Gravity, Geoid and Height Systems 2016

Stochastic modeling of altimetric sea surface height measurements — refined AR models from iterative residual analysis

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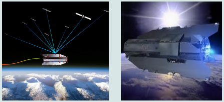
Thessaloniki, 23/09/2016

Outline

Dynamic Ocean Topography (DOT) Model

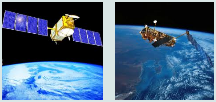
Estimation of the DOT from along track altimetry

Gravity field model



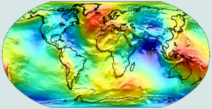
- ⋆ GOCE-based
- ⋆ band-limited
- ⇒ full VCM

Altimetry

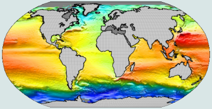


- ⋆ along-track SSH
- ⋆ e.g. AVISO Cor-SSH product

separation ⇒ joint adjustment



Geoid
global spherical harmonics

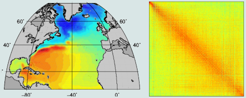


Dyn. Topography
local finite elements

spherical harmonics:

- ⋆ here: buffer for high frequency altimetry
- ⋆ eliminated from NEQ
- ⋆ future: target quantity

DT on predefined grid + VCM



finite elements:

- ⋆ target quantity
- ⋆ currently: local focus
- ⋆ resolution defined by grid & base functions

Methods, algorithms & implementation: Becker (2012), Müller (2014), Müller et al. (2015), Becker et al. (2014a,b), Brockmann (2014)

Parameterization used in this Study

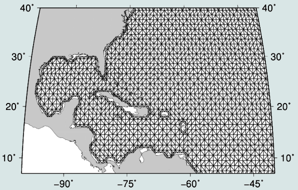
Gravity Field

- spherical harmonic base functions, physically motivated
- global support
- here up to degree 540

Dynamic Topography

- finite element base functions
- local support
- here: linear elements, triangulated $1^\circ \times 1^\circ$ grid (IFEOM), static

here: linear finite elements are set up for an local area (North Atlantic), constant in time $\rightarrow$ MDT as linear combination of base functions



$$\text{MDT}(\theta,\lambda) = \sum_{k\in K} a_k b_k(\theta,\lambda), \quad \mathbf{x}_{FE} = [a_k]$$

- nodal points predefined by ocean model
- unknowns $a_k \Rightarrow$ MDT at the nodal points
- **direct access to target quantities**

Focus Here: Modeling of Uncertainties

Gravity Field

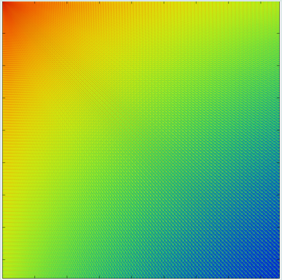
- use of unconstrained models and full covariance matrix: $\mathbf{x}_{\text{gf}}$, $\mathbf{\Sigma}_{\mathbf{x}_{\text{gf}}\mathbf{x}_{\text{gf}}}$
- NEQs as sufficient statistics of original observations (inversion)

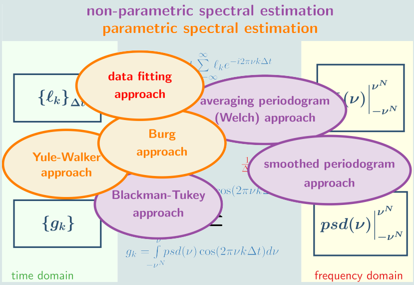
$\Rightarrow$ direct use of VCM, no special action required

Altimetric Observations

- measurements: time series along the orbit
- model describes: stochastic measurement errors, model errors

$\Rightarrow$ AR(p) processes iteratively estimated from residuals





AR(q) Processes to Describe Uncertainties

- Robust estimation of AR(q) process coefficients from residuals $\mathcal{V}_{\mathsf{ssh}_{m,c,p}}$
- Iterative refinement: after full adjustment (gravity field and MDT)

$$\mathcal{V}_{\mathsf{ssh}_{m,c,p}}(k) = \mathcal{S}_{m,k} + \mathcal{N}_{m,k}$$

$$\mathcal{S}_k = \sum_{j=1}^{q} \alpha_j \mathcal{S}_{k-j} + \mathcal{E}_k$$

$\mathcal{S}_k$ ... stochastic process in AR representation
$\mathcal{E}_k$ ... innovation process
$\mathcal{N}_k$ ... additive process (noise)
cf. Schuh et al. (2015)

