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Introduction

Why are currency unions formed and why do they break up? Motivated by that question, the first paper of my dissertation "The Political Economy of Currency Unions" asks how a currency union can be sustained with fiscal and monetary policy when member states have an exit option. In particular, I am asking: Can fiscal transfers between countries save the currency union? And: Can monetary policy through interest rate setting alone save the common currency? To tackle that issue, I set up a two-country open economy model in which there are two different monetary regimes, namely a currency union and a regime with national currencies. In a currency union, trade is assumed to increase, while the exchange rate as an important shock absorber is missing. The lack of an exchange rate is costly when an asymmetric shock hits the union. I embed this set-up in an environment with a two-sided lack of commitment from governments to the currency union. This means that the government can decide to leave the currency union whenever it wants. I run an experiment in this model setup and simulate the economy to compare the outcome of different planners who have different policy instruments at hand to sustain the union. The first planner that I consider is a purely national planner who does nothing to sustain the currency union. Governments simply decide to stay or to leave. Monetary policy and the outside option are taken as given. I show that in the majority of the simulations, the currency union would eventually collapse. The next scenario is a union-wide Ramsey planner who sets lump-sum transfers between countries. The planner sets transfers and makes promises about future transfers in such a way that none of the countries would leave the currency union at any point in time. The planner takes monetary policy as given. The third scenario considers monetary policy as an instrument: A union-wide central bank uses interest rate setting and takes the governments' exit option into account. Optimally, the central bank sets interest rates in such a way that none of the governments want to leave the union.

The paper has three main findings. First, I show how a central bank can sustain a currency union by following an interest rate rule that features time-varying country weights. By partly departing from the original objective of price stability, the central bank emphasizes stabilization of crisis countries and in turn can sustain the union. The simulation shows however – and that leads to the second result - that the ability of the central bank to sustain the currency union is limited. Interest rates are distortionary for the economy of at least one country, especially compared to lump-sum transfers. Furthermore, a certain asymmetry in the business cycle between both countries is needed. Otherwise, interest rates cannot redistribute in the first place. The dependence of the central bank on the business cycle severely limits its ability to intervene. Therefore, the third result demonstrates that the central bank can only temporarily extend the lifetime of a currency union. The union would eventually collapse in the simulation, if only the central bank tries to sustain the union. I also show that in those situations in which the central bank falls short, fiscal policy can step in and save the union.

The next paper is joint work with Ricardo Duque Gabriel. Building on my namesake Master's thesis it asks whether the exchange rate regime matters for inflation, interest rates and economic activity. While in the first paper, gains in a currency union were simply assumed, "Inflation, Interest Rates and the Choice of the Exchange Rate Regime" discusses how large these gains are. In the paper, we consider fixed exchange rate regimes as currency unions without fiscal transfers. We document three empirical regularities in a dataset that covers more than 400 shifts of the exchange rate regime between 1950 and 2016 for 178 countries. First, we document that inflation and interest rates are persistently lower in a fixed exchange rate regime by around four percentage points. The second finding indicates that the variability of inflation and interest rates tends to be lower for countries that enter a fixed exchange rate regime. This is not only true for countries that decide to peg, but also for the anchor countries themselves. The third and last finding shows that GDP growth tends to go up temporarily by two percent when a country decides to peg its currency. We confront these empirical findings with an estimated open economy model in which countries can be in one of two monetary regimes: Either with perfectly flexible exchange rates or in a currency union with a fixed exchange rate. In both regimes, monetary policy acts under discretion and reacts to temptation shocks that drive up inflation permanently. Countries with high inflation rates can then peg their currency to a stable anchor to bring down inflation. Even relatively stable countries can benefit in such a setup if they form a currency union with other stable countries, whose temptation shocks are negatively correlated. In this way, inflation variability for the stable countries can be brought down as the common central bank only reacts to the average temptation shock of the union. In a next step we estimate a two-country version of this model using data from Italy and Germany. We show in a simulation that inflation in Italy goes down by a similar magnitude as in the data when it joins a currency union with Germany. Not only does the level of inflation go down, but also its volatility. Germany on the other hand benefits from the currency union as inflation variability is reduced. Last, we show that reduced inflation also induces GDP for Italy to rise. The reason for that is that high inflation entails costs as households need cash to buy goods.

In the last part of my dissertation, I discuss one of the defining features of our time: Digitalization. In the paper "Consumption Inequality in the Digital Age" my co-author Katja Mann and I ask in how far the increased usage of digital technology impacts consumption inequality. To answer that question the paper proceeds in two steps. We first provide some empirical evidence about the digital content of the consumption basket of American households along the income distribution. For this, we measure how much digital capital is used in the production process on an industry level. After considering the input output structure between industries, we link more than 150 final goods to 809 consumption categories of the Consumer Expenditure Survey. The survey also reports on household's income which allows us to analyze the digital content of the consumption basket of a certain income group. We demonstrate that the digital share rises the richer a household is. While for the poorest households the digital share is at 13 %, the richest households have a share of 14.7%. This difference is driven by several categories such as food or textile manufacturing that are relatively important for poor households but only use little digital technology in the production process. Consumption categories such as education or finance and insurance are important for the rich and use a lot of digital technology. Last, we document that products with a lot of digital technology have experienced less price inflation than other goods. Together with our first finding, this indicates that the relative price effect of digitalization has been more beneficial for rich households. Just how beneficial this relative price change was is at the center of the second part of the paper: We set up a model in which sector-biased technological progress in IT impacts households not only via the income channel, but also via the price channel. The key feature of the model is that rich households consume relatively more products that rely on the sector that uses IT capital more intensely. When there is technological progress in IT, production of the IT intensive sector disproportionally increases, and its product prices go down. At the same time, rich households benefit from technological progress more as their labor input is more complementary to IT capital. Therefore, our model combines the well-studied income polarizing effect of digitalization together with the relative price effect. Using our findings from the empirical study, we calibrate the model to match key moments of the data, such as the digital share of poor and rich households, relative wages of high- and low-income households and the relative good price movements over time. In the simulation of the fully calibrated model, we demonstrate that between 1960 and 2017, rich households have experienced a welfare increase equivalent to 23% of their initial income due to digitalization. In a decomposition we show that around one fourth of that increase is due to the price effect. If rich households had the same digital share than the poor, their welfare increase would only be equivalent to 17.4% of their initial income. At the same time, poor households hardly benefit at all, as their relative wages decline, and their consumption basket has a low digital share.

Chapter 1

The Political Economy of Currency Unions*

1.1 Introduction

Currency unions, such as the eurozone, are inherently unstable. One reason for this instability is the fact that member states are still sovereign nations that can decide to leave the union. The eurozone crisis has forcefully shown this. Next to fiscal policy, the role of the European Central Bank for the currency union has been debated extensively. This poses the question what monetary policy can actually do if the union is confronted with the threat of a break-up. How can a central bank help to make a currency union sustainable?

This paper sheds new light on that question by considering fiscal and monetary policy in a two-country open economy model in which governments have the option to choose between being in a currency union and having an own national currency. With an own currency, the central bank can focus on price stability and let the exchange rate float freely. In a currency union there is only one central bank for both countries. The benefit of a common currency is that it facilitates trade. By assumption, if both countries use the same currency, trade costs are reduced and bilateral trade increases. The downside of the currency union is that macroeconomic stabilization is less effective for certain states of the world since a common central bank sets interest rates for the whole union. Therefore, the costs of a currency union are time-varying and in some situations these costs might outweigh the benefits.

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I use this setup to run an experiment in which I calibrate the economy to simulate and then look at the outcome of four scenarios. In the first scenario both governments decide freely when they want to leave the currency union. That is the only decision. They take monetary policy and the outside option as given. Once a government leaves the currency union, the union is destroyed forever. In the second scenario, I add a union-wide Ramsey planner who sets lump-sum transfers between countries. The planner takes the member states' exit option into account. The idea is to set transfers in such a way that no government wants to leave the union. In the end, under the veil of ignorance, both countries are better off with this transfer scheme as the union is sustained. As in the first scenario, monetary policy is taken as given by the Ramsey planner. The third scenario considers a union-wide central bank, that sets interest rates and takes the exit option of both countries into account. No transfers take place in this scenario. As with the union-wide Ramsey planner, the idea is to set interest rates in such a way that no country wants to leave the union at any point in time. In the fourth and last scenario, I consider a joint monetary and fiscal response with a one-time monetary intervention in the crisis period itself and systematic transfers afterwards. All these four scenarios are run with different amounts of trade gains in a currency union that are consistent with the range of estimates from the literature[1]. The goal is to check which policy works depending on the amount of gains coming from the currency union.

The paper has three main findings: First I show how a central bank can prevent a break-up of the currency union by following an interest rule that puts more weight on stabilizing crisis countries that would otherwise exit the union. Second, I highlight that interest rate policy alone is a poor tool to redistribute between countries, as it relies on business cycles being not perfectly synchronized. Furthermore, compensation through interest rates is distortionary. Therefore- and this leads to the third result- the central bank alone can only sustain the union for some time, but if a sequence of sufficiently large asymmetric shocks emerges the union will eventually collapse. I demonstrate how fiscal transfers can sustain the union in the experiment in those situations in which interest rate setting alone cannot.

The first finding shows how a central bank can use an interest rate rule to sustain the currency union when member states want to exit. The central bank does this by following a rule that features *time-varying* country weights. When a country wants to leave the currency union, the central bank promises this country to put a greater emphasis on stabilizing its economy. This way the central bank gives more weight to that country and makes the currency union for it relatively more attractive than the outside option with national currencies. Which country is stabilized more by the central bank is determined ex post, after shocks have materialized. Therefore, with

1. See the literature review at the end of the section and the calibration in section 1.4

the interest rate rule derived in this paper the central bank can in principle factor in exit options of member states.

The second finding relates to the strength of this policy instrument to redistribute and in turn to sustain the union. The central bank can only promise to favor a certain country in the future, if the business cycles of the member states are not perfectly synchronized. This means that a certain degree of asymmetry between both countries is needed for interest rates to be an effective tool. If business cycles are expected to be perfectly synchronized in the future, the central bank has no way to favor a specific country because both countries want to have the same interest rates. This puts a limit to the ability of the central bank to make promises to countries that are willing to leave, as compared to a planner who can promise transfers.

This leads to the third result, namely that the currency union will eventually break up if monetary policy is the only tool considered to preserve the union. The experiment shows that an actual break-up of the union is rather likely if fiscal transfers and monetary accommodation are absent. In the simulation, the central bank can increase the average duration of the currency union, but she cannot totally suppress the possibility of a break-up. With a monetary policy intervention, the union can be sustained for a while until a sequence of exceptionally large asymmetric shocks hit the union. I furthermore demonstrate in the experiment that fiscal transfers can sustain the currency union also in those simulations in which monetary policy alone fails to achieve that.

In conclusion, the central bank can help to sustain the union and reduce the probability of a break-up. This is done by partly departing from the original objective of union-wide price stability and emphasizing stabilization of crisis countries. The central bank however is only able to buy some time for the currency union. The option of using fiscal transfers is a more effective policy tool and ensures that the union is permanently sustained.

Related Literature

The first strand of literature that this paper relates to goes back to the optimum currency literature, pioneered by Mundell (1961). Currency unions are vulnerable to so called asymmetric shocks, especially when factor mobility is low and a common fiscal policy is missing, as noted by McKinnon (1963) and Kenen (1969). Eichengreen (1992) and Shambaugh (2012) have discussed if the eurozone constitutes an optimal currency area and noted several vulnarabilities. These vulnerabilities are in fact so large that markets price in a positive probability of a eurozone break-up, as shown by Bayer, Kim, and Kriwoluzky (2018). My paper explicitly microfounds the costs of a monetary union and models when a break-up occurs. It also discusses how such a break-up can be prevented. I use a two-country model based on Corsetti and Pesenti (2002). This kind of model is part of the new open economy literature that has been established over the last decades [2]. An important issue that this literature addresses is the question which monetary regime is optimal depending on the invoicing regime. Conclusion reach from letting the exchange rate float freely, as proposed by Friedman (1953) and Clarida, Galí, and Gertler (2002), to pegging the exchange rate as in Devereux and Engel (2003). Optimal cooperation between monetary authorities has also been extensively discussed by the literature, see Benigno and Benigno (2003), Corsetti and Pesenti (2002), Corsetti, Dedola, and Leduc (2018), Bodenstein, Guerrieri, and Labriola (2019) and Egorov and Mukhin (2020). Historically, the world has seen many different exchange rate regimes, as shown by Ilzetzki, Reinhart, and Rogoff (2019). How exchange rate regimes are chosen and why they evolve in the way we observe it, is not well understood and has been discussed recently by Mukhin (2018). I contribute to this literature and show under which conditions a currency union, seen as a fixed exchange rate regime, can collapse and be sustained. Why such unions are formed in the first place is an open debate. My paper considers trade advantages in a currency union as the main benefit, as a common currency is thought to reduce trade costs (Alesina and Barro, 2002). Evidence of more trade inside a currency union has been given by Baldwin, Di Nino, Fontagne, De Santis, and Taglioni (2008) and Micco, Stein, and Ordonez (2003) who find trade increases between 4% to 16%. Even higher estimates have been found by Rose (2000), Frankel and Rose (2002) and Glick and Rose (2002). Baier, Bergstrand, and Feng (2014) highlight that those large increases in bilateral trade of economic unions arise if other economic trade agreements such as customs union and common markets are considered as well. Another potential benefit of entering a currency union is the reduction of inflationary biases in some countries, when a new credible central bank is created, see for example Alesina and Barro (2002). A similar point has been made by Chari, Dovis, and Kehoe (2020) who points out that an inflationary bias can be reduced in a currency union even if the newly created central bank is not credible. My paper therefore combines the good and bad sides of a currency union in one model. The costs of the currency union are time-varying and might exceed the benefits when a big asymmetric shock emerges. Such a situation gives rise to the possibility of a break-up of the union, that is discussed in the second part of the literature review.

Forming and disrupting political and economic unions has been analyzed by Balassa (1961), Haas (1958) and Bolton and Roland (1997). As noted by Cohen (1993), a currency union consisting out of sovereign nations can break up. Fuchs and Lippi (2006) formally establish an exit option in a reduced-form model of a monetary union. They embed this union into a dynamic contract with limited commitment[3]

2. See for example Benigno and Benigno (2003), Gali and Monacelli (2005) Clarida, Galí, and Gertler (2002), Corsetti and Pesenti (2005),Corsetti, Dedola, and Leduc (2010) and Engel (2011)

3. The literature of dynamic contracts with commitment problems was pioneered by Thomas and Worrall (1988), Kocherlakota (1996) and Marcet and Marimon (2019).

of member states to the union. They find that with such an exit option, the union-wide central bank optimally uses time-varying country weights. I contribute to that by explicitly modeling the macroeconomics of a currency union and deriving an interest rate setting rule that features time-varying country weights as well. In addition to that, I compare this policy to fiscal transfers that aim to make the currency union sustainable. Auclert and Rognlie (2014) show that a monetary union can favor the creation of a fiscal union. They demonstrate how a central bank departs from its traditional role of price stability for the union to encourage more political integration with its policy. In a similar way, my paper shows how a central bank can prevent political *dis*integration with its policy. Ferrari, Marimon, and Simpson-Bell (2020) have demonstrated how fiscal policy can be used as a tool to deal with exit options and significantly reduce the costs of a currency union. Compared to them, I introduce aggregate risks and provide a framework to jointly analyze fiscal and monetary policy. How fiscal policy can improve welfare in a currency union has been shown by Farhi and Werning (2017). They establish that even in the presence of perfect financial markets, as in Cole and Obstfeld (1991), fiscal policy plays an important role in stabilizing a currency union. Recently, the literature discussed fiscal policy in the context of moral hazard in Europe, see for example Ábrahám, Carceles-Poveda, Liu, and Marimon (2019) and Müller, Storesletten, and Zilibotti (2019). In my paper, fiscal policy can improve the outcome by ensuring that governments do not exert the exit option. This way, the currency union is sustained and both countries benefit from trade costs over a longer horizon. Other papers consider exit options as well, such as Kriwoluzky, Müller, and Wolf (2019) who find that a sovereign debt crisis can be amplified by exit expectations, or Eijffinger, Kobielarz, and Uras (2018) highlighting crisis contagion to other member states in the presence of exit options. Another result of my paper relates to political integration more generally. I show how countries decide to join a currency union with no transfers in the beginning. As the threat of a break-up looms, both countries voluntarily enter a primitive fiscal union with transfers between countries. The threat of a break-up serves as a driver of a deeper political and economic union, since countries automatically climb the 'staircase' of political integration, as in Auclert and Rognlie (2014).

The work is organized as follows. Section 1.2 presents the two-country model that gives rise to different monetary regimes and the benchmark allocations. In section 1.3 I describe the political economy, where governments choose the monetary regime. Section 1.4 discusses the calibration of the model, while section 1.5 runs the experiment and shows the results. Section 1.6 concludes.

1.2 Model of the Economy

This section outlines a model based on Corsetti and Pesenti (2002), and Corsetti and Pesenti (2005). I establish a dynamic two-country general equilibrium model with trade and stochastic productivity shocks. I extend the baseline by Corsetti and Pesenti (2002) to allow for trade costs and to explicitly give the governments the option to choose between a currency union and national currencies.

1.2.1 Households, Consumption Bundles and Price Indices

There are two countries, a Home country (H) and a foreign country (F). Each is populated by a mass one of identical individuals. Lifetime utility of the representative household in H is given by:

$$\mathbb{E}_t\Big[\sum_{j=t}^{\infty}\beta^{j-t}\Big(\ln(C_j)-\kappa L_j\Big)\Big], \tag{1.1}$$

where C_t is a basket of consumption goods and L_t are working hours for the individual with κ being a coefficient for disutility of labor. $\beta \in (0,1)$ is the time discount factor which is assumed to be the same for individuals in both countries. In addition to that, utility is quasi-linear in labor to simplify the aggregation in later steps. Preferences of agents in F are described analogously with all variables being denoted with a *. The consumption basket consists of consumption of Home goods $C_{H,t}$ and foreign goods $C_{F,t}$ with an elasticity of substitution of 1. It can be written as a Cobb Douglas function:

$$C_t = (C_{H,t})^{\gamma}(C_{F,t})^{1-\gamma}, \quad C_t^* = (C_{H,t}^*)^{1-\gamma}(C_{F,t}^*)^{\gamma}, \tag{1.2}$$

where γ governs the taste of households for goods from country H or F. In contrast to Corsetti and Pesenti (2002), I assume that both countries have a Home bias and that every country weights its own good with γ. The individual's consumption index for goods from country H is an aggregator of different brands h with elasticity of substitution θ:

$$C_{H,t} = \Big[\int_0^1 C(h)^{\frac{\theta-1}{\theta}}dh\Big]^{\frac{\theta}{\theta-1}}, \quad C_{F,t} = \Big[\int_0^1 C(f)^{\frac{\theta^*-1}{\theta^*}}df\Big]^{\frac{\theta^*}{\theta^*-1}} ; \quad \theta,\theta^* > 1$$

Each country hence specializes in the production of a single type of good. Each brand h is produced by a single Home firm and sold in all countries in a monopolistic market. The utility-based price index P_t of H is the consumption-based price index that can be obtained by minimizing expenditures to buy one unit of composite real consumption C_t

$$P_t = \frac{P_{F,t}^{1-\gamma}P_{H,t}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}, \quad P_t^* = \frac{P_{F,t}^{*\gamma}P_{H,t}^{*1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}.$$

The price indexes $P_{H,t}$ and $P_{F,t}$ for Home goods and foreign goods respectively in the Home country can be derived in a similar way

$$P_{H,t} = \left(\int_0^1 p_t(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}}, \quad P_{F,t} = \left(\int_0^1 p_t(f)^{1-\theta^*} df \right)^{\frac{1}{1-\theta^*}}.$$

Household's portfolio consists of several components. Agents can access financial markets in order to sell and buy Home bonds[4] $B_{H,t}$ and foreign bonds $B_{F,t}$. Foreign bonds have to be converted into Home currency. The exchange rate $\mathcal{E}_t$ is defined as Home currency over foreign currency [5]. In addition, the households own the firms and supply labor on a competitive market. Therefore, they receive wages, firms' profits $\Pi_{H,t}$ and interest rates from bonds. Furthermore, they pay non-distortionary net taxes T_t to the government. As in Woodford (2003), I consider the limiting case of a cashless economy. The nominal flow budget constraint of individual j at time t is given by the following inequality:

$$B_{H,t} + \mathcal{E}_t B_{F,t} + P_{H,t} C_{H,t} + P_{F,t} C_{F,t} + T_t \leq (1+i_t) B_{H,t-1} + (1+i_t^*) \mathcal{E}_t B_{F,t-1} + W_t L_t + \Pi_{H,t} \tag{1.3}$$

the short-term nominal interest rate i_t is paid out at the beginning of period t and known in $t-1$. The household's optimization problem is to maximize lifetime utility (1.1) subject to the consumption aggregator (1.2) and the budget constraint (1.3). Demand for brand h and f by the representative consumer can then be expressed as a function of the relative price and total consumption of Home and foreign goods:

$$C_t(h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t}, \quad C_t(f) = \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\theta^*} C_{F,t} \tag{1.4}$$

Consumption of goods produced in the Home country is a function of its price relative to the overall price index and total consumption:

$$C_{H,t} = \gamma \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t, \quad C_{F,t} = (1-\gamma) \left(\frac{P_{F,t}}{P_t} \right)^{-1} C_t$$

Demand can also be expressed as a function of international relative prices. Let the terms of trade $\mathcal{T}_t$ be defined as the price of foreign export goods over Home export goods

$$\mathcal{T}_t = \mathcal{E}_t P_{F,t}^* / P_{H,t}. \tag{1.5}$$

4. $B_{H,t-1}$ are accumulated bonds until the period t that are carried over to period t. Households choose in t how many bonds to hold.

5. A higher $\mathcal{E}_t$ means that one unit of a foreign currency can now buy more units of the Home currency. We say the Home currency depreciates.

The Euler equation determines agent's intertemporal allocation

$$\frac{1}{P_t C_t} = (1 + i_{t+1})\mathbb{E}_t\Big[\beta \frac{1}{P_{t+1} C_{t+1}}\Big]. \tag{1.6}$$

The stochastic discount factor is defined as $Q_{t,t+1} \equiv \beta \frac{P_t C_t}{P_{t+1} C_{t+1}}$. The optimality condition for labor $W_t = \kappa P_t C_t$ implies that $Q_{t,t+1}$ is the same for every individual. In addition, the law of one price holds. Thus $p_t(h) = \epsilon_t p_t(f)$.

1.2.2 Production, Good Transport and Prices

Production in the model is a function of labor input and a stochastic technology parameter a_t. Supply of brand h is given by

$$Y_t(h) = L_t(h) a_t . \tag{1.7}$$

The technology parameter determines aggregate productivity in the economy and is the only source of uncertainty in the model. a_t and its foreign analog a_t^* follow an identical stochastic process. Let $s_t = (a_t, a_t^*)$ denote the state of the world. a_t is a random variable with support A, its history is described by $s^t = (\{a_t, a_t^*\}, \{a_{t-1}, a_{t-1}^*\}, ..., \{a_0, a_0^*\})$. The process is Markov with transition matrix $p(s^t)$. Higher values of a_t correspond to greater productivity ('boom') while lower values indicate lower productivity ('recession'). In this setup, one country can be in a boom, while the other is in a recession. Such a state is considered as an asymmetric shock. A firm faces demand for brand h by consumers in H and in F, as given by (1.4). Total demand for firm h is

$$\Big(\frac{p_t(h)}{P_{H,t}}\Big)^{-\theta} C_{H,t} + (1 + \varpi)\Big(\frac{p_t^*(h)}{P_{H,t}^*}\Big)^{-\theta} C_{H,t}^*. \tag{1.8}$$

At this point, I extend the model and assume that a certain fraction ϖ of goods in the non-domestic market are lost. Like in Alesina and Barro (2002), iceberg trade costs occur when transporting a good to the non-domestic market. It is necessary to ship $1 + \varpi$ units from H to F if one unit of h shall arrive in F. Crossing the border between two countries entails transport costs reflecting for example currency conversion costs. These expenses are lost for the economy. I assume that the adoption of a common currency reduces these costs. For the calibration in section 1.4, a range of empirical estimates from the literature discipline ϖ.

Labor markets are competitive. Let W_t denote the nominal wage. Nominal marginal costs are identical across firms:

$$MC_t(h) = MC_t = a_t^{-1} W_t$$

Profits generated in the foreign market need to be converted into the Home currency. The firm knows that a certain fraction of goods is lost when selling them in the non-domestic market. Knowing overall demand (1.8), profits are given by

$$\begin{aligned} \Pi_t(h) =& \Big((1-\tau)p_t(h)-MC_t\Big)\Big(\frac{p_t(h)}{P_{H,t}}\Big)^{-\theta} C_{H,t} \\ &+\Big((1-\tau)\mathscr{E}_t p_t^*(h)-(1+\varpi)MC_t\Big)\Big(\frac{p_t^*(h)}{P_{H,t}^*}\Big)^{-\theta} C_{H,t}^*. \end{aligned} \tag{1.9}$$

$p_t(h)$ is the nominal price of brand h in H and $\mathscr{E}_t$ is the nominal exchange rate between both countries defined as units of Home currency per unit of foreign currency. $p_t^*(h)$ is the price of brand h in the foreign market. As in Benigno and Benigno (2003), τ is a country specific proportional tax on firms' revenues that is rebated to households via lump-sum transfers. This tax eliminates monopolistic markups.

The model features nominal rigidities: Firms set prices $p_t(h)$ one period in advance, in $t-1$. They form expectations about productivity in the next period and maximize the present discounted value of profits. For given prices, firms satisfy demand for their good[6]. Firms optimally set prices equal to expected marginal nominal costs multiplied with the equilibrium markup Φ.

$$p_t(h) = P_{H,t} = \Phi\mathbb{E}_{t-1}[MC_t], \tag{1.10}$$

where Φ is the level of monopolistic markup corrected by distortionary taxation:

$$\Phi = \frac{\theta}{(\theta-1)(1-\tau)}, \quad \Phi^* = \frac{\theta^*}{(\theta^*-1)(1-\tau^*)}$$

$\frac{\theta}{\theta-1}$ is the markup that arises due to monopolistic competition. For $\Phi = 1$, monopolistic distortions are completely eliminated by taxes. If they are not completely eliminated Φ is greater than 1 and makes prices greater than their marginal costs.

Firms selling abroad also set their prices one period in advance. I assume that these prices are set according to the Producer Currency Pricing (PCP) model. This means that exported goods are sold in the currency of the producer. For example, goods produced in H and sold in F are priced in H's currency. The price firms receive from selling goods to a foreign country is not affected by exchange rate movements. For given quantities, exchange rate variations have no impact on profits, because prices move one to one. For consumers however, the price of non-domestic goods depends on the exchange rate. Let the price for exports that firms choose in their currency

6. Firms only sell goods, if their prices are higher than their marginal costs, that is $P_{H,t} \geq MC_t$ and $P_{H,t}^* \geq \frac{MC_t}{\mathscr{E}_t}(1+\varpi)$ Firms that do not met the participation constraint will not sell goods. I only look at versions of the model, where prices are higher than marginal costs.

be denoted by $\tilde{p}_t(h)$. The actual price that consumers face in their currency is $p_t^*(h)$. Both prices are linked via the exchange rate:

$$p_t^*(h) = \frac{\tilde{p}_t(h)}{\mathcal{E}_t}$$

Firms choose the price of their export goods $\tilde{p}_t(h)$ such that their profits (1.9) are maximized

$$p_t^*(h) = P_{H,t}^* = \Phi(1+\varpi)\frac{\mathbb{E}_{t-1}[MC_t]}{\mathcal{E}_t}. \tag{1.11}$$

The transportation costs ϖ increase prices of h in F. Prices of foreign brands in country H are analogous:

$$p_t(f) = \tilde{p}_t(f)\mathcal{E}_t$$

For foreign goods we have

$$P_{F,t}^* = \Phi^*\mathbb{E}_{t-1}[MC_t^*], \quad P_{F,t} = \Phi^*(1+\varpi)\mathcal{E}_t\mathbb{E}_{t-1}[MC_t^*]. \tag{1.12}$$

1.2.3 Government and Central Bank

The government runs a balanced budget every period

$$T_t = \tau p_t(h)\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} C_{H,t} + \tau\mathcal{E}_t p_t^*(h)\left(\frac{p_t^*(h)}{P_{H,t}^*}\right)^{-\theta} C_{H,t}^*.$$

The model also features a central bank that controls the interest rate i_t and provides a nominal anchor for market expectations. Furthermore, the central bank has an inflation target Π. Inflation Π_t is defined as

$$\Pi_t = \frac{P_t}{P_{t-1}}.$$

Monetary policy can be useful by closing output and employment gaps in the presence of price stickiness. The central bank uses interest rates to operate via the Euler equation. As in Corsetti and Pesenti (2002), I introduce a monetary stance $\mu_t = P_t C_t$ that controls nominal expenditures in the economy. This stance links the nominal interest rate in the Euler equation such that

$$\frac{1}{\mu_t} = \beta(1+i_{t+1})\mathbb{E}_t\left[\frac{1}{\mu_{t+1}}\right].$$

μ_{t+1}/μ_t determines inflation Π_t, the steady state nominal interest rate is $1+i = \Pi/\beta$. In equilibrium one obtains that $\mu_t = P_t C_t = W_t/\kappa$[7]. An expansionary monetary policy in H corresponds to interest rates cuts today or households' expectations about interest rate cuts in the future. In this case μ_t lies above the trend, it coincides with increased nominal spending $P_t C_t$ in the economy.

7. Inspect the Euler equation with logarithmic utility for that

1.2.4 Market Clearing

The labor market in H and F is cleared:

$$L_t = \int_0^1 L_t(h)dh, \quad L_t^* = \int_0^1 L_t(f)df.$$

International financial markets for bonds are cleared, all bonds are in zero net supply

$$B_{H,t} + B_{H,t}^* = 0, \quad B_{F,t} + B_{F,t}^* = 0.$$

Supply of each brand (1.7) equals its aggregate demand (1.8)

1.2.5 Benchmark Allocations

This section discusses monetary policy in a currency union and with national currencies. I derive the allocation of consumption and labor in those two regimes with sticky prices.[8] In section 1.3, the governments will choose between these two regimes.

1.2.5.1 National Currency

Consider a central bank that commits to pre-announced rules in country H. The national authority in the Home country chooses its monetary stance μ_t and maximizes expected utility of the representative agent. The central bank takes the information set of last period as given. As in Corsetti and Pesenti (2005), the central bank of H does not resort to time-inconsistent discretionary monetary policies, rather it acts under commitment:

$$\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} \left[\sum_{t=k}^{\infty} \sum_{s^t \in A} \beta^{t-k} p(s^t \mid s^0) \Big(\ln(C_t) - \kappa L_t \Big) \right]$$

The problem is subject to the equilibrium conditions of the economy. For further details, see section 1.A.5.1. The optimal policy of the central bank ensures price stability:

$$MC_t = \mathbb{E}_{t-1}[MC_t] \tag{1.13}$$

This means, that the central bank chooses interest rates in such a way, that actual marginal costs for domestic firms always equal expected marginal costs. With this policy, the central bank replicates the flex-price equilibrium[9] and eliminates any distortion coming from rigid prices. This implies that monetary policy is completely

8. The benchmark allocations of a social planner is discussed in the Appendix 1.A.1.1, as well as the allocation in an economy with flexible prices Appendix 1.A.4.

9. see 1.A.4

inward looking. The central bank stabilizes the domestic price index only. As noted by Benigno and Benigno (2003) this is a very special result and relies on the PCP assumption and that the trade elasticity of substitution (Cobb Douglas aggregator) as well as the intertemporal elasticity (log consumption) are both set to 1. An inward-looking monetary stance also means that only domestic productivity shocks are considered, and the central bank does not want to manipulate the terms of trade.

Consider an example: In one period, productivity in H is higher than previously expected. This means that marginal costs of home firms fall. In the presence of price stickiness, prices cannot fall in the same period. This means that prices of home goods are too high, implying inefficiently low demand for home goods. Optimal monetary policy cuts interest rates in such a situation. This boosts nominal expenditures of the economy and causes the exchange rate of the home country to depreciate. As the home currency gets cheaper, foreign households can now buy more home goods with their own currency. The exchange rate movement mimics the price fall that would have occurred in a flexible price world. This way, domestic and foreign demand is put to its efficient flex price level. With this policy in place, actual marginal costs always equal their expected value, implying price stability for the whole economy.

The central bank in the Foreign country operates in the same way as in the Home country. The optimal policy of the central bank in F implies price stability for F and is completely inward looking as well. As a result, the exchange rate is flexible.

With both central banks following their policy rules, I can analytically compute consumption and labor as in Corsetti and Pesenti (2002). These variables have the superscript 'N' for national

$$
\begin{aligned}
C_{Ht}^{N} &= \frac{\gamma a_t}{\Phi\kappa}, & C_{Ht}^{*N} &= \frac{(1-\gamma)\left(\frac{1}{1+\varpi}\right)a_t}{\Phi\kappa},\\
C_{Ft}^{N} &= \frac{(1-\gamma)\left(\frac{1}{1+\varpi}\right)a_t^*}{\Phi^*\kappa^*}, & C_{Ft}^{*N} &= \frac{\gamma a_t^*}{\Phi^*\kappa^*},\\
L_t^{N} &= \frac{1}{\Phi\kappa}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right), & L_t^{*N} &= \frac{1}{\Phi^*\kappa^*}\left(\frac{\gamma}{1+\varpi} + 1 - \gamma\right).
\end{aligned}
\tag{1.14}
$$

Consumption moves together with productivity, while labor does not, as in the efficient allocation of the social planner (1.A.1). Trade costs ϖ decrease consumption and employment and cannot be eliminated by the central bank. There is also no other state variable, such as wealth. As in Corsetti and Pesenti (2002) the current account is always balanced and households of a country do not accumulate any debt or wealth. The reason for that is that endogenous terms of trade movements offset productivity shocks, if the inter- and intratemporal elasticity of substitution are both set to 1. For further details, see section 1.A.8. I also consider the possibility of a non-credible central bank in F that is not able to commit to any policies. If such a central bank is in charge, an inflationary bias can arise. The policy problem and the implied allocation is described in 1.A.5.4. For now, we focus on a situation in which both central banks can commit to policies as the main benchmark.

1.2.5.2 Currency Union

In a currency union, monetary policy is conducted by a union-wide central bank that sets interest rates for the whole union. I assume that there are no trade costs in a currency union, as both countries use the same currency. The central bank of the union maximizes the weighted sum of both countries' representative agents' lifetime utility. Let ξ be the weight for country H and $1-\xi$ be the weight for country F. The objective function for the union-wide central bank is

$$\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} \left[\xi \sum_{t=k}^{\infty}\sum_{s^t\in A}\beta^{t-k}p(s^t|s^0)\Big(\ln(C_t)-\kappa L_t\Big)+(1-\xi)\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta^{t-k}p(s^t|s^0)\Big(\ln(C_t^*)-\kappa^* l_t^*\Big)\right]$$

subject to the equilibrium conditions in a currency union, see section 1.A.5.2. Price stability is the optimal policy, as the central bank stabilizes the weighted average of both countries' marginal costs:

$$1 = \left(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}+\big(\xi(1-\gamma)+(1-\xi)\gamma\big)\frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]}\right)^{-1} \tag{1.15}$$

Let $\Psi=\big(\xi\gamma+(1-\xi)(1-\gamma)\big)$ and $1-\Psi=\big(\xi(1-\gamma)+(1-\xi)\gamma\big)$ be the effective weights in front of marginal costs. To illustrate the intuition for this monetary rule, consider the case in which there is no Home bias ($\gamma=0.5$), e.g. Home and foreign goods are equally important to all. In that case, the effective weight is 0.5 as well, independent from the Pareto-weight ξ. As both countries like both goods in the same way, the central bank also stabilizes both marginal costs in the same way and Pareto-weights are irrelevant. Another interesting case is the scenario in which there is an equal weight $\xi=0.5$ for both countries. In this case, the effective weight is 0.5 as well. The central bank has to stabilize both countries equally, as both are equally important to the central bank and both have a *symmetric* Home bias to their own goods.[10] Note, that if the weight for the Home country is 1 ($\xi=1$), the effective weight in front of the Home country's marginal costs equals γ. The effective weights for marginal costs are therefore in line with Home's weight for the corresponding goods in its own consumption bundle. For $\xi=0$, the effective weights for Foreign marginal costs would be γ, in line with Foreign's taste for Foreign goods. In section 1.3, I derive how these effective weights become state dependent when there are

10. This would not be the case, if both countries have different Home biases. For example, if Foreign has a Home bias γ^*, the effective weight in front of Home marginal costs would be $0.5\gamma+0.5(1-\gamma^*)$.

exit options and how the central bank can use this to favor a specific country of the union to prevent it from exiting.
When both countries have the same productivity, monetary policy coincides with the policy, that a national central bank would have chosen in (1.13). Actual and expected marginal costs are the same for both countries in that case. If there is an asymmetric shock, optimal union-wide monetary policy differs from national monetary policy. If one country experiences a boom with high productivity and the other a recession with low productivity, the central bank only stabilizes the economy on average.
Consumption and labor in a monetary union have the superscript ’U’ for Union

$$\begin{aligned}
C_{Ht}^{U} &= \frac{\gamma(\Psi a_t^{-1} + (1-\Psi)a_t^{*-1})^{-1}}{\Phi\kappa}, & C_{Ht}^{*U} &= \frac{(1-\gamma)(\Psi a_t^{-1} + (1-\Psi)a_t^{*-1})^{-1}}{\Phi\kappa}, \\
C_{Ft}^{U} &= \frac{(1-\gamma)(\Psi a_t^{-1} + (1-\Psi)a_t^{*-1})^{-1}}{\Phi^{*}\kappa^{*}}, & C_{Ft}^{*U} &= \frac{\gamma(\Psi a_t^{-1} + (1-\Psi)a_t^{*-1})^{-1}}{\Phi^{*}\kappa^{*}}, \\
L_t^{U} &= \frac{1}{\Phi\kappa}\frac{a_t^{-1}}{\Psi a_t^{-1} + (1-\Psi)a_t^{*-1}}, & L_t^{*U} &= \frac{1}{\Phi^{*}\kappa^{*}}\frac{a_t^{*-1}}{\Psi a_t^{-1} + (1-\Psi)a_t^{*-1}}.
\end{aligned} \tag{1.16}$$

The amount of labor does depend on productivity in the monetary union. Agents of the recession country work more than agents in the boom country, since the central bank is not able to close all output and employment gaps. As a result, utility of agents in the recession country is lower than in the boom country, making it more attractive in a recession to leave the monetary union. As with national currencies, there is no other state variable, as households do not accumulate any debts or wealth. In a currency union, labor adjusts as a response to productivity shocks in such a way, that it offsets the movement in productivity. Production and consumption for both countries are always the same, implying a balanced current account and no debts dynamic. Note that labor is at the (efficient) flex-price level when productivity is the same for both countries. Trade costs are completely eliminated in a currency union.

1.3 Political Economy in a Currency Union

Consider now the political economy of currency unions. The goal is to model the decision process of a break-up of a currency union. I model the currency union as a dynamic contract, that each government is free to walk away from. This is based on Ligon, Thomas, and Worrall (2002) and draws from work by Ljungqvist and Sargent (2004), chapter 18-20 and Thomas and Worrall (1988).
Suppose both countries are initially in a currency union. In every period, the governments of both countries decide if they want to leave. That is the only decision of the government. They base this decision on lifetime utility of the representative agent in the country given a certain state today. The allocation in the corresponding regimes are taken as given. If a representative agent is better off in a currency

union than with national currencies, the government decides to stay in the union. This is the case if utility as a function of consumption and labor (1.16) in a currency union plus the continuation value of the union is higher than utility with national currencies (1.14). In contrast, a country leaves the union if an agent obtains higher lifetime utility with national currencies. In this case the *participation constraint* of the country is violated. I assume, that once a government has decided against a currency union, no further currency union can be formed in the future and everyone keeps national currencies for the rest of the time.

I use this to set up the scenarios discussed in the introduction: First I consider a model environment in which both countries start with a common currency and the governments decide in each period if they leave the union. After that, I discuss a union-wide Ramsey planner with transfers who takes the lack of commitment of both countries into account. In a next step, I discuss if a central bank with interest rate setting only is able to sustain the union as well. Last I consider interest rates and transfers combined.

1.3.1 National Social Planner with Exit Option

The monetary union is modeled as a contract that both governments are free to walk away from whenever they want to. The history s^t summarizes past and present shocks and -conditional on the model- monetary regimes. Let $u^i(s^t) = \ln(C^i(s^t)) - \kappa L^i(s^t)$ denote the period utility of country H and $v^i(s^t)$ the corresponding per period utility of country F in regime $i \in \{N, U\}$ for history s^t. Consumption and labor are as in the allocation of (1.16) for the union and as in (1.14) with national currencies. The utility gain from a monetary union over national currencies from period t onward is defined as

$$U_t(s^t) \;=\; u^U(s^t) - u^N(s^t) + \mathbb{E}_t\Big[\sum_{j=t+1}^{\infty} \beta^j\big(u^U(s^j) - u^N(s^j)\big)\Big]. \tag{1.17}$$

The first term is the short-run gain from the union and the last term in expectation the long-run continuation gain from the union. The utility gain $V_t(s^t)$ for country F is defined in an analogous way.

From an economic perspective, a national planner (for example the government) decides to leave the union as soon as the expected utility gain of the representative agent is negative. When this happens, the monetary union breaks up, even if the other country has a positive gain. More formally, a government has no incentive to leave the union, if

$$U_t(s^t) \;\geq\; 0, \quad V_t(s^t) \;\geq\; 0. \tag{1.18}$$

These two participation constraints are central for the political economy of currency unions. An allocation in a currency union is said to be *sustainable*, if both inequalities hold. Whether they hold, depend on the specific history s^t that summarizes: The current state of the economy, how volatile the economy is expected to be and the transfer history in the contract. Remember that in a monetary union, the central bank struggles to effectively stabilize output if an asymmetric shock occurs. The more asymmetric the shock is, the larger is the welfare loss in a monetary union. With these participation constraints, the allocation of the national social planner can be computed for any sequence of shocks. Before doing this, let us compare this to a union-wide social planner with transfers.

1.3.2 Union-wide Social Planner with Transfers amid Exit Option

In a next step, I consider a union-wide planner that sets transfers (the Ramsey planner) taking the lack of commitment from member states into account. Therefore, the contract also includes transfers between countries. These transfers correspond to the lump-sum transfers in the two-country model before, see (1.3). A contract $T(\cdot)$ now specifies for all histories s^t a transfer $T(s^t)$ from H to F. Consumption in a monetary union is therefore $C^U(s_t) - T(s^t)$ for H and $C^{*U}(s_t) + T(s^t)$ for F. Let $u^i(s^t) = \ln(C^i(s_t) - T(s^t)) - \kappa L^i(s_t)$ denote the period utility of country H and $v^i(s^t)$ for F as before that include transfers. If transfers are always zero, the situation is the same as in the allocation of a national social planner in section 1.3.1. To solve for optimal transfers, it is helpful to consider the Markov structure of the problem. The optimization problem of finding an efficient contract is always the same, when the same state occurs. Furthermore, an efficient contract has after every history s^t an efficient continuation contract. As both participation constraints are therefore forward-looking, the set of sustainable continuation values depends only on the current state of the world. The challenge therefore is that the optimization is subject to forward looking and occasionally binding constraints (the participation constraints). A tool for solving this model is the promised utility approach. By introducing an additional state variable, promised utility, the planner obtains a policy instrument to solve this problem.[11] To get all efficient contracts, the Pareto frontier and its domain of definition must be known. This depends on the convexity of the set of sustainable allocations and the set of sustainable discounted surplus. It can also be shown that the set of sustainable surpluses is a compact interval $[\underline{U}(s^t), \bar{U}(s^t)]$ for H and for F $[\underline{V}(s^t), \bar{V}(s^t)]$, see Appendix 1.A.12.1. The minimum surplus is $\underline{U}(s^t) = 0$, meaning that a currency union and national currencies yield the same utility.

Next define $V(s^t, U(s^t))$ to be the ex post Pareto frontier which solves the following problem: Maximize F's surplus discounted to period t by choosing a transfer today $T(s^t)$ for state s^t and making state-contingent promises about future utility $U(s^{t+1})$.

11. Marcet and Marimon (2019) sideline the promised utility approach by studying a recursive Lagrangian instead. This provides a straightforward method to compute the solution. As promised utility in the application of this paper has an important interpretation and the set of feasible promised utility is easy to compute, I use this approach.

This problem is subject to giving H at least $U(s^t)$. $U(s^t)$ is promised utility in state s^t that was given by the planner to the country H in the period before. Since the new contract chosen at state s^t must be sustainable, both participation constraints are required to be satisfied for all future states s^{t+1}. Thomas and Worrall (1988) show that the Pareto frontier is decreasing, strictly concave and differentiable on the interval. This will also be the case here. It can also be shown that the constraint $U(s^{t+1}) \leq \bar{U}(s^{t+1})$ is equivalent to $V(s^{t+1}, U(s^{t+1})) \geq \underline{V}(s^{t+1})$. The bounds of the interval and the relationship between V and U are intuitive: When H receives the maximum surplus $\bar{U}(s^{t+1})$ of the union in state s^{t+1} , F must receive the minimum surplus of the union in state s^{t+1}, $\underline{V}(s^{t+1}) = V(s^{t+1}, \bar{U}(s^{t+1}))$. If that is not fulfilled, one could either lower or increase one country's surplus and still have a sustainable contract.

The Pareto frontier is defined by

$$
\begin{aligned}
&V(s^t, U(s^t)) \;=\\
&\max_{T(s^t),(U(s^{t+1}))_{s^{t+1}}^{S}} \ln\left(C^{*U}(s_t)+T(s^t)\right)-\kappa^* l^{*U}(s_t)-v^N(s_t)+\beta\sum_{s^{t+1}}^{S} p(s^{t+1}\mid s^t)V(s^{t+1},U(s^{t+1}))\\
&\text{s.t. } [\lambda(s^t)]\ \ln\left(C^U(s_t)-T(s^t)\right)-\kappa l^U(s_t)-u^N(s_t)+\beta\sum_{s^{t+1}}^{S} p(s^{t+1}|s^t)U(s^{t+1}) \geq U(s^t)\\
&[\beta p(s^{t+1}|s^t)\phi(s^{t+1})]\ U(s^{t+1}) \;\geq\; 0\\
&[\beta p(s^{t+1}|s^t)\zeta(s^{t+1})]\ V(s^{t+1}, U(s^{t+1})) \;\geq\; 0\\
&C(s_t) \;=\; C_H^{\gamma}(s_t)C_F^{1-\gamma}(s_t)\\
&Y_H(s_t) \;=\; C_H(s_t) + C_H^*(s_t)\\
&Y_F(s_t) \;=\; C_F(s_t) + C_F^*(s_t)
\end{aligned}
\tag{1.19}
$$

The first constraint is the promise keeping constraint for H. The Lagrange multiplier $\lambda(s^t)$ is attached to that constraint. As in Marcet and Marimon (2019), $\lambda(s^t)$ can be interpreted as the planner's weight for H. The next two conditions are the participation constraints, they receive the Lagrange multipliers $\beta p(s^{t+1}|s^t)\phi(s^{t+1})$ and $\beta p(s^{t+1}|s^t)\zeta(s^{t+1})$ respectively. Notice the timing of the social planner in this setup: The planner chooses a transfer $T(s^t)$ given the overall history and makes a state contingent plan of continuation values for all states in the next period. I show in the Appendix 1.A.12.2 that the Pareto frontier $V_s(\cdot)$ is concave. Therefore, the following first order conditions are necessary and sufficient:

$$
-\frac{\frac{d}{dT(s^t)}u^{*U}(s_t)}{\frac{d}{dT(s^t)}u^U(s_t)} = \frac{C^U(s_t) - T(s^t)}{C^{*U}(s_t) + T(s^t)} = \lambda(s^t) \tag{1.20}
$$

and

$$
\frac{\lambda(s^t) + \phi(s^{t+1})}{1 + \zeta(s^{t+1})} = -V'(s^{t+1}, U(s^{t+1})). \tag{1.21}
$$

In addition, the envelope condition is

$$\lambda(s^t) = -V'(s^t, U(s^t)). \tag{1.22}$$

The optimal contract is therefore characterized by the evolution of $\lambda(s^t)$ over time. $\lambda(s^t)$, according to (1.22), measures the rate of transformation of the social planner: At which rate can H's surplus be traded ex post against that of F's surplus? The first order conditions also trace out a positively sloped relationship between $U(s^{t+1})$ and actual consumption in H. If promised utility is increased for H, the social planner optimally also increases consumption for the same period[12]. Once the state of nature s^{t+1} in the next period is known, the new value of $\lambda(s^{t+1})$ which equals $V(s^{t+1}, U(s^{t+1}))$ is determined by (1.21). In that case it is important to consider, if the participation constraints bind. As $\lambda(s^t)$ also equals the ratio of marginal utilities of consumption, this pins down the current optimal transfer together with the aggregate resource constraint.

The role of the participation constraints for the allocation of consumption can be illustrated by combining (1.22) and (1.21)

$$\frac{-V_s'(U(s^t)) + \phi(s^{t+1})}{1 + \zeta(s^{t+1})} = -V'(s^{t+1}, U(s^{t+1})). \tag{1.23}$$

There are three regions of interest for state s^{t+1}:

1. Neither participation constraint binds.

No participation constraint binds. This is the case for example when both countries are equally productive. This implies that both Lagrange multipliers are 0 ($\zeta(s^{t+1}) = 0, \phi(s^{t+1}) = 0$):

$$V'(s^t, U(s^{t+1})) = V'(s^{t+1}, U(s^{t+1}))$$

Therefore the country's relative weight for the planner stays the same, $\lambda(s^{t+1}) = \lambda(s^t)$. The intuition is, if no country wants to leave the union, no change in the contract is necessary. The relative weight stays the same, promised utility is unchanged and the ratio of marginal utilities is unchanged as well.

2. F's participation constraint binds.

F wants to leave the union, the participation constraint binds. Therefore $\zeta(s^{t+1}) > 0, \phi(s^{t+1}) = 0$.

$$\frac{-V'(s^t, U(s^t))}{1+\zeta(s^{t+1})} = -V'(s^{t+1}, U(s^{t+1}))$$
$$\Rightarrow -V'(s^t, U(s^t)) > -V'(s^{t+1}, U(s^{t+1})) \Rightarrow U(s^t) > U(s^{t+1}).$$

12. This means that if the social planner becomes active and changes the allocation, she uses both policy instruments to increase utility. Current consumption and promised utility.

Remember that $V'(\cdot) < 0$. If F's participation constraint binds in state s^{t+1}, promised utility $U(s^{t+1})$ for H decreases, compared to the initial promise $U(s^t)$. As a result, H's relative consumption in that period decreases as well. This is done to make F stay in the union, as consumption and expected future utility of F increase to ensure that its participation constraint holds with equality.

3. H's participation constraint binds.

H's participation constraint binds. In that case $\zeta(s^{t+1}) = 0, \phi(s^{t+1}) > 0$ and

$$\begin{aligned} -V'(s^t, U(s^t)) + \phi(s^{t+1}) &= -V'(s^{t+1}, U(s^{t+1})) \\ \Rightarrow -V_s(U(s^t)) < -V(s^{t+1}, U(s^{t+1})) &\Rightarrow U(s^t) < U(s^{t+1}). \end{aligned}$$

Promised utility and relative consumption is increased in state s^{t+1} to make H stay in the monetary union. H's utility level is given by the binding participation constraint. As in Ligon, Thomas, and Worrall (2002), an equation summarizes the dynamics for consumption: There exist state-dependent intervals $[\underline{\lambda}(s^{t+1}), \bar{\lambda}(s^{t+1})]$ $\forall s^{t+1} \in S$, such that $\lambda(s^t)$ evolves according to the following rule: Let s^t be given and s^{t+1} be the state at time $t+1$, then

$$\lambda(s^{t+1}) \begin{cases} = \underline{\lambda}(s^{t+1}) & \text{if } \lambda(s^t) < \underline{\lambda}(s^{t+1}) \\ = \lambda(s^t) & \text{if } \lambda(s^t) \in [\underline{\lambda}(s^{t+1}), \bar{\lambda}(s^{t+1})] \\ = \bar{\lambda}(s^{t+1}) & \text{if } \lambda(s^t) > \bar{\lambda}(s^{t+1}). \end{cases} \tag{1.24}$$

where $\underline{\lambda}(s^{t+1}) \equiv -V'(\underline{U}(s^{t+1}))$ and $\bar{\lambda}(s^{t+1}) \equiv -V'(\bar{U}(s^{t+1}))$ are the endpoints of the equation, indicating whether the participation constraints bind, if the old contract $\lambda(s^t)$ is still in place.

The intuition behind the evolution for $\lambda(s^t)$ is the following: An optimal contract requires that the ratios of marginal utilities of both countries stay constant over time, whenever possible. Transfers are therefore chosen such that the old ratio $\lambda(s^t)$ is the same as the new ratio $\lambda(s^{t+1})$ if all constraints are satisfied. Whenever one of the participation constraints is violated for a certain state and for a given old contract, a new contract is put into place, that engineers the minimum change necessary in marginal utilities to satisfy both participation constraints. That is, put the country that wants to leave the union at its participation constraint by choosing the appropriate transfer. This new contract with its transfer system and its marginal utility ratio is in place as long as possible but will change again when one country is at is participation constraint. In the context of the two-country model, the evolution of $\lambda(s^t)$ has a remarkable feature: As long as no new participation constraint binds transfers in % of GDP are constant over time.[13] This provides a simple rule that helps to sustain the monetary union.

13. See the Appendix 1.A.10 for the proof.

Furthermore, there will be effects on output, employment and prices, as transfers shift consumption from one country to another. These general equilibrium effects are present, because countries have a preference for domestic goods due to their Home bias[14]. For a further discussion of these effects, see section 1.A.10 in the Appendix. Starting with a certain state s^0, the Pareto frontier [15] can be traced out by letting the initial value $\lambda(s^0)$ vary between the minimum value $\underline{\lambda}(s^0)$ and maximum value $\bar{\lambda}(s^0)$. These contracts correspond to transfers that are chosen in such a way, that the gain of a currency union compared to national currencies is zero or its maximum possible value. A natural starting point are zero transfers with an equal gain split between both countries in the benchmark simulation.

I now outline the algorithm that solves for transfers and the overall allocation in the economy. Given the process for productivity a_t and a_t^*, a sequence of shocks $a(s^t)$ is simulated. Consumption and labor in a currency union and outside a currency union are then computed according to (1.16) and (1.14). Starting with zero transfers, the gain (1.17) is computed and the algorithm checks for which t any of the gains are negative. The algorithm computes the set of feasible promised utilities and in the first period when the participation constraint binds for one country, transfers and the promise for that country are chosen such that the gain is set to zero. The Pareto weight is set to the corresponding endpoint of the state. Future utility (the promise) is explicitly written in state contingent form that include future transfers. These future transfers obey (1.20) and (1.24). The condition is, that marginal rate of transformation of the social planner stays the same in all states, except for the other asymmetric state when $\lambda(s^t)$ is inversed. $\lambda(s^t)$ is updated in the period with negative gains. With that, the new ratio of marginal rates of utility is computed that includes transfers, obeying (1.24), as long as the next country has a positive gain. The promise keeping constraint is checked for all new transfer schemes. The updated lambda is then used, to compute new gains from that moment onward. As soon as another participation constraint binds, the algorithm computes a new $\lambda(s^t)$ as before and updates the allocation.

1.3.3 Union-wide Central Bank with Exit Option

Now consider the setup as before. Both countries can exit in every period and a social planner maximizes union-wide welfare, taking the lack of commitment of both member states into account. The only difference is that the planner uses monetary policy $\mu(s^t)$ as an instrument instead of transfers. $\mu(s^t)$ summarizes the history of monetary policy until now, if today's state is s. It reflects the path of interest rate

14. This goes back to an old debate between Keynes and Ohlin in 1929, the so-called Transfer debate. Back then the debate centered around transfers (debt repayments) of Germany to the Allied nations after its defeat in World War I and the general equilibrium effects of these transfers.

15. See Figure1.A.1

that the central bank chooses. The central bank chooses the monetary stance today and promises future utility:[16]

$$
\begin{aligned}
V_s(U(s^t)) = \max_{\mu(s^t),(U(s^{t+1}))^S_{s^{t+1}}} & \ln\left(C^{*U}(\mu(s^t))\right) - \kappa^* l^{*U} - v^N(s_t) + \beta \sum_{s^{t+1}}^{S} p(s^{t+1}|s^t) V(s^{t+1}, U(s^{t+1})) \\
\text{s.t. } [\lambda(s^t)] \ & \ln\left(C^{U}(\mu(s^t))\right) - \kappa l^{U} - u^N(s_t) + \beta \sum_{s^{t+1}}^{S} p(s^{t+1}|s^t) U(s^{t+1}) \geq U(s^t) \\
[\beta p(s^{t+1} \mid s^t)\phi(s^{t+1})] \ & U(s^{t+1}) \geq 0 \\
[\beta p(s^{t+1} \mid s^t)\zeta(s^{t+1})] \ & V(s^{t+1}, U(s^{t+1})) \geq 0 \\
& C(s_t) = C_H^{\gamma}(s_t) C_F^{1-\gamma}(s_t) \\
& l(\mu(s^t))a(s_t) = C_H(\mu(s^t)) + C_H^*(\mu(s^t)) \\
& l(\mu(s^t))^* a(s_t) = C_F(\mu(s^t)) + C_F^*(\mu(s^t))
\end{aligned}
\tag{1.25}
$$

The first order conditions with respect to promised utility $U(s^{t+1})$ are the same, the only difference is the first order condition with respect to the policy instrument $\mu(s^t)$:

$$
\begin{aligned}
&-\left[\frac{1}{\mu(s^t)} - \frac{(1-\gamma)a^{-1}(s_t)}{\sum_{s^t}^{S} p(s^t \mid s^{t-1}) a^{-1}(s_t)\mu(s^t)} - \frac{\gamma a^{*-1}(s_t)}{\sum_{s^t}^{S} p(s^t \mid s^{t-1}) a^{*-1}(s^t)\mu(s^t)}\right] \cdot \\
&\left[\frac{1}{\mu(s^t)} - \frac{\gamma a^{-1}(s_t)}{\sum_{r=1}^{S} p(s^t \mid s^{t-1}) a^{-1}(s_t)\mu(s^t)} - \frac{(1-\gamma)a^{*-1}(s_t)}{\sum_{s^t}^{S} p(s^t \mid s^{t-1}) a^{*-1}(s_t)\mu(s^t)}\right]^{-1} = \lambda(s^t)
\end{aligned}
$$

Writing in terms of marginal costs of both countries:

$$
1 = \left(\frac{1-\gamma+\lambda(s^t)\gamma}{1+\lambda(s^t)} \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{\gamma+\lambda(s^t)(1-\gamma)}{1+\lambda(s^t)} \frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]}\right)^{-1} \tag{1.26}
$$

The new monetary stance features *time-varying* Pareto weights $\lambda(s^t)$. This means that compared to the initial stance without exit option (1.15), the central bank stabilizes a time-varying weighted average of marginal costs of both countries. Monetary policy in both regimes is exactly the same if $\lambda(s^t) = \frac{1-\xi}{\xi}$. Hence, the central bank can stick to the structure of the old monetary rule but announce that the effective weights for both countries become state-dependent. If the participation constraint of one country is binding, the central bank puts more weight on stabilizing marginal costs of the crisis country today and promises to do that in the future as well. The

16. Monetary policy under commitment implies that only consumption is targeted, but not employment. I adopt the same notion here.

mechanism at work is the following. Imagine the Home country is in a severe recession and wants to leave the union. As hours worked are too high, H wants to use national currencies to increase interest rates. A union-wide central bank recognizes that the participation constraint of H binds and increases the effective weight $\lambda(s^t)$ for H. This implies that the central bank favors H when setting interest rates. Union-wide interest rates are higher than they would be without the threat of a break-up. This way the central bank stabilizes marginal costs of the home country more and in turn puts the employment level of home closer to optimum. In addition, the central bank announces to conduct monetary policy in favor of H in the future. The Pareto weight $\lambda(s^t)$ has persistently changed and the gains of H are exactly zero during the crisis period, such that it does not leave the union. $\lambda(s^t)$ stays the same, until another participation constraint will bind.

