommendation if she benefits on average. Finally, the external bidder receives $\frac{1}{8}(1-s)$ without any information provision. When the shareholder follows management's recommendation, he obtains $(1-s)(\frac{1}{6} - \frac{1}{2}b_I + \frac{1}{4}b_I^2)$. He thus prefers no information whenever $b_I > 1 - \sqrt{\frac{5}{6}} \approx 0.087$. Cheap talk is costly to the bidder for high biases because takeovers become scarce and expensive.

Even though aggregate welfare is the same without cheap talk and with a fully informed shareholder, the distribution of payoffs differs substantially. When the shareholder is fully informed, her payoff amounts to $\mathbb{E}[v] = (1-s)(\mathbb{E}[\omega_I \mathbf{1}_{\{\omega_I > \frac{1}{2}\omega_E\}}] + \frac{1}{2}\mathbb{E}[\omega_E \mathbf{1}_{\{\frac{1}{2}\omega_E \geq \omega_I\}}]) = (1-s)(\frac{11}{24} +$

[30] $\frac{2}{3}$ equals the expected value of the first-order statistic of two random variables distributed uniformly on the unit interval.

[31] Uniqueness stems from the fact that $\lambda = 1 - s$ and $J = 1$.

[32] Of course, if B_I becomes larger, this effect is outperformed by the private benefits.

$\frac{1}{2}\frac{4}{24}) = (1-s)\frac{13}{24}$. She prefers to be informed by the manager over being fully informed if $b_I \in [0.12, 0.28]$.[33] The intuition is as follows: For low values of b_I, the shareholder only receives a small part of the payoff increase created by the takeover. Increasing b_I induces the bidder to post higher prices and the shareholder prefers cheap talk. However, if b_I becomes very large, takeovers become too scarce and full information is again preferred by the shareholder.

With a fully informed shareholder, E obtains $\mathbb{E}[u_E] = (1-s)\mathbb{E}[(\omega_E - p_E^*)\mathbf{1}_{\{\frac{1}{2}\omega_E \geq \omega_I\}}] = \frac{1}{2}(1-s)\mathbb{E}[\omega_E \mathbf{1}_{\{\frac{1}{2}\omega_E \geq \omega_I\}}] = (1-s)\frac{2}{24}$. Consequently, E prefers the manager's recommendation over the shareholder learning the current firm value if $b_I \leq 0.18$. Cheap talk helps E to extract full gains of trade if $b_I = 0$. As b_I increases, however, takeovers become too scarce and he prefers the shareholder being fully informed. Observe that E always prefers an uninformed over a fully informed shareholder. Finally, in the latter case, I receives $\mathbb{E}[u_I] = \frac{5}{8}s + \frac{1}{4}B_I$ which is worse than in the other two cases. Table 2.1 provides an overview for all these cases.

Information	E	I	S
Full Information	$\frac{2}{24}(1-s)$	$\frac{5}{8}s + \frac{1}{4}B_I$	$\frac{13}{24}(1-s)$
Cheap Talk	$(\frac{1}{6} - \frac{1}{2}b_I + \frac{1}{4}b_I^2)(1-s)$	$\frac{2}{3}s + \frac{1}{2}B_I$	$(\frac{1}{2} + \frac{1}{2}b_I - \frac{5}{4}b_I^2)(1-s)$
No Information	$\frac{1}{8}(1-s)$	$\frac{5}{8}s + \frac{1}{2}B_I$	$\frac{1}{2}(1-s)$

Table 2.1: Distribution of Expected Welfare Across Players.

6. Managerial Compensation and Golden Parachutes

In our model, efficient management advice can only be provided during a takeover if I possesses some share endowment. One can interpret this result as an additional argument for equity compensation beyond the classical moral hazard rationale (Jensen and Meckling 1976). Furthermore, it is important that the manager obtains security benefits of the company after the bidder gains control over the target firm. Hence, frequently observed[34] vested share schemes can also be rationalized by our model.

Recall that $b_I = \frac{B_I}{s}$. From Corollary 2.1, we know that $b_I = 0$ implements the first-best control allocation and that small biases are welfare-superior to full information and no information on the shareholder side. To obtain a small b_I, one can either try to lower the private benefit from being in charge, B_I, or to increase the incumbent's share endowment s. With positive B_I, the first-best allocation of control rights may still be attainable because one can compensate the manager in case of a takeover for his loss of B_I. The practice of golden

[33] These are rounded values.

[34] See Edmans et al. (2017) for a recent summary of data regarding executive compensation and vesting methods.

parachutes[35], which are often subject to public criticism as they seemingly reward executives for failure, may be optimal in our model as they enable the manager to increase welfare via his advisory role.

To be precise, denote the amount the golden parachute pays in case of a takeover by $G \in \mathbb{R}_+$. Then, I is indifferent between a takeover and remaining in charge if and only if

$$s\mathbb{E}[\omega_E|p_E] + G = s\omega_I + B_I,$$

and it directly follows that $G = B_I$ implements the first-best outcome. Hence, in the likely scenario that private benefits of control are non-negative, golden parachutes enable the manager to fulfill his advisory role during takeovers. In our model, golden parachutes have no downside as we abstract from the classical moral hazard problem of the manager. Inderst and Mueller (2010) show how severance pay after terminating a bad CEO's contract rewards failure and thus makes incentivizing effort more difficult. In their model, steep incentives (high equity compensation) alleviate the problem because this makes continuation costly for bad CEOs. In our model, equity compensation and severance pay are substitutes regarding the manager's advisory role (high s or G make I more willing to endorse a takeover). It is important to stress that our model provides a rationale for golden parachutes that are triggered if management is let go within a takeover process. This only represents a small fraction of management turnover which squares with empirical findings that, as noted in the Introduction, companies frequently adopt golden parachutes *conditional* on takeovers.

7. An Equivalence of Cheap Talk and Auctions

An interesting connection between auctions and cheap talk arises in our model. To see this, suppose that there are three potential managers: two external bidders E_1 and E_2 and one unbiased incumbent manager I. All potential managers have i.i.d. private values for the firm value under their management distributed according to a continuous cdf F on $[0, 1]$. E_i's private value is ω_{E_i} for $i = 1, 2$. For ease of exposition, further suppose that $\lambda = 1 - s$ and $J = 1$.

First, suppose the company was auctioned off among the two external bidders E_1 and E_2 in a sealed-bid first-price auction such that the bidder with the higher bid receives the fraction λ of shares and thus control over the target firm. Further, assume that the manager remains silent (m_I is uninformative). Then, we know from standard auction results (see e.g. Krishna

[35] We are by no means the first to consider the problem of golden parachutes or severance pay (see, for example, Eisfeldt and Rampini (2008), Bebchuk and Fried (2004), Lambert and Larcker (1985), Harris (1990), Knoeber (1986), Almazan and Suarez (2003)). None of these papers considers, however, how golden parachutes influence management's advisory role in takeovers.

(2009)) that each bidder will bid according to

$$p^*_{E_i}(\omega_{E_i}) = \mathbb{E}[\omega_{Ej}|\omega_{Ej} \leq \omega_{E_i}], \text{ for } i \neq j.$$

Now compare this setting with our model with exactly one bidder E_1 and a cheap talk message by the incumbent. We know from Theorem 2.1 that there is an equilibrium where E_1 bids according to

$$p^*_{E_1}(\omega_{E_1}) = \mathbb{E}[\omega_I|\omega_I \leq \omega_{E_1}],$$

and one can immediately see that the external bidder's bid is the same as if he faced a competitor from outside the target firm. In both cases, the good is allocated to the potential manager (E_1, E_2 or I) with the higher type. It follows that the expected firm value in our model with a single bidder facing an incumbent manager who sends a cheap talk message is the same as if the allocation mechanism was a first-price auction among two external bidders. Further, by revenue equivalence, the same holds true if we substitute the first-price auction with any other standard auction format that yields the same allocation rule and gives the lowest type the same expected utility (Krishna 2009). Of course, this relies on all potential managers having i.i.d. types. Hence, our model shows that the competition induced by a simple cheap talk message by the incumbent is as powerful (with respect to allocative efficiency) as bidding competition.

Interestingly, as the incumbent has the toehold s in our model, Burkart (1995) shows that if he gave a bid, he might overbid. This is why a counterbid by the incumbent may systematically differ from a cheap talk message by the incumbent in terms of allocative efficiency.

8. Concluding Remarks

We investigate the optimal control allocation in corporate takeovers. In our model, a bidder posts a tender offer and the incumbent manager reacts by sending a cheap talk recommendation to the shareholders. We show that with an informative message by the (potentially biased) manager, there exists an equilibrium in which the bidder fully reveals his type and that, for vanishing bias, the efficient control allocation is implemented. In practice, takeovers often involve costly provision of fairness opinions by outside parties such as investment banks. In our model, initial shareholders always prefer more information about the firm value than management is willing to provide. We show that the strategic and only partially informative recommendation by the manager is superior to a fully informative signal about the firm value under current management. This gives rise to two policy implications.

First, managerial salary is crucial to enable informative management recommendations. Our model rationalizes several features prevalent in reality: abstracting from moral hazard, steep incentives for the manager via equity compensation are useful as they enable communication in our model. Further, retention periods for managers' equity position after a takeover benefit the incumbent's capability to credibly communicate with shareholders. In our model, it is crucial for effective strategic communication that the manager's bias (private benefit per share) of remaining in charge is sufficiently small. Golden parachutes, often criticized, may actually be beneficial for allocative efficiency because they reduce management's bias and may strengthen its advisory role. Of course, they should be contingent on a successful takeover and not be triggered when management is replaced due to poor performance.

Second, legally prescribed fairness opinions and mandatory disclosure are generally not efficient as they can prevent value-increasing takeovers. As shareholders always prefer more information when they make their tendering decision, they are inclined to force management to disclose additional information to increase their rents from a successful takeover. Similar to Grossman and Hart (1980) who advocate (partial) exclusion of shareholders from post-takeover security benefits, excluding shareholders from obtaining excessive information may thus increase allocative efficiency.

Appendices

A. Proofs of Chapter 2

Proof of Proposition 2.1. **Step 1:** If E does not fully separate in an equilibrium, then first-best is not achieved in this equilibrium.

Suppose, on the way to a contradiction that this was not true, i.e. there exist some bidder types ω_E, ω'_E with $\omega_E > \omega'_E$ but $p_E(\omega_E) = p_E(\omega'_E)$. By the common support assumption, there exists an open interval of incumbent types $(\underline{\omega}_I, \overline{\omega}_I) \neq \emptyset$ such that $(\underline{\omega}_I, \overline{\omega}_I) \subset (\omega'_E, \omega_E)$. For all $\omega_I \in (\underline{\omega}_I, \overline{\omega}_I)$, first-best requires that a takeover does not occur at ω'_E, but at ω_E. But since $p_E(\omega_E) = p_E(\omega'_E)$, either a takeover occurs at both types or at none. Hence, whenever the bidder does not fully separate, first-best cannot be achieved.

Step 2: If E fully separates, first-best requires zero profits for all bidder types.

Whenever E fully reveals his type, the shareholder prefers a takeover whenever there is some $\gamma \geq \frac{\lambda}{(1-s)} > 0$ such that $\gamma p_E + (1-\gamma)\omega_E \geq \omega_I$. This coincides with the optimal allocation rule (that a takeover occurs if and only if $\omega_E \geq \omega_I$) if and only if $p_E = \omega_E$. Of course, $p_E = \omega_E$ implies zero profits for E.

Step 3: Suppose an equilibrium was fully separating and implements first-best, then there is a non-degenerate interval of bidder types with a profitable deviation.

Suppose all bidder types make zero profits, so $\omega_E = p_E$ (strictly negative profits can of course never be part of an equilibrium). Then, any type $\omega_E > 0$ could deviate to some type $\omega_E'' \in (0, \omega_E)$ and the takeover probability at $p_E = \omega_E''$ is $F_I(\omega_E'') > 0$. $F_I(\omega_E'')$ is strictly positive because first-best requires that a takeover occurs for all $\omega_I \in [0, \omega_E'')$. Therefore, the proposed deviation yields strictly positive profits of $[\omega_E - \omega_E'']\ F_I(\omega_E'') > 0$. Hence, we obtain a contradiction and can conclude that first-best is not attainable with fully informed shareholders.

□

Proof of Theorem 2.1. We start by establishing that given the incumbent sends a cheap talk message according to

$$m_I \in \begin{cases} [0, \omega_I^*], & \text{if } \omega_I \leq \omega_I^* \\ (\omega_I^*, 1], & \text{otherwise} \end{cases}$$

and the shareholder follows this message, the bidder finds it indeed optimal to post $p_E^* = \mathbb{E}[\omega_I | \omega_I \leq \omega_I^*(\omega_E)]$. Afterwards, we verify that, given m_I^*, p_E^* and her posteriors, the shareholder optimally tenders $\gamma^* = \frac{\lambda}{(1-s)}$ shares if $m_I^* \in [0, \omega_I^*]$ and zero otherwise.

In $t = 3$, as she plays a pure strategy, given any p_E, m_I and the respective posteriors of ω_I, ω_E, a takeover occurs with probability one or zero. Hence, the incumbent can send at most two-non outcome equivalent messages.

Step 0: Single crossing and I's equilibrium message.

In $t = 2$, for a fixed p_E and posterior of ω_E, I's utility from retaining control is $s\omega_I$ and thus strictly increasing in ω_I. His expected utility from a takeover is $s\mathbb{E}[\omega_E | p_E]$ and thus independent of ω_I. Therefore, the difference in his expected utility from sending a message m_I that induces a takeover and a message m_I' that does not is given by $\mathbb{E}[u_I | p_E, m_I, \omega_I] - \mathbb{E}[u_I | p_E, m_I', \omega_I] = s\mathbb{E}[\omega_E | p_E] - s\omega_I$. This is strictly decreasing in ω_I.

By this single crossing argument, all types below $\omega_I^* = \mathbb{E}[\omega_E | p_E]$ prefer a takeover. In the conjectured equilibrium, the shareholder always follows the incumbent's message. Hence, I has no incentive to deviate as he obtains his maximal payoff.

Step 1: Necessary condition for a fully separating bidder strategy.

Suppose the bidder plays a fully separating strategy, i.e. p_E is strictly increasing in ω_E (and thus invertible). As noted in the proof of Proposition 2.1, in any fully separating equilibrium, $p_E^* < \omega_E$ holds and thus $\gamma^* = \frac{\lambda}{1-s}$ independent of p_E (below, we show this more formally). Then, given his true type ω_E, the bidder's optimal bid p is given by

$$\underset{p \in \mathbb{R}_+}{argmax} \;\; F_I[\omega_I^*(p_E^{-1}(p)]\, \lambda \,[\omega_E - p].$$

The first-order condition (FOC) is

$$f_I[\omega_I^*(p_E^{-1}(p)]\; \omega_I^{*'}(p_E^{-1}(p))\; p_E^{-1}(p)'\; [\omega_E - p] - F_I[\omega_I^*(p_E^{-1}(p))] = 0.$$

Observe that p_E is strictly increasing and it follows that $\omega_I^* = \mathbb{E}[\omega_E|p_E] = \omega_E$. Further, at the equilibrium bid $p = p_E(\omega_E)$, this can be rewritten as the following ODE:

$$p_E'(\omega_E) = \frac{f_I[\omega_I^*(\omega_E)]}{F_I[\omega_I^*(\omega_E)]}\Big(\omega_E - p_E(\omega_E)\Big) = \frac{f_I(\omega_E)}{F_I(\omega_E)}\Big(\omega_E - p_E(\omega_E)\Big). \tag{A.1}$$

Notice that equation (A.1) is reminiscent to the symmetric two player first-price auction where both players have i.i.d. private values distributed according to F_I (for comments on the relation of our results to auction theory, we refer to Section 7). It can be shown that the general solution to (A.1) is given by[36]

$$p_E(\omega_E) = \frac{\int_0^{\omega_E} f_I(z) z dz + C}{F_I(\omega_E)}, \tag{A.2}$$

where C is a constant that pins down the solution depending on the initial value. As the lowest bidder type $\omega_E = 0$ can only bid zero in equilibrium, we know that $C = 0$. Hence,

$$p_E^*(\omega_E) = \frac{\int_0^{\omega_E} f_I(z) z dz}{F_I(\omega_E)} = \mathbb{E}[\omega_I | \omega_I \le \omega_E].$$

Step 2: Sufficiency.

We now show that the bidder's objective function is concave evaluated at the price function derived above and that any bidder type ω_E optimally chooses $p = p_E^*(\omega_E)$, i.e. $p_E^*(\omega_E)$ indeed constitutes an equilibrium price function. The objective of the bidder (up to the amount

[36]Applying Leibniz's integral rule and taking the derivative with respect to ω_E yields $p_E'(\omega_E) = \frac{f_I(\omega_E)\omega_E F_I(\omega_E) - \left(\int_0^{\omega_E} f_I(z) z dz + C\right) f_I(\omega_E)}{F^2(\omega_E)}$ which can be written as $\frac{f_I(\omega_E)\omega_E}{F_I(\omega_E)} - \frac{f_I(\omega_E)}{F^2(\omega_E)}\left(\int_0^{\omega_E} f_I(z) z dz + C\right)$. Comparing (A.1) with (A.2) shows the claim.

of shares he acquires that is independent of p_E), evaluated at $p_E^*(\omega_E)$ becomes

$$F_I[p_E^{-1}(p)]\,[\omega_E - \frac{\int_0^{p_E^{-1}(p)} \omega_I f_I(\omega_I)d\omega_I}{F_I[p_E^{-1}(p)]}] = \omega_E\, F_I[p_E^{-1}(p)] - \int_0^{p_E^{-1}(p)} \omega_I f_I(\omega_I)d\omega_I. \tag{A.3}$$

To see that it is indeed optimal to post $p = p_E^*(\omega_E)$, take the derivative of (A.3) with respect to p to arrive at

$$\omega_E - p_E^{-1}(p),$$

which is zero at $p = p_E^*(\omega_E)$, strictly positive whenever $p < p_E(\omega_E)$ and strictly negative for $p > p_E^*(\omega_E)$. Hence, the bidder indeed finds it optimal to post $p_E^*(\omega_E)$ given the other players expect him to play $p_E^*(\omega_E)$.

Step 3: Shareholder does sell after $(p_E^*, m_I^*(\omega_I \leq \omega_I^*))$.

For p_E^* and $m_I^*(\omega_I \leq \omega_I^*)$, it has to hold that there is a $\gamma \geq \frac{\lambda}{1-s}$ such that

$$\gamma p_E^*(\omega_E) + (1-\gamma)\mathbb{E}[\omega_E | p_E^*(\omega_E)] \geq \mathbb{E}[\omega_I | \omega_I \leq \omega_I^*(\omega_E)].$$

Plugging in p_E^* and ω_I^*, this becomes

$$\gamma \mathbb{E}[\omega_I | \omega_I \leq \omega_E] + (1-\gamma)\omega_E \geq \mathbb{E}[\omega_I | \omega_I \leq \omega_E],$$

which holds true for any $\gamma \in [0,1]$ since $\mathbb{E}[\omega_I | \omega_I \leq \omega_E] < \omega_E$ by full support.

Step 4: Shareholder does not sell after $(p_E^*, m_I^*(\omega_I > \omega_I^*))$.

For p_E^* and $m_I^*(\omega_I > \omega_I^*)$, there is no $\gamma \geq \frac{\lambda}{1-s}$ such that

$$\gamma p_E^*(\omega_E) + (1-\gamma)\mathbb{E}[\omega_E | p_E^*(\omega_E)] \geq \mathbb{E}[\omega_I | \omega_I > \omega_I^*(\omega_E)].$$

To see this, plug in p_E^* and the latter inequality becomes

$$\gamma \mathbb{E}[\omega_I | \omega_I \leq \omega_E] + (1-\gamma)\omega_E \geq \mathbb{E}[\omega_I | \omega_I > \omega_E].$$

The right-hand side is strictly larger than the left-hand side by the full support assumption. Hence, the shareholder does not want to sell *any amount of shares* if current management does not recommend to do so.

Step 5: Shareholder does not sell more than $\gamma^* = \frac{\lambda}{1-s}$ shares.

Suppose this was not true and she sells, after observing p_E^* and $m_I^*(\omega_I \leq \omega_I^*)$, a fraction of $\hat{\gamma} > \gamma^* = \frac{\lambda}{1-s}$. It must then hold that

$$\hat{\gamma} p_E^*(\omega_E) + (1 - \hat{\gamma})\omega_E \geq \frac{\lambda}{1-s} p_E^*(\omega_E) + (1 - \frac{\lambda}{1-s})\omega_E.$$

As $p_E^* < \omega_E$, the left-hand side is strictly smaller than the right hand-side. Thus, the inequality is violated and we can conclude that $\gamma^* = \frac{\lambda}{1-s}$ whenever a takeover occurs.

Step 6: Individual rationality.

Since $p_E^*(\omega_E) = \mathbb{E}[\omega_I | \omega_I \leq \omega_E] < \omega_E$ implies strictly positive expected profits for $\omega_E > 0$ and zero for $\omega_E = 0$, $p_E^*(\omega_E)$ is individually rational.

Step 7: There are no profitable deviations to prices not played on the equilibrium path.

As $F_I(p_E^*(1)) = 1$, a takeover occurs with certainty when the bidder posts the highest equilibrium price. Posting any price above $p_E^*(1)$ can thus never be profitable as it only increases the costs of a takeover. Further, as $p_E^*(0) = 0$, there are no downward deviations to off-path prices. This completes the proof. □

Proof of Proposition 2.2. We want to establish that, in any babbling equilibrium, there exists a single price such that a takeover occurs with certainty at this price and that all types above this price post it. We perform the proof in four steps.

Step 1: If there is a $p_E < 1$ such that all $\omega_E \geq p_E$ post p_E and $\gamma^*(p_E) \geq \frac{\lambda}{(1-s)}$, then $\gamma^*(p_E) = \frac{\lambda}{(1-s)}$.

Suppose, on the way to a contradiction, this was not true, i.e. $\exists p_E < 1$ such that $\gamma^*(p_E) > \frac{\lambda}{(1-s)}$ and all $\omega_E \geq p_E$ post p_E. Then, $\mathbb{E}[\omega_E | p_E] > p_E$ by full support. As a consequence, the shareholder could lower γ^* to $\gamma' := \gamma^* - \epsilon$ for an $\epsilon > 0$ such that $\gamma' \geq \frac{\lambda}{(1-s)}$ still holds. As $\mathbb{E}[\omega_E | p_E] > p_E$, this is a strictly profitable deviation.

Step 2: $\exists p_E < 1$ such that $\gamma^*(p_E) \geq \frac{\lambda}{(1-s)}$.

As I does not provide any information, the shareholder's tendering decision is

$$\gamma p_E + (1-\gamma)\mathbb{E}[\omega_E | p_E] \geq \mu_I, \tag{A.4}$$

for $\gamma \geq \frac{\lambda}{(1-s)}$ to make the takeover successful. From the full support assumption, we know that $\mu_I < 1$. Now suppose, on the way to a contradiction, there is an equilibrium where no takeover occurs for all bidder types. In this equilibrium, all bidder types post prices $p_E < \mu_I$ as otherwise a takeover would occur. There are now two possibilities: after some deviation to $p'_E \in [\mu_I, 1)$, either off-path beliefs yield $\mathbb{E}[\omega_E|p'_E] \geq p'_E$ or $\mathbb{E}[\omega_E|p'_E] < p'_E$. In the former case, the shareholder would tender a fraction $\frac{\lambda}{1-s}$ (or any $\gamma \geq \frac{\lambda}{1-s}$ in case of strict inequality) of her shares. Any bidder type $\omega_E > p'_E$ makes strictly positive profits by deviating to p'_E as opposed to zero on the proposed equilibrium path.