- $k\sigma$ rejection in iterative Least squares adjustment
- AR process coefficients are computed *A) per mission, B) per cycle, C) per pass*
- choice of process order q: $q = 16$ (A), $q = 16$ (B), $q =$pass length/50 (C)
- AR process is converted to error covariance function (ecf) via Yule-Walker equation

1. Input data and information
   - predefined finite element setup (triangulation/base function type)
   - predefined spherical harmonic setup (maximal resolution)
   - NEQs of gravity field model: $\mathbf{N}_{\mathsf{gf}}$, $\mathbf{n}_{\mathsf{gf}}$,
   - OEQs of ssh observations of a mission m: $\mathbf{A}_{\mathsf{ssh}_m}$, $\boldsymbol{\ell}_{\mathsf{ssh}_m}$, and initial covariance model $\mathbf{Q}^{(\mathbf{0})}_{\mathsf{ssh}_\mathbf{m}}$
   - initial weights for all observation groups (w_m and w_{gf})
2. Compute NEQs for ssh observations

$$\mathbf{N}_{\mathsf{ssh}_m} = \mathbf{A}^T_{\mathsf{ssh}_m} {\mathbf{Q}^{(\mathbf{0})}}^{-1}_{\mathsf{ssh}_m} \mathbf{A}_{\mathsf{ssh}_m}, \quad \mathbf{n}_{\mathsf{ssh}_m} = \mathbf{A}^T_{\mathsf{ssh}_m} {\mathbf{Q}^{(\mathbf{0})}}^{-1}_{\mathsf{ssh}_m} \mathbf{A}_{\mathsf{ssh}_m}, \tag{1}$$

3. Combine NEQs (different parameter spaces!)

$$\mathbf{N} = w_{\mathsf{gf}}\mathbf{N}_{\mathsf{gf}} + \sum_m w_m \mathbf{N}_{\mathsf{ssh}_m}, \qquad \mathbf{n} = w_{\mathsf{gf}}\mathbf{n}_{\mathsf{gf}} + \sum_m w_m \mathbf{n}_{\mathsf{ssh}_m} \tag{2}$$

4. Rigorous parameter estimation (huge dimensional) $\tilde{\mathbf{x}} = \mathbf{N}^{-1}\mathbf{n}$
5. Estimate weights for all (w_m and w_{gf}), continue with 3.
6. Compute SSH residuals $\boldsymbol{v}_{\mathsf{ssh}_m} = \mathbf{A}_{\mathsf{ssh}_m}\tilde{\mathbf{x}} - \boldsymbol{\ell}_{\mathsf{ssh}_m}$

1. Input data and information
2. Compute NEQs for ssh observations

$$\mathbf{N}_{\mathsf{ssh}_m} = \mathbf{A}_{\mathsf{ssh}_m}^T {\mathbf{Q}^{(0)}}_{\mathsf{ssh}_m}^{-1} \mathbf{A}_{\mathsf{ssh}_m}, \; \mathbf{n}_{\mathsf{ssh}_m} = \mathbf{A}_{\mathsf{ssh}_m}^T {\mathbf{Q}^{(0)}}_{\mathsf{ssh}_m}^{-1} \mathbf{A}_{\mathsf{ssh}_m}, \quad (1)$$

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$$\mathbf{N} = w_{\mathrm{gf}} \mathbf{N}_{\mathrm{gf}} + \sum_m w_m \mathbf{N}_{\mathsf{ssh}_m}, \qquad \mathbf{n} = w_{\mathrm{gf}} \mathbf{n}_{\mathrm{gf}} + \sum_m w_m \mathbf{n}_{\mathsf{ssh}_m} \quad (2)$$

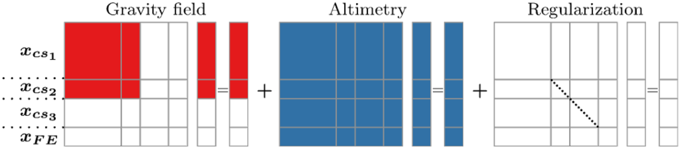
4. Rigorous parameter estimation (huge dimensional) $\tilde{\mathbf{x}} = \mathbf{N}^{-1}\mathbf{n}$
5. Estimate weights for all (w_m and w_{gf}), continue with 3.
6. Compute SSH residuals $\boldsymbol{v}_{\mathsf{ssh}_m} = \mathbf{A}_{\mathsf{ssh}_m} \tilde{\mathbf{x}} - \boldsymbol{\ell}_{\mathsf{ssh}_m}$
7. Analyse $\boldsymbol{v}_{\mathsf{ssh}_m}$ and adjust a stochastic model to $\boldsymbol{v}_{\mathsf{ssh}_m}$, update $\mathbf{Q}_{\mathsf{ssh}_m}^{(1)}$ and continue with 2.