Are there limitations for the central bank to redistribute resources between countries with interest rate setting? Yes, there is for example no way the central bank can put more weight on one country than on another in a symmetric current state of the economy. The best that monetary policy can do in such a situation is to close output and employment gaps of both countries. Only when there are asymmetric states, the central bank can alter the weights to favor a specific country. Therefore, the ability to make credible promises about future utility is limited for the central bank. The paper considers an example, in which transfers can sustain the union, while interest rate setting alone cannot.

1.3.4 Union-wide Central Bank and Transfers with Exit Option

Here I consider a joint response of both, monetary policy and fiscal policy. In the period in which the participation constraint of one country is binding, given the policy in place from the past, the central bank re-calibrates the weight of the country only for this period. In a next step, the fiscal authority, taking the new monetary policy today into account, sets fiscal transfers as in section 1.3.2 and tries to sustain the union. I will consider two possibilities for the central bank. The first features an increase of the weight for the crisis country to one. The second option includes a drop in the weight for the crisis country to zero, this coincides with an increased economic activity for the whole union. In the experiment I will check if any of these two options increases the survival rate of the currency union, compared to other policy interventions.

1.4 Calibration

The section calibrates the model. The model seeks to highlight conditions under which a currency union such as the eurozone can break up. Towards that aim, I focus on two large members of the eurozone, namely Germany and Italy. The choice of Italy and Germany as our countries of interest has a reason: Both are the largest

countries of their respective block: Germany being part of the so-called core (or the northern) block in the currency union, where the economy in the last twenty years expanded significantly. And Italy as the largest country of the so-called periphery (or the southern block) that experienced large economic downturns. I will use data from these two countries to calibrate trade openness and real interest rates. One period in the model taken to be a year.
Other parameters are calibrated based on the outside literature. Furthermore, a range of trade costs parameters will be considered, implying different amounts of gains coming from the union.

1.4.1 Calibration of Preferences and Technology

Both Home and Foreign are assumed to be symmetric in their parameters. The discount factor β of the representative household is set to 0.98 to match a yearly real interest rate of about 2 % in line with Brand, Bielecki, and Penalver (2018) for the eurozone. The Home bias parameter γ is set to 0.75 which is in line with Italy's trade openness in 2015 measured as imports relative to GDP[17]. The elasticity of substitution between domestic goods is set to 6 as in Galí (2008) implying a markup of 20%. I have made the following implicit assumptions by choosing preferences as in equation (1.1). The intertemporal consumption elasticity is set to 1 so that consumption utility is log. As in Corsetti and Pesenti (2002), labor is just linear implying an infinite Frisch elasticity of labor supply, such that household satisfy labor demand. κ is set to $8/3$ so that household spend one third of their time with labor. The trade elasticity of substitution between Home and foreign goods is set to 1, so that the consumption aggregator in (1.2) is Cobb Douglas. Together with the assumption of log consumption, this implies that the current account is always balanced which is numerically convenient, see also section 1.A.8. A trade elasticity of 1 is at the lower end of available estimates surveyed by Head and Mayer (2014). Estimates vary widely. Lower values of the trade elasticities are in most cases related to measurements of short-run elasticities. Low values of trade elasticity imply for the model, that a reduction in trade costs has a smaller effect on the trade volume[18].

Table 1.1. Calibration

Symbol	Value	Description	Target
β	0.98	Time discount rate	Real rate of 2% p.a.
γ	0.75	Home bias for each country	Trade openness Italy 2015
$\theta, (\theta^*)$	6	Elasticity of subst. of Home goods	Galí (2008)

17. According to Eurostat imports relative to GDP in 2015 for Italy is 26.7%.
18. As discussed below, I calibrate the trade costs in such a way, that bilateral trade increases between 3% and 10 %.

Next, I discuss choices for parameters that are central for the motives of forming a currency union. They are summarized in table 2. The calibration of ϖ is crucial, as it determines the trade gains from a currency union. If gains are very large, a currency union would always be formed and would never break up. Instead, if the gains are low, the monetary union becomes fragile. For example, Micco, Stein, and Ordonez (2003) find that bilateral trade increases by around 4-16% if a common currency is adopted. This is a bit higher than estimates by Baldwin et al. (2008) (5%), but much lower than Rose (2000). Therefore, the paper considers several specifications with large, medium and small trade gains that are in line with the wide range of estimates that the literature finds.

Table 1.2. Calibration of union trade gains

Symbol	Large gains	Medium gains	Small gains	Description
ϖ	0.1	0.066	0.05	Transportation costs.
ξ	0.5	0.5	0.5	Weight of H
τ	-0.2	-0.2	-0.2	Subsidy, no markup.

As gains come from trade costs reduction only, benefits of a currency union are symmetric between both countries. In the "large gains" scenario I set ϖ in such a way that with national currencies 10% of all exported goods are lost. The elimination of trade costs generates an increase of bilateral trade of 10 % in good times. Taken the productivity process into account, the currency union would never break up. Gains are so large that no country would voluntarily leave the union
Consider the "medium gains" calibration. Given the same productivity process, trade costs reduction ϖ is 6.5%. In this specification with lower trade gains, the currency union can actually break up when the biggest possible asymmetric shock emerges, see section 1.5.
Last I will discuss low trade gains of 5% in line with estimates from Baldwin et al. (2008). The union is more likely to break up in that specification, as governments decide to leave the union also in those states in which relatively small asymmetric shocks occur.

1.5 Model Experiment and Results

I want to capture, how each of the planners in section 1.3 fares when productivity fluctuates stochastically over time. For this purpose I run a simulation of the model and compare the outcome of each planner in the simulation. This table reminds of the policy instruments used by each planner

Table 1.3. Planners and which policy instruments is used to prevent a breakup

Planner Allocation	Transfers are used	Interest rates are used
National Planner	-	-
Union-wide Ramsey Planner	✓	-
Union-wide Central Bank	-	✓
Transfers & Mon Pol	✓	✓

Consider the model with overall 25 possible states, in which each country can have 5 different productivity values: $\mathscr{A} = \{a^{bb}, a^{b}, a^{n}, a^{r}, a^{rr}\}$, where $a^{bb} = 1.04$ indicates a big boom with very high productivity. It indicates GDP growth of 4%. $a^{b} = 1.02$ is a normal boom with higher productivity, $a^{n} = 1$ a neutral state and a^{r}, a^{rr} indicate recessions of equal size as the boom. In that setup, there are 5 symmetric states, in which both countries have the same productivity, 20 are asymmetric. I assume that productivity is independent between countries and over time. The probability for each country to have productivity $\mathscr{A}$ is $Prob = \{0.15, 0.4, 0.25, 0.15, 0.05\}$. The first entry corresponds to the probability to go get productivity $a^{bb} = 1.04$. Each simulation has 100 periods. Overall, I run 2500 simulations. In a next step, I use the baseline calibration discussed in section 1.4 for the simulation. In addition to that, I use different amount of gains from trade for the simulations.

1.5.1 Trade Gains 6.5%

The simulation is used to compute average consumption and employment with trade gains of 6.5%. This is done by considering the pure allocations of a national currency (1.14), a currency union (1.16) and the first best allocation (1.A.1). No planner intervention is considered yet.

Table 1.4. Allocation under different regimes, trade costs reduction of 6.5%

Planner Allocation	Average Consumption	Average labor
National Currency	14.144	24.61
Currency Union	14.3703 (+1.6%)	25 (+1.57%)
First Best	14.3723 (+1.61%)	25 (+1.57%)

With trade costs reduced by 6.5%, consumption of both countries increases by around 1.6% in a currency union. I take one simulation out as an example. Consider how productivity evolves over time, starting from a point in which both countries are in a boom:

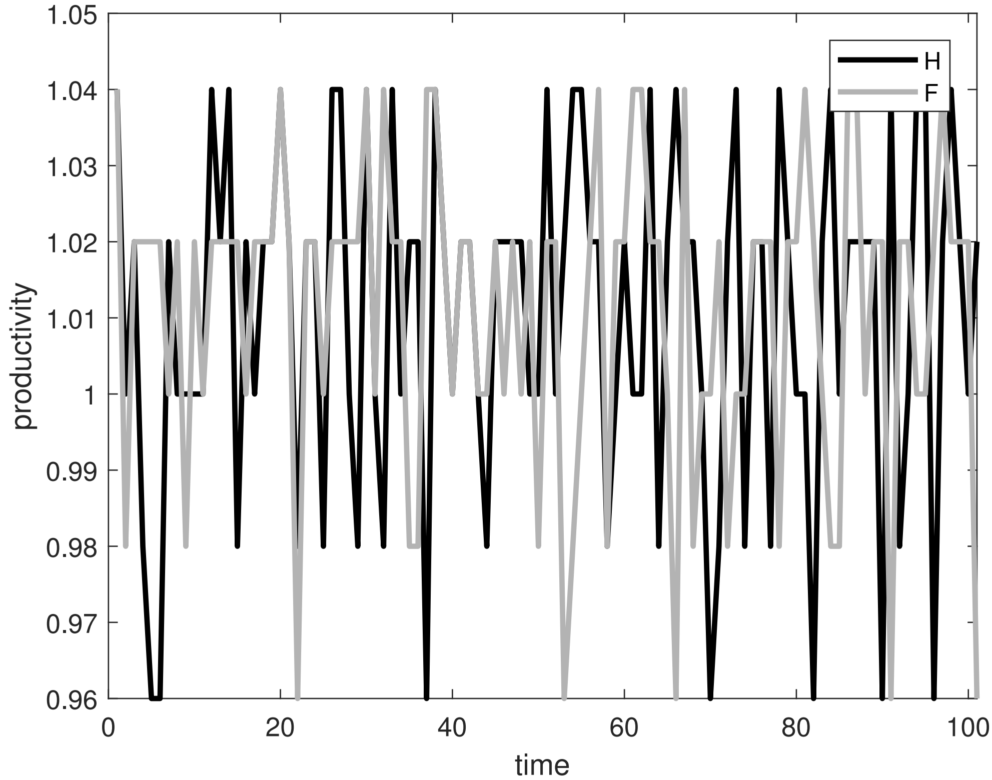


Figure 1.1. Productivity of both countries over time.

Given the evolution of productivity, I compute consumption and employment over time. The gains from the currency union in (1.18) are then computed. This allows us to check if the participation constraint of the union holds in this specific simulation.

National Planner:

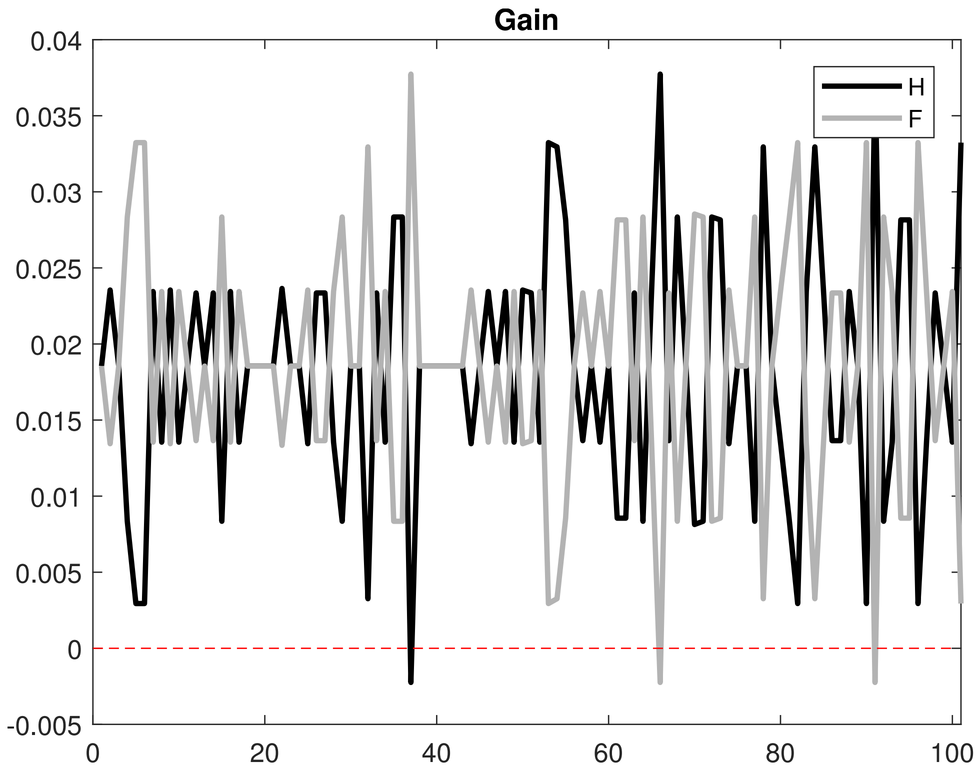


Figure 1.2. Gain of both countries over time.

The values for H and F fluctuate around 0.2. Both countries' gains are exactly identical when productivity is the same. If there are asymmetric shocks, the recession country's gain goes down while the boom country's gain goes up. In that example, only the biggest possible asymmetric state can endanger the currency union. For trade gains larger than 6.7% gains are always positive and the union would never collapse. For the specification in this simulation, gains turn negative in period 36. At that point in time, there is a huge asymmetric shock with the Home country being in a deep recession, while the Foreign country is in a big boom. The gain of the Home country is negative and the government of that country wants to leave the currency union. The next point in time, when gains turn negative is in period 65. Then, the Foreign country's gain is negative and its government wants to leave the union. As discussed in 1.3.1, the union breaks up as soon as the first gain turns negative. Both countries have zero gain from that moment onward.

Transfers: A union-wide Ramsey planner with transfers between countries, as in section 1.3.2 sets transfers in the following way to prevent that break-up:

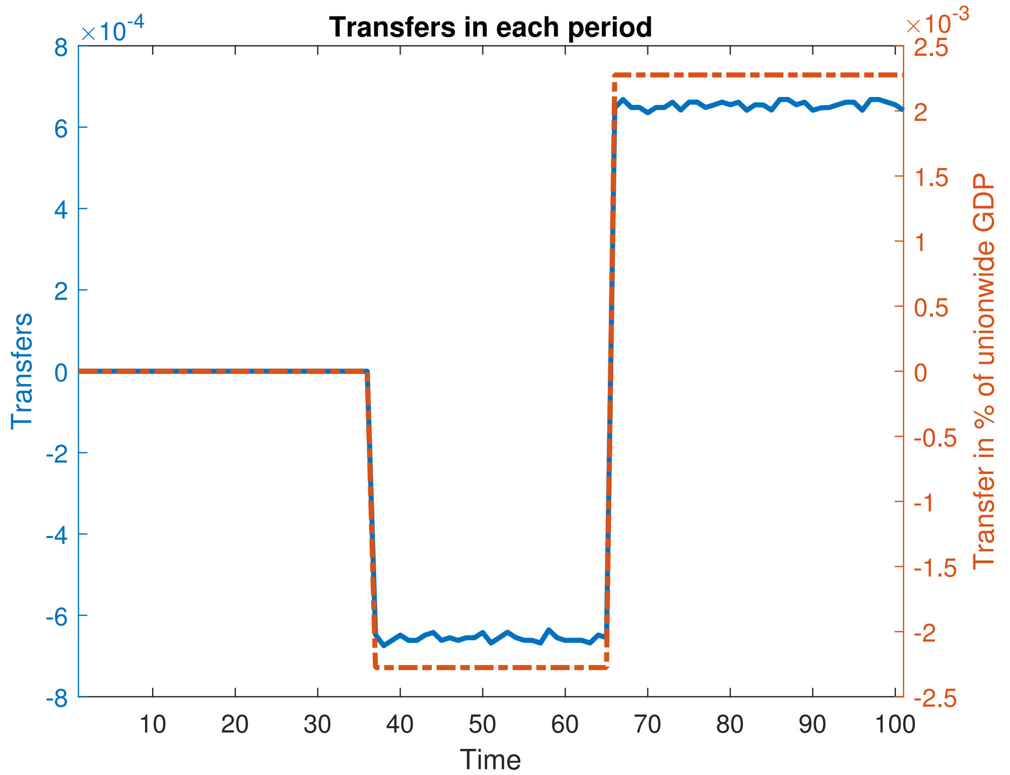


Figure 1.3. Transfers by the Ramsey planner over time. The solid blue solid line are real transfers in terms of consumption units (scale on the left axis), while the red dashed line are transfers in percent of union-wide GDP (scale on the right axis).

When the huge asymmetric shock emerges in period 36, the Ramsey planner gives transfers to the recession country H, T_t is negative. Furthermore she makes a promise that a constant fraction of union-wide GDP is redistributed to the Home country in every period, until another participation constraint binds again. This happens in period 66, as the Foreign country enters a severe recession and the Home country experience a strong boom. Transfers turn positive in that period to prevent the Foreign country to exit. The transfer scheme reverses. In this example, a transfer of 0.0024% of union-wide GDP every period sustains the union. As in Ferrari, Marimon, and Simpson-Bell (2020), the relative amount stays constant whenever possible, reflecting a persistent increase in the Pareto-weight. As there are also aggregate fluctuations in my model, the absolute amount of transfers (solid blue line) varies over time together with the economy.

Central Bank: If fiscal transfers between countries are not feasible, can a union-wide central bank with interest rate setting, as in section 1.3.3 prevent a break-up? In this example, the answer is yes. In period 36, the central bank alters its monetary stance to favor the Home country. The central bank does not only favor the Home country in the crisis period, but also in the future. The following figure illustrates the new behavior of interest rates around the first crisis in period 36.

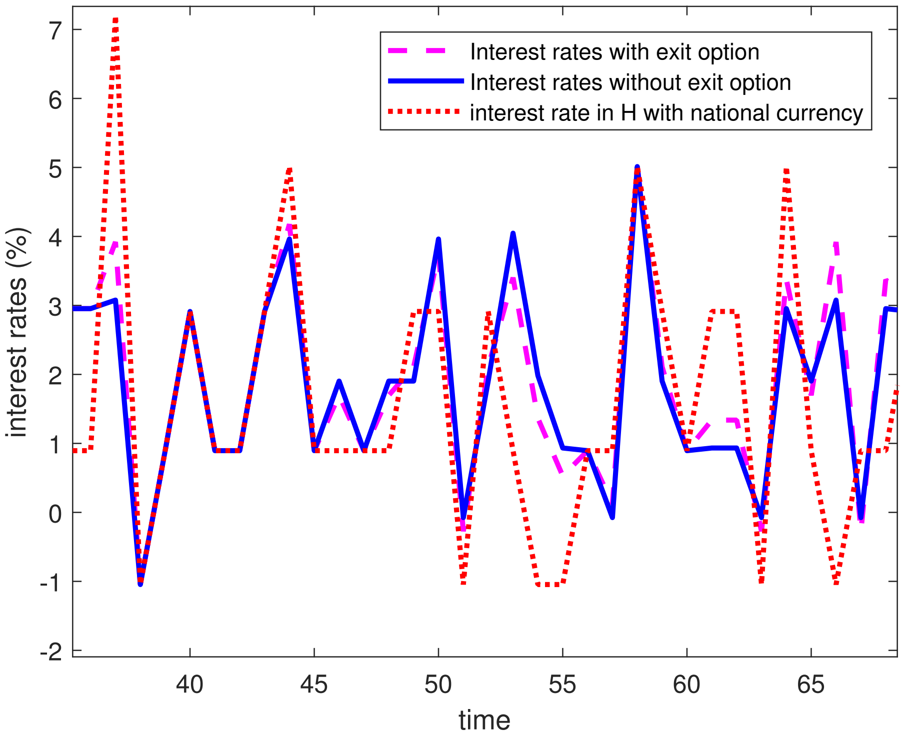


Figure 1.4. Interest rates over time in different regimes.

The new interest rate (dashed magenta line) is closer to what interest rates would be for H with national currencies (the dotted red line). Remember that in a currency union without exit options (the solid blue line) both countries have an equal weight in the central bank's objective function. When there are exit options, the weight in the objective function becomes time-varying and state-dependent to take care of the participation constraints. The big asymmetric shock makes H's participation constraint binding, which leads to a persistently higher weight of H in the central bank's objective function. By increasing H's weight, the central bank makes sure that H stays in the currency union. The higher weight of H persists until F's participation constraint binds. Therefore, in the crisis period 36 interest rates are closer to what H wants with national currencies: With very low productivity in H, the central bank increases interest rates to lower aggregate demand, which is in H's favor. In period 36, interest rates are at 4% rather than at 3% as they would normally be. In period 37 to 42 all interest rates align, as productivity is the same in both countries. This means that the union-wide central bank has no room to set interest rates in one country's favor, as both want exactly the same interest rate. In other asymmetric states after the big shock in 36, the central bank systematically sets interest rates in favor of the Home country. The Pareto weight changes in period 66, when F hits its participation constraint. From that moment on, the central bank favors F in its policy stance. This example highlights the conditions necessary for the central bank to succeed to sustain the union: There have to be sufficiently many 'small' asymmetric shocks, that the central bank can use to favor a country without endangering the union. If there are no such states with small asymmetric shocks, the central bank cannot credibly promise to give the crisis country more utility in the future. In this example, there are sufficiently many small asymmetric shocks that are also likely to occur.

The following graph illustrates this point. I plot interest rates in a union over time in this simulation, together with the set of all possible interest rates that would favor one or another country. The set of possible interest rates is computed by considering all possible weights $\xi \in [0, 1]$ in the central bank's monetary stance (1.15). In the extreme, the central bank puts full weight on H or F respectively. The weights are reflected in consumption (1.16) and with the Euler equation (1.6) the set of all possible interest rates are computed.

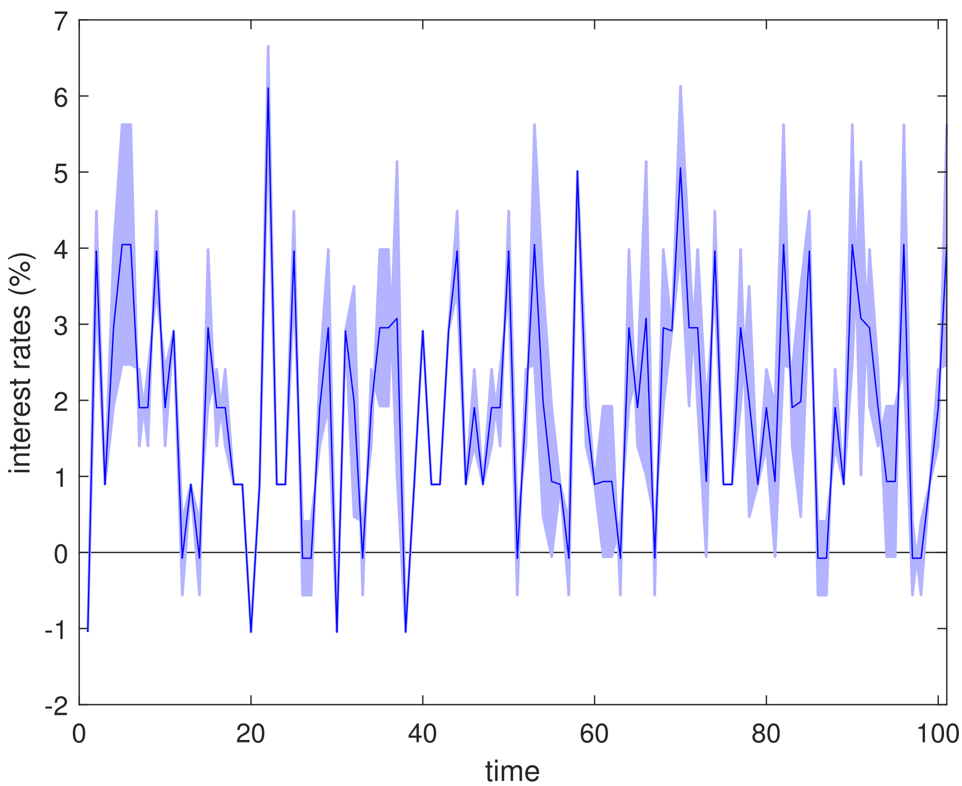


Figure 1.5. Interest rates over time. The shaded area illustrates the range of interest rates that a union-wide central bank can optimally implement, when putting full weight on H or F respectively. The solid blue line indicates interest rates with equal weight.

There are periods, in which the set of possible interest rates is just one point. In these periods, productivity is the same for both countries, implying that both want the same interest rates. This leaves no room for the central bank to favor a country.
Central Bank and Transfers: In the three periods 36, 66 and 91 with huge asymmetric shocks the central bank puts full weight on the countries that want to leave the union. Emphasizing stabilization of crisis countries during an asymmetric shock alters interest rates in those periods and reduces the amount of necessary transfers to sustain the union from 0.0025% to 0.0017%, see figures 1.A.4 and 1.A.5.
Summary: Turn to the statistics that describe the likelihood of a break-up for different planner intervention. Given the productivity process in the simulation, the currency union experiences in 1.5 % of the time a huge asymmetric shock that endangers the union. In 79.8% of all 2500 simulations, such a shock actually occurs within the first 100 periods and the currency union breaks up if national planners are in charge. The average break-up period is 81.8. The last column summarizes average gains in this simulation. With a national planner, the average gain is lowest as the currency union breaks up relatively often in the simulation. All other planners are able to increase the average gain substantially, as they succeed to sustain the union and the trade gains. Monetary policy fairs slightly worse than other interventions, as interest rates are a distortionary policy instrument. The following table summarizes these results.

Table 1.5. Break-up under different planners, trade costs 6.5%

Planner Allocation	Prob. of a union break-up next period	Average break-up period	Prob. of a break-up within 100 periods	Average Gain
National	1.5 %	81.8	79.8%	0.150
Fiscal	0 %	-	0%	0.189
Monetary	0 %	-	0%	0.188
Fiscal & Mon	0 %	-	0%	0.189
First Best	0 %	-	0%	0.189

Overall, a union-wide transfer scheme always succeeds to sustain the currency union, as does a common central bank.
In a next step, I consider lower trade gains coming from a currency union.

1.5.2 Trade Gains 5%

This section discusses how lower trade gains affect the effectiveness of policy instruments that aim to sustain the union. First I consider a specific example with lower trade gains, then I show for which ranges of gains in a currency which policy works. The following table summarizes the effect of a trade costs reduction of 5%:

Table 1.6. Allocation under different regimes, trade costs reduction of 5%

Regime	Average Consumption	Average labor
National Currency	14.1957	24.6984
Currency Union	14.3705 (+1.23%)	25 (+1.22%)
First Best	14.3725 (+1.24%)	25 (+1.22%)

A 5 % decrease in trade costs in a currency union increases consumption in the simulation by around 1.23 %, employment by around 1.22%. The starting point of the simulation is again a strong boom for both countries (a^{bb}). Consider a random simulation that I have taken out as an example. As before I consider first the outcome of the experiment of the national planner, then the Ramsey planner with transfers, then the union-wide central bank and then the joint intervention.

National Planner First I plot the evolution of gains, as in (1.18) to check in which point in time a national planner decides to leave the currency union. As productivity diverges, so do gains. The Foreign country experiences a recession and its gains from the currency union go down. They turn negative in period 8, when an asymmetric shock emerges. Afterwards, each countries' gains get closer to each other, as productivity of both countries aligns again. The union would collapse in period 8 and both countries receive zero gains from that moment onward.

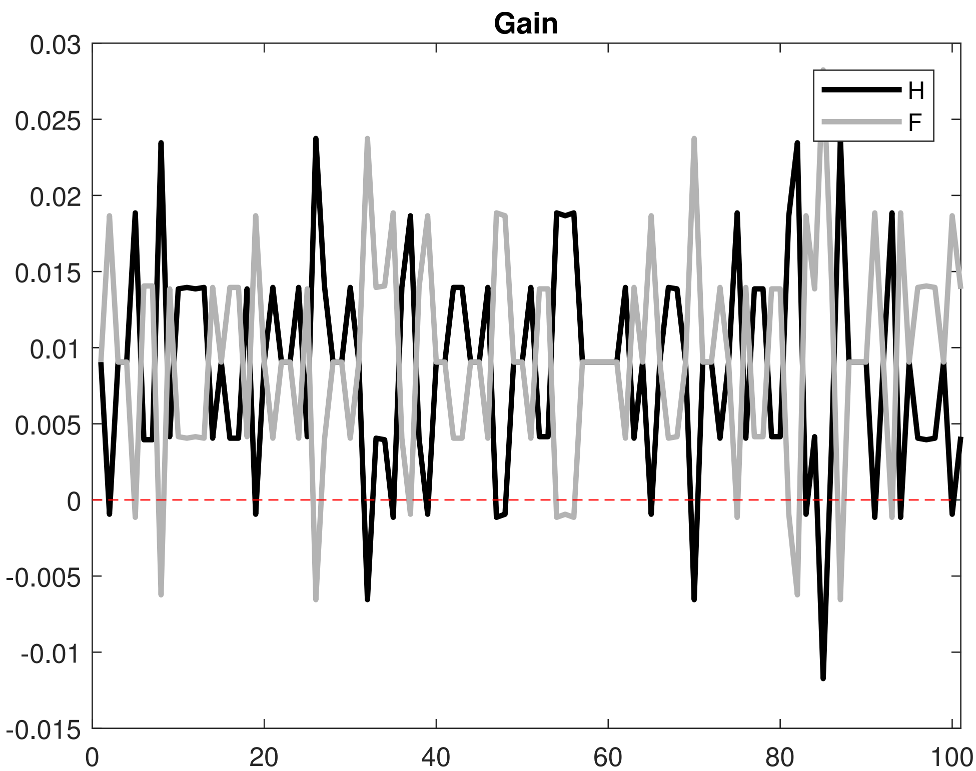


Figure 1.6. Gains over time .

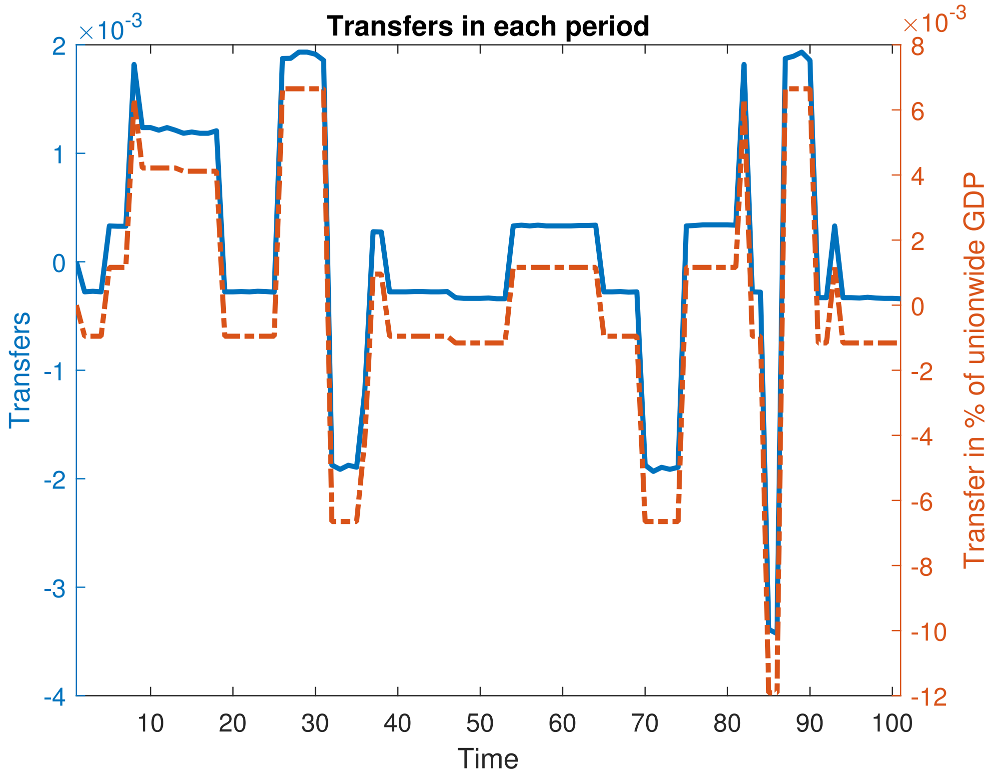


Figure 1.7. Transfers over time. The solid blue solid line are real transfers in terms of consumption units (scale on the left axis), while the red dashed line are transfers in percent of union-wide GDP (scale on the right axis).

Transfers A union-wide social planner with transfers, as in 1.3.2 sets transfers as in Figure 1.7. Compared to figure 1.3, transfers fluctuate stronger as the participation constraints of both countries are hit more frequently. In addition to that, transfers have to be changed as well if one country would leave the union, because the current transfer scheme puts it into a disadvantage. This is true for example in period 15. Still transfers can always sustain the union by ensuring that gains are not negative.

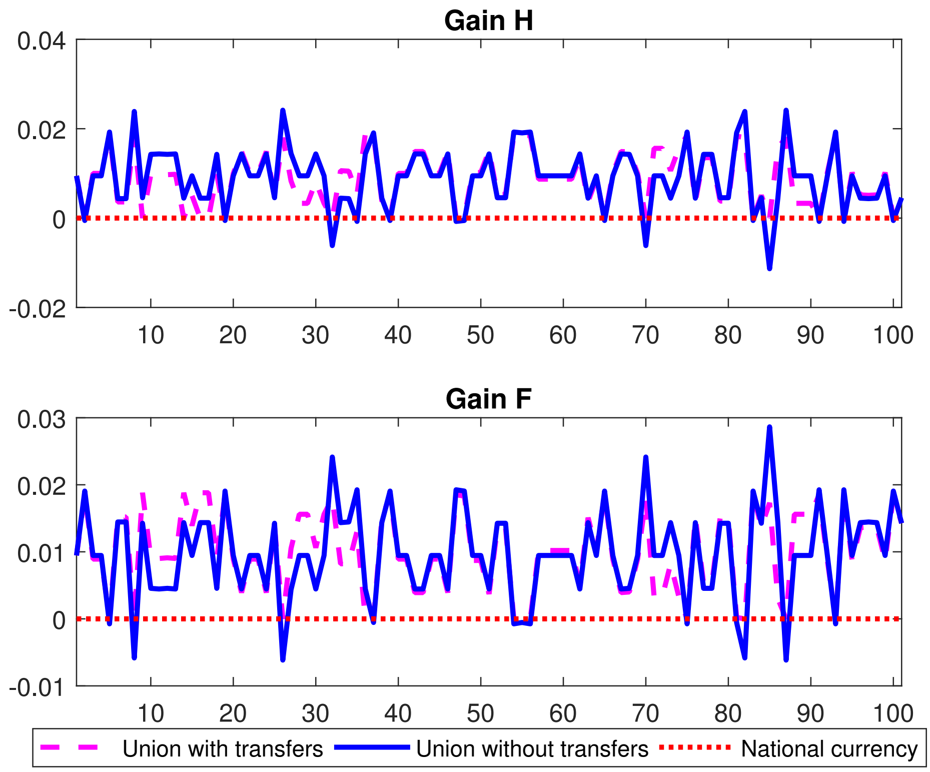


Figure 1.8. Gains over time under different policy regimes.

Central Bank: Can the central bank in this simulation sustain the union? Only for some time. The following graph zooms into the first 10 periods of the simulation and illustrate this point. The first two asymmetric shocks that would destroy the union under national planners, can be addressed with interest rate setting by the central bank. First in period 2, there is an asymmetric shock that makes H want to leave the Union and in period 5, in which F wants to leave the Union. In both cases, the central bank steps in by accommodating the corresponding crisis country during the crisis period and afterwards. In period 8 however, F is hit again by an asymmetric shock, but this time the shock is so large that the central bank cannot sustain the currency union, even if she puts full weight on F. Despite the central bank's best effort to keep F in the union, the gain is still negative and the government decides to leave the union. In this simulation, the central bank is able to extend the survival of the currency union for 6 periods, but not to permanently sustain it.

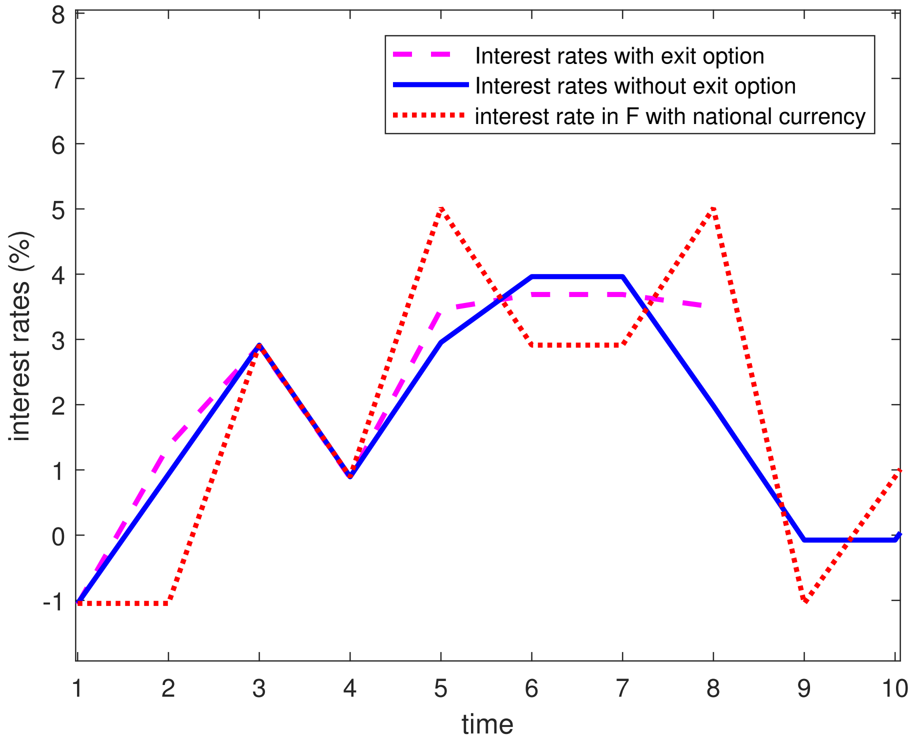


Figure 1.9. Interest rates over time under different policy regimes, trade gains are 5%.

Transfers and Central Bank: A central bank that puts a full focus on stabilizing the crisis country in the crisis periods reduces the amount of transfers necessary only by a very small margin, see Figure 1.A.9.
Summary: Overall, a union-wide transfer scheme always succeeds to sustain the currency union in the benchmark simulation, while a common central bank fails to achieve that. What a central bank can do is to address the threat of a break-up in some states. This reduces the probability of a shock that destroys the currency union in the next period from 32 % to 10%. With a common central bank that tries to prevent a break-up the average duration of a currency union is increased from 2.7 years to 8.4 years.

Table 1.7. Break-up under different planners, trade costs 5%

Planner Allocation	Prob. of a union break-up next period	Average break-up period	Prob. of a break-up within 100 periods	Average Gain
National	32 %	2.7	100%	0.0002
Fiscal	0 %	-	0%	0.0076
Monetary	10 %	8.4	100%	0.0007
Fiscal & Mon	0 %	-	0%	0.0076
First Best	0 %	-	0%	0.0091

The next table summarizes the policy options that manage to sustain the union, depending on how large trade gains are.

Table 1.8. Break-up under trade gains

Trade Gains	Probability of dangerous shocks	Transfers can always sustain the union	Central bank can sustain the union
$> 6.7\%$	0%	yes	yes
$[6.6\%, 6.4\%]$	1.5 %	yes	yes
$[6.3\%, 3.3\%]$	$[1.5\%, 73\%]$	yes	no
$3.3\% >$	$73\% >$	no	no

If trade gains are larger than 6.7% no country would ever decide to leave the union, no policy interventions are necessary and therefore the union is sustained forever. For trade gains between 6.6% and 6.4% there is a possibility that the union breaks up if the biggest possible asymmetric shock hits the union. Both, fiscal and monetary policy succeed in sustaining the union. If trade gains are lower than 6.4% monetary policy will not always sustain the union, as the costs of stabilization in the union are too large when a big asymmetric shock hits the union. The gains of the union cannot be sufficiently redistributed with interest rate setting alone. Transfers however always manage to sustain the union, up to trade gains to 3.3%. If trade gains are lower than this, even transfers between countries cannot sustain the union. A joint fiscal and monetary intervention does not increase the survival rate of a currency union, independent if the central bank puts full weight on crisis countries in the crisis period or induces an economic boom in the currency union to increase the available amount of fiscal transfers between countries.

1.6 Conclusion

This paper shows how a currency union can be sustained with fiscal and monetary policies when member states have an exit option. If there is a big asymmetric shock, trade gains in a union are outweighed by less effective monetary policy. The recession country is severely affected, as gaps in the level of employment are more hurtful in a recession than in a boom. Therefore, the recession country exits in a severe crisis and the union collapses. The paper discusses, how the currency union can be sustained via fiscal or monetary policies. The first option is a fiscal intervention by a union-wide Ramsey planner: A simple and credible transfer rule gives the crisis country a constant fraction of union-wide GDP over time. This is enough to prevent a breakup of the union. These transfers are in place as long as the other country of the currency union is not in a crisis. If a crisis happens and the other country wants to exit, the rule is reversed and the new crisis country gets transfers. In the benchmark simulation of the model, the currency union can always be sustained with transfers. Both countries are better off ex post and ex ante compared to a situation when no policies are in place that sustain the currency union. The second option that the paper considers is monetary policy. If there are no fiscal transfers, the central bank can take the lack of commitment from the countries into account. In normal times, the central bank stabilizes a weighted average of the economy from both countries. The weights depend on the size of the economy and on the Pareto weights for the country. This paper derives that these weights become state-dependent when there are participation constraints: As soon as one country hits the participation constraint, the weight of that country increases and the central bank systematically favors the crisis country in its policy. The greater weight persists, until another participation constraint binds. In some situations, the central bank can sustain the union with that policy, but not in all. The central bank needs sufficiently many small asymmetric shocks in the future that can be used to favor a specific country. In addition to that, large trade gains from the currency union are needed. If this is not the case, the central bank has not enough room to favor one country and fails to sustain the union. A joint intervention of the union-wide central bank and fiscal transfers does not increase the survival rate of a currency union compared to a situation when only transfers are used.

Appendix 1.A Appendix

1.A.1 Derivations

1.A.1.1 Allocation of the Social Planner

An interesting benchmark allocation for the model of the economy is the allocation of the social planner. I assume that the social planner can freely allocate labor and consumption and faces no trade costs. She maximizes welfare of all agents subject to the resource constraints of the economy:

$$\begin{aligned}
\max_{\{C_t, L_t, C_t^*, L_t^*\}} \quad & \mathbb{E}_t\Big[\sum_{\tau=t}^{\infty}\beta^{\tau}\Big(\ln(C_\tau)-\kappa L_\tau\Big)\Big]+\mathbb{E}_t\Big[\sum_{\tau=t}^{\infty}\beta^{\tau}\Big(\ln(C_\tau^*)-\kappa^* L_\tau^*\Big)\Big] \\
s.t. \quad Y_t(h) &= L_t(h)a_t = \underbrace{\int_0^1 C_t(h,j)dj}_{C_t(h)} + \underbrace{\int_0^1 C_t^*(h,j^*)dj^*}_{C_t^*(h)} \\
Y_t(f) &= L_t(f)a_t = \underbrace{\int_0^1 C_t(f,j)dj}_{C_t(f)} + \underbrace{\int_0^1 C_t^*(f,j^*)dj^*}_{C_t^*(f)}
\end{aligned}$$

This problem can be written as:

$$\begin{aligned}
\max_{\{C_t, L_t, C_t^*, L_t^*\}} \quad & \mathbb{E}_t\Big[\sum_{\tau=t}^{\infty}\beta^{\tau}\Big(\ln(C_\tau)-\kappa L_\tau\Big)\Big]+\mathbb{E}_t\Big[\sum_{\tau=t}^{\infty}\beta^{\tau}\Big(\ln(C_\tau^*)-\kappa^* L_\tau^*\Big)\Big] \\
s.t. \quad Y_t &= L_t a_t = C_{H,t} + C_{H,t}^* \\
Y_t^* &= L_t^* a_t^* = C_{F,t} + C_{F,t}^* \\
C_t &= C_{H,t}^{\gamma} C_{F,t}^{1-\gamma} \\
C_t^* &= C_{H,t}^{*1-\gamma} C_{F,t}^{*\gamma}
\end{aligned}$$

She then determines the optimal amount of labor, which produces the goods given the technological constraints and then allocates the goods to each consumer. The Lagrangian is given by:

$$\begin{aligned}
\max_{C_{H,t}, C_{F,t}, C_{H,t}^*, C_{F,t}^*, L_t, L_t^*} L =& \gamma\ln(C_{H,t})+(1-\gamma)\ln(C_{F,t})-\kappa L_t+(1-\gamma)\ln(C_{H,t}^*)+\gamma\ln(C_{F,t}^*)-\kappa^* L_t^* \\
&+\lambda_{1t}(a_t L_t - C_{H,t} - C_{H,t}^*) + \lambda_{2t}(a_t^* L_t^* - C_{F,t} - C_{F,t}^*)
\end{aligned}$$

The first order conditions are:

$$
\begin{aligned}
L_{L_t} &: \quad \kappa = \lambda_{1t} a_t & L_{L_t^*} &: \quad \kappa^* = \lambda_{2t} a_t^* \\
L_{C_{H,t}} &: \quad \frac{\gamma}{C_{H,t}} = \lambda_{1t} & L_{C_{F,t}} &: \quad \frac{1-\gamma}{C_{F,t}} = \lambda_{2t} \\
L_{C_{H,t}^*} &: \quad \frac{1-\gamma}{C_{H,t}^*} = \lambda_{1t} & L_{C_{F,t}^*} &: \quad \frac{\gamma}{C_{F,t}^*} = \lambda_{2t}
\end{aligned}
$$

Combining these conditions, the allocation of the social planner is:

$$
C_{H,t} = \frac{\gamma}{\kappa} a_t \qquad C_{H,t}^* = \frac{1-\gamma}{\kappa} a_t \tag{1.A.1}
$$

$$
C_{F,t} = \frac{1-\gamma}{\kappa^*} a_t^* \qquad C_{F,t}^* = \frac{\gamma}{\kappa^*} a_t^* \tag{1.A.2}
$$

$$
L_t = \frac{1}{\kappa} \qquad L_t^* = \frac{1}{\kappa^*} \tag{1.A.3}
$$

1.A.1.2 Market Economy: Consumer's Problem

In the market economy, each individual maximizes her own utility. the Lagrangian of that maximization problem is given by:

$$
\begin{aligned}
L(h = j) = \mathbb{E}_t\Big[\sum_{\tau=t}^{\infty} \beta^{\tau} \Big(& \ln(C_\tau) - \kappa L_\tau \\
& + \lambda_\tau \big(- B_{H,\tau} + (1 + i_{\tau-1}) B_{H,\tau-1} - \mathscr{E} B_{F,\tau} \\
& + (1 + i_{\tau-1}^*) \mathscr{E}_\tau B_{F,\tau-1} + \int \Pi_{t-1}(h) dh - P_{H,\tau} C_{H,\tau} - P_{F,\tau} C_{F,\tau} + W_\tau L_\tau \big) \Big) \Big]
\end{aligned}
$$

Consumption C_t consists of a combination of a Home and foreign consumption bundle given by:

$$
C_t = C_{H,t}^{\gamma} C_{F,t}^{1-\gamma}
$$

We can obtain the first order conditions (focs) with respect to $C_{H,\tau}, C_{F,\tau}, L_\tau, B_{H,\tau}, B_{F,\tau}$:

$$
\begin{aligned}
L_{C_{H,t}} &: & \frac{\gamma}{C_{H,t}} &= \lambda_t P_{H,t} \\
L_{C_{F,t}} &: & \frac{1-\gamma}{C_{F,t}} &= \lambda_t P_{F,t} \\
L_{L_t} &: & \kappa &= \lambda_t W_t \\
L_{B_{H,t}} &: & \lambda_t &= \beta \mathbb{E}_t[\lambda_{t+1}(1 + i_t)] \\
L_{B_{F,t}} &: & \mathscr{E}_t \lambda_t &= \beta \mathbb{E}_t[\mathscr{E}_{t+1} \lambda_{t+1}(1 + i_t^*)]
\end{aligned}
$$

and the budget constraint

$$B_{H,t} + \mathcal{E}_t B_{F,t} \le (1 + i_{t-1})B_{H,t-1} - T_t + W_t L_t$$
$$+ (1 + i^*_{t-1})\mathcal{E}_t B_{F,t-1} + \int_0^1 \Pi_{t-1}(h)dh - \int_0^1 p_t(h)C_t(h,j)dh - \int_0^1 p_t(f)C_t(f,j)df$$

Using the first two focs and taking a geometric average with weights γ and $1-\gamma$ gives:

$$\gamma^\gamma(1-\gamma)^{1-\gamma} = \lambda_t(P_{H,t}C_{H,t})^\gamma(P_{F,t}C_{F,t})^{1-\gamma}$$

which yields

$$\lambda_t = \frac{1}{P_t C_t}$$

where

$$P_t \equiv \frac{P^\gamma_{H,t}P^{1-\gamma}_{F,t}}{\gamma^\gamma(1-\gamma)^{1-\gamma}}$$

is defined as the utility-based price index in country H. Therefore, Home and foreign consumption are just the corresponding fraction of overall consumption:

$$P_t C_t = \frac{1}{\gamma}P_{H,t}C_{H,t} = \frac{1}{1-\gamma}P_{F,t}C_{F,t}$$

Foreign country

For F, the optimization problem is the same, except that

$$C^*_t = C^{*1-\gamma}_{H,t}C^{*\gamma}_{F,t}$$

This changes the first two first order condition with respect to Home and foreign good consumption:

$$L_{C_{H,t}}: \quad \frac{1-\gamma}{C^*_{H,t}} = \lambda^*_t P^*_{H,t}$$
$$L_{C_{F,t}}: \quad \frac{\gamma}{C^*_{F,t}} = \lambda^*_t P^*_{F,t}$$

Those two first order conditions can be combined to:

$$\gamma^\gamma(1-\gamma)^{1-\gamma} = \lambda^*_t(P^*_{H,t}C^*_{H,t})^{1-\gamma}(P^*_{F,t}C^*_{F,t})^\gamma$$

which yields

$$\lambda^*_t = \frac{1}{P^*_t C^*_t}$$

where

$$P_t^* \equiv \frac{P_{H,t}^{*1-\gamma} P_{F,t}^{*\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}$$

is defined as the utility-based price index in country F. Therefore, Home and foreign consumption are just the corresponding fraction of overall consumption:

$$P_t^* C_t^* = \frac{1}{1-\gamma} P_{H,t}^* C_{H,t}^* = \frac{1}{\gamma} P_{F,t}^* C_{F,t}^*$$

1.A.1.3 Intertemporal Allocation

Combining both consumption focs with the bond foc gives the Euler equation:

$$\frac{1}{C_t} = \beta(1+i_t)\mathbb{E}_t\Big[\frac{P_t}{P_{t+1}} \frac{1}{C_{t+1}}\Big]$$

Now let's take a closer look at the financial market of the model. Let the variable $Q_{t,t+1}$ be the stochastic discount rate for j:

$$Q_{t,t+1} \equiv \beta \frac{P_t C_t}{P_{t+1} C_{t+1}}$$

The expected stochastic discount factor is related to the inverse nominal interest rate (from the bond foc)

$$\mathbb{E}_t[Q_{t,t+1}] = \frac{1}{1+i_t} \quad \mathbb{E}_t[Q_{t,t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}] = \frac{1}{1+i_t^*}.$$

In a symmetric model in which all agent can access the same domestic financial markets the individual discount factors are the same ($Q_{t,t+1} = Q_{t,t+1}$). Therefore, the nominal interest rates parity is given by:

$$(1+i_t) = \mathbb{E}_t\Big[\frac{\mathcal{E}_{t+1}}{P_{t+1}C_{t+1}}\Big]\mathbb{E}_t\Big[\frac{\mathcal{E}_t}{P_{t+1}C_{t+1}}\Big]^{-1}(1+i_t^*)$$

Finally, bonds are in zero net supply:

$$\int_0^1 B_{H,t-1} dj + \int_0^1 B_{H,t-1}^* dj^* = 0$$
$$\int_0^1 B_{F,t-1} dj + \int_0^1 B_{F,t-1}^* dj^* = 0$$

The first order condition for labor gives a condition that determines wages W_t for that period.
In addition, a transversality condition is imposed in order to ensure that consumers really exhaust their resources.

1.A.1.4 Prices

Firms selling brand h maximize profits:

$$\max \mathbb{E}_{t-1}[Q_{t-1,t}((1-\tau)p_t(h)-MC_t)\int_0^1 C_t(h,j)dj+ \\ (\frac{\mathcal{E}_t(1-\tau)\tilde{p}_t(h)}{\mathcal{E}_t}-(1+\varpi)MC_t))\int_0^1 C_t^*(h,j^*)dj^*)]$$

Accounting for consumer's demand (1.4) they choose prices such that they maximize their profits:

$$\max_{p_t(h),\tilde{p}_t(h)} \mathbb{E}_{t-1}[Q_{t-1,t}(((1-\tau)p_t(h)-MC_t)\Big(\frac{p_t(h)}{P_{H,t}}\Big)^{-\theta} C_{H,t} \\ +((1-\tau)\tilde{p}_t(h)-(1+\varpi)MC_t))\Big(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}}\Big)^{-\theta} C_{H,t}^*]$$

For a firm the optimal domestic price is equal to marginal costs augmented by the equilibrium markup and an appropriate discount.

$$p_t(h) = \frac{1}{(1-\tau)}\frac{\theta}{\theta-1}\frac{\mathbb{E}_{t-1}[Q_{t-1,t}p_t(h)^{-\theta}P_{h,t}^{\theta}C_{H,t}MC_t]}{\mathbb{E}_{t-1}[Q_{t-1,t}p_t(h)^{-\theta}P_{h,t}^{\theta}C_{H,t}]}$$

Plugging in the stochastic discount rate and the relationship between expenditures for goods H and overall expenditures gives the price as in the main text:

$$p_t(h) = \frac{1}{(1-\tau)}\frac{\theta}{\theta-1}\mathbb{E}_{t-1}[MC_t]$$

The optimal price of Home goods in the foreign market can be obtained by differentiating the firm's objective function with respect to $\tilde{p}_t(h)$:

$$\mathbb{E}_{t-1}[Q_{t-1,t}\big((1-\tau)(1-\theta)\Big(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}}\Big)^{-\theta} C_{H,t}^* + \theta(1+\varpi)MC_t\Big(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}}\Big)^{-\theta}\tilde{p}_t(h)^{-1}C_{H,t}^*\big)] = 0$$

$$\mathbb{E}[Q_{t-1,t}\big((1-\tau)(\theta-1)\Big(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}}\Big)^{-\theta} C_{H,t}^*\big)] = \mathbb{E}[Q_{t-1,t}\big(\theta(1+\varpi)MC_t\Big(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}}\Big)^{-\theta}\tilde{p}_t(h)^{-1}C_{H,t}^*\big)]$$

$$\tilde{p}_t(h) = \frac{1}{(1-\tau)}\frac{\theta(1+\varpi)}{\theta-1}\frac{\mathbb{E}_{t-1}[Q_{t-1,t}\tilde{p}_t(h)^{-\theta}\tilde{P}_{H,t}^{\theta}C_{H,t}^*MC_t]}{\mathbb{E}_{t-1}[Q_{t-1,t}\tilde{p}_t(h)^{-\theta}\tilde{P}_{H,t}^{\theta}C_{H,t}^*]}$$

Plug in the stochastic discount factor.

$$\tilde{p}_t(h) = \frac{1}{(1-\tau)}\frac{\theta(1+\varpi)}{\theta-1}\frac{\mathbb{E}_{t-1}[\frac{P_{t-1}C_{t-1}}{P_tC_t}\tilde{p}_t(h)^{-\theta}\tilde{P}_{H,t}^{\theta}C_{H,t}^*MC_t]}{\mathbb{E}_{t-1}[\frac{P_{t-1}C_{t-1}}{P_tC_t}\tilde{p}_t(h)^{-\theta}\tilde{P}_{H,t}^{\theta}C_{H,t}^*]}$$

$$\tilde{p}_t(h) = \frac{1}{(1-\tau)}\frac{\theta(1+\varpi)}{\theta-1}\frac{\mathbb{E}_{t-1}[\frac{C_{H,t}^*}{P_tC_t}MC_t]}{\mathbb{E}_{t-1}[\frac{C_{H,t}^*}{P_tC_t}]}$$

Plug in demand for $C^*_{H,t}$

$$\tilde{p}_t(h) = \frac{1}{(1-\tau)}\frac{\theta(1+\varpi)}{\theta-1}\frac{\mathbb{E}_{t-1}[\frac{(1-\gamma)P^*_tC^*_t/P^*_{H,t}}{P_tC_t}MC_t]}{\mathbb{E}_{t-1}[\frac{(1-\gamma)P^*_tC^*_t/P^*_{H,t}}{P_tC_t}]}$$

$$\tilde{p}_t(h) = \frac{1}{(1-\tau)}\frac{\theta(1+\varpi)}{\theta-1}\frac{\mathbb{E}_{t-1}[\frac{C^*_t}{C_t}MC_t]}{\mathbb{E}_{t-1}[\frac{C^*_t}{C_t}]}$$

Consumption of both countries is always the same in a symmetric calibration, since terms of trade movements ensure perfect risk sharing. Therefore

$$\tilde{p}_t(h) = \frac{1}{(1-\tau)}\frac{\theta(1+\varpi)}{\theta-1}\mathbb{E}_{t-1}[MC_t]$$

$$p^*_t(h) = P^*_{H,t} = \frac{1}{(1-\tau)}(1+\varpi)\frac{\theta}{\theta-1}\frac{\mathbb{E}_{t-1}[MC_t]}{\mathcal{E}_t}$$

The firm then supplies for the given prices (wages and good prices) the amount of goods demanded by the consumers. This in the end determines the amount of work in the economy. With flexible prices, the expectations operator just drops and firms choose prices such that they match actual marginal costs, augmented with the equilibrium mark up.