If off-path beliefs are such that $\mathbb{E}[\omega_E|p'_E] < p'_E$, then the shareholder optimally tenders (as $p'_E \geq \mu_I$) all of her shares and the takeover succeeds. Again this is a profitable deviation for $\omega_E > p'_E$. It is then clear that there exists at least one price $p_E < 1$ such that a takeover occurs with probability one, i.e. $\gamma^*(p_E) \geq \frac{\lambda}{(1-s)}$. Denote $\hat{p}_E$ as the minimal price such that the takeover succeeds. By (A.4), such a minimal price exists.

Step 3: All types $\omega_E \geq \hat{p}_E$ post $\hat{p}_E$.

We show that there is no price $p'_E > \hat{p}_E$ such that some bidder type posts p'_E. If this was true, bidder types need to be compensated by receiving a larger fraction of shares, i.e. we need $\gamma^*(p'_E) > \gamma^*(\hat{p}_E) \geq \frac{\lambda}{(1-s)}$. Suppose this was the case. It follows that $p'_E = \mathbb{E}[\omega_E|p'_E]$ because if it were true that $p'_E < \mathbb{E}[\omega_E|p'_E]$ and $\gamma^*(p'_E) > \frac{\lambda}{(1-s)}$, the shareholder would have a profitable deviation to tendering fewer shares but still making the takeover successful. Since $p'_E = \mathbb{E}[\omega_E|p'_E]$ holds, one can infer that $p'_E = \omega_E$. The shareholder's decision becomes $p'_E > \mu_I$ and they may tender a fraction larger than $\frac{\lambda}{(1-s)}$. This, however, yields zero profits for E who has now an incentive to deviate and post the price $\hat{p}_E$. Hence, all types above $\hat{p}_E$ post $\hat{p}_E$.

Step 4: For all $p_E < \hat{p}_E$, no takeover occurs.

Suppose this was not true, i.e. $\exists p_E < \hat{p}_E$ and $\gamma^* \geq \frac{\lambda}{1-s}$ at p_E. Then, all types above $\hat{p}_E$ would deviate to p_E. □

Proof of Proposition 2.4. As we consider babbling equilibria, suppose $m_I^*(p_E)$ is uninformative for all $p_E \in \mathbb{R}_+$.

Step 1: Suppose $s_j < \lambda, \forall j$. Then, there always exists an equilibrium in which no takeover ever occurs.

We show by construction that the following equilibrium always exists provided no shareholder is pivotal on her own.

1. $\gamma_j^*(p_E, m_I) = 0, \forall j, p_E, m_I$,
2. $p_E^* = 0, \forall \omega_E$,
3. $m_I^* = 1, \forall \omega_I, p_E$.

Given $\gamma_j^*(p_E, m_I) = 0 \; \forall j, p_E, m_I$, no shareholder j has an incentive to deviate as she cannot induce a takeover unilaterally. And as $\gamma_j^* = 0$ independent of m_I and p_E, the incumbent knows that shareholders will not react on his message and therefore it is optimal for him to send an uninformative message, e.g. $m_I^* = 1$ for all ω_I.

As all prices lead to no takeover and thus zero profits, any bidder type finds it optimal to post, for example, $p_E^* = 0$. Off-path beliefs regarding ω_I and ω_E are irrelevant given this coordination failure.

Step 2: There exists an equilibrium with a cutoff price $\hat{p}_E < 1$ such that:
if $\omega_E < \hat{p}_E$, a takeover occurs with probability zero;
if $\omega_E \geq \hat{p}_E$, E posts $\hat{p}_E$ and a takeover occurs with probability one.
Finally, it holds that $T^*(\hat{p}_E) = \lambda$.

The message sent by I is still uninformative. Then, there is a price $\hat{p}_E \in (0, 1)$ such that all shareholders tender $\gamma_j^* = \gamma^* = \frac{\lambda}{1-s}$ whenever $p_E \geq \hat{p}_E$. For $p_E < \hat{p}_E$, shareholders tender zero shares. $\hat{p}_E$ is the price that makes shareholders exactly indifferent between tendering and not tendering given the (on-path) posterior expected bidder type, i.e.

$$\frac{\lambda}{1-s}\hat{p}_E + (1 - \frac{\lambda}{1-s})\mathbb{E}[\omega_E|\omega_E \geq \hat{p}_E] = \mu_I.$$

This equilibrium is, for instance, supported by an off-path belief that assigns all mass to $\mathbb{E}[\omega_E|\omega_E \leq p_E]$ for $p_E < \hat{p}_E$ and $\mathbb{E}[\omega_E|\omega_E \geq p_E]$ for $p_E > \hat{p}_E$.

By their symmetric tendering strategy $\gamma^* = \frac{\lambda}{1-s}$, each shareholder is pivotal at any $p_E \geq \hat{p}_E$. Further, at $\hat{p}_E$, each shareholder is indifferent between tendering γ^* shares and not tendering thereby letting the takeover fail. Hence, it is (weakly) optimal for shareholders to tender exactly a fraction of $\frac{\lambda}{1-s}$.

For any $p_E > \hat{p}_E$, any shareholder strictly prefers a takeover to occur and tendering at least γ^* shares. No shareholder has a (strict) incentive to tender more than γ^* shares because according to above off-path beliefs: $\mathbb{E}[\omega_E|\omega_E \geq p_E]$ for $p_E > \hat{p}_E$, and expected security benefits weakly exceed the price.[37] As $\sum_j^J s_j = 1 - s$, it follows that $T^* = \sum_j^J s_j \gamma_j^* = \lambda$.

For E, deviating to a price above $\hat{p}_E$ yields to a purchase of λ shares with certainty but at a higher cost. Deviating to a price smaller than $\hat{p}_E$ yields no takeover and zero profits.

Step 3: Suppose $s_j < \lambda$ for all $j \in \{1, \ldots, J\}$. Then, there is an equilibrium where $p_E^*(\omega_E = 1) = 1$ and $\omega_E = 1$ is the only bidder type who secures a takeover. Further, $T^*(p_E^*(1)) \geq \lambda$.

Suppose $\gamma_j^*(p_E) = 0$ for all $p_E < 1$ and $\gamma_j^*(p_E = 1) = \frac{\lambda}{1-s}$ for all $j = 1, \ldots, J$. Further suppose that $p_E^*(\omega_E) = 0$ for all $\omega_E < 1$ and $p_E^*(\omega_E = 1) = 1$. In the conjectured equilibrium, a takeover occurs only after $p_E^* = 1$. Any $T^*(p_E^* = 1) \geq \lambda$ can be supported in equilibrium because $p_E = \omega_E = 1$ and shareholders are thus indifferent between security benefits after a successful takeover and the tender price. If a shareholder was pivotal at $p_E^* = 1$, i.e. she could block the takeover by not tendering she would refrain from doing so as $\mu_I < 1 = p_E = \omega_E$ by the full support assumption. Therefore, $T^*(p_E^*(1)) \geq \lambda$.

No bidder type $\omega_E < 1$ has an incentive to deviate to $p_E = 1$ as this would imply strictly negative profits. Independent of off-path beliefs, it is optimal for any shareholder not to tender after any price $p_E < 1$ because she is not pivotal ($s_j < \lambda$ for all $j \in \{1, \ldots, J\}$). Bidder type $\omega_E = 1$ does not want to deviate downwards as this would also imply zero profits.

Step 4: In any equilibrium in which a takeover occurs with non-zero probability, there exists a unique price $\hat{p}_E \leq 1$ such that $\mathbb{P}[takeover|\hat{p}_E] = 1$.

Suppose, on the way to a contradiction, this was not the case, i.e., there are at least two prices $\hat{p}_E \neq p'_E$ s.t. $\mathbb{P}[takeover|\hat{p}_E] = \mathbb{P}[takeover|p'_E] = 1$. W.l.o.g. assume $\hat{p}_E < p'_E$. Then, for bidder types that post p'_E on the equilibrium path, it must hold that $T^*(p'_E) > T^*(\hat{p}_E) \geq \lambda$ as otherwise p'_E implies higher costs but leaves the takeover probability and the amount of shares acquired constant.

For $T^*(p'_E) > \lambda$ to be part of an equilibrium and conditional on making the takeover successful, shareholders must be indifferent between selling and keeping their shares at p'_E, i.e. $p'_E = \mathbb{E}[\omega_E|p'_E]$ must hold true. Otherwise, if $p'_E < \mathbb{E}[\omega_E|p'_E]$, $T^*(p'_E)$ cannot be an equilibrium object because any shareholder tendering a positive amount would sell less shares to enjoy the larger security benefits and still making the takeover succeed. By the full support

[37] Except for $p_E = 1$ at which E makes at most zero profits. Hence, this can never be a profitable deviation.

assumption and incentive compatibility, $p'_E = \mathbb{E}[\omega_E|p'_E]$ is only possible if type $\omega_E = p'_E$ alone posts p'_E. But this implies zero profits, so this type has a profitable deviation to $\hat{p}_E$.

Step 5: All types $\omega_E \geq \hat{p}_E$ post $\hat{p}_E$.

Since there is a unique price on the equilibrium path that leads to a takeover, the only other possibility is that these types post a price that does not realize a takeover. This, however, would imply zero profits and is therefore no profitable deviation.

Step 6: All $\omega_E < \hat{p}_E$ post a price that does not realize a takeover.

Posting $p_E \geq \hat{p}_E$ implies strictly negative profits. Any $p_E < \hat{p}_E$ cannot yield $T^*(p_E) \geq \lambda$ as otherwise $\hat{p}_E$ would not be the unique price after which a takeover is implemented.

Step 7: $T^*(\hat{p}_E) = \lambda$ for $\hat{p}_E < 1$.

Suppose not. However, we know that $\hat{p}_E$ is unique and that all $\omega_E \geq \hat{p}_E$ post $\hat{p}_E$ on the equilibrium path. Hence, $\mathbb{E}[\omega_E|\hat{p}_E] > \hat{p}_E$ for all $\hat{p}_E < 1$. Thus, if $T^*(\hat{p}_E) > \lambda$, any shareholder could profitably deviate and tender strictly less shares but make the takeover still succeed. □

Proof of Theorem 2.2. We want to establish that the following constitutes an equilibrium:

1. The bidder fully reveals ω_E via p^*_E, where

$$p^*_E = \begin{cases} \mathbb{E}[\omega_I|\omega_I \leq \omega^*_I(\omega_E)] + b_I, & \text{if } \omega_E \geq b_I \\ \omega_E, & \text{otherwise.} \end{cases}$$

2. Given p^*_E, the incumbent's belief assigns probability one to $\omega_E = p^{*-1}_E(\omega_E)$. Hence, $\omega^*_I = max\{\omega_E - b_I; 0\}$ and I sends

$$m_I \in \begin{cases} [0, \omega^*_I], & \text{if } \omega_I \leq \omega^*_I \\ (\omega^*_I, 1], & \text{otherwise.} \end{cases}$$

3. Given p^*_E and m^*_I, shareholder j assigns probability one to $\omega_E = p^{*-1}_E(\omega_E)$ and updates his belief about the incumbent's type conditional on m^*_I to $f_I(\omega_I|\omega_I \leq \omega^*_I)$ and $f_I(\omega_I|\omega_I > \omega^*_I)$, respectively.

   If $m^*_I \in [0, \omega^*_I]$, then $\sum_j^J \gamma^*_j s_j = \lambda$. If $m^*_I \in (\omega^*_I, 1]$, then $\gamma^*_j = 0$ for all j.

4. Off-path beliefs by the incumbent and shareholders after some price offer p_E that is not played on the equilibrium path are restricted to those surviving the intuitive criterion by Cho and Kreps (1987).

In $t = 3$, as shareholders play pure strategies, given any p_E, m_I and the respective posteriors of ω_I, ω_E, a takeover still occurs with probability one or zero. Hence, the incumbent can send at most two-non outcome equivalent messages.

In $t = 2$, for a fixed p_E and posterior of ω_E, the incumbent's utility from no takeover is $s\omega_I + B_I$ and thus strictly increasing in ω_I. His expected utility from a takeover is $s\mathbb{E}[\omega_E|p_E]$ and thus independent of ω_I. Therefore, the difference in his expected utility from sending a message m_I inducing a takeover and a message m_I' not inducing a takeover is given by $\mathbb{E}[u_I|p_E, m_I, \omega_I] - \mathbb{E}[u_I|p_E, m_I', \omega_I] = s\mathbb{E}[\omega_E|p_E] - s\omega_I - B_I$ and thus strictly decreasing in ω_I. All types above ω_I^* prefer keeping control over the company. In the conjectured equilibrium, shareholders always follow the incumbent's message. Hence, I has no incentive to deviate as he obtains his maximal payoff.

Now consider $t = 1$ and the bidder's choice of p_E. For ease of exposition, we start by solving the bidder's problem for the special case of $J = 1$ and $\lambda = 1 - s$. Hence, a shareholder tenders all of her shares if and only if $p_E \geq \mathbb{E}[\omega_I|p_E, m_I(p_E)]$. By restricting attention to $J = 1$ and $\lambda = 1 - s$, we can focus on E's equilibrium price and leave the shareholders' tender weights γ_j aside. Afterwards we generalize our proof.

Step 1: Necessary condition for a fully separating bidder strategy.

Suppose the bidder plays a fully separating strategy p_E, i.e. p_E is strictly increasing in ω_E (and thus invertible). In any fully separating equilibrium, $\gamma^* = \frac{\lambda}{1-s}$ must hold. The reason is that, as in the case without bias, the equilibrium has to entail $p_E^*(\omega_E) < \omega_E$. To see this, recall that in the conjectured equilibrium, all types larger than b_I have a positive takeover probability. Thus, all bidder types $\omega_E \geq b_I$ can imitate the equilibrium price offer by some type $\omega_E' \in [b_I, \omega_E)$ yielding a profitable deviation. Therefore, in any fully separating equilibrium, $p_E^*(\omega_E) < \omega_E$ must hold. Hence, if $\gamma^* > \frac{\lambda}{1-s}$, the shareholder has a profitable deviation to tender fewer shares, still making the takeover possible and gain on the expected increase in firm value.

Let ω_E be the bidder's true type. As γ^* is independent of p_E, the bidder's optimal bid price p is given by

$$\underset{p\in\mathbb{R}_+}{argmax}\ \ F_I[\omega_I^*(p_E^{-1}(p)]\ \lambda\ [\omega_E - p],$$

where $\omega_I^* = \omega_E - b_I$ for $\omega_E \geq b_I$ and zero, otherwise.

Suppose $\omega_E \geq b_I$. Replicating the same steps as in the proof of Theorem 2.1 (with $b_I = 0$) yields

$$p_E'(\omega_E) = \frac{f_I(\omega_E - b_I)}{F_I(\omega_E - b_I)}\Big(\omega_E - p_E(\omega_E)\Big). \tag{A.5}$$

It can be shown that the general solution to (A.5) is given by

$$p_E^*(\omega_E) = \frac{\int_{b_I}^{\omega_E} f_I(z - b_I)zdz + C}{F_I(\omega_E - b_I)}, \tag{A.6}$$

where $C = 0$ in equilibrium because the type $\omega_E = b_I$ has a takeover probability of zero.

Observe that we can further rewrite the price function stated in (A.6):

$$\begin{aligned}
&\frac{\int_{b_I}^{\omega_E} f_I(z - b_I)zdz}{F_I(\omega_E - b_I)} \\
&= \frac{\int_0^{\omega_E - b_I} f_I(z)(z + b_I)dz}{F_I(\omega_E - b_I)} = \frac{\int_0^{\omega_E - b_I} f_I(z)zdz}{F_I(\omega_E - b_I)} + b_I\frac{\int_0^{\omega_E - b_I} f_I(z)dz}{F_I(\omega_E - b_I)} \\
&= \mathbb{E}[\omega_I|\omega_I \leq \omega_I^*] + b_I.
\end{aligned}$$

Hence, $p_E^*(\omega_E) = \mathbb{E}[\omega_I|\omega_I \leq \omega_E - b_I] + b_I$ for $\omega_E \geq b_I$.

For $\omega_E < b_I$, a takeover never occurs in equilibrium because $\omega_I^* = 0$. Thus, all types below b_I do not want to deviate to a price posted by some $\omega_E \geq b_I$ since this would yield strictly negative profits. Hence, offering the true type $p_E = \omega_E < b_I$ is optimal.

Step 2: Sufficiency.

This step is identical to the case with $b_I = 0$.

Step 3: Verification of Constraints.

We must check that the shareholder follows I's recommendation and that individual rationality holds for the bidder. To be precise, we must verify that the following constraints hold given p_E^*, m_I^*:

$$\begin{aligned} &[I] \qquad p_E^*(\omega_E) \geq \mathbb{E}[\omega_I|\omega_I \leq \omega_I^*], \\ &[II] \qquad p_E^*(\omega_E) < \mathbb{E}[\omega_I|\omega_I > \omega_I^*], \\ &[III] \qquad \omega_E \geq p_E^*(\omega_E). \end{aligned}$$

We show that none of the constraints are binding and that the solution to the unconstrained problem derived above is also the solution to the constrained optimization problem.

We begin with the case that $\omega_E \leq b_I$. Notice that we do not need to check constraint $[I]$ for $\omega_E \leq b_I$ because for these types a takeover occurs with probability zero. Similarly, constraint $[III]$ only has to hold if, from E's perspective, the takeover probability is strictly positive. Thus, we do not need to check it for $\omega_E \leq b_I$.

Claim: Suppose $\omega_E \leq b_I$. Then, $b_I \leq \mu_I$ is a necessary and sufficient condition for constraint $[II]$ to hold.

1. $[II]$ holds only if $b_I \leq \mu_I$: Suppose, on the way to a contradiction, this was not true, i.e. $b_I > \mu_I$. Then, there exists $\omega_E' \in (\mu_I, b_I)$ by full support. As $\omega_E' < b_I$ it follows that $\omega_I^*(\omega_E') = 0$ and hence $[II]$ requires that $p_E(\omega_E') < \mathbb{E}[\omega_I|\omega_I > \omega_I^*(\omega_E') = 0] = \mu_I$. But then there is a profitable deviation for ω_E' by posting a price p_E' such that $\mu_I < p_E' < \omega_E' < b_I$ which generates a strictly positive profit because $\omega_E' > \mu_I$ by assumption. Since $p_E' > \mathbb{E}[\omega_I|\omega_I > (\omega_I^*(\omega_E') = 0)] = \mu_I$ the second constraint cannot be fulfilled and we have a contradiction.

2. Sufficiency: Assume $b_I \leq \mu_I$. Then, $\omega_E \leq b_I \leq \mu_I = \mathbb{E}[\omega_I|\omega_I > \omega_I^*(\omega_E) = 0]$. $[II]$ follows immediately because posting any p_E can generate at most zero profits: for any price inducing a takeover, we need $p_E \geq \mu_I = \mathbb{E}[\omega_I|\omega_I > \omega_I^*(\omega_E) = 0]$ which yields strictly negative profits and hence $p_E < \mathbb{E}[\omega_I|\omega_I > \omega_I^*(\omega_E)]$.

We now turn to $\omega_E > b_I$ and verify constraints $[I]$, $[II]$ and $[III]$. We begin with constraint $[I]$:

$$p_E^* \geq \mathbb{E}[\omega_I|\omega_I \leq \omega_E - b_I].$$

Plugging in p_E^* yields

$$\mathbb{E}[\omega_I|\omega_I \leq \omega_E - b_I] + b_I \geq \mathbb{E}[\omega_I|\omega_I \leq \omega_E - b_I],$$

which is trivially true because $b_I > 0$. In particular, the constraint is never binding for any $b_I > 0$.
We now turn to [II], i.e. we want to show that

$$p_E^* < \mathbb{E}[\omega_I | \omega_I > \omega_I^*],$$

or

$$\mathbb{E}[\omega_I | \omega_I \leq \omega_E - b_I] + b_I < \mathbb{E}[\omega_I | \omega_I > \omega_E - b_I],$$

which can be written as

$$b_I < \mathbb{E}[\omega_I | \omega_I > \omega_E - b_I] - \mathbb{E}[\omega_I | \omega_I \leq \omega_E - b_I],$$

and the right-hand side is strictly positive by full support. By continuity, there exists a bias $\overline{b}_I^1$ such that the constraint is fulfilled for any $b_I \leq \overline{b}_I^1$.

Finally, we check [III]. Plugging in the price function yields $p_E^* = \mathbb{E}[\omega_I | \omega_I \leq \omega_E - b_I] + b_I < \omega_E - b_I + b_I = \omega_E$ and individual rationality obtains.

All in all, the solution to the unconstrained problem is also the solution to the constrained problem for sufficiently small bias b_I.
Although maximizing expected utility gives the optimal p_E^* on the interval of equilibrium prices $[0, p_E^*(1)]$, we have yet to check whether there exist profitable deviations by posting off-path prices above this interval (deviations to downward off-path prices are not possible because $p_E \in \mathbb{R}_+$).

Step 4: Off-path Upward Deviation.

To prove that there are no profitable upward deviations, we must show the following:

$$\forall \omega_E \in [0,1] \nexists \epsilon > 0 : \tag{A.7}$$
$$p_E^*(1) + \epsilon \geq \mathbb{E}[\omega_I | \omega_I > \omega_I^*(p_E^*(1) + \epsilon)] \text{ and } \mathbb{P}(\omega_I \leq \omega_I^*) u_E(\omega_E, p_E^*(1)) < u_E(\omega_E, p_E^*(1) + \epsilon).$$

Condition (A.7) requires that it is not profitable for any bidder type to post a price above $p_E^*(1)$, the price the highest type would post, to secure the takeover with probability one. This will not be profitable since ϵ, the premium paid beyond $p_E^*(1)$ to convince the shareholder to always tender, will be too large – at least for small b_I. We call this deviation price p^{dev}. After inserting ω_I^*, the inequality in condition (A.7) can be written as

$$p^{dev} \geq \mathbb{E}[\omega_I|\omega_I > \mathbb{E}[\omega_E|p^{dev}] - b_I].$$

By the intuitive criterion, off-path beliefs assign all probability mass to $\omega_E \geq p^{dev}$ because all other types would make strictly negative profits by such a deviation. It follows:

$$p^{dev} \geq \mathbb{E}[\omega_I|\omega_I > \mathbb{E}[\omega_E|p^{dev}] - b_I] \geq \mathbb{E}[\omega_I|\omega_I > p^{dev} - b_I].$$

Now, by continuity and full support, there is a $\overline{b}_I^2 > 0$ such that $\mathbb{E}[\omega_I|\omega_I > p^{dev} - \overline{b}_I^2] > p^{dev}$ which yields a contradiction and no upward deviation is profitable for $b_I \leq \overline{b}_I^2$. Take $\min\{\overline{b}_I^1, \overline{b}_I^2, \mu_I\}$ and the claim follows.

Step 5: General Case.

