Soil resistance to the penetration

effect of soil bulk density and organic matter on soil strength

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1 Soil model

1.1 Effect of soil organic matter on soil bulk density

The volume occupied by the soil is composed of solid, liquid and gaseous phases:

$$V_{Soil} = V_s + V_l + V_g \tag{1}$$

The solid phase represents the volume occupied by mineral particles and organic materials:

$$V_s = V_m + V_o \tag{2}$$

Thus, the volume of a dry soil with organic matter ($V_l = 0$ and $F_o > 0$) is:

$$V_{Soil} = V_m + V_o + V_g \tag{3}$$

As the soil is dry, we can consider that $V_g = \theta_{sat}$. Then:

$$V_{Soil} = V_m + V_o + \theta_{sat} \tag{4}$$

Additionally, if we set $V_{Soil} = 1$ cm^3, then

$$\theta_{sat} = 1 - (V_m + V_o) \tag{5}$$

As $\rho_m = \frac{M_m}{V_m}$ and $\rho_o = \frac{M_o}{V_o}$, and replacing in (5):

$$\theta_{sat} = 1 - (\frac{M_m}{\rho_m} + \frac{M_o}{\rho_o}) \tag{6}$$

As the soil bulk density is expressed by:

$$D_b = \frac{M_{Soil}}{V_{Soil}} \tag{7}$$

The soil mineral and organic mass fractions are given by:

$$M_m = M_{Soil} \cdot F_m \tag{8}$$

$$M_o = M_{Soil} \cdot F_o \tag{9}$$

If $V_{Soil} = 1$ cm^3, then $M_{Soil} = D_b$, $M_m = D_b \cdot F_m$ and $M_o = D_b \cdot F_o$, which, replacing in (6), yields:

$$\theta_{sat} = 1 - (\frac{D_b \cdot F_m}{\rho_m} + \frac{D_b \cdot F_o}{\rho_o}) \tag{10}$$

Isolating D_b:

$$D_b = (1 - \theta_{sat}) \times (\frac{\rho_m \cdot \rho_o}{\rho_o \cdot F_m + \rho_m \cdot F_o}) \tag{11}$$

According to Saxton and Rawls (2006 [6]), θ_{sat} is a function of S, C and OM. Therefore, we expand this to D_b, which can be estimated from S, C, OM, θ_{sat}, ρ_o and F_o (assuming ρ_m to be constant: $\rho_m = 2.65$ g $\cdot$ cm^{-3}).

When ρ_o is unknown, it can be estimated according the following empirical model (Rühlmann et al. 2006 [4]):

$$\rho_o = 1.127 + 0.373 \cdot Fo \tag{12}$$

In the same way, when $F_o = 0$ (soil with no organic matter):

$$\theta_{sat} = 1 - \frac{D_b}{\rho_m} \tag{13}$$

$$D_b = (1 - \theta_{sat}) \times \rho_m \tag{14}$$

1.2 Soil-water characteristic curves

The soil-water characteristic curve (SWCC) is defined as the relationship between the soil volumetric water content and the soil water potential. In our case, between θ and ψ_M. The shape of SWCC may be described as (Saxton and Rawls 2006 [6]):

$$\psi_M = \begin{cases} A(\theta)^{-B}, & \text{if } \theta \in [\theta_{1500}, \theta_{33}) \\ 33 - \frac{(\theta - \theta_{33}) \cdot (33 - \psi_e)}{\theta_{sat} - \theta_{33}}, & \text{otherwise} \end{cases} \tag{15}$$

Where:

$$A = \exp(\ln(33) + B \cdot \ln(\theta_{33})) \tag{16}$$

$$B = \frac{\ln(1500) - \ln(33)}{\ln(\theta_{33}) - \ln(\theta_{1500})} \tag{17}$$

$$\theta_{sat} = \theta_{33} + \theta_{s\text{-}33} + 0.097 \times S + 0.043 \tag{18}$$

θ_{33}, $\theta_{s\text{-}33}$, θ_{1500} and ψ_e are functions of S, C and OM and calculated according the Saxton and Rawls equations (2006 [6]). When the effect of Db on SWCC (15) is considered, θ_{sat} is calculated according to equations (10) or (13). Then, θ_{33} and $\theta_{s\text{-}33}$ are also recalculated using the *ad hoc* density adjustment equations (see Saxton and Rawls 2006 [6]). Thus, for each $\theta \in [\theta_{1500}, \theta_{sat}]$ a ψ_M is obtained.