1.A.1.5 Consumption

The first order condition of the consumer's problem yields, when optimizing w.r.t $C_{H,t}$ and $C_{F,t}$

$$1 = \lambda_t * P^{\gamma}_{H,t}P^{1-\gamma}_{F,t}\underbrace{(C_{H,t})^{\gamma}(C_{F,t})^{1-\gamma}}_{C_t}\underbrace{\frac{1}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}}_{\frac{1}{\gamma_w}}$$

This gives:

$$\lambda_t = \frac{1}{P_tC_t}$$

using the solution for the prices, consumption C_t is given by ($\lambda_t = 1/P_tC_t = 1/P_tC_t = 1/\mu_t$):

$$C_t = \frac{\gamma_w\mu_t}{P^{\gamma}_{H,t}P^{1-\gamma}_{F,t}}$$

or more explicit

$$C_t = \left(\frac{1}{1+\varpi}\right)^{1-\gamma}\frac{\gamma_w\Phi^{-\gamma}\Phi^{*-(1-\gamma)}\mu_t\mathcal{E}_t^{-(1-\gamma)}}{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC^*_t])^{1-\gamma}}$$

$$C^*_t = \left(\frac{1}{1+\varpi}\right)^{1-\gamma}\frac{\gamma_w\Phi^{*-\gamma}\Phi^{*-(1-\gamma)}\mu^*_t\mathcal{E}_t^{1-\gamma}}{(\mathbb{E}_{t-1}[MC_t])^{1-\gamma}(\mathbb{E}_{t-1}[MC^*_t])^{\gamma}}$$

1.A.1.6 Labor

The firm chooses labor such that it meets global demand for the brand:

$$L_t(h) = a_t^{-1}(p_t(h)^{-\theta}P_{H,t}^{\theta}C_{H,t} + p_t^*(h)^{-\theta}(P_{H,t}^{*}{}^{\theta}C_{H,t}^*)$$

$MC_t = a_t^{-1}W_t$. Rearranging the labor foc, plugging in $\lambda = 1/\mu$ and you arrive at:

$$MC_t = a_t^{-1}\mu_t\kappa$$

In a symmetric equilibrium $p_t(h) = P_{H,t}$. Since households consume a constant fraction of foreign and Home goods ($P_tC_t\gamma = P_{H,t}C_{H,t}$), one can plug in $C_{H,t}$ and $C_{H,t}^*$ respectively to obtain:

$$L_t(h) = a_t^{-1}\Big(\gamma\frac{\overbrace{P_tC_t}^{\mu_t}}{\underbrace{P_{H,t}}_{\Phi\mathbb{E}_{t-1}[MC_t]}} + (1-\gamma)\frac{P_t^*C_t^*}{P_{H,t}^*}\Big)$$

Plugging in $P_{H,t}^*$ and the monetary stance and assuming that the degree of monopolistic distortion is the same in both countries

$$L_t(h) = \frac{1}{\Phi}a_t^{-1}\Big(\gamma\frac{\mu_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{\mu_t^*}{\frac{\mathbb{E}_{t-1}[MC_t]}{\mathcal{E}_t}}\Big)$$

Augment the expression and use the relationship between both monetary stances and the exchange rate:

$$L_t(h) = \frac{1}{\Phi\kappa}\Big(\gamma\frac{\overbrace{a_t^{-1}\kappa\mu_t}^{MC_t}}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{a_t^{-1}\kappa\mu_t}{\mathbb{E}_{t-1}[MC_t]}\Big)$$

Demand of for every good in F and H is a function of the marginal costs of the firm producing that good:

$$\begin{aligned} L_t(h) &= \frac{1}{\Phi\kappa}\Big(\gamma\frac{MC_t}{\mathbb{E}_{t-1}(MC_t)} + \frac{(1-\gamma)}{1+\varpi}\frac{MC_t}{\mathbb{E}_{t-1}(MC_t)}\Big) \\ L_t^*(f) &= \frac{1}{\Phi^*\kappa^*}\Big(\frac{1-\gamma}{1+\varpi}\frac{MC_t^*}{\mathbb{E}_{t-1}(MC_t^*)} + \gamma\frac{MC_t^*}{\mathbb{E}_{t-1}(MC_t^*)}\Big) \end{aligned}$$

1.A.2 Solution Free Market and Flexible Prices

1.A.2.1 National Currency

The consumer solves the lifetime optimization problem. All variables can be expressed as a function of shocks a_t, a_t^* and economic parameter. The expectations operator drops when prices are flexible.

$$\mathcal{E}_t = \frac{\mu_t}{\mu_t^*} \tag{1.A.4}$$

$$MC_t = \kappa a_t^{-1} \mu_t \tag{1.A.5}$$

$$MC_t^* = \kappa^* a_t^{*-1} \mu_t^* \tag{1.A.6}$$

$$P_{H,t} = \Phi MC_t \tag{1.A.7}$$

$$P_{F,t} = \Phi^*(1+\varpi)\mathcal{E}_t MC_t^* \tag{1.A.8}$$

$$P_{F,t}^* = \Phi^* MC_t^* \tag{1.A.9}$$

$$P_{H,t}^* = \Phi(1+\varpi)\frac{1}{\mathcal{E}_t} MC_t \tag{1.A.10}$$

$$C_t = \left(\frac{1}{1+\varpi}\right)^{1-\gamma} \frac{\gamma_w \mu_t \mathcal{E}_t^{-1(1-\gamma)}}{(\Phi MC_t)^{\gamma}(\Phi^* MC_t^*)^{1-\gamma}} \tag{1.A.11}$$

$$C_t^* = \left(\frac{1}{1+\varpi}\right)^{1-\gamma} \frac{\gamma_w \mu_t^* \mathcal{E}_t^{1-\gamma}}{(\Phi MC_t)^{1-\gamma}(\Phi^* MC_t^*)^{\gamma}} \tag{1.A.12}$$

$$L_t(h) = \frac{1}{\Phi\kappa}\left(\gamma + \frac{(1-\gamma)}{1+\varpi}\right) \tag{1.A.13}$$

$$L_t^*(f) = \frac{1}{\Phi^*\kappa^*}\left(\frac{1-\gamma}{1+\varpi} + \gamma\right) \tag{1.A.14}$$

1.A.2.2 Currency Union

$$\mathcal{E}_t = 1 \tag{1.A.15}$$

$$MC_t = \kappa a_t^{-1} \mu_t \tag{1.A.16}$$

$$MC_t^* = \kappa^* a_t^{*-1} \mu_t^* \tag{1.A.17}$$

$$P_{H,t} = \Phi MC_t \tag{1.A.18}$$

$$P_{F,t} = \Phi^* MC_t^* \tag{1.A.19}$$

$$P_{F,t}^* = \Phi^* MC_t^* \tag{1.A.20}$$

$$P_{H,t}^* = \Phi MC_t \tag{1.A.21}$$

$$C_t = \frac{\gamma_w \mu_t \mathcal{E}_t^{-1(1-\gamma)}}{(\Phi MC_t)^\gamma (\Phi^* MC_t^*)^{1-\gamma}} \tag{1.A.22}$$

$$C_t^* = \frac{\gamma_w \mu_t^* \mathcal{E}_t^{1-\gamma}}{(\Phi MC_t)^{1-\gamma} (\Phi^* MC_t^*)^\gamma} \tag{1.A.23}$$

$$L_t(h) = \frac{1}{\Phi \kappa} \tag{1.A.24}$$

$$L_t^*(f) = \frac{1}{\Phi \kappa^*} \tag{1.A.25}$$

1.A.3 Solution Central Bank and Sticky Prices

1.A.3.1 National Currency

The consumer solves the lifetime optimization problem. All variables can be expressed as a function of shocks a_t, a_t^*, monetary stances μ_t, μ_t^* and economic parameter.

$$\mathscr{E}_t = \frac{\mu_t}{\mu_t^*} \tag{1.A.26}$$
$$MC_t = \kappa a_t^{-1}\mu_t \tag{1.A.27}$$
$$MC_t^* = \kappa^* a_t^{*-1}\mu_t^* \tag{1.A.28}$$
$$P_{H,t} = \Phi\mathbb{E}_{t-1}[MC_t] \tag{1.A.29}$$
$$P_{F,t} = \Phi^*(1+\varpi)\mathscr{E}_t\mathbb{E}_{t-1}[MC_t^*] \tag{1.A.30}$$
$$P_{F,t}^* = \Phi^*\mathbb{E}_{t-1}[MC_t^*] \tag{1.A.31}$$
$$P_{H,t}^* = \Phi(1+\varpi)\frac{1}{\mathscr{E}_t}\mathbb{E}_{t-1}[MC_t] \tag{1.A.32}$$
$$C_t = \Big(\frac{1}{1+\varpi}\Big)^{1-\gamma}\frac{\gamma_w\mu_t\mathscr{E}_t^{-1(1-\gamma)}}{(\Phi\mathbb{E}_{t-1}[MC_t])^{\gamma}(\Phi^*\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}} \tag{1.A.33}$$
$$C_t^* = \Big(\frac{1}{1+\varpi}\Big)^{1-\gamma}\frac{\gamma_w\mu_t^*\mathscr{E}_t^{1-\gamma}}{(\Phi\mathbb{E}_{t-1}[MC_t])^{1-\gamma}(\Phi^*\mathbb{E}_{t-1}[MC_t^*])^{\gamma}} \tag{1.A.34}$$
$$L_t(h) = \frac{1}{\Phi\kappa}\Big(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\Big) \tag{1.A.35}$$
$$L_t^*(f) = \frac{1}{\Phi^*\kappa^*}\Big(\frac{1-\gamma}{1+\varpi}\frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]} + \gamma\frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]}\Big) \tag{1.A.36}$$

1.A.3.2 Currency Union

$$\mathscr{E}_t = 1 \tag{1.A.37}$$
$$MC_t = \kappa a_t^{-1}\mu_t^U \tag{1.A.38}$$
$$MC_t^* = \kappa^* a_t^{*-1}\mu_t^U \tag{1.A.39}$$
$$P_{H,t} = \Phi\mathbb{E}_{t-1}[MC_t] \tag{1.A.40}$$
$$P_{F,t} = \Phi^*\mathscr{E}_t\mathbb{E}_{t-1}[MC_t^*] \tag{1.A.41}$$
$$P_{F,t}^* = \Phi^*\mathbb{E}_{t-1}[MC_t^*] \tag{1.A.42}$$
$$P_{H,t}^* = \Phi\frac{1}{\mathscr{E}_t}\mathbb{E}_{t-1}[MC_t] \tag{1.A.43}$$
$$C_t = \frac{\gamma_w\mu_t^U\mathscr{E}_t^{-1(1-\gamma)}}{(\Phi\mathbb{E}_{t-1}[MC_t])^{\gamma}(\Phi^*\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}} \tag{1.A.44}$$
$$C_t^* = \frac{\gamma_w\mu_t^U\mathscr{E}_t^{1-\gamma}}{(\Phi\mathbb{E}_{t-1}[MC_t])^{1-\gamma}(\Phi^*\mathbb{E}_{t-1}[MC_t^*])^{\gamma}} \tag{1.A.45}$$
$$L_t(h) = \frac{1}{\Phi\kappa}\Big(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + (1-\gamma)\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\Big) \tag{1.A.46}$$
$$L_t^*(f) = \frac{1}{\Phi^*\kappa^*}\Big(\gamma\frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]} + (1-\gamma)\frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]}\Big) \tag{1.A.47}$$

1.A.4 Free Market and Flexible Prices

Now consider a decentralized economy, in which market forces determine the allocation. I show here that the flex price allocation is an important welfare benchmark. I consider two regimes, one with national currencies and one in a currency union.

1.A.4.1 National Currency

Households maximize their lifetime utility by choosing consumption and supplying labor:

$$\max_{\{C_t,L_t,B_t\}} \mathbb{E}_t\Big[\sum_{j=t}^{\infty}\beta^j\Big(\ln\big((C_{H,j})^{\gamma}(C_{F,j})^{1-\gamma}\big)-\kappa L_j\Big]$$
$$\text{s.t. } B_{H,t}+\mathcal{E}_t B_{F,t}+P_{H,t}C_{H,t}+P_{F,t}C_{F,t} = (1+i_{t-1})B_{H,t-1}-T_t+W_tL_t+(1+i^*_{t-1})\mathcal{E}_t B_{F,t-1}+\Pi_{H,t}$$

Firms selling brand h maximize profits given the marginal costs, accounting for consumers' demand and the pricing strategy and trade costs with national currencies:

$$\max_{p_t(h),\tilde{p}_t(h)}\Big((1-\tau)p_t(h)-MC_t\Big)\Big(\frac{p_t(h)}{P_{H,t}}\Big)^{-\theta}C_{H,t}+\Big(\frac{\mathcal{E}_t(1-\tau)\tilde{p}_t(h)}{\mathcal{E}_t}-(1+\varpi)MC_t)\Big)\Big(\frac{\tilde{p}_t(h)}{\tilde{P}_{H,t}}\Big)^{-\theta}C^*_{H,t}$$

The solution steps of that problem are in the appendix. Consumption and labor here have a superscript n:

$$\begin{aligned}
C^n_{Ht} &= \frac{\gamma a_t}{\Phi\kappa} & C^{*n}_{Ht} &= \frac{(1-\gamma)\big(\frac{1}{1+\varpi}\big)a_t}{\Phi\kappa}\\
C^n_{Ft} &= \frac{(1-\gamma)\big(\frac{1}{1+\varpi}\big)a^*_t}{\Phi^*\kappa^*} & C^{*n}_{Ft} &= \frac{\gamma a^*_t}{\Phi^*\kappa^*}\\
L^n_t &= \frac{1}{\Phi\kappa}\Big(\gamma+\frac{1-\gamma}{1+\varpi}\Big) & L^{*n}_t &= \frac{1}{\Phi^*\kappa^*}\Big(\frac{\gamma}{1+\varpi}+1-\gamma\Big)
\end{aligned} \tag{1.A.48}$$

The distribution of consumption in a decentralized allocation is the same, except that monopolistic markups lower consumption and employment, while trade costs lower consumption of non-domestic goods and overall employment.

1.A.4.2 Currency Union

Households face the same problem as before:

$$\max_{\{C_t,L_t,B_t\}} \mathbb{E}_t\Big[\sum_{j=t}^{\infty}\beta^j\Big(\ln\big((C_{H,j})^{\gamma}(C_{F,j})^{1-\gamma}\big)-\kappa L_j\Big]$$
$$\text{s.t. } B_{H,t}+\mathcal{E}_t B_{F,t}+P_{H,t}C_{H,t}+P_{F,t}C_{F,t} = (1+i_t)B_{H,t-1}-T_t+W_tL_t+(1+i^*_t)\mathcal{E}_t B_{F,t-1}+\Pi_{H,t}$$

In contrast to the case with national currencies, there are no trade costs and no exchange rate in a currency union:

$$\max_{p_t(h),p_t^*(h)} \Big((1-\tau)p_t(h) - MC_t\Big)\Big(\frac{p_t(h)}{P_{H,t}}\Big)^{-\theta} C_{H,t} + \Big((1-\tau)p_t^*(h) - MC_t)\Big)\Big(\frac{p_t^*(h)}{P_{H,t}^*}\Big)^{-\theta} C_{H,t}^*$$

Consumption and labor are not a function of trade costs anymore and have superscript u:

$$\begin{aligned} C_{Ht}^u &= \frac{\gamma a_t}{\Phi\kappa} & C_{Ht}^{*u} &= \frac{(1-\gamma)a_t}{\Phi\kappa} \\ C_{Ft}^u &= \frac{(1-\gamma)a_t^*}{\Phi^*\kappa^*} & C_{Ft}^{*u} &= \frac{\gamma a_t^*}{\Phi^*\kappa^*} \\ L_t^u &= \frac{1}{\Phi\kappa} & L_t^{*u} &= \frac{1}{\kappa^*\Phi^*} \end{aligned} \tag{1.A.49}$$

Overall, employment and consumption in a currency union with flexible prices are the same as in the social planner's allocation, except for the monopolistic distortion.

1.A.5 Monetary Policy

For analytic convenience, let's introduce a monetary stance μ_t that controls nominal expenditures P_tC_t in the economy. It links the nominal interest rate in the Euler equation such that

$$\frac{1}{\mu_t} = \beta(1+i_t)\mathbb{E}_t[\frac{1}{\mu_{t+1}}]$$

μ_{t+1}/μ_t determines the inflation target π, the steady state nominal interest rate is $1+i=\pi/\beta$. In equilibrium one obtains that $\mu_t = P_tC_t = W_t/\kappa$[19]. An expansionary monetary policy in H corresponds with interest rates cuts today or households' expectations about interest rate cuts in the future. In this case μ_t lies above the trend, it coincides with increased nominal spending P_tC_t in the economy.

1.A.5.1 Optimal National Monetary Policy under Commitment

A national authority maximizes expected utility of the representative agent. I use a state-contingent notation:

19. Inspect the Euler equation with logarithmic utility for that

$$
\begin{aligned}
\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} \Big[\sum_{t=k}^{\infty} \sum_{s^t \in A} \beta^{t-k} p(s^t \mid s^0) \Big(\ln(C_t) - \kappa L_t \Big) \Big] \\
\text{s.t.} C_t &= \Big(\frac{1}{1+\varpi}\Big)^{1-\gamma} \frac{\Upsilon_w \big(\frac{\theta-1}{\theta}\big)^{\gamma} \big(\frac{\theta^*-1}{\theta^*}\big)^{1-\gamma} \mu_t(s^t) \mathcal{E}_t^{-1(1-\gamma)}}{(\mathbb{E}_{t-1}[MC_t])^{\gamma} (\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}} \\
L_t(h) &= \frac{\theta-1}{\theta\kappa} \Big(\gamma \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi} \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} \Big) \\
MC_t &= \kappa a_t^{-1} \mu_t(s^t) \\
MC_t^* &= \kappa^* a_t^{*-1} \mu_t^*(s^t) \\
\mathcal{E}_t &= \frac{1-\gamma}{\gamma} \frac{\mu_t(s^t)}{\mu_t^*(s^t)} \\
\mathbb{E}_{t-1}[MC_t] &= \sum_{s^t \in A} p(s^t \mid s^0) MC_t \\
\mathbb{E}_{t-1}[MC_t^*] &= \sum_{s^t \in A} p(s^t \mid s^0) MC_t^*
\end{aligned}
$$

Plugging in:

$$
\begin{aligned}
&\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} \Big[\sum_{t=k}^{\infty} \sum_{s^t \in A} \beta^{t-k} p(s^t \mid s^0) \\
&\Big(\ln\Big(\Big(\frac{1}{1+\varpi}\Big)^{1-\gamma} \frac{\Upsilon_w \big(\frac{\theta-1}{\theta}\big)^{\gamma} \big(\frac{\theta^*-1}{\theta^*}\big)^{1-\gamma} \mu_t(s^t) \big(\frac{1-\gamma}{\gamma} \frac{\mu_t(s^t)}{\mu_t^*(s^t)}\big)^{-1(1-\gamma)}}{(\sum_{s^t \in A} p(s^t \mid s^0) \kappa a_t^{-1} \mu_t(s^t))^{\gamma} (\sum_{s^t \in A} p(s^t \mid s^0) \kappa^* a_t^{*-1} \mu_t^*(s^t))^{1-\gamma}}\Big) \\
&- \kappa \frac{\theta-1}{\theta\kappa} \Big(\gamma \frac{\kappa a_t^{-1} \mu_t(s^t)}{\sum_{s^t \in A} p(s^t \mid s^0) \kappa a_t^{-1} \mu_t(s^t)} + \frac{(1-\gamma)}{1+\varpi} \frac{\kappa a_t^{-1} \mu_t(s^t)}{\sum_{s^t \in A} p(s^t \mid s^0) \kappa a_t^{-1} \mu_t(s^t)} \Big) \Big) \Big]
\end{aligned}
$$

Dissolve the ln expression

$$
\begin{aligned}
&\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} \sum_{s^t \in A} p(s^t | s^0) \Big[\ln\Big(\Big(\frac{1}{1+\varpi}\Big)^{1-\gamma} \Upsilon_w \Big(\frac{\theta-1}{\theta}\Big)^{\gamma} \Big(\frac{\theta^*-1}{\theta^*}\Big)^{1-\gamma} \Big(\frac{1-\gamma}{\gamma}\Big)^{-(1-\gamma)} \Big) + \ln(\mu_t(s^t)) \\
&- (1-\gamma)(\ln(\mu_t(s^t)) - \ln(\mu_t^*(s^t))) - \gamma \ln\Big(\sum_{s^t \in A} p(s^t \mid s^0) \kappa a_t^{-1} \mu_t(s^t)\Big) \\
&- (1-\gamma) \ln\Big(\sum_{s^t \in A} p(s^t \mid s^0) \kappa^* a_t^{*-1} \mu_t^*(s^t)\Big) \Big] \\
&- \sum_{s^t \in A} p(s^t \mid s^{t-1}) \Big[\kappa \frac{\theta-1}{\theta\kappa} \Big(\gamma \frac{\kappa a_t^{-1} \mu_t(s^t)}{\sum_{s^t \in A} p(s^t \mid s^{t-1}) \kappa a_t^{-1} \mu_t(s^t)} + \\
&\frac{1-\gamma}{1+\varpi} \frac{\kappa a_t^{-1} \mu_t(s^t)}{\sum_{s^t \in A} p(s^t \mid s^{t-1}) \kappa a_t^{-1} \mu_t(s^t)} \Big) \Big].
\end{aligned}
$$

Note, that the last term representing labor is just a constant under monetary policy under commitment, the first order condition is

$$
\frac{1}{\mu_t(s^t)} - \frac{(1-\gamma)}{\mu_t(s^t)} - \gamma \frac{\kappa a_t^{-1}}{\sum_{s^t \in A} p(s^t \mid s^0) \kappa a_t^{-1} \mu_t(s^t)} = 0
$$

This can be rewritten to get the optimal monetary policy as in the main text:

$$\frac{1}{\mu_t(s^t)} = \frac{\kappa a_t^{-1}}{\sum_{s^t \in A} p(s^t \mid s^0)\kappa a_t^{-1}\mu_t(s^t)}$$
$$\mu_t(s^t)a_t^{-1}(s^t) = \mathbb{E}_{t-1}\left[\mu_t(s^t)a_t^{-1}(s^t)\right]$$

Alternatively, we can also use the utility gap approach as in Corsetti and Pesenti (2005)

$$\min \mathbb{E}_{t-1}[W_t^{flex} - W_t] = \min \mathbb{E}_{t-1}\Big[\ln\left(C_t^{flex}/C_t\right) - \kappa L_t^{flex} + \kappa L_t\Big]$$
$$\min \mathbb{E}_{t-1}\Bigg[\ln \frac{\left(\left(\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\frac{\gamma_w \mu_t \mathcal{E}_t^{-1(1-\gamma)}}{MC_t^{\gamma}(MC_t^*)^{1-\gamma}}\right)\right.}{\left(\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\frac{\gamma_w \mu_t \mathcal{E}_t^{-1(1-\gamma)}}{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}\right)\Big)} - \kappa L_t^{flex} + \kappa L_t\Bigg]$$
$$\min \mathbb{E}_{t-1}\Bigg[\ln\left(\left(\frac{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}{MC_t^{\gamma}(MC_t^*)^{1-\gamma}}\right)\right) - \kappa L_t^{flex} + \kappa L_t\Bigg]$$

Now plug in labor

$$\min \mathbb{E}_{t-1}\Bigg[\ln\left(\frac{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}{MC_t^{\gamma}(MC_t^*)^{1-\gamma}}\right) -$$
$$\frac{1}{\kappa}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right) + \kappa\frac{1}{\kappa}\left(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]}\right)\Bigg]$$
$$\min \mathbb{E}_{t-1}\Bigg[\ln\left(\frac{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}{MC_t^{\gamma}(MC_t^*)^{1-\gamma}}\right)\Bigg] -$$
$$\frac{1}{\kappa}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right) + \kappa\frac{1}{\kappa}\left(\gamma\frac{\mathbb{E}_{t-1}MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{\mathbb{E}_{t-1}MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]}\right)\Bigg]$$

Under Monetary Policy under commitment, labor is not actively targeted for by monetary policy and trade costs do not play a role. Therefore, monetary policy optimally minimizes:

$$\min \mathbb{E}_{t-1}\Bigg[\ln\left(\left(\frac{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}{MC_t^{\gamma}(MC_t^*)^{1-\gamma}}\right)\right)\Bigg]$$

Note that, according to Jensen's Inequality

$$\ln\left((\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}\right) - \mathbb{E}_{t-1}\Big[\ln(MC_t^{\gamma}(MC_t^*)^{1-\gamma})\Big]$$
$$\geq \mathbb{E}_{t-1}\Big[\ln((MC_t])^{\gamma}([MC_t^*])^{1-\gamma}\Big] - \mathbb{E}_{t-1}\Big[\ln(MC_t^{\gamma}(MC_t^*)^{1-\gamma})\Big] = 0$$

The best monetary policy could do is to set the gap to 0. Rewrite the objective function to:

$$\begin{aligned}
&\min \mathbb{E}_{t-1}\Big[\gamma \ln\Big(\frac{\mathbb{E}_{t-1}[MC_t]}{MC_t}\Big) + (1-\gamma)\ln\Big(\frac{(\mathbb{E}_{t-1}[MC_t^*]}{MC_t^*}\Big)\Big]\\
=&\min_{\mu_t} \mathbb{E}_{t-1}\Big[\gamma \ln\Big(\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}\mu_t}\Big)\\
=&\min_{\mu_t} \sum_{s^t\in A} p(s^t \mid s^0)\Big[\gamma \ln\Big(\frac{\sum_{s^t\in A} p(s^t \mid s^0)[a_t^{-1}\mu_t]}{a_t^{-1}\mu_t}\Big)\Big]\\
=&\min_{\mu_t} \sum_{s^t\in A} p(s^t \mid s^0)\Big[\gamma(\ln(\sum_{s^t\in A} p(s^t \mid s^0)[a_t^{-1}\mu_t]) - \ln(a_t^{-1}\mu_t))\Big]
\end{aligned}$$

Differentiate for specific state $\bar{A}$, then the first order condition is:

$$p(\bar{A})\Big[\frac{a_t^{-1}(\bar{A})}{\sum_{s^t\in A} p(s^t \mid s^0)a_t^{-1}(s_t)\mu_t(s_t)}(\sum_{s^t\in A} p(s^t \mid s^0))\Big] - p(\bar{A})\Big[\frac{a_t^{-1}(\bar{A})}{a_t^{-1}(\bar{A})\mu_t(\bar{A})}\Big] = 0$$

The policy rule for state $\bar{A}$ is:

$$a_t^{-1}(\bar{A})\mu_t(\bar{A}) = \mathbb{E}_{t-1}[a_t^{-1}\mu_t]$$

The same can be done for the foreign country. Note that under commitment, the central bank can not resort to negative monetary surprises to push the gap below zero.

We can also differentiate with respect to μ_t making use of the result: $\frac{\partial f(\mathbb{E}_{t-1}[x_t\mu_t^{\pi}])}{\partial \mu_t} = f'(\mathbb{E}_{t-1}[x_t\mu_t^{\pi}]) \cdot x_t\pi\mu_t^{\pi-1}$

$$\begin{aligned}
0 &= \frac{1}{\mu_t} - \frac{(1-\gamma)}{\mu_t} - \gamma\frac{a_t^{-1}}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\\
\Rightarrow \mu_t &= \frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}
\end{aligned}$$

This way we can avoid the state contingent notation.

Alternative version: Try to avoid using μ_t as a policy instrument and add time discount shock:

$$\begin{aligned}
\max_{\{i_t(s^t)\}_{t=k}^{\infty}} & \Big[\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta_t^{t-k}p(s^t \mid s^0)\Big(\ln(C_t)-\kappa L_t\Big)\Big] \\
\text{s.t.} C_t &= \mu_t^{-1}/P_t \\
\mu_t &= (\beta_t(1+i_t))\left(\mathbb{E}_t\left[\frac{1}{P_{t+1}C_{t+1}}\right]\right) \\
P_t &= \frac{P_{H,t}^{\gamma}P_{F,t}^{1-\gamma}}{\gamma_w} \\
L_t(h) &= \frac{\theta-1}{\theta\kappa}\Big(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}+\frac{(1-\gamma)}{1+\varpi}\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\Big) \\
P_{H,t} &= \Phi\mathbb{E}_{t-1}[MC_t] \\
P_{F,t} &= \Phi^*\mathscr{E}_t(1+\varpi)\mathbb{E}_{t-1}[MC_t^*] \\
MC_t &= \kappa a_t^{-1}\mu_t(s^t)^{-1} \\
MC_t^* &= \kappa^* a_t^{*-1}\mu_t^*(s^t)^{-1} \\
\mathscr{E}_t &= \left(\frac{\mu_t(s^t)}{\mu_t^*(s^t)}\right)^{-1} \\
\mathbb{E}_{t-1}[MC_t] &= \sum_{s^t\in A}p(s^t \mid s^0)MC_t \\
\mathbb{E}_{t-1}[MC_t^*] &= \sum_{s^t\in A}p(s^t \mid s^0)MC_t^*
\end{aligned}$$

Plugging in everything except the Euler equation and considering the expectations operator in front:

$$\max_{\{i_t(s^t)\}_{t=k}^{\infty}}\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta_t^{t-k}p(s^t|s^{t-1})\left(\ln\left(\frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w\left(\frac{\theta-1}{\theta}\right)^{\gamma}\left(\frac{\theta^*-1}{\theta^*}\right)^{1-\gamma}\mu_t(s^t)^{-\gamma}\mu_t^*(s^t)^{-1+\gamma}}{\left(\mathbb{E}_{t-1}\left[\kappa a_t^{-1}\mu_t(s^t)^{-1}\right]\right)^{\gamma}\left(\mathbb{E}_{t-1}\left[\kappa a_t^{*-1}\mu_t^*(s^t)^{-1}\right]\right)^{1-\gamma}}\right)\right)$$

$$\begin{aligned}
\text{s.t.}\quad \mu_t &= (\beta_t(1+i_t))\left(\mathbb{E}_t\left[\frac{1}{P_{t+1}C_{t+1}}\right]\right) \\
\mu_t^* &= (\beta_t^*(1+i_t^*))\left(\mathbb{E}_t\left[\frac{1}{P_{t+1}^*C_{t+1}^*}\right]\right)
\end{aligned}$$

Plugging in both Euler equation, I obtain the following maximization problem

$$\max_{\{i_t(s^t)\}_{t=k}^{\infty}}\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta_t^{t-k}p(s^t \mid s^{t-1})$$

$$\ln\left(\frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w\Phi^{\gamma}(\Phi^*)^{1-\gamma}\Big((\beta_t(1+i_t))\Big(\mathbb{E}_t\Big[\frac{1}{P_{t+1}C_{t+1}}\Big]\Big)\Big)^{-\gamma}\Big((\beta_t^*(1+i_t^*))\Big(\mathbb{E}_t\Big[\frac{1}{P_{t+1}^*C_{t+1}^*}\Big]\Big)\Big)^{\gamma-1}}{\left(\mathbb{E}_{t-1}\left[\kappa a_t^{-1}\Big(\beta_t(1+i_t)\mathbb{E}_t\Big[\frac{1}{P_{t+1}C_{t+1}}\Big]\Big)^{-1}\right]\right)^{\gamma}\left(\mathbb{E}_{t-1}\left[\kappa a_t^{*-1}\Big(\beta_t^*(1+i_t^*)\mathbb{E}_t\Big[\frac{1}{P_{t+1}^*C_{t+1}^*}\Big]\Big)^{-1}\right]\right)^{1-\gamma}}\right)$$

In an iid case $\left(\mathbb{E}_t\left[\frac{1}{P_{t+1}^*C_{t+1}^*}\right]\right)$ cancels out. We are left with:

$$\max_{\{i_t(s^t)\}_{t=k}^{\infty}}\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta_t^{t-k}p(s^t|s^{t-1})\ln\left(\frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w\Phi^{\gamma}(\Phi^*)^{1-\gamma}\left((\beta_t(1+i_t))\right)^{-\gamma}\left((\beta_t^*(1+i_t^*))\right)^{\gamma-1}}{\left(\mathbb{E}_{t-1}\left[\kappa a_t^{-1}(\beta_t(1+i_t))^{-1}\right]\right)^{\gamma}\left(\mathbb{E}_{t-1}\left[\kappa a_t^{*-1}(\beta_t^*(1+i_t^*))^{-1}\right]\right)^{1-\gamma}}\right)$$

Derivative with respect to i_t:

$$-\gamma\frac{1}{(1+i_t)}+\gamma\frac{\kappa a_t^{-1}\beta_t^{-1}\frac{1}{1+i_t}^2}{\mathbb{E}_{t-1}\left[\kappa a_t^{-1}(\beta_t(1+i_t))\right]} = 0$$

The monetary interest rate rule is described by:

$$a_t^{-1}\beta_t^{-1}(1+i_t)^{-1} = \mathbb{E}_{t-1}\left[a_t^{-1}(\beta_t(1+i_t))^{-1}\right]$$

Supply shock; a_t goes up (expansionary) implies that country is more productive. Central bank optimally lowers interest rates.

Demand shock; β_t goes up (contractionary) implies that households want to save more. Central bank optimally lowers interest rates.

1.A.5.2 Optimal Monetary Policy in a Currency Union under Commitment

Now take a look at the monetary optimization problem:

$$\min \xi\Big(\mathbb{E}\Big[\ln\Big(\frac{(\mathbb{E}_{t-1}[MC_t])^{\gamma}(\mathbb{E}[MC_t^*])^{1-\gamma}}{MC_t^{\gamma}(MC_t^*)^{1-\gamma}}\Big)\Big]\Big)+(1-\xi)\Big(\mathbb{E}\Big[\ln\Big(\frac{(\mathbb{E}[MC_t])^{1-\gamma}(\mathbb{E}_{t-1}[MC_t^*])^{\gamma}}{MC_t^{1-\gamma}(MC_t^*)^{\gamma}}\Big)\Big]\Big)$$

$$\min \mathbb{E}\Big[\big(\xi\gamma+(1-\xi)(1-\gamma)\big)\ln\Big(\frac{\mathbb{E}[MC_t]}{MC_t}\Big)+\big(\xi(1-\gamma)+(1-\xi)\gamma\big)\ln\Big(\frac{\mathbb{E}[MC_t^*]}{MC_t^*}\Big)\Big]$$

Weights do not matter if $\gamma = 1/2$, every country values Home and foreign goods equally. The state contingent objective function is

$$\begin{aligned}\min\sum_{s^t\in A}p(s^t \mid s^0)\Big[&\big(\xi\gamma+(1-\xi)(1-\gamma)\big)\ln\Big(\frac{\sum_{s^t\in A}p(s^t \mid s^0)[a_t^{-1}(s^t)\mu_t(s^t)]}{a_t^{-1}(s^t)\mu_t(s^t)}\Big)\\ &+\big(\xi(1-\gamma)+(1-\xi)\gamma\big)\ln\Big(\frac{\sum_{s^t\in A}p(s^t \mid s^0)[a_t^{*-1}(s^t)\mu_t(s^t)]}{a_t^{*-1}(s^t)\mu_t(s^t)}\Big)\Big]\end{aligned}$$

The first order condition with respect to $\mu_t(\bar{A})$ is

$$\begin{aligned}&\big(\xi\gamma+(1-\xi)(1-\gamma)\big)p(\bar{A})\Big[\frac{a_t^{-1}(\bar{A})}{\sum_{s^t\in A}p(s^t \mid s^{t-1})a_t^{-1}(s_t)\mu_t(s_t)}\sum_{s^t\in A}p(s^t|s^0)-\frac{a_t^{-1}(\bar{A})}{a_t^{-1}(\bar{A})\mu_t(\bar{A})}\Big]\\ &+\big(\xi(1-\gamma)+(1-\xi)\gamma\big)p(\bar{A})\Big[\frac{a_t^{*-1}(\bar{A})}{\sum_{s^t\in A}p(s^t|s^{t-1})a_t^{*-1}(s_t)\mu_t(s_t)}\sum_{s^t\in A}p(s^t|s^{t-1})-\frac{a_t^{*-1}(\bar{A})}{a_t^{*-1}(\bar{A})\mu_t(\bar{A})}\Big]\\ &= 0\end{aligned}$$

Solving for $\mu_t(\bar{A})$:

$$\mu_t(\bar{A}) = \Big(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)\frac{a_t^{-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}+\big(\xi(1-\gamma)+(1-\xi)\gamma\big)\frac{a_t^{*-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}\Big)^{-1}$$

For symmetric consumption baskets without Home bias as in Corsetti and Pesenti (2002), the objective functions boils down to:

$$\min_{\mu_t} \sum_{s^t \in A} p(s^t \mid s^0)\Big[\gamma(\ln(\sum_{s^t \in A} p(s^t \mid s^0)[a_t^{-1}\mu_t]) - \ln(a_t^{-1}\mu_t))\Big]$$
$$+ \sum_{s^t \in A} p(s^t \mid s^0)\Big[(1-\gamma)(\ln(\sum_{s^t \in A} p(s^t \mid s^0)[a_t^{*-1}\mu_t]) - \ln(a_t^{*-1}\mu_t))\Big]$$

giving the same optimal monetary stance as in Corsetti and Pesenti (2002).

Maximizing expected lifetime utility ex ante leads to the same monetary rule:

$$\max_{\mu_s} \xi \sum \beta^t \Big(\sum p_s\Big[\ln(C_s) - \kappa L_s\Big]\Big) + (1-\xi)\sum \beta^t\Big(\sum p_s\Big[\ln(C_s^*) - \kappa l_s^*\Big]\Big)$$

Plugging in consumption and labor, the foc for the monetary stance is:

$$\xi p_s\Big[\frac{1}{\mu_s} - \frac{\gamma\kappa a^{-1}}{\mathbb{E}[MC_t]} - \frac{(1-\gamma)\kappa^* a^{*-1}}{\mathbb{E}[MC_t^*]}\Big] + (1-\xi)p_s\Big[\frac{1}{\mu_s} - \frac{(1-\gamma)\kappa a^{-1}}{\mathbb{E}[MC_t]} - \frac{(\gamma\kappa^* a^{*-1}}{\mathbb{E}[MC_t^*]}\Big]$$
$$+\xi\sum_{p_A^-}\Big(\frac{-\gamma p_s \kappa a^{-1}}{\mathbb{E}[MC_t]} - \frac{(1-\gamma)p_s\kappa^* a^{*-1}}{\mathbb{E}[MC_t^*]}\Big) + (1-\xi)\sum_{p_A^-}\Big(\frac{-(1-\gamma)p_s\kappa a^{-1}}{\mathbb{E}[MC_t]} - \frac{\gamma p_s \kappa^* a^{*-1}}{\mathbb{E}[MC_t^*]}\Big) = 0$$

Rearranging a bit gives

$$\frac{\xi p_s}{\mu_s} + \frac{(1-\xi)p_s}{\mu_s} - \frac{\xi\gamma p_s\kappa a^{-1}}{\mathbb{E}[MC_t]} - \frac{(1-\xi)(1-\gamma)p_s\kappa a^{-1}}{\mathbb{E}[MC_t]} - \frac{\xi(1-\gamma)p_s\kappa^* a^{*-1}}{\mathbb{E}[MC_t^*]}$$
$$-\frac{(1-\xi)\gamma p_s\kappa^* a^{*-1}}{\mathbb{E}[MC_t^*]} = 0$$
$$\mu_t(\bar{A}) = \Big(\big(\xi\gamma + (1-\xi)(1-\gamma)\big)\frac{a_t^{-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \big(\xi(1-\gamma) + (1-\xi)\gamma\big)\frac{a_t^{*-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}\Big)^{-1}$$

Avoid using μ_t and introduce demand shocks:

$$\begin{aligned}
\max_{\{i_t(s^t)\}_{t=k}^{\infty}} \xi\Big[\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta_t^{t-k}p(s_t \mid s^{t-1})\Big(\ln(C_t)-\kappa L_t\Big)\Big] \\
+ (1-\xi)\Big[\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta_t^{t-k}p(s_t \mid s^{t-1})\Big(\ln(C_t)-\kappa L_t\Big)\Big]
\end{aligned}$$

$$\begin{aligned}
\text{s.t.} C_t &= \lambda_{1,t}^{-1}/P_t & C_t^* &= \lambda_{1,t}^{*-1}/P_t^* \\
\lambda_{1t} &= (\beta_t(1+i_t))\left(\mathbb{E}_t\left[\frac{1}{P_{t+1}C_{t+1}}\right]\right) & \lambda_{1t}^* &= (\beta_t^*(1+i_t))\left(\mathbb{E}_t\left[\frac{1}{P_{t+1}^*C_{t+1}^*}\right]\right) \\
P_t &= \frac{P_{H,t}^{\gamma}P_{F,t}^{1-\gamma}}{\gamma_w} & P_t^* &= \frac{P_{H,t}^{*1-\gamma}P_{F,t}^{*\gamma}}{\gamma_w} \\
L_t(h) &= \frac{\theta-1}{\theta\kappa}\left(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + (1-\gamma)\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\right) \\
P_{H,t} &= \Phi\mathbb{E}_{t-1}[MC_t] \\
P_{F,t} &= \Phi^*\mathbb{E}_{t-1}[MC_t^*] \\
MC_t &= \kappa a_t^{-1}\lambda_{1,t}^{-1} \\
MC_t^* &= \kappa^* a_t^{*-1}\lambda_{1,t}^{*-1} \\
\mathbb{E}_{t-1}[MC_t] &= \sum_{s^t\in A} p(s^t \mid s^0)MC_t \\
\mathbb{E}_{t-1}[MC_t^*] &= \sum_{s^t\in A} p(s^t \mid s^0)MC_t^*
\end{aligned}$$

1.A.5.3 Optimal Discretion with National Currencies

Now consider optimal monetary policy under discretion, the monetary authority maximizes

$$\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}}\Big[\sum_{t=k}^{\infty}\sum_{s^t\in A}\beta^{t-k}p(s^t \mid s^t)\Big(\ln(C_t)-\kappa L_t\Big)\Big]$$

The decisive difference to the optimization problem before is that the information set (inside the probability function) is for period t not $t-1$. The problem is subject to all equilibrium conditions. Plugging these in as before, the central bank has to maximize

$$\begin{aligned}
\max_{\mu_t(s^t)} \quad & \ln\Big(\Big(\frac{1}{1+\varpi}\Big)^{1-\gamma}\gamma_w\big(\Phi\big)^{-\gamma}\big(\Phi\big)^{-(1-\gamma)}\Big(\frac{1-\gamma}{\gamma}\Big)^{-(1-\gamma)}\Big) + \ln(\mu_t(s^t)) \\
& - (1-\gamma)(\ln(\mu_t(s^t)) - \ln(\mu_t^*(s^t))) - \gamma\ln\Big(\sum_{s^t\in A} p(s^t \mid s^0)\kappa a_t^{-1}\mu_t(s^t)\Big) \\
& - (1-\gamma)\ln\Big(\sum_{s^t\in A} p(s^t \mid s^0)\kappa^* a_t^{*-1}\mu_t^*(s^t)\Big) \\
& - \kappa\frac{1}{\Phi\kappa}\frac{1+\gamma\varpi}{1+\varpi}\Big(\frac{\kappa a_t^{-1}\mu_t(s^t)}{\sum_{s^t\in A}p(s^t \mid s^{t-1})\kappa a_t^{-1}\mu_t(s^t)}\Big).
\end{aligned}$$

In this case labor is not just a constant and the focs with respect to monetary policy in state $\bar{A}$ are

$$\frac{1}{\mu_t(\bar{A})} - \frac{(1-\gamma)}{\mu_t(\bar{A})} - \gamma \frac{\kappa a_t(\bar{A})^{-1}}{\sum_{s^t \in A} p(s^t \mid s^0) \kappa a_t^{-1} \mu_t(s^t)}$$

$$-\kappa \frac{1}{\Phi\kappa} \frac{1+\gamma\varpi}{1+\varpi} \left[\frac{\kappa a_t(\bar{A})^{-1}}{\sum_{s^t \in A} p(s^t \mid s^0) \kappa a_t^{-1} \mu_t(s^t)} - \frac{\kappa a_t(\bar{A})^{-1} \mu_t(\bar{A}) \kappa a_t(\bar{A})^{-1}}{\left(\sum_{s^t \in A} p(s^t \mid s^0) \kappa a_t^{-1} \mu_t(s^t)\right)^2} \right] = 0$$

Rearrange to get

$$\frac{\gamma}{\mu_t(\bar{A})} - \gamma \frac{a_t(\bar{A})^{-1}}{\sum_{s^t \in A} p(s^t \mid s^0) a_t^{-1} \mu_t(s^t)}$$

$$= \underbrace{\frac{1+\gamma\varpi}{\Phi(1+\varpi)}}_{\Theta^N} \frac{a_t(\bar{A})^{-1}}{\sum_{s^t \in A} p(s^t \mid s^0) a_t^{-1} \mu_t(s^t)} \left[1 - \frac{a_t(\bar{A})^{-1} \mu_t(\bar{A})}{\sum_{s^t \in A} p(s^t \mid s^0) a_t^{-1} \mu_t(s^t)} \right]$$

The solution of this problem in general differs from $a_t^{-1} \mu_t(s^t) = \sum_{s^t \in A} p(s^t \mid s^0) a_t^{-1} \mu_t(s^t)$. Rearrange a bit and use the notation with the expectation operator again:

$$\frac{\gamma}{\mu_t(\bar{A})} = \gamma \frac{a_t(\bar{A})^{-1}}{\mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)} + \Theta^N \frac{a_t(\bar{A})^{-1}}{\mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)} \left[1 - \frac{a_t(\bar{A})^{-1} \mu_t(\bar{A}))}{\mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)} \right]$$

$$\Rightarrow \frac{\gamma}{\Theta^N} = \left(1 - \frac{a_t(\bar{A})^{-1} \mu_t(\bar{A})}{\mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)} + \frac{\gamma}{\Theta^N} \right) \frac{a_t(\bar{A})^{-1}}{\mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)}$$

Optimal monetary policy in state $\bar{A}$ is hence characterized by

$$\frac{a_t(\bar{A})^{-1} \mu_t(\bar{A})}{\mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)} = \frac{\gamma}{\Theta^N}$$

If

$$\frac{\gamma}{\Theta^N} = 1 \qquad \text{then} \qquad a_t^{-1} \mu_t(s^t) = \mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)$$

There is no bias in the monetary policy decision rule and output and employment gaps are closed. Even under discretion monetary policy puts the economy to its first best. If

$$\frac{\gamma}{\Theta^N} > 1, \qquad \text{then} \qquad a_t^{-1} \mu_t(s^t) > \mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)$$

Monetary policy has an inflationary bias, as the size of the domestic economy (or the preference for domestic goods consumption, depending on your interpretation) γ is so great, that the central bank cares more about domestic markups. If

$$\frac{\gamma}{\Theta^N} < 1 \qquad \text{then} \qquad a_t^{-1} \mu_t(s^t) < \mathbb{E}_{t-1} a_t^{-1} \mu_t(s^t)$$

Monetary policy has a deflationary bias. The domestic economy is relatively unimportant for consumers welfare and the central bank tries to make foreign goods cheaper via terms of trade movements. (explanation via markups is analogous, noting that smaller values for $\Theta^N < 1$ imply higher markups.

The reason for the bias under monetary policy under discretion is that firms anticipate that monetary policy wants to use surprise policies. In the case of an inflationary bias monetary policy tries to inflate away the domestic markup when firms cannot react anymore. Anticipating that, domestic firms already increase the price before. The deflationary bias stems from the desire of the central bank to use surprise terms of trade movements to make non-domestic goods cheaper. Under PCP foreign firms still receive the same price, but domestic consumers have to pay less.

1.A.5.4 Optimal Discretion in a Union

Now consider the central bank in F, that acts under discretion. This means that the information set of the expectation operator in the maximization problem is for period t and not for period $t-1$. The objective function is therefore:

$$\min \mathbb{E}_t[W_t^{flex} - W_t] = \min \mathbb{E}_t\left[\ln\left(C_t^{flex}/C_t\right) - \kappa L_t^{flex} + \kappa L_t\right]$$

Under discretion monetary policy is characterized by the following rule:

$$\mu_t^* = \frac{\gamma}{\Theta^{*N}} \frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}$$

where $\frac{\gamma}{\Theta^{*N}}$ is a bias[20] stemming from discretionary policy. As discussed by Corsetti and Pesenti (2001), this bias can either be inflationary or deflationary. If $\frac{\gamma}{\Theta^{*N}} = 1$, then there is no bias, if $\frac{\gamma}{\Theta^{*N}} > 1$ there is an inflationary bias. As domestic markups are very important for the welfare of the agents in the economy, the central bank tries to inflate away the monopolistic markups. That is, when Θ^{*N} is small and/or when γ is very large. In contrast a deflationary bias arises, if $\frac{\gamma}{\Theta^{*N}} < 1$. In that case domestic markups and domestic goods in general are less important and the central bank tries to deflate the value of the currency such that domestic consumers can buy more non-domestic goods. This case is in particular relevant, if γ is low. That is if consumers have a strong preference for non-domestic goods.

A common central bank maximizes a weighted sum of both countries' lifetime utility

$$\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} \left[\xi \sum_{t=k}^{\infty}\sum_{s^t\in A} \beta^{t-k} p(s^t|s^t)\left(\ln(C_t) - \kappa L_t\right) + (1-\xi)\sum_{t=k}^{\infty}\sum_{s^t\in A} \beta^{t-k} p(s^t|s^t)\left(\ln(C_t^*) - \kappa^* l_t^*\right)\right]$$

20. $\Theta^{*N} = \frac{1+\gamma\varpi}{\Phi^*(1+\varpi)}$, $\Phi^* = \frac{\theta^*}{(\theta^*-1)(1-\tau)}$

subject to the equilibrium conditions in a currency union:

$$\begin{aligned}
MC_t &= \kappa a_t^{-1}\mu_t^U \\
MC_t^* &= \kappa^* a_t^{*-1}\mu_t^U \\
C_t &= \frac{\gamma_w \mu_t^U \mathcal{E}_t^{-1(1-\gamma)}}{(\Phi\mathbb{E}_{t-1}[MC_t])^{\gamma}(\Phi^*\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}} \\
C_t^* &= \frac{\gamma_w \mu_t^U \mathcal{E}_t^{1-\gamma}}{(\Phi\mathbb{E}_{t-1}[MC_t])^{1-\gamma}(\Phi^*\mathbb{E}_{t-1}[MC_t^*])^{\gamma}} \\
L_t(h) &= \frac{1}{\kappa\Phi}\Big(\gamma\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + (1-\gamma)\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\Big) \\
L_t^*(f) &= \frac{1}{\kappa^*\Phi^*}\Big(\gamma\frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]} + (1-\gamma)\frac{MC_t^*}{\mathbb{E}_{t-1}[MC_t^*]}\Big)
\end{aligned}$$

Recall that $\Phi\frac{1+\gamma\varpi}{1+\varpi} = \Theta^N$ and let $\Phi = \Theta^U$. As the markups in the union do not contain any trade costs $\Theta^U < \Theta^N$. The central bank maximizes

$$\begin{aligned}
\max_{\{\mu_t(s^t)\}_{t=k}^{\infty}} \Big[\xi\Big(&\ln(\frac{\gamma_w\mu_t^U}{(\sum_{s^t\in A}p(s^t \mid s^{t-1})\kappa a_t^{-1}\mu_t^U)^{\gamma}(\sum_{s^t\in A}p(s^t \mid s^{t-1})\kappa^* a_t^{*-1}\mu_t^U)^{1-\gamma}}) \\
&-\Theta^U\Big(\frac{\kappa a_t^{-1}\mu_t^U}{\sum_{s^t\in A}p(s^t \mid s^{t-1})\kappa a_t^{-1}\mu_t^U}\Big)\Big) \\
&+(1-\xi)\Big(\ln(\frac{\gamma_w\mu_t^U}{(\sum_{s^t\in A}p(s^t \mid s^{t-1})\kappa a_t^{-1}\mu_t^U)^{1-\gamma}(\sum_{s^t\in A}p(s^t \mid s^{t-1})\kappa^* a_t^{*-1}\mu_t^U)^{\gamma}}) \\
&-\Theta^{*U}\Big(\frac{\kappa^* a_t^{*-1}\mu_t^U}{\sum_{s^t\in A}p(s^t \mid s^{t-1})\kappa^* a_t^{*-1}\mu_t^U}\Big)\Big)\Big]
\end{aligned}$$

The first order conditions are

$$\begin{aligned}
&\xi\Big[\frac{1}{\mu(\bar{A})} - \frac{\gamma a_t^{-1}(\bar{A})}{\sum_{s^t\in A}p(s^t \mid s^{t-1})a_t^{-1}\mu_t^U} - \frac{(1-\gamma)a_t^{*-1}(\bar{A})}{\sum_{s^t\in A}p(s^t \mid s^{t-1})a_t^{*-1}\mu_t^U} \\
&-\Theta^U\Bigg(\frac{a_t^{-1}(\bar{A})}{\sum_{s^t\in A}p(s^t \mid s^{t-1})a_t^{-1}\mu_t^U} - \frac{a_t^{-1}(\bar{A})\mu_t(\bar{A})a_t^{-1}(\bar{A})}{(\sum_{s^t\in A}p(s^t \mid s^{t-1})a_t^{-1}\mu_t^U)^2}\Bigg) \\
&+(1-\xi)\Big[\frac{1}{\mu(\bar{A})} - \frac{(1-\gamma)a_t^{-1}(\bar{A})}{\sum_{s^t\in A}p(s^t \mid s^{t-1})a_t^{-1}\mu_t^U} - \frac{\gamma a_t^{*-1}(\bar{A})}{\sum_{s^t\in A}p(s^t \mid s^{t-1})a_t^{*-1}\mu_t^U} \\
&-\Theta^{*U}\Bigg(\frac{a_t^{*-1}(\bar{A})}{\sum_{s^t\in A}p(s^t \mid s^{t-1})a_t^{*-1}\mu_t^U} - \frac{a_t^{*-1}(\bar{A})\mu_t(\bar{A})a_t^{*-1}(\bar{A})}{(\sum_{s^t\in A}p(s^t \mid s^{t-1})a_t^{*-1}\mu_t^U)^2}\Bigg) = 0
\end{aligned}$$

Rearrange and compare to the solution before

$$\begin{aligned}
\frac{1}{\mu_t(\bar{A})} &= \big(\xi\gamma+(1-\xi)(1-\gamma)\big)\frac{a_t^{-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)\frac{a_t^{*-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]} \\
&+\Theta^U\xi\left(1-\frac{a_t^{-1}(\bar{A})\mu_t(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\right)\frac{a_t^{-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \Theta^{*U}(1-\xi)\left(1-\frac{a_t^{*-1}(\bar{A})\mu_t(\bar{A})}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}\right)\frac{a_t^{*-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}
\end{aligned}$$

The first row is the same as under commitment in a monetary union, while the second one represents the inflationary or deflationary bias .

Consider a state where both countries have the same productivity: $a_t(\bar{A}) = a_t^*(\bar{A})$,

$$\frac{1}{\mu_t(\bar{A})} = \frac{a_t^{-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \left(\xi\Theta^U + (1-\xi)\Theta^{*U}\right)\frac{a_t^{-1}(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\left[1 - \frac{a_t^{-1}(\bar{A})\mu_t(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\right]$$

This can be rearranged in the same way as before for $\Theta^U = \Theta^{*U}$

$$\frac{1}{\Theta^U} = \frac{a_t^{-1}}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\left[1 - \frac{a_t^{-1}\mu_t}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \frac{1}{\Theta^U}\right]$$

The solution is

$$\frac{1}{\Theta^U} = \frac{a_t^{-1}(\bar{A})\mu_t(\bar{A})}{\mathbb{E}_{t-1}a_t^{-1}\mu_t}$$

Compare this to the discretionary monetary policy in H outside the union:

$$\frac{\gamma}{\Theta^N} = \frac{a_t(\bar{A})^{-1}\mu_t(\bar{A})}{\mathbb{E}_{t-1}a_t^{-1}\mu_t}$$

For the first best allocation we know that the LHS must be one. We know that $\Theta^N < \Theta^U$, but $\gamma < 1$. This means that there are only gains of a union, if the drop in markups is sufficiently large. As the deflationary bias stemming from incentives to manipulate the exchange rate is removed, the mitigating effect for the inflationary bias disappears. then markup is lower because of lower trade costs + if asymmetric markup shocks, bias of MP is lower.

Consider an asymmetric shock. In such a case the bias from markups of the boom country leads to and inflationary bias as $\left(1 - \frac{a_t^{-1}(\bar{A})\mu_t(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\right) > 0$ while the recession country induces a deflationary bias s $\left(1 - \frac{a_t^{-1}(\bar{A})\mu_t(\bar{A})}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\right) < 0$

1.A.6 Closed form solution of Consumption and Labor

1.A.6.1 National Currency under Commitment

Plug in monetary policy in a world with national currencies only:

$$C_t = \left(\frac{1}{1+\varpi}\right)^{1-\gamma} \frac{\gamma_w \frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}} \left(\frac{\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}}\right)^{-1(1-\gamma)}}{(\Phi\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t])^{\gamma}(\Phi^*\mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu_t^*])^{1-\gamma}}$$

$$\Rightarrow C_t = \frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w a_t \left(\frac{a_t}{a_t^*}\right)^{\gamma-1}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa^*)^{(1-\gamma)}} = \frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w a_t^{\gamma} a_t^{*(1-\gamma)}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa^*)^{(1-\gamma)}}$$

Foreign Consumption

$$C_t^* = \frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma} \gamma_w a_t^{1-\gamma} a_t^{*\gamma}}{(\Phi\kappa)^{1-\gamma}(\Phi^*\kappa)^{\gamma}}$$

If you plug int both forms of consumption into the exchange rate condition, this equation is true, because the exchange rate is augmented by $(1-\gamma)/\gamma$ labor is given by:

$$L_t(h) = \frac{1}{\Phi\kappa}\left(\gamma \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\right)$$

Plugging in monetary policy:

$$\begin{aligned}
L_t(h) &= \frac{1}{\Phi\kappa}\left(\gamma \frac{\kappa a_t^{-1}\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{\kappa a_t^{-1}\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t]}\right) \\
\Rightarrow L_t(h) &= \frac{1}{\Phi\kappa}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right)
\end{aligned}$$

1.A.6.2 National Currency under Discretion

Plug in monetary policy in a world with national currencies only:

$$\begin{aligned}
C_t &= \left(\frac{1}{1+\varpi}\right)^{1-\gamma} \frac{\gamma_w \frac{\gamma}{\Theta^N}\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}\left(\frac{\frac{\frac{\gamma}{\Theta^N}\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\frac{\frac{\gamma}{\Theta^{*N}}\mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}}\right)^{-1(1-\gamma)}}{(\Phi\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t])^{\gamma}(\Phi^*\mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu_t^*])^{1-\gamma}} \\
\Rightarrow C_t &= \frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w \frac{\gamma}{\Theta^N} a_t \left(\frac{a_t}{a_t^*}\right)^{\gamma-1}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa^*)^{(1-\gamma)}} = \left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w \frac{\gamma}{\Theta^N}\frac{a_t^{\gamma} a_t^{*(1-\gamma)}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa)^{(1-\gamma)}}
\end{aligned}$$

To keep the expression tractable, I assumed that $\frac{\gamma}{\Theta^N} = \frac{\gamma}{\Theta^{*N}}$. Foreign Consumption is

$$C_t^* = \left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma_w \frac{\gamma}{\Theta^N}\frac{a_t^{1-\gamma} a_t^{*\gamma}}{(\Phi\kappa)^{1-\gamma}(\Phi^*\kappa)^{*\gamma}}$$

If you plug int both forms of consumption into the exchange rate condition, this equation is true, because the exchange rate is augmented by $(1-\gamma)/\gamma$. Labor is given by:

$$L_t(h) = \frac{1}{\Phi\kappa}\left(\gamma \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\right)$$

Plugging in monetary policy:

$$\begin{aligned}
L_t(h) &= \frac{1}{\Phi\kappa}\left(\gamma \frac{\kappa a_t^{-1}\frac{\gamma}{\Theta^N}\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t]} + \frac{(1-\gamma)}{1+\varpi}\frac{\kappa a_t^{-1}\frac{\gamma}{\Theta^N}\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t]}\right) \\
\Rightarrow L_t(h) &= \frac{1}{\Phi\kappa}\frac{\gamma}{\Theta^N}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right) \\
\Rightarrow L_t^*(f) &= \frac{1}{\Phi^*\kappa}\frac{\gamma}{\Theta^{*N}}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right)
\end{aligned}$$

1.A.6.3 National Currency, Commitment in H and Discretion in F

Plug in monetary policy in a world with national currencies only:

$$C_t = \Big(\frac{1}{1+\varpi}\Big)^{1-\gamma} \frac{\gamma_w \frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}} \left(\frac{\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\frac{\frac{\gamma}{\Theta^{*N}} \mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}} \right)^{-1(1-\gamma)}}{(\Phi \mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t])^{\gamma} (\Phi^* \mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu_t^*])^{1-\gamma}}$$

$$\Rightarrow C_t = \frac{\Big(\frac{1}{1+\varpi}\Big)^{1-\gamma} \gamma_w a_t \Big(\frac{a_t}{a_t^* \frac{\gamma}{\Theta^{*N}}}\Big)^{\gamma-1}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa^*)^{(1-\gamma)}} = \Big(\frac{1}{1+\varpi}\frac{\gamma}{\Theta^{*N}}\Big)^{1-\gamma} \gamma_w \frac{a_t^{\gamma} a_t^{*(1-\gamma)}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa^*)^{(1-\gamma)}}$$

If only one country has a bias, it is transmitted through the exchange rate. Foreign Consumption is

$$C_t^* = \Big(\frac{1}{1+\varpi}\Big)^{1-\gamma} \frac{\gamma_w \frac{\gamma}{\Theta^{*N}} \frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}} \left(\frac{\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\frac{\frac{\gamma}{\Theta^{*N}} \mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}} \right)^{1-\gamma}}{(\Phi \mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t])^{1-\gamma} (\Phi^* \mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu_t^*])^{\gamma}}$$

$$\Rightarrow C_t^* = \frac{\Big(\frac{1}{1+\varpi}\Big)^{1-\gamma} \gamma_w \frac{\gamma}{\Theta^{*N}} a_t \Big(\frac{a_t}{a_t^* \frac{\gamma}{\Theta^{*N}}}\Big)^{1-\gamma}}{(\Phi\kappa)^{1-\gamma}(\Phi^*\kappa^*)^{\gamma}} = \Big(\frac{1}{1+\varpi}\Big)^{1-\gamma} \gamma_w \left(\frac{\gamma}{\Theta^{*N}}\right)^{\gamma} \frac{a_t^{1-\gamma} a_t^{*\gamma}}{(\Phi\kappa)^{1-\gamma}(\Phi^*\kappa^*)^{\gamma}}$$

Both countries end up consuming less of the non-domestic good. If you plug in both forms of consumption into the exchange rate condition, this equation is true, because the exchange rate is augmented by $(1-\gamma)/\gamma$. Labor is given by:

$$L_t(h) = \frac{1}{\Phi\kappa}\Big(\gamma \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]} + \frac{(1-\gamma)}{1+\varpi} \frac{MC_t}{\mathbb{E}_{t-1}[MC_t]}\Big)$$

Plugging in monetary policy for H

$$L_t(h) = \frac{1}{\Phi\kappa}\Big(\gamma \frac{\kappa a_t^{-1} \frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t]} + \frac{(1-\gamma)}{1+\varpi} \frac{\kappa a_t^{-1} \frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}}{\mathbb{E}_{t-1}[\kappa a_t^{-1}\mu_t]}\Big)$$

$$\Rightarrow L_t(h) = \frac{1}{\Phi\kappa}\Big(\gamma + \frac{1-\gamma}{1+\varpi}\Big)$$

and for F Plugging in monetary policy:

$$L_t^*(f) = \frac{1}{\Phi^*\kappa^*}\Big(\gamma \frac{\kappa^* a_t^{*-1} \frac{\gamma}{\Theta^{*N}} \frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t^*]}{a_t^{*-1}}}{\mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu_t^*]} + \frac{(1-\gamma)}{1+\varpi} \frac{\kappa^* a_t^{*-1} \frac{\gamma}{\Theta^{*N}} \frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}{a_t^{*-1}}}{\mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu_t]}\Big)$$

$$\Rightarrow L_t^*(f) = \frac{1}{\Phi^*\kappa^*}\frac{\gamma}{\Theta^{*N}}\Big(\gamma + \frac{1-\gamma}{1+\varpi}\Big)$$

1.A.6.4 Currency Union

Plug in monetary policy in a world with a currency union.