We now extend the last result to a general condition λ and multiple shareholder ownership $j \in \{1, \ldots, J\}$. We conjecture that

$$p_E^* = \begin{cases} \mathbb{E}[\omega_I|\omega_I \leq \omega_I^*(\omega_E)] + b_I, & \text{if } \omega_E \geq b_I \\ \omega_E, & \text{otherwise.} \end{cases}$$

is an optimal price. Given this price function, we know that in the proposed equilibrium $\omega_E > p_E^*$ holds for all $\omega_E > b_I$, i.e. for all bidder types who have a strictly positive probability of taking over the company.

We claim that shareholders will jointly tender $T^* = \lambda$ if a takeover occurs. Suppose this was not true, i.e. $T^* > \lambda$. Consider some shareholder j who tenders a fraction $\hat{\gamma}_j > 0$ of her shares. Then, shareholder j can lower $\hat{\gamma}_j$ by some strictly positive amount and the takeover would still occur. This is a strictly profitable deviation because $\omega_E > p_E^*$ given the proposed price function.

Thus, for any λ, the amount of shares tendered cancels out of the first-order condition and the optimal p_E^* remains $\mathbb{E}[\omega_I|\omega_I \leq \omega_I^*] + b_I$, formally:

$$\max_{p \in \mathbb{R}_+} \; F_I[\omega_I^*(p_E^{-1}(p)] \, \lambda \, [\omega_E - p] = \max_{p \in \mathbb{R}_+} \; F_I[\omega_I^*(p_E^{-1}(p)] \, [\omega_E - p],$$

where $\omega_I^* = \omega_E - b_I$ for $\omega_E > b_I$ and zero otherwise. We now establish that all shareholders tendering $\gamma_j^* > 0$ still want to follow m_I^*. This is sufficient because all shareholders with $\gamma_j^* = 0$ do not tender any shares and the constraints do not have to hold for them.

As argued above, the solution to the unconstrained problem remains $p_E^* = \mathbb{E}[\omega_I|\omega_I \leq$

$\omega_I^*] + b_I$. We now verify E's constraints.

Constraint $[I]$ becomes $\gamma_j^* p_E^* + (1-\gamma_j^*)\mathbb{E}[\omega_E|p_E^*] \geq \mathbb{E}[\omega_I|\omega_I \leq \omega_I^*]$. Again we know that in a fully revealing equilibrium, it must hold that $\mathbb{E}[\omega_E|p_E^*] = \omega_E$. By the same reasoning as in the case with $J = 1$, we know that $\omega_E \geq p_E^*$. Thus, we can rewrite constraint $[I]$ as $\gamma_j^* p_E^* + (1-\gamma_j^*)\omega_E \geq p_E^* \geq \mathbb{E}[\omega_I|\omega_I \leq \omega_I^*]$. The last inequality is true because of the same argument as in the single shareholder case.

Now observe that if $\gamma_{-j}^*(p_E^*, m_I^*(\omega_I > \omega_I^*)) = (0, \dots, 0)$, then for any individual shareholder j it is a best response not to tender as well if she is not pivotal on her own (i.e. $s_j < \lambda$). Consequently, obedience in the multiple shareholder case is easier to support in equilibrium. We will show, however, that for sufficiently small bias, we need not exploit the coordination failure but can show that even if a shareholder was pivotal with some $\gamma_j^* > 0$, she would not like to tender. To see this, note that constraint $[II]$ becomes $\gamma_j^* p_E^* + (1-\gamma_j^*)\mathbb{E}[\omega_E|p_E^*] < \mathbb{E}[\omega_I|\omega_I > \omega_I^*]$. We focus on the case where b_I becomes small and plug in our expression for p_E^* to arrive at

$$\gamma_j^*(\mathbb{E}[\omega_I|\omega_I \leq \omega_I^*] + b_I) + (1-\gamma_j^*)\mathbb{E}[\omega_E|p_E^*] < \mathbb{E}[\omega_I|\omega_I > \omega_I^*].$$

The left-hand side converges to $\gamma_j^*\mathbb{E}[\omega_I|\omega_I \leq \omega_E] + (1-\gamma_j^*)\omega_E$ and the right-hand side becomes $\mathbb{E}[\omega_I|\omega_I > \omega_E]$ as b_I goes to zero. Thus, in the limit we have

$$\gamma_j^*\mathbb{E}[\omega_I|\omega_I \leq \omega_E] + (1-\gamma_j^*)\omega_E < \omega_E < \mathbb{E}[\omega_I|\omega_I > \omega_E],$$

where the strict inequalities follow from the full support assumption. Again, by continuity, there is a bias $\overline{b}_I^{J1} > 0$ such that for all smaller biases the constraint is fulfilled.

Constraint $[III]$ can be shown to hold in the same fashion as in the case where all shares are tendered.

Step 6: Off-path Upward Deviation for $J > 1$.

By definition, there exists no off-path upward deviation if

$$\begin{aligned}
&\forall \omega_E \in [0,1] \nexists \epsilon > 0 : \\
&\gamma_j^*(p_E^*(1) + \epsilon) + (1-\gamma_j^*)\mathbb{E}[\omega_E|p_E^*(1) + \epsilon] \geq \mathbb{E}[\omega_I|\omega_I > \omega_I^*(p_E^*(1) + \epsilon)] \\
&\text{ and } \mathbb{P}(\omega_I \leq \omega_I^*)u_E(\omega_E, p_E^*(1)) < u_E(\omega_E, p_E^*(1) + \epsilon).
\end{aligned}$$

The argument is similar to the single shareholder case. Again define the deviation price $p^{dev} := p_E^*(1) + \epsilon$. Suppose such a deviation is profitable, then it holds

$$\gamma_j^* p^{dev} + (1-\gamma_j^*)\mathbb{E}[\omega_E|p^{dev}] \geq \mathbb{E}[\omega_I|\omega_I > \mathbb{E}[\omega_E|p^{dev}] - b_I]. \tag{A.8}$$

The intuitive criterion excludes off-path beliefs assigning positive probability to types $\omega_E < p^{dev}$ as they would make a strict loss by such a deviation. Thus, $\mathbb{E}[\omega_E|p^{dev}] \geq p^{dev}$. As $\gamma_j^* \in (0,1]$, the LHS in (A.8) is weakly smaller than $\mathbb{E}[\omega_E|p^{dev}]$. Hence,

$$\mathbb{E}[\omega_E|p^{dev}] \geq \mathbb{E}[\omega_I|\omega_I > \mathbb{E}[\omega_E|p^{dev}] - b_I].$$

But by continuity and full support, there exists a $\overline{b}_I^{J2} > 0$ such that for all $b_I \leq \overline{b}_I^{J2}$: $\mathbb{E}[\omega_E|p^{dev}] < \mathbb{E}[\omega_I|\omega_I > \mathbb{E}[\omega_E|p^{dev}] - b_I]$ which yields a contradiction. Now define $\overline{b}_I := \min\{\overline{b}_I^{J1}, \overline{b}_I^{J2}, \overline{b}_I^1, \overline{b}_I^2, \mu_I\}$ and the equilibrium exists for every $b_I \leq \overline{b}_I$. □

Proof of Proposition 2.5. In the fully revealing equilibrium of Theorem 2.2, a takeover occurs whenever $\omega_I \leq \omega_I^* = \mathbb{E}[\omega_E|p_E^*] - b_I = \omega_E - b_I$ and $\lim_{b_I \to 0} \omega_I^* = \omega_E$. The decision rule whether a takeover occurs or not is thus the optimal allocation rule in the sense of Definition 2.1. Hence, in the limit we attain first-best firm value. The existence of an upper bound $\overline{b}_I^{FV}$ on b_I follows from continuity of ω_I^* in b_I. □

B. Information Structures and Shareholder Learning

Let X be a signal about ω_I with realization $x \in [0,1]$ and suppose the shareholder can choose an information structure G at zero costs as follows. Given the prior $F_I \in \Delta([0,1])$, the distribution of X induces a joint distribution over signals and states $G : [0,1] \times [0,1] \to [0,1]$. Given x, the shareholder forms a posterior mean $\mathbb{E}[\omega_I|x]$. At the time of tendering, her decision whether to tender or to keep the shares depends only on $\mathbb{E}[\omega_I|x]$. Hence, without loss of generality, we identify the signal with its induced posterior mean: $\mathbb{E}[\omega_I|x] = x$. Thus, the shareholder is only interested in the marginal distribution of the signal X. Doing so, we identify each signal with the cdf of its marginal distribution and denote it by G_X.[38] We define the set of admissible information structures as mean-preserving spreads (MPS) of the prior

[38] This is equivalent to saying that each signal x provides the shareholder with an unbiased estimate about ω_I. For two papers that model signals in the same way, see Roesler and Szentes (2017) and Ravid et al. (2019).

F_I:

$$\mathcal{G} := \Big\{ G_X \text{ cdf over } [0,1] : \int_0^y F_I(\omega_I)d\omega_I \geq \int_0^y G_X(x)dx \; \forall \; y \in \; [0,1], \\ \int_0^1 F_I(\omega_I)d\omega_I = \int_0^1 G_X(x)dx \Big\}.$$

Lemma 2.1

Let X be a signal about ω_I with realization $x \in [0,1]$ and suppose the shareholder can choose any information structure from $\mathcal{G}$ at zero costs. Then, the shareholder chooses the fully informative signal structure $\overline{G}_X$.

Proof of Lemma 2.1. Define $z := \frac{\lambda p_E + (1-s-\lambda)\mathbb{E}[\omega_E|p_E]}{(1-s)}$. As $\gamma^* = \frac{\lambda}{1-s}$, the shareholder tenders whenever $z \geq x$. Given some $G_X \in \mathcal{G}$, the expected utility per share of the shareholder is then given by

$$\int_0^z z dG_X(x) + \int_z^1 x dG_X(x) = zG_X(z) + 1 - zG_X(z) - \int_z^1 G_X(x)dx = 1 - \int_z^1 G_X(x)dx. \tag{B.1}$$

Now take $\overline{G}_X$ which is an MPS of any $G_X \in \mathcal{G}$ and it follows from (B.1) that her utility under $\overline{G}_X$ minus her utility under G_X equals

$$1 - \int_z^1 \overline{G}_X dx - 1 + \int_z^1 G_X dx = \int_z^1 G_X - \overline{G}_X dx = \int_0^z \overline{G}_X - G_X dx \geq 0.$$

The inequality follows from $\overline{G}_X$ being an MPS of G_X. To see this, note that

$$\int_z^1 \overline{G}_X dx = \int_0^1 \overline{G}_X dx - \int_0^z \overline{G}_X dx,$$

and recall that $\int_0^1 \overline{G}_X dx = \int_0^1 G_X dx$. □

By Lemma 2.1, the shareholder wants to become perfectly informed. Therefore, by Proposition 2.1, first-best is not attainable if she can acquire additional information. This result also holds if the shareholder could acquire information about both states of the world:

Lemma 2.2

Suppose the shareholder is perfectly informed about ω_E and ω_I. Then, the first-best allocation is never implemented.

Proof of Lemma 2.2. Suppose the shareholder can choose information structures H_E and H_I at zero costs as follows: there are two independent signals $X_E, X_I \in [0,1]$ inducing joint distributions over signals and states $H_I : [0,1]\times[0,1] \to [0,1]$ and $H_E : [0,1]\times[0,1] \to [0,1]$. As before we focus on signals that fulfill $\mathbb{E}[\omega_E|x_E] = x_E$ and $\mathbb{E}[\omega_I|x_I] = x_I$. We denote the marginals by H_{X_E} and H_{X_I}. Now, the shareholder can acquire any information $(H_{X_E}, H_{X_I}) \in \mathcal{H}$ where

$$\mathcal{H} := \Big\{(H_{X_E}, H_{X_I}) \text{ cdfs over } [0,1] :$$

$$\int_0^y F_I(\omega_I)d\omega_I \geq \int_0^y H_{X_I}(x_I)dx_I \;\forall\, y \in [0,1], \int_0^1 F_I(\omega_I)d\omega_I = \int_0^1 H_{X_I}(x_I)dx$$

$$\text{and} \int_0^y F_E(\omega_E)d\omega_E \geq \int_0^y H_{X_E}(x_E)dx_E \;\forall\, y \in [0,1], \int_0^1 F_E(\omega_E)d\omega_E = \int_0^1 H_{X_E}(x_E)dx\Big\}.$$

In the same way as in Lemma 2.1, one can show that it is optimal for her to acquire full information about ω_E, as well. Her tendering decision becomes $\gamma p_E + (1-\gamma)\omega_E \geq \omega_I$ and suppose first-best is implementable, so it follows that $p_E = \omega_E$. Given full separation, we obtain the result with the same arguments as in the proof of Proposition 2.1. □

Chapter 3

Teamwork and Information Transmission: Complements and Substitutes

1. Introduction

A decision-maker (DM) must choose who of her staff should work on a task when they can transmit non-verifiable information about the return from their work.[1] Existing models can be used to study her optimal decision when one agent must work alone (Li et al. 2016). This paper, however, investigates her optimal decision when agents are able to work in a team – which happens frequently in real-world applications. Indeed, a supervisor may pick one of her staff to work alone or she could ask both agents to work together. Finally, if a task is very time-consuming and distracting from more important things, no one at all should be working. Examples for organizations and tasks like these are: (a) companies that want to hire a new employee via a job interview. Frequently, a superior selects members of her staff to join the interview because they are better in assessing the fit to the requirements of daily business and to get a second opinion. (b) University departments that aim at raising funds. The econ department chair needs to choose someone who writes a research grant; she may pick one from the micro or one from the macro department – or both of them.[2]

The model consists of two agents who have private information about the return of their own task[3] and their type positively affects their payoff. Agents' information is non-verifiable and the DM cannot commit to a decision rule so her decision who must work is based on simultaneous cheap talk. In the job interview example, an agent's type is the assessment of the new colleague measured in terms of increase in sales figures and this may positively affect the own variable salary. Moreover, the agent attending such an interview may receive

[1] Throughout the paper, I refer to the DM with the female and to the agents with the male pronoun.

[2] More broadly, my setting applies to any superior who can assign tasks to two staff members and who can instruct agents to work, for instance through her authority.

[3] Alternatively, one can follow the interpretation that there is a single task on which two agents can work simultaneously.

reputation for doing a good job. In the research grant example, the type is the money raised and a private benefit from being successful. Working is costly because of the opportunity costs of not working on other tasks, such as meeting customers to sell a product or doing research. But as part of the same organization, an agent can benefit if someone else works. Agents then receive a positive externality equal to a fraction $\alpha \in [0, 1]$ of the other type. This measures the level of integration between tasks across agents. If, for instance, a member from the macro department raises funds for his department only, this will not have a direct effect on the micro department. But if another staff member attends a job interview and hires a good applicant, this may lead to an increase in sales and the own bonus. In short, my model features interdependent values with an agent's own type being weakly more valuable to him than that of the other agent.

As novel feature, agents can be instructed to work in a team. In the above examples, they can prepare and attend a job interview together; and university staff members can jointly write research grant proposals. In the model, teamwork increases the agents' payoff additively so they receive their own return plus the externality. But compared with the costs of working alone, costs of teamwork are re-scaled with a parameter larger (agents are substitutes) or smaller than one (agents are complements). The economic interpretation is that, net of the externality, in the substitutes (complements) case, working on the own task becomes less (more) valuable when the other agent works on his task, as well. The scaling parameter can therefore be interpreted as a match value that measures the impact of teamwork on individual costs which can go into two different directions: for a (bad) match value larger than one, teamwork leads to proportionately higher costs because agents must coordinate their work and this coordination effort is larger than potential synergies. If the match value is smaller than one, the opposing effect is stronger and teamwork saves time. The DM has efficiency preferences and, as an outside option for her, she can decide that no one has to work on his specific task.[4]

As main contribution of this paper, I show that the equilibrium message structure in both polar cases is fundamentally different. In the complements case, information transmission is limited by adding teamwork into her choice set. The intuition is as follows. The most preferred outcome for agents is either teamwork (if the own return is sufficiently large), or free-riding on the other agent working alone (if the own return is sufficiently small). Working alone is never the preferred outcome because if the own return is smaller than the costs, one is better off with the externality; and if the own return is larger, teamwork is preferred because of the lower costs. Moreover, for $\alpha > 0$, the outside option is never the preferred outcome because of the externality. Notably, as the teamwork technology affects payoffs additively, all relevant information about the most preferred outcome is in the own type and it turns

[4]In practice, agents then work on other tasks of day-to-day business.

out that the type space can be divided into three distinct intervals: a high one in which the preferred outcome is teamwork, a low one in which it is free-riding; and an intermediate one in which it is also teamwork, but where, if an agent had to work alone, he would suffer from a payoff smaller than from the outside option. Communication equilibria therefore exhibit at most three non-outcome equivalent messages per agent. I characterize all equilibria for the complements case and if the match value between agents is good (or small) enough, equilibria consist of simple binary messages in favor or against work. If the externality and the match value are both large, then no information at all can be transmitted.

In the substitutes case, equilibrium messages are related to the setting where the DM cannot choose teamwork and the outside option, so one agent must work alone (see Section 3). In this case, agents have an incentive to free-ride on the externality by pretending to be a low type. Reporting to be a low type, however, is risky because even net of costs, the own type may be larger than the externality from the other agent. Therefore, understating the own type becomes costlier for higher types. This setting is conceptually the same as in Li et al. (2016) and I call communication with the same structure as in this paper *competitive cheap talk* (henceforth CCT).[5] If a communication equilibrium has the CCT form, then both agents endogenously possess a finite set of equilibrium messages which decrease in size because low types have the largest incentive to free-ride and thus information is very imprecise here. In equilibrium, the agent who sends the higher message works on his task (up to a tie-breaking rule if both send the same message).

This trade-off between working alone and free-riding carries over to the case *with teamwork*. Here, high types either prefer to work alone (if the other type is sufficiently small) or in a team (if the other type is sufficiently high). But low types never prefer teamwork and have the incentive to free-ride. Hence, without teamwork, CCT is played on the whole message space as in Li et al. (2016). And with teamwork, I show that the message structure is the same, but only on an endogenous subset of the message space. Additionally, there is one highest (lowest) message according to which the DM chooses teamwork (the outside option) provided both agents send this message.

Interestingly, in both polar cases, there exist task-separating equilibria with binary messages meaning that in practice, all four outcomes could be implemented with simple yes/no questions. The fact that sometimes agents can only send such a simple message in favor or against working provides an appealing interpretation for simple yes/no questions that can be observed in daily life. Not only do such questions save time because they make the questioned person choose from a binary set; endogenously, more credible information can often

[5]The term is directly borrowed from Li et al. (2016). Although there are several papers in which agents compete via their messages (such as Schmidbauer (2017) and Schmidbauer (2019)), I use the term CCT whenever incentives and message structure are like in Li et al. (2016). My model builds on this work. For more details I refer to the review of related literature.

not be obtained from the respondent.

In terms of welfare I show that many informative equilibria outperform equilibria in which the DM ignores the agents' messages. To be precise, in the complements case, I show that binary equilibria yield a larger expected welfare than uninformative equilibria. In the substitutes case, I provide numerical examples when this is the case. This provides a new rationale in favor of investing in teamwork skills for organizations: having staff members who are skilled in teamwork can improve the precision of information and thus achieve more efficient allocations of costly tasks.[6] I further show that the rules of the game are decisive when the DM can make both agents work. Because of the binary decision, simultaneous and sequential communication are outcome-equivalent in Li et al. (2016), and all equilibria under delegation are a subset of those. Due to the similarities of my model with theirs, this applies here when exactly one agent has to work. If teamwork is possible, however, this is no longer true and welfare decreases when departing from simultaneous communication. Thus, my model rationalizes the use of simultaneous communication when assigning costly tasks to subordinates.

The rest of this paper is organized as follows. In the remainder of this section, I highlight related literature. Section 2 introduces the model. In Section 3, I analyze the setup without teamwork and outside option. In the main Section 4, I investigate teamwork; the complements case is treated in Subsection 4.2 and Subsection 4.3 deals with the substitutes case. In Section 5, I discuss extensions. Section 6 concludes. All proofs are delegated to Appendix A.

Literature

In the broadest sense, my paper contributes to the literature on communication without commitment and verifiable information which goes back to the seminal work by Crawford and Sobel (1982). In this paper, one informed and biased sender faces an uninformed receiver who must choose an action from a continuous space. Surprisingly, although preferences are misaligned and information is not verifiable, meaningful information can be transmitted if the sender is not too biased. In a narrower sense, my paper contributes to the communication literature with more than one sender. In particular, it is related to Li et al. (2016) who investigate a model with two agents who can carry out an agent-specific project, and an uninformed decision-maker who must choose exactly one of both projects. Agents are privately informed about the return of their own project and the decision is made on the basis of cheap talk messages. If the other project is implemented, an agent receives the corresponding (full) return as externality. In one variant of the model, agents possess different positive and additive biases

[6]Teamwork is a skill that can be learned and taught (Oakley et al. 2004). There is a vast research on teamwork skills (see, for example, Senior et al. (2010), and Ballantine and McCourt Larres (2007)) and numerous companies invest in workshops to foster teamwork skills.

so they compete for their projects via their messages and have an incentive to overstate the own return. Endogenously, the DM chooses the project with the higher posterior induced by agents' messages. My model is similar to theirs in the following dimensions: agents incur (equal) costs when they have to work so they are biased *against* working. Moreover, they receive an externality from the work by the other agent and information transmission is non-binding and costless. The major difference is that the decision set of the DM is larger as she can choose both agents to work or none of them. Without teamwork and outside option, my model is identical to Li et al. (2016) with homogeneous and negative bias (see Section 3). When agents are substitutes, the same message structure as in their paper reappears as part of an equilibrium. On the other hand, in the complements case, introducing teamwork limits information transmission and the message structure is fundamentally different. A further important difference is that in Li et al. (2016), only ordinal information matters because the DM simply chooses the agent with the higher message. In my model, however, cardinal information matters because players' preferences can generally not be ordered in one dimension.

Other papers in which agents compete for their own project to be implemented via cheap talk are Schmidbauer (2017) and Schmidbauer (2019). Both models differ from the present one in that they analyze a dynamic and private value setting.

Goldlücke and Tröger (2018) analyze the problem of finding an agent to perform a tedious task in a mechanism design setup. Exactly one of many agents has to provide the costly service. Working, however, benefits everyone as in my model. In a two-type-setting (high or low quality), the optimal mechanism is a threshold rule that asks agents the binary question whether they want to volunteer or not. Then, the task is assigned at random among volunteers if there are enough of them. Otherwise, the task must be done by a non-volunteer.

One can interpret teamwork in my model as contribution to one large joint project. In that sense, my paper also relates to the literature on the provision of public goods. This strand of literature is mainly concerned with differences in the costs of the goods provision and focuses on the interplay between group size and the provision probability (Bergstrom 2017). My paper, on the other hand, introduces the public good teamwork to study information transmission between providers of the good and the decision-maker.[7]

[7]Psychological experiments suggest that communication among agents can increase contributions in volunteer dilemma experiments (Dawes et al. 1977). My results explain how *independent* communication can increase allocative efficiency in organizations.