1. Input data and information
2. Compute NEQs for ssh observations

$$\mathbf{N}_{\mathsf{ssh}_m} = \mathbf{A}^T_{\mathsf{ssh}_m} \mathbf{Q}^{(0)}{}^{-1}_{\mathsf{ssh}_m} \mathbf{A}_{\mathsf{ssh}_m}, \; \mathbf{n}_{\mathsf{ssh}_m} = \mathbf{A}^T_{\mathsf{ssh}_m} \mathbf{Q}^{(0)}{}^{-1}_{\mathsf{ssh}_m} \mathbf{A}_{\mathsf{ssh}_m}, \quad (1)$$

3. Combine NEQs (different parameter spaces!)

$$\mathbf{N} = w_{\mathsf{gf}} \mathbf{N}_{\mathsf{gf}} + \sum_m w_m \mathbf{N}_{\mathsf{ssh}_m}, \qquad \mathbf{n} = w_{\mathsf{gf}} \mathbf{n}_{\mathsf{gf}} + \sum_m w_m \mathbf{n}_{\mathsf{ssh}_m} \quad (2)$$



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7. Analyse $\boldsymbol{v}_{\mathsf{ssh}_m}$ and adjust a stochastic model to $\boldsymbol{v}_{\mathsf{ssh}_m}$, update $\mathbf{Q}^{(1)}_{\mathsf{ssh}_m}$

1. Input data and information
2. Compute NEQs for ssh observations

$$\mathbf{N}_{\mathsf{ssh}_m} = \mathbf{A}^T_{\mathsf{ssh}_m} {\mathbf{Q}^{(0)}}^{-1}_{\mathsf{ssh}_m} \mathbf{A}_{\mathsf{ssh}_m}, \; \mathbf{n}_{\mathsf{ssh}_m} = \mathbf{A}^T_{\mathsf{ssh}_m} {\mathbf{Q}^{(0)}}^{-1}_{\mathsf{ssh}_m} \mathbf{A}_{\mathsf{ssh}_m}, \quad (1)$$

3. Combine NEQs (different parameter spaces!)

$$\mathbf{N} = w_{\mathsf{gf}} \mathbf{N}_{\mathsf{gf}} + \sum_m w_m \mathbf{N}_{\mathsf{ssh}_m}, \qquad \mathbf{n} = w_{\mathsf{gf}} \mathbf{n}_{\mathsf{gf}} + \sum_m w_m \mathbf{n}_{\mathsf{ssh}_m} \quad (2)$$

4. Rigorous parameter estimation (huge dimensional) $\tilde{\mathbf{x}} = \mathbf{N}^{-1}\mathbf{n}$
5. Estimate weights for all (w_m and w_{gf}), continue with 3.
6. Compute SSH residuals $\boldsymbol{v}_{\mathsf{ssh}_m} = \mathbf{A}_{\mathsf{ssh}_m} \tilde{\mathbf{x}} - \boldsymbol{\ell}_{\mathsf{ssh}_m}$
7. Analyse $\boldsymbol{v}_{\mathsf{ssh}_m}$ and adjust a stochastic model to $\boldsymbol{v}_{\mathsf{ssh}_m}$, update $\mathbf{Q}^{(1)}_{\mathsf{ssh}_m}$ and continue with 2.

Iterative Refinement

1. Input data and information
2. Compute NEQs for ssh observations

$$\mathbf{N}_{\mathsf{ssh}_m} = \mathbf{A}^T_{\mathsf{ssh}_m} {\mathbf{Q}^{(\mathbf{0})}}^{-1}_{\mathsf{ssh}_m} \mathbf{A}_{\mathsf{ssh}_m}, \ \mathbf{n}_{\mathsf{ssh}_m} = \mathbf{A}^T_{\mathsf{ssh}_m} {\mathbf{Q}^{(\mathbf{0})}}^{-1}_{\mathsf{ssh}_m} \mathbf{A}_{\mathsf{ssh}_m}, \quad (1)$$