Now calculate consumption

$$C_t = \frac{\gamma_w\Big(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)\frac{a_t^{-1}}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)\frac{a_t^{*-1}}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}\Big)^{-1}}{(\Phi\mathbb{E}_{t-1}[MC_t])^{\gamma}(\Phi^*\mathbb{E}_{t-1}[MC_t^*])^{1-\gamma}}$$

$$C_t = \frac{\gamma_w\Big(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)a_t^{-1} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)a_t^{*-1}\Big)^{-1}}{(\Phi\kappa)^{\gamma}(\Phi^*\kappa^*)^{1-\gamma}}$$

The last step only works, if shocks are iid, such that $\mathbb{E}_{t-1}[a_t^{*-1}\mu_t] = \mathbb{E}_{t-1}[a_t^{-1}\mu_t]$. If not, keep it and compute numerically. With $C_t = C_{H,t}^{\gamma}C_{F,t}^{1-\gamma}$, consumption of Home and foreign goods is:

$$C_{H,t} = \frac{\gamma\Big(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)a_t^{-1} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)a_t^{*-1}\Big)^{-1}}{\Phi\kappa}$$

$$C_{F,t} = \frac{(1-\gamma)\Big(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)a_t^{-1} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)a_t^{*-1}\Big)^{-1}}{\Phi^*\kappa^*}$$

Labor in a currency union is:

$$L_t(h) = \frac{1}{\Phi\kappa}\Big(\gamma\frac{a_t^{-1}\mu_t}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + (1-\gamma)\frac{a_t^{-1}\mu_t}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\Big)$$

$$L_t(h) = \frac{1}{\Phi\kappa}\left(\frac{a_t^{-1}\Big(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)\frac{a_t^{-1}}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)\frac{a_t^{*-1}}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}\Big)^{-1}}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}\right)$$

$$L_t(h) = \frac{1}{\Phi\kappa}\left(\frac{a_t^{-1}}{\big(\xi\gamma+(1-\xi)(1-\gamma)\big)a_t^{-1} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)a_t^{*-1}}\right)$$

1.A.7 Allocation and Monetary Policy in Corsetti and Pesenti (2002)

In Corsetti and Pesenti (2002), the consumption basket is symmetric and both countries weight good H with γ:

$$C_t = C_{H,t}^{\gamma}C_{F,t}^{1-\gamma}, \quad C_t^* = C_{H,t}^{*\gamma}C_{F,t}^{*1-\gamma}$$

As a result, the benchmark allocations are different:

Social Planner:

$$\text{Consumption} \qquad C_{H,t} = \frac{1}{2\kappa}a_t \; C^*_{H,t} = \frac{1}{2\kappa}a_t$$

$$C_{F,t} = \frac{1}{2\kappa^*}a^*_t \; C^*_{F,t} = \frac{1}{2\kappa^*}a^*_t$$

$$\text{Labor} \qquad L_t = \frac{1}{\kappa} \; L^*_t = \frac{1}{\kappa^*}$$

Flexible Prices (National Currencies)

$$\text{Consumption} \qquad C_{H,t} = \frac{\gamma}{\kappa}a_t \; C^*_{H,t} = \frac{1-\gamma}{\kappa}a_t$$

$$C_{F,t} = \frac{\gamma}{\kappa^*}a^*_t \; C^*_{F,t} = \frac{1-\gamma}{\kappa^*}a^*_t$$

$$\text{Labor} \qquad L_t = \frac{1}{\kappa} \; L^*_t = \frac{1}{\kappa^*}$$

Sticky Prices (National Currencies)

$$\text{Monetary Policy} \qquad \mu_t = \frac{\mathbb{E}[a_t^{-1}\mu_t]}{a_t^{-1}}$$

$$\text{Consumption} \qquad C_t = \frac{\left(\frac{1}{1+\varpi}\right)^{1-\gamma}\gamma a_t^{1-\gamma}a_t^{*\gamma}}{\kappa^{1-\gamma}\kappa^{*\gamma}} \; C^*_t = \frac{\left(\frac{1}{1+\varpi}\right)^{\gamma}(1-\gamma)a_t^{1-\gamma}a_t^{*\gamma}}{\kappa^{1-\gamma}\kappa^{*\gamma}}$$

$$\text{Labor} \qquad L_t = \frac{1}{\kappa}\left(\gamma + \frac{1-\gamma}{1+\varpi}\right) \; L^*_t = \frac{1}{\kappa}\left(\frac{\gamma}{1+\varpi} + (1-\gamma)\right)$$

Sticky Prices (Currency Union)

$$\text{Monetary Policy} \qquad \mu_t = \left(\gamma\frac{a_t^{-1}}{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]} + \frac{(1-\gamma)a_t^{*-1}}{\mathbb{E}_{t-1}[a_t^{*-1}\mu_t]}\right)^{-1}$$

$$\text{Consumption} \qquad C_t = \frac{\gamma a_t^{1-\gamma}a_t^{*\gamma}}{\kappa^{1-\gamma}\kappa^{*\gamma}} \; C^*_t = \frac{(1-\gamma)a_t^{1-\gamma}a_t^{*\gamma}}{\kappa^{1-\gamma}\kappa^{*\gamma}}$$

$$\text{Labor} \qquad L_t = \frac{1}{\kappa}\frac{a_t^{-1}}{\gamma a_t^{-1} + (1-\gamma)a_t^{*-1}} \; L^*_t = \frac{1}{\kappa}\frac{a_t^{*-1}}{\gamma a_t^{-1} + (1-\gamma)a_t^{*-1}}$$

1.A.8 The Current Account and Risk Sharing

As in the model of Corsetti and Pesenti (2001) the current account is balanced in all points in time, if the model is initialized with zero wealth. This goes back to work by Cole and Obstfeld (1991) and is reflected in the model: Consumption risk is shared efficiently, there is no need for debts or savings in certain states. Intuitively, risk sharing is ensured via endogenous terms of trade movements. The terms of trade are defined as the relative price of domestic imports in terms of domestic exports, in case of PCP, as in (1.5) $\mathcal{T} = \mathcal{E}P^*_F/P_H$ Consider a productivity boom in H. In such a situation an expansionary policy is optimal for the central bank in H.

Therefore, the exchange rate depreciates, terms of trade depreciate as well. With one unit of F's currency, more units of H's currency can now be bought. Productivity has increased the production of h-type goods, therefore the nominal value of H's exports measured in its currency has increased. At the same time, it has become more expensive for H to buy non-domestic goods. In this special setup[21] the nominal value of exports always equals the nominal value of imports due to that mechanism. Note that F has to pay less for h-type goods in terms of F's currency due to H's exchange rate depreciation. Therefore, even though F does not produce more goods, it can afford to buy more h-type goods without running a current account deficit. With this mechanism in place H's productivity increase spills over to the other country.
In a currency union the exchange rate is fixed and terms of trade movements cannot absorb any asymmetric shocks hitting the economy. Another mechanism of the model makes sure that in such a situation the current account is balanced: As the central bank stabilizes the average of the economy, wedges in the labor market occur. For the boom country monetary policy is not expansionary enough creating a negative wedge, while for the recession country it is too expansionary creating a positive wedge. As a result, employment in the recession country is higher and in the boom country it is lower. With the special setup considered in this paper, overall production of both countries in the currency union is the same. Current accounts are therefore also balanced with asymmetric shocks.

1.A.9 International Relative Prices

Balanced Current Accounts

The current account is balanced all the time for both monetary regimes, value of imports equal values of exports:

$$
\begin{aligned}
P_{F,t}C_{F,t} &= \mathscr{E}_t P^*_{H,t}C^*_{H,t} \\
\Phi^*(1+\varpi)\mathscr{E}_t\mathbb{E}_{t-1}[MC^*_t]\frac{(1-\gamma)a^*_t}{\Phi^*(1+\varpi)\kappa^*} &= \Phi\mathscr{E}_t(1+\varpi)\frac{1}{\mathscr{E}_t}\mathbb{E}_{t-1}[MC_t]\frac{(1-\gamma)a_t}{\Phi(1+\varpi)\kappa}
\end{aligned}
$$

Trade costs cancel each other out, they do not matter for a balanced current account. Plugging in marginal costs and the equilibrium exchange rate gives

$$
\begin{aligned}
\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}/\frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu^*_t]}{a_t^{*-1}}\mathbb{E}_{t-1}[\kappa^* a_t^{*-1}\mu^*_t]\frac{(1-\gamma)a^*_t}{\kappa^*} &= \mathbb{E}_{t-1}[a_t^{-1}\mu_t\kappa]\frac{(1-\gamma)a_t}{\kappa} \\
\Big(\frac{\mathbb{E}_{t-1}[a_t^{*-1}\mu^*_t]}{a_t^{*-1}}\Big)^{-1}\mathbb{E}_{t-1}[a_t^{*-1}\mu^*_t]a^*_t &= \left(\frac{\mathbb{E}_{t-1}[a_t^{-1}\mu_t]}{a_t^{-1}}\right)^{-1}\mathbb{E}_{t-1}[a_t^{-1}\mu_t]a_t
\end{aligned}
$$

21. The elasticity of substitution between Home and foreign goods is 1, as is the intertemporal elasticity. Furthermore, firms use producer currency pricing.

which is true. Intuitively, in a world with producer currency pricing and elasticity of substitution of 1 between Home and foreign goods, terms of trade movements make sure that risk is perfectly pooled in that economy. This means that the current account between both countries is balanced all the time.

Consumption Risk Sharing

Each country consumes a constant fraction of the produced good in all regimes, as given by the analytic expression for consumption.

1.A.10 Consumption, Prices and Labor with Transfers

With transfers from the union-wide planner (superscript P) that benefit country F, consumption of Home agents is lower with transfers: $C_t^P = C_t^U - T_t$. Production needs to satisfy this new demand

$$
\begin{aligned}
y_t^P(h) &= \left(\frac{\gamma}{\gamma_w}(C_t^U - T_t) + \frac{1-\gamma}{\gamma_w}(C_t^{*U} + T_t)\right) \\
y_t^P(f) &= \left(\frac{1-\gamma}{\gamma_w}(C_t^U - T_t) + \frac{\gamma}{\gamma_w}(C_t^{*U} + T_t)\right)
\end{aligned}
$$

With transfers going from H to F ($T_t > 0$) overall consumption is shifted from Home goods to foreign goods. As a result, employment in the foreign country increases while it decreases in the Home country, as long as each country has a Home bias ($\gamma > 0.5$).

$$
L_t^P = L_t^U + a_t^{-1}\frac{1-2\gamma}{\gamma_w}T_t, \qquad L_t^{*P} = L_t^{*U} - a_t^{*-1}\frac{1-2\gamma}{\gamma_w}T_t
$$

There is also an effect on prices, as firms expect the transfer scheme to be in place for the immediate future for most possible states of the world, see also Appendix 1.A.10. In the end, Home firms lower their prices for the next period as the demand for these goods gets lower, while prices of foreign goods increase. The terms of trade (1.5), defined as prices of foreign exports times the exchange rate over prices of Home exports permanently increase when transfers go to F. Recall that with a recession in F and a boom in H, the exchange rate immediately increases with national currencies: As H's monetary policy is optimally more expansive, the exchange rate (Home currency per foreign currency) goes up (H's currency becomes less valuable) and the terms of trade go up as well *permanently*. This might be an unwanted side effect. In the benchmark calibration, the effects are quantitatively very small, as transfers are very small as well. The Euler equation only changes, because of price changes. Lump-sum transfers do not directly distort the intertemporal decision of households. I assume that households do not anticipate the possibility of a 'regime change' in transfers before. That means, if there are zero transfers before, the model is solved as if households do not expect any changes in the transfer scheme before. As soon as the transfers are announced by the social planner, households take the transfers as given and form expectations about it. In the period of announcement, firms adjust their prices for next period taking the future transfers into account. Therefore in the period of transfer announcement, inflation jumps. Note however that this effect is also very small: If transfers go to F from H, prices of F goods rise, while prices of H goods fall. In the aggregate price index these effects partially offset each other. There are only minor effects, for the Foreign country that receives transfers, the aggregate price index goes slightly up, as for F Foreign goods are more important. For H the opposite holds.

Consumption

With transfers from the union-wide planner (superscript P), consumption becomes $C_t^P = C_t^U + T_t$ and $C_t^{*P} = C_t^{*U} - T_t$. The transfers are used by consumers such that the consumption of h goods and f goods changes. The ratio of h goods to the overall consumption bundle is still the same with that specification of preferences. Lump-sum transfers are not distortionary. The ratio is given by:

$$\begin{aligned}\frac{C_{H,t}^U}{C_t^U} &= \frac{\gamma\Big(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)a_t^{-1} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)a_t^{*-1}\Big)^{-1}}{\Phi\kappa} \\ &\left(\frac{\gamma\Big(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)a_t^{-1} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)a_t^{*-1}\Big)^{-1}}{\Phi\kappa}\right)^{-\gamma} \\ &\left(\frac{(1-\gamma)\Big(\big(\xi\gamma+(1-\xi)(1-\gamma)\big)a_t^{-1} + \big(\xi(1-\gamma)+(1-\xi)\gamma\big)a_t^{*-1}\Big)^{-1}}{\Phi^*\kappa^*}\right)^{-(1-\gamma)} \\ &= \frac{\gamma}{\gamma_w}\end{aligned}$$

Therefore, consumption of h by a Home agent is given by

$$\begin{aligned} C_{H,t}^P &= \frac{\gamma}{\gamma_w}(C_t^U - T_t), & C_{H,t}^{*P} &= \frac{1-\gamma}{\gamma_w}(C_t^{*U} + T_t), \\ C_{F,t}^P &= \frac{1-\gamma}{\gamma_w}(C_t^U - T_t), & C_{F,t}^{*P} &= \frac{\gamma}{\gamma_w}(C_t^{*U} + T_t) \end{aligned}$$

Prices

Firms know that in a transfer union demand will change. They maximize their profits, accounting for consumer's new demand including transfers. Note that the stochastic discount factor and marginal costs do not change, as lump-sum transfers do not distort the decision of households:

$$\begin{aligned}\max_{p_t^P(h),\tilde{p}_t^P(h)} \mathbb{E}_{t-1}[Q_{t-1,t}(((1-\tau)p_t^P(h) - MC_t)\left(\frac{p_t^P(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}^P + \\ ((1-\tau)\tilde{p}_t^P(h) - MC_t))\left(\frac{\tilde{p}_t^P(h)}{\tilde{P}_{H,t}}\right)^{-\theta} C_{H,t}^{*P}]\end{aligned}$$

Plug in demand

$$\max_{p_t^P(h),\tilde{p}_t^P(h)} \mathbb{E}_{t-1}\Big[Q_{t-1,t}\Big(((1-\tau)p_t^P(h)-MC_t)\Big(\frac{p_t^P(h)}{P_{H,t}}\Big)^{-\theta}\frac{\gamma}{\gamma_w}(C_t^U-T_t)$$
$$+((1-\tau)\tilde{p}_t^P(h)-MC_t))\Big(\frac{\tilde{p}_t^P(h)}{\tilde{P}_{H,t}}\Big)^{-\theta}\frac{1-\gamma}{\gamma_w}(C_t^{*U}+T_t)\Big)\Big]$$

Write the problem in state-contingent form, dropping the time index for simplicity:

$$\max_{p^P(h),\tilde{p}^P(h)}\sum_s p(s|s^{-1})\Big[Q_{t-1,t}(s_t)\Big(((1-\tau)p^P(h)-MC(s_t))\Big(\frac{p^P(h)}{P_H}\Big)^{-\theta}\frac{\gamma}{\gamma_w}(C^U(s_t)-T(s^t))$$
$$+((1-\tau)\tilde{p}(h)-MC(s_t)))\Big(\frac{\tilde{p}(h)}{\tilde{P}_H}\Big)^{-\theta}\frac{1-\gamma}{\gamma_w}(C^{*U}(s_t)+T(s^t))\Big)\Big]$$

The first order condition is with respect to $p^P(h)$

$$\sum_s p(s\mid s^{-1})\Big[Q_{t-1,t}(s_t)\left((1-\tau)(1-\theta)\Big(\frac{p^P(h)}{P_H}\Big)^{-\theta}+MC(s_t)\theta p^P(h)^{-1}\Big(\frac{p^P(h)}{P_H}\Big)^{-\theta}\right)$$
$$\left(\frac{\gamma}{\gamma_w}(C^U(s_t)-T(s^t))\right)\Big]=0$$

Due to symmetric firms we have $P_{H,t}=p_t(h)$. One period price stickiness means that the price $p^P(h)$ is predetermined and does not depend on the state. Therefore, we arrive at

$$p^P(h)=\frac{\theta}{(\theta-1)(1-\tau)}\frac{\sum_s p(s\mid s^{-1})\Big[Q_{t-1,t}(s_t)\left(MC(s_t)\left(\frac{\gamma}{\gamma_w}\left(C^U(s_t)-T(s^t)\right)\right)\right)\Big]}{\sum_s p(s\mid s^{-1})\Big[Q_{t-1,t}(s_t)\left(\left(\frac{\gamma}{\gamma_w}\left(C^U(s_t)-T(s^t)\right)\right)\right)\Big]}$$

Turn to the stochastic discount factor $Q_{t-1,t}(s_t)=\beta\frac{P_{t-1}C_{t-1}(s_t)}{P_tC_t(s_t)}$. Note, that this discount factor was derived from the Euler equation and is not a function of Transfers. The Transfers are lump-sum and do not distort household's intertemporal decision. Therefore, we arrive at

$$p^P(h)=\frac{\theta}{(\theta-1)(1-\tau)}\frac{\sum_s p(s\mid s^{-1})\Big[(MC(s_t)(\frac{\frac{\gamma}{\gamma_w}(C^U(s_t)-T(s^t))}{(C^U(s_t))}\Big]}{\sum_s p(s\mid s^{-1})\Big[\left(\frac{\frac{\gamma}{\gamma_w}(C^U(s_t)-T(s^t))}{(C^U(s_t))}\right)\Big]}$$

With $T(s^t)=0$, we arrive at the same condition for prices as before. As shown before, Transfers are a constant fraction of GDP, therefore $T(s^t)/C^U(s_t)$ is the same value for all states, except for the state, in which transfers reverse.

$$p^P(h)=\frac{\theta}{(\theta-1)(1-\tau)}\sum_s p(s\mid s^{-1})MC(s_t)$$

setting prices equal to expected marginal costs times the markup. For the price in the foreign market $\tilde{p}^P(h)$, the firm has the following first order condition

$$\sum_s p(s|s^{-1})\Big[Q_{t-1,t}(s_t)\left((1-\tau)(1-\theta)\Big(\frac{\tilde{p}^P(h))}{\tilde{P}_H^P}\Big)^{-\theta}+MC(s_t)\theta\tilde{p}^P(h)^{-1}\Big(\frac{\tilde{p}^P(h)}{\tilde{P}_H^P}\Big)^{-\theta}\right)\cdot \left(\frac{1-\gamma}{\gamma_w}(C^{*U}(s_t)+T(s^t))\right)\Big] = 0$$

Rearrange

$$-\sum_s p(s\mid s^{-1})\Big[Q_{t-1,t}(s_t)\left((1-\tau)(1-\theta)\right)\left(\frac{1-\gamma}{\gamma_w}(C^{*U}(s_t)+T(s^t))\right)\Big]$$
$$=\sum_s p(s\mid s^{-1})\Big[Q_{t-1,t}(s_t)\left(MC(s_t)\theta\tilde{p}^P(h)^{-1}\right)\cdot\left(\frac{1-\gamma}{\gamma_w}(C^{*U}(s_t)+T(s^t))\right)\Big]$$

As $\tilde{p}^P(h)$ is predetermined we can draw it out and arrive at

$$\tilde{p}^P(h) = \frac{\theta}{(\theta-1)(1-\tau)}\frac{\sum_s p(s\mid s^{-1})\Big[Q_{t-1,t}(s_t)\left(MC(s_t)\left(\frac{1-\gamma}{\gamma_w}\left(C^{*U}(s_t)+T(s^t)\right)\right)\right)\Big]}{\sum_s p(s\mid s^{-1})\Big[Q_{t-1,t}(s_t)\left(\left(\frac{1-\gamma}{\gamma_w}\left(C^{*U}(s_t)+T(s^t)\right)\right)\right)\Big]}$$
$$= \frac{\theta}{(\theta-1)(1-\tau)}\frac{\sum_s p(s\mid s^{-1})\Big[\left(MC(s_t)\left(\frac{\frac{1-\gamma}{\gamma_w}(C^{*U}(s_t)+T(s^t))}{C^U(s_t)}\right)\right)\Big]}{\sum_s p(s\mid s^{-1})\Big[\left(\frac{\frac{1-\gamma}{\gamma_w}(C^{*U}(s_t)+T(s^t))}{C^U(s_t)}\right)\Big]}$$

$C^{*U}(s_t)/C^U(s_t)$ are still the same in all states, $T(s^t)/C^U(s_t)$ is also the same in all states, except for the state with a huge asymmetric shock.

$$\max_{p_t^{*P}(f),p_t(f)} \mathbb{E}_{t-1}\left[Q^*_{t-1,t}(((1-\tau)p_t^{*P}(f)-MC_t^{*P})\Big(\frac{p_t^{*P}(f)}{P^*_{F,t}}\Big)^{-\theta}C^{*P}_{F,t}+((1-\tau)p_t(f)-MC_t^{*P}))\Big(\frac{p_t(f)}{P_{F,t}}\Big)^{-\theta}C^P_{F,t}\right]$$

The first order condition with respect to $p_t^{*P}(f)$ is

$$\sum_s p(s\mid s^{-1})\Big[Q^*_{t-1,t}(s_t)\left((1-\tau)(1-\theta)\Big(\frac{p^{*P}(f)}{P_F^*}\Big)^{-\theta}+MC(s_t)\theta p^{*P}(f)^{-1}\Big(\frac{p^{*P}(f)}{P_F^*}\Big)^{-\theta}\right)\cdot \left(\frac{\gamma}{\gamma_w}(C^{*U}(s_t)+T(s^t))\right)\Big] = 0$$

$$
\begin{aligned}
p^{*P}(f) &= \frac{\theta}{(\theta-1)(1-\tau)} \frac{\sum_s p(s \mid s^{-1})\Big[Q^*_{t-1,t}(s_t)\Big(MC^{*P}(s_t)\Big(\frac{\gamma}{\gamma_w}\big(C^{*U}(s_t)+T(s^t)\big)\Big)\Big)\Big]}{\sum_s p(s \mid s^{-1})\Big[Q^*_{t-1,t}(s_t)\Big(\Big(\frac{\gamma}{\gamma_w}\big(C^{*U}(s_t)+T(s^t)\big)\Big)\Big)\Big]} \\
&= \frac{\theta}{(\theta-1)(1-\tau)} \frac{\sum_s p(s \mid s^{-1})\Big[\Big(MC^{*P}(s_t)\Big(\frac{\gamma}{\gamma_w}\frac{(C^{*U}(s_t)+T(s^t))}{C^{*U}(s_t)+T(s^t)}\Big)\Big)\Big]}{\sum_s p(s \mid s^{-1})\Big[\Big(\Big(\frac{\gamma}{\gamma_w}\frac{(C^{*U}(s_t)+T(s^t))}{C^{*U}(s_t)+T(s^t)}\Big)\Big)\Big]} \\
&= \frac{\theta}{(\theta-1)(1-\tau)}\mathbb{E}[MC^{*P}_t]
\end{aligned}
$$

for foreign good prices in the Home country, we have

$$
\begin{aligned}
p^{P}(f) &= \frac{\theta}{(\theta-1)(1-\tau)} \frac{\sum_s p(s \mid s^{-1})\Big[Q^*_{t-1,t}(s_t)\Big(MC^{*P}(s_t)\Big(\frac{1-\gamma}{\gamma_w}\big(C^{U}(s_t)-T(s^t)\big)\Big)\Big)\Big]}{\sum_s p(s \mid s^{-1})\Big[Q^*_{t-1,t}(s_t)\Big(\Big(\frac{1-\gamma}{\gamma_w}\big(C^{U}(s_t)-T(s^t)\big)\Big)\Big)\Big]} \\
&= \frac{\theta}{(\theta-1)(1-\tau)} \frac{\sum_s p(s \mid s^{-1})\Big[\Big(MC^{*P}(s_t)\Big(\frac{1-\gamma}{\gamma_w}\frac{(C^{U}(s_t)-T(s^t))}{C^{*U}(s_t)+T(s^t)}\Big)\Big)\Big]}{\sum_s p(s \mid s^{-1})\Big[\Big(\Big(\frac{1-\gamma}{\gamma_w}\frac{(C^{U}(s_t)-T(s^t))}{C^{*U}(s_t)+T(s^t)}\Big)\Big)\Big]}
\end{aligned}
$$

With that we can calculate the corresponding national price indices under the planner regime

$$
P^P_t = \frac{P^{1-\gamma}_{F,t}P^{\gamma}_{H,t}}{\gamma^\gamma(1-\gamma)^{1-\gamma}}, \qquad P^{*P}_t = \frac{P^{*\gamma}_{F,t}P^{*1-\gamma}_{H,t}}{\gamma^\gamma(1-\gamma)^{1-\gamma}}.
$$

Labor

Firm stand ready to satisfy demand

$$
\begin{aligned}
L_t(h) &= a_t^{-1}\left(C^P_{H,t} + C^{*P}_{H,t}\right) \\
&= a_t^{-1}\left(\frac{\gamma}{\gamma_w}(C^U_t - T_t) + \frac{1-\gamma}{\gamma_w}(C^{*U}_t + T_t)\right)
\end{aligned}
$$

1.A.11 Nominal Equilibrium

The optimal monetary rules do not pin down the nominal equilibrium. To address this issue, I follow Corsetti and Pesenti (2005) and define two rules $\hat{\mu}_t$ and $\hat{\mu}^*_t$ such that:

$$
\hat{\mu}_t = \mu_t\left(\frac{P_{H,t}}{\bar{P}_H}\right)^{\delta}, \quad \hat{\mu}^*_t = \mu^*_t\left(\frac{P^*_{F,t}}{\bar{P}^*_F}\right)^{\delta^*}
$$

where $\delta, \delta^* < 0$ are two small negative constants and $\bar{P}_H, \bar{P}_F^*$ are nominal targets of the government. Consider the price of the Home good $P_{H,t}$:

$$\begin{aligned}
P_{H,t} &= \Phi \mathbb{E}_{t-1}[\kappa a_t^{-1} \hat{\mu}_t] \\
P_{H,t} &= \Phi \mathbb{E}_{t-1}\left[\kappa a_t^{-1} \mu_t \left(\frac{P_{H,t}}{\bar{P}_H}\right)^{\delta}\right] \\
P_{H,t} &= P_{H,t} \left(\frac{P_{H,t}}{\bar{P}_H}\right)^{\delta}
\end{aligned}$$

The solution to that is $P_{H,t} = \bar{P}_H$. Therefore, the governments set an anchor and credibly threatens to adjust monetary policy, if the price deviates from the target. Given the target for domestically produced goods, the prices for imported goods can be computed: Under PCP we have

$$\begin{aligned}
P_{F,t} &= \Phi^* \mathbb{E}[\kappa^* a_t^{*-1} \mu_t^*] \mathscr{E}_t \\
&= \Phi^* \mathbb{E}[\kappa^* a_t^{*-1} \mu_t^*] \frac{\mu_t}{\mu_t^*} \\
&= \Phi^* \mathbb{E}[\kappa^* a_t^{*-1} \mu_t^*] \frac{\frac{\mathbb{E}[a_t^{-1} \mu_t]}{a_t^{-1}}}{\frac{\mathbb{E}[a_t^{*-1} \mu_t^*]}{a_t^{*-1}}} \\
&= \Phi^* \kappa^* \frac{\frac{\frac{\kappa \Phi}{\kappa \Phi} \mathbb{E}[a_t^{-1} \mu_t]}{a_t^{-1}}}{\frac{1}{a_t^{*-1}}} \\
&= \Phi^* \kappa^* \frac{\frac{\frac{\kappa \Phi}{\kappa \Phi} \mathbb{E}[a_t^{-1} \mu_t \left(\frac{P_{H,t}}{\bar{P}_H}\right)^{\delta}]}{a_t^{-1}}}{\frac{1}{a_t^{*-1}}} \\
&= \Phi^* \kappa^* \frac{\frac{\frac{1}{\kappa \Phi} \bar{P}_H}{a_t^{-1}}}{\frac{1}{a_t^{*-1}}} \\
P_{F,t} &= \frac{\Phi^* \kappa^*}{\Phi \kappa} \bar{P}_H \frac{a_t^{*-1}}{a_t^{-1}}
\end{aligned}$$

Note that the non-domestic good price fluctuates because of the flexible exchange rate. For the currency union, the central bank just sets the anchor $\bar{P}_H$?

1.A.12 Problem with Two-Sided Limited Commitment

1.A.12.1 Functional Equation

Why can the Pareto frontier (1.19) be described by the recursive problem. The histories of the constraints are potentially large dimensional objects. Thomas and Worrall

(1988) show in their work that the problem can indeed be written as a recursive program. The dimensions are contained by using an accounting system cast solely in terms of promised utility. Promised utility is a state variable and summarizes all relevant aspects of an agent's history. With this we can formulate the problem recursively. Expected Lifetime utility for F is rewritten in utility today plus expected lifetime utility in the future. It is a function of promised utility u_s, the state variable of the problem. The constraints are also rewritten in this form. Bellman's principal of optimality states that if a program is optimal in t onwards for state s, it is also optimal in t+1 onward for all possible states.

A remarkable result is that the appropriate state variable (promised utility) equals future expected utility $u_s = \mathbb{E}_{t-1}[\sum_{j=0}^{\infty} \beta^j u(c_{t+j})]$. Why does promised utility equal the continuation value? Lemma 1 of Thomas and Worrall (1988) states that for each promised value $u_a \in [W^{*N}, W^{*Max}]$ there exists a unique continuation value of the contract δ at time t in which $W(\delta;(h^{t-1}, s_t)) = u_s$ and $W^*(\delta;(h^{t-1}, s_t)) = W^*(u_s)$. The proof is the following: Existence follows from the compactness of all possible future promises. Uniqueness from the convexity of all self-enforcing allocations Γ and the concavity of utility.

Utility in this setup is concave, increasing and continuously differentiable.

Γ is convex: Consider two self-enforcing contracts δ δ' that promise a sequence of consumption $\{C_t(\delta, s^t)\}_{t=0}^{\infty}, \{C_t(\delta', s^t)\}_{t=0}^{\infty}$. Let the convex combinations of both contracts be denoted by δ^{λ} with consumption streams $\{C_t(\delta^{\lambda}.s^t)\}_{t=0}^{\infty}$ By the concavity of utility, it holds that: $W(\delta^{\lambda}, s^t) \geq \lambda W(\delta, s^t) + (1-\lambda)W(\delta', s^t)$. Therefore, the convex combination of both sustainable contracts is sustainable as well.

Promised utility is compact: Proof for that: Let I_s b the set of feasible values of u_s. If $u_s \in I_s$, then $u'_s \in I_s \forall u'_s \in [W^N, u_s)$ is I_s closed? Consider a sequence $u_s^{\nu} \in I_s$ such that $\lim_{\nu\to\infty} u_s^{\nu} = u_s$ with a corresponding consumption stream (contract δ^{ν}). For a given parameterization, consumption is contained in an interval, say $[a, b]$, therefore the contract specifies only a countable number of consumption streams, the space of contracts is sequentially compact on the product topology. So, there is a subsequence of contracts converging pointwise to a limiting contract δ^{∞}. Since utility V is continuous and $\beta \in (0,1)$, by the dominated convergence theorem after any history the limit of the gain to an agent equals the gain from the limiting contract, for both agents. Therefore δ^{∞} is self-enforcing since each δ^{ν} is and gives promise utility of u_s.

1.A.12.2 Pareto Frontier is concave

Take any two sustainable surpluses $U(s^t)$ and $\hat{U}_s$. Following the same argument as made above, the convex combination $\alpha U_S + (1-\alpha)\hat{U}_s$ will offer H more than the average of these contracts and household F strictly more than the average of the original surpluses, because v is concave. Therefore each V_s is concave.

$V_s(U(s^t))$ is strictly decreasing in $U(s^t)$ on the whole interval $[\underline{U}_s, \bar{U}_s]$, since starting from any $U(s^t) > \underline{U}_s$, there must be some history h_t, such that $U_t(h_t) > 0$. A small increase in $T(s^t)$ leads to an increase in F's utility and a decrease in H's, while not violating the participation constraint. It follows that $U(s^{t+1}) \leq \bar{U}_r$ can be written as $V(s^{t+1}, U(s^{t+1})) \geq \underline{V}_r$.

1.A.12.3 The Lagrangian of the Dynamic Problem

The Lagrangian of the problem (1.19) is

$$\begin{aligned}
\mathscr{L} = \max_{T(s^t),(U(s^{t+1}))_{r=1}^S} & \ln\left(C^{*U}(s_t)+T(s^t)\right) - \kappa^* l^{*U}(s_t) - v^N(s_t) + \beta \sum_{r=1}^S p(s^{t+1} \mid s^t) V(s^{t+1}, U(s^{t+1})) \\
& + \lambda(s^t)\Big[\ln\left(C^U(s_t)-T(s^t)\right) - \kappa l^U(s_t) - u^N(s_t) + \beta \sum_{r=1}^S p(s^{t+1} \mid s^t) U(s^{t+1}) \geq U(s^t)\Big] \\
& + \beta p(s^{t+1} \mid s^t)\phi(s^{t+1}) U_r + \beta p(s^{t+1} | s^t)\zeta(s^{t+1}) V(s^{t+1}, U(s^{t+1}))
\end{aligned}$$

The first order conditions are

$$\begin{aligned}
\mathscr{L}_{T(s^t)} \; : \quad & v'_{T(s^t)} - \lambda u'_{T(s^t)} \; = \; 0 \\
& \Rightarrow \frac{u'}{v'} \; = \; \lambda \qquad \frac{1}{C^{*U}(s_t) + T(s^t)} - \lambda \frac{1}{C^U(s_t) - T(s^t)} \; = \; 0 \\
\mathscr{L}_{U(s^{t+1})} \; : \quad & \beta p(s^{t+1} \mid s^t) V'_r(U(s^{t+1})) + \lambda \beta p(s^{t+1} \mid s^t) + \beta p(s^{t+1} \mid s^t)\phi(s^{t+1}) \\
& + \beta p(s^{t+1} \mid s^t) V'_r(U(s^{t+1})) \; = \; 0 \\
& \Rightarrow \frac{\lambda(s^t) + \phi(s^{t+1})}{1 + \zeta(s^{t+1})} \; = \; -V'_r(U(s^{t+1})) \\
\text{Envelope Condition} \quad & \lambda(s^t) \; = \; V_s(U(s^t))
\end{aligned}$$

1.A.12.4 Intuition for Transfers

Note that when no new participation constraint binds, (1.24) tells us, that transfers are given by:

$$T(s^{t+1}) \; = \; \frac{C^U(s_{t+1}) - \lambda(s^t) C^{*U}(s_{t+1})}{1 + \lambda(s^t)}$$

Recall the perfect risk sharing property of the model, which tells us, that consumption of the Home and the foreign country in the Union are always the same. This means that as soon as the economy is in a synchronized boom (or a synchronized recession), both consumption values simultaneously increase (or decrease) by the same amount. Therefore transfers, that were obtained with the help of (1.24) increase (or decrease in a recession) by the same amount as consumption does. Therefore, transfers relative to GDP stay always the same, as long as no new participation constraint binds.

1.A.12.5 Differentiability of the Pareto Frontier

see Koeppl (2004)

1.A.13 Monetary Policy under two-sided limited Commitment

$$
\begin{aligned}
&V_s(U(s^t)) = \\
&\max_{\mu(s^t),(U(s^{t+1}))_{r=1}^S} \ln\left(C^{*U}(\mu(s^t))\right) - \kappa^* l^{*U}(\mu(s^t)) - v^N(s_t) + \beta \sum_{r=1}^{S} p(s^{t+1}|s^t) V(s^{t+1}, U(s^{t+1})) \\
&\qquad \text{s.t.}[\lambda(s^t)]\ \ln\left(C^U(\mu(s^t))\right) - \kappa l^U(\mu(s^t)) - u^N(s_t) + \beta \sum_{r=1}^{S} p(s^{t+1}|s^t) U(s^{t+1}) \geq U(s^t) \\
&[\beta p(s^{t+1} \mid s^t)\phi(s^{t+1})]\ U(s^{t+1}) \geq 0 \\
&[\beta p(s^{t+1} \mid s^t)\zeta(s^{t+1})]\ V(s^{t+1}, U(s^{t+1})) \geq 0 \\
&\qquad C(s_t) = C_H^{\gamma}(s_t) C_F^{1-\gamma}(s_t) \\
&\qquad l(\mu(s^t)) a_s = C_H(\mu(s^t)) + C_H^*(\mu(s^t)) \\
&\qquad l(\mu(s^t)) a_s = C_F(\mu(s^t)) + C_F^*(\mu(s^t))
\end{aligned}
$$

The Lagrangian is

$$
\begin{aligned}
\mathscr{L} = &\max_{\mu(s^t),(U(s^{t+1}))_{r=1}^S} \ln\left(C^{*U}(\mu(s^t))\right) - \kappa^* l^{*U}(\mu(s^t)) - v^N(s_t) + \beta \sum_{r=1}^{S} p(s^{t+1}|s^t) V(s^{t+1}, U(s^{t+1})) \\
&+ \lambda(s^t)\left(\ln\left(C^U(\mu(s^t))\right) - \kappa l^U(\mu(s^t)) - u^N(s_t) + \beta \sum_{r=1}^{S} p(s^{t+1}|s^t) U(s^{t+1}) - U(s^t)\right) \\
&+ \beta p(s^{t+1} \mid s^t)\phi(s^{t+1}) U(s^{t+1}) + \beta p(s^{t+1} \mid s^t)\zeta(s^{t+1}) V(s^{t+1}, U(s^{t+1}))
\end{aligned}
$$

The first order condition with respect to the monetary stance today $\mu(s^t)$ is given by

$$
\begin{aligned}
\mathscr{L}'_{\mu(s^t)} :\quad & \frac{C^{*U'}(\mu(s^t))}{C^{*U}(\mu(s^t))} - \kappa^* l^{*U'}(\mu(s^t)) + \lambda(s^t)\left(\frac{C^{U'}(\mu(s^t))}{C^U(\mu(s^t))} - \kappa l^{U'}(\mu(s^t))\right) = 0 \\
& \Rightarrow -\frac{\frac{C^{*U'}(\mu(s^t))}{C^{*U}(\mu(s^t))} - \kappa^* l^{*U'}(\mu(s^t))}{\frac{C^{U'}(\mu(s^t))}{C^U(\mu(s^t))} - \kappa l^{U'}(\mu(s^t))} = \lambda(s^t)
\end{aligned}
$$

Plugging in consumption and labor as a function of the monetary stance, one arrives at

$$
\begin{aligned}
&-\Bigg[\frac{1}{\mu(s^t)} - \frac{(1-\gamma)a^{-1}(s_t)}{\sum_{r=1}^{S} p(s^{t+1}\mid s^t)a^{-1}(r)\mu_r^U} - \frac{\gamma a^{*-1}(s_t)}{\sum_{r=1}^{S} p(s^{t+1}\mid s^t)a^{*-1}(r)\mu_r^U}\\
&-\Theta^{*U}\left(\frac{a^{*-1}(s_t)}{\sum_{r=1}^{S} p(s^{t+1}\mid s^t)a^{*-1}(r)\mu_r^U} - \frac{a^{*-1}(s_t)\mu(s^t)a_r^{*-1}(s_t)}{(\sum_{r=1}^{S} p_s r a^{*-1}(r)\mu_r^U)^2}\right)\Bigg]\\
&\Bigg[\frac{1}{\mu(s^t)} - \frac{\gamma a^{-1}(s_t)}{\sum_{r=1}^{S} p(s^{t+1}\mid s^t)a^{-1}(r)\mu_r^U} - \frac{(1-\gamma)a^{*-1}(s_t)}{\sum_{r=1}^{S} p(s^{t+1}\mid s^t)a^{*-1}(r)\mu_r^U}\\
&-\Theta^{U}\left(\frac{a^{-1}(s_t)}{\sum_{r=1}^{S} p(s^{t+1}\mid s^t)a^{-1}(r)\mu_r^U} - \frac{a^{-1}(s_t)\mu(s^t)a^{-1}(s_t)}{(\sum_{r=1}^{S} p(s^{t+1}\mid s^t)a^{-1}(r)\mu_r^U)^2}\right)\Bigg]^{-1} = \lambda(s^t)
\end{aligned}
$$

If monetary policy announces not to consider employment in their objective function to avoid any inflationary bias, the optimal rule is

$$
\begin{aligned}
&-\Bigg[\frac{1}{\mu(s^t)} - \frac{(1-\gamma)a^{-1}(s_t)}{\sum_{r=1}^{S} p(s^{t+1}\mid s^t)a^{-1}(r)\mu_r^U} - \frac{\gamma a^{*-1}(s_t)}{\sum_{r=1}^{S} p(s^{t+1}\mid s^t)a^{*-1}(r)\mu_r^U}\Bigg].\\
&\Bigg[\frac{1}{\mu(s^t)} - \frac{\gamma a^{-1}(s_t)}{\sum_{r=1}^{S} p(s^{t+1}\mid s^t)a^{-1}(r)\mu_r^U} - \frac{(1-\gamma)a^{*-1}(s_t)}{\sum_{r=1}^{S} p(s^{t+1}\mid s^t)a^{*-1}(r)\mu_r^U}\Bigg]^{-1} = \lambda(s^t)
\end{aligned}
$$

Taking the derivative with respect to $U(s^{t+1})$ into account give

$$
\begin{aligned}
&+\beta p(s^{t+1}\mid s^t)V'(s^{t+1},U(s^{t+1})) + \lambda(s^t)\beta p(s^{t+1}\mid s^t) + \beta p(s^{t+1}\mid s^t)\phi_{sr}\\
&+\beta p(s^{t+1}\mid s^t)\zeta(s^{t+1})V'(s^{t+1},U(s^{t+1})) = 0\\
&\frac{\lambda(s^t)+\phi(s^{t+1})}{1+\zeta(s^{t+1})} = -V_r'(U(s^{t+1}))
\end{aligned}
$$

The envelope condition gives $\lambda(s^t) = -V_s'(U(s^t))$. It states that the relative weight today $\lambda(s^t)$ (the Lagrange multiplier) is equal to the Marginal rate of transformation of the social planner. This transformation states how much marginal utility loss occurs for F when marginal utility for H is increased marginally. Linking these three conditions together with the complementary slackness conditions (this condition shows when a constraint is binding or not) gives an equation that described the evolution for the relative weight $\lambda(s^t)$:

$$
\lambda(s^{t+1})\begin{cases} = \underline{\lambda}(s^{t+1}) & \text{if } \lambda(s^t) < \underline{\lambda}(s^{t+1})\\ = \lambda(s^t) & \text{if } \lambda(s^t) \in [\underline{\lambda}(s^{t+1}),\bar{\lambda}(s^{t+1})]\\ = \bar{\lambda}(s^{t+1}) & \text{if } \lambda(s^t) > \bar{\lambda}(s^{t+1}).\end{cases}
$$

Note the following: The path of $\lambda(s^t)$ should be the same for both policy instruments. The only difference is how the ratio is achieved.

1.A.14 Figures

The following graph depicts the Pareto frontier for the initial state bb (boom in both countries).

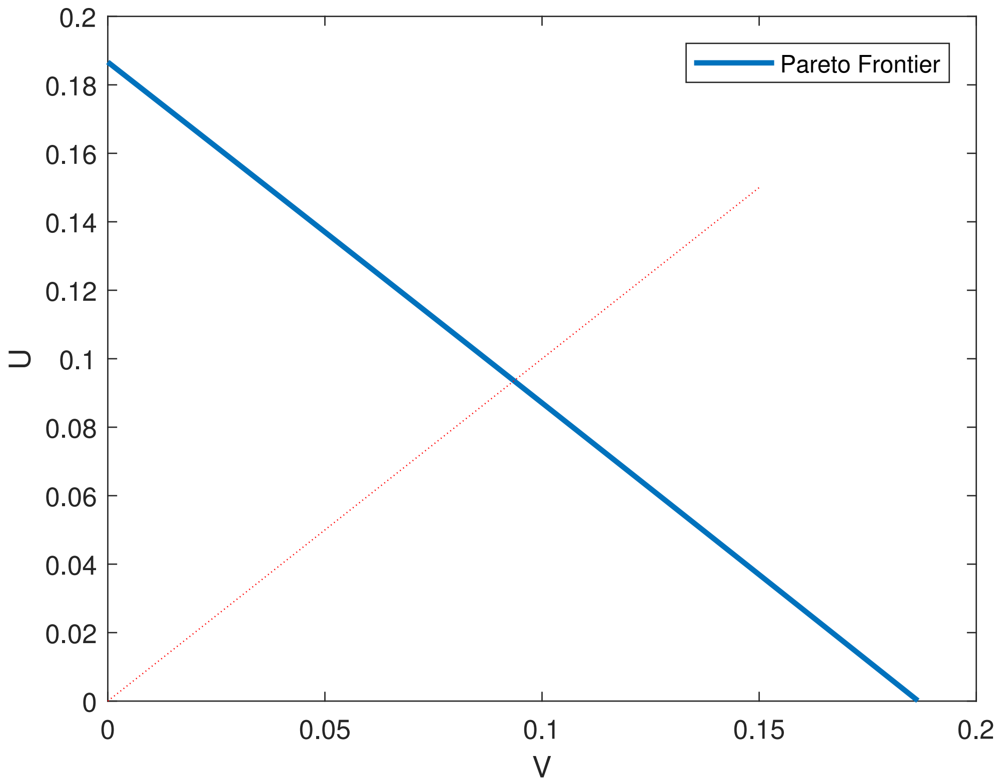
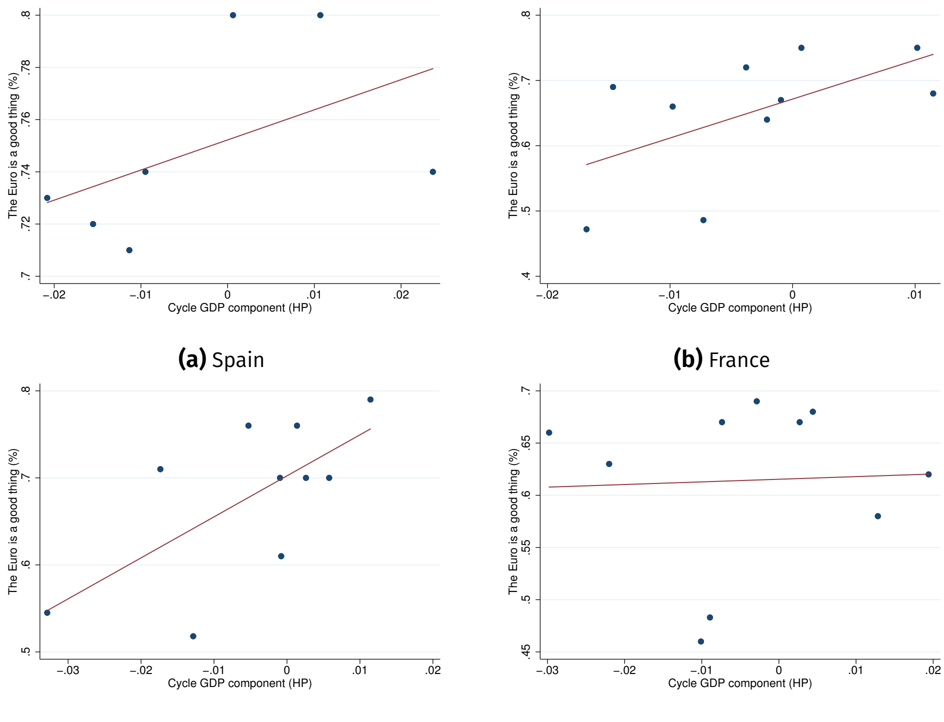


Figure 1.A.1. Pareto frontier, when both countries are initially in a boom. The red dashed line is the 45-degree line.

The Pareto frontier is indeed concave. If V is zero U reaches its maximum value, meaning that all the gains go to country H.

Empirical evidence for recession countries leaving the union



(a) Spain

(b) France

(c) Germany

(d) Italy

Figure 1.A.2. Cyclical HP GDP component and Eurobarometer: Is the Euro a good thing?

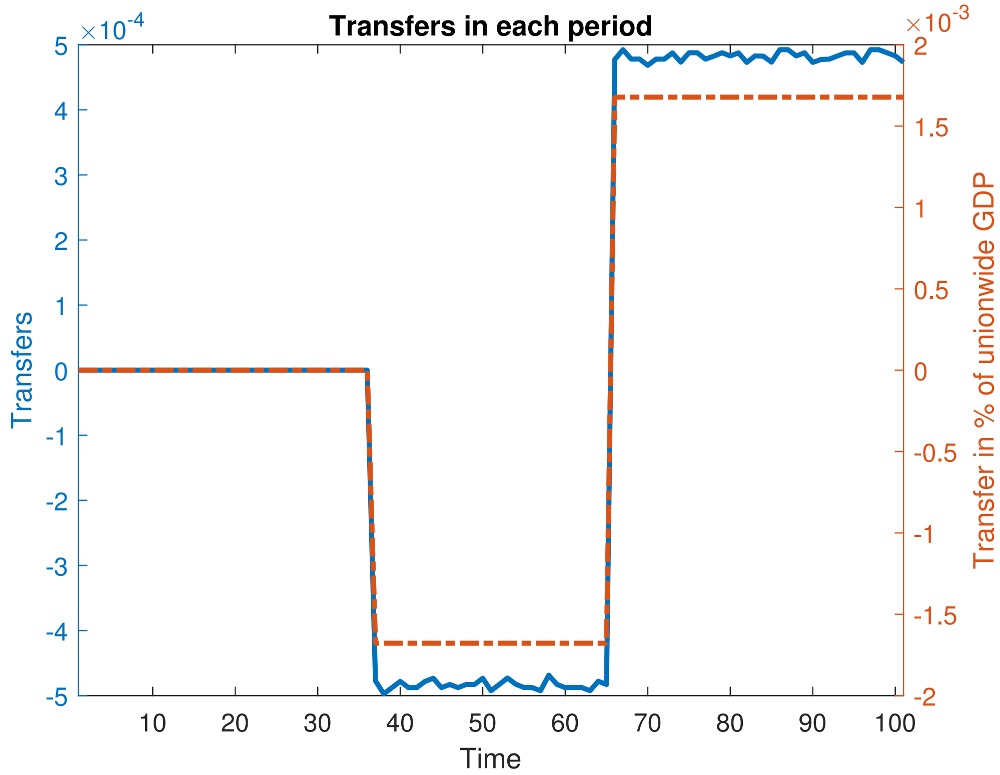


Figure 1.A.4. Evolution of transfers with one-time monetary intervention, trade costs reduction of 6.5%.

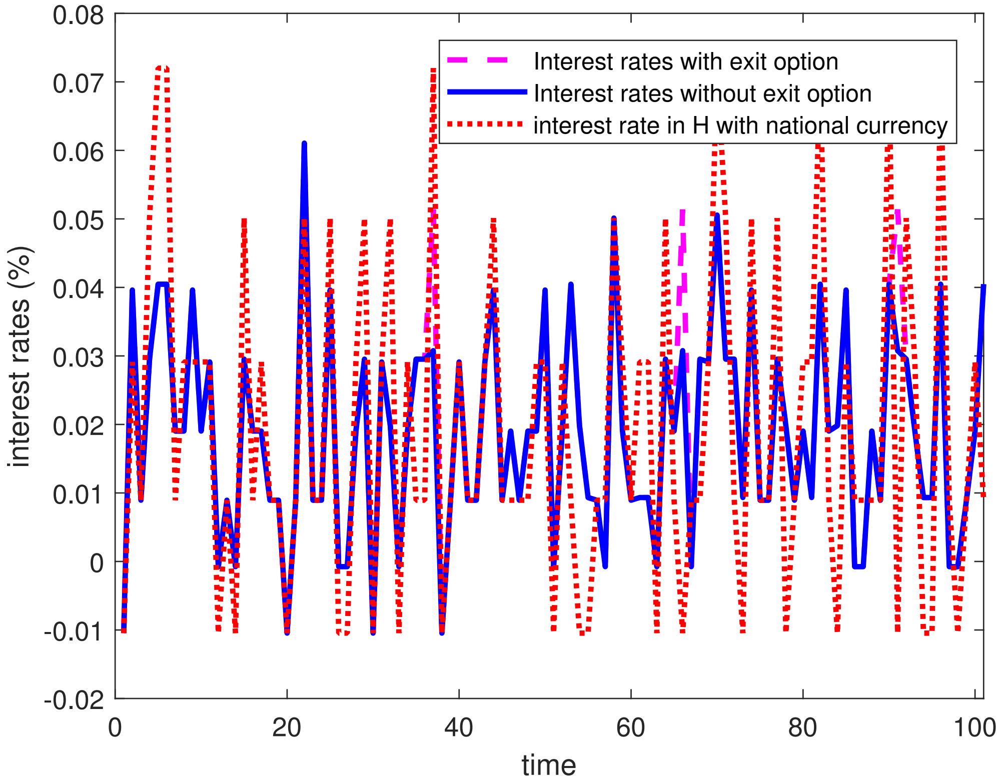


Figure 1.A.5. Evolution of interest rates with one-time monetary intervention, trade costs reduction of 6.5%

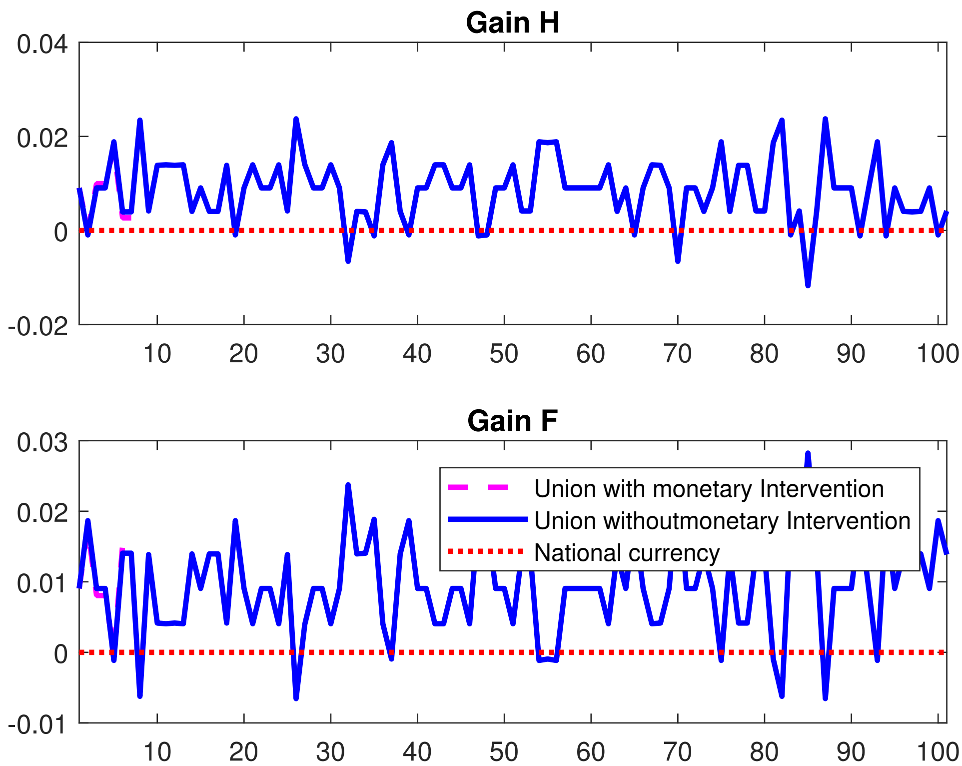


Figure 1.A.6. Evolution of gains with union-wide central bank only, trade costs 5%

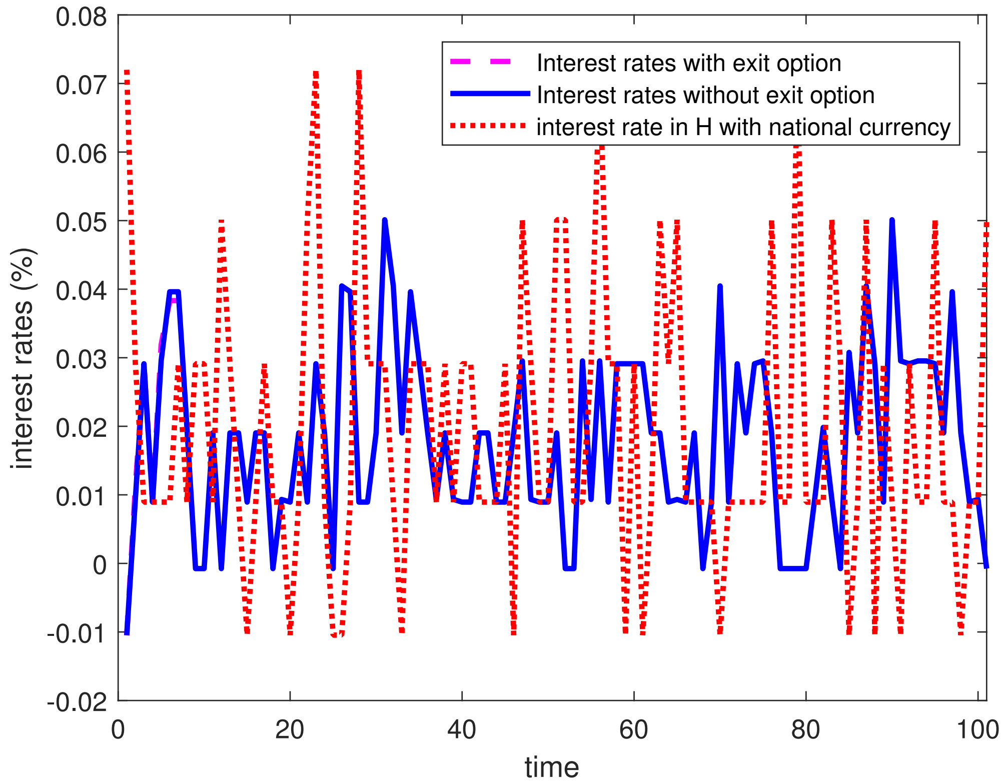


Figure 1.A.7. Evolution of interest rates with a trade cost reduction of 5% and a permanent union-wide central bank intervention.