2. Model

Tasks

There are two agents, one decision-maker and two agent-specific tasks. The DM decides who has to work and her decision is denoted by d. Tasks are written in parentheses to distinguish them from agents. To be precise, the following work assignments can appear: one agent works alone, $d = (i)$, where $i = 1, 2$; both work together, $d = (1, 2)$, or none of them has to work: $d = (0)$. I call $(1, 2)$ *teamwork* and (0) is an *outside option* for the DM. Agents do not have an outside option. Agent i possesses private information θ_i about the return of task (i). Types θ_1 and θ_2 are independent random variables that are distributed uniformly on $[0, 1]$.

Payoffs

Working on the own task is time-consuming and leads to costs $c \in (0, 1)$. Agents receive their return and pay c if they work alone. In contrast, they receive an externality (and incur no costs) if only the other agent works: if $d = (j)$, agent i receives $\alpha\theta_j$. The parameter $\alpha \in [0, 1]$ is commonly known and indicates the level of integration of the organization formed by agents and the DM: $\alpha = 1$ corresponds to large spillover effects across agents, whereas $\alpha = 0$ means that working on the own task has no effect on the other agent.[8] If she implements $d = (1, 2)$, agents receive their own return plus the externality and incur costs of βc. The parameter β is a commonly known match value between agents and I will distinguish between two regimes: $0 < \beta < 1$ and $\beta > 1$. This models the fact that working together leads to two opposing effects: on the one hand, agents benefit from synergies, for example through knowledge and skills the other agent possesses leading to less time spent for the own task. On the other hand, agents must agree on a timetable, an agenda and priorities. Accordingly, in the case $\beta < 1$, synergies are higher than the costs of coordination and if $\beta > 1$, the opposite is true.[9] It follows that net of the externality, working on the own task can become more or less valuable when the other agent also works on his task and I call agents *complements* for $\beta < 1$ and *substitutes* for $\beta > 1$.

If no one has to work, agents receive a utility of zero.[10] The DM has efficiency preferences

[8] In the job interview example with agent 1 joining the interview and $\alpha = 0$, a new employee has no impact on 2, for example because he works in a different regional division that is irrelevant for 2's salary.

[9] More broadly, the DM may have a pool of employees from which she can choose at most two agents. Any pair k of two agents has one specific match value β_k. It may be that there are no matches creating net synergies and all β_k's are larger than one. Assuming she has knowledge about the teamwork skills of her staff, one can interpret the game as the DM disregarding all pairs except for the minimal β.

[10] There is no outside option for agents so their ex post payoff can be negative. Hence, realizing that the DM's decision yields an agent a negative payoff, he could refrain from working. To circumvent this issue, one could assume that agents are punished with a payoff of $-\infty$. In German labor law there are written warnings for refusal to work and several warnings lead to dismissal. And in the US, employers can often dismiss an employee at any time and for any reason.

so her utility equals the sum of agents' payoffs. All in all, the payoffs for agent i and the DM are[11]

$$u_i = \begin{cases} 0, & \text{if } d = (0) \\ \theta_i - c, & \text{if } d = (i) \\ \alpha\theta_j, & \text{if } d = (j) \\ \theta_i + \alpha\theta_j - \beta c, & \text{if } d = (i, j), \end{cases} \qquad u_{DM} = \begin{cases} 0, & \text{if } d = (0) \\ (1+\alpha)\theta_i - c, & \text{if } d = (i) \\ (1+\alpha)(\theta_i + \theta_j) - 2\beta c, & \text{if } d = (i, j). \end{cases}$$

Timing and Strategies

The agents' information is non-verifiable and the DM has no commitment power. Incentive contracts are thus not feasible.[12] The only basis for her decision are messages sent by the agents. To be precise, agents communicate with the DM via simultaneous cheap talk. The timing of the game is as follows: first, agents observe their type, then they communicate to the DM. And finally, the DM makes a decision d. I assume that the agents' message space is $[0, 1]$, so agent i chooses a type-dependent message strategy $m_i : [0, 1] \to [0, 1]$. Given message pair $m := (m_1, m_2)$, the DM's (pure) strategy is a decision $d(m) \in \{(0), (1), (2), (1, 2)\}$. The solution concept is Perfect Bayesian Equilibrium in pure strategies (henceforth "PBE" or just "equilibrium") which requires that

1. given the decision rule d and agent j's message m_j, agent i chooses m_i optimally.
2. $d(m)$ is optimal given her belief about (θ_1, θ_2) induced by (m_1, m_2).
3. All players form their belief according to Bayes' rule whenever possible.

I assume a tie-breaking in favor of teamwork, that is the DM always chooses $d = (1, 2)$ when she is indifferent between this decision and $d = (0)$ or (i), for $i = 1, 2$. Moreover, I assume that she chooses $d = (0)$ when she is indifferent between the outside option and one agent working alone. This tie-breaking rule is made to circumvent openness problems and to ensure existence of equilibria. I will refer to the conditional expected value $\mathbb{E}[\theta_i|m_i]$ as *posterior of* θ_i. For convenience, I will further refer to a typical agent as $i = 1$ unless there is cause for confusion. Finally, one will see that equilibrium messages can be ordered on the message space. I denote this order with a superscript so that agent i choosing the n-th message will be written as m_i^n. I ignore the agent's subscript and write m^n if it is clear (or not important) who sends the message.

[11]The additive functional form is surely restrictive. It allows, however, to conclude that all results come from incentives and not from the technological change through teamwork.

[12]This is a plausible assumption because writing a contract for special tasks costs a lot of time and resources.

3. Competitive Cheap Talk

In this section, I analyze the case where the DM can neither choose $d = (1, 2)$ nor $d = (0)$ so that one agent must work alone. If agent i works he receives a payoff of $\theta_i - c$, whereas j receives $\alpha\theta_i$. The DM wants the agent with the higher type to work so she chooses the agent with the higher posterior given his message. If both posteriors are identical, she randomizes (without loss of generality) with equal probability. This setting is very close to Li et al. (2016) and all arguments in this subsection are adapted from that paper. The only difference is that here, agents pay (equal) costs if they have to work, whereas in Li et al. (2016), agents possess an upward (agent-dependent) project bias.[13] Investigating this setup will make the analysis of the substitutes case ($\beta > 1$) easier. To simplify the analysis, assume $\alpha = 1$.[14] The DM's and agent 1's most preferred decisions in the type space are depicted in Figure 3.1.

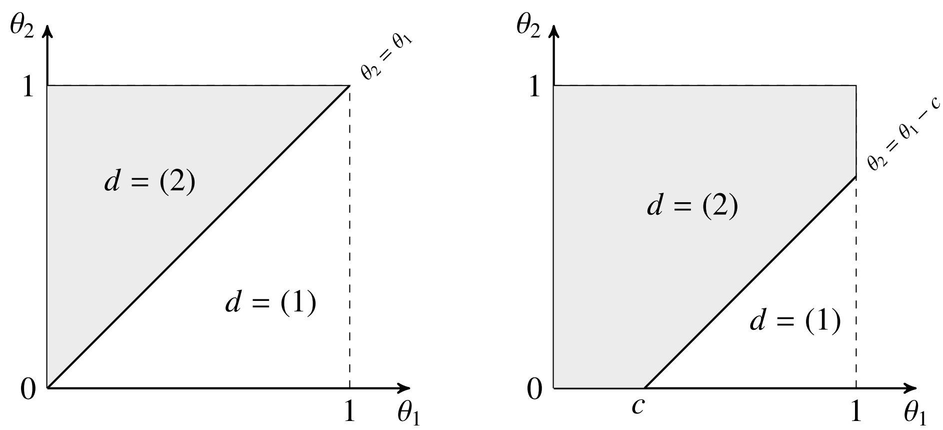

Figure 3.1: Preferred Decisions for DM (left panel) and Agent 1 (right panel) without $d = (1, 2)$ and (0).

Obviously, an agent's preferences depend on the other agent's type which will justify the notion of competitive cheap talk. Agent 1 has an incentive to free-ride because the externality θ_2 may be larger than his payoff from working.

Surprisingly, equilibria must be of the interval form à la Crawford and Sobel (1982) although in that paper, the action space is continuous and players have quadratic loss functions.

[13]One can easily introduce heterogeneous costs in my model. Li et al. (2016) show that in this case agents possess different sets of equilibrium messages. In particular, one agent has a sure option – the highest possible message after which he is revealed to be the better agent. The same holds at the bottom. I abstract from these observations and focus on the effects that stem from the introduction of teamwork.

[14]Li et al. (2016) perform two different analyses of their model: one with pure additive bias and one with pure multiplicative bias. Different from Melumad and Shibano (1991), combining both does not allow for an explicit representation of cheap talk equilibria. Therefore, the case with $\alpha < 1$ *and* $c > 0$ would not yield explicit results here and later if $\beta > 1$.

The reason is that the single-crossing condition is fulfilled, so higher types send higher messages. To see this, write i's expected payoff for type θ_i sending message m_i^n given the message strategy by j and the decision rule by the DM as

$$\mathbb{E}_{\theta_j}[u_i|\theta_i, m_i^n] = \mathbb{P}(d = (i)|m_i^n)(\theta_i - c) + \mathbb{P}(d = (j)|m_i^n)\mathbb{E}[\theta_j|d = (j), m_i^n]. \tag{3.1}$$

Now take two messages $m_i^n, m_i^{n'}$ such that $\mathbb{P}(d = (i)|m_i^n) > \mathbb{P}(d = (i)|m_i^{n'})$. Obviously, the second summand in equation (3.1) is independent of θ_i and it follows that

$$\frac{d(\mathbb{E}_{\theta_j}[u_i|\theta_i, m_i^n] - \mathbb{E}_{\theta_j}[u_i|\theta_i, m_i^{n'}])}{d\theta_i} = \mathbb{P}(d = (i)|m_i^n) - \mathbb{P}(d = (i)|m_i^{n'}) > 0,$$

which shows the claim. As $c > 0$, full information revelation is impossible: if it were possible, suppose that j actually reveals his type truthfully and that the DM expects both agents to do so. The best response of i is then to send $m_i = \theta_i - c < \theta_i$ which yields a contradiction.

More precisely, because of the costs they pay if they have to work, agents have the incentive to free-ride by understating their type. On the other hand, there are implicit costs of understating because the own type net of c may be larger than the externality. Endogenously, these implicit costs are increasing in types. To see this, consider the cutoff type a_i^n of agent i who is indifferent between sending message $m_i^n := [a_i^{n-1}, a_i^n]$ and $m_i^{n+1} := [a_i^n, a_i^{n+1}]$. As costs are the same for agents, the cutoff points between messages are identical and I can drop the agents' subscript. The key argument to derive this cutoff point is now as follows. If, say agent 1, is indifferent between message m^n and message m^{n+1}, then this is only relevant for his message strategy when agent 2's message is either m^n or m^{n+1}: if 2's message is higher than m^{n+1}, he will work for sure and irrespective of m_1. Otherwise, if m_2 is lower than m^n, agent 1 will work. Given these considerations and recalling that the DM randomizes with equal probability after hearing $m_1 = m_2$, $\mathbb{E}[u_1|m_1 = m^n]$ must be equal to $\mathbb{E}[u_1|m_1 = m^{n+1}]$ at a^n. It follows:

$$\begin{aligned}&\mathbb{P}(m_2 = m^n)(\frac{1}{2}(a^n - c) + \frac{1}{2} \times \frac{1}{2}(a^{n-1} + a^n)) + \mathbb{P}(m_2 = m^{n+1})\frac{1}{2}(a^n + a^{n+1})\\ =&\mathbb{P}(m_2 = m^n)(a^n - c) + \mathbb{P}(m_2 = m^{n+1})(\frac{1}{2}(a^n - c) + \frac{1}{2} \times \frac{1}{2}(a^n + a^{n+1})).\end{aligned}$$

This can be simplified to the difference equation

$$(a^{n+1} - a^n) - (a^n - a^{n-1}) = -2c. \tag{3.2}$$

Following Li et al. (2016), I call communication that evolves according to this difference equation *competitive cheap talk*. From equation (3.2), it follows that equilibrium messages are decreasing in size at the constant rate equal to twice the costs c. Intuitively, CCT balances off the incentive to free-ride with the risk of giving up a larger payoff from the own task. As low types have a higher incentive to free-ride, information transmission is more imprecise at the bottom. The intuition is as follows. Consider two different types of agent 1 pretending to be the same lower type. Compared to the lower type, understating by the higher type involves a higher cost because this type is – net of costs – more likely to be larger than a given type of agent 2. Put differently, if agent 1 is indifferent between sending m^n and m^{n+1}, with decreasing interval size the other agent is less likely to send the higher message and to be a higher type. Therefore, this indifferent type has higher costs of sending the lower message m^n.[15]

Now, one can solve difference equation (3.2) recursively.[16] Rewrite the equation as $a^{n+1} = 2a^n - a^{n-1} - 2c$ with boundary conditions $a^0 = 0$ and $a^N = 1$, where N denotes the maximal number of intervals (or messages) in a given equilibrium. One can show[17] that $a^n = na^1 - n(n-1)c$, or

$$a^n = \frac{n}{N} + (N - n)nc. \tag{3.3}$$

Since $a^0 = 0$, it must hold that $-N(N-1)c > -1$ and the number of intervals must fulfill $1 \leq N < \overline{N}(c) = \left\lceil \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4}{c}} \right\rceil$ where $\overline{N}(c)$ is the largest natural such that $N(N-1)c - 1 \leq 0$. Intuitively, for vanishing costs, the number of supportable intervals increases and information revelation becomes easier. Conversely, for c converging to 1, the only equilibrium is a babbling equilibrium and no information can be credibly transmitted.

An equilibrium is then characterized as follows. It consists of (a) interval messages with interior boundary points evolving according to (3.3), (b) $N < \overline{N}(c)$, and (c) the resulting induced beliefs that assign probability one to the event that the agents' types fall into the reported intervals which makes the DM choose the agent with the highest message with probability one (and randomize with 50-50 probability if $m_1 = m_2$). All these equilibria are symmetric in that agents have the same set of equilibrium messages. Figure 3.2 shows the partition points of such an equilibrium with $N = 4$ (which exists for $c < \frac{1}{12}$) and $c = 0.05$.

Figure 3.2: Equilibrium CCT Messages with $c = 0.05$.

It is worth noticing that there are no obedience constraints for the DM to check because

[15]If the partition size was constant, this type would strictly prefer to send the lower message.

[16]Here, $\alpha = 1$ is needed. For $\alpha \in (0, 1)$, the difference equation becomes $a^{n+1} = \frac{2}{\alpha}a^n - a^{n-1} - \frac{2}{\alpha}c$ which has no explicit solution.

[17]This is a standard argument which is why I skip several steps.

she wants the agent with the higher return to work and simply assigns working to the agent with the larger posterior. The only constraint with bite here is the one on the supportable number of equilibrium messages mirrored by $\overline{N}$.

Interestingly, these equilibria are ex post (or regret-free) equilibria, that is no agent would change his message even if he had heard the message by the other agent. To see this, rewrite equation (3.2) as $a^n = \frac{1}{2}(a^n + a^{n+1}) + \frac{1}{2}(a^{n-1} + a^n) + c = \frac{1}{2}\mathbb{E}[\theta_i|m^{n+1}] + \frac{1}{2}\mathbb{E}[\theta_i|m^n] + c$ and suppose, without loss of generality, that agent 2 sends a higher message adjacent to 1's message: m_2^{n+1} and m_1^n so that agent 2 is chosen to work. For agent 1, it follows that $\theta_1 \leq a^n$, or $\theta_1 - c \leq \frac{1}{2}\mathbb{E}[\theta_2|m^{n+1}] + \frac{1}{2}\mathbb{E}[\theta_2|m^n] < \mathbb{E}[\theta_2|m^{n+1}]$ and he would not change his message. Similarly, for agent 2 it holds that $\theta_2 \geq a^n$ and thus $\theta_2 - c \geq \frac{1}{2}\mathbb{E}[\theta_1|m^{n+1}] + \frac{1}{2}\mathbb{E}[\theta_1|m^n] > \mathbb{E}[\theta_1|m^n]$ and agent 2 indeed prefers to work.

The last argument also indicates that if the DM chose to communicate sequentially with the agents, the outcome would not change.[18] Even under simultaneous communication, an agent's message is only pivotal if the other message is adjacent and the optimal message conditions on this event. It follows that under sequential cheap talk, there is no informational advantage for the second sender. The only difference is that the second mover has now only two non-outcome equivalent messages: he either sends a message in favor or against his own task. The outcome equivalence between sequential and simultaneous communication thus stems from the binary decision that only conditions on ordinal information. In Section 5, I show that when cardinal information matters, as it is the case in my model, this equivalence breaks down.

4. Optimal Allocation with Teamwork

I now turn the focus back to the main model. It will turn out that the substitutes case ($\beta > 1$) is qualitatively similar to the setup without teamwork. On the other hand, the complements case ($\beta < 1$) is structurally different in that there exists no CCT in equilibrium.

4.1 Preliminary Analysis

Before I start with the analysis of either case, I first show that also in my model, all equilibria must be of the interval form. It directly follows that truthful communication cannot be part of a PBE.

[18] The formal argument can be found in Li et al. (2016). They even show that if the DM delegated the decision right to one agent, there exists an equilibrium that is outcome-equivalent to sequential communication. I will explain in Section 5 how this changes in my model.

Proposition 3.1
With $d \in \{(0),(1),(2),(1,2)\}$, all equilibria of the game are interval equilibria.

According to this result, in equilibrium, the type space is partitioned into intervals and agents reveal to which interval their type belongs. This rationalizes the assumption that the message space is equal to the type space. It follows that off-path beliefs never matter. Further, in any equilibrium below, the DM's belief will assign all probability mass to the event that an agent's type falls into the reported interval.[19]

4.2 The Complements Case: $\frac{1}{2} < \beta < 1$

In this subsection, I resume the assumption that $\alpha \in [0,1]$. Comparing payoffs readily yields Figure 3.3 that shows her first-best decision for $\beta > \frac{1}{2}$ and the outcomes agent 1 prefers.[20] For $\beta \leq \frac{1}{2}$, the regions in the left panel where $d = (1)$ and (2) are preferred vanish and the DM either wants teamwork or the outside option, depending on whether $\theta_1 + \theta_2 \geq \frac{2\beta c}{1+\alpha}$ or not. This case can be solved in the same way as below, but here, I want to focus on equilibria where all four decisions can be an equilibrium outcome.

As noted above, perfect revelation of information is not possible. Likewise, Figure 3.3 indicates that first-best is not attainable because of misaligned preferences: the DM prefers one agent working alone if his return is large enough and the other is not, whereas she wants both agents (none of them) to work if both types are large (small) enough.

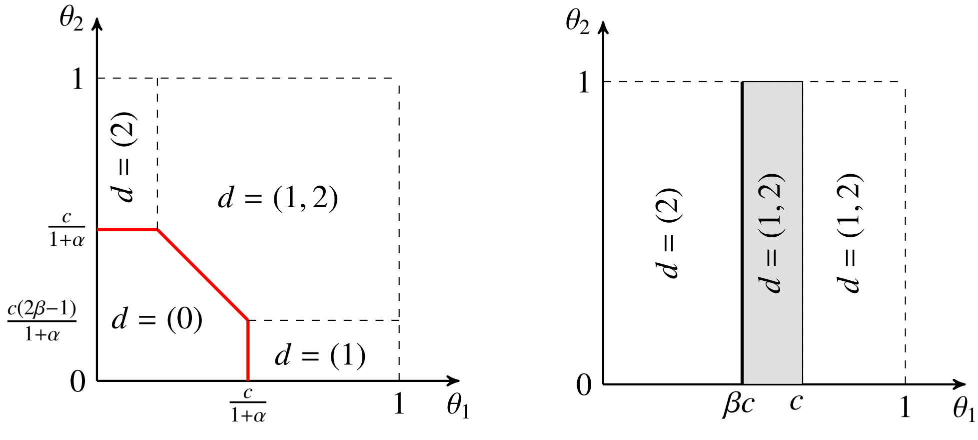

Figure 3.3: $\frac{1}{2} < \beta < 1$: First-Best Decisions (left panel) and Preferred Decisions for Agent 1 (right panel).

Agents' most preferred decisions, on the other hand, are independent of the other agent's type. This differs from the case without teamwork (and the substitutes case below). Agents

[19] I omit this repetitive argument in the description of the equilibria below.
[20] If $\beta = 1$, the region where she prefers $d = (0)$ is a square of length $\frac{c}{1+\alpha}$.

never want to work alone because teamwork yields the externality and leads to lower cost. Comparing agent 1's payoff from teamwork with that from $d = (2)$ shows that teamwork is preferred if and only if $\theta_1 \geq \beta c$. If the own return is smaller than βc, then agents prefer the other agent to work because of the externality (if $\alpha > 0$; otherwise they are indifferent between free-riding and the outside option). It follows that low types have the incentive to understate the own type in order to free-ride, but high types have the incentive to overstate to make the DM implement teamwork. All in all, for $\alpha > 0$ there are exactly three different intervals of the type space that correspond to different preference orders over decisions:

1. For $\theta_1 \in [0, \beta c)$: agent 1 prefers $d = (2)$ over (0) and $(1, 2)$ (the order depends on θ_2) over (1).
2. For $\theta_1 \in [\beta c, c)$: agent 1 prefers $d = (1, 2)$ over (2) over (0) over (1).
3. For $\theta_1 \in [c, 1]$: agent 1 prefers $d = (1, 2)$ over (1) or (2) (depending on whether $\theta_1 > \alpha\theta_2$) over (0).

In particular, for $\theta_1 \in [\beta c, c)$ and if $d = (1)$, then $u_1 < 0$, a payoff smaller than from the outside option. Before I solve this game, I proceed with a benchmark. As a cheap talk game, there always exist babbling equilibria where no information is transmitted.

Benchmark: Uninformative Messages
Suppose messages are uninformative (or the DM ignores them). Then, $\mathbb{E}[\theta_i|m_i] = \frac{1}{2}$ for $i = 1, 2$. Her ex ante payoff from teamwork is $(1 + \alpha) - 2\beta c$, and $\frac{(1+\alpha)}{2} - c$ if one agent works alone.

Proposition 3.2
Suppose agents' messages are uninformative. Then,

1. *if* $1 + \alpha \geq 2c$*, she chooses* $d = (1, 2)$ *– independent of* β*;*
2. *if* $1 + \alpha < 2c$*, she chooses* $d = (0)$ *for* $\beta > \frac{1+\alpha}{2c}$ *and* $d = (1, 2)$ *otherwise.*

   She never chooses one agent to work alone.

With uninformative messages, all equilibria are trivial in that she either chooses teamwork or lets no one work – depending on the primitives. The reason why no agent ever works alone is as follows: by the distributional assumption, it holds that $\mathbb{E}[\theta_i] = \frac{1}{2}$. Further, by the left panel in Figure 3.3, her preferences for one agent working alone are determined by the cutoff $\frac{c}{1+\alpha}$ while the other type must be smaller than $\frac{c(2\beta-1)}{1+\alpha}$. With uninformative messages, however, she can never detect whether a type falls below the latter cutoff. She only knows whether $\frac{c}{1+\alpha}$ is larger or smaller than $\frac{1}{2}$. If messages are informative, I now show that she can use the information to implement all four outcomes.