3. Combine NEQs (different parameter spaces!)

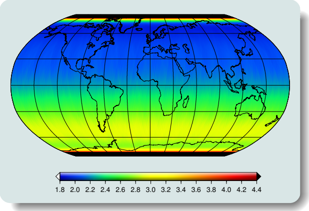
$$\mathbf{N} = w_{\mathsf{gf}} \mathbf{N}_{\mathsf{gf}} + \sum_m w_m \mathbf{N}_{\mathsf{ssh}_m}, \qquad \mathbf{n} = w_{\mathsf{gf}} \mathbf{n}_{\mathsf{gf}} + \sum_m w_m \mathbf{n}_{\mathsf{ssh}_m} \quad (2)$$

4. Rigorous parameter estimation (huge dimensional) $\tilde{\mathbf{x}} = \mathbf{N}^{-1}\mathbf{n}$
5. Estimate weights for all (w_m and w_{gf}), continue with 3.
6. Compute SSH residuals $\boldsymbol{v}_{\mathsf{ssh}_m} = \mathbf{A}_{\mathsf{ssh}_m} \tilde{\mathbf{x}} - \boldsymbol{\ell}_{\mathsf{ssh}_m}$
7. Analyse $\boldsymbol{v}_{\mathsf{ssh}_m}$ and adjust a stochastic model to $\boldsymbol{v}_{\mathsf{ssh}_m}$, update $\mathbf{Q}^{(\mathbf{1})}_{\mathsf{ssh}_\mathbf{m}}$ and continue with 2.

Data Used in the Study

⋆ **Geoid data: Mainly EGM_TIM_RL05 NEQs** (Brockmann et al., 2014)

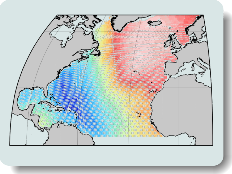
i	$l_{\min}$	$l_{\max}$
SGG	2	281
SST	2	150
KAULA	181	540
KAULA	zonal coeff.	



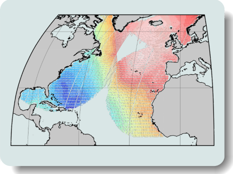
Propagated Geoid accuracy of the EGM_TIM_RL05 model at 100 km resolution

⋆ **SSH data: ≈ 1 year, 4 missions,** mainly from 2007 (AVISO CorSSH)

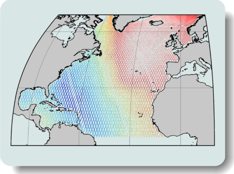
Envisat, cy 53–64, 1 972 985 obs



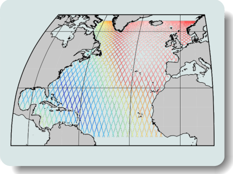
ERS2, cy 121, 157 512 obs



GFO, cy 184–208, 1 724 166 obs



Jason1, cy 179–222, 2 747 694 obs



Computed Models/Setups

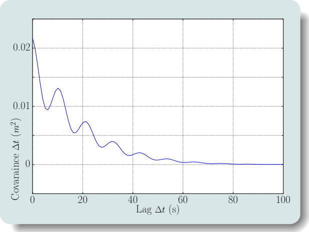
A) mean analytical ecf per mission m
$\Rightarrow$ 4 analytical ecfs

B) discrete ecf per m & cycle c
$\Rightarrow$ 82 discrete ecfs

C) discrete ecf per m, c & pass p
$\Rightarrow$ 10k discrete ecfs

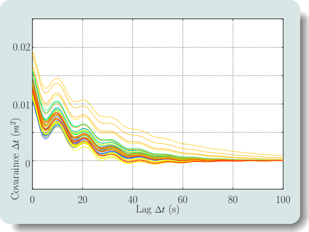
Example: **Jason 1**, error covariance functions

J1,



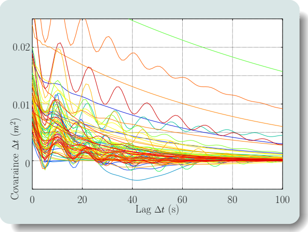
- mean analytical function

cycle 179–222



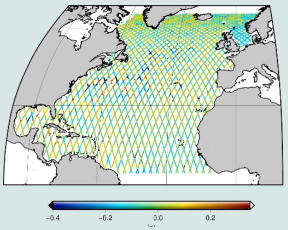
- stable estimates per cycle
- changes during cycle?