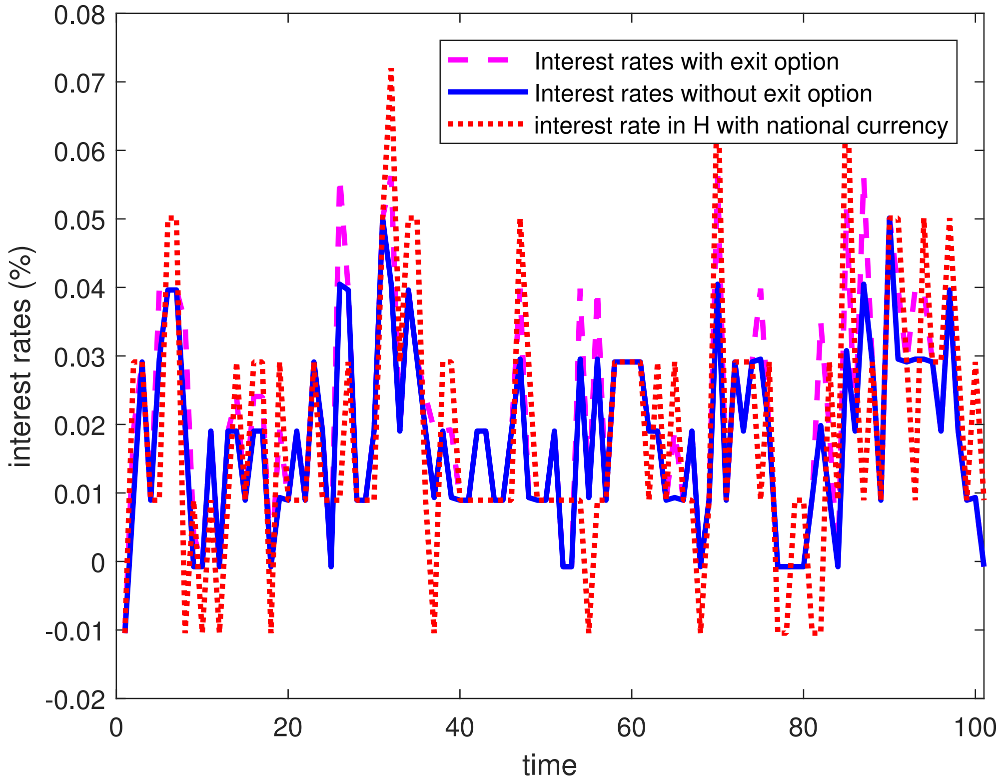


Figure 1.A.8. Evolution of interest rates with a trade cost reduction of 5% and a one-time monetary intervention.

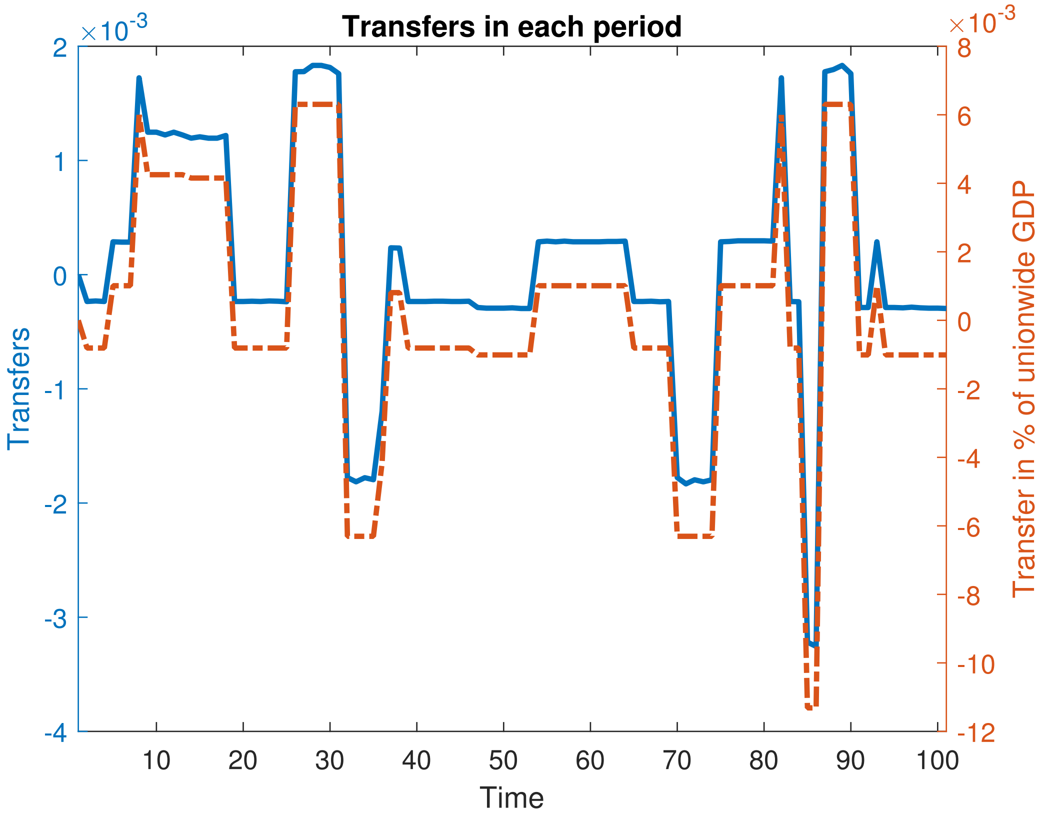


Figure 1.A.9. Evolution of transfers with one-time monetary intervention, trade cost reduction of 5%.

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Chapter 2

Inflation, Interest Rates and the Choice of the Exchange Rate Regime*

Joint with Ricardo Duque Gabriel

2.1 Introduction

What is the impact of the exchange rate regime on inflation, interest rates and economic activity? This question relates to a central topic in international macroeconomics that discusses benefits and costs of giving up monetary autonomy and pegging the exchange rate. One benefit is thought to be a significant reduction of inflation. Indeed, a reason for the formation of the eurozone was the hope that a common and stable currency for the whole continent could bring down inflation and interest rates of all countries to the low level of Germany.

This paper sheds new light on this question by providing empirical evidence about the impact of the exchange rate regime on the behavior of inflation, interest rates and economic activity. We establish three main observations in our extensive data set: First we find that fixing the exchange rate regime leads to a persistent reduction of inflation and nominal interest rates of around 4 percentage points for the median

* Thanks to Donghai Zhang, Pavel Brendler and Keith Kuester for comments on the early stage of this project. This paper is based on a Master's thesis (Arvai, 2021) submitted by one of the authors in April 2021 and shares the idea, the model and part of the write-up. Some of the graphs are identical to what has been shown in the Master's thesis. Compared to the thesis, the current work expands on it in three dimensions. First, by extending the descriptive empirical evidence from some selected European countries to a broader set of 179 countries. Second, the empirical analysis is further improved by using the inverse probability weighting methodology of Jordà and Taylor (2016) to measure the effect of a shift of the exchange rate regime. This helps in a regression analysis since the decision to change the exchange rate might be confounded by other factors. Last, the calibration of the model is improved by using the simulated method of moments to better match the model with the data. This way, the empirical impulse response functions can be directly compared to the model impulse response functions.

country that decides to peg. Furthermore, nominal rates of the anchor country also go down slightly. Second, volatility of inflation goes down not only for the country that pegs its currency, but also for the anchor country. Last, we demonstrate that GDP tends to rise after the exchange rate regime becomes more fixed by around 2 percentage points.

In a next step, we follow the model of Chari, Dovis, and Kehoe (2020a) to rationalize these findings. We calibrate a two-country version of the model to match the behavior of inflation in Italy and Germany during the time of flexible exchange rates. We then demonstrate that Italy's inflation goes down by a similar magnitude as in the data when it pegs its currency and forms a currency union with Germany. For Germany, the model predicts a small decline in inflation as well. In line with our empirical observation, volatility of inflation for both countries goes down as well. The reason why inflation declines in a currency union is that the inflationary bias that arises under monetary policy under discretion becomes less pronounced. When central banks react to country-specific temptation shocks, the currency union's central bank only reacts to the average temptation shock of the union. If temptation shocks are not perfectly correlated the currency union helps to mitigate the lack of commitment. Last, our simulation also shows that GDP for Italy increases and inflation decreases after pegging the exchange rate. The reason for an increase of GDP is that on average lower inflation reduces the opportunity costs of holding money that is needed to buy goods. This increases consumption and, as a consequence, production.

Section 2.2 provides descriptive empirical evidence that links inflation, interest rates and economic activity to movements of the exchange rate and to shifts of the overall exchange rate regime. In a first step we take a look at a country comparison centering around Germany and Italy. We demonstrate that episodes of flexible exchange rates in the 70s were associated with periods of persistently high inflation and interest rates for Italy, but not for Germany. Reversely, we show that inflation went down after the exchange rate was pegged to the German currency. Germany in contrast, was only slightly affected by a completely flexible exchange rate. Entering a more fixed regime helped to bring down inflation by a small margin in Germany as well. Furthermore we emphasize the behavior of volatility of inflation and interest rates under different regimes. For both countries it is true that the variability of inflation and interest rates is high under a regime with flexible exchange rates. Entering a fixed exchange rate regime, as for example in 1985 or with the creation of the Euro at the end of the 90s, brought down inflation variability of both countries by a meaningful margin.

In a next step, we generalize these observations by extending the considered country set to 178 and by considering over 400 floating and pegging episodes from the last decades using the dataset from Ilzetzki, Reinhart, and Rogoff (2019). We confirm that episodes of floating exchange rate regimes were associated with persistently higher nominal interest rates and inflation rates, while fixed exchange rate regimes coincided with lower inflation and interest rates. Our second observation regarding the behavior of inflation variability is also confirmed as countries with flexible exchange rate display a larger variety of inflation rates. This is true not only for those countries that decide to peg their currency to a stable anchor, but also for the anchor countries themselves. Last, our third observation indicates that pegging the exchange rate is associated with an increase of GDP growth by around 2 percent. Motivated by these three observations, we then put forward an open economy model relying on the analysis in Chari, Dovis, and Kehoe (2020a) in Section 2.3. The goal is to estimate the model and to compare the model outcome with our empirical findings. The setup of the model is the following: There is a mass of countries each populated with representative agents. Two goods are produced, a non-traded good and a traded good. While the traded good is produced under perfect competition and flexible prices, the non-traded good sector is subject to imperfect competition and inflexible prices. The timing in the model is such that monetary policy moves after non-traded goods have set their prices. This implies that non-credible central banks are tempted to use surprise inflation to lower markups of firms ex post. This generates high inflation rates when a country has independent monetary policy that acts under discretion. Markups follow a certain stochastic process that we calibrate in a later step. At the same time, surprise inflation is costly as households need to hold money from last period to buy traded goods. Ultimately, the central bank trades off costs of inflation with costs of markups. Firms anticipate the attempt of the central bank to inflate away their markups and simply rise their prices. In equilibrium, the economy ends up with higher prices and inflation. A credible central bank in contrast can commit to low inflation policies beforehand. It would credibly promise to not react to firm's markups. In that case firms do not increase their prices before and the central bank does not use surprise inflation in the first place. How do these institutional differences in credibility of the central bank relate to the exchange rate regime? If the inflationary bias is too costly for a country, one way to reduce inflation and interest rates could be to give up monetary autonomy and peg the exchange rate to a stable anchor. The anchor can be stable in the sense that either the temptation shocks are less pronounced or that monetary policy is able to act under commitment. By pegging, the high-inflation country binds its monetary policy to the anchor. It is not possible to systematically pursue more expansionary monetary policy than the anchor country without endangering the peg. If the announcement of the peg is not credible, the country could also join a currency union with the anchor country. Chari, Dovis, and Kehoe (2020a) show that even if the union-wide central bank acts under discretion, countries with dissimilar temptation shocks can benefit from forming a currency union, as the central bank then reacts only to the average temptation shock of the union. Inflation becomes less volatile and is lower on average. As inflation in this model setup is costly and reduces consumption, lower inflation increases consumption and therefore production as well. There are also Mundellian costs of forming a currency union when productivity fluctuates. At this stage of our work, we focus on markup shocks only.

After illustrating the basic mechanism, we revisit the German and Italian data to calibrate the model. The goal is to match the behavior of inflation in Italy and Germany. We use the method of simulated moments to match the markup shock process of both countries with the behavior of inflation in Italy and Germany between 1972 and 1985 -the time in which the currency of both countries was flexible. In a next step we simulate the estimated model under three different monetary regimes: First a regime with flexible exchange rates under commitment. In this regime there is no inflationary bias, as all central banks are credible. The second regime that we consider is discretionary and has flexible exchange rates as well. Last, we simulate a discretionary regime in a currency union. The currency union consists out of Italy and a block of stable countries four times the size of Germany. We show in the simulation, that in a currency union inflation for Italy goes down by 10 percentage points and approaches the average level of Germany. For Germany, volatility of inflation under this regime is cut in half, while average inflation is only slightly reduced. Lower and less volatile inflation increases consumption in Italy, leading to an increase in GDP of around 4 percent.

Related Literature:

Part of the literature review is taken from Arvai (2021). The paper relates to the open economy literature that examines the relationship of exchange rate regimes and the economy. Mussa (1986) showed in a seminal work that the decision to let the exchange rate regime float freely after Bretton Woods did not only have an impact on the nominal exchange rate, but also on the *real* exchange rate. The real exchange rate is commonly defined as the nominal exchange rate times the relative price index. The fact that a movement of the nominal exchange rate impacted also the real exchange rate was taken as evidence for price rigiditity: Relative prices did not react accordingly to offset movements in the nominal exchange rate. Indeed, Nakamura (2018) cites Mussa (1986) as one of the most convincing evidence for monetary non-neutrality. In more recent work, Mukhin (2018) reconfirm the findings of Mussa (1986) and emphasize that changes in the exchange rate regime fail to show up in other real macroeconomic variables such as GDP or consumption. They also find that there is no systematic change of cyclical properties in inflation, after a shift of the exchange rate regime. This paper redirects the focus from cyclical properties towards level shifts of nominal macroeconomic variables. We show in our dataset that inflation and interest rates are persistently lower in a fixed exchange rate regime. This is in line with findings of Bordo and Schwartz (1999) who report that inflation is historically lower for countries during periods of fixed exchange rate regimes (e.g., Gold Standard and Bretton Woods); Ghosh, Qureshi, and Tsangarides (2014) argue that inflation is lower when the central bank both *de jure* commits and *de facto* pegs the exchange rate. Two more findings stand out in our work: First, we demonstrate that volatility of inflation is clearly reduced after entering a fixed exchange rate regime. Second, countries subsequently tend to experience more economic growth. We obtain these results by considering an extensive historical record of exchange rate regimes for countries around the world for a long-time horizon, provided by Ilzetzki, Reinhart, and Rogoff (2019). They classify different exchange rate regimes that range from currency unions to fully flexible exchange rates. Furthermore, they also identify countries that serve as anchor currencies for other countries. We use their classification in the empirical part of Section 2.2 to link changes of the exchange rate regime to inflation. We then confront our empirical findings with an estimated version of the Chari, Dovis, and Kehoe (2020a) model. They set up an open economy model and link it to discretionary monetary policy in the Barro and Gordon (1983) tradition. Models in the tradition of Barro and Gordon (1983) point to the signaling content of the regime choice. Governments and monetary authorities that suffer from a credibility deficit can signal their commitment to tough policies by appropriately choosing the exchange rate regime Giavazzi and Pagano (1988). Such a shift in credibility is able to mitigate the inflation bias arising from a monetary authority with the incentive to conduct expansionary monetary policy to raise output. This decrease in inflation is persistent if the credibility change is also perceived as persistent.

2.2 Data and Stylized Facts

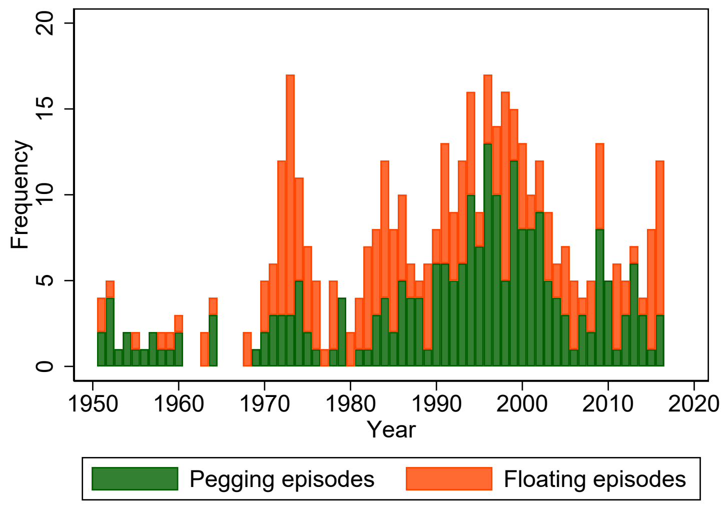
In this section, we start by describing the details of the global data set that we compiled for our analysis. Then, motivated by the case-study of Italy and Germany, we report a set of stylized facts that focus on the dynamics of inflation, GDP and interest rates before and after a pegging episode occurred.

2.2.1 Data

We base our analysis on an unbalanced panel with annual data for 178 economies, including both Advanced Economies (AEs) and Emerging Economies (EMEs), from 1950 to 2016. The data used in this paper mainly relies on the IMF's International Financial Statistics (IFS) database, which we complement with information from the Macrohistory Database (Schularick and Taylor, 2017). We assemble data from the IFS on consumer price index (CPI) inflation, short-term interest rates (bills), long-term interest rates (bonds), gross domestic product (GDP), and bilateral exchange rate of the sample countries to the US dollar, the German Mark and the British Pound. In order to perform a consistent analysis, we only use observations for which we have data on, at least, CPI inflation and real GDP, rendering a total of 6,742 country-year observations between 1950 and 2016.

We further complement the resulting dataset with the exchange rate regime classification from Ilzetzki, Reinhart, and Rogoff (2019). They identify the exchange rate regime in place for all countries in our sample based on both *de jure* and *de facto* classifications. Throughout the study, we will rely on their coarse episode classification which arguably identifies significant changes in the regime.

In a first step, let us look at the events in our sample where exchange rate regimes changed. Figure 2.1 illustrates how many times countries moved towards a more pegged or flexible regime over time. In our sample, we observe 223 pegging episodes and 211 floating episodes.



Notes: Number of the changes of the exchange rate regime classification from Ilzetzki, Reinhart, and Rogoff (2019). Green bars: Move towards a "more pegged" regime ($N = 223$). Red bars: Move towards a "more float" regime ($N = 211$).

Figure 2.1. Frequency of flexible and fixed regime changes

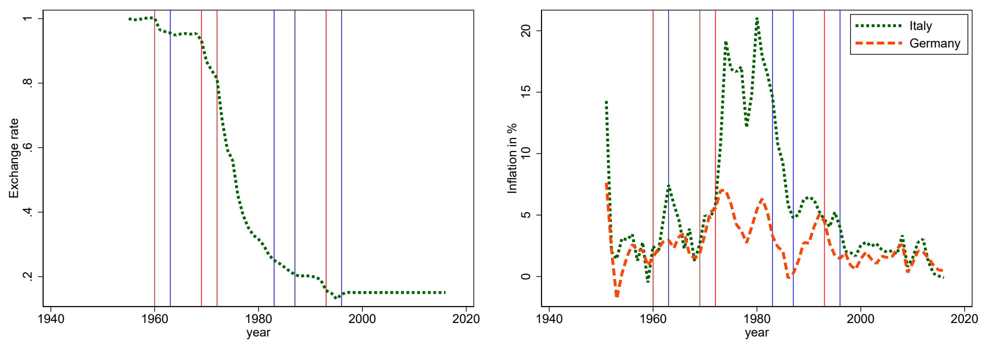
There are two big waves of regime adjustment episodes: a) following the Bretton Woods collapse in 1971, pegged countries were forced to float their currency or peg it to another anchor currency; and b) after 1990 there was a surge on pegging episodes (green bars) preceding both the Euro creation and the dollarization of emerging economies. Such variation is important to motivate both the econometric analysis and the next sections. We motivate our research question by looking at the relation between Italy and Germany as a case study and then we generalize it by using the global panel dataset introduced in this section.

2.2.2 Case Study: Italy and Germany

We start by providing descriptive evidence about the relationship between the exchange rate regime and nominal macroeconomic variables in Italy (as the pegging country) and Germany (as the anchor country). We take Germany as the benchmark

because it is the largest economy in Europe and plays a pronounced role for the continent's economy. With this assessment, we follow Ilzetzki, Reinhart, and Rogoff (2019) who identify Germany as the anchor country for most continental (western) European countries following the breakdown of the Bretton Woods agreement.
Figure 2.3a shows the bilateral exchange rate of the Italian Lira to the German Mark between 1954 and 2016. The exchange rate is indexed to 1 in 1955, the data are taken from the Bundesbank. Figure 2.3b shows the inflation rate of Italy and Germany. The exchange rate regime changes are identified by a vertical blue (peg) and red (float) lines:

(a) Exchange Rate (1955=1) **(b)** Inflation (in %)



Notes: Graph (a) shows the evolution of the bilateral exchange rate of the Italian Lira to the German Mark normalized to 1 in 1955. Graph (b) shows how inflation in Germany (dashed red line) and Italy (dotted green line) co-moved over time. According to the fine classification of Ilzetzki, Reinhart, and Rogoff (2019), the vertical red lines indicate a fall of the exchange rate or a shift towards a floating exchange rate regime, the blue vertical lines a shift towards an exchange rate regime that is more pegged and that was followed by a stabilization of the exchange rate. The graph is taken from Arvai (2021). Sources: Bundesbank, IFS, and Ilzetzki, Reinhart, and Rogoff (2019).

Figure 2.2. Exchange rate and inflation in Italy

At the beginning of our sample Italy and Germany were both in a fixed exchange rate regime. There is almost no movement in the exchange rate and inflation moves below 5% for both countries. After the end of Bretton Woods, Italy's currency experiences a large depreciation. This coincides with a large increase of Italy's inflation. Inflation peaks at over 20 percent after 1980. After 1985, as the exchange rate gets pegged to the German Mark, the behavior of Italy's inflation changes: Fixing the exchange rate to Germany coincides with a convergence of inflation to the relatively low and stable German level.
Moreover, there seems to be a change in the behavior of the variability of inflation: During the time of a flexible exchange rate regime - between 1972 and 1985 - inflation displayed higher volatility. In contrast, volatility decreased from the 90's onward, marking the arrival of the Euro. This decline in volatility was very pronounced for Italy, but also clearly visible for Germany. Furthermore, when comparing Ger-

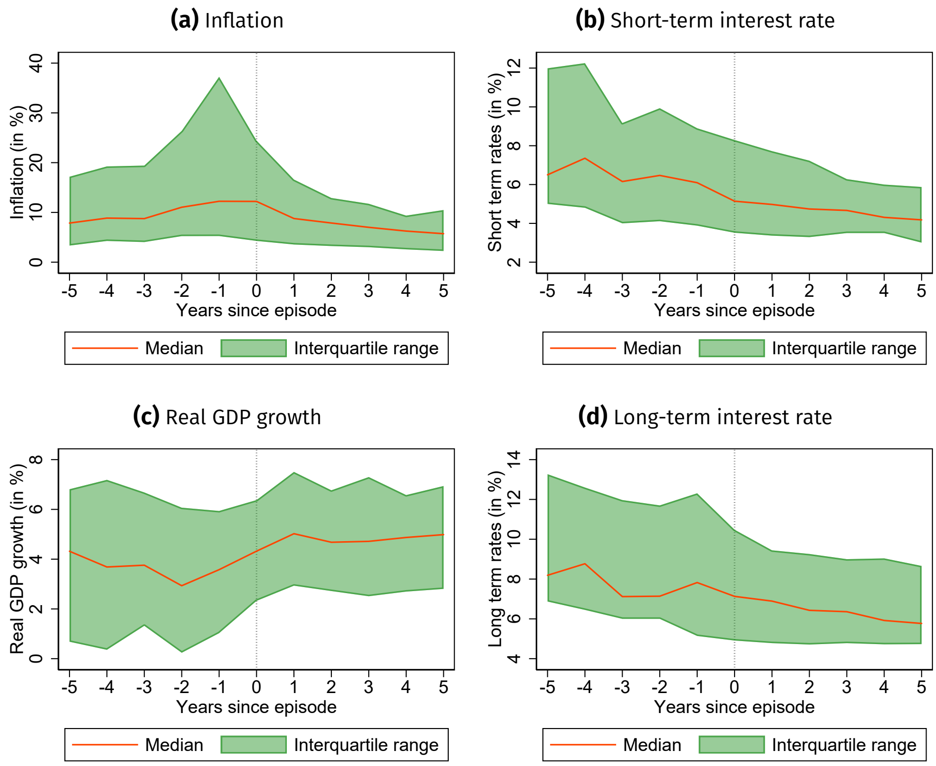
many's inflation during the episodes of flexible exchange rates with those episodes with fixed exchange rates, it seems that average inflation is also slightly lower with fixed rates.

Other southern European countries, like Spain or Portugal (Figure 2.A.9), experienced similar patterns: A stable exchange rate to the German Mark coincided with similar inflation rates, but when monetary policy was conducted independently without any exchange rate goal, the exchange rate depreciated, inflation substantially increased compared to Germany and the variability went up. Contrarily, countries like Austria and the Netherlands had their inflation closely tracking Germany's inflation (Figure 2.A.7). The evolution of nominal interest rates over time show a similar picture, see Figure 2.A.6.

2.2.3 Broader Evidence

To derive stylized facts from the previous case-study we revisit our data and perform an event study for all possible countries in order to analyze how key economic variables varied before and after a change in the exchange rate regime. Figure 2.4 illustrates inflation, short- and long-term interest rates, and real GDP growth before and after a pegging episode at $t = 0$, for the cross-section of countries in our sample that went through at least one such episode.[1]

1. For completeness, we present the same event study for a floating exercise in Appendix, Figure 2.A.2.



Notes: The figure shows the event-study for median inflation and median interest rates in percentage points, and median real GDP growth in percent before and after a pegging episode, when the exchange rate regime becomes more pegged. $N \approx 2000$.

Figure 2.4. Event-study for a pegging episode

It establishes three main observations: First, the median of inflation and interest rates decreases after a pegging episode. While median inflation is above 10 percent before the exchange rate gets pegged, it decreases to around 6 percent a couple of years after the decision to peg. Second, the variability of inflation and interest rates goes down, as the interquartile range of countries gets wider under a float and more narrow under a peg. Third, we also find that real GDP slightly increases when the currency gets pegged. There is not only an increase in real GDP growth by almost 2 percentage points for the median country, but also a reduction in its volatility, as the interquartile range becomes more narrow as well.[2] These three features - the behavior of the level of inflation, its volatility and the reaction of real GDP growth - can be rationalized by the estimated model presented in Section 2.3.

2. It is worth emphasizing that we derive opposite observations after a floating episode as we can see from Figure 2.A.2 in the Appendix.

2.3 Model

This Section 2.3 entirely follows Chari, Dovis, and Kehoe (2020a). The model setup and the corresponding model results are identical to Chari, Dovis, and Kehoe (2020a). This section is also partly identical to the write-up of one of the author's Master's thesis Arvai (2021). The goal is to use this model setup to link it to the data presented before in the calibration in Section 2.4.

In the model, the absence of commitment (or lack of credibility) leads to an inflationary bias in the central bank's policy as the central bank is subject to temptation shocks. This keeps inflation systematically higher and more volatile than in a country with less pronounced temptation shocks. Countries can either conduct monetary policy independently without any regard of the exchange rate, or they can give up monetary policy and peg the exchange rate to a stable anchor. They can even form a currency union with each other. The goal is then to provide an estimated version of that model and to simulate it under different regimes. We then compare the behavior of nominal and real variables under flexible and fixed exchange rates and compare the outcome with the data.

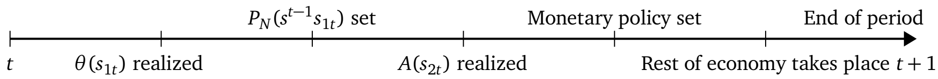
2.3.1 Setup

The model closely follows Chari, Dovis, and Kehoe (2020a). The economy consists out of a continuum of countries. Each country produces traded and non-traded goods. The traded good sector is assumed to be perfectly competitive while the non-traded good sector has imperfect competition and sticky prices. This assumption reflects the notion that flexible exchange rates are desirable as they ensure that the relative prices of traded goods to non-traded goods move as if all prices were flexible.

There are two different sources of shocks that hit the non-traded sector only: A markup shock and a productivity shock. Each of these shocks can happen on an aggregate level that hits the whole world equally and on a country-specific level. We adopt the same notation as in Chari, Dovis, and Kehoe (2020a) and denote $z_t = (z_{1t}, z_{2t}) \in Z$ as an aggregate shock in time t where the subindex 1 refers to the markup shock and the subindex 2 to the productivity shock. The country-specific shock $v_t = (v_{1t}, v_{2t}) \in V$ is drawn each period. All of the shocks are i.i.d. over time and across country [3]. The probability of aggregate shocks is $f(z_{1t}, z_{2t}) = f^1(z_{1t})f^2(z_{2t})$, while the probability for country-specific shocks is given by $g(v_{1t}, v_{2t}) = g^1(v_{1t})g^2(v_{2t})$. Let $s_t = (s_{1t}, s_{2t})$ summarize the current state of the world with $s_{it} = (z_{it}, v_{it})$ and let $h(s_t) = h^1(s_{1t})h^2(s_{2t})$ denote the probability of that specific state with $h^i(s_{it}) = f^i(z_{it})g^i(v_{it})$. In particular let $A(s_{2t})$ denote the produc-

3. This keeps the model tractable, as it becomes static. There is no persistence such that a large shock today affects future states. The calibration discusses the shock process in more detail.

tivity shock and $\theta(s_{1t})$ denote the markup shocks to the non-traded sector. The conditional mean of the shocks is given by $E_v(\theta \mid z) = \sum_{v_1} g^1\left(v_1\right)\theta\left(z_1, v_1\right)$ and $E_v(A \mid z) = \sum_{v_2} g^2\left(v_2\right)A\left(z_2, v_2\right)$. The timing is as in Chari, Dovis, and Kehoe (2020a). First the markup shock is realized, then non-traded good firms set their prices, then productivity is realized, then monetary policy reacts and last the rest of the economy takes places where traded good firms set their prices and households make their decision.



The important feature in this setup is that a discretionary monetary authority has an incentive to use surprise-inflation to inflate away the socially inefficient markups of firms. Firms anticipate the attempt of the central bank to inflate and raise their prices for non-traded goods before. In equilibrium, the economy ends up with higher prices. A lack of commitment by the central bank results in an inflationary bias for the economy. In contrast, a central bank that commits to policies realizes that it cannot inflate away the markups. Hence it promises ex ante to focus on productivity shocks only when using monetary policy and successfully avoids the inflationary bias.

Countries can be identified by the history of country-specific shocks $v^t = (v_0, v_1, ..., v_t)$ and are therefore symmetric with respect to their parameters, technology and preferences. We first consider how the economy works for one single "home" country and then consider country blocks and unions in Section 2.3.6.

2.3.2 Production

Firms are owned by households. Production of traded goods is given by

$$Y_T(s^t) \;=\; L_T(s^t).$$

Production is linear in the labor input $L_T(s^t)$. Traded good firms maximize their profits $P_T\left(s^t\right)L_T\left(s^t\right) - W\left(s^t\right)L_T\left(s^t\right)$. Optimally firms set the price of traded goods $P_T\left(s^t\right)$ equal to the wage $W\left(s^t\right)$. $W(s^t)$ can therefore be replaced by $P_T\left(s^t\right)$.

Production of non-traded goods is subject to two frictions, namely monopolistic markets and rigid prices. This gives rise to markups that increase prices of non-traded goods. A microfoundation for markups can be given by closely following the setup of Smets and Wouters (2007) which is also described in the Appendix of Chari, Dovis, and Kehoe (2020a). The non-traded good is produced by a competitive final producer who uses differentiated inputs $y_N(j, s^t)$ from input firms of mass $j \in [0,1]$ to produce the final good $Y_N(s^t)$:

$$Y_N\left(s^t\right) = \left[\int y_N\left(j, s^t\right)^{\theta(s_{1t})}\, dj\right]^{1/\theta(s_{1t})}, \quad \theta\left(s_{1t}\right) \in (0,1).$$

where $\theta\left(s_{1t}\right)$ is the time-varying substitution parameter between the inputs [4]. $\theta\left(s_{1t}\right) \in (0,1)$ implies that demand for inputs is elastic. If $\theta\left(s_{1t}\right)$ is very close to 1 goods are almost perfect substitutes and firms are not able to use any monopolistic power. The closer $\theta\left(s_{1t}\right)$ is to 0, the more monopolistic power a firm has. The final good firm maximizes

$$P_N\left(s^{t-1}, s_{1t}\right) Y_N\left(s^t\right) - \int P_N\left(j, s^{t-1}, s_{1t}\right) y_N\left(j, s^t\right) dj.$$

Demand for intermediate goods is therefore

$$y_N\left(j, s^t\right) = \left(\frac{P_N\left(s^{t-1}, s_{1t}\right)}{P_N\left(j, s^{t-1}, s_{1t}\right)}\right)^{\frac{1}{1-\theta\left(s_{1t}\right)}} Y_N\left(s^t\right).$$

Intermediate goods are produced by monopolistic firms who use a linear production function: $y_N(j, s^t) = A(s_{2t})L_N(j, s^t)$. Intermediate good firms choose their prices $P = P(j, s^{t-1}, s_{1t})$ to maximize their expected profits:

$$\max_P \sum_{s_{2t}} Q\left(s^t\right)\left[P - \frac{W\left(s^t\right)}{A\left(s_{2t}\right)}\right]\left(\frac{P_N\left(s^{t-1}, s_{1t}\right)}{P}\right)^{\frac{1}{1-\theta\left(s_{1t}\right)}} Y_N\left(s^t\right)$$

where $Q(s^t)$ is the discount factor, the price of a state-contingent claim to local currency units at s^t in units of local currency in s^{t-1}. Optimally, intermediate good producer j sets the price on non-traded goods as a time-varying markup over a weighted average of marginal costs:

$$P_N\left(j, s^{t-1}, s_{1t}\right) = \frac{1}{\theta\left(s_{1t}\right)} \frac{\sum_{s_{2t}} Q\left(s^t\right) Y_N\left(s^t\right) \frac{W\left(s^t\right)}{A\left(s_t\right)}}{\sum_{s_{2t}} Q\left(s^t\right) Y_N\left(s^t\right)}$$

where $\frac{1}{\theta\left(s_{1t}\right)}$ is the markup that increases prices. Note that the price equation is not a function of j such that the price is the same for all intermediate firms. Plugging in $W(s^t) = P_T(s^t)$ gives the pricing equation

$$P_N\left(s^{t-1}, s_{1t}\right) = \frac{1}{\theta\left(s_{1t}\right)} \sum_{s_{2t}} \left(\frac{Q\left(s^t\right) Y_N\left(s^t\right)}{\sum_{\tilde{s}_{2t}} Q\left(\tilde{s}^t\right) Y_N\left(\tilde{s}^t\right)}\right) \frac{P_T\left(s^t\right)}{A\left(s_t\right)}. \tag{2.1}$$

This implies that all intermediate firms hire the same amount of labor and their production function is then simply given by

$$Y_N(s^t) = A(s_{2t})L_N(s^t).$$

4. The elasticity of substitution between the inputs is $\frac{1}{1-\theta(s^t)}$

2.3.3 Households

Households derive utility from consumption of traded goods $C_T(s^t)$ and from consumption of non-traded goods $C_N(s^t)$. In addition, they experience disutility from labor $L(s^t)$: $\sum_{t=0}^{\infty}\sum_{s^t}\beta^t h_t\left(s^t\right)U\left(C_T\left(s^t\right),C_N\left(s^t\right),L\left(s^t\right)\right)$. As in Chari, Dovis, and Kehoe (2020a), we specialize preferences as

$$U\left(C_T,C_N,L\right) \;=\; \alpha\log C_T + (1-\alpha)\log C_N - \psi L.$$

This specification entails several simplifying assumptions, first it assumes that the elasticity of substitution between traded and non-traded goods is 1. Second, log-utility in consumption means that the inter-temporal elasticity of substitution is 1 as well. Those assumptions imply that households do not have an incentive to borrow or save across countries, as the willingness to substitute goods across time is exactly offset by the willingness to substitute traded goods to non-traded goods. α reflects the weight of traded goods in the overall consumption basket, large values imply that the countries in the economy have a very high degree of trade openness. Finally, the preferences are quasi-linear in labor, which simplifies aggregation results[5]. The budget constraint of households is given by

$$\begin{aligned} &P_T\left(s^t\right)C_T\left(s^t\right)+P_N\left(s^{t-1},s_{1t}\right)C_N\left(s^t\right)+M_H\left(s^t\right)+B\left(s^t\right)\\ &\leq P_T\left(s^t\right)L\left(s^t\right)+M_H\left(s^{t-1}\right)+R\left(s^t\right)B\left(s^{t-1}\right)+T\left(s^t\right)+\Pi\left(s^t\right)\end{aligned} \tag{2.2}$$

where $T(s^t)$ are nominal transfers. $\Pi\left(s^t\right)=P_N(s^{t-1},s_{1t})Y_N(s^t)-P_T(s^t)L_N(s^t)$ are profits from the traded-goods sectors. As households own the firms in their corresponding country, these profits go to the households. Firms themselves are not traded on international markets. $R(s^t)$ is the interest rate paid on the non-contingent one-period nominal bond in the domestic currency and $B(s^t)$ are the nominal government bonds. Compared to Chari, Dovis, and Kehoe (2020a), we replaced the price that is paid to buy new bonds with interest rates that are paid on existing bonds. We show in the Appendix 2.B.1 that the price of bonds in Chari, Dovis, and Kehoe (2020a) is simply the inverse of interest rates used here. The model abstracts from international capital markets, as households do not have an incentive to borrow or lend across countries, given the considered preferences.
There is also a cash-in-advance constraint for consumers, that requires domestic money brought from period $t-1$ to be used to purchase traded goods:

$$P_T\left(s^t\right)C_T\left(s^t\right) \;\leq\; M_H\left(s^{t-1}\right)$$

Under flexible exchange rates, consumers use their local currency $M_H\left(s^{t-1}\right)$ to pay for these goods. The superscript H denotes the individual holding of money. Domestic money is only hold by domestic households. Even though money is dominated

5. Quasi-linear utility eliminates any wealth effects in the demand for money, which makes all agents choose the same amount of money. See Ricardo and Wright (2005)

by bonds as they pay interest on the existing stock, households need money to buy traded-goods. The assumption of cash-in-advance makes surprise inflation costly, as they can only use cash from the last period. In addition, the assumption that only traded goods are affected by this is for simplicity. This assumption can also be interpreted as a trade friction that requires to commit a certain amount of cash beforehand when internationally traded goods are bought from a foreign country. Note that current money injection that increase the nominal price of traded goods cannot be used for the cash in advance constraint. In a currency union they use the common currency to pay for the traded goods.
The first order conditions for the households imply

$$\begin{aligned}
\frac{U_N\left(s^t\right)}{P_N\left(s^{t-1},s_{1t}\right)} &= -\frac{U_L\left(s^t\right)}{W\left(s^t\right)},\\
\frac{U_T\left(s^t\right)}{P_T\left(s^t\right)} &= -\frac{U_L\left(s^t\right)}{W\left(s^t\right)} + \phi\left(s^t\right),\\
-\frac{U_L\left(s^t\right)}{W\left(s^t\right)} &= \beta\sum_{s^{t+1}} h\left(s^{t+1} \mid s^t\right)\frac{U_T\left(s^{t+1}\right)}{P_T\left(s^{t+1}\right)},\\
1 &= \beta\sum_{s^{t+1}} h\left(s^{t+1} \mid s^t\right)R(s^{t+1})\frac{U_N\left(s^{t+1}\right)}{P_N\left(s^t,s_{1t+1}\right)}\frac{P_N\left(s^{t-1},s_{1t}\right)}{U_N\left(s^t\right)},
\end{aligned}$$

where $\phi(s^t)$ is the normalized multiplier of the cash-in-advance constraint. The Euler equation can be obtained by combining the home bonds first order condition with the consumption first order condition. It governs the household's intertemporal decision:

$$\frac{1}{C_N(s^t)} = \beta\mathbb{E}_t\left[\frac{1}{C_N(s^{t+1})}\frac{P_N(s^t)}{P_N(s^{t+1})}R(s^{t+1})\right] \tag{2.3}$$

The nominal stochastic discount factor is defined as

$$Q\left(s^{t+1}\right) = \beta h\left(s^{t+1} \mid s^t\right)U_N\left(s^{t+1}\right)P_N\left(s^{t-1},s_{1t}\right)/\left(P_N\left(s^t,s_{1t+1}\right)U_N\left(s^t\right)\right).$$

This discount factor is also used by firms to discount their profits.

2.3.4 Government

The government budget constraint for each country under flexible exchange rates is given by

$$B\left(s^t\right) = R\left(s^t\right)B\left(s^{t-1}\right) + T\left(s^t\right) - \left(M(s^t) - M(s^{t-1})\right),$$

where $M(s^t)$ denotes the money supply in the economy. The last term is seignorage income from the growth in money supply. In a currency union, union-wide seignorage is equally split across countries according to their size. The initial money supply for each consumer in each country is set to M_{-1} and the initial bond holding B_{-1} are zero. The central bank specifies nominal interest rates, the quantity of debt and taxes for each country, satisfying the budget constraint. Note that there are no externalities for the central banks. This ensures that monetary policy does not have any incentive to set monetary policy in a non-cooperative way and to use its monopoly on its currency to manipulate the terms of trade.

2.3.5 Market Clearing and Equilibrium

Labor markets clear, which means that the demand for non-traded goods labor and traded goods labor equals overall labor supply

$$L_N\left(s^t\right)+L_T\left(s^t\right) = L\left(s^t\right).$$

Good markets clear for traded and non-traded goods.

$$C_T\left(s^t\right) = Y_T\left(s^t\right), \quad C_N\left(s^t\right) = A\left(s^t\right)Y_N\left(s^t\right).$$

GDP in this model is defined as the sum of consumption of traded and non-traded goods. Money demand from households $M_H(s^t)$ is met by money supply of the central bank

$$M_H\left(s^t\right) = M\left(s^t\right).$$

An equilibrium under flexible exchange rates is defined as an allocation in which 1) consumers behave optimally, 2) firms behave optimally, 3) the government's budget constraint holds and 4) markets clear.

As the law of one price holds in this model, the multilateral exchange rate can be defined as the price of traded goods in the considered country relative to the average price of traded goods in the rest of the world:

$$e\left(s^t\right) = \frac{P_T\left(s^t\right)}{\sum_{v^t} P_T\left(z^t, v^t\right) g^t\left(v^t\right)},$$

where $g^t(v^t) = g(v_0)...g(v_t)$ is simply the average over all countries. With a sufficiently large rest of the world, only country-specific shocks of the considered country can change the exchange rate, as the common shocks are the same and the average of the price of traded goods in the rest of the world is independent of shocks to small countries in the rest of the world.

In a monetary union money supply is chosen by the union-wide central bank. The nominal exchange rate is fixed for all states: $e\left(s^t\right) = 1 \quad \forall s^t$ and consequently, the price of traded goods is the same everywhere. This means that only aggregate shocks can change the price of traded goods. Formally, if the state of the world in one country is $s^t = z^t, v^t$ and $\tilde{s}^t = z^t, \tilde{v}^t$ in the other country, then prices of traded goods are still the same

$$P_T(s^t) = P_T(\tilde{s}^t).$$

An equilibrium in a monetary union is defined in the same way as with flexible exchange rates, the only difference being that the exchange rate is set to 1 for all states and that total money holding in a union adds up to the overall money supply

$$\sum_{v^t} M_H\left(z^t, v^t\right) g_t\left(v^t\right) = \bar{M}\left(z^t\right).$$

In this model, shocks to markups lead to distortions in the economy that vary over time. This can be seen when combining the first order conditions of households with the first order condition of firms. Suppose productivity is constant, then the marginal rate of substitution (MRS) between labor and non-traded goods equals the marginal rate of transformation (MRT) of labor times the inverse markup

$$-\frac{U_L}{U_N} = A\theta\left(s_t\right) < A.$$

This means that the markup drives a wedge $1 - \theta(s^t)$ between the MRS and the MRT. The larger the markup $1/\theta(s^t)$, the greater the distortions resulting from imperfect competition. The next section explains how monetary policy deals with that issue and how a lack of commitment can lead to an inflationary bias in that environment.

2.3.6 Monetary Regimes

This subsection discusses benchmark allocations under different monetary regimes and compares them. We consider three monetary regimes: In the first regime, every country conducts its own monetary policy under commitment. The exchange rate floats freely, and the central bank is credible. In the second regime, all countries conduct monetary policy independently under discretion. This means that the exchange rate floats freely, and the central bank is not credible. In the last regime, a currency union is formed in which a common central bank conducts monetary policy under discretion. We describe how monetary policy operates in each regime. These results reproduce those in Chari, Dovis, and Kehoe (2020a). Then we show how this difference is reflected in consumption and other nominal variables.

2.3.6.1 Flexible Exchange Rates: Monetary Policy under Commitment

The central bank conducts monetary policy under commitment. This means that the central bank maximizes ex ante lifetime utility of its representative household. It chooses an appropriate state-contingent path of prices subject to the consumer and firm first order conditions, the resource constraint, as well as the production function [6]. The central bank sets its policy after productivity has realized.

$$
\begin{aligned}
\max_{\{P_T(s^t),P_N(s^t)\}_{t=0}^{\infty}} \quad & \mathbb{E}_0\left[\sum_{\tau=t}\beta^t\left(\alpha\log(C_T(s^\tau)+(1-\alpha)\log(C_N(s^\tau))-\psi L(s^\tau)\right)\right]\\
\text{s.t.} \quad L(s^t) &= \frac{C_N(s^t)}{A(s_{2t})}+C_T(s^t),\\
C_T(s^t) &= \frac{\alpha}{\psi},\\
C_N(s^t) &= \frac{1-\alpha}{\psi}\frac{P_T(s^t)}{P_N(s^{t-1},s_{1t})},\\
\sum_{s_{2t}} h\left(s^t \mid s^{t-1},s_{1t}\right)C_N\left(s^t\right)&\left[U_N\left(s^t\right)+\frac{1}{\theta\left(s_{1t}\right)}\frac{U_L\left(s^t\right)}{A\left(s_{2t}\right)}\right] = 0,
\end{aligned}
$$

where the first constraint is the resource constraint combined with the production functions, the next two are the consumers first order conditions and the last constraint is the optimality condition of firms combined with the stochastic discount factor and $W(s^t)=P_T(s^t)$. Importantly, the central bank realizes that firms will set their relative prices equal to expected productivity times the markup. In a world under discretion, in which the central bank would take $P_N(s^{t-1},s_{1t})$ as given, it would try to inflate away the markup, to set $P_T(s^t)/P_N(s^{t-1},s_{1t})=A(s_{2t})$. Under commitment the central bank realizes that this attempt of surprise inflation will not work. Therefore, optimal policy does not respond to markup shocks. It only responds to productivity shocks. Intuitively, the monetary authority has to live with the distortions from markup shocks and attempts to accommodate productivity shocks. Therefore, the optimal policy of the central bank implies

$$
\frac{P_T(s^t)}{P_N(s^{t-1},s_{1t})} = \theta(s_{1t})A(s_{2t}).
$$

The interpretation of that policy rule is straightforward: After productivity has realized the central bank makes sure that relative prices move in such a way that they replicate the outcome as if non-traded good prices were flexible. This way the central bank can eliminate any distortions coming from rigid prices. The central bank engineers a movement of the exchange rate in such a way that relative prices align. For example, if productivity of the non-traded goods sector is high today, P_N should decrease as it is easier to produce that good. As prices of that good do not adjust, the central bank instead uses the exchange rate to let the currency depreciate so such P_T rises, which means that the relative price for P_N falls. The movement of the

6. The central banks could also jointly maximize a weighted sum of all countries using their policy instrument for each country. As there are no externalities in the model of Chari, Dovis, and Kehoe (2020a), cooperative and non-cooperative equilibria coincide.

exchange rate aims to replicate the outcome of relative prices as if all prices were flexible.

Note also, that optimal monetary policy would never cause consumers to lose consumption because they do not have enough cash. Therefore, the cash in advance constraint is never binding in a way that would lower the household's consumption. That is the reason why the consumer first order condition with respect to C_T has a multiplier from the cash in advance constraint equal to zero.

2.3.6.2 Flexible Exchange Rates: Monetary Policy under Discretion

Now consider how a non-credible central bank sets monetary policy. The important difference when a central bank acts under discretion is that it takes the price of non-traded goods as given, as firms have set their prices before the central bank acts. As a consequence, the central bank will try to use monetary policy to inflate away the inefficient monopolistic markups and implement an allocation, that equalizes household's marginal rate of substitution with the marginal rate of transformation of the economy. That is $P_T(s^t)/P_N(s^{t-1}, s_{1t}) = A(s_{2t})$. In order to do that the central bank will go so far to make the cash in advance constraint binding. As long as this constraint is slack, the central bank can use more inflation to reduce the markups. Therefore, the central bank makes the cash in advance constraint binding and ultimately trades off the costs of markups with the costs of surprise inflation that lower the household's purchasing power. A central bank under discretion therefore chooses $p_T(s^t) = P_T(s^t)/M(s^{t-1})$ to maximize the following problem:

$$\begin{aligned} \max_{p_T(s^t)} \quad & \mathbb{E}_t\left[\sum_{\tau=t} \beta^t \left(\alpha \log(C_T(s^\tau) + (1-\alpha)\log(C_N(s^\tau)) - \psi L(s^\tau)\right)\right] \\ \text{s.t.} \quad C_T(s^t) &= \frac{1}{p_T(s^t)}, \\ C_N(s^t) &= \frac{1-\alpha}{\psi}\frac{p_T(s^t)}{p_N(s^{t-1}, s_{1t})}, \\ L(s^t) &= C_T(s^t) + \frac{C_N(s^t)}{A(s^t)}. \end{aligned}$$

Note the following differences to the problem before: The central bank's objective function has an expectation operator that starts in t as the central bank acts under discretion and does not commit beforehand. The consumption constraint for traded goods is also altered, as the cash in advance constraint is binding. In addition, the central bank does not take the firm's first order condition into account as it acts under discretion. Chari, Dovis, and Kehoe (2020a) show that the dynamic dimension of this problem does not play a role, so the central bank simply acts as maximizing the per period utility of its household. The best response of the monetary authority is to set the price of traded goods as:

$$p_T(s_t) = p_N(s_{1t})A(s_{2t}) \underbrace{\frac{1}{2(1-\alpha)}\left[(1-2\alpha)+\sqrt{(1-2\alpha)^2+4(1-\alpha)\frac{1}{A(s_{2t})}\frac{\psi}{p_N(s_{1t})}}\right]}_{F\left(\frac{1}{A(s_{2t})p_N(s_{1t})}\right)}$$

where the first part on the right-hand side $p_N(s_{1t})A(s_{2t})$ captures the willingness of the central bank to put the marginal rate of transformation equal to the marginal rate of substitution and $F(\cdot)$ captures the costs from surprise inflation. If p_N increases by one, p_T increases less than one-to-one. In the following we assume as in Chari, Dovis, and Kehoe (2020a) that $\frac{1}{\theta(s)} < \frac{1-\alpha}{1-2\alpha}$ so that there is a point where marginal costs of surprise inflation equal their marginal benefits. This simply bounds markups from above, meaning that it is not possible that reducing markup distortions always exceed the costs of reducing trade goods consumption.
Another aspect that needs to be mentioned is, when productivity is stochastic and is sufficiently low compared to its average value, it can happen that the cash in advance constraint is not binding despite the central bank's policy. That is if $Ap_N < C_T$ then $p_T = p_N A$. Taken this into account as well, it implies that the price of traded goods is described by $p_T(s_t) = \max\{p_N(s_{1t})A(s_{2t}), p_N(s_{1t})A(s_{2t})F(\cdot)\}$.
For policy under discretion, it is also important to consider the firms. They take into account that the central bank will try to inflate away their markups. Optimally firms still set prices of traded goods as in (2.1). Remember that firms observe the markup shock and then set their price taking their expectation for future productivity into account. Overall, the price of traded goods in the equilibrium solves the fixed-point problem of equaling the optimal price firms would set and what the central bank wants to implement. So, in equilibrium, any attempt of the central bank to inflate away the markup is frustrated, as firms anticipate the central bank's move and set their prices accordingly. The only thing the central bank achieves is an inflationary bias with higher volatility of prices and consumption.

2.3.6.3 Currency Union: Monetary Policy under Discretion

In a monetary union, the exchange rate is fixed and set to $e(s^t) = 1$ for all states. This implies that P_T cannot vary across countries and is only a function of aggregate union-wide shocks. This gives rise to the "Union constraint"

$$\frac{U_T(s^t)}{U_N(s^t)} = \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)}.$$

The union consists out of many blocks, each block i having a mass of countries n^i. The relative weight of block i is $\lambda^i = \frac{n^i}{\sum_i n^i}$. Countries are all the same across blocks, except for the shock process of their markup. The central bank acts under discretion and chooses the union-wide price of traded goods to maximize an equally weighted average of all countries of the world. The union-wide central bank chooses a traded good price for the union taking the non-traded good prices as given.

$$
\begin{aligned}
\max_{p_T} \sum_{\lambda^i} \lambda^i \sum_{v^t} g(v^t) \left[\alpha \log C_T^i(s^t) + (1-\alpha) \log C_N^i(s^t) - \psi \left(L^i(s^t) \right) \right] \\
\text{s.t.} \quad L^i(s^t) &= \frac{C_N^i(s^t)}{A^i(s_{2t})} + C_L^i(s^t), \\
C_T^i(s^t) &= \frac{1}{p_T(s^t)}, \\
C_N^i(s^t) &= \frac{1-\alpha}{\psi} \frac{p_T(s^t)}{p_N^i(s^{t-1}, s_{1t})}, \\
\frac{U_T(s^t)}{U_N(s^t)} &= \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)},
\end{aligned}
$$

where $g(v)$ gives the average state of all countries within a block, given the aggregate state. The policy of the central bank implies to set the price of traded goods such that:

$$
p_T\left(z, \left\{p_N^i\left(z_1, v_1\right)\right\}\right) = \frac{(1-2\alpha) + \sqrt{(1-2\alpha)^2 + 4\sum_i \lambda^i \sum_v g(v) \frac{(1-\alpha)}{A^i(z_2, v_2)} \frac{\psi}{p_N^i(z_1, v_1)}}}{\sum_{i=N,S} \lambda^i \sum_v g(v) \frac{2(1-\alpha)}{A^i(z_2, v_2)} \frac{1}{p_N^i(z_1, v_1)}}.
$$

Compared to the policy rule under discretion with an independent national central bank single country-specific shocks are replaced by the average shock realization of the union.

As before, firms anticipate the policy of the central bank and take this into account when setting their prices. In a currency union however, they realize that the central bank will only react to the average temptation shock, not the country-specific one. The result is still more inflation. The next section discusses how the policy under discretion in a currency union can still yield some benefits compared to discretion of a single country.

2.3.7 Overview

This section summarizes key real and nominal variables given the policy rules under different monetary regimes. We still closely follow Chari, Dovis, and Kehoe (2020a), the model results are the same as in their Appendix 7 and 10 I (Chari, Dovis, and Kehoe, 2020b) with blocks of countries that form a currency union. For simplicity again, we focus on a model solution with non-stochastic productivity such that the cash in advance constraint is binding in discretion. First consider how consumption compares across different regimes:

Table 2.1. Consumption under different monetary regimes.

Symbol	Commit & float	Discretion & float	Discretion and union
C_T	$\frac{\alpha}{\psi}$	$\frac{\alpha}{\psi}-\frac{1-\alpha}{\psi}(1-\theta(s))$	$\frac{\alpha}{\psi}-\frac{1-\alpha}{\psi}\left(1-\sum_i \lambda^i \mathbb{E}_v\left(\theta^i(s)\right)\right)$
C_N	$\frac{1-\alpha}{\psi}\theta(s)A$	$\frac{1-\alpha}{\psi}\theta(s)A$	$\frac{1-\alpha}{\psi}\frac{\theta(s)}{\sum_i \lambda^i \mathbb{E}_v(1/A^i)}$

In general, consumption of traded goods is larger the larger the trade openness α. Large values of disutility from work $\psi > 0$ lower consumption. Under both discretionary regimes, C_T is lower than with commitment, as the central bank follows an inflationary policy. With high inflation, the household's cash in advance constraint is binding such that traded good consumption is lower. Larger markups increase the inflationary bias and hence decrease the amount of traded goods consumption under discretion. That is, if $\theta \in (0,1)$ is relatively small. Consumption of non-traded goods is a function of actual productivity and markups, as long as the exchange rate can float. As soon as there is a currency union non-traded good consumption is a function of the average productivity of the union: With a fixed exchange rate the central bank can only ensure that relative prices align for the average of the union, not for each individual country. If productivity is stochastic, there will be welfare losses in a peg: Prices for very productive countries are too high compared to a flex price world, while prices of low productivity countries are too low. This is inefficient as this implies employment gaps that lower welfare.
Next turn to the nominal variables of the model. This table summarizes key rates under different regimes[7].