Addtionally, the water conductivity (at saturation) is calculated as:

$$k_{sat} = 4632 \times (\theta_{sat} - \theta_{33})^{(3-\lambda)} \tag{19}$$

with $\lambda = 1/B$.

After getting ψ_M values for each $\theta \in [\theta_{1500}, \theta_{sat}]$, the van Genuchten model (1980 [7]) is fitted to obtain the θ_{sat} (according to van Genuchten model), θ_r, α and n coefficients:

$$\theta = \theta_r + \frac{\theta_{sat} - \theta_r}{[1 + (\alpha \cdot \psi_M)^n]^m} \tag{20}$$

with $m = 1 - 1/n$.

Therefore, SWCC, θ_{sat}, ψ_M and k_s can be expressed as functions of S, C, OM, ρ_o, F_o and Db.

1.3 Soil water potential

The water potential of the soil (ψ) is a function of:

$$\psi = \psi_M + \psi_O + \psi_G \tag{21}$$

ψ_M can be calculated using the SWCC equations: (15) or (20). For each $\theta \in [\theta_{1500}, \theta_{sat}]$, a ψ_O can be calculated as (Saxton and Rawls 2006 [6]):

$$\psi_O = \frac{\theta_{sat}}{\theta} \times 36 \cdot EC \tag{22}$$

Finally, ψ_G is expressed as (Nobel 2004 [3]):

$$\psi_G = \Gamma \cdot z \tag{23}$$

Where,

$$\Gamma = \rho_w \cdot g \tag{24}$$

As $\rho_w = 997$ kg·m^{-3} and $g = 9.81$ m·s^{-2}, then $\Gamma = 9.78$ kN·m^{-3} (at 25°C).

1.4 Soil resistance to the penetration

The degree of resistance to the penetration of a soil not only depends on Db but also on other physicochemical properties. The equations proposed by Whalley et al. (2007 [8]) and Gao et al. (2012[1], 2016[2]) take into account ψ and θ_{sat}, which are functions of C, S, OM a Db as shown previously.

Some of the Whalley et al. (2007 [8]) equations considered in this work are:

$$log_{10}Q = 0.27 \cdot log_{10}\psi + 2.38 \cdot Db + 0.14 \cdot OC \tag{25}$$

$$log_{10}Q = 0.32 \cdot log_{10}\left(\psi\frac{\theta}{\theta_{sat}}\right) + 2.01 \cdot Db \tag{26}$$

$$log_{10}Q = 0.76 \cdot log_{10}\left(\psi\frac{\theta}{\theta_{sat}}\right) + log_{10}62 \tag{27}$$

$$log_{10}Q = 0.35 \cdot log_{10}\left(\psi\frac{\theta}{\theta_{sat}}\right) + 0.93 \cdot Db + 1.2623 \tag{28}$$

The percentage of organic carbon in soil mass can be approximated as $OC = OM/1.724$ (Rühlmann and Körschens 2009 [5]).

The Gao et al. (2012 [1]) equations are:

$$Q = Db \cdot g \cdot \left(A^*\frac{(F-e)^2}{1+e}\left(\sigma_s^p - \psi\left(\frac{\psi}{\psi_e}\right)^b\right)^f\right)^2 \tag{29}$$

$$Q = Db \cdot g \cdot \left(A^*\frac{(F-e)^2}{1+e}\left(\sigma_s^p - \psi\frac{\theta}{\theta_{sat}}\right)^f\right)^2 \tag{30}$$

Where A^*, b, F, f and p are empirical coefficients. $\frac{\theta}{\theta_{sat}}$ is the degree of saturation, which is set equals to 0.5 as long as $\frac{\theta}{\theta_{sat}} \leq 0.5$ (Gao et al. 2012 [1]; Whalley et al. 2012 [9]). σ_s is the net stress, which can be intepreted as the applied stress at soil surface when the penetrometer resistance is being measured (Gao et al. 2012 [1]). Finally, the soild void ratio can be caculated as follows:

$$e = \frac{\theta_{sat}}{1-\theta_{sat}} \tag{31}$$

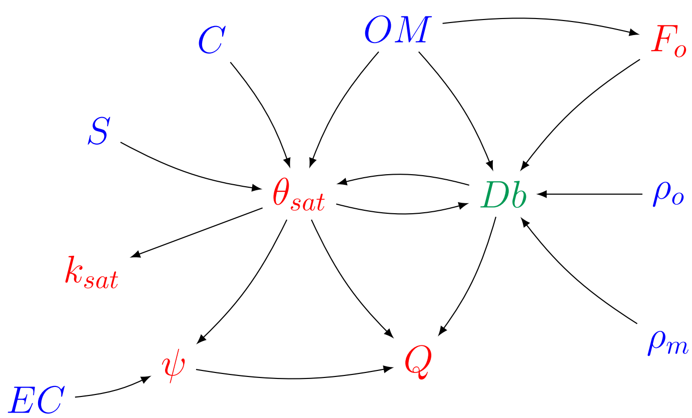

Figure 1: Relationship among soil properties. Simplified diagram showing the connection among the most important soil properties used in this work. Color code: inputs, outputs and input/output. Db is a input/output parameter: if it is unknown, this is calculated according to (11).

References

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Table 1: List of equation symbols

symbol	unit	description
A	$-$	coefficient of Saxton and Rawls (2006) SWCC[1]
A^*	$-$	coefficient of Gao et al. (2012) penetration resistance model
B	$-$	coefficient of Saxton and Rawls (2006) SWCC[1]
b	$-$	coefficient of Gao et al. (2012) penetration resistance model
C	$\%$	percentage of clay in soil mass
Db	$\text{g} \cdot \text{cm}^{-3}$	soil bulk density
EC	$\text{dS} \cdot \text{m}^{-1}$	soil electrical conductivity
e		soil void ratio
F	$-$	coefficient of Gao et al. (2012) penetration resistance model
F_m	$-$	mineral fraction of soil mass
F_o	$-$	organic fraction of soil mass
f	$-$	coefficient of Gao et al. (2012) penetration resistance model
g	$\text{m} \cdot \text{s}^{-2}$	gravitational acceleration
k_{sat}	$\text{cm} \cdot \text{h}^{-1}$	water conductivity at saturation
m	$-$	coefficient of van Genuchten's (1980) SWCC[1]
MAD	$\%$	management allowed depletion
M_m	g	mass of soil mineral matter fraction
M_o	g	mass of soil organic matter fraction
M_{Soil}	g	soil mass
n	$-$	coefficient of van Genuchten's (1980) SWCC[1]
OC	$\%$	percentage of organic carbon in soil mass
OM	$\%$	percentage of organic matter in soil mass
p	$-$	coefficient of Gao et al. (2012) penetration resistance model
Q	kPa	soil penetration resistance
S	$\%$	percentage of sand in soil mass
V_g	cm^3	volume of soil gas fraction
V_l	cm^3	volume of soil liquid fraction
V_m	cm^3	volume of soil mineral fraction
V_o	cm^3	volume of soil organic matter fraction
V_s	cm^3	volume of soil solid fraction
V_{Soil}	cm^3	soil volume
z	m	rooting depth

[1]Soil-water characteristic curve

Table 1: List of equation symbols *(contd)*

symbol	unit	description
α	$-$	coefficient of van Genuchten's (1980) SWCC[1]
Γ	$\text{kN} \cdot \text{m}^{-3}$	specific weight of water at 25°C
λ	$-$	coefficient of Saxton and Rawls (2006) SWCC[1]
θ	$\text{cm}^3 \cdot \text{cm}^{-3}$	soil volumetric water content
θ_{33}	$\text{cm}^3 \cdot \text{cm}^{-3}$	soil volumetric water content at 33 kPa
$\theta_{s\text{-}33}$	$\text{cm}^3 \cdot \text{cm}^{-3}$	soil volumetric water content between 0 to 33 kPa
θ_{1500}	$\text{cm}^3 \cdot \text{cm}^{-3}$	soil volumetric water content at 1500 kPa
θ_r	$\text{cm}^3 \cdot \text{cm}^{-3}$	residual volumetric water content
θ_{sat}	$\text{cm}^3 \cdot \text{cm}^{-3}$	soil volumetric water content at saturation
ρ_m	$\text{g} \cdot \text{cm}^{-3}$	soil mineral matter density
ρ_o	$\text{g} \cdot \text{cm}^{-3}$	soil organic matter density
ρ_w	$\text{g} \cdot \text{cm}^{-3}$	water density at 25°C
σ_s	kPa	net stress
ψ	kPa	soil water potential
ψ_e	kPa	tension at air entry (bubbling pressure)
ψ_G	kPa	soil gravitational potential
ψ_M	kPa	soil matric potential
ψ_O	kPa	soil osmotic potential

[1]Soil-water characteristic curve