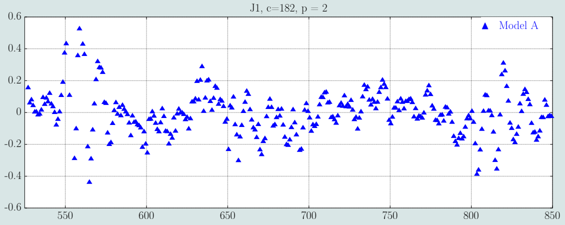
pass 1–84 of cycle 182



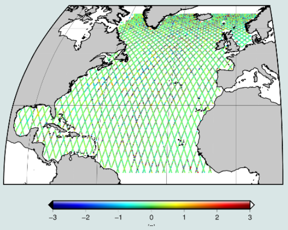
- strong variations
- unstable?
- AR order changing

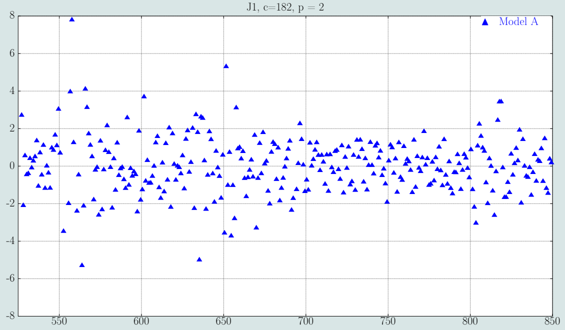
Residuals, $\sigma = 0.101\,\mathrm{m}$



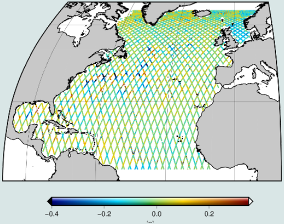


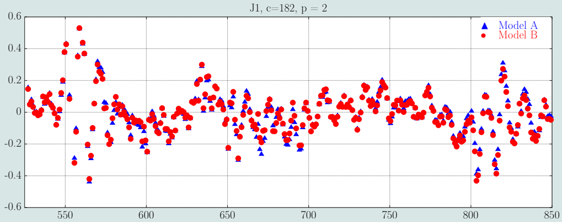
Decorrelated residuals, $\sigma = 1.16$



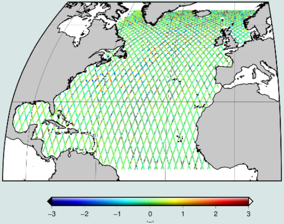


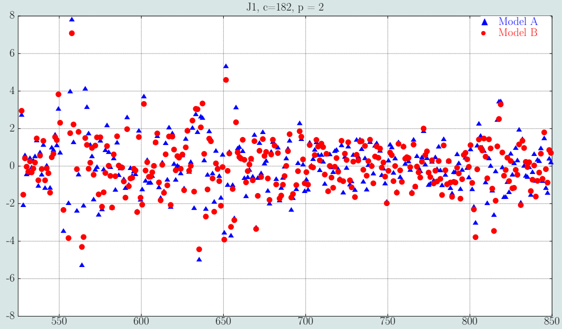
Residuals, $\sigma = 0.099\,\mathrm{m}$



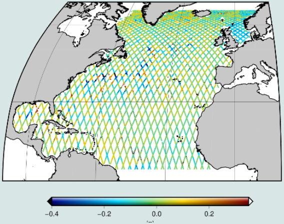


Decorrelated residuals, $\sigma = 1.10$

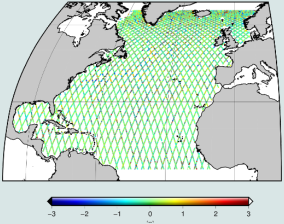


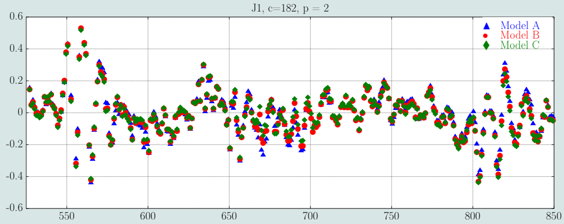


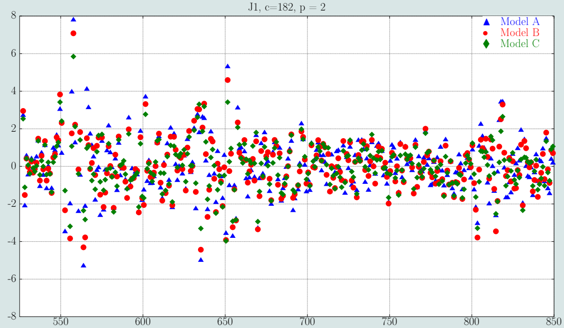
Residuals, $\sigma = 0.098\,\text{m}$



Decorrelated residuals, $\sigma = 0.96$

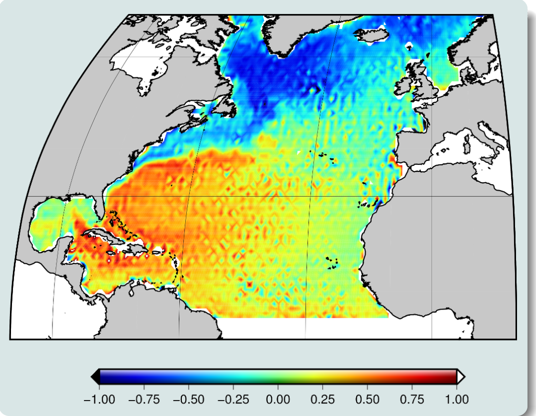




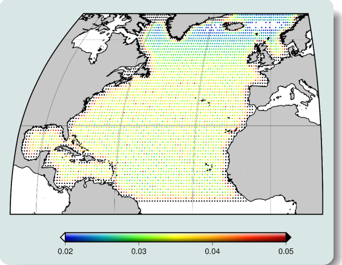


Computed models A

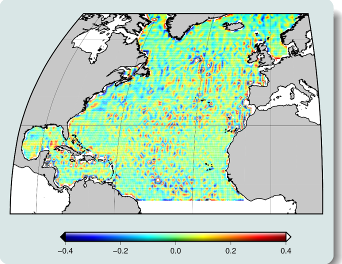