Informative Messages: Necessary Conditions

I start with the general structure of possible outcomes after messages. This helps to exclude non-supportable assignments of decisions to messages. The remaining candidate equilibria are then proven to exist. Recall that agents' type space can be divided into three distinct intervals with respect to their preference order over outcomes. Hence, in equilibrium there cannot be more information transmitted than three non-outcome equivalent messages per agent – or nine message pairs in total (I formally prove this claim at the end of this subsection). I denote these potential messages by m^1, m^2, m^3 and the cutoffs between them as x and y, so sending m_1^2 reveals that θ_1 lies in the interval $[x, y]$. If equilibrium messages are binary I denote the unique cutoff by z. Figure 3.4 shows the general message structure in the two-dimensional message space.

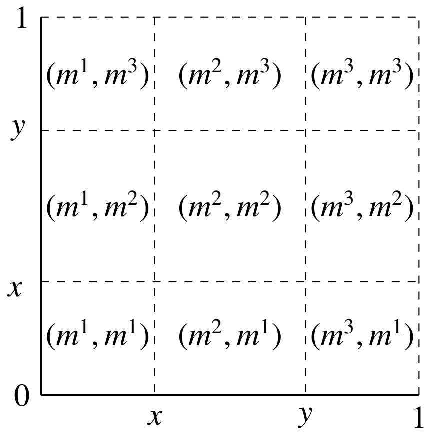

Figure 3.4: General Message Structure for $\frac{1}{2} < \beta < 1$.

Neither agents nor the DM face a trade-off between $d = (1)$ and $d = (2)$. This indicates the major difference to the setup without teamwork: for $\beta < 1$, there exists no equilibrium with competitive cheap talk and information transmission is limited by agents' preferences. To be precise, comparing the DM's preferences from Figure 3.3 with Figure 3.4 yields that after (m^1, m^1), her decision is $d = (0)$ (I show below that this is indeed optimal). Similarly, after (m^3, m^3), she chooses teamwork. The other seven areas in Figure 3.4 can be filled with any decision such that the DM is obedient. By symmetry one can deduct that, if she chooses teamwork after (m^2, m^3), then she must do so after (m^3, m^2). But then, she does not choose $d = (i)$ after (m^2, m^2) because her payoff from $d = (1, 2)$ after hearing (m^2, m^3) is

$$(1 + \alpha)(\frac{x + y}{2} + \frac{y + 1}{2}) - 2\beta c \geq (1 + \alpha)\frac{y + 1}{2} - c,$$

where the inequality follows because she prefers teamwork over $d = (2)$. Rewriting this yields $x + y \geq \frac{2c}{1+\alpha}(2\beta - 1)$. Now, if she chose $d = (i)$ after (m^2, m^2), then it must hold that[21]

$$(1 + \alpha)\frac{x + y}{2} - c > (1 + \alpha)(x + y) - 2\beta c,$$

or $\frac{2c}{1+\alpha}(2\beta - 1) > x + y$, a contradiction. Hence, after (m^2, m^2) she either chooses teamwork or the outside option (at the end of this subsection, I show that $d = (0)$ after (m^2, m^2) cannot be part of an equilibrium). Similar arguments show that after hearing message (m^1, m^3) or (m^3, m^1), she will not choose $d = (1)$ or $d = (2)$, respectively. Given these preliminaries, the number of candidate equilibria is limited.

I begin with constructing an assignment of decisions to messages without $d = (i)$. Then, by symmetry, either she chooses $d = (1, 2)$ after (m^1, m^2), (m^1, m^3) and after (m^2, m^1), (m^3, m^1); or she chooses $d = (0)$ in all these cases. But these candidate equilibria are then determined by one single cutoff type, say z. The upper left and upper right panel of Figure 3.5 illustrate this.[22]

There are also candidate equilibria with individual work. By symmetry, it must then hold that either both (m^1, m^2) and (m^1, m^3) yield $d = (2)$ or that only (m^1, m^3) yields $d = (2)$ with (m^1, m^2) leading to $d = (0)$ – and similarly for $d = (1)$ in the lower right of the message space. These two *task-separating* candidate equilibria are shown in the lower two panels of Figure 3.5. The former also has a binary message structure, while the latter entails three messages per agent.

Finally, I now argue that it is indeed optimal for her to choose the outside option if both agents send the lowest message. If this was not true and she chose, say $d = (1)$,[23] her expected payoff after (m^1, m^1) is $(1 + \alpha)\frac{x}{2} - c$ and this must be strictly larger than zero, so: $x > \frac{2c}{1+\alpha}$. But then by symmetry, she will not choose teamwork after (m^2, m^1), but $d = (1)$. This yields $x < \frac{2}{1+\alpha}(2\beta c - c)$. Combining both inequalities yields $2\beta c > 2c$ or $\beta > 1$, a contradiction.[24]

[21] By symmetry, this argument remains true if she randomizes over $d = (1)$ and $d = (2)$.

[22] These candidate equilibria consist of only two messages. Qualitatively, the panel in the upper left (right) of Figure 3.5 is the same if teamwork (the outside option) is allocated to the four rectangles in the upper right (lower left).

[23] If she chooses teamwork here, then this must be true for all messages because her payoff is monotone in types. This is, however, outcome-equivalent to the trivial equilibrium with uninformative messages.

[24] This reasoning only depends on the lowest message and thus holds for binary messages. A similar argument can be used to show that her decision is teamwork if both agents send the highest message. This will help to argue that I have found all equilibria at the end of this subsection.

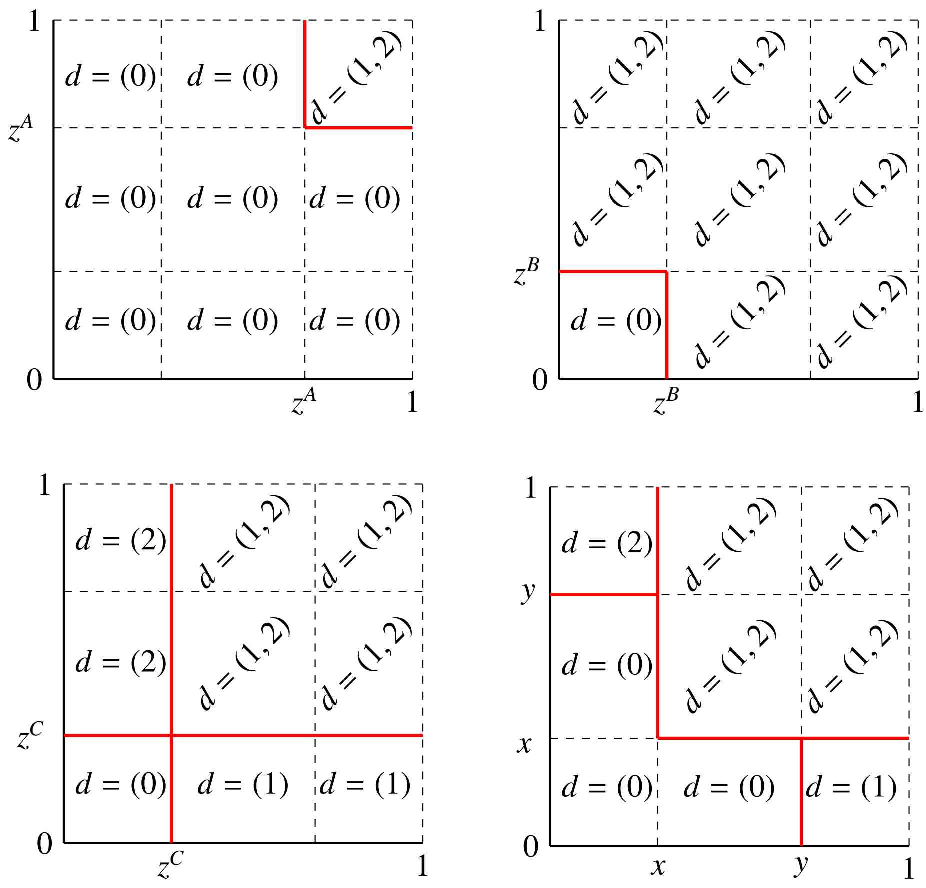

Figure 3.5: Possible Equilibria for $\frac{1}{2} < \beta < 1$.

Binary Equilibria

With binary messages, denote the high message as $m_i = 1$ and the low message as $m_i = 0$. Therefore, there exist four pairs of messages: $m \in \{(0,0), (1,0), (0,1), (1,1)\}$. Such binary equilibria have an intuitive interpretation: 1 and 0 simply encode the answers "yes" and "no" to the question: "Do you want to work?". The three candidate equilibria with binary message pair are depicted in the the first row and the lower left panel of Figure 3.5 and assign decisions to messages in the following way:

A $d = (0)$ for $m \in \{(0,0), (0,1), (1,0)\}$ and $d = (1,2)$ after $m = (1,1)$.

B $d = (0)$ after $m = (0,0)$ and $d = (1,2)$ for $m \in \{(0,1), (1,0), (1,1)\}$.

C $d = (0)$ after $m = (0,0)$, $d = (1)$ after $m = (1,0)$, $d = (2)$ after $m = (0,1)$ and $d = (1,2)$ after $m = (1,1)$.

Denoting the respective cutoff in the three cases with z^A, z^B and z^C, the following result shows that these indeed constitute equilibria of the communication game. Moreover, the cutoff types are given and restrictions for existence are made.

Proposition 3.3

The cases A-C constitute equilibria. These have the following properties.

- *Equilibrium A exists if and only if* $\frac{(1+\alpha)(2-\alpha)}{4c} < \beta \leq \frac{2+\alpha}{1+\alpha} - \frac{1}{c} < 1$.
- *Equilibrium B exists if and only if* $\beta \leq \min\left\{\frac{(2+\alpha)(1+\alpha)}{4c}; \frac{2+\alpha}{3+\alpha}\right\}$.
- *Equilibrium C exists if and only if*

$$\beta \geq -\frac{4-6c-(1+\alpha)}{8c} + \frac{1}{8c}\sqrt{9-4c+4c^2-6\alpha+\alpha^2+12c\alpha}.$$

The cutoffs between messages $m_i = 0$ *and* $m_i = 1$ *are*

$$z^A = \frac{2\beta c - \alpha}{2+\alpha} \leq z^B = \frac{2\beta c}{2+\alpha} \leq \beta c < z^C = \frac{\beta c}{1-c(1-\beta)} < c.$$

The proof is constructive: agents must be indifferent between sending the high and the low message at the cutoff $z^k, k \in \{A, B, C\}$. Having found the cutoff for any case, it remains to be shown that her payoff is indeed maximized by following the proposed equilibrium path.

Equilibrium C is particularly interesting because here, two messages suffice to reach task separation and individual work is implemented if the respective agent sends $m_i = 1$ alone. For equilibrium A, notice that $\frac{(1+\alpha)(2-\alpha)}{4c} > \frac{\alpha}{2c}$ holds for all $\alpha < 1$ (with equality at $\alpha = 1$). Hence, if equilibrium A exists, then $\beta > \frac{(1+\alpha)(2-\alpha)}{4c} > \frac{\alpha}{2c}$ and $z^A > 0$. Moreover, $z^A = z^B = \beta c$ for $\alpha = 0$.

I now proceed with a discussion and comparative statics of the binary equilibria. The order of the three cutoff types is intuitive because in equilibrium A and B, there is no risk of working alone. The fact that the largest cutoff z^C is smaller than c stems from the fact that types larger than the cost parameter never incur a negative ex post payoff and would not send $m_i = 0$.

In equilibrium A, after all but one message pair no one works. Hence, z^A is the lowest cutoff and relatively many types send the high message because they are perfectly insured by her outside option. Moreover, the equilibrium exists only if c is large (see Table 3.1 and 3.2 for an overview).

Equilibrium B is the polar case. Teamwork is implemented here unless both messages are low, so less types send the high message (if $\alpha > 0$). Both z^A and z^B are increasing in β and in c, so a worse match value and higher costs of working make agents send $m_i = 1$ less often to balance out the larger expected costs. On the other hand, both z^A and z^B are decreasing in α and a larger externality makes them send the high message more often.

Finally, also z^C is strictly increasing in β and c. The fact that equilibrium C exists and individual work is supportable with such a coarse message strategy may be surprising. After all, $d = (i)$ is a decision that is never preferred by agents. Observe, however, that z^C lies in the interval $(\beta c, c)$. Moreover, β must be large for this equilibrium to exist. But if β converges to one from below, then z^C approaches c and the probability that a type falls into the interval (z^C, c) becomes small. This risk of working alone and receiving a negative payoff represents the costs of the possibility to free-ride. Further, z^C is independent of α because whether an agent receives the externality is independent of the own message: if agent 1 sends the high message, he will work in a team only if agent 2 sends the high message. And if agent 1 sends the low message, he will free-ride only if agent 2 sends the high message.

The restrictions on β in any equilibrium stem from the DM's obedience constraints. For equilibrium A, it is shown in the proof of Proposition 3.3 that there are two constraints with bite: she must choose (0) after hearing one high and one low message. Hence, β must be large enough to deter her from choosing teamwork, but it must also be small enough to make her choose teamwork after $m = (1, 1)$.

In equilibrium B, β must be small enough to make teamwork possible except for when both agents send the low message. Moreover, equilibrium B is more easily attained if α is large and c is small.

To attain task separation in equilibrium C, β must be large enough to deter the DM from choosing teamwork after one high and one low message. Observe, however, that for $\alpha = 1$ equilibrium C does not exist. This is because for larger α she prefers, all else unchanged, teamwork. But task separation is only supportable for large β which makes teamwork less desirable and equilibrium C breaks down.

Eq.	m	d	Cutoff z	Restriction on β
A	$(0,0),(1,0),(0,1)$ $(1,1)$	(0) $(1,2)$	$z^A = \frac{2\beta c-\alpha}{2+\alpha}$	$\frac{(1+\alpha)(2-\alpha)}{4c} < \beta \leq \frac{2+\alpha}{1+\alpha} - \frac{1}{c}$
B	$(0,0)$ $(1,0),(0,1),(1,1)$	(0) $(1,2)$	$z^B = \frac{2\beta c}{2+\alpha}$	$\beta \leq \min\left\{\frac{(2+\alpha)(1+\alpha)}{4c}; \frac{2+\alpha}{3+\alpha}\right\}$
C	$(0,0)$ $(1,0)$ $(0,1)$ $(1,1)$	(0) (1) (2) $(1,2)$	$z^C = \frac{\beta c}{1-c(1-\beta)}$	$\beta \geq -\frac{4-6c-(1+\alpha)}{8c}$ $+\frac{1}{8c}\sqrt{9-4c+4c^2-6\alpha+\alpha^2+12c\alpha}$

Table 3.1: Equilibria with Binary Message Pair.

The following Table 3.2 provides the restrictions on β for different values of α. For equilibrium A to exist and $\alpha = 0$ ($\alpha = 0.5$), combining both restrictions on β yields that c must be larger than $\frac{3}{4}$ ($\frac{15}{16}$).

If c converges to zero and for $\alpha = 0$ ($\alpha = 0.5$), the task-separating equilibrium exists for $\beta \geq \frac{2}{3}$ ($\beta \geq \frac{4}{5}$). And for $c \to 1$: $\beta \geq \frac{3}{4}$ and $\beta \geq \frac{7}{8}$, respectively. Finally, for large externality, the only equilibrium with binary messages is type B (if β is small enough).

Eq.	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
A	$\frac{1}{2c} < \beta \leq 2 - \frac{1}{c}$	$\frac{9}{16c} < \beta \leq \frac{5}{3} - \frac{1}{c}$	Does not exist
B	$\beta \leq \min\{\frac{1}{2c}; \frac{2}{3}\}$	$\beta \leq \frac{5}{7}$	$\beta \leq \frac{3}{4}$
C	$\beta \geq \frac{3}{4} - \frac{3}{8c} + \frac{1}{8c}\sqrt{9 - 4c + 4c^2}$	$\beta \geq \frac{3}{4} - \frac{5}{16c} + \frac{1}{8c}\sqrt{\frac{25}{4} + 2c + 4c^2}$	Does not exist

Table 3.2: Restrictions on β for Equilibrium A-C and $\alpha = 0, 0.5$ and 1.

Three Messages: The Most Informative Equilibrium

The next result proves existence of a 3-message equilibrium with task separation.

Proposition 3.4

There exists an $\overline{\alpha} > 0$ such that for all $\alpha \leq \overline{\alpha}$, there exist an upper and a lower bound $\overline{\beta} := \overline{\beta}(\alpha, c)$, $\underline{\beta} := \underline{\beta}(\alpha, c)$, and for all $\beta \in (\underline{\beta}, \overline{\beta}]$, the equilibrium from the lower right panel of Figure 3.5 exists. It is characterized by the messages $m^1 := [0, x]$, $m^2 := [x, y]$ and $m^3 := [y, 1]$, with $y = c$ and

$$x = \frac{1 + \beta c}{2 + \alpha} - \frac{1}{2 + \alpha}\sqrt{(1 + \beta c)^2 + (2 + \alpha)(\alpha c^2 - 2\beta c)}.$$

If it exists, this equilibrium exhibits an undistorted cutoff at the top equal to the costs of working alone: $y = c$. Moreover, if $\alpha = 0$, then agents are indifferent between her outside option and free-riding and $x = \beta c$ which is the agents' undistorted cutoff at the bottom. In this case and from the fact that she chooses $d = (1)$ after (m^3, m^1), it follows that $4\beta c - 2c > \beta c$ or $\beta > \frac{2}{3} = \underline{\beta}$; the upper bound $\overline{\beta}$ is equal to one and irrelevant in this case (the details can be found in Appendix A). For $\alpha > 0$ it holds that $x < \beta c$ and agents send the intermediate message more often because of the externality gain if teamwork is implemented. There is, however, no such equilibrium for large externalities. To see this, suppose $\alpha = 1$. Then, after message (m^2, m^1), she must choose $d = (0)$, so: $0 \geq x + y - c$, but as $y = c$, this cannot be true in a 3-message equilibrium. Further, it is intuitive that β must be large enough to deter the DM from choosing teamwork after (m^3, m^1) and (m^1, m^3).

It remains to be shown that other assignments of decisions to messages cannot be part of an equilibrium. First, recall that in any informative equilibrium, teamwork and the outside option are chosen. Second, the 3-message structure in Figure 3.6 is not an equilibrium because incentive compatibility prescribes that intermediate types larger than $x = \beta c$ send a message in favor of $d = (0)$. But then, after (m^2, m^2), the DM would know that both types are larger than βc and she would prefer teamwork.

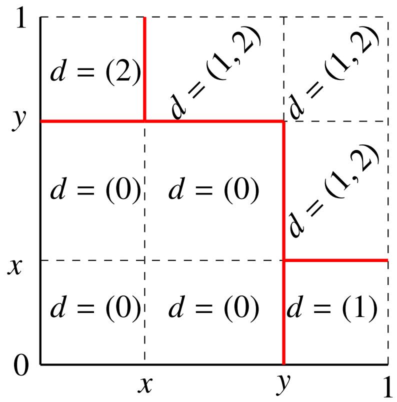

Figure 3.6: No Equilibrium Structure.

To see this, suppose it were an equilibrium with partition points $0 < x < y < 1$. Then, type x of agent 1 is indifferent between message m^1 and m^2, hence:

$$\mathbb{P}(m_2 = m^3)\frac{\alpha}{2}(y+1) = \mathbb{P}(m_2 = m^3)(x + \frac{\alpha}{2}(y+1) - \beta c) \iff x = \beta c.$$

After (m^2, m^2), it must hold that $\frac{2\beta c}{1+\alpha} > x + y = \beta c + y$. Rearranging yields $\beta c > \frac{1+\alpha}{1-\alpha} y$ and thus $x = \beta c > y$, a contradiction.

So far, I have dealt with communication equilibria that exhibit one, two and three distinct messages. Finally, not more information than encoded in three messages can be revealed.

Proposition 3.5
With $\beta < 1$, there is no equilibrium with more than three different messages per agent.

It follows that I have found all equilibria for the complements case. The reason that not more information can be transmitted is that there is no trade-off between working alone and free-riding. But for the DM, there is one between her outside option and teamwork. Agents, however, never want her outside option because of the externality (if $\alpha > 0$). More precise information transmission by intermediate types would make it easier for the DM to assign $d = (0)$, but this cannot be incentive-compatible. And if $\alpha = 0$, the cutoffs in Proposition 3.4 are $x = \beta c$ and $y = c$ which perfectly corresponds to agents' preferences and three messages are the maximum an agent reveals. Finally, as there is no informative equilibrium if α and β are both large, the only equilibrium in this case is an uninformative one in the sense of Proposition 3.2.

Welfare

Denote by U^k_{DM} her ex ante payoff in equilibrium k from Proposition 3.3 and by U^{3M}_{DM} her payoff in the 3-message equilibrium from Proposition 3.4. A welfare comparison across all equilibria is not viable because their existence depends on the primitives α, c and β and most cases are mutually exclusive. So given a fixed β, a certain equilibrium exists. It holds:[25]

$$\begin{aligned}
U^A_{DM} &= (1-(z^A)^2)(1-z^A)(1+\alpha) - 2\beta c(1-z^A)^2,\\
U^B_{DM} &= (1+\alpha)(1-(z^B)^3) - 2\beta c(1-(z^B)^2),\\
U^C_{DM} &= (1+\alpha)(1-(z^C)^2) - 2c(1-z^C)(\beta+(1-\beta)z^C).
\end{aligned}$$

Proposition 3.2 showed that uninformative equilibria always exist, and Proposition 3.3 characterized informative, binary equilibria. In particular, their existence and structure depends on the match value β. If there is an informative equilibrium, the next result shows that in the binary cases information transmission is better for efficiency.

Proposition 3.6

If there is exactly one informative equilibrium with binary messages, then her ex ante payoff is larger in this equilibrium than without information transmission.
If multiple binary equilibria coexist, then at least one exhibits a larger ex ante welfare compared with the corresponding uninformative equilibrium.

Proving general welfare dominance of informative equilibria over their uninformative counterparts seems to be hard. This result, however, ensures that in case of multiplicity of equilibria, there is at least one equilibrium with binary messages that outperforms no information transmission. Furthermore, ex ante welfare in the most informative equilibrium is as follows.

$$U^{3M}_{DM} = 2x(1-y)((1+\alpha)\frac{y+1}{2} - c) + (1-x)^2[(1+\alpha)(x+1) - 2\beta c].$$

A welfare comparison between this equilibrium and uninformative equilibria is intricate in general terms.[26] But suppose, for example, that $\alpha = 0$. In this case, the equilibrium exists

[25] For $c \to 0$, $z^C = \frac{\beta c}{1-c(1-\beta)}$ converges to zero and U^C_{DM} becomes $1+\alpha$, which is her payoff if she always chooses teamwork. Similarly, for $c \to 1$, z^C becomes 1 and $U^C_{DM} = 0$ – her payoff if she always chooses the outside option (see Proposition 3.2).

[26] Also a welfare comparison between both task-separating equilibria is not practical because the lower bound on β in Proposition 3.4 is only shown to exist, but not derived explicitly.

with $x = \beta c$ and $y = c$. Her payoff becomes

$$U_{DM}^{3M} = (1 - c^2)\beta c + (1 - \beta c)^3,$$

which is obviously larger than zero. Recall that she would never choose one agent to work with uninformative messages. So setting $(1 - c^2)\beta c + (1 - \beta c)^3 > 1 - 2\beta c$ yields

$$0 > 2 + \beta^2 c - 3\beta - c. \tag{4.1}$$

The right-hand side has roots $\beta_{R,L} = \frac{3}{2c} \pm \frac{1}{2c}\sqrt{4c^2 - 8c + 9}$. Recall that $\frac{2}{3}$ is the lower bound on β such that this equilibrium exists. One readily shows that $\beta_R > 1$ and that $\beta_L < \frac{2}{3}$, so (4.1) is true for all relevant β's.