Table 2.2. Interest, Inflation and Money Growth rates under different monetary regimes with no stochastic productivity.

Symbol	Commit & flexible	Discretion & flexible	Discretion & union
R	1	$\mathbb{E}\left[\frac{\alpha-(1-\alpha)(1-\theta(s'))}{\alpha}\right]^{-1}$	$\mathbb{E}\left[\frac{\alpha-(1-\alpha)(1-\sum_i \lambda^i \mathbb{E}_v \theta^i(s))}{\alpha}\right]^{-1}$
π_N	$\frac{\theta(s)}{\theta(s')}\beta$	$\frac{\theta(s)}{\theta(s')}\frac{\beta\alpha}{\alpha-(1-\alpha)(1-\theta(s))}\Theta(s')$	$\frac{\theta(s)}{\theta(s')}\frac{\beta\alpha}{\alpha-(1-\alpha)(1-\sum_i \lambda^i \theta^i(s))}\Theta^U(s')$
ΔM	β	$\beta\frac{\alpha}{\alpha-(1-\alpha)(1-\theta(s))}$	$\beta\frac{\alpha}{\alpha-(1-\alpha)(1-\sum_i \lambda^i \mathbb{E}_v \theta^i(s))}$

where $\Theta(s') = \frac{\frac{\alpha}{\psi}-\frac{1-\alpha}{\psi}(1-\theta(s))}{\frac{\alpha}{\psi}-\frac{1-\alpha}{\psi}(1-\theta(s'))}$ and $\Theta^U(s') = \frac{\frac{\alpha}{\psi}-\frac{1-\alpha}{\psi}(1-\sum_i \lambda^i \mathbb{E}_v \theta^i(s))}{\frac{\alpha}{\psi}-\frac{1-\alpha}{\psi}(1-\sum_i \lambda^i \mathbb{E}_v \theta^i(s'))}$. Under commitment, the gross nominal interest rate is one, which means that nominal interest rates are zero. The central bank follows the Friedman (1969) rule implying a negative money growth rate. The intuition why zero interest rates are optimal under commitment is the following. For households, nominal bonds dominate money holding as long as they pay an interest on its stock, Money does not pay any returns for its holder. Nevertheless, households need to hold money to buy traded goods. Therefore, the central bank optimally implements zero interest rates to make the necessary money

7. For a derivation see the Appendix 2.B.4

holding as good as the bond holding. In addition, deflation ensures that the cash in advance constraint is never binding for households.

In contrast inflation, interest rates and money growth rates are larger in both discretionary regimes. As discussed before, the central bank has an incentive to use surprise inflation to inflate away markups. Ultimately, the central bank trades off costs of inflation in from of a binding cash in advance constraint with reduced markups. Firms anticipate this attempt and simply raise their prices. In equilibrium, the economy ends up with higher inflation. The size of the inflationary bias depends on $\frac{\alpha}{\alpha-(1-\alpha)(1-\theta(s))}$. Values of that term close to one imply no inflationary bias. This means that larger markups (small $\theta(s_{1t})$) correspond to a larger inflationary bias. The larger trade-openness (large α) the lower is the inflationary bias. As internationally traded goods are more important to households, the central bank is careful not to induce too much inflation that lowers consumption of internationally traded goods. The central bank achieves higher inflation by inducing a positive growth rate for money supply. The Euler equation then dictates that nominal interest rates have to be higher as well. Next consider the role of $\Theta(s)$ that impacts inflation: This term simply adds more volatility in the inflation process under discretion. For π_N, if the markup rises in the future, this also increases inflation of this good by a larger amount. If markups are lower than usual, then inflation decreases more than without this term. It is simply an amplifier. Together with the higher money growth rate, inflation rates *are higher on average and more volatile*!

Comparing both discretionary regimes, a currency union can ensure that Θ is more stable over time when countries with the same shock process form a union. Country-specific markup shocks vary more than the average of all markup shocks. Therefore, a currency union is able to significantly reduce the volatility of inflation. Furthermore average money growth rate in a currency union must be lower as well, as $\frac{\alpha}{\alpha-(1-\alpha)(1-\theta(s))}$ is a convex function in θ. The average value of that term with country-specific shocks is larger as the average value of that term with average union-wide shocks. This implies that -absent stochastic productivity- forming a currency union for countries with the same stochastic process yields benefits, as inflation is lower on average. The next chapter calibrates the shock process in more detail.

2.4 Calibration

The section calibrates the model. The model here seeks to highlight benefits of creating a currency union that fixes the exchange rate and as a consequence lowers inflation. Towards that aim, we focus on two large members of the eurozone, namely Germany and Italy between 1972 and 1985. The reason for that time horizon is that the model predicts substantially different inflation rates only if the exchange rate of both countries is flexible. That sample includes the time after the breakdown of Bretton Woods in which the exchange rate of Italy moved by a great margin. In

1985 Italy decided to peg its currency to the German Mark. We also consider an alternative time horizon from 1960-1999 that accounts for some of the temporary currency devaluations in Italy that took place before the arrival of the Euro. The choice of Italy and Germany as our countries of interest has also a reason: Both are the largest countries of their respective block: Germany being part of the core (or the northern) block in the currency union, with low and stable inflation rates. And Italy as the largest country of the periphery (or the southern block) that experienced large increases in inflation during the mid 70s and 80s. One period in the model taken to be a year. The calibration proceeds in two steps. First, we calibrate parameters based on long-run moments in the data and the outside literature. Thereafter, taking these as given, we calibrate the markup process to match key stylized facts on business cycle movements of Germany and Italy.
Some parts of the calibration are identical to Arvai (2021). The model is kept relatively simple, therefore only a couple of parameters need to be calibrated. This table summarizes the key calibration values:

Table 2.3. Calibration

Symbol	Value	Description	Target
β	0.98	Time discount rate	Real rate of 2% p.a.
ψ	8/3	Disutility of work	1/3 of time spent working
α	0.25	Share traded goods in consumption	Trade openness Italy 2015

The time discount factor is chosen to replicate a real interest rate of around 2% per year, in line with estimates for European countries by Brand, Bielecki, and Penalver (2018). Next, we choose the trade openness α to be 25 % in line with the trade openness (imports over GDP) for Italy in 2015. We also consider the impact of smaller and larger values of trade-openness in Figure 2.B.1. The trade elasticity and intertemporal elasticity is already chosen to be 1 in the specification of preferences. Next, we turn to the heart of the calibration, that aims to match cyclical inflation movements in Europe with the evolution of markups in the model. The calibration of the markups process $1/\theta(s)$ is crucial. It determines how large and volatile the inflationary bias is on average for those countries. This is relevant for potential benefits of entering a currency union. The range of estimates of markups varies widely, see for example De Loecker and Warzynski (2012), Christopoulou and Vermeulen (2012), Kuester (2010) or Midrigan (2011). In most applications, as for instant in Gomes, Jacquinot, and Pisani (2012), markups vary between 15% and 50%. The higher the markup, the higher inflation under discretion. For more open economies -larger α- inflation is lower. We calibrate the markup process for Italy and Germany between 1972 and 1985. The goal is to match the behavior of inflation for those countries using the simulated method of moments: The model generates certain moments of inflation given a process for $\theta(s)$, like the mean and volatility of inflation in a discretionary float. The model predicts that countries in a float have potentially different inflation rates, depending on their shock process. We then assume that the country-specific component of θ is uniformly distributed between two values $\underline{\theta}$ and $\bar{\theta}$. The global component is muted for this exercise, the process is still assumed to be iid. This is an important assumption as there is no persistent component in the process that we estimate. Large shocks today do not have an impact on future shocks. Even though this assumption limits the behavior of markups, it keeps the model simple and tractable. For now, we also impose zero correlation of these shocks between countries. We aim to include correlation between countries in the future, as the correlation of markup shocks is a crucial component for the desirability of currency unions. Our method will then choose the two parameters of the uniform distribution for each country separately such that the simulated moments of inflation (mean and standard deviation) in the model match the empirical moments in the data well. A country with very volatile inflation will require $\underline{\theta}$ and $\bar{\theta}$ to be relatively far away from each other. Low average inflation values would correspond to relatively low markups, that is a value of θ that is closer to 1. We also impose that certain model assumptions still hold, such as $\theta \in (0,1)$ or $\theta > 1 - \alpha/(1-2\alpha)$.

Formally, let x be the data and $m(x)$ the moments of the data. The corresponding moments of the model are denoted by $m(\tilde{x}, \upsilon)$ where υ are the parameters of the model. We simulate the model S times, such that there are S simulations of the model data $\tilde{x} = \{\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_S\}$. The vector of moments in one simulation s of length T consists -in a first step- out of two expressions. The standard deviation and the mean of a country's inflation rate during a discretionary float in simulation s

$$std_{\pi,s} = \sqrt{\frac{1}{T}\sum_{t}^{T}(\pi_t - \bar{\pi}_s)^2}, \quad \mu_{\pi,s} = \frac{1}{T}\sum_{t}^{T}\pi_t.$$

The estimated model moments from the simulation are

$$\hat{m}(\tilde{x}, \upsilon) = \frac{1}{S}\sum_{s=1}^{S} m\left(\tilde{x}_s \mid \upsilon\right).$$

The SMM approach estimates the parameter vector $\hat{\upsilon}_{SMM}$ to choose υ in such a way that it minimizes the L^2 norm of the sum of squared errors in moments. We define the moment error function as the percent difference in the vector of simulated model moments from the data moments

$$e(\tilde{x}, x \mid x) = \frac{\hat{m}(\tilde{x} \mid \upsilon) - m(x)}{m(x)}.$$

The SMM estimator is now the following:

$$\hat{\upsilon}_{SMM} = \upsilon : \min_{\upsilon} e(\tilde{x}, x \mid x)^T W e(\tilde{x}, x \mid x)$$

where W is a weighting matrix, in a first step it is the identity matrix, implying equal weights for all moments.

Next, the table summarizes the estimation, the moments of the data and the moments of the model under a discretionary float for both countries.

Table 2.4. SMM for 1972-1985

Country	$\underline{\theta}$	$\bar{\theta}$	μ_π data	μ_π model	σ_π data	σ_π model
Italy	0.939	0.962	15.21%	15.10%	0.036	0.036
Germany	0.972	0.985	4.57%	4.68%	0.017	0.017

Given that the process follows a uniform distribution, markups vary between 4% and 6.5% for Italy and between 1.5% and 2.9% for Germany. In this model, inflation in a discretionary regime gets very large as soon as markups are greater than 5 %, see also Figure 2.B.1. This is because the only costs of inflation come from the cash in advance constraint. There is no price dispersion, that arises when prices are set as in a model with Calvo price stickiness. There is also no money in utility, which implies high costs of inflation. As these additional costs of inflation are absent in this model the central bank is willing to tolerate high inflation in order to reduce markups. Therefore these relatively low values for markups are enough to generate average inflation of around 15.1% in Italy and 4.7 % in Germany. We also estimated a process for θ for 1960-1999 in Table 2.B.1. For time horizons with fixed exchange rate regimes, average inflation is lower.
Last, we also need to take a stand about the block size λ^i, or more precisely about the welfare weight of the block for the union-wide central bank. For now, we assume that the German block is very large compared to Italy. If Italy were to join in the currency union with a large weight, the average temptation shock of the union would be greater for Germany. This would imply that Germany's inflation rate rises when joining such a currency union. We therefore assume that the central bank only reacts to the average temptation shock of the German block, and that the German block consists out of four Germanys. This matches the relative size of Germany to the eurozone. Italy then simply joins the currency union and has no weight in the central bank's policy function. Furthermore, we do not yet include productivity shocks. This means that there are no Mundellian costs of a currency union[8].

2.5 Results

With the calibration as in Section 2.4, we simulate the model for both countries under the three monetary regimes. The following table summarizes mean and standard deviation of consumption and inflation for 100 simulations, each with 1000 periods. The first table shows the values for Italy:

8. We aim to include and estimate a productivity component in future work.

Table 2.5. Mean and standard deviation of consumption and inflation for Italy under different monetary regimes with no stochastic productivity.

Symbol	Commit & float	Discretion & float	Discretion and union
$\bar{C}_T$	0.0313	0.0266	0.0293
σ_{C_T}	0	0.0006	0.0002
$\bar{C}_N$	0.0891	0.0891	0.0891
σ_{C_N}	0.0006	0.0006	0.0006
$\bar{\pi}_N$	0.9800	1.1510	1.0466
σ_{π_N}	0.0096	0.0355	0.0124

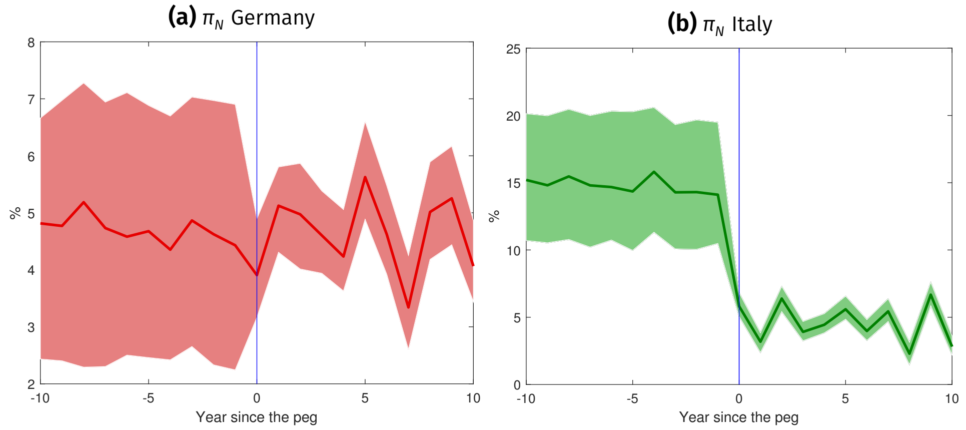
Average inflation is lowest in the regime under commitment. The central bank follows the Friedman (1969) rule, implementing zero interest rates and a continued contraction of the money supply. As a consequence, consumption of traded goods is large as the cash in advance constraint never binds. In a discretionary float inflation averages about 15% in Italy, causing a substantial reduction in the consumption of traded goods. In addition, volatility of inflation and of consumption has increased. Moving from a discretionary float to a discretionary currency union under the leadership of a block of Germanys leads to a large reduction of inflation. The central bank only reacts to the average temptation shock of the German block. Inflation volatility also goes down. As a consequence the mean of consumption of traded good goes up, while its volatility declines. Consumption of non-traded goods is unaffected as productivity is kept constant in this simulation. There are no Mundellian cost of forming a currency union. This ensures that Italy benefits a lot by joining a currency union that does policy for the average German block and not for Italy.

Table 2.6. Mean and standard deviation of consumption and inflation for Germany under different monetary regimes with no stochastic productivity.

Symbol	Commit & float	Discretion & float	Discretion and union
$\bar{C}_T$	0.0313	0.0293	0.0293
σ_{C_T}	0	0.0003	0.0002
$\bar{C}_N$	0.0918	0.0918	0.0918
σ_{C_N}	0.0003	0.0003	0.0003
$\bar{\pi}_N$	0.9800	1.0468	1.0466
σ_{π_N}	0.0052	0.0166	0.0089

Turning to Germany, inflation under commitment is -2% on average as well. Inflation volatility under commitment is slightly lower than Italy's as markup shocks are less volatile in Germany. A discretionary float yields average inflation rates of 4.68%, reducing consumption of traded goods. In addition volatility of both inflation and consumption increases. When Germany forms a currency union with another block of Germanys four times the size of itself, then volatility of inflation is cut in half while volatility of traded goods consumption is also reduced. Average inflation is only reduced by a small margin, going down from 4.68 % to 4.66%. The simulation suggests that the effect on inflation volatility is very large, while there is barely a reduction of average inflation.

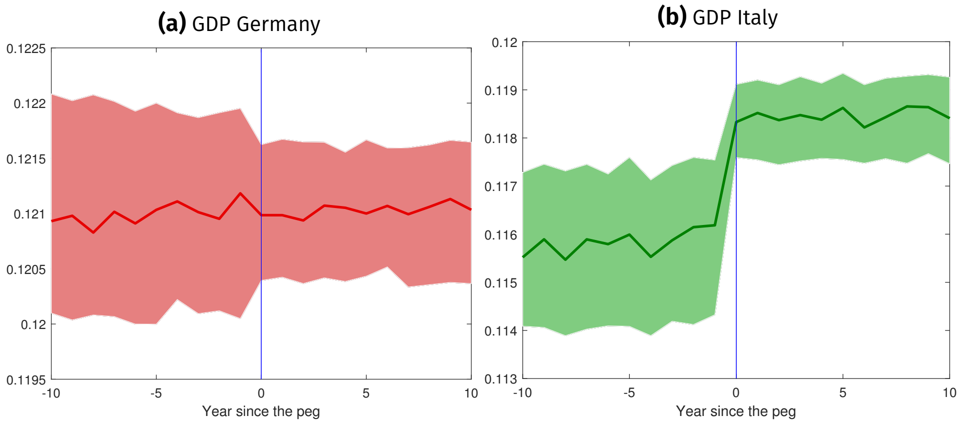
Let us turn to one specific simulation, in which we replicate Figure 2.4 of the event study. The simulation starts in a discretionary regime under flexible exchange rate at $t = -10$ for both Italy and Germany. The regime changes to a discretionary currency union between these two countries in $t = 0$, indicated by a vertical blue line. This regime stays until the end of the simulation. Consider how inflation looks like for Germany and Italy.



Notes: Inflation of non-traded goods in the model for Italy and Germany. Both countries initially start in a discretionary regime with flexible exchange rates. The vertical blue line indicates when the peg gets effective. The shaded areas indicate the 10th and 90th percentile of highest and lowest values of inflation in the simulations.

Figure 2.6. π_N in the model for Italy and Germany.

During the time in a discretionary float, the median value of Italy's inflation rate fluctuates around its steady state value of 15%, the variability of inflation is large. This is indicated by the shaded areas that show the 10th and 90th percentile of the highest and lowest values of inflation in the simulations. The German inflation rate fluctuates around 4% while its central bank reacts to its country-specific markup shocks. As soon as Germany and Italy form a currency union, Italy's inflation and volatility goes down to the German level. Variability of the German inflation rate, indicated by the shaded area also gets more narrow as well mirroring the observation in Section 2.2. Next consider what happens to GDP:



Notes: GDP over time. GDP is defined as the sum of consumption of traded and non-traded goods. Italy initially starts in a discretionary float, the vertical blue line indicates the peg. The shaded areas indicate the 10th and 90th percentile of highest and lowest values of GDP in the simulations.

Figure 2.8. GDP in the model for Italy and Germany.

GDP in the discretionary float is smaller for Italy, but increases as soon as inflation goes down in a currency union. Because households are able to consume more traded goods with lower inflation rates, production of traded goods in Italy increases, leading to an increase of GDP of around 4 %. For both countries it is true that the variability of GDP goes down as well when a currency union is formed.

2.5.1 Discussion

A calibrated version of the model of Chari, Dovis, and Kehoe (2020a) indeed manages to generate a reduction of average inflation of around 10 percentage points and a substantial decline in volatility for Italy when a currency union is formed. This is in line with our evidence in Section 2.2 in which we show that average inflation decreases permanently. The model also predicts that inflation volatility of Germany will go down by a large margin, but it fails to generate the substantial decline of the level of inflation in Germany that we observe in the data. The decline of inflation in Germany might be due to several factors that we do not account for in our thought experiment. The global component of the markup shock process is muted, there is also no trend component that we consider, neither for markups nor for productivity. In addition, we assume that central banks in Germany, Italy and of the currency union act under full discretion. If central banks gains credibility when forming a currency union, inflation would also go down as the central bank can credibly pursue low-inflation policies. Turning to GDP growth: Our simulation shows that Italy experiences a substantial increase of GDP of around 4% when pegging its currency, while Germany only experiences a reduction in GDP volatility. The data suggest that GDP grows only by around 2 percentage points for the median country. Comparing this to other work that estimates the costs of inflation, a GDP loss of 4% with inflation of around 15% is relatively large. Lucas (2000) estimate in a model with money in utility, that a permanent level of 10% nominal interest rates entails welfare costs in the steady state equivalent to 1.5% of steady state consumption[9]. Furthermore, our evidence that GDP increases after a pegging is in contrast to other work. Mukhin (2018) for instant (and many others) emphasize that there are no effects on macroeconomic variables after a change in the exchange rate regime. We therefore want to provide more systematic econometric evidence for the effects exchange rate regimes have on inflation and real economic activity in the next section.

2.6 Econometric Evidence

In this section, we test whether the model's implications apply to episodes across the globe. We base our empirical analysis on annual data for 178 economies, including both AEs and EMEs, from 1950 to 2016 as presented in Section 2.2.1. With this dataset at hands we test the following model implications. Do inflation and interest rates decrease, and real GDP growth increase, following a change in the exchange rate regime towards a more fixed regime?

To estimate the impact of changing the exchange rate regime (ERR), we need to compare two counterfactual scenarios: One where the representative country in our sample effectively changed the ERR and the other where it did not. If the ERR change decision was random, it would be sufficient to compare the average performance of changers to non-changers. But do countries randomly change their exchange rate regime?

Historically, there are two well studied episodes that offer quasi-random variation. First, the United States unilateral decision of terminating the convertibility of the US dollar to gold on 15 August 1971. This event effectively led to the collapse of the Bretton Woods agreement, and thus forced countries to change their exchange rate regime (Bordo, 1993). While some were forced to immediately float their currency, others decided to peg to another anchor currency, with the German Mark being one of the preferred currencies (Ilzetzki, Reinhart, and Rogoff, 2019). Second, the Euro creation. Eurozone accession was driven mainly by political rather than economic factors (Feldstein, 1997). In fact, by not satisfying the requirements of an Optimum Currency Area, many economists believed that countries adopting the euro would face economic losses (Jonung and Drea, 2009), belief that was later corroborated as argued by Puzzello and Gomis-Porqueras (2018) and Gabriel and Pessoa (2020). Notwithstanding, it is not given that all such events in our sample are as good as random.

9. In the specification used by Lucas (2000) interest rates of 15% entail welfare costs equivalent to 1.7% of steady state consumption.

We thus accept that some changes in the ERR decisions in our dataset are more endogenous than others, but we seek to explicitly model this endogenous decision process and account for it in our estimation. By modeling the ERR change decision, we can effectively reverse-engineer it and rebalance the sample "as if" it were taken at random. To do this, we use the inverse propensity score weighting methodology as in Jordà and Taylor (2016), described in the following Section 2.6.1. Just as in the theoretical section, throughout this section, we are going to explore pegging episodes and provide the complementary results to a floating episode in appendix.

2.6.1 Methodology

As we argued before, it is possible that policy makers choose a specific exchange rate regime due to current economic circumstances or because they wanted to achieve a certain economic outcome (such as lower inflation or interest rates). Those changes in the exchange rate regime cannot be seen as exogenous and are hence uninformative in inferring causal effects of a fixed or a flexible regime.

To estimate the causal response, we thus employ an inverse probability weighted regression-adjusted (IPWRA) estimator. An inverse probability weighted (IPW) estimator gives more weight to those events that are difficult to predict based on observables and less weight to those instances that are endogenous due to the other factors. This estimator will thus rebalance the sample to mimic a setting where the ERR change decision was random. Applications of such method study, for example, the economic response to austerity (Jordà and Taylor, 2016), to macroprudential policy changes (Richter, Schularick, and Shim, 2019), and to sovereign defaults (Kuvshinov, 2019).

Let $d_{i,t}$ be a dummy variable that takes value 1 if there was a change towards a more fixed exchange rate regime and zero otherwise. The estimation proceeds in two stages. In the first stage, we model the ERR change decision by estimating a propensity score for each observation in our sample. Such score is obtained by a logit model which estimates the probability that the ERR is going to change as follows:

$$log\left(\frac{P[d_{i,t} = 1|Z_{i,t-1}]}{P[d_{i,t} = 0|Z_{i,t-1}]}\right) = \alpha_i + \beta Z_{i,t-1} + \varepsilon_{i,t} \tag{2.4}$$

where $Z_{i,t-1}$ is a vector of macroeconomic controls at time $t-1$, where we include the lagged growth rates of real GDP, CPI inflation, and short-term interest rates. In addition, we include country-fixed effects to account for country-specific trends. We refer to the probability of a tightening as the propensity score and its estimate from Equation (2.4) is denoted by $p_{i,t}$.

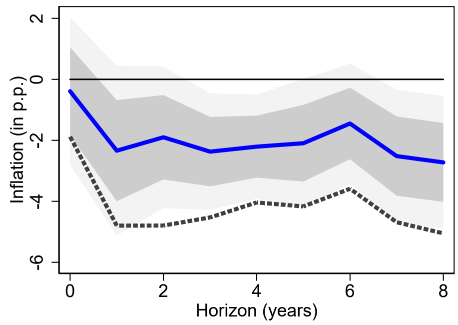
In the second stage, we estimate local projections using regression weights given by the inverse of $p_{i,t}$. To be precise, the weights are defined by $w_{i,t} = \dfrac{d_{i,t}}{p_{i,t}} + \dfrac{1-d_{i,t}}{1-p_{i,t}}$, where we truncate $w_{i,t}$ at 10. Weighting by the inverse of the propensity score puts more weight on those observations that were difficult to predict and thereby re-randomises the treatment. In our application, this implies putting more weight on exchange rate regime changes that were taken as a surprise based on observables, and putting less weight on those changes that could be predicted. Once the sample is rebalanced, the impact of an ERR change is measured as its "average treatment effect", that is, the average difference in potential outcomes of changers and non-changers across the sample. Potential outcomes are computed using a conditional local projection forecast over a horizon of 8 years (Jordà, 2005). To implement the second stage, we thus run the following specification using weighted least squares:

$$\Delta_h y_{i,t+h} = \alpha_i^h + \gamma_t^h + \Gamma^h d_{i,t} + \sum_{k=0}^{2} \phi_k^h \Delta Z_{i,t-k} + \sum_{k=1}^{2} \kappa_k^h \Delta y_{i,t-k} + \epsilon_{i,t+h}, \quad h \in \{0, \dots, 8\} \tag{2.5}$$

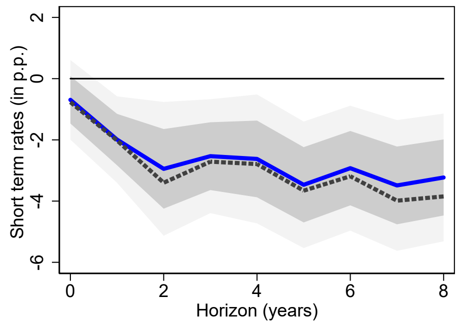
where $\Delta_h y_{i,t+h} = y_{i,t+h} - y_{i,t-1}$ is the conditional forecast of the cumulative growth in percentage points in one of the outcome variables (short term interest rates or inflation), or $\Delta_h y_{i,t+h} = (y_{i,t+h} - y_{i,t-1})/y_{i,t-1}$ in percent for real GDP, in country i between base year $t-1$ and year $t+h$ over varying prediction horizons $h = 0, 1, \dots, 8$ years. $d_{i,t}$ is the treatment dummy variable as before, taking a value of 1 whenever there is a pegging (floating) episode and thus Γ^h is our coefficient of interest.
We include a rich set of covariates in each specification including country dummies to control for country-specific growth rates α_i^h as well as time-fixed effects γ_t^h to control for global trends. Moreover, we include $Z_{i,t-k}$ which is a vector consisting of up to $k = 2$ lags of real GDP growth, inflation, and changes in the interest rates. Finally, $\epsilon_{i,t+h}$ is the error term, and the standard errors are clustered by country. This procedure now assigns a higher weight to the treated observations that were less likely to be treated based on this analysis, i.e. those observations with very low probabilities. Further details on the methodology can be found in Jordà and Taylor (2016).

2.6.2 Results

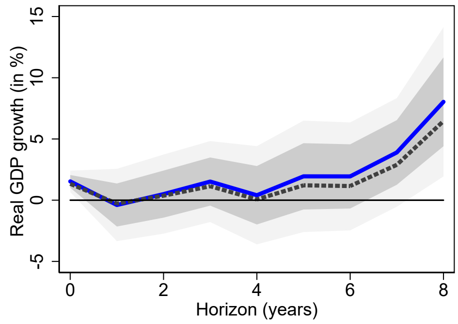
Figure 2.10 presents the main results. To put our findings in perspective, we estimate Equation (2.5) using OLS besides the estimating it with WLS as explained in the previous sub-section. This way we can evaluate the correction of the expected bias.



(a) IRF CPI inflation



(b) IRF short term rates



(c) IRF real GDP growth

Notes: The figure shows the impulse response functions for inflation and interest rates in percentage points, and real GDP growth in percent over time, when the exchange rate regime becomes more pegged. Equation (2.5) has been estimated with weighted least squares. The weights correspond to the inverse estimated probability of an exchange rate regime change from (2.4). The (dark) gray shaded areas indicate a confidence interval of (68%) 90%. The black dashed line shows the OLS estimates. $N \approx 1200$.

Figure 2.10. IPWRA Results of a pegging event

The estimates are in line with the proposed model and suggest that pegging episodes seem to have significant and persistent effects on inflation and interest rates. Let us first consider what happens to inflation. If a country moves towards a more pegged exchange rate regime, inflation goes persistently down by around 4 percentage points (Figure 2.11a). This decrease is significant and permanent.

Next, we analyze the impact on short term nominal interest rates of government bonds. After pegging, nominal interest rates experience a similar decline as inflation does. Interest rates go down by around 4 percentage points, the decline is significant and persistent. Reassuringly, the magnitude of the decline is the same for interest rates and inflation in the long-run. This means that the Fisher equation holds even though we did not require it in our estimation.

In contrast, a shift towards a more floating regime, leads to a strong and significant increase in both inflation and interest rates in the short and medium-term (Figures

2.A.12a 2.A.12b). Just like with the real GDP response, the increase becomes eventually insignificant thus pointing to a more temporary impact of a floating episode.

2.7 Conclusion

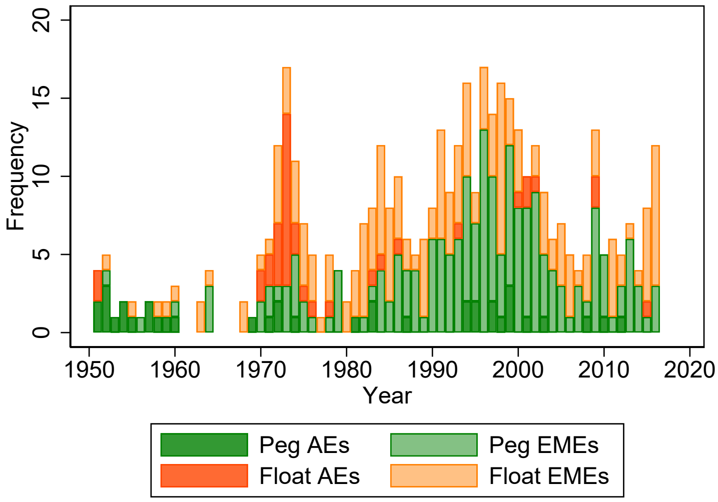
In this paper, we reassess the gains of adopting a fixed exchange rate regime. The underlying problem with a flexible exchange rate regime is that independent central banks with a lack of credibility tend to use surprise inflation to promote economic growth. This surprise inflation leads to an inflationary bias that increases the level of inflation and interest rates permanently. Entering a fixed exchange rate regime helps to mitigate this commitment problem. We show this via an event-study: Before a pegging episode, pegging countries displayed persistently higher inflation and nominal interest rates and lower economic growth. In the European context, this finding is especially true for countries with traditionally weak institutions over the last 60 years like Italy, Spain and Portugal, standing in contrast to Germany, Austria or the Netherlands which do not experience rising inflation nor rates when their exchange rate floated freely.

Our contribution is two folded. First, we provide an estimated version of the model by Chari, Dovis, and Kehoe (2020a) to show that countries displaying such an inflationary bias can solve such bias by giving up monetary autonomy and pegging the exchange rate to a stable anchor country. We simulate the model under different regimes (fixed and flexible exchange rate regimes under commitment or discretion) and compare the behavior of nominal and real variables with the one observed in the data.

At the empirical level, we produce novel evidence by testing the model's implications and by studying the impact of the adoption of a fixed exchange rate regime on the same nominal and real variables. We show that after a pegging episode both inflation and nominal short-term interest rates persistently decrease by around 4 percentage points. At the same time, volatility of inflation and interest rates goes down substantially. This is true for the country that pegs its currency to an anchor country and it is also true for the anchor country itself. Last we provide evidence that pegging the exchange rate tends to be followed by an increase in GDP. We rationalize the last finding in the model, by emphasizing costs of inflation in a setup where cash is needed to buy goods.

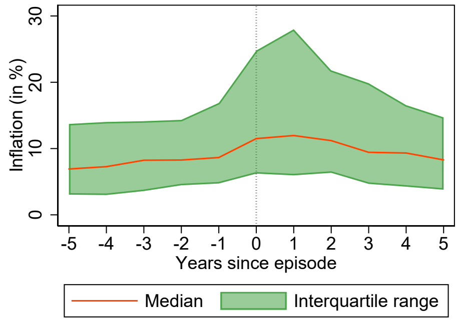
Appendix 2.A Data

Some of the graphs in this appendix are identical to the graphs in Arvai (2021).

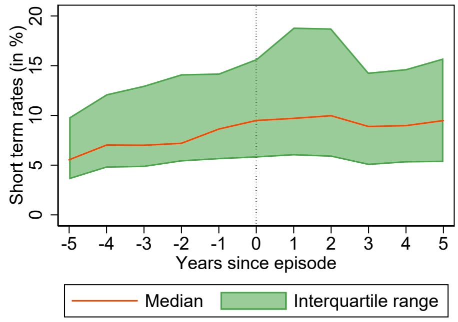


Notes: Number of the changes of the exchange rate regime classification from Ilzetzki, Reinhart, and Rogoff (2019). Green bars: Move towards a "more pegged" regime ($N = 223$). Red bars: Move towards a "more float" regime ($N = 211$).

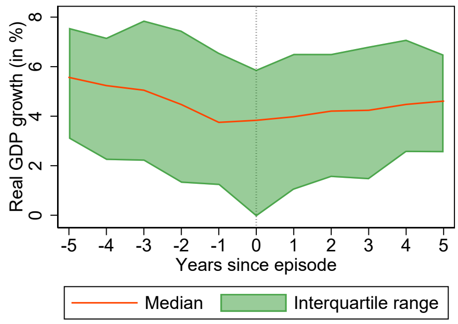
Figure 2.A.1. Frequency of flexible and fixed regime changes



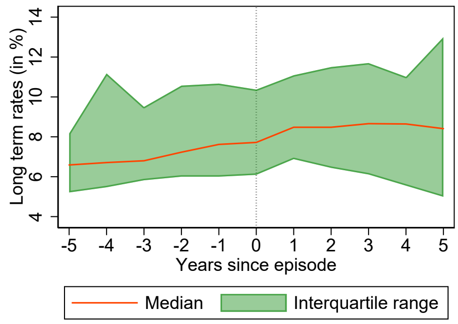
(a) Event-study inflation



(b) Event-study short-term interest rate



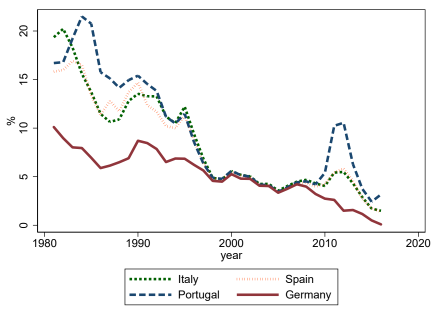
(c) Event-study real GDP growth



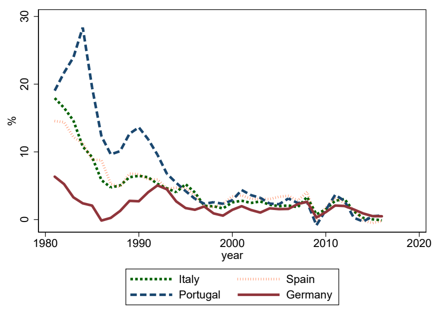
(d) Event-study long-term interest rate

Notes: The figure shows the event-study for median inflation and median interest rates in percentage points, and median real GDP growth in percent before and after a floating episode, when the exchange rate regime becomes more pegged. $N \approx 2000$.

Figure 2.A.2. Event-study for a floating episode



(a) Interest rate on 10-year government bonds



(b) Inflation (in %)

Notes: Graph (a) shows the evolution of the interest rate for 10-year government bonds while Graph (b) shows how inflation co-moved over time. The graphs are taken from Arvai (2021). Source: IFS.

Figure 2.A.4. Interest rates and Inflation in Europe

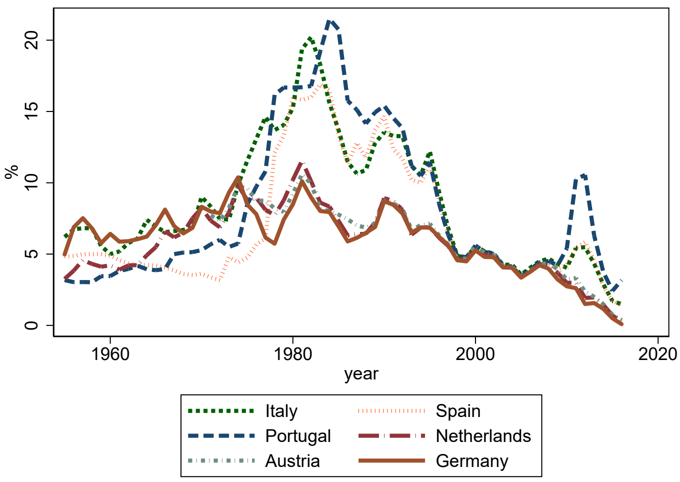
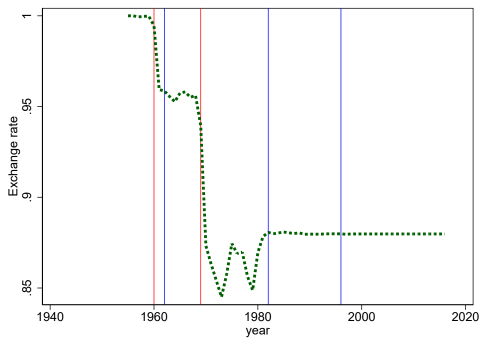
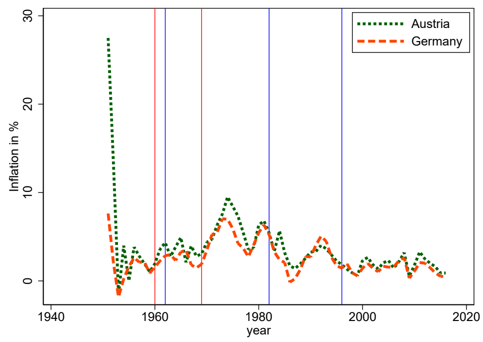


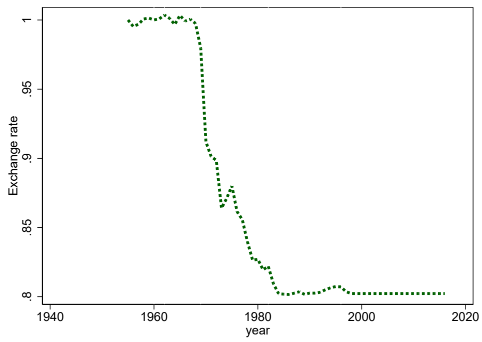
Figure 2.A.6. Interest rate of 10-year government bonds in Europe. The graph is taken from Arvai (2021). Source: IFS



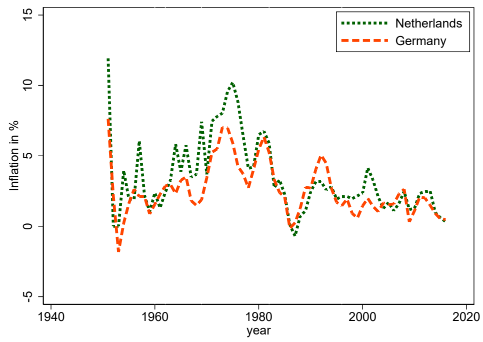
(a) Exchange Rate AUT-DEU (1955=1)



(b) Inflation (in %)



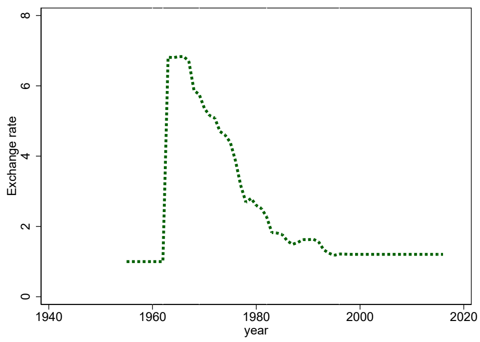
(c) Exchange Rate NLD-DEU (1955=1)



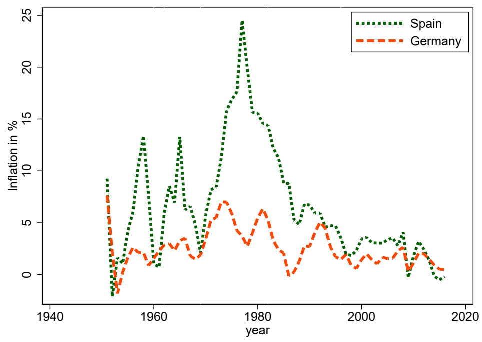
(d) Inflation (in %)

Notes: Graph (a) shows the evolution of the bilateral exchange rate of the Austrian Schilling to the German Mark normalized to 1 in 1955. Graph (b) shows how inflation in Germany (dashed red line) and Austria (dotted green line) co-moved over time. The vertical red lines indicate a fall of the exchange rate or a shift towards a floating exchange rate regime, the blue vertical lines a shift towards an exchange rate regime that is more pegged and that was followed by a stabilization of the exchange rate. Inflation and Exchange rate Netherlands Graph (c) shows the evolution of the bilateral exchange rate of the Dutch currency to the German Mark normalized to 1 in 1955. Graph (d) shows how inflation in Germany (dashed red line) and the Netherlands (dotted green line) co-moved over time. The graphs are taken from Arvai (2021). Sources: Bundesbank and IFS

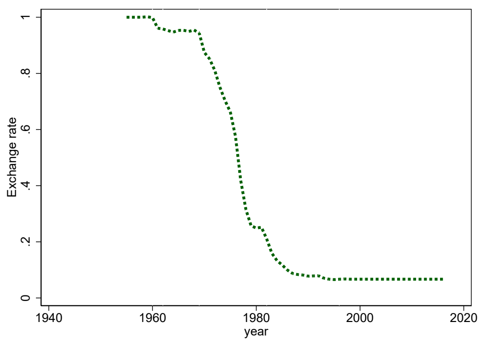
Figure 2.A.7. Exchange rate and inflation in Austria and the Netherlands



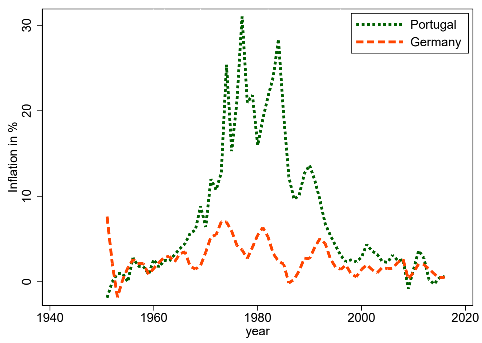
(a) Exchange Rate ESP-DEU (1955=1)



(b) Inflation (in %)



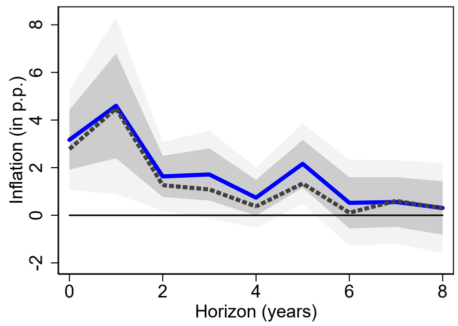
(c) Exchange Rate PRT-DEU (1955=1)



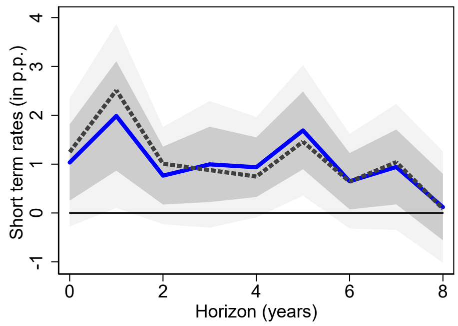
(d) Inflation (in %)

Notes: Graph (a) shows the evolution of the bilateral exchange rate of the Spanish currency to the German Mark normalized to 1 in 1955. Graph (b) shows how inflation in Germany (dashed red line) and the Spain (dotted green line) co-moved over time. Inflation and Exchange rate Portugal Graph (c) shows the evolution of the bilateral exchange rate of the Portuguese currency to the German Mark normalized to 1 in 1955. Graph (d) shows how inflation in Germany (dashed red line) and the Portugal (dotted green line) co-moved over time. The graphs are taken from Arvai (2021).

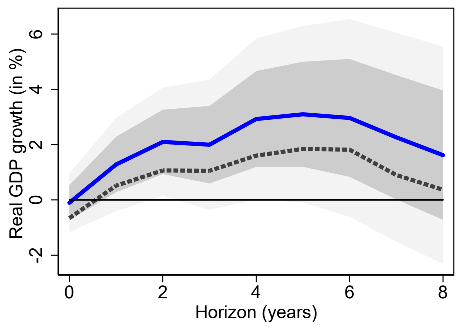
Figure 2.A.9. Exchange rate and inflation in Spain and Portugal



(a) IRF after a floating episode



(b) IRF after a floating episode



(c) IRF after a floating episode

Notes: The figure shows the impulse response function for inflation and interest rates in percentage points, and real GDP growth in percent over time, when the exchange rate regime becomes more flexible. Equation (2.5) has been estimated with weighted least squares. The weights correspond to the inverse estimated probability of an exchange rate regime change from (2.4). The (dark) gray shaded areas indicate a confidence interval of (68%) 90%. The black dashed line shows the OLS estimates. $N \approx 1200$.

Figure 2.A.11. IPWRA Results for a float

Appendix 2.B Model

Part of the derivations are identical to Arvai (2021).

2.B.1 Consumer Optimization

The Lagrangean is

$$\max_{C_T,C_N,L,B,B^*,M_H} \mathscr{L} = \sum_{t=0}^{\infty}\sum_{s^t}\beta^t h_t\left(s^t\right)\Big[\alpha\log C_T\left(s^t\right)+(1-\alpha)\log C_N\left(s^t\right)-\psi L\left(s^t\right)-$$
$$\lambda(s^t)\Big(P_T\left(s^t\right)C_T\left(s^t\right)+P_N\left(s^{t-1},s_{1t}\right)C_N\left(s^t\right)+M_H\left(s^t\right)+B\left(s^t\right)+e\left(s^t\right)B^*\left(s^t\right)-$$
$$\left(P_T\left(s^t\right)L\left(s^t\right)+M_H\left(s^{t-1}\right)+R\left(s^t\right)B\left(s^{t-1}\right)+e\left(s^t\right)R^*\left(s^t\right)B^*\left(s^{t-1}\right)+T\left(s^t\right)+\Pi\left(s^t\right)\right)\Big)$$
$$-\mu(s^t)\left(P_T(s^t)C_T(s^t)-M_H(s^{t-1})\right)\Big]$$

The first order conditions are

$$\frac{\alpha}{C_T(s^t)} = \lambda(s^t)P_T(s^t)+\mu(s^t)P_T(s^t) \tag{2.B.1}$$
$$\frac{1-\alpha}{C_N(s^t)} = \lambda(s^t)P_N(s^t) \tag{2.B.2}$$
$$\psi = \lambda(s^t)P_T(s^t) \tag{2.B.3}$$
$$\lambda(s^t) = \beta\mathbb{E}_t\left[\lambda(s^{t+1})R(s^{t+1})\right] \tag{2.B.4}$$
$$\lambda(s^t)e(s^t) = \beta\mathbb{E}_t\left[\lambda(s^{t+1})e(s^{t+1})R^*(s^{t+1})\right] \tag{2.B.5}$$
$$\lambda(s^t) = \beta\mathbb{E}_t\left[\lambda(s^{t+1})\right]+\beta\mathbb{E}_t\left[\phi(s^{t+1})\right] \tag{2.B.6}$$

Combining (2.B.2) and (2.B.4) gives the Euler equation:

$$\frac{1}{C_N(s^t)} = \beta\mathbb{E}_t\left[\frac{1}{C_N(s^{t+1})}\frac{P_N(s^t)}{P_N(s^{t+1})}R(s^{t+1})\right]$$

Combining (2.B.4) and(2.B.5) gives the uncovered interest parity condition:

$$\beta\mathbb{E}_t\left[\lambda(s^{t+1})R(s^{t+1})\right] = \beta\mathbb{E}_t\left[\lambda(s^{t+1})\frac{e(s^{t+1})}{e(s^t)}R^*(s^{t+1})\right]$$

The standardized multiplier on the cash in advance constraint is $\phi(s^t) = \mu(s^t)P_T(s^t)$.

If we use Chari, Dovis, and Kehoe (2020a) framework of prices on bonds instead of interest rates, the budget constraint changes to

$$P_T\left(s^t\right)C_T\left(s^t\right)+P_N\left(s^{t-1},s_{1t}\right)C_N\left(s^t\right)+M_H\left(s^t\right)+\bar{Q}(s^t)B\left(s^t\right)+\bar{Q}^*(s^t)e\left(s^t\right)B^*\left(s^t\right)$$
$$\leq P_T\left(s^t\right)L\left(s^t\right)+M_H\left(s^{t-1}\right)+B\left(s^{t-1}\right)+e\left(s^t\right)B^*\left(s^{t-1}\right)+T\left(s^t\right)+\Pi\left(s^t\right)$$

The first order condition is then

$$\lambda(s^t) \;=\; \beta\mathbb{E}_t\Big[\lambda(s^{t+1})\underbrace{\frac{1}{\bar{Q}(s^t)}}_{R(s^{t+1})}\Big]$$

So, using a framework with bond prices instead of interest rates on one period government bonds means that the price of a new bond is the inverse nominal interest rate on bonds that are being hold. $R(s^{t+1})$ is known in t.

2.B.2 International Capital Markets

The budget constraint is extended to allow households to buy non-domestic bonds as well. These bonds B^* are risk free and denoted in foreign currency:

$$\begin{aligned} P_T\left(s^t\right)C_T\left(s^t\right) + P_N\left(s^{t-1}, s_{1t}\right)C_N\left(s^t\right) + M_H\left(s^t\right) + \bar{Q}\left(s^t\right)B\left(s^t\right) + e(s^t)\bar{Q}^*\left(s^t\right)B^*\left(s^t\right) \\ \le P_T\left(s^t\right)L\left(s^t\right) + M_H\left(s^{t-1}\right) + B\left(s^{t-1}\right) + e(s^t)B^*\left(s^{t-1}\right) + T\left(s^t\right) + \Pi\left(s^t\right) \end{aligned} \tag{2.B.7}$$

The exchange rate $e(s^t)$ has to be taken into account. As households can now choose non-domestic bonds, there is a a new first order condition:

$$\bar{Q}^*(s^t)\lambda(s^t)e(s^t) \;=\; \beta\mathbb{E}_t\left[\lambda(s^{t+1})e(s^{t+1})\right]$$

Combining it with the old conditions

$$\begin{aligned} \bar{Q}(s^t)\lambda(s^t) \;&=\; \beta\mathbb{E}_t\left[\lambda(s^{t+1})\right] \\ \lambda(s^t) \;&=\; \frac{\alpha}{P_T(s^t)C(s^t)} \end{aligned}$$

gives the so-called uncovered interest rate parity that relates domestic with foreign interest rates:

$$\frac{\bar{Q}^*(s^t)e(s^t)}{\bar{Q}(s^t)} \;=\; \frac{\mathbb{E}_t\left[\lambda(s^{t+1})e(s^{t+1})\right]}{\mathbb{E}_t\left[\lambda(s^{t+1})\right]}$$

with iid shocks we have

$$\mathbb{E}_t\left[Q\left(s^{t+1}\right)R(s^{t+1})\right] \;=\; \mathbb{E}_t\left[Q\left(s^{t+1}\right)\frac{e(s^{t+1})}{e(s^t)}R^*(s^{t+1})\right]$$

The nominal interest rate spread is offset by a continuous devaluation of the home currency vis-a-vis to the rest of the world. The rest of the model is not altered by the introduction of international capital markets, as households do not have an incentive to borrow or lend across countries given their current preference structure (log utility and Cobb Douglas consumption aggregator).

2.B.3 Firm Optimization

A microfoundation for markups can be given by following the setup of Smets and Wouters (2007). The non-traded good is produced by a competitive final producer who uses differentiated inputs $y_N(j, s^t)$ from input firms of mass $j \in [0, 1]$ to produce the final good $Y_N(s^t)$:

$$Y_N\left(s^t\right) = \left[\int y_N\left(j, s^t\right)^{\theta(s_{1t})} dj\right]^{1/\theta(s_{1t})}$$

This firm maximizes

$$P_N\left(s^{t-1}, s_{1t}\right) Y_N\left(s^t\right) - \int P_N\left(j, s^{t-1}, s_{1t}\right) y_N\left(j, s^t\right) dj$$

Demand for intermediate goods is therefore

$$y_N\left(j, s^t\right) \;=\; \left(\frac{P_N\left(s^{t-1}, s_{1t}\right)}{P_N\left(j, s^{t-1}, s_{1t}\right)}\right)^{\frac{1}{1-\theta(s_{1t})}} Y_N\left(s^t\right)$$

Intermediate goods are produced by monopolistic firms who use a linear production function: $y_N(j, s^t) = A(s_{2t})L_N(j, s^t)$. Intermediate good firms choose their prices $P = P(j, s^{t-1}, s_{1t})$ to maximize their profits:

$$\max_P \sum_{s_{2t}} Q\left(s^t\right)\left[P - \frac{W\left(s^t\right)}{A\left(s_{2t}\right)}\right]\left(\frac{P_N\left(s^{t-1}, s_{1t}\right)}{P}\right)^{\frac{1}{1-\theta(s_{1t})}} Y_N\left(s^t\right)$$

where $Q(s^t)$ is the discount factor as before. We assume that $\theta(s_{1t} \in (0, 1)$ implying elastic demand and finite prices. Optimally, intermediate good producer j sets the price in the following way:

$$P_N\left(j, s^{t-1}, s_{1t}\right) \;=\; \frac{1}{\theta\left(s_{1t}\right)} \frac{\sum_{s_{2t}} Q\left(s^t\right) Y_N\left(s^t\right) \frac{W(s^t)}{A(s_t)}}{\sum_{s_{2t}} Q\left(s^t\right) Y_N\left(s^t\right)}$$

Where $\frac{1}{\theta(s_{1t}}$ is the markup that increases prices. Note that the price equation is not a function of j such that the price is the same for all intermediate firms. Plugging in $W(s^t) = P_T(s^t)$ gives the same formula as in equation (2.1). This implies that all intermediate firms hire the same amount of labor and their production function is then simply $Y_N(s^t) = A(s_{2t}L_N(s^t)$.

2.B.4 Monetary Policy Optimization

Commitment and Float. The central bank makes a state-contingent plan for prices of traded and non traded goods to maximize the representative households ex ante utility

$$
\begin{aligned}
\max_{\{P_T(s^t),P_N(s^t)\}_{t=0}^{\infty}} \quad & \mathbb{E}_0\left[\sum_{\tau=t}\beta^t\left(\alpha\log(C_T(s^\tau)+(1-\alpha)\log(C_N(s^\tau))-\psi L(s^\tau)\right)\right] \\
\text{s.t.} \quad L(s^t) &= \frac{C_N(s^t)}{A(s_{2t})}+C_T(s^t) \\
C_T(s^t) &= \frac{\alpha}{\psi} \\
C_N(s^t) &= \frac{1-\alpha}{\psi}\frac{P_T(s^t)}{P_N(s^{t-1},s_{1t})} \\
\sum_{s_{2t}} h\left(s^t \mid s^{t-1},s_{1t}\right)C_N\left(s^t\right)&\left[U_N\left(s^t\right)+\frac{1}{\theta\left(s_{1t}\right)}\frac{U_L\left(s^t\right)}{A\left(s_{2t}\right)}\right] = 0
\end{aligned}
$$

Looking at the plugged in firm's first order condition:

$$
\sum_{s_{2t}} h\left(s^t \mid s^{t-1},s_{1t}\right)C_N\left(s^t\right)\left[\frac{1-\alpha}{C_N\left(s^t\right)}-\frac{1}{\theta\left(s_{1t}\right)}\frac{\psi}{A\left(s_{2t}\right)}\right] = 0
$$

Plugging in C_N

$$
\sum_{s_{2t}} h\left(s^t \mid s^{t-1},s_{1t}\right)C_N\left(s^t\right)\left[\frac{1-\alpha}{\frac{1-\alpha}{\psi}\frac{P_T(s^t)}{P_N(s^{t-1},s_{1t})}}-\frac{1}{\theta\left(s_{1t}\right)}\frac{\psi}{A\left(s_{2t}\right)}\right] = 0
$$

This can only be zero if

$$
\frac{P_T(s^t)}{P_N(s^{t-1},s_{1t})} = A\left(s_{2t}\right)\theta\left(s_{1t}\right)
$$

The best the central bank can do is to ensure that this condition holds. The central bank realizes that it is not possible to reduce markups by manipulating relative prices with inflation. Therefore it focuses to stabilize productivity shocks.

Nominal variables can be computed as well, using the following trick: First normalize all variables with their money supply of the last period, $p_T=\frac{P_T(s^t)}{M(s^{t-1})}$ and $p_N(s^{t-1},s_{1t})=\frac{P_N(s^{t-1},s_{1t})}{M(s^{t-1})}$. Then construct prices in such a way, that the cash in advance constraint is *exactly* binding in the highest possible productivity state[10]. Then

10. This way, no consumption is lost.

use that $p_T(s^t)/p_N(s^{t-1}, s_{1t}) = A(s_{2t})\theta(s_{1t})$ and $p_T(s^t) = C_T(s^t)$ if the cash in advance constraint binds in the highest state to get:

$$\begin{aligned} p_N(s^{t-1}, s_{1t}) &= \frac{1}{\theta(s_{1t})} \frac{\psi}{\alpha} \frac{1}{\max\{A(s_{2t})\}} \\ p_T(s^t) &= A(s_{2t})\theta(s_{1t}) p_N(s^{t-1}, s_{1t}) \\ \frac{M\left(s^t\right)}{M\left(s^{t-1}\right)} &= \beta \sum_{s^{t+1}} h\left(s^{t+1} \mid s^t\right) \frac{A\left(s_{2t}\right)}{A\left(s_{2t+1}\right)} \end{aligned}$$

Together with an initial level for $M(s^0)$, the nominal equilibrium is pinned down. The per period money growth rate equals productivity today times the discounted expected inverse productivity in the future. If productivity today is relatively large, money growth will also be relatively large, reflecting expansionary monetary policy and a depreciating exchange rate from the example before. If productivity is not stochastic, money gross growth rate is $\beta < 1$.