a) MDT model estimate (m)



b) Standard deviation (m)



c) Diff. to CNES-CLS13 (m)



Statistics (m)

a) $\sigma_{\text{rms}} = 0.280\,\text{m}$ (variability)

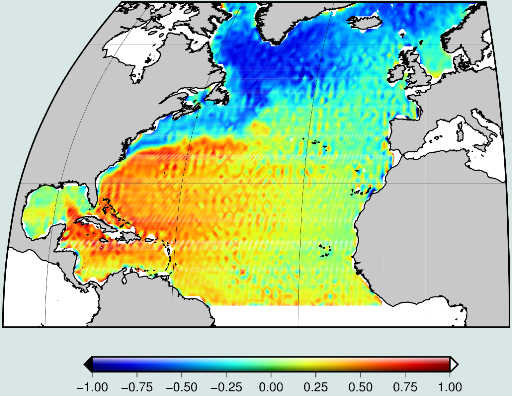
b) $\sigma_{\text{mean}} = 0.053\,\text{m}$ (mean error)

c) $\sigma_{\text{rms}} = 0.155\,\text{m}$, $\sigma_{\text{mad}} = 0.148\,\text{m}$

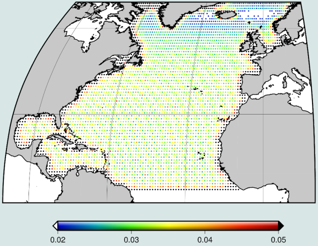
MDT_CNES-CLS13 was produced by CLS Space Oceanography Division and distributed by Aviso, with support from Cnes (http://www.aviso.altimetry.fr/)

Computed models B

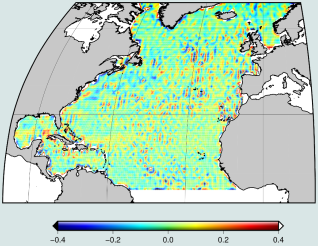
a) MDT model estimate (m)



b) Standard deviation (m)



c) Diff. to CNES-CLS13 (m)



Statistics (m)

a) $\sigma_{\text{rms}} = 0.277\,\text{m}$ (variability)

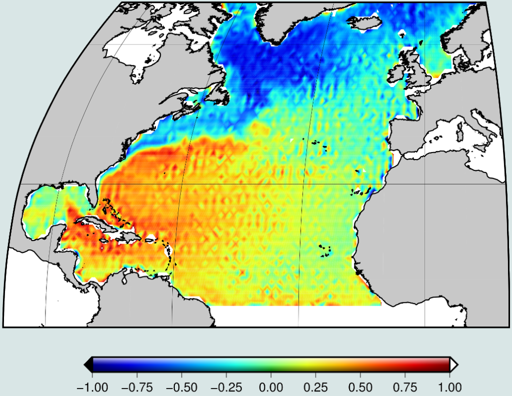
b) $\sigma_{\text{mean}} = 0.051\,\text{m}$ (mean error)

c) $\sigma_{\text{rms}} = 0.141\,\text{m}$, $\sigma_{\text{mad}} = 0.135\,\text{m}$