4.3 The Substitutes Case: $\beta > 1$

I now analyze the case in which costs of teamwork are higher than the costs of working alone and show that information transmission is improved by the trade-off between working alone and free-riding. Two restrictions are made. First, $\alpha = 1$ holds. As in Section 3, this assumption is made to derive explicit results. Second, I assume that teamwork is efficient if both agents are the highest possible type. Comparing her payoff from all outcomes for this case, it follows that $1 < \beta \leq \frac{1}{c} + \frac{1}{2}$. Figure 3.7 illustrates what the first-best decision is and which decision agent 1 prefers in the type space. This is the polar case compared with $\beta < 1$. In particular, there is no trade-off between teamwork and her outside option, but between $d = (1)$ and $d = (2)$. This observation provides a first intuition why CCT reappears here as part of an equilibrium.

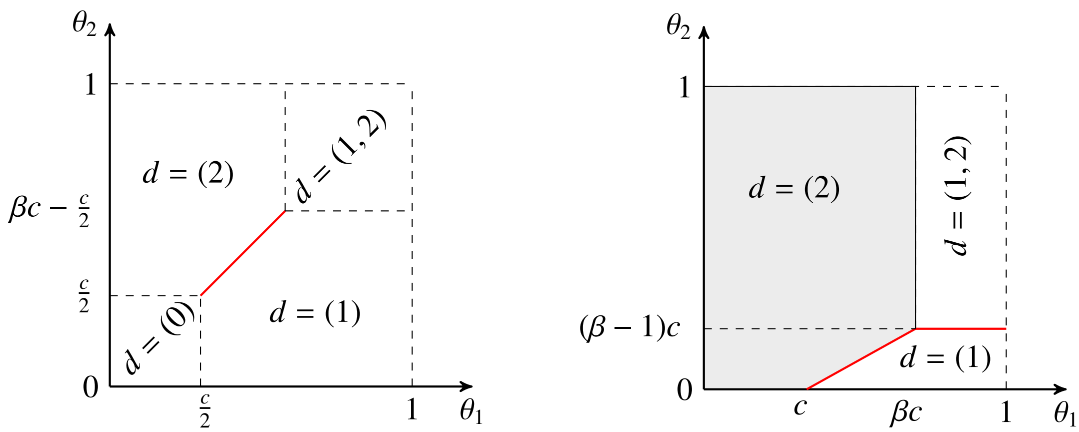

Figure 3.7: $\beta > 1$: First-Best Decisions (left panel) and Preferred Decisions for Agent 1 (right panel).

Recall that in the complements case, agents' most preferred outcomes were independent of the other agent's type. Here if the own type, say θ_1, is larger than the costs of teamwork βc, the opposite is true and preferences are independent of the own type: teamwork is preferred if and only if θ_2 is large enough to exceed the higher costs. Otherwise, agent 1 prefers working alone. For $\theta_1 < \beta c$, agents' preferences mirror the case without teamwork and outside option, so low types have an incentive to free-ride (see Figure 3.1). Before I solve this game, I want to emphasize that – like in the complements case – her optimal decision with uninformative messages is trivial.

Benchmark: Uninformative Messages

Proposition 3.7
Suppose agents' messages are uninformative. Then,

1. *if* $\beta \leq \frac{1}{2c} + \frac{1}{2}$, *she chooses* $d = (1, 2)$*;*
2. *if* $\frac{1}{2c} + \frac{1}{2} < \beta \leq \frac{1}{c} + \frac{1}{2}$, *she randomizes over* (1) *and* (2) *with equal probability.*

The proof is the same as in the polar case in Proposition 3.2 (and thus omitted). Her payoff with uninformative messages is $2(1 - \beta c)$ if she chooses $d = (1, 2)$ and $1 - c$ otherwise.[27] Comparing outcomes pairwise shows the claim.

Informative Messages
I now turn to the original game and begin with showing existence of an equilibrium in which all outcomes can be implemented. Based on such a task-separating equilibrium, I explain afterwards which other outcomes can occur if task separation is not possible.

The preferences of the DM and agents suggest the following equilibrium structure. I show that there exists an endogenous interval $[x, y] \subset [0, 1]$ in which communication is like in the case without teamwork and outside option. I call this the *CCT interval*. Further, there exists a lowest message $m^0 := [0, x]$ such that she chooses $d = (0)$ if and only if both agents send this message; and there is a highest message $m^{N+1} := [y, 1]$ that leads to teamwork if and only if both agents send this message. I denote $a^{-1} := 0, a^0 := x, a^N := y, a^{N+1} := 1$ to be consistent with the CCT framework from Section 3. Hence, m^N denotes the largest equilibrium message in $[x, y]$. Before I show that this is indeed an equilibrium, one must adapt these messages to the CCT interval. In $[x, y]$, the decision is made between $d = (1)$ and (2) only and thus, the difference equation according to which interval partitions evolve is the same as in Section 3:

[27] The fact that she never chooses the outside option here follows from the assumption that $\alpha = 1$. For $\alpha < 1$, she would choose $d = (0)$ if $1 + \alpha \leq 2c$ and irrespective of β.

$a^{n+1} = 2a^n - a^{n-1} - 2c$. The boundary conditions, however, are now $a^0 = x$ and $a^N = y$. Using this, a standard argument yields that

$$a^n = na^1 - (n-1)x - n(n-1)c, \tag{4.2}$$

and $a^N = Na^1 - (N-1)x - N(N-1)c$. Since $y - a^N = 0$, it follows that $(N-1)x + N(N-1)c < y - x$, the length of the CCT interval.[28] The upper bound on partition elements fulfils $\overline{N}(c) = \left\lceil \frac{1}{2} - \frac{x}{2c} + \sqrt{\frac{x^2}{4c^2} - \frac{x}{2c} + \frac{1}{4} + \frac{y}{c}} \right\rceil$, and for $x = 0$ and $y = 1$, $\overline{N}(c)$ becomes $\left\lceil \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4}{c}} \right\rceil$ as in Section 3. Also here, for $c \to 1$, not more information than babbling (with respect to the interval $[x, y]$) can be credibly revealed.[29] Conversely, if c approaches zero, then partitions become finer and communication becomes easier. Finally, using $a^N = y$ and (4.2) yields

$$a^n = \frac{n}{N}(y - x) + ncN + x - n^2c. \tag{4.3}$$

The following main result of this subsection shows existence of the proposed equilibria with non-trivial CCT played by intermediate types.

Proposition 3.8
Suppose $2 \leq N < \overline{N}(c)$. *Then, there exist* $\underline{\beta}(c)$ *and* $\overline{\beta}(c)$ *such that for* $\beta \in (\underline{\beta}(c), \overline{\beta}(c)]$, *the following constitutes an equilibrium:*

- *If* $m = (m_1^0, m_2^0)$, *then* $d = (0)$.
- *If* $m = (m_1^{N+1}, m_2^{N+1})$, *then* $d = (1, 2)$.
- *If* $m_1 \neq m_2$, *then the agent with the higher message works alone. If* $m_1 = m_2$ *and* $m_i \notin \{m_i^0, m_i^{N+1}\}$, *then she chooses* $d = (1)$ *or* $d = (2)$ *with equal probability.*

Messages $m_i \in \{m_i^1, \ldots, m_i^N\}$ *have the CCT structure on an interval* $[x, y] \subset [0, 1]$, *where* y *is a function of* x *with* $x \leq c$ *and*

$$y(x) = \frac{1 + \beta c - c}{2} - \sqrt{\frac{1}{4}(1 + \beta c - c)^2 - \beta c + x^2}. \tag{4.4}$$

Discussion and Example
The result says that for $\beta > 1$, CCT is supportable in equilibrium. By symmetry, the outside option is only chosen if both agents send the lowest message m^0 and both agents reveal

[28] This is the same argument as in the workhorse example of Crawford and Sobel (1982), only with a negative bias and different type space.

[29] Babbling on the whole interval $[x, y]$ is the only possible communication strategy if $\frac{1}{2} - \frac{x}{2c} + \sqrt{\frac{1}{4} + \frac{x^2}{4c^2} + \frac{y}{c} - \frac{1}{2c}} < 2$ or $\frac{y-2x}{2} < c$.

that their return is lower than x.[30] The same applies at the top for the highest message m^{N+1} and teamwork (and revealing that the return is larger than y). If both agents send different messages, individual work is allocated to the agent with the higher message. Assuming that she randomizes with equal probability is again without loss of generality. It is intuitive that such an equilibrium can only exist if β is not too large because, if it were, teamwork would never be implementable. Further and as in the complements case, there is a lower bound on the admissible β's, $\underline{\beta}(c)$. This stems from the fact that she chooses individual work after hearing, for example, $m = (m_1^N, m_2^{N+1})$. So β must not be too small to deter the DM from implementing teamwork here. It is shown in Appendix A that this deterrence constraint is given by $2\beta c > Nc + \frac{x}{N} + y(2 - \frac{1}{N})$ and the RHS is increasing in N so that an equilibrium with more information transmission is harder to sustain.

To give an intuition for the result, I outline the equilibrium construction for $N = 2$ – the simplest case without babbling in the middle. Then, $a^{N-1} = a^1$ and there are two messages in the CCT interval. To be precise, all equilibrium messages are $m^0 = [0, x], m^1 = [x, a^1], m^2 = [a^1, y], m^3 = [y, 1]$.

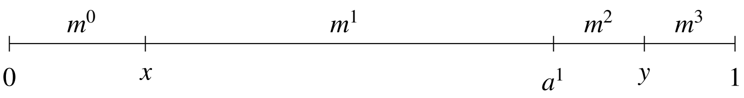

Figure 3.8: Equilibrium Messages for $N = 2$.

According to (4.3), the only interior CCT cutoff is $a^1 = \frac{1}{2}x + \frac{1}{2}y + c$.[31] The first step is to derive the boundaries of the CCT interval. At x, agent 1 is indifferent between m^0 and m^1. Recalling that $\alpha = 1$, it must hold that

$$\begin{aligned}&\mathbb{P}(m_2 = m^0) \times 0 + \mathbb{P}(m_2 > m^0)\frac{1}{2}(x + 1) = \frac{1}{2}(1 - x^2)\\ =&\mathbb{P}(m_2 = m^0)(x - c) + \mathbb{P}(m_2 = m^1)(\frac{1}{2} \times \frac{1}{2}(x + a^1) + \frac{1}{2}(x - c)) + \mathbb{P}(m_2 > m^1)\frac{1}{2}(1 + a^1).\end{aligned}$$

Simplifying and inserting a^1 results in

$$0 = \frac{15}{16}x^2 - \frac{1}{16}y^2 + \frac{1}{8}xy - \frac{1}{2}yc - \frac{1}{2}xc - \frac{3}{4}c^2. \tag{4.5}$$

[30] Other assignments at the bottom are outcome-equivalent: suppose one divides the lowest message m^0 into, say, two messages after which the outside option is chosen if both agents send one of them. Then one can merge these messages into m^0. The same holds at the top for m^{N+1} and teamwork.

[31] Observe that the length of m^1 is $\frac{1}{2}y - \frac{1}{2}x + c$ which is larger than the one of m^2 (equal to $\frac{1}{2}y - \frac{1}{2}x - c$) and they differ by a length of $2c$. This is reminiscent to the decreasing interval size of CCT messages which reflects the increasing costs of understating the own type (see Section 3).

Similarly, at the cutoff type y, agents are indifferent between m^2 and m^3. As teamwork is only chosen if both agents send the highest message, it must hold:

$$
\begin{aligned}
&= (1-y)(y + \frac{1}{2}(1+y) - \beta c) + y(y-c) \\
&= (1-y)\frac{1}{2}(1+y) + (y-a^1)(\frac{1}{2}\times\frac{1}{2}(y+a^1) + \frac{1}{2}(y-c)) + a^1(y-c),
\end{aligned}
$$

or $0 = \frac{3}{4}y^2 + y(\frac{1}{2}a^1 + \frac{1}{2}c - 1 - \beta c) + \beta c - \frac{1}{2}a^1(\frac{1}{2}a^1 + c)$. After inserting a^1, one receives

$$0 = \frac{15}{16}y^2 - \frac{1}{16}x^2 + \frac{1}{8}xy + \frac{1}{2}yc - y - y\beta c - \frac{1}{2}xc - \frac{3}{4}c^2 + \beta c. \tag{4.6}$$

Now, equating (4.5) with (4.6) gives $0 = y^2 + y(c - 1 - \beta c) + \beta c - x^2$, which has the root $y(x)$. For y to exist, the square root term in (4.4) must be positive which is fulfilled whenever $\beta \leq \overline{\beta}(c) = 1 + \frac{1}{c} - \frac{2}{c}\sqrt{c - x^2}$ – the upper bound on the admissible β's.

In equilibrium, the DM must be obedient[32] and after hearing $m = (m_1^0, m_2^0)$, she must choose $d = (0)$, so:

$$0 \geq 2 \times \frac{x}{2} - c \Longleftrightarrow c \geq x.$$

Similarly, after $m = (m_1^3, m_2^3)$, she must prefer $(1, 2)$ over (i):

$$2 \times 2\frac{1+y}{2} - 2\beta c \geq 2 \times \frac{1+y}{2} - c \Longleftrightarrow y + 1 \geq 2\beta c - c.$$

And after (m_1^2, m_2^3):

$$2 \times \frac{1+y}{2} - c > 2 \times (\frac{y+a^1}{2} + \frac{1+y}{2}) - 2\beta c \Longleftrightarrow 2\beta c - c > y + a^1.$$

Inserting a^1 gives $2\beta c - 2c > \frac{1}{2}x + \frac{3}{2}y$. Finally, for this equilibrium to exist, $N = 2 < \overline{N}(c)$ must hold which reduces to $y > 2c + 2x$. The following list summarizes the constraints for $N = 2$.

1. $x \leq c$,
2. $y \geq 2\beta c - c - 1$,
3. $2\beta c - 2c > \frac{1}{2}x + \frac{3}{2}y$,
4. $y > 2c + 2x$.

[32] I skip redundant constraints here. For the complete argument, I refer to the proof in Appendix A.

Combining 3. and 4. yields $\beta > \underline{\beta}(c) = \frac{5}{2} + \frac{7x}{4c}$. I now provide an exemplary equilibrium where I require that the lowest message is as large as possible, that is $x = c$.

Example 3.1
Suppose $N = 2$, $c = \frac{1}{30}$ and $\beta = 20$. For $x = c$ all of the constraints 1.-4. are fulfilled and $y(\frac{1}{30}) \approx 0.779$.

The last arguments do not only show existence of an equilibrium with CCT, but also interesting properties: if feasible, setting $x = c$ provides the agents' undistorted cutoff for working alone. Sending m^0 means that the sender will not work, irrespective of the other message and no agent incurs a negative payoff from working alone here.[33]

Comparative Statics
x determines both boundary points of the CCT interval. For any admissible x that specifies the lowest message m^0, it fixes the highest message m^{N+1}. Thus, $y(x)$ is independent of N as it determines the largest equilibrium message beyond CCT. One easily shows that $y'(x) < 0$ and the teamwork message $[y, 1]$ grows in size if x becomes large. Hence, agents trade off the certainty of a non-negative ex post payoff from her outside option for low types with a larger likelihood of teamwork resulting in higher costs of working. Further, y is larger than βc, so agents never incur a negative ex post payoff from teamwork. Similarly, one readily shows that $\frac{\partial y}{\partial \beta} > 0$ and teamwork becomes harder to sustain if the match value becomes worse.

Other Equilibria
If $y(x)$ is larger than one, teamwork cannot be part of an equilibrium but there still exists an equilibrium with m^0 at the bottom of the type space followed by CCT on $[x, 1]$. Conversely, if $x = 0$, the outside option is not chosen and in this case, CCT is played on $[0, y]$.

Similarly, if $\beta > \overline{\beta}(c)$, then y does not exist and teamwork cannot be part of a PBE. And if $\beta < \underline{\beta}$, choosing $d = (2)$ after (m_1^N, m_2^{N+1}) cannot be supported in equilibrium. Hence, task separation breaks down and she either chooses teamwork (if both agents send a high message) or the outside option (if both agents send a low message). More formally, let β converge to 1 from above. Setting $x = c$ in Proposition 3.8 leads to $y(c) = c$. This is a binary message structure in which she chooses teamwork (the outside option) if both agents send the high (low) message.

[33] Depending on β, the efficient cutoff *at the bottom* equal to $x = \frac{c}{2}$ may also be feasible (see Figure 3.7). Efficiency *at the top*, however, is never feasible because this would require $y(x) = \beta c - \frac{c}{2}$. But y is – if it exists – larger than βc because $\frac{1+\beta c-c}{2} - \sqrt{\frac{1}{4}(1+\beta c-c)^2 - \beta c + x^2} > \beta c$ reduces to $\beta > (\frac{x}{c})^2$.

There are interesting connections between the complements and the substitutes case: in the former case, let β converge to 1 from below. If additionally $\alpha = 1$, the equilibrium in Proposition 3.4 degenerates, a single interior cutoff equal to c remains and both equilibria coincide.

Further, if β converges to 1 from above and additionally, c goes to 0, then also $x = y = 0$. This is an equilibrium in which teamwork is always implemented irrespective of any information transmission (which also exists in Subsection 4.2).

Another interesting connection is as follows. Setting x equal to y in Proposition 3.8 yields the single cutoff $\frac{\beta c}{1-c(1-\beta)}$. In the complements case, the two messages induced by this cutoff are sufficient to implement task separation. It can readily be shown that if c is sufficiently small, such a binary equilibrium with the same cutoff exists here, as well ($\alpha = 1$ is not needed to derive this equilibrium).[34]

Welfare

Denote the DM's expected payoff in an N-partition equilibrium within $[x, y]$ as U_{DM}^N. Then, U_{DM}^N equals

$$2(1-y)^2(1+y-\beta c) + 2y(1-y)(y+1-c) + \sum_{n=1}^{N} ((a^n)^2 - (a^{n-1})^2)((a^n + a^{n-1}) - c).$$

The fist summand reflects the case in which both agents send the highest message. y is larger than βc so this term is clearly positive. The second term captures the case where exactly one agent sends the highest message. Further, her payoff increases in the amount of information transmission by intermediate types N: $(a^n)^2 - (a^{n-1})^2$ is positive by construction. And from difference equation (4.3), it follows that $a^n > a^1 > c$ for every n.[35]

Using the values from Example 3.1, it follows that $a^1 \approx 0.4395$ and $U_{DM}^2 \approx 1.284$. Her optimal decision without information transmission according to Proposition 3.7 is randomizing over agents to work alone because $\beta = 20 \in (15.5, 30.5]$. This gives her an expected payoff of $1 - c = 0.9\overline{6}$. It follows that information transmission increases expected welfare because the more precise information allows for separation of outcomes.

[34] The obedience constraints are of course the same as in the proof of Proposition 3.3 which is why I do not want to prove this claim here. The only difference is that – as β is now larger than 1 – different constraints are decisive. Further, it can be shown that the binary equilibria A and B from Proposition 3.3 cannot be supported in the substitutes case because the DM is not obedient.

[35] It is not clear what her payoff-maximizing choice of x is. Recall the diverging preferences from Figure 3.7: she prefers an agent to work alone if his return is larger than $\frac{c}{2}$, whereas agents want to do so only if their return exceeds the costs. On the other hand, increasing x leads to a smaller y and more teamwork on average, so the optimal x will depend on β. y and a^n are functions of x, so the FOC is already very intricate for $N = 2$ and calculating $\frac{\partial U_{DM}^N}{\partial x}$ is not practical.

5. Discussion

In this section, I discuss two variants of my model and investigate what happens to the equilibria above if the agents communicate sequentially, and if the DM delegates the decision right to one agent. By doing so, I show that the role of the DM is crucial in my model, even beyond her obedience constraints. But before, I want to point out that commitment is not needed in my model, except for equilibrium selection.[36] The reason is that in any equilibrium, agents behave optimally when sending their messages and it would thus be Bayesian incentive compatible to send these messages if the DM could commit to a decision rule ex ante. Moreover, the DM is obedient given these messages, so her payoff is maximized at her choice.

Sequential Communication

As noted in Section 3, without teamwork and outside option, equilibria are regret-free in that agents would not change their message even if they had learned the other agent's message. In my model, this is generally no longer true. To see this, suppose without loss of generality that first, agent 1 sends m_1 publicly, then agent 2 sends m_2 (which is now a function of θ_2 and m_1) and then, the DM makes her decision.

For $\beta < 1$, recall the binary task-separating equilibrium with cutoff $z^C \in (\beta c, c)$ where all types above z^C send $m_i = 1$. After hearing message $m_1 = 0$, types $\theta_2 \in (z^C, c)$ will no longer send $m_2 = 1$ because then they will work alone and receive a negative payoff. To see this more formally, suppose agent 1 adheres to the equilibrium message determined by z^C. After $m_1 = 0$, agent 2 will either work alone (and receive $\theta_2 - c$) or no one works (which gives him a payoff of 0). Comparing both yields the new cutoff type $\theta_2 = c$ and types above (below) this cutoff send $m_2 = 1$ ($m_2 = 0$). Similarly, if he hears $m_1 = 1$, he compares $d = (1, 2)$ (if he sends $m_2 = 1$) with $d = (1)$ (if he sends $m_2 = 0$) which leads to the new cutoff type $\theta_2 = \beta c$. As these new cutoffs are the ones agent 2 would choose in a situation where he only faces the outcomes $d = (0)$ or $d = (2)$ (after $m_1 = 0$), or $d = (1)$ and $d = (1, 2)$ (after $m_1 = 1$), he basically takes the decision on his own now.[37]

In contrast, equilibrium A and B are still ex post equilibria: consider equilibrium B (the argument for equilibrium A is the same) and suppose that $\theta_2 \in (z^B, \beta c)$ so that the equilibrium calls for $m_2 = 1$. If $m_1 = 1$, then $d = (1, 2)$ is implemented for sure and m_2 has no effect on the decision. If $m_1 = 0$, one readily sees that agent 2 sends $m_2 = 0$ if and only if $\theta_2 < \frac{2\beta c}{2+\alpha} = z^B$ – the same cutoff as with simultaneous communication. Hence, he will not change his message after hearing m_1. The reason is that, if agent 1 sends the low (high) message in equilibrium A

[36]But many of the equilibria do not exist simultaneously.

[37]For welfare implications, I refer to the analysis on delegation below.

(B), then the message by agent 2 becomes irrelevant and cannot alter the outcome.