The derivation from the money growth rate comes from the consumer's first order condition, that combines the labor and traded goods first order condition with the money first order condition. As $p_T(s^t) = P_T(s^t)/M(s^{t-1})$, we can draw out the money growth rate as follows

$$-\frac{M(s^t)}{M(s^{t-1})} \frac{U_L}{p_T} = \beta \sum_{s'} h\left(s'\right) \frac{U_T\left(b', 1, x'_T, S'_T\right)}{p_T\left(x'_T, S'_T\right)}$$

If you rearrange and plug in, you arrive at

$$\frac{M(s^t)}{M(s^{t-1})} = \beta \sum_{s'} h\left(s'\right) \frac{p_T(s)}{p_T(s')} \frac{\alpha/\psi}{C_T(s')}$$

Plugging in $p_T = A\theta p_N$ with $p_N = \frac{1}{\theta}\frac{\psi}{\alpha}\frac{1}{max(A)}$ at a binding cash in advance constraint and $C_T = \frac{\alpha}{\psi}$ gives the money growth rate as above, only a function of productivity.

Nominal interest rates can then be computed via the Euler equation, see Appendix 2.B.5 for a derivation

$$R\left(s^t\right) = \frac{\max\{A(s_{2t})\}}{\max\{A(s_{2t+1})\}}$$

Interest rates are simply the ratio of the maximum value of productivity today and tomorrow. If productivity stays always the same, then $R(s^t) = 1$ and $M(s^t)/M(s^{t-1}) = \beta < 1$. This means that nominal interest rates are zero and the central bank follows the Friedman rule implying a negative money growth rate. The intuition why zero interest rates are optimal is the following. Nominal bonds dominate money holding as they pay an interest on its stock, while money does not. Nevertheless, households need to hold money to buy traded goods. Therefore, the central bank optimally implements zero interest rates to make the necessary money holding as good as the bond holding. Inflation rates of both goods are given by

$$\pi_N(s^t, s_{1t+1}) = \frac{P_N(s^t, s_{1t+1})}{P_N(s^{t-1}, s_{1t})} = \frac{\theta(s_{1t})\max\{A(s_{2t}\}}{\theta(s_{1t+1})\max\{A(s_{2t+1})\}}\frac{M(s^t)}{M(s^{t-1})}$$

$$\pi_T(s^{t+1}) = \frac{P_T(s^{t+1})}{P_T(s^t)} = \frac{A(s^{t+1})\theta(s_{1t+1})P_N(s^t, s_{1t+1})}{A(s^t)\theta(s_{1t})P_N(s^{t-1}, s_{1t})} = \frac{A(s_{2t+1})\max\{A(s_{2t})\}}{A(s_{2t})\max\{A(s_{2t+1}\}}\frac{M(s^t)}{M(s^{t-1})}$$

Markups influence prices of non-traded goods only. The bigger the markup ($1/\theta$ is high) compared to last period, the higher is inflation. Higher productivity of the non-traded good increases prices of traded goods, the relative price adjusts. Higher money growth rates increase both inflation rates. In a world with no stochastic components, inflation is determined by the money growth rate which is then simply $\beta < 1$. This implies deflation. The Friedman rule is a solution for the nominal equilibrium, a continued contraction of the money supply implies deflation which ensures that the cash in advance constraint is never binding.

Discretion and Float. Chari, Dovis, and Kehoe (2020a) show, that there is no intertemporal dimension of the problem for the central bank. The reason is that in equilibrium there is no bond holding and that lump-sum transfers are always available to the government. In addition, households do not derive utility out of money, such that the growth rate of money supply is not intertwined with the static problem. The optimal problem of the central bank can then be thought of as choosing the price of the traded good subject to the first order conditions of households. As the cash in advance constraint optimally binds for the traded good, the primal problem of the central bank is to maximize

$$\begin{aligned}
\max_{P_T(s^t)} \quad & \alpha \ln C_T(s^t) + (1-\alpha)\ln C_N(s^t) - \psi(C_T(s^t) + C_N(s^t)/A(s^t)) \\
\text{s.t.} \quad C_T(s^t) &= \frac{M(s^{t-1})}{P_T(s^t)} \\
C_N(s^t) &= \frac{1-\alpha}{\psi}\frac{P_T(s^t)}{P_N(s^t)}
\end{aligned}$$

The first order condition is (already divided by $M(s^{t-1})$

$$-\frac{\alpha}{p_T(s^t)} + \frac{1-\alpha}{p_T(s^t)} - \psi\left(-\frac{1}{p_T(s^t)^2} + \frac{1-\alpha}{\psi}\frac{1}{A(s^t)}\frac{1}{p_N(s^t)}\right) = 0$$

Solving for $p_T(s^t)$ gives the optimal reaction function of the central bank under discretion:

$$p_T(s_t) = p_N(s_{1t})A(s_{2t})\underbrace{\frac{1}{2(1-\alpha)}\left[(1-2\alpha) + \sqrt{(1-2\alpha)^2 + 4(1-\alpha)\frac{1}{A(s_{2t})}\frac{\psi}{p_N(s_{1t})}}\right]}_{F\left(\frac{1}{A(s_{2t})p_N(s_{1t})}\right)}$$

If you consider the firm's price setting equation 2.1, then you can compute prices:

$$p_N\left(s^{t-1},s_{1t}\right) = \frac{1}{\theta\left(s_{1t}\right)}\sum_{s_{2t}}\left(\frac{Q\left(s^t\right)Y_N\left(s^t\right)}{\sum_{\tilde{s}_{2t}}Q\left(\tilde{s}^t\right)Y_N\left(\tilde{s}^t\right)}\right)$$
$$\frac{p_N\left(s_{1t}\right)A\left(s_{2t}\right)\frac{1}{2(1-\alpha)}\left[(1-2\alpha)+\sqrt{(1-2\alpha)^2+4(1-\alpha)\frac{1}{A\left(s_{2t}\right)}\frac{\psi}{p_N\left(s_{1t}\right)}}\right]}{A\left(s_t\right)}$$

If p_N rises, p_T will then in general rise by less than 1 to 1, reflecting the costs of higher inflation.

If A is not stochastic the cash in advance constraint never binds (implicit assumption here). We can then write

$$1 = \frac{1}{\theta\left(s_{1t}\right)}\frac{A\left(s_{2t}\right)\frac{1}{2(1-\alpha)}\left[(1-2\alpha)+\sqrt{(1-2\alpha)^2+4(1-\alpha)\frac{1}{A\left(s_{2t}\right)}\frac{\psi}{p_N\left(s_{1t}\right)}}\right]}{A\left(s_t\right)}$$

Solving for $p_N(s_{1t})$ gives

$$(2(1-\alpha)\theta-(1-2\alpha))^2 = (1-2\alpha)^2+4(1-\alpha)\frac{\psi}{A(s_{2t})p_N(s_{1t})}$$

With this we get p_N as in the main text:

$$p_N(s_t) = \frac{1}{\theta(s_{1t})}\frac{1}{A(s_{2t})}\frac{\psi}{\alpha-(1-\alpha)(1-\theta(s_{1t}))}$$

Discretion and Currency Union. There is a mass of $\bar{n}^N$ northern countries and n^S southern countries. The relative weight of north is then $\lambda^N = \frac{\bar{n}^N}{\bar{n}^N+n^S}$. The union-wide central bank chooses a traded good price for the union taking the non-traded good prices as given. The current state is $s=(z,p_N^i(z,v))$, the primal problem is then

$$\max_{p_T}\sum_{\lambda^i\in(N,S)}\lambda^i\sum_{v^t}g(v^t)\left[\alpha\log C_T^i(s^t)+(1-\alpha)\log C_N^i(s^t)-\psi\left(L^i(s^t)\right)\right]$$
$$\begin{aligned}
\text{s.t.}\quad L^i(s^t) &= \frac{C_N^i(s^t)}{A^i(s_{2t})}+C_L^i(s^t)\\
C_T^i(s^t) &= \frac{1}{p_T}\\
C_N^i(s^t) &= \frac{1-\alpha}{\psi}\frac{p_T(s^t)}{p_N^i(s^{t-1},s_{1t})}\\
\frac{U_T(s^t)}{U_N(s^t)} &= \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)}
\end{aligned}$$

where $g(v)$ is again just the average of the union, given the aggregate state. The first order condition is given by:

$$0 = (1-2\alpha)\frac{1}{p_T} + \psi\frac{1}{p_T^2} - (1-\alpha)\sum_{i=N,S}\lambda^i\sum_v g(v)\frac{1}{p_N^i(z,v)A^i}$$

We can solve the quadratic equation to get the monetary authorities best response:

$$p_T\left(z,\left\{p_N^i\left(z_1,v_1\right)\right\}\right) = \frac{(1-2\alpha)+\sqrt{(1-2\alpha)^2+4\sum_{i=N,S}\lambda^i\sum_v g(v)\frac{(1-\alpha)}{A^i\left(z_2,v_2\right)}\frac{\psi}{p_N^i\left(z_1,v_1\right)}}}{\sum_{i=N,S}\lambda^i\sum_v g(v)\frac{2(1-\alpha)}{A^i\left(z_2,v_2\right)}\frac{1}{p_N^i\left(z_1,v_1\right)}},$$

As a next step consider again the pricing equation of firms in country i: $p_N^i = \mathbb{E}_v\left(\frac{1}{A^i}\right)\frac{1}{\theta^i}p_T^i$. As with a single country under discretion, we can solve the problem by plugging in the reaction functions into each other, this gives a fixed point problem and can explicitly be solved for p_T. Let $\sum_v g(v)\frac{1}{A^i\left(z_2,v_2\right)p_N^i\left(z_1,v_1\right)} = \mathbb{E}_v[\frac{1}{A^ip_N^i}\mid z]$. Then

$$p_N^j = \mathbb{E}_v\left(\frac{1}{A^j}\right)\frac{1}{\theta^j}\frac{\psi}{(1-\alpha)p_N^j\sum_{i=N,S}\lambda^i\mathbb{E}_v[\frac{1}{A^ip_N^i}]\theta^jA^j-(1-2\alpha)}$$

For the p_N^i on the right hand side of the equation, plug in $p_N^i = \mathbb{E}\left(\frac{1}{A^i}\right)\frac{p_T}{\theta^i}$

$$p_N^j = \mathbb{E}_v\left(\frac{1}{A^j}\right)\frac{1}{\theta^j}\frac{\psi}{(1-\alpha)\frac{p_T}{A^j\theta^j}\sum_{i=N,S}\lambda^i\mathbb{E}_v[\frac{1}{A^i\frac{p_T}{A^i\theta^i}}\mid z]\theta^jA^j-(1-2\alpha)}$$

$$p_N^j = \mathbb{E}_v\left(\frac{1}{A^j}\right)\frac{1}{\theta^j}\frac{\psi}{(1-\alpha)\sum_{i=N,S}\lambda^i\mathbb{E}_v\left(\theta^i\mid z\right)-(1-2\alpha)}$$

This gives p_T

$$p_T = \frac{\psi}{(1-\alpha)\sum_{i=N,S}\lambda^i\mathbb{E}_v[\theta^i\mid z]-(1-2\alpha)}$$

C_T is then given by:

$$C_T = \frac{1}{p_T} = \frac{\alpha}{\psi} - \frac{1-\alpha}{\psi}\left(1-\sum_{i=N,S}\lambda^i\mathbb{E}_v\left(\theta^i\mid z\right)\right)$$

and C_N

$$C_N^i = \frac{1-\alpha}{\psi}\mathbb{E}_v\left(\frac{1}{A^i}\right)^{-1}\theta^i(s)$$

Money growth rate is

$$\Delta M = \beta\frac{\alpha}{\psi}p_T$$

Commitment and Currency Union. In a monetary union, the exchange rate is fixed and set to $e(s^t) = 1$ for all states. This implies that P_T cannot vary across countries and is only a function of aggregate union-wide shocks. This gives rise to the "Union constraint"

$$\frac{U_T(s^t)}{U_N(s^t)} = \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)}$$

The central bank acts under commitment and chooses the union-wide price of traded goods and the prices of non-traded goods to maximize an equally weighted average of all countries of the world:

$$\begin{aligned}
\max_{P_T, P_N(v)} \mathbb{E}_0 & \left[\sum_{\tau=t} \sum_{v^\tau} \beta^\tau g(v^\tau) \left[\alpha \log C_T(z^\tau, v^\tau) + (1-\alpha) \log C_N(z^\tau, v^\tau) - \psi\left(L(z^\tau, v^\tau)\right)\right]\right] \\
\text{s.t.} \quad L(s^t) &= \frac{C_N(s^t)}{A(s_{2t})} + C_L(s^t) \\
C_T(s^t) &= \frac{\alpha}{\psi} \\
C_N(s^t) &= \frac{1-\alpha}{\psi} \frac{P_T(s^t)}{P_N(s^{t-1}, s_{1t})} \\
\sum_{s_{2t}} h\left(s^t \mid s^{t-1}, s_{1t}\right) C_N\left(s^t\right) & \left[U_N\left(s^t\right) + \frac{1}{\theta\left(s_{1t}\right)} \frac{U_L\left(s^t\right)}{A\left(s_t\right)}\right] = 0 \\
\frac{U_T(s^t)}{U_N(s^t)} &= \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)}
\end{aligned}$$

where $\sum_{v^\tau} g(v^\tau)$ simply sums up all the countries. Remember that $s^t = (z^t, v^t)$ where z^t is the aggregate shock and v^t is the country-specific shock. Optimally, the cash in advance constraint does not bind to avoid losses in consumption. The central bank sets prices such that it stabilizes the average of the whole union:

$$\frac{P_T(s^t)}{P_N(s^{t-1}, s_{1t})} = \theta(s_{1t}) \left(\sum_{v_{2t}} g(v_{2t}) \frac{1}{A(z_{2t}, v_{2t})}\right)^{-1}$$

As the exchange rate is fixed, prices of traded goods are the same for all countries and the only thing the union-wide central bank can do is to set relative prices equal to the markup times the *average* productivity of the union.

Consumption and labor are then given by

$$C_T(s^t) = \frac{\alpha}{\psi}, \quad C_N(s^t) = \frac{1-\alpha}{\psi} \frac{\theta\left(s_{1t}\right)}{E_v(1/A(v_{2t} \mid z_{2t})}, \quad L(s^t) = \frac{C_N(s^t)}{A(s_{2t})} + C_L(s^t)$$

Consumption of traded goods is as with a flexible exchange rate under commitment (Section 2.3.6.1) as the cash in advance constraint is not binding. The difference is that consumption of non-traded goods now depends on average productivity in the currency union, as the central bank now conditions its policy on the average of the union and not on each individual country. This will in general be costly for the economy, as the central bank is not able to eliminate all costs coming from rigid prices. For some countries, monetary policy will be too expansionary, for some it will be too contractionary.

Nominal prices, interest rates and money growth rates are computed by resolving the indeterminacy problem in the same way as before. Let $X\left(z_{2t}\right) = \sum_{v_{2t}} g\left(v_{2t}\right) \frac{1}{A\left(s_{2t}\right)}$. Consider the lowest possible value of X. That corresponds to the highest possible aggregate productivity value and assume that the cash in advance constraint is exactly binding in this state. Prices are again standardized by their last period's money holding.

$$\begin{aligned} p_N\left(s^{t-1}, s_{1t}\right) &= \tfrac{1}{\theta\left(s_{1t}\right)} \tfrac{\psi}{\alpha} \min_{z_2} \left\{X\left(z_{2t}\right)\right\} \textstyle\sum_{s_2} h^2\left(s_2\right) \tfrac{1/A\left(s_{2t}\right)}{X\left(z_{2t}\right)} \\ p_T\left(s^t\right) &= A\left(s_{2t}\right) \theta\left(s_{1t}\right) p_N\left(s^{t-1}, s_{1t}\right) = \tfrac{\psi}{\alpha} \tfrac{\min_{z_2}\left\{X\left(z_{2t}\right)\right\}}{X\left(z_{2t}\right)} \\ \tfrac{M\left(s^t\right)}{M\left(s^{t-1}\right)} &= \beta \textstyle\sum_{s^{t+1}} h\left(s^{t+1} \mid s^t\right) X\left(z_{2t+1}\right) / X\left(z_{2t}\right) \end{aligned}$$

The nominal interest rate in the currency union is given by the Euler equation as before:

$$R\left(s^t\right) = \left(\beta \sum_{s^{t+1}} h\left(s^{t+1} \mid s^t\right) \frac{\frac{\min_{z_2}\left\{X\left(z_{2t}\right)\right\}}{X\left(z_{2t}\right)}}{\frac{\min_{z_2}\left\{X\left(z_{2t+1}\right)\right\}}{X\left(z_{2t+1}\right)}} M\left(s^{t-1}\right) / M\left(s^t\right)\right)^{-1}$$

Inflation rates are:

$$\pi_N\left(s^t, s_{1t+1}\right) = \frac{P_N\left(s^t, s_{1t+1}\right)}{P_N\left(s^{t-1}, s_{1t}\right)} = \frac{\theta\left(s_{1t}\right)}{\theta\left(s_{1t+1}\right)} \frac{\frac{\psi}{\alpha} \min_{z_2} \left\{X\left(z_{2t+1}\right)\right\} \sum_{s_2} h^2\left(s_2\right) \frac{1/A\left(s_{2t+1}\right)}{X\left(z_{2t+1}\right)}}{\frac{\psi}{\alpha} \min_{z_2} \left\{X\left(z_{2t}\right)\right\} \sum_{s_2} h^2\left(s_2\right) \frac{1/A\left(s_{2t}\right)}{X\left(z_{2t}\right)}} \frac{M\left(s^t\right)}{M\left(s^{t-1}\right)}$$

$$\pi_T\left(s^{t+1}\right) = \frac{P_T\left(s^{t+1}\right)}{P_T\left(s^t\right)} = \frac{A\left(s^{t+1}\right) \theta\left(s_{1t+1}\right) P_N\left(s^t, s_{1t+1}\right)}{A\left(s^t\right) \theta\left(s_{1t}\right) P_N\left(s^{t-1}, s_{1t}\right)} = \frac{A\left(s_{2t+1}\right)}{A\left(s_{2t}\right)} \frac{\min_{z_2}\left\{X\left(z_{2t+1}\right)\right\} \mathbb{E} \frac{\frac{1}{A\left(s_{2t+1}\right)}}{X\left(z_{2t+1}\right)}}{\min_{z_2}\left\{X\left(z_{2t}\right)\right\} \mathbb{E} \frac{\frac{1}{A\left(s_{2t}\right)}}{X\left(z_{2t}\right)}} \frac{M\left(s^t\right)}{M\left(s^{t-1}\right)}$$

If productivity is not stochastic, then money growth is simply $\beta < 1$. For inflation this means $\pi_N = \Delta\theta\beta$, $\pi_T = \beta$. Nominal interest rates are then $R = 1$. As in the case with monetary policy under commitment with flexible exchange rates, the union-wide central bank follows the Friedman rule as well. This implies a continued contraction in money supply, zero interest rates and deflation.

2.B.5 Computation of Interest Rates

Flexible exchange rate and commitment.

$$
\begin{aligned}
\bar{Q}(s^t) &= \beta \sum_{s^{t+1}} h(s^{t+1} \mid s^t) \frac{U_N(s^{t+1})}{P_N(s^t, s_{1t+1})} \frac{P_N(s^{t-1}, s_{1t})}{U_N(s^t)} \\
\bar{Q}(s^t) &= \beta \sum_{s^{t+1}} h(s^{t+1} \mid s^t) \frac{\frac{1-\alpha}{C_N(s^{t+1})}}{\frac{1-\alpha}{C_N(s^t)}} \frac{P_N(s^{t-1}, s_{1t})}{P_N(s^t, s_{1t+1})} \frac{M(s^{t-1})/M(s^{t-1})}{M(s^t)/M(s^t)} \\
\bar{Q}(s^t) &= \beta \sum_{s^{t+1}} h(s^{t+1} \mid s^t) \frac{C_N(s^t)}{C_N(s^{t+1})} \frac{\frac{1}{\theta(s_{1t})} \frac{\psi}{\alpha} \frac{1}{\max\{A(s_{2t})\}} M(s^{t-1})}{\frac{1}{\theta(s_{1t+1})} \frac{\psi}{\alpha} \frac{1}{\max\{A(s_{2t+1})\}} M(s^t)} \\
\bar{Q}(s^t) &= \beta \sum_{s^{t+1}} h(s^{t+1} \mid s^t) \frac{\frac{1-\alpha}{\psi} \theta(s_{1t}) A(s_{2t})}{\frac{1-\alpha}{\psi} \theta(s_{1t+1}) A(s_{2t+1})} \frac{\frac{1}{\theta(s_{1t+1})} \frac{\psi}{\alpha} \frac{1}{\max\{A(s_{2t})\}} M(s^{t-1})}{\frac{1}{\theta(s_{1t+1})} \frac{\psi}{\alpha} \frac{1}{\max\{A(s_{2t+1})\}} M(s^t)} \\
\bar{Q}(s^t) &= \beta \sum_{s^{t+1}} h(s^{t+1} \mid s^t) \frac{A(s_{2t})}{A(s_{2t+1})} \frac{\frac{1}{\max\{A(s_{2t})\}}}{\frac{1}{\max\{A(s_{2t+1})\}}} \frac{M(s^{t-1})}{M(s^t)} \\
\bar{Q}(s^t) &= \frac{\max\{A(s_{2t+1})\}}{\max\{A(s_{2t})\}}
\end{aligned}
$$

Interest rates are zero if productivity is not stochastic.

Flexible exchange rates and discretion.

$$
\begin{aligned}
\bar{Q}(s^t) &= \beta \sum_{s^{t+1}} h(s^{t+1} \mid s^t) \frac{C_N(s^t)}{C_N(s^{t+1})} \frac{P_N(s^{t-1}, s_{1t})}{P_N(s^t, s_{1t+1})} \\
\bar{Q}(s^t) &= \beta \sum_{s^{t+1}} h(s^{t+1} \mid s^t) \frac{\frac{1-\alpha}{\psi} A(s^t)\, \theta(s^t)}{\frac{1-\alpha}{\psi} A(s^{t+1})\, \theta(s^{t+1})} \frac{p_N(s^{t-1}, s_{1t}) M(s^{t-1})}{p_N(s^t, s_{1t+1}) M(s^t)} \\
\bar{Q}(s^t) &= \beta \sum_{s^{t+1}} h(s^{t+1} | s^t) \frac{\frac{1-\alpha}{\psi} A(s^t)\, \theta(s^t)}{\frac{1-\alpha}{\psi} A(s^{t+1})\, \theta(s^{t+1})} \frac{p_N(s^{t-1}, s_{1t})}{p_N(s^t, s_{1t+1})} \frac{\psi}{\beta \alpha A(s_{2t}) \theta(s_{1t}) p_N(s^{t-1}, s_{1t})} \\
\bar{Q}(s^t) &= \beta \sum_{s^{t+1}} h(s^{t+1} \mid s^t) \frac{1}{A(s^{t+1})\, \theta(s^{t+1})} \frac{1}{p_N(s^t, s_{1t+1})} \frac{\psi}{\beta \alpha} \\
\bar{Q}(s^t) &= \beta \sum_{s^{t+1}} h(s^{t+1} | s^t) \frac{1}{A(s^{t+1})\, \theta(s^{t+1})} \frac{1}{\frac{1}{\theta(s_{1t+1})} \frac{1}{A(s_{2t+1})} \frac{\psi}{\alpha-(1-\alpha)(1-\theta(s_{1t+1}))}} \frac{\psi}{\beta \alpha} \\
\bar{Q}(s^t) &= \beta \sum_{s^{t+1}} h(s^{t+1} | s^t) \frac{\alpha-(1-\alpha)(1-\theta(s_{1t+1}))}{\psi} \frac{\psi}{\beta \alpha} < \beta \sum_{s^{t+1}} h(s^{t+1} | s^t) \frac{\alpha}{\psi} \frac{\psi}{\beta \alpha} \\
R(s^{t+1})^{-1} &= \mathbb{E}_t \left[\frac{\alpha - (1-\alpha)(1-\theta(s_{1t+1}))}{\alpha} \right]
\end{aligned}
$$

which implies that $\bar{Q}^{disc}(s^t) < \bar{Q}^{Commit}(s^t)$ and therefore $(1+i)^{disc} > (1+i)^{commit}$.

In a currency union with commitment.

$$
\begin{aligned}
\bar{Q}\left(s^t\right) &= \beta \sum_{s^{t+1}} h\left(s^{t+1} \mid s^t\right) \frac{C_N\left(s^t\right)}{C_N\left(s^{t+1}\right)} \frac{P_N\left(s^{t-1}, s_{1t}\right)}{P_N\left(s^t, s_{1t+1}\right)} \\
\bar{Q}\left(s^t\right) &= \beta \sum_{s^{t+1}} h\left(s^{t+1} \mid s^t\right) \frac{\frac{1-\alpha}{\psi} \frac{P_T(s^t)}{P_N(s^{t-1}, s_{1t})}}{\frac{1-\alpha}{\psi} \frac{P_T(s^{t+1})}{P_N(s^t, s_{1t+1})}} \frac{P_N\left(s^{t-1}, s_{1t}\right)}{P_N\left(s^t, s_{1t+1}\right)} \\
\bar{Q}\left(s^t\right) &= \beta \sum_{s^{t+1}} h\left(s^{t+1} \mid s^t\right) \frac{P_T(s^t)}{P_T(s^{t+1})} \\
\bar{Q}\left(s^t\right) &= \beta \sum_{s^{t+1}} h\left(s^{t+1} \mid s^t\right) \frac{\frac{\psi}{\alpha} \frac{\min_{z_2}\left\{X\left(z_{2t}\right)\right\}}{X\left(z_{2t}\right)}}{\frac{\psi}{\alpha} \frac{\min_{z_2}\left\{X\left(z_{2t+1}\right)\right\}}{X\left(z_{2t+1}\right)}} M(s^{t-1})/M(s^t)
\end{aligned}
$$

2.B.6 Model Graphs and Estimation

Table 2.B.1. SMM for 1960-1999

Country	$\underline{\theta}$	$\bar{\theta}$	μ_θ data	μ_θ model	σ_θ data	σ_θ model
Italy	0.9517	0.9928	7.03%	7.04%	0.057	0.057
Germany	0.976	0.991	2.96%	3.16%	0.019	0.020

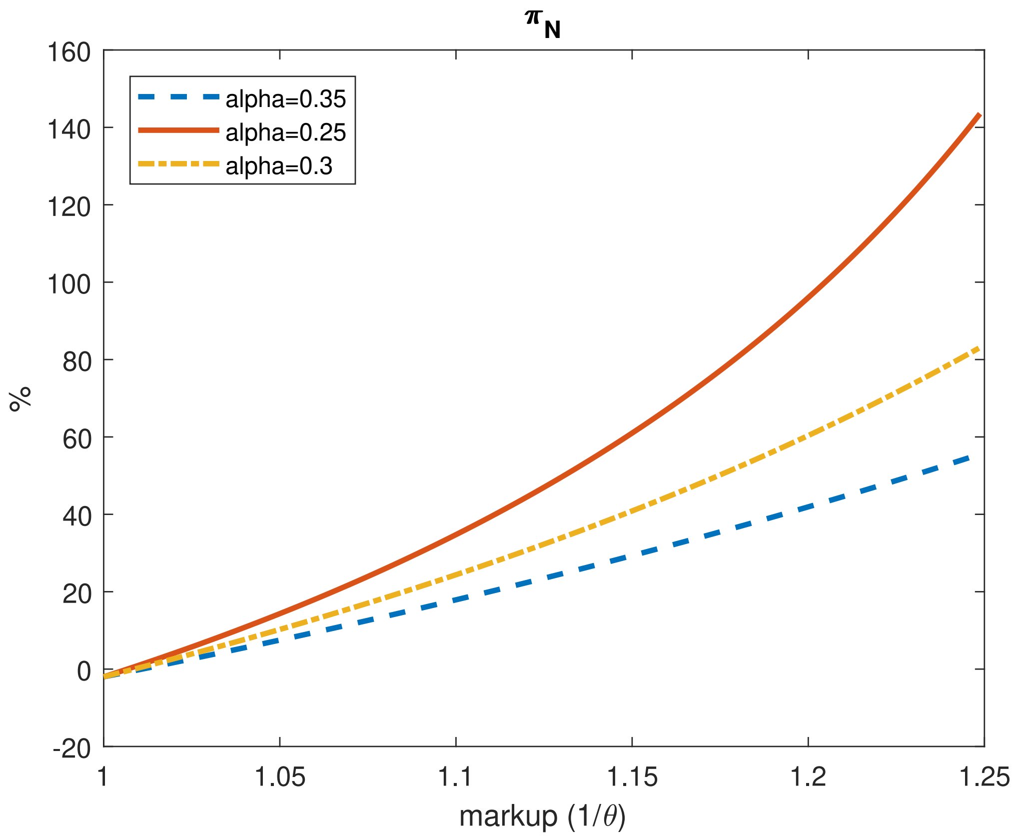


Figure 2.B.1. π_N as a function ot the markup in a monetary regime under discretion. The markup is defined as $\frac{1}{\theta}$. High markups correspond to a low elasticity of substitution between intermediate goods, allowing those firms to charge high prices. The dashed blue line corresponds to a trade openness of 35 %, the solid red line of 25% and the dashed yellow line of 30%. The same graph can be found in Arvai (2021)

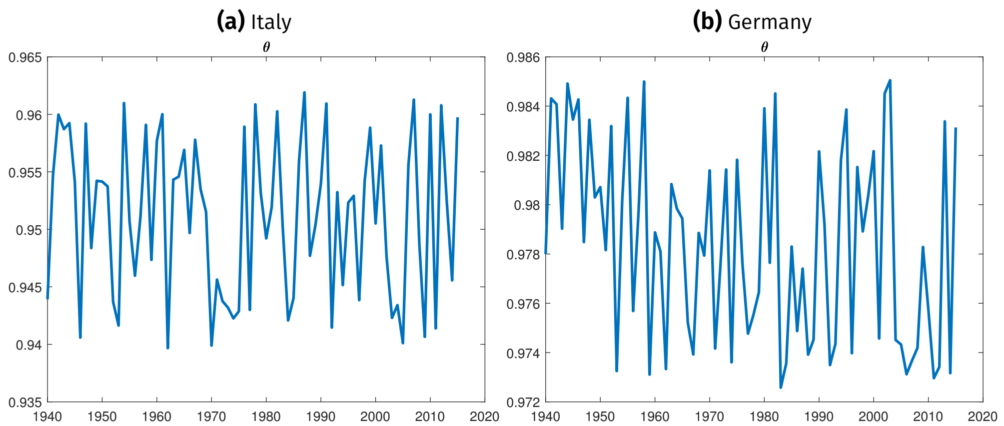


Figure 2.B.2. θ over time in the simulation

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Chapter 3

Consumption Inequality in the Digital Age*

Joint with Katja Mann

3.1 Introduction

Digital technology is transforming our economy, as it fundamentally changes the way we consume and produce. Digitalization and automation more broadly have been associated with increasing income inequality in the Western world. The literature documents that the increased usage of automation capital, such as robots and computers, has contributed to wage and employment polarization (Acemoglu and Autor, 2011; Autor and Dorn, 2013; Hemous and Olsen, 2021) and a fall in the labor share (Acemoglu and Restrepo, 2018; Eden and Gaggl, 2018; Martinez, 2018), even though it may have been beneficial for the overall number of jobs (Mann and Püttmann, 2018; Gaggl and Wright, 2017). While the debate over rising levels of inequality centers around income, what ultimately matters for welfare is consumption. A comprehensive assessment therefore has to consider not just the income effect of digitalization, but also its effect on prices of different types of goods.
A priori, it is unclear how this price effect should impact consumption inequality. If increased use of digital technology makes some consumption goods cheaper than others, it will benefit those income groups that consume relatively more of these goods. Depending on what these goods are, either rich or poor households could be the beneficiaries. This effect may then either work in the same direction or in the opposite direction of the income effect. The current paper quantifies the effects of digitalization on consumption inequality taking into account both channels. We combine a rigorous empirical analysis using household-level and sector-level data with a structural model calibrated to the U.S. economy. We find that that richer

* Thanks to Keith Kuester and Moritz Kuhn for comments on the early stage of this project.

households benefit more from the relative price decline as they consume more digital products, which exacerbates the income channel.

The paper consists of two parts. In the first part, we construct a new measure of the digitization content of goods and services. Using data from the U.S. Bureau of Economic Analysis (BEA), we identify assets that relate to Information and Communication Technology (ICT) and measure their share in industry-level capital stocks. The resulting digitalization measure covers 61 industries between 1960 and 2017. We find that the share of ICT assets in the overall capital stock has increased from almost 0% in 1960 to over 16% in 2017. There is substantial heterogeneity in the usage of ICT assets across industries. We then account for the digitalization content of intermediate products by relying on the input-output structure of the production network. As a next step, we link these industry-level digitalization measure of final goods to consumption categories in the Consumption Expenditure Survey (CEX). Using CEX data on household income, we construct the overall ICT share of consumption baskets along the income distribution. We find that rich households have a larger ICT share in consumption than poor households. In particular, consumption categories such as food manufacturing or textiles, which are consumed disproportionately by poor households, originate from industries that have a low ICT share. Categories that tend to be more important for rich households, such as finance and insurance or education, have higher ICT shares. We also document that consumer price inflation is weaker for ICT-intensive commodities, which means that digitalization benefits consumers of ICT-intensive goods.

In the second part of the paper, we construct a structural model building on these findings. The model features a two-sector economy with two types of capital, ICT capital and non-ICT capital. Sector 2 uses ICT capital more intensively than sector 1. The economy is populated by two types of agents that differ by skill endowment. High-skill labor is complementary to ICT capital, whereas a composite good constructed from these two inputs is substitutable to low-skill labor. Digitalization is modeled as an increase in the rate of transformation of output into ICT capital. The ICT-intensive sector benefits relatively more from this technology trend, which means that the relative price decreases. At the same time, the skill premium increases. In a setting with non-homothetic preferences, the effect of changing relative prices depends on the position in the income distribution of the agent. In line with our empirical finding, we assume that the ICT-intensive good is the luxury good, which is consumed more heavily by the high-skill, high-earner households. To assess the relative importance of income and price changes for consumption, we calibrate the model to the U.S. economy between 1960 and 2017. The simulated increase in consumption inequality is exacerbated by the decline of prices for ICT intensive goods. Overall, the high-skill household's welfare gain is equivalent to an increase of 23% of their income, while the low-skill households hardly benefit. If high-skill households had the same share of digital products in their consumption than low-skill households, their welfare increase in terms of income would only be equivalent to 17.4%. This means that a quarter of the overall welfare increase for the rich high-skill households can be attributed to the change in relative good prices.

Related literature: This paper is related to the literature on automation and wage polarization. Most of these papers define automation more broadly than digitalization, also comprising automated machines like robots. Autor, Katz, and Kearney (2008), Acemoglu and Autor (2011), Autor and Dorn (2013) and Hemous and Olsen (2021), among many others, show that recent technological change goes to the detriment of low-skill or routine-intensive occupations. Computers and robots substitute for low-skill workers and complement high-skill workers. As automation technology becomes more productive, this leads to an increase in the skill-premium. Moll and Rachel (2021) show that capital owners also benefit from automation, such that not just the income distribution, but also the wealth distribution becomes more unequal. Related to this finding, Eden and Gaggl (2018), Acemoglu and Restrepo (2018) and Martinez (2018) argue that automation depresses the labor share in value added. The production side of our model is most closely related to Eden and Gaggl (2018), who build on Krusell, Ohanian, Ríos-Rull, and Violante (2000) to show that automation exacerbates income inequality by increasing the skill-premium and the income share of capital. We rely on the literature on sector-biased technological change (as summarized recently by Herrendorf, Rogerson, and Valentinyi, 2014) to extend the framework to a two-sector economy where technological progress has unequal effects across sectors. We then continue to emphasize the effect of digitalization on goods prices and its impact on consumption inequality.

In studying the effect of digitalization on consumption, we also relate to the literature on consumption inequality. Aguiar and Bils (2015) and Attanasio and Pistaferri (2016) document that consumption inequality closely tracks income inequality when correcting for measurement error in survey data. While consumption inequality has not yet been in the focus of the automation literature, it has been studied by the trade literature, e.g. Fajgelbaum and Khandelwal (2016), Nigai (2014) and Borusyak and Jaravel (2018). The effect of automation is similar and different to that of trade: Similar because in both cases, a change in the production process alters factor returns as well as the price of output, and different because automation works through the capital stock and thus attributes a crucial role to the complementarity between capital and labor. Our paper adopts the approach taken by the trade literature and in particular Borusyak and Jaravel (2018) and is the first – to our knowledge – to study the distributional consequences of automation technology in consumption.

In what follows, Section 3.2 presents the empirical analysis. In Section 3.3 we introduce our model. Section 3.4 explains the calibration, followed by the simulation results in Section 3.5. Section 3.6 concludes.

3.2 Empirical analysis

This section motivates our focus on the relative price channel. We assemble a novel dataset and establish that households along the income distribution differ in the digitalization share of their consumption basket. Our data show that the rich consume more ICT-intensive goods than the poor. At the same time we find that prices of ICT goods have grown at a slower pace than prices of non-ICT goods, making rich households beneficiaries of digitalization.

We assemble our dataset by focusing on industry asset data and household-level consumption data and proceed along the following steps: (1) Create an industry-level measure of ICT intensity by computing the ICT vs. non-ICT capital used in production. (2) Trace linkages across industries to create an ICT intensity at the level of final commodities. (3) Match commodities to consumption good categories and calculate the digitalization share of consumption baskets along the income distribution. (4) Using supplementary data on prices, show the association between ICT intensity and consumer prices.

3.2.1 ICT capital share by industry

The BEA provides data on the stock and investment of 96 different types of assets for 61 private industries in the Detailed Data for Fixed Assets. We use this yearly dataset between 1960-2017 to construct the stock of digital assets for each industry. We define **ICT assets** to be the following asset categories: Mainframes, PCs, DASDs, printers, terminals, tape drives, storage devices, system integrators and intellectual property products, such as: prepackaged software, custom software, own-account software, semiconductor and other component manufacturing, computers and peripheral equipment manufacturing, other computer and electronic manufacturing, n.e.c., software publishers and computer systems design and related services. We also refer to these assets as digital assets. All other assets are defined as non-ICT assets.

Figure 3.1 plots the aggregate stock of ICT and non-ICT capital in the U.S. economy by year, both stocks indexed to 1 in 1995. Over the whole time horizon, ICT capital has grown faster than non-ICT capital. Since 1995, the ICT capital stock has increased by more than 700%, while non-ICT capital has increased by around 300%.

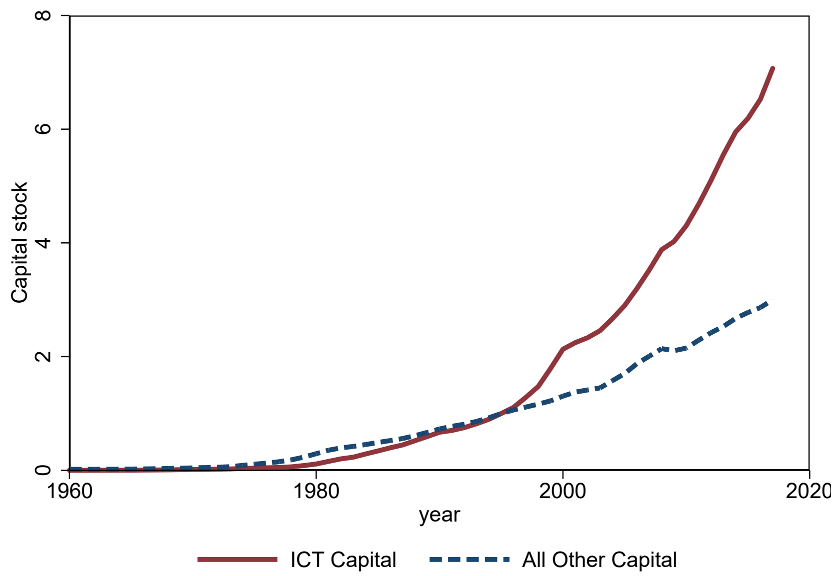


Figure 3.1. Aggregate ICT- and non-ICT capital stock, 1995=1
Note: The graph shows ICT and non-ICT capital in the U.S. economy between 1960 and 2017. Both data series show the capital stock relative to its 1995 value, which we have set to 1. *Source:* BEA and own calculations.

Throughout the paper, we will measure the degree of digitalization as the share of ICT capital in the overall capital stock of an industry or of the whole economy. We refer to this measure as **ICT intensity**. In adopting this definition, we focus on (relative) inputs in the production process, rather than, for example, the income shares of different factors of production. We argue that the ICT intensity best reflects the structure of the production process, thereby capturing the role that digital assets play in producing certain goods and services. Since both types of capital are evaluated at their current prices, the measure describes how much the ICT capital is worth to producers relative to the non-ICT capital. The relative valuation reflects how productive each capital type is, i.e. the level of ICT- and non-ICT technology. An additional advantage of our digitalization measure is that all data series are directly observable.[1]

Ignoring any interlinkages between industries, Figure 3.2 shows the ICT intensity by industry and focuses on the aggregate ICT intensity as well as industries in the 10th and 90th percentile. The average ICT capital share has risen substantially from almost zero in 1960 to 16% in 2017. Underlying the aggregate measures is a large

1. This is not the case when focusing on the ICT capital income share in value added, which has been used in some papers to measure digitalization, see e.g. Eden and Gaggl (2018) and Karabarbounis and Neiman (2019). These measures rely on estimated rates of return to ICT assets, which differ widely across papers.

degree of heterogeneity across industries. Some industries have barely accumulated any ICT capital, while in other industries, more than half of the capital stock consists of ICT capital by 2017. Industries in the top 10% of the ICT intensity mostly belong to the finance and insurance industry or to the computer and electronic products manufacturing industry. The least ICT-intensive industries are for example agriculture, the plastic and rubber industry or the textile industry. The difference in ICT intensity between industries will become relevant later on. This matters, because some industries are more relevant to produce certain consumption goods that are more or less important along the income distribution.

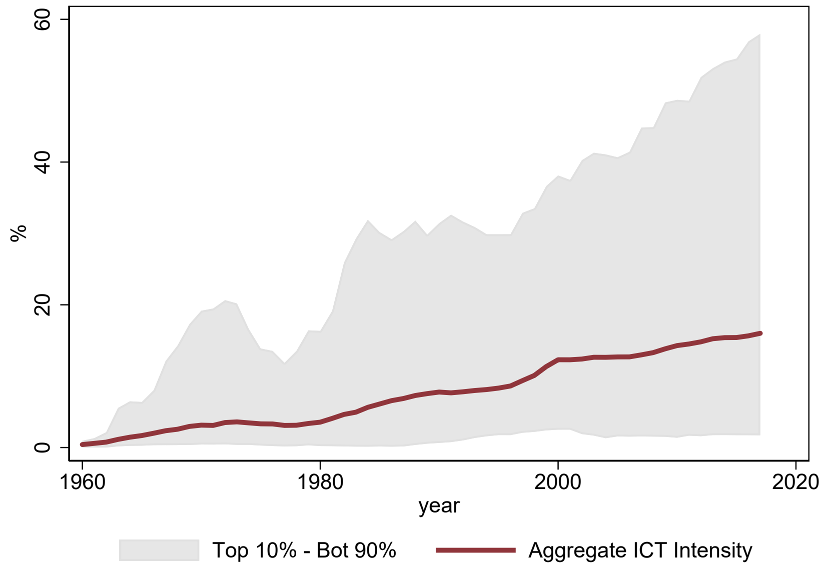
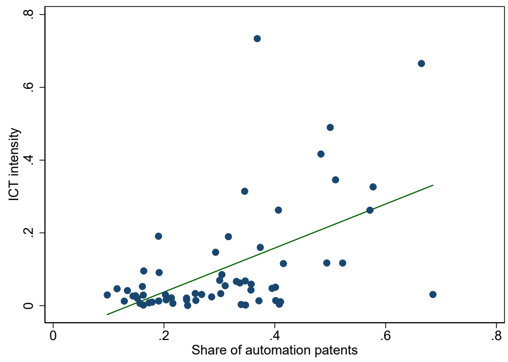


Figure 3.2. Industry-level ICT intensity

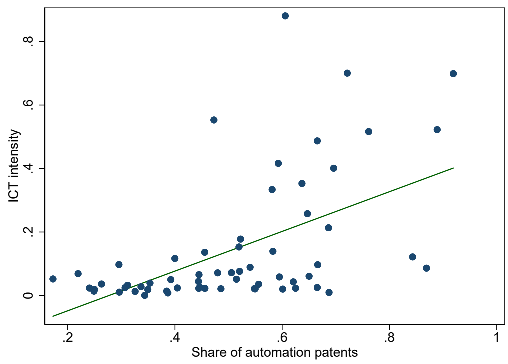
Note: The graph shows the share of ICT capital in the total capital stock (ICT intensity) by BEA industry. The solid line shows the average and the gray area the industries between the 10th and 90th percentile of the distribution. *Source:* BEA and own calculations.

3.2.2 Comparison to established measures of digitalization

This section compares our ICT intensity measure with other measures established in the literature. The first comparison is with automation patent data from Mann and Püttmann (2018). Mann and Püttmann (2018) classify US patents as automation or non-automation patents via a text search algorithm and assign patents to the industry of their likely use. While this measure defines automation more broadly, also including robots, computers and communication technology make for the largest share of automation patents. In Figure 3.3, we plot the correlation of the share of automation patents to our ICT intensity measure for 2010 (plots would look similar for other years). The positive correlation hints that actual investment in ICT assets mirrors new automation technology, which makes us positive that our measure is picking up technological progress in ICT.



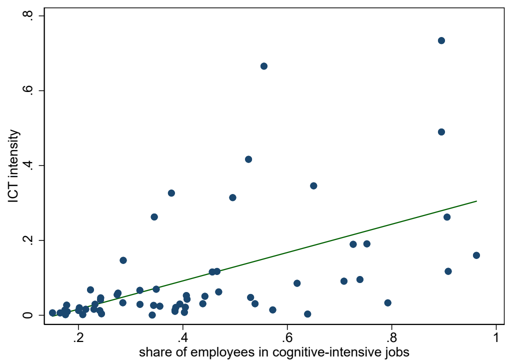
(a) 1990



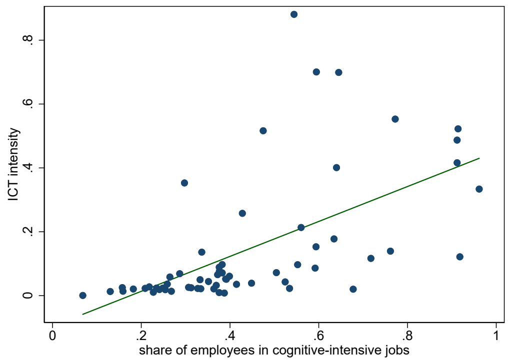
(b) 2010

Figure 3.3. Relationship between share of automation patents and ICT-intensity measure in 2010
Note: The graph plots ICT intensity against the share of automation patents in the total number of patents by BEA industry. Each dot represents one industry, the line shows a linear prediction. The left panel shows raw data for 1990, the right panel for 2010. *Source:* Mann and Püttmann (2018), BEA and own calculations.

Another established procedure in the literature is to consider the input of different tasks across occupations in each industry. Autor, Levy, and Murnane (2003) and Autor and Dorn (2013) focus on routine tasks as a measure of automation potential of production processes. Gaggl and Wright (2017) and Adao, Beraja, and Pandalai-Nayar (2020), among others, argue that in the context of ICT, it is more relevant to focus on cognitive tasks, which are presumingly complementary to ICT capital. We define a cognitive-task intensity analogous to the routine-task intensity of Autor and Dorn (2013), by taking the log of abstract tasks divided by routine and manual tasks by occupation and defining a job as cognitive-intensive when this number is in the upper third of the distribution.
Figure 3.5 plots the relationship between the share of cognitive tasks and our new measure of ICT intensity for 1990 and 2010. There is a clear positive correlation



(a) 1990



(b) 2010

Figure 3.5. Relationship between share of cognitive employment and the share of ICT capital on overall capital

Note: The graph plots ICT intensity against the share of employees that work in cognitive-intensive jobs by BEA industry for 1990 (left panel) and 2010 (right panel). Each dot represents one industry, the line shows a linear prediction. *Source:* Census, American Community Survey, Autor and Dorn (2013), BEA and own calculations.

and this correlation increases over time. Higher ICT intensity coincides with a larger share of employees in cognitive-intensive jobs.

3.2.3 Digitalization share of final output

Industries may not only use digital capital in their own production, but also use intermediate inputs that have been produced with digital capital. In order to calculate the share of digital capital used in the production of final goods, we take input-output linkages among industries into account.

The BEA's Input Output Accounts show how industries provide input to or use output from each other. These accounts provide detailed information on the flows of the goods and services that comprise the production process of industries. We use the detailed Input-Output tables after redefinitions and focus on private industries. The tables contain 394 private industries in 2012 and 473 private industries in 1997, which can be matched to the 61 BEA industries[2]. We create a commodity-by-commodity direct requirements matrix, which we use to calculate the digitalization share of final goods and services as a weighted sum of the digitalization shares of its intermediate inputs and value added. See Appendix 3.A.1 for details.

We pursue an iterative approach. We initially assign each commodity the digitalization share of the industry that is the ultimate producer of the commodity. Then, we consider all commodities that use this specific commodity as intermediate input. We update the output digitalization share by calculating a weighted sum of the digitalization shares of all inputs plus the digitalization share of value added of the final producer. We again assign this share to the inputs used to produce other commodities, and update the output digitalization share again, etc. We continue this procedure until the commodity digitalization shares have converged to fixed values. We construct the ICT-intensity measure for 1996-2017 for all I-O commodities.

To illustrate the importance of the input-output structure, Figure 3.7 shows the ICT shares for 2012 before and after taking input-output linkages into account. Commodities that initially have a low degree of digitalization when considering only the ICT intensity of the final goods-producing industry tend to be more digitized after the inputs from other industries are considered. They lie above the dotted red 45 degree line. The reverse is true for very digitized industries that also use inputs from less digitized industries.

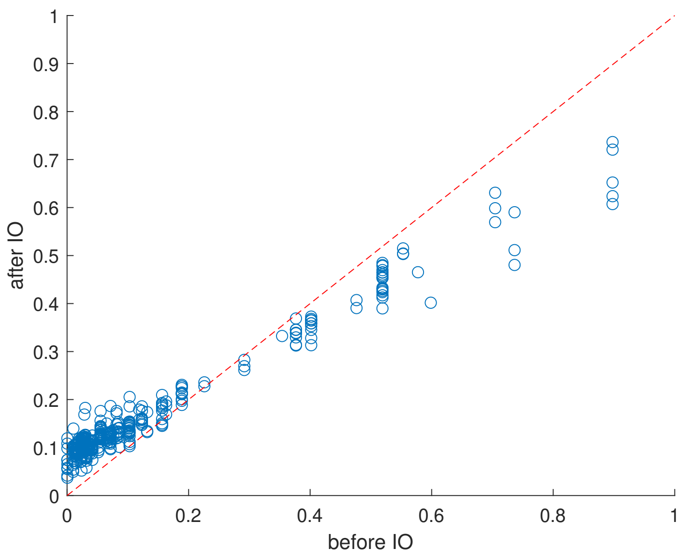
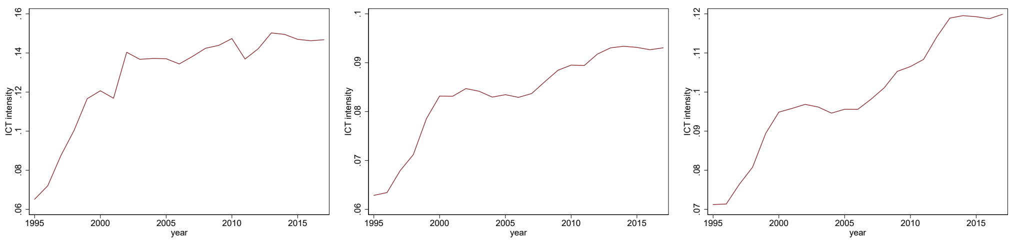


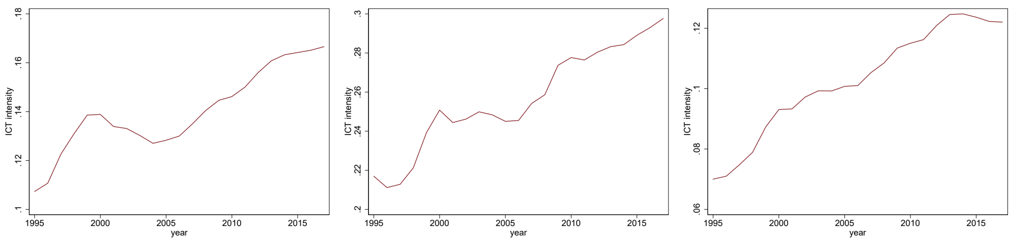
Figure 3.7. ICT intensity with and without considering the I-O structure
Note: The graph shows ICT intensities for 2012 before the I-O structure has been taken into account (x-axis) and afterwards (y-axis). Each dot represents an I-O industry. The red dotted line is the 45-degree line. *Source:* BEA and own calculations.

2. For 1996-2004 we use the 1997 matrix for 2005-2017 we use the 2012 matrix.

We group commodities into 25 broader categories and in Figure 3.8 plot the ICT intensity of six of them over time. In anticipation of the future link to consumption categories in the CEX, we focus on the time period 1996-2017 and consider the six most important categories for consumption: Real estate, food manufacturing, textile manufacturing, transport, finance & insurance and restaurants.[3]. There has been an increase in the ICT intensity in all of the commodity categories over time and the pattern often looks similar, e.g. reflecting the build-up and subsequent burst of the dotcom bubble. However, commodities are characterized by large differences in the average value of ICT intensity and notable finance and insurance has a much higher ICT intensity throughout the whole sample period while food manufacturing has a low ICT intensity. These differences will become relevant later on.



(a) Real estate **(b)** Food manufacturing **(c)** Textile manufacturing



(d) Transport equipment manuf. **(e)** Finance and insurance **(f)** Restaurants

Figure 3.8. ICT intensity by commodity

Note: The graph shows ICT intensities for six broad commodity categories over time. The ICT intensity in these categories is calculated as a weighted average of ICT intensity of the relevant I-O commodities. The weights correspond to their share in value added. *Source:* BEA and own calculations.

3.2.4 Consumption patterns across households

We measure expenditures of U.S. households using the Consumer Expenditure Survey (CEX). The CEX is the most detailed expenditure survey in the United States, carried out at the household level. Next to data about the purchase of hundreds

3. The following I-O commodities are in these categories: real estate: 5310HS; food manufacturing: 311*, 312*; textile manufacturing: 313*, 314*, transport: 336*; finance and insurance: 52*; restaurants: 722*.

of disaggregate goods and services, it also contains a large amount of demographic and income information of the households. This enables us to study the expenditure pattern of households by income. We use the public-use microdata for 1996-2017. We combine the interview survey, which contains information across all expenditure categories, with the diary survey, in which households report purchases on only a subset of goods and services, but in a more detailed way. There are around 2,500-3,000 households per year in each of the surveys.

As explained in more detail in Appendix 3.A.1, we divide households into ten equal-sized bins based on labor income. For each income group, we create a weighted average of expenditure by year. Using the concordance table of Borusyak and Jaravel (2018), we link 809 CEX commodities to 159 I-O commodities. Each commodity in the CEX is matched to a unique I-O industry.[4] Note that the concordance table treats housing as a service from the real estate sector rather than as a good from the construction sector.

Income groups vary by their spending pattern. Table 3.1 shows for the 25 commodity categories the expenditure share of households in the first and of the tenth decile of the labor income distribution as well as their ratio, both for 1996-99 and for 2010-2014. These are ranked by their importance for low-income households. The largest share of income – around 25% – is being spent on housing and therefore gets allocated to real estate. There is little difference across time or across income deciles. Other important categories with spending patterns being similar across deciles are transport equipment – which most notably includes expenditure on cars –, restaurants as well as information – comprising of telecommunication, IT services, broadcasting, audio and video recordings. Some categories are consumed more extensively by the rich, in particular accommodation, arts, entertainment and recreation, and education. The consumption basket of the poor is more tilted towards food, beverages and tobacco, utilities and agricultural products. There has been little change in expenditure patterns over time.

One dimension of inequality that we cannot capture with our data are quality differences in the goods consumed. A rare exception is the sub-category restaurants (broad industry 722, corresponding to “food away from home" in CEX). Households in the CEX are asked to report separately money spent at fast food, take-out, delivery and vending machines, at full-service restaurants, and at the workplace. Households in the lowest decile spend about 60% of their total restaurant expenses on fast food, but only 30% in full-service restaurants. In contrast, households in the highest decile spend 40% of their total restaurant expenses on fast food, but 55% in full-service restaurants. As full-service restaurants are presumingly more labor-intensive than fast food restaurants, the restaurant expenses of high-income households will likely

4. In contrast, a commodity would often be matched to several consumption categories. Therefore we convert consumption categories into commodities rather than the other way round. These 159 I-O commodities are the ones that are relevant for consumption in the CEX.

Table 3.1. Expenditure shares for 1st and 10th decile by commodity category in %

industry	1996-99			2010-14		
	1st	10th	**ratio**	1st	10th	**ratio**
real estate (incl. construction)	22.12	24.86	**0.89**	24.80	27.36	**0.91**
food, beverages, tobacco	17.19	8.79	**1.96**	16.03	7.33	**2.19**
transport equipment	8.12	9.15	**0.89**	8.26	10.45	**0.79**
restaurants	7.24	8.15	**0.89**	6.96	7.32	**0.95**
textile, apparel, leather	7.05	5.49	**1.28**	6.61	4.73	**1.40**
chemicals, petroleum	6.22	4.83	**1.29**	6.72	4.94	**1.36**
finance,insurance	5.20	7.32	**0.71**	5.15	6.85	**0.75**
utilities	4.88	2.97	**1.64**	5.19	3.02	**1.72**
information	4.33	3.59	**1.21**	4.49	4.04	**1.11**
other services	2.99	4.29	**0.70**	2.87	4.76	**0.60**
misc. manufacturing	2.94	3.80	**0.77**	2.87	4.76	**0.60**
machinery,electrics,electronics	2.45	2.92	**0.84**	1.95	2.60	**0.75**
agriculture	2.25	1.21	**1.86**	2.13	1.08	**1.97**
health	1.73	1.73	**1.00**	1.38	1.92	**0.72**
trade,transport,warehousing	0.89	1.68	**0.53**	0.96	1.51	**0.64**
wood,furniture	0.88	1.60	**0.55**	0.84	1.29	**0.65**
professional services	0.65	0.65	**1.00**	0.35	0.67	**0.52**
rental and leasing	0.64	1.54	**0.42**	0.67	1.31	**0.51**
paper, printing	0.62	0.47	**1.32**	0.52	0.38	**1.37**
education	0.57	1.39	**0.41**	0.55	2.32	**0.24**
rubber, nonmetallic minerals	0.42	1.01	**0.42**	0.42	0.57	**0.74**
admin support	0.24	0.48	**0.50**	0.31	0.52	**0.60**
arts, entertainment, recreation	0.22	1.02	**0.22**	0.28	1.10	**0.25**
accommodation	0.15	0.85	**0.18**	0.14	0.86	**0.16**
primary and fabricated metal	0.02	0.04	**0.50**	0.01	0.03	**0.33**

be less automation-intensive than the restaurant expenses of low-income households. Advances in digitalization technology will thus affect them differently.

3.2.5 Digitalization share of consumption

Before considering the ICT intensity of different consumption baskets as a whole, we illustrate which commodities are particularly relevant for determining the ICT intensity of high- vs. low-income consumption. In Figure 3.10, we plot the ICT intensity against the relative importance of a commodity (in terms of expenditure share) for high-income vs. low-income households. We consider data for over 159 final commodities in the year 2017[5]. Each dot corresponds to an I-O commodity and the size reflects the importance of the commodity (in terms of expenditure shares for the median household) in 2017.