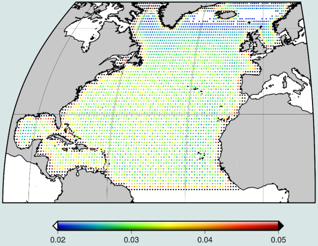
MDT_CNES-CLS13 was produced by CLS Space Oceanography Division and distributed by Aviso, with support from Cnes (http://www.aviso.altimetry.fr/)

Computed models C

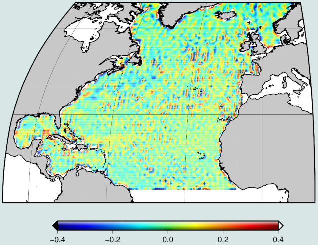
a) MDT model estimate (m)



b) Standard deviation (m)



c) Diff. to CNES-CLS13 (m)



Statistics (m)

a) $\sigma_{\text{rms}} = 0.280\,\text{m}$ (variability)
b) $\sigma_{\text{mean}} = 0.049\,\text{m}$ (mean error)
c) $\sigma_{\text{rms}} = 0.146\,\text{m}$, $\sigma_{\text{mad}} = 0.138\,\text{m}$

MDT_CNES-CLS13 was produced by CLS Space Oceanography Division and distributed by Aviso, with support from Cnes (http://www.aviso.altimetry.fr/)

Summary and Conclusions

- AR processes can be used as stochastic model for temporal correlations of along-track SSH observations
- Conversion to error covariance functions is possible (computational advantages)
- Estimation can be performed per mission, per cycle and per pass
- To deal with non-stationarity of residuals: estimation per pass was tested $\Rightarrow$ no obvious gain yet

Outlook

- Finite elements describing the MDT remain noisy (bumpy)

$\Rightarrow$ Averaging over one year not sufficient? $\rightarrow$ extend analysis to several years

$\Rightarrow$ Spatial correlations are neglected $\rightarrow$ model spatial correlations within a cycle

$\Rightarrow$ Sufficient seperation? (some MDT signal goes to the gravity field and vice versa, smooth MDT/rough gravity)

$\Rightarrow$ Derived standard deviations of MDT inconsistent to rms (factor 2-5)

$\Rightarrow$ Spatial correlations are neglected, other systematic effects?

References

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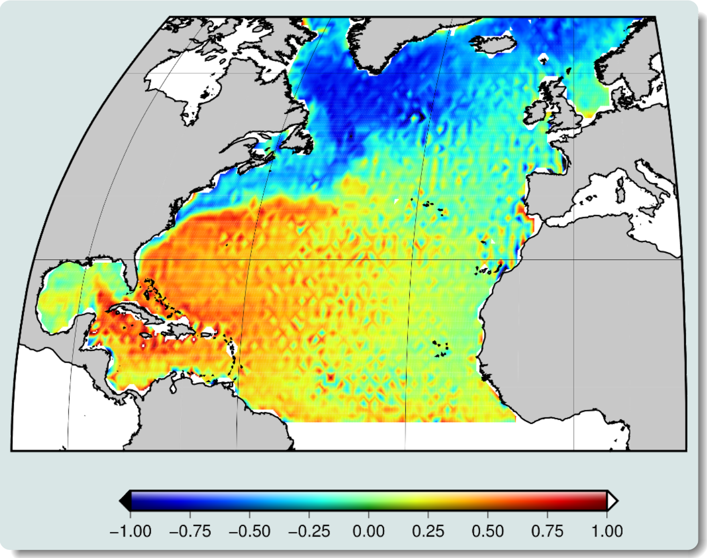
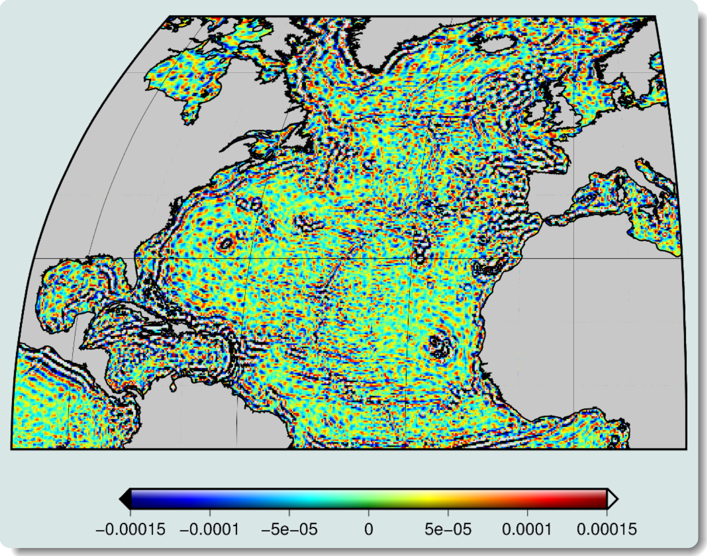
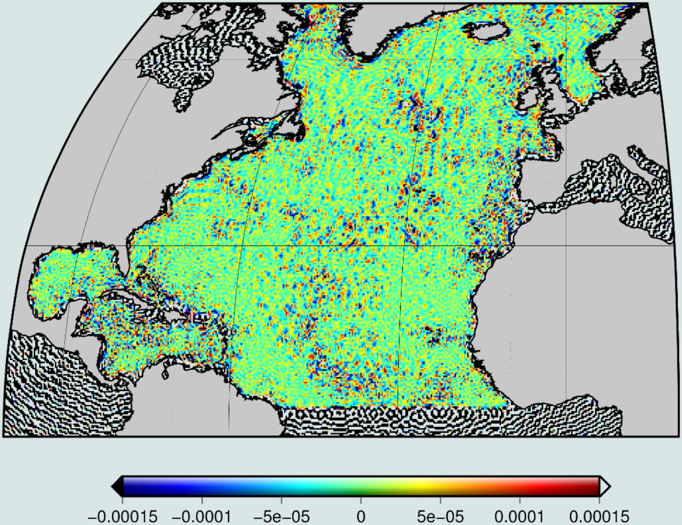
Computed Gravity Field Models A