Now consider the case $\beta > 1$ in which CCT is played on the endogenous interval $[x, y]$. This equilibrium is not regret-free either: suppose $N = 2$ and parameter values as in Example 3.1, so $\beta c > a^1$. Further assume that agent 1 sends m_1^3 (see Figure 3.8) and that $\theta_2 \in (\beta c, y]$, so that agent 2 would send m_2^2 under simultaneous communication. Assessing his options after m_1^3, he now compares sending m_2^2 and receiving $\frac{y+1}{2}$ from $d = (1)$ with sending m_2^3 and receiving $\theta_2 + \frac{y+1}{2} - \beta c$. This yields a new type who is indifferent between m_2^2 and m_2^3 equal to $\beta c < y$, so agent 2 sends m_2^3. Teamwork becomes a more frequent outcome here because agents' preferences depend solely on the other type if the own type is sufficiently large. Therefore, agents benefit from learning the other agent's message and the outcome of the game is not robust to changing the rules from simultaneous to sequential communication when her choice set contains teamwork.

Delegation to one Agent

If one agent must work alone, the DM is basically inactive and Li et al. (2016) show that the set of equilibria when delegating the decision right to one agent is a subset of the equilibria under sequential communication and these are outcome-equivalent to the ones with simultaneous communication. In my model, this is not true either. The reason is that with binary decision only ordinal information is important and her preferences are not very different from agents' preferences (see Figure 3.1). In my model, however, cardinal information is decisive and since her preferences are fundamentally different from the agents', her involvement in the subsequent game is of utmost importance.

To shed light on this, assume that the DM performs simple delegation in the sense of Aghion and Tirole (1997): first, agent 2 sends m_2 to agent 1. Then, agent 1 takes decision $d(\theta_1, m_2(\theta_2))$. Suppose $\beta < 1$. From Figure 3.3, it is already clear that agent 1 will never choose $d = (0)$ (for $\alpha > 0$) nor (1). In particular, m_2 becomes void for agent 1's decision and he will choose $d = (1, 2)$ if and only if his own type is larger than βc and $d = (2)$ otherwise. In terms of efficiency, agent 2 can send any message, but the informational content is zero because agent 1 always ignores m_2. It follows that $\mathbb{E}[\theta_2 | m_2] = \frac{1}{2}$ and welfare equals

$$\begin{aligned}&\mathbb{P}(d = (1, 2))((1 + \alpha)(\mathbb{E}[\theta_1|\theta_1 \geq \beta c] + \mathbb{E}[\theta_2]) - 2\beta c) + \mathbb{P}(d = (2))((1 + \alpha)\mathbb{E}[\theta_2] - c)\\ =&(1 + \alpha)(1 - \frac{1}{2}(\beta c)^2) + 2\beta c(\beta c - 1) - \beta c^2. \qquad (5.1)\end{aligned}$$

To stress the efficiency loss, recall equilibrium B under simultaneous communication and suppose $\alpha = 0$, so that $z^B = \beta c$. The DM's expected payoff then equals $1 - 2\beta c + (\beta c)^3$. This is larger than her payoff from delegating to agent 1 (see (5.1)) if $\beta^2 c + 1 > \frac{3}{2}\beta$. Further recall that equilibrium B exists if and only if β is smaller than $\frac{2}{3}$ and $\frac{1}{2c}$. But either it holds that $c \leq \frac{3}{4}$

and $\beta \leq \frac{2}{3} \leq \frac{1}{2c}$ and then the inequality is certainly true. Or, $c > \frac{3}{4}$ and $\beta < \frac{1}{2c}$ must hold. But then $\frac{3}{2}\beta < \frac{3}{4c} < 1 < \beta^2 c + 1$, and welfare under delegation is smaller.

For $\beta > 1$, the conclusion is less unambiguous because if the own type is larger than c, the comparison with the other type is important for the most preferred outcome (see Figure 3.7). But the first-best cutoff for θ_2 according to which the DM would implement teamwork is larger than agent 1's: $\beta c - \frac{c}{2} > \beta c - c$. Therefore, compared with simultaneous communication, teamwork will again be a more frequent outcome and this benefits the agent with the decision right (and harms the other one). Teamwork is chosen inefficiently often at the expense of less individual work done by agent 1.

6. Concluding Remarks

In virtually all organizations, a decision-maker must assign costly tasks to her staff. Non-verifiable private information about the performance and organizational structures make cheap talk a natural way to study such problems. The analysis of this setting can be performed with *competitive cheap talk*, coined by Li et al. (2016), if one agent must work alone. In this case, credible information can be transmitted because agents have the incentive to free-ride which makes them compete via their messages. So far, however, it remained an open question what changes if agents can work together.

In this paper, I show that by adding teamwork to the set of outcomes the amount of meaningful information transmission depends on the match value between agents, that is on how the teamwork technology affects preferences. If people are instructed to work in a team, two effects can occur: the team members create net synergies (complements) or working becomes more costly because of a high coordination effort (substitutes). I show that information transmission in both cases is fundamentally different: if agents are complements, teamwork is always preferred over working alone. For large types, this introduces the incentive to overstate the own type. Meaningful information transmission is limited to at most three messages. In the polar substitutes case, if the own type is sufficiently small, teamwork is never preferred and agents have the incentive to free-ride so that competition via cheap talk reappears in equilibrium.

My model can be extended to the general case with N agents. Then, however, the team size matters. If her decision remains between one agent working alone and a team that contains all agents, the analysis is similar. If smaller team sizes than the grand coalition are feasible, the analysis becomes more difficult.

A. Appendix: Proofs of Chapter 3

Proof of Proposition 3.1. By symmetry, the proof is performed from agent 1's perspective. First denote the posterior of θ_1 induced by a message m_1 as p_1. From the specified tie-breaking rule it follows that she chooses $d = (1,2)$ if

$$(1+\alpha)(p_1 + \mathbb{E}[\theta_2|m_2(\cdot)]) - 2\beta c \geq \max\{(1+\alpha)p_1 - c; (1+\alpha)\mathbb{E}[\theta_2|m_2(\cdot)] - c; 0\}.$$

Similarly, she chooses $d = (1)$ if

$$(1+\alpha)p_1 - c > \max\{(1+\alpha)(p_1 + \mathbb{E}[\theta_2|m_2(\cdot)]) - 2\beta c; (1+\alpha)\mathbb{E}[\theta_2|m_2(\cdot)] - c; 0\},$$

and $d = (2)$ if

$$(1+\alpha)\mathbb{E}[\theta_2|m_2(\cdot)] - c > \max\{(1+\alpha)(p_1 + \mathbb{E}[\theta_2|m_2(\cdot)]) - 2\beta c; (1+\alpha)p_1 - c; 0\}.$$

Agent 1's expected utility given θ_1, induced posterior p_1, own message strategy m_1 and any $m_2(\cdot)$ is then

$$\begin{aligned}\mathbb{E}_{\theta_2}[u_1|\theta_1, p_1] = {} & \mathbb{P}(d = (1,2)|m_1, m_2(\cdot))(\theta_1 + \alpha\mathbb{E}[\theta_2|d = (1,2)] - \beta c) \\ & + \mathbb{P}(d = (1)|m_1, m_2(\cdot))(\theta_1 - c) \\ & + \mathbb{P}(d = (2)|m_1, m_2(\cdot))\alpha\mathbb{E}[\theta_2|d = (2)],\end{aligned} \tag{A.1}$$

where $\mathbb{P}(d = (1,2)|m_1, m_2(\cdot))$ and $\mathbb{P}(d = (1)|m_1, m_2(\cdot))$ are given by

$$\mathbb{P}((1+\alpha)(p_1 + \mathbb{E}[\theta_2|m_2(\cdot)]) - 2\beta c \geq \max\{(1+\alpha)p_1 - c; (1+\alpha)\mathbb{E}[\theta_2|m_2(\cdot)] - c; 0\}),$$

$$\mathbb{P}((1+\alpha)p_1 - c \geq \max\{(1+\alpha)(p_1 + \mathbb{E}[\theta_2|m_2(\cdot)]) - 2\beta c; (1+\alpha)\mathbb{E}[\theta_2|m_2(\cdot)] - c; 0\}).$$

It follows that $\mathbb{P}(d = (1,2)|m_1, m_2(\cdot))$ and $\mathbb{P}(d = (1)|m_1, m_2(\cdot))$ are both weakly increasing in p_1. Therefore, $\frac{\partial \mathbb{E}_{\theta_2}[u_1|\theta_1, p_1]}{\partial \theta_1}$ is also increasing in p_1 (the third summand in (A.1) is irrelevant).

Now consider $\overline{p}_1 > \underline{p}_1$. Then, there is at most one type who is indifferent between both posteriors. Further assume, on the way to a contradiction, that there is a PBE that is not of the interval form. To be precise, suppose there are two types $\theta_1' > \theta_1''$ such that $\mathbb{E}_{\theta_2}[u_1|\theta_1'', \overline{p}_1] \geq \mathbb{E}_{\theta_2}[u_1|\theta_1'', \underline{p}_1]$, but $\mathbb{E}_{\theta_2}[u_1|\theta_1', \overline{p}_1] < \mathbb{E}_{\theta_2}[u_1|\theta_1{}', \underline{p}_1]$. But then

$$\begin{aligned} & \mathbb{E}_{\theta_2}[u_1|\theta_1', \overline{p}_1] - \mathbb{E}_{\theta_2}[u_1|\theta_1', \underline{p}_1] < \mathbb{E}_{\theta_2}[u_1|\theta_1'', \overline{p}_1] - \mathbb{E}_{\theta_2}[u_1|\theta_1'', \underline{p}_1] \\ \iff & \mathbb{E}_{\theta_2}[u_1|\theta_1', \overline{p}_1] - \mathbb{E}_{\theta_2}[u_1|\theta_1'', \overline{p}_1] < \mathbb{E}_{\theta_2}[u_1|\theta_1', \underline{p}_1] - \mathbb{E}_{\theta_2}[u_1|\theta_1'', \underline{p}_1],\end{aligned}$$

which violates the fact that $\frac{\partial \mathbb{E}_{\theta_2}[u_1|\theta_1, p_1]}{\partial \theta_1}$ is increasing in p_1. Hence, all PBE must be of the interval form. □

Proof of Proposition 3.2. Comparing her payoff from $d = (1, 2)$ with that from $d = (i)$ yields that she prefers $(1, 2)$ if $\beta \leq \frac{1+\alpha}{4c} + \frac{1}{2}$. Further, β must be smaller than $\frac{1+\alpha}{2c}$ for her choosing $(1, 2)$ over (0). If $1 + \alpha \geq 2c$, both upper bounds are larger than 1 so both constraints are trivially fulfilled as $\beta < 1$. The case where $1 + \alpha < 2c$ works in the same way. □

Proof of Proposition 3.3. Consider agent 1. The proof consists of two steps per case: deriving the agents' indifference type z^k, $k \in \{A, B, C\}$, and then showing that the DM is obedient with her decision.

Case A:

Type $\theta_1 = z$ is indifferent between $m_1 = 1$ and $m_1 = 0$, it follows:

$$\mathbb{E}[u_1|m_1 = 1] = \mathbb{P}(m_2 = 1)(z + \alpha\frac{1+z}{2} - \beta c) + \mathbb{P}(m_2 = 0) \times 0$$
$$= \mathbb{E}[u_1|m_1 = 0] = \mathbb{P}(m_2 = 1) \times 0 + \mathbb{P}(m_2 = 0) \times 0,$$

or

$$(1 - z)(z + \alpha\frac{1+z}{2} - \beta c) = 0.$$

Since $z > 0$ holds in an informative equilibrium, the last equality reduces to $z^A := \frac{2\beta c - \alpha}{2+\alpha}$. It remains to be shown that the DM is indeed obedient given this cutoff. The obedience constraints will give restrictions on β. After each message pair, there are two relevant deviations because the cases $d = (1)$ and $d = (2)$ are symmetric. As the DM's posterior is the same after hearing message $(1, 0)$ or $(0, 1)$, there are three relevant message pairs and thus six constraints per equilibrium to check.

1(a): I begin with $m = (1, 1)$ and after this message, she must choose $d = (1, 2)$ given the cutoff z^A, hence:

$$\mathbb{E}[u_{DM}(d = (1, 2))|m = (1, 1)] = (1 + \alpha)2 \times \frac{1+z^A}{2} - 2\beta c$$
$$\geq \mathbb{E}[u_{DM}(d = (0))|m = (1, 1)] = 0,$$

or $z^A \geq \frac{2\beta c}{1+\alpha} - 1$. Plugging z^A in and simplifying yields $\frac{1+\alpha}{c} \geq \beta$ which is always true since the left-hand side (LHS) is larger than 1.

1(b):

$$(1 + \alpha)2\frac{1+z^A}{2} - 2\beta c \geq (1 + \alpha)\frac{1+z^A}{2} - c = \mathbb{E}[u_{DM}(d = (i))|m = (1, 1)],$$

which reduces to

$$\frac{2+\alpha}{3+\alpha}+\frac{1+\alpha}{c(3+\alpha)}\geq\beta.$$

The LHS is minimized at $c = 1$ and in this case, the inequality reduces to $\frac{3+2\alpha}{3+\alpha}\geq\beta$ which is true since the LHS is weakly larger than 1.

2(a):

I proceed with message $m = (1, 0)$ (and $m = (0, 1)$ by symmetry) after which equilibrium A calls for $d = (0)$. Hence, it must hold that

$$0>(1+\alpha)(\frac{z^A}{2}+\frac{1+z^A}{2})-2\beta c=\mathbb{E}[u_{DM}(d=(1,2))|m=(1,0)].$$

This is equivalent to

$$\beta>\frac{(1+\alpha)(2-\alpha)}{4c}. \tag{A.2}$$

2(b): Moreover,

$$0\geq(1+\alpha)\frac{1+z^A}{2}-c,$$

or

$$\beta\leq\frac{2+\alpha}{1+\alpha}-\frac{1}{c}. \tag{A.3}$$

3(a): Finally, consider $m = (0, 0)$ It must hold that

$$0>(1+\alpha)2\times\frac{z^A}{2}-2\beta c=\mathbb{E}[u_{DM}(d=(1,2))|m=(0,0)],$$

which reduces to $2\beta c > -\alpha(1 + \alpha)$. This is trivially fulfilled.

3(b):

$$0\geq(1+\alpha)\frac{z^A}{2}-c=\mathbb{E}[u_{DM}(d=(i))|m=(0,0)],$$

which is equivalent to

$$\beta\leq\frac{2+\alpha}{1+\alpha}+\frac{\alpha}{2c}.$$

This constraint is redundant because the RHS is larger than 1.

Case B:
The condition $\mathbb{E}[u_1|m_1 = 1] = \mathbb{E}[u_1|m_1 = 0]$ becomes

$$
\begin{aligned}
&\mathbb{P}(m_2 = 1)(z + \alpha\frac{1+z}{2} - \beta c) + \mathbb{P}(m_2 = 0)(z + \alpha\frac{z}{2} - \beta c)\\
=&\mathbb{P}(m_2 = 1)(z + \alpha\frac{1+z}{2} - \beta c) + \mathbb{P}(m_2 = 0) \times 0,
\end{aligned}
$$

or

$$(1-z)(z + \alpha\frac{1+z}{2} - \beta c) + z(z + \alpha\frac{z}{2} - \beta c) = (1-z)(z + \alpha\frac{1+z}{2} - \beta c),$$

which reduces to $z^B = \frac{2\beta c}{2+\alpha}$. The obedience constraints for the DM are as follows. I start with $m = (1, 1)$.
1(a):

$$\mathbb{E}[u_{DM}(d = (1, 2))|m = (1, 1)] = 2(1 + \alpha)\frac{1+z^B}{2} - 2\beta c \geq 0,$$

or $\beta \leq \frac{2+3\alpha+\alpha^2}{2c}$. The RHS is always larger than 1.

1(b):

$$2(1 + \alpha)\frac{1+z^B}{2} - 2\beta c \geq (1 + \alpha)\frac{1+z^B}{2} - c = \mathbb{E}[u_{DM}(d = (i)|m = (1, 1)],$$

which can be written as

$$\beta \leq \frac{2+\alpha}{3+\alpha} + \frac{(2+\alpha)(1+\alpha)}{2c(3+\alpha)},$$

and this becomes – for $c = 1$:

$$\beta \leq \frac{2+\alpha}{3+\alpha} + \frac{(2+\alpha)(1+\alpha)}{6+2\alpha} = 1 + \frac{\alpha}{2},$$

and thus the constraint is redundant.

2(a): I proceed with message $m = (1, 0)$ and $(0, 1)$, respectively, so it must hold:

$$(1 + \alpha)(\frac{z^B}{2} + \frac{1+z^B}{2}) - 2\beta c \geq 0,$$

or

$$\beta \leq \frac{(2+\alpha)(1+\alpha)}{4c}. \tag{A.4}$$

2(b):

$$(1+\alpha)(\frac{z^B}{2}+\frac{1+z^B}{2})-2\beta c \geq (1+\alpha)\frac{1+z^B}{2}-c,$$

or

$$\beta \leq \frac{2+\alpha}{3+\alpha}. \tag{A.5}$$

3(a): Finally, after $m=(0,0)$, it must hold

$$0 > 2(1+\alpha)\frac{z^B}{2}-2\beta c,$$

which is equivalent to $2+\alpha > 1+\alpha$.

3(b):

$$0 \geq (1+\alpha)\frac{z^B}{2}-c,$$

or $\beta \leq \frac{2+\alpha}{1+\alpha}$ which is redundant, too.

Hence, inequalities (A.4) and (A.5) together yield

$$\beta \leq \min\Big\{\frac{(2+\alpha)(1+\alpha)}{4c}; \frac{2+\alpha}{3+\alpha}\Big\}.$$

In particular, the minimum operator is necessary because $\frac{(2+\alpha)(1+\alpha)}{4c} > \frac{2+\alpha}{3+\alpha}$ reduces to $(3+\alpha)(1+\alpha) > 4c$ which does not hold for $\alpha = 0$ and $c \approx 1$.

Case C:

$z^C = \frac{\beta c}{1-c(1-\beta)}$ is derived via $\mathbb{E}[u_1|m_1=1] = \mathbb{E}[u_1|m_1=0]$ or

$$(1-z)\big(z+\alpha\frac{1+z}{2}-\beta c\big)+z(z-c)=(1-z)\alpha\frac{1+z}{2}+z\times 0.$$

1(a): I begin with $m=(0,0)$ which prescribes $d=(0)$, hence

$$0 \geq (1+\alpha)\frac{z^C}{2}-c,$$

or $\beta \leq \frac{2(1-c)}{1+\alpha-2c}$ – a redundant constraint.

1(b):

$$0 > 2(1+\alpha)\frac{z^C}{2}-2\beta c,$$

or

$$\beta > 1-\frac{1-\alpha}{2c}. \tag{A.6}$$

2(a): After $m=(1,0)/(0,1)$, it follows

$$(1+\alpha)\frac{1+z^C}{2} - c > 0,$$

or

$$\beta > \frac{(1-c)(2c-(1+\alpha))}{2c((1+\alpha)-c)}. \tag{A.7}$$

One can readily show that the RHS of (A.7) is smaller than $\frac{1}{2}$ for all $c < 1$ and this constraint is redundant.

2(b):

$$(1+\alpha)\frac{1+z^C}{2} - c > (1+\alpha)(\frac{1+z^C}{2} + \frac{z^C}{2}) - 2\beta c.$$

This reduces to

$$\beta^2 + \beta\frac{4-6c-(1+\alpha)}{4c} - \frac{1-c}{2c} > 0.$$

The LHS is a quadratic function that opens upward and its roots are

$$\beta_{R,L} = -\frac{4-6c-(1+\alpha)}{8c} \pm \frac{1}{8c}\sqrt{9-4c+4c^2-6\alpha+\alpha^2+12c\alpha}.$$

First observe that the square root term is always positive, so the roots exist. To see this, write $6+3+4c^2+\alpha^2+12c\alpha \geq 4c+6\alpha$, and obviously $6 \geq 6\alpha$. Moreover, $3+4c^2+\alpha^2+12c\alpha \geq 4c$ reduces to

$$c^2 - c(1-3\alpha) + \frac{3+\alpha^2}{4} \geq 0,$$

which is true for all $c < 1$ and all $\alpha \in [0, 1]$.

If $-\frac{4-6c-(1+\alpha)}{8c} \leq 0$, then β_L is obviously smaller than zero. If this term is positive, however, it is straightforward to show that $\beta_L < \frac{1}{2}$ and for $\beta < \beta_L$ this cannot be part of equilibrium C.

Moreover, $\beta_R \leq 1$ reduces to $16c\alpha \leq 16c$ which is also always true. Hence, the important constraint here is

$$\beta \geq \beta_R = -\frac{4-6c-(1+\alpha)}{8c} + \frac{1}{8c}\sqrt{9-4c+4c^2-6\alpha+\alpha^2+12c\alpha}. \tag{A.8}$$

Furthermore, $\beta_R \geq \frac{1}{2}$ reduces to $c + c\alpha \geq 0$ which is always true. Moreover, β_R is always larger than $1 - \frac{1-\alpha}{2c}$, so (A.6) is irrelevant.

3(a): Now turn the focus to message $m = (1, 1)$ after which $d = (1, 2)$ must be made, hence:

$$2(1+\alpha)\frac{1+z^C}{2} - 2\beta c \geq 0,$$

which simplifies to

$$\beta^2 - \beta\frac{\alpha+c}{c} - \frac{(1+\alpha)(1-c)}{2c^2} \leq 0.$$

The quadratic term on the LHS opens upwards and has the roots

$$\beta_{R,L} = \frac{\alpha + c}{2c} \pm \frac{1}{2c}\sqrt{\alpha^2 + 2\alpha + 2 - 2c + c^2}.$$

It can readily be shown that $\beta_L < 0$ and $\beta_R > 1$ so that the above inequality is always fulfilled and the constraint becomes void.

3(b): Finally, she must prefer $d = (1, 2)$ over $d = (i)$ after $m = (1, 1)$:

$$2(1 + \alpha)\frac{1 + z^C}{2} - 2\beta c \geq (1 + \alpha)\frac{1 + z^C}{2} - c,$$

which becomes

$$\beta^2 - \beta\frac{2(1 + \alpha) + (6c - 4)}{4c} - \frac{2c(1 - c) + (1 + \alpha)(1 - c)}{4c^2} \leq 0,$$

and the graph of the LHS opens upward. The LHS has the roots

$$\beta_{R,L} = \frac{2(1 + \alpha) + (6c - 4)}{8c} \pm \frac{1}{8c}\sqrt{20 + 8\alpha + 4\alpha^2 - 8c + 8\alpha c + 4c^2}.$$

It can easily be shown that $\beta_L < 0$ and that $\beta_R > 1$, so also this constraint is always fulfilled. □

Proof of Proposition 3.4. Consider agent 1 and recall the lower right panel of Figure 3.5 for the assignment of decisions to messages in this equilibrium. To find the cutoff type $\theta_1 = y$, it must hold

$$\begin{aligned}
&\mathbb{E}[u_1|m_1 = m^2] = \mathbb{P}(m_2 = m^2)(y + \frac{\alpha}{2}(x + y) - \beta c) + \mathbb{P}(m_2 = m^3)(y + \frac{\alpha}{2}(y + 1) - \beta c) \\
&= \mathbb{E}[u_1|m_1 = m^3] = \mathbb{P}(m_2 = m^1)(y - c) \\
&\qquad + \mathbb{P}(m_2 = m^2)(y + \frac{\alpha}{2}(x + y) - \beta c) + \mathbb{P}(m_2 = m^3)(y + \frac{\alpha}{2}(y + 1) - \beta c),
\end{aligned}$$

which reduces to $0 = x(y - c)$ from which follows that $y = c$ because if $x = 0$, then this cannot constitute an equilibrium with three messages. Similarly, for $\theta_1 = x$:

$$\begin{aligned}
&\mathbb{E}[u_1|m_1 = m^1] = \mathbb{P}(m_2 = m^3)\frac{\alpha}{2}(y + 1) \\
&= \mathbb{E}[u_1|m_1 = m^2] = \mathbb{P}(m_2 = m^2)(x + \frac{\alpha}{2}(x + y) - \beta c) + \mathbb{P}(m_2 = m^3)(x + \frac{\alpha}{2}(y + 1) - \beta c),
\end{aligned}$$

which reduces to

$$0 = (y - x)(x + \frac{\alpha}{2}(x + y) - \beta c) + (1 - y)(x - \beta c).$$

Inserting $y = c$ and simplifying yields

$$x^2 - x\frac{2(1+\beta c)}{2+\alpha} + \frac{2(\beta c - \frac{\alpha}{2}c^2)}{2+\alpha} = 0.$$

The roots of this quadratic term are

$$x_{R,L} = \frac{1+\beta c}{2+\alpha} \pm \frac{1}{2+\alpha}\sqrt{(1+\beta c)^2 + (2+\alpha)(\alpha c^2 - 2\beta c)}.$$

It can readily be shown that the larger root x_R is strictly larger than c which cannot be part of this proposed equilibrium. Hence, write $x := x_L$.