5. for data from 1997 and 2007 see figure 3.A.1.

There is a positive correlation between the ICT intensity and the relative expenditure share of the rich. On the left side of the graph, where categories are more important for poor households, most ICT intensities are below the average value of 0.16. Around the center, where commodities are equally important for both households, we find industries with very different ICT intensities. The largest category, real estate, has an average ICT intensity, whereas for example the information industry has a very high ICT intensity. Moving to the right part of the graph which depicts commodities that are relatively more important for the rich, we have some categories of average ICT intensities, but also some with very large ICT intensities such as education.

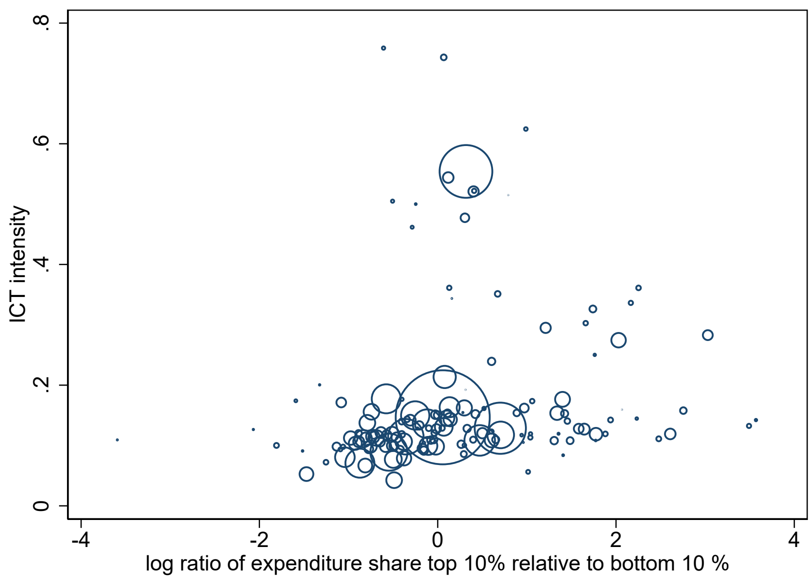


Figure 3.10. ICT intensity of vs. relative expenditure shares by commodity in 2017
Note: Each circle corresponds to one of the 159 I-O commodities. The size of the circle reflects the share of expenditures for that category of the median household. The x-axis shows the log ratio of the expenditure share of the top 10 % relative to the bottom 10 %. The more to the right the data are, the more important that category for the rich. Values around zero indicate that a category is equally important to the top 10% as it as for the bottom 10 %. The y-axis shows ICT intensity of the industry. Data are for 2017. *Source:* BEA, CEX and own calculations.

To put labels on the different commodities, we aggregate them into the 25 commodity categories of Table 3.1. The left panel of Figure 3.11 puts labels on the industries, while the right graph emphasizes the expenditure shares of the median household by means of circle size. Some industries that are relatively important for the low-income households, such as food manufacturing (311) and agriculture (11), are characterized by low ICT shares. Industries linked to educational service (61) matter more for rich households and feature a high ICT share. There are also industries with high ICT shares that are equally important for both households, such as the manufacturing products (333). The most important category for both households, real estate (531) has an average ICT intensity and will not play a decisive role for differences in ICT intensity across income groups.

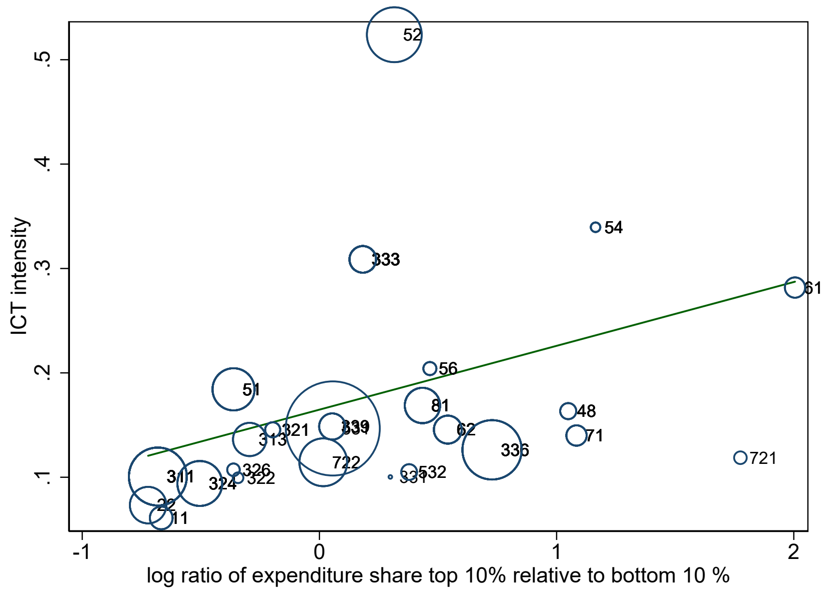


Figure 3.11. ICT intensity of consumption top 10% relative to bottom 90 % aggregate consumption categories, 2017.

Note: The graph shows the same data as Figure 3.10 in more aggregated form. The consumption categories are: 11=agriculture, 22 = utilities, 311=food,beverages, tobacco, 313=textile,apparal,leather, 321= wood, furniture, 322=paper,printing, 324= chemicals,petroleum, 326=rubber, nonmetallic minerals, 331= primary and fabricated metal, 333= machinery, electrics and electronics. 336 = transport equipment, 339=misc. manufactoring, 48 = trade, transport, warehousing, 51=information, 52= finance, insurace, 531= real estate, 532 rental and leasing, 54= professional services, 56 admin support, 61 = education, 62=health, 71 = arts, 7210= accomodation, 722=restaurants, 81= other services. The line shows a linear prediction. *Source:* BEA, CEX and own calculations.

As the final step, we compute the ICT intensity of the consumption basket by income decile. These reflect the expenditure patterns across income levels and the industry-level ICT intensities.

In Figure 3.12, we show two lines, a solid post I-O line that considers the input-output structure of the economy and a dashed pre I-O line based on the raw data. In both cases, the ICT intensity is substantially lower for the poorest 30% of the households than for richer households. Post I-O ICT shares are higher, because those industries that are directly matched to consumption categories are themselves characterized by low ICT shares in capital. The digital share increases when inputs from other industries are considered. This is true for example for consumption categories linked to crop production or motor vehicles.

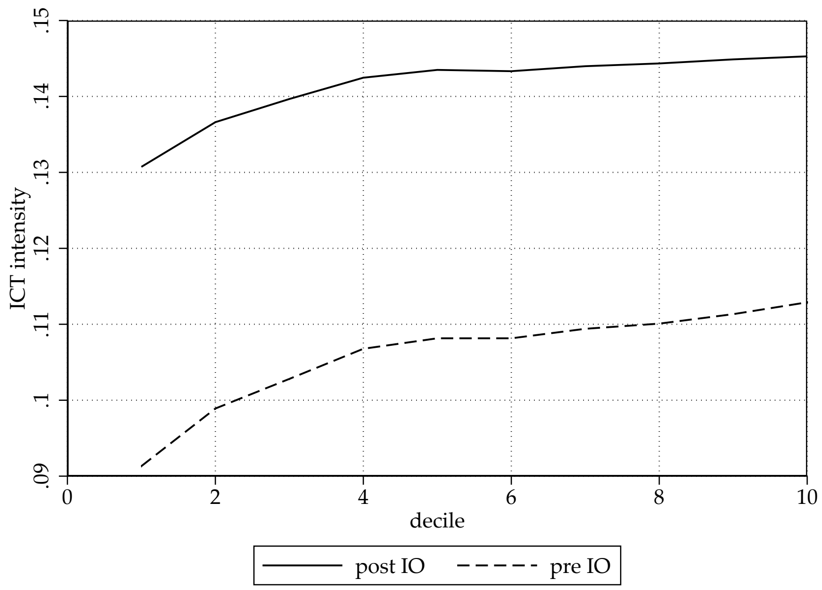


Figure 3.12. ICT intensity of consumption along the income distribution
Note: The graph shows the ICT share of the consumption basket by income decile. Data points are average values between 1996 and 2017. *Source:* BEA, CEX and own calculations.

Figure 3.13 exploits the time series dimension of the data by showing the ratio of ICT consumption of the top 10% relative to the bottom 10% of the income distribution. This number has increased gradually from around 1.08 o 1.15. So the strong increase in the ICT share over time that we documented in Section 3.2.1 affects the consumption bundle of the rich slightly more. The inequality in digital consumption has thus been increasing over the last 20 years.

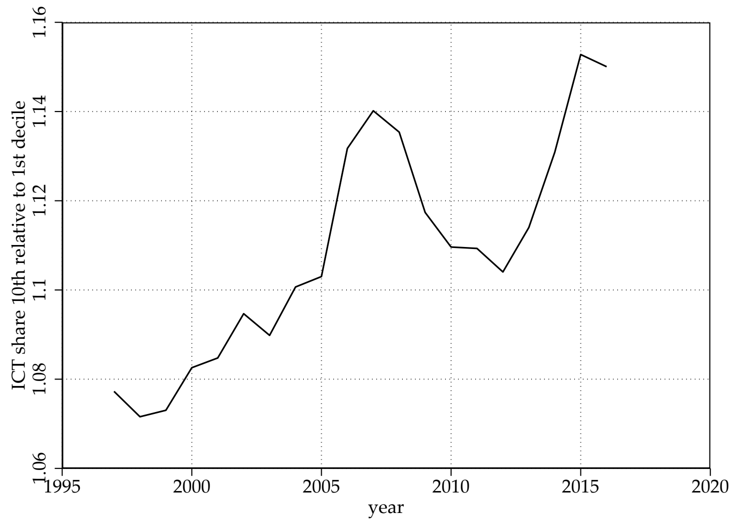


Figure 3.13. ICT intensity of consumption top 10% relative to bottom 90 %
Note: The graph shows the ICT share of the consumption basket of the 10th decile relative to the 1st decile of the income distribution. Each data point represents a three-year moving average. *Source:* BEA, CEX and own calculations.

Is the different ICT share for high- and low-income households consequential? The households at the lower end of the income distribution have an ICT share of 13% in their consumption bundle, while those at the upper end have an ICT share of around 14.5%. Considering lifetime consumption, these numbers could potentially have strong welfare effects of digitalization. To quantify the effect, we need to carry out a more structured analysis. This is why we introduce a model framework in Section 3.3.

3.2.6 Digitalization and consumer prices

The CEX does not provide separate information on the quantities and prices of the goods purchased. We therefore supplement our dataset using consumer price indices from the BLS, similar to Jaravel (2019). We match by hand 632 CEX product categories with 207 BLS price data series between 1996 and 2017.[6] We then create aggregate price indices at the level of 159 different IO commodities using the Törnqvist price index and our CEX-IO concordance. The Törnqvist index is a chain-weighted price index that considers product substitutions made by consumers and other changes in their spending habits, and is therefore well suited for our purposes. The growth rate of the Törnqvist index is a weighted average of the growth rates of the disaggregated price series of J individual goods, p_j, where the weights are nom-

6. Some disaggregated price series start later than 1996. In these cases, we match the CEX category to the series at the next higher hierarchical level in the BLS.

inal expenditure shares of the median household s_j,

$$\Delta p_t = \sum_{j=1}^{J} \ln\left(\frac{p_{j,t}}{p_{j,t-1}}\right)\left(\frac{s_{j,t} + s_{j,t-1}}{2}\right)$$

The level is recovered recursively relative to a base year 0 with $p_0 = 1$ as

$$p_t = p_{t-1} \exp(\Delta p_t).$$

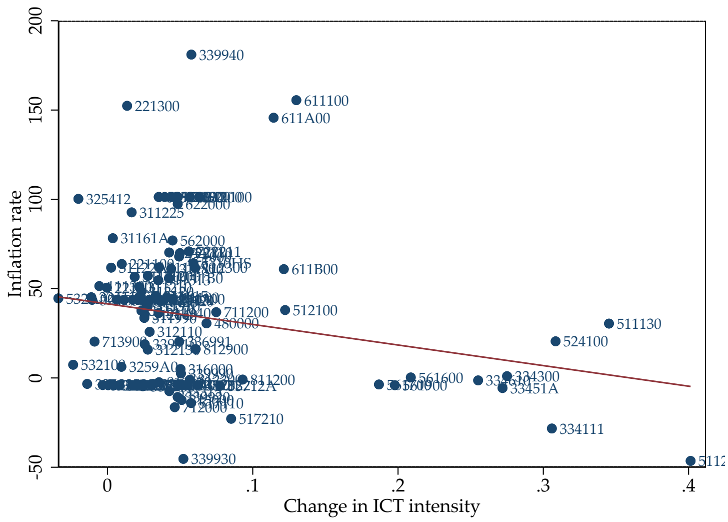


Figure 3.14. Consumer price inflation and ICT intensity

Note: The graph plots percentage point changes in ICT intensity (post-IO) against inflation in consumer prices between 1997 and 2017. Each dot represents an I-O commodity. *Source:* BEA, CEX, BLS and own calculations.

As Figure 3.14 shows, between 1997 and 2017 consumer price inflation has been lower for commodities that have a higher ICT intensity. Among the commodities with the largest price decline are software publishers (511200) and electronic computer manufacturing (334111), which have a high ICT intensity. (A large price decline was also observed in doll, toy and game manufacturing (339930), which has a low ICT intensity but possible a large share of imports.) The largest price increases are in low-ICT intensity commodities like office supply manufacturing (339940), 221300 (water, sewage and other systems) and 611100 and 611A00, which both refer to education. The results are similar when using shorter time periods. The coefficient of correlation is -0.21. Thus, consumer with a consumption basket tilted towards ICT-intensive goods fare better. This finding aligns with Aghion, Antonin, Bunel, and Jaravel (2020), who show that automation reduces the producer price index in French manufacturing industries and, Mann (2020), who documents a negative association between an industry's share of automation patents and the producer price index. Graetz Georg and Michaels (2018) find that the use of robots also leads to lower prices.

3.3 Model

In the following, we model the effect of an increase in digital technology on consumption inequality, working through changes in relative prices and income. The model covers two types of households with non-homothetic preferences, two capital goods (ICT and non-ICT) and two sectors with different demands for ICT technology. This setting allows us to quantify the effect of digitalization on consumption inequality and to analyze the mechanisms at work.

3.3.1 Sectoral production functions

There are two sectors $i = 1, 2$, which each produce a final output using as inputs two types of capital and two types of labor. We follow Krusell et al. (2000) and Eden and Gaggl (2018) in setting up the following nested production structure:

Inner nest: high-skill labor H_i is partially complementary to ICT capital IT_i, producing what we call skilled work (SW)

$$SW_i = \left[\gamma_i H_i^{\frac{\epsilon_i - 1}{\epsilon_i}} + (1-\gamma_i) IT_i^{\frac{\epsilon_i - 1}{\epsilon_i}}\right]^{\frac{\epsilon_i}{\epsilon_i - 1}}. \tag{3.1}$$

Middle nest: the skilled output is partially substitutable to low-skill work L_i, jointly producing total work (TW):

$$TW_i = \left[\phi_i L_i^{\frac{\eta_i - 1}{\eta_i}} + (1-\phi_i) SW_i^{\frac{\eta_i - 1}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i - 1}}, \tag{3.2}$$

with $\eta_i > 1$ and $\eta_i > \epsilon_i$.

Outer nest: Total work and non-ICT capital are combined in a Cobb-Douglas production function, owing to the fact that the share of non-IT capital in revenue has been constant over the last decades (Eden and Gaggl, 2018):

$$Y_i = K_i^{\alpha_i} TW_i^{1-\alpha_i} \tag{3.3}$$

There is perfect competition in each sector. First-order conditions yield the following equations for factor returns

$$K_i : \quad p_i \alpha_i \frac{Y_i}{K_i} = r_K, \tag{3.4}$$

$$L_i : \quad p_i (1-\alpha_i) \phi_i \frac{Y_i}{TW_i^{\frac{\eta_i - 1}{\eta_i}} L_i^{\frac{1}{\eta_i}}} = w_L, \tag{3.5}$$

$$H_i : \quad p_i (1-\alpha_i)(1-\phi_i) \gamma_i \frac{Y_i SW_i^{\frac{\eta_i - 1}{\eta_i}}}{SW_i^{\frac{\epsilon_i - 1}{\epsilon_i}} TW_i^{\frac{\eta_i - 1}{\eta_i}} H_i^{\frac{1}{\epsilon_i}}} = w_H, \tag{3.6}$$

$$IT_i \; : \quad p_i(1-\alpha_i)(1-\phi_i)(1-\gamma_i)\frac{Y_i SW_i^{\frac{\eta_i-1}{\eta_i}}}{SW_i^{\frac{\epsilon_i-1}{\epsilon_i}} TW_i^{\frac{\eta_i-1}{\eta_i}} IT_i^{\frac{1}{\epsilon_i}}} \;=\; r_{IT}. \tag{3.7}$$

3.3.2 Technological progress and capital formation

The output of sector 1 can be used for consumption as well as to produce non-ICT capital K. The output of sector 2 can be used for consumption as well as to produce ICT capital. While we assume that Y_1 can be transformed at the same rate (of 1) into the investment good and the consumption good, we define μ as the rate of transformation of Y_2 into IT capital. Then, the resource constraints are

$$Y_1 \;=\; C_1 + I_K, \qquad Y_2 \;=\; C_2 + \mu I_{IT}. \tag{3.8}$$

μ is the relative price of IT capital and at the same time measures progress in ICT technology (see e.g. Karabarbounis and Neiman, 2014; Eden and Gaggl, 2018 for similar set-ups). A decline in μ will make ICT technology more productive.
Capital follows the standard law of motion

$$K' \;=\; (1-\delta_K)K + I_K, \qquad IT' \;=\; (1-\delta_{IT})IT + I_{IT}. \tag{3.9}$$

3.3.3 Households

There are two types of households in this economy, high-skill and low-skill. Think about skill as level of education, which is determined before entering the labor market. Both types $j = H, L$ have non-homothetic preferences over the two goods 1 and 2. The period utility function takes the Stone-Geary form

$$u_j \;=\; \ln\left(\left[(1-\omega)\left(C_{1,j}-\bar{C}_1\right)^{\frac{\sigma-1}{\sigma}} + \omega C_{2,j}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right) \;\equiv\; \ln(C_j), \tag{3.10}$$

where $\bar{C}_1$ can be either larger than 0, making good 1 the necessity good, or smaller than zero, making it the luxury good. For our puprose, the less ICT intensive good 1 is going to be the necessity good. This implies that poor households (the low-skill households) have larger share of good 1 in their basket than rich high-skill households. If σ is relatively small then good 1 and 2 are partial complements to the consumer. If σ is large then they tend to be substituts[7]. We are going to use our data regarding expenditure shares for consumption goods over time to discipline the parameters of the utility function.

7. For $\bar{C} = 0$ and $\sigma = 1$ the consumption aggregator collapses to the familiar Cobb Douglas function with share ω for good 2.

The high-skill households jointly provide $\bar{H}$ units of labor, the low-skill households $\bar{L}$. Households can work in any sector and can switch at no cost. All non-IT and IT capital is owned by type H households. This assumption is motivated by empirical evidence, as we show based on the Survey of Consumer Finances (SCF) in Appendix 3.A.3.[8] In the survey years 1998-2013, net wealth of a median household where the head holds a college degree was more than five times that of a household without college degree. The wealth-to-income ratio is 2.4 for college graduates and 1.0 for non-college graduates. Capital income makes for a less than 1% of income for the median household in both groups. The budget constraint for type H households is

$$C_{1,H}P_1 + C_{2,H}P_2 + I_K P_1 + I_{IT}P_2 = \bar{H}w_H + Kr_K + ITr_{IT}. \tag{3.11}$$

L households are excluded from capital markets and are hand-to-mouth consumers. The budget constraint for type L households is

$$C_{1,L}P_1 + C_{2,L}P_2 = \bar{L}w_L. \tag{3.12}$$

Both types solve an intratemporal optimization problem, deciding between consumption of goods 1 and 2. Define the aggregate price index

$$P = \left[(1-\omega)^\sigma P_1^{1-\sigma} + \omega^\sigma P_2^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{3.13}$$

If we solve via expenditure minimization, the Lagrangian of household j is

$$\mathscr{L} = P_1 C_{1,j} + P_2 C_{2,j} - \lambda(\left(\left[(1-\omega)\left(C_{1,j} - \bar{C}_1\right)^{\frac{\sigma-1}{\sigma}} + \omega C_{2,j}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right) - U_j), \tag{3.14}$$

and results in the following Hicksian demand functions

$$C_{1,j} = C_j\left(\frac{P}{P_1}\right)^\sigma (1-\omega)^\sigma + \bar{C}_1, \qquad C_{2,j} = C_j\left(\frac{P}{P_2}\right)^\sigma \omega^\sigma. \tag{3.15}$$

The L-type households cannot save, so they consume all of their income in each period. The H-type households also solve an intertemporal optimization problem, choosing consumption vs. investment in the two sectors. We can express the Bellman equation as

$$V(K, IT) = \max(\ln(C_H)) + \beta\mathbb{E}\left[V(K', IT')\right], \tag{3.16}$$

which we solve subject to the household budget constraint eq (3.11). This results in the Euler equations

$$\frac{P_2}{C_H P} = \beta \frac{P_2'}{C_H' P'}\left(\frac{\frac{R_{IT}'}{P_2'} + \mu'(1-\delta_{IT}')}{\mu}\right), \tag{3.17}$$

8. Jaimovich, Saporta-Eksten, Siu, and Yedid-Levi (2020) make the same assumption about capital ownership.

$$\frac{P_1}{C_H P} = \beta \frac{P_1'}{C_H' P'} \left(\frac{R_K'}{P_1'} + (1 - \delta_K) \right), \tag{3.18}$$

which we can also write jointly as a no-arbitrage condition

$$\frac{P_2'}{P_2} \left(\frac{\frac{R_{IT}'}{P_2'} + \mu'(1 - \delta_{IT}')}{\mu} \right) = \frac{P_1'}{P_1} \left(\frac{R_K'}{P_1'} + (1 - \delta_K) \right). \tag{3.19}$$

3.3.4 Market clearing

The two capital markets clear

$$K = K_1 + K_2, \qquad IT = IT_1 + IT_2. \tag{3.20}$$

The goods markets clear

$$C_1 = C_{1,L} + C_{1,H}, \qquad C_2 = C_{2,L} + C_{2,H}. \tag{3.21}$$

The labor markets clear

$$\bar{L} = L_1 + L_2, \qquad \bar{H} = H_1 + H_2. \tag{3.22}$$

In solving the model, we normalize $P_2 = 1$ in all periods.

3.4 Calibration

3.4.1 Industry classification

In order to define ICT-intensive industries, we cluster the 61 private BEA industries according to our main ICT intensity measure and let the algorithm classify how to divide the industries. For the average years of 1996-1998 10 industries [9] are classified to be part of the ICT-intensive sector that correspond to our sector 2. This sector has an ICT intensity of 40% and combines a value added share of around 18%. The non-ICT intensive sector 1 has an ICT share of 9%.
The following table summarizes key numbers of the two sectors in the data for the year 2000 combining data from the BEA and the ACS.

9. These are: BEA codes 3340 (Computer and electronic products), 5110 Publishing industries, , 5140 Data processing, internet publishing, and other information services , 5230 Securities, commodity contracts, investments, and related activities , 5240 Insurance carriers and related activities , 5250 Funds, trusts, and other financial vehicles , 5411 Legal services , 5412 Accounting and bookkeeping services , 5415 Computer systems design and related services , 5500 Management of companies and enterprises

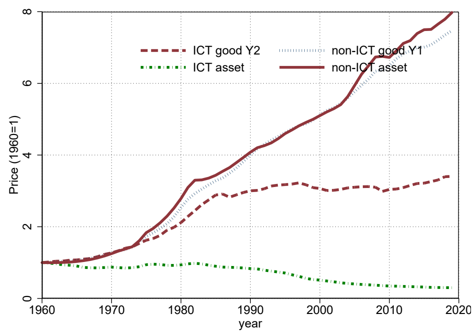
Table 3.2. ICT and non-ICT intensive sector

Data in %	Sector 1 (non-ICT intensive)	Sector 2 (ICT intensive)
Value added share	82%	18%
Cons. Expenditure share	91%	9%
Labor share	50%	70%
ICT intensity	9%	40%
Cognitive empl. share	59%	79%

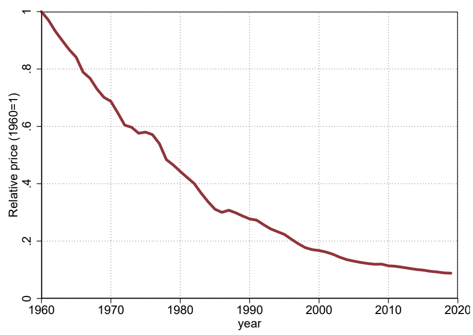
Our model assumption that the second ICT intensive sector is also the sector that produces ICT capital in in line with the data, as the industries summarized in that sector are either clearly producers of ICT capital (such as Computer and electronic Products, Computer systems and design) or service providers (such as as Funds, Legal services) that do not produce any other capital.

3.4.2 Advances in Digital Technology

According to eq.(3.8), progress in ICT technology works through a decline in μ, the price of ICT capital relative to output of good 2. Panel (a) of Figure 3.15 shows the evolution of the prices of Y_1 and Y_2 as well as ICT and non-ICT assets. The series are Törnqvist price indices based on NIPA price series for individual assets and commodities. While the prices of Y_1 and I_K move in parallel, justifying the assumption of a constant rate of transformation, there is a strong decline in the price of I_{IT} relative to Y_2. The resulting μ is shown in panel (b). We will feed (a smoothed version of) this series into our simulation as exogenous change in ICT technology.



(a) Individual price series



(b) μ

Figure 3.15. Prices of final consumption goods and assets

3.4.3 Other parameters

We need to calibrate σ, ω and $\bar{C}$ in the utility function. In addition, ten parameters in the production function need to be calibrated: four elasticities of substitution (ϵ_i, η_i $\forall i = 1, 2$) and six weights (α_i,γ_i, ϕ_i). We choose these to minimize the difference between model and data with respect to the relative price, the wage premium, the expenditure shares for high-skill and low-skill households in each good, and the ICT intensity in each sector. To generate for example the rising wage premium for high-skill households we need the elasticity of substitution between high-skill work and ICT capital to be much lower than the elasticity of substitution between low-skill work and specialized work, which means that $\epsilon_i < \eta_i$. To generate the difference in consumption pattern, the model needs to generate a sufficient difference in disposable income and choose an appropriate subsistence level of consumption $\bar{C}$.

Table 3.3 summarizes how we currently calibrate the parameter values.

Table 3.3. Calibration, values from 1996-1998

Symbol	Value	Description	Target
Preferences			
ω	0.23	Weight for ICT good 2	estimated
$\bar{C}_1$	50	Subsistence consumption	ICT share rich vs poor (CEX)
σ	1.51	El. of subst. ICT non-ICT	estimated
Skilled Work			
γ_1	0.94	Weight H inner nest in 1	Wage bill high-skill (ACS)
γ_2	0.55	Weigh H inner nest in 2	Wage bill high-skill (ACS)
ϵ_1	1.15	El. of Subst H and ICT in 1	Imperfect substitutes
ϵ_2	1.2	El. of Subst H and ICT in 2	Imperfect substitutes
Total Work			
ϕ_1	0.43	Weight L middle nest in 1	Wage bill low-skill (ACS)
ϕ_2	0.54	Weight L middle nest in 2	Wage bill low-skill (ACS)
η_1	3.9	El. of Subst L and SW in 1	Strong substitutes
η_2	3.3	El. of Subst L and SW in 2	Strong substitutes
Final Good			
$\alpha 1$	0.55	Capital share in 1	BEA
$\alpha 2$	0.28	Capital share in 2	BEA
δ_{ICT}	0.15	Depreciation ICT capital	BEA
δ_K	0.08	Depreciation non-ICT capital	BEA

3.5 Model Simulation

We simulate the model economy starting from an initial steady state of $\mu = 1$ in 1960. We feed in the series shown in Figure 3.15 between 1960 and 2017 and assume that the economy converges to a new steady state at the 2017 value, $\mu = 0.087$, afterwards. Along the transition, we assume a perfect foresight equilibrium.
The advances in ICT technology affect consumption of high- and low-skill households through two channels, which we study in detail below. First, labor demand changes, since high-skill labor and ICT capital are complements, whereas low-skill labor is a substitute to SW, the demand for high-skill workers rises, whereas the demand for low-skill workers declines. As a result, the skill premium increases. High-skill households gain purchasing power and boost their consumption relative to low-skill households. Second, the price of good 1 relative to good 2 increases, which implies that a larger share of consumption expenditure falls on good 2. Since good 2 is a luxury good, the price changes benefit the high-income households relatively more, whose consumption increases relative to low-skill households.

3.5.1 Income channel

Let us first study the income channel. Figure 3.17 shows the evolution of the skill premium W_H/W_L, normalized to 1 in 1960.

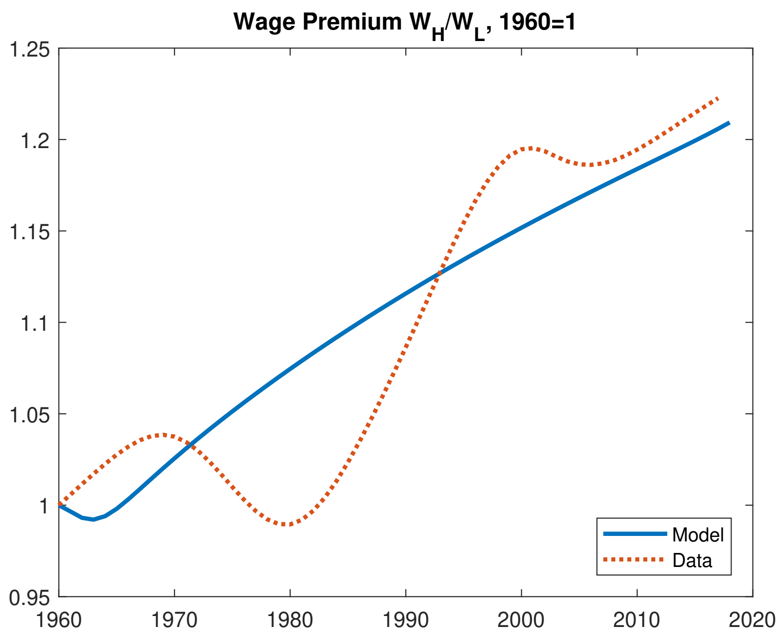


Figure 3.17. Skill premium, 1960=1

Note: The figure shows the evolution of the skill premium W_H/W_L in the model (solid blue line) and in the data (dotted red line). The data are taken from the ACS.

The skill premium increases substantially both in the data and in the model. The increase is slightly larger in the data (22% vs. 21%). The model does not capture some of the short-run fluctuations during the transition, but this is not surprising given that we focus on long-run trends in the skill demand and abstract from factors that create fluctuations at business-cycle frequencies. Advances in the ICT technology affect the skill premium in the following way: As ICT capital becomes cheaper and more efficient in the production, firms accumulate more ICT capital and increase their ICT intensity (see Figure 3.B.1). This means that demand for high skill labor rises. This is due to the assumption of partial complementarity between ICT capital and high-skill labor. Simply put, if a firm uses more ICT capital such a computer hardware or software, it also needs a high-skill worker who can work with that new form of capital. At the same time, low-skill labor can be substituted by the composite input of high-skill labor and ICT technology. This reflects the assumption that ICT technology together with high-skill labor is able to substitute low-skill labor. Spoken simply again, certain tasks can be executed more efficiently by computers operated by high-skill workers, rather than by low-skill workers.

Total income of the high-skill households includes both wage and capital income, whereas low-skill households only earn wage income. However, it is worth emphasizing that the capital share in total income increases only slightly. So while both labor and capital income increase for high-skill households, the large increase in overall income inequality is mostly driven by shifts in relative wages. Overall, the technology-induced shifts in factor incomes translates into a higher income for high-skill and low-skill households, as Figure 3.18 shows.

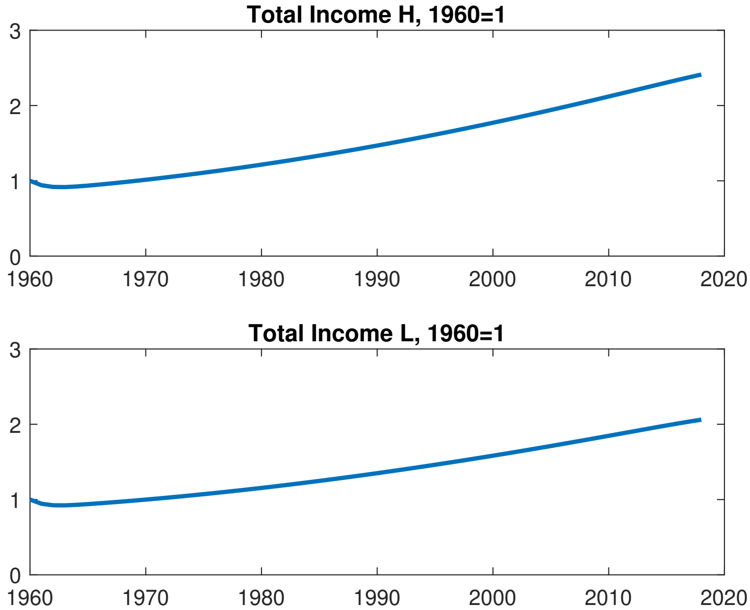


Figure 3.18. Evolution of the total income in the model (solid blue line).

Note: The figure shows the evolution of total income for high-skill households (top) and low-skill households (bottom). Values are normalized to 1 in 1960.

Until 2017, the income of the high-skilled households has increased to 241% of the 1960 value, whereas low-skilled households' income increases to 206% of the 1960 value. As we only consider one source of technological progress, this increase in entirely driven by ICT technology.

3.5.2 Price channel

Next, we consider the price channel. As Figure 3.19 shows, the relative price P_1/P_2 increases in response to the technology shock. As progress in digitalization favors the ICT intensive good more, the relative price of good 1 rises. Within our time horizon the relative price of good 1 increased by 220 % compared to the price of good 2. In the model, the final increase is 216% with some smaller deviations during the transition.

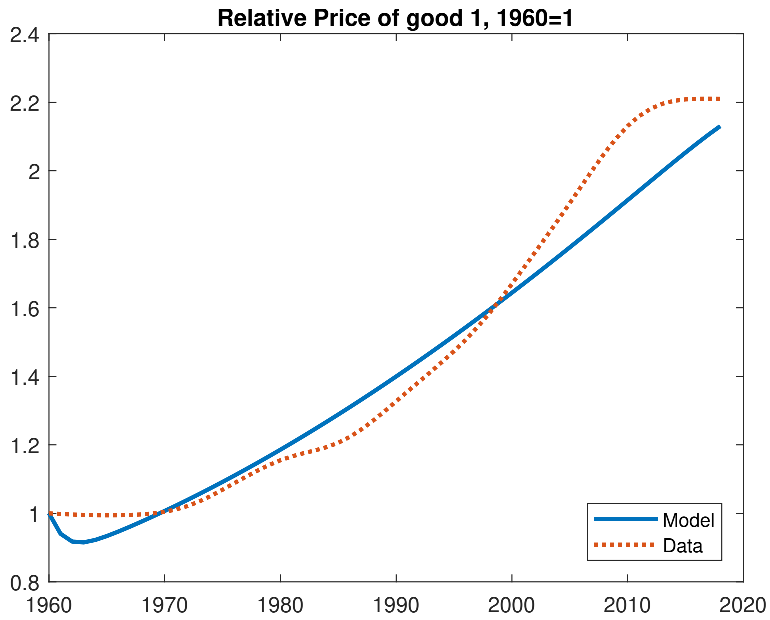


Figure 3.19. Relative price of good 1, 1960=1

Note: The relative price is the price of the non-ICT intensive good 1 relative to the ICT-intensive good 2. The figure shows the model (solid blue line) and the data (dotted red line). The data for good prices are taken from the NIPA price series for assets and commodities. The sorting in the non-ICT intensive sector 1 and the ICT-intensive sector 2 was done as in Section (3.4.1).

3.5.3 The Effect on Consumption

As we have seen, technological progress in ICT increases income and changes relative prices. Figure 3.20 shows how relative consumption is impacted in the model:

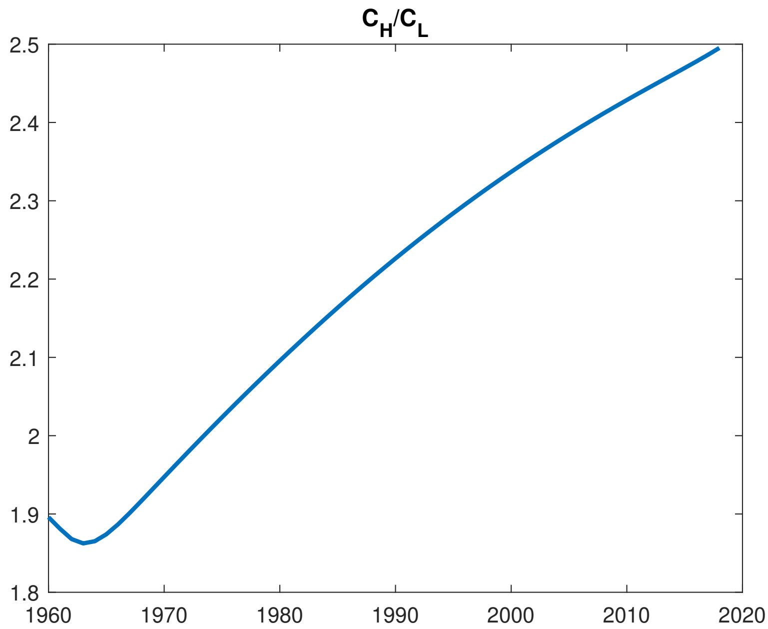


Figure 3.20. Evolution of consumption of both type of goods
Note: The figure shows relative consumption of high-skill households relative to low-skill households.

In real terms consumption of both high-skill households and low-skill households increases. As Figure 3.20 illustrates however, the increase in consumption is larger for high-skill households than for low-skill households. Initially, high-skill households have a 1.9 times larger consumption composite than low-skill households. This number increases to around 2.5 by the end of the simulation period.

An important feature of the model are non-homothetic preferences. These preferences imply that households with more income consume a larger share of ICT intensive goods 2. We calibrated the preferences such that they match the difference in consumption behavior between both households. Figure 3.21 illustrates this:

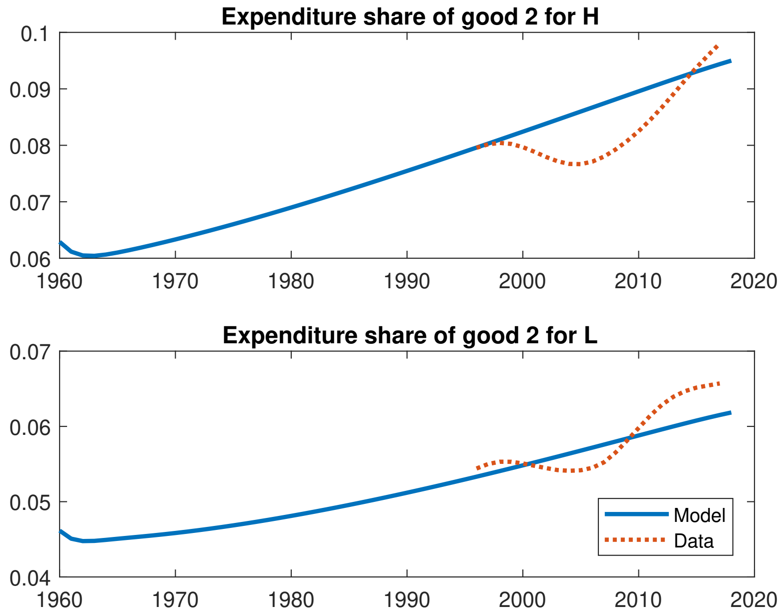


Figure 3.21. Expenditure shares for good 2

Note: The figure shows the evolution of consumption expenditure shares of good 2 of high-skill and low-skill household in the model (solid blue line) and in the data (dotted red line). Consumption data in the CEX start in 1996. In constructing the data series, we define low-skill households as the bottom income decile and high-skill households as the top income decile of the CEX. Consumption goods are classified into goods 1 and 2 using our industry clusters.

High-skill households have a larger expenditure share of the ICT intensive good 2. In the data, those households have increased the expenditure share of those goods from around 8% in 1996 to 10% in 2017. The model generates a slightly lower expenditure share for good 2. For low-skill households, the expenditure share was around 5.5% in 1996 and has risen to around 6.6% in 2017.
The next paragraph aims to decompose the income and the relative price effect that drive the differential increase in consumption for high- and low-skill households. Obviously, the income channel works in favor of the high-skill households. But how relevant is the relative price increase of good 1, which makes for a larger share of consumption of low-skill households?

3.5.4 Quantifying the importance of each channel

In our thought experiment we do a compensatory variation: We ask how much additional income we need to give to households in order to compensate them for the price increase that happens due to digitalization. Remember that we normalized the price of good 2 (the ICT good), therefore digitalization leads to an increase in the price of good 1, the non-ICT good. In a next step we compare this compensatory income of both skill types with each other and then compare it with the actual increase in income that results from digitalization.

We derive demand for good 1 and 2 as a function of disposable income Inc_j/P_2 and the relative price P_1/P_2

$$
\begin{aligned}
C_{1j} &= \left(\frac{Inc_j}{P_2} + \bar{C}_1\left(\frac{\omega}{1-\omega}\right)^{\sigma}\left(\frac{P_1}{P_2}\right)^{\sigma}\right) \Big/ \left(\frac{P_1}{P_2} + \left(\frac{\omega}{1-\omega}\right)^{\sigma}\left(\frac{P_1}{P_2}\right)^{\sigma}\right) \\
C_{2j} &= \frac{Inc_j}{P_2} - \left(\frac{P_1}{P_2}\right) C_{1j}
\end{aligned}
$$

For low-skill households income is simply the wage income. For high-skill households it is their wage income and their capital income minus the investments they undertake.

The first step is to compute welfare in the pre-digitalization steady state. This equals the utility from eq. (3.10), using the actual amounts consumed in 1960, C_{1j} and C_{2j}. Then we consider the prices at the end of our simulation and ask how much income both households need to receive in order to get the same utility as before.

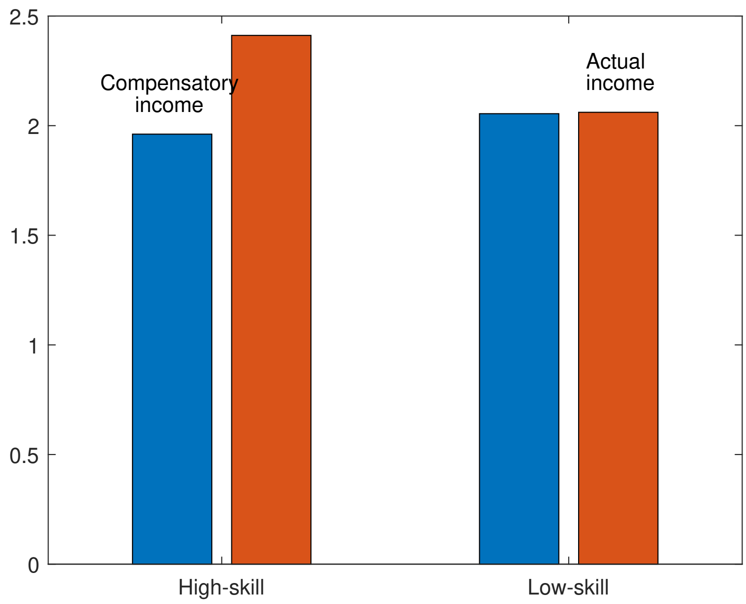


Figure 3.22. Compensatory income (blue) actual income (red)

Note: The figure shows how much income households get with the compensatory variation (the blue bar) and how much income households actually get post-digitalization (the red bar) relative to their initial income. Income is shown for both High-skill households and for Low-skill households. The compensatory variation asks, how much income households need to have in order to achieve the same utility as after the price change of digitalization.

Figure 3.22 compares the compensatory income with the actual income. The blue bar shows how much income households need to receive in the second steady state in order to achieve the same level of welfare as in the initial steady state. This is the amount of income required to compensate them for the price increase. As low-skill households rely more on consumption of good 1 than high-skill households, their income needs to be increased by a larger margin (by 205.5 %) than for rich households (196.1 %). The red bar shows how much more income households actually receive at the end of the simulation relative to the initial steady state. As digitalization increases wages of high-skill households more, their disposable income increases by 241.2 %. They are clearly better off thanks to digitalization. In contrast, low-skill household income increases by 206.1%, making them only slightly better off as in the initial steady state.

This means that high-skill households experience a substantial increase in welfare due to digitalization while low-skill households gain almost nothing. In terms of additional income high-skill households gain 23% (241.2/196.1), while low-skill households gain only 0.3 %. This large difference is because of an increase in wage polarization but also due to the price response of goods that disproportionally favors the rich. Our decomposition has shown, that the price effect is worth around 4.8% (205.5/196.1) in terms of income. If rich households have the same good 2 share in their consumption basket than poor households, digitalization would have only been worth 17.4% (241.2/205.5) additional lifetime income and not 23%. Vice versa, if low-skill households had the same exposure to ICT goods in their consumption basket than the rich, digitalization would bring an increase in welfare worth around 5.1% (206.1/196.1). The decomposition for the high-skill households show that the price effect together with the different consumption pattern across high and low skill households accounts for around 25 % of the overall increase in inequality and is therefore sizable.

3.6 Conclusion

Households differ in the digital share for their consumption basket. This paper proposes a measure of in final goods production and combines it with consumption data to show that rich households tend to have a larger digitalization share than poor households. We present a structural model and calibrate it to assess the effect of the massive decline in ICT asset prices over the last decades on consumption inequality. We find that consumption inequality has increased by a meaningful margin. These results point out that the effect of digitalization on inequality could be even stronger than the literature has previously estimated. Declining prices for ICT assets disproportionately benefit rich households consumption basket, aggravating trends in income inequality that arise due to digitalization.

Appendix 3.A Data Appendix

3.A.1 Details on data sources and data construction

Input-Output Accounts

We ue the BEA's Input Output Accounts and focus on the detailed Input-Output tables after redefinitions. We use producer-value tables, which means that the distribution margin (i.e. the cost of wholesaling, retailing and transportation) is modeled as a flow from the distribution industries to the final consumer rather than as a flow from the producing industry to the final consumer. This is the standard approach and seems appropriate here as we have no reason to believe that the digitalization content of distribution differs between the different goods and services.

The Input-Output accounts are presented in a set of different tables, among them *use, make* and *direct requirements* tables. We start from the commodity-by-industry direct requirements table, which shows the amount of each commodity that is required by an industry to produce one dollar of the industry's output. The problem with this table is that the same commodity can be produced by different industries, e.g. ice-cream can be produced by the dairy product manufacturing industry and the ice-cream manufacturing industry. We follow Horrowitz and Planting (2009) in creating a commodity-by-commodity direct requirements matrix that takes the market shares of each industry in producing certain commodities into account. With this matrix, we can calculate the digitalization share of final goods and services as a weighted sum of the digitalization shares of its intermediate inputs and of value added.

CEX

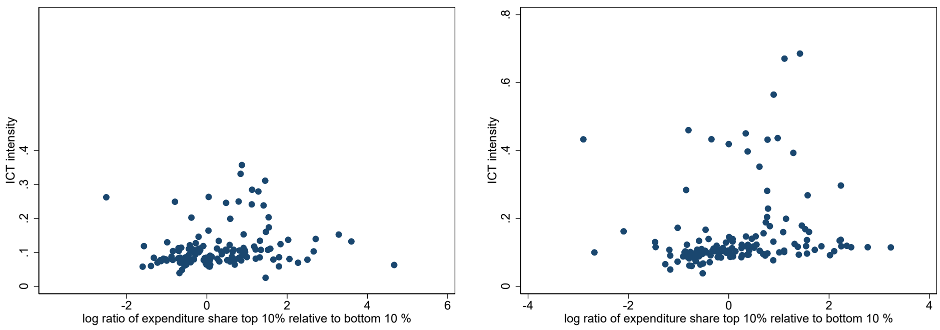
We convert all purchases into annual values at constant 2010 US Dollars. Our income measure is gross labor income, which is captured by the variable FSALARYX (FSALARYM in later vintages) in the interview survey and FWAGEX (FWAGEXM) in the diary survey.[10] Both income and expenditure in the CEX is at the household level. We create individual-equivalent observations by dividing the values by the square root of the number of households and multiplying the sample weights with the number of household members (see e.g. Fernández-Villaverde and Krueger, 2007; Fisher,

10. An alternative would be to consider total before-tax household income, which corresponds to FINCBTAX (FINCBTXM in later vintages) for the interview survey and FINCBEFX (FINCBEFM) for the diary survey. In addition to labor income, these broader measures include farm and non-farm business income, social security income, interest on savings accounts or bonds, income from dividends, royalties, estates and trusts and rental income. However, for the group of households we consider – households that participate in the labor market and are not self-employed – labor income makes for around 94% of total income on average (less at lower points of the income distribution because of the higher dependence on social security).

Johnson, Marchand, Smeeding, and Torrey, 2008). This is important because households at different points in the income distribution vary in their size. In particular, poorer households tend to have more children. The average household size in the bottom quintile of the income distribution is 3.6, whereas it is 2.3 at the top quintile.

We drop households that do not stay in the survey for the entire four quarters of the interview or two weeks of the diary. We consider only households where the head is between 16 and 64 years old and in the labor force, and we drop self-employed. This is because we want to focus on households where labor income is the main source of income. In defining the head, we divert from the CEX convention by making the head the man in mixed couples. We drop the top and bottom 5% of the income distribution in order to mitigate the effect of outliers and top-coding. The CEX is known for under-reporting of expenditure, in particular by richer households (Aguiar and Bils, 2015; Attanasio and Pistaferri, 2016). This problem has been increasing over time. In consequence, inequality measures like the total consumption expenditure of rich relative to poor households are biased downwards. For the purpose of this analysis, we consider the spending of households of a specific income bin on different products relative to their total expenditure. As long as rich households under report all expenditures to an equal extent, our results should not be biased.[11]

3.A.2 Additional figures



(a) 1997 **(b)** 2007

Figure 3.A.1. ICT intensity of consumption, top 10% relative to bottom 90 %

Note: The graph plots ICT intensity against the share of employees that work in cognitive-intensive jobs by BEA industry for 1990 (left panel) and 2010 (right panel). Each dot represents one industry, the line shows a linear prediction with 95% confidence interval. *Source:* Census, American Community Survey, Autor and Dorn (2013), BEA and own calculations.

11. An additional problem is that the CEX only captures about 70% of total household expenditure. But other surveys like the PSID are not detailed enough and do not offer a long-enough time series dimension.

3.A.3 Income and asset data from the SCF

We use data from the Survey of Consumer Finances (SCF) between 1998 and 2013. We focus on households where the head is between 20 and 64 years old and in the labor force. We define incomes as in Ríos-Rull and Kuhn (2016): labor income is the wage and salary income plus a share of business and farm income. This share corresponds to the share of unambiguous labor income (i.e. wage and salary income) in the sum of unambiguous capital income (interest, dividends and capital gains) and labor income. Capital income is interest income, dividends and capital gains plus the remaining share of business and farm income. Total income is the sum of labor income, capital income and transfer income (e.g. social security). It is approximately equal to the payments to the factors of production owned by the household plus transfers, with the exception that it does not include income imputed from owner-occupied housing. Wealth is the household net worth, i.e. financial and non-financial income minus debt. All variables are before taxes.

Table 3.A.1. Income and wealth by education group, SCF

	Labor income	Capital income	Total income	Net wealth
college graduates	56,443	50	59,300	142,044
non-college graduates	27,873	0	29,707	27,634

All numbers are in 2019 US Dollars and refer to the median household in each education group. Total income is the sum of labor income, capital income and transfer income.

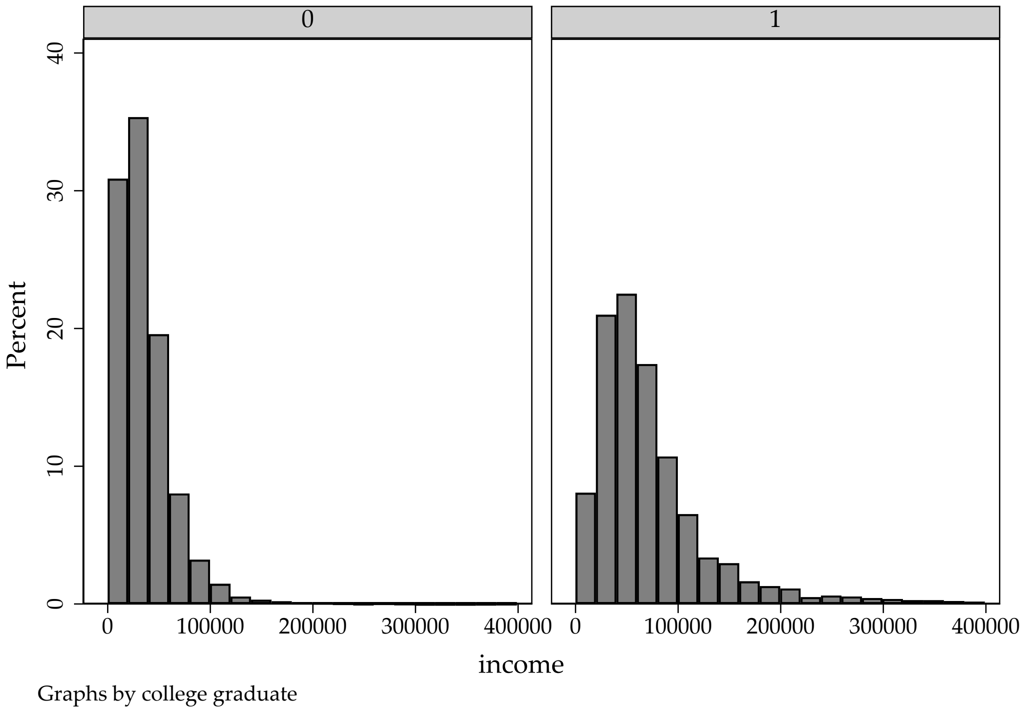


Figure 3.A.3. Income distribution by skill
Note: The graph shows the distribution of income for incomes lower than 400,000 USD.
Source: SCF.

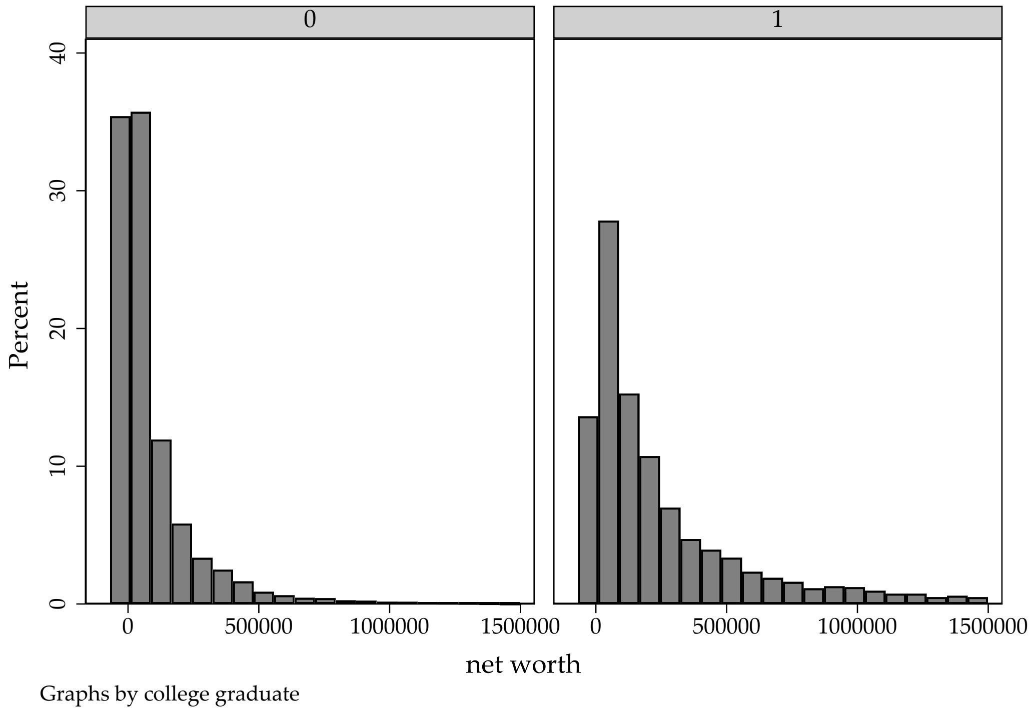


Figure 3.A.4. Net wealth distribution by skill
Note: The graph shows the distribution of net worth for net worth lower than 1.5 mio USD.
Source: SCF.

Appendix 3.B Model Appendix

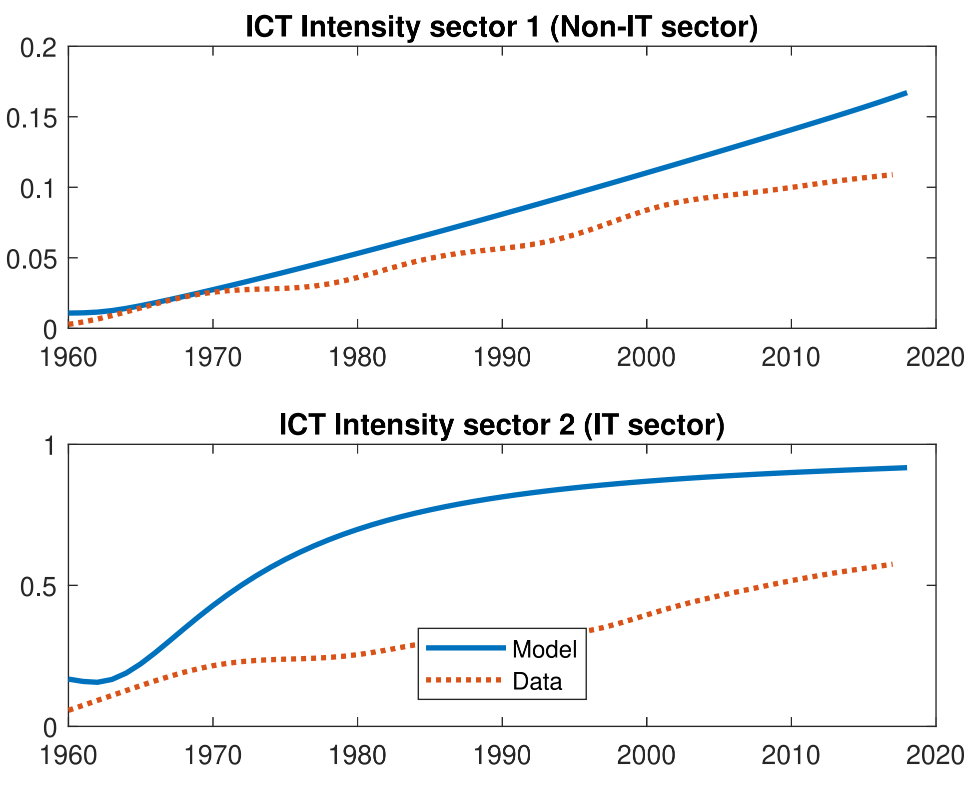


Figure 3.B.1. ICT intensity in the model

Note: The graph shows ICT intensity in the model for both sectors.
ICT intensity is defined as $P_2IT/(P_2IT + P_1K)$.
Source: BEA.

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