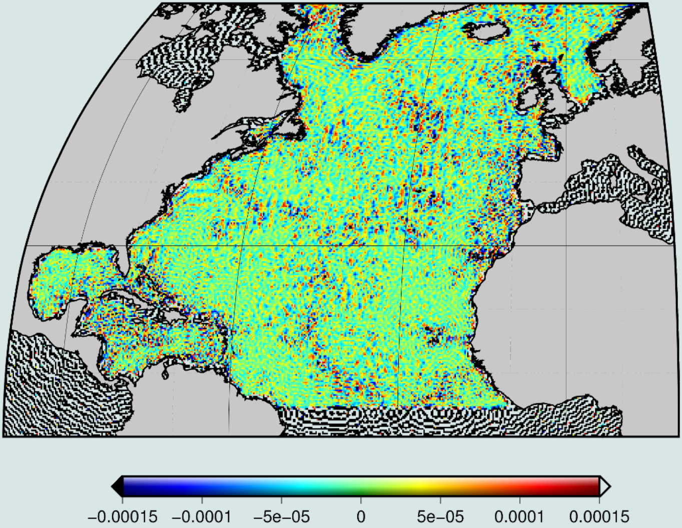
Anomalies EGM_TIM_RL05 (d/o 240)- EGM2008 (d/o 540) (m/s^2)



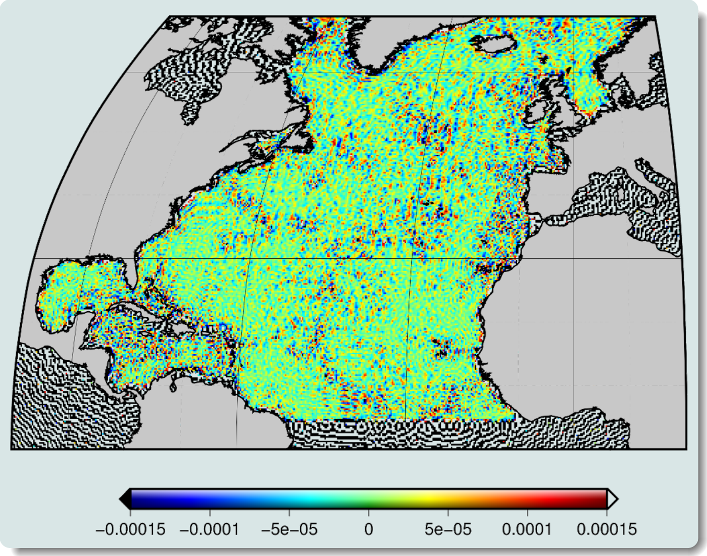
Anomalies GF A (d/o 540)- EGM2008 (d/o 540) (m/s^2)



Anomalies GF B (d/o 540)- EGM2008 (d/o 540) (m/s^2)



Anomalies GF C (d/o 540)- EGM2008 (d/o 540) (m/s^2)



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