Now observe that the square root term of x is always positive, so x exists for all α, β and c. To see this, rewrite this term as

$$\beta^2 - 2\beta\frac{1+\alpha}{c} + (2+\alpha)\alpha + \frac{1}{c^2} \geq 0.$$

This is a quadratic function that opens upward so showing that the left root is larger than 1 shows that the term is positive for all $\beta < 1$. The left root is

$$\frac{1+\alpha}{c} - \frac{1}{c}\sqrt{(1+\alpha)^2 - \alpha c^2(2+\alpha) - 1},$$

which is larger than 1 if and only if

$$\frac{1+\alpha}{c} - \frac{c}{c} \geq \frac{1}{c}\sqrt{(1+\alpha)^2 - \alpha c^2(2+\alpha) - 1},$$

or

$$\left(c(1+\alpha) - 1\right)^2 \geq 0.$$

Hence, x always exists.

For existence of this equilibrium, the DM must be obedient. The important obedience constraints can be identified from the red lines in the lower right panel of Figure 3.5. By symmetry and since payoffs are monotone in returns, I concentrate on the lower right four rectangles. These are:

1. after (m^2, m^1): $d = (0)$, hence

$$0 > (1+\alpha)\left(\frac{x+y}{2} + \frac{x}{2}\right) - 2\beta c \iff \frac{4\beta c}{1+\alpha} > 2x + y, \tag{A.9}$$

and

$$0 \geq (1+\alpha)\frac{x+y}{2} - c \Longleftrightarrow \frac{2c}{1+\alpha} \geq x+y. \tag{A.10}$$

2. After (m^3, m^1): $d = (1)$, hence

$$(1+\alpha)\frac{y+1}{2} - c > 0 \Longleftrightarrow y+1 > \frac{2c}{1+\alpha}, \tag{A.11}$$

and

$$(1+\alpha)\frac{y+1}{2} - c > (1+\alpha)(\frac{y+1}{2} + \frac{x}{2}) - 2\beta c \Longleftrightarrow \frac{2c}{1+\alpha}(2\beta - 1) > x. \tag{A.12}$$

3. After (m^2, m^2): $d = (1, 2)$, hence

$$(1+\alpha)(x+y) - 2\beta c \geq 0 \Longleftrightarrow x+y \geq \frac{2\beta c}{1+\alpha}, \tag{A.13}$$

and

$$(1+\alpha)(x+y) - 2\beta c \geq (1+\alpha)\frac{x+y}{2} - c \Longleftrightarrow x+y \geq \frac{2c}{1+\alpha}(2\beta - 1), \tag{A.14}$$

where the latter is implied by (A.13).

4. Finally, after (m^3, m^2): $d = (1, 2)$, hence

$$(1+\alpha)(\frac{x+y}{2} + \frac{y+1}{2}) - 2\beta c \geq 0 \Longleftrightarrow x + 2y + 1 \geq \frac{4\beta c}{1+\alpha}. \tag{A.15}$$

All other constraints are redundant. In the following, I do not check all constraints in general terms as this is computationally very involved. Instead, I show that the equilibrium exists for $\alpha = 0$ and β large enough as the DM is obedient for this specification. If $\alpha = 0$, then $x = \beta c$ and recall that $y = c$. Then, constraints (A.9)-(A.15) are trivially true except for (A.12) which becomes $4\beta c - 2c > \beta c$ or $\beta > \frac{2}{3}$. Hence, for small α and large enough β, this equilibrium exists.

For $\alpha = 1$, constraint (A.10) becomes $c \geq x + c$ or $0 \geq x$ which can obviously not be true in an equilibrium with three messages and the equilibrium does not exist for any β. By continuity, there exists an upper bound $\overline{\alpha}$ and the equilibrium only exists for $\alpha \leq \overline{\alpha}$ and for β larger than some lower bound $\underline{\beta}(\alpha, c)$. Moreover, after inserting x and y and rearranging, constraint (A.10) becomes

$$(1+\alpha)[(1+\alpha)(\alpha c - 2\beta) + 2(1+\beta c)(1-\alpha)] \geq c(1-\alpha)^2(2+\alpha),$$

which in turn yields the upper bound

$$\beta \leq \overline{\beta}(\alpha, c) := \frac{(1+\alpha)^2 \alpha c + 2(1-\alpha^2) - c(1-\alpha)^2(2+\alpha)}{2(1+\alpha)[(1+\alpha) - c(1-\alpha)]}.$$

□

Proof of Proposition 3.5. Intuitively, there might exist an equilibrium with finer partitions in $[x, y]$, and following the 3-message equilibrium in Proposition 3.4, one could try to approximate the DM's preferences.

Figure A.1 illustrates this idea. Here, three additional interior cutoffs a^1, a^2, a^3 (and four messages) between x and y are depicted and, for an interval size converging to zero, the green straight line is approached resembling the DM's preferences.

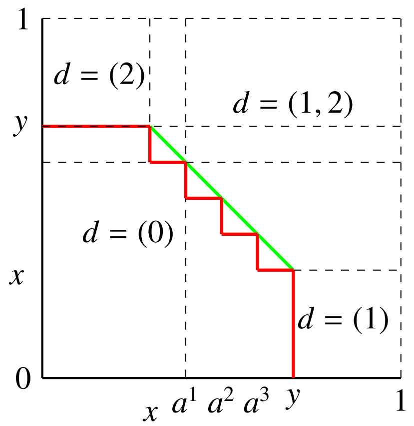

Figure A.1: Approximation of DM's Preferences.

I claim, however, that this is not incentive-compatible. To see this, suppose there exists a partition $x = a^0 < a^1 < a^2 < \cdots < a^N = y$ such that message (m^k, m^k) yields $d = (0)$, but message (m^{k+1}, m^k) yields $(1, 2)$, for $k = 1, \ldots, N$. As the trade-off within the interval $[x, y]$ is solely between teamwork and the outside option and because of the symmetric structure, agent 1's indifference condition between message m^k and m^{k+1} becomes

$$0 = \mathbb{P}(m_2 = m^k)(a^k + \frac{\alpha}{2}(a^k + a^{k-1}) - \beta c),$$

or

$$a^k = \frac{2\beta c}{2+\alpha} - \frac{\alpha}{2+\alpha} a^{k-1}.$$

This is a first-order difference equation with general solution

$$a^n = \frac{2\beta c}{2+\alpha}\sum_{k=0}^{n-2}(-1)^k(\frac{\alpha}{2+\alpha})^k + (-1)^{n-1}(\frac{\alpha}{2+\alpha})^{n-1}a^1, \tag{A.16}$$

that has an alternating structure. In particular, $a^2 > a^1$ holds if and only if $\frac{\beta c}{1+\alpha} > a^1$, but

$$a^2 = \frac{2\beta c}{2+\alpha} - \frac{\alpha}{2+\alpha}a^1 < \frac{2\beta c}{2+\alpha}(1-\frac{\alpha}{2+\alpha}) + (\frac{\alpha}{2+\alpha})^2 a^1 = a^3 \iff \frac{\beta c}{1+\alpha} < a^1,$$

a contradiction. □

Remark 3.1

Equation (A.16) *offers the same insight from a different angle. The expression converges to* $\frac{\beta c}{1+\alpha}$. *Hence, if this were an equilibrium, in the limit the agent would send four different messages* $m^1 = [0, x]$, $m^2 = [x, \frac{\beta c}{1+\alpha}]$, $m^3 = [\frac{\beta c}{1+\alpha}, y]$ *and* $m^4 = [y, 1]$, *where x and y are the same as in Proposition* 3.4. *Comparing the DM's payoff from* $d = (1, 2)$ *with that from* $d = (0)$ *yields that* $\frac{\beta c}{1+\alpha}$ *is the type for which she is just indifferent between both decisions:* $(1+\alpha)(\frac{\beta c}{1+\alpha} + \frac{\beta c}{1+\alpha}) - 2\beta c = 0$. *Recall that agents never want the outside option because of the externality. But for* $\alpha > 0$, $\frac{\beta c}{1+\alpha} < \beta c$. *Therefore, if both agents truthfully revealed that their type lies in the interval* $[x, \frac{\beta c}{1+\alpha}]$, *then the DM would choose* $d = (0)$ *which cannot be incentive-compatible. Finally, for* $\alpha = 0$, *it follows that* $x = \beta c$ *and one receives the 3-message equilibrium of Proposition 3.4.*

Proof of Proposition 3.6. According to Proposition 3.2, there are three cases without information transmission.

1) First, suppose $1 + \alpha \geq 2c$, so that she always chooses $d = (1, 2)$ for all admissible β and consider equilibrium A with

$$\begin{aligned} U^A_{DM} &= \mathbb{P}(m = (1,1))\mathbb{E}[(1+\alpha)(\theta_1+\theta_2) - 2\beta c | \theta_1, \theta_2 \geq z^A] \\ &= (1-z^A)^2((1+\alpha)(1+z^A) - 2\beta c), \end{aligned} \tag{A.17}$$

with $z^A = \frac{2\beta c - \alpha}{2+\alpha}$. Setting $U^A_{DM} > (1+\alpha) - 2\beta c$ yields, after basic algebra,

$$10\alpha\beta c + 12\beta c + \alpha^3 > 4 + 6\alpha + \alpha^2 + 4(\beta c)^2, \tag{A.18}$$

or

$$0 > 4\beta^2 c^2 - \beta c(10\alpha + 12) - \alpha^3 + 6\alpha + \alpha^2 + 4.$$

The RHS is quadratic and has roots

$$\beta_{R,L} = \frac{10\alpha + 12}{8c} \pm \frac{1}{8c}\sqrt{(10\alpha + 12)^2 + 16\alpha^3 - 16(6\alpha + \alpha^2 + 4)},$$

and one easily shows that $\beta_R > 1$. Moreover, equilibrium A only exists if $\beta > \frac{(1+\alpha)(2-\alpha)}{4c}$ and one can show that $\beta_L < \frac{(1+\alpha)(2-\alpha)}{4c}$, so that (A.18) is true for all β such that equilibrium A exists.

If equilibrium B (with $z^B = \frac{2\beta c}{2+\alpha}$) exists, then her payoff is $U^B_{DM} = (1 + \alpha)(1 - (z^B)^3) - 2\beta c(1 - (z^B)^2)$. This is larger than $1 + \alpha - 2\beta c$ if and only if $2\beta c > z^B(1 + \alpha)$. Inserting z^B yields $2 + \alpha > 1 + \alpha$.

2) If $\frac{1+\alpha}{2c} < 1$ and $\beta > \frac{1+\alpha}{2c}$, she would choose $d = (0)$ without information transmission. Observe that in this case $\frac{(1+\alpha)(2-\alpha)}{4c} \leq \frac{1+\alpha}{2c} < \beta$ holds, and if $\beta \leq \frac{2+\alpha}{1+\alpha} - \frac{1}{c}$, then equilibrium A exists, as well. Moreover, if $\frac{(2+\alpha)(1+\alpha)}{4c} \leq \frac{2+\alpha}{3+\alpha} < \frac{2+\alpha}{1+\alpha} - \frac{1}{c}$, then equilibrium B also exists. Hence, I focus on equilibrium A. But from (A.17), it readily follows that U^A_{DM} is always larger than zero.

After some basic algebra, one shows that the case $\frac{2+\alpha}{1+\alpha} - \frac{1}{c} < \frac{2+\alpha}{3+\alpha} < \frac{(2+\alpha)(1+\alpha)}{4c}$ is equivalent to $\frac{2+\alpha}{3+\alpha} < \frac{1+\alpha}{2c}$ and thus irrelevant here.

Similarly, $\frac{2+\alpha}{1+\alpha} - \frac{1}{c} < \frac{(2+\alpha)(1+\alpha)}{4c} \leq \frac{2+\alpha}{3+\alpha}$ is not possible. Hence, if equilibrium B exists for $\frac{1+\alpha}{2c} < 1$, then also does equilibrium A.

If β is large enough such that she would choose $d = (0)$ with uninformative messages and equilibrium C exists, then

$$U^C_{DM} = (1 + \alpha)(1 - (z^C)^2) - 2c(1 - z^C)(\beta + (1 - \beta)z^C) > 0$$
$$\Longleftrightarrow z^C(1 + \alpha - 2c(1 - \beta)) + 1 + \alpha > 2\beta c.$$

Inserting $z^C = \frac{\beta c}{1-c(1-\beta)}$ and rearranging yields $(1+\alpha)(1-c)+2\alpha\beta c > 0$, which is obviously true.

3) If $1 + \alpha < 2c$ and β is small, she would always choose $d = (1, 2)$ without information transmission. If equilibrium B exists, the same argument as above shows that this equilibrium is payoff-dominant for her. □

Proof of Proposition 3.8. As in the complements case, the proof is constructive. First, I derive the cutoffs x and $y(x)$. Second, I derive the DM's obedience constraints and the constraint for the existence of CCT. These yield the bounds for the admissible β's, $\underline{\beta}(c)$ and $\overline{\beta}(c)$. Finally, I show that given these bounds, the DM follows the agents' messages determined by x and

$y(x)$.

To find x, observe that here, agent 1 is indifferent between message m^0 and m^1 (see Figure A.2).

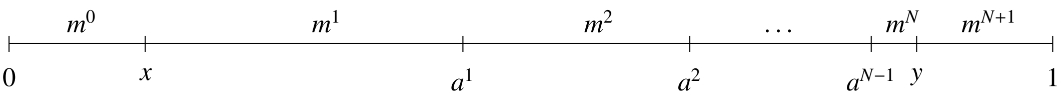

Figure A.2: $\beta > 1$: Equilibrium Messages.

Recalling that $\alpha = 1$, it must hold that

$$\begin{aligned}
\mathbb{E}[u_1|m_1 = m^0] &= \mathbb{P}(m_2 = m^0) \times 0 + \mathbb{P}(m_2 > m^0)\frac{1}{2}(x+1) = \frac{1}{2}(1-x^2) \\
= \mathbb{E}[u_1|m_1 = m^1] &= \mathbb{P}(m_2 = m^0)(x-c) + \mathbb{P}(m_2 = m^1)(\frac{1}{2} \times \frac{1}{2}(x+a^1) + \frac{1}{2}(x-c)) \\
&\quad + \mathbb{P}(m_2 > m^1)\frac{1}{2}(1+a^1) \\
&= x(x-c) + \frac{1}{4}((a^1)^2 - x^2) + \frac{1}{2}(a^1 - x)(x-c) + \frac{1}{2}(1-(a^1)^2).
\end{aligned}$$

Simplifying yields $0 = \frac{3}{4}x^2 + \frac{x}{2}(a^1 - c) - \frac{1}{2}a^1(c + \frac{1}{2}a^1)$. Recall from (4.3) that $a^1 = \frac{y}{N} + (N-1)(c + \frac{x}{N})$. This results in the identity

$$\begin{aligned}
0 = \frac{3}{4}x^2 + \frac{x}{2}(\frac{y}{N} + (N-1)(c+\frac{x}{N})) - \frac{x}{2}c - \frac{1}{2}c(\frac{y}{N} + (N-1)(c+\frac{x}{N})) \\
- \frac{1}{4}(\frac{y}{N} + (N-1)(c+\frac{x}{N}))^2.
\end{aligned} \tag{A.19}$$

Similarly, at $\theta_i = y$, agent i is indifferent between message $m^N = [a^{N-1}, y]$ and $m^{N+1} = [y, 1]$. Since teamwork is only chosen if both agents send the highest message, it holds:

$$\begin{aligned}
\mathbb{E}[u_1|m_1 = m^{N+1}] &= (1-y)(y + \frac{1}{2}(1+y) - \beta c) + y(y-c) \\
= \mathbb{E}[u_1|m_1 = m^N] &= (1-y)\frac{1}{2}(1+y) + (y - a^{N-1})(\frac{1}{2} \times \frac{1}{2}(y + a^{N-1}) + \frac{1}{2}(y-c)) + a^{N-1}(y-c),
\end{aligned}$$

or

$$0 = \frac{3}{4}y^2 + y(\frac{1}{2}a^{N-1} + \frac{1}{2}c - 1 - \beta c) + \beta c - \frac{1}{4}(a^{N-1})^2 - \frac{1}{2}a^{N-1}c.$$

Using again (4.3) and inserting $a^{N-1} = \frac{x}{N} + (N-1)(c + \frac{y}{N})$, one receives

$$0 = \frac{3}{4}y^2 + \frac{y}{2}(\frac{x}{N} + (N-1)(c + \frac{y}{N})) + \frac{1}{2}yc - y - y\beta c + \beta c - \frac{1}{4}(\frac{x}{N} + (N-1)(c + \frac{y}{N}))^2$$
$$- \frac{1}{2}c(\frac{x}{N} + (N-1)(c + \frac{y}{N})). \tag{A.20}$$

Now, equating (A.19) with (A.20) gives

$$0 = y^2 + y(c - 1 - \beta c) + \beta c - x^2,$$

which has the following roots:

$$y_{1/2}(x) = \frac{1}{2}(1 + \beta c - c) \pm \sqrt{\frac{[c(1-\beta) - 1]^2}{4} - \beta c + x^2}. \tag{A.21}$$

The larger root y_1 can be neglected. The reason is as follows: for $c \to 0$, one receives $x = 0$ (below, I show that $x \leq c$ has to hold), $y_1 = 1$ and $y_2 = 0$. $y = 1$, however, is implausible as this would dictate to always send a message against teamwork, irrespective of β. This cannot be part of an equilibrium when costs are infinitesimally small. Therefore, I focus on the smaller root and denote this by $y(x)$.

Observe that $\frac{1}{2}(1 + \beta c - c)$ is positive. Moreover, for $y(x)$ to exist, the square root term must be positive. This is the case for

$$\beta \leq \overline{\beta}(c) := 1 + \frac{1}{c} - \frac{2}{c}\sqrt{c - x^2}. \tag{A.22}$$

Now, I check whether the DM is obedient. Accordingly, after (m_1^0, m_2^0), she must prefer the outside option which means that $2 \times \frac{x}{2} - c \leq 0$ from which follows $x \leq c$.[38] This inequality is a degree of freedom on m^0 such that any $x \leq c$ is admissible and leads to a different m^{N+1}. Moreover, it must hold what I call *deterrence at the bottom*, that is after $m = (m_1^0, m_2^1)$, she prefers $d = (2)$ so that $2 \times \frac{x+a^1}{2} - c > 0$. Plugging in a^1 yields

$$x + \frac{N-1}{N}x + \frac{y}{N} + (N-2)c > 0,$$

which is always true because $y \geq c$. Moreover, she prefers agent 2 working alone over teamwork because this yields $2\beta c - c > x$ and this is true because $2\beta c - c > c \geq x$.

After $m = (m_1^{N+1}, m_2^{N+1})$ with $m^{N+1} = [y, 1]$, she prefers $d = (1, 2)$ over $d = (0)$ and (i)

[38]This also implies $x < \beta c$ which is the constraint for the DM preferring $d = (0)$ over $d = (1, 2)$.

which yields

$$y \geq 2\beta c - c - 1. \tag{A.23}$$

For, $m = (m_1^N, m_2^{N+1})$, deterrence at the top (or $d = (2)$ instead of teamwork) prescribes

$$\begin{aligned} 2 \times \frac{y+1}{2} - c &> 2 \times (\frac{y+1}{2} + \frac{a^{N-1}+y}{2}) - 2\beta c \\ \Longleftrightarrow 2\beta c - c &> a^{N-1} + y, \end{aligned} \tag{A.24}$$

or

$$2\beta c > Nc + \frac{x}{N} + y(2 - \frac{1}{N}). \tag{A.25}$$

I claim that deterrence at the top is sufficient for any other deterrence at a lower message. To be precise, if she chooses $d = (2)$ after $m = (m_1^N, m_2^{N+1})$, then she chooses $d = (2)$ after $m = (m_1^{k-1}, m_2^k)$. To see this, let both messages be from the set $\{m^1, \ldots, m^N\}$ (so a CCT message within $[x, y]$) and suppose without loss that 2 sends m_2^k and 1 sends m_1^{k-1} with $1 < k \leq N$. As she chooses $d = (2)$, it holds:

$$\begin{aligned} 2 \times \frac{a^{k-1}+a^k}{2} - c &> 2 \times (\frac{a^{k-2}+a^{k-1}}{2} + \frac{a^{k-1}+a^k}{2}) - 2\beta c \\ \Longleftrightarrow 2\beta c - c &> a^{k-1} + a^{k-2}. \end{aligned}$$

But the latter inequality is implied by deterrence at the top (A.24).

If $m = (m_1^j, m_2^l)$ with $j \leq l$, it must hold that

$$2 \times \frac{a^{l-1}+a^l}{2} - c \geq 0,$$

which is always true because $a^l \geq a^1 > c$ holds by (4.3).

Finally, CCT must exist which means that $N < \overline{N}(c) = \left\lceil \frac{1}{2} - \frac{x}{2c} + \sqrt{\frac{1}{4} + \frac{x^2}{4c^2} + \frac{y}{c} - \frac{x}{2c}} \right\rceil$, or

$$y > N(N-1)c + Nx. \tag{A.26}$$

Hence, the proposed equilibrium can be summarized as $a^0 = x$, $a^N = y$, where $x \leq c$, $y = \frac{1}{2}(1 + \beta c - c) - \sqrt{\frac{[c(1-\beta)-1]^2}{4} - \beta c + x^2}$ subject to

$$
\begin{aligned}
y &\geq 2\beta c - c - 1 && \text{(symmetry at the top)},\\
2\beta c &> Nc + \frac{x}{N} + y(2 - \frac{1}{N}) && \text{(deterrence at the top)},\\
y &> N(N-1)c + Nx. && \text{(existence of CCT)}.
\end{aligned}
$$

Now combine the constraint *deterrence at the top* with *existence of CCT* to receive a lower bound on β:

$$\beta > \underline{\beta}(c) := \Big(N(N-1)c + Nx\Big)\frac{2N-1}{2cN} + \frac{N}{2} + \frac{x}{2cN}.$$

